Targeting Additionality: A Theory of Blended Finance*

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Abstract

Blended finance combines commercial capital with scarce concessional capital to fund socially valuable projects. I develop a mechanism-design model in which impact potential is privately known and impact effort is hidden. Concessional capital can generate additionality on two margins: by bringing otherwise unfunded projects to market, or by improving impact in projects commercial finance would already support. Under competitive commercial finance, no-loss impact finance cannot generate additionality; loss-absorbing capital is necessary. Optimal contracts use impact-contingent payments to screen high-impact projects and induce costly impact effort. At the portfolio level, both margins are governed by a common shadow value of concessional funds. The model characterizes when blended finance should operate on each margin and why mobilization is an incomplete measure of impact.

Keywords: Development finance, blended finance, concessional capital, additionality, optimal contracts, moral hazard, adverse selection.

JEL Codes: D82, D86, G23, O16, Q56.

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1 Introduction

The transition to a sustainable economy requires financing a large set of projects whose social value is not fully reflected in private returns. New technologies, unfamiliar business models, long horizons, and execution in high-risk markets all create gaps between social and private profitability.[1] Commercial finance alone cannot close this gap. One reason is that some socially valuable projects are simply not privately profitable enough to be funded. A second reason arises even when projects are commercially viable, but firms may still have weak incentives to undertake costly actions that raise social value but do not increase pledgeable cash flows. The market therefore fails on two margins. On the *extensive margin*, the market does not bring socially valuable projects to market. On the *intensive margin*, financed projects do not generate their full social potential.

Blended finance has recently emerged as the institutional response to these two failures. It combines commercial capital, subject to fiduciary obligations or break-even returns, with loss-absorbing *concessional capital* from public or philanthropic sources—capital that does not require commercial returns and can be deployed at expected loss. The concessional layer can absorb losses, lower financing costs, provide repayment flexibility, or reward verified outcomes. But this layer is extremely scarce. The central design problem in blended finance is therefore to decide where scarce loss-absorbing capital generates *additionality*—changes in outcomes relative to what the market would otherwise deliver—rather than simply mobilizing commercial funds. The central question of the paper is whether scarce loss-absorbing capital should prioritize the extensive margin, where concessional resources bring otherwise unfunded projects to market, or the intensive margin, where they improve impact in projects that commercial finance would already support.

Two projects illustrate the two margins. The first is a sustainable palm oil expansion in Mexico, whose growth faced financing frictions related to its early stage, long crop-maturation horizons, and the limited experience of local financiers with the asset class. The structure combined commercial and concessional loans with repayment flexibility and outcome-based interest reductions tied to climate milestones, including biochar production, carbon-credit issuance, and cogeneration from industrial waste. This is *extensive additionality*: blended finance brings a sustainable business model to market when conventional financing is difficult to obtain. By contrast, a wind farm in Chile illustrates *intensive addi-*

[1] Early solar deployment illustrates this logic: future costs and scalability were uncertain, yet learning, emissions, and proof-of-concept benefits justified sustained public and philanthropic support. Similar challenges arise today in clean energy, sustainable agriculture, water and sanitation, climate adaptation, biodiversity protection, and resilient infrastructure in developing economies.

tionality. The sponsor was a financially strong energy company and could finance the asset on commercial terms. The blended-finance instrument instead used concessional capital to accelerate the retirement of two profitable coal units and link avoided emissions to a reduction in borrowing costs. The relevant additionality was not that the project became financeable; it was that concessional capital changed the timing and intensity of decarbonization.[2]

Two frictions complicate any allocation rule. First, even when impact outcomes are measurable ex post—through verified emissions reductions, certified biodiversity gains, or audited social indicators—achieving them requires costly actions that are not contractible ex ante. Realized impact thus depends on hidden effort. Second, impact potential is often privately known to the firm. A local firm may know more than outside investors about the realistic emissions displacement of a biomass facility, the feasibility of a water-treatment investment, or the achievable scope for female workforce participation in an infrastructure project. Existing theory has not studied these frictions inside a layered contract in which commercial and concessional capital coexist and compete for scarce loss-absorbing capacity.

I develop a tractable mechanism-design model to study this problem. Projects differ along two dimensions. Financial viability, denoted by p, summarizes the risk-adjusted monetary attractiveness of the project and is publicly observed. Impact potential, denoted by q, summarizes expected social value conditional on successful operation and is privately known to the entrepreneur. There are two financiers: a competitive commercial sector that values only monetary repayments and must break even, and an impact investor that values social impact and has access to a limited budget of concessional capital. After financing is provided, the entrepreneur chooses hidden effort on two margins: financial effort, which affects the probability of project success, and impact effort, which affects the probability of social impact conditional on success. The specification captures a basic feature of real projects: impact is generated only if the project remains operational.

The first result establishes the commercial-finance benchmark and formalizes why loss-absorbing capital is necessary. If the impact investor must break even, it cannot generally improve on commercial finance. For bankable projects, inducing higher impact effort requires compensating the entrepreneur for giving up private benefits from low-impact behavior; under competitive finance there are no rents available to fund that compensation. For non-bankable projects, a no-loss contract cannot both finance the project and satisfy entrepreneur participation. No-loss impact finance therefore cannot generate additionality

[2]The two transactions correspond, respectively, to IDB Invest's blended-finance support for Prolade in Mexico (Convergence Blended Finance, 2026) and to IDB Invest's blended-finance facility for Engie's Calama wind farm and accelerated coal retirement in Chile (Convergence Blended Finance, 2025).

on either margin.

The second result characterizes the blended-finance contracts that use concessional capital to relax this constraint. For commercially bankable projects, the optimal intervention is an impact-contingent overlay: the project remains financeable by the market, and concessional capital funds incentives that reward realized impact. For commercially excluded projects, the optimal intervention is a rescue contract: concessional capital makes the project financeable and induces both financial and impact effort. In both cases, the same instrument must screen impact potential and provide incentives. Higher-impact firms value impact-contingent rewards more, which makes these rewards useful for selection; but they are also easier to incentivize, which pushes toward weaker rewards. The resulting contract pools accepted impact types at the spread required by the marginal type, generating informational rents for inframarginal high-impact firms.

The third result turns project-level blended finance into a portfolio allocation problem. The impact investor chooses an impact-potential cutoff for each level of financial viability. Commercially viable projects with low impact potential are left to the market; those with high impact potential receive impact incentives. Non-bankable projects with high impact potential are rescued; those with low impact potential remain unfunded, even if they generate positive net surplus. Both forms of additionality draw on the same concessional budget. The investor therefore faces a single trade-off—every unit of concessional capital used on one margin is unavailable on the other—captured by a common shadow value of scarce funds. The optimal cutoff need not be continuous at the commercial-finance boundary, because the marginal role of concessional capital changes from improving behavior in existing projects to creating projects that would not otherwise exist.

This portfolio perspective clarifies why mobilization is an incomplete metric. Mobilized private capital—the standard success metric for blended-finance programs—is generated only on the extensive margin. When concessional capital is sufficiently scarce, it may be optimal to use it exclusively to improve impact in already-financed projects, generating additionality without mobilizing new capital. Conversely, mobilizing capital is not sufficient for impact if support flows to low-impact projects or fails to change firm behavior. The relevant policy object is therefore not the volume of capital mobilized, but additionality per unit of fiscal cost.

Two extensions sharpen the message. First, when entrepreneurs have low internal liquidity, rescue contracts must leave implementation rents to induce financial effort. These rents can attract lower-impact firms that enter the program without exerting impact effort, generating leakage on precisely the extensive margin where blended finance is meant to

bring excluded projects to market. This provides a contractual foundation for *greenwashing*: subsidies needed to solve one incentive problem may attract firms that exploit another. Second, when commercial financiers have market power, the entrepreneur's commercial outside option is less attractive. The impact investor can then require more repayment on the participation margin while still attracting the entrepreneur into the impact contract. Market power lowers the fiscal cost of intensive additionality and, through the common shadow value of concessional capital, can free resources to rescue additional non-bankable projects.

Literature. This paper contributes to the emerging literature on blended finance. Existing work, exemplified by Flammer, Giroux, and Heal (2024, 2025), emphasizes the extensive role of concessional capital: improving the risk-return profile of projects that private investors would otherwise not finance, with mobilization as the main metric. I take this agenda as a starting point but study blended finance as a contract in which commercial and concessional capital coexist within the same transaction. This makes it possible to analyze two uses of scarce loss-absorbing capital simultaneously: bringing excluded projects to market and changing behavior in projects that the market would already finance. It also separates mobilization from additionality: mobilized capital arises only on the extensive margin and is neither necessary nor sufficient for welfare-relevant impact.

The paper also relates to the sustainable-finance literature. Oehmke and Opp (2024) show that impact must be evaluated relative to the counterfactual allocation and derive a Social Profitability Index for ranking scarce socially responsible capital across firms. I share their counterfactual principle, but study a different object. Their framework allocates a single pool of socially responsible capital across firms to change technology choices; here, the planner designs a layered contract with a no-loss commercial tranche and a concessional overlay that funds incentives the market will not price. Other related work studies how socially responsible investors affect firm behavior (Bolton, Kacperczyk, and Samama, 2021; Martin and Pindyck, 2021), and Barbalau and Zeni (2025) examine green debt under private information about externalities. These papers inform the model's treatment of impact incentives and greenwashing, but do not study the contractual coexistence of commercial and concessional capital that defines blended finance.

Finally, the paper builds on mechanism-design work on financing externalities under informational frictions, especially Lach, Neeman, and Schankerman (2021), who study optimal government financing of R&D under adverse selection and moral hazard. I inherit their targeting logic: scarce public finance must screen unobservable types and induce hidden effort. The blended-finance environment differs in three respects. First, there are two ef-

fort margins, financial and impact effort. Second, impact potential is privately known on a dimension commercial lenders do not price. Third, the instrument is not a public loan but a state-contingent contract that layers loss-absorbing concessional capital on top of no-loss commercial finance. These differences generate the paper's main results: no-loss impact finance cannot generate additionality under competitive commercial finance; concessional capital plays different roles on the intensive and extensive margins; and both margins are linked through a common shadow value of scarce concessional funds.

Roadmap. The rest of the paper is organized as follows. Section 2 presents the economic environment. Section 3 characterizes the first-best and commercial-finance benchmarks, together with the limitations of no-loss impact investing. Section 4 studies optimal blended-finance contracts and derives the fiscal cost of implementing a given allocation. Section 5 formulates and solves the impact investor's global allocation problem. Section 6 studies two extensions: low entrepreneurial liquidity, which generates leakage, and commercial market power, which lowers the fiscal cost of intensive additionality. Section 7 presents numerical illustrations. Section 8 concludes.

2 Model

Consider an economy with a continuum of entrepreneurs, each with access to one project. Each project requires one unit of investment and is characterized by a pair

$$(p, q) \in [0, 1]^2.$$

The parameter p denotes the project's baseline probability of financial success and is publicly observed before contracting. It is distributed according to G, which is common knowledge. The parameter q denotes the baseline probability of impact conditional on financial success and is privately known to the entrepreneur. Conditional on p, outside investors know that q is distributed according to $F(\cdot \mid p)$, with density $f(\cdot \mid p)$ on $[0, 1]$; this distribution is also common knowledge. The model thus allows financial viability and impact potential to be correlated.

If the project succeeds financially, it generates a verifiable monetary return $R > 1$. Conditional on financial success, the project may also generate a verifiable non-monetary impact of size $\sigma > 0$. The parameter σ is the impact investor's monetary-equivalent valuation of impact and is taken as exogenous; the framework accommodates either a private valuation

by the funder or a social value set by a planner. There are therefore three verifiable output states: no financial success, financial success without impact, and financial success with impact. Denote these states by 0, R, and $R+\sigma$, where the latter is a label for the joint-success state: pledgeable monetary repayment in that state remains R, and σ is the verifiable but non-pledgeable impact value.

After financing is in place, the entrepreneur simultaneously chooses effort in two dimensions. Effort choices are not observable or verifiable. The first dimension is financial effort: high financial effort delivers financial success with probability p, while diversion lowers it to kp, where $k \in (0,1)$, and yields the entrepreneur a private payoff $c > 0$. The second dimension is impact effort: high impact effort delivers impact with conditional probability q, while diversion lowers it to kq and yields the same private payoff c. Diversion on one margin therefore gives private payoff c, while diversion on both margins gives $2c$. These private payoffs may represent side activities, private perks, diverted attention, or cost savings from lower-quality practices. They enter the entrepreneur's payoff but are not verifiable project output, pledgeable income, or measurable impact.

Denote the four possible action profiles by GG, GB, BG, and BB. The first letter refers to financial effort and the second to impact effort.[3] The induced probabilities of the three output states are

	$R+\sigma$	R	0
GG	pq	$p(1-q)$	$1-p$
GB	pkq	$p(1-kq)$	$1-p$
BG	kpq	$kp(1-q)$	$1-kp$
BB	k^2pq	$kp(1-kq)$	$1-kp.$

The entrepreneur has internal funds $E \in [0,1)$ and therefore requires external finance in the amount $1-E$; a financed entrepreneur contributes her available internal funds. There are two sources of external finance, each with a distinct mandate. The first is a competitive commercial financial sector. Commercial financiers value only verifiable monetary repayments and must break even in expectation on every project. They compete in posted contracts: each financier offers a take-it-or-leave-it repayment schedule, the entrepreneur selects the most attractive offer, and free entry drives expected profits to zero on accepted contracts. This Bertrand-style protocol delivers competitive pricing and is the equilibrium concept used throughout the paper. Section 6 relaxes the zero-profit assumption to study the case where commercial financiers extract part of the pledgeable surplus.

[3]The labels can be read as Good/Bad on each margin, or equivalently as Green/Brown when the impact dimension represents environmental performance.

The second financier is an impact investor that values realized impact and is endowed with a budget $K \geq 0$ of loss-absorbing concessional capital with no fiduciary obligation to break even. The impact investor's instrument is a state-contingent contract that may incorporate—and sit alongside—commercial finance within the same transaction. This contractual coexistence is the structural feature of blended finance that the model is built to capture: concessional and commercial capital are not parallel pools; they are layered within a single contract.

If the impact investor finances a tranche $I \in [0, 1 - E]$, the commercial sector finances the residual $1 - E - I$. Repayments depend on the verifiable output state and cannot condition on the entrepreneur's type or effort choice. The entrepreneur is protected by limited liability: in any state, total entrepreneur receipts must be non-negative, and in the no-success state the entrepreneur pays nothing. In the financial-success states (with or without impact), aggregate repayments to outside financiers are bounded above by the pledgeable monetary return R; the non-pledgeable impact value σ accrues to the impact investor through verification, not through monetary repayment.

All agents are risk neutral. The entrepreneur's payoff is expected monetary income, net of her contribution E, plus any private payoffs from diversion. Commercial financiers earn zero expected profits on accepted contracts. The impact investor's objective is total additionality, defined as expected social surplus generated relative to the counterfactual allocation in which only competitive commercial finance is available. Entrepreneur surplus enters as a participation constraint but not as a welfare term, consistent with the institutional reality that impact investing mandates evaluate projects by social impact rather than entrepreneur welfare. We make the additionality objective formal in Sections 4 and 5, after deriving the relevant counterfactuals.

Main modeling assumptions. Three modeling choices are worth flagging. First, the diversion technology is symmetric, with identical private payoff c and identical multiplier k on both margins. This delivers a transparent screening structure but is not load-bearing: the qualitative results extend to heterogeneous (c_F, c_I) and (k_F, k_I), with thresholds and rents adjusted accordingly. Second, all agents are risk neutral. The paper's contribution concerns information frictions and the scarcity of loss-absorbing capacity, not risk reallocation; risk aversion would generate additional channels, notably a second role for concessional capital as risk insurance, that we view as a separate inquiry. Third, effort is binary. Continuous effort would smooth several of the kinks the model exploits, but the central qualitative results survive.

The timing of the model is summarized in Figure 1.

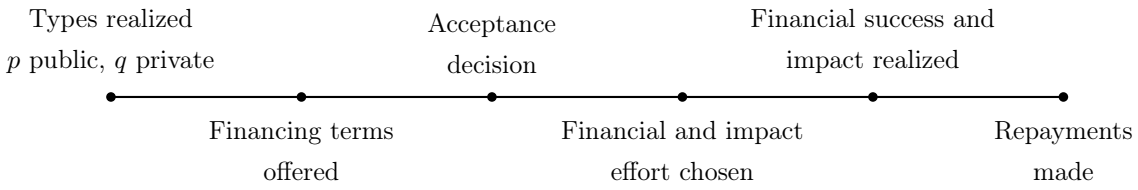


Figure 1: Timing of the model.

3 Benchmarks

First-best. Suppose that project types are observable, effort choices are contractible, and the impact investor faces no financing, fiduciary, or budget constraints. This benchmark characterizes the allocation an unconstrained impact investor would choose when it can finance any project with nonnegative project surplus, valuing impact in monetary-equivalent units.

The private payoff from diversion, c, is not included in the project's surplus. Diversion captures resources, attention, or effort that would otherwise have been devoted to the project, so c represents value extracted from the project rather than newly created by it; including it in surplus would double-count, since the resources that generate c are the same resources whose absence depresses expected output and impact. From a project-surplus perspective, c is a transfer of value from financiers to the entrepreneur, not an addition to total output. It enters the entrepreneur's payoff and creates an agency problem, but it does not enter the surplus the impact investor seeks to generate.

Conditional on financing, the unconstrained allocation implements high effort on both dimensions. Under GG, a type (p, q) generates project surplus

$$pR + pq\sigma - 1.$$

Any diversion weakly lowers expected monetary output, expected impact, or both. Hence the first-best impact-finance region is

$$pR + pq\sigma - 1 \geq 0,$$

or equivalently

$$p \geq p^{FB}(q) := \frac{1}{R + q\sigma}.$$

Thus, the first-best allocation depends jointly on financial viability and impact potential. Projects with low financial viability may be worth financing when impact potential is sufficiently high, while projects with low impact potential must rely primarily on financial returns to justify investment.

Commercial financing. We next characterize the allocation implemented by competitive commercial finance. Commercial lenders value only verifiable monetary repayments and do not value impact. They could in principle reward impact realization in their contracts. Doing so under break-even, however, requires raising the base repayment to compensate for the expected payout in the impact state, which leaves the entrepreneur with a strictly lower payoff than under a simple debt contract that ignores impact. The entrepreneur therefore prefers the impact-blind debt contract, and impact incentives go unprovided in the commercial market. Any commercially financed entrepreneur consequently diverts on the impact dimension.

In the commercial benchmark, if the lender expects high financial effort, competition implies

$$D^C(p) = \frac{1-E}{p},$$

where $D^C(p)$ is the repayment due after financial success. The entrepreneur's payoff from commercial finance is therefore

$$U^B(p) = p(R - D^C(p)) - E + c = pR - 1 + c,$$

where c is the private payoff from diverting on the impact dimension.

The baseline analysis assumes that entrepreneurs have enough internal funds for liquidity constraints not to bind on financial effort.

Assumption 1 (Sufficient entrepreneurial skin in the game)**.** *The parameters and the entrepreneur's internal funds satisfy*

$$\bar{E} := c + \frac{c}{1-k} < 1, \qquad E \geq \bar{E}.$$

Assumption 1 isolates the role of impact-oriented finance from the additional distortions created by limited entrepreneurial liquidity. Under this assumption, the entrepreneur's full contribution E is sufficient to discipline financial effort across all contracts considered in the baseline analysis. Section 6 relaxes the assumption and studies the case $E < \bar{E}$, where rescue contracts must leave implementation rents and may generate leakage.

Commercial finance must satisfy participation and financial-effort incentive compatibility. Participation requires $pR - 1 + c \geq 0$. Under Assumption 1, the financial-effort constraint is weaker than participation: the entrepreneur's own contribution provides enough skin in the game to discipline financial effort whenever the project is privately profitable. Thus, in the baseline economy, commercial finance is limited by private profitability rather than by financial moral hazard.

Lemma 1 (Commercial finance)**.** *Under Assumption 1, a project can be financed by the competitive commercial sector while inducing high financial effort if and only if*

$$p \geq p^C := \frac{1-c}{R}.$$

The entrepreneur's outside option under commercial finance is

$$U^B(p) = \begin{cases} pR - 1 + c, & p \geq p^C, \\ 0, & p < p^C. \end{cases}$$

The cutoff p^C is the central object of the analysis that follows. Above p^C, commercial finance brings projects to market and disciplines financial effort but does not reward impact; blended finance can only operate intensively, by adding impact-contingent incentives to projects already financed. Below p^C, entrepreneurs do not accept commercial financing as it delivers negative net financial value; blended finance must rescue projects through a contract that both finances and incentivizes them. The two regimes differ in the role concessional capital plays, and the analysis in the rest of the paper traces this difference through the contracting, allocation, and policy implications of the model.

Since Assumption 1 implies $c < 1$, the cutoff satisfies $p^C > 0$. I focus on the nondegenerate case $R > 1 - c$, so that the commercial-finance region is nonempty. Limited liability is not binding either: under Assumption 1, $D^C(p) = (1-E)/p \leq R$ for any $p \geq (1-E)/R$, and $(1-E)/R < p^C$ since $E > c$, so the high-effort commercial repayment is always feasible whenever participation holds.

Commercial contracts that knowingly tolerate low financial effort are also dominated under Assumption 1. Below p^C, even a low-effort commercial contract that breaks even fails the entrepreneur's participation constraint; above p^C, it is strictly dominated by the high-effort commercial contract.[4] Commercial finance therefore sorts projects only by financial viability:

[4]The argument follows the verification in the proof of Lemma 1: under low effort, zero profits require $kpD^S = 1 - E$, yielding entrepreneur payoff $U^S(p) = kpR - 1 + 2c$, which is strictly negative for any $p < p^C$ under Assumption 1, and is dominated by $U^B(p)$ for $p \geq p^C$.

projects with $p \geq p^C$ are financed and exert high financial effort, and projects with $p < p^C$ are not implemented.

This allocation leaves room for additionality on two distinct margins. Among commercially financed projects, an impact investor may improve incentives on the impact dimension; this is *intensive additionality*. Among projects with $p^{FB}(q) \leq p < p^C$, commercial finance excludes projects that would generate positive surplus if financed under the right incentives, and an impact investor may bring such projects to market; this is *extensive additionality*. The rest of the paper studies whether these two forms of additionality can be achieved without losses and, if not, how scarce concessional capital should be allocated across them.

Impact financing without blended finance. The next result is the central impossibility theorem of the paper. It establishes why blended finance, as a category, requires loss-absorbing capital—not merely additional capital. An impact investor that satisfies a no-loss constraint, in which expected repayment is at least as large as the amount it finances, cannot improve on commercial finance on either margin.

Lemma 2 (No-loss impact finance)**.** *Under Assumption 1, suppose the impact investor must break even in expectation. Then, regardless of which action profile a contract induces, it cannot generate additionality on either margin:*

(i) for any project with $p \geq p^C$, no break-even contract can induce high impact effort while making the entrepreneur weakly prefer the contract to commercial finance;

(ii) for any project with $p < p^C$, no break-even contract can finance the project and satisfy entrepreneur participation.

Lemma 2 pins down the economic role of concessional capital. Impact-contingent contracts are not the constraint; expected losses are. For bankable projects, inducing impact effort eliminates the entrepreneur's private payoff from impact diversion and therefore requires compensation relative to commercial finance, which a break-even contract cannot deliver. For non-bankable projects, break-even finance cannot leave the entrepreneur enough surplus to participate. The blended-finance problem studied below uses scarce concessional capital to relax the no-loss constraint, allowing additionality on both margins.

4 Optimal Blended Finance Contracts

The previous section established that any improvement on the commercial allocation requires loss-absorbing capital. This section characterizes how a blended-finance contract uses such capital. The contractual environment is the structural feature that distinguishes blended finance from generic targeted public finance: concessional and commercial capital coexist within the same financing arrangement, and the impact investor's instrument is a state-contingent overlay layered on top of no-loss commercial finance.

For each observable financial type p, take as given a cutoff $\hat{q}$, such that the impact investor seeks to support all types $q \geq \hat{q}$ and exclude types $q < \hat{q}$. The implementation problem differs across two regions. If $p \geq p^C$, the project is commercially financeable. The market already brings the project to market and disciplines financial effort, but it does not induce impact effort. The relevant intervention is an *intensive-additionality contract*: the impact investor leaves financing in place and adds impact-contingent incentives that move the entrepreneur from impact diversion to high impact effort.

If $p < p^C$, the project is not commercially financeable. Under pure commercial financing the project is not implemented, so the impact investor must both finance the project and induce effort along both dimensions. The relevant intervention is a *rescue contract*: the impact investor uses concessional capital to make the project financeable and incentive compatible. Under Assumption 1, the entrepreneur's contribution E is sufficient to discipline financial effort without leaving implementation rents to the marginal rescued type, so the baseline model admits clean cutoff allocations in the extensive region. Section 6 relaxes this assumption.

Contracting environment. Every financed entrepreneur contributes her internal funds E. Outside investors finance $1-E$. If the impact investor finances a tranche $I \in [0, 1-E]$, the commercial sector finances the residual $1-E-I$. Since the commercial sector is competitive and expects high financial effort, its repayment after financial success is

$$D_B = \frac{1-E-I}{p}.$$

The impact investor receives d_N in the state of financial success without impact and d_I in the state of financial success with impact. Decompose these into a no-loss component and a concessional overlay:

$$d_N = \frac{I}{p} + \tau_N, \qquad d_I = \frac{I}{p} + \tau_I.$$

Define the impact-contingent spread by

$$s = \tau_N - \tau_I.$$

This decomposition is the structural separation that defines the blended-finance contract: I/p is the fair repayment on the impact investor's no-loss tranche and breaks even in expectation, while (τ_N, τ_I) form the concessional overlay that affects incentives and generates fiscal cost. Crucially, the local implementation problem pins down the concessional overlay but *not* the split of the no-loss tranche between the impact investor and the commercial sector: any $I \in [0, 1 - E]$ can be paired with the same overlay. The two capital layers are therefore visible in the contract structure itself—the no-loss component and the concessional overlay are separable instruments within a single transaction.

For a type q exerting high effort on both dimensions, the expected fiscal cost of the overlay is

$$L(p, q; \tau_N, s) = p(qs - \tau_N).$$

For later use, define

$$S(p, \hat{q}) = 1 - F(\hat{q} \mid p), \qquad M(p, \hat{q}) = \int_{\hat{q}}^{1} q\, f(q \mid p)\, dq.$$

Thus, $S(p, \hat{q})$ is the mass of types above the cutoff and $M(p, \hat{q})$ is their aggregate impact potential.

Limited liability. Before turning to the contract derivation, it is useful to verify that the blended-finance contracts considered below respect the limited-liability bounds set out in Section 2.

Lemma 3 (Feasibility of blended-finance contracts)**.** *Under Assumption 1, the intensive-additionality contract characterized in Proposition 1 and the rescue contract characterized in Proposition 3 both satisfy the limited-liability bound $D_B + d_N \leq R$ in the no-impact-success state. Feasibility in the impact-success state, $D_B + d_I \leq R$, follows automatically since $s > 0$ implies $d_I < d_N$.*

The lemma confirms that the contracts proposed below remain feasible across the entire parameter region covered by Assumption 1. The structural feature of blended finance shows up in the proof: feasibility holds because the concessional overlay—not the no-loss tranche—

is what consumes pledgeable repayment, and the entrepreneur's contribution E bounds how aggressive the overlay can be.

Entrepreneur payoffs and cutoff pooling. Conditional on high financial effort, the entrepreneur's payoff under high impact effort is

$$U_{GG}(p, q) = pR - 1 - p(\tau_N - qs),$$

and under impact diversion

$$U_{GB}(p, q) = pR - 1 - p(\tau_N - kqs) + c.$$

Thus high impact effort is incentive compatible if and only if

$$pq(1 - k)s \geq c.$$

For a type q, the minimum spread that induces high impact effort is

$$s(q) = \frac{c}{pq(1 - k)}.$$

The contracts below pool all accepted impact types at the spread required by the marginal type. This is the standard direct-mechanism implementation in adverse-selection-with-effort settings (Laffont and Martimort, 2002; Myerson, 1981): incentive compatibility requires the spread schedule to be weakly increasing in q (a consequence of single-crossing in (q, s)), while cost minimization on the moral-hazard side prefers a uniform spread set at the level needed to incentivize the marginal type. Pooling at $s^*(p, \hat{q}) = c/[p\hat{q}(1-k)]$ is therefore the cheapest implementation. The screening conflict and its pooling resolution are familiar; what is new is the contractual environment in which they sit: the implementing contract must coexist with no-loss commercial finance, must respect limited liability layer-by-layer, and is funded out of a bounded budget of loss-absorbing capital. Sections 5 and onward exploit precisely these constraints.

4.1 Intensive Additionality: $p \geq p^C$

For $p \geq p^C$, the project is already commercially financed. The impact investor does not need to bring the project to market; it only needs to change incentives on the impact dimension.

Commercial finance already induces high financial effort, so the blended-finance contract operates as an impact-contingent overlay. Under Assumption 1, this overlay does not upset financial-effort incentives: the entrepreneur's contribution E remains sufficient to make financial diversion unprofitable. The entrepreneur's outside option is commercial finance,

$$U^B(p) = pR - 1 + c.$$

Proposition 1 (Intensive-additionality contract)**.** *For any $p \geq p^C$ and any $\hat{q} \in (0, 1]$, the no-loss tranche $I \in [0, 1 - E]$ is payoff-irrelevant for implementation. The minimum-cost concessional overlay that implements high impact effort for all $q \geq \hat{q}$, while leaving types $q < \hat{q}$ under commercial finance, is*

$$s^*(p, \hat{q}) = \frac{c}{p\hat{q}(1-k)}, \qquad \tau_N^*(p, \hat{q}) = \frac{ck}{p(1-k)}, \qquad \tau_I^*(p, \hat{q}) = \frac{ck}{p(1-k)} - \frac{c}{p\hat{q}(1-k)}.$$

Among all overlays implementing this cutoff, this overlay minimizes expected fiscal cost.

The marginal accepted type is $q = \hat{q}$. Both its impact incentive constraint and its participation constraint bind: if either had slack, the impact investor could lower transfers and reduce fiscal cost. These two binding conditions pin down s^*, τ_N^*, and τ_I^*. Since $\tau_I^* < \tau_N^*$, realized impact is rewarded through a lower repayment.

The contract gives an accepted type $q \geq \hat{q}$ payoff

$$U_{GG}^I(p, q \mid \hat{q}) = pR - 1 + \frac{c}{1-k}\left(\frac{q}{\hat{q}} - k\right).$$

At the cutoff, $U_{GG}^I(p, \hat{q} \mid \hat{q}) = pR - 1 + c = U^B(p)$, so the marginal type is indifferent between the impact contract and commercial finance. Higher-impact types earn information rents:

$$U_{GG}^I(p, q \mid \hat{q}) - U^B(p) = \frac{c}{1-k}\left(\frac{q}{\hat{q}} - 1\right).$$

Types below the cutoff do not take the impact overlay: if a type $q < \hat{q}$ accepts the overlay and diverts on the impact dimension, its payoff relative to commercial finance is $\frac{kc}{1-k}(q/\hat{q} - 1) < 0$.

Proposition 2 (Additionality and fiscal cost in the intensive region)**.** *The intensive-additionality contract characterized in Proposition 1 generates additionality*

$$B_I(p, \hat{q}) = p(1-k)\sigma M(p, \hat{q}),$$

and fiscal cost

$$L_I(p,\hat{q}) = cS(p,\hat{q}) + \frac{c}{1-k}\left[\frac{M(p,\hat{q})}{\hat{q}} - S(p,\hat{q})\right].$$

The expression for B_I reflects that the project would be financed even without intervention. Blended finance creates only the incremental impact from moving the project from impact diversion to high impact effort. For type q, expected impact rises from $pkq\sigma$ to $pq\sigma$, so the gain is $p(1-k)q\sigma$. Integrating gives $B_I(p,\hat{q})$.

The fiscal cost L_I has two components. The first term, $cS(p,\hat{q})$, is the implementation cost: each accepted entrepreneur must be compensated for giving up the private payoff from diverting on the impact dimension. The second term is the screening-rent cost generated by pooling. Lowering the cutoff requires a larger spread to incentivize the marginal type, and this larger spread is paid to all inframarginal accepted types.

Information rents and the boundedness of the cutoff. The information rent $\frac{c}{1-k}(q/\hat{q}-1)$ grows without bound as the cutoff $\hat{q}$ approaches zero. This is not pathological. The cutoff is endogenous: the impact investor chooses it to maximize additionality net of fiscal cost subject to the budget constraint. As shown in Section 5, the marginal-cost term from inframarginal rents enters the first-order condition with weight proportional to $1/\hat{q}^2$, so the optimal cutoff is bounded away from zero except in measure-zero parameter regions. Information rents along any optimal allocation are therefore bounded.

4.2 Extensive Additionality: $p < p^C$

For $p < p^C$, the project is not commercially financeable. The impact investor must therefore do more than provide an impact overlay: it must bring the project to market and induce both financial and impact effort. The entrepreneur's outside option is zero. Under Assumption 1, the entrepreneur's contribution E is sufficient to discipline financial effort while making the marginal rescued type indifferent.

Proposition 3 (Rescue contract)**.** *For any $p < p^C$ and any $\hat{q} \in (0,1]$, suppose the entrepreneur contributes her internal funds E. The no-loss tranche $I \in [0, 1-E]$ is payoff-irrelevant for implementation. Under Assumption 1, the minimum-cost concessional overlay that finances the project, induces high effort along both dimensions for all $q \geq \hat{q}$, and excludes all $q < \hat{q}$, is*

$$s^*(p,\hat{q}) = \frac{c}{p\hat{q}(1-k)}, \quad \tau_N^*(p,\hat{q}) = \frac{pR - 1 + \frac{c}{1-k}}{p}, \quad \tau_I^*(p,\hat{q}) = \frac{pR - 1 + \frac{c}{1-k}}{p} - \frac{c}{p\hat{q}(1-k)}.$$

Among all overlays implementing this cutoff, this overlay minimizes expected fiscal cost.

The impact incentive constraint is unchanged relative to the intensive region. The lowest accepted type is $q = \hat{q}$, so the required spread is $s^*(p, \hat{q})$. The difference is that, in the extensive region, the contract must also discipline financial effort. The entrepreneur's contribution E provides skin in the game for financial effort, while the spread provides incentives for impact effort and screens impact types.

The cutoff type's participation constraint pins down $p\tau_N^* = pR - 1 + c/(1-k)$. The relevant financial-effort constraint is the joint-deviation constraint that prevents the cutoff type from diverting on both dimensions, requiring $p\tau_N^* \leq pR - 1 + E - c$. Substituting gives $c/(1-k) \leq E - c$, or $E \geq \bar{E}$—which is precisely Assumption 1. If $E = \bar{E}$, the joint-deviation constraint binds; if $E > \bar{E}$, it holds with slack.

The contract gives an accepted type $q \geq \hat{q}$ payoff

$$U_{GG}^E(p, q \mid \hat{q}) = \frac{c}{1-k}\left(\frac{q}{\hat{q}} - 1\right).$$

At the cutoff, $U_{GG}^E(p, \hat{q} \mid \hat{q}) = 0$, so the marginal rescued type is indifferent between accepting the rescue contract and not undertaking the project. Higher-impact types earn information rents identical in form to those in the intensive region. This is not a coincidence: the information rent is governed entirely by the impact-effort incentive constraint at the marginal type, which has the same form in both regions. The two contracts differ only in the level transfer τ_N^*, not in how rents scale across types.

For a fixed cutoff $\hat{q}$, the local contracts therefore coincide in their entrepreneur-payoff structure at $p = p^C$: using $1 - p^C R = c$, one verifies that τ_N^* is continuous across the boundary. The local implementation problem is continuous at p^C for any fixed cutoff. This local continuity, however, does not imply that the *globally optimal* cutoff schedule is continuous: as Corollary 1 below establishes, the optimal cutoff jumps at p^C because the marginal role of concessional capital changes discretely across the boundary.

Types below the cutoff do not enter. For any $q < \hat{q}$, the spread $s^*(p, \hat{q})$ is too small to induce high impact effort. If such a type enters and chooses GB, its payoff is $\frac{kc}{1-k}(q/\hat{q} - 1) < 0$. If it chooses BB, its payoff is bounded above by $\frac{k^2 c}{1-k}(q/\hat{q} - 1) < 0$ under Assumption 1. The BG deviation is also unprofitable. The rescue contract therefore implements a clean cutoff in the baseline economy.

Proposition 4 (Additionality and fiscal cost in the extensive region)**.** *The rescue contract*

characterized in Proposition 3 generates additionality

$$B_E(p, \hat{q}) = (pR - 1)S(p, \hat{q}) + p\sigma M(p, \hat{q}),$$

and fiscal cost

$$L_E(p, \hat{q}) = (1 - pR)S(p, \hat{q}) + \frac{c}{1-k}\left[\frac{M(p, \hat{q})}{\hat{q}} - S(p, \hat{q})\right].$$

The extensive additionality term is the full project surplus created by rescued projects. Absent blended finance, these projects are not financed; with the rescue contract, each type $q \geq \hat{q}$ is financed and operated under GG, generating project surplus $pR-1+pq\sigma$. Integrating over $q \geq \hat{q}$ gives B_E.

The fiscal cost L_E has two components. The first, $(1-pR)S(p, \hat{q})$, is the expected financial shortfall from rescuing projects that are not commercially financeable. The second is the screening-rent cost generated by pooling. Financially stronger projects are cheaper to rescue because expected cash flows offset part of the concessional cost; lowering the cutoff remains costly because it raises rents for all inframarginal high-q types.

4.3 Information Revelation at Vanishing Cost

The pooling result of Sections 4.1 and 4.2 concerns payoff-relevant incentive contracts. It does not imply that the impact investor cannot learn q. The following result shows that information about accepted types can be elicited at vanishing fiscal cost without altering the cost-minimizing structure of the contract.

Proposition 5 (Information revelation at vanishing cost)**.** *For any $\varepsilon > 0$, the impact investor can append a reporting layer to the pooled contract such that:*

(i) conditional on financial success, the entrepreneur reports $r \in [0, 1]$ before impact is realized, and reports truthfully $r = q$;

(ii) the allocation and impact-effort incentives induced by the contract coincide with those of the pooled contract, in the limit $\varepsilon \to 0$;

(iii) the additional fiscal cost of the reporting layer is bounded above by ε.

The result clarifies that the pooling structure of Propositions 1 and 3 is about the structure of *incentives*, not about information availability. An impact investor who values learning the

realized distribution of q can do so essentially for free, by appending a small reporting layer that does not disturb the cost-minimizing contract. The pooling outcome is therefore a feature of the cost-minimizing implementation, not a substantive informational restriction.

5 Portfolio Allocation

The previous section solves the local implementation problem. For each financial type p and cutoff $\hat{q}(p)$, Propositions 1–4 characterize the minimum-cost concessional overlay that implements the desired allocation and the additionality it generates. We now solve the global allocation problem: how should an impact investor with a broad additionality mandate choose the cutoff schedule $p \mapsto \hat{q}(p)$ when concessional capital is scarce?

The object of choice is no longer an individual contract but a portfolio rule. Both forms of additionality—intensive and extensive—draw from the same scarce concessional budget, so the impact investor cannot solve them independently. The shadow value of concessional capital provides the pricing instrument that links the two margins, and the optimal cutoff in either region depends on what concessional capital could otherwise achieve in the other.

Aggregate additionality and fiscal cost. The impact investor selects a measurable cutoff function $\hat{q} : [0,1] \to [0,1]$. For $p \geq p^C$, types $q \geq \hat{q}(p)$ receive the intensive-additionality contract; for $p < p^C$, types $q \geq \hat{q}(p)$ receive the rescue contract. Types below the cutoff either remain under commercial finance, when $p \geq p^C$, or remain unfunded, when $p < p^C$. Throughout, the convention $\hat{q} = 1$ corresponds to no support, and $L_r(p, 0) = +\infty$ since inducing impact effort for $q = 0$ would require an infinite spread.

Total additionality and total fiscal cost are

$$B[\hat{q}] = \int_{p^C}^{1} B_I(p, \hat{q}(p))\, dG(p) + \int_{0}^{p^C} B_E(p, \hat{q}(p))\, dG(p),$$

$$L[\hat{q}] = \int_{p^C}^{1} L_I(p, \hat{q}(p))\, dG(p) + \int_{0}^{p^C} L_E(p, \hat{q}(p))\, dG(p).$$

The impact investor solves

$$\max_{\hat{q}(\cdot)} B[\hat{q}] \quad \text{s.t.} \quad L[\hat{q}] \leq K.$$

The budget is written in terms of L, not gross investment: L nets out repayments received by the impact investor and captures only the expected fiscal cost of the concessional overlay.

This is the natural formulation for blended finance, since the binding scarcity is loss-absorbing capacity, not gross capital deployment.

Proposition 6 (Optimal portfolio allocation)**.** *There exists a multiplier $\mu \geq 0$ such that an optimal cutoff schedule satisfies, for almost every p,*

$$\hat{q}(p;\mu) \in \arg\max_{\hat{q}\in[0,1]} \left\{ B_r(p,\hat{q}) - \mu L_r(p,\hat{q}) \right\},$$

where $r = I$ if $p \geq p^C$ and $r = E$ if $p < p^C$. The multiplier μ satisfies the complementary-slackness condition

$$\mu \geq 0, \qquad L[\hat{q}] \leq K, \qquad \mu(K - L[\hat{q}]) = 0.$$

For each financial type p, the impact investor chooses the cutoff that maximizes additionality net of fiscal cost valued at the shadow price μ. The same multiplier applies to all projects because all uses of concessional capital draw from the same loss-absorbing budget. The investor does not solve independent problems for each p; the common value of μ coordinates the allocation across margins. When concessional capital is scarce, $\mu > 0$ and the budget binds; if the unconstrained allocation would already satisfy $L[\hat{q}] < K$, then $\mu = 0$ and the investor stops at the cutoff that maximizes pointwise additionality without fiscal-cost discipline.

The argmax formulation accommodates corner solutions. For some p the investor may set $\hat{q}(p) = 1$, so that no type receives support. For other p the investor may set $\hat{q}$ at the lowest interior level. Both possibilities matter when concessional capital is scarce: the optimal portfolio may exclude entire regions of the type space rather than thin support uniformly.

Regularity. The interior characterization that follows requires standard regularity on the conditional distribution of impact potential.

Assumption 2 (Regularity)**.** *For each $p \in [0,1]$, the conditional density $f(\cdot \mid p)$ is strictly positive and continuously differentiable on $[0,1]$, and the conditional hazard rate $f(q \mid p)/[1 - F(q \mid p)]$ is monotone non-decreasing in q.*

Assumption 2 is a standard monotone-hazard-rate condition used in screening models with one-dimensional types. It guarantees that the pointwise objective $B_r(p,\hat{q}) - \mu L_r(p,\hat{q})$ admits a unique interior maximum for each p, and that the resulting interior cutoff schedule is well-defined and continuous in μ. The condition is satisfied by standard families of distributions, including the truncated normal, beta, and uniform distributions used in the simulations of Section 7.

Cutoff rule. When the solution is interior and $f(\hat{q}(p) \mid p) > 0$, the optimal cutoff satisfies

$$\frac{\partial B_r(p, \hat{q})}{\partial \hat{q}} = \mu \frac{\partial L_r(p, \hat{q})}{\partial \hat{q}}.$$

Marginal additionality from lowering the cutoff equals marginal fiscal cost, scaled by the shadow value of concessional capital. Define the conditional mean above the cutoff

$$\bar{q}(p, \hat{q}) = \mathbb{E}[q \mid q \geq \hat{q}, p] = \frac{M(p, \hat{q})}{1 - F(\hat{q} \mid p)}.$$

Proposition 7 (Cutoff rule)**.** *Suppose Assumption 2 holds, the solution is interior,* $f(\hat{q}(p) \mid p) > 0$*, and* $1 - F(\hat{q}(p) \mid p) > 0$*. For* $p \geq p^C$*, the optimal cutoff satisfies*

$$p(1-k)\sigma\, \hat{q}_I = \mu \left[c + \frac{c}{1-k} \frac{\bar{q}(p, \hat{q}_I)}{\hat{q}_I^2} \frac{1 - F(\hat{q}_I \mid p)}{f(\hat{q}_I \mid p)} \right].$$

For $p < p^C$*, the optimal cutoff satisfies*

$$pR - 1 + p\sigma\, \hat{q}_E = \mu \left[1 - pR + \frac{c}{1-k} \frac{\bar{q}(p, \hat{q}_E)}{\hat{q}_E^2} \frac{1 - F(\hat{q}_E \mid p)}{f(\hat{q}_E \mid p)} \right].$$

The two equations have the same structural form. The left-hand side is the marginal additionality from lowering the cutoff to include a marginal type. The right-hand side is the marginal fiscal cost, scaled by μ. The final term on the right captures the screening-rent cost of pooling: lowering the cutoff requires a larger spread for the marginal type, which is paid to all inframarginal accepted types, with weight proportional to the inverse hazard rate.

What differs between the two regions is the *economic content* of each side. In the intensive region, the marginal benefit is the incremental impact from moving a bankable project from impact diversion to high impact effort, and the direct marginal fiscal cost is c—the impact diversion private benefit the cutoff type must be compensated for forgoing. In the extensive region, the marginal benefit is the full project surplus, including the financial component $pR - 1$ that turns negative for non-bankable projects, and the direct marginal fiscal cost is $1 - pR$—the expected financial shortfall absorbed by concessional capital. The two regions are linked only through the common shadow value μ: the cost structures and counterfactuals are different, but their cutoffs draw from the same budget.

Discontinuity at the commercial-finance boundary. The two cutoff schedules in Proposition 7 solve different optimization problems on either side of p^C. Their values at

the boundary need not agree. The next result formalizes the conditions under which they could agree, and shows that those conditions are satisfied only in a measure-zero parameter region.

Corollary 1 (Discontinuity at the commercial-finance boundary)**.** *Suppose Assumption 2 holds. Fix a shadow value $\mu > 0$, and suppose the cutoff schedules are interior in a neighborhood of p^C. Let*

$$\hat{q}_I^+(\mu) := \lim_{p \downarrow p^C} \hat{q}_I(p;\mu), \qquad \hat{q}_E^-(\mu) := \lim_{p \uparrow p^C} \hat{q}_E(p;\mu),$$

whenever these limits exist. If the two cutoffs coincide at the boundary, $\hat{q}_I^+(\mu) = \hat{q}_E^-(\mu) = q^0$, then necessarily

$$q^0 = \frac{c}{kp^C\sigma}.$$

The optimal cutoff is therefore generically discontinuous at p^C: equality requires both $c/(kp^C\sigma) \in (0,1)$ and a knife-edge value of μ at which this candidate cutoff also satisfies the common first-order condition. If $c/(kp^C\sigma) \notin (0,1)$, no continuous interior cutoff is possible at p^C.

The discontinuity is a structural feature of blended finance, not an artifact of binary effort. The marginal role of concessional capital changes *discretely* at p^C: just above the boundary, blended finance changes incentives in a project the market already finances; just below, blended finance creates the project itself. This regime change in the counterfactual is what produces the discontinuity. Continuous-effort versions of the model preserve this feature: the discontinuity arises from the fact that the entrepreneur's outside option drops from $pR-1+c$ to zero at p^C and the marginal additionality jumps from $p(1-k)\sigma\hat{q}$ to $pR-1+p\sigma\hat{q}$, neither of which depends on the discreteness of effort.

The structure of the candidate continuity threshold is informative. Under uniform conditional impact potential, $F(q \mid p) = q$, the first-order conditions of Proposition 7 reduce at $p = p^C$ to cubic equations in $\hat{q}$ that share a common right-hand side but differ in their left-hand sides by exactly $c - kp^C\sigma\hat{q}$. The two cubics have a common root only when this difference vanishes—that is, at $\hat{q} = q^0 = c/(kp^C\sigma)$—and only at the unique value of μ at which q^0 is itself the optimal cutoff. The closed-form critical multiplier under uniform F is

$$\mu^0 = \frac{2(p^C)^2\sigma^2(1-k)^2(q^0)^3}{c\big[(1-2k)(q^0)^2+1\big]},$$

which is generically distinct from the equilibrium multiplier solving the budget constraint. For any other admissible μ, the two cutoffs disagree at p^C. The discontinuity is therefore not

approximate: it is a structural property that can be verified pointwise from the parameters of the model, and is illustrated numerically in Section 7.

Welfare and policy interpretation. The discontinuity has implications beyond a feature of the cutoff schedule. Two projects with arbitrarily close financial viability p but on opposite sides of p^C receive structurally different treatment: one may be supported intensively, the other rescued extensively, with different cutoff values, different fiscal costs, and different welfare consequences. A policy that allocates concessional capital under a continuous heuristic—for example, by ranking projects on a continuous Social Profitability Index—will systematically misallocate near the boundary. The discontinuity is the model's microfoundation for why blended-finance policy must distinguish, in practice, between intensive and extensive interventions rather than treat them as a single ranking problem.

Proposition 8 (Market sorting under blended finance)**.** *Let $\hat{q}_I(p;\mu)$ and $\hat{q}_E(p;\mu)$ denote the optimal cutoff schedules characterized in Proposition 6. The optimal allocation partitions the type space $\Omega = [0,1]^2$ into four regions:*

$$\Omega_I(\mu) = \{(p,q) \in \Omega : p \geq p^C,\ q \geq \hat{q}_I(p;\mu)\},$$

$$\Omega_C(\mu) = \{(p,q) \in \Omega : p \geq p^C,\ q < \hat{q}_I(p;\mu)\},$$

$$\Omega_E(\mu) = \{(p,q) \in \Omega : p < p^C,\ q \geq \hat{q}_E(p;\mu)\},$$

$$\Omega_0(\mu) = \{(p,q) \in \Omega : p < p^C,\ q < \hat{q}_E(p;\mu)\}.$$

Types in $\Omega_I(\mu)$ are financed commercially and receive the intensive-additionality contract; types in $\Omega_C(\mu)$ are financed commercially without intervention; types in $\Omega_E(\mu)$ receive the rescue contract; types in $\Omega_0(\mu)$ remain unfunded.

Proposition 8 describes how blended finance sorts projects across markets. Commercial finance determines bankability through p^C; conditional on this market status, concessional capital is allocated according to impact potential. Among bankable projects, the impact investor uses concessional capital only to improve impact incentives, leaving the financing decision to the market. Among non-bankable projects, the impact investor must use concessional capital both to bring the project to market and to induce high effort. Higher-q projects are targeted first because they generate more additionality per unit of fiscal cost.

The boundary at p^C is economically meaningful. To its right, blended finance operates on the intensive margin; to its left, on the extensive margin. The two cutoff schedules need not

meet at p^C, because the outside option, the marginal benefit, and the marginal fiscal cost all change discretely at the bankability threshold (Corollary 1).

The optimal policy is therefore complementary to the market, not substitutive. Blended finance leaves commercially viable, low-impact projects to the market; improves commercially viable, high-impact projects through impact-contingent overlays; and uses scarce concessional capital to rescue only those non-bankable projects whose impact potential is high enough to justify the fiscal cost of bringing them to market.

Mobilized non-concessional finance. Among the central policy questions about blended finance is how much non-concessional capital the model mobilizes alongside the concessional layer. In the intensive region, projects would be financed commercially regardless, so any no-loss tranche provided by the impact investor merely substitutes for commercial finance and does not increase total financing. In the extensive region, by contrast, projects would remain unfunded absent the rescue contract, so no-loss capital deployed alongside the concessional overlay is financially additional.

Every rescued entrepreneur contributes her internal funds E, so outside no-loss finance equals $1 - E$ per rescued project. This outside finance can be split between a commercial tranche and a non-concessional impact-investor tranche; the split is irrelevant for incentives and is not pinned down by the model.

Proposition 9 (Mobilization of non-concessional finance)**.** *Let $\hat{q}_E(p;\mu)$ denote the optimal rescue cutoff. The total amount of non-concessional outside finance mobilized on the extensive margin is*

$$\mathcal{M}(\mu) = (1 - E) \int_0^{p^C} S(p, \hat{q}_E(p;\mu))\, dG(p), \qquad S(p, \hat{q}_E) = 1 - F(\hat{q}_E \mid p).$$

If concessional capital is scarce and the budget binds, the amount of non-concessional finance mobilized per unit of concessional capital is $\mathcal{M}(\mu)/K$. If the budget is slack, the relevant ratio is $\mathcal{M}(\mu)/L[\hat{q}]$.

Proposition 9 clarifies the relationship between the model's mobilization measure and the OECD-DAC mobilization metric used in policy practice. The OECD-DAC measure counts gross private capital deployed alongside concessional funds, including capital deployed in projects the market would have financed under the same terms. Such capital appears in the headline metric but generates no additionality, since it does not change which projects are undertaken or how they perform. The model's measure $\mathcal{M}(\mu)$ restricts attention to the

extensive margin, where no-loss capital is genuinely additional. The two measures coincide only when no-loss tranches are confined to rescued projects; in general, the OECD-DAC measure overstates mobilization-relevant private capital by including capital that would have been deployed commercially regardless.

This decoupling has a sharper implication that the simulations in Section 7 make explicit: when concessional capital is allocated optimally toward intensive additionality, mobilization can be *zero* even though additionality is strictly positive. The mobilization metric is therefore an incomplete diagnostic for blended-finance success—blind both to projects that would have been financed anyway and to interventions that change behavior in already-financed projects. The relevant policy object is additionality per unit of fiscal cost, with mobilization a correlated but distinct outcome.

6 Extensions

The baseline analysis isolates the contractual frictions that motivate blended finance: hidden impact effort, private impact potential, and the no-loss constraint. Two further frictions are central to blended-finance practice but absent from the baseline. First, entrepreneurs in many target sectors are liquidity-constrained, with insufficient internal funds to discipline financial effort on their own. Second, commercial finance in the institutional settings where blended finance operates—emerging-market infrastructure, frontier-market lending, sectors served by a small number of regional financiers—is not always competitive. This section relaxes each assumption in turn and shows that they generate, respectively, a leakage problem on the extensive margin and a portfolio spillover that lowers the fiscal cost of intensive additionality.

6.1 Low Entrepreneurial Liquidity

The baseline analysis assumes that entrepreneurs have enough internal funds to discipline financial effort in rescue contracts. We now relax this assumption and consider the low-liquidity case

$$E < \bar{E} := c + \frac{c}{1-k}.$$

This extension has two purposes. First, it shows how the commercial-finance benchmark changes when financial moral hazard binds. Second, it shows why rescue contracts may generate leakage: when entrepreneurs lack enough skin in the game, the impact investor must leave implementation rents to induce financial effort, and these rents may attract

lower-impact types. The leakage mechanism is structural to the contractual environment of blended finance: subsidies needed to solve one incentive problem (financial effort) attract types that exploit another (impact effort).

To keep the extension focused on high-effort commercial finance, we rule out commercial lending under anticipated financial diversion.

Assumption 3 (No low-effort commercial finance)**.** *The pledgeable return under financial diversion is insufficient to cover outside finance:*

$$1 - E > kR.$$

Indeed, if lenders expected financial diversion, zero profits would require repayment $D^S = (1-E)/(kp)$. Since the monetary return after success is at most R, such a contract is feasible only if $1 - E \leq kpR$. Assumption 3 rules this out for all $p \leq 1$.

Commercial finance when financial moral hazard binds. The entrepreneur's payoff from high-effort commercial finance remains

$$U^B(p) = pR - 1 + c.$$

What changes is the cutoff. Commercial finance must now satisfy both participation and financial-effort incentive compatibility. Participation requires $p \geq (1-c)/R$; financial-effort incentive compatibility requires

$$p \geq \frac{1 - E + \frac{c}{1-k}}{R}.$$

When $E < \bar{E}$, the financial-effort constraint is tighter. The high-effort commercial-finance cutoff becomes

$$p_L^C = \frac{1 - E + \frac{c}{1-k}}{R} > \frac{1-c}{R}.$$

Commercial lending is no longer restricted only by private profitability but also by financial moral hazard. There is now a range of projects,

$$\frac{1-c}{R} \leq p < p_L^C,$$

that would be privately attractive under high financial effort but cannot be financed commercially while preserving financial-effort incentives. The commercial-finance boundary moves outward when entrepreneurs are liquidity-constrained, expanding the set of projects that require concessional support to be brought to market.

Intensive additionality. For $p \geq p_L^C$, the project is commercially financeable with high financial effort. The market already brings the project to market and disciplines financial effort, so the impact investor only needs to induce impact effort. The intensive-additionality contract and its associated additionality and fiscal cost are therefore unchanged. The only change is that the intensive region now begins at p_L^C rather than at p^C: low entrepreneurial liquidity shrinks the intensive region but does not change its contractual structure.

Rescue contracts and implementation rents. The extensive region changes more substantially. When $p < p_L^C$, the project is not commercially financeable with high financial effort. A rescue contract must therefore bring the project to market and induce both financial and impact effort. However, because $E < \bar{E}$, the entrepreneur's own contribution is no longer sufficient to discipline financial effort while making the marginal rescued type indifferent.

Fix a desired cutoff $\hat{q}$. Within the same contractual space as the baseline, accepted types are pooled at the spread required to induce high impact effort for the marginal type, and the impact incentive constraint pins down

$$s^*(p, \hat{q}) = \frac{c}{p\hat{q}(1-k)}.$$

The no-impact repayment is set as high as possible subject to financial-effort incentives, giving

$$\tau_N^*(p) = \frac{pR - 1 + E - c}{p}, \qquad \tau_I^*(p, \hat{q}) = \tau_N^*(p) - s^*(p, \hat{q}).$$

The cutoff type then obtains payoff

$$U_{GG}^{E,L}(p, \hat{q} \mid \hat{q}) = \bar{E} - E \equiv \Delta_E > 0.$$

Unlike in the baseline, the marginal rescued type must receive a strictly positive implementation rent Δ_E. This is not an informational rent; it is the rent required to discipline financial effort when the entrepreneur lacks sufficient skin in the game. For any accepted type $q \geq \hat{q}$,

$$U_{GG}^{E,L}(p, q \mid \hat{q}) = \Delta_E + \frac{c}{1-k}\left(\frac{q}{\hat{q}} - 1\right).$$

Accepted types receive both the implementation rent Δ_E and, for $q > \hat{q}$, the usual informational rents generated by pooling.

Leakage. The implementation rent Δ_E creates a leakage problem. A lower-impact type $q < \hat{q}$ is not induced to exert high impact effort because the spread is designed for the marginal type. Nevertheless, the contract may still be attractive: the implementation rent makes participation attractive even without delivering impact effort. Within the baseline contractual space, such a type's best deviation is to divert on both dimensions.

A lower-impact type enters whenever $q \geq q^{\ell}(E, \hat{q})$, where

$$q^{\ell}(E, \hat{q}) = \min\left\{\hat{q},\, \max\left\{0,\, \hat{q}\,\frac{(1-k)\big(E-(2+k)c\big)}{k^2 c}\right\}\right\}.$$

Since $E < \bar{E}$, the leakage interval $[q^{\ell}(E, \hat{q}), \hat{q})$ is nonempty whenever there is positive mass of types below the intended cutoff. These entrants are not intended beneficiaries of the rescue contract; they enter because the subsidy and implementation rents make participation attractive, but they do not deliver impact effort. This is greenwashing leakage: the rescue subsidy attracts firms that exploit it without delivering the additionality that justified the subsidy in the first place.

Proposition 10 (Low liquidity and leakage)**.** *Suppose $E < \bar{E}$ and Assumption 3 holds. Under the baseline rescue contract designed to implement cutoff $\hat{q}$, the marginal rescued type must receive implementation rent $\Delta_E = \bar{E} - E > 0$. Moreover, absent additional eligibility screening, lower-impact types $q < \hat{q}$ enter the rescue program and divert on both dimensions whenever*

$$q \in [q^{\ell}(E, \hat{q}), \hat{q}).$$

Since $E < \bar{E}$, this interval is nonempty. Low entrepreneurial liquidity therefore generates leakage in the extensive region.

The implication is twofold. With an additional eligibility-screening technology, the intended rescue allocation can still be implemented, but the fiscal cost includes the implementation rent:

$$L_E^L(p, \hat{q}) = (1 - pR + \Delta_E)S(p, \hat{q}) + \frac{c}{1-k}\left[\frac{M(p, \hat{q})}{\hat{q}} - S(p, \hat{q})\right].$$

Without eligibility screening, the portfolio problem must account for leakage explicitly: lower-impact entrants absorb concessional resources and may undermine the no-loss nature of mobilized capital by diverting on the financial dimension. The leakage-adjusted portfolio problem is characterized in Appendix B.

The low-liquidity case reinforces the central message of the paper. Blended finance can

create additionality, but when the contractual environment leaves implementation rents on the extensive margin, those rents distort the screening problem the rescue contract is meant to solve. Scarce concessional capital must therefore be targeted not only to projects with high social value, but to settings where eligibility and incentive problems can be effectively controlled. The model thus provides a contractual rationale for two patterns emphasized in blended-finance practice: the importance of eligibility verification in donor-funded programs, and the heightened risk of greenwashing in rescue-style interventions targeting capital-constrained sectors.

6.2 Commercial Market Power

The baseline analysis assumes commercial finance is competitive. This is the appropriate first cut for a theory of blended finance because it makes the impossibility result in Lemma 2 as sharp as possible: when commercial financiers break even, an impact investor that also breaks even cannot improve on the market allocation. We now relax the competitive assumption. In many of the institutional settings where blended finance operates—emerging-market infrastructure, frontier-market project finance, sectors with a small number of regional financiers—commercial lenders earn rents on the projects they finance. The question is whether and how these rents change the role of concessional capital.

Equilibrium concept under market power. We model commercial market power as the ability of commercial lenders to extract a fraction $\eta \in [0, 1]$ of the pledgeable monetary surplus generated by commercially financed projects. The case $\eta = 0$ is the competitive benchmark; larger values capture relationship-lending rents, search frictions, geographic concentration, or other departures from perfect competition. We assume the same posted-contract protocol as in the baseline: each commercial financier offers a contract; the entrepreneur accepts the most attractive offer that satisfies break-even. Under $\eta > 0$, the most attractive feasible commercial offer leaves the entrepreneur with $pR - 1 + c - \eta\Pi(p)$, where $\Pi(p) := (pR - 1)^+$ denotes the positive pledgeable surplus. Banks cannot extract the entrepreneur's private payoff from diversion c; this payoff is not pledgeable income but the value of diverting attention or resources from the project. Commercial rent extraction therefore applies only to monetary surplus.

The impact investor enters the same posted-contract market as a competing financier. Crucially, the impact investor can offer a no-loss financing package that delivers strictly more entrepreneur surplus than the rent-extracting commercial contract whenever rents are

positive: the impact investor needs only to break even, while the commercial lender extracts $\eta\Pi(p)$ on top. The entrepreneur therefore strictly prefers any no-loss offer from the impact investor that matches break-even pricing without rent extraction. This is the displacement mechanism: at $\eta > 0$, the impact investor can substitute itself for the rent-extracting commercial lender on bankable projects without reducing the entrepreneur's effort incentives.

Outside option and intensive cost. Under rent extraction η, the entrepreneur's commercial outside option becomes

$$U_\eta^B(p) = pR - 1 + c - \eta\Pi(p).$$

For $p < 1/R$ there is no positive monetary surplus to extract, so the commercial contract coincides with the competitive one. For $p \geq 1/R$, rent extraction increases repayments to commercial lenders; under Assumption 1, financial-effort incentives remain satisfied for all projects that were commercially financeable in the baseline. The commercial-finance cutoff p^C is therefore unchanged.

What changes is the cost of intensive additionality. In the competitive benchmark, the impact investor must compensate the entrepreneur for giving up the private payoff from impact diversion. With commercial market power, part of this compensation can be financed by displacing rents that would otherwise accrue to commercial lenders.

Proposition 11 (Commercial market power and intensive additionality)**.** *Suppose commercial lenders extract a fraction $\eta \in [0,1]$ of pledgeable monetary surplus $\Pi(p) = (pR-1)^+$, and the impact investor displaces the rent-extracting commercial contract with a no-loss financing package. For $p \geq p^C$, the minimum fiscal cost of implementing intensive additionality is*

$$L_I^\eta(p,\hat{q}) = L_I(p,\hat{q}) - \eta\Pi(p)S(p,\hat{q}).$$

Commercial market power therefore reduces the fiscal cost of intensive additionality by the amount of commercial rents displaced on supported projects.

When $L_I^\eta(p,\hat{q}) < 0$, the intervention generates net financial surplus by displacing commercial rents. We treat this surplus as available to relax the aggregate concessional-capital constraint, consistent with the budgetary interpretation in Section 5.

The proposition shows why the competitive benchmark is the hardest case for no-loss impact finance. Under perfect competition, there are no financial-sector rents to redirect toward impact incentives. Under commercial market power, the impact investor can offer

a less extractive financing package and use the displaced rents to fund impact-contingent rewards. If displaced rents are large enough, intensive additionality may require little or no concessional capital.

Portfolio spillover. Commercial market power also has a portfolio effect. The extensive-margin rescue technology is unchanged: projects with $p < p^C$ still require concessional capital to be brought to market. However, because intensive additionality becomes cheaper, the aggregate concessional-capital constraint is relaxed. The same shadow value of concessional funds governs both margins. By lowering the cost of intervention above p^C, commercial market power can therefore indirectly expand rescue finance below p^C.

The portfolio problem under market power becomes

$$\max_{\hat{q}(\cdot)} \left\{ \int_{p^C}^{1} B_I(p, \hat{q}(p))\, dG(p) + \int_{0}^{p^C} B_E(p, \hat{q}(p))\, dG(p) \right\}$$

subject to

$$\int_{p^C}^{1} L_I^{\eta}(p, \hat{q}(p))\, dG(p) + \int_{0}^{p^C} L_E(p, \hat{q}(p))\, dG(p) \leq K.$$

Proposition 12 (Commercial market power raises attainable additionality)**.** *Fix K, and let $V^{\eta}(K)$ denote the maximized additionality under commercial rent extraction $\eta \in [0, 1]$. Then $V^{\eta}(K)$ is weakly increasing in η. If the budget constraint binds and the additional fiscal slack created by a higher η can be used to expand support on a set of positive measure, then $V^{\eta}(K)$ is strictly increasing.*

Holding the concessional budget fixed, a higher η expands the set of implementable cutoff schedules because some impact incentives can be financed by displacing bank rents rather than by drawing on concessional capital.

Proposition 13 (Portfolio spillover of commercial market power)**.** *Under Assumption 2, for a given shadow value μ, commercial market power lowers the fiscal cost of intensive additionality and therefore lowers the intensive cutoff relative to the competitive benchmark. If the budget constraint binds, the reduction in intensive-region fiscal costs weakly lowers the equilibrium shadow value of concessional capital. Through this lower shadow value, commercial market power can also lower the rescue cutoff below p^C, expanding extensive additionality.*

Imperfect competition in commercial finance therefore has two effects on the blended-finance allocation. First, it directly reduces the concessional cost of improving impact among

already-financed projects: a no-loss tranche from the impact investor can substitute for a rent-extracting commercial loan, and the displaced rents fund impact incentives. Second, because concessional capital is allocated at the portfolio level, this cost reduction spills over to the extensive margin: cheaper intensive additionality frees scarce concessional resources that can be used to rescue additional non-bankable projects. The two effects together mean that commercial market power, viewed as a friction in conventional finance, becomes a partial substitute for concessional capital in blended-finance practice.

7 Simulations

This section presents a simple numerical exercise to illustrate the structure of the optimal allocation. The objective is not to estimate the model but to choose a transparent parameterization grounded in observable market quantities and study how the allocation responds to the availability of concessional capital. The exercise also makes the discontinuity result of Corollary 1 visible: the optimal cutoff schedule jumps at the commercial-finance boundary even under a uniform conditional distribution of impact types.

Parameterization. I assume p is uniformly distributed on $[0, 1]$, and that conditional on p, the impact type q is also uniformly distributed on $[0, 1]$. This conditional distribution satisfies Assumption 2 (the uniform hazard rate is monotone non-decreasing) and provides a neutral benchmark in which no region of the type space is mechanically favored. The simulation uses

$$R = 2.25, \qquad E = 0.5, \qquad k = 0.5, \qquad c = 0.15, \qquad \sigma = 4.5.$$

The first two parameters are chosen to be broadly consistent with observable market quantities. Since $\mathbb{E}[p] = 1/2$, setting $R = 2.25$ implies an average gross monetary return of $\mathbb{E}[p]R = 1.125$, or a net return of approximately 12.5%.[5] This is close to estimated average returns for Colombia, Chile, and Brazil obtained using Damodaran's country risk premium data: 12.6% for Colombia, 10.9% for Chile, and 13.0% for Brazil (Damodaran, 2026). I interpret E as a stylized measure of inside equity, and set $E = 0.5$ inspired by average equity shares in these markets (approximately 57% for Brazil, 54.4% for Chile, 59.6% for Colombia in the same data). These values should be read as illustrative moments aligned with observed

[5]The model has no time dimension, so any annualization is informal. The 12.5% figure should be read as a single-period gross-to-net comparison, not as a literal annual rate of return.

market magnitudes, not as a formal calibration.

The remaining parameters are structural. I set $k = 0.5$, so diversion halves the success probability on the relevant dimension; $c = 0.15$, which generates a meaningful agency problem while satisfying the high-equity assumption,

$$\bar{E} = c + \frac{c}{1-k} = 0.15 + \frac{0.15}{0.5} = 0.45 < E = 0.5;$$

and $\sigma = 4.5$, so impact value is quantitatively important relative to financial returns. Under these values, the commercial-finance cutoff is

$$p^C = \frac{1-c}{R} = \frac{0.85}{2.25} \simeq 0.378.$$

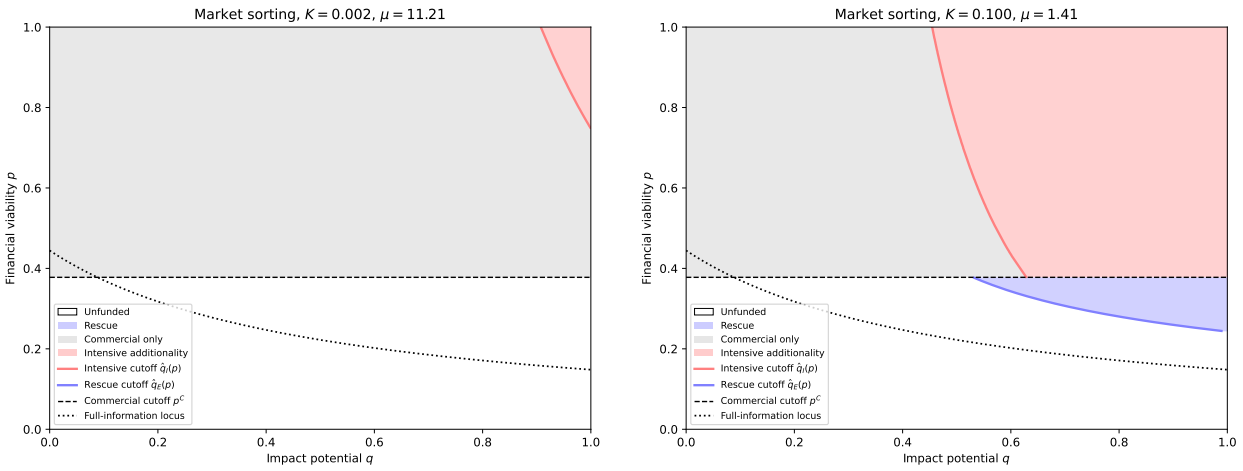
Projects with $p \geq 0.378$ are commercially bankable; projects below this threshold require concessional support to be financed.

Market sorting and the discontinuity. Figure 2 plots the optimal allocation in (q, p) space for two values of the concessional budget, $K = 0.002$ and $K = 0.10$. The horizontal axis is impact potential q; the vertical axis is financial viability p. The figure also plots the commercial-finance cutoff p^C and the full-information efficiency locus

$$p = \frac{1}{R + q\sigma},$$

which separates projects with positive surplus from those that are inefficient even under GG.

The two cutoff schedules—intensive (above p^C) and extensive (below p^C)—are plotted separately, and the visible jump at p^C is the empirical counterpart of Corollary 1. The intensive cutoff sits well above the extensive cutoff at p^C for both budget levels, with the gap reflecting the discrete change in the marginal role of concessional capital across the boundary. At $K = 0.002$, concessional capital is extremely scarce: the impact investor concentrates only on the cheapest form of additionality, and in this calibration that is intensive additionality. No non-bankable projects are rescued. At $K = 0.10$, the shadow value of concessional capital is close to one and both intervention types are active; the impact investor continues to provide intensive additionality among bankable projects and also begins to rescue high-impact projects below p^C. The larger budget expands the allocation along both margins simultaneously, with the discontinuity at p^C preserved.



(a) $K = 0.002$ *(b)* $K = 0.10$

Figure 2: Market sorting under blended finance. The dashed horizontal line is the commercial-finance cutoff p^C, and the dotted curve is the full-information efficiency locus $p = 1/(R + q\sigma)$. The intensive cutoff $\hat{q}_I(p)$ (above p^C) and rescue cutoff $\hat{q}_E(p)$ (below p^C) are drawn separately; the visible jump at p^C is the discontinuity established in Corollary 1.

Shadow value and coverage of welfare-generating projects. Figure 3 summarizes how the optimal allocation responds to the budget. The left panel plots $\mu(K)$, which is decreasing as predicted by Proposition 6: when concessional capital is scarce, its shadow value is high and the investor applies strict selection criteria; as the budget expands, the marginal value of an additional unit falls.

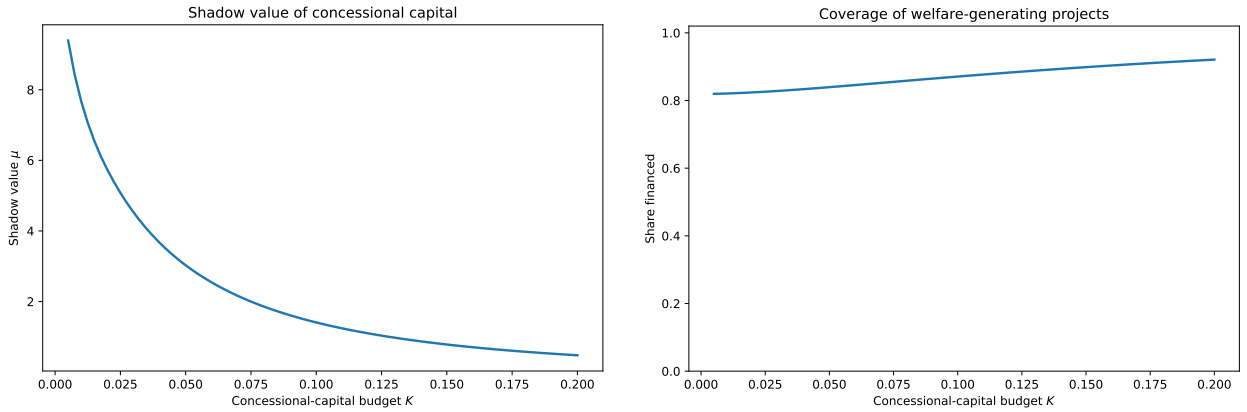
The right panel plots the share of welfare-generating projects financed under the optimal allocation. Define

$$\mathcal{W} = \{(p, q) \in [0, 1]^2 : p(R + q\sigma) - 1 \geq 0\}$$

as the set of types that would be financed under full information, and let

$$\Psi(K) = \frac{\text{mass of funded projects in } \mathcal{W}}{\text{mass of projects in } \mathcal{W}}$$

be the share covered. $\Psi(K)$ is increasing in the budget: more concessional capital allows the investor to lower the relevant cutoffs and expand support. The figure also clarifies the role of contractual frictions. Even when a project lies in $\mathcal{W}$, it may remain unfunded if its impact potential is too low relative to the screening, self-selection, and effort-provision costs that scarce concessional capital must cover. The gap between the full-information allocation and the blended-finance allocation therefore reflects not only the scarcity of capital but also the information rents and incentive costs of implementing additionality.



(a) Shadow value μ *(b) Share of welfare-generating projects financed*

Figure 3: Response of the optimal allocation to the concessional-capital budget. The left panel plots the shadow value of concessional capital. The right panel plots the share of welfare-generating projects financed under the optimal allocation.

Mobilization and additionality. Figure 4 plots both the mobilized non-concessional finance $\mathcal{M}(K)$ and the mobilization ratio $\mathcal{M}(K)/K$ as functions of the budget. Mobilized capital is increasing in K: more concessional resources allow the investor to rescue more non-bankable projects and therefore deploy more no-loss capital alongside the concessional layer.

The mobilization ratio is non-monotone, however, and this is the central diagnostic the figure provides. At very low budgets, all concessional capital is allocated intensively and mobilization is zero—additionality is strictly positive, but $\mathcal{M}(K)/K = 0$. As the budget rises and the rescue cutoff falls, the ratio increases. At higher budgets, the ratio plateaus or declines because the marginal concessional dollar is increasingly absorbed by intensive interventions and screening rents on inframarginal rescued projects. The ratio is therefore a noisy diagnostic for blended-finance success: it can be zero when additionality is positive (intensive allocations only), and it can rise without proportional increases in additionality (rescues that absorb fiscal slack with low impact potential).

This non-monotonicity makes precise the disconnect between the OECD-DAC mobilization metric and welfare-relevant additionality discussed in Section 5. A blended-finance program scored on mobilization alone would prefer the budget regions where rescues are abundant, regardless of whether the marginal rescue generates more additionality than an intensive intervention. The model therefore predicts that mobilization-targeted policy will systematically underweight intensive additionality and overweight extensive interventions whose marginal additionality is small.

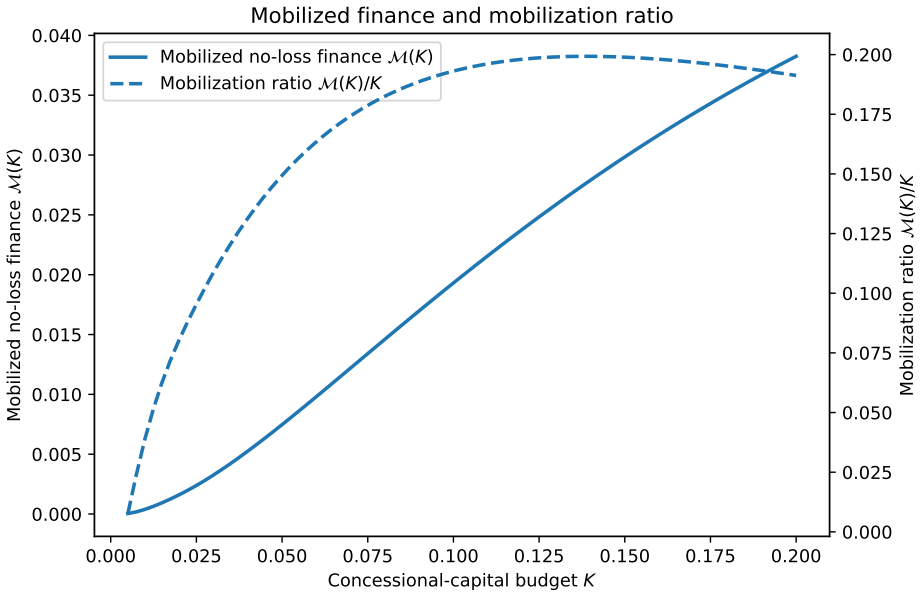


Figure 4: Mobilized non-concessional finance and mobilization ratio. The solid line is $\mathcal{M}(K)$, the total non-concessional finance mobilized on the extensive margin. The dashed line is the ratio $\mathcal{M}(K)/K$. The non-monotonicity of the ratio reflects shifts in the composition of the optimal allocation between intensive and extensive interventions.

Discussion. Three lessons from the simulations connect back to the analytical results. First, blended finance generates a sharp sorting pattern: concessional capital is targeted toward high-impact projects, and the form of intervention depends discretely on the project's commercial bankability. Second, the same shadow value governs allocation across margins, so the order in which intensive and extensive additionality activate as K rises depends on parameters; in this calibration, intensive activates first because it is the cheapest form of additionality at small budgets, but the order is parameter-dependent. Third, mobilization and additionality are distinct outcomes. Mobilization can be zero when additionality is strictly positive, and it can rise without proportional gains in additionality. The simulations therefore reinforce the central message of the paper: the role of blended finance is not to maximize private capital deployed, but to allocate scarce loss-absorbing capital where contractual frictions otherwise prevent real additionality.

8 Conclusion

This paper develops a contract-theoretic model of blended finance. The contractual environment—in which loss-absorbing concessional capital coexists with no-loss commercial capital inside a single state-contingent transaction, when impact potential is privately known and impact

effort is hidden—requires structural features absent from existing models of impact-oriented finance. Within this environment, scarce concessional capital must be allocated across two distinct margins of additionality: intensive interventions in projects already financed by the market, and extensive rescues of projects the market excludes.

The first result establishes that no-loss impact finance cannot generate additionality on either margin under competitive commercial finance: loss-absorbing capital is necessary, not merely useful. The second result characterizes the contracts that use this loss-absorbing capital—an impact-contingent overlay on bankable projects, and a rescue contract on excluded ones. The third result turns the project-level problem into a portfolio allocation problem in which a common shadow value of concessional capital coordinates the two margins. The optimal cutoff schedule is generically discontinuous at the commercial-finance boundary, reflecting the discrete change in what concessional capital does on either side: above the boundary it changes incentives; below, it creates the project itself. This discontinuity is structural to blended finance, not an artifact of binary effort, and it is reproduced numerically under a uniform conditional distribution of impact types.

The framework distinguishes mobilization from additionality. Mobilized capital is generated only on the extensive margin, but real additionality can come from intensive interventions that mobilize no new capital. Conversely, rescue finance may mobilize capital while delivering little additionality once incentive costs, informational rents, and leakage risks are taken into account. The relevant policy object is therefore additionality per unit of fiscal cost, not the volume of private capital mobilized—a result with direct implications for the OECD-DAC mobilization metric used in blended-finance practice.

Two extensions reinforce the central message. When entrepreneurs have low liquidity, rescue contracts must leave implementation rents to induce financial effort, and these rents can attract lower-impact firms into the program; this provides a contractual foundation for greenwashing leakage on the extensive margin. When commercial lenders have market power, displaced financial-sector rents reduce the fiscal cost of intensive additionality and, through the portfolio shadow value, free resources to rescue additional non-bankable projects. Both cases show that the value of blended finance depends on the contractual frictions behind the market failure, not on whether private capital is present.

Limitations and extensions. Several modeling choices keep the analysis tractable but bound the questions it can answer. The model is static: it does not capture multi-period financing, the dynamic learning that justifies early support for unproven technologies, or the commitment problems that arise when concessional support is provided in stages. The con-

tractual environment has a single impact investor; competition or coordination among multiple impact investors—a feature of blended-finance practice in markets with several DFIs—would change the structure of mobilization and the bargaining position of entrepreneurs. The model treats impact as a single one-dimensional outcome; many real blended-finance transactions target bundled outcomes (emissions reductions, biodiversity gains, social inclusion) whose joint contractibility raises additional questions. Finally, agents are risk-neutral, so the model does not address the risk-allocation channel that practitioners often emphasize as a primary function of blended finance. Each of these is a natural direction for follow-up work that would extend rather than displace the contractual logic developed here.

Implications for blended-finance practice. The paper's results carry concrete implications for how blended-finance programs are designed and evaluated. Targeting decisions should be made conditional on financial viability: bankable projects warrant intensive overlays, not rescue financing that substitutes for commercial capital and generates no additionality. Mobilization metrics should be supplemented with measures of additionality per unit of fiscal cost; otherwise, intensive interventions—which mobilize no new capital but can generate substantial additionality—will be systematically underweighted. Eligibility verification matters most where rescue contracts must leave implementation rents, as in capital-constrained sectors; without it, leakage erodes the additionality that justified the subsidy. And the value of concessional capital is sensitive to the competitive structure of the commercial sector it is layered with: where commercial finance extracts rents, concessional capital can do more with less.

Blended finance is not valuable because it mechanically multiplies money. Its value lies in the allocation of scarce loss-absorbing capital to the contractual frictions that prevent real additionality. Sometimes this means making an excluded project commercially viable. Sometimes it means changing behavior in a project the market would already finance. And sometimes, when incentive rents or leakage risks are too costly, it means withholding concessional capital altogether.

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A Proofs

Proof of Lemma 1

Proof. Under high financial effort, zero-profit pricing gives $D^C(p) = (1-E)/p$. Since the contract does not reward impact, the entrepreneur diverts on the impact margin and obtains $U^B(p) = p(R - D^C(p)) - E + c = pR - 1 + c$. Participation requires $p \geq p^C = (1-c)/R$.

Financial-effort incentive compatibility requires $pR - (1-E) \geq c/(1-k)$. At $p = p^C$ the left-hand side equals $E - c \geq c/(1-k)$ by Assumption 1, and the left-hand side is increasing in p; the constraint is therefore slack on $[p^C, 1]$. Hence high-effort commercial finance is feasible iff $p \geq p^C$, with outside option $U^B(p) = pR - 1 + c$.

Low-effort commercial contracts require $kpD^S = 1-E$, giving entrepreneur payoff $U^S(p) = kpR - 1 + 2c$. For $p < p^C$, $U^S(p) < k(1-c) - 1 + 2c = c(2-k) - (1-k) < 0$ by $\bar{E} < 1$. Above p^C, $U^S(p) < U^B(p)$. The outside option is therefore zero for $p < p^C$ and $pR - 1 + c$ for $p \geq p^C$. ■

Proof of Lemma 2

Proof. Part (i). Under any break-even contract on a project with $p \geq p^C$, expected repayments equal $1 - E$ and the entrepreneur retains expected pledgeable income $pR - (1-E) = pR - 1 + E$ net of her contribution E, giving total payoff $pR - 1$ when high impact effort is induced and no diversion payoff is collected. The commercial outside option is $pR - 1 + c > pR - 1$. No no-loss contract inducing high impact effort can match it.

Part (ii). For $p < p^C$, under any break-even contract: under GG or GB the entrepreneur's payoff is at most $pR - 1 + c < 0$ by $p < p^C$; under BG or BB it is at most $kpR - 1 + 2c < k(1-c) - 1 + 2c = c(2-k) - (1-k) < 0$ by $\bar{E} < 1$. No action profile yields nonnegative payoff. ■

Proof of Lemma 3

Proof. The total repayment to outside financiers in the no-impact-success state is $D_B + d_N = (1-E)/p + \tau_N$ (the I/p terms cancel between D_B and d_N).

Intensive contract. $\tau_N = ck/[p(1-k)]$, so $p(D_B + d_N) = (1-E) + ck/(1-k)$. Under Assumption 1, $E \geq \bar{E}$ gives the bound $(1-E) + ck/(1-k) \leq 1 - c - c/(1-k) + ck/(1-k) =$

$1-2c$. For $p \geq p^C = (1-c)/R$, $pR \geq 1-c > 1-2c$, so $D_B + d_N \leq R$ with strict slack.

Rescue contract. $\tau_N = [pR - 1 + c/(1-k)]/p$, so $D_B + d_N = R + (c/(1-k) - E)/p$. Under Assumption 1, $E \geq \bar{E} = c + c/(1-k) > c/(1-k)$, giving $D_B + d_N \leq R$ strictly.

Since $s > 0$, $D_B + d_I = D_B + d_N - s < D_B + d_N \leq R$, so feasibility in the impact state follows. ■

Proof of Proposition 1

Proof. The no-loss tranche is payoff-irrelevant: a tranche of I generates impact-investor repayment I/p in the success states, exactly offsetting the reduction in commercial repayment from $(1-E)/p$ to $(1-E-I)/p$. Payoffs and incentives therefore depend only on the concessional overlay (τ_N, s).

Conditional on high financial effort, $U_{GG}(p,q) = pR - 1 - p(\tau_N - qs)$ and $U_{GB}(p,q) = pR - 1 - p(\tau_N - kqs) + c$. High impact effort is incentive compatible iff $pq(1-k)s \geq c$. Implementation for all $q \geq \hat{q}$ requires the constraint at $\hat{q}$; since fiscal cost is increasing in s, it binds: $s^* = c/[p\hat{q}(1-k)]$.

The cutoff type's participation constraint, $p(\hat{q}s - \tau_N) \geq c$, binds (fiscal cost decreasing in τ_N). Substituting $s = s^*$ gives $\tau_N^* = ck/[p(1-k)]$ and $\tau_I^* = \tau_N^* - s^*$.

For $q < \hat{q}$, accepting and diverting on impact gives payoff relative to commercial finance equal to $\frac{ck}{1-k}(q/\hat{q} - 1) < 0$, so lower types stay under commercial finance. Financial-effort incentives are preserved by Assumption 1. Cost minimization follows from the binding constraints. ■

Proof of Proposition 2

Proof. For $q \geq \hat{q}$, the intervention raises expected impact from $pkq\sigma$ to $pq\sigma$, giving $B_I(p,\hat{q}) = \int_{\hat{q}}^1 p(1-k)q\sigma f(q \mid p)\, dq = p(1-k)\sigma M(p,\hat{q})$.

The expected fiscal cost of the overlay for type q is $p(qs^* - \tau_N^*) = \frac{c}{1-k}(q/\hat{q} - k)$. Integrating gives

$$L_I(p,\hat{q}) = \frac{c}{1-k}\left[\frac{M(p,\hat{q})}{\hat{q}} - kS(p,\hat{q})\right] = cS(p,\hat{q}) + \frac{c}{1-k}\left[\frac{M(p,\hat{q})}{\hat{q}} - S(p,\hat{q})\right]. \qquad \blacksquare$$

Proof of Proposition 3

Proof. The no-loss tranche is payoff-irrelevant by the same argument as in Proposition 1. Under GG, $U_{GG}(p,q) = pR - 1 - p\tau_N + pqs$; under GB, $U_{GB}(p,q) = pR - 1 - p\tau_N + pkqs + c$. Impact-effort incentive compatibility for all $q \geq \hat{q}$ binds at $\hat{q}$: $s^* = c/[p\hat{q}(1-k)]$.

The marginal type's participation constraint $pR - 1 - p\tau_N + p\hat{q}s \geq 0$ binds. Using $p\hat{q}s^* = c/(1-k)$, we obtain $\tau_N^* = [pR - 1 + c/(1-k)]/p$ and $\tau_I^* = \tau_N^* - s^*$.

Financial-effort deviations are tightest at $\hat{q}$. The joint deviation BB at the marginal type yields payoff $(2-k)c - (1-k)E \leq 0$ iff $E \geq \bar{E} = (2-k)c/(1-k)$, which holds by Assumption 1. The BG deviation is then unprofitable; the impact-only deviation is ruled out by construction. For $q \geq \hat{q}$, the spread is at least s^*, so all incentive constraints hold.

For $q < \hat{q}$: under GB, payoff is $\frac{kc}{1-k}(q/\hat{q} - 1) < 0$; under BB, bounded by $\frac{k^2 c}{1-k}(q/\hat{q} - 1) < 0$ using Assumption 1; BG is also unprofitable. So lower types stay out. Cost minimization follows from the binding marginal constraints. ■

Proof of Proposition 4

Proof. Each accepted type $q \geq \hat{q}$ is financed and operates under GG, generating project surplus $pR - 1 + pq\sigma$. Integrating, $B_E(p,\hat{q}) = (pR-1)S(p,\hat{q}) + p\sigma M(p,\hat{q})$.

The fiscal cost of the overlay for type $q \geq \hat{q}$ is $p(qs^* - \tau_N^*) = 1 - pR + \frac{c}{1-k}(q/\hat{q} - 1)$. Integrating gives

$$L_E(p,\hat{q}) = (1-pR)S(p,\hat{q}) + \frac{c}{1-k}\left[\frac{M(p,\hat{q})}{\hat{q}} - S(p,\hat{q})\right].$$ ■

Proof of Proposition 5

Proof. After financial success, let the impact investor pay $\varepsilon S(r,y)$ where $y \in \{0,1\}$ is the realized impact outcome and S is a strictly proper scoring rule normalized so realized payments lie in $[0,\varepsilon]$. Under high impact effort, y is Bernoulli with mean q, so a strictly proper scoring rule elicits truthful reports $r = q$; for accepted types, truthful reports lie in $[\hat{q},1]$. The reporting layer perturbs the binding impact-effort incentive constraint by an amount of order ε, and adds expected fiscal cost of order ε per accepted type. As $\varepsilon \to 0$, the allocation, the cutoff structure, and the fiscal cost converge to the pooling outcome. ■

Proof of Proposition 6

Proof. The Lagrangian $\mathcal{L}(\hat{q}, \mu) = B[\hat{q}] + \mu(K - L[\hat{q}])$ is separable in p:

$$\mathcal{L} = \mu K + \int_{p^C}^{1} [B_I - \mu L_I]\, dG + \int_{0}^{p^C} [B_E - \mu L_E]\, dG.$$

For each p, the optimal cutoff therefore solves the pointwise argmax, and μ satisfies the standard complementary-slackness conditions. ■

Proof of Proposition 7

Proof. Let $f = f(\hat{q} \mid p)$, $S = S(p, \hat{q})$, $M = M(p, \hat{q})$, so $\partial S/\partial \hat{q} = -f$ and $\partial M/\partial \hat{q} = -\hat{q}f$.

Intensive region. $B_I = p(1-k)\sigma M$ and $L_I = \frac{c}{1-k}(M/\hat{q} - kS)$. Differentiating,

$$\frac{\partial B_I}{\partial \hat{q}} = -p(1-k)\sigma \hat{q} f, \qquad \frac{\partial L_I}{\partial \hat{q}} = -cf - \frac{c}{1-k}\frac{M}{\hat{q}^2}.$$

The first-order condition $\partial B_I/\partial \hat{q} = \mu \partial L_I/\partial \hat{q}$, simplified using $M = \bar{q}S$, yields

$$p(1-k)\sigma \hat{q}_I = \mu \left[c + \frac{c}{1-k} \frac{\bar{q}(p, \hat{q}_I)}{\hat{q}_I^2} \frac{1 - F(\hat{q}_I \mid p)}{f(\hat{q}_I \mid p)} \right].$$

Extensive region. $B_E = (pR - 1)S + p\sigma M$ and $L_E = (1 - pR)S + \frac{c}{1-k}(M/\hat{q} - S)$. Differentiating,

$$\frac{\partial B_E}{\partial \hat{q}} = -(pR - 1 + p\sigma \hat{q}) f, \qquad \frac{\partial L_E}{\partial \hat{q}} = -(1 - pR) f - \frac{c}{1-k}\frac{M}{\hat{q}^2}.$$

The first-order condition yields

$$pR - 1 + p\sigma \hat{q}_E = \mu \left[1 - pR + \frac{c}{1-k} \frac{\bar{q}(p, \hat{q}_E)}{\hat{q}_E^2} \frac{1 - F(\hat{q}_E \mid p)}{f(\hat{q}_E \mid p)} \right].$$ ■

Proof of Corollary 1

Proof. Suppose both interior cutoffs are continuous at p^C with common value q^0. At $p = p^C$, $p^C R = 1 - c$, so $1 - p^C R = c$, and the right-hand sides of the two first-order conditions in

Proposition 7 coincide. Equating the left-hand sides,

$$p^C(1-k)\sigma q^0 = p^C R - 1 + p^C \sigma q^0 = -c + p^C \sigma q^0,$$

which gives $q^0 = c/(kp^C\sigma)$. Continuity therefore requires $q^0 \in (0,1)$ and the knife-edge value of μ at which q^0 satisfies the common first-order condition. Generically, the optimal cutoff is discontinuous at p^C. ■

Proof of Proposition 8

Proof. Direct from Proposition 6 and the implementation results: for $p \geq p^C$, types with $q \geq \hat{q}_I(p;\mu)$ accept the intensive overlay (region Ω_I) and lower types remain under commercial finance (Ω_C); for $p < p^C$, types with $q \geq \hat{q}_E(p;\mu)$ are rescued (Ω_E) and lower types remain unfunded (Ω_0). ■

Proof of Proposition 9

Proof. Mobilization is generated only on the extensive margin. Each rescued project requires outside no-loss finance $1-E$, and the mass of rescued projects of type $p < p^C$ is $S(p, \hat{q}_E(p;\mu))$. Integrating,

$$\mathcal{M}(\mu) = (1-E)\int_0^{p^C} S(p, \hat{q}_E(p;\mu))\, dG(p).$$

The mobilization ratios follow by dividing by K (when the budget binds) or by $L[\hat{q}]$ (when it is slack). ■

Proof of Proposition 10

Proof. Under $E < \bar{E}$ the rescue contract sets $s^* = c/[p\hat{q}(1-k)]$ and $\tau_N^* = [pR-1+E-c]/p$. The cutoff type's payoff under GG is

$$U_{GG}^{E,L}(p, \hat{q} \mid \hat{q}) = pR - 1 - p\tau_N^* + p\hat{q}s^* = \bar{E} - E \equiv \Delta_E > 0.$$

For $q < \hat{q}$, the spread s^* does not induce high impact effort: $pq(1-k)s^* = c(q/\hat{q}) < c$. Among after-entry deviations, BB dominates for $q < \hat{q}$, with payoff

$$U_{BB}^{E,L}(p, q \mid \hat{q}) = -E + (2+k)c + \frac{k^2 c}{1-k}\frac{q}{\hat{q}}.$$

Setting this ≥ 0 and truncating to $[0, \hat{q}]$ gives the leakage threshold $q^{\ell}(E, \hat{q})$ in the proposition.

Finally, $(2+k)c + k^2c/(1-k) = (2-k)c/(1-k) = \bar{E}$, so $E < \bar{E}$ implies $(1-k)(E-(2+k)c)/(k^2c) < 1$. Hence $q^{\ell}(E, \hat{q}) < \hat{q}$, and the leakage interval is nonempty whenever there is positive mass below $\hat{q}$. ■

Proof of Proposition 11

Proof. Under market power η, the entrepreneur's outside option is $U_{\eta}^{B}(p) = pR - 1 + c - \eta\Pi(p)$. The impact incentive constraint is unchanged, so $s^{\eta} = c/[p\hat{q}(1-k)]$. The marginal type's participation constraint $p(\hat{q}s - \tau_N) \geq c - \eta\Pi(p)$ binds, yielding $\tau_N^{\eta} = ck/[p(1-k)] + \eta\Pi(p)/p$ and $\tau_I^{\eta} = \tau_N^{\eta} - s^{\eta}$.

The fiscal cost for type $q \geq \hat{q}$ is $p(qs^{\eta} - \tau_N^{\eta}) = \frac{c}{1-k}(q/\hat{q} - k) - \eta\Pi(p)$. Integrating gives $L_I^{\eta}(p, \hat{q}) = L_I(p, \hat{q}) - \eta\Pi(p)S(p, \hat{q})$. ■

Proof of Proposition 12

Proof. For $\eta' > \eta$, $L_I^{\eta'} \leq L_I^{\eta}$ pointwise on $p \geq p^C$, while L_E is unchanged. Hence the feasible set under η' contains the feasible set under η, and $V^{\eta'}(K) \geq V^{\eta}(K)$. Strictness holds whenever the additional fiscal slack permits positive-measure expansion of support. ■

Proof of Proposition 13

Proof. For fixed μ, the intensive pointwise objective becomes $B_I - \mu L_I + \mu\eta\Pi(p)S(p, \hat{q})$. Since S is decreasing in $\hat{q}$, market power raises the marginal benefit of lowering the intensive cutoff. Under Assumption 2, the intensive cutoff is therefore weakly lower than at $\eta = 0$.

When the budget binds, lower L_I^{η} weakly relaxes the equilibrium shadow value μ. The extensive pointwise objective $B_E - \mu L_E$ is unchanged in form, but a lower μ weakly lowers the rescue cutoff, expanding extensive additionality. ■

B Leakage-Adjusted Portfolio Problem

This appendix characterizes the portfolio problem when entrepreneurial liquidity is low and the impact investor cannot exclude leakage types through additional eligibility screening. The analysis stays within the baseline rescue-contract space: the investor chooses an intended cutoff $\hat{q}$, pools accepted types at the spread that induces impact effort for the marginal intended type, and bears the fiscal and welfare consequences of any lower-impact types who find the contract attractive.

Throughout, assume $E < \bar{E} := c + c/(1-k)$. Under Assumption 3, the high-effort commercial-finance cutoff becomes

$$p_L^C = \frac{1 - E + \frac{c}{1-k}}{R} > p^C.$$

For $p \geq p_L^C$, the project is commercially financeable with high financial effort, and the intensive-additionality contract is unchanged; the only difference relative to the baseline is that the intensive region begins at p_L^C. For $p < p_L^C$, the project is not commercially financeable, and any rescue must operate within the constraints described in Section 6.

Induced allocation. For an intended cutoff $\hat{q}$ and $p < p_L^C$, the rescue contract sets $s^* = c/[p\hat{q}(1-k)]$ and $\tau_N^* = [pR - 1 + E - c]/p$, leaving the marginal intended type with implementation rent $\Delta_E := \bar{E} - E > 0$ (Proposition 10). A lower-impact type $q < \hat{q}$ is not induced to exert high impact effort; if it enters, its best deviation is BB, with payoff $U_{BB}^{E,L}(p, q \mid \hat{q}) = -E + (2+k)c + \frac{k^2 c}{1-k}(q/\hat{q})$. Setting this nonnegative and truncating to $[0, \hat{q}]$ defines the leakage frontier

$$q^\ell(E, \hat{q}) = \min\left\{\hat{q},\, \max\left\{0,\, \hat{q}\,\frac{(1-k)\big(E - (2+k)c\big)}{k^2 c}\right\}\right\}.$$

The induced allocation is therefore: types $q \geq \hat{q}$ accept the contract and operate under GG; types $q \in [q^\ell(E, \hat{q}), \hat{q})$ enter and operate under BB (leakage); types $q < q^\ell(E, \hat{q})$ do not enter.

For notational convenience, define the mass and aggregate impact potential of intended rescued types as $S_H(p, \hat{q}) = 1 - F(\hat{q} \mid p)$ and $M_H(p, \hat{q}) = \int_{\hat{q}}^1 q\, f(q \mid p)\, dq$, and of leakage types as $S_\ell(p, \hat{q}) = F(\hat{q} \mid p) - F(q^\ell \mid p)$ and $M_\ell(p, \hat{q}) = \int_{q^\ell}^{\hat{q}} q\, f(q \mid p)\, dq$.

Leakage-adjusted additionality and fiscal cost. Intended rescued types operate under GG and generate project surplus $pR - 1 + pq\sigma$. Leakage types operate under BB, generating

project surplus $kpR - 1 + k^2pq\sigma$, which can be negative. The leakage-adjusted extensive additionality is therefore

$$\widetilde{B}_E(p, \hat{q}) = (pR - 1)S_H + p\sigma M_H + (kpR - 1)S_\ell + k^2 p\sigma M_\ell.$$

For intended rescued types, fiscal cost equals the financial shortfall plus the implementation and informational rents needed to induce GG, giving $(1 - pR + \Delta_E)S_H + \frac{c}{1-k}(M_H/\hat{q} - S_H)$. For leakage types, the contract was priced for GG but they choose BB; the concessional layer absorbs the resulting shortfall, with cost per type $1 - E - kpR + kc + \frac{k^2 c}{1-k}(q/\hat{q})$. Integrating gives the leakage-adjusted extensive fiscal cost

$$\widetilde{L}_E(p, \hat{q}) = (1 - pR + \Delta_E)S_H + \frac{c}{1-k}\left[\frac{M_H}{\hat{q}} - S_H\right] + (1 - E - kpR + kc)S_\ell + \frac{k^2 c}{1-k}\frac{M_\ell}{\hat{q}}.$$

Leakage-adjusted portfolio problem. The impact investor chooses a measurable intended cutoff $p \mapsto \hat{q}(p)$ to solve

$$\max_{\hat{q}(\cdot)} \left\{ \int_{p_L^C}^1 B_I(p, \hat{q})\, dG(p) + \int_0^{p_L^C} \widetilde{B}_E(p, \hat{q})\, dG(p) \right\} \quad \text{s.t.} \quad \int_{p_L^C}^1 L_I\, dG + \int_0^{p_L^C} \widetilde{L}_E\, dG \le K.$$

Proposition 14 (Leakage-adjusted optimal portfolio)**.** *Suppose $E < \bar{E}$ and that leakage types cannot be excluded by additional eligibility screening. Within the baseline rescue-contract space, there exists $\mu \ge 0$ such that the optimal intended cutoff satisfies, for almost every p,*

$$\hat{q}(p; \mu) \in \arg\max_{\hat{q} \in [0,1]} \big\{ \mathcal{B}_r(p, \hat{q}) - \mu \mathcal{L}_r(p, \hat{q}) \big\},$$

with $(\mathcal{B}_r, \mathcal{L}_r) = (B_I, L_I)$ for $p \ge p_L^C$ and $(\widetilde{B}_E, \widetilde{L}_E)$ for $p < p_L^C$. The multiplier satisfies the standard complementary-slackness conditions.

Proof. The Lagrangian is separable in p by the same argument as in Proposition 6. The functions $\widetilde{B}_E$ and $\widetilde{L}_E$ summarize the actual induced allocation under intended cutoff $\hat{q}$, accounting for both intended rescues and leakage entrants. The pointwise maximization and complementary-slackness conditions follow. ■

Interior cutoff rule. The cutoff rule is less transparent than in the baseline because changing $\hat{q}$ shifts both the intended rescue region and the leakage band. When the leakage

frontier is interior, $q^{\ell}(E, \hat{q}) = \lambda(E)\hat{q}$ with

$$\lambda(E) = \max\left\{0,\ \frac{(1-k)\big(E-(2+k)c\big)}{k^2 c}\right\} \in [0,1).$$

Let $f_h = f(\hat{q} \mid p)$ and $f_\ell = f(\lambda(E)\hat{q} \mid p)$. Differentiating,

$$\frac{\partial \widetilde{B}_E}{\partial \hat{q}} = -p\big[(1-k)R + (1-k^2)\sigma\hat{q}\big]f_h + \lambda(E)\big[1 - kpR - k^2 p\sigma\lambda(E)\hat{q}\big]f_\ell,$$

$$\frac{\partial \widetilde{L}_E}{\partial \hat{q}} = -(1-pR+\Delta_E)f_h - \frac{c}{1-k}\frac{M_H}{\hat{q}^2} + (1-E-kpR+kc)\big[f_h - \lambda(E)f_\ell\big] + \frac{k^2 c}{1-k}\left[f_h - \lambda(E)^2 f_\ell - \frac{M_\ell}{\hat{q}^2}\right].$$

The interior cutoff rule $\partial \widetilde{B}_E/\partial \hat{q} = \mu\, \partial \widetilde{L}_E/\partial \hat{q}$ therefore internalizes both the additionality created by intended rescues and the fiscal and welfare cost generated by leakage entrants. Lowering $\hat{q}$ expands the intended rescue region but also shifts the mass and composition of leakage entrants, so the optimal intended cutoff is not a simple modification of the baseline cutoff rule.

The leakage-adjusted problem reinforces the central message of the paper. When the contractual environment leaves implementation rents on the extensive margin, those rents distort the screening problem the rescue contract was designed to solve. Concessional capital must be targeted not only to projects with high social value, but to settings where eligibility and incentive problems can be effectively controlled.