Stablecoins, Currency Digitalization and Optimal Policy*

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Abstract

We study how stablecoins and digital payments reshape taxation, seigniorage, and public debt in a two-sector monetary framework (formal/TradFi vs. informal/crypto). Extending a baseline cash economy to include government bonds, fully reserved stablecoins, or a central bank digital currency (CBDC), four main results emerge. (i) The economy exhibits a strict Laffer-curve tradeoff between extracting inflation seigniorage and preserving the formal tax base. (ii) Endogenous sector choices generate multiple macroeconomic steady states. (iii) Private stablecoins create a profound macro-fiscal tension: tokens cannibalize fiat seigniorage, but issuers' massive demand for reserve assets compresses nominal yields, subsidizing Treasury borrowing. Systemically, however, this reserve hoarding starves commercial banks of collateral, crowding out formal trade. Furthermore, a monopolistic issuer restricts token supply, yielding a safer intrinsic reserve ratio and easing bank collateral scarcity, but substantially diminishing the fiscal yield subsidy. (iv) Conversely, a public CBDC preserves seigniorage and eliminates TradFi collateral frictions, but lacks captive safe-asset demand. This exposes the sovereign to a stricter debt volume burden,

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where high debt necessitates a uniform inflation tax that depresses all trade, risking an inflation-driven sovereign debt spiral.

Key words: Stablecoins, Currency Digitalization, Fiscal-Monetary Policy Mix

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1 Introduction

In recent years, advanced economies have faced persistently high government debt, with ratios for the U.S. and Euro-area reaching 120.8% and 87.4% of GDP, respectively, by the end of 2024.[1] Simultaneously, the global payments landscape has been transformed by two powerful forces: the maturation of fiat-redeemable stablecoins and the scaling of national digital payment systems. The stablecoin market has achieved systemic macroeconomic relevance, with total market capitalization surpassing US$ 300 billion by late 2025 and annual transfer volumes in the trillions. Critically, this growth has forged a direct link between private digital money and public finance, as major stablecoin issuers have become significant holders of short-term government bonds in the U.S.. A recent BIS study confirms this safe-asset channel, finding that stablecoin inflows measurably reduce 3-month T-bill yields (BIS, 2025). Concurrently, fast payment rails like India's UPI and Brazil's Pix now process billions of transactions daily, while the U.S. FedNow system continues its expansion. Not withstanding the fast growing digital payments in China: although the mobile payments of Wechat Pay, Alipay still dominate retail transactions, the pilot project of China's CBDC (Central Bank Digital Currency; called e-CNY) has been steadily expanded since late 2019. China's central bank announced that, by the end of September 2025, the cumulative volume of e-CNY transactions had reached RMB 14.2 trillion (approximately US$ 2 trillion), stemming from 3.32 billion cumulative transactions and 225 million wallets (Xinhua, 2025). With new regulatory frameworks like the EU's MiCA and the U.S. GENIUS Act taking effect, these developments motivate a macro framework in which

[1] Data source: IMF World Economic Outlook, 2025.

various transaction technologies and safe-asset demand jointly determine the optimal monetary-fiscal policy mix.

Motivated by the above stylized facts, our paper aims to address *three questions. First*, as stablecoins scale and issuers accumulate government bonds as reserves, how does this voracious safe-asset demand affect bond yields, commercial bank collateral, and the government's seigniorage-debt mix? Second, how do information frictions and the market structure of stablecoin issuers shape this macro-fiscal tension? *Third*, what are the diverging macroeconomic and welfare implications of the two dominant currency digitalization paths: private stablecoins versus a sovereign CBDC? To answer these questions, we build a tractable monetary framework, introducing progressive layers of financial and technological sophistication.

To answer the above questions, we begin with a baseline pure Cash Economy characterized by two sectors. With probability ρ, agents enter Sector 1 (Formal/Digital/TradFi economy), where transactions are non-anonymous and subject to proportional taxes. With probability $1-\rho$, agents enter Sector 2 (Informal/Offline/Crypto economy), where trading is anonymous and untaxed, but subject to an idiosyncratic disutility cost. In the second step, we endogenize ρ, allowing agents to optimally choose their sector considering the trade-offs of the two sectors, thereby creating an endogenous boundary between the taxable formal economy and the informal/shadow economy. In the third step, we introduce a traditional banking sector and government bonds to model Stablecoin Economy. Banks provide deposit contracts to invest in a portfolio of fiat money and government bonds to facilitate Sector-1 trading. Simultaneously, stablecoin issuers cater to Sector 2 by issuing digital tokens backed by government bonds, in a narrow banking arrangement. We introduce information frictions—the ability of issuers to misrepresent reserves at a verification cost—which elegantly pins down the endogenous reserve ratio and token pledgeability. In this environment, the consolidated government budget constraint (GBC) comprises three dynamic funding sources: seigniorage, tax revenues, and a debt-service yield subsidy generated by the bond premium. Finally, we model a CBDC Economy by substituting cash in the formal sector with a public digital fiat money, to evaluate the competing paradigm

of public currency digitalization.

Our analysis yields **four main theoretical and policy results**. *First,* in the baseline cash economy, the government faces a strict Laffer-curve trade-off between generating seigniorage via inflation and preserving the purchasing power of money to stimulate trade. *Second*, when sector choice becomes endogenous, the model generically produces multiple steady states. Small variations in institutional quality or tax rates can shift the economy between an "efficient" equilibrium characterized by high formalization, low inflation, and a broad tax base, and an "inefficient" equilibrium plagued by high inflation and rampant informality. *Third*, the introduction of stablecoins generates a profound macroeconomic tension: the "safe-asset subsidy" versus "seigniorage erosion." As stablecoins displace cash in the informal sector, the central bank's traditional seigniorage base shrinks. However, the stablecoin issuers' massive demand for reserve assets drives up the price of government bonds and lowers nominal yields. This generates a direct yield subsidy that significantly eases the Treasury's borrowing costs. Yet, this mechanism is not without systemic costs; stablecoin reserve hoarding severely crowds out the traditional banking sector, starving banks of the collateral required to facilitate formal economic trade. Furthermore, we uncover a novel *Macroeconomic Trilemma* driven by moral hazard and information frictions. We show that a monopolistic stablecoin issuer deliberately restricts token supply to suppress its reserve acquisition costs. Consequently, the monopoly operates with a technically safer (higher) intrinsic reserve ratio and leaves more collateral available for the traditional banking sector, but it provides a substantially smaller fiscal yield subsidy to the Treasury compared to a competitive market.

Lastly, our structural comparison of currency digitalization paths reveals that private stablecoins and public CBDCs carry fundamentally different macro-fiscal vulnerabilities. A CBDC directly preserves the public seigniorage base and enhances formal sector efficiency by eliminating the collateral constraints faced by traditional banks. However, because a CBDC system lacks the captive safe-asset demand generated by private stablecoin reserves, it exposes the Treasury to a severe "debt volume burden". Under a CBDC path, a high sovereign debt load forces the government to finance its

debt at fundamental market rates. To satisfy the budget constraint, the sovereign must rely heavily on a uniform inflation tax, which indiscriminately destroys purchasing power across both the formal CBDC and informal cash sectors, crowding out trade and risking a severe sovereign debt spiral. To formally evaluate these competing paradigms, we establish a rigorous welfare criterion that unifies real, constrained, and first-best social surpluses, providing a clear map for optimal regulatory intervention. Following the passage of the GENIUS Act in July 2025, the U. S. has committed to a regulatory approach that promotes private USD-pegged stablecoins, whereas China has explored issuing its digital fiat money (e-CNY) in pilot areas since late 2019. We can exactly use the framework to quantify the macro-fiscal consequences of these diverging paths – seigniorage, tax incidence, public-debt composition, and the fiscal-monetary policy mix – and to evaluate the welfare implications.

Literature Review This paper connects to three strands of literature. *The first strand* is the work that treats fiat-redeemable stablecoins as payment technology and as large users of safe assets, and also the work on digital payments and CBDC. On stablecoin design and regulation, Gorton and Zhang (2021, 2023) argue for bank-like oversight of redeemable tokens; Lyons and Viswanath-Natraj (2023) analyze peg stabilization and arbitrage; policy syntheses from the BIS, FSB, and IMF emphasize run vulnerability, reserve quality, and same-risk–same-rule approaches; and recent empirical work documents that stablecoin flows move safe-asset yields in the short run. Li and Mayer (2025) presents a dynamic model of a single, profit-maximizing stablecoin issuer in a continuous-time framework, to study the issuer's stablecoin-management strategy, equity valuation, and responses to regulations. The ideas of Azzimonti and Quadrini (2025) is related to our paper, but the focus is very different. It develops a multi-country general equilibrium model to study how the rise of a "Digital Economy" (i.e., crypto world), particularly stablecoins, affects the global financial system and the U.S.. It focuses on stablecoins not as a means of payment, but primarily as a store of value that competes with and complements traditional reserve assets like US government debt. Hence, all of these stablecoin papers are *not* about the macro implications of rising stablecoins on seigniorage and optimal

fiscal-monetary policy mix, particularly from the perspective of a stablecoin issuing country. And this is one main contribution of our paper.

On digital payments and CBDC, Cong and Mayer (2025) studies the competition between CBDC and private digital money, but it focuses more on the strategic choices of various countries on digitization of payment methods. Auer and Bohme (2021) discuss minimally-invasive architectures. Niepelt (2020), Andolfatto (2021), Keister and Sanches (2023), and Dong and Xiao(2024) all analyze banking and intermediation effects of issuing CBDC, from different perspectives. BIS Annual Economic Reports (2023–2025) evaluate system-wide implications of tokenized money and payments. Compared to these papers and reports, our paper provides a more general theoretical framework to cover CBDC, digital money, government bonds and stablecoins, and focuses more on the macro implications of rising digital payments and stablecoins.

The second strand is, also a methodological one, is the New Monetarism literature in which money and assets are captured with explicit micro-foundations in macro models, to study monetary and macroeconomic topics (Lagos and Wright 2005, and Lagos et al. 2017). Particularly, our modellings of government bonds and open market operations (OMOs) are originated from Rocheteau et al. (2018), and Dong and Xiao (2019). This literature clarifies how bond yields embed both return and acceptability premium, why OMOs and helicopter drops differ in general equilibrium, and what are the impacts of unconventional monetary policy (like QE) on real economy. We use similar logic to keep track of both public and private bond demand, and to study instrument choices when stablecoins and digital payments coexist as media of exchange.

A third strand of literature covers payment-system economics and fast-payment adoption. Classic work on two-sided platforms and payment cards (Rochet and Tirole, 2002, 2006) establishes how network structure and pricing shape adoption and usage; Kahn and Roberds (2009) provide an overview of payments economics and finality; CPMI and central-bank reports document the rollout and design of fast-payment systems worldwide and the implications for cross-border retail payments. We build on these insights by modeling digital payments and stablecoins as competing transaction

technologies that alter the government budget constraint, the equilibrium composition of money and safe-asset holdings, and the optimal monetary–fiscal mix, which is absent in the current literature.

The rest of paper is organized as follows. Section 2 provides a baseline cash economy, where cash circulates in both sectors, given ρ, the prob. agents choose a sector. We discuss equilibrium existence and uniqueness, and fiscal-monetary policy mix. Then we endogenize the sector dichotomy in Section 3, by allowing free sector choices of agents, and find more dynamics on the fiscal-monetary policy mix. To capture the public finance implications of fast-rising stablecoins, we move on to a stablecoin economy in Section 4, by adding government bonds and stablecoins to the baseline. We model banks and stablecoins issuers, discuss about equilibrium deposit contracts and stablecoin demand to close the model, when stablecoin reserves are taken as given. Then we endogenize the reserve pledgeability and reserve holdings of issuers by incorporating the information frictions that issuers misrepresent reserve holdings. Furthermore, in Section 5, we discuss about currency digitalization paths and the macroeconomic implications, by comparing a CBDC path to the stablecoin path and conducting welfare analysis. Finally, Section 6 provides discussion and concludes the paper.

2 Cash Economy

We start from a baseline model, where cash is the only medium of exchange, and circulates in the whole economy.

2.1 Environment

Time is discrete and continues forever. There are two subperiods each period: a decentralized market, or DM, with frictions discussed below, in the first subperiod, a frictionless centralized market, or CM in the second. Agents carry cash (m_m) in the beginning of every period. The measure of agents is normalized to one. Here m_m is fiat money, and the *direct liability* of the central bank.

In the beginning of every DM, there are idiosyncratic shocks, for agents to choose to enter Sector 1, or Sector 2. We can interpret *the dichotomy of Sector 1 and Sector 2 as Formal v.s. Informal sectors, or Online v.s. Offline transactions, or even TradFi v.s. crypto transactions.* The common rational for the dichotomy is that the former is for non-anonymous trading (Formal Sector), or trading that can generate digital footprints (from online trading or TradFi), while the later is either anonymous or trading that can't generate digital footprints.[2]

Due to a lack of enforcement/commitment plus imperfect information, agents in DM need to trade with money, m_m, which is also the only medium of exchange in the economy. For simplicity, we assume, once choosing a sector, agents have a half of the probability to become a buyer, and another a half to become a seller. In Sector 1, since the tradings are non-anomymous, agents need to pay taxes at a proportional tax rate $\tau \in (0, 1)$. In Sector 2, the tradings are anonymous, so agents do not pay any taxes. However, due to lack of legal protection or other opportunity costs, each agent choosing Sector 2 (Informal), need to bear an individual specific cost $\kappa \geqslant 0$, with CDF $F(\kappa)$.[3] Suppose the measure of agents is normalized to "1". Then the measure of agents choosing Sector 1 is $\rho \in (0, 1)$, and that choosing Sector 2 (Informal) is $1-\rho$. As usual, for DM trading q, $u(q)$ is the utility function of buyers and $c(q)$ is the production function for sellers, with $u(0) = c(0) = c'(0) = 0$, $u'(0) = +\infty$, $u'(q) > 0$, $u''(q) < 0$, $c'(q) > 0$, $c''(q) > 0$.[4]

In CM, all agents work or consume CM goods x, which can also serve as a numeraire for the whole economy, and rebalance their portfolios. The government is mainly active in CM, except collecting taxes from Sector 1 in DM. And it is a consol-

[2] And there is literature showing the growth of online trading has decreased the informality level of the economy (Aguilar et al. 2024, Lv et al. 2024, and Micha et al. 2024), which provides a justification for the above broad interpretations of non-anonymous trading either in Formal Sector, or from online trading.

[3] We can also interprete κ as an individual specific cost relating to offline trading, when Sector 2 is interpreted as "Offline trading". Particularly, for a buyer, κ can be the opportunity cost of choosing offline trading, thus forgoing the benefits of online shoping (more convinient, higher matching rates of buying what he wants); for a seller, κ can be the opportunity cost of choosing offline trading, thus forgoing the potential benefits of online trading (a broader customer base, or a high matching rate α with customers.

[4] For some analytical and numerical results in the paper, for simplicity, we let $c(q) = q$, and consider functional forms of $u(q)$ like log utility, or CRRA.

idated authority of the treasury and central bank. The central bank issues fiat money (or cash), M, and

$$\frac{M}{M_-} = 1 + \mu_m, \tag{1}$$

where M_- refers to total money supply of previous period (the subscript "$-$" means "$t-1$"), and μ_m is the money growth rate ("m" means fiat money). In contrary to most New Monetarism models, in this paper, μ_m is an endogenous policy variable, instead of just an exogenous policy parameter (although it *can be* treated as exogenous, when discussing endogenous fiscal policy).

In addition, the government budget constraint (GBC) is

$$G = \phi(M - M_-) + (\frac{1}{2}\tau_b D_1 + \frac{1}{2}\tau_s D_1),$$

where $G \equiv g + T$, with g denoting normal government expenditures, T denoting lump-sum transfer to agents, all in real terms; ϕ is the real price of money; and $D_1 \equiv \sum_i \alpha_1^i d_1$ denotes aggregate transaction amounts in Sector 1, from the sum of after-matching transaction amounts of type-i payment, with α_1^i, being the matching rate, and d_1 is the pre-tax transaction amount in Sector 1. For simplicity, we let $\tau_b = \tau_s \equiv \tau$. For the current cash economy, $i = m$, $D_1 \equiv \rho\alpha_1 d_1$, d_1 is the payment from Sector 1, by cash. Using (1), we can further rewrite GBC as

$$G = \mu\phi M + \tau \cdot D_1, \tag{2}$$

Here we can see that the RHS shows two main sources of government revenues to support G: the first term is seigniorage from money growth μ_m (or "inflation tax"), and the second term is tax revenues from Sector 1, with ρ being the measure of this "taxable" sector. In the baseline model, we treat ρ as *exogenous*, but endogenize it in Section 3.

2.2 Model

Next we present the baseline model, with the timeline that CM starts first, then DM next period. All of the terms are real, except stating otherwise. To ease notations, we omit time subscripts, but use "ˆ" to denote variables next period.

In CM, an agent holding real balances of money, $z_m \equiv \phi m_m$, need to decide CM consumption x, labor hours h and how much $\hat{z}_m$ to next period:

$$W(z_m) = \max_{x,h,\hat{z}_m} \{U(x) - h + \beta V(\hat{z}_m)\}$$
$$\text{st. } x + (1+\mu)\hat{z}_m = z_m + h + T,$$

where β is time discount factor, T is lump-sum transfer from the government. Hence,

$$W(z_m) = z_m + T + \max_x\{U(x) - x\} + \max_{\hat{z}_m}\{-(1+\mu)\hat{z}_m + \beta \mathbb{E}V(\hat{z}_m)\}.$$

Then we get the envelope condition:

$$\frac{\partial W(z_m)}{\partial z_m} = 1.$$

For the FOCs, we easily have x^* satisfying $U'(x^*) = 1$, and the Euler equations for $\hat{z}_m$:

$$\frac{1+\mu}{\beta} = \frac{\partial \mathbb{E}V(\hat{z}_m)}{\partial \hat{z}_m}.$$

In DM next period, an agent can choose to enter Sector 1 (subscript "1") or Sector 2 ("2"). Given the probability he choose the former is ρ, we have

$$\mathbb{E}V(\hat{z}_m) = \rho V_1(\hat{z}_m) + (1-\rho)V_2(\hat{z}_m, \kappa).$$

Once entering a sector, an agent has a half of the probability to become a buyer (superscript "b") and another half to become a seller (superscript "s"). Hence, in Sector 1,

$$V_1(\hat{z}_m) = \frac{1}{2}V_1^b(\hat{z}_m) + \frac{1}{2}V_1^s(\hat{z}_m),$$

where

$$V_1^b(\hat{z}_m) = \alpha[u(q_1) - (1+\tau)d_1] + W(\hat{z}_m)$$
$$V_1^s(\hat{z}_m) = \alpha[-c(q_1) + (1-\tau)d_1] + W(\hat{z}_m),$$

with $(1+\tau)d_1 \leqslant \hat{z}_m$, i.e., the post-tax payments of buyers shall not exceed the money holdings $\hat{z}_m$, and both buyers and sellers need to pay taxes at the rate τ. Here α is the matching rate, (q_i, d_i), $i = \{1, 2\}$, is the terms of trade for Sector i.

Similarly, in Sector 2,

$$V_2(\hat{z}_m, \kappa) = \frac{1}{2}V_2^b(\hat{z}_m, \kappa) + \frac{1}{2}V_2^s(\hat{z}_m, \kappa),$$

where agents still trade with cash, do not pay taxes but need to bear the disutility cost κ, then

$$V_2^b(\hat{z}_m, \kappa) = -\kappa + \alpha[u(q_2) - d_2] + W(\hat{z}_m)$$
$$V_2^s(\hat{z}_m, \kappa) = -\kappa + \alpha[-c(q_2) + d_2] + W(\hat{z}_m),$$

with $d_2 \leqslant \hat{z}_m$.

2.3 Equilibrium Analysis

Next we sort out the bargaining solutions, then discuss about the general equilibrium.

2.3.1 Bargaining Solutions

We start from the bargaining problem in Sector 1. Suppose buyers and sellers bargain to determine the terms of trade (q_1, d_1). We consider Kalai bargaining, and the bargaining power of buyers is θ. Hence, in Sector 1,

$$\begin{aligned} &\max_{q_1, d_1}\{u(q_1) - (1+\tau)d_1\} \\ \text{st. } &\frac{u(q_1) - (1+\tau)d_1}{-c(q_i) + (1-\tau)d_1} = \frac{\theta}{1-\theta} \\ &(1+\tau)d_1 \leq \hat{z}_m. \end{aligned}$$

Since it's costly to carry money over periods, WLOG, agents choose to spend all money they carry. Hence, when $(1+\tau)d_1 \leq \hat{z}_m$ binds, we have $d_1 = \hat{z}_m/(1+\tau)$,

$$\upsilon(q_1) = A(\tau,\theta)\hat{z}_m, \tag{3}$$

where we define $\upsilon(q_j) \equiv (1-\theta)u(q_j) + \theta c(q_j)$, $j = \{1,2\}$, and

$$A(\tau,\theta) \equiv \frac{1+(1-2\theta)\tau}{1+\tau}.$$

Here we can easily have $0 < A(\tau,\theta) < 1$, with $\tau \in (0,1)$, $\theta \in (0,1)$. We also have $\upsilon'(q_1) > 0$, and

$$\frac{\partial q_1}{\partial \hat{z}_m} = \frac{A(\tau,\theta)}{\upsilon'(q_1)} > 0.$$

Similarly, in Sector 2, we have

$$\begin{aligned} &\max_{q_2,d_2}\{u(q_2) - d_2\} \\ \text{st. } &\frac{u(q_2)-d_2}{-c(q_2)+d_2} = \frac{\theta}{1-\theta} \\ &d_2 \leq \hat{z}_m. \end{aligned}$$

When $d_2 \leq \hat{z}_m$ binds, we have

$$\upsilon(q_2) \equiv (1-\theta)u(q_2) + \theta c(q_2) = \hat{z}_m. \tag{4}$$

Since $\upsilon'(q) > 0$, with $\upsilon(q_1) < \hat{z}_m = \upsilon(q_2)$, we have $q_1 < q_2$. Similarly, we have $\upsilon'(q_2) > 0$, and

$$\frac{\partial q_2}{\partial \hat{z}_m} = \frac{1}{\upsilon'(q_2)} > 0.$$

2.3.2 General Equilibrium

With the terms of trade determined, we can now embed these bargaining outcomes into the broader economy to analyze the general equilibrium dynamics of money demand and others. First, substituting bargaining solutions into the FOC against

$\hat{z}_m$, we can derive the Euler equation for cash as,

$$\frac{1+\mu}{\beta} - 1 = \rho\frac{\alpha}{2}\lambda_1(q_1) + (1-\rho)\frac{\alpha}{2}\lambda_2(q_2), \tag{5}$$

where

$$\lambda_1(q_1) \equiv \frac{\theta[(1-\tau)u'(q_1)/(1+\tau) - c'(q_1)]}{[(1-\theta)u'(q_1) + \theta c'(q_1)]} \tag{6}$$

$$\lambda_2(q_2) \equiv \frac{\theta[u'(q_2) - c'(q_2)]}{(1-\theta)u'(q_2) + \theta c'(q_2)}. \tag{7}$$

We can also easily prove that $\lambda_j'(q_j) < 0$, $j = \{1, 2\}$, which means higher tradings (higher q_j) in either sector lead to more holdings of money $\hat{z}_m$, then lower the liquidity premium $\lambda_j(q_j)$.

Furthermore, we can rewrite GBC as,

$$G = \mu\hat{z}_m + \rho\tau\frac{\alpha\hat{z}_m}{1+\tau}. \tag{8}$$

Here the RHS clearly shows there are two funding sources for the government spending G: the first term represents seigniorage, determined by money growth rate μ, and total money supply (in real terms) $\hat{z}_m$; the second term is total tax revenues, determined by the measure of Sector 1, ρ, proportional tax rate τ, and *the post-matching tax base* $D_1 \equiv \alpha d_1$, i.e., pre-tax sales revenues from the transactions in Sector 1, which is further determined by τ, $\hat{z}_m$ and the matching rates α.

More interestingly, we can see the interplays between these two terms, or more implicitly, the interplays between monetary policy, i.e., directly by changing μ_m, or indirectly by changing $\hat{z}_m$ (also determined by μ_m), and fiscal policy, i.e., directly by changing τ, or indirectly by changing D_1. When μ_m changes, it will not only affect the real balances of money $\hat{z}_m$, but also the tax base for Sector 1, D_1, since $\hat{z}_m$ is one of the important factors there. When τ changes, obviously it will affect total tax revenues, including the tax base D_1, then $\hat{z}_m$, which is part of the monetary base of seigniorage. Therefore, no matter changing μ, or τ, it will always cause interplays between seigniorage (the central bank) and tax revenues (the Treasury). Although

our baseline model uses an abstract and simple way to model the government, it is already rich enough to capture the interplays between the monetary and fiscal authorities in the real world.

Now we are ready to define general equilibrium as follows.

Definition 1 *Given G, τ and ρ, a stationary monetary equilibrium is a list $\{q_1, q_2, \hat{z}_m, \mu\}$,that satisfies [i]bargaining solutions in Sector 1 and Sector 2, shown by (3)- (4); [ii]Euler equations for money, shown by (5); [iii] the government budget constraint, shown by (8); [iv] market clearing conditions for fiat money (m_m).*

Explicitly, given G, τ and ρ, equilibrium conditions for $\{q_1, q_2, \hat{z}_m, \mu\}$ are (3) - (8). And we know $\mu \geq \beta - 1$ from Friedman Rule (or a non-arbitrage condition). Alternatively, if G and μ are given, (3) - (8) become equilibrium conditions for $\{q_1, q_2, \hat{z}_m, \tau\}$. This is relevant to the discussion of optimal policy later.

For the RHS of (8), we let

$$S(q_2) \equiv \hat{z}_m(\mu + \frac{\alpha\rho\tau}{1+\tau}) = \upsilon(q_2)[\mu(q_2) + \frac{\alpha\rho\tau}{1+\tau}], \tag{9}$$

where

$$\mu(q_2) \equiv \beta - 1 + \frac{\beta\alpha}{2}\{\rho\lambda_1[q_1(q_2)] + (1-\rho)\lambda_2(q_2)\}, \tag{10}$$

is derived from (5). When $\mu = \beta - 1$, it is at the Friedman Rule, we then define $q_{2|\mu=1-\beta} \equiv q_{FR}$, and

$$S(q_{FR}) = \upsilon(q_{FR})[\beta - 1 + \frac{\alpha\rho_m\tau}{1+\tau}].$$

Notice that we have $\lambda_1(q_1) = \lambda_1[q_1(q_2)]$, since (3) and (4) lead to

$$\upsilon(q_1) = A(\tau, \theta)\upsilon(q_2). \tag{11}$$

And we can also easily derive $q_1 < q_2$, since $0 < A(\tau, \theta) < 1$, and $\upsilon'(q) > 0$. Then we have

$$q_1 = \upsilon^{-1}[A(\tau, \theta)\upsilon(q_2)] \equiv q_1(q_2).$$

Obviously, we have

$$q_1'(q_2) = \frac{dq_1}{dq_2} = \frac{A(\tau,\theta)\upsilon'(q_2)}{\upsilon'(q_1)} > 0. \tag{12}$$

Using (5), we have the RHS decreases in q_2, since $\lambda_j'(q_j) < 0$, $j = \{1,2\}$, and $dq_1/dq_2 > 0$. This means the LHS should also decrease in q_2 (or q_1), i.e.,

$$\frac{\partial \mu}{\partial q_j} < 0.$$

That is, with more trading in either Sector 1 or 2, or equivalently more money to support the trading, the liquidity premium of money goes down, which further suppresses money growth rate μ. Easily, we know, when $\mu = \beta - 1$, by (5), we have $\lambda_1(q_1) = 0$, and $\lambda_2(q_2) = 0$, then

$$q_{1|\mu=\beta-1} = q_1^*, \text{ with } \frac{u'(q_1^*)}{c'(q_1^*)} = \frac{1+\tau}{1-\tau} > 1$$
$$q_{2|\mu=\beta-1} \equiv q_{FR} = q^*, \text{ with } \frac{u'(q^*)}{c'(q^*)} = 1.$$

And we can easily prove $q_1^* < q^*$ since $(1+\tau)/(1-\tau) > 1$. Hence, $q_1 \in (0, q_1^*]$, $q_2 \in (0, q_{FR}]$, with $\mu \geq \beta - 1$.

Now we check the monotonicity of $S(q_2)$. By (9), we have

$$S'(q_2) = \upsilon'(q_2)[\mu(q_2) + \frac{\alpha\rho_m\tau}{1+\tau}] + \upsilon(q_2)\mu'(q_2) \lesseqgtr 0,$$

since $\upsilon'(q_2) > 0$, but $\mu'(q_2) < 0$. The above shows, in general, the sign of $S'(q_2)$ is ambiguous. In addition, we can define

$$\bar{S} \equiv \max_{q_2}\{\upsilon(q_2)[\mu(q_2) + \frac{\alpha\rho_m\tau}{1+\tau}]\},$$

where $q_2 \in (0, q^*]$. The FOC is

$$\upsilon'(q_2)[\mu(q_2) + \frac{\alpha\rho_m\tau}{1+\tau}] + \upsilon(q_2)\mu'(q_2) = 0.$$

Using (10) and (12), we have

$$\begin{aligned}\mu'(q_2) &= \frac{\beta\alpha}{2}[\rho_m\lambda_1'(q_1)\cdot q_1'(q_2) + (1-\rho_m)\lambda_2'(q_2)] \\ &= \frac{\beta\alpha}{2}\{\rho_m\lambda_1'(q_1)\frac{A(\tau,\theta)\upsilon'(q_2)}{\upsilon'(q_1)} + (1-\rho_m)\lambda_2'(q_2)\}.\end{aligned}$$

Then we can rewrite the above FOC, to derive a maximizer $q_2 \equiv \tilde{q}_2(\tau)$ satisfying,

$$\mu(q_2) + \frac{\alpha\rho_m\tau}{1+\tau} = -\frac{\beta\alpha}{2}\upsilon(q_2)\{\rho_m\frac{A(\tau,\theta)\lambda_1'(q_1)}{\upsilon'(q_1)} + (1-\rho_m)\frac{\lambda_2'(q_2)}{\upsilon'(q_2)}\}. \tag{13}$$

Since $\lambda_j'(q_j) < 0$ and $\upsilon'(q_j) > 0$, the RHS is positive as a whole. Hence, the maximum $\tilde{q}_2(\tau)$ is such that, at the fiscal limit the government sits exactly where the converted marginal benefit of holding money (the LHS) is equal with the marginal effect on the "money-demand" elasticity term (the RHS). Then

$$\bar{S} = S(\tilde{q}_2(\tau)).$$

Since $q_2 \in (0, q^*]$, we have

$$\tilde{q}_2 < q^* = q_{FR}.$$

We *conject* that $\bar{S} = S(\tilde{q}_2(\tau)) > S(q_{FR})$, with $S'(q_2) < 0$, and $q_2 \in [\tilde{q}_2, q_{FR}]$.

Now we conduct *numerical analysis* for the monotonicity of $S(q_2)$, general equilibrium existence and uniqueness. We use standard parameters and functional forms: $\alpha = 0.5$, $\beta = 0.95$, $\theta = 0.5$, $\tau = 0.20$, $G = 0.08$, $\rho = 0.8$, $u(q) = a \cdot q^{0.5}/0.5$ (let $a = 1$ for now), $c(q) = 0.5q^2$.[5] As shown in Figure 1, we can see $S(q_2)$ is a hump-shaped curve, and also plot the maximum govt. revenue $\bar{S} = 0.0867$, and the revenue at Friedman Rule, $S(q_{FR}) = 0.016$, by solving for $\tilde{q}_2(\tau) = 0.381$, $q_{FB} = 1$. Figure 1 also shows two equilibria, i.e., $q_2 = \{0.233, 0.548\}$, since $S(q_{FR}) = 0.016 < G = 0.08 < \bar{S} = 0.087$, and the $G = 0.08$ line crosses the hump-shaped $S(q_2)$ curve twice. We have $S'(q_2) < 0$, with $q_2 \in [\tilde{q}_2, q_{FB}]$, so the numerical analysis also verifies our previous *conjecture*.

[5] We use the same parameter values and functional forms through this section, except clearly stated, otherwise.

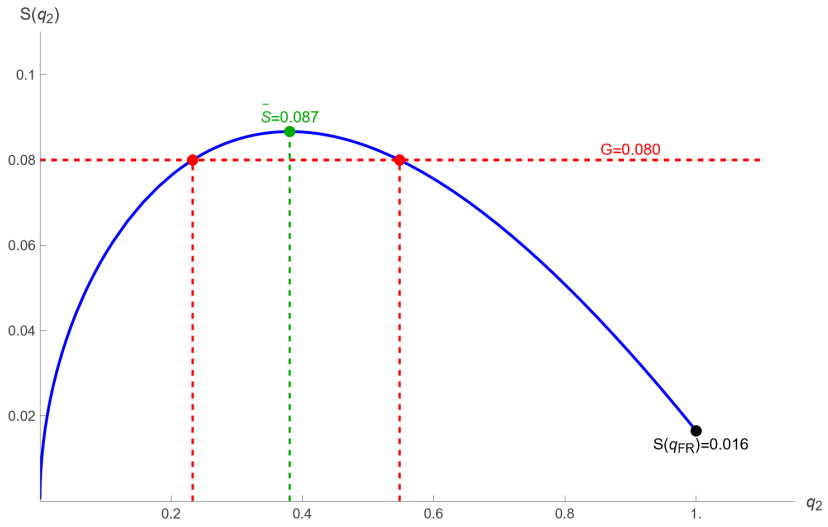


Figure 1: Seigniorage Laffer Curve & Multiple Equilibria

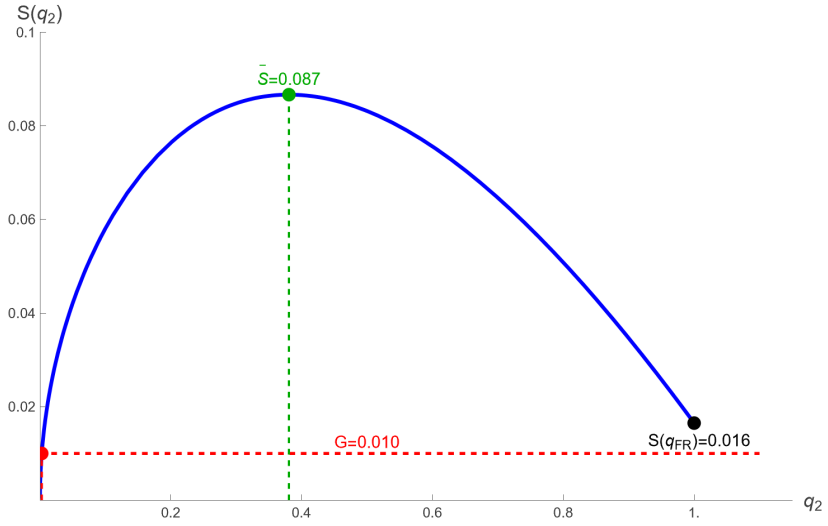


Figure 2: Seigniorage Laffer Curve & Unique Equilibrium

In addition, as shown in Figure 2, when $G = 0.01 < S(q_{FR})$, there is only one equilibrium at the left "bad" equilibrium side, since the $G = 0.01$ line crosses the $S(q_2)$ curve only once. Then we use Proposition 1 below to summarize the equilibrium existence and uniqueness condition.

Proposition 1 *Given a hump-shaped $S(q_2)$, a stationary monetary equilibrium exists if $G \leq \bar{S}$. If $G = \bar{S}$, there is a knife-edge case of a unique equilibrium, i.e., $q_2 = \tilde{q}_2$, so that $S(\tilde{q}_2) = \bar{S} = G$. To have an equilibrium existing at the Friedman Rule, we need to have $\tau_{LB} \equiv (1-\beta)/[\alpha\rho_m - (1-\beta)] < \tau < 1$, then $S(q_{FR}) > 0$. As for uniqueness, if $G < S(q_{FR})$, there exists a unique monetary equilibrium, which is on the inefficient side of the Laffer Curve, with $S'(q_2) > 0$; and if $S(q_{FR}) \leq G < \bar{S}$, there exists two equilibria: one "inefficient" equilibrium, with $S'(q_2) > 0$, and another "efficient" one, with $S'(q_2) < 0$.*

Proof: Given a hump-shaped $S(q_2)$, first, we need to have $G \leq \bar{S}$ for the existence of general equilibrium. Next, we can rewrite (8) to have

$$\mu = \frac{G}{\hat{z}_m} - \frac{\rho_m \alpha \tau}{1+\tau} > \beta - 1,$$

where we know $\hat{z}_m = \upsilon(q_2)$. Then the LHS of (5) changes to

$$LHS = \frac{1}{\beta}[1 + (\frac{G}{\hat{z}_m} - \frac{\rho_m \alpha \tau}{1+\tau})] - 1 > 0.$$

When $\hat{z}_m \to 0$, LHS $\to \infty$; and when $\hat{z}_m \to \infty$, $LHS \to \frac{1}{\beta}(1 - \frac{\rho_m \alpha \tau}{1+\tau}) - 1$, which should be nonnegative, to be consistent with $LHS \geq 0$. This implies

$$\frac{\alpha \rho_m \tau}{1+\tau} \geq 1 - \beta$$

$\to$

$$\tau > \frac{1-\beta}{\alpha\rho_m - (1-\beta)} \equiv \tau_{LB} \tag{14}$$

In addition, since $\tau \in (0,1)$, τ_{LB} should also be in the same range, i.e., $0 <$

$\frac{1-\beta}{\alpha\rho_m-(1-\beta)} < 1$. Then we can derive

$$\rho_m > \frac{2(1-\beta)}{\alpha}. \tag{15}$$

Furthermore, at the Friedman Rule, i.e., $\mu = 1-\beta$, we have $q_2 \equiv q_{FR} = q^*$, with $u'(q^*) = c'(q^*)$. Then

$$S(q_{FR}) \equiv \upsilon(q_{FR})[\beta - 1 + \rho_m \tau\alpha/(1+\tau)] = G_{|\mu=\beta-1} > 0,$$

where we use $\hat{z}_m = \upsilon(q_2) = \upsilon(q_{FR}) > 0$. Then we can derive the same $\tau > \tau_{LB}$ as in (14), to have $S(q_{FR}) > 0$. It means, even at Friedman Rule, to have GBC satisfied, the RHS of (8), $S(q_{FR})$ need to be positive, with $\tau > \tau_{LB}$. Hence, the conditions $G \leq \bar{S}$, (14) plus (15) provide the parameter space of the equilibrium existence at the Friedman Rule.

Lastly, we discuss about the uniqueness of general equilibrium. As shown in the numerical analysis of Figure 1 and 2, with a CRRA $u(q)$ and a quadratic $c(q)$, we get a hump-shaped government revenue function, $S(q_2)$, or a standard seigniorage Laffer Curve. When $S(q_{FR}) \leq G \leq \bar{S}$, there usually exists two equilibria: a "good" equilibrium with a low inflation rate μ and high q_2 (then high $\hat{z}_m$ and also high q_1), and a "bad" one with a high inflation rate μ and low q_2 (then low $\hat{z}_m$ and also low q_1). If $G < S(q_{FR})$ at the good side, i.e., $S'(q_2) < 0$, then there will an equilibrium at the bad side, which is also a unique one. Of course, if $G > \bar{S}$, there will no equilibrium, since the fiscal upper limit $\bar{S}$ can't support the government spending G.

Q.E.D.

2.4 Policy Analysis

After the analysis on equilibrium existence and uniqueness, we are ready for policy analysis, changing G, τ and ρ.

1. Standard Inflation Laffer Curve: effects on changing μ on govt. revenue, S

We plot a standard inflation Laffer Curve in Figure 3. We still use the same parameter values and functional forms as before. With $\beta = 0.95$, we have $\mu_{FR} =$

-5%. We can derive the inflation rate $\mu_{peak} = 6.6\%$ that maximizes total government revenues with $\bar{S} = 0.087$, at $\tilde{q}_2 = 0.381$. Hence, this is also the level of inflation which supports the "fiscal limit" of the model economy. Furthermore, we can also calculate $\mu_{\max} = 12.4\%$ which leads to $q_2 \to 0$, and $S(q_2) = 0.0191$. And we particularly show the three critical values of the inflation rate, i.e., μ_{FR}, μ_{peak}, $\mu_{\max}$ in Figure 3.

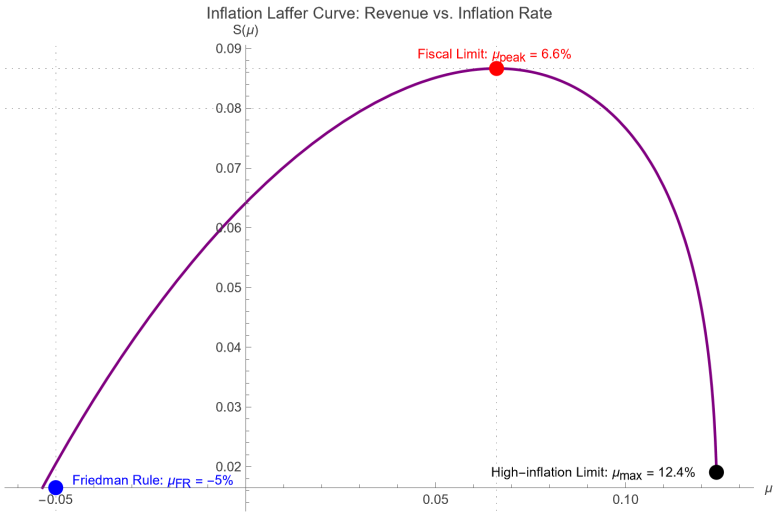


Figure 3: Inflation Laffer Curve

2. Comparative Statics (CS): effects of changing G, τ, ρ_m, α, θ on $\{q_2, q_1, \hat{z}_m, \mu\}$

(1) Effects of changing G

First, let's provide theoretical analysis for the effects of changing G. Using (9), we have

$$\frac{\partial q_2}{\partial G} = \frac{1}{S'(q_2)} \cong S'(q_2) \gtrless 0,$$

where we know, in general, the sign of $S'(q_2)$ is ambiguous, so is that of $\partial q_2/\partial G$. Using (4), (12) and (10), we also have

$$\frac{\partial \hat{z}_m}{\partial G} = \upsilon'(q_2)\frac{\partial q_2}{\partial G} \cong S'(q_2),\ \frac{\partial q_1}{\partial G} = q_1'(q_2)\frac{\partial q_2}{\partial G} \cong S'(q_2)$$
$$\frac{\partial \mu}{\partial G} = \frac{\beta\alpha}{2}[\rho_m \lambda_1'(q_1)\cdot q_1'(q_2) + (1-\rho_m)\lambda_2'(q_2)]\frac{\partial q_2}{\partial G} \cong -S'(q_2).$$

On the other hand, from Proposition 1, we know, for the efficient equilibrium (low-

inflation equilibrium), $S'(q_2) < 0$, then

$$\frac{\partial q_2}{\partial G} < 0, \frac{\partial \hat{z}_m}{\partial G} < 0, \frac{\partial q_1}{\partial G} < 0, \frac{\partial \mu}{\partial G} > 0.$$

And $S'(q_2) > 0$ for the inefficient equilibrium (high-inflation equilibrium), so

$$\frac{\partial q_2}{\partial G} > 0, \frac{\partial \hat{z}_m}{\partial G} > 0, \frac{\partial q_1}{\partial G} > 0, \frac{\partial \mu}{\partial G} < 0.$$

Next, we provide the numerical plotting, for both "Efficient Equilibrium" (blue curves), and "Inefficient Equilibrium" (red curves) explicitly, in Figure 4. We use the same parameters and functional forms as before: $\alpha = 0.5$, $\beta = 0.95$, $\theta = 0.5$, $\tau = 0.20$, $u(q) = a \cdot q^{0.5}/0.5$ (let $a = 1$ for now), $c(q) = 0.5q^2$, except we change G from $G = S(q_2) = 0.0191$ at $q_2 = 0.01$, to that of $G = \bar{S} = 0.0867$ at $q_2 = \tilde{q}_2 = 0.381$.

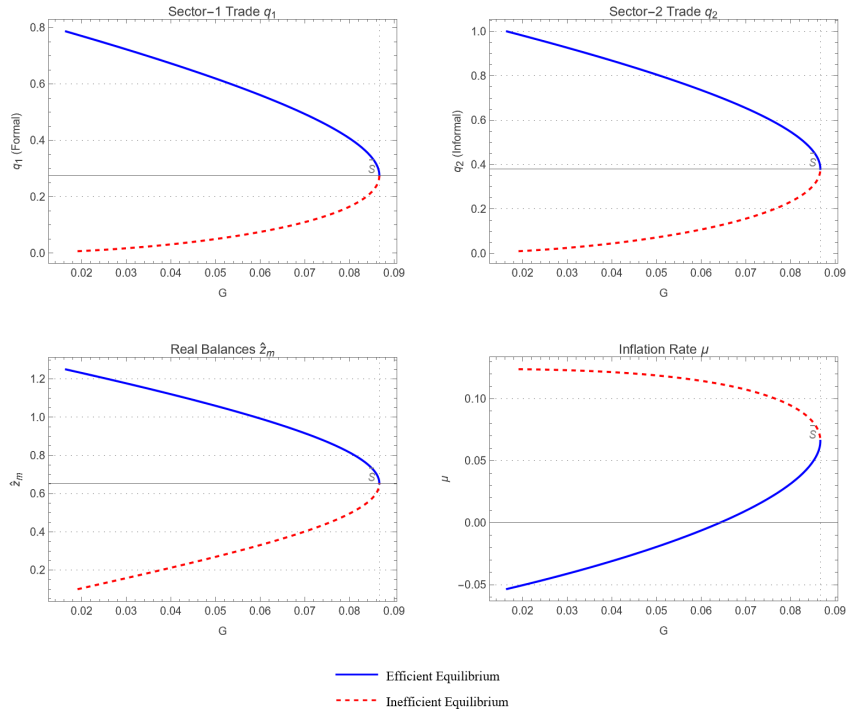


Figure 4: Effects of Changing G

(2) Effects of changing τ

Similarly, let's first make theoretical analysis for the effects of changing τ. From

(8) and (9), we know

$$G = S(q_2;\tau) = \upsilon(q_2)(\mu + \frac{\alpha\rho_m\tau}{1+\tau}),$$

and $\hat{z}_m = \upsilon(q_2)$, $q_1 = q_1(q_2;\tau)$, $\mu = \mu(q_2;\tau)$. Hence,

$$\frac{\partial q_2}{\partial \tau} = -\frac{S_\tau(q_2;\tau)}{S_{q_2}(q_2;\tau)} = -\frac{\upsilon(q_2)[\mu_\tau + \alpha\rho_m/(1+\tau)^2]}{\upsilon'(q_2)[\mu(q_2) + \alpha\rho_m\tau/(1+\tau)] + \upsilon(q_2)\mu'(q_2)}.$$

Here

$$\mu_\tau = -\frac{\theta\beta\alpha\rho_m\lambda_1'[q_1(q_2)]u'(q_1)}{(1+\tau)^2[(1-\theta)u'(q_1) + \theta c'(q_1)]} > 0,$$

so $S_\tau(q_2;\tau) < 0$. Then we have

$$\frac{\partial q_2}{\partial \tau} \cong -S_{q_2}(q_2;\tau) = -\{\upsilon'(q_2)[\mu(q_2) + \frac{\alpha\rho_m\tau}{1+\tau}] + \upsilon(q_2)\mu'(q_2)\} \lessgtr 0,$$

since $\upsilon'(q_2) > 0$, $\mu'(q_2) < 0$. Furthermore, using (4), (11) and (10), we have

$$\begin{aligned}
\frac{\partial \hat{z}_m}{\partial \tau} &= \upsilon'(q_2)\frac{\partial q_2}{\partial \tau} \cong -S_{q_2}(q_2;\tau) \\
\frac{\partial q_1}{\partial \tau} &= A(\tau,\theta)\frac{\upsilon'(q_2)}{\upsilon'(q_1)}\frac{\partial q_2}{\partial \tau} - \frac{2\theta\upsilon(q_2)}{(1+\tau)^2\upsilon'(q_1)} \\
\frac{\partial \mu}{\partial \tau} &= \frac{\beta\alpha}{2}\rho_m\lambda_1'(q_1)[\frac{\partial q_1}{\partial \tau} - \frac{2\theta u'(q_1)}{(1+\tau)^2\upsilon'(q_1)}] + \frac{\beta\alpha}{2}(1-\rho_m)\lambda_2'(q_2)\frac{\partial q_2}{\partial \tau}.
\end{aligned}$$

It seems that all CS regarding changing τ is ambiguous.

Hence, we now resort to numerical analysis to explicitly show the effects of changing τ. From Proposition 1, we conject there can be a regime switch when changing τ: $\tau_{LB} \equiv (1-\beta)/[\alpha\rho_m - (1-\beta)$ is the lower bound of τ, and there is another critical value of τ when $G = S(q_{FR})$, then the model economy switches from two equilibrium, with $S(q_{FR}) \leq G < \bar{S}$, to one equilibria, with $G < S(q_{FR})$. Indeed, using the same parameter values and functional forms as before, except now we allow τ change from $\tau_{LB} = 0.143$, to $\tau^* = 0.454$ (when $G = 0.08 = S(q_{FR})$) until τ is close to 1. We label the first regime with 2 equilibria, as Regime A (see Figure 5), with $\tau_{LB} = 0.143 < \tau < \tau^* = 0.454$, and the second one with 1 unique equilibrium as Regime B (see Figure 6), with $\tau^* = 0.454 < \tau < 1$.

Figure 5 shows that, in a "good" efficient equilibrium (*blue* curves), a higher tax

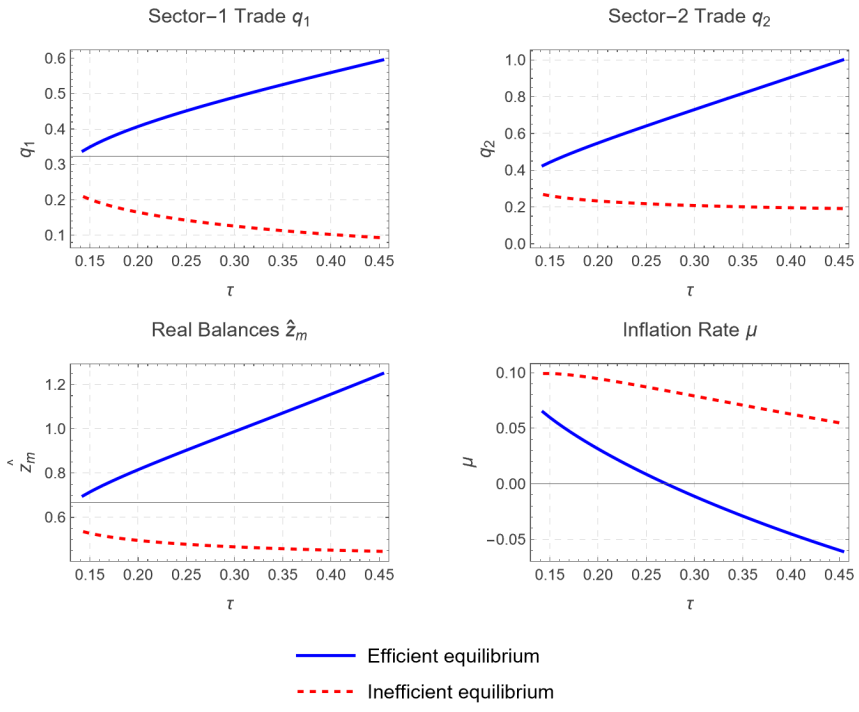


Figure 5: Effects of Changing τ (Regime A)

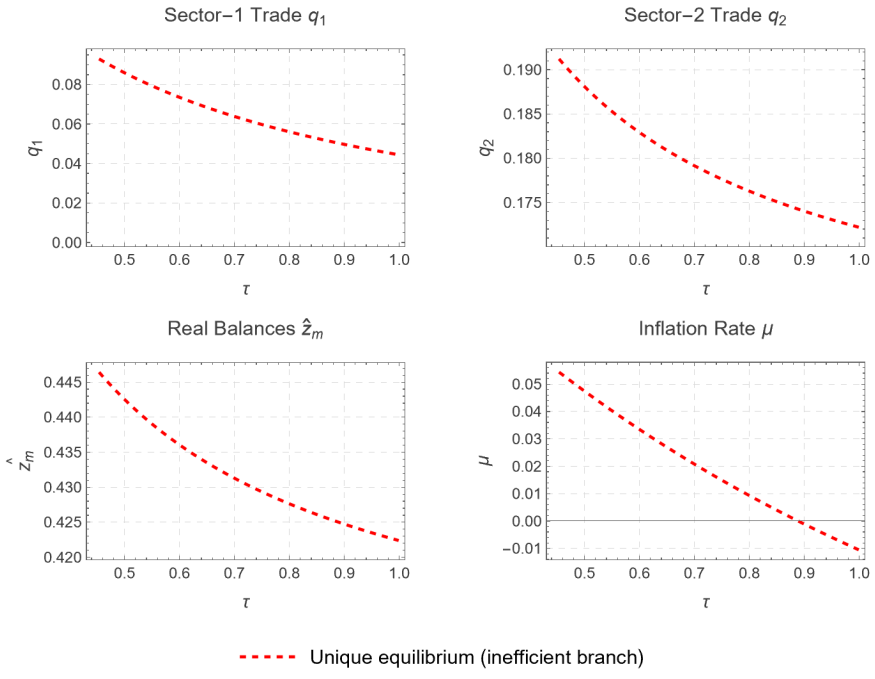


Figure 6: Effects of Changing τ (Regime B)

rate τ generates higher tax revenues so the equilibrium inflation rate μ decreases, which then supports bigger amounts of money holdings ($\hat{z}_m$ increases), and higher tradings in both Sector 1 and 2 (q_1, q_2 both increases). In contrast, if the economy is in a "bad" inefficient equilibrium (red dashed curves), a higher τ still leads to a lower μ, but can't help the real economy due to possibly too high inflation, so $\hat{z}_m$, q_1, q_2 all decreases. It is the same in Regime B.

Notice that the effects of changing τ on q_1, q_2 have the same patterns: in an efficient equilibrium (like in Regime A), a higher τ increases both q_1 and q_2, which seems strange, since there is no taxation in Sector 2, then a higher τ should benefit Sector 2, but hurt Sector 1. From GBC, a higher τ means higher tax revenues so a lower inflation rate is needed to generate seigniorage, which further benefits Sector 2, but the equilibrium effect on Sector 1 will depend on the net effect from a negative impact from the taxation channel and a positive one from the inflation channel. However, we have the sector size ρ_m fixed, then there is no extensive-margin reallocation channel, from Sector 1 to Sector 2. This explains the "weird" patterns for both q_1 and q_2, no matter in an efficient equilibrium (in Regime A), or an inefficient one (no matter in Regime A or Regime B). We will explore more on this, once we allow the sector size to be endogenous later.

(3) Effects of changing ρ_m

First, we conduct the theoretical analysis regarding the effects of changing ρ_m. By GBC, we have

$$\frac{\partial q_2}{\partial \rho_m} = -\frac{S_{\rho_m}(q_2;\rho_m)}{S_{q_2}(q_2;\rho_m)} = -\frac{\upsilon(q_2)[\frac{\beta\alpha}{2}(\lambda_1-\lambda_2)+\frac{\alpha\tau}{1+\tau}]}{\upsilon'(q_2)[\mu(q_2)+\frac{\alpha\rho_m\tau}{1+\tau}]+\upsilon(q_2)\mu'(q_2)} \cong -S_{q_2}(q_2;\rho_m),$$

where we use $q_1 < q_2$ so $\lambda_1 > \lambda_2$, then $S_{\rho_m}(q_2;\rho_m) > 0$, but we also know $S_{q_2}(q_2;\rho_m) =$

$S'(q_2) \gtrless 0$. Furthermore, using the equilibrium conditions, we have

$$\frac{\partial \hat{z}_m}{\partial \rho_m} = \upsilon'(q_2)\frac{\partial q_2}{\partial \rho_m} \cong \frac{\partial q_2}{\partial \rho_m}$$
$$\frac{\partial q_1}{\partial \rho_m} = \frac{A(\tau,\theta)\upsilon'(q_2)}{\upsilon'(q_1)}\frac{\partial q_2}{\partial \rho_m} \cong \frac{\partial q_2}{\partial \rho_m}$$
$$\frac{\partial \mu}{\partial \rho_m} = \mu'(q_2)\frac{\partial q_2}{\partial \rho_m} + \frac{\beta\alpha}{2}(\lambda_1 - \lambda_2).$$

We already know $\mu'(q_2) < 0$, $\lambda_1 - \lambda_2 > 0$, so the sign of the first term is ambiguous and that of the second one is positive, for the sign of $\partial\mu/\partial\rho_m$.

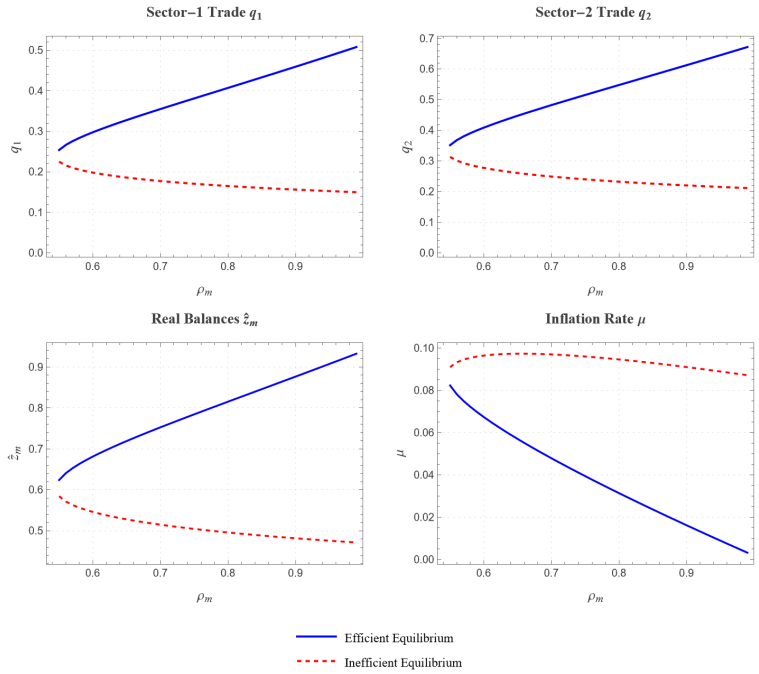


Figure 7: Effects of Changing ρ_m

In general, the signs of the above CS are ambiguous, although we do know the CS of changing ρ_m on q_1, q_2 and $\hat{z}_m$ have the same signs. Hence, we now resort to numerical analysis for explicit effects of changing ρ_m. And the numerical results are shown in Figure 7. Given the same parameter values and functional forms (except now changing ρ_m), first we can derive $\rho_m > 0.2$, by (15), then set the range $\rho_m \in (0.2, 0.99)$.[6] Indeed, Figure 7 shows, no matter in a "good"/efficient equilibrium, or

[6]Figure 7 still shows ρ_m starts from a much bigger value than 0.2, because $\rho_m = 0.2$ can't generate equilibria for $G = 0.08$.

a "bad"/inefficient one, the curves showing the effects of changing ρ_m on q_1, q_2 and $\hat{z}_m$ move in the same direction: they all increases in the former, and all decreases in the latter.

3 Endogenous Dichotomy

Now we consider free choices of agents to endogenize ρ. Cash still circulates in both sectors as in Section 2 , but now agents can choose to enter Sector 1 or Sector 2. As mentioned before, if an agent chooses Sector 2, he need to bear an individual-specific cost, κ, with CDF $F(\kappa)$. Suppose there exists a threshold value $\hat{\kappa}$, so that the agent is indifferent to choose Sector 1 or 2. That is,

$$V_1(\hat{z}_m) = V_2(\hat{z}_m, \hat{\kappa}).$$

Hence,

$$\frac{\alpha}{2}[u(q_1) - c(q_1) - 2\tau d_1] = -\hat{\kappa} + \frac{\alpha}{2}[u(q_2) - c(q_2)].$$

Then we have

$$\hat{\kappa} = \frac{\alpha}{2}[u(q_2) - c(q_2)] - \frac{\alpha}{2}[u(q_1) - c(q_1)] + \frac{\alpha\tau\upsilon(q_2)}{1+\tau} \equiv \Delta(q_1, q_2), \tag{16}$$

where we use $d_1 = \hat{z}_m/(1+\tau) = \upsilon(q_2)/(1+\tau)$, and $\Delta(q_1, q_2)$ denotes the surplus gap between Sector 2 and Sector 1. Notice that here the threshold value $\hat{\kappa}$ is not exogenous or constant any more. Instead, in equilibrium, $\hat{\kappa} = \Delta(q_1, q_2)$, i.e., the threshold value of the disutility cost choosing Sector 2 is exactly equal with the surplus gap of these two sectors. It is equivalent to rewrite $\Delta(q_1, q_2) \equiv \Delta(q_1(q_2), q_2)$, since q_1 is also a function of q_2, with $\upsilon(q_1) = A(\tau, \theta)\upsilon(q_2)$ from the bargaining solutions. As aforementioned, with τ, $\theta \in (0, 1)$ then $0 < A(\tau, \theta) < 1$, we always have $q_1 < q_2$, which also means $\hat{\kappa} = \Delta(q_1, q_2) > 0$, by (16).

More importantly, with $1 - \rho_m = F(\hat{\kappa})$, then

$$1 - \rho_m = F \circ \Delta(q_1(q_2), q_2). \tag{17}$$

Suppose κ is in a uniform distribution of the support $[\sigma, \gamma]$, $\sigma, \gamma > 0$, then $F(\kappa) = \kappa/(\gamma - \sigma)$, where $\gamma - \sigma$ determines the range of the distribution, and economically, can be interpreted as the range of a disutility cost, or "resistance" an agent faces when choosing Sector 2. For simplicity, we let $\sigma = 0$, so the support is $[0, \gamma]$.[7] Thus we can rewrite (17) as

$$1 - \rho_m = \frac{\hat{\kappa}}{\gamma} = \frac{\Delta(q_1(q_2), q_2)}{\gamma}.$$

Notice that, if

$$0 < \Delta(q_1(q_2), q_2) < \gamma, \tag{18}$$

we have $0 < \rho_m < 1$, i.e., there is an interior solution for ρ_m. As explained earlier, we always have $\Delta(q_1(q_2), q_2) = \hat{\kappa} > 0$, for τ, $\theta \in (0, 1)$, then rule out the case $\Delta(q_1(q_2), q_2) < 0$ (then $\rho_m = 1$), which means we always have $\rho_m < 1$. But it is possible to have the corner solution $\rho_m = 0$, i.e., all agents choose Sector 2, if $\gamma \leq \Delta(q_1(q_2), q_2)$. Equivalently, if we define

$$\hat{q}_2 \equiv \arg\max \Delta(q_1(q_2), q_2),$$

then we need to have

$$\gamma > \hat{\gamma} \equiv \max \Delta(\hat{q}_2), \tag{19}$$

to guarantee $0 < \rho_m < 1$.

In addition, *ceteris paribus*, a smaller γ, representing lower inherent resistance or disutility for agents choosing Sector 2, means a higher measure of Sector 2, or a lower measure of Sector 1, i.e., a lower ρ_m. Conversely, a higher $\hat{\kappa}$ or Δ, means a higher measure of Sector 2 (or a lower ρ_m). Since $\hat{\kappa} = \Delta(q_1, q_2)$ represents the surplus gap between Sector 2 and Sector 1 (before accounting for the cost of choosing Sector 2), a larger $\hat{\kappa}$ signifies a wider surplus gap, thus raising the "tolerating" threshold for

[7] In math, a small γ indicates that the distribution of κ is more concentrated around zero, and the range of possible κ values is narrow. In the uniform distribution, it means most or all agents have relatively low values of κ, and the average κ value is also small ($\gamma/2$). In contrast, if γ is large, the range of possible κ values is wide. This means that while some agents might still have low κ values, there are also many agents with high κ values, extending all the way up to γ. The average κ value would be much larger ($\gamma/2$).

agents opting for Sector 2. Furthermore, given $\gamma > 0$, we have

$$\frac{\partial \hat{\kappa}}{\partial \rho_m} = -\gamma < 0,$$

which means, with the increasing of ρ_m, the threshold $\hat{\kappa}$ decreases. This well captures the reality: an increase in the formal-sector share ρ_m is accompanied by ongoing development and the consolidation of the judiciary, regulatory enforcement, tax administration, and the welfare system; as the disparity between Sector 1 and Sector 2 diminishes, the threshold value $\hat{\kappa}$ for choosing Sector 2 declines.

3.1 Equilibrium Analysis

Now we can rewrite the equilibrium conditions as

$$\begin{aligned} \rho_m &= 1 - \frac{1}{\gamma}\Delta(q_1(q_2), q_2) \\ G &= \upsilon(q_2)[\mu(q_2) + \rho_m \frac{\alpha\tau}{1+\tau}], \end{aligned} \tag{20}$$

where $q_1 = q_1(q_2)$ from (11), and $\mu(q_2)$ from (10). Furthermore, we can substitute ρ_m into GBC to have,

$$G = \upsilon(q_2)\{\mu(q_2) + \frac{\alpha\tau}{1+\tau}[1 - \frac{1}{\gamma}\Delta(q_1(q_2), q_2)]\} \equiv S(q_2), \tag{21}$$

which can solve for q_2, given G, τ, and other parameters. Using (20) and other equilibrium conditions, we can further solve for $\{\rho_m, q_1, \hat{z}_m, \mu\}$. Notice that, in (21), again we rewrite the RHS of GBC as $S(q_2)$, and

$$S'(q_2) = \upsilon'(q_2)[\mu(q_2) + \frac{\alpha\tau\rho_m(q_2)}{1+\tau}] + \upsilon(q_2)[\mu'(q_2) + \frac{\alpha\tau\rho'_m(q_2)}{1+\tau}], \tag{22}$$

where

$$\mu'(q_2) = \frac{\beta\alpha}{2}[\rho_m\lambda'_1(q_1)\cdot q'_1(q_2) + (1-\rho_m)\lambda'_2(q_2) - \frac{\Delta'(q_2)}{\gamma}(\lambda_1(q_1) - \lambda_2(q_2))], \tag{23}$$

with $\rho'_m(q_2) = -\frac{\Delta'(q_2)}{\gamma}$, and

$$\begin{aligned}\Delta'(q_2) &= \frac{\alpha}{2}\{[u'(q_2) - c'(q_2)] - [u'(q_1) - c'(q_1)]q'_1(q_2)\} + \frac{\alpha\tau}{1+\tau}\upsilon'(q_2) \\ &= \frac{\alpha}{2}\{[u'(q_2) - c'(q_2)] - [u'(q_1) - c'(q_1)]\frac{A(\tau,\theta)\upsilon'(q_2)}{\upsilon'(q_1)}\} + \frac{\alpha\tau}{1+\tau}\upsilon'(q_2). \quad (24)\end{aligned}$$

We already know $q_2 > q_1$, then $\upsilon'(q_2) > \upsilon'(q_1)$; but $A(\tau,\theta) < 1$, then we can't know $q'_1(q_2) = A(\tau,\theta)\upsilon'(q_2)/\upsilon'(q_1)$ is greater or smaller than 1. Although we do now $[u'(q_2) - c'(q_2)] < [u'(q_1) - c'(q_1)]$, with $H(q) \equiv u'(q) - c'(q)$ being decreasing in q, we can't know for sure the sign of the term in the big bracket, nor that of $\Delta'(q_2)$. Furthermore, we can't sign $\mu'(q_2)$ since $\Delta'(q_2)$ appears in the third term, by (23). In the end of the day, we can't sign $S'(q_2)$, so are not sure of the monotonicity of $S(q_2)$. This also explains why multiple equilibria are natural: $\rho'_m(q_2)$ feeds back to $\mu'(q_2)$, then into the tax term, i.e., the second term in the big bracket of $S(q_2)$.

In addition, if we look more carefully at (21) and (22), γ, τ are important parameters that determine the monotonicity of $S(q_2)$, then determine equilibrium existence and uniqueness, except the model primitives like β, α, θ. The former, γ, can be interpreted as an "institutional" constraint, with a small γ representing weak institutions so small disutility cost to choose Sector 2 (Informal), while a big one representing a large disutility cost. The latter, τ, is basically a policy parameter, which is the tax rate only levied in Sector 1 (Formal), hence changing τ can affect agents' choices to Sector 1 and 2. Using (20) and (10), we have

$$\begin{aligned}\rho_{m,\gamma} &= \frac{d\rho_m}{d\gamma} = \frac{1}{\gamma^2}\Delta(q_1(q_2), q_2) > 0 \\ \mu_\gamma &= \frac{d\mu}{d\gamma} = \frac{\beta\alpha}{2}\frac{d\rho_m}{d\gamma}[\lambda_1(q_1) - \lambda_2(q_2)] = \frac{\beta\alpha}{2\gamma^2}\Delta(q_1(q_2), q_2)[\lambda_1(q_1) - \lambda_2(q_2)] > 0,\end{aligned}$$

where we use $\Delta(q_1(q_2), q_2) = \hat{\kappa} > 0$, and $q_1 < q_2$, then $\lambda_1(q_1) > \lambda_2(q_2)$. Therefore, we

have

$$S_\gamma = \frac{dS(q_2;\gamma,\tau)}{d\gamma} = \upsilon(q_2)(\mu_\gamma + \rho_{m,\gamma}\frac{\alpha\tau}{1+\tau})$$
$$= \frac{\upsilon(q_2)}{\gamma^2}\Delta(q_1(q_2),q_2)\{\frac{\beta\alpha}{2}[\lambda_1(q_1)-\lambda_2(q_2)] + \frac{\alpha\tau}{1+\tau}\} > 0, \tag{25}$$

which means, *ceteris paribus*, a higher γ leads to a larger $S(q_2)$. Then

$$\frac{\partial q_2}{\partial \gamma} = -\frac{S_\gamma}{S'(q_2)} \cong -S'(q_2).$$

Using other equilibrium conditions, we can derive

$$\frac{\partial q_1}{\partial \gamma} = q_1'(q_2)\frac{\partial q_2}{\partial \gamma} \cong -S'(q_2)$$
$$\frac{\partial \rho_m}{\partial \gamma} = -\frac{1}{\gamma}\Delta'(q_2)\frac{\partial q_2}{\partial \gamma} + \frac{1}{\gamma^2}\Delta(q_1(q_2),q_2)$$
$$\frac{\partial \hat{z}_m}{\partial \gamma} = \upsilon'(q_2)\frac{\partial q_2}{\partial \gamma} \cong -S'(q_2)$$
$$\frac{\partial \mu}{\partial \gamma} = \mu'(q_2)\frac{\partial q_2}{\partial \gamma} + \frac{\beta\alpha}{2\gamma^2}\Delta(q_1(q_2),q_2)[\lambda_1(q_1)-\lambda_2(q_2)],$$

where $\Delta'(q_2)$ is from (24), and $\mu'(q_2)$ from (23). To sum up, although their signs are ambiguous, we can see the signs of the effects of changing γ on q_1, q_2 and $\hat{z}_m$ are all the same. We leave the effects of changing τ to policy analysis later.

Furthermore, we can use (21) to check the condition to generate maximum government revenues. Similarly, we can solve for $q_2 \equiv \tilde{q}_2$ to support the fiscal limit $\bar{S} = S(\tilde{q}_2)$, with

$$\bar{S} = \max_{q_2} S(q_2).$$

Then the FOC is,

$$S'(\tilde{q}_2) = 0.$$

Using (22), we then derive

$$\frac{\upsilon'(\tilde{q}_2)}{\upsilon(\tilde{q}_2)} = -\frac{\mu'(\tilde{q}_2) + \frac{\alpha\tau}{1+\tau}\rho_m'(\tilde{q}_2)}{\mu(\tilde{q}_2) + \frac{\alpha\tau}{1+\tau}\rho_m(\tilde{q}_2)}. \tag{26}$$

That is,

$$\frac{(1-\theta)u'(\tilde{q}_2)+\theta c'(\tilde{q}_2)}{(1-\theta)u(\tilde{q}_2)+\theta c(\tilde{q}_2)} = -\frac{\frac{\beta\alpha}{2}\rho_m\lambda_1'(q_1)\cdot q_1'(\tilde{q}_2)+\frac{\beta\alpha}{2}(1-\rho_m)\lambda_2'(\tilde{q}_2)-\frac{\Delta'(q_2)}{\gamma}[\frac{\beta\alpha}{2}(\lambda_1(q_1)-\lambda_2(\tilde{q}_2))+\frac{\alpha\tau}{1+\tau}]}{\beta-1+\frac{\beta\alpha}{2}[\rho_m\lambda_1(q_1(\tilde{q}_2))+(1-\rho_m)\lambda_2(\tilde{q}_2)]+\frac{\alpha\tau}{1+\tau}\rho_m(\tilde{q}_2)}. \tag{27}$$

Since $\rho_m = \rho_m(q_2;\gamma)$, we can use the above explicit expression of the FOC, (27), to derive a threshold value $\tilde{\gamma}$: if $\gamma \geq \tilde{\gamma}$, $G \leq \bar{S} = S(\tilde{q}_2)$, then general equilibrium exists. With the inequality by (19), we have

$$\gamma > \max\{\hat{\gamma}, \tilde{\gamma}\}. \tag{28}$$

Due to nonlinearity of (27), we provide a numerical analysis for the effects of changing γ, by Table 1 and Figure 9. Given $G = 0.08$, other parameter values and standard functional forms as in Section 1, we can derive $\tilde{\gamma} = 0.19$, to generate $\bar{S} = G = 0.08$. We can also derive $\hat{\gamma} = 0.11$, using (19). Hence, we can derive the final range of γ: $\gamma \geq \max\{\hat{\gamma}, \tilde{\gamma}\} = 0.19$. Furthermore, we also provide Table 1 for various values of γ, $\bar{S}$ and endogenous ρ_m.

Table 1. Fiscal Limit Frontier - Numerical

γ **Value**	**Fiscal Limit** $\bar{S}$	ρ_m **(at peak)**
0.19	0.08	0.55
0.50	0.0872	0.818
1.00	0.0899	0.907
2.00	0.0914	0.953
5.00	0.0923	0.981
15.00	0.0928	0.994
20.00	0.0928	0.995
100	0.0929	0.999

We also use Figure 8 to provide a "saturation" analysis for the "Fiscal Limit Frontier": with minimum $\gamma \geq 0.19$, the equilibrium exists above that threshold (below which is the "Collapse Zone", no equilibrium), and we can see that, although the fiscal

limit $\bar{S}$ increases with bigger γ, consistent with $S_\gamma > 0$ by (25), there is a "Saturation effect" (or diminishing returns of changing γ). The fiscal limit frontier rises sharply from $\gamma = 0.2$ to (possibly) $\gamma = 2.0$, but is almost flat from $\gamma = 5.0$ to $\gamma = 20$. This confirms that marginal institutional improvements in already developed economies (high γ) have negligible effects on fiscal capacity, whereas they can be critical for developing economies (low γ).

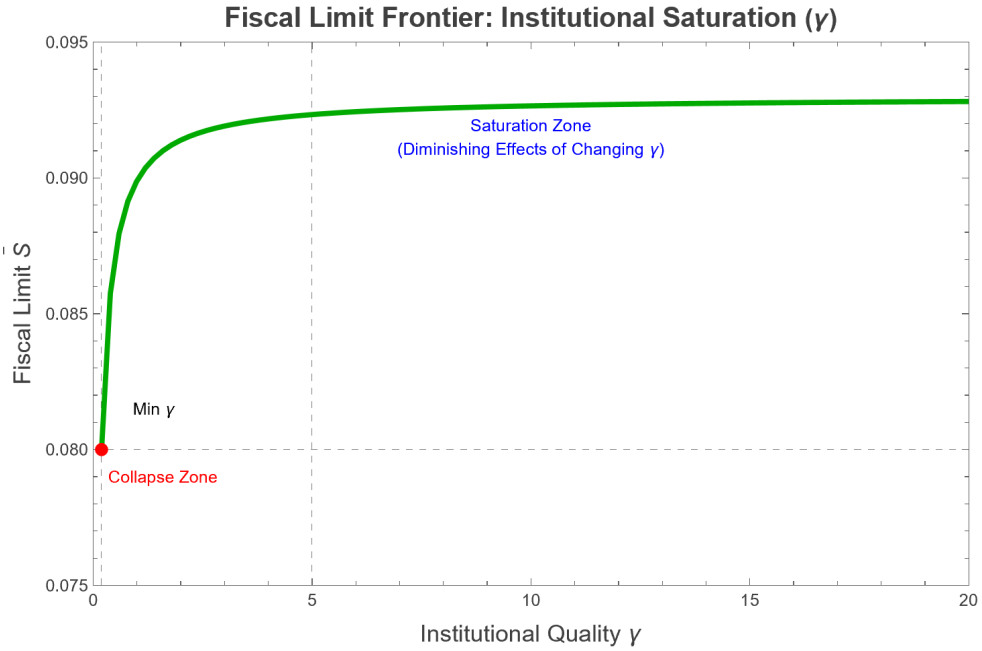


Figure 8: Fiscal Limit Frontier: $\bar{S}$ & γ

In addition, we also provide the numerical analysis of changing γ on all variables, $\{q_1, q_2, \hat{z}_m, \mu, \rho_m\}$, plus on $\hat{\kappa} = \Delta(q_2)$, in Figure 9. We vary γ for the range of (0,5), and plot the results in both "Efficient Equilibrium" (blue) and "Inefficient Equilibrium" (red, dashed).

We can also check the case at Friedman Rule, i.e., $\mu = \beta - 1$, then $q_1 = q_1^*$, $q_2 = q_{FR} = q^*$, where q_1^*, q_{FR} are the same as in Section 2. Furthermore, using (21), we have

$$S(q_{FR}) = \upsilon(q_{FR})[\beta - 1 + \frac{\alpha\tau}{1+\tau}\rho_m(q_{FR})],$$

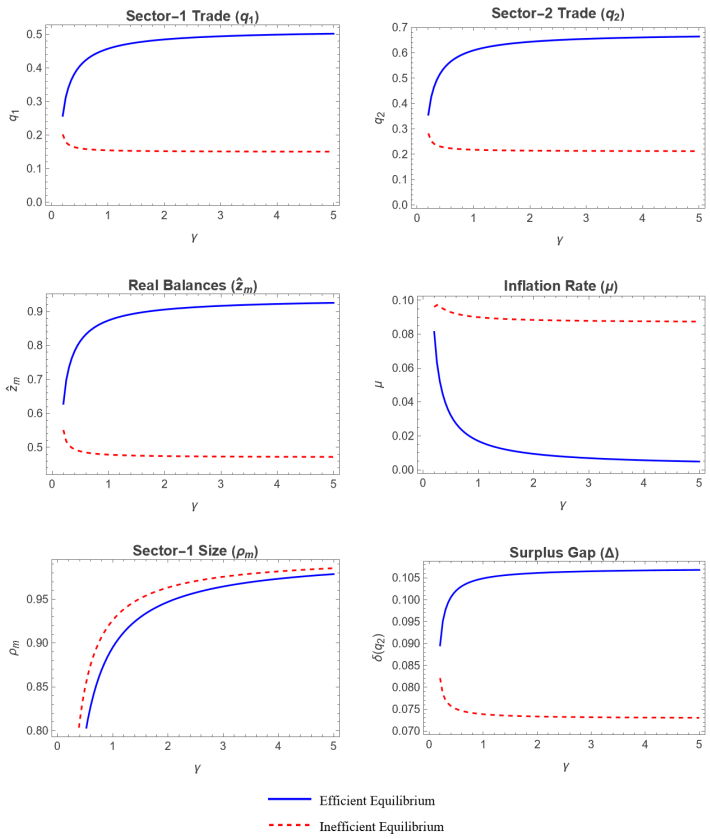


Figure 9: Effects of Changing γ

with

$$\rho_m(q_{FR}) = 1 - \frac{1}{\gamma}\{\frac{\alpha}{2}[u(q_{FR}) - c(q_{FR})] - \frac{\alpha}{2}[u(q_1^*) - c(q_1^*)] + \frac{\alpha\tau}{1+\tau}\upsilon(q_{FR})\}.$$

Hence, to have an equilibrium existing at Friedman Rule, we need to have

$$S(q_{FR}) = G_{|\mu=\beta-1} > 0,$$

which means

$$\rho_m(q_{FR}) > \frac{(1-\beta)(1+\tau)}{\alpha\tau}.$$

That is,

$$\Delta(q_{FR}) < \gamma[1 - \frac{(1-\beta)(1+\tau)}{\alpha\tau}], \tag{29}$$

where

$$\Delta(q_{FR}) \equiv \frac{\alpha}{2}[u(q_{FR}) - c(q_{FR})] - \frac{\alpha}{2}[u(q_1^*) - c(q_1^*)] + \frac{\alpha\tau\upsilon(q_{FR})}{1+\tau}.$$

Therefore, if the above condition (29) is satisfied, there exists an equilibrium at the Friedman Rule.

Figure 10 provides the numerical results of different equilibrium regimes, given $\gamma = 2.0$, $G = 0.08$, $\tau = 0.20$, other parameter values and functional forms as in Section 1. Notice that there is a major difference (from Section 1) is that now ρ_m is endogenous, not fixed any more. Figure 10 shows there are two equilibrium solutions, $q_2 = \{0.214, 0.644\}$; the fiscal limit is $\bar{S} = 0.091$ at $\tilde{q}_2 = 0.413$, the endogenous $\rho_m = 0.953$; and at Friedman Rule, $S(q_{FR}) = 0.031$, $q_{FR} = 1$, $\rho_m = 0.943$. It verifies that, with $S(q_{FR}) = 0.031 < G = 0.08 < \bar{S} = 0.091$, there are dual equilibria. We can also see that, if $G < S(q_{FR})$, there will be a unique equilibrium at the inefficient side (the left branch).

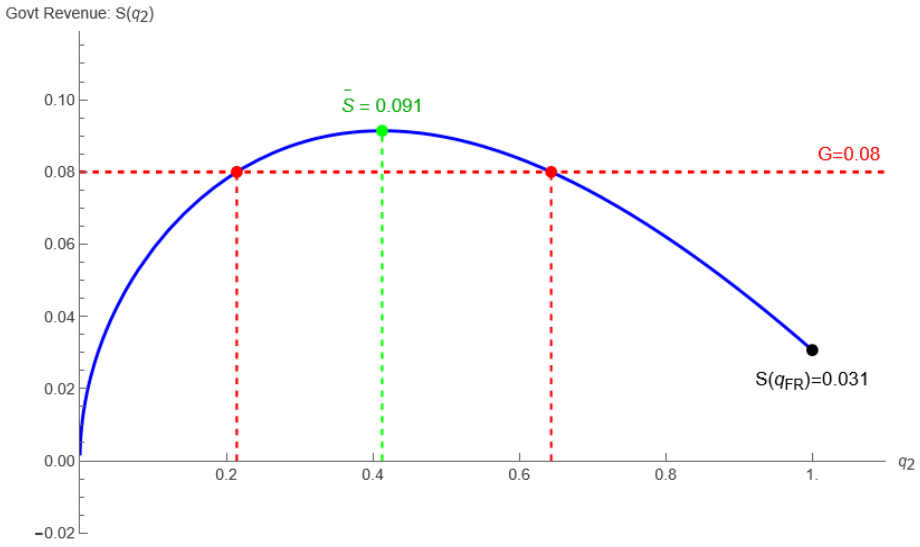


Figure 10: Endogenous Dichotomy: Laffer Curve and Equilibrium Regimes

We sum up the *equilibrium existence and uniqueness conditions* in Proposition 2:

Lemma 1 ***Proposition 2*** *If $G \leq \bar{S} = S(\tilde{q}_2)$, a monetary equilibrium exists. There is a knife-edge case of equilibrium at $G = \bar{S}$, then $q_2 = \tilde{q}_2$. If (29) is satisfied, there exists an equilibrium at the Friedman Rule, i.e., $\mu = \beta - 1$, $q_2 = q_{FR}$, and $G = S(q_{FR})$. As for the equilibrium uniqueness, if $S(q_{FR}) \leq G < \bar{S}$, there are two equilibria: an "Efficient Equilibrium" with high q_2 (and low μ), and an "Inefficient Equilibrium"*

with with low q_2 (and high μ); if $G < S(q_{FR})$, there is an unique equilibrium at the "Inefficient" branch with low q_2 (and high μ).

3.2 Policy Analysis

1. Effects of changing G

Using (21) and other equilibrium conditions, we have,

$$\frac{\partial q_2}{\partial G} = \frac{1}{S'(q_2)} \cong S'(q_2),\ \frac{\partial q_1}{\partial G} = \frac{A(\tau,\theta)\upsilon'(q_2)}{\upsilon'(q_1)}\frac{1}{S'(q_2)} \cong S'(q_2),\ \frac{\partial \hat{z}_m}{\partial G} = \frac{\upsilon'(q_2)}{S'(q_2)} \cong S'(q_2)$$

$$\frac{\partial \rho_m}{\partial G} = -\frac{\Delta'(q_2)}{\gamma}\frac{\partial q_2}{\partial G} = -\frac{\alpha}{2\gamma S'(q_2)}\{[u'(q_2)-c'(q_2)]-[u'(q_1)-c'(q_1)]q_1'(q_2)+\frac{2\tau\upsilon'(q_2)}{1+\tau}\}$$

$$\frac{\partial \mu}{\partial G} = \frac{\beta\alpha}{2}[\rho_m\lambda_1'(q_1)\cdot q_1'(q_2)+(1-\rho_m)\lambda_2'(q_2)-\frac{\Delta'(q_2)}{\gamma}(\lambda_1(q_1)-\lambda_2(q_2))]\frac{1}{S'(q_2)}.$$

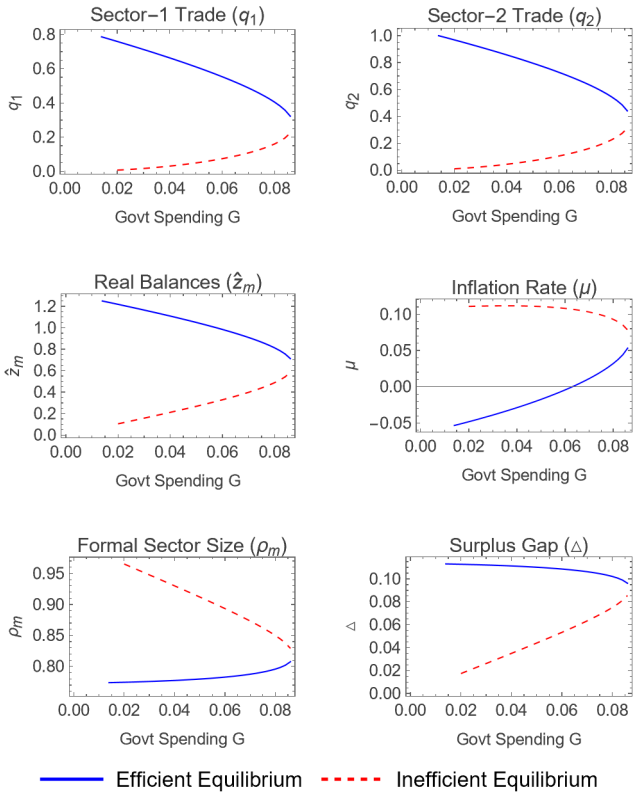


Figure 11: Effects of Changing G ($\gamma = 0.5$, $\tau = 0.20$)

Figure 11 provides numerical analysis regarding the effects of changing G from 0.01 to $\bar{S} = 0.0872$, given $\gamma = 0.50$, $\tau = 0.20$, other parameters and functional forms

same as before. In an "Efficient Equilibrium", with the tax rate τ unchanged, a higher G need to have higher seigniorage revenues to support, which need to have higher μ. But higher inflation rates causes agents to hold less money, and trade less in both sectors, so $\hat{z}_m$, q_1, q_2 all decreases, which also decreases the surplus gap Δ, so the endogenous size of Sector 1, ρ_m, increases instead (although not too much), by (20). In contrast, in an "Inefficient Equilibrium", we get exactly the opposite effects.

2. Effects of changing τ

Using (21), we have

$$\frac{\partial q_2}{\partial \tau} = -\frac{S_\tau(q_2;\tau)}{S_{q_2}(q_2;\tau)},$$

where $S_{q_2}(q_2;\tau) = S'(q_2)$, as in (22), and

$$\begin{aligned} S_\tau(q_2;\tau) = \upsilon(q_2)\frac{\beta\alpha}{2}[(\lambda_1(q_1) - \lambda_2(q_2))\frac{d\rho_m}{d\tau} - \frac{2\theta u'(q_1)\rho_m}{(1+\tau)^2\upsilon'(q_1)}] \\ + \upsilon(q_2)[\frac{\alpha\rho_m}{(1+\tau)^2} + \frac{\alpha\tau}{1+\tau}\frac{d\rho_m}{d\tau}]. \end{aligned}$$

Notice that, here we use the explicit expressions of $\mu(q_2;\tau)$ by (10), and that of $\lambda_1(q_1;\tau)$ by (6), $\rho_m(q_2;\tau)$ from (20), etc. Using (16) and (20), we have

$$\frac{d\rho_m}{d\tau} = -\frac{\alpha\upsilon(q_2)}{\gamma(1+\tau)^2} < 0.$$

Therefore,

$$S_\tau(q_2;\tau) = \frac{\alpha\upsilon(q_2)}{(1+\tau)^2}\{\rho_m - \frac{\alpha\tau\upsilon(q_2)}{\gamma(1+\tau)} - \frac{\beta}{2}[\frac{\alpha}{\gamma}(\lambda_1(q_1) - \lambda_2(q_2))\upsilon(q_2) + \frac{2\theta u'(q_1)}{\upsilon'(q_1)}\rho_m]\}.$$

It seems the signs of $S_{q_2}(q_2;\tau)$ and $S_\tau(q_2;\tau)$ are both ambiguous, so is that of $\partial q_2/\partial\tau$.

Using other equilibrium conditions, we can also derive

$$\frac{\partial q_1}{\partial \tau} = \frac{A(\tau,\theta)\upsilon'(q_2)}{\upsilon'(q_1)}\frac{\partial q_2}{\partial \tau} - \frac{2\theta\upsilon(q_2)}{(1+\tau)^2}$$
$$\frac{\partial \rho_m}{\partial \tau} = -\frac{\Delta'(q_2)}{\gamma}\frac{\partial q_2}{\partial \tau} - \frac{1}{\gamma}\frac{\alpha\upsilon(q_2)}{(1+\tau)^2}$$
$$\frac{\partial \hat{z}_m}{\partial \tau} = \upsilon'(q_2)\frac{\partial q_2}{\partial \tau} \cong \frac{\partial q_2}{\partial \tau}$$
$$\frac{\partial \mu}{\partial \tau} = \frac{\beta\alpha}{2}\{\rho_m\lambda_1'(q_1)\cdot q_1'(q_2) + (1-\rho_m)\lambda_2'(q_2) - \frac{\Delta'(q_2)}{\gamma}[\lambda_1(q_1)-\lambda_2(q_2)]\}\frac{\partial q_2}{\partial \tau}$$
$$- \frac{\beta\alpha}{2(1+\tau)^2}[\frac{\alpha}{\gamma}\upsilon(q_2)\lambda_1(q_1) + \frac{2\rho_m\theta u'(q_1)}{\upsilon'(q_1)}].$$

We conject that the signs of $\partial q_1/\partial\tau$, $\partial\rho_m/\partial\tau$, $\partial\hat{z}_m/\partial\tau$, $\partial\mu/\partial\tau$ are also ambiguous, so resort to numerical analysis, to fully show the effects of changing τ.

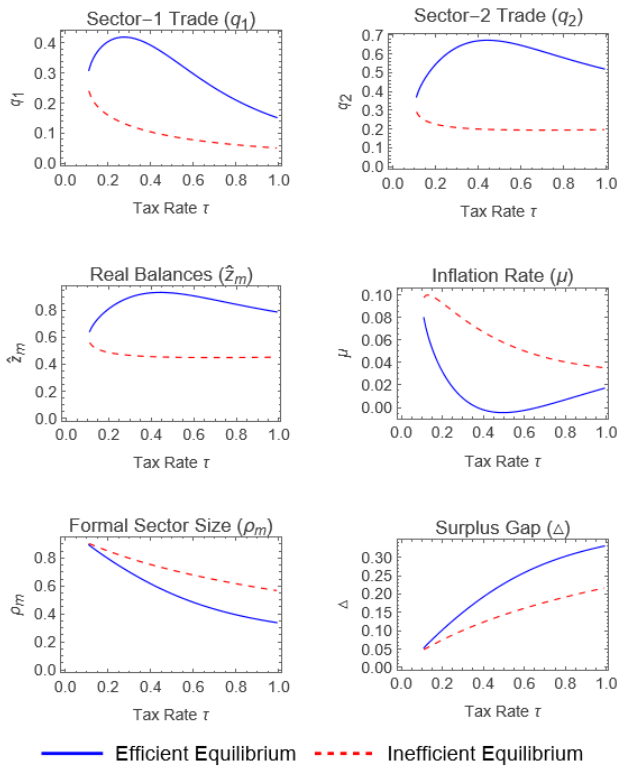


Figure 12: Effects of Changing τ ($\gamma = 0.5$)

Figure 12 provides the results on the effects of changing τ from $\tau_{\min} = 0.11$ to $\tau_{\max} = 0.99$,[8] given $G = 0.08$, $\gamma = 0.5$, other parameters and functional forms being

[8]Here we choose $\tau_{\min} = 0.11$ because we can derive $\tau \geq 0.104$, to have $G \leq \bar{S}$, then a general equilibrium exists.

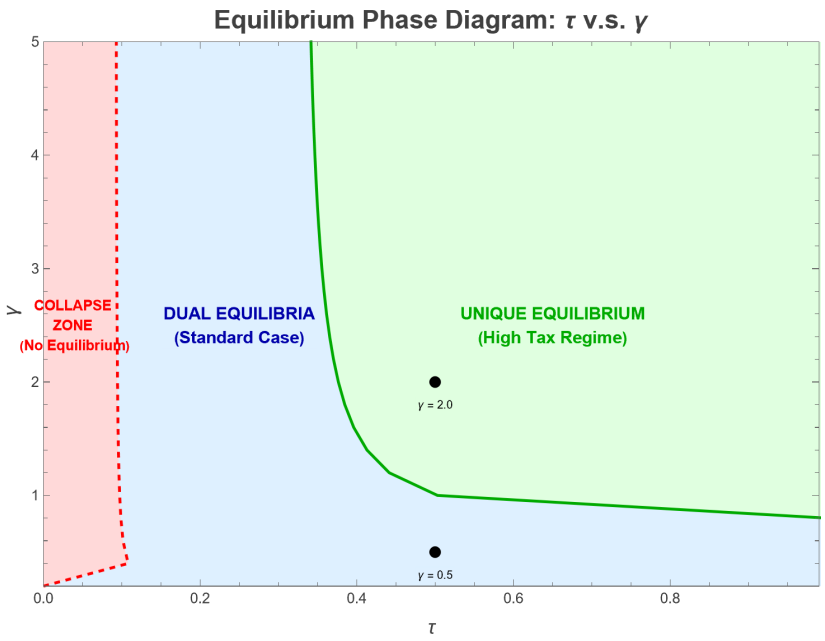


Figure 13: Equilibrium Phase Diagram: τ & γ

the same as before. We can see that, even just in an "*Efficient Equilibrium*" (blue), the effects of changing τ, particularly on q_1, q_2, $\hat{z}_m$, and μ, can be non-monotone. That is, with a higher τ, first, it contributes more to GBC, then not much seigniorage is needed for a fixed G, so the inflation rate μ decreases, which motivates agents to hold more money then $\hat{z}_m$ increases, benefiting trading in both Sector 1 and Sector 2 (thus q_1, q_2 both increase); however, when τ keeps increasing, it hurts the tradings in Sector 1, shrinks the tax base at the aggregate level, then higher seigniorage is needed to maintain the same G, so μ starts to increases, which furthermore hurts $\hat{z}_m$, q_1 and q_2. As for ρ_m, a higher τ increases the surplus gap Δ, then decreases ρ_m, by (20), which is shown in the last panels of Figure 12. In addition, in an "Inefficient Equilibrium" (red, dashed), there are not so many non-monotone effects from changing τ, but the mechanisms are similar.

3. Phase Diagram: τ and γ

Figure 13 provides an equilibrium phase diagram for $\tau \in (0, 0.99)$, $\gamma \in (0.2, 5.0)$, and different colors represent different equilibrium zones: no equilibrium (red), dual equilibrium (blue), and unique equilibrium (green).

4 Stablecoin Economy

To capture the public finance implication of fast-rising stablecoins, we add govt. bonds b, stablecoins m_s, to the baseline model developed in Section 2. We assume fiat money m_m (cash) initially circulates across the entire economy but faces direct technological competition from private stablecoins, in Sector 2. Ultimately, we focus on the regime where m_s replace cash in Sector 2. This setup transitions our framework from an economy reliant solely on seigniorage and taxes to a modern monetary-fiscal regime capturing government debt, bank intermediation, and stablecoin safe-asset demand.

4.1 Environment

We build upon the baseline environment outlined in Section 2.1, but augment the model with a banking sector, a representative stablecoin issuer, and a government bond market.

To microfound the coexistence of fiat money and interest-bearing bonds, we introduce a Diamond-Dybvig (1983) liquidity insurance motive of banks. Competitive banks, with the measure of "1", operate in CM, offering deposit contracts to agents entering Sector 1 (TradFi). Let $z_m \equiv \phi_m m_m$, $z_a \equiv \phi_m a$, $z_s \equiv \phi_m m_s$, and $z_b \equiv b$ denote the real balances of fiat money m_m, bank deposits a„ stablecoins m_s, and one-period government bonds b (real bonds), respectively. Notice that we set the same real price of fiat money (cash), ϕ_m as that of stablecoins, since stablecoins issuers make sure 1:1 pegging between m_s and m_m in CM (will discuss about depegging later). Banks pool all deposits, then acquire a portfolio of money (equivalently, reserves, z_m) and government bonds (z_b), and need to satisfy a reserve requirement: $\varepsilon z_a \leq z_m + z_b$, where $\varepsilon \in (0, 1)$ is the required reserve ratio, or more broadly, a parameter denoting capital requirement constraint of banks, e.g., a liquidity buffer ratio. Here $z_m + z_b$ can also be broadly interpreted as the value of reserve holdings by commerical banks. The required reserve constraint then means, a fraction ε of total deposits z_a issued by banks shall not be greater than the value of reserves $(z_m + z_b)$. Banks are compet-

itive, hence free entry drives bank profits to zero. Furthermore, following Williamson (2024), we assume banks have the technology of posting collateral to secure their deposit liabilities, and these claims can be costlessly verified and transferred via a communication network, granting banks the unique ability to create liquid inside money that ordinary agents cannot replicate. Due to limited commitment and lack of record-keepings, cash and bank deposits (but not government bonds) can serve as payment methods in Sector 1, while stablecoins and (or) cash facilitate trading in Sector 2.

As for Sector 2, the key feature is that the payment methods are anonymous, or will not generate any digital footprints. Suppose cash m_m still circulates in Sector 2, but now there is also another type of meetings that trades with stablecoins m_s. Basically these are two permanent types of meetings: with prob. α_2^m, agents trade with cash, as in any informal or small-amount transactions; with prob. α_2^s, agents trade with stablecoins, as in any crypto-related or big-amount transactions nowadays. If $\alpha_2^m = 0 < \alpha_2^s$, only stablecoins are accepted in Sector 2. This can be interpreted as the scenario that the innovation or quick development of payment technology leads to stablecoins totally phasing out cash in Sector 2.

As before, every period DM starts first, then CM opens. To explain things clearly, we describe the *timing and events* as follows.

1. In the end of every CM, after the idiosyncratic shocks are realized, agents make the choice to enter Sector 1 (pay τ taxes ex post) or Sector 2 (bear κ disutility cost ex ante) in DM. Simutanously, agents decide to hold fiat money or stablecoins. As mentioned below, excepts stablecoin transactions in Sector 2, agents all hold fiat money m_m, no matter choosing Sector 1 or Sector 2 when the shocks are realized. Agents choosing Sector 1 will contact banks to deposit money and sign deposit contracts with banks, and they can access to banking services before DM starts (*not* after), but they do not know they will be in type-m or type-a meetings in DM later. As for agents choosing Sector 2, they know their types immediatly: with prob. α_2^m, they trade with cash directly later in DM; with prob. $\alpha_2^s = 1 - \alpha_2^m$, they trade with stablecoins so contact stablecoin issuers f to acquire m_s. Notice that stablecoin issuers f are private

entrepreneurs, and they issue stablecoins m_s which can maintain anonymity to get around with government taxation, as cash.

For agents choosing Sector 1, they can deposit CM goods (or the equivalent in assets) at banks, to purchase deposit contracts from them: with prob. ω, they enter a type-m meetings of DM, then withdraw cash m_m in the end of CM; in type-a meetings, depositors just use deposit claims a for DM trading later. As in Williamson (2012), banks acquires enough deposits from depositors to hold a portfolio of z_m units of money and z_b units of real govt. bonds, which bears a nominal return $\iota_b \leq \iota_m$ due to non-arbitrage conditions, and ι_m is the nominal returns of an illiquid bonds next CM if agents choose to invest one dollar at the bonds today (see more details from Rocheteau et al., 2018).[9] Therefore, different from Williamson (2012), deposit claims a are inside money issued by banks (due to the fractional reserve system, or the required reserve constraint), and are the liability of banks, which can always keep 1:1 convertibility with fiat money, when measured by CM goods x (here we assume away the issues of bank runs and others).

2. Stablecoin issuers, f, are also active in CM: each f issues stablecoins m_s, holding govt. bonds z_b^s as reserves, following a narrow banking arrangement. Agents can use CM goods (or or the equivalent in assets) to purchase m_s from f, and then redeem stablecoins next CM (after DM tradings next period).[10] Although issuers claim it is a "narrow banking" arrangement, i.e., all stablecoins issued are backed up by 100% reserves (govt. bonds), due to moral hazard problems, they always have a motive to misrepresent their reserve holdings. Suppose they hold genuine reserves (bonds) up to a ratio $\chi \in (0, 1]$. If $\chi = 1$, there is no misrepresentation. If they misrepresent z_b^s bond holding information, they need to produce fake bonds in CM first before issuing new stablecoins, by incurring cost φz_b^s, $0 < \varphi < 1$. And fake bonds generate zero returns, but can't be distinguished by the public. Hence, it is a private information to f.

[9] That is, $\iota_m = (1 + \mu_m)/\beta - 1 \geq 0$, the Friedman Rule is equivalent to $\iota_m = 0$.

[10] This setup allows different ways to capture the roles of stablecoins, mainly as a medium of exchange in the crypto industry, but also increasingly as the bridge between the crypto industry and TradFi.

3. In DM, agents enter Sector 1 or Sector 2. In Sector 1, with prob. ω, sellers only accepts m_m, so buyers withdraw money from banks, paying by m_m (type-m meetings); with the rest prob., sellers accept bank deposits, and buyers just pay with deposits a (type-a meetings). In Sector 2, with prob. α_2^s, agents trade with stablecoins m_s, which they can acquire from f in the end of CM previous period; with the rest prob. $\alpha_2^m = 1 - \alpha_2^s$, agents directly trade with cash m_m. For DM pricing protocols, we can still adopt Kalai bargaining ($\theta \in (0,1)$ is the bargaining power of buyers) as before, but focus on the case $\theta = 1$, $c(q) = q$ a lot, i.e., buyers make T-I-O-L-T offers, due to analytical tractability.

4. In CM, all settlements and portfolio rebalancing happens, all of the debt (from the government, or banks, or f) get settled or redeemed, and agents and banks work or consume. Stablecoin issuers f maximize their profits then consume all profits (or work) in CM, and roll over reserves (bond holdings) to issue new stablecoins for next period.

Suppose govt. bonds b are one-period real bonds, and ι_b is the nominal return of bonds, then

$$1 + \iota_b \equiv \frac{1/\phi_{m,+}}{\phi_b/\phi_m} = \frac{1+\mu_m}{\phi_b},$$

where ϕ_m is the real price of fiat money, and ϕ_b is the issuing price of one-period real bonds. The government is still active in CM, as before. But now there is a bond market in CM, and

$$B = B_p + B_s,$$

where B is aggregate bond supply at the current period, B_p denote bond holdings by the public (subscript "p", also bond holdings by the banking sector) and B_s denotes the holdings by stablecoin issuers (subscript "s"). At the period t, we define the new GBC as,

$$G_t + B_{t-1} = \phi_{b,t} B_t + \phi_{m,t}(M_{m,t} - M_{m,t-1}) + \tau \cdot D_{1,t},$$

where B_{t-1} is bond maturing from the previous period, $M_{m,t}$ denotes aggregate supply of fiat money (to differentiate from stablecoin M_s), and $D_{1,t}$ is the formal tax base,

also denoting post-matching transaction amount of Sector 1, i.e., $D_{1,t} = \rho\alpha_1[\omega d^m_{1,t} + (1-\omega)d^a_{1,t}] + I_s \cdot \rho\alpha^s_1 d^s_{1,t}$, with $d^m_{1,t}$, $d^a_{1,t}$ denoting transaction amounts (in real terms), that accept cash and deposits, respectively, and $d^s_{1,t}$ denoting that accept stablecoins (if $I_s = 1$), respectively. At steady states, we have

$$G = \phi_m \mu_m M_m + \tau \cdot D_1 + (\phi_b - 1)\bar{B}, \tag{30}$$

where $\bar{B} = B_p + B_s$, meaning aggregate bonds outstanding ($\bar{B}$) come from two components: holdings by the public (B_p) and by stablecoin issuers (B_s), at steady states. This expanded GBC encapsulates the core macroeconomic tension of the digital era: while stablecoin adoption displaces the fiat seigniorage base (the first term), the issuer's voracious demand for reserve assets can exert downward pressure on bond yields, generating a debt-service subsidy (the third term) for the government.

4.2 Decentralized Trading

As mentioned before, by the end of CM, after the idiosyncratic shocks are realized, an agent knows which sector he will choose and but does not yet know which types of meetings he will be in Sector 1. Those choosing Sector 1, sign the deposit contracts with banks, so that he can trade with deposit claims $\hat{z}_a$ or cash $\hat{z}_m$ for DM trading. As for those choosing Sector 2, they will contact stablecoin issuers f, to acquire stablecoins $\hat{z}_s$, for DM trading later.

Now we focus on DM. Since ρ is the probability agents choose Sector 1, we have

$$\mathbb{E}V(\hat{z}_a, \hat{z}_m, \hat{z}_s) = \rho V_1(\hat{z}_a, \hat{z}_m) + (1-\rho)V_2(\hat{z}_s, \hat{z}_m, \kappa).$$

In Sector 1 (Formal/TradeFi),

$$V_1(\hat{z}_a, \hat{z}_m) = \frac{1}{2}V^b_1(\hat{z}_a, \hat{z}_m) + \frac{1}{2}V^s_1(\hat{z}_a, \hat{z}_m),$$

where

$$V_1^b(\hat{z}_a, \hat{z}_m) = \alpha_1\{\omega[u(q_1^m) - (1+\tau)d_1^m] + (1-\omega)[u(q_1^a) - (1+\tau)d_1^a]\} + W_1(\hat{z}_a, \hat{z}_m)$$
$$V_1^s(\hat{z}_a, \hat{z}_m) = \alpha_1\{\omega[-c(q_1^m) + (1-\tau)d_1^m] + (1-\omega)[-c(q_1^a) + (1-\tau)d_1^a]\} + W_1(\hat{z}_a, \hat{z}_m),$$

where ω is the prob. buyers meet with a seller who only accepts $\hat{z}_m$, then withdraws from banks, to pay with money; $1-\omega$ is the prob. a buyer meets with a seller who only accepts bank deposits $\hat{a}$; and (q_1^j, d_1^j), $j = \{m, a\}$, denotes the terms of trade in type-j meetings of Sector 1, with $(1+\tau)d_1^m \leqslant \hat{z}_m/\omega$, $(1+\tau)d_1^a \leq \hat{z}_a$.[11] Here we use $W_1(\hat{z}_a, \hat{z}_m)$ to denote the value function when agents from Sector 1 enter CM, which is not the value function of those agents in the end of CM. As mentioned in the timing and events, agents will bear idiosyncratic shocks in the end of CM, then will make the choices of sectors and of asset/money holdings.

In Sector 2 (Informal/Crypto),

$$V_2(\hat{z}_s, \hat{z}_m, \kappa) = \frac{1}{2}V_2^b(\hat{z}_s, \hat{z}_m, \kappa) + \frac{1}{2}V_2^s(\hat{z}_s, \hat{z}_m, \kappa),$$

where

$$V_2^b(\hat{z}_s, \hat{z}_m, \kappa) = -\kappa + \alpha_2^s[u(q_2^s) - d_2^s] + \alpha_2^m[u(q_2^m) - d_2^m] + W_2(\hat{z}_s, \hat{z}_m)$$
$$V_2^s(\hat{z}_s, \hat{z}_m, \kappa) = -\kappa + \alpha_2^s[-c(q_2^s) + d_2^s] + \alpha_2^m[-c(q_2^m) + d_2^m] + W_2(\hat{z}_s, \hat{z}_m),$$

with $\alpha_2^s + \alpha_2^m = 1$. Again κ is the ex ante disutility cost for agents choosing Sector 2, and $(\alpha_2^i, q_2^i, d_2^i)$, means agents trade with the matching rate α_2^i, and the terms of trade are (q_2^i, d_2^i) in the permanent type-i meetings, $i = \{m, s\}$. For the type-m meetings, $d_2^m \leq \hat{z}_m$, meaning agents trade with cash, and the payment shall not exceed the cash holdings $\hat{z}_m$. As for type-s meetings, agents trade with stablecoins, and $d_2^s \leq \hat{z}_s = \chi\hat{z}_b^s$, i.e., the payment d_2^s can't exceed the stablecoin holdings $\hat{z}_s$, or equivalently $\chi\hat{z}_b^s$, $0 < \chi \leq 1$. Here we use $\chi\hat{z}_b^s$ to denote the value of reserves (bond

[11] For the timeline of events, agents choosing Sector 1, purchase deposit contracts from banks in the end of previous CM, before knowing their DM types (type-m, then withdraw fiat money, or type-a meetings, then just use deposit claims a to trade). That is, banking services are available to them in the end of CM, but before DM starts.

holdings, up to the genuine ratio χ), for $\hat{z}_s$ units of stablecoins. Again $W_2(\hat{z}_s, \hat{z}_m)$ is only the value function when agents from Sector 2 enter CM, not the one in the end of CM.

Now we consider the endogenous dichotomy, as in Section 3. Suppose $I_s = 0$, then, in equilibrium, agents should be indifferent to enter Sector 1 or Sector 2, i.e.,

$$V_1(\hat{z}_a, \hat{z}_m) = V_2(\hat{z}_s, \hat{z}_m, \kappa).$$

Then we have,

$$\frac{\alpha_1}{2}\{\omega[u(q_1^m)-c(q_1^m)-2\tau d_1^m]+(1-\omega)[u(q_1^a)-c(q_1^a)-2\tau d_1^a]\} = -\hat{\kappa}+\frac{\alpha_2^m}{2}[u(q_2^m)-c(q_2^m)]+\frac{\alpha_2^s}{2}[u(q_2^s)-c(q_2^s)].$$

Hence,

$$\hat{\kappa} = \Delta(q_1^m, q_1^a, q_2^s),$$

where $\Delta(q_1^m, q_1^a, q_2^s)$ is the trading surplus between Sector 2 and Sector 1, with

$$\begin{aligned}\Delta(q_1^{mb}, q_2^s) \equiv\ & \frac{\alpha_2^m}{2}[u(q_2^m) - c(q_2^m)] + \frac{\alpha_2^s}{2}[u(q_2^s) - c(q_2^s)] - \frac{\alpha_1}{2}\{\omega[u(q_1^m) - c(q_1^m)] + (1-\omega)[u(q_1^a) - c(q_1^a)] \\ & +\alpha_1\tau[\omega d_1^m + (1-\omega)d_1^a].\end{aligned}$$

With $1 - \rho = F(\hat{\kappa})$, κ following the same uniform distribution, we have

$$\rho = 1 - \frac{\Delta(q_1^m, q_1^a, q_2^s, q_2^m)}{\gamma}. \tag{31}$$

4.3 Sector 1 - Deposit Contracts

As mentioned before, in the end of CM, once agents know they will go to Sector 1 or Sector 2, those choosing Sector 1 will purchase deposit contracts from banks. Here we consider $\theta = 1$ (buyers make T-I-O-L-T offers) and $c(q) = q$, then banks, on behalf of buyers, maximize their trading surplus in an equilibrium deposit contract,

$\{\hat{z}_b, \hat{z}_m, d_1^m, d_1^a\}$, which solves for

$$\max_{\hat{z}_b, \hat{z}_m, d_1^m, d_1^a} \{-\phi_b \hat{z}_b - (1+\mu_m)\hat{z}_m + \beta \frac{\alpha_1}{2} \omega [u(q_1^m(d_1^m)) - (1+\tau)d_1^m]$$
$$+\beta \frac{\alpha_1}{2}(1-\omega)[u(q_1^a(d_1^a)) - (1+\tau)d_1^a] + \beta(\hat{z}_b + \hat{z}_m)\} \tag{32}$$

$$\text{st. } (1+\tau)d_1^m \leq \frac{\hat{z}_m}{\omega} \tag{33}$$

$$(1+\tau)d_1^a \leq \frac{\hat{z}_a}{1-\omega} \tag{34}$$

$$\varepsilon \hat{z}_a \leq \hat{z}_m + \hat{z}_b \tag{35}$$

We can see that, in type-m meetings, each depositor (outside of ω total) receives q_1^m units of goods to exchange for cash d_1^m withdrawn from banks; and a cash constraint (33) means he can withdraw cash to finish post-taxation payment up to the per-capita share of the bank' currency reserves.[12] As for type-a meetings, each depositor (outside of $1-\omega$ total) receives q_1^a units of goods to exchange for deposit claims d_1^a; and the deposit constraint (34) means he can make payments by deposits up to the per-capita share of deposit claims. They all need to pay taxes at the rate τ, in both meetings. Furthermore, as mentioned before, banks need to satisfy a *required reserve constraint* shown by (35): a ratio ε of total deposit $\hat{z}_a$ shall be no greater than the total value of reserves, $\hat{z}_m + \hat{z}_b$.

Depending on the abundance status of bank bond holding levels, we need to consider two cases as follows. Below we use B_p^* to represent bond holdings of banks at an abundant level.

1. When $B_p < B_p^*$

Then bond holdings B_p (by ibanks, representing public holdings of bonds) are scarce, so that the required reserve constraint binds, then

$$\hat{z}_m + \hat{z}_b = \varepsilon(1-\omega)\hat{z}_a.$$

[12] In theory, both q_1^m and q_1^a can go to first-best levels if agents have enough money and deposits. However, WLOG, we assume way the senario that $q_1^m = q_1^*$, to stay focused on stablecoins. Equivalently, it means the cash constraint (33) always binds.

This means the deposit payment constraint also binds, so that $q_1^a < q_1^*$. Hence,

$$d_1^a = \frac{\hat{z}_a}{1+\tau} = \frac{\hat{z}_m + \hat{z}_b}{\varepsilon(1-\omega)(1+\tau)}$$

then $q_1^a < q_1^*$,

$$\hat{z}_a = \frac{\hat{z}_m + \hat{z}_b}{\varepsilon(1-\omega)}, \tag{36}$$

and both bonds and money provide liquidity premium at the margin. And we know

$$\rho \hat{z}_b = B_p,$$

in equilibrium. Using the bargaining solutions at $\theta = 1$, $c(q_1^j) = q_1^j = (1-\tau)d_1^j$, we have

$$\begin{aligned} d_1^m &= \frac{\hat{z}_m}{\omega(1+\tau)}, \; d_1^a = \frac{\hat{z}_a}{1+\tau} = \frac{\hat{z}_m + \hat{z}_b}{\varepsilon(1-\omega)(1+\tau)} && (37) \\ q_1^m &= C(\tau)\frac{\hat{z}_m}{\omega}, \; q_1^a = \frac{C(\tau)}{\varepsilon(1-\omega)}(\hat{z}_m + \hat{z}_b) && (38) \end{aligned}$$

where $C(\tau) \equiv (1-\tau)/(1+\tau)$. Using all of these, we can rewrite the above deposit contract problem as

$$\begin{aligned} &\max_{\hat{z}_b, \hat{z}_m} \{-\phi_b \hat{z}_b - (1+\mu_m)\hat{z}_m + \frac{\beta}{2}\alpha_1 \omega [u(C(\tau)\frac{\hat{z}_m}{\omega})) - \frac{\hat{z}_m}{\omega}] \\ &+ \frac{\beta}{2}\alpha_1(1-\omega)[u(C(\tau)\frac{\hat{z}_m + \hat{z}_b}{\varepsilon(1-\omega)}) - \frac{\hat{z}_m + \hat{z}_b}{\varepsilon(1-\omega)}] + \beta(\hat{z}_b + \hat{z}_m)\} \end{aligned}$$

Then we can derive the Euler equations for $\hat{z}_b$, $\hat{z}_m$ as

$$\begin{aligned} \frac{\phi_b}{\beta} - 1 &= \frac{\alpha_1}{2}\frac{1}{\varepsilon}[C(\tau)u'(C(\tau)\frac{\hat{z}_m + \hat{z}_b}{\varepsilon}) - 1] && (39) \\ \frac{1+\mu_m}{\beta} - 1 &= \frac{\alpha_1}{2}\{[C(\tau)u'(C(\tau)\hat{z}_m) - 1] + \frac{1}{\varepsilon}[C(\tau)u'(C(\tau)\frac{\hat{z}_m + \hat{z}_b}{\varepsilon}) - 1]\}. && (40) \end{aligned}$$

Previously we use $\lambda_1(\cdot)$ to denote liquidity premium, so now we have

$$\lambda_1(q_1^m) \equiv C(\tau)u'(C(\tau)\hat{z}_m) - 1, \; \lambda_1(q_1^a) \equiv C(\tau)u'(C(\tau)\frac{\hat{z}_m + \hat{z}_b}{\varepsilon}) - 1.$$

And we have $\lambda_1'(q_1^j) < 0$. Hence, we have $q_1^a = q_1^*$, when $u'(q_1^*) = 1/C(\tau)$. Hence, we

can rewrite the RHS of (40) as

$$\frac{\alpha_1}{2}[\lambda_1(q_1^m) + \frac{1}{\varepsilon}\lambda_1(q_1^a)] \equiv \mathcal{L}_1, \tag{41}$$

where $\mathcal{L}_1$ is the liquidity premium of cash provided for Sector 1, coming from a weighted sum of type-m and type-a meetings.

Case 2: $B_p \geq B_p^*$, bond holdings are at an abundant level, then $q_1^a = q_1^*$,

$$\rho\hat{z}_b = B_p \geq \rho[\frac{\varepsilon(1-\omega)}{C(\tau)}q_1^* - \hat{z}_m] \equiv B_p^*, \tag{42}$$

so that $d_1^m = \hat{z}_m/[\omega(1+\tau)]$, $d_1^a = q_1^*/(1-\tau)$. Hence, cash still provides the liquidity premium at the margain (for type-m meetings), but the trading for type-a goes to the first best level q_1^*, and bonds do not provide the liquidity premium at the margain. Hence,

$$\frac{1+\mu_m}{\beta} - 1 = \frac{\alpha_1}{2}\lambda_1(q_1^m) = \frac{\alpha_1}{2}[C(\tau)u'(C(\tau)\hat{z}_m) - 1]. \tag{43}$$

4.4 Sector 2 - "Money" Demand

There are two types of "money" in Sector 2: cash $\hat{z}_m$ and stablecoins $\hat{z}_s$, which also means, for agents choosing Sector 2, there are two permanent types of meetings: cash or stablecoin meetings. Hence, given $\theta = 1$, $c(q) = q$, the maximization problem of agents in cash meetings is

$$\max_{\hat{z}_m}\{-(1+\mu_m)\hat{z}_m + \beta\frac{\alpha_2^m}{2}[u(q_2^m) - q_2^m] + \beta\hat{z}_m\}$$
$$\text{st. } d_1^m = q_2^m \leq \hat{z}_m$$

Since it is costly to carry cash across periods (or the same reason mentioned in Footnote **??**), we have

$$d_1^m = q_2^m = \hat{z}_m. \tag{44}$$

Then the Euler eq. for $\hat{z}_m$ is

$$\frac{1+\mu_m}{\beta} - 1 = \frac{\alpha_2^m}{2}\lambda_2(q_2^m) \equiv \mathcal{L}_2, \tag{45}$$

where $\lambda_2(q_2^m) \equiv u'(q_2^m) - 1$, and $\mathcal{L}_2$ denotes the liquidity premium that cash provides for Sector 2.

Similarly, the maximization problem of agents in stablecoin meetings of Sector 2 is

$$\max_{\hat{z}_s}\{-(1+\mu_s)\hat{z}_s + \beta\frac{\alpha_2^s}{2}[u(q_2^s) - d_2^s] + \beta\hat{z}_s\}$$
$$\text{st. } d_2^s \leq \hat{z}_s.$$

When the payment constraint $d_2^s \leq \hat{z}_s$ binds, we can derive

$$\frac{1+\mu_s}{\beta} - 1 = \frac{\alpha_2^s}{2}\lambda_2(q_2^s) = \frac{\alpha_2^s}{2}[u'(q_2^s) - 1].$$

However, this is *not* the Euler equation for stablecoins because the narrow banking arrangement, $\hat{z}_s \leq \chi\hat{z}_b^s$, already provides a "cap" at the supply side, to determine $\hat{z}_s$. Instead, we can interprete the LHS, $\iota_s \equiv \frac{1+\mu_s}{\beta} - 1$, as the "shadow" yields of stablecoins. To clarify, with $\theta = 1$, $c(q) = q$, the demand of stablecoins is specified as

$$\max_{q_2^s, d_2^s}\{u(q_2^s) - q_2^s\} \tag{46}$$
$$\text{st. } -q_2^s + d_2^s = 0 \tag{47}$$
$$d_2^s \leq \hat{z}_s \tag{48}$$
$$\hat{z}_s \leq \chi\hat{z}_b^s, \tag{49}$$

where $\hat{z}_b^s$ is the bond holdings by a representative f (with superscript "s"), differentiated from $\hat{z}_b$ – bond holdings in the investment portfolio of banks, and χ is the pledgeability of stablecoin reserves (bonds). For simplicity, we could just let $\chi = 1$, but just carry it for general cases. Using the zero surplus constraint for sellers (47),

when the second constraint (48) binds, we have

$$d_2^s = q_2^s = \hat{z}_s. \tag{50}$$

As for the last constraint (49), this is the constraint for the "narrow banking" arrangement of stablecoins: each unit of stablecoins is backed up by reserves, so that the value of $\hat{z}_s$ should not be greater than the values of reserves (bonds) given the genuine bond ratio χ. In equilibrium, the narrow banking constraint binds, so that

$$\begin{aligned} \hat{z}_s &= \chi \hat{z}_b^s \\ (1-\rho)\alpha_2^s \cdot \chi \hat{z}_b^s &= \chi B_s, \end{aligned}$$

where $1-\rho$ is the measure of agents choosing Sector 2. Hence, the aggregate stablecoin demand, $(1-\rho)\alpha_2^s \hat{z}_s$, is equal with the value of equivalent aggregate reserves (bonds), i.e., B_s times χ (genuine bond holding ratio). Suppose B_s^* is the threshold level of issuers' abundant bond holdings, then we also define

$$q_2^s \equiv q_2^* = \frac{\chi B_s^*}{(1-\rho)\alpha_2^s},$$

where $u'(q_2^*) = 1$. Then, there are two cases to sort out the demand for stablecoins as follows.

1. When $B_s < B_s^*$, the issuers' bond holdings are scarce, then $q_2^s < q_2^*$, and

$$d_2^s = \hat{z}_s = \frac{\chi B_s}{(1-\rho)\alpha_2^s} = q_2^s. \tag{51}$$

2. When $B_s \geq B_s^*$, the issuers' bond holdings are abundant, then agents can always acquire enough stablecoins to support DM tradings, i.e., $q_2^s \geq q_2^* = \chi B_s^*/[(1-\rho)\alpha_2^s]$. Hence,

$$B_s = \frac{(1-\rho)\alpha_2^s}{\chi} q_2^s \geq \frac{(1-\rho)\alpha_2^s}{\chi} q_2^* \equiv B_s^*. \tag{52}$$

In this case, changing B_s has no effects to the tradings in Sector 2, or to the whole macroeconomy.

To sum up, given B_s, i.e., aggregate bond holding levels of f, the demand of

stablecoins is mainly determined by the reserve holding status of f, instead of the shadow yield ι_s.

4.5 General Equilibrium

Next we solve for the equilibrium, by taking χ, ρ as given, i.e., we do not consider stablecoin misrepresentation or endogenous dichotomy, for now. Depending on the bond abandunce levels of banks and f, there can be four cases of general equilibria. Before that, we discuss about several important issues.

First, at s.s., cash demand should be aggregated using (40), (41) and (45), since a fraction ρ of agents enter Sector 1 and use cash to trade in type-m meetings, and $(1-\rho)$ enter Sector 2 and trade with cash in the permanent type of cash meetings there. That is,

$$\frac{1+\mu_m}{\beta} - 1 = \rho\mathcal{L}_1 + (1-\rho)\mathcal{L}_2, \tag{53}$$

where L1 and L2 are from (41) and (45), respectively.

Second, to avoid confusion, we define the mapping between aggregate variables (in capital letters) and per-capita variables as follows,

$$\begin{aligned}
Z_m &= \rho\hat{z}_m + (1-\rho)\alpha_2^m\hat{z}_m = [\rho + (1-\rho)\alpha_2^m]\hat{z}_m \\
Z_a &= \rho\hat{z}_a = \rho\frac{\hat{z}_m + \hat{z}_b}{\varepsilon} \\
Z_s &= (1-\rho)\alpha_2^s\hat{z}_s \\
B_s &= (1-\rho)\alpha_2^s\hat{z}_b^s,\ B_p = \rho\hat{z}_b \\
\bar{B} &= B_p + B_s = \rho\hat{z}_b + (1-\rho)\alpha_2^s\hat{z}_b^s \\
D_1 &= \rho\alpha_1[\omega d_1^m + (1-\omega)d_1^a].
\end{aligned}$$

Easily, we have $B_p = \bar{B} - B_s$, then B_s increases means B_p decreases (given $\bar{B}$ is constant at s.s.). Here we use lowercase $\hat{z}_m$ for per-capita real balances held by a representative participant in that sector, and uppercase for the corresponding aggregate real stocks; with sector measures, ρ, $1-\rho$, aggregates equal the measure times per-capita holdings. For example, Z_m *is aggregate fiat money stock (or the monetary base)*,

which is fiat money circulating in Sector 1 plus Sector 2, so $Z_m = \rho\hat{z}_m + (1-\rho)\alpha_2^m\hat{z}_m$, etc. We also use Z_a to denote aggregate deposit claims that is equal with total deposits circulating in type-a meetings of Sector 1, while Z_s is aggregate stablecoins that is equal with total stablecoins circulating in stablecoin transacations of Sector 2, etc. As for D_1, we need to consider the two cases of TradFi bond holding status. **When** $B_p < B_p^*$, we can rewrite to have

$$D_1 = \frac{\rho\alpha_1}{1+\tau}(\hat{z}_m + \frac{\hat{z}_m + \hat{z}_b}{\varepsilon}) = \frac{\rho\alpha_1}{1+\tau}[(1+\frac{1}{\varepsilon})\hat{z}_m + \frac{B_p}{\varepsilon\rho}],$$

where we use $d_1^m = \hat{z}_m/[\omega(1+\tau)]$, $d_1^a = \hat{z}_a/(1+\tau) = (\hat{z}_m + \hat{z}_b)/[\varepsilon(1-\omega)(1+\tau)]$. But **when** $B_p \geq B_p^*$, with $d_1^m = \hat{z}_m/[\omega(1+\tau)]$, $d_1^a = q_1^*/(1-\tau)$, we have

$$D_1 = \rho\alpha_1[\frac{\hat{z}_m}{1+\tau} + (1-\omega)\frac{q_1^*}{1-\tau}].$$

Third, we can rewrite GBC as

$$G = \mu_m Z_m + \tau D_1 + (\phi_b - 1)\bar{B}.$$

If $B_p < B_p^*$, after substituting Z_m, D_1, we have

$$G = \mu_m[\rho + (1-\rho)\alpha_2^m]\hat{z}_m + \frac{\tau\alpha_1}{1+\tau}[\rho(1+\frac{1}{\varepsilon})\hat{z}_m + \frac{\bar{B} - B_s}{\varepsilon}] + (\phi_b - 1)\bar{B} \tag{54}$$

where we use $B_p = \bar{B} - B_s$. From (54), we can see that, the effects of increasing B_s can push up the bond price ϕ_b but decrease bond yields ι_b, which lower govt. borrowing cost. It can also suppresses type-a meetings in Sector 1 by the bond liquidity channel (q_1^a decreases), and generate multiple equilibrium effects to type-m meetings and the whole macroeconomy (through the GBC channel). We will discuss more in the four cases of general equilibria as follows.

Now we define the general equilibrium as follows:

Definition 2 *Given $(G, \tau, B_s, \bar{B})$, a stationary monetary equilibrium is a list $\{q_1^m, q_1^a, q_2^m, q_2^s, \hat{z}_m, \hat{z}_s, \mu_m, \phi_b\}$ such that [i] with T-I-O-L-T offers, for type-m and type-a meetings in Sector 1, bargaining solutions are shown by (37), (38); for the cash and stable-*

coin meetings in Sector 2, shown by (44) and (50); [ii] banks solve the deposit contract problem in (32); when $B_p < B_p^*$, Euler equations for bonds and money are shown by (39), (40); and when $B_p \geq B_p^*$, bond holdings B_p satisfy (42), and are abundant to support $q_1^a = q_1^*$ for type-a meetings, and Euler equation for money becomes (43); [iii] when $B_s < B_s^*$, Euler equation for stablecoins is shown by (51); and when $B_s \geq B_s^*$, bond holdings of f satisfy (52), and are abundant to support $q_2^s = q_2^*$; [iv] the aggregate Euler equation for cash is shown by (53); [v] the government budget constraint is shown by (54); [vi] market clearing conditions for fiat money (m_m), govt. bonds (b) and stablecoins (m_s).

Similarly as before, if given $(G, \tau, B_s, \bar{B})$, then τ becomes a policy choice variable, and we can discuss more about optimal policy. In addition, notice that we have not put μ_s in the list as one of the variables, but $\mu_s = \mu_m$ in equilibrium. So far we model stablecoins as pegging to fiat money through a narrow banking arrangement, then at steady states, the carrying cost of stablecoins should be exactly the same at that of fiat money.

Depending on the bond abundance levels of banks and f, there are four cases of general equilibria as follows.

Case 1: when $B_s < B_s^*$ and $B_p < B_p^*$

GE conditions:

$$
\begin{aligned}
G &= \mu_m[\rho + (1-\rho)\alpha_2^m]\hat{z}_m + \frac{\tau\alpha_1}{1+\tau}[\rho(1+\frac{1}{\varepsilon})\hat{z}_m + \frac{\bar{B}-B_s}{\varepsilon}] + (\phi_b - 1)\bar{B} \\
s_b &\equiv \frac{\phi_b}{\beta} - 1 = \frac{\alpha_1}{2\varepsilon}\lambda_1(q_1^a) \\
\frac{1+\mu_m}{\beta} - 1 &= \rho\mathcal{L}_1 + (1-\rho)\mathcal{L}_2 = \rho\frac{\alpha_1}{2}[\lambda_1(q_1^m) + \frac{\lambda_1(q_1^a)}{\varepsilon}] + (1-\rho)\frac{\alpha_2^m}{2}\lambda_2(q_2^m) \\
q_1^m &= \frac{C(\tau)}{\omega}\hat{z}_m,\ q_1^a = \frac{C(\tau)}{\varepsilon(1-\omega)}(\hat{z}_m + \frac{\bar{B}-B_s}{\rho}) \\
q_2^m &= \hat{z}_m,\ q_2^s = \hat{z}_s = \frac{\chi B_s}{(1-\rho)\alpha_2^s},
\end{aligned}
$$

where we use $B_p = \bar{B} - B_s$.

When $B_s < B_s^* = (1-\rho)\alpha_2^s q_2^*/\chi$, and $B_p < B_p^* = \rho[\frac{\varepsilon(1-\omega)}{C(\tau)}q_1^* - \hat{z}_m]$, bond holdings by banks and by f are both scarce, then bonds provide liquidity premium for tradings in both sectors. The above Definition 2 has provided all the equilibrium conditions for the list of $\{q_1^m, q_1^a, q_2^m, q_2^s, \hat{z}_m, \mu_m, \phi_b\}$. In the end, all equilibrium conditions can collapse to one single equations for $\hat{z}_m$ as follows,

$$G = S(\hat{z}_m; B_s, \bar{B}), \tag{55}$$

where we define

$$S(\hat{z}_m; B_s, \bar{B}) \equiv \mu_m(\hat{z}_m)[\rho+(1-\rho)\alpha_2^m]\hat{z}_m + \frac{\tau\alpha_1}{1+\tau}[\rho(1+\frac{1}{\varepsilon})\hat{z}_m + \frac{\bar{B}-B_s}{\varepsilon}] + [\phi_b(\hat{z}_m)-1]\bar{B},$$

with

$$\mu_m(\hat{z}_m) \equiv \beta - 1 + \beta\{\rho\frac{\alpha_1}{2}[\lambda_1(\frac{C(\tau)}{\omega}\hat{z}_m) + \frac{1}{\varepsilon}\lambda_1(\frac{C(\tau)}{\varepsilon(1-\omega)}(\hat{z}_m + \frac{\bar{B}-B_s}{\rho}))] + (1-\rho)\frac{\alpha_2^m}{2}\lambda_2(\hat{z}_m)\} \tag{56}$$

$$\phi_b(\hat{z}_m) \equiv \beta + \beta\frac{\alpha_1}{2\varepsilon}\lambda_1(q_1^a). \tag{57}$$

As for the equilibrium existence and uiqueness, similarly as Lemma 1, given $\alpha_1 > 0$ or $\alpha_2^m > 0$, a stationary monetary equilibrium exists if $G \leq \max S(\hat{z}_m)$; if $S(\hat{z}_m(q_{FR})) \leq G \leq \max S(\hat{z}_m)$, there are generically two equilibria; and if $0 < G < S(\hat{z}_m(q_{FR}))$, there is a unique equilibrium on the inefficient branch. Here q_{FR} refers to the trading quantities at the Friedman Rule, i.e., $\mu_m = \beta - 1$, $\hat{z}_m(q_{FR})$ thus denotes the real balances of money at the trading quantities q_{FR}.

It is interesting to investigate how the bond holdings by f will affect the tradings in Sector 1, and possibly also have public finance implications through the GBC channel. First, we will check the effects of changing B_s, then that of changing $\bar{B}$. Using (57), we have

$$\frac{\partial\phi_b}{\partial B_s} = -\frac{\beta\alpha_1 C^2(\tau)}{2\varepsilon^2\rho(1-\omega)}u''(q_1^a) > 0.$$

Then, using (56), we have

$$\frac{\partial\mu_m}{\partial B_s} = \rho\frac{\partial\phi_b}{\partial B_s} > 0.$$

Hence, using (55), we have

$$S_{B_s} = \frac{\partial S}{\partial B_s} = [\rho(\rho + (1-\rho)\alpha_2^m)\hat{z}_m + \bar{B}]\frac{\partial \phi_b}{\partial B_s} - \frac{\tau\alpha}{(1+\tau)\varepsilon} \lessgtr 0.$$

where the first term is positive, representing the yield subsidy from more reserve (bond) holdings of f, but the second term is negative, representing the tax base erosion, since higher B_s crowd out the collateral of TradFi (banks), strictly eroding formal tax revenues from type-a transactions of Sector 1. Hence, this explicitly formalizes the *macroeconomic tension* from f holding more reserves (bonds). Since S_{B_s} is theoretically ambiguous,

$$\frac{\partial \hat{z}_m}{\partial B_s} = -\frac{S_{B_s}}{S_{\hat{z}_m}},$$

dictates a strict macro-fiscal trade-off, where

$$S_{\hat{z}_m} = [\rho + (1-\rho)\alpha_2^m]\mu_m + \frac{\rho\tau\alpha_1}{1+\tau}(1+\frac{1}{\varepsilon}) + [\rho + (1-\rho)\alpha_2^m]\hat{z}_m\frac{d\mu_m}{d\hat{z}_m} + \bar{B}\frac{d\phi_b}{dd\hat{z}_m} \lessgtr 0.$$

We can see that the signs of $S_{\hat{z}_m}$, S_{B_s} are both ambigous, so is the sign of $\partial\hat{z}_m/\partial B_s$. We conject the effects of changing B_s are theoretically ambiguous, so will use numerical work to show more explicit results later.

As for the effects of changing $\bar{B}$, since

$$\frac{\partial \phi_b}{\partial \bar{B}} = -\frac{\partial \phi_b}{\partial B_s},$$

we have

$$S_{\bar{B}} = -S_{B_s} + (\phi_b - 1).$$

This means, given Bs unchanged, increasing $\bar{B}$ (aggregate bonds outstanding) reverses the stablecoin crowding-out effect, i.e., expanding TradFi tax capacity but destorying the yield subsidy, leaving only the fundamental pure-debt carrying cost $(\phi_b - 1)$.

Furthermore, we have

$$\frac{\partial \hat{z}_m}{\partial \bar{B}} = -\frac{S_{\bar{B}}}{S_{\hat{z}_m}},$$

where we already know $S_{\hat{z}_m}$ is ambiguous, so is that of $\partial \hat{z}_m / \partial \bar{B}$. Again we will resort to numerical analysis to show more explicit results later.

Case 2: when $B_s < B_s^*$, and $B_p \geq B_p^*$

When $B_s < B_s^* = (1-\rho)\alpha_2^s q_2^*/\chi$, and $B_p \geq B_p^* = \rho(\frac{\varepsilon(1-\omega)}{C(\tau)} q_1^* - \hat{z}_m)$, banks hold enough bonds to satiate formal trade, i.e., $q_1^a = q_1^*$, driving the liquidity premium on public bond holdings to zero, which means $\phi_b = \beta$; but bond holdings of f are still scarce, so $q_2^s < q_2^*$. Then the equilibrium conditions are

$$\begin{aligned}
G &= \mu_m[\rho + (1-\rho)\alpha_2^m]\hat{z}_m + \rho\tau\alpha_1[\frac{\hat{z}_m}{1+\tau} + (1-\omega)\frac{q_1^*}{1-\tau}] + (\beta - 1)\bar{B} \equiv S(\hat{z}_m) \\
q_1^a &= q_1^* = \frac{C(\tau)}{\varepsilon(1-\omega)}(\hat{z}_m + \frac{B_p^*}{\rho}),\ q_1^m = \frac{C(\tau)}{\varepsilon}\hat{z}_m \\
\frac{1+\mu_m}{\beta} - 1 &= \rho\frac{\alpha_1}{2}\lambda_1(q_1^m) + (1-\rho)\frac{\alpha_2^m}{2}\lambda_2(q_2^m) \\
q_2^m &= \hat{z}_m,\ q_2^s = \hat{z}_s = \frac{\chi B_s}{(1-\rho)\alpha_2^s}.
\end{aligned}$$

Again notice that q_2^s is now pinned down by B_s, given χ, ρ and α_2^s. From the GBC, given $B_p \geq B_p^*$, the bond holdings of banks are abundant, then bonds do not provide liquidity premium at the margin, i.e., $\phi_b = \beta$. Consequently, B_s vanishes from GBC, and changing it has no impact on $\hat{z}_m$, nor does it impact μ_m, q_1^m, q_1^a, or q_2^m. Hence, changing B_s solely affects Sector 2: it increases q_2^s, driving down the "shadow" liquidity premium $\lambda_2(q_2^s)$, which in turn lowers the unobserved or "shadow" convenience yield of stablecoins, ι_s.

As for the effects of changing $\bar{B}$, we have

$$\frac{\partial \hat{z}_m}{\partial \bar{B}} = -\frac{S_{\bar{B}}}{S_{\hat{z}_m}} = \frac{1-\beta}{S_{\hat{z}_m}} \cong S_{\hat{z}_m}.$$

Case 3: when $B_s \geq B_s^*$ and $B_p < B_p^*$

Since the reserves (bonds) are abundant for f, we have $q_2^s = q_2^*$, and $B_s \geq B_s^* =$

$(1-\rho)\alpha_2^s q_2^*/\chi$. Then the equilibrium conditions are

$$
\begin{aligned}
G &= \mu_m[\rho + (1-\rho)\alpha_2^m]\hat{z}_m + \frac{\tau\alpha_1}{1+\tau}[\rho(1+\frac{1}{\varepsilon})\hat{z}_m + \frac{\bar{B}-B_s}{\varepsilon}] + (\phi_b - 1)\bar{B} \\
s_b &\equiv \frac{\phi_b}{\beta} - 1 = \frac{\alpha_1}{2\varepsilon}\lambda_1(q_1^a) \\
\frac{1+\mu_m}{\beta} - 1 &= \rho\frac{\alpha_1}{2}[\lambda_1(q_1^m) + \frac{1}{\varepsilon}\lambda_1(q_1^a)] + (1-\rho)\frac{\alpha_2^m}{2}\lambda_2(q_2^m) \\
q_1^m &= \frac{C(\tau)}{\omega}\hat{z}_m,\ q_1^a = \frac{C(\tau)}{\varepsilon(1-\omega)}(\hat{z}_m + \frac{\bar{B}-B_s}{\rho}) \\
q_2^m &= \hat{z}_m,\ q_2^s = q_2^*.
\end{aligned}
$$

Since $B_s \geq B_s^*$, we can see that stablecoin issuers cap their reserves at satiation. Hence increasing B_s beyond B_s^* has no effects on Sector 2 (instead incurs a spread cost for f since $\phi_b > \beta$). Furthermore, it will make bond scarcity become more severe for banks, since $B_p = \bar{B} - B_s$ will decrease, then the "spillover" effects to Sector 1 will be similar to an expansionary OMO by the central bank (see Rocheteau et al., 2018): q_1^m increases, q_1^a decreases, but μ_m may decrease or increase, depending on the marginal effect on seignirage v.s. taxation, which one is bigger.

As for the effects of changing $\bar{B}$, given $B_s \geq B_s^*$, $B_p = \bar{B} - B_s^*$, then any changing on aggregate bond supply flows one-for-one directly into the banking sector, and the effects on Sector 1 will exactly be the same as the above Case 1.

Case 4: when $B_s \geq B_s^*$ and $B_p \geq B_p^*$

This is the case safe assets are globally abundant, i.e., $\bar{B}$ is massive. Then we have $q_1^a = q_1^*$, $q_2^s = q_2^*$, and the liquidity premia of bonds and stablecoins both collapse, so that $\phi_b = \beta$, $\mu_s = \beta - 1$ Therefore, there are no effects from changing B_s or $\bar{B}$. Massive $\bar{B}$ implies a huge debt burden for the government, which may trigger a pure debt-spiral vulnerability, requiring compound tax/seigniorage financing.

4.6 Numerical Analysis

For the numerical analysis, except $\theta = 1$, $c(q) = q$, we keep the same functional forms and parameter values as in Section 2: $\alpha_1 = 0.5$, $\beta = 0.95$, $\tau = 0.20$, $\rho = 0.8$, $u(q) = q^{1-\sigma}/(1-\sigma)$ with $\sigma = 0.5$, but we let $G = 0.015$ to make sure the equilibrium

exists in this low-curvature economy, plus add new parameters $\varepsilon = 0.1$, $\chi = 1$, $\omega = 0.3$ (to match a majority of tradings in Sector 1 pay by bank deposits), $\alpha_2^m = 0.3$, $\alpha_2^s = 0.7$ here. Then we can derive $B_s^* = (1-\rho)\alpha_2^s q_2^*/\chi = 0.2 * 0.7 * 1 = 0.14$, but $B_p^* = (1+\tau)q_1^*/(1-\tau) - \hat{z}_m$ is a bit tricky: first we can derive $\hat{z}_m$ endogenously, then B_p^* will dynamically change with $\hat{z}_m$.

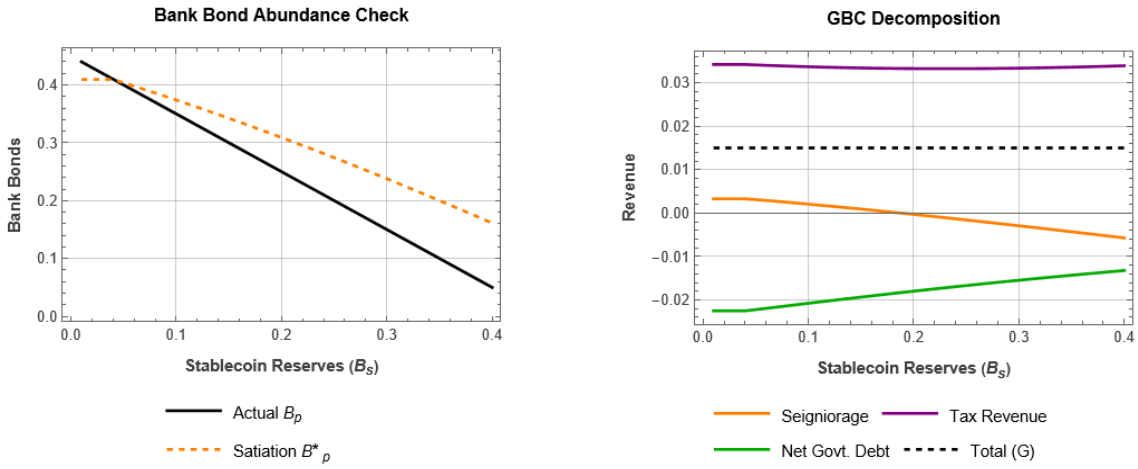


Figure 14: Effects of Changing B_s: Equilibrium Cases & Decomposition

First, we provide the big-picture regarding the effects of changing stablecoin reserves (B_s) in Figure 14. The left pane shows our numerical analysis starts from abundant bond holdings of banks, i.e., $B_p \geq B_p^*$ (Case 2), and transitions to scarce bond holdings (Case 1) for equilibrium analysis. Actually it also switches to the equilibrium Case 3 since $B_s^* = 0.2$, and the figure shows the range of B_s as $(0, 0.40)$. The right pane shows the effects of changing B_s on different components of the GBC: the green curve shows that the (negative) net government debt cost slightly decreases in absolute value, meaning the government receives a larger yield subsidy. The orange curve shows that seigniorage revenue decreases, while the purple curve shows that tax revenue also slightly decreases due to the crowding out of the trade in Sector 1.

Then, Figure 15 and 16 explicitly detail the effects of increasing B_s on Sector 2 (q_2^s, μ_s), on Sector 1 (q_1^m, q_1^a, z_m, μ_m), and on bond markets (bond price ϕ_b and nominal bond yields ι_b). As stablecoin issuers accumulate more reserves, stablecoin trading

(q_2^s) mechanically expands until B_s reaches to B_s^*, then q_2^s is capped at the relevant level, driving the effective stablecoin growth rate (μ_s) down toward the negative regime. However, this reserve accumulation creates a pronounced crowding-out effect in the traditional banking sector, starving banks of the collateral or investment needed to facilitate type-a trade (q_1^a) in Sector 1. Remarkably, this systemic collateral scarcity pushes the real bond price (ϕ_b) up and significantly lowers nominal bond yields (ι_b). Buoyed by this "yield subsidy" and cheaper borrowing costs, the government reduces its reliance on the inflation tax, allowing the inflation rate (μ_m) to keep declining, even reaching a negative inflation zone. This subsequently incentives agents to hold higher fiat real balances (z_m), which boosts fiat-based type-m trade (q_1^m).

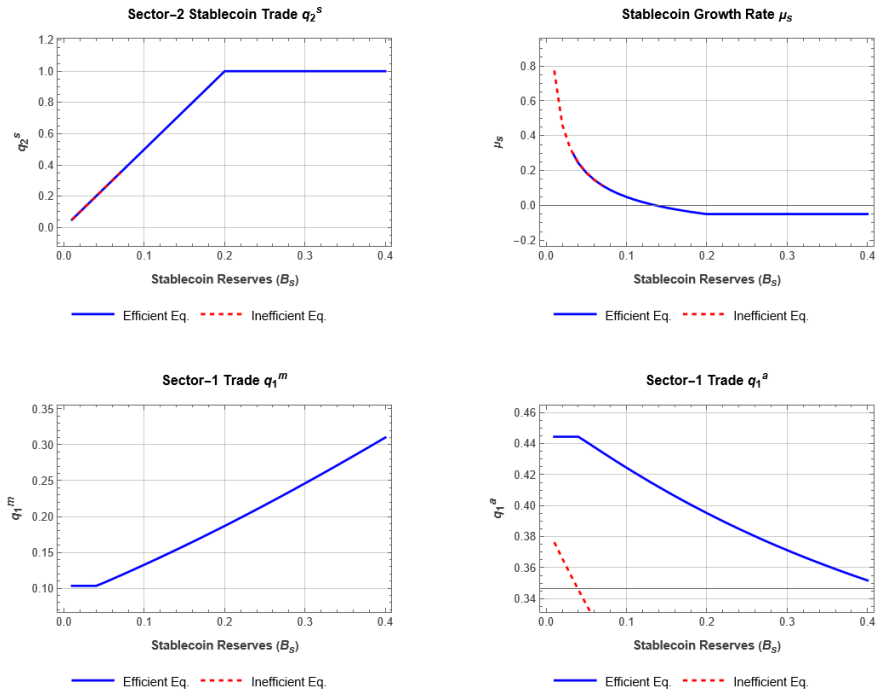


Figure 15: Effects of Changing B_s: Sector 2 & 1

Next, we provide the results on changing aggregate bond stocks $(\bar{B})$ in Figures 17 - 19. Initially, injecting bonds into the economy relieves collateral scarcity, allowing formal Sector 1 trade (q_1^a) to expand. Consequently, the liquidity premium on bonds dissipates, causing the bond price (ϕ_b) to fall toward its fundamental value and nominal yields (ι_b) to normalize upward. However, this normalization of yields on a larger

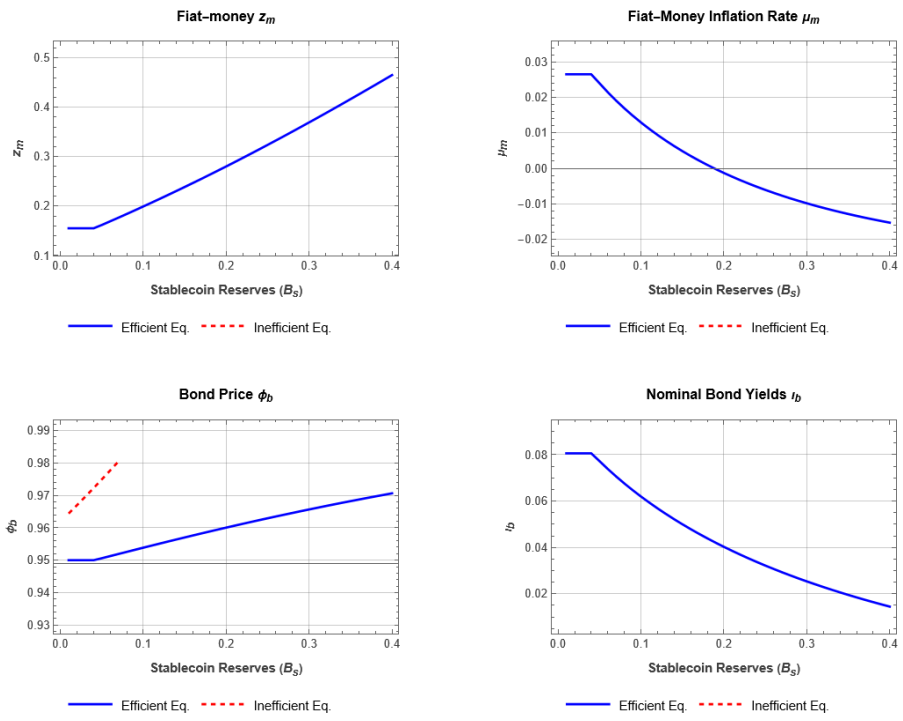


Figure 16: Effects of Changing B_s: Financial Variables

debt stock sharply increases the government's debt-servicing burden (as seen by the plunging green curve in the GBC decomposition). To satisfy the budget constraint, the government is forced to aggressively raise fiat inflation (μ_m), which destroys fiat real balances (z_m). Because banks require a portfolio of both fiat and bonds to facilitate formal transactions, the violent collapse in z_m overpowers the abundance of bonds. This throws the banking sector back into severe scarcity, triggering a sovereign debt spiral where formal trade (q_1^a) crashes once again.

Last but not the least, we can summarize the *equilibrium existence and uniqueness conditions* for Stablecoin Economy. Different from Proposition 1 in Cash Economy, in this stablecoin economy, general equilibrium is case by case, but the regime inequalities are endogenous. That is, bank bond scarcity condition is equivalently

$$B_p < B_p^* \iff q_1^a < q_1^* \iff \hat{z}_m < \frac{q_1^*}{C(\tau)} - \frac{\bar{B} - B_s}{\rho}.$$

Therefore, in each equilibrium case $i \in \{1, 2, 3, 4\}$, we can define a case-specific net

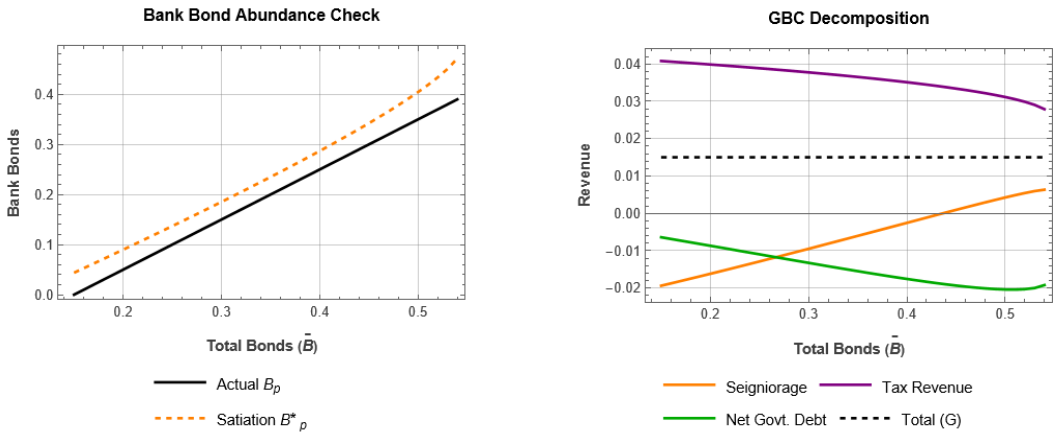


Figure 17: Effects of Changing $\bar{B}$: Equilibrium Cases & Decomposition

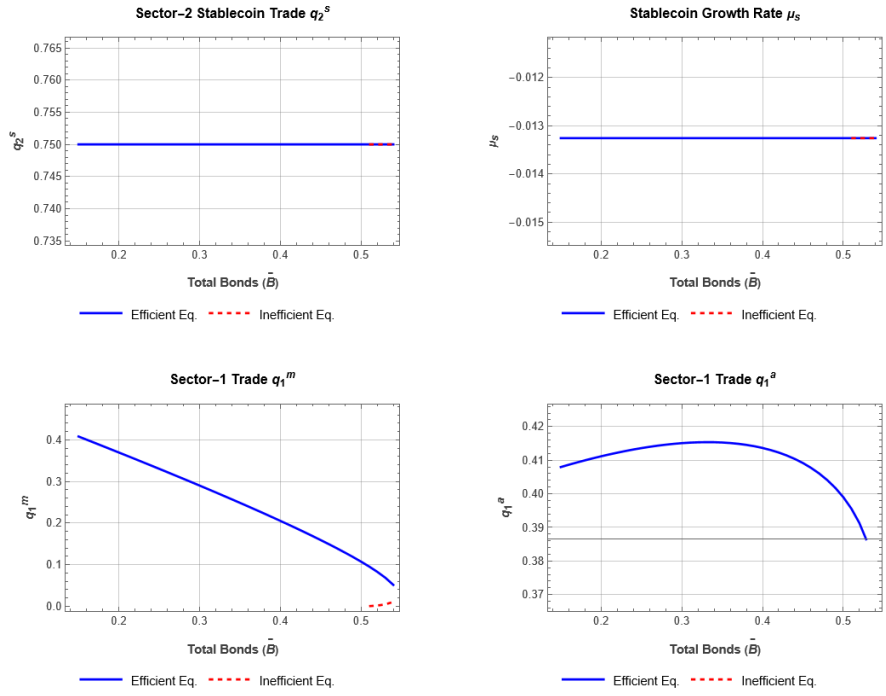


Figure 18: Effects of Changing $\bar{B}$: Sector 2 & 1

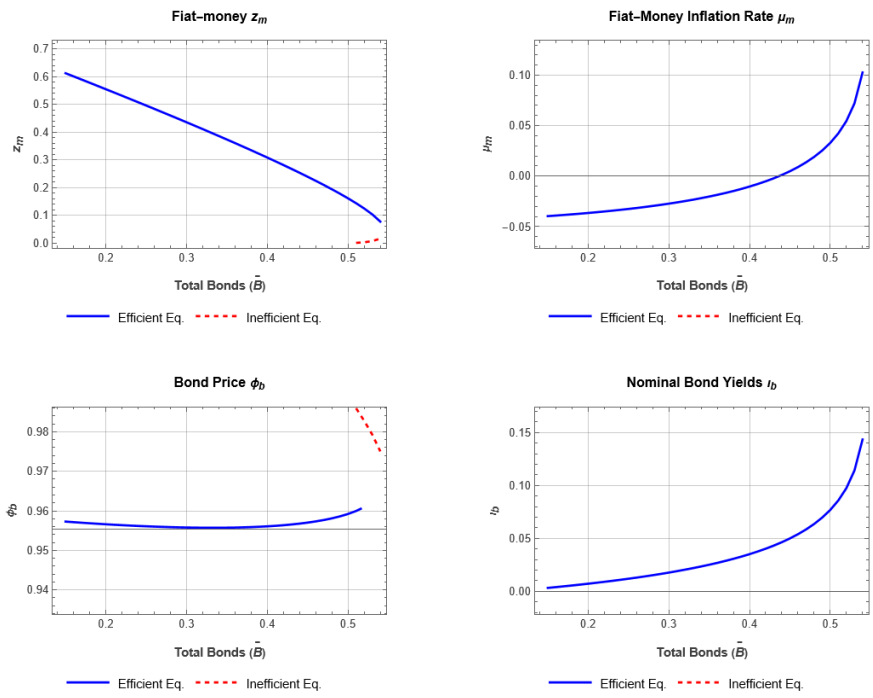


Figure 19: Effects of Changing $\bar{B}$: Financial Variables

government revenue function $S_i(\hat{z}_m)$ obtained by substituting the relevant Euler equations and terms of trade solutions. A stationary monetary equilibrium of case i is a root of $G = S_i(\hat{z}_m)$ that also satisfies the above case inequalities.

4.7 Stablecoin Issuer Misrepresentation

We now endogenize the genuine reserve ratio χ and stablecoin supply B_s by addressing the issuers' moral hazard problem. As mentioned before, to misrepresent bond holdings, f need to produce fake bonds $\hat{z}_b^s$ in CM in advance (as reserves to issue stablecoins used for DM next period), at the cost $\varphi \hat{z}_b^s$. Suppose the ratio of genuine bonds is $\chi \in (0, 1]$, then $d_2^s \leq \hat{z}_s = \chi \hat{z}_b^s$. As is standard in signalling models, we let $\theta = 1$. When holding genuine bonds, the present-value of f profits are

$$\Pi^1 = \hat{z}_s - \phi_b \hat{z}_b^s + \beta(\hat{z}_b^s - d_2^s),$$

where d_2^s is stablecoins used for DM trading later. When holding fake bonds, the profits are

$$\Pi^0 = \hat{z}_s - \varphi \hat{z}_b^s.$$

Therefore, the incentive condition for issuers is

$$\Pi^1 \geq \Pi^0,$$

which can be rewritten as

$$\phi_b \hat{z}_b^s - \beta(\hat{z}_b^s - d_2^s) \leq \varphi \hat{z}_b^s.$$

We then derive

$$d_2^s \leq (\frac{\varphi}{\beta} - s_b)\hat{z}_b^s,$$

where $s_b \equiv \phi_b/\beta - 1$, as defined before. Since the collateral/pledgeability constraint is $d_2^s \leq \hat{z}_s = \chi \hat{z}_b^s$, we should have

$$\chi = \frac{\varphi}{\beta} - s_b. \tag{58}$$

The above shows the endogenous pledgeability of bond holdings as stablecoin reserves, which is also the incentive-compatible bond holding level for f. In addition, we need to have

$$\beta s_b < \varphi \leq \beta(1 + s_b),$$

to have $\chi \in (0, 1]$.

When $d_2^s \leq \hat{z}_s = \chi \hat{z}_b^s$ binds, and χ is from (58), we can now rewrite the profits of f as

$$\Pi^1 = \hat{z}_s - \phi_b \hat{z}_b^s + \beta(\hat{z}_b^s - d_2^s) = (\frac{1-\beta}{\beta}\varphi - s_b)\frac{B_s}{1-\rho},$$

where we can use $\hat{z}_b^s = (1-\rho)B_s$ in equilibrium. The participation constraint for f to hold genuine bonds is,

$$\Pi^1 \geq 0.$$

Hence, we can derive $\varphi \geq \beta s_b/(1-\beta)$. Previously, we already have $\beta s_b < \varphi \leq$

$\beta(1+s_b)$, therefore, in the end,

$$\frac{\beta s_b}{1-\beta} \leq \varphi \leq \beta(1+s_b), \tag{59}$$

which is a general condition for the range of φ, with the upper bound determined by s_b. Later we discuss about the ranges of φ under different market structure of stablecoin issuers.

Now we discuss about the *market structure of stablecoin markets*: should the policy maker regulate the stablecoin markets competitive (superscript "C"), or monopolistic (superscript "M")? Suppose there is free entry for the stablecoin market, so f is a competitive issuer, then the profits will go to zero, i.e., $\Pi^1 = 0$. Then

$$s_b^C = \frac{1-\beta}{\beta}\varphi. \tag{60}$$

Furthermore, we can also derive

$$\chi^C = \frac{\varphi}{\beta} - s_b^C = \varphi, \tag{61}$$

which means, under the competitive market for f, free entry and competition will drive the genuine bond holdings ratio χ^C exactly to the marginal cost of faking bonds, φ. Using (59), we can also update the boundary for φ for a competitive f as

$$0 < \varphi < \frac{\beta s_b^{\max}}{1-\beta},$$

where $s_b^{\max}$ is the maximum feasible crowding-out spread, since stablecoin supply cannot physically exceed total available bonds (when $B_s \to \bar{B}$).

Therefore, with a competitive f, now we can solve for the equilibrium B_s as follows. First, since we have (60), then we have

$$\frac{1-\beta}{\beta}\varphi = \frac{\phi_b}{\beta} - 1 = \frac{\alpha}{2}(1-\omega)\lambda_1(q_1^a)$$

to solve for

$$\phi_b = (1-\beta)\varphi + \beta$$

and q_1^a from

$$u'(q_1^a) = [\frac{2(1-\beta)\varphi}{\rho\alpha(1-\omega)\beta} + 1]/C(\tau)$$

where we use $\lambda_1(q_1^a) \equiv C(\tau)u'(q_1^j) - 1$. Then we can solve for $(\hat{z}_m, B_s)$ from

$$\begin{aligned} G &= \rho\mu_m(\hat{z}_m, B_s)\cdot\hat{z}_m + \frac{\tau\alpha}{1+\tau}[\rho\hat{z}_m + (1-\omega)(\bar{B} - B_s)] + (1-\beta)(\varphi-1)\bar{B} \quad (62) \\ q_1^a &= C(\tau)\hat{z}_m + C(\tau)\frac{\bar{B} - B_s}{\rho}. \quad (63) \end{aligned}$$

Once we get $(\hat{z}_m, B_s)$, we can solve for other variables like q_1^m, q_2^s, $\hat{z}_m$, μ_m, ϕ_b.

If f is a monopoly, we let $s_b^M \equiv s_b^M(B_s)$, with $s^{M\prime}(B_s) > 0$, which means f is a monopoly issuer that can determine bond pricing, the higher B_s is, the more demand for bonds, then ϕ_b increases, and $s_b = \phi_b/\beta - 1$ increases as well. Then

$$\Pi^1 = \max_{B_s}\{[\frac{1-\beta}{\beta}\varphi - s_b^M(B_s)]\frac{B_s}{1-\rho}\}.$$

The FOC against B_s is

$$\frac{1-\beta}{\beta}\varphi - B_s\cdot s_b^{M\prime}(B_s) = s_b^M(B_s). \quad (64)$$

Using (64), we can solve for $s_b^M(B_s)$, and furthermore can solve for ϕ_b^M and q_1^a since $s_b^M = \phi_b^M/\beta - 1$, $s_b^M = \rho\alpha(1-\omega)\lambda_1(q_1^a)/2$. We can also derive

$$\begin{aligned} \frac{\partial q_1^a}{\partial s_b^M} &= \frac{2}{\rho\alpha(1-\omega)\lambda_1'(q_1^a)} < 0, \\ \frac{\partial q_1^a}{\partial B_s} &= \frac{2}{\rho\alpha(1-\omega)\lambda_1'(q_1^a)}\frac{ds_b^M}{dB_s} < 0, \end{aligned}$$

since we know $s_b^{M\prime}(B_s) > 0$, $\lambda_1'(q_1^a) < 0$. Furthermore, once we get ϕ_b^M and q_1^a, similarly, we can use the two equations as follows to solve for $(\hat{z}_m, B_s)$,

$$\begin{aligned} G &= \rho\mu_m(\hat{z}_m, B_s)\cdot\hat{z}_m + \frac{\tau\alpha}{1+\tau}[\rho\hat{z}_m + (1-\omega)(\bar{B} - B_s)] + (\phi_b^M - 1)\bar{B} \quad (65) \\ q_1^a &= C(\tau)\hat{z}_m + C(\tau)\frac{\bar{B} - B_s}{\rho}. \quad (66) \end{aligned}$$

Since $s^{M\prime}(B_s) > 0$, given the same φ, (60) and (64) show that

$$s_b^M(B_s) < s_b^C,$$

i.e., the monopoly spread is strictly lower than the competitive spread. It means the monopolist issuer can deliberately restrict stablecoins issuance to suppress its reserve acquisition costs. Given $s_b^{M\prime}(B_s) > 0$, we have

$$B_s^M < B_s^C,$$

Hence, the spread function $s_b(\cdot)$ is monotonically increasing, the inequality $s_b^M(B_s) < s_b^C(B_s)$ mathematically guarantees that reserve holdings by monopoly f is lower than that of competitive f, given any feasible φ. Aforementioned, the Euler equations shows q_1^a decreases with s_b^M, hence a lower bond spread means a higher type-a trading quantities, i.e.,

$$q_1^{a,M} > q_1^{a,C}.$$

Since $\phi_b = \beta(1 + s_b)$, a lower spread means a lower bond price, then higher bond yields, i.e.,

$$\phi_b^M < \phi_b^C,\ \iota_b^M > \iota_b^C.$$

Therefore, from a policy perspective, a stablecoin monopoly issuer can forces the government to pay higher interest rates on its debt, compared to a competitive stablecoin market. Considering the misrepresentation frictions, using (58) and (64), we can derive

$$\chi^M = \frac{\varphi}{\beta} - s_b^M = \varphi + B_s \cdot s_b^{M\prime}(B_s) > \varphi = \chi^C. \tag{67}$$

That is, a lower bond spread $s_b^M(B_s)$ leads to a bigger genuine reserve ratio. We summarize the main results as a "Macroeconomic Trilemma" Proposition as follows,

Proposition 3 *Compared to a competitive market, with information frictions captured by φ, a monopolistic stablecoin issuer deliberately restricts stablecoin supply ($B_s^M < B_s^C$), to suppress its reserve acquisition costs, resulting in a strictly lower*

equilibrium bond spread, i.e., $s_b^M(B_s) < s_b^C$*. In equilibrium, the monopolistic market structure yields the following "macroeconomic trilemma": (i) Enhanced reserve safety: A strictly higher intrinsic genuine reserve ratio:* $\chi^M > \chi^C$*; (ii) Greater bank intermediation: eased collateral scarcity for the traditional banking sector, yielding higher formal deposit-backed trade:* $q_1^{a,M} > q_1^{a,C}$*; (iii) Smaller fiscal subsidy: lower bond prices and higher nominal yields on public debt, reflecting higher sovereign borrowing costs:* $\phi_b^M < \phi_b^C$ *and* $\iota_b^M > \iota_b^C$*.*

To provide an explicit comparision between competitive and monopoly f, we also provide numerical analysis here, with the same functional forms and parameter values as before: $\theta = 1$, $c(q) = q$, $\alpha = 0.5$, $\beta = 0.95$, $\tau = 0.20$, $\rho = 0.8$, $u(q) = q^{1-\sigma}/(1-\sigma)$ with $\sigma = 0.5$, $G = 0.015$, $\omega = 0.3$, except that we set $\bar{B} = 0.40$, and now χ and B_s is endogenous. Under a competitive market of stablecoins, we can derive the feasibilty bound of s_b when $B_s \to \bar{B}$, i.e., stablecoin issuers hold all of the govt. bond as collateral, basically crowding out all trading of TradFi (type-a meetings in Sector 1), i.e., $s_b^{\max} \approx 0.0182$**, when** $B_s = 0.399$**.** And we can also compute the corresponding upper bound for φ as 0.3775. Furthermore, under a monopoly stablecoin issuer, using (**??**), we can derive the upper bound of φ is 0.9698.

Figure 20 shows the effects of changing φ on stablecoin reserves (B_s) and on type-a trade quantities q_1^a, which can be interpreted as the trade quantities representing TradFi (wholly supported by bank deposits, and banks hold the portfolio of fiat money and govt. bonds), and the decreasing of q_1^a with increasing φ can be regarded as the crowding-out effects of fake bond holdings by f on bank collateral (genuine bonds). More interestingly, Figure 20 particularly compares the difference responses under monopoly f (red dashed line) and competitive f (blue line). Figure 21, furthermore, shows the effects of changing φ on the genuine bond holding ratio χ, and the ratio under monopoly f is higher than that under competitive f.

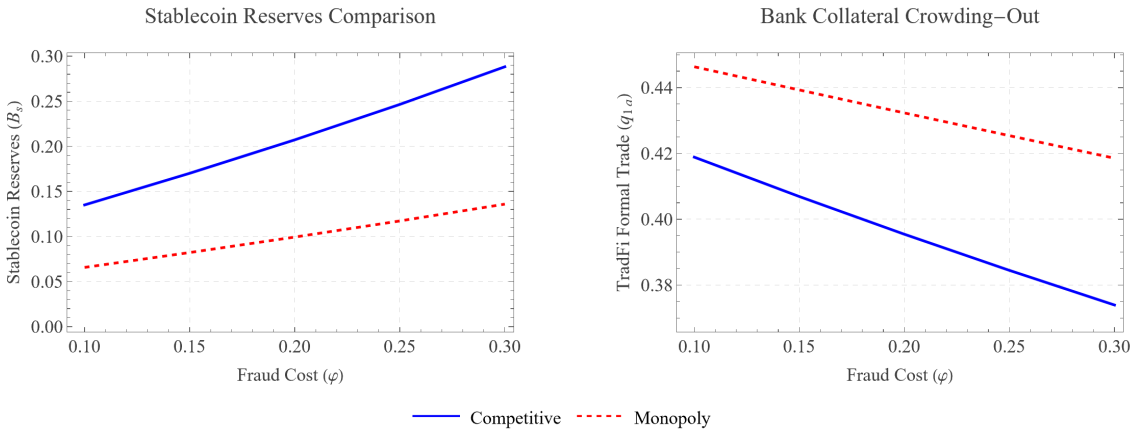


Figure 20: Effects of Changing φ on B_s and q_1^a

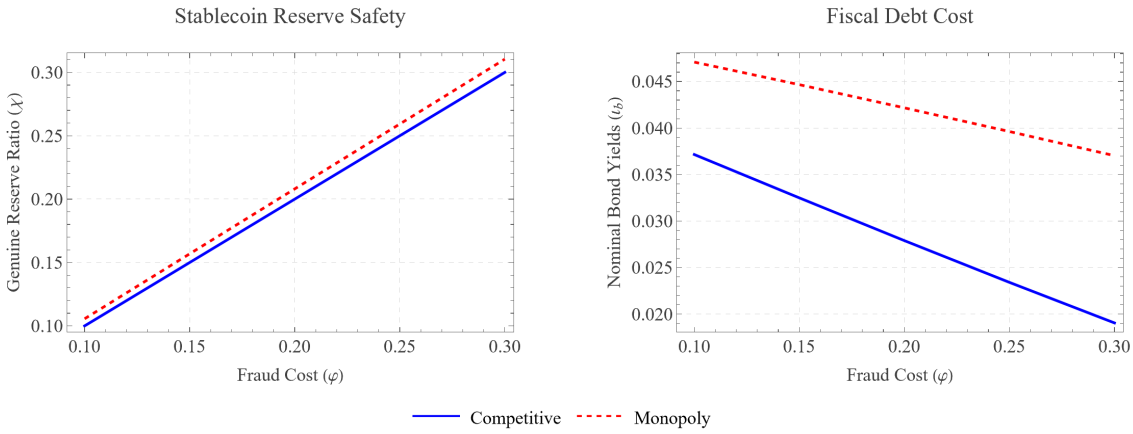


Figure 21: Effects of Changing φ on χ and ι_b

5 Currency Digitalization Paths

Having established the profound macroeconomic and public-finance mechanism of a private stablecoin regime, we now broaden our scope to evaluate the diverging trajectories of global currency digitalization. Currently, the international financial system stands at a strategic crossroads between two dominant paradigms: the proliferation of privately issued, fiat-backed stablecoins—a trajectory recently codified by U.S. regulatory frameworks like the GENIUS Act—and the state-led introduction of CBDCs, exemplified by China's e-CNY pilot. To formally contrast these strategies, this section extends our nested framework to model the "CBDC Path," wherein public digital fiat directly replaces physical cash in Sector 1 (Formal). By juxtaposing this centralized public architecture against the decentralized stablecoin economy analyzed in Section 4, we systematically assess how the choice of digital payment rails differentially shapes the central bank's seigniorage base, the systemic demand for safe assets, commercial bank intermediation, and the ultimate optimal monetary-fiscal policy mix.

5.1 CBDC Path

To capture the currency digitalization path of adopting CBDC, we add govt. bonds b and digital fiat money m_c, and also add banks to the baseline model in Section 2. We assume CBDC m_c replaces cash m_m in Sector 1, but cash still circulates in Sector 2, due to anonymity and tax evasion.[13] As in Section 4, there are banks who can accept CM goods (or assets in equivalence) from agents as deposits a, then hold the portfolio of (m_c, b) for liquidity management, or investment on behalf of agents. Hence, there are both CBDC m_c and deposits a circulating in Sector 1, to settle DM trade, while cash still circulates in Sector 2. The bond pricing, ϕ_b, ι_b, s_b is the same as in the stablecoin model of Section 4.1. And the government is still active in CM,

[13]This modelling assumption is based on stylized facts on CBDC and cash usage as follows: In China (and many other countries), cash is used little in ordinary transacations, due to well-developed mobile payment or instant payment systems; moreover, as pointed out by PBOC (China's Central Bank), "cash and e-CNY will coexist in the long run".

and the GBC changes to

$$G + B_{-1} = \phi_b B + \phi_m(M - M_{-1}) + \tau \cdot D_1,$$

where M_- refers to total money supply of previous period (the subscript "$-$" means "$t-1$"), $M = M_m + M_c$ including both cash and CBDC, and D_1 again is post-matching total transactsion amounts of Sector 1.[14] At steady states, we have

$$G = \mu_m \phi_m M + \tau \cdot D_1 + (\phi_b - 1)B. \tag{68}$$

Here B is, actually bond holdings by the public, hence an one-time change of B can be interpreted as OMOs (or IORB, ON RRP): bond sales of the central bank will increase B, then lead to contractionary monetary policy; bond purchases of the central bank will decrease B, then lead to expansionary policy (Rocheteau et al., 2018). Of course, here our focus is different: when B changes, how will it affect CBDC tradings in DM, and what are the implications to public finance? How does it differ from the effects of changing B_s, or $\bar{B}$ in Stablecoin Economy? We will discuss all of these later.

5.1.1 Model

In the end of CM, after the idiosyncratic shocks are realized, agents know which sector they will enter for DM trading. Those choosing Sector 1, contact banks to purchase deposit contracts, then hold a portfolio of CBDC and deposits (m_c, a) to enter DM. And those choosing Sector 2, just pay with cash m_m. Similarly, due to complicated activities in CM, here we focus on DM, and leave more description and analysis of CM for later.

In **DM** next period, agents choosing Sector 1 hold a portfolio of CBDC and deposits $(\hat{z}_a, \hat{z}_c)$. Then

$$V_1(\hat{z}_a, \hat{z}_c) = \frac{1}{2} V_1^b(\hat{z}_a, \hat{z}_c) + \frac{1}{2} V_1^s(\hat{z}_a, \hat{z}_c),$$

[14] Suppose the fraction of CBDC is $\varepsilon \in (0, 1)$, so $M_c/M = \varepsilon$. Then ε can be another policy parameter, and we could discuss about the policy implications of changing ε.

where

$$V_1^b(\hat{z}_a, \hat{z}_c) = \alpha_1\{\omega[u(q_1^c) - (1+\tau)d_1^c] + (1-\omega)[u(q_1^a) - (1+\tau)d_1^a]\} + W_1(\hat{z}_a, \hat{z}_c)$$
$$V_1^s(\hat{z}_a, \hat{z}_c) = \alpha_1\{\omega[-c(q_1^c) + (1-\tau)d_1^c] + (1-\omega)[-c(q_1^a) + (1-\tau)d_1^a]\} + W_1(\hat{z}_a, \hat{z}_c),$$

with (q_1^j, d_1^j), $j = \{a, c\}$, denoting the terms of trade in type-j meetings in Sector 1. Similarly as in Stablecoin Economy, there are deposit contracts between banks and agents for $\{\hat{z}_b, \hat{z}_c, d_1^c, d_1^a\}$ by the end of CM in the previous period, and agents carry $\hat{z}_a, \hat{z}_c$ for DM trading, with $(1+\tau)d_1^c \leqslant \hat{z}_c$, $(1+\tau)d_1^a \leqslant \hat{z}_a$.

As for agents choosing Sector 2, they bear the disutility cost κ ex ante, and pay by cash for DM trading,

$$V_2(\hat{z}_m, \kappa) = \frac{1}{2}V_2^b(\hat{z}_m, \kappa) + \frac{1}{2}V_2^s(\hat{z}_m, \kappa),$$

where

$$V_2^b(\hat{z}_m, \kappa) = -\kappa + \alpha_2[u(q_2^m) - d_2^m] + W_2(\hat{z}_m)$$
$$V_2^s(\hat{z}_m, \kappa) = -\kappa + \alpha_2[-c(q_2^m) + d_2^m] + W_2(\hat{z}_m).$$

Here (q_2^m, d_2^m) denotes the terms of trade in Sector 2, and $d_2^m \leqslant \hat{z}_m$.

Now we consider the endogenous dichotomy, as in Section **??**. In equilibrium, agents should be indifferent to enter Sector 1 or Sector 2. Then

$$\frac{\alpha_1}{2}\{\omega[u(q_1^c) - c(q_1^c) - 2\tau d_1^c] + (1-\omega)[u(q_1^a) - c(q_1^a) - 2\tau d_1^a]\} = -\hat{\kappa} + \frac{\alpha_2}{2}[u(q_2^m) - c(q_2^m)].$$

Hence,

$$\hat{\kappa} = \Delta(q_1^m, q_1^a, q_2^s),$$

where $\Delta(q_1^m, q_1^a, q_2^s)$ is the trading surplus between Sector 2 and Sector 1, with

$$\Delta(q_1^{mb}, q_2^s) \equiv \frac{\alpha_2}{2}[u(q_2^m) - c(q_2^m)] - \frac{\alpha_1}{2}\{\omega[u(q_1^m) - c(q_1^m)] + (1-\omega)[u(q_1^a) - c(q_1^a)]\}$$
$$+ \alpha_1\tau[\omega d_1^m + (1-\omega)d_1^a].$$

With $1-\rho = F(\hat{\kappa})$, κ following the same uniform distribution, we have

$$\rho = 1 - \frac{\Delta(q_1^c, q_1^a, q_2^m)}{\gamma}. \tag{69}$$

Again, as in Section **??**, there are corner cases: if $\Delta(q_1^m, q_1^a, q_2^s) \leq 0$, then $\rho = 1$; and if $\Delta(q_1^m, q_1^a, q_2^s) \geq \gamma$, then $\rho = 0$. Hence, if $0 < \Delta(q_1^m, q_1^a, q_2^s) < 1$, we have $0 < \rho < 1$.

5.1.2 Equilibrium Analysis

In this subsection, we sort out the problem of banks' deposit contracts for Sector 1, then the cash demand problems for Sector 2, and make explicit equilibrium analysis of two cases, in the end. We also check the effects of changing B, and compare the mechanism and results to that of changing B_s, $\bar{B}$ in Stablecoin Economy.

I. Deposit Contracts

The equilibrium deposit contracts, $\{\hat{z}_b, \hat{z}_c, d_1^c, d_1^a\}$, solve for

$$\begin{aligned}
\max_{\hat{z}_b, \hat{z}_c, d_1^c, d_1^a} \{&-(1+\mu_m)\hat{z}_c - \phi_b \hat{z}_b + \beta\alpha_1\omega[u(q_1^c(d_1^c) - (1+\tau)d_1^c] \\
&+\beta\alpha_1(1-\omega)[u(q_1^a(d_1^c)) - (1+\tau)d_1^a]\} \\
&\text{st. } (1+\tau)d_1^c \leq \hat{z}_c, \\
&(1+\tau)d_1^a \leq \hat{z}_a, \\
&\varepsilon\hat{z}_a \leq \frac{\hat{z}_c + \hat{z}_b}{1-\omega}
\end{aligned}$$

Using the bargaining solutions, $c(q_1^j) = q_1^j = (1-\tau)d_1^j$, $j = \{c, a\}$, we discuss about bond abundance status of banks, and two cases of equilibria as follows.

1. When $B < B^*$,

$$d_1^c = \frac{\hat{z}_c}{\omega(1+\tau)}, \quad d_1^a = \frac{\hat{z}_c + \hat{z}_b}{\varepsilon(1-\omega)(1+\tau)} \tag{70}$$

$$q_1^c = C(\tau)\frac{\hat{z}_c}{\omega}, \quad q_1^a = \frac{C(\tau)}{\varepsilon(1-\omega)}(\hat{z}_c + \hat{z}_b), \tag{71}$$

where still $C(\tau) \equiv (1-\tau)/(1+\tau)$.

Then we can derive the Euler equations for $\hat{z}_b$, $\hat{z}_c$ as

$$\frac{\phi_b}{\beta} - 1 = \frac{\alpha}{2\varepsilon}\lambda_1(q_1^a) \tag{72}$$

$$\frac{1+\mu_m}{\beta} - 1 = \frac{\alpha}{2}[\lambda_1(q_1^c) + \frac{1}{\varepsilon}\lambda_1(q_1^a)], \tag{73}$$

where we have, for $j = \{a, c\}$,

$$\lambda_1(q_1^j) \equiv C(\tau)u'(q_1^j) - 1,$$

and $\lambda_1'(q_1^j) < 0$. We have $q_1^j = q_1^*$, when $u'(q_1^*) = (1+\tau)/(1-\tau) = 1/C(\tau)$. Notice that the RHS of (73) shows that CBDC $\hat{z}_c$ provides a weighted sum of liquidity premium from both type-c and type-a meetings. The former comes from $\hat{z}_c$ serves as a medium of exchange, while the latter is more like $\hat{z}_c$ serves as reserves or other liquidity management facilities for banks. It provides an interesting mix of outside money and inside money roles of CBDC, which captures China's e-CNY now has dual roles of these two monies too.

2. When $B \geq B^*$, banks' bond holdings B are at an abundant level, then

$$B = \rho\hat{z}_b \geq \rho(\frac{1+\tau}{1-\tau}q_1^* - \hat{z}_c) \equiv B^*, \tag{74}$$

so that $d_1^c = \hat{z}_c$, $d_1^a = q_1^*/(1-\tau)$, with $u'(q_1^*) = (1+\tau)/(1-\tau)$, and $\phi_b = \beta$. Hence, CBDC still provides the liquidity premium at the margin (for type-c meetings), but the trading for type-a goes to the first best level q_1^*, and bonds do not provide the liquidity premium at the margin. Hence, CBDC $\hat{z}_c$ only provides liquidity premium for type-c meetings.

$$\frac{1+\mu_m}{\beta} - 1 = \frac{\alpha}{2}\lambda_1(q_1^c). \tag{75}$$

II. Cash Demand

As is standard in New Monetarism models, the cash demand problem is,

$$\max_{\hat{z}_m}\{-(1+\mu_m)\hat{z}_m+\beta\frac{\alpha}{2}[u(q_2^m)-d_2^m]\}$$
$$\text{st. } d_2^m\leq\hat{z}_m.$$

When $d_2^m\leq\hat{z}_m$ binds, with T-I-O-L-T offers, we have

$$d_2^m=\hat{z}_m=q_2^m. \tag{76}$$

Then we can derive the Euler eq. as,

$$\frac{1+\mu_m}{\beta}-1=\frac{\alpha}{2}\lambda_2(q_2^m), \tag{77}$$

where

$$\lambda_2(q_2^m)=u'(q_2^m)-1.$$

III. Equilibrium Analysis

Now we define a stationary monetary equilibrium as follows.

Definition 3 *Given the policy parameters $\{G,B,\tau\}$, a stationary monetary equilibrium is a list of $\{q_1^c,q_1^a,q_2^m,\hat{z}_m,\hat{z}_c,\mu_m,\phi_b\}$, satisfying: [i] bargaining solutions (70), (71), (76); [ii] Euler equations for bonds, CBDC and cash, by (**??**), (73) and (77); [iii] the government budget constraint (68); [iv] market clearing conditions for money and bonds, by $\phi_m M=\rho\hat{z}_c+(1-\rho)\hat{z}_m$, $B=\rho\hat{z}_b$, and bond pricing equations for ϕ_b and s_b.*

Next we discuss two cases of general equilibrium, depending on the abundant level of bond holdings by banks.

Case 1: When $B<B^*$

In this case, bonds are scarce as banks' assets (or collateral). Since $B<B^*$, we have $q_1^a<q_1^*$, hence $\hat{z}_b+\hat{z}_c<q_1^*/C(\tau)$. And we can rewrite the post-matching

transaction amounts D_1 as

$$\begin{aligned} D_1 &= \rho\alpha[\omega d_1^c + (1-\omega)d_1^c] \\ &= \frac{\alpha_1}{1+\tau}\left[\rho\left(1+\frac{1}{\varepsilon}\right)\hat{z}_c + \frac{B}{\varepsilon}\right]. \end{aligned}$$

Then we can rewrite GBC as

$$G = \mu_m[\rho\hat{z}_c + (1-\rho)\hat{z}_m] + \frac{\tau\alpha_1}{1+\tau}\left[\rho\left(1+\frac{1}{\varepsilon}\right)\hat{z}_c + \frac{B}{\varepsilon}\right] - (1-\phi_b)B, \qquad (78)$$

where we substitute $\hat{z}_b = B/\rho$. And at the RHS of (78), the first term still represents seigniorage, with real balances of money stock $\phi_m M = \rho\hat{z}_c + (1-\rho)\hat{z}_m$; the second term is tax revenues collected from Sector 1; and the third term is net government debt cost (usually negative, due to bond issuing discount), or the absolute values represents the net cost.

We also have

$$q_1^c = C(\tau)\hat{z}_c, q_1^a = C(\tau)(\hat{z}_b + \hat{z}_c).$$

We know $\hat{z}_b = B/\rho$, then

$$q_1^a = C(\tau)(\hat{z}_c + \frac{B}{\rho}).$$

Then we can use the Euler eq. for CBDC and money and rewrite to have

$$u'(\hat{z}_m) - 1 = [C(\tau)u'(C(\tau)\frac{\hat{z}_c}{\omega}) - 1] + \frac{1}{\varepsilon}[\frac{C(\tau)}{\varepsilon(1-\omega)}u'(C(\tau)(\hat{z}_c + \frac{B}{\rho})) - 1] \equiv R(\hat{z}_c; B).$$

which shows the key links between two fiat monies, $\hat{z}_c$ and $\hat{z}_m$. Hence,

$$\hat{z}_m \equiv \hat{z}_m(\hat{z}_c),$$

which means $\hat{z}_m$ can be rewritten as a function of $\hat{z}_c$. In addition, given the same money growth rate μ_m, these two monies are supposed to provide the same liquidity premium, although they are used in Sector 1 (Formal/TradFi) and Sector 2(Crypto), respectively. If we define the above RHS as a function $R(\hat{z}_c; B)$. Since $\lambda_1'(q_1^j) < 0$, $j = \{c, a\}$, $dR(\hat{z}_c; B)/d\hat{z}_c < 0$. The LHS $\lambda_2(q_m) = u'(\hat{z}_m) - 1$ is fixed when $\hat{z}_m$ is fixed.

Hence, there is at most one $\hat{z}_c = h(\hat{z}_m; B)$ solving the above condition. Moreover, by implicit differentiation of the above condition, we have $h_{\hat{z}_m} > 0$, i.e., higher $\hat{z}_m$ lowers $\lambda_2(q_m)$, so $\hat{z}_c$ must rise to lower the RHS; $h_B < 0$, i.e., more bonds raise q_1^a, lowering the RHS, then to restore equality, $\hat{z}_c$ must fall.

Furthermore, we can rewrite to have

$$\begin{aligned} \mu_m(\hat{z}_m) &= \beta - 1 + \frac{\beta\alpha}{2}[u'(q_2^m) - 1] \\ \phi_b(\hat{z}_m; B) &\equiv \beta[1 + \frac{\alpha}{2\varepsilon}[\frac{C(\tau)}{\varepsilon(1-\omega)}u'(C(\tau)(\hat{z}_c + \frac{B}{\rho})) - 1]. \end{aligned}$$

In the end, we rewrite (78) as

$$G = S(\hat{z}_m; B),$$

where

$$\begin{aligned} S(\hat{z}_m; B) \equiv\ & \mu_m(\hat{z}_m)[\rho\hat{z}_c + (1-\rho)\hat{z}_m] + \frac{\alpha\tau}{1+\tau}\left[\rho\left(1 + \frac{1}{\varepsilon}\right)\hat{z}_c + \frac{B}{\varepsilon}\right] \\ & + [\phi_b(\hat{z}_m; B) - 1]B. \end{aligned}$$

For the equilibrium existence, since $S(\hat{z}_m; B)$ is continuous in $\hat{z}_m$, a stationary equilibrium exists if G lies in the range of $S(\hat{z}_m; B)$ over the feasible set where $B < B^*$. As before, the model usually delivers a Laffer-Curve shape (non-monotone $S(\hat{z}_m; B)$), there can be 0, 1, or 2 equilibria depending on G. As for local uniqueness, at a candidate $\hat{z}_m$, we have local uniqueness iff

$$\frac{\partial}{\partial \hat{z}_m}(G - S(\hat{z}_m; B)) \neq 0.$$

In economic intuitions, there will be a unique equilibrium if it is not sitting exactly at the peak or valley of the net-revenue. curve.

Case 2: When $B \geq B^*$

Since bond are plentiful enough to support the first best q_1^* in type-a meetings, we have $q_1^a = q_1^*$, with $u'(q_1^*) = 1/C(\tau)$, and $\phi_b = \beta$. And we have $B = \rho\hat{z}_b \geq B^*$,

with

$$B^* = \rho(\frac{\varepsilon(1-\omega)}{C(\tau)}q_1^* - \hat{z}_c).$$

Given $d_1^a = q_1^a/(1-\tau) = q_1^*/(1-\tau)$, we have

$$D_1 = \rho\alpha_1 \left[\frac{\hat{z}_c}{1+\tau} + (1-\omega)\frac{q_1^*}{1-\tau}\right].$$

Then we can rewrite GBC as

$$G = \mu_m[\rho\hat{z}_c(\hat{z}_m) + (1-\rho)\hat{z}_m] + \rho\alpha_1\tau\left[\frac{\hat{z}_c}{1+\tau} + (1-\omega)\frac{q_1^*}{1-\tau}\right] + (\beta-1)B,$$

where again we get $\hat{z}_c \equiv \hat{z}_c(\hat{z}_m)$ from using the Euler eq. of CBDC and cash,

$$\left[\lambda_1\left(C(\tau)\frac{\hat{z}_c}{\omega}\right) + \frac{1}{\varepsilon}\lambda_1\left(\frac{C(\tau)}{\varepsilon(1-\omega)}\left(\hat{z}_c + \frac{B}{\rho}\right)\right)\right] = \frac{\alpha_2^m}{2}\lambda_2(\hat{z}_m).$$

For the effects of changing B, since the theoretical results are ambiguous, we provide numerical analysis as follows. All of the parameter and functional forms are the same as Section 4.3.3: again we have $\theta = 1$, $c(q) = q$, and try to use the same functional forms and parameter values as in Section 2: $\alpha = 0.5$, $\beta = 0.95$, $\tau = 0.20$, $\rho = 0.8$, $u(q) = q^{1-\sigma}/(1-\sigma)$ with $\sigma = 0.5$, except that we let $G = 0.018$ to make sure the equilibrium exists and equilibrium regime switches (when B changes), plus add new parameters $\chi = 1$, $\omega = 0.3$. Then the effect of changing B are shown in Figure 22 and Figure 23.

Figure 22 and 23 illustrate the macroeconomic effects of expanding the public debt supply B when a CBDC is introduced to Sector 1 and replaces cash. Initially, when government bonds are scarce ($B < B^* \approx 0.34$), injecting bonds into the economy effectively relieves collateral constraints for commercial banks. This allows deposit-backed formal trade q_1^a to expand and dissipates the liquidity premium on bonds, pushing the real bond price ϕ_b down toward its fundamental value β. However, this normalization of yields imposes a severe "debt volume burden" on the Treasury. Because the public CBDC framework does not benefit from the artificial reserve hoarding of private stablecoin issuers, the government must bear the full fundamental interest cost of its

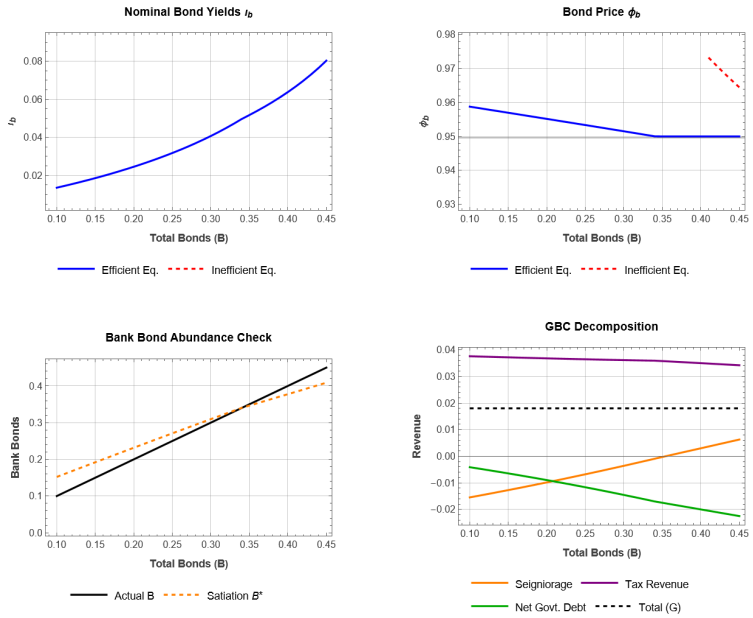


Figure 22: Effects of Changing B: Decomposition

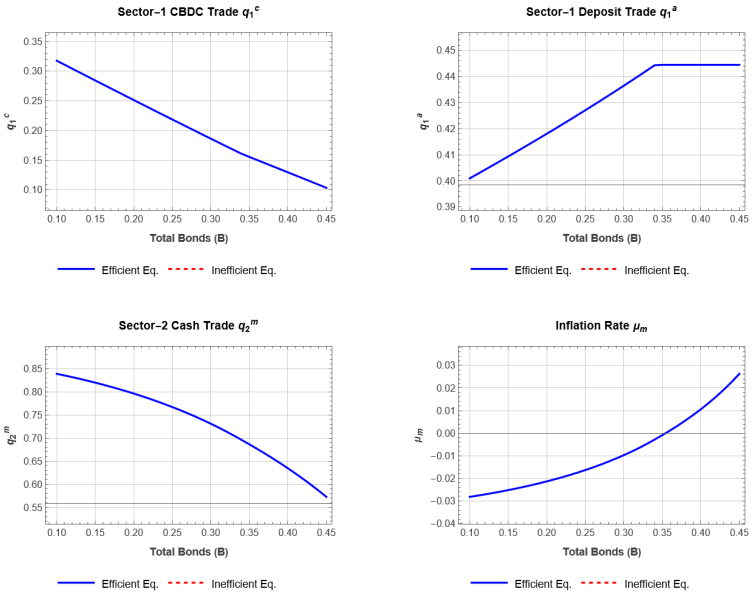


Figure 23: Effects of Changing B: Sector 1&2

expanding debt. To satisfy the consolidated budget constraint under a high fiscal target, the sovereign is forced to extract higher seigniorage by aggressively raising the inflation rate μ_m. A defining feature of the CBDC path is the strict "non-arbitrage" Euler link: because CBDC and physical cash are both direct central bank liabilities, they are subject to the exact same inflation tax. Consequently, as the debt burden rises, this uniform inflation tax simultaneously erodes the purchasing power of public money, strictly crowding out both formal CBDC trade (q_1^c) and informal cash trade (q_2^m). Once the bond market satiates ($B \geq 0.34$), the economy enters a sovereign debt spiral: new debt provides zero marginal liquidity benefits to the banking sector but acts as a pure deadweight interest burden, forcing inflation to accelerate exponentially and ultimately destroying real economic activity across both sectors.

5.2 Welfare Analysis: Stablecoin v.s. CBDC

Comparing the economy's response to debt issuance under the two currency digitalization paths reveals a profound macro-fiscal dichotomy. The fundamental difference lies in who holds the safe assets and how seigniorage is collected.

In the Stablecoin Economy, private token issuers act as massive, captive buyers of government debt to collateralize their tokens. This voracious private demand artificially suppresses nominal bond yields, granting the Treasury a vital "yield subsidy" that delays the onset of the sovereign debt spiral. However, this subsidy comes at a steep systemic cost: stablecoin issuers hoard bonds away from the TradFi sector, starving commercial banks of the collateral necessary to facilitate formal trade. Furthermore, because stablecoins displace physical cash in the informal sector, the central bank's traditional seigniorage base is severely eroded.

Conversely, in the CBDC Economy, the central bank preserves its monetary sovereignty and its full seigniorage base across both the formal (CBDC) and informal (cash) sectors. By eliminating the stablecoin issuers' reserve hoarding, public bonds flow entirely to commercial banks, drastically reducing TradFi collateral frictions. Yet, without the artificial safe-asset demand generated by stablecoin reserves, the

Treasury must finance its debt at fundamental market rates. Consequently, a CBDC economy is vastly more sensitive to debt overhang; high levels of government debt translate directly into a higher baseline inflation requirement, acting as a universal tax on all means of payment.

To explicitly show the difference of these currency digitalization paths, next we make welfare analysis, and check the Ramsey planner solutions.

I. Welfare analysis:

We can define three types of welfare for the economies: the first one is a real welfare/surplus, $\mathcal{W}_s(q_1^m, q_1^a, q_2^s, \tau, \rho, \kappa)$ for Stablecoin Econ., $\mathcal{W}_c(q_1^c, q_1^a, q_2^m, \tau, \rho, \kappa)$ for CBDC Econ., with the equilibrium trading quantities for two sectors, given τ, ρ, κ; the second one is a "constrained" (capital "C") social welfare, $\mathcal{W}_s^C(q_1^*, q_1^*, q_2^*, , \tau, \rho, \kappa)$ and $\mathcal{W}_c^C(q_1^*, q_1^*, q_2^*, , \tau, \rho, \kappa)$, with $q_1^m = q_1^*$, $q_1^a = q_1^*$, $q_2^s = q_2^*$ with $\tau > 0$ as given, and also ρ, κ; and the last one is a social welfare at the "first best" ("$*$"), $\mathcal{W}_s^*(q^*, \rho, \kappa)$, $\mathcal{W}_c^*(q^*, \rho, \kappa)$, without any tax distortion ($\tau = 0$), but still ρ, κ.

For Stablecoin Econ., the real welfare is

$$\begin{aligned}
\mathcal{W}_s(q_1^m, q_1^a, q_2^s) &= \frac{\rho}{2}\alpha\{\omega[u(q_1^m) - q_1^m - 2\tau d_1^m] + (1-\omega)[u(q_1^a) - q_1^a - 2\tau d_1^a]\} \\
&+ \frac{1-\rho}{2}[\alpha[u(q_2^s) - q_2^s] - \int_0^{\hat{\kappa}} \kappa dF(\kappa) \\
&= \frac{\rho}{2}\alpha\{\omega[u(C(\tau)\hat{z}_m) - C(\tau)\hat{z}_m - 2\tau\frac{\hat{z}_m}{1+\tau}] \\
&+ (1-\omega)[u(C(\tau)(\hat{z}_m + \frac{\bar{B} - B_s}{\rho})) - C(\tau)(\hat{z}_m + \frac{\bar{B} - B_s}{\rho}) - 2\tau\frac{\hat{z}_m + \frac{\bar{B} - B_s}{\rho}}{1+\tau}]\} \\
&+ \frac{1-\rho}{2}[\alpha[u(\frac{\chi B_s}{1-\rho}) - \frac{\chi B_s}{1-\rho}] - (1-\rho)\bar{\kappa},
\end{aligned}$$

where we substitute the equilibrium money holdings given $B_p < B_p^*$, $B_s < B_s^*$, and $\bar{\kappa}$ represents average κ among those who choose Sector 2. For a uniform distribution of κ with the CDF $F(\kappa)$, we refer to the scenario with endogenous ρ (Section 31): given $\hat{\kappa}$ being the threshold value of agents choosing between two sectors, we have $\bar{\kappa} = E[\kappa \leq \hat{\kappa}] = \hat{\kappa}/2$, which we use to match the value of $\bar{\kappa}$ in numerical analysis in the current subsection.

For CBDC Econ.,

$$\begin{aligned}\mathcal{W}_c(q_1^c, q_1^a, q_2^m) &= \frac{\rho}{2}\alpha\{\omega[u(q_1^c) - q_1^c - 2\tau d_1^c] + (1-\omega)[u(q_1^a) - q_1^a - 2\tau d_1^a]\} \\ &+ \frac{1-\rho}{2}\alpha[u(q_2^m) - q_2^m] - \int_0^{\hat{\kappa}} \kappa dF(\kappa) \\ &= \frac{\rho}{2}\alpha\{(1-\omega)[u(C(\tau)(\hat{z}_c + \frac{B}{\rho})) - C(\tau)(\hat{z}_c + \frac{B}{\rho}) - \frac{2\tau}{1+\tau}(\hat{z}_c + \frac{B}{\rho})] \\ &+ \omega[u(C(\tau)\hat{z}_c) - C(\tau)\hat{z}_c - \frac{2\tau\hat{z}_c}{1+\tau}]\} + \frac{1-\rho}{2}\alpha[u(\hat{z}_m) - \hat{z}_m] - (1-\rho)\bar{\kappa},\end{aligned}$$

where we substitute $q_1^a = C(\tau)(\hat{z}_c + \frac{B}{\rho}) = (1-\tau)d_1^a$, $q_1^c = C(\tau)\hat{z}_c$, $q_2^m = (1-\tau)d_1^c$, $q_2^m = \hat{z}_m$ when $B < B^*$.

Numerical results for welfare analysis, and the Ramsey Planner solutions are to be added soon.

6 Discussion and Conclusion

This paper develops a nested, multi-sector monetary framework to evaluate how the transition from a physical cash economy to digitized payment regimes—specifically via fully reserved stablecoins and CBDCs—reshapes a nation's taxation capacity, seigniorage, public debt costs, and optimal monetary-fiscal policy mix. We methodically build this environment by beginning with a pure cash baseline divided between Sector 1 (Formal/TradFi) and Sector 2 (Informal/Crypto). By allowing agents to endogenously choose their sectors, we demonstrate that the boundary of the formal tax base is highly elastic to policy, with multiple steady states emerging based on the severity of the inflation tax and formal tax rates.

Building upon this foundation, we embed a traditional commercial banking sector, interest-bearing government bonds, and stablecoin issuers to capture the defining macroeconomic feature of the modern crypto-economy: the safe-asset demand channel. Our core theoretical contribution is formalizing the resulting macro-fiscal trade-off. As stablecoins scale and displace physical cash as a medium of exchange, they inevitably cannibalize the central bank's traditional fiat seigniorage base. How-

ever, because stablecoin issuers operate under narrow-banking constraints—holding short-term government bonds as reserves—their voracious demand for safe assets compresses nominal bond yields. This dynamic generates a vital "yield subsidy" that directly eases the Treasury's debt-servicing burden. Crucially, we show that this subsidy comes at a systemic equilibrium cost: as stablecoin issuers hoard government bonds, they exacerbate collateral scarcity for commercial banks, inadvertently crowding out formal sector trade.

Furthermore, we demonstrate that market structure and information frictions are highly macro-relevant. When issuers face moral hazard regarding the misrepresentation of reserves, a monopolistic issuer will strategically restrict token issuance to suppress its reserve acquisition costs. As established in our Macroeconomic Trilemma, this results in a safer peg ratio and relieves TradFi collateral scarcity, but deprives the sovereign of the maximum possible yield subsidy.

Finally, our comparison of digitalization paths provides stark structural warnings. The public CBDC path reinforces the singleness of money, preserves central bank seigniorage, and removes the collateral frictions that hamper formal banking trade. In contrast, it removes the artificial safe-asset demand of the stablecoin regime. We show that this subjects the sovereign to a strict debt volume burden; without private token issuers absorbing debt, high levels of public borrowing force the central bank to levy a uniform inflation tax that severely depresses both CBDC and physical cash transactions, risking a total economic contraction and a sovereign debt spiral. The private stablecoin path—exemplified by emerging regulatory frameworks like the U.S. GENIUS Act—effectively outsources seigniorage to private platforms in exchange for a massive, captive buyer of sovereign debt.

For policymakers, our framework proves that regulatory parameters such as reserve mandates, auditing frequency, and restrictions on issuer monopolies are no longer isolated microprudential issues. Rather, they are first-order macroeconomic levers that determine the cost of national debt and the efficacy of monetary policy.

Moving forward, the immediate next step in this research agenda is to finalize the normative welfare analysis. Having established the theoretical welfare functions across

real, constrained, and first-best allocations, we plan to computationally characterize the Ramsey planner's optimal policy mix. Extending this framework quantitatively—mapping observable stablecoin adoption rates, reserve disclosures, and CBDC pilot data into our welfare criteria—will ultimately provide a rigorous apparatus for evaluating the long-term welfare and debt-sustainability impacts of the ongoing digital currency revolution.

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