Macroprudential Policy According to HANK *

Javier Bianchi
Federal Reserve Bank of Minneapolis

Louphou Coulibaly
University of Wisconsin-Madison and NBER

Xing Xu
University of Minnesota

October 2025

PRELIMINARY AND INCOMPLETE

Link to Latest Draft

Abstract

This paper studies the design of macroprudential policy in a heterogeneous agent New Keynesian (HANK) model. We present an open economy model with downward nominal wage rigidity and uninsurable unemployment risk. We show that the presence of unemployment risk at the extensive margin amplifies the welfare costs of aggregate demand externalities. We solve for the optimal schedule of ex-ante and ex-post taxes on borrowing that maximize households' welfare.

*NOT FOR CIRCULATION. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis, the Federal Reserve System.

1 Introduction

Over the last 15 years, there has been a growing consensus on the need for macroprudential policies. A key theoretical justification for these policies is based on aggregate demand externalities, which lead households to consume excessively during booms and insufficiently during recessions (e.g., Schmitt-Grohé and Uribe, 2016; Farhi and Werning, 2016). However, much of the normative literature on macroprudential policy has relied on models with limited heterogeneity, where agents can share idiosyncratic risk. At the same time, a large literature has employed Heterogeneous Agent New Keynesian (HANK) models to study monetary transmission in environments with uninsurable idiosyncratic shocks (e.g., Kaplan, Moll and Violante, 2018; Auclert, Rognlie and Straub, 2018). However, this literature has largely abstracted from the role of macroprudential policy. A key challenge in extending macroprudential analysis to HANK is the need to capture the non-linear response to changes in policy and the wealth distribution.

In this paper, we study optimal macroprudential policy in an economy with idiosyncratic unemployment risk and downward nominal wage rigidity. The presence of household heterogeneity generates a two-way interaction: macroprudential policy affects households asymmetrically depending on their financial and employment position, while heterogeneity itself shapes aggregate demand and alters the transmission of macroprudential interventions. We show that unemployment risk reinforces the case for ex-ante borrowing restrictions through two channels. First, there is a risk-sharing channel, as macroprudential policy reduces the probability of a low consumption in an unemployment state. Second, there is an aggregate demand channel, as a more dispersed distribution of income lowers aggregate consumption and raises unemployment.

Literature: Schmitt-Grohé and Uribe (2021); Farhi and Werning (2016); Acharya, Challe and Dogra (2023); Farhi and Werning (2016); Heathcote and Perri (2018); Ravn and Sterk (2021); Bhandari, Evans, Golosov and Sargent (2021); Bianchi and Lorenzoni (2021); Bianchi and Coulibaly (2021) [to be completed]

2 Model

We consider a small open economy populated by infinitely-lived households of measure one subject to idiosyncratic unemployment risk, a representative firm and a government. Our focus will be on an economy that is a member of a monetary union and thus cannot dictate monetary policy.

2.1 Households

Households are ex ante identical. Each household j in our small open economy has preferences given by the following utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_{j,t})$$

where $\beta \in (0,1)$ is the discount factor and the composite consumption $c_{j,t}$ is a composite of tradable consumption $c_{j,t}^T$ and non-tradable consumption $c_{j,t}^N$ according to a constant elasticity of substitution aggregator

$$c_{j,t} = \left[\phi\left(c_{j,t}^T\right)^{\frac{\gamma-1}{\gamma}} + (1-\phi)\left(c_{j,t}^N\right)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}},$$

where $\phi \in (0,1)$. The elasticity of substitution between tradable and non-tradable consumption is γ. For convenience, we use $u(c_{j,t}^T, c_{j,t}^N)$ to denote the utility as a function of the two consumption goods.

Each period, household j is endowed with y_t^T units of tradable goods and receives labor endowment $\overline{h}_j$. In the absence of rationing in the labor market, the household supplies her full labor endowment inelastically, and, the economy operates at full employment with aggregate employment being $H_t = \overline{h} \equiv \int \overline{h}_j dj$. However, due to the presence of nominal wage rigidities (to be described below), the economy might experience involuntary unemployment $H_t < \overline{h}$ and the household might be able to sell only a fraction $\lambda_{j,t}$ of his full labor endowment.

We assume that λ_{jt} is drawn independently and identically from a Bernoulli distribution with probability $\frac{H_t}{\overline{h}}$, specifically,

$$\lambda_{j,t} = \begin{cases} \lambda^L, & \text{with probability } 1 - \frac{H_t}{\overline{h}}, \\ \lambda^H, & \text{with probability } \frac{H_t}{\overline{h}}, \end{cases} \tag{1}$$

where $\frac{H_t}{\overline{h}}$ represents the aggregate impact of labor demand shortfall and λ^L and λ^H are such that $\mathbb{E}[\lambda_{j,t}] = \frac{H_t}{\overline{h}}$ to ensure labor market clearing. In particular, the parameterization we consider is

$$\lambda^L = (1-\xi)\frac{H_t}{\overline{h}} \quad \text{and} \quad \lambda^H = \xi + (1-\xi)\frac{H_t}{\overline{h}}, \tag{2}$$

where $\xi \in [0,1]$ governs the cross-sectional variance, which takes the form

$$\text{Var}(\lambda_{jt}) = \xi^2 \frac{H_t}{\overline{h}}\left(1 - \frac{H_t}{\overline{h}}\right),$$

A higher ξ gives rise to higher uncertainty over individual labor supply. Importantly, the two extremes of the value ξ entails zero uncertainty or full unemployment risks. When $\xi = 0$, both outcomes collapse to $\lambda^L = \lambda^H = \frac{H_t}{\overline{h}}$; that is, $\lambda_{j,t}$ is degenerate and the model reduces to a representative agent framework (hereby referred to as '**RANK**'), as in Schmitt-Grohé and Uribe (2016). When $\xi = 1$, we have that

$$\lambda_{j,t} = \begin{cases} 0, & \text{with probability } 1 - \frac{H_t}{\overline{h}}, \\ 1, & \text{with probability } \frac{H_t}{\overline{h}}, \end{cases}$$

Under this formulation, when labor is rationed and there is involuntary unemployment, $H_t < \overline{h}$, households are randomly selected into full unemployment and the variance of the cross-sectional income distribution increases endogenously (hereby referred to as '**HANK**').

We assume full liability dollarization by considering that each household can only trade a one-period non-state-contingent bonds denominated in units of tradables. The bond is potentially subject to a tax τ_t (subsidy if negative) on debt issuance. Household j's budget constraint expressed in units of tradables can therefore be expressed as

$$c^T_{j,t} + \frac{P^N_t}{P^T_t} c^N_{j,t} + \frac{1}{1+\tau_t}\frac{b_{j,t+1}}{R^*_t} = \frac{W_t}{P^T_t}\lambda_{j,t}\overline{h}_j + \phi^N_t + y^T_t + T_{j,t} + b_{j,t} \tag{3}$$

where $b_{j,t+1}$ denote household's j holding of the bond which pays an exogenous gross return R^*_t units of tradables, T_{jt} is the transfer received from the government, W_t is the wage rate in domestic currency, P^N_t is the price of the non-tradable good and P^T_t denote the price of tradable goods in units of domestic currency. The nominal exchange rate e_t is defined as the price of the foreign currency in terms of the domestic currency.

We assume that the law of one price holds for the tradable good, that is $P^T_t = e_t P^{T*}_t$,

where P_t^{T*} is the price of the tradable good denominated in foreign currency which we normalize to unity. Finally, each household faces an exogenous borrowing constraint,

$$\frac{b_{j,t+1}}{R_t^*} \geq -\bar{b}. \tag{4}$$

The optimal allocation of resources to the consumption of the two goods implies that the household equate her marginal rate of substitution between tradable and nontradable goods to the relative price of these goods. Because of homothetic preferences, this implies that the consumption of tradables relative to that of non-tradables for each household depends only on the relative prices of these two goods,

$$\frac{P_t^N}{e_t} = \frac{1-\phi}{\phi}\left(\frac{c_{j,t}^T}{c_{j,t}^N}\right)^{\frac{1}{\gamma}}$$

Hence, the ratio of $c_{j,t}^T / c_{j,t}^N$ is equated for all agents.

2.2 Firms and Nominal Rigidities

Firms. The non-tradable good is produced by a continuum of firms in a perfectly competitive market. Each firm produces a non-tradable good according to a production technology given by $y_t^N = H_t$ and obtain profits given by

$$\phi_t^N = P_t^N H_t - W_t H_t.$$

Given the homogeneous production function, we obtain that in equilibrium $W_t = P_t^N$. An individual firm is therefore indifferent between any levels of employment.

Nominal rigidities. We introduced nominal rigidities by assuming that the nominal wage is downwardly rigid. In particular, we follow Schmitt-Grohé and Uribe (2016) and consider that the wage can't fall by more than a fraction $\rho_w \in [0,1]$ of the previous period wage,

$$W_t \geq \rho_w W_{t-1}$$

where ρ_w indicates the degree of the downward nominal rigidity. Because of this downward nominal wage rigidity, the labor market might experience rationing. The equilibrium

in the labor market can then be summarized by the following condition

$$(W_t - \rho_w W_{t-1})(H_t - \overline{h}) = 0. \tag{5}$$

This condition states if the market clearing wage exceeds the minimum wage in nominal terms $\rho_w W_{t-1}$, the equilibrium employment must equal the aggregate endowment of labor $H_t = \overline{h}$. Otherwise, hours are determined by labor demand $H_t < \overline{h}$, and $W_t = W_{t-1}$. When there is rationing in the labor market and the economy features involuntary unemployment, the impact on the allocation of hours across households is heterogeneous and given by (1).

2.3 Government

We assume that the economy is under a currency peg, that is the monetary authority commits to fix the nominal exchange rate $e_t = \bar{e}$ for all t. The government sets a tax on bond issuances τ_t and rebates it back to households. As is common in the literature, this tax can be interpreted as a capital control or as a macroprudential policy (see e.g., Bianchi, 2011; Schmitt-Grohé and Uribe, 2016; and Fornaro and Romei, 2019). The government transfer is then set to avoid any redistributive effect on wealth, i.e. for all households

$$T_{j,t} = -\frac{\tau_t}{1+\tau_t}\frac{b_{j,t+1}}{R_t^*} \tag{6}$$

That is, each agent takes as given the lump-sum transfer received by the government, but it receives in equilibrium an amount equal to the amount paid.

Note that this nests the case when there are no inequalities in household bond holdings, the transfer is lump sum and identical across households.

2.4 Competitive Equilibrium

An equilibrium requires market clearing in the non-tradable sector, as well as market clearing for labor. We define a competitive equilibrium as follows

Definition 1. Given an initial wage W_{-1} and an initial distribution of bond holdings, an equilibrium is defined by a set of government policies $\{\tau_t, T_{j,t}\}_{t\geq 0}$, a sequence of allocation for individual households $\{c_{j,t}^T, c_{j,t}^N, b_{j,t+1}\}_{t\geq 0}$ for any j, firms decisions $\{H_t\}_{t\geq 0}$, measures $\{\mu_{j,t}\}_{t\geq 0}$ and prices $\{W_t, P_t^N, P_t^T\}_{t\geq 0}$ such that

(i) households and firms optimize given prices and government policies;

(ii) the sequence of distributions satisfies aggregate consistency conditions

$$B_{t+1} = \int b_{j,t+1} d\mu_{j,t}, \quad c_T^T = \int c_{j,t}^T d\mu_{j,t}, \quad c_T^N = \int c_{j,t}^N d\mu_{j,t}; \tag{7}$$

(iii) the law of one price holds, $P_t^T = \bar{e}$ for all t;

(iv) the government budget constraint (6) is satisfied;

(v) labor market clears, $H_t = \int \lambda_{j,t} \bar{h}_j dj$ and (5) is satisfied;

(vi) market for the non-tradable good clears, $c_t^N = H_t$.

These conditions imply that the following resource constraint for tradables or balance of payment condition:

$$y_t - c_t^T = \frac{B_{t+1}}{R^*} - B_t. \tag{8}$$

3 Optimal Policy in a Tractable HANK

In this section, we investigate the optimal macroprudential policy in an analytically tractable model where households are ex-ante identical $\bar{h}_j = \bar{h}$ for all j and face idiosyncratic unemployment risk ex-post when there is labor rationing as described by (1). To shed light on the role of ex-post uninsurable idiosyncratic risk in the design of macroprudential policies, we start by presenting the timing of actions and characterize analytically the optimal policy. We then consider welfare implications in a quantitative model.

Timing. To make progress toward an analytical characterization, we consider a simple scenario where prior to date $t = 0$, the economy is assumed to be in a steady state with ex-ante identical households, that is $b_{j,t} = B_0$ for all j and there is aggregate uncertainty only in at date $t = 1$. The timeline is as follows:

i) **date $t = 0$:** an unexpected, transitory MIT negative interest rate shock occurs, reducing R^* for one period. Simultaneously, households learn that a persistent negative endowment shock may occur at $t = 1$ with probability π;

ii) **date $t = 1$:** with probability Π, the economy experiences a negative y^T shock. In this period, nominal wage rigidity may bind;

iii) **date $t \geq 2$:** the economy transitions into a new steady state.

Figure 1 presents a graphical illustration of the timeline.

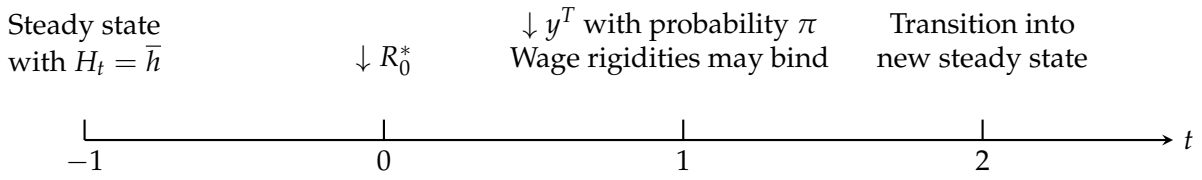


Figure 1: Timeline in simplified Setup

3.1 Government's optimization

We now turn to characterize analytically the optimal macroprudential policy when the planner only intervenes at $t = 0$ by choosing the tax rate on borrowing and lets markets clear competitively for $t \geq 0$.

To examine the optimal policy problem in a more intuitive way, we adopt the recursive formulation. The aggregate state vector of the economy at date t is $(\mathbf{B}_t, W_{t-1}, y_t^T)$ where $\mathbf{B}_t$ is the aggregate bond holdings an W_{-1} is the period period wage and y_t^T is the realization of the endowment shock. It follows that the state variables for an household's optimization problem are the individual bond holdings $b_{j,t}$ and the aggregate state $(\mathbf{B}_t, W_{t-1}, y_t^T)$. Letting $V_t^j(b_{j,t}; \mathbf{B}_t, W_{t-1}, y_t^T)$ denote the value of an individual household j in period t, we can write the Ramsey-optimal macroprudential policy problem in period 0 as

$$\mathcal{V}_0(\mathcal{B}_0; \mathbf{B}_0, W_{-1}, y^T) = \max_{\mathcal{B}_1, c_0^T, c_0^N} u(c_0^T, c_0^N) + \beta \mathbb{E}_{y_1^T} \int_j V_1^j \left(\mathcal{B}_1; \mathbf{B}_1, W_0, y_1^T\right) dj \tag{9}$$

subject to

$$W_0 = \bar{e} \frac{1-\phi}{\phi} \left(\frac{c_0^T}{c_0^N}\right)^{\frac{1}{\gamma}} \tag{10}$$

$$c_0^T = y^T + \mathcal{B}_0 - \frac{\mathcal{B}_1}{R_0^*} \tag{11}$$

$$c_0^N = y^N \tag{12}$$

$$\mathbf{B}_1 = \mathcal{B}_1 \tag{13}$$

In effect, by choosing tax on borrowing in period 0, the Ramsey-planner controls for house-

holds' credit operations.[1] Thus, the optimal macroprudential policy problem amounts to the problem of choosing optimally bond holdings at the end of period 0 on behalf of households and letting households choose the mix of consumption and markets clear as described by (10), (11) and (12). It is important to note that since households are ex-ante identical $b_{j,0} = \mathcal{B}_0$ and $b_{j,1} = \mathcal{B}_1$ for all j.

As we show in Appendix , when the realization of the tradable endowment shock in period 1 is $y_t^T = y_H^T > y^T$, the wage rigidity constraint does not bind in equilibrium and all households are employed and ex-post identical. In contrast, when the realization of the tradable endowment shock in period 1 is $y_t^T = y_L^T < y^T$, the wage rigidity constraint binds and a fraction $\lambda(H_1)$ of households are unemployed while the remaining $1 - \lambda(H_1)$ are employed. Letting $V_1^e(\mathcal{B}_1; \mathbf{B}_1, W_0, y_1^T)$ denote the value of an employed household in period 1 and $V_1^u(\mathcal{B}_1; \mathbf{B}_1, W_0, y_1^T)$ denote the value of an unemployed household in period 1, we can rewrite the Ramsey-planner problem as

$$\begin{aligned}\mathcal{V}_0(\mathcal{B}_0; \mathbf{B}_0, W_{-1}, y^T) = \max_{\mathcal{B}_1, c_0^T} \; & u(c_0^T, y^N) + \beta(1-\pi) V_1^e(\mathcal{B}_1; \mathbf{B}_1, W_0(\mathbf{B}_1), y_H^T) \\ & + \beta\pi \Big[\lambda\left(H(\mathbf{B}_1)\right) V_1^u(\mathcal{B}_1; \mathbf{B}_1, W_0(\mathbf{B}_1), y_L^T) \\ & + \left(1 - \lambda\left(H(\mathbf{B}_1)\right)\right) V_1^e(\mathcal{B}_1; \mathbf{B}_1, W_0(\mathbf{B}_1), y_L^T) \Big] \qquad (14)\end{aligned}$$

subject to

$$c_0^T = y^T + \mathcal{B}_0 - \frac{\mathcal{B}_1}{R_0^*} \qquad (15)$$

$$\mathbf{B}_1 = \mathcal{B}_1 \qquad (16)$$

where we used (10) and (12). To simplify notations, we will denote by $s_H \equiv (\mathbf{B}_1, W_0(\mathbf{B}_1), y_H^T)$ the aggregate state when $y_t^T = y_H^T$ and by $s_L \equiv (\mathbf{B}_1, W_0(\mathbf{B}_1), y_L^T)$ the aggregate state when $y_t^T = y_L^T$.

3.2 Optimal ex-ante policy with ex-post heterogeneity

As noted earlier, the Ramsey-planner problem consists in choosing borrowing in period 0 to maximize households' welfare subject to the resource constraints while internalizing how current borrowing affect current wage, future employment and the fraction of employed

[1] Technically, households' Euler equation for bonds is not a binding implementability constraint for the Ramsey-planner (i.e., the Lagrange multiplier on that constraint is zero). This occurs because τ_0 only enters directly households' Euler equation. See Appendix for more details.

household next period.

To understand the key between the borrowing decisions of the Ramsey-planner relative to the borrowing decisions of households, we compare the planner's Euler equation for bonds given by

$$
\begin{aligned}
u_T(c_0^T, c_0^N) = \beta R_0^* \Bigg\{ & (1-\pi)\frac{\partial V_1^e(\mathcal{B}_1; s_H)}{\partial \mathcal{B}_1} + \pi \left[\lambda(H_1) \frac{\partial V_1^u(\mathcal{B}_1; s_L)}{\partial \mathcal{B}_1} + (1-\lambda(H_1)) \frac{\partial V_1^e(\mathcal{B}_1; s_L)}{\partial \mathcal{B}_1} \right] \\
& + \underbrace{\pi \left[\lambda(H_1) \frac{\partial \mathcal{C}_1^{N,u}(\mathcal{B}_1; s_L)}{\partial \mathbf{B}_1} u_N(\mathcal{C}_1^{T,u}, \mathcal{C}_1^{N,u}) + (1-\lambda(H_1)) \frac{\partial \mathcal{C}_1^{N,e}(\mathcal{B}_1; s_L)}{\partial \mathbf{B}_1} u_N(\mathcal{C}_1^{T,e}, \mathcal{C}_1^{N,e}) \right]}_{\text{aggregate demand externality: } \Omega^{\text{AD}}} \\
& + \underbrace{\pi \frac{\partial W_0(\mathbf{B}_1)}{\partial \mathbf{B}_1} \left[\lambda(H_1) \frac{\partial V_1^u(\mathcal{B}_1; s_L)}{\partial W_0} + (1-\lambda(H_1)) \frac{\partial V_1^e(\mathcal{B}_1; s_L)}{\partial W_0} \right]}_{\text{redistributive externality: } \Omega^{\text{Redist}}} \\
& + \underbrace{\pi \frac{\partial \lambda(H_1)}{\partial H_1} \cdot \frac{\partial H(\mathbf{B}_1)}{\partial \mathbf{B}_1} \left[V^u(\mathcal{B}_1; s_L) - V^e(\mathcal{B}_1; s_L) \right]}_{\text{change in risk exposure (insurance): } \Omega^{\text{Risk}}} \Bigg\}
\end{aligned} \tag{17}
$$

with the corresponding equation for households in the competitive equilibrium with no government intervention

$$
u_T(c_0^T, c_0^N) = \beta R_0^* \left\{ (1-\pi)\frac{\partial V_1^e(b_1; s_H)}{\partial b_1} + \pi \left[\lambda_1 \frac{\partial V_1^u(b_1; s_L)}{\partial b_1} + (1-\lambda_1) \frac{\partial V_1^e(b_1; s_L)}{\partial b_1} \right] \right\}
$$

The Ramsey-planner borrowing choices and households' borrowing choices differs in three aspects: First, because they are atomistic, individual households' fail to take into account that a reduction in borrowing made at date $t = 0$ raises consumption of tradables at date $t = 1$ and helps support aggregate demand for non-tradables. This in turn leads to an increase aggregate employment and non-tradable output at date $t = 1$ in the state in which the wage constraint binds. This aggregate demand externalities is common to both **RANK** and **HANK** economies and stems from the presence of nominal rigidities (see e.g., Schmitt-Grohé and Uribe, 2016 and Farhi and Werning, 2016).

Second, the Ramsey-planner takes into account that a reduction in borrowing at date $t = 0$ lowers current wages which implies lower relative price of non-tradables at date $t = 1$ in the state in which the wage constraint binds. This in helps the households that are net buyers of non-tradable goods (i.e., poor unemployed households $\bar{h}_j = 0$) and hurts the other households that are net sellers of non-tradables (i.e., rich employed households

$\bar{h}_j = \bar{h}$). To the extent that the mass of unemployed household is large enough, average welfare increases.

Finally, individual households do not internalize how the increase in aggregate employment at date $t = 1$ induced by a reduction in borrowing at date $t = 0$ reduces the probability of being unemployed in the state where the wage constraint binds in period 1. This in turn helps increase welfare to the extent that the continuation value to the extent that employed agents have a higher value than the unemployed.

The next Proposition characterizes the optimal macroprudential tax on borrowing.

Proposition 1 (Optimal macroprudential policy in **HANK**). *The optimal macroprudential tax on debt in the presence of ex-post uninsurable unemployment risk is given by*

$$\tau_0^{\mathbf{HA}} = \pi \left[\mathbb{E}_0 \int_j \frac{\partial V_1 \left(\mathcal{B}_1, s_1 \right)}{\partial \mathcal{B}_1} dj \right]^{-1} \left[\Omega^{AD} + \Omega^{Redist} + \Omega^{Risk} \right] \tag{18}$$

where Ω^{AD}, Ω^{Redist} and Ω^{Risk} are defined in (17) with

$$\Omega^{AD} > 0, \quad \Omega^{Redist} \gtreqless 0 \quad \textit{and} \quad \Omega^{Risk} > 0.$$

Equation (18) provides an analytical characterization of the optimal tax on borrowing and sheds lights on the different motives for macroprudential policy interventions.

4 Quantitative Analysis

In this section, we provide a quantitative analysis of the model under the simple timeline described in Figure 1. Our main goal is to highlight the role of uninsurable idiosyncratic risk, contrasting the **RANK** and **HANK** economies.

Calibration. We calibrate the model using Spanish data in 2007. The weight on tradable consumption in the CES function ϕ is calibrated to match an average XX% share of tradable output in the total value of production as observed in Spain. This implies $\phi = 0.27$. We set the real interest rate R^* to 1.015 The endowment of tradable y^T, the endowment of hours $\bar{h}$ and the level of the nominal exchange rate $\bar{e}$ are normalized to one. In period 0, the shocks to the real rate R^* is set to imply a 10% decline in the interest rate. In period 1, the endowment of tradable output can take two values $y^T = \{1, 0.8\}$ where the probability that $y^T = 0.8$ is realized is set to $\pi = 10\%$. Table 1 describes the parameter values.

Table 1: Calibration

Description	Value	Target
Intertemporal elasticity	$\sigma = 2$	Standard value
Elasticity of substitution T-N	$\gamma = 0.83$	Schmitt-Grohé and Uribe (2016)
Degree of downward wage rigidity	$\rho_w = 0.96$ (currently 0.9)	Schmitt-Grohé and Uribe (2016)
Risk-free interest rate	$R^* = 1.02$	Spanish real rate in 2007
Discount factor	$\beta = 1/R^*$	Implied discount rate
Weight on tradables in CES	$\phi = 0.27$	INE Spanish National Accounts

We solve for the optimal Ramsey policies under two scenarios. In the ex-ante case, the government chooses $\{\tau_0, T_0\}$ at $t = 0$. In the ex-post case, the government selects $\{\tau_1, T_1\}$ at $t = 1$ contingent on the realization of the y^T shock. By definition of the Ramsey policy, households fully anticipate these policy paths, and the government commits to them perfectly.

4.1 Optimal prudential policies

In this section, we study ex-ante policies at $t = 0$ under two alternative specifications of the nominal wage rigidity constraint with different implications for aggregate unemployment. First, we consider the case where the nominal wage rigidity constraint may bind only in period 1 and as a involuntary unemployment can only be only *temporary*. Then, we relax this assumption allow for the nominal wage rigidity constraint to possibly bind in every period resulting in *persistent* involuntary unemployment.

4.1.1 Temporary unemployment

We first study the ex-ante policies at $t = 0$ under different specifications of the unemployment risk. Figure 2 presents the **RANK** transition paths under both the no-policy benchmark and the optimal policies.[2]

Optimal policy in RANK. Under no macroprudential policies, the wage rises modestly in period 0, but falls sharply in period 2 when the y^T shock materializes. This decline triggers aggregate involuntary unemployment of 17.2% at $t = 1$. With $\beta R = 1$ and no further uncertainty, the economy settles after $t = 2$. While tradable consumption rises

[2]For all plots of transition paths, we only plot outcomes conditional on the realization of the y^T shock.

slightly in period 0, it declines by more than 20% in period 1. There is an increase in private savings due to consumption smoothing motives from period 0 to period 1, but this response is insufficient to offset the wage drop and the ensuing aggregate demand externality.

As shown in the yellow line in Figure 2, under the optimal ex-ante policy, the government imposes a 9.8% ex-ante tax to stimulate saving at $t = 0$, which leads to a lower level of debt at the start of period 1 compared to the decentralized equilibrium. The trade-off the social planner faces here is between lowering aggregate demand externality and raising consumption distortions. As a result, the optimal policy only partially neutralizes the aggregate demand externality and unemployment still presents when the endowment shock realizes. In consumption-equivalent terms–that is, by the percentage of consumption required to compensate for the welfare loss–the optimal policy recovers 0.2% of the ex-ante welfare losses.

These results under **RANK** underscore the importance of preemptive policy intervention as in Bianchi (2011) and Schmitt-Grohé and Uribe (2016): by adjusting the ex-ante macroprudential instruments, the government can mitigate aggregate unemployment risks in the future and stabilize key economic aggregates.

Optimal policy in HANK. Turning to the **HANK** specification ($\xi = 1$), again we show the transition paths under no policies and optimal policies in Figure 3. Under **HANK**, where the aggregate unemployment risk is allocated heterogeneously across households, the adverse effects of the persistent negative y^T shock are even more pronounced. In the absence of policy interventions, despite lower unemployment rate due to higher ex-ante saving motive, the dispersion in labor market outcomes leads to 1.07% higher ex-ante welfare losses.

To counteract the amplified negative effects, the optimal ex-ante policy adopts a more aggressive response. In our calibration, the optimal policy features a 31.1% ex-ante tax to offset the aggregate demand externality by reducing household debt accumulation from $t = 0$ to $t = 1$. The unemployment is lowered by 11.7% due to the policy, which is 7.8% larger than the **RANK** case. Notably, this tax is significantly larger than the optimal tax under the **RANK** specification, reflecting the greater severity of the shock's impact when heterogeneity is present. The welfare gain of the optimal ex-ante macroprudential policies is larger under **HANK** and able to recover 0.7% of the ex-ante welfare losses.

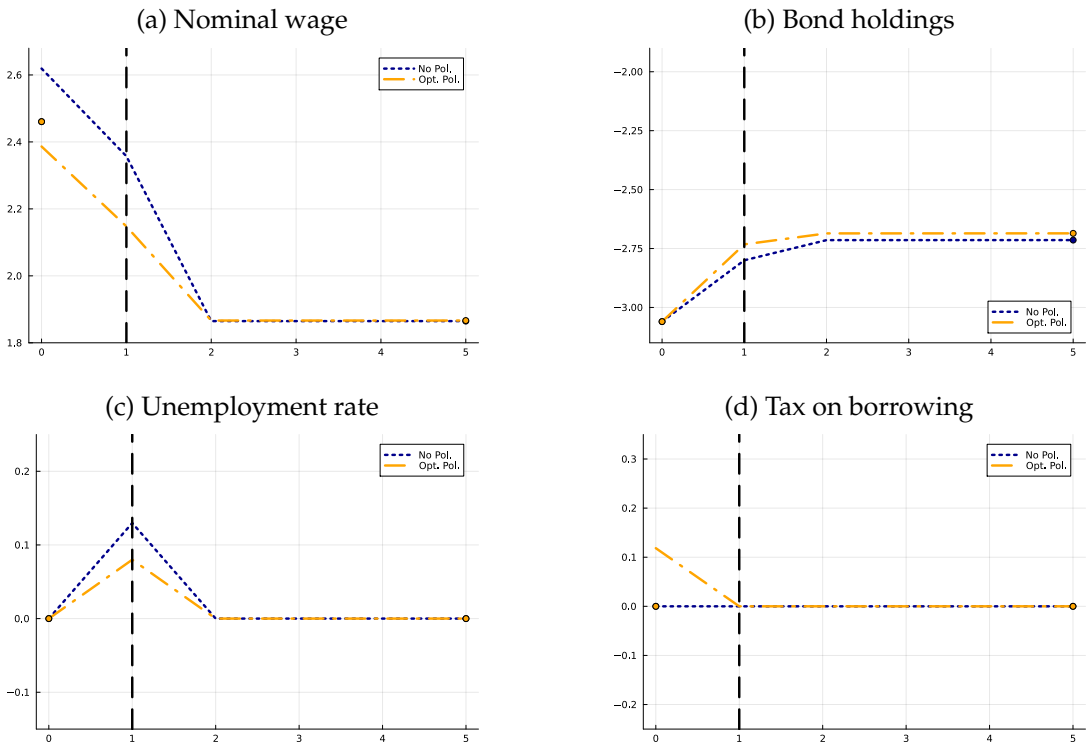


Figure 2: **RANK** transitions and policy paths with temporary unemployment

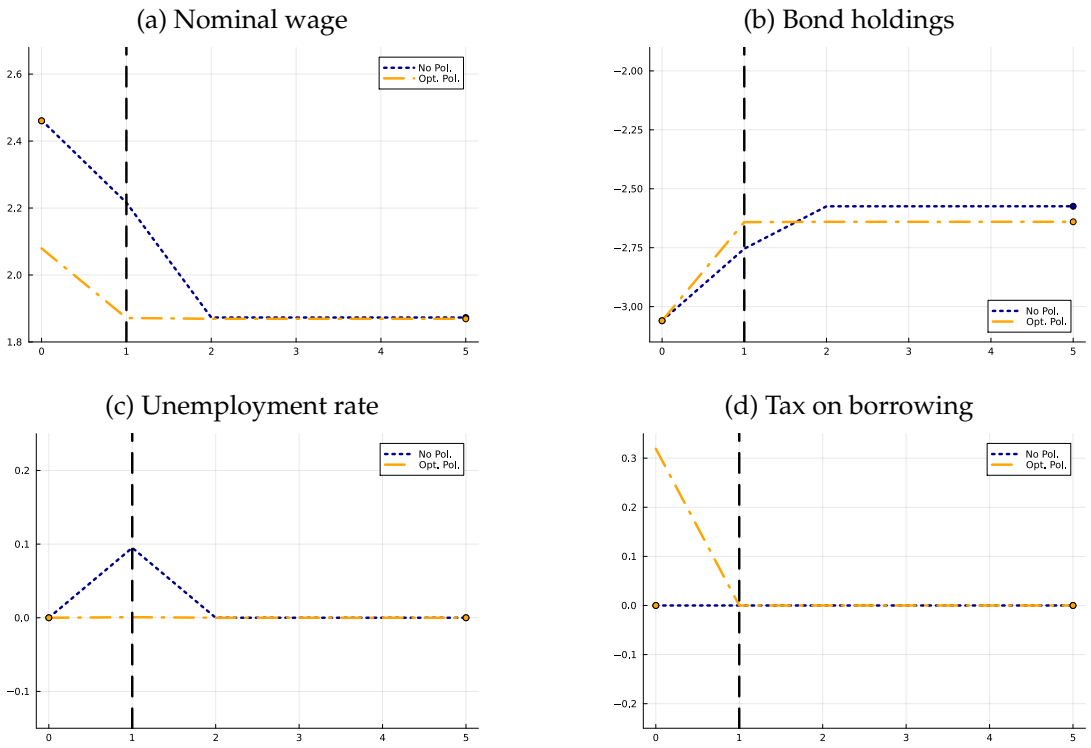
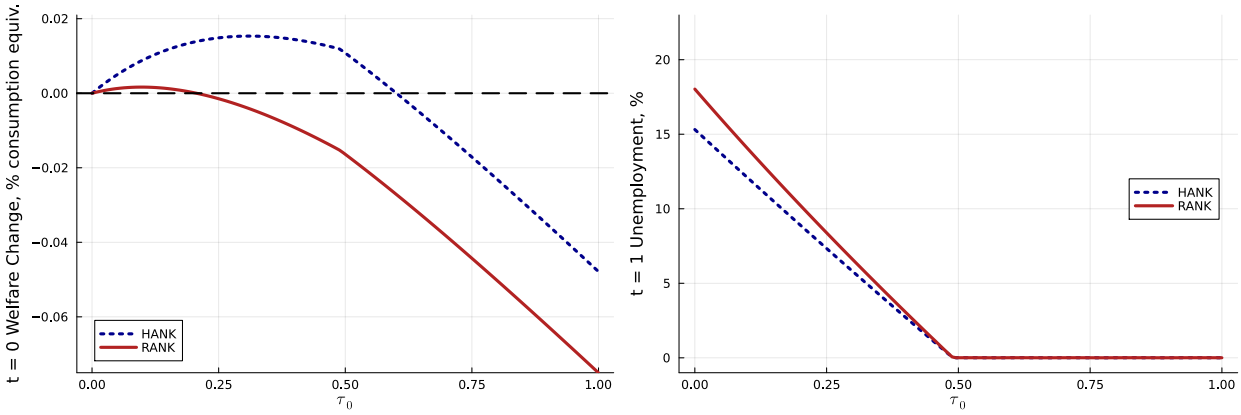


Figure 3: **HANK** transitions and policy paths with temporary unemployment

Welfare implications. We hereby vary the ex-ante taxes to look at the welfare implications under both **RANK** and **HANK**. Figure 4 shows the welfare analysis with different level of ex-ante taxes with $\Pi = 10\%$. As previously discussed, the optimal level of ex-ante taxes are higher under **HANK**. Compared to the no-policy benchmark ($\tau_0 = 0$), larger welfare gains can be obtained using macroprudential policy under **HANK**.



(a) Welfare change relative to no policy

(b) Unemployment Rate at $t = 1$

Figure 4: Welfare analysis with different levels of ex-ante taxes

4.1.2 Persistent unemployment

We now repeat the experiments while relaxing our previous assumption on wage rigidity. Under this new specification, wage rigidity can bind in **every period**. That is, given the initial wage W_{-1}, unemployment may occur at any $t \geq 0$ whenever $W_t = \Psi W_{t-1}$. Figures 5 and 6 present the transition dynamics and optimal policy paths under the **RANK** and **HANK** specifications, respectively.

In the benchmark (no-policy) scenario, the key difference from our earlier experiment is the prolonged periods of unemployment and larger welfare costs. As shown in Figure 5, in the absence of policy interventions, wages take three periods to adjust, and the economy does not restore full employment until $t = 4$.

Under the optimal ex-ante policy in the **RANK** specification, the government imposes a 22.3% tax. The intuition is that since the nominal rigidity can also bind at $t = 0$, an additional policy trade-off here is between lowering unemployment at $t = 0$ and after $t = 1$. Too much taxation on ex-ante borrowing depresses consumption in period 0, which in turn triggers wage rigidity to bind prematurely. Thus, the policy will only alleviate the

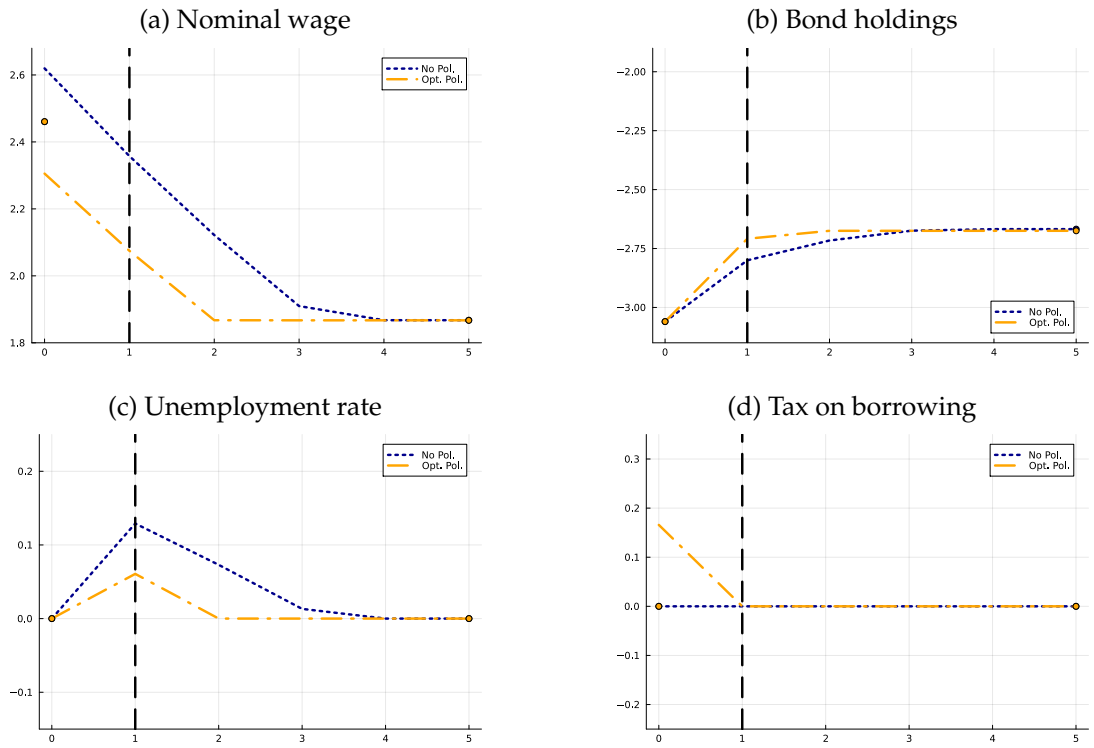


Figure 5: **RANK** transitions and policy paths with persistent unemployment

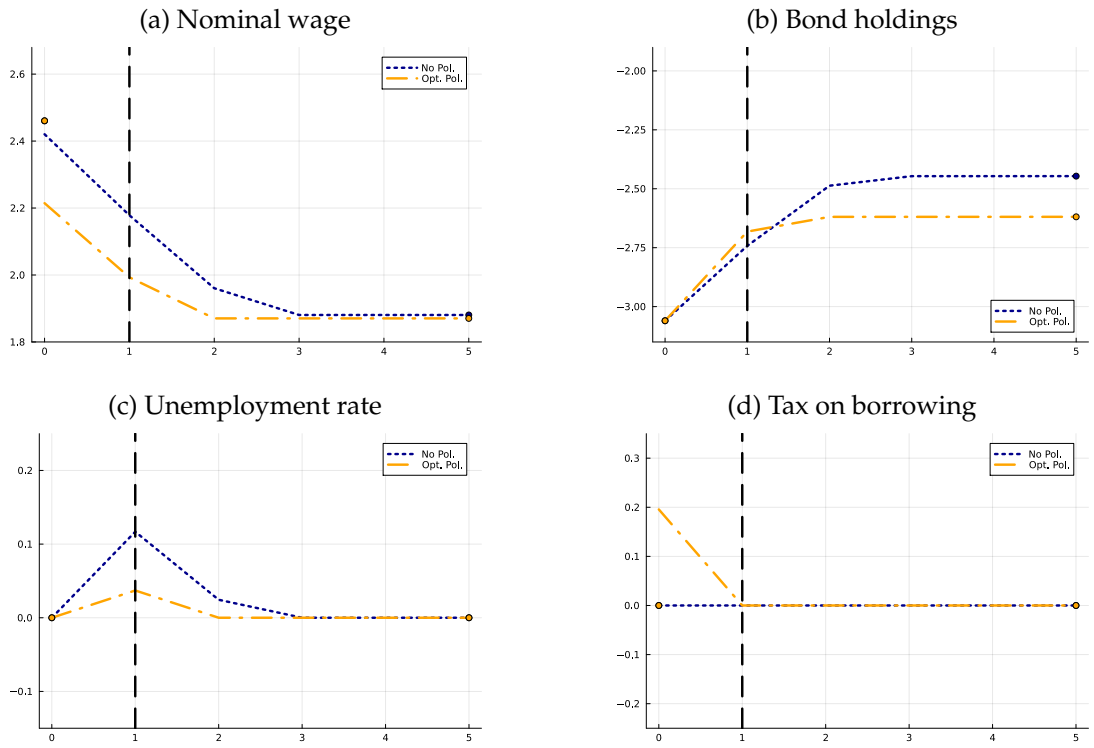


Figure 6: **HANK** transitions and policy paths with with persistent unemployment

aggregate unemployment cost rather than fully eliminating it. In this case, the optimal ex-ante tax reduces the involuntary unemployment at period 1 by 8.58% and recovers 7.3% of the welfare losses from the no-policy benchmark.

Figure 6 displays the corresponding aggregate dynamics under the **HANK** specification. Similar to the **RANK** model, unemployment is prolonged to 3 periods, and the overall welfare costs are higher. The households hold a higher private incentive for ex-ante savings due to the larger ex-post welfare cost from going into full unemployment compared to **RANK**.

The optimal ex-ante policy under **HANK** requires a 31.0% tax, which is still more aggressive than under **RANK**, although the gap between the two cases is narrowed. The policy again is wary of the unemployment tradeoff between $t = 0$ and $t = 1$. The government no longer eliminates the ex-post unemployment and instead reduces it by 13.3% at period 1, while avoiding unemployment at period 0. In consumption equivalent terms, the optimal policy under **HANK** mitigates 10.8% of the welfare loss.

4.2 Optimal ex-post policies

Now, on top of $\{\tau_0, T_0\}$, assume that the government can also choose the ex-post policies $\{\tau_1, T_1\}$. Again, we compare the optimal policies under **RANK** and **HANK**.

Figure 7 shows the transition paths with both optimal ex-ante and ex-post policies with unemployment only at $t = 1$. The ex-post policy takes the form of a subsidy on borrowing and the additional policy instrument is able to fully eliminate unemployment at $t = 1$. Thus, the two specifications of the unemployment risk coincide and so do the optimal policies. With the introduction of ex-post policy intervention, the level of ex-ante taxes decreased compared to Figure 2.

Figure 7 shows the **RANK** transition paths with rigidity constraints in all periods. With the policy instruments being only τ_0 and τ_1, the government no longer fully eliminate ex-post unemployment. The optimal levels policies are $19.x\%$ for ex-ante taxes and $-21.x\%$ for ex-post subsidies (TBA).

Figure 8 shows the **HANK** transition paths with optimal ex-ante and ex-post policies. The optimal policies consist of a 25.1% ex-ante tax and a 15.3% ex-post subsidy. The ex-post subsidy again doesn't eliminates $t = 1$ unemployment but lowers it to 3.2% and thus promote welfare gains. The **HANK** features a higher ex-ante tax and a lower ex-post

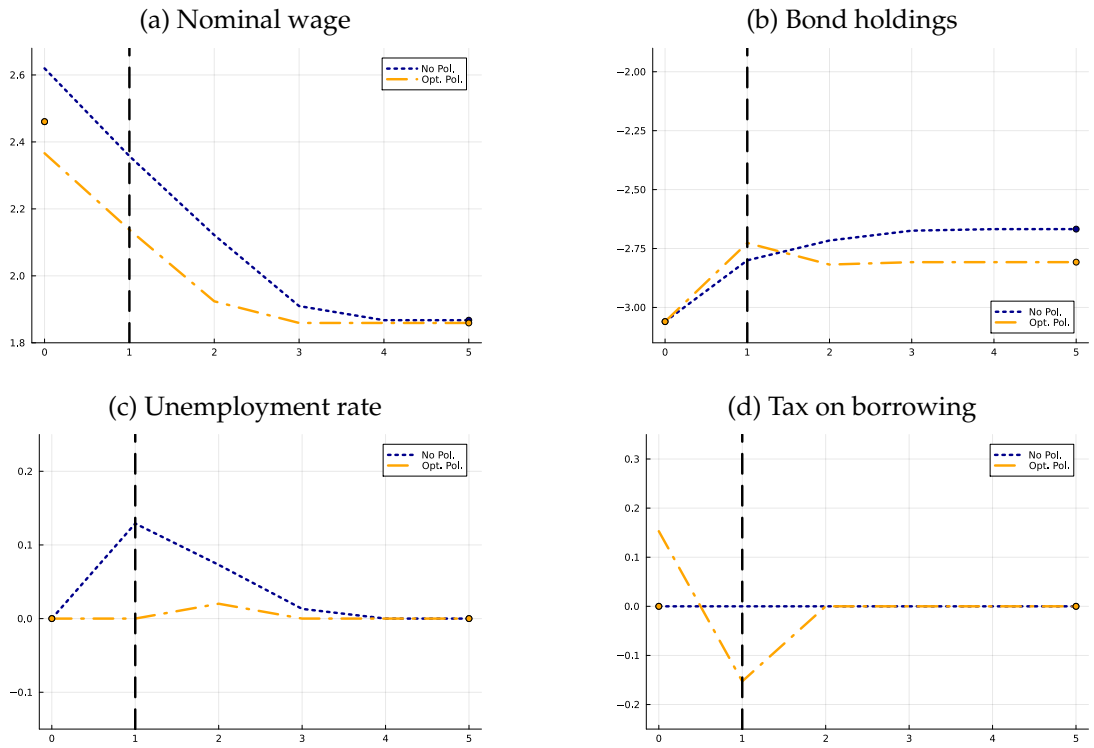


Figure 7: **RANK** transitions and policy paths with rigidity constraints in any period

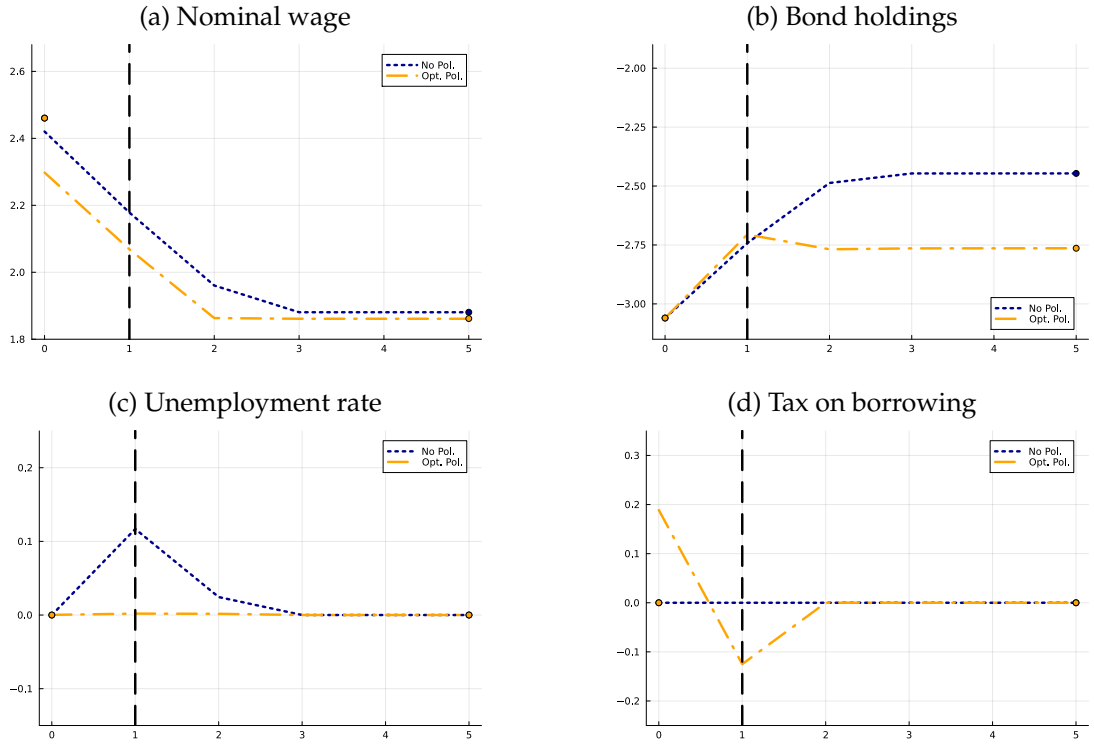


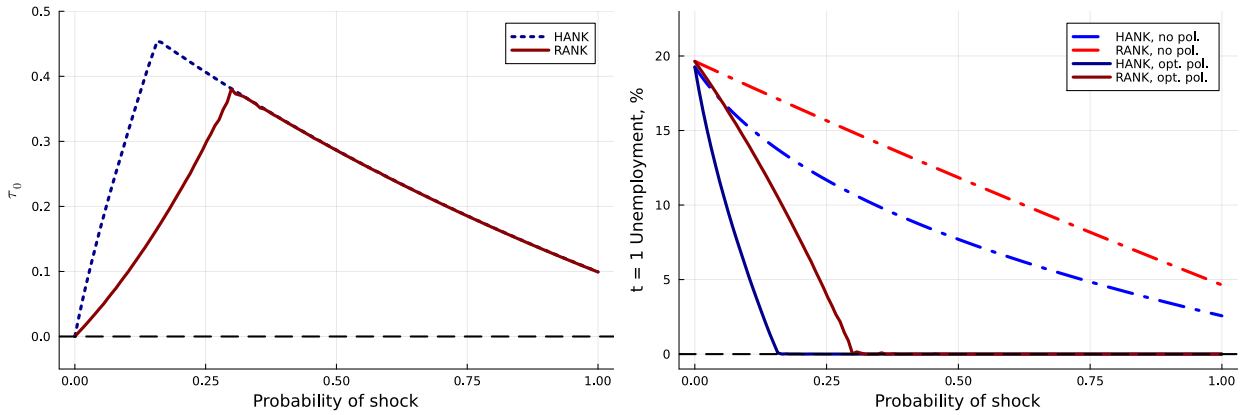
Figure 8: **RANK** transitions and policy paths with rigidity constraints in any period

subsidy compared to **RANK** (Figure 7).

It should also be noted that the ex-ante tax is 5.9% lower than the case without ex-post subsidies (Figure 8). The introduction of ex-post subsidies lowers the aggregate demand externality and thus only requires a smaller ex-ante tax.

4.3 The role of shock probability

How does the optimal policy change when the probability of the endowment shock Π changes? We consider $t = 1$ unemployment only, and look at the ex-ante policy.

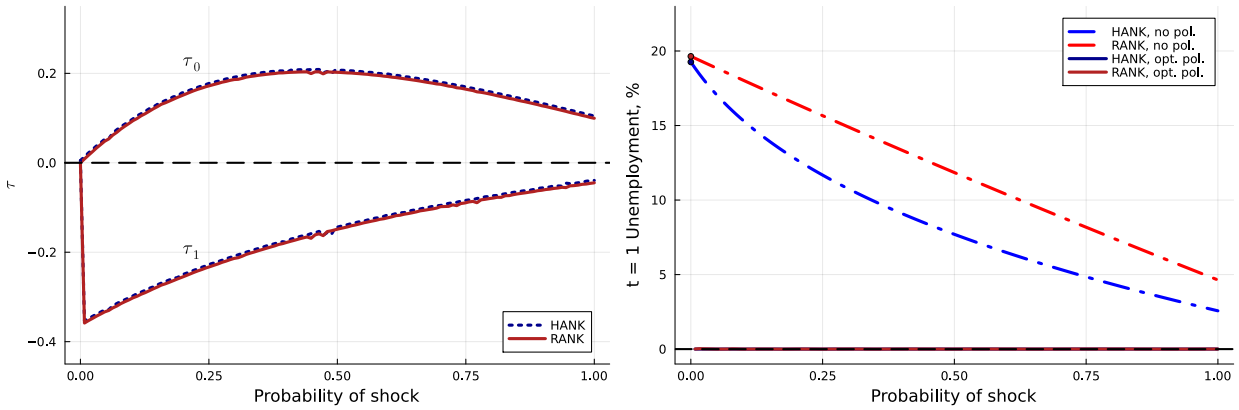


(a) Optimal $t = 0$ Policies

(b) Unemployment Rate at $t = 1$

Figure 9: Optimal ex-ante policies with temporary unemployment

Figure 9 plots the optimal ex-ante policies and ex-post unemployment for different probability of shock Π. When Π is low, in panel (a) we obtain our previous results as in Figure 2 and 3 where the Ramsey planner under **HANK** chooses a higher macroprudential policy than under **RANK**, and cuts unemployment further than **RANK**, as shown in panel (b). When Π goes up and aggregate unemployment is more likely to happen under the decentralized equilibrium, the social planner always want to fully eliminate ex-post unemployment, and the



(a) Optimal $t = 0$ Policies

(b) Unemployment Rate at $t = 1$

Figure 10: Optimal ex-ante and ex-post policies with temporary unemployment

Figure 10 shows the optimal ex-ante and ex-post policies when we vary Π. As previously shown, the optimal ex-ante policies are the combination of an ex-ante tax and an ex-post subsidy. With only one period of wage rigidity, the optimal policies fully eliminate the ex-post unemployment risk and **RANK** and **HANK** coincide.

5 Idiosyncratic Productivity and Wealth Heterogeneity

The literature on **HANK** highlights the case for ex-ante income and wealth inequality of households. We take into consideration the standard idiosyncratic labor productivity shock on top of the unemployment risk. In particular, we assume households receive $\overline{h}_{jt}$ units of labor endowment instead of $\overline{h}$. $\overline{h}_{jt}$ has both a persistent and a transient component:

$$\begin{aligned} \overline{h}_{j,t} &= z_{j,t} + e_{j,t}, \\ z_{j,t} &= \rho\, z_{j,t-1} + \nu_{j,t}, \qquad \nu_{j,t} \sim \mathcal{N}(0, \sigma_\nu^2), \\ e_{j,t} &\sim \mathcal{N}(0, \sigma_e^2), \end{aligned}$$

where $z_{j,t}$ is the persistent component, with autocorrelation ρ and standard error of the innovation σ_ν; $e_{j,t}$ is the i.i.d. transitory component with standard error σ_e.

As a result, household j supplies in total $\overline{\lambda}_{j,t} h_{j,t}$ units of labor. We also assume that the borrowing constraint $-\overline{b}$ in equation 4 can be potentially binding in accordance with the literature.

We calibrate the model to the Spanish economy in 2007. Spain had the lowest unemployment rate in decades following a prolonged asset and credit boom, fueled by

rapid housing construction and abundant capital inflows. Table 2 provides externally calibrated model parameters, and Table 3 reports the remaining parameters estimated through simulated methods of moments (SMM). Appendix A provides additional details on the construction of these parameters.

Table 2: Calibrated Parameters

Parameter	Value	Source
σ	2	Standard value
γ	0.83	Adopted from Schmitt-Grohé and Uribe (2016)
ϕ	0.27	INE Spanish National Accounts
R^*	1.015	Spanish real rate in 2007
ρ_w	0.96	Adopted from Schmitt-Grohé and Uribe (2016)
ρ_ε	0.75	GRID
σ_ε	0.49	GRID
$\underline{y}^T$	1	Normalization
$\overline{h}$	1	Full employment benchmark

Table 3: Estimated Parameters

Parameter	Value	Target	Source	Data	Model
β	0.91	Asset Gini-coefficient	Bover (2008)	0.59	0.61
$\overline{b}$	5.15	Net NFA to GDP ratio	Eurostat	-0.78	-0.78

The choice of 2007 as the calibration year is deliberate: it represents the pre-crisis steady state, when macroeconomic indicators were broadly stable and unemployment had fallen below 9 percent–the lowest in decades. Since 2008, Spain experienced a severe recession triggered by the global financial crisis. Unemployment surged to historically high levels–peaking at 26.08% in 2013. Real wages stagnated, output contracted sharply, and the economy entered a prolonged adjustment process under fiscal consolidation and labor market reforms.

The specifications give rise to the standard steady state distribution of bonds in Figure 11, with around 11.6% of hand-to-mouth households on the borrowing constraint and a fat tail.

Again, we study an experiment of an anticipated negative shock on the tradable endowment y_T with probability Π. When realized, the shock generates ex-post unemployment.

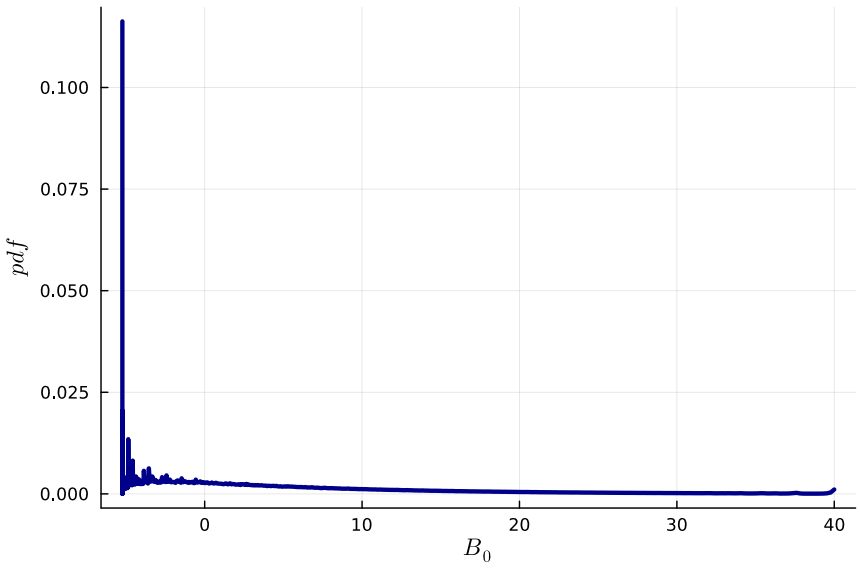


Figure 11: Steady State Distribution of Bonds

We solve for the optimal macroprudential policies. To avoid confusion, we refer to the case where $\xi = 0$ (previously **RANK**) as aggregate λ_{jt} and the case where $\xi = 1$ (previously **HANK**) as idiosyncratic λ_{jt}.

Figure 13 performs the welfare analysis for households with different assets and labor endowment. We plot the ex-ante welfare change of the optimal policy against the no policy benchmark. Household with a low level of asset and labor endowment gains most from the implementation of policy.

6 Conclusion

To be added.

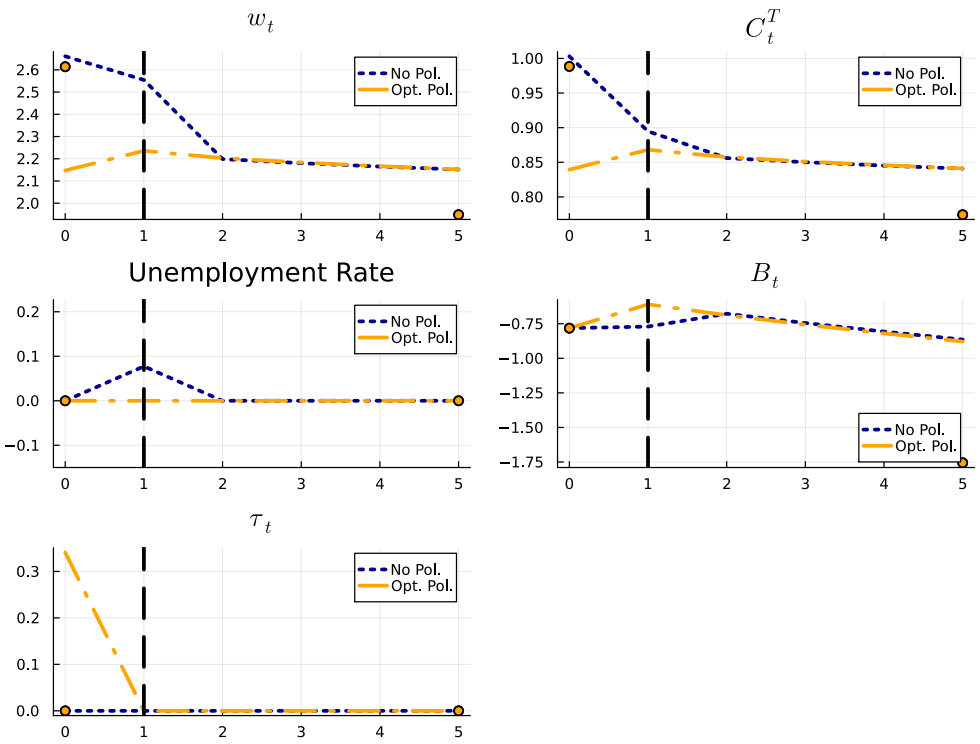


Figure 12: Optimal ex-ante policies with only $t = 0$ unemployment

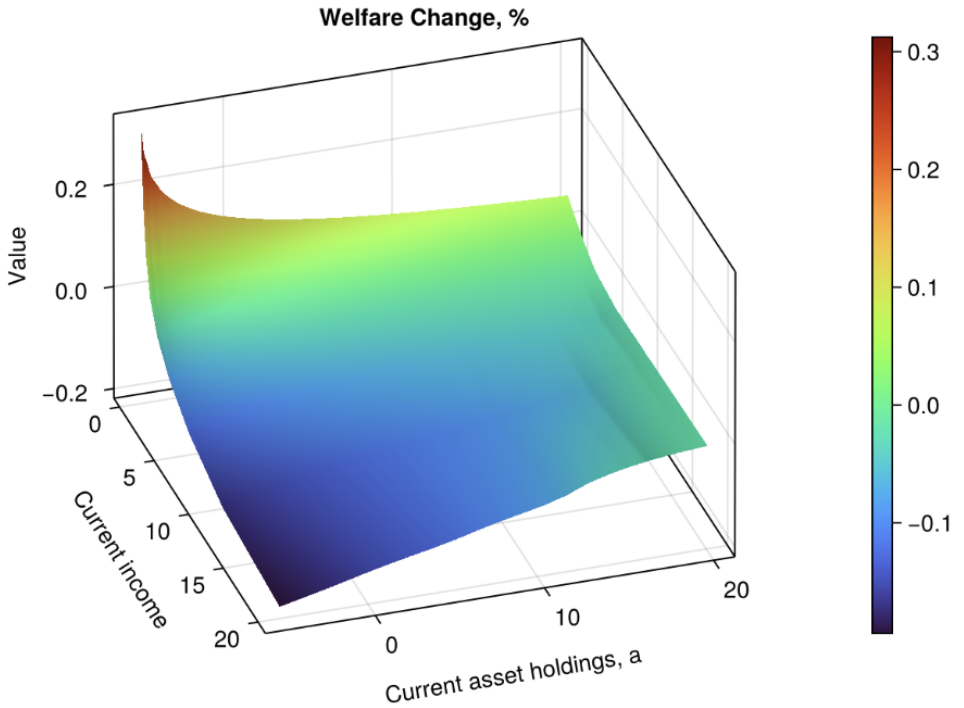


Figure 13: Welfare comparison

References

Acharya, Sushant, Edouard Challe, and Keshav Dogra, "Optimal monetary policy according to HANK," *American Economic Review*, 2023, *113* (7), 1741–1782.

Auclert, Adrien, Matt Rognlie, and Ludwig Straub, "The Intertemporal Keynesian Cross," Working Paper 25020, NBER 2018.

Bhandari, Anmol, David Evans, Mikhail Golosov, and Thomas J Sargent, "Inequality, Business Cycles, and Monetary-Fiscal Policy," *Econometrica*, 2021, *89* (6), 2559–2599.

Bianchi, Javier, "Overborrowing and Systemic Externalities in the Business Cycle," *American Economic Review*, 2011, *101* (7), 3400–3426.

_ **and Guido Lorenzoni**, "The Prudential Use of Capital Controls and Foreign Currency Reserves," 2021. Forthcoming, Handbook of International Economics, Volume V, edited by Gita Gopinath, Elhanan Helpman and Kenneth Rogoff, North Holland.

_ **and Louphou Coulibaly**, "Liquidity Traps, Prudential Policies, and International Spillovers," 2021. Mimeo, Federal Reserve Bank of Minneapolis.

Bover, Olympia, "Wealth Inequality and Household Structure: US vs. Spain," Documento de Trabajo 0804, Banco de España, Madrid 2008.

Dávila, Eduardo and Andreas Schaab, "Optimal monetary policy with heterogeneous agents: A timeless ramsey approach," *Available at SSRN 4102028*, 2022.

Farhi, Emmanuel and Iván Werning, "A Theory of Macroprudential Policies in the Presence of Nominal Rigidities," *Econometrica*, 2016, *84* (5), 1645–1704.

Fornaro, Luca and Federica Romei, "The Paradox of Global Thrift," *American Economic Review*, 2019, *109* (11), 3745–3779.

Heathcote, Jonathan and Fabrizio Perri, "Wealth and volatility," *The Review of Economic Studies*, 2018, *85* (4), 2173–2213.

Kaplan, Greg, Benjamin Moll, and Giovanni L. Violante, "Monetary Policy According to HANK," *American Economic Review*, 2018, *108* (3), 697–743.

Ravn, Morten O and Vincent Sterk, "Macroeconomic fluctuations with HANK & SAM: An analytical approach," *Journal of the European Economic Association*, 2021, *19* (2), 1162–1202.

Schmitt-Grohé, Stephanie and Martın Uribe, "Downward nominal wage rigidity, currency pegs, and involuntary unemployment," *Journal of Political Economy*, 2016, *124* (5), 1466–1514.

_ **and** _ , "Multiple equilibria in open economies with collateral constraints," *Review of Economic Studies*, 2021, *88* (2), 969–1001.

A Data appendix

We calibrate our model to Spanish data in 2007. This section provides details on the data and methodology for the calibration.

A.1 Expenditure share ϕ

To discipline the tradable share parameter ϕ, we follow the standard production-based approach using Spanish National Accounts (INE, Table 69069).

We classify industries at the one-letter CNAE level:

- **Tradables (T):** Agriculture, forestry and fishing (A), Mining and quarrying (B), and Manufacturing (C).
- **Non-tradables (N):** All remaining sectors (D–U), including utilities, construction, services, real estate, and public sectors.

For each year, we compute the total production of tradables (Y^T) and non-tradables (Y^N), and define the tradable share as

$$\phi_t = \frac{p_t^T Y_t^T}{p_t^T Y_t^T + p_t^N Y_t^N}.$$

Table 4: Tradable and Non-tradable Production (Spain, million Euros)

Year	Tradables ($p_t^T Y_t^T$)	Non-tradables ($p_t^N Y_t^N$)	Share (ϕ_t)
2005	527,689	1,291,684	0.2900
2006	566,667	1,436,214	0.2829
2007	606,119	1,545,482	0.2817
2008	605,110	1,577,803	0.2772
2009	485,297	1,508,722	0.2434
2010	518,395	1,455,486	0.2626
2011	541,477	1,424,669	0.2754
2012	522,193	1,375,337	0.2752
2013	512,190	1,340,354	0.2765

Table 4 displays the aggregates of tradable and non-tradable production and the computed shares from 2005 to 2013. We take the average share 0.2739 as our baseline. Note that

imputed rents (item L-68a) are included in the baseline measure, consistent with national account aggregates;[3] using gross value added instead of total production yields a lower ϕ_t (about 0.16), the qualitative results from the model are identical.

A.2 Income Process

We discipline the idiosyncratic earnings risk using harmonized moments from the *Global Repository of Income Dynamics (GRID)* constructed by Fatih Guvenen, Luigi Pistaferri, and Gianluca Violante from administrative data of different countries. GRID reports, by country–year–demographic cell, both cross-sectional dispersion and serial dependence of residual log earnings for working-age workers (ages 25–55). Throughout, we work with GRID's *residual* log earnings, constructed by removing predictable life-cycle and time effects.

Residual earnings moments in GRID. Let $u_{i,t}$ denote residual log earnings. GRID constructs $u_{i,t}$ as the residual from a regression of log earnings on a full set of age dummies (and fixed controls) estimated year-by-year, thereby removing predictable age (life-cycle) and time effects. The variables we use are: (i) the one-year autocorrelation $\text{AC1}_t \equiv \text{Corr}(u_{i,t}, u_{i,t-1})$ (variable `autocorr_lag1_res_log_inc`);[4] (ii) the two-year autocorrelation $\text{AC2}_t \equiv \text{Corr}(u_{i,t}, u_{i,t-2})$ (variable `autocorr_lag2_res_log_inc`); and (iii) the cross-sectional standard deviation $s_t \equiv \sqrt{\text{Var}(u_{i,t})}$ (variable `std_res_log_inc`).

Income Process Consistent with the income-dynamics literature, we map GRID moments into an idiosyncratic income process with a persistent component and an i.i.d. transitory component:

$$u_{i,t} = z_{i,t} + e_{i,t}, \tag{19}$$

$$z_{i,t} = \rho\, z_{i,t-1} + \nu_{i,t}, \qquad \nu_{i,t} \sim \mathcal{N}(0, \sigma_\nu^2), \tag{20}$$

$$e_{i,t} \sim \mathcal{N}(0, \sigma_e^2), \qquad \text{i.i.d.}. \tag{21}$$

Under (19)–(21), the population autocovariances of $u_{i,t}$ satisfy

$$\text{Cov}(u_{i,t}, u_{i,t-1}) = \text{Cov}(z_{i,t}, z_{i,t-1}) = \rho\ \text{Var}(z), \qquad \text{Cov}(u_{i,t}, u_{i,t-2}) = \rho^2\ \text{Var}(z),$$

[3] Excluding imputed rents raises the share slightly; results are robust.

[4] GRID computes AC1_t as the cross-sectional correlation between $u_{i,t}$ and $u_{i,t-1}$ among individuals observed in both years.

and therefore the corresponding autocorrelations obey

$$\rho \;=\; \frac{\mathrm{AC2}}{\mathrm{AC1}}. \tag{22}$$

Moreover, since $\mathrm{Var}(u) = \mathrm{Var}(z) + \mathrm{Var}(e)$ and $\mathrm{AC1} = \mathrm{Cov}(u_t, u_{t-1})/\,\mathrm{Var}(u) = \rho\;\mathrm{Var}(z)/\,\mathrm{Var}(u)$, we obtain the implied decomposition

$$\mathrm{Var}(z) \;=\; \frac{\mathrm{AC1}}{\rho}\;\mathrm{Var}(u), \qquad \mathrm{Var}(e) \;=\; \mathrm{Var}(u) - \mathrm{Var}(z), \qquad \mathrm{Var}(u) = s^2. \tag{23}$$

Implementation for Spain. We use Spanish GRID moments for year 2007 and set $\mathrm{Var}(u) \;=\; s^2_{2007}$, $\mathrm{AC1} \;=\; \mathrm{AC1}_{2007}$, and $\mathrm{AC2} \;=\; \mathrm{AC2}_{2007}$. Applying (22)–(23) yields the parameter values

$$\rho \;=\; 0.918, \qquad \mathrm{Var}(z) \;=\; 0.332, \qquad \sigma_e^2 \equiv \mathrm{Var}(e) \;=\; 0.209,$$

so that $\mathrm{Var}(u) \;=\; \mathrm{Var}(z) + \sigma_e^2 \;=\; 0.541$ and the implied dispersion of one-year residual log-earnings changes is

$$\mathrm{sd}(\Delta u_{i,t}) \;=\; \sqrt{2\,\mathrm{Var}(u)\big(1 - \mathrm{AC1}\big)} \;\approx\; 0.5,$$

consistent with the time series pattern of $\mathrm{sd}(\Delta u)$ in the data.

Mapping into the HANK model and discretization. Household labor efficiency is given by

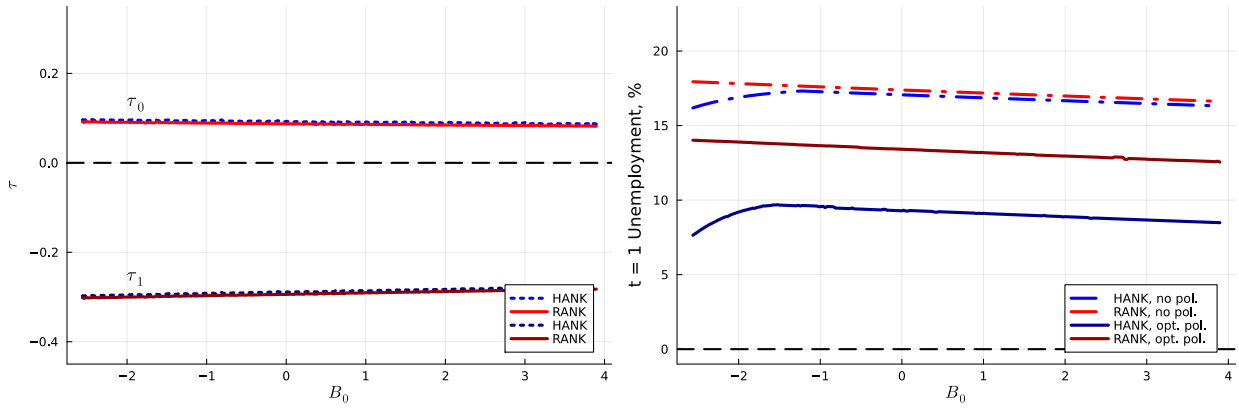
$$h_{i,t} \;=\; \exp\big(z_{i,t} + e_{i,t} - \mu\big), \tag{24}$$

where μ is a normalization constant chosen so that $\mathbb{E}[h_{i,t}] \;=\; 1$. We discretize the persistent component $z_{i,t}$ using the Rouwenhorst method with n_z states. We approximate the i.i.d. transitory shock $e_{i,t} \sim \mathcal{N}(0, \sigma_e^2)$ using Gauss–Hermite quadrature with n_e nodes $\{e_k\}_{k=1}^{n_e}$ and weights $\{\omega_k\}_{k=1}^{n_e}$. The composite earnings state is the pair (z, e) with $n_z \times n_e$ realizations and transition matrix

$$\Pr(z', e' \mid z, e) \;=\; \Pr(z' \mid z)\,\Pr(e'),$$

i.e. the Kronecker product of the Rouwenhorst transition for z and the i.i.d. quadrature weights for e.

B Additional results

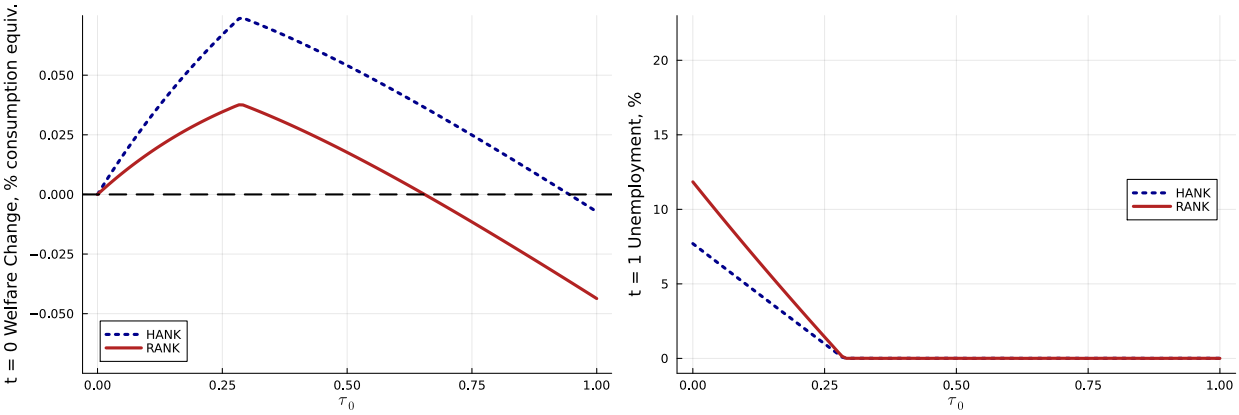


(a) Optimal Policies

(b) Unemployment Rate at $t = 1$

Figure 14: Comparison of optimal policies with only $t = 1$ unemployment

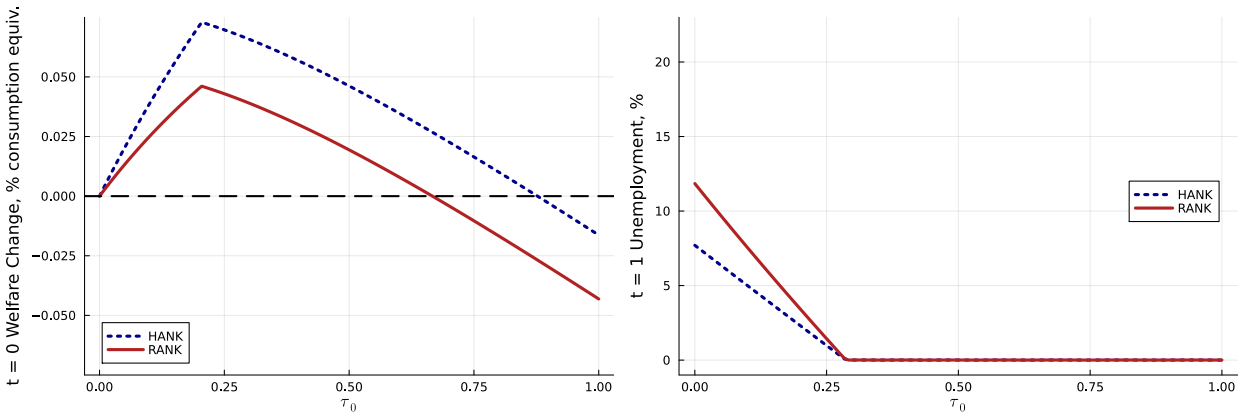
Figure 14 shows the optimal ex-ante and ex-post policies when the unemployment can only take place in period 1. The optimal policies always fully eliminate the $t = 1$ unemployment that the policies under RANK and HANK coincide with each other.



(a) Welfare change relative to no policy

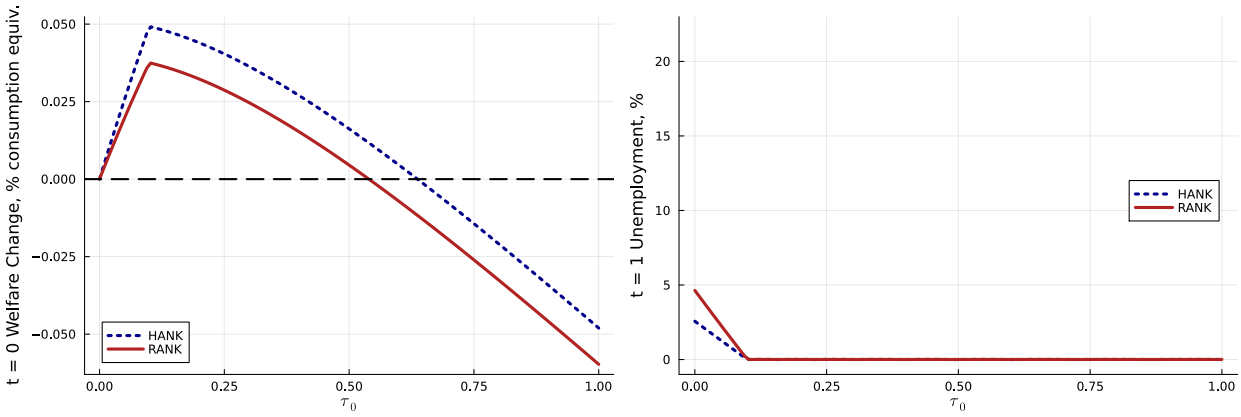
(b) Unemployment Rate at $t = 1$

Figure 15: Welfare analysis with different levels of ex-ante taxes ($\Pi = 50\%$)



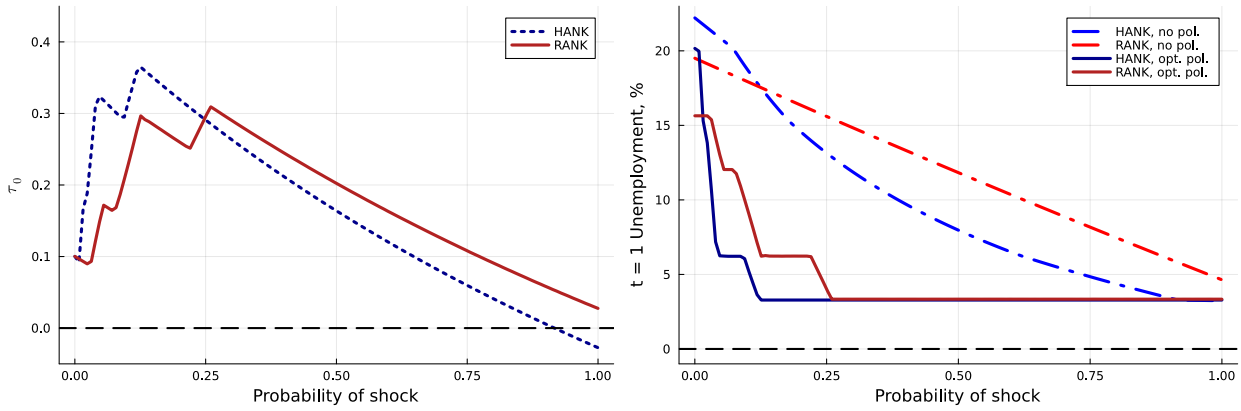
(a) Welfare change relative to no policy (b) Unemployment Rate at $t = 1$

Figure 16: Welfare analysis with different levels of ex-ante taxes ($\Pi = 70\%$)



(a) Welfare change relative to no policy (b) Unemployment Rate at $t = 1$

Figure 17: Welfare analysis with different levels of ex-ante taxes ($\Pi = 100\%$)



(a) Optimal $t = 0$ and $t = 1$ Policies (b) Unemployment Rate at $t = 1$

Figure 18: Optimal ex-ante and ex-post policies with wage rigidities in all periods

C Computational notes

The Euler equation for this model when the household is away from its borrowing limit is

$$U'(c_{jt})c_{jt}^{\frac{1}{\gamma}}(c_{jt}^T)^{-\frac{1}{\gamma}} = \beta R_t^*(1+\tau_t)\mathbb{E}_t\left\{U'(c_{j,t+1})c_{j,t+1}^{\frac{1}{\gamma}}(c_{j,t+1}^T)^{-\frac{1}{\gamma}}\right\}$$

In particular, assuming CRRA utility with relative risk aversion σ and substituting in the intratemporal condition, we can rewrite it as

$$\left[\phi + (1-\phi)^{\gamma}\phi^{1-\gamma}w_t^{1-\gamma}\right]^{\frac{1-\gamma\sigma}{\gamma-1}}(c_{jt}^T)^{-\sigma} = \beta R_t^*(1+\tau_t)\mathbb{E}_t\left\{\left[\phi + (1-\phi)^{\gamma}\phi^{1-\gamma}w_{t+1}^{1-\gamma}\right]^{\frac{1-\gamma\sigma}{\gamma-1}}(c_{j,t+1}^T)^{-\sigma}\right\} \tag{25}$$

Next, we will describe the algorithm to solve the stationary equilibria and optimal policies along the transition paths using the endogenous-grid method (EGM).

C.1 Recursive stationary equilibrium

Consider a stationary equilibrium where the interest rate is fixed, there is no macroprudential policy and wage rigidity is not binding. In a stationary equilibrium without aggregate uncertainties, wage is fixed and we can simplify equation 25 to

$$(c_{jt}^T)^{-\sigma} = \beta R^*\mathbb{E}_t\left\{(c_{j,t+1}^T)^{-\sigma}\right\} \tag{26}$$

Household problem via an EGM algorithm. We exploit equation 26 and solve the household j's problem from above given some wages. The algorithm goes as follows

1. Use a power grid for asset that puts more points near the borrowing limit. Guess a tradable consumption function $c^T(b,h;w)$ for each b,h and value function $v(b,h;w)$.
2. Given the consumption policy for tradables, compute the RHS of equation 26.
3. Then, invert to find the new updated consumption choice, i.e.

$$c^T(b^{*\prime},h;w) = \left[\beta R^*\mathbb{E}_h\left\{(c^{T\prime}(b^{*\prime},h';w))^{-\sigma}\right\}\right]^{-\frac{1}{\sigma}}$$

that is, the tradable consumption policy today given tomorrow's debt position.

4. Next, we invert the budget constraint given $b^{*\prime}$ and w to find the corresponding debt position today $\tilde{b^*}$,

$$\tilde{b^*} = \left[\left(\frac{\phi}{1-\phi}\right)^{-\gamma} w^{1-\gamma} + 1\right] c^T(b^{*\prime}, h; w) + \frac{b^{*\prime}}{R^*} - w\lambda h - y^T$$

5. We have found $c^T(\tilde{b^*}, h; w)$, which however is in general not on the specified asset grid. The mapping from $\tilde{b^*}$ to b^* gives the bond policy function on the correct grid, which can be constructed with interpolation. Note that here we also need to impose the borrowing limit $\frac{b^{*\prime}}{R^*} \geq -\kappa$.

6. With the bond policy function $b^{*\prime}(b^*, h; w)$, update the consumption policy function $c^T(b^*, h; w)$ using the budget constraint again.

7. Go back to step 2 and iterate until $c^T(b^*, h; w)$ converges. From this, the policy functions for non-tradable consumption and bond follows.

8. Iterate and find the value function with the result from step 7.

Stationary distribution and equilibrium wage. From the solution to the dynamic programming problem, we can apply the following algorithm to solve the stationary distribution of households, aggregates, and equilibrium wages

1. Guess wage w and household distribution $\psi(b^*, h)$ [5].

2. Solve household problem given wage w and obtain the household bond policy function $b^{*\prime}(b^*, h; w)$.

3. Now we want to find the invariant distribution of household. The subtlety here is that $b^{*\prime}(b^*, h; w)$ does not lie exactly on the asset grid in general. Let's say for some b^* and h, $b^{*\prime}(b^*, h; w)$ lies between $[b^*_{[i]}, b^*_{[i+1]})$, where $b_{[i]}$ and $b_{[i+1]}$ are the two nearest asset grids. The approach we take here is to assign a fraction of household weighed

[5] The distribution can in general take on a different grid from b^*. For example, if adding grids when solving the household problem is more costly and the distribution is not smooth enough due to insufficient grids, we can interpolate the policy function onto a finer grid and obtain the stationary distribution on that.

by the inverse of relative distance to the grids. That is, for all i,

$$\psi_{new}(b^*_{[i]}, h') = \sum_{b,h} \mathbb{1}_{b^{*\prime}(b^*,h,w)\in[b^*_{[i]},b^*_{[i+1]}]} \frac{b^*_{[i+1]} - b^{*\prime}(b^*,h;w)}{b^*_{[i+1]} - b^*_{[i]}} \pi(h'|h)\psi_{old}(b^*,h)$$

$$\psi_{new}(b^*_{[i+1]}, h') = \sum_{b^*,h} \mathbb{1}_{b^{*\prime}(b^*,h,w)\in[b^*_{[i]},b^*_{[i+1]}]} \frac{b^{*\prime}(b^*,h;w) - b^*_{[i]}}{b^*_{[i+1]} - b^*_{[i]}} \pi(h'|h)\psi_{old}(b^*,h)$$

where ψ_{old} is the distribution from last iteration and ψ_{new} is the updated distribution [6]. Iterate on this step until the distribution converges.

4. Compute the equilibrium aggregates $B^*(\psi;w)$, $C^T(\psi;w)$ and $C^N(\psi;w)$ using the invariant distribution and household policy functions.

5. Since full employment is always obtained at the stationary equilibrium, update the guess of wage by

$$w^{new} = \tilde{\theta}\frac{1-\phi}{\phi}\left(\frac{c^T(\psi;w^{old})}{\bar{h}}\right)^{\frac{1}{\gamma}} + (1-\tilde{\theta})w^{old}$$

where $\tilde{\theta}$ determines the weight for the update rule.

6. Use the updated wage, go back to step 2 and iterate until the wage converges.

C.2 Transition dynamics and welfare comparisons

In order to solve for optimal policies, we need to compute the entire transition paths for correct welfare comparisons. We consider various shocks on fundamentals and policies. The shocks we consider can be an arbitrary combination of unanticipated (MIT) shocks and perfectly anticipated shocks [7].

As an example, here is the algorithm for solving the transition dynamics of an unanticipated one-time credit crunch (say at period 1, κ_a suddenly jumps to κ_b)

1. Fix some policy path $\{\tau_t\}_{t=0}^T$ and some level of nominal rigidity Ψ.

2. Pick a sufficiently large T such that the economy have reached the new stationary equilibrium by period T. Solve for the initial and final stationary equilibria (corresponding to κ_a and κ_b)

[6] If $b^{*\prime}(b^*,h;w)$ goes beyond the last grid of b^*, we assign all the households to the last grid.
[7] We abstract away from aggregate uncertainties for simplicity

3. Guess a transition path, which consists of $\{\psi_t(b^*,h), w_t, \lambda_{jt}, T_t, C_t^N, C_t^T\}_{t=0}^T$. Note that from the stationary equilibria, we know these objects at period 0 and period T. In addition, from the final stationary equilibrium, we know the value function and policy function at period T.

4. Using the tradable consumption policy at period T and the guessed transition path, iterate backwards until period 1 with step 2-6 in the household problem. Note that the borrowing limit, the wage and the unemployment level can be different across time now.

5. Using the policy along the path and the initial distribution, iterate forward to find $\{\psi_t(b^*,h), C_t^N, C_t^T, T_t\}$ along the path. The approach here is same as in step 3-4 in the stationary distribution.

6. From period 1 to $T-1$, update the wage w_t consecutively by

$$w_t^{new} = \tilde{\theta} \max\left\{ \frac{1-\phi}{\phi} \left(\frac{c^T(\psi; w_t^{old})}{\bar{h}} \right)^{\frac{1}{\gamma}}, \Psi w_{t-1}^{new} \right\} + (1-\tilde{\theta}) w_t^{old}$$

   where the maximum is to check if the nominal rigidity constraint binds.

7. If the nominal rigidity constraint were to bind, i.e., $w_t = \Psi w_{t-1}$ for some t, update λ_{jt} with its functional form of C_t^N and $\bar{h}$.

8. Go back to step 4, iterate until the path converges.

9. With the path $\{\psi_t(b^*,h), C_t^N, C_t^T, T_t\}$ and household policies, iterate backwards and get the new household value at period 1. This will be the proper welfare to make comparisons.

C.3 Optimal macroprudential policies along the transition path

We consider the Ramsey macroprudential policy along the transition path. That is, the government perfectly commits to a policy path $\{\tau_t\}_{t=0}^T$ along the transition path to maximize ex-ante consumer welfare. The social planner is assumed to be utilitarian, who puts uniform Pareto weights on each household and chooses to maximize

We conjecture that at stationary distributions with no active macroprudential policies and external shocks, the government will keep the macroprudential policy to be inactive. This is due to the fact that there are no externalities at play and there is no redistribution

motive from the policy by construction. Further, conjecture that when the nominal rigidity is not expected to bind, the government will not wield the policy tool and the equilibrium will reach the same new stationary distribution.

Again consider the MIT shock (κ_a unexpectedly jumps to κ_b at period 1) for simplicity, the algorithm to solve the Ramsey policy is as follows

1. Conjecture a "relevant" window for policy. For instance, the government might not have the incentive to use the policy after some period T_r and we know the equilibrium will converge to the final stationary equilibrium. Here for the MIT shock, with nominal rigidity, the relevant window is just the first period
2. Construct a grid for possible policy paths $\{\tau_t\}_{t=1}^{T_r}$. In particular, take a range of possible values of τ and take the T_rth Cartesian product of it.
3. Solve the initial and final stationary equilibria according to the shock.
4. For each possible policy path, solve the transition paths and the consumer welfare at the initial period.
5. Find the maximized initial welfare and the corresponding Ramsey policy $\{\tau_t\}_{t=1}^{T_r}$.

Note that this process is in general computationally costly. For a grid of τ with n grid points and T_r relevant periods. We need to evaluate n^{T_r} transition paths. Parallelization methods like multi-threading is a must, but the dimensionality of the problem remains. It is thus wise to take a coarser grid first and find the relevant range of the policy. Then try to add more precisions by putting more grid points.

Apart from grid search, we can apply a solver on the τ_t path, which can increase accuracy with a cost of the threat of local maximum and boundary problems (τ_t might, say, go below -100%).

Thus, a two step approach is recommended. One shall first narrow the potential range of the optimal policy by grid search and then apply box constrained optimization to tackle down the exact policy path.

D Summary of results under different shock to R_0

Table 5: Results under no R_0 shock

Instrument	Wage Rigidity	Optimal Policy	Unemployment with Policy
τ_0 only	$t = 1$ only	$\tau_0^{HANK} \gg \tau_0^{RANK} > 0$	$\lambda_1^{RANK} < 1, \lambda_1^{HANK} = 1$
τ_0 and τ_1	$t = 1$ only	$\tau_0^{HANK} = \tau_0^{RANK} > 0,\ \tau_1^{HANK} = \tau_1^{RANK} < 0$	$\lambda_1^{RANK} = 1, \lambda_1^{HANK} = 1$
τ_0 only	all periods	$\tau_0^{RANK} > 0 \gg \tau_0^{HANK}$	$\lambda_1^{RANK} < 1, \lambda_1^{HANK} < 1$
τ_0 and τ_1	all periods	$\tau_0^{HANK} \approx \tau_0^{RANK} > 0,\ \tau_1^{HANK} \approx \tau_1^{RANK} < 0$	$\lambda_1^{RANK} = 1, \lambda_1^{HANK} = 1$

Table 6: Results under Large R_0 shock (R_0 drop 20%)

Instrument	Wage Rigidity	Optimal Policy	Unemployment with Policy
τ_0 only	$t = 1$ only	$\tau_0^{HANK} \gg \tau_0^{RANK} > 0$	$\lambda_1^{RANK} < 1, \lambda_1^{HANK} < 1$
τ_0 and τ_1	$t = 1$ only	$\tau_0^{HANK} = \tau_0^{RANK} > 0,\ \tau_1^{HANK} = \tau_1^{RANK} < 0$	$\lambda_1^{RANK} = 1, \lambda_1^{HANK} = 1$
τ_0 only	all periods	$\tau_0^{HANK} > \tau_0^{RANK} > 0$	$\lambda_1^{RANK} < 1, \lambda_1^{HANK} < 1$
τ_0 and τ_1	all periods	$\tau_0^{HANK} > \tau_0^{RANK} > 0,\ \tau_1^{RANK} < \tau_1^{HANK} < 0$	$\lambda_1^{RANK} < 1, \lambda_1^{HANK} < 1$