Do production frictions affect the impact of sustainable investing?

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April 2026†

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Abstract

Prior studies focus on how investors' sustainability preferences incentivize firms to reallocate capital from dirty to clean production. However, the impact depends critically on the production technology. I provide the first dynamic structural estimates and find that dirty and clean capital are highly complementary (elasticity of substitution ≈ 0.56), dirty capital is more productive (productivity share ≈ 0.81), and clean capital is much costlier to adjust than dirty. Together these features make ESG demand pressure ineffective or even counterproductive. Counterfactual analysis of carbon taxes and R&D subsidies further highlights this fundamental production friction as a binding constraint.

JEL Codes: G11, G12, D24, C61

Keywords: Sustainable investing, capital complementarity, ESG demand, climate finance, structural estimation

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†I am deeply grateful to Itzhak Ben-David, Andrei Gonçalves, René Stulz (Co-Chair), and Lu Zhang (Co-Chair) for their continuous guidance. I am grateful for the helpful comments received from Justin Birru, Yongyang Cai, Aditya Chaudhry, Rebecca De Simone (discussant), Thomas Flanagan, Deeksha Gupta, Sangeun Ha, Byungwook Kim, Hyung Joo Kim, Minsu Ko, Kai Li (discussant), Yicheng Liu, Tom Meling, Alessandro Melone, Lasse Pedersen, Christoph Schiller, Sergey Sarkisyan, Kris Shen, Daisy Wang, Michael Weisbach, Ingrid Werner, Michael Wittry, Yufeng Wu, Qiping Xu (discussant), and Shaojun Zhang. I thank the conference and seminar participants at the Ohio State University, Texas Christian University, the University of Pittsburgh, the University of Notre Dame, Michigan State University, the University of Minnesota, Cornell University, the University of Michigan, the London Business School, the University of Cincinnati, the Chinese University of Hong Kong, the 2024 Financial Management Association Conference, the 37th Australian Finance and Banking Conference, the 2024 Colorado Finance Summit Job Market Session, the 2025 World Symposium on Investment Research and the 20th Early Career Women in Finance Conference for their helpful comments and suggestions. I thank Jack Fan for his excellent research assistance. All errors are my own. **Disclosure:** This research received no external funding. The author has no financial or other potential conflicts of interest to disclose.

1 Introduction

Investors representing over $121 trillion in assets have signed onto the UN Principles for Responsible Investment (PRI, 2025), yet global carbon emissions continue to set new records (Global Carbon Project, 2025). How effective is sustainable investment in making firms cleaner? Prior studies have focused on investors' nonpecuniary preferences for green stocks or their risk aversion to climate change, arguing these raise the cost of capital for dirty firms and redirect investment toward cleaner technologies (Pástor, Stambaugh, and Taylor, 2021; Dangl et al., 2023; Hong, Wang, and Yang, 2023), while a separate set of studies finds that these cost-of-capital effects have little measurable impact on firm-level emission outcomes (Hartzmark and Shue, 2022; Heath et al., 2023). An implicit assumption across both literatures is a frictionless transition from dirty to clean capital in the production function, so that firms' investment decisions depend only on investors' sustainability demands. This paper shows that the production technology itself acts as a binding friction: dirty and clean capital are highly complementary and strongly asymmetric in productivity and adjustment cost, so even when sustainable investing successfully raises the cost of capital for dirty firms, it fails to reallocate capital toward cleaner assets.

The argument depends on what "dirty capital" and "clean capital" mean and on how readily firms can substitute one for the other in production. In my framework, "dirty capital" refers to physical assets that contribute to a firm's production while generating carbon emissions, such as factories and machinery. "Clean capital" refers to physical assets that contribute to a firm's production without generating carbon emissions. To illustrate the difference between complementarity and substitutability, consider two examples. Tesla requires clean gigafactories as well as dirty factories and supply chains that produce the lithium and other inputs for its electric car batteries. Without either the clean or dirty components, production halts, demonstrating high complementarity.[1] In contrast, consider an AI company that relies on supercomputers that generate

[1]This example illustrates complementarity and does not imply that Tesla is harmful to the environment or that electric cars should be abandoned. Rather, this example underscores that dirty capital is complementary to clean capital and thus vital to production across many industries and the entire economy.

carbon emissions (dirty) and computer algorithms (clean). If the AI company had an ample supply of clean energy, it could easily switch to sources such as wind or water to power its supercomputers, demonstrating high substitutability between dirty capital and clean capital.

In the standard constant elasticity of substitution (CES) production function, several features determine whether ESG demand pressure can tilt capital composition toward cleaner assets. One is ν, the elasticity of substitution between dirty and clean capital, which measures the complementarity level: when ν is high, firms can swap clean for dirty easily; when ν is low, the two types are gross complements and locked together. Another is α, the productivity share of dirty capital: when α is high, dirty capital is the more productive input and firms are reluctant to reduce it regardless of investor pressure. A further feature is the relative cost of adjusting each capital type: when clean is costlier to adjust than dirty, firms preferentially shrink clean under shrinkage pressure. Prior studies either assume $\nu = 1$ (Cobb-Douglas) or close to $\nu = \infty$ (perfect substitutes) and calibrate α to assumed steady-state ratios when modeling clean and dirty capital stocks (Barnett et al., 2023a; Gormsen, Huber, and Oh, 2024; Lanteri and Rampini, 2023; Li, Wang, and Xu, 2024); related static estimates exist on the *energy-input* margin (Jo, 2025) but not on the within-firm physical-capital margin. I structurally estimate these features from firm-level data. The headline finding is high complementarity, reinforced by strong asymmetries between dirty and clean capital. These features operate within the two financial mechanisms described in the next paragraph.

This paper examines the real effects of sustainable investing through two mechanisms, corresponding to the two main lines of models in the climate finance literature. The first follows the classical climate economics tradition (Nordhaus and Sztorc, 2013; Barnett, Brock, and Hansen, 2020; Barnett et al., 2023a): dirty capital generates pollution that reduces firm output through a damage function, and investors with sustainability concerns price this damage risk in the stochastic discount factor.[2] This first mechanism operates primarily through firm cash flows: climate damage

[2]Standard climate-economics models often include additional layers that link the damage function to an increased frequency or severity of natural disasters due to global warming. This paper abstracts from those extra layers, allowing for a broader interpretation of the damage function that encompasses, for example, the reputational cost of high carbon emissions. The effect of the environment on productivity is better understood as an externality;

directly reduces output. The second follows the portfolio-tilt and flow-driven investor demand tradition (Pástor, Stambaugh, and Taylor, 2021; Pastor, Stambaugh, and Taylor, 2023; Koijen and Yogo, 2019; Van der Beck, 2023): sustainability-conscious investors tilt their portfolios toward green firms and away from brown firms, and the resulting demand wedge raises the relative cost of capital for high-emission firms through the Koijen-Yogo demand system. This second mechanism operates primarily through firm valuations: portfolio tilts shift the relative price of brown versus green firms without altering their underlying cash flows. Both mechanisms are developed in a tractable two-period analytical model and in a fully dynamic calibrated model.

In both models, sustainable investing makes dirty capital less attractive at the margin, but the route differs. In the damage model, the firm sees lower expected cash flows on dirty capital because the damage shock hits dirty-firm output, and a higher risk-adjusted discount rate because that shock correlates with bad aggregate states. The first-order condition for dirty investment therefore tilts toward clean. In the flow-driven model, cash flows are unchanged: instead, ESG portfolio tilts give green firms a valuation premium and brown firms a discount, so the firm directly faces a higher cost of capital on dirty assets. The first-order condition again tilts toward clean. In both cases, the size of the firm's response depends primarily on the substitutability ν. When clean and dirty are gross complements (low ν), the firm cannot reallocate much regardless of the route through which the price signal arrives; the productivity and adjustment-cost asymmetries then determine which capital type shrinks the most.

Whether the U.S. economy actually sits in this regime is an empirical question. To answer it, I estimate ν from firm-level data using two independent strategies. For the reduced-form estimate of ν, I transform the CES first-order condition into a panel regression following Chirinko and Mallick (2017) and estimate it on a firm-year panel using the Bai (2009) interactive fixed-effects estimator, which accounts for time-varying unobservable productivity. Within-firm covariation

full internalization of the damage function is therefore best interpreted through the lens of shareholder welfare maximization with sustainable investors (see Hart and Zingales, 2017 and Edmans, 2021). Hart and Zingales (2017) demonstrates that if profit and damage are inseparable, shareholder welfare is not equivalent to market value, making shareholder welfare the appropriate objective function in this context.

between emission intensity and the marginal productivity of emissions, after the interactive fixed effects strip out time-varying productivity, identifies the substitutability parameter. For the structural estimates of ν, I combine a dynamic structural model of corporate investment in the tradition of Hennessy and Whited (2007), Strebulaev and Whited (2012), and Bazdresch, Kahn, and Whited (2018) with the Koijen and Yogo (2019) demand system, and estimate the full parameter vector by SMM, matching moments on capital allocation, equity returns, and the cross-sectional relationship between ESG flows and emission intensity. I use the Koijen and Yogo (2019) demand system because it generates a firm-specific cost-of-capital wedge from observable investor holdings, allowing ESG flows to translate into firm-level price impacts that vary across firms with their emission intensity and other characteristics.[3] The two strategies use distinct sources of variation in the data, so the fact that they deliver the same estimate of ν is strong evidence that the result is not driven by any single identification choice.

Both estimation approaches deliver $\nu \approx 0.56$ with tight confidence intervals that exclude Cobb-Douglas ($\nu = 1$) at the 1% level: dirty and clean capital are gross complements by a wide margin. The structural estimate of the productivity share is $\alpha \approx 0.81$, meaning dirty capital is by a substantial margin the more productive factor in U.S. production.[4] Clean capital is also estimated to be much costlier to adjust than dirty. Industry-level estimates vary in level but remain below $\nu = 1$ in all ten Fama-French sectors; high-tech and energy are relatively more substitutable than the others.[5] Over the sample period, the industry-average ν rises from 0.36 in 2006 to 0.61 in 2022,

[3]The Koijen-Yogo framework delivers a firm-by-firm cost-of-capital wedge as a function of estimated investor characteristics, which is what allows ESG flow tilts and firm-level emission intensity to be jointly informative about ν. Choi et al. (2025) use the same demand system for related joint-estimation purposes; this paper extends that framework to a CES production block with two physical capital types.

[4]This α estimate is consistent with external benchmarks. Globally, fossil fuels supply roughly 82% of primary energy as of 2023 (International Energy Agency, 2024), and at the model-implied benchmark, the steady-state dirty-capital ratio $K_D/(K_C + K_D) \approx 70\%$ closely matches the share of physical capital (PPEGT) held by firms with above-median emission intensity in Compustat-Trucost ($\approx 69\%$; see Appendix B.2 for the construction).

[5]Macro-level estimates of the elasticity of substitution between clean and dirty *energy* inputs are typically above one. Papageorgiou, Saam, and Schulte (2017), for example, estimate values around 1.8 across OECD country-sector panels, and Jo (2025) obtains a similarly elastic estimate from French manufacturing firms. These estimates are at a different unit of analysis: they measure substitution at the energy-flow margin (where switching from coal to wind generators is comparatively easy) and aggregate over within- and across-firm variation, while my estimates are at the within-firm physical-capital margin (where adjustment is sticky). Qualitatively my estimates agree that energy sectors exhibit relatively high substitutability between clean and dirty inputs; quantitatively, substitutability

but substantial complementarity remains. The estimated (ν, α) pair places the U.S. economy in the corner where ESG demand pressure cannot reallocate capital as intended.

Why does ESG pressure fail to reallocate capital under these estimates? Consider how the flow-driven model maps ESG pressure to firm capital allocation. As ESG fund demand grows, the Koijen-Yogo system assigns a valuation premium to cleaner firms and a discount to dirtier ones, raising the relative cost of capital for high-emission firms and creating an incentive in the firm's first-order conditions to shift investment toward clean capital. Under perfect substitution, this mechanism works as designed: the firm tilts toward clean capital and emission intensity falls as ESG demand strengthens from no demand pressure to a strong tilt against high-emission firms. The damage-function model shows the same dampening through a different channel, the cash flow channel: under the estimated complementarity, the dirty-capital response is roughly one-third of its perfect-substitution magnitude.

Surprisingly, the same ESG pressure can be counterproductive in the flow-driven model: under the benchmark estimates ($\nu = 0.56$ and $\alpha = 0.81$), the sign of the response flips, and emission intensity rises rather than falls as ESG demand strengthens.[6] Several structural features of the production technology interact to produce this reversal. Complementarity ($\nu = 0.56$) means that when dirty capital contracts in response to a higher cost of capital, its marginal productivity rises sharply, largely offsetting the demand penalty. Because neither capital type can fully substitute for the other, both must contract together. Most important, clean capital is roughly five times more costly to adjust than dirty in the SMM estimates, so firms preferentially let clean depreciate while continuing to maintain dirty. The productivity asymmetry ($\alpha = 0.81$) reinforces this asymmetric contraction but is not what flips the sign of the response: the perverse positive response is

is naturally lower at the within-firm *capital-stock* margin than at the aggregate *energy-input* margin.

[6] Both Hartzmark and Shue (2022) and this paper find that sustainable investing can produce a counterproductive capital allocation, but the mechanisms differ. In Hartzmark and Shue (2022), the counterproductive result arises from the financing channel itself: ESG price pressure can misdirect capital by undervaluing brown stocks relative to green. In this paper, the financing channel works as designed (ESG demand correctly raises the relative cost of capital for high-emission firms), but the production friction overrides it. The distinction matters for policy: correcting the ESG pricing signal addresses the mechanism in Hartzmark and Shue (2022), while addressing the production-side mechanism requires changing the production technology itself.

even larger in counterfactuals with $\alpha < 0.5$ as long as the adjustment-cost asymmetry remains. Conversely, equalizing the adjustment costs of clean and dirty capital substantially relieves the perverse response, though the sign remains positive at the estimated complementarity.

Now consider the reverse question: what if substitutability were raised, holding everything else equal? The same production friction delivers a second result that runs opposite to the standard intuition. Holding all else equal, directly raising the substitutability between dirty and clean capital, for example through R&D that lets clean stand in for dirty, raises the level of dirty capital and emission intensity rather than lowering them. Once firms are freed from complementarity, they concentrate even more in dirty capital because it is much cheaper to adjust than clean capital and (at the SMM benchmark) more productive as well. The cost asymmetry is what flips the sign of the response: even when clean is the marginally more productive input ($\alpha < 0.5$), the high cost of maintaining clean still leads firms to shrink it faster than dirty. At the benchmark, raising substitutability from $\nu = 0.56$ to perfect substitution roughly triples dirty capital and total emissions and pushes the dirty share above 90%. ESG pressure becomes more effective at higher substitutability (its percentage reduction in dirty capital and emissions is larger), but not enough to overturn the level rise from the productivity and adjustment-cost asymmetries. Both findings point to the same conclusion: in addition to addressing the complementarity issue, making clean capital both more productive and cheaper to adjust is critical for delivering meaningful reductions in dirty capital and emissions.

Turning to asset prices, the model also delivers predictions for the greenium, the expected-return spread between brown and green firms. The greenium is not a targeted moment in the SMM objective because the empirical literature has not agreed on its sign or magnitude.[7] At the

[7]Empirical estimates of the brown-minus-green return spread span roughly −3% to +4% per year. Bolton and Kacperczyk (2021) and Hsu, Li, and Tsou (2023) document positive spreads of about 2–4% in U.S. equities. Pástor, Stambaugh, and Taylor (2022) argue that the realized brown-minus-green spread of recent years is negative (green outperformed) due to unexpected ESG-sentiment shocks, but the underlying *ex ante* risk premium is positive. Zhang (2024) document that much of the apparent realized brown-minus-green spread disappears after correcting for ex-post emissions data revisions. Eskildsen et al. (2024) use forward-looking expected returns and estimate a small ex ante spread, about −25 basis points per standard-deviation increase in greenness (equivalently, +25 basis points in brown-minus-green terms).

benchmark estimates, the raw value-weighted brown-minus-green spread is about 5% per year and rises sharply with ESG demand pressure. A surprising feature comes from the SMM itself. The estimated demand coefficient on emission intensity is zero, so investors do not directly penalize dirty firms in the demand system; yet the greenium is nevertheless positive. It is driven by the correlation between emission intensity and other firm characteristics (size, profitability, investment rate) that do load on investor demand. After controlling for the Fama-French factors, the brown-minus-green alpha shrinks substantially, to about 3% per year at the benchmark. This suggests that much of the raw greenium reflects exposure to systematic factors rather than a direct ESG penalty on emissions.

At the estimated production technology, neither ESG demand pressure nor a carbon tax alone delivers a clean reallocation toward green. ESG pressure shrinks firms but does not reverse the dirty share at $\nu = 0.56$ and $\alpha = 0.81$. A carbon tax at the EU Emissions Trading System rate (around €70–90 per tonne CO_2 in 2024–2025, equivalent to about 1.6% of revenue in the model) is effectively indistinguishable from no tax. Even a tax ten times larger (equivalent to 16% of revenue in the model) actually raises the level of emission intensity at every value of ESG demand pressure: from 244 to 274 tCO_2/\$M in the no-ESG baseline, and from 300 to 331 tCO_2/\$M under strong ESG demand. The reason is that complementarity locks dirty and clean capital together while clean capital is much more costly to adjust than dirty. A tax that targets dirty therefore also forces clean to contract, and the cost asymmetry leads firms to shrink clean even faster, so output falls faster than emissions and emission intensity rises. ESG pressure on top of the tax adds further cost that again falls disproportionately on clean, raising emission intensity further. The bottleneck is the production technology itself: ESG demand and a carbon tax can shift firms' incentives, but they cannot change ν and α. Real reallocation requires changes to the production technology, not just to the price signal.

Can firms themselves change the production technology? To examine this, I extend the model to let firms choose their substitutability endogenously, paying an R&D cost to upgrade. Tracing outcomes across a wide range of R&D costs and ESG pressure intensities, the result confirms

the earlier counterintuitive finding. At high R&D cost, firms do not raise their substitutability, and the production friction persists. At low R&D cost, firms do raise substitutability, but given that dirty capital is more productive and cheaper to adjust than clean, they then endogenously choose a higher dirty-capital ratio and higher emission intensity. With the current asymmetry in productivity and adjustment cost, increasing ESG pressure actually weakens the firm's incentive to upgrade substitutability: complementarity itself keeps the firm cleaner, so staying locked in is preferable to upgrading and then paying the ESG penalty for the dirtier endogenous mix. The right policy bundle is therefore to change the production function itself, after which other levers such as ESG demand, carbon pricing, or R&D subsidies can work as intended.

Related Literature

This paper contributes to two strands of literature. The first is the rapidly growing climate finance literature.[8] A growing literature documents that sustainable investors affect firm value and corporate policies through the cost of equity. Sustainable investors demonstrate preferences for green firms and construct portfolios on the basis of firms' environmental sustainability, thereby influencing the equity costs of these firms through the required returns (Chava, 2014; Starks, Venkat, and Zhu, 2017; Choi, Gao, and Jiang, 2020; Engle et al., 2020; Krueger, Sautner, and Starks, 2020; Ilhan, Sautner, and Vilkov, 2021; Pástor, Stambaugh, and Taylor, 2021; Pedersen, Fitzgibbons, and Pomorski, 2021; Sautner et al., 2023). More specifically, my work contributes to the carbon premium and carbon emissions literature (Bolton and Kacperczyk, 2021; Gregory, 2021; Bartram, Hou, and Kim, 2022; Bolton and Kacperczyk, 2023; Huij et al., 2023; Van der Beck, 2023; Zhang, 2024). I assume that the carbon risk price can be influenced by investor preferences and focus on the implications of this risk price for firms' investment decisions and carbon emissions. Previous studies documented that sustainable investors or sustainability demands may or may not

[8] The climate finance and ESG literature is growing rapidly and is too extensive to be fully summarized in the main text. For excellent review articles that summarize these areas of study and highlight directions for future research, see Martin, Muûls, and Wagner (2016), Hong, Karolyi, and Scheinkman (2020), Giglio, Kelly, and Stroebel (2021), Gillan, Koch, and Starks (2021), Edmans and Kacperczyk (2022), Starks (2023), Cepni, Grigoli, and Stroebel (2024), and Kräussl, Oladiran, and Stefanova (2024).

alter firm behaviors (Heinkel, Kraus, and Zechner, 2001; Schillebeeckx, Kautonen, and Hakala, 2022; De Angelis, Tankov, and Zerbib, 2023; Heath et al., 2023; Kahn, Matsusaka, and Shu, 2023; De Simone, Naaraayanan, and Sachdeva, 2024; Gormsen, Huber, and Oh, 2024; Li et al., 2024; Bellon, 2025). Favilukis, Garlappi, and Uppal (2023) showed that the impact of portfolio mandates crucially depends on firms' demand elasticity of capital in a production economy. Both Hartzmark and Shue (2022) and this paper find that sustainable investing can produce a counterproductive capital allocation, though through different mechanisms (the financing channel itself in Hartzmark and Shue, the production friction in this paper). More broadly, the production technology this paper estimates can account for the divergent empirical findings on ESG real effects: the optimistic and skeptical views of sustainable investing can both be consistent, depending on the underlying production technology.

Within this strand, the paper also contributes to the climate policy and directed technical change literature. Much of this literature has focused on studying the interactions between social welfare and financial quantities through macrofinance models (Daniel, Litterman, and Wagner, 2018; Bansal, Ochoa, and Kiku, 2017; Donadelli, Grüning, and Hitzemann, 2019; Barnett, Brock, and Hansen, 2020; Lemoine, 2021; Van den Bremer and Van der Ploeg, 2021; Barnett, 2023; Hong, Wang, and Yang, 2023; Lontzek et al., 2023). Some recent models have examined the real effects of sustainable investors on firms (Hambel, Kraft, and van der Ploeg, 2020; Campbell and Martin, 2021; Pástor, Stambaugh, and Taylor, 2021; Dangl et al., 2023; Lanteri and Rampini, 2023; Bellon and Boualam, 2024; Bustamante and Zucchi, 2024). These theories focus primarily on investor-side influences and explore how they propagate to firms and the broader economy. In contrast, my model centers on firm-side decisions and the trade-off between green and brown inputs in the production function. I emphasize the importance of technological innovation for capital substitutability, which aligns with the findings of Barnett et al. (2023b), who demonstrated the value of R&D investments.

The second is the investment-based asset pricing literature (Zhang, 2005; Liu, Whited, and Zhang, 2009; Lin, 2012; Eisfeldt and Papanikolaou, 2013; Frank and Shen, 2016). This body of work

has examined equity value and returns by considering endogenous investment decisions. Previous papers have also highlighted the implications of heterogeneous capital and labor sources (Belo, Lin, and Vitorino, 2014; Belo et al., 2017; Gonçalves, Xue, and Zhang, 2020; Belo et al., 2022). I extend the standard investment-based model by incorporating both dirty and clean capital and show that the endogenous trade-off between capital allocation and productivity limits the effectiveness of sustainable investment. The elasticity of substitution between dirty and clean capital has been treated in several ways in the prior literature. Models in the DICE tradition and many production asset-pricing models with carbon assume Cobb-Douglas ($\nu = 1$) (Nordhaus and Sztorc, 2013; Cai and Lontzek, 2019; Gormsen, Huber, and Oh, 2024). The directed-technical-change literature assumes high substitutability to generate a clean-growth transition (Aghion et al., 2016; Barnett et al., 2023b). Many climate-finance models with a single capital stock implicitly assume perfect substitution. This paper provides the first joint dynamic structural estimate of the physical-capital elasticity of substitution ν, the productivity share α, and the adjustment-cost asymmetry between clean and dirty capital, using SMM to match firm-level investment, return, and ESG-flow moments. Whereas prior work calibrates this technology to steady-state targets, this paper estimates it structurally; the resulting estimates are consistent with external benchmarks on the dirty-capital share of physical capital and the fossil-fuel share of primary energy. I further marry this literature with the demand-based asset pricing literature (Koijen and Yogo, 2019; Van der Beck, 2023), which maps investor demand to asset prices and firm investment. I show that even a demand system that correctly prices emission intensity at the firm level cannot redirect real capital allocation under the empirical production technology.

The remainder of this paper is organized as follows. Section 2 develops two-period models that build intuition under both damage-driven and flow-driven sustainability incentives. Section 3 presents the reduced-form estimation of ν. Section 4 develops the dynamic flow-driven model and reports the SMM estimates. Section 5 presents the quantitative counterfactuals. Section 6 endogenizes complementarity. Section 7 concludes.

2 Two-Period Model

Sustainability concerns reach firms through two distinct channels: a physical channel, in which climate damage reduces output and raises the effective cost of dirty capital; and a financial channel, in which ESG investor demand raises the cost of equity for high-emission firms through security pricing. The damage-function model (Section 2.1) formalizes the physical channel following the DICE tradition; the flow-driven model (Section 2.2) formalizes the financial channel using the Koijen and Yogo (2019) demand system, which is also the basis for the SMM estimation in Section 3. Both models share the same production technology, capital accumulation, adjustment costs, and pricing kernel. They differ in how production complementarity interacts with the sustainability incentive: in the damage model, the cost of dirty capital operates through cash flows; in the flow model, it operates through market valuation. Section 2.3 compares the two mechanisms directly.

2.1 Damage-Function Mechanism

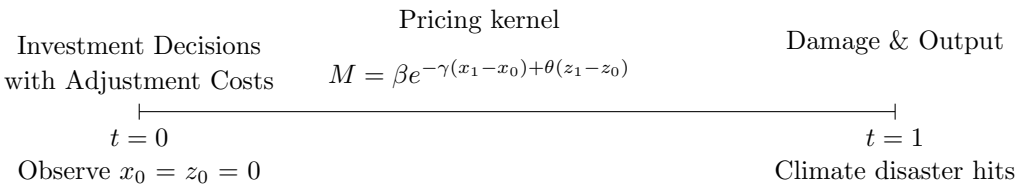
Three simplifying assumptions keep the model tractable. First, capital evolves with partial depreciation: $K_{1,j} = (1-\delta)K_{0,j}+I_j$ for $j \in \{C, D\}$, where $\delta \in (0, 1)$ is the depreciation rate. Second, clean and dirty capital are identical except for their contribution to damage, isolating the pure effects of sustainability preferences. Third, the representative firm starts with $K_{0,C} = K_{0,D} = 1$, so the initial dirty-capital share is 50%. Initial wealth is W_0.

The terminal period captures a discrete climate event rather than long-run gradual warming.[9] Dirty capital generates emissions that cause damage in the next period, so the damage function is increasing in K_D. This link between dirty capital and output loss is standard in the macro-climate literature (Nordhaus and Sztorc, 2013; Cai and Lontzek, 2019; Barnett, Brock, and Hansen, 2020) and accommodates several interpretations: direct output losses from physical climate risk, cash flow reductions from lower demand for high-emission products, or shareholder welfare losses from negative externalities (Hart and Zingales, 2017). The richer dynamics between temperature

[9] Long-run climate change has only mild per-period effects, which would be negligible in a two-period setting and would obscure the mechanisms of interest.

accumulation and dirty capital accumulation are developed in the full infinite-horizon model in Section 4.

The timeline of the model is as follows:



The production output is given by

$$\Pi = (1 - D)X\Phi \tag{1}$$

where D represents the damage function, X denotes aggregate productivity, and Φ represents the constant elasticity of substitution (CES) production function. Specifically, the damage function increases with the level of dirty capital stock K_D:

$$D = \eta Z K_D \tag{2}$$

where η denotes the damage intensity level, and Z represents a stochastic variable for capturing the uncertainty of the realized damage in the next period. The CES production function incorporates both types of capital:

$$\Phi = \left((1 - \alpha)(K_C)^{\frac{\nu-1}{\nu}} + \alpha(K_D)^{\frac{\nu-1}{\nu}}\right)^{\frac{\nu}{\nu-1}} \tag{3}$$

where α captures the productivity share of dirty capital, and ν is the elasticity of substitution between clean and dirty capital. For the two-period model, I set $\alpha = 0.5$ for expositional symmetry; the quantitative implications of the estimated $\alpha = 0.81$ are explored in the full dynamic model in Section 4.

The pricing kernel captures investors' preferences for less dirty capital or aversion to pollution or damage.[10] Following the investment-based asset pricing literature, I use an exogenous log-linear

[10]Investors may be willing to hold claims to clean capital at a lower expected return, while remaining risk-neutral

pricing kernel.

$$\ln(M) = \ln(\beta) - \gamma(x_1 - x_0) + \theta(z_1 - z_0) \tag{4}$$

where $x_0 = \ln(X_0)$ represents the initial aggregate productivity level, and $z_0 = \ln(Z_0)$ denotes the initial damage level at time $t = 0$. x_1 and z_1 are, respectively, the next period realized productivity and damage shocks. γ and θ measure the price of risk associated with aggregate productivity shocks and damage shocks, respectively. A higher θ can be interpreted as investors having stronger preferences for less pollution or damage.

The shocks follow a binomial tree. The productivity shock x_1 is as follows:

$$\ln(X_1) = x_1 = \begin{cases} x_0 + \Delta x, & \text{Prob } = \frac{1}{2} \\ x_0 - \Delta x, & \text{Prob } = \frac{1}{2} \end{cases} \tag{5}$$

and damage shock z_1 follows

$$\ln(Z_1) = z_1 = \begin{cases} z_0 + \Delta z, & \text{Prob } = \frac{1}{2} \\ z_0 - \Delta z, & \text{Prob } = \frac{1}{2} \end{cases} \tag{6}$$

The specific values of Δx and Δz do not change the qualitative results or the economic intuitions. The investment adjustment cost is

$$\phi^j = \frac{1}{2} I_j^2 \tag{7}$$

where $j \in \{C, D\}$. The net cash flow is thus

$$CF_0 = W_0 - I_C - I_D - \phi^C - \phi^D \tag{8}$$

$$CF_1 = \Pi_1 \tag{9}$$

The firm maximizes value

$$V = \max_{I_C, I_D} CF_0 + E[MCF_1] \tag{10}$$

to damage risk, meaning that the cost of capital wedge between dirty and clean assets is not necessarily risk-based. I do not take a stance on whether this wedge is due to risk or other factors. Under no-arbitrage conditions, any cost of capital effect can be represented by a stochastic discount factor (see Harrison and Kreps, 1979; Hansen and Richard, 1987). Thus, the economic setup is equivalent whether I model damage as a priced factor or directly impose a wedge between the costs of dirty capital and those of clean capital.

The first-order conditions are therefore

$$[I_C] \quad 1 + I_C = \quad E\big[M(1-D)X_1 \frac{\partial \Phi}{\partial I_C}\big] \tag{11}$$

$$[I_D] \quad 1 + I_D = \underbrace{E\big[M(1-D)X_1 \frac{\partial \Phi}{\partial I_D}\big]}_{\text{marginal contribution to production}} - \underbrace{E[M \frac{\partial D}{\partial I_D} X_1 \Phi]}_{\text{marginal contribution to damage}} \tag{12}$$

To understand the economic intuitions more explicitly, consider two special cases. First, when $\nu \to \infty$, the two capital stocks are perfect substitutes, and the first-order conditions become

$$1 + I_C = E\big[M(1-D)X_1 \cdot 0.5\big] \tag{13}$$

$$1 + I_D = E\big[M(1-D)X_1 \cdot 0.5\big] - E[M \frac{\partial D}{\partial I_D} X_1 \Phi] \tag{14}$$

The wedge between the two types of investments is

$$I_C - I_D = E[M \frac{\partial D}{\partial I_D} X_1 \Phi] \tag{15}$$

which is driven purely by the potential damage generated by dirty capital. The greater the potential damage and aversion to it (θ), the less the firm invests in dirty capital. If the damage is sufficiently severe, the firm will rely on a substantially higher share of clean capital for production.[11]

However, when $\nu = 1$, the production function is Cobb-Douglas. The first-order conditions become

$$1 + I_C = E\big[M(1-D)X_1 \cdot 0.5 (\frac{K_D}{K_C})^{0.5}\big] \tag{16}$$

$$1 + I_D = E\big[M(1-D)X_1 \cdot 0.5 (\frac{K_C}{K_D})^{0.5}\big] - E[M \frac{\partial D}{\partial I_D} X_1 \Phi] \tag{17}$$

If the economy entirely discards dirty capital ($I_D = K_D = 0$), it will not be able to produce. In this case, as shown in equations (16) and (17), the marginal product of dirty capital approaches infinity, whereas the marginal product of clean capital falls to zero.

[11] Perfect substitutability implies a potentially wide wedge between clean and dirty investment but does not necessarily mean a zero share of dirty investment. Because the investment adjustment cost is convex, it is less costly to invest one dollar in each form of capital than to invest two dollars in clean capital.

The wedge between clean and dirty investments depends on two factors: marginal production and marginal damage. When the two types of capital are perfectly substitutable, clean capital can fully replace the marginal production of dirty capital, so damage and shareholder preferences largely disincentivize investment in dirty capital. If the potential damage is substantial or if investors place a very high concern on damage (high θ), the firm can achieve a very low (or even zero) share of dirty capital. However, when the two types of capital are complementary, production concerns dominate the damage and investor preference channels in firm investment decisions, making it challenging to attain a very low level of dirty capital.

Numerical results. Figure 1 plots the optimal $t = 1$ dirty-capital ratio $K_{1,D}/(K_{1,C} + K_{1,D})$ for the representative firm ($K_{0,C} = K_{0,D} = 1$, $\delta = 0.5$) as a function of θ. At $\theta = 0$ both scenarios start slightly below 0.5 because, even without risk pricing, the expected damage $E[\eta Z K_D]$ reduces expected output and induces a mild tilt toward clean capital. As θ grows the tilt strengthens: by $\theta = 5$ the dirty-capital ratio falls to about 0.13 under perfect substitution, while Cobb-Douglas only falls to about 0.39. The gap reflects the production complementarity: under Cobb-Douglas a marginal unit of dirty capital has a high marginal product (scarcity effect), which partly offsets the damage penalty and keeps the allocation close to balance.

2.2 Flow-Driven Sustainable Incentives

This model replaces the damage function with an investor demand system: the firm maximizes market value, and ESG fund flows create a wedge between market value and fundamentals that directly alters investment incentives.[12] Without damage, the pricing kernel simplifies to $\ln(M) = \ln(\beta) - \gamma(x_1 - x_0)$. I denote fundamental output at $t = 1$ by $F_1 = X_1\Phi(K_{1,C}, K_{1,D})$ to keep it distinct from market value V.

Investor demand for firm j follows a logit system in firm characteristics. I keep two

[12] This flow-based channel is motivated by the empirical evidence in Pastor, Stambaugh, and Taylor, 2023 and Van der Beck, 2023, which shows that sustainable fund flows tilt portfolio weights toward green firms and move green prices relative to brown prices. The channel operates through valuation, not through any direct effect on production.

characteristics: the fundamental value F_j and the emission intensity e_j, where

$$e_j = \frac{\varepsilon_D K_{1,D,j}}{F_j} \tag{18}$$

and ε_D is the emission rate per unit of dirty capital.[13] Emission intensity rises with the dirty capital share and falls with output. The investor's demand index for firm j is

$$\mu_j = \exp\left(b_{\text{mkt}} \log F_j + b_{\text{emit}} \log e_j + \log v\right) \tag{19}$$

where $b_{\text{emit}} < 0$ reflects investors' distaste for dirty firms, and $v > 0$ is ESG sentiment. A higher v or a more negative b_{emit} both amplify the penalty applied to dirty firms. I use lower-case b for the demand coefficients to keep them distinct from the discount factor β and the risk quantities β^C, β^D in Section 2.1. The portfolio weight on firm j is

$$w_j = \frac{\mu_j}{\mu_j + (J-1)\bar{\mu} + 1} \tag{20}$$

where J is the number of investable firms and $\bar{\mu}$ is the average demand index. Market clearing implies that the market price is

$$P_j = F_j(1 + \iota_j), \quad \iota_j = Jw_j - 1 \tag{21}$$

A clean firm with low e_j attracts excess demand ($\iota_j > 0$) and trades at a green premium above fundamentals. A dirty firm with high e_j faces low demand ($\iota_j < 0$) and trades at a brown discount.

The $t = 0$ cash flow is unchanged from Section 2.1. At $t = 1$, the firm receives fundamental output $CF_1 = F_1 = X_1\Phi(K_C, K_D)$, but the market clears at price $P_1 = F_1(1 + \iota)$. The firm maximizes market value:

$$V = \max_{I_C, I_D} CF_0 + E\big[M \cdot F_1(1 + \iota)\big] \tag{22}$$

[13] Under partial depreciation, $K_{1,D,j} = (1 - \delta)K_{0,D,j} + I_{D,j}$ is bounded away from zero whenever the initial endowment $K_{0,D,j}$ is positive, so this emission formula is well behaved. The full dynamic SMM additionally uses the full set of Koijen-Yogo characteristics (book value, profitability, and investment rate), which I suppress here for clarity.

Because the firm maximizes market value rather than fundamental value, the demand-induced markup ι enters the objective directly. This is the key departure from Section 2.1: dirty capital is not less productive; it is simply priced at a discount by investors.

The first-order conditions are

$$[I_C] \quad 1 + I_C = \underbrace{E\big[M(1+\iota)X_1 \frac{\partial \Phi}{\partial I_C}\big]}_{\text{production channel}} + \underbrace{E\big[MF_1 \frac{\partial \iota}{\partial I_C}\big]}_{\text{demand channel}} \tag{23}$$

$$[I_D] \quad 1 + I_D = \underbrace{E\big[M(1+\iota)X_1 \frac{\partial \Phi}{\partial I_D}\big]}_{\text{production channel}} + \underbrace{E\big[MF_1 \frac{\partial \iota}{\partial I_D}\big]}_{\text{demand channel}} \tag{24}$$

The left side is the marginal cost of investment (purchase price plus adjustment cost). The right side has two terms. The production channel is the marginal product of capital, discounted by the pricing kernel and scaled by the demand markup $(1 + \iota)$. The demand channel is the marginal effect of the capital choice on the demand-induced price ι.

The demand channel creates a wedge in favor of clean investment. Since ι depends on emission intensity $e = \varepsilon_D K_{1,D}/F$, increasing clean capital raises F in the denominator and lowers e, raising investor demand ($\partial\iota/\partial I_C > 0$); increasing dirty capital raises $K_{1,D}$ in the numerator and raises e, lowering investor demand ($\partial\iota/\partial I_D < 0$). The demand channel therefore adds a positive term to the first-order condition for clean capital and a negative term to the first-order condition for dirty capital. The size of these terms depends on how strongly investors penalize emissions: a more negative b_{emit} makes the wedge larger.

To understand the economic intuitions more explicitly, consider the same two special cases as in Section 2.1. First, when $\nu \to \infty$, the two capital stocks are perfect substitutes, and $\partial\Phi/\partial I_C = \partial\Phi/\partial I_D = 0.5$. The first-order conditions become

$$1 + I_C = E\big[M(1+\iota)X_1 \cdot 0.5\big] + E\big[MF_1 \tfrac{\partial \iota}{\partial I_C}\big] \tag{25}$$

$$1 + I_D = E\big[M(1+\iota)X_1 \cdot 0.5\big] + E\big[MF_1 \tfrac{\partial \iota}{\partial I_D}\big] \tag{26}$$

The production channel makes the same contribution to both conditions, so the investment wedge is driven entirely by the demand channel:

$$I_C - I_D = E\big[MF_1\big(\tfrac{\partial \iota}{\partial I_C} - \tfrac{\partial \iota}{\partial I_D}\big)\big] > 0 \tag{27}$$

With perfect substitutability, the firm can freely shift its capital mix toward clean capital without any loss in output. If ESG sentiment v is sufficiently high, the firm invests mostly in clean capital, and the economy can achieve a very low share of dirty capital.

Second, when $\nu = 1$, the production function is Cobb-Douglas. The first-order conditions become

$$1 + I_C = E\big[M(1+\iota)X_1 \cdot 0.5(\tfrac{K_D}{K_C})^{0.5}\big] + E\big[MF_1 \tfrac{\partial \iota}{\partial I_C}\big] \tag{28}$$

$$1 + I_D = E\big[M(1+\iota)X_1 \cdot 0.5(\tfrac{K_C}{K_D})^{0.5}\big] + E\big[MF_1 \tfrac{\partial \iota}{\partial I_D}\big] \tag{29}$$

If the firm discards dirty capital entirely ($K_D \to 0$), the marginal product of dirty capital goes to infinity and the marginal product of clean capital falls to zero. The production channel therefore pushes back hard against any tilt toward clean capital. The demand channel still favors clean investment, but the production channel dominates.

The investment wedge now has two opposing forces. The demand channel, created by investor distaste for emissions, pushes toward more clean investment. The production channel, created by complementarity, pushes back toward balanced investment. When ν is large, the production channel is weak and the demand channel dominates. When ν is small (close to or below 1), the production channel is strong and the demand channel is largely offset. In the gross-complements region ($\nu < 1$) that the SMM estimates target, the production channel is even stronger: a 1% decrease in dirty capital requires more than a 1% increase in clean capital to maintain output, so the firm cannot respond much to investor demand.

Numerical results. Figure 2 plots the representative firm's optimal $t = 1$ dirty-capital ratio $K_{1,D}/(K_{1,C} + K_{1,D})$ as a function of $|b_{\text{emit}}|$. At $|b_{\text{emit}}| = 0$ the ESG channel is switched off and both scenarios start at exactly 0.5. As $|b_{\text{emit}}|$ grows, both tilt toward clean capital, but the perfect-substitution firm tilts much more sharply: by $|b_{\text{emit}}| \approx 0.2$ it hits the I^D lower bound and its dirty-capital share settles near 0.13, while Cobb-Douglas continues to decline gradually to about 0.38 at $|b_{\text{emit}}| = 0.6$. Complementarity keeps the Cobb-Douglas firm from abandoning dirty capital even under strong ESG pressure.

2.3 Comparing the Two Mechanisms

Sections 2.1 and 2.2 deliver the same structural message (the effectiveness of sustainable investing is governed by ν) but differ in how that message plays out for specific observables. This subsection compares the two side by side; the detailed results are in the preceding subsections.

The two models produce the same qualitative pattern for the dirty-capital ratio. The ratio falls as the sustainable incentive grows, with perfect substitution tilting much more than Cobb-Douglas. The economic force differs (the damage model operates through reduced cash flow, the flow-driven model through a discounted market price) but the first-order-condition resolution is structurally similar, and both channels are moderated by the same production complementarity.

The two mechanisms are complementary rather than mutually exclusive: the damage function captures the physical transmission of climate risk through cash flows, while the flow-driven channel captures the financial transmission of sustainability preferences through prices. The full dynamic model in Section 4 retains the damage function; the SMM estimation exploits firm-level flow-driven moments to separately identify ν and the demand elasticity b_{emit}.

3 A Simple Empirical Analysis

The results from the two-period model highlight the importance of substitution between dirty and clean capital. How high is this elasticity in the data? This study is the first to estimate it directly

using firm-level carbon emission data, motivated by theoretical foundations.[14] Previous papers focus on capital-labor substitution because both inputs have straightforward empirical measures: Chirinko, Fazzari, and Meyer (2011) and Chirinko and Mallick (2017) estimate $\nu = 0.4$, and Gechert et al. (2022) finds an average of $\nu = 0.3$ in a meta-analysis conditional on no publication bias. My estimates indicate that the dirty-clean capital margin is also far from perfect substitution.

3.1 Empirical Strategy

The empirical strategy combines the methods in Chirinko, Fazzari, and Meyer (2011) and Chirinko and Mallick (2017) with the model presented in Section 2. Recall the CES production function in equation (3)

$$\Phi_{it} = \left((1-\alpha)(K_{C,it})^{\frac{\nu-1}{\nu}} + \alpha(K_{D,it})^{\frac{\nu-1}{\nu}}\right)^{\frac{\nu}{\nu-1}}$$

This functional form is standard in empirical studies estimating the CES substitution elasticity, as it links the regression coefficient directly to ν.

This production function has two broad interpretations. First, and most intuitively, it captures the two types of capital within the production process of a single firm. The focal firm trades off between dirty capital and clean capital to produce its goods. This interpretation can be captured by using Scope 1 carbon emissions data, which reflect the carbon emissions that are directly generated by the production process of the firm, as a proxy for dirty capital. Second, the production function can be interpreted as the focal firm using capital inputs acquired from both the dirty sector and the clean sector. In this case, other firms within each sector may produce intermediate capital inputs, and the intermediate inputs from the two sectors complement each other during the production of the focal firm's final goods, which consumers ultimately consume. This interpretation can be captured by using Scope 2 or Scope 3 carbon emissions data, as these measures include carbon emissions from energy purchases and supply chains.

Following Chirinko, Fazzari, and Meyer (2011) and Chirinko and Mallick (2017), a firm-specific

[14] In this paper, I focus on capital inputs. Some papers have focused on dirty and clean energy inputs, such as Li, Wang, and Xu (2024), Jo (2020), and Jo and Miftakhova (2024).

productivity, A_{it}, is added. Now, firm i has an output of

$$Y_{it} = A_{it}\big((1-\alpha)(K_{C,it})^{\frac{\nu-1}{\nu}} + \alpha(K_{D,it})^{\frac{\nu-1}{\nu}}\big)^{\frac{\nu}{\nu-1}} \tag{30}$$

Take the partial derivative with respect to K_{it}^D

$$\frac{\partial Y_{it}}{\partial K_{D,it}} = \alpha A_{it}^{\frac{\nu-1}{\nu}} Y_{it}^{\frac{1}{\nu}} K_{D,it}{}^{-\frac{1}{\nu}} \tag{31}$$

Rearranging the equation[15]

$$\frac{K_{D,it}}{Y_{it}} = \alpha^{\nu} A_{it}^{\nu-1} (\frac{\partial Y_{it}}{\partial K_{D,it}})^{-\nu} \tag{32}$$

This equation does not rely on specific inputs in the production function. Although there may be other separately modeled inputs, such as labor and intangible capital, the above equation does not change because everything is included in the total output Y_{it} (see Internet Appendix Section B.1 for more details). Since carbon emissions are directly observable, I use them as a proxy for the dirty capital stock.

$$G_{it} = \eta K_{D,it} \tag{33}$$

where G_{it} represents carbon emissions, I assume a linear relationship between dirty capital stock and carbon emissions for simplicity and to remain consistent with my dynamic model assumption. This assumption aligns with the functional form in Barnett et al. (2023a) and Hong, Wang, and Yang (2023). The reasoning is that carbon emissions are a linear function of output (Nordhaus and Sztorc, 2013), and output can be assumed to be a linear function of capital, resulting in a linear relationship between carbon emissions and capital.

The linear relation between carbon emission and output is empirically supported in Zhang (2024). Empirically,

$$log(Emissions) = log(c) + \beta \cdot log(Sales) + \epsilon \tag{34}$$

[15] I rearrange the equation to place the partial derivative, $\partial Y_{it}/\partial K_{D,it}$, on the right-hand side because the firm-specific TFP is correlated with the dirty capital-to-output ratio, $K_{D,it}/Y_{it}$. In contrast, the partial derivative, which is a function of $\partial \ln Y_{it}/\partial \ln K_{D,it}$, does not depend on the firm-specific TFP by definition of the polynomial function used to approximate the nonlinear function, $\ln Y_{it}$.

Zhang (2024) Table 3 shows that β is very close to one, with $R^2 \approx 0.66 - 0.71$ (Scope 2, U.S. and international). This finding implies $Emission = e^c \cdot Sales$, a linear mapping from emissions to output. This specific functional form does not affect the estimation strategy for ν (see Internet Appendix Section B.1 for more details), but it does affect the estimates of α. I plug the equation above into equation (32), and obtain

$$\frac{G_{it}}{Y_{it}} = \alpha^{\nu} \eta^{1-\nu} A_{it}^{\nu-1} (\frac{\partial Y_{it}}{\partial G_{it}})^{-\nu} \tag{35}$$

Following Chirinko and Mallick (2017), the firm-specific productivity has the following functional form:

$$A_{it}^{\nu-1} = e^{u_i + u_t + u_{it}} \tag{36}$$

The portion of the firm TFP that is allowed to correlate with other firm variables is represented by the sum of firm and time fixed effects, plus an idiosyncratic residual u_{it}.

Following equation (35), the economic meaning of the empirical strategy is as follows: the elasticity of substitution, or the complementarity level, quantifies the relation between the marginal productivity of emissions and the carbon emission intensity.

$$\ln(\frac{\partial Y_{it}}{\partial G_{it}}) \sim -\frac{1}{\nu} \ln(\frac{G_{it}}{Y_{it}})$$

The marginal productivity of emissions to the total output, $\partial Y / \partial G$, represents how productive carbon emissions are, indicating how much total output can be generated per marginal unit of carbon emissions. If dirty capital and clean capital are highly complementary (ν is low), a lower carbon emission intensity is associated with a higher marginal productivity of carbon emission. For example, if Tesla has no lithium batteries but many gigafactories, a slight increase in lithium production is associated with a high level of marginal productivity of electric cars due to the concavity of the production function. In contrast, if dirty capital and clean capital are a perfect substitutes (ν is $+\infty$), the marginal productivity of carbon emission does not depend on the carbon

emission intensity, because the production process does not rely on the input of dirty capital to maintain its production output.

Take the log on both sides and rearrange

$$\ln(\frac{G_{it}}{Y_{it}}) \triangleq a - \nu \cdot \ln(\frac{\partial Y_{it}}{\partial G_{it}}) + u_i + u_t + u_{it} \tag{37}$$

where $a = \nu \cdot \ln(\alpha) + (1-\nu) \cdot \ln(\eta)$ is the intercept. I put the marginal productivity of carbon emission on the right-hand side because, by construction, it is less subject to endogeneity due to firm-specific productivity shocks.

Equation (37) assumes that firm-specific TFP enters the residual additively through separable firm and time fixed effects, plus an idiosyncratic component u_{it}. This assumption is restrictive: productivity shocks are typically time-varying and loaded heterogeneously across firms, so if u_{it} is correlated with $\ln(\partial Y_{it}/\partial G_{it})$ through unobserved common factors that hit firms with different intensities, the OLS estimator of ν is biased. I address this concern using the interactive-fixed-effects methodology of Bai (2009), which replaces the separable $u_i + u_t$ term with a latent factor structure estimated by principal component analysis (PCA). The modified regression equation is

$$\ln(\frac{G_{it}}{Y_{it}}) \triangleq a - \nu \cdot \ln(\frac{\partial Y_{it}}{\partial G_{it}}) + \lambda_i^\top f_t + \epsilon_{it}, \tag{38}$$

where f_t is a vector of latent common factors and λ_i is the corresponding vector of firm-specific loadings, both extracted from the data by PCA. Bai's estimator jointly estimates (ν, f_t, λ_i) by iterating between the OLS step that updates ν and the PCA step that updates the factors and loadings. The resulting residual ϵ_{it} is orthogonal to any linear combination of the latent factors, so the part of firm-specific TFP absorbed by the factor structure no longer contaminates the slope estimate. Under the standard regularity conditions in Bai (2009), the estimator is consistent and asymptotically normal.

The emission-to-output ratio, G_{it}/Y_{it}, is straightforward to calculate. However, calculating the firm-level marginal contribution of emissions to output, $\partial Y_{it}/\partial G_{it}$, is challenging. Therefore, I

estimate it via sector-year regressions. In the model, the output Y is a nonlinear function of the productivity, A, dirty capital, K_D, and clean capital, K_C. I capture the productivity via firm and time fixed effects. I use total assets, AT, as a proxy for total capital, K_D+K_C, and carbon emissions, G, as a proxy for K_D (an implication from equation (31)). Thus, for each sector j, I assume that output Y is a nonlinear function of emissions G and total assets AT. I use cross-sectional firm data to estimate the nonlinear function within each sector. I then apply the sector-year-specific coefficients to calculate the firm-level $\partial Y_{it}/\partial G_{it}$. Motivated by Gala, Gomes, and Liu (2022), the nonlinear function can be approximated by a polynomial function

$$\ln Y_{it} = c_{j0,t}+c_{j1,t}\cdot\ln G_{it}+c_{j2,t}\cdot(\ln G_{it})^2+c_{j3,t}\cdot\ln AT_{it}+c_{j4,t}\cdot(\ln AT_{it})^2+c_{j5,t}\cdot\ln G_{it}\times\ln AT_{it}+\epsilon_{it} \quad (39)$$

where AT_{it} is the total assets. The equation is estimated for each combination of sector j and time t, with coefficients that are specific for each sector-year combination.[16] Because the levels of the output and emissions data are highly right-skewed, the logarithmic form provides a more stable relationship, following Gala, Gomes, and Liu (2022) and Zhang (2024). Using total assets as the primary independent variable is appropriate due to the functional form of the CES production function, where total output depends on dirty capital and clean capital stock. Total assets are most directly related to total capital stock.[17] Taking the partial derivative of (39), I obtain

$$\frac{\partial \ln Y_{it}}{\partial \ln G_{it}} = c_{j1,t} + 2\cdot c_{j2,t}\cdot\ln G_{it} + c_{j5,t}\cdot\ln AT_{it} \quad (40)$$

The partial derivative of the logarithm can be easily converted back to the original partial derivative in levels: $\partial Y_{it}/\partial G_{it} = (\partial \ln Y_{it}/\partial \ln G_{it})\cdot(Y_{it}/G_{it})$. Equation (38) is then used to estimate ν.

[16] By definition, the output Y_{it} is a function of firm-specific productivity, A_{it}, which depends on firm and time fixed effects. To estimate the polynomial function, I use sector and year combinations, which capture the year fixed effect. Because the variation within a sector comes from firm-level variables, I cannot include firm fixed effects.

[17] The results are robust if I replace the total assets with physical capital (PPEGT or PPENT from COMPUSTAT,) or the sum of PPEGT and intangible capital. See Internet Appendix Table C.3, Table C.4, and Table C.5 for more details.

3.2 Data

The data used for the empirical analysis are merged from the S&P Trucost Environmental and Compustat Fundamentals Annual (North America and Global) datasets from WRDS. The Trucost dataset covers global firm-level carbon emissions in tons of carbon dioxide equivalent (tCO2e) at an annual frequency (from 2002 to 2023). The Trucost data are merged with Compustat data via GVKEY and year. The sample is restricted to firms with positive total assets, carbon emissions, and total revenue. In the data, total revenue represents total output Y in the model. More detailed descriptions of the data variables are provided in Internet Appendix Table C.1.

I study Scope 1, 2, and 3 carbon emissions. The Scope 1 GHG emissions originate from sources owned or controlled by the company. The Scope 2 GHG emissions result from the consumption of purchased electricity, heat, or steam by the company. The Scope 3 GHG emissions (upstream and downstream emissions) are indirect carbon emissions from other activities linked to the firm's production that are not covered in Scope 2.

The nonlinear function (39) is estimated at the 2-, 3-, and 4-digit SIC levels. To mitigate the impact of large firms, the regression is weighted by the inverse of the logarithm of total assets. Since there are six independent variables, for each combination of sector and year, samples with fewer than six observations are excluded. The corresponding control variables and estimates are winsorized at the 1st and 99th percentiles of the distribution.

3.3 Sample and Summary Statistics

The matched sample between Trucost and Compustat covers both developed and emerging markets at an annual frequency. I drop countries with fewer than 20 observations and exclude financial firms (SIC codes between 6000 and 6999).[18] Table 1 reports the number of observations, start year, end year, and averages of the key variables used in the empirical analysis. The sample predominantly

[18]Financial firms might rely on polluting physical capital, such as buildings that use electricity. However, their production functions can fundamentally differ from those of other industries. I report estimates of ν with financial firms in Internet Appendix Table C.2, which are quantitatively similar to estimates without financial firms but slightly higher overall.

comprises developed markets, accounting for approximately 90% of the total observations. The leading countries are the U.S. (75%), Canada (5%), China (3%), and the United Kingdom (2%). The United States comprises the majority of the sample. I include the average log ratios of total revenue, Scope 1 carbon emissions, Scope 2 carbon emissions, and total assets. These are the critical variables used in estimating the marginal productivity of carbon emissions and dirty capital substitutability, ν.

Table 2 presents key summary statistics for firm-level carbon measures and fundamentals. Carbon emission measures in the U.S. do not differ substantially from those in the global sample. In the U.S. sample, across the three scopes of carbon emissions, Scope 3 has the highest intensity (4.88) compared with Scope 1 (2.28) and Scope 2 (2.56). Carbon intensity is measured by the logarithm of carbon emissions scaled by total revenue. Scope 1 has the highest standard deviation (2.02) compared with Scope 2 (1.36) and Scope 3 (1.36). For the logarithm of the carbon emission levels, Scope 3 has the highest value (11.52), which is higher than those of Scope 1 (8.92) and Scope 2 (9.20). Like carbon intensity, Scope 1 has the highest standard deviation (3.09), whereas Scope 2 (2.64) and Scope 3 (2.69) have similar standard deviations.

Total revenue and total assets are important components in the empirical analysis. In the U.S. sample, the average log of total revenue (in millions) is 6.66, and the average log of total assets (in millions) is 7.83. The summary statistics of the estimated marginal productivity of carbon emissions, $\partial Y/\partial G$, are calculated via Scope 1 carbon emissions and industry proxies on the basis of 3-digit SIC codes. The logarithm of the Scope 3 emissions-to-total-asset ratio is the highest (3.70), followed by Scope 2 (1.37) and Scope 1 (1.10) in the U.S. sample.

3.4 Empirical Results

I estimate equation (38) using the interactive-fixed-effects estimator of Bai (2009) with 5 factors. Table 3 panels A, B, and C use Scope 1, 2, and 3 carbon emissions, respectively, as proxies for G. Within each carbon emission proxy, I estimate the marginal productivity of carbon emissions

using 2-, 3-, and 4-digit SIC codes as the sector classifications. After obtaining the industry-level coefficients, I use them to estimate firm-level $\partial Y/\partial G$ before the final pooled firm-level regressions.

At the benchmark of 5 factors, the estimates range from 0.519 to 0.717 across scope and SIC-digit specifications. Scope 1 estimates cluster around 0.68–0.72, Scope 2 around 0.54, and Scope 3 around 0.52–0.56, indicating that the within-firm production margin (Scope 1) is somewhat more substitutable than the energy and supply-chain margins (Scopes 2 and 3) but that all three lie well below one. These values indicate gross complementarity: the production function is more complementary than Cobb-Douglas ($\nu = 1$) and far from perfect substitution ($\nu \to +\infty$).

The empirical proxy for dirty capital is firm-level annual emissions. Emissions are a flow, whereas capital is a stock; utilization, fuel mix, and abatement can add noise. I therefore (i) use 3-year moving averaged emissions and (ii) re-estimate equation (38) as robustness. As in Table C.6 the estimated ν remains well below one. Importantly, our ν-estimator is functional-form invariant (Appendix B), so the mapping between emissions and dirty capital does not drive identification.

The dependence on dirty capital in the overall production function can vary over time. Van der Beck (2023) suggests the emergence of sustainable investors after 2012, which is also the midpoint of my data period. I split the sample at two thresholds, 2012 and 2016 (the year of the Paris Agreement), to examine whether the elasticity of dirty capital changes across subperiods.

Table 4 presents both sets of subsample estimates for Scope 1 and Scope 2 emissions with 3- and 4-digit SIC classifications. When splitting by 2012, the estimates are not statistically different between subperiods: Scope 1 with 3-digit SIC yields ν of 0.739 before 2012 and 0.753 after, and all p-values exceed 0.14. In contrast, the 2016 split reveals a pronounced and highly significant increase: Scope 1 with 3-digit SIC yields ν of 0.710 before 2016 and 0.816 after (z-statistic $= -9.834$, p-value $= 0.000$), and Scope 2 with 3-digit SIC yields 0.499 and 0.608 (p-value $= 0.000$). The pattern is consistent across all Scope 2 and 4-digit SIC specifications. These results suggest that the Paris Agreement marked a structural shift in the complementarity between clean and dirty capital, whereas the 2012 threshold is not associated with a significant change.

Finally, I split the sample by industry and estimate the heterogeneous elasticity parameter within each industry. I follow Fama and French's definition of ten industries, using the 4-digit SIC codes to divide the sample.[19] Table 5 presents the results across different specifications, covering Scope 1, Scope 2, and Scope 3 emissions with 3- and 4-digit SIC industry classifications. The results are reasonably consistent across different specifications.

Compared with other industries, energy is consistently among the most substitutable across all three emission scopes, and high-tech is also relatively elastic. Using Scope 1 emissions with 3-digit SIC classifications, high-tech is at 0.593 and energy at 0.520, both above the cross-industry average of 0.41; under Scope 2 the values are 0.547 and 0.457, and under Scope 3 they are 0.634 and 0.769. The high ν in high-tech could reflect that R&D- and intangible-intensive industries rely less on dirty capital or have a better chance of finding substitutes for it. The high ν in energy is consistent with the transition to renewable generation, as wind and solar can substitute for fossil-fuel inputs at the energy-flow margin.

In contrast, consumer goods (durables and nondurables), telecom, shops, and health industries have relatively lower elasticity of dirty capital. These are traditional or service-oriented industries with limited room for innovations in their production functions and are also less active in finding replacements for existing dirty technologies. This may be due to lower R&D investments or a lack of affordable and viable alternatives. For example, consumer-facing industries rely on long-lived physical infrastructure (stores, distribution networks, hospitals) for which clean alternatives are limited or costly to deploy.

4 Structural Estimation of the Flow-Driven Model

The reduced-form analysis in Section 3 suggests that clean and dirty capital are highly complementary, with elasticities of substitution well below one across a range of specifications. That analysis, however, has two limitations I have not yet resolved. First, the right-hand-side

[19] For more details, see Kenneth French's data library.

variable in the regression (the marginal productivity of emissions, $\partial Y/\partial G$) is not directly observed and is estimated from sector-year polynomial regressions, which introduces measurement error. Second, firms' emission choices can respond to unobserved shocks that are correlated with output, so the right-hand-side variable is partly endogenous. Both concerns can bias the reduced-form estimate of ν. In this section I address them by estimating a structural model using simulated method of moments (SMM). The structural approach imposes more economic discipline than the reduced-form regression: it uses the firm's optimal investment response to *exogenous* variation in ESG fund flows as the main source of identification, and it jointly matches a battery of moments that tie down the full set of production, financing, and demand parameters.

I base the structural estimation on the flow-driven model of Section 2.2, not the damage-function model of Section 2.1. There are two reasons. First, the flow-driven model's identifying moment (the correlation between the time series of ESG fund flows and the time series of aggregate emission intensity) is directly observable in firm-level data on 13F institutional holdings and carbon emissions. The damage-function model's analogous identifying moment (the covariance between a damage shock and output) would require separate measurement of physical climate damage, which is not available at the annual firm-level frequency that matches the investment decisions I want to discipline. Second, the two mechanisms are not mutually exclusive, and the flow-driven channel is the one that has been most cleanly identified in the recent empirical literature (Pastor, Stambaugh, and Taylor, 2023; Van der Beck, 2023); the damage channel remains useful as a comparative-statics benchmark, which I return to in Section 5.

The structure of the section follows Choi et al. (2025). Section 4.1 sets up the dynamic model, which is the infinite-horizon analog of the two-period flow-driven model of Section 2.2. Section 4.2 describes the data used to construct the empirical moments. Section 4.3 presents the SMM objective function. Section 4.4 discusses identification. Section 4.5 reports the parameter estimates and standard errors. Section 4.6 reports model fit. The solution method is described in Appendix B.3.

4.1 The Dynamic Model

Time is discrete and infinite, $t = 0, 1, 2, \ldots$. The economy is populated by a representative investor and a cross-section of firms indexed by $j = 1, \ldots, J$. The investor prices all assets through a pricing kernel M_{t+1}. Firms produce a single consumption good using two types of capital, clean $K_{C,jt}$ and dirty $K_{D,jt}$, and choose investment in each type to maximize market value. The market value deviates from the fundamental value because the investor's portfolio demand depends on a firm's characteristics, including its emission intensity, via the Koijen and Yogo (2019) demand system. The model extends the Section 2.2 flow-driven block to infinite horizon and shares production technology, capital accumulation, and adjustment costs with the dynamic damage model of Section 5.

State variables. Each firm's state is $(K_{C,jt}, K_{D,jt}, \zeta_{jt}, X_t, v_t)$, where $K_{C,jt}, K_{D,jt}$ are the current clean and dirty capital stocks (using the same capital-letter convention as in Section 2 and the dynamic damage model in Section 5), ζ_{jt} is an idiosyncratic log-productivity shock, X_t is the aggregate productivity shock (as in Section 2.1), and v_t is an aggregate shifter on the Koijen-Yogo demand index (as in Section 2.2). v_t multiplies the demand index uniformly across all firm characteristics; whether it acts as ESG sentiment in particular depends on the magnitude of the emission loading b_{emit}, since with $b_{\text{emit}} = 0$ a higher v_t raises demand for all firms proportionally rather than tilting toward greener ones. I drop the subscript j when referring to a representative firm.

Aggregate shocks. Aggregate productivity, in log form $x_t = \log X_t$, follows an AR(1):

$$x_{t+1} = \rho_x x_t + \sigma_x \epsilon_{x,t+1}, \qquad \epsilon_{x,t+1} \sim N(0,1). \tag{41}$$

The aggregate demand shifter v_t follows an AR(1) in logs:

$$\log v_{t+1} = \rho_v \log v_t + \sigma_v \epsilon_{v,t+1}, \qquad \epsilon_{v,t+1} \sim N(0,1). \tag{42}$$

The two innovations are independent. The persistence and volatility parameters $(\rho_x, \sigma_x, \rho_v, \sigma_v)$ are part of the estimation (except for ρ_x and σ_x, which I calibrate externally using aggregate TFP data).

Idiosyncratic productivity. Firm-level log-productivity ζ_{jt} follows an AR(1):

$$\zeta_{j,t+1} = \rho_\zeta \zeta_{jt} + \sigma_\zeta \epsilon_{\zeta,j,t+1}, \qquad \epsilon_{\zeta,j,t+1} \sim N(0,1), \tag{43}$$

independent across firms and across time, and independent of the aggregate shocks. I use ζ rather than the letter Z used in Section 5 because the flow-driven dynamic model has no emission-uncertainty shock, so there is no risk of confusion.

Production. Firms produce using a CES production function over clean and dirty capital, with decreasing returns to scale, adding a homogeneity exponent μ to the Section 2.2 specification:

$$F_{jt} = X_t \cdot \exp(\zeta_{jt}) \cdot \left[(1-\alpha)\, K_{C,jt}^{\omega} + \alpha\, K_{D,jt}^{\omega}\right]^{\mu/\omega}, \tag{44}$$

where F_{jt} is the firm's fundamental output, $\alpha \in (0,1)$ is the dirty-capital productivity share, and $\mu < 1$ is the homogeneity exponent. The parameter $\omega \equiv (\nu-1)/\nu \in (-\infty, 1)$ is the CES exponent, where $\nu = 1/(1-\omega) > 0$ is the elasticity of substitution between clean and dirty capital introduced in Sections 2 and 3. I parameterize the production function in terms of ω rather than ν for the structural estimation because ω is bounded above by 1 (the perfect-substitution limit, $\nu \to \infty$), whereas ν is unbounded, making ω the better-behaved parameter for numerical optimization in the SMM (see Figure 3 for the monotone mapping). The homogeneity exponent μ pins down firm size but does not affect the clean-dirty capital mix, so identifying ω from capital mix behavior is not contaminated by μ.

Emission intensity. Dirty capital produces emissions at rate ε_D per unit, as in Section 2.2:

$$e_{jt} = \frac{\varepsilon_D\, K_{D,jt}}{F_{jt}}. \tag{45}$$

Emission intensity is a firm characteristic observed by investors and used as an input in the demand system.

Capital accumulation and adjustment costs. Capital accumulates with partial depreciation:

$$K_{\ell,j,t+1} = (1 - \delta_\ell) K_{\ell,jt} + I_{\ell,jt}, \qquad \ell \in \{C, D\}. \tag{46}$$

Adjustment costs are Hayashi quadratic:

$$\psi^\ell(I_{\ell,jt}, K_{\ell,jt}) = \frac{\phi^\ell \, I_{\ell,jt}^2}{2 \, K_{\ell,jt}}. \tag{47}$$

The adjustment-cost parameters ϕ^C, ϕ^D can differ across clean and dirty capital, capturing the possibility that installing new clean technology is more costly than installing new dirty capital (or the reverse).

Cash flows. The firm's after-investment cash flow at t is

$$\pi_{jt} = F_{jt} - I_{C,jt} - I_{D,jt} - \psi^C(I_{C,jt}, K_{C,jt}) - \psi^D(I_{D,jt}, K_{D,jt}). \tag{48}$$

I abstract from taxes, leverage, and external financing frictions; the model focuses on the real investment decision and its interaction with the demand side of capital markets.

Pricing kernel. Extending the static Section 2.2 pricing kernel $\ln M = \ln \beta - \gamma(x_1 - x_0)$ to the dynamic setting, I use a CRRA-in-x specification with state-dependent price of risk following Choi et al. (2025):

$$\ln M_{t+1} = \ln \beta - \gamma_t(x_{t+1} - \rho_x x_t) - \tfrac{1}{2}\gamma_t^2 \sigma_x^2, \tag{49}$$

with $\gamma_t = \gamma_0 + \gamma_1 x_t$. This functional form lets the model match both the average equity premium and its time-series volatility without introducing habit or long-run risk preferences. The preference parameters $(\beta, \gamma_0, \gamma_1)$ are calibrated externally using standard equity-premium targets; see Section 4.5.

Investor demand. The investor's portfolio weight on firm j at time t follows the logit specification of Koijen and Yogo (2019), extending Section 2.2's static demand system to the full set of Koijen-Yogo characteristics:

$$w_{jt} = \frac{m_{jt}}{m_{jt} + \sum_{k \neq j} m_{kt} + 1}, \tag{50}$$

where the demand index m_{jt} is

$$m_{jt} = \exp\big(b_0 + b_{\text{mkt}} \log V_{jt} + b_{\text{size}} \log B_{jt} + b_{\text{profit}} \pi_{jt}/B_{jt} + b_{\text{invest}} I_{jt}/B_{jt} + b_{\text{emit}} \log e_{jt} + \log v_t\big). \tag{51}$$

The coefficients follow the lowercase-b convention of Section 2.2 (so as not to collide with the discount factor β or with the damage-model risk quantities β^C, β^D of Section 2.1). I use m_{jt} for the demand index to avoid collision with the homogeneity exponent μ in (44). The characteristics are the firm's fundamental value V_{jt} (the *ex ante* market value under the demand system), book value $B_{jt} = K_{C,jt} + K_{D,jt}$, profitability π_{jt}/B_{jt}, investment rate I_{jt}/B_{jt} (with $I_{jt} = I_{C,jt} + I_{D,jt}$), and emission intensity e_{jt}. The aggregate demand shifter v_t enters as a scalar multiplier on the demand index, as in Section 2.2.

Market clearing requires that the total portfolio weight across firms plus the outside (risk-free) option sums to one, which fixes the "+1" in the denominator of (50). The resulting equilibrium price impact (the demand markup ι of Section 2.2) is

$$\iota_{jt} = J w_{jt} - 1, \tag{52}$$

where J is the number of investable firms. When $b_{\text{emit}} < 0$, a firm with below-average emission intensity has m_{jt} higher than the average, which raises w_{jt} above $1/J$ and hence $\iota_{jt} > 0$ (a green premium), while a firm with high emission intensity faces $\iota_{jt} < 0$ (a brown discount). The shifter v_t scales the demand index uniformly across all characteristics; its impact on the green-versus-brown wedge in ι therefore operates through whichever priced characteristics happen to differ between green and brown firms, including emission intensity itself when b_{emit} is materially nonzero.

Firm's problem. Each firm chooses clean and dirty investment to maximize the expected present discounted value of market prices. Extending the two-period objective $V = \max_{I_C, I_D} CF_0 + E[M \cdot F_1(1+\iota)]$ from (22) to infinite horizon:

$$V(K_C, K_D, \zeta; X, v) = \max_{I_C, I_D} \; \pi - I_C - I_D - \psi^C - \psi^D + E_t\big[M_{t+1} \cdot P_{t+1}\big], \tag{53}$$

where $P_{t+1} = V(K_{C,t+1}, K_{D,t+1}, \zeta_{t+1}; X_{t+1}, v_{t+1}) \cdot (1+\iota_{t+1})$ is the market price at $t+1$. The capital accumulation equations close the problem. As in Section 2.2, the firm maximizes *market* value, not fundamental value, because the investor demand system drives a wedge ι between the two. This wedge is the economic channel through which sustainable investing affects real investment.

The solution is a pair of policy functions $I_C(K_C, K_D, \zeta; X, v)$ and $I_D(K_C, K_D, \zeta; X, v)$, and a value function $V(K_C, K_D, \zeta; X, v)$. The demand system (50) creates a fixed-point problem because each firm's ι depends on the cross-section of fundamental values $\{V_{kt}\}$, which in turn depend on the distribution of capital $\{(K_{C,k}, K_{D,k})\}$. The shock processes are discretized using the Tauchen method for the SMM model, while the damage block in Section 5.2 uses Rouwenhorst, which is more accurate for highly persistent shocks such as the emission process. The numerical solution method is described in Appendix B.3.

4.2 Data

The empirical moments are constructed from three data sources: (i) firm-level emissions from S&P Trucost; (ii) firm-level accounting and market data from Compustat; and (iii) institutional portfolio holdings from the SEC Form 13F filings aggregated by Thomson-Reuters. These sources overlap on a sample of U.S. publicly listed non-financial firms covered by Trucost, matched to Compustat by GVKEY and to 13F by CUSIP. Accounting and emissions moments are constructed from the full Compustat-Trucost panel (2000–2023). Holdings-based moments (the ESG flow tilt, the GMB series, and the Koijen-Yogo auxiliary regression) use 2012–2023, the period of active ESG investing identified in Van der Beck (2023) and Pastor, Stambaugh, and Taylor (2023). The same sample is

used for the reduced-form analysis in Section 3.

Emission intensity. I compute firm-level emission intensity $e_{jt} = G_{jt}/F_{jt}$, where G_{jt} is the sum of Scope 1 and Scope 2 carbon emissions in tons of CO_2 equivalent (from Trucost) and F_{jt} is total revenue (from Compustat, used as the empirical proxy for fundamental output). The cross-sectional and time-series moments of e_{jt} are used to identify ε_D and the idiosyncratic productivity process.

Investment and capital. Investment rates I_{jt}/B_{jt} are computed from Compustat capital expenditures scaled by lagged physical capital. Book value B_{jt} is total tangible capital ($PPEGT_{jt}$). The clean/dirty split is a model object: $K_{C,jt}$ and $K_{D,jt}$ are not separately observed in the data and are recovered from the structural model at the SMM estimates of α and ω.

ESG fund flows and the green-minus-brown tilt. Following Pastor, Stambaugh, and Taylor (2023), I construct an aggregate green-minus-brown (GMB) holdings tilt from 13F filings. For each institution-quarter, I compute the portfolio's average emission intensity weighted by dollar holdings. I then compute the cross-sectional difference in portfolio-average emission intensity between the top- and bottom-decile "green" and "brown" institutions, call it GMB_t. A rising GMB_t reflects tilt toward green firms. The key empirical moment for identifying ω is the correlation between ΔGMB_t and $\Delta\bar{e}_t$, where $\bar{e}_t$ is the cross-sectional average of firm-level emission intensities.

Returns and risk premia. Stock returns are monthly CRSP returns aggregated to the annual frequency. The green and brown return series are constructed from value-weighted portfolios of firms in the top and bottom deciles of emission intensity. The equity premium and its volatility are computed from the CRSP value-weighted market portfolio over the sample period.

4.3 SMM Strategy

Let m denote the vector of empirical moments constructed from the data, and $m(\theta)$ the corresponding simulated moments from the model at parameter vector θ. The SMM estimator

solves

$$\hat{\theta} = \arg\min_{\theta} \left[m - m(\theta)\right]' W \left[m - m(\theta)\right], \tag{54}$$

where W is a positive-definite weighting matrix. The estimator targets equality between the model and data moments at $\hat{\theta}$.

The parameter vector has dimension 15:

$$\theta = (\omega, \alpha, \mu, \varepsilon_D, \phi^C, \phi^D, \rho_\zeta, \sigma_\zeta, b_{\text{mkt}}, b_{\text{size}}, b_{\text{profit}}, b_{\text{invest}}, b_{\text{emit}}, \rho_v, \sigma_v). \tag{55}$$

The remaining parameters are calibrated externally. The pricing-kernel preference parameters are calibrated sequentially following Choi et al. (2025): γ_0 is solved to match a 7.1% mean equity premium, γ_1 to match a 19.1% equity-premium volatility, and the subjective discount factor $\beta = 0.9652$ is then pinned in closed form to deliver a 1.6% annual risk-free rate, all targets computed from the CRSP value-weighted market portfolio over the sample period. The aggregate productivity process (ρ_x, σ_x) is set from measured aggregate TFP; the depreciation rates are $\delta_C = \delta_D = 0.13$, calibrated to average investment rates in the data; and the investor taste-shock standard deviations are set from preliminary empirical passes on the holdings data.

The moments are organized in three blocks:

1. **Firm-level cross-sectional moments**: mean, standard deviation, and autocorrelation of investment rates, emission intensities, and profitability; the cross-sectional correlation between investment and emission intensity; the Tobin's q spread between the highest and lowest deciles of emission intensity. These moments identify ε_D, ϕ^C, ϕ^D, μ, α, and $(\rho_\zeta, \sigma_\zeta)$.

2. **Aggregate time-series moments**: the mean and volatility of the equity premium; the mean and autocorrelation of the GMB tilt; the volatility of ESG fund flows. These moments identify (ρ_v, σ_v) and verify the externally calibrated SDF parameters.

3. **Cross-equation moment**: the correlation between ΔGMB_t and $\Delta \bar{e}_t$. This moment is the primary identifier for ω.

A full list of moments and data values is provided in Table 7.

Indirect inference on demand loadings. For the Koijen-Yogo demand coefficients $(b_{\text{mkt}}, b_{\text{size}}, b_{\text{profit}}, b_{\text{invest}}, b_{\text{emit}})$, I follow the indirect-inference approach of Koijen and Yogo, 2019. At each candidate θ, I simulate a panel from the model and run the same auxiliary regression (holdings weight on firm characteristics) on the simulated panel that Koijen and Yogo, 2019 run on the real data. On the data side, the market-value regressor is observed market capitalization; on the model side, the corresponding regressor is the firm's continuation value, which plays the role of fundamental market value in the demand system. Because the model has a single representative investor, the data regression's manager-by-year demeaning collapses to year demeaning on the model side. The difference between the simulated auxiliary-regression coefficients and the data coefficients enters the SMM objective, allowing the demand loadings to be identified jointly with the production and adjustment-cost parameters.

4.4 Identification

Identification of ω draws on multiple moments rather than a single time-series correlation. A numerical sensitivity analysis of the SMM Jacobian, scale-normalized so each entry represents a proportional change in the moment per unit change in the parameter, points to three firm-level panel moments with the largest loadings on ω: the firm-level AR(1) of emission intensity, the cross-sectional standard deviation of emission intensity, and the within-firm correlation between the investment rate and emission intensity. The economic logic is the same across these moments: ω governs how quickly and flexibly the clean/dirty capital mix adjusts to shocks. When ω is close to one (near-perfect substitutes), firms reallocate easily, so emission intensity exhibits low firm-level persistence, wider cross-sectional dispersion across firm-specific shock realizations, and a tighter within-firm comovement between investment and emission intensity. When ω is far below zero (strong complements), the production friction locks the capital mix in place, raising persistence and compressing cross-sectional dispersion. Identification therefore comes from the full firm-level

panel, not from any single moment.

Why the ESG-flow moment plays a special role. The correlation between ΔGMB_t and $\Delta\bar{e}_t$ also plays an important role for the identification strategy because it addresses the endogeneity concern from the reduced-form regression. The reduced-form regression relies on within-firm covariation between emission intensity and the marginal productivity of emissions. Both variables are endogenous objects chosen by the firm, which creates the identification concerns summarized in the opening of this section. The SMM moment, by contrast, uses *aggregate* ESG fund flows as the driving variable. These flows primarily reflect investor sentiment about climate, which is plausibly exogenous to firm-level productivity shocks and production-function parameters. Taking first differences also eliminates any fixed firm-level unobservables. The identifying restriction comes from the model's cross-equation mapping between flows and emission intensity, not from a direct measure of $\partial Y/\partial G$. This is exactly the source of variation that the reduced-form regression lacks.

Identification of the remaining parameters. The other parameters are identified as follows. Mean emission intensity pins down ε_D. The mean, volatility, and autocorrelation of investment rates identify the adjustment-cost parameters ϕ^C, ϕ^D and the homogeneity exponent μ. Cross-sectional dispersion and firm-level autocorrelation of investment and emission intensity identify $(\rho_\zeta, \sigma_\zeta)$. The cross-sectional correlation between investment and emission intensity identifies α. The clean-minus-dirty Tobin's q spread (the price-level green premium at each date) identifies b_{emit}, because a stronger investor distaste for emissions produces a larger cross-sectional spread in valuations. The mean and volatility of the equity premium verify the externally calibrated pricing-kernel preference parameters. The Koijen-Yogo demand loadings $(b_{\text{mkt}}, b_{\text{size}}, b_{\text{profit}}, b_{\text{invest}})$ are identified by the indirect-inference auxiliary regression. The persistence and volatility of the aggregate demand-shifter process (ρ_v, σ_v) are identified by the autocorrelation of the GMB tilt and the standard deviation of ESG flows.

4.5 Parameter Estimates

Table 6 reports the SMM estimates and their standard errors. Standard errors are computed using the sandwich formula:

$$\mathrm{Var}(\hat{\theta}) = T_{\text{eff}}^{-1} \left(G'WG\right)^{-1} G'WSWG \left(G'WG\right)^{-1}, \tag{56}$$

where $G = \partial m(\hat{\theta})/\partial\theta$ is the numerical Jacobian of model moments at $\hat{\theta}$, W is the weighting matrix used in the objective function, and S is the moment-covariance matrix computed analytically by the delta method. The effective sample size is set to $T_{\text{eff}} = 30$, the length of the full Compustat-Trucost panel underlying the accounting and emissions moments.

Headline estimate of ω. The estimated CES substitution parameter is $\omega = -0.80$ (SE 0.18), with the 95% confidence interval $[-1.15, -0.46]$ lying entirely below zero. This maps to an elasticity of substitution $\nu = 1/(1-\omega) \approx 0.56$, which confirms the reduced-form finding that dirty and clean capital are gross complements. The estimate excludes perfect substitution ($\omega = 1$) at well below the 1% level, and also excludes Cobb-Douglas ($\omega = 0$) at the 1% level. The SMM and Bai (2009) interactive-fixed-effects estimates (Table 3) therefore agree on the headline finding of gross complementarity.

The estimated $\nu = 0.56$ is markedly below the elasticities reported in existing empirical work: Papageorgiou, Saam, and Schulte (2017) estimate $\nu \approx 1.8$ in electricity-generating sectors across 26 countries, and Jo (2025) recover a similarly elastic estimate from French manufacturing firms. Both prior estimates are static and identified at the energy-input margin; the present paper estimates ν dynamically and at the within-firm physical-capital margin. Theoretical calibrations assume even larger values, with Acemoglu et al. (2012) using $\nu \in \{3, 10\}$. The divergence reflects the difference in margin: existing estimates identify substitution from variation in energy consumption, capturing short-run fuel-switching within a given capital stock, whereas the present paper identifies ν from the long-run capital investment response to ESG-driven demand shocks. Because physical capital

is slow to adjust and dirty capital has no clean substitute for many production processes, this longer-run margin is naturally less elastic than fuel-switching.

Estimate of α. The estimated CES dirty-capital share is $\alpha = 0.81$, indicating that dirty capital accounts for a substantial share of production once adjustment costs and the demand channel are accounted for. Most existing work calibrates α to match observed energy or revenue shares rather than estimating it from investment data (Acemoglu et al., 2012; 2016); the present paper recovers α from the cross-sectional correlation between firm-level investment rates and emission intensity. This estimate is broadly consistent with fossil fuels supplying approximately 82% of global primary energy as of 2023 (International Energy Agency, 2024).

At the estimated parameters, the model's equilibrium dirty-capital ratio is $K_D/(K_C + K_D) \approx 70\%$ at the benchmark ($\theta = 0$; Table 8, Panel A), lower than α because gross complementarity ($\nu < 1$) forces firms to hold more clean capital alongside dirty capital than a purely productivity-based allocation would dictate. This equilibrium ratio is consistent with the data. To see this, define emission intensity empirically as E_i/Y_i (Scope 1 CO_2 in metric tons divided by revenue in USD), the direct counterpart to $\varepsilon_D K_{D,i}/Y_i$ in the model. Because E_i/Y_i is monotone in $K_{D,i}$ holding output fixed, ranking firms by E_i/Y_i ranks them by dirty-capital intensity, and the cross-sectional median is the natural threshold separating dirty-dominant ($K_D > K_C$) from clean-dominant ($K_C > K_D$) firms. In the Compustat-Trucost sample, summing gross property, plant, and equipment (PPEGT) for above-median firms and dividing by aggregate PPEGT yields 69%, close to the model's 70.5%. The construction of this empirical share is detailed in Appendix B.2. Note that this 69% is the PPEGT share of dirty-dominant *firms*, not aggregate $K_D/(K_C + K_D)$ itself, since those firms also hold some K_C; it is therefore an upper bound on the true dirty-capital share in the data, bracketing the model's equilibrium from above.

Other production parameters. The homogeneity exponent is $\mu = 0.71$, in the standard range for firm-level decreasing-returns estimates. The per-unit emission rate is $\varepsilon_D = 69.97$, which matches

the sample mean of emission intensity after scaling. Adjustment costs are asymmetric: $\phi^C = 1.90$ and $\phi^D = 0.37$. Clean capital is substantially more costly to adjust than dirty capital. This is consistent with the interpretation that clean technology is newer and scaling it up requires building and deploying unfamiliar production systems, whereas dirty-capital adjustments draw on long-established industrial infrastructure. The asymmetry is economically meaningful: it represents a second friction that slows the green transition, distinct from the CES complementarity captured by ω. In counterfactuals, this distinction matters because a policy that lowers the adjustment cost on clean capital (an R&D subsidy, say) would have a different effect from one that raises the complementarity elasticity (a technology-transition policy).

ESG demand and sentiment. The Koijen-Yogo demand coefficient on emission intensity is b_{emit}, close to zero. This suggests that, once other firm characteristics are controlled for in the holdings regression, the pure cross-sectional demand penalty on high-emission firms is modest. The SMM result is consistent with the data moment itself: the auxiliary-regression loading on $\log e_{jt}$ in the data is small in magnitude (-0.011) and not statistically distinguishable from zero once standard errors account for serial correlation within firm and within manager.

With $b_{\text{emit}} \approx 0$, v_t does not act as an emission-specific sentiment shock. It enters the demand index (51) as a scalar multiplier on *all* characteristics, so the cross-sectional spread in ι_{jt} between high- and low-emission firms is generated by the correlation between emission intensity and the other priced characteristics (size, profitability, investment rate), not by a direct emission-based penalty. The aggregate process driving v_t is highly persistent, $\rho_v = 0.93$, and moderately volatile, $\sigma_v = 0.44$. The idiosyncratic productivity process has $\rho_\zeta = 0.63$ and $\sigma_\zeta = 0.13$, similar to values in the standard investment-based asset-pricing literature.

4.6 Model Fit

Table 7 reports each model-implied moment alongside its data target. The fit is strongest for the moments that the literature has identified as critical. The ESG-flow-to-emission-intensity

correlation (the primary identifier for ω) matches almost exactly: -0.151 in the model versus -0.153 in the data. The equity-premium mean and volatility also match well: 0.075 versus 0.071 for the mean, and 0.183 versus 0.191 for the volatility. Both moments match the data within a half percentage point.

Mean greenium. The mean green-minus-brown return premium is not a targeted moment in the SMM objective because there is no consensus in the empirical literature about its sign or magnitude. Bolton and Kacperczyk (2021) find a positive carbon premium (brown minus green) of roughly 2–3% in U.S. equities; Hsu, Li, and Tsou (2023) report that carbon-intensive firms earn about 4% higher returns per year. In contrast, Pástor, Stambaugh, and Taylor (2022) argue that the *ex ante* premium is negative and that realized positive spreads reflect unexpected shocks to ESG sentiment, and Zhang (2024) document that much of the apparent premium disappears after correcting for ex-post revisions to emissions data. Point estimates in the literature span roughly -3% to $+4\%$ per year. At the estimated parameters, the model produces a realized greenium of $+4.8\%$ per year (time-series mean of the top-minus-bottom emission-intensity decile spread, winsorized at the 1%/99% tails; standard deviation 2.0%) and a physical-measure expected-return greenium of $+5.0\%$. Both fall in the upper part of the empirical range. The positive sign is consistent with Bolton and Kacperczyk (2021) and Hsu, Li, and Tsou (2023); the magnitude is close to the Hsu, Li, and Tsou (2023) estimate. The greenium is not driven by a direct ESG demand penalty (the estimated b_{emit} is essentially zero) but by the correlation between emission intensity and other firm characteristics that enter the demand system (size, investment rate, profitability), which opens a cross-sectional wedge in the price impact ι across emission deciles.

Under-shoots. The model under-shoots some cross-sectional dispersion. The standard deviation of firm-level investment is 0.05 in the model versus 0.15 in the data, and the cross-sectional standard deviation of emission intensity is 200 versus 420. The model also under-shoots the magnitude of some of the Koijen-Yogo demand loadings (notably b_{mkt} and b_{profit}) and the mean of the

GMB tilt. These tensions reflect sources of firm-level heterogeneity that the current specification does not capture, such as heterogeneity in financing frictions, tax treatment, or industry-specific adjustment-cost structure. Adding such heterogeneity would likely improve the fit on dispersion moments without affecting the identification of ω, since the ESG-flow-to-emission-intensity correlation is an aggregate time-series moment that is insensitive to cross-sectional dispersion.

5 Quantitative Results

This section reports benchmark and counterfactual results from the two dynamic mechanisms. The flow-driven model (Section 5.1) builds on the SMM-estimated specification of Section 4.1 and traces how capital allocation responds to ESG demand pressure. The damage-function model (Section 5.2) is the infinite-horizon analog of Section 2.1: it shares the firm technology of the flow-driven model but replaces the Koijen-Yogo demand wedge with a DICE-style temperature-and-damage block, so the same complementarity friction is evaluated through cash flows rather than valuations. Section 5.3 then turns to the greenium, which the two models generate through distinct mechanisms.

5.1 Flow-Driven Model

This subsection reports counterfactuals from the flow-driven dynamic model of Section 4.1, evaluated at the SMM estimates in Table 6. ESG demand pressure is parameterized as $\theta \equiv -b_{\text{emit}}$, the (sign-flipped) Koijen-Yogo loading on emission intensity, so that $\theta = 0$ corresponds to no emission-based demand penalty and larger θ corresponds to stronger investor distaste for high-emission firms. I sweep the elasticity of substitution $\nu \in \{0.56, 1, 2, 5, +\infty\}$ across $\theta \in \{0, 0.3, 0.5, 0.7\}$, varying one structural parameter at a time relative to the benchmark and holding the remaining SMM-estimated parameters fixed. Table 8 reports the time-series average dirty-capital ratio $K_D/(K_C + K_D)$ and Table 9 reports firm-level emission intensity $\bar{e}$. The underlying mean clean and dirty capital stocks are in Internet Appendix Tables C.9 and C.10.

The headline result is that complementarity reverses the sign of the response to ESG demand.

In the benchmark (Panel A), at the SMM-estimated $\nu = 0.56$ the dirty-capital ratio *rises* from 70.48% at $\theta = 0$ to 76.56% at $\theta = 0.7$ (+8.62%), whereas at $\nu = +\infty$ it falls from 92.99% to 84.50% (−9.13%). The cross-over is monotone in ν: at $\nu = 1$ the change is −3.84%, at $\nu = 2$ it is −6.72%, and at $\nu = 5$ it is −7.64%. Emission intensity moves with the same pattern (Table 9, Panel A): under complementarity it climbs from 243.7 to 299.9 tCO_2/\$M (+23.0%), while under perfect substitution it falls from 295.3 to 201.5 tCO_2/\$M (−31.8%). At empirically plausible levels of complementarity, ESG demand pressure does not redirect capital from dirty to clean—it pushes the economy in the opposite direction.

The mechanism is visible in the underlying capital stocks. At $\nu = 0.56$ in the benchmark, both $\bar{K}_C$ and $\bar{K}_D$ decline as θ rises, but clean capital falls faster: $\bar{K}_C$ drops from 14.67 to 7.94 (−46%, Table C.9) while $\bar{K}_D$ drops from 35.85 to 27.75 (−23%, Table C.10). Complementarity locks the two factors together in production, so firms cannot expand clean to substitute away from the demand-penalized dirty input. The price wedge against dirty firms instead propagates to clean capital through complementarity, and firms scale down both. They scale down clean more aggressively because the productivity asymmetry $\alpha = 0.81$ makes clean the marginally less valuable factor, so the bundle becomes dirtier. At the opposite end ($\nu = +\infty$), $\bar{K}_C$ stays essentially flat near the lower bound of the policy grid while $\bar{K}_D$ collapses from 123.5 to 27.8 (−77%). Substitutable firms simply shed dirty capital, so the dirty share falls sharply.

Three structural features of the estimated model interact to produce this reversal. The first is the demand channel itself, the second is complementarity, and the third is the productivity-and-adjustment-cost asymmetry between clean and dirty capital.

First, the ESG demand system does create the intended differential in cost of capital. The Koijen-Yogo logit framework assigns a valuation premium to cleaner firms ($\iota > 0$) and a discount to dirtier firms ($\iota < 0$), so the firm's first-order conditions include a demand channel that adds a positive incentive for clean investment ($\partial\iota/\partial I_C > 0$) and a negative one for dirty investment ($\partial\iota/\partial I_D < 0$). The demand channel is therefore pushing toward clean capital, not absent. However,

the estimated cross-sectional loading on emission intensity, b_{emit}, is near zero, so the magnitude of this within-firm composition incentive is modest; the primary ESG-related variation enters through the aggregate demand shifter v_t as a scaling of overall demand rather than a sharply targeted cross-sectional penalty on emission intensity.

Second, the estimated complementarity ($\nu = 0.56$) prevents the demand channel from translating into a compositional tilt. With $\nu < 1$, the marginal product of dirty capital rises sharply as K_D contracts, largely offsetting the demand penalty in the $[I_D]$ first-order condition. The demand wedge therefore propagates from dirty to clean through the production technology: firms cannot expand K_C to substitute for the penalized K_D, and both stocks must contract together.

Third, the productivity asymmetry ($\alpha = 0.81$) determines which factor absorbs the bulk of the contraction. With dirty capital contributing roughly four times as much output per unit as clean, the firm treats K_C as the marginally less valuable input and scales it down more aggressively. The asymmetric adjustment costs ($\phi^C = 1.90$ versus $\phi^D = 0.37$) amplify this: rebuilding clean capital is approximately five times more costly than replacing dirty, so the firm allows K_C to depreciate without full reinvestment. Together, these three features explain why the demand channel, while pushing in the right direction, is overwhelmed by the production friction: K_C falls twice as fast as K_D in percentage terms and the dirty share rises rather than falls as ESG pressure strengthens.

The remaining panels of Tables 8 and 9 confirm that the sign reversal is robust to perturbations of the structural environment, and that emission intensity tracks the dirty-capital ratio closely across panels. Panel B (equal productivity shares, $\alpha = 0.49$) flips the productivity asymmetry by making clean capital as productive as dirty in the CES bundle. One might expect this to fix the perverse response, since the firm no longer treats K_C as the marginally less valuable factor. The opposite happens at $\nu = 0.56$: the wrong-sign response in the dirty-capital ratio is preserved and amplified, with Diff. rising from +8.62% in the benchmark to +34.2%. Emission intensity moves even more sharply, climbing from 350.2 to 605.4 tCO_2/\$M (+72.9%, Table 9, Panel B), because output also contracts as the firm shrinks both stocks. The reason for the amplification is that the

starting dirty share at $\theta = 0$ is now near 50% rather than 70% in Panel A, so as ESG pressure forces both stocks to contract, the higher clean adjustment cost $\phi^C = 1.90$ alone is enough to push the dirty share upward; the productivity channel was previously offsetting this and is now neutral. Panel B is the cleanest demonstration that complementarity plus asymmetric adjustment costs, even without productivity asymmetry, are sufficient to deliver the sign reversal.

Panels C and D add a carbon tax on top of ESG demand. Panel C ($\tau_D = 1.6\%$ of revenue, the EU ETS rate around 2024) leaves the cross-over essentially unchanged from the benchmark: at $\nu = 0.56$, Diff. is +9.04% in the dirty-capital ratio (versus +8.62% without the tax) and +24.6% in emission intensity (versus +23.0%). Panel D pushes the tax to $\tau_D = 16\%$ of revenue, an order of magnitude larger than any policy currently on the books, and the perverse pattern persists: the dirty-capital ratio still rises with θ at $\nu = 0.56$ (Diff. = +11.3%) and emission intensity still climbs from 274.4 to 331.4 tCO_2/\$M (+20.8%). Notably, the level of emission intensity in Panel D is uniformly higher than in the no-tax benchmark across all θ values at $\nu = 0.56$ (e.g., 274.4 versus 243.7 at $\theta = 0$), because the tax shrinks output faster than emissions when complementarity ties the two capital stocks together. A carbon price therefore does not solve the friction it was designed to address; under complementarity, it can even raise the level of emission intensity.

Panel E equalizes the adjustment costs ($\phi^C = \phi^D$), removing one of the three structural features identified above. The cross-over between $\nu = 0.56$ and $\nu = +\infty$ is preserved (Diff. at $\nu = 0.56$ is +6.79% in the dirty-capital ratio and +5.03% in emission intensity, both compressed but not flipped), confirming that complementarity and the productivity asymmetry alone are enough to generate the perverse response; the adjustment-cost gap amplifies but does not create it. Across all four perturbations, the empirically estimated complementarity places the U.S. economy on the side of the cross-over where ESG flows raise both the dirty share and emission intensity. The perverse response does not rely on any single parameter assumption. It survives all four perturbations (Panels B through E), and only a much higher value of ν would reverse it.

5.2 Dynamic Damage Model

This subsection sets up and solves the dynamic damage-function model, the infinite-horizon analog of Section 2.1. The model combines elements of the classical investment-based model with an exogenous log-linear SDF (Zhang, 2005) and standard elements from the dynamic integrated climate economy (DICE) model (Nordhaus and Sztorc, 2013; Cai and Lontzek, 2019). Unlike the two-period model, it uses a more realistic representation of temperature change as a proxy for climate concerns (Nordhaus and Sztorc, 2013; Balvers, Du, and Zhao, 2017; Bansal, Ochoa, and Kiku, 2017; Hugon and Law, 2019). Investors' preferences regarding climate change are captured by a positive price on the carbon emission shock in the pricing kernel. Firms make optimal dirty and clean investment decisions in response to the exogenous pricing kernel. Time is discrete and infinite.

Following the infinite-horizon model in Section 4.1, the economy is populated by a cross-section of firms indexed by $j = 1, \ldots, J$, all sharing the same technology and pricing kernel but drawing idiosyncratic shocks independently. This panel structure lets the damage model be simulated as a panel of J firms in a common aggregate economy, directly comparable to the flow-driven panel in Section 4.1. Production, capital accumulation, and adjustment-cost parameters are shared with the flow-driven model; damage-function and temperature parameters are calibrated in the DICE tradition; the aggregate emission-shock process (ρ_Z, σ_Z) is estimated from the Compustat-Trucost panel. The emission-risk price θ is not a calibration target; it is varied across the sweep to trace how sustainable-investing preferences affect firms' investment decisions. For each value of θ, γ and β are re-calibrated to match the same equity-premium and risk-free-rate targets as the SMM. Calibration details are in Section 5.2.3.

5.2.1 The Firm

Each firm j at time t has a state $(K_{C,jt}, K_{D,jt}, \zeta_{jt}, X_t, \Delta T_t, Z_{jt})$, where $K_{C,jt}, K_{D,jt}$ are clean and dirty capital stocks, ζ_{jt} is an idiosyncratic log-productivity shock, X_t is aggregate productivity (common to all firms), ΔT_t is the cumulative temperature change, and Z_{jt} is a firm-specific

emission-multiplier shock.[20] As in the DICE framework (Nordhaus and Sztorc, 2013), each firm's total output is

$$\Pi_{jt} = (1 - D_t)\, F_{jt}, \tag{57}$$

where D_t is the damage function and F_{jt} is the pre-damage fundamental output given by the same CES production function (44) as in Section 4.1. ω, α, and μ are fixed at their SMM-estimated values; I discuss why technological substitutability is plausibly exogenous over the sample and a preliminary endogenous-ω extension in Section 6.

I follow the climate economics literature in introducing a damage function for total output. This damage function can be interpreted as either a loss in product demand because of the firm's reputation loss or as a shareholder's welfare loss induced by the negative externalities that are internalized in the firm's objective function, as in Hart and Zingales (2017).[21] As in previous studies, the damage increases with temperature relative to preindustrial levels (Cai and Lontzek, 2019), and temperature change is driven by aggregate emissions: a firm with more dirty capital emits more, contributing more to future warming and therefore to future damage. The specific functional form of the damage is

$$D_t = 1 - \frac{1}{1 + a_1 \Delta T_t + a_2 \Delta T_t^2 + a_3 \Delta T_t^{\eta_c}}, \tag{58}$$

where ΔT_t is the temperature at time t relative to its preindustrial level (Nordhaus and Sztorc, 2013; Nordhaus, 2014). The third term $a_3 \Delta T_t^{\eta_c}$ captures catastrophic, accelerating damage at high temperatures following Weitzman (2012) and Cai and Lontzek (2019); with $\eta_c = 6.754$ (Weitzman's calibrated exponent) and $a_3 = 5 \times 10^{-5}$ this term is negligible near $2°C$ but contributes meaningfully

[20] I use ζ (not Z) for the idiosyncratic productivity to keep it distinct from the damage-specific emission shock Z. The SMM simulation in Section 4.1 uses the same letter ζ for idiosyncratic productivity, so the two panels have directly comparable firm-level heterogeneity.

[21] If global warming passes certain thresholds, the climate system may enter an irreversible state with significantly more severe disasters. According to the Federal Emergency Management Agency (FEMA), the most frequent natural disasters are fires, severe storms, and floods, all of which are directly linked to global warming. Alternatively, the damage can be seen as a negative externality internalized in shareholder welfare. As in Hart and Zingales, 2017, the firm should maximize welfare rather than market value if profit and damage are inseparable, especially when firms can pollute but individuals cannot offset pollution by themselves.

above $4°C$. Capital accumulates with the same partial-depreciation rule and quadratic adjustment costs as in Section 4.1. The firm's emission flow is

$$G_{jt} = \sigma_D \, K_{D,jt} \, Z_t, \tag{59}$$

where σ_D is the per-unit emission rate of dirty capital (calibrated separately for this block; see Section 5.2.3) and Z_t is an emission-multiplier shock. The shock is drawn at the firm level in simulation but is calibrated from its aggregate component (the cross-sectional mean of $\log Z_{jt}$ year by year), so the systematic piece is what drives the pricing kernel. A positive emission shock is considered bad news by investors, who impose a positive price on it; a higher positive price indicates stronger aversion. Aggregate temperature rises proportionally to aggregate emissions (Matthews et al., 2009; Barnett, Brock, and Hansen, 2020):

$$\Delta T_{t+1} = \Delta T_t + \lambda \, \bar{G}_t, \qquad \bar{G}_t = \frac{1}{J} \sum_{j=1}^{J} G_{jt}. \tag{60}$$

ΔT_t is a common aggregate state, and each firm's output is reduced by the damage $D(\Delta T_t)$. In the value function iteration, I use a representative-firm approximation in which next-period ΔT is computed from the firm's own G_{jt}; in the panel simulation, ΔT evolves according to the aggregate emission flow $\bar{G}_t$ as in the equation above.

Aggregate log-productivity x_t and firm-level log-productivity ζ_{jt} follow the same independent AR(1) processes as in Section 4.1. The aggregate emission shock $z_t = \log Z_t$ follows an AR(1):

$$z_{t+1} = \rho_Z z_t + \sigma_Z \, \epsilon^Z_{t+1}, \qquad \epsilon^Z_{t+1} \sim N(0,1), \tag{61}$$

common to all firms. The contemporaneous cash flow of firm j is

$$CF_{jt} = \Pi_{jt} - \tau_D \, G_{jt} - I_{C,jt} - I_{D,jt} - \psi^C_{jt} - \psi^D_{jt}, \tag{62}$$

where τ_D is a carbon tax applied per unit of emissions. Because the carbon tax usually captures the social welfare loss from carbon emissions, it enters shareholders' welfare and the firm's objective

function if the firm maximizes shareholders' welfare, as in Hart and Zingales (2017).[22] Taking the pricing kernel as given, the firm's Bellman equation is:

$$V(K_C, K_D, \zeta; X, \Delta T, Z) = \max_{I_C, I_D} \left\{ CF + E\left[M_{t+1} V(K_C', K_D', \zeta'; X', \Delta T', Z') \right] \right\} \tag{63}$$

subject to capital accumulation, temperature law, shock AR(1)'s.

(Firm subscripts j are suppressed when referring to the representative firm's value function.)

Greenium. I define the greenium using a tercile-based construction. At each period t, firms are ranked by their contemporaneous dirty-capital ratio $K_{D,jt}/(K_{C,jt} + K_{D,jt})$; the greenium is the time-series average of the expected-return spread between the top-tercile (high dirty-capital-ratio, i.e. high-emission) and bottom-tercile firms. Because the emission flow $G_{jt} = \sigma_D K_{D,jt} Z_t$ is proportional to $K_{D,jt}$ given the aggregate shock Z_t, sorting by dirty-capital ratio is equivalent to sorting by emission intensity in any period. The sign and magnitude of the greenium depend on (i) the gap in ΔT-sensitivity between the two terciles, which in turn depends on the gap in K_D; and (ii) the CES elasticity ω: when ω is close to 1 (perfect substitution), the two terciles' capital stocks diverge sharply and the greenium can open wide; when ω is strongly negative (complements), production forces drive each firm toward a balanced (K_C, K_D) mix regardless of idiosyncratic shocks, compressing the tercile gap and the greenium.

5.2.2 The Pricing Kernel

The exogenous log-linear SDF, or pricing kernel, has the following form:

$$\ln M_{t+1} = \ln \beta - \gamma(x_{t+1} - x_t) + \theta(z_{t+1} - \rho_Z z_t) \tag{64}$$

where β is the time discount factor, γ is the price of aggregate-productivity risk, and θ is the price of emission risk. For each value of θ, (γ, β) are re-calibrated so that the simulated panel

[22] I adopt a reduced-form treatment of the carbon tax and take it as exogenous. The optimal-tax literature typically defines it as the "social cost of carbon," the welfare loss induced by emissions (Weitzman, 2014; Bansal, Ochoa, and Kiku, 2017; Cai and Lontzek, 2019; Lemoine, 2021; Olijslagers and van Wijnbergen, 2024). Introducing τ_D lets me explore how capital substitutability interacts with carbon-tax policy and matches the real-world practice in an increasing number of countries.

matches the same two moments as the SMM: an annual equity premium of 7% and an annual risk-free rate of 1.6%. Because β enters $E[M]$ multiplicatively, its calibration is closed-form given γ; γ is solved by bisection.[23] Because a positive emission shock is bad news for investors, it should correlate positively with M_{t+1}. The emission term loads on the AR(1) innovation $z_{t+1} - \rho_Z z_t$ (the surprise component of z relative to its conditional expectation $\rho_Z z_t$) rather than on the level or a deviation from the unconditional mean. This innovation-based specification has the important practical advantage of making $E_t[M_{t+1}]$ state-independent (given ρ_Z close to unity, the conditional expectation of $\exp(\theta(z_{t+1} - \rho_Z z_t))$ is nearly constant across states), which is critical for numerical stability of the VFI when $\rho_Z = 0.946$ is large. In the continuous Gaussian limit, the innovation specification and the level specification are equivalent up to a constant that is absorbed by β; in the discretized Rouwenhorst grid, the innovation specification delivers substantially more uniform pricing-kernel weights across grid points and faster convergence.

5.2.3 Calibration

The model is solved and simulated at an annual frequency over 200 years. Table 12 reports the full parameter vector. Production-side parameters (CES shape ω and its elasticity ν, dirty-capital share $\alpha = 0.81$, returns-to-scale μ, adjustment-cost scales $\phi^C = 1.90$ and $\phi^D = 0.37$, depreciation $\delta^C = \delta^D = 0.13$, idiosyncratic productivity $(\rho_\zeta, \sigma_\zeta)$, and aggregate productivity $(\rho_x, \sigma_x) = (0.81, 0.014)$) are taken directly from the SMM estimates and external calibration in Section 4, so the damage model and the flow-driven model share the same firm technology. The per-unit emission rate is calibrated separately for the damage block at $\sigma_D = 60$, chosen so that the simulated cross-sectional mean emission intensity falls in the range observed in the Compustat-Trucost panel. The benchmark carbon tax rate is $\tau_D = 0$, consistent with most U.S. states; the counterfactuals in Section 5.2.4 explore $\tau_D > 0$.

[23] This is the standard structural-calibration pattern of pinning the closed-form parameter analytically per cell and searching only over the non-separable one. It lets the SDF deliver the same aggregate pricing moments as the SMM estimation regardless of where in the θ-grid we are, so that cross-θ comparisons of the real-side outcomes (dirty-capital share, greenium) are clean of variation in the aggregate risk-free benchmark.

The damage function follows Cai and Lontzek, 2019, combining a Nordhaus-style quadratic term with a Weitzman-style catastrophic tail. The linear coefficient a_1 is set to zero by convention. The quadratic coefficient $a_2 = 0.06$ is calibrated so that damage at $\Delta T = 2°C$ is approximately 20%, consistent with the empirical estimate of Burke, Hsiang, and Miguel (2015). This is substantially more aggressive than Nordhaus's DICE specification (1% at $2°C$), reflecting the post-2010 literature's view that DICE understates damages once non-market losses, tipping risks, and growth-rate effects are incorporated (Weitzman, 2012; Dietz and Stern, 2015). The catastrophic-damage coefficient $a_3 = 5 \times 10^{-5}$ and exponent $\eta_c = 6.754$ are taken from Weitzman (2012). The emission-to-temperature sensitivity $\lambda = 5 \times 10^{-5}$ is calibrated so that the simulated benchmark ΔT path reaches approximately $4°C$ by year 100, in the IPCC RCP 4.5–8.5 range (IPCC, 2018). The initial temperature $\Delta T_0 = 1°C$ corresponds to the observed global mean warming in the early 2020s.

To isolate the reallocation effect of emission risk pricing from any scale effect, I impose a capital-budget constraint $K'_C + K'_D \leq \bar{K}$, where $\bar{K} = 5$ is set to exceed the unconstrained optimal total capital at $\theta = 0$. The constraint is therefore non-binding at the benchmark and binds only when higher θ would otherwise inflate total capital accumulation; the cross-θ comparisons in Tables 13 and 14 thus reflect reallocation between clean and dirty capital rather than changes in the total capital stock.

The aggregate emission-shock process (ρ_Z, σ_Z) is estimated from the same Compustat-Trucost panel used to build the SMM moments. From the emission equation $G_{jt} = \sigma_D K_{D,jt} Z_{jt}$, the log emission shock satisfies

$$\log Z_{jt} = \log G_{jt} - \log \sigma_D - \log K_{D,jt}. \tag{65}$$

Using $\sigma_D = 60$ and proxying the firm's dirty capital stock as $K_{D,jt} = 0.70 \cdot PPEGT_{jt}$ (where 0.70 is the average long-run share $K_D/(K_C + K_D)$ produced by the SMM-estimated model simulated at its point estimates) I compute $\log Z_{jt}$ for each firm-year with positive Scope-1+Scope-2 emissions and positive tangible capital. Because the pricing kernel prices only the systematic

(non-diversifiable) component of the emission shock, I extract the aggregate piece $\bar{z}_t = \frac{1}{N_t}\sum_j \log Z_{jt}$ (the cross-sectional mean at year t), demean it, and estimate the AR(1) by OLS:

$$\bar{z}_t - \bar{\bar{z}} = \rho_Z\left(\bar{z}_{t-1} - \bar{\bar{z}}\right) + u_t, \qquad \sigma_Z^2 = \mathrm{Var}(u_t). \tag{66}$$

The resulting point estimates are $\rho_Z = 0.946$ and $\sigma_Z = 0.087$, based on 23 annual observations (aggregated from 44,115 firm-year rows across 5,618 firms). The unconditional mean $\bar{z}$ is set to zero so that $E[Z] \approx 1$ and the shock acts as a pure multiplicative residual on emissions.

5.2.4 Results

The economy is simulated annually for 200 years with a cross-section of 3,000 firms evolving under a common aggregate temperature path $\Delta T_{t+1} = \Delta T_t + \lambda \bar{G}_t$, starting at $\Delta T_0 = 1°C$. The first 50 years are excluded as a burn-in; all reported statistics are time-series averages over the remaining 150 years.[24] For each value of θ, the aggregate risk price γ and discount factor β are re-calibrated by bisection to maintain an annualized equity premium of 7% and a risk-free rate of 1.6%, the SMM targets from Section 4. Table 13 reports the average dirty-capital ratio $K_D/(K_C + K_D)$ and Table 14 reports the greenium (expected-return spread between the top- and bottom-dirty-capital-share tercile), both across the emission risk price $\theta \in \{0, 5, 10, 15\}$ and the two benchmark substitutability values $\nu \in \{0.56, +\infty\}$. Each table presents six panels that vary one parameter at a time from the benchmark (Panel A).

The headline result is that complementarity dampens the response of capital allocation to emission risk pricing by roughly a factor of three. In the benchmark (Panel A of Table 13), at $\theta = 0$ the dirty-capital ratio is 69.6% under the estimated complementarity ($\nu = 0.56$) and 99.6% under perfect substitution ($\nu = +\infty$), where the latter reflects the productivity asymmetry $\alpha = 0.81$. As θ rises to 15, the ratio falls 3.47% under complementarity (to 67.2%), compared with 10.87% under perfect substitution (to 88.8%). Temperature change moves with the same

[24] To reduce simulation variance, 10 independent aggregate-productivity histories are drawn and the 3,000-firm cross-section is simulated separately for each; all statistics are averaged across the $10 \times 3{,}000$ firm-history observations. Technical details of the VFI temperature convention and simulation procedure are in Appendix B.4.

pattern (Table C.11, Internet Appendix): in the benchmark, ΔT falls from 1.91°C to 1.48°C under $\nu = 0.56$ (-22.7%) and from 2.53°C to 1.90°C under $\nu = +\infty$ (-24.9%). The 1.91°C benchmark value lies in the IPCC RCP 4.5–8.5 range, broadly matching the calibration target. Note that the percentage temperature reduction is similar across ν even though the dirty-capital-ratio response differs sharply: the perfect-substitution case starts from a much hotter baseline, so it has more room to cut. The dampening therefore shows up in capital composition rather than in total cooling.

The mechanism is parallel to the flow-driven model but operates through cash flows. The damage function reduces dirty firms' output, raising their effective cost of capital. Complementarity then prevents the firm from substituting clean for dirty in response: the marginal product of dirty capital rises sharply as K_D contracts, partially offsetting the damage penalty. The productivity asymmetry $\alpha = 0.81$ and the asymmetric adjustment costs ($\phi^C = 1.90$ versus $\phi^D = 0.37$) determine which factor absorbs the shrinkage, just as in Section 5.1.

Panel B (near-equal productivity shares, $\alpha = 0.49$) is the exception. Under complementarity, the dirty-capital ratio actually *rises* with θ (+13.13%), the same sign reversal as in the flow-driven model. The reason is that $\alpha = 0.49$ puts the production balance point near $K_D \approx K_C$, so firms start at 52% dirty rather than the benchmark 70%. As emission risk pressure forces both stocks to contract, the higher adjustment cost on clean capital makes K_C decumulate faster, mechanically pushing the dirty-capital ratio upward. Temperature still falls in this panel (-16.6% at $\nu = 0.56$), because the absolute level of K_D drops along with K_C.

Panels C ($\tau_D > 0$) and D (high damage) reinforce the cash-flow penalty on dirty capital. The dirty-capital ratio is lower at all θ values than in the benchmark, but the marginal effect of raising θ at $\nu = 0.56$ remains small (-2.31% in Panel C, -2.86% in Panel D) compared with $\nu = +\infty$ (-12.65% and -13.99%). Both panels also produce $\sim$ 16–18% further temperature reductions under maximum ESG pressure, starting from a lower ΔT baseline (1.60–1.72°C at $\theta = 0$). Adding a carbon tax or amplifying the damage function lowers the level of warming but does not unlock the capital-composition response under complementarity.

Panel E (equal adjustment costs, $\phi^C = \phi^D$) is the most revealing of this group. Removing the clean-capital adjustment disadvantage causes the complementarity case to respond much more sharply, with the dirty-capital ratio falling 11.64% versus 3.47% in the benchmark, and the temperature reduction widening to -21.0%. The gap between $\nu = 0.56$ and $\nu = +\infty$ narrows substantially. This confirms that the adjustment-cost asymmetry in the SMM estimates is an important amplifier of the complementarity friction; complementarity alone is not sufficient to dampen the response by a factor of three.

Panel F (constant returns to scale) produces the largest temperature swings: ΔT falls from 2.20°C to 1.52°C (-30.7%) under complementarity, because firms can scale down total capital more freely without diminishing returns to scale. The dirty-capital ratio at $\nu = 0.56$ falls 7.03%, more than in the benchmark but still well below the 16.24% response under perfect substitution.

The damage model therefore tells the same story as the flow-driven model but through a different channel: emission risk pricing meaningfully cools the simulated economy, but the magnitude of the cooling and the size of the capital reallocation depend jointly on ν, α, and the adjustment-cost structure. At the SMM-estimated $\nu = 0.56$, the dirty-capital response is roughly one-third of what it would be under perfect substitution, and the only way to recover a sharper response is to either equalize adjustment costs (Panel E) or move ν well above the SMM estimate.

5.3 Greenium

A central object in the ESG asset-pricing literature is the *greenium*: the expected-return spread between brown and green firms (Pástor, Stambaugh, and Taylor, 2021; Bolton and Kacperczyk, 2021; Hsu, Li, and Tsou, 2023). Using the no-arbitrage condition $1 = E_t[M_{t+1}\, R_{j,t+1}]$, the greenium admits the general decomposition

$$E_t[R_{\text{brown},t+1}] - E_t[R_{\text{green},t+1}] = -\frac{Cov_t(M_{t+1},\, R_{\text{brown},t+1}) - Cov_t(M_{t+1},\, R_{\text{green},t+1})}{E_t[M_{t+1}]}, \tag{67}$$

which expresses the greenium entirely as the cross-firm gap in SDF covariance. The two models fill in this covariance through distinct mechanisms, derived in turn below.

5.3.1 Flow-Driven Model

Risk channel. In the flow-driven model, the covariance in (67) arises from the procyclicality of the demand markup ι_{jt}. From equation (45), emission intensity $e_{jt} = \varepsilon_D K_{D,jt}/F_{jt}$ is *countercyclical*: when aggregate productivity X_t rises, the same dirty capital stock generates higher output, so e_{jt} falls. Through the demand system (50), a lower e_{jt} raises investor demand and lifts ι_{jt}. The brown firm, with large $K_{D,jt}$, has a wider e_{jt} swing over the cycle and hence a more procyclical ι_{jt}: the brown discount compresses in booms and deepens in recessions. Because market value $P_{jt} = V_{jt}(1 + \iota_{jt})$ therefore falls more sharply in bad states for brown firms,

$$Cov_t(M_{t+1},\, R_{\text{brown},t+1}) < Cov_t(M_{t+1},\, R_{\text{green},t+1}) < 0, \tag{68}$$

and the risk channel generates a positive expected-return premium. Crucially, this channel is operative even when the aggregate demand shifter v_t is held constant; it is a property of the interaction between K_D intensity and the demand system, not of v_t dynamics.

Flow channel. The flow channel arises from *changes* in the aggregate demand shifter v_t. Rewriting the market return as

$$R_{j,t+1} = \frac{V_{j,t+1}}{V_{j,t}} \cdot \frac{1 + \iota_{j,t+1}}{1 + \iota_{j,t}}, \tag{69}$$

the second factor captures the change in the demand markup. When sentiment rises ($v_{t+1} > v_t$), the green premium ι_{green} expands and the brown discount ι_{brown} deepens, generating a price-level capital gain for green firms and a capital loss for brown firms. Its contribution to the realized brown-minus-green spread is *negative* during periods of rising sentiment, consistent with the outperformance of green firms documented in Pastor, Stambaugh, and Taylor, 2023 and Van der Beck, 2023 over 2012–2022, and *positive* during periods of falling sentiment. Unlike the

risk channel, the flow channel is entirely absent when v_t is constant.

Tables 10 and 11 report the simulated greenium from the flow-driven model across the same panels as the dirty-capital ratio results in Section 5.1. Following the convention in equation (67), the greenium is the value-weighted brown-minus-green return spread, sorted by dirty-capital share into top and bottom terciles. Table 10 reports the raw expected-return spread and Table 11 reports the FF5-adjusted alpha (the residual after stripping out exposure to the Fama-French five factors), both in percent per year.

The benchmark greenium is small at $\theta = 0$ but rises sharply as θ increases. In Panel A of Table 10, at $\nu = 0.56$ the raw spread climbs from 4.82% at $\theta = 0$ to 79.24% at $\theta = 0.7$; under perfect substitution it rises from 0.53% to 64.26%. The positive sign reflects the risk-channel mechanism described above: brown firms have more procyclical demand markups, so their returns load more negatively on the SDF, and they earn a higher risk premium in equilibrium. The level of the spread and its rate of increase in θ are both larger under complementarity than under perfect substitution, reflecting two reinforcing effects: a wider equilibrium dispersion in K_D across the brown and green terciles, and the inability of brown firms to reallocate away from the demand penalty, which keeps their procyclical exposure intact. The FF5 alphas in Table 11 are smaller in magnitude than the raw spreads—the five Fama-French factors absorb a significant share of the cross-sectional return variation—but the qualitative pattern is identical: positive, rising in θ, and larger under complementarity (e.g., 33.15% at $\nu = 0.56$ versus 9.31% at $\nu = +\infty$ at $\theta = 0.7$ in the benchmark).

The remaining panels confirm the pattern. Panel B (equal productivity shares, $\alpha = 0.49$) does not amplify the spread at low ν: at $\theta = 0.7$, the greenium under $\nu = 0.56$ is 72.5%, slightly *below* the benchmark of 79.2%, while at $\nu \geq 1$ the spread is materially higher than the benchmark (peaking at 93.3% for $\nu = 1$). Removing the productivity asymmetry weakens the complementarity-specific amplification because brown firms no longer hold a productivity rent on K_D. Panels C and D add carbon taxes: the level of the greenium rises with τ_D because direct emission costs hit brown cash flows harder in bad states, deepening the brown discount and steepening the SDF covariance

differential. Panel D ($\tau_D = 16\%$ of revenue) generates the largest spreads in the table, ranging from 244 to 274% per year at $\theta = 0.7$ across ν, an extreme magnitude that would not be supportable in a calibrated empirical setting but is informative about the scale of the risk-channel forces the model can in principle deliver. Panel E ($\phi^C = \phi^D$) compresses the spreads modestly relative to the benchmark, consistent with the benchmark's asymmetric adjustment costs reinforcing brown firms' procyclical exposure.

5.3.2 Damage-Function Model

In the damage-function model of Section 5.2, the flow channel is absent by construction: there is no investor demand system, $\iota_{jt} = 0$ for all firms, and returns are purely fundamental. The risk channel operates through cash flows. Two forces with opposing signs are at work. The first is the standard climate-risk channel: brown firms have higher emissions, so when Z_t surprises upward (and the SDF M_{t+1} is high), brown firms contribute more to future warming, raising future ΔT, future damage D_{t+1}, and lowering their future cash flows in bad states. A positive carbon tax (Panels C, D) reinforces the same direction by hitting brown cash flows directly through higher tax payments. Together these effects push the greenium positive. The second, opposing force operates through the marginal product of dirty capital. Under gross complementarity, K_D is hard to substitute away from in production, so the marginal product of each remaining unit rises sharply when the aggregate stock contracts. As emission risk becomes more heavily priced (higher θ) or damage compounds, all firms cut dirty investment, the aggregate K_D stock falls, and the per-unit return on existing K_D rises. Brown firms hold most of this stock and benefit. Their continuation values therefore rise precisely in states where the constraint on K_D tightens, making brown returns more positively correlated with M_{t+1} than green returns and pushing the greenium negative. Which force dominates depends on the panel.

Table 14 shows the greenium across the same counterfactual panels as Table 13 in Section 5.2. Panel A (benchmark, $\alpha = 0.81$) yields a very small greenium at all θ values, with magnitudes below

2.4% and mixed signs (e.g. +1.19% at $\nu = 0.56$ versus −2.36% at $\nu = +\infty$ at $\theta = 15$). Both top and bottom terciles hold predominantly dirty capital in this panel because $\alpha = 0.81$ pushes essentially all firms toward K_D-heavy allocations, so cross-firm dispersion in K_D share is limited and neither force can generate a large spread.

Panels C through F, which either add direct emission costs (carbon tax, higher damage) or relax the production technology (equal adjustment costs, constant returns to scale), produce substantially larger negative greenium magnitudes as θ rises, ranging from approximately −48% to −98% at $\theta = 15$. The negative sign indicates that the marginal-product channel dominates the standard climate-risk channel in these panels. Each of these perturbations shrinks the equilibrium K_D stock (Panels C and D through carbon taxes or higher damage; Panel F's constant returns let firms scale down total capital more freely), making the marginal unit of K_D more valuable, and brown firms hold most of those marginal units. Panel B (near-equal productivity shares, $\alpha = 0.49$) generates only modest greenium spreads because, although it widens cross-firm dispersion in the K_D share, it does not also shrink the aggregate K_D stock the way the cash-flow penalties in Panels C and D do.

Panel E ($\phi^C = \phi^D$) is informative because it isolates the role of the adjustment-cost asymmetry. Without the clean-capital adjustment disadvantage, green firms can build K_C as cheaply as brown firms can maintain K_D, so the advantage of already holding K_D is weaker. At $\theta = 0$ the greenium is small and positive (1.72% under $\nu = 0.56$), reflecting the residual climate-risk channel. As θ rises and the equilibrium K_D stock contracts, the marginal-product channel takes over and the greenium turns negative. Across most panels, the greenium is larger in magnitude under perfect substitution than under complementarity at high θ, consistent with the dirty-capital ratio results: when firms can reallocate easily, cross-firm dispersion in K_D holdings widens, and the per-unit returns on the contracting K_D stock are concentrated more sharply on the brown tercile.

Risk versus non-risk interpretation. The risk channel is compensation for systematic risk and represents, in the language of Pástor, Stambaugh, and Taylor, 2021, an *equilibrium* premium for

exposure to climate risk. The flow channel is non-risk: it reflects price-level adjustments driven by shifts in v_t that do not correspond to changes in expected future cash flows, and can generate large realized return spreads during periods of expanding institutional demand for green firms but does not represent a durable risk premium. Critically, both channels are modulated by ω: when clean and dirty capital are strong complements (low ω), firms cannot reallocate away from dirty capital, so the cross-firm gap in emission exposure and damage exposure persists, and both channels can remain large even as policy pressure intensifies.

6 Endogenizing Complementarity Decisions

The previous sections treat ω as fixed at its SMM estimate. Over a long horizon, firms can invest in R&D to make clean capital a closer substitute for dirty, raising ω at a cost. This section endogenizes that decision in two steps. Section 6.1 extends the flow-driven dynamic model of Section 4 so that firms choose ω jointly with investment, subject to a convex R&D cost indexed by friction κ, and traces how ω evolves under different combinations of κ and ESG demand pressure. Section 6.2 provides empirical support: industry-level ν has been rising over the sample, and the rate of increase covaries with intangible-capital investment. The two pieces together suggest that ω is plausibly fixed over short horizons (justifying the structural estimation) but slowly responsive over long horizons (motivating R&D-subsidy counterfactuals).

6.1 Dynamic Model

All model elements other than ω remain unchanged from Section 4: the CES production function (44), capital accumulation, quadratic adjustment costs, the CRRA pricing kernel (49), and the Koijen-Yogo demand system (50). Firms start at the SMM-estimated value $\omega = -0.803$ and choose ω' from the discrete grid $\{-0.803, 0, 0.5, 0.8, 1.0\}$ each period. Increasing ω requires an R&D investment with an asymmetric convex cost:

$$\mathcal{C}(\Delta\omega) = \frac{\kappa}{2}\max(\Delta\omega, 0)^2, \qquad \Delta\omega \equiv \omega' - \omega, \tag{70}$$

so firms can freely reduce substitutability but pay a quadratic cost to raise it. At the optimum, the firm equates the marginal R&D cost $\mathcal{C}'(\Delta\omega)$ to the expected discounted marginal benefit of greater substitutability; the latter rises with ESG investor pressure, since a more flexible production function relaxes the constraint on clean capital allocation. The firm's problem (53) is extended to include ω in the state and ω' in the choice, with $\mathcal{C}(\Delta\omega)$ deducted from cash flows. I vary R&D friction $\kappa \in \{10, 50, 80, 100, 150, 500, 1000\}$ and ESG-demand loading $\theta = -b_{\text{emit}} \in \{0, 0.1, 0.3, 0.5\}$, holding all other parameters at their SMM estimates.

The slow pace of observed technological change motivates treating high κ as the empirically realistic baseline. The industry-level evidence in Section 6.2 shows that the cross-industry average ν has risen only modestly over 2006–2022 even as ESG pressure has grown, implying that production functions change gradually even when investor incentives are present. Quantitatively, the one-time cost of upgrading from the SMM estimate to Cobb-Douglas ($\Delta\omega = 0.803$) is approximately $2.8\kappa\%$ of firm value.[25] At $\kappa = 100$ the cost reaches 280% of firm value; at $\kappa = 500$ it exceeds 1,400%. These are not extreme parameter choices: industries whose core production technology has not changed materially in decades plausibly face κ in this range, which is consistent with why firms do not voluntarily upgrade their production functions in response to ESG pressure alone. Against this high-κ baseline, existing R&D subsidy programs provide only partial relief. The U.S. Inflation Reduction Act (2022) provides 30–45% investment tax credits for clean-energy capital: at the 45% rate, a firm facing $\kappa = 100$ still confronts over 150% of firm value. Direct per-company grants from the U.S. DOE Hydrogen Hubs program and manufacturing-decarbonisation awards cover 1–10% of a large industrial firm's equity value, bridging the full upgrade cost only for $\kappa \lesssim 4$. The most ambitious instruments on record, EU Carbon Contracts for Difference at EUR 0.5–2 billion per facility, correspond to $\kappa \approx 4$–18 in model terms. Current policy instruments can therefore reduce the effective κ, but do not move most industries out of the high-κ regime.

[25] Upgrade cost $= \frac{\kappa}{2}(0.803)^2 \approx 0.32\kappa$ in annual output units. Firm value $\approx$ annual output $/\, r^e$, where $r^e = 1.6\% + 7\% = 8.6\%$ is the model's implied cost of equity (risk-free rate plus equity premium). Hence cost/firm-value $= 0.32\kappa \times 0.086 \approx 0.028\kappa$.

Figure 4 illustrates the consequences of the high-κ baseline: firms near $\kappa \geq 150$ remain at $\omega \approx -0.803$ regardless of θ, so the complementarity friction persists even under strong ESG pressure. As R&D subsidies reduce the effective κ, Figure 5 reports steady-state outcomes across the (κ, θ) grid, and two findings stand out. First, subsidies without ESG incentives ($\theta = 0$) are counterproductive: as κ falls, low-friction firms converge quickly to $\omega = 1$, but freed from the complementarity constraint they optimally concentrate in dirty capital. Second, when ESG demand is present ($\theta > 0$), the investor penalty on high-emission firms suppresses the incentive to raise ω unconditionally, so ω remains near its SMM estimate while the green reallocation is achieved through the demand channel. Technical details are in Appendix B.3.2.

Two policy implications follow. First, R&D subsidies alone, without ESG pressure, can be counterproductive. By making it cheap to raise ω, the subsidy frees firms to concentrate in whichever input is more productive, and at $\alpha = 0.81$ that input is dirty capital. Complementarity had been holding firms back from over-concentrating in K_D; removing that brake without imposing an ESG penalty raises the dirty share rather than lowering it. Second, R&D subsidies are most effective when combined with ESG pressure: the subsidy provides the technological flexibility to reallocate, and ESG demand points that flexibility toward clean capital instead of dirty.

6.2 Empirical Analysis

On the empirical side, I first explore the time-series trend of the elasticity of substitution. Figure 6 plots the cross-industry average of ν, estimated from equation (37) with a five-year rolling window, 3-digit SIC industry classifications, and Scope 1 GHG emissions. The average ν rises steadily from roughly 0.36 in 2006 to 0.61 in 2022, with a noticeable acceleration after 2018, although the level remains well below one (perfect substitution). The upward trend is consistent with the model's prediction that ESG pressure pushes firms to raise ω over time, while the slow pace and remaining distance from perfect substitution match the high-κ baseline used in Section 6.1.

Next, I explore the correlation between the industry-level substitutability estimates and

innovation activities. For each industry-year, I compute the 3-year moving average of ν and of scaled intangible capital. I use intangible capital rather than R&D expenditure because the R&D expenditure variable (XRD from COMPUSTAT) has many missing values. The intangible capital data is from Peters and Taylor (2017). I then run the following regression:

$$\nu_{i,t} = \beta_0 + \beta_1 \text{Intangible_Capital_Scaled}_{i,t} + \text{IndustryFE} + \text{YearFE} + \epsilon_{i,t} \tag{71}$$

where i denotes industry and t denotes year. Intangible capital is scaled by total assets or sales. I include both industry and year fixed effects. For the sales-scaled specification, I additionally control for industry-level output growth ($\Delta \log Y$) and use wild cluster bootstrap inference (999 replications, Rademacher weights, clustered by Fama-French 10-industry classification). The output growth control matters because fast-growing industries tend to accumulate intangible capital and raise ν simultaneously, creating a spurious positive correlation driven by common expansion trends rather than a structural link between innovation and substitutability. Controlling for $\Delta \log Y$ isolates variation in intangible intensity that is orthogonal to output scale. Wild cluster bootstrap addresses the well-known downward bias in asymptotic cluster-robust standard errors when the number of clusters is small (here only ten Fama-French industries), guarding against over-rejection.

Table 15 and Table 16 report the regression results, with intangible capital scaled by total assets and by sales respectively. Each table contains three panels corresponding to the three GHG scopes used to estimate ν (Scope 1 in Panel A, Scope 2 in Panel B, Scope 3 in Panel C), and three columns corresponding to industry classifications at the 2-, 3-, and 4-digit SIC level. Sample sizes are around 180–188 industry-year observations across the three panels, with industries dropped when the underlying ν estimate cannot be computed reliably from the SIC classification at hand. Standard errors are reported in parentheses; significance is denoted by $*$, $**$, and $***$ at the 10%, 5%, and 1% levels.

In the assets-scaled regression (Table 15), seven of nine coefficients are positive but most lack statistical significance: only Panel B Scope 2 SIC4 (significant at 10%) and Panel C

Scope 3 SIC4 (significant at 5%) reach conventional thresholds. The two negative coefficients (Panel A Scope 1 SIC2 and Panel B Scope 2 SIC2) are small in magnitude and not significant. Coefficients are generally larger and more often significant at finer SIC classifications, suggesting the innovation-substitutability relationship is more visible at the more granular industry level.

In the sales-scaled regression (Table 16), all nine coefficients are positive and statistically significant at the 5% level or better, ranging from 0.05 to 0.25, again with magnitudes generally rising as the SIC classification becomes finer. The pattern is consistent across Scope 1, Scope 2, and Scope 3 emissions, indicating that the innovation-substitutability link is broad rather than confined to direct production technology.[26]

The two specifications point in the same direction qualitatively, but the magnitudes and significance differ. One reading of the gap is that total assets includes the book value of tangible capital, so the intangible-to-assets ratio is mechanically compressed for capital-intensive industries even when those industries invest heavily in innovation, attenuating the estimated correlation with ν. Sales, by contrast, reflects productive output and is not directly inflated by tangible capital accumulation, so $\overline{K_{\text{Intangible}}/Y}$ varies more cleanly with shifts toward intangible-intensive processes. Both measures have legitimate interpretations, and the consistent positive sign across both is more informative than the precision of any single coefficient.

7 Conclusion

This paper structurally estimates a model of firm investment in clean and dirty capital to study how production frictions shape the real effects of sustainable investing. The elasticity of substitution between clean and dirty capital is $\nu = 0.56$, well below one, indicating gross complementarity. The estimate, from both a reduced-form panel estimator and full-information SMM, contrasts with the values $\nu \approx 1.8$ to 10 assumed in macro-climate models calibrated to energy-consumption data: those models identify substitutability from short-run fuel switching, while this paper identifies it

[26]The sales-scaled results are robust to using asymptotic cluster-robust standard errors in place of wild cluster bootstrap.

from capital investment responses, a distinct and slower-moving margin. The SMM also estimates dirty-capital productivity share $\alpha = 0.81$, consistent with IEA data and Compustat-Trucost evidence, and a much higher adjustment cost of clean capital.

The flow-driven model delivers the paper's central result. The standard intuition, and the result under perfect substitution, is that stronger ESG investor demand should reduce both emission intensity and the dirty-capital ratio. Under the estimated production technology, the sign of the response reverses: as ESG demand strengthens, both emission intensity and the dirty-capital ratio increase rather than decrease. Several structural features drive this reversal: gross complementarity, the productivity asymmetry favoring dirty capital, and an adjustment-cost asymmetry that makes clean capital substantially more costly to adjust than dirty. Once ESG pressure shrinks both stocks together, the higher cost of maintaining clean capital lets it depreciate faster, and the dirty share rises. The damage-function model delivers a parallel finding through cash flows rather than valuations: as the emission risk price rises, the dirty-capital response under the estimated complementarity is roughly one-third of its perfect-substitution magnitude. Carbon pricing does not resolve the complementarity friction: in both models, the dirty-capital response under the estimated technology remains weak or perverse even with a carbon tax imposed. Taken together, both models point to the same conclusion: financial incentives, whether through investor preferences or carbon pricing, must be paired with changes to the underlying production technology.

Several extensions are worth pursuing. First, embedding this production structure in a general equilibrium model with firm entry, exit, and directed innovation would allow study of how complementarity shapes creative destruction during the low-carbon transition, and whether ESG pressure accelerates or delays the entry of clean-capital-intensive firms. Second, incorporating green debt alongside equity would clarify the interaction between capital structure and clean investment: whether green bonds complement or substitute for the equity-channel ESG pressure modeled here, and how leverage constraints interact with the adjustment cost asymmetry between K_C and K_D. Third, the finding that complementarity dampens policy effectiveness suggests an empirical question

for ESG rating providers: whether existing scores already covary with firm-level substitutability, and if not, whether incorporating it would improve the alignment between sustainable investment flows and real capital reallocation.

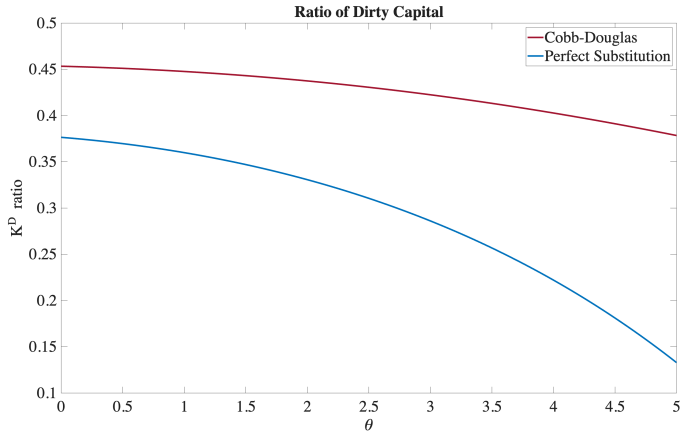


Figure 1: Optimal ratio of dirty capital in the damage-function two-period model (Section 2.1). Dirty investment is denoted by I^D, and clean investment by I^C. The ratio is defined as $I^D/(I^C + I^D)$. Because of full depreciation, the investment I is equivalent to the capital stock K. This figure shows the optimal ratio in relation to different prices of the damage shock, θ, under two scenarios of substitutability.

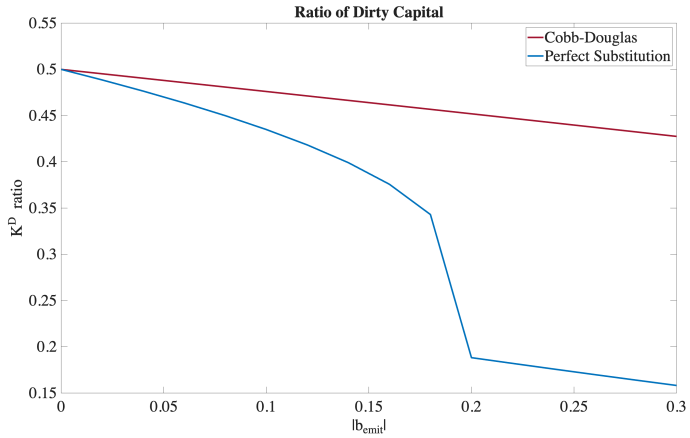


Figure 2: Optimal ratio of dirty capital in the flow-driven two-period model (Section 2.2). The x-axis is the magnitude of the emission-demand coefficient $|b_{\text{emit}}|$, which measures how sharply investors penalize firms with high emission intensity. At $|b_{\text{emit}}| = 0$ the ESG channel is off and both scenarios start at the production-only 50/50 baseline; as $|b_{\text{emit}}|$ grows, firms tilt toward clean capital, with perfect substitution tilting far more strongly than Cobb-Douglas.

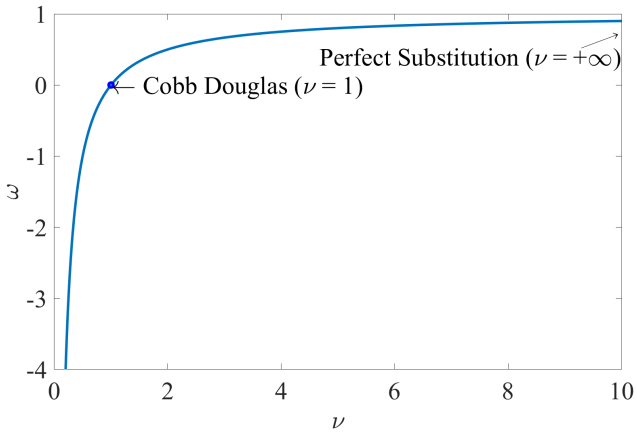
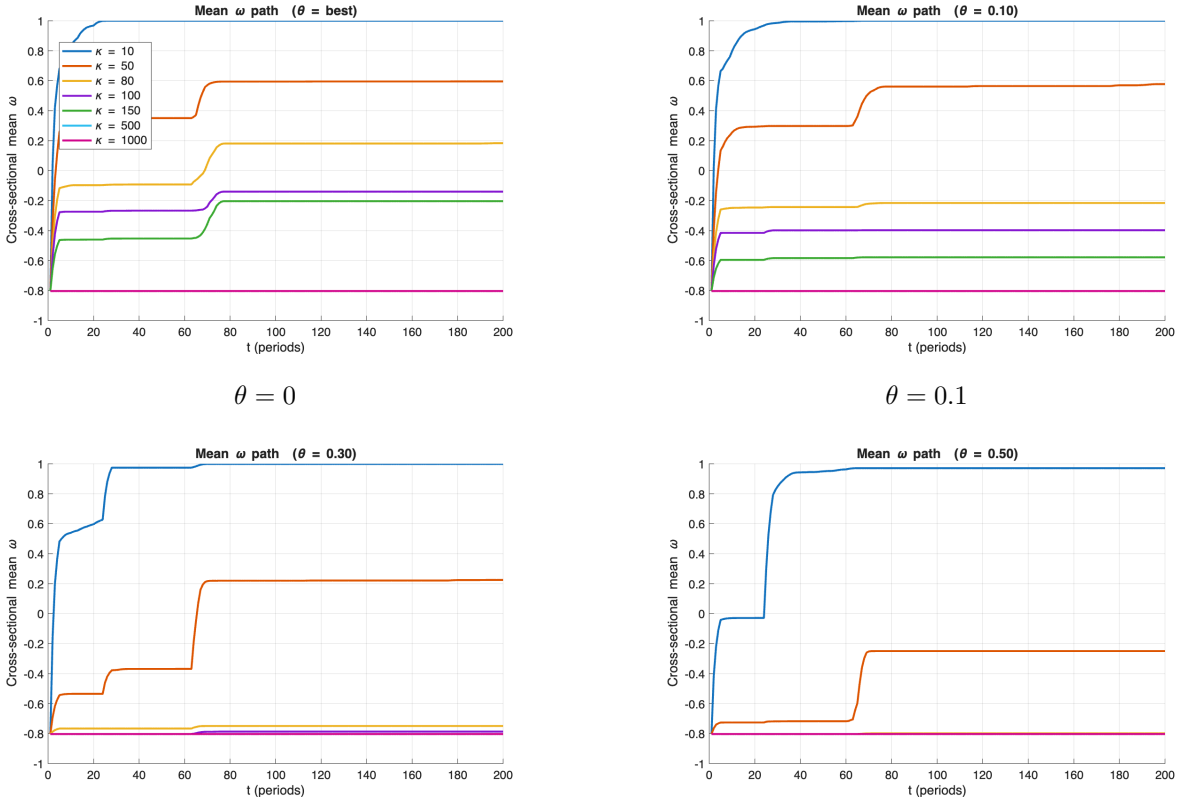


Figure 3: Relation between ω and ν, where $\omega = (\nu - 1)/\nu$. The production function is Cobb-Douglas at $\nu = 1$ ($\omega = 0$), gross complements for $\nu < 1$ ($\omega < 0$), and approaches perfect substitution as $\nu \to +\infty$ ($\omega \to 1$). The structural estimation uses ω because it is bounded above by 1, whereas ν is unbounded.



$\theta = 0$ $\theta = 0.1$

$\theta = 0.3$ $\theta = 0.5$

Figure 4: Mean ω path over time for different values of R&D friction κ, shown separately for each ESG-demand loading θ. All firms start at the SMM estimate $\omega = -0.803$. Each line corresponds to a different $\kappa \in \{10, 50, 80, 100, 150, 500, 1000\}$ (legend in top-left panel only). At $\theta = 0$, low-friction firms converge quickly toward $\omega = 1$; as θ rises, ESG demand suppresses the incentive to raise ω, flattening all trajectories toward the SMM estimate.

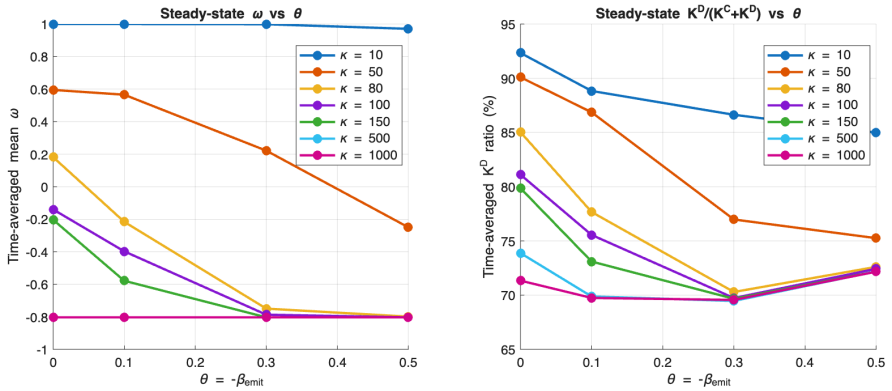


Figure 5: Steady-state ω (top) and dirty capital share $K_D/(K_C + K_D)$ (bottom) as a function of ESG-demand loading $\theta = -b_{\text{emit}}$, for each value of R&D friction κ. Steady-state averages are computed over the last 50 simulation periods. Higher θ represents stronger ESG investor pressure. All other parameters are held at their SMM estimates.

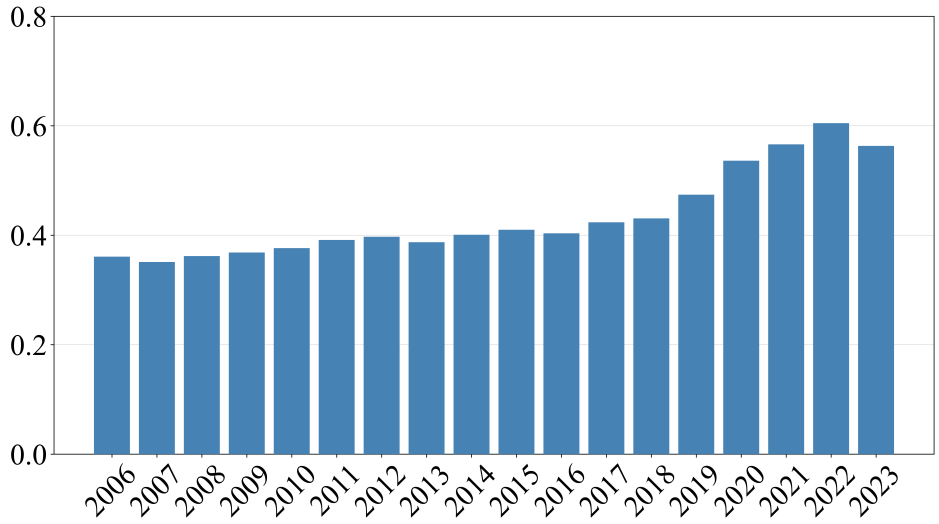


Figure 6: Cross-industry average estimates of the elasticity of substitution, ν, using a five-year rolling window. Each year's bar shows the average ν across industries, estimated from equation (37) using 3-digit SIC industry classifications and Scope 1 GHG emissions as the proxy for carbon emissions.

Table 1: Summary Statistics by Country

This table presents the number of observations, sample time range, and averages of key variables by country. Ln(Y) is the log ratio of the total revenue, where the total revenue is measured in million U.S. dollars. Ln(GHG1) and Ln(GHG2) are the log ratios of the Scope 1 and Scope 2 GHG emissions, respectively, with carbon emissions measured in tCO2e. Ln(AT) is the log ratio of the total assets, with the total assets measured in million U.S. dollars.

Panel A: Developed Markets							
Country	Observations	Start Year	End Year	Ln(Y)	Ln(GHG1)	Ln(GHG2)	Ln(AT)
Australia	52	2004	2022	7.06	10.46	10.69	8.94
Belgium	47	2005	2022	6.82	10.70	9.52	8.28
Canada	1073	2002	2022	7.18	10.94	10.56	8.62
Denmark	46	2002	2022	6.78	8.99	9.45	7.96
France	152	2002	2022	8.13	11.26	11.25	9.00
Germany	114	2002	2022	7.99	10.25	10.61	9.19
Hong Kong	93	2002	2022	8.08	10.05	10.68	9.18
Ireland	256	2002	2022	7.78	10.34	10.38	8.86
Israel	346	2005	2022	5.52	7.89	8.08	6.48
Italy	44	2002	2022	10.04	13.81	12.64	10.64
Japan	164	2002	2022	9.78	11.17	12.64	10.93
Netherlands	136	2002	2022	7.74	9.33	10.27	8.93
Norway	31	2002	2022	9.28	13.25	11.69	9.89
Singapore	63	2005	2022	6.03	9.04	8.43	6.81
Spain	42	2002	2022	9.97	11.34	12.41	11.67
Sweden	46	2002	2022	7.86	9.84	10.96	8.44
Switzerland	150	2002	2022	8.06	10.18	10.63	9.32
United Kingdom	423	2002	2022	8.54	11.20	11.33	9.74
United States	16133	2002	2022	6.66	8.94	9.28	7.83
Panel B: Emerging Markets							
Argentina	42	2005	2022	7.99	10.32	9.89	9.01
Bermuda	165	2002	2022	7.47	9.01	9.18	8.94
Brazil	175	2002	2022	8.90	12.59	11.44	9.88
Cayman Islands	42	2010	2022	5.87	9.04	8.49	7.09
Chile	89	2005	2022	7.75	11.14	9.68	9.10
China	681	2002	2022	6.83	9.25	9.50	7.57
Colombia	21	2008	2022	8.24	13.60	10.75	9.02
Greece	114	2002	2022	7.99	10.25	9.05	9.19
India	99	2002	2022	7.85	9.57	10.06	8.92
Indonesia	31	2002	2022	8.37	10.03	11.02	9.00
Jersey	23	2002	2022	5.86	10.68	9.56	6.73
Luxembourg	83	2002	2022	8.09	11.30	11.95	8.40
Mexico	63	2003	2022	8.58	11.34	11.60	9.29
Monaco	37	2014	2022	5.65	13.16	8.62	7.52
Peru	21	2005	2022	7.22	9.55	10.39	9.11
Russia	33	2005	2021	8.53	12.32	12.11	8.85
South Africa	89	2004	2022	7.46	12.12	13.44	8.00
South Korea	78	2002	2022	9.07	11.27	11.77	10.87
Taiwan	103	2002	2022	7.97	11.34	12.46	8.55

Table 2: Summary Statistics

This table presents summary statistics of the key variables. SD represents the standard deviation. Scope 1 Intensity, Scope 2 Intensity, and Scope 3 Intensity are the logarithms of GHG1, GHG2, and GHG3 emissions divided by the total revenue, respectively. Ln(G) is the logarithm of carbon emissions. Ln(Y) is the log ratio of the total revenue, and Ln(AT) is the log ratio of the total assets. Ln($\frac{\partial Y}{\partial G}$) represents the log ratio of the marginal productivity of emissions, estimated using the 3-digit SIC codes as the sector classification. GHG1/AT, GHG2/AT, and GHG3/AT are ratios of Scope 1, Scope 2, and Scope 3 emissions to the total assets, respectively. The total assets and revenue are reported in nominal U.S. dollars.

	US				Global			
	Mean	SD	Min	Max	Mean	SD	Min	Max
Scope 1 Intensity	2.28	2.02	-9.37	10.04	3.15	1.89	-6.89	10.35
Scope 2 Intensity	2.56	1.36	-6.42	8.47	2.97	1.14	-6.42	8.64
Scope 3 Intensity	4.88	1.36	2.68	13.79	5.23	1.30	2.84	13.79
Scope 1 ln(G)	8.92	3.09	-3.47	19.51	10.05	3.22	-3.47	19.78
Scope 2 ln(G)	9.20	2.64	-3.65	16.92	9.87	2.68	-3.08	17.62
Scope 3 ln(G)	11.52	2.69	-2.21	20.73	12.13	2.87	-1.65	21.12
Ln(Y)	6.66	2.12	0.37	11.72	6.92	2.28	0.37	11.72
Ln(AT)	7.83	1.91	3.72	13.49	7.65	1.90	3.72	13.49
Ln($\frac{\partial Y}{\partial G}$)	-3.14	2.35	-15.89	9.14	-4.11	2.23	-16.33	7.13
GHG1/AT	62.25	291.39	0.00	7704.66	105.25	425.24	0.00	12718.73
Ln(GHG1/AT)	1.10	2.78	-10.88	8.95	2.42	2.15	-8.35	9.45
GHG2/AT	18.64	46.81	0.00	2304.09	27.83	81.49	0.00	3078.91
Ln(GHG2/AT)	1.37	2.23	-9.53	7.74	2.24	1.60	-8.22	8.03
GHG3/AT	451.65	5292.29	0.00	658896.60	547.95	5541.43	0.00	658896.60
Ln(GHG3/AT)	3.70	2.19	-6.23	13.40	4.49	1.83	-6.23	13.40

Table 3: Estimates of ν with Time-Varying Interactive Fixed Effects

This table presents the estimates of ν, the degree of substitution of dirty capital, following equation (37). The benchmark setting, the separate firm and time fixed effects are replaced by time-varying interactive fixed effects, following Bai (2009). Proxies for carbon emissions, G, include Scopes 1, 2, and 3. The 2-, 3-, and 4-digit SIC codes are used to classify sectors to estimate equation (39) and the marginal productivity of carbon emissions, $\frac{\partial Y}{\partial G}$. The dependent variable is $\ln(G/Y)$, and the independent variable is $\ln(\frac{\partial Y}{\partial G})$. The estimates and standard errors (in parentheses) are bootstrapped from 10,000 iterations.

Panel A: Scope 1									
	2-digit SIC			3-digit SIC			4-digit SIC		
	r = 1	5	10	1	5	10	1	5	10
$\ln(\frac{\partial Y}{\partial G})$	0.682	0.676	0.685	0.709	0.710	0.730	0.716	0.717	0.722
	(0.004)	(0.004)	(0.006)	(0.004)	(0.005)	(0.009)	(0.004)	(0.006)	(0.012)
Constant	0.154	0.213	0.212	0.250	0.277	0.240	0.336	0.357	0.385
	(0.016)	(0.020)	(0.029)	(0.017)	(0.024)	(0.041)	(0.017)	(0.026)	(0.055)
Firm Fixed	Y	Y	Y	Y	Y	Y	Y	Y	Y
Time Fixed	Y	Y	Y	Y	Y	Y	Y	Y	Y
Panel B: Scope 2									
	2-digit SIC			3-digit SIC			4-digit SIC		
	r = 1	5	10	1	5	10	1	5	10
$\ln(\frac{\partial Y}{\partial G})$	0.548	0.543	0.533	0.543	0.547	0.540	0.545	0.552	0.547
	(0.005)	(0.010)	(0.012)	(0.006)	(0.010)	(0.017)	(0.006)	(0.010)	(0.017)
Constant	0.741	0.765	0.773	0.888	0.850	0.857	0.909	0.853	0.860
	(0.020)	(0.046)	(0.053)	(0.023)	(0.047)	(0.069)	(0.024)	(0.045)	(0.069)
Firm Fixed	Y	Y	Y	Y	Y	Y	Y	Y	Y
Time Fixed	Y	Y	Y	Y	Y	Y	Y	Y	Y
Panel C: Scope 3									
	2-digit SIC			3-digit SIC			4-digit SIC		
	r = 1	5	10	1	5	10	1	5	10
$\ln(\frac{\partial Y}{\partial G})$	0.596	0.561	0.525	0.610	0.534	0.466	0.598	0.519	0.450
	(0.003)	(0.005)	(0.009)	(0.005)	(0.008)	(0.014)	(0.006)	(0.008)	(0.015)
Constant	1.630	1.827	2.049	1.710	2.092	2.472	1.799	2.163	2.575
	(0.018)	(0.027)	(0.047)	(0.027)	(0.041)	(0.073)	(0.032)	(0.040)	(0.076)
Firm Fixed	Y	Y	Y	Y	Y	Y	Y	Y	Y
Time Fixed	Y	Y	Y	Y	Y	Y	Y	Y	Y

Table 4: Estimates of ν: Sample Splits by Year 2012 and 2016

This table presents subsample estimates of ν, the degree of substitution of dirty capital, following the Bai (2009) Interactive Fixed Effects estimator (5 factors, levels specification). Each panel contains eight data columns: columns 2–5 split the sample at Year 2012 (subperiods ≤2012 and >2012 under 3-digit and 4-digit SIC classification), and columns 6–9 repeat the same structure splitting at the year of the Paris Agreement, 2016. Proxies for carbon emissions, G, include Scope 1 (Panel A) and Scope 2 (Panel B) emissions. The 3- and 4-digit SIC codes are used for sector classification in estimating equation (39) and calculating the marginal productivity of carbon emissions, $\frac{\partial Y}{\partial G}$. The independent variable is $\ln(\frac{\partial Y}{\partial G})$ and the dependent variable is $\ln(G/Y)$. The estimates and standard errors (in parentheses) are bootstrapped from 10,000 iterations. The Adj-R-squared and N values (shown only for the 2016 split columns) are from the OLS levels specification for reference. The z-statistics and p-values test equality of the two subperiod estimates using bootstrapped standard errors.

Panel A: Scope 1

	Split by Year 2012				Split by Year 2016			
	3-digit SIC		4-digit SIC		3-digit SIC		4-digit SIC	
	≤2012	>2012	≤2012	>2012	≤2016	>2016	≤2016	>2016
$\ln(\frac{\partial Y}{\partial G})$	0.739	0.753	0.731	0.752	0.710	0.816	0.709	0.821
	(0.012)	(0.007)	(0.012)	(0.009)	(0.010)	(0.005)	(0.012)	(0.006)
Constant	0.276	-0.013	0.390	0.146	0.343	-0.188	0.446	-0.102
	(0.049)	(0.028)	(0.053)	(0.037)	(0.041)	(0.021)	(0.050)	(0.023)
Firm Fixed	Y	Y	Y	Y	Y	Y	Y	Y
Time Fixed	Y	Y	Y	Y	Y	Y	Y	Y
Adj-R-squared	0.95	0.93	0.96	0.94	0.96	0.96	0.97	0.96
N	5162	16182	4000	14184	8927	12417	7173	11011
z-stats	-1.059		-1.404		-9.834		-8.573	
p-value	0.289		0.160		0.000		0.000	

Panel B: Scope 2

	Split by Year 2012				Split by Year 2016			
	3-digit SIC		4-digit SIC		3-digit SIC		4-digit SIC	
	≤2012	>2012	≤2012	>2012	≤2016	>2016	≤2016	>2016
$\ln(\frac{\partial Y}{\partial G})$	0.502	0.541	0.566	0.523	0.499	0.608	0.516	0.603
	(0.026)	(0.010)	(0.027)	(0.011)	(0.014)	(0.008)	(0.014)	(0.008)
Constant	1.052	0.875	0.830	0.972	1.045	0.682	1.004	0.720
	(0.101)	(0.038)	(0.104)	(0.042)	(0.059)	(0.028)	(0.057)	(0.032)
Firm Fixed	Y	Y	Y	Y	Y	Y	Y	Y
Time Fixed	Y	Y	Y	Y	Y	Y	Y	Y
Adj-R-squared	0.95	0.91	0.95	0.91	0.88	0.90	0.89	0.89
N	5657	17282	4334	14834	9810	13129	7682	11486
z-stats	-1.393		1.458		-6.683		-5.236	
p-value	0.164		0.145		0.000		0.000	

Table 5: Estimates of ν By Industry

This table presents the estimates of ν, the degree of substitution of dirty capital, across different industries. Estimates follow the Bai (2009) Interactive Fixed Effects estimator (5 factors, levels specification). Scope 1, Scope 2, and Scope 3 emissions are used as proxies for carbon emissions G. The 3- and 4-digit SIC codes are applied to classify sectors for estimating equation (39) and to calculate the marginal productivity of carbon emissions, $\frac{\partial Y}{\partial G}$. The industry classification follows the ten-industry classification available at Ken French's Website.

Panel A: Scope 1		
	3-digit SIC	4-digit SIC
1 Consumer Nondurables	0.275	0.232
2 Consumer Durables	0.012	0.044
3 Manufacturing	0.604	0.614
4 Energy	0.520	0.499
5 Hightech	0.593	0.556
6 Telecom	0.268	0.270
7 Shops	0.235	0.237
8 Health	0.319	0.258
9 Utilities	0.539	0.489
10 Others	0.755	0.763
Panel B: Scope 2		
	3-digit SIC	4-digit SIC
1 Consumer Nondurables	0.115	0.068
2 Consumer Durables	0.125	0.134
3 Manufacturing	0.365	0.385
4 Energy	0.457	0.425
5 Hightech	0.547	0.477
6 Telecom	0.390	0.332
7 Shops	0.228	0.179
8 Health	0.310	0.235
9 Utilities	0.670	0.656
10 Others	0.662	0.682
Panel C: Scope 3		
	3-digit SIC	4-digit SIC
1 Consumer Nondurables	0.032	0.056
2 Consumer Durables	0.453	0.218
3 Manufacturing	0.248	0.265
4 Energy	0.769	0.790
5 Hightech	0.634	0.611
6 Telecom	0.243	0.179
7 Shops	0.315	0.282
8 Health	0.601	0.408
9 Utilities	0.353	0.320
10 Others	0.417	0.412

Table 6: SMM Parameter Estimates

This table reports the parameters of the dynamic flow-driven model in Section 4. Panel A lists the 15 parameters estimated by SMM. Panel B lists the externally calibrated parameters. ω is the CES substitution parameter, with implied elasticity of substitution $\nu = 1/(1-\omega)$; α is the dirty-capital share; μ is the homogeneity (returns-to-scale) exponent; ε_D is the per-unit emission rate; ϕ^C, ϕ^D are the quadratic adjustment-cost scales on clean and dirty capital; ρ_ζ, σ_ζ govern the idiosyncratic log-productivity process; the b's are the Koijen-Yogo demand coefficients; and ρ_v, σ_v govern the aggregate demand-shifter process. In the identifying-moment column, $\bar{e}$ denotes the cross-sectional mean of firm-level emission intensity and GMB denotes the aggregate green-minus-brown holdings tilt (the dollar-weighted difference in portfolio emission intensity between green- and brown-tilted institutional investors, following Pastor, Stambaugh, and Taylor, 2023). Standard errors are computed using the standard SMM sandwich formula with firm-clustered bootstrap moment covariance.

Parameter	Description	Estimate	SE	Identifying moment
	Panel A: Estimated parameters			
ω (ν)	CES substitution (elasticity of substitution)	-0.80 (0.56)	0.175	AR(1) of e; CS std of e Corr($\Delta GMB, \Delta\bar{e}$)
α	CES dirty-capital share	0.81	0.017	CS std of e; Corr($I/K, e$)
μ	Homogeneity (returns to scale)	0.71	0.013	Investment moments
ε_D	Emission rate per unit of K_D	69.97	4.941	Mean emission intensity
ϕ^C	Adj. cost, clean capital	1.90	0.387	Inv. volatility (clean)
ϕ^D	Adj. cost, dirty capital	0.37	0.140	Inv. volatility (dirty)
ρ_ζ	Idiosync. prod. persistence	0.63	0.121	Firm-level AR(1) of I/K
σ_ζ	Idiosync. prod. volatility	0.13	0.073	Cross-sec. std of I/K
b_{mkt}	KY loading on $\log V$	0.05	0.006	Indirect inf.: logME
b_{size}	KY loading on $\log B$	-0.01	0.002	Indirect inf.: logBE
b_{profit}	KY loading on profitability	0.23	0.002	Indirect inf.: ROA
b_{invest}	KY loading on investment rate	-0.13	0.002	Indirect inf.: I/K
b_{emit}	KY loading on $\log e$	0.00	0.005	Tobin's q spread
ρ_v	ESG sentiment persistence	0.93	0.003	AR(1) of GMB
σ_v	ESG sentiment volatility	0.44	0.030	Std of ESG flows
	Panel B: Calibrated parameters			
β	Discount factor	0.965		$r_f = 1.6\%$
γ_0	Price of risk, constant	34.73		Equity premium mean
γ_1	Price of risk, state loading	-1.00		Equity premium volatility
ρ_x	Aggregate TFP persistence	0.81		U.S. aggregate TFP
σ_x	Aggregate TFP volatility	0.014		U.S. aggregate TFP
$\delta_C = \delta_D$	Depreciation rate	0.13		Mean investment rate
J	Number of investable firms	1,000		KY sample
b_0	KY demand intercept	-3.07		KY outside-asset share

Note. The "Identifying moment" column lists the moment(s) with the largest scaled sensitivity for each parameter in the SMM Jacobian. For ω, the ESG-flow-to-emission-intensity correlation is the primary economic identifier; the firm-level AR(1) and cross-sectional standard deviation of emission intensity provide additional identification from the firm-level panel. See Section 4.4 for discussion.

Table 7: SMM Moment Fit

This table reports each empirical moment used in the SMM objective, its data target, and its model-implied value at the estimated parameter vector. GMB_t denotes the aggregate green-minus-brown holdings tilt, the dollar-weighted difference in portfolio emission intensity between green- and brown-tilted institutional investors (following Pastor, Stambaugh, and Taylor, 2023). $\bar{e}_t$ is the cross-sectional mean of firm-level emission intensity. Δ denotes the annual first difference. e is a firm's emission intensity (tons of CO_2 per dollar of sales). The primary identifier for ω is the Panel B moment, $\text{Corr}(\Delta GMB_t, \Delta\bar{e}_t)$. Panel C moments are indirect-inference auxiliary-regression coefficients from Koijen and Yogo (2019): OLS holdings on log market equity (logME), log book equity (logBE), return on assets (ROA), investment rate (I/K), and $\log e$.

Moment	Data	Model
Panel A: Cross-sectional firm moments		
Mean emission intensity ($\bar{e}$, tCO_2/\$M sales)	197.39	245.28
Mean investment rate I/K	0.128	0.132
Std of investment rate	0.148	0.048
Firm-level AR(1) of investment rate	0.436	0.340
Corr(I/K, e) (within firm)	−0.096	−0.067
Cross-sectional std of e	419.9	199.6
Firm-level AR(1) of e	0.907	0.718
Tobin's q spread (clean − dirty decile)	1.627	1.443
Panel B: Aggregate time-series moments		
AR(1) of aggregate $\bar{e}_t$	0.805	0.740
Std of aggregate ESG fund flows	0.117	0.003
Mean green-minus-brown holdings tilt (GMB)	0.142	0.005
Mean equity premium	0.071	0.075
Std of equity premium	0.191	0.183
$\text{Corr}(\Delta GMB_t, \Delta\bar{e}_t)$	−0.153	−0.151
Panel C: Indirect inference on demand loadings		
KY holdings loading on logME	0.668	0.050
KY holdings loading on logBE	−0.014	−0.012
KY holdings loading on ROA	0.814	0.232
KY holdings loading on I/K	−0.237	−0.128
KY holdings loading on $\log e$	−0.011	0.000

Table 8: Flow-Driven Model: Dirty-Capital Ratio (%)

This table reports the time-series average dirty-capital ratio $K_D/(K_C + K_D)$ (%) from the dynamic flow-driven model of Section 4, evaluated at the SMM estimates in Table 6. Each panel sweeps the elasticity of substitution $\nu \in \{0.56, 1, 2, 5, +\infty\}$ and the emission risk price $\theta \in \{0, 0.3, 0.5, 0.7\}$, holding all other parameters fixed. Panel A is the benchmark. Panel B sets $\alpha = 0.49$ (equal productivity shares). Panels C and D add a carbon tax τ_D; the 1.6% and 16% figures denote the carbon-tax bill as a share of firm revenue (i.e., $\tau_D \cdot \varepsilon_D K_D/Y = 1.6\%$ in Panel C and 16% in Panel D). Panel E equalizes the adjustment costs ($\phi^C = \phi^D$). "Diff. (%)" is the percentage change in the dirty-capital ratio from $\theta = 0$ to $\theta = 0.7$.

	Emission risk price θ				
ν	0	0.3	0.5	0.7	Diff. (%)
Panel A: Benchmark					
0.56	70.48	68.99	72.32	76.56	+8.62
1	80.81	74.25	76.45	77.71	−3.84
2	90.56	85.97	84.67	84.48	−6.72
5	91.48	86.35	84.65	84.50	−7.64
$+\infty$	92.99	86.72	84.68	84.50	−9.13
Panel B: $\alpha = 0.49$					
0.56	56.27	57.96	67.21	75.50	+34.18
1	58.72	63.13	68.33	76.94	+31.02
2	64.97	66.49	71.42	79.19	+21.90
5	77.08	69.55	73.03	79.27	+2.84
$+\infty$	85.62	70.21	72.92	79.39	−7.27
Panel C: $\tau_D = 1.6\%$					
0.56	70.24	69.17	72.37	76.58	+9.04
1	80.47	74.68	76.60	77.71	−3.44
2	90.58	85.97	84.65	84.47	−6.74
5	91.27	86.36	84.66	84.49	−7.43
$+\infty$	92.59	86.64	84.68	84.50	−8.74
Panel D: $\tau_D = 16\%$					
0.56	70.58	70.82	73.95	78.56	+11.32
1	79.16	76.21	77.46	78.81	−0.45
2	88.00	84.83	84.54	84.45	−4.03
5	88.77	84.93	84.58	84.46	−4.85
$+\infty$	89.06	84.96	84.59	84.47	−5.15
Panel E: $\phi_C = \phi_D$					
0.56	68.71	65.71	68.78	73.38	+6.79
1	78.22	71.24	73.83	76.03	−2.80
2	90.27	77.85	79.63	81.30	−9.94
5	91.44	86.25	84.65	84.72	−7.35
$+\infty$	92.73	86.66	84.89	84.90	−8.44

Table 9: Flow-Driven Model: Emission Intensity (tCO_2/\$M sales)

This table reports the time-series average firm-level emission intensity $\bar{e}$ (tCO_2 per \$M of sales) from the dynamic flow-driven model of Section 4, evaluated at the SMM estimates in Table 6. Panel and column definitions follow Table 8: in Panels C and D, the 1.6% and 16% figures denote the carbon-tax bill as a share of firm revenue. "Diff. (%)" is the percentage change in emission intensity from $\theta = 0$ to $\theta = 0.7$.

	Emission risk price θ				
ν	0	0.3	0.5	0.7	Diff. (%)
Panel A: Benchmark					
0.56	243.7	269.4	281.4	299.9	+23.04
1	251.7	214.6	216.4	218.4	−13.25
2	279.0	218.5	213.9	213.1	−23.64
5	285.5	212.2	206.0	205.2	−28.12
$+\infty$	295.3	210.7	202.3	201.5	−31.77
Panel B: $\alpha = 0.49$					
0.56	350.2	404.1	532.2	605.4	+72.85
1	238.5	290.8	322.3	365.6	+53.30
2	235.7	249.2	270.1	291.9	+23.82
5	268.6	243.5	251.5	266.3	−0.87
$+\infty$	290.4	233.6	239.4	254.4	−12.38
Panel C: $\tau_D = 1.6\%$					
0.56	240.8	270.7	281.6	300.1	+24.61
1	247.0	215.1	216.6	218.4	−11.57
2	275.2	218.4	213.9	213.0	−22.59
5	279.0	212.6	205.9	205.2	−26.44
$+\infty$	287.5	209.8	202.3	201.5	−29.93
Panel D: $\tau_D = 16\%$					
0.56	274.4	275.7	293.7	331.4	+20.81
1	227.1	215.8	218.0	222.0	−2.22
2	234.3	214.9	213.4	212.9	−9.13
5	234.6	207.4	205.6	205.0	−12.60
$+\infty$	234.4	203.7	201.9	201.3	−14.09
Panel E: $\phi_C = \phi_D$					
0.56	222.8	204.5	216.7	234.0	+5.03
1	247.4	206.1	208.9	212.6	−14.05
2	278.5	209.0	208.5	210.0	−24.59
5	284.6	212.6	206.0	205.4	−27.82
$+\infty$	304.0	210.3	202.9	202.5	−33.40

Table 10: Flow-Driven Model: Greenium (% per year, value-weighted)

This table reports the time-series average greenium (value-weighted brown-minus-green expected-return spread, in % per year) from the dynamic flow-driven model of Section 4, evaluated at the SMM estimates in Table 6. Firms are sorted into terciles by dirty-capital share $K_D/(K_C+K_D)$; the spread is the value-weighted return on the top tercile (brown) minus the value-weighted return on the bottom tercile (green). Panel and column definitions follow Table 8: in Panels C and D, the 1.6% and 16% figures denote the carbon-tax bill as a share of firm revenue.

	Emission risk price θ			
ν	0	0.3	0.5	0.7
Panel A: Benchmark				
0.56	4.82	32.52	62.51	79.24
1	3.19	33.49	58.99	70.64
2	0.97	30.45	53.05	65.88
5	0.00	28.29	51.23	64.66
$+\infty$	0.53	26.97	50.27	64.26
Panel B: $\alpha = 0.49$				
0.56	5.91	18.41	25.28	72.52
1	6.53	33.59	66.88	93.33
2	7.33	61.09	93.58	84.57
5	7.85	50.68	83.33	77.38
$+\infty$	3.65	46.55	81.72	78.54
Panel C: $\tau_D = 1.6\%$				
0.56	5.51	35.07	71.76	97.69
1	3.90	35.47	63.74	81.29
2	1.66	31.87	56.94	73.37
5	0.54	29.23	54.38	71.05
$+\infty$	0.94	27.98	53.18	70.20
Panel D: $\tau_D = 16\%$				
0.56	16.67	36.26	105.17	253.12
1	12.84	80.94	124.74	274.00
2	8.26	68.10	175.54	244.01
5	5.83	56.75	134.60	270.60
$+\infty$	5.01	52.65	123.99	245.44
Panel E: $\phi_C = \phi_D$				
0.56	4.63	34.42	64.67	77.27
1	2.91	32.18	55.10	67.26
2	1.15	29.67	54.05	65.54
5	0.20	28.26	50.78	63.09
$+\infty$	0.52	26.77	49.33	62.00

Table 11: Flow-Driven Model: Greenium FF5 Alpha (% per year, value-weighted)

This table reports the value-weighted Fama-French five-factor alpha of the greenium portfolio (top-minus-bottom dirty-capital-share tercile, in % per year) from the dynamic flow-driven model of Section 4, evaluated at the SMM estimates in Table 6. The alpha is the intercept from a time-series regression of the simulated brown-minus-green spread on the five Fama-French factors. Panel and column definitions follow Table 8: in Panels C and D, the 1.6% and 16% figures denote the carbon-tax bill as a share of firm revenue.

	Emission risk price θ			
ν	0	0.3	0.5	0.7
Panel A: Benchmark				
0.56	2.77	4.09	10.40	33.15
1	2.43	5.61	12.62	29.41
2	0.65	3.78	4.92	16.78
5	−0.11	3.02	1.18	10.42
$+\infty$	−0.01	1.48	−0.56	9.31
Panel B: $\alpha = 0.49$				
0.56	0.63	8.13	6.97	13.81
1	0.58	5.84	19.82	47.81
2	2.18	27.74	32.46	25.03
5	4.30	11.79	14.38	13.36
$+\infty$	1.86	7.99	12.87	41.30
Panel C: $\tau_D = 1.6\%$				
0.56	3.43	3.73	11.69	37.59
1	2.69	3.97	11.84	31.71
2	0.74	4.15	5.94	18.75
5	0.16	2.88	0.34	11.71
$+\infty$	−0.06	1.56	−1.19	10.23
Panel D: $\tau_D = 16\%$				
0.56	10.65	16.28	21.38	30.71
1	12.76	43.23	10.32	43.42
2	7.79	4.90	3.49	67.87
5	2.75	6.97	3.75	27.23
$+\infty$	1.98	6.74	1.97	29.82
Panel E: $\phi_C = \phi_D$				
0.56	2.84	3.71	12.93	35.98
1	2.35	7.63	10.30	27.40
2	0.61	4.85	10.60	24.26
5	0.20	2.86	1.54	9.33
$+\infty$	−0.01	2.04	−0.02	5.15

Table 12: Dynamic Damage Model Parameters

This table lists the parameter values used to solve and simulate the dynamic damage-function model of Section 5.2, at annual frequency. Panel A reports parameters shared with the flow-driven SMM model of Section 4; these are taken directly from the SMM estimates in Table 6 or externally calibrated from U.S. aggregate TFP data. Panel B reports damage-model-specific parameters calibrated either from the Compustat-Trucost panel (ρ_Z, σ_Z) or from the climate-economics literature (a_1, a_2, λ) and the SMM emission-intensity moment (σ_D). Panel C reports policy/calibration knobs: the carbon tax τ_D (exogenous), the emission-risk price θ (the sweep variable), the price of aggregate-productivity risk γ (fixed at 20, calibrated to match the equity premium at the benchmark estimates), and the discount factor β (re-calibrated at each θ to pin the annual risk-free rate to 1.6%). The Source column indicates where each value is obtained.

Parameter	Value	Meaning	Source
		Panel A: Shared with SMM (Section 4)	
ω (ν)	-0.80 (0.56)	CES substitution (elasticity of substitution)	SMM (Table 6)
α	0.81	Productivity share of dirty capital	SMM
μ	0.71	Homogeneity (returns-to-scale) exponent	SMM
ϕ^C	1.90	Adjustment cost, clean capital	SMM
ϕ^D	0.37	Adjustment cost, dirty capital	SMM
$\delta^C = \delta^D$	0.13	Depreciation rate (annual)	Calibrated to mean investment rate
ρ_ζ	0.63	Idiosyncratic productivity persistence	SMM
σ_ζ	0.13	Idiosyncratic productivity volatility	SMM
ρ_x	0.81 ($= 0.95^4$)	Aggregate TFP persistence	U.S. TFP data
σ_x	0.014	Aggregate TFP volatility	U.S. TFP data
		Panel B: Damage-model-specific	
a_1	0	Damage function, linear coefficient	Convention (Cai and Lontzek, 2019)
a_2	0.06	Damage function, quadratic coefficient	Burke, Hsiang, and Miguel, 2015
a_3	5×10^{-5}	Damage function, catastrophic coefficient	Weitzman, 2012; Cai and Lontzek, 2019
η_c	6.754	Catastrophic-damage exponent	Weitzman, 2012
λ	5×10^{-5}	Emission-to-temperature sensitivity	IPCC RCP 4.5–8.5
σ_D	60	Emission rate per unit of dirty capital	Match $\bar{e} \approx 197$ tCO_2/\$M (SMM)
ρ_Z	0.946	Aggregate emission-shock persistence	Compustat-Trucost panel
σ_Z	0.087	Aggregate emission-shock volatility	Compustat-Trucost panel
$\bar{z}$	0	Unconditional mean of $\log Z$	Convention
ΔT_0	$1^\circ C$	Initial temperature (above pre-industrial)	IPCC, 2018, observed in early 2020s
		Panel C: Policy and calibration knobs	
τ_D	0	Carbon tax rate	Policy input
θ	$\{0, 5, 10, 15\}$	Price of emission-shock risk	Swept (main x-axis)
γ	20	Price of aggregate-productivity risk	Calibrated to match EP at benchmark
β	Calibrated per θ	Subjective discount factor	Pinned to match $r_f = 1.6\%$

Table 13: Dynamic Damage Model: Counterfactual Average Dirty-Capital Ratio (%)

This table reports the time-series average dirty-capital ratio $K_D/(K_C + K_D)$ (%) from the dynamic damage model. Panel A uses the benchmark parameters. Panels B through F each vary one parameter relative to the benchmark: Panel B sets $\alpha = 0.49$ (near-equal productivity shares); Panel C adds a positive carbon tax $\tau_D > 0$; Panel D uses a higher damage coefficient; Panel E equalizes adjustment costs $\phi^C = \phi^D$; Panel F imposes constant returns to scale. Each panel reports results for the estimated elasticity $\nu = 0.56$ and for perfect substitution ($\nu = +\infty$). "Diff." is the percentage change in the dirty-capital ratio from $\theta = 0$ to $\theta = 15$.

	Emission risk price θ				
ν	0	5	10	15	Diff. (%)
Panel A: Benchmark					
0.56	69.563	70.062	68.140	67.151	−3.47
$+\infty$	99.612	99.606	99.083	88.782	−10.87
Panel B: $\alpha = 49\%$					
0.56	51.696	52.283	53.529	58.487	+13.13
$+\infty$	99.332	99.324	96.618	82.952	−16.49
Panel C: Carbon tax ($\tau_D > 0$)					
0.56	65.837	66.116	62.123	64.317	−2.31
$+\infty$	99.436	99.396	98.351	86.861	−12.65
Panel D: High damage					
0.56	69.091	69.623	65.660	67.117	−2.86
$+\infty$	99.222	99.214	97.531	85.344	−13.99
Panel E: $\phi^C = \phi^D$					
0.56	67.740	67.793	64.945	59.857	−11.64
$+\infty$	99.611	99.605	99.081	88.679	−10.97
Panel F: Constant returns to scale					
0.56	67.944	68.137	62.628	63.169	−7.03
$+\infty$	99.748	99.739	96.217	83.545	−16.24

Table 14: Dynamic Damage Model: Counterfactual Greenium (%)

This table reports the time-series average greenium (expected-return spread between the top- and bottom-dirty-capital-share tercile, in %) from the dynamic damage model. Panel and parameter definitions follow Table 13.

	Emission risk price θ			
ν	0	5	10	15
Panel A: Benchmark				
0.56	−0.059	−0.088	−0.482	1.187
$+\infty$	−0.013	−0.035	−0.049	−2.360
Panel B: $\alpha = 49\%$				
0.56	−0.088	−0.151	−2.041	−4.765
$+\infty$	−0.021	−0.041	−1.256	−0.844
Panel C: Carbon tax ($\tau_D > 0$)				
0.56	−17.000	−18.962	−23.703	−80.192
$+\infty$	−23.467	−24.293	−39.734	−98.378
Panel D: High damage				
0.56	−14.222	−15.444	−20.040	−73.897
$+\infty$	−16.244	−16.914	−27.587	−88.693
Panel E: $\phi^C = \phi^D$				
0.56	1.716	0.697	−12.878	−48.369
$+\infty$	−21.807	−22.040	−34.874	−96.279
Panel F: Constant returns to scale				
0.56	−6.693	−6.146	−24.314	−81.704
$+\infty$	−11.635	−12.450	−26.360	−63.747

Table 15: Correlation Between ν and Industry Intangible Capital Share

This table presents the correlation between industry-level average 3-year rolling estimates of ν and the industry's intangible capital to total assets ratio, $\overline{K_{Intangible}/AT}$. Complemetarity parameter, ν, are estimated from original equation (37). Scope 1, Scope 2, and Scope 3 emissions are used as proxies for carbon emissions G. The 2-, 3-, and 4-digit SIC codes are applied to classify sectors for estimating equation (39) and to calculate the marginal productivity of carbon emissions, $\frac{\partial Y}{\partial G}$. The industry classification follows the ten-industry classification available at Ken French's Website. Standard errors are clustered at the industry level.

Panel A: Scope 1			
	SIC2	SIC3	SIC4
$\overline{K_{Intangible}/AT}$	-0.0058	0.1530	0.1814
	(0.1154)	(0.1981)	(0.3829)
Constant	0.5654***	0.1742	0.0233
	(0.0866)	(0.1284)	(0.2344)
Industry Fixed	Y	Y	Y
Time Fixed	Y	Y	Y
Adj-R-squared	0.561	0.818	0.815
N	188	185	181
Panel B: Scope 2			
	SIC2	SIC3	SIC4
$\overline{K_{Intangible}/AT}$	-0.1400	0.2936	0.5238*
	(0.1178)	(0.2909)	(0.3053)
Constant	0.4668***	-0.0702	-0.2364
	(0.1150)	(0.1714)	(0.1685)
Industry Fixed	Y	Y	Y
Time Fixed	Y	Y	Y
Adj-R-squared	0.595	0.737	0.749
N	188	183	182
Panel C: Scope 3			
	SIC2	SIC3	SIC4
$\overline{K_{Intangible}/AT}$	0.1519	0.1202	0.5193**
	(0.1739)	(0.1753)	(0.2052)
Constant	0.6613***	-0.0587	-0.2059
	(0.1266)	(0.1435)	(0.1338)
Industry Fixed	Y	Y	Y
Time Fixed	Y	Y	Y
Adj-R-squared	0.587	0.654	0.732
N	188	184	182

Table 16: Correlation Between ν and Industry Intangible Capital to Sales

This table presents the correlation between industry-level average 3-year rolling estimates of ν and industry intangible capital-to-sales ratio, $\overline{K_{Intangible}/Y}$ (scaled $\times 100$). Complemetarity parameter, ν, are estimated from original equation (37). Scope 1, Scope 2, and Scope 3 emissions are used as proxies for carbon emissions G. The 2-, 3-, and 4-digit SIC codes are applied to classify sectors for estimating equation (39) and to calculate the marginal productivity of carbon emissions, $\frac{\partial Y}{\partial G}$. The industry classification follows the ten-industry classification available at Ken French's Website. The regression controls for industry-level output growth ($\Delta \log Y$). Standard errors (in parentheses) are wild cluster bootstrap with 999 replications and Rademacher weights, clustered by Fama-French 10-industry classification.

Panel A: Scope 1			
	SIC2	SIC3	SIC4
$\overline{K_{Intangible}/Y}$	0.0665**	0.1264***	0.1670***
	(0.0261)	(0.0312)	(0.0402)
Constant	56.7897***	27.1100***	14.0592***
	(5.7441)	(2.9120)	(5.3225)
Industry Fixed	Y	Y	Y
Time Fixed	Y	Y	Y
Output Growth	Y	Y	Y
Adj-R-squared	0.573	0.826	0.825
N	188	185	181
Panel B: Scope 2			
	SIC2	SIC3	SIC4
$\overline{K_{Intangible}/Y}$	0.0499***	0.2205***	0.2466***
	(0.0187)	(0.0347)	(0.0390)
Constant	38.9212***	10.7390*	8.3150
	(5.6701)	(6.1524)	(7.5450)
Industry Fixed	Y	Y	Y
Time Fixed	Y	Y	Y
Output Growth	Y	Y	Y
Adj-R-squared	0.592	0.759	0.759
N	188	183	182
Panel C: Scope 3			
	SIC2	SIC3	SIC4
$\overline{K_{Intangible}/Y}$	0.1321***	0.0830***	0.2397***
	(0.0572)	(0.0169)	(0.0170)
Constant	75.4517***	1.9090	10.0544
	(6.4994)	(6.7612)	(7.4462)
Industry Fixed	Y	Y	Y
Time Fixed	Y	Y	Y
Output Growth	Y	Y	Y
Adj-R-squared	0.595	0.662	0.743
N	188	184	182

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Appendix A Additional Examples

To better illustrate the meanings of dirty capital and clean capital, as well as complementarity versus substitutability, this section provides real-world examples from various industries.

A.1 Complementarity

Steel Manufacturing:

This industry uses two main types of assets: blast furnaces and electric arc furnaces. Blast furnaces emit significant amounts of carbon but produce the high-purity steel required in sectors such as aerospace. Electric arc furnaces are cleaner but have limitations in controlling steel purity because of their reliance on scrap steel. These two furnace types complement each other by producing steel with different purity levels necessary for diverse end products.

Wind Turbine Manufacturing:

Producing renewable energy equipment, such as wind turbines, depends on steel mills and smelting facilities for turbine materials (dirty) and renewable energy facilities to power the manufacturing process (clean). Without either steel production or renewable energy inputs, turbine production is infeasible.

Semiconductor Manufacturing:

Semiconductor production relies on "dirty" fabrication tools, such as lithography machines, and "clean" facilities, such as cleanrooms. Fabrication equipment is necessary for processing silicon wafers, while cleanrooms prevent dust and temperature fluctuations from damaging microcircuitries. Production cannot proceed without either capital type.

A.2 Substitutability

Steel Manufacturing:

As noted in complementarity, blast and electric arc furnaces both produce steel. For products that do not need high-purity steel, electric arc furnaces can fully substitute for blast furnaces, resulting in reduced emissions. However, the industry continues to use blast furnaces because of factors such as installation costs and economies of scale.

Utilities:

Utility companies can use wind, water, coal, or gas to generate electricity, substituting energy sources on the basis of availability. However, high reliance on coal and gas persists due to supply constraints. For example, solely using wind energy to power New York City is infeasible because of limited local wind resources.

Transportation:

Transportation companies such as UPS and Amazon use diesel trucks (dirty) and electric trucks (clean), which can substitute for each other to fulfill the same transportation function.

Building Construction:

Green building projects employ sustainable materials such as cross-laminated timber, recycled steel, and low-carbon cement alternatives to replace traditional concrete, which relies on CO_2-intensive cement production.

Appendix B Technical Details

B.1 Robustness of the Empirical Strategy

B.1.1 Other Economic Inputs

This subsection demonstrates that the empirical strategy is not contingent upon specific economic inputs within the production function. Consider a more general production function with N additional economic inputs apart from the dirty capital K_D. For simplicity, the firm and time indicators i and t are omitted. The firm's output is

$$Y = A\big(\alpha(K_D)^{\frac{\nu-1}{\nu}} + \sum_{j=1}^{N} \alpha_j K_j^{\frac{\nu-1}{\nu}}\big)^{\frac{\nu}{\nu-1}}$$

where the total production shares satisfy $\alpha + \sum_{j=1}^{N} \alpha_j = 100\%$. Taking the partial derivative with respect to K_D yields:

$$\begin{aligned}
\frac{\partial Y}{\partial K_D} &= A \cdot \frac{\nu}{\nu-1} \cdot \big(\alpha(K_D)^{\frac{\nu-1}{\nu}} + \sum_{j=1}^{N} \alpha_j K_j^{\frac{\nu-1}{\nu}}\big)^{\frac{1}{\nu-1}} \cdot \alpha \cdot \frac{\nu-1}{\nu} \cdot {K_D}^{-\frac{1}{\nu}} \\
&= A \cdot \big(\alpha(K_D)^{\frac{\nu-1}{\nu}} + \sum_{j=1}^{N} \alpha_j K_j^{\frac{\nu-1}{\nu}}\big)^{\frac{1}{\nu-1}} \cdot \alpha \cdot {K_D}^{-\frac{1}{\nu}} \\
&= A^{\frac{\nu-1}{\nu}} \cdot Y^{\frac{1}{\nu}} \cdot \alpha \cdot {K_D}^{-\frac{1}{\nu}} \\
&= \alpha \cdot A^{\frac{\nu-1}{\nu}} \cdot \big(\frac{K_D}{Y}\big)^{-\frac{1}{\nu}}
\end{aligned}$$

Rearranging both sides, I obtain equation (32) again:

$$\frac{K_D}{Y} = \alpha^{\nu} A^{\nu-1} \big(\frac{\partial Y}{\partial K_D}\big)^{-\nu}$$

B.1.2 Other Functional Forms of Dirty Capital

This subsection shows that the estimation strategy of ν does not rely on the specific functional form between dirty capital and carbon emissions. For simplicity, I omit firm and time indicators.

Consider the dirty capital as a function of carbon emissions as follows:

$$K_D = f(G)$$

Take the derivative with respect to G:

$$\frac{\partial K_D}{\partial G} = f'(G)$$

Plugging into equation (32), I obtain

$$\begin{aligned}
\frac{f(G)}{Y} &= \alpha^{\nu} A^{\nu-1} \left(\frac{\partial Y}{\partial K_D} \right)^{-\nu} \\
&= \alpha^{\nu} A^{\nu-1} \left(\frac{\partial Y}{\partial G} \frac{\partial G}{\partial K_D} \right)^{-\nu} \\
&= \alpha^{\nu} A^{\nu-1} \left(\frac{\partial Y}{\partial G} \cdot \frac{1}{f'(G)} \right)^{-\nu}
\end{aligned}$$

The left-hand side can be rewritten as

$$\frac{f(G)}{Y} = \frac{G}{Y} \cdot \frac{f(G)}{G}$$

Then, I obtain

$$\frac{G}{Y} = \frac{G}{f(G)} \alpha^{\nu} A^{\nu-1} \left(\frac{\partial Y}{\partial G} \cdot \frac{1}{f'(G)} \right)^{-\nu}$$

Take the log on both sides, add back the firm and time indicators, and assume that $A^{\nu-1}$ can be captured by firm and time fixed effects

$$\ln \left(\frac{G_{it}}{Y_{it}} \right) = a - \nu \cdot \ln \left(\frac{\partial Y_{it}}{\partial G_{it}} \right) + u_i + u_t + u_{it} \tag{72}$$

where $a = \ln(G_{it}) - \ln(f(G_{it})) + \nu \ln(\alpha) + \nu \ln(f'(G_{it}))$

Equation (72) shows that the specific functional form of $f(\cdot)$ does not affect the estimation of the coefficient ν, but does affect the information contained in the intercept a. The intercept is a nonlinear function of other parameters and carbon emissions. In the next two paragraphs, I show how the results change given different functional forms of $f(\cdot)$.

Linear Function. In Section 3, I assume $G = \eta K_D$. Equivalently, $K_D = \frac{G}{\eta}$. Then, plugging the functional form into equation (72), I obtain

$$\begin{aligned} a &= \ln(G_{it}) - \ln\left(\frac{G_{it}}{\eta}\right) + \nu \ln(\alpha) + \nu \ln\left(\frac{1}{\eta}\right) \\ &= \nu \ln(\alpha) + (1-\nu)\ln(\eta) \end{aligned}$$

In this case, the intercept is a function of the elasticity of substitution ν, the productivity share of dirty capital α, and the carbon emission intensity η.

Power Function. Suppose that $K_D = G^b$. Then, plugging the functional form into equation (72), I obtain

$$\begin{aligned} a &= \ln(G_{it}) - \ln(G_{it}^b) + \nu \ln(\alpha) + \nu \ln(b \cdot G_{it}^{b-1}) \\ &= (1-\nu)(1-b)\ln(G_{it}) + \nu \ln(\alpha) + \nu \ln(b) \end{aligned}$$

In this case, the intercept is a function of the elasticity of substitution ν, the productivity share of dirty capital α, the carbon emission intensity η, the parameter b that governs the functional form of $f(\cdot)$, and the carbon emissions G.

B.2 Empirical Dirty-Capital Share

This subsection documents the calculation of the empirical dirty-capital share referenced in the introduction and in Section 4.5. The procedure is:

1. Merge S&P Trucost (Scope 1 emissions, in tCO_2) to Compustat North America by GVKEY-year over 2002–2023, restricting to U.S. non-financial firms with positive total revenue (REVT), positive gross property, plant, and equipment (PPEGT), and non-missing Scope 1 emissions.

2. For each firm-year, compute emission intensity $E_i/Y_i = \text{Scope1}_i/\text{REVT}_i$.

3. Within each year, compute the cross-sectional median of E_i/Y_i. Label firm-years with E_i/Y_i above the median as "dirty-dominant" and at-or-below the median as "clean-dominant."

4. Sum PPEGT separately within each label and across all firm-years.

5. The dirty-dominant share equals (sum of PPEGT for dirty-dominant firm-years) / (sum of PPEGT for all firm-years) = 0.69.

Because dirty-dominant firms also hold some clean capital, this 69% is an upper bound on the true aggregate $K_D/(K_C + K_D)$ in the data, bracketing the model's equilibrium share from above.

B.3 Solution Methods for the Flow-Driven Model

B.3.1 Baseline Model

The flow-driven model of Section 4.1 is solved by two nested loops, following the methodology of Choi et al. (2025). The *outer loop* imposes demand equilibrium: for a given distribution of firms' fundamental values, the demand index m_{jt} and the price impact ι_{jt} are computed, and fundamental values are updated until a fixed point is reached. The *inner loop*, nested inside each outer-loop iteration, solves the firm's Bellman equation (53) by value function iteration (VFI) with Howard improvement for speed.

State-space discretization. I discretize the continuous state variables on a grid. The two capital stocks (K_C, K_D) are placed on a 20×20 log-spaced grid. The idiosyncratic log-productivity ζ is discretized onto a 7-point grid using the Tauchen (1986) procedure. The aggregate log-productivity x is discretized onto a 5-point grid using the same procedure. The aggregate demand shifter v is discretized onto a 5-point grid. Policy and value functions are evaluated on the product of these grids, and linear interpolation is used for off-grid states.

Outer loop: demand equilibrium. Given an initial guess of the firm distribution and fundamental values, the outer loop iterates:

1. Compute each firm's demand index m_{jt} using the current fundamental values and characteristics.

2. Compute market-clearing portfolio weights w_{jt} and price impacts ι_{jt} from (50).

3. Update the firm's value function, treating ι_{jt} as given (inner loop, below).

4. Compare the updated fundamental values to the initial guess. If the change exceeds a tolerance, repeat.

I use an adaptive step size with a 2-cycle detector to stabilize convergence, following Choi et al. (2025).

Inner loop: value function iteration. Given the current ι, the firm's Bellman equation (53) is solved by VFI. At each grid point, the firm searches over the (I_C, I_D) choice to maximize the right-hand side of (53). Howard improvement iterates the policy function between value updates, which reduces the number of costly optimization steps. The inner loop stops when the change in the value function across successive iterations is below a tolerance.

Simulation. After convergence, I simulate a panel of $N_{\text{sim}} = 3{,}000$ firms for $T_{\text{sim}} = 200$ periods, with the first $T_{\text{burn}} = 50$ periods discarded as burn-in. The simulated panel produces the firm-level series needed to construct the empirical moments. The number of independent aggregate histories is 10, and results are averaged across histories.

B.3.2 Endogenous-ω Extension

Extended state space. The state is $(K_C, K_D, \zeta; X, v, \omega)$. The substitutability parameter ω is placed on a five-point discrete grid $\{-0.803, 0, 0.5, 0.8, 1.0\}$, where -0.803 is the SMM point estimate from Section 4. All other grids and discretization procedures are identical to the Baseline Model above.

Extended Bellman equation. The endogenous-ω extension augments (53) with the ω' choice and the R&D cost:

$$V(K_C, K_D, \zeta; X, v, \omega) = \max_{I_C, I_D, \omega'} \Big\{ \pi - I_C - I_D - \psi^C - \psi^D - \mathcal{C}(\omega' - \omega) + E_t\big[M_{t+1} \cdot P_{t+1}\big] \Big\}, \quad (73)$$

where $P_{t+1} = V(K_{C,t+1}, K_{D,t+1}, \zeta_{t+1}; X_{t+1}, v_{t+1}, \omega') \cdot (1 + \iota_{t+1})$ and $\mathcal{C}$ is defined in (70). Because ω' enters both the current R&D cost and the next-period state, the Bellman operator is evaluated over the joint (I_C, I_D, ω') choice.

Solution. The model is solved by the same two-loop procedure described in the Baseline Model above. The inner VFI loop is extended to include the discrete ω' choice: at each state $(K_C, K_D, \zeta, X, v, \omega)$, the firm evaluates all five ω' values, computing the R&D cost $\mathcal{C}(\omega' - \omega)$ and the corresponding continuation value, and selects the maximum. Parallelization proceeds over the (X, ζ, v, ω) dimensions ($5 \times 7 \times 5 \times 5 = 875$ states). The simulation panel tracks each firm's ω path in addition to capital stocks, with $N_{\text{sim}} = 3{,}000$ firms and $T_{\text{sim}} = 100$ periods (no burn-in discarded, steady-state averages computed over the last 50 periods).

B.4 Solution Methods for the Dynamic Damage Model

B.4.1 VFI Temperature Convention

Inside the Bellman equation, each firm treats temperature ΔT as a state variable and advances it according to its own emissions: $\Delta T' = \Delta T + \lambda G_j$. This is the standard representative-firm approximation for partial-equilibrium dynamic models—each atomistic firm internalizes its own marginal emission contribution when forming the value function, even though it is too small to move the aggregate.

In the panel simulation, by contrast, temperature evolves with the cross-sectional mean:

$$\Delta T_{t+1} = \Delta T_t + \lambda \bar{G}_t, \qquad \bar{G}_t = \frac{1}{N} \sum_{j=1}^{N} G_{jt}. \tag{74}$$

Every firm then faces the same common aggregate ΔT_{t+1} at the next date—exactly as stated in the main text. The two conventions are mutually consistent: the policy functions, solved under the representative-firm approximation, are evaluated in the simulation at the aggregate ΔT_t, so each firm correctly responds to the economywide temperature level. The approximation affects only the marginal incentive embedded in the VFI (which uses the firm's own G_j rather than the aggregate $\bar{G}$); at the simulation stage, all firms face the same ΔT path regardless of their individual emission levels.

B.4.2 Panel Simulation

To reduce Monte Carlo variance, I run $n_{\text{hist}} = 10$ independent draws of the aggregate productivity path $\{X_t\}_{t=1}^{T}$, each paired with $N = 3{,}000$ independent sets of idiosyncratic (ζ_j, Z_j) paths.

The 10 histories share the same policy functions (solved once per (θ, γ, ω) cell) but draw from separate random-number streams so they are statistically independent. Reported statistics—the dirty-capital ratio and the greenium—are averages over the $10 \times 3{,}000 = 30{,}000$ firm-history observations after the $T_{\text{burn}} = 50$-period burn-in. The greenium is computed as the expected-return spread between the top and bottom thirds of the cross-sectional distribution of dirty-capital share $K_D/(K_C + K_D)$—rather than of emission intensity—because dirty-capital share is the cleanest and most structurally consistent measure of brown-firm identity in this model, where firms are differentiated primarily by their capital-stock composition. Realized and expected returns are winsorized at the pooled 1st–99th percentiles before computing the spread, and firm-years with near-zero ex-dividend value ($V_{\text{ex}} < 0.0001$) are excluded as effectively exited.

Appendix C Additional Tables

Table C.1: Data Variable Definitions

This table lists the data variable names, sources, identifiers in the original data source (ID), and descriptions.

Name	Source	ID	Description
GHG Scope 1	Trucost	di_319413	Greenhouse gas (GHG) emissions from sources that are owned or controlled by the company (categorised by the greenhouse gas protocol)
GHG Scope 2	Trucost	di_319414	Greenhouse gas (GHG) emissions from the consumption of purchased electricity, heat, or steam by the company (categorized by the greenhouse gas protocol). Emissions are calculated using a location-based methodology i.e. using grid emission factors for each region.
GHG Scope 3 Upstream	Trucost	di_319415	Greenhouse gas (GHG) emissions from other upstream activities not covered in scope 2 (categorised by the greenhouse gas protocol)
GHG Scope 3 Downstream	Trucost	di_326737	Total downstream indirect greenhouse gas (GHG) emissions associated with the use of sold goods and services
Total Revenue	Trucost	di_319522	For holding companies, revenue is calculated by apportioning each subsidiary's revenue plus adding dividend, share of equity profits and gain on derivatives. For all other companies the value derives from CIQ data.
Total Asset	Compustat	AT	Total value of assets reported on the Balance Sheet
Property, Plant and Equipment (Gross)	Compustat	PPEGT	The cost and/or valuation of tangible fixed assets used in the production of revenue
Property, Plant and Equipment (Net)	Compustat	PPENT	The cost, less accumulated depreciation, of tangible fixed property used in the production of revenue
Intangible Capital	Peters & Taylor	K_INT	Replacement cost of firms intangible capital

Table C.2: Estimates of ν with Financial Firms

Estimates of ν, the degree of substitution of dirty capital, including financial firms (SIC code 6000 - 6999). The estimates follow the Bai (2009) Interactive Fixed Effects estimator (5 factors, levels specification). Proxies for carbon emissions, G, include Scope 1, 2, and 3. The 2-, 3-, and 4-digit SIC codes are used to classify sectors to estimate equation (39) and the marginal productivity of carbon emission, $\frac{\partial Y}{\partial G}$. The independent variable is $\ln(\frac{\partial Y}{\partial G})$ and the dependent variable is $\ln(G/Y)$. The estimates and standard errors (in parentheses) are bootstrapped from 10,000 iterations. The Adj-R-squared and N values are from the OLS levels specification for reference.

Panel A: Scope 1			
	2-digit SIC	3-digit SIC	4-digit SIC
$\ln(\frac{\partial Y}{\partial G})$	0.730***	0.779***	0.797***
	(0.003)	(0.004)	(0.006)
Constant	-0.117***	-0.173***	-0.188***
	(0.013)	(0.015)	(0.027)
Firm Fixed	Y	Y	Y
Time Fixed	Y	Y	Y
Adj-R-squared	0.96	0.97	0.97
N	35436	29194	25633
Panel B: Scope 2			
	2-digit SIC	3-digit SIC	4-digit SIC
$\ln(\frac{\partial Y}{\partial G})$	0.624***	0.638***	0.636***
	(0.006)	(0.006)	(0.006)
Constant	0.376***	0.372***	0.407***
	(0.027)	(0.022)	(0.023)
Firm Fixed	Y	Y	Y
Time Fixed	Y	Y	Y
Adj-R-squared	0.92	0.92	0.92
N	37332	30737	26528
Panel C: Scope 3			
	2-digit SIC	3-digit SIC	4-digit SIC
$\ln(\frac{\partial Y}{\partial G})$	0.621***	0.628***	0.614***
	(0.004)	(0.006)	(0.007)
Constant	1.424***	1.494***	1.549***
	(0.021)	(0.027)	(0.031)
Firm Fixed	Y	Y	Y
Time Fixed	Y	Y	Y
Adj-R-squared	0.93	0.91	0.90
N	38662	32204	28117

Table C.3: Estimates of ν using PPEGT

This table presents the estimates of ν, the degree of substitution of dirty capital, following the Bai (2009) Interactive Fixed Effects estimator (5 factors, levels specification). In equation (39), the total asset is replaced by property, plant, and equipment (gross). Proxies for carbon emissions, G, include Scope 1, 2, and 3. The 2-, 3-, and 4-digit SIC codes are used to classify sectors to estimate equation (39) and the marginal productivity of carbon emissions, $\frac{\partial Y}{\partial G}$. The independent variable is $\ln(\frac{\partial Y}{\partial G})$ and the dependent variable is $\ln(G/Y)$. The estimates and standard errors (in parentheses) are bootstrapped from 10,000 iterations. The Adj-R-squared and N values are from the OLS levels specification for reference.

Panel A: Scope 1			
	2-digit SIC	3-digit SIC	4-digit SIC
$\ln(\frac{\partial Y}{\partial G})$	0.693***	0.701***	0.705***
	(0.004)	(0.006)	(0.006)
Constant	0.305***	0.455***	0.532***
	(0.018)	(0.028)	(0.030)
Firm Fixed	Y	Y	Y
Time Fixed	Y	Y	Y
Adj-R-squared	0.95	0.95	0.96
N	31497	25902	22279
Panel B: Scope 2			
	2-digit SIC	3-digit SIC	4-digit SIC
$\ln(\frac{\partial Y}{\partial G})$	0.589***	0.567***	0.563***
	(0.006)	(0.007)	(0.007)
Constant	0.704***	0.853***	0.896***
	(0.023)	(0.029)	(0.029)
Firm Fixed	Y	Y	Y
Time Fixed	Y	Y	Y
Adj-R-squared	0.87	0.88	0.87
N	33767	27537	23572
Panel C: Scope 3			
	2-digit SIC	3-digit SIC	4-digit SIC
$\ln(\frac{\partial Y}{\partial G})$	0.614***	0.587***	0.564***
	(0.005)	(0.008)	(0.007)
Constant	1.667***	1.877***	1.993***
	(0.025)	(0.042)	(0.038)
Firm Fixed	Y	Y	Y
Time Fixed	Y	Y	Y
Adj-R-squared	0.93	0.91	0.90
N	36174	30069	25879

Table C.4: Estimates of ν using PPENT

This table presents the estimates of ν, the degree of substitution of dirty capital, following the Bai (2009) Interactive Fixed Effects estimator (5 factors, levels specification). In equation (39), the total asset is replaced by property, plant, and equipment (net). Proxies for carbon emissions, G, include Scope 1, 2, and 3. The 2-, 3-, and 4-digit SIC codes are used to classify sectors to estimate equation (39) and the marginal productivity of carbon emissions, $\frac{\partial Y}{\partial G}$. The independent variable is $\ln(\frac{\partial Y}{\partial G})$ and the dependent variable is $\ln(G/Y)$. The estimates and standard errors (in parentheses) are bootstrapped from 10,000 iterations. The Adj-R-squared and N values are from the OLS levels specification for reference.

Panel A: Scope 1			
	2-digit SIC	3-digit SIC	4-digit SIC
$\ln(\frac{\partial Y}{\partial G})$	0.725***	0.718***	0.703***
	(0.004)	(0.007)	(0.006)
Constant	0.189***	0.416***	0.571***
	(0.017)	(0.034)	(0.028)
Firm Fixed	Y	Y	Y
Time Fixed	Y	Y	Y
Adj-R-squared	0.96	0.95	0.96
N	33005	26578	23031
Panel B: Scope 2			
	2-digit SIC	3-digit SIC	4-digit SIC
$\ln(\frac{\partial Y}{\partial G})$	0.617***	0.579***	0.597***
	(0.005)	(0.007)	(0.008)
Constant	0.628***	0.823***	0.782***
	(0.021)	(0.029)	(0.030)
Firm Fixed	Y	Y	Y
Time Fixed	Y	Y	Y
Adj-R-squared	0.88	0.88	0.88
N	34707	28154	23945
Panel C: Scope 3			
	2-digit SIC	3-digit SIC	4-digit SIC
$\ln(\frac{\partial Y}{\partial G})$	0.661***	0.624***	0.597***
	(0.004)	(0.008)	(0.009)
Constant	1.432***	1.699***	1.830***
	(0.019)	(0.040)	(0.045)
Firm Fixed	Y	Y	Y
Time Fixed	Y	Y	Y
Adj-R-squared	0.94	0.92	0.91
N	36511	30429	26158

Table C.5: Estimates of ν using PPEGT and Intangible

This table presents the estimates of ν, the degree of substitution of dirty capital, following the Bai (2009) Interactive Fixed Effects estimator (5 factors, levels specification). In equation (39), the total asset is replaced by the sum of property, plant, and equipment (gross) and intangible capital. Proxies for carbon emissions, G, include Scope 1, 2, and 3. The 2-, 3-, and 4-digit SIC codes are used to classify sectors to estimate equation (39) and the marginal productivity of carbon emissions, $\frac{\partial Y}{\partial G}$. The independent variable is $\ln(\frac{\partial Y}{\partial G})$ and the dependent variable is $\ln(G/Y)$. The estimates and standard errors (in parentheses) are bootstrapped from 10,000 iterations. The Adj-R-squared and N values are from the OLS levels specification for reference.

Panel A: Scope 1			
	2-digit SIC	3-digit SIC	4-digit SIC
$\ln(\frac{\partial Y}{\partial G})$	0.680***	0.711***	0.710***
	(0.004)	(0.005)	(0.006)
Constant	0.282***	0.328***	0.445***
	(0.019)	(0.023)	(0.031)
Firm Fixed	Y	Y	Y
Time Fixed	Y	Y	Y
Adj-R-squared	0.95	0.95	0.96
N	31497	25902	22279
Panel B: Scope 2			
	2-digit SIC	3-digit SIC	4-digit SIC
$\ln(\frac{\partial Y}{\partial G})$	0.525***	0.555***	0.572***
	(0.006)	(0.008)	(0.009)
Constant	0.900***	0.840***	0.814***
	(0.027)	(0.035)	(0.042)
Firm Fixed	Y	Y	Y
Time Fixed	Y	Y	Y
Adj-R-squared	0.87	0.88	0.87
N	33767	27537	23572
Panel C: Scope 3			
	2-digit SIC	3-digit SIC	4-digit SIC
$\ln(\frac{\partial Y}{\partial G})$	0.594***	0.552***	0.503***
	(0.005)	(0.007)	(0.007)
Constant	1.693***	1.996***	2.261***
	(0.028)	(0.036)	(0.038)
Firm Fixed	Y	Y	Y
Time Fixed	Y	Y	Y
Adj-R-squared	0.93	0.91	0.90
N	36174	30069	25879

Table C.6: Estimates of ν using 3-year Moving Average of Carbon Emissions

This table presents the estimates of ν, the degree of substitution of dirty capital, following the Bai (2009) Interactive Fixed Effects estimator (5 factors, levels specification). Proxies for carbon emissions, $\bar{G} = \frac{1}{3}(G_{t-2} + G_{t-1} + G_t)$, are a moving average from the previous three years, and include Scope 1, 2, and 3. The 2-, 3-, and 4-digit SIC codes are used to classify sectors to estimate equation (39) and the marginal productivity of carbon emissions, $\frac{\partial Y}{\partial \bar{G}}$. The dependent variable is $\ln(\bar{G}/Y)$ and the independent variable is $\ln(\frac{\partial Y}{\partial \bar{G}})$. The estimates and standard errors (in parentheses) are bootstrapped from 10,000 iterations.

Panel A: Scope 1			
	2-digit SIC	3-digit SIC	4-digit SIC
$\ln(\frac{\partial Y}{\partial \bar{G}})$	0.663	0.703	0.711
	(0.005)	(0.007)	(0.006)
Constant	0.248	0.286	0.376
	(0.026)	(0.033)	(0.025)
Firm Fixed	Y	Y	Y
Time Fixed	Y	Y	Y
Panel B: Scope 2			
	2-digit SIC	3-digit SIC	4-digit SIC
$\ln(\frac{\partial Y}{\partial \bar{G}})$	0.521	0.530	0.533
	(0.009)	(0.008)	(0.009)
Constant	0.819	0.881	0.895
	(0.044)	(0.035)	(0.035)
Firm Fixed	Y	Y	Y
Time Fixed	Y	Y	Y
Panel C: Scope 3			
	2-digit SIC	3-digit SIC	4-digit SIC
$\ln(\frac{\partial Y}{\partial \bar{G}})$	0.542	0.516	0.506
	(0.005)	(0.007)	(0.008)
Constant	1.903	2.159	2.215
	(0.026)	(0.036)	(0.040)
Firm Fixed	Y	Y	Y
Time Fixed	Y	Y	Y

Table C.7: Estimates of ν by Country

This table presents the estimates of ν by country using the Bai (2009) Interactive Fixed Effects estimator (5 factors, levels specification). GHG1 and GHG2 represent Scope 1 and Scope 2 carbon emissions, respectively. SIC3 and SIC4 correspond to the 3-digit and 4-digit SIC codes. The Average column shows the average across the four specifications.

Country	GHG1 SIC3	GHG1 SIC4	GHG2 SIC3	GHG2 SIC4	Average
Panel A: Developed Markets					
Australia	0.563	0.673	0.663	0.820	0.680
Canada	0.637	0.661	0.596	0.519	0.603
Denmark	0.449	0.932	-0.117	-0.084	0.295
France	0.838	0.767	0.141	0.337	0.521
Germany	0.662	0.513	0.342	0.154	0.418
Hong Kong	0.097	0.186	0.179	0.201	0.166
Ireland	0.232	0.476	0.202	0.256	0.291
Israel	0.391	0.299	0.326	0.270	0.321
Italy	0.982	0.866	1.113	0.114	0.769
Japan	0.162	0.064	0.411	0.061	0.175
Netherlands	0.345	0.300	0.300	0.257	0.300
Singapore	0.893	0.903	0.624	0.109	0.632
Spain	0.850	0.850	0.267	0.782	0.687
Sweden	-0.240	0.037	0.451	-0.038	0.052
Switzerland	0.780	0.726	0.554	0.574	0.658
United Kingdom	0.721	0.747	0.603	0.621	0.673
United States	0.637	0.637	0.345	0.354	0.493
Panel B: Emerging Markets					
Bermuda	0.692	0.663	0.230	0.247	0.458
Brazil	0.668	0.727	0.511	0.424	0.583
Chile	0.391	0.417	0.774	0.815	0.599
China	0.719	0.641	0.572	0.516	0.612
Greece	0.565	0.594	-0.020	-0.020	0.280
India	0.154	0.084	0.236	0.177	0.163
Luxembourg	0.812	0.952	0.615	0.681	0.765
Mexico	0.430	0.954	0.569	0.672	0.656
South Africa	0.715	1.055	0.717	0.832	0.830
South Korea	0.425	0.331	0.248	0.299	0.326
Taiwan	0.323	0.192	0.511	0.498	0.381

Table C.8: Estimates of ν By Industry

This table presents the estimates of ν, the degree of substitution of dirty capital, by industries. Estimates follow the Bai (2009) Interactive Fixed Effects estimator (5 factors, levels specification). The carbon emissions of Scope 1 (GHG1), Scope 2 (GHG2), and Scope 3 (GHG3) are used to proxy carbon emissions G. The 2-, 3-, and 4-digit SIC codes (SIC2, SIC3, SIC4) are used to classify sectors to estimate equation (39) and the marginal productivity of carbon emissions $\frac{\partial Y}{\partial G}$. Industry classification follows the ten industry classifications from Ken French's Website. The ID column indicates the industry ID: 1-consumer nondurables, 2-consumer durables, 3-manufacturing, 4-energy, 5-hightech, 6-telecom, 7-shops, 8-health, 9-utilities, 10-others. The average column (row) represents the average across different carbon emissions and industry classifications (industries).

Panel A: Original Equation (37)										
ID	GHG1 SIC2	GHG1 SIC3	GHG1 SIC4	GHG2 SIC2	GHG2 SIC3	GHG2 SIC4	GHG3 SIC2	GHG3 SIC3	GHG3 SIC4	Average
1	0.61	0.27	0.23	0.25	0.11	0.07	0.74	0.03	0.06	0.26
2	0.25	0.01	0.04	0.27	0.13	0.13	0.36	0.45	0.22	0.21
3	0.62	0.60	0.61	0.49	0.37	0.38	0.36	0.25	0.27	0.44
4	0.31	0.52	0.50	0.37	0.46	0.43	0.72	0.77	0.79	0.54
5	0.63	0.59	0.56	0.53	0.55	0.48	0.63	0.63	0.61	0.58
6	0.36	0.27	0.27	0.48	0.39	0.33	0.51	0.24	0.18	0.34
7	0.29	0.24	0.24	0.35	0.23	0.18	0.36	0.32	0.28	0.28
8	0.37	0.32	0.26	0.43	0.31	0.23	0.56	0.60	0.41	0.39
9	0.65	0.54	0.49	0.68	0.67	0.66	0.51	0.35	0.32	0.54
10	0.77	0.76	0.76	0.67	0.66	0.68	0.52	0.42	0.41	0.63
Average	0.49	0.41	0.40	0.45	0.39	0.36	0.53	0.41	0.36	

Table C.9: Flow-Driven Model: Mean Clean Capital ($\bar{K}_C$)

This table reports the cross-sectional, time-series average of clean capital $\bar{K}_C$ from the dynamic flow-driven model of Section 4, evaluated at the SMM estimates in Table 6. Panel and column definitions follow Table 8: in Panels C and D, the 1.6% and 16% figures denote the carbon-tax bill as a share of firm revenue.

	Emission risk price θ			
ν	0	0.3	0.5	0.7
Panel A: Benchmark				
0.56	14.67	12.75	10.21	7.94
1	9.97	9.85	8.03	7.36
2	4.84	4.75	4.75	4.75
5	4.74	4.74	4.74	4.74
$+\infty$	4.74	4.74	4.74	4.74
Panel B: $\alpha = 0.49$				
0.56	23.55	21.07	14.30	8.72
1	21.59	18.07	13.71	8.00
2	16.22	15.75	12.27	7.11
5	9.69	13.83	11.55	7.09
$+\infty$	5.75	13.79	11.75	7.08
Panel C: $\tau_D = 1.6\%$				
0.56	14.28	12.67	10.18	7.92
1	9.77	9.63	7.95	7.35
2	4.80	4.75	4.75	4.75
5	4.74	4.74	4.74	4.74
$+\infty$	4.74	4.74	4.74	4.74
Panel D: $\tau_D = 16\%$				
0.56	12.54	11.22	9.21	6.98
1	8.20	8.24	7.46	6.83
2	4.75	4.75	4.74	4.74
5	4.74	4.74	4.74	4.74
$+\infty$	4.74	4.74	4.75	4.74
Panel E: $\phi_C = \phi_D$				
0.56	16.75	14.02	11.63	9.07
1	12.27	10.85	9.19	8.03
2	5.16	7.87	6.47	5.73
5	4.74	4.74	4.75	4.75
$+\infty$	4.74	4.74	4.75	4.76

Table C.10: Flow-Driven Model: Mean Dirty Capital ($\bar{K}_D$)

This table reports the cross-sectional, time-series average of dirty capital $\bar{K}_D$ from the dynamic flow-driven model of Section 4, evaluated at the SMM estimates in Table 6. Panel and column definitions follow Table 8: in Panels C and D, the 1.6% and 16% figures denote the carbon-tax bill as a share of firm revenue.

	Emission risk price θ			
ν	0	0.3	0.5	0.7
Panel A: Benchmark				
0.56	35.85	28.56	27.91	27.75
1	48.07	28.89	27.93	27.75
2	73.91	29.39	28.08	27.79
5	102.37	30.15	28.14	27.83
$+\infty$	123.48	31.27	28.23	27.84
Panel B: $\alpha = 0.49$				
0.56	29.80	28.12	27.93	27.79
1	30.46	28.78	28.23	27.98
2	30.80	28.84	28.33	28.28
5	32.39	28.66	28.36	28.25
$+\infty$	39.35	28.50	28.20	28.23
Panel C: $\tau_D = 1.6\%$				
0.56	34.32	28.59	27.91	27.75
1	44.88	28.86	27.92	27.75
2	68.84	29.38	28.07	27.79
5	92.58	30.23	28.14	27.83
$+\infty$	111.70	30.75	28.23	27.84
Panel D: $\tau_D = 16\%$				
0.56	29.83	28.10	27.79	27.72
1	31.59	28.13	27.80	27.71
2	36.70	28.41	27.90	27.75
5	43.94	28.67	27.99	27.77
$+\infty$	48.01	28.75	28.02	27.79
Panel E: $\phi_C = \phi_D$				
0.56	37.54	28.19	27.84	27.74
1	50.02	28.58	27.99	27.78
2	74.23	29.41	28.05	27.81
5	101.19	30.35	28.16	27.88
$+\infty$	131.09	31.04	28.35	28.05

Table C.11: Damage Model: Temperature Increase ΔT (°C)

This table reports the time-series average end-of-horizon cumulative temperature change ΔT (°C above pre-industrial) from the dynamic damage model of Section 5.2, computed under the law of motion $\Delta T_{t+1} = \Delta T_t + \lambda \bar{G}_t$ with $\Delta T_0 = 1$°C and $\lambda = 5 \times 10^{-5}$. Panel and parameter definitions follow Table 13. "Diff. (%)" is the percentage change in ΔT from $\theta = 0$ to $\theta = 15$.

	Emission risk price θ				
ν	0	5	10	15	Diff. (%)
Panel A: Benchmark					
0.56	1.909	1.879	1.533	1.475	−22.72
$+\infty$	2.527	2.507	2.136	1.898	−24.89
Panel B: $\alpha = 49\%$					
0.56	1.594	1.571	1.307	1.329	−16.62
$+\infty$	2.030	2.009	1.715	1.687	−16.88
Panel C: Carbon tax ($\tau_D > 0$)					
0.56	1.631	1.585	1.332	1.370	−15.99
$+\infty$	2.119	2.073	1.811	1.729	−18.43
Panel D: High damage					
0.56	1.716	1.697	1.422	1.448	−15.62
$+\infty$	2.177	2.166	1.920	1.801	−17.28
Panel E: $\phi^C = \phi^D$					
0.56	1.878	1.857	1.587	1.484	−20.99
$+\infty$	2.451	2.435	2.117	1.885	−23.12
Panel F: Constant returns to scale					
0.56	2.195	2.180	1.697	1.522	−30.68
$+\infty$	2.839	2.833	2.558	1.999	−29.59