The Micro Effects of Aggregate Shocks in Endogenous Trade Networks *

Carlos Bolivar

University of Minnesota

March 27, 2026

Link to Latest Draft

Abstract

This paper studies the effect of trade liberalization on aggregate risk. I develop a multi-country model where firms choose suppliers before shocks realize, trading off cost against risk. Lower trade barriers concentrate sourcing in productive central suppliers, raising efficiency but amplifying shock propagation through the network. Under incomplete markets, firms price risk against domestic conditions. Each buyer ignores how its sourcing raises aggregate risk for others. Calibrating to WIOD data (24 countries, 1967–2014), I show that trade liberalization increased aggregate volatility, and I quantitatively evaluate the implications of incomplete markets in the production network.

Keywords: Aggregate shocks, production networks, international trade, uncertainty, global financial crisis.

JEL classification: F44, F14, E32, D85, C68.

*I thank Manuel Amador, Timothy J. Kehoe, Javier Bianchi, and Marco Basseto for all their continuous support, guidance, and helpful comments on various stages of the project. I also thank Zhen Huo, Ilenin Kondo, Doireann Fitzgerald and Joseph Mullins for helpful conversations, as well as seminar participants of the Vigo 2025 conference, Wisconsin-Minnesota workshop, and the trade workshop at the University of Minnesota.

1 Introduction

Trade increases productivity by connecting producers to the most efficient suppliers in the world. As trade barriers fall, production concentrates in a smaller set of highly productive hubs that serve as key input suppliers to many countries, such as Taiwan in semiconductor manufacturing. This concentration is at the core of the gains from trade: the most productive producers expand their reach to different locations, and buyers benefit from lower costs. But concentration comes at a price. As shown by Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012), when a small number of industrys account for a disproportionate share of input-output linkages, granular shocks generate stronger aggregate fluctuations. The idea is that, a hub that supplies a large share of the world market can propagate shocks through its extensive input-output connections, amplifying aggregate volatility across its trading partners. The force that makes trade beneficial, the emergence of productive, central suppliers, is the same force that makes the global economy more volatile. This paper argues that this tension between the productivity gains from concentration and the aggregate risk it creates has first-order consequences for volatility, international comovement, and the welfare effects of trade policy.

To study this tension, I first document that network concentration and aggregate risk co-move over fifty years of global input-output data. I then build a multi-country model in which the network and aggregate risk are jointly determined, calibrate it to the observed global network, and use it to evaluate the effects of trade liberalization on aggregate volatility and welfare.

The environment is a multi-country model that extends Kopytov, Mishra, Nimark, and Taschereau-Dumouchel (2024) to an open economy with heterogeneous trade costs, endogenous labor supply, and incomplete markets. The key equilibrium object is a *risk-adjusted price* that combines expected costs, trade costs, and a covariance term capturing the insurance value of each supplier. Firms face uncertainty about global, country-specific, and industry-specific productivity shocks. Before these shocks are realized, they choose suppliers to minimize risk-adjusted costs, so network formation is equivalent to a portfolio choice over global suppliers. In this environment, the elasticity of substitution is endogenous: it depends on trade costs and on the covariance structure of prices. Also, because financial markets are incomplete, firms discount future payoffs using the domestic pricing kernel, implying that the competitive equilibrium is generally inefficient. I establish existence and uniqueness of equilibrium under sufficient conditions, and then use the

model to study how trade liberalization affects aggregate risk.

The main mechanism of the model works as follows. Trade distortions fragment the global production network into regional clusters. Reducing them has two effects: it allows firms to diversify sourcing across more countries, as in Caselli, Koren, Lisicky, and Tenreyro (2020), but it also enables the emergence of globally central suppliers—highly productive industries that become deeply interconnected because lower barriers make them accessible to everyone. These two effects have opposing implications for aggregate volatility. Diversification reduces exposure to country-specific shocks, but central suppliers propagate shocks more strongly through the network.

The net effect depends on firms' optimal response, which changes over the business cycle. Under uncertainty, firms trade off low expected prices against insurance: central suppliers offer lower prices but become poor hedges when uncertainty spikes, since their prices co-move strongly with global conditions. In episodes like 2008, firms shift sourcing away from central suppliers, partially offsetting the amplification mechanism. Under incomplete markets, however, this reallocation is distorted: each firm prices risk against its own domestic conditions rather than world conditions, so sourcing decisions do not fully account for their effect on aggregate network risk. This creates a wedge that reduces the welfare gains from trade liberalization

The model is estimated using WIOD data for 23 countries and 23 industries from 1967 to 2014. Bilateral trade costs are recovered as structural residuals from observed network shares and risk-adjusted prices, extending the inversion approach of Eaton and Kortum (2002) and Waugh (2010) to a setting where time variation in expenditure shares reflects changes in trade costs, changes in productivity and aggregate risk. Next, I validate the reallocation mechanism with a direct empirical test: in a panel regression of changes in expenditure shares on network centrality interacted with the estimated variance aggregate shocks, central industrys lose market share when uncertainty rises. Also, network concentration is negatively correlated with the estimated variance of aggregate shocks.

Finally, I present two counterfactuals. First, fixing trade costs at their 1967 levels reduces both aggregate volatility and welfare, confirming that trade liberalization over the sample period amplified risk transmission through the network. Second, trade liberalization experiment under complete markets yields welfare gains roughly twice as large as under the competitive equilibrium, with the gap driven entirely by the distortion in how firms price network risk.

Related Literature The main precursors of this paper are Kopytov et al. (2024) and Huo, Levchenko, and Pandalai-Nayar (2024). I build on Huo et al. (2024) by endogenizing the production network that in their framework is fixed. This paper belongs to the emerging literature on how risk shapes trade and the global production network. Fan and Luo (2025) study how tariff uncertainty reshapes international sourcing. Kleinman, Liu, and Redding (2025) model portfolio choice in final goods production. My model adds portfolio choice in intermediate goods, where input-output links generate an additional channel: aggregate shocks penalize central industries disproportionately, shifting demand from central to peripheral suppliers. The direction of the reallocation depends on network position and on the covariance of suppliers' shocks with the Domar-weighted aggregate, a heterogeneity that has no analog in models with fixed networks.

The paper also connects to a growing literature on how incomplete markets modify the gains from trade. Fitzgerald (2025) shows that for volatile developing economies, balanced-trade gains are only one-third of complete-markets gains because agents cannot smooth consumption across states ex ante. Allen and Atkin (2022) document that incomplete insurance halves the gains from trade integration for Indian farmers by distorting ex-ante crop portfolio choices. Adamopoulos and Leibovici (2024) find a pecuniary externality in food trade under incomplete markets that justifies large import tariffs for food-importing nations. My paper contributes a distinct mechanism: incomplete markets prevent firms from pricing the aggregate risk consequences of their sourcing decisions, leading to a misallocation of risk across the network rather than insufficient ex-post consumption smoothing.

The empirical motivation connects to work on time-varying volatility and production networks. Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018) document that plant-level TFP dispersion roughly doubles during recessions; I find an analogous pattern in an international panel of 43 countries and 56 industries. The time variation in network concentration relates most closely to Carvalho and Gabaix (2013), who show that changing Domar weights account for the Great Moderation and its partial reversal. I document analogous variation in the global production network at business cycle frequency. On the comovement side, di Giovanni and Levchenko (2010) and Kose and Yi (2001) establish that bilateral trade flows are among the strongest predictors of GDP comovement across countries; my model provides a structural account of why this relationship strengthens during downturns. vom Lehn and Winberry (2022) provide complementary evidence that the U.S. investment network has changed over time with consequences for business cycle

comovement.

The paper also relates to several broader literatures: on endogenous production networks (Arkolakis, Huneeus, and Miyauchi, 2025; Acemoglu and Azar, 2020; Elliott, Golub, and Leduc, 2022), on aggregation and distortions in networks (Jones, 2011; Baqaee and Farhi, 2019; Bigio and La'O, 2020; Baqaee and Farhi, 2024), on trade and aggregate volatility (Caselli et al., 2020; di Giovanni, Levchenko, and Méjean, 2018), on tail risk in production networks (Dew-Becker, 2023; Dew-Becker, Tahbaz-Salehi, and Vedolin, 2021), and on the granular origins of aggregate fluctuations (Gabaix, 2011; di Giovanni, Levchenko, and Méjean, 2014).

Outline. Section 2 presents the empirical motivation. Section 3 presents the model. Section 4 studies the optimal network choice and Section 5 the effect of aggregate shocks. Section 7 describes the calibration. Section 8 present the quantitative results. Section 10 concludes.

2 Empirical Motivation

This section documents two sets of empirical patterns using the Long-run WIOD (Woltjer, Gouma, and Timmer, 2021; Timmer, Dietzenbacher, Los, Stehrer, and De Vries, 2015), which covers 24 countries, 23 industries, and 48 years from 1967 to 2014.[1] First, aggregate risk is time-varying and driven by a common global component. Second, the global production network changed substantially over the same period, both in its degree of concentration and in the geographic distribution of central nodes.

2.1 Time-Varying Aggregate Risk

I measure aggregate risk using a variance decomposition of value-added growth into a global component $\tilde{f}_t$ common to all country-industry pairs, a country component $\tilde{f}_{c,t}$, an industry component $\tilde{f}_{i,t}$, and a granular residual $\tilde{f}_{ci,t}$, estimated by PCA following Caselli et al. (2020), with time-varying variances computed via GARCH(1,1) applied to each extracted series. Figure 1 shows the variance decomposition over 1967–2014.

[1]These economies account for over 85% of world GDP throughout the sample. See Appendix A.1 for country and industry coverage and data construction details.

The global component dominates throughout the sample and shows larger time variation in its variance, while the variance of country, industry, and granular components remains relatively stable. Episodes of high aggregate volatility are therefore global events, where the variance of the common factor affecting all country-industry pairs rises.

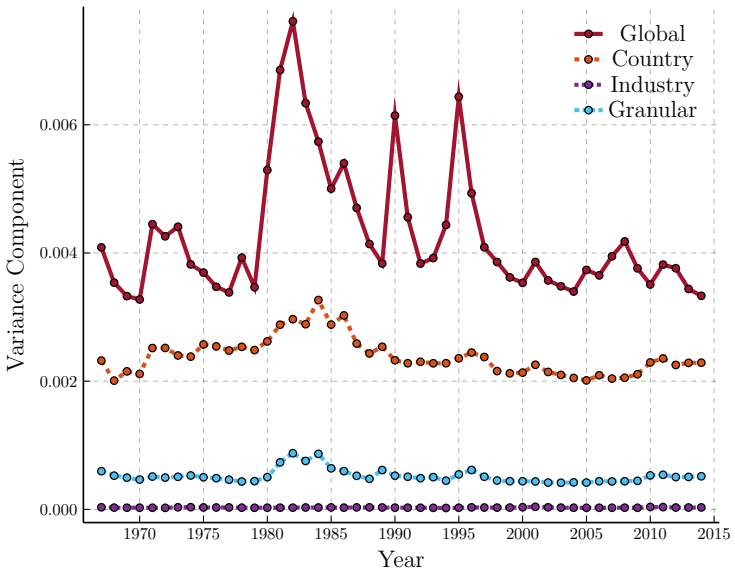


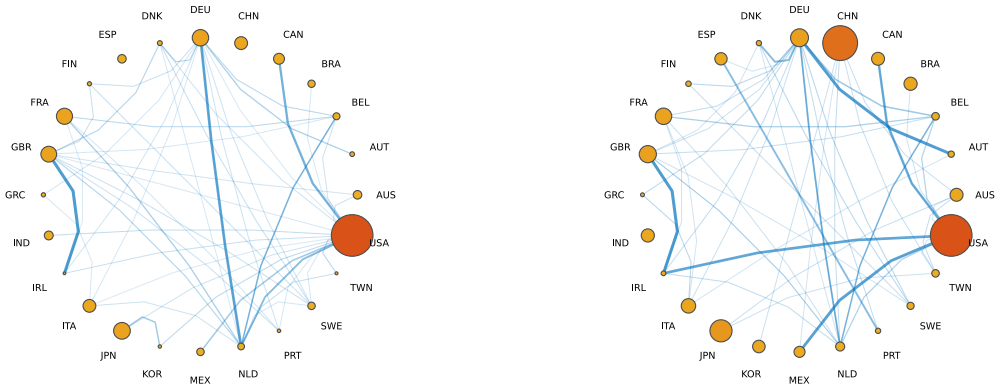
Figure 1: Time-Varying Aggregate Risk

Notes: The plot shows the time-varying variance of each component in $a_{ci,t} = g_t + \chi_{c,t} + \zeta_{i,t} + u_{ci,t}$, estimated as in Caselli et al. (2020) using a five-year rolling window. WIOD data: 23 countries, 23 industries, 1967–2014.

The decomposition of value-added reveals two model-relevant patterns. First, the country component remains quantitatively important, consistent with Caselli et al. (2020), who show that country-specific productivity are more important to explain aggregate volatility compare to industry-specfic shocks. Second, a global component emerges clearly, spiking at each crisis. Huo et al. (2024) show that production-network transmission alone cannot match the observed degree of international comovement, so correlated shocks are required to match the observed comovement in the data. The global component in value-added growth thus motivates studying the effect of changes in the variance of a global shock.

2.2 A Changing Global Production Network

The geographic distribution of central nodes also shifted substantially over the period. Figure 2 shows the global trade network at two points in time, 1967 and 2014. The node size is proportional to each country's sale share (aggregate Domar weight) and edge thickness is proportional to bilateral trade flows. The United States is the dominant hub in both years, but the surrounding structure changed markedly: Japan rose as a second major hub through the 1980s before partially retreating, China grew from a peripheral node in 1967 to one of the largest hubs by 2014, and the overall network became more multipolar.



(a) Trade Network, 1967

(b) Trade Network, 2014

Figure 2: Global Trade Network at Two Points in Time

Notes: Node size proportional to country Domar weight $\omega_{c,t} = \sum_i \omega_{ci,t}$. Edge thickness proportional to bilateral intermediate trade flows. WIOD data, 23 countries, 1967 and 2014.

In addition, I measure network concentration using the Herfindahl-Hirschman Index of sales shares (Domar weights), $H_t = \sum_{c,i} \omega_{ci,t}^2$, where $\omega_{ci,t}$ denotes the ratio of country-industry (c, i) gross output to world GDP; Acemoglu et al. (2012) show that aggregate volatility is proportional to H_t, so movements in concentration directly translate into movements in how strongly the network amplifies productivity shocks.

Figure 3 plots H_t over 1967–2014. The index oscillates between roughly 0.012 and 0.020 over fifty years, compressing and expanding repeatedly at business-cycle, with sharp reversals at three identifiable episodes around 1982, 1991, and 2008. The network is

therefore an actively adjusting object that moves over the business cycle.

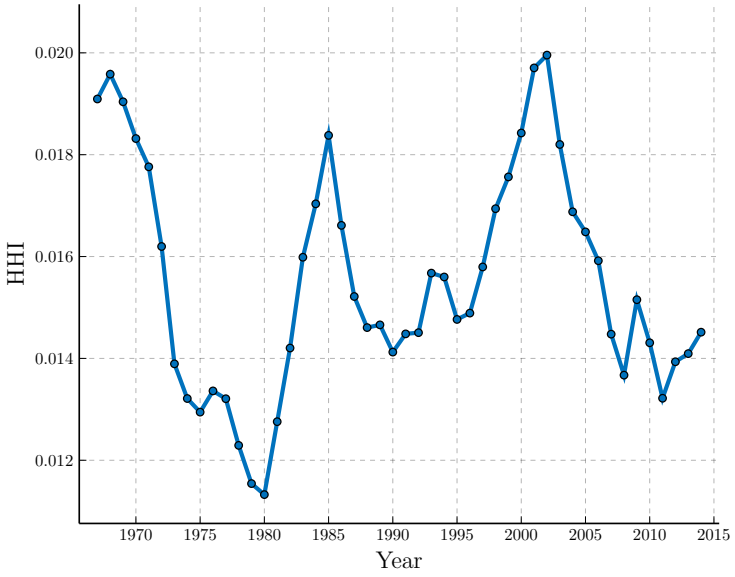


Figure 3: Herfindahl Index of Global Sales Shares, 1967–2014

Notes: The figure plots $H_t = \sum_{c,i} \omega_{ci,t}^2$ where $\omega_{ci,t}$ is the ratio of country-industry (c, i) gross output to world GDP. Dashed vertical lines mark 1982, 1991, and 2008. WIOD data, 23 countries, 23 industries, 1967–2014.

2.3 Discussion

The two patterns are the joint object this paper seeks to explain. Aggregate risk shapes the network through firms' sourcing decisions; the network in turn determines how risk propagates. The model is designed to disentangle this feedback.

In particular, the model decomposes changes in the network into two sources: changes in trade costs and changes in aggregate risk. I then use the model's equilibrium conditions to identify both objects from the data, and use them to quantify the effects of trade liberalization.

3 Environment

There is a set of C countries. In each country, there are J producers that belong to a global industry. In total, there are $N = C \times J$ representative firms that produce differentiated intermediate goods. Each country has a representative firm that combines intermediate

goods from all N suppliers into a final good that is sold to a representative household. Finally, each household supplies labor L_c to the J industries within its country c, consumes, and owns the firms.

The timing is as follows. At the beginning of the period, firms update their beliefs about productivity shocks and choose the production technique, facing uncertainty about productivity and prices. Next, after shocks are realized, markets clear. All firms and households choose the demand and supply of all intermediate and final goods. There are no dynamic choices linking equilibria across periods.

3.1 Households

The representative household in the country c derives its utility from consumption and leisure. Preferences have the following form:

$$W_c = \sum_t \frac{\left(Z_{c,t} - \sum_i^J \left(L_{ci,t}\right)^{1+\frac{1}{\psi}}\right)^{1-\gamma}}{1-\gamma} \tag{1}$$

I follow Greenwood, Hercowitz, and Huffman (1988) in defining quasi-linear preferences, which eliminates any income effect on labor supply. Households choose consumption after uncertainty is realized. Thus, in each period and for each state of the world, the budget constraint is:

$$P_{c,t}Z_{c,t} = \sum_i W_{ci,t}L_{ci,t} + \delta_{c,t} \tag{2}$$

where $P_{c,t}$ is the price index of the final good, $W_{ci,t}$ is the nominal wage in industry (c,i). Also, I allow for trade imbalances by introducing exogenous, time-varying trade deficits $\delta_{c,t}$, following the approach of Dekle, Eaton, and Kortum (2007) and Caliendo and Parro (2015), among others. Trade deficits are predetermined at the time households make their consumption decision and are financed externally, so that markets are incomplete in the sense of Heathcote and Perri (2002): there is no state-contingent asset trade, and households cannot insure against aggregate risk through financial markets.[2] A positive $\delta_{c,t}$ means the country imports more than it exports, financed by an external transfer; a

[2]Heathcote and Perri (2002) show that models with financial autarky perform well in accounting for business-cycle comovement. Allowing for trade deficits while maintaining incomplete markets preserves this property while matching the observed patterns in the data.

negative value corresponds to a trade surplus. The optimality condition for labor in each industry is given by the following expression:

$$L_{ci,t} = \psi \frac{W_{ci,t}}{P_{c,t}} \tag{3}$$

Equation (3) gives the standard optimal condition that relates labor supply to the real wage. Finally, I define the household's price kernel as follows:

$$\Lambda_{c,t} = \frac{\left(Z_{c,t} - \sum_{i=1}^{J} \left(L_{ci,t}\right)^{1+\frac{1}{\psi}}\right)^{-\gamma}}{P_{c,t}}$$

which measures the marginal utility of consumption for households in each state of the world.

3.2 Intermediate Goods Producers

Firs Stage Before productivity shocks are realized, the representative firm in country c, industry i chooses a production technique $\eta_{ci,t}$ to maximize expected profits discounted by the household pricing kernel $\Lambda_{c,t}$:

$$\eta^{\star}_{ci,t} \in \arg\max_{\eta_{ci,t} \in \mathcal{A}} \mathbf{E}\left[\Lambda_{c,t}\left(P_{ci,t}Y_{ci,t} - \sum_{\hat{c},k} P_{c,\hat{c}k,t}X_{ci,\hat{c}k,t}\right)\right] \tag{4}$$

The firm operates a Cobb-Douglas technology:

$$F_{ci,t}(\eta_{ci,t}) = e^{A_{ci,t}(\eta_{ci,t})+\epsilon_{ci,t}} L_{ci,t}^{\mu_{ci,t}} \left(\prod_{\hat{c},k} X_{ci,\hat{c}k,t}^{\eta_{ci,\hat{c}k,t}}\right)^{1-\mu_{ci,t}} \tag{5}$$

where $X_{ci,\hat{c}k,t}$ is the intermediate input from industry k in country $\hat{c}$, $L_{ci,t}$ is labor, and $\epsilon_{ci,t}$ is an exogenous productivity shock. The parameter $\mu_{ci,t} \in (0,1)$ is the Cobb-Douglas labor share, which varies over time. The vector $\eta_{ci,t} = \{\eta_{ci,\hat{c}k,t}\}$ specifies the production technique, constrained to the unit simplex $\sum_{\hat{c},k} \eta_{ci,\hat{c}k,t} = 1$. Deviating from the ideal technique η^{0}_{ci} reduces productivity:

$$A_{ci,t}(\eta_{ci,t}) = -\kappa^{I}_{ci} \sum_{\hat{c},k} \eta_{ci,\hat{c}k,t} \log \frac{\eta_{ci,\hat{c}k,t}}{\eta^{0}_{ci,\hat{c}k}} \tag{6}$$

The adjustment cost is buyer-specific parameter, and $\kappa^{I}_{ci} > 0$ governs the cost of deviating from the ideal sourcing pattern.

Intermediate goods are subject to iceberg trade costs. The price of good $(\hat{c}, k)$ paid by buyers in country c is $P_{c,\hat{c}k,t} = P_{\hat{c},\hat{c}k,t}\,\tau^{I}_{c,\hat{c}k,t}$.

Second Stage The first-order conditions yield the unit cost function:

$$K_{ci,t}(\eta_{ci}, P) = e^{-A_{ci,t}(\eta_{ci,t})-\epsilon_{ci,t}}\; W_{ci,t}^{\mu_{ci,t}} \left(\prod_{\hat{c},k} P_{c,\hat{c}k,t}^{\eta_{ci,\hat{c}k,t}} \right)^{1-\mu_{ci,t}} \tag{7}$$

so the first-stage problem becomes:

$$\eta^{\star}_{ci,t} \in \arg\min_{\eta_{ci,t}\in\mathcal{A}} \; \mathbf{E}\left[\Lambda_{c,t}\, Y_{ci,t}\, K_{ci,t}(\eta_{ci}, P)\right] \tag{8}$$

3.3 Final Goods Producers

Each country has a representative final goods producer that combines intermediate goods into a consumption bundle. The production function is:

$$F_{c,t}(\alpha_{c,t}) = e^{A_{c,t}(\alpha_{c,t})} \prod_{\hat{c},k} Z_{c,\hat{c}k,t}^{\alpha_{c,\hat{c}k,t}} \tag{9}$$

where $\alpha_{c,t}$ lies in the unit simplex and deviating from the ideal technique α^0_c reduces productivity as in equation (6), with buyer-specific adjustment cost κ^F_c. The technique choice problem is analogous to that of intermediate goods producers: final goods firms choose $\alpha_{c,t}$ before shocks are realized to minimize expected costs discounted by $\Lambda_{c,t}$. Trade costs in final goods are $\tau^F_{c,\hat{c},t}$.

3.4 Stochastic Structure

Uncertainty comes from TFP shocks. Following Caselli et al. (2020), the productivity of each industry decomposes into global, country, industry, and granular components:

$$\epsilon_{ci,t} = g_t + \chi_{c,t} + \zeta_{i,t} + u_{ci,t} \tag{10}$$

Each component follows an independent AR(1) process with persistence ρ_k and time-varying innovation variance $\sigma^2_{k,t}$ for $k \in \{g, \chi, \zeta, u\}$.

3.5 Equilibrium

We now define a competitive equilibrium. Since all firms are competitive, equilibrium prices equal costs. The equilibrium of the economy with the endogenous network is defined as follows:

Definition 1. (Competitive Equilibrium) An equilibrium is a sequence of allocations for intermediate firms $\{Y_{ci,t}, Z_{ci,\hat{c}k,t}, \eta_{ci,t}\}_{i,c}$; a sequence of allocations for final good producers $\{Y_{c,t}, Z_{c,\hat{c}k,t}, \alpha_{c,t}\}$; a sequence of allocations for households $\{Z_{c,t}, L_{ci,t}\}_c$; and a sequence of prices $\{P_{ci,t}, W_{ci,t}, P_{c,t}\}_{ci}$ such that all agents in the economy solve their problem.

3.6 Definitions and Notation

Before characterizing the solution, I introduce some key objects.

Input-Output and Final Goods Matrices. The IO matrix $\boldsymbol{\eta}_t$ is the $N \times N$ matrix whose $(ci, \hat{c}k)$th element is the Cobb-Douglas coefficient controlling the intensity at which industry ci uses inputs from industry $\hat{c}k$. The Final Goods matrix $\boldsymbol{\alpha}_t$ stores the Cobb-Douglas coefficients in the final goods production function; columns for industries within the same country share the same values. Let $\boldsymbol{\mu}_t$ be the diagonal $N \times N$ matrix of labor shares $\mu_{ci,t}$.

Domar Weights. The Domar weight of industry ci is its share of world GDP measured in sales:

$$\omega_{ci,t} = \frac{P_{ci,t} Y_{ci,t} Y_{ci,t}}{GDP} \tag{11}$$

I also define the country Domar weight $\Omega_c = \sum_i \omega_{ci,t}$ and the industry Domar weight $\hat{\Omega}_i = \sum_c \omega_{ci,t}$.

4 Network Formation and Equilibrium

I analyze the model backwards. First, I characterize prices given a network. Next, I use these results to characterize the optimal choice of network.

4.1 Equilibrium Given a Network

Given a the aggregate state in the first-stage $s = \{\alpha^\star, \eta^\star, \Sigma, \theta, \tau\}$, I derive the equilibrium and the distribution of prices that firms use to solve the optimal sourcing problem in the next subsection. Throughout, I denote $\boldsymbol{x}$ as the vector containing all values of log of $X_{ci,t}$.

Domar Weights. The relative size of each industry in the global economy is summarized by its Domar weight—the ratio of gross output to world GDP—which determines how expenditure flows through the network.

Lemma 1. *Define $\boldsymbol{D}_\delta$ the deficit as a share of income. The Domar weight vector $\boldsymbol{\omega}(s)$ is the unit eigenvector with eigenvalue 1 of:*

$$\boldsymbol{\omega}(s) = \left[\boldsymbol{\mu}\left(I + \boldsymbol{D}_\delta\right)\boldsymbol{\alpha}^\star + \left(I - \boldsymbol{\mu}\right)\boldsymbol{\eta}^\star\right]\boldsymbol{\omega}(s)$$

Proof. See Appendix B.1. □

The term $\boldsymbol{\mu}\boldsymbol{\alpha}^\star$ accounts for revenue from sales of final goods, while $(1-\boldsymbol{\mu})\boldsymbol{\eta}^\star$ accounts for revenue from sales of intermediate goods. Market shares increase either because other intermediate-good producers increase their use of the good, or because final-good producers increase their expenditure shares. Given the Cobb-Douglas structure, Domar weights do not change with the stochastic state ϵ_t.

The relative size of final versus intermediate goods trade depends on the labor share vector $\boldsymbol{\mu}$. Higher labor shares direct more production income to households, increasing the role of the final goods network $\boldsymbol{\alpha}^\star$ relative to the intermediate input network $\boldsymbol{\eta}^\star$. The deficit matrix $\boldsymbol{D}_\delta$ adjusts for trade imbalances in the data so that countries spending more than they earn are accounted for in the model's expenditure flows.

Prices. Equilibrium prices are determined by the network and the realization of productivity shocks.

Lemma 2. *Let $\boldsymbol{p}$ be the log price vector and ϵ the vector of TFP shocks. Then:*

$$\boldsymbol{p}(s, \epsilon) = \mathcal{L}(s)\left(B(s) - \epsilon\right),$$

where the extended Leontief inverse is:

$$\mathcal{L}(s) = \left(I - \boldsymbol{\mu} \frac{\psi}{1+\psi} \boldsymbol{\alpha}^\star - (1 - \boldsymbol{\mu}) \boldsymbol{\eta}^\star \right)^{-1},$$

and the deterministic price component is:

$$B(s) = -A(\eta_{ci}^\star) - \frac{\psi}{1+\psi} \mu A(\alpha_c^\star) + \text{diag}\left(\tfrac{\psi}{1+\psi} \left[\mu\alpha^\star + (I-\mu)\eta^\star \right] \right) \log \tau + \frac{1}{1+\psi} (\log \mu + \log \omega(s)).$$

Proof. See Appendix B.2. □

Whereas the matrices $\boldsymbol{\alpha}^\star$ and $\boldsymbol{\eta}^\star$ record *direct* links between producers, the Leontief inverse records *direct and indirect* exposures through the entire network. This Leontief inverse extends the classical one with the additional term $\boldsymbol{\mu} \frac{\psi}{1+\psi} \boldsymbol{\alpha}^\star$, which arises from the household labor supply condition and captures how a TFP shock in any industry feeds into wages and therefore into costs everywhere else.[3] The elasticity ψ governs the relative weight of the two transmission channels: when labor supply is inelastic, wage responses are muted and intermediate input prices are the dominant channel; when labor supply is elastic, final-good prices have a stronger effect on wages and final-good trade becomes the key transmission channel.

The vector $B(s)$ summarizes how the network tilts prices in the absence of shocks. The terms $A(\alpha_c^\star)$ and $A(\eta_{ci}^\star)$ capture how costly supplier reallocation lowers effective productivity and raises prices in industries that deviate from their ideal technology. The trade cost component makes prices higher in industries that source intensively from expensive foreign inputs. The $\log \boldsymbol{\mu}$ and $\log \boldsymbol{\omega}(s)$ terms reflect that industrys using labor more intensively and industrys with larger market shares pay higher wages to compensate households, who supply labor elasticity ψ.

The final-good price index aggregates supplier prices using the optimal network weights $\alpha_c^\star$, adjusted for trade costs.

Corollary 1 (Final-good price index)**.** *The log price index of the final good in country*

[3] Huo et al. (2024) derive this extended Leontief inverse in a model with CES production.

c is:

$$p_c(s,\epsilon) = A(\alpha_c^\star) + \boldsymbol{\alpha}_c^\star \left(\boldsymbol{p}(s,\epsilon) + \log \tau\right).$$

This price index is the object that affect household welfare and the pricing kernel, so it summarizes each country's exposure, direct and indirect, to shocks anywhere in the global economy.

Pricing Kernel. The pricing kernel connects price realizations to the household's marginal utility of consumption and is the key object for evaluating the insurance value of each supplier.

Lemma 3. *The log pricing kernel of the representative household in country c is:*

$$\log \Lambda_c(s,\epsilon) = (\gamma - 1)\, p_c(s,\epsilon) + C_c(s),$$

where $C_c(s) = -\gamma\left(1 + \frac{1}{\psi}\right) \log\left(\sum_i (\mu_{ci}\, \omega_{ci}(s))^{\frac{\psi}{1+\psi}}\right) + \gamma \log \psi$ *depends on the network but is independent of shocks.*

Proof. See Appendix B.6. □

The separation in Lemma 3 is what keeps the first-stage problem tractable. Both $C_c(s)$ and $\mathrm{Cov}(p_c, p_{\hat{c}k})$ are determined by the network, but the insurance value of each supplier depends only on the covariance term, not on $C_c(s)$. Endogenous labor supply therefore does not add a new channel to the first-stage problem compare to the baseline model Kopytov et al. (2024).

Expectations. The final ingredient for the optimal network problem is the distribution of prices,

Lemma 4. *Let* $\epsilon_t \sim \mathcal{N}(\theta_t, \Sigma_{\epsilon,t})$. *Then* $\boldsymbol{p}(s,\epsilon)$ *is normally distributed with:*

$$\mathbf{E}\left[\boldsymbol{p}(s,\epsilon)\right] = \mathcal{L}(s)\,(B(s) - \theta),$$
$$\mathbf{V}\left[\boldsymbol{p}(s,\epsilon)\right] = \mathcal{L}(s)^\top\, \Sigma_\epsilon\, \mathcal{L}(s).$$

Proof. See Appendix B.4. □

The Lemma 4 determines both expected costs and the covariance term entering the risk-adjusted price. Log-normality follows from the Cobb-Douglas production function combined with normally distributed shocks, which together ensure that prices are linear in ϵ through the Leontief inverse. The variance formula $\mathcal{L}(s)^\top \Sigma_\epsilon \mathcal{L}(s)$ shows how the network amplifies and transmits the primitive shock variances: an industry's price variance is the sum of all shock variances weighted by the square of the corresponding Leontief inverse elements, so industries that sit downstream of many others inherit more risk.

4.2 Optimal Production Networks

Firms in both final and intermediate industries choose a supplier vector to minimize expected discounted production costs, using the domestic pricing kernel derived in Lemma 3 the problem of firms in the first stage can be characterized as follows.

Proposition 1 (Risk-Adjusted Prices). *Let $m = \alpha_c$ for final-good producers and $m = \eta_{ci}$ for intermediate-good producers. In equilibrium, their choice satisfies:*

$$m^\star(s) \in \arg\max_{m \in \Delta} \; -\kappa_b^m \sum_{\hat{c},k} m_{\hat{c}k} \log \frac{m_{\hat{c}k}}{m^0_{\hat{c}k}} \; - \; \sum_{\hat{c},k} m_{\hat{c}k} \, R_{\hat{c}k}(s),$$

where the risk-adjusted price of supplier $(\hat{c}, k)$ is:

$$R_{\hat{c}k}(s) = E[p_{\hat{c}k}(s,\epsilon)] + \log \tau_{\hat{c}k} + (\gamma - 1)\,\mathrm{Cov}(p_c(s,\epsilon),\; p_{\hat{c}k}(s,\epsilon))$$

and p_c is the domestic final-good price index.

Proof. See Appendix B.6. □

The risk-adjusted price $R_{\hat{c}k}(s)$ combines three components. The first, $E[p_{\hat{c}k}]$, captures the expected productivity of the supplier: firms with lower expected prices reflect either higher own productivity or access to productive upstream inputs. The second, $\log \tau_{\hat{c}k}$, introduces a standard gravity channel: holding productivity fixed, firms prefer geographically close or domestically sourced suppliers. The third, $(\gamma - 1)\mathrm{Cov}(p_c, p_{\hat{c}k})$, is the insurance value of the supplier: as shown in the proof, household consumption in country c is a decreasing function of p_c, so firms prefer suppliers whose prices are low precisely when

the domestic price index is high—suppliers that are cheap in bad times provide valuable insurance and have a lower risk-adjusted price.

Proposition 1 nests the risk-adjusted cost in Kopytov et al. (2024) as a special case: with a single country and no trade costs ($\tau_{\hat{c}k} = 1$).[4] The open-economy extension adds two forces absent from their framework. First, trade costs introduce heterogeneous barriers that break the symmetry across potential suppliers, generating the gravity structure needed to match observed bilateral trade patterns. Second, the covariance term $\text{Cov}(p_c, p_{\hat{c}k})$ now reflects cross-country heterogeneity: in a multi-country network with incomplete markets, it depends on each country's equilibrium final-good network $\alpha_c^\star$, so the insurance value of the same supplier differs across buyers depending on how exposed their domestic economy is to that supplier's price movements. The solution of the optimal network is:

Lemma 5 (Optimal Network). *Let $m \in \{I, F\}$. The optimal network satisfies the softmax:*

$$m^\star_{\hat{c}k}(s) = \frac{m^0_{\hat{c}k} \exp\left(-R_{\hat{c}k}(s)/2\kappa_b^m\right)}{\sum_{(\hat{c}',k')} m^0_{\hat{c}'k'} \exp\left(-R_{\hat{c}'k'}(s)/2\kappa_b^m\right)}, \tag{12}$$

where κ_b^m is the buyer-specific adjustment cost (κ_{ci}^I for intermediates, κ_c^F for final goods).

Proof. See Appendix B.7. □

The equilibrium network is a softmax function of risk-adjusted prices, with κ_b^m acting as the temperature: large κ_b^m keeps shares close to the ideal m^0, while small κ_b^m concentrates sourcing on the cheapest supplier. The equilibrium solves a fixed point: risk adjustments $R_{\hat{c}k}$ depend on the optimal networks, and optimal networks depend on risk-adjusted costs. Under the entropy cost, all shares are strictly positive—there are no corner solutions—so every supplier in the ideal network remains active. The degree of substitutability is endogenous: it depends on the temperature κ_b^m and the dispersion of risk-adjusted prices, which in turn is determined by trade costs and the covariance structure of prices.

[4] Under $\tau = 1$, Lemma 4 and the single-country restriction imply $R_{\hat{c}k}(s) = E[p_{\hat{c}k}(s,\epsilon)] + (\gamma - 1)\mathcal{L}(s)^\top \Sigma_\epsilon \mathcal{L}(s)\, m^\star$, which coincides with Lemma 2 in Kopytov et al. (2024).

4.3 Existence and Uniqueness

The competitive equilibrium is a fixed point between the aggregate network and the best responses of all firms, where each firm's sourcing decision changes the covariance structure of prices, which in turn affects the optimal decisions of all other firms. This general equilibrium feedback can in principle generate multiple equilibria, and the problem does not admit a planner's representation under incomplete markets—so the approach of Kopytov et al. (2024), who establish uniqueness via the planning problem, is not available here. I instead cast the competitive equilibrium as a static game and establish existence and conditions for uniqueness.

Proposition 2 (Existence and Uniqueness). *Let $\kappa_c^F > 0$ and $\kappa_{ci}^I > 0$ for all c, i. Then the equilibrium $s^*, p(s^*, \epsilon)$ exists. Moreover, s^* is unique if:*

$$\bar{\kappa} \;>\; \frac{(1+\psi)^2}{\underline{\mu}^3}\sqrt{(1-\underline{\mu})^2 + \left(\frac{\psi\,\bar{\mu}}{1+\psi}\right)^2}\left[2(\gamma-1)(1+\psi)\,\bar{\sigma}^2 \;+\; (\bar{\theta}+\bar{B})\,\underline{\mu}\right]$$

where $\bar{B} := \frac{\psi}{1+\psi}\bar{\mu}\,\bar{A} + \frac{\psi}{1+\psi}\overline{\log\tau} + \frac{1}{1+\psi}(\overline{\log\mu} + 1)$, and $\bar{\kappa} = \min_{c,i}\{\kappa_c^F, \kappa_{ci}^I\}$, $\bar{\sigma}^2$ is the maximum eigenvalue of Σ_ϵ, and $\underline{\mu}, \bar{\mu}$ are the minimum and maximum labor shares.

Proof. See Appendix D. □

Existence follows from Brouwer's theorem: the strategy space is a product of simplices, prices are uniquely pinned down by the Leontief inverse for any given network, and the entropy adjustment costs make each firm's best response continuous. So the fixed-point mapping from the aggregate network to best responses is continuous on a compact convex set and must have a fixed point.

Uniqueness requires controlling the general equilibrium feedbacks. The right-hand side of the condition measures the maximum sensitivity of risk-adjusted prices to changes in the aggregate network: it is increasing in risk aversion γ and in aggregate shock variance $\bar{\sigma}^2$. The condition says that if adjustment costs $\bar{\kappa}$ exceed this bound, firms respond weakly enough to changes in the covariance structure that the best-response map is a contraction and the equilibrium is unique. Intuitively, large adjustment costs anchor firms near their ideal technology m^0, preventing multiple equilibria.

4.4 Welfare and Production

I close this section by describing the effect of changes in the network $\{\alpha^\star, \eta^\star\}$ in macroeconomic outcomes: production and welfare.

Real Value Added. I characterize the log of real value added in each country-industry pair as a function of the network and the realization of productivity shocks.

Lemma 6. *The log of real value added a_{ci} is:*

$$a_{ci}(s,\epsilon) = \epsilon_{ci} + A(\eta^\star_{ci}) + \mu_{ci}\, l_{ci}(s,\epsilon)$$

where log labor supply $l_{ci}(s,\epsilon)$ is:

$$l_{ci}(s,\epsilon) = \frac{\psi}{1+\psi}\Big[-p_c(s,\epsilon) + \log\mu_{ci} + \log\omega_{ci}(s)\Big]$$

Proof. See Appendix B.3. □

Lemma 6 decomposes value added into three components. The first, ϵ_{ci}, is the direct TFP shock. The second, $A(\eta^\star_{ci})$, is the endogenous productivity determined by the choice of network. The third is the endogenous labor supply response: a TFP shock anywhere in the network propagates into the domestic price index p_c, changing the real wage and shifting labor supply and value added across all country-industry pairs—the channel first emphasized by Backus, Kehoe, and Kydland (1992) as the source of international comovement. Lemma 6 closely related to Huo et al. (2024), however departs form their result because in their they characterize deviations form the steady state in a CES model.

Welfare. I express welfare for the representative household in country c directly in terms of the equilibrium network, the expected price index, and the variance of prices.

Lemma 7. *At each period t, welfare in country c is:*

$$W_c(s) = b + \left(1+\tfrac{1}{\psi}\right)\log\left(\sum_i \big(\omega_{ci}(s)\,\mu_{ci}\big)^{\frac{\psi}{1+\psi}}\right) - \mathbb{E}\big[p_c(s,\epsilon)\big] + \tfrac{1}{2}(1-\gamma)\,\mathrm{Var}\big(p_c(s,\epsilon)\big)$$

where b is a constant.

Proof. See Appendix B.8. □

Welfare aggregates three forces that all operate through the network. The first term captures the relative size of the domestic economy: households in larger economies face higher labor demand, work more, and consume more. The second term, $\mathbb{E}[p_c]$, measures the cost of the consumption basket: lower expected prices and raises purchasing power and increases consumption. The third term captures country-specific risk: under incomplete markets, $\text{Var}(p_c)$ measures the volatility of consumption and reduces welfare when $\gamma > 1$.

Sourcing decisions by any producer in the economy affect welfare by reshuffling production across countries, changing the efficiency of the consumption basket through $\mathbb{E}[p_c]$, and changing country-specific risk through $\text{Var}(p_c)$.

5 Trade an Aggregate Risk

I now characterize how firms reallocate sourcing away from central suppliers when aggregate risk rises.

5.1 Centrality

I begin by providing the definition of centrality in this economy.

Definition 2 (Centrality)**.** The centrality of country-industry pair (c, i) is the row sum of the Leontief inverse:

$$v_{ci} = \sum_{\hat{c}k} \mathcal{L}_{ci,\hat{c}k}$$

The exposure of country c to central industries is:

$$v_c = \sum_{\hat{c}k} \alpha^{\star}_{c,\hat{c}k} \, v_{\hat{c}k}$$

In contrast to Acemoglu et al. (2012), With endogenous labor supply, centrality is not characterize by the Domar weight. Here the the relevant measure of an industry's importance for shock transmission, arises because the Leontief inverse weights final and

intermediate sales asymmetrically by ψ, while Domar weights treat them symmetrically.[5]

5.2 An Example Economy

To build intuition, I consider a two-country economy — Home (H) and Foreign (F) — each with a single intermediate good producer that uses only labor. Foreign is the star: $\theta_F > \theta_H$, so Foreign's intermediate good is cheaper in expectation. Final good producers in each country choose sourcing shares $\alpha_c = \{\alpha_{c,H}, \alpha_{c,F}\}$ over the two intermediate goods subject to iceberg trade cost τ. Let $\epsilon_c = g + u_c$. The variance of the Home final good price decomposes as:

$$\text{Var}(p_H^F) = \underbrace{\sigma_g^2 \, \nu_H^2}_{\text{Global}} + \underbrace{\sigma_u^2 \sum_{c'} \alpha_{H,c'}^2}_{\text{Granular}} \tag{13}$$

There are two sources of uncertainty that generate aggregate risk in this economy. The first source consists of granular shocks, denoted by σ_u. The contribution of a granular shock depends on the exposure of the final-good producer to each supplier. The second source is the variance of the global shock, σ_g. A global shock hits all suppliers simultaneously, but it affects more central sectors more strongly, as measured by ν_H^2. As a result, the contribution of the global shock depends on the exposure of the final-good producer to central sectors, measured by

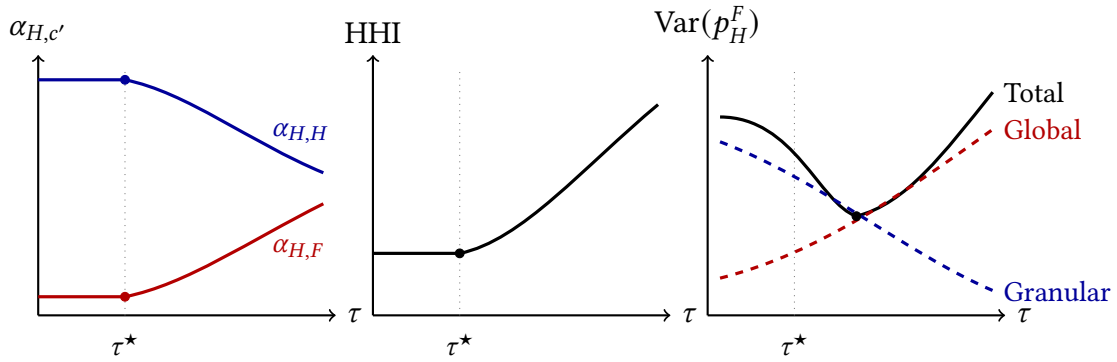
$$\nu_H = \sum_{c'} \alpha_{H,c'} \nu_{c'}.$$

Figure 4 illustrates what happens during episodes of trade liberalization. Panel (a) shows the sourcing shares. There is a level of trade distortions, $\tau^\star$, at which the economy is in autarky and Home sources entirely from itself ($\alpha_{H,H} = 1$, $\alpha_{H,F} = 0$). As τ falls below the threshold $\tau^\star$, Home gradually shifts its sourcing toward the more productive Foreign sector. The result is that trade allows the final-good producer in the Home country to buy from the more productive Foreign supplier, which is the classical mechanism that delivers gains from trade.

However, trade also changes the structure of the economy. Panel (b) shows the HHI rising as sourcing becomes more concentrated in the productive Foreign sector. In this simple economy, the HHI coincides with the exposure to centrality, so changes in con-

[5]The result is related to Baqaee and Farhi (2024) who show that revenue-based and cost-based transmission measures also differ in economies with wedges.

centration have implications for the propagation of shocks. Panel (c) shows the effect of trade on risk, which has a U-shape in $\text{Var}(p_H^F)$. Close to autarky, greater trade allows the final-good producer to diversify the risk coming from granular shocks, as in Caselli et al. (2020). In this region, diversification reduces aggregate risk. But as $\alpha_{H,F}$ grows, Home's centrality exposure ν_H rises and the increase in the global due to higher concentration component eventually dominates.



(a) Sourcing shares (b) HHI (c) Variance decomposition

Figure 4: Trade Liberalization: Sourcing, Concentration, and Volatility

Notes: Two-country economy. Foreign is the star: $\theta_F > \theta_H$, $\sigma_{u_c}^2 = \sigma_u^2$ for both countries. $\tau^\star$ is the threshold below which Home sources from Foreign. Arrows indicate the direction of decreasing τ.

The example takes σ_g^2 as fixed. But in equilibrium, when aggregate risk rises, firms respond by shifting sourcing away from central suppliers. I characterize this response in the next section.

5.3 Centrality-Risk Reallocation

I now show how firms reallocate sourcing away from central suppliers when aggregate risk rises. In the general model, the effect of a rise in σ_g^2 on equilibrium network is:

Proposition 3. *The first-order effect of a change in aggregate risk σ_g^2 on expenditure shares is:*

$$\frac{\partial m^\star_{ci,\hat{c}k}}{\partial \sigma_g^2} = -\frac{(\gamma-1)}{2\kappa^m}\left(\nu_{\hat{c}k} - \bar{\nu}_{ci}\right)\nu_c \frac{\psi}{1+\psi} \tag{14}$$

where $\bar{\nu}_{ci} = \frac{1}{N^m}\sum_{\hat{c}k' \in N^m} \nu_{\hat{c}k'}$ is the average centrality among active suppliers.

Proof. See Appendix E. □

The sign of $\partial m^\star / \partial \sigma_g^2$ depends on whether the supplier's centrality $v_{\hat{c}k}$ exceeds the buyer's average $\bar{v}_{ci}$: suppliers with above-average centrality lose market share when global risk rises, and suppliers with below-average centrality gain. The adjustment is amplified by the buyer's own exposure v_c. A buyer that is itself central faces a stronger incentive to insure through its supplier choice, creating a hierarchy of responses across buyers.

This result extends Kopytov et al. (2024), who show that industries with higher variance lose market share. Here the channel is different: an industry can have low own-variance $\sigma^2_{u_{ci}}$ and still be a risky input if it is highly central, because its price loads on the global factor through the Leontief inverse. The flight to safety is therefore driven by network position.

This mechanism applies in general for any common shock, like country-specific (σ_χ) and industry specific shocks (σ_ζ). For example, when the variance of country-specific shocks increases central industries inside the country will have higher impact and as a result will lose market shares. In the quantitative section I will test this mechanism in the data.

6 The Planner's Problem and Decentralization

A benevolent global planner chooses the intermediate and final-goods networks $s = \{\eta_{ci,\hat{c}k}, \alpha_{c,\hat{c}k}\}$ to maximize a Pareto-weighted sum of countries' welfare:

$$\mathcal{W}(s) \equiv \sum_c \lambda_c W_c(s), \tag{15}$$

where $\lambda_c > 0$ and $\sum_c \lambda_c = 1$. Relative to the competitive equilibrium, the planner has one additional instrument: state-contingent lump-sum transfers $T_c(\epsilon)$ across countries.

Proposition 4 (Planner's Risk-Adjusted Price)**.** *At the planner's optimum, the risk-adjusted price of supplier $\hat{c}k$ is:*

$$\mathcal{R}^\star_{\hat{c}k}(s) \;=\; \mathbb{E}\big[p_{\hat{c}k}\big] \;+\; \log \tau_{\hat{c}k} \;+\; (\gamma - 1)\,\mathrm{Cov}\big(p_{\hat{c}k},\; p^*\big) \tag{16}$$

where $p^*(s,\epsilon) \equiv \sum_c \lambda_c p_c(s,\epsilon)$ is the Pareto-weighted world price index.

Proof. See Appendix D.4. □

Comparing (16) to the competitive firm's risk-adjusted price in Proposition 1, the expected cost and trade cost terms are identical. The only difference is that the CE firm discounts supplier risk against its own domestic price index p_c, while the planner discounts against the world price index p^*. The Proposition 1 shows how this result reflects the effect of transfers. At the planner's optimum, all countries share aggregate risk collectively, so the relevant covariance for any sourcing decision is with the world price, not the domestic one. The planner's optimum can therefore be decentralized. Every firm then solves its standard problem with p^* replacing p_c in the risk-adjusted price, and the resulting equilibrium coincides with the planner's optimum.

The Centrality Wedge. Using equation (13) to expand both covariances, the gap between the planner's and the competitive firm's risk-adjusted prices is:

$$\mathcal{R}^{\star}_{\hat{c}k} \;-\; \mathcal{R}^{\mathrm{CE}}_{c,\hat{c}k} \;=\; (\gamma - 1)\,\sigma_g^2\,\nu_{\hat{c}k}\left(\nu^* - \nu_c\right) \tag{17}$$

where $\nu_c = \sum_{\hat{c}k} \alpha^{\star}_{c,\hat{c}k}\nu_{\hat{c}k}$ is country c's consumption-basket centrality exposure and $\nu^* = \sum_c \lambda_c \nu_c$ is the world average. The sign of the wedge depends on whether ν_c exceeds ν^*. For **central countries** with $\nu_c > \nu^*$, the wedge is negative: the CE firm perceives any supplier $\hat{c}k$ as riskier than the planner does, because its own consumption basket is more exposed to global shocks than the world average. CE firms therefore over-diversify away from central suppliers relative to the efficient allocation. For **peripheral countries** with $\nu_c < \nu^*$, the wedge is positive: CE firms under-diversify, remaining excessively concentrated in central suppliers. An average country with $\nu_c = \nu^*$ faces no wedge, its private insurance incentive exactly coincides with the globally efficient one.

7 Quantitative Analysis

I calibrate the model using WIOD data for 24 countries and 23 industries from 1967 to 2014; labor shares $\mu_{ci,t}$ vary over time and are taken directly from the data period by period and the remaining parameters are estimated as described below.

7.1 Trade Elasticities and Trade Costs

I identify κ^m and the time-varying bilateral trade costs $\tau_{b,k,t}$ by combining the model's optimality conditions with observed input-output and value-added data.

Trade elasticities. I estimate κ^I_{ci} and κ^F_c via the GMM described in Appendix H.2, which exploits the cross-sectional structure of the entropy FOC: differencing the log-share optimality condition across suppliers and over time absorbs trade costs and ideal technologies, and the covariance-based instrument $Z_{b,k,t}$ identifies γ separately from κ.

Inverting for trade costs. The identification of bilateral trade costs works as follows.

Proposition 5 (Identification of Time-Varying Trade Costs). *Let $(\kappa^m_b, \gamma, \boldsymbol{\mu}, \psi)$ be given. Normalize $\log \tau^m_{c,\hat{c}k,t} = 0$ for $c = \hat{c}$, the trade costs solve the linear system:*

$$\begin{pmatrix} \mathbf{M}_F\left(\mathcal{L}_F\, d^F + S_F\right) & \mathbf{M}_F\, \mathcal{L}_F\, d^I \\ \mathbf{M}_I\, \mathcal{L}_I\, d^F & \mathbf{M}_I\left(\mathcal{L}_I\, d^I + S_I\right) \end{pmatrix} \begin{pmatrix} \log \tau^F \\ \log \tau^I \end{pmatrix} = \begin{pmatrix} -2\,\kappa^F_c\, \mathbf{M}_F\, (\log \alpha^\star - \log \alpha^0) - \mathbf{M}_F\, \mathcal{L}_F\, (B(0) - \theta) \\ -2\,\kappa^I_{ci}\, \mathbf{M}_I\, (\log \eta^\star - \log \eta^0) - \mathbf{M}_I\, \mathcal{L}_I\, (B(0) - \theta) \end{pmatrix}$$

where $\mathbf{M}_m$ is the demeaning matrix over the active supplier set.

Proof. See Appendix H. □

Proposition 5 extends the standard gravity inversion of Eaton and Kortum (2002) and Arkolakis, Costinot, and Rodríguez-Clare (2012) by filtering trade costs through risk. In the standard approach, all cross-country variation in expenditure shares is attributed to bilateral frictions. Here, the right-hand side of Proposition 5 decomposes that variation into two forces: the first term captures differences in sourcing intensity between buyer c and the producing-country reference $\hat{c}$, which under frictionless risk-neutral sourcing would be zero; the second term nets out the differential insurance value of supplier $(\hat{c}, k)$ for the two buyers. The estimated $\hat{\tau}$ therefore absorbs only the component of bilateral price wedges that cannot be explained by risk — a standard inversion would attribute the entire wedge to frictions, overstating trade barriers in crisis periods when the covariance

term spikes.

Incomplete markets are essential for this separation. Under complete markets, the pricing kernel is equalized across countries, so $(\gamma-1)\text{Cov}(p_c, p_{\hat{c}k})$ becomes buyer-invariant and drops from the cross-country difference — risk and frictions are then observationally equivalent in the cross-section. Incomplete markets break this degeneracy: because households cannot insure across borders, p_c reflects country-specific exposure to global shocks, and the same supplier provides different insurance value to buyers in different locations. It is this buyer-country heterogeneity in $\text{Cov}(p_c, p_{\hat{c}k})$ that allows the model to separately identify frictions from insurance motives.

7.2 Stochastic Structure

Productivity shocks and common factors. Given the observed network, I invert the VA-price mapping of Lemma 6 following Huo et al. (2024) to recover productivity shocks $\epsilon_{ci,t}$ from observed VA, then decompose them into four orthogonal factors:

$$\epsilon_{ci,t} = g_t + \xi_{c,t} + \zeta_{i,t} + u_{ci,t} \tag{18}$$

where g_t is a global factor, $\xi_{c,t}$ a country factor, $\zeta_{i,t}$ an industry factor, and $u_{ci,t}$ an idiosyncratic residual; I estimate the persistence ρ_k of each factor by OLS on the extracted series.

Estimating time-varying risk. I estimate time-varying variances for each common factor using a GARCH(1,1) model applied to each extracted factor series $\hat{f}_{k,t} \in \{\hat{g}_t, \hat{\xi}_{c,t}, \hat{\zeta}_{i,t}, \hat{\sigma}^2_{u,ci}\}$:

$$\hat{\sigma}^2_{k,t} = \bar{\omega}_k + \alpha_{1,k}\,\hat{f}^2_{k,t-1} + \beta_{1,k}\,\hat{\sigma}^2_{k,t-1} \tag{19}$$

with $\bar{\omega}_k > 0$, $\alpha_{1,k} \geq 0$, $\beta_{1,k} \geq 0$, and $\alpha_{1,k} + \beta_{1,k} < 1$ ensuring covariance stationarity; parameters are estimated by maximum likelihood separately for each factor. The full time-varying shock covariance matrix is then:

$$\hat{\Sigma}_{\epsilon,t} = \hat{\sigma}^2_{g,t}\,\mathbf{1}\mathbf{1}^\top + \sum_c \hat{\sigma}^2_{\xi,c,t}\,\mathbf{1}_c\mathbf{1}_c^\top + \sum_i \hat{\sigma}^2_{\zeta,i,t}\,\mathbf{1}_i\mathbf{1}_i^\top + \text{diag}\big(\hat{\sigma}^2_{u,ci}\big) \tag{20}$$

8 Quantitative Results

This section presents the main quantitative results, organized around the estimated structural primitives and a panel test of whether the centrality-risk mechanism of Proposition 3 operates in the data.

Parameters. Table 1 reports the calibrated parameters; the Frisch elasticity is set to $\psi = 1$, $\gamma = 2$ and (κ^I, κ^F) are calibrated as described in Appendix H.4.

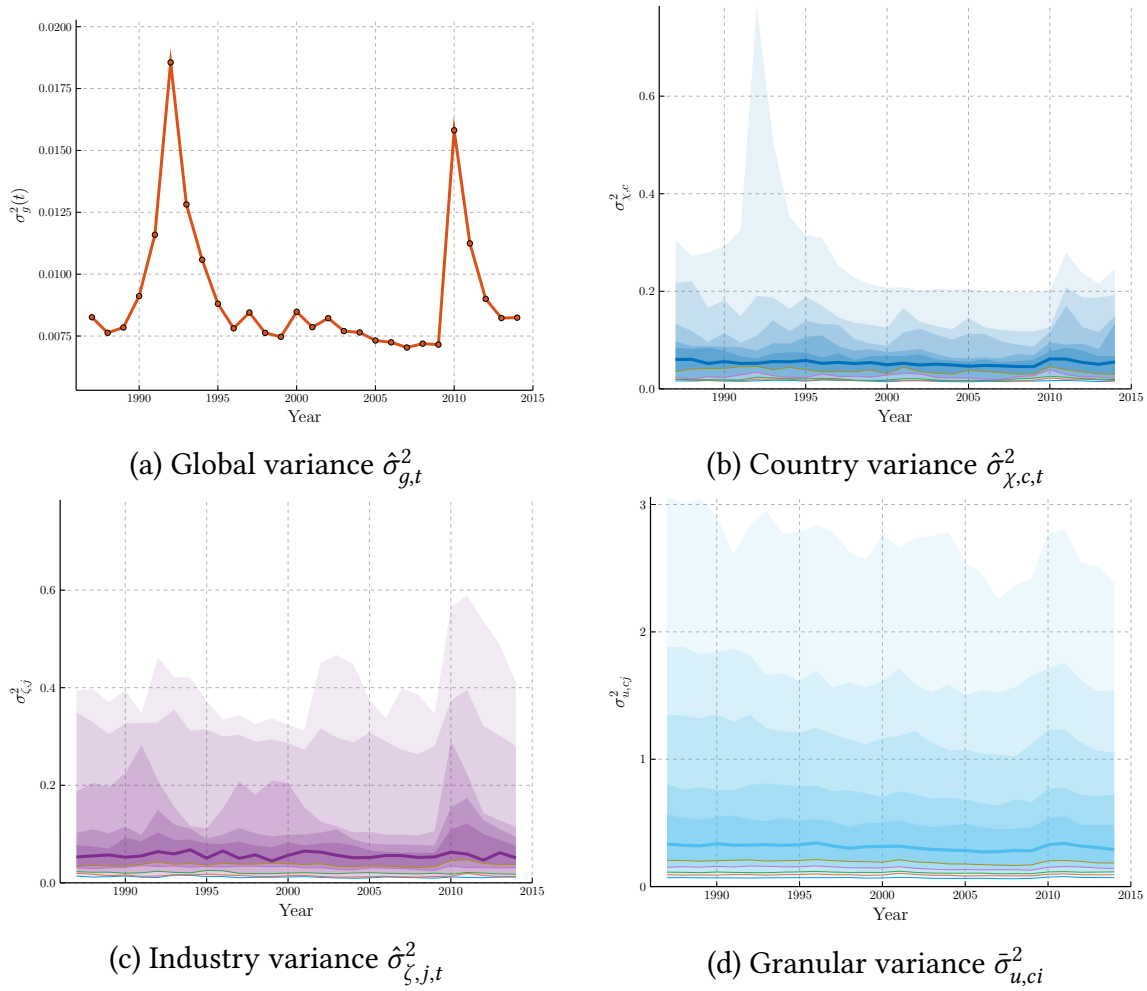
Table 1: Parameters

Parameter	Value	Source	Related to
External			
ψ	1.0	Standard	Frisch elasticity
γ	2.0	Standard	Risk aversion
μ_j	[0.2, 0.68]	WIOT	Labor shares
Calibrated			
κ^I_{ci}	365	IV	Adjustment cost – Intermediate (median)
κ^F_c	189	IV	Adjustment cost – Final (median)
τ	[1.0, 3.2]	–	Trade cost
σ^2_g	0.0011	–	Global risk
σ^2_c	0.0008	–	Country risk – Average
σ^2_i	0.0018	–	Industry risk – Average

Notes: Ranges report the 10th and 90th percentiles of the distribution.

8.1 Estimated Structural Primitives

Figure 5 plots the four estimated conditional variance series. Panel (a) shows $\hat{\sigma}^2_{g,t}$: the series fluctuates between 0.008 and 0.016 in log VA units, with a sharp spike around 2008–09 that roughly doubles the tranquil-period baseline, making it the dominant source of time variation in aggregate risk across the sample.



(a) Global variance $\hat{\sigma}^2_{g,t}$

(b) Country variance $\hat{\sigma}^2_{\chi,c,t}$

(c) Industry variance $\hat{\sigma}^2_{\zeta,j,t}$

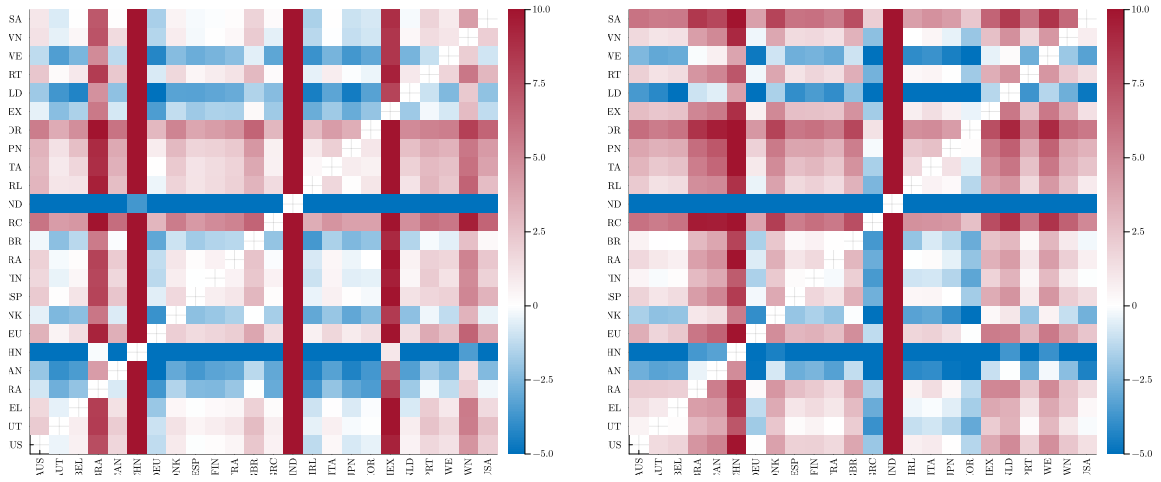
(d) Granular variance $\bar{\sigma}^2_{u,ci}$

Figure 5: Changes in Uncertainty

Panel (b) shows the fan chart of country-specific variances $\hat{\sigma}^2_{\chi,c,t}$, which range between 0.06 and 0.10 in log VA units. The cross-sectional distribution is wide but stable in shape, with the 2008 spike visible as a uniform upward shift of the entire distribution rather than a divergence between countries. Panel (c) shows the industry fan chart $\hat{\sigma}^2_{\zeta,j,t}$: industry variances are larger on average than country variances (0.09–0.16 vs 0.06–0.10) but display less persistent time variation. Panel (d) shows the fan chart of granular variances $\hat{\sigma}^2_{u,ci,t}$, which are an order of magnitude larger (0.6–0.7) but exhibit less time variation than the common factors, consistent with the finding that the dominant source of time-varying aggregate risk is the global component.

Figure 6 shows the time-averaged bilateral heat maps. Intra-European pairs have low

estimated trade costs (median $\log \hat{\tau}^I \approx 0.9$, $\log \hat{\tau}^F \approx 0.6$), while intercontinental pairs display substantially higher frictions: Europe-to-Asia final-good costs average 7.9 and intermediate-good costs 1.8. The US–Canada corridor is asymmetric, with $\log \hat{\tau}^I = 7.9$ for US exports to Canada but -4.5 in the reverse direction, suggesting that the structural residual captures not only transport costs but also regulatory and institutional asymmetries. Negative entries indicate bilateral pairs that trade more than the frictionless benchmark, consistent with preferential trade agreements or deep institutional integration.



(a) Final goods (b) Intermediate goods

Figure 6: Heterogeneity in Trade Cost

8.2 The Centrality-Risk Mechanism in the Data

Proposition 3 predicts that when aggregate risk rises, buyers substitute away from above-average-centrality suppliers, reducing concentration. I test this prediction using changes in risk:

$$\Delta\omega_{cj,t} = \sum_{k\in\{g,\chi,\zeta\}} \beta_k \, \hat{\ell}_{cj,t-1} \times \Delta\hat{\sigma}^2_{k,t} + \mathbf{x}'_{cj,t}\,\delta + \varepsilon_{cj,t} \tag{21}$$

where $\hat{\ell}_{cj,t-1} = \sum_{nk} \mathcal{L}_{cj,nk,t-1}$ is the row sum of the Leontief inverse (network exposure centrality), globally demeaned, and $\mathbf{x}_{cj,t}$ includes the variance levels $\hat{\sigma}^2_{\chi,t-1}$, $\hat{\sigma}^2_{\zeta,t-1}$, $\hat{\sigma}^2_{u,t-1}$ and the factor realizations as controls.

The coefficient of interest is $\beta_g < 0$: above-average-centrality industries should lose market share when global risk increases. Table 2 reports the results. The global interaction $\hat{\ell}_{t-1} \times \Delta\sigma^2_{g,t}$ is negative and significant at the one-percent level: when global uncertainty

rises, industries with above-average network centrality lose a disproportionate share of economic activity. The country and industry risk interactions carry no significant signal, consistent with the model's prediction that reallocation is driven by common shocks that penalize central suppliers through their network position. The within-country Herfindahl index also responds negatively to increases in global risk, consistent with the aggregate pattern. Full results including the HHI regressions, level-based specifications, and progressive controls appear in Appendix H.7.

	$\Delta\omega_{cj,t} \times 10^4$
$\hat{\ell}_{t-1} \times \Delta\sigma^2_{g,t}$	-6.7084^{***}
	(2.2244)
$\hat{\ell}_{t-1} \times \Delta\sigma^2_{\chi,t}$	0.0312
	(0.1010)
$\hat{\ell}_{t-1} \times \Delta\sigma^2_{\zeta,t}$	-0.0300
	(0.0836)
Controls	Yes
Observations	14,904
R^2	0.0039

Notes: Dependent variable is $\Delta\omega_{cj,t} \times 10^4$, winsorized at the 1st/99th percentile. $\hat{\ell}_{cj,t-1} = \sum_{nk} \mathcal{L}_{cj,nk,t-1}$ is the row sum of the full Leontief inverse, globally demeaned. Controls include variance levels (σ^2_u, σ^2_χ, σ^2_ζ) and factor realizations ($\chi_{c,t-1}$, $\zeta_{j,t-1}$, $u_{ci,t-1}$). All RHS variables lagged one period and standardized. $N = 24 \times 23 \times 27 = 14{,}904$. SE clustered at year level (27 clusters). $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table 2: The Centrality-Risk Mechanism

9 Counterfactual Analysis

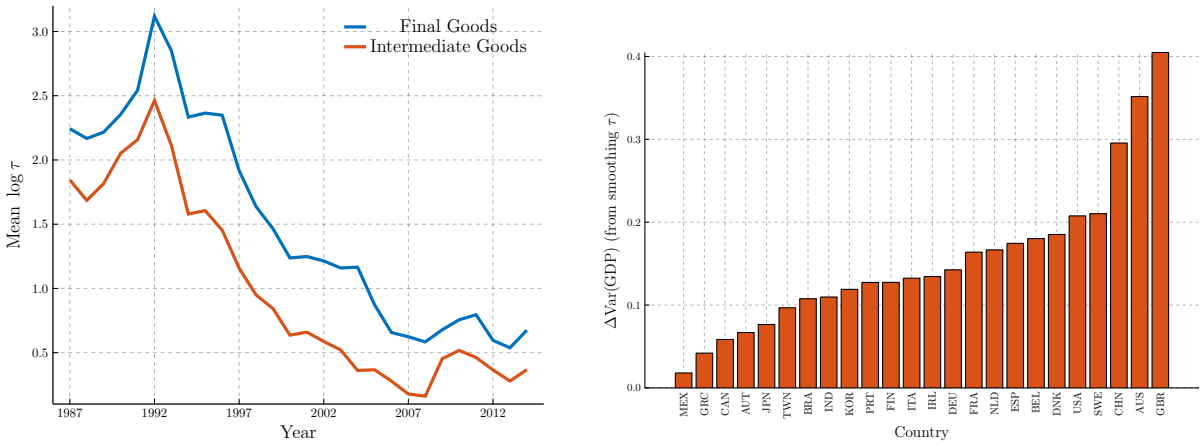
The baseline results established that the centrality-risk mechanism operates in the data. This section asks what it implies quantitatively for aggregate volatility and welfare. The analysis proceeds in three steps. First, we compare aggregate volatility between two points in the data to assess whether trade liberalization raises or lowers GDP variance, revisiting the question of Caselli et al. (2020). Second, we compare the competitive equilibrium network to the planner's optimum to quantify the concentration externality. Third, we

trace how the welfare gap between the CE and the planner evolves as trade costs fall, showing that the externality grows with openness.

9.1 Trade Liberalization and Aggregate Volatility

The first experiment compares aggregate volatility at $t_0 = 1967$ and $t_1 = 2014$, the two end points of our sample. The question is whether the net effect on aggregate volatility is positive or negative, which is the central empirical question of Caselli et al. (2020).

Figure 7 reports thecross-country distribution of the variance change. The net effect is an increase in average GDP variance of XX percent for all countries in the sample. This contrast the finding of Caselli et al. (2020): under Cobb-Douglas production with endogenous network formation, trade liberalization raises rather than lowers aggregate volatility, because firms concentrate in productive central suppliers rather than diversify toward less correlated ones.



(a) Change in Trade Costs.

(b) Change in Aggregate Risk.

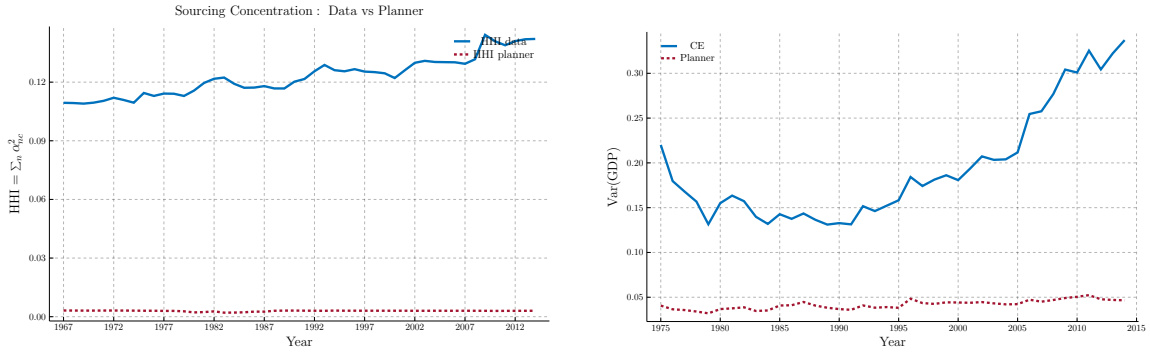
Figure 7: Trade cost and Aggregate Risk.

9.2 Competitive Equilibrium vs. Planner

The second experiment compares the sourcing concentration chosen by the competitive equilibrium to the constrained-efficient allocation chosen by a planner who internalizes the network amplification externality. Individual buyers in the CE do not internalize how their concentration in central suppliers raises the aggregate variance of prices faced by all

other buyers downstream. The planner corrects this externality by tilting sourcing toward less central, more diversified suppliers.

Figure 8 plots the HHI of sourcing concentration in the CE and at the planner's optimum over the sample period 2000–2011. The two series diverge sharply. The CE HHI averages 0.12 over the sample and trends upward as trade costs fall, reflecting the progressive concentration of the global network in central suppliers. The planner's HHI is close to zero throughout, implying that the constrained-efficient allocation calls for near-complete diversification at every point in time. The gap between the two series is the sourcing-concentration counterpart of the welfare cost of the externality: the CE systematically over-concentrates production because individual buyers do not internalize how their sourcing decisions feed back into aggregate risk.



(a) Concentration

(b) Average GDP variance

Figure 8: Trade liberalization and aggregate volatility.

10 Conclusions

This paper studies how aggregate shocks reshape production networks through endogenous choice of suppliers, generating two-way feedback between network formation and shock propagation. I develop a quantitative model in which firms choose suppliers before productivity shocks are realized, trading off price efficiency against risk exposure.

The analysis yields three main insights. First, when global uncertainty rises, trade shifts from central country-industry pairs, which serve many buyers and source from many producers, toward peripheral suppliers, as firms seek suppliers less exposed to global shocks. This reallocation reflects a change in comparative advantage: well-connected central suppliers offer the worst insurance when aggregate risk is high. Second, network

reorganization explains 16-20% of GDP growth during and after the 2009 crisis, with heterogeneous country effects. Third, lower trade costs can raise aggregate volatility: while cheaper transportation boosts expected welfare, it also leads firms to concentrate on riskier but more productive suppliers, increasing exposure to productivity shocks.

References

Acemoglu, D. and P. D. Azar (2020): "Endogenous production networks," *Econometrica*, 88, 33–82.

Acemoglu, D., V. M. Carvalho, A. Ozdaglar, and A. Tahbaz-Salehi (2012): "The network origins of aggregate fluctuations," *Econometrica*, 80, 1977–2016.

Acemoglu, D., A. Ozdaglar, and A. Tahbaz-Salehi (2017): "Microeconomic origins of macroeconomic tail risks," *American Economic Review*, 107, 54–108.

Adamopoulos, T. and F. Leibovici (2024): "Trade Risk and Food Security," *Federal Reserve Bank of St. Louis Working Paper*.

Allen, T. and D. Atkin (2022): "Volatility and the Gains from Trade," *Econometrica*, 90, 2053–2092.

Arkolakis, C., A. Costinot, and A. Rodríguez-Clare (2012): "New Trade Models, Same Old Gains?" *American Economic Review*, 102, 94–130.

Arkolakis, C., F. Huneeus, and Y. Miyauchi (2025): "Production Network Formation, Trade, and Welfare," Tech. rep., CESifo Working Paper.

Backus, D. K., P. J. Kehoe, and F. E. Kydland (1992): "International real business cycles," *Journal of political Economy*, 100, 745–775.

Baqaee, D. R. and E. Farhi (2019): "The Macroeconomic Impact of Microeconomic Shocks: Beyond Hulten's Theorem," *Econometrica*, 87, 1155–1203.

——— (2024): "Networks, barriers, and trade," *Econometrica*, 92, 505–541.

Bigio, S. and J. La'O (2020): "Distortions in Production Networks," *Quarterly Journal of Economics*, 135, 2187–2253.

Bloom, N., M. Floetotto, N. Jaimovich, I. Saporta-Eksten, and S. J. Terry (2018): "Really Uncertain Business Cycles," *Econometrica*, 86, 1031–1065.

Caliendo, L. and F. Parro (2015): "Estimates of the Trade and Welfare Effects of NAFTA," *Review of Economic Studies*, 82, 1–44.

Carvalho, V. M. and X. Gabaix (2013): "The Great Diversification and Its Undoing," *American Economic Review*, 103, 1697–1727.

Caselli, F., M. Koren, M. Lisicky, and S. Tenreyro (2020): "Diversification through trade," *The Quarterly Journal of Economics*, 135, 449–502.

Dekle, R., J. Eaton, and S. Kortum (2007): "Unbalanced Trade," *American Economic Review*, 97, 351–355.

Dew-Becker, I. (2023): "Tail risk in production networks," *Econometrica*, 91, 2089–2123.

Dew-Becker, I., A. Tahbaz-Salehi, and A. Vedolin (2021): "Skewness and time-varying second moments in a nonlinear production network: theory and evidence," Tech. rep., National Bureau of Economic Research.

di Giovanni, J. and A. A. Levchenko (2010): "Putting the Parts Together: Trade, Vertical Linkages, and Business Cycle Comovement," *American Economic Journal: Macroeconomics*, 2, 95–124.

di Giovanni, J., A. A. Levchenko, and I. Méjean (2014): "Firms, Destinations, and Aggregate Fluctuations," *Econometrica*, 82, 1303–1340.

——— (2018): "The Micro Origins of International Business-Cycle Comovement," *American Economic Review*, 108, 82–108.

Eaton, J. and S. Kortum (2002): "Technology, geography, and trade," *Econometrica*, 70, 1741–1779.

Elliott, M., B. Golub, and M. V. Leduc (2022): "Supply network formation and fragility," *American Economic Review*, 112, 2701–2747.

Fan, J. and W. Luo (2025): "Global Production Networks with Global Uncertainty," *Available at SSRN 5282371*.

Fitzgerald, D. (2025): "3-D Gains from Trade," *Federal Reserve Bank of Minneapolis Working Paper*.

Gabaix, X. (2011): "The granular origins of aggregate fluctuations," *Econometrica*, 79, 733–772.

Greenwood, J., Z. Hercowitz, and G. W. Huffman (1988): "Investment, Capacity Utilization and the Real Business Cycle," *American Economic Review*, 78, 402–417.

Heathcote, J. and F. Perri (2002): "Financial autarky and international business cycles," *Journal of monetary Economics*, 49, 601–627.

Huo, Z., A. Levchenko, and N. Pandalai-Nayar (2024): "International comovement in the global production network," *Review of Economic Studies*, rdae033.

Jones, C. I. (2011): "Intermediate Goods and Weak Links in the Theory of Economic Development," *American Economic Journal: Macroeconomics*, 3, 1–28.

Kleinman, B., E. Liu, and S. J. Redding (2025): "International Trade in an Uncertain World," Manuscript, Stanford University and Princeton University.

Kopytov, A., B. Mishra, K. Nimark, and M. Taschereau-Dumouchel (2024): "Endogenous production networks under supply chain uncertainty," *Econometrica*, 92, 1621–1659.

Kose, M. A. and K.-M. Yi (2001): "International Trade and Business Cycles: Is Vertical Specialization the Missing Link?" *American Economic Review*, 91, 371–375.

Timmer, M. P., E. Dietzenbacher, B. Los, R. Stehrer, and G. J. De Vries (2015): "An illustrated user guide to the world input–output database: the case of global automotive production," *Review of International Economics*, 23, 575–605.

vom Lehn, C. and T. Winberry (2022): "Investment Network, Sectoral Comovement, and the Changing U.S. Business Cycle," *Quarterly Journal of Economics*, 137, 387–433.

Waugh, M. E. (2010): "International Trade and Income Differences," *American Economic Review*, 100, 2093–2124.

Woltjer, P., R. Gouma, and M. P. Timmer (2021): "The Long-Run WIOD: A Global Input–Output Database Covering the Years 1965–2000," GGDC Research Memorandum 186, Groningen Growth and Development Centre.

A Additional Empirical Evidence

A.1 Data Construction

The empirical analysis uses the Long-WIOD dataset, a balanced panel of world input-output tables spanning 1967–2014. The dataset covers 24 major economies and 23 sectors at the ISIC Rev. 3 level, yielding 552 country-sector pairs observed over 48 years. Tables 4 and 3 list the countries and sectors included. These economies account for over 85% of world GDP throughout the sample period.

Sources and chain-linking. The Long-WIOD combines two underlying datasets. The historical tables for 1967–2000 are drawn from Woltjer et al. (2021), who construct a retrospective world input-output database by harmonizing national accounts and bilateral trade data for the pre-WIOD period. The tables for 2001–2014 come from the WIOD 2016 release (**?**). The two series are chain-linked at the 2000–2001 overlap year by rescaling the historical tables so that the level of each bilateral flow matches the WIOD 2016 value in 2001, ensuring consistency of the intermediate input shares across the splice point.

Key variables. From the input-output tables I construct three main objects used in the empirical analysis and in the structural estimation. *Domar weights* $\omega_{ci,t}$ are the ratio of gross output of country-sector (c, i) to world GDP in year t, computed directly from the supply-use accounts. *Expenditure shares* $\eta_{ci,c'i',t}$ and $\alpha_{c,c'i',t}$ are intermediate and final demand shares, respectively, obtained by dividing each bilateral flow by the total purchases of the buying sector. *Value added* $a_{ci,t}$ is log real value added, deflated using the gross output price index from the socioeconomic accounts following Huo et al. (2024).

Trade deficit series. Country-level trade deficits $\delta_{c,t}$ are computed as the difference between total expenditure (absorption) and total income (value added plus net taxes) in each year, expressed as a share of world GDP. The series is taken directly from the supply-use accounts and requires no additional assumptions.

Sample coverage. The 24 economies in the sample include all G7 members, the four largest emerging markets represented in the WIOD (Brazil, China, India, Mexico), and the major European economies. The 23 sectors span agriculture, mining, manufacturing (14

subsectors), and services (7 subsectors), covering all tradeable and non-tradeable industries at a level of disaggregation sufficient to identify heterogeneous trade costs across country-sector pairs. The balanced panel structure—all 552 pairs observed in all 48 years—is essential for the factor decomposition in Section 2.1, which requires a complete cross-section in each period to identify the global and country-specific components of VA growth.

Code	Description
AtB	Agriculture, Hunting, Forestry & Fishing
C	Mining & Quarrying
D15t16	Food, Beverages & Tobacco
D17t19	Textiles, Leather & Footwear
D21t22	Pulp, Paper, Printing & Publishing
D23	Coke, Refined Petroleum & Nuclear Fuel
D24	Chemicals & Chemical Products
D25	Rubber & Plastics
D26	Other Non-Metallic Mineral Products
D27t28	Basic Metals & Fabricated Metal Products
D29	Machinery, nec
D30t33	Electrical & Optical Equipment
D34t35	Transport Equipment
Dnec	Manufacturing, nec; Recycling
E	Electricity, Gas & Water Supply
F	Construction
G	Wholesale & Retail Trade
H	Hotels & Restaurants
I60t63	Transport & Storage
I64	Post & Telecommunications
J	Financial Intermediation
K	Real Estate, Renting & Business Activities
LtQ	Public Administration & Other Services

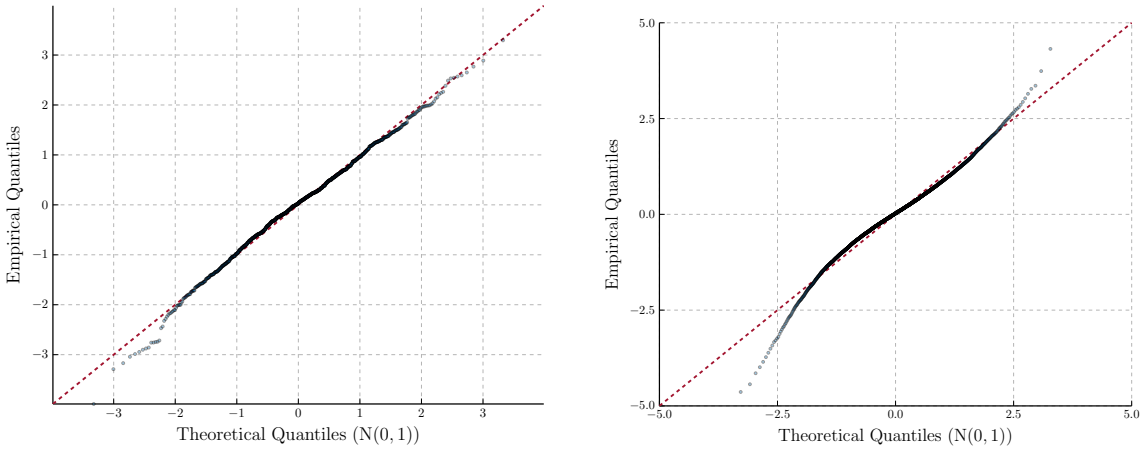
Table 3: Sectors in the Long-WIOD Panel (ISIC Rev. 3)

AUS – Australia	AUT – Austria	BEL – Belgium	BRA – Brazil
CAN – Canada	CHN – China	DEU – Germany	DNK – Denmark
ESP – Spain	FIN – Finland	FRA – France	GBR – United Kingdom
GRC – Greece	IND – India	IRL – Ireland	ITA – Italy
JPN – Japan	KOR – South Korea	MEX – Mexico	NLD – Netherlands
PRT – Portugal	SWE – Sweden	TWN – Taiwan	USA – United States

Table 4: Countries in the Long-WIOD Panel

A.2 QQ Plots and the Nature of Aggregate Risk

Figure 9 compares the empirical quantiles of growth rates against the normal distribution. Panel 9a shows that GDP growth exhibits heavy tails: extreme negative outcomes occur far more frequently than a constant-variance normal distribution predicts. Panel 9b shows that sectoral value-added growth is well approximated by normality at all quantiles. Individual sectors are approximately normal, but the aggregate is not. This discrepancy points to episodes of abnormal synchronization: in most years the cross-sectional standard deviation of sectoral VA growth is approximately 12% and roughly 25% of sectors experience negative growth, but in 2009 dispersion nearly doubled to 18% and the fraction of declining sectors rose to over 80%. The joint increase in dispersion and synchronization is the signature of a large common factor, not of large idiosyncratic shocks (see also Acemoglu, Ozdaglar, and Tahbaz-Salehi, 2017).



(a) Country GDP growth (b) Sectoral VA growth

Figure 9: Q-Q Plots of Growth Rates Against the Normal Distribution

Notes: Each panel plots empirical quantiles of annual growth rates against theoretical normal quantiles with the same mean and variance. WIOD data, 23 countries, 23 industries, 1967–2014.

A.3 Country-Level Variance Decomposition

Figure 10 reports the share of each country's GDP variance accounted for by each factor component, averaged over the full sample. The global factor dominates for most countries but with substantial heterogeneity: it accounts for over 75% of GDP variance in France,

the United Kingdom, and Germany—open economies deeply integrated into the global production network—but under 35% in China and Brazil, whose large domestic markets make them less exposed to global fluctuations. This heterogeneity foreshadows the model's prediction that the flight-to-safety mechanism has heterogeneous effects depending on each country's position in the global network.

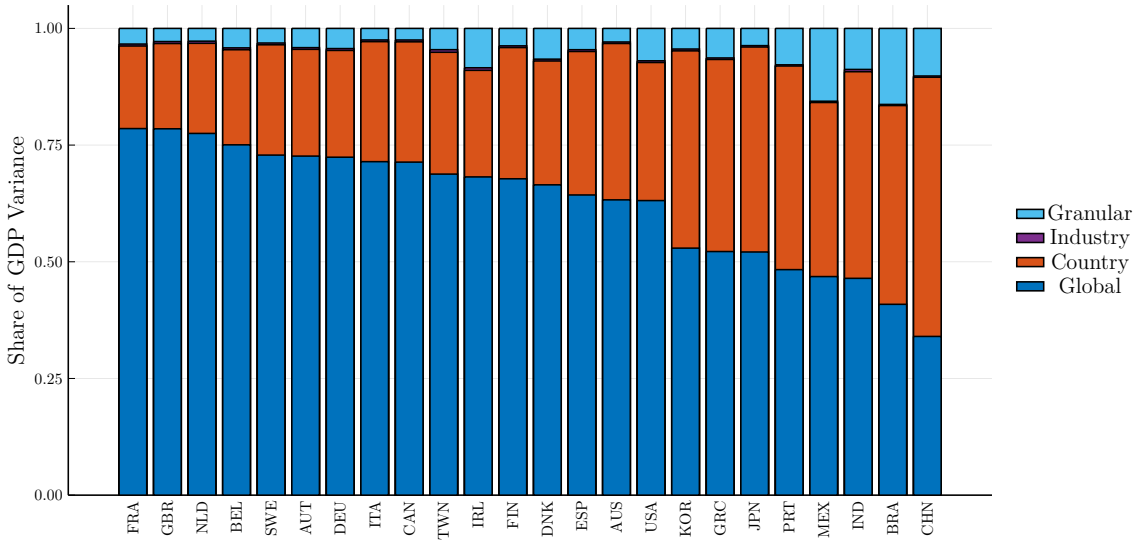


Figure 10: Share of GDP Variance by Component, by Country

Notes: Each bar shows the share of a country's GDP variance accounted for by the global (σ_g^2), country (σ_χ^2), industry (σ_ζ^2), and idiosyncratic (σ_u^2) components of the factor decomposition, averaged over 1967–2014. Countries sorted by global component share. WIOD data.

A second complementary statistic is the average pairwise correlation of VA growth rates across all country-industry pairs in each year:

$$\bar{\rho}_t = \frac{2}{N(N-1)} \sum_{(ci)<(c'i')} \rho_{ci,c'i',t} \tag{22}$$

where $\rho_{ci,c'i',t}$ is the correlation between the VA growth rates of pairs (ci) and $(c'i')$ computed over a rolling window centered at t. When $\bar{\rho}_t$ is high, country-industry units move together more than usual. This is a measure of synchronization.

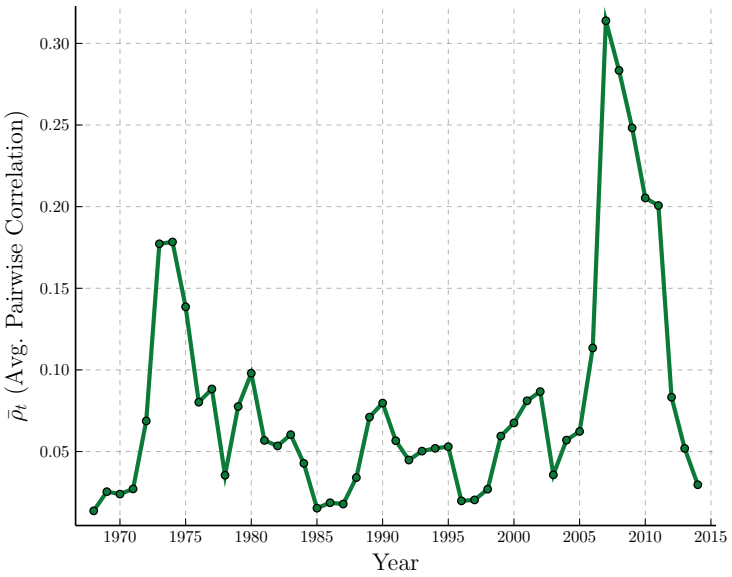


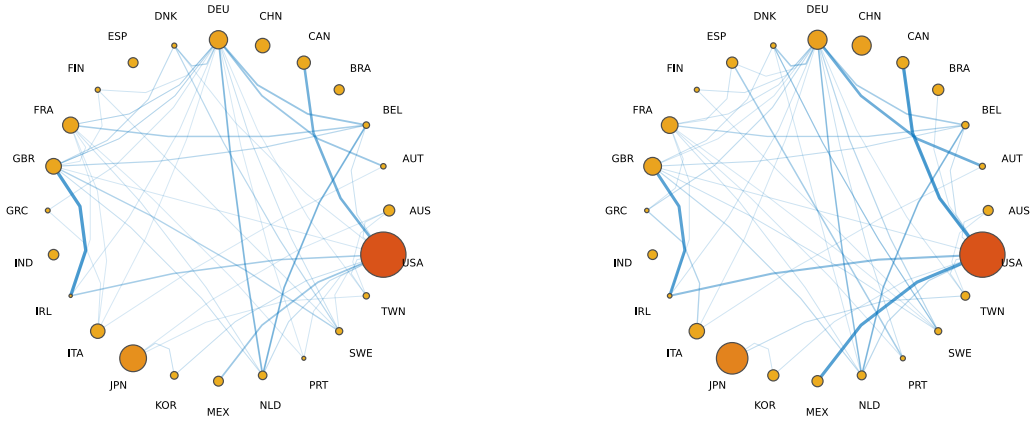
Figure 11: Average pairwise correlation $\bar{\rho}_t$

Notes: Shows the average pairwise correlation $\bar{\rho}_t$ of VA growth across all $N(N-1)/2$ country-industry pairs, computed with a five-year rolling window. WIOD data: 23 countries, 23 industries, 1967-2014.

Figure 11 shows that $\bar{\rho}_t$ is time-varying, with pronounced spikes around 1982, 1991, and 2008: in normal years the average pair of country-industry units moves with a correlation of approximately 0.32, rising to 0.45 at the peak of the 1982 episode and spiking again in 2008.

A.4 Intermediate Network Snapshots

Figure 12 shows the global trade network in 1984 and 2000. The 1984 network shows Japan's emergence as a major hub: its node has grown substantially relative to 1967, reflecting the expansion of Japanese manufacturing during the 1970s and early 1980s. By 2000, Japan remains large but China begins to appear as a non-trivial node, while the US retains its dominant position. These intermediate snapshots confirm that the structural transformation of the network was gradual, driven primarily by Japan's rise and China's emergence rather than by any discrete event.



(a) Trade Network, 1984 (b) Trade Network, 2000

Figure 12: Global Trade Network: Intermediate Snapshots

Notes: Node size proportional to country Domar weight $\omega_{c,t}$. Edge thickness proportional to bilateral intermediate trade flows above the 75th percentile. WIOD data, 23 countries, 1984 and 2000.

B Proofs of Theoretical Sections

B.1 Lemma 1

Proof. We begin with the resource constraint at each sector i;

$$Y_{ci} = \sum_{\hat{c}} Z_{\hat{c},ci}\tau_{\hat{c},ci} + \sum_{\hat{c}}\sum_{k} Z_{ci,\hat{c}k}\tau_{\hat{c},ci} \tag{23}$$

Multiplying everything by P_{ci} we get:

$$P_{ci}Y_{ci} = \sum_{\hat{c}} P_{ci}Z_{ci,\hat{c}}\tau_{ci,\hat{c}} + \sum_{\hat{c}}\sum_{k} P_{ci}Z_{ci,\hat{c}k}\tau_{ci,\hat{c}} \tag{24}$$

In addition, the optimality conditions for the final and intermediate goods produced are as follows:

$$\tau_{ci,\hat{c}}P_{ci}Z_{ci,\hat{c}} = P_{\hat{c}}Z_{\hat{c}}\alpha_{ci,\hat{c}} \tag{25}$$

$$\tau_{ci,\hat{c}}P_{ci}Z_{ci,\hat{c}k} = P_{\hat{c}k}Y_{\hat{c}k}\eta_{ci,\hat{c}k} \tag{26}$$

Combining them, we have:

$$P_{ci}Y_{ci} = \sum_{\hat{c}} P_{\hat{c}}Z_{\hat{c}}\alpha_{ci,\hat{c}} + \sum_{\hat{c}}\sum_{k}(1-\mu_{\hat{c}k})P_{\hat{c}k}Y_{\hat{c}k}\eta_{ci,\hat{c}k} \tag{27}$$

Next, we use the budget constraint of the households to get:

$$P_{\hat{c}}Z_{\hat{c}} = \sum_{k} \mu_{\hat{c}k}P_{\hat{c}k}Y_{\hat{c}k} \tag{28}$$

So:

$$P_{ci}Y_{ci} = \sum_{\hat{c}}\sum_{k} \mu_{\hat{c}k}\alpha_{ci,\hat{c}}P_{\hat{c}k}Y_{\hat{c}k} + \sum_{\hat{c}}\sum_{k}(1-\mu_{\hat{c}k})P_{\hat{c}k}Y_{\hat{c}k}\eta_{ci,\hat{c}k} \tag{29}$$

In vector notation, we have:

$$\boldsymbol{PY} = (\boldsymbol{\mu\alpha} + (I-\boldsymbol{\mu})\boldsymbol{\eta})\boldsymbol{PY} \tag{30}$$

The solution of the system is the eigenvalue, with $\lambda = 1$, of the matrix $\boldsymbol{\mu\alpha} + (I - \boldsymbol{\mu})\boldsymbol{\eta}$. We need to solve the following.

$$det(\boldsymbol{\mu\alpha} + (I - \boldsymbol{\mu})\boldsymbol{\eta} - I) = 0 \tag{31}$$

Because the rows of the matrix sum one by construction, we know that $\lambda = 1$ is an eigenvalue of the system, so the solution of the system exist. Finally, I impose normalization $\sum \omega_{ci} = 1$ to identify a unique solution. During this normalization, I choose the global GDP to be numeraire.

□

B.2 Lemma 2

Proof. **B.2.1 Nominal GDP**

The starting point of the proof is the Domar weights that we characterize in Lemma **??**.

$$P_{ci}Y_{ci} = \omega_{ci} \tag{32}$$

Taking logs we have:

$$\log P_{ci} + \log Y_{ci} = \log(\omega_{ci}) \tag{33}$$

B.2.2 Labor Demand

The FOC of firms for labor are as follows:

$$W_c L_{ci} = \mu_{ci} P_{ci} Y_{ci} \tag{34}$$

$$\log P_{ci} + \log Y_{ci} + \log \mu_{ci} = \log W_c + \log L_{ci} \tag{35}$$

Then:

$$\log L_{ci} = -\log W_c + \log \mu_{ci} + \log(\omega_{ci}) \tag{36}$$

B.2.3 Labor supply

Also, FOC of households implies:

$$P_c(L_{ci})^{\frac{1}{\psi}} = W_c \tag{37}$$

$$\log L_{ci} = [\log W_c - \log P_c]\psi \tag{38}$$

Combining both:

$$\log W_c = \frac{\psi}{1+\psi}\log P_c + \frac{1}{1+\psi}[\log \mu_{ci} + \log(\omega_{ci})] \tag{39}$$

B.2.4 Demand of Inputs

From FOC of intermediate inputs of firms:

$$\log P_{ci} = -\epsilon_i - A(\eta_i^\star) + \mu_{ci}\log W_c + (1-\mu_{ci})\sum_{\hat{c}}\sum_j \eta_{ci,\hat{c}j}[\log P_{\hat{c}j} + \log(\tau_{c,\hat{c}j})] \tag{40}$$

$$\begin{aligned}\log P_{ci} = -\epsilon_i - A(\eta_i^\star) + \mu_{ci}\left[\frac{\psi}{1+\psi}\log P_c + \frac{1}{1+\psi}(\log \mu_{ci} + \log(\omega_{ci}))\right] + \\ (1-\mu_{ci})\sum_{\hat{c}}\sum_j \eta_{ci,\hat{c}j}[\log P_{\hat{c}j} + \log(\tau_{c,\hat{c}j})]\end{aligned} \tag{41}$$

$$\begin{aligned}\log P_{ci} = -\epsilon_i - A(\eta_i^\star) + \mu_{ci}\left[\frac{\psi}{1+\psi}\left[A(\alpha_c^\star) + \sum_{\hat{c}}\sum_j \alpha^\star_{c,\hat{c}j}[\log P_{\hat{c}j} + \log(\tau_{c,\hat{c}j})]\right] + \right. \\ \left. \frac{1}{1+\psi}(\log \mu_{ci} + \log(\omega_{ci}))\right] + (1-\mu_{ci})\sum_{\hat{c}}\sum_j \eta_{ci,\hat{c}j}[\log P_{\hat{c}j} + \log(\tau_{c,\hat{c}j})]\end{aligned} \tag{42}$$

Turning in matrix notation:

$$\begin{aligned}\log P[I - \frac{\psi}{1+\psi}\mu\alpha^\star - (I-\mu)\eta] = \\ -\epsilon - A(\eta_i^\star) + \frac{\psi}{1+\psi}\mu A(\alpha_c^\star) + \text{diag}\left[\left(\frac{\psi}{1+\psi}\mu\alpha^\star + (I-\mu)\eta\right)\log(\tau)\right] + \frac{1}{1+\psi}(\log\mu + \log(\omega))\end{aligned} \tag{43}$$

Using the definition of the Leontief Inverse:

$$\log P = \mathcal{L}(\alpha^\star)\left(-\epsilon - A(\eta_i^\star) + \frac{\psi}{1+\psi}\mu A(\alpha_c^\star) + \text{diag}\left[\left(\frac{\psi}{1+\psi}\mu\alpha^\star + (I-\mu)\eta\right)\log(\tau)\right] + \frac{1}{1+\psi}(\log\mu + \log(\omega)\right. \tag{44}$$

So, applying the definition in the lemma:

$$\boldsymbol{p} = -\mathcal{L}\,\epsilon + \overline{p}(s) \tag{45}$$

□

B.3 Proof of Lemma 6

Proof. The proof starts from the definition of VA:

$$VA = e^{\epsilon A(\eta_{ci})} L_{ci}^{\mu_{ci}} \tag{46}$$

Taking logs lead to the first part of the Lemma. Also from the proof of Lemma2 we have:

$$l_{ci} = \frac{\psi}{1+\psi}[-p_c + \log\mu_{ci} + \log(\omega_{ci})] \tag{47}$$

Let us denote by a star the prices at the initial equilibrium. The real GDP can be expressed as:

$$g_{ci} = \frac{d\,P_{ci}^\star Y_{ci}}{VA_{ci}} - \sum_{\hat{c}}\sum_{k}\frac{d\,P_{c,\hat{c}k}^\star X_{ci,\hat{c}k}}{VA_{ci}} \tag{48}$$

$$= \frac{d\,P_{ci}^\star Y_{ci}}{P_{ci}^\star Y_{ci}}\frac{P_{ci}^\star Y_{ci}}{VA_{ci}}\frac{\mu}{\mu} - \sum_{\hat{c}}\sum_{k}\frac{d\,P_{c,\hat{c}k}^\star X_{ci,\hat{c}k}}{P_{c,\hat{c}k}^\star X_{ci,\hat{c}k}}\frac{P_{c,\hat{c}k}^\star X_{ci,\hat{c}k}}{VA_{ci}} \tag{49}$$

$$= \frac{1}{\mu}\left(y_{ci} - \sum_{\hat{c}}\sum_{k}\frac{P_{c,\hat{c}k}^\star X_{ci,\hat{c}k}}{VA_{ci}}\,x_{ci,\hat{c}k}\right) \tag{50}$$

I used $VA_{ci} = \mu_{ci}P_{ci}Y_{ci}$. Then using the production function of sector i, we have:

$$g_{ci} = \frac{1}{\mu_{ci}} \left(\epsilon_{ci} + \mu_{ci} l_{ci} + (1 - \mu_{ci}) x_{ci} - \frac{p_{x,ci} X_{ci}}{p_{ci} y_{ci}} \, x_{ci} \right) \tag{51}$$

$$= \frac{1}{\mu_{ci}} \left(\epsilon_{ci} + \mu_{ci} l_{ci} \right) \tag{52}$$

Here, the last line uses the FOC of the firm. Now, from (47) and Lemma 2 we get:

$$L = \frac{\psi}{1 + \psi} \hat{\Omega} \, \mathcal{L} \, \epsilon \tag{53}$$

Because the rows of $\hat{\Omega}$ are the same for sectors in the same country, $L_i = L_j$ for any $i, j \in c$. Replace it in (52) to get:

$$g_{ci} = \frac{1}{\mu_{ci}} \left(\epsilon_{ci} \right) + l_{ci} \tag{54}$$

$$= \frac{1}{\mu_{ci}} \epsilon_{ci} + \frac{\psi}{1 + \psi} \sum_{c'} \sum_{j} \alpha^{\star}_{c,c'j} \left(\sum_{\hat{c}} \sum_{k} \mathcal{L}_{ik}(\boldsymbol{\alpha}^{\star}) \, \epsilon_{\hat{c}k} \right) \tag{55}$$

□

B.4 Proof of Corollary 4

Proof. The result follows directly from taking derivatives of:

$$\mathbf{E}[p_{ci}] = \mathcal{L} \left(B(s) - \theta \right) \tag{56}$$

$$\mathbf{V}[p_{ci}] = \left(\mathcal{L} \right)^{\mathrm{T}} \Sigma \left(\mathcal{L} \right) \tag{57}$$

□

B.5 Proof of Lemma 9

Proof. Let us denote by a star the prices at the initial equilibrium. The real GDP can be expressed as:

$$g_c = \frac{dG_c}{G_c} \tag{58}$$

$$d\log(GDP_c) = \sum_{i \in N_c} \frac{d\, p_i^\star y_i}{G_c} - \frac{d\, p_{xi}^\star x_i}{G_c} \tag{59}$$

$$= \sum_{i \in N_c} \frac{d\, p_i^\star y_i}{p_i^\star y_i} \frac{p_i^\star y_i}{G_c} \frac{\mu_{ci}}{\mu_{ci}} - \frac{d\, p_{xi}^\star x_i}{p_{xi}^\star x_i} \frac{p_{xi}^\star x_i}{G_c} \tag{60}$$

$$= \sum_{i \in N_c} \frac{s_{ci}}{\mu_{ci}} \left(d\log(y_i) - \frac{p_{xi}^\star x_i}{p_i^\star y_i}\, d\log(x_i) \right) \tag{61}$$

In the last line I multiplied and divided by $p_i y_i$ the last term. Also, I use $G_c = \mu_{ci} \sum P_{ci} Y_{ci}$. Then using the production function of sector i, we have:

$$G_c = \sum_{i \in N_c} s_{ci} \left(\epsilon_i + \mu L_i + (1-\mu)\log(x_i) - \frac{p_{xi} x_i}{p_i y_i}\, d\log(x_i) \right) \tag{62}$$

$$= \sum_{i \in N_c} s_{ci} \left(\epsilon_i + \mu L_i \right) \tag{63}$$

Here, the last line uses the FOC of the firm. Now, from (47) and Lemma 2 we get:

$$L = \frac{\psi}{1+\psi} \alpha^\star \mathcal{L}\, \epsilon \tag{64}$$

Because the rows of $\hat{\Omega}$ are the same for sectors in the same country, $L_i = L_j$ for any $i, j \in c$. Replace it in (63) to get:

$$G_c = \sum_{i \in N_c} \frac{s_{ci}}{\mu} (\epsilon_i) + L_c \tag{65}$$

$$= \sum_{i \in N_c} \frac{s_{ci}}{\mu} (\epsilon_i) + \frac{\psi}{1+\psi} \sum_{i \in N} \sum_{k \in N} \alpha_{ci} \, \lambda_{ik} \, \epsilon_k \tag{66}$$

□

B.6 Proof of Proposition 1

Proof. The proof has three steps:

Step 1: Firms Objective Starting from equation (6). For any input-bundle shares, the problem is the following.

$$\max \quad E\left[\Lambda_c \, Y \left(P_{\text{rev}} - K_{\text{cost}}\right)\right],$$

with $(P_{\text{rev}}, K_{\text{cost}}) = (P_c, K_c(\alpha, P))$ for the finals and $(P_{\text{rev}}, K_{\text{cost}}) = (P_{ci}, K_{ci}(\eta, P))$ for the intermediates. Since P_{rev} does not depend on the shares, choosing the shares is equivalent to *minimizing* the discounted expected cost:

$$\min \quad E[\Lambda_c \, Y \, K_{\text{cost}}] \, .$$

Log-moment step. Write $k_{\text{cost}} = \ln K_{\text{cost}}$ and $\lambda_c = \ln \Lambda_c$, $y = \ln Y$. By log-normal identity,

$$\ln E\left[e^{\lambda_c + y + k_{\text{cost}}}\right] \;=\; E[\lambda_c + y + k_{\text{cost}}] \;+\; \tfrac{1}{2} \, \text{Var}(\lambda_c + y + k_{\text{cost}}).$$

Only the part that varies with the shares matters. Using $\frac{1}{2} \, \partial \text{Var}(X) = \text{Cov}(X, \partial X)$ and $\partial k_{\text{cost}} / \partial(\text{share on } (c', k)) = p_{c'k}$, the share-wise index that multiplies each supplier (c', k) is

$$\underbrace{E[p_{c'k}] + \ln \tau_{\bullet, c'k}}_{\text{mean delivered price}} \quad + \quad \underbrace{\text{Cov}\big(\lambda_c + y + k_{\text{cost}}, \; p_{c'k}\big)}_{\text{risk term}},$$

where $\tau_{\bullet, c'k}$ denotes the relevant iceberg factor (final or intermediate).

Final-good problem (to get (23)). Here $y = y_c$ and $k_{\text{cost}} = k_c$. Under equilibrium

conditions for finals,

$$y_c = -\,p_c, \qquad k_c = p_c.$$

Therefore

$$\mathrm{Cov}(\lambda_c + y_c + k_c,\ p_{c'k}) = \mathrm{Cov}(\lambda_c,\ p_{c'k}) + \underbrace{\mathrm{Cov}(-p_c,\ p_{c'k}) + \mathrm{Cov}(p_c,\ p_{c'k})}_{=0}.$$

Hence the per-supplier index for finals is

$$\boxed{E[p_{c'k}] + \ln \tau_{c,c'k} \ +\ \mathrm{Cov}(\lambda_c,\ p_{c'k})}.$$

Intermediate-good problem. For intermediates we must use sectoral (own-good) demand Y_{ci}; then $y = \ln Y_{ci} = y_{ci}$ and $k_{\text{cost}} = k_{ci}$. Under assumptions for intermediates,

$$y_{ci} = -\,p_{ci}, \qquad k_{ci} = p_{ci}.$$

Thus

$$\mathrm{Cov}(\lambda_c + y_{ci} + k_{ci},\ p_{c'k}) = \mathrm{Cov}(\lambda_c,\ p_{c'k}) + \underbrace{\mathrm{Cov}(-p_{ci},\ p_{c'k}) + \mathrm{Cov}(p_{ci},\ p_{c'k})}_{=0}.$$

Hence the per-supplier index for intermediates is

$$\boxed{E[p_{c'k}] + \ln \tau_{ci,c'k} \ +\ \mathrm{Cov}(\lambda_c,\ p_{c'k})}.$$

Substituting the same pricing-kernel mapping for λ_c (exactly as stated in the text immediately before (24)) yields Eq. (24).

Stacking. Summing these per-supplier indexes with weights $\alpha_{c,c'k}$ (finals) and $\eta_{ci,c'k}$ (intermediates) gives the objective indexes used in (23) and (24), respectively.

Step2, Pricing kernel

Household pricing kernel in terms of labor only. From the labor FOC we have

$$z_c = L_c^{1+\frac{1}{\psi}},$$

Since the disutility of labor is $\frac{L_c^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}}$, its derivative is

$$\frac{\mathrm{d}}{\mathrm{d}x}\frac{x^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} = x^{\frac{1}{\psi}} \quad \Longrightarrow \quad z_c = \left(\tfrac{1}{\psi}+1\right)L_c^{1/\psi+1} = \left(\tfrac{1}{\psi}+1\right)\frac{L_c^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}}.$$

Hence

$$z_c - \frac{L_c^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} = \left(\tfrac{1}{\psi}+1\right)\frac{L_c^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} - \frac{L_c^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} = \frac{1}{\psi}\,\frac{L_c^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}}.$$

The stochastic discount factor is

$$\Lambda_c = \frac{u'\left(z_c - \frac{L_c^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}}\right)}{P_c} = \frac{\left(z_c - \frac{L_c^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}}\right)^{-\gamma}}{P_c} = \frac{\left(\frac{1}{\psi}\frac{L_c^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}}\right)^{-\gamma}}{P_c} = \psi^{\gamma}\left(\frac{L_c^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}}\right)^{-\gamma} P_c^{-1}.$$

$$\log\Lambda_c = -\gamma\left[\left(1+\tfrac{1}{\psi}\right)\log L_c - \log\left(1+\tfrac{1}{\psi}\right)\right] + \gamma\log\psi - p_c(s,\epsilon) = -\gamma\left(1+\tfrac{1}{\psi}\right)\log L_c + \gamma\log\psi - \log P_c.$$

Also from lemma 3:

$$\log L_c(s,\epsilon) = -\frac{\psi}{1+\psi}\,p_c(s,\epsilon) + \log\left(\sum_i(\omega_{ci}\mu_{ci})^{\frac{\psi}{1+\psi}}\right).$$

$$\log L_c(s,\epsilon) = -\frac{\psi}{1+\psi}\,p_c(s,\epsilon) + \log\chi_c.$$

Substituting,

$$\begin{aligned}\log\Lambda_c &= -\gamma\left(1+\tfrac{1}{\psi}\right)\left[-\frac{\psi}{1+\psi}\,p_c + \log\left(\sum_i(\mu_{ci}\omega_{ci})^{\frac{\psi}{1+\psi}}\right)\right] + \gamma\log\psi - p_c \\ &= \left[\gamma(1+\tfrac{1}{\psi})\tfrac{\psi}{1+\psi} - 1\right]p_c \;-\; \gamma\left(1+\tfrac{1}{\psi}\right)\log\left(\sum_i(\mu_{ci}\omega_{ci})^{\frac{\psi}{1+\psi}}\right) \;+\; \gamma\log\psi.\end{aligned}$$

Since

$$\gamma\left(1+\tfrac{1}{\psi}\right)\tfrac{\psi}{1+\psi} = \gamma,$$

the coefficient on p_c is $\gamma - 1$. Thus

$$\log\Lambda_c(s,\epsilon) = (\gamma-1)\,p_c(s,\epsilon) + C_c(s),$$

where

$$C_c(s) = -\gamma\left(1 + \tfrac{1}{\psi}\right)\log\left(\sum_i (\mu_{ci}\omega_{ci})^{\frac{\psi}{1+\psi}}\right) + \gamma\log\psi$$

depends only on the network s (through ω_{ci}) and parameters, but not on shocks.

Step 3: Combining both For final-good producers in country c, the (log) cost index is

$$k_c(\alpha_c;\alpha^\star) = -A_c(\alpha_c) + \sum_{\hat{c},k} \alpha_{c,\hat{c}k}\left(p_{\hat{c}k} + \log\tau_{c,\hat{c}k}\right),$$

while the equilibrium final-good price index satisfies

$$p_c(\alpha^\star) = -A_c(\alpha_c^\star) + \sum_{\hat{c},k} \alpha^\star_{c,\hat{c}k}\left(p_{\hat{c}k} + \log\tau_{c,\hat{c}k}\right).$$

Since $C_c(s)$, $A_c(\cdot)$ and $\log\tau_{c,\hat{c}k}$ are deterministic, they drop out of covariances, and we obtain

$$\begin{aligned}
\operatorname{Cov}\left(\Lambda_c, k_c(\alpha_c;\alpha^\star)\right) &= (\gamma-1)\operatorname{Cov}\left(p_c(\alpha^\star), k_c(\alpha_c;\alpha^\star)\right) \\
&= (\gamma-1)\operatorname{Cov}\left(\sum_{\hat{c}',j}\alpha^\star_{c,\hat{c}'j}p_{\hat{c}'j}, \sum_{\hat{c},k}\alpha_{c,\hat{c}k}p_{\hat{c}k}\right) \\
&= (\gamma-1)\sum_{\hat{c},k}\sum_{\hat{c}',j}\alpha_{c,\hat{c}k}\,\alpha^\star_{c,\hat{c}'j}\operatorname{Cov}\left(p_{\hat{c}k}, p_{\hat{c}'j}\right).
\end{aligned}$$

The final-good producer in country c chooses α_c to maximize expected (log) cost adjusted for risk:

$$\max_{\alpha_c\in\mathcal{A}} \mathbb{E}\left[k_c(\alpha_c;\alpha^\star)\right] + \operatorname{Cov}\left(\Lambda_c, k_c(\alpha_c;\alpha^\star)\right).$$

Substituting the expressions above and rearranging,

$$\begin{aligned}
&\mathbb{E}\left[k_c(\alpha_c;s)\right] + \operatorname{Cov}\left(\Lambda_c, k_c(\alpha_c;s)\right) \\
&= -A_c(\alpha_c) + \sum_{\hat{c},k}\alpha_{c,\hat{c}k}\left(\mathbb{E}[p_{\hat{c}k}] + \log\tau_{c,\hat{c}k} + (\gamma-1)\sum_{\hat{c}',j}\alpha^\star_{c,\hat{c}'j}\operatorname{Cov}\left(p_{\hat{c}k}, p_{\hat{c}'j}\right)\right).
\end{aligned}$$

Defining the *risk-adjusted delivered price* as

$$\mathcal{R}_{c,\hat{c}k}(s) := \mathbb{E}[p_{\hat{c}k}] + \log \tau_{c,\hat{c}k} + (\gamma - 1) \sum_{\hat{c}',j} \alpha^{\star}_{c,\hat{c}'j} \operatorname{Cov}\big(p_{\hat{c}k}, p_{\hat{c}'j}\big),$$

and using $A_c(\alpha_c) = \sum_{\hat{c},k} \kappa_{\hat{c}k}(\alpha_{c,\hat{c}k} - \alpha^0_{\hat{c}k})^2$, the problem of the final-good producer is

$$\alpha^{\star}_c \in \arg\max_{\alpha_c \in \mathcal{A}} \; - \sum_{\hat{c},k} \kappa_{\hat{c}k} \big(\alpha_{c,\hat{c}k} - \alpha^0_{\hat{c}k}\big)^2 \; - \sum_{\hat{c},k} \alpha_{c,\hat{c}k}\, \mathcal{R}^F_{c,\hat{c}k}(s).$$

For an intermediate producer in sector (c, i), the (log) cost index is

$$k_{ci}(\eta_{ci}; s) = -A_{ci}(\eta_{ci}) + \sum_{\hat{c},k} \eta_{ci,\hat{c}k} \big(p_{\hat{c}k} + \log \tau_{ci,\hat{c}k}\big),$$

while the household SDF for country c is the same Λ_c as above. Using again that constants drop from covariances, we obtain

$$\begin{aligned} \operatorname{Cov}\big(\Lambda_c, k_{ci}(\eta_{ci}; s)\big) &= (\gamma - 1) \operatorname{Cov}\Big(p_c(\alpha^{\star}), \sum_{\hat{c},k} \eta_{ci,\hat{c}k} p_{\hat{c}k}\Big) \\ &= (\gamma - 1) \sum_{\hat{c},k} \sum_{\hat{c}',j} \eta_{ci,\hat{c}k}\, \alpha^{\star}_{c,\hat{c}'j} \operatorname{Cov}\big(p_{\hat{c}k}, p_{\hat{c}'j}\big). \end{aligned}$$

Thus the intermediate producer solves

$$\max_{\eta_{ci} \in \mathcal{E}} \; \mathbb{E}\big[k_{ci}(\eta_{ci}; s)\big] \; + \; \operatorname{Cov}\big(\Lambda_c, k_{ci}(\eta_{ci}; \eta^{\star})\big),$$

that is,

$$- A_{ci}(\eta_{ci}) + \sum_{\hat{c},k} \eta_{ci,\hat{c}k} \Big(\mathbb{E}[p_{\hat{c}k}] + \log \tau_{ci,\hat{c}k} + (\gamma - 1) \sum_{\hat{c}',j} \alpha^{\star}_{c,\hat{c}'j} \operatorname{Cov}\big(p_{\hat{c}k}, p_{\hat{c}'j}\big)\Big).$$

Replacing the risk-adjusted prices and writing $A_{ci}(\eta_{ci}) = \sum_{\hat{c},k} \kappa_{\hat{c}k}(\eta_{ci,\hat{c}k} - \eta^0_{ci,\hat{c}k})^2$, the problem of the intermediate producer is

$$\eta^{\star}_{ci} \in \arg\max_{\eta_{ci} \in \mathcal{E}} \; - \sum_{\hat{c},k} \kappa_{\hat{c}k} \big(\eta_{ci,\hat{c}k} - \eta^0_{ci,\hat{c}k}\big)^2 \; - \sum_{\hat{c},k} \eta_{ci,\hat{c}k}\, \mathcal{R}_{ci,\hat{c}k}(s).$$

Which closes the proof

□

B.7 Proof of Lemma 5

We consider the problem of finding

$$\alpha_c^\star \in \arg\max_{\alpha_c \in \mathcal{A}_c} \left\{ a_c(\alpha_c) - \sum_i \alpha_{ci}\, \mathcal{R}_i \right\},$$

where

$$a_c(\alpha_c) = -\sum_i \kappa_{ci} \left(\alpha_{ci} - \alpha_{ci}^0\right)^2.$$

In addition, we impose the normalization constraint

$$\sum_i \alpha_{ci} = 1,$$

and the non-negativity constraint

$$\alpha_{ci} \geq 0 \quad \forall\, i.$$

We assume that

$$\sum_i \alpha_{ci}^0 = 1.$$

Step 1: Forming the Lagrangian We introduce a Lagrange multiplier λ for the equality constraint and additional multipliers $\mu_i \geq 0$ for the inequality constraints. The Lagrangian is given by

$$\mathcal{L}(\alpha_c, \lambda, \mu) = -\sum_i \kappa_{ci} \left(\alpha_{ci} - \alpha_{ci}^0\right)^2 - \sum_i \alpha_{ci}\, \mathcal{R}_i + \lambda \left(\sum_i \alpha_{ci} - 1 \right) - \sum_i \mu_i\, \alpha_{ci}.$$

Step 2: First-Order Conditions Differentiate $\mathcal{L}$ with respect to each α_{ci}:

$$\frac{\partial \mathcal{L}}{\partial \alpha_{ci}} = -2\kappa_{ci} \left(\alpha_{ci} - \alpha_{ci}^0\right) - \mathcal{R}_i + \lambda - \mu_i = 0.$$

This yields

$$-2\kappa_{ci} \left(\alpha_{ci} - \alpha_{ci}^0\right) = \mathcal{R}_i - \lambda + \mu_i,$$

or, equivalently,

$$\alpha_{ci} = \alpha_{ci}^0 + \frac{1}{2\kappa_{ci}} \left(\lambda - \mathcal{R}_i - \mu_i\right).$$

Step 3: Complementary Slackness The Karush-Kuhn-Tucker (KKT) conditions impose that for each i

$$\mu_i \geq 0, \quad \alpha_{ci} \geq 0, \quad \mu_i \, \alpha_{ci} = 0.$$

Thus, for any index i where $\alpha_{ci} > 0$ (the active set I), we have $\mu_i = 0$ and

$$\alpha_{ci} = \alpha_{ci}^0 + \frac{1}{2\kappa_{ci}} \left(\lambda - \mathcal{R}_i\right).$$

For indices where the above expression would yield $\alpha_{ci} < 0$, the optimal solution sets $\alpha_{ci} = 0$ and the corresponding $\mu_i > 0$.

Step 4: Enforcing the Constraint Let I denote the active set of indices for which $\alpha_{ci} > 0$. Then the normalization constraint becomes

$$\sum_{i \in I} \left[\alpha_{ci}^0 + \frac{1}{2\kappa_{ci}} \left(\lambda - \mathcal{R}_i\right)\right] = 1.$$

Expanding the sum, we have

$$\sum_{i \in I} \alpha_{ci}^0 + \lambda \sum_{i \in I} \frac{1}{2\kappa_{ci}} - \sum_{i \in I} \frac{\mathcal{R}_i}{2\kappa_{ci}} = 1.$$

Since $\sum_i \alpha_{ci}^0 = 1$, if all indices are active then $\sum_{i \in I} \alpha_{ci}^0 = 1$ and we obtain

$$1 + \lambda \sum_i \frac{1}{2\kappa_{ci}} - \sum_i \frac{\mathcal{R}_i}{2\kappa_{ci}} = 1.$$

Thus,

$$\lambda \sum_i \frac{1}{2\kappa_{ci}} = \sum_i \frac{\mathcal{R}_i}{2\kappa_{ci}},$$

so that

$$\boxed{\lambda = \sum_i \mathcal{R}_i}$$

In the general case, where only a subset I is active, the normalization condition becomes

$$\lambda \sum_{i \in I} \frac{1}{2\kappa_{ci}} = 1 - \sum_{i \in I} \alpha_{ci}^0 + \sum_{i \in I} \frac{\mathcal{R}_i}{2\kappa_{ci}}.$$

Thus, the formula for λ is

$$\boxed{\lambda = 1 - \sum_{i \in I} \alpha_{ci}^0 + \sum_{i \in I} \frac{\mathcal{R}_i}{2\kappa_{ci}}}$$

Since $\sum_i \alpha_{ci}^0 = 1$, if all indices are active (I is the full index set) then the numerator simplifies to $\sum_i \frac{\mathcal{R}_i}{2\kappa_{ci}}$, recovering the previous expression.

B.8 Proof of Lemma 7

From the household first-order condition we have

$$\frac{L_c^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} = \frac{\psi}{1+\psi} Z_c,$$

so that net consumption is a fixed fraction of Z_c:

$$Z_c - \frac{L_c^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} = Z_c - \frac{L_c^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} = Z_c - \frac{\psi}{1+\psi} Z_c = \frac{1}{1+\psi} Z_c.$$

With CRRA utility,

$$U(Z_c - \frac{L_c^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}}) = \frac{(Z_c - \frac{L_c^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}})^{1-\gamma}}{1-\gamma} = \frac{1}{1-\gamma} \left(\frac{Z_c}{1+\psi} \right)^{1-\gamma}.$$

Using the relation between consumption and labor,

$$Z_c = \left(1 + \frac{1}{\psi}\right) L_c^{1+\frac{1}{\psi}},$$

and the labor-supply expression from Lemma 6,

$$\log L_c(s,\epsilon) = -\frac{\psi}{1+\psi} p_c(s,\epsilon) + \log \left(\sum_i (\omega_{ci}\mu_{ci})^{\frac{\psi}{1+\psi}} \right),$$

we obtain

$$\begin{aligned}\log Z_c(s,\epsilon) &= \log\Big(1+\tfrac{1}{\psi}\Big) + \Big(1+\tfrac{1}{\psi}\Big)\log L_c(s,\epsilon)\\ &= \log\Big(1+\tfrac{1}{\psi}\Big) + \Big(1+\tfrac{1}{\psi}\Big)\left[-\frac{\psi}{1+\psi}\,p_c(s,\epsilon) + \log\left(\sum_i (\omega_{ci}\mu_{ci})^{\frac{\psi}{1+\psi}}\right)\right]\\ &= -\,p_c(s,\epsilon) + \log\Big(1+\tfrac{1}{\psi}\Big) + \Big(1+\tfrac{1}{\psi}\Big)\log\left(\sum_i (\omega_{ci}\mu_{ci})^{\frac{\psi}{1+\psi}}\right).\end{aligned}$$

Since $Z_c - \frac{L_c^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} = Z_c/(1+\psi)$, we have

$$\begin{aligned}\log\big(Z_c - \frac{L_c^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}}\big) &= \log Z_c - \log(1+\psi)\\ &= -\,p_c(s,\epsilon) + \log\Big(1+\tfrac{1}{\psi}\Big) + \Big(1+\tfrac{1}{\psi}\Big)\log\left(\sum_i (\omega_{ci}\mu_{ci})^{\frac{\psi}{1+\psi}}\right) - \log(1+\psi).\end{aligned}$$

Assume $\log(Z_c - \frac{L_c^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}})$ is (approximately) normal with mean $\mathbb{E}[\log(Z_c - \frac{L_c^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}})]$ and variance $\mathrm{Var}(\log(Z_c - \frac{L_c^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}}))$. By the lognormal identity,

$$\log\mathbb{E}\big[U(Z_c - \frac{L_c^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}})\big] = \log\Big(\tfrac{1}{1-\gamma}\Big) + (1-\gamma)\,\mathbb{E}\big[\log(Z_c - \frac{L_c^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}})\big] + \frac{1}{2}(1-\gamma)^2\,\mathrm{Var}\big(\log(Z_c - \frac{L_c^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}})\big).$$

From the expression above,

$$\mathbb{E}\big[\log(Z_c - \frac{L_c^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}})\big] = -\,\mathbb{E}[p_c(s,\epsilon)] + \log\Big(1+\tfrac{1}{\psi}\Big) + \Big(1+\tfrac{1}{\psi}\Big)\log\left(\sum_i (\omega_{ci}\mu_{ci})^{\frac{\psi}{1+\psi}}\right) - \log(1+\psi),$$

and, since the remaining terms are deterministic,

$$\mathrm{Var}\big(\log(Z_c - \frac{L_c^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}})\big) = \mathrm{Var}\big(p_c(s,\epsilon)\big).$$

Putting these together,

$$\log \mathbb{E}\Big[U(Z_c - \frac{L_c^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}})\Big] = \underbrace{\log\Big(\tfrac{1}{1-\gamma}\Big) + (1-\gamma)\left[\log\Big(1+\tfrac{1}{\psi}\Big) - \log(1+\psi)\right]}_{=:\,c} + \Big(1+\tfrac{1}{\psi}\Big)\log\left(\sum_i (\omega_{ci}\mu_{ci})^{\frac{\psi}{1+\psi}}\right) +$$
$$- (1-\gamma)\,\mathbb{E}[p_c(s,\epsilon)] + \frac{1}{2}(1-\gamma)^2\ \mathrm{Var}\big(p_c(s,\epsilon)\big).$$

Thus welfare for country c can be written as

$$\log \mathbb{E}\Big[U(Z_c - \frac{L_c^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}})\Big] = C_c(\mu,\psi)\ -\ (1-\gamma)\,\mathbb{E}[p_c(s,\epsilon)]\ +\ \frac{1}{2}(1-\gamma)^2\ \mathrm{Var}\big(p_c(s,\epsilon)\big),$$

where $C_c(\mu,\psi)$ collects all terms that depend on the labor-share parameters μ_{ci}, Domar weights ω_{ci}, and the Frisch elasticity ψ, but not on the realization of shocks.

C Example Economy

This appendix derives the effective elasticity of substitution implied by the network problem and compares it to CES, using the two-country, two-sector economy of Section 4.

Setup

There are four country-sector pairs $\{HM, HS, FM, FS\}$. Foreign manufacturing is the star sector with productivity parameter $\theta_{FM} > \theta_{HM} = \theta_{HS} = \theta_{FS}$; granular variances are equal, $\sigma^2_{u_{ci}} = \sigma^2_u$ for all (c, i). We focus on the Home final-good producer allocating shares $(\eta_{H,FM}, \eta_{H,HM})$ between FM and HM, suppressing the shares on HS and FS for brevity (the constraint $\sum_{\hat{c}k} \eta_{H,\hat{c}k} = 1$ is maintained throughout).

Definition of the Elasticity

Let $q_{ci} \equiv e^{R_{ci}}$ be the exponentiated risk-adjusted price of supplier (c, i). The Hicks–Allen elasticity of substitution between FM and HM from the perspective of the Home producer is

$$\sigma_{FM,HM} \;\equiv\; -\frac{d\log(\eta_{H,FM}/\eta_{H,HM})}{d\log(q_{H,FM}/q_{H,HM})} \;=\; -\frac{d\log(\eta_{H,FM}/\eta_{H,HM})}{d\,\Delta R_H}, \tag{67}$$

where $\Delta R_H \equiv R_{H,FM} - R_{H,HM}$.

Risk-Adjusted Prices in the Example

By Proposition 1, the risk-adjusted price of supplier $\hat{c}k$ for the Home final-good producer is

$$R_{H,\hat{c}k} \;=\; E[p_{\hat{c}k}] + \log \tau_{H,\hat{c}k} + (\gamma - 1)\,\mathrm{Cov}(p_H,\, p_{\hat{c}k}).$$

The price of each sector log-linearizes as $p_{\hat{c}k} = -\theta_{\hat{c}k} + v_{\hat{c}k}\, g + u_{\hat{c}k}$, where $g \sim N(0, \sigma^2_g)$ is the global shock and $u_{\hat{c}k}$ is the granular shock. Hence

$$E[p_{\hat{c}k}] = -\theta_{\hat{c}k}, \qquad \mathrm{Cov}(p_H,\, p_{\hat{c}k}) = v_H\, v_{\hat{c}k}\, \sigma^2_g + \sigma^2_u\, \mathbf{1}\{(\hat{c}k) = (H, \cdot)\},$$

where $\nu_H = \sum_{\hat{c}k} \alpha^{\star}_{H,\hat{c}k} \nu_{\hat{c}k}$ is the exposure of the Home country. The relative risk-adjusted price becomes

$$\Delta R_H = \underbrace{(\theta_{HM} - \theta_{FM})}_{\text{productivity gap } <0} + \underbrace{\log \tau_{H,FM}}_{\text{trade cost}} + \underbrace{(\gamma - 1)\nu_H(\nu_{FM} - \nu_{HM})\sigma_g^2}_{\text{risk premium}}. \tag{68}$$

Since $\theta_{FM} > \theta_{HM}$, the productivity gap is negative (FM is attractive on cost grounds). Since FM is the star sector, $\nu_{FM} > \nu_{HM}$, so the risk premium is positive for $\gamma > 1$: FM's prices covary more with the domestic price index, making it costly to insure with. Trade costs $\log \tau_{H,FM}$ shift ΔR_H upward, working against concentration in FM.

Optimal Shares

By Lemma 5, the interior solution (assuming both suppliers are active) satisfies

$$\eta^{\star}_{H,FM} = \eta^{0}_{H,FM} - \frac{1}{4\kappa^I}\Delta R_H, \qquad \eta^{\star}_{H,HM} = \eta^{0}_{H,HM} + \frac{1}{4\kappa^I}\Delta R_H, \tag{69}$$

where $\eta^{0}_{H,FM} + \eta^{0}_{H,HM} = 1$ (normalizing the two-supplier subproblem). The share of FM is decreasing in ΔR_H, which in turn is increasing in both σ_g^2 and $\tau_{H,FM}$. Comparative statics follow directly:

$$\frac{\partial \eta^{\star}_{H,FM}}{\partial \sigma_g^2} = -\frac{(\gamma - 1)\nu_H(\nu_{FM} - \nu_{HM})}{4\kappa^I} < 0, \qquad \frac{\partial \eta^{\star}_{H,FM}}{\partial \log \tau_{H,FM}} = -\frac{1}{4\kappa^I} < 0.$$

Both higher aggregate risk and higher trade costs reduce the share of the star sector.

The Effective Elasticity of Substitution

Differentiating $\log(\eta_{H,FM}/\eta_{H,HM})$ with respect to ΔR_H,

$$\frac{d}{d\Delta R_H} \log\left(\frac{\eta_{H,FM}}{\eta_{H,HM}}\right) = \frac{-4\kappa^I}{(4\kappa^I \eta^{0}_{H,FM} - \Delta R_H)(\Delta R_H + 4\kappa^I \eta^{0}_{H,HM})}. \tag{70}$$

Using (67), the effective elasticity is

$$\boxed{\sigma^{\text{net}}_{FM,HM}(\Delta R_H) = \frac{4\kappa^I}{(4\kappa^I \eta^{0}_{H,FM} - \Delta R_H)(4\kappa^I \eta^{0}_{H,HM} + \Delta R_H)}} \tag{71}$$

This elasticity has three notable properties.

State-dependence. σ^{net} depends on ΔR_H and hence on σ_g^2 and $\tau_{H,FM}$. At the baseline portfolio ($\Delta R_H = 0$),

$$\sigma^{\text{net}}(0) = \frac{1}{4\kappa^I \, \eta^0_{H,FM} \, \eta^0_{H,HM}},$$

which is highest when the portfolio is balanced ($\eta^0 = 0.5$) and decreasing in κ^I.

Collapse at corners. As $\Delta R_H \to 4\kappa^I \eta^0_{H,FM}$ (the Home producer concentrates fully in HM) or $\Delta R_H \to -4\kappa^I \eta^0_{H,HM}$ (full concentration in FM), the denominator approaches zero from above and $\sigma^{\text{net}} \to +\infty$ on approach but $s_{FM} \to 0$ or $s_{FM} \to 1$, at which point the KKT conditions bind and further substitution is zero. In practice, the effective elasticity falls toward zero once concentration is high: small changes in R produce no further reallocation.

Asymmetry between risk and trade liberalization. From (68), a rise in σ_g^2 and a fall in $\tau_{H,FM}$ both shift ΔR_H but in *opposite* directions, starting from the same equilibrium. When risk rises (crisis), the network moves away from FM and the elasticity evaluated at the new ΔR_H is higher — producers are still near the interior of the portfolio simplex and respond elastically. When τ falls (trade liberalization), the network concentrates in FM and the elasticity evaluated at the new ΔR_H is lower — further reductions in τ produce diminishing reallocation. A CES aggregator with a constant elasticity σ cannot capture this asymmetry: it implies the same marginal response to risk as to trade costs, missing both the rapid flight to safety in crises and the self-limiting nature of concentration under liberalization.

Comparison with CES

With a CES aggregator of elasticity σ and delivered prices $P_{\hat{c}k} = \tau_{H,\hat{c}k} \, e^{p_{\hat{c}k}}$, expenditure shares satisfy

$$\log\left(\frac{\eta^{\text{CES}}_{H,FM}}{\eta^{\text{CES}}_{H,HM}}\right) = \text{const} + (1-\sigma)\,\Delta R_H,$$

so

$$\sigma^{\text{CES}}_{FM,HM} = \sigma \quad \forall\, \Delta R_H.$$

The CES elasticity is a global constant, independent of the current allocation, aggregate risk, or trade costs. This implies two counterfactual predictions. First, trade liberalization (a fall in $\tau_{H,FM}$) produces a reallocation toward FM that is *proportional to* σ, with no self-limiting dynamics. Second, a rise in σ_g^2 produces an identical elastic reallocation away from FM,

with no acceleration near the baseline. The network model corrects both: concentration in FM under liberalization is self-limiting because the effective elasticity falls as $\eta_{H,FM}$ rises, while the flight to safety under risk is more elastic precisely because the portfolio has not yet concentrated.

D Existence and Uniqueness

This appendix provides detailed derivations for existence and uniqueness of the equilibrium. The strategy is to cast the equilibrium as a static game (MFG) to analyze existence and uniqueness of equilibrium.

Assumptions Let us begin by stating the assumptions we require in the proof

Assumption 1 (Assumption)**.**

1. *The strategy set $\mathcal{A}$ is the unit simplex, and the productivity function $A_{ci,t}$ is bounded below.*
2. *Input shares satisfy $1 > \mu_{ci,t} > 0$ for all country-industries ci and periods t.*
3. ***Curvature:** $\kappa_k^I, \kappa_c^F > 0$*
4. ***Finite frictions:** $\tau_{b,k,t} \geq 1$ for all bilateral pairs (b,k) and periods t.*
5. *There exists $\bar{A} > -\infty$ such that, for all feasible α_c, $A(\alpha_c) \geq \bar{A}$*

It is important to highlight that no new assumptions are introduced here. This statement serves merely as a summary of previously established assumptions needed for this proof.

Notation for uniform bounds. For any vector x, let $\bar{x} := \|x\|_\infty$ denote its ℓ_∞ norm. Also note:

$$\|x\|_\infty = \max_i |x_i| \leq \max\{|\bar{x}|, |\underline{x}|\},$$

Then, I define

$$\underline{\mu} := \min_k \mu_k, \overline{\mu} := \max_k \mu_k, \overline{\log \tau} := \|\log \tau\|_\infty, \overline{\log \mu} := \|\log \mu\|_\infty, \overline{\log \omega} := \|\log \omega\|_\infty.$$

D.1 Game Formulation

Players, Strategies I formulate the equilibrium as a game. Using the notation of my model, the players are intermediate goods producers $k = 1, \ldots, N$ and final goods producers $c = 1, \ldots, C$. Since there is a representative firm for each sector, each k and each c can be understood as a type of producer. The actions of each producer are the share vectors

$\eta_k \in \mathcal{A}$ (intermediate goods) and $\alpha_c \in \mathcal{A}$ (final goods), respectively. The strategy space is

$$\mathcal{M} := \left(\prod_{k=1}^{N} \mathcal{A}\right) \times \left(\prod_{c=1}^{C} \mathcal{A}\right),$$

Given that $\mathcal{A}$ is the simplex, $\mathcal{M}$ is nonempty, compact, and convex. The state is the matrix of risk-adjusted prices $R(s)$.

Aggregate State For any $s = (\eta, \alpha) \in \mathcal{M}$. As Lemma 2 proves, the Leontief inverse exists up to normalization. That guarantees the function $p(s, \epsilon)$ denote the unique normalized solution to the prices. $E[p(s, \epsilon)]$ and $\mathrm{Cov}[p_i(s, \epsilon), p_i(s, \epsilon)]$ are unique functions. Thus, the aggregate state $R(s)$ from Proposition 1 is a unique function of the aggregate network.

Payoff As shown in Proposition 1, the cost of each type of producer can be defined as:

$$J(c, \alpha_c, s) = -\sum_j \kappa^F (\alpha_{c,j} - \alpha^0_{c,j})^2 - \sum_j \alpha_{c,j} R_{cj}(s),$$

$$J(k, \eta_k, s) = -\sum_j \kappa^I (\eta_{k,j} - \eta^0_{k,j})^2 - \sum_j \eta_{k,j} R_{kj}(s),$$

Best Response Given s, let $\eta^*(s)$ and $\alpha^*(s)$ be the solution of Lemma 5. Then, best-response map is:

$$\Phi : \mathcal{M} \to \mathcal{M}, \qquad \Phi(s) := (\eta^*(s), \alpha^*(s)).$$

D.2 Existence via Fixed Point

We are ready to prove the first claim of the proposition, existence:

Claim (Continuity of Φ). For every $s \in \mathcal{M}$: (i) $R(s)$ is well defined and continuous in s; (ii) Lemma 5 has a unique solution in $R(s)$; (iii) the map $s \mapsto (\eta^*(s), \alpha^*(s))$ is continuous. Hence Φ is a continuous self-map of $\mathcal{M}$.

> *Proof.* (i) By Lemma 1 the normalized solution $p(s, \epsilon)$ exists, is unique, and continuously depends on s. Then $R(s)$ is also unique and continuous. (ii) The quadratic penalty makes each objective strongly convex on the compact convex constraint set, so a unique minimizer exists. (iii) The continuity of $R(s)$ and strong convexity of the cost J yield the

continuity of the best responses according to Berge's maximum theorem. Therefore, Φ is continuous and maps $\mathcal{M}$ to itself. □

Claim (Existence via fixed point). There exists a competitive equilibrium $(s^\star, R(s^\star))$ with $s^\star = (\eta^*, \alpha^*) \in \mathcal{M}$.

Proof. By Claim 1, $\Phi : \mathcal{M} \to \mathcal{M}$ is a continuous self-map of a compact convex non-empty set. By Brouwer's fixed point theorem, there exists $s^* = (\eta^*, \alpha^*) \in \mathcal{M}$ with $\Phi(s^*) = s^*$. Given s^*, solve Lemma 2 to obtain the unique normalized price vector $p(s^\star, \epsilon)$. By construction, each η_k^* and α_c^* optimize their respective problems at $R(s^*)$ and prices satisfy unit costs; hence $(s^*, p(s^*, \epsilon))$ is a competitive equilibrium. □

D.3 GE Effects

To prove uniqueness, we show that the best response map Φ is a contraction in the space of networks. Here, prices, covariances and the risk adjusted costs depend on the whole network. Instead of relying on monotonicity, I bound how much mean prices and covariances can change when the network moves. The maximum change in expected prices and covariances are defined as:

$$L_{pp} := \sup_{s_1 \neq s_2} \frac{\| \operatorname{Cov}(p(s_1), p(s_1)) - \operatorname{Cov}(p(s_2), p(s_2)) \|_\infty}{\|s_1 - s_2\|_\infty} \tag{72}$$

$$L_p := \sup_{s_1 \neq s_2} \frac{\left\| \mathbb{E}[p(s_1)] - \mathbb{E}[p(s_2)] \right\|_\infty}{\|s_1 - s_2\|_\infty}, \qquad s = (\alpha, \eta). \tag{73}$$

Bound on the Change of Price covariance. Recall from Corollary 1.

$$\operatorname{Cov}(p(s_1), p(s_1)) = \mathcal{L} \Sigma_\epsilon \mathcal{L}^\top \tag{74}$$

Claim. There exists a finite constant L_p such that (72) holds. Moreover, one can choose

$$L_{pp} \;\leq\; \frac{2\,\bar{\sigma}^2 (1+\psi)^3}{\underline{\mu}^3} \sqrt{(1-\underline{\mu})^2 + \left(\tfrac{\psi}{1+\psi}\,\overline{\mu}\right)^2} \tag{75}$$

Proof. Let $s_1 = (\alpha_1, \eta_1)$ and $s_2 = (\alpha_2, \eta_2)$ be two feasible networks, and denote

$$\Delta s := s_1 - s_2, \quad \Delta \mathcal{L} := \mathcal{L}(s_1) - \mathcal{L}(s_2), \quad \Delta B := B(s_1) - B(s_2).$$

Then,

$$\mathrm{Cov}(p(s_1), p(s_1)) - \mathrm{Cov}(p(s_2), p(s_2)) = \mathcal{L}_1 \Sigma_\epsilon (\Delta \mathcal{L})^\top + (\Delta \mathcal{L}) \Sigma_\epsilon \mathcal{L}_1^\top, \tag{76}$$

Step 1: Bound on $\mathcal{L}(s)$ Recall

$$\mathcal{L}(s) = \left(I - (I - \mu)\eta - \tfrac{\psi}{1+\psi} \mu \alpha \right)^{-1}, \qquad s = (\eta, \alpha),$$

and define

$$H(s) := (I - \mu)\eta + \tfrac{\psi}{1+\psi} \mu \alpha, \qquad \mathcal{L}(s) = (I - H(s))^{-1}.$$

The row sum of row k of $H(s)$ is

$$\sum_j H_{kj}(s) = (1 - \mu_k) + \tfrac{\psi}{1+\psi} \mu_k = 1 - \tfrac{\psi}{1+\psi} \mu_k.$$

Hence, using $\mu_k \geq \underline{\mu}$,

$$\|H(s)\|_\infty = \max_k \sum_j |H_{kj}(s)| = \max_k \sum_j H_{kj}(s) \leq 1 - \tfrac{\psi}{1+\psi} \underline{\mu} =: x < 1.$$

By the Neumann series,

$$\mathcal{L}(s) = (I - H(s))^{-1} = \sum_{n=0}^{\infty} H(s)^n,$$

and thus

$$\|\mathcal{L}(s)\|_\infty \leq \sum_{n=0}^{\infty} \|H(s)\|_\infty^n \leq \sum_{n=0}^{\infty} x^n = \frac{1}{1-x} = \frac{1}{\frac{\psi}{1+\psi} \underline{\mu}} = \frac{1+\psi}{\underline{\mu}\, \psi},$$

Step 2: Bounds on $\Delta \mathcal{L}$. Write $H_\ell := H(s_\ell)$ and $\mathcal{L}_\ell := \mathcal{L}(s_\ell)$ for $\ell = 1, 2$. Using the matrix identity

$$A^{-1} - B^{-1} = A^{-1}(B - A)B^{-1},$$

with $A = I - H_1$ and $B = I - H_2$, we obtain

$$\mathcal{L}_1 - \mathcal{L}_2 = (I - H_1)^{-1} - (I - H_2)^{-1} = \mathcal{L}_1 \, (H_1 - H_2) \, \mathcal{L}_2.$$

Taking $\| \cdot \|_\infty$ norms and applying the submultiplicativity of the operator norm,

$$\|\mathcal{L}_1 - \mathcal{L}_2\|_\infty \le \|\mathcal{L}_1\|_\infty \, \|H_1 - H_2\|_\infty \, \|\mathcal{L}_2\|_\infty.$$

By step 1,

$$\|\mathcal{L}_1\|_\infty, \ \|\mathcal{L}_2\|_\infty \le \left(\frac{1 + \psi}{\underline{\mu} \, \psi} \right)^2,$$

so

$$\|\mathcal{L}_1 - \mathcal{L}_2\|_\infty \le \left(\frac{1 + \psi}{\underline{\mu} \, \psi} \right)^2 \|H_1 - H_2\|_\infty. \tag{77}$$

We now bound $\|H_1 - H_2\|_\infty$ in terms of Δs. By definition,

$$H_1 - H_2 = (I - \mu)(\eta_1 - \eta_2) + \tfrac{\psi}{1+\psi} \, \mu(\alpha_1 - \alpha_2) = (I - \mu)\Delta\eta + \tfrac{\psi}{1+\psi} \, \mu \Delta\alpha.$$

For any row k,

$$\sum_j |(H_1 - H_2)_{kj}| \le (1 - \mu_k) \sum_j |\Delta\eta_{kj}| + \tfrac{\psi}{1+\psi} \mu_k \sum_j |\Delta\alpha_{kj}|.$$

Thus,

$$\|H_1 - H_2\|_\infty \le (1 - \underline{\mu}) \, \|\Delta\eta\|_\infty + \tfrac{\psi}{1+\psi} \, \overline{\mu} \, \|\Delta\alpha\|_\infty.$$

If we define

$$\|\Delta s\|_\infty := \max\{\|\Delta\eta\|_\infty, \ \|\Delta\alpha\|_\infty\},$$

then

$$\|H_1 - H_2\|_\infty \le \left[(1 - \underline{\mu}) + \tfrac{\psi}{1+\psi} \, \overline{\mu} \right] \|\Delta s\|_\infty = K_H \, \|\Delta s\|_\infty.$$

Substituting this into (77) yields

$$\|\mathcal{L}(s_1) - \mathcal{L}(s_2)\|_\infty \le \left(\frac{1 + \psi}{\underline{\mu} \, \psi} \right)^2 K_H \, \|\Delta s\|_\infty,$$

Replace into 76 to get the result □

Bound on the Change of Mean Prices Recall from Corollary 1. that expected prices can be written as

$$\mathbb{E}[p(s)] = -\mathcal{L}(s)\,\theta + \mathcal{L}(s)\,B(s). \tag{78}$$

where

$$B(s) = \frac{\psi}{1+\psi}\,\mu\,A(\alpha^\star) + \operatorname{diag}\Big(\tfrac{\psi}{1+\psi}\,[\mu\alpha^\star + (I-\mu)\eta^\star]\Big)\log\tau + \frac{1}{1+\psi}(\log\mu + \log\omega) \tag{79}$$

Claim (Bounded sensitivity of mean prices). There exists a finite constant L_p such that (73) holds. Moreover, one can choose

$$L_p \;\le\; \left(\overline{\theta} + \bar{B}\right)\left(\frac{1+\psi}{\underline{\mu}}\right)^2\sqrt{(1-\underline{\mu})^2 + \left(\tfrac{\psi\,\overline{\mu}}{1+\psi}\right)^2}, \tag{80}$$

where

$$\bar{B} := \frac{\psi}{1+\psi}\,\overline{\mu}\,\bar{A} + \frac{\psi}{1+\psi}\,\log\overline{\tau} + \frac{1}{1+\psi}\big(\log\overline{\mu} + \log\overline{\omega}\big). \tag{81}$$

Proof. Let $s_1 = (\alpha_1, \eta_1)$ and $s_2 = (\alpha_2, \eta_2)$ be two feasible networks, and denote

$$\Delta s := s_1 - s_2, \quad \Delta\mathcal{L} := \mathcal{L}(s_1) - \mathcal{L}(s_2), \quad \Delta B := B(s_1) - B(s_2).$$

From (78),

$$\mathbb{E}[p(s_1)] - \mathbb{E}[p(s_2)] = -\Delta\mathcal{L}\,\theta + \Delta\mathcal{L}\,B(s_2) + \mathcal{L}(s_1)\,\Delta B. \tag{82}$$

Taking $\|\cdot\|_\infty$ norms and using the triangle inequality,

$$\big\|\mathbb{E}[p(s_1)] - \mathbb{E}[p(s_2)]\big\|_\infty \le \|\Delta\mathcal{L}\|_\infty\,\big(\|\theta\|_\infty + \|B(s_2)\|_\infty\big) + \|\mathcal{L}(s_1)\|_\infty\,\|\Delta B\|_\infty. \tag{83}$$

Step 1 and 2: Bounds on $\mathcal{L}(s)$ and $\Delta\mathcal{L}$. Take steps 1 and 2 from the proof of claim D.3

Step 3: uniform bound on $B(s)$. From (79)

$$\|\mu A(\alpha^\star)\|_\infty \le \overline{\mu}\,\|A(\alpha^\star)\|_\infty \le \overline{\mu}\,\bar{A}.$$

The diagonal term is bounded by

$$\left\| \operatorname{diag}\left(\tfrac{\psi}{1+\psi} [\mu\alpha^\star + (I-\mu)\eta^\star] \right) \log \tau \right\|_\infty \le \frac{\psi}{1+\psi} \|\log \tau\|_\infty,$$

since $\mu\alpha^\star + (I-\mu)\eta^\star$ is a vector of shares in $[0,1]$. Finally,

$$\left\| \frac{1}{1+\psi} (\log \mu + \log \omega) \right\|_\infty \le \frac{1}{1+\psi} \left(\|\log \mu\|_\infty + \|\log \omega\|_\infty \right).$$

Combining these, we obtain the uniform bound

$$\|B(s)\|_\infty \le \bar{B} \quad \text{for all } s, \tag{84}$$

with $\bar{B}$ given by (81).

Step 3: bound on ΔB. The mapping $s \mapsto B(s)$ in (79) is affine in (α, η), up to the adjustment term $A(\alpha^\star)$, which is bounded. Hence there exists a finite constant C_B such that

$$\|\Delta B\|_\infty \le C_B \, \|\Delta s\|_\infty, \tag{85}$$

and we can choose the crude bound $C_B \le \bar{B}$.

Step 4: collecting bounds. Substituting (84), and (85) into (83), we obtain

$$\left\| \mathbb{E}[p(s_1)] - \mathbb{E}[p(s_2)] \right\|_\infty \le \left(\frac{1+\psi}{\underline{\mu}} \right)^2 K_H \left(\|\theta\|_\infty + \bar{B} \right) \|\Delta s\|_\infty + \frac{1+\psi}{\underline{\mu}\,\psi} \, C_B \, \|\Delta s\|_\infty.$$

Absorbing the second term into the first by enlarging $\bar{B}$ if necessary, we obtain the simpler bound

$$\left\| \mathbb{E}[p(s_1)] - \mathbb{E}[p(s_2)] \right\|_\infty \le \left(\overline{\theta} + \bar{B} \right) \left(\frac{1+\psi}{\underline{\mu}} \right)^2 \sqrt{(1-\underline{\mu})^2 + \left(\tfrac{\psi \overline{\mu}}{1+\psi} \right)^2},$$

which implies (80) by the definition (73) of L_p. □

D.4 Contraction and Uniqueness

The dependence of risk adjusted costs on the network s is summarized by

$$\|R(s_1) - R(s_2)\|_\infty \;\le\; \Lambda \, \|s_1 - s_2\|_\infty, \qquad \Lambda := |1-\gamma| \, L_{pp} + L_p,$$

for all $s_1, s_2 \in \mathcal{M}$, where $\|\cdot\|_\infty$ denotes the sup-norm and $\|s\|_\infty := \max\{\|\eta\|_\infty, \|\alpha\|_\infty\}$ for $s = (\eta, \alpha)$.

Claim (Uniqueness). If

$$\min\{\kappa^F, \kappa^I\} > \Lambda,$$

then the best response operator Φ is a contraction on $\mathcal{M}$. Hence, $s^* = (\eta^*, \alpha^*)$ and the corresponding normalized price vector p^* are unique.

Proof. Step 1 (Inner problems via Lemma 6). By Lemma 6, for each $m \in \{I, F\}$ and each pair (c, k) the optimal share $s^m_{ck}(s)$ satisfies

$$s^m_{ck}(s) = \max\left\{ s^{0,m}_{ck} - \frac{1}{2\kappa^m}\big(\lambda^m(s) - R_{ck}(s)\big),\; 0 \right\},$$

where $\lambda^m(s)$ is the average of $R_{ck}(s)$ over the active set and is linear in $R(s)$. Define

$$x^m_{ck}(s) := s^{0,m}_{ck} - \frac{1}{2\kappa^m}\big(\lambda^m(s) - R_{ck}(s)\big).$$

Then, for any s_1, s_2,

$$|x^m_{ck}(s_1) - x^m_{ck}(s_2)| \le \frac{1}{2\kappa^m}\Big(|\lambda^m(s_1) - \lambda^m(s_2)| + |R_{ck}(s_1) - R_{ck}(s_2)| \Big) \le \frac{1}{\kappa^m} \, \|R(s_1) - R(s_2)\|_\infty,$$

since λ^m is an average of components of R. The scalar map $z \mapsto \max\{z, 0\}$ is 1-Lipschitz, so

$$|s^m_{ck}(s_1) - s^m_{ck}(s_2)| \le |x^m_{ck}(s_1) - x^m_{ck}(s_2)|.$$

Taking the sup over (c, k) gives

$$\|s^m(s_1) - s^m(s_2)\|_\infty \;\le\; \frac{1}{\kappa^m} \, \|R(s_1) - R(s_2)\|_\infty, \qquad m \in \{I, F\}. \tag{86}$$

Identifying $s^F(s) \equiv \alpha^*(s)$ and $s^I(s) \equiv \eta^*(s)$, (86) becomes

$$\|\alpha^*(s_1)-\alpha^*(s_2)\|_\infty \le \frac{1}{\kappa^F}\,\|R(s_1)-R(s_2)\|_\infty, \qquad \|\eta^*(s_1)-\eta^*(s_2)\|_\infty \le \frac{1}{\kappa^I}\,\|R(s_1)-R(s_2)\|_\infty.$$

Step 2 (Sensitivity of risk-adjusted costs). By the bounds on mean prices and covariances,

$$\|R(s_1) - R(s_2)\|_\infty \;\le\; \Lambda\,\|s_1 - s_2\|_\infty \qquad \text{for all } s_1, s_2 \in \mathcal{M}.$$

Step 3 (Contraction of Φ). The aggregate best response operator is

$$\Phi(s) := (\eta^*(s), \alpha^*(s)).$$

Using the product sup-norm on $\mathcal{M}$ and the bounds above,

$$\begin{aligned}\|\Phi(s_1) - \Phi(s_2)\|_\infty &= \max\left\{\|\eta^*(s_1) - \eta^*(s_2)\|_\infty,\ \|\alpha^*(s_1) - \alpha^*(s_2)\|_\infty\right\} \\ &\le \max\left\{\frac{1}{\kappa^I}, \frac{1}{\kappa^F}\right\}\|R(s_1) - R(s_2)\|_\infty \\ &\le \frac{\Lambda}{\min\{\kappa^F, \kappa^I\}}\,\|s_1 - s_2\|_\infty.\end{aligned}$$

If $\min\{\kappa^F, \kappa^I\} > \Lambda$,

$$\frac{\Lambda}{\min\{\kappa^F, \kappa^I\}} < 1,$$

so Φ is a contraction on the complete metric space $(\mathcal{M}, \|\cdot\|_\infty)$. By Banach's fixed-point theorem, Φ admits a unique fixed point s^*. The corresponding normalized price vector $p^* = p(s^*, \epsilon)$ is unique by Lemma 2, which yields uniqueness of the competitive equilibrium. □

The Planner's Problem

Setup. The planner maximizes:

$$\max_{s \in \mathcal{S}} \mathcal{W}(s) \equiv \sum_c \lambda_c W_c(s) \tag{87}$$

where, from Lemma 7:

$$W_c(s) = b + \left(1 + \tfrac{1}{\psi}\right) \log\left(\sum_i (\omega_{ci}(s)\mu_{ci})^{\frac{\psi}{1+\psi}}\right) - \mathbb{E}\big[p_c(s,\epsilon)\big] + \tfrac{1}{2}(1-\gamma)\,\mathrm{Var}\big(p_c(s,\epsilon)\big) \tag{88}$$

and $\mathcal{S}$ is the set of feasible networks satisfying market clearing and the Domar weight fixed point.

Complete Markets and SDF Equalization. With GHH preferences, the stochastic discount factor of country c is:

$$m_c(s,\epsilon) = -\gamma\, p_c(s,\epsilon) \tag{89}$$

Suppose the planner implements state-contingent lump-sum transfers $T_c(\epsilon)$ across countries. The first-order condition for these transfers at an interior optimum equates Pareto-weighted marginal utility of wealth across all countries in every state:

$$\lambda_c\, P_c(s,\epsilon)^{-\gamma} = \mu(\epsilon) \qquad \forall\, c,\ \forall\, \epsilon \tag{90}$$

where $\mu(\epsilon)$ is the shadow price of world resources. Taking logs:

$$p_c(s,\epsilon) = p^*(s,\epsilon) + \delta_c \qquad \forall\, c,\ \forall\, \epsilon \tag{91}$$

where $\delta_c = \frac{1}{\gamma}\log\lambda_c - \bar{\delta}$ is a country-specific constant absorbed by the transfers, and the world price index is:

$$p^*(s,\epsilon) \equiv \sum_c \lambda_c\, p_c(s,\epsilon) \tag{92}$$

Condition (91) states that with complete markets all countries' price indices move together stochastically, differing only by the constants δ_c that transfers offset. All SDFs are therefore

equalized up to constants:

$$m_c(s,\epsilon) \;=\; -\gamma\, p^*(s,\epsilon) + \text{const}_c \;\equiv\; m^*(s,\epsilon) + \text{const}_c \tag{93}$$

The world SDF $m^* = -\gamma p^*$ is the common pricing kernel used by all households at the planner's optimum.

Proof of Proposition 4. Differentiating $\mathcal{W}(s)$ with respect to $\eta_{ci,\hat{c}k}$:

$$\begin{aligned}\frac{\partial \mathcal{W}}{\partial \eta_{ci,\hat{c}k}} &= -\sum_{c'} \lambda_{c'} \frac{\partial \mathbb{E}[p_{c'}]}{\partial \eta_{ci,\hat{c}k}} + \frac{1-\gamma}{2} \sum_{c'} \lambda_{c'} \frac{\partial \operatorname{Var}(p_{c'})}{\partial \eta_{ci,\hat{c}k}} + \frac{\partial \mathcal{W}_\omega}{\partial \eta_{ci,\hat{c}k}} \\ &= -\frac{\partial \mathbb{E}[p^*]}{\partial \eta_{ci,\hat{c}k}} + (1-\gamma) \sum_{c'} \lambda_{c'} \operatorname{Cov}\left(p_{c'}, \frac{\partial p_{c'}}{\partial \eta_{ci,\hat{c}k}}\right) + \frac{\partial \mathcal{W}_\omega}{\partial \eta_{ci,\hat{c}k}}\end{aligned} \tag{94}$$

where $\mathcal{W}_\omega$ collects the Domar-weight terms. These cancel at the competitive equilibrium by the envelope theorem: the planner's and the firm's assessments of the level effects coincide to first order.

The key step is evaluating the covariance sum. At the planner's optimum, condition (91) holds, so $p_{c'} = p^* + \delta_{c'}$. Since constants do not affect covariances and $\sum_{c'} \lambda_{c'} \frac{\partial p_{c'}}{\partial \eta} = \frac{\partial p^*}{\partial \eta}$ by linearity:

$$\sum_{c'} \lambda_{c'} \operatorname{Cov}\left(p_{c'}, \frac{\partial p_{c'}}{\partial \eta_{ci,\hat{c}k}}\right) \;=\; \operatorname{Cov}\left(p^*, \frac{\partial p^*}{\partial \eta_{ci,\hat{c}k}}\right) \tag{95}$$

Substituting (95) into (94) and setting the FOC to zero:

$$0 \;=\; -\frac{\partial \mathbb{E}[p^*]}{\partial \eta_{ci,\hat{c}k}} + (1-\gamma) \operatorname{Cov}\left(p^*, \frac{\partial p^*}{\partial \eta_{ci,\hat{c}k}}\right) \tag{96}$$

This is precisely the FOC of a competitive firm whose SDF is $m^* = -\gamma p^*$, which gives the risk-adjusted price (16). ■

Relation to Kopytov et al.. In the notation of the present paper, the welfare function of Kopytov et al. (2024) (their equation 26) is:

$$W^K = \underbrace{\boldsymbol{\omega}^\top \bar{\boldsymbol{\epsilon}} \;+\; \boldsymbol{\omega}^\top \mathbf{A}(\boldsymbol{\eta}^\star)}_{\mathbb{E}[y]} \;-\; \frac{1}{2}(\gamma - 1)\, \underbrace{\boldsymbol{\omega}^\top \Sigma_\epsilon\, \boldsymbol{\omega}}_{V[y]} \tag{97}$$

where $\bar{\boldsymbol{\epsilon}}$ is the vector of mean TFP shocks, $\mathbf{A}(\boldsymbol{\eta}^\star)$ collects the network productivity terms, and Σ_ϵ is the shock covariance matrix. Compare this to each country's welfare in the present model:

$$W_c = b + \left(1 + \tfrac{1}{\psi}\right) \log\left(\sum_i (\omega_{ci}\mu_{ci})^{\frac{\psi}{1+\psi}}\right) - \mathbb{E}[p_c] \;-\; \frac{1}{2}(\gamma - 1)\, \boldsymbol{\alpha}_c^{\star\top}\, \mathcal{L}\, \Sigma_\epsilon\, \mathcal{L}^\top\, \boldsymbol{\alpha}_c^\star \tag{98}$$

Both are mean-variance problems with variance as a quadratic form in Σ_ϵ weighted by endogenous network objects. The two expressions are identical in the single-country, inelastic-labor limit ($C = 1$, $\psi \to 0$). At that limit the Domar weight fixed point gives $\boldsymbol{\omega} = \mathcal{L}^\top \boldsymbol{\alpha}_c^{\star\top}$, so:

$$\boldsymbol{\alpha}_c^{\star\top}\, \mathcal{L}\, \Sigma_\epsilon\, \mathcal{L}^\top\, \boldsymbol{\alpha}_c^\star \;=\; \boldsymbol{\omega}^\top\, \Sigma_\epsilon\, \boldsymbol{\omega} \;=\; V[y] \tag{99}$$

and the welfare measure collapses to Kopytov et al. (2024). The present model extends Kopytov et al. (2024) in two dimensions that break this equivalence. First, with endogenous labor supply ($\psi > 0$), the Leontief inverse acquires an additional term (Lemma 2), so $\mathcal{L}^\top \boldsymbol{\alpha}_c^{\star\top} \neq \boldsymbol{\omega}$ and price variance and GDP variance diverge. Second, with multiple countries ($C > 1$), there are C separate variance objects that the planner tracks jointly, weighted by λ_c. The scalar reduction to $\boldsymbol{\omega}$-space is therefore special to the closed-economy, inelastic-labor environment of Kopytov et al. (2024).

E Effect of Aggregate TFP Shocks

E.1 Proof of Proposition 3

Step 1: Common Risk Structure

Both intermediate and final goods producers in country c face the same consumption price index:

$$p_{c,t+1} = \sum_{c'j} \alpha_{c,c'j} p_{c'j,t+1} \tag{100}$$

Under aggregate shocks: $p_{c,t+1} = -g_{t+1}\nu_c$ + other shocks, where $\nu_c = \sum_{c'j} \alpha_{c,c'j}\nu_{c'j}$.

Step 2: Risk-Adjusted Prices

For any producer in country c, the risk-adjusted price of supplier $(c\hat{k})$ is:

$$R_{c,c\hat{k}} = E_t[p_{c\hat{k},t+1} + \log \tau_{c,c\hat{k},t}] - (\gamma - 1)\mathrm{Cov}_t[p_{c\hat{k},t+1}, p_{c,t+1}]\frac{\psi}{1+\psi} \tag{101}$$

The covariance with consumption prices:

$$\mathrm{Cov}_t[p_{c\hat{k},t+1}, p_{c,t+1}] = \nu_{c\hat{k}}\nu_c\sigma_g^2 \tag{102}$$

Step 3: Effect of Risk on Covariance

Taking the derivative:

$$\frac{\partial \mathrm{Cov}_t[p_{c\hat{k},t+1}, p_{c,t+1}]}{\partial \sigma_g^2} = \nu_{c\hat{k}}\nu_c \tag{103}$$

Therefore:

$$\frac{\partial R_{c,c\hat{k}}}{\partial \sigma_g^2} = -(\gamma - 1)\nu_{c\hat{k}}\nu_c\frac{\psi}{1+\psi} \tag{104}$$

Similarly for the average: $\frac{\partial \bar{R}_c}{\partial \sigma_g^2} = -(\gamma - 1)\bar{\nu}_c\nu_c\frac{\psi}{1+\psi}$

Step 4: Network Optimization

For intermediate goods with adjustment cost κ^I:

$$\frac{d\eta^*_{ci,c\hat{k}}}{d\sigma_g^2} = -\frac{1}{2\kappa^I}\left[\frac{\partial R_{c,c\hat{k}}}{\partial \sigma_g^2} - \frac{\partial \bar{R}_c}{\partial \sigma_g^2}\right] = -\frac{(\gamma-1)}{2\kappa^I}(\nu_{c\hat{k}} - \bar{\nu}_c)\nu_c\frac{\psi}{1+\psi} \tag{105}$$

For final goods with adjustment cost κ^F, the identical risk structure yields:

$$\frac{d\alpha^*_{c,c\hat{k}}}{d\sigma_g^2} = -\frac{(\gamma-1)}{2\kappa^F}(\nu_{c\hat{k}} - \bar{\nu}_c)\nu_c \frac{\psi}{1+\psi} \quad \square \tag{106}$$

F GDP and Value Added Characterization

F.1 Output

The nominal output or Gross Domestic Product (GDP) for country c is the sum of the value of the goods produced in the country minus the value of intermediate inputs. This coincides with the total value added earned by producers located in the country:

$$GDP_c = \sum_i \left(P_{ci} y_{ci} - \sum_k \sum_{\hat{c}} p_{ci,\hat{c}k} X_{ci,\hat{c}k} \right) = \sum_{i \in N_c} W_{ci} L_{ci} \tag{107}$$

Following the convention on national accounts, the real GDP of country c is:

$$G_c = \sum_i^J \left(\overline{P}_{ci} Y_{ci} - \sum_{\hat{c}} \sum_k \overline{P}_{\hat{c}k} Z_{ci,\hat{c}k} \right) \tag{108}$$

F.2 Real Value Added

Lemma 8. *The log of real VA, a_{ci}, is:*

$$a_{ci}(s,\epsilon) = \epsilon_{ci} + A(\eta^\star_{ci}) + \mu_{ci}\, l_{ci}(s,\epsilon)$$

where $l_{ci}(s,\epsilon)$ is the log of labor supply:

$$l_{ci}(s,\epsilon) = \frac{\psi}{1+\psi}\Big[- p_c(s,\epsilon) + \log \mu_{ci} + \log(\omega_{ci}(s)) \Big]$$

Proof. See Appendix B.3. □

Lemma 8 decomposes value added into productivity and labor supply. The stochastic component of labor supply is driven by the transmission of TFP shocks through wages: a productivity shock in any (c,i) changes the price index and the real wage, affecting labor supply and value added in all other pairs. This channel, first emphasized by Backus et al. (1992), generates international comovement.[6] The strength of the response depends on the

[6] See also Huo et al. (2024), who characterize deviations from steady-state VA using the Leontief inverse.

exposure of the final-good firm to each good, summarized by $\alpha_c^\star$.

The deterministic part depends on (i) sales and labor shares (Domar weights), since larger or more labor-intensive industries employ more labor in equilibrium; and (ii) the deterministic component of final-good prices, reflecting expenditures on trade costs. Changes in the production network affect the deterministic component of labor across sectors.

F.3 Influence Matrix

Corollary 2. *The influence matrix M, measuring the effect of a TFP shock in sector $\hat{c}k$ on real VA in sector ci, is:*

$$M_{ci,\hat{c}k}(s) = \mathbf{1}_{\{ci=\hat{c}k\}} + \frac{\psi}{1+\psi} \sum_{c'} \sum_{j} \alpha^\star_{c,c'j} \mathcal{L}_{c'j,\hat{c}k}(s)$$

The first term captures the direct effect of TFP shocks; the second captures the effect on endogenous labor supply. Define also:

$$v_{ci} = \sum_{\hat{c}} \sum_{k} M_{ci,\hat{c}k} \tag{109}$$

which captures the total effect of a global shock on the VA of sector ci, both directly and through network connections.

F.4 Real GDP Growth

Lemma 9. *Let $\hat{\omega}_{ci}(s)$ be the local Domar weight and $g_{ci,t} = a_{ci,t} - a_{ci,t-1}$ the growth in VA. Then GDP growth is:*

$$g_{c,t}(s,\epsilon) = \sum_{i} \omega_{ci,t-1}(s) \left(\Delta\epsilon_{ci,t} + \Delta A(\eta^\star_{ci,t}) + \mu_{ci,t}\, \Delta l_{ci,t} \right)$$

Proof. See Appendix B.5. □

Lemma 9 aggregates sectoral VA growth into GDP growth using base-year Domar weights. The first two terms inside the brackets provide a Hulten aggregation of changes in

exogenous and endogenous TFP, while the last term captures how the equilibrium network generates growth through changes in labor supply.

F.5 Growth Decomposition

Let $g_{ci,t}$ denote the growth in VA of country c, sector i from $t-1$ to t. Growth decomposes as:

$$g_{ci,t} = \underbrace{M_t \left(\Delta\hat{\epsilon}_t + \Delta\theta_{ci,t}\right)}_{\text{Transmission of shocks}} + \underbrace{\Delta M_t \, \epsilon_{ci,t-1} + \Delta l_{ci,t}}_{\text{Network Adjustment}} \tag{110}$$

The first component is the standard Acemoglu et al. (2012) propagation of contemporaneous innovations ($\hat{\epsilon}_t$) and revisions in expected productivity ($\Delta\theta_t$) through the existing production structure. The second component captures endogenous reallocation of the network: when uncertainty increases, ΔM_t may tilt exposure towards safer suppliers, potentially lowering expected growth while reducing risk. The last source of growth is the deterministic component of labor supply $\Delta l_{ci,t}$: changes in the network affect relative sector sizes (Domar weights) and the final-good price index, both of which feed into labor supply.

G Variance and Covariance of GDP

This appendix characterizes the variance and covariance of GDP as functions of the production network and the distribution of TFP shocks. The results extend the decomposition in Huo et al. (2024) by isolating the interaction of aggregate shocks with the network.

G.1 Covariance Decomposition

Using the influence vectors v and M defined in Appendix F, the covariance of GDP between two countries decomposes as follows:

Proposition 6. *Let $\sigma_{c'j}$ be the standard deviation of sector-specific shocks. Then, given*

the network $\alpha^\star, \eta^\star$, the covariance in GDP between any two countries is:

$$\text{Cov}(g_c, g_{\hat{c}}) = \sum_i \sum_k \omega_{ci}\omega_{\hat{c}k}\Bigg(\underbrace{\sigma_g^2}_{\text{Global Shock}} + \underbrace{\sum_{c'} \sum_j \sigma^2_{c'j} M_{ci,c'j}\, M_{\hat{c}k,c'j}}_{\text{Sector-Specific Transmission}} + \underbrace{\sigma_g^2\ [v_{ci}\, v_{\hat{c}k} - 1]}_{\text{Global Transmission}} \Bigg)$$

Proof. See below. □

Proposition 6 extends the decomposition proposed by Huo et al. (2024) by separating the effect of global shocks on comovement into two components. The first, σ_g^2, is a mechanical effect: comovement increases because a common shock affects sectors simultaneously. The second, $\sigma_g^2[v_{ci}v_{\hat{c}k} - 1]$, captures how global shocks interact with the network: a global shock changes prices and wages throughout the network, and the vectors v_{ci} and $v_{\hat{c}k}$ measure the total exposure of each sector to these indirect effects.

Comovement between two countries increases when the largest sectors in both economies are exposed to the same sector-specific shocks, and when both countries are exposed to the aggregate shock through central sectors (high v_{ci} weighted by high ω_{ci}).

G.2 Variance of GDP

As a special case of the covariance decomposition, the variance of GDP for country c is:

$$\text{Var}(g_c) = \sum_i \sum_k \omega_{ci}\omega_{ck}\Big(\sigma_g^2 + \sum_{c',j} \sigma^2_{c'j} M_{ci,c'j} M_{ck,c'j} + \sigma_g^2[v_{ci}v_{ck} - 1]\Big)$$

This can be written more compactly as:

$$\text{Var}(g_c) = \sigma_g^2 \left(\sum_i \omega_{ci} v_{ci}\right)^2 + \sum_{c',j} \sigma^2_{c'j} \left(\sum_i \omega_{ci} M_{ci,c'j}\right)^2$$

The first term shows that aggregate volatility depends on the Domar-weighted average of global influence v_{ci}: countries whose large sectors are also highly exposed to the global factor have higher GDP volatility. The second term captures the contribution of sector-specific shocks, amplified by the network through the influence matrix M.

G.3 Full Decomposition with Four Shock Components

In the full model with global (g_t), country ($\chi_{c,t}$), industry ($\zeta_{i,t}$), and idiosyncratic ($u_{ci,t}$) shocks, the covariance decomposes into:

$$\text{Cov}(g_c, g_{\hat{c}}) = \underbrace{\sigma_g^2 \left(\sum_i \omega_{ci} v_{ci}\right)\left(\sum_k \omega_{\hat{c}k} v_{\hat{c}k}\right)}_{\text{Global}} + \underbrace{\sum_{c'} \sigma_{c'}^2 \left(\sum_i \omega_{ci} V^c_{ci,c'}\right)\left(\sum_k \omega_{\hat{c}k} V^c_{\hat{c}k,c'}\right)}_{\text{Country}}$$

$$+ \underbrace{\sum_j \sigma_j^2 \left(\sum_i \omega_{ci} V^i_{ci,j}\right)\left(\sum_k \omega_{\hat{c}k} V^i_{\hat{c}k,j}\right)}_{\text{Industry}} + \underbrace{\sum_{c',j} \sigma_{c'j}^2 \left(\sum_i \omega_{ci} M_{ci,c'j}\right)\left(\sum_k \omega_{\hat{c}k} M_{\hat{c}k,c'j}\right)}_{\text{Idiosyncratic}}$$

where $V^c_{ci,c'}$ and $V^i_{ci,j}$ are the influence vectors for country-specific and industry-specific shocks respectively.

G.4 Asymptotic Behavior

This section extends Acemoglu et al. (2012) by considering aggregate shocks. The following result characterizes the behavior of GDP comovement as the number of sectors grows.

As $N \to \infty$ with all sectors becoming arbitrarily small, the production network becomes irrelevant for GDP comovement. In this limiting case, comovement depends only on aggregate and sector-specific shocks:

$$\lim_{N\to\infty} \text{Cov}(g_c, g_{\hat{c}}) = \sigma_g^2$$

Similarly, the variance of GDP in the limit is explained by country-specific and global shocks alone. The network transmission terms vanish because with infinitely many small sectors, the diversification argument of Acemoglu et al. (2012) applies.

H Trade Cost Identification

H.1 Preliminaries

Factor Structure of Value Added

Analogously to the decomposition of productivity shocks (Assumption 1 in the main text), the log of real value added admits a factor structure.

Assumption 2 (Factor Structure of Value Added). *The log of real value added $a_{ci,t}$ admits the decomposition*

$$a_{ci,t} = g^a_t + \chi^a_{c,t} + \zeta^a_{i,t} + u^a_{ci,t}, \tag{111}$$

where each component $k \in \{g^a, \chi^a, \zeta^a, u^a\}$ follows an independent $AR(1)$ process with persistence ρ^a_k. The conditional mean is $\theta^a_{ci,t} \equiv \mathbf{E}_t[a_{ci,t}]$, the innovation is $\tilde{a}_{ci,t} \equiv a_{ci,t} - \theta^a_{ci,t}$, and the time-varying variances $\Sigma^a_t = \{\sigma^a_g(t), \sigma^a_c(t), \sigma^a_i(t), \sigma^a_{ci}\}$ are estimated using GARCH(1,1) models applied directly to VA.

This assumption mirrors the structure applied to productivity shocks. All components—conditional means θ^a_t, factor realizations, and time-varying variances—are estimated directly from the panel $\{a_{ci,t}\}$ without requiring knowledge of trade costs τ.

H.2 Proof of Proposition 5 (Identification of Trade Costs)

Proof. The proof recovers bilateral trade costs $(\log \hat{\boldsymbol{\tau}}^I_t, \log \hat{\boldsymbol{\tau}}^F_t)$ jointly from observed expenditure shares and value added under restrictions (??)–(??). The key step is differencing the first-order conditions against the *producing-country* reference and averaging across buying sectors, which eliminates both the buyer-level Lagrange multiplier and the expected-price terms.

Step 1 (First-order conditions). By Lemma 5, the risk-adjusted price of supplier $(\hat{c}, k)$ for buyer (c, i) in market m satisfies:

$$R^m_{ci,\hat{c}k,t} = \lambda^m_{ci,t} + 2\kappa^m \left(s^m_{ci,\hat{c}k,t} - s^{m,0}_{ci,\hat{c}k}\right), \tag{112}$$

where $\lambda^m_{ci,t}$ is the buyer-level Lagrange multiplier (average risk-adjusted price over active suppliers).

Step 2 (Producing-country differencing). By Proposition 1:

$$R^m_{ci,\hat{c}k,t} = \mathbb{E}_t[p_{\hat{c}k,t}] + \log \tau^m_{ci,\hat{c}k,t} + (\gamma - 1)\,\mathrm{Cov}_t(p_{c,t},\, p_{\hat{c}k,t}). \tag{113}$$

Write the same expression for the producing-country buyer $(\hat{c}, i)$ purchasing from the same supplier $(\hat{c}, k)$. Under (**??**), $\tau^I_{ci,\hat{c}k,t} = \tau^I_{c,\hat{c}k,t}$ and the normalization $\log \tau^I_{\hat{c},\hat{c}k,t} = 0$ applies to all sectors inside the producing country. Taking the difference:

$$R^I_{ci,\hat{c}k,t} - R^I_{\hat{c}i,\hat{c}k,t} = \log \tau^I_{c,\hat{c}k,t} + \underbrace{\left(\mathbb{E}_t[p_{\hat{c}k,t}] - \mathbb{E}_t[p_{\hat{c}k,t}]\right)}_{=0} + (\gamma - 1)\big(\mathrm{Cov}_t(p_c, p_{\hat{c}k}) - \mathrm{Cov}_t(p_{\hat{c}}, p_{\hat{c}k})\big). \tag{114}$$

The expected-price terms cancel exactly: both buyers purchase from the same supplier $(\hat{c}, k)$, so $\mathbb{E}_t[p_{\hat{c}k}]$ appears identically for both and drops out.

Step 3 (Eliminate Lagrange multipliers by differencing the FOC). Using (112) for both buyer (c, i) and reference buyer $(\hat{c}, i)$:

$$R^I_{ci,\hat{c}k,t} - R^I_{\hat{c}i,\hat{c}k,t} = (\lambda^I_{ci,t} - \lambda^I_{\hat{c}i,t}) + 2\kappa^I\left[(\eta^{data}_{ci,\hat{c}k,t} - \eta^0_{k,\hat{k}}) - (\eta^{data}_{\hat{c}i,\hat{c}k,t} - \eta^0_{k,\hat{k}})\right], \tag{115}$$

where restriction (**??**) ensures $\eta^0_{ci,\hat{c}k} = \eta^0_{\hat{c}i,\hat{c}k} = \eta^0_{k,\hat{k}}$, so the ideal-technology terms cancel:

$$R^I_{ci,\hat{c}k,t} - R^I_{\hat{c}i,\hat{c}k,t} = (\lambda^I_{ci,t} - \lambda^I_{\hat{c}i,t}) + 2\kappa^I\big(\eta^{data}_{ci,\hat{c}k,t} - \eta^{data}_{\hat{c}i,\hat{c}k,t}\big). \tag{116}$$

Step 4 (Sector averaging). Under (**??**), $\log \tau^I_{c,\hat{c}k,t}$ does not depend on the buying sector i. Averaging (114) and (116) across all active buying sectors $i \in \mathcal{I}_{c,t}$ eliminates the Lagrange-multiplier difference (which averages to a buyer-level constant absorbed by the averaging) and yields:

$$\bar{r}^I_{c,\hat{c}k,t} \equiv \frac{2\kappa^I}{J_{c,t}} \sum_{i \in \mathcal{I}_{c,t}} \big(\eta^{data}_{ci,\hat{c}k,t} - \eta^{data}_{\hat{c}i,\hat{c}k,t}\big) - (\gamma - 1)\left[\mathrm{Cov}_t(p_c, p_{\hat{c}k}) - \mathrm{Cov}_t(p_{\hat{c}}, p_{\hat{c}k})\right], \tag{117}$$

where $J_{c,t} = |\mathcal{I}_{c,t}|$. The RHS contains only observables, κ^I from Step 1, and the risk term.

No $\mathbb{E}_t[\varepsilon_{t+1}]$, no ρ, no $\eta^0_{k,\hat{k}}$.

Step 5 (Express $\mathbb{E}_t[p]$ through the Leontief, identifying both τ channels). The price of sector (c,i) satisfies the zero-profit condition:

$$p_{ci,t} = -\varepsilon_{ci,t} + \mu_{ci}\, w_{c,t} + (1-\mu_{ci}) \sum_{\hat{c}k} \eta^{\star}_{ci,\hat{c}k,t} \big(p_{\hat{c}k,t} + \log \tau^{I}_{c,\hat{c}k,t}\big).$$

Wages depend on the final-good price through labor supply (Lemma 6): $w_{c,t} = \frac{\psi}{1+\psi}\, p_{c,t} +$ terms free of τ, where $p_{c,t} = A(\boldsymbol{\alpha}^{\star}_{c,t}) + \sum_{\hat{c}k} \alpha^{\star}_{c,\hat{c}k,t}(p_{\hat{c}k,t} + \log \tau^{F}_{c,\hat{c}k,t})$. Taking expectations and iterating through the Leontief inverse $\mathcal{L}_t$:

$$\mathbb{E}_t[\mathbf{p}_t] = \mathcal{L}_t \Big[\underbrace{B^0_t - \mathbb{E}_t[\boldsymbol{\varepsilon}_{t+1}]}_{\text{free of } \tau} + \underbrace{C^I_t \log \boldsymbol{\tau}^I_t}_{\tau^I \text{ channel}} + \underbrace{C^F_t \log \boldsymbol{\tau}^F_t}_{\tau^F \text{ channel}} \Big], \tag{118}$$

where

$$B^0_{ci,t} \equiv -A(\eta^{\star}_{ci,t}) - \tfrac{\psi}{1+\psi} \mu_{ci}\, A(\alpha^{\star}_{c,t}) + \tfrac{1}{1+\psi} \big(\log \mu_{ci} + \log \omega_{ci,t}\big), \tag{119}$$

$$C^I_{ci,t} \equiv \tfrac{\psi}{1+\psi}(1-\mu_{ci})\, \boldsymbol{\eta}^{\star}_{ci,t}, \tag{120}$$

$$C^F_{c,t} \equiv \tfrac{\psi}{1+\psi} \mu_{ci}\, \boldsymbol{\alpha}^{\star}_{c,t}, \tag{121}$$

with $A(\eta^{\star}_{ci}) \equiv -\kappa^I \sum_{\hat{c}k} (\eta^{\star}_{ci,\hat{c}k} - \eta^0_{k,\hat{k}})^2$ and $A(\alpha^{\star}_{c}) \equiv -\kappa^F \sum_{\hat{c}k} (\alpha^{\star}_{c,\hat{c}k} - \alpha^0_{c,\hat{c}k})^2$. Note that $\mathbb{E}_t[\mathbf{p}_t]$ depends on $\log \tau$ through the GE pass-through terms C^I_t and C^F_t. However, the τ-free component $B^0_t - \mathbb{E}_t[\varepsilon_{t+1}]$ has already been eliminated from the RHS of (117) by the producing-country differencing in Step 2.

Step 6 (Collect $\log \tau$ terms and stack). Equating the structural decomposition (Step 2) with the averaged FOC residual (Step 4), the only remaining dependence on $\log \tau$ in the RHS comes from the GE pass-through of trade costs through prices. Collecting all $\log \tau$ terms on the left-hand side and stacking both market types:

$$\begin{pmatrix} I + \mathcal{L}_t C^I_t & \mathcal{L}_t C^F_t \\ \mathcal{L}_t C^I_t & I + \mathcal{L}_t C^F_t \end{pmatrix} \begin{pmatrix} \log \hat{\boldsymbol{\tau}}^I_t \\ \log \hat{\boldsymbol{\tau}}^F_t \end{pmatrix} = \begin{pmatrix} \bar{r}^I_{c,\hat{c}k,t} \\ \bar{r}^F_{c,\hat{c}k,t} \end{pmatrix}, \tag{122}$$

which matches (**??**). The system is linear in the stacked unknowns $(\log \boldsymbol{\tau}^I_t, \log \boldsymbol{\tau}^F_t)$ and is solved period by period for each t. The normalization $\log \tau^I_{c,\hat{c}k,t} = 0$ for $c = \hat{c}$ (all sectors inside the producing country) pins the domestic rows, and the sector averaging

eliminates the Lagrange multiplier. □

Appendix: Trade Elasticity Estimation

H.3 GMM Estimation of κ^I and κ^F

The curvature parameters κ^I (intermediate goods) and κ^F (final demand) are estimated jointly by generalized method of moments (GMM). The identification strategy exploits the cross-sectional structure of the network optimality condition: at the observed equilibrium, trade-share deviations from a reference pair are proportional to bilateral price differences, with proportionality coefficients κ^m.

Moment conditions. Fix $\gamma = 1$ (so that the risk premium vanishes from first-order conditions). For each bilateral pair (b, k) with an active link, the network optimality condition implies:

$$s_{b,k,t} - s_k^0 = -\frac{1}{2\kappa^m}\left(\log \tau_{b,k,t} - \bar{\lambda}_{b,t}^D\right) + u_{b,k,t}, \tag{123}$$

where $\bar{\lambda}_{b,t}^D$ is the buyer's average log-adjusted price across active suppliers and $u_{b,k,t}$ is a zero-mean residual. Bilateral log-trade costs $\log \tau_{b,k,t}$ are parameterized as a linear function of observable gravity proxies (log bilateral distance, common language, common border, and time fixed effects):

$$\log \tau_{b,k,t} = \delta_0 + \delta_1 \log \text{dist}_{bk} + \delta_2 \mathbf{1}[\text{lang}_{bk}] + \delta_3 \mathbf{1}[\text{border}_{bk}] + f_{b,t} + f_{k,t}, \tag{124}$$

with buyer and supplier-time fixed effects absorbing common price components. The GMM objective minimizes the squared residuals from (123) using the gravity variables as instruments, separately for intermediate-good links (κ^I) and final-demand links (κ^F).

Uniqueness constraint. The estimator imposes the sufficient condition for equilibrium existence and uniqueness from Proposition 2 as a constraint: the estimated $\hat{\kappa}^m$ must satisfy

$$\hat{\kappa}^m < \frac{(1+\psi)^2}{\underline{\omega}^3}\sqrt{(1-\underline{\omega})^2 + \left(\frac{\bar{\sigma}^2}{\bar{B}}\right)^2}, \tag{125}$$

where $\underline{\omega}$, $\bar{B}$, $\bar{\sigma}^2$ are the sample counterparts defined in Proposition 2. This constraint ensures that the implied equilibrium is well-defined at the estimated parameters.

H.4 Fixed-Point Algorithm for τ_t and ε_t

Given estimated $(\hat{\kappa}^I, \hat{\kappa}^F)$ and restrictions (??)–(??), the time series of trade costs $\{\tau_t\}$ and TFP shocks $\{\varepsilon_t\}$ are recovered in two sequential phases. Under the restrictions, the expected-price terms cancel from the τ inversion (Proposition 5), eliminating the outer loop over ρ that would otherwise be required. Phase A sets $\gamma = 1$ and recovers $(\hat{\tau}_t^A, \hat{\varepsilon}_t^A, \hat{\rho}, \hat{\Sigma}_t^\varepsilon)$ in a single forward pass; Phase B sets $\gamma = \gamma^\star$ and performs one additional forward pass using fixed inputs from Phase A.

Price formula. Throughout, prices are given by $\mathbf{p}(s, \varepsilon) = \mathcal{L}(s)[B(s) - \varepsilon]$, where the deterministic component decomposes as

$$B_{ci}(s) = B^0_{ci}(s) + C^I_{ci}(s) \log \tau^I_{c,\hat{c}k} + C^F_c(s) \log \tau^F_c, \tag{126}$$

with B^0, C^I, C^F defined in (119)–(121). The price moments are:

$$\mathbb{E}_t[\mathbf{p}_t] = \mathcal{L}_t(B_t - \mathbb{E}_t[\varepsilon_{t+1}]), \qquad \mathrm{Cov}_t(\mathbf{p}_t) = \mathcal{L}_t \, \Sigma^\varepsilon_t \, \mathcal{L}_t^\top. \tag{127}$$

VA equation and shock recovery. Value added satisfies:

$$\mathbf{a}_t = M_t \, \varepsilon_t + \mathbf{D}_t, \qquad M_t = I + \tfrac{\psi}{1+\psi} \, \boldsymbol{\mu} \, \boldsymbol{\alpha}^\star_t \, \mathcal{L}_t, \tag{128}$$

where $\mathbf{D}_t$ collects deterministic terms. Shocks are recovered by inversion:

$$\hat{\varepsilon}_t = M_t^{-1}(\mathbf{a}_t - \mathbf{D}_t). \tag{129}$$

Both M_t and $\mathbf{D}_t$ are evaluated at the observed data network s_t^{data} and remain fixed throughout Phases A and B. Note that $\eta^0_{k,\hat{k}}$ does not appear in (128)–(129).

Normalization. Trade costs are identified in relative terms:

- *Intermediate goods:* $\log \tau^I_{c,\hat{c}k,t} = 0$ for $c = \hat{c}$ (all sectors inside the producing country).
- *Final goods:* $\log \tau^F_{c,c1,t} = 0$ (sector 1 of the same country).

τ **inversion.** By Proposition 5, the joint linear system is:

$$\begin{pmatrix} \log \hat{\tau}_t^I \\ \log \hat{\tau}_t^F \end{pmatrix} = \begin{pmatrix} I + \mathcal{L}_t C_t^I \mathcal{L}_t C_t^F \\ \mathcal{L}_t C_t^I I + \mathcal{L}_t C_t^F \end{pmatrix}^{-1} \begin{pmatrix} \bar{r}^I_{c,\hat{c}k,t} \\ \bar{r}^F_{c,\hat{c}k,t} \end{pmatrix}. \quad (130)$$

Factor structure. $\varepsilon_{ci,t} = g_t + \chi_{c,t} + \zeta_{j,t} + u_{ci,t}$, each factor recovered by cross-sectional demeaning.

Phase A ($\gamma = 1$). With $\gamma = 1$, the risk term in (??) vanishes. The RHS of the τ system reduces to:

$$\bar{r}^{I,A}_{c,\hat{c}k,t} = \frac{2\kappa^I}{J_{c,t}} \sum_{i \in \mathcal{I}_{c,t}} \left(\eta^{data}_{ci,\hat{c}k,t} - \eta^{data}_{\hat{c}i,\hat{c}k,t} \right),$$

which contains only observables and κ^I from Step 1. No ε_t, no $\mathbb{E}_t[\varepsilon_{t+1}]$, no ρ, no $\eta^0_{k,\hat{k}}$.

Single forward pass over $t = 1, \ldots, T$:

1. Compute $B_t^0(s_t^{data})$, C_t^I, C_t^F, $\mathcal{L}_t$ at s_t^{data}.
2. Compute $\bar{r}_t^{I,A}$ and $\bar{r}_t^{F,A}$.
3. Solve (130) for $(\log \hat{\tau}_t^{I,A}, \log \hat{\tau}_t^{F,A})$.
4. Recover $\hat{\varepsilon}_t^A = M_t^{-1}(\mathbf{a}_t - \mathbf{D}_t(\hat{\tau}_t^A))$.

After the forward pass, estimate $\hat{\rho}_k$ by AR(1) on each extracted factor series $\{\hat{\varepsilon}_t^A\}_{t=1}^T$. No outer loop over ρ is required because $\mathbb{E}_t[\varepsilon_{t+1}]$ does not appear in the τ system.

GARCH step. Run GARCH(1,1) on factor innovations $\hat{v}_{\cdot,t}$. Assemble the factor covariance structure:

$$\hat{\Sigma}_t^\varepsilon = \hat{\sigma}_g^2(t)\, \mathbf{1}\mathbf{1}^\top + \sum_c \hat{\sigma}_\chi^2(c,t)\, \mathbf{1}_c \mathbf{1}_c^\top + \sum_j \hat{\sigma}_\zeta^2(j,t)\, \mathbf{1}_j \mathbf{1}_j^\top + \mathrm{diag}(\hat{\sigma}_u^2(ci)). \quad (131)$$

By Lemma 10 below, $\hat{\Sigma}_t^\varepsilon$ is invariant to γ and is used without modification in Phase B and the counterfactual solver.

Phase B ($\gamma = \gamma^\star$). Single forward pass. Takes $\hat{\rho}$, $\hat{\varepsilon}_t^A$, and $\hat{\Sigma}_t^\varepsilon$ from Phase A as fixed inputs. The risk term in (??) is now active:

$$\mathrm{Cov}_t(p_c, p_{\hat{c}k}) - \mathrm{Cov}_t(p_{\hat{c}}, p_{\hat{c}k}) = \left(\boldsymbol{\alpha}_{c,t}^{data\top} - \boldsymbol{\alpha}_{\hat{c},t}^{data\top}\right) \mathcal{L}_t \, \hat{\Sigma}_t^\varepsilon \, \mathcal{L}_t^\top \, e_{\hat{c}k}.$$

This is nonzero whenever the final-goods networks of countries c and $\hat{c}$ differ.

Forward pass over $t = 1, \ldots, T$:

1. Compute $B_t^0(s_t^{data})$, C_t^I, C_t^F, $\mathcal{L}_t$ at s_t^{data}.
2. Compute $\bar{r}_t^{I,B}$ via (??) using $\gamma^\star$ and the covariance term. Compute $\bar{r}_t^{F,B}$ analogously.
3. Solve (130) for $(\log \hat{\tau}_t^{I,B}, \log \hat{\tau}_t^{F,B})$.
4. Recover $\hat{\varepsilon}_t^B = M_t^{-1}(\mathbf{a}_t - \mathbf{D}_t(\hat{\tau}_t^B))$.

No iteration is required: $\hat{\Sigma}_t^\varepsilon$ is fixed from Phase A, so there is no feedback between τ and ε in Phase B.

η^0 inversion. Given $\hat{\tau}_t^B$, recover $\hat{\eta}_{k,\hat{k}}^0$ via Proposition ??: compute $\hat{R}_{\hat{c}k,t}(c)$ and $\hat{\lambda}_{ci,t}$ at the Phase B estimates, then average $\hat{\eta}_{k,\hat{k}}^0(c,t)$ across all $|C_k| \times T$ observations. This runs after Phase B with no feedback to the τ system.

Invariance of Σ_t^ε to γ.

Lemma 10. *$\Sigma_t^\varepsilon(\gamma) = \Sigma_t^\varepsilon(\gamma')$ for all γ, γ' and all t.*

Proof. From (128), $\mathbf{a}_t = M_t \varepsilon_t + \mathbf{D}_t(\tau(\gamma))$ where $M_t = M(s_t^{data})$ and $\mathbf{D}_t$ is $\mathcal{F}_{t-1}$-measurable. Therefore $\mathrm{Var}_{t-1}(\mathbf{a}_t) = M_t \Sigma_t^\varepsilon M_t^\top$, so $\Sigma_t^\varepsilon = M_t^{-1} \mathrm{Var}_{t-1}(\mathbf{a}_t) M_t^{-\top}$. Neither M_t nor $\mathrm{Var}_{t-1}(\mathbf{a}_t)$ depends on γ. □

This lemma implies that $\hat{\Sigma}_t^\varepsilon$ is computed once after Phase A and reused in Phase B and the counterfactual solver without modification. If the numerical estimates differ between phases, it signals that τ_t is being treated as correlated with ε_t, violating the timing assumption.

Appendix: Counterfactual Equilibrium Computation

Given the estimated fundamentals $\{\hat{\tau}_t, \hat{\varepsilon}_t, \hat{\Sigma}^{\varepsilon}_t, \hat{\eta}^0_{k,\hat{k}}, \hat{\alpha}^0_{c,\hat{k}}\}$ and parameters $(\gamma^\star, \kappa^I, \kappa^F, \psi, \boldsymbol{\mu})$, we solve for the equilibrium network $s^\star_t = (\eta^\star_t, \alpha^\star_t)$. This algorithm is used both to verify internal consistency of the estimation (at the data equilibrium) and to compute counterfactual allocations (at alternative trade costs or under the planner's objective).

H.5 Competitive Equilibrium Solver

Network-dependent objects. At each candidate network $s^{(j)}_t$, recompute:

1. Full Leontief: $\mathcal{L}^{(j)}_t = [I - \frac{\psi}{1+\psi}\boldsymbol{\mu}\,\alpha^{(j)}_t - (I - \boldsymbol{\mu})\,\eta^{(j)}_t]^{-1}$.
2. $B^{0,(j)}_t$ via (119) at $s^{(j)}_t$ (updating Domar weights $\omega^{(j)}_{ci,t}$ and adjustment costs).
3. $B^{(j)}_t = B^{0,(j)}_t + C^{I,(j)}_t \log \hat{\tau}^I_t + C^{F,(j)}_t \log \hat{\tau}^F_t$ with $\hat{\tau}_t$ held fixed.

Expected prices.

$$\mathbb{E}^{(j)}_t[\mathbf{p}_t] = \mathcal{L}^{(j)}_t \big(B^{(j)}_t - \hat{\rho}\,\hat{\varepsilon}_t\big). \tag{132}$$

Risk-adjusted prices. For each active pair (b,k) in market m:

$$R^{m,(j)}_{b,k,t} = \mathbb{E}^{(j)}_t[p_{k,t}] + \log \hat{\tau}^m_{b,k,t} + (\gamma^\star - 1)\,\boldsymbol{\alpha}^{\star(j)\top}_{c(b),t}\,\mathcal{L}^{(j)}_t\,\hat{\Sigma}^{\varepsilon}_t\,(\mathcal{L}^{(j)}_t)^\top\,e_k. \tag{133}$$

Lagrange multiplier.

$$\lambda^{D(j)}_{b,t} = \frac{1}{N^m_{b,t}} \sum_{k \in \mathcal{N}^m_{b,t}} R^{m,(j)}_{b,k,t}. \tag{134}$$

Network update.

$$\eta^{(j+1)}_{ci,\hat{c}k,t} = \hat{\eta}^0_{k,\hat{k}} - \frac{1}{2\kappa^I}\Big(R^{I,(j)}_{ci,\hat{c}k,t} - \lambda^{D(j)}_{ci,t}\Big), \tag{135}$$

$$\alpha^{(j+1)}_{c,\hat{c}k,t} = \hat{\alpha}^0_{c,\hat{k}} - \frac{1}{2\kappa^F}\Big(R^{F,(j)}_{c,\hat{c}k,t} - \lambda^{D(j)}_{c,t}\Big). \tag{136}$$

Project onto the simplex: set negative shares to zero, remove the corresponding suppliers from the active set, recompute λ^D, and repeat until no new violations occur.

Equilibrium loop. Iterate (132)–(136) until $\|s_t^{(j+1)} - s_t^{(j)}\|_\infty < \text{tol}$.

- *Data equilibrium:* Initialize at $s_t^{(0)} = s_t^{data}$.
- *Counterfactuals:* Initialize at the converged data equilibrium, then perturb $\hat{\tau}_t$ or $\hat{\Sigma}_t^\varepsilon$ as required.

Internal consistency test. At $s_t^{(0)} = s_t^{data}$ with the estimated fundamentals $(\hat{\tau}_t, \hat{\varepsilon}_t)$, the loop must converge in one iteration: $s_t^{(1)} = s_t^{(0)}$. By construction, $\hat{\tau}_t$ satisfies the FOC at the observed shares, so (133)–(136) produce no update. Failure of this test indicates a mismatch between the objects computed in the estimation phase (B_t^0, C_t) and those computed in the CE solver.

H.6 Planner's Equilibrium Solver

The planner internalizes how each buyer's sourcing decision affects all other countries' price risk through the network. By Proposition 4, at the planner's optimum, all countries' price indices move together (SDF equalization), and the risk-adjusted price faced by each buyer uses the *world* price index $p^\star = \sum_c \lambda_c p_c$ rather than the individual country's index p_c. The planner's risk-adjusted price for pair (b, k) in market m is:

$$R_{b,k,t}^{m,P(j)} = \mathbb{E}_t^{(j)}[p_{k,t}] + \log \hat{\tau}_{b,k,t}^m + (\gamma^\star - 1)\, \boldsymbol{\lambda}^\top \mathcal{A}_t^{(j)}\, \mathcal{L}_t^{(j)}\, \hat{\Sigma}_t^\varepsilon\, (\mathcal{L}_t^{(j)})^\top\, e_k, \tag{137}$$

where $\mathcal{A}_t^{(j)}$ maps sector prices to country price indices via $p_c = \boldsymbol{\alpha}_c^{\star(j)\top}\mathbf{p}$, and $\boldsymbol{\lambda}$ are the Pareto weights.

The planner's solver proceeds identically to the CE solver above, replacing $R_{b,k,t}^{m,(j)}$ in (133) with $R_{b,k,t}^{m,P(j)}$ from (137). All other steps—Leontief, Domar weights, expected prices, Lagrange multiplier, network update, simplex projection, and convergence check—are unchanged.

H.7 Counterfactual Exercises

The counterfactual analysis in Section 9 uses the CE and planner solvers under three scenarios:

1. **Trade liberalization and aggregate volatility** (Section 9.1). Compare the data equilibrium at $t_0 = 1967$ and $t_1 = 2014$: solve the CE at each period's estimated fundamentals $(\hat{\tau}_{t_0}, \hat{\Sigma}^{\varepsilon}_{t_0})$ and $(\hat{\tau}_{t_1}, \hat{\Sigma}^{\varepsilon}_{t_1})$ and compute the change in $\mathrm{Var}(\Delta \log \mathrm{GDP}_c)$.

2. **Competitive equilibrium vs. planner** (Section 9.2). At each period's estimated fundamentals, solve both the CE and the planner's problem. Compare sourcing concentration (HHI) and welfare.

3. **Openness and the externality** (Section **??**). Reduce all bilateral intermediate trade costs proportionally from their estimated 2000 values toward free trade, holding $(\hat{\varepsilon}, \hat{\Sigma}^{\epsilon})$ fixed. At each trade-cost level, solve both the CE and the planner's problem, tracing HHI and welfare as functions of the proportional reduction in τ^I.

In all cases, the equilibrium solver is initialized at the converged data equilibrium and the perturbation is applied incrementally to aid convergence. Welfare is computed using Lemma 7.

Appendix: Additional Empirical Results

This appendix reports full regression results for the centrality-risk mechanism discussed in Section 8.2.

Centrality-Risk: Full $\Delta\sigma^2$ Specification

Table 5 reports the complete specification underlying Table 2, including all controls.

	$\Delta\omega_{cj,t} \times 10^4$
$\hat{\ell}_{t-1} \times \Delta\sigma^2_{g,t}$	-6.7084^{***}
	(2.2244)
$\hat{\ell}_{t-1} \times \Delta\sigma^2_{\chi,t}$	0.0312
	(0.1010)
$\hat{\ell}_{t-1} \times \Delta\sigma^2_{\zeta,t}$	-0.0300
	(0.0836)
Variance controls	
$\sigma^2_{u,t-1}$	0.0940^{***}
	(0.0178)
$\sigma^2_{\chi,t-1}$	0.0683
	(0.1151)
$\sigma^2_{\zeta,t-1}$	-0.4128^{***}
	(0.1315)
Level controls	
$\chi_{c,t-1}$	-0.0218^{*}
	(0.0111)
$\zeta_{j,t-1}$	-0.0070^{***}
	(0.0021)
$u_{ci,t-1}$	-0.0037
	(0.0029)
Observations	14,904
R^2	0.0039

Notes: Dep. var. is $\Delta\omega_{cj,t} \times 10^4$, winsorized at $1^{\text{st}}/99^{\text{th}}$ percentile. $\hat{\ell}_{cj,t-1} = \sum_{nk} \mathcal{L}_{cj,nk,t-1}$ is the row sum of the full Leontief inverse (network exposure centrality), globally demeaned. All RHS variables lagged one period and standardized. $N = 24 \times 23 \times 27 = 14{,}904$. SE clustered at year level (27 clusters). $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table 5: Centrality-Risk Mechanism ($\Delta\sigma^2$ Interactions, Full Controls)

Centrality-Risk: Level Specification

Table 6 uses variance levels $\sigma^2_{k,t-1}$ rather than changes $\Delta\sigma^2_{k,t}$ in the interaction terms, with progressive addition of controls and fixed effects.

	(1)	(2)	(3)	(4)
$(\hat{\ell} - \bar{\ell})_{t-1} \times \sigma^2_{g,t-1}$	0.0012	0.0011	0.0011	0.0017
	(0.0244)	(0.0240)	(0.0240)	(0.0267)
$(\hat{\ell} - \bar{\ell})_{t-1} \times \sigma^2_{\chi,t-1}$	0.0138	0.0138	0.0139	0.0124
	(0.0195)	(0.0197)	(0.0196)	(0.0215)
$(\hat{\ell} - \bar{\ell})_{t-1} \times \sigma^2_{\zeta,t-1}$	-0.0094^{*}	-0.0083^{*}	-0.0083^{*}	-0.0066
	(0.0049)	(0.0048)	(0.0048)	(0.0054)
Variance controls				
$\sigma^2_{u,t-1}$	0.1563^{***}	0.1625^{***}	0.1619^{***}	0.1106
	(0.0468)	(0.0431)	(0.0427)	(0.1054)
$\sigma^2_{\chi,t-1}$	0.0399	0.0361	0.0361	-0.0050
	(0.0278)	(0.0270)	(0.0270)	(0.0430)
$\sigma^2_{\zeta,t-1}$	-0.0543	-0.0992^{**}	-0.0989^{**}	-0.1480^{*}
	(0.0472)	(0.0455)	(0.0454)	(0.0866)
Level controls				
$\chi_{c,t-1}$		0.0207	0.0207	-0.4169^{***}
		(0.0497)	(0.0497)	(0.0847)
$\zeta_{j,t-1}$		-0.0638	-0.0637	0.5735^{*}
		(0.0494)	(0.0494)	(0.3003)
$u_{ci,t-1}$			-0.0123	0.0465
			(0.0172)	(0.0374)
FE	No	No	No	Pair
Obs.	15,456	15,456	15,456	15,456
R^2	0.0057	0.0063	0.0063	0.0118

Notes: Dep. var. is $\Delta\omega_{cj,t} \times 10^4$, winsorized at $1^{\text{st}}/99^{\text{th}}$ percentile. $\hat{\ell}_{cj,t-1} = \sum_{nk} \mathcal{L}_{cj,nk,t-1}$ is the row sum of the full Leontief inverse (network exposure centrality), globally demeaned. All RHS variables lagged one period and standardized. $N = 24 \times 23 \times 28 = 15{,}456$. SE clustered at year level (28 clusters). $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table 6: Centrality-Risk Mechanism (σ^2 Level Interactions)

The level specification yields no significant global or country risk interactions, con-

firming that it is *changes* in global uncertainty—not its level—that drive reallocation away from central industries.

Concentration and Risk: HHI Regressions

Table 7 tests whether sourcing concentration, measured by the Herfindahl index of Domar weights, responds to risk at the country and global levels.

	Panel A: Country HHI			*Panel B: Global HHI*	
	(1)	(2)	(3)	(4)	(5)
$\sigma^2_{\chi,c,t-1}$	−0.4847	0.4758	0.3081		
	(0.8596)	(0.9834)	(0.9970)		
$\sigma^2_{g,t-1}$	−1.6740	−1.7837	−1.7646	−0.0166	−0.2554
	(2.5635)	(2.5444)	(2.5477)	(2.0394)	(1.6991)
$\chi_{c,t-1}$			−3.1125*		
			(1.7384)		
g_{t-1}					−2.7326*
					(1.4185)
Country FE	No	Yes	Yes	–	–
Obs.	672	672	672	28	28
R^2	0.0071	0.0072	0.0121	0.0000	0.1089

Notes: Panel A dep. var. is $\Delta\text{HHI}_{c,t} \times 10^4$, where $\text{HHI}_{c,t} = \sum_j (\omega_{cj,t}/\omega_{c\cdot,t})^2$ is the within-country Herfindahl index, winsorized at $1^{\text{st}}/99^{\text{th}}$ percentile ($N = 24 \times 28 = 672$ obs.). Panel B dep. var. is $\Delta\text{HHI}_{\text{global},t} \times 10^4$, winsorized at $5^{\text{th}}/95^{\text{th}}$ percentile, where weights are world Domar shares ($N = 28$ obs.). All RHS variables are lagged one period and standardized. SE clustered at year level. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table 7: Concentration and Risk

The variance coefficients in the HHI regressions are uniformly negative but not significant, reflecting the low power of the aggregate specification (28 time periods at the global level, 672 at the country level). The level of the global shock g_{t-1} is marginally significant in Panel B, column (5): periods with low realizations of the global factor are followed by increases in global concentration, consistent with the model's prediction that positive shocks relax the risk motive and allow the network to concentrate in productive hubs.