Matthieu Pegon
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Development finance and climate finance executive experienced in:

- Leading organizational change and institutional development, including restructurings and establishment of new functions, and design and implementation of operational and governance frameworks to support scale-up and transformation
- Overseeing the deployment of concessional finance through a wide array of instruments (debt, bonds, equity, guarantees, etc.), over the full project lifecycle, from fundraising to execution, monitoring and evaluation
- Managing multicultural teams in public sector and private sector organizations
- Strategic and thought leadership in concessional finance, financial innovation in climate, and private capital mobilization
- First-hand debt capital markets experience and track record in mobilizing institutional investors towards emerging markets

EXPERIENCE

2025-Present **Deputy Chief Executive Officer – Climate Investment Funds (CIF), Washington DC**

CIF is a $13 billion multilateral climate fund hosted by the World Bank that supports 82 developing countries to adapt to and mitigate climate change, working with 6 multilateral development banks as implementing entities.

Strategic leadership, stakeholders' management and representation

- Managing the programming, finance, risk, portfolio, results, and administrative functions of the CIF
- Leading policies making and engagement with Board and relevant stakeholders for overseen functions
- Supporting the CEO in matters relating to general strategy, external relations and stakeholders' engagement
- Representing the CIF in high level events and meetings

Programming and deployment of CIF resources

- Overseeing the implementation of CIF programs (~$1.8 billion CIF funding approved in 49 projects in 2025)
- Leading the development of new financial products and modalities, and initiatives to enhance disbursement performance and operational efficiency (e.g. development and approval of a new $500 million high leverage modality in 2025)
- Integrating private capital mobilization into programming, risk, and results frameworks.

Risk, financial and administrative management

- Providing strategic oversight and supervision over CIF's risk, financial management, and debt capital markets functions (AA+ bond program), ensuring alignment with CIF's risk appetite, financial sustainability objectives, and Boards directions
- Leading administrative budget preparation, approval, and execution (~$48 million p.a.)
- Overseeing work program management, workforce planning, and people systems (60 full time equivalent employees and ~40 consultants)

Results and portfolio management

- Overseeing a portfolio of 463 projects (~$9 billion CIF funding, ~$56 billion public co-financing, ~$40 billion private capital mobilized, ~43 $MtCO_2e$ GHG emission abated annually, 65+ million climate adaption beneficiaries, ~38 million hectares under sustainable management)
- Leading portfolio management including endorsement and escalation of material amendment, and use of portfolio analytics to inform strategic allocation, capital planning, and risk decisions
- Guiding the design and continuous improvement of integrated results frameworks, supervision of results tracking and monitoring, and use of results evidence for learning, reporting and evaluation

2017-2025 **Director of Blended Finance – IDB Invest, Washington DC**

IDB Invest is the private-sector arm of the Inter-American Development Bank (IDB) Group. The Blended Finance Division oversees the deployment of IDB Invest's concessional funds.

Setting-up IDB Invest Blended Finance Division

- Built an in-house asset management infrastructure, created operational policies and procedures, integrated the management of third-party funds in IDB Invest systems
- Developed strategy for concessional finance, led internal strategic dialogue with Board
- Recruited, managed, and mentored a team of 10 blended finance investment officers
- Oversaw administrative budget from donors' fees

Execution of low carbon and climate resilient blended finance investments

- Managed a pool of ~$1.1 billion of concessional resources from 18 programs/funds
- Oversaw the structuring, approval, closing, disbursement, monitoring and evaluation of blended finance investments through a wide array of financing instruments (101 projects closed for an aggregate ~$22.2 billion in investments, ~133 $MtCO_2e$ expected total GHG emission abated)
- Developed innovative climate finance products (e.g., carbon credits put options, climate outcomes' carry, technology risk principal deferral, gender outcome-based interest rate reduction in climate projects, etc.)

Fundraising, partnerships and thought leadership

- Maintained a strategic dialogue with bilateral and multilateral donors, fundraised concessional resources (7 new funds raised for an aggregate ~$520 million), negotiated donors' arrangements and results frameworks
- Represented the IDB Group in events on blended finance (e.g., OECD, CGD, Brookings, Convergence, COPs)
- Co-developed the DFIs Enhanced Principles for Blended Concessional Finance
- Wrote, and contributed to publications on concessional finance, lectured at the University of Zurich

2015-2016 **Senior Structured Finance Specialist – Green Climate Fund (GCF), Incheon (Republic of Korea)**

GCF is the world's largest multilateral climate finance fund.

Strategic planning and business development for Latin America and the Caribbean

- Strategized deployment of concessional resources based on identified priorities in term of climate impact areas
- Partnered with recipient countries' governments and GCF's accredited entities to develop a pipeline of projects/programs (pipeline >$1.7 billion, GCF share ~$600 million)

Investments execution

- Assessed risks and impacts, interfaced with the relevant internal teams to assess consistency of proposals with GCF's safeguards and policies, drafted internal reviews
- Negotiated terms and conditions, presented proposals to GCF's Management, Independent Technical Advisory Panel and Board, approved 3 investments (~$1.5 billion, GCF share ~$350 million)

2007-2015 **BNP Paribas Corporate & Investment Banking**

BNP Paribas is an international banking group.

2012-2015, Vice-President, Fixed Income – Structured Debt Capital Markets, New York

Originated, structured and executed project bonds, securitizations and other debt capital market solutions in emerging markets. Sectors covered included Energy & Infrastructure, Shipping & Offshore industries

- Wrote term sheets, built financial models, led credit rating processes, conducted market soundings, negotiated and reviewed legal documentations and offering materials, prepared investors' presentations, organized and participated to roadshows, supported investors' due diligence
- Led ~40 proposals to prospective clients, won 4 mandates (~$4.5 billion)
- Developed new asset classes for institutional investors (e.g., renewable energy project bonds, royalties' securitizations) and led 10 award-winning offerings (>$8 billion issuance in aggregate)
- Maintained an ongoing relationship with key institutional investors (e.g. PacLife, AIG, Dodge & Cox)
- Managed, trained and mentored a pool of 4-5 Analysts and Associates

2007-2012, Credit Analyst and Relationship Manager, Shipping & Offshore Finance, Rome and New York

- Drafted and negotiated terms and conditions for new financings, built financial models, analysed and rated risks negotiated transactions' legal documentation, coordinated closings and disbursements
- Managed a portfolio of ~10 ship owning companies
- Executed ~10 transactions in Africa, Europe and the Americas (~$1.5 billion, BNP Paribas share ~$500 million)
- Led the workout of 2 syndicated loans (>$2 billion, BNP Paribas share ~$400 million, lenders pools >10 banks)
- Drafted credit proposals and annual reviews (~40 credit proposals, aggregate portfolio of ~$750 million)

EDUCATION

2023-Present **ESCP Business School**, Paris

- Executive Ph.D. in Financial Economics (research topic: blended finance optimization)

2003-2007 **Grenoble Ecole de Management**, Grenoble (France)

- Master's Degree in Management, Majors in Finance and Energy Markets
- One-year exchange MBA Program at NUCB Graduate School, Nagoya (Japan)

2001-2003 **Lycée Jacques Decour**, Classe Préparatoire Economique et Commerciale option scientifique, Paris

- Two-year degree course emphasised on Mathematics preparing for the national competitive entry examinations to French Graduate Business Schools

1988-2001 **European School of Varese**, Varese (Italy)

- European Baccalaureate, specialisation in Mathematics

SKILLS AND INTERESTS

Languages **French** native, **English**, **Italian** and **Spanish** fluent

IT skills MS Office (Excel, Power Point & Word), Bloomberg Terminal, Stata

Interests Woodworking, fencing, skiing, surfing, triathlon (pro-bono swim coach), hiking, History, science fiction, music

Donors' Blended Finance Portfolio Optimization

A Variation on Modern Portfolio Theory

July 2026

Matthieu Pegon[1]

Abstract

For a blended finance donor, is extending a grant different than extending a concessional investment of the same subsidy equivalent? If not, what drives a donor's choice of instrument and participation amount? More broadly, in the context of constrained aid budgets, is there room for donor-side financial optimization in blended finance, holding other decision-making factors constant?

To address these questions, I draw on Modern Portfolio Theory to formulate and solve a donor optimization problem. I assume donors' decision-making is sequential: aid envelopes, broad policy objectives and project selection are set before the financial terms of individual concessional financings are determined.

Assuming stochastic and correlated projects' returns, donor preferences at this final stage can be represented in terms of the donor's expected present cost and the variance of that cost. The model yields five main results. First, grants are preferred when donor risk tolerance is zero or close to it. Second, greater donor risk tolerance lowers the expected donor's cost, but only by increasing nominal donor participation and thereby reducing private capital leverage. Third, grants are also preferred for any asset for which the donor's cost of funding exceeds the investor's hurdle rate. Fourth, when optimal donor participation exceeds the aggregate subsidy equivalent, the same optimum can be implemented through different combinations of grants and investments. Finally, in the unconstrained interior case, donor risk tolerance affects the amount of the donor's participation but not the relative portfolio weights across assets.

Keywords: *Blended Finance, Concessional Finance, Official Development Assistance, Donors Optimization, Private Capital Leverage*

JEL Codes: *G11, D81, F35, O19, O22, Q01*

[1] Executive PhD candidate at ESCP Business School and Deputy CEO at Climate Investment Funds, formerly Director of Blended Finance at IDB Invest. The views expressed are solely those of the author and do not necessarily reflect those of the aforementioned organizations. I would like to thank without implication Bertrand Badré, Paddy Carter, Santiago Barraza, Joan Larrea and Ugo Panizza for helpful comments and feedback. ChatGPT was used for language editing and as an aid to literature search, and Refine for consistency. All resulting text revisions and references were reviewed and validated by the author. The mathematical notation of the optimization problem is partly drawn from Thierry Roncalli's Handbook of Sustainable Finance.

1. Introduction

Official Development Assistance (ODA), the standard measure of donor countries' financial efforts to support development in low- and middle-income countries expressed on a grant-equivalent basis, is shrinking. ODA fell by 6.1% in 2024 and 23.1% in 2025 to US$174.3 billion amid mounting political and fiscal pressures (OECD, 2026). Faced with a growing funding gap to meet the objectives of the Sustainable Development Goals (UN, 2024), the efficient use of scarce donor resources is increasingly relevant (G20, 2024).

In contrast, annual investments supported by blended finance have grown by 12% per year for the 2020-2024 period to reach US$ 18.3 billion in 2024 (Convergence, 2025).[2] From a policy standpoint, blended finance has increasingly been at the forefront of the international development agenda, as shown by the 2025 outcome text of the Fourth International Conference on Financing for Development (UN, 2025).

This paper examines the financial optimization of donors' concessional resources under budget constraints. It extends two strands of literature: donor-side aid optimization and instrument choice, and the burgeoning blended finance literature. It builds on the claim that existing donor-side optimization models, while useful for analyzing sovereign-to-sovereign grants and loans, do not fully capture the allocation problem raised by blended finance, where payoffs are uncertain and concessional resources are deployed across portfolios of correlated assets. Once concessional finance is understood in those terms, the donor's problem is naturally one of mean-variance optimization. The paper therefore develops, to my knowledge, the first donor-side mean-variance framework for concessional finance. In doing so, it offers a generalization of earlier optimization models. The traditional sovereign-to-sovereign case is retained as a limiting case, while the framework is extended to blended finance, where donor allocation takes place across risky and correlated portfolios. It also differs from the existing blended finance literature by avoiding a normative assumption about donor motivation – particularly the assumption that private capital mobilization is the dominant objective – and instead allowing for multiple, potentially conflicting donor objectives. At the same time, the model shows that these objectives may interact, notably through a trade-off between reducing expected donor cost and maintaining private capital leverage.

In the paper, I develop a model that relies on two modelling assumptions. First, the donors' selection and design of blended finance interventions follow a sequential, top-down process that starts with donors defining an overarching ODA envelope with broad policy objectives and ends in the financial structuring of individual blended finance interventions. This assumption allows the model to abstract from the earlier phases of the decision chain and focus specifically on the financial structuring stage, where the donor's concessional financing is structured. Restricting the analysis to this last phase allows the second assumption. At this final stage, the donor's preferences can be narrowed down and represented over the mean and variance of the donor's present cost, consistent with a mean–variance utility framework.

For both assumptions, academic literature is limited. I therefore rely primarily on scholarly work and policy documents from official sources, informed by my own experience as a blended finance practitioner. For illustration, I have drawn from the Government of Canada's international climate finance deployed through blended finance. In some instances, I have introduced additional simplifying assumptions which may depart from empirical evidence and my own direct observations. Those are documented accordingly.

Building on these two assumptions, I formulate and solve a donor optimization problem drawn from Markowitz's Modern Portfolio Theory and derive properties that may be of practical use for donors and blended finance practitioners. The paper is organized as follows. Section 2 provides an overview of the relevant literature. Section 3 sets up the model by defining the staged nature of donor decision-making, the characteristics of the portfolio and its assets, the investor's hurdle rate, the subsidy equivalent, and the donor's concessional financing instruments. Section 4 then formulates the donor's optimization problem in portfolio terms and derives the key expressions for the mean and variance of the donor's present cost. Section 5 solves the optimization problem and presents the main properties of the donor's optimal allocation, including the conditions under which grants are preferred, the role of the

[2] Compounded annual growth rate calculated by the author based on Convergence, 2025.

donor's cost of funding, the scope for combining grants and concessional investments, the implications for portfolio weights, and the trade-off with private capital leverage objectives. Section 6 summarizes the main results accompanied by a stylized representation and discusses limitations. Finally, section 7 concludes with policy recommendations. Annexes include references, list of model's variables, and mathematical demonstrations.

2. Literature Review

The paper intersects with two strands of literature: donor-side aid optimization and instrument choice, and the emerging blended finance literature. The first strand of literature has long been a matter of interest, intertwined with efforts of normalizing rules of accounting for ODA across financial instruments (Scott, 2017). Pincus (1963) was the first to raise explicitly the question of the real cost of aid to donors, arguing that a loan with the same face value as a grant should count for less in ODA terms. He introduced the concept of grant equivalence, defined as the face value of an instrument minus the discounted present value of expected reflows, using an appropriate opportunity cost of capital as discount rate.[3] Schmidt (1964) extended this approach to compare the efficiency of grants and concessional loans while holding recipient benefits constant. He showed that concessional loans are more efficient when the donor's opportunity cost of capital is lower than the recipient's, whereas grants are more efficient in the opposite case.

Frank (1970) generalized these results by distinguishing between grant equivalence measured at the donor's opportunity cost of capital, which captures the present cost to the donor, and grant equivalence measured at the recipient's opportunity cost of capital, which captures the present benefit to the recipient. He further showed that when the donor's nominal financing envelope is constrained, and the donor's opportunity cost of capital is lower than the recipient's, the Pareto-optimal outcome may be to extend the full nominal amount as a concessional loan rather than a grant. At the same time, where recipient absorptive capacity is limited or risk aversion is high, optimal financing terms lie closer to the grant end of the spectrum and lending volumes should be reduced.

More recent literature reframes the grant-loan choice as a broader donor optimization problem defined by a more heterogeneous set of objectives and constraints, including poverty reduction, economic growth, debt sustainability, and the recipient country's policy environment (Koeda, 2004, Odedokun, 2004, Cordella and Ulku, 2007, Iimi and Ojima, 2008). This literature also emphasizes the importance of budgetary considerations. Kawai and Takagi (2001) show that Japan's extensive use of ODA loans reflected a budgetary structure that allowed aid volumes to expand with limited direct budgetary cost. In the multilateral context, Sanford (2002) and Sanford (2004) show that resistance to expanding IDA grants reflected concern over the loss of loan reflows that financed future commitments.

The foregoing literature is framed around individual sovereign-to-sovereign transactions and therefore abstracts from uncertainty and portfolio interactions. Loans are treated as deterministic obligations, so risk does not enter the donor's optimization problem in the way it does in blended finance, where payoffs are stochastic and allocated across portfolios of correlated investments. Even so, this literature provides several principles that are retained to the present analysis. It highlights the importance of distinguishing between donor present cost, based on the donor's opportunity cost of funds, and recipient economic benefit, which in blended finance is more appropriately captured through the construct of subsidy equivalence. The principles underpinning grant-equivalence calculations are also retained as the basis for defining donor present cost and subsidy equivalence. In addition, this literature suggests testable propositions that can be revisited in a generalized setting, including whether grants remain preferable when the donor's cost of capital is relatively high, and whether the role played by recipient risk aversion in the existing literature has a mirror in donor risk aversion under uncertainty. Finally, it establishes two ideas that are retained directly: that the degree of concessionality affects the nominal amount of the donor's financing, and that donor choice between financial instruments is shaped not only by developmental objectives but also by budgetary considerations.

[3] In the case of Pincus (1963), marginal domestic long-term return of capital, international lending rates, and rates that private investors would have earned in the area of investment are used. Note that the grant equivalent of a grant will equal the face value of the grant under that methodology.

The second strand of literature concerns blended finance itself. I define blended finance as the use of ODA extended directly or indirectly to private agents through concessional financing (IDB Invest, 2023). Concessional financing is in turn defined as subsidized financing extended at terms and/or conditions that are more favorable than those available in the market (ADB et al., 2017). Implicit in this definition is that the underlying assets financed through blended finance are non-sovereign and therefore characterized by uncertain and correlated returns.

Blended finance is associated with several stated policy objectives, which may overlap but need not coincide. Historically, Multilateral Development Banks (MDBs) and Development Finance Institutions (DFIs) have framed blended finance as a market-creation instrument for investments yielding positive externalities. Concessional resources are used to address market failures and move markets toward commercial viability (Buiter and Schankerman, 2002, ADB et al., 2013, ADB et al., 2017).[4] By contrast, the OECD gives greater prominence to the mobilization of commercial finance and private to public resources leverage ratio (OECD, 2018, OECD, 2025), possibly driven by increasing funding commitments – such as the US$ 300 billion annual New Collective Quantified Goals – amid decreasing ODA volumes.[5]

This duality in policy objectives is reflected in the nascent blended finance academic literature. Flammer et al. (2024) formalizes a blended-finance investment selection framework in which donors allocate concessionality across projects to maximize social returns. The model yields two main predictions. First, higher concessionality should be directed to projects with greater sustainability impact, because subsidies are costly and must be justified by higher social returns. Second, concessionality requirements should increase with risk frictions. Flammer et al. (2025) extend this analysis. In the case of certain returns, they show that required concessionality is proportional to the project's shortfall from the private hurdle rate, implying that projects with less-monetizable, more public benefits require more concessionality. Where returns are uncertain, the paper relies on stochastic dominance to characterize how contract structure can make investments more attractive. Increasing the share of concessional capital (reducing the private capital required) mechanically improves the private return distribution in a way that first-order stochastically dominates alternatives (preferred by any expected-utility maximizer). Separately, "de-risking" that reduces returns distribution (a mean-preserving contraction, e.g., trimming tails) makes the investment more attractive for risk-averse investors (second-order dominance logic). These papers are thus best read as operating in a market-creation and social-returns objectives, but they remain investor-centric, since the structure of concessional financing is informed by investor participation conditions rather than donor preferences.

Conversely, Ben Slimane et al. (2025) and Panizza (2026) emphasize private capital mobilization as the main policy objective of blended finance. Ben Slimane et al. (2025) develop a structuring framework for donor-funded junior tranches in emerging market aggregation vehicles designed to crowd in risk-constrained investors. A central result is that diversification lowers the subsidy intensity of the donor tranche. But here too the optimization is investor-centric: the junior tranche design is determined by private investors' risk-return preferences and portfolio characteristics, not by donor preferences. Panizza (2026) is anchored around the policy objective of catalytic multiplier, understood as leverage per unit of expected public cost. His first result is that the catalytic multiplier decreases with the severity of the market failure, creating a fundamental tension: the interventions that target the largest distortions achieve the lowest leverage. Panizza also derives instrument choice results depending on underlying market failures. Production externalities call for subsidized loans, while financial frictions are best addressed with guarantees. Importantly, however, the comparison is between guarantees and subsidized loans modeled through an interest-rate subsidy, rather than between returnable donor concessional investments and guarantees.

[4] MDBs and DFIs also emphasize private capital mobilization, but the latter is framed as a mean to an end.

[5] At the 29th Conference of the Parties (COP29) of the United Nations Framework Convention on Climate Change (UNFCCC), Parties adopted a New Collective Quantified Goal (NCQG) on climate finance to contribute to accelerating the achievement of the goals of the Paris Agreement. The US$300 billion refers to the annual climate finance mobilization goal, to be led by developed country Parties (Falduto and Jachnik, 2025).

This distinction in policy objectives matters because the literature points to a possible tension between maximizing social returns and maximizing leverage. Kotchen and Vogt (2025), in the related context of environmental multilateral funds, show theoretically that introducing cofinancing as an independent objective is redundant at best and potentially counterproductive when the primary objective is environmental impact. Empirically, however, they do not find trade-offs strong enough to conclude that current cofinancing preferences are already reducing environmental benefits. The broader implication is that the empirical case remains unsettled, but the possibility of tradeoffs between policy objectives is real. This helps motivate the approach taken in this paper. Rather than imposing normative policy objectives, it treats those as potentially heterogeneous and determined upstream and focuses instead on the donor's financial optimization problem at the structuring stage. In that sense, the paper complements existing literature that is either investor-centric or organized around a single policy objective ex ante.

3. Model Set-up

The practice of allocating ODA is structured as a sequence of decision stages: budgetary allocation to official aid, high-level resource programming, partner selection, project identification, and downstream financing instrument design. Although academic literature rarely maps this entire chain as a formal model, scholarly work and operational documents from official sources provide useful pointers. In his *The Law of Development Cooperation*, law scholar Philipp Dann argues that ODA transfers *take place in four stages: first, the general budget decision to reserve a certain amount for ODA expenses; second, the multi-year planning for one country or one sector; third, the negotiation and agreement on a concrete development intervention (project or program); and fourth, the implementation of such intervention* (Dann, 2013). A European Commission management guide for the deployment of the European Union's external assistance presents the process as explicitly sequential: multiannual programming sets strategic priorities and indicative allocations, which then feed into a five-phase project cycle: programming, identification, formulation, implementation and evaluation (EC, 2004).

A useful illustration of how blended finance is operationalized through a staged donor process comes from the deployment of Canada's international climate finance. In 2015, Canada announced CAD$2.65 billion in international climate finance over five years for the 2015–2021 period and subsequently allocated these resources across 130 initiatives (GAC, 2022). Within that envelope, CAD$956.4 million was channeled to five blended finance trust funds managed by four MDBs, representing 36% of the total envelope. Each fund operated under its own eligibility rules and impact objectives (GAC, 2022). For instance, in 2019, Canada contributed CAD$223.5 million to the second phase of the Canadian Climate Fund for the Private Sector in the Americas through the Inter-American Development Bank (IDB) Group, with expectations to reduce greenhouse gas emissions by 25 million tons and to leverage US$1 billion in private-sector investment for renewable energy, sustainable agriculture, and forestry (IDB Invest, 2019). In this architecture, Canada sets the overall budgetary commitment and channels resources into partner-managed vehicles, while the partner MDB is delegated authority to select projects within the fund's mandate and to structure the concessional finance at the transaction level. Consistent with this delegated model, the IDB Group has executed 40 fund-supported investments (IDB Invest, 2025).[6]

For the purpose of the model, let the **donor** be a donor country's government that is looking to use ODA to finance a portfolio of blended finance assets. For simplification, the term donor includes the donor country's government and any entities that are delegated allocation decision on behalf of that government. Consistent with the modelling assumption of a phased approach to allocating ODA, the model assumes that in earlier stages, the donor already determined the most desirable portfolio of assets to finance, reflecting its priorities, strategic interests or development philosophies. The choice of portfolio is treated as exogenous to the model. In this final stage, which is the focus of this paper, the donor is only concerned with the financial structuring of its concessional financings.[7]

[6] The list includes projects funded through the first and second phase of the fund.

[7] This is somewhat consistent with Markowitz (1952) phased approach to portfolio selection: *the process of selecting a portfolio may be divided into two stages. The first stage starts with observation and experience and ends with beliefs about the future performances of available securities. The second stage starts with the relevant beliefs about future performances and ends with the choice of portfolio. This paper is concerned with the second stage.*

By abstracting from the earlier stage, the model is only normative towards donor financial preferences. It does not attempt to evaluate whether non-financial preferences that guide the earlier stages are altruistic, strategic, economic, political, historical, or otherwise motivated, nor does it impose an ideal preference structure. This is incidentally consistent with the literature that emphasizes the plurality and heterogeneity of preferences, with each donor prioritizing a distinct combination of motives.[8]

A limitation is that both the academic literature (Flammer et al., 2024) and practitioner literature (IDB Invest, 2023) suggest that project selection and financial structuring may, at least in part, be determined simultaneously, which may contradict the assumptions of sequential decision-making. The contradiction may be solved by distinguishing the construct of subsidy equivalency (or concessionality) from the donor's present cost (following the distinction established by Frank, 1970). In the model, subsidy equivalency is considered an exogenous variable, while the donor's present cost is endogenous. This allows a donor to hypothetically select a portfolio based on its development impact relative to its subsidy equivalency intensity (per Flammer et al., 2024) as a prior step to the model, which in turn focuses on the donor's present cost optimization.

3.1. The Portfolio and its Assets

Let p be the portfolio of n blended finance investments or projects indexed by i that the donor wishes to support (those are referred to as **assets**). For simplicity, each asset requires a nominal investment of 1 at t_0 and yields a random rate of return of ra_i at t_1 under a normal distribution. The normality assumption allows the donor's preferences to be represented by a quadratic mean–variance utility function that is consistent with expected utility under normally distributed returns. It is, however, an approximation rather than a literal description of most blended-finance transactions. A meaningful share of blended finance is structured through loans or loan-like instruments (ADB et al. 2023). Payoffs are typically asymmetric. Upside is capped, whereas downside is concentrated in default states. Relative to such distributions, a normal approximation tends to understate negative skewness and tail risk. In addition, the model assumes no transactions costs and no information asymmetry. Those assumptions and their underlying limitations are discussed in more detail in §6.

Let μ_{ra_i}, $\sigma^2_{ra_i}$, and σ_{ra_i}, be respectively the mean, variance and standard deviation of an asset's rate of return, as follows:

Definition 1 Assets' Rate of Return Mean, Variance and Standard Deviation

$$Mean\,rate\,of\,return = E\left[ra_i\right] = \mu_{ra_i}$$

$$Variance = Var\left[ra_i\right] = E\left[\left(ra_i - \mu_{ra_i}\right)^2\right] = \sigma^2_{ra_i}$$

$$Standard\,deviation = \sqrt{Var\left[ra_i\right]} = \sigma_{ra_i}$$

3.2. The Investor and its Commercial Investment

The **investor** is a private agent that would hypothetically finance an asset i should it receive a minimum mean rate of return μ_{rm_i} (a **commercial investment**). Thus, μ_{rm_i} represents the investor's opportunity cost for that particular asset.[9] For each asset, μ_{rm_i} is assumed to be strictly higher than the mean rate return μ_{ra_i}. Therefore, in absence of concessional finance, the investor does not invest in the asset. This framing is broadly consistent with Flammer et al. (2024), Warner (2013) and IDB Invest (2023).

[8] Canonical works include Alesina & Dollar (2000) and Berthélemy (2006), among many others.

[9] Although the model refers to a single investor, hypothetically for each asset i there might be a different marginal investor with the lowest opportunity cost (μ_{rm_i}) for that particular asset.

Like Flammer et al. (2024 and 2025), the model assumes that the investor's hurdle rate μ_{rm_i} is exogenous and does not vary with the investor's share in each asset. In other words, whether the investor is offered the full exposure or only a fraction of the asset, it is assumed to require the same return. This assumption relies on observed practice: in individual blended finance transactions, the marginal investors are typically large local and international banks (and, less frequently, institutional investors), whose required returns are driven mainly by counterparty risk, tenor, currency, and regulatory capital treatment rather than by differences in ticket size. Except for unusually large transactions where concentration limits or balance-sheet constraints may be binding, allocation size would therefore have limited influence on the hurdle rate.

3.3. The Donor's Concessional Financing

Flammer et al. (2025) proposes two donor's concessional financing contracts to enhance the investor's risk-return paradigm: increase the share of the concessional capital to further increase the expected return of the investor or reduce the spread of the distribution of return. The model considers a contract akin to the first option, whereby through the donor's concessional financing, the investor receives a mean rate of return μ_{rm_i}. The model assumes the principle of minimum subsidy is applied: the donor receives the highest rate of return that enables the asset to be financed by the investor.[10]

The donor can extend concessional financing through two instruments, a **concessional investment**, or a **grant**. A **concessional investment** is a concessional financing from the donor that generates a mean rate of return μ_{rc_i} on the donor's participation v_i, so that the investor receives the mean rate of return μ_{rm_i} on its participation $1 - v_i$. A **grant** is a cash subsidy from the donor to the investor that yields a rate of return of -100% so that the investor receives the mean rate of return μ_{rm_i}.

3.4. The Donor's Concessional Investment

Suppose that regardless of the rate ra_i returned by the asset at t_1, the investor's rate of returns is increased by the difference between the investor's required mean rate of return μ_{rm_i} and the asset's actual mean rate of return μ_{ra_i}. Thus, the mean rate of return received by the investor would be the desired mean rate of return μ_{rm_i}:

Definition 2 Investor's Subsidized Rate of Return

$$rm_i = ra_i + \mu_{rm_i} - \mu_{ra_i}$$
$$\Rightarrow E\left[rm_i\right] = E\left[ra_i + \mu_{rm_i} - \mu_{ra_i}\right] = \mu_{rm_i}$$

Recall that v_i is the face value of the donor's concessional financing (the **nominal participation**). Following Definition 2, the donor's rate of return rc_i is a function of the asset's rate of return, the investor's required returns, and the donor's nominal participation.

Lemma 1 Donor's Rate of Return

$$rc_i = ra_i + \mu_{rm_i} - \mu_{ra_i} + \frac{\mu_{ra_i} - \mu_{rm_i}}{v_i}$$

[10] Otherwise known as the principle of minimum concessionality which is one of the five DFIs and MDBs principles for the use of blended finance (ADB et al 2017).

Proof. See Annex 31.

Because the investor's required rate of return is higher than the asset's rate of return, the higher the donor's nominal participation, the higher the donor's rate of return. Under the model, the donor's rate of return is therefore less than or equal to the asset rate of return, which is strictly inferior to the investor's rate of return:

$$rc_i \leq ra_i < rm_i$$

It follows from the linearity of expectation that the mean rate of return of the donor μ_{rc_i} is a function of its participation v_i, the mean rate of return of the asset μ_{ra_i} and the investor's required mean rate of return μ_{rm_i}.

Definition 3 Donor's Mean Rate of Return

$$\mu_{rc_i} = E\left[rc_i\right] = E\left[ra_i + \mu_{rm_i} - \mu_{ra_i} + \frac{\mu_{ra_i} - \mu_{rm_i}}{v_i}\right] = \mu_{rm_i} + \frac{\mu_{ra_i} - \mu_{rm_i}}{v_i}$$

Because of the structure of the donor's concessional investment, the variances of the rates of return of the asset ($\sigma^2_{ra_i}$), the investor ($\sigma^2_{rm_i}$) and the donor ($\sigma^2_{rc_i}$) are the same (in the case where the donor extends a grant, the variance is equal to zero). The same applies for the standard deviations:

Lemma 2 Equality of Investments Rates of Return Variances

$$\sigma^2_{ra_i} = \sigma^2_{rm_i} = \sigma^2_{rc_i}$$
$$\sigma_{ra_i} = \sigma_{rm_i} = \sigma_{rc_i}$$

Proof. See Annex 32.

3.5. The Asset's Subsidy Equivalent and the Donor's Grant

Consistent with academic (Pincus, 1963, Schmidt 1964, Chang et al., 1998) and practitioner (ADB et al., 2018) literature, an asset's **subsidy equivalent** is the present benefit to the investor of receiving concessional financing expressed in grant equivalent. To determine the t_0 value for the investor of that additional return $\mu_{rm_i} - \mu_{ra_i}$ introduced in Definition 2, it is discounted by the opportunity cost μ_{rm_i}.

Definition 4 Assets' Subsidy Equivalent

$$Subsidy\ \ equivalent\ \ of\ \ asset\ \ i = \ s_i = \frac{\mu_{rm_i} - \mu_{ra_i}}{1 + \mu_{rm_i}}$$

If the investor receives the subsidy equivalent s_i as a grant at t_0, it would extend a commercial investment in asset i, since it would now receive the mean rate of return μ_{rm_i} it desires:

Lemma 3 Extending the Subsidy Equivalent as Donor's Grant

$$Investor's\ mean\ rate\ of\ return = \frac{Mean\ returns\ at\ t_0 - Investment\ at\ t_1\ (net\ of\ s_i)}{Investment\ at\ t_1\ (net\ of\ s_i)}$$

$$= \frac{(1+\mu_{ra_i}) - (1-s_i)}{(1-s_i)} = \mu_{rm_i}$$

Proof. See Annex 33.[11]

The following can be extrapolated.

Proposition 1 Investor's Indifference to Concessional Finance Instruments

The investor would be indifferent between receiving a grant at t_0 equal to the subsidy equivalent or an additional return $\mu_{rm_i} - \mu_{ra_i}$ equal to the subsidy equivalent at t_1.

Proof. Derived from Lemma 3.

Importantly, if the donor extends a grant, the rate of return on the instrument is -100%. However, the rate of return is certain. The variance of the donor's rate of return when extending a grant is therefore equal to zero.

Having established the investor's indifference to type of concessional financing, the choice of instrument and the participation amount becomes a function of the donor's preferences.

3.6. Donor's Preferences

To explain the donor's preferences formally, recall that the donor has already identified a portfolio p of assets i that requires concessional financing. In the model, the donor decides its nominal participation v_i in each asset – which in turn drives the choice of instruments between grant and concessional investment.

The donor's decision is modeled as a single, ex ante assessment in which the expected cost and uncertainty of its concessional financing are weighed jointly. In other words, the donor forms a view about both how much an intervention is expected to cost on average and how uncertain that cost is likely to be.

Lower expected cost is valued because it preserves scarce budgetary space and allows the donor to undertake a larger number of interventions, generally within a predetermined fixed ODA envelope, consistent with Kawai and Takagi (2001), Sanford (2002) and Sanford (2004). In a note discussing the use of concessional investments – as opposed to grants – in blended finance, IFC (2019) states that the model *appeals to governments because they regularly receive the reflows, and can redeploy the funds for other programs or priorities*. Canada's 2021–2026 CAD$5.3 billion international climate finance envelope provides an illustration: 40% is expected to be delivered through grants and 60% through *Unconditionally Repayable Contributions* described by the Government of Canada as akin to concessional loans (GAC, 2025a).[12]

Aversion to uncertainty reflects the fact that both unusually bad and unusually good financial outcomes can be institutionally problematic. Large, realized costs can trigger unforeseen budgetary losses, fiduciary concerns, political scrutiny, and reputational damage. Conversely, unusually small cost or net gains can also generate disutility by complicating delivery against planned ODA volumes, raising questions about the appropriateness of profit-making with aid resources, or undermining expectations

[11] Note that rate of return for the investor may be different, only the mean rate of return is the same. However, as shown in Annex 33, the investor's net present values are the same.

[12] Where, in principle, Canada could pursue the same policy objective through a pure grant calibrated to the implied subsidy of the concessional loan.

about how concessional resources will be deployed.[13] Importantly, in this setting the donor is assumed to care about the size of deviations from what was expected, not their direction. What matters is the extent to which realized cost depart from *ex-ante* expected cost, regardless of whether this departure takes the form of higher-than-expected cost or lower-than-expected cost.[14]

This logic aligns with a Markowitz mean-variance perspective applied to public finance. Donors prefer interventions that are expected to consume fewer resources and that deliver those outcomes in a more predictable manner. Interventions that combine low expected cost with low uncertainty are therefore strictly preferred, while trade-offs arise when an intervention reduces expected cost at the cost of greater volatility, or vice versa.

3.7. The Present Cost of the Donor's Concessional Financing

In the model, the donor's cost is calculated as the present value of the cash flows from the donor's concessional financing discounted by rd, a constant discount rate representing the donor's cost of funding for the period t (the donor's **present cost**).[15]

The use of the donor's cost of funding as a proxy for its opportunity cost is practice-motivated: donors often assess the fiscal exposure of returnable assistance in net present value terms, discounting expected cash flows at a rate intended to reflect their cost of funding – that is, the rate at which the government can raise resources today (or the return it forgoes by committing those resources) over the relevant horizon, as hypothesized in Pincus, 1963. Carter (2015), however, questions whether borrowing costs are the right way to represent the donor's true opportunity cost. Government borrowing rates do not capture the wider economic costs created by the taxation required to service debt. From a welfare perspective, Carter argues that a more relevant benchmark may be the marginal cost of public funds, which incorporates the behavioral distortions associated with raising public revenue.

In addition, treating rd as constant abstracts from the possibility that, as governments borrow more to finance the portfolio of assets, marginal funding costs can rise because investors demand a higher risk premium as debt burdens rise. In the extreme case where a donor would rely heavily on borrowing – beyond what its taxation capacity could credibly support – while expecting repayments from risky blended-finance assets, one may expect borrowing costs to converge toward the market-required return for those assets. An intuition Carter makes explicit when noting that the appropriate discount rate *may not be so very different* from private investors' required returns.

In practice, however, existing government support for blended finance remains small relative to overall sovereign borrowing volumes, making a constant rd a reasonable working assumption for the purposes of the model.[16]

Nevertheless, the model considers in §5.3 the case in which rd exceeds the investor's hurdle rate and discusses how this changes the donor's optimum. For concessional investments, the donor's present cost is calculated as follows.

Definition 5 Donor's Present Cost for Concessional Investments

$$Present\ \ cost\ \ of\ \ donor's\ \ subsidized\ \ financing \in asset\ \ i = g_i$$

[13] Anecdotally, I have witnessed in a several instances donors commenting that their returns on certain blended finance programs were "too high" and this was generating "internal scrutiny".

[14] GAC (2025b) explicitly recognizes this possibility: *In the context of international development assistance, it is not anticipated that GAC would receive a financial return over and above the repayment of its contribution except in rare instances where the underlying investment(s) supported perform exceptionally well. In this case, the amount paid to GAC could exceed the initial contribution (on a net present value basis). That is, while ex ante, the funding is expected to be concessional (as per the OECD Blended finance guidance & principles), ex post, there is a possibility that the Government of Canada could realize a positive return.*

[15] Note that in the calculation, the present cost is a positive number (in case of gains, the present cost becomes negative).

[16] For instance, Canada central government debt stands at CAD$1.99 trillion in 2024 (IMF, 2026), compared to an estimated CAD$ 636 million of yearly *Unconditionally Repayable Contributions* deployed under Canada's 2021–2026 international climate finance envelope (see §3.6).

$$= v_i - \frac{v_i(1+rc_i)}{1+rd}$$

$$= \frac{v_i(rd - rc_i)}{1+rd}$$

$$= \frac{v_i(rd - ra_i - \mu_{rm_i} + \mu_{ra_i}) + \mu_{rm_i} - \mu_{ra_i}}{1+rd}$$

Having defined g_i, let μ_{g_i}, $\sigma^2_{g_i}$ and σ_{g_i}respectively be the mean, variance and standard deviation of the donor's present cost for each asset.

Lemma 4 Mean, Variance and Standard Deviation of Donor's Present Cost for Concessional Investments

$$Mean\ \ present\ \ cost = \mu_{g_i} = E[g_i] = \frac{v_i(rd - \mu_{rm_i}) + \mu_{rm_i} - \mu_{ra_i}}{1+rd}$$

$$Variance\ of\ present\ \ cost = \sigma^2_{g_i} = Var[g_i] = E\left[(g_i - \mu_{g_i})^2\right] = \left(\frac{v_i}{1+rd}\right)^2 \sigma^2_{ra_i}$$

$$Standard\ deviation\ of\ present\ \ cost = \sigma_{g_i} = \frac{v_i}{1+rd}\sigma_{ra_i}$$

Proof. See Annex 34.

Suppose the donor extends a concessional investment equal to the subsidy equivalent s_i. It follows that the donor's mean present cost μ_{g_i} is equal to the subsidy equivalent s_i.

Lemma 5 Donor's Mean Present Cost for Concessional Investment and Asset's Subsidy Equivalent

$$v_i = s_i$$

$$\Rightarrow \mu_{g_i} = s_i$$

Proof. See Annex 35.

4. Formulating the Donor's Optimization Problem

4.1. Donor's Disutility Function and Optimization Problem

Following §3.6 and §3.7, the donor's preferences can be defined over the mean and variance space of the donor's present cost, consistent with a mean-variance utility framework. In this framework, the donor would optimally allocate its concessional financings to minimize a present cost disutility function that incorporates present cost mean and variance, analogous to Markowitz's Modern Portfolio Theory (Markowitz, 1959) but applied to blended finance.

Let D be the donor's disutility function from its $(v_1, \ldots, v_n)$ nominal participations in each asset, based on a Markowitz (1956) modified mean-variance utility function:

Definition 6 Donor's Disutility Function

$$D(v_1,\ldots,v_n)=\gamma\,\mu_{g_p}+\frac{1}{2}\sigma^2_{g_p}$$

Where μ_{g_p} and $\sigma^2_{g_p}$ are respectively the donor's present cost mean and variance of its concessional financing of the portfolio p, and γ is the donor's risk-tolerance coefficient.

Suppose the donor wants to optimally allocate its nominal participations v_i in each asset i of the portfolio p up to a maximum aggregate nominal budget of b_{max}. According to §3.3, for each asset i, the donor may invest v_i, between a minimum of s_i and a maximum of 1, subject to the sum of v_i not exceeding b_{max}. The optimization problem for the donor takes the form of a standard quadratic programming problem:

Definition 7 Donor's Optimization Problem

$$(v_1^*,\ldots,v_n^*)(\gamma)=arg\ \ min\ \left\{D(v_1,\ldots,v_n)=\gamma\,\mu_{g_p}+\frac{1}{2}\sigma^2_{g_p}\right\}$$

$$s.t.\ \begin{cases}\sum_{i=1}^{n} v_i \le b_{max}\\ s_i \le v_i \le 1\end{cases}$$

4.2. The Donor's Portfolio of Concessional Investments

Let $V=(v_1,\ldots,v_n)$ be the vector of the donor's nominal participation in the concessional investment for each asset i. The donor's total participation is $\sum_{i=1}^{n} v_i = 1_n^T V \le b_{max}$.

The variances of asset returns are assumed to be strictly positive, asset returns are assumed to be linearly independent, and pairwise correlations are assumed to be strictly positive. The pairwise correlation positivity assumption is analytically convenient because in cases that support the grant results (see §5.2 and §5.3), it helps ensure that the relevant partial derivatives of the donor's disutility with respect to nominal participations are strictly positive over the feasible set. In practice, it can be rationalized for portfolios exposed to common macroeconomic, exchange-rate, commodity-price, or sovereign-risk factors. This is arguably the case for blended finance assets, which are focused on developing countries eligible for ODA and concentrated in the sectors of financial services, energy and infrastructure and agribusiness. Even so, the restriction is strong and possible limitations are discussed in more detail in §6.

4.3. The Mean Present Cost of the Donor's Portfolio

Let μ_{g_p} be the donor's mean portfolio present cost. From the linearity of expectation, it follows that:

$$\mu_{g_p}=E[g_p]=\sum_{i=1}^{n} E[g_i]=\sum_{i=1}^{n}\frac{v_i(rd-\mu_{rm_i})+\mu_{rm_i}-\mu_{ra_i}}{1+rd}$$

Let $A=\left(\mu_{ra_1},\ldots,\mu_{ra_n}\right)$ be the vector of the mean rates of return for each asset i, $M=\left(\mu_{rm_1},\ldots,\mu_{rm_n}\right)$ the vector of the minimum mean rate of return threshold required by the investor for each asset i and $D=\left(rd,\ldots,rd\right)$ the n dimension vector of the donor's cost of funding rd. μ_{g_p} is rewritten isolating the linear term from the constant term:

Definition 8 Mean Present Cost of the Donor's Portfolio

$$\mu_{g_p}=\frac{1_n^T\left(M-A\right)}{1+rd}+\frac{V^T\left(D-M\right)}{1+rd}$$

4.4. The Present Cost Variance of the Donor's Portfolio

Let $\sigma_{g_p}^2$ be the present cost variance of the donor's portfolio of concessional financings. Since covariance is bilinear:

$$\sigma_{g_p}^2=E\left[\left(\sum_{i=1}^{n}g_i-E\left[\sum_{i=1}^{n}g_i\right]\right)^2\right]=E\left[\left(\sum_{i=1}^{n}g_i-\mu_{g_i}\right)^2\right]=\sum_{i=1}^{n}\sum_{i'=1}^{n}Cov\left(g_i,g_{i'}\right)$$

Which can be expressed as a function of v_i and ra_i:

Lemma 6 Present Cost Variance of the Donor's Portfolio

$$\sigma_{g_p}^2=\sum_{i=1}^{n}\sum_{i'=1}^{n}\frac{v_iv_{i'}}{\left(1+rd\right)^2}Cov\left(ra_i,ra_{i'}\right)$$

Proof. See Annex 36.

Let $\rho_{ii'}=\frac{Cov\left(ra_i,ra_{i'}\right)}{\sigma_{ra_i}\sigma_{ra_{i'}}}$ be the correlation between the rates of return ra_i and $ra_{i'}$:

$$\sigma_{g_p}^2=\sum_{i=1}^{n}\sum_{i'=1}^{n}\frac{v_iv_{i'}}{\left(1+rd\right)^2}\sigma_{ra_i}\sigma_{ra_{i'}}\rho_{ii'}$$

Let $\sum$ be the covariance matrix of the rates of return ra_i so that:

Definition 9 Assets Rate of Return Covariance Matrix

$$\sum=\begin{bmatrix}\sigma_{ra_1}^2 & \cdots & \sigma_{ra_n}\sigma_{ra_1}\rho_{n1}\\ \vdots & \ddots & \vdots\\ \sigma_{ra_a}\sigma_{ra_n}\rho_{1n} & \cdots & \sigma_{ra_n}^2\end{bmatrix}$$

A covariance matrix is always positive semi-definite. Since the rates of returns of the assets i are assumed to be linearly independent, $\sum$ is positive definite.

Definition 10 Present Cost Variance of the Donor's Portfolio

$$\sigma_{g_p}^2 = \left(\frac{1}{1+rd}\right)^2 V^T \sum V$$

The donor's optimization problem can be expanded as follows:

Definition 11 Expanded Donor's Optimization Problem

$$(V^*)(\gamma) = arg\,min\left\{ D(V) = \gamma \frac{1_n^T (M - A)}{1+rd} + \gamma \frac{V^T (D - M)}{1+rd} + \frac{1}{2}\left(\frac{1}{1+rd}\right)^2 V^T \sum V \right\}$$

$$s.t. \begin{cases} 1_n^T V \le b_{max} \\ S \le V \le 1 \end{cases}$$

Where $S = (s_1, \ldots, s_n)$ is the vector of subsidy equivalents for each asset i. From a mathematical standpoint, the above optimization problem features a quadratic objective function $D(V)$ with linear inequality constraints.[17] Per the assumptions §4.1 on the rates of return ra_i, $\sum$ is a positive definite matrix. Therefore, $D(V)$ admits a global minimum V^*. If the donor was only allowed to extend concessional financing in the form of concessional investments, its optimal vector of nominal participations would be V^*. However, the possibility for the donor to extend concessional financing in the form of grants, introduces several complexities. These are assessed in the following section, notably the subsidy-equivalent floor and the interior solutions set.

5. Solving the Donor's Optimization Problem

5.1. Gradient of Donor's Disutility Function

Let $\nabla D(V)$ be the gradient of the disutility function $D(V)$.

Lemma 7 Gradient of Donor's Disutility Function

$$\nabla D(V) = \gamma \frac{(D - M)}{1+rd} + \left(\frac{1}{1+rd}\right)^2 \sum V$$

Proof. See Annex 37.

5.2. Donor Extending Grants Equal to the Subsidy Equivalents

Suppose the donor's risk tolerance coefficient γ is equal to zero. The gradient of the donor's disutility function is updated as follows:

[17] The feasible region defined by the linear inequality constraints is nonempty.

$$\nabla D_{\gamma=0}(V)=\left(\frac{1}{1+rd}\right)^2 \sum V$$

From the model's assumptions, all v_i are strictly positive (§3.3) and all correlations $\rho_{ii'}$ are strictly positive (§4.1). Therefore, the gradient's components are strictly positive. The optimal vector V^* of nominal participations is equal to the lower bound S, the vector of subsidy equivalents (since all $v_i \geq s_i$).

Recall from Lemma 3 that the investor is agnostic between the donor extending S as a grant or as a concessional investment and from Lemma 5 that the donor's mean present cost of extending a concessional investment is equal to the subsidy equivalent (mean present cost is the same for the donor). Let $D(S_{grant})$ be the disutility functions where the donor extends S as a grant. The donor's mean present cost is the sum of the subsidy equivalents for each asset multiplied by the risk-tolerance coefficient: $\gamma 1_n^T S$. The present cost variance is equal to zero, since the variance for each asset's present cost is equal to zero. In other words, the cost is certain, since the concessional financing is extended as grant.

Definition 12 Donor's Utility Function when Extending a Grant S_{grant}

$$D(S_{grant})=\gamma 1_n^T S$$

Let $D(S_{invest})$ be the disutility functions where the donor extends S as concessional investments. From Lemma 5:

$$\begin{aligned} D(S_{invest}) &= \gamma \frac{1_n^T(M-A)}{1+rd} + \gamma \frac{S^T(D-M)}{1+rd} + \frac{1}{2}\left(\frac{1}{1+rd}\right)^2 S^T \sum S \\ &= \gamma 1_n^T S + \frac{1}{2}\left(\frac{1}{1+rd}\right)^2 S^T \sum S \\ &\Rightarrow D(S_{grant}) < D(S_{invest}) \end{aligned}$$

Proposition 2 Extending Grants S_{grant} when $\gamma = 0$

The donor will always prefer to extend a nominal participation in the form of grants S_{grant} when its risk tolerance coefficient $\gamma = 0$.

Proof. See above.

Proposition 3 Extending a Grant S_{grant} instead of Concessional Investments S_{invest}

The donor will always prefer to extend a nominal participation in the form of grants S_{grant} rather than concessional investments S_{invest}.[18]

Proof. See above.

The above suggests that where the donor's risk tolerance coefficient is close to zero, it may still prefer to extend a grant S_{grant}. For this to be case, $D(S_{grant})$ must be strictly inferior to $D(V^*)$, where $D(V^*)$ is

[18] The same is true for any $v_i^* = s_i$.

the donor's lowest disutility from supporting the portfolio through concessional investments. Let $D\left(S_{grant}\right) = \gamma \mu_{g_S}$ and $D\left(V^*\right) = \gamma \mu_{g_{V^*}} + \frac{1}{2}\sigma^2_{g_{V^*}}$ with:

$$\mu_{g_S} = 1_n^T S$$

$$\mu_{g_{V^*}} = \gamma \frac{1_n^T \left(M - A\right)}{1+rd} + \gamma \frac{V^{*T}\left(D - M\right)}{1+rd}$$

$$\sigma^2_{g_{V^*}} = \left(\frac{1}{1+rd}\right)^2 V^{*T} \sum V^*$$

μ_{g_S} is strictly positive per Definition 4. Per the above, $\mu_S > \mu_{V^*}$, thus, $\mu_S - \mu_{V^*}$ is strictly positive. Finally, $\sigma^2_{g_{V^*}}$ is strictly positive. Hence the following inequality:

$$\gamma \mu_{g_S} < \gamma \mu_{g_{V^*}} + \frac{1}{2}\sigma^2_{g_{V^*}}$$

$$\Rightarrow \gamma < \frac{1}{2}\frac{\sigma^2_{V^*}}{\mu_S - \mu_{V^*}}$$

Proposition 4 Extending a Grant S_{grant} when $\gamma > 0$

The donor will always prefer to extend a nominal participation in the form of a grant S_{grant} when its risk tolerance coefficient $\gamma < \frac{1}{2}\frac{\sigma^2_{V^*}}{\mu_S - \mu_{V^*}}$

Proof. See above.

5.3. Donor's Cost of Funding and Optimal Choice

For a given i, suppose that $rd > \mu_{rm_i}$. Based on Lemma 7, the partial derivative of the disutility function with respect to v_i is:

$$\frac{\partial D}{\partial v_i} = \gamma \frac{rd - \mu_{rm_i}}{1+rd} + \frac{\sum_{i'=1}^{n} v_{i'} \sigma_{ra_i} \sigma_{ra_{i'}} \rho_{ii'}}{\left(1+rd\right)^2} > 0$$

Since $rd > \mu_{rm_i}$ and $rd > 0$, the first term is positive. From the assumptions above, all $v_{i'}$ are strictly positive (§3.3) and all correlations $\rho_{ii'}$ are strictly positive (§4.2). Thus, the partial derivative is strictly positive. In this case, the optimal v_i^* that minimizes $D\left(V\right)$ is the lower bound s_i. From Proposition 3, where $v_i^* = s_i$, the donor prefers to extend a grant rather than a concessional investment.

Therefore, for any asset i where $rd > \mu_{rm_i}$, the optimal donor concessional financing is to extend a grant equal to the subsidy equivalent for that asset (consistent with Schmidt, 1964 and Frank, 1970 but

expanded to a portfolio of risk bearing assets). For the remaining assets in the portfolio p, the optimal vector of nominal participations is given by solving the optimization problem excluding the assets i where $rd > \mu_{rm_i}$.

Proposition 5 Donor's Cost of Funding and Optimal Participations

For assets where the donor's cost of funding rd is strictly superior to the market's opportunity cost μ_{rm_i}, the donor's concessional financing is a grant equal to the required subsidy equivalent s_i for that asset.

Proof. See above.

5.4. Combining Grants and Concessional Investments

The cases where $v_i^* = s_i$ under Proposition 2, Proposition 4 and Proposition 5 are now excluded, so that $v_i^* > s_i$. Therefore, the donor's optimal vector of nominal participations V^* is given by solving the optimization problem from Definition 11.

For each asset i, suppose the donor is allowed to provide a portion of its optimal nominal participation as grant, and the remainder as an investment. Let x_i be the portion of the required subsidy equivalent s_i that is extended as a grant (with a rate of return of -100%), and y_i the portion extended as investment so that $x_i + y_i = v_i$.

Let rc_{y_i} be the required rate of return of the donor concessional investment. rc_{y_i} is a function of the asset's return minus the investor's required returns, based on the donor's nominal participation in the concessional investment (i.e. the total nominal participation net of the portion extended as grant).

Lemma 8 Rate of Return of the Donor Concessional Investment when Partly Extending a Grant

$$1 + ra_i = x_i(1 - 100\%) + y_i(1 + rc_{y_i}) + (1 - v_i)(1 + rm_i)$$

$$\Rightarrow rc_{y_i} = \frac{v_i(1 + ra_i + \mu_{rm_i} - \mu_{ra_i}) - \mu_{rm_i} + \mu_{ra_i}}{y_i} - 1$$

Proof. See Annex 38.

Let g_{x_i} be the present cost of the donor's concessional financing when partly extending it as a grant and partly as a concessional investment. It can be demonstrated that it is the same than if the donor's concessional financing is solely extended as a concessional investment.

Lemma 9 Donor's Present Cost with Grant Portion

$$g_{x_i} = g_i$$

Proof. See Annex 39.

It can also be demonstrated that the mean, variance and standard deviation of the donor's concessional financing when partly extending it as a grant and partly as a concessional investment, is the same as if extended solely as a concessional investment.

Let $\mu_{g_{x_i}}$, $\sigma^2_{g_{x_i}}$ and $\sigma_{g_{x_i}}$ be the mean, variance and standard deviation of the donor's concessional financing when partly extending it as a grant portion.

Lemma 10 Mean, Variance and Standard Deviation of Donor's Present Cost with Grant Portion

$$\mu_{g_{x_i}} = \mu_{g_i} = \frac{v_i\left(rd - \mu_{rm_i}\right) + \mu_{rm_i} - \mu_{ra_i}}{1+rd}$$

$$\sigma^2_{g_{x_i}} = \sigma^2_{g_i} = \left(\frac{v_i}{1+rd}\right)^2 \sigma^2_{ra_i}$$

$$\sigma_{g_{x_i}} = \sigma_{g_i} = \frac{v_i}{1+rd}\sigma_{ra_i}$$

Proof. See Annex 310.

From Lemma 10, it follows that for any optimal vector of nominal participations $V^* > S$, the donor can break down its participations v_i^* between a grant x_i up to the subsidy equivalent s_i and an investment for $y_i = v_i^* - x_i$ without affecting the donor's optimum.

Proposition 6 Combining Grants and Concessional Investments

For any optimal vector of nominal participations $V^* > S$, the donor can break down its participations v_i^* between a grant x_i up to the subsidy equivalent s_i and an investment $y_i = v_i^* - x_i$ without affecting the donor's optimum.

Proof. Derived from Lemma 10.

5.5. Combining Grants and Commercial Investments

For this section, the cases where $v_i^* = s_i$ under Proposition 2, Proposition 4 and Proposition 5 are excluded, so that $v_i^* > s_i$. From Proposition 6, suppose the donor decides to extend $x_i = s_i$ as grant for all assets, and the balance $y_i = v_i^* - x_i$ as investment.

From Lemma 3, if the donor extends a grant s_i, then the assets will yield the commercial mean rate of return μ_{rm_i}. In this case, both the investor and the donor – on its investment y_i – will receive the commercial mean return threshold.

Proposition 7 Combining Grants and Commercial Investments

For any optimal vector of nominal participations $V^* > S$, the donor can break down its participations v_i^* between a grant s_i and an investment $y_i = v_i^* - s_i$ that will yield the commercial mean return threshold.

Proof. Derived from Lemma 3.

5.6. Donor's Indifference Point

Suppose $\gamma = \frac{1}{2}\frac{\sigma^2_{V^*}}{\mu_S - \mu_{V^*}}$. From Proposition 4, the donor will be donor be indifferent between extending a grant equal to the aggregate subsidy equivalent or a vector of nominal participations V^* given by solving the optimization problem from Definition 11.

Proposition 8 Donor's Indifference Point

Where the donor's risk tolerance coefficient $\gamma = \frac{1}{2}\frac{\sigma_{V^*}^2}{\mu_S - \mu_{V^*}}$ *the donor's is indifferent between extending a grant* S_{grant} *or a participation* V^*.

Proof. Derived from Proposition 4.

5.7. Donor's Portfolio Weights within the Interior Solutions Set

Suppose no constraints in both the maximum face value of the donors' portfolio (b_{max} is infinite) and the size of the donor's investment in each asset i. The cases where $v_i^* = s_i$ under Proposition 2, Proposition 4 and Proposition 5 are excluded, so that $v_i^* > s_i$. In such a setting, the donor is looking to minimize the following unconstrained disutility function:

Definition 13 Donor's Optimization Problem within the Interior Solutions Set

$$\left(V^*\right)(\gamma) = arg\,min\left\{D(V) = \gamma\frac{1_n^T(M - A)}{1+rd} + \gamma\frac{V^T(D - M)}{1+rd} + \frac{1}{2}\left(\frac{1}{1+rd}\right)^2 V^T \sum V\right\}$$

Recall the gradient $\nabla D(V)$ defined in Lemma 7. The gradient is set to zero and solved for V. Since $\sum$ is positive definite, it admits an inverse matrix $\sum^{-1}$ (where the product of both matrices is equal to the identity matrix):

$$\gamma\frac{(D - M)}{1+rd} + \left(\frac{1}{1+rd}\right)^2 \sum V = 0$$

$$\Rightarrow \sum\sum^{-1} V = -\gamma(1+rd)\sum^{-1}(D - M)$$

$$\Rightarrow V^* = -\gamma(1+rd)\sum^{-1}(D - M)$$

$\sum$ is positive definite and therefore $D(V)$ admits a unique minimum V^*.

Proposition 9 Donor's Optimization Problem within the Interior Solutions Set

Within boundaries constraints, there is a unique vector V^* *of participation in each asset that minimizes the donor's disutility given by setting the gradient of the disutility function to zero as follows:*

$$\left(V^*\right)(\gamma) = arg\ \ min\ \left\{D(V) = \gamma\frac{1_n^T(M - A)}{1+rd} + \gamma\frac{V^T(D - M)}{1+rd} + \frac{1}{2}\left(\frac{1}{1+rd}\right)^2 V^T \sum V\right\}$$

$$\exists!\, V^* = -\gamma(1+rd)\sum^{-1}(D - M)$$

Proof. See above.

The above shows the linear relationship between V^* and the risk-tolerance coefficient γ. Let b^* be the aggregate nominal participation of the donor in the concessional investments and $W^* = \left(w_1^*, \ldots, w_n^*\right)$ the

vector of weights of the donor's portfolio of concessional investments so that $b^* = \sum_{i=1}^{n} v_i^*$ and $V^* = b^* W^*$.

Suppose a second donor with the same cost of funding but a different risk-tolerance coefficient γ' is interested in financing the same portfolio p. γ' can be expressed as a multiple k of γ so that $\gamma' = k\gamma$. It follows that V'^* (the vector of optimal nominal participations of the second donor) can be expressed as follows:

$$\begin{aligned} V'^* &= -\gamma'(1+rd)\sum^{-1}(D-M) \\ &= -k\gamma(1+rd)\sum^{-1}(D-M) \\ &= k\,b^* W^* \end{aligned}$$

Thus, regardless of risk tolerance, the weights of the donor's participation in the assets in the portfolio are unchanged (within boundaries conditions).

Proposition 10 Unique Vector of Weights Within Boundaries Conditions

Within boundaries conditions, regardless of the donor's risk-tolerance coefficient there is a unique vector W^* of weights in the composition of the donor's nominal participations.

Proof. See above.

5.8. Private Capital Leverage

For simplicity, assumptions from §5.7 are maintained. Let l be the private capital leverage such that:

Definition 14 Private Capital Leverage

$$\begin{aligned} l &= \frac{Total\ \ Investment - Donor's\ \ Nominal\ \ Participation}{Donor's\ \ Nominal\ \ Participation} \\ &= \frac{n - 1_n^T V^*}{1_n^T V^*} = n\left(1_n^T V^*\right)^{-1} - 1 = n\left[-\gamma(1+rd)1_n^T\sum^{-1}(D-M)\right]^{-1} - 1 \end{aligned}$$

The partial derivative of l with respect to γ is:

$$\frac{\partial l}{\partial \gamma} = n\gamma^{-2}\left[(1+rd)1_n^T\sum^{-1}(D-M)\right]^{-1}$$

Recall $V^* = -\gamma(1+rd)\sum^{-1}(D-M)$, $\gamma > 0$, $rd > 0$, and $v_i^* > 0$. Therefore $\frac{\partial l}{\partial \gamma}$ is negative. This shows that, within the present model, maximizing leverage is not generally aligned with minimizing donor expected cost. Greater donor risk tolerance leads the donor to take a larger nominal share of the financing structure, which lowers expected present cost but mechanically leaves less room for private capital participation.

Proposition 11 Private Capital Leverage Tradeoff

The higher the donor's risk-tolerance coefficient, the higher its aggregate nominal participation and the lower the private capital leverage.

Proof. See above.

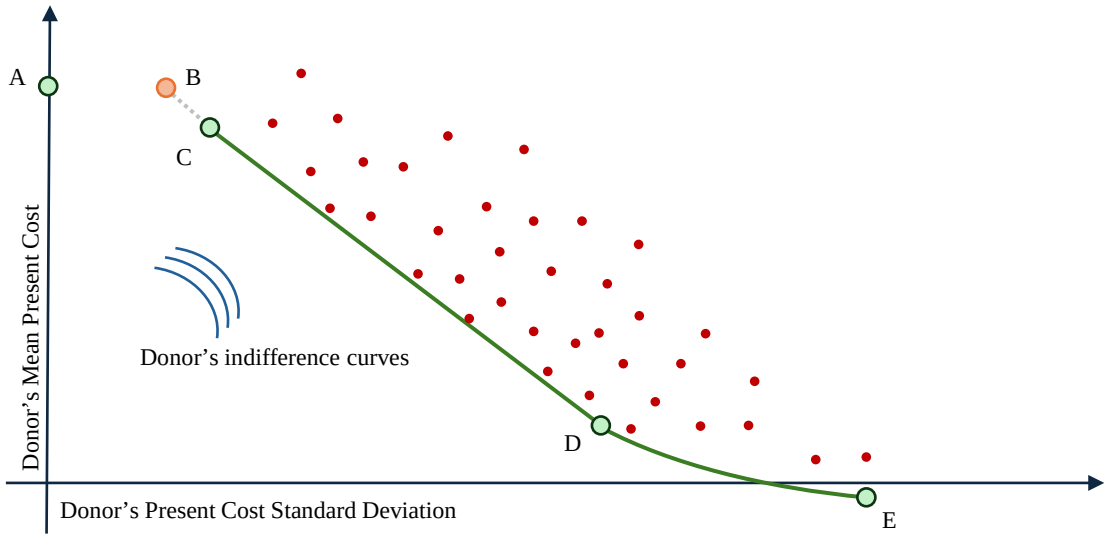
6. Summary of Results and Limitations

This paper models blended finance as a donor-side portfolio optimization problem. It rests on several assumptions. First, donor decision-making is treated as sequential: the aid envelope, broad policy objectives, and project selection are determined upstream, while the structuring of the donor's concessional financing is determined at a later stage. The portfolio of assets the donor wishes to support is therefore treated as given. Second, the returns from the portfolio's assets are stochastic, following a normal distribution, and correlated. Third, investors are assumed to have fixed exogenous hurdle rates for financing these assets. Fourth, at the structuring stage, donor preferences are represented over the expected present cost of support and the variance of that cost, using a fixed opportunity cost – the donor's cost of funding – as discount rate. Fifth, the model assumes no transaction costs and no information asymmetry.

Based on the model, the investor is indifferent between receiving the subsidy equivalent upfront as a grant and receiving the same support through an increased share of the assets returns. This implies that the investor is indifferent towards the donor's choice of concessional financing.

For each asset in the portfolio, the donor can choose the type and nominal amount of concessional financing. It may extend a grant equal to the subsidy equivalent, a concessional investment up to the full financing amount of the asset (the donor's rate of return is a function of its participation), or a combination of grants and concessional investments.

For a given portfolio of assets and a given donor cost of funding, the figure below plots the ordered pairs of the donor's expected present cost and the standard deviation of that cost, using the calculations developed in the paper. The vertical axis shows the donor's expected present cost. The horizontal axis shows the standard deviation of present cost. Each red dot represents a possible intervention available to the donor.



The donor prefers interventions that use fewer resources and produce more predictable outcomes. Any intervention located below and to the left of another will always be preferred, regardless of the donor's risk tolerance. Interventions that combine lower expected cost with lower uncertainty are therefore strictly preferred.[19]

[19] This first order stochastic dominance also applies for any coordinate that is either at the same level vertically but below another coordinate, or at the same level horizontally but to the left of another coordinate (that is, for an equal mean, the intervention with a lower variance will be preferred, and vice-versa).

Point A represents the case in which the donor provides its concessional support entirely as a grant. The donor's present cost is then equal to the aggregate subsidy equivalent required for the portfolio. Since the cost is certain, the standard deviation is equal to zero. Point B represents the lowest cost boundary among concessional investments, where the donor's nominal participation is equal to the aggregate subsidy equivalent but is provided as concessional investment rather than as a grant. This shows that the donor will always prefer a grant equal to the subsidy equivalent over a concessional investment of the same amount.[20] Point E represents the opposite boundary case, in which the donor fully finances the entire portfolio (assuming the donor is still within its maximum budget).

Points A and C–D–E, shown by the green line, form the donor's efficiency frontier. Whatever the donor's preferences, its preferred intervention must lie on this frontier. Points away from the frontier are inefficient because another feasible intervention offers either a lower expected cost for the same level of risk, a lower level of risk for the same expected cost, or an improvement in both.

The donor's risk tolerance is represented on the chart by indifference curves. At any two coordinates on the same curve, the donor is indifferent between the corresponding interventions. The optimal intervention is the point at which the lowest attainable indifference curve is tangent to the efficiency frontier.

Assume that, for all assets in the portfolio, the investor's required rate of return exceeds the donor's cost of funding.[21] A donor with no tolerance for risk will choose point A, that is, a grant equal to the aggregate subsidy equivalent. As risk tolerance increases, the donor becomes willing to accept greater volatility in exchange for a lower expected cost. Its optimal nominal participation then increases and moves along the efficiency frontier from the upper left to the lower right. As the donor's nominal participation rises, it's expected cost decreases, but the share of financing left to private investors falls.

Donors with risk tolerance close to zero will still choose a grant equal to the aggregate subsidy equivalent rather than a point between B and C.[22] There is also a particular case in which the donor is indifferent between providing a grant equal to the aggregate subsidy equivalent and choosing the concessional investment represented by point C.[23]

Between points C and D, the donor's nominal participation increases as risk tolerance rises, but the composition of that participation across assets remains unchanged. That is, the share of each asset in the donor's nominal participation is fixed, only the size of that participation varies. Point D marks the point at which the donor becomes willing to fully finance at least one asset in the portfolio. Beyond that point, further increases in risk tolerance no longer preserve the same diversification pattern, because the donor can no longer maintain the optimal combination of asset weights. At point E, the donor fully finances the portfolio.[24] This is the upper boundary of participation, so further increases in risk tolerance have no additional effect.

These results should be interpreted with care as the model abstracts from several features of "real-world" blended finance. First, the normality assumptions should be read as an approximation. If donor preferences remain mean-variance, normality matters mainly for calibration. A more realistic default-state distribution may alter the estimated means, variances, and covariances of donor cost, and therefore the quantitative location of donor optima. However, the underlying optimization logic is unchanged. A separate question is whether mean-variance is an adequate description of donor preferences. If donors care about higher-order features of donor cost, such as skewness, fat tails, or tail dependence, then mean and variance may not be sufficient to rank risky allocations.

Second, the assumption that pairwise correlations are strictly positive is useful because, in the cases that support the grant results, it helps ensure that the relevant partial derivatives of the donor's disutility with

[20] See Proposition 3 in §5.2.

[21] For all assets where the donor's cost of funding is higher than the investor's mean rate of return, the donor will choose to extend a grant equal to the subsidy equivalent for that asset.

[22] See Proposition 4 in §5.2.

[23] See Proposition 8 in §5.6.

[24] Note that there might be sets of participation that lead to a mean profit for the donor, as shown in the chart. Also, the donor may be subject to a nominal envelop cap – as modeled in the paper – which may prevent higher participations.

respect to nominal participations are strictly positive over the feasible set. The optimum is then attained at the lower bound, a grant equal to the subsidy equivalent. If some correlations were zero or negative, diversification could be stronger than in the present model, and those results would no longer hold as consistently.

Third, the results abstract from implementation frictions. In practice, transaction costs, screening, monitoring, and legal structuring may alter the donor's optimum. Information asymmetries may do the same, especially where public and private actors price risk differently.

Fourth, the model considers only funded instruments and therefore excludes guarantees and other contingent instruments. This matters because guarantees may be preferred, especially where public and private actors perceive risk differently.

Finally, the paper is deliberately non-normative about donor policy objectives. It isolates the financial optimization problem conditional on an upstream portfolio choice. The results should therefore not be read as prescribing how donors ought to trade off development impact, market creation, and mobilization. Rather, they show what follows once that prior choice has been made.

Overall, the model value lies in clarifying the structure of the problem under a transparent set of assumptions, not in reproducing the full complexity of observed blended-finance practice. The analysis also suggests scope to deepen the framework in ways that would bring it closer to observed practice, including non-normal return distributions, dynamic settings, implementation frictions, guarantees, and a fuller treatment of the interaction between financial structuring, portfolio selection, and donor policy objectives.

7. Policy Recommendations

A useful way to interpret the model is through the idea of an efficient frontier for the donor. For a given portfolio, some allocations of concessional support are clearly inferior to others. They deliver either a higher expected cost for the same level of risk, or a higher level of risk for the same expected cost. The efficient frontier is made up of the allocations that cannot be improved on both dimensions at once. In practical terms, donors should seek structures that lie on this frontier rather than inside it. This means choosing the combination of instruments and the size of nominal participations that best matches their tolerance for risk and their opportunity cost proxy – whether it is cost of funding or other benchmarks.

The efficient frontier also gives a clear role to diversification. A donor portfolio should not be treated as a collection of isolated transactions. Its risk depends not only on the volatility of each asset, but also on how those assets move together. Better diversification improves the donor's efficient set by reducing risk for a given expected cost or reducing expected cost for a given level of risk.

The more diversified the portfolio, the better this trade-off is likely to be. In practice, this suggests that donor portfolios and pooled concessional vehicles should be designed with explicit diversification rules. These may include industry limits, country exposure limits, regional balance, and objective-based allocation rules built around observed or estimated correlations. Where correlation data are available, they should inform portfolio construction more directly than is often the case in current practice. Notably, the benefits of diversification may extend beyond financial considerations to development impact, as the latter may be predicated on the financial success of the project.

This does not mean that diversification should override policy purpose. Donors may still choose concentrated portfolios because of development mandates, strategic priorities, or impact objectives. But the cost of doing so should be understood clearly. A concentrated portfolio may move the donor away from a better feasible allocation and weaken the trade-off between expected cost and risk. For that reason, concentration should be the result of policy choice, not a by-product of weak portfolio design.

Under the model, the central policy trade-off for the donor is primarily between budget certainty and reducing expected donor cost. A secondary trade-off is that allocations that reduce expected cost also reduce private capital mobilization. These trade-offs depend on donor preferences. Greater donor risk tolerance leads the donor to take a larger nominal share of the financing structure, which lowers expected present cost but mechanically leaves less room for private capital participation. Whether the donor

should internalize this trade-off depends on its motivations. If leverage is valued only insofar as it allows scarce concessional resources to reach more projects, then the donor may rationally accept lower leverage when the reduction in expected donor cost is sufficiently valuable. If, however, private mobilization carries independent policy value, then it should enter the donor's objective function explicitly.

The donor's own cost of funds also matters (or alternative opportunity cost proxies). Where the donor's funding cost exceeds the investor's required rate of return for a given asset, the preferred instrument is a grant equal to the subsidy equivalent for that asset. This may be especially relevant for donors that face high funding costs or shadow costs of public funds. Perhaps counterintuitively, grants may also be more appropriate in lower-risk settings, where investor return requirements are lower.

At the portfolio level, the most important factors are the variance in rates of returns and the extent to which assets move together. This does not imply that mean rates of return are irrelevant in general. They may matter at the earlier stage of portfolio selection. But once the donor has decided which assets to support, the model implies that the optimal financing structure is determined by the portfolio's variances and correlations, together with the donor's risk tolerance. In practice, a higher participation in a high-risk investment that requires high concessionality may be preferrable, to the extent it correlates less with other assets in the portfolio.

The model also points to a degree of instrument flexibility. For cases in which the optimal nominal participation exceeds the subsidy equivalent, that participation can be split between a grant component and an investment component without changing the donor's optimum. In practice, some blended finance transactions could therefore be designed in a simpler way, with one component delivered as a grant and the other as a share in commercial returns.

There may also be a case for pooling donors. If several donors share the same objectives, support the same portfolio, and apply similar funding benchmarks, they may be able to invest through a common vehicle even if their tolerance for risk differs. Where such pooling also reduces transaction and administrative costs, it may improve overall efficiency.

Taken together, these last two results point to a possible model for pooled concessional finance. Donor resources could be combined in a common vehicle designed to accept both grant and returnable contributions. That vehicle could then deploy support through a combination of grants and a share in commercial returns. Under the assumptions of the model, such a structure could accommodate donors with similar objectives, a common target portfolio, and comparable opportunity cost benchmarks, even where their tolerance for risk differs.

Overall, the paper suggests that the financial structuring of blended finance deserves more explicit optimization on the donor side. Even where the level of subsidy is fixed by investor requirements and the portfolio is fixed by policy choice, donors may still improve financial efficiency through better choices on instrument mix, nominal participation, and diversification. In a world of tighter aid budgets, that imperative is likely to become more important rather than less.

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Annex 2. Overview of Model's Variables

Name	Description	Type	Dimension	Mathematical representation
i	An asset in need of subsidy to be financed. The value of each i is assumed to be equal to 1	Exogenous	Index	$i=\{1,2\ldots,n\}$
n	Amount of assets i	Exogenous	Quantity	$n \in \mathbb{Z}_*^+$
p	The portfolio of n assets i	Exogenous	Identifier	n/a
t	t is a fixed time interval between t_0 and t_1, two discrete points in time	Exogenous	Period	n/a
ra_i	The rate of return of asset i over the period t	Exogenous	Rate	$ra_i \in \mathbb{R}, \forall i: ra_i < rm_i$
rm_i	The rate of return received by the investor under the donor's contract	Endogenous	Rate	See Definition 2
rd	The donor's cost of funding over the period t	Exogenous	Rate	$rd \in \mathbb{R}_*^+$
rc_i	The subsidized rate of return for the donor for asset i over the period t	Endogenous	Rate	See Lemma 1
s_i	The required subsidy equivalent for asset i over the period t	Endogenous	Discounted amount	See Definition 4
g_i	The present cost of the donor's concessional financing of asset i over the period t	Endogenous	Discounted amount	See Definition 5
g_p	The present cost of the donor's concessional financing of the portfolio p of assets i over the period t	Endogenous	Discounted amount	$g_p = \sum_{i=1}^{n} g_i$
b	The donor's aggregate participation in the in the portfolio p	Exogenous	Amount	$b \in \mathbb{R}$, $\sum_{i=1}^{n} s_i \le b_{max} \le n$
b_{max}	The maximum donor's	Exogenous	Amount	$b_{max} \in \mathbb{R}$,

	nominal participation in p at t_0			$\sum_{i=1}^{n} s_i \leq b_{max} \leq n$
v_i	The donor's nominal participation in asset i at t_0	Exogenous	Amount	$v_i \in \mathbb{R}, \forall i: s_i \leq v_i \leq 1$ $\sum_{i=1}^{n} v_i = b \leq b_{max}$
x_i	The amount of grant in the donor's concessional financing of asset i at t_0 when partly extending it as grant and as investment	Exogenous	Amount	$x_i \in \mathbb{R}_*^+, \forall i: 0 \leq x_i \leq s_i$
y_i	The nominal amount of investment from the donor in asset at t_0 when partly extending it as grant and as investment	Exogenous	Amount	$y_i \in \mathbb{R}_*^+, \forall i: 0 \leq y_i \leq v_i$
w_i	The weight of the donor's participation in asset i relative to its participation in the portfolio p	Exogenous	Weight	$w_i \in \mathbb{R}, \forall i: 0 \leq w_i \leq 1$ $\sum_{i=1}^{n} w_i = 1$
μ_{ra_i}	The mean rate of return of ra_i over the period t	Exogenous	Rate	$\mu_{ra_i} \in \mathbb{R}$
μ_{rm_i}	The investor's required mean return	Exogenous	Rate	Defined in §3.2
μ_{rc_i}	The mean rate of return of rc_i over the period t	Endogenous	Rate	$\mu_{rc_i} \in \mathbb{R}$
μ_{g_i}	The donor's mean present cost at t_0	Endogenous	Discounted amount	$\mu_{g_i} \in \mathbb{R}$
$\sigma^2_{ra_i} / \sigma_{ra_i}$	Variance and standard deviation of ra_i over the period t	Exogenous	Rate	$\sigma^2_{ra_i}, \sigma_{ra_i} \in \mathbb{R}$
$\sigma^2_{rm_i} / \sigma_{rm}$	Variance and standard deviation of rm_i over the period t	Exogenous	Rate	$\sigma^2_{rm_i}, \sigma_{rm_i} \in \mathbb{R}$
$\sigma^2_{rc_i} / \sigma_{rc_i}$	Variance and standard deviation of rc_i over the period t	Endogenous	Rate	$\sigma^2_{rc_i}, \sigma_{rc_i} \in \mathbb{R}$
$\sigma^2_{g_i} / \sigma_{g_i}$	Variance and standard deviation of g_i over the	Endogenous	Discounted amount	$\sigma^2_{g_i}, \sigma_{g_i} \in \mathbb{R}$

	period t			
$\rho_{ii'}$	Correlation between ra_i and $ra_{i'}$ over the period t	Exogenous	Percentage	$\rho_{ii'} \in \mathbb{R}, \forall a, b : -1 \leq \rho$ $\rho_{ii'} = \frac{Cov(ra_i, ra_{i'})}{\sigma_{ra_i}\sigma_{ra_{i'}}}$
$D¿$	Donor's disutility function	Exogenous	Function	Also expressed as $D(v_1, \ldots, v_n)$
$\Delta D¿$	Gradient of $D¿$	Exogenous	Function	n/a
l	Private capital leverage	Exogenous	Percentage	See Definition 14
A	The vector of the mean rates of return for each asset i over the period t	Exogenous	Vector	$A = (\mu_{ra_1}, \ldots, \mu_{ra_n})$
M	The vector of the mean rate of return threshold required by the investor for each asset i over the period t	Exogenous	Vector	$M = (\mu_{rm_1}, \ldots, \mu_{rm_n})$
D	the n dimension vector of the donor's cost of funding rd over the period t	Exogenous	Vector	$D = (rd, \ldots, rd)$
V	The vector of nominal participations of the donor in each asset i at t_0	Exogenous	Vector	$V = (v_1, \ldots, v_n)$
X	The vector of grant participation x_i in each asset i at t_0	Exogenous	Vector	$X = (x_1, \ldots, x_n)$
Y	The vector of the donor's nominal participation in commercial investments for each asset i at t_0	Endogenous	Vector	$Y = (y_1, \ldots, y_n)$
S	The vector of subsidy equivalents for each asset i over the period t	Endogenous	Vector	$S = (s_1, \ldots, s_n)$
$\sum$	The covariance matrix of the rates of return ra_i over the period t	Exogenous	Matrix	See Definition 9

Annex 3. Mathematical Demonstrations

Annex 31 Rate of Return of the Donor Concessional Investment

Recall from Definition 2:

$$rm_i = ra_i + \mu_{rm_i} - \mu_{ra_i}$$

Therefore:

$$rc_i = \frac{1 + ra_i - (1 - v_i)(1 + rm_i) - v_i}{v_i}$$

$$= \frac{1 + ra_i - (1 - v_i)(1 + ra_i + \mu_{rm_i} - \mu_{ra_i}) - v_i}{v_i}$$

$$= \frac{1 + ra_i - 1 - ra_i - \mu_{rm_i} + \mu_{ra_i} + v_i + v_i ra_i + v_i \mu_{rm_i} - v_i \mu_{ra_i} - v_i}{v_i}$$

$$= ra_i + \mu_{rm_i} - \mu_{ra_i} + \frac{\mu_{ra_i} - \mu_{rm_i}}{v_i} \blacksquare$$

Annex 32 Variance of Investor's and Donor's Rate of Return

Recall that:

$$rm_i = ra_i + \mu_{rm_i} - \mu_{ra_i}$$

Therefore:

$$\sigma^2_{rm_i} = E\left[\left(rm_i - \mu_{rm_i}\right)^2\right] = E\left[\left(ra_i - \mu_{ra_i}\right)^2\right] = \sigma^2_{ra_i} \blacksquare$$

Recall that:

$$rc_i = ra_i + \mu_{rm_i} - \mu_{ra_i} + \frac{\mu_{ra_i} - \mu_{rm_i}}{v_i}$$

And:

$$\mu_{rc_i} = \mu_{rm_i} + \frac{\mu_{ra_i} - \mu_{rm_i}}{v_i}$$

Thus:

$$
\begin{aligned}
& rc_i - \mu_{rc_i} \\
&= ra_i + \mu_{rm_i} - \mu_{ra_i} + \frac{\mu_{ra_i} - \mu_{rm_i}}{v_i} - \mu_{rm_i} - \frac{\mu_{ra_i} - \mu_{rm_i}}{v_i} \\
&= ra_i - \mu_{ra_i}
\end{aligned}
$$

And:

$$\sigma^2_{rc_i} = E\left[\left(rc_i - \mu_{rc_i}\right)^2\right] = E\left[\left(ra_i - \mu_{ra_i}\right)^2\right] = \sigma^2_{ra_i} \blacksquare$$

Annex 33 Investor's Mean Rate of Return when Receiving s_i as Grant

Recall that:

$$s_i = \frac{\mu_{rm_i} - \mu_{ra_i}}{1 + \mu_{rm_i}}$$

Therefore:

$$
\begin{aligned}
& \frac{\left(1 + \mu_{ra_i}\right) - \left(1 - s_i\right)}{\left(1 - s_i\right)} \\
&= \frac{\mu_{ra_i} + \dfrac{\mu_{rm_i} - \mu_{ra_i}}{1 + \mu_{rm_i}}}{\left(1 - \dfrac{\mu_{rm_i} - \mu_{ra_i}}{1 + \mu_{rm_i}}\right)} \\
&= \frac{\dfrac{\mu_{ra_i} + \mu_{ra_i}\mu_{rm_i} + \mu_{rm_i} - \mu_{ra_i}}{1 + \mu_{rm_i}}}{\dfrac{1 + \mu_{rm_i} - \mu_{rm_i} + \mu_{ra_i}}{1 + \mu_{rm_i}}} \\
&= \frac{\mu_{ra_i}\mu_{rm_i} + \mu_{rm_i}}{1 + \mu_{ra_i}} \\
&= \mu_{rm_i} \blacksquare
\end{aligned}
$$

Note that rate of return for the investor when receiving a grant s_i at t_0 may be different than when receiving the subsidized return $rm_i = \mu_{rm_i} - \mu_{ra_i}$ at t_1. Only the mean rate of return is the same. Note however that the investor's net present values are always the same, as shown below.

Let NPV_{rm_i} and NPV_{s_i} be the investor's net present value when respectively receiving the subsidized return $rm_i = \mu_{rm_i} - \mu_{ra_i}$ at t_1 and receiving a grant s_i at t_0

$$NPV_{rm_i} = -1 + \frac{1 + ra_i + \mu_{rm_i} - \mu_{ra_i}}{1 + \mu_{rm_i}}$$

$$NPV_{s_i} = -1 + s_i + \frac{1 + ra_i}{1 + \mu_{rm_i}} = -1 + \frac{\mu_{rm_i} - \mu_{ra_i}}{1 + \mu_{rm_i}} + \frac{1 + ra_i}{1 + \mu_{rm_i}} = NPV_{rm_i} \blacksquare$$

Annex 34 Mean, Variance and Standard Deviation of Donor's Present Cost

Through the linearity of expectation:

$$\mu_{g_i} = E\left[g_i\right] = E\left[\frac{v_i\left(rd - ra_i - \mu_{rm_i} + \mu_{ra_i}\right) + \mu_{rm_i} - \mu_{ra_i}}{1 + rd}\right] = \frac{v_i\left(rd - \mu_{rm_i}\right) + \mu_{rm_i} - \mu_{ra_i}}{1 + rd} \blacksquare$$

Recall that:

$$\sigma^2_{rc_i} = E\left[\left(rc_i - \mu_{rc_i}\right)^2\right] = E\left[\left(ra_i - \mu_{ra_i}\right)^2\right] = \sigma^2_{ra_i}$$

Since:

$$g_i - \mu_{g_i} = \frac{v_i\left(rd - rc_i\right)}{1 + rd} - \frac{v_i\left(rd - \mu_{rc_i}\right)}{1 + rd} = \frac{v_i\left(\mu_{rc_i} - rc_i\right)}{1 + rd}$$

Then:

$$\sigma^2_{g_i} = E\left[\left(g_i - \mu_{g_i}\right)^2\right] = E\left[\left(\frac{v_i\left(\mu_{rc_i} - rc_i\right)}{1 + rd}\right)^2\right] = \left(\frac{v_i}{1 + rd}\right)^2 E\left[\left(rc_i - \mu_{rc_i}\right)^2\right]$$

$$= \left(\frac{v_i}{1 + rd}\right)^2 \sigma^2_{ra_i} \blacksquare$$

Annex 35 Mean Donor Present Cost and Asset's Subsidy Equivalent

Suppose $v_i = s_i$ and recall that:

$$s_i = \frac{\mu_{rm_i} - \mu_{ra_i}}{1 + \mu_{rm_i}}$$

And:

$$\mu_{g_i} = \frac{v_i\left(rd - \mu_{rm_i}\right) + \mu_{rm_i} - \mu_{ra_i}}{1 + rd}$$

Then:

$$\mu_{g_i} = \frac{\frac{\mu_{rm_i} - \mu_{ra_i}}{1 + \mu_{rm_i}}\left(rd - \mu_{rm_i}\right) + \mu_{rm_i} - \mu_{ra_i}}{1 + rd}$$

$$= \frac{\left(\mu_{rm_i} - \mu_{ra_i}\right)\left(rd - \mu_{rm_i}\right) + \left(\mu_{rm_i} - \mu_{ra_i}\right)\left(1 + \mu_{rm_i}\right)}{\left(1 + rd\right)\left(1 + \mu_{rm_i}\right)}$$

$$= \frac{\mu_{rm_i} rd - \mu_{rm_i}^2 - \mu_{ra_i} rd + \mu_{ra_i}\mu_{rm_i} + \mu_{rm_i} + \mu_{rm_i}^2 - \mu_{ra_i} - \mu_{ra_i}\mu_{rm_i}}{\left(1 + rd\right)\left(1 + \mu_{rm_i}\right)}$$

$$= \frac{\mu_{rm_i} rd - \mu_{ra_i} rd + \mu_{rm_i} - \mu_{ra_i}}{\left(1 + rd\right)\left(1 + \mu_{rm_i}\right)}$$

$$= \frac{\left(1 + rd\right)\left(\mu_{rm_i} - \mu_{ra_i}\right)}{\left(1 + rd\right)\left(1 + \mu_{rm_i}\right)}$$

$$= s_i \blacksquare$$

Annex 36 Present Cost Variance of the Donor's Portfolio

Since:

$$Cov\left(a, b\right) = E\left[\left(a - E\left[a\right]\right)\left(b - E\left[b\right]\right)\right]$$

Then:

$$Cov\left(g_i, g_{i'}\right) = E\left[\left(g_i - E\left[g_i\right]\right)\left(g_{i'} - E\left[g_{i'}\right]\right)\right]$$

Recall that:

$$g_i - E\left[g_i\right] = \frac{v_i\left(\mu_{rc_i} - rc_i\right)}{1 + rd}$$

Then:

$$Cov(g_i, g_{i'}) = E\left[\frac{v_i(\mu_{rc_i} - rc_i)}{1+rd} * \frac{v_{i'}(\mu_{rc_{i'}} - rc_{i'})}{1+rd}\right]$$

$$= \frac{v_i v_{i'}}{(1+rd)^2} E\left[(-1)(rc_i - \mu_{rc_i})(-1)(rc_{i'} - \mu_{rc_{i'}})\right]$$

Recall that:

$$rc_i - \mu_{rc_i} = ra_i - \mu_{ra_i}$$

Therefore:

$$Cov(g_i, g_{i'}) = \frac{v_i v_{i'}}{(1+rd)^2} E\left[(ra_i - \mu_{ra_i})(ra_{i'} - \mu_{rs_{i'}})\right] = \frac{v_i v_{i'}}{(1+rd)^2} Cov(ra_i, ra_{i'}) \blacksquare$$

Annex 37 Gradient of Donor's Disutility Function

Recall that:

$$D(V) = \gamma \frac{1_n^T(M - A)}{1+rd} + \gamma \frac{V^T(D - M)}{1+rd} + \frac{1}{2}\left(\frac{1}{1+rd}\right)^2 V^T \sum V$$

Where the first term is constant, the second term linear and the third term is quadratic. Thus:

$$\nabla D(V) = \gamma \frac{(D - M)}{1+rd} + \frac{1}{2}\left(\frac{1}{1+rd}\right)^2 (\sum + \sum^T) V$$

$\sum$ is symmetric, so $\sum = \sum^T$. This simplifies to:

$$\nabla D(V) = \gamma \frac{(D - M)}{1+rd} + \frac{1}{2}\left(\frac{1}{1+rd}\right)^2 2\sum V = \gamma \frac{(D - M)}{1+rd} + \left(\frac{1}{1+rd}\right)^2 \sum V \blacksquare$$

Annex 38 Rate of Return of the Donor Concessional Investment when Partly Extending a Grant

Recall from Definition 2:

$$rm_i = ra_i + \mu_{rm_i} - \mu_{ra_i}$$

Therefore:

$$rc_{y_i} = \frac{1 + ra_i - (1 - v_i)(1 + rm_i) - y_i}{y_i}$$

$$= \frac{1 + ra_i - (1 - v_i)(1 + ra_i + \mu_{rm_i} - \mu_{ra_i}) - y_i}{y_i}$$

$$= \frac{1 + ra_i - 1 - ra_i - \mu_{rm_i} + \mu_{ra_i} + v_i + v_i ra_i + v_i \mu_{rm_i} - v_i \mu_{ra_i} - y_i}{y_i}$$

$$= \frac{v_i(1 + ra_i + \mu_{rm_i} - \mu_{ra_i}) - \mu_{rm_i} + \mu_{ra_i}}{y_i} - 1 \blacksquare$$

Annex 39 Donor Present Cost with Grant Portion

The same discounting exercise than for the donor's present cost is performed, but this time the cash flows at t_1 are limited to the investment portion of the donor's concessional financing, returning the rate of rc_{y_i} (since the grant portion x_i doesn't return cash flows). Recall that:

$$rc_{y_i} = \frac{v_i(1 + ra_i + \mu_{rm_i} - \mu_{ra_i}) - \mu_{rm_i} + \mu_{ra_i}}{y_i} - 1$$

$$= \frac{v_i(1 + ra_i + \mu_{rm_i} - \mu_{ra_i}) - \mu_{rm_i} + \mu_{ra_i} - y_i}{y_i}$$

Then:

$$g_{x_i} = v_i - y_i \frac{1 + rc_{y_i}}{1 + rd}$$

$$\Rightarrow g_{x_i} = v_i - y_i \frac{1 + \frac{v_i(1 + ra_i + \mu_{rm_i} - \mu_{ra_i}) - \mu_{rm_i} + \mu_{ra_i} - y_i}{y_i}}{1 + rd}$$

$$\Rightarrow g_{x_i} = v_i - \frac{v_i(1 + ra_i + \mu_{rm_i} - \mu_{ra_i}) - \mu_{rm_i} + \mu_{ra_i}}{1 + rd}$$

$$\Rightarrow g_{x_i} = \frac{v_i + v_i rd - v_i - v_i ra_i - v_i \mu_{rm_i} + v_i \mu_{ra_i} + \mu_{rm_i} - \mu_{ra_i}}{1 + rd}$$

$$\Rightarrow g_{x_i} = \frac{v_i(rd - ra_i - \mu_{rm_i} + \mu_{ra_i}) + \mu_{rm_i} - \mu_{ra_i}}{1 + rd}$$

$$\Rightarrow g_{x_i} = g_i \blacksquare$$

Annex 310 Donor Present Cost Mean, Variance and Standard Deviation with Grant Portion

For the donor's mean present cost, based on the previous demonstration:

$$\mu_{g_{x_i}} = E\left[g_{x_i}\right] = E\left[g_i\right] = \mu_{g_i} = \frac{v_i\left(rd - \mu_{rm_i}\right) + \mu_{rm_i} - \mu_{ra_i}}{1 + rd} \blacksquare$$

For the donor's present cost variance, based on the previous two demonstrations:

$$\sigma^2_{g_{x_i}} = E\left[\left(g_{x_i} - \mu_{g_{x_i}}\right)^2\right] = E\left[\left(g_i - \mu_{g_i}\right)^2\right] = \sigma^2_{g_i} = \left(\frac{v_i}{1 + rd}\right)^2 \sigma^2_{ra_i} \blacksquare$$