Market Fragmentation, Distributional Uncertainty, and Asset Pricing*

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Abstract

In the U.S. equities market, liquidity has improved significantly over the past three decades across traditional metrics; however, uncertainty in the *distribution* of liquidity across fragmented venues has risen markedly. We show that while the "cost of a trade" may have fallen, the "cost of finding a trade" has become a primary driver of risk premiums. We develop a model of directed search across exchanges with discriminatory pricing and prove that distributional uncertainty unambiguously reduces welfare and trading volume, and can widen the bid–ask spread. Empirical evidence is consistent with these predictions: investors demand substantial compensation for higher distributional uncertainty (9.38% average annual excess return over the Fama–French five factors plus momentum), and this premium operates primarily through the transaction cost channel. The impact of fragmentation on returns depends on market structure (pre- versus post-Regulation NMS), which we explain using a decomposition of market shares into stable and noisy components. We also show that the illiquidity premium is conditioned by distributional uncertainty and propose a fragmentation-adjusted measure.

Keywords: Distributional Liquidity Uncertainty, Market Fragmentation, Asset Pricing, Illiquidity.

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1 Introduction

Over the past three decades, liquidity in U.S. equity markets has improved significantly and uniformly across a wide array of traditional metrics. Quoted, effective, and realized spreads have trended downward, while aggregate trading activity—measured by dollar volume, number of transactions, and share turnover—has surged. Even traditional illiquidity measures, such as the Amihud (2002) ratio, indicate a long-term structural improvement in market liquidity. On the surface, these trends suggest a more efficient and accessible marketplace for all participants.

However, this period of aggregate improvement coincided with a fundamental shift in market structure. The Securities and Exchange Commission ("SEC") established Regulation NMS in April 2005 to modernize U.S. equity markets. An unintended consequence of Regulation NMS is that it dispersed trading across exchanges and created a highly fragmented market structure. For stocks listed on the New York Stock Exchange (NYSE), the listing exchange's market share of total share volume plummeted from over 80% in early 2005 to approximately 20% by 2009. Concurrently, the Herfindahl–Hirschman Index (HHI) for these stocks fell from the 8000s to the 2000s, signaling a substantial and permanent increase in market fragmentation.

While the rise of fragmentation is well-documented, we demonstrate that a more critical friction has emerged: a sharp increase in **uncertainty in liquidity distribution**. In the modern U.S. equities markets, traders face not only a multitude of platforms but also significant uncertainty regarding *where* liquidity is located at any given moment. Our paper emphasizes that while the "cost of a trade" (spread) may have fallen, the "cost of finding a trade" (distributional uncertainty) has risen, becoming a primary driver of risk premiums in the 21st century. This friction differs fundamentally from aggregate market-wide volatility: it is the cost of finding a trade, not the cost of the trade itself. Our data reveal a stark divergence: while uncertainty in aggregate market-wide liquidity is significantly lower today than in the 1990s and has remained fairly stable in the post-Reg NMS era, uncertainty in liquidity distribution has soared.

We formalize this friction in a model of directed search across N exchanges with discriminatory pricing (Section 3). Buyers route orders before observing exchange-level sell-side liquidity; the short side of each market trades at the long side's reservation values. The model

yields two key results. First, the total gains from trade are strictly concave in exchange-level liquidity flow, so any increase in distributional uncertainty (a mean-preserving spread in liquidity across venues) unambiguously reduces welfare and trading volume (Proposition 4). Second, under reasonable distributional assumptions, liquidity uncertainty widens the bid–ask spread (Proposition 5). The average transaction price, by contrast, can respond ambiguously to uncertainty, underscoring the distinction between allocative efficiency and commonly used price-based measures of market quality.

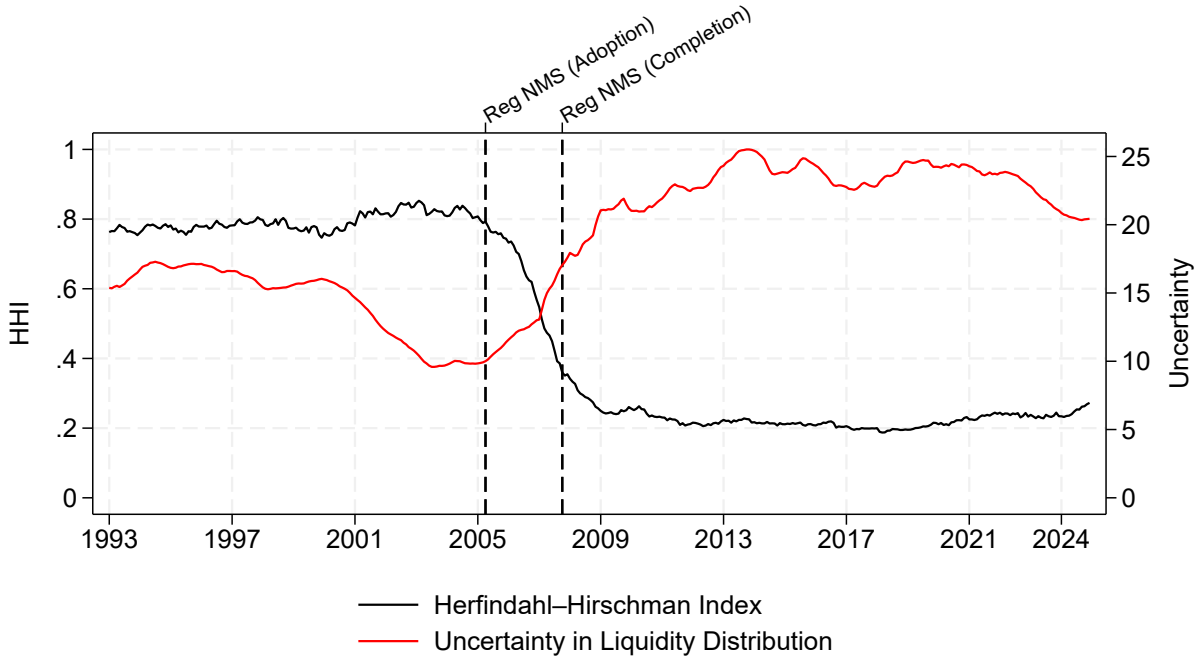


Figure 1: **Fragmentation and Uncertainty in Liquidity Distribution** The sample covers stock-months that have non-missing data from January 1993 to December 2024, encompassing ordinary common shares listed on the NYSE. The left y-axis displays the average HHI in the cross-section, with each stock's HHI calculated as the sum of the squared exchange-specific market shares based on their share volumes. The right y-axis displays the uncertainty in liquidity distribution, defined as the monthly ratio of the average absolute daily deviations of exchange market shares from their long-term averages. The left dashed line corresponds to April 2005, the time of the adoption of Regulation NMS. The right dashed line corresponds to July 2007, the completion of Regulation NMS.

The empirical results are consistent with our model's predictions. We first show that investors require significant compensation for holding stocks with higher distributional un-

certainty. A long-short portfolio of stocks with high versus low average absolute short-term noise—our proxy for daily distributional uncertainty—yields an average annual excess return of 9.38%. Furthermore, we demonstrate that uncertainty is not necessarily a new independent risk factor; rather, it affects asset prices primarily via the **transaction cost channel**. Investors demand higher returns for high-uncertainty stocks because uncertainty significantly intensifies the effective cost of execution.

This mechanism explains why the impact of fragmentation on asset pricing is heavily dependent on the prevailing market structure. In the concentrated markets (pre-Reg NMS), high fragmentation (low HHI) was the primary source of uncertainty, and a long-short portfolio of high- versus low-fragmentation stocks yielded an average annual excess return of 8.7%. However, in the fragmented markets (post-Reg NMS), this relationship reversed to -12.0%. This reversal occurs because, in the modern structure, a higher HHI no longer signals stable concentration on a listing exchange. Instead, a higher HHI typically indicates either greater concentration on noisy non-listing venues or larger short-term noise—both of which increase the distributional uncertainty for which investors demand a higher risk premium.

When we decompose market shares into stable long-term components and noisy short-term fluctuations, the results are telling. A long-short portfolio based on average absolute short-term noise—our proxy for daily distributional uncertainty—yields an average annual excess return of 9.38%. Conversely, investors value stable liquidity on listing exchanges, accepting 6.42% lower returns for stocks with high long-term listing exchange concentration, while demanding a 10.43% premium for stocks dispersed across non-listing venues. We also show that the conventional illiquidity premium is conditioned by distributional uncertainty (rising from 3.91% to 28.32% across uncertainty levels) and develop a fragmentation-adjusted illiquidity measure (15.66% annual excess return); the full analysis is in the Appendix.

Our paper contributes to the literature on market microstructure and asset pricing by shifting the focus from the amount of liquidity to the uncertainty in its distribution. We contribute to three strands of literature. First, we depart from studies on market fragmentation that focus solely on market quality by focusing on asset pricing implications. We shift the focus from the *amount* of fragmentation to the *uncertainty* it generates. Second, we link microstructure to the cross-section of expected returns, a connection often overlooked in traditional asset pricing. Finally, we contribute to the illiquidity literature by demonstrating that the illiquidity premium is fundamentally shaped by the predictability of liquidity

distribution.

Our paper contributes to three strands of literature: market fragmentation, the cross-section of expected stock returns, and (il)liquidity. The literature on market fragmentation typically examines how fragmentation affects market quality, with contributions from, among others, Bennett and Wei (2006), Foucault and Menkveld (2008), O'Hara and Ye (2011), Degryse et al. (2015), Kwan et al. (2015), Pagnotta and Philippon (2018), Baldauf and Mollner (2021), Chen and Duffie (2021), Guo and Jain (2023), and Haslag and Ringgenberg (2023). Departing from these studies on market microstructure, we focus on the implications of market fragmentation for asset prices, linking our research to the literature examining the cross-section of expected stock returns, which are extensively covered by Harvey et al. (2016), Green et al. (2017), Linnainmaa and Roberts (2018), Feng et al. (2020), and Jensen et al. (2023). This body of work usually ignores the evolving dynamics of U.S. equities markets and overlooks microstructure aspects (O'Hara, 2003). Our paper bridges this gap and establishes a connection between microstructure and asset pricing. Specifically, we illustrate that market fragmentation, as well as substantial changes in securities regulations affecting market structure, significantly influence asset prices.

We also contribute to the literature studying the effect of illiquidity on asset pricing: among others, Amihud (2002), Pastor and Stambaugh (2003), Acharya and Pedersen (2005), Brennan et al. (2013), Bali et al. (2014), Lou and Shu (2017), Amihud and Noh (2021). This strand of research intersects with the broader asset pricing literature, as illiquidity is often examined as a priced risk factor. We demonstrate that the impact of illiquidity on asset prices heavily depends on market fragmentation, and to account for this variation, we develop a new fragmentation-adjusted illiquidity metric.

Our study also holds policy relevance. Policy discussions regarding trading and markets generally focus on their impacts on market quality. However, we emphasize that frictions stemming from market microstructure and its regulations can also influence asset prices. Regulations can have unintended consequences, which can lead to significant changes in market structure and consequently shape investors' risk perceptions, potentially to varying degrees across stocks. We demonstrate that market fragmentation, an unintended consequence of Regulation NMS, can serve as such an impetus. In this respect, the SEC's recent amendments in September 2024, detailed in 89 FR 81620, to reduce exchange access fees and minimum price increments, might affect asset prices, as they are likely to influence

competition between market centers and, consequently, could impact market structure.

The rest of the paper is organized as follows. Section 2 covers institutional details. Section 3 develops the theoretical framework. Section 4 describes the data and empirical methodology. Section 5 constructs our measures of distributional uncertainty and tests their effect on expected returns and transaction costs. Section 6 examines how market structure shapes the fragmentation–return relationship and documents the amplification effect of Regulation NMS. Section 7 concludes. Robustness analyses and the illiquidity-fragmentation unifying measure are in the Appendix.

2 Institutional Details

Regulation NMS, adopted in April 2005, has significantly transformed U.S. equities markets: listing exchanges have lost their dominance and trading has become increasingly fragmented. Although Regulation NMS has affected all U.S.-listed stocks, this transformation is most noticeable in stocks listed on the NYSE (75 FR 3594, pp. 3594), as previously displayed in Figure 1, due to its extensive reliance on manual trading practices until late 2006. Regulation NMS encompasses four major interrelated changes. Two of these changes are primarily responsible for the shift in trading from the NYSE to other market centers, resulting in greater fragmentation: namely, Sub-Penny Pricing Rule (Rule 612) and Order Protection Rule (Rule 611). The former redirected trading from the NYSE to off-exchange market centers, while the latter shifted it from the NYSE to other competing national securities exchanges. Figure 2 offers an in-depth view of the timing of these rules and other changes in market structure. Contrary to the common tendency in the literature to use the full implementation date of Regulation NMS (July-August 2007) as a shock to market structure, as shown in Figure 2, the market structure had already undergone significant changes by the time Regulation NMS was fully implemented.[1] By July 2007, the NYSE had already lost over 30% of its market share, and the market had become 150% more fragmented.

The primary reason why Regulation NMS began affecting markets even before its implementation is the Sub-Penny Pricing Rule, which set the minimum price increment (i.e., tick size) to \$0.01 for all quotations priced at or above \$1.00. This rule applies to both

[1]For example, Haslag and Ringgenberg (2023) compares the market quality of small and large stocks in a difference-in-differences framework, using July-August 2007 to delineate pre- and post-policy periods.

national securities exchanges and off-exchange market centers—alternative trading systems (ATSs) and non-ATSs. It often creates artificially larger quoted bid-ask spreads, even for actively traded stocks, as quoted spreads frequently remain at or just above a penny (Kwan et al. (2015)). This dynamic, together with exchanges' price-time priority rules according to which limit orders are filled, results in longer queues on exchanges' limit order books, which, in turn, has led investors to off-exchange market centers to jump these long queues. This migration is likely to be reinforced by network externalities: as more investors switch to off-exchange market centers, execution becomes more likely, resulting in further migration.

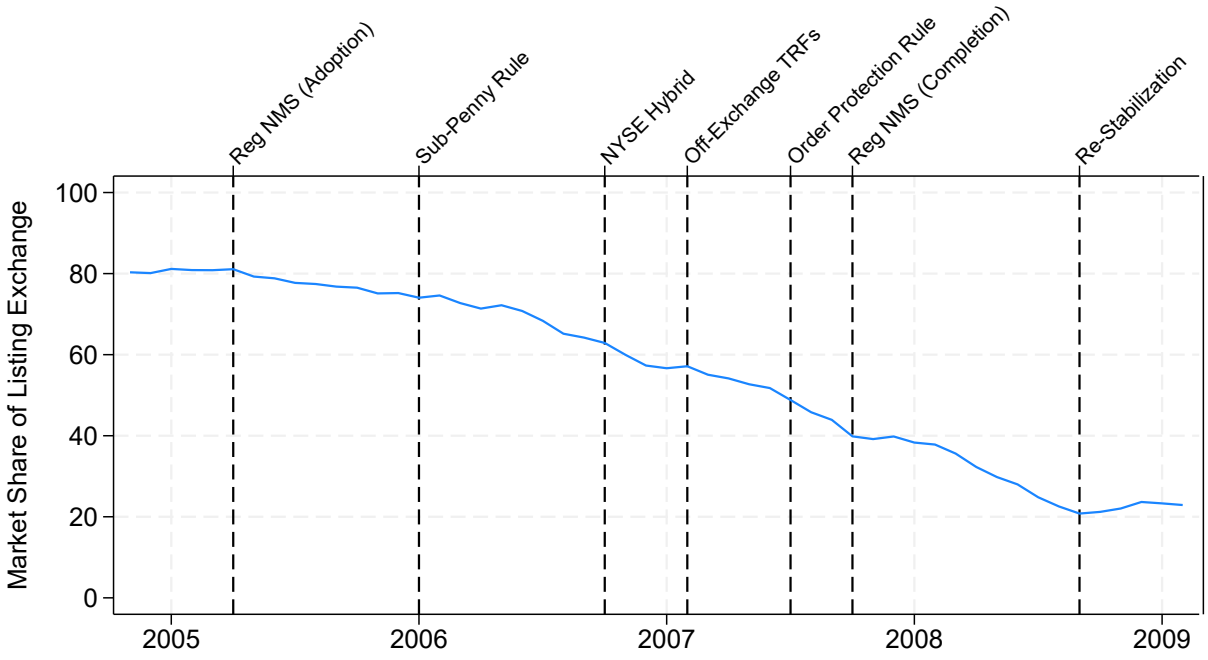


Figure 2: **Timing of Regulation NMS and Other Changes in Market Structure.** The sample covers stock-months with non-missing data from November 2004 to February 2009, encompassing ordinary common shares listed on the NYSE. The left y-axis represents the percentage of monthly consolidated share volume executed on the NYSE. The seven dashed lines correspond to the following events and dates, from left to right: **(1)** the adoption of Regulation NMS in April 2005; **(2)** the implementation of the Sub-Penny Pricing Rule (Rule 612 of Regulation NMS) in January 2006; **(3)** the NYSE's implementation of the NYSE Hybrid in October 2006 to expand its electronic trading; **(4)** the requirement to report off-exchange trades separately from on-exchange trades through trade reporting facilities (TRFs) in March 2007; **(5)** the implementation of the Order Protection Rule (Rule 611 of Regulation NMS) in July 2007; **(6)** the completion of phased-in compliance with Regulation NMS in October 2007; **(7)** the period around when market shares across exchanges stabilized again, September 2008.

Larger quoted spreads have also facilitated off-exchange trading by providing more incentive for investors to seek sub-penny price improvements (Bartlett and McCrary (2015)). Although the Sub-Penny Pricing Rule does not prohibit any market center from executing trades in sub-penny increments, it has led to a more frequent occurrence of sub-penny trading on off-exchange market centers, especially non-ATSs, compared to exchanges.[2] Exchanges match multilateral orders on an agency basis in accordance with publicly disclosed rules and procedures, without the discretion of their operators or trading parties, where orders submitted must be in penny increments as per Rule 612. In this environment, sub-penny trading occurs on exchanges only for mid-point or volume-weighted executions or for executions falling under exemptions outlined in Rule 612(c). In contrast, it is easier for non-ATSs to trade in sub-penny increments because they typically trade in a principal capacity and exercise discretion on a per-order basis in determining price improvements, without needing to establish publicly disclosed pre-determined rules and procedures. In this regard, the Sub-Penny Pricing Rule has provided non-ATSs with a competitive edge relative to exchanges and helped them attract additional order flow.

The initial compliance date of the Sub-Penny Pricing Rule was set for August 29, 2005, giving investors and their brokers less than five months to adjust their routing matrices. Figure 2 clearly reveals that investors started to adjust their routing immediately after the adoption of the Sub-Penny Pricing Rule instead of waiting for its implementation. This rapid movement in trading is likely due to the fact that making such a significant change in routing and re-optimizing routing matrices requires time, prompting market participants to begin acting sooner rather than later. The compliance date was later extended to January 31, 2006, on August 2, 2005. Figure 2 reveals that the drop in the NYSE's market share accelerated after the official implementation of the Sub-Penny Pricing Rule in January 2006.

The Order Protection Rule has further contributed to the shift of trading from the NYSE to other market centers, especially to other national securities exchanges. It was originally scheduled to be implemented towards mid-2006; however, its implementation was delayed for more than a year because market centers needed additional time to modify their trading systems and conduct quality assurance tests beforehand. The rule began to be implemented

[2]For instance, in the first half of 2022, of the total dollar volume of sub-penny trades, excluding mid-point and volume-weighted average price executions, 11% occurred on exchanges while 89% occurred on off-exchange market centers (87 FR 80266, p. 80273).

for 250 NMS stocks on July 9, 2007, and for all remaining NMS stocks on August 20, 2007. This rule prohibits any market center from executing orders that are worse than protected quotations—lower than a protected bid or higher than a protected offer. It protects only the best bid and best offer at the top of each market center's order book. The remaining quotations lower in a market center's book are not guaranteed to receive price priority, even if they are better than the best available protected prices in other market centers.[3] This situation has created an opportunity for other exchanges to draw order flow from the NYSE, as each exchange has its top quotes protected, consequently intensifying competition against the NYSE. Larger orders have typically begun to be executed across multiple exchanges, resulting in a more fragmented market structure and the decline of the NYSE's previously dominant position as a trading venue (Spatt (2018)).[4]

Under Regulation NMS, for a quote to be protected, it must be an automated quotation displayed by an automated trading center. These automation aspects generally require that quotations be immediately accessible and automatically executable without any delay. Combined with increased competition from fully electronic market centers, such as electronic communications networks (ECNs), these regulatory developments have placed significant pressure on the NYSE to automate its trade execution processes.[5] The NYSE officially adopted a new market system, known as the Hybrid Market, combining a floor-based auction with electronic trading, on March 22, 2006. It then gradually expanded its automatic order routing and execution practices between October 6, 2006 and January 24, 2007. This initiative forfeited a key competitive advantage of the old specialist system, which relied heavily on manual elements (Spatt (2018)). As a result, the NYSE became more similar to other exchanges in its trade execution processes, likely accelerating its loss of market share.

[3]For instance, in the spirit of Stoll (2006)'s example, suppose there are only two market centers: A and B. A bids \$50.05 for 300 shares and \$50.00 for 200 shares, while market center B bids \$49.95 for 400 shares. The Order Protection Rule protects A's bid of \$50.05 but not its bid of \$50.00. An incoming market order to sell 500 shares would be required to execute at the best available "top of the book" prices, meaning it would sell 300 shares in A at \$50.05 and 200 shares in B at \$49.95.

[4]In Form 424B3 filing on May 5, 2006, the NYSE stated its expectation that Regulation NMS could adversely impact their business by intensifying competition between market centers following the implementation of the Order Protection Rule.

[5]Another key feature for a quote to be protected is that it must be disseminated through the consolidated market data feeds, implying that no quotes are protected when the consolidated market data feeds are not in operation. Please refer to the SEC's memorandum on April 30, 2015 for more details regarding the Order Protection Rule and other key elements of a quote required to receive protection.

3 A Theory of Fragmentation and Asset Pricing

This section develops a model of fragmented trading in which exchange-specific liquidity is uncertain. The model delivers three testable predictions: (i) uncertainty in the distribution of liquidity across venues reduces expected trading volume and total gains from trade; (ii) under reasonable distributional assumptions, uncertainty widens the bid–ask spread; and (iii) the average transaction price responds ambiguously to uncertainty, underscoring the distinction between allocative efficiency and measured transaction costs.

3.1 Environment

Consider a static model of a fragmented secondary market in which investors trade a single asset across $N \geq 2$ exchanges, as in the U.S. equity market setting described in Section 2. There is a unit measure of buyers, each seeking to purchase at most one share. Buyers choose which exchange to route their orders to.

Valuations. Buyers have heterogeneous valuations: a buyer's valuation V_B is drawn i.i.d. from a continuous CDF F with strictly positive density f on $[\underline{v}, \bar{v}]$. Sellers have heterogeneous reservation values: each seller's reservation value V_S is drawn i.i.d. from a continuous CDF H with strictly positive density h on $[\underline{w}, \bar{w}]$. We denote $\mu_B \equiv \mathbb{E}[V_B]$ and $\mu_S \equiv \mathbb{E}[V_S]$. The supports do not overlap: $\bar{w} < \underline{v}$, so there are always gains from trade between any buyer–seller pair.

Seller flow. The mass of sell orders on exchange i is S_i, an exogenous nonnegative random variable. In a symmetric environment, S_i is i.i.d. across exchanges with common CDF G having mean $\mathbb{E}[S_i] > 0$. Sellers do not observe exchange-level realizations before submitting.[6]

Timing. The model proceeds in three stages: (1) Buyers choose an exchange $i \in \{1, \ldots, N\}$ and submit a buy order. (2) Sell order flow S_i is realized on each exchange, and each buyer

[6] This is a simplification: we do not model strategic routing by sellers. The seller mass can be endogenized in a dynamic extension with stationary type-switching; we focus on the static setting for clarity.

draws her valuation V_B from F.[7] (3) Each exchange allocates shares and clears using the pricing rule described below.

Pricing rule and institutional motivation. On each exchange, the short side of the market trades at the long side's reservation values. We refer to this as a *discriminatory pricing rule*: when there are more buyers than sellers, each executed trade occurs at the matched buyer's valuation; when there are more sellers than buyers, each executed trade occurs at the matched seller's reservation value. This rule differs from a uniform-price call auction and captures a key feature of modern limit order markets. In practice, a marketable buy order that walks up the offer side of the book pays a price that depends on its own size and urgency; similarly, a marketable sell order that walks down the bid side receives a price that depends on its own characteristics. The effective price paid by each investor thus reflects her own willingness to trade, not a single market-clearing price. Our discriminatory rule is a tractable reduced form for this mechanism.

3.2 Allocation and prices

On exchange i, let B_i denote the mass of buy orders and S_i the mass of sell orders. The allocation rule is efficient conditional on the order flow: the highest-valuation buyers trade with the lowest-reservation-value sellers.

- **Buyer-heavy state** ($S_i < B_i$): All S_i sellers trade and the S_i highest-valuation buyers receive shares. Each trade occurs at the matched buyer's valuation V_B. The marginal transacting buyer has valuation v_B^* satisfying $\bar{F}(v_B^*) = S_i/B_i$, where $\bar{F} \equiv 1 - F$.
- **Seller-heavy state** ($S_i > B_i$): All B_i buyers trade and the B_i lowest-reservation-value sellers supply shares. Each trade occurs at the matched seller's reservation value V_S. The marginal transacting seller has reservation value v_S^* satisfying $H(v_S^*) = B_i/S_i$.

Realized volume on exchange i is $\min(B_i, S_i)$.

[7] The assumption that buyers learn their valuation after choosing a venue captures the idea that investors route based on ex ante exchange characteristics, while the realized urgency or private value that governs willingness to pay is revealed only at execution.

Lemma 1 (Exchange-level prices) *The average transaction price on exchange i is*

$$\bar{P}_i = \begin{cases} \mathbb{E}[V_B \mid V_B \geq v_B^*] = \dfrac{1}{\bar{F}(v_B^*)} \displaystyle\int_{v_B^*}^{\bar{v}} v \, dF(v) & \text{if } S_i < B_i, \\ \mathbb{E}[V_S \mid V_S \leq v_S^*] = \dfrac{1}{H(v_S^*)} \displaystyle\int_{\underline{w}}^{v_S^*} v \, dH(v) & \text{if } S_i > B_i, \end{cases}$$

where $v_B^* = \bar{F}^{-1}(S_i/B_i)$ and $v_S^* = H^{-1}(B_i/S_i)$.

The average transaction price depends on the *magnitude* of the order imbalance through the cutoffs v_B^* and v_S^*, not only on which side is long. We define the *bid–ask spread* as the difference between the expected average price in buyer-heavy and seller-heavy states: $\text{Spread} \equiv \mathbb{E}[\bar{P}_i \mid S_i < B_i] - \mathbb{E}[\bar{P}_i \mid S_i > B_i]$.

3.3 Equilibrium

Fix an exchange with buy mass $B > 0$ and seller flow S. Under the discriminatory rule, buyer surplus takes a simple form. If $S < B$, executed buyers pay their own valuation, so buyer surplus is zero. If $S > B$, all buyers trade at seller reservation values, yielding conditional expected surplus

$$w(B, S) = \begin{cases} 0 & \text{if } S < B, \\ \mu_B - \mathbb{E}[V_S \mid V_S \leq v_S^*(B, S)] & \text{if } S > B. \end{cases}$$

The function $W(B) \equiv \mathbb{E}[w(B, S_i)]$ gives the ex ante expected surplus of a buyer routed to an exchange with buy mass B. Since W is strictly decreasing in B—higher buy-side crowding reduces both the probability of trading and the surplus conditional on trading—all exchanges must offer the same expected surplus in equilibrium.

Definition 2 (Symmetric equilibrium) *A* symmetric equilibrium *is an allocation $B_i = 1/N$ for all $i \in \{1, \ldots, N\}$ such that $W(1/N) \geq W(B')$ for any buy mass $B' > 1/N$ achievable by a positive-measure deviation of buyers to a single exchange.*

In the symmetric equilibrium, each exchange receives buy mass $B \equiv 1/N$, and the

expected price on each exchange is

$$\mathbb{E}[\bar{P}_i] = \mathbb{E}\big[\mathbb{E}[V_B \mid V_B \geq v_B^*]\,\mathbf{1}\{S_i < B\}\big] + \mathbb{E}\big[\mathbb{E}[V_S \mid V_S \leq v_S^*]\,\mathbf{1}\{S_i > B\}\big]. \tag{1}$$

3.4 Liquidity uncertainty and market outcomes

We study how the dispersion of exchange-specific sell-side liquidity affects execution, the gains from trade, and the bid–ask spread. Throughout this subsection, the economy is in the symmetric equilibrium with buy mass $B = 1/N$ on each exchange, and changes in liquidity uncertainty are represented by mean-preserving spreads of S_i.

3.4.1 Execution and welfare

Expected trading volume is $\text{Vol} = N\,\mathbb{E}\min(B, S_i)$, and the expected fill rate for a buyer is $\mathbb{E}[\min(1, S_i/B)]$. Since $s \mapsto \min(B, s)$ is concave, a mean-preserving spread in S_i reduces both objects by Jensen's inequality. The following result establishes a stronger welfare conclusion.

Definition 3 (Gains from trade) *Fix buy mass $B > 0$. The* total gains from trade *realized on an exchange with seller flow $S \geq 0$ are*

$$\Gamma(S) \equiv \begin{cases} S\big[\mathbb{E}[V_B \mid V_B \geq v_B^*] - \mu_S\big] & \text{if } S \leq B, \\ B\big[\mu_B - \mathbb{E}[V_S \mid V_S \leq v_S^*]\big] & \text{if } S > B, \end{cases}$$

where $v_B^* = \bar{F}^{-1}(S/B)$ and $v_S^* = H^{-1}(B/S)$.

When $S \leq B$, the S highest-valuation buyers trade with all S sellers; $\Gamma(S)$ is the total buyer valuation minus the total seller reservation value of executed trades. When $S > B$, all B buyers trade with the B lowest-reservation-value sellers, and $\Gamma(S)$ is defined analogously.

Proposition 4 (Execution and welfare) *Fix buy mass $B > 0$.*

1. *Γ is continuous, strictly increasing, and strictly concave on $(0, \infty)$.*
2. *For any mean-preserving spread of S_i, expected trading volume $\mathbb{E}[\min(B, S_i)]$ and expected total gains from trade $\mathbb{E}[\Gamma(S_i)]$ both strictly decrease.*

Proof. For part 1, we compute derivatives separately on $(0, B)$ and (B, ∞). On $(0, B)$: $\Gamma(S) = B \int_{v_B^*}^{\bar{v}} v \, dF(v) - S\mu_S$, where $v_B^* = \bar{F}^{-1}(S/B)$. Differentiating, $\Gamma'(S) = \bar{F}^{-1}(S/B) - \mu_S > 0$ and $\Gamma''(S) = -\frac{1}{Bf(v_B^*)} < 0$. On (B, ∞): $\Gamma(S) = B\mu_B - S \int_{\underline{w}}^{v_S^*} v \, dH(v)$, where $v_S^* = H^{-1}(B/S)$. Differentiating, $\Gamma'(S) = \int_{\underline{w}}^{v_S^*} (v_S^* - v) \, dH(v) > 0$ and $\Gamma''(S) = -\frac{B^2}{S^3 \, h(v_S^*)} < 0$. At $S = B$, the left derivative is $\underline{v} - \mu_S$ and the right derivative is $\bar{w} - \mu_S$. Since $\underline{v} > \bar{w}$, the left derivative strictly exceeds the right derivative, so Γ has a downward kink at $S = B$. Combining, Γ is globally strictly concave on $(0, \infty)$. Part 2 follows from Jensen's inequality: for any strictly concave function, a mean-preserving spread reduces the expectation. The volume result follows identically from the strict concavity of $s \mapsto \min(B, s)$. ■

Proposition 4 is the central welfare result: the coordination failure induced by liquidity uncertainty does not merely reduce the number of executed trades; it reduces the total surplus that trade generates. The mechanism is that extreme imbalances are costly: when S is very low, few trades occur and the ones that do involve the highest-valuation buyers; when S is very high, all buyers trade but sellers receive prices close to their reservation values. The strict concavity of Γ captures the fact that gains from an excess of sellers cannot compensate for the losses from a shortfall.

3.4.2 Bid–ask spread

Whereas the welfare result is unambiguous, the effect of liquidity uncertainty on the bid–ask spread depends on the shape of the valuation distributions.

Proposition 5 (Bid–ask spread) *Define the upper-tail mean of F and lower-tail mean of H as $\phi_B(\alpha) \equiv \mathbb{E}[V_B \mid V_B \geq \bar{F}^{-1}(\alpha)]$, $\alpha \in (0, 1]$, and $\psi_S(\beta) \equiv \mathbb{E}[V_S \mid V_S \leq H^{-1}(\beta)]$, $\beta \in (0, 1]$. In a symmetric equilibrium with $B = 1/N$: (1) When $S_i < B$, the average transaction price is $\phi_B(S_i/B)$; when $S_i > B$, it is $\psi_S(B/S_i)$. (2) Suppose $\mathbb{E}[S_i] = B$ and the mean-preserving spread preserves symmetry of S_i around B. If ϕ_B is convex on $(0, 1]$ and ψ_S is concave on $(0, 1]$, then the bid–ask spread weakly increases under the mean-preserving spread.*

Proof. Part 1 follows from Lemma 1 by substituting the cutoff definitions. For part 2, under the symmetry assumption the conditional distribution of S_i/B given $S_i < B$ (respectively B/S_i given $S_i > B$) becomes more dispersed in the convex order under a symmetric mean-preserving spread. Convexity of ϕ_B then implies $\mathbb{E}[\phi_B(S_i/B) \mid S_i < B]$ weakly increases;

concavity of ψ_S implies $\mathbb{E}[\psi_S(B/S_i) \mid S_i > B]$ weakly decreases. Hence the spread weakly increases. ■

Remark 6 *The convexity and concavity conditions have natural interpretations. The upper-tail mean $\phi_B(\alpha)$ is convex when higher-valuation buyers are sufficiently right-skewed (e.g., Pareto-type tails). The lower-tail mean $\psi_S(\beta)$ is concave when the seller distribution H has a sufficiently increasing density. For uniform distributions, both ϕ_B and ψ_S are linear, so the sufficient conditions do not hold and the sign of the spread response is parametrization-dependent. In contrast, the welfare result in Proposition 4 holds for all continuous F and H with non-overlapping supports.*

Remark 7 (Average transaction price) *The expected average transaction price $\mathbb{E}[\bar{P}_i]$ in (1) mixes buyer-side and seller-side transaction prices. A mean-preserving spread in S_i changes both the probability of each imbalance state and the state-contingent cutoffs, so the net effect on $\mathbb{E}[\bar{P}_i]$ is in general ambiguous. This ambiguity reflects the fact that the average transaction price is not a welfare-relevant statistic: it conflates the surplus accruing to buyers and sellers through different pricing regimes. The appropriate welfare measure is Γ in Proposition 4, which is unambiguously reduced by liquidity uncertainty.*

Empirical predictions. The model maps directly to our empirical design. The dispersion of S_i across exchanges corresponds to *uncertainty in liquidity distribution*: in the data, we proxy this with $|\bar{\varepsilon}|^{\text{total}}/\gamma^{\text{pex}}$, the ratio of average absolute short-term noise in market shares to the long-term market share of the primary listing exchange (Section 5). Proposition 4 implies that higher distributional uncertainty reduces expected volume and gains from trade, which motivates our finding that distributional uncertainty raises effective transaction costs (Section 5, Transaction Cost Analysis) and that investors demand higher expected returns for high-uncertainty stocks. Proposition 5 provides sufficient conditions under which uncertainty widens the bid–ask spread, consistent with a risk premium for distributional uncertainty. Remark 7 explains why a single summary measure of "fragmentation" (e.g., the Herfindahl–Hirschman Index) can produce opposing return premiums in different market structures: the average transaction price responds ambiguously to liquidity uncertainty, so the mapping from concentration to expected returns depends on whether long-term listing exchange share or short-term noise dominates (Section 6).

4 Data and Empirical Framework

We use two main data sources: the Center for Research in Securities Prices (CRSP) and NYSE Trades and Quotes (TAQ). We obtain stock prices, returns, shares outstanding, primary listing exchange information, and other stock characteristics from CRSP. The main analysis includes all ordinary common stocks (CRSP share code: 10-11) that are listed on the NYSE (CRSP primary exchange: N). TAQ contains intraday transactions data (trades and quotes) for all stocks listed on the New York Stock Exchange (NYSE), American Stock Exchange (AMEX), Nasdaq Stock Market (NASDAQ), and SmallCap issues. We obtain exchange-specific daily share volumes from TAQ. We match CRSP and TAQ using the WRDS Daily TAQ CRSP Link. We utilize similar filters as Amihud (2002), requiring the total number of trading days for stock i in month t (D_{it}) to be greater than or equal to 10 and excluding stock-months with monthly average share prices less than \$5.

Our sample spans the period from January 1993 to December 2024. We conduct univariate and bivariate portfolio analyses, as well as perform stock-level regressions following Fama and MacBeth (1973) to confirm that our results remain robust after accounting for other risk factors and control variables. In portfolio analyses, we report excess returns over the three Fama-French factors (MKT, SMB, and HML) and the momentum factor (UMD), which we call four-factor alphas. In Fama-MacBeth regressions, we conduct monthly cross-sectional regressions and report the time-series averages of the coefficients, along with their associated Newey and West (1987) standard errors with six lags. These analyses are primarily conducted for the entire sample, as well as two sub-periods, 1993–2004 and 2009–2023, representing two distinct yet stable market structures. The former period corresponds to the pre-Regulation NMS environment, where the NYSE heavily dominated trading in its listed stocks. The latter period corresponds to the post-Regulation NMS environment, where trading is highly fragmented across exchanges, with no single exchange dominating. Despite these differences, the NYSE's long-term market share remains stable in both periods, as shown in Figure 1.

Our dependent variable in Fama-MacBeth regressions is the monthly four-factor adjusted return.[8] Factor loadings for a stock in month t are estimated using its monthly excess returns

[8]We select this dependent variable to align our study with existing research on liquidity, facilitating comparisons of our new fragmentation-adjusted illiquidity measure with those commonly employed in such studies. See, for instance, Pastor and Stambaugh (2003), Brennan et al. (2013), and Lou and Shu (2017). Our results are robust to using monthly excess stock returns over the risk-free rate, as adopted in Amihud

over the risk-free rate and the three Fama-French factors from the previous 60 months ($t-60$ to $t-1$), with a minimum requirement of 24 observations for the estimation. Our main independent variables of interest are fragmentation and fragmentation-adjusted illiquidity measures, which are introduced in their respective sections.

In addition to these primary independent variables, we include two sets of control variables in our regressions. The first set of variables encompasses those that are correlated with our fragmentation or fragmentation-adjusted illiquidity measures and have been established to affect expected stock returns in the literature. Foucault and Menkveld (2008) show that fragmentation affects liquidity supply, and it has been well established in the literature that liquidity affects expected returns.[9] To this end, we control for the logarithm of the original Amihud illiquidity measure lnA, with A calculated as the monthly average of the daily ratio of absolute return to dollar trading volume in millions, based on daily trading volume data from TAQ. Fragmentation is also related to price impact through changing traders' incentives to send more aggressive orders (Chen and Duffie (2021)) and price impact affects expected stock returns (Lou and Shu (2017)). Therefore, we control for monthly average of daily price impact coefficients, where daily values are directly obtained from WRDS Intraday Indicators. WRDS estimates daily price impact coefficients using regressions of five-minute changes in the midpoints of consolidated best bid and offer prices on the square root of the signed absolute net volume (absolute difference between buyer- and seller-initiated share volume times sign of this difference). In the regression for month t, these two independent variables along with our primary independent variables are taken from month $t-2$.[10]

The second set comprises standard control variables, as discussed in Lou and Shu (2017) and Amihud and Noh (2021), which include idiosyncratic return volatility and past returns. Idiosyncratic volatility is estimated as the standard deviation of the residuals from regressions of daily stock returns against the daily Fama-French three factors over the previous year. Additionally, we include the previous month's stock return to control for short-term price reversals and cumulative stock returns over the past twelve months, excluding the previous month, to control for momentum.

and Noh (2021). These results are not reported for the sake of brevity and to avoid repetition.

[9] Among others, for instance, Amihud (2002), Pastor and Stambaugh (2003), Brennan et al. (2013), Lou and Shu (2017) and Amihud and Noh (2021).

[10] We do not include lnA when examining how our new fragmentation-adjusted illiquidity measure affects expected stock returns in Appendix B since our new measure embeds A, as will be demonstrated there.

5 Uncertainty in Liquidity Distribution and Expected Returns

Having established that distributional uncertainty impairs welfare and can widen spreads (Section 3), we now construct empirical measures of this uncertainty and test whether it is priced in the cross-section of expected returns. We introduce a framework that decomposes exchange-specific market shares into stable long-term and noisy short-term components, then build a proxy for liquidity distribution uncertainty and examine its effect on returns and transaction costs.

5.1 Stable and Noisy Parts of Market Shares

Let J_{it} represent the number of platforms where stock i is traded during month t, and let $\mathrm{V}^j_{i,d,t}$ and $\mathrm{V}_{i,d,t} \equiv \sum_{j=1}^{J_{it}} \mathrm{V}^j_{i,d,t}$ denote share volume on platform j and total consolidated share volume, respectively.[11] Budish et al. (2024) document that exchanges have very stable market shares, which is consistent with the trends in market shares observed in our sample. The stability of market shares across exchanges implies that the total market share of off-exchange market centers is also stable. In this context, the daily market share of platform j for a stock can be decomposed into stable long-term and noisy short-term components as follows:

$$\frac{\mathrm{V}^j_{i,d,t}}{\mathrm{V}_{i,d,t}} = \gamma^j_{i,-t} + \frac{\varepsilon^j_{i,d,t}}{\mathrm{V}_{i,d,t}} \quad \text{for } j = 1, \ldots, J_{it}. \tag{2}$$

This formulation asserts that each platform's market share in total trading volume for a stock remains stable over longer periods, while experiencing noisy fluctuations over shorter ones. The parameter $\gamma^j_{i,-t}$ represents the *long-term market share* of platform j for stock i. It is predetermined and reflects stable trading patterns from past periods. It remains constant across the days of a month but can vary from one month to another. Specifically, we compute $\gamma^j_{i,-t}$ for month t as the average daily market share of stock i on platform j using data over the preceding three months. The term $\varepsilon^j_{i,d,t}/\mathrm{V}_{i,d,t}$ captures *short-term noise* in stock i's daily market share on platform j, which quantifies transient daily deviations in stock i's

[11] We use "platform" to refer to each reporting market participant in TAQ. Before March 2007, J_{it} equals the number of exchanges trading stock i; from March 2007 onward, off-exchange market centers are reported separately, so J_{it} includes one additional platform. See Appendix D.

market share on platform j from its long-term value $\gamma^j_{i,-t}$, due to unpredictable daily market conditions. These conditions include factors, among others, traders' preferences, company announcements, or macroeconomic news. It is calculated as the residual of platform j's daily market share, derived by subtracting $\gamma^j_{i,-t}$ from $\left(\mathrm{V}^j_{i,d,t}/\mathrm{V}_{i,d,t}\right)$.

$\gamma^j_{i,-t}$ can be interpreted as the expected probability of matching with a counterparty on platform j, which holds true on average in the long term. $\varepsilon^j_{i,d,t}/\mathrm{V}_{i,d,t}$ quantifies deviations in actual matching probabilities from their long-term values due to unpredictable daily factors. In this context, it can be interpreted as the degree of uncertainty in daily liquidity shares across exchanges. To illustrate how short-term noise affects expected returns, let us start with the primary listing exchange *pex*. Let $\varepsilon^{\text{pex}}_{i,t}$ represent the monthly average of daily short-term noises for stock i on its primary listing exchange:

$$\varepsilon^{\text{pex}}_{i,t} := \frac{1}{D_{it}} \sum_{d=1}^{D_{it}} \frac{\varepsilon^{\text{pex}}_{i,d,t}}{\mathrm{V}_{i,d,t}} \tag{3}$$

where D_{it} is the total number of days that stock i is traded in month t, with an index $d = 1, 2, \ldots, D_{it}$. At the beginning of each month t from 1993 to 2023, we sort stocks into quintiles based on their ε^{pex} from month $t-2$ and report their four-factor adjusted returns (alphas). Table 1 presents equally weighted four-factor alphas of these portfolios. Short-term noise on the listing exchange does not seem to exhibit any excess return over four factors. The difference between the highest and lowest quintile returns is only -0.07%, economically small and statistically insignificant.

	Low	2	3	4	High	H-L	t-stat	p-val
ε^{pex}	0.62	0.40	0.30	0.29	0.55	-0.07	-0.86	0.39

Table 1: **Monthly Excess Portfolio Returns Sorted on Short-Term Noise of the Primary Listing Exchange (ε^{pex})**

Portfolios are constructed from January 1993 to December 2024 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. At the beginning of each month t, from January 1993 to December 2024, stocks are sorted into quintile portfolios based on ε^{pex} from two months earlier, month $t-2$. We then calculate monthly equal-weighted excess returns over four factors and report time-series averages for each quintile (Fama-French three factors—MKT, SMB, and HML—and the momentum factor—UMD), where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. The differences between the top and bottom quintiles are also reported, along with the associated t-statistics and p-values. $\boldsymbol{\varepsilon}^{\text{pex}}$ is the monthly average of the daily deviations in the listing exchange's market share from its long-term value γ^{pex}, and is defined in Equation 3.

To capture this relationship, let $|\varepsilon_{i,t}|^{\text{pex}}$ denote the monthly average of the *absolute* daily short-term noises for stock i on its primary listing exchange:

$$|\varepsilon_{i,t}|^{\text{pex}} := \frac{1}{D_{it}} \sum_{d=1}^{D_{it}} \left| \frac{\varepsilon^{j}_{i,d,t}}{\text{V}_{i,d,t}} \right| \tag{4}$$

Similarly, let $|\bar{\varepsilon_{i,t}}|^{\text{other}}$ and $|\bar{\varepsilon_{i,t}}|^{\text{total}}$ denote the monthly averages of the *absolute* daily short-term noises for stock i on non-listing exchanges and across all platforms, respectively:

$$|\bar{\varepsilon_{i,t}}|^{\text{other}} := \frac{1}{D_{it}} \frac{1}{J_{it}-1} \sum_{d=1}^{D_{it}} \sum_{j \neq \textbf{pex}} \left| \frac{\varepsilon^{j}_{i,d,t}}{\text{V}_{i,d,t}} \right| \tag{5}$$

$$|\bar{\varepsilon_{i,t}}|^{\text{total}} := \frac{1}{D_{it}} \frac{1}{J_{it}} \sum_{d=1}^{D_{it}} \sum_{j=1}^{J_{it}} \left| \frac{\varepsilon^{j}_{i,d,t}}{\text{V}_{i,d,t}} \right| \tag{6}$$

where, as previously defined, J_{it} is the number of platforms on which stock i is traded during month t.[12] We construct similar variables for long-term market shares, namely $\gamma^{\text{pex}}_{i,-t}$ and $\bar{\gamma}^{\text{other}}_{i,-t}$, with:

$$\bar{\gamma}^{\text{other}}_{i,-t} = \begin{cases} \frac{1}{J_{it}-1} \sum_{j \neq \text{pex}} \gamma^{j}_{i,-t} \text{ if } \gamma^{\text{pex}}_{i,-t} < 1 \\ 0 \text{ if } \gamma^{\text{pex}}_{i,-t} = 1 \end{cases} \tag{7}$$

There is no need to take monthly averages of these variables, as they are already at a monthly frequency due to our decomposition method. Additionally, because $\bar{\gamma}^{\text{total}}_{i,-t} = \frac{1}{J_{it}} \sum_{j=1}^{J_{it}} \gamma^{j}_{i,-t} = \frac{1}{J_{it}}$, we examine J_{it} instead of $\bar{\gamma}^{\text{total}}_{i,-t}$. We begin by constructing six univariate portfolios to examine how different market share components affect expected stock returns. At the beginning of each month t from 1993 to 2023, we sort stocks into quintiles based on their γ^{pex}, $\bar{\gamma}^{\text{other}}$, J, $|\varepsilon|^{\text{pex}}$, $|\bar{\varepsilon}|^{\text{other}}$, and $|\bar{\varepsilon}|^{\text{total}}$ from month $t-2$ and report their four-factor adjusted returns (alphas).

Table 2 presents equally weighted four-factor alphas of these portfolios. Firstly, alpha monotonically decreases with γ^{pex}, from 0.80% per month in the lowest quintile to 0.28% in

[12]The separation of the listing exchange, NYSE, from other platforms accounts for the unique features available only on the NYSE, such as Designated Market Makers (DMMs), formerly known as specialists, and floor trading. The NYSE is the only major U.S. stock exchange that has consistently offered these features.

the highest quintile, resulting in a statistically and economically significant average annual excess return of -6.42% between the two extreme portfolios. In contrast, alpha monotonically increases with $\bar{\gamma}^{\text{other}}$, from 0.19% per month in the lowest quintile to 1.02% in the highest quintile, resulting in a statistically and economically significant average annual excess return of 10.43% between the two extreme portfolios. Finally, the number of platforms, J, on which a stock is traded per se has no effect on expected returns. It is the distribution of liquidity across listing and non-listing exchanges that matters for asset pricing.

These findings indicate that investors do not consider listing and non-listing exchanges as substitutes. They require compensation for holding stocks that are consistently traded away from their listing exchange—the NYSE. One explanation is that investors value unique features available only on the NYSE, such as DMMs and floor trading. The NYSE assigns DMMs to each of its listed stocks, who are required to quote bids and offers on a regular basis, maintain price continuity, and stabilize prices for their assigned stocks. This feature distinguishes it from other market centers in terms of maintaining a continuous level of liquidity throughout a trading day.[13]

[13]Clark-Joseph et al. (2017) use two technological glitches as a natural experiment and show that DMMs are still highly useful in the current market structure and significantly enhance liquidity. More information about DMMs can be found in a white paper by Steven W. Poser: "Market Makers in Financial Markets: Their Role, How They Function, Why They Are Important, and the NYSE DMM Difference," which can be accessed at: https://www.nyse.com/publicdocs/nyse/NYSE_Paper_on_Market_Making_Sept_2021.pdf. Similarly, Brogaard et al. (2024) demonstrate that floor traders are still significant contributors to market quality.

	Low	2	3	4	High	H-L	t-stat	p-val
Long-Term M.S. (γ^{pex})	0.80	0.40	0.38	0.29	0.28	-0.52	-3.29	0.00
Long-Term M.S. ($\bar{\gamma}^{\text{other}}$)	0.19	0.25	0.31	0.40	1.02	0.83	6.16	0.00
Number of Platforms (J)	0.65	0.35	0.35	0.19	0.72	0.18	0.34	0.74
Short-Term Noise ($\lvert\varepsilon\rvert^{\text{pex}}$)	0.20	0.20	0.38	0.54	0.84	0.64	5.52	0.00
Short-Term Noise ($\lvert\bar{\varepsilon}\rvert^{\text{other}}$)	0.15	0.27	0.31	0.51	0.93	0.78	8.18	0.00
Short-Term Noise ($\lvert\bar{\varepsilon}\rvert^{\text{total}}$)	0.16	0.24	0.32	0.53	0.92	0.75	7.41	0.00

Table 2: **Monthly Excess Portfolio Returns Sorted on Long-Term and Short-Term Components of Market Shares**

Portfolios are constructed from January 1993 to December 2024 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. At the beginning of each month t, from January 1993 to December 2024, stocks are sorted into quintile portfolios based on γ^{pex}, $\bar{\gamma}^{\text{other}}$, J, $|\varepsilon|^{\text{pex}}$, $|\bar{\varepsilon}|^{\text{other}}$, and $|\bar{\varepsilon}|^{\text{total}}$ from two months earlier, month $t-2$. We then calculate monthly equal-weighted excess returns over four factors and report time-series averages for each quintile (Fama-French three factors—MKT, SMB, and HML—and the momentum factor—UMD), where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. The differences between the top and bottom quintiles are also reported, along with the associated t-statistics and p-values. $\boldsymbol{\gamma}^{\text{pex}}$ is defined as the average daily market share of the primary listing exchange over the past three months, representing the listing exchange's long-term market share. $\bar{\gamma}^{\text{other}}$ represents the average market share of all platforms outside the primary listing exchange over the past three months, as defined in Equation 7. **J** is the number of platforms that a stock is traded in during a month, where each exchange and total off-exchange market centers are referred to as separate platforms. $|\varepsilon|^{\text{pex}}$ denotes the monthly average of the absolute daily deviations in the listing exchange's market share from its long-term value, as defined in Equation 4. $|\bar{\boldsymbol{\varepsilon}}|^{\text{other}}$ and $|\bar{\boldsymbol{\varepsilon}}|^{\text{total}}$ are defined as the monthly averages of the absolute deviations in platforms' market shares from their long-term values on non-listing platforms and across all platforms, respectively, as defined in Equations 5 and 6.

In contrast, short-term deviations in market shares always drive a positive return premium. There is a monotonic increase in alpha with $|\bar{\varepsilon}|^{\text{total}}$, ranging from 0.16% per month in the lowest quintile to 0.92% in the highest quintile, leading to a statistically and economically significant average annual excess return of 9.38%. $|\varepsilon|^{\text{pex}}$ and $|\bar{\varepsilon}|^{\text{other}}$ yields similar numbers, generating average annual excess return of 7.96% and 9.77%, respectively. These results indicate that the stability of trading volume, and equivalently, the stability of liquidity, across platforms is an important driver of asset prices. Deviations of market shares from their long-term averages are positively associated with expected returns, regardless of whether they occur in or out of the listing exchange. Investors value smaller day-to-day fluctuations between exchanges and demand compensation for higher uncertainty in daily liquidity shares across exchanges.

	γ^{pex}:1	γ^{pex}:2	γ^{pex}:3	γ^{pex}:4	γ^{pex}:5
Short-Term ($\lvert\bar{\varepsilon}\rvert^{\text{total}}$)					
Low	0.33	0.18	0.17	0.03	0.33
2	0.56	0.29	0.17	0.24	0.15
3	0.54	0.30	0.38	0.22	0.22
4	1.11	0.29	0.53	0.31	0.16
High	1.48	0.97	0.67	0.66	0.56
H-L	1.15	0.78	0.50	0.63	0.24
t-stat	4.27	3.61	3.78	5.74	2.04
p-val	0.00	0.00	0.00	0.00	0.04

Table 3: **Monthly Excess Portfolio Returns Double Sorted on Long-Term and Short-Term Components of Market Shares**

Portfolios are constructed from January 1993 to December 2024 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. At the beginning of each month t, from January 1993 to December 2024, using values from two months earlier, month $t-2$, stocks are first sorted into quintile portfolios based on γ^{pex}, and then, within each γ^{pex} quintile, sorted on $\lvert\bar{\varepsilon}\rvert^{\text{total}}$. We then calculate monthly equal-weighted excess returns over four factors and report time-series averages for each portfolio (Fama-French three factors—MKT, SMB, and HML—and the momentum factor—UMD), where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. The differences between the top and bottom $\lvert\bar{\varepsilon}\rvert^{\text{total}}$ quintiles within a given γ^{pex} quintile are also reported, along with the associated t-statistics and p-values. $\boldsymbol{\gamma}^{\text{pex}}$ is defined as the average daily market share of the primary listing exchange over the past three months, representing the listing exchange's long-term market share. $\lvert\bar{\boldsymbol{\varepsilon}}\rvert^{\text{total}}$ is the monthly average of the absolute daily deviations in platforms' market shares from their long-term values, and is defined in Equation 6.

In light of these findings, we can summarize the market share forces affecting asset prices into three variables: γ^{pex}, $\bar{\gamma}^{\text{other}}$, and $\lvert\bar{\varepsilon}\rvert^{\text{total}}$. We now conduct bivariate portfolio analysis to gain deeper insight into how long-term and short-term components interact. Given that γ^{pex} and $\bar{\gamma}^{\text{other}}_{i,-t}$ are closely related, it is sufficient to examine how expected returns behave with $\lvert\bar{\varepsilon}\rvert^{\text{total}}$ under varying levels of γ^{pex}. Table 3 presents equally weighted four-factor alphas of these portfolios, revealing two important dynamics. Firstly, regardless of the extent of long-term market share concentration on the primary listing exchange, short-term deviations in market shares from their long-term values are consistently associated with higher returns in both statistical and economic terms. In each γ^{pex} quintile, alpha typically increases with $\lvert\bar{\varepsilon}\rvert^{\text{total}}$, always resulting in statistically and economically significant return spreads between stocks with the highest and lowest levels of short-term noise. For instance, in the lowest quintile of γ^{pex}, alpha almost monotonically increases from 0.33% per month in the lowest

$|\bar{\varepsilon}|^{\text{total}}$ quintile to 1.48% in the highest $|\bar{\varepsilon}|^{\text{total}}$ quintile, generating an average annual excess return of 14.71%. Secondly, this average annual premium is almost monotonic in γ^{pex}, decreasing from 14.71% in the smallest γ^{pex} quintile to 2.92% in the largest γ^{pex} quintile. In the largest γ^{pex} quintile, the premium is significant but exhibits a lower p-value compared to other quintiles of γ^{pex}.

The decomposition above delivers distinct roles for long-term market shares and short-term noise. We now combine them into a single empirical measure of liquidity distribution uncertainty that corresponds to the dispersion of exchange-level liquidity in the model (Section 3).

5.2 Uncertainty in Liquidity Distribution

The preceding results show that short-term noise is most impactful on expected returns for stocks consistently traded away from their listing exchange. This relationship can be summarized by the following ratio: $\frac{|\bar{\varepsilon}_{i,t}|^{\text{total}}}{\gamma_{i,-t}^{\text{pex}}}$. It proxies the degree of uncertainty in daily liquidity distribution across exchanges due to unpredictable daily factors, benchmarked to the long-term market share of the primary listing exchange. Larger values of this ratio correspond to higher uncertainty, which increases when short-term fluctuations become larger or when the listing exchange starts executing less trading consistently.[14] Table 4 presents equally weighted four-factor alphas of portfolios sorted on this ratio, revealing that alpha monotonically increases from 0.11% per month in the lowest quintile to 1.03% in the highest quintile. A long-short portfolio with high versus low uncertainty yields an average annual excess return of 13.08%.

[14] We also derive two alternative fragmentation measures to proxy for uncertainty in daily liquidity distributions, one replacing the denominator with a weighted average of long-term market shares across all platforms, and another replacing the numerator with the short-term noise of the primary listing exchange. These alternatives are discussed in Appendix E.2.

	Low	2	3	4	High	H-L	t-stat	p-val
$\lvert\bar{\varepsilon}\rvert^{\text{total}}/\gamma^{\text{pex}}$	0.11	0.18	0.29	0.55	1.03	0.91	7.43	0.00

Table 4: **Monthly Excess Portfolio Returns Sorted on Liquidity Uncertainty**

Portfolios are constructed from January 1993 to December 2024 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. At the beginning of each month t, from January 1993 to December 2024, stocks are sorted into quintile portfolios based on $|\bar{\boldsymbol{\varepsilon}}|^{\text{total}}/\boldsymbol{\gamma}^{\text{pex}}$ from two months earlier, month $t-2$. We then calculate monthly equal-weighted excess returns over four factors and report time-series averages for each quintile (Fama-French three factors—MKT, SMB, and HML—and the momentum factor—UMD), where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. The differences between the top and bottom quintiles are also reported, along with the associated t-statistics and p-values. $\boldsymbol{\gamma}^{\text{pex}}$ is defined as the average daily market share of the primary listing exchange over the past three months, representing the listing exchange's long-term market share. $|\bar{\boldsymbol{\varepsilon}}|^{\text{total}}$ is the monthly average of the absolute daily deviations in platforms' market shares from their long-term values, and is defined in Equation 6.

We now conduct regression analyses to control for additional variables and conduct further analyses that are impractical in univariate and bivariate portfolio analysis settings, and test whether our results remain robust. Table 5 presents the results from these regressions. We include standard control variables in all six regressions. Additionally, we control for the Amihud illiquidity measure and price impact starting from the second and third regression, respectively. The results from these regressions are consistent with our portfolio analyses.

Regressions 1 to 3 reveal that including illiquidity reduces the coefficient of liquidity uncertainty from 0.091 to 0.067, while adding price impact causes almost no change. The coefficient remains positive, economically and statistically significant, demonstrating that liquidity uncertainty is not priced solely due to its interaction with illiquidity, price impact, or other controlled risk factors. After accounting for these additional variables and four factors, a one standard deviation increase in liquidity uncertainty generates an annual excess return of 3.17%, based on Regression 3.

In Regression 4, we test whether the long-term market share of the primary listing exchange and the average absolute short-term noise across all platforms is still priced. The results are consistent with our portfolio analyses. Long-term concentration of market share on the listing exchange reduces the expected returns for its listed stocks: a one standard deviation increase in γ^{pex} reduces average annual excess returns by 8.34%, after controlling for illiquidity, price impact, and other standard control variables.

	(1)	(2)	(3)	(4)	(5)	(6)
$\lvert\bar{\varepsilon}\rvert^{\text{total}}/\gamma^{\text{pex}}$	0.091***	0.067***	0.065***		0.074***	
	(0.014)	(0.013)	(0.011)		(0.012)	
$1(\varepsilon^{pex} \geq 0) \times (\lvert\bar{\varepsilon}\rvert^{total}/\gamma^{pex})$					-0.016***	
					(0.006)	
γ^{pex}				-0.022***		-0.025***
				(0.007)		(0.007)
$\lvert\bar{\varepsilon}\rvert^{\text{total}}$				0.264***		0.282***
				(0.057)		(0.059)
$1(\varepsilon^{pex} \geq 0) \times \lvert\bar{\varepsilon}\rvert^{total}$						-0.045***
						(0.016)
λ			0.021***	0.022***	0.021***	0.022***
			(0.004)	(0.004)	(0.004)	(0.004)
lnA		0.299***	0.141***	0.131***	0.142***	0.139***
		(0.046)	(0.037)	(0.034)	(0.037)	(0.035)
Observations	448924	448924	448924	448924	448924	448924
Adjusted R^2	0.041	0.044	0.048	0.049	0.048	0.050
Other Controls	✓	✓	✓	✓	✓	✓

Table 5: **Fama-MacBeth Regressions of Excess Returns on Liquidity Uncertainty**

Monthly Fama-MacBeth regressions are estimated from January 1993 to December 2024 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. We estimate a cross-sectional regression in each month t and then report the time-series averages of the coefficients and standard errors, as well as the adjusted R^2. The **dependent variable** is the monthly excess return over four factors (Fama-French three factors—MKT, SMB, and HML—and the momentum factor—UMD), where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. $\boldsymbol{\gamma}^{\text{pex}}$ is defined as the average daily market share of the primary listing exchange over the past three months, representing the listing exchange's long-term market share. $\lvert\bar{\boldsymbol{\varepsilon}}\rvert^{\text{total}}$ is the monthly average of the absolute daily deviations in platforms' market shares from their long-term values, and is defined in Equation 6. $\mathbf{1(\varepsilon^{pex} \geq 0)}$ is a dummy variable that is equal to one when ε^{pex} is non-negative, where $\boldsymbol{\varepsilon^{pex}}$ is the monthly average of the daily deviations in the listing exchange's market share from its long-term value γ^{pex}, and is defined in Equation 3. $\boldsymbol{\lambda}$ is the monthly average of daily price impact coefficients. **lnA** is the natural logarithm of the original Amihud illiquidity measure A, where A is calculated as the monthly average of the daily ratio of absolute return to dollar trading volume in millions. In the regression for month t, these independent variables are taken from month $t-2$. Additionally, we include other standard control variables, which are defined explicitly in Section 4, with coefficients not presented to save space: idiosyncratic return volatility, previous month's stock return, and cumulative stock returns over the past twelve months excluding the previous one. t-statistics are calculated using Newey-West robust standard errors with six lags. **Standard errors** are in parentheses; * $p < 0.10$ ** $p < 0.05$ *** $p < 0.01$.

Our findings strongly indicate that investors place substantial value on the centrality of trading on the listing exchange, accepting lower returns for holding stocks traded more heavily on this platform. Short-term noise is also similarly priced as before; a one standard deviation increase in $|\bar{\varepsilon}|^{\text{total}}$ raises average annual excess returns by 7.96%, after controlling for illiquidity, price impact, and other standard control variables.

The last two regressions in Table 5 test whether short-term noise has an asymmetric effect, examining whether positive and negative noise on the primary listing exchange have heterogeneous effects on expected stock returns. Regression 5 indicates that stocks experiencing short-term noises that redirect trading from other platforms to the listing exchange achieve approximately 22% lower liquidity uncertainty premium (0.016/0.074) compared to stocks experiencing short-term noises in the opposite direction. However, this premium remains economically and statistically significant regardless of the direction of short-term noises, indicating that investors require compensation for any deviations in market shares, whether they occur in or out of the listing exchange. Regression 6 provides similar conclusions.

Whether distributional uncertainty is priced because it raises effective transaction costs or because it represents a distinct risk factor has important implications for interpretation and policy. We next test the transaction cost channel directly.

5.3 Transaction Cost Analysis

We test whether uncertainty in liquidity distribution affects asset prices primarily through the transaction cost channel or instead serves as an independent risk factor. Table 6 provides empirical support for the transaction cost channel using Fama-MacBeth regressions where the dependent variable is the excess return over the Fama-French six factors. In our baseline, parsimonious specifications (Columns 1 and 2), the coefficient on distributional uncertainty is positive and statistically significant at 0.014 and 0.015, respectively. However, as we progressively introduce controls for trading frictions and market structure—more specifically, after adding effective spread—the coefficient on distributional uncertainty severely attenuates. The point estimates drop to values between 0.004 and -0.003 and entirely lose their statistical significance. This attenuation aligns perfectly with this hypothesis: the premium investors demand for holding high-uncertainty stocks is fundamentally due to the increased effective cost of execution.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Dist. Uncertainty	0.015**	0.014**	0.003	0.004	0.001	-0.002	-0.003
	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)
Turnover (CV)		0.256**	0.066	0.084	0.082	0.068	0.108
		(0.100)	(0.109)	(0.108)	(0.103)	(0.106)	(0.110)
Effective Spread			0.012***	0.014***	0.013***	0.003	0.005*
			(0.002)	(0.002)	(0.002)	(0.003)	(0.003)
Execution Time				-3.924	-3.640	-7.178*	-3.370
				(2.859)	(2.561)	(4.257)	(3.381)
Fragmentation					-0.508	0.399	0.282
					(1.168)	(1.235)	(1.237)
Illiquidity						4.081***	1.673**
						(0.934)	(0.830)
Kyle's λ							0.010***
							(0.002)
Observations	458918	458918	458918	458918	458918	458918	458918
Adjusted R^2	0.025	0.026	0.028	0.028	0.030	0.031	0.033
Other Controls	✓	✓	✓	✓	✓	✓	✓

Table 6: **Effect of Uncertainty in Liquidity Distribution on Excess Returns under Different Control Variables**

Monthly Fama-MacBeth regressions are estimated for stock-months with non-missing data. **Dist. Uncertainty** is the monthly ratio of the average absolute daily deviations of exchange market shares from their long-term averages. **Turnover (CV)** is the coefficient of variation of share turnover. **Effective Spread** is the average cost of trading accounting for execution price relative to the midpoint. **Execution Time** is the average time to complete a trade. **Fragmentation** is calculated as 10,000 − HHI, where HHI is the Herfindahl-Hirschman Index of market shares. **Illiquidity** is the natural logarithm of the Amihud illiquidity measure. **Kyle's** λ is the monthly average of daily price impact coefficients. Standard errors are reported in parentheses, with $^{*}p < 0.10$, $^{**}p < 0.05$, and $^{***}p < 0.01$.

To formally establish that distributional uncertainty drives transaction costs, we next turn to addressing the potential endogeneity of market share instability. Specifically, analyzing this relationship presents a reverse causality concern, as wider spreads might mechanically discourage complex order routing across different venues. Alternatively, both market share instability and transaction costs could be jointly determined by omitted latent variables. We mitigate these endogeneity concerns using a Two-Stage Least Squares (2SLS) approach with a size-based instrumental variable. We construct this instrument using a two-step procedure that isolates idiosyncratic variation. First, we classify all firms into size deciles based

on their market capitalization lagged by two months ($t-2$) to ensure the groupings are strictly predetermined. We define a systematic overall market instability factor (u_t^{market}) as the average uncertainty across all stocks in a given month. For each month t, using the prior 60 months of data ($[t-60, t-1]$), we estimate the firm's systematic exposure to this market-wide factor. We then extract the idiosyncratic residual ($\epsilon_{i,t}$), which represents the component of a stock's instability ($u_{i,t}^{total}$) that cannot be explained by concurrent overall market movements, calculated as:

$$\epsilon_{i,t} = u_{i,t}^{total} - (\hat{\alpha}_{i,t} + \hat{\beta}_{i,t} u_t^{market})$$

In the second step, we aggregate these idiosyncratic residuals to construct the instrument for the focal stock i ($Z_{i,t}^{Resid}$). We average the residuals across the stock's predetermined size decile g while maintaining a strict leave-one-out structure:

$$Z_{i,t}^{Resid} = \frac{1}{N_g - 1} \sum_{j \in g, j \neq i} \epsilon_{j,t}$$

Table 7 presents the results of this transaction cost analysis, directly comparing ordinary least squares (OLS) estimates with our 2SLS (IV) specifications. The OLS regressions severely underestimate the friction: the coefficient for distributional uncertainty on Quoted Spreads (QS) is 0.005 and lacks statistical significance, while its effect on Effective Spreads (ES) is only marginally significant at 0.026. In stark contrast, instrumenting for endogeneity reveals a profound impact. The IV estimate for distributional uncertainty surges to 1.404 for QS and 1.974 for ES, both highly significant at the 1% level. A first-stage F-statistic of 54.460 confirms the strength of our size-based instrument. Furthermore, the coefficients for other variables, such as Turnover (CV) and Fragmentation, are substantially larger in magnitude under the IV specification compared to OLS. These results cement our core argument: in modern equity markets, the soaring "cost of finding a trade" directly inflates execution costs, which in turn necessitates the higher risk premium demanded by investors.

	QS		ES	
	OLS	IV	OLS	IV
Dist. Uncertainty	0.005	1.404***	0.026*	1.974***
	(0.021)	(0.455)	(0.015)	(0.351)
Turnover (CV)	3.080***	47.256***	18.502***	45.207***
	(0.283)	(8.864)	(0.492)	(7.586)
Fragmentation	22.958***	271.503***	32.072***	174.600***
	(2.406)	(19.787)	(1.799)	(15.681)
Observations	454745	454714	454745	454714
Adjusted R^2	0.689	-0.490	0.517	-0.253
Other Controls	✓	✓	✓	✓
Stock FE	✓	✓	✓	✓
Time FE	✓	✓	✓	✓
First-Stage F-stat		54.460		54.460

Table 7: **Effect of Uncertainty in Liquidity Distribution and Uncertainty in Overall Liquidity on Quote and Effective Spreads**

Dist. Uncertainty refers to the uncertainty in liquidity distribution, measured as the monthly ratio of the average absolute daily deviations of exchange market shares from their long-term averages. **QS** denotes the quoted spread, while **ES** denotes the effective spread. **Turnover (CV)** is the coefficient of variation of share turnover. **Fragmentation** is calculated as 10,000−HHI, where HHI is the Herfindahl-Hirschman Index of market shares.

6 Market Structure and Expected Returns

This section examines how market structure shapes the relationship between fragmentation and expected returns. We first document that a simple HHI-based fragmentation measure produces regime-dependent return premiums (Section 6.1). We then use our decomposition framework to explain why (Section 6.2) and show that Regulation NMS amplified distributional uncertainty (Section 6.3).

6.1 Fragmentation Premium

The Herfindahl–Hirschman Index (HHI) for stock i on day d in month t is

$$\mathrm{HHI}_{i,d,t} := \sum_{j=1}^{J_{it}} \left(\frac{\mathrm{V}^{j}_{i,d,t}}{\mathrm{V}_{i,d,t}} \right)^2, \tag{8}$$

where J_{it}, $\mathrm{V}^{j}_{i,d,t}$, and $\mathrm{V}_{i,d,t}$ are as defined in Section 5.1. Higher values of HHI correspond to lower fragmentation. We construct univariate portfolios based on monthly HHI values (monthly HHI is the average of daily HHI). At the beginning of each month t from 1993 to 2023, we sort stocks into quintiles based on their HHI from month $t-2$ and report four-factor alphas.

	Low	2	3	4	High	H-L	t-stat	p-val
Entire Sample	0.44	0.29	0.36	0.34	0.74	0.29	1.37	0.17
Pre-Reg NMS	0.90	0.25	0.16	0.09	0.19	-0.70	-2.49	0.01
Post-Reg NMS	0.14	0.20	0.34	0.35	1.09	0.95	4.46	0.00

Table 8: **Monthly Excess Portfolio Returns Sorted on HHI**

Portfolios are constructed from January 1993 to December 2024 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. At the beginning of each month t, from January 1993 to December 2024, stocks are sorted into quintile portfolios based on HHI from two months earlier, month $t-2$. We then calculate monthly equal-weighted excess returns over four factors and report time-series averages for each quintile (Fama-French three factors—MKT, SMB, and HML—and the momentum factor—UMD), where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. The differences between the top and bottom quintiles are also reported, along with the associated t-statistics and p-values. Pre-Reg NMS and Post-Reg NMS correspond to 1993–2004 and 2009–2023, respectively. **HHI** is calculated as the monthly average of daily Herfindahl–Hirschman indices, where the daily index is as the sum of the squares of daily market shares on each exchange and off-exchanges as a whole, based on their share volumes.

Table 8 presents equally weighted four-factor alphas. In the entire sample, stocks in the highest HHI quintile have higher alphas than stocks in the lowest HHI quintile, but this difference is not statistically significant. However, in two sub-periods with more homogeneous market structure, fragmentation generates opposing premiums. Prior to Regulation NMS, alpha typically decreases with HHI: the most fragmented (lowest HHI) quintile provides an average annual excess return of 8.73% relative to the most concentrated. After Regulation NMS, alpha monotonically increases in HHI: the lowest quintile earns 12.01% less average annual excess return than the highest quintile.

We also conduct regression analysis controlling for other variables. Table 9 presents the results. HHI exhibits a positive premium in the entire sample that is statistically significant; a one standard deviation increase in HHI is associated with a 4.16% higher alpha annually (Regression 4). The sub-sample results match the portfolio analysis: HHI has negative and positive coefficients before and after Regulation NMS, respectively, both economically and statistically significant.

	(1)	(2)	(3)	(4)	(5)	(6)
	Entire Sample	Pre-Reg NMS	Post-Reg NMS	Entire Sample	Pre-Reg NMS	Post-Reg NMS
HHI	1.154**	-1.643**	3.676***	1.188**	-1.390**	3.421***
	(0.565)	(0.660)	(0.750)	(0.519)	(0.628)	(0.674)
lnA	0.297***	0.162***	0.445***	0.138***	0.068	0.222***
	(0.041)	(0.052)	(0.056)	(0.036)	(0.062)	(0.050)
λ				0.021***	0.005***	0.034***
				(0.004)	(0.001)	(0.006)
Observations	448924	197015	192808	448924	197015	192808
Adjusted R^2	0.045	0.036	0.055	0.049	0.038	0.061
Other Controls	✓	✓	✓	✓	✓	✓

Table 9: **Fama-MacBeth Regressions of Excess Returns on HHI**

Monthly Fama-MacBeth regressions are estimated from January 1993 to December 2024 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. We estimate a cross-sectional regression in each month t and then report the time-series averages of the coefficients and standard errors, as well as the adjusted R^2. The **dependent variable** is the monthly excess return over four factors (Fama-French three factors—MKT, SMB, and HML—and the momentum factor—UMD), where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. Pre-Reg NMS and Post-Reg NMS correspond to 1993–2004 and 2009–2023, respectively. **HHI** is calculated as the logarithm of the monthly average of daily Herfindahl–Hirschman indices, where the daily index is calculated as the sum of the squares of daily market shares on each exchange and off-exchange centers as a whole, based on their share volumes. $\boldsymbol{\lambda}$ is the monthly average of daily price impact coefficients. **lnA** is the natural logarithm of the original Amihud illiquidity measure A, where A is calculated as the monthly average of the daily ratio of absolute return to dollar trading volume in millions. In the regression for month t, these independent variables are taken from month $t-2$. Additionally, we include other standard control variables, which are defined explicitly in Section 4, with coefficients not presented to save space: idiosyncratic return volatility, previous month's stock return, and cumulative stock returns over the past twelve months excluding the previous one. t-statistics are calculated using Newey-West robust standard errors with six lags. **Standard errors** are in parentheses; * $p < 0.10$ ** $p < 0.05$ *** $p < 0.01$.

The regime-dependent premium documented above raises a natural question: does there exist a systematic positive bias towards the prevailing market structure? We now use our decomposition to address this question.

6.2 Is There A Positive Bias Towards Market Structure?

We have shown that there are three market share forces affecting expected returns: γ^{pex}, $\bar{\gamma}^{\text{other}}$, and $|\bar{\varepsilon}|^{\text{total}}$. These forces are included in the HHI, but to varying degrees depending on the market structure. The impact of HHI on expected returns depends on whether γ^{pex} dominates the other two forces or vice versa. To see this effect, let's rewrite HHI using our decomposition:

$$\text{HHI}_{i,d,t} = \sum_{j=1}^{J_{it}} \left(\gamma^j_{i,-t} + \frac{\varepsilon^j_{i,d,t}}{\text{V}_{i,d,t}} \right)^2 \tag{9}$$

Consider platform j's contribution to the HHI: $\left(\gamma^j_{i,-t}\right)^2 \left(1 + \left[\frac{\varepsilon^j_{i,d,t}}{\text{V}_{i,d,t}}\right] / \gamma^j_{i,-t}\right)^2$. If the short-term noise in platform j's market share is sufficiently small relative to its long-term counterpart, that is, if $\frac{\varepsilon^j_{i,d,t}}{\text{V}_{i,d,t}} / \gamma^j_{i,-t}$ is small, then j's contribution to the HHI is approximately equal to $(\gamma^j_{i,-t})^2$. This scenario describes the era before Regulation NMS, where short-term noise had negligible impacts on fragmentation, and the HHI was roughly equal to $\sum_{j=1}^{J_{it}} (\gamma^j_{i,-t})^2$. Moreover, this approximate sum primarily reflected the NYSE's long-term market share in this era, as it dominated trading in its listed stocks with a median long-term market share of 88%.[15] To sum up, an increase in HHI before Regulation NMS is almost equivalent to a higher concentration of trading on the primary listing exchange. The correlation of HHI with $\gamma^{\text{nyse}}_{i,-t}$ is 0.83 in this period.

However, if the short-term noise in platform j's market share is relatively large compared to its long-term counterpart, then this short-term noise can also make a meaningful contribution to the HHI. This situation applies to the post-Regulation NMS era. Just as crucially, the NYSE lost its dominance in trading its listed stocks following Regulation NMS. The correlation of HHI with $\gamma^{\text{nyse}}_{i,-t}$ decreased to 0.42 during this period, indicating that an increase in HHI after Regulation does not necessarily reflect a higher concentration on the listing exchange. To summarize, in the post-Regulation NMS period, the HHI captures not only

[15] To further illustrate how this domination affects the HHI, consider a stock where $\gamma^{\text{nyse}}_{i,-t} = 0.88$. This approximated sum takes its maximum value when another exchange executes the rest, and all others have zero market shares. In this case, the NYSE's contribution to the HHI is $\frac{0.88^2}{0.88^2+0.12^2} = 98.2\%$, which provides a lower bound for the NYSE's contribution to the HHI. In any other allocation of the 0.12 market share among other exchanges, the NYSE's contribution to the HHI would be higher.

the long-term market share of the primary listing exchange but also those of other exchanges and short-term noises, and thus can approximately be expressed as:

$$\mathrm{HHI}_{i,d,t} \approxeq \underbrace{\left(\gamma_{i,-t}^{\mathrm{pex}}\right)^2}_{\mathrm{HHI(LT)}_{i,-t}^{\mathrm{pex}}} + \underbrace{\sum_{j\neq \mathrm{pex}} \left(\gamma_{i,-t}^{j}\right)^2}_{\mathrm{HHI(LT)}_{i,-t}^{\mathrm{other}}} + \underbrace{\left(\frac{\varepsilon_{i,d,t}^{\mathrm{pex}}}{\mathrm{V}_{i,d,t}}\right)^2}_{\mathrm{HHI(ST)}_{i,d,t}^{\mathrm{pex}}} + \underbrace{\sum_{j\neq \mathrm{pex}} \left(\frac{\varepsilon_{i,d,t}^{j}}{\mathrm{V}_{i,d,t}}\right)^2}_{\mathrm{HHI(ST)}_{i,d,t}^{\mathrm{other}}} \tag{10}$$

where *pex* and *other* represent the primary listing exchange and other platforms (other exchanges and off-exchange market centers collectively), respectively. Table 10 presents how each of these four components, as well as the total long-term and short-term components, affect expected returns in a univariate monthly portfolio setting. At the beginning of each month t from 1993 to 2023, we sort stocks into quintiles based on these variables' values from month $t-2$ and report four-factor adjusted returns (alphas).

	Low	2	3	4	High	H-L	t-stat	p-val
HHI(LT)	0.52	0.31	0.33	0.29	0.71	0.19	0.90	0.37
$\mathrm{HHI(LT)}^{\mathrm{pex}}$	0.80	0.40	0.38	0.29	0.28	-0.52	-3.29	0.00
$\mathrm{HHI(LT)}^{\mathrm{other}}$	0.20	0.26	0.24	0.36	1.10	0.91	5.23	0.00
HHI(ST)	0.16	0.26	0.34	0.51	0.90	0.74	6.99	0.00
$\mathrm{HHI(ST)}^{\mathrm{pex}}$	0.20	0.22	0.41	0.53	0.81	0.61	5.26	0.00
$\mathrm{HHI(ST)}^{\mathrm{other}}$	0.16	0.28	0.29	0.53	0.91	0.75	7.89	0.00

Table 10: **Monthly Excess Portfolio Returns Sorted on HHI Components**

Portfolios are constructed from January 1993 to December 2024 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. At the beginning of each month t, from January 1993 to December 2024, stocks are sorted into quintile portfolios based on HHI components from two months earlier, month $t-2$. We then calculate monthly equal-weighted excess returns over four factors and report time-series averages for each quintile (Fama-French three factors—MKT, SMB, and HML—and the momentum factor—UMD), where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. The differences between the top and bottom quintiles are also reported, along with the associated t-statistics and p-values. Daily HHI is decomposed into long-term (LT) and short-term (ST) components, where each component is explicitly defined in Equation 10. $\mathbf{HHI(ST)^{pex}}$ and $\mathbf{HHI(ST)^{other}}$ are calculated as the monthly average of their daily values. $\mathrm{HHI(LT)} = \mathrm{HHI(LT)}^{\mathrm{pex}} + \mathrm{HHI(LT)}^{\mathrm{other}}$ and $\mathrm{HHI(ST)} = \mathrm{HHI(ST)}^{\mathrm{pex}} + \mathrm{HHI(ST)}^{\mathrm{other}}$.

The results demonstrate the intricate nature of HHI. Firstly, without explicit distinction between the primary listing exchange and other platforms, one might erroneously conclude

that long-term market share concentration is of no importance for asset pricing, given that the resulting total sum, HHI(LT), does not produce any return premium. However, long-term concentration of share volume on the primary listing exchange is negatively associated with expected stock returns, while long-term concentration on other exchanges is positively correlated with expected stock returns. Alpha monotonically decreases in $\text{HHI(LT)}^{\text{pex}}$ from 0.80% in the lowest quintile, stocks with the lowest concentration on the primary listing exchange, to 0.28% in the highest quintile, the most highly concentrated stocks, yielding a spread of -0.52%, translating into an average annual excess return of -6.42%. In contrast, alpha typically increases with long-term market share concentration in platforms outside the primary listing exchange, $\text{HHI(LT)}^{\text{other}}$. This result is not surprising given the near-perfect negative correlation between $\text{HHI(LT)}^{\text{pex}}$ and $\text{HHI(LT)}^{\text{other}}$. Secondly, short-term noises are consistently positively associated with expected stock returns.

These two results together indicate that there are two opposing forces in HHI affecting asset prices: $\text{HHI(LT)}^{\text{pex}}$ versus HHI(ST) and $\text{HHI(LT)}^{\text{other}}$. Before Regulation NMS, HHI is heavily dominated by $\text{HHI(LT)}^{\text{pex}}$, resulting in a negative association between HHI and expected stock returns. After Regulation NMS, HHI primarily reflects $\text{HHI(LT)}^{\text{other}}$ and HHI(ST), both of which are positively associated with expected stock returns, leading HHI to have a positive association as well. HHI does not explicitly take account these market dynamics, resulting in seemingly contradictory conclusions regarding how fragmentation affects expected stock returns before and after Regulation NMS.

Having identified the sources of HHI's regime dependence, we now quantify how Regulation NMS amplified distributional uncertainty and its pricing implications.

6.3 Amplification of Liquidity Uncertainty with Regulation NMS

We demonstrate that investors place substantial value on the centrality of trading on listing exchanges. They demand compensation for stocks consistently traded away from their listing exchanges and for larger short-term deviations in platforms' daily market shares from their long-term counterparts. Discussions in policy or academic studies on fragmentation generally focus on how it affects market quality. Our paper highlights that fragmentation, as well as market structure, are important determinants of asset prices. Thus, regulations that intensify fragmentation or alter market structure can have long-term repercussions for publicly listed

companies, as higher expected returns typically translate into higher costs of capital.

Regulation NMS has undoubtedly increased market fragmentation. However, as shown in Figure 1, except for a transition period, market shares remain quite stable under the market structure where the listing exchange executes more than 80% of trades, as well as under the structure where trading is dispersed across numerous exchanges with no single exchange dominating. Despite this apparent stability in market shares over longer horizons, the stability disappears over shorter horizons. Figure 3 illustrates the average monthly deviations of daily market shares from their long-term averages for the listing exchange. Although volatile, these deviations were still not large before Regulation NMS. However, following Regulation NMS, liquidity uncertainty has significantly increased. Thus, Regulation NMS has not only dispersed trading across numerous exchanges, leading to smaller market shares on each exchange but also significantly increased uncertainty about each exchange's daily liquidity share, resulting in larger short-term deviations.

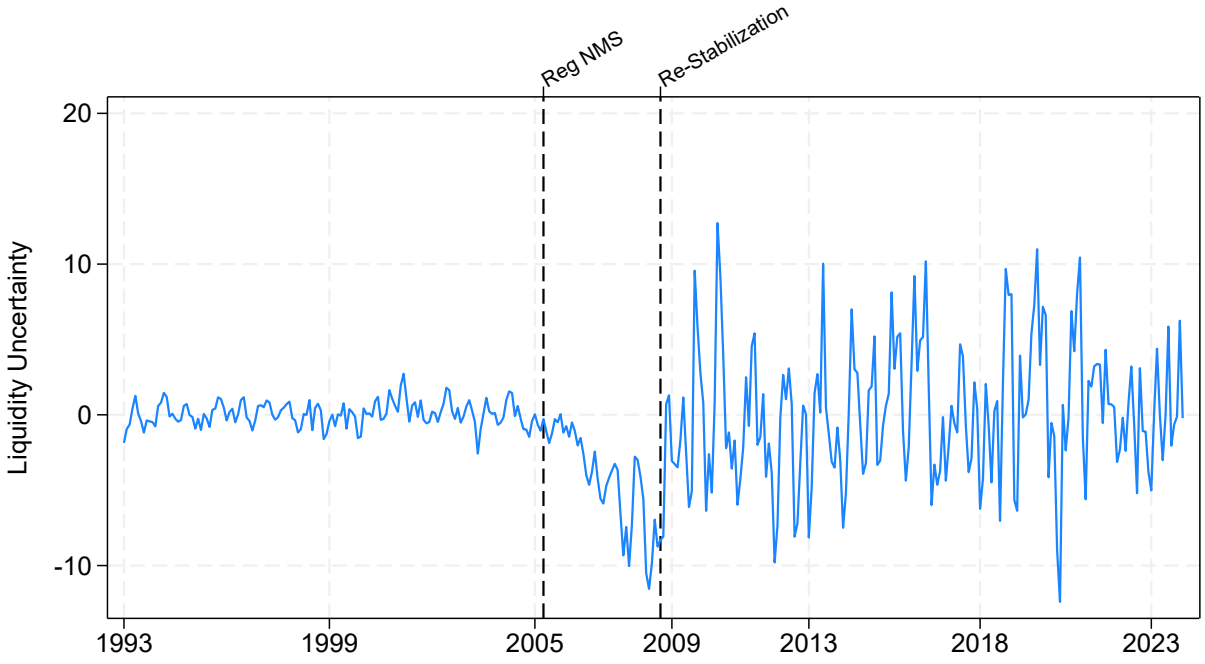


Figure 3: Liquidity Uncertainty On The Listing Exchange ($\varepsilon^{\text{pex}}/\gamma^{\text{pex}}$)

This figure displays the monthly cross-sectional averages of the ratio of ε^{pex} to $\boldsymbol{\gamma}^{\text{pex}}$ from January 1993 to December 2024 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. We decompose market shares (in share volumes) into long-term (γ) and short-term (ε) components. $\boldsymbol{\gamma}^{\text{pex}}$ is defined as the average daily market share of the primary listing exchange over the past three months, representing the listing exchange's long-term market share. $\boldsymbol{\varepsilon}^{\text{pex}}$ is the monthly average of the daily deviations in the listing exchange's market share from its long-term value γ^{pex}, and is defined in Equation 3. The left dashed line corresponds to April 2005, the time of adoption of Regulation NMS. The right dashed line corresponds to September 2008, around when market shares across exchanges stabilized again.

We now demonstrate that liquidity uncertainty has become more significant for asset pricing in the second market structure. To this end, we conduct regression analysis in two sub-samples, 1993–2004 versus 2009–2023, representing these stable periods before and after Regulation NMS. Table 11 presents the results from these regressions. The coefficient of $|\bar{\varepsilon}_{i,t}|^{\text{total}}/\gamma^{\text{pex}}_{i,-t}$ increases more than three-fold from 1993–2004 to 2009–2023. One standard deviation increase in $|\bar{\varepsilon}_{i,t}|^{\text{total}}/\gamma^{\text{pex}}_{i,-t}$ results in a 0.12% alpha per month during 1993–2004 compared to an alpha of 0.36% during 2009–2023, indicating that liquidity uncertainty requires three times more compensation after Regulation NMS. The difference translates into an average annual excess return of 2.94%.

	(1)	(2)	(3)	(4)
	Pre-Reg NMS	Post-Reg NMS	Pre-Reg NMS	Post-Reg NMS
$\lvert\bar{\varepsilon}\rvert^{\text{total}}/\gamma^{\text{pex}}$	0.033***	0.104***	0.037***	0.120***
	(0.010)	(0.017)	(0.010)	(0.020)
$1(\varepsilon^{pex} \geq 0) \times (\lvert\bar{\varepsilon}\rvert^{total}/\gamma^{pex})$			-0.012	-0.022***
			(0.010)	(0.007)
λ	0.006***	0.035***	0.006***	0.035***
	(0.001)	(0.006)	(0.001)	(0.006)
lnA	0.011	0.265***	0.013	0.265***
	(0.067)	(0.040)	(0.067)	(0.040)
Observations	197015	192808	197015	192808
Adjusted R^2	0.036	0.060	0.036	0.061
Other Controls	✓	✓	✓	✓

Table 11: **Fama-MacBeth Regressions of Excess Returns on Liquidity Uncertainty Before/After Regulation NMS**

Monthly Fama-MacBeth regressions are estimated from January 1993 to December 2024 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. We estimate a cross-sectional regression in each month t and then report the time-series averages of the coefficients and standard errors, as well as the adjusted R^2. The **dependent variable** is the monthly excess return over four factors (Fama-French three factors—MKT, SMB, and HML—and the momentum factor—UMD), where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. $\boldsymbol{\gamma}^{\text{pex}}$ is defined as the average daily market share of the primary listing exchange over the past three months, representing the listing exchange's long-term market share. $\lvert\bar{\boldsymbol{\varepsilon}}\rvert^{\text{total}}$ is the monthly average of the absolute daily deviations in platforms' market shares from their long-term values, and is defined in Equation 6. $\mathbf{1}(\boldsymbol{\varepsilon}^{\boldsymbol{pex}} \geq \mathbf{0})$ is a dummy variable that is equal to one when ε^{pex} is non-negative, where $\boldsymbol{\varepsilon}^{\boldsymbol{pex}}$ is the monthly average of the daily deviations in the listing exchange's market share from its long-term value γ^{pex}, and is defined in Equation 3. $\boldsymbol{\lambda}$ is the monthly average of daily price impact coefficients. **lnA** is the natural logarithm of the original Amihud illiquidity measure A, where A is calculated as the monthly average of the daily ratio of absolute return to dollar trading volume in millions. In the regression for month t, these independent variables are taken from month $t-2$. Additionally, we include other standard control variables, which are defined explicitly in Section 4, with coefficients not presented to save space: idiosyncratic return volatility, previous month's stock return, and cumulative stock returns over the past twelve months excluding the previous one. t-statistics are calculated using Newey-West robust standard errors with six lags. **Standard errors** are in parentheses; * $p < 0.10$ ** $p < 0.05$ *** $p < 0.01$.

This result is consistent with the idea that when a single market center dominates trading, seeking liquidity outside this platform is more dependent on trader preferences than on necessity. In fragmented markets, like the current U.S. equities markets, due to significant uncertainty about daily liquidity distribution across trading platforms, seeking liquidity becomes a necessity rather than a choice. Table 11 indicates that this type of uncertain and required liquidity search is compensated more than a liquidity search likely resulting from trader preferences.

The SEC approved a rule on September 18, 2024 to lower access fees that exchanges can

charge brokers. While this rule applies to all exchanges, it is likely to impact smaller or newer exchanges more significantly. Larger exchanges, such as the NYSE, are less reliant on access fees due to their higher listing fees and market data revenues from consolidated tape plans. Reflecting these concerns, some financial entities highlighted potential competitive issues in their comment letters to the SEC. For example, Virtu stated that "limitation on exchange rebates and fee tiers would reduce incentives to provide liquidity and harm competition between exchanges, especially new exchanges." Similarly, Themis Trading argued that "[l]ower access fees will not harm the major exchanges." This rule change could increase the NYSE's market power for its listed stocks, which might potentially mitigate some adverse effects of fragmentation on asset prices for stocks listed on the NYSE.[16]

7 Conclusion

Liquidity in U.S. equity markets has improved along traditional dimensions over the past three decades, but the nature of frictions has changed. Uncertainty in the *distribution* of liquidity across fragmented venues has risen sharply: the "cost of finding a trade" has become a primary driver of risk premiums, distinct from the cost of the trade itself. Our theoretical analysis shows that this distributional uncertainty is inherently costly. In a model of directed search across exchanges with discriminatory pricing, the total gains from trade are strictly concave in exchange-level liquidity flow (Proposition 4), so any mean-preserving spread in liquidity across venues unambiguously reduces welfare and trading volume. This result does not depend on specific distributional assumptions. Under reasonable conditions, liquidity uncertainty also widens the bid–ask spread (Proposition 5).

Empirically, we decompose market shares into stable long-term and noisy short-term components and show that investors demand substantial compensation for higher distributional uncertainty (9.38% average annual excess return). The premium operates primarily through the transaction cost channel rather than as an independent risk factor. Market structure shapes the relationship between fragmentation and returns: the HHI produces regime-dependent premiums (negative pre-Reg NMS, positive post-Reg NMS), which we ex-

[16]The comment letters are available at the following links: Virtu and Themis Trading. The SEC also approved another rule on September 18, 2024, to reduce tick sizes for many stocks, which will likely further affect fragmentation. A summary of both changes can be found in SEC Press Release, 2024-137, with the full text available in 89 FR 81620.

plain via our decomposition. Regulation NMS amplified distributional uncertainty and its pricing implications. We also show that the conventional illiquidity premium is conditioned by distributional uncertainty and develop a fragmentation-adjusted measure; the full analysis is in the Appendix.

Our findings have policy relevance. Regulation NMS was intended to enhance competition and price protection; fragmentation emerged as an unintended consequence, impacting asset prices in addition to market quality. Policies that reduce the dispersion of exchange-specific liquidity or improve pre-trade transparency could generate welfare gains even when their effects on commonly reported price-based measures are ambiguous. The SEC's recent amendments to exchange access fees and tick sizes may further influence market structure and, consequently, asset prices.

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A Robustness of Results

We conduct two main sets of robustness analyses. The first set examines the robustness of our results in Section 5 concerning the relationship between fragmentation and asset prices. Specifically, we first analyze whether fragmentation is priced due to its association with transaction costs. We measure transaction costs using either the effective spread or the quoted spread.[17] Secondly, instead of using the four-factor adjusted returns as the dependent variable, we utilize excess stock returns over the risk-free rate and control separately for four factors in the regression, as in Amihud and Noh (2021). Under all these specifications, our results regarding how fragmentation affects expected returns remain robust.

The second set includes robustness analyses for our fragmentation-adjusted illiquidity measure developed in Appendix B. We derive two alternative measures and demonstrate that our results are robust to these alternative specifications. Firstly, we develop a measure by not adjusting for daily liquidity discrepancies with $1-\gamma^{\text{pex}}_{i,-t}$ and define $\delta_{i,d,t}$ in Equation 13 without including $1-\gamma^{\text{pex}}_{i,-t}$ at the beginning. This alternative measure is highly correlated with our main measure and also shows significant pricing effects. However, there is an important nuance regarding the aspect of long-term fragmentation it captures: our main measure reflects the extent to which long-term market share is consolidated on a stock's primary listing exchange, whereas this alternative captures how equally long-term market shares are distributed across the listing exchange versus other market centers. Secondly, we construct a measure without absolute values. We directly use monthly averages of $\delta_{i,d,t}$, the approximated daily liquidity discrepancies given in Equation 15, to gauge liquidity with fragmentation. The advantage of this approach is that the correlation between $|r_{i,d,t}|\,/\mathrm{V}^{\$}_{i,d,t}$ and $\varepsilon^{\text{pex}}_{i,d,t}/\mathrm{V}_{i,d,t}$ is nearly zero (0.0175), allowing for a more precise decomposition. The downside is that $\varepsilon^{\text{pex}}_{i,d,t}$ can frequently alternate in sign for some stocks, leading to cancellations between positive and negative terms when calculating $\frac{1}{D_{it}}\sum_{d=1}^{D_{it}}\frac{\varepsilon^{\text{pex}}_{i,d,t}}{\mathrm{V}_{i,d,t}}$. Both of these alternative measures are provided in detail in Appendix F.

[17] Fragmentation can positively influence transaction costs by intensifying competition among market centers, or negatively by reducing liquidity in each market center.

B Illiquidity and Fragmentation: A Unifying Measure

As previously illustrated in Figure 1, trading has shifted from stocks' listing exchange to other market centers following Regulation NMS. This transition moved the market from a state where the NYSE dominated trading with more than 80% of market share, to a more fragmented market structure where the NYSE is responsible for roughly 20% of trading for its listed stocks. This change in market structure poses a challenge to traditional liquidity measures, such as those proposed by Amihud (2002) and Pastor and Stambaugh (2003), which do not consider the evolving nature of the U.S. equities market.

These measures rely on trading volumes that are now fragmented across numerous trading platforms. The core principle behind them is to assess how the trading volume of a stock influences its return. When a stock achieves a significant absolute return with minimal trading activity, it suggests a lack of liquidity, as a small number of trades have a substantial impact on its price. However, the interpretation of trading volume and its impact on returns can differ significantly when trading is concentrated in a single dominant exchange versus when it is distributed across multiple platforms. In the latter market structure, there is significant uncertainty in liquidity distribution across exchanges, which can amplify the adverse effects of illiquidity on asset prices.

	$\lvert\bar{\varepsilon}\rvert^{\text{total}}/\gamma^{\text{pex}}$:1	$\lvert\bar{\varepsilon}\rvert^{\text{total}}/\gamma^{\text{pex}}$:2	$\lvert\bar{\varepsilon}\rvert^{\text{total}}/\gamma^{\text{pex}}$:3	$\lvert\bar{\varepsilon}\rvert^{\text{total}}/\gamma^{\text{pex}}$:4	$\lvert\bar{\varepsilon}\rvert^{\text{total}}/\gamma^{\text{pex}}$:5
Illiquidity (A)					
Low	-0.01	-0.02	0.01	0.17	0.36
2	0.13	0.06	0.17	0.15	0.43
3	0.13	0.19	0.21	0.39	0.78
4	0.02	0.19	0.47	0.70	1.14
High	0.31	0.48	0.58	1.37	2.46
H-L	0.32	0.50	0.57	1.20	2.10
t-stat	2.44	2.99	3.08	5.69	10.44
p-val	0.02	0.00	0.00	0.00	0.00

Table 12: **Monthly Excess Portfolio Returns Double Sorted on Liquidity Uncertainty and Amihud Illiquidity**

Portfolios are constructed from January 1993 to December 2024 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. At the beginning of each month t, from January 1993 to December 2024, using values from two months earlier, month $t-2$, stocks are first sorted into quintile portfolios based on $|\bar{\boldsymbol{\varepsilon}}|^{\text{total}}/\boldsymbol{\gamma}^{\text{pex}}$, and then, within each $|\bar{\boldsymbol{\varepsilon}}|^{\text{total}}/\boldsymbol{\gamma}^{\text{pex}}$ quintile, sorted on A. We then calculate monthly equal-weighted excess returns over four factors and report time-series averages for each portfolio (Fama-French three factors—MKT, SMB, and HML—and the momentum factor—UMD), where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. The differences between the top and bottom illiquidity quintiles within a given $|\bar{\boldsymbol{\varepsilon}}|^{\text{total}}/\boldsymbol{\gamma}^{\text{pex}}$ quintile are also reported, along with the associated t-statistics and p-values. $\boldsymbol{\gamma}^{\text{pex}}$ is defined as the average daily market share of the primary listing exchange over the past three months, representing the listing exchange's long-term market share. $|\bar{\boldsymbol{\varepsilon}}|^{\text{total}}$ is the monthly average of the absolute daily deviations in platforms' market shares from their long-term values, and is defined in Equation 6. **A** is the Amihud illiquidity measure, calculated as the monthly average of the daily ratio of absolute return to dollar trading volume in millions.

The interaction between illiquidity and fragmentation can easily be observed in a bivariate portfolio setting analyzing how illiquidity, measured by Amihud (2002), affects expected stock returns under varying levels of liquidity distribution uncertainty, $|\bar{\varepsilon}_{i,t}|^{\text{total}}/\gamma^{\text{pex}}_{i,-t}$. Table 12 presents equally weighted four-factor alphas of these portfolios. Regardless of the degree of uncertainty, illiquidity is consistently associated with higher returns in both statistical and economic terms. In each $|\bar{\varepsilon}_{i,t}|^{\text{total}}/\gamma^{\text{pex}}_{i,-t}$ quintile, alpha typically increases with illiquidity, and illiquidity consistently leads to statistically and economically significant return spreads between stocks with the lowest and highest levels of illiquidity. However, this spread level heavily depends on uncertainty, increasing monotonically from 0.32% per month in the portfolio containing the stocks with the lowest uncertainty to 2.10% per month in the portfolio with the highest uncertainty. This result demonstrates that illiquidity has a more significant impact on expected returns when there is higher uncertainty in stocks' liquidity distributions across exchanges, which are features more commonly observed in fragmented market

structures. The difference in the illiquidity premium between the lowest and highest quintiles of uncertainty is 1.78% per month, resulting in an annual excess return of 23.6% over four-factor adjusted returns.

These results demonstrate that the impact of a stock's trading volume on its return depends on whether its volume is concentrated or fragmented. To address this limitation, we construct a new illiquidity metric that is a variant of the original Amihud (2002) measure, taking into account both illiquidity and fragmentation.[18] The original Amihud (2002) illiquidity measure is defined for stock i in month t as

$$\mathrm{A}_{i,t} := \frac{1}{D_{it}} \sum_{d=1}^{D_{it}} \frac{|r_{i,d,t}|}{\mathrm{V}^{\$}_{i,d,t}}, \tag{11}$$

where $\mathrm{V}^{\$}_{i,d,t}$ is dollar trading volume. Multiplying each volume term in Equation 2 by stock i's closing price on day d yields

$$\frac{\mathrm{V}^{\$,j}_{i,d,t}}{\mathrm{V}^{\$}_{i,d,t}} = \gamma^{j}_{i,-t} + \frac{\varepsilon^{\$,j}_{i,d,t}}{\mathrm{V}^{\$}_{i,d,t}} \tag{12}$$

The discrepancy $\delta_{i,d,t}$ and the fragmentation-adjusted measure $|\mathrm{LDA}_{i,t}|$ are

$$\delta_{i,d,t} := (1 - \gamma^{\mathrm{pex}}_{i,-t}) \times \left(\frac{|r_{i,d,t}|}{\mathrm{V}^{\$,\mathrm{other}}_{i,d,t}/(1 - \gamma^{\mathrm{pex}}_{i,-t})} - \frac{|r_{i,d,t}|}{\mathrm{V}^{\$,\mathrm{pex}}_{i,d,t}/\gamma^{\mathrm{pex}}_{i,-t}} \right) \tag{13}$$

and

$$|\mathrm{LDA}_{i,t}| := \frac{1}{D_{it}} \sum_{d=1}^{D_{it}} |\delta_{i,d,t}|. \tag{14}$$

A first-order approximation yields

$$\delta_{i,d,t} \approx \frac{|r_{i,d,t}|}{\mathrm{V}^{\$}_{i,d,t}} \times \frac{\varepsilon^{\mathrm{pex}}_{i,d,t}/\mathrm{V}_{i,d,t}}{\gamma^{\mathrm{pex}}_{i,-t}}. \tag{15}$$

Thus, the measure is a function of the original Amihud measure and uncertainty in liquidity

[18] A similar measure could easily be constructed using Pastor and Stambaugh (2003).

distribution, which is compared with average uncertainty across all platforms in Appendix E.2. This formulation establishes that the impact of traditional illiquidity is amplified as market fragmentation increases.

	Low	2	3	4	High	H-L	t-stat	p-val
LDA	0.89	0.13	0.14	0.25	0.75	-0.14	-1.47	0.14
|LDA|	0.04	0.13	0.25	0.49	1.25	1.22	8.72	0.00

Table 13: **Monthly Excess Portfolio Return Sorted on LDA and |LDA|**

Portfolios are constructed from January 1993 to December 2024 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. At the beginning of each month t, from January 1993 to December 2024, stocks are sorted into quintile portfolios based on LDA and |LDA| from two months earlier, month $t-2$. We then calculate monthly equal-weighted excess returns over four factors and report time-series averages for each quintile (Fama-French three factors—MKT, SMB, and HML—and the momentum factor—UMD), where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. The differences between the top and bottom quintiles are also reported, along with the associated t-statistics and p-values. **LDA** (|**LDA**|) is the (absolute) illiquidity spread of A, defined as the product of $1-\gamma^{\text{pex}}$ and the monthly average of the (absolute) difference between two ratios: the daily ratio of absolute return to dollar trading volume, in millions, outside the primary listing exchange, divided by $1-\gamma^{\text{pex}}$, and a similar measure on the primary exchange divided by γ^{pex}. $\boldsymbol{\gamma}^{\text{pex}}$ is defined as the average daily market share of the primary listing exchange over the past three months, representing the listing exchange's long-term market share.

At the beginning of each month t from 1993 to 2023, we sort stocks into quintiles based on their LDA and |LDA| from month $t-2$ and report their four-factor adjusted returns (alphas). Table 13 presents equally weighted four-factor alphas. LDA does not exhibit any excess return over four factors; the difference between the highest and lowest quintile returns is -0.14% and statistically insignificant. The portfolios sorted on |LDA| reveal that alpha is monotonic in |LDA|, increasing from 0.04% in the lowest quintile to 1.25% in the highest quintile, yielding a spread of 1.22% per month, or an average annual excess return of 15.66%. Table 14 presents Fama-MacBeth regressions. The coefficient of |LDA| remains positive, economically and statistically significant after controlling for price impact and other variables; a one standard deviation increase in |LDA| raises average annual excess return by 7.70% (Regression 2). The last regression indicates that the fragmentation-adjusted illiquidity premium remains positive and significant regardless of the direction of the liquidity discrepancy.

	(1)	(2)	(3)
|LDA|	1.414***	0.953***	1.145***
	(0.233)	(0.159)	(0.164)
$1(\text{LDA} \geq 0) \times \|\text{LDA}\|$			-0.352***
			(0.116)
λ		0.021***	0.021***
		(0.003)	(0.003)
Observations	448924	448924	448924
Adjusted R^2	0.043	0.047	0.048
Other Controls	✓	✓	✓

Table 14: **Fama-MacBeth Regressions of Excess Returns on |LDA|**

Monthly Fama-MacBeth regressions are estimated from January 1993 to December 2024 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. We estimate a cross-sectional regression in each month t and then report the time-series averages of the coefficients and standard errors, as well as the adjusted R^2. The **dependent variable** is the monthly excess return over four factors (Fama-French three factors—MKT, SMB, and HML—and the momentum factor—UMD), where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. **LDA** (**|LDA|**) is the (absolute) illiquidity spread of A, defined as the product of $1 - \gamma^{\text{pex}}$ and the monthly average of the (absolute) difference between two ratios: the daily ratio of absolute return to dollar trading volume, in millions, outside the primary listing exchange, divided by $1 - \gamma^{\text{pex}}$, and a similar measure on the primary exchange divided by γ^{pex}. $\boldsymbol{\gamma}^{\text{pex}}$ is defined as the average daily market share of the primary listing exchange over the past three months, representing the listing exchange's long-term market share. $\mathbf{1(\text{LDA} \geq 0)}$ is a dummy variable that is equal to one when LDA is non-negative. $\boldsymbol{\lambda}$ is the monthly average of daily price impact coefficients. In the regression for month t, these independent variables are taken from month $t-2$. Additionally, we include other standard control variables, which are defined explicitly in Section 4, with coefficients not presented to save space: idiosyncratic return volatility, previous month's stock return, and cumulative stock returns over the past twelve months excluding the previous one. t-statistics are calculated using Newey-West robust standard errors with six lags. **Standard errors** are in parentheses; * $p < 0.10$ ** $p < 0.05$ *** $p < 0.01$.

C Summary Statistics

	Mean	St.D.	1st	25th	Median	75th	99th
HHI	0.51	0.29	0.16	0.21	0.55	0.80	0.95
γ^{pex}	57.74	30.53	14.52	24.98	70.36	87.60	96.09
$\bar{\gamma}^{\text{other}}$	5.08	2.46	1.03	2.92	5.24	6.71	11.13
J	8.72	4.04	2.00	6.00	7.00	12.00	17.00
ε^{pex}	-0.35	3.98	-11.57	-2.44	-0.20	1.91	9.62
$\lvert\varepsilon\rvert^{\text{pex}}$	6.59	3.78	1.88	3.93	5.56	8.19	19.51
$\lvert\bar{\varepsilon}\rvert^{\text{other}}$	2.53	2.08	0.68	1.37	1.90	2.87	11.15
$\lvert\bar{\varepsilon}\rvert^{\text{total}}$	3.16	2.44	0.87	1.67	2.34	3.69	12.78
$\lvert\bar{\varepsilon}\rvert^{\text{total}}/\gamma^{\text{pex}}$	6.52	3.94	1.08	3.53	5.99	8.55	19.43
λ	37.31	73.71	-5.93	4.79	13.87	37.70	379.44
ME (bn)	9.45	28.58	0.06	0.63	1.94	6.11	149.61
B/M	0.62	0.54	-0.22	0.30	0.52	0.81	2.51
Observations	448924						

Table 15: **Summary Statistics I**

Summary statistics are constructed from January 1993 to December 2024 for stock-months that have non-missing data, encompassing ordinary common shares listed on the NYSE. **HHI** is the monthly average of daily Herfindahl–Hirschman indices, where the daily index is calculated as the sum of the squares of the daily market shares of each exchange and off-exchanges as a whole, based on their share volumes. $\boldsymbol{\gamma}^{\text{pex}}$ is defined as the average daily market share of the primary listing exchange over the past three months, representing the listing exchange's long-term market share. $\bar{\gamma}^{\text{other}}$ represents the average market share of all platforms outside the primary listing exchange over the past three months, as defined in Equation 7. **J** is the number of platforms that a stock is traded in during a month, where each exchange and total off-exchange market centers are referred to as separate platforms. $\boldsymbol{\varepsilon}^{\text{pex}}$ is the monthly average of the daily deviations in the listing exchange's market share from its long-term value γ^{pex}. $|\varepsilon|^{\text{pex}}$ denotes the monthly average of the absolute daily deviations in the listing exchange's market share from its long-term value, as defined in Equation 4. $|\bar{\boldsymbol{\varepsilon}}|^{\text{other}}$ and $|\bar{\boldsymbol{\varepsilon}}|^{\text{total}}$ are defined as the monthly averages of the absolute deviations in platforms' market shares from their long-term values on non-listing platforms and across all platforms, respectively, as defined in Equations 5 and 6. $\boldsymbol{\lambda}$ is the monthly average of daily price impact coefficients. **ME (bn)** is market capitalization, expressed in billions of dollars, calculated using the closing prices and shares outstanding on the last trading day of the previous year. **B/M** is the ratio of book value of equity to the market value of equity, where the ratio in months from July of year $y+1$ to June of year $y+2$ is calculated using the book value from the fiscal year ending in calendar year y and the market value at the end of year y.

	Mean	St.D.	1st	25th	Median	75th	99th
A	0.0300	0.1369	0.0000	0.0003	0.0014	0.0083	0.6073
LDA	0.1437	1.4087	-2.0594	-0.0022	0.0008	0.0252	4.9093
|LDA|	0.5687	2.6320	0.0002	0.0036	0.0186	0.1428	11.3494
Observations	448924						

Table 16: **Summary Statistics II**

Summary statistics are constructed from January 1993 to December 2024 for stock-months that have non-missing data, encompassing ordinary common shares listed on the NYSE. **A** is the original Amihud illiquidity measure, calculated as the monthly average of the daily ratio of absolute return to dollar trading volume in millions. **LDA** (|**LDA**|) is the (absolute) illiquidity spread of A, defined as the product of $1 - \gamma^{\text{pex}}$ and the monthly average of the (absolute) difference between two ratios: the daily ratio of absolute return to dollar trading volume, in millions, outside the primary listing exchange, divided by $1 - \gamma^{\text{pex}}$, and a similar measure on the primary exchange divided by γ^{pex}.

	HHI	γ^{pex}	$\bar{\gamma}^{\text{other}}$	J	ε^{pex}	$\lvert\varepsilon\rvert^{\text{pex}}$	$\lvert\bar{\varepsilon}\rvert^{\text{other}}$	$\lvert\bar{\varepsilon}\rvert^{\text{total}}$	$\lvert\bar{\varepsilon}\rvert^{\text{total}}/\gamma^{\text{pex}}$	λ	A	LDA	|LDA|	lnME
HHI	1.00													
γ^{pex}	0.97	1.00												
$\bar{\gamma}^{\text{other}}$	-0.64	-0.65	1.00											
J	-0.88	-0.89	0.30	1.00										
ε^{pex}	0.09	-0.04	0.08	0.02	1.00									
$\lvert\varepsilon\rvert^{\text{pex}}$	0.33	0.34	0.11	-0.45	-0.16	1.00								
$\lvert\bar{\varepsilon}\rvert^{\text{other}}$	0.31	0.29	0.35	-0.49	-0.10	0.78	1.00							
$\lvert\bar{\varepsilon}\rvert^{\text{total}}$	0.39	0.37	0.25	-0.56	-0.11	0.86	0.98	1.00						
$\lvert\bar{\varepsilon}\rvert^{\text{total}}/\gamma^{\text{pex}}$	-0.45	-0.51	0.74	0.22	-0.01	0.41	0.55	0.49	1.00					
λ	0.26	0.25	0.07	-0.32	0.04	0.38	0.45	0.46	0.17	1.00				
A	0.20	0.16	0.09	-0.24	0.02	0.33	0.46	0.46	0.23	0.42	1.00			
LDA	0.12	0.09	0.09	-0.13	0.23	0.17	0.18	0.20	0.07	0.29	0.03	1.00		
|LDA|	0.13	0.09	0.19	-0.17	0.08	0.43	0.47	0.47	0.34	0.43	0.61	0.30	1.00	
lnME	-0.31	-0.29	-0.09	0.42	-0.02	-0.57	-0.58	-0.61	-0.33	-0.49	-0.36	-0.13	-0.34	1.00
Observations	448924													

Table 17: **Correlations**

Correlations are constructed from January 1993 to December 2024 for stock-months that have non-missing data, encompassing ordinary common shares listed on the NYSE. **HHI** is the monthly average of daily Herfindahl–Hirschman indices, where the daily index is calculated as the sum of the squares of the daily market shares of each exchange and off-exchanges as a whole, based on their share volumes. $\boldsymbol{\gamma}^{\text{pex}}$ is defined as the average daily market share of the primary listing exchange over the past three months, representing the listing exchange's long-term market share. $\bar{\gamma}^{\text{other}}$ represents the average market share of all platforms outside the primary listing exchange over the past three months, as defined in Equation 7. **J** is the number of platforms that a stock is traded in during a month, where each exchange and total off-exchange market centers are referred to as separate platforms. $\boldsymbol{\varepsilon}^{\text{pex}}$ is the monthly average of the daily deviations in the listing exchange's market share from its long-term value γ^{pex}. $|\varepsilon|^{\text{pex}}$ denotes the monthly average of the absolute daily deviations in the listing exchange's market share from its long-term value, as defined in Equation 4. $|\bar{\boldsymbol{\varepsilon}}|^{\text{other}}$ and $|\bar{\boldsymbol{\varepsilon}}|^{\text{total}}$ are defined as the monthly averages of the absolute deviations in platforms' market shares from their long-term values on non-listing platforms and across all platforms, respectively, as defined in Equations 5 and 6. $\boldsymbol{\lambda}$ is the monthly average of daily price impact coefficients. **A** is the original Amihud illiquidity measure, calculated as the monthly average of the daily ratio of absolute return to dollar trading volume in millions. **LDA** (|**LDA**|) is the (absolute) illiquidity spread of A, defined as the product of $1-\gamma^{\text{pex}}$ and the monthly average of the (absolute) difference between two ratios: the daily ratio of absolute return to dollar trading volume, in millions, outside the primary listing exchange, divided by $1-\gamma^{\text{pex}}$, and a similar measure on the primary exchange divided by γ^{pex}. **lnME** is the logarithm of market capitalization, calculated using the closing prices and shares outstanding on the last trading day of the previous year.

	A (CRSP)	A (TAQ)	AT (CRSP)	AT (TAQ)
A (CRSP)	1.00			
A (TAQ)	0.99	1.00		
AT (CRSP)	0.71	0.71	1.00	
AT (TAQ)	0.69	0.70	0.99	1.00
Observations	687754			

Table 18: **Correlations Between CRSP and TAQ Amihud Illiquidity Measures**

Correlations are constructed from January 1993 to December 2024 for stock-months that have non-missing data, encompassing ordinary common shares listed on the NYSE. **A** is the original Amihud (2002) illiquidity measure, calculated as the monthly average of the daily ratio of absolute return to dollar trading volume in millions. **AT** is the turnover-version of the original Amihud (2002) illiquidity measure, which is calculated as the monthly average of the daily ratio of absolute return to share turnover. Share turnover is calculated as the daily share volume divided by the number of shares outstanding. **CRSP** and **TAQ** indicate that daily trading volume data are obtained from CRSP and TAQ, respectively.

D Off-Exchange Reporting Regulations

Calculating market shares, and thus the HHI, is more empirically challenging than it might initially appear due to off-exchange trades. Prior to March 5, 2007, off-exchange market centers reported their trades to exchanges, which then reported these trades to the consolidated tape along with their on-exchange transactions, as described in Bartlett and McCrary (2015). Furthermore, during this time, consolidated data revenue was allocated among exchanges—the only eligible entities—primarily in proportion to their reported trades. This market structure incentivized exchanges to establish arrangements with off-exchange market centers, mainly electronic communication networks, to increase their reported trades, effectively boosting data revenue. These arrangements created noticeable jumps in the market shares of certain exchanges, including NASDAQ, Boston, Cincinnati, and Pacific Stock Exchanges, as demonstrated in Caglio and Mayhew (2016). This market structure artificially distorted HHIs prior to March 5, 2007, due to reporting practices rather than actual changes in market concentration.

Starting from March 5, 2007, off-exchange transactions began to be reported to designated trade reporting facilities (TRFs), making them separately distinguishable from exchange trades in the TAQ data. Although this change in reporting has made the market shares of exchanges free from off-exchange trades, it has complicated the construction of a consistent HHI for samples spanning both before and after this date, as in our case, since the market share of an exchange is not directly comparable before and after this change. Before March 2007, all reporting market participants in TAQ are categorized as exchanges, yet they include off-exchange transactions. One cannot differentiate how much of the reported volume corresponds to an exchange's own trades and how much to off-exchange trades. After March 2007, reporting market participants began to be classified into individual exchanges and aggregate off-exchange market centers as distinct entities. The reported volumes of individual exchanges now contained only their own trades.

For the period before March 2007, researchers had no choice but to overlook these data limitations and calculate the HHI among all reporting exchanges, despite the inherent problems. Afterward, the HHI can either be calculated solely among exchanges, which are free from off-exchange trades, as utilized in Chao et al. (2017) and Guo and Jain (2023), or by treating aggregate off-exchange market centers as a single entity akin to a giant exchange, and

calculating the HHI among all reporting market participants, as in Haslag and Ringgenberg (2023).

The first approach focuses on national securities exchanges, which share more similar characteristics with each other than with off-exchange market centers. However, this approach excludes a significant portion of total trading volume. Due to the reporting structure, econometricians are forced to incorporate this volume into pre-March 2007 HHI calculations, leading to inconsistencies across periods in terms of trading volume coverage. The second approach captures the entire volume in either period, thereby ensuring a more uniform HHI metric in terms of coverage across different periods. Yet, it groups approximately 200 off-exchange market centers as a single entity and treats this entity like a giant exchange. Ultimately, econometricians relying on TAQ data to calculate the HHI over extended periods face a trade-off: calculating the HHI among more homogeneous entities in each reporting period or ensuring higher consistency across periods in terms of volume coverage. We adopt the second approach to maintain a more comparable metric throughout our sample period.

E Alternative Constructions of Liquidity Uncertainty

E.1 A Measure Based on Only Primary Listing Exchange

In this section, we present a liquidity uncertainty measure similar to the one developed and used in our main analysis, but with a key difference: it utilizes only the primary listing exchange. Specifically, it is defined as $\varphi_{i,t}^{\text{pex}} := \frac{|\varepsilon_{i,t}|^{\text{pex}}}{\gamma_{i,-t}^{\text{pex}}}$. This measure offers an advantage in that it better mitigates problems resulting from off-exchange reporting regulations. The NYSE market shares are less susceptible to off-exchange reporting practices and can therefore be estimated in a more consistent manner throughout the entire TAQ data period. This stability results from the NYSE's lesser inclination to make arrangements with ECNs, given its longstanding tradition of manual floor trading until the implementation of the NYSE Hybrid in October 2006.[19] Even though the NYSE's market share prior to March 2007 includes some off-exchange transactions with utmost certainty, Figure 2 illustrates that the changes in off-exchange reporting regulations appear to cause only a slight decline in the NYSE's market share, not a jump, when passing from February to March 2007. Moreover, this decline is relatively modest compared to the decreases observed in other exchanges, such as NASDAQ. The downside of this alternative is that it ignores short-term market share deviations in other markets, which are important in fragmented market structures.

	Low	2	3	4	High	H-L	t-stat	p-val
$\lvert\varepsilon\rvert^{pex}/\gamma^{\text{pex}}$	0.18	0.16	0.35	0.46	1.01	0.83	6.69	0.00

Table 19: **Monthly Excess Portfolio Returns Sorted on Alternative Measure**

Portfolios are constructed from January 1993 to December 2024 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. At the beginning of each month t, from January 1993 to December 2024, stocks are sorted into quintile portfolios based on φ^{pex} from two months earlier, month $t-2$. We then calculate monthly equal-weighted excess returns over four factors and report time-series averages for each quintile (Fama-French three factors—MKT, SMB, and HML—and the momentum factor—UMD), where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. The differences between the top and bottom quintiles are also reported, along with the associated t-statistics and p-values. $\boldsymbol{\gamma}^{\text{pex}}$ is defined as the average daily market share of the primary listing exchange over the past three months, representing the listing exchange's long-term market share. $|\boldsymbol{\varepsilon}|^{\text{pex}}$ is the monthly average of the absolute daily deviations in the listing exchange's market share from its long-term value γ^{pex}.

[19] For instance, Anderson and Dyl (2007) discuss differences in reported volumes of NYSE and NASDAQ, noting no arrangements between the NYSE and ECNs, but reporting NASDAQ's acquisitions of two ECNs. Similarly, Caglio and Mayhew (2016) examine arrangements between exchanges and ECNs and their effects on exchanges' market shares, but do not report any such arrangements for the NYSE.

	(1)	(2)	(3)	(4)	(5)	(6)
$\lvert\varepsilon\rvert^{\text{pex}}/\gamma^{\text{pex}}$	0.042***	0.029***	0.029***		0.034***	
	(0.005)	(0.005)	(0.004)		(0.005)	
$1(\varepsilon^{pex} \geq 0) \times (\lvert\varepsilon\rvert^{pex}/\gamma^{pex})$					-0.007**	
					(0.003)	
γ^{pex}				-0.029***		-0.034***
				(0.007)		(0.008)
$\lvert\varepsilon\rvert^{\text{pex}}$				0.067***		0.080***
				(0.014)		(0.016)
$1(\varepsilon^{pex} \geq 0) \times \lvert\varepsilon\rvert^{pex}$						-0.024***
						(0.007)
λ			0.022***	0.022***	0.022***	0.022***
			(0.004)	(0.004)	(0.004)	(0.004)
lnA		0.295***	0.134***	0.157***	0.134***	0.164***
		(0.045)	(0.038)	(0.037)	(0.037)	(0.037)
Observations	448924	448924	448924	448924	448924	448924
Adjusted R^2	0.040	0.044	0.047	0.049	0.048	0.049
Other Controls	✓	✓	✓	✓	✓	✓

Table 20: **Fama-MacBeth Regressions of Excess Returns on Alternative Measure and Its Components**

Monthly Fama-MacBeth regressions are estimated from January 1993 to December 2024 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. We estimate a cross-sectional regression in each month t and then report the time-series averages of the coefficients and standard errors, as well as the adjusted R^2. The **dependent variable** is the monthly excess return over four factors (Fama-French three factors—MKT, SMB, and HML—and the momentum factor—UMD), where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. $\boldsymbol{\gamma}^{\text{pex}}$ is defined as the average daily market share of the primary listing exchange over the past three months, representing the listing exchange's long-term market share. $\lvert\boldsymbol{\varepsilon}\rvert^{\text{pex}}$ is the monthly average of the absolute daily deviations in the listing exchange's market share from its long-term value γ^{pex}. $\mathbf{1}(\boldsymbol{\varepsilon^{pex}} \geq \mathbf{0})$ is a dummy variable that is equal to one when ε^{pex} is non-negative, where $\boldsymbol{\varepsilon^{pex}}$ is the monthly average of the daily deviations in the listing exchange's market share from its long-term value γ^{pex}, and is defined in Equation 3. $\boldsymbol{\lambda}$ is the monthly average of daily price impact coefficients. **lnA** is the natural logarithm of the original Amihud illiquidity measure A, where A is calculated as the monthly average of the daily ratio of absolute return to dollar trading volume in millions. In the regression for month t, these independent variables are taken from month $t-2$. Additionally, we include other standard control variables, which are defined explicitly in Section 4, with coefficients not presented to save space: idiosyncratic return volatility, previous month's stock return, and cumulative stock returns over the past twelve months excluding the previous one. t-statistics are calculated using Newey-West robust standard errors with six lags. **Standard errors** are in parentheses; * $p < 0.10$ ** $p < 0.05$ *** $p < 0.01$.

E.2 A Measure Based on All Platforms

We can extend our main liquidity uncertainty measure and include long-term market shares of other platforms, using a weighted average of long-term market shares in the denominator, $\omega\gamma_{i,-t}^{\text{pex}} + (1-\omega)\bar{\gamma}_{i,-t}^{\text{other}}$ or $(\gamma_{i,-t}^{\text{pex}})^{\omega}(\bar{\gamma}_{i,-t}^{\text{other}})^{(1-\omega)}$, where $(\omega, 1-\omega)$ represent weights. This alternative might provide a clearer picture of long-term market shares when measuring market liquidity uncertainty, as it includes all platforms. However, as demonstrated earlier, $\gamma_{i,-t}^{\text{pex}}$ and $\bar{\gamma}_{i,-t}^{\text{other}}$ affect asset pricing in opposing ways, thus combining them mixes components with different implications for asset pricing. Moreover, our measure can be rewritten as $\frac{|\bar{\varepsilon}_{i,t}|^{\text{total}}}{1-(J_{it}-1)\bar{\gamma}_{i,-t}^{\text{other}}}$, and can therefore be interpreted based on long-term market shares of other platforms as well. Given that $\gamma_{i,-t}^{\text{pex}}$ and $\bar{\gamma}_{i,-t}^{\text{other}}$ have a correlation of -0.65, there is little need to introduce the complexity of determining weights, which would ideally balance the opposing effects of $\gamma_{i,-t}^{\text{pex}}$ and $\bar{\gamma}_{i,-t}^{\text{other}}$. Our results are robust to this alternative specification across a broad range of weights. These results are available upon request.

F Illiquidity and Fragmentation

Here, we first repeat the regression analysis of Appendix B using the turnover-version of the Amihud illiquidity measure, which we denote as |LDAT|. We then derive two alternative measures of illiquidity that also accounts for how returns respond to aggregate volume as well as liquidity uncertainty, and demonstrate that our results are robust to these alternative specifications. Even though these alternative measures are highly correlated with our main measure that we develop in Equation 14, they slightly capture different aspects of liquidity.

F.1 Turnover-Based Fragmentation-Adjusted Illiquidity Measure

	(1)	(2)	(3)
|LDAT|	0.071***	0.041***	0.056***
	(0.011)	(0.008)	(0.008)
$1(\text{LDAT} \geq 0) \times \|\text{LDAT}\|$			-0.019***
			(0.005)
λ		0.024***	0.024***
		(0.004)	(0.004)
Observations	448924	448924	448924
Adjusted R^2	0.041	0.046	0.046
Other Controls	✓	✓	✓

Table 21: **Fama-MacBeth Regressions of Excess Returns on |LDAT|**

Monthly Fama-MacBeth regressions are estimated from January 1993 to December 2024 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. We estimate a cross-sectional regression in each month t and then report the time-series averages of the coefficients and standard errors, as well as the adjusted R^2. The **dependent variable** is the monthly excess return over four factors (Fama-French three factors—MKT, SMB, and HML—and the momentum factor—UMD), where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. **LDAT** (**|LDAT|**) is the (absolute) illiquidity spread of AT, defined as the product of $1-\gamma^{\text{pex}}$ and the monthly average of the (absolute) difference between two ratios: the daily ratio of absolute return to share turnover outside the primary listing exchange divided by $1-\gamma^{\text{pex}}$, and a similar measure on the primary exchange divided by γ^{pex}. $\boldsymbol{\gamma}^{\text{pex}}$ is defined as the average daily market share of the primary listing exchange over the past three months, representing the listing exchange's long-term market share. $\mathbf{1(\text{LDAT} \geq 0)}$ is a dummy variable that is equal to one when LDAT is non-negative. $\boldsymbol{\lambda}$ is the monthly average of daily price impact coefficients. In the regression for month t, these independent variables are taken from month $t-2$. Additionally, we include other standard control variables, which are defined explicitly in Section 4, with coefficients not presented to save space: idiosyncratic return volatility, previous month's stock return, and cumulative stock returns over the past twelve months excluding the previous one. t-statistics are calculated using Newey-West robust standard errors with six lags. **Standard errors** are in parentheses; * $p<0.10$ ** $p<0.05$ *** $p<0.01$.

F.2 Alternative Measure I

We develop a measure without adjusting daily liquidity discrepancies with $1-\gamma_{i,-t}^{\text{pex}}$ and define $\delta_{i,d,t}$ in Equation 13 without $1-\gamma_{i,-t}^{\text{pex}}$, such that

$$\hat{\delta}_{i,d,t} := \left(\frac{|r_{i,d,t}|}{\mathrm{V}_{i,d,t}^{\$,\text{other}}/(1-\gamma_{i,-t}^{\text{pex}})} - \frac{|r_{i,d,t}|}{\mathrm{V}_{i,d,t}^{\$,\text{nyse}}/\gamma_{i,-t}^{\text{pex}}} \right) \tag{16}$$

We denote our first alternative measure as $\widehat{|\mathrm{LDA}_{i,t}|}$ and define it as

$$\widehat{|\mathrm{LDA}_{i,t}|} := \frac{1}{D_{it}} \sum_{d=1}^{D_{it}} \left| \hat{\delta}_{i,d,t} \right|$$

Following similar steps, this setup results in an equation similar to that of Equation **??**,

$$\widehat{|\mathrm{LDA}_{i,t}|} \approx \left[\frac{1}{D_{it}} \sum_{d=1}^{D_{it}} \frac{|r_{i,d,t}|}{\mathrm{V}_{i,d,t}^{\$}} \times \left| \frac{\varepsilon_{i,d,t}^{\text{pex}}}{\mathrm{V}_{i,d,t}} \right| \right] \times \frac{1}{\gamma_{i,-t}^{\text{pex}}(1-\gamma_{i,-t}^{\text{pex}})} \tag{17}$$

which can be further decomposed as

$$\widehat{|\mathrm{LDA}_{i,t}|} \approx \underbrace{\left[\frac{1}{D_{it}} \sum_{d=1}^{D_{it}} \frac{|r_{i,d,t}|}{\mathrm{V}_{i,d,t}^{\$}} \right]}_{\text{Amihud Illiquidity}} \times \underbrace{\left[\frac{1}{D_{it}} \sum_{d=1}^{D_{it}} \left| \frac{\varepsilon_{i,d,t}^{\text{pex}}}{\mathrm{V}_{i,d,t}} \right| \right]}_{\text{Liquidity Uncertainty}} \times \underbrace{\left[\frac{1}{\gamma_{i,-t}^{\text{pex}}(1-\gamma_{i,-t}^{\text{pex}})} \right]}_{\text{Long-Term Market Share}} \tag{18}$$

It is not surprising that this alternative measure is highly correlated with our main measure and also exhibits significant pricing. It is identical to our main measure except for the last term. However, there is an important nuance concerning the aspect of long-term fragmentation it captures. Our main measure reflects the extent to which long-term market share is consolidated on a stock's primary listing exchange, the NYSE. In contrast, this alternative measure captures how equally long-term market shares are distributed across the listing exchange and other market centers. Keeping all else constant, $\widehat{|\mathrm{LDA}_{i,t}|}$ takes its minimum value when $\gamma_{i,-t}^{\text{pex}} = \frac{1}{2}$—indicating that the listing exchange versus other market centers hold equal amounts of long-term market shares. If we loosely interpret long-term market share as the probability of matching with a counterparty in a market center, this

alternative measure suggests that long-term fragmentation is least harmful to liquidity when market centers are more alike in terms of execution probabilities.

	(1)	(2)	(3)
|LDA|	1.063***	0.703***	0.794***
	(0.202)	(0.135)	(0.144)
$1(\text{LDA} \geq 0) \times \|\text{LDA}\|$			-0.204**
			(0.094)
λ		0.021***	0.022***
		(0.003)	(0.003)
Observations	448924	448924	448924
Adjusted R^2	0.043	0.047	0.048
Other Controls	✓	✓	✓

Table 22: **Fama-MacBeth Regressions of Stock Returns on $|\widehat{\mathbf{LDA}}_{i,t}|$**

Monthly Fama-MacBeth regressions are estimated from January 1993 to December 2024 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. We estimate a cross-sectional regression in each month t and then report the time-series averages of the coefficients and standard errors, as well as the adjusted R^2. The **dependent variable** is the monthly excess return over the Fama-French three-factor model, where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. **LDA** (**|LDA|**) is the (absolute) illiquidity spread of A, defined as the monthly average of the (absolute) difference between two ratios: the daily ratio of absolute return to dollar trading volume, in millions, outside the primary listing exchange divided by $1 - \gamma^{\text{pex}}$, and a similar measure on the primary exchange divided by γ^{pex}. $\boldsymbol{\gamma}^{\text{pex}}$ is defined as the average daily market share of the primary listing exchange over the past three months, representing the listing exchange's long-term market share. **1(LDA $\geq$ 0)** is a dummy variable that is equal to one when LDA is non-negative. $\boldsymbol{\lambda}$ is the monthly average of daily price impact coefficients. In the regression for month t, these independent variables are taken from month $t-2$. Additionally, we include other standard control variables, which are defined explicitly in Section 4, with coefficients not presented to save space: idiosyncratic return volatility, previous month's stock return, and cumulative stock returns over the past twelve months excluding the previous one. t-statistics are calculated using Newey-West robust standard errors with six lags. **Standard errors** are in parentheses; * $p < 0.10$ ** $p < 0.05$ *** $p < 0.01$.

F.3 Alternative Measure II

We also develop a measure that directly uses monthly average $\delta_{i,d,t}$ from Equation 13 to assess liquidity with fragmentation. Instead of initially taking the absolute values, we first calculate the monthly average $\delta_{i,d,t}$ and then use the absolute value of this monthly average. We denote our second alternative measure as $|\widetilde{\text{LDA}_{i,t}}|$ and define it as

$$|\widetilde{\text{LDA}_{i,t}}| := \left| \frac{1}{D_{it}} \sum_{d=1}^{D_{it}} \delta_{i,d,t} \right|$$

Following similar steps, this setup results in an equation very similar to our main measure developed in Equation **??**,

$$|\widetilde{\text{LDA}_{i,t}}| \approx \left| \left[\frac{1}{D_{it}} \sum_{d=1}^{D_{it}} \frac{|r_{i,d,t}|}{\text{V}^{\$}_{i,d,t}} \times \frac{\varepsilon^{\text{pex}}_{i,d,t}}{\text{V}_{i,d,t}} \right] \times \frac{1}{\gamma^{\text{pex}}_{i,-t}} \right| \tag{19}$$

which can be further decomposed as

$$|\widetilde{\text{LDA}_{i,t}}| \approx \underbrace{\left[\frac{1}{D_{it}} \sum_{d=1}^{D_{it}} \frac{|r_{i,d,t}|}{\text{V}^{\$}_{i,d,t}} \right]}_{\text{Amihud Illiquidity } (\text{A}_{i,t})} \times \underbrace{\left| \left[\frac{1}{D_{it}} \sum_{d=1}^{D_{it}} \frac{\varepsilon^{\text{pex}}_{i,d,t}}{\text{V}_{i,d,t}} \right] \right|}_{\text{Short-Term Noise } (|\varepsilon^{\text{pex}}_{i,t}|)} \times \underbrace{\left[\frac{1}{\gamma^{\text{pex}}_{i,-t}} \right]}_{\text{Long-Term Market Share}} \tag{20}$$

This measure is very similar to our main measure developed in 14 except that the second term takes averages of $\frac{\varepsilon^{\text{pex}}_{i,d,t}}{\text{V}_{i,d,t}}$ instead of $\left|\frac{\varepsilon^{\text{pex}}_{i,d,t}}{\text{V}_{i,d,t}}\right|$. The advantage of this approach is that the correlation between $|r_{i,d,t}|/\text{V}^{\$}_{i,d,t}$ and $\varepsilon^{\text{pex}}_{i,d,t}/\text{V}_{i,d,t}$ is nearly zero (0.0175), allowing for more precise approximation when transitioning from Equation 19 to 20. The downside is that $\varepsilon_{i,d,t}$ can frequently alternate in sign for some stocks, leading to cancellations between positive and negative terms when calculating $\frac{1}{D_{it}} \sum_{d=1}^{D_{it}} \frac{\varepsilon_{i,d,t}}{\text{V}_{i,d,t}}$, which may underestimate short-term fragmentation in some cases.

	(1)	(2)	(3)
|LDA|	1.989***	1.207***	1.494***
	(0.337)	(0.213)	(0.232)
$1(\text{LDA} \geq 0) \times \|\text{LDA}\|$			-0.287
			(0.182)
λ		0.022***	0.022***
		(0.004)	(0.004)
Observations	448924	448924	448924
Adjusted R^2	0.042	0.046	0.047
Other Controls	✓	✓	✓

Table 23: **Fama-MacBeth Regressions of Stock Returns on $|\widetilde{\text{LDA}}_{i,t}|$**

Monthly Fama-MacBeth regressions are estimated from January 1993 to December 2024 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. We estimate a cross-sectional regression in each month t and then report the time-series averages of the coefficients and standard errors, as well as the adjusted R^2. The **dependent variable** is the monthly excess return over the Fama-French three-factor model, where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. **LDA** is the illiquidity spread of A, defined as the monthly average of the difference between two ratios: the daily ratio of absolute return to dollar trading volume, in millions, outside the primary listing exchange divided by $1 - \gamma^{\text{pex}}$, and a similar measure on the primary exchange divided by γ^{pex}. $|\textbf{LDA}|$ is the absolute value of LDA. $\boldsymbol{\gamma}^{\text{pex}}$ is defined as the average daily market share of the primary listing exchange over the past three months, representing the listing exchange's long-term market share. $\mathbf{1}(\text{LDA} \geq \mathbf{0})$ is a dummy variable that is equal to one when LDA is non-negative. $\boldsymbol{\lambda}$ is the monthly average of daily price impact coefficients. In the regression for month t, these independent variables are taken from month $t-2$. Additionally, we include other standard control variables, which are defined explicitly in Section 4, with coefficients not presented to save space: idiosyncratic return volatility, previous month's stock return, and cumulative stock returns over the past twelve months excluding the previous one. t-statistics are calculated using Newey-West robust standard errors with six lags. **Standard errors** are in parentheses; * $p < 0.10$ ** $p < 0.05$ *** $p < 0.01$.

G Figures

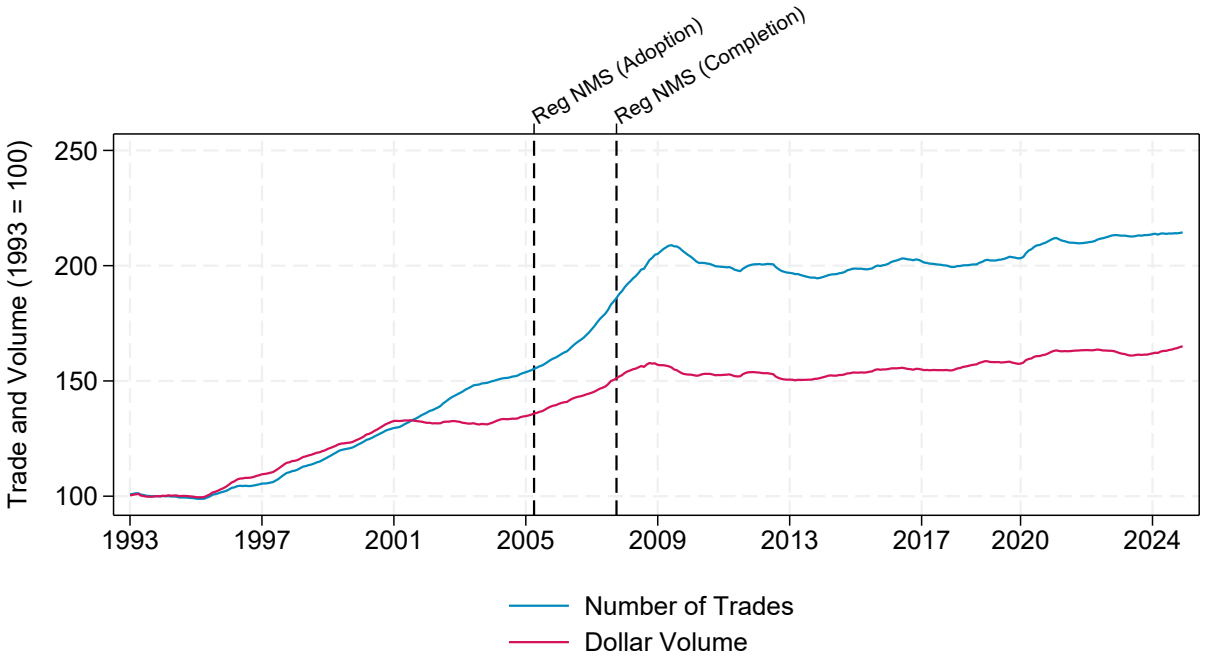


Figure 4: **Number of Trades and Dollar Volume**

This figure displays the monthly cross-sectional averages of trading activity. The sample covers stock-months that have non-missing data from January 1993 to December 2024, encompassing ordinary common shares listed on the NYSE. The figure plots the total number of trades and the total dollar volume, both normalized by taking their respective values in 1993 as 100. The left dashed line corresponds to April 2005, the time of the adoption of Regulation NMS. The right dashed line corresponds to July 2007, the completion of Regulation NMS.

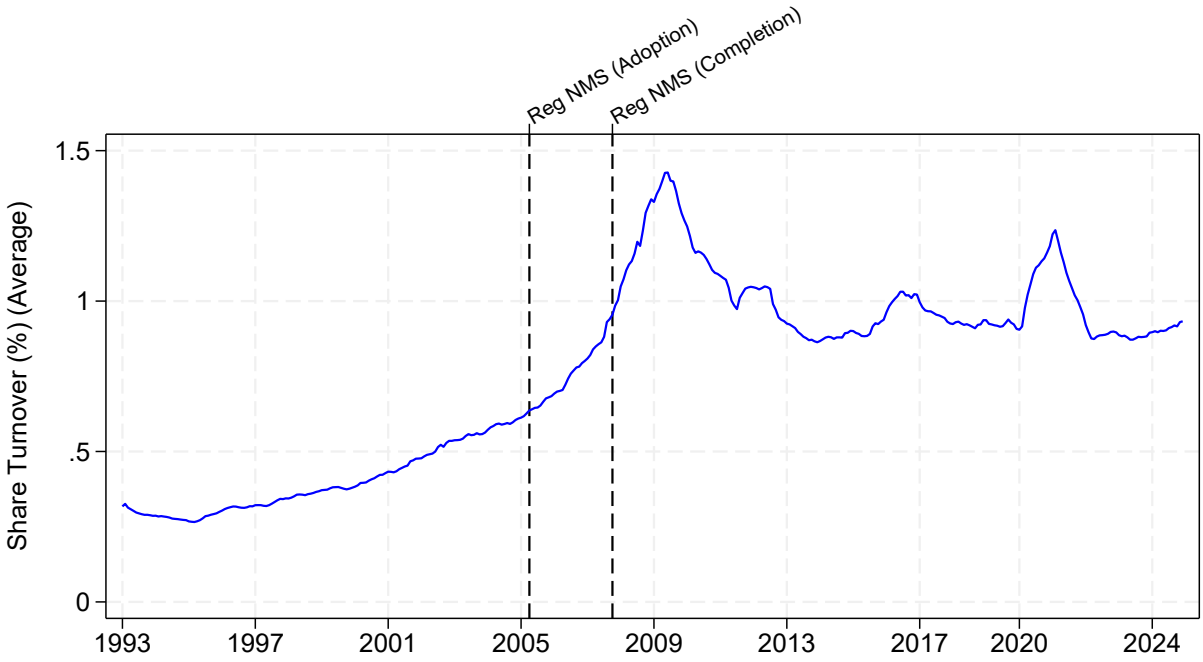


Figure 5: **Average Share Turnover**

This figure displays the monthly cross-sectional averages of share turnover. The sample covers stock-months that have non-missing data from January 1993 to December 2024, encompassing ordinary common shares listed on the NYSE. **Turnover** is defined as the daily share volume divided by the total number of shares outstanding, expressed as a percentage. The left dashed line corresponds to April 2005, the time of the adoption of Regulation NMS. The right dashed line corresponds to July 2007, the completion of Regulation NMS.

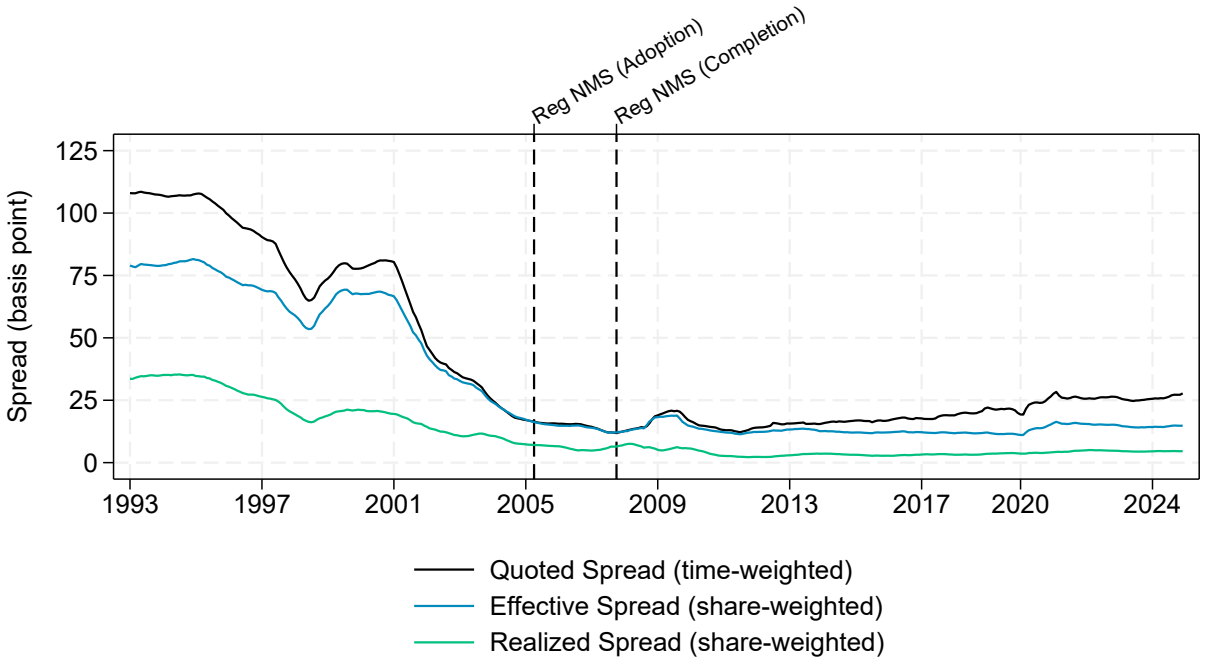


Figure 6: **Quoted, Effective, and Realized Spreads**

This figure displays the monthly cross-sectional averages of trading spreads. The sample covers stock-months that have non-missing data from January 1993 to December 2024, encompassing ordinary common shares listed on the NYSE. The figure plots the time-weighted quoted spread (**QS**), the share-weighted effective spread (**ES**), and the share-weighted realized spread (**RS**), all expressed in basis points. **QS** captures the visible cost of trading, while **ES** captures the actual execution cost relative to the midpoint, and **RS** captures the revenue to liquidity providers net of adverse selection. The left dashed line corresponds to April 2005, the time of the adoption of Regulation NMS. The right dashed line corresponds to July 2007, the completion of Regulation NMS.

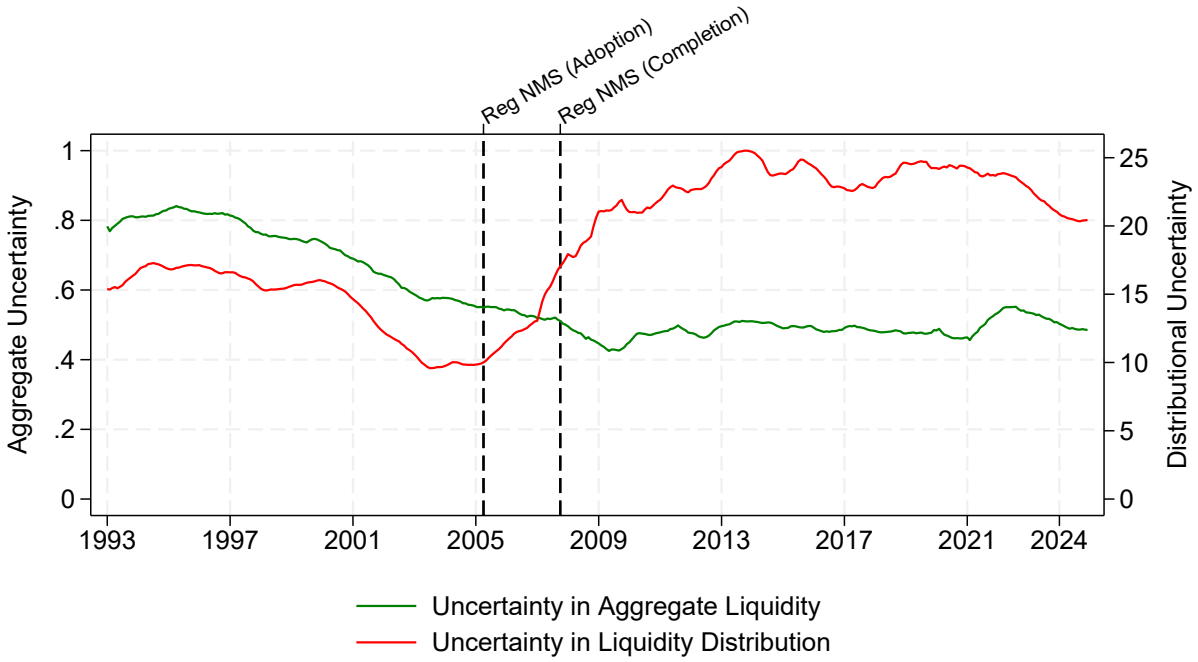


Figure 7: **Aggregate versus Distributional Uncertainty in Liquidity**

This figure compares aggregate liquidity uncertainty with distributional uncertainty. The sample covers stock-months that have non-missing data from January 1993 to December 2024, encompassing ordinary common shares listed on the NYSE. The left y-axis displays the uncertainty in aggregate liquidity, measured as the coefficient of variation of share turnover. The right y-axis displays the distributional uncertainty in liquidity, defined as the monthly average of the absolute daily deviations in platforms' market shares from their long-term values. The left dashed line corresponds to April 2005, the time of the adoption of Regulation NMS. The right dashed line corresponds to July 2007, the completion of Regulation NMS.

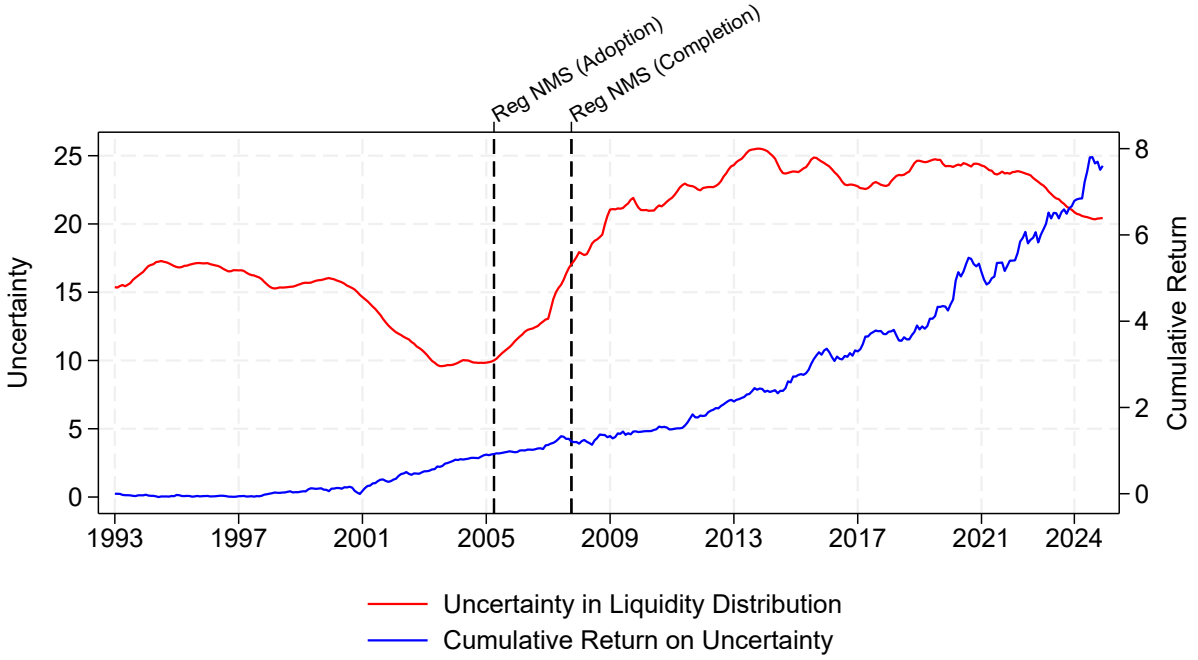


Figure 8: **Uncertainty in Liquidity Distribution and Cumulative Return**

This figure displays the relationship between the uncertainty in liquidity distribution and the corresponding cumulative returns. The sample covers stock-months that have non-missing data from January 1993 to December 2024, encompassing ordinary common shares listed on the NYSE. The left y-axis displays the uncertainty in liquidity distribution. The right y-axis displays the cumulative return of a long-short portfolio, sorted based on this uncertainty measure. The left dashed line corresponds to April 2005, the time of the adoption of Regulation NMS. The right dashed line corresponds to July 2007, the completion of Regulation NMS.

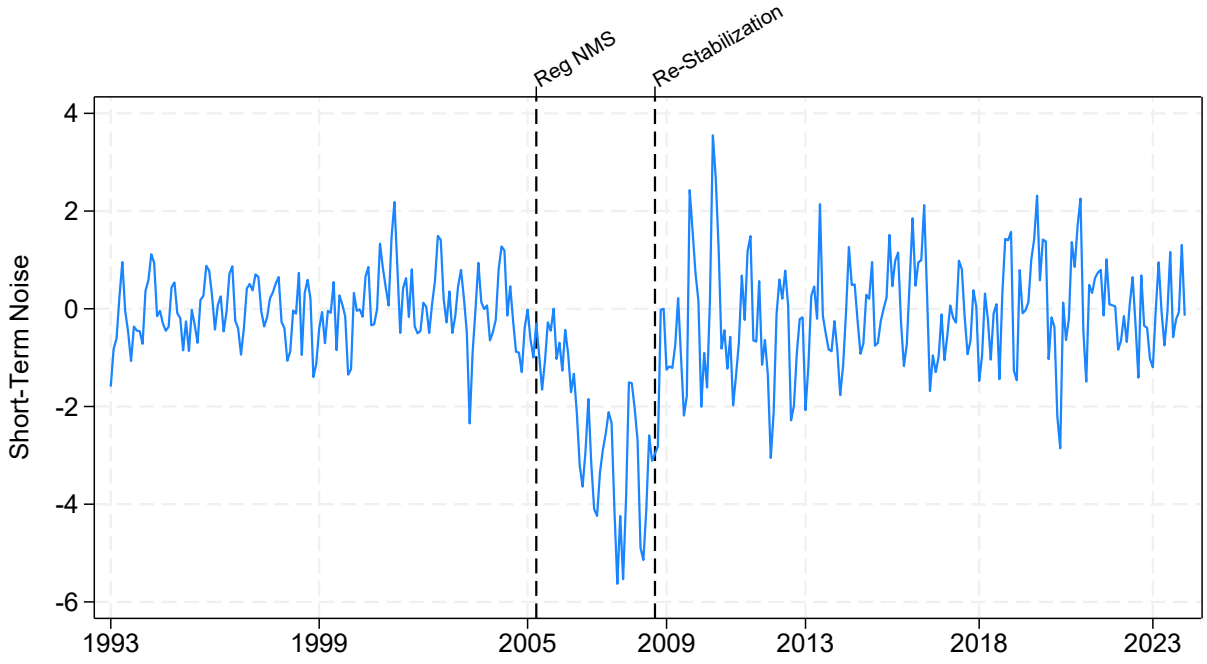


Figure 9: Short-Term Market Share Deviations from the Listing Exchange to Other Market Centers (ε^{pex})

This figure displays the monthly cross-sectional averages of ε^{pex} from January 1993 to December 2023 for stock-months that have non-missing data, encompassing ordinary common shares listed on the NYSE. We decompose market shares (in share volumes) into long-term (γ) and short-term (ε) components. $\boldsymbol{\gamma}^{\text{pex}}$ is defined as the average daily market share of the primary listing exchange over the past three months, representing the listing exchange's long-term market share. $\boldsymbol{\varepsilon}^{\text{pex}}$ is the monthly average of the daily deviations in the listing exchange's market share from its long-term value γ^{pex}, and is defined in Equation 3. The left dashed line corresponds to April 2005, the time of adoption of Regulation NMS. The right dashed line corresponds to September 2008, around when market shares across exchanges stabilized again.

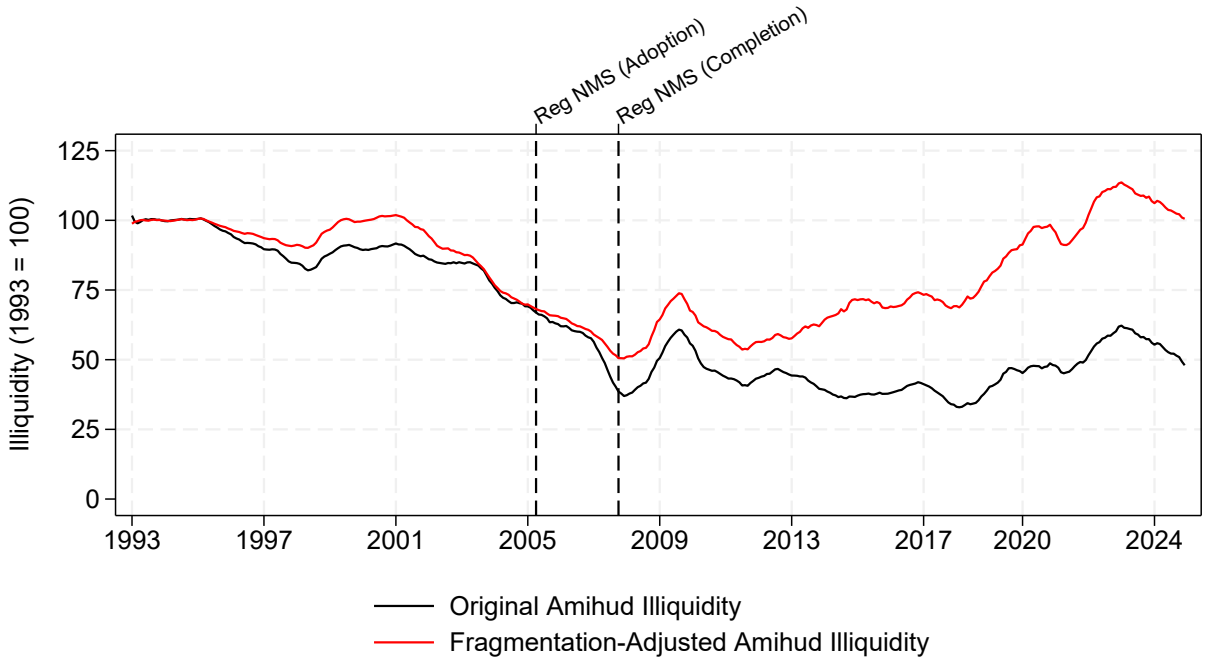


Figure 10: **Original versus Fragmentation-Adjusted Amihud Illiquidity**

This figure displays the monthly cross-sectional averages of the original Amihud (2002) illiquidity measure **A** and the fragmentation-adjusted Amihud illiquidity measure |**LDA**|. The sample covers stock-months that have non-missing data from January 1993 to December 2024, encompassing ordinary common shares listed on the NYSE. Both illiquidity series are normalized by taking their respective values in 1993 as 100. **A** of a stock for a specific month is calculated as the monthly average of the daily ratio of absolute return to dollar trading volume in millions. |**LDA**| is the fragmentation-adjusted (absolute) illiquidity spread. The left dashed line corresponds to April 2005, the time of the adoption of Regulation NMS. The right dashed line corresponds to July 2007, the completion of Regulation NMS.