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Donors' Blended Finance Portfolio Optimization

A Variation on Modern Portfolio Theory

by

Matthieu PEGON[1]

under the supervision of

Prof. Santiago BARRAZA – ESCP Business School

[1] Executive PhD candidate at ESCP Business School and Deputy CEO at Climate Investment Funds, formerly Director of Blended Finance at IDB Invest. I am grateful to [●] for their review and feedback on this paper. Generative AI was used for language editing and as an aid to literature search. All resulting text revisions and references were reviewed and validated by the author. The mathematical notation of the optimization problem is partly drawn from Thierry Roncalli's Handbook of Sustainable Finance.

Abstract

For a blended finance donor, is extending a grant different than extending a concessional investment of the same subsidy equivalent? If not, what drives a donor's choice between the two instruments? More broadly, in the context of constrained aid budgets, is there room for donor-side financial optimization in blended finance, holding other decision-making factors constant?

To address these questions, I draw on seminal Finance theory to formulate and solve a donor optimization problem. I assume donors' decision-making is sequential: aid envelopes, broad policy objectives and project selection are set before the financial terms of individual concessional financings are determined.

Assuming stochastic and partly correlated project returns, donor preferences at this final stage can be represented in terms of the donor's expected present cost and the variance of that cost. The model yields four main results. First, grants are preferred when donor risk tolerance is zero or close to it. Second, grants are also preferred for any asset for which the donor's cost of funding exceeds the investor's hurdle rate. Third, when optimal donor participation exceeds the aggregate subsidy equivalent, the same optimum can be implemented through different combinations of grants and investments. Fourth, in the unconstrained interior case, donor risk tolerance affects the amount of participation but not the relative portfolio weights across assets. Fifth, the model identifies a trade-off between donor-side financial efficiency and private capital mobilization: greater donor risk tolerance lowers expected donor losses, but only by increasing nominal donor participation and thereby reducing private capital leverage.

1. Introduction

Official Development Assistance (ODA), the standard international measure of donor countries' financial efforts to support development in low- and middle-income countries expressed on a grant-equivalent basis, is shrinking. ODA fell by 6.1% in 2024 and 23.1% in 2025 to US$ 174.3 billion amid mounting political and fiscal pressures (OECD, 2026). Faced with a growing funding gap to meet the objectives of the Sustainable Development Goals (UN, 2024), the efficient use of scarce donor resources is increasingly relevant (G20, 2024).

In contrast, investments supported by blended finance have grown from US$60 billion to US$250 billion annually over the last decade (Panizza, 2026). From a policy standpoint, blended finance has increasingly been at the forefront of the international development agenda, as shown by the 2025 outcome text of the Fourth International Conference on Financing for Development (UN, 2025).

This paper examines the financial optimization of donors' concessional resources under budget constraints. It extends two strands of literature: donor-side aid efficiency and instrument choice, and the burgeoning blended finance literature. It builds on the claim that existing donor-side financial optimization models, while useful for analyzing sovereign-to-sovereign grants and loans, do not fully capture the allocation problem raised by blended finance, where payoffs are uncertain and concessional resources are deployed across portfolios of correlated assets. Once concessional finance is understood in those terms, the donor's problem is naturally one of mean-variance optimization. The paper therefore develops, to my knowledge, the first donor-side mean-variance framework for concessional finance. In doing so, it offers a generalization of earlier optimization models: the traditional sovereign-to-sovereign case is retained as a limiting case, while the framework is

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extended to blended finance, where donor allocation takes place across risky and correlated portfolios. It also differs from the existing blended finance literature by avoiding a normative assumption about donor motivation – particularly the assumption that private capital mobilization is the dominant objective – and instead allowing for multiple, potentially conflicting donor objectives. At the same time, the model shows that these objectives may interact, notably through a trade-off between reducing expected donor losses and maintaining private capital leverage.

The first strand of literature – economic and financial optimization of donors' ODA – has long been a matter of interest, intertwined with efforts of normalizing rules of accounting for ODA across financial instruments (Scott, 2017). Pincus (1963) was the first to raise explicitly the question of the real cost of aid to donors, arguing that a loan with the same face value as a grant should count for less in ODA terms. He introduced the concept of grant equivalence, defined as the face value of an instrument minus the discounted present value of expected reflows, using an appropriate opportunity cost of capital.[2] Schmidt (1964) extended this approach to compare the efficiency of grants and concessional loans while holding recipient benefits constant. He showed that concessional loans are more efficient when the donor's opportunity cost of capital is lower than the recipient's, whereas grants are more efficient in the opposite case.

Frank (1970) generalized these results by distinguishing between grant equivalence measured at the donor's opportunity cost of capital, which captures the present cost to the donor, and grant equivalence measured at the recipient's opportunity cost of capital, which captures the present benefit to the recipient. He further showed that when the donor's nominal financing envelope is constrained, and the donor's opportunity cost of capital is lower than the recipient's, the Pareto-optimal outcome may be to extend the full nominal amount as a concessional loan rather than a grant. At the same time, where recipient absorptive capacity is limited or risk aversion is high, optimal financing terms lie closer to the grant end of the spectrum and lending volumes should be reduced.

More recent literature recasts the grant-loan choice as a broader donor optimization problem under budget, incentive, and debt-sustainability constraints. Cordella and Ulku (2007) show that, for a given aid envelope, greater concessionality is more appropriate in poorer, more indebted, and weaker policy environments. Koeda (2004) similarly shows that concessional loans can dominate grants when they ease financing constraints without generating excessive over-borrowing or aid dependence, but that grants become preferable when those distortions are strong. Odedokun (2004) empirically finds that the grant-loan mix varies systematically with donor and recipient characteristics

Finally, donor optimization is also institutional and financial. Kawai and Takagi (2001) show that Japan's extensive use of ODA loans reflected a budgetary structure that allowed aid volumes to expand with limited direct budgetary cost. In the multilateral context, Sanford (2002) and Sanford (2004) show that resistance to expanding IDA grants reflected concern over the loss of loan reflows that financed future commitments.

[2] In the case of Pincus (1963), marginal domestic long-term return of capital, international lending rates, and rates that private investors would have earned in the area of investment are used. Note that the grant equivalent of a grant will equal the face value of the grant under that methodology.

The foregoing literature is framed around individual sovereign-to-sovereign transactions and therefore abstracts from uncertainty and portfolio interactions. Loans are treated as deterministic obligations, so risk does not enter the donor's optimization problem in the way it does in blended finance, where payoffs are stochastic and allocated across portfolios of potentially correlated investments. Even so, this literature provides several principles that are retained to the present analysis. It highlights the importance of distinguishing between donor present cost, based on the donor's opportunity cost of funds, and recipient benefit, which in blended finance is more appropriately captured through the construct of subsidy equivalence. The principles underpinning grant-equivalence calculations are also retained as the basis for defining donor present cost and subsidy equivalence. In addition, this literature suggests testable propositions that can be revisited in a generalized setting, including whether grants remain preferable when the donor's cost of capital is relatively high, and whether the role played by recipient risk aversion in the existing literature has a mirror in donor risk aversion under uncertainty. Finally, it establishes two ideas that are retained directly: that the degree of concessionality affects the nominal amount of the donor's financing, and that donor choice between financial instruments is shaped not only by developmental objectives but also by budgetary considerations.

The second strand of literature concerns blended finance itself. I define blended finance as the use of ODA extended directly or indirectly to private agents through concessional financing (IDB Invest, 2023). Concessional financing is in turn defined as subsidized financing extended at terms and/or conditions that are more favorable than those available in the market (ADB et al., 2017). Implicit in this definition is that the underlying assets financed through blended finance are non-sovereign and therefore characterized by uncertain and potentially correlated returns.

Blended finance is associated with several stated policy objectives, which may overlap but need not coincide. Historically, Multilateral Development Banks (MDBs) and Development Finance Institutions (DFIs) have framed blended finance as a market-creation instrument for investments yielding positive externalities. Concessional resources are used to address market failures and move markets toward commercial viability (Buiter and Schankerman, 2002, ADB et al., 2013, ADB et al., 2017). By contrast, the OECD gives greater prominence to the mobilization of commercial finance (OECD, 2018, OECD, 2025), possibly driven by increasing funding commitments – such as the US$ 300 billion annual New Collective Quantified Goals – amid decreasing ODA volumes.[3]

This duality in policy objectives is reflected in the nascent blended finance academic literature. Flammer et al. (2024) formalizes a blended-finance investment selection framework in which donors allocate concessionality across projects to maximize social returns. The model yields two main predictions. First, higher concessionality should be directed to projects with greater sustainability impact, because subsidies are costly and must be justified by higher social returns. Second, concessionality requirements should increase with risk frictions. Flammer et al. (2025) extend this analysis. In the case of certain returns, they show that required

[3] At the 29th Conference of the Parties (COP29) of the United Nations Framework Convention on Climate Change (UNFCCC), Parties adopted a New Collective Quantified Goal (NCQG) on climate finance to contribute to accelerating the achievement of the goals of the Paris Agreement. The USD 300 billion refers to the annual climate finance mobilization goal, to be led by developed country Parties (Falduto and Jachnik, 2025).

concessionality is proportional to the project's shortfall from the private hurdle rate, implying that projects with less-monetizable, more public benefits require more concessionality. Where returns are uncertain, the paper relies on stochastic dominance to characterize how contract structure can make investments more attractive. Increasing the share of concessional capital (reducing the private capital required) mechanically improves the private return distribution in a way that first-order stochastically dominates alternatives (preferred by any expected-utility maximizer). Separately, "de-risking" that reduces returns distribution (a mean-preserving contraction, e.g., trimming tails) makes the investment more attractive for risk-averse investors (second-order dominance logic). These papers are thus best read as operating in a market-creation and social-returns objectives, but they remain investor-centric, since the structure of concessional financing is informed by investor participation conditions rather than donor preferences.

Conversely, Ben Slimane et al. (2025) and Panizza (2026) emphasize private-capital-mobilization as the main policy objective of blended finance. Ben Slimane et al. (2025) develop a structuring framework for donor-funded junior tranches in emerging-market aggregation vehicles designed to crowd in risk-constrained investors. A central result is that diversification lowers the subsidy intensity of the donor tranche. But here too the optimization is investor-centric: the junior tranche design is determined by private investors' risk-return preferences and portfolio characteristics, not by donor preferences. Panizza (2026) is anchored around the policy objective of catalytic multiplier, understood as leverage per unit of expected public cost. His first result is that the catalytic multiplier decreases with the severity of the market failure, creating a fundamental tension: the interventions that target the largest distortions achieve the lowest leverage. Panizza also derives instrument-choice results depending on underlying market failures. Production externalities call for subsidized loans, while financial frictions are best addressed with guarantees. Importantly, however, the comparison is between guarantees and subsidized loans modeled through an interest-rate subsidy, rather than between returnable donor concessional investments and guarantees.

This distinction in policy objectives matters because the literature points to a possible tension between maximizing social returns and maximizing leverage. Kotchen and Vogt (2025), in the related context of environmental multilateral funds, show theoretically that introducing cofinancing as an independent objective is redundant at best and potentially counterproductive when the primary objective is environmental impact. Empirically, however, they do not find trade-offs strong enough to conclude that current cofinancing preferences are already reducing environmental benefits. The broader implication is that the empirical case remains unsettled, but the theoretical possibility of tradeoffs between policy objectives is real. This helps motivate the approach taken in this paper. Rather than imposing normative policy objectives, it treats those as potentially heterogeneous and determined upstream and focuses instead on the donor's financial optimization problem at the structuring stage. In that sense, the paper complements existing literature that is either investor-centric or organized around a single policy objective ex ante.

In the paper, I develop a model that relies on two axioms. First, the donors' selection and design of blended finance interventions follow a sequential, top-down process that starts with donors defining an overarching Official Development Assistance (ODA) envelope with broad policy objectives and ends in the financial structuring of individual blended finance interventions. This assumption allows the model to

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abstract from the earlier phases of the decision chain and focus specifically on the financial structuring stage, where the donor's concessional financing is structured. Restricting the analysis to this last phase allows the second axiom. At this final stage, the donor's preferences can be narrowed down and represented over the mean and variance of the donor's present cost, consistent with a mean–variance utility framework.

For both axioms, academic literature is limited. I therefore rely primarily on scholarly work and policy documents from official sources, informed by my own experience as a blended finance practitioner. For illustration, I have drawn from the Government of Canada's international climate finance deployed through blended finance. In some instances, I have introduced simplifying assumptions which may depart from empirical evidence and my own direct observations. Those are documented accordingly.

Building on these two axioms, I formulate and solve a donor optimization problem drawn from Markowitz's Modern Portfolio Theory and derive properties that may be of practical use for donors and blended finance practitioners. The remainder of the paper is therefore organized as follows. Section 2 sets up the model by defining the staged nature of donor decision-making, the characteristics of the portfolio and its assets, the investor's hurdle rate, the subsidy equivalent, and the donor's concessional financing instruments. Section 3 then formulates the donor's optimization problem in portfolio terms and derives the key expressions for the mean and variance of the donor's present cost. Section 4 solves the optimization problem and presents the main properties of the donor's optimal allocation, including the conditions under which grants are preferred, the role of the donor's cost of funding, the scope for combining grants and concessional investments, the implications for portfolio weights, and the trade-off with private capital leverage objectives. Section 5 concludes with a stylized representation of the main results and a discussion of their limitations and practical policy implications for donor and blended finance practitioners. The annexes provide references, model's variables, and mathematical demonstrations.

2. Model Set-up

The practice of allocating ODA is typically structured as a sequence of decision stages: budgetary allocation to official aid, high-level resource programming, partner selection, project identification, and downstream financing instrument design. Although academic literature rarely maps this entire chain as a formal model, scholarly work and operational documents from official sources provide useful pointers. In his *The Law of Development Cooperation*, law scholar Philipp Dann argues that ODA transfers *take place in four stages: first, the general budget decision to reserve a certain amount for ODA expenses; second, the multi-year planning for one country or one sector; third, the negotiation and agreement on a concrete development intervention (project or program); and fourth, the implementation of such intervention* (Dann, 2013). A European Commission management guide for the deployment of the European Union's external assistance presents the process as explicitly sequential: multiannual programming sets strategic priorities and indicative allocations, which then feed into a five-phase project cycle: programming, identification, formulation, implementation and evaluation (EC, 2004).

A useful illustration of how blended finance is operationalized through a staged donor process comes from the deployment of Canada's international climate

finance. In 2015, Canada announced CAD$2.65 billion in international climate finance over five years for the 2015–2021 period and subsequently allocated these resources across 130 initiatives (GAC, 2022). Within that envelope, CAD$956.4 million was channeled to five blended finance trust funds managed by four MDBs. Each fund operated under its own eligibility rules and impact objectives (GAC, 2022). For instance, in 2019, Canada contributed CAD$223.5 million to the second phase of the Canadian Climate Fund for the Private Sector in the Americas through the Inter-American Development Bank (IDB) Group, with expectations to reduce greenhouse gas emissions by 25 million tons and to leverage US$1 billion in private-sector investment for renewable energy, sustainable agriculture, and forestry (IDB Invest, 2019). In this architecture, Canada sets the overall budgetary commitment and channels resources into partner-managed vehicles, while the partner MDB is delegated authority to select projects within the fund's mandate and to structure the concessional finance at the transaction level. Consistent with this delegated model, the IDB Group has executed 40 fund-supported investments (IDB Invest, 2025).[4]

For the purpose of the model, let the **donor** be a donor country's government that is looking to use ODA to finance a portfolio of blended finance assets. For simplification, the term donor includes the donor country's government and any entities that are delegated allocation decision on behalf of that government. Consistent with the axiom of a phased approach to allocating ODA, the model assumes that in earlier stages, the donor already determined the most desirable portfolio of assets to finance, reflecting its priorities, strategic interests or development philosophies. The choice of portfolio is treated as exogenous to the model. In this final stage, which is the focus of this paper, the donor is only concerned with the financial structuring of its concessional financings.[5]

By abstracting from the earlier stage, the model is only normative towards donor financial preferences. It does not attempt to evaluate whether non-financial preferences that guide the earlier stages are altruistic, strategic, economic, political, historical, or otherwise motivated, nor does it impose an ideal preference structure. This is incidentally consistent with the literature that emphasizes the plurality and heterogeneity of preferences, with each donor prioritizing a distinct combination of motives.[6]

A limitation to this axiom is that both academic (Flammer et al., 2024) and practitioner (IDB Invest, 2023) literatures suggest some level of trade-offs between financial and non-financial preferences, which may contradict the axiom of sequential decision-making that separates financial from non-financial decisions. The contradiction may be solved by distinguishing the construct of subsidy equivalency (or concessionality) from the donor's present cost (following Frank, 1970). In the model, subsidy equivalency is considered an exogenous variable, while the donor's present cost is endogenous. This allows a donor to hypothetically select a portfolio based on its development impact relative to its subsidy equivalency intensity (per Flammer et al., 2024) as a prior step to the model, which in turn

[4] The list includes projects funded through the first and second phase of the fund.

[5] This is somewhat consistent with Markowitz (1952) phased approach to portfolio selection: *the process of selecting a portfolio may be divided into two stages. The first stage starts with observation and experience and ends with beliefs about the future performances of available securities. The second stage starts with the relevant beliefs about future performances and ends with the choice of portfolio. This paper is concerned with the second stage.*

[6] Canonical works include Alesina & Dollar (2000) and Berthélemy (2006), among many others.

focuses on the donor's present cost optimization. A similar tradeoff may arise where private capital mobilization becomes a stand-alone policy objective. The matter is analyzed specifically in §4.8.

2.1. *The Portfolio and its Assets*

Let p be the portfolio of n blended finance investments or projects indexed by i that the donor wishes to support (those are referred to as **assets**). For simplicity, each asset requires a nominal investment of 1 at t_0 and yields a random rate of return of ra_i at t_1 under a normal distribution. The normality assumption allows the donor's preferences to be represented by a quadratic mean–variance utility function that is consistent with expected utility under normally distributed returns. It is, however, an approximation rather than a literal description of most blended-finance cash flows. A meaningful share of blended finance is structured through loans or loan-like instruments (ADB et al. 2023). Payoffs are typically asymmetric. Upside is capped, whereas downside is concentrated in default states. Relative to such distributions, a normal approximation tends to understate negative skewness and tail risk. In addition, the model assumes no transactions costs and no information asymmetry. Those assumptions and their underlying limitations are discussed in more detail in §5.3.

Let μ_{ra_i}, $\sigma^2_{ra_i}$, and σ_{ra_i}, be respectively the mean, variance and standard deviation of an asset's rate of return, as follows:

Definition 1 Assets' Rate of Return Mean, Variance and Standard Deviation

$$Mean\ rate\ of\ return = \mathbb{E}[ra_i] = \mu_{ra_i}$$

$$Variance = Var[ra_i] = \mathbb{E}\left[\left(ra_i - \mu_{ra_i}\right)^2\right] = \sigma^2_{ra_i}$$

$$Standard\ deviation = \sqrt{Var[ra_i]} = \sigma_{ra_i}$$

2.2. *The Investor and its Commercial Investment*

The **investor** is a private agent that would hypothetically finance an asset i should it receive a minimum mean rate of return μ_{rm_i}. For each asset, μ_{rm_i} is assumed to be strictly higher than the mean rate return μ_{ra_i}. This framing is broadly consistent with Flammer et al. (2024), Warner (2013) and IDB Invest (2023).

For example, the mean returns for investing in a battery charging infrastructure in a developing country may be insufficient to compensate for the risk of financing it. Thus, without any public intervention, the infrastructure would not be financed by the market, which incidentally justifies the donor's additionality in financing the asset.

Although the model refers to a single investor, hypothetically for each asset i there might be a different marginal investor with the lowest opportunity cost (μ_{rm_i}) for that particular asset.

Like Flammer et al. (2024 and 2025), the model assumes that the investor's hurdle rate is exogenous and does not vary with the investor's share in each asset. In other words, whether the investor is offered the full exposure or only a fraction of the asset, it is assumed to require the same return. This assumption relies on observed

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practice: in individual blended finance transactions, the marginal investors are typically large local and international banks (and, less frequently, institutional investors), whose required returns are driven mainly by counterparty risk, tenor, currency, and regulatory capital treatment rather than by differences in ticket size. Except for unusually large transactions where concentration limits or balance-sheet constraints may be binding, allocation size would therefore have limited influence on the hurdle rate.

A **commercial investment** is an investment yielding the minimum mean rate of return desired by the investor. Suppose that regardless of the rate ra_i returned by the asset at t_1, the investor's rate of returns is increased by the difference between the investor's required mean rate of return μ_{rm_i} and the asset's actual mean rate of return μ_{ra_i}. Thus, the mean rate of return received by the investor would be the desired mean rate of return μ_{rm_i}:

Definition 2 Investor's Rate of Return Threshold

$$rm_i = ra_i + \mu_{rm_i} - \mu_{ra_i}$$
$$\Leftrightarrow \mathbb{E}[rm_i] = \mathbb{E}\left[ra_i + \mu_{rm_i} - \mu_{ra_i}\right] = \mu_{rm_i}$$

2.3. *The Subsidy Equivalent*

Consistent with academic (Pincus, 1963, Schmidt 1964, Chang et al., 1998) and practitioner (ADB et al., 2018) literature, an asset's **subsidy equivalent** is the present benefit to the investor of receiving concessional financing expressed in grant equivalent. To determine the t_0 value for the investor of that additional return $\mu_{rm_i} - \mu_{ra_i}$, it is discounted by the opportunity cost μ_{rm_i}.

Definition 3 Assets' Subsidy Equivalent

$$Subsidy\ equivalent\ of\ asset\ i = \ s_i = \frac{\mu_{rm_i} - \mu_{ra_i}}{1 + \mu_{rm_i}}$$

If the investor receives the subsidy equivalent s_i as a grant at t_0, it would extend a commercial investment in asset i, since it would now receive the mean rate of return μ_{rm_i} it desires:

Lemma 1 Extending the Subsidy Equivalent as Grant

$$Investor's\ mean\ rate\ of\ return = \frac{Mean\ returns\ at\ t_0 - Investment\ at\ t_1\ (net\ of\ s_i)}{Investment\ at\ t_1\ (net\ of\ s_i)}$$
$$= \frac{\left(1 + \mu_{ra_i}\right) - (1 - s_i)}{(1 - s_i)} = \mu_{rm_i}$$

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Proof. See Annex 3.1.[7]

The following can be extrapolated.

Proposition 1 Investor's Indifference to Concessional Finance Instruments

The investor would be indifferent between receiving a grant at t_0 equal to the subsidy equivalent or an additional return $\mu_{rm_i} - \mu_{ra_i}$ equal to the subsidy equivalent at t_1.

Proof. Derived from Lemma 1.

2.4. The Donor's Concessional Financing

Flammer et al. (2025) proposes two donor's concessional financing contracts to enhance the investor's risk-return paradigm: increase the share of the concessional capital to further increase the expected return of the investor or reduce the spread of the distribution of return. The model considers a contract akin to the first option, whereby through the donor's concessional financing, the investor receives a mean rate of return μ_{rm_i}. The model assumes the principle of minimum subsidy is applied: the donor receives the highest rate of return that enables the asset to be financed by the investor.[8]

In the model, the donor may extend financing through the following instruments: grants, concessional investments and commercial investments (together, the **financing instruments**).[9] The donor's rate of return is a function of its nominal participation into the asset. For instance, a **grant** is a cash subsidy from the donor to the investor that yields a rate of return of -100%. A **concessional investment** is a concessional financing that generates a mean rate of return μ_{rc_i} below the investor's minimum mean rate of return μ_{rm_i}.[10] A commercial investment – defined in §2.2 – features a mean rate of return of μ_{rm_i}. Both grants and concessional investments constitute a form of **concessional financing**. An **investment** is either a concessional investment or a commercial investment.

Let v_i be the face value of the donor's financing instrument for asset i (the **nominal participation**).

For a concessional investment, the donor's rate of return rc_i is a function of the asset's return minus the investor's required returns, based on the donor's nominal participation.

Lemma 2 Donor's Rate of Return

$$rc_i = ra_i + \mu_{rm_i} - \mu_{ra_i} + \frac{\mu_{ra_i} - \mu_{rm_i}}{v_i}$$

Proof. See Annex 3.2.

[7] Note that rate of return for the investor may be different, only the mean rate of return is the same. However, as shown in Annex 3.1, the investor's net present values are the same.

[8] Otherwise known as the principle of minimum concessionality which is one of the five DFIs and MDBs principles for the use of blended finance (ADB et al 2017).

[9] I develop subsequently the case in which the donor would extend a commercial investment (see §2.4)

[10] See §4.6 for the exception where the donor receives μ_{rm_i}.

Under the model, the donor's rate of return is less than or equal to the asset rate of return, which is strictly inferior to the investor's rate of return:

$$rc_i \leq ra_i < rm_i$$

It follows from the linearity of expectation that the mean rate of return of the donor μ_{rc_i} is a function of its participation v_i, the mean rate of return of the asset μ_{ra_i} and the investor's required mean rate of return μ_{rm_i}.

Definition 4 Donor's Mean Rate of Return

$$\mu_{rc_i} = \mathbb{E}[rc_i] = \mathbb{E}\left[ra_i + \mu_{rm_i} - \mu_{ra_i} + \frac{\mu_{ra_i} - \mu_{rm_i}}{v_i}\right] = \mu_{rm_i} + \frac{\mu_{ra_i} - \mu_{rm_i}}{v_i}$$

Because of the structure of the donor's concessional investment, the variances of the rates of return of the asset ($\sigma^2_{ra_i}$), the investor ($\sigma^2_{rm_i}$) and the donor ($\sigma^2_{rc_i}$) are the same (in the case where the donor extends a grant, the variance is equal to zero). The same applies for the standard deviations:

Lemma 3 Equality of Investments Rates of Return Variances

$$\sigma^2_{ra_i} = \sigma^2_{rm_i} = \sigma^2_{rc_i}$$
$$\sigma_{ra_i} = \sigma_{rm_i} = \sigma_{rc_i}$$

Proof. See Annex 3.3.

2.5. Donor's Preferences

To explain the donor's preferences formally, recall that the donor has already identified a portfolio p of assets i that requires concessional financing. In the model, the donor decides its nominal participation v_i in each asset – which in turn determines the choice of financing instruments (defined in §2.4).

The donor's decision is modeled as a single, ex ante assessment in which expected losses and uncertainty are weighted jointly. In other words, the donor forms a view about both how much of financial loss – a proxy for ODA – an intervention is expected to "consume" on average and how uncertain that "consumption" is likely to be.

Lower expected losses are valued because they preserve scarce budgetary space and allow the donor to undertake a larger number of interventions, generally within a predetermined fixed ODA envelope. This preference is arguably revealed by the fact that donors are extending ODA through returnable instruments even in cases where, in principle, they could pursue the same policy objective through a pure grant calibrated to the implied subsidy of the transaction. Canada's 2021–2026 CAD$5.3 billion international climate finance envelope provides an illustration: 40% is expected to be delivered through grants and 60% through *Unconditionally Repayable Contributions* described by the Government of Canada as akin to

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concessional loans (GAC, 2025a). In a note discussing the use of returnable instruments in blended finance, IFC (2019) states that the model *appeals to governments because they regularly receive the reflows, and can redeploy the funds for other programs or priorities*.

Aversion to uncertainty reflects the fact that both unusually bad and unusually good financial outcomes can be institutionally problematic. Large realized losses can trigger unforeseen budgetary losses, fiduciary concerns, political scrutiny, and reputational damage. Conversely, unusually small losses or net gains can also generate disutility by complicating delivery against planned ODA volumes, raising questions about the appropriateness of profit-making with aid resources, or undermining expectations about how concessional resources will be deployed.[11] Importantly, in this setting the donor is assumed to care about the size of deviations from what was expected, not their direction. What matters is the extent to which realized losses depart from *ex-ante* expected losses, regardless of whether this departure takes the form of higher-than-expected losses or lower-than-expected losses.[12]

This logic aligns with a Markowitz-style mean–variance perspective applied to public finance. Donors prefer interventions that are expected to consume fewer resources and that deliver those outcomes in a more predictable manner. Interventions that combine low expected losses with low uncertainty are therefore strictly preferred, while trade-offs arise when an intervention reduces expected losses at the cost of greater volatility, or vice versa.

2.6. The Present Cost of the Donor's Financing Instrument

In the model, losses are calculated as the present value of the cash flows from the donor's concessional financing discounted by rd, a constant discount rate representing the donor's cost of funding for the period t (the donor's **present cost**).[13]

The use of the donor's cost of funding as a proxy for its opportunity cost is practice-motivated: donors often assess the fiscal exposure of returnable assistance in net present value terms, discounting expected cash flows at a rate intended to reflect their cost of funding – that is, the rate at which the government can raise resources today (or the return it forgoes by committing those resources) over the relevant horizon, as hypothesized in Pincus, 1963. Carter (2015), however, questions whether borrowing costs are the right way to represent the donor's true opportunity cost. Government borrowing rates do not capture the wider economic costs created by the taxation required to service debt. From a welfare perspective, Carter argues that a more relevant benchmark may be the marginal cost of public funds, which incorporates the behavioral distortions associated with raising public revenue.

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[11] Anecdotally, I have witnessed in a several instances donors commenting that their returns on certain blended finance programs were "too high" and this was generating "internal scrutiny".

[12] GAC (2025b) explicitly recognizes this possibility: *In the context of international development assistance, it is not anticipated that GAC would receive a financial return over and above the repayment of its contribution except in rare instances where the underlying investment(s) supported perform exceptionally well. In this case, the amount paid to GAC could exceed the initial contribution (on a net present value basis). That is, while ex ante, the funding is expected to be concessional (as per the OECD Blended finance guidance & principles), ex post, there is a possibility that the Government of Canada could realize a positive return*.

[13] Note that in the calculation, the present cost is a positive number (in case of gains, the present cost becomes negative).

In addition, treating rd as constant abstracts from the possibility that, as governments borrow more to finance the portfolio of assets, marginal funding costs can rise because investors demand a higher risk premium as debt burdens rise. In the extreme case where a donor would rely heavily on borrowing – beyond what its taxation capacity could credibly support – while expecting repayments from risky blended-finance assets, one may expect borrowing costs to drift toward the market-required return for those assets. An intuition Carter makes explicit when noting that the appropriate discount rate *may not be so very different* from private investors' required returns.

In practice, however, existing government support for blended finance remains small relative to overall sovereign borrowing volumes, making a constant rd a reasonable working assumption for the purposes of the model.[14]

Nevertheless, the model considers in §4.4 the case in which rd exceeds the investor's hurdle rate and discusses how this changes the donor's optimum.

Definition 5 Donor's Present Cost

$$\begin{aligned} Present\ cost\ of\ donor's\ subsidized\ financing\ in\ asset\ i = \ g_i \\ = v_i - \frac{v_i(1+rc_i)}{1+rd} \\ = \frac{v_i(rd-rc_i)}{1+rd} \\ = \frac{v_i\left(rd-ra_i-\mu_{rm_i}+\mu_{ra_i}\right)+\mu_{rm_i}-\mu_{ra_i}}{1+rd} \end{aligned}$$

Having defined g_i, let μ_{g_i}, $\sigma_{g_i}^2$ and σ_{g_i} respectively be the mean, variance and standard deviation of the donor's present cost for each asset.

Lemma 4 Mean, Variance and Standard Deviation of Donor's Present Cost

$$Mean\ present\ cost = \mu_{g_i} = \mathbb{E}[g_i] = \frac{v_i\left(rd-\mu_{rm_i}\right)+\mu_{rm_i}-\mu_{ra_i}}{1+rd}$$

$$Variance\ of\ present\ cost = \sigma_{g_i}^2 = Var[g_i] = \mathbb{E}\left[\left(g_i-\mu_{g_i}\right)^2\right] = \left(\frac{v_i}{1+rd}\right)^2 \sigma_{ra_i}^2$$

$$Standard\ deviation\ of\ present\ cost = \sigma_{g_i} = \frac{v_i}{1+rd}\sigma_{ra_i}$$

Proof. See Annex 3.4.

2.7. Donor's Present Cost and Asset's Subsidy Equivalent

[14] For instance, Canada central government debt stands at CAD$1.99 trillion in 2024 (IMF, 2026), compared to an estimated CAD$ 636 million of yearly *Unconditionally Repayable Contributions* deployed under Canada's 2021–2026 international climate finance envelope (see §2.5).

Suppose the donor extends a concessional investment equal to the subsidy equivalent s_i. It follows that the donor's mean present post μ_{g_i} is equal to the subsidy equivalent s_i.

Lemma 5 Donor's Mean Present Cost and Asset's Subsidy Equivalent

$$v_i = s_i$$
$$\Rightarrow \mu_{g_i} = s_i$$

Proof. See Annex 3.5.

3. Formulating the Donor's Optimization Problem

3.1. Donor's Disutility Function and Optimization Problem

Following §2.5 and §2.6, the donor's preferences can be defined over the mean and variance space of the donor's present cost, consistent with a mean-variance utility framework. In this framework, the donor would optimally allocate financing instruments to minimize a present cost disutility function that incorporates present cost mean and variance, analogous to Markowitz's Modern Portfolio Theory (Markowitz, 1959) but applied to blended finance.

Let $\mathcal{D}$ be the donor's disutility function from its $(v_1, \dots, v_n)$ nominal participations in each asset, based on Markowitz (1956) tweaked mean-variance utility function:

Definition 6 Donor's Disutility Function

$$\mathcal{D}(v_1, \dots, v_n) = \gamma \mu_{g_p} + \frac{1}{2}\sigma_{g_p}^2$$

Where μ_{g_p} and $\sigma_{g_p}^2$ are respectively the donor's present cost mean and variance of its concessional financing of the portfolio p, and γ is the donor's risk-tolerance coefficient.

Suppose the donor wants to optimally allocate its nominal participations v_i in each asset i of the portfolio p up to a maximum aggregate nominal budget of b_{max}. According to §2.4, for each asset i, the donor may invest v_i, between a minimum of s_i and a maximum of 1, subject to the sum of v_i not exceeding b_{max}. The optimization problem for the donor takes the form of a standard quadratic programming problem:

Definition 7 Donor's Optimization Problem

$$(v_1^*, \dots, v_n^*)(\gamma) = arg\ min \left\{ \mathcal{D}(v_1, \dots, v_n) = \gamma \mu_{g_p} + \frac{1}{2}\sigma_{g_p}^2 \right\}$$
$$s.t. \begin{cases} \sum_{i=1}^{n} v_i \leq b_{max} \\ s_i \leq v_i \leq 1 \end{cases}$$

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3.2. The Donor's Portfolio of Concessional Investments

Let $V = (v_1, \dots, v_n)$ be the vector of the donor's nominal participation in the concessional investment for each asset i. The donor's total participation is $\sum_{i=1}^{n} v_i = 1_n^T V \leq b_{max}$.

The variances of asset returns are assumed to be strictly positive, asset returns are assumed to be linearly independent, and pairwise correlations are assumed to be strictly positive. The pairwise correlation positivity assumption is analytically convenient because in cases that support the grant results (see §4.3 and §4.4), it helps ensure that the relevant partial derivatives of the donor's disutility with respect to nominal participations are strictly positive over the feasible set. In practice, it can be rationalized for portfolios exposed to common macroeconomic, exchange-rate, commodity-price, or sovereign-risk factors. This is arguably the case for blended finance assets, which are focused on developing countries eligible for ODA and concentrated in the sectors of financial services, energy and infrastructure and agribusiness. Even so, the restriction is strong and possible limitations are discussed in more detail in §5.3.

3.3. The Mean Present Cost of the Donor's Portfolio of Concessional Investments

Let μ_{g_p} be the donor's mean portfolio present cost. From the linearity of expectation, it follows that:

$$\mu_{g_p} = \mathbb{E}[g_p] = \sum_{i=1}^{n} \mathbb{E}[g_i] = \sum_{i=1}^{n} \frac{v_i(rd - \mu_{rm_i}) + \mu_{rm_i} - \mu_{ra_i}}{1 + rd}$$

Let $A = (\mu_{ra_1}, \dots, \mu_{ra_n})$ be the vector of the mean rates of return for each asset i, $M = (\mu_{rm_1}, \dots, \mu_{rm_n})$ the vector of the minimum mean rate of return threshold required by the investor for each asset i and $D = (rd, \dots, rd)$ the n dimension vector of the donor's cost of funding rd. μ_{g_p} is rewritten using matrix algebra and isolating the linear term from the constant term:

Definition 8 Mean Present Cost of the Donor's Portfolio

$$\mu_{g_p} = \frac{1_n^T(M - A)}{1 + rd} + \frac{V^T(D - M)}{1 + rd}$$

3.4. The Present Cost Variance of the Donor's Portfolio of Concessional Investments

Let $\sigma_{g_p}^2$ be the present cost variance of the donor's portfolio of financing instruments. Since covariance is bilinear:

$$\sigma_{g_p}^2 = \mathbb{E}\left[\left(\sum_{i=1}^{n} g_i - \mathbb{E}\left[\sum_{i=1}^{n} g_i\right]\right)^2\right] = \mathbb{E}\left[\left(\sum_{i=1}^{n} g_i - \mu_{g_i}\right)^2\right] = \sum_{i=1}^{n}\sum_{i'=1}^{n} Cov(g_i, g_{i'})$$

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Which can be expressed as a function of v_i and ra_i:

Lemma 6 Present Cost Variance of the Donor's Portfolio

$$\sigma_{g_p}^2 = \sum_{i=1}^{n}\sum_{i'=1}^{n} \frac{v_i v_{i'}}{(1+rd)^2} Cov(ra_i, ra_{i'})$$

Proof. See Annex 3.6.

Let $\rho_{ii'} = \frac{Cov(ra_i, ra_{i'})}{\sigma_{ra_i}\sigma_{ra_{i'}}}$ be the correlation between the rates of return ra_i and $ra_{i'}$:

$$\sigma_{g_p}^2 = \sum_{i=1}^{n}\sum_{i'=1}^{n} \frac{v_i v_{i'}}{(1+rd)^2} \sigma_{ra_i}\sigma_{ra_{i'}}\rho_{ii'}$$

Let $\sum$ be the covariance matrix of the rates of return ra_i so that:

Definition 9 Assets Rate of Return Covariance Matrix

$$\sum = \begin{bmatrix} \sigma_{ra_1}^2 & \cdots & \sigma_{ra_n}\sigma_{ra_1}\rho_{n1} \\ \vdots & \ddots & \vdots \\ \sigma_{ra_a}\sigma_{ra_n}\rho_{1n} & \cdots & \sigma_{ra_n}^2 \end{bmatrix}$$

Note that a covariance matrix is always positive semi-definite. Since the rates of returns of the assets i are linearly independent, $\sum$ is positive definite. $\sigma_{g_p}^2$ is rewritten using matrix algebra:

Definition 10 Present Cost Variance of the Donor's Portfolio

$$\sigma_{g_p}^2 = \left(\frac{1}{1+rd}\right)^2 V^T \sum V$$

4. Solving the Donor's Optimization Problem

4.1. Solving the Donor's Optimization Problem (Concessional Investments Only)

Following §3.3 and §3.4, the donor's optimization problem can be expanded as follows:

Definition 11 Expanded Donor's Optimization Problem

$$(V^*)(\gamma) = \arg\min\left\{\mathcal{D}(V) = \gamma\frac{1_n^T(M-A)}{1+rd} + \gamma\frac{V^T(D-M)}{1+rd} + \frac{1}{2}\left(\frac{1}{1+rd}\right)^2 V^T\sum V\right\}$$

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$$s.t.\begin{cases} 1_n^T V \leq b_{max} \\ S \leq V \leq 1 \end{cases}$$

Where $S = (s_1, \dots, s_n)$ is the vector of subsidy equivalents for each asset i. The above optimization problem features a quadratic objective function $\mathcal{D}(V)$ with linear inequality constraints.[15] Per the assumptions §3.1 on the rates of return ra_i, $\sum$ is a positive definite matrix. Therefore, $\mathcal{D}(V)$ admits a global minimum V^*.

Proposition 2 Donor's Optimal Vector of Nominal Participations in Concessional Investments

Where the donor's nominal participation in a portfolio of risky assets is constrained by b_{max} , there is a unique vector V^* of nominal participations in concessional investments that minimizes the donor's disutility.

Proof. See above.

Proposition 2 establishes that for each donor with a given risk tolerance, there is a unique set of nominal participations from the donor in each asset of the portfolio. The donor's nominal participation in each asset in turn drives both the donor's rate of return and the percentage of the investor's participation.

4.2. Gradient of Donor's Disutility Function

Let $\nabla\mathcal{D}(V)$ be the gradient of the disutility function $\mathcal{D}(V)$.

Lemma 7 Gradient of Donor's Disutility Function

$$\nabla\mathcal{D}(V) = \gamma\frac{(D-M)}{1+rd} + \left(\frac{1}{1+rd}\right)^2 \sum V$$

Proof. See Annex 3.7.

While the optimal participation of the donor V^* is a function of the variance and correlation of the rate of returns on the assets i, the mean rate of return of the asset μ_{ra_i} has no incidence on the donor's optimal participation. In other words, once the donor has determined that given assets should be part of its portfolio, the mean rates of return on these assets have no incidence in the donor's choice of instrument, only the variances and correlations do.

4.3. Donor Extending Grants Instead of Concessional Investments

Suppose the donor's risk tolerance coefficient γ is equal to zero. The gradient of the donor's disutility function is updated as follows:

$$\nabla\mathcal{D}_{\gamma=0}(V) = \left(\frac{1}{1+rd}\right)^2 \sum V$$

From the model's assumptions, all v_i are strictly positive (§2.4) and all correlations $\rho_{ii'}$ are strictly positive (§3.1). Therefore, the gradient's components are strictly

[15] The feasible region defined by the linear inequality constraints is nonempty.

positive. The optimal vector V^* of nominal participations is equal to the lower bound S, the vector of subsidy equivalents (since all $v_i \geq s_i$).

Recall from Lemma 1 that the investor is agnostic between the donor extending S as a grant or as a concessional investment and from Lemma 5 that the donor's mean present cost of extending a concessional investment is equal to the subsidy equivalent (mean present cost is the same for the donor). Let $\mathcal{D}(S_{grant})$ be the disutility functions where the donor extends S as a grant. The donor's mean present cost is the sum of the subsidy equivalents for each asset multiplied by the risk-tolerance coefficient: $\gamma 1_n^T S$. The present cost variance is equal to zero, since the variance for each asset's present cost is equal to zero. In other words, the loss is certain, since the concessional financing is extended as grant.

Definition 12 Donor's Utility Function when Extending a Grant S_{grant}

$$\mathcal{D}(S_{grant}) = \gamma 1_n^T S$$

Let $\mathcal{D}(S_{invest})$ be the disutility functions where the donor extends S as concessional investments. From Lemma 5:

$$\begin{aligned}\mathcal{D}(S_{invest}) &= \gamma \frac{1_n^T(M-A)}{1+rd} + \gamma \frac{S^T(D-M)}{1+rd} + \frac{1}{2}\left(\frac{1}{1+rd}\right)^2 S^T \sum S \\ &= \gamma 1_n^T S + \frac{1}{2}\left(\frac{1}{1+rd}\right)^2 S^T \sum S \\ &\Rightarrow \mathcal{D}(S_{grant}) < \mathcal{D}(S_{invest})\end{aligned}$$

Proposition 3 Extending Grants S_{grant} when $\gamma = 0$

The donor's will always prefer to extend a nominal participation in the form of grants S_{grant} when its risk tolerance coefficient $\gamma = 0$.

Proof. See above.

Proposition 4 Extending a Grant S_{grant} instead of Concessional Investments S_{invest}

The donor's will always prefer to extend a nominal participation in the form of grants S_{grant} rather than concessional investments S_{invest}.[16]

Proof. See above.

The above suggests that where the donor's risk tolerance coefficient is close to zero, it may still prefer to extend a grant S_{grant}. For this to be case, $\mathcal{D}(S_{grant})$ must be strictly inferior to $\mathcal{D}(V^*)$. Let $\mathcal{D}(S_{grant}) = \gamma \mu_{g_S}$ and $\mathcal{D}(V^*) = \gamma \mu_{g_{V^*}} + \frac{1}{2}\sigma^2_{g_{V^*}}$ with:

$$\mu_{g_S} = 1_n^T S$$

[16] The same is true for any $v_i^* = s_i$.

$$\mu_{g_{V^*}} = \gamma \frac{1_n^T (M - A)}{1 + rd} + \gamma \frac{V^{*T}(D - M)}{1 + rd}$$

$$\sigma^2_{g_{V^*}} = \left(\frac{1}{1 + rd}\right)^2 V^{*T} \sum V^*$$

μ_{g_S} is strictly positive per Definition 3. Per the above, $\mu_S > \mu_{V^*}$, thus, $\mu_S - \mu_{V^*}$ is strictly positive. Finally, $\sigma^2_{g_{V^*}}$ is strictly positive. Hence the following inequality:

$$\gamma \mu_{g_S} < \gamma \mu_{g_{V^*}} + \frac{1}{2}\sigma^2_{g_{V^*}}$$

$$\Rightarrow \gamma < \frac{1}{2} \frac{\sigma^2_{V^*}}{\mu_S - \mu_{V^*}}$$

Proposition 5 Extending a Grant S_{grant} when $\gamma > 0$

The donor's will always prefer to extend a nominal participation in the form of a grant S_{grant} when its risk tolerance coefficient $\gamma < \frac{1}{2} \frac{\sigma^2_{V^*}}{\mu_S - \mu_{V^*}}$

Proof. See above.

4.4. Donor's Cost of Funding and Optimal Choice

For a given i, suppose that $rd > \mu_{rm_i}$. Based on Lemma 7, the partial derivative of the disutility function with respect to v_i is:

$$\frac{\partial \mathcal{D}}{\partial v_i} = \gamma \frac{rd - \mu_{rm_i}}{1 + rd} + \frac{\sum_{i'=1}^{n} v_{i'} \sigma_{ra_i} \sigma_{ra_{i'}} \rho_{ii'}}{(1 + rd)^2} > 0$$

Since $rd > \mu_{rm_i}$ and $rd > 0$, the first term is positive. From the assumptions above, all $v_{i'}$ are strictly positive (§2.4) and all correlations $\rho_{ii'}$ are strictly positive (§3.2). Thus, the partial derivative is strictly positive. In this case, the optimal v_i^* that minimizes $\mathcal{D}(V)$ is the lower bound s_i. From Proposition 4, where $v_i^* = s_i$, the donor prefers to extend a grant rather than a concessional investment.

Therefore, for any asset i where $rd > \mu_{rm_i}$, the optimal donor concessional financing is to extend a grant equal to the subsidy equivalent for that asset (consistent with Schmidt, 1964 but expanded to a portfolio of risk bearing assets). For the remaining assets in the portfolio p, the optimal vector of nominal participations is given by solving the optimization problem excluding the assets i where $rd > \mu_{rm_i}$.

Proposition 6 Donor's Cost of Funding and Optimal Participations

Where the donor's nominal participation in a portfolio of risky assets is constrained by b_{max} , there is a unique vector V^* of nominal participations that minimizes the donor's disutility. For assets where the donor's cost of funding rd is strictly superior to the market's opportunity cost μ_{rm_i}, the donor's concessional financing is a grant equal to the required subsidy equivalent s_i for that asset.

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Proof. See above.

4.5. Combining Grants and Concessional Investments

For each asset i, suppose the donor is allowed to provide a portion of its nominal participation as grant, and the remainder as an investment. Let x_i be the portion of the required subsidy equivalent s_i that is extended as a grant (with a rate of return of -100%), and y_i the portion extended as investment so that $x_i + y_i = v_i$.

The cases where $v_i^* = s_i$ under Proposition 3, Proposition 5 and Proposition 6 are excluded, so that $v_i^* > s_i$.

Let rc_{y_i} be the required rate of return of the donor concessional investment. rc_{y_i} is a function of the asset's return minus the investor's required returns, based on the donor's nominal participation in the concessional investment (i.e. the total nominal participation net of the portion extended as grant).

Lemma 8 Rate of Return of the Donor Concessional Investment when Partly Extending a Grant

$$1 + ra_i = x_i(1 - 100\%) + y_i\left(1 + rc_{y_i}\right) + (1 - v_i)(1 + rm_i)$$

$$\Rightarrow rc_{y_i} = \frac{v_i(1 + ra_i + \mu_{rm_i} - \mu_{ra_i}) - \mu_{rm_i} + \mu_{ra_i}}{y_i} - 1$$

Proof. See Annex 3.8.

Let g_{x_i} be the present cost of donor's concessional financing when partly extending it as a grant and partly as a concessional investment. It can be demonstrated that it is the same than if the donor's concessional financing is solely extended as a concessional investment.

Lemma 9 Donor's Present Cost with Grant Portion

$$g_{x_i} = g_i$$

Proof. See Annex 3.9.

It can also be demonstrated that the mean, variance and standard deviation of the donor's concessional financing when partly extending it as a grant and partly as a concessional investment, is the same as if extended solely as a concessional investment.

Let $\mu_{g_{x_i}}$, $\sigma^2_{g_{x_i}}$ and $\sigma_{g_{x_i}}$ be the mean, variance and standard deviation of the donor's concessional financing when partly extending it as a grant portion.

Lemma 10 Mean, Variance and Standard Deviation of Donor's Present Cost with Grant Portion

$$\mu_{g_{x_i}} = \mu_{g_i} = \frac{v_i\left(rd - \mu_{rm_i}\right) + \mu_{rm_i} - \mu_{ra_i}}{1 + rd}$$

$$\sigma_{g_{x_i}}^2 = \sigma_{g_i}^2 = \left(\frac{v_i}{1+rd}\right)^2 \sigma_{ra_i}^2$$

$$\sigma_{g_{x_i}} = \sigma_{g_i} = \frac{v_i}{1+rd}\sigma_{ra_i}$$

Proof. See Annex 3.10.

From Lemma 10, it follows that for any optimal vector of nominal participations $V^* > S$, the donor can break down its participations v_i^* between a grant x_i up to the subsidy equivalent s_i and an investment for $y_i = v_i^* - x_i$ without affecting the donor's optimum.

Proposition 7 Combining Grants and Concessional Investments

For any optimal vector of nominal participations V^*, the donor can break down its participations v_i^* between a grant x_i up to the subsidy equivalent s_i and an investment $y_i = v_i^* - x_i$ without affecting the donor's optimum.

Proof. Derived from Lemma 10.

4.6. Combining Grants and Commercial Investments

For this section, the cases where $v_i^* = s_i$ under Proposition 3, Proposition 5 and Proposition 6 are excluded, so that $v_i^* > s_i$. From Proposition 7, suppose the donor decides to extend $x_i = s_i$ as grant for all assets, and the balance $y_i = v_i^* - x_i$ as investment.

From Lemma 1, if the donor extends a grant s_i, then the assets will yield the commercial mean rate of return μ_{rm_i}. In this case, both the investor and the donor – on its investment y_i – will receive the commercial mean return threshold.

Proposition 8 Combining Grants and Commercial Investments

For any optimal vector of nominal participations $V^* > S$, the donor can break down its participations v_i^* between a grant s_i and an investment $y_i = v_i^* - s_i$ that will yield the commercial mean return threshold.

Proof. Derived from Lemma 1.

4.7. Donor's Portfolio Weights within Boundaries Constraints

Suppose no constraints in both the maximum face value of the donors' portfolio (b_{max} is infinite) and the size of the donor's investment in each asset i. The cases where $v_i^* = s_i$ under Proposition 3, Proposition 5 and Proposition 6 are excluded, so that $v_i^* > s_i$. In such a setting, the donor is looking to minimize the following unconstrained disutility function:

Definition 13 Donor's Optimization Problem within Boundaries Constraints

$$(V^*)(\gamma) = \arg\min\left\{\mathcal{D}(V) = \gamma\frac{1_n^T(M-A)}{1+rd} + \gamma\frac{V^T(D-M)}{1+rd} + \frac{1}{2}\left(\frac{1}{1+rd}\right)^2 V^T\sum V\right\}$$

Recall the gradient $\nabla\mathcal{D}(V)$ defined in Lemma 7. The gradient is set to zero and solved for V. Since $\sum$ is positive definite, it admits an inverse matrix $\sum^{-1}$ (where the product of both matrices is equal to the identity matrix):

$$\gamma \frac{(D-M)}{1+rd} + \left(\frac{1}{1+rd}\right)^2 \sum V = 0$$
$$\Rightarrow \sum \sum^{-1} V = -\gamma(1+rd)\sum^{-1}(D-M)$$
$$\Rightarrow V^* = -\gamma(1+rd)\sum^{-1}(D-M)$$

$\sum$ is positive definite and therefore $\mathcal{D}(V)$ admits a unique minimum V^*.

Proposition 9 Donor's Optimization Problem within Boundaries Constraints

Within boundaries constraints, there is a unique vector V^* of participation in each asset that minimizes the donor's disutility given by setting the gradient of the disutility function to zero as follows:

$$(V^*)(\gamma) = arg\ min \left\{ \mathcal{D}(V) = \gamma \frac{1_n^T(M-A)}{1+rd} + \gamma \frac{V^T(D-M)}{1+rd} + \frac{1}{2}\left(\frac{1}{1+rd}\right)^2 V^T \sum V \right\}$$
$$\exists!\, V^* = -\gamma(1+rd)\sum^{-1}(D-M)$$

Proof. See above.

The above shows the linear relationship between V^* and the risk-tolerance coefficient γ. Let b^* be the aggregate nominal participation of the donor in the concessional investments and $W^* = (w_1^*, \dots, w_n^*)$ the vector of weights of the donor's portfolio of concessional investments so that $b^* = \sum_{i=1}^n v_i^*$ and $V^* = b^* W^*$.

Suppose a second donor with the same cost of funding but a different risk-tolerance coefficient γ' is interested in financing the same portfolio p. γ' can be expressed as a multiple k of γ so that $\gamma' = k\gamma$. It follows that V'^* (the vector of optimal nominal participations of the second donor) can be expressed as follows:

$$V'^* = -\gamma'(1+rd)\sum^{-1}(D-M)$$
$$= -k\gamma(1+rd)\sum^{-1}(D-M)$$
$$= -kb^* W^*$$

Thus, regardless of risk tolerance, the weights of the donor's participation in the assets in the portfolio are unchanged (within boundaries conditions).

Proposition 10 Unique Vector of Weights Within Boundaries Conditions

Within boundaries conditions, regardless of the donor's risk-tolerance coefficient there is a unique vector W^* of weights in the composition of the donor's nominal participations.

Proof. See above.

4.8. Private Capital Leverage

For simplicity, assumptions from §4.7 are maintained. Let l be the private capital leverage such that:

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Definition 14 Private Capital Leverage

$$l = \frac{Total\ Investment - Donor's\ Nominal\ Participation}{Donor's\ Nominal\ Participation}$$

$$= \frac{n - 1_n^T V^*}{1_n^T V^*} = n(1_n^T V^*)^{-1} - 1 = n[-\gamma(1+rd)1_n^T \sum^{-1}(D-M)]^{-1} - 1$$

The partial derivative of l with respect to γ is:

$$\frac{\partial l}{\partial \gamma} = n\gamma^{-2}[(1+rd)1_n^T \sum^{-1}(D-M)]^{-1}$$

Recall $V^* = -\gamma(1+rd)\sum^{-1}(D-M)$, $\gamma > 0$, $rd > 0$, and $v_i^* > 0$. Therefore $\frac{\partial l}{\partial \gamma}$ is negative. There is a tradeoff between reducing the donor's expected losses and maximizing private capital leverage.

Proposition 11 Private Capital Leverage Tradeoff

The higher the donor's risk-tolerance coefficient, the higher its aggregate nominal participation and the lower the private capital leverage.

Proof. See above.

The proposition shows that, within the present model, maximizing leverage is not generally aligned with minimizing donor expected losses. Greater donor risk tolerance leads the donor to take a larger nominal share of the financing structure, which lowers expected present cost but mechanically leaves less room for private capital participation.

5. Conclusion

5.1. Summary and Stylized Representation

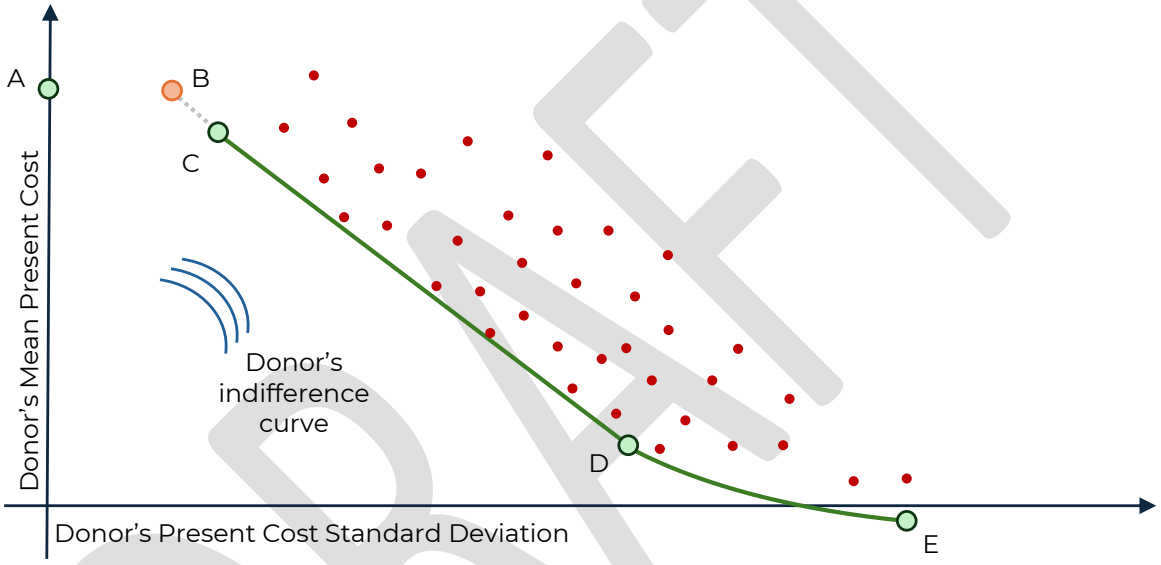
This paper models blended finance as a donor-side portfolio optimization problem. It rests on several assumptions. Those include, first, the returns from private sector investments that require concessional finance are stochastic and partly correlated. Second, private capital investors are assumed to have fixed exogenous hurdle rates for financing these investments. Third, donor decision-making is treated as sequential: the aid envelope, broad policy objectives, and project selection are determined upstream, while the structuring of the donor's concessional financing is determined at a later stage. The portfolio of assets the donor wishes to support is therefore treated as given. Fourth, at the structuring stage, donor preferences are represented over the expected present cost of support and the variance of that cost, using a fixed opportunity cost – the donor's cost of funding – as discount rate. Fifth, the model assumes no transaction costs and no information asymmetry.

For each asset in the portfolio, the donor can choose the type and nominal amount of concessional financing. It may extend a grant up to the subsidy equivalent, a concessional investment up to the full financing amount of the asset (the donor's rate of return is a function of its participation), or a combination of grants and concessional investments.

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Based on the model, the investor is indifferent between receiving the subsidy equivalent upfront as a grant and receiving the same support through higher returns of equal present value. This means that the required subsidy is fixed by the investor's participation constraint and the investor is indifferent towards the donor's choice.

For a given portfolio of assets and a given donor cost of funding, the figure below plots the ordered pairs of the donor's expected present cost and the standard deviation of that cost, using the calculations developed in the paper. The vertical axis shows the donor's expected present cost. The horizontal axis shows the standard deviation of present cost. Each red dot represents a possible intervention available to the donor.



The donor prefers interventions that use fewer resources and produce more predictable outcomes. Any intervention located below and to the left of another will always be preferred, regardless of the donor's risk tolerance. Interventions that combine lower expected cost with lower uncertainty are therefore strictly preferred.[17]

Point A represents the case in which the donor provides its concessional support entirely as a grant. The donor's present cost is then equal to the aggregate subsidy equivalent required for the portfolio (and since the loss is certain, the standard deviation is equal to zero).[18] Point B represents the lowest cost boundary among concessional investments, where the donor's nominal participation is equal to the aggregate subsidy equivalent but is provided as concessional investment rather than as a grant. This shows that the donor will always prefer a grant equal to the subsidy equivalent over a concessional investment of the same amount.[19] Point E represents the opposite boundary case, in which the donor fully finances the entire portfolio (assuming the donor is still within its maximum budget).

[17] This first order stochastic dominance also applies for any coordinate that is either at the same level vertically but below another coordinate, or at the same level horizontally but to the left of another coordinate (that is, for an equal mean, the intervention with a lower variance will be preferred, and vice-versa).
[18] See Lemma 5 in §2.7.
[19] See Proposition 4 in §4.3.

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Points A and C–D–E, shown by the green line, form the donor's efficiency frontier. Whatever the donor's preferences, its preferred intervention must lie on this frontier. Points away from the frontier are inefficient because another feasible intervention offers either a lower expected cost for the same level of risk, a lower level of risk for the same expected cost, or an improvement in both.

The donor's risk tolerance is represented on the chart by indifference curves. At any two coordinates on the same curve, the donor is indifferent between the corresponding interventions. The optimal intervention is the point at which the lowest attainable indifference curve is tangent to the efficiency frontier.[20]

Assume that, for all assets in the portfolio, the investor's required rate of return exceeds the donor's cost of funding.[21] A donor with no tolerance for risk will choose point A, that is, a grant equal to the aggregate subsidy equivalent. As risk tolerance increases, the donor becomes willing to accept greater volatility in exchange for a lower expected cost. Its optimal nominal participation then increases and moves along the efficiency frontier from the upper left to the lower right. As the donor's nominal participation rises, the share of financing left to private investors falls.

Donors with risk tolerance close to zero will still choose a grant equal to the aggregate subsidy equivalent rather than a point between B and C. There is also a particular case in which the donor is indifferent between providing a grant equal to the aggregate subsidy equivalent and choosing the concessional investment represented by point C.

Between points C and D, the donor's nominal participation increases as risk tolerance rises, but the composition of that participation across assets remains unchanged. That is, the share of each asset in the donor's nominal participation is fixed, only the size of that participation varies. Point D marks the point at which the donor becomes willing to fully finance at least one asset in the portfolio. Beyond that point, further increases in risk tolerance no longer preserve the same diversification pattern, because the donor can no longer maintain the optimal combination of asset weights. At point E, the donor fully finances the portfolio.[22] This is the upper boundary of participation, so further increases in risk tolerance have no additional effect.

5.2. Policy Recommendations

A useful way to interpret the model is through the idea of an efficient frontier for the donor. For a given portfolio, some allocations of concessional support are clearly inferior to others. They deliver either a higher expected cost for the same level of risk, or a higher level of risk for the same expected cost. The efficient frontier is made up of the allocations that cannot be improved on both dimensions at once. In practical terms, donors should seek structures that lie on this frontier rather than inside it. This means choosing the combination of instruments and the size of nominal participations that best matches their tolerance for risk and their opportunity cost proxy – whether it is cost of funding or other benchmarks.

[20] See Proposition 2 in §4.1.

[21] For all assets where the donor's cost of funding is higher than the investor's mean rate of return, the donor will choose to extend a grant equal to the subsidy equivalent for that asset.

[22] Note that there might be sets of participation that lead to a mean profit for the donor, as shown in the chart. Also, the donor may be subject to a nominal envelop cap – as modeled in the paper – which may prevent higher participations.

The efficient frontier also gives a clear role to diversification. A donor portfolio should not be treated as a collection of isolated transactions. Its risk depends not only on the volatility of each asset, but also on how those assets move together. Better diversification improves the donor's efficient set by reducing risk for a given expected cost or reducing expected cost for a given level of risk.

The more diversified the portfolio, the better this trade-off is likely to be. In practice, this suggests that donor portfolios and pooled concessional vehicles should be designed with explicit diversification rules. These may include industry limits, country exposure limits, regional balance, and objective-based allocation rules built around observed or estimated correlations. Where correlation data are available, they should inform portfolio construction more directly than is often the case in current practice. Notably, the benefits of diversification may extend beyond financial considerations to development impact, as the latter may be predicated on the financial success of the project.

This does not mean that diversification should override policy purpose. Donors may still choose concentrated portfolios because of development mandates, strategic priorities, or impact objectives. But the cost of doing so should be understood clearly. A concentrated portfolio may move the donor away from a better feasible allocation and weaken the trade-off between expected cost and risk. For that reason, concentration should be the result of policy choice, not a by-product of weak portfolio design.

Under the model, the central policy trade-off for the donor is primarily between budget certainty and reducing expected donor losses. A secondary trade-off is that allocations that reduce expected losses also reduce private capital mobilization. These trade-offs depend on donor preferences. Greater donor risk tolerance leads the donor to take a larger nominal share of the financing structure, which lowers expected present cost but mechanically leaves less room for private capital participation. Whether the donor should internalize this trade-off depends on its motivations. If leverage is valued only insofar as it allows scarce concessional resources to reach more projects, then the donor may rationally accept lower leverage when the reduction in expected donor cost is sufficiently valuable. If, however, private mobilization carries independent policy value, then it should enter the donor's objective function explicitly.

The donor's own cost of funds also matters (or alternative opportunity cost proxies). Where the donor's funding cost exceeds the investor's required rate of return for a given asset, the preferred instrument is a grant equal to the subsidy equivalent for that asset. This may be especially relevant for donors that face high funding costs or shadow costs of public funds. Perhaps counterintuitively, grants may also be more appropriate in lower-risk settings, where investor return requirements are lower.

At the portfolio level, the most important factors are the variance in rates of returns and the extent to which assets move together. This does not imply that mean rates of return are irrelevant in general. They may matter at the earlier stage of portfolio selection. But once the donor has decided which assets to support, the model implies that the optimal financing structure is determined by the portfolio's variances and correlations, together with the donor's risk tolerance.

The model also points to a degree of instrument flexibility. For cases in which the optimal nominal participation exceeds the subsidy equivalent, that participation can be split between a grant component and an investment component without

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changing the donor's optimum. In practice, this suggests that some blended finance transactions could be designed in a simpler way, with one component delivered as a grant and the other as a share in commercial returns.

There may also be a case for pooling donors. If several donors share the same objectives, support the same portfolio, and apply similar funding benchmarks, they may be able to invest through a common vehicle even if their tolerance for risk differs. Where such pooling also reduces transaction and administrative costs, it may improve overall efficiency.

Taken together, these last two results point to a possible model for pooled concessional finance. Donor resources could be combined in a common vehicle designed to accept both grant and returnable contributions. That vehicle could then deploy support through a combination of grants and a share in commercial returns. Under the assumptions of the model, such a structure could accommodate donors with similar objectives, a common target portfolio, and comparable opportunity cost benchmarks, even where their tolerance for risk differs.

Overall, the paper suggests that the financial structuring of blended finance deserves more explicit optimization on the donor side. Even where the level of subsidy is fixed by investor requirements and the portfolio is fixed by policy choice, donors may still improve financial efficiency through better choices on instrument mix, nominal participation, and diversification. In a world of tighter aid budgets, that imperative is likely to become more important rather than less.

5.3. Limitations

These results should be interpreted with care as the model abstracts from several features of "real-world" blended finance.

First, the normality assumptions should be read as an approximation. If donor preferences remain mean-variance, normality matters mainly for calibration. A more realistic default-state distribution may the estimated means, variances, and covariances of donor cost, and therefore the quantitative location of donor optima. However, the underlying optimization logic is unchanged. A separate question is whether mean-variance is an adequate description of donor preferences. If donors care about higher-order features of donor cost, such as skewness, fat tails, or tail dependence, then mean and variance may not be sufficient to rank risky allocations. The model may then mis-rank structures that have similar expected cost and variance but differ in their exposure to rare severe losses or clustered defaults.

Second, the assumption that pairwise correlations are strictly positive is useful because, in the cases that support the grant results, it helps ensure that the relevant partial derivatives of the donor's disutility with respect to nominal participations are strictly positive over the feasible set. The optimum is then attained at the lower bound, a grant equal to the subsidy equivalent. If some correlations were zero or negative, diversification could be stronger than in the present model, and those results would no longer hold as consistently.

Third, the model is best interpreted as conditional on given market pricing. If investor hurdle rates reflect regulation, concentration limits, or systematically pessimistic risk perceptions, the estimated subsidy need not map cleanly into underlying fundamentals. In that case, the model still characterizes donor optimization, but around market prices rather than around a deeper measure of economic efficiency.

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Fourth, the results are static. They identify an optimal structuring choice for one period, not an intertemporal strategy. They therefore say little about learning, refinancing, recycling of reflows, or how current concessionality affects future investor behavior.

Fifth, the results abstract from implementation frictions. In practice, transaction costs, screening, monitoring, and legal structuring may alter the donor's optimum. Information asymmetries may do the same, especially where public and private actors price risk differently.

Sixth, the model considers only funded instruments and therefore excludes guarantees and other contingent instruments. This matters because guarantees may be preferred, especially where public and private actors perceive risk differently.

Finally, the paper is deliberately non-normative about donor policy objectives. It isolates the financial optimization problem conditional on an upstream portfolio choice. The results should therefore not be read as prescribing how donors ought to trade off development impact, market creation, and mobilization. Rather, they show what follows once that prior choice has been made.

Overall, the model is best read as a benchmark for donor-side financial optimization. Its value lies in clarifying the structure of the problem under a transparent set of assumptions, not in reproducing the full complexity of observed blended-finance practice. The analysis also suggests scope to deepen the framework in ways that would bring it closer to observed practice, including non-normal return distributions, dynamic settings, implementation frictions, guarantees, and a fuller treatment of the interaction between financial structuring, portfolio selection, and donor policy objectives.

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Annex 2. Overview of Model's Variables

Name	Description	Type	Dimension	Mathematical representation
i	An asset in need of subsidy to be financed. The value of each i is assumed to be equal to 1	Exogenous	Index	$i = \{1,2 \dots, n\}$
n	Amount of assets i	Exogenous	Quantity	$n \in \mathbb{Z}_*^+$
p	The portfolio of n assets i	Exogenous	Identifier	n/a
t	t is a fixed time interval between t_0 and t_1, two discrete points in time	Exogenous	Period	n/a
ra_i	The rate of return of asset i over the period t	Exogenous	Rate	$ra_i \in \mathbb{R}, \forall i: ra_i < rm_i$
rm_i	The rate of return received by the investor under the donor's contract	Endogenous	Rate	See Definition 2
rd	The donor's cost of funding over the period t	Exogenous	Rate	$rd \in \mathbb{R}_*^+$
rc_i	The subsidized rate of return for the donor for asset i over the period t	Endogenous	Rate	See Lemma 2
s_i	The required subsidy equivalent for asset i over the period t	Endogenous	Discounted amount	See Definition 3
g_i	The present cost of the donor's concessional financing of asset i over the period t	Endogenous	Discounted amount	See Definition 5
g_p	The present cost of the donor's concessional financing of the portfolio p of assets i over the period t	Endogenous	Discounted amount	$g_p = \sum_{i=1}^{n} g_i$

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b	The donor's aggregate participation in the in the portfolio p	Exogenous	Amount	$b \in \mathbb{R}$, $\sum_{i=1}^{n} s_i \leq b_{max} \leq n$
b_{max}	The maximum donor's nominal participation in p at t_0	Exogenous	Amount	$b_{max} \in \mathbb{R}$, $\sum_{i=1}^{n} s_i \leq b_{max} \leq n$
v_i	The donor's nominal participation in asset i at t_0	Exogenous	Amount	$v_i \in \mathbb{R}, \forall i\colon s_i \leq v_i \leq 1$ $\sum_{i=1}^{n} v_i = b \leq b_{max}$
x_i	The amount of grant in the donor's concessional financing of asset i at t_0 when partly extending it as grant and as investment	Exogenous	Amount	$x_i \in \mathbb{R}_*^+, \forall i\colon 0 \leq x_i \leq s_i$
y_i	The nominal amount of investment from the donor in asset at t_0 when partly extending it as grant and as investment	Exogenous	Amount	$y_i \in \mathbb{R}_*^+, \forall i\colon 0 \leq y_i \leq v_i$
w_i	The weight of the donor's participation in asset i relative to its participation in the portfolio p	Exogenous	Weight	$w_i \in \mathbb{R}, \forall i\colon 0 \leq w_i \leq 1$ $\sum_{i=1}^{n} w_i = 1$
μ_{ra_i}	The mean rate of return of ra_i over the period t	Exogenous	Rate	$\mu_{ra_i} \in \mathbb{R}$
μ_{rm_i}	The investor's required mean return	Exogenous	Rate	Defined in §2.2
μ_{rc_i}	The mean rate of return of rc_i over the period t	Endogenous	Rate	$\mu_{rc_i} \in \mathbb{R}$
μ_{g_i}	The donor's mean present cost at t_0	Endogenous	Discounted amount	$\mu_{g_i} \in \mathbb{R}$

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$\sigma^2_{ra_i}$ $/\sigma_{ra_i}$	Variance and standard deviation of ra_i over the period t	Exogenous	Rate	$\sigma^2_{ra_i}, \sigma_{ra_i} \in \mathbb{R}$
$\sigma^2_{rm_i}$ $/\sigma_{rm_i}$	Variance and standard deviation of rm_i over the period t	Exogenous	Rate	$\sigma^2_{rm_i}, \sigma_{rm_i} \in \mathbb{R}$
$\sigma^2_{rc_i}$ $/\sigma_{rc_i}$	Variance and standard deviation of rc_i over the period t	Endogenous	Rate	$\sigma^2_{rc_i}, \sigma_{rc_i} \in \mathbb{R}$
$\sigma^2_{g_i}/\sigma_{g_i}$	Variance and standard deviation of g_i over the period t	Endogenous	Discounted amount	$\sigma^2_{g_i}, \sigma_{g_i} \in \mathbb{R}$
$\rho_{ii'}$	Correlation between ra_i and $ra_{i'}$ over the period t	Exogenous	Percentage	$\rho_{ii'} \in \mathbb{R}, \forall a, b: -1 \leq \rho_{ii'} \leq 1$ $\rho_{ii'} = \frac{Cov(ra_i, ra_{i'})}{\sigma_{ra_i}\sigma_{ra_{i'}}}$
$\mathcal{D}(V)$	Donor's disutility function	Exogenous	Function	Also expressed as $\mathcal{D}(v_1, \dots, v_n)$
$\Delta\mathcal{D}(V)$	Gradient of $\mathcal{D}(V)$	Exogenous	Function	n/a
l	Private capital leverage	Exogenous	Percentage	See Definition 14
A	The vector of the mean rates of return for each asset i over the period t	Exogenous	Vector	$A = (\mu_{ra_1}, \dots, \mu_{ra_n})$
M	The vector of the mean rate of return threshold required by the investor for each asset i over the period t	Exogenous	Vector	$M = (\mu_{rm_1}, \dots, \mu_{rm_n})$
D	the n dimension vector of the donor's cost of funding rd over the period t	Exogenous	Vector	$D = (rd, \dots, rd)$
V	The vector of nominal participations of the donor in each asset i at t_0	Exogenous	Vector	$V = (v_1, \dots, v_n)$

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X	The vector of grant participation x_i in each asset i at t_0	Exogenous	Vector	$X = (x_1, \dots, x_n)$
Y	The vector of the donor's nominal participation in commercial investments for each asset i at t_0	Endogenous	Vector	$Y = (y_1, \dots, y_n)$
S	The vector of subsidy equivalents for each asset i over the period t	Endogenous	Vector	$S = (s_1, \dots, s_n)$
$\sum$	The covariance matrix of the rates of return ra_i over the period t	Exogenous	Matrix	See Definition 9

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Annex 3. Mathematical Demonstrations

Annex 3.1 Investor's Mean Rate of Return when Receiving s_i as Grant

Recall that:

$$s_i = \frac{\mu_{rm_i} - \mu_{ra_i}}{1 + \mu_{rm_i}}$$

Therefore:

$$\frac{(1 + \mu_{ra_i}) - (1 - s_i)}{(1 - s_i)}$$

$$= \frac{\mu_{ra_i} + \frac{\mu_{rm_i} - \mu_{ra_i}}{1 + \mu_{rm_i}}}{\left(1 - \frac{\mu_{rm_i} - \mu_{ra_i}}{1 + \mu_{rm_i}}\right)}$$

$$= \frac{\frac{\mu_{ra_i} + \mu_{ra_i}\mu_{rm_i} + \mu_{rm_i} - \mu_{ra_i}}{1 + \mu_{rm_i}}}{\frac{1 + \mu_{rm_i} - \mu_{rm_i} + \mu_{ra_i}}{1 + \mu_{rm_i}}}$$

$$= \frac{\mu_{ra_i}\mu_{rm_i} + \mu_{rm_i}}{1 + \mu_{ra_i}}$$

$$= \mu_{rm_i} \blacksquare$$

Note that rate of return for the investor when receiving a grant s_i at t_0 may be different than when receiving the subsidized return $rm_i = \mu_{rm_i} - \mu_{ra_i}$ at t_1. Only the mean rate of return is the same. Note however that the investor's net present values are always the same, as shown below.

Let NPV_{rm_i} and NPV_{s_i} be the investor's net present value when respectively receiving the subsidized return $rm_i = \mu_{rm_i} - \mu_{ra_i}$ at t_1 and receiving a grant s_i at t_0

$$NPV_{rm_i} = -1 + \frac{1 + ra_i + \mu_{rm_i} - \mu_{ra_i}}{1 + \mu_{rm_i}}$$

$$NPV_{s_i} = -1 + s_i + \frac{1 + ra_i}{1 + \mu_{rm_i}} = -1 + \frac{\mu_{rm_i} - \mu_{ra_i}}{1 + \mu_{rm_i}} + \frac{1 + ra_i}{1 + \mu_{rm_i}} = NPV_{rm_i} \blacksquare$$

Annex 3.2 Rate of Return of the Donor Concessional Investment

Recall from Definition 2:

$$rm_i = ra_i + \mu_{rm_i} - \mu_{ra_i}$$

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Therefore:

$$rc_i = \frac{1 + ra_i - (1 - v_i)(1 + rm_i) - v_i}{v_i}$$

$$= \frac{1 + ra_i - (1 - v_i)\left(1 + ra_i + \mu_{rm_i} - \mu_{ra_i}\right) - v_i}{v_i}$$

$$= \frac{1 + ra_i - 1 - ra_i - \mu_{rm_i} + \mu_{ra_i} + v_i + v_i ra_i + v_i \mu_{rm_i} - v_i \mu_{ra_i} - v_i}{v_i}$$

$$= ra_i + \mu_{rm_i} - \mu_{ra_i} + \frac{\mu_{ra_i} - \mu_{rm_i}}{v_i} \blacksquare$$

Annex 3.3 Variance of Investor's and Donor's Rate of Return

Recall that:

$$rm_i = ra_i + \mu_{rm_i} - \mu_{ra_i}$$

Therefore:

$$\sigma^2_{rm_i} = \mathbb{E}\left[\left(rm_i - \mu_{rm_i}\right)^2\right] = \mathbb{E}\left[\left(ra_i - \mu_{ra_i}\right)^2\right] = \sigma^2_{ra_i} \blacksquare$$

Recall that:

$$rc_i = ra_i + \mu_{rm_i} - \mu_{ra_i} + \frac{\mu_{ra_i} - \mu_{rm_i}}{v_i}$$

And:

$$\mu_{rc_i} = \mu_{rm_i} + \frac{\mu_{ra_i} - \mu_{rm_i}}{v_i}$$

Thus:

$$rc_i - \mu_{rc_i}$$

$$= ra_i + \mu_{rm_i} - \mu_{ra_i} + \frac{\mu_{ra_i} - \mu_{rm_i}}{v_i} - \mu_{rm_i} - \frac{\mu_{ra_i} - \mu_{rm_i}}{v_i}$$

$$= ra_i - \mu_{ra_i}$$

And:

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$$\sigma_{rc_i}^2 = \mathbb{E}\left[\left(rc_i - \mu_{rc_i}\right)^2\right] = \mathbb{E}\left[\left(ra_i - \mu_{ra_i}\right)^2\right] = \sigma_{ra_i}^2 \blacksquare$$

Annex 3.4 Mean, Variance and Standard Deviation of Donor's Present Cost

Through the linearity of expectation:

$$\mu_{g_i} = \mathbb{E}[g_i] = \mathbb{E}\left[\frac{v_i\left(rd - ra_i - \mu_{rm_i} + \mu_{ra_i}\right) + \mu_{rm_i} - \mu_{ra_i}}{1 + rd}\right] = \frac{v_i\left(rd - \mu_{rm_i}\right) + \mu_{rm_i} - \mu_{ra_i}}{1 + rd} \blacksquare$$

Recall that:

$$\sigma_{rc_i}^2 = \mathbb{E}\left[\left(rc_i - \mu_{rc_i}\right)^2\right] = \mathbb{E}\left[\left(ra_i - \mu_{ra_i}\right)^2\right] = \sigma_{ra_i}^2$$

Since:

$$g_i - \mu_{g_i} = \frac{v_i(rd - rc_i)}{1 + rd} - \frac{v_i\left(rd - \mu_{rc_i}\right)}{1 + rd} = \frac{v_i\left(\mu_{rc_i} - rc_i\right)}{1 + rd}$$

Then:

$$\sigma_{g_i}^2 = \mathbb{E}\left[\left(g_i - \mu_{g_i}\right)^2\right] = \mathbb{E}\left[\left(\frac{v_i\left(\mu_{rc_i} - rc_i\right)}{1 + rd}\right)^2\right] = \left(\frac{v_i}{1 + rd}\right)^2 \mathbb{E}\left[\left(rc_i - \mu_{rc_i}\right)^2\right]$$
$$= \left(\frac{v_i}{1 + rd}\right)^2 \sigma_{ra_i}^2 \blacksquare$$

Annex 3.5 Mean Donor Present Cost and Asset's Subsidy Equivalent

Suppose $v_i = s_i$ and recall that:

$$s_i = \frac{\mu_{rm_i} - \mu_{ra_i}}{1 + \mu_{rm_i}}$$

And:

$$\mu_{g_i} = \frac{v_i\left(rd - \mu_{rm_i}\right) + \mu_{rm_i} - \mu_{ra_i}}{1 + rd}$$

Then:

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$$\mu_{g_i} = \frac{\frac{\mu_{rm_i} - \mu_{ra_i}}{1 + \mu_{rm_i}}(rd - \mu_{rm_i}) + \mu_{rm_i} - \mu_{ra_i}}{1 + rd}$$

$$= \frac{(\mu_{rm_i} - \mu_{ra_i})(rd - \mu_{rm_i}) + (\mu_{rm_i} - \mu_{ra_i})(1 + \mu_{rm_i})}{(1 + rd)(1 + \mu_{rm_i})}$$

$$= \frac{\mu_{rm_i} rd - \mu_{rm_i}^2 - \mu_{ra_i} rd + \mu_{ra_i}\mu_{rm_i} + \mu_{rm_i} + \mu_{rm_i}^2 - \mu_{ra_i} - \mu_{ra_i}\mu_{rm_i}}{(1 + rd)(1 + \mu_{rm_i})}$$

$$= \frac{\mu_{rm_i} rd - \mu_{ra_i} rd + \mu_{rm_i} - \mu_{ra_i}}{(1 + rd)(1 + \mu_{rm_i})}$$

$$= \frac{(1 + rd)(\mu_{rm_i} - \mu_{ra_i})}{(1 + rd)(1 + \mu_{rm_i})}$$

$$= s_i \blacksquare$$

Annex 3.6 Present Cost Variance of the Donor's Portfolio

Since:

$$Cov(a, b) = \mathbb{E}[(a - \mathbb{E}[a])(b - \mathbb{E}[b])]$$

Then:

$$Cov(g_i, g_{i'}) = \mathbb{E}[(g_i - \mathbb{E}[g_i])(g_{i'} - \mathbb{E}[g_{i'}])]$$

Recall that:

$$g_i - \mathbb{E}[g_i] = \frac{v_i(\mu_{rc_i} - rc_i)}{1 + rd}$$

Then:

$$Cov(g_i, g_{i'}) = \mathbb{E}\left[\frac{v_i(\mu_{rc_i} - rc_i)}{1 + rd} * \frac{v_{i'}\left(\mu_{rc_{i'}} - rc_{i'}\right)}{1 + rd}\right]$$

$$= \frac{v_i v_{i'}}{(1 + rd)^2}\mathbb{E}\left[(-1)(rc_i - \mu_{rc_i})(-1)\left(rc_{i'} - \mu_{rc_{i'}}\right)\right]$$

Recall that:

$$rc_i - \mu_{rc_i} = ra_i - \mu_{ra_i}$$

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Therefore:

$$Cov(g_i, g_{i'}) = \frac{v_i v_{i'}}{(1+rd)^2}\mathbb{E}\left[\left(ra_i - \mu_{ra_i}\right)\left(ra_{i'} - \mu_{rs_{i'}}\right)\right] = \frac{v_i v_{i'}}{(1+rd)^2}Cov(ra_i, ra_{i'})\blacksquare$$

Annex 3.7 Gradient of Donor's Disutility Function

Recall that:

$$\mathcal{D}(V) = \gamma\frac{1_n^T(M-A)}{1+rd} + \gamma\frac{V^T(D-M)}{1+rd} + \frac{1}{2}\left(\frac{1}{1+rd}\right)^2 V^T\sum V$$

Where the first term is constant, the second term linear and the third term is quadratic. Thus:

$$\nabla\mathcal{D}(V) = \gamma\frac{(D-M)}{1+rd} + \frac{1}{2}\left(\frac{1}{1+rd}\right)^2(\sum + \sum^T)V$$

$\sum$ is symmetric, so $\sum = \sum^T$. This simplifies to:

$$\nabla\mathcal{D}(V) = \gamma\frac{(D-M)}{1+rd} + \frac{1}{2}\left(\frac{1}{1+rd}\right)^2 2\sum V = \gamma\frac{(D-M)}{1+rd} + \left(\frac{1}{1+rd}\right)^2\sum V\blacksquare$$

Annex 3.8 Rate of Return of the Donor Concessional Investment when Partly Extending a Grant

Recall from Definition 2:

$$rm_i = ra_i + \mu_{rm_i} - \mu_{ra_i}$$

Therefore:

$$rc_{y_i} = \frac{1 + ra_i - (1-v_i)(1+rm_i) - y_i}{y_i}$$

$$= \frac{1 + ra_i - (1-v_i)(1+ra_i+\mu_{rm_i}-\mu_{ra_i}) - y_i}{y_i}$$

$$= \frac{1 + ra_i - 1 - ra_i - \mu_{rm_i} + \mu_{ra_i} + v_i + v_i ra_i + v_i\mu_{rm_i} - v_i\mu_{ra_i} - y_i}{y_i}$$

$$= \frac{v_i(1 + ra_i + \mu_{rm_i} - \mu_{ra_i}) - \mu_{rm_i} + \mu_{ra_i}}{y_i} - 1\blacksquare$$

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Annex 3.9 Donor Present Cost with Grant Portion

The same discounting exercise than for the donor's present cost is performed, but this time the cash flows at t_1 are limited to the investment portion of the donor's concessional financing, returning the rate of rc_{y_i} (since the grant portion x_i doesn't return cash flows). Recall that:

$$rc_{y_i} = \frac{v_i(1 + ra_i + \mu_{rm_i} - \mu_{ra_i}) - \mu_{rm_i} + \mu_{ra_i}}{y_i} - 1$$
$$= \frac{v_i(1 + ra_i + \mu_{rm_i} - \mu_{ra_i}) - \mu_{rm_i} + \mu_{ra_i} - y_i}{y_i}$$

Then:

$$g_{x_i} = v_i - y_i \frac{1 + rc_{y_i}}{1 + rd}$$
$$\Rightarrow g_{x_i} = v_i - y_i \frac{1 + \frac{v_i(1 + ra_i + \mu_{rm_i} - \mu_{ra_i}) - \mu_{rm_i} + \mu_{ra_i} - y_i}{y_i}}{1 + rd}$$
$$\Rightarrow g_{x_i} = v_i - \frac{v_i(1 + ra_i + \mu_{rm_i} - \mu_{ra_i}) - \mu_{rm_i} + \mu_{ra_i}}{1 + rd}$$
$$\Rightarrow g_{x_i} = \frac{v_i + v_i rd - v_i - v_i ra_i - v_i \mu_{rm_i} + v_i \mu_{ra_i} + \mu_{rm_i} - \mu_{ra_i}}{1 + rd}$$
$$\Rightarrow g_{x_i} = \frac{v_i(rd - ra_i - \mu_{rm_i} + \mu_{ra_i}) + \mu_{rm_i} - \mu_{ra_i}}{1 + rd}$$
$$\Rightarrow g_{x_i} = g_i \blacksquare$$

Annex 3.10 Donor Present Cost Mean, Variance and Standard Deviation with Grant Portion

For the donor's mean present cost, based on the previous demonstration:

$$\mu_{g_{x_i}} = \mathbb{E}[g_{x_i}] = \mathbb{E}[g_i] = \mu_{g_i} = \frac{v_i(rd - \mu_{rm_i}) + \mu_{rm_i} - \mu_{ra_i}}{1 + rd} \blacksquare$$

For the donor's present cost variance, based on the previous two demonstrations:

$$\sigma^2_{g_{x_i}} = \mathbb{E}\left[\left(g_{x_i} - \mu_{g_{x_i}}\right)^2\right] = \mathbb{E}\left[(g_i - \mu_{g_i})^2\right] = \sigma^2_{g_i} = \left(\frac{v_i}{1 + rd}\right)^2 \sigma^2_{ra_i} \blacksquare$$

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