Market Concentration, Variable Markups and the Welfare Gains from Trade

Alejandro G. Graziano *

University of Nottingham & CEPR

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Abstract

In recent decades, large firms have become more dominant in domestic markets across different countries. How does this increase in market concentration affect the welfare gains from trade and, thus, the incentives to maintain lower barriers? How does it interact with the role of foreign competition in disciplining domestic firms' markups? In this paper, I study how observed market structures and trade openness shape the welfare gains from trade using a sufficient statistic approach. I construct a general equilibrium model in which granular firms from different origins compete in domestic markets by supplying products that are imperfect substitutes. I show that, under both quantity and price competition, there exists a welfare-consistent concentration statistic that captures the direct consumer welfare cost from markups due to granularity. On the producer side, I uncover a domestic aggregate markup that determines domestic producers' aggregate profits. These measures give rise to two sufficient statistics measuring the consumer and producer gains from trade, together defining the total gains. I leverage these statistics to study the process of trade opening experienced by Chile at the turn of the century in the context of increasing secular concentration. Despite rising domestic concentration, I find that markup costs declined as trade openness increased, reflecting the disciplining effect of foreign competition on domestic firms.

Keywords: Gains from Trade, Welfare, Market Concentration, Competition, Market Power

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*School of Economics, University of Nottingham. Email: alejandro.graziano@nottingham.ac.uk.
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1 Introduction

A key theoretical gain from opening to international trade is the reduction in domestic firms' market power from heightened competition—usually called the *pro-competitive gains from trade*. In a world with increasingly concentrated markets and greater exposure to foreign competition, this mechanism is especially relevant: closing an economy today may lead to a less competitive environment than in earlier periods of lower import penetration. In this paper, I develop a theoretical framework that connects market concentration, variable markups, and trade exposure via a family of sufficient statistics to study how the presence of large firms shapes the welfare gains from trade.

In recent years, the international trade literature has developed rich sufficient statistic approaches to measure different aspects of the gains from trade. The prime example is the formula developed by Arkolakis, Costinot and Rodriguez-Clare (2012)—the ACR formula—which collapses the aggregate gains from trade implied by different widely employed trade models into two statistics: the share of expenditure on domestic goods and the elasticity of import values with respect to variable trade costs—the trade elasticity. However, one of the key conditions for this result to hold with heterogeneous firms is that firms are small relative to the economy. In other words, individual firms are assumed to have no market power beyond monopolistic competition.

I build on that framework by introducing firms that can individually affect industry variables such as price and quantity indices. These so-called granular firms break a key necessary condition behind the ACR formula: a fixed, exogenous share of profits in aggregate income. I show how to exploit the structure of the CES demand system to overcome this issue and derive sufficient statistics for the changes in the distribution of markups and aggregate profits. In doing so, I adjust the statistic for the gains from trade to a world where large firms are dominant (c.f. Bernard et al., 2018). Centrally, the factor of correction that I introduce to deal with granularity only depends on domestic firms' market shares, making it a convenient statistic that can be easily computed with the same type of information needed to compute widely used market concentration measures such as the Herfindahl-Hirschman Index (HHI) and the Concentration Ratios (CR). This correction can be useful given recent macroeconomic discussions showing rising markups and concentration (e.g. De Loecker et al., 2020; Autor et al., 2020; Gutierrez and Philippon, 2017).

I derive two statistics to capture the two main features of market power in the presence of granular firms: consumer costs from variable markups and producer gains from higher profits. In a general equilibrium setting, changes in market power arising from any change in fundamentals affect welfare through these two channels. First, they modify the price level and its distribution due to changes in firms' markups. Such changes are often unobserved and difficult to estimate. I derive

a welfare-consistent domestic concentration measure that captures the aggregate deviation from the optimal efficient price distribution. Conveniently, this measure only needs firms' market shares in a given market in the case of quantity competition, plus the elasticity of substitution across firms' varieties in the case of price competition. Second, I derive a function to measure aggregate profits closely related to the level of misallocation in the economy. This measure is a function of widely employed concentration functions and can also be easily computed using firms' market shares. To be sure, versions of this result exist in the industrial organization literature (Syverson, 2019, Berry et al., 2019). I embed it in an international trade framework and show how to implement it in a setting where domestic firms also export to foreign markets. These two families of measures can be understood as sufficient statistics, useful for international economists and macroeconomists studying market power.

Many papers have evaluated the pro-competitive gains from trade using different strategies. One of them has been to maintain the assumption of monopolistic competitive firms but abandon CES preferences, firm heterogeneity or both (e.g. Neary, 2016; Feenstra and Weinstein, 2017; Arkolakis et al., 2019). In these papers, the usual goal is to get analytical expressions to perform comparative statistics and derive sufficient statistics. A different strategy has been to keep CES preferences but introduce granular firms to quantitatively estimate the gains from trade (e.g. Edmond, Midrigan and Xu, 2015). In this case, obtaining tractable analytical expressions becomes challenging. I show how to restore the ability to measure local welfare changes to this last setting using the aforementioned sufficient statistics, which summarize welfare costs arising from markups of firms from all exporting origins and welfare gains from domestic firms earning profits in all markets where they are active. In a nutshell, the key to restoring the ability to measure welfare gains from trade under granularity is to employ the partial elasticities of the two market power measures—the welfare-consistent concentration function and aggregate profits—with respect to the price index, allowing all firms to fully update the markups.

Have the gains from trade changed as globalization and market concentration increased across markets over the last few decades? Answering this question is key to evaluating the opportunity cost of reducing trade openness, as some recent events may suggest. To do so, I put the formulae to work to measure welfare changes in a period of a steep increase in import penetration and market concentration in Chile. Between 1998 and 2007, this country signed free trade agreements with the US, the EU and China, uniformly decreased the Most Favored Nation (MFN) tariff applied to all WTO members, and experienced an appreciation of its exchange rate, making the Chilean peso 50% more expensive in dollars in 2007 relative to 2003. Jointly, these events had a likely role in the

increase in import penetration from 59% in 2000 to about 66% in 2007.

In contrast to what the usual market concentration indices show, I find that the average welfare-consistent domestic concentration measure across sectors remained unchanged throughout this period. This implies that the direct cost from granular domestic markups stayed the same in spite of an increase in the HHI and CR. The intuition is that, while these firms appeared more concentrated based on the allocation of expenditure on domestic goods alone, increased foreign competition forced them to lower their markups overall. The horse race between higher import penetration, and thus lower domestic markups, and higher domestic concentration resulted in the former dominating and generating lower consumer welfare gains from trade in 2007. In other words, domestic firms were more disciplined by foreign competition and thus their markups were less elastic to small changes in trade openness. The consumer welfare losses from a 10 log-point increase in the share of total domestic expenditure in domestic goods were 0.3 log points lower in 2007 vs 1998.

A key difference in the presence of granular firms is that profits are higher than under monopolistic competition when entry is limited. In other words, the producer gains from trade, in the form of higher markups, are positive compared to when firms are assumed to be in a continuum. From an aggregate perspective, changing trade conditions also modify the source of profits across sectors. In other words, sectors that are less affected by foreign competition are more likely to increase their share in aggregate profits. Therefore, the resulting producer gains indicator results from the interaction between industry profit shares and markup increases. I find that the local producer gains from closing the economy were relatively the same at the end of the period of trade opening. In the end, the aggregate welfare effect from closing the economy uniformly in 10 log points when considering both producer gains and consumer losses were slightly lower in 2007 vs 2003. The takeaway is that the aggregate local gains from trade depend on the domestic market structure, which at the same time depends on foreign competition. Therefore, the opportunity cost of closing the economy is likely to vary substantially across countries as the degree of firm granularity in economies increases.

The main contribution of this paper is to provide a family of sufficient statistics for the gains from trade in the presence of granular firms. As mentioned above, Arkolakis, Costinot and Rodriguez-Clare (2012)—ACR—was foundational in showing that gravity trade models, ranging from those focusing on Ricardian comparative advantages to those with firm heterogeneity and monopolistic competition, all fall into a single sufficient statistic when the goal is evaluating the welfare gains from trade. In a subsequent paper, Arkolakis, Costinot, Donaldson and Rodriguez-Clare (2019)—ACDR—extended this framework to allow for a wide range of preference structures that yielded variable markups, including those delivering non-homothetic demand. In that setting, they main-

tained monopolistic competition and an ex-ante approach, which allowed them to correct the ACR statistic by a multiplicative factor that only depends on model parameters. Other papers also employed similar approaches to study the gains from trade from firm selection (Melitz and Redding, 2015), variety and markups in the context of monopolistic competition (Feenstra and Weinstein, 2017; Feenstra, 2018), with worker heterogeneity (Galle et al., 2022), and innovation (Impullitti et al., 2022), among others. My paper delivers a stylized formula to analyze the ex-post, short-run welfare gains in settings with high concentration.[1] Moreover, this formula allows for decomposing welfare changes into producers' and consumers' surplus changes, an analysis usually performed in the classic international trade literature that is standard in today's international trade undergraduate courses (e.g. Feenstra and Taylor, 2014).

My second contribution is to inform the literature studying sectoral and aggregate changes in market power in recent years by providing welfare-consistent concentration measures in open economies. Different papers have documented a rise in market power recently (e.g. De Loecker et al., 2020; Autor et al., 2020; Benkard et al., 2021), while others have cast doubts on the actual implication of the measures used for market power measurement (e.g. Rossi-Hansberg et al., 2021, Amiti and Heisse, 2024, Shapiro and Yurukoglu, 2024). I microfoundate a family of concentration measures by exploiting the convex relationship between markups and market shares when the latter account for the entire market, e.g. foreign exporters active in domestic markets. By doing so, my measures become most valuable in periods of changing trade conditions, given that domestic market concentration and markups alone cannot account for the entire state of competition.

Finally, I also contribute to a deeper understanding of the structure of trade and macroeconomic models with firm heterogeneity and granularity. These models feature complex interactions between firms, which makes working with them analytically challenging. They have been employed in quantitative settings to study international price deviations from relative purchasing power parity (Atkeson and Burstein, 2008), the gains from trade under misallocation (Edmond et al., 2016), the strength of firms' strategic complementarities in prices (Amiti et al., 2019), the role of granular firms in determining comparative advantages (Gaubert and Itskhoki, 2021), labor and input market power (Berger et al., 2022; Macedoni and Tyazhelnikov, 2024), the aggregate welfare costs of markups (Edmond et al., 2023) and its relationship to firms' product scope and development (Macedoni, 2022), the interrelation between trade, innovation and markups (Cavenaile et al., 2023), how granularity modifies the trade gravity equation (Breinlich et al., 2023), the bargaining power between buyers and sellers (Alviarez et al., 2023), macroeconomic fragility (Ferrari and Queirós,

[1] Independent work by Antras et al. (2025) also looks at the welfare gains from trade in the context of oligopolistic model although without a focus on sufficient statistics.

2024), and the pro-competitive effects of trade agreements (Crowley et al., 2024), among others. To the best of my knowledge, I am the first to theoretically study the aggregation of firm-level markups in a unified way encompassing quantity and price competition. Doing so allows me to cast light on the model anatomy and its general equilibrium implications.

The rest of the paper is organized as follows. Section 2 presents the industry model with domestic and foreign granular firms, and shows how industry concentration and the CES price index are jointly determined in equilibrium, to then conclude by deriving the statistic for the consumer welfare gains from trade. Section 3 builds up a multi-sector, multi-country model to define the aggregate domestic markup and the producer welfare gains from trade. It ends by defining the local aggregate welfare gains from trade that extends ACR. Section 4 studies changes in trade exposure, market concentration and the gains from trade in a period of trade opening in Chile. Section 5 concludes.

2 Model

In this section, I present the model and derive the key industry and country aggregates. I start with the characterization of a typical industry and finish with the aggregate economy.

2.1 Industry Structure

I focus on a specific market defined as an industry-country, in which an exogenous number of granular firms $N_i = \sum_{i=1}^{I} N_{ji}$ from multiple origins indexed by i are active.

2.1.1 Consumers

Consumers have CES preferences over the different varieties ω of an industry good v, with elasticity of substitution σ. The industry quantity index is:

$$Q_i^v = \left[\sum_{j=1}^{J} \sum_{\omega \in \Omega_{ji}^v} q(\omega)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},$$

where $q(\omega)$ is the ω-specific quantity.

Expenditure on variety ω is $s_{ji}^v(\omega)E_i^v$, where $s_{ji}^v(\omega) = \frac{p_{ji}^v(\omega)^{1-\sigma}}{(P_i^v)^{1-\sigma}}$ is the expenditure share on ω, which I will refer to as the market share, and E_i^v is total consumer expenditure on industry v. $P_i^v = [\sum_{j=1}^{J} \sum_{\omega \in \Omega_{ij}} p(\omega)^{1-\sigma}]^{\frac{1}{1-\sigma}}$, is the CES price index.

2.1.2 Firms

I assume there are N_{ji} firms from j active in country i in sector v (I omit the industry superindex for the rest of the section). These firms own a blueprint of a variety that can be produced with productivity $\varphi(\omega)$ drawn from a continuous distribution $F(\varphi)$ with a lower bound φ_L. The set of productivities in this industry is $\{\varphi_1, \varphi_2, ..., \varphi_{N_{ji}}\}$.[2].

Firms use only labor to produce, which can be hired at wage w_j. In order to export to country i, a firm in j has to ship τ_{ji} per unit of a product—i.e. there are iceberg trade costs. Therefore, a firm with productivity φ in country j has a marginal cost of $c_{ij}(\varphi) = w_j\tau_{ji}/\varphi$ to sell in i. Firms do not pay a trade cost to ship domestically (i.e. $\tau_{ii} = 1$). I will identify individual firms with their marginal costs, c.

2.1.3 Type of Competition and Optimal Prices

Each variety's optimal price depends on firms' marginal costs c, and the type of competition $\mathscr{M}$, which can be quantity, price or monopolistic competition. Omitting subindices for simplicity, we get:

$$p(c;\mathscr{M}) = c \times \underbrace{\bar{\mu} \times \varepsilon(c;\mathscr{M})}_{\text{Mark-up } \mu}, \tag{1}$$

where $\bar{\mu} \equiv \frac{\sigma}{\sigma-1}$ is the fixed mark-up component from product differentiation, and $\varepsilon(c;\mathscr{M}) \geq 1$ is the *ad-valorem excess markup* from granular market power.

The optimal price depends on the type of competition only through the excess markup. Specifically, it takes the following functional forms for each type of competition:

$$\varepsilon(c;\mathscr{M}) = \begin{cases} 1 + \frac{s(c)}{1-s(c)} & \text{if } \mathscr{M} \text{ is quantity competition} \\ 1 + \frac{s(c)}{1-s(c)}\frac{1}{\sigma} & \text{if } \mathscr{M} \text{ is price competition} \\ 1 & \text{if } \mathscr{M} \text{ is monopolistic competition} \end{cases} \tag{2}$$

There are different aspects of the excess markup that are worth mentioning. The first one is that this formulation shows that monopolistic competition does not generate misallocation across firms given trade costs. Secondly, both quantity and price competition make $\varepsilon(c;\mathscr{M})$ endogenous, as it depends on the firm's market share. Thirdly, the excess markup is parameter-free under quantity

[2] $|\Omega_{ji}| = N_{ji}$

competition. Finally, we can think of both monopolistic and quantity competition as special cases of price competition when $\sigma \to 1$ and $\sigma \to \infty$, respectively.[3]

The CES industry price index can then be written as follows:

$$P_i(\mathscr{M}) = \left[\sum_{j=1}^{J} \sum_{c(\varphi)\in\Omega_{ji}} \left[(c\bar{\mu})^{1-\sigma} \times \varepsilon(c;\mathscr{M})^{1-\sigma}\right]\right]^{\frac{1}{1-\sigma}}. \quad (3)$$

2.1.4 Industry Equilibrium

The industry equilibrium is a set of N optimal price equations, N market shares equations and the CES industry price index equation, which depends on market shares through each excess markup term as shown in equation 3. These $2N + 1$ equations depend on the productivity draws, the iceberg trade costs, the wage and the total expenditure in the industry, and determine $\left\{\{s(c), p(c)\}_{c\in\Omega_i}, P_i\right\}$.[4]

2.2 Aggregate Economy

The economy is comprised of a continuum of industries. Consumers have Cobb-Douglas preferences over industries, with expenditure share α^v. I analyze an economy that has firms that are large domestically but small in foreign markets.

2.2.1 Aggregate Total Income and Markup

Aggregate domestic income originates from labor income and domestic profits from domestic and foreign sales in all industries:

$$Y_i(\mathscr{M}) = w_i L_i + \int [\Pi_{ii}^v(\mathscr{M}) + \sum_{j\neq i} \Pi_{ij}^v] dv, \quad (4)$$

where L_i is total labor, which is supplied inelastically. Aggregate income depends on the type of competition via granular domestic profits only due to the assumption of domestic firms being small in the world.

Given that aggregate profits are endogenous and depend on market structure, the share of

[3]Note that this is only true for the excess markup only and not the full markup μ, which is a function of σ too. Markups under monopolistic competition and quantity competition are, though, special cases of price competition in log changes, i.e. $\hat{\mu} = \hat{\varepsilon}$.

[4]$\Omega_i \equiv \bigcup_j \Omega_{ji}$.

profits in aggregate income is variable. Specifically, we can write such a share as the inverse of an aggregate markup, i.e. $\mu_i^{-1} = \frac{w_i L_i}{Y_i}$. Is there a closed-form solution for such an aggregate markup in this economy? The following proposition shows that there is:

Proposition 1. ***Aggregate Domestic Markup.***

Under trade balance, the aggregate markup μ_i *in this economy is as follows:*

$$\mu_i = \bar{\mu} \times \mathcal{E}_i \tag{5}$$

where $\mathcal{E}_i \equiv \left[1 + \frac{\int \alpha^v \Xi(\lambda_{ii}^v, H_{ii}^v)}{1 - \int \alpha^v \Xi(\lambda_{ii}^v, H_{ii}^v)}\right]$ *is the Aggregate Domestic Excess Markup, and the function* $\Xi \in [0, 1)$ *captures the industry-specific degree of domestic granularity, which positively depends on* λ_{ii}^v*, domestic absorption, and* H_{ii}^v*, a competition-specific proper measure of domestic concentration.*

Proof: See Appendix .

There are different points to stress about Proposition 1. First, it shows that the aggregate markup can be constructed analogously to the firm-level markup: a fixed markup that does not distort the relative prices times an ad-valorem term that increases it and depends on the industry-specific market structures and import competition. The key assumption to reach this result is trade balance, which allows to use the economy-wide budget constraint.[5]

A second observation is that both domestic absorption and domestic market concentration directly determine the aggregate excess markup. In the Appendix, I show that such a function takes the following functional forms depending on the type of competition:

$$\Xi(\lambda_{ii}^v, H_{ii}^v) = \begin{cases} (\lambda_{ii}^v)^2 HHI_{ii}^v & \text{if } \mathscr{M} \text{ is quantity competition} \\ \frac{(\lambda_{ii}^v)^2}{\sigma(1-\lambda_{ii}^v)+\lambda_{ii}^v} \overline{HKI}_{ii}^v & \text{if } \mathscr{M} \text{ is price competition} \\ 0 & \text{if } \mathscr{M} \text{ is monopolistic competition,} \end{cases} \tag{6}$$

where:

- $HHI_{ii}^v \equiv \sum_c z_{ii}^2(c)$ is the domestic Herfindahl-Hirschman Index over firms' market shares on the expenditure on domestic foreign variaties, z;[6] and

[5] This result can also be derived under an exogenous trade deficit or surplus, with the difference that the aggregate markup is not only on top of labor income but also on top of the exogenous international transfers that explain the imbalance.

[6] $z(c) \equiv s(c)/\lambda_{ii}^v$

- $\overline{HKI}^v_{ii} \equiv \sum_{k=2}^{\infty} \theta(k, \lambda^v_{ii}; \sigma) HKI^v_{ii}(k)$ is the weighted average of an infinite sum of Hannah and Kay indices over all its discrete degrees ($HKI^v_{ii}(k) = \sum_c z^k_{ii}(c)$), where weights depend on industry-specific domestic absorptions, σ, and the degrees ($\sum_{k=2}^{\infty} \theta(k, \lambda^v_{ii}; \sigma) = 1$).[7]

Under quantity competition, the markup positively depends on the average sectoral absorption, λ^v_{ii}, and domestic concentration, HHI. In the case of price competition, the expression is analogous although more involved, as market shares do not convey the same degree of market power. Specifically, the absorption term is multiplied by $[\sigma(1-\lambda^v_{ii})+\lambda^v_{ii}]^{-1} < 1$ and the well-defined concentration function is a weighted average across all the degrees of the Hannah and Key indices.[8]

Given that we have the aggregate markup, we can back up the aggregate demand elasticity η_i in this economy using the Lerner Index. Specifically, we know that $\eta_i^{-1} = 1 - \frac{1}{\mu_i}$, so:

$$\eta_i = \left[1 + (\sigma - 1) \int \alpha^v \Xi(\lambda^v_{ii}, H^v_{ii})\right]^{-1}. \tag{7}$$

The higher the degree of domestic granularity, the lowest the aggregate demand elasticity, and thus the higher the profit income share.

2.2.2 General Equilibrium

There are two endogenous variables that need to be determined in equilibrium: the domestic wage w_i and aggregate income Y_i. I use w_i as a numeraire, so the aggregate budget constraint that delivers the aggregate markup determines aggregate income:

$$Y_i = L_i \bar{\mu} \times \mathcal{E}_i. \tag{8}$$

In this economy, domestic income depends on the degree of market power firms can excersice over domestic consumers, which is determined by the aggregate domestic elasticity, η_i.[9]

3 A Welfare-Consistent Concentration Measure under Granularity

In this section, I derive a Welfare-consistent concentration measure under granularity and CES preferences across firms' varieties. Changes in this index capture changes in the CES industry price

[7] Note that $HKI(2) = HHI$ and $HKI(k) < HHI$ for all $k > 2$, so $HHI \geq \overline{HKI}$.

[8] As σ approaches one under price competition, Ξ tends to the expression under quantity competition: the factor of correction tends to one, and $\overline{HKI}$ tends to HHI.

[9] Note that $\mathcal{E}_i = \frac{\sigma - 1}{\sigma - \frac{\eta^{MC}}{\eta_i}}$, where $\eta^{MC} = \sigma > \eta_i$ is the aggregate demand elasticity under monopolistic competition.

index only due to first-order changes in the excess markup and, as such, it can be interpreted as changes in consumer welfare. In addition, this measure will be a necessary step towards deriving the welfare gains from trade for consumers in Section 4.

3.1 The Industry Excess Markup As a Concentration Measure

When firms compete in quantities and prices, the CES price index in equation 3 is endogenous, as it depends on the specific market structure $\{s(c)\}_{c=c_1,...c_{N_i}}$ through the ad-valorem excess markup $\varepsilon(c)$. In order to make progress in understanding the relationship between $P_i(\mathscr{M})$, the market structure, and competition, I log-differentiate equation 3:

$$\hat{P}_i(\mathscr{M}) \quad = \quad \hat{P}_i^m(\{c\},\mathscr{M}) + \hat{\mathcal{C}}_i(\{s(c)\},\mathscr{M}), \tag{9}$$

where $\hat{P}_i^m(\{c\}) \equiv \sum_{c(\omega)\in\Omega_{ji}} s(c)\hat{p}^m(c)$ is the log change in the CES price index that takes place due to changes in the efficient prices and $\hat{\mathcal{C}}_i(\{s(c)\},\mathscr{M}) \equiv \sum_{l=1}^{N_{ji}} s_{ji,l}\hat{\varepsilon}_{ji,l}(s(c_{j,l});\mathscr{M})$ is the log change that originates from changes in the industry ad-valorem excess markup. Given that the differentiation takes place around an initial equilibrium and preferences are CES, each firm-specific change is weighted by the equilibrium market share.

Both terms defining the change in prices in Equation 9 depend on the type of competition, but in different ways. In the case of $\hat{P}_i^m$, it works through individual firms' weights and thus it does not have a first-order impact. On the contrary, the market structure has a first-order effect on $\hat{\mathcal{C}}_i(\{s(c)\},\mathscr{M})$ because shares modify individual firms' markups and not only their shares. Therefore, this term is key to understanding how any change in market and policy conditions (e.g. import tariffs) first-orderly affects the price index and thus consumer welfare when we depart from monopolistic competition.

Is there a closed-form solution for $\mathcal{C}_i$ given that both individual firms' excess markups and their weights depend on market shares? Proposition 2 proves that there is and provides the expression.

Proposition 2. ***Welfare-Consistent Concentration Measure under Granularity and CES Preferences.***

There exists an industry-specific closed-form concentration measure that captures the consumer welfare cost from granularity when consumers have CES preferences over varieties, and it takes the following functional form depending on the type of competition:

$$\mathcal{C}_i(\{s(c)\}, \mathscr{M}) = \begin{cases} -\sum_{c\in\Omega_i}\left[\log\left(1-s(c)\right)+s(c)\right] & \text{if } \mathscr{M} \text{ is quantity competition} \\ -\sum_{c\in\Omega_i}\left[\log\left(1-s(c)\right)-\bar{\mu}\log\left(1-s(c)\bar{\mu}^{-1}\right)\right] & \text{if } \mathscr{M} \text{ is price competition} \\ 0 & \text{if } \mathscr{M} \text{ is monopolistic competition,} \end{cases} \tag{10}$$

where $\bar{\mu} \equiv \frac{\sigma}{\sigma-1}$ *is the markup under monopolitic competition.*

Proof: See Appendix .

The function $\mathcal{C}_i$ inherites several properties from the variety-specific excess markup: (i) It is zero under monopolistic competition, given that there are no welfare costs from granularity in such a scenario, (ii) it is endogenous, as it depends on market shares and thus on the market structure, (iii) it is parameter-free under quantity competition which makes it a potentially attractive empirical measure, and (iv) the monopolistic and quantity competition variants are special cases of the price competition variant.[10]

3.2 Comparison to Other Market Concentration Measures

Economists use a series of market concentration measures to understand how market conditions change over time. The most well-known measures share the same property: they are weakly convex in firms' market shares. Two examples are the Herfindahl-Hirschman Index (HHI) and the share of top N^T firms, often called Concentration Ratios ($CR(N^T)$, where N^T is the number of top firms considered).[11] Given that the function $\mathcal{C}_i$ is convex on market shares as well, it can also be interpreted as a concentration measure.

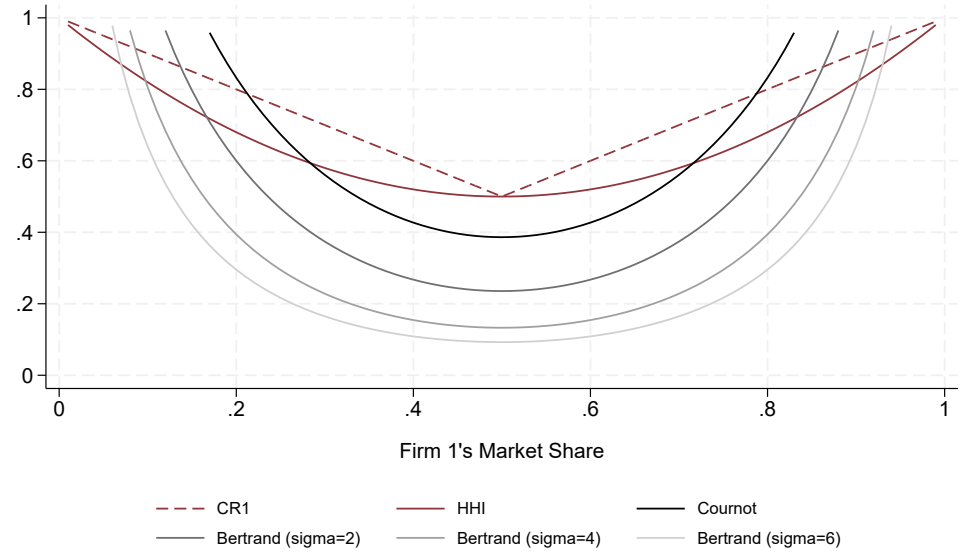
Figure 1 compares $\mathcal{C}$ to HHI and $CR(1)$ in a market with two firms under different scenarios.

[10] $\mathcal{C} \to 0$ and $\mathcal{C} \to \sum_{c(\omega)\in\Omega_{ji}}[-s(c) - \ln(1-s(c))]$ when $\sigma \to +\infty$ and $\sigma \to 1$, respectively.

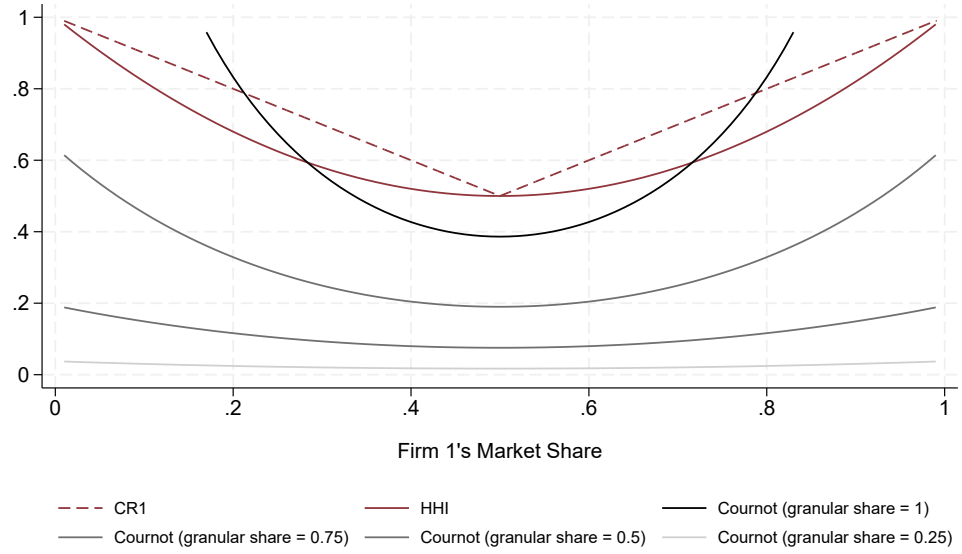
[11] For example, the US Census Bureau reports the HHI of the top 50 firms and the Concentration Ratio of 4, 8, 20 and 50 firms.

Figure 1: Concentration Measures with Two Firms.

(a) Different competition and σ



(b) Different Granular Shares



In Figure 1a, I include $\mathcal{C}$ under quantity competition and price competition.[12] In the last case, $\mathcal{C}$ is parametrized by σ, so I include it for different values of σ. Note that as σ increases, the concentration measure shifts down, implying that the same market structure delivers lower excess markups. This is intuitive since a higher σ implies higher substitutability across varieties and, therefore, more direct competition across firms. In addition, as σ approaches one, the concentration measure under price competition pointwise converges to the one under quantity competition.

[12] $\mathcal{C}$ is trivially zero under monopolistic competition.

3.3 Concentration with a Varying Granular Market Segment

It is relevant to explore how the concentration function changes when we only measure a segment of the market. This can arise under different scenarios. For instance, it may be that only a subset of firms in the market are granular. In this case, only the share of granular firms matters since non-granular firms would charge the monopolistic competitive markup or no markup at all, depending on the nature of competition. Alternatively, we may want to measure only the concentration of a specific segment of the market, such as domestic varieties, to study gains from trade.[13]

In Figure 1b, I show the concentration measure under a different share of granular firms in the case of quantity competition. As the granular share decreases, the concentration measure becomes less convex and shifts downwards, capturing the decrease in their excess markups and thus in their market power.

The fact that $\mathcal{C}$'s convexity depends on the granular segment of the market is an advantage of this measure over others, such as the HHI, as it can help to distinguish between efficient and inefficient concentration. Suppose that we are interested in the degree of market power of domestic firms during a period in which import penetration increased. Standard trade models with heterogeneous firms and endogenous markups would show under standard productivity distributions that an increase in foreign competition can imply an increase in the HHI when computed only over domestic firms. Instead, $\mathcal{C}$ would decrease because it uses the right market shares, i.e. the ones that account for foreign firms as well.[14]

Last but not least, the numerical evaluation of $\mathcal{C}$ has a direct interpretation through the lens of this model: it measures the first-order consumer welfare costs from granular markups. In the absence of it, this function is equal to zero, as shown by Equation 10.

3.4 An Aggregate Welfare-Consistent Concentration Measure

We can compute an aggregate welfare-consistent concentration index in the context of the model presented in Section 2. The Cobb-Douglas aggregation across industries imply that the aggregate price index is a weighted geometric average across industries, where weights are expenditure shares. Therefore, the aggregate welfare-consistent concentration in logs is as follows:

$$\hat{\mathcal{C}}_i \quad = \quad \int_v \alpha_v \hat{\mathcal{C}}_i^v dv. \tag{11}$$

[13] This can be done given that $\mathcal{C}$ is additively separable across firms.

[14] We could compute the HHI using the right market shares as well, but that would be against the common use of HHI and not deliver a welfare-consistent measure.

In practical terms, this measure can capture the first-order consumer welfare costs from granularity at the economy-wide level, provided that there is no expenditure reallocation across sectors.

4 The Welfare Gains from Trade under Granularity

In this section, I employ the model in Section 2 and the welfare-consistent concentration measure from Section 3 to derive sufficient statistics that can be employed to measure the welfare gains from trade under granularity.

Granularity affects welfare through two channels in this model: the prices paid by consumers (P_i) and the income made by workers and firms ($Y_i = w_i L_i + \Pi_i$). In the first case, consumers face markups that depend on industries' market structures. The more concentrated these markets are, the higher the excess markup from granularity and thus the higher the consumer welfare costs. Second, granular firms make profits that contribute to aggregate income.[15]

In a closed economy, these two channels perfectly offset each other: the markups paid by consumers are the profits made by firms. This does not mean there are no welfare costs. With firm heterogeneity, firms face different demand elasticities and thus charge different markups, distorting relative prices and thus reducing aggregate productivity.

With foreign firms competing against domestic firms in domestic markets, the origin-specific degree of granularity matters, as it regulates who bears the burden of misallocation. The theoretical results below capture this.

4.1 Consumer Welfare Gains from Trade

In order to understand how consumer welfare is affected by changes in trade openness via granularity, I employ the welfare-consistent concentration measure. I start with the analysis at the industry level and aggregate the resulting statistics in the end. For illustrative purposes, I start by assuming industries are homogeneous.[16]

4.1.1 The Change in the CES Price Index

Changes in consumer surplus are determined by changes in the CES price index, P_i. In order to derive a sufficient statistic that captures the relationship between welfare and trade openness, I follow the same strategy as in Arkolakis et al. (2012). In their paper, the statistic for the

[15] Note that, in reality, markups also affect labor demand and thus labor income. Given the assumption of an inelastic labor supply, the labor market is not distorted in this model, allowing me to focus on the goods markets.

[16] This implies that each industry in the continuum has the same number of domestic and foreign industries, each with the same unit cost.

gains from trade associated to a model with a fixed number of varieties but without granularity is $\hat{W}_i^{ACR} = -\hat{P}_i^{ACR} = -\frac{\hat{\lambda_{ii}}}{\sigma-1}$. I will call this expression the ACR term or formula. In Proposition 3, I derive the change in consumer welfare when we allow for granularity.

Proposition 3. ***Sufficient Statistic for the Industry CES Price Index Changes from Changes in Trade Costs under Granularity.***

The industry CES price index log change as a result of changes in iceberg trade costs is an additive function of domestic absorption rate and a CES-consistent concentration measure under both quantity and price competition, normalized by the equilibrium absorption rate.

$$\hat{P}_i = \frac{\hat{\lambda_{ii}}}{\sigma - 1} + \frac{\hat{\mathcal{C}}_{ii}}{\lambda_{ii}}. \tag{12}$$

Proof: See Appendix .

Equation 12 augments the ACR formula to accommodate granular firms with an additive term. The extra term is the change in the proper concentration function in equation 10, but only computed on the domestic segment of the market. Therefore, this statistic can be computed by using the same data employed for HHIs and concentration ratios, plus the absorption rate.

The reason why the term $\frac{\hat{\mathcal{C}}_{ii}}{\lambda_{ii}}$ functionally captures both changes in domestic and foreign markups is clearer if we think of a single-sector economy in which country-specific wages are determined only by labor demand.[17] In this setting, an increase in trade costs raises the relative price of foreign firms, reducing their market share and therefore their markups. This is a partial equilibrium result. The increase in trade costs also reduces foreign labor demand, lowering w_j. This opposes the direct effect, increasing foreign firms' market shares, and therefore raising their markups to the point that it fully offsets the direct effect. This is why $\mathcal{C}_{ji}$, which is the statistic for the aggregate welfare cost of foreign firms' markups, is not featured in equation 12. On the other hand, the degree of domestic concentration reduces the increase in domestic wages from higher labor demand, so the change in domestic markups is magnified, as shown by the domestic absorption level $\lambda_{ii} < 1$ in $\mathcal{C}_{ji}$'s denominator. As a result, the total change in the price index positively depends on the increase in domestic concentration, and more so, the initial higher rate of absorption, since the magnification mechanism would be higher.

In conclusion, an increase in trade costs raises the price index via the direct effect, captured by the "ACR term", and the indirect effect from market power reallocation, the $\frac{\hat{\mathcal{C}}_{ii}}{\lambda_{ii}}$ term.

[17] I ignore the potential monopsonistic power of granular firms on the labor market that would arise in a single-sector setting in this example.

4.1.2 Endogenous Concentration Changes

In order to understand the total consumer welfare costs from changes in trade costs, we would need the counterfactual level of concentration. In other words, Equation 12 is not sufficient since $\frac{\hat{\mathcal{C}}_{ii}}{\hat{\lambda}_{ii}}$ is not determined. In this section, I focus on analyzing changes in domestic concentration and derive an expression that depends on the industry CES price index and the domestic absorption rate.

Any change to trade costs affecting only foreign firms can modify the relative price between domestic firms, given that each firm faces a different perceived demand elasticity depending on its initial market share. This means that when trade costs change, they will impose a different competitive pressure on firms' prices, affecting their market shares. Proposition 4 formalizes this result.

Proposition 4. ***Sufficient Statistic for the (normalized) Domestic CES-Consistent Concentration Measure from Changes in Trade Costs.***

The normalized domestic CES-consistent concentration measure change as a result of changes in iceberg trade costs only affecting foreign firms depends on the CES industry price index and the rate of absorption under both quantity and price competition.

$$\frac{\hat{\mathcal{C}}_{ii}}{\lambda_{ii}} = \Gamma^{c}_{ii} \times \left[\tilde{\Phi}_{ii} \hat{P}_i - \frac{\hat{\lambda_{ii}}}{\sigma - 1} \right] \tag{13}$$

where:

- $\Gamma^{c}_{ii} \equiv \frac{\partial \log e^{\mathcal{C}_{ii}}}{\partial \log P_i} / \lambda_{ii} < 0$ *is the (normalized) partial elasticity of concentration with respect to* P_i.
- $\tilde{\Phi}^{c}_{ii} \equiv \frac{\sum_{l=1}^{N_{ii}} \frac{d \log e^{\mathcal{C}_{ii}}}{d \log p_l}}{\frac{\partial \log e^{\mathcal{C}_{ii}}}{\partial \log P_i}} \in (0,1)$ *is the relative strength of individual firms' equilibrium responses.*

Proof: See Appendix .

Equation 13 captures the two mechanisms through which a change in trade costs affects the distribution of domestic market shares and thus domestic markups. The first one is the positive relationship between $\frac{\hat{\lambda_{ii}}}{\sigma - 1}$ and concentration. This captures the direct shift in market share from foreign to domestic varieties due to the foreign firms' loss in competitiveness. The elasticity Γ^{c}_{ii} regulates the strength of this change, being greater the higher the degree of domestic granularity.

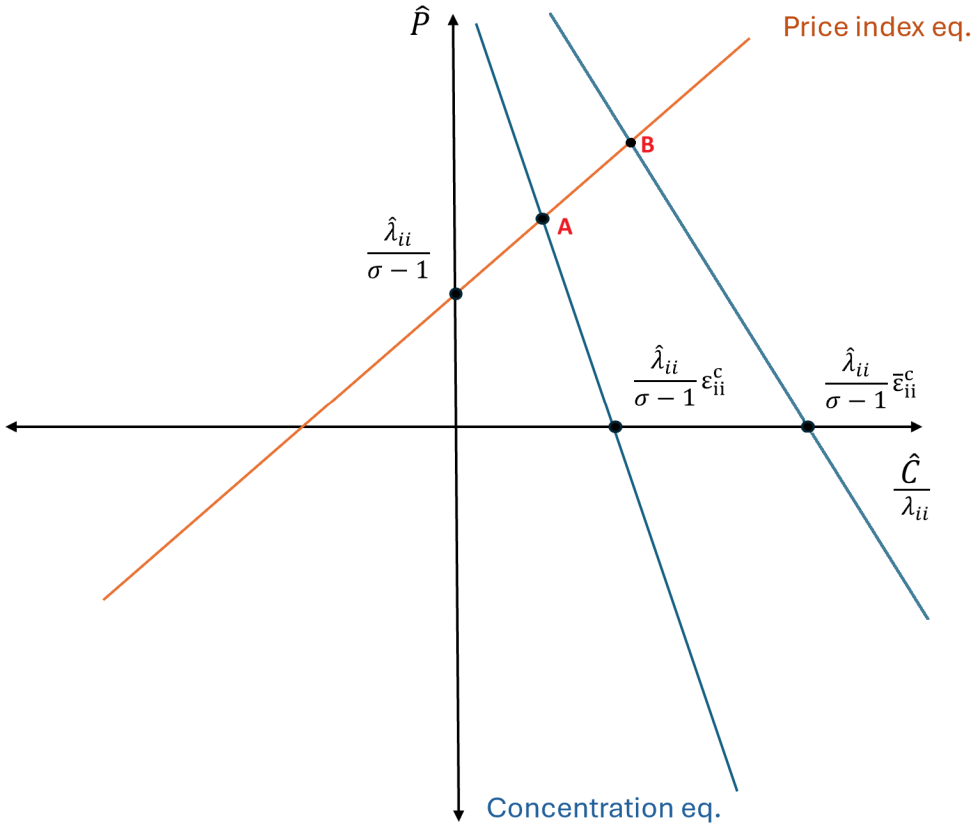
An increase in trade costs also implies stronger competition between domestic firms to capture a greater share. This implies an opposing force that reduces the degree of domestic misallocation induced by higher trade costs. The negative relationship between the CES price index and concentration captures this and is also regulated by the relative strength of individual firms' equilibrium

responses summarized by $\tilde{\Phi}_{ii}$. The more concentrated these firms are, the greater the strength of these equilibrium responses.

4.1.3 The Joint Determination of Changes in the Domestic Price Index and Concentration

The *Price Index Equation* 12 and the *Concentration Equation* 13 fully characterize the change in the CES price index as a function of the change in the degree of trade openness, as captured by changes in absorption rate. Figure 2 shows the first-order log-change in P_i and $\mathcal{C}_{ii}$ that results from an increase in domestic absorption after a rise in trade costs.

Figure 2: Price Index and Domestic Concentration Change with Increasing Trade Costs



In a model with monopolistic competition, only the price equation determines the change in consumer welfare given that $\frac{\hat{\mathcal{C}}_{ii}}{\lambda_{ii}} = 0$. This delivers the well-known ACR formula under monopolistic competition with a fixed number of varieties. Therefore, the equilibrium change in that case is a point (0 on Figure 2). Under granularity, the price index positively depends on changes in markups, which is captured by the linear relationship of the Price Index Equation.

Concentration, on the other hand, negatively depends on the CES price index through the

misallocation channel. The negative linear relationship of the *Concentration Equation* shows this. The joint equilibrium change in $\mathcal{C}_{ii}$ and P_{ii} is found at point A in Figure 2.

The equilibrium change in the CES price index under granularity is greater than under no granularity. An increase in trade costs raises the domestic absorption rate, and domestic markups increase due to a lower demand elasticity, amplifying the increase in P_{ii}, causing consumer welfare losses. In other words, under granularity, there is a pro-competitive effect of trade that needs to be considered.

How does the degree of concentration affect the equilibrium change in the price index? When there is high concentration, the partial elasticity Γ^c_{ii} is higher, so the *Concentration Equation* is shifted right. Therefore, the joint equilibrium in this case is at point B, which shows a greater increase in the price index due to a larger increase in markups.

4.1.4 A Granular-corrected Consumer Welfare Formula

We can solve for $\hat{P}_i$ analytically using both the *Price Index* and *Concentration Equations*. The resulting expression summarizes the change in consumer welfare from changes in trade opening:

$$\hat{P}_i \quad = \quad \mathbf{g}^{\text{cs}}_{ii} \times \frac{\hat{\lambda}_{ii}}{\sigma - 1}, \tag{14}$$

with:

$$\mathbf{g}^{\text{cs}}_{ii} \quad \equiv \quad \frac{\text{Magnification from } \uparrow \text{ markups}}{\text{Attenuation from } \uparrow \text{ misallocation}} \equiv \frac{1 + \Gamma^c_{ii}}{1 + \Gamma^c_{ii}\tilde{\Phi}^c_{ii}} > 1.$$

The $\mathbf{g}^{\text{cs}}_{ii}$ factor augments the original ACR-type formula to account for changes in consumer surplus due to changes in markups. It arises because closing the economy magnifies the increase in the price index by decreasing the perceived demand elasticity by domestic firms and thus domestic markups. Closing the economy also generates misallocation across domestic firms, given that the demand elasticity declines more for larger firms, transferring demand to smaller, less productive firms. This is captured by the denominator in Equation 15. The factor $\mathbf{g}^{\text{cs}}_{ii}$ summarizes both mechanisms. Importantly for measurement, this factor depends only on the domestic market structure, the type of competition, and the elasticity of substitution.

4.1.5 A Sufficient Statistic for the Consumer Welfare Gains from Trade under Granularity

In ACR, the welfare formula is understood as a sufficient statistic for the gains from trade given that absorption is observed and the denominator is the trade elasticity—a widely estimated parameter in the trade literature. Can we interpret equation 14 in a similar way?

This consumer welfare formula with granularity has three elements: $\hat{\lambda}_{ii}$, $\sigma - 1$ and $\mathbf{g}_{ii}^{\text{cs}}$. The first one can be assumed or observed in the data. For instance, what is the increase in the CES price index as a result of an increase in the domestic absorption rate of 10 log points?[18] The second element, $\sigma - 1$, can be interpreted as the trade elasticity only in the limit in which there is enough competition that granular firms compete as in monopolistic competition.

The last element, $\mathbf{g}_{ii}^{\text{cs}}$, depends on the observed distribution of domestic market shares, the absorption rate and σ. Therefore, the only additional information needed for computing this factor is the distribution of domestic market shares. This information is the same used to calculate other concentration measures such as HHI, which makes it relatively simple to compute.

4.1.6 Relationship to Other Welfare Formulas

The welfare formula in equation 14 is similar to others that use the structure of trade models, such as ACR and ACDR, but captures the pro-competitive gains from trade via $\mathbf{g}_{ii}^{\text{cs}}$. It shows explicitly how granularity can enhance the consumer gains from trade given that increasing foreign competition disciplines domestic firms' markups.

In ACDR, the authors allow for variable markups that arise due to non-homothetic preferences. In their case, the welfare gains from trade are lower than those with CES, as captured by a factor that can be lower or greater than one, depending on the equilibrium response of the markup distribution. They find empirically that the factor is lower than one by estimating the preference parameters present in the factor. Under CES preferences, trade always decreases markups for a fixed number of firms, causing the factor to be greater than one. Although this may seem more restrictive, the advantage is that it takes an ex-post approach, so it varies substantially depending on the specific market structure in which trade conditions change. Intuitively, the welfare impact of trade can be higher in a highly concentrated economy versus a more competitive one. As a result, this statistic can be implemented in concrete empirical examples to gauge consumer gains.

Edmond et al. (2016) employ a model with domestic and foreign granular firms to compute how the welfare gains from trade are determined in this scenario. They find that the original ACR

[18] Note that this expression captures local and not global changes in consumer welfare.

formula approximates the total gains from trade reasonably well. They measure total gains as a change in aggregate productivity, which is not limited to changes in consumption. In this regard, it is worth mentioning that Equation 14 only captures changes in the CES price index, i.e. in consumers' welfare given total income. This is another advantage of having an explicit correction of the ACR formula that accounts separately for changes in consumer and producer surplus.

4.2 Producer Welfare Gains from Reducing Trade Openness

In this model, trade opening affects aggregate income by reducing domestic markups. Therefore, domestic producers always prefer a less open economy. In reality, there are many other reasons why domestic producers may prefer an open economy, such as access to cheaper inputs or enhanced productivity via scale. However, the focus of this paper is to analyze the role of domestic concentration and the pro-competitive gains from trade in depth.[19]

In this subsection, I reintroduce industry heterogeneity to stress the importance of welfare-consistent industry-weights.

4.2.1 Aggregate Markup Changes

In Equation 6, we can see that the aggregate excess markup directly depends on two domestic industry-specific endogenous objects: the industry absorption rate, λ_{ii}^{v} and concentration (HHI_{ii}^{v} or $\overline{HKI}_{ii}^{v}$ depending on quantity or price competition respectively). An increase in either of them increases industry profits by lowering the elasticity of demand to all domestic firms.

In Proposition 5, I characterize the change in the aggregate markup as a function of the absorption rate and the CES price index, as I did for the change in the consumer costs from markups.

Proposition 5. ***Aggregate Markup Change as a Function of Openness and Competition Changes.***

The aggregate domestic markup $\mathcal{E}_{ii}$ changes as a result of changes in iceberg trade costs only affecting foreign firms additively depends on the CES industry price index and the rate of absorption under both quantity and price competition.

$$\hat{\mathcal{E}}_{ii} = \Gamma_{ii}^{H} \int \tilde{\alpha}_{iiv} \left[\frac{\hat{\lambda}_{iiv}}{\sigma - 1} - \tilde{\Phi}_{iiv}^{H} \hat{P}_{iiv} \right] dv \tag{15}$$

where:

[19] Domestic firms do not get granular profits in foreign countries, which may be another reason why domestic producers may prefer an open economy in reality.

- $\tilde{\alpha}_{iiv} \equiv \frac{\Gamma_{iiv}}{\int \Gamma_{iiv} dv}$ *are the expenditure shares corrected by the sector-specific degrees of domestic granularity,*
- $\Gamma^H_{ii} \equiv \frac{\partial \log HHI_{ii}}{\partial \log P_i} < 0$ *is the partial elasticity of* HHI_{iiv} *with respect to* P_i*, where* HHI_{iiv} *is replaced by* $\overline{HKI}_{iiv}$ *under price competition, and*
- $\tilde{\Phi}^H_{iiv} \equiv \frac{\sum_{l=1}^{N_{iiv}} \frac{d \log HHI_{iiv}}{d \log p_l}}{\frac{\partial \log HHI_{iiv}}{\partial \log P_i}} \in (0,1)$ *is the relative strength of individual firms' equilibrium responses, where* HHI_{iiv} *is replaced by* $\overline{HKI}_{iiv}$ *under price competition.*

Proof: See Appendix .

Proposition 5 specifies how the aggregate excess markup changes and thus how aggregate income does. As with the welfare-consistent concentration index, an increase in absorption raises the aggregate markup, increasing income, in this case, via higher profits.

4.2.2 A Producer Welfare Formula under Granularity

Domestic firms gain if the economy closes to foreign competition because they can charge markups above monopolistic competition. Such gains depend on how much the domestic absorption rate and concentration change when trade costs rise. We know that $\hat{Y}_{ii} = \hat{\mathcal{E}}_{ii}$, so we can use Equation 15 to derive the producer gains and replace $\hat{P}_{iiv}$ by the industry-specific expression in Equation 15.

The following equation shows the changes in producer welfare as a function of the domestic absorption rate and the price index:

$$\hat{Y}_i = \int_v \left[\tilde{\alpha}_{iiv} \times \mathbf{g}^{\text{ps}}_{iiv} \times \frac{\hat{\lambda}_{iiv}}{\sigma - 1}\right] dv, \tag{16}$$

with:

$$\begin{aligned} \mathbf{g}^{\text{ps}}_{iiv} &\equiv \left[\uparrow \text{Income from} \uparrow \text{markups} - \downarrow \text{Income from} \uparrow \text{misallocation}\right] \\ &\equiv \left[\Gamma^H_{ii} - \Gamma^H_{ii} \tilde{\Phi}^H_{iiv} \frac{1 + \Gamma^c_{iiv}}{1 + \Gamma^c_{iiv} \tilde{\Phi}^c_{iiv}}\right]. \end{aligned}$$

Equation 17 directly captures how industry producer surplus increases when the economy closes. This effect is attenuated by misallocation, as in the case of consumer surplus. In this case, this is captured by $\tilde{\Phi}^{\text{m},v}_{ii}$.

4.3 A Sufficient Statistic for the Welfare Gains from Trade under Granularity

We can put together the consumer and producer surplus expressions to get an expression for the total welfare gains from trade under granularity:

$$\hat{\mathbf{W}}_i = \int_v \mathcal{G}_{iiv} \times \frac{\hat{\lambda}_{iiv}}{\sigma - 1} dv \tag{17}$$

with:

$$\mathcal{G}_{iiv} \equiv \tilde{\alpha}_{iiv} \mathbf{g}^{\text{ps}}_{iiv} - \alpha^v \mathbf{g}^{\text{cs}}_{iiv}.$$

Equation 17 shows that welfare gains depend heavily on the interaction between the industry-specific changes in openness and its contribution to producer and consumer surplus based on specific industrial market structures. These contributions depend on the functionally different concentration functions required in each case, as well as on the relative degree of granularity across sectors via $\tilde{\alpha}_{iiv}$.

It is worth noting that this is a local result. In other words, it captures the first-order impact on welfare of small changes in trade opening. Given that this model involves markups that are highly nonlinear in market shares, this expression is unlikely to capture global changes in welfare accurately, e.g. closing the economy to autarky.

Finally, this expression makes explicit why the ACR formula approximates well the welfare gains from trade for small countries under granularity, as shown by Edmond et al. (2015). The fact that changes in consumer surplus only require domestic firms' market shares, as in the case of producer surplus, implies that the departure from a monopolistic competitive setting due to granularity will tend to offset each other. Specifically, we can re-express $\hat{\mathbf{W}}_i$ as follows:

$$\mathcal{G}_{iiv} = -\Delta_{1,iiv}(1 + \Gamma^c_{iiv}) - \Delta_{2,iiv}\Gamma^c_{iiv}, \tag{18}$$

where $\Delta_{1,iiv} \equiv \frac{1+\Gamma^H_{iiv}\Phi^h_{iiv}}{1+\Gamma^c_{iiv}\Phi^c_{iiv}}$, and $\Delta_{2,iiv} \equiv \frac{\Gamma^H_{iiv}}{\Gamma^c_{iiv}}$ are the ratio of sectoral equilibrium responses and concentration elasticities, respectively. Given that, in both cases, both the numerator and denominator capture the reaction of two different but closely related concentration functions, $HHI_{iiv}/\overline{HKI}_{iiv}$ and $\mathcal{C}_{iiv}$ respectively, these ratios will tend to be close to one under relatively low granularity. Therefore, $\mathcal{G}_{iiv} \approx 1$ at low levels of domestic concentration and high trade openness, returning the

welfare gains from trade statistic to the original ACR formula.

5 Putting the Formulae to Work: Trade Opening, Market Concentration and the Gains from Trade in Chile

In this section, I characterize the key trade and concentration statistics behind the sufficient statistics derived in the theory section and construct them for Chile between 1995 and 2007, a period of rapid increase in import penetration.

5.1 Data

There are two families of statistics that I need to compute the welfare formula. The first one is related to the individual domestic firm's market shares in the Chilean market. I use data from the Chilean Annual Manufacturing Survey ENIA (by its Spanish acronym to *Encuesta Nacional Industrial Anual*). This survey includes the 4-digit ISIC Revision 3.1 sector and revenue from domestic and foreign sales at the establishment level with a firm identifier. This allows me to construct market shares at the firm-level by defining a market as a 4-digit industry.[20] I also use this data to construct the industry domestic revenue shares.

I also employ import data to construct the domestic absorption rates, i.e. the sectoral share of domestic firms in domestic expenditure. I employ import data from COMTRADE at the same level of aggregation.

5.2 The Increase in Trade Exposure

The period leading up to 2003 was very active for Chile regarding changes in trade exposure. A series of trade policy changes generated the conditions for an increase in import penetration: (i) China accessed the WTO in 2001, (ii) Chile signed two major trade agreements with the United States and the European Union in 2003, which were in force by January 2004, (iii) Chile also reduced its flat MFN tariff by 5 percentage points over the 1998-2003 period to reach a 6% uniform tariff in more than 95% of its tariff lines, and (iv) a period of currency depreciation reverted sharply, making importing cheaper. In addition to these changes, a free trade agreement with China came into force in 2006. My goal is not to identify the individual effect of these events described above on domestic trade exposure, but instead to provide a useful context for the increase in import penetration over this period.

[20] I do not employ sectors in the 23, and 27 to 29 divisions in the analysis, as they pertain exclusively to oil and mineral manufacturing.

5.2.1 Import Penetration

Import penetration is a key statistic for welfare in the context of oligopolistic competition given that its inverse enters directly into concentration and the standard ACR formula. I estimate the following equation to capture the changes in this variable.

$$\log \text{IP}_t^v = \sum_{s \in S} \delta_s \times \mathbb{1}\{s = t\} + \theta^v + \nu_t^v, \tag{19}$$

where IP_t^v is import penetration in industry v and year t, θ^v is an industry fixed effect and ν_t^v is the error term. The omitted period is 1995.

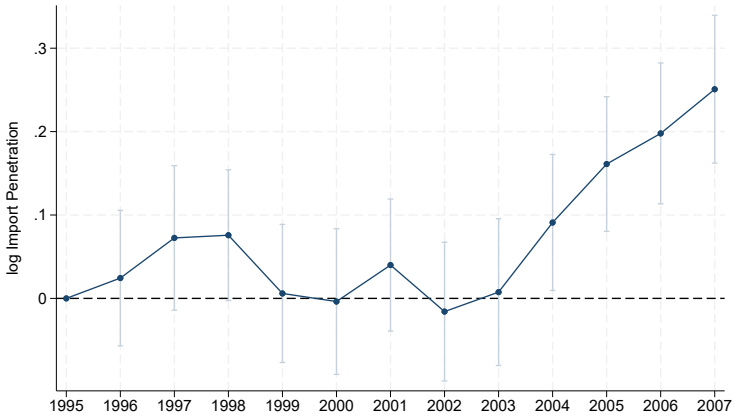
In Figure 3a, I plot coefficients from equation 19. Import penetration started increasing sharply after 2003, in coincidence with the trade opening events listed above. In 2007, it was about 20 log points higher than in the 1995-2003 period.[21].[22].

[21] Aggregate import penetration was about 59% in 1995 and ends up above 65% in 2007

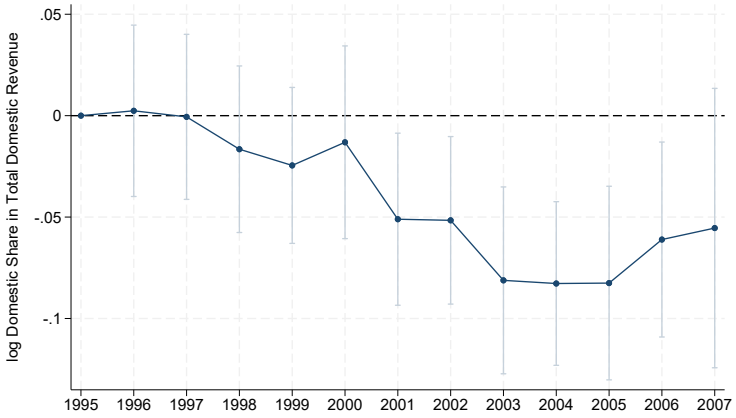
[22] Import penetration from China increased faster and from earlier, reflecting possibly secular changes not associated with the events above. This is shown in Appendix Figure A1

Figure 3: Trade Exposure Indicators. 1995-2007.

(a) Average Import Penetration over Time



(b) Average Domestic Share in Total Domestic Revenue



What does it imply for the welfare effect of domestic firms' market power under the lens of the model? Since the increase in import penetration is the inverse of a decrease in the domestic absorption rate, domestic firms lost market power at home over this period. When computing the sufficient statistics, this will be a downward force for the local consumer welfare gains from trade. The reason is that the disciplinary role of trade on domestic firms was more important at the beginning than at the end of the period given domestic markups convexity in market shares.

5.2.2 Industry Share of Domestic Sales in Total Domestic Revenue

During this period, Chilean firms increased the revenue derived from export markets. To see this, I estimate equation 19 but using instead the log of the domestic revenue share as a dependent variable. In Figure 3b, we can see a steady decline in the importance of domestic revenue of about 5 log points.[23] In this case, most of the decline took place before the above-mentioned changes in

[23]The average industry share of domestic sales in total revenue went down from 87% to about 82% in 2007.

trade policy.

The decrease in the share of domestic sales in total revenue is a downward force on Chilean profits in my model, given the domestic firms' smallness assumption in foreign markets. Although this seems plausible in this case, it is expected to be a lower bound as a force pushing down aggregate profits, as Chile may operate as a large economy in some regional markets.

5.3 Domestic Market Concentration

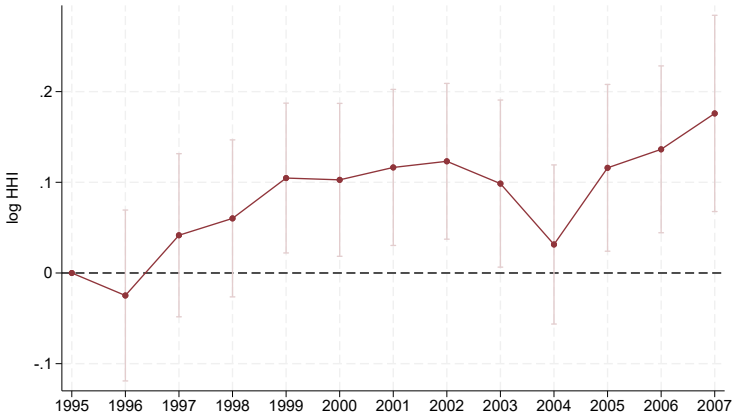
Market concentration indices are often used to draw conclusions about the state of competition (e.g. Gutierrez and Phillipon, 2017). As discussed above, these indices cannot work as sufficient statistic because they (i) can confound increases in concentration that are actually caused by an increase in efficiency (i.e. lower misallocation), and (ii) employ sales shares of domestic firms only. Still, it is useful to compute and compare them to the existing evidence in other countries and the welfare-consistent concentration index. Moreover, the HHI, also partially determines the domestic aggregate markup by domestic firms in the context of the model, so it also has a microfounded interpretation.

5.3.1 Usual Market Concentration Indices

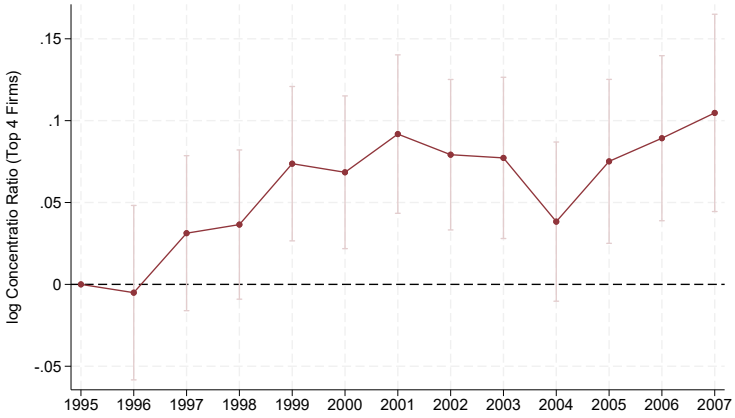
In Figure A8, I plot the coefficients from estimating equation 19 for two commonly used concentration indices: The Herfindahl-Hirschman Index (HHI) and the market share of the top 4 firms ($CR(4)$). Regardless of the index used, this period saw an increase in domestic concentration in line with evidence for the US.

Figure 4: Domestic Market Concentration

(a) HHI



(b) Share of Top Four Firms



5.3.2 Welfare-Consistent Concentation Measure

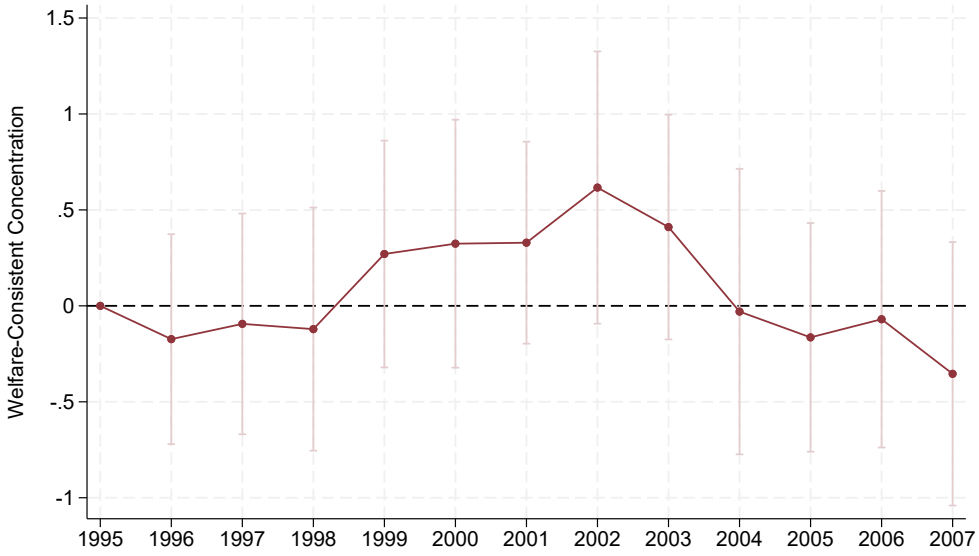
I compute the welfare-consistent concentration measure $\mathcal{C}$ for each industry and year using the formula for quantity competition, which is parameter-free. In contrast to the concentration measures in the previous section, this statistic is microfounded by the model as it captures the first-order consumer welfare costs associated with the excess markup due to oligopolistic behaviour.

In Figure 5, I plot the coefficients from equation 19. The evaluation of index shows that there was not an increase in concentration that could be associated with an increase in market power. In fact, the index stayed the same throughout the period, indicating that the direct welfare costs from granularity did not change.[24][25]

[24] Assuming price competition shifts the function down, maintaining the over-time pattern.

[25] This result capturing relatively stable domestic markups is in line with estimates using the production approach for Chile over this period (e.g. De Loecker and Eeckhout, 2018; Garcia-Marin, 2021).

Figure 5: Welfare-Consistent Concentration Measure. 1995-2007.



The evaluation of this index cannot be associated with the pro-competitive gains from trade as the latter is associated with changes in markups around the existing equilibrium. Below, I check the evolution of this result.

5.4 The Gains From Trade

In this section, I compute the local welfare gains from trade for each year. This indicator is useful because it captures the opportunity cost of marginally closing the economy. In the computations below, I assume quantity competition, although price competition delivers exactly the same qualitative predictions.

5.4.1 A Local Decrease in Trade Opening

My goal is to show that the opportunity cost of closing the economy is time-varying via the pro-competitive and profit gains from trade channels. To show this, I assume a uniform increase of 10 log points in the domestic absorption rate $\hat{\lambda}_{ii}^{v}$ and domestic revenue share $\hat{r}_{ii}^{v}$ in all industries. I also assume $\sigma = 5$, in line with previous estimates in the literature. These assumptions imply that the local change in the gains from trade at a time t is as follows:

$$\hat{\mathbf{W}}_{it} = \left\{ \underbrace{\sum_{v} \left[\tilde{\alpha}_{iit}^{v} \mathbf{g}_{iit}^{\text{ps},v} \right]}_{\text{Producer Gains}} - \underbrace{\sum_{v} \left[\alpha^{v} \mathbf{g}_{iit}^{\text{cs},v} \right]}_{\text{Consumer Losses}} \right\} \times 2.5. \tag{20}$$

5.4.2 Calibration

The ex-post approach employed to derive the statistics requires very few parameters. The most important one is the elasticity of substitution across varieties. As mentioned above, I assume the value of 5 as in Gaubert and Itskhoki (2021) given that they employ a similar setting with granular firms and a continuum of sectors. This delivers a trade elasticity of 4, which is within the common interval found in the literature (e.g. Fontagné et al., 2022), and a monopolistic competitive markup of 1.25, which is usually at the lower end in the empirical markup estimation literature (e.g. De Locker et al., 2020). It is worth recalling that this will be also the lower bound for markups in my framework.

I also need the Cobb-Douglas elasticities. I employ the sectoral sales from the ENIA and import values from COMTRADE to proxy for the sectoral total expenditure and use this information to compute sectoral expenditure shares. I compute the average shares using the 1995-1997 period—i.e. before the main trade opening events took place in my sample.

5.4.3 Industry Profit Weights

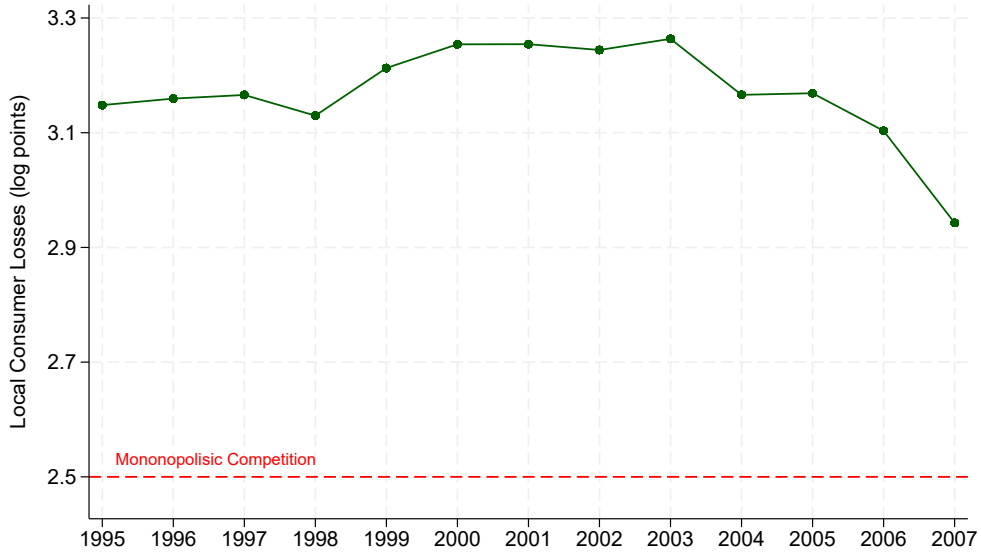
A key statistic for aggregation is the industry-specific profit weight $\tilde{\alpha}_{iit} \equiv \frac{\alpha^v r_{ii}^v \lambda_{ii}^v HHI_{ii}^v}{\sum_v \alpha^v r_{ii}^v \lambda_{ii}^v HHI_{ii}^v}$. These weights vary over time as the across-industry profit profile due to differential changes in competition. In Appendix Figure A4, I show the distribution of these weights along with their relation with the expenditure shares. The correlation between these two was 0.61 in 1995.

5.4.4 Consumer Welfare Gains From Trade

I graph the consumer component of the local gains from trade $\sum_v \left[\alpha_{iit}^v \mathbf{g}_{iit}^{\text{cs},v}\right] \times 2.5$ in Figure 6. In the 1995-2003 period, the consumer losses from closing the economy by 10 log points was about 3.2 log points. This is explained by a relatively closer economy in which domestic firms have more leeway to increase markups under lower foreign competition.[26] After 2003, consumer losses declined to about 2.9 log points in 2007, reflecting the increase in foreign competition and therefore a lower scope for domestic firms to increase markups. This implied a lower opportunity cost via the pro-competitive market from closing the economy, as domestic firms were more disciplined by competition in 2007 relative to before 2003.

[26] In fact, this index rose in the 1995-2003 period, reflecting the increase in domestic concentration without an increase in trade opening.

Figure 6: Consumer Losses from Closing the Economy (10 log points). 1995-2007.

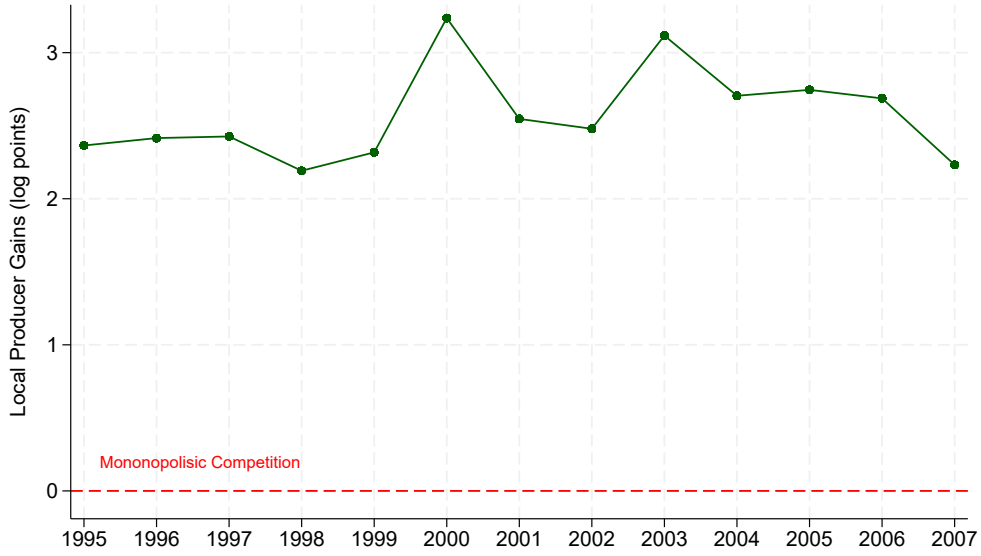


In Figure 6, I also added the consumer losses from closing the economy under monopolistic competition as a benchmark. Given that markups are constant in such a setting, the statistic collapses to the constant standard ACR. Including this indicator also helps in visualizing that under higher competition, losses from oligopolistic markups go down and tend to the monopolistic competitive benchmark.

5.4.5 Producer Welfare Gains From Trade

In Figure 7, I graph the producer component of the local gains from trade $\sum_v \left[\alpha_{iit}^v \mathbf{g}_{iit}^{\text{cs},v}\right] \times 2.5$. This term behaved differently over this period. In 1996-2003, it had substantial swings, being as low as 2.2 in 1998 and as high as 3.2 in 2000. Second, the statistic is virtually identical in 2007 to 1995. Note that the monopolistic competitive benchmark is zero in this case, as the profit share in income is constant in such a setting.

Figure 7: Producer Gains from Closing the Economy (10 log points). 1995-2007.



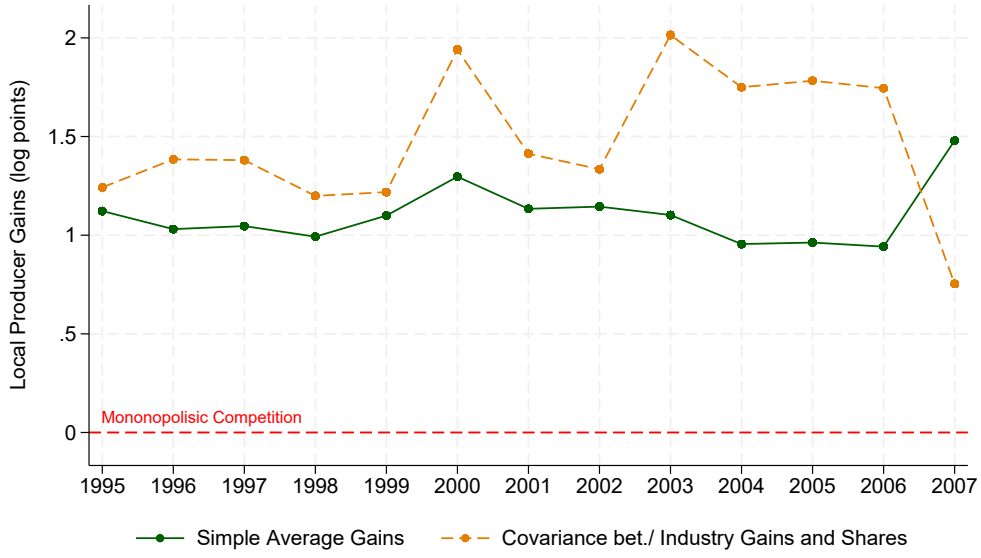
A key difference in this case is that the industry shares vary over time. Therefore, it is relevant to understand the evolution of the producer gains and the shares separately. To distinguish between these two, I employ the Olley-Pakes decomposition:

$$\mathcal{G}_{it}^{\text{ps}} = \overline{\mathbf{g}_{it}^{\text{ps}}} + COV_{it}(\tilde{\alpha}_{iit}^{v}, \mathbf{g}_{iit}^{\text{ps},v}), \tag{21}$$

where the first term is the simple average across sectors and the second one is the covariance between industry shares and producer gains. An increase in the covariance indicates a more concentrated pattern of specialization in sectors yielding higher profits.

In Figure 8, I show both terms from the decomposition in Equation 21. The simple average across industries was fairly constant in the entire period over time, except in 2007, when it jumped from 1 log point to 1.5. The covariance term increased over time from 1.25 to 1.7 in 2006, but then it collapsed in 2007 to 0.75. From this graph, it seems clear that 2007, the year in which China had full duty-free access to Chilean markets, strongly affected Chilean markets.

Figure 8: Decomposition of the Producer Gains from Closing the Economy (10 log points). 1995-2007.

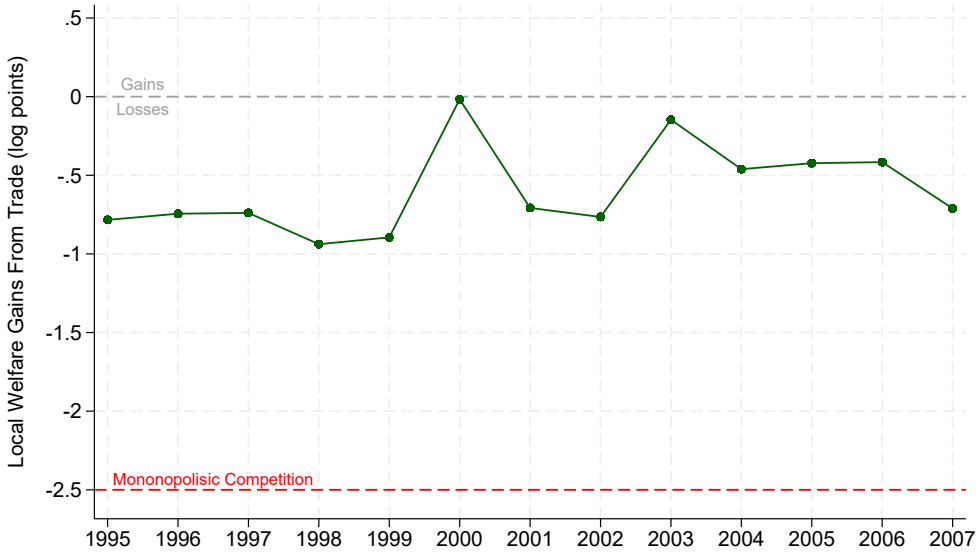


5.4.6 Aggregate Welfare Losses From Closing the Economy

What are the (local) aggregate welfare losses from closing the economy, and how did they evolve over time? In this section, I put together the consumer and producer statistics to compute Equation 20. In the monopolistic competitive case, such losses would be 2.5 log points given the assumptions, and it would only arise due to consumer losses through a higher domestic price index. Since there are no endogenous markups in such a case, the pro-competitive margin is zero and constant and there are no producer gains. With oligopolistic competition, markups (domestic and foreign) and profits are endogenous so this statistic changes over time and also has the potential to show aggregate welfare gains.

In Figure 9, I show the welfare statistic along with the monopolistic competitive benchmark and the gains-losses division. The average over the entire period was -0.6 log points—1.9 lower welfare losses than under monopolistic competition. This is not surprising given that domestic producers would benefit substantially in the short run from a closer economy via the profit channel. The statistic is never positive but gets close to it in 1998-2003, a period in which domestic concentration was already higher and the economy stayed relatively unchanged in terms of openness.

Figure 9: Local Welfare Losses from Closing the Economy (10 log points). 1995-2007.



In conclusion, the welfare gains from trade are variable and can differ substantially across periods when firms internalize the impact of their choices on industry aggregates. For this reason, it is important to consider the market structure conditions when a process of trade opening or closing takes place and how such market conditions would affect domestic consumers and producers differently. In a world with highly concentrated markets, the cost for consumers when the economy gets closed may be higher, given the higher possibility of increasing markups under weaker foreign competition for domestic firms. On the contrary, domestic producers may gain more in this situation. The aggregate effect depends on how the relative market power across sectors changes as the economy closes.

6 Secular Market Concentration and the Welfare Gains from Trade

The sufficient statistics studied so far answer a specific question: How do small changes in trade costs affect the welfare gains of consumers and producers from trade? These changes can be interpreted as short-run effects, since entry is restricted. In this section, I introduce large changes to trade costs to argue that, in the context of increasing secular concentration, closing the economy is more costly in terms of consumer surplus due to markups. In doing so, I allow non-granular domestic firms to enter and impose additional competitive pressure on granular firms.

6.1 The Average Industry Before and After Trade Opening and Increasing Concentration

I analyze the consumer impact of increasing trade costs in the average industry in Chile in two moments in time: 1995 and 2007. In 1995, the typical industry was less concentrated and open than in 2007, as shown in Section 5. There are two main channels through which concentration may have experienced secular concentration: (1) A change in technology that favors a more skewed distribution of firms,i.e. the superstar hypothesis, or (2) an increase in entry costs that generates market structures with few, larger firms.[27]

In order to analyze which cause, if any, is behind the increase in secular concentration, I introduce the following additional structure to the model. I assume that the typical industry has four granular domestic and four granular foreign firms, which are the most productive ones within each origin. Firms less productive than the fourth one are assumed to be non-granular.[28] All non-granular firms produce varieties of the same good as the granular ones, with elasticity of substitution σ.

Each granular firm receives a unit cost draw c from a Pareto distribution $G(c) = c^k$, with support $(0, 1)$, and shape parameter $k > 1$.[29] Non-granular firms are assumed to be homogeneous in terms of productivity and to receive the highest cost draw.[30] The lower the k, the more skewed are the typical draws and thus the higher the resulting concentration. Therefore, I will assume that changes in k capture secular motives favoring the rise of superstars in a reduced form way.

I introduce free entry into the domestic and foreign continuum, assuming that the respective potential entrants pay a fixed entry cost F to be active in the industry.[31] Firms enter until non-granular profits are zero. Granular firms continue to make positive firms, since their blueprints cannot be acquired by potential entrants. An increase in F would also increase concentration, since fewer firms would be able to enter. Therefore, this parameter will capture the conjecture that barriers to entry increased, reducing competition.

6.2 Quantitative Exercise

I employ Simulated Method of Moments to calibrate the typical industry in Chile in 1995 and in 2007. I found that the typical industry had a lower shape parameter in 2007 than in 1995,

[27] See (e.g. Autor et al., 2020, De Loecker et al., 2020, for discussions on the matter.

[28] The typical fifth domestic firm has an average market share within domestic firms of 5%, which under an import penetration of 60%, represents a market share in expenditure of 3% and an excess markup on top of the monopolistic competitive markup of the same magnitude under quantity competition.

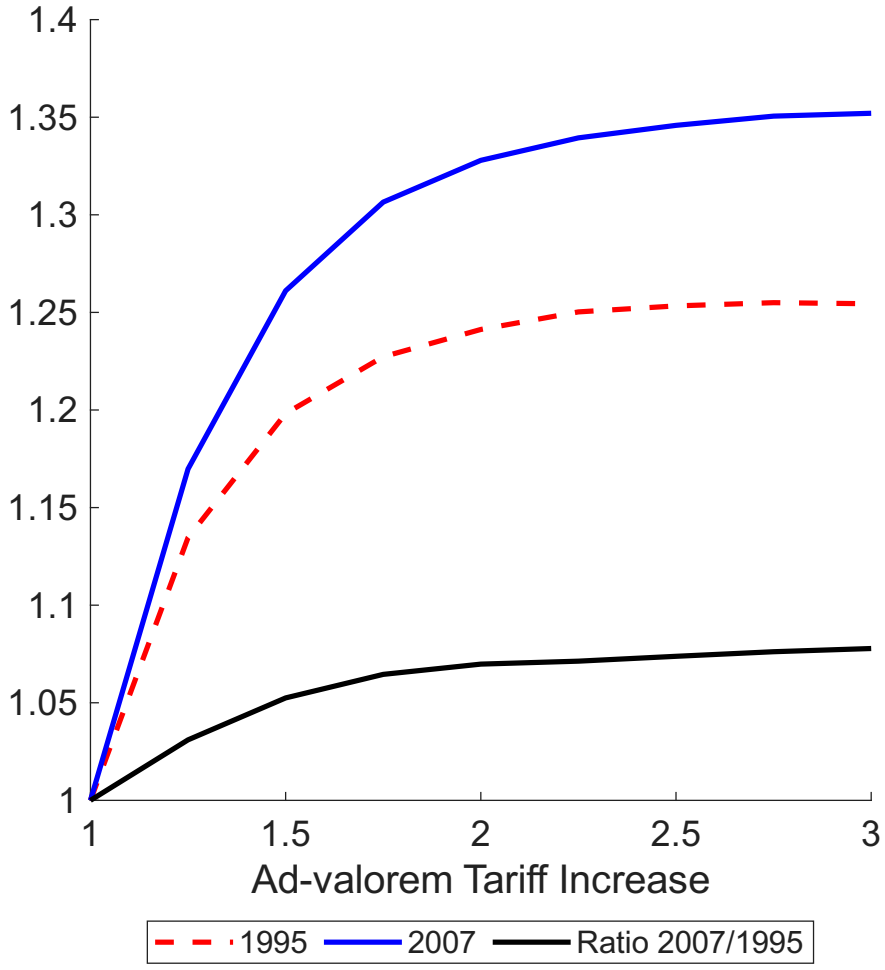
[29] $G(c)$ implies the typical Pareto productivity distribution $F(1/c) = 1 - (\frac{1}{1/c})^k$

[30] In a given economy, there are five cost draws: the lowest four to granular firms, and the fifth to non-granular firms.

[31] For simplicity, I assume the same entry cost for domestic and foreign firms.

accounting for changes in trade costs.[32] Entry costs did not experinced changes. This implied higher welfare losses from closing the economy in 2007 than in 1996 as shown in the figure below.

Figure 10: Closing the Economy After a Secular Increase in Concentration.



7 Conclusion

In this paper, I studied the role of market concentration in determining the welfare gains from trade in the presence of granular heterogeneous firms producing different varieties. I showed that this family of models gives rise to welfare-consistent concentration measures that (i) acknowledge the existence of foreign firms competing in domestic markets, and (ii) impose the right functional form from individual firms' perceived elasticities of demand. This allowed me to extend the canonical welfare formula of the gains from trade by including two factors of correction, capturing changes in consumer and producer surplus separately when trade conditions change.

I applied the statistics to study the case of Chile in the 1995-2007 period. During that time, Chile

[32] k declined from 3.28 to 3.17.

experienced an increase in import penetration and domestic market concentration, as measured by standard indices. When I employ the microfounded welfare-consistent concentration index, I found that the cost from markups due to granularity stayed the same throughout this period. In terms of welfare, I find that the consumer gains from trade were lower in 2007 than before. This result is a consequence of the important role of foreign competition in disciplining domestic markups at lower rather than higher levels of trade opening. The welfare gains from trade when also considering changes in producers' surplus were fairly constant throughout the period and lower than under monopolistic competition, thanks to the offsetting effect of the increasing domestic profits in closer markets.

This paper provides a set of sufficient statistics that are useful to characterize the strength of market power in the context of granular economies and interrelated markets. Understanding the local welfare gains from trade is important to characterize the opportunity cost of a closing economy depending on the specific short-run market circumstances in which this process takes place. In this sense, these statistics can be useful for policy and academic work alike.

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Online Appendices

A Theoretical Results

A.1 Optimal Prices and Ad-valorem Excess Markups

Consumers have CES preferences $Q = \left[\sum_i^N \delta^{\frac{1}{\sigma}} q_i^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$.

N firms produce one variety each.

Firm i unit cost of producing q_i is c_i (so its productivity is $1/c_i$).

A.1.1 Quantity Competition

When firms compete in quantities, we need to take an extra step, which is to invert the demand function:

$$\begin{aligned} q_i &= \delta p_i^{-\sigma} P^{\sigma-1} \\ p_i &= \delta^{-1} q_i^{-\frac{1}{\sigma}} P^{\frac{\sigma-1}{\sigma}} \\ p_i &= \delta^{-1} q_i^{-\frac{1}{\sigma}} Q^{-\frac{\sigma-1}{\sigma}} \end{aligned}$$

where in the last step I used the CES expenditure function result ($Q \times P = 1$).

Therefore, each firm solves the following problem:

$$\begin{aligned} &\max_{q_i} (p_i - c_i) q_i \\ \text{s.t. } &p_i = \delta^{-1} q_i^{-\frac{1}{\sigma}} Q^{-\frac{\sigma-1}{\sigma}} \end{aligned}$$

The FOC is:

$$\begin{aligned} (p_i - c_i) + q_i p_i' &= 0 \\ p_i + q_i \delta^{-1} [-\frac{1}{\sigma} q_i^{-\frac{1}{\sigma}-1} Q^{-\frac{\sigma-1}{\sigma}}] - q_i \delta^{-1} \frac{\sigma-1}{\sigma} [q_i^{-\frac{2}{\sigma}} Q^{-2\frac{\sigma-1}{\sigma}}] &= c_i \\ p_i - p_i \frac{1}{\sigma} - \frac{\sigma-1}{\sigma} p_i q_i^{1-\frac{1}{\sigma}} Q^{-\frac{\sigma-1}{\sigma}} &= c_i \\ 1 - \frac{1}{\sigma} - \frac{\sigma-1}{\sigma} s_i &= \frac{1}{\mu_i^c} \\ \mu_i^c &= \frac{\sigma}{(\sigma-1)(1-s_i)} \end{aligned}$$

where I used that $\frac{q_i^{\frac{\sigma-1}{\sigma}}}{Q^{\frac{\sigma-1}{\sigma}}} = \frac{p_i^{1-\sigma}}{P^{1-\sigma}} = s_i$.

In this case, the excess markup is parameter-free as shown below:

$$\begin{aligned}\mu_i^c &= \bar{\mu}\left[\frac{\sigma}{(\sigma-1)(1-s_i)}\frac{\sigma-1}{\sigma}\right] \\ &= \bar{\mu}\left[\frac{1}{1-s_i}\right] \\ &= \bar{\mu}\left[1+\frac{s_i}{(1-s_i)}\right]\end{aligned}$$

Hence:

$$e^c = \frac{s_i}{(1-s_i)}$$

A.1.2 Price Competition

In price competition, firm i chooses their optimal price by solving the following problem:

$$\begin{aligned}&\max_{p_i}(p_i - c_i)q_i \\ &\text{s.t. } q_i = \delta p_i^{-\sigma} P^{\sigma-1}\end{aligned}$$

where $P = \left[\sum_i^N \delta p_i^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$ is the CES ideal price index, and we normalize total consumer expenditure to one.

The FOC is:

$$\begin{aligned}q_i + (p_i - c_i)q_i' &= 0 \\ \delta p_i^{-\sigma}P^{\sigma-1} + (p_i - c_i)\left[-\delta\sigma p_i^{-\sigma-1}P^{\sigma-1} + \delta(\sigma-1)p_i^{-2\sigma}P^{2(\sigma-1)}\right] &= 0 \\ 1 + (p_i - c_i)\left[-\sigma p_i^{-1} + (\sigma-1)p_i^{-\sigma}P^{\sigma-1}\right] &= 0 \\ 1 - \sigma(1-1/\mu_i^b) + (\sigma-1)(1-1/\mu_i^b)s_i &= 0 \\ 1 &= (1-1/\mu_i^b)(\sigma-(\sigma-1)s_i) \\ \frac{1}{\sigma-(\sigma-1)s_i} &= (1-1/\mu_i^b) \\ 1/\mu_i^b &= 1 - \frac{1}{\sigma-(\sigma-1)s_i} \\ \mu_i^b &= \frac{\sigma-(\sigma-1)s_i}{\sigma-(\sigma-1)s_i-1}\end{aligned}$$

In order to derive the excess markup, we use the definition of $\bar{\mu}$:

$$\begin{aligned}\mu_i^b &= \bar{\mu}\left[\frac{\sigma-(\sigma-1)s_i}{\sigma-(\sigma-1)s_i-1}\frac{\sigma-1}{\sigma}\right] \\ &= \bar{\mu}\left[\frac{\sigma-(\sigma-1)s_i}{(1-s_i)\sigma}\right] \\ &= \bar{\mu}\left[1+\frac{s_i}{\sigma(1-s_i)}\right]\end{aligned}$$

Therefore:

$$e^b = \frac{s_i}{\sigma(1-s_i)}$$

As mentioned, this excess markup depends on both the marker share and the elasticity of substitution.

A.2 Welfare-Consistent Concentration Measure

I prove this result in the case of quantity competition. Let's log differentiate the price index in the case of quantity competition:

$$\begin{aligned}d\log P_j^{1-\sigma} &= d\log\Big[\sum_{v\in\Omega_j}[c^v\times\bar{\mu}\times(1-s_j^v)^{-1}]^{1-\sigma}\Big] \\ &= \frac{1}{P_i^{1-\sigma}}\Big[\sum_{v\in\Omega_j}[c^v\times\bar{\mu}]^{1-\sigma}\times d(1-s_j^v)^{\sigma-1}\Big] \\ &= \frac{1}{P_j^{1-\sigma}}\Big[\sum_{v\in\Omega_j}[c^v\times\bar{\mu}\times(1-s_j^v)^{-1}]^{1-\sigma}(\sigma-1)d\log(1-s_j^v)\Big] \\ &= \frac{1}{P_j^{1-\sigma}}\Big[\sum_{v\in\Omega_j}[c^v\times\bar{\mu}\times(1-s_j^v)^{-1}]^{1-\sigma}\times\frac{1-\sigma}{1-s_j^v}ds_j^v\Big] \\ &= (1-\sigma)\Big[\sum_{v\in\Omega_i}\frac{s_j^v}{1-s_j^v}ds_j^v\Big],\end{aligned}$$

where in the last step I used CES, i.e. $s_j^v = [c^v\bar{\mu}(1-s_j^v)^{-1}]^{1-\sigma}/P_j^{1-\sigma}$.

Gradient Theorem (also known as the Fundamental Theorem of Calculus for Line Integrals):

If $\varphi : U \subseteq R^n \to R$ is a differentiable function and γ a differentiable curve in U which starts at a point p and ends at a point q, then:

$$\int_\gamma \nabla\varphi(\mathbf{s})\cdot d\mathbf{s} = \varphi(q)-\varphi(p)$$

where $\nabla\varphi(\mathbf{s})$ denotes the gradient vector field of φ.

If we think of φ as a function $\mathcal{C}(\mathbf{s})$, differentiable in market shares, and U as $[0,1]^n$, which is equivalent to $\log P_j$) but does not depend on marginal costs and other parameters, then we can apply this theorem:

$$
\begin{aligned}
\int_\gamma \nabla \mathcal{C}(\mathbf{s}) \cdot d\mathbf{s} &= \int_\gamma \sum_{v=1}^{n} \frac{s_{ij}^v}{1-s_{ij}^v} ds_{ij}^v \\
&= \sum_{v=1}^{n-1} \int_0^{\bar{s}_v} \frac{s_{ij}^v}{1-s_{ij}^v} ds_{ij}^v + \int_0^{1-\sum_{u=1}^{n-1}\bar{s}_u} \frac{s_{ij}^n}{1-s_{ij}^n} ds_{ij}^n \\
&= \sum_{v=1}^{n-1} \Big\{ \Big[-\bar{s}_v - \log(|1-\bar{s}_v|) \Big] - \Big[-0 - \log(|1-0|) \Big] \Big\} + \\
&+ \Big\{ \Big[-(1-\sum_{u=1}^{n-1}\bar{s}_u) - \log(|1-(1-\sum_{u=1}^{n-1}\bar{s}_u)|) \Big] - \Big[-0 - \log(|1-0|) \Big] \Big\} \\
&= \underbrace{\sum_{v=1}^{n} \Big[-\bar{s}_v - \log(|1-\bar{s}_v|) \Big]}_{\mathcal{C}(\bar{\mathbf{s}})} - \underbrace{\sum_{v=1}^{n} \Big[-0 - \log(|1-0|) \Big]}_{\mathcal{C}(0)=0}]
\end{aligned}
$$

where note that I defined γ as a curve mapping n to the $n-1$ dimensional simplex and I omitted the constant of integration.

Therefore, we get the expression that we needed:

$$
\mathcal{C}(\bar{\mathbf{s}}) = \sum_{v=1}^{n} \Big[-\bar{s}_v - \log(|1-\bar{s}_v|) \Big]
$$

In the case of price competition, it is the same procedure.

A.3 Concentration Equation

$$
\hat{\mathcal{C}_{ii}} = \sum_{l=1}^{N_{ii}} \frac{s_l}{1-s_l} ds_l
$$

The key to understanding counterfactual concentration when trade costs change is understanding how market shares change: which firms would grow and which firms would shrink?

We can re-write it as follows:

$$
\hat{\mathcal{C}_{ii}} = \sum_{l=1}^{N_{ii}} \frac{s_l^2}{1-s_l} \Big[\underbrace{\hat{p}_l^{1-\sigma} - \hat{P}_{ii}^{1-\sigma}}_{\substack{\text{Changes in domestic} \\ \text{market share } z_l}} + \underbrace{\hat{P}_{ii}^{1-\sigma} - \hat{P}_i^{1-\sigma}}_{\text{Changes in absorption } \lambda_{ii}} \Big]
$$

We know how to compute counterfactual absorption λ_{ii}, but we need the counterfactual distribution of domestic market shares, i.e. $z_l \equiv \frac{p_l^{1-\sigma}}{P_{ii}^{1-\sigma}}$ for each l.

Each individual price will react to a different level of competition as captured by P_i.

$$
\hat{p}_l = \frac{\psi_l}{1+\psi_l} \hat{P}_i,
$$

where $\psi_l \equiv (\sigma-1)\frac{s_l}{1-s_l}$ is what Amiti et al. (2019) call the "firm l own price markup elasticity".

ψ_l is increasing in the size of firms, so $\frac{\psi_l}{1+\psi_l} \in (0,1)$ is also increasing in the the size of firms.

Large firms prices are more elastic to changes in competition (as measured by changes in the CES price index).

We can write the domestic firm market share elasticity to the distribution of prices as follows:

$$\begin{aligned}
\hat{z}_l = \hat{p}_l^{1-\sigma} - \hat{P}_{ii}^{1-\sigma} &= \hat{p}_l^{1-\sigma} - \sum_{l=1}^{N_{ii}} z_l \hat{p}_l^{1-\sigma} \\
&= \frac{\psi_l}{1+\psi_l}\hat{P}_i^{1-\sigma} - \sum_{l=1}^{N_{ii}} z_l \frac{\psi_l}{1+\psi_l}\hat{P}_i^{1-\sigma} \\
&= (1-\sigma)\ \underbrace{\Big[\frac{\psi_l}{1+\psi_l} - \sum_{l=1}^{N_{ii}} z_l \frac{\psi_l}{1+\psi_l}\Big]}_{\substack{\text{Firm's l markup response relative} \\ \text{to aggregate market response}}}\ \hat{P}_i
\end{aligned}$$

The larger the firm, the more likely the response is greater than the average, e.g. an increase in P_i increases misallocation (reduces market shares of large firms and increases the one of small firms).

We can re-write the concentration function as follows:

$$\begin{aligned}
\hat{\mathcal{C}_{ii}} &= \sum_{l=1}^{N_{ii}} \frac{s_l^2}{1-s_l}\left\{(1-\sigma)\Big[\frac{\psi_l}{1+\psi_l} - \sum_{l=1}^{N_{ii}} z_l \frac{\psi_l}{1+\psi_l}\Big]\hat{P}_i + \hat{\lambda_{ii}}\right\} \\
&= -\mathcal{E}_{ii}^c \times \lambda_{ii} \times \left[\sum_{l=1}^{N_{ii}} [c_l - z_l]\frac{\psi_l}{1+\psi_l}\hat{P}_i + \frac{\hat{\lambda_{ii}}}{1-\sigma}\right] \\
&= -\mathcal{E}_{ii}^c \times \lambda_{ii} \times \left[\tilde{\Phi}_{ii}\hat{P}_i + \frac{\hat{\lambda_{ii}}}{1-\sigma}\right]
\end{aligned}$$

Where all the defined objects have a clear interpretation:

$\implies \mathcal{E}_{ii}^c \equiv \sum_{l=1}^{N_{ii}} z_l \frac{s_l}{1-s_l}(\sigma-1) > 0$ is the domestic industry's price index markup elasticity.

$\implies c_i \equiv z_l \frac{s_l}{1-s_l}(\sigma-1)/\mathcal{E}_{ii}^c$ is firm's l contribution to the price index markup elasticity.

$\implies \tilde{\Phi}_{ii} \equiv \Phi_{ii}^c - \Phi_{ii}^z \equiv \sum_{l=1}^{N_{ii}} [c_l - z_l]\frac{\psi_l}{1+\psi_l} \in (0,1)$ is captures misallocation (positive by proof).

Concentration (normalized to domestic size) depends on the CES industry price index and foreign competition changes, as captured by absorption:

$$\frac{\hat{\mathcal{C}_{ii}}}{\lambda_{ii}} = -\mathcal{E}_{ii}^c \tilde{\Phi}_{ii}\hat{P}_i + \mathcal{E}_{ii}^c \frac{\hat{\lambda_{ii}}}{\sigma-1}$$

An increase in the price index (less competition) reduces domestic concentration via misallocation.

An increase in absorption (less foreign competition) increases concentration by reallocating market shares relatively more to large firms.

A.4 Equilibrium Changes

Now, we can employ the following system of equations to understand how the CES price index would change when trade costs change, at the observed equilibrium concentration:

$$\textbf{Price Index Equation} \implies \hat{P}_i = \frac{\hat{\lambda_{ii}}}{\sigma - 1} + \frac{\hat{\mathcal{C}_{ii}}}{\lambda_{ii}}$$

$$\textbf{Concentration Equation} \implies \frac{\hat{\mathcal{C}_{ii}}}{\lambda_{ii}} = -\mathcal{E}^c_{ii}\tilde{\Phi}_{ii}\hat{P}_i + \mathcal{E}^c_{ii}\frac{\hat{\lambda_{ii}}}{\sigma - 1}$$

We can solve for the ACR-type formula with these two equations, getting:

$$\hat{P}_i = \left[\frac{1 + \mathcal{E}^c_{ii}}{1 + \mathcal{E}^c_{ii}\tilde{\Phi}_{ii}}\right] \times \frac{\hat{\lambda}_{ii}}{\sigma - 1}$$

Where $[\frac{1+\mathcal{E}^c_{ii}}{1+\mathcal{E}^c_{ii}\tilde{\Phi}_{ii}}] > 1$, i.e. when trade costs increase, domestic granular firms are able to charge higher markups due to lower competition.

A.5 Price Index Equation

Define $p^m(c) \equiv c \times \bar{\mu}$ as the price under monopolistic competition.

The CES industry price index can be written as follows:

$$P_i(\mathcal{M}) = \left[\sum_{j=1}^{J}\sum_{l=1}^{N_{ij}} p^m_{ij,l}(c_{i,l})^{1-\sigma}\varepsilon_{ij,l}(c_{i,l};\mathcal{M})^{1-\sigma}\right]^{\frac{1}{1-\sigma}},$$

where I index firms with l and keep explicitly the number of firms N_{ij} fixed.

Recall that firms i, l market share is sufficient to compute the wedge $\varepsilon_{ij,l}(c_{j,l};\mathcal{M})$ (plus σ if $\mathcal{M}$ is price competition).

The fact that market shares only enter into the wedge and not the first-best price is convenient.

Let's log differentiate $P_i(\mathcal{M})$ (indicated by the hats) and make explicit that ε depends on marginal costs via market shares:

$$\hat{P}_i(\mathcal{M}) = \hat{P}_j(\{c\}) + \hat{\mathcal{C}}_i(\{s(c)\},\mathcal{M}),$$

where $\hat{\mathcal{C}}_i(\{s(c)\},\mathcal{M}) \equiv \sum_{l=1}^{N_{ij}} s_{ij,l}d\varepsilon_{ij,l}(s(c_{i,l});\mathcal{M})$.

Changes in the family of functions $\mathcal{C}$ measure the first-order welfare costs of changes in the distribution of markup wedges.

A.5.1 ACR Strategy

Not being specific about the type of competition from now on for simplicity.

Assuming no changes in the distribution of marginal costs and the same number of firms, we have that under changes in trade costs:

$$\hat{P}_i = \lambda_{ii}\hat{w}_i + \lambda_{ji}\hat{w}_j + \hat{\mathcal{C}}_{ii} + \hat{\mathcal{C}}_{ji},$$

where I separated the concentration function $\hat{\mathcal{C}}_i$ into domestic and foreign, i.e. $\hat{\mathcal{C}}_i = \hat{\mathcal{C}}_{ii} + \hat{\mathcal{C}}_{ji}$, where e.g. $\hat{\mathcal{C}}_{ji}$ sums only firms from j in i.

Following strategy in ACR:

$$\hat{\lambda_{ji}} - \hat{\lambda_{ii}} = (1-\sigma)[\hat{w}_j - \hat{w}_i] + \frac{(1-\sigma)}{\lambda_{ji}}\hat{\mathcal{C}}_{ji} - \frac{(1-\sigma)}{\lambda_{ii}}\hat{\mathcal{C}}_{ii},$$

and thus $\hat{w}_j = \frac{1}{1-\sigma}[\hat{\lambda_{ji}} - \hat{\lambda_{ii}}] + \frac{1}{\lambda_{ii}}\hat{\mathcal{C}}_{ii} - \frac{1}{\lambda_{ji}}\hat{\mathcal{C}}_{ji}$.

Replacing $\hat{w}_j$ in $\hat{P}_i$ and using $\hat{w}_i = 0$, we get:

$$\begin{aligned}
\hat{P}_i &= \lambda_{ii}\Big[\frac{\hat{\lambda_{ii}} - \hat{\lambda_{ii}}}{1-\sigma} + \frac{1}{\lambda_{ii}}\hat{\mathcal{C}}_{ii} - \frac{1}{\lambda_{ii}}\hat{\mathcal{C}}_{ii}\Big] + \lambda_{ji}\Big[\frac{\hat{\lambda_{ji}} - \hat{\lambda_{ii}}}{1-\sigma} + \frac{1}{\lambda_{ii}}\hat{\mathcal{C}}_{ii} - \frac{1}{\lambda_{ji}}\hat{\mathcal{C}}_{ji}\Big] + \hat{\mathcal{C}}_{ii} + \hat{\mathcal{C}}_{ji} \\
&= \frac{\hat{\lambda_{ii}}}{\sigma - 1} + \frac{\hat{\mathcal{C}}_{ii}}{\lambda_{ii}}
\end{aligned}$$

The ACR term needs to be augmented by the domestic concentration only in the presence of granularity.

A.6 Aggregate Profits and Markup

A.6.1 Ad-valorem Excess Profits

Quantity Competition

$$
\begin{aligned}
\Pi &= \sum_{v=1}^{N} \pi_v \\
&= \sum_{v=1}^{N} (p_v - c_v) q_v \\
&= \sum_{v=1}^{N} (1 - c_v/p_v) q_v p_v \\
&= \sum_{v=1}^{N} \Big[1 - \frac{1}{\tilde{\mu}(1-s_v)^{-1}}\Big] r_v \\
&= \sum_{v=1}^{N} \Big[1 - \frac{(\sigma-1)(1-s_v)}{\sigma}\Big] r_v \\
&= \frac{R}{\sigma} \sum_{v=1}^{N} \Big[\sigma - (\sigma-1)(1-s_v)\Big] s_v \\
&= \frac{R}{\sigma} \sum_{v=1}^{N} \Big[1 + (\sigma-1) s_v\Big] s_v \\
&= \frac{\lambda R}{\sigma} \sum_{v=1}^{N} \Big[z_v + (\sigma-1)\lambda z_v\Big] \\
&= \frac{\lambda R}{\sigma} \Big[1 + \sum_{v=1}^{N} (\sigma-1)\lambda (z_i)^2\Big] \\
&= \frac{\lambda R}{\sigma} \Big[1 + (\sigma-1)\lambda HHI\Big]
\end{aligned}
$$

$$H_{ii} = 1 + (\sigma-1)\lambda_{ii} HHI_{ii}$$

Price Competition

$$
\begin{aligned}
\Pi &= \sum_{v=1}^{N} \pi_v \\
&= \sum_{v=1}^{N} (p_v - c_v) q_v \\
&= \sum_{v=1}^{N} (1 - c_v/p_v) q_v p_{vi} \\
&= \sum_{v=1}^{N} \Big[1 - \frac{c_v}{(\tilde{\mu} - s_v)(1 - s_v)^{-1} c_v} \Big] r_v \\
&= \sum_{v=1}^{N} \Big[1 - \frac{1 - s_v}{\tilde{\mu} - s_v} \Big] r_v \\
&= R \sum_{v=1}^{N} \Big[1 - \frac{(\sigma - 1)(1 - s_v)}{\sigma - s_v(\sigma - 1)} \Big] s_v \\
&= R \sum_{v=1}^{N} \frac{\sigma - s_v(\sigma - 1) - (\sigma - 1)(1 - s_v)}{\sigma - s_v(\sigma - 1)} s_v \\
&= R \sum_{v=1}^{N} \frac{s_v}{\sigma - s_v(\sigma - 1)} \\
&= \frac{R}{\sigma} \sum_{v=1}^{N} \frac{s_v}{1 - s_v \frac{\sigma - 1}{\sigma}}
\end{aligned}
$$

Now, let's work on the term in parenthesis:

$$
\begin{aligned}
\sum_{v=1}^{N} \frac{s_v}{1 - s_v \frac{\sigma-1}{\sigma}} &= \sum_{v=1}^{N} s_v \frac{1}{1 - s_v \frac{\sigma-1}{\sigma}} \\
&= \sum_{v=1}^{N} s_v \sum_{k=1}^{\infty} [s_v \frac{\sigma-1}{\sigma}]^{k-1} \\
&= \sum_{v=1}^{N} \sum_{k=1}^{\infty} s_v^k [\frac{\sigma-1}{\sigma}]^{k-1} \\
&= \sum_{k=1}^{\infty} \sum_{v=1}^{N} s_v^k [\frac{\sigma-1}{\sigma}]^{k-1} \\
&= \sum_{k=1}^{\infty} [\frac{\sigma-1}{\sigma}]^{k-1} \sum_{v=1}^{N} s_v^k \\
&= \sum_{k=1}^{\infty} [\frac{\sigma-1}{\sigma}]^{k-1} \sum_{v=1}^{N} \lambda^k z_v^k \\
&= \sum_{k=1}^{\infty} [\frac{\sigma-1}{\sigma}]^{k-1} \lambda^k \sum_{v=1}^{N} z_v^k \\
&= \lambda + \sum_{k=2}^{\infty} [\frac{\sigma-1}{\sigma}]^{k-1} \lambda^k \sum_{v=1}^{N} z_v^k \\
&= \lambda \Big[1 + \sum_{k=2}^{\infty} [\frac{\sigma-1}{\sigma}]^{k-1} \lambda^{k-1} \sum_{v=1}^{N} z_v^k \Big] \\
&= \lambda \Big[1 + \sum_{k=2}^{\infty} [\frac{\sigma-1}{\sigma} \lambda]^{k-1} HKI(k) \Big]
\end{aligned}
$$

where the third to last step follows from noting that $\sum_{v=1}^{N} z_v^k = 1$ and $[\frac{\sigma-1}{\sigma}]^{k-1} = 0$ when $k = 1$.

Note that $[\frac{\sigma-1}{\sigma}\lambda]^{k-1} \in (0, 1)$, so we can treat this term as weights to the different HKI degrees greater than 2 if we divide it by its sum.

$$
\begin{aligned}
\sum_{k=2}^{\infty} [\frac{\sigma-1}{\sigma}\lambda]^{k-1} &= \sum_{k=1}^{\infty} [\frac{\sigma-1}{\sigma}\lambda]^{k-1} - 1 \\
&= \frac{1}{1 - \frac{\sigma-1}{\sigma}\lambda} - 1 \\
&= \frac{\sigma}{\sigma - (\sigma-1)\lambda} - 1 \\
&= \frac{\sigma - (\sigma - (\sigma-1)\lambda)}{\sigma - (\sigma-1)\lambda} \\
&= \frac{(\sigma-1)\lambda}{\sigma - (\sigma-1)\lambda}
\end{aligned}
$$

So we use $\frac{(\sigma-1)\lambda}{\sigma-(\sigma-1)\lambda}$ as the denominator to define the k-specific weights $\iota_k \equiv [\frac{\sigma-1}{\sigma}\lambda]^{k-1} / \frac{(\sigma-1)\lambda}{\sigma-(\sigma-1)\lambda}$:

$$
\begin{aligned}
&= \lambda\Big[1+\sum_{k=2}^{\infty}[\frac{\sigma-1}{\sigma}\lambda]^{k-1}HKI(k)\Big] \\
&= \lambda\Big[1+\frac{\frac{(\sigma-1)\lambda}{\sigma-(\sigma-1)\lambda}}{\frac{(\sigma-1)\lambda}{\sigma-(\sigma-1)\lambda}}\sum_{k=2}^{\infty}[\frac{\sigma-1}{\sigma}\lambda]^{k-1}HKI(k)\Big] \\
&= \lambda\Big[1+\frac{(\sigma-1)\lambda}{\sigma-(\sigma-1)\lambda}\sum_{k=2}^{\infty}\iota_k HKI(k)\Big] \\
&= \lambda\Big[1+\frac{(\sigma-1)\lambda}{\sigma-(\sigma-1)\lambda}\overline{HKI}\Big]
\end{aligned}
$$

where $\overline{HKI} \equiv \sum_{k=2}^{\infty}\iota_k HKI(k)$ is the weighted average concentration index HKI over all the degrees (note that as $k \to \infty$, $HKI \to 0$, so the more weight on higher degrees, the lower the $\overline{HKI}$. Graphically, I can show that higher σ imply higher weight on higher degrees and $\sigma \to 1$ implies $\overline{HKI} \to HHI$, capturing the idea that quantity competition is a special case of price competition when $\sigma \to 1$.

We can come back to the original expression to have:

$$
\Pi = \frac{R}{\sigma}\sum_{v=1}^{N}\frac{s_v}{1-s_v\frac{\sigma-1}{\sigma}} = \frac{\lambda R}{\sigma}\Big[1+\frac{(\sigma-1)\lambda}{\sigma-(\sigma-1)\lambda}\overline{HKI}\Big]
$$

This implies that the following:

$$
H_{ii} = 1+\frac{(\sigma-1)\lambda}{\sigma-(\sigma-1)\lambda}\overline{HKI}
$$

A.6.2 Aggregate Markup

$$\begin{aligned}
Y_i &= L_i + \int_V [\Pi_{ii}^V + \Pi_{ij}^V] dV \\
&= L_i + \int_V [\frac{R_{ii}^V}{\sigma} H_{ii}^V + \frac{R_{ij}^V}{\sigma}] dV \\
&= L_i + \int_V [\frac{Y_i \alpha^V \lambda_{ii}^V}{\sigma} H_{ii}^V + \frac{Y_j \alpha^V \lambda_{ji}^V}{\sigma}] dV \\
&= L_i + \int_V \frac{Y_i \alpha^V \lambda_{ii}^V}{\sigma} H_{ii}^V dV + \int_V \frac{Y_j \alpha^V \lambda_{ji}^V}{\sigma} dV \\
&= L_i + \int_V \frac{Y_i \alpha^V \lambda_{ii}^V}{\sigma} H_{ii}^V dV + \int_V \frac{Y_i \alpha^V (1 - \lambda_{ii}^V)}{\sigma} dV \\
&= L_i + \frac{Y_i}{\sigma} \Big[\int_V \alpha^V \lambda_{ii}^V H_{ii}^V dV + \int_V \alpha^V (1 - \lambda_{ii}^V) dV \Big] \\
&= L_i + \frac{Y_i}{\sigma} \Big[1 + \int_V \alpha^V \lambda_{ii}^V (H_{ii}^V - 1) dV \Big] \\
&= L_i \times \frac{\sigma}{\sigma - 1 - \int_V \alpha^V \lambda_{ii}^V (H_{ii}^V - 1) dV}
\end{aligned}$$

Quantity Competition

$$\begin{aligned}
\mathbf{m}_{ii} &\equiv \frac{\sigma}{\sigma - 1 - \int_V \alpha^V \lambda_{ii}^V (H_{ii}^V - 1) dV} \\
&= \frac{\sigma}{\sigma - 1 - \int_V \alpha^V \lambda_{ii}^V (\sigma - 1) \lambda_{ii}^V HHI_{ii}^V dV} \\
&= \frac{\sigma}{(\sigma - 1)(1 - \int_V \alpha^V (\lambda_{ii}^V)^2 HHI_{ii}^V dV)} \\
&= \frac{\sigma}{\sigma - 1} \times [1 - \int_V \alpha^V (\lambda_{ii}^V)^2 HHI_{ii}^V dV]^{-1}
\end{aligned}$$

This aggregate excess markup can be computed as an ad-valorem term:

$$\mathbf{m}_{ii} = \frac{\sigma}{\sigma - 1} \times [1 + \frac{\int_V \alpha^V (\lambda_{ii}^V)^2 HHI_{ii}^V dV}{1 - \int_V \alpha^V (\lambda_{ii}^V)^2 HHI_{ii}^V dV}]$$

Price Competition

$$\begin{aligned}
\mathbf{m}_{ii} &\equiv \frac{\sigma}{\sigma - 1 - \int_V \alpha^V \lambda_{ii}^V (H_{ii}^V - 1) dV} \\
&= \frac{\sigma}{\sigma - 1 - \int_V \alpha^V \lambda_{ii}^V [\frac{(\sigma - 1)\lambda^V}{\sigma - (\sigma - 1)\lambda^V} \overline{HKI}^V] dV} \\
&= \frac{\sigma}{(\sigma - 1)(1 - \int_V \alpha^V (\lambda_{ii}^V)^2 \frac{\overline{HKI}^V}{\sigma - (\sigma - 1)\lambda^V} dV)} \\
&= \frac{\sigma}{\sigma - 1} \times [1 - \int_V \alpha^V (\lambda_{ii}^V)^2 \frac{\overline{HKI}^V}{\sigma - (\sigma - 1)\lambda^V} dV]^{-1}
\end{aligned}$$

This aggregate excess markup can be computed as an ad-valorem term:

$$\mathbf{m}_{ii} = \frac{\sigma}{\sigma-1} \times [1 + \frac{\int_V \alpha^V (\lambda_{ii}^V)^2 \frac{\overline{HKI}^V}{\sigma-(\sigma-1)\lambda^V} dV}{1 - \int_V \alpha^V (\lambda_{ii}^V)^2 \frac{\overline{HKI}^V}{\sigma-(\sigma-1)\lambda^V} dV}]$$

A.7 Equilibrium Aggregate Markup and Income Changes

A.7.1 Aggregate Markup Changes

Quantity Competition

$$\begin{aligned}
\hat{\mathbf{m}}_{ii} &= -(1 - \int_V \widehat{\alpha^V (\lambda_{ii}^V)^2 HHI_{ii}^V} dV) \\
&= \frac{1}{1 - \int_V \alpha^V (\lambda_{ii}^V)^2 HHI_{ii}^V dV} \Big[d \int_V \alpha^V (\lambda_{ii}^V)^2 HHI_{ii}^V dV \Big] \\
&= \frac{1}{1 - \int_V \alpha^V (\lambda_{ii}^V)^2 HHI_{ii}^V dV} \int_V \alpha^V d\Big[(\lambda_{ii}^V)^2 HHI_{ii}^V \Big] dV \\
&= \frac{1}{1 - \int_V \alpha^V (\lambda_{ii}^V)^2 HHI_{ii}^V dV} \int_V \alpha^V (\lambda_{ii}^V)^2 HHI_{ii}^V \Big[2\hat{\lambda}_{ii}^V + \widehat{HHI}_{ii}^V \Big] dV \\
&= \frac{1}{1 - \int_V \alpha^V (\lambda_{ii}^V)^2 HHI_{ii}^V dV} \Big[\int_V \alpha^V (\lambda_{ii}^V)^2 HHI_{ii}^V 2\hat{\lambda}_{ii}^V dV + \int_V \alpha^V (\lambda_{ii}^V)^2 HHI_{ii}^V \widehat{HHI}_{ii}^V dV \Big] \\
&= \frac{\int_V \alpha^V (\lambda_{ii}^V)^2 HHI_{ii}^V dV}{1 - \int_V \alpha^V (\lambda_{ii}^V)^2 HHI_{ii}^V dV} \Big[\int_V \frac{\alpha^V (\lambda_{ii}^V)^2 HHI_{ii}^V}{\int_V \alpha^V (\lambda_{ii}^V)^2 HHI_{ii}^V dV} 2\hat{\lambda}_{ii}^V dV + \int_V \frac{\alpha^V (\lambda_{ii}^V)^2 HHI_{ii}^V}{\int_V \alpha^V (\lambda_{ii}^V)^2 HHI_{ii}^V dV} \widehat{HHI}_{ii}^V dV \Big] \\
&= \frac{\int_V \alpha^V (\lambda_{ii}^V)^2 HHI_{ii}^V dV}{1 - \int_V \alpha^V (\lambda_{ii}^V)^2 HHI_{ii}^V dV} \Big[\int_V \frac{\alpha^V (\lambda_{ii}^V)^2 HHI_{ii}^V}{\int_V \alpha^V (\lambda_{ii}^V)^2 HHI_{ii}^V dV} 2\hat{\lambda}_{ii}^V dV \Big] + \\
&+ \frac{\int_V \alpha^V (\lambda_{ii}^V)^2 HHI_{ii}^V dV}{1 - \int_V \alpha^V (\lambda_{ii}^V)^2 HHI_{ii}^V dV} \Big[\int_V \frac{\alpha^V (\lambda_{ii}^V)^2 HHI_{ii}^V}{\int_V \alpha^V (\lambda_{ii}^V)^2 HHI_{ii}^V dV} \widehat{HHI}_{ii}^V dV \Big] \\
&= 2\theta_{ii}^Q \Big[\int_V \tilde{\alpha}^{q,V} \hat{\lambda}_{ii}^V dV \Big] + \theta_{ii}^Q \Big[\int_V \tilde{\alpha}^{q,V} \widehat{HHI}_{ii}^V dV \Big]
\end{aligned}$$

where $\tilde{\alpha}^{q,V} \equiv \frac{\alpha^V (\lambda_{ii}^V)^2 HHI_{ii}^V}{\int_V \alpha^V (\lambda_{ii}^V)^2 HHI_{ii}^V dV}$, is a modified sector share by the relative degree of domestic market power.

This expression means that we can use the computation for each sector to construct the local changes. These local changes will depend heavily on the initial state of competition across industries.

Deriving an expression for $\widehat{HHI}_{ii}$ is analogous to what I did for concentration on the consumer side. It implies using the firm-specific markup elasticity (that depends on shares) and then aggregating it based on the HHI function.

$$
\begin{aligned}
\widehat{HHI}_{ii} &= \frac{1}{HHI_{ii}} \sum_{v=1}^{N_{ii}} dz_v^2 \\
\widehat{HHI}_{ii} &= \frac{1}{HHI_{ii}} \sum_{v=1}^{N_{ii}} z_v^2 2\hat{z}_v \\
\widehat{HHI}_{ii} &= 2 \sum_{v=1}^{N_{ii}} h_v \hat{z}_v
\end{aligned}
$$

This expression depends on log changes in individual domestic market shares as $\mathcal{C}$.
In those results, we have $\hat{z_v} = (1-\sigma)[\frac{\psi_v}{1+\psi_v} - \sum_{v=1}^{N_{ii}} z_v \frac{\psi_v}{1+\psi_v}]\hat{P}_i$, so we can use this expression:

$$
\begin{aligned}
\widehat{HHI}_{ii} &= 2 \sum_{v=1}^{N_{ii}} h_v (1-\sigma)[\frac{\psi_v}{1+\psi_v} - \sum_{v=1}^{N_{ii}} z_v \frac{\psi_v}{1+\psi_v}]\hat{P}_i \\
&= 2(1-\sigma) \sum_{v=1}^{N_{ii}} h_v \frac{\psi_v}{1+\psi_v} \hat{P}_i - \sum_{v=1}^{N_{ii}} z_v \frac{\psi_v}{1+\psi_v} \hat{P}_i \\
&= 2(1-\sigma) \sum_{v=1}^{N_{ii}} [h_v - z_v] \frac{\psi_v}{1+\psi_v} \hat{P}_i \\
&= 2(1-\sigma) \tilde{\Phi}_{ii}^q \hat{P}_i
\end{aligned}
$$

Here we cannot use the aggregate price index for this expression as the weights in the markup use $\tilde{\alpha}$ rather than α. Therefore:

$$
\begin{aligned}
\hat{\mathbf{m}}_{ii} &= 2\theta_{ii}^Q \Big[\int_V \tilde{\alpha}^{q,V} \hat{\lambda}_{ii}^V dV \Big] + \theta_{ii}^Q \Big[\int_V \tilde{\alpha}^{q,V} \widehat{HHI}_{ii}^V dV \Big] \\
&= 2\theta_{ii}^Q \Big[\int_V \tilde{\alpha}^{q,V} \hat{\lambda}_{ii}^V dV \Big] + \theta_{ii}^Q \Big[\int_V \tilde{\alpha}^{q,V} 2(1-\sigma) \tilde{\Phi}_{ii}^{q,V} \hat{P}_i^V dV \Big] \\
&= 2\theta_{ii}^Q (1-\sigma) \Big[\int_V \tilde{\alpha}^{q,V} \frac{\hat{\lambda}_{ii}^V}{1-\sigma} dV \Big] + \Big[\int_V \tilde{\alpha}^{q,V} \tilde{\Phi}_{ii}^{q,V} \hat{P}_i^V dV \Big] \\
&= 2\theta_{ii}^Q (1-\sigma) \Big[\int_V \tilde{\alpha}^{q,V} [\frac{\hat{\lambda}_{ii}^V}{1-\sigma} + \tilde{\Phi}_{ii}^{q,V} \hat{P}_i^V] dV \Big]
\end{aligned}
$$

Price Competition Thanks to having the exact same formulation, we can follow the exact same strategy but also accounting for concentration degrees relative distortions:

$$
\begin{aligned}
\hat{\mathbf{m}}_{ii} &= -(1-\int_V \alpha^V (\lambda_{ii}^V)^2 \widehat{\frac{\overline{HKI}^V}{\sigma-(\sigma-1)\lambda^V}} dV) \\
&= \frac{1}{1-\int_V \alpha^V (\lambda_{ii}^V)^2 \frac{\overline{HKI}^V}{\sigma-(\sigma-1)\lambda^V} dV} \Big[d\int_V \alpha^V (\lambda_{ii}^V)^2 \frac{\overline{HKI}^V}{\sigma-(\sigma-1)\lambda^V} dV \Big] \\
&= \frac{1}{1-\int_V \alpha^V (\lambda_{ii}^V)^2 \frac{\overline{HKI}^V}{\sigma-(\sigma-1)\lambda^V} dV} \int_V \alpha^V d\Big[(\lambda_{ii}^V)^2 \frac{\overline{HKI}^V}{\sigma-(\sigma-1)\lambda^V} \Big] dV \\
&= \frac{1}{1-\int_V \alpha^V (\lambda_{ii}^V)^2 \frac{\overline{HKI}^V}{\sigma-(\sigma-1)\lambda^V} dV} \int_V \alpha^V (\lambda_{ii}^V)^2 \frac{\overline{HKI}^V}{\sigma-(\sigma-1)\lambda^V} \Big[\frac{\widehat{(\lambda_{ii}^V)^2}}{\sigma-(\sigma-1)\lambda^V} + \widehat{\overline{HKI}^V} \Big] dV \\
&= \frac{\int_V \alpha^V (\lambda_{ii}^V)^2 \frac{\overline{HKI}^V}{\sigma-(\sigma-1)\lambda^V} dV}{1-\int_V \alpha^V (\lambda_{ii}^V)^2 \frac{\overline{HKI}^V}{\sigma-(\sigma-1)\lambda^V} dV} \int_V \frac{\alpha^V (\lambda_{ii}^V)^2 \frac{\overline{HKI}^V}{\sigma-(\sigma-1)\lambda^V}}{\int_V \alpha^V (\lambda_{ii}^V)^2 \frac{\overline{HKI}^V}{\sigma-(\sigma-1)\lambda^V} dV} \Big[\frac{\widehat{(\lambda_{ii}^V)^2}}{\sigma-(\sigma-1)\lambda^V} + \widehat{\overline{HKI}^V} \Big] dV \\
&= \frac{\int_V \alpha^V (\lambda_{ii}^V)^2 \frac{\overline{HKI}^V}{\sigma-(\sigma-1)\lambda^V} dV}{1-\int_V \alpha^V (\lambda_{ii}^V)^2 \frac{\overline{HKI}^V}{\sigma-(\sigma-1)\lambda^V} dV} \int_V \tilde{\alpha}^{p,V} \Big[\frac{\widehat{(\lambda_{ii}^V)^2}}{\sigma-(\sigma-1)\lambda^V} + \widehat{\overline{HKI}^V} \Big] dV
\end{aligned}
$$

****

As before, we know how $\hat{\lambda}_{ii}$ changes with autarky, so we need $\widehat{\overline{HKI}_{ii}}$

Deriving an expression for $\widehat{\overline{HKI}_{ii}}$ is more involved now because weights depend on λ as well. First, let's re-write $\widehat{\overline{HKI}}$ as follows.

$$
\begin{aligned}
\overline{HKI}_{ii} &= \sum_{k=2}^{\infty} \frac{[\frac{\sigma-1}{\sigma}\lambda]^{k-1}}{\frac{(\sigma-1)\lambda}{\sigma-(\sigma-1)\lambda}} HKI(k) \\
&= \sum_{k=2}^{\infty} \frac{[\frac{\sigma-1}{\sigma}\lambda]^{k-1}}{\frac{(\sigma-1)\lambda}{\sigma-(\sigma-1)\lambda}} \sum_{v=1}^{N} z_v^k \\
&= \sum_{k=2}^{\infty} \sum_{v=1}^{N} \frac{[\frac{\sigma-1}{\sigma}\lambda]^{k-1}}{\frac{(\sigma-1)\lambda}{\sigma-(\sigma-1)\lambda}} z_v^k \\
&= \sum_{v=1}^{N} \sum_{k=2}^{\infty} \frac{[\frac{\sigma-1}{\sigma}\lambda]^{k-1}}{\frac{(\sigma-1)\lambda}{\sigma-(\sigma-1)\lambda}} z_v^k
\end{aligned}
$$

If we log-differentiate but keep λ constant (in the weights), we get:

$$
\begin{aligned}
\widehat{\overline{HKI}_{ii}}\big|_{\bar{\lambda}} &= \frac{1}{\overline{HKI}}\sum_{v=1}^{N}\sum_{k=2}^{\infty}\frac{[\frac{\sigma-1}{\sigma}\lambda]^{k-1}}{\frac{(\sigma-1)\lambda}{\sigma-(\sigma-1)\lambda}}kz_v^k\hat{z}_v \\
&= \frac{1}{\overline{HKI}}\sum_{k=2}^{\infty}\iota(k)k\sum_{v=1}^{N}z_v^k\hat{z}_v \\
&= \frac{1}{\overline{HKI}}\left[\iota(2)2\sum_{v=1}^{N}z_v^2\hat{z}_v+\iota(3)3\sum_{v=1}^{N}z_v^3\hat{z}_v+\iota(4)4\sum_{v=1}^{N}z_v^4\hat{z}_v+...\right] \\
&= \frac{1}{\overline{HKI}}\left[2HKI(2)\iota(2)\sum_{v=1}^{N}h_v(2)\hat{z}_v+3HKI(3)\iota(3)\sum_{v=1}^{N}h_v(3)\hat{z}_v+4HKI(4)\iota(4)\sum_{v=1}^{N}h_v(4)\hat{z}_v+...\right] \\
&= \sum_{v=1}^{N}\left[\frac{2\iota(2)HKI(2)}{\sum_{k=2}^{\infty}\iota(k)HKI(k)}h_v(2)+\frac{3\iota(3)HKI(3)}{\sum_{k=2}^{\infty}\iota(k)HKI(k)}h_v(3)+\frac{4\iota(4)HKI(4)}{\sum_{k=2}^{\infty}\iota(k)HKI(k)}h_v(4)+...\right]\hat{z}_v \\
&= \sum_{v=1}^{N}\left[2o(2)h_v(2)+3o(3)h_v(3)+4o(4)h_v(4)+...\right]\hat{z}_v \qquad o(k)\textbf{: contribution of } k \textbf{ degree to } \overline{HKI} \\
&= \sum_{v=1}^{N}\left[\sum_{k=2}^{\infty}ko(k)h_v(k)\right]\hat{z}_v
\end{aligned}
$$

My goal here is to get to an expression $\widehat{\overline{HKI}_{ii}} = \rho\sum_{v=1}^{N}\rho_v\hat{z}_v$ such that $\sum_{v=1}^{N}h_v = 1$ and $0 < \varrho < C < \infty$, i.e. ϱ is a finite bounded number. In the quantity case $\varrho = 2$.

$$
\begin{aligned}
\widehat{\overline{HKI}_{ii}}\big|_{\bar{\lambda}} &= \sum_{v=1}^{N}\left[\sum_{k=2}^{\infty}ko(k)h_v(k)\right]\hat{z}_v \\
&= \sum_{v=1}^{N}\left[\sum_{k=2}^{\infty}o(k)\times\Big[kh_v(k)\Big]\right]\hat{z}_v \qquad \textbf{Note that } \sum_{k=2}^{\infty}o(k)=1\textbf{, so it is a weighted average} \\
&= \sum_{v=1}^{N}\tilde{h}_v\hat{z}_v \qquad\qquad \tilde{h}_v \equiv \sum_{k=2}^{\infty}o(k)\times\Big[kh_v(k)\Big] \textbf{ is the weighted average} \\
&= \varrho\sum_{v=1}^{N}\rho_v\hat{z}_v \qquad\qquad \rho_v\textbf{: contribution of the } v \textbf{ firm to } \varrho
\end{aligned}
$$

It is worth noting the following of ϱ:

$$\begin{aligned}
\varrho &= \sum_{v=1}^{N}\sum_{k=2}^{\infty} o(k)kh_v(k) \\
&= \sum_{k=2}^{\infty} o(k)k\sum_{v=1}^{N} h_v(k) \\
&= \sum_{k=2}^{\infty} o(k)k\sum_{v=1}^{N} \frac{z_v^k}{HKI(k)} \\
&= \sum_{k=2}^{\infty} o(k)k
\end{aligned}$$

So closed form for $\widehat{\overline{HKI}_{ii}}\big|_{\bar{\lambda}}$ depends on ϱ is finite, i.e. that $\sum_{k=2}^{\infty} o(k)k$ converges. I conjecture this is true given that: $o(k) \in (0,1)$ for all k and goes to zero convexly as $k \to \infty$, whereas k goes to infinity linearly. This should be proven though.

I will also need that ρ_v is convex on z_v too to have as a result that the markup's own elasticity to prices is positive. For now, I will assume this is true.

Now, we need to know $\widehat{\overline{HKI}_{ii}}\big|_{\{\bar{z}_v\}_{v=1}^N}$ i.e. the log difference change due to changes in λ keeping the firms' market shares constant.

$$\begin{aligned}
\widehat{\overline{HKI}_{ii}}\big|_{\{\bar{z}_v\}_{v=1}^N} &= \frac{1}{\overline{HKI}}\sum_{k=2}^{\infty}\iota(k)HKI(k)\frac{\hat{\iota(k)}}{\hat{\lambda}}\hat{\lambda} \\
&= \sum_{k=2}^{\infty} o(k)\frac{\hat{\iota(k)}}{\hat{\lambda}}\hat{\lambda}
\end{aligned}$$

We need $\frac{\hat{\iota(k)}}{\hat{\lambda}}$, i.e. $\frac{d\log\iota(k)}{d\log\lambda}$, the elasticity of of the weights with respect to λ. Note that $\log\iota = (k-1)\log\frac{\sigma}{\sigma-1} + (k-1)\log\lambda + \log[\sigma-(\sigma-1)\lambda] - \log(\sigma-1) - \log\lambda$.

$$\frac{d\log\iota(k)}{d\log\lambda} = (k-2) - \frac{(\sigma-1)\lambda}{\sigma-(\sigma-1)\lambda}$$

We can replace this back:

$$\begin{aligned}
\widehat{\overline{HKI}_{ii}}\big|_{\{\bar{z}_v\}_{v=1}^N} &= \sum_{k=2}^{\infty} o(k)[(k-2) - \frac{(\sigma-1)\lambda}{\sigma-(\sigma-1)\lambda}]\hat{\lambda} \\
&+ \left[\Big[\sum_{k=2}^{\infty} o(k)k\Big] - [2 + \frac{(\sigma-1)\lambda}{\sigma-(\sigma-1)\lambda}]\right]\hat{\lambda} \qquad \textbf{I can the } \varrho \textbf{ definition and its finite assumptio} \\
&+ \left[\varrho - [2 + \frac{(\sigma-1)\lambda}{\sigma-(\sigma-1)\lambda}]\right]\hat{\lambda}
\end{aligned}$$

Then, we can use that $\widehat{\overline{HKI}_{ii}} = \widehat{\overline{HKI}_{ii}}\big|_{\bar{\lambda}} + \widehat{\overline{HKI}_{ii}}\big|_{\{\bar{z}_v\}_{v=1}^N} = \varrho\sum_{v=1}^{N}\rho_v\hat{z}_v + \left[\varrho - [2 + \frac{(\sigma-1)\lambda}{\sigma-(\sigma-1)\lambda}]\right]\hat{\lambda}$

****

$$
\begin{aligned}
&= \theta_{ii}^{P}\int_{V}\tilde{\alpha}^{p,V}\varrho_{ii}^{V}(1-\sigma)\Big[\frac{\hat{\lambda}_{ii}^{V}}{1-\sigma}+\tilde{\Phi}_{ii}^{p,V}\hat{P}_{i}\Big]dV\\
&= \varrho_{ii}\theta_{ii}^{P}(1-\sigma)\int_{V}\tilde{\alpha}^{p,V}\tilde{\varrho}_{ii}^{V}\Big[\frac{\hat{\lambda}_{ii}^{V}}{1-\sigma}+\tilde{\Phi}_{ii}^{p,V}\hat{P}_{i}\Big]dV
\end{aligned}
$$

where I defined objects analogous to the single sector case, but also needed to define $\varrho_{ii}\equiv\int_{V}\tilde{\alpha}^{p,V}\varrho_{ii}^{V}dV$ and $\tilde{\varrho}_{ii}^{V}\equiv\frac{\varrho_{ii}^{V}}{\int_{V}\tilde{\alpha}^{p,V}\varrho_{ii}^{V}dV}$.

A.7.2 Aggregate Income Changes

Quantity Competition

$$
\begin{aligned}
\hat{Y}_{i}=\hat{\mathbf{m}}_{ii} &= 2\theta_{ii}^{Q}(1-\sigma)\Big[\int_{V}\tilde{\alpha}^{q,V}[\frac{\hat{\lambda}_{ii}^{V}}{1-\sigma}+\tilde{\Phi}_{ii}^{q,V}\left[\frac{1+\mathcal{E}_{ii}^{c,V}}{1+\mathcal{E}_{ii}^{c,V}\tilde{\Phi}_{ii}^{c,V}}\right]\frac{\hat{\lambda}_{ii}^{V}}{\sigma-1}]dV\Big]\\
&= 2\theta_{ii}^{Q}(\sigma-1)\Big[\int_{V}\tilde{\alpha}^{q,V}[\frac{\hat{\lambda}_{ii}^{V}}{\sigma-1}-\tilde{\Phi}_{ii}^{q,V}\left[\frac{1+\mathcal{E}_{ii}^{c,V}}{1+\mathcal{E}_{ii}^{c,V}\tilde{\Phi}_{ii}^{c,V}}\right]\frac{\hat{\lambda}_{ii}^{V}}{\sigma-1}]dV\Big]\\
&= 2\theta_{ii}^{Q}(\sigma-1)\Big[\int_{V}\tilde{\alpha}^{q,V}[1-\tilde{\Phi}_{ii}^{q,V}\left[\frac{1+\mathcal{E}_{ii}^{c,V}}{1+\mathcal{E}_{ii}^{c,V}\tilde{\Phi}_{ii}^{c,V}}\right]]\frac{\hat{\lambda}_{ii}^{V}}{\sigma-1}dV\Big]
\end{aligned}
$$

As before, this expression is helpful, as it shows how aggregate profits depend on changes in absorption.

In this case, I will also add a different formulation employing an index of distortions:

$$
\begin{aligned}
\hat{Y}_{i} &= 2\theta_{ii}^{Q}(\sigma-1)\Big[\int_{V}\tilde{\alpha}^{q,V}[1-\tilde{\Phi}_{ii}^{q,V}\left[\frac{1+\mathcal{E}_{ii}^{c,V}}{1+\mathcal{E}_{ii}^{c,V}\tilde{\Phi}_{ii}^{c,V}}\right]]\frac{\hat{\lambda}_{ii}^{V}}{\sigma-1}dV\Big]\\
&= \Xi_{ii}^{Q}\int_{V}\xi_{ii}^{q}\frac{\hat{\lambda}_{ii}^{V}}{\sigma-1}dV
\end{aligned}
$$

where $\Xi_{ii}^{Q}\equiv 2\theta_{ii}^{Q}(\sigma-1)\Big[\int_{V}\tilde{\alpha}^{q,V}[1-\tilde{\Phi}_{ii}^{q,V}\left[\frac{1+\mathcal{E}_{ii}^{c,V}}{1+\mathcal{E}_{ii}^{c,V}\tilde{\Phi}_{ii}^{c,V}}\right]]dV\Big]$ and $\xi_{ii}^{q}\equiv 2\theta_{ii}^{Q}(\sigma-1)\Big[\tilde{\alpha}^{q,V}[1-\tilde{\Phi}_{ii}^{q,V}\left[\frac{1+\mathcal{E}_{ii}^{c,V}}{1+\mathcal{E}_{ii}^{c,V}\tilde{\Phi}_{ii}^{c,V}}\right]]\Big]/\Xi_{ii}^{Q}$.

Price Competition

$$
\begin{aligned}
\hat{Y}_{i}=\hat{\mathbf{m}}_{ii} &= \varrho_{ii}\theta_{ii}^{P}(1-\sigma)\Big[\int_{V}\tilde{\alpha}^{p,V}\tilde{\varrho}_{ii}^{V}[\frac{\hat{\lambda}_{ii}^{V}}{1-\sigma}+\tilde{\Phi}_{ii}^{p,V}\left[\frac{1+\mathcal{E}_{ii}^{c,V}}{1+\mathcal{E}_{ii}^{c,V}\tilde{\Phi}_{ii}^{c,V}}\right]\frac{\hat{\lambda}_{ii}^{V}}{\sigma-1}]dV\Big]\\
&= \varrho_{ii}\theta_{ii}^{P}(\sigma-1)\Big[\int_{V}\tilde{\alpha}^{p,V}\tilde{\varrho}_{ii}^{V}[1-\tilde{\Phi}_{ii}^{p,V}\left[\frac{1+\mathcal{E}_{ii}^{c,V}}{1+\mathcal{E}_{ii}^{c,V}\tilde{\Phi}_{ii}^{c,V}}\right]]\frac{\hat{\lambda}_{ii}^{V}}{\sigma-1}dV\Big]
\end{aligned}
$$

where I defined objects analogous to the single sector case, but also needed to define $\varrho_{ii}\equiv\int_{V}\tilde{\alpha}^{p,V}\varrho_{ii}^{V}dV$ and $\tilde{\varrho}_{ii}^{V}\equiv\frac{\varrho_{ii}^{V}}{\int_{V}\tilde{\alpha}^{p,V}\varrho_{ii}^{V}dV}$.

B Additional Empirical Results

Figure A1: Chinese Import Penetration. 1995-2007.

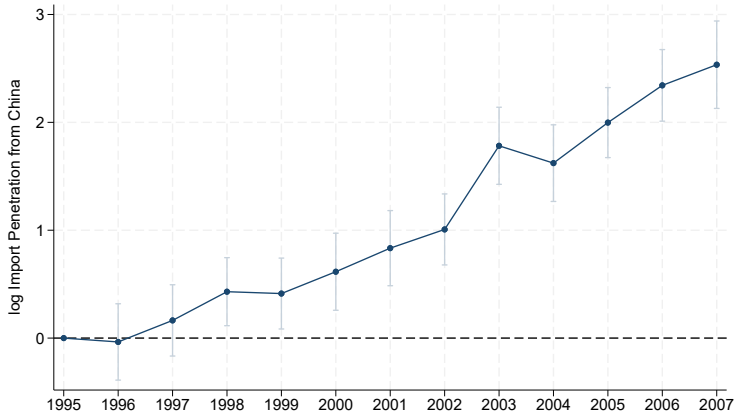


Figure A2: Sectoral Import Penetration. 1995 vs 2007.

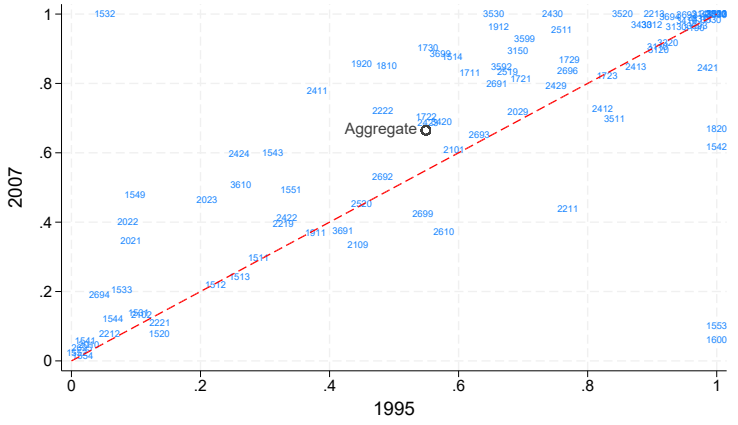


Figure A3: Sectoral Domestic Revenue Share. 1995 vs 2007.

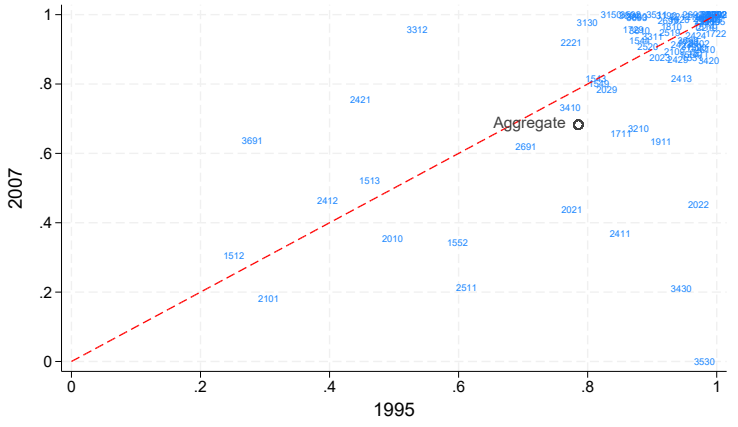


Figure A4: Profit and Expenditure Shares. 1995.

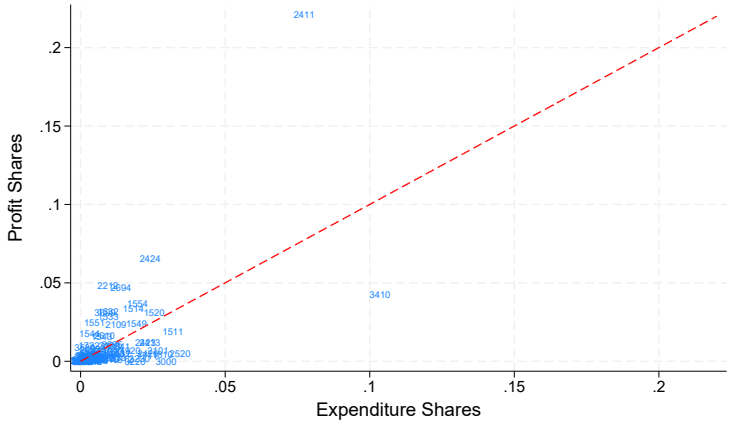
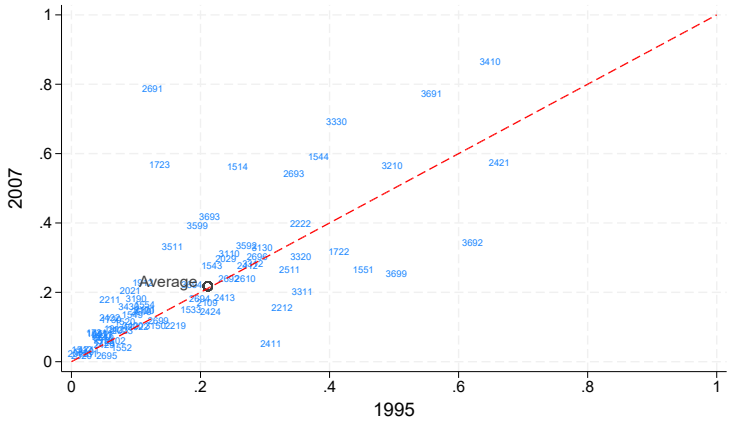


Figure A5: Sectoral Domestic Market Concentration.1995 vs 2007

(a) HHI



(b) Share of Top Four Firms

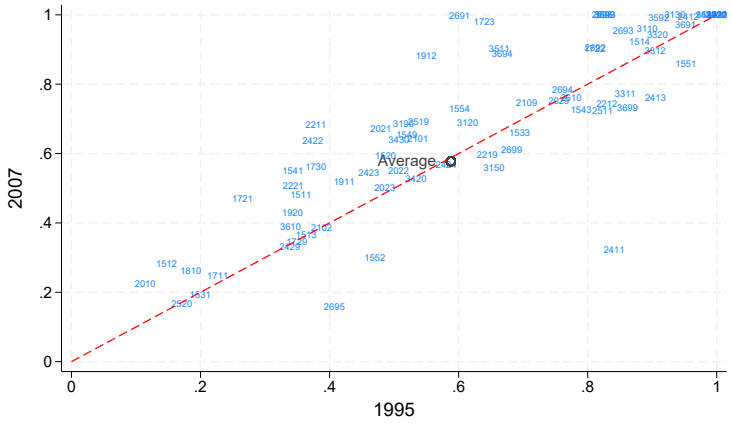


Figure A6: Welfare-Consistent Concentration Measure. Price Competition. 1995-2007.

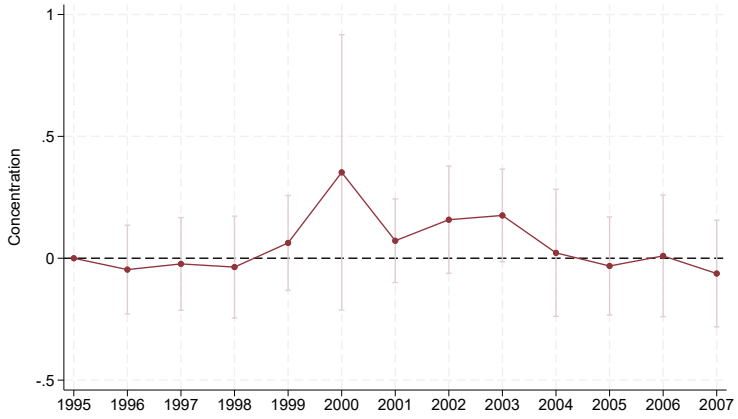


Figure A7: Welfare-Consistent Concentration Measure. 1995 vs 2007.

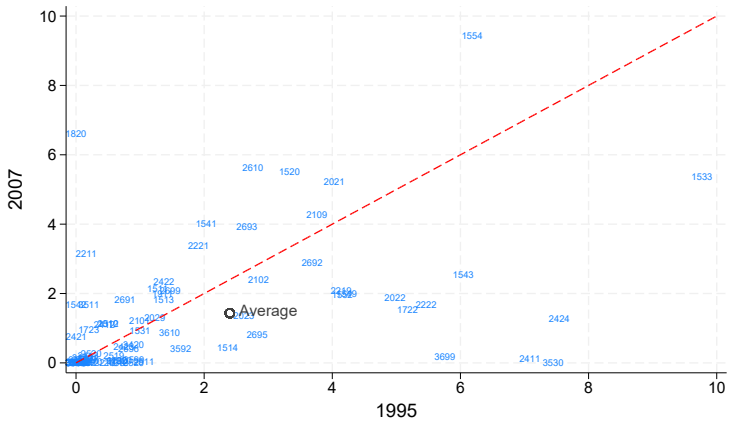
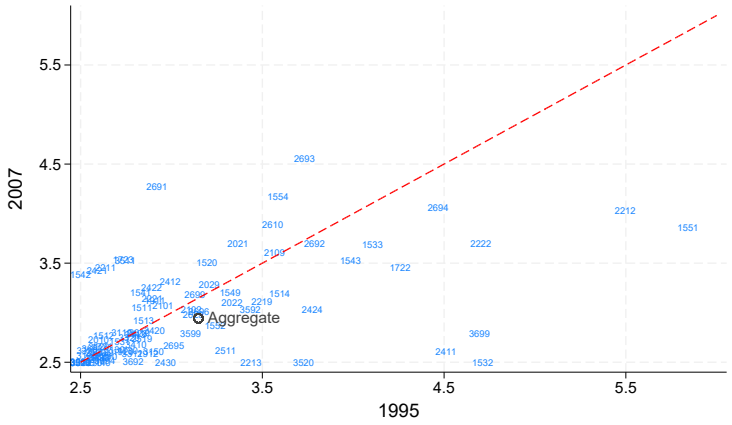


Figure A8: Sectoral Consumer Losses and Producer Gains from Closing the Economy (10 log points).1995 vs 2007

(a) Consumer Losses



(b) Producer Gains

