Mis-measuring Externalities: Carbon Accounting and Equilibrium Capital Allocation*

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Abstract

We study how carbon accounting affects capital allocation when firms generate both direct and avoided emissions. In an equilibrium model, recycling raises reported emissions while displacing dirtier production elsewhere. Carbon accounting therefore changes the signal priced by investors and shifts equilibrium capital allocation. We show that accounting for avoided emissions can reverse sector rankings, mismeasurement creates a wedge between priced and welfare-relevant emissions, and static firm-level metrics generally fail to implement the efficient allocation.

Keywords: Carbon Accounting, ESG, Capital Allocation, Externalities, Avoided Emissions, Consequential Accounting

JEL Classification: D58, D62, G11, Q52, Q56

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1 Introduction

Capital markets increasingly condition capital allocation on reported climate metrics. This is relatively straightforward when environmental performance is well summarized by realized emissions alone. It is less straightforward when economic activity also generates avoided emissions by displacing more carbon-intensive production elsewhere in the economy. In that case, the reported metric is no longer a neutral summary statistic: once capital is conditioned on it, the accounting rule helps determine which environmental object is priced.

We study carbon accounting as a determinant of decentralized capital allocation. The main question is simple: when investors, regulators, or financial institutions allocate capital using reported emissions metrics, what exactly is the environmental object that enters private payoffs? In the environment studied here, that object need not coincide with the welfare-relevant externality. As a result, markets may allocate capital efficiently conditional on the reported metric and still generate an inefficient allocation from a welfare perspective.

To study this mechanism, we develop a stylized equilibrium model with circular production. Recycling provides a useful benchmark because it generates, in a particularly transparent way, a divergence between firm-level realized emissions and welfare-relevant net emissions once displacement is taken into account. In the model, recycling raises realized emissions but displaces more carbon-intensive virgin production elsewhere in the economy. Avoided emissions are therefore not treated as a primitive firm-level attribute, but as an equilibrium displacement effect. This allows us to distinguish three objects that are often conflated in practice: realized emissions, the welfare-relevant externality, and the accounting metric observed and priced by decentralized agents. More generally, the paper studies capital allocation under measured externalities, that is, environments in which decentralized agents price a reportable firm-level object while welfare depends on an equilibrium counterfactual. Accounting is modeled as a generic rule that credits recy-

cled input at coefficient ϕ, allowing the analysis to compare realized, measured adjusted, and true consequential emissions metrics within a common equilibrium framework.

The paper's central point is that carbon accounting is not allocatively neutral: by determining the emissions object embedded in decentralized payoffs, the accounting rule affects equilibrium capital allocation. The analysis delivers three main results. First, changing the accounting rule changes equilibrium capital allocation because it changes the environmental object priced by decentralized agents. Second, when avoided emissions are mismeasured, the priced environmental signal differs from the welfare-relevant one, generating a wedge between decentralized allocation and the planner benchmark. Third, static firm-level accounting cannot generally implement the planner allocation, because the planner's marginal condition depends on aggregate equilibrium responses that are not contained in contemporaneous firm-level accounting levels.

These results place the paper at the intersection of equilibrium sustainable-finance theory and the literature on carbon-accounting design. The paper is closest to Landier and Lovo (2025), whose two-sector equilibrium approach provides a natural starting point for thinking about capital allocation under environmental objectives. We build on that allocative logic but depart from it in one central respect: the environmental signal guiding decentralized allocation is not taken as given. Instead, carbon accounting determines which emissions object is priced by investors and regulators. The paper contributes to the equilibrium literature on sustainable finance, including Pástor, Stambaugh, and Taylor (2021), Oehmke and Opp (2025), Berk and van Binsbergen (2025), and Landier and Lovo (2025). It also contributes to the literature on carbon accounting and avoided emissions, especially the distinction between attributional and consequential accounting and the measurement of displacement effects (Weidema, 2003; Ekvall and Weidema, 2004; Brander and Wylie, 2011; WBCSD, 2025), by embedding that distinction in an equilibrium capital-allocation problem. While recycling provides the cleanest microfoundation in our setting, the underlying logic is broader: the same measurement problem arises whenever private allocation relies on reportable firm-level metrics while welfare depends

on system-wide counterfactual effects. The paper is therefore complementary to work on environmental policy and transition dynamics, such as Acemoglu et al. (2012), although our focus is not directed technical change but the measurement of the environmental object entering decentralized capital allocation.

The broader implication is that measurement rules matter not only because they affect disclosure, but also because they affect allocation. Once reported carbon metrics enter investment mandates, lending decisions, portfolio construction, or benchmarking rules, accounting conventions influence the environmental signal transmitted to capital markets. Carbon accounting is therefore not only a reporting technology; it is also part of the allocative environment faced by decentralized capital markets.

The remainder of the paper proceeds as follows. Section 2 presents the model. Section 3 characterizes decentralized equilibrium under alternative accounting rules. Section 4 derives the planner benchmark and identifies the measurement wedge. Section 5 develops the main comparative-statics and welfare implications. Section 6 discusses empirical and institutional implications. Section 7 concludes.

2 Model

The model is designed to isolate how alternative carbon-accounting rules affect decentralized capital allocation. The circular-production structure should be viewed as a tractable microfoundation for a broader class of environments in which firms are evaluated on observable accounting objects, while the relevant social externality depends on equilibrium displacement or substitution effects outside the firm's reporting boundary. This microfoundation disciplines the interpretation of recycling, displacement, and avoided emissions, while the main results are stated in reduced-form equilibrium terms to keep the measurement channel transparent.

We consider a static two-sector economy indexed by $i \in \{1, 2\}$, with $j \neq i$. A unit mass of capital is allocated across sectors. One unit of capital activates one project,

so that K_i denotes both the mass of active firms in sector i and the amount of capital allocated to that sector. Sector sizes satisfy

$$K_1 + K_2 = 1. \tag{1}$$

All firms within a sector are identical. The economy is otherwise frictionless. The only distortion studied in the paper is that decentralized agents price an accounting metric that need not coincide with the welfare-relevant externality.

From this point onward, we index all endogenous firm-level and aggregate objects by K_1, with $K_2 = 1 - K_1$. This reduced-form notation captures the fact that equilibrium prices, output, recycled-input use, and emissions depend on the sectoral allocation of capital.

2.1 Technology and Circular Production

Production is circular in the following sense. Each sector combines virgin material purchased from the other sector with recycled material recovered from its own waste. This structure creates a natural source of displacement: greater recycled-input use reduces the need for virgin production elsewhere in the economy. The model therefore links firm-level recycling activity to an economy-wide consequential emissions effect.

A representative firm in sector i produces output using a materials bundle m_i according to

$$y_i = A_i \, m_i^{\alpha_i}, \qquad A_i > 0, \qquad \alpha_i \in (0, 1). \tag{2}$$

The parameter A_i captures sector-specific production efficiency, while α_i governs the elasticity of output with respect to the materials bundle m_i and ensures diminishing returns at the project level. The materials bundle combines a virgin input purchased from the other sector with a recycled input supplied by a competitive recycling market

operating in sector i:[1]

$$m_i = \left(\omega\, n_i^{\frac{\sigma-1}{\sigma}} + (1-\omega)\, r_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \qquad \omega \in (0,1), \qquad \sigma > 0, \qquad \sigma \neq 1. \tag{3}$$

Here n_i denotes virgin material used by sector i and purchased from sector $j \neq i$, while r_i denotes recycled material used by sector i and purchased from a competitive recycling market supplied out of sector-i waste. This specification captures the idea that recycled and virgin materials are imperfect substitutes. This imperfect substitutability is important because it makes recycling an economically meaningful margin of adjustment rather than a perfect one-for-one replacement technology. The elasticity parameter σ governs the strength of substitution, while ω captures the technological weight on virgin material.

Let p_i^c denote the price of sector i's output, which is the price paid by final consumers for good i, and let p_i^r denote the price of recycled input used by sector i. Since the virgin input used by sector i is good j, its price is p_j^c. Standard CES cost minimization implies that the dual price index of the materials bundle is

$$q_i = \left[\omega^\sigma (p_j^c)^{1-\sigma} + (1-\omega)^\sigma (p_i^r)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{4}$$

The composite price q_i is the relevant marginal cost object for the firm: it embeds the substitution margin between virgin and recycled inputs and therefore governs how recycling affects production decisions in equilibrium.[2]

The representative firm in sector i therefore solves

$$\max_{m_i \geq 0} \;\; p_i^c\, A_i\, m_i^{\alpha_i} - q_i\, m_i. \tag{5}$$

[1]The CES specification is close in spirit to the Armington-type aggregation used by Bongers and Casas (2022) in a circular-economy framework. In our setting, it provides a parsimonious way to model imperfect substitution between virgin and recycled inputs while keeping the equilibrium structure tractable.

[2]Appendix D collects the closed-form blocks underlying the reduced-form representation used in the main text, including household demand, sectoral profit erosion, and a benchmark welfare decomposition.

Conditional on m_i, the cost-minimizing input demands are

$$n_i = m_i \, \omega^{\sigma} \left(\frac{p_j^c}{q_i} \right)^{-\sigma}, \tag{6}$$

$$r_i = m_i \, (1-\omega)^{\sigma} \left(\frac{p_i^r}{q_i} \right)^{-\sigma}. \tag{7}$$

Because recycling services are purchased from a competitive secondary-input market at price p_i^r, the resource cost of recycling is fully embedded in q_i. No separate recycling term is needed in firm profits.

Production generates waste. At the firm level, waste is proportional to output

$$\Theta_i = \theta_i \, y_i, \qquad \theta_i \in [0,1]. \tag{8}$$

At the sector level, aggregate output and aggregate waste are

$$Y_i = K_i \, y_i, \qquad \Theta_i^{agg} = \theta_i \, Y_i. \tag{9}$$

A competitive recycling sector transforms a fraction of sector i's waste into usable recycled input. In the core model, the recovery rate $\rho_i \in [0,1]$ is exogenous. Aggregate recycled-input supply therefore satisfies

$$R_i \leq \rho_i \, \theta_i \, Y_i. \tag{10}$$

The recycled-input market clears when

$$K_i \, r_i = R_i. \tag{11}$$

In a symmetric equilibrium with a binding recovery constraint, (10)–(11) imply

$$r_i = \rho_i \, \theta_i \, y_i. \tag{12}$$

Accordingly, the baseline model should be interpreted as a sector-level recycling economy: firms choose recycled-input demand, while the supply of recycled material is generated collectively from sectoral waste and cleared on a competitive secondary-input market. This formulation resolves an important modeling issue. Firms choose recycled input through standard cost minimization, while the economy-wide recycling technology determines equilibrium supply. The identity $r_i = \rho_i \, \theta_i \, y_i$ is thus an equilibrium outcome of the recycling market, not a private technological constraint imposed directly inside the firm's static problem.

This distinction is important for the interpretation of avoided emissions. Firms choose recycled-input use through standard cost minimization, while the quantity of recycled material available in the economy depends on sectoral waste generation and recycling capacity. As a result, the displacement effect associated with recycling is an equilibrium object, not a purely technological coefficient imposed at the firm level.[3]

2.2 Households and Welfare

The representative household consumes the two final goods and suffers disutility from aggregate emissions. Preferences are Cobb-Douglas over consumption and separable in the environmental externality

$$U(C_1, C_2, E^*) = \frac{C_1^{\gamma_1} \, C_2^{\gamma_2}}{(1 + E^*)^{\delta}}, \qquad \gamma_1, \gamma_2 \in (0, 1), \qquad \gamma_1 + \gamma_2 = 1, \qquad \delta > 0. \tag{13}$$

Here E^* denotes the true welfare-relevant externality, defined below. Let W denote aggregate household income. Standard Cobb-Douglas demand implies

$$C_i = \frac{\gamma_i \, W}{p_i^c}, \qquad i \in \{1, 2\}. \tag{14}$$

[3]A schematic representation of the circular production structure is provided in Appendix D.

The environmental term shifts welfare but does not affect relative consumption demands conditional on income and prices.[4]

The paper adopts a consequential welfare criterion. Welfare depends on the true net emissions effect of economic activity once system-wide displacement is taken into account. This object need not coincide with realized emissions, and it need not coincide with the accounting metric used in decentralized allocation.

2.3 Emissions, Displacement, and Accounting

The model distinguishes realized emissions, displaced emissions, the true social externality, and the accounting metric used by decentralized agents. The key issue is that these objects coincide only under restrictive conditions: in general, what is physically emitted, what is socially relevant, and what is reported to markets are distinct environmental objects.

Realized emissions. Realized emissions of a representative firm in sector i are

$$e_i^R(K_1) = \varepsilon_i^{prod}\, y_i(K_1) + \varepsilon_i^{waste}\big(\theta_i\, y_i(K_1) - r_i(K_1)\big) + \varepsilon_i^{rec}\, r_i(K_1), \tag{15}$$

where ε_i^{prod} is the emissions intensity of production, ε_i^{waste} is the emissions intensity of unrecycled waste, and ε_i^{rec} is the emissions intensity of recycling. The dependence on K_1 reflects general-equilibrium effects through prices, output, and recycled-input use. Aggregate realized emissions are

$$E^R(K_1) = K_1\, e_1^R(K_1) + (1 - K_1)\, e_2^R(K_1), \tag{16}$$

Displacement. One unit of recycled input displaces virgin production elsewhere in the economy. Let $f > 0$ denote the true emissions intensity of the marginal activity displaced

[4] Appendix D collects the closed-form blocks underlying the reduced-form representation used in the main text, including household demand, sectoral profit erosion, and a benchmark welfare decomposition.

by one unit of recycled input. In the core model, f is treated as a reduced-form constant. Aggregate displacement capacity is

$$B(K_1) = K_1\, r_1(K_1) + (1 - K_1)\, r_2(K_1). \tag{17}$$

The corresponding aggregate displaced emissions are $f\, B(K_1)$. In the present setting, $B(K_1)$ can be interpreted as aggregate recycled-input use, that is, the economy-wide scale at which circular production displaces virgin activity. Economically, f is a consequential parameter: it captures the emissions intensity of the marginal virgin activity displaced by recycling. Accordingly, $f\, B(K_1)$ represents displaced virgin emissions, that is, the counterfactual emissions avoided elsewhere in equilibrium, rather than an additional physical emissions inventory attached to recycling itself. Realized emissions already include the emissions generated by recycling activity through the term $\varepsilon_i^{rec}\, r_i$. The consequential adjustment therefore subtracts only the emissions of the marginal virgin activity displaced elsewhere in equilibrium. This convention rules out double counting and makes clear that the welfare object is realized emissions minus displaced virgin emissions.

True welfare-relevant externality. The consequential emissions object relevant for welfare is

$$E^*(K_1) = E^R(K_1) - f\, B(K_1). \tag{18}$$

It combines observed emissions with the emissions that are avoided elsewhere in equilibrium through displacement. With constant f, the same decomposition can be written at the firm level as

$$e_i^*(K_1) = e_i^R(K_1) - f\, r_i(K_1). \tag{19}$$

Using Equation (12) and Equation (15), this becomes

$$e_i^*(K_1) = y_i(K_1)\left[\varepsilon_i^{prod} + \varepsilon_i^{waste}\,\theta_i + \left(\varepsilon_i^{rec} - \varepsilon_i^{waste} - f\right)\rho_i\,\theta_i\right]. \tag{20}$$

This expression makes clear that recycling affects welfare through two channels at once: it changes realized emissions directly, and it changes the amount of dirtier activity displaced elsewhere in the economy. Equation (20) clarifies a point that will be central throughout the paper. Recycling enters realized emissions once, through $\varepsilon_i^{rec}\,r_i$. The displaced-emissions term subtracts the emissions of the marginal virgin activity displaced elsewhere in the economy. It does not subtract recycling emissions a second time. The accounting object relevant for welfare is therefore realized emissions minus displaced virgin emissions.

Nothing in the wedge identified here is conceptually specific to recycling per se. Recycling is simply the benchmark case in which the distance between realized and consequential emissions can be microfounded transparently through displaced virgin production. More generally, the same structure arises whenever reported firm-level emissions differ from the welfare-relevant externality because part of the environmental effect operates through equilibrium counterfactuals outside the reporting boundary.

Accounting rules. In practice, the coefficient embedded in the reported metric may reflect reporting conventions, standard-setting choices, or estimated avoided-emissions factors rather than the true consequential benchmark. This is consistent with the broader consequential-accounting view that avoided emissions depend on assumptions about marginal displacement and system boundaries rather than on a purely attributional inventory of realized emissions (Weidema, 2003; Ekvall and Weidema, 2004; WBCSD, 2025). Decentralized agents do not observe the true consequential coefficient f directly. Instead, they price an implementable accounting metric constructed from reportable firm-level objects. We represent accounting with a generic coefficient $\phi \in \mathbb{R}_+$ and define the emissions metric

attributed to sector i as

$$e_i^{\phi}(K_1) = e_i^R(K_1) - \phi\, r_i(K_1). \tag{21}$$

Aggregating across sectors gives

$$E^{\phi}(K_1) = E^R(K_1) - \phi\, B(K_1). \tag{22}$$

This formulation nests the accounting regimes of interest:

$$\phi = 0 \quad \Rightarrow \quad \text{realized accounting,} \tag{23}$$

$$\phi = \widetilde{f} \quad \Rightarrow \quad \text{measured adjusted accounting,} \tag{24}$$

$$\phi = f \quad \Rightarrow \quad \text{true consequential accounting.} \tag{25}$$

The key distinction is that $E^*(K_1)$ is the welfare object, whereas $E^{\phi}(K_1)$ is the priced accounting object. The distinction between $\widetilde{f}$ and f is central. The coefficient $\widetilde{f}$ is the avoided-emissions factor embedded in the accounting rule that investors or regulators can actually observe and price. By contrast, f is the true consequential coefficient relevant for welfare. The two coincide only when avoided emissions are measured without error. We study the allocative consequences of the wedge between the two.

Table 1 summarizes the main emissions objects and accounting concepts used throughout the paper.

The previous subsections define the technological and environmental objects of the economy. We now turn to the decentralized pricing channel through which accounting affects capital allocation.

2.4 Decentralized Capital Allocation

One unit of capital activates one firm, so sectoral capital allocation is summarized by K_1 and $K_2 = 1 - K_1$. Investors evaluate projects using financial profitability adjusted

Table 1: Main objects of the model

Object	**Interpretation**
ϕ	Generic accounting coefficient applied to recycled input. It indexes the accounting regime.
$r_i(K_1)$	Recycled input used per project in sector i. This is the per-project accounting base for avoided-emissions credits.
$B(K_1)$	Aggregate displacement capacity, equal to aggregate recycled-input use. It determines the scale of displaced emissions in the economy.
f	True emissions intensity of the marginal activity displaced by one unit of recycled input.
$\widetilde{f}$	Measured avoided-emissions coefficient used by an implementable adjusted-accounting rule.
$E^R(K_1)$	Aggregate realized emissions in the economy.
$E^\phi(K_1)$	Aggregate accounting metric priced by decentralized agents under accounting rule ϕ.
$E^*(K_1)$	True welfare-relevant aggregate externality, equal to realized aggregate emissions minus true displaced emissions.

for the priced emissions metric. Under accounting rule ϕ, the perceived payoff of a representative sector-i project is

$$\widetilde{\Pi}_i^\phi(K_1) = \pi_i(K_i) - \tau\, e_i^\phi(K_1), \tag{26}$$

where $\pi_i(K_i)$ denotes financial profit per project and $\tau \geq 0$ is the shadow value assigned to measured emissions. The term $\tau\, e_i^\phi(K_1)$ can be interpreted as a reduced-form pricing wedge applied to the accounting-based emissions metric.

This wedge should be understood as a reduced-form representation of investor behavior, not as a literal policy tax. In the model, investors are the decentralized agents who allocate capital across projects, while τ summarizes the extent to which reported emissions enter financing conditions. In that sense, τ governs the intensity of environmental pricing, while ϕ governs its base.

Laissez-faire is the special case $\tau = 0$, in which capital is allocated solely on the

basis of financial profitability. If $\tau > 0$, accounting affects capital allocation because investors price the reported object $e_i^{\phi}(K_1)$. The role of investors is therefore to transmit accounting into capital allocation, whereas the role of the accounting rule is to determine which environmental object is transmitted.

The microfoundation developed above determines equilibrium profits, recycled-input use, and emissions as functions of sectoral capital allocation. The next section uses this structure to characterize decentralized equilibrium through reduced-form mappings that summarize these equilibrium relationships without carrying the full system of prices and quantities.

3 Decentralized Equilibrium

This section characterizes decentralized capital allocation under a generic accounting rule ϕ. The key object is the equilibrium allocation K_1^{ϕ}, that is, the share of capital allocated to sector 1 when investors price the emissions metric induced by ϕ.

The underlying microfoundation in Section 2 implies that, for any sectoral allocation K_1, equilibrium prices, output, recycled-input use, and emissions are jointly determined by firms' input choices, household demand, and market-clearing conditions. For the allocative results of this section, it is sufficient to summarize those equilibrium relationships through reduced-form objects indexed by K_1: per-project profits $\pi_i(K_i)$, accounting bases $b_i(K_1)$, and firm-level accounting emissions $e_i^{\phi}(K_1)$. These objects should therefore be read as equilibrium mappings induced by the benchmark microfounded economy, rather than as independent primitives.

In the benchmark circular-production economy, sectoral crowding and goods-market clearing imply that project-level profitability declines with sector size, while recycled-input use and accounting emissions vary with equilibrium output and recycling activity. The reduced-form objects used below summarize these equilibrium responses.

In particular, Appendix D.3 shows that, in the benchmark closed-form system, per-

project profits satisfy

$$\pi_i(K_i) = \frac{K_i^*}{K_i}, \tag{27}$$

so profitability is strictly decreasing in sector size. Likewise, the accounting bases $b_i(K_1)$ summarize the recycling margins generated by the circular-production block, and the accounting emissions objects $e_i^{\phi}(K_1)$ inherit their dependence on K_1 from equilibrium output and recycled-input use.

For any allocation K_1, define

$$G(K_1, \phi) := \widetilde{\Pi}_1^{\phi}(K_1) - \widetilde{\Pi}_2^{\phi}(K_1), \tag{28}$$

that is, the difference in perceived net payoffs between sector 1 and sector 2 under accounting rule ϕ.

Equivalently,

$$G(K_1, \phi) = \pi_1(K_1) - \pi_2(1 - K_1) - \tau\Big(e_1^{\phi}(K_1) - e_2^{\phi}(K_1)\Big), \tag{29}$$

where

$$e_i^{\phi}(K_1) = e_i^{R}(K_1) - \phi\, b_i(K_1), \qquad b_i(K_1) := r_i(K_1). \tag{30}$$

The distinction is conceptual: $r_i(K_1)$ is the underlying equilibrium recycling margin, while $b_i(K_1)$ denotes the accounting base used by decentralized agents. In the benchmark model, the two coincide.

The assumptions imposed below on $\pi_i(K_i)$, $b_i(K_1)$, and $G(K_1, \phi)$ should be interpreted as reduced-form regularity conditions on the equilibrium mappings generated by the underlying microfounded economy.

Definition 1 (Decentralized equilibrium)**.** *A decentralized equilibrium under accounting*

rule ϕ is an allocation $K^\phi = (K_1^\phi, K_2^\phi)$, together with prices and quantities, such that firms optimize, households optimize, goods and recycling markets clear, and investors allocate capital across sectors according to perceived payoffs.

For an interior allocation, the equilibrium condition is

$$G(K_1^\phi, \phi) = 0, \qquad K_2^\phi = 1 - K_1^\phi. \tag{31}$$

To obtain a sharp characterization of decentralized allocation, we impose standard reduced-form regularity conditions: per-project profits are decreasing in sector size, and the relative-payoff function $G(\cdot, \phi)$ crosses zero exactly once for each accounting rule in the range of interest. These conditions are sufficient to deliver a unique interior equilibrium and transparent comparative statics.

Proposition 1 (Existence, uniqueness, and accounting non-neutrality)**.** *Suppose that, for each sector $i \in \{1, 2\}$, the per-project profit function $\pi_i : (0, 1) \to \mathbb{R}$ is continuously differentiable and strictly decreasing. Suppose also that, for every accounting rule ϕ in the range of interest, the function $K_1 \mapsto G(K_1, \phi)$ is strictly decreasing on $(0, 1)$ and satisfies*

$$\lim_{K_1 \to 0} G(K_1, \phi) > 0, \qquad \lim_{K_1 \to 1} G(K_1, \phi) < 0. \tag{32}$$

Then, for each accounting rule ϕ, there exists a unique interior decentralized equilibrium $K_1^\phi \in (0, 1)$ satisfying Equation (31).

Moreover, if G is continuously differentiable in (K_1, ϕ), the equilibrium allocation is differentiable in ϕ, and

$$\frac{\partial K_1^\phi}{\partial \phi} = -\frac{G_\phi(K_1^\phi, \phi)}{G_{K_1}(K_1^\phi, \phi)} = -\frac{\tau\big(b_1(K_1^\phi) - b_2(K_1^\phi)\big)}{G_{K_1}(K_1^\phi, \phi)}. \tag{33}$$

In particular, since $G_{K_1}(K_1^\phi, \phi) < 0$,

$$b_1(K_1^\phi) > b_2(K_1^\phi) \qquad \Longrightarrow \qquad \frac{\partial K_1^\phi}{\partial \phi} > 0, \tag{34}$$

and the inequality is reversed when $b_1(K_1^\phi) < b_2(K_1^\phi)$.

Proposition 1 summarizes the reduced-form allocative implication of the microfounded circular-production environment developed in Section 2. The proposition abstracts from closed-form equilibrium details in order to isolate the measurement channel: changing the accounting rule changes the environmental object embedded in perceived payoffs, which in turn changes equilibrium capital allocation.

Proposition 1 formalizes the paper's first core mechanism: accounting is not neutral. The key object is the relative accounting base $b_1(K_1^\phi) - b_2(K_1^\phi)$. Sectors that receive more avoided-emissions credit per project become more attractive when the accounting rule places greater weight on displacement. If sector 1 receives more avoided-emissions credit per project than sector 2, increasing ϕ raises sector 1's relative attractiveness and shifts capital toward it.

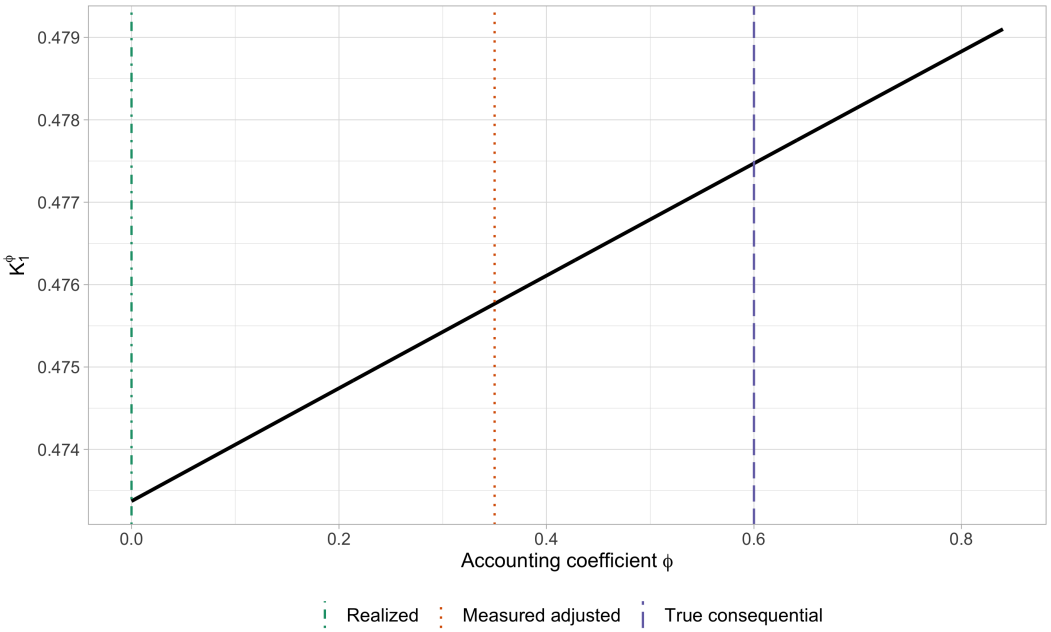
In the underlying circular-production economy, this relative accounting base is rooted in sectoral differences in recycled-input use and therefore in displacement capacity. Proposition 1 is a positive equilibrium result: it characterizes how accounting changes decentralized allocation, not whether the resulting allocation is welfare-improving.

Figure 1 provides a benchmark illustration of Proposition 1. It plots the decentralized equilibrium allocation K_1^ϕ as a function of the accounting coefficient ϕ under the benchmark parameterization used in the numerical illustration.

4 Planner Benchmark

We now turn to the normative benchmark. Welfare depends on the true consequential externality $E^*(K_1)$, not on the accounting metric priced by decentralized investors. In that sense, our planner benchmark follows the standard public-economics logic of evaluat-

Figure 1: Equilibrium allocation under alternative accounting rules



Notes: The figure plots the decentralized equilibrium allocation K_1^ϕ as a function of the accounting coefficient ϕ. The vertical markers identify realized accounting ($\phi = 0$), measured adjusted accounting ($\phi = \widetilde{f}$), and true consequential accounting ($\phi = f$). In the benchmark calibration, avoided-emissions credit primarily benefits sector 1, so increasing ϕ shifts capital away from sector 2 and therefore increases K_1^ϕ. The figure is based on the benchmark parameterization $A_1 = A_2 = 1$, $\alpha_1 = \alpha_2 = 0.5$, $\gamma_1 = \gamma_2 = 0.5$, $\theta_1 = \theta_2 = 0.5$, $\rho_1 = 0.5$, $\rho_2 = 0$, $\varepsilon_1^{prod} = 1.2$, $\varepsilon_2^{prod} = 0.2$, $\varepsilon_1^{waste} = \varepsilon_2^{waste} = 0.5$, $\varepsilon_1^{rec} = \varepsilon_2^{rec} = 0.5$, $\delta = 6$, $f = 0.6$, and $\widetilde{f} = 0.35$. The quantitative variation in K_1^ϕ is small in this benchmark, but the figure highlights the direction of the allocative response.

ing decentralized allocations against a welfare criterion based on the true social externality, even though our focus is not optimal taxation per se but the allocative consequences of alternative accounting bases (Pigou, 1920; Golosov et al., 2014).

For the normative analysis, we work with the reduced-form welfare object $\mathcal{W}(K_1)$ induced by the underlying equilibrium allocation. The results below require only standard regularity conditions on this object.[5]

Table 2 summarizes the main regimes considered in the paper. The key distinction is whether capital allocation is determined by decentralized investors or by the planner, and whether the relevant environmental object is realized, measured adjusted, or true

[5]Appendix D.4 provides a closed-form welfare decomposition for the benchmark microfoundation.

consequential emissions.

Table 2: Regimes considered in the analysis

Regime	**Allocation rule**	**Environmental object relevant for decisions**
Laissez-faire	Capital is allocated according to financial profitability only ($\tau = 0$).	No environmental object is priced in decentralized allocation.
Realized accounting	Capital is allocated by decentralized investors under accounting rule $\phi = 0$.	Realized emissions, $E^R(K_1)$.
Measured adjusted accounting	Capital is allocated by decentralized investors under accounting rule $\phi = \widetilde{f}$.	Measured adjusted emissions, $E^{\widetilde{f}}(K_1) = E^R(K_1) - \widetilde{f}\, B(K_1)$.
True consequential accounting	Capital is allocated by decentralized investors under accounting rule $\phi = f$.	True consequential accounting metric, $E^f(K_1) = E^R(K_1) - f\, B(K_1)$.
First best	The planner chooses K_1 to maximize true welfare.	True welfare-relevant externality, $E^*(K_1) = E^R(K_1) - f\, B(K_1)$.

As presented in Table 2, the paper compares two distinct margins. The first is the allocation mechanism: decentralized capital allocation versus planner choice. The second is the environmental object entering decisions. The measurement wedge arises because decentralized agents generally respond to $E^{\phi}(K_1)$, while the planner evaluates allocations using $E^*(K_1)$.

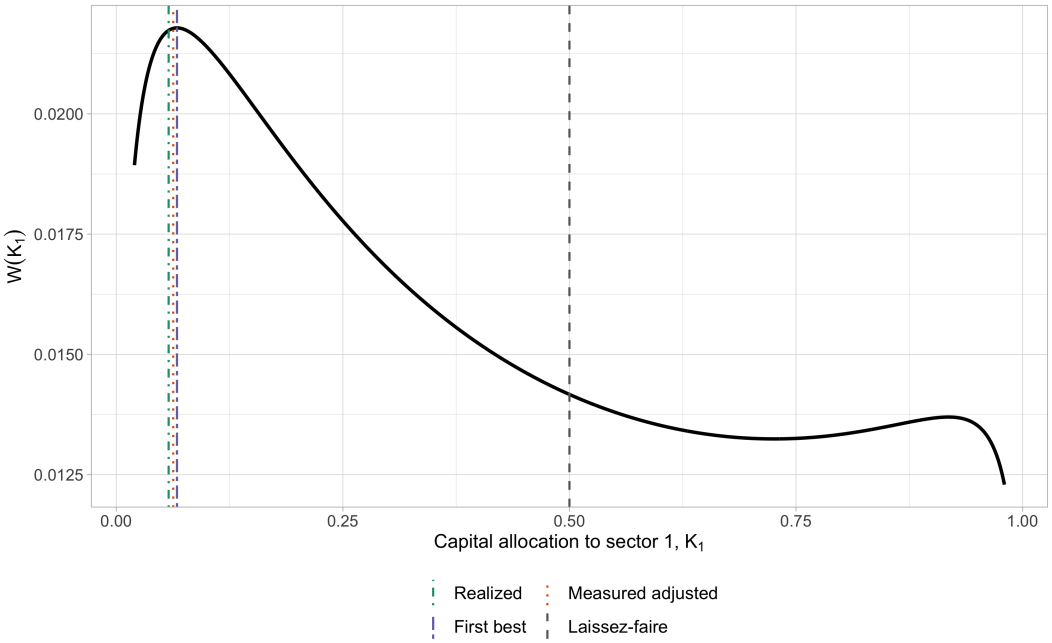
Figure 2 provides a benchmark illustration of how true welfare varies with the capital allocation K_1 and how the main regimes considered in the paper map into different allocations along that welfare schedule.[6]

Given the equilibrium allocation K_1, define true social welfare as

$$\mathcal{W}(K_1) := \frac{C_1(K_1)^{\gamma_1}\, C_2(K_1)^{\gamma_2}}{\big(1 + E^*(K_1)\big)^{\delta}}, \tag{35}$$

[6]Figure 2 is constructed using the closed-form welfare representation derived in Proposition 7 in Appendix D.4, under the benchmark parameterization described in the notes.

Figure 2: True welfare across capital allocations



Notes: The figure plots true social welfare $\mathcal{W}(K_1)$ as a function of the capital allocation to sector 1, K_1, using the closed-form benchmark welfare representation derived in Proposition 7 in Appendix D.4. The vertical markers identify the allocations associated with realized accounting, measured adjusted accounting, the first best, and laissez-faire. The figure is based on the benchmark parameterization $A_1 = A_2 = 1$, $\alpha_1 = \alpha_2 = 0.5$, $\gamma_1 = \gamma_2 = 0.5$, $\theta_1 = \theta_2 = 0.5$, $\rho_1 = 0.5$, $\rho_2 = 0$, $\varepsilon_1^{prod} = 1.2$, $\varepsilon_2^{prod} = 0.2$, $\varepsilon_1^{waste} = \varepsilon_2^{waste} = 0.5$, $\varepsilon_1^{rec} = \varepsilon_2^{rec} = 0.5$, $\delta = 6$, $f = 0.6$, and $\widetilde{f} = 0.35$. The figure is included for intuition and does not represent a general analytical result.

where

$$E^*(K_1) = E^R(K_1) - f\,B(K_1), \qquad B(K_1) = K_1\,b_1(K_1) + (1 - K_1)\,b_2(K_1). \tag{36}$$

The planner chooses K_1 to maximize $\mathcal{W}(K_1)$, taking into account the effect of capital allocation on output, consumption, recycling, and true aggregate emissions.

To state the planner's first-order condition compactly, define

$$\Lambda(K_1) := \frac{d}{dK_1}\left[\gamma_1 \log C_1(K_1) + \gamma_2 \log C_2(K_1)\right] \tag{37}$$

$$= \gamma_1 \frac{C_1'(K_1)}{C_1(K_1)} + \gamma_2 \frac{C_2'(K_1)}{C_2(K_1)}. \tag{38}$$

This term summarizes the marginal effect of reallocating capital on the consumption component of welfare.

The next result isolates the paper's normative core: the marginal environmental gradient relevant for welfare generally differs from the one embedded in decentralized accounting-based capital allocation.

Proposition 2 (Planner FOC and the measurement wedge)**.** *Suppose that $\mathcal{W}$ is continuously differentiable and that the first-best allocation $K_1^{FB} \in (0,1)$ is interior. Then K_1^{FB} satisfies*

$$\Lambda(K_1^{FB}) = \delta \, \frac{E^{*\prime}(K_1^{FB})}{1 + E^{*}(K_1^{FB})}. \tag{39}$$

Moreover, true aggregate emissions satisfy

$$E^{*\prime}(K_1) = E^{R\prime}(K_1) - f \, B'(K_1), \tag{40}$$

whereas the environmental gradient induced by accounting rule ϕ is

$$E^{\phi\prime}(K_1) = E^{R\prime}(K_1) - \phi \, B'(K_1). \tag{41}$$

Hence the measurement wedge is

$$\mathcal{M}(K_1; \phi) := E^{*\prime}(K_1) - E^{\phi\prime}(K_1) = (f - \phi) \, B'(K_1). \tag{42}$$

Equation (42) identifies the paper's central wedge. When $\phi \neq f$, the environmental gradient embedded in decentralized allocation differs from the welfare-relevant one by $(f - \phi) \, B'(K_1)$. Measurement error therefore changes the marginal reallocation signal, not only the level of measured emissions. Put differently, the accounting rule determines the environmental base to which decentralized markets attach an implicit shadow price, thereby acting as an implicit Pigouvian pricing system over the reported emissions object.

The term $B'(K_1)$ is economically important. It measures how the economy's displacement capacity changes when capital is reallocated across sectors. This is not merely a mechanical recycling term: in the underlying equilibrium, it reflects both a composition effect and an equilibrium-response effect through changes in output and input demand.[7]

The next result states the paper's central implementation result. Even true consequential accounting need not implement the planner allocation, because the planner's marginal condition depends on an aggregate derivative of emissions rather than on contemporaneous firm-level accounting levels alone.

Proposition 3 (Static firm-level accounting cannot generally implement the planner allocation)**.** *Suppose decentralized agents allocate capital by pricing firm-level accounting objects of the form*

$$e_i^{\phi}(K_1) = e_i^{R}(K_1) - \phi\, b_i(K_1), \tag{43}$$

for some accounting coefficient ϕ and some firm-level accounting base $b_i(K_1)$.

Suppose also that true aggregate consequential emissions can be written as

$$E^*(K_1) = K_1\, e_1^*(K_1) + (1 - K_1)\, e_2^*(K_1), \tag{44}$$

with $e_1^*(K_1)$ and $e_2^*(K_1)$ differentiable. Then the planner-relevant aggregate gradient satisfies

$$E^{*\prime}(K_1) = e_1^*(K_1) - e_2^*(K_1) + K_1 \frac{de_1^*(K_1)}{dK_1} + (1 - K_1)\frac{de_2^*(K_1)}{dK_1}. \tag{45}$$

Hence, the planner's marginal condition depends not only on contemporaneous firm-level emissions levels, but also on equilibrium-response terms describing how sectoral emissions change when capital is reallocated.

Therefore, any decentralized allocation rule based solely on static firm-level accounting

[7] Appendix E decomposes $B'(K_1)$ into these two margins.

levels generally fails to implement the planner allocation. This failure persists even under true consequential accounting ($\phi = f$), except under knife-edge restrictions that collapse the aggregate marginal condition into a function of contemporaneous firm-level accounting levels alone.

Proposition 3 identifies a structural implementation failure. The issue is not only that decentralized agents may price the wrong coefficient on avoided emissions. More fundamentally, decentralized allocation is based on contemporaneous project-level accounting levels, whereas the planner's condition depends on an aggregate marginal response. Setting $\phi = f$ removes the coefficient wedge identified in Proposition 2, but it does not generally eliminate the implementation wedge, because equilibrium-response terms remain outside static firm-level accounts. Static accounting levels are therefore generally not sufficient statistics for the planner's derivative-based problem.

5 Comparative Statics and Welfare

This section derives the paper's main implications for sector rankings and welfare.

5.1 Ranking Reversals

The first implication is that displacement can overturn environmental rankings based on realized emissions alone. What matters for marginal reallocation is not only direct emissions, but the comparison between direct emissions and marginal displacement capacity.

The next result shows that environmental rankings depend on displacement as well as realized emissions, and that the two can point in opposite directions.

Proposition 4 (Threshold for environmental ranking reversals)**.** *Fix any benchmark allocation $K_1^\circ \in (0,1)$ such that $B'(K_1^\circ) \neq 0$. Define the threshold*

$$f^\circ := \frac{E^{R\prime}(K_1^\circ)}{B'(K_1^\circ)}. \tag{46}$$

The threshold f° is the avoided-emissions coefficient at which the realized-emissions margin and the displacement margin exactly offset each other.

Then

$$E^{*\prime}(K_1^\circ) = E^{R\prime}(K_1^\circ) - f\,B'(K_1^\circ) = B'(K_1^\circ)\big(f^\circ - f\big), \tag{47}$$

so that

$$\operatorname{sign}\big(E^{*\prime}(K_1^\circ)\big) = \operatorname{sign}\big(B'(K_1^\circ)\big)\,\operatorname{sign}\big(f^\circ - f\big). \tag{48}$$

In particular, if $B'(K_1^\circ) > 0$ corresponding to reallocations toward sector 1 increasing aggregate displacement capacity at the margin, then:

$$f > f^\circ \qquad \Longrightarrow \qquad E^{*\prime}(K_1^\circ) < 0, \tag{49}$$

so a marginal increase in K_1 reduces true aggregate emissions. If instead $f < f^\circ$, the environmental effect is reversed.

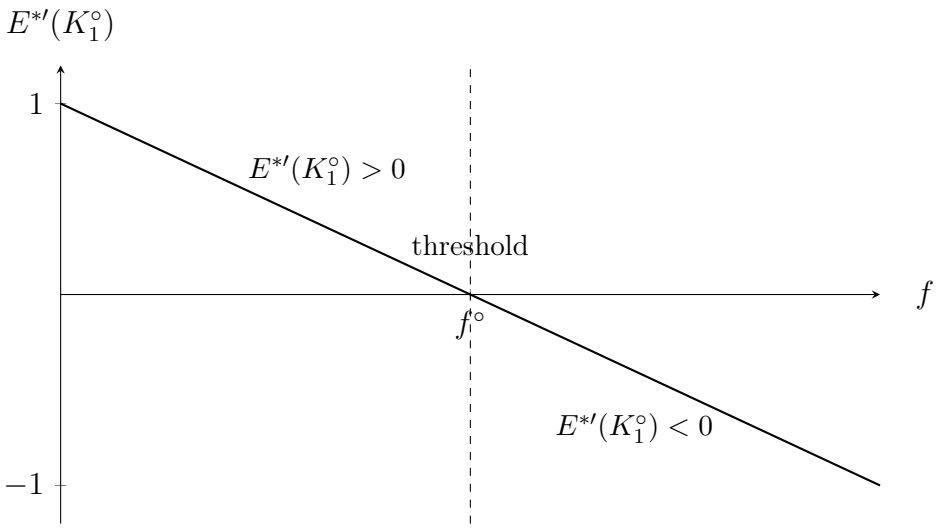
Proposition 4 is best understood as a ranking result. Under realized accounting, sectors are ranked only by the direct emissions consequences of reallocating capital. Under consequential accounting, sectors are ranked by the net effect of reallocation once displaced emissions are taken into account. The threshold f° marks the point at which these two ranking criteria switch places. In particular, a sector can appear environmentally worse under realized emissions and yet be socially preferable once its displacement capacity is sufficiently large. The relevant comparison is therefore not realized emissions alone, but realized emissions relative to marginal displacement.

Accounting standards matter because they determine whether capital markets respond only to the realized-emissions side of this comparison or to the displacement side.

Figure 3 illustrates Proposition 4. It shows how the sign of the true environmental gradient depends on the avoided-emissions factor f relative to the threshold f°. The

figure makes clear that the ranking of sectors can reverse when the avoided-emissions factor is sufficiently large. Below the threshold, realized emissions dominate the environmental comparison. Above the threshold, displacement dominates and the sign of the true environmental gradient changes. This threshold logic is the key to understanding when adjusted accounting improves welfare relative to realized accounting: welfare rises only when the induced capital reallocation moves in the planner-preferred direction.

Figure 3: Threshold for environmental ranking reversals



Notes: The figure plots the true environmental gradient $E^{*\prime}(K_1^\circ)$ as a function of the avoided-emissions factor f, for the case $B'(K_1^\circ) > 0$. The threshold is $f^\circ = \frac{E^{R\prime}(K_1^\circ)}{B'(K_1^\circ)}$. Below the threshold, the gradient is positive; above it, the gradient is negative.

5.2 Mismeasurement and Welfare

The second implication is that measured adjusted accounting can improve welfare relative to realized accounting only when the induced reallocation moves capital in the planner-preferred direction. Because true welfare depends on the allocation K_1 but not directly on the accounting parameter ϕ, the welfare effect of changing accounting works entirely through induced capital reallocation.

The result below is deliberately local: its purpose is to identify the sign of the welfare effect of moving away from realized accounting when the accounting-induced reallocation

is small.

Proposition 5 (Local welfare effect of adjusted accounting)**.** *Suppose that $\mathcal{W}$ is differentiable and that the decentralized equilibrium allocation K_1^ϕ is differentiable in ϕ around $\phi = 0$.*

Then

$$\frac{d}{d\phi}\mathcal{W}(K_1^\phi)\Big|_{\phi=0} = \mathcal{W}'(K_1^0)\,\frac{\partial K_1^\phi}{\partial \phi}\Big|_{\phi=0}. \tag{50}$$

Hence, if

$$\mathcal{W}'(K_1^0) > 0 \qquad \textit{and} \qquad b_1(K_1^0) > b_2(K_1^0), \tag{51}$$

then

$$\frac{d}{d\phi}\mathcal{W}(K_1^\phi)\Big|_{\phi=0} > 0, \tag{52}$$

so a marginal increase in the accounting weight on avoided emissions raises true welfare. Conversely, if $\mathcal{W}'(K_1^0) < 0$ while $b_1(K_1^0) > b_2(K_1^0)$, then increasing ϕ lowers welfare locally.

Equation (50) in Proposition 5 makes clear that accounting has no direct welfare effect in the model: its welfare consequences operate entirely through the capital-allocation response it induces. Proposition 4 identifies when accounting can reverse environmental rankings; Proposition 5 shows when such accounting-induced reallocation is actually welfare-improving. Proposition 5 gives a disciplined local version of the partial-correction logic. Adjusted accounting improves welfare when it changes capital allocation in the same direction as the planner would. But this is not automatic: if avoided-emissions credits are attached to the wrong margin, or if the planner locally prefers reallocation in the opposite direction, increasing the accounting weight on displacement can lower welfare instead.

A direct local implication is that, if the accounting-induced reallocation moves capital in the planner-preferred direction at $\phi = 0$, then sufficiently small positive avoided-emissions weights improve welfare relative to realized accounting. Measured adjusted accounting should therefore be understood as a potentially useful but imperfect proxy for true consequential accounting, rather than as a guarantee of welfare improvement.

6 Implications

The model has a direct empirical interpretation. In practice, capital markets do not allocate capital on the basis of the welfare-relevant externality itself, but on the basis of an emissions metric that is observed, reported, and embedded in financing conditions. The distinction developed in the model between realized emissions E^R, the welfare-relevant consequential externality E^*, and the priced accounting object E^ϕ therefore maps naturally into an empirical distinction between physical emissions inventories, welfare-relevant net emissions effects, and the emissions metric that enters decentralized financial decisions.

This distinction matters because these objects need not coincide. Standard emissions inventories are designed to record realized emissions within a reporting boundary. By contrast, avoided-emissions claims are inherently counterfactual: they depend on assumptions about what activity is displaced, at what emissions intensity, and under which system boundary. In the language of the model, this is the difference between the true consequential coefficient f, which governs the welfare-relevant displacement effect, and the implementable accounting coefficient $\widetilde{f}$, which governs the emissions object that investors and regulators can actually observe and price. This interpretation is consistent with recent practitioner guidance, which treats avoided emissions as conceptually distinct from the core GHG inventory footprint (WBCSD, 2025; PCAF, 2025).

The framework therefore provides a simple interpretation of why alternative accounting rules can generate different allocative outcomes even when markets process disclosed

information efficiently. If financing conditions respond primarily to realized emissions measured within narrow reporting boundaries, activities such as recycling may be penalized because they increase directly measured processing emissions. If instead financing conditions respond to an adjusted metric that credits displacement, the same activities may be rewarded. What changes across these cases is not technology, preferences, or informational efficiency. What changes is the environmental object that accounting makes available to be priced.

Although the model is written around circular production and recycling, the underlying logic extends beyond that application. Similar wedges arise whenever capital allocation is based on reportable firm-level metrics while the relevant environmental effect depends on system-wide counterfactual responses, for instance because cleaner activities displace dirtier incumbents, because transition activities raise direct emissions while reducing higher-emitting production elsewhere, or because environmental impact depends on baseline choice, market-mediated substitution, or rebound effects. More broadly, any carbon metric used in capital allocation embeds a choice of accounting base and, implicitly or explicitly, a coefficient on avoided emissions or displacement. Differences across reporting standards, methodologies, or data providers may therefore generate different capital allocations not because markets disagree about how to price a common signal, but because they are pricing different signals. Measurement differences are therefore not merely disclosure differences. They are differences in the allocative environment faced by decentralized capital markets.

Implications for empirical work. The model implies that empirical work on carbon metrics in finance should distinguish between two separate questions: whether markets price reported environmental information, and whether the reported information corresponds to the welfare-relevant environmental margin. These questions should not be conflated. Markets may price reported carbon data efficiently and still generate inefficient capital allocation when the reported metric is misaligned with the true consequential

externality.

Implications for standard-setting and sustainable finance. The model has implications for the design of accounting standards and sustainability-related financial practices. Once reported carbon metrics enter portfolio construction, lending decisions, mandates, or benchmarking rules, accounting conventions influence which environmental signal is transmitted into capital allocation. Better accounting may move priced metrics closer to the welfare-relevant margin and thereby improve decentralized allocation. But it does not, in general, eliminate the gap with the first best, because the planner's condition depends on aggregate marginal responses that are not contained in static firm-level accounting objects. Carbon accounting should therefore be evaluated not only as a disclosure technology, but as an allocative institution.

7 Conclusion

We study how carbon accounting affects capital allocation when firms generate both realized and avoided emissions. In the model, recycling raises realized emissions but displaces more carbon-intensive production elsewhere in the economy, so the environmental object priced by decentralized agents need not coincide with the welfare-relevant externality.

Three results follow. First, accounting is not allocatively neutral: changing the accounting rule changes equilibrium capital allocation because it changes the emissions object entering decentralized payoffs. Second, mismeasured avoided emissions create a wedge between the priced signal and the welfare-relevant one. Third, static firm-level accounting cannot generally implement the planner allocation, because the planner's condition depends on aggregate marginal responses that are not contained in contemporaneous firm-level emissions levels.

The paper thus clarifies both the role and the limit of carbon accounting in capital allocation. Better accounting can improve decentralized allocation by moving the priced

metric closer to the welfare-relevant margin. But accounting remains a partial instrument: even under true consequential accounting, static firm-level metrics generally do not contain the derivative information required for first-best implementation. More broadly, the paper highlights a general limitation of accounting-based allocation systems: whenever the welfare-relevant externality is an equilibrium object rather than a directly reportable firm-level attribute, measurement rules become part of the allocation mechanism itself.

A natural extension would be to allow the emissions intensity of displaced activity to vary with the scale of recycling. Another would be to embed the accounting rule in a richer asset-pricing environment with heterogeneous investors, mandates, or disclosure constraints.

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A Notation Summary

Table 3 summarizes the main notation used in the paper.

Table 3: Summary of notation used in the paper

Notation	**Interpretation**	**Location / role**
Allocation and equilibrium		
$i \in \{1, 2\}$	Sector index.	Model setup
$j \neq i$	The other sector.	Model setup
K_i	Capital allocated to sector i; equivalently, mass of active projects/firms in sector i.	Main text, Section 2
$K_1 + K_2 = 1$	Aggregate capital endowment normalized to one.	Main text, Section 2
K_1^{ϕ}	Decentralized equilibrium allocation to sector 1 under accounting rule ϕ.	Section 3
K_1^{FB}	First-best allocation chosen by the planner.	Section 4
K_1°	Benchmark allocation used for local ranking comparisons.	Proposition 4
$\pi_i(K_i)$	Financial profit per project in sector i.	Decentralized equilibrium
$\widetilde{\Pi}_i^{\phi}(K_1)$	Investor-perceived payoff of a representative project in sector i under accounting rule ϕ.	Decentralized allocation
$\tau \geq 0$	Reduced-form shadow cost assigned by investors to measured emissions.	Decentralized allocation
$G(K_1, \phi)$	Difference in perceived net payoffs between sectors 1 and 2 under accounting rule ϕ.	Equilibrium characterization

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Notation	Interpretation	Location / role
$G_{K_1}(K_1, \phi)$	Partial derivative of G with respect to K_1.	Proposition 1
$G_{\phi}(K_1, \phi)$	Partial derivative of G with respect to ϕ.	Proposition 1
Technology and recycling		
A_i	Sector-specific productivity parameter.	Technology block
$\alpha_i \in (0, 1)$	Output elasticity with respect to the materials bundle; ensures diminishing returns.	Technology block
y_i	Output per project in sector i.	Technology block
$Y_i = K_i\, y_i$	Aggregate output in sector i.	Technology block
m_i	Materials bundle used by a representative firm in sector i.	Technology block
n_i	Virgin material input used by sector i, purchased from sector j.	Technology block
r_i	Recycled input used per project in sector i.	Technology / accounting base
R_i	Aggregate recycled-input supply in sector i.	Recycling market
$\omega \in (0, 1)$	CES weight on virgin material in the materials bundle.	Technology block
$\sigma > 0$	Elasticity of substitution between virgin and recycled inputs in the CES materials bundle.	Technology block
q_i	Dual price index of the materials bundle in sector i.	Technology / cost minimization
p_i^c	Consumer price of final good i.	Goods market / household demand
p_i^r	Price of recycled input used by sector i.	Recycling market

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Notation	Interpretation	Location / role
Θ_i	Waste generated per project in sector i.	Technology block
Θ_i^{agg}	Aggregate waste generated in sector i.	Technology block
$\theta_i \in [0,1]$	Waste intensity of output in sector i.	Technology block
$\rho_i \in [0,1]$	Recovery rate converting waste into recycled material in sector i.	Recycling block
Households, welfare, and planner		
C_i	Consumption of final good i.	Household side / welfare
$U(C_1, C_2, E^*)$	Household utility as a function of consumption and the true externality.	Household side
W	Aggregate household income.	Household demand / appendix closed form
$\gamma_i \in (0,1)$	Cobb-Douglas expenditure share on good i.	Household side
$\gamma_1 + \gamma_2 = 1$	Total expenditure shares sum to one.	Household side
$\delta > 0$	Weight of environmental disutility in welfare.	Household side / planner
$\mathcal{W}(K_1)$	True social welfare as a function of the capital allocation.	Section 4
$\Lambda(K_1)$	Marginal effect of reallocation on the consumption component of log welfare.	Proposition 2
$\mathcal{M}(K_1; \phi)$	Measurement wedge between the true environmental gradient and the accounting-induced gradient.	Proposition 2

Continued on next page

Notation	Interpretation	Location / role
Emissions and accounting		
ε_i^{prod}	Emissions intensity of production in sector i.	Emissions block
ε_i^{waste}	Emissions intensity of unrecycled waste in sector i.	Emissions block
ε_i^{rec}	Emissions intensity of recycling activity in sector i.	Emissions block
$e_i^R(K_1)$	Realized emissions per project in sector i.	Emissions block
$E^R(K_1)$	Aggregate realized emissions in the economy.	Emissions block
f	True consequential emissions intensity of the marginal activity displaced by one unit of recycled input.	Emissions / welfare benchmark
$\widetilde{f}$	Measured avoided-emissions coefficient embedded in an implementable accounting rule.	Accounting block
ϕ	Generic accounting coefficient applied to recycled input; indexes the accounting regime.	Accounting block
$e_i^\phi(K_1)$	Firm-level accounting metric priced by decentralized agents under rule ϕ.	Accounting block
$E^\phi(K_1)$	Aggregate accounting metric priced by decentralized agents under rule ϕ.	Accounting block
$e_i^*(K_1)$	True consequential emissions per project in sector i, equal to realized emissions net of true displacement.	Welfare-relevant emissions
$E^*(K_1)$	True welfare-relevant aggregate externality, equal to realized aggregate emissions net of displaced emissions.	Welfare / planner
$b_i(K_1)$	Firm-level accounting base for avoided-emissions credit; in the core model, $b_i(K_1) = r_i(K_1)$.	Decentralized equilibrium

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Notation	Interpretation	Location / role
$B(K_1)$	Aggregate displacement capacity, equal to aggregate recycled-input use.	Emissions / displacement
$B'(K_1)$	Marginal change in aggregate displacement capacity when capital is reallocated.	Planner wedge / threshold result
f°	Threshold value of the true avoided-emissions coefficient at which realized-emissions and displacement margins exactly offset at K_1°.	Proposition 4
Appendix closed-form objects		
X_i	Aggregate intermediate demand for good i from the other sector in the appendix closed-form block.	Appendix D.3
K_i^*	Closed-form benchmark level of aggregate sectoral profit in sector i.	Appendix D.3
Z_i	Closed-form exponent appearing in the welfare decomposition; equal to $\frac{K_i^*}{1-\alpha_i}$.	Appendix D.4
B_i	Auxiliary closed-form object linking prices, productivity, and sector size in the welfare derivation.	Appendix D.4
β	Positive constant collecting technology and preference parameters in the closed-form welfare decomposition.	Appendix D.4

B Summary of Main Results

Table 4 summarizes the propositions and lemmas in the main text and appendices.

Table 4: Summary of propositions and lemmas

Result	**Location**	**Role**
Main-text results		
Proposition 1	Section 3	Establishes existence and uniqueness of decentralized equilibrium under accounting rule ϕ, and characterizes how equilibrium allocation changes with the accounting coefficient.
Proposition 2	Section 4	Derives the planner's first-order condition and identifies the measurement wedge between the welfare-relevant environmental gradient and the accounting-induced one.
Proposition 3	Section 4	Shows that static firm-level accounting cannot generally implement the planner allocation, because the planner's marginal condition depends on equilibrium-response terms absent from project-level accounting levels.
Proposition 4	Section 5	Defines the threshold value of the true avoided-emissions coefficient at which realized-emissions and consequential-emissions rankings switch locally.
Proposition 5	Section 5	Characterizes the local welfare effect of increasing the accounting weight on avoided emissions through the induced capital reallocation.

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Result	Location	Role
Appendix microfoundations and closed-form blocks		
Proposition 6	Appendix D.1	Derives the CES materials block, its dual price index, Hicksian input demands, optimal materials demand, output, and per-project operating profit.
Lemma 1	Appendix D.2	Derives Marshallian demand and indirect utility for the representative household under Cobb-Douglas preferences with environmental disutility.
Lemma 2	Appendix D.3	Solves the benchmark goods-market-clearing system and delivers the reduced-form profit-erosion expression $\pi_i(K_i) = \frac{K_i^*}{K_i}$.
Proposition 7	Appendix D.4	Provides a closed-form welfare decomposition separating productivity, allocative, and environmental components.
Appendix decompositions and auxiliary results		
Lemma 3	Appendix E	Decomposes the displacement gradient into a composition effect and an equilibrium-response effect, clarifying the structure of the welfare-relevant emissions gradient.

C Proofs for the Main Text

C.1 Proof of Proposition 1

Proof. The proof proceeds in three steps.

Step 1. Existence of an interior equilibrium.

Fix an accounting rule ϕ. By the conditions stated in Proposition 1, the function $K_1 \mapsto G(K_1, \phi)$ is continuously differentiable on $(0, 1)$, hence continuous on $(0, 1)$.

Moreover, the assumption imposes the boundary sign conditions

$$\lim_{K_1 \to 0} G(K_1, \phi) > 0, \qquad \lim_{K_1 \to 1} G(K_1, \phi) < 0. \tag{53}$$

Therefore, there exist $\underline{K}, \overline{K} \in (0, 1)$ with $\underline{K} < \overline{K}$ such that

$$G(\underline{K}, \phi) > 0, \qquad G(\overline{K}, \phi) < 0. \tag{54}$$

By continuity of $G(\cdot, \phi)$, the Intermediate Value Theorem implies that there exists at least one $K_1^\phi \in (\underline{K}, \overline{K}) \subset (0, 1)$ such that

$$G(K_1^\phi, \phi) = 0. \tag{55}$$

Hence an interior decentralized equilibrium exists.

Step 2. Uniqueness.

The same conditions also imply that, for every ϕ in the relevant range, the function $K_1 \mapsto G(K_1, \phi)$ is strictly decreasing on $(0, 1)$.

Suppose, toward a contradiction, that there exist two distinct interior equilibria $K_1^a \neq K_1^b$ satisfying

$$G(K_1^a, \phi) = 0, \qquad G(K_1^b, \phi) = 0. \tag{56}$$

Without loss of generality, let $K_1^a < K_1^b$. Since $G(\cdot, \phi)$ is strictly decreasing, it follows that

$$G(K_1^a, \phi) > G(K_1^b, \phi), \tag{57}$$

which contradicts the fact that both values equal zero. Therefore, the equilibrium is unique.

This proves the first claim: for each accounting rule ϕ, there exists a unique interior equilibrium allocation $K_1^\phi \in (0, 1)$ satisfying Equation (31).

Step 3. Comparative statics with respect to the accounting rule.

Assume now that G is continuously differentiable in both arguments. Define the equilibrium correspondence implicitly by

$$G(K_1^\phi, \phi) = 0. \tag{58}$$

Because the equilibrium is interior and unique, and because

$$G_{K_1}(K_1^\phi, \phi) < 0 \tag{59}$$

by strict monotonicity, the Implicit Function Theorem applies. Hence there exists a differentiable map $\phi \mapsto K_1^\phi$ in a neighborhood of any given ϕ, and

$$\frac{\partial K_1^\phi}{\partial \phi} = -\frac{G_\phi(K_1^\phi, \phi)}{G_{K_1}(K_1^\phi, \phi)}. \tag{60}$$

It therefore remains to compute G_ϕ. Recall from Equation (29) that

$$G(K_1, \phi) = \pi_1(K_1) - \pi_2(1 - K_1) - \tau\Big(e_1^\phi(K_1) - e_2^\phi(K_1)\Big), \tag{61}$$

and that

$$e_i^{\phi}(K_1) = e_i^R(K_1) - \phi\, b_i(K_1). \tag{62}$$

Substituting into G yields

$$G(K_1, \phi) = \pi_1(K_1) - \pi_2(1 - K_1) - \tau\Big[(e_1^R(K_1) - \phi\, b_1(K_1)) - (e_2^R(K_1) - \phi\, b_2(K_1))\Big] \tag{63}$$

$$= \pi_1(K_1) - \pi_2(1 - K_1) - \tau\big(e_1^R(K_1) - e_2^R(K_1)\big) + \tau\,\phi\big(b_1(K_1) - b_2(K_1)\big). \tag{64}$$

Differentiating with respect to ϕ gives

$$G_{\phi}(K_1, \phi) = \tau\big(b_1(K_1) - b_2(K_1)\big). \tag{65}$$

Substituting into Equation (60) yields

$$\frac{\partial K_1^{\phi}}{\partial \phi} = -\frac{\tau\big(b_1(K_1^{\phi}) - b_2(K_1^{\phi})\big)}{G_{K_1}(K_1^{\phi}, \phi)}, \tag{66}$$

which is exactly Equation (33).

Since $G_{K_1}(K_1^{\phi}, \phi) < 0$, the sign of $\frac{\partial K_1^{\phi}}{\partial \phi}$ coincides with the sign of $b_1(K_1^{\phi}) - b_2(K_1^{\phi})$. Therefore:

$$b_1(K_1^{\phi}) > b_2(K_1^{\phi}) \quad \Longrightarrow \quad \frac{\partial K_1^{\phi}}{\partial \phi} > 0, \tag{67}$$

and conversely if $b_1(K_1^{\phi}) < b_2(K_1^{\phi})$.

This completes the proof. □

C.2 Proof of Proposition 2

Proof. The proof proceeds in two parts. We first derive the planner's first-order condition. We then derive the measurement wedge.

Step 1. Rewriting the planner's problem in logs.

The planner chooses $K_1 \in (0,1)$ to maximize

$$\mathcal{W}(K_1) = \frac{C_1(K_1)^{\gamma_1}\, C_2(K_1)^{\gamma_2}}{\big(1 + E^*(K_1)\big)^{\delta}}. \tag{68}$$

Because the logarithm is strictly increasing, maximizing $\mathcal{W}(K_1)$ is equivalent to maximizing

$$\log \mathcal{W}(K_1) = \gamma_1 \log C_1(K_1) + \gamma_2 \log C_2(K_1) - \delta \log\big(1 + E^*(K_1)\big). \tag{69}$$

Step 2. Differentiating the log objective.

By assumption, $\mathcal{W}$ is continuously differentiable, so all derivatives below are well defined. Differentiating Equation (69) with respect to K_1 yields

$$\frac{d}{dK_1} \log \mathcal{W}(K_1) = \frac{d}{dK_1} \left[\gamma_1 \log C_1(K_1) + \gamma_2 \log C_2(K_1)\right] - \delta \frac{E^{*\prime}(K_1)}{1 + E^*(K_1)}. \tag{70}$$

By the definition of $\Lambda(K_1)$ in Equation (38), this becomes

$$\frac{d}{dK_1} \log \mathcal{W}(K_1) = \Lambda(K_1) - \delta \frac{E^{*\prime}(K_1)}{1 + E^*(K_1)}. \tag{71}$$

Step 3. First-order condition at an interior optimum.

Because K_1^{FB} is assumed to be an interior optimum, the standard first-order condition applies:

$$\frac{d}{dK_1} \log \mathcal{W}(K_1)\Big|_{K_1 = K_1^{FB}} = 0. \tag{72}$$

Substituting Equation (71) gives

$$\Lambda(K_1^{FB}) = \delta \frac{E^{*\prime}(K_1^{FB})}{1 + E^*(K_1^{FB})}, \tag{73}$$

which is Equation (39).

Step 4. Derivation of the true environmental gradient.

By Equation (36), true aggregate emissions satisfy

$$E^*(K_1) = E^R(K_1) - f\,B(K_1). \tag{74}$$

Since f is constant in the core model, differentiating with respect to K_1 yields

$$E^{*\prime}(K_1) = E^{R\prime}(K_1) - f\,B'(K_1), \tag{75}$$

which is Equation (40).

Step 5. Derivation of the accounting-induced gradient.

Under accounting rule ϕ, aggregate priced emissions are

$$E^{\phi}(K_1) = E^R(K_1) - \phi\,B(K_1). \tag{76}$$

Again, treating ϕ as a constant coefficient, differentiation yields

$$E^{\phi\prime}(K_1) = E^{R\prime}(K_1) - \phi\,B'(K_1), \tag{77}$$

which is Equation (41).

Step 6. Measurement wedge.

By definition,

$$\mathcal{M}(K_1;\phi) := E^{*\prime}(K_1) - E^{\phi\prime}(K_1). \tag{78}$$

Substituting the previous two expressions gives

$$\mathcal{M}(K_1;\phi) = \big(E^{R\prime}(K_1) - f\,B'(K_1)\big) - \big(E^{R\prime}(K_1) - \phi\,B'(K_1)\big) \tag{79}$$

$$= (f-\phi)B'(K_1), \tag{80}$$

which is Equation (42).

This completes the proof. □

C.3 Proof of Proposition 3

Proof. The proof compares the object priced by decentralized agents with the marginal object required by the planner.

Step 1. Decentralized allocation prices static firm-level accounting levels.

By assumption, decentralized agents evaluate projects using firm-level accounting objects of the form

$$e_i^{\phi}(K_1) = e_i^{R}(K_1) - \phi\, b_i(K_1). \tag{81}$$

Hence decentralized capital allocation is based on contemporaneous project-level accounting levels.

Step 2. The planner internalizes an aggregate marginal condition.

By Proposition 2, an interior planner allocation satisfies

$$\Lambda(K_1^{FB}) = \delta \frac{E^{*\prime}(K_1^{FB})}{1 + E^{*}(K_1^{FB})}. \tag{82}$$

Thus the planner's problem depends on the aggregate marginal emissions response $E^{*\prime}(K_1)$ rather than on contemporaneous firm-level accounting levels alone.

Step 3. Decomposition of the planner-relevant aggregate gradient.

Using

$$E^*(K_1) = K_1\, e_1^*(K_1) + (1 - K_1)\, e_2^*(K_1), \tag{83}$$

differentiate each term with respect to K_1. For sector 1,

$$\frac{d}{dK_1}\Big[K_1\, e_1^*(K_1)\Big] = e_1^*(K_1) + K_1 \frac{de_1^*(K_1)}{dK_1}. \tag{84}$$

For sector 2,

$$\frac{d}{dK_1}\Big[(1 - K_1)\, e_2^*(K_1)\Big] = -\, e_2^*(K_1) + (1 - K_1)\frac{de_2^*(K_1)}{dK_1}. \tag{85}$$

Adding the two terms yields

$$E^{*\prime}(K_1) = e_1^*(K_1) - e_2^*(K_1) + K_1 \frac{de_1^*(K_1)}{dK_1} + (1 - K_1)\frac{de_2^*(K_1)}{dK_1}. \tag{86}$$

Step 4. Static firm-level accounting levels do not generally encode equilibrium responses.

Equation (86) shows that the planner-relevant aggregate gradient contains two components: a contemporaneous level difference, $e_1^*(K_1) - e_2^*(K_1)$, and equilibrium-response terms, $K_1 \frac{de_1^*(K_1)}{dK_1}$ and $(1 - K_1)\frac{de_2^*(K_1)}{dK_1}$.

A decentralized accounting rule based solely on static firm-level objects of the form $e_i^{\phi}(K_1)$ can price contemporaneous project-level emissions levels, but does not generally encode how sectoral emissions change when capital is reallocated in equilibrium. Those derivative terms depend on equilibrium output, prices, and recycling responses, and therefore are not, in general, recoverable from contemporaneous accounting levels alone.

Step 5. Correct calibration does not generally restore implementation.

Setting $\phi = f$ aligns the accounting coefficient with the true consequential displacement parameter. This removes the coefficient wedge identified in Proposition 2. However,

it does not eliminate the equilibrium-response terms in Equation (86). Hence matching the coefficient on displacement is not sufficient, in general, to make the decentralized allocation satisfy the planner's marginal condition.

Step 6. Generic failure of implementation.

Therefore, any decentralized allocation rule based solely on static firm-level accounting levels generally fails to implement the planner allocation. Implementation can succeed only under knife-edge restrictions under which the equilibrium-response terms either vanish or are exactly summarized by contemporaneous firm-level accounting levels, so that the planner-relevant aggregate derivative collapses to a function of static accounting levels alone.

This completes the proof. □

C.4 Proof of Proposition 4

Proof. The proof is immediate once the true emissions gradient is written in terms of realized emissions and displacement.

Step 1. Start from the definition of the true emissions gradient.

By Proposition 2,

$$E^{*\prime}(K_1) = E^{R\prime}(K_1) - f\, B'(K_1). \tag{87}$$

Evaluating this identity at the benchmark allocation K_1° gives

$$E^{*\prime}(K_1^\circ) = E^{R\prime}(K_1^\circ) - f\, B'(K_1^\circ). \tag{88}$$

Step 2. Define the threshold.

Since the proposition assumes $B'(K_1^\circ) \neq 0$, the ratio

$$f^\circ := \frac{E^{R\prime}(K_1^\circ)}{B'(K_1^\circ)} \tag{89}$$

is well defined.

Substituting $E^{R\prime}(K_1^\circ) = f^\circ B'(K_1^\circ)$ into Equation (88) yields

$$E^{*\prime}(K_1^\circ) = f^\circ B'(K_1^\circ) - f B'(K_1^\circ) \tag{90}$$

$$= B'(K_1^\circ)\big(f^\circ - f\big). \tag{91}$$

Step 3. Derive the sign formula.

Because $E^{*\prime}(K_1^\circ)$ is the product of $B'(K_1^\circ)$ and $(f^\circ - f)$, its sign is

$$\operatorname{sign}\big(E^{*\prime}(K_1^\circ)\big) = \operatorname{sign}\big(B'(K_1^\circ)\big)\ \operatorname{sign}\big(f^\circ - f\big), \tag{92}$$

which is exactly Equation (48).

Step 4. Interpretation when $B'(K_1^\circ) > 0$.

If $B'(K_1^\circ) > 0$, then the sign of $E^{*\prime}(K_1^\circ)$ is the sign of $(f^\circ - f)$. Hence:

$$f > f^\circ \quad \Longrightarrow \quad f^\circ - f < 0 \quad \Longrightarrow \quad E^{*\prime}(K_1^\circ) < 0. \tag{93}$$

Thus a marginal increase in K_1 reduces true aggregate emissions. Conversely, if $f < f^\circ$, then $E^{*\prime}(K_1^\circ) > 0$, so the environmental effect is reversed.

This completes the proof. □

C.5 Proof of Proposition 5

Proof. The proof is an application of the chain rule.

Step 1. Welfare depends on accounting only through equilibrium allocation.

The true welfare function is

$$\mathcal{W}(K_1) = \frac{C_1(K_1)^{\gamma_1}\, C_2(K_1)^{\gamma_2}}{(1 + E^*(K_1))^{\delta}}. \tag{94}$$

Importantly, the accounting parameter ϕ does not appear directly in this expression. Its only effect on welfare operates indirectly through the equilibrium allocation K_1^ϕ.

Therefore, the composite map of interest is

$$\phi \mapsto \mathcal{W}(K_1^\phi). \tag{95}$$

Step 2. Differentiate by the chain rule.

By assumption, the equilibrium allocation K_1^ϕ is differentiable in ϕ around $\phi = 0$. Assuming that $\mathcal{W}$ is differentiable in its argument, the chain rule gives

$$\frac{d}{d\phi}\mathcal{W}(K_1^\phi) = \mathcal{W}'(K_1^\phi)\,\frac{\partial K_1^\phi}{\partial \phi}. \tag{96}$$

Evaluating at $\phi = 0$ yields

$$\frac{d}{d\phi}\mathcal{W}(K_1^\phi)\Big|_{\phi=0} = \mathcal{W}'(K_1^0)\,\frac{\partial K_1^\phi}{\partial \phi}\Big|_{\phi=0}, \tag{97}$$

which is Equation (50).

Step 3. Sign of the local welfare effect.

By Proposition 1,

$$\frac{\partial K_1^\phi}{\partial \phi} = -\frac{\tau\big(b_1(K_1^\phi) - b_2(K_1^\phi)\big)}{G_{K_1}(K_1^\phi, \phi)}. \tag{98}$$

Evaluating at $\phi = 0$ gives

$$\frac{\partial K_1^\phi}{\partial \phi}\Big|_{\phi=0} = -\frac{\tau\big(b_1(K_1^0) - b_2(K_1^0)\big)}{G_{K_1}(K_1^0, 0)}. \tag{99}$$

Since $\tau \geq 0$ and $G_{K_1}(K_1^0, 0) < 0$ by assumption, it follows that

$$b_1(K_1^0) > b_2(K_1^0) \quad \Longrightarrow \quad \frac{\partial K_1^\phi}{\partial \phi}\Big|_{\phi=0} > 0. \tag{100}$$

Hence, if in addition

$$\mathcal{W}'(K_1^0) > 0, \tag{101}$$

then both terms on the right-hand side of Equation (50) are positive, so

$$\frac{d}{d\phi}\mathcal{W}(K_1^\phi)\Big|_{\phi=0} > 0. \tag{102}$$

Therefore, a marginal increase in the accounting weight on avoided emissions raises true welfare locally.

Likewise, if

$$\mathcal{W}'(K_1^0) < 0 \qquad \text{and} \qquad b_1(K_1^0) > b_2(K_1^0), \tag{103}$$

then

$$\frac{d}{d\phi}\mathcal{W}(K_1^\phi)\Big|_{\phi=0} < 0, \tag{104}$$

so increasing ϕ lowers welfare locally.

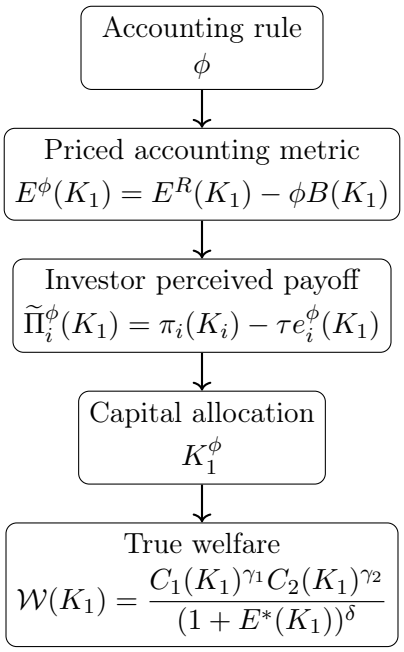
This proves the proposition. □

D Microfoundation and Closed-Form Equilibrium Blocks

This appendix restates the closed-form equilibrium blocks underlying the reduced-form objects used in the main text. It provides the corresponding derivations for the production block, household demand, sectoral profitability, and welfare.

Figure 4 summarizes the mechanism studied in the paper.

Figure 4: Mechanism of the paper

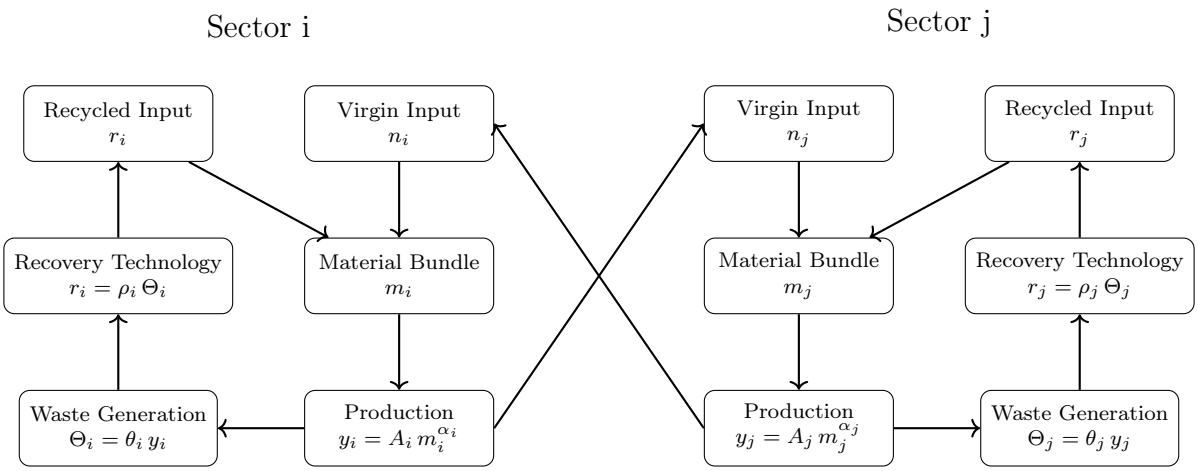


Notes: The accounting rule ϕ determines the environmental object priced by decentralized investors. This priced metric affects perceived payoffs and therefore equilibrium capital allocation. Welfare, however, depends on the true consequential externality $E^*(K_1)$ rather than on the priced metric $E^{\phi}(K_1)$.

Figure 4 highlights the central wedge: decentralized agents respond to the priced accounting metric $E^{\phi}(K_1)$, whereas welfare depends on the true consequential externality $E^*(K_1)$.

Figure 5 provides a schematic overview of the circular production structure that underlies the microfoundation.

Figure 5: Circular production structure



Notes: Each sector produces a final good using a materials bundle that combines a virgin input purchased from the other sector with a recycled input sourced from a competitive recycling market. Production generates waste at the sector level, a fraction of which is recovered and supplied back as recycled material. Circularity arises because each sector both supplies the other sector's virgin input and generates the waste base from which its recycled-input market is supplied.

D.1 CES input block, composite pricing, and operating profits

We begin with the firm's static problem. The next result derives the CES materials block, the associated dual price index, and the corresponding expressions for materials demand, output, and operating profit.

Proposition 6 (CES input block and composite input pricing)**.** *Consider a representative firm in sector $i \in \{1, 2\}$ with production technology*

$$y_i = A_i m_i^{\alpha_i}, \qquad A_i > 0, \qquad \alpha_i \in (0, 1), \tag{105}$$

where the materials bundle m_i is a CES composite of virgin input n_i and recycled input r_i:

$$m_i = \left(\omega n_i^{\frac{\sigma-1}{\sigma}} + (1-\omega) r_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \qquad \omega \in (0, 1), \qquad \sigma > 0, \qquad \sigma \neq 1. \tag{106}$$

Let p_i^c denote the output price in sector i, let p_j^c denote the price of the virgin input purchased from sector $j \neq i$, and let p_i^r denote the price of recycled input used by sector i. Then:

(i) The dual price index of the materials bundle is

$$q_i = \left[\omega^\sigma (p_j^c)^{1-\sigma} + (1-\omega)^\sigma (p_i^r)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{107}$$

(ii) Conditional on m_i, the Hicksian input demands are

$$n_i = m_i\, \omega^\sigma \left(\frac{p_j^c}{q_i}\right)^{-\sigma}, \tag{108}$$

$$r_i = m_i\, (1-\omega)^\sigma \left(\frac{p_i^r}{q_i}\right)^{-\sigma}. \tag{109}$$

(iii) The firm's optimal materials-bundle demand solves

$$\max_{m_i \geq 0} \; p_i^c A_i m_i^{\alpha_i} - q_i m_i, \tag{110}$$

and is given by

$$m_i = \left(\frac{\alpha_i A_i p_i^c}{q_i}\right)^{\frac{1}{1-\alpha_i}}. \tag{111}$$

(iv) Output is

$$y_i = A_i \left(\frac{\alpha_i A_i p_i^c}{q_i}\right)^{\frac{\alpha_i}{1-\alpha_i}}. \tag{112}$$

(v) Per-project operating profit is

$$\pi_i = (1-\alpha_i)\, q_i \left(\frac{\alpha_i A_i p_i^c}{q_i}\right)^{\frac{1}{1-\alpha_i}}. \tag{113}$$

D.2 Household demand

We next turn to the household side. The following lemma states the standard Marshallian demand system and indirect utility implied by the preference specification in the main text.

Lemma 1 (Marshallian demand and indirect utility)**.** *Consider the representative household with preferences*

$$U(C_1, C_2, E^*) = \frac{C_1^{\gamma_1} C_2^{\gamma_2}}{(1+E^*)^{\delta}}, \qquad \gamma_1, \gamma_2 \in (0,1), \qquad \gamma_1 + \gamma_2 = 1, \tag{114}$$

and budget constraint

$$p_1^c C_1 + p_2^c C_2 \leq W, \tag{115}$$

where W denotes aggregate household income. Taking E^* as given, the Marshallian demand for good $i \in \{1,2\}$ is

$$C_i = \frac{\gamma_i W}{p_i^c}. \tag{116}$$

Indirect utility is

$$U^*(W; p_1^c, p_2^c, E^*) = \frac{\left(\frac{\gamma_1}{p_1^c}\right)^{\gamma_1} \left(\frac{\gamma_2}{p_2^c}\right)^{\gamma_2}}{(1+E^*)^{\delta}} W. \tag{117}$$

D.3 Goods market clearing and profit erosion

The next result derives the closed-form relation between goods-market clearing and sectoral profitability. In particular, it yields the benchmark profit-erosion expression used to motivate the reduced-form profit functions in the main text.

Lemma 2 (Goods market clearing and closed-form profit erosion)**.** *Suppose that all firms*

in sector $i \in \{1,2\}$ are identical, with production technology

$$y_i = A_i\, m_i^{\alpha_i}, \qquad \alpha_i \in (0,1), \tag{118}$$

and let sector sizes satisfy

$$K_1 + K_2 = 1. \tag{119}$$

Let Y_i denote aggregate output of good i, C_i aggregate final consumption of good i, and let sector $j \neq i$ use good i as its virgin material input. Then goods-market clearing requires

$$Y_i = C_i + K_j\, n_j. \tag{120}$$

In the closed-form benchmark inherited from the microfounded two-sector block, per-project profit in sector i can be written as

$$\pi_i(K_i) = \frac{K_i^*}{K_i}, \tag{121}$$

where

$$K_i^* := \frac{(\gamma_i + \alpha_j\,\gamma_j)(1-\alpha_i)}{1-\alpha_i\alpha_j}, \qquad j \neq i. \tag{122}$$

Hence aggregate sectoral profit is constant and equal to K_i^*, while per-project profit is strictly decreasing in sector size.

D.4 Closed-form welfare decomposition

We finally combine the previous building blocks to obtain a closed-form welfare decomposition. The resulting expression separates the productivity, allocative, and environmental components of welfare.

Proposition 7 (Closed-form welfare decomposition)**.** *Suppose that the closed-form equilibrium representation of Lemma 2 holds. Let sector sizes satisfy* $K_1 + K_2 = 1$*, and let true firm-level consequential emissions be*

$$e_i^*(K_1) = y_i(K_1)\left[\varepsilon_i^{prod} + \varepsilon_i^{waste}\,\theta_i + \left(\varepsilon_i^{rec} - \varepsilon_i^{waste} - f\right)\rho_i\,\theta_i\right]. \tag{123}$$

Then aggregate consequential emissions are

$$E^*(K_1) = K_1\,e_1^*(K_1) + (1 - K_1)\,e_2^*(K_1). \tag{124}$$

Under the closed-form benchmark, true welfare can be written as

$$\mathcal{W}(K_1) = \beta\,\frac{A_1^{Z_1}\,A_2^{Z_2}}{(1 + E^*(K_1))^{\delta}}\left(\frac{K_1}{K_1^*}\right)^{K_1^*}\left(\frac{K_2}{K_2^*}\right)^{K_2^*}, \tag{125}$$

where

$$Z_i := \frac{K_i^*}{1 - \alpha_i}, \tag{126}$$

and $\beta > 0$ *collects technological and preference parameters.*

This decomposition separates welfare into three components: a productivity term, a misallocation term, and an environmental term driven by the true consequential externality $E^*(K_1)$*.*

Together, these results provide the closed-form benchmark underlying the reduced-form representation used in the main text.

D.5 Proofs

D.5.1 Proof of Proposition 6

Proof. The proof proceeds in two steps. We first solve the firm's inner cost-minimization problem for the composition of the materials bundle. We then solve the firm's outer problem for total materials demand, output, and operating profit.

Step 1. Inner CES cost minimization and composite input pricing.

Fix a target materials bundle $m_i > 0$ and input prices $(p_j^c, p_i^r) \gg 0$. The representative firm in sector i solves

$$\min_{n_i, r_i \geq 0} \; p_j^c \, n_i + p_i^r \, r_i \qquad \text{s.t.} \qquad \left(\omega \, n_i^{\frac{\sigma-1}{\sigma}} + (1-\omega) \, r_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \geq m_i. \tag{127}$$

By monotonicity of the CES aggregator, the constraint binds at the optimum. Let

$$\rho := \frac{\sigma - 1}{\sigma}, \qquad \rho < 1, \qquad \sigma \neq 1.$$

Then the problem can be rewritten as

$$\min_{n_i, r_i \geq 0} \; p_j^c \, n_i + p_i^r \, r_i \qquad \text{s.t.} \qquad \omega \, n_i^\rho + (1-\omega) \, r_i^\rho = m_i^\rho. \tag{128}$$

The Lagrangian is

$$\mathcal{L} = p_j^c \, n_i + p_i^r \, r_i + \lambda \left[m_i^\rho - \omega \, n_i^\rho - (1-\omega) \, r_i^\rho \right], \qquad \lambda > 0. \tag{129}$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial n_i} = p_j^c - \lambda\,\omega\,\rho\,n_i^{\rho-1} = 0, \tag{130}$$

$$\frac{\partial \mathcal{L}}{\partial r_i} = p_i^r - \lambda\,(1-\omega)\,\rho\,r_i^{\rho-1} = 0, \tag{131}$$

$$\omega\,n_i^\rho + (1-\omega)\,r_i^\rho = m_i^\rho. \tag{132}$$

Dividing Equation (130) by Equation (131) yields

$$\frac{p_j^c}{p_i^r} = \frac{\omega}{1-\omega}\left(\frac{n_i}{r_i}\right)^{\rho-1}. \tag{133}$$

Since

$$\rho - 1 = -\frac{1}{\sigma},$$

we obtain

$$\frac{p_j^c}{p_i^r} = \frac{\omega}{1-\omega}\left(\frac{n_i}{r_i}\right)^{-\frac{1}{\sigma}}. \tag{134}$$

Next, multiply Equation (130) by n_i and Equation (131) by r_i, then add the two expressions:

$$p_j^c\,n_i + p_i^r\,r_i = \lambda\,\rho\left[\omega\,n_i^\rho + (1-\omega)\,r_i^\rho\right]. \tag{135}$$

Using the binding constraint Equation (132), this becomes

$$p_j^c\,n_i + p_i^r\,r_i = \lambda\,\rho\,m_i^\rho. \tag{136}$$

Let q_i denote the unit expenditure function associated with one unit of the materials

bundle:

$$q_i := \min_{n_i, r_i \geq 0} \left\{ p_j^c \, n_i + p_i^r \, r_i : \left(\omega \, n_i^{\frac{\sigma-1}{\sigma}} + (1-\omega) \, r_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} = 1 \right\}. \tag{137}$$

By homogeneity of degree one of the CES aggregator, total cost is $q_i \, m_i$, so Equation (136) implies

$$q_i \, m_i = \lambda \, \rho \, m_i^{\rho} \qquad \Longrightarrow \qquad \lambda = \frac{q_i}{\rho} \, m_i^{1-\rho}. \tag{138}$$

Substituting Equation (138) into Equation (130) yields

$$p_j^c = \omega \, q_i \, m_i^{1-\rho} \, n_i^{\rho-1}. \tag{139}$$

Using again $\rho - 1 = -\frac{1}{\sigma}$, we obtain

$$\frac{p_j^c}{q_i} = \omega \left(\frac{n_i}{m_i} \right)^{-1/\sigma} \qquad \Longrightarrow \qquad \frac{n_i}{m_i} = \omega^{\sigma} \left(\frac{p_j^c}{q_i} \right)^{-\sigma}. \tag{140}$$

Hence

$$n_i = m_i \, \omega^{\sigma} \left(\frac{p_j^c}{q_i} \right)^{-\sigma}. \tag{141}$$

Similarly, substituting Equation (138) into Equation (131) gives

$$\frac{p_i^r}{q_i} = (1-\omega) \left(\frac{r_i}{m_i} \right)^{-\frac{1}{\sigma}} \qquad \Longrightarrow \qquad \frac{r_i}{m_i} = (1-\omega)^{\sigma} \left(\frac{p_i^r}{q_i} \right)^{-\sigma}, \tag{142}$$

so

$$r_i = m_i \, (1-\omega)^{\sigma} \left(\frac{p_i^r}{q_i} \right)^{-\sigma}. \tag{143}$$

To derive the dual price index, use the identity

$$q_i\, m_i = p_j^c\, n_i + p_i^r\, r_i,$$

and substitute Equations (141)–(143):

$$q_i\, m_i = p_j^c\, m_i\, \omega^\sigma \left(\frac{p_j^c}{q_i}\right)^{-\sigma} + p_i^r\, m_i\, (1-\omega)^\sigma \left(\frac{p_i^r}{q_i}\right)^{-\sigma} \tag{144}$$

$$= m_i\, q_i^\sigma \left[\omega^\sigma (p_j^c)^{1-\sigma} + (1-\omega)^\sigma (p_i^r)^{1-\sigma}\right]. \tag{145}$$

Dividing by $m_i\, q_i^\sigma$ gives

$$q_i^{1-\sigma} = \omega^\sigma (p_j^c)^{1-\sigma} + (1-\omega)^\sigma (p_i^r)^{1-\sigma}, \tag{146}$$

hence

$$q_i = \left[\omega^\sigma (p_j^c)^{1-\sigma} + (1-\omega)^\sigma (p_i^r)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{147}$$

This proves items (i) and (ii).

Step 2. Outer problem: optimal materials demand, output, and profit.

Given the dual unit cost q_i, the representative firm solves

$$\max_{m_i \geq 0} \;\; p_i^c\, A_i\, m_i^{\alpha_i} - q_i\, m_i. \tag{148}$$

The first-order condition is

$$p_i^c\, A_i\, \alpha_i\, m_i^{\alpha_i - 1} = q_i. \tag{149}$$

Rearranging gives

$$m_i^{1-\alpha_i} = \frac{\alpha_i \, A_i \, p_i^c}{q_i}, \tag{150}$$

so optimal materials demand is

$$m_i = \left(\frac{\alpha_i \, A_i \, p_i^c}{q_i} \right)^{\frac{1}{1-\alpha_i}}. \tag{151}$$

This proves item (iii).

Substituting Equation (151) into the production function yields

$$y_i = A_i \, m_i^{\alpha_i} = A_i \left(\frac{\alpha_i \, A_i \, p_i^c}{q_i} \right)^{\frac{\alpha_i}{1-\alpha_i}}, \tag{152}$$

which proves item (iv).

Finally, profit is

$$\pi_i = p_i^c \, y_i - q_i \, m_i. \tag{153}$$

Using Equation (149), we have

$$q_i \, m_i = p_i^c \, A_i \, \alpha_i \, m_i^{\alpha_i} = \alpha_i \, p_i^c \, y_i,$$

so

$$\pi_i = (1 - \alpha_i) \, p_i^c \, y_i. \tag{154}$$

Substituting Equation (152) gives

$$\pi_i = (1-\alpha_i) p_i^c \, A_i \left(\frac{\alpha_i \, A_i \, p_i^c}{q_i} \right)^{\frac{\alpha_i}{1-\alpha_i}} \tag{155}$$

$$= (1-\alpha_i) \, q_i \, \left(\frac{\alpha_i \, A_i \, p_i^c}{q_i} \right)^{\frac{1}{1-\alpha_i}}, \tag{156}$$

which proves item (v).

This completes the proof. □

D.5.2 Proof of Lemma 1

Proof. Consider the representative household's problem:

$$\max_{C_1, C_2} \; \frac{C_1^{\gamma_1} \, C_2^{\gamma_2}}{(1+E^*)^{\delta}} \qquad \text{s.t.} \qquad p_1^c \, C_1 + p_2^c \, C_2 \leq W. \tag{157}$$

The Lagrangian is

$$\mathcal{L} = \frac{C_1^{\gamma_1} \, C_2^{\gamma_2}}{(1+E^*)^{\delta}} + \lambda \left(W - p_1^c \, C_1 - p_2^c \, C_2 \right). \tag{158}$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial C_1} = \frac{\gamma_1 \, C_1^{\gamma_1 - 1} \, C_2^{\gamma_2}}{(1+E^*)^{\delta}} - \lambda \, p_1^c = 0, \tag{159}$$

$$\frac{\partial \mathcal{L}}{\partial C_2} = \frac{\gamma_2 \, C_1^{\gamma_1} \, C_2^{\gamma_2 - 1}}{(1+E^*)^{\delta}} - \lambda \, p_2^c = 0, \tag{160}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = W - p_1^c \, C_1 - p_2^c \, C_2 = 0. \tag{161}$$

Dividing Equation (159) by Equation (160) gives

$$\frac{\gamma_1}{\gamma_2} \frac{C_2}{C_1} = \frac{p_1^c}{p_2^c}, \tag{162}$$

which implies

$$C_2 = \frac{\gamma_2 p_1^c}{\gamma_1 p_2^c} C_1. \tag{163}$$

Substituting Equation (163) into the budget constraint Equation (161), we obtain

$$p_1^c\, C_1 + p_2^c \left(\frac{\gamma_2\, p_1^c}{\gamma_1\, p_2^c} C_1 \right) = W, \tag{164}$$

that is,

$$p_1^c C_1 \left(1 + \frac{\gamma_2}{\gamma_1} \right) = W. \tag{165}$$

Since $\gamma_1 + \gamma_2 = 1$, we have

$$1 + \frac{\gamma_2}{\gamma_1} = \frac{\gamma_1 + \gamma_2}{\gamma_1} = \frac{1}{\gamma_1}.$$

Therefore

$$p_1^c\, C_1\, \frac{1}{\gamma_1} = W, \tag{166}$$

so

$$C_1 = \frac{\gamma_1\, W}{p_1^c}. \tag{167}$$

By symmetry,

$$C_2 = \frac{\gamma_2\, W}{p_2^c}. \tag{168}$$

This proves the Marshallian demand system.

We now derive indirect utility. Substitute Equations(167)–(168) into the utility func-

tion:

$$U^*(W; p_1^c, p_2^c, E^*) = \frac{\left(\frac{\gamma_1 W}{p_1^c}\right)^{\gamma_1} \left(\frac{\gamma_2 W}{p_2^c}\right)^{\gamma_2}}{(1+E^*)^{\delta}} \tag{169}$$

$$= \frac{\gamma_1^{\gamma_1}\, \gamma_2^{\gamma_2}\, W^{\gamma_1+\gamma_2}}{(p_1^c)^{\gamma_1}\, (p_2^c)^{\gamma_2}\, (1+E^*)^{\delta}}. \tag{170}$$

Using $\gamma_1 + \gamma_2 = 1$, this simplifies to

$$U^*(W; p_1^c, p_2^c, E^*) = \frac{\left(\frac{\gamma_1}{p_1^c}\right)^{\gamma_1} \left(\frac{\gamma_2}{p_2^c}\right)^{\gamma_2}}{(1+E^*)^{\delta}}\, W. \tag{171}$$

This completes the proof. □

D.5.3 Proof of Lemma 2

Proof. **Step 1. Goods-market clearing.**

Let Y_i denote aggregate output of sector i, C_i aggregate final consumption of good i, and let sector $j \neq i$ use good i as virgin material input. Goods-market clearing requires

$$Y_i = C_i + K_j\, n_j. \tag{172}$$

Under the benchmark closed-form block, virgin material demand by a representative firm in sector j is proportional to output through cost minimization and the firm's first-order condition. In particular, the aggregate intermediate demand for good i by sector j is

$$X_i := K_j\, n_j = \alpha_j\, \frac{p_j^c\, Y_j}{p_i^c}. \tag{173}$$

Using household demand from Lemma 1,

$$C_i = \frac{\gamma_i\, W}{p_i^c}. \tag{174}$$

Substituting Equations (173)–(174) into Equation (172) yields

$$Y_i = \frac{\gamma_i W}{p_i^c} + \alpha_j \frac{p_j^c Y_j}{p_i^c}. \tag{175}$$

Similarly,

$$Y_j = \frac{\gamma_j W}{p_j^c} + \alpha_i \frac{p_i^c Y_i}{p_j^c}. \tag{176}$$

Step 2. Sectoral revenues.

Define sectoral revenues

$$Z_i := p_i^c Y_i, \qquad Z_j := p_j^c Y_j. \tag{177}$$

Multiplying Equation (175) by p_i^c and Equation (176) by p_j^c gives

$$Z_i = \gamma_i W + \alpha_j Z_j, \tag{178}$$

$$Z_j = \gamma_j W + \alpha_i Z_i. \tag{179}$$

In the closed-form benchmark, aggregate household income equals aggregate profits. Since only relative allocations matter here, we normalize

$$W = 1.$$

Then Equations (178)–(179) become

$$Z_i = \gamma_i + \alpha_j Z_j, \tag{180}$$

$$Z_j = \gamma_j + \alpha_i Z_i. \tag{181}$$

Substitute the second equation into the first:

$$Z_i = \gamma_i + \alpha_j(\gamma_j + \alpha_i\, Z_i) \tag{182}$$

$$= \gamma_i + \alpha_j\, \gamma_j + \alpha_i\, \alpha_j\, Z_i. \tag{183}$$

Hence

$$(1 - \alpha_i\, \alpha_j)\, Z_i = \gamma_i + \alpha_j\, \gamma_j, \tag{184}$$

so

$$Z_i = \frac{\gamma_i + \alpha_j\, \gamma_j}{1 - \alpha_i\, \alpha_j}. \tag{185}$$

By symmetry,

$$Z_j = \frac{\gamma_j + \alpha_i\, \gamma_i}{1 - \alpha_i\, \alpha_j}. \tag{186}$$

Step 3. Per-project profits.

From Proposition 6, per-project operating profit in sector i is

$$\pi_i = (1 - \alpha_i)\, p_i^c\, y_i. \tag{187}$$

Aggregate profit in sector i is therefore

$$K_i\, \pi_i = (1 - \alpha_i)\, p_i^c\, K_i\, y_i = (1 - \alpha_i)\, p_i^c\, Y_i = (1 - \alpha_i)\, Z_i. \tag{188}$$

Using Equation (185), aggregate sectoral profit is

$$K_i\, \pi_i = (1 - \alpha_i)\frac{\gamma_i + \alpha_j \gamma_j}{1 - \alpha_i \alpha_j}. \tag{189}$$

Define

$$K_i^* := \frac{(\gamma_i + \alpha_j\gamma_j)(1-\alpha_i)}{1-\alpha_i\alpha_j}, \qquad j \neq i. \tag{190}$$

Then aggregate sectoral profit is exactly K_i^*, so

$$K_i\pi_i = K_i^*. \tag{191}$$

Dividing by K_i yields per-project profit

$$\pi_i(K_i) = \frac{K_i^*}{K_i}. \tag{192}$$

Step 4. Monotonicity.

Since $K_i^* > 0$ is constant with respect to sector size, it follows immediately that

$$\frac{d\pi_i(K_i)}{dK_i} = K_i^* \frac{d}{dK_i}\left(K_i^{-1}\right) = K_i^*\left(-K_i^{-2}\right) \tag{193}$$

$$= -\frac{K_i^*}{K_i^2} < 0. \tag{194}$$

Hence per-project profit is strictly decreasing in sector size.

This completes the proof. □

D.5.4 Proof of Proposition 7

Proof. The proof derives the closed-form welfare representation from the household block, the closed-form benchmark for sectoral revenues, and the profit representation of Lemma 2.

Step 1. Start from indirect utility.

From Lemma 1, true welfare is

$$\mathcal{W}(K_1) = \frac{\left(\frac{\gamma_1}{p_1^c}\right)^{\gamma_1}\left(\frac{\gamma_2}{p_2^c}\right)^{\gamma_2}}{(1+E^*(K_1))^{\delta}}\, W. \tag{195}$$

Thus the key task is to express the price term $(p_1^c)^{-\gamma_1}(p_2^c)^{-\gamma_2}$ in terms of fundamentals and sector sizes.

Step 2. Relate prices to sector sizes through profits.

From Proposition 6,

$$\pi_i = (1-\alpha_i)\, q_i \left(\frac{\alpha_i\, A_i\, p_i^c}{q_i}\right)^{\frac{1}{1-\alpha_i}}. \tag{196}$$

Equivalently,

$$\pi_i = (1-\alpha_i)\,(A_i\, p_i^c)^{\frac{1}{1-\alpha_i}}\, \alpha_i^{\frac{\alpha_i}{1-\alpha_i}}\, q_i^{-\frac{\alpha_i}{1-\alpha_i}}. \tag{197}$$

Rearranging,

$$\frac{p_i^c}{q_i^{\alpha_i}}\, A_i = \left(\frac{\pi_i}{(1-\alpha_i)\,\alpha_i^{\frac{\alpha_i}{1-\alpha_i}}}\right)^{1-\alpha_i}. \tag{198}$$

Using Lemma 2, $\pi_i(K_i) = \frac{K_i^*}{K_i}$, we obtain

$$\frac{p_i^c}{q_i^{\alpha_i}}\, A_i = \left(\frac{K_i^*}{K_i\,(1-\alpha_i)}\right)^{1-\alpha_i} \frac{1}{\alpha_i^{\alpha_i}}. \tag{199}$$

Define

$$B_i := \frac{p_i^c}{q_i^{\alpha_i}}\, A_i = \left(\frac{K_i^*}{K_i\,(1-\alpha_i)}\right)^{1-\alpha_i} \frac{1}{\alpha_i^{\alpha_i}}. \tag{200}$$

Similarly,

$$B_j := \frac{p_j^c}{q_j^{\alpha_j}}\, A_j = \left(\frac{K_j^*}{K_j\,(1-\alpha_j)}\right)^{1-\alpha_j} \frac{1}{\alpha_j^{\alpha_j}}. \tag{201}$$

In the benchmark two-sector block, the materials-bundle price of sector i depends on

the output price of sector j, so we can write the system

$$p_i^c = \frac{B_i}{A_i} (p_j^c)^{\alpha_i}, \tag{202}$$

$$p_j^c = \frac{B_j}{A_j} (p_i^c)^{\alpha_j}. \tag{203}$$

Step 3. Solve for the equilibrium price terms.

Substitute Equation (202) into Equation (203):

$$p_j^c = \frac{B_j}{A_j} \left(\frac{B_i}{A_i} (p_j^c)^{\alpha_i} \right)^{\alpha_j} \tag{204}$$

$$= \frac{B_j}{A_j} \left(\frac{B_i}{A_i} \right)^{\alpha_j} (p_j^c)^{\alpha_i \, \alpha_j}. \tag{205}$$

Hence

$$(p_j^c)^{1-\alpha_i \alpha_j} = \frac{B_j}{A_j} \left(\frac{B_i}{A_i} \right)^{\alpha_j}, \tag{206}$$

so

$$p_j^c = \left[\frac{B_j}{A_j} \left(\frac{B_i}{A_i} \right)^{\alpha_j} \right]^{\frac{1}{1-\alpha_i \, \alpha_j}}. \tag{207}$$

Similarly,

$$p_i^c = \left[\frac{B_i}{A_i} \left(\frac{B_j}{A_j} \right)^{\alpha_i} \right]^{\frac{1}{1-\alpha_i \, \alpha_j}}. \tag{208}$$

Therefore

$$(p_i^c)^{-\gamma_i} (p_j^c)^{-\gamma_j} = \left(\frac{A_i}{B_i} \right)^{\frac{\gamma_i + \alpha_j \, \gamma_j}{1-\alpha_i \, \alpha_j}} \left(\frac{A_j}{B_j} \right)^{\frac{\gamma_j + \alpha_i \, \gamma_i}{1-\alpha_i \, \alpha_j}}. \tag{209}$$

Define

$$Z_i := \frac{\gamma_i + \alpha_j\,\gamma_j}{1 - \alpha_i\,\alpha_j}, \qquad Z_j := \frac{\gamma_j + \alpha_i\,\gamma_i}{1 - \alpha_i\,\alpha_j}. \tag{210}$$

Then

$$(p_i^c)^{-\gamma_i}\,(p_j^c)^{-\gamma_j} = \left(\frac{A_i}{B_i}\right)^{Z_i}\left(\frac{A_j}{B_j}\right)^{Z_j}. \tag{211}$$

Step 4. Substitute into welfare.

Using Equation (195) and Equation (211),

$$\mathcal{W}(K_1) = \frac{\gamma_1^{\gamma_1}\,\gamma_2^{\gamma_2}\,W}{(1 + E^*(K_1))^{\delta}}\left(\frac{A_1}{B_1}\right)^{Z_1}\left(\frac{A_2}{B_2}\right)^{Z_2}. \tag{212}$$

Now substitute Equations (200)–(201):

$$\left(\frac{A_i}{B_i}\right)^{Z_i} = A_i^{Z_i}\left[\left(\frac{K_i^*}{K_i(1-\alpha_i)}\right)^{1-\alpha_i}\frac{1}{\alpha_i^{\alpha_i}}\right]^{-Z_i} \tag{213}$$

$$= A_i^{Z_i}\left(\frac{K_i}{K_i^*}\right)^{Z_i(1-\alpha_i)}(1-\alpha_i)^{Z_i(1-\alpha_i)}\,\alpha_i^{Z_i\alpha_i}. \tag{214}$$

Since

$$K_i^* = Z_i(1-\alpha_i),$$

therefore

$$\left(\frac{A_i}{B_i}\right)^{Z_i} = A_i^{Z_i}\left(\frac{K_i}{K_i^*}\right)^{K_i^*}(1-\alpha_i)^{K_i^*}\,\alpha_i^{Z_i\alpha_i}. \tag{215}$$

Applying the same expression to sector 2 and substituting into Equation (212), we obtain

$$\mathcal{W}(K_1) = \frac{A_1^{Z_1}\,A_2^{Z_2}}{(1 + E^*(K_1))^{\delta}}\left(\frac{K_1}{K_1^*}\right)^{K_1^*}\left(\frac{K_2}{K_2^*}\right)^{K_2^*} \tag{216}$$

$$\times\,\gamma_1^{\gamma_1}\,\gamma_2^{\gamma_2}\,W\,\alpha_1^{Z_1\alpha_1}\,(1-\alpha_1)^{K_1^*}\,\alpha_2^{Z_2\alpha_2}\,(1-\alpha_2)^{K_2^*}. \tag{217}$$

Collect all terms independent of K_1 and $E^*(K_1)$ into the positive constant β:

$$\beta := \gamma_1^{\gamma_1}\,\gamma_2^{\gamma_2}\,W\,\alpha_1^{Z_1\alpha_1}\,(1-\alpha_1)^{K_1^*}\alpha_2^{Z_2\alpha_2}\,(1-\alpha_2)^{K_2^*}. \tag{218}$$

Then welfare can be written as

$$\mathcal{W}(K_1) = \beta\,\frac{A_1^{Z_1}\,A_2^{Z_2}}{(1+E^*(K_1))^{\delta}}\left(\frac{K_1}{K_1^*}\right)^{K_1^*}\left(\frac{K_2}{K_2^*}\right)^{K_2^*}. \tag{219}$$

This is exactly the claimed closed-form decomposition. It separates welfare into a productivity component, an allocative component, and an environmental component driven by the true consequential externality $E^*(K_1)$.

This completes the proof. □

E Decomposition of the Displacement Gradient

This appendix unpacks the marginal displacement term appearing in the main text. In particular, it shows that the derivative of the displacement base $B'(K_1)$ contains both a composition effect and an equilibrium-response effect through output adjustments.

Lemma 3 (Decomposition of the displacement gradient)**.** *Let*

$$B(K_1) = K_1\,\rho_1\,\theta_1\,y_1(K_1) + (1-K_1)\,\rho_2\,\theta_2\,y_2(K_1). \tag{220}$$

Then

$$B'(K_1) = \underbrace{\rho_1\,\theta_1\,y_1(K_1) - \rho_2\,\theta_2\,y_2(K_1)}_{\textit{composition effect}} + \underbrace{K_1\,\rho_1\,\theta_1\,\frac{dy_1}{dK_1} + (1-K_1)\,\rho_2\,\theta_2\,\frac{dy_2}{dK_1}}_{\textit{equilibrium-response effect}}. \tag{221}$$

Moreover, since

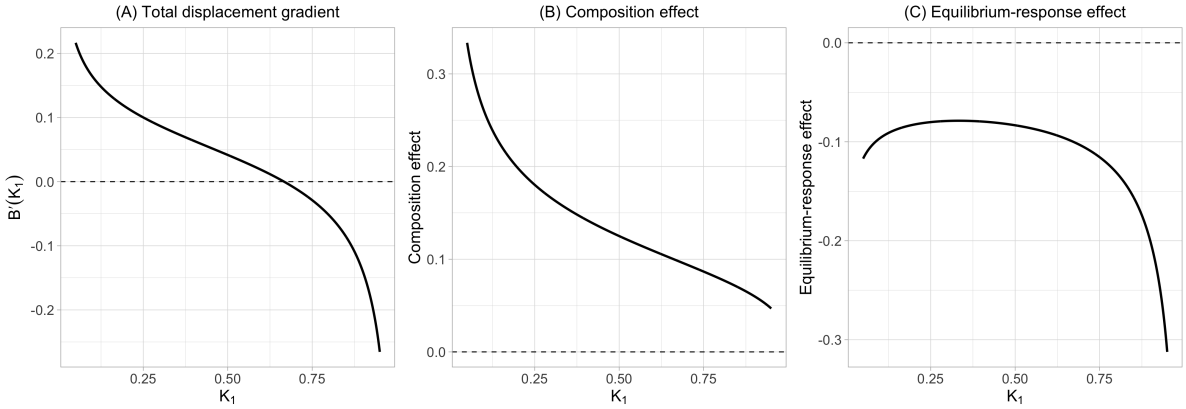
$$E^*(K_1) = E^R(K_1) - f\,B(K_1), \tag{222}$$

it follows that

$$E^{*\prime}(K_1) = E^{R\prime}(K_1) - f\,B'(K_1). \tag{223}$$

Figure 6 provides a benchmark illustration of the decomposition of the displacement gradient developed in Lemma 3. It separates the marginal displacement term into the total displacement gradient, a composition effect, and an equilibrium-response effect.

Figure 6: Decomposition of the displacement gradient



Notes: Panel A plots the total displacement gradient $B'(K_1)$. Panel B plots the composition effect, that is, the direct effect of reallocating capital across sectors while holding per-firm output responses fixed. Panel C plots the equilibrium-response effect, which captures how sectoral output adjustments contribute to displacement capacity. The figure is based on the benchmark parameterization $A_1 = A_2 = 1$, $\alpha_1 = \alpha_2 = 0.5$, $\gamma_1 = \gamma_2 = 0.5$, $\theta_1 = \theta_2 = 0.5$, $\rho_1 = 0.5$, $\rho_2 = 0$, $\varepsilon_1^{prod} = 1.2$, $\varepsilon_2^{prod} = 0.2$, $\varepsilon_1^{waste} = \varepsilon_2^{waste} = 0.5$, $\varepsilon_1^{rec} = \varepsilon_2^{rec} = 0.5$, $\delta = 6$, $f = 0.6$, and $\widetilde{f} = 0.35$. The figure is included to visualize the logic of Appendix E and does not represent a general analytical result.

E.1 Proof

Proof. By definition,

$$B(K_1) = K_1\,\rho_1\,\theta_1\,y_1(K_1) + (1 - K_1)\,\rho_2\,\theta_2\,y_2(K_1). \tag{224}$$

Differentiating with respect to K_1 yields

$$B'(K_1) = \rho_1\,\theta_1\,y_1(K_1) - \rho_2\,\theta_2\,y_2(K_1) + K_1\,\rho_1\,\theta_1\,\frac{dy_1}{dK_1} + (1 - K_1)\,\rho_2\,\theta_2\,\frac{dy_2}{dK_1}. \tag{225}$$

This gives the decomposition into a composition effect and an equilibrium-response effect.

The second claim follows immediately by differentiating

$$E^*(K_1) = E^R(K_1) - f\,B(K_1). \tag{226}$$

□