# The Distributional Effects of Trade across Local Labor Markets and Heterogeneous Consumers*

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**Abstract**

I study how global trade and technology shocks propagate across local labor markets and affect the welfare of consumers with heterogeneous consumption baskets along the income distribution. I propose a tractable general equilibrium Ricardian-Roy framework that allows for non-homothetic expenditure patterns. Methodologically, it reconciles heterogeneous workers and non-homothetic demand with the *exact-hat algebra* approach to counterfactual analysis, which imposes minimal data requirements without separately identifying unobserved structural fundamentals. I also derive a generalized welfare gains formula for the distributional impact of trade shocks that applies to any demand system across sectors. Beyond its distributional implications, the framework yields a novel – the Consumption Inequality – channel of the aggregate welfare gains. I apply the model to revisit the general equilibrium effects of the China shock for 722 US commuting zones and a sample of other countries. It is found that the China shock led to aggregate welfare gains for the US of 0.71%, about one-third of which is attributed to the Consumption Inequality channel. The shock disproportionately benefited low-income US consumers and commuting zones by reducing the relative prices of low-income elastic sectors. The distribution of welfare gains across income quantiles varied substantially between countries.

Keywords: aggregate and distributional effects of trade, income inequality, non-homothetic demand, heterogeneous consumers, local labor markets.

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# 1 Introduction

Global trade and technology shocks are known to create winners and losers at the country level. Although the aggregate welfare effects are often found to be modest, there is growing empirical evidence of sizable distributional impacts of trade and trade policies across local labor markets.[1] Generally, once a country is exposed to a global shock, it may generate winners and losers along several dimensions. For instance, consider the impact of an arbitrary shock that has heterogeneous sectoral exposure. Based on differences in sectoral employment patterns, some local labor markets may significantly gain from trade openness and changes in a country's specialization, while others may be adversely affected. While affecting the distribution of nominal wages across workers via the earnings channel, the shock may also alter relative prices across sectors. This creates winners and losers among heterogeneous consumers – the expenditure channel of welfare change. So, the resulting welfare impact of any shock for a particular group of individuals tends to be jointly determined by the interaction of earnings and expenditure channels in general equilibrium.

To provide a specific example, consider a low-income individual who lives in a region that specializes in import-competing manufacturing industries (e.g., the US Rust Belt). On the one hand, the rising Chinese import competition is likely to reduce the individual's nominal wage. On the other hand, low-income individuals may disproportionally benefit from the expenditure channel if the shock reduces the relative prices of low-income elastic goods/sectors. Depending on which of the two effects is dominating, it may result in increasing or decreasing the country's welfare inequality in response to the shock.

In this paper, I aim to study how global trade and technology shocks propagate across local labor markets and affect the welfare of local consumers with heterogeneous consumption baskets across the income distribution. Such a goal essentially gives rise to several underlying questions: (i) How do non-homothetic consumption patterns affect equilibrium outcomes in general equilibrium? (ii) What are the distributional consequences of shocks across individuals at the local level, and what are the implications for aggregate welfare gains? (iii) What is the relative importance of non-homothetic effects compared with traditional channels of welfare gains?

To study these questions theoretically and quantitatively, I propose a tractable general equilibrium Ricardian–Roy framework that allows for non-homothetic expenditure patterns. In particular, I introduce the Almost Ideal Demand System (Deaton and Muellbauer 1980) – which provides an arbitrary first-order approximation to any demand system derived from utility-maximizing behavior – into a multi-sectoral extension of the Eaton and Kortum (2002) model that is enriched with Roy (1951)-type allocation of workers across sectors and multiple

[1] See, e.g., Autor, Dorn, and Hanson (2013) for the effect of rising Chinese import competition across US local labor markets.

local markets as in Galle, Rodríguez-Clare, and Yi (2023). The model's flexible demand side implies that the aggregate expenditure shares for each sector may depend on other sectors' price indices and the income distribution across individual consumers. In particular, the Almost Ideal Demand System implies that the impact of income distribution on the aggregate expenditure shares can be summarized by sufficient statistics – mean income adjusted by the Theil index of income inequality. As a result, expenditure shares may change in response to shocks through both homothetic (through changes in sectoral price indices) forces and non-homothetic (through changes in mean income and the Theil index of income inequality) forces. Apart from affecting the equilibrium outcomes of aggregate variables, non-homothetic effects also generate distributional welfare consequences of any shock along the income distribution. In this context, the Theil index governs the inequality margin of adjustment to trade and technology shocks across local labor markets and heterogeneous consumers.

Deviation from the standard Cobb–Douglas case of constant expenditure shares gives rise to two main methodological challenges. First, to deliver theory-consistent predictions of the impacts of any shock, it is imperative to reconcile heterogeneous consumers and non-homothetic demand with the counterfactual analysis based on relative changes – the *exact-hat algebra* in the spirit of Dekle, Eaton, and Kortum (2008). By not requiring the separate identification of unobserved structural fundamentals, the exact-hat algebra approach is a cornerstone of the quantitative analysis in the corresponding literature. The second challenge is that the well-known welfare change formula by Arkolakis, Costinot, and Rodríguez-Clare (2012) inevitably ceases to hold in such a setup and requires adjustments. I address these methodological issues in Sections 2 and 3, respectively.

In particular, I combine the convenient market share aggregation properties of the Almost Ideal Demand System, which is based on sufficient statistics, with the results from the supply side of the model, which endogenously generates the Fréchet income distribution. The supply side of the model gives rise to a closed-form expression for the Theil index of income inequality in each local labor market. I also show that solving for changes in the aggregate expenditure shares for each country boils down to solving a system of linear equations for each country. It nests counterfactual changes in the aggregate consumption shares obtained under the homothetic Translog expenditure function and constant (Cobb-Douglas) sectoral expenditure shares as special cases. Exploiting the supply-side properties of the model, I also derive analytical expressions for counterfactual values of the country-level Theil indices. As a result, the model remains highly tractable, and the data requirements to solve for any counterfactual equilibrium based on the exact-hat algebra approach are similar to the standard case of constant expenditure shares across sectors.

As for the second methodological challenge, given a non-homothetic demand side, the welfare analysis based on the standard tools of the exact price index cannot be applied in

general. So, I use *Equivalent Variation* as a measure, to a first-order approximation, of welfare changes associated with changes in prices and income. I derive a parsimonious welfare change formula for the distributional impact of any foreign/non-local shock for consumers with a given income level that applies to any demand system across sectors. It generalizes the well-known ACR (Arkolakis, Costinot, and Rodríguez-Clare 2012) formula for the Ricardian model and the one derived by Galle, Rodríguez-Clare, and Yi (2023), both of which are based on Cobb–Douglas/CES frameworks. In particular, it shows that an income group's local welfare change in response to any trade shock can be decomposed into a country-level shift in trade openness, the local market's change in the degree of specialization, and the covariance of these terms with the expenditure patterns of the income group. It also complements the formula derived by Arkolakis, Costinot, Donaldson, and Rodriguez-Clare (2019) (ACDR) for infinitesimal trade shocks.[2]

In addition to the traditional Ricardian and Roy channels of welfare gains, the framework also gives rise to *the Consumption Inequality* channel of the aggregate welfare effects of trade and technology shocks.[3] It suggests the importance of taking into account non-homothetic expenditure patterns for studying both the distributional and aggregate impact of shocks. I also analytically decompose this channel at the local labor market level into three different margins: *i*) the *Pro-poor* margin, *ii*) the *Mean income* margin, and *iii*) the *Inequality* margin.[4] I show that the Pro-poor margin amplifies the aggregate local welfare gains when the nature of a shock is "pro-poor" and, conversely, dampens the aggregate local welfare gains if the nature of a shock is "pro-rich". The Mean income margin implies that if the nature of a shock is "pro-poor", local markets with relatively lower mean income than the country-level one gain from this margin and lose otherwise. Similarly, the Inequality margin implies that local markets with a relatively lower Theil index of income inequality gain if the nature of a shock is "pro-poor". These results are driven by the demand side property that a country's aggregate expenditure shares across sectors depend on the mean income adjusted by the exponent of the Theil index of income inequality. So, local labor markets with relatively lower average income and lower income inequality tend to spend disproportionally more on low-income elastic than their relatively more affluent counterparts. As a result, for example, a "pro-poor" shock, by reducing the prices of low-income elastic sectors, tends to benefit low-income and low-inequality local labor markets. In Section 4, I quantify the contribution of the Consumption

[2]The relation between my formula and ACDR (2019) can be summarized as follows. First, while the ACDR (2019) formula is derived from a flexible demand system within a sector, I also allow for an arbitrary demand system across sectors when studying the distributional welfare effects. Second, while the ACDR (2019) formula is derived from a monopolistically competitive framework, it does not take into account heterogeneous consumers or Roy-type comparative advantage and its corresponding channel of welfare gains.

[3]I also analytically decompose the welfare impact of any shock into homothetic vs. non-homothetic channels and discuss the difference between the Consumption Inequality channel and the non-homothetic channel.

[4]Notably, the spatial dimension represented by heterogeneous local labor markets differing in their patterns of comparative advantage gives rise to the Mean income and Inequality margins of adjustment to trade shocks.

Inequality channel as well as each of its margins.

From a research and policymaker's perspective, the framework possesses several advantages. First, it allows one to study the heterogeneous impact of trade policy on individuals jointly via the earnings and expenditure channels in general equilibrium. Second, despite the very flexible supply and demand sides, the model remains analytically tractable. In particular, the Almost Ideal Demand System – combined with the convenient supply-side properties – provides closed-form expressions for the aggregate expenditure shares and a local market's welfare response to any foreign/non-local shock. It also allows for analytically tractable decomposition of the Consumption Inequality channel that isolates its different margins. Third, as noted above, given the quantitative nature of the model in line with recent literature built on Eaton and Kortum (2002), it allows for counterfactual analysis in relative changes – the exact-hat algebra – which imposes minimal data requirements for counterfactual predictions and welfare analysis without separately identifying unobserved structural fundamentals like trade costs or technological parameters. So, it conveniently allows for many potential applications studied in the recent literature, such as implications of "Tariff Wars", the impact of AI/Automation, and Climate Change, among others.

I apply the model to revisit the impact of the China shock on a sample of 40 countries and the universe of US local labor markets represented by 722 US Commuting Zones. I find that while the China shock led to aggregate gains for most countries, some countries experienced net losses. As for the United States, the aggregate welfare impact of the shock is estimated to be 0.71%. It suggests larger country-level gains than most previous findings in the quantitative trade literature (see, e.g., Galle, Rodríguez-Clare, and Yi 2023), which mainly rely on the simplifying assumption of homothetic expenditure shares. In particular, a notable part of this increase (0.26%) is attributed to the Consumption Inequality channel driven by non-homothetic expenditure patterns across sectors. For the US, I also quantify the contribution of each margin of the Consumption Inequality channel. I find that each of these margins accounts for a sizable part of the distributional impact of the China shock across local labor markets and that Commuting Zones with lower mean income were more likely to gain from the shock. At the individual level, the China shock favored relatively poor consumers by reducing the relative prices of low-income elastic sectors. I also find that the distribution of welfare gains across income quantiles varied substantially between countries.

**Related Literature** This paper is related to several strands of the literature. First of all, it complements Ricardian(-Roy) models and models with trade and sectoral reallocation (see, e.g., Adão 2016, Costinot and Vogel 2015, Caliendo, Dvorkin, and Parro 2019, Galle, Rodríguez-

Clare, and Yi 2023, Kim and Vogel 2020, etc.).[5] While these papers focus on the distributional impact of trade through the earnings channel, I also study the expenditure effects in general equilibrium that occur under non-homothetic demand. In particular, I derive the Consumption Inequality channel and analytically decompose it into several margins. I also quantify the contribution of each of these margins as well as the traditional Ricardian and Roy channels to the total welfare change.

Another related strand of the literature is trade models with non-homothetic demand and/or heterogeneous consumers (see, e.g., Arkolakis, Costinot, Donaldson, and Rodriguez-Clare 2019, Artuc, Porto, and Rijkers 2019, Borusyak and Jaravel 2021, Faber 2014, Faber and Fally 2017, Fajgelbaum, Grossman, and Helpman 2011, Fajgelbaum and Khandelwal 2016, Fan, Li, Xu, and Yeaple 2020, Feenstra and Romalis 2014, Fieler 2011, Flam and Helpman 1987, Nigai 2016, Sharapudinov 2022, Simonovska 2015, Waugh 2023 etc.).[6] In contrast to these papers, I reconcile the flexible non-homothetic demand side represented by a first-order approximation to any demand system, with a quantitative Ricardian model enriched with Roy-like heterogeneous workers and heterogeneous local markets differing in their patterns of comparative advantage. Within this flexible framework, I also conduct a counterfactual analysis of any foreign/non-local shock based on *the exact-hat algebra* approach, which imposes only mild restrictions on data availability and does not require identifying the actual levels of the model's fundamentals, such as trade costs and technology parameters. Thus, the paper bridges the literature on the distributional effects of trade through the expenditure channel and that on quantitative trade and spatial models built on Eaton and Kortum (2002).

The rest of the paper is organized as follows. In Section 2, I propose a theoretical framework and derive the main theoretical results for counterfactual analysis. In Section 3, I provide a welfare analysis to study the aggregate and distributional impact of an arbitrary foreign shock. In Section 4, I apply the model to revisit the distributional impact of the China shock. Section 5 provides concluding remarks. The Appendix A contains auxiliary derivations and proofs. In the (Online) Appendix B, I discuss supplementary results for the paper.

## 2 Theory

In this theory part, I derive and discuss the main theoretical results of the paper. In particular, in Section 2.1, I describe the baseline environment and the supply side of the baseline

[5] It also complements the recent literature on trade and inequality and models with heterogeneous workers (see, e.g., Antràs, De Gortari, and Itskhoki 2017, Dix-Carneiro and Traiberman 2023, Helpman, Itskhoki, and Redding 2010, Sampson 2014, Yeaple 2005, etc.).

[6] By allowing for a non-homothetic demand side, it is also related to recent papers on structural change (see, e.g., Comin, Lashkari, and Mestieri 2021, Eckert and Peters 2022), spatial models with non-homotheticity (see, e.g., Couture, Gaubert, Handbury, and Hurst 2024), and the literature on inflation inequality (see, e.g., Argente and Lee 2021, Atkin, Faber, Fally, and Gonzalez-Navarro 2024, Cravino and Levchenko 2017, Jaravel 2021, etc.).

Ricardian-Roy model. Section 2.2 describes equilibrium conditions and comparative statics equations for an unspecified demand side. I introduce non-homothetic demand in Section 2.3. Derivations of counterfactual changes in the aggregate shares of consumption are presented in Section 2.4. Section 2.4.1 derives counterfactual changes for the mean income and income inequality.

## 2.1 Environment and the Supply side

While the demand side and its implications are the main novel parts of the framework, it is instructive to start with the supply side of the model that closely follows a standard Ricardian-Roy multi-sector quantitative trade model as in Galle, Rodríguez-Clare, and Yi 2023, which also relies on Eaton and Kortum (2002) and Roy (1951)-like frameworks (see, e.g., Burstein, Hanson, Tian, and Vogel 2020, Burstein, Morales, and Vogel 2019, Hsieh, Hurst, Jones, and Klenow 2019, Lagakos and Waugh 2013, etc.). I assume that there are *i*) $N$ countries indexed by $n, i \in \mathcal{N} \equiv \{1, 2, ..., N\}$, *ii*) $J$ sectors indexed by $j, k \in \mathcal{J} \equiv \{1, 2, ..., J\}$, and *iii*) $G_n$ country-specific local markets indexed by $g, r \in \mathcal{G}_n \equiv \{1, 2, ..., G_n\}$.

**Consumers** In each country $n \in \mathcal{N}$, consumers derive utility from consuming $J$ sectoral (aggregate) goods. Each sector $j \in \mathcal{J}$ resembles Eaton and Kortum (2002), and each consumer derives utility from consumption of a continuum of varieties with (sector-specific) constant elasticity of substitution $\sigma_j > 1$. While preferences within sectors can be aggregated to the standard CES consumption index, preferences across sectors are allowed to be non-homothetic.[7] As has been mentioned, deviation from the standard Cobb-Douglas case gives rise to two main methodological challenges. First, one has to reconcile heterogeneous consumers and non-homothetic demand with the exact-hat algebra approach (Dekle, Eaton, and Kortum 2008) for counterfactual analysis. Second, the well-known exact welfare change formula by Arkolakis, Costinot, and Rodríguez-Clare (2012) ceases to hold in such a setup, thereby requiring a more complex welfare analysis of trade shocks. I will address both of these methodological issues in Sections 2 and 3, respectively.

Trade is subject to iceberg costs, implying that for one unit of a variety from sector $j$ to arrive in country $n$, country $i$ must ship $\tau_{inj} \geq 1$ units. Technologies to produce a variety $\omega$ in each country $n$ and sector $j$ exhibit constant returns to scale with productivity $\varphi_{nj}$ drawn from the Fréchet distribution:

$$F_{nj}(\varphi) \equiv \Pr\left\{\varphi_{nj}(\omega) \leq \varphi\right\} = \exp\left(-T_{nj}\varphi^{-\theta_j}\right),$$

*where $\theta_j > \sigma_j - 1$ and $T_{nj} > 0$ are the distribution's shape and scale parameters, respectively.*

[7] The demand side will be discussed in detail in Section 2.3.

Note that $T_{nj}$ denotes the level of aggregate productivity of country $n$ in the production of goods in sector $j$. The parameter $\theta_j$ governs the sector-specific dispersion of productivities across goods within each country, where a higher value of $\theta_j$ implies less heterogeneity.

**Workers and Labor supply** Each country has its own set of labor markets $\mathcal{G}_n = \{1, 2, ..., G_n\}$. In the model's baseline version, I assume that labor markets are local, implying that workers are mobile across sectors but cannot migrate across labor markets. Each worker from labor market $g \in \mathcal{G}_n$ independently draws a number of efficiency units $z_{ngj}$ (or, equivalently, a productivity level) in each sector $j$ from the following Fréchet distribution:

$$\mathcal{F}_{ngj}(z) \equiv \Pr\{z_{ngj} \leq z\} = \exp\left(-A_{ngj} z^{-\alpha_{ng}}\right),$$

*where* $\alpha_{ng} > 1$ *and* $A_{ngj} > 0$ *are the shape and scale parameters of the corresponding distribution.*

The scale parameter $A_{ngj}$ governs the level of aggregate productivity of workers in sector $j$ in local market $g \in \mathcal{G}_n$. Note also that $\alpha_{ng}$ governs the dispersion of local productivities for $g \in \mathcal{G}_n$, where a higher value of $\alpha_{ng}$ corresponds to a smaller dispersion. In contrast to the baseline multi-sectoral extensions of Eaton and Kortum (2002), which feature a common wage per efficiency unit within a country (see, e.g., Caliendo and Parro 2015 and Costinot, Donaldson, and Komunjer 2012), the model implies that wages are country-sector specific.[8] In what follows, $\mathbf{w} \equiv \{w_{nj}\}_{n \in \mathcal{N}, j \in \mathcal{J}}$, where $w_{nj}$ denotes the *wage per efficiency unit* in sector $j$ of country $n$. Each worker from local market $g \in \mathcal{G}_n$ draws a vector of sectoral productivities $\boldsymbol{z} = (z_1, z_2, .., z_J)$ and makes a choice regarding which sector to apply for employment. For given wages, let $\Omega_{nj} \equiv \{\boldsymbol{z} \text{ s.t. } w_{nj} z_j \geq w_{nk} z_k \; \forall k \in \mathcal{J}\}$ denote the set of (sectoral) productivity vectors under which a worker ends up applying to sector $j$. One can readily verify that the share of workers in labor market $g \in \mathcal{G}_n$ who are employed in sector $j$ is given by[9]

$$\pi_{ngj} \equiv \int_{\Omega_{nj}} \mathrm{d}\mathcal{F}_{ng}(\boldsymbol{z}) = \frac{A_{ngj} w_{nj}^{\alpha_{ng}}}{\sum_{k=1}^{J} A_{ngk} w_{nk}^{\alpha_{ng}}}. \tag{2.1}$$

In what follows, $Y_{ng}$ and $L_{ng}$ denote labor market $g$'s total income and total amount of workers, respectively. So, given 2.1, one can readily verify that labor market's $g$'s total supply of efficiency units of labor in sector $j$ is given by

$$Z_{ngj} \equiv L_{ng} \int_{\Omega_{nj}} z_j \mathrm{d}\mathcal{F}_{ng}(\boldsymbol{z}) = \Gamma\left(1 - 1/\alpha_{ng}\right) L_{ng} A_{ngj} \left(\frac{w_{nj}}{\Upsilon_{ng}}\right)^{\alpha_{ng}-1},$$

[8] As noted by Galle, Rodríguez-Clare, and Yi (2023), sector-specific wage per efficiency unit is one of the implications of a Roy (1951)-like supply-side featuring an upward-sloping sectoral labor supply curve.

[9] See also Galle, Rodríguez-Clare, and Yi (2023).

*where* $\Upsilon_{ng} \equiv \left(\sum_{k=1}^{J} A_{ngk} w_{nk}^{\alpha_{ng}}\right)^{1/\alpha_{ng}}$ *and* $\Gamma(\cdot)$ *is the gamma function.*

Similarly, total income in labor market $g \in \mathcal{G}_n$ can be derived as:

$$Y_{ng} \equiv \sum_{j \in \mathcal{J}} w_{nj} Z_{ngj} = \Gamma\left(1 - 1/\alpha_{ng}\right) L_{ng} \Upsilon_{ng}.$$

While the share of workers in labor market $g \in \mathcal{G}_n$ who are employed in sector $j$ is given by $\pi_{ngj}$, the Fréchet distribution of worker productivity implies that the share of income earned by workers in local market $g \in \mathcal{G}_n$ in sector $j$ is also equal to $\pi_{ngj}$:

$$\pi_{ngj} = \frac{w_{nj} Z_{ngj}}{Y_{ng}}.$$

## 2.2 General equilibrium and Comparative statics

In this subsection, I derive the equilibrium and comparative statics equations. For expositional purposes, I leave the demand side unspecified and postpone the corresponding discussion until Section 2.3.

Note that the baseline version of the model assumes that there are no trade costs across local labor markets within a country. Hence, technologies and prices of goods are set at the country level. It also implies that while wages can differ across sectors, they are equal across local labor markets for a particular sector $j$.

One can readily infer that the excess demand for efficiency units of labor in sector $j$ of country $i$ is given by

$$\mathcal{D}_{ij} \equiv \underbrace{\frac{1}{w_{ij}} \sum_{n \in \mathcal{N}} \lambda_{inj} S_{nj} X_n}_{Demand} - \underbrace{\frac{1}{w_{ij}} \sum_{g \in \mathcal{G}_i} \pi_{igj} Y_{ig}}_{Supply}, \tag{2.2}$$

*where:*

– $\lambda_{inj} = \frac{T_{ij}(\tau_{inj} w_{ij})^{-\theta_j}}{\sum_{n'} T_{n'j}\left(\tau_{n'nj} w_{n'j}\right)^{-\theta_j}}$ *is the share of country* $i$ *in country* $n$*'s spending in sector* $j$*;*

– $X_n = Y_n + D_n$ *is the total expenditures of country* $n$ *with* $Y_n \equiv \sum_{g \in \mathcal{G}_n} Y_{ng}$ *and* $D_n$ *denoting its total income and trade imbalance (via transfers), respectively;*

– $S_{nj}$ *is the aggregate share of sector* $j$ *in expenditures of country* $n$*.*[10]

Note that $\frac{1}{w_{ij}} \sum_{n \in \mathcal{N}} \lambda_{inj} S_{nj} X_n$ represents the demand for efficiency units of labor of sector $j$ in country $i$. Similarly, $\sum_{g \in \mathcal{G}_i} Z_{igj}$ represents the supply of efficiency units of labor in sector $j$ in country $i$. Note also that each $\mathcal{D}_{ij}$ from (2.2) is a function of the $N \times J$ wage matrix

[10] As discussed later, the aggregate expenditure shares $\{S_{nj}\}$ are allowed to be non-constant and dependent on sectoral prices and the whole distribution of income across individual consumers.

$\mathbf{w} \equiv \{w_{ij}\}_{i\in\mathcal{N},j\in\mathcal{J}}$, with an arbitrary $w_{ij}$ chosen as a numéraire. So, there is a system of $N \times J$ equations: $\{\mathcal{D}_{ij}(\mathbf{w}) = 0\}_{i\in\mathcal{N},j\in\mathcal{J}}$.

### 2.2.1 Comparative statics

A counterfactual equilibrium analysis is of primary interest. For this purpose, one can solve the model in *relative changes* (the *exact-hat algebra* approach by Dekle, Eaton, and Kortum 2008). In what follows, $\hat{x} \equiv x'/x$ for any $x$, where $x'$ is the counterfactual value and $x$ is the current value. The salient advantage of this approach is that one does not need to identify current levels of the model's "fundamentals" like trade costs $\{\tau_{inj}\}$ and technology parameters $\{T_{nj}\}$, $\{A_{ngj}\}$.

One can readily verify that for each equilibrium condition, $\mathcal{D}_{ij}(\mathbf{w}) = 0$, its counterfactual counterpart, $\mathcal{D}'_{ij}(\mathbf{w}') = 0$, can be re-written in "relative changes" as follows:

$$\mathcal{D}'_{ij}(\hat{\mathbf{w}}) = 0 \Longleftrightarrow \sum_{n\in\mathcal{N}} \hat{\lambda}_{inj}\lambda_{inj}\hat{S}_{nj}S_{nj}\left(\sum_{g\in\mathcal{G}_n}\hat{Y}_{ng}Y_{ng} + \hat{D}_n D_n\right) = \sum_{g\in\mathcal{G}_i}\hat{\pi}_{igj}\pi_{igj}\hat{Y}_{ig}Y_{ig}, \tag{2.3}$$

*where:*

– $\hat{\lambda}_{inj} \equiv \frac{\lambda'_{inj}}{\lambda_{inj}} = \frac{\hat{T}_{ij}(\hat{\tau}_{inj}\hat{w}_{ij})^{-\theta_j}}{\sum_{n'}\lambda_{n'nj}\hat{T}_{n'j}\left(\hat{\tau}_{n'nj}\hat{w}_{n'j}\right)^{-\theta_j}}$ *is the relative change in country $i$'s trade share in country $n$'s imports of sector $j$;*

– $\hat{Y}_{ig} \equiv \frac{Y'_{ig}}{Y_{ig}} = \left(\sum_{j=1}^{J}\pi_{igj}\hat{A}_{igj}\hat{w}_{ij}^{\alpha_{ig}}\right)^{\frac{1}{\alpha_{ig}}}$ *is the relative change in total income of workers in local market $g$ in country $i$;*

– $\hat{\pi}_{igj} \equiv \frac{\pi'_{igj}}{\pi_{igj}} = \frac{\hat{A}_{igj}\hat{w}_{ij}^{\alpha_{ig}}}{\sum_k \pi_{igk}\hat{A}_{igk}\hat{w}_{ik}^{\alpha_{ig}}}$ *is the relative change in sector $j$'s employment share of workers in local market $g$ in country $i$.*

Note that (2.3) implies a system of $N \times J$ equations, $\left\{\mathcal{D}'_{ij}(\hat{\mathbf{w}}) = 0\right\}$, that solves for an equilibrium matrix of relative changes in country-sector-specific wages, $\hat{\mathbf{w}} \equiv \{\hat{w}_{ij}\}_{i\in\mathcal{N},j\in\mathcal{J}}$, subject to an arbitrary normalization (e.g., $\hat{w}_{11} = 1$).

The demand side of the model will imply that the aggregate consumption shares may change in response to shocks: $\hat{S}_{nj}S_{nj} \neq S_{nj}$. Note that in contrast to the standard Ricardian(–Roy) models (see, e.g., Galle, Rodríguez-Clare, and Yi 2023), individual expenditure shares of each consumer are now non-homothetic and allowed to depend on: i) sectoral price indices in country $n$, $\mathbf{P}_n \equiv (P_{n1}, P_{n2}, ..., P_{nJ})$, ii) the individual expenditure/income level $y$ of this consumer. As a result, the aggregate expenditure shares for each sector depend on sectoral prices and the whole distribution of income across individual consumers. Importantly, these non-homothetic expenditure patterns, apart from affecting equilibrium $\hat{\mathbf{w}} \equiv \{\hat{w}_{ij}\}_{i\in\mathcal{N},j\in\mathcal{J}}$ and the aggregate outcomes, also generate distributional consequences of any shock along the income distribution. The rest of this section derives the main results and discusses novel

implications driven by the non-homothetic demand side.

## 2.3 Non-homothetic demand

In the previous sections, the demand side has been intentionally left unspecified to disentangle the supply and demand forces. In this part of the paper, I introduce a demand system into the framework and derive expressions crucial for comparative static and welfare analysis. Specifically, I use the Almost Ideal Demand System (Deaton and Muellbauer 1980) across sectors. Apart from allowing non-homothetic effects, one of the salient advantages of this demand system is its flexibility.[11] In particular, as noted by Deaton and Muellbauer (1980), it gives an arbitrary first-order approximation to any demand system derived from utility-maximizing behavior.[12] So, its implications allow us to examine the general role of homothetic and non-homothetic effects in Ricardian(-Roy) models. Moreover, it will be shown that it nests several popular demand specifications in the literature as special cases.

The indirect utility of a consumer with income $y$ function is given by

$$\mathcal{V}(\mathbf{P}_n, y) = \ln\left(\left(\frac{y}{a(\mathbf{P}_n)}\right)^{\frac{1}{b(\mathbf{P}_n)}}\right), \tag{2.4}$$

*where:*

– $a(\mathbf{P}_n) \equiv \exp\left(\underline{a}_n + \sum_{j=1}^{J} a_{nj} \ln P_{nj} + \frac{1}{2}\sum_{j=1}^{J}\sum_{k=1}^{J} \gamma_{njk} \ln P_{nj} \ln P_{nk}\right)$ *is the homothetic price index (Translog);*

– $b(\mathbf{P}_n) \equiv \exp\left(\sum_{j=1}^{J} \beta_{nj} \ln P_{nj}\right)$ *is the non-homothetic component of preferences.*

For any $n \in \mathcal{N}$, restrictions on parameters are given by: *i)* $\sum_{j\in\mathcal{J}} a_{nj} = 1$; *ii)* $\sum_{j\in\mathcal{J}} \beta_{nj} = 0$; *iii)* $\sum_{k\in\mathcal{J}} \gamma_{njk} = 0\ \forall j \in \mathcal{J}$ and $\gamma_{njk} = \gamma_{nkj}\ \forall j, k \in \mathcal{J}$.

Applying Roy's identity to the indirect utility function yields the individual sectoral share of consumption for a consumer with income $y$ as follows[13]

$$s_{nj}(\mathbf{P}_n, y) = a_{nj} + \sum_{k=1}^{J} \gamma_{njk} \ln P_{nk} + \beta_{nj} \ln\left(\frac{y}{a(\mathbf{P}_n)}\right). \tag{2.5}$$

Note that parameters $\beta_{nj}$ are semi-elasticities of sectoral market shares with respect to

[11] The Almost Ideal Demand System also permits exact aggregation over consumers. In contrast to non-homothetic CES preferences (see, e.g., Fieler 2011, Comin, Lashkari, and Mestieri 2021), the presence of the representative consumer facilitates the exact-hat algebra approach for counterfactuals in a Ricardian(-Roy) model.

[12] Specifically, they show that the *price-independent generalized logarithmic* (PIGLOG) class of expenditure functions provides a first-order approximation to any cost function arising from utility maximization. The Almost Ideal Demand System itself is a specific parameterization within this class.

[13] Alternatively, one can apply Shephard's lemma to the expenditure function. The corresponding expenditure function is given by $\ln e(\mathbf{P}_n, u) = \ln a(\mathbf{P}_n) + u \cdot b(\mathbf{P}_n)$.

income. In particular, sectors with $\beta_{nj} > 0$ have positive income elasticities of individual market shares. Analogously, sectors with $\beta_{nj} < 0$ have negative income elasticities of individual market shares.

The supply side of the model implies that the (endogenous) income distribution in local labor market $g$ in country $n$ obeys the Fréchet distribution and is given by $H_{ng}(y) = \exp\left(-\left(y/\Upsilon_{ng}\right)^{-\alpha_{ng}}\right)$, where $\Upsilon_{ng} \equiv \left(\sum_{j=1}^{J} A_{ngj} w_{nj}^{\alpha_{ng}}\right)^{1/\alpha_{ng}}$. As a result, one can analytically derive the mean local income, $y_{ng} = Y_{ng}/L_{ng}$, and the Theil index, $\mathcal{T}_{ng}$, for a local market $g \in \mathcal{G}_n$:[14]

$$y_{ng} = \int_{\mathscr{Y}} y \, \mathrm{d}H_{ng}(y) = \Gamma\left(1 - 1/\alpha_{ng}\right) \left(\sum_{j=1}^{J} A_{ngj} w_{nj}^{\alpha_{ng}}\right)^{1/\alpha_{ng}}, \tag{2.6}$$

$$\mathcal{T}_{ng} = \int_{\mathscr{Y}} \frac{y}{y_{ng}} \ln\left(\frac{y}{y_{ng}}\right) \mathrm{d}H_{ng}(y) = \ln\left(\Gamma\left(1 - 1/\alpha_{ng}\right)\right)^{-1} - \frac{\psi\left(1 - 1/\alpha_{ng}\right)}{\alpha_{ng}}, \tag{2.7}$$

*where $\Gamma(\cdot)$ is the gamma function, and $\psi(\cdot) \equiv \Gamma'(\cdot)/\Gamma(\cdot)$ is the digamma function.*

Now, let $H_n(\cdot)$ denote the c.d.f. of income in country $n$. This distribution is continuous and represents a (finite) mixture distribution. As a result, aggregating across heterogeneous consumers and exploiting the Theil index decomposition properties yield the aggregate expenditure shares of the following form:

**Proposition 1.** *The aggregate expenditure share of sector $j$ in country $n$ is given by:*

$$S_{nj} \equiv \int_{\mathscr{Y}} \frac{y}{y_n} s_{nj}(\mathbf{P}_n, y) \, \mathrm{d}H_n(y) = s_{nj}(\mathbf{P}_n, \, y_n \cdot \exp(\mathcal{T}_n)), \tag{2.8}$$

*where country $n$'s mean income, $y_n$, and the Theil index of the income distribution, $\mathcal{T}_n$, are given by:*

$$y_n \equiv \int_{\mathscr{Y}} y \, \mathrm{d}H_n(y) = \sum_{g \in \mathcal{G}_n} \ell_{ng} y_{ng},$$

$$\mathcal{T}_n \equiv \int_{\mathscr{Y}} \frac{y}{y_n} \ln\left(\frac{y}{y_n}\right) \mathrm{d}H_n(y) = \sum_{g \in \mathcal{G}_n} \ell_{ng} \frac{y_{ng}}{y_n} \mathcal{T}_{ng} + \sum_{g \in \mathcal{G}_n} \ell_{ng} \frac{y_{ng}}{y_n} \ln\left(\frac{y_{ng}}{y_n}\right).$$

*Proof.* See Appendix A.1. □

In formula (2.8), $y_n \cdot \exp(\mathcal{T}_n)$ represents the "inequality-adjusted" mean income for country $n$. It implies that the aggregate expenditure shares are equal to the expenditure shares of an "inequality-adjusted representative consumer," in the sense that this representative consumer has mean income, adjusted by the (country-level) Theil index of income inequality. Proposition 1 states that if there is any level of income heterogeneity, then the aggregate expenditure

[14] The Lorenz curve for a local market $g \in \mathcal{G}_n$ is given by $\mathscr{L}_{ng}(u) = \frac{\int_0^u (-\ln t)^{-1/\alpha_{ng}} \mathrm{d}t}{\Gamma(1-1/\alpha_{ng})}$, where $u$ denotes a particular quantile.

shares of high-income elastic sectors are higher than in the case of no income inequality (i.e., $\mathcal{T}_n = 0$). Analogously, aggregate shares of low-income elastic sectors are lower than in the income homogeneity scenario.

As Proposition 1 also states, the country-level Theil index can be represented as the sum of its *within* and *between* components:

$$\mathcal{T}_n = \underbrace{\sum_{g \in \mathcal{G}_n} \ell_{ng} \frac{y_{ng}}{y_n} \mathcal{T}_{ng}}_{\mathcal{T}_{n,\,within}} + \underbrace{\sum_{g \in \mathcal{G}_n} \ell_{ng} \frac{y_{ng}}{y_n} \ln \frac{y_{ng}}{y_n}}_{\mathcal{T}_{n,\,between}}, \tag{2.9}$$

*where $\ell_{ng} \equiv L_{ng}/L_n$ is the labor share, and $y_{ng}$ is the mean income of a local labor market $g$.*

This decomposition implies that the country-level Theil index is given by the (weighted) mean inequality within each local labor market plus the inequality between local labor markets. Hence, given $\mathcal{T}_{ng}$ and data on $y_{ng}$ and $\ell_{ng}$ for all $g \in \mathcal{G}_n$, one can readily infer the country-level Theil index, $\mathcal{T}_n$.

## 2.4 Counterfactual changes in the aggregate shares of consumption

In this part, I provide an analysis of counterfactual changes in the aggregate shares of consumption. In particular, to solve the system (2.3) for the equilibrium wage matrix of relative changes, $\hat{\mathbf{w}} \equiv \{\hat{w}_{ij}\}$, we need to derive counterfactual values of aggregate sectoral shares $S'_{nj} \equiv \hat{S}_{nj} S_{nj}$. To proceed with the analysis, it is convenient to represent each counterfactual aggregate expenditure share as the sum of its current sectoral share and an absolute change $S'_{nj} \equiv \Delta S_{nj} + S_{nj}$, where $\Delta S_{nj} \equiv S'_{nj} - S_{nj}$. Recall that the current levels of the aggregate consumption shares $S_{nj}$ are available from the data. So, for counterfactual predictions, we need to derive $\Delta S_{nj} \equiv S'_{nj} - S_{nj}$ for any $n \in \mathcal{N}$ and $j \in \mathcal{J}$.

Let $\hat{y}_n \equiv y'_n/y_n$ denote the relative change in the country-level mean income and $\Delta \mathcal{T}_n \equiv \mathcal{T}'_n - \mathcal{T}_n$ denote the difference between counterfactual and actual values of the country-level Theil index. One can readily verify that

$$\Delta S_{nj} = \underbrace{\sum_{k=1}^{J} \gamma_{njk} \ln \hat{P}_{nk}}_{homothetic} + \underbrace{\beta_{nj} \ln \left( \frac{\hat{y}_n \cdot \exp(\Delta \mathcal{T}_n)}{\hat{a}_n} \right)}_{non-homothetic}, \tag{2.10}$$

*where $\hat{P}_{nk} = \hat{w}_{nk} \hat{\lambda}_{nnk}^{1/\theta_k} / \hat{T}_{nk}^{1/\theta_k}$ for any sector $k \in \{1, .., J\}$.*

As one can see from (2.10), any changes in aggregate expenditure shares can be decomposed into homothetic and non-homothetic parts. In particular, $\beta_{nj} \ln (\hat{y}_n \cdot \exp(\Delta \mathcal{T}_n)/\hat{a}_n)$ represents a change in $S_{nj}$ driven by non-homotheticity. One can readily verify that if $\beta_{nj} = 0$

for any $n \in \mathcal{N}$ and $j \in \mathcal{J}$, then changes in aggregate sectoral shares of consumption may still change due to homothetic forces captured by changes in relative sectoral prices. Note also that if we additionally impose that $\gamma_{njk} = 0$ for any $n \in \mathcal{N}$ and $j, k \in \mathcal{J}$, then the model reduces to the basic Cobb-Douglas case, implying that aggregate expenditure shares do not change in any counterfactual scenario (i.e., $\Delta S_{nj} = 0$ for any $n \in \mathcal{N}$ and $j \in \mathcal{J}$).

Now, a matter of primary interest for the exact-hat algebra counterfactual analysis is deriving tractable expressions for relative changes in the inequality-adjusted mean income, $y_n \cdot \exp(\mathcal{T}_n)$, and in the homothetic price index, $\hat{a}_n$. In particular, it can be shown that the relative change in the homothetic price index component, $\ln \hat{a}_n$, can be represented as:

$$\ln \hat{a}_n \equiv \ln(a'_n/a_n) = \sum_{k=1}^{J} S_{nk} \ln \hat{P}_{nk} + \sum_{k=1}^{J} \frac{\Delta S_{nk}}{2} \ln \hat{P}_{nk}. \tag{2.11}$$

Combining (2.11) with (2.10), I derive counterfactual absolute changes in the aggregate expenditure shares in a matrix form.

**Proposition 2.** *Given the matrix of relative changes in wages, $\hat{\mathbf{w}} \equiv \{\hat{w}_{ij}\}$, counterfactual changes in the aggregate sectoral expenditure shares, $\mathbf{\Delta S}_n \equiv (\Delta S_{n1}, \Delta S_{n2}, ..., \Delta S_{nJ})^T$, in country $n$ are given by:*

$$\mathbf{\Delta S}_n = (\text{diag}(\mathbf{1}) + \mathbf{\Omega}_n)^{-1} \left( \tilde{\mathbf{P}}_n + \tilde{\mathbf{b}}_n \right), \tag{2.12}$$

*where:*

– $\text{diag}(\mathbf{1})$ *is the identity matrix of size $J$;*

– $\tilde{\mathbf{P}}_n$ *is a $J \times 1$ vector with its $j$-th element given by $\tilde{P}_{nj} = \sum_k \gamma_{njk} \ln \hat{P}_{nk}$;*

– $\tilde{\mathbf{b}}_n$ *is a $J \times 1$ vector with its $j$-th element given by $\tilde{b}_{nj} = \beta_{nj} \left( \ln \hat{y}_n + \Delta \mathcal{T}_n - \sum_k S_{nk} \ln \hat{P}_{nk} \right)$;*

– $\mathbf{\Omega}_n \equiv \begin{pmatrix} \frac{\beta_{n1}}{2} \ln \hat{P}_{n1} & ... & \frac{\beta_{n1}}{2} \ln \hat{P}_{nJ} \\ \vdots & \ddots & \vdots \\ \frac{\beta_{nJ}}{2} \ln \hat{P}_{n1} & ... & \frac{\beta_{nJ}}{2} \ln \hat{P}_{nJ} \end{pmatrix}$ *is a country-specific $J \times J$ matrix of cross-interactions between sectoral (semi-)elasticities and changes in sectoral prices.*

Proposition 2 demonstrates that solving for changes in the aggregate expenditure shares for each country $n$ boils down to solving a country-specific linear system. It is also straightforward to see that (2.12) nests counterfactual changes in the aggregate consumption shares

under the homothetic Translog expenditure function and fixed (Cobb-Douglas) sectoral expenditure shares. It is instructive to discuss both of these special cases separately.

**Special Case 1 (Homothetic Translog expenditure function)** The case of the Translog expenditure function arises when $\{\beta_{nj}\} = 0$ and implies no cross-interactions between sectoral (semi-)elasticities and changes in sectoral prices (i.e., $\mathbf{\Omega}_n = \mathbf{0}_{J\times J}$) and no interactions between sectoral (semi-)elasticities and the inequality-adjusted mean income (i.e., $\tilde{\mathbf{b}}_n = \mathbf{0}_{J\times 1}$). As a result, the formula (2.12) reduces to $\mathbf{\Delta S}_n = \tilde{\mathbf{P}}_n$, implying that changes in the aggregate expenditure shares in any country $n$ do not depend on the distribution of income across consumers and are driven only by relative changes in sectoral homothetic price indices and substitution patterns across them captured by the matrix $\{\gamma_{njk}\}$.

**Special Case 2 (Cobb-Douglas expenditure shares)** Another special case arises when, in addition to the restrictions imposed by the Translog case, each element of the matrix of substitution patterns across sectoral price indices is zero (i.e., $\gamma_{njk} = 0$ for any $n \in \mathcal{N}$ and $j, k \in \mathcal{J}$). In this case $\tilde{\mathbf{P}}_n = \mathbf{0}_{J\text{x}1}$, implying also that $\mathbf{\Delta S}_n = \mathbf{0}_{J\times 1}$. It can readily be inferred that this corresponds to the standard case of constant aggregate expenditure shares, implying Cobb-Douglas preferences across sectors as in Galle, Rodríguez-Clare, and Yi (2023).

### 2.4.1 Counterfactual changes in income inequality and mean income

This subsection provides details on counterfactual changes in the inequality-adjusted mean income. A direct corollary of Proposition 1 is that the impact of any changes in the distribution of income within a country on the aggregate sectoral expenditure shares is leveraged by changes in the inequality-adjusted mean income. Recall that the inequality-adjusted mean income is given by $y_n \cdot \exp(\mathcal{T}_n)$. So, its relative change is given by $\hat{y}_n \cdot \exp(\Delta\mathcal{T}_n)$. It suggests that any trade or technology shock may affect both mean income and inequality. It is straightforward to show that the relative change in the country-level mean income, $\hat{y}_n$, is given by

$$\hat{y}_n \equiv \frac{y_n'}{y_n} = \sum_{g\in\mathcal{G}_n} \frac{y_{ng}}{y_n}\hat{Y}_{ng}, \tag{2.13}$$

*where* $\hat{Y}_{ng} = \left(\sum_{j=1}^{J} \pi_{ngj}\hat{A}_{ngj}\hat{w}_{nj}^{\alpha_{ng}}\right)^{\frac{1}{\alpha_{ng}}}$ *is the relative change in total income of workers in local market* $g$ *in country* $n$ *(given fixed* $L_{ng}$*, it also coincides with* $\hat{y}_{ng}$*).*

Note that, given $L_{ng}$ fixed, $\hat{Y}_{ng}$ also coincides with the relative change in mean income ($\hat{y}_{ng}$) of workers in local market $g$ in country $n$. Now, recall from (2.7) that the local Theil index depends exclusively on $\alpha_{ng}$ and can be analytically expressed via the *gamma* and *digamma*

functions as $\mathcal{T}_{ng} = \ln\left(\Gamma\left(1-1/\alpha_{ng}\right)\right)^{-1} - \frac{\psi(1-1/\alpha_{ng})}{\alpha_{ng}}$. Given that $\alpha_{ng}$ is constant, the local Theil index does not change in a counterfactual scenario, $\mathcal{T}'_{ng} = \mathcal{T}_{ng}$ for any $g \in \mathcal{G}_n$. Yet, in the case of many local labor markets, the country-level Theil index is subject to change in a counterfactual equilibrium due to changes in both the *within* and *between* components. So, in general, $\mathcal{T}'_n \neq \mathcal{T}_n$, as the following proposition states.

**Proposition 3.** *A counterfactual value of the country-level Theil index is given by:*

$$\mathcal{T}'_n = \underbrace{\sum_{g\in\mathcal{G}_n} \frac{\hat{Y}_{ng}Y_{ng}}{\hat{Y}_n Y_n}\mathcal{T}_{ng}}_{\mathcal{T}'_{n,\,within}} + \underbrace{\sum_{g\in\mathcal{G}_n} \frac{\hat{Y}_{ng}Y_{ng}}{\hat{Y}_n Y_n} \ln\left(\frac{\hat{y}_{ng}y_{ng}}{\hat{y}_n y_n}\right)}_{\mathcal{T}'_{n,\,between}}. \tag{2.14}$$

*where*

- $\mathcal{T}_{ng}$ *is a current value of the Theil index in local market* $g$ *in country* $n$*;*
- $\mathcal{T}'_{n,\,within}$ *denotes a counterfactual "within" component of the Theil index in country* $n$*;*
- $\mathcal{T}'_{n,\,between}$ *denotes a counterfactual "between" component of the Theil index in country* $n$*;*
- $\hat{Y}_n \equiv Y'_n/Y_n = \sum_{g\in\mathcal{G}_n} \frac{Y_{ng}}{Y_n}\hat{Y}_{ng}$ *is the relative change in total income in country* $n$*.*

*Proof.* See Appendix A.2. □

Note that the expression (2.14) makes it clear that if we consider only one labor market $g$ for a country $n$, then the country-level Theil index is necessarily left unchanged in any counterfactual scenario, $\mathcal{T}'_n = \mathcal{T}_n$. Thus, apart from non-homotheticities, enriching a Ricardian-Roy model with the spatial dimension gives rise to the inequality margin of adjustment to trade shocks. This change in the Theil index, $\Delta\mathcal{T}_n$, translates to an additional source of Consumption Inequality between sectors through the non-homothetic mechanism.

**Data requirements for counterfactual analysis based on the exact-hat algebra** To perform counterfactual analysis, one needs to solve for an equilibrium matrix of relative changes in country-sector-specific wages $\hat{\mathbf{w}} \equiv \{\hat{w}_{nj}\}_{n\in\mathcal{N},j\in\mathcal{J}}$ implicitly defined by (2.3). To solve for it, one needs structural parameters $\{\alpha_{ng}, \theta_j, \gamma_{njk}, \beta_{nj}\}$ as well as data on:

*i) sectoral expenditure, trade, and employment shares* $\{S_{nj}, \lambda_{inj}, \pi_{ngj}\}$*;*
*ii) total income and employment levels across local labor markets* $\{Y_{ng}, L_{ng}\}$*;*
*iii) current levels of trade imbalances and their relative changes* $\{D_n, \hat{D}_n\}$*;*
*iv) relative changes in (trade cost and technology) fundamentals* $\{\hat{\tau}_{inj}, \hat{T}_{nj}, \hat{A}_{ngk}\}$*.*

Given an equilibrium matrix $\hat{\mathbf{w}} \equiv \{\hat{w}_{nj}\}$, one can readily restore the other endogenous variables of interest and make welfare predictions. Note that the solution for the changes in the model's endogenous variables does not require information on the level of fundamentals $\boldsymbol{T} \equiv \{T_{nj}\}$, $\boldsymbol{A} \equiv \{A_{ngj}\}$, and $\boldsymbol{\tau} \equiv \{\tau_{nij}\}$. Surprisingly, the data requirements to

solve for a counterfactual equilibrium are almost the same as in the standard case of constant (Cobb-Douglas) sectoral consumption shares.[15] In particular, given structural parameters $\{\alpha_{ng}, \theta_j, \gamma_{njk}, \beta_{nj}\}$, the only additional data one needs is information on the local-market-specific total amount of labor, $\{L_{ng}\}$. Data on $\{L_{ng}\}$ are used to recover $\{y_{ng}\}$ that, in turn, allow us to derive counterfactual values of the country-level mean income and the Theil index.

# 3 Welfare Analysis

In this Section, I provide the main results of the welfare analysis. Given a non-homothetic demand side, the analysis based on the exact price index cannot generally be applied. So, I use the *Equivalent Variation* ($EV$) as a measure of welfare changes associated with changes in prices and income.[16] It represents the amount of additional income, given the current prices, that would have the same effect on consumer welfare as if the economic environment had changed. Formally, the Equivalent Variation is defined by the following condition:

$$\frac{\partial \ln \mathcal{V}_n}{\partial \ln y}\frac{EV(y)}{y} = \frac{\partial \ln \mathcal{V}_n}{\partial \ln y}\mathrm{d}\ln y + \sum_{j=1}^{J}\frac{\partial \ln \mathcal{V}_n}{\partial \ln P_{nj}}\mathrm{d}\ln P_{nj}, \tag{3.1}$$

where the right-hand side of the equation above is obtained by totally differentiating the indirect utility function $\mathcal{V}_n(\mathbf{P}_n, y)$ of a consumer with individual income $y$ in country $n$.

In what follows, let $\mathrm{d}\ln \mathcal{W}_n(y)$ denote the corresponding change in the welfare measure for an individual with income level $y$. One can readily infer that rearranging terms in (3.1) and applying Roy's identity directly yields

$$\mathrm{d}\ln \mathcal{W}_n(y) \equiv \frac{EV(y)}{y} = \mathrm{d}\ln y - \sum_{j=1}^{J} s_{nj}(\mathbf{P}_n, y)\mathrm{d}\ln P_{nj}. \tag{3.2}$$

Note that (3.2) leaves the demand and supply features of the model unspecified. However, to make counterfactual predictions on the distributional impacts of trade or technology shocks, one needs to infer $\mathrm{d}\ln y$ and $\mathrm{d}\ln P_{nj}$ based on the model's structure. In the next subsection, I propose a generalized welfare change formula based on the supply-side properties of the Ricardian-Roy model.

[15] Note that one also does not need to estimate some structural demand parameters of the model: $\{\underline{a}_n\}$ and $\{a_{nj}\}$.

[16] It is common in the literature to exploit the Equivalent Variation as a measure of welfare changes under non-homothetic demand (see, e.g., Fajgelbaum and Khandelwal 2016, Borusyak and Jaravel 2021, etc.).

## 3.1 The distributional welfare impact of shocks: a generalized formula

In this subsection, I derive a generalized welfare change formula for the distributional impact of trade that applies to any demand system across sectors. It combines the Ricardian- and Roy-like terms and also gives rise to a novel – *Consumption Inequality* – channel of the aggregate impact of trade or technology shocks.

The formula is amenable to examining the distributional impact of any *foreign/non-local* shock. In the spirit of Arkolakis, Costinot, and Rodríguez-Clare (2012), I first introduce a formal definition of these shocks using $\boldsymbol{L} \equiv \{L_{ng}\}$, $\boldsymbol{T} \equiv \{T_{nj}\}$, $\boldsymbol{A} \equiv \{A_{ngj}\}$, and $\boldsymbol{\tau} \equiv \{\tau_{nij}\}$.

**Definition 1.** A foreign/non-local shock for a labor market $g$ in country $n$ is given by any change from $(\boldsymbol{L}, \boldsymbol{T}, \boldsymbol{A}, \boldsymbol{\tau})$ to $(\boldsymbol{L'}, \boldsymbol{T'}, \boldsymbol{A'}, \boldsymbol{\tau'})$ such that $L_{ng} = L'_{ng}$ and $\{T_{nj}, A_{ngj}, \tau_{nnj}\}_{j\in\mathcal{J}} = \{T'_{nj}, A'_{ngj}, \tau'_{nnj}\}_{j\in\mathcal{J}}$.

The definition above imposes mild restrictions on the nature of a shock. In particular, it implies that a shock is foreign/non-local in the sense that it does not affect: *i*) labor endowments of the local labor market, *ii*) the domestic country's technological levels, *iii*) the local labor market's technological levels, and *iv*) the domestic country's ability to serve its own market.[17] The proposition below states the welfare impact of such a shock on a particular group of individuals with a common income level $y$.

**Proposition 4.** *Consider any foreign/non-local shock. A first-order approximation to the welfare change of income-$y$ individuals in local market $g$ in country $n$ is given by*

$$\hat{\mathcal{W}}_{ng}(y) \approx \underbrace{\prod_{j\in\mathcal{J}} \hat{\lambda}_{nnj}^{-s_{ngj}(y)/\theta_j}}_{\text{consumer Ricardian-type gains}} \cdot \underbrace{\prod_{j\in\mathcal{J}} \hat{\pi}_{ngj}^{-s_{ngj}(y)/\alpha_{ng}}}_{\text{consumer Roy-type gains}}, \tag{3.3}$$

*where* $\hat{\lambda}_{nnj} = \frac{\hat{w}_{nj}^{-\theta_j}}{\sum_{n'} \lambda_{n'nj} \hat{T}_{n'j} \left(\hat{\tau}_{n'nj} \hat{w}_{n'j}\right)^{-\theta_j}}$, $\hat{\pi}_{ngj} = \frac{\hat{w}_{nj}^{\alpha_{ng}}}{\sum_k \pi_{ngk} \hat{w}_{nk}^{\alpha_{ng}}}$, *with relative changes in wages,* $\hat{\mathbf{w}} \equiv \{\hat{w}_{nj}\}_{n\in\mathcal{N}, j\in\mathcal{J}}$, *implicitly given by the solution of (2.3).*

*Proof.* See Appendix A.3. □

As one can see, this parsimonious formula applies to any demand system across sectors. Recall that, depending on preferences,$\{s_{ngj}(y)\}$ may take different forms.[18] Below, I provide

[17] Note also that, to examine welfare change implications aggregated to the country level, one needs to additionally impose the assumption that $L_{ng} = L'_{ng}$ for any $g \in \mathcal{G}_n$.

[18] Note that Proposition 4 also allows for a further decomposition of the Ricardian- and Roy-type gains into the individual, local, and aggregate components (see Appendix B.1).

an alternative representation of this Proposition to better grasp the intuition of the distributional effects at play.

**Proposition 5** (Alternative (decomposition) representation)**.** *Consider any foreign/non-local shock. A first-order approximation to the welfare change of income-$y$ individuals in local market $g$ in country $n$ is given by*

$$\ln \hat{\mathcal{W}}_{ng}(y) \approx \mathbb{E}_j \left[ \ln \hat{\lambda}_{nnj}^{\frac{-1}{\theta_j}} \right] + \mathbb{E}_j \left[ \ln \hat{\pi}_{ngj}^{\frac{-1}{\alpha_{ng}}} \right] + J \cdot \left( Cov_j \left[ \ln \hat{\lambda}_{nnj}^{\frac{-1}{\theta_j}}, s_{ngj}(y) \right] + Cov_j \left[ \ln \hat{\pi}_{ngj}^{\frac{-1}{\alpha_{ng}}}, s_{ngj}(y) \right] \right), \quad (3.4)$$

*where:*

***i)*** $\mathbb{E}_j \left[ \ln \hat{\lambda}_{nnj}^{-\frac{1}{\theta_j}} \right] \equiv \frac{1}{J} \sum_j \ln \hat{\lambda}_{nnj}^{-\frac{1}{\theta_j}}$ *controls for the impact of changes in trade openness across sectors in country $n \in \mathcal{N}$;*

***ii)*** $\mathbb{E}_j \left[ \ln \hat{\pi}_{ngj}^{-\frac{1}{\alpha_{ng}}} \right] \equiv \frac{1}{J} \sum_j \ln \hat{\pi}_{ngj}^{-\frac{1}{\alpha_{ng}}}$ *controls for the impact of change in sectoral specialization of a local market $g \in \mathcal{G}_n$ in country $n \in \mathcal{N}$;*

***iii)*** $Cov_j \left[ \ln \hat{\lambda}_{nnj}^{\frac{-1}{\theta_j}}, s_{ngj}(y) \right]$ *is the covariance between changes in trade openness of a country $n \in \mathcal{N}$ across sectors and individual expenditure shares;*

***iv)*** $Cov_j \left[ \ln \hat{\pi}_{ngj}^{\frac{-1}{\alpha_{ng}}}, s_{ngj}(y) \right]$ *is the covariance between changes in the specialization pattern of a local market $g \in \mathcal{G}_n$ across sectors and individual expenditure shares.*

The proof is straightforward given Proposition 4. It is instructive to discuss each of the terms separately.

**i**) First, consider the term $\mathbb{E}_j \left[ \ln \hat{\lambda}_{nnj}^{-1/\theta_j} \right]$ that determines the component of welfare gains from an increase in a country's trade openness that is uncorrelated with the sectoral consumption profile $\{s_{ngj}(y)\}$ of income–$y$ individuals. In other words, this term represents the standard ACR (2012) term for welfare gains, stating that a country tends to benefit from increased trade openness across sectors. Note that this term can be tractably unpacked and decomposed into: **a)** the *Average Openness effect* , **b)** the *Specialization effect*, and **c)** the *Alignment effect*:

$$\mathbb{E}_j \left[ \ln \hat{\lambda}_{nnj}^{-\frac{1}{\theta_j}} \right] = \underbrace{\mathbb{E}_j \left[ \frac{1}{\theta_j} \right] \cdot \ln \left( \frac{\bar{\lambda}_{nn}}{\bar{\lambda}'_{nn}} \right)}_{Average\ Openness} + \underbrace{\mathbb{E}_j \left[ \frac{1}{\theta_j} \right] \cdot \left( \mathrm{MLD}_j \left( \lambda'_{nnj} \right) - \mathrm{MLD}_j \left( \lambda_{nnj} \right) \right)}_{Specialization} + \underbrace{Cov_j \left[ \frac{1}{\theta_j}, \ln \left( \frac{\lambda_{nnj}}{\lambda'_{nnj}} \right) \right]}_{Alignment}, \quad (3.5)$$

where $\bar{\lambda}'_{nn} \equiv \frac{1}{J} \sum_j \lambda'_{nnj}$, $\bar{\lambda}_{nn} \equiv \frac{1}{J} \sum_j \lambda_{nnj}$, while $\mathrm{MLD}_j \left( \lambda'_{nnj} \right) = \ln \bar{\lambda}'_{nn} - \frac{1}{J} \sum_j \ln \lambda'_{nnj}$ and $\mathrm{MLD}_j \left( \lambda_{nnj} \right) = \ln \bar{\lambda}_{nn} - \frac{1}{J} \sum_j \ln \lambda_{nnj}$ are the *mean log deviations* of $\lambda'_{nnj}$ and $\lambda_{nnj}$, respectively.

**The *Average Openness* effect** compares the average shares of expenditure on domestic goods in the factual and counterfactual scenarios, adjusted by the inverse of the average sectoral trade elasticities. This effect captures the fact that it is beneficial for a country to have a lower average share of expenditure on domestic goods in a counterfactual scenario, $\bar{\lambda}'_{nn} < \bar{\lambda}_{nn}$, regardless of the nature of the shock. This is the dominant term because one can readily infer that it is still present even in the case of "uniform" trade liberalization ($\hat{\lambda}_{nnj} = \text{const}$ for all $j \in \mathcal{J}$) and when all trade elasticities are identical across sectors ($\theta_j = \theta$ for all $j \in \mathcal{J}$).

**The *Specialization* effect** compares the mean log deviations of the shares of expenditure on domestic goods between the counterfactual and factual scenarios. It captures whether trade openness across sectors becomes more uneven, or, equivalently, whether a country's specialization becomes stronger in response to a shock. As in the previous case, the effect is multiplied by the inverse of the average sectoral trade elasticities. Note that for "uniform" trade liberalization ($\hat{\lambda}_{nnj} = \text{const}$ for all $j \in \mathcal{J}$) this effect vanishes.

**The *Alignment* effect** captures whether more heterogeneous sectors (i.e., sectors with lower $\theta_j$) tend to experience larger changes in trade openness (i.e., higher values of $1/\hat{\lambda}_{nnj}$) following a shock. In other words, gaining easier access to foreign varieties in more differentiated sectors magnifies gains from trade at the country level. Note that this term vanishes if $\theta_j = \theta$ for all $j \in \mathcal{J}$.

**ii**) Second, consider the role of the term $\mathbb{E}_j\left[\ln \hat{\pi}_{ngj}^{-1/\alpha_{ng}}\right]$ that operates at the local level and affects all local individuals uniformly. It captures the change in specialization within local market $g \in G_n$. To see this explicitly, note that

$$\mathbb{E}_j\left[\ln \hat{\pi}_{ngj}^{-1/\alpha_{ng}}\right] = \frac{D_{KL}\left(\pi^{\mathcal{U}}_{ng} \parallel \pi'_{ng}\right) - D_{KL}\left(\pi^{\mathcal{U}}_{ng} \parallel \pi_{ng}\right)}{\alpha_{ng}}, \tag{3.6}$$

where $D_{KL}\left(\pi^{\mathcal{U}}_{ng} \parallel \pi'_{ng}\right) \equiv \sum_j \pi^{\mathcal{U}}_{ngj} \ln\left(\frac{\pi^{\mathcal{U}}_{ngj}}{\pi'_{ngj}}\right)$ denotes the Kullback–Leibler (KL) divergence from $\pi'_{ng} \equiv \left(\pi'_{ng1}, \pi'_{ng2}, .., \pi'_{ngJ}\right)$ to the vector $\pi^{\mathcal{U}}_{ng} \equiv \left(\pi^{\mathcal{U}}_{ng1}, \pi^{\mathcal{U}}_{ng2}, .., \pi^{\mathcal{U}}_{ngJ}\right)$ corresponding to the uniform distribution: $\pi^{\mathcal{U}}_{ngj} = 1/J$ for any $j \in \mathcal{J}$. Similarly, $D_{KL}\left(\pi^{\mathcal{U}}_{ng} \parallel \pi_{ng}\right) \equiv \sum_j \pi^{\mathcal{U}}_{ngj} \ln\left(\frac{\pi^{\mathcal{U}}_{ngj}}{\pi_{ngj}}\right)$ denotes the KL divergence from $\pi_{ng} \equiv (\pi_{ng1}, \pi_{ng2}, .., \pi_{ngJ})$ to the vector $\pi^{\mathcal{U}}_{ng}$ .

As a result, conditioning on the consumption/income profile, individuals within a given local market tend to benefit if the labor market becomes more specialized:

$$\mathbb{E}_j\left[\ln \hat{\pi}_{ngj}^{-\frac{1}{\alpha_{ng}}}\right] > 0 \Longleftrightarrow D_{KL}\left(\pi^{\mathcal{U}}_{ng} \parallel \pi'_{ng}\right) > D_{KL}\left(\pi^{\mathcal{U}}_{ng} \parallel \pi_{ng}\right).$$

Alternatively, one can use the notion of a geometric mean to demonstrate that increas-

ing a labor market's specialization tend to uniformly benefit its individuals. Specifically, let $GM_j(\pi_{ng}) \equiv \left(\prod_{j\in\mathcal{J}} \pi_{ngj}\right)^{1/J}$ and $GM_j(\pi'_{ng}) \equiv \left(\prod_{j\in\mathcal{J}} \pi'_{ngj}\right)^{1/J}$ denote the corresponding geometric means. Thus, one may obtain $\mathbb{E}_j\left[\ln \hat{\pi}_{ngj}^{-1/\alpha_{ng}}\right] = \frac{1}{\alpha_{ng}} \ln \frac{GM_j(\pi_{ng})}{GM_j(\pi'_{ng})}$. The properties of the geometric mean imply that it falls toward zero as the labor market concentrates. So, similar to the case of the KL divergence decomposition, individuals within a given local market tend to benefit if the labor market becomes more specialized, as captured by a lower value of the geometric mean:

$$\mathbb{E}_j\left[\ln \hat{\pi}_{ngj}^{-\frac{1}{\alpha_{ng}}}\right] > 0 \Longleftrightarrow GM_j(\pi_{ng}) > GM_j(\pi'_{ng}).$$

Note also from both approaches imply that the size of the aggregate effect is adjusted by the factor of skill heterogeneity $1/\alpha_{ng}$, implying that the total effect of increasing a labor market's specialization on its aggregate welfare gains is larger if workers are more heterogeneous in their productivity (i.e., low $\alpha_{ng}$).

**iii**) Third, consider the term $Cov_j\left[\ln \hat{\lambda}_{nnj}^{-1/\theta_j}, s_{ngj}(y)\right]$ that determines the component of welfare gains from an increase in a country's trade openness that is correlated with the sectoral consumption profile $\{s_{ngj}(y)\}$ of income–$y$ individuals. It simply states that a group of consumers with a particular level of income is going to benefit from a shock if it results in higher trade openness in sectors that have relatively larger shares in the group's consumption basket.

**iv**) Last, consider the term $Cov_j\left[\ln \hat{\pi}_{ngj}^{-1/\alpha_{ng}}, s_{ngj}(y)\right]$, which captures the component of welfare gains based on the correlation of the sectoral consumption profile of income–$y$ individuals with the pattern of changes in labor market $g$'s specialization. In particular, it states that consumers tend to benefit from the shock if they consume relatively more in sectors that experience a relative decline in local employment.

## 3.2 A novel channel of the aggregate welfare gains

Compared to the case of Cobb-Douglas or any other homothetic preferences across sectors, I show that the present framework gives rise to a novel "*Consumption Inequality*" (CI) channel of the aggregate impact of trade and technology shocks. To demonstrate it more directly, it is instructive to move from an unspecified demand system considered in Section 3.1 back to the Almost Ideal Demand System. I also start with discussing an individual's welfare gains formula. In particular, combining Propositions 4 and 5, the welfare impact of any foreign/non-local shock for income-$y$ individuals in local market $g \in \mathcal{G}_n$ is now given by

$$\hat{\mathcal{W}}_{ng}(y) \approx \prod_{j\in\mathcal{J}} \hat{\lambda}_{nnj}^{-\frac{S_{nj}}{\theta_j}} \cdot \prod_{j\in\mathcal{J}} \hat{\pi}_{ngj}^{-\frac{S_{nj}}{\alpha_{ng}}} \cdot \left(\prod_{j\in\mathcal{J}} \hat{\lambda}_{nnj}^{-\frac{\beta_{nj}}{\theta_j}} \cdot \prod_{j\in\mathcal{J}} \hat{\pi}_{ngj}^{-\frac{\beta_{nj}}{\alpha_{ng}}}\right)^{\ln(y/y_n)-\mathcal{T}_n}. \tag{3.7}$$

Note that, apart from the traditional Ricardian/ACR channel, $\prod_{j\in\mathcal{J}} \hat{\lambda}_{nnj}^{-S_{nj}/\theta_j}$, and Roy channel, $\prod_{j\in\mathcal{J}} \hat{\pi}_{ngj}^{-S_{nj}/\alpha_{ng}}$, an individual's welfare change depends on the consumer's income level relative to the inequality-adjusted mean income in the economy. Whether high- or low-income consumers benefit from a shock is determined by whether this term is higher or lower than 1.

Now, let $\varepsilon \in [-1, \infty)$ denote a measure of inequality-aversion. Then, the aggregate welfare gains in local market $g$ in country $n$ can be defined as follows:

$$\hat{\mathcal{W}}_{ng}^{(\varepsilon)} \equiv \frac{\int_{\mathscr{Y}} y^{-\varepsilon}\, \hat{\mathcal{W}}_{ng}(y)\, \mathrm{d}H_{ng}(y)}{\int_{\mathscr{Y}} y^{-\varepsilon}\, \mathrm{d}H_{ng}(y)}.$$

One can readily verify that integrating across the corresponding income distribution yields the following parsimonious formula for aggregate welfare gains in local market $g \in \mathcal{G}_n$.

**Proposition 6.** *Consider any foreign/non-local shock. A first-order approximation to the aggregate welfare change in local market $g$ in country $n$ is given by*

$$\hat{\mathcal{W}}_{ng}^{(\varepsilon)} = \underbrace{\underbrace{\prod_{j\in\mathcal{J}} \hat{\lambda}_{nnj}^{-S_{nj}/\theta_j}}_{\text{ACR (2012)}} \cdot \prod_{j\in\mathcal{J}} \hat{\pi}_{ngj}^{-S_{nj}/\alpha_{ng}}}_{\text{GRY (2023)}} \cdot \left( \prod_{j\in\mathcal{J}} \hat{\lambda}_{nnj}^{-\beta_{nj}/\theta_j} \cdot \prod_{j\in\mathcal{J}} \hat{\pi}_{ngj}^{-\beta_{nj}/\alpha_{ng}} \right)^{\Theta_{ng}^{(\varepsilon)}}, \tag{3.8}$$

*where:*

- $\Theta_{ng}^{(\varepsilon)} \equiv \tilde{\mathcal{P}}^{(\varepsilon)} + \ln(y_{ng}/y_n) + (\mathcal{T}_{ng} - \mathcal{T}_n)$, *where* $\tilde{\mathcal{P}}^{(\varepsilon)} \equiv \ln\left( \frac{\left(\mathbb{E}\left[y^{\Phi_{ng}-\varepsilon}\right]/\mathbb{E}\left[y^{-\varepsilon}\right]\right)^{1/\Phi_{ng}}}{y_{ng}\exp(\mathcal{T}_{ng})} \right)$ *and* $\Phi_{ng} \equiv \sum_{j\in\mathcal{J}} \beta_{nj} \left( \ln \hat{\lambda}_{nnj}^{-1/\theta_j} + \ln \hat{\pi}_{ngj}^{-1/\alpha_{ng}} \right)$;
- $\hat{\lambda}_{nnj} = \frac{\hat{w}_{nj}^{-\theta_j}}{\sum_{n'} \lambda_{n'nj} \hat{T}_{n'j} \left(\hat{\tau}_{n'nj}\hat{w}_{n'j}\right)^{-\theta_j}}$ *is the change in domestic share in sector $j$ in country $n$;*
- $\hat{\pi}_{ngj} = \frac{\hat{w}_{nj}^{\alpha_{ng}}}{\sum_k \pi_{ngk} \hat{w}_{nk}^{\alpha_{ng}}}$ *is the change in employment share of sector $j$ at local labor market $g \in \mathcal{G}_n$;*
- $\hat{\mathbf{w}} \equiv \{\hat{w}_{nj}\}_{n\in\mathcal{N}, j\in\mathcal{J}}$ *is implicitly given by the solution of (2.3).*

It directly follows from (3.8) that, in addition to the standard Ricardian and Roy channels of welfare gains, the model gives rise to a novel – *the Consumption Inequality* – channel at the aggregate (local) level.[19]

[19] Note that it is straightforward to aggregate this formula further to the country level.

### 3.3 Decomposing the Consumption Inequality channel

Note that the total local impact of the CI channel, $\left(\prod_j \hat{\lambda}_{nnj}^{-\beta_{nj}/\theta_j} \cdot \prod_j \hat{\pi}_{ngj}^{-\beta_{nj}/\alpha_{ng}}\right)^{\Theta_{ng}^{(\varepsilon)}}$, can be decomposed into three different margins: **i**) the *Pro-poor* margin; **ii**) the *Mean income* margin; and **iii**) the *Inequality* margin. To better grasp the mechanics of the CI channel, it is instructive to consider each of these margins separately.

**The *Pro-poor* margin** corresponds to the term $\left(\prod_j \hat{\lambda}_{nnj}^{-\beta_{nj}/\theta_j} \cdot \prod_j \hat{\pi}_{ngj}^{-\beta_{nj}/\alpha_{ng}}\right)^{\ln \frac{\left(\mathbb{E}\left[y^{\Phi_{ng}-\varepsilon}\right]/\mathbb{E}\left[y^{-\varepsilon}\right]\right)^{\frac{1}{\Phi_{ng}}}}{y_{ng}\exp(\mathcal{T}_{ng})}}$. One can readily verify that, under the Fréchet supply side, the exponent can be rewritten in the following closed-form:[20]

$$\ln\left(\frac{\left(\mathbb{E}\left[y^{\Phi_{ng}-\varepsilon}\right]/\mathbb{E}\left[y^{-\varepsilon}\right]\right)^{\frac{1}{\Phi_{ng}}}}{y_{ng}\exp\left(\mathcal{T}_{ng}\right)}\right) = \frac{\psi\left(1-1/\alpha_{ng}\right)}{\alpha_{ng}} + \frac{1}{\Phi_{ng}}\ln\left(\frac{\Gamma\left(1+\frac{\varepsilon-\Phi_{ng}}{\alpha_{ng}}\right)}{\Gamma\left(1+\varepsilon/\alpha_{ng}\right)}\right).$$

Note that the Pro-poor margin tends to amplify aggregate local welfare gains when the nature of a shock is "pro-poor" (i.e., $\prod_j \hat{\lambda}_{nnj}^{-\beta_{nj}/\theta_j} \cdot \prod_j \hat{\pi}_{ngj}^{-\beta_{nj}/\alpha_{ng}} < 1$) and, conversely, dampens aggregate local welfare gains if the nature of a shock is "pro-rich" (i.e., $\prod_j \hat{\lambda}_{nnj}^{-\beta_{nj}/\theta_j} \cdot \prod_j \hat{\pi}_{ngj}^{-\beta_{nj}/\alpha_{ng}} > 1$). It can be shown that the Pro-poor margin is primarily driven by the "wedge" between the welfare criterion ($\varepsilon$) and the demand aggregation (i.e., the aggregation of individual expenditure shares into aggregate shares). It is also straightforward to see that the nonlinear pro-poor term vanishes when the distribution of skills and, thereby, the (endogenous) distribution of income within local market $g \in \mathcal{G}_n$ becomes homogeneous ($\alpha_{ng} \to \infty$).[21]

**The *Mean income* margin** corresponds to the term $\left(\prod_j \hat{\lambda}_{nnj}^{-\beta_{nj}/\theta_j} \cdot \prod_j \hat{\pi}_{ngj}^{-\beta_{nj}/\alpha_{ng}}\right)^{\ln(y_{ng}/y_n)}$. The sign of this effect is driven by the "nature" of a shock captured by $\prod_j \hat{\lambda}_{nnj}^{-\beta_{nj}/\theta_j} \cdot \prod_j \hat{\pi}_{ngj}^{-\beta_{nj}/\alpha_{ng}}$ and the deviation of the local market's mean income from the country-level mean income captured by $\ln\left(y_{ng}/y_n\right)$. In particular, it implies that if a shock is "pro-poor", local markets with relatively lower mean income than the country-level mean gain from this margin and lose otherwise. Recall from Proposition 1 that the aggregate expenditure shares across sectors depend on the mean income adjusted by the exponent of the Theil index of income inequality. The same mechanism applies to the consumption pattern of local labor markets. So, conditional on the Theil indices, local labor markets with relatively lower mean income spend dispro-

[20] To ensure that $\mathbb{E}\left[y^{\Phi_{ng}-\varepsilon}\right]$ is finite under the (local) Fréchet distribution of income, I impose the assumptions that $\Phi_{ng} - \varepsilon < \alpha_{ng}$ and $\varepsilon < \alpha_{ng}$ for any $g \in \mathcal{G}_n$ and $n \in \mathcal{N}$.

[21] Formally, $\lim_{\alpha_{ng}\to\infty}\left(\frac{\gamma+\psi(1-1/\alpha_{ng})}{\alpha_{ng}} + \frac{1}{\Phi_{ng}}\ln\left(\frac{\Gamma(1+(\varepsilon-\Phi_{ng})/\alpha_{ng})}{\Gamma(1+\varepsilon/\alpha_{ng})}\right)\right) = 0$ for all values of $\Phi_{ng}$ and $\varepsilon$.

portionally more on low-income elastic sectors than their relatively richer counterparts. As a result, for example, a "pro-poor" shock, by reducing the relative prices of low-income elastic sectors, tends to benefit low-income local labor markets.

**The *Inequality* margin** corresponds to the term $\left(\prod_j \hat{\lambda}_{nnj}^{-\beta_{nj}/\theta_j} \cdot \prod_j \hat{\pi}_{ngj}^{-\beta_{nj}/\alpha_{ng}}\right)^{\mathcal{T}_{ng}-\mathcal{T}_n}$. It is driven by the deviation of the local market's Theil index from the country-wide Theil index of income inequality, $\mathcal{T}_{ng} - \mathcal{T}_n$. For instance, it implies that if the nature of a shock is "pro-poor", then local markets with relatively lower income inequality gain from this margin. To better grasp the intuition, again recall from Proposition 1 that the aggregate expenditure shares across sectors depend on the mean income adjusted by the exponent of the Theil index of income inequality. In other words, if there is any level of income heterogeneity, then the aggregate expenditure shares of high-income elastic sectors are higher than in the case of no income inequality (i.e., $\mathcal{T}_n = 0$). Analogously, aggregate shares of low-income elastic sectors are lower than in the income homogeneity scenario. Note that the same logic applies to the expenditure patterns at the level of local labor markets. So, conditional on the mean incomes, local labor markets with lower inequality than the economy-wide Theil index tend to spend disproportionally less on high-income elastic sectors while having disproportionally larger expenditure shares of low-income elastic sectors. As a result, "pro-poor" shocks, by reducing the relative prices of low-income elastic sectors, tend to benefit local labor markets with lower levels of income inequality. It is straightforward to see that the opposite mechanism applies to "pro-rich" shocks.

## 3.4 The role of geography and skill heterogeneity in welfare implications

The formula (3.8) and the CI channel decomposition shed light on the local market's incidence of any shock. For example, if a country-level shock is "pro-poor", then it tends to favor local labor markets with *i*) a higher level of workers' skill heterogeneity (i.e., when $\alpha_{ng}$ is low), *ii*) lower mean income ($y_{ng} < y_n$) and *iii*) lower income inequality ($\mathcal{T}_{ng} < \mathcal{T}_n$).

Now, note that if one imposes the assumption that workers in each local labor market within a country draw their sectoral productivities from a distribution with a common shape parameter (i.e., $\alpha_{ng} = \alpha_n$ for any $g \in \mathcal{G}_n$, while allowing for local markets' "absolute advantages" captured by differences in $\{A_{ngj}\}$), then the Theil index of income inequality is the same for all labor markets within a country. In this case, the "Inequality" margin of (3.8) that is driven by differences in Theil indices is equal to zero for all labor markets. If, in addition, workers are assumed to be homogeneous within each local labor market (i.e., $\alpha_{ng} \to \infty$), then differences in the welfare response to a shock across labor markets are driven solely by differences in the mean local incomes (i.e., the "Mean income" margin) and the formula (3.8)

reduces to:

$$\hat{\mathcal{W}}_{ng}^{(\varepsilon)} = \prod_{j\in\mathcal{J}} \hat{\lambda}_{nnj}^{-\frac{S_{nj}}{\theta_j}} \cdot \left( \prod_{j\in\mathcal{J}} \hat{\lambda}_{nnj}^{-\frac{\beta_{nj}}{\theta_j}} \right)^{\ln(y_{ng}/y_n)}.$$

Alternatively, if one abstracts away from the geographical distribution of economic activity and assumes only one labor market within country $n$ (i.e., $G_n = 1$), then both the "Mean income" and the "Inequality" margins disappear, implying that only the "Pro-poor" margin remains operative. So, in this particular case, the country-level welfare response to a shock is given by

$$\hat{\mathcal{W}}_{n}^{(\varepsilon)} = \prod_{j\in\mathcal{J}} \hat{\lambda}_{nnj}^{-S_{nj}/\theta_j} \cdot \prod_{j\in\mathcal{J}} \hat{\pi}_{nj}^{-S_{nj}/\alpha_n} \cdot \left( \prod_{j\in\mathcal{J}} \hat{\lambda}_{nnj}^{-\beta_{nj}/\theta_j} \cdot \prod_{j\in\mathcal{J}} \hat{\pi}_{nj}^{-\beta_{nj}/\alpha_n} \right)^{\ln \frac{\left(\mathbb{E}\left[y^{\Phi_{ng}-\varepsilon}\right]/\mathbb{E}\left[y^{-\varepsilon}\right]\right)^{\frac{1}{\Phi_{ng}}}}{y_{ng}\exp(\mathcal{T}_{ng})}}.$$

Note that the expression above implies that the CI channel is driven by the term $\left(\mathbb{E}_{-\varepsilon}\left[y_{ng}^{\Phi_{ng}}\right]\right)^{\frac{1}{\Phi_{ng}}}$ that magnifies the aggregate welfare gains when the nature of a shock is "pro-poor" and, conversely, dampens the aggregate effects if the nature of a shock is "pro-rich".

In Section 4, I quantitatively study the contribution of each traditional channel and each margin of the CI channel to the total welfare change in response to the China shock.

## 3.5 Relation to other welfare gains formulas

As has been mentioned, Propositions (4) and (5) apply to any demand system across sectors. To highlight the difference from some existing well-known results, it is enough to consider special cases of the aggregate (local) welfare gains formula for the Almost Ideal Demand System (3.8). Specifically, I will separately consider four different special cases.

**Special Case 1 (Homothetic Translog expenditure function)** It can be readily inferred that if one shuts down non-homotheticities (i.e., $\beta_{nj} = 0$ for any $j \in \mathcal{J}$ and $n \in \mathcal{N}$), then $\left(\prod_j \hat{\lambda}_{nnj}^{-\beta_{nj}/\theta_j} \cdot \prod_j \hat{\pi}_{ngj}^{-\beta_{nj}/\alpha_{ng}}\right)^{\Theta_{ng}^{(\varepsilon)}} = 1$ for any $g \in \mathcal{G}_n$ and the CI channel disappears for any local labor market. As a result, (3.8) reduces to

$$\hat{\mathcal{W}}_{ng}^{(\varepsilon)} = \prod_{j\in\mathcal{J}} \hat{\lambda}_{nnj}^{-\frac{S_{nj}}{\theta_j}} \cdot \prod_{j\in\mathcal{J}} \hat{\pi}_{ngj}^{-\frac{S_{nj}}{\alpha_{ng}}}.$$

The expression above represents a first-order approximation to the welfare gains for the particular case of a homothetic Translog expenditure function across sectors, implying variable elasticities of substitution across them. As has been discussed in Section 2.4, $\{\beta_{nj}\} = 0$

implies no cross-interactions between sectoral income (semi-)elasticities and changes in sectoral prices (i.e., $\mathbf{\Omega}_n = \mathbf{0}_{J\text{x}J}$) as well as no interactions between sectoral income elasticities and the inequality-adjusted mean income (i.e., $\tilde{\mathbf{b}}_n = \mathbf{0}_{J\text{x}1}$). As a result, changes in the aggregate expenditure shares in any country $n$ are driven only by homothetic forces through (relative) changes in sectoral price indices and substitution patterns across them captured by the matrix of $\{\gamma_{njk}\}$: $\mathbf{\Delta S}_n = \tilde{\mathbf{P}}_n$, where $\tilde{\mathbf{P}}_n$ is a $J$x1 vector with $j$'s element given by $\tilde{P}_{nj} = \sum_{k=1} \gamma_{njk} \ln \hat{P}_{nk}$. Note also that the formula becomes invariant to the value of $\varepsilon$, and this superscript can be omitted.

**Special Case 2 (Constant expenditure shares as in Galle, Rodríguez-Clare, and Yi 2023)** If one additionally imposes that the matrix of substitution patterns across sectoral price indices is zero (i.e., $\{\gamma_{njk}\} = 0$), then the model's welfare change predictions reduce to the case of Cobb-Douglas preferences across sectors studied by Galle, Rodríguez-Clare, and Yi (2023). So, the formula above now represents the exact welfare gains in local labor market $g$ in country $n$:

$$\hat{\mathcal{W}}_{ng} = \prod_{j \in \mathcal{J}} \hat{\lambda}_{nnj}^{-\frac{S_{nj}}{\theta_j}} \cdot \prod_{j \in \mathcal{J}} \hat{\pi}_{ngj}^{-\frac{S_{nj}}{\alpha_{ng}}}. \tag{3.9}$$

Note that while this expression is observationally equivalent to the case of the first-order approximation to the welfare gains under a homothetic Translog expenditure function, these two special cases deliver quantitatively different predictions, even conditional on observed trade shares, $\{S_{nj}\}$, and structural parameters, $\{\theta_j\}$ and $\{\alpha_{ng}\}$. In particular, recall that both counterfactual trade shares ($\hat{\lambda}_{nnj}$) and employment shares ($\hat{\pi}_{ngj}$) depend on the matrix of counterfactual (relative) changes in wages, $\hat{\mathbf{w}} \equiv \{\hat{w}_{ij}\}$. This matrix, in turn, is indirectly affected by model-implied changes in aggregate expenditure shares (see Proposition 2):

$$\mathbf{\Delta S}_n = (\text{diag}(\mathbf{1}) + \mathbf{\Omega}_n)^{-1} \left( \tilde{\mathbf{P}}_n + \tilde{\mathbf{b}}_n \right)$$

via the system of labor market clearing conditions (2.3). In the case of fixed expenditure shares, apart from the absence of non-homothetic forces ($\mathbf{\Omega}_n = \mathbf{0}_{J \times J}$ and $\tilde{\mathbf{b}}_n = \mathbf{0}_{J \times 1}$), there are also no changes in the aggregate expenditure shares that are driven by homothetic forces via changes in relative prices across sectors (that is, $\tilde{\mathbf{P}}_n = \mathbf{0}_{J \times 1}$, in contrast to the Translog case). As a result, $\mathbf{\Delta S}_n = (\text{diag}(\mathbf{1}) + \mathbf{\Omega}_n)^{-1} \left( \tilde{\mathbf{P}}_n + \tilde{\mathbf{b}}_n \right)$ reduces to the case of constant aggregate expenditure shares: $\mathbf{\Delta S}_n = \mathbf{0}_{J \times 1}$.

**Special Case 3 (Multi-sector Ricardian models)** Note also that if, in addition to the assumption of Cobb-Douglas preferences across sectors, one also dispenses with Roy-type worker heterogeneity, then the corresponding welfare term vanishes (i.e., $\prod_{j \in \mathcal{J}} \hat{\pi}_{ngj}^{-S_{nj}/\alpha_{ng}} = 1$ for any

$g \in \mathcal{G}_n$), and welfare gains are equal across local labor markets within a country:

$$\hat{\mathcal{W}}_n = \prod_{j \in \mathcal{J}} \hat{\lambda}_{nnj}^{-\frac{S_{nj}}{\theta_j}}. \tag{3.10}$$

The formula above determines the exact welfare gains in multi-sector extensions of the standard Eaton and Kortum (2002) model (see, e.g., Costinot, Donaldson, and Komunjer 2012 or Caliendo and Parro 2015 under the case of no input-output linkages).

**Special Case 4 (Eaton and Kortum 2002)** Lastly, if one assumes a single-sector case (i.e., $J = 1$), then the formula (3.10) reduces to

$$\hat{\mathcal{W}}_n = \hat{\lambda}_{nn}^{-\frac{1}{\theta}}. \tag{3.11}$$

One can readily infer that it corresponds to the exact welfare gains in the baseline Eaton and Kortum (2002) model. As a result, the generalized formula derived in this Section nests the main existing cases in the corresponding literature that tend to rely heavily on the assumption of homothetic preferences. So, in addition to the general tractability of the model, the welfare analysis also remains analytically tractable, and one can predict *ex-ante* the welfare consequences of any foreign/non-local shock. In Section 4, I examine the quantitative implications of the CI channel and other model features by revisiting the impact of the China shock.

# 4 Application: the China shock

## 4.1 Data

To quantify the aggregate and distributional impacts of the China shocks, data from several sources are collected. In particular, data on the aggregate expenditure shares $\{S_{nj}\}_{n \in \mathcal{N}, j \in \mathcal{J}}$ and trade shares $\{\lambda_{inj}\}_{i,n \in \mathcal{N}, j \in \mathcal{J}}$ are constructed from the World Input-Output Database (WIOD). Data on employment shares across different US commuting zones $\{\pi_{gj}\}_{g \in \mathcal{G}_{US}, j \in \mathcal{J}}$ are constructed based on the County Business Patterns database (CBP) and the WIOD database. Country-level employment shares for other countries, $\{\pi_{nj}\}_{n \in \mathcal{N}, j \in \mathcal{J}}$, are constructed using the WIOD database. As for the US CZ-level income $\{Y_g\}_{g \in \mathcal{G}_{US}}$ and labor force $\{L_g\}_{g \in \mathcal{G}_{US}}$ data, they are obtained from the American Community Survey (ACS). The corresponding country-level data on total income $\{Y_n\}_{n \in \mathcal{N}}$ and labor force $\{L_n\}_{n \in \mathcal{N}}$ for the sample of other countries are obtained from the World Bank Open Data. Based on the data collected, the following subsection describes the calibration strategy.

## 4.2 Calibration

### 4.2.1 Geographical distribution of skill heterogeneity

Recall that the supply side of the model implies that the Theil index of income inequality at the local market $g$ in country $n$ is given by $\mathcal{T}_{ng} = \ln\left(\Gamma\left(1-1/\alpha_{ng}\right)\right)^{-1} - \frac{\psi(1-1/\alpha_{ng})}{\alpha_{ng}}$, where $\alpha_{ng}$ is the shape parameter of the Fréchet distribution of income and also a Roy-Fréchet (shape) parameter of the local-level distribution of worker productivity. It suggests that one can calibrate parameters $\{\alpha_{ng}\}$ based on information about the distribution of Theil indices across the economy. In terms of data availability, however, Theil indices are rarely available, especially at the level of local markets. Yet, one can alternatively infer $\{\alpha_{ng}\}$ from data on the Gini indices of income inequality. Specifically, in our case of Fréchet-distributed local incomes, the Gini index of income inequality at the local market $g$ in country $n$ is given by $2^{1/\alpha_{ng}} - 1$. So, given the Fréchet income distribution, there is a direct map between the Gini and Theil indices of income inequality.

Figure 1: Calibrated values of $\alpha_{US,g}$ for 722 CZs.

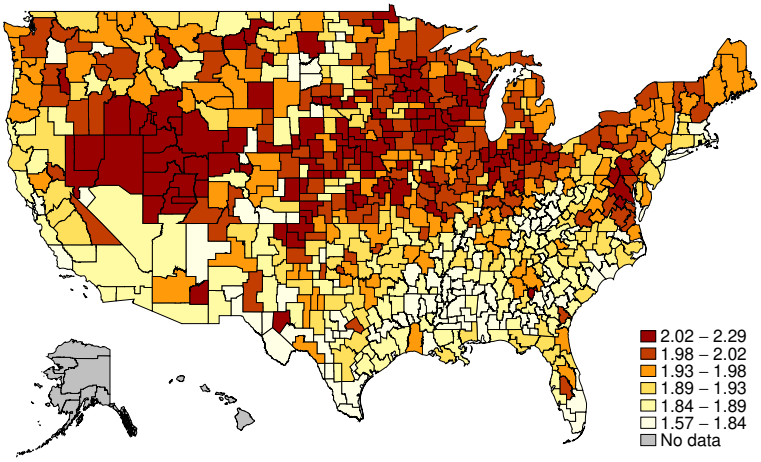



Figure 1 shows calibrated values of $\alpha_{US,g}$ based on information about Gini indices for each of the 722 Commuting Zones across the mainland US. Recall that the higher the value of $\alpha_{ng}$, the lower the heterogeneity of workers within the local labor market $g \in \mathcal{G}_n$, also implying lower income inequality. So, Figure 1 suggests that Commuting Zones from the northern part of the US generally tend to have lower income inequality than their southern counterparts. However, as one can see from Figure 2, there is no clear relationship between the mean income of Commuting Zones and the income inequality captured by the inferred local Theil index, $\mathcal{T}_{ng} = \ln\left(\Gamma\left(1-1/\alpha_{ng}\right)\right)^{-1} - \frac{\psi(1-1/\alpha_{ng})}{\alpha_{ng}}$.

Figure 2: Local income inequality vs. the local mean income across the US CZs.

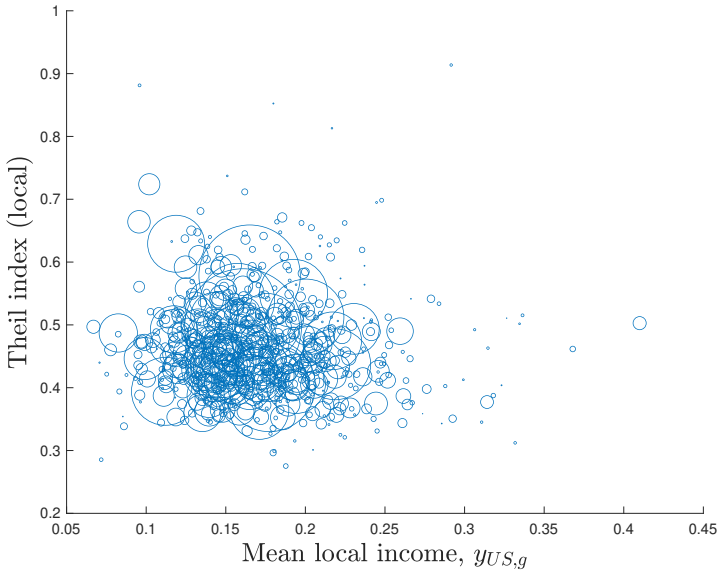



Figure 2 and the direct mapping between $\mathcal{T}_{ng}$ and $\alpha_{ng}$ suggest that, while both "Pro-poor" and "Inequality" margins may contribute a sizable part to the aggregate gains of a particular local labor market, differences in magnitudes of the CI channel across local labor markets are going to be mainly driven by differences in the "Mean income" margin. I will discuss it in more detail in the corresponding subsection.

### 4.2.2 Calibration of the China shock

I calibrate the China shock in the spirit of Caliendo, Dvorkin, and Parro (2019) and Galle, Rodríguez-Clare, and Yi (2023). Specifically, to cope with potential endogeneity driven by the US demand shocks and isolate that part of the China shock that is driven by China's supply forces, I obtain predicted 2000-2007 relative changes in the China-US trade shares by running the ("first-stage") regression in the spirit of the IV-strategy by Autor, Dorn, and Hanson (2013):

$$\frac{\lambda^{2007}_{Ch,US,j}}{\lambda^{2000}_{Ch,US,j}} = \delta_0 + \delta_1 \cdot \frac{\sum_{n \in Other} \lambda^{2007}_{Ch,n,j}}{\sum_{n \in Other} \lambda^{2000}_{Ch,n,j}} + \varepsilon_j,$$

where $Other = \{\text{Australia, Denmark, Finland, Germany, Japan, Spain}\}$ is a set of developed countries similar to that used by Autor, Dorn, and Hanson (2013).

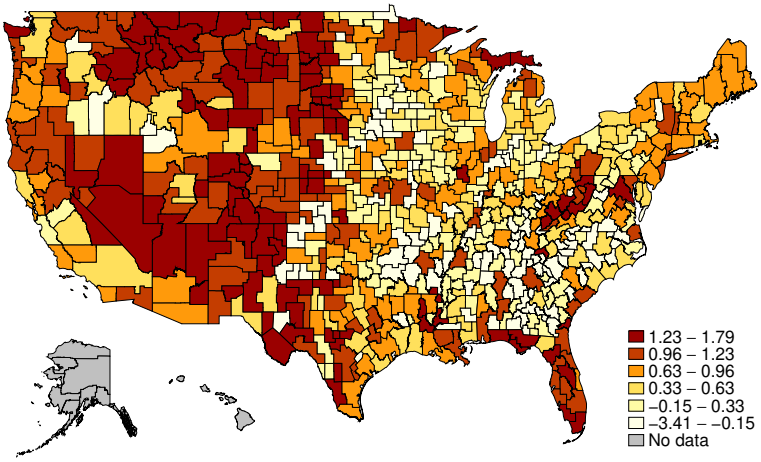
After obtaining the predicted (relative) changes in trade shares, $\hat{\lambda}^{data}_{Ch,US,j} \equiv \frac{\lambda^{2007}_{Ch,US,j}}{\lambda^{2000}_{Ch,US,j}}$, China's sector-specific changes in technology parameters, $\left\{\hat{T}_{China,j}\right\}_{j \in \mathcal{J}}$, are calibrated such that the model-predicted changes in trade shares, $\left\{\hat{\lambda}^{model}_{Ch,US,j}\right\}_{j \in \mathcal{J}}$, match those predicted by the regres-

sion analysis, $\left\{\hat{\lambda}^{data}_{Ch,US,j}\right\}_{j\in\mathcal{J}}$.[22]

## 4.3 The aggregate and local market effects of the China shock

In this subsection, I discuss the results for the aggregate effects of the China shock as well as how the shock is distributed across local labor markets (CZs).[23] The aggregate welfare impact of the China shock across Commuting Zones averages 0.71%. It implies larger country-level gains than most of the previous findings in the quantitative trade literature based on the simplifying assumption of homothetic expenditure shares. The reason is that a significant part of this increase (0.26%) is attributed to the Consumption Inequality channel that operates via non-homothetic expenditure patterns across sectors. In the following subsection, I will discuss the CI channel in more detail and will quantify each margin of this channel. As for the traditional channels, the Ricardian channel operates at the country level and contributes 0.28% to the aggregate welfare gains of the US economy. The Roy channel, in contrast to the Ricardian one, is heterogeneous across local labor markets, and its aggregate impact across Commuting Zones averaged 0.17%. Figure 3 shows the geographical distribution of welfare changes across different Commuting Zones.

Figure 3: The aggregate (local) welfare gains across the US CZs, in %.




As one can note from Figure 3, the geographical distribution of the China shock across local labor markets tends to demonstrate spatially correlated patterns. The question is whether Commuting Zones, which, for instance, gained from the shock, tended to have some features

[22]Calibration of $\{\theta_j\}$ is based on Head and Mayer (2014), while demand parameters $\{\beta_{nj}\}$ are calibrated to match sectoral income elasticities in Fajgelbaum and Khandelwal (2016).

[23]The results below are based on the previous version of the paper, which, in particular, used slightly different formulas for the aggregate and distributional welfare effects. The quantitative section will be updated in future drafts.

in common. Figure 4 helps to shed light on this by plotting the aggregate (local) welfare response versus mean local income for each Commuting Zone.

Figure 4: The aggregate welfare change vs. the local mean income across the US CZs.

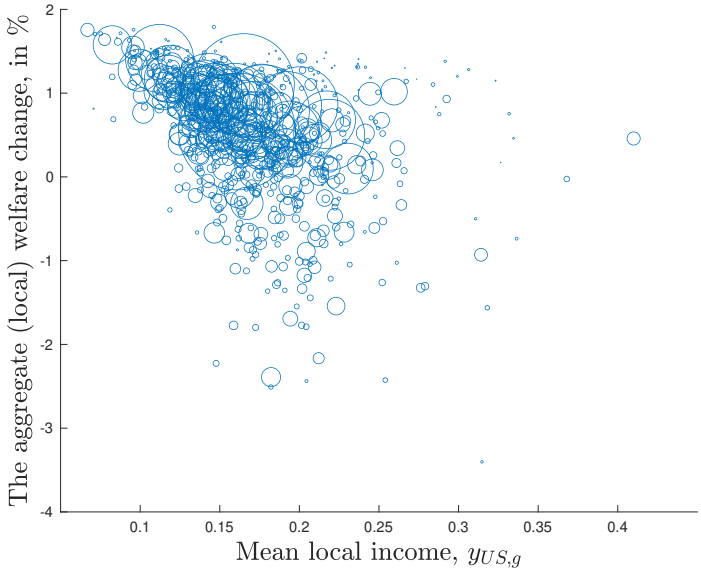
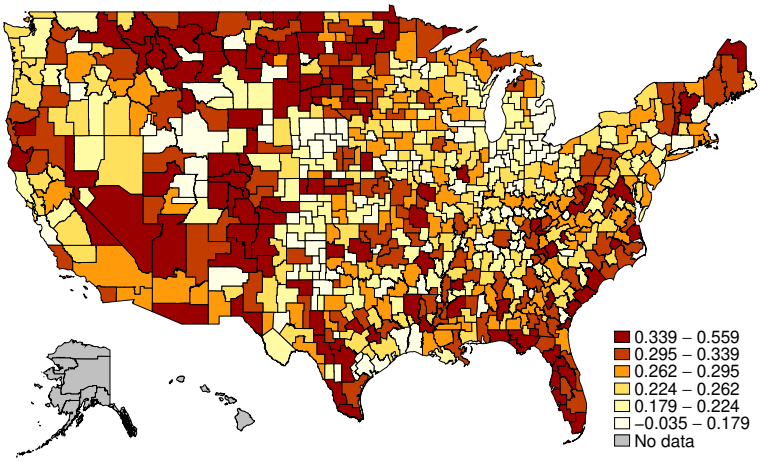



Figure 4 shows that Commuting Zones with lower mean income were more likely to gain from the shock. In contrast, relatively more affluent Commuting Zones tended to bear welfare losses from the China shock, along with a large variance in the aggregate local impact. The following subsection uncovers the mechanism behind this pattern via the presence of the CI channel.

## 4.4 The Consumption Inequality channel

In this subsection, I quantitatively study the aggregate and distributional impacts of the CI channel. First, note that while its aggregate impact across Commuting Zones is mainly positive, averaging 0.26%, there is significant heterogeneity in its geographical incidence. As Figure 5 demonstrates, its aggregate local impact ranges from slightly negative (–0.04%) to strictly positive (0.56%).

Figure 5: The Consumption Inequality channel (in %) across CZs.




As before, the question is whether Commuting Zones, which gained relatively more from the shock, tended to have some features in common. Figure 6 shows that low-income Commuting Zones were more likely to gain from the CI channel, implying that the incidence of the shock was mainly "pro-poor".

Figure 6: The CI channel vs. the local mean income across the US CZs.

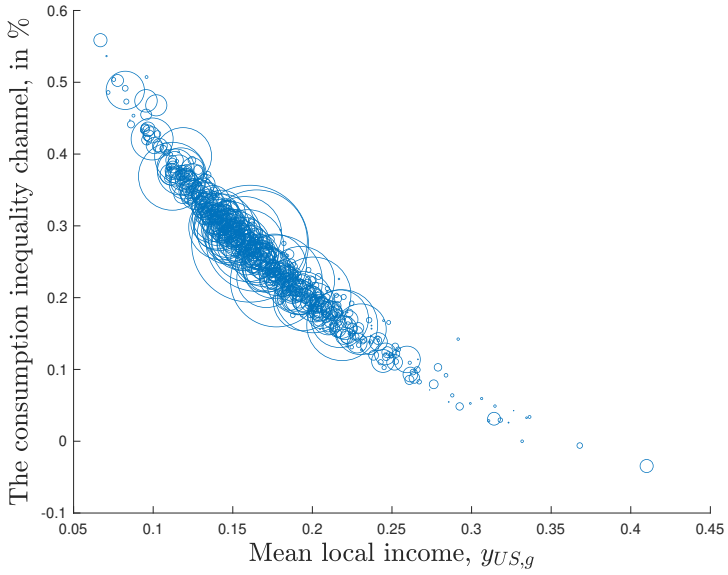



Figure 6 also clearly demonstrates that the CI channel was the main driving force behind the pattern that Commuting Zones with lower mean income were more likely to gain from the China shock (as was previously shown by Figure 4). To better understand the mechanism through which it operates, it is instructive to consider the effects of each of the three margins of the CI channel.

### 4.4.1 The "Pro-poor" margin of the Consumption Inequality channel

Recall that the "Pro-poor" margin amplifies the aggregate local welfare gains when the nature of a shock is "pro-poor" and, conversely, dampens the aggregate local welfare gains if the nature of a shock is "pro-rich" Figure 7 shows that the resulting effect of the "Pro-poor" margin is positive for each CZ in response to the China shock. It suggests that the China shock generally favored relatively low-income consumers via the reduction in relative prices of low-income elastic sectors.

Figure 7: The "Pro-poor" margin vs. the local mean income across the US CZs.

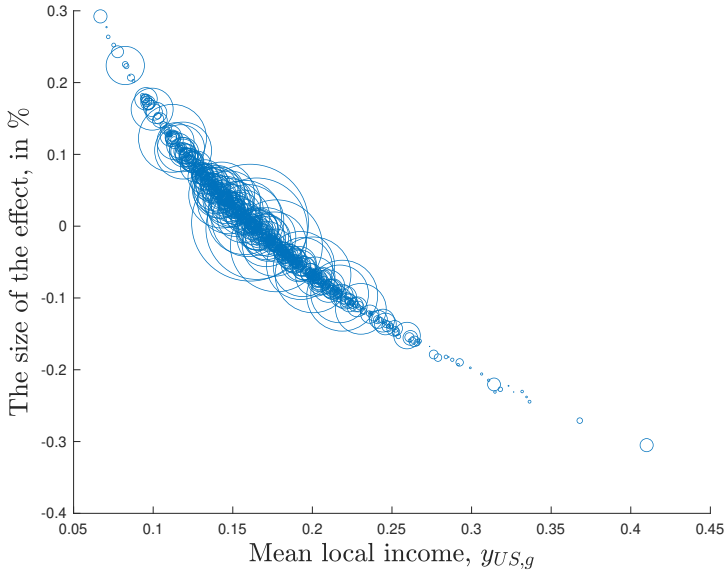



Figure 7 shows that there is no correlation between mean local income and the size of the "Pro-poor" margin effect. The reason is that this margin heavily hinges on the local market's level of skill heterogeneity captured by $\alpha_{ng}$. However, recalling from Figure 2, there is no clear relationship between the mean local income and the Theil index of income inequality (as well as the level of skill heterogeneity). Despite this, the contribution of this margin to the aggregate local welfare gains is sizable and ranges from 0.15% to 0.48%.

### 4.4.2 The "Mean income" margin of the Consumption Inequality channel

The "Mean income" margin creates winners and losers from the CI channel across Commuting Zones. In particular, as Figure 8 demonstrates, CZs with relatively lower average income gain from this margin (up to 0.29%), while others – lose (up to –0.30%).

Figure 8: The "Mean income" margin vs. the local average income across the US CZs.

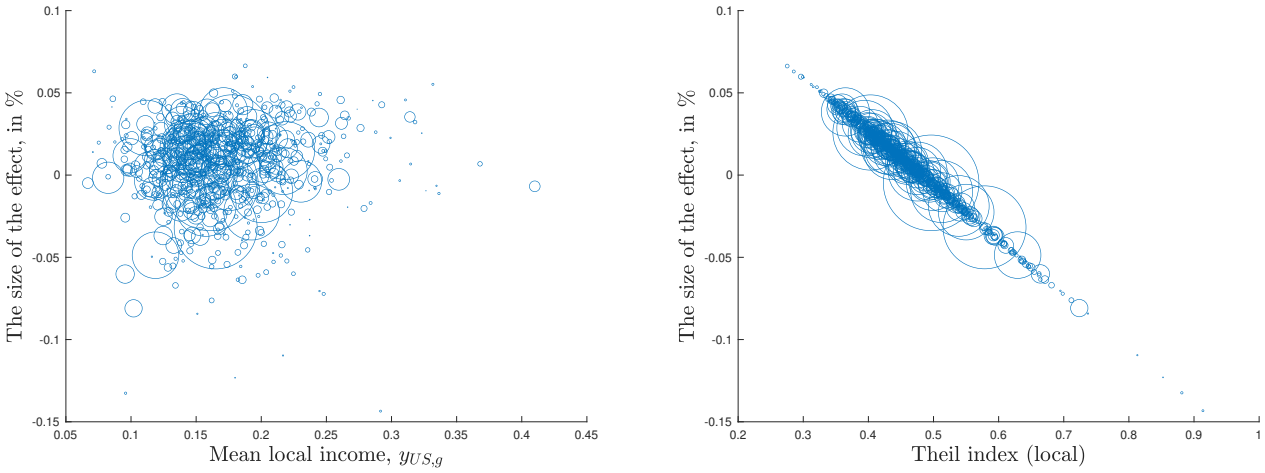



To better understand this pattern, recall from the theory part that the aggregate expenditure shares across sectors depend on the mean income adjusted by the exponent of the Theil index of income inequality. A similar mechanism applies to the consumption pattern of local labor markets. So, conditional on the Theil indices, local labor markets with relatively lower average income spend disproportionally more on low-income elastic sectors than their relatively more affluent counterparts. As a result, the China shock, by reducing relative prices of low-income elastic sectors, benefited low-income Commuting Zones through this margin.

#### 4.4.3 The "Inequality" margin of the Consumption Inequality channel

The "Inequality" margin also creates winners (up to 0.07%) and losers (up to –0.14%) across Commuting Zones. First, note that the left panel of Figure 9 shows that there is no correlation between mean local income and the size of the "Inequality" margin effect. The reason is similar to that described for the "pro-poor" margin. In particular, one can recall from Figure 2 that there is no clear relation between the mean local income and the Theil index of income inequality.

Figure 9: The "Inequality" margin vs. the local mean income and Theil index across the US CZs.




The right panel of Figure 9 shows that the size of the effect is strictly correlated with the value of the local Theil index. It is not surprising, given the definition of the "Inequality" margin. Recall that it is driven by a deviation of the local market's Theil index from the country-wide Theil index of income inequality. Given that the nature of the China shock is "pro-poor", local markets with relatively lower income inequality gain from this margin. To better grasp the intuition, again recall that the aggregate expenditure shares across sectors depend on mean income adjusted by the exponent of the Theil index of income inequality. So, conditional on the mean incomes, local labor markets with relatively lower income inequality spend disproportionally more on low-income elastic sectors than their relatively wealthier counterparts. So, the China shock, by reducing relative prices of low-income elastic sectors, benefited low-income Commuting Zones through this margin.

## 4.5 The impact of the China shock on other countries

In this subsection, I apply the welfare change formula (3.8) to quantify the aggregate (country-level) impact of the China shock on other countries in the sample. Recall that the formula (3.8) applies to examine the welfare consequences of any foreign/non-local shock. So, it cannot be directly applied to quantify the impact of the China shock on China's economy. Table 1 summarizes the welfare changes for the sample of 38 other countries and the "Rest of the World" (RoW).

Table 1: The aggregate impact of the China shock across the sample countries.

| Country | $\Delta W_n/W_n$ | Country | $\Delta W_n/W_n$ | Country | $\Delta W_n/W_n$ |
|---|---|---|---|---|---|
| AUS | 1.61% | GRC | –0.04% | NLD | 0.65% |
| AUT | 0.78% | HUN | 0.04% | POL | 2.23% |
| BEL | 0.11% | IND | 2.69% | PRT | 0.57% |
| BRA | 1.88% | IDN | 2.06% | ROU | 1.42% |
| BGR | 0.95% | IRL | 0.46% | RUS | 1.37% |
| CAN | 0.61% | ITA | 0.78% | SVK | 1.79% |
| CYP | 0.43% | JPN | 1.75% | SVN | 1.45% |
| CZE | 2.14% | KOR | 0.05% | ESP | 2.06% |
| DNK | % | LVA | 0.34% | SWE | –0.91% |
| EST | 0.19% | LTU | 2.15% | TUR | 0.04% |
| FIN | 1.26% | LUX | 2.21% | UK | 0.40% |
| FRA | 1.77% | MLT | 3.72% | TWN | 2.62% |
| GER | 0.45% | MEX | 1.27% | RoW | 2.20% |

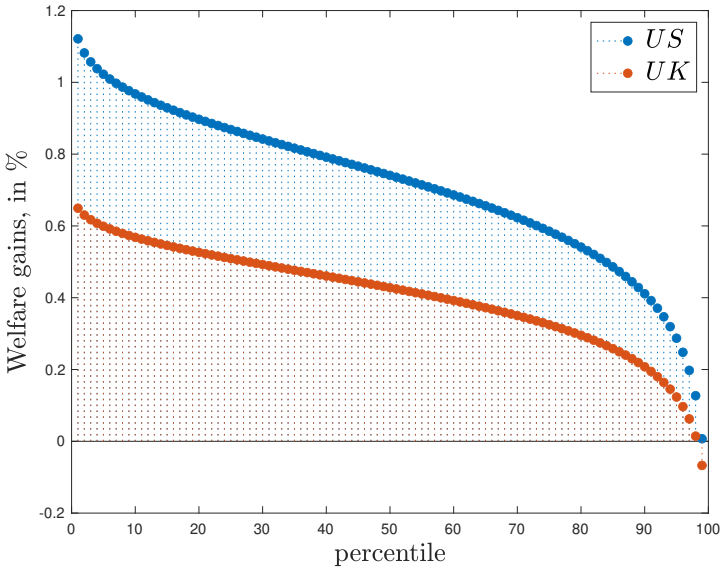
Note that while the China shock led to aggregate gains for most countries, a few countries experienced net losses. To grasp the intuition, recall that for countries other than the US, I follow the corresponding quantitative trade literature by imposing the simplifying assumption that there is only one labor market within each country. Recall also that if one abstracts away from the geographical distribution of economic activity, then both the "Mean income" and the "Inequality" margins of the CI channel disappear for this country, and only the "Pro-poor" margin remains operative. It implies that the CI channel magnifies the aggregate welfare gains when the China shock was "pro-poor" for this country, and, conversely, dampens the aggregate welfare effects if the nature of a shock was "pro-rich". So, even if a country increases its foreign trade shares for each sector $j$ in response to the China shock, a country may still bear aggregate losses through, for example, *i*) the Roy channel via a decrease in the country's specialization, *ii*) the Consumption Inequality channel if, for example, the country's trade openness pattern has been shifted towards high-income elastic sectors, thereby benefiting relatively affluent consumers.

## 4.6 The distributional effects across income quantiles

In the previous sections, the distributional effects of the China shock were studied through the lens of heterogeneous exposure across local labor markets. However, given the presence of heterogeneous workers and non-homothetic expenditure patterns, the model also implies notable distributional consequences along the income distribution. In this subsection, I provide

a comparison of the US welfare gains patterns with some other countries.

Figure 10: The distribution of welfare gains across income quantiles in the US vs. the UK.




As one can see from Figure 10, the China shock disproportionately benefited low-income US consumers. This aligns with the previous findings that low-income commuting zones gained from the shock via reductions in the relative prices of low-income elastic sectors. This Figure also compares the distribution of welfare gains across income quantiles in the US versus that in the UK. One can readily see that the lower the percentile, the larger the divergence in welfare gains between the two countries. In particular, while low-income individuals in the UK gain more than high-income individuals in the country, they benefit relatively less than their counterparts in the US.

Figure 11: The distribution of welfare gains across income quantiles in the US vs. Germany.

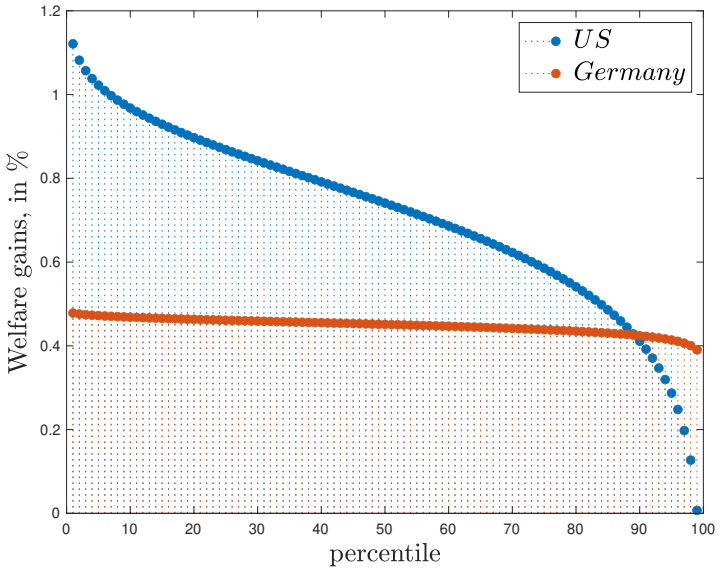
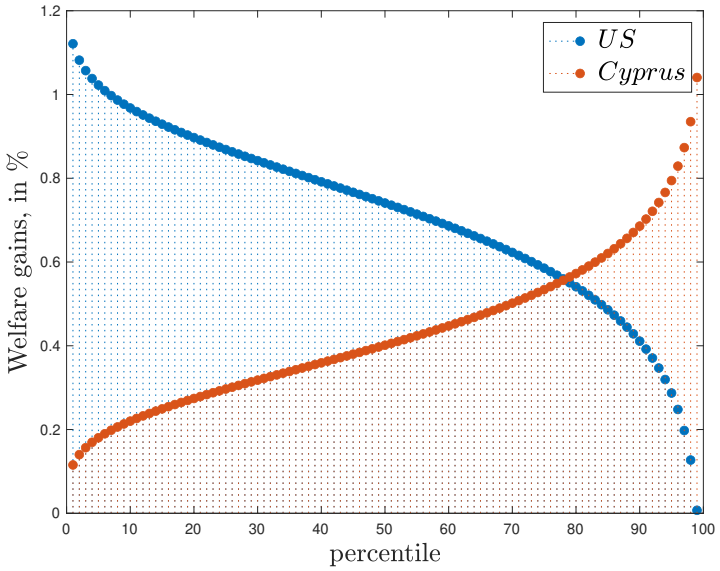



Figure 11 demonstrates that, compared to the US, the distributional effect of the China shock across income quantiles in Germany was quite neutral. It suggests that the gains through non-homothetic forces for low-income individuals were not as pronounced in Germany as in the US.

Figure 12: The distribution of welfare gains across income quantiles in the US vs. Cyprus.




As shown in Figure 12, the China shock may disproportionately benefit high-income consumers in Cyprus. In particular, in Cyprus, the nature of the China shock was "pro-rich"

in that it reduced the relative prices of high-income elastic sectors. As a result, high-income consumers there disproportionately benefited from the shock.

# 5 Conclusion

In this paper, I study the aggregate and distributional consequences of trade shocks across different local labor markets and heterogeneous consumers in general equilibrium. I propose a tractable theoretical framework that combines Ricardian-Roy supply forces and comparative advantage with non-homothetic demand in a unified quantitative model. Methodologically, I show how to reconcile heterogeneous consumers characterized by an arbitrary first-order approximation to any demand system derived from utility-maximizing behavior with counterfactual analysis based on relative changes – namely, the exact-hat algebra approach – that does not require identifying the model's unobserved fundamentals. I also derive a generalized welfare change formula that applies to any demand system. Apart from its rich distributional implications, the framework gives rise to the Consumption Inequality channel of aggregate welfare gains. From a research and policymaker's perspective, the framework's main advantages are that it (i) allows for studying the heterogeneous impact of trade policy across consumers with different expenditure patterns, (ii) remains analytically tractable, and (iii) imposes only mild data requirements for counterfactual predictions and welfare analysis.

I apply the model to revisit the impact of the China shock on a sample of 40 countries and the universe of US local labor markets, represented by 722 US Commuting Zones. I find that while the China shock led to aggregate gains for most countries, some countries experienced net losses. The aggregate welfare impact of the shock on the US economy is estimated to be 0.71%, implying larger country-level gains than those reported in much of the previous quantitative trade literature, which is based on the simplifying assumption of constant expenditure shares. A significant part of this increase (0.26%) is attributed to the Consumption Inequality channel that operates via non-homothetic expenditure patterns across sectors. For the US, I also quantify the contribution of each margin of the CI channel. I find that each of these margins accounts for a sizable part of the distributional impact of the China shock across local labor markets and that Commuting Zones with lower mean income were more likely to gain from the shock. At the individual level, the China shock favored relatively poor consumers by reducing the relative prices of low-income elastic sectors. Moreover, the distribution of welfare gains across income quantiles varied substantially between countries.

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# Appendix

## A Proofs

In Appendix A, I provide proofs of the main propositions as well as some auxiliary derivations.

### A.1 Proof of Proposition 1

*Proof.* The proof consists of two parts. The first part applies the aggregation result from Deaton and Muellbauer (1980) to the case of a continuous distribution. The second part exploits the decomposition properties of the Theil index.

**Part i**

By definition, the share of sector $j$ in the aggregate expenditures of country $n$ is the weighted integral of individual market shares:

$$S_{nj} \equiv \frac{\int_{\mathscr{Y}} y s_{nj}(\mathbf{P}_n, y)\, \mathrm{d}H_n(y)}{\int_{\mathscr{Y}} y\, \mathrm{d}H_n(y)} = \int_{\mathscr{Y}} \frac{y}{y_n} s_{nj}(\mathbf{P}_n, y)\, \mathrm{d}H_n(y).$$

So, if $s_{nj}\left(\mathbf{P}_n, y\right) = \alpha_{nj} + \sum_{k=1}^{J} \gamma_{njk} \ln P_{nk} + \beta_{nj} \ln\left(y/a(\mathbf{P}_n)\right)$, we obtain the following:

$$S_{nj} = \int_{\mathscr{Y}} \frac{y}{y_n} s_j\left(\mathbf{P}_n, y\right) \mathrm{d}H_n(y) = \alpha_{nj} + \sum_{k=1}^{J} \gamma_{njk} \ln P_{nk} - \ln a(\mathbf{P}_n) + \beta_{nj} \int_{\mathscr{Y}} \frac{y}{y_n} \ln y\, \mathrm{d}H_n(y).$$

Now, consider the last term of this expression. One can readily verify that

$$\int_{\mathscr{Y}} \frac{y}{y_n} \ln y\, \mathrm{d}H_n(y) = \ln y_n + \int_{\mathscr{Y}} \frac{y}{y_n} \ln y\, \mathrm{d}H(y) - \int_{\mathscr{Y}} \frac{y}{y_n} \ln y_n\, \mathrm{d}H_n(y)$$

$$= \ln y_n + \underbrace{\int_{\mathscr{Y}} \ln\left(y/y_n\right)^{y/y_n}\, \mathrm{d}H_n(y)}_{=\mathcal{T}_n} = \ln y_n + \mathcal{T}_n = \ln\left(y_n \cdot \exp(\mathcal{T}_n)\right).$$

**Part ii**

Next, we derive the mean income in an arbitrary country $n$, $y_n$, and the corresponding country-level Theil index of income inequality.

First, recall that the income distribution in local market $g$ in country $n$ is given by

$$H_{ng}(y) = \exp\left(-\left(y/\Upsilon_{ng}\right)^{-\alpha_{ng}}\right),$$

where $\Upsilon_{ng} \equiv \left(\sum_{j \in \mathcal{J}} A_{ngj} w_{nj}^{\alpha_{ng}}\right)^{1/\alpha_{ng}}$.

Hence, the country-level distribution of income can be represented as a (finite) mixture distribution:

$$H_n(y) = \sum_{g \in \mathcal{G}_n} \ell_{ng} H_{ng}(y),$$

where $\ell_{ng}$ is the labor share of the market $g$ within country $n$ (i.e., $\sum_{g \in \mathcal{G}_n} \ell_{ng} = 1$).

As a result, exploiting the properties of moments of the mixture distribution yields

$$y_n \equiv \int_{\mathscr{Y}} y \, \mathrm{d}H_n(y) = \sum_{g \in \mathcal{G}_n} \ell_{ng} \int_{\mathscr{Y}} y \, \mathrm{d}H_{ng}(y) = \sum_{g \in \mathcal{G}_n} \ell_{ng} \Upsilon_{ng} \Gamma\left(1 - 1/\alpha_{ng}\right)$$

$$= \sum_{g \in \mathcal{G}_n} \ell_{ng} \Gamma\left(1 - 1/\alpha_{ng}\right) \left( \sum_{j \in \mathcal{J}} A_{ngj} w_{nj}^{\alpha_{ng}} \right)^{1/\alpha_{ng}}.$$

Now, I derive the country-level Theil index. By definition of the Theil index we obtain

$$\mathcal{T}_n \equiv \int_{\mathscr{Y}} \frac{y}{y_n} \ln\left(\frac{y}{y_n}\right) \mathrm{d}H_n(y) = \int_{\mathscr{Y}} \frac{y}{y_n} \ln\left(\frac{y}{y_n}\right) \sum_{g \in \mathcal{G}_n} \ell_{ng} \mathrm{d}H_{ng}(y) = \sum_{g \in \mathcal{G}_n} \ell_{ng} \int_{\mathscr{Y}} \frac{y_{ng}}{y_n} \frac{y}{y_{ng}} \ln\left(\frac{y}{y_{ng}} \frac{y_{ng}}{y_n}\right) \mathrm{d}H_{ng}(y)$$

$$= \sum_{g \in \mathcal{G}_n} \ell_{ng} \int_{\mathscr{Y}} \frac{y_{ng}}{y_n} \frac{y}{y_{ng}} \ln\left(\frac{y}{y_{ng}}\right) \mathrm{d}H_{ng}(y) + \sum_{g \in \mathcal{G}_n} \ell_{ng} \int_{\mathscr{Y}} \frac{y_{ng}}{y_n} \frac{y}{y_{ng}} \ln\left(\frac{y_{ng}}{y_n}\right) \mathrm{d}H_{ng}(y)$$

$$= \sum_{g \in \mathcal{G}_n} \ell_{ng} \frac{y_{ng}}{y_n} \mathcal{T}_{ng} + \sum_{g \in \mathcal{G}_n} \ell_{ng} \frac{y_{ng}}{y_n} \ln\left(\frac{y_{ng}}{y_n}\right) \underbrace{\int_{\mathscr{Y}} \frac{y}{y_{ng}} \mathrm{d}H_{ng}(y)}_{=1}.$$

Thus, we obtain

$$\mathcal{T}_n \equiv \int_{\mathscr{Y}} \frac{y}{y_n} \ln\left(\frac{y}{y_n}\right) \mathrm{d}H_n(y) = \underbrace{\sum_{g \in \mathcal{G}_n} \ell_{ng} \frac{y_{ng}}{y_n} \mathcal{T}_{ng}}_{\text{within}_n} + \underbrace{\sum_{g \in \mathcal{G}_n} \ell_{ng} \frac{y_{ng}}{y_n} \ln \frac{y_{ng}}{y_n}}_{\text{between}_n}.$$

Note also that the expression above reflects the decomposability property of the Theil index into the *within*– and *between*–group components. □

## A.2 Proof of Proposition 3

*Proof.* As was previously derived, the Theil index is given by

$$\mathcal{T}_n = \sum_{g \in \mathcal{G}_n} \ell_{ng} \frac{y_{ng}}{y_n} \mathcal{T}_{ng} + \sum_{g \in \mathcal{G}_n} \ell_{ng} \frac{y_{ng}}{y_n} \ln \frac{y_{ng}}{y_n}.$$

Recall that $\ell'_{ng} = \ell_{ng}$ and $\mathcal{T}'_{ng} = \mathcal{T}_{ng}$ for any $n \in \mathcal{N}$ and $g \in \mathcal{G}_n$. Hence, applying *the*

*exact-hat algebra* approach yields

$$\mathcal{T}_n' = \sum_{g\in\mathcal{G}_n} \frac{\ell_{ng}\hat{y}_{ng}y_{ng}}{\hat{y}_n y_n}\mathcal{T}_{ng} + \sum_{g\in\mathcal{G}_n} \frac{\ell_{ng}\hat{y}_{ng}y_{ng}}{\hat{y}_n y_n}\ln\left(\frac{\hat{y}_{ng}y_{ng}}{\hat{y}_n y_n}\right)$$

$$= \sum_{g\in\mathcal{G}_n} \frac{\frac{L_{ng}}{L_n}\hat{y}_{ng}\frac{Y_{ng}}{L_{ng}}}{\hat{y}_n\frac{Y_n}{L_n}}\mathcal{T}_{ng} + \sum_{g\in\mathcal{G}_n} \frac{\frac{L_{ng}}{L_n}\hat{y}_{ng}\frac{Y_{ng}}{L_{ng}}}{\hat{y}_n\frac{Y_n}{L_n}}\left(\ln\frac{\hat{y}_{ng}\frac{Y_{ng}}{L_{ng}}}{\hat{y}_n\frac{Y_n}{L_n}}\right).$$

As a result, a counterfactual value of the Theil index is given by

$$\mathcal{T}_n' = \underbrace{\sum_{g\in\mathcal{G}_n} \frac{\hat{Y}_{ng}Y_{ng}}{\hat{Y}_n Y_n}\mathcal{T}_{ng}}_{\mathcal{T}'_{n,\,within}} + \underbrace{\sum_{g\in\mathcal{G}_n} \frac{\hat{Y}_{ng}Y_{ng}}{\hat{Y}_n Y_n}\ln\left(\frac{\hat{y}_{ng}y_{ng}}{\hat{y}_n y_n}\right)}_{\mathcal{T}'_{n,\,between}}.$$

□

## A.3 Proof of Proposition 4

*Proof.* Consider the equivalent variation formula that applies to any demand system,

$$\frac{\mathrm{d}\mathcal{W}_{ng}(y)}{\mathcal{W}_{ng}(y)} \equiv \frac{EV}{y} = \underbrace{\frac{\mathrm{d}y}{y}}_{\substack{earnings\\ channel}} \underbrace{-\sum_{j=1}^{J} S_{nj}\frac{\mathrm{d}P_{nj}}{P_{nj}}}_{agg.\ expenditure\ effect} \underbrace{-\sum_{j=1}^{J}\left(\tilde{s}_{ngj}-S_{nj}\right)\frac{\mathrm{d}P_{nj}}{P_{nj}}}_{local\ expenditure\ effect} \underbrace{-\sum_{j=1}^{J}\left(s_{ngj}(y)-\tilde{s}_{ngj}\right)\frac{\mathrm{d}P_{nj}}{P_{nj}}}_{individual\ expenditure\ effect}.$$

In this model, the earnings channel is expressed as changes in the expected income of individuals with current income level $y$. Given that labor is perfectly substitutable in production, the earnings channel is equivalent to the (expected changes in the mean income $y_{ng}$). So, we have

$$\mathrm{d}\ln\mathcal{W}_{ng}(y) = \mathrm{d}\ln y_{ng} - \sum_{j=1}^{J} S_{nj}\mathrm{d}\ln P_{nj} - \sum_{j=1}^{J}\left(\tilde{s}_{ngj}-S_{nj}\right)\mathrm{d}\ln P_{nj} - \sum_{j=1}^{J}\left(s_{ngj}(y)-\tilde{s}_{ngj}\right)\mathrm{d}\ln P_{nj}.$$

Now, we want to expand the expression for $\mathrm{d}\ln P_{nj}$. Recall that

$$P_{nj} = \zeta_j\,\Phi_{nj}^{-1/\theta_j} = \zeta_j\left(\sum_{i\in\mathcal{N}} T_{ij}\left(\tau_{inj}w_{ij}\right)^{-\theta_j}\right)^{-1/\theta_j},$$

where $\zeta_j = \left[\Gamma\left(1-\frac{\sigma_j-1}{\theta_j}\right)\right]^{1/(1-\sigma_j)}$ is a sector-specific constant.

Now, note that taking logs and totally differentiating this expression yields

$$\mathrm{d}\ln P_{nj} = -(1/\theta_j)\frac{\sum_{i\in\mathcal{N}}\mathrm{d}\left[T_{ij}(\tau_{inj}w_{ij})^{-\theta_j}\right]}{\sum_{i\in\mathcal{N}}T_{ij}(\tau_{inj}w_{ij})^{-\theta_j}} \iff \mathrm{d}\ln P_{nj} = -\frac{1}{\theta_j}\frac{\sum_{i\in\mathcal{N}}T_{ij}(\tau_{inj}w_{ij})^{-\theta_j}\frac{\mathrm{d}\left[T_{ij}(\tau_{inj}w_{ij})^{-\theta_j}\right]}{T_{ij}(\tau_{inj}w_{ij})^{-\theta_j}}}{\sum_{i\in\mathcal{N}}T_{ij}(\tau_{inj}w_{ij})^{-\theta_j}}$$

$$\mathrm{d}\ln P_{nj} = -(1/\theta_j)\frac{\sum_{i\in\mathcal{N}}T_{ij}(\tau_{inj}w_{ij})^{-\theta_j}\mathrm{d}\ln\left(T_{ij}(\tau_{inj}w_{ij})^{-\theta_j}\right)}{\sum_{i\in\mathcal{N}}T_{ij}(\tau_{inj}w_{ij})^{-\theta_j}}$$

$$\mathrm{d}\ln P_{nj} = \sum_{i\in\mathcal{N}}\frac{T_{ij}(\tau_{inj}w_{ij})^{-\theta_j}}{\sum_{n'\in\mathcal{N}}T_{n'j}(\tau_{n'nj}w_{n'j})^{-\theta_j}}\mathrm{d}\ln(T_{ij}^{-1/\theta_j}\tau_{inj}w_{ij})$$

Recall that $\lambda_{inj} = \frac{T_{ij}(\tau_{inj}w_{ij})^{-\theta_j}}{\sum_{n'}T_{n'j}\left(\tau_{n'nj}w_{n'j}\right)^{-\theta_j}}$. Hence, the formula above is equivalent to

$$\mathrm{d}\ln P_{nj} = \sum_{i\in\mathcal{N}}\lambda_{inj}\mathrm{d}\ln(T_{ij}^{-1/\theta_j}\tau_{inj}w_{ij}).$$

Hence, the formula for $\mathrm{d}\ln W_{ng}$ can be re-written as follows:

$$\mathrm{d}\ln\mathcal{W}_{ng}(y) = \mathrm{d}\ln y_{ng} - \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}}S_{nj}\lambda_{inj}\mathrm{d}\ln(T_{ij}^{-1/\theta_j}\tau_{inj}w_{ij})$$

$$-\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}}\left(\tilde{s}_{ngj}-S_{nj}\right)\lambda_{inj}\mathrm{d}\ln(T_{ij}^{-1/\theta_j}\tau_{inj}w_{ij}) - \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}}\left(s_{ngj}(y)-\tilde{s}_{ngj}\right)\lambda_{inj}\mathrm{d}\ln(T_{ij}^{-1/\theta_j}\tau_{inj}w_{ij}). \tag{A.1}$$

Now, we want to expand the first term of the expression above, $\mathrm{d}\ln y_{ng}$. First, note that $\mathrm{d}\ln y_{ng} = \mathrm{d}\ln Y_{ng}$. Recall that $y_{ng} = \frac{Y_{ng}}{L_{ng}} \propto \left(\sum_{j\in\mathcal{J}}A_{ngj}w_{nj}^{\alpha_{ng}}\right)^{1/\alpha_{ng}}$. Local market $g$'s productivity levels, $\{A_{ngj}\}_{j=1}^{J}$, are assumed to remain constant in a counterfactual scenario. As a result, we obtain

$$\ln Y_{ng} = const + (1/\alpha_{ng})\ln\left(\sum_{j\in\mathcal{J}}A_{ngj}w_{nj}^{\alpha_{ng}}\right),$$

Taking the total differential yields

$$\mathrm{d}\ln Y_{ng} = (1/\alpha_{ng})\,\mathrm{d}\ln\left(\sum_{j\in\mathcal{J}}A_{ngj}w_{nj}^{\alpha_{ng}}\right)$$

$$\mathrm{d}\ln Y_{ng} = (1/\alpha_{ng})\frac{\sum_{j\in\mathcal{J}}A_{ngj}\mathrm{d}\left(w_{nj}^{\alpha_{ng}}\right)}{\sum_{k\in\mathcal{J}}A_{ngk}w_{nk}^{\alpha_{ng}}} \iff \mathrm{d}\ln Y_{ng} = (1/\alpha_{ng})\frac{\sum_{j\in\mathcal{J}}A_{ngj}w_{nj}^{\alpha_{ng}}\frac{\mathrm{d}\left(w_{nj}^{\alpha_{ng}}\right)}{w_{nj}^{\alpha_{ng}}}}{\sum_{k\in\mathcal{J}}A_{ngk}w_{nk}^{\alpha_{ng}}}$$

$$\mathrm{d}\ln Y_{ng} = \frac{1}{\alpha_{ng}} \frac{\sum_{j\in\mathcal{J}} A_{ngj} w_{nj}^{\alpha_{ng}} \mathrm{d}\ln\left(w_{nj}^{\alpha_{ng}}\right)}{\sum_{k\in\mathcal{J}} A_{ngk} w_{nk}^{\alpha_{ng}}}$$

$$\iff \mathrm{d}\ln Y_{ng} = \sum_{j\in\mathcal{J}} \frac{A_{ngj} w_{nj}^{\alpha_{ng}}}{\sum_{k\in\mathcal{J}} A_{ngk} w_{nk}^{\alpha_{ng}}} \mathrm{d}\ln w_{nj}.$$

Recall that $\pi_{ngj} = \frac{A_{ngj} w_{nj}^{\alpha_{ng}}}{\sum_k A_{ngk} w_{nk}^{\alpha_{ng}}}$, so the expression above is equivalent to

$$\implies \mathrm{d}\ln Y_{ng} = \sum_{j\in\mathcal{J}} \pi_{ngj} \mathrm{d}\ln w_{nj}.$$

Hence, we can re-write the expression for $\mathrm{d}\ln W_{ng}$ as follows

$$\mathrm{d}\ln \mathcal{W}_{ng}(y) = \sum_{j\in\mathcal{J}} \pi_{ngj} \mathrm{d}\ln w_{nj} - \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}} S_{nj} \lambda_{inj} \mathrm{d}\ln(T_{ij}^{-1/\theta_j} \tau_{inj} w_{ij})$$

$$- \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}} \left(\tilde{s}_{ngj} - S_{nj}\right) \lambda_{inj} \mathrm{d}\ln(T_{ij}^{-1/\theta_j} \tau_{inj} w_{ij}) - \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}} \left(s_{ngj}(y) - \tilde{s}_{ngj}\right) \lambda_{inj} \mathrm{d}\ln(T_{ij}^{-1/\theta_j} \tau_{inj} w_{ij}). \tag{A.2}$$

Now, note that the formula for trade share $\lambda_{inj} = \frac{T_{ij}(\tau_{inj} w_{ij})^{-\theta_j}}{\sum_{n'} T_{n'j}\left(\tau_{n'nj} w_{n'j}\right)^{-\theta_j}}$. Recall that domestic technology levels, $\{T_{nj}\}_{j=1}^{J}$, are assumed to remain constant. Hence,

$$\ln\left(\lambda_{inj}/\lambda_{nnj}\right) = \ln\left(\frac{T_{ij}^{-1/\theta_j} \tau_{inj} w_{ij}}{T_{nj}^{-1/\theta_j} w_{nj}}\right)^{-\theta_j} \implies \frac{\mathrm{d}\ln\left(\lambda_{inj}/\lambda_{nnj}\right)}{\mathrm{d}\ln\left(T_{ij}^{-1/\theta_j} \tau_{inj} w_{ij}/w_{nj}\right)} = -\theta_j,$$

Similarly, recall that local market $g$'s productivity levels, $\{A_{ngj}\}_{j=1}^{J}$, are assumed to remain constant in a counterfactual scenario. So, one can readily infer that the formula for employment share $\pi_{ngj} = \frac{A_{ngj} w_{nj}^{\alpha_{ng}}}{\sum_k A_{ngk} w_{nk}^{\alpha_{ng}}}$ implies

$$\ln\left(\pi_{ngj}/\pi_{ngk}\right) = \ln\left(\frac{A_{ngj}}{A_{ngk}} \left(\frac{w_{nj}}{w_{nk}}\right)^{\alpha_{ng}}\right) \implies \frac{\mathrm{d}\ln\left(\pi_{ngj}/\pi_{ngk}\right)}{\mathrm{d}\ln\left(\frac{w_{nj}}{A_{ngk}^{1/\alpha_{ng}} w_{nk}}\right)} = \alpha_{ng}.$$

Now, solving for $\mathrm{d}\ln\left(T_{ij}^{-1/\theta_j} \tau_{inj} w_{ij}\right)$ and $\mathrm{d}\ln w_{nj}$, we obtain:

$$\mathrm{d}\ln\left(T_{ij}^{-1/\theta_j} \tau_{inj} w_{ij}\right) = \frac{\mathrm{d}\ln\left(\lambda_{inj}/\lambda_{nnj}\right)}{-\theta_j} + \mathrm{d}\ln w_{nj},$$

$$\mathrm{d}\ln w_{nj} = \frac{\mathrm{d}\ln\left(\pi_{ngj}/\pi_{ngk}\right)}{\alpha_{ng}} + \mathrm{d}\ln\left(A_{ngk}^{1/\alpha_{ng}} w_{nk}\right).$$

Combining these two equations yields

$$\Longrightarrow \mathrm{d}\ln\left(T_{ij}^{-1/\theta_j}\tau_{inj}w_{ij}\right) = \frac{\mathrm{d}\ln\left(\lambda_{inj}/\lambda_{nnj}\right)}{-\theta_j} + \frac{\mathrm{d}\ln\left(\pi_{ngj}/\pi_{ngk}\right)}{\alpha_{ng}} + \mathrm{d}\ln\left(A_{ngk}^{1/\alpha_{ng}}w_{nk}\right)$$

For brevity, I temporarily re-write the expression for $\mathrm{d}\ln\mathcal{W}_{ng}(y)$ in an equivalent, yet a more compact way without decomposing the expenditure channel on different effects:

$$\mathrm{d}\ln\mathcal{W}_{ng}(y) = \sum_{j\in\mathcal{J}}\pi_{ngj}\mathrm{d}\ln w_{nj} - \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}}s_{ngj}(y)\lambda_{inj}\mathrm{d}\ln\left(T_{ij}^{-1/\theta_j}\tau_{inj}w_{ij}\right).$$

Now, combining it with derived expressions for $\mathrm{d}\ln\left(T_{ij}^{-1/\theta_j}\tau_{inj}w_{ij}\right)$ and $\mathrm{d}\ln w_{nj}$ yields

$$\mathrm{d}\ln\mathcal{W}_{ng}(y) =$$

$$\sum_{j\in\mathcal{J}}\pi_{ngj}\left(\frac{\mathrm{d}\ln\frac{\pi_{ngj}}{\pi_{ngk}}}{\alpha_{ng}} + \mathrm{d}\ln(A_{ngk}^{\frac{1}{\alpha_{ng}}}w_{nk})\right) - \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}}s_{ngj}(y)\lambda_{inj}\left(\frac{\mathrm{d}\ln\frac{\lambda_{inj}}{\lambda_{nnj}}}{-\theta_j} + \frac{\mathrm{d}\ln\frac{\pi_{ngj}}{\pi_{ngk}}}{\alpha_{ng}} + \mathrm{d}\ln(A_{ngk}^{\frac{1}{\alpha_{ng}}}w_{nk})\right).$$

One can readily verify that

$$\sum_{j\in\mathcal{J}}\pi_{ngj}\mathrm{d}\ln(A_{ngk}^{\frac{1}{\alpha_{ng}}}w_{nk}) = \mathrm{d}\ln(A_{ngk}^{\frac{1}{\alpha_{ng}}}w_{nk})\left(\sum_{j\in\mathcal{J}}\pi_{ngj}\right) = \mathrm{d}\ln(A_{ngk}^{\frac{1}{\alpha_{ng}}}w_{nk})$$

and

$$-\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}}s_{ngj}(y)\lambda_{inj}\mathrm{d}\ln(A_{ngk}^{\frac{1}{\alpha_{ng}}}w_{nk}) = -\mathrm{d}\ln(A_{ngk}^{\frac{1}{\alpha_{ng}}}w_{nk})\left(\sum_{j\in\mathcal{J}}s_{ngj}(y)\sum_{i\in\mathcal{N}}\lambda_{inj}\right) = -\mathrm{d}\ln(A_{ngk}^{\frac{1}{\alpha_{ng}}}w_{nk}).$$

So, we obtain

$$\mathrm{d}\ln\mathcal{W}_{ng}(y) = \sum_{j\in\mathcal{J}}\pi_{ngj}\left(\frac{\mathrm{d}\ln\frac{\pi_{ngj}}{\pi_{ngk}}}{\alpha_{ng}}\right) - \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}}s_{ngj}(y)\lambda_{inj}\left(\frac{\mathrm{d}\ln\frac{\lambda_{inj}}{\lambda_{nnj}}}{-\theta_j} + \frac{\mathrm{d}\ln\frac{\pi_{ngj}}{\pi_{ngk}}}{\alpha_{ng}}\right).$$

Next, recalling that $\sum_{j=1}^{J}\pi_{ngj} = 1$ and $\sum_{j=1}^{J}\mathrm{d}\pi_{ngj} = 0$, one can readily verify that

$$\sum_{j\in\mathcal{J}}\pi_{ngj}\left(\frac{\mathrm{d}\ln\frac{\pi_{ngj}}{\pi_{ngk}}}{\alpha_{ng}}\right) = \frac{1}{\alpha_{ng}}\left[\sum_{j\in\mathcal{J}}\mathrm{d}\pi_{ngj} - \sum_{j\in\mathcal{J}}\pi_{ngj}\mathrm{d}\ln\pi_{ngk}\right] = -\frac{\mathrm{d}\ln\pi_{jgk}}{\alpha_{ng}}.$$

Hence, rearranging terms, formula for $\mathrm{d}\ln\mathcal{W}_{ng}(y)$ can be re-written as follows

$$\mathrm{d}\ln\mathcal{W}_{ng}(y) = -\frac{\mathrm{d}\ln\pi_{ngk}}{\alpha_{ng}} - \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}}s_{ngj}(y)\lambda_{inj}\left(\frac{\mathrm{d}\ln\frac{\lambda_{inj}}{\lambda_{nnj}}}{-\theta_j} + \frac{\mathrm{d}\ln\frac{\pi_{ngj}}{\pi_{ngk}}}{\alpha_{ng}}\right)$$

$$= -\frac{\mathrm{d}\ln \pi_{ngk}}{\alpha_{ng}} - \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}} s_{ngj}(y)\lambda_{inj}\left(\frac{\mathrm{d}\ln \lambda_{nnj}}{\theta_j} + \frac{\mathrm{d}\ln \pi_{ngj}}{\alpha_{ng}}\right) - \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}} s_{ngj}(y)\lambda_{inj}\left(\frac{\mathrm{d}\ln \lambda_{inj}}{-\theta_j} - \frac{\mathrm{d}\ln \pi_{ngk}}{\alpha_{ng}}\right)$$

$$= -\frac{\mathrm{d}\ln \pi_{ngk}}{\alpha_{ng}} - \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}} s_{ngj}(y)\lambda_{inj}\left(\frac{\mathrm{d}\ln \lambda_{nnj}}{\theta_j} + \frac{\mathrm{d}\ln \pi_{ngj}}{\alpha_{ng}}\right) + \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}} s_{ngj}(y)\lambda_{inj}\left(\frac{\mathrm{d}\ln \lambda_{inj}}{\theta_j} + \frac{\mathrm{d}\ln \pi_{ngk}}{\alpha_{ng}}\right).$$

Now, note that the third term $\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}} s_{ngj}(y)\lambda_{inj}\left(\frac{\mathrm{d}\ln \lambda_{inj}}{\theta_j} + \frac{\mathrm{d}\ln \pi_{ngk}}{\alpha_{ng}}\right)$ is equal to

$$\sum_{j\in\mathcal{J}} \frac{s_{ngj}(y)}{\theta_j}\sum_{i\in\mathcal{N}} \mathrm{d}\lambda_{inj} + \frac{\mathrm{d}\ln \pi_{ngk}}{\alpha_{ng}}\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}} s_{ngj}(y)\lambda_{inj} = \frac{\mathrm{d}\ln \pi_{ngk}}{\alpha_{ng}}.$$

As a result, we obtain the following expression for $\mathrm{d}\ln \mathcal{W}_{ng}(y)$:

$$\mathrm{d}\ln \mathcal{W}_{ng}(y) = -\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}} s_{ngj}(y)\lambda_{inj}\left(\frac{\mathrm{d}\ln \lambda_{nnj}}{\theta_j} + \frac{\mathrm{d}\ln \pi_{ngj}}{\alpha_{ng}}\right).$$

Recalling that $\sum_{i=1}^{N}\lambda_{inj} = 1$ yields

$$\mathrm{d}\ln \mathcal{W}_{ng}(y) = -\sum_{j\in\mathcal{J}} s_{ngj}(y)\left(\frac{\mathrm{d}\ln \lambda_{nnj}}{\theta_j} + \frac{\mathrm{d}\ln \pi_{ngj}}{\alpha_{ng}}\right).$$

$$\iff \mathrm{d}\ln \mathcal{W}_{ng}(y) = \sum_{j\in\mathcal{J}}\left(\mathrm{d}\ln \lambda_{nnj}^{-s_{ngj}(y)/\theta_j} + \mathrm{d}\ln \pi_{ngj}^{-s_{ngj}(y)/\alpha_{ng}}\right)$$

$$\iff \mathrm{d}\ln \mathcal{W}_{ng}(y) = -\sum_{j\in\mathcal{J}} \frac{s_{ngj}(y)}{\theta_j}\mathrm{d}\ln \lambda_{nnj} - \sum_{j\in\mathcal{J}} \frac{s_{ngj}(y)}{\alpha_{ng}}\mathrm{d}\ln \pi_{ngj}. \tag{A.3}$$

As a result, keeping the expenditure shares fixed, we obtain the following first-order approximation for the exact welfare change:[24]

$$\ln \hat{\mathcal{W}}_{ng}(y) \approx -\sum_{j\in\mathcal{J}} \frac{s_{ngj}(y)}{\theta_j}\ln \hat{\lambda}_{nnj} - \sum_{j\in\mathcal{J}} \frac{s_{ngj}(y)}{\alpha_{ng}}\ln \hat{\pi}_{ngj}. \tag{A.4}$$

Note that one can equivalently rewrite the expression above as

$$\hat{\mathcal{W}}_{ng}(y) \approx \underbrace{\prod_{j\in\mathcal{J}} \hat{\lambda}_{nnj}^{-s_{ngj}(y)/\theta_j}}_{consumer\ Ricardian-type\ gains} \cdot \underbrace{\prod_{j\in\mathcal{J}} \hat{\pi}_{ngj}^{-s_{ngj}(y)/\alpha_{ng}}}_{consumer\ Roy-type\ gains}, \tag{A.5}$$

[24] See also Galle, Rodríguez-Clare, and Yi (2023) for derivations of the welfare gains formula under Cobb-Douglas preferences across sectors.

*where* $\hat{\lambda}_{nnj} = \frac{\hat{w}_{nj}^{-\theta_j}}{\sum_{n'} \lambda_{n'nj} \hat{T}_{n'j} \left(\hat{\tau}_{n'nj} \hat{w}_{n'j}\right)^{-\theta_j}}$ *and* $\hat{\pi}_{ngj} = \frac{\hat{w}_{nj}^{\alpha_{ng}}}{\sum_k \pi_{ngk} \hat{w}_{nk}^{\alpha_{ng}}}$. □

## A.4 Proof of Proposition 7

*Proof.* The proof is based on a general formula (B.2) for decomposing a welfare change as a response to small price changes. Here, I specify the implications of this formula under the model's supply and demand side.

First, note that $\sum_{j=1}^{J} S_{nj} = 1$ implies that $-\sum_{j=1}^{J} S_{nj} \frac{\mathrm{d}P_{nj}}{P_{nj}} = 1 - \sum_{j=1}^{J} S_{nj} \hat{P}_{nj}$. Note also that the Almost Ideal Demand System implies $(\tilde{s}_{ngj} - S_{nj}) = \ln\left(\frac{y_{ng} \exp(\mathcal{T}_{ng})}{y_n \exp(\mathcal{T}_n)}\right) \beta_{nj}$ and $(s_{ngj}(y) - \tilde{s}_{ngj}) = \ln\left(\frac{y/y_{ng}}{\exp(\mathcal{T}_{ng})}\right) \beta_{nj}$. Moreover, by definition of sectoral income (semi-)elasticities, $\sum_{j=1}^{J} \beta_{nj} = 0$. Employing these results yields

$$\sum_{j=1}^{J} (\tilde{s}_{ngj} - S_{nj}) \frac{\mathrm{d}P_{nj}}{P_{nj}} = -\ln\left(\frac{y_{ng} \exp(\mathcal{T}_{ng})}{y_n \exp(\mathcal{T}_n)}\right) \sum_{j=1}^{J} \beta_{nj} \hat{P}_{nj},$$

$$\sum_{j=1}^{J} (s_{ngj}(y) - \tilde{s}_{ngj}) \frac{\mathrm{d}P_{nj}}{P_{nj}} = -\ln\left(\frac{y/y_{ng}}{\exp(\mathcal{T}_{ng})}\right) \sum_{j=1}^{J} \beta_{nj} \hat{P}_{nj}.$$

Now, recalling that $\hat{P}_{nj} = \hat{w}_{nj} \left(\hat{\lambda}_{nnj} / \hat{T}_{nj}\right)^{1/\theta_j}$, yields the following expression:

$$\frac{\Delta \mathcal{W}_{ng}(y)}{\mathcal{W}_{ng}(y)} \approx$$

$$\underbrace{\hat{y}_{ng} - 1}_{\substack{earnings \\ channel}} + \underbrace{1 - \sum_{j=1}^{J} S_{nj} \frac{\hat{w}_{nj} \hat{\lambda}_{nnj}^{1/\theta_j}}{\hat{T}_{nj}^{1/\theta_j}}}_{agg.\ expenditure\ effect} - \underbrace{\ln\left(\frac{y_{ng} \exp(\mathcal{T}_{ng})}{y_n \exp(\mathcal{T}_n)}\right) \sum_{j=1}^{J} \beta_{nj} \frac{\hat{w}_{nj} \hat{\lambda}_{nnj}^{1/\theta_j}}{\hat{T}_{nj}^{1/\theta_j}}}_{local\ expenditure\ effect} - \underbrace{\ln\left(\frac{y/y_{ng}}{\exp(\mathcal{T}_{ng})}\right) \sum_{j=1}^{J} \beta_{nj} \frac{\hat{w}_{nj} \hat{\lambda}_{nnj}^{1/\theta_j}}{\hat{T}_{nj}^{1/\theta_j}}}_{individual\ expenditure\ effect},$$

*where* $\hat{y}_{ng} = \left(\sum_{j=1}^{J} \pi_{ngj} \hat{A}_{ngj} \hat{w}_{nj}^{\alpha_{ng}}\right)^{1/\alpha_{ng}}$ *and* $\hat{\lambda}_{nnj} = \frac{\hat{T}_{nj} \hat{w}_{nj}^{-\theta_j}}{\sum_{n'}^{N} \lambda_{n'nj} \hat{T}_{n'j} \left(\hat{\tau}_{n'nj} \hat{w}_{n'j}\right)^{-\theta_j}}$. □

# B Online Appendix

In this Appendix, I discuss some supplementary results of the main part of the paper.

## B.1 Decomposition of the individual welfare gains

As noted in the main body of the paper, Proposition 4 also implies the possibility a further decomposition of the Ricardian- and Roy-type gains into the individual, local, and aggregate components, respectively:

$$\ln \hat{\mathcal{W}}_{ng}(y) \approx \underbrace{\sum_{j=1}^{J} \left(s_{ngj}(y) - \tilde{s}_{ngj}\right) \ln \hat{\lambda}_{nnj}^{-1/\theta_j}}_{\textit{individual Ricardian gains}} + \underbrace{\sum_{j=1}^{J} \left(s_{ngj}(y) - \tilde{s}_{ngj}\right) \ln \hat{\pi}_{ngj}^{-1/\alpha_{ng}}}_{\textit{individual Roy gains}} + \underbrace{\sum_{j=1}^{J} \left(\tilde{s}_{ngj} - S_{nj}\right) \ln \hat{\lambda}_{nnj}^{-1/\theta_j}}_{\textit{local Ricardian gains}}$$

$$+ \underbrace{\sum_{j=1}^{J} \left(\tilde{s}_{ngj} - S_{nj}\right) \ln \hat{\pi}_{ngj}^{-1/\alpha_{ng}}}_{\textit{local Roy gains}} + \underbrace{\sum_{j=1}^{J} S_{nj} \ln \hat{\lambda}_{nnj}^{-1/\theta_j}}_{\textit{aggregate Ricardian gains}} + \underbrace{\sum_{j=1}^{J} S_{nj} \ln \hat{\pi}_{ngj}^{-1/\alpha_{ng}}}_{\textit{aggregate Roy gains}}, \tag{B.1}$$

where $\tilde{s}_{ngj}$ denotes a local aggregate expenditure share on sector $j \in \mathcal{J}$ for consumers at a labor market $g \in \mathcal{G}_n$.

## B.2 Earnings vs. Expenditure effects of welfare change

In this subsection, I decompose the total welfare change into the *earnings/wage channel* and several distinct types of *expenditure effects*. In particular, one can readily verify that the welfare change of income-$y$ individuals in local market $g$ in country $n$ can be decomposed into four channels:

$$\frac{\mathrm{d}\mathcal{W}_{ng}(y)}{\mathcal{W}_{ng}(y)} \equiv \frac{EV}{y} = \underbrace{\frac{\mathrm{d}y}{y}}_{\substack{\textit{earnings} \\ \textit{channel}}} \underbrace{- \sum_{j=1}^{J} S_{nj} \frac{\mathrm{d}P_{nj}}{P_{nj}}}_{\textit{agg. expenditure effect}} \underbrace{- \sum_{j=1}^{J} \left(\tilde{s}_{ngj} - S_{nj}\right) \frac{\mathrm{d}P_{nj}}{P_{nj}}}_{\textit{local expenditure effect}} \underbrace{- \sum_{j=1}^{J} \left(s_{ngj}(y) - \tilde{s}_{ngj}\right) \frac{\mathrm{d}P_{nj}}{P_{nj}}}_{\textit{individual expenditure effect}}, \tag{B.2}$$

where $\tilde{s}_{ngj}$ denotes an aggregate share of sector $j$ in local market $g$ in country $n$.

The proof is straightforward given the formula (3.2). In particular, it is just an alternative way of representing the expenditure effect as a sum of the aggregate, local, and individual expenditure effects, respectively.

Now, I revisit the implications of the formula above under the Almost Ideal Demand System. Recall that the model's Roy-like labor supply implies that workers self-select into differ-

ent sectors based on their comparative advantage. As a result, when considering a particular worker, any trade or technology shock may force this worker to change their employment sector. Thus, when studying the welfare effects of a shock, I study the expected welfare change for individuals with income level $y$ in country $n$ in local market $g$. Specifically, the earnings/wage channel of welfare change represents the expected change in the income of individuals with current income $y$, i.e., the change averaged (weighted) across all sectors. The following proposition provides a formal statement.

**Proposition 7.** *Given a shock to trade costs and/or technology levels, a first-order approximation to the (expected) welfare change of income-$y$ individuals in local market $g$ in country $n$ is given by*

$$\frac{\Delta \mathcal{W}_{ng}(y)}{\mathcal{W}_{ng}(y)} \approx$$

$$\underbrace{\hat{y}_{ng} - 1}_{\substack{earnings \\ channel}} + \underbrace{1 - \sum_{j=1}^{J} S_{nj} \frac{\hat{w}_{nj} \hat{\lambda}_{nnj}^{1/\theta_j}}{\hat{T}_{nj}^{1/\theta_j}}}_{agg.\ expenditure\ effect} - \underbrace{\ln\left(\frac{y_{ng}\exp(\mathcal{T}_{ng})}{y_n \exp(\mathcal{T}_n)}\right) \sum_{j=1}^{J} \beta_{nj} \frac{\hat{w}_{nj} \hat{\lambda}_{nnj}^{1/\theta_j}}{\hat{T}_{nj}^{1/\theta_j}}}_{local\ expenditure\ effect} - \underbrace{\ln\left(\frac{y/y_{ng}}{\exp(\mathcal{T}_{ng})}\right) \sum_{j=1}^{J} \beta_{nj} \frac{\hat{w}_{nj} \hat{\lambda}_{nnj}^{1/\theta_j}}{\hat{T}_{nj}^{1/\theta_j}}}_{individual\ expenditure\ effect}, \tag{B.3}$$

*where* $\hat{y}_{ng} = \left(\sum_{j=1}^{J} \pi_{ngj} \hat{A}_{ngj} \hat{w}_{nj}^{\alpha_{ng}}\right)^{1/\alpha_{ng}}$ *and* $\hat{\lambda}_{nnj} = \frac{\hat{T}_{nj}(\hat{\tau}_{nnj}\hat{w}_{nj})^{-\theta_j}}{\sum_{n'}^{N} \lambda_{n'nj} \hat{T}_{n'j} \left(\hat{\tau}_{n'nj}\hat{w}_{n'j}\right)^{-\theta_j}}$.

*Proof.* See Appendix A.4. □

Proposition 7 exploits the result that changes in a sectoral price index are driven by changes in sectoral wages, trade shares, and productivity: $\hat{P}_{nj} = \hat{w}_{nj}\left(\hat{\lambda}_{nnj}/\hat{T}_{nj}\right)^{1/\theta_j}$ for any $n \in N$ and $j \in J$. One can readily verify that i) *the earnings channel* (EC) operates at the local level, ii) *the aggregate expenditure effect* (AEE) operates at the country level, iii) *the local expenditure effect* (LEE) operates at the local level, and iv) *the individual expenditure effect* (IEE) operates at the local level along the income distribution.

Note that nothing requires any of these channels to be always positive or negative. In particular, the earnings channel is positive (negative) if $\hat{y}_{ng} > 1\ (< 1)$. The aggregate expenditure effect, in turn, is positive (negative) if $\sum_{j=1}^{J} S_{nj} \hat{w}_{nj} \hat{\lambda}_{nnj}^{1/\theta_j} / \hat{T}_{nj}^{1/\theta_j} < 1\ (> 1)$. Put it differently, it is positive if the "weighted" average of relative changes of sectoral prices is lower than 1.[25]

As for the local and individual expenditure effects, note that $\sum_{j=1}^{J} \beta_{nj} = 0$ directly implies that $\sum_{j=1}^{J} \beta_{nj} \frac{\hat{w}_{nj} \hat{\lambda}_{nnj}^{1/\theta_j}}{\hat{T}_{nj}^{1/\theta_j}} = J \cdot Cov_j\left(\beta_{nj}, \frac{\hat{w}_{nj} \hat{\lambda}_{nnj}^{1/\theta_j}}{\hat{T}_{nj}^{1/\theta_j}}\right)$, or, equivalently, $\sum_{j=1}^{J} \beta_{nj} \hat{P}_{nj} = J \cdot Cov_j\left(\beta_{nj}, \hat{P}_{nj}\right)$. Hence, depending on a particular shock, the covariance between the vector of sectoral price changes and income semi-elasticities of market share, $Cov_j\left(\beta_{nj}, \hat{P}_{nj}\right)$,

[25] Note also that $\sum_{j=1}^{J} S_{nj} \hat{w}_{nj} \left(\hat{\lambda}_{nnj}/\hat{T}_{nj}\right)^{1/\theta_j} = J \cdot Cov_j\left(S_{nj}, \hat{P}_{nj}\right) + \frac{1}{J} \sum_{j=1}^{J} \hat{P}_{nj}$.

can be either positive or negative. Moreover, the term $\ln\left(\frac{y/y_{ng}}{\exp(\mathcal{T}_{ng})}\right)$ in the expression for the individual expenditure effect directly implies that the sign of the effect for relatively rich local consumers (i.e., those consumers who are richer than the local inequality-adjusted mean income, $y_{ng}\exp(\mathcal{T}_{ng})$) is the opposite of the sign of the expenditure effect for relatively poor consumers. Similarly, the term $\ln\left(\frac{y_{ng}\exp(\mathcal{T}_{ng})}{y_n\exp(\mathcal{T}_n)}\right)$ in the expression for the local expenditure effect states that the sign of the effect for relatively rich local labor markets (i.e., commuting zones with the local inequality-adjusted mean income higher than the country-level one, $y_n\exp(\mathcal{T}_n)$) is the opposite of the sign of the local effect for relatively poor local labor markets.

### B.2.1 Homothetic vs. Non-homothetic channels of welfare change

Apart from the decomposition of welfare changes into the earning and expenditure effects discussed earlier, the welfare change can also be alternatively decomposed into just two separate channels: the *homothetic* channel and the *non-homothetic* channel, respectively. Note that the non-homothetic channel does not entirely absorb all of the expenditure effects. In particular, a part of the aggregate expenditure effect (AEE) is driven by the homothetic channel. To isolate the non-homothetic part of the aggregate expenditure effect, I rewrite it as follows

$$\text{AEE}_n = 1 - \sum_{j=1}^{J} S_{nj}\hat{P}_{nj} = 1 - \sum_{j=1}^{J}\left(S_{nj} - \beta_{nj}\ln\frac{y}{a(\mathbf{P}_n)}\right)\hat{P}_{nj} - \ln\left(\frac{y}{a(\mathbf{P}_n)}\right)\sum_{j=1}^{J}\beta_{nj}\hat{P}_{nj}, \quad \text{(B.4)}$$

where $\hat{P}_{nj} = \hat{w}_{nj}\left(\hat{\lambda}_{nnj}/\hat{T}_{nj}\right)^{1/\theta_j}$.

One can readily verify that the local expenditure effect and individual expenditure effects are entirely absorbed by the non-homothetic channel. As a result, one can derive the following first-order welfare decomposition of a given shock into the homothetic and non-homothetic channels.

$$\frac{\Delta\mathcal{W}_{ng}(y)}{\mathcal{W}_{ng}(y)} \approx \underbrace{\hat{y}_{ng} - \sum_{j=1}^{J}\left(S_{nj} - \beta_{nj}\ln\left(\frac{y}{a(\mathbf{P}_n)}\right)\right)\frac{\hat{w}_{nj}\hat{\lambda}_{nnj}^{1/\theta_j}}{\hat{T}_{nj}^{1/\theta_j}}}_{homothetic\ channel} \underbrace{- \left(\ln\frac{y^2}{a(\mathbf{P}_n)y_n\exp(\mathcal{T}_n)}\right)\sum_{j=1}^{J}\beta_{nj}\frac{\hat{w}_{nj}\hat{\lambda}_{nnj}^{1/\theta_j}}{\hat{T}_{nj}^{1/\theta_j}}}_{non-homothetic\ channel} \quad \text{(B.5)}$$

The proof is straightforward given Proposition 7 and formula (B.4).