Directed Innovation across Space

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Abstract

I introduce directed innovation into a quantitative spatial model. Profit incentives direct innovation across regions through the interaction of research productivity, gradual technology diffusion, trade, and worker mobility – extending the concept of directed technical change (Acemoglu, 2002) to a spatial setting. I show that any steady-state distribution of innovation is consistent with balanced growth and determines the economy's long-run growth rate. I then analytically decompose the welfare impact of any shock to economic fundamentals into its transitory and long-run growth components. I apply the model to the post-1990 rise in the spatial concentration of American innovation, which was concentrated in high-skill regions and driven primarily by the growth of innovation in information and communication technologies (ICT). The model decomposes the contributions of demand-side forces – changing innovation incentives across regions – and supply-side forces – the sorting of skilled workers – and quantifies the aggregate consequences for growth and welfare.

Keywords: directed innovation; quantitative spatial models; spatial growth; trade and growth; technology diffusion; information and communication technologies

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1 Introduction

How does the geography of innovation respond to changes in economic fundamentals, and what are the consequences for aggregate growth and welfare? Several forces make the spatial dimension of innovation essential. First, workers differ in skills and face mobility frictions, so certain regions accumulate the human capital that enables innovation. Second, technology diffuses gradually across space – even within a country – so the location of innovation determines where productivity and wage gains occur first. Third, trade costs mean that where productivity improvements happen affects relative prices and the distribution of welfare gains across regions. If workers were perfectly mobile, technology diffused instantaneously, and trade were frictionless, the spatial dimension would collapse and one could work with an aggregate model of innovation and growth. It is precisely the interaction of these frictions that makes the geography of innovation consequential for aggregate outcomes.

Answering these questions requires a framework in which innovation is directed across space by profit incentives – extending the concept of directed technical change (Acemoglu, 2002) from sectors to regions. Building such a framework poses three challenges. First, innovators must earn profits that vary across regions, which requires introducing imperfect competition without losing the tractability needed for a quantitative spatial model. Existing quantitative spatial models with endogenous innovation have not achieved this. Second, the returns to innovation should ideally be connected to the spatial network of trade and technology diffusion, so that profit incentives reflect the full spatial structure of the economy. Even in the broader trade and growth literature – which typically abstracts from worker mobility – existing approaches disconnect innovation profits from the technology diffusion network. Third, innovation must feed back into equilibrium outcomes – shaping trade flows and wages – so that it is integral to the determination of growth and welfare.

In this paper, I introduce *directed innovation across space*. I develop a quantitative framework that overcomes these challenges: profit incentives endogenously direct innovation across regions and depend on the interaction of gradual technology diffusion, trade, and worker mobility, while innovation in turn shapes productivity, trade, wages, and hence growth and welfare.

The framework achieves this through two main components. In the first component, I build on techniques from Eaton and Kortum (2001) and Lind and Ramondo (2023a, 2024) and integrate endogenous innovation and gradual technology diffusion at the level of *individual ideas*, the fundamental unit of the Eaton-Kortum *world*[1] (Eaton and Kortum, 2024). Innovation workers in each region and sector discover new ideas according to a Poisson process, and the diffusion of each idea to other regions is governed by independent Poisson processes with heterogeneous bilateral diffusion speeds. Each idea has stochastic quality (drawn upon discovery) and stochastic applicability (drawn upon arrival in each new region), with productivity given by their product. This microfounded structure generates a multivariate productivity distribution for goods at

[1]Throughout the paper, "Eaton-Kortum *world*" refers not to Eaton and Kortum (2002) alone but to the broader class of models developed by Eaton and Kortum (Kortum, 1997; Eaton and Kortum, 1999, 2001; Bernard et al., 2003; Dekle et al., 2007; Eaton et al., 2011, 2016, 2022) from probabilistic microfoundations of technology, which can flexibly model trade, firm-to-firm matching, innovation, diffusion, and growth. See Eaton and Kortum (2024) for how the ingredients of this world connect across their papers.

each instant, with Fréchet marginals à la Eaton and Kortum (2002) whose endogenous scale parameters depend on the history of innovation in all regions and the entire technology diffusion network. Bertrand competition à la Bernard et al. (2003) generates differential profits across regions and sectors: because innovation and diffusion are connected at the idea level, the expected value of an idea depends on the entire trade and technology diffusion network.

In the second component, I introduce dynamic worker mobility à la Caliendo et al. (2019), allowing workers to respond to these differential incentives by sorting into research in regions where innovation is more profitable, so the spatial distribution of innovation evolves gradually along the transition path.

The framework delivers three main results. First, I characterize the spatial direction of innovation – the spatial analog of the sectoral direction of technical change in Acemoglu (2002) – which governs how profit incentives direct innovation across regions along the transition path. Second, I show that any steady-state distribution of innovation is consistent with balanced growth, where the long-run growth rate is the rate at which prices fall. This result means the framework imposes no restrictions on which regions innovate in the long run, and can accommodate any driver of the geography of innovation. This also connects quantitative spatial and endogenous growth models: regional wages are determined by the spatial distribution of workers, trade, and technology diffusion as in quantitative spatial models, while the aggregate growth rate depends on the geography of innovation as in growth models. Third, I analytically decompose the welfare impact of any shock to economic fundamentals into its transitory and long-run growth components. This decomposition applies to a wide class of shocks, making the framework applicable to a variety of empirical settings.

Because innovation is directed by profit incentives across space, the framework can address questions that existing models cannot. For example: how does a shock to research productivity in one sector reshape innovation across all regions through trade and technology diffusion networks? How much of the resulting welfare effect operates through the transition path versus changes in long-run growth? What role does worker mobility play in amplifying or dampening the spatial reallocation of innovation? More concretely, the framework can assess whether place-based policies raise aggregate innovation or merely relocate it; whether falling communication costs concentrate or disperse innovation; and how the welfare consequences of rising innovation concentration differ across regions.

I apply the framework to study the changing geography of American innovation, where innovation became increasingly concentrated in high-skill regions after 1990. Drawing on facts and decompositions documented in Tan (2026), I specify the information and communication technology (ICT) shock through two exogenous components – a one-time increase in national ICT research productivity and an increase in bilateral diffusion speeds reflecting falling communication costs. The model decomposes the contributions of demand-side forces – changing innovation incentives across regions – and supply-side forces – the sorting of skilled workers – and quantifies the aggregate consequences for growth and welfare.

Contributions to the Literature

Within the quantitative trade and innovation literature, the closest antecedents are Eaton and Kortum (2001) and Lind and Ramondo (2023b,a, 2024). Lind and Ramondo generalize the gradual technology diffusion model of Eaton and Kortum (1999) and embed it in the Eaton-Kortum trade framework. I build on their microfoundations to endogenize innovation and connect it to the technology diffusion network. Existing approaches miss or simplify this connection. Buera and Oberfield (2020) model technology diffusion as knowledge transfer from existing goods to new ones, disconnecting the diffusion network from innovation profits. Somale (2021) incorporates endogenous innovation but excludes technology diffusion. Cai et al. (2022) incorporate both endogenous innovation and technology diffusion, but assume perfect substitutability of ideas from different locations, resulting in what Eaton and Kortum call "the problem of flats", where small wage changes cause large shifts in idea adoption. Xiang (2025) uses the multivariate Fréchet productivity distribution from Ramondo and Rodríguez-Clare (2013) to introduce instantaneous diffusion through multinational production in Eaton and Kortum (2001)'s dynamic model. In my framework, gradual diffusion links the value of an idea to where it will spread over time, making the location of innovation more consequential and creating persistent spatial inequality along the transition path. This literature also assumes workers are immobile across regions but freely reallocate between production and research within each region, and focuses on the balanced growth path; by incorporating dynamic worker mobility with frictions across regions, sectors, and occupations, my framework can additionally characterize the transition path.

I also extend the class of quantitative dynamic spatial models with rich transition dynamics that incorporate worker migration (Caliendo et al., 2019), capital accumulation (Kleinman et al., 2023), and knowledge diffusion (Cai et al., 2025) by introducing endogenous and directed innovation. Desmet et al. (2018) is perhaps the only workhorse quantitative spatial model with endogenous innovation, but competitive bidding for land absorb all profits, so innovation is not directed. In contrast, Bertrand competition in my framework generates spatially differential profits that direct innovation.

More broadly, relatively few models examine innovation across regions. My framework combines directed innovation with trade, diffusion, and worker mobility in a quantitative spatial setting. Berkes et al. (2025) provide a transparent quantitative framework that focuses on modeling technological waves and knowledge diffusion across regions, but without trade or firms, innovation occurs through a stochastic max operation over productivity draws rather than deliberate investment directed by profit incentives. Berkes and Gaetani (2021) develop a theoretical urban model that offers valuable insights into the composition of innovation across space, distinguishing conventional from unconventional innovation, with a focus on qualitative predictions rather than a quantitative framework for counterfactuals.

Finally, my analytical characterization of the spatial direction of innovation advances the directed technical change literature in macroeconomics (e.g. Acemoglu, 1998, 2002, 2007), which has focused exclusively on the sectoral direction – how profit incentives direct innovation toward technologies that complement particular factors of production, for example between automation and the creation of new tasks (Acemoglu and Restrepo, 2018) or between clean and dirty technologies (Acemoglu et al., 2016). In that literature,

the direction of innovation is governed by factor endowments, the elasticity of substitution between factors, and the structure of innovation costs. I extend this concept to the spatial dimension, where the direction of innovation additionally depends on the spatial network of trade and technology diffusion, creating a richer set of forces that shape where innovation occurs and how it affects aggregate growth.

2 A Spatial Growth Model with Endogenous and Directed Innovation, Technology Diffusion, and Worker Mobility

This section introduces directed innovation into a quantitative spatial model. The model has two main components. First, I integrate endogenous innovation and technology diffusion at the level of *individual ideas*, the fundamental unit of the Eaton-Kortum world. Specifically, I leverage tools from Eaton and Kortum (2024) to introduce imperfect competition and endogenous innovation in Lind and Ramondo (2023a, 2024)'s quantitative trade model with idea diffusion. This structure allows equilibrium trade and technology diffusion flows to determine innovation incentives. Second, I incorporate dynamic worker mobility across regions, sectors, and between production and research, extending Caliendo et al. (2019). This component allows workers to respond to differential innovation incentives across regions and shape the spatial distribution of innovation. In turn, innovation raises productivity and influences trade, wages, and welfare through the first component. Thus, in the model, shocks to economic fundamentals reshape spatial patterns of innovation, which then influence aggregate growth and welfare.

Formally, the economy consists of N regions, indexed by r,o,d, corresponding to the locations of innovation or research (r), production (o for origin), and consumption (d for destination). Sectors are indexed by k and s, and there are two types of economic activity: goods production (G) and research (R). Time is continuous: t^* denotes the time at which ideas are produced through research, and t denotes the time at which goods are produced and consumed. The equilibrium objects and central mechanisms of the model are presented as lemmas, propositions, and corollaries, with proofs provided in Appendix A.

2.1 Microfounded Innovation and Technology Diffusion

In each region and sector, there is a unit continuum of immobile firms. At each time t^*, these firms hire local innovation workers to conduct research, discovering ideas from a stochastic process with rate $\lambda^k_{r,t^*}dt^*$[2]. I assume that λ^k_{r,t^*} depends on the region's fundamental research productivity, sector-specific national research productivity, and the local supply of inventors. Importantly, all subsequent results in the model hold for an arbitrary specification of λ^k_{r,t^*}.

Each idea pertains to the production of a specific good ν on the unit interval [0,1] and has stochastic quality determined on discovery, such that the aggregate number of ideas discovered in region r with quality above

[2] λ^k_{r,t^*} may be thought of as the output of the research production function.

q by time t is drawn from a Poisson distribution with mean parameter[3]:

$$\lambda^{Q}_{r,t}(q) = q^{-\theta} \int_{-\infty}^{t} \lambda_{r,t^*} dt^*. \tag{1}$$

After an idea is discovered in region r at time t^*, applications of the idea arrive via independent stochastic processes in every region[4]. Each application has stochastic applicability determined on arrival, such that the aggregate number of applications of this idea in region o at time t with applicability above a is drawn from a separate Poisson distribution with mean parameter[5]:

$$\lambda^{A}_{ro,t^*t}(a) = a^{-\sigma} \cdot \Omega_{ro,t^*}(t - t^*) \cdot \Gamma\left(1 - \frac{\theta}{\sigma}\right)^{-\frac{\sigma}{\theta}}. \tag{2}$$

From this distribution, the expected number of applications of the idea above a to ever arrive in region o is $\Gamma\left(1 - \frac{\theta}{\sigma}\right)^{-\frac{\sigma}{\theta}} a^{-\sigma}$. $\Omega_{ro,t^*}(t - t^*)$[6] is the share of applications that have arrived in region o by time t and represents any function of the idea's age $t - t^*$ with $\Omega : [0, \infty) \to [0, 1]$ and $\lim_{t \to t^*} \Omega_{ro,t^*}(t - t^*) = 0$, $\lim_{t \to \infty} \Omega_{ro,t^*}(t - t^*) = 1$. Intuitively, $\frac{d\Omega_{ro,t^*}(t-t^*)}{dt}$ captures the density or fertility of the idea in creating new applications. I assume that idea fertility declines exponentially with idea age, following Eaton and Kortum (1999); Lind and Ramondo (2024), as formalized below:

Assumption 1. *Idea diffusion occurs exponentially over the idea's age, such that:*

$$\Omega_{ro,t^*}(t - t^*) = 1 - e^{-\delta_{ro,t^*}(t-t^*)}, \tag{3}$$

with $\Omega_{rr,t^*}(t - t^*) = 1$ *(i.e. by setting* $e^{-\delta_{rr,t^*}(t-t^*)}$ *as* 0 *for all* $t \geq t^*$*).*

This assumption strikes a balance between two extremes: instantaneous diffusion, where new ideas are disproportionately weighted, and linear diffusion, where older ideas continue to generate numerous new applications over time. In particular, the speed of exponential diffusion is governed by the diffusion lags δ_{ro,t^*}. A decrease in the common component δ_{t^*} across all region-pairs over time captures, for example, reduced communication costs.

[3]Note that I omit the sector superscript k in this equation and what follows since the microfounded structure of innovation and technology diffusion is independent across sectors.

[4]This arrival of applications captures technology diffusion at the level of individual ideas.

[5]I provide microfoundations for the Poisson distributions of idea quality and applicability in Appendix A.1.

[6]Notice that $\Omega_{ro,t^*}(t - t^*)$ could be parameterized as a function of trade and migration flows at time t, mimicking Buera and Oberfield (2020) and Cai et al. (2025). Because innovation and technology diffusion are fundamentally connected at the level of individual ideas in my model, for any *arbitrary* function $\Omega_{ro,t^*}(t - t^*)$, the technology diffusion network then influences innovation rates through the expected value of individual ideas, as shown in Section 2.3 below. This impact cannot be obtained when innovation and technology diffusion are modeled as separate processes.

2.2 Evolution of Technology Levels, Production, and Trade

The discovery and diffusion of individual ideas generate aggregate trade and technology adoption flows across all region-pairs.

2.2.1 Production of Individual Goods

In each region o, local firms produce each individual good ν using only labor and the most efficient idea application for that good. The efficiency of applying idea i to good ν is the product of its quality and its applicability in that region. Accordingly, the productivity of producing good ν in region o at time t is:

$$z_{o,t}(\nu) = \max_i \{q_i \cdot a_{i,o,t}\} \tag{4}$$

where q_i denotes the quality of idea i, and $a_{i,o,t}$ denotes its best applicability in region o at time t.

Given the stochastic microfoundations of innovation and diffusion detailed in equations (1)-(2), the productivity of each individual good $z_{o,t}(\nu)$ is drawn from an endogenous multivariate Fréchet distribution. Formally:

Lemma 1. *Given that each idea is discovered at time t^* at a unique discovery location, idea applications have quality and applicability drawn from the Poisson distributions in equations* (1)-(2) *respectively, and their efficiency is multiplicative in quality and applicability [equation 4], the* ***joint productivity distribution*** *of individual goods across regions at each time t is multivariate Fréchet and given by:*

$$\mathbb{P}\left[Z_{1,t} \leq z_1, \ldots, Z_{N,t} \leq z_N\right] = \exp\left[-\sum_{o=1}^{N} \int_{-\infty}^{t} \left[\sum_{r=1}^{N} \Omega_{ro,t^*}(t-t^*)\ z_o^{-\frac{\theta}{1-\rho}}\right]^{1-\rho} \lambda_{r,t^*} dt^*\right] \tag{5}$$

where $\rho = 1 - \frac{\theta}{\sigma} < 1$, and the ***marginal productivity distribution*** *in each region is Fréchet and given by:*

$$\mathbb{P}\left[Z_{o,t} \leq z_o\right] = \exp\left[-T_{o,t} z_o^{-\theta}\right] \tag{6}$$

with shape parameter $\theta > 0$ and scale parameter:

$$T_{o,t} = \sum_{r=1}^{N} T_{ro,t} = \sum_{r=1}^{N} \int_{-\infty}^{t} \underbrace{\Omega_{ro,t^*}(t-t^*)^{1-\rho}}_{\text{technology diffusion}} \cdot \underbrace{\lambda_{r,t^*}}_{\text{innovation}}\ dt^*. \tag{7}$$

Notice from equation (6) that the marginal productivity distribution in each region is Fréchet, like in the canonical Eaton and Kortum (2002) model [henceforth, EK]. As shown below, this distribution facilitates closed form expressions for trade shares, enhancing tractability. The region's mean productivity in producing individual goods is given by the scale parameter, $T_{o,t}$. Unlike EK, this scale parameter is endogenously determined from the *dynamics* of innovation and technology diffusion, as shown by equation (7). Thus, changes in innovation and technology diffusion patterns following any shock to economic fundamentals

impact trade shares and, consequently, production worker wages and aggregate welfare. Because technology diffusion is modeled as the arrival of different applications of the same idea, the impact of technology diffusion on trade shares is governed by the relative variability of idea applicabilities against idea quality, $\rho = \frac{\theta}{\sigma}$. This elasticity is captured by the exponent $1-\rho$ in the marginal (equation 6) and joint productivity distributions (equation 5), ensuring that technology diffusion does not lead to large fluctuations in technology levels and, hence, trade shares over time.

2.2.2 Aggregation across Individual Goods and Equilibrium Trade Shares

In each region and sector, a representative final goods producer aggregates across these individual goods ν via a Cobb-Douglas function[7]:

$$Y_{d,t} = \exp \int_0^1 \ln Y_{d,t}(\nu) d\nu. \tag{8}$$

Each individual good ν need not be produced in the same region d. Instead, the final goods producer purchases each good from its cheapest production location, taking into account trade costs. Trade costs are of the standard iceberg type, such that delivering one unit of any good from region o to region d at time t requires shipping $\tau_{od,t} \geq 1$ units of the good, with $\tau_{oo,t} = 1$ for all o, and $\tau_{od,t} \leq \tau_{od',t}\tau_{d'd,t}$ for all o, d, and d'. Thus, the cost of each good ν in region d is the minimum unit cost across all regions:

$$c_{d,t}(\nu) = \min_o \left\{ \frac{\tau_{od,t} w_{o,t}}{z_{o,t}(\nu)} \right\} \tag{9}$$

where $w_{o,t}$ is the wage of production workers in region o at time t.

Consequently, the equilibrium trade shares are given by the share of goods in each destination that are purchased from different production locations. Because the productivity of each good is drawn from a multivariate Fréchet distribution, as shown in equation (5), these trade shares can be expressed in closed form. Formally:

Lemma 2. *Given that the productivity of each good is drawn from the multivariate Fréchet distribution in equation* (5)*, equilibrium* ***trade shares*** *are given by:*

$$\pi_{od,t} = \sum_{r=1}^{N} \pi_{rod,t} = \sum_{r=1}^{N} \int_{-\infty}^{t} \underbrace{\phi_{rd,t^*t}}_{\substack{\textit{share of goods in } d \textit{ at } t \\ \textit{using ideas from } r \textit{ at } t^* \\ \textbf{\textit{(idea market shares)}}}} \cdot \underbrace{\varphi_{o|rd,t^*t}}_{\substack{\textit{share of goods in } d \\ \textit{produced in } o \textit{ at } t \\ \textit{given ideas from } r \textit{ at } t^* \\ \textbf{\textit{(conditional idea}} \\ \textbf{\textit{adoption shares)}}}} dt^* \tag{10}$$

[7]The Cobb-Douglas aggregator implies that every good is essential for the production of the local sectoral final good, so the monopoly price for each individual good is infinite. Thus, under Bertrand competition introduced in the next subsection, the price charged for every good is a markup on the second lowest cost, simplifying the computation for the expected value of an individual idea. With a CES aggregator, the price of each good is the minimum between the second lowest cost and the monopoly markup of the lowest cost, as shown in Bernard et al. (2003).

*with the **idea market shares** given by:*

$$\phi_{rd,t^*t} = \frac{\Phi_{rd,t^*t}^{1-\rho}\lambda_{r,t^*}}{\sum_{r'}\int_{-\infty}^{t}\Phi_{r'd,t't}^{1-\rho}\lambda_{r',t'}dt'}, \tag{11}$$

*where I define the **idea market access** term Φ_{rd,t^*t} as:*

$$\Phi_{rd,t^*t} \equiv \sum_{o}\Omega_{ro,t^*}(t-t^*)\left(w_{o,t}\tau_{od,t}\right)^{-\frac{\theta}{1-\rho}}, \tag{12}$$

*and the **conditional idea adoption shares** given by:*

$$\varphi_{o|rd,t^*t} = \frac{\Omega_{ro,t^*}(t-t^*)\left(w_{o,t}\tau_{od,t}\right)^{-\frac{\theta}{1-\rho}}}{\sum_{o'=1}^{N}\Omega_{ro',t^*}(t-t^*)\left(w_{o',t}\tau_{o'd,t}\right)^{-\frac{\theta}{1-\rho}}}. \tag{13}$$

The trade shares in equation (10) illustrate the interconnections between innovation, diffusion, and trade[8]. Unlike standard trade models, where all goods are produced where their ideas originate, this model separates innovation and production locations. Consequently, trade shares are represented as the sum of trilateral shares, $\pi_{rod,t}$, across all innovation regions r. These trilateral trade shares denote the share of goods sold in destination region d that were produced in origin region o using ideas discovered in region r. Because the discovery time of an idea, t^*, may differ from the time of production, trade, and consumption, t, and because ideas are not perfect substitutes, it is necessary to distinguish goods produced from ideas developed at different times. Accordingly, the trilateral trade shares at time t are expressed as an integral over all idea cohorts $t^* \leq t$: $\pi_{rod,t} = \int_{-\infty}^{t}\pi_{rod,t^*t}\,dt^*$. The trilateral share for each idea cohort, π_{rod,t^*t}, exhibits a nested structure because productivity is drawn from a multivariate Fréchet distribution with correlation across idea discovery locations. Specifically, the trilateral share for each idea cohort equals the share of goods ϕ_{rd,t^*t} sold in destination region d at time t using ideas developed in region r at time t^* (idea market shares) multiplied by the conditional probability $\varphi_{o|rd,t^*t}$ that region o is the lowest-cost location for producing these goods (conditional idea adoption shares).

By aggregating the conditional idea adoption shares in equation (13) across different destination markets d, we obtain the (unconditional) idea adoption shares:

$$\varphi_{ro,t^*t} = \sum_{d}\pi_{o|rd,t^*t} = \sum_{d}\frac{\Omega_{ro,t^*}(t-t^*)\left(w_{o,t}\tau_{od,t}\right)^{-\frac{\theta}{1-\rho}}}{\sum_{o'=1}^{N}\Omega_{ro',t^*}(t-t^*)\left(w_{o',t}\tau_{o'd,t}\right)^{-\frac{\theta}{1-\rho}}}. \tag{14}$$

This expression represents the share of successful ideas discovered in region r at time t^* that were adopted in region o for production of the corresponding goods at time t. A "successful idea" here refers to one that

[8]Note that the trade shares depend only on the probabilistic assumptions of technology (i.e. the Poisson distributions for idea quality and applicability in equations 1 and 2 respectively) and are independent of market structure. This is because each destination always sources goods from their cheapest location, a fundamental result highlighting the flexibility of the Eaton-Kortum world.

resulted in a good sold in at least one destination market at time t.

2.3 Market Structure, Prices, Profits, and Innovation Worker Wages

I now introduce incentives to innovate through imperfect competition. Specifically, in each region and sector, firms hire innovation workers and own their ideas. These firms engage in Bertrand competition, where the lowest cost producer of each good claims the entire market for that good, charging the highest markup that deters any competitor from entering.

2.3.1 Prices and the Markup Distribution

Since all goods are essential in the production of the local final good (equation 8), the price of each good is its second lowest cost. Aggregating across all goods yields the price index in each region. Formally:

Lemma 3. *Given Bertrand competition and the Cobb-Douglas aggregation of goods in each region and sector (equation 8), the* ***price index*** *in region d at time t is:*

$$P_{d,t} = \gamma \left[\sum_{r'=1}^{N} \int_{-\infty}^{t} \Phi_{r'd,t^*t}^{1-\rho} \lambda_{r',t^*} dt^* \right]^{-\frac{1}{\theta}}, \tag{15}$$

where γ is the Euler-Mascheroni constant.

Additionally, the markup for each good is the ratio of the second lowest cost to the lowest cost. Since idea quality is drawn from a Pareto distribution, the ratio of the highest to second highest quality draw is Pareto and scale invariant. Consequently, the markup distribution is Pareto and invariant across regions and time. Formally:

Lemma 4. *Given Bertrand competition and the Cobb-Douglas aggregation of goods in each region and sector (equation 8), the* ***markup distribution*** *is Pareto and invariant across idea discovery time t^*, production time t, idea discovery location r, origin o, destination d, and sector k:*

$$G^{(2)/(1)}(m) \equiv \mathbb{P}\left[\frac{C^{(2)}}{C^{(1)}} \leq m | C^{(2)} = c_2 \right] = 1 - m^{-\theta}. \tag{16}$$

This distribution coincides with that in a setting without technology diffusion because I assume that only the most efficient application of each idea can be used to produce its corresponding good (see equation 4). Consequently, Bertrand competition occurs across ideas, not across multiple applications of the same idea.[9] This invariant markup distribution is crucial for deriving closed-form expressions for aggregate profits and the expected value of individual ideas within each region-sector.

[9] I impose this simplifying assumption because I interpret technology diffusion in my model as primarily occurring *within* firms, to capture the effects of firm spatial expansion. This interpretation is also consistent with strong intellectual property protections. Allowing for both within- and across-firm diffusion, and hence competition across different applications of the same idea, may be more appropriate in an international or developing-country context.

2.3.2 Profits and Innovation Worker Wages

Specifically, the invariant markup distribution implies that all firms in sector k selling in destination d charge a markup drawn from $G^{(2)/(1)}(m)$[10]. Consequently, the total profits earned from selling in destination d, irrespective of the locations of innovation and production, are given by:

$$\Pi_{d,t} = X_{d,t} \int_0^1 1 - \frac{1}{m(\nu)} d\nu = X_{d,t} \int_1^\infty 1 - \frac{1}{m(\nu)} dG^{(2)/(1)}(m) = \frac{X_{d,t}}{1+\theta}, \tag{17}$$

where $X_{d,t}$ is the total spending by production workers and inventors in region d at time t and given by equation (21). Because the markup distribution is identical whether conditional or unconditional on the production location, profits earned by firms from selling their goods can be arbitrarily assigned to production, innovation, or a combination of both[11]. I now make assumptions on how these profits are allocated and subsequently paid to individual inventors.

Assumption 2 (Allocation of Profits from Sales)**.**
(i) There is perfect intellectual property protection, such that all profits from sales are allocated to innovation; (ii) At every instant, firms invest their profits in risk-free assets, which are produced using the same technology as final goods in each region[12] *and do not depreciate over time. Firms compensate the inventors they hire with wages equal to the expected return of their innovation efforts.*

Given Assumption 2(i), the expected value of an idea is formalized in the following lemma:

Lemma 5. *Given Bertrand competition, the Cobb-Douglas aggregation across individual goods (equation 8), the time- and region-invariant markup distribution (equation 16), and the allocation of profits from sales (Assumption 2(i)), the* ***expected value of an idea*** *in region r and sector k is:*

$$\check{V}_{r,t^*} = \int_{t^*}^\infty e^{-\zeta(t-t^*)} \sum_{d=1}^N \underbrace{\frac{\phi_{rd,t^*t}}{\lambda_{r,t^*}}}_{\substack{\text{share of profits earned} \\ \text{in region } d \text{ at time } t \\ \text{by an idea discovered} \\ \text{in region } r \text{ at time } t^*}} \cdot \underbrace{\frac{X_{d,t}}{1+\theta}}_{\substack{\text{profits earned} \\ \text{in region } d \\ \text{at time } t \\ \text{by all ideas}}} \cdot \underbrace{\frac{P_{rt^*}}{P_{rt}}}_{\substack{\text{accounting for} \\ \text{changes in} \\ \text{purchasing power} \\ \text{over time}}} dt \tag{18}$$

where θ is the trade elasticity, ζ is the discount rate, and ϕ_{rd,t^*t} is the idea market share.

The expected value of an idea is given by the present discounted value of the trajectory of profits for all $t \geq t^*$ during which the idea is used. The first term captures the probability the idea developed in region r at time t^* results in a good sold in destination market d at time t, the second term represents the total

[10] Formally, to derive the unconditional markup distribution across all ideas, we need to integrate the conditional markup distribution $G^{(2)/(1)}(m)$ over all values of the second lowest cost c_2. However, since this conditional distribution is independent of c_2, it is exactly equal to the unconditional markup distribution.

[11] Consequently, my model is consistent with any structure of intellectual property protection or technology licensing, nesting Hemous et al. (2025).

[12] This structure mirrors the investment good technology in Kleinman et al. (2023), ensuring that the market clearing condition in each period (see equation 20) is solely determined by contemporaneous variables.

profits earned in destination market d at time t by all ideas, while the third term accounts for changes in purchasing power over time.

Given Assumption 2(ii), wages of inventors are simply the product of the expected number of ideas they discover, as described by the idea production function in equation (27), and the expected value of each idea in equation (18). Formally:

$$w_{r,t^*}^{k,R} = \frac{\lambda_{r,t^*}^k}{L_{r,t^*}^{k,R}} \check{V}_{r,t^*}^k. \tag{19}$$

2.4 Consumption, Market Clearing, and Production Worker Wages

At each time t, production workers and inventors have identical Cobb-Douglas preferences over local final goods from each sector, with ι^k capturing the expenditure share on goods produced in sector k. Given these preferences, the market clearing condition at each time t is formalized in the following lemma:

Lemma 6. *Given the allocation of profits from sales in Assumption 2, the combined goods and innovation* ***market clearing*** *condition at time t is given by:*

$$\frac{1+\theta}{\theta} w_{o,t}^k L_{o,t}^k = \sum_d \pi_{od,t}^k \iota^k \left[\sum_s \left(w_{d,t}^s L_{d,t}^s + \sum_r \varphi_{dr,t}^s \frac{1}{\theta} w_{r,t}^s L_{r,t}^s \right) \right] \tag{20}$$

where $\pi_{od,t}^k$ are the trade shares from equation (10)*, $\varphi_{dr,t}^s$ are the idea adoption shares from equation* (14)*, $w_{o,t}^k$ are the production worker wages, and $L_{o,t}^k$ is the number of production workers.*

In particular, total expenditure in region d on sector k goods is given by:

$$X_{d,t}^k = \iota^k \left[\sum_s \left(w_{d,t}^s L_{d,t}^s + \sum_r \varphi_{dr,t}^s \frac{1}{\theta} w_{r,t}^s L_{r,t}^s \right) \right]. \tag{21}$$

Since profits earned from production are a constant multiple of the income earned by production workers in the region-sector, and firms reinvest their profits in the same period to produce assets, neither profits nor wages of innovation workers appear explicitly in the market clearing condition. Consequently, I omit the superscript G for wages and labor to reduce notational burden.

2.5 Dynamic Worker Mobility

To allow innovation and production wages (equations 19 and 20 respectively) to shape the spatial and sectoral directions of innovation, I introduce worker mobility across regions and sectors and between production and research. Mobility decisions are dynamic, capturing how anticipated future opportunities influence current mobility decisions. I denote worker occupations h, n to represent either production (G) or research (R). Because continuous-time mobility complicates the separation of current and future payoffs, I impose the following assumption on the timing of migration:

Assumption 3. *A Poisson arrival process with rate 1 governs when* ***all*** *workers can move.*

In anticipation of when a move arrival occurs at some t', workers decide where to migrate to, which sector to work in, and whether to engage in production or research based on the present value stream of utility minus the associated mobility costs from their current region, sector, and occupation. With perfect foresight, the optimization problem of a worker in region d, sector k, and occupation h at time t is:

$$v_{d,t}^{k,h} = \max_{o,s,n} \quad \mathbb{E}_t \left(\int_t^{t'} \frac{w_{d,\check{t}}^{k,h}}{P_{d,\check{t}}} d\check{t} \right) + \frac{1}{1+\zeta} \mathbb{E}_t \left(\mathbb{E}_\epsilon \left[v_{o,t'}^{s,n} \right] \right) - \kappa_{do,t}^{ks,hn} + \epsilon_{o,t}^{s,n} \tag{22}$$

where $\kappa_{do,t}^{ks,hn}$ are the costs of moving from region d, sector k, and occupation h to region o, sector s, and occupation n, ζ is the discount rate, $\epsilon_{o,t}^{s,n}$ is an individual-specific idiosyncratic shock in each potential destination region-sector-occupation, $\mathbb{E}_t(\cdot)$ is the time-t expectation over future state variables, and $\mathbb{E}_\epsilon(\cdot)$ is the expectation over the agent's future realizations of the idiosyncratic shock. At each time t, I assume each individual-specific idiosyncratic shock $\epsilon_{o,t}^{s,n}$ is drawn from a multivariate Gumbel distribution with the following cumulative distribution function:

$$\check{F}\left(\{\epsilon_{o,t}^{s,n}\}_{o=1,\ldots,N}^{s=1,\ldots,K;\ n=\{G,R\}} \right) = \exp\left\{ - \left[\sum_o \sum_s \left(\sum_n \exp\left(-\epsilon_{o,t}^{s,n} \right)^{\frac{\Upsilon}{\upsilon}} \right)^{\upsilon} \right] \right\} \tag{23}$$

where υ is the elasticity of worker mobility across regions and sectors, and $\frac{\Upsilon}{\upsilon}$ is the elasticity of worker mobility between production and research.

Lemma 7. *Given individual-level worker mobility decisions defined by equations* (22)-(23)*, the* ***expected value*** *or lifetime utility of a representative worker in labor market* (d,k,h) *is given by:*

$$V_{d,t}^{k,h} \equiv \mathbb{E}_\epsilon \left[v_{d,t}^{k,h} \right] = \mathbb{E}_t \left(\int_t^{t'} \frac{w_{d,\check{t}}^{k,h}}{P_{d,\check{t}}} d\check{t} \right) + \frac{1}{\Upsilon} \log \left[\sum_o \sum_s \left(\sum_n \exp\left(\frac{1}{1+\zeta} V_{o,t'}^{s,n} - \kappa_{do,t}^{ks,hn} \right)^{\frac{\Upsilon}{\upsilon}} \right)^{\upsilon} \right] \tag{24}$$

aggregate ***mobility shares*** *of workers from* (d,k,h) *to* (o,s,n) *are given by:*

$$\begin{aligned} \mu_{do,t}^{ks,hn} \equiv \mathbb{E}_t \left[\check{\mu}_{do,t'}^{ks,hn} \right] &\equiv \mu_{do,t}^{ks,hn} | \mu_{do,t}^{ks} \cdot \mu_{do,t}^{ks} \\ &= \underbrace{\frac{\exp\left(\frac{1}{1+\zeta} V_{o,t'}^{s,n} - \kappa_{do,t}^{ks,hn} \right)^{\frac{\Upsilon}{\upsilon}}}{\sum_{n'} \exp\left(\frac{1}{1+\zeta} V_{o,t'}^{s,n'} - \kappa_{do,t}^{ks,hn'} \right)^{\frac{\Upsilon}{\upsilon}}}}_{\textit{switching between production and research}} \cdot \underbrace{\frac{\left[\sum_{n'} \exp\left(\frac{1}{1+\zeta} V_{o,t'}^{s,n'} - \kappa_{do,t}^{ks,hn'} \right)^{\frac{\Upsilon}{\upsilon}} \right]^{\upsilon}}{\sum_{o'} \sum_{s'} \left[\sum_{n'} \exp\left(\frac{1}{1+\zeta} V_{o',t'}^{s',n'} - \kappa_{do',t}^{ks',hn'} \right)^{\frac{\Upsilon}{\upsilon}} \right]^{\upsilon}}}_{\textit{migration across regions and sectors}} \end{aligned} \tag{25}$$

and the worker population in (o,s,n) *evolves as follows:*

$$L_{o,t'}^{s,n} = \sum_h \sum_k \sum_d \mu_{do,t}^{ks,hn} L_{d,t}^{k,h}. \tag{26}$$

2.6 Endogenous and Directed Innovation

Up to this point, the Poisson arrival rate of new ideas (or innovation levels) λ^k_{r,t^*} has been treated as exogenous. I now endogenize it, allowing the supply of innovation workers to shape local innovation levels and thereby completing the link between worker mobility and the spatial direction of innovation.

Formally, I specify:

$$\lambda^k_{r,t^*} = \underbrace{A_{r,t^*}}_{\substack{\text{fundamental}\\ \text{research}\\ \text{productivity}\\ \text{in region } r}} \cdot \underbrace{A^k_{t^*}}_{\substack{\text{sector-specific}\\ \text{national research}\\ \text{productivity}}} \cdot \underbrace{T^k_{r,t^*}}_{\substack{\text{sector-specific}\\ \text{technology level}\\ \text{in region } r}} \cdot \underbrace{L^{k,R}_{r,t^*}}_{\substack{\text{number}\\ \text{of inventors}}} \tag{27}$$

The first term represents the region's fundamental research productivity. The second term captures sector-specific research productivity. The third term is the region–sector technology level T^k_{r,t^*}; linearity in this term connects the model to single-region endogenous growth models such as Romer (1990)[13]. The fourth term is the number of innovation workers in the region and sector.

2.7 Definition of Equilibrium

Given an initial distribution of technology levels $\{T^k_{o,0}\}^{N;K}_{o=1;k=1}$ and workers $\{L^{k,h}_{o,0}\}^{N;K;\{G,R\}}_{o=1;k=1;h}$, trajectories of bilateral trade costs $\{\tau_{od,t}\}^{N;N;\infty}_{o=1;d=1;t=0}$, bilateral migration costs $\{\kappa^{ks,ha}_{od,t}\}^{N;N;\infty;K;K;\{G,R\}}_{o=1;d=1;t=0;k=1;s=1;h,a}$, bilateral diffusion lags $\{\delta_{od,t^*}\}^{N;N;\infty}_{o=1;d=1;t^*=0}$, fundamental productivities in idea production $\{A_{r,t^*}, A^k_{t^*}\}^{N;K;\infty}_{r=1;k=1;t^*=0}$ and fundamental parameters and elasticities $\{\theta, \sigma, \upsilon, \Upsilon, \iota, \alpha, \zeta\}$, the **dynamic competitive equilibrium** is defined by a trajectory of values, wages, prices and labor allocations $\{V, w, P, L\}$ that satisfy the bilateral migration shares and evolution of worker populations (Lemma 7), innovation levels (equation 27), evolution of technology levels (Lemma 1 equation 7), bilateral trade and idea adoption shares (Lemma 2 equations 10 and 14), price indices (Lemma 3), returns to innovation (Lemma 5) and market clearing condition (Lemma 6).

[13] Alternatively, replacing T_{r,t^*} with T^{β}_{r,t^*} with $\beta < 1$ would yield semi-endogenous growth, as in many single-region growth models pioneered by Jones (1995); Kortum (1997).

3 The Spatial Direction of Innovation: Drivers and Aggregate Consequences

The model integrates two building blocks – idea-level endogenous innovation and technology diffusion, and dynamic worker mobility – that together generate *directed innovation across space*. Here I formally characterize this concept in partial equilibrium, extending directed technical change (Acemoglu, 2002) from sectors to space. I then derive the balanced growth and transition paths, providing a system of equations that characterize the spatial direction of innovation in general equilibrium. Finally, I use these paths to decompose the welfare impact of any shock to economic fundamentals into its transitory and long-run growth components.

3.1 Spatial Direction of Innovation

I combine the research production function (equation 27) with the expression for inventor wages (equation 19) to characterize how profit incentives direct innovation across regions:

Proposition 1. *The **spatial direction of innovation** is governed by the ratio of sector-specific inventor real wages across regions, given as follows:*

$$\frac{\omega_{r,t^*}^{k,R}}{\omega_{r',t^*}^{k,R}} = \underbrace{\frac{A_{r,t^*}}{A_{r',t^*}}}_{\substack{\text{fundamental} \\ \text{research} \\ \text{productivity}}} \cdot \underbrace{\frac{T_{r,t^*}^{k}}{T_{r',t^*}^{k}}}_{\substack{\text{relative} \\ \text{technology} \\ \text{levels}}} \cdot \underbrace{\frac{\int_{t^*}^{\infty} e^{-\zeta(t-t^*)} \sum_{d=1}^{N} \frac{\boldsymbol{\phi_{rd,t^*t}}}{\boldsymbol{\lambda_{r,t^*}}} \frac{Y_{d,t}}{1+\theta} \frac{1}{P_{r,t}} dt}{\int_{t^*}^{\infty} e^{-\zeta(t-t^*)} \sum_{d=1}^{N} \frac{\boldsymbol{\phi_{r'd,t^*t}}}{\boldsymbol{\lambda_{r',t^*}}} \frac{Y_{d,t}}{1+\theta} \frac{1}{P_{r',t}} dt}}_{\substack{\text{relative expected} \\ \text{value of an idea}}} \tag{28}$$

The higher the ratio in equation (28), the greater the returns to innovation directed toward region r relative to region r'. Within each sector, three main factors shape the incentives to innovate across space: (i) time-varying fundamental research productivity; (ii) relative technology levels; and (iii) the relative expected value of an idea discovered in that region and sector. The third factor comes from equation (18) and depends on the region's idea market shares, which capture the probability that a locally discovered idea is used in each destination market, and on the size and price level of those markets. Intuitively, an idea's market share is higher when it diffuses to many low-cost production locations that have good trade access to large destination markets. Any increase in the ratio in equation (28) following a shock to fundamentals – driven by one or more of these factors – induces innovation workers to move from region r' to region r, thereby raising innovation in r relative to r' through equation (27).

Because this ratio captures inventor real wages in partial equilibrium, it highlights the key forces shaping where innovation is directed. However, inventor real wages are jointly determined in general equilibrium together with technology levels, labor allocations, prices, and idea market shares in equation (28). Consequently, inventor real wages – and thus the spatial direction of innovation – are also indirectly influenced by many other factors in the economy, such as production worker wages across all regions. Appendix A.8

presents the corresponding wage ratio that governs the sectoral direction of innovation.

3.2 Balanced Growth Path

The balanced growth path has a block-recursive structure that distinguishes and connects the *causes* and *consequences* of the geography of innovation, while establishing a direct link between quantitative trade and spatial models and endogenous growth models in macroeconomics. The following proposition formalizes this structure:

Proposition 2. *Along the* ***balanced growth path*** *where all equilibrium variables grow at constant (but possibly different) rates:*
(i) The growth rate of technology in each sector g^k is identical across regions and determined by the solution to the following system of equations:

$$\dot{T}_o^k(t) = \sum_r \gamma_r^k T_r^k(t) \int_{-\infty}^{t} g^k e^{-g^k(t-t^*)} \Omega_{ro}(t-t^*)^{1-\rho} dt^* \tag{29}$$

where γ_r^k is the endogenous region-sector-specific innovation rate. In matrix form, this equation is given by:

$$g\mathbf{T}^k = \mathbf{\Delta}^k(g)\mathbf{T}^k \tag{30}$$

where $\mathbf{T}^k$ is an $N \times 1$ vector with representative element T_o^k and $\mathbf{\Delta}^k(g)$ is an $N \times N$ matrix with representative element:

$$\Delta_{ro}^k(g^k) = \gamma_r^k \int_0^{\infty} g^k e^{-g^k a} \Omega_{ro}(a)^{1-\rho} da.$$

Thus, g^k is the Perron-Frobenius root of equation (30) *with relative technology levels $\mathbf{T}$ corresponding to the Perron-Frobenius eigenvector that is defined up to a scalar multiple;*
(ii) Production worker wages, inventor wages, and the distribution of workers across regions, sectors, and occupations are constant, prices are falling at rate $g_p = \frac{1}{\theta}\sum_k \iota^k g^k$, and the expected value of workers is rising at rate $g_v = \frac{1+\zeta}{\zeta}\frac{1}{\theta}\sum_k \iota^k g^k$.

Regarding aggregate *consequences*, part (i) of the proposition shows how any arbitrary steady-state geography of innovation rates ($\gamma_r^k = \frac{\lambda_r^k}{T_r^k}$) and region-pair idea diffusion speeds (δ_{ro}) can generate a balanced growth path characterized by parallel growth at the sectoral rate g^k, alongside persistent level differences in technology T^k across regions for each sector[14]. Part (ii) shows how the sector-specific technology growth rates (g^k) determine the rate at which prices fall and, consequently, the growth rate of workers' expected value on the balanced growth path. This price decline parallels standard macroeconomic growth models. However, unlike those models, the rate at which prices fall here depends on the entire spatial distribution of innovation, rather than solely on the aggregate innovation rate.

Regarding *causes*, part (ii) implies that the steady-state geography of innovation rates (γ_r^k) is determined by the steady-state distribution of workers, which in turn is shaped by the exogenous fundamentals of the

[14]This result arises because the Poisson arrival rate of ideas depends linearly on technology levels (see equation 27).

economy, including bilateral idea diffusion speeds, migration costs, trade costs, and research productivities. The equilibrium expressions for innovation rates, inventor wages and other variables are provided in Appendix A.9. In steady state, the trade and production block of the model mirrors a standard quantitative trade model. Since any arbitrary steady-state geography of innovation rates is consistent with the balanced growth path described above, my framework can readily accommodate various mechanisms that drive the geography of innovation in different contexts.

3.3 Transition Path

I now characterize the transition path, which governs how the spatial distribution of workers and hence the spatial direction of innovation evolve following shocks to fundamentals. I assume the economy is initially on a balanced growth path[15] and characterize its transition to a new balanced growth path following shocks to fundamentals at time $t = 0$. To discretize time and align the model with observed annual data, I strengthen Assumption 3 as follows:

Assumption 4. *A Poisson arrival process with rate 1 governs when innovation, production, consumption, and migration occur across all agents and all regions*[16]*.*

This reinterprets λ^k_{r,t^*} as the total number of ideas discovered in region r and sector k during the discrete time period t^*, and allows me to extend the **dynamic hat algebra** methodology of Caliendo et al. (2019) to simulate the transition path without information on levels of migration and trade costs. I introduce the following notation. Let $\mathcal{T}$ denote the set of times at which innovation, production, consumption, and migration occur. For any variable x, let $\widetilde{x}_t = x_t e^{-g_x t}$ denote its detrended value with growth rate g_x, and $\dot{x}_{t'} = \frac{x_{t'}}{x_t}$ its change over time. Let $u = \exp(V)$. The transition path can then be obtained given the trajectory of *changes* in migration and trade costs – as opposed to levels – alongside levels of research productivity and diffusion lags. This characterization is formalized in the following proposition:

Proposition 3. *Given an initial distribution of workers,* ***wages****, technology levels, and migration and trade shares* $\left\{L^k_{o,0}, T^k_{o,0}, \mu^{ks,hn}_{od,0}, \pi^k_{od,0}\right\}^{N;N;K;K;\{G,R\}}_{o=1;d=1;k=1;s=1;h,n}$*, exogenous trajectories of changes in migration and trade costs, research productivity, and bilateral diffusion lags* $\left\{\dot{\kappa}^{ks,hn}_{od,t}, \dot{\tau}_{od,t}, A^k_{r,t}, \delta_{ro,t}\right\}^{N;N;N;K;K;\{G,R\};\infty}_{o=1;d=1;r=1;k=1;s=1;h,n;t=0}$*, the* ***transition path*** *of the economy is characterized by the evolution of the distribution of workers for all* $t^*, t, t' \geq 0$*:*

[15] This assumption substitutes for historical data on innovation levels and diffusion speeds from time $-\infty$ to 0, enabling computation of trade shares at all time periods.

[16] This assumption also ensures that workers' expected values are equalized across regions, sectors, and between production and research when innovation and production occur. An alternative, stronger assumption is that all these activities occur only at the beginning or end of each year. In expectation, both assumptions are equivalent.

$$\log\left(\dot{\widetilde{u}}_{d,t}^{k,h}\right)=\log\left(\frac{\dot{w}_{d,t}^{k,h}}{\dot{\widetilde{P}}_{d,t}}\right)+\frac{1}{\Upsilon}\log\left[\sum_{n}\left[\sum_{s}\sum_{o}\mu_{do,t'}^{ks,hn}\left(\dot{\widetilde{u}}_{o,t'}^{s,n}\right)^{\frac{\Upsilon}{(1+\zeta)\upsilon}}\left(\dot{\kappa}_{do,t}^{ks,hn}\right)^{\frac{\Upsilon}{\upsilon}}\right]^{\upsilon}\right] \tag{31}$$

$$\mu_{od,t'}^{ks,hn}=\frac{\mu_{od,t}^{ks,hn}\left(\dot{\widetilde{u}}_{d,t'}^{s,n}\right)^{\frac{\Upsilon}{(1+\zeta)\upsilon}}\left(\dot{\kappa}_{od,t}^{ks,hn}\right)^{\frac{\Upsilon}{\upsilon}}}{\sum_{n'}\mu_{od,t}^{ks,hn'}\left(\dot{\widetilde{u}}_{d,t'}^{s,n'}\right)^{\frac{\Upsilon}{(1+\zeta)\upsilon}}\left(\dot{\kappa}_{od,t}^{ks,hn'}\right)^{\frac{\Upsilon}{\upsilon}}}\cdot\frac{\left[\sum_{n'}\mu_{od,t}^{ks,hn'}\left(\dot{\widetilde{u}}_{d,t'}^{s,n'}\right)^{\frac{\Upsilon}{(1+\zeta)\upsilon}}\left(\dot{\kappa}_{od,t}^{ks,hn'}\right)^{\frac{\Upsilon}{\upsilon}}\right]^{\upsilon}}{\sum_{n'}\left[\sum_{d'}\sum_{s'}\mu_{od',t}^{ks',hn'}\left(\dot{\widetilde{u}}_{d',t'}^{s',n'}\right)^{\frac{\Upsilon}{(1+\zeta)\upsilon}}\left(\dot{\kappa}_{od',t}^{ks',hn'}\right)^{\frac{\Upsilon}{\upsilon}}\right]^{\upsilon}} \tag{32}$$

$$L_{d,t'}^{k,h}=\sum_{n}\sum_{s}\sum_{o}\mu_{od,t}^{ks,hn}L_{o,t}^{s,n} \tag{33}$$

where at each time t innovation levels are given by equation (27) *and technology levels by:*

$$\begin{aligned}\lambda_{r,t}^{k}&=A_{t^*}^{k}A_{r,t^*}T_{r,t^*}^{k}L_{r,t^*}^{k,R}\\ T_{o,t}^{k}&=\sum_{r=1}^{N}\sum_{t^*\in\Im_{-\infty}^{t}}\Omega_{ro,t^*}(t-t^*)^{1-\rho}\cdot\lambda_{r,t^*}^{k},\end{aligned} \tag{34}$$

the trade equilibrium is given by:

$$\pi_{od,t}^{k}=\sum_{r=1}^{N}\sum_{t^*\in\Im_{-\infty}^{t}}\frac{\Omega_{ro,t^*}(t-t^*)\left(w_{o,t}^{k,G}\tau_{od,t}^{k}\right)^{-\frac{\theta}{1-\rho}}}{\sum_{o'}\Omega_{ro',t^*}(t-t^*)\left(w_{o',t}^{k,G}\tau_{o'd,t}^{k}\right)^{-\frac{\theta}{1-\rho}}}\frac{\left[\sum_{o'}\Omega_{ro',t^*}(t-t^*)\left(w_{r,t}^{k,G}\tau_{ro',t}^{k}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}\lambda_{r,t^*}^{k}}{\sum_{r'}\sum_{\check{t}\in\Im_{-\infty}^{t}}\left[\sum_{o'}\Omega_{r'o',\check{t}}(t-\check{t})\left(w_{o',t}^{k,G}\tau_{o'd,t}^{k}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}\lambda_{r',\check{t}}^{k}} \tag{35}$$

$$\varphi_{ro,t}^{k}=\sum_{d=1}^{N}\sum_{t^*\in\Im_{-\infty}^{t}}\frac{\Omega_{ro,t^*}(t-t^*)\left(w_{o,t}^{k,G}\tau_{od,t}^{k}\right)^{-\frac{\theta}{1-\rho}}}{\sum_{o'}\Omega_{ro',t^*}(t-t^*)\left(w_{o',t}^{k,G}\tau_{o'd,t}^{k}\right)^{-\frac{\theta}{1-\rho}}}\frac{\left[\sum_{o'}\Omega_{ro',t^*}(t-t^*)\left(w_{r,t}^{k,G}\tau_{ro',t}^{k}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}\lambda_{r,t^*}^{k}}{\sum_{r'}\sum_{\check{t}\in\Im_{-\infty}^{t}}\left[\sum_{o'}\Omega_{r'o',\check{t}}(t-\check{t})\left(w_{o',t}^{k,G}\tau_{o'd,t}^{k}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}\lambda_{r',\check{t}}^{k}} \tag{36}$$

$$P_{d,t}^{k}=\Gamma\left[\sum_{r'=1}^{N}\sum_{t^*\in\Im_{-\infty}^{t}}\left[\sum_{o'=1}^{N}\Omega_{r'o',t^*}(t-t^*)\left(w_{o',t}^{k,G}\tau_{o'd,t}^{k}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}\lambda_{r',t^*}^{k}\right]^{-\frac{1}{\theta}} \tag{37}$$

$$\frac{1+\theta}{\theta}w_{o,t}^{k,G}L_{o,t}^{k,G}=\sum_{d}\pi_{od,t}^{k}\iota^{k}\left[\sum_{s}\left(w_{d,t}^{s,G}L_{d,t}^{s,G}+\sum_{r}\varphi_{dr,t}^{s}\frac{1+\theta}{\theta}w_{r,t}^{s,G}L_{r,t}^{s,G}\right)\right]$$

where the market clearing condition comes from equation (20)*, and the wages of inventors, returns to innovation and the probability that goods sold in destination d at time t' use ideas discovered in region r at time t are given by:*

$$w_{r,t}^{k,R} = \frac{\check{V}_{r,t}^{k} \lambda_{r,t}^{k}}{L_{r,t}^{k,R}} \tag{38}$$

$$\check{V}_{r,t}^{k} = \sum_{t' \in \mathfrak{T}_{t}^{\infty}} \left(\frac{1}{1-\rho} \right)^{t'-t} \sum_{d} \frac{X_{d,t'}^{k}}{1+\theta} \cdot \frac{P_{r,t}}{P_{r,t'}} \cdot \frac{\phi_{rd,tt'}^{k}}{\lambda_{r,t}^{k}} \tag{39}$$

$$X_{d,t'}^{k} = \iota^{k} \left[\sum_{s} \left(w_{d,t'}^{s,G} L_{d,t'}^{s,G} + \sum_{r} \varphi_{dr,t'}^{s} \frac{1+\theta}{\theta} w_{r,t'}^{s,G} L_{r,t'}^{s,G} \right) \right] \tag{40}$$

$$\phi_{rd,tt'}^{k} = \frac{\left[\sum_{o'} \Omega_{ro',t}(t'-t) \left(w_{r,t'}^{k,G} \tau_{ro',t'}^{k} \right)^{-\frac{\theta}{1-\rho}} \right]^{1-\rho} \lambda_{r,t}^{k}}{\sum_{r'} \sum_{\check{t} \in \mathfrak{T}_{-\infty}^{t'}} \left[\sum_{o'} \Omega_{r'o',\check{t}}(t'-\check{t}) \left(w_{o',t'}^{k,G} \tau_{o'd,t'}^{k} \right)^{-\frac{\theta}{1-\rho}} \right]^{1-\rho} \lambda_{r',\check{t}}^{k}}, \tag{41}$$

the growth rate of prices is $g_p = -\frac{1}{\theta} \sum_k \iota^k g^k$ *and worker value is* $g_v = \frac{1+\zeta}{\zeta} \frac{1}{\theta} \sum_k \iota^k g^k$*, with* g^k *as the Perron-Frobenius root of equation* (30)*, as described in Proposition 2.*

The equilibrium conditions on the transition path reveal a recursive structure: worker mobility (equations 31–33) determines innovation levels (equation 27), which determine technology levels and the trade equilibrium (equations 34–35), which determine profits and inventor wages (equations 38–39), which in turn shape worker mobility. This recursive loop, which links worker mobility to innovation to trade and back, is what allows this model to capture endogenous and directed innovation in a spatial setting.

A key implication of gradual technology diffusion is that trade shares depend on the entire history of past innovations (equations 35–37), not just the contemporaneous technology stock. To maintain the practical advantage of dynamic hat algebra – requiring only changes in trade costs rather than levels – this history dependence requires two additional sets of initial conditions. First, assuming that the economy is on a balanced growth path for $t \leq 0$, the technology stock at $t = 0$ serves as a sufficient statistic for the trajectory of innovation levels for $t \leq 0$. Second, data on wages at $t = 0$, along with trade shares and technology levels, allow for the recovery of initial trade costs from equation (70). Simulating the transition path for $t \geq 0$ then requires only information on changes in trade costs over time, since these changes can be converted to absolute levels given initial trade costs.[17]

3.4 Welfare Impacts of Shocks to Economic Fundamentals

Given the balanced growth and transition paths, I now decompose the welfare impact of an arbitrary anticipated sequence of counterfactual changes in fundamentals into its transitory and long-run growth components. Let $\acute{x}$ denote the counterfactual path and $\widehat{x}_{t'} = \frac{\acute{x}_{t'}}{\dot{x}_{t'}} = \frac{\acute{x}_{t'}/\acute{x}_{t}}{x_{t'}/x_{t}}$ denote counterfactual changes for any variable x. Define the welfare impact in **market** (d, k, h) of an anticipated sequence of counterfactual changes in fundamentals from time $t = 0$ as the compensating variation in consumption for market (d, k, h),

[17] The trade equilibrium does not incorporate detrended variables because production worker wages remain constant on the balanced growth path and can be determined independently of the sectoral price index, which declines over time. As a result, prices can be detrended *after* computing the trajectory of trade equilibria, which streamlines the numerical algorithm.

$\log \delta_d^{k,h}$, given by the following equation:

$$\acute{V}_{d,0}^{k,h} = V_{d,0}^{k,h} + \sum_{t' \in \mathcal{T}_0^{\infty}} \left(\frac{1}{1+\zeta}\right)^{t'} \log \delta_d^{k,h}.$$

Using this notation and definition of welfare, the impact of an anticipated sequence of counterfactual changes in fundamentals is given by the following proposition:

Proposition 4. *Given the transition and balanced growth paths, the* ***welfare*** *effects in each market (i.e., region-sector-occupation) of an anticipated sequence of counterfactual changes in fundamentals are given by:*

$$\log\left(\delta_d^{k,h}\right) = \sum_{t' \in \mathcal{T}_{\mathbb{R}+}} \left(\frac{1}{1+\zeta}\right)^{t'} \log\left(\underbrace{\frac{\widehat{w_{d,t}^{k,h}}}{\widehat{\widetilde{P}_{d,t}}}}_{\substack{\text{change in future detrended real wages}}} \underbrace{\frac{1}{\left(\widehat{\mu_{dd,t}^{kk}}\right)^{1/\Upsilon}\left(\widehat{\mu_{dd,t}^{kk,hh}|\mu_{dd,t}^{kk}}\right)^{\upsilon/\Upsilon}}}_{\text{change in option value of moving}} \right) + \underbrace{\frac{1}{\theta}\sum_k \iota^k\left(\acute{g}^k - g^k\right)}_{\text{growth effects}}. \quad (42)$$

Local and aggregate welfare are defined as the population-weighted average of the welfare impacts in the relevant markets.

There are two key differences in this welfare expression compared to Caliendo et al. (2019). First, the option value of moving is determined by both the own-migration share across region-sectors and the conditional own-migration share across occupations, because workers' idiosyncratic preferences for each occupation are correlated across region-sectors. Second, the welfare expression accounts for long-run growth, since a counterfactual change in fundamentals affects both transitory and growth outcomes in my model.

3.5 Model Extensions

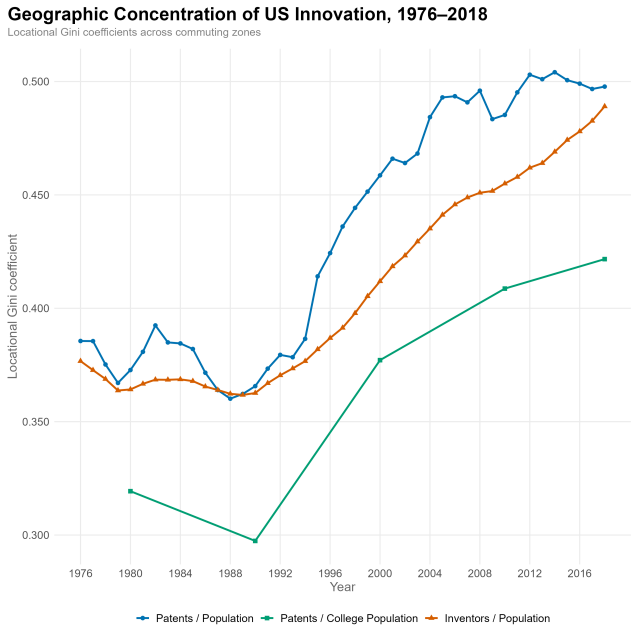
My dynamic spatial model microfounds innovation and technology diffusion *within* the Eaton-Kortum world. Consequently, it is highly tractable and can accommodate a broad range of extensions. These include dynamic worker sorting, input-output loops, capital accumulation, and the inclusion of amenities with congestion and agglomeration in goods production, as demonstrated in Appendix B.

4 Application: The Changing Geography of American Innovation

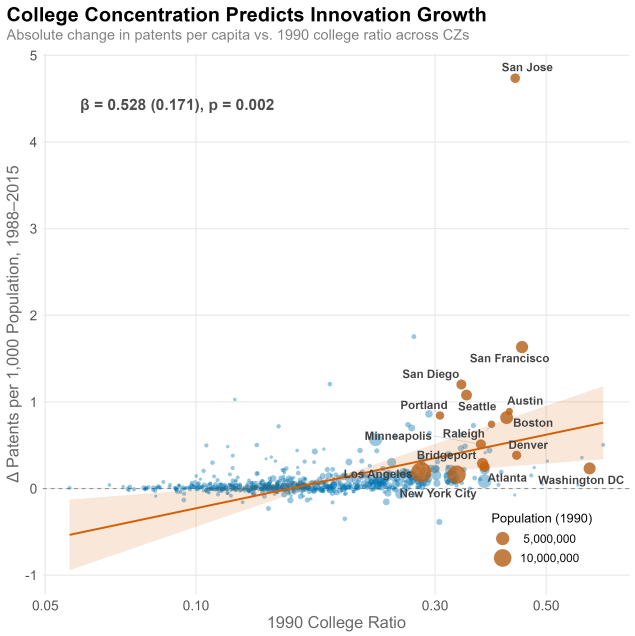
I now apply the quantitative framework to examine the causes and consequences of the changing geography of innovation in the United States in general equilibrium.

4.1 Empirical Context

A large literature has studied the geography of innovation, documenting where patents and inventors are located and the role of agglomeration economies, knowledge spillovers, and human capital in shaping these patterns (Jaffe et al., 1993; Audretsch and Feldman, 1996; Moretti, 2021). A striking finding, documented by Andrews and Whalley (2022) and Tan (2026), is that the spatial concentration of patents across Commuting Zones (CZs) in the US remained approximately constant or fell slightly between 1976 and 1990 but rose sharply thereafter. Tan (2026) further shows that this rise is concentrated in CZs with high initial shares of college-educated workers, and conducts progressive decompositions – by technology field, subfield, and firm – to trace the aggregate trend to the compositional shift in innovation toward ICT and the growth of high-skill CZ firms, i.e., firms that initially concentrated innovation in high-skill CZs. Figure 1 reproduces two key patterns from Tan (2026): the left panel plots the locational Gini coefficient of patents over time, showing the sharp post-1990 rise, and the right panel plots changes in patents per capita against the 1990 college ratio across CZs, showing that the increase is concentrated in high-skill regions.



(a) Trends in locational Gini coefficients



(b) Change in patents per capita vs. 1990 college ratio

Figure 1: The rising spatial concentration of innovation in high-skill regions after 1990. The left panel plots annual locational Gini coefficients of patents and inventors relative to population from 1976 to 2018. The right panel plots the absolute change in patents per 1,000 population between 1986–1990 and 2013–2017 against the 1990 college ratio across CZs. Source: Tan (2026).

The rising concentration of innovation in high-skill CZs reflects both demand- and supply-side forces. On the demand side, the growth of ICT and the expansion of firms that initially concentrated innovation in these regions created innovation incentives in high-skill CZs (Tan, 2026). On the supply side, college-educated workers increasingly sorted into cities with already high college shares from 1980 to 2000 (Moretti, 2013; Diamond, 2016; Diamond and Gaubert, 2022), expanding the pool of potential inventors. Figure 2 shows that this sorting occurred primarily before 1990 – before the sharp rise in innovation concentration. The correlation between changes in the college ratio and the initial college ratio peaked sharply around 1990 (left panel), driven by college-educated workers disproportionately moving to high-skill CZs during the 1980s (right panel); both correlations fell substantially after 1990. This supply of skilled workers may have created the human-capital foundations that enabled the subsequent growth of ICT-oriented firms and their innovation in these regions. Because supply and demand are jointly determined in equilibrium, disentangling their contributions requires the general equilibrium model.

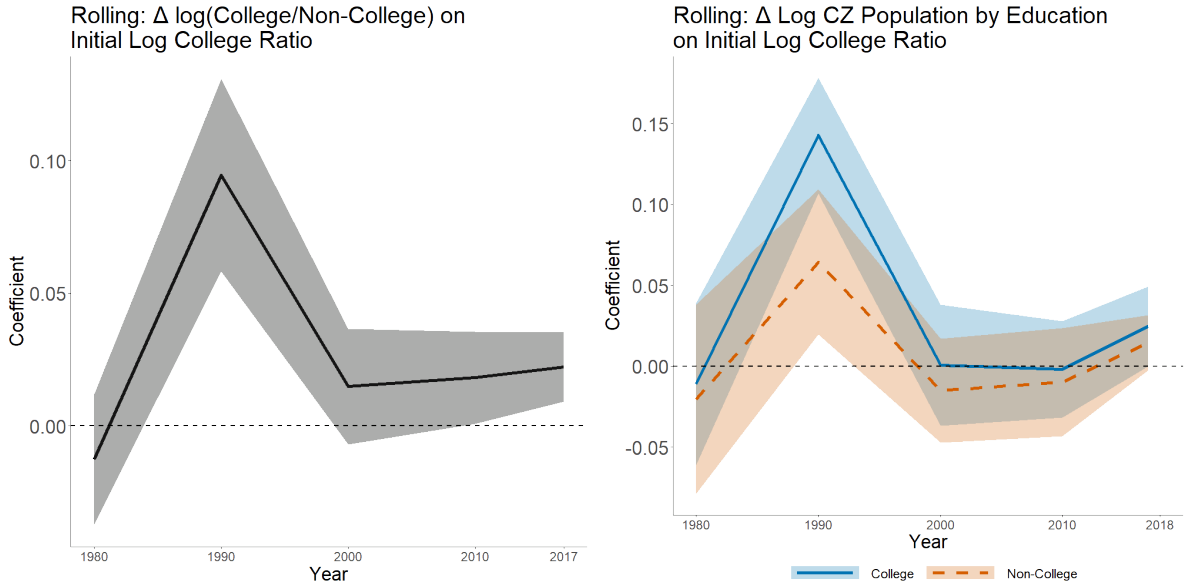


Figure 2: Worker sorting toward high-skill CZs. The left panel plots the rolling correlation of changes in the log college ratio with the initial college ratio for each decade from 1980 to 2018. The right panel plots the same rolling correlation separately for changes in log CZ population of college-educated and non-college workers.

Beyond decomposing the drivers of the post-1990 rise, a key open question is what the aggregate consequences of the rising spatial concentration of innovation have been for growth and welfare. Greater concentration could raise aggregate innovation output if agglomeration economies amplify research productivity (Moretti, 2021), but it could also reduce welfare in regions left behind. The general equilibrium model allows us to move beyond aggregate innovation output to quantify effects on aggregate growth and welfare.

4.2 Quantification Strategy and Data

To apply the general framework of Section 2 to the changing geography of American innovation, I specify the ICT shock through two exogenous components, describe how the model endogenously generates the

observed spatial patterns, and summarize the data and calibration strategy.

The ICT Shock. The ICT shock enters the model through two exogenous components: (i) a one-time increase in the national productivity of ICT innovation A_{1990}^{ICT}, reflected in the Poisson arrival rate of new ideas in the ICT sector (second term of equation 27). This ensures that ICT innovation levels rise in all regions relative to a scenario without the shock, generating a compositional shift of innovation toward ICT; and (ii) a decrease in the national component of bilateral diffusion speeds δ_{t^*}, representing reduced communication costs (application of Assumption 1).

Model Mechanisms. The two exogenous components endogenously generate the observed spatial patterns through three channels. First, the one-time increase in ICT research productivity raises innovation disproportionately in regions where ICT production is concentrated, because idea adoption shares (equation 14) are higher for locally produced goods. This captures the *colocation* of ICT innovation with ICT production in high-skill regions. Second, reduced communication costs – modeled as a uniform increase in bilateral diffusion speeds – disproportionately raise idea market access in high-skill (high-wage) regions, generating an *asymmetric scale effect* (Corollary 2). This captures how ICT-induced firm spatial expansion concentrated innovation further in high-skill regions. Third, dynamic worker mobility captures how *worker sorting* toward high-skill CZs in the 1980s provided the human-capital foundations that enabled the subsequent growth of ICT-oriented firms, and how the ICT shock further attracted workers to high-skill regions. Technology diffusion in the model is interpreted as primarily occurring *within* firms, consistent with strong intellectual property protections in the US and the role of firm spatial expansion following the ICT shock.

Data. The model can be simulated with several sets of bilateral data and a few key elasticities, as in standard quantitative trade and spatial models. Patent data from the USPTO provides innovation levels by CZ and sector, as well as mobility flows between production and research. Bilateral worker mobility flows by education come from IPUMS USA (Ruggles et al., 2023) and the Decennial Census, supplemented by IRS county-to-county migration data and IPUMS NHGIS (Schroeder et al., 2025) for population by education. Intersectoral mobility flows come from IPUMS CPS (Flood et al., 2025). Bilateral technology diffusion flows are constructed from Dun & Bradstreet establishment data, using the plant network of multi-establishment firms as a proxy for within-firm knowledge flows. Bilateral trade flows come from the Commodity Flow Survey, supplemented by BEA county-by-industry wage data. Initial technology levels are recovered by inverting the model's trade share equation given observed wages, employment, and trade shares. Appendix C.2 provides full details on data construction.

Calibration and Estimation. Most parameters are calibrated from the existing literature: the trade elasticity $\theta = 5$ (Costinot and Rodríguez-Clare, 2014), the migration elasticity across regions and sectors $\upsilon = 3$ (Caliendo et al., 2019), and the diffusion elasticity $\rho = 0.9$ (Lind and Ramondo, 2024). Since the diffusion elasticity was estimated in an international setting, I simulate the model across a range of values for robustness. The elasticity of worker mobility between production and research Υ can be set at $\Upsilon = 3$ mirroring the migration elasticity, or estimated using the constructed mobility flows and inventor wages; I examine robustness across a range of values. The two exogenous components of the ICT shock – the

increase in ICT research productivity A_{1990}^{ICT} and the trajectory of bilateral diffusion speeds – are disciplined by the observed time series of ICT patent counts and the bilateral firm establishment network, respectively. Appendix C.3 provides further details on parameter estimation. The simulation algorithm, which iterates on the transition and balanced growth paths using Propositions 2 and 3, is described in Appendix C.4.

4.3 Quantitative Results

[To be completed.]

4.3.1 Drivers of the Rising Spatial Concentration of US Innovation

4.3.2 Consequences on Aggregate Growth and Welfare

5 Conclusion

This paper introduces directed innovation into a quantitative spatial model. I integrate endogenous innovation with gradual technology diffusion at the level of *individual ideas* within the Eaton-Kortum world, alongside dynamic worker mobility. Profit incentives endogenously direct innovation across regions through the interaction of research productivity, gradual technology diffusion, trade, and worker mobility.

The framework delivers three main results. First, I characterize the *spatial direction of innovation* – the spatial analog of the sectoral direction of technical change in Acemoglu (2002) – which governs how profit incentives direct innovation across regions along the transition path. Second, I show that any steady-state distribution of innovation is consistent with balanced growth and determines the economy's long-run growth rate, connecting quantitative spatial models with endogenous growth theory. Third, I analytically decompose the welfare impact of any shock to economic fundamentals into its transitory and long-run growth components.

I apply the framework to the post-1990 rise in the spatial concentration of American innovation, which was concentrated in high-skill regions and driven primarily by the growth of ICT innovation (Tan, 2026). The model decomposes the contributions of demand-side forces – changing innovation incentives across regions through the trade and technology diffusion networks – and supply-side forces – the sorting of skilled workers toward high-skill regions – and quantifies the aggregate consequences for growth and welfare.

More broadly, the framework's tractability and generality make it applicable to a variety of empirical settings beyond the ICT shock studied here.

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A Proofs and Extensions of Propositions and Lemmas

Lemmas 1 to 6 provide the equilibrium conditions that govern endogenous innovation and technology diffusion; Lemma 7 details the equilibrium conditions on dynamic worker mobility; Proposition 1 characterizes the *spatial direction of innovation* and; Propositions 2 to 4 *decomposes the welfare impacts of any shock to economic fundamentals into its transitory and long-run growth components.*

Specifically, in the proofs of Lemmas 1 and 2, I leverage techniques from Eaton and Kortum (2024) to provide a more general derivation of the multivariate productivity distribution, minimum cost distribution, and trade shares relative to Lind and Ramondo (2024). This generalization allows me to introduce imperfect competition, characterized by the price index (Lemma 3) and markup distribution (Lemma 4). Along with worker mobility (Lemma 7), I thus *introduce directed innovation in a dynamic spatial model*, characterized by the ratio of inventor wages across regions (Proposition 1) which incorporate the expected value of individual ideas (Lemma 5).

A.1 Lemma 1 (Productivity Distribution)

Proof. I first derive the conditional distribution of applicabilities of an idea, then the goods productivity distribution across all ideas.

(i) Distribution of Applicabilities of an Idea

Microfoundations After an idea is discovered in region r at time t^*, independent stochastic processes of rate $\frac{\tilde{\Omega}}{t-t^*} \cdot \Omega'_{ro,t^*}(t-t^*)dt$ govern its diffusion to each of the other regions $o \neq r$. $\Omega'_{ro,t^*}(t-t^*)$ captures how the density or fertility of an idea in creating applications evolves with the age of the idea, $t-t^*$.

At each instance the idea arrives in region o, its applicability a is stochastic and drawn from a Pareto distribution:

$$H^A(a) = 1 - \left(\frac{a}{\underline{a}}\right)^{-\sigma}, \qquad a \geq \underline{a}, \quad \sigma > \theta, \quad \underline{a} = \tilde{a} \cdot \Gamma\left(1 - \frac{\theta}{\sigma}\right)^{-\frac{1}{\theta}}.$$

Thus, the probability that each application is greater than a is:

$$p_a = 1 - H^A(a) = \Gamma\left(1 - \frac{\theta}{\sigma}\right)^{-\frac{\sigma}{\theta}} a^{-\sigma}.$$

The number of applications of this idea in region o that is greater than a by time t follows a binomial distribution with expectation:

$$\mathbb{E} = \Omega_{ro,t^*}(t-t^*) \cdot \Gamma\left(1 - \frac{\theta}{\sigma}\right)^{-\frac{\sigma}{\theta}} \cdot a^{-\sigma} \cdot \tilde{\Omega}\tilde{a}^{\sigma}.$$

I assume that as the number of applications get arbitrarily large ($\tilde{\Omega} \to \infty$) and the applicability of a typical idea becomes arbitrarily small ($\tilde{a}^{\sigma} \to 0$), its product converges to 1 ($\tilde{\Omega} \cdot \tilde{a}^{\sigma} \to 1$). In this limit, the number

of applications greater than a by time t converges to a Poisson distribution with the same expectation. Thus, the number of applications with applicability $A > a$ that have arrived in region o by time t of an idea discovered in region r at time t^* is distributed Poisson with parameter:

$$\lambda^A(a) = M_{ro,tt^*} a^{-\sigma}, \qquad M_{ro,tt^*} \equiv \Omega_{ro,t^*}(t-t^*) \cdot \Gamma\left(1-\frac{\theta}{\sigma}\right)^{-\frac{\sigma}{\theta}}, \quad a > 0,$$

which corresponds to equation (2) in the main text. The expected number of applications of this idea with applicability above a to ever arrive in region o is $\Gamma\left(1-\frac{\theta}{\sigma}\right)^{-\frac{\sigma}{\theta}} a^{-\sigma}$. The expected share of these applications that arrives by time t is $\Omega_{ro,t^*}(t-t^*)$.

Distribution of Applicabilities Using this Poisson distribution, the distribution of the best applicability of this idea in region o by time t is given by the probability that there is no application more efficient than a:

$$F^A_{ro,t^*t}(a) = \mathbb{P}\left[A_o \leq a\right] = e^{-M_{ro,tt^*} a^{-\sigma}} = \exp\left[-\Gamma\left(1-\frac{\theta}{\sigma}\right)^{-\frac{\sigma}{\theta}} \Omega_{ro,t^*}(t-t^*) \cdot a^{-\sigma}\right], \tag{43}$$

which is a Frechet distribution. Across all regions, the joint distribution of the best application in time t of an idea discovered in region r at time t^* is:

$$\begin{aligned} F^A_{r,t^*t}(a_1,\ldots,a_N) &= \mathbb{P}\left[A_1 \leq a_1,\ldots,A_N \leq a_N\right] = e^{-\sum_o M_{ro,tt^*} a_o^{-\sigma}} \\ &= \exp\left[-\Gamma\left(1-\frac{\theta}{\sigma}\right)^{-\frac{\sigma}{\theta}} \sum_o \Omega_{ro,t^*}(t-t^*) \cdot a_o^{-\sigma}\right], \end{aligned} \tag{44}$$

which is the product of independent Frechet distributions in each region o.

(ii) Goods Productivity Distribution

Microfoundations In region r at time t^*, ideas are discovered from a stochastic process at rate $\lambda_{r,t^*}dt$. At discovery, each idea i has stochastic quality drawn from a Pareto distribution:

$$H^Q(q) = 1 - \left(\frac{q}{\underline{q}}\right)^{-\theta}, \qquad q \geq \underline{q}, \quad \theta > 0,$$

and pertains to the production of a specific good drawn from the unit interval. I assume that as the number of ideas gets arbitrarily large ($\int_0^{t^*} \lambda_{r,t}dt \to \infty$), such that their quality converges to 0 ($\underline{q}^\theta \to 0$), their product $\underline{q}^\theta \int_0^{t^*} \lambda_{r,t}dt$ remains constant. Following a similar argument for the arrival of idea applications above, the number of ideas with quality greater than q that have been discovered in region r by time t^* is thus distributed Poisson with parameter:

$$\lambda^Q_{r,t}(q) = q^{-\theta}\underline{q}^\theta \int_0^{t^*} \lambda_{r,t}dt,$$

which corresponds to equation (1) in the main text.

After an idea is discovered in region r at time t^*, its diffusion to each of the other regions $o \neq r$ is described in part (i) above. The efficiency of an idea i to produce the corresponding good ν in region o at time t is the product of its quality and best applicability in region o at time t:

$$z_i = q_i a^*_{i,o,t}, \tag{45}$$

where $a^*_{i,o,t}$ is drawn from equation (43). Each region o produces each good ν with the most efficient idea:

$$Z_{o,t} = \max_i \left\{ q_i a^*_{i,o,t} \right\}. \tag{46}$$

Marginal Productivity Distribution In Each Region Using the definition of idea efficiency in equation 45, I derive the Poisson process of idea application arrivals in region o incorporating both idea quality on its discovery in region r and idea applicability on its arrival in region o. The expected number of ideas with efficiency $Z > z$ in region o at time t is the total number of ideas produced across all regions with quality above $q = \frac{z}{A_{o,t}}$, where $A_{o,t}$ is the best application of the idea in region o by time t and held fixed when computing q:

$$\begin{aligned}
\lambda^Z_{o,t}(z) &= z^{-\theta} \sum_{r=1}^{N} \int_0^t \int_0^\infty A^\theta dF^A_{ro,tt^*}(a)\ \lambda_{r,t^*} dt^* \\
&= z^{-\theta} \sum_{r=1}^{N} \int_0^t \Gamma\left(1 - \frac{\theta}{\sigma}\right) \cdot \left[\Gamma\left(1 - \frac{\theta}{\sigma}\right)^{-\frac{\sigma}{\theta}} \Omega_{ro,t^*}(t - t^*) \right]^{\frac{\theta}{\sigma}} \lambda_{r,t^*} dt^* \\
&= z^{-\theta} \sum_{r=1}^{N} \int_0^t \left[\Omega_{ro,t^*}(t - t^*)\right]^{\frac{\theta}{\sigma}} \lambda_{r,t^*} dt^*
\end{aligned} \tag{47}$$

where the second equality comes from the mean of the Frechet distribution $F^A_{ro,tt^*}(a)$ in equation (43). Thus the total number of ideas with efficiency above z in region o at time t is distributed Poisson with parameter $\lambda^Z_{o,t}(z)$:

$$P\left[I = i\right] = e^{-\lambda^Z_{o,t}(z)} \frac{\left[\lambda^Z_{o,t}(z)\right]^i}{i!}.$$

The probability that the i'th most efficient idea has efficiency below z is the probability that at most $i - 1$ ideas have efficiency greater than z:

$$F^{Z(i)}_{o,t}(z) = e^{-\lambda^Z_{o,t}(z)} \sum_{l=0}^{i-1} \frac{\left[\lambda^Z_{o,t}(z)\right]^l}{l!}.$$

The marginal productivity distribution in region o at time t is given by the distribution of the most efficient idea:

$$F^{Z(1)}_{o,t}(z) = \mathbb{P}\left[Z^{(1)}_{o,t} \leq z\right] = \exp\left[-\lambda^Z_{o,t}(z)\right] = \exp\left[-T_{o,t} z^{-\theta}\right]$$

where:

$$T_{o,t} = \sum_{r=1}^{N} \int_0^t \left[\Omega_{ro,t^*}(t-t^*)\right]^{\frac{\theta}{\sigma}} \lambda_{r,t^*} dt^*$$

which correspond to equations (6) and (7) in the main text.

Joint Productivity Distribution I now derive the Poisson process for idea application arrivals across all regions incorporating quality and applicability, and consequently the joint productivity distribution. The expected number of ideas with efficiency above z_o in at least one region at time t is equivalent to the expected number of ideas with quality $q > \min_o \frac{z_o}{A_{o,t}}$, where $A_{o,t}$ is the best application of the idea in region o at time t and held fixed when computing q:

$$\begin{aligned}
\lambda_t^Z(z_1,\ldots,z_N) &= \sum_{r=1}^{N} \int_0^t \int_0^\infty \left[\min_o \left\{\frac{z_o}{A_{o,t}}\right\}\right]^{-\theta} dF^A_{r,tt^*}(a_1,\ldots,a_N)\ \lambda_{r,t^*} dt^* \\
&= \sum_{r=1}^{N} \int_0^t \int_0^\infty \max_o \left\{A_o^\theta z_o^{-\theta}\right\} dF^A_{r,tt^*}(a_1,\ldots,a_N)\ \lambda_{r,t^*} dt^* \\
&= \sum_{r=1}^{N} \int_0^t \Gamma\left(1-\frac{\theta}{\sigma}\right) \cdot \left[\Gamma\left(1-\frac{\theta}{\sigma}\right)^{-\frac{\sigma}{\theta}} \sum_{o=1}^{N} \Omega_{ro,t^*}(t-t^*)\cdot z_o^{-\sigma}\right]^{\frac{\theta}{\sigma}} \lambda_{r,t^*} dt^* \\
&= \sum_{r=1}^{N} \int_0^t \left[\sum_{o=1}^{N} \Omega_{ro,t^*}(t-t^*)\cdot z_o^{-\sigma}\right]^{\frac{\theta}{\sigma}} \lambda_{r,t^*} dt^*
\end{aligned} \tag{48}$$

where the second equality comes from solving the integral:

$$\int_0^\infty \max_o \left\{A_o^\theta z_o^{-\theta}\right\} dF^A_{r,tt^*}(a_1,\ldots,a_N) = \mathbb{E}\left[\max_o \left\{A_o^\theta Z_o^{-\theta}\right\}\right] \tag{49}$$

via three steps: (i) computing the distribution of the transformed variable $B_o \equiv \frac{a_o}{z_o}$:

$$\mathbb{P}(B_o \le b) = \mathbb{P}\left(\frac{A_o}{z_o} \le b\right) = \mathbb{P}\left(A_o \le b z_o\right) = \exp\left[-\Gamma\left(1-\frac{\theta}{\sigma}\right)^{-\frac{\sigma}{\theta}} \Omega_{ro,t^*}(t-t^*)\cdot b^{-\sigma} z_o^{-\sigma}\right],$$

(ii) computing the distribution of the maximum of the transformed variable across regions o, $\max_o B_o$:

$$\mathbb{P}\left(\max_o B_o \le b\right) = \exp\left[-\Gamma\left(1-\frac{\theta}{\sigma}\right)^{-\frac{\sigma}{\theta}} \sum_o \Omega_{ro,t^*}(t-t^*)\cdot z_o^{-\sigma}\cdot b^{-\sigma}\right],$$

and (iii) computing the moment $\mathbb{E}\left[\max_o B_o^\theta\right]$ using the known formula for Frechet random variables:

$$\mathbb{E}\left[\max_o B_o^\theta\right] = \mathbb{E}\left[\left(\max_o B_o\right)^\theta\right] = \Gamma\left(1-\frac{\theta}{\sigma}\right)\left[\Gamma\left(1-\frac{\theta}{\sigma}\right)^{-\frac{\sigma}{\theta}} \sum_o \Omega_{ro,t^*}(t-t^*)\cdot z_o^{-\sigma}\right]^{\frac{\theta}{\sigma}}.$$

Using this Poisson process, the number of ideas with efficiency above z_o in at least one region by time t is distributed Poisson with parameter $\lambda_t^Z(z_1, \ldots, z_N)$:

$$\mathbb{P}\left[I = i\right] = e^{-\lambda_t^Z(z_1,\ldots,z_N)} \frac{\left[\lambda_t^Z(z_1, \ldots, z_N)\right]^i}{i!}.$$

The probability that the i'th most efficient idea has efficiency below z is the probability that at most $i-1$ ideas have efficiency greater than z:

$$F_t^{Z(i)}(z_1, \ldots, z_N) = e^{-\lambda_t^Z(z_1,\ldots,z_N)} \sum_{l=0}^{i-1} \frac{\left[\lambda_t^Z(z_1, \ldots, z_N)\right]^l}{l!}.$$

The joint productivity distribution across all regions at time t is given by the distribution of the most efficient idea:

$$\mathbb{P}[Z_1 \leq z_1, \ldots, Z_N \leq z_n] = F_t^{Z(1)}(z_1, \ldots, z_N) = \exp\left[-\sum_{r=1}^{N} \int_0^t \left[\sum_{o=1}^{N} \Omega_{ro,t^*}(t-t^*) \cdot z_o^{-\sigma}\right]^{\frac{\theta}{\sigma}} \lambda_{r,t^*} dt^*\right],$$

which corresponds to equation (5) in the main text. □

A.2 Lemma 2 (Distribution of the Minimum Costs and Trade Shares)

Proof. I first derive the unconditional cost distribution, then the conditional cost distribution in each region, the joint cost distribution, and finally the equilibrium trade shares.

(i) Distribution of the Minimum Cost in each Destination

Each destination sources goods from the cheapest location. Thus, what matters is the number of ideas whose goods can be delivered to region d below cost c by some region o by time t. The expected number of these ideas is the expected number of ideas with efficiency greater than $z_o = \frac{w_{o,t}\tau_{od,t}}{c}$ in at least one region o:

$$\lambda_{d,t}^C(c) = \lambda_t^Z\left(\frac{w_{1,t}\tau_{1d,t}}{c}, \ldots, \frac{w_{N,t}\tau_{Nd,t}}{c}\right) = c^\theta \sum_{r=1}^{N} \int_0^t \left[\sum_{o=1}^{N} \Omega_{ro,t^*}(t-t^*) \cdot (w_{o,t}\tau_{od,t})^{-\sigma}\right]^{\frac{\theta}{\sigma}} \lambda_{r,t^*} dt^*. \tag{50}$$

Thus, the number of ideas whose goods can be delivered to region d below cost c by time t – regardless of production and innovation location – is distributed Poisson with parameter $\lambda_{d,t}^C(c)$. Ranking these ideas by their costs in destination d, $C^{(1)} < C^{(2)} < C^{(3)} < \ldots$, the probability that the i'th idea is above cost c is given by:

$$e^{-\lambda_{d,t}^C(c)} \sum_{l=0}^{i-1} \frac{\left[\lambda_{d,t}^C(c)\right]^l}{l!}$$

Thus the probability that the i'th idea is below cost c is given by:

$$G_{d,t}^{(i)}(c) = 1 - e^{-\lambda_{d,t}^{C}(c)} \sum_{l=0}^{i-1} \frac{\left[\lambda_{d,t}^{C}(c)\right]^{l}}{l!} = 1 - \mathbb{P}\left[C_{1d,t}^{(i)} > c, \ldots, C_{Nd,t}^{(i)} > c\right]. \tag{51}$$

In particular, the distribution of the lowest cost is given by:

$$G_{d,t}^{(1)}(c) = 1 - e^{-\lambda_{d,t}^{C}(c)} = 1 - \exp\left\{ - \left[\sum_{r=1}^{N} \int_{0}^{t} \left[\sum_{o=1}^{N} \Omega_{ro,t}(t - t^{*}) \left(w_{o,t} \tau_{od,t} \right)^{-\sigma} \right]^{\frac{\theta}{\sigma}} \lambda_{r,t^{*}} dt^{*} \right] c^{\theta} \right\}.$$

(ii) Equilibrium Trade Shares

Because different production locations may potentially produce goods from the same idea, the number of potential goods produced by each region o is not independent. This correlation across production locations implies that the straightforward way of computing trade shares does not hold, $\pi_{od,t} \neq \frac{\lambda_{od,t}^{C}(c)}{\lambda_{d,t}^{C}(c)}$. Instead, trade shares take on a nested form reflecting this correlation.

Since the number of ideas is independent across research locations, the share of goods purchased by destination d that were produced from ideas discovered in research location r is given by:

$$\phi_{rd,t} = \frac{\lambda_{rd,t}^{C}(c)}{\lambda_{d,t}^{C}(c)} = \frac{\int_{0}^{t} \left[\sum_{o=1}^{N} \Omega_{ro,t^{*}}(t - t^{*}) \cdot \left(w_{o,t} \tau_{od,t} \right)^{-\sigma} \right]^{\frac{\theta}{\sigma}} \lambda_{r,t^{*}} dt^{*}}{\sum_{r'=1}^{N} \int_{0}^{t} \left[\sum_{o=1}^{N} \Omega_{r'o,t^{*}}(t - t^{*}) \cdot \left(w_{o,t} \tau_{od,t} \right)^{-\sigma} \right]^{\frac{\theta}{\sigma}} \lambda_{r',t^{*}} dt^{*}}.$$

For each idea discovered in region r at time t^{*} whose good is eventually sold to region d at time t, its quality q and cost c are fixed and the number of applications in each production location o is independent. Thus, the share of these ideas whose goods are produced in region o is given by the number of applications with applicability above $a = \frac{w_{o,t} \tau_{od,t}}{q \cdot c}$ in region o divided by the total number of applications with applicability above $a = \frac{w_{o',t} \tau_{o'd,t}}{q \cdot c}$ for all regions o':

$$\varphi_{o|rd,t^{*}t} = \frac{\lambda_{o|rd,t}^{A}\left(\frac{w_{o,t} \tau_{od,t}}{q \cdot c} \right)}{\sum_{o'} \lambda_{o'|rd,t}^{A}\left(\frac{w_{o',t} \tau_{o'd,t}}{q \cdot c} \right)} = \frac{\Omega_{ro,t^{*}}(t - t^{*}) \left(w_{o,t} \tau_{od,t} \right)^{-\sigma}}{\sum_{o'=1}^{N} \Omega_{ro',t^{*}}(t - t^{*}) \left(w_{o',t} \tau_{o'd,t} \right)^{-\sigma}}.$$

where the function for the Poisson arrival of applications $\lambda^{A}(a)$ above a comes from equation (2) in the main text. Since this share is independent of q and hence independent across ideas, the share of goods sold in destination d at time t whose ideas were discovered in region r and goods produced in region o (the trilateral

share) is given by:

$$\pi_{rod,t} = \int_0^t \frac{\left[\sum_{o'=1}^{N} \Omega_{ro',t^*}(t-t^*)\cdot\left(w_{o',t}\tau_{o'd,t}\right)^{-\sigma}\right]^{\frac{\theta}{\sigma}}\lambda_{r,t^*}}{\sum_{r'=1}^{N}\int_0^t\left[\sum_{o'=1}^{N}\Omega_{r'o',t^*}(t-t^*)\cdot\left(w_{o',t}\tau_{o'd,t}\right)^{-\sigma}\right]^{\frac{\theta}{\sigma}}\lambda_{r',t^*}dt^*}\frac{\Omega_{ro,t^*}(t-t^*)\left(w_{o,t}\tau_{od,t}\right)^{-\sigma}}{\sum_{o'=1}^{N}\Omega_{ro',t^*}(t-t^*)\left(w_{o',t}\tau_{o'd,t}\right)^{-\sigma}}dt^*.$$

The trade shares are obtained by summing these trilateral shares across research locations: $\pi_{od,t} = \sum_r \pi_{rod,t}$. These expressions correspond to equations (10)-(13) in the main text. □

A.3 Lemma 3 (Price Index)

Proof. Under Bertrand competition, consumers regard different varieties (i.e. different idea applications) of a good as perfect substitutes. Since all goods are essential for the production of the local final good, the price charged for each good is its second lowest cost, drawn from the following distribution:

$$G_{d,t}^{(2)}(c) = 1 - e^{-\lambda_{d,t}^{C}(c)}\left[1+\lambda_{d,t}^{C}(c)\right] \tag{52}$$

where $\lambda_{d,t}^{C}(c)$ comes from equation (50). Notice that this is a special case of equation (51) with $i = 2$. To simplify the subsequent computations, I define:

$$\tilde{\Phi}_{d,t} \equiv \sum_{r=1}^{N}\int_0^t\left[\sum_{o=1}^{N}\Omega_{ro,t^*}(t-t^*)\cdot\left(w_{o,t}\tau_{od,t}\right)^{-\sigma}\right]^{\frac{\theta}{\sigma}}\lambda_{r,t^*}dt^*, \tag{53}$$

so that $\lambda_{d,t}^{C}(c) = \tilde{\Phi}_{d,t}c^{\theta}$ and $G_{d,t}^{(2)}(c) = 1 - e^{-\tilde{\Phi}_{d,t}c^{\theta}}\left[1+\tilde{\Phi}_{d,t}c^{\theta}\right]$.

Aggregating prices across all varieties yields the price index in each region:

$$P_{d,t} = \exp\int_0^1 \ln p_{d,t}(\nu)d\nu = \exp\int_0^{\infty}\ln c\; dG_{d,t}^{(2)}(c) = \gamma\tilde{\Phi}_{d,t}^{-\frac{1}{\theta}}$$

where γ is the Euler-Mascheroni constant. This corresponds to equation (15) in the main text. □

A.4 Lemma 4 (Markup Distribution)

Proof. Under Bertrand competition, the lowest cost producer of each good claims the entire market for that good, charging the highest markup that deters any competitor from entering. With the Cobb-Douglas aggregation across individual goods to produce the local sectoral final good (equation 8), the price of each good is given by the second lowest cost. Consequently, the markup of each good is the ratio of the lowest cost to the second lowest cost. I now apply the general derivation of the price gap distribution from Eaton and Kortum (2024) in my setting with idea applications, then use it to obtain the ratio of the lowest cost to the second lowest cost.

(i) Distribution of Transformed Costs

To derive the price gap distribution, it is helpful to work with transformed costs $U \equiv \tilde{\Phi} C^{\theta}$ where $\tilde{\Phi}$ is given by equation (53). I first derive the distribution for $U^{(i)}$. Ranking the $U^{(i)}$'s like the $C^{(i)}$'s, the distribution of the lowest transformed cost is the probability that there is no transformed cost below u:

$$\mathbb{P}\left[U^{(1)} \leq u\right] = 1 - e^{-u},$$

which is the unit exponential distribution. Given a lower transformed cost $U^{(i)} = u_i$, the probability that the next lowest transformed cost $U^{(i+1)}$ is less than u is simply the probability that there is no transformed cost between u_i and u:

$$\mathbb{P}\left[U^{(i+1)} \leq u | U^{(i)} = u_i\right] = 1 - e^{-(u-u_i)}. \tag{54}$$

Thus, the distribution of the gap conditional on the lower transformed cost is unit exponential. This implies that the set of ordered transformed costs can be drawn from independent unit exponential distributions V_i where:

$$U^{(1)} = V_1, \qquad U^{(2)} = U^{(1)} + V_2, \qquad U^{(3)} = U^{(2)} + V_3, \qquad \ldots. \tag{55}$$

Since $U^{(i)}$ is the sum of i independent unit exponential distributions, it follows an Erlang distribution with shape parameter i and rate parameter 1. Its probability density function is:

$$f^{(i)}(u) = \frac{u^{i-1} e^{u}}{(i-1)!} \tag{56}$$

and cumulative distribution function is:

$$F^{U(i)}(u) = \mathbb{P}\left[U^{(i)} \leq u\right] = 1 - e^{-u} \sum_{l=0}^{i-1} \frac{u^i}{i!} = \int_0^u \frac{x^{i-1} e^{x}}{(i-1)!}\, dx. \tag{57}$$

Notice that the distribution of transformed costs can also be directly obtained from the cost distribution (equation 51). This derivation provides intuition on why the cost and transformed cost distributions are Erlang.

(ii) Conditional Distribution of Lower Transformed Costs Given Higher Transformed Costs

Using the distribution of transformed costs $U^{(i)}$, I now derive the probability of the lower cost given a higher cost. The joint probability density function of $U^{(i)}$ and $U^{(i+1)}$ is given by:

$$\begin{aligned} f^{(i,i+1)}(u_i, u_{i+1}) &= \frac{d}{du_{i+1}} \left\{ \mathbb{P}\left[U^{(i+1)} \leq u_{i+1} | U^{(i)} = u_i\right] \cdot \mathbb{P}\left[U^{(i)} = u_i\right] \right\} \\ &= \frac{d}{du_{i+1}} \left\{ \left(1 - e^{-(u_{i+1}-u_i)}\right) \frac{u_i^{i-1} e^{u_i}}{(i-1)!} \right\} \\ &= \frac{u_i^{i-1} e^{u_{i+1}}}{(i-1)!} \end{aligned} \tag{58}$$

Thus the conditional distribution of the lower transformed costs (i.e. higher quality draw) is given by:

$$\mathbb{P}\left[U^{(i)} \leq u | U^{(i+1)} = u_{i+1}\right] = \int_0^u \frac{f^{(i,i+1)}(x, u_{i+1})}{f^{(i+1)}(u_{i+1})} dx = \int_0^u \frac{x^{i-1} e^{u_{i+1}}}{(i-1)!} \frac{i!}{u_{i+t}^i e^{u_{i+1}}} dx = \left(\frac{u}{u_{i+1}}\right)^i \tag{59}$$

which is a Pareto distribution with shape parameter i and scale parameter u_{i+1}.

(iii) Markup Distribution under Bertrand Competition

Using this result, the markup distribution conditional on the second lowest cost is given by:

$$\begin{aligned} G^{(2)/(1)}(m) &\equiv \mathbb{P}\left[\frac{C^{(2)}}{C^{(1)}} \leq m | C^{(2)} = c_2\right] = \mathbb{P}\left[\frac{U^{(2)}}{U^{(1)}} \leq m^\theta | U^{(2)} = u_2\right] \\ &= \mathbb{P}\left[U^{(1)} \geq u_2 m^{-\theta} | U^{(2)} = u_2\right] = 1 - \left(\frac{m^{-\theta} u_2}{u_2}\right) = 1 - m^{-\theta}. \end{aligned} \tag{60}$$

Remarkably, this distribution is independent of the second lowest cost c_2. Intuitively, this result arises because the conditional distribution of the higher quality draw is Pareto and hence scale invariant. That this, the ratio of the highest quality to the second highest quality idea is independent of the second highest quality idea.

Note that the markup distribution conditional on the lowest cost is different and given by:

$$\mathbb{P}\left[\frac{C^{(2)}}{C^{(1)}} \leq m | C^{(1)} = c_1\right] = \mathbb{P}\left[U^{(1)} \leq u_1 m^\theta | U^{(1)} = u_1\right] = 1 - \exp\left[-\Phi c_1^\theta \left(m^{1/\theta} - 1\right)\right] \tag{61}$$

In contrast to equation (60), this distribution depends on the lowest cost. Intuitively, a higher value of the higher quality draw implies that the cutoff/second highest quality draw must have been higher. □

A.5 Lemma 5 (Expected Value of an Idea)

Proof. Total profits earned at time s in region d is given by $\Pi_{d,t}^k = \frac{X_{d,t}^k}{1+\theta}$, as described in equation (17). The share of these profits from ideas discovered in region r at time t^* is ϕ_{rd,t^*t}^k. Since the flow rate of ideas in region r at time t^* is λ_{r,t^*}^k, the expected flow of profits at time s in region d of an idea discovered in region r at time t^* is $\frac{\phi_{rd,t^*t}^k}{\lambda_{r,t^*}^k} \frac{X_{d,t}^k}{1+\theta}$. Accounting for changes in the purchasing power in region r over time and discounting future flows yields equation (18). Notice that ϕ_{ld,t^*t}^k conditions on idea cohort and hence is a direct measure on whether the idea remains the lowest cost one in destination d. □

A.6 Lemma 6 (Market Clearing)

Proof. The market clearing condition is slightly more complex relative to standard trade models due to the transfer of profits across regions. To account for these transfers, I distinguish across profits earned from innovation $\overline{\Pi}$, production Π^*, and sales Π in each region. Income in each region $Y_{d,t}$ equals total expenditure

from the region, which comes from two sources: final spending by production workers and profits earned from innovation by firms. Thus the market clearing condition is given by:

$$\begin{aligned} w_{o,t}^{k,G} L_{o,t}^{k,G} + \Pi_{o,t}^{k\ *} &= \sum_d \pi_{od,t}^k \iota^k \left[\sum_k w_{d,t}^{k,G} L_{d,t}^{k,G} + \overline{\Pi_{d,t}^k} \right] \\ \implies \frac{1+\theta}{\theta} w_{o,t}^{k,G} L_{o,t}^{k,G} &= \sum_d \pi_{od,t}^k \iota^k \left[\sum_k \left(w_{d,t}^{k,G} L_{d,t}^{k,G} + \sum_r \varphi_{dr,t}^k \Pi_{r,t}^{k\ *} \right) \right] \\ \implies \frac{1+\theta}{\theta} w_{o,t}^{k,G} L_{o,t}^{k,G} &= \sum_d \pi_{od,t}^k \iota^k \left[\sum_k \left(w_{d,t}^{k,G} L_{d,t}^{k,G} + \sum_r \varphi_{dr,t}^k \frac{1}{\theta} w_{r,t}^{k,G} L_{r,t}^{k,G} \right) \right] \end{aligned} \tag{62}$$

where $\varphi_{dr,t}^k$ are the idea adoption shares and $\pi_{od,t}^k$ are the trade shares. Notice that I eliminate the profit terms in the market clearing condition by exploiting the fact that profits earned from production is a constant multiple of income earned by production workers in the region-sector:

$$w_{o,t}^{k,G} L_{o,t}^{k,G} + \frac{1}{1+\theta} \sum_d \pi_{od,t}^k X_{d,t}^k = \sum_d \pi_{od,t}^k X_{d,t}^k.$$

This enhances the tractability and simplifies the potential quantification of the model. □

A.7 Lemma 7 (Expected Worker Value and Worker Mobility Shares)

Proof. Note that the individual worker mobility problem, as defined by equations 22-23, represents a continuous time extension of Caliendo et al. (2019) (henceforth CDP) and incorporates switching between production and research alongside mobility across regions and sectors. In what follows, I demonstrate how the equilibrium mobility shares, as expressed in equation 25, can be derived under these extensions.

First, transitioning from discrete to continuous time with the possibility of worker movement complicates the distinction between current versus future payoffs. To overcome this challenge, I assume that there is a Poisson arrival process governing when *all* workers can move, with an arrival rate of 1, as described in Assumption 3 in the main text. This setup is a simplified version of Arcidiacono et al. (2016). With a Poisson arrival process of rate 1, the time until the next arrival at t' follows an exponential distribution with rate 1, where the probability density function is given by $e^{-(t'-t)}$. The present value at time t of the payoff at time t' is given by: $V_{t'} e^{-\zeta(t'-t)}$, where ζ is the discount rate. Thus, the expected value at time t of the payoff at time t' is given by:

$$\text{Expected Payoff} = V_{t'} \cdot \int_0^\infty e^{-\zeta(t'-t)} e^{-(t'-t)}\ d(t'-t) = \frac{1}{1+\rho} V_{t'}.$$

This is why the individual worker mobility problem in equation 22 has the discount factor $\frac{1}{1+\rho}$. Note that my formulation is a strict generalization of CDP. Though my discount factor $\frac{1}{1+\rho}$ is isomorphic to β in the migration problem in CDP and $\mathbb{E}_t(t') = t+1$, the actual migration time t' is stochastic and depends on the exact realization of the Poisson process for move arrivals.

Second, I incorporate switching between production and research, where individual-specific idiosyncratic shocks for each market are drawn from a multivariate Gumbel or generalized extreme value distribution with symmetric correlation between production and research across all region-sectors. The functional form of this distribution is provided in equation 23, with the correlation function defined as:

$$\check{F}\left(\left\{\exp\left(-\epsilon_{o,t}^{s,n}\right)\right\}_{o=1,\ldots,N}^{s=1,\ldots,K;\ n=\{G,R\}}\right) = \sum_{o}\sum_{s}\left(\sum_{n}\exp\left(-\epsilon_{o,t}^{s,n}\right)^{\frac{\Upsilon}{\upsilon}}\right)^{\upsilon}, \tag{63}$$

where υ captures the correlation between production and research across region-sectors, and Υ is the scale parameter that adjusts the sensitivity of the expected value in production or research to differences in the deterministic components of value in these activities. Note that this correlation function is homogeneous of degree $\log\Upsilon$ (instead of 1), similar to Ben-Akiva and Francois (1983), but otherwise retains the same properties listed in McFadden (1978).

I now use this correlation function to derive the option value of moving and the resulting mobility shares. Since the option value of moving is defined as:

$$\Phi_{d,t}^{k,h} = \mathbb{E}_{\epsilon}\left[\max_{o,s,n}\left\{\frac{1}{1+\rho}V_{o,t'}^{s,n} - \kappa_{do,t}^{ks,hn} + \epsilon_{o,t}^{s,n}\right\}\right],$$

I first derive the distribution of the maximum of random variables whose joint distribution follows the GEV distribution in equation (23). Let $U_{o,t}^{s,n} = \frac{1}{1+\rho}V_{o,t'}^{s,n} - \kappa_{do,t}^{ks,hn} + \epsilon_{o,t}^{s,n}$ and $U = \max_{o,s,n} U_{o,t}^{s,n}$. Extending Proposition 1 in Choi and Moon (1997), the cumulative distribution function is:

$$\begin{aligned}
&F_U(u) \equiv \mathbb{P}(U \leq u)\\
&= \mathbb{P}(U_1^{1,G} \leq u, \ldots, U_N^{K,R} \leq u)\\
&= \exp\left\{-\check{F}\left(\exp\left[\frac{1}{1+\rho}V_{1,t'}^{1,G} - \kappa_{d1,t}^{k1,hG} - u\right], \ldots, \exp\left[\frac{1}{1+\rho}V_{N,t'}^{K,R} - \kappa_{dN,t}^{kK,hR} - u\right]\right)\right\}\\
&= \exp\left\{-\exp(-\Upsilon u)\check{F}\left(\exp\left[\frac{1}{1+\rho}V_{1,t'}^{1,G} - \kappa_{d1,t}^{k1,hG}\right], \ldots, \exp\left[\frac{1}{1+\rho}V_{N,t'}^{K,R} - \kappa_{dN,t}^{kK,hR}\right]\right)\right\}\\
&= \exp\left\{-\exp\left(-\Upsilon\left[u - \frac{1}{\Upsilon}\log\left[\check{F}\left(\exp\left[\frac{1}{1+\rho}V_{1,t'}^{1,G} - \kappa_{d1,t}^{k1,hG}\right], \ldots, \exp\left[\frac{1}{1+\rho}V_{N,t'}^{K,R} - \kappa_{dN,t}^{kK,hR}\right]\right)\right]\right]\right)\right\}
\end{aligned}$$

where the second last equality comes from the correlation function being homogeneous of degree $\log\Upsilon$. Thus, the distribution of the maximum is a Gumbel distribution with scale parameter Υ and location parameter $\frac{1}{\Upsilon}\log\left[\check{F}\left(\exp\left[\frac{1}{1+\rho}V_{1,t'}^{1,G} - \kappa_{d1,t}^{k1,hG}\right], \ldots, \exp\left[\frac{1}{1+\rho}V_{N,t'}^{K,R} - \kappa_{dN,t}^{kK,hR}\right]\right)\right]$. The mean of this distribution is $\frac{1}{\Upsilon}\log\check{F} + \Upsilon\gamma$, where γ is Euler's constant. Thus the option value of moving – also known as the inclusive value of the entire choice set in the discrete choice literature – is given by:

$$\Phi_{d,t}^{k,h} = \frac{1}{\Upsilon}\log\left[\sum_{o,s}\left(\sum_{n}\exp\left(\frac{1}{1+\zeta}V_{o,t'}^{s,n} - \kappa_{do,t}^{ks,hn}\right)^{\frac{\Upsilon}{\upsilon}}\right)^{\upsilon}\right], \tag{64}$$

where the term $\Upsilon\gamma$ is constant over time and markets and hence can be normalized to 0. Substituting the

option value of moving into the individual worker migration problem in equation 22 yields the expected worker value in equation 24.

Mobility shares are given by the probability of choosing a specific market (o, s, n) to move to:

$$\begin{aligned}\mu_{do,t}^{ks,hn} &= \frac{\partial \Phi_{d,t}^{k,h}}{\partial \left(\frac{1}{1+\rho} V_{o,t'}^{s,n} - \kappa_{do,t}^{ks,hn}\right)} \\ &= \underbrace{\frac{\exp\left(\frac{1}{1+\zeta} V_{o,t'}^{s,n} - \kappa_{do,t}^{ks,hn}\right)^{\frac{\Upsilon}{\upsilon}}}{\sum_{n'} \exp\left(\frac{1}{1+\zeta} V_{o,t'}^{s,n'} - \kappa_{do,t}^{ks,hn'}\right)^{\frac{\Upsilon}{\upsilon}}}}_{\text{switching between production and research}} \cdot \underbrace{\frac{\left[\sum_{n'} \exp\left(\frac{1}{1+\zeta} V_{o,t'}^{s,n'} - \kappa_{do,t}^{ks,hn'}\right)^{\frac{\Upsilon}{\upsilon}}\right]^{\upsilon}}{\sum_{o'} \sum_{s'} \left[\sum_{n'} \exp\left(\frac{1}{1+\zeta} V_{o',t'}^{s',n'} - \kappa_{do',t}^{ks',hn'}\right)^{\frac{\Upsilon}{\upsilon}}\right]^{\upsilon}}}_{\text{migration across regions and sectors}}\end{aligned}$$

where the first equality comes from McFadden (1978). Note that given the correlation function, the individual worker mobility problem can also be recast as a nested discrete choice problem: each worker first chooses which region-sector to move to and then whether to supply their labor for research or production in that region-sector. In this formulation, we can interpret the second term as the probability of choosing region o and sector s among all region-sector alternatives and the first term as the conditional probability of choosing production or research given the region-sector choice. The conditional option value of choosing between production and research given the choice of region o and sector s is:

$$\Phi_{do,t}^{ks,h} = \frac{\upsilon}{\Upsilon} \log\left[\sum_{n} \exp\left(\frac{1}{1+\zeta} V_{o,t'}^{s,n} - \kappa_{do,t}^{ks,hn}\right)^{\frac{\Upsilon}{\upsilon}}\right].$$

The unconditional option value across all regions, sectors, and occupations – as shown in equation 64 – is related to this conditional option value as follows:

$$\Phi_{d,t}^{k,h} = \frac{1}{\upsilon} \log\left[\sum_{o,s} \exp\left(\Phi_{do,t}^{ks,h}\right)^{\upsilon}\right]. \tag{65}$$

Notice that when $\upsilon = 1$ in the correlation function, draws of ϵ are independent across markets, and the worker mobility shares and option value collapses to expressions of the form in Caliendo et al. (2019). By leveraging the correlation function and techniques from the foundational discrete choice literature, this proof *clarifies and generalizes dynamic migration in general equilibrium to a setting where preference shocks are correlated across markets*, i.e. drawn from a generalized extreme value distribution with any arbitrary correlation function $\check{F}$ that satisfies the properties listed in McFadden (1978) and μ-homogeneity in Ben-Akiva and Francois (1983). □

A.8 Proposition 1 (Spatial and Sectoral Direction of Innovation)

The ratio of inventor real wages across regions in the same sector follows directly from the text. Additionally, the sectoral direction of innovation is governed by:

$$\frac{\omega^{k,R}_{r,t^*}}{\omega^{k',R}_{r,t^*}} = \underbrace{\frac{A^k_{t^*}}{A^{k'}_{t^*}}}_{\substack{\text{differences in}\\ \text{fundamental}\\ \text{research}\\ \text{productivity}\\ \text{across sectors}}} \cdot \underbrace{\frac{T^k_{r,t^*}}{T^{k'}_{r,t^*}}}_{\substack{\text{relative}\\ \text{technology}\\ \text{levels}}} \cdot \underbrace{\left(\frac{L^{k,G}_{r,t^*}}{L^{k',G}_{r,t^*}}\right)^{\chi}}_{\substack{\text{benefits of}\\ \text{colocation of}\\ \text{innovation and}\\ \text{production}}} \cdot \underbrace{\frac{\int_{t^*}^{\infty} e^{-\zeta(t-t^*)} \sum_{d=1}^{N} \frac{\phi^k_{rd,t^*t}}{\lambda^k_{r,t^*}} \frac{X^k_{d,t}}{1+\theta} \frac{1}{P_{r,t}} dt}{\int_{t^*}^{\infty} e^{-\zeta(t-t^*)} \sum_{d=1}^{N} \frac{\phi^{k'}_{rd,t^*t}}{\lambda^{k'}_{r,t^*}} \frac{X^{k'}_{d,t}}{1+\theta} \frac{1}{P_{r,t}} dt}}_{\substack{\text{relative expected}\\ \text{value of an idea}}}. \tag{66}$$

The first term captures the one-time increase in ICT research productivity at a national level (the first exogenous component of the ICT shock), the second term captures relative technology levels across sectors, the third term anchors the field of innovative activity to the sectoral composition of production in the region, and the fourth term captures differences in the expected market potential of ideas across sectors.

A.9 Proposition 2 (Balanced Growth Path)

Proof. **Part (i)**: On the balanced growth path, technology levels in all regions and sectors grow at the same rate $g = \frac{\dot{T}^k_r(t)}{T^k_r} \forall r, k$, the exogenous bilateral diffusion lags δ_{ro} are constant over time and the innovation rate is given by:

$$\lambda^k_r(t) = \gamma^k_r T^k_r(t) \implies \dot{\lambda}^k_r(t) = \gamma^k_r \dot{T}^k_r(t). \tag{67}$$

From equation 7 we know that the technology level at each time t is given by:

$$T^k_{o,t} = \sum_{r=1}^{N} T^k_{ro,t} = \sum_{r=1}^{N} \int_{-\infty}^{t} \Omega_{ro}(t-t^*)^{1-\rho} \cdot \lambda^k_r(t^*) dt^*.$$

Taking the derivative w.r.t. t yields:

$$\dot{T}^k_{o,t} = \sum_{r=1}^{N} \int_{-\infty}^{t} \frac{d\Omega_{ro}(t-t^*)^{1-\rho}}{dt} \cdot \lambda^k_r(t^*) dt^* + \Omega_{ro}(0)^{1-\rho} \lambda^k_r(t).$$

Now using integration by parts with $u = \lambda^k_r(t^*)$ and $dv = \frac{d\Omega_{ro}(t-t^*)^{1-\rho}}{dt}$ yields:

$$\begin{aligned}
\dot{T}^k_{o,t} &= \sum_{r=1}^{N} \left[-\lambda^k_r(t^*)\Omega_{ro}(t-t^*)^{1-\rho}\right]_{t^*=-\infty}^{t^*=t} + \int_{-\infty}^{t} \Omega_{ro}(t-t^*)^{1-\rho} \dot{\lambda}^k_r(t^*) dt^* + \Omega_{ro}(0)^{1-\rho}\lambda_l(t) \\
&= \sum_{r=1}^{N} \lim_{t^* \to -\infty} \lambda^k_r(t^*)\Omega_{ro}(t-t^*)^{1-\rho} + \int_{-\infty}^{t} \Omega_{ro}(t-t^*)^{1-\rho} \gamma^k_r \dot{T}_r(t^*) dt^* \\
&= \sum_{r=1}^{N} \gamma^k_r \int_{-\infty}^{t} \Omega_{ro}(t-t^*)^{1-\rho} \dot{T}^k_r(t^*) dt^*.
\end{aligned}$$

Thus we have that:

$$\begin{aligned}\dot{T}^k_{o,t} &= \sum_{r=1}^{N}\gamma^k_r\int_{-\infty}^{t}\Omega_{ro}(t-t^*)^{1-\rho}\frac{\dot{T}^k_r(t^*)}{T^k_l(t^*)}T^k_r(t^*)dt^* \\ &= g\sum_{r=1}^{N}\gamma^k_r\int_{-\infty}^{t}\Omega_{ro}(t-t^*)^{1-\rho}T^k_r(t)e^{-g(t-t^*)}dt^* \\ &= \sum_{r}\gamma^k_r T^k_{r,t}\int_{-\infty}^{t}ge^{-g(t-t^*)}\left[1-e^{-\delta_{ro}(t-t^*)}\right]^{1-\rho}dt^* \\ &= \sum_{r}\gamma^k_r T^k_{r,t}\int_{0}^{\infty}ge^{-ga}\left[1-e^{-\delta_{ro}(a)}\right]^{1-\rho}da\end{aligned} \tag{68}$$

where $e^{-\delta_{rr}(t-t^*)} \equiv 0$ and the last equality follows from a change of variable $a = t - t^*$. Note that $\int_0^\infty ge^{-ga}\left[1-e^{-\delta_{ro}(a)}\right]^{1-\rho}da$ is a constant.

Now building on Eaton and Kortum (2024), in matrix form we have:

$$g\mathbf{T}^k = \mathbf{\Delta}^k(g)\mathbf{T}^k \tag{69}$$

where $\mathbf{T}^k$ is an $N\times 1$ vector with representative element T^k_r and $\mathbf{\Delta}^k(g)$ is an $N\times N$ matrix with representative element:

$$\Delta^k_{ro}(g) = \gamma^k_r\int_0^\infty ge^{-ga}\left[1-e^{-\delta_{ro}(a)}\right]^{1-\rho}da.$$

The aggregate growth rate is the Perron-Frobenius root of equation 69 with relative technology levels $\mathbf{T}$ corresponding to the Perron-Frobenius eigenvector that is defined up to a scalar multiple. Thus, any arbitrary set of exogenous diffusion speeds δ_{ro} and endogenous innovation rates γ^k_r delivers a balanced growth path with parallel growth at rate g with level differences in technology T across regions.

Part (ii): I now solve for the remaining variables in the economy on the BGP. Suppose that the distribution of workers across regions, sectors, and occupations are constant. Since the relative technology levels are also constant on the BGP, the trade shares given by equation 10 and the market condition given by equation 20 imply that relative wages across regions and sectors in production is constant. In particular, trade shares on the BGP are given by:

$$\pi^k_{od} = \sum_{r=1}^{N}\int_0^\infty \frac{\Omega_{ro}(a)\left(w^k_o\tau^k_{od}\right)^{-\frac{\theta}{1-\rho}}}{\sum_{o'=1}^{N}\Omega_{ro'}(a)\left(w^k_{o'}\tau^k_{o'd}\right)^{-\frac{\theta}{1-\rho}}}\cdot\frac{\gamma^k_r T^k_r e^{-g^k\cdot a}\left[\sum_{o'=1}^{N}\Omega_{ro'}(a)\left(w^k_{o'}\tau^k_{o'd}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}}{\sum_{r'=1}^{N}\gamma^k_{r'}T^k_{r'}\int_0^\infty\left[\sum_{o'=1}^{N}\Omega_{r'o'}(a')\left(w^k_{o'}\tau^k_{o'd}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}e^{-g^k\cdot a'}da'}da. \tag{70}$$

where T^k_r are the relative technology levels determined by equation 69.

On the BGP, the price index in each region and sector is given by:

$$P_{d,t}^{k\ BGP} = \Gamma \left[\sum_{r'=1}^{N} \gamma_{r'}^{k} T_{r',t}^{k} \int_{0}^{\infty} e^{-ga} \left[\sum_{o'=1}^{N} \Omega_{r'o'}(a) \left(w_{o'}^{k} \tau_{o'd}^{k} \right)^{-\frac{\theta}{1-\rho}} \right]^{1-\rho} da \right]^{-\frac{1}{\theta}} \tag{71}$$

Differentiating this expression w.r.t time, the growth rate of the sectoral price index is:

$$g_{P^k} = \frac{\dot{P_{d,t}^{k}}^{BGP}}{P_{d,t}^{k\ BGP}} = -\frac{1}{\theta} \sum_{r'} \check{\pi}_{r'd} \frac{\dot{T_{r',t}^{k}}}{T_{r',t}^{k}} = -\frac{g^k}{\theta} \tag{72}$$

Since preferences are Cobb-Douglas across local sectoral final goods for each region, the aggregate price index is:

$$P_{d,t} = \check{\Gamma} \prod_{k} \left(P_{d,t}^{k} \right)^{\iota^k} \tag{73}$$

where $\check{\Gamma}$ is a constant. The growth rate of the aggregate price index on the BGP is:

$$g_p = -\frac{1}{\theta} \sum_{k} \iota^k g^k \tag{74}$$

On the BGP, the expected value of an idea is:

$$\begin{aligned} \check{V}_{r,t^*}^{k} &= \frac{P_{r,t^*}}{1+\theta} \int e^{-\zeta(t-t^*)} \sum_{d} \frac{\phi_{rd,t^*t}}{\gamma_r^k T_{r,t^*}^k} \frac{X_{d,t}}{P_{r,t}} dt \\ &= \frac{\sum_d \phi_{rd} X_d}{1+\theta} \frac{1}{\gamma_r^k T_{r,t^*}^k} \int e^{-\zeta(t-t^*)} e^{-g_P(t-t^*)} dt \\ &= \frac{\sum_d \phi_{rd} X_d}{1+\theta} \frac{1}{\gamma_r^k T_{r,t^*}^k} \frac{1}{\zeta - g_P/\theta}. \end{aligned} \tag{75}$$

Thus $\check{V}_{r,t^*}^{k}$ is falling at rate g^k on the BGP while inventor wages are constant and given by:

$$w_r^{k,R} = \frac{\sum_d \check{\pi}_{rd} X_d}{1+\theta} \frac{1}{L_r^{k,R}} \frac{1}{\zeta - g_P/\theta}. \tag{76}$$

I now solve for the growth rate of worker expected value. Let $\exp\left(V_{d,t}^{k,h}\right) = \exp\left(\widetilde{V}_{d}^{k,h}\right) e^{g_V t}$ and $P_{d,t} = \widetilde{P}_d e^{g_P t}$, where $\widetilde{V}_{d}^{k,h}$ and $\widetilde{P}_d$ are the detrended value and price respectively. Thus given production worker

wages, inventor wages and local aggregate prices, worker expected value is given by:

$$\begin{aligned} V_{d,t}^{k,h} &= \widetilde{V}_d^{k,h} + g_V t \\ &= \log\left(\frac{w_d^{k,h}}{\widetilde{P}_d}\right) - g_P t + \frac{1}{\Upsilon}\log\left[\sum_o \sum_s \left(\sum_n \exp\left(\frac{1}{1+\zeta}\left[\widetilde{V}_o^{s,n} + g_V t'\right] - \kappa_{do}^{ks,hn}\right)^{\frac{\Upsilon}{\upsilon}}\right)^{\upsilon}\right] \\ &= \log\left(\frac{w_d^{k,h}}{\widetilde{P}_d}\right) - g_P t + \frac{1}{1+\zeta} g_V t' + \frac{1}{\Upsilon}\log\left[\sum_o \sum_s \left(\sum_n \exp\left(\frac{1}{1+\zeta}\widetilde{V}_o^{s,n} - \kappa_{do}^{ks,hn}\right)^{\frac{\Upsilon}{\upsilon}}\right)^{\upsilon}\right] \end{aligned} \tag{77}$$

On the BGP, the growth rate must be the same on both sides of the equation. Thus, the growth rate of expected value is given by:

$$\begin{aligned} g_v &= -g_p + \frac{1}{1+\zeta} g_v \\ \implies g_v &= -\frac{1+\zeta}{\zeta} g_p = \frac{1+\zeta}{\zeta}\frac{1}{\theta}\sum_k \iota^k g^k \end{aligned} \tag{78}$$

where the first equality comes from $\frac{1}{1+\zeta} g_v t' = \frac{1}{1+\zeta} g_v t + \frac{1}{1+\zeta} g_v$ in equation 77 since $\mathbb{E}_t(t'-t) = 1$, because the Poisson arrival rate of move possibilities is 1. The detrended expected value of workers is given by:

$$\widetilde{V}_d^{k,h} = \log\left(\frac{w_d^{k,h}}{\widetilde{P}_d}\right) + \frac{1}{1+\zeta} g_v + \frac{1}{\Upsilon}\log\left[\sum_o \sum_s \left(\sum_n \exp\left(\frac{1}{1+\zeta}\widetilde{V}_o^{s,n} - \kappa_{do}^{ks,hn}\right)^{\frac{\Upsilon}{\upsilon}}\right)^{\upsilon}\right]. \tag{79}$$

Substituting the decomposition of expected worker value in equation 77 into equation 25 in the main text, worker mobility shares are alternately given by:

$$\mu_{do}^{ks,hn} = \frac{\exp\left(\frac{1}{1+\zeta}\widetilde{V}_o^{s,n} - \kappa_{do}^{ks,hn}\right)^{\frac{\Upsilon}{\upsilon}}}{\sum_{n'} \exp\left(\frac{1}{1+\zeta}\widetilde{V}_o^{s,n'} - \kappa_{do,t}^{ks,hn'}\right)^{\frac{\Upsilon}{\upsilon}}} \cdot \frac{\left[\sum_{n'} \exp\left(\frac{1}{1+\zeta}\widetilde{V}_o^{s,n'} - \kappa_{do}^{ks,hn'}\right)^{\frac{\Upsilon}{\upsilon}}\right]^{\upsilon}}{\sum_{o'}\sum_{s'}\left[\sum_{n'} \exp\left(\frac{1}{1+\zeta}\widetilde{V}_{o'}^{s',n'} - \kappa_{do'}^{ks',hn'}\right)^{\frac{\Upsilon}{\upsilon}}\right]^{\upsilon}} \tag{80}$$

and are constant on the BGP. Hence, from equation 26, the distribution of workers across regions, sectors, and occupations are constant on the BGP.

Thus the balanced growth path of the economy is obtained, where workers, wages, migration shares, trade shares and innovation rates are constant, technology in each sector k and region is growing at rate g^k determined from equation 69, prices in each region is growing at rate $g_p = -\frac{1}{\theta}\sum_k \iota^k g^k$ and expected worker value is growing at rate $g_v = \frac{1+\zeta}{\zeta}\frac{1}{\theta}\sum_k \iota^k g^k$. □

Special Cases:

(i) When $\theta = \sigma$ (such that $\rho = 0$), we have the case of exponential diffusion where idea applicabilities do

not matter (**case NA** for no applicabilities). Equation (68) collapses to:

$$\begin{aligned}\dot{T}_{o,t}^{k} &= \sum_{r} \gamma_{r}^{k} T_{r,t}^{k} \int_{0}^{\infty} g e^{-ga} \left[1 - e^{-\delta_{ro}(a)}\right] da \\ &= \sum_{r} g \gamma_{r}^{k} T_{r,t}^{k} \left(\frac{1}{g} + \frac{1}{g + \delta_{ro}}\right) \\ &= \sum_{r} \left(\frac{\delta_{ro}}{g + \delta_{ro}}\right) \gamma_{r}^{k} T_{r,t}^{k}\end{aligned}$$

yielding equation (49) in Eaton and Kortum (2024). The trade shares and price index collapse to the canonical Eaton and Kortum (2002) expressions:

$$\pi_{od}^{NA} = \frac{T_{o,t} \left(w_o \tau_{od}\right)^{-\theta}}{\sum_{o'=1}^{N} T_{o',t} \left(w_{o'} \tau_{o'd}\right)^{-\theta}}, \quad P_{d,t}^{NA} = \Gamma \left[\sum_{o'=1}^{N} T_{o',t} \left(w_{o'} \tau_{o'd}\right)^{-\theta}\right]^{-\frac{1}{\theta}} \tag{81}$$

and idea diffusion shares are given by:

$$\varphi_{ro}^{NA} = \sum_{d=1}^{N} \frac{T_{ro,t} \left(w_o \tau_{od}\right)^{-\theta}}{\sum_{o'=1}^{N} T_{o',t}^{k} \left(w_{o'} \tau_{o'd}\right)^{-\theta}}. \tag{82}$$

All the other variables and growth rates remain the same as the full model.

(ii) When $\Omega_{ro,t^*}(t - t^*) = \delta_{ro,t}$ for $r \neq o$ and $t \geq t^*$, we have the case of instantaneous diffusion with idea applicabilities (**case ID**) as in Xiang (2023), equation (68) collapses to:

$$\begin{aligned}\dot{T}_{o,t}^{k} &= \sum_{r} \gamma_{r}^{k} T_{r,t}^{k} \int_{0}^{\infty} g e^{-ga} \delta_{ro}^{1-\rho} da \\ &= \sum_{r} \delta_{ro}^{1-\rho} \gamma_{r}^{k} T_{r,t}^{k}.\end{aligned}$$

The trade shares and price index collapse to the expressions in Ramondo and Rodríguez-Clare (2013):

$$\pi_{od}^{ID} = \sum_{r=1}^{N} \frac{\left(T_{ro,t}\right)^{\frac{1}{1-\rho}} \left(w_o \tau_{od}\right)^{-\frac{\theta}{1-\rho}}}{\sum_{o'=1}^{N} \left(T_{ro',t}\right)^{\frac{1}{1-\rho}} \left(w_{o'} \tau_{o'd}\right)^{-\frac{\theta}{1-\rho}}} \cdot \frac{\left[\sum_{o'=1}^{N} \left(T_{ro',t}\right)^{\frac{1}{1-\rho}} \left(w_{o'} \tau_{o'd}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}}{\sum_{r'=1}^{N} \left[\sum_{o'=1}^{N} \left(T_{ro',t}\right)^{\frac{1}{1-\rho}} \left(w_{o'} \tau_{o'd}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}} \tag{83}$$

$$P_{d,t}^{ID} = \Gamma \left[\sum_{r'=1}^{N} \left[\sum_{o'=1}^{N} \left(T_{r'o',t}\right)^{\frac{1}{1-\rho}} \left(w_{o'} \tau_{o'd}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}\right]^{-\frac{1}{\theta}} \tag{84}$$

and the idea diffusion shares are:

$$\varphi_{od,t}^{ID} = \frac{\left(T_{ro,t}\right)^{\frac{1}{1-\rho}} \left(w_o \tau_{od}\right)^{-\frac{\theta}{1-\rho}}}{\sum_{o'=1}^{N} \left(T_{ro',t}\right)^{\frac{1}{1-\rho}} \left(w_{o'} \tau_{o'd}\right)^{-\frac{\theta}{1-\rho}}} \cdot \frac{\left[\sum_{o'=1}^{N} \left(T_{ro',t}\right)^{\frac{1}{1-\rho}} \left(w_{o'} \tau_{o'd}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}}{\sum_{l'=1}^{N} \left[\sum_{o'=1}^{N} \left(T_{ro',t}\right)^{\frac{1}{1-\rho}} \left(w_{o'} \tau_{o'd}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}} \tag{85}$$

All the other variables and growth rates remain the same as the full model.

A.10 Proposition 3 (Transition Path)

Proof. I first characterize the transition path with Assumption 3, then with Assumption 4 in order to apply dynamic hat algebra and map my model to data on innovation levels, trade shares, migration shares, and firm establishment networks.

(i) Transition Path with Continuous Innovation, Production, and Consumption; and Migration at Discrete Moments in Time (Assumption 3)

The transition path towards balanced growth is characterized by a trajectory of detrended expected values given by equation 79 and worker mobility shares given by equation 80 but with all variables varying over time, alongside the evolution of worker population given by equation 26:

$$\begin{aligned}
\widetilde{V}_{d,t}^{k,h} &= \int_t^{t'} \log\left(\frac{w_{d,\check{t}}^{k,h}}{\widetilde{P}_{d,\check{t}}}\right) d\check{t} + \frac{1}{1+\zeta} g_v + \frac{1}{\Upsilon} \log\left[\sum_o \sum_s \left(\sum_n \exp\left(\frac{1}{1+\zeta} \widetilde{V}_{o,t'}^{s,n} - \kappa_{do}^{ks,hn}\right)^{\frac{\Upsilon}{\upsilon}}\right)^{\upsilon}\right] \\
\mu_{do,t}^{ks,hn} &= \frac{\exp\left(\frac{1}{1+\zeta} \widetilde{V}_{o,t'}^{s,n} - \kappa_{do,t}^{ks,hn}\right)^{\frac{\Upsilon}{\upsilon}}}{\sum_{n'} \exp\left(\frac{1}{1+\zeta} \widetilde{V}_{o,t'}^{s,n'} - \kappa_{do,t}^{ks,hn'}\right)^{\frac{\Upsilon}{\upsilon}}} \cdot \frac{\left[\sum_{n'} \exp\left(\frac{1}{1+\zeta} \widetilde{V}_{o,t'}^{s,n'} - \kappa_{do,t}^{ks,hn'}\right)^{\frac{\Upsilon}{\upsilon}}\right]^{\upsilon}}{\sum_{o'} \sum_{s'} \left[\sum_{n'} \exp\left(\frac{1}{1+\zeta} \widetilde{V}_{o',t'}^{s',n'} - \kappa_{do',t}^{ks',hn'}\right)^{\frac{\Upsilon}{\upsilon}}\right]^{\upsilon}} \\
L_{o,t'}^{s,n} &= \sum_h \sum_k \sum_d \mu_{do,t'}^{ks,hn} L_{d,t}^{k,h},
\end{aligned}$$

where at each time t the innovation and technology levels are given by equations 27 and 7 respectively:

$$\begin{aligned}
\lambda_{r,t}^k &= A_{t^*}^k A_{r,t^*} \left(L_{r,t^*}^{k,G}\right)^{\chi} L_{r,t^*}^{k,R} T_{r,t^*}^k \\
T_{o,t}^k &= \sum_{r=1}^{N} \int_{-\infty}^{t} \Omega_{ro,t^*} (t - t^*)^{1-\rho} \cdot \lambda_{r,t^*}^k dt^*,
\end{aligned}$$

the trade shares, idea adoption shares, price index, and market clearing condition are given by equations 10,

14, 15, and 20 respectively:

$$\pi_{od,t}^{k}=\sum_{r=1}^{N}\int_{-\infty}^{t}\frac{\Omega_{ro,t^{*}}(t-t^{*})\left(w_{o,t}^{k,G}\tau_{od,t}^{k}\right)^{-\frac{\theta}{1-\rho}}}{\sum_{o'}\Omega_{ro',t^{*}}(t-t^{*})\left(w_{o',t}^{k,G}\tau_{o'd,t}^{k}\right)^{-\frac{\theta}{1-\rho}}}\frac{\left[\sum_{o'}\Omega_{ro',t^{*}}(t-t^{*})\left(w_{r,t}^{k,G}\tau_{ro',t}^{k}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}\lambda_{r,t^{*}}^{k}}{\sum_{r'}\int_{-\infty}^{t}\left[\sum_{o'}\Omega_{r'o',\check{t}}(t-\check{t})\left(w_{o',t}^{k,G}\tau_{o'd,t}^{k}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}\lambda_{r',\check{t}}^{k}d\check{t}}dt^{*}$$

$$\varphi_{ro,t}^{k}=\sum_{d=1}^{N}\int_{-\infty}^{t}\frac{\Omega_{ro,t^{*}}(t-t^{*})\left(w_{o,t}^{k,G}\tau_{od,t}^{k}\right)^{-\frac{\theta}{1-\rho}}}{\sum_{o'}\Omega_{ro',t^{*}}(t-t^{*})\left(w_{o',t}^{k,G}\tau_{o'd,t}^{k}\right)^{-\frac{\theta}{1-\rho}}}\frac{\left[\sum_{o'}\Omega_{ro',t^{*}}(t-t^{*})\left(w_{r,t}^{k,G}\tau_{ro',t}^{k}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}\lambda_{r,t^{*}}^{k}}{\sum_{r'}\int_{-\infty}^{t}\left[\sum_{o'}\Omega_{r'o',\check{t}}(t-\check{t})\left(w_{o',t}^{k,G}\tau_{o'd,t}^{k}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}\lambda_{r',\check{t}}^{k}d\check{t}}dt^{*}$$

$$P_{d,t}^{k}=\gamma\left[\sum_{r'=1}^{N}\int_{-\infty}^{t}\left[\sum_{o'=1}^{N}\Omega_{r'o',t^{*}}(t-t^{*})\left(w_{o',t}^{k,G}\tau_{o'd,t}^{k}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}\lambda_{r',t^{*}}^{k}dt^{*}\right]^{-\frac{1}{\theta}}$$

$$\frac{1+\theta}{\theta}w_{o,t}^{k,G}L_{o,t}^{k,G}=\sum_{d}\pi_{od,t}^{k}\iota^{k}\left[\sum_{k}\left(w_{d,t}^{k,G}L_{d,t}^{k,G}+\sum_{r}\varphi_{dr,t}^{k}\frac{1+\theta}{\theta}w_{r,t}^{k,G}L_{r,t}^{k,G}\right)\right],$$

the wages of inventors are given by equation (19), the returns to innovation by equation (18), total expenditures by equation (40), and idea market shares by equation (41):

$$w_{r,t}^{k,R}=\frac{\check{V}_{r,t}^{k}\lambda_{r,t}^{k}}{L_{r,t}^{k,R}}$$

$$\check{V}_{r,t}^{k}=\int_{t}^{\infty}e^{-\zeta(t'-t)}\sum_{d=1}^{N}\frac{X_{d,t'}^{k}}{1+\theta}\cdot\frac{P_{r,t}}{P_{r,t'}}\cdot\frac{\check{\pi}_{ld}^{k}(t,t')}{\lambda_{l,t}^{k,\check{R}}}\;dt'$$

$$X_{d,t'}^{k}=\iota^{k}\left[\sum_{s}\left(w_{d,t'}^{s,G}L_{d,t'}^{s,G}+\sum_{r}\varphi_{dr,t'}^{s}\frac{1+\theta}{\theta}w_{r,t'}^{s,G}L_{r,t'}^{s,G}\right)\right]$$

$$\phi_{rd,tt'}^{k}=\frac{\left[\sum_{o'}\Omega_{lo',t}(t'-t)\left(w_{l',t'}^{k}\tau_{lo',t'}^{k}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}\lambda_{l,t}^{k,\check{R}}}{\sum_{l'}\int_{-\infty}^{t'}\left[\sum_{o'}\Omega_{l'o',\check{t}}(t'-\check{t})\left(w_{o',t'}^{k}\tau_{o'd,t'}^{k}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}\lambda_{l',\check{t}}^{k,\check{R}}d\check{t}}.$$

(ii) Transition Path with Innovation, Production, Consumption, and Migration Occuring Simultaneously at Discrete Moments in Time (Assumption 4)

I now apply Assumption 4 to characterize the transition path in time changes. Let $\mathcal{T}$ be the set of times where innovation, production, consumption, and migration all occur simultaneously, $\dot{x}_t=\frac{x_{t'}}{x_t}$ for any variable x, and $u=\exp(V)$. Then the detrended equations for the migration equilibrium (i.e. equations 79, 80, 26)

can be expressed in time changes:

$$\log\left(\widehat{u}_{d,t}^{k,h}\right) = \log\left(\frac{\widehat{w}_{d,t}^{k,h}}{\widehat{P}_{d,t}}\right) + \frac{1}{\Upsilon}\log\left[\sum_{n}\left[\sum_{s}\sum_{o}\mu_{do,t'}^{ks,hn}\left(\widehat{u}_{o,t'}^{s,n}\right)^{\frac{\Upsilon}{(1+\zeta)\upsilon}}\left(\widehat{\kappa}_{do,t}^{ks,hn}\right)^{\frac{\Upsilon}{\upsilon}}\right]^{\upsilon}\right] \tag{86}$$

$$\mu_{od,t'}^{ks,hn} = \frac{\mu_{od,t}^{ks,hn}\left(\widehat{u}_{d,t'}^{s,n}\right)^{\frac{\Upsilon}{(1+\zeta)\upsilon}}\left(\widehat{\kappa}_{od,t}^{ks,hn}\right)^{\frac{\Upsilon}{\upsilon}}}{\sum_{n'}\mu_{od,t}^{ks,hn'}\left(\widehat{u}_{d,t'}^{s,n'}\right)^{\frac{\Upsilon}{(1+\zeta)\upsilon}}\left(\widehat{\kappa}_{od,t}^{ks,hn'}\right)^{\frac{\Upsilon}{\upsilon}}} \cdot \frac{\left[\sum_{n'}\mu_{od,t}^{ks,hn'}\left(\widehat{u}_{d,t'}^{s,n'}\right)^{\frac{\Upsilon}{(1+\zeta)\upsilon}}\left(\widehat{\kappa}_{od,t}^{ks,hn'}\right)^{\frac{\Upsilon}{\upsilon}}\right]^{\upsilon}}{\sum_{n'}\left[\sum_{d'}\sum_{s'}\mu_{od',t}^{ks',hn'}\left(\widehat{u}_{d',t'}^{s',n'}\right)^{\frac{\Upsilon}{(1+\zeta)\upsilon}}\left(\widehat{\kappa}_{od',t}^{ks',hn'}\right)^{\frac{\Upsilon}{\upsilon}}\right]^{\upsilon}} \tag{87}$$

$$L_{d,t'}^{k,h} = \sum_{n}\sum_{s}\sum_{o}\mu_{od,t}^{ks,hn}L_{o,t}^{s,n}. \tag{88}$$

Technology levels is now given by:

$$T_{o,t}^{k} = \sum_{r=1}^{N}\sum_{t^*\in\mathcal{T}_{-\infty}^{t}}\Omega_{ro,t^*}(t-t^*)^{1-\rho}\cdot\lambda_{r,t^*}^{k}, \tag{89}$$

where the diffusion term $\Omega_{ro,t^*}(t-t^*)$ remains unchanged, while innovation levels remain unchanged from equation (27). The trade shares, idea adoption shares, and price index is now given by:

$$\pi_{od,t}^{k} = \sum_{r=1}^{N}\sum_{t^*\in\mathcal{T}}\frac{\Omega_{ro,t^*}(t-t^*)\left(w_{o,t}^{k}\tau_{od,t}^{k}\right)^{-\frac{\theta}{1-\rho}}}{\sum_{o'}\Omega_{ro',t^*}(t-t^*)\left(w_{o',t}^{k}\tau_{o'd,t}^{k}\right)^{-\frac{\theta}{1-\rho}}}\frac{\left[\sum_{o'}\Omega_{ro',t^*}(t-t^*)\left(w_{o',t}^{k}\tau_{o'd,t}^{k}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}\lambda_{r,t^*}^{k}}{\sum_{r'}\sum_{t^*\in\mathcal{T}}\left[\sum_{o'}\Omega_{r'o',\check{t}}(t-\check{t})\left(w_{o',t}^{k}\tau_{o'd,t}^{k}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}\lambda_{r',\check{t}}^{k}} \tag{90}$$

$$\varphi_{ro,t}^{k} = \sum_{d=1}^{N}\sum_{t^*\in\mathcal{T}}\frac{\Omega_{ro,t^*}(t-t^*)\left(w_{o,t}^{k}\tau_{od,t}^{k}\right)^{-\frac{\theta}{1-\rho}}}{\sum_{o'}\Omega_{ro',t^*}(t-t^*)\left(w_{o',t}^{k}\tau_{o'd,t}^{k}\right)^{-\frac{\theta}{1-\rho}}}\frac{\left[\sum_{o'}\Omega_{ro',t^*}(t-t^*)\left(w_{o',t}^{k}\tau_{o'd,t}^{k}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}\lambda_{r,t^*}^{k}}{\sum_{r'}\sum_{t^*\in\mathcal{T}}\left[\sum_{o'}\Omega_{r'o',\check{t}}(t-\check{t})\left(w_{o',t}^{k}\tau_{o'd,t}^{k}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}\lambda_{r',\check{t}}^{k}} \tag{91}$$

$$P_{d,t}^{k} = \Gamma\left[\sum_{r'=1}^{N}\sum_{t^*\in\mathcal{T}}\left[\sum_{o'=1}^{N}\Omega_{r'o',t^*}(t-t^*)\left(w_{o',t}^{k}\tau_{o'd,t}^{k}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}\lambda_{r',t^*}^{k}\right]^{-\frac{1}{\theta}}, \tag{92}$$

where $\mathcal{T}\equiv\mathcal{T}_{-\infty}^{t}$ are the set of times from $-\infty$ to t where innovation, production, consumption, and migration occurs. The market clearing condition remains unchanged from equation (20). Inventor wages are given by equation (19), total expenditures by equation (40), and the expected value of an idea and the idea market shares are now given by:

$$\check{V}_{r,t}^{k} = \sum_{t'\in\mathcal{T}_{t}^{\infty}}\left(\frac{1}{1+\zeta}\right)^{t'-t}\sum_{d=1}^{N}\frac{X_{d,t'}^{k}}{1+\theta}\cdot\frac{P_{r,t}}{P_{r,t'}}\cdot\frac{\phi_{rd,tt'}^{k}}{\lambda_{r,t}^{k}} \tag{93}$$

$$\phi_{rd,tt'}^{k} = \frac{\left[\sum_{o'}\Omega_{ro',t}(t'-t)\left(w_{r',t'}^{k}\tau_{ro',t'}^{k}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}\lambda_{r,t}^{k}}{\sum_{r'}\sum_{\check{t}\in\mathcal{T}_{-\infty}^{t'}}\left[\sum_{o'}\Omega_{r'o',\check{t}}(t'-\check{t})\left(w_{o',t'}^{k}\tau_{o'd,t'}^{k}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}\lambda_{r',\check{t}}^{k}}. \tag{94}$$

□

Special Cases:

When idea applicabilities are not relevant (**case NA**), only changes in trade costs – as opposed to levels – along with the other fundamentals are required to simulate the transition path. This is because the trade shares, idea diffusion shares, and price index depend only on the contemporaneous technology stock rather than the trajectory of innovation in all past periods, and are given by equations 81 and 82:

$$\pi_{od,t}^{k,NA} = \frac{T_{o,t}^k \left(w_{o,t}^k \tau_{od,t}^k\right)^{-\theta}}{\sum_{o'=1}^{N} T_{o',t}^k \left(w_{o',t}^k \tau_{o'd,t}^k\right)^{-\theta}},\ \varphi_{ro,t}^{k,NA} = \sum_{d=1}^{N} \frac{T_{ro,t}^k \left(w_{o,t}^k \tau_{od,t}^k\right)^{-\theta}}{\sum_{o'=1}^{N} T_{o',t}^k \left(w_{o',t}^k \tau_{o'd,t}^k\right)^{-\theta}},\ P_{d,t}^{k,NA} = \Gamma \left[\sum_{o'=1}^{N} T_{o',t}^k \left(w_{o',t}^k \tau_{o'd,t}^k\right)^{-\theta}\right]^{-\frac{1}{\theta}}.$$

Thus, the production side of the economy can also be expressed using dynamic hat algebra. Changes in trade shares, idea diffusion shares and the price index are given by:

$$\widehat{\pi}_{od,t}^{k,NA} = \frac{\widehat{T}_{o,t}^k \left(\widehat{w}_{o,t}^k \widehat{\tau}_{od,t}^k\right)^{-\theta}}{\sum_{o'=1}^{N} \widehat{T}_{o',t}^k \left(\widehat{w}_{o',t}^k \widehat{\tau}_{o'd,t}^k\right)^{-\theta}} \tag{95}$$

$$\widehat{\varphi}_{ro,t}^{k,NA} = \sum_{d=1}^{N} \frac{\widehat{T}_{ro,t}^k \left(\widehat{w}_{o,t}^k \widehat{\tau}_{od,t}^k\right)^{-\theta}}{\sum_{o'=1}^{N} \widehat{T}_{o',t}^k \left(\widehat{w}_{o',t}^k \widehat{\tau}_{o'd,t}^k\right)^{-\theta}} \tag{96}$$

$$\widehat{P}_{d,t}^{k,NA} = \left[\sum_{o'=1}^{N} \widehat{T}_{o',t}^k \left(\widehat{w}_{o',t}^k \widehat{\tau}_{o'd,t}^k\right)^{-\theta}\right]^{-\frac{1}{\theta}} \tag{97}$$

and the market clearing condition remains unchanged from equation 20. Innovation levels remain unchanged from equation 27 and changes in technology levels are given by:

$$\begin{aligned} T_{o,t'}^k - T_{o,t}^k &= \sum_r \delta_{ro,t} \left(\Lambda_{l,t}^k - T_{lo,t}^k\right) \\ \implies \widehat{T}_{o,t'}^k &= 1 - \delta_{ro,t} + \sum_l \frac{\delta_{lo,t} \Lambda_{l,t}^k}{T_{o,t}^k} \end{aligned} \tag{98}$$

where $\Lambda_{r,t}^k = \sum_{t^* \in \mathcal{T}} \lambda_{r,t^*}^k$ is the stock of innovations produced in region r at time t. This recursive formulation of exponential idea diffusion comes from Eaton and Kortum (2024). Since changes in technology are a function of the previous technology level, data on initial technology levels are still required to solve the transition path quantitatively.

Similarly, when there is instantaneous diffusion (**case ID**), only changes in trade costs – as opposed to levels – along with the other fundamentals are required to simulate the transition path. Trade shares, idea

diffusion shares, and price index are given by equations 83 -85:

$$\pi_{od,t}^{k,ID} = \sum_{r=1}^{N} \frac{\left(T_{ro,t}^{k}\right)^{\frac{1}{1-\rho}} \left(w_{o,t}^{k}\tau_{od,t}^{k}\right)^{-\frac{\theta}{1-\rho}}}{\sum_{o'=1}^{N}\left(T_{ro',t}^{k}\right)^{\frac{1}{1-\rho}} \left(w_{o',t}^{k}\tau_{o'd,t}^{k}\right)^{-\frac{\theta}{1-\rho}}} \cdot \frac{\left[\sum_{o'=1}^{N}\left(T_{ro',t}^{k}\right)^{\frac{1}{1-\rho}} \left(w_{o',t}^{k}\tau_{o'd,t}^{k}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}}{\sum_{r'=1}^{N}\left[\sum_{o'=1}^{N}\left(T_{ro',t}^{k}\right)^{\frac{1}{1-\rho}} \left(w_{o',t}^{k}\tau_{o'd,t}^{k}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}}$$

$$P_{d,t}^{k,ID} = \Gamma\left[\sum_{r'=1}^{N}\left[\sum_{o'=1}^{N}\left(T_{r'o',t}^{k}\right)^{\frac{1}{1-\rho}} \left(w_{o',t}^{k}\tau_{o'd,t}^{k}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}\right]^{-\frac{1}{\theta}}$$

$$\varphi_{od,t}^{k,ID} = \frac{\left(T_{ro,t}^{k}\right)^{\frac{1}{1-\rho}} \left(w_{o,t}^{k}\tau_{od,t}^{k}\right)^{-\frac{\theta}{1-\rho}}}{\sum_{o'=1}^{N}\left(T_{ro',t}^{k}\right)^{\frac{1}{1-\rho}} \left(w_{o',t}^{k}\tau_{o'd,t}^{k}\right)^{-\frac{\theta}{1-\rho}}} \cdot \frac{\left[\sum_{o'=1}^{N}\left(T_{ro',t}^{k}\right)^{\frac{1}{1-\rho}} \left(w_{o',t}^{k}\tau_{o'd,t}^{k}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}}{\sum_{r'=1}^{N}\left[\sum_{o'=1}^{N}\left(T_{ro',t}^{k}\right)^{\frac{1}{1-\rho}} \left(w_{o',t}^{k}\tau_{o'd,t}^{k}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}}$$

Thus changes in trade shares, idea diffusion shares and the price index are given by:

$$\widehat{\pi}_{od,t}^{k,ID} = \sum_{r=1}^{N} \frac{\left(\widehat{T}_{ro,t}^{k}\right)^{\frac{1}{1-\rho}} \left(\widehat{w}_{o,t}^{k}\widehat{\tau}_{od,t}^{k}\right)^{-\frac{\theta}{1-\rho}}}{\sum_{o'=1}^{N}\left(\widehat{T}_{ro',t}^{k}\right)^{\frac{1}{1-\rho}} \left(\widehat{w}_{o',t}^{k}\widehat{\tau}_{o'd,t}^{k}\right)^{-\frac{\theta}{1-\rho}}} \cdot \frac{\left[\sum_{o'=1}^{N}\left(\widehat{T}_{ro',t}^{k}\right)^{\frac{1}{1-\rho}} \left(\widehat{w}_{o',t}^{k}\widehat{\tau}_{o'd,t}^{k}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}}{\sum_{r'=1}^{N}\left[\sum_{o'=1}^{N}\left(\widehat{T}_{ro',t}^{k}\right)^{\frac{1}{1-\rho}} \left(\widehat{w}_{o',t}^{k}\widehat{\tau}_{o'd,t}^{k}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}} \tag{99}$$

$$\widehat{P}_{d,t}^{k,ID} = \left[\sum_{r'=1}^{N}\left[\sum_{o'=1}^{N}\left(\widehat{T}_{r'o',t}^{k}\right)^{\frac{1}{1-\rho}} \left(\widehat{w}_{o',t}^{k}\widehat{\tau}_{o'd,t}^{k}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}\right]^{-\frac{1}{\theta}} \tag{100}$$

$$\widehat{\varphi}_{od,t}^{k,ID} = \frac{\left(\widehat{T}_{ro,t}^{k}\right)^{\frac{1}{1-\rho}} \left(\widehat{w}_{o,t}^{k}\widehat{\tau}_{od,t}^{k}\right)^{-\frac{\theta}{1-\rho}}}{\sum_{o'=1}^{N}\left(\widehat{T}_{ro',t}^{k}\right)^{\frac{1}{1-\rho}} \left(\widehat{w}_{o',t}^{k}\widehat{\tau}_{o'd,t}^{k}\right)^{-\frac{\theta}{1-\rho}}} \cdot \frac{\left[\sum_{o'=1}^{N}\left(\widehat{T}_{ro',t}^{k}\right)^{\frac{1}{1-\rho}} \left(\widehat{w}_{o',t}^{k}\widehat{\tau}_{o'd,t}^{k}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}}{\sum_{r'=1}^{N}\left[\sum_{o'=1}^{N}\left(\widehat{T}_{ro',t}^{k}\right)^{\frac{1}{1-\rho}} \left(\widehat{w}_{o',t}^{k}\widehat{\tau}_{o'd,t}^{k}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}} \tag{101}$$

and the market clearing condition remains unchanged from equation 20. Changes in technology levels are given by:

$$\begin{aligned} T_{o,t'}^{k} - T_{o,t}^{k} &= \sum_{r} \delta_{ro,t}^{1-\rho}\gamma_{r,t}^{k}T_{r,t}^{k} \\ \implies \widehat{T}_{o,t'}^{k} &= 1 + \sum_{r} \delta_{ro,t}^{1-\rho}\gamma_{r,t}^{k}\frac{T_{r,t}^{k}}{T_{o,t}^{k}} \end{aligned} \tag{102}$$

and innovation levels remain unchanged from equation 27.

A.11 Proposition 4 (Regional and Aggregate Welfare)

Proof. I extend the welfare derivation and expression in Caliendo et al. (2019) to my setting, where: (i) preference shocks are correlated across markets; (ii) there is endogenous and microfounded innovation and technology diffusion, and (iii) the endogenous distribution of innovation and technology diffusion across markets drives parallel growth in all regions in the long run.

The expected worker value in equation (24) is given by:

$$V_{d,t}^{k,h} = U\left(C_{d,t}^{k,h}\right) + \frac{1}{\Upsilon}\log\left[\sum_{o}\sum_{s}\left(\sum_{n}\exp\left(\frac{1}{1+\zeta}V_{o,t'}^{s,n} - \kappa_{do,t}^{ks,hn}\right)^{\frac{\Upsilon}{\upsilon}}\right)^{\upsilon}\right] = \log\left(\frac{w_{d,t}^{k,h}}{P_{d,t}}\right) + \Phi_{d,t}^{k,h}.$$

I now express the option value $\Phi_{d,t}^{k,h}$ in terms of own-mobility shares:

$$\begin{aligned}
\Phi_{d,t}^{k,h} &= -\frac{1}{\Upsilon}\log\mu_{dd,t}^{kk} + \frac{\upsilon}{\Upsilon}\log\left[\sum_{n}\exp\left(\frac{1}{1+\zeta}V_{d,t'}^{k,n} - \kappa_{dd,t}^{kk,hn}\right)^{\frac{\Upsilon}{\upsilon}}\right] \\
&= -\frac{1}{\Upsilon}\log\mu_{dd,t}^{kk} - \frac{\upsilon}{\Upsilon}\log\left(\mu_{dd,t}^{kk,hh}|\mu_{dd,t}^{kk}\right) + \frac{\upsilon}{\Upsilon}\log\left[\exp\left(\frac{1}{1+\zeta}V_{d,t'}^{k,h}\right)^{\frac{\Upsilon}{\upsilon}}\right] \\
&= \frac{1}{1+\zeta}V_{d,t'}^{k,h} - \frac{1}{\Upsilon}\log\mu_{dd,t}^{kk} - \frac{\upsilon}{\Upsilon}\log\left(\mu_{dd,t}^{kk,hh}|\mu_{dd,t}^{kk}\right).
\end{aligned}$$

Hence, expected worker value can be alternately expressed as:

$$V_{d,t}^{k,h} = \log\left(\frac{w_{d,t}^{k,h}}{P_{d,t}}\right) + \frac{1}{1+\zeta}V_{d,t'}^{k,h} - \frac{1}{\Upsilon}\log\mu_{dd,t}^{kk} - \frac{\upsilon}{\Upsilon}\log\left(\mu_{dd,t}^{kk,hh}|\mu_{dd,t}^{kk}\right).$$

Iterating this equation forward, we have:

$$\begin{aligned}
V_{d,t}^{k,h} &= \sum_{t'\in\mathcal{T}_t^{\infty}}\left(\frac{1}{1+\zeta}\right)^{t'-t}\log\left(\frac{w_{d,t}^{k,h}}{P_{d,t}\left(\mu_{dd,t}^{kk}\right)^{1/\Upsilon}\left(\mu_{dd,t}^{kk,hh}|\mu_{dd,t}^{kk}\right)^{\upsilon/\Upsilon}}\right) \\
&= \sum_{t'\in\mathcal{T}_t^{\infty}}\left(\frac{1}{1+\zeta}\right)^{t'-t}\left\{\underbrace{\log\left(\frac{w_{d,t}^{k,h}}{\widetilde{P}_{d,t}}\right)}_{\substack{\text{detrended}\\ \text{future value}}} - \underbrace{\log\left[\left(\mu_{dd,t}^{kk}\right)^{1/\Upsilon}\left(\mu_{dd,t}^{kk,hh}|\mu_{dd,t}^{kk}\right)^{\upsilon/\Upsilon}\right]}_{\text{option value of movement}}\right\} + \underbrace{\frac{1+\zeta}{\zeta}\frac{1}{\theta}\sum_{k}\iota^{k}g^{k}}_{\text{long-run growth}}.
\end{aligned}$$

Denote $\acute{x}$ as the counterfactual of any variable x. My measure of the **welfare** impact in **market** (d,k,h) of an anticipated sequence of counterfactual changes in fundamentals from time $t=0$ is the compensating

variation in consumption for market (d,k,h) at $t=0$, $\log \delta_d^{k,h}$ given by:

$$\acute{V}_{d,0}^{k,h} = V_{d,0}^{k,h} + \sum_{t' \in \mathcal{T}_0^\infty} \left(\frac{1}{1+\zeta}\right)^{t'} \log \delta_d^{k,h}.$$

Rearranging this equation and substituting the expressions for expected worker value, we have:

$$\begin{aligned}
\log\left(\delta_d^{k,h}\right) &= \frac{\zeta}{1+\zeta}\left[\acute{V}_{d,0}^{k,h} - V_{d,0}^{k,h}\right] \\
&= \left(1-\frac{1}{1+\zeta}\right)\sum_{t' \in \mathcal{T}_0^\infty}\left(\frac{1}{1+\zeta}\right)^{t'} \log\left(\frac{\acute{w}_{d,t}^{k,h}}{w_{d,t}^{k,h}}\frac{\widetilde{P}_{d,t}}{\acute{\widetilde{P}}_{d,t}}\frac{\left(\mu_{dd,t}^{kk}\right)^{1/\Upsilon}\left(\mu_{dd,t}^{kk,hh}|\mu_{dd,t}^{kk}\right)^{\upsilon/\Upsilon}}{\left(\mu_{dd,t}^{\acute{k}k}\right)^{1/\Upsilon}\left(\mu_{dd,t}^{k\acute{k},h\acute{h}}|\mu_{dd,t}^{kk}\right)^{\upsilon/\Upsilon}}\right) + \frac{1}{\theta}\sum_k \iota^k\left(\acute{g}^k - g^k\right) \\
&= \sum_{t' \in \mathcal{T}_0^\infty}\left(\frac{1}{1+\zeta}\right)^{t'} \log\Bigg(\underbrace{\frac{\widehat{w_{d,t}^{k,h}}}{\widehat{\widetilde{P}_{d,t}}}}_{\substack{\text{change in}\\ \text{future}\\ \text{detrended}\\ \text{real wages}}} \underbrace{\frac{1}{\left(\widehat{\mu_{dd,t}^{kk}}\right)^{1/\Upsilon}\left(\widehat{\mu_{dd,t}^{kk,hh}|\mu_{dd,t}^{kk}}\right)^{\upsilon/\Upsilon}}}_{\text{change in option value of movement}}\Bigg) + \underbrace{\frac{1}{\theta}\sum_k \iota^k\left(\acute{g}^k - g^k\right)}_{\text{growth effects}},
\end{aligned}$$

where $\widehat{x}_{t'} = \frac{\dot{x}_{t'}}{\dot{x}_t}$ denotes the counterfactual change in any variable x.

Using this measure, I define **local and aggregate welfare** as population-weighted averages of welfare in the relevant markets:

$$\log(\delta_d) = \sum_{k,h} \frac{L_d^{k,h}}{\sum_{k,h} L_d^{k,h}} \log\left(\delta_d^{k,h}\right)$$

$$\log(\delta) = \sum_{d,k,h} \frac{L_d^{k,h}}{\sum_{d,k,h} L_d^{k,h}} \log\left(\delta_d^{k,h}\right).$$

□

B Model Extensions

My quantitative spatial growth model in the main text is deliberately parsimonious to capture the main drivers of the rising spatial concentration of innovation from the data. Nonetheless, the central feature of my spatial model is that I introduce endogenous innovation and integrate it with technology diffusion at the *idea* level, the fundamental unit of the Eaton-Kortum world. Thus, my model requires minimal assumptions and can flexibly incorporate other components in quantitative dynamic spatial models.

B.1 Dynamic Worker Sorting by Skill

To incorporate college-educated H and non-college educated S workers, equation 24-26 can be duplicated for each worker type. Equilibrium migration shares would now capture worker sorting patterns in the data as opposed to aggregate bilateral migration flows across both worker types. In the production of goods, labor

is now a composite of college-educated and non-college educated workers. For instance, one could assume a CES agggregate between both types of labor:

$$L_{o,t}^k = \left[\left(L_{o,t}^{H,k} \right)^{\frac{\varphi-1}{\varphi}} + \left(L_{o,t}^{S,k} \right)^{\frac{\varphi-1}{\varphi}} \right]^{\frac{\varphi}{\varphi-1}} \tag{103}$$

such that wages $w_{o,t}^k$ in the market clearing condition is now an aggregate of wages of each worker type, with no other required changes. Note that I drop the superscript G for production workers in this section of the appendix for notational clarity, since there is no discussion on innovation workers.

B.2 Multiple Factors of Production and Input-Output Loops

Input-output loops can be easily incorporated into the trade equilibrium at each t following Alvarez and Lucas (2007); Caliendo and Parro (2015). Instead of just using labor, production of each variety ν in each region is now given by a two-tier Cobb-Douglas constant returns to scale technology:

$$Y_{o,t}^k(\nu) = z_{o,t}^k(\nu) \left[(K_{o,t}^k)^{\psi} (L_{o,t}^k)^{1-\psi} \right]^{\chi} M_{o,t}^{1-\chi} \tag{104}$$

where $z_{o,t}(\nu)$ is the productivity drawn from the multivariate Fréchet distribution given by equation 5, $K_{o,t}$ is capital used in production, referring to commercial structures such as local buildings, $L_{o,t}$ is labor, and $M_{o,t}$ is intermediate inputs purchased from the final goods producer in the same region, ψ is the share of local structures in value added, and χ is the share of value added. The unit cost of an input bundle is given by:

$$x_{o,t}^k = \check{\Gamma} \left[(\check{r}_{o,t})^{\psi} \left(w_{o,t}^k \right)^{1-\psi} \right]^{\chi} {P_{o,t}^k}^{1-\chi} \tag{105}$$

where $\check{\Gamma}_o$ is a constant, $\check{r}_{o,t}$ is the rental rate of capital from local capitalists, and $P_{o,t}$ is also the price of the local industry aggregate of varieties. Replacing $w_{o,t}$ with $x_{o,t}$ in equation 10 yields the equilibrium trade shares.

Capital market clearing is given by:

$$\check{r}_{o,t} K_{o,t}^k = \frac{1-\psi}{\psi} w_{o,t}^k L_{o,t}^k \tag{106}$$

Since capital income is a constant multiple of production worker income, the combined capital and labor market clearing condition is still given by equation 20 in the main paper. With input-output loops of the form in equation 105 – where goods producers purchase the final good only in that sector[18] – intermediate good spending is a constant multiple of production worker and capital income:

$$X_{o,t}^I = \frac{1-\chi}{\chi} \left(w_{o,t}^k L_{o,t}^k + r_{o,t} K_{o,t}^k \right) = \frac{1-\chi}{\chi} \frac{1}{\psi} w_{o,t}^k L_{o,t}^k. \tag{107}$$

Thus, the combined capital and labor market clearing condition is still given by equation 20 in the main

[18] If goods producers purchase the final goods from all sectors, the market clearing condition will be slightly modified, as shown in Caliendo and Parro (2015), but remains highly tractable.

paper. All the other equations in the model also remain the same.

B.3 Capital Accumulation

Capital accumulation can be added following Kleinman et al. (2023). Apart from workers and local immobile firms, we can introduce local immobile capitalists. In each region, local immobile capitalists build durable local structures $K_{o,t}$ and rent to firms in different sectors at a nominal rate $\check{r}_{o,t}$. With their rental income, capitalists choose their consumption and investment to maximize intertemporal utility:

$$\check{V}_{o,t}^{K} = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^{t+s} \frac{\left(C_{o,t+s}^{K}\right)^{1-1/\eta}}{1-1/\eta} \tag{108}$$

subject to their budget constraint:

$$r_{o,t} K_{o,t} = P_{o,t} \left(C_{o,t}^{K} + K_{o,t+1} - (1-\delta_{o,t}) K_{o,t} \right) \tag{109}$$

where rental income can be used for consumption (first term), or saving to increase future capital (last two terms). Per period consumption expenditures $P_{o,t} C_{o,t}^{K}$ are allocated via the same utility function as households.

The optimal consumption and saving decisions are given by:

$$C_{o,t}^{K} = \varsigma_{o,t} R_{o,t} K_{o,t} \tag{110}$$

$$K_{o,t+1} = (1-\varsigma_{o,t}) R_{o,t} K_{o,t} \tag{111}$$

$$\varsigma_{o,t}^{-1} = 1 + \beta^{\eta} \left(\mathbb{E}_t \left[R_{o,t+1}^{\frac{\eta-1}{\eta}} \varsigma_{o,t+1}^{-\frac{1}{\eta}} \right] \right)^{\eta} \tag{112}$$

where the consumption rate $\varsigma_{o,t}$ is defined recursively and the gross return on capital is $R_{o,t} \equiv 1-\delta+r_{o,t}/P_{o,t}$. The other equations in the model remain the same.

B.4 Amenities with Congestion and Agglomeration in Production

Amenities, explicit congestion forces, and agglomeration in production can be introduced following Allen and Arkolakis (2014). The instantaneous utility function is now given by:

$$U\left(C_{o,t}, B_{o,t}\right) = \log\left(B_{o,t} C_{o,t}\right) \tag{113}$$

with $B_{o,t} = \check{B}_{o,t} L_{o,t}^{-\xi}$ where $\check{B}_{o,t}$ are fundamental amenities in region o at time t and ξ captures congestion forces. The goods production function in equation 104 now becomes:

$$Y_{o,t}^{k}(\nu) = z_{o,t}^{k}(\nu) (L_{o,t}^{k})^{\check{\alpha}} \left[(K_{o,t}^{k})^{\psi} (L_{o,t}^{k,G})^{1-\psi} \right]^{\chi} M_{o,t}^{1-\chi} \tag{114}$$

where $\check{\alpha}$ captures agglomeration in production. All the other equations in the model remain the same.

C Details of Model Quantification

C.1 Mechanisms and Connection to the Model

C.1.1 Breakthrough ICT Patents

The first component of the ICT shock is the sudden emergence of influential ICT breakthroughs. I identify breakthrough ICT patents as those in the top 1% of the *originality* distribution across all patents, where originality is measured as the ratio of forward to backward textual similarity following Kelly et al. (2021). Between 1976 and 2018, 880,927 ICT patents were produced, of which 27,419 qualify as breakthroughs. The left panel of Figure 3 traces the evolution of their influence – defined as the number of patents that cite each breakthrough as their first breakthrough, following citations up to two degrees. The figure reveals a sharp rise in the influence of breakthrough ICT patents between 1984 and 1993. These breakthroughs catalyzed the subsequent surge in ICT patenting. The right panel of Figure 3 displays the corresponding trend in the number of ICT patents that cite these breakthroughs (within two degrees). Across all years, 78% of all ICT patents cite a breakthrough ICT patent within two degrees.

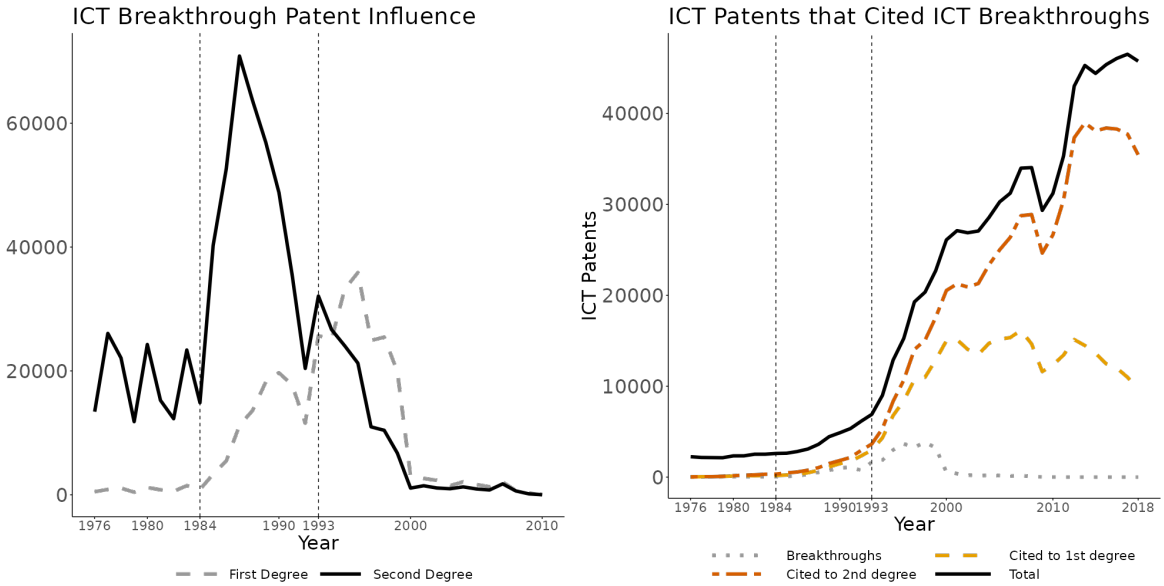


Figure 3: Origins of the ICT shock. The left panel shows the influence of these breakthrough patents, defined as the number of patents that cite each breakthrough as their first breakthrough, tracing citations up to two degrees. The right panel depicts trends in the number of ICT patents that cited these patents.

C.1.2 Colocation of ICT Innovation and Production

Descriptive Evidence. This surge in ICT innovation productivity disproportionately increased innovation in *ICT-oriented firms*, rather than uniformly raising ICT patenting across all firms. Most ICT-oriented firms operate plants in the following industries in the Information Sector (NAICS 51): Software Publishers (5112);

Telecommunications (517), and; Data Processing, Hosting, and Related Services (518)[19]. This suggests that having an ICT service plant is a key input in the production of ICT innovation.

ICT service plants disproportionately employ high-skill workers and are therefore heavily concentrated in high-skill CZs. The left panel of Figure 4 shows that the share of college-educated workers in the ICT service industry (solid black line) is substantially higher than that in other industries (dashed blue line) across all years. The right panel of Figure 4 plots the relationship between a CZ's 1990 college ratio and its pre-1985 ICT employment share; the positive slope indicates that high-skill CZs already had disproportionately large ICT service employment before 1985.

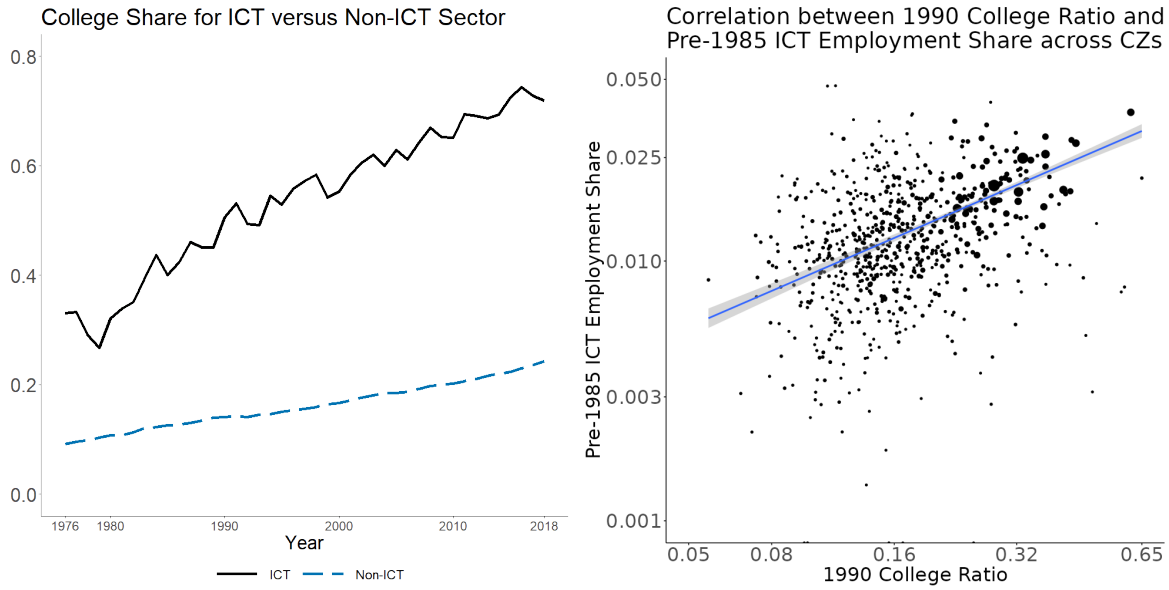


Figure 4: The ICT sector disproportionately employs college-educated workers and is concentrated in high-skill CZs. The left panel displays trends in the college share for the ICT and non-ICT sectors. The right panel shows the correlation of the pre-1985 ICT employment share against the 1990 college ratio. Each dot represents a commuting zone (CZ), where the size of the dot captures the CZ's population in 1990.

The growth of ICT patents primarily occurred in CZs where the ICT industry was initially more concentrated. To show this, I estimate the following CZ-year specification, reminiscent of the first stage of a Bartik regression:

$$\text{Outcome}_{r,t} = \beta \cdot \text{ Pre-1985 ICT Employment Share}_r \times \text{Growth of National ICT Patents}_t + \gamma_r + \gamma_t + \varepsilon_{r,t} \quad (115)$$

where Growth of National ICT Patents$_t$ is the growth of national patents in year t relative to 1985, and γ_r and γ_t are CZ and year fixed effects. Specification (1) shows that CZs with a higher initial ICT employment share experienced substantially larger increases in ICT patenting. A 1 p.p. higher pre-1985 ICT employment share is associated with 8.3 more ICT patents in years when national ICT patenting is 10% above its 1985

[19]In contrast to Fort et al. (2020), I categorize these industries as ICT *production*, rather than *innovation*, because establishments that focus on R&D activities are instead classified as Scientific Research and Development Services (5417) and occasionally Corporate, Subsidiary, and Regional Managing Offices (551114) under NAICS.

level. Specification (2) confirms that the geographical incidence of ICT patents is not driven merely by population growth: the same 1 p.p. difference predicts 1.5 additional ICT patents per thousand residents under a 10% national ICT growth shock. Turning to employment outcomes, Specification (3) shows that ICT employment also increased disproportionately in CZs where the ICT industry was initially more concentrated: a 1 p.p. higher pre-1985 ICT employment share is associated with 77 additional ICT workers when national ICT patenting rises by 10%. Taken together, the parallel rise in ICT employment and ICT patents per capita in the same CZs provides evidence of the colocation of ICT innovation and ICT production in regions with strong pre-existing ICT presence. Finally, Specification (4) shows that these CZs did not experience larger increases in the share of local employment in ICT, likely because employment in other industries also expanded.

Dependent Variables:	ICT Patents	ICT Patents per capita	ICT Employment	ICT Employment Share
Model:	(1)	(2)	(3)	(4)
Pre-1985 ICT Employment Share$_r$×	8,334.9***	1.474***	77,372.4***	-0.0247
Growth of National ICT Patents$_t$	(2,735.3)	(0.5416)	(28,464.3)	(0.0204)
Fixed-effects				
CZ	Yes	Yes	Yes	Yes
Year	Yes	Yes	Yes	Yes
Observations	20,938	20,938	20,938	20,934
R^2	0.81282	0.81900	0.96195	0.83343
Within R^2	0.15934	0.06459	0.05407	0.01308

Clustered (CZ) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table 1: The geographical incidence of the post-1985 growth of national ICT patents.

Connection to Model. In the model, this channel operates through the *colocation* mechanism characterized in the equilibrium idea adoption shares (equation 14): a positive shock to ICT research productivity raises innovation disproportionately in regions where ICT production is concentrated, because idea adoption shares are higher for locally produced goods. Using the idea adoption shares, I formally characterize the degree of *colocation* between innovation and production in the following corollary:

Corollary 1. *For goods sold at time t using ideas discovered at time t^*, the **degree of colocation between innovation and production** for any region r relative to an alternative production location $o \neq r$ is given by:*

$$\frac{\varphi_{rr,t^*t}}{\varphi_{ro,t^*t}} = \frac{1}{\Omega_{ro,t^*}(t-t^*)} \cdot \left(\frac{w_{r,t}}{w_{o,t}}\right)^{-\frac{\theta}{1-\rho}} \cdot \frac{\sum_d (\tau_{rd,t})^{-\frac{\theta}{1-\rho}}}{\sum_d (\tau_{od,t})^{-\frac{\theta}{1-\rho}}}. \tag{116}$$

This ratio captures the relative share of goods produced in the idea's discovery region r versus an alternative production region o, providing a direct measure of how closely innovation and production are colocated. The different components reflect key factors driving colocation, including the share of all ideas discovered in region r at time t^* that have diffused to o (first term), the cost competitiveness between r and o (second term),

and the relative accessibility of destination markets d from r and o (third term). In my application to U.S. innovation, this ratio provides two reasons why ICT production became more concentrated in high-skill regions following a one-time increase in ICT research productivity (the first exogenous component of the ICT shock): frictions in technology diffusion (the first term), and the greater accessibility of ICT destination markets from high-skill regions. These colocation and asymmetric scale effect mechanisms arise only in a model where innovation and technology diffusion are connected at the level of individual ideas.[20]

C.1.3 Asymmetric Scale Effect of Firm Spatial Expansion

Descriptive Evidence. ICT-induced reductions in communication costs facilitated firm spatial expansion. Firms with R&D in high-skill CZs expanded into substantially more regions and increased their innovation output, predominantly in high-skill CZs.

Connection to Model. The idea market shares (equation 11) give rise to what I term the ***asymmetric scale effect***: high-skill regions experience a larger increase in idea market access (equation 12) from a uniform rise in bilateral diffusion speeds. Substituting Assumption 1 into equation 12 yields the following corollary:

Corollary 2. *If bilateral diffusion speeds are symmetric ($\delta_{rr',t^*} = \delta_{r'r,t^*} \equiv \delta_{rr'}$) and trade costs are identical across all region pairs ($\tau_{rd,t} = \tau_{r'd,t} \equiv \tau_t$), then an increase in the bilateral diffusion speed raises idea market access more for the higher-wage region, i.e.:*

$$\frac{\partial \Phi_{rd,t^*t}}{\partial \delta_{rr',t^*}} - \frac{\partial \Phi_{r'd,t^*t}}{\partial \delta_{rr',t^*}} = \delta_{rr',t^*} e^{-\delta_{rr',t^*}(t-t^*)} \left[\left(w_{r',t} \tau_{r'd,t} \right)^{-\theta} - \left(w_{r,t} \tau_{rd,t} \right)^{-\theta} \right] > 0 \quad \textit{if} \quad w_{r,t} > w_{r',t}.$$

Intuitively, faster diffusion raises idea market access more in high-wage (and high-skill) regions because it allows them to exploit the lower production costs of other regions when applying their ideas. By contrast, the low-wage regions gain less: their ideas already benefit from relatively low local production costs, so improved access to higher cost regions for production yields only modest additional gains. This asymmetric scale effect helps explain why firms with R&D in high-skill CZs expanded geographically far more than firms with R&D in low-skill CZs.

C.2 Data for Quantification

Using the analytical characterizations of the balanced growth and transition paths above, my quantitative spatial growth model can be simulated with just several sets of bilateral data and a few key elasticities, as

[20] When innovation and technology diffusion are modeled at an aggregate level (e.g. Desmet et al., 2018) or as independent processes (e.g. Buera and Oberfield, 2020; Cai et al., 2025), the idea *adoption* shares would exactly coincide with the idea *diffusion* shares, so the degree of colocation between innovation and production does not depend on wages and trade costs. Additionally, idea market access would remain unchanged when the speed of technology diffusion changes, so the asymmetric scale effect cannot be obtained. When innovation and technology diffusion are connected at the level of individual ideas but ideas have identical efficiency across different production locations (Cai et al., 2022; Eaton and Kortum, 2024), colocation and the asymmetric effect can be obtained, but technology diffusion results in large fluctuations in idea market shares and hence trade shares (as seen by setting $\rho = 0$ in equation 11).

in standard quantitative trade and spatial models.

C.2.1 Patent Data

Patent data may be used to compute worker mobility flows between production and research $\mu_{l,t}^{PR}, \mu_{l,t}^{RP}$. Specifically, the share of workers in research that moved to production $\mu_{l,t}^{RP}$ at time t can be computed as the number of inventors that patented in year $t-1$ but not in year t divided by the total number of inventors that patented in year $t-1$, while the share of workers in production that moved to research $\mu_{l,t}^{PR}$ at time t can be computed as the number of inventors that patent in year t but not in year $t-1$ divided by the total number of college-educated workers from IPUMS NHGIS (see below for details), with all variables referring to the respective CZ and sector.

Given these mobility flows between production and innovation, as well as the initial labor allocation, the number or the quality-adjusted number of patents in each CZ, sector, and year serves as sufficient statistics for the vector of fundamental research productivities $\check{A}_{l,t}^{R}$.

C.2.2 Bilateral Worker Mobility Flows

IPUMS USA (Ruggles et al., 2023) provides a representative longitudinal worker sample of the American Community Survey (ACS) at an annual frequency from 2005, and the Decennial Census for 1990 and 2000. Each observation is a worker-year, and contains information on migration status and education level. Using this data, one can construct county-to-county and state-to-state migration flows for college educated and non-college educated workers annually from 2005-2018, and in 5-year windows from 1990 to 2005. The Internal Revenue Service (IRS) provides annual county-to-county migration flows from 1990. Note, however, that county-to-county migration flows in both datasets are often surpressed for neighboring counties. IPUMS NHGIS (Schroeder et al., 2025) provides the college and non-college educated worker population for each county in 1980, 1990, 2000, and annually from 2005 to 2018. Using these three datasets, one can impute the annual CZ-to-CZ migration flows for college educated and non-college educated workers from 1990 to 2018 as follows:

IPUMS CPS (Flood et al., 2025) provides annual worker mobility flows between all industry-pairs from 1977-2018. One can use these flows to compute annual worker mobility flows between the ICT and non-ICT sectors from 1990-2018. These mobility flows between sectors and regions, along with the flows between production and research above, are sufficient statistics for the trajectory of bilateral migration costs $\{\kappa\}_{t=0}^{\infty}$ across regions, sectors, and between production and research.

C.2.3 Bilateral Firm Establishment Networks

The raw D&B files provides a comprehensive annual list of firms and their establishments from 1990-2020. Using this data, annual technology diffusion flows $\breve{\pi}_{lo,t} \equiv \sum_d \breve{\pi}_{lod|ld,t}$ for all CZ-pairs can be computed as

follows:

$$\breve{\pi}_{lo,t} = \frac{\text{Employment in region } l \text{ of firms with HQ in region } l \text{ and at least a plant in region } o}{\text{Employment in region } l \text{ of all Multi-Estab Firms with HQ in region } l}. \tag{117}$$

These are sufficient statistics for the trajectory of bilateral diffusion lags $\{\delta_{lo,t^*}\}_{t^*=0}^{\infty}$.

C.2.4 Bilateral Trade Flows

Domestic trade data at the county-pair or CZ-pair level are unavailable. However, following Monte et al. (2018), one can construct model-implied CZ-pair trade flows and validate them using bilateral trade data reported at more aggregated geographic units.

The Commodity Flows Survey (CFS) provides trade flows for all state-pairs and CFS-area-pairs by NAICS industry in 2002, 2007, 2012, 2017, as well as aggregated state-pair trade flows for 1993 and 1997. The 1977 Commodity Transportation Survey (CTS) reports the value of shipments from each state of origin to each census division destination. Following Kleinman et al. (2023), one can downscale each census division's 1977 shipment value to the state level by using the shares of state-level shipment values from the 1993 CFS. Similarly, one can disaggregate aggregate state-pair and CFS-area pair trade flows into the ICT and non-ICT sectors using the each sector's shipment value shares from the 2002 CFS. Interpolating these data yields annual sectoral trade flows for all state pairs and CFS-area pairs from 1990 to 2018.

Model-implied CZ-pair trade flows can be generated using three data inputs and equation (10). First, bilateral trade costs may be parameterized as a constant-elasticity function of distance with elasticity -1.29. Second, annual county-by-industry wage data for 1969–2018 are available from the Bureau of Economic Analysis (BEA). Third, trade deficits for each CFS area can be downscaled to counties then upscaled to CZs using the corresponding regional income shares.

C.2.5 Initial Technology Levels, Labor Allocation and Wages

The number of college and non-college workers in production in each CZ-sector can be obtained from IPUMS NHGIS, while the number of innovation workers in each CZ-sector can be constructed from patent data. County-by-industry wages come from the BEA, as described above. Technology levels in the initial period can be recovered from the model's implied productivity residuals: given observed wages, employment, and trade shares in the initial year, one can invert equation (10) to obtain sector- and region-specific technology levels that rationalize the baseline allocation, similar to (Caliendo and Parro, 2015; Monte et al., 2018; Cai et al., 2025).

C.2.6 Elasticities

Most parameters can be calibrated using estimated values from the existing literature. One can set the trade elasticity at $\theta = 5$ following Costinot and Rodríguez-Clare (2014), the migration elasticity across regions and sectors at $\upsilon = 3$ following Caliendo et al. (2019), and the diffusion elasticity at $\rho = 0.9$ following Lind

and Ramondo (2024). Since the diffusion elasticity was estimated in an international setting, it is important to simulate the model using a range of values for ρ for robustness. The elasticity of worker mobility between production and research can be set at $\Upsilon = 3$ mirroring the migration elasticity, or estimated using the mobility flows between production and research in Section C.2.1, production wages from , and constructed inventor wages as described in Section C.2.5. Because the mobility flows between production and research and the computation of inventor wages rely on specific assumptions, it is important to examine the robustness of the model by simulating it across a range of Υ values.

C.3 Estimation of Parameters

C.4 Simulation Algorithm

Using Propositions 2 and 3, we can solve for the transition and balanced growth paths given an initial allocation of the economy and an anticipated convergent sequence of fundamentals. Since I use annual data on trade flows, migration flows, and innovation levels, the intervals between two innovation, production, or migration times are 1. Let superscripts in parentheses (i) denote guesses for variables I search over in iteration i through the loop. The numerical algorithm is as follows:

- **Step 1**: Guess a path of $\left\{\widehat{u}_{o,t+1}^{k,h,(i)}\right\}_{t=0}^{T}$ that converges to $\widehat{u}_{o,T+1}^{k,h,(i)} = 1$.
- **Step 2**: For all $t \geq 0$, use equation (32), the guesses for $\left\{\widehat{u}_{o,t+1}^{k,h,(i)}\right\}_{t=0}^{T}$ and data on initial migration shares $\mu_{od,-1}^{ks,ha}$ and the trajectory of exogenous changes in migration costs $\left\{\widehat{\kappa}_{od,t}^{ks,ha}\right\}_{t=0}^{T}$ to solve for the trajectory of migration shares $\left\{\mu_{od,t}^{ks,ha}\right\}_{t=0}^{T}$.
- **Step 3**: Use equation (33), the trajectory of migration shares $\left\{\mu_{od,t}^{ks,ha}\right\}_{t=0}^{T}$ and the initial labor allocation $\left\{L_{o,0}^{k,h}\right\}$ to solve for the trajectory of labor allocations $\left\{L_{o,t}^{k,h}\right\}_{t=0}^{T}$.
- **Step 4**: Use equations (27) and (34), the trajectory of inventors $\left\{L_{o,t}^{k,\breve{R}}\right\}_{t=0}^{T}$ and data on initial technology levels $\left\{T_{o,0}^{k}\right\}$ and the trajectory of exogenous fundamental research productivities $\left\{A_{o,t}^{k,\breve{R}}\right\}$ to solve for the trajectory of innovation and technology levels $\left\{\lambda_{o,t}^{k,\breve{R}}, T_{o,t}^{k}\right\}_{t=0}^{T}$.
- **Step 5**: Use equation (30), innovation levels in the final period $\lambda_{o,T}^{k,\breve{R}}$, and data on the exogenous diffusion lag in the final period $\delta_{lo,T}$ to solve for the growth rate of technology in each sector g^k and hence the growth rate of prices $g_p = \frac{1}{\theta}\sum_k \iota^k g^k$ on the **balanced growth path**.
- **Step 6**: Use the trajectory of labor allocations and innovation levels $\left\{L_{o,t}^{k,\breve{R}}, \lambda_{o,t}^{k,\breve{R}}\right\}_{t=0}^{T}$ from Steps 3 and 4 respectively and data on the trajectory of exogenous idea diffusion speeds and changes in trade costs $\left\{\delta_{lo,t}, \widehat{\tau}_{od,t}^{k}\right\}_{t=0}^{T}$ to solve the **trade equilibrium** at each t:

- **Step 6A**: Guess a value for $w_{o,t}^{k,(j)}$, and use equations (35), (36) and (20) to get $w_{o,t}^{k,(j+1)}$ and repeat until convergence

- **Step 6B**: Use the converged value for wages $w_{o,t}^{k}$ to compute the sectoral price index from equation (37), total expenditures in each sector $X_{d,t}^{k}$ from equation (40) and idea market shares $\check{\pi}_{ld}^{k}(t^*,t)$ from equation (11).

- **Step 7**: Use the trajectory of total expenditures and idea market shares $\left\{X_{d,t}^{k}, \check{\pi}_{ld}^{k}(t^*,t)\right\}_{t=0,t^*\leq t}^{T}$ and equation (39) to compute the trajectory of the value of ideas $\left\{\check{V}_{o,t}^{k}\right\}_{t=0}^{T}$, then equation (38) to compute the trajectory of inventor wages $\left\{w_{o,t}^{k,\check{R}}\right\}_{t=0}^{T}$.

- **Step 8**: For each $t \geq 1$, use equation (31), $\left\{\widehat{u}_{o,t+2}^{L(i)}\right\}$ from the previous outer loop iteration, migration shares $\left\{\mu_{od,t}^{ks,ha}\right\}_{t=0}^{T}$ from Step 2, production worker wages and sectoral price indices from Step 6, inventor wages $\left\{w_{o,t}^{k,\check{R}}\right\}_{t=0}^{T}$ from Step 7, and the growth rate of prices g_p from Step 4 to solve backwards in time for $\left\{\widehat{u}_{o,t+1}^{k(i+1)}\right\}_{t=0}^{T}$.

- **Step 9**: If $\left\{\widehat{u}_{o,t+1}^{k,h,(i)}\right\}_{t=0}^{T} \approx \left\{\widehat{u}_{o,t+1}^{k,h,(i+1)}\right\}_{t=0}^{T}$, i.e. the maximum difference across all t is less than some pre-specified tolerance level, $\left\{\widehat{u}_{o,t+1}^{k,h,(i)}, \mu_{od,t}^{ks,ha}, L_{o,t}^{k,h}\right\}_{t=0}^{T}$ is the solution to the problem. Otherwise update the initial guess and repeat the steps until convergence.