Politics, Finance, and Environmental Policy *

Jonathan Payne and Yinshan Shang†

March 26, 2026

Abstract

How do political and financial frictions shape environmental policy? We build a general equilibrium asset pricing model where investors with heterogeneous preferences for carbon reduction ("green" or "brown") negotiate over a carbon tax, while internalizing their impact of the real economy. This leads to a "Political-Climate CAPM", in which green investment is determined by mean-variance tradeoffs, emission hedging, and political risk hedging. Political frictions lead to inefficiently low carbon taxes, with more political polarization leading to more conservative investments and higher household risk aversion leader carbon taxes. A green fund helps, but does not completely resolve the problems unless it can unbundle capital allocation and risk sharing.

Keywords: Carbon taxes, political economy, green finance.

*We thank Markus Brunnermeier, Sylvain Chassang, Daniel Chen, Ran Duchin, Stefano Giglio, Moritz Lenel, Ernest Liu, Tom Sargent, Balint Szoke, Wei Xiong, Leeat Yariv, Motohiro Yogo, seminar participants at NBER Climate Finance PhD workshop and Princeton Finance Student Workshop for helpful comments and suggestions.

†Payne: Princeton, Department of Economics, Bendheim Center for Finance, jepayne@princeton.edu. Shang: Princeton, Department of Economics, yinshans@princeton.edu.

1 Introduction

Many researchers have argued that we can resolve global warming by taxing carbon emissions and letting the market respond to the resulting financial incentive to use "green" technologies. The reality is more complicated. Political frictions make environmental policies difficult to implement, financial frictions distort reactions to policies, and international capital markets allow investors to evade taxation policy. For all these reasons, economists need to study how politics and finance interact to determine environmental policy. What is the largest carbon tax that is feasible within our political system? How do political frictions and risks show up in asset pricing? Is there a role for financial policy or green investment funds to help address climate change? In this paper, we build a general equilibrium model with environmental, financial, and political frictions to address these questions.

Our economy is populated by two types of investors: "green" investors who care more about carbon emissions and "brown" investors who care less. The investors can purchase equity in two different sectors: a "green" sector with low pollution and returns, and a "brown" production sector with high pollution and returns. When choosing portfolios, investors face risk to productivity, emissions, and carbon taxes. All markets are competitive, so investor portfolio decisions determine equilibrium production and carbon emissions in the economy. This means that green investment is determined by the investors' standard mean-variance trade-off and their desire to hedge both emissions risk and political risk. For this reason, we call our model a "Political-Climate CAPM".

The carbon tax is set endogeneously through a risky political process. The climate externality in our economy is that no investors internalize the impact of their individual financial decisions on carbon emissions when they choose portfolios. Instead, before financial decisions are made, the different types of investors collectively negotiate to set a target carbon tax on emissions. Throughout the process, each investor group internalizes the impact of carbon taxes on both general equilibrium output and carbon emissions. To capture the risk involved in the political process, we impose that the realized carbon tax is the negotiated target rate plus a shock with volatility that is larger when green and brown agents have more heterogeneous ideal tax rates. The government carbon tax rule closes our Political-Climate CAPM so that general equilibrium green investment comes from the feedback between private sector responses to carbon taxes *and* political policy response to private sector portfolio choices.

We characterize how the equilibrium tax rate and policy uncertainty depend on economic environment in the political-market equilibrium, which then determines the market outcomes in investment and emissions. We show that the outcome of the political bargaining problem is not generically efficient. This is due to two frictions. First, as is standard in public finance models, the carbon tax distorts the labor market, leading to inefficiently low production. Second, political bargaining is incomplete in the sense that the different groups can only agree upon a carbon tax rate; they cannot commit to future portfolio choices or future lump sum transfers between groups. Moreover, we can characterize how changes to our environment impact the political-financial equilibrium. An increase political polarization—as measured by an increase in preference heterogeneity between green and brown groups—leads to more conservative investment. The only leads to higher green investment if green investments have a higher bargaining weight and so green investment is lower risk. An increase in household risk aversion reduces both the equilibrium tax and political disagreement.

Our model generates a "political risk premium", whereby investors demand higher expected returns as compensation for bearing this non-diversifiable political risk. Whether the political risk premium appears on green or brown equity depends on which type of investments are most exposed to political risk. In our baseline model, the agents bargain over carbon taxes so the risk is primarily born by brown firms and a "brown" premium emerges on brown equity. However, if the agents instead bargained over subsidies (or negative taxes) for non-polluting production, then the green firms primarily bear the risk and so a "green" premium emerges on green equity.

The difficulties in the political process have led to the emergence of green-finance or environmental, social, and governance (ESG) funds that attempt to use the financial sector to address climate change. To explore the implications of this, in Section 4, we introduce a fund that explicitly internalizes the aggregate impact of their investment decisions on environmental outcomes. This means that green investors act as a coordinated bloc that internalizes its effect on aggregate emissions, while brown investors remain atomistic. We show that the fund improves the equilibrium level of green investment because the coordinated green investors optimally tilt their portfolios toward green assets. However, we also demonstrate that the equilibrium remains generically Pareto inefficient because portfolio choice bundles together two distinct functions—capital allocation and risk bearing—that a social planner would

optimally separate.

In Section 5, we discuss extensions of the model. We show that incorporating workers into the bargaining problem creates additional barriers to carbon policy because the welfare costs of carbon tax policies are disproportionately borne by workers. We also discuss how the model extends to a multi-country setting with integrated financial markets, where international capital mobility introduces a *financial leakage* channel that can amplify the domestic political friction and undermine unilateral climate policy.

Literature review: Our paper brings together two strands of the literature: asset pricing with climate concerns and the political economy of climate policy.

There is a recent and growing finance literature on asset pricing with climate concerns (e.g., Heinkel et al. (2001); Engle et al. (2020); Bolton and Kacperczyk (2021); Flammer (2021); Giglio et al. (2021); Pástor et al. (2021); Pedersen et al. (2021); Goldstein et al. (2022); Pástor et al. (2022); Hsu et al. (2023); Acharya et al. (2025); Green and Roth (2025); Hambel and Van Der Ploeg (2025)). Recent work in finance has investigated how investor preferences and anticipated climate policies influence asset returns. While these studies emphasize market responses to exogenous climate risks or static policy measures, we introduce endogenous political bargaining over carbon taxes within a general equilibrium framework. Specifically, our model incorporates investors' heterogeneous preferences explicitly into policy formation rather than into a "warm glow" utility from holding green asset. This provides insights into how political frictions shape policy choices, subsequent asset allocations, and ultimately green or brown risk premia.

We also contribute to the political economy literature on climate policy, which examines domestic and international challenges in environmental policy cooperation (See Oates and Portney (2003); Nordhaus (2015); Battaglini and Harstad (2016, 2020); Cassidy Jr (2023)). We expand upon these works by highlighting the role of integrated financial markets as channels for leakage and international free-riding. By integrating a Nash bargaining framework with general equilibrium asset pricing, we reveal how domestic political frictions combined with international capital mobility exacerbate policy inefficiencies, underscoring the limitations of unilateral carbon taxes and the potential need for coordinated financial market regulations.

We outline the market equilibrium in a single-country general equilibrium model with given carbon tax policy in Section 2. We then describe the political process that endogenizes the carbon policy and the equilibrium outcomes in Section 3, where

we also establish the generic inefficiency of the political–market equilibrium (with a formal comparison to a social planner's benchmark in Appendix A). In Section 4 we ask whether coordination by green investors can restore efficiency. We discuss extensions, including labor market considerations and the international setting, in Section 5 and conclude in Section 6.

2 Green Investments Under Climate and Political Risk

This section characterizes the competitive market equilibrium in a single-country production economy with three sources of risk: stochastic productivity, uncertain emission intensity, and political risk over the carbon tax. We derive a closed-form equilibrium in which the market portfolio and asset prices are determined by an *climate pricing matrix* $\widetilde{M}$ that encodes the interactions among risk compensation, emission hedging, and political risk hedging. The equilibrium asset pricing equation takes the form of a three-factor model—a Climate Policy CAPM—with market, emission, and political risk factors.

Throughout this section, the carbon tax process is treated as exogenous so we can focus on the general equilibrium financial market equilibrium. We endogenize the carbon tax through political bargaining in the next section and show how political decision making and asset pricing interact.

2.1 Environment

The economy lasts for two periods, $t \in \{0, 1\}$, and is populated by a unit mass of investors and a collection of workers who provide labor for production. There is one consumption good and one pollution bad.

Agent preferences. There are two types of investors of equal mass, $i \in \{G, B\}$ ("green" and "brown"). Type i is endowed with goods y_i at $t = 0$ and ranks outcomes at $t = 1$ according to

$$u_i(c_i; E) = -\exp\big(-\gamma\, c_i + \eta_i\, E\big),$$

where c_i is consumption, E is aggregate emissions, $\gamma > 0$ is the common coefficient of absolute risk aversion, and η_i is the marginal disutility for an agent of type i. We impose that $\eta_G > \eta_B$, so green investors get more disutility from pollution.

Labor is supplied by a collection of workers who incur a constant marginal disutility w_s from supplying labor to sector s.[1] Workers do not receive endowments or own assets and consume their wage income for linear utility.

Production and emissions. There are two sectors, $s \in \{g, b\}$, each with a representative firm. Sector s uses capital K_s and labor L_s at $t = 0$ to produces goods and emissions at $t = 1$ according to:

$$f_s(K_s, L_s) = A_s K_s^{\alpha} L_s^{1-\alpha},$$
$$h_s(K_s, L_s) = P_s K_s^{\alpha} L_s^{1-\alpha}.$$

where A_s is productivity in sector s, P_s is emission intensity in sector s, and $\alpha \in (0, 1)$ is the capital share. Sector productivity and emission intensity are stochastic variables respectively following:

$$A_s = \bar{A}_s + \epsilon_A + \epsilon_{A_s},$$
$$P_s = \bar{P}_s + \epsilon_{P_s}$$

where $\epsilon_A \sim \mathcal{N}(0, \sigma_A^2)$ is an aggregate productivity shock common to both sectors, $\epsilon_{A_s} \sim \mathcal{N}(0, \sigma_{A,s}^2)$ is a sector-specific shock, and $\epsilon_{P_s} \sim \mathcal{N}(0, \sigma_{P_s}^2)$ is a sector specific emissions shock. We allow within-sector covariance $\sigma_{AP,s} := \mathrm{Cov}(\epsilon_{A_s}, \epsilon_{P_s})$ to be non-zero, potentially capturing whether high-productivity states coincide with high emissions (i.e. "dirty booms") if $\sigma_{AP,s} > 0$. The shocks ϵ_A, $(\epsilon_{A_g}, \epsilon_{P_g})$, and $(\epsilon_{A_b}, \epsilon_{P_b})$ are mutually independent. We impose that the brown sector produces more expected emissions per unit of capital at the no-tax benchmark:

$$\bar{P}_b \left(\frac{\bar{A}_b}{w_b} \right)^{(1-\alpha)/\alpha} > \bar{P}_g \left(\frac{\bar{A}_g}{w_g} \right)^{(1-\alpha)/\alpha}.$$

Since firms choose labor optimally, output per unit of capital in sector s scales with $(\bar{A}_s/w_s)^{(1-\alpha)/\alpha}$; the condition adjusts emission intensity for equilibrium production scale, and reduces to $\bar{P}_b > \bar{P}_g$ when sectors share the same productivity and labor costs. Aggregate emissions are $E = \sum_s h_s(K_s, L_s)$.

Capital creation. Firms in sector s can use endowment goods to produce capital

[1] In our baseline model, workers do not make strategic decisions or participate in political bargaining because they earn zero surplus and are always indifferent at the margin. In Section 5.1, we discuss extensions where workers have positive value functions and so play a role in the political bargaining.

at $t = 0$ but face an adjustment cost. In particular, producing K_s capital requires $K_s + \frac{\phi_s}{2} K_s^2$ goods with parameter $\phi_s > 0$. The potential asymmetry $\phi_g \neq \phi_b$ allows the two sectors to differ in the ease of scaling investment—an interpretation we discuss in Section 2.5.

Assets and financial markets. There are three financial assets: equity in each sector and a risk-free bond in zero net supply. All assets are traded competitively. We denote the risk-free rate by r_f, the return on sector-s equity by $r_s = \pi_s / K_s$ (where π_s is the firm dividend), and write $r = (r_g, r_b)'$.

Environmental policy. The government levies a carbon tax on emissions and rebates a fraction $\lambda_R \in (0, 1]$ of the revenue equally to all investors. The parameter λ_R captures frictions in the rebate process: administrative costs, political earmarking of revenue, or redistribution to non-investor groups. When $\lambda_R = 1$, investors receive the full tax revenue; when $\lambda_R < 1$, only a fraction is returned.

Since realized emission intensity P_s is not directly verifiable by the regulator, the tax is assessed on expected emission intensity $\bar{P}_s$, so that firm s pays $\tilde{\tau}\, \bar{P}_s$ per unit of output. Each investor therefore receives a rebate of $\lambda_R\, \tilde{\tau}\, \bar{E}$, where $\bar{E} = \sum_s \bar{P}_s D_s K_s$ is expected aggregate emissions.[2]

The carbon tax $\tilde{\tau}$ is a random variable centered on an average rate τ^* which will later be determined through political bargaining:

$$\tilde{\tau} = \tau^* + \epsilon_\tau, \qquad \epsilon_\tau \sim \mathcal{N}(0, \sigma_\tau^2),$$

with ϵ_τ independent of all productivity and emission shocks. The shock ϵ_τ represents implementation uncertainty, electoral surprises, or last-minute political shifts.

Timing. At $t = 0$: the expected tax τ^* is determined (Section 3 endogenizes this); firms receive capital K_s and hire labor L_s; investors choose portfolios. At $t = 1$: shocks $(A_s, P_s, \epsilon_\tau)$ are realized; firms produce, pay dividends, and pay taxes; the government rebates revenue; investors consume.

[2]This is consistent with carbon pricing systems such as the EU ETS, where permits are allocated against expected emission intensities rather than realized emissions. In Appendix **??**, we show that an alternative tax structure—where realized emission deviations are assessed at the base rate τ^* while political noise applies to expected intensities—preserves the same equilibrium formulas while also capturing the interaction between the tax rate and emission uncertainty.

2.2 Agent Problems

Investor problem. Given an outcome of the political process, prices, and emission outcome, a type-i investor chooses a portfolio $\theta_i = (\theta_{ig}, \theta_{ib})'$ to solve:

$$\max_{\theta_i} \quad \mathbb{E}[u_i(c_i; E)] \qquad \text{s.t.} \qquad c_i = y_i\big[r_f + \theta_i'(r - r_f\mathbf{1})\big] + \lambda_R\,\tilde{\tau}\,\bar{E}. \tag{2.1}$$

Worker problem. A worker in sector s has constant marginal disutility w_s of supplying labor. In a competitive labor market, the equilibrium wage equals w_s, so workers are indifferent over the quantity supplied and earn zero surplus. Labor supply is therefore perfectly elastic at wage w_s.

Firm problem. Each firm in sector s takes its capital K_s (determined by investors' portfolio choices and market clearing), the wage w_s, and the distribution of $\tilde{\tau}$ as given, and chooses labor L_s to maximize expected payout to equity holders:

$$\max_{L_s \geq 0} \quad \mathbb{E}\big[(A_s - \tilde{\tau}\,\bar{P}_s)\,K_s^{\alpha}\,L_s^{1-\alpha}\big] - w_s\,L_s - \tfrac{\phi_s}{2}\,K_s^2. \tag{2.2}$$

2.3 Equilibrium Definition and Characterization

Definition 1 (Competitive Equilibrium)**.** Given an expected carbon tax τ^* and political risk σ_τ^2, a competitive equilibrium consists of prices (r_f, r), portfolios $(\theta_i)_{i\in\{G,B\}}$, capital allocations (K_g, K_b), and labor choices (L_g, L_b) such that: (i) investors solve (2.1); (ii) firms solve (2.2); (iii) workers supply labor elastically at wage w_s; and (iv) equity and bond markets clear:

$$K_s = \bar{y}\,\bar{\theta}_s \quad \forall s, \qquad \sum_s \bar{\theta}_s = 1,$$

where $\bar{\theta} = \frac{1}{\bar{y}}\int y_i\theta_i\,di$ is the market portfolio and $\bar{y} = \frac{1}{2}(y_G + y_B)$ is the average endowment.

The competitive equilibrium admits a closed-form characterization. Since the tax $\tilde{\tau}\bar{P}_s$ is linear in the normal shock ϵ_τ and independent of the emission shock ϵ_{P_s}, sector returns are exactly jointly normal (see Lemma 2). The detailed derivation is in Appendix B.

Theorem 1 (Market Equilibrium)**.** *Any competitive equilibrium with positive investment in both sectors ($\bar{\theta}_s > 0$) is characterized as follows. Define each sector's*

effective production scale

$$D_s := \left[\frac{(1-\alpha)(\bar{A}_s - \tau^* \bar{P}_s)}{w_s}\right]^{(1-\alpha)/\alpha}, \tag{2.3}$$

which is decreasing in τ^* *and* $\bar{P}_s$*. Then:*

(i) Labor, capital, and expected returns:

$$L_s = D_s^{1/(1-\alpha)} K_s, \qquad K_s = \bar{y}\bar{\theta}_s, \qquad \bar{r}_s = \bar{R}_s - \tfrac{\phi_s}{2} K_s, \qquad \bar{R}_s := \alpha(\bar{A}_s - \tau^* \bar{P}_s)\, D_s.$$

(ii) Market portfolio and risk-free rate:

$$\bar{\theta} = \widetilde{M}^{-1}(\bar{R} - r_f \mathbf{1}), \tag{2.4}$$

$$r_f = \frac{\mathbf{1}' \widetilde{M}^{-1} \bar{R} - 1}{\mathbf{1}' \widetilde{M}^{-1} \mathbf{1}}, \tag{2.5}$$

where the climate pricing matrix *is:*

$$\widetilde{M} = \underbrace{\gamma \bar{y}\, \Sigma_r + \tfrac{\bar{y}}{2}\, \Phi}_{M\ (standard)} - \underbrace{\bar{y}\bar{\eta}\; diag(D_s^2 \sigma_{AP,s})}_{emission\ hedging} - \underbrace{\gamma \lambda_R \bar{y} \sigma_\tau^2\, (D\bar{P})(D\bar{P})'}_{political\ risk\ hedging}.$$

Here $D = diag(D_g, D_b)$*,* $\Phi = diag(\phi_g, \phi_b)$*,* $\bar{P} = (\bar{P}_g, \bar{P}_b)'$*, and:*

- $\Sigma_r = D(V + \sigma_\tau^2 \bar{P}\bar{P}')D$ *is the return covariance, where* $V = \begin{pmatrix} v_g & \sigma_A^2 \\ \sigma_A^2 & v_b \end{pmatrix}$ *with* $v_s := \sigma_A^2 + \sigma_{A,s}^2$*;*
- $\bar{\eta} = \frac{1}{2}(\eta_G + \eta_B)$ *is the average emission disutility;*
- $\sigma_{AP,s} := Cov(\epsilon_{A_s}, \epsilon_{P_s})$ *governs the direction of the emission hedge for sector* s *(the "dirty boom" covariance).*

The climate pricing matrix $\widetilde{M}$ extends the standard pricing matrix $M = \gamma\bar{y}\Sigma_r + \frac{\bar{y}}{2}\Phi$ with two corrections: an emission hedging term (reflecting investors' desire to hedge emission risk) and a political risk hedging term (reflecting the partial offset between return losses and rebate gains from tax shocks). Positive definiteness of $\widetilde{M}$ ensures that equilibria with positive investment exist, which holds when the hedging corrections are small relative to M.

Corollary 1 (Portfolio Holdings)**.** *In the equilibrium of Theorem 1, the optimal portfolio for investor i decomposes as:*

$$\theta_i = \underbrace{\frac{1}{\gamma\, y_i}\,\Sigma_r^{-1}(\bar{r} - r_f\mathbf{1})}_{\theta_i^{MV}:\ \textit{mean-variance}} + \underbrace{\frac{\eta_i}{\gamma\, y_i}\,\Sigma_r^{-1}\,\Sigma_{rE}}_{\theta_i^{E}:\ \textit{emission hedging}} + \underbrace{\frac{\lambda_R \bar{E}\,\sigma_\tau^2}{y_i}\,\Sigma_r^{-1}\,D\bar{P}}_{\theta_i^{\tau}:\ \textit{political risk hedging}} \tag{2.6}$$

where $\Sigma_{rE} := \big(Cov(r_s, E)\big)_s \in \mathbb{R}^2$ and $\bar{E} = \bar{y}(D\bar{P})'\bar{\theta}$ is expected aggregate emissions.

With equal wealth $y_G = y_B = y$, the portfolio difference between green and brown investors is:

$$\theta_G - \theta_B = \frac{\eta_G - \eta_B}{\gamma\, y}\,\Sigma_r^{-1}\,\Sigma_{rE}. \tag{2.7}$$

Remark 1 (Portfolio Heterogeneity)**.** The mean-variance demand θ_i^{MV} and the political risk hedging demand θ_i^{τ} are identical across investor types (up to wealth scaling). Only the emission hedging demand θ_i^{E} is type-specific, through η_i. Without emission risk ($\Sigma_{rE} = 0$), all investors hold identical portfolios regardless of environmental preferences—the only channel for expressing green preferences is the political process. With correlated emission risk, the individual hedging demand θ_i^{E} depends on η_i, which differs across types. The *average* emission hedging (through $\bar{\eta}$ in $\widetilde{M}$) affects equilibrium prices and capital allocation, while the individual deviations from the average are zero-sum trades between green and brown investors that change risk exposures without altering $\bar{\theta}$.

Remark 2 (Nesting Special Cases)**.** Setting $\phi_s = 0$, $\sigma_\tau^2 = 0$, $\sigma_{AP,s} = 0$, and $\sigma_A^2 = 0$ eliminates adjustment costs, political risk, and emission-productivity correlation, leaving only sector-specific productivity risk. The climate pricing matrix reduces to $\widetilde{M} = \gamma\bar{y}\Sigma_r$ with $\Sigma_r = D\,\mathrm{diag}(\sigma_{A,s}^2)\,D$, and the equilibrium takes the standard CARA-normal form (modified by adjustment costs when $\phi_s > 0$) with heterogeneous emission preferences as the only departure from the classical CAPM. When $\lambda_R = 1$, the tax rebate perfectly hedges political risk, and as we show in Corollary 2(b), political uncertainty has no effect on capital allocation.

Discussion of Environmental Externality. The fundamental environmental problem in this economy is that some households care about aggregate carbon emissions, E, but no households believe they personally impact aggregate emissions. This means that agents need to coordinate to reduce pollution. Ultimately, coordi-

nation is complicated by the heterogeneity across the population in how much they care about reducing emissions. As Remark 1 shows, without emission risk the only channel for expressing green preferences is the political process, underscoring the central role of political bargaining in Section 3.

2.4 A Climate Policy CAPM

The equilibrium pricing equation can be rewritten in a form that generalizes the CAPM to incorporate climate-specific state variables. We call this the *Climate Policy CAPM*.

Proposition 1 (Three-Factor Asset Pricing)**.** *In the equilibrium of Theorem 1, the expected excess return on sector s satisfies*

$$\bar{r}_s - r_f = \underbrace{\gamma\bar{y}\, Cov(r_s, r_M)}_{\textit{market risk premium}} - \underbrace{\bar{\eta}\, Cov(r_s, E)}_{\textit{emission risk premium}} + \underbrace{\gamma\lambda_R\bar{E}\, Cov(r_s, \epsilon_\tau)}_{\textit{political risk premium}}, \quad (2.8)$$

where $r_M = \bar{\theta}' r$ is the market portfolio return. Equivalently, in beta form:

$$\bar{r}_s - r_f = \lambda_M\, \beta_{s,M} + \lambda_E\, \beta_{s,E} + \lambda_\tau\, \beta_{s,\tau},$$

where:

Factor	***Beta***	***Price of risk***
Market	$\beta_{s,M} = \frac{Cov(r_s, r_M)}{Var(r_M)}$	$\lambda_M = \gamma\bar{y}\, Var(r_M)$
Emission	$\beta_{s,E} = \frac{Cov(r_s, E)}{Var(E)}$	$\lambda_E = -\bar{\eta}\, Var(E)$
Political	$\beta_{s,\tau} = \frac{Cov(r_s, \epsilon_\tau)}{\sigma_\tau^2} = -D_s\bar{P}_s$	$\lambda_\tau = \gamma\lambda_R\bar{E}\,\sigma_\tau^2$

Interpretation. The market factor is the standard CAPM channel, now incorporating all three sources of return variation. The emission factor parallels the "climate beta" in recent empirical work; our model microfounds its price of risk $\lambda_E = -\bar{\eta}\,\mathrm{Var}(E)$ as a function of preferences ($\bar{\eta}$) and emission variance. Since $\bar{\eta} > 0$, $\lambda_E < 0$: assets with positive emission beta earn *lower* expected returns because they hedge high-emission states.

Political risk affects required returns through two channels that work in opposite directions. First, political uncertainty raises total return variance (since σ_τ^2 enters Σ_r), increasing the market risk premium. Second, the tax rebate provides a partial

hedge: when the tax rises, returns fall but the rebate $\lambda_R \tilde{\tau} \bar{E}$ increases, partially offsetting the loss. This hedge is reflected in the third factor, where $\lambda_\tau > 0$ and $\beta_{s,\tau} = -D_s \bar{P}_s < 0$ produce a negative contribution that *reduces* required returns (equivalently, $\text{Cov}(r_s, \epsilon_\tau) < 0$). When $\lambda_R = 1$ (full rebate), the two channels exactly offset and political risk has no net effect on capital allocation (Corollary 2(b)). When $\lambda_R < 1$, the variance channel dominates *for capital allocation*: political uncertainty shifts investment from brown to green (Corollary 2(b)(iii)), with the brown sector bearing greater exposure to the unhedged variance.

Remark 3 (Policy Instruments and Risk Premia)**.** The risk premium—whether "brown" or "green"—depends on the policy instrument creating the uncertainty. The policy in our model is a carbon tax that punishes polluters; uncertainty in this regime makes brown assets risky, generating a brown premium. If, instead, climate policy were implemented via a green subsidy (e.g., a per-unit subsidy on green output), policy uncertainty would be concentrated in the green sector. In that case, our model would predict a *green premium*, as investors would demand compensation for bearing the risk of fluctuating green subsidies.

Comparative statics with respect to (τ^*, σ_τ^2) and the risk parameters $(\sigma_{AP,s}, \sigma_{P_s}^2, \sigma_A^2)$ are collected in Corollaries 2–3 in Appendix B.

2.5 Asymmetric Adjustment Costs

The adjustment cost parameters (ϕ_g, ϕ_b) allow for asymmetry in the ease of scaling investment. If the green sector represents newer technologies with more flexible capital deployment, one natural specification is $\phi_g < \phi_b$; if green technologies require costly specialized infrastructure, then $\phi_g > \phi_b$.

Proposition 2 (Effect of Asymmetric Adjustment Costs)**.** *In any equilibrium with positive investment in both sectors, a decrease in ϕ_g (lower green adjustment cost) has the following effects:*

1. *Capital allocation: K_g increases and K_b decreases; the green share $K_g/(K_g + K_b)$ rises.*

2. *Expected returns: If, in addition, the variance of the cross-sector return differential exceeds the combined hedging corrections—i.e.,*

$$\gamma \bar{y} \begin{pmatrix} 1 \\ -1 \end{pmatrix}' \Sigma_r \begin{pmatrix} 1 \\ -1 \end{pmatrix} > \bar{y}\bar{\eta}(D_g^2 \sigma_{AP,g} + D_b^2 \sigma_{AP,b}) + \gamma \lambda_R \bar{y} \sigma_\tau^2 (D_g \bar{P}_g - D_b \bar{P}_b)^2, \quad (2.9)$$

then both $\bar{r}_g$ and $\bar{r}_b$ increase, with $\bar{r}_g$ increasing more.

3. *Return spread: Under the same condition* (2.9)*, the brown-green expected return spread $\bar{r}_b - \bar{r}_g$ narrows.*

Interpretation. Lower green adjustment costs channel more capital into the green sector without incurring steep marginal costs. While capital flows to green, both sectors' expected returns rise: the green sector benefits directly from the lower cost penalty, and the brown sector benefits from reduced capital (less DRS pressure). The green sector gains more, narrowing the spread. Thus, asymmetric scalability affects the *level* of returns but does not by itself generate a "green premium" (lower green expected returns). In our model, the green premium is driven primarily by the emission hedging channel (Corollary 1), which compresses green expected returns through investors' willingness to accept lower returns on assets that hedge emission risk.

3 Domestic Political Bargaining

The preceding analysis characterized the market equilibrium for a given carbon tax process (τ^*, σ_τ^2). Section 2 established that, absent emission intensity risk ($\sigma_{AP,s} = 0$), all investors hold identical risky portfolios regardless of their environmental preferences (Remark 1). This motivates our shift to the political arena: if investors cannot express their heterogeneous environmental values through portfolio choice, they must do so through collective policy choice.

We endogenize the carbon tax by modeling its determination as the outcome of a political process between the two groups of capital owners.[3] Throughout, we work under the simplification $\sigma_{P_s} = 0$ and $\sigma_{AP,s} = 0$ (no stochastic emission intensity), so that all investors hold identical portfolios in the market equilibrium of Section 2. We maintain general productivity risk ($\sigma_A, \sigma_{A,s}$), adjustment costs (ϕ_s), rebate fraction (λ_R), and capital share (α). The fully general case is treated in Appendix **??**.

[3]This approach isolates the political conflict that arises purely from the investor channel. We abstract from the political influence of labor by assuming a perfectly elastic supply at a fixed wage for each sector, rendering laborers indifferent to the policy and focusing the analysis on the preferences of capital owners.

3.1 Weighted Welfare Maximization

We model the political process as a weighted welfare maximization. The two investor blocs—green (G) and brown (B)—negotiate over the expected carbon tax rate $\tau \geq 0$. The politically determined tax rate τ^* solves

$$\tau^* = \arg\max_{\tau \geq 0} \quad \beta\, W_G(\tau) + (1-\beta)\, W_B(\tau), \tag{3.1}$$

where $\beta \in [0,1]$ is the political weight of the green bloc and $W_i(\tau)$ is the indirect welfare (certainty-equivalent) of type i when the expected tax is τ.[4]

When bargaining, each bloc internalizes the full general equilibrium impact of τ on market outcomes. The indirect welfare $W_i(\tau)$ is the certainty-equivalent utility of type i evaluated at the competitive equilibrium induced by expected tax τ and political risk σ_τ^2:

$$W_i(\tau) := \mathbb{E}[c_i^*(\tau)] - \frac{\gamma}{2}\,\mathrm{Var}(c_i^*(\tau)) - \frac{\eta_i}{\gamma}\,\bar{E}(\tau), \tag{3.2}$$

where $c_i^*(\tau)$ is the equilibrium consumption of type i and $\bar{E}(\tau) = \bar{y}(D\bar{P})'\bar{\theta}$ is aggregate benchmark emissions.[5]

3.2 Welfare Structure and Ideal Points

Lemma 1 (Welfare Decomposition)**.** *Under equal wealth ($y_G = y_B = \bar{y}$) and no emission intensity risk ($\sigma_{P_s} = 0$, $\sigma_{AP,s} = 0$), the indirect welfare decomposes as*

$$W_i(\tau) = \Psi(\tau) - \frac{\eta_i}{\gamma}\,\bar{E}(\tau),$$

where $\Psi(\tau)$ is a common financial welfare function, identical across investor types:

$$\Psi(\tau) = \bar{y}\, r_f + \frac{\gamma \bar{y}^2}{2}\,\bar{\theta}'\Sigma_r\,\bar{\theta} + \lambda_R\,\tau\,\bar{E} - \frac{\gamma}{2}\,\lambda_R^2\,\sigma_\tau^2\,\bar{E}^2. \tag{3.3}$$

[4]This objective has a natural microfoundation through probabilistic voting. In such models, two office-seeking candidates choose policy platforms, and the equilibrium platform maximizes a weighted sum of group utilities, with weights determined by the density of swing voters in each group. The parameter β captures the green bloc's effective political influence: a larger β means the green bloc is more pivotal.

[5]The full CARA-normal certainty equivalent of $u_i = -\exp(-\gamma c_i + \eta_i E)$ is $\mathbb{E}[c_i] - \frac{\gamma}{2}\mathrm{Var}(c_i) - \frac{\eta_i}{\gamma}\bar{E} - \frac{\eta_i^2}{2\gamma}\mathrm{Var}(E) + \eta_i \mathrm{Cov}(c_i, E)$. When $\sigma_{P_s} = 0$, emissions are deterministic ($E = \bar{E}$), so the last two terms vanish and (3.2) is exact.

Proof. See Appendix B. □

The decomposition implies that the only source of political disagreement is the emission disutility term. Both types benefit equally from financial returns and tax revenue; they differ solely in how much they value a reduction in $\bar{E}$. Differentiating:

$$W_i'(\tau) = \Psi'(\tau) - \frac{\eta_i}{\gamma}\,\bar{E}'(\tau), \tag{3.4}$$

so the marginal welfare difference between the two types is

$$W_G'(\tau) - W_B'(\tau) = -\frac{\eta_G - \eta_B}{\gamma}\,\bar{E}'(\tau). \tag{3.5}$$

Whenever the tax reduces emissions ($\bar{E}'(\tau) < 0$), the green bloc has a strictly higher marginal value from increasing the tax. Under the assumption $D_b^0 \bar{P}_b > D_g^0 \bar{P}_g$ from Section 2, we have $\bar{E}'(\tau) < 0$ for $\tau \in [0, \bar{\tau})$ with $\bar{\tau} := \min_s \bar{A}_s/\bar{P}_s$ (Lemma 4 in the appendix).

We now state the ordering of ideal policies. Define the ideal tax rate for type i as $\tau_i^* := \arg\max_{\tau \geq 0} W_i(\tau)$.

Proposition 3 (Ordering of Ideal Tax Rates)**.** *Suppose $W_i(\tau)$ is strictly quasi-concave in τ for each $i \in \{G, B\}$. Then*

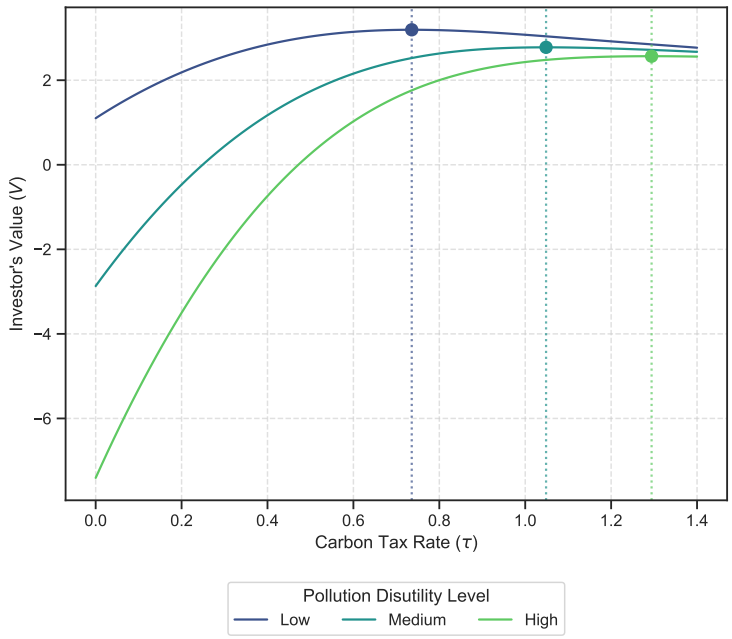
$$\tau_G^* \geq \tau_B^* \geq 0.$$

The inequality is strict ($\tau_G^* > \tau_B^*$) whenever τ_B^* is interior or $W_G'(0) > 0$. A sufficient condition for the latter is that the green bloc's environmental benefit exceeds the financial cost of the first unit of tax: $\frac{\eta_G}{\gamma}(-\bar{E}'(0)) > -\Psi'(0)$.

Proof. See Appendix B. The argument proceeds from the single-crossing condition (3.5): since $\eta_G > \eta_B$ and $\bar{E}'(\tau) < 0$ (Lemma 4), we have $W_G'(\tau) > W_B'(\tau)$ for all τ. When τ_B^* is interior, $W_B'(\tau_B^*) = 0$ implies $W_G'(\tau_B^*) > 0$, so $\tau_G^* > \tau_B^*$ by quasi-concavity. At a corner ($\tau_B^* = 0$), the strict inequality $\tau_G^* > 0$ requires $W_G'(0) > 0$. □

Equation (3.5) captures the political conflict. The carbon tax generates three competing effects: (i) portfolio distortion—the tax reduces expected returns $\bar{R}_s$ (since $\partial \bar{R}_s/\partial \tau = -D_s \bar{P}_s < 0$), lowering financial welfare; (ii) revenue rebate—tax revenue $\lambda_R \tau \bar{E}$ provides income, following a Laffer curve as the tax base $\bar{E}$ shrinks; (iii) environmental benefit—reduced emissions yield utility gain $-\frac{\eta_i}{\gamma}\bar{E}$, proportional to η_i. The first two effects enter $\Psi'(\tau)$ and are identical for both types. Only

Figure 1: Welfare and Ideal Policies under Different Pollution Preferences



the environmental benefit is type-specific. Green investors tolerate greater financial distortion because they value the environmental improvement more highly.

Under the quadratic approximation introduced below, W_i is concave in τ if and only if $-\Psi''(0) + \frac{\eta_i}{\gamma}\bar{E}''(0) > 0$. The financial curvature $-\Psi''(0) > 0$ captures the accelerating deadweight loss from tax distortion (Appendix B.2). When $\bar{E}''(0) \geq 0$—which holds whenever the intensive curvature dominates, as is the case for $\alpha \leq 1/2$—both terms are positive and concavity is automatic. When $\bar{E}''(0) < 0$, concavity requires $-\Psi''(0) > \frac{\eta_G}{\gamma}|\bar{E}''(0)|$, i.e., the financial curvature dominates the emission curvature. We maintain this as a regularity condition.

3.3 Closed-Form Equilibrium Tax Rate

Computing τ^* in closed form requires resolving the nonlinear dependence of all equilibrium objects $(D_s, \bar{\theta}, r_f, \Sigma_r, \bar{E})$ on τ. These are rational functions of τ of high degree (through $D_s \propto (\bar{A}_s - \tau\bar{P}_s)^{(1-\alpha)/\alpha}$), precluding exact closed forms. We therefore approximate the welfare functions by their second-order Taylor expansions around the no-policy benchmark $\tau = 0$, which is accurate when τ^* is small relative to $\bar{\tau}$.

Under the quadratic approximation, $W_i(\tau) \approx W_i(0) + W_i'(0)\tau + \frac{1}{2}W_i''(0)\tau^2$, where from (3.4):

$$W_i'(0) = \Psi'(0) - \frac{\eta_i}{\gamma}\bar{E}'(0), \tag{3.6}$$
$$W_i''(0) = \Psi''(0) - \frac{\eta_i}{\gamma}\bar{E}''(0).$$

The derivatives $\Psi'(0)$, $\bar{E}'(0)$, $\Psi''(0)$, $\bar{E}''(0)$ are evaluated at the no-policy benchmark ($\tau = 0$, $\sigma_\tau = 0$), holding σ_τ^2 fixed; the fixed-point condition on σ_τ^2 is closed separately in Section 3.5. Explicit computation of these derivatives from the equilibrium system is in Appendix B.2.

Theorem 2 (Closed-Form Political Equilibrium)**.** *Under the quadratic approximation, suppose $-W_i''(0) > 0$ for $i \in \{G, B\}$ (welfare is concave for both types). If the interior condition holds—*

$$W_{\bar{\eta}_\beta}'(0) := \Psi'(0) - \frac{\bar{\eta}_\beta}{\gamma}\bar{E}'(0) > 0, \tag{3.7}$$

where $\bar{\eta}_\beta := \beta\eta_G + (1-\beta)\eta_B$ is the politically-weighted average emission sensitivity—then the political equilibrium tax rate is

$$\tau^* = \frac{\Psi'(0) - \frac{\bar{\eta}_\beta}{\gamma}\bar{E}'(0)}{-\Psi''(0) + \frac{\bar{\eta}_\beta}{\gamma}\bar{E}''(0)}. \tag{3.8}$$

When (3.7) fails, the constrained optimum is $\tau^* = 0$. The ideal tax rate for each investor type is

$$\tau_i^* = \frac{\Psi'(0) - \frac{\eta_i}{\gamma}\bar{E}'(0)}{-\Psi''(0) + \frac{\eta_i}{\gamma}\bar{E}''(0)}. \tag{3.9}$$

Proof. Setting $W_i'(\tau_i^*) = 0$ and using the linear approximation $W_i'(\tau) \approx W_i'(0) + W_i''(0)\tau$ gives $\tau_i^* = -W_i'(0)/W_i''(0) = W_i'(0)/(-W_i''(0))$, which is (3.9). For the political equilibrium, the FOC of (3.1) is $\beta W_G'(\tau^*) + (1-\beta)W_B'(\tau^*) = 0$. Since $W_i'(\tau)$ is affine in η_i (equation (3.4)), the weighted FOC reduces to $W_{\bar{\eta}_\beta}'(\tau^*) = 0$, where $W_\eta(\tau) := \Psi(\tau) - \frac{\eta}{\gamma}\bar{E}(\tau)$. Concavity of this function requires $-W_{\bar{\eta}_\beta}''(0) > 0$. Since $\bar{\eta}_\beta \in [\eta_B, \eta_G]$ and $-W_i''(0) > 0$ for $i \in \{G, B\}$, linearity in η guarantees

$-W''_{\bar{\eta}_\beta}(0) > 0$. The interior condition (3.7) ensures $\tau^* > 0$; we maintain $\tau^* < \bar{\tau}$ so the quadratic approximation is applied within the economically relevant range. □

The equilibrium tax τ^* coincides with the ideal policy of a "representative investor" whose emission sensitivity is $\bar{\eta}_\beta$. This is a consequence of the linearity of $W'_i(\tau)$ in η_i: the weighted FOC is equivalent to a single-agent FOC with the averaged preference. When $\beta = 1/2$ (equal weights), $\bar{\eta}_\beta = \bar{\eta} = \frac{1}{2}(\eta_G + \eta_B)$, which equals the average emission disutility appearing in the climate pricing matrix $\widetilde{M}$ of Section 2. As β increases toward 1, the effective preference shifts toward η_G and τ^* rises.

The numerator of (3.8) is the marginal benefit of introducing the tax: the financial benefit $\Psi'(0)$ (rebate revenue net of portfolio distortion) plus the environmental benefit $-\frac{\bar{\eta}_\beta}{\gamma}\bar{E}'(0)$ (weighted emission abatement, positive since $\bar{E}'(0) < 0$). The denominator is the rate at which marginal costs increase: the financial curvature $-\Psi''(0)$ (how rapidly the portfolio distortion grows) plus the emission curvature correction $\frac{\bar{\eta}_\beta}{\gamma}\bar{E}''(0)$ (how the marginal environmental benefit changes). The marginal financial benefit $\Psi'(0)$ itself decomposes as

$$\Psi'(0) = \underbrace{\Psi'_{\text{fin}}(0)}_{\substack{\text{portfolio distortion} \\ \text{cost } (\leq 0)}} + \underbrace{\lambda_R \bar{E}^0}_{\substack{\text{rebate} \\ \text{benefit } (>0)}},$$

where Ψ_{fin} denotes the financial welfare excluding the direct rebate term, and $\bar{E}^0 := \bar{E}(0)$ is baseline emissions. When the rebate fraction λ_R is large enough, $\Psi'(0) > 0$: there is a "revenue motive" for taxation beyond environmental considerations.

3.4 Political Risk

The bargaining process also determines the political risk σ_τ^2. The variance of the realized tax around τ^* reflects the intensity of political conflict—contested elections, legislative uncertainty, and policy reversals all contribute to higher variance when the two blocs disagree. We model this as:

$$\sigma_\tau^2 = \kappa^2 \, (\tau_G^* - \tau_B^*)^2, \qquad \kappa > 0,$$

where κ governs the transmission from political disagreement to implementation noise. This is part of the environment: the mapping from disagreement to uncertainty is a primitive of the political system, not a result of optimization.

From (3.9), the ideal-point gap is

$$\tau_G^* - \tau_B^* = \frac{(\eta_G - \eta_B)/\gamma \cdot \left[\bar{E}'(0)\,\Psi''(0) - \bar{E}''(0)\,\Psi'(0)\right]}{\left[-\Psi''(0) + \frac{\eta_G}{\gamma}\bar{E}''(0)\right]\left[-\Psi''(0) + \frac{\eta_B}{\gamma}\bar{E}''(0)\right]}.$$

Political risk is increasing in preference heterogeneity $(\eta_G - \eta_B)$ and policy effectiveness $(-\bar{E}'(0))$. It does not depend on the bargaining weight β: political power determines the expected tax level but not the uncertainty around it.

3.5 The Political–Market Equilibrium

We now combine the market equilibrium of Section 2 with the political equilibrium to close the model.

Definition 2 (Political–Market Equilibrium)**.** A political–market equilibrium consists of a tax process (τ^*, σ_τ^2) and a competitive market equilibrium $(r_f, r, \bar{\theta}, K, L)$ such that:

(i) The market equilibrium satisfies Theorem 1 given (τ^*, σ_τ^2);

(ii) The expected tax τ^* solves the weighted welfare problem (3.1), where W_i is evaluated at the market equilibrium in (i);

(iii) The political risk satisfies $\sigma_\tau^2 = \kappa^2(\tau_G^* - \tau_B^*)^2$, where τ_i^* maximizes W_i evaluated at the same market equilibrium.

The equilibrium is a fixed point: the market depends on (τ^*, σ_τ^2), which in turn are determined by welfare in that market equilibrium. Specifically, given (τ^*, σ_τ^2), the equilibrium capital and emissions are determined by Theorem 2.1:

$$K_s = \bar{y}\,\bar{\theta}_s(\tau^*, \sigma_\tau^2), \qquad \bar{E} = \bar{y}\,(D\bar{P})'\bar{\theta}(\tau^*, \sigma_\tau^2),$$

where D_s, $\widetilde{M}$, $\bar{R}_s$, r_f, and $\bar{\theta}$ are all evaluated at (τ^*, σ_τ^2). The change from the no-policy benchmark decomposes into a *tax-level channel* and a *political-risk channel*:

$$K_g - K_g^0 = \underbrace{K_g(\tau^*, 0) - K_g^0}_{\Delta K_g^\tau} + \underbrace{K_g(\tau^*, \sigma_\tau^2) - K_g(\tau^*, 0)}_{\Delta K_g^\sigma},$$
$$\bar{E} - \bar{E}^0 = \underbrace{\bar{E}(\tau^*, 0) - \bar{E}^0}_{\Delta \bar{E}^\tau} + \underbrace{\bar{E}(\tau^*, \sigma_\tau^2) - \bar{E}(\tau^*, 0)}_{\Delta \bar{E}^\sigma},$$

where ΔK_g^τ and $\Delta \bar{E}^\tau$ capture the effect of the average tax rate at zero political risk, and ΔK_g^σ and $\Delta \bar{E}^\sigma$ capture the additional effect of policy noise. The tax-level channel reduces emissions ($\Delta \bar{E}^\tau < 0$) for any interior $\tau^* \in (0, \bar{\tau})$, since $\bar{E}'(\tau) < 0$ on this interval (Lemma 4). The political-risk channel shifts capital toward green ($\Delta K_g^\sigma \geq 0$) and further reduces emissions ($\Delta \bar{E}^\sigma \leq 0$), with equality when $\lambda_R = 1$: full revenue rebate perfectly hedges political risk, shutting down the risk channel. The two channels have distinct welfare implications: the tax channel corrects the externality by pricing pollution, while the risk channel adds uncompensated uncertainty.

Proposition 4 (Comparative Statics)**.** *Under the conditions of Definition 2 and the quadratic welfare approximation,*

(a) Bargaining power β. *An increase in the green bloc's political weight raises the equilibrium tax τ^*; Political risk σ_τ^2 is independent of β. Through the tax-level channel, K_g increases and $\bar{E}$ decreases.*

(b) Preference heterogeneity ($\eta_G - \eta_B$), holding $\bar{\eta} = \frac{1}{2}(\eta_G + \eta_B)$ fixed. *The expected tax increases when $\beta > 1/2$, is unchanged when $\beta = 1/2$, and decreases when $\beta < 1/2$. Political risk always increases, regardless of β. When $\beta \geq 1/2$, or when risk channel dominates, capital shifts toward green and emission drops.*

(c) Risk aversion γ. *Holding $\Psi'(0)$, $\Psi''(0)$, $\bar{E}'(0)$, $\bar{E}''(0)$ constant, an increase in γ reduces both the equilibrium tax and political disagreement.*

The proof is in Appendix B.1.

Efficiency of the political–market equilibrium. The political–market equilibrium is generically Pareto inefficient. Comparing the decentralized outcome to a social planner who directly controls capital allocation and consumption transfers, two frictions emerge. First, the carbon tax distorts labor demand below its efficient level, since firms perceive their marginal revenue product as $\bar{A}_s - \tau \bar{P}_s$ rather than the social productivity $\bar{A}_s$. Second, the political bargain is over an incomplete contract: agents negotiate a single tax rate τ but cannot commit to portfolio choices or lump-sum transfers between groups. A planner can independently optimize capital allocation and distribution, but the decentralized process collapses this two-dimensional problem into a one-dimensional path parameterized by τ. The result is systematic under-investment in green capital relative to the Pareto frontier—even a "brown-friendly" planner would allocate more to the green sector than the market out-

come. We develop this argument in Appendix A; a formal quantitative comparison is straightforward given the closed-form equilibrium but omitted for brevity.

4 Coordinated Green Investors: A Solution?

The preceding analysis assumed that all investors are atomistic: each takes market prices and aggregate emissions as given when choosing portfolios. In practice, institutional investors often coordinate their actions, particularly around environmental, social, and governance (ESG) objectives. Green investment funds, pension funds with climate mandates, and shareholder activism groups explicitly internalize the aggregate impact of their investment decisions on environmental outcomes.

In this section, we explore whether such coordination can remedy the inefficiency identified in Section 3 (see also Appendix A). We begin by formalizing an environment where green investors act as a coordinated bloc that internalizes its effect on aggregate emissions, while brown investors remain atomistic. We then characterize the resulting market equilibrium, showing how coordinated green investors optimally tilt their portfolios toward green assets. Despite this internalization of the environmental externality, we demonstrate that the equilibrium remains generically Pareto inefficient. The source of this inefficiency is market incompleteness: portfolio choice bundles together two distinct functions—capital allocation and risk bearing—that a social planner would optimize separately. We identify the missing markets (state-contingent transfers) that would restore efficiency, clarifying the precise friction that coordination alone cannot overcome.

4.1 Setup: Coordinated Green Investors

and the political-market equilibrium from Section 3, with one key modification: green investors act as a single coordinated bloc that internalizes the effect of its portfolio choice on aggregate emissions E, while brown investors remain atomistic. Let Y_i denote the total wealth of type-i investors, with $Y_G + Y_B = \bar{y}$. For simplicity, we also make the following assumptions:

(i) *No emission intensity risk:* $\sigma_{P_s} = 0$ and $\sigma_{AP,s} = 0$ for each sector s. This is the same restriction as in Section 3. In the atomistic benchmark, all investors hold identical risky portfolios.

(ii) *Constant returns to scale:* $\phi_s = 0$ for each sector. Under CRS, the return distribution $(\bar{r}, \Sigma_r)$ is invariant to the scale of production.

(iii) *Equal wealth:* $y_G = y_B = \bar{y}$.

Political risk is retained: the realized carbon tax is $\tilde{\tau} = \tau^* + \epsilon_\tau$ with $\epsilon_\tau \sim \mathcal{N}(0, \sigma_\tau^2)$, where (τ^*, σ_τ^2) are determined by the political-market equilibrium of Section 3.

Atomistic Brown Investors. Brown investors remain price-takers who do not internalize their effect on emissions. Since they take E as given, their first-order condition yields the standard mean-variance portfolio from Theorem 1:

$$\theta_B = \frac{1}{\gamma \bar{y}} \Sigma_r^{-1} (\bar{r} - r_f \mathbf{1}). \tag{4.1}$$

Market Clearing. The bond market is in zero net supply. Total risky investment equals total wealth: $\frac{\bar{y}}{2}(\mathbf{1}'\theta_G + \mathbf{1}'\theta_B) = \bar{y}$, so $\mathbf{1}'\theta_G + \mathbf{1}'\theta_B = 2$. Substituting (4.1), the equilibrium risk-free rate becomes a function of the green portfolio:

$$r_f(\theta_G) = \frac{\mathbf{1}'\Sigma_r^{-1}\bar{r} + \gamma\bar{y}\,\mathbf{1}'\theta_G - 2\gamma\bar{y}}{\mathbf{1}'\Sigma_r^{-1}\mathbf{1}}, \tag{4.2}$$

with derivative $\frac{\partial r_f}{\partial \theta_G} = \frac{\gamma\bar{y}}{\mathbf{1}'\Sigma_r^{-1}\mathbf{1}}\,\mathbf{1}$. As the green bloc increases its risky asset holdings, the risk-free rate rises to clear the bond market.

Coordinated Green Investors. Each green investor has consumption

$$c_G = \bar{y}\big[r_f + (r - r_f\mathbf{1})'\theta_G\big] + \lambda_R \tilde{\tau}\,\bar{E},$$

where the tax rebate $\lambda_R \tilde{\tau} \bar{E}$ is stochastic through the political shock ϵ_τ. The green bloc maximizes each investor's certainty-equivalent welfare while internalizing its effect on aggregate emissions:

$$\max_{\theta_G} \quad \mathbb{E}[c_G] - \frac{\gamma}{2}\mathrm{Var}(c_G) - \frac{\eta_G}{\gamma}\,\bar{E}(\theta_G).$$

The green bloc understands that its choice of θ_G affects: (i) the risk-free rate $r_f(\theta_G)$ through market clearing; (ii) brown investors' portfolio $\theta_B(r_f)$ through their optimality condition (4.1); and (iii) capital allocation $K(\theta_G) = \frac{\bar{y}}{2}(\theta_G + \theta_B(r_f(\theta_G)))$ and thus emissions $\bar{E}(\theta_G) = v^{P\prime} K(\theta_G)$.

4.2 Equilibrium with Green Coordination

We now characterize equilibrium when green investors coordinate. Throughout, we impose short-selling constraints $\theta_{ik} \geq 0$ for all investors. In the atomistic benchmark of Section 2, these constraints do not bind: all investors hold positive weights in both sectors. With coordination, however, green investors may wish to short brown assets to reduce emissions, making the constraint potentially binding.

The first-order condition for the coordinated green bloc takes the form (derived in Appendix B.3):

$$\underbrace{\bar{y}(\bar{r} - r_f \mathbf{1}) - \gamma \bar{y}^2 \Sigma_r \theta_G + \frac{\gamma \bar{y}^2 (1 - \mathbf{1}'\theta_G)}{\mathbf{1}'\Sigma_r^{-1}\mathbf{1}} \mathbf{1}}_{\text{mean-variance + interest rate internalization}} + \underbrace{\mu_E^{\text{eff}} \cdot \frac{\bar{y}}{2} \Delta v^P}_{\text{emission internalization}} + \underbrace{\gamma \lambda_R \sigma_\tau^2 \bar{y} \bar{E}\, v^P}_{\text{political risk hedging}} = 0, \tag{4.3}$$

where:

- $\mu_E^{\text{eff}} := \lambda_R \tau^* - \frac{\eta_G}{\gamma} + \gamma \lambda_R \sigma_\tau^2 H_G$ is the *effective emission price* for the green bloc, combining the expected tax rebate, the environmental disutility, and a political risk hedging correction;
- $H_G := \bar{y}\, \theta_G' v^P - \lambda_R \bar{E}$ measures the green bloc's *hedging gap*: the difference between its portfolio's exposure to the tax shock (through returns) and its rebate exposure;
- $\Delta v^P := v^P - \bar{v}^P \mathbf{1}$ is the demeaned emission intensity vector, with $\bar{v}^P := \frac{\mathbf{1}'\Sigma_r^{-1} v^P}{\mathbf{1}'\Sigma_r^{-1}\mathbf{1}}$.

Compared to the atomistic benchmark, three additional terms appear beyond the standard mean-variance demand. The first group (interest rate internalization) is identical in structure to the case without political risk; the green bloc internalizes its effect on r_f. The emission internalization term captures the environmental motive: the green bloc tilts its portfolio in the direction Δv^P, which shifts capital from high-emission brown to low-emission green. Since brown has higher emission intensity ($D_b P_b > D_g P_g$, the standing assumption from Section 3), $\Delta v_g^P < 0 < \Delta v_b^P$, so when $\mu_E^{\text{eff}} < 0$ (environmental disutility exceeds the rebate benefit), the emission term pushes toward green.

The political risk term captures a hedging motive: because the tax rebate $\lambda_R \tilde{\tau} \bar{E}$ is stochastic, and returns are negatively correlated with the tax shock (higher $\tilde{\tau}$ reduces returns), holding more risky assets partially hedges the rebate risk. Since

brown assets are more exposed to the tax shock ($D_b P_b > D_g P_g$), this hedging motive pushes the green bloc *toward brown*, counteracting the environmental tilt.

Theorem 3 (Equilibrium with Green Coordination)**.** *Consider the coordinated green investor problem with short-selling constraints $\theta_{G,s} \geq 0$.*

(i) Portfolio scale. If both constraints are slack, the coordinated green portfolio satisfies:

$$\mathbf{1}'\theta_G^* = 1 + \frac{\lambda_R \sigma_\tau^2 \bar{E} \cdot \mathbf{1}'\Sigma_r^{-1} v^P}{3\bar{y}}.$$

When $\sigma_\tau^2 = 0$, this reduces to $\mathbf{1}'\theta_G^* = 1$: coordinated green investors neither borrow nor lend. With political risk ($\sigma_\tau^2 > 0$) and a positive rebate ($\lambda_R > 0$), the correction is generically nonzero; its sign is determined by $\mathbf{1}'\Sigma_r^{-1} v^P$, which is positive when political risk is moderate (Appendix B.4). In this case, $\mathbf{1}'\theta_G^* > 1$: green investors slightly leverage into risky assets to hedge rebate risk.

(ii) Portfolio tilt. Green investors tilt toward the green sector: $\theta_{G,g}^* > \theta_{B,g}$ and $\theta_{G,b}^* < \theta_{B,b}$. The magnitude of the tilt is determined by the effective emission price μ_E^{eff} and the emission differential $D_b P_b - D_g P_g$, and is dampened by political risk through the hedging counterforce.

(iii) Corner solution. There exists a threshold $\bar{\eta}_G > 0$ such that if $\eta_G > \bar{\eta}_G$, the constraint $\theta_{G,b} \geq 0$ binds. In this case, $\theta_{G,b}^* = 0$ and green investors borrow to finance additional green investment. Relative to the case without political risk, the threshold $\bar{\eta}_G$ is higher: political risk raises the cost of extreme green tilts (through increased consumption variance), making the corner solution harder to reach.

In all cases, green and brown investors hold different portfolios, in contrast to the portfolio homogeneity of the atomistic equilibrium (Remark 2.3).

The proof is in Appendix B.4.

The theorem reveals that coordinated green investors have two margins: scale (total risky investment $\mathbf{1}'\theta_G$) and tilt (the ratio $\theta_{G,g}/\theta_{G,b}$). Only tilts affect emissions: borrowing to invest proportionally in both sectors triggers an offsetting response from brown investors through the risk-free rate. Moreover, borrowing is costly: it raises r_f, which the green bloc internalizes. The preferred strategy is

therefore to tilt the portfolio to go long green and short brown, while maintaining $\mathbf{1}'\theta_G = 1$. When short-selling constraints prevent this, green investors resort to borrowing, accepting the interest rate cost to achieve emission reductions.

4.3 Efficiency of the Coordinated Equilibrium

Does coordination by green investors achieve an efficient allocation? When brown investors have no environmental preferences ($\eta_B \equiv 0$), the only agents who care about emissions are the coordinated green bloc, who fully internalize their effect. Standard intuition suggests this should yield efficiency. We show this intuition is wrong: the coordinated equilibrium remains generically Pareto inefficient. However, we also identify the missing market that would restore efficiency.

The source of inefficiency is not a failure to internalize externalities, but rather market incompleteness. Portfolio choice serves two functions: it determines capital allocation (and hence emissions), and it determines each investor's exposure to consumption risk. A social planner can optimize these objectives separately. The market cannot—the only way to redirect capital toward green production is to hold more green equity, which mechanically increases exposure to green-sector risk.

Efficient Risk Sharing. Consider a planner who chooses capital allocation $K = (K_g, K_b)'$ and state-contingent consumption (c_G, c_B) to maximize weighted utilities $\omega_G \mathbb{E}[u_G(c_G; \bar{E})] + \omega_B \mathbb{E}[u_B(c_B)]$ subject to feasibility $c_G + c_B = C$, where $C = r'K$ is aggregate consumption. With CARA utility and common risk aversion γ, the planner's optimal consumption allocation satisfies (Appendix B.7):

$$c_i = a_i + \frac{1}{2}(C - \bar{C}) \quad \text{for each } i \in \{G, B\}, \tag{4.4}$$

where a_i is mean consumption for group i and $\bar{C} = \mathbb{E}[C]$. Both groups bear *equal* consumption risk, regardless of environmental preferences or Pareto weights; these affect only the level a_i, not the loading on aggregate risk.

(i). Market equilibrium risk allocation. Define each investor's dollar risk exposure as $\xi_i := \bar{y}\theta_i \in \mathbb{R}^2$, the dollar position in each sector's equity. From (4.1), brown investors' exposure is $\xi_B = \frac{1}{\gamma}\Sigma_r^{-1}(\bar{r} - r_f\mathbf{1})$. For coordinated green investors,

rearranging (4.3) yields (Appendix B.5):

$$\xi_G - \xi_B = \frac{1}{2\gamma} \Sigma_r^{-1} \left(\lambda_R \tau - \frac{\eta_G}{\gamma} \right) \Delta v^P. \tag{4.5}$$

This wedge is zero in two cases: (i) when $\eta_G = \gamma \lambda_R \tau$, so the marginal environmental benefit exactly equals the marginal tax rebate benefit; or (ii) when $D_g P_g = D_b P_b$, meaning both sectors have identical emission intensity and tilting the portfolio has no effect on emissions. In either case, green investors have no incentive to deviate from the standard mean-variance portfolio, and $\theta_G = \theta_B$.

Outside these knife-edge cases, the wedge is nonzero: green investors tilt toward green assets to reduce emissions, bearing more green-sector risk than efficient sharing prescribes.

Proposition 5 (Inefficiency of Coordinated Equilibrium)**.** *Suppose $\eta_B = 0$, both investor types share risk aversion γ, and $y_G = y_B = \bar{y}$. If $\eta_G \neq \gamma \lambda_R \tau$ and $D_g P_g \neq D_b P_b$, then the coordinated equilibrium is Pareto inefficient.*

The proof (Appendix B.6) follows from the observation that efficient risk sharing requires $\xi_G = \xi_B$ (equation (4.4)), while equation (4.5) implies $\xi_G \neq \xi_B$ under the stated conditions. The planner could improve welfare by maintaining the same capital allocation while redistributing consumption risk to equalize exposures.

Restoring Efficiency with Complete Markets. The inefficiency arises because portfolio choice bundles capital allocation and risk bearing into a single instrument. The missing market is a state-contingent transfer that reallocates consumption risk independently of capital flows.

Consider augmenting the financial market with a *return swap*: a contract paying $(r_g - r_b)$ per dollar of position at $t = 1$. Investors can now separately choose their equity holdings θ_i (which determine capital allocation) and their swap positions ϕ_i (which adjust risk exposure). This separates the two functions that portfolio choice previously bundled.

Theorem 4 (Efficiency with Complete Markets)**.** *Suppose $\eta_B = 0$, both investor types share risk aversion γ, and green investors coordinate. If a return swap market is available, any competitive equilibrium is Pareto efficient.*

The proof (Appendix B.8) shows that with swaps, both groups achieve equal risk exposure ($\xi_G = \xi_B$) while green investors still tilt their equity portfolios to reduce

emissions. The equilibrium allocation solves the social planner's problem for welfare weights determined by initial endowments and equilibrium prices.

Proposition 5 and Theorem 4 together identify the two frictions that cause inefficiency:

1. *Uninternalized externalities.* When investors are atomistic, no one accounts for their effect on aggregate emissions. Coordination by green investors removes this friction. With $\eta_B = 0$, brown investors impose no uninternalized externality.

2. *Incomplete risk markets.* Without swaps, portfolio choice bundles capital allocation and risk bearing. Green investors who tilt portfolios for environmental reasons must accept suboptimal risk exposure. Adding swap markets separates these functions: equity holdings determine capital flows, swaps determine risk exposure.

When both frictions are removed, the First Welfare Theorem applies. Efficiency obtains without carbon taxation when green investors fully coordinate and complete risk markets are feasible. Carbon taxation becomes necessary when investors cannot fully coordinate or when risk-sharing markets are incomplete, conditions that likely hold in practice.

5 Extensions

Our framework provides a foundation for exploring several critical dimensions of the interplay between climate policy, finance, and politics. We briefly outline two promising extensions below.

5.1 The Role of Labor and Distributional Conflict

Our main analysis abstracts from the welfare of labor by assuming a perfectly elastic supply at a fixed wage, rendering laborers indifferent to the policy outcome. This simplification focuses the core conflict on capital owners with heterogeneous environmental preferences.

A straightforward and important extension is to consider the distributional consequences for labor. One could, for instance, assume the exogenous wage w_s represents a binding minimum wage that is above labor's true reservation wage (e.g., unemployment benefits). In this case, aggregate employment $L = \sum_s L_s$ becomes

a direct, albeit imperfect, proxy for labor's welfare. Since our model predicts that labor demand L_s is decreasing in the tax rate τ, any positive carbon tax would create an explicit conflict between environmental goals and labor interests. The policy bargain would then involve a trade-off not only between green and brown investors but also between capital owners and workers.

A richer model would endogenize wage determination and include labor as a third strategic actor in the political bargain. The resulting policy would depend on the coalition formation between labor and the two capital-owning groups. For instance, labor in the brown sector might align with brown investors to resist high taxes to preserve jobs; if not properly compensated, laborers with green preference may side with brown investors due to high marginal cost of losing jobs. Alternatively, labor could be induced to support green policy if a sufficient portion of the carbon tax revenue is used to fund programs like targeted wage subsidies or job retraining. Analyzing these dynamics would provide deeper insights into the political coalitions that can either enable or block effective climate action.

5.2 International Capital Mobility and Financial Leakage

Our single-country framework extends naturally to a multi-country setting with integrated financial markets. In such an extension, each country j sets its own carbon tax τ_j through domestic political bargaining, while investors can allocate capital to green and brown firms in any jurisdiction.

The key new force is *financial leakage*. Because portfolio homogeneity carries over to the international setting, all investors hold the same global market portfolio regardless of nationality or environmental preference. A unilateral increase in one country's carbon tax lowers expected returns on its domestic assets, causing the global market portfolio to rebalance toward the untaxed jurisdiction. This capital reallocation stimulates foreign production and pollution, potentially offsetting (or even reversing) the environmental benefits of the domestic tax.

Financial leakage also compounds the domestic political friction. When countries set policy sequentially, each country's bargainers must anticipate how the other will respond. A higher domestic tax may provoke strategic substitution abroad, as the foreign country lowers its own tax to attract the inflowing capital. Anticipating this policy neutralization weakens the domestic green bloc's bargaining position, pushing the equilibrium toward inefficiently low taxes. At the same time, open capital markets may ease domestic agreement on a carbon tax by allowing brown

investors to evade its cost through foreign investment, but this comes at the expense of higher global emissions.

These dynamics suggest that a carbon tax alone may be insufficient when capital is internationally mobile. Capital controls or cross-border investment frictions could serve as a second-best policy by segmenting financial markets and restoring the potency of domestic environmental policy, though at the cost of sacrificing the benefits of financial integration such as efficient global capital allocation and risk sharing. We leave the formal analysis of optimal international policy coordination for future work.

6 Conclusion

This paper develops a general equilibrium framework that nests endogenous policy determination, political and environmental risk, and standard production risk into a tractable structure with closed-form equilibrium. Investors with heterogeneous preferences over carbon emissions negotiate a carbon tax, which feeds back into portfolio decisions, real investment, and asset prices. The equilibrium pricing equation takes the form of a Climate Policy CAPM: a three-factor model with market, emission, and political risk factors that nests the standard CAPM as a special case.

We endogenize the carbon tax through political bargaining between green and brown investor blocs. The political process operates through two separable channels: a tax-level channel, through which the bargained tax shifts capital from brown to green, and a political-risk channel, through which disagreement generates policy uncertainty that further tilts portfolios toward the less exposed sector. The resulting equilibrium is generically Pareto inefficient, because the political bargain is over an incomplete contract: agents negotiate a tax rate but cannot commit to portfolio choices or transfers between groups, leading to systematic under-investment in green capital relative to the Pareto frontier. Coordination among green investors does not resolve this inefficiency; the residual friction is market incompleteness, whereby portfolio choice bundles capital allocation and risk bearing into a single instrument. Adding a return swap market that separates these two functions restores efficiency.

The tractability of the framework makes it a natural platform for extensions. Incorporating workers into the political process introduces distributional conflict between capital and labor. An international extension with integrated capital markets introduces financial leakage, whereby unilateral taxes trigger capital reallocation to

untaxed jurisdictions. More broadly, the framework provides a unified setting for studying how financial regulation, green industrial policy, and international coordination interact with the political process to shape environmental outcomes.

References

Acharya, V. V., Giglio, S., Pastore, S., Stroebel, J., Tan, Z., and Yong, T. (2025). Climate transition risks and the energy sector. Technical report, CESifo.

Battaglini, M. and Harstad, B. (2016). Participation and duration of environmental agreements. *Journal of Political Economy*, 124(1):160–204.

Battaglini, M. and Harstad, B. (2020). The political economy of weak treaties. *Journal of Political Economy*, 128(2):544–590.

Bolton, P. and Kacperczyk, M. (2021). Do investors care about carbon risk? *Journal of financial economics*, 142(2):517–549.

Cassidy Jr, W. M. (2023). Elections have consequences: The impact of political agency on climate policy and asset prices. Technical report.

Engle, R. F., Giglio, S., Kelly, B., Lee, H., and Stroebel, J. (2020). Hedging climate change news. *The Review of Financial Studies*, 33(3):1184–1216.

Flammer, C. (2021). Corporate green bonds. *Journal of financial economics*, 142(2):499–516.

Giglio, S., Kelly, B., and Stroebel, J. (2021). Climate finance. *Annual review of financial economics*, 13(1):15–36.

Goldstein, I., Kopytov, A., Shen, L., and Xiang, H. (2022). On esg investing: Heterogeneous preferences, information, and asset prices. Technical report, National Bureau of Economic Research.

Green, D. and Roth, B. N. (2025). The allocation of socially responsible capital. *The Journal of Finance*, 80(2):755–781.

Hambel, C. and Van Der Ploeg, F. (2025). Policy transition risk, carbon premiums, and asset prices. *Journal of Monetary Economics*, page 103780.

Heinkel, R., Kraus, A., and Zechner, J. (2001). The effect of green investment on corporate behavior. *Journal of financial and quantitative analysis*, 36(4):431–449.

Hsu, P.-H., Li, K., and Tsou, C.-Y. (2023). The pollution premium. *The Journal of Finance*, 78(3):1343–1392.

Nordhaus, W. (2015). Climate clubs: Overcoming free-riding in international climate policy. *American Economic Review*, 105(4):1339–1370.

Oates, W. E. and Portney, P. R. (2003). The political economy of environmental policy. In *Handbook of environmental economics*, volume 1, pages 325–354. Elsevier.

Pástor, L., Stambaugh, R. F., and Taylor, L. A. (2021). Sustainable investing in equilibrium. *Journal of financial economics*, 142(2):550–571.

Pástor, L., Stambaugh, R. F., and Taylor, L. A. (2022). Dissecting green returns. *Journal of financial economics*, 146(2):403–424.

Pedersen, L. H., Fitzgibbons, S., and Pomorski, L. (2021). Responsible investing: The esg-efficient frontier. *Journal of financial economics*, 142(2):572–597.

A (In)efficiency of the Decentralized Equilibrium

This appendix discusses the inefficiency of the political–market equilibrium defined in Section 3. We argue that the decentralized equilibrium is generically Pareto inefficient by comparing it to a social planner's benchmark.

A.1 Planner's benchmark

Consider a benevolent social planner who directly controls the allocation of capital to each sector, (K_g, K_b), and the distribution of final consumption across investor types, (c_G, c_B). The planner maximizes a weighted sum of certainty-equivalent utilities, $\omega_G W_G + \omega_B W_B$ with $\omega_G + \omega_B = 1$, subject to the economy's resource and goods-market feasibility constraints. Varying the Pareto weight $\omega_G \in (0, 1)$ traces out the Pareto frontier.

The planner differs from the decentralized economy in two respects. First, the planner chooses labor to maximize physical net output $\mathbb{E}[f_s(K_s, L_s) - w_s L_s]$, so labor is not distorted by the carbon tax. Second, the planner chooses capital allocation and consumption transfers independently, whereas the decentralized economy determines both through a single instrument τ.

A.2 Sources of inefficiency

The political–market equilibrium of Definition 2 is generically Pareto inefficient. The inefficiency stems from a dimensionality mismatch. The planner navigates a two-dimensional choice space—capital allocation (K_g, K_b) and distributional outcomes (via ω_G)—while the political process collapses both into a single dimension parameterized by τ. Two frictions drive the wedge:

Friction 1: Tax-induced production distortion. A firm facing tax $\tau > 0$ perceives its marginal revenue product of labor as $\bar{A}_s - \tau \bar{P}_s < \bar{A}_s$, employing less labor than a planner who internalizes that tax payments are transfers, not real costs. This wedge reduces output for any given capital stock.

Friction 2: Incomplete political contracting. The political process bargains over a tax rate but cannot commit to portfolio allocations or lump-sum transfers between investor blocs. A planner can implement the efficient capital allocation and compensate the brown group via transfers; the political process cannot decouple

allocation from distribution. The tax must simultaneously price the externality and indirectly steer capital, but atomistic investors respond only to the financial risk-return profile conditional on τ. The result is under-investment in green capital: even a planner with $\omega_G \to 0$ would allocate more to the green sector than the decentralized equilibrium, because the planner can compensate the brown group directly rather than through a lower tax rate.

Together, these frictions imply that the decentralized equilibrium produces less total output (Friction 1) and allocates it inefficiently across sectors (Friction 2), placing both investor types strictly inside the Pareto frontier.

B Proofs

Proof of Theorem 1

Lemma 2 (Return Distribution)**.** *Sector returns satisfy* $r \sim \mathcal{N}(\bar{r}, \Sigma_r)$ *with:*

$$\bar{r}_s = \bar{R}_s - \tfrac{\phi_s}{2} K_s, \qquad \bar{R}_s := \alpha(\bar{A}_s - \tau^* \bar{P}_s)\, D_s,$$
$$(\Sigma_r)_{ss} = D_s^2(v_s + \bar{P}_s^2\, \sigma_\tau^2), \qquad (\Sigma_r)_{gb} = D_g D_b(\sigma_A^2 + \bar{P}_g \bar{P}_b\, \sigma_\tau^2),$$

where $v_s = \sigma_A^2 + \sigma_{A,s}^2$*. In matrix form,*

$$\Sigma_r = D(V + \sigma_\tau^2\, \bar{P}\bar{P}')D, \qquad V = \begin{pmatrix} v_g & \sigma_A^2 \\ \sigma_A^2 & v_b \end{pmatrix}.$$

Proof. Substituting the optimal labor (2.3) into the firm's dividend $\pi_s = (A_s - \tilde{\tau}\bar{P}_s)K_s^\alpha L_s^{1-\alpha} - w_s L_s - \frac{\phi_s}{2}K_s^2$, we obtain $K_s^\alpha L_s^{1-\alpha} = D_s\, K_s$ and $w_s L_s = (1-\alpha)(\bar{A}_s - \tau^* \bar{P}_s) D_s\, K_s$, so the return $r_s = \pi_s / K_s$ is

$$r_s = (A_s - \tilde{\tau}\bar{P}_s)\, D_s - (1-\alpha)(\bar{A}_s - \tau^* \bar{P}_s)\, D_s - \tfrac{\phi_s}{2} K_s.$$

Expanding around means:

$$r_s - \bar{r}_s = D_s(\epsilon_A + \epsilon_{A_s} - \bar{P}_s\, \epsilon_\tau). \tag{B.1}$$

This is a linear function of the jointly normal shocks $(\epsilon_A, \epsilon_{A_s}, \epsilon_\tau)$, so returns are

exactly jointly normal. Computing the variance:

$$\mathrm{Var}(r_s) = D_s^2[\mathrm{Var}(\epsilon_A) + \mathrm{Var}(\epsilon_{A_s}) + \bar{P}_s^2\sigma_\tau^2] = D_s^2(v_s + \bar{P}_s^2\sigma_\tau^2).$$

For the cross-sector covariance, using independence of $(\epsilon_{A_g}, \epsilon_{P_g})$ and $(\epsilon_{A_b}, \epsilon_{P_b})$:

$$\mathrm{Cov}(r_g, r_b) = D_g D_b[\mathrm{Var}(\epsilon_A) + \bar{P}_g\bar{P}_b\sigma_\tau^2] = D_g D_b(\sigma_A^2 + \bar{P}_g\bar{P}_b\sigma_\tau^2).$$

□

Lemma 3 (Covariance Structure)**.** *The return-emission and return-political covariances are:*

$$\begin{aligned} Cov(r_s, E) &= D_s^2 K_s\, \sigma_{AP,s}, \\ Cov(r_s, \epsilon_\tau) &= -D_s\bar{P}_s\, \sigma_\tau^2. \end{aligned}$$

Expected aggregate emissions are $\bar{E} = \sum_s \bar{P}_s D_s K_s = \bar{y}(D\bar{P})'\bar{\theta}$.

Proof. Substituting optimal labor, realized emissions are $E = \sum_s P_s D_s K_s$, so $E - \bar{E} = \sum_s \epsilon_{P_s} D_s K_s$. From (B.1) and using $\epsilon_A \perp \epsilon_{P_j}$ for all j, $\epsilon_{A_s} \perp \epsilon_{P_j}$ for $j \neq s$, and $\epsilon_\tau \perp \epsilon_{P_s}$:

$$\mathrm{Cov}(r_s, E) = D_s\,\mathrm{Cov}(\epsilon_{A_s},\, \epsilon_{P_s} D_s K_s) = D_s^2 K_s\, \sigma_{AP,s}.$$

For the political risk covariance, only the $-D_s\bar{P}_s\epsilon_\tau$ term in (B.1) is correlated with ϵ_τ:

$$\mathrm{Cov}(r_s, \epsilon_\tau) = -D_s\bar{P}_s\sigma_\tau^2.$$

□

Proof of Theorem 1. The tax rebate is $\lambda_R\tilde{\tau}\bar{E} = \lambda_R(\tau^* + \epsilon_\tau)\bar{E} = \lambda_R\tau^*\bar{E} + \lambda_R\epsilon_\tau\bar{E}$. Substituting into the CARA exponent $Z_i := -\gamma c_i + \eta_i E$:

$$Z_i = -\gamma y_i[r_f + \theta_i'(r - r_f\mathbf{1})] + \eta_i E - \gamma\lambda_R\bar{E}\,\epsilon_\tau + \text{const.} \tag{B.2}$$

Since (r, E, ϵ_τ) are jointly normal, Z_i is normal and maximizing $\mathbb{E}[u_i]$ is equivalent

to minimizing $\mathbb{E}[Z_i] + \frac{1}{2}\text{Var}(Z_i)$. The FOC with respect to θ_i yields

$$\theta_i = \frac{1}{\gamma y_i}\Sigma_r^{-1}\big[(\bar{r} - r_f\mathbf{1}) + \eta_i\Sigma_{rE} + \gamma\lambda_R\bar{E}\,\sigma_\tau^2\,D\bar{P}\big],$$

where we used $\Sigma_{r\tau} = \text{Cov}(r, \epsilon_\tau) = -\sigma_\tau^2 D\bar{P}$ from Lemma 3.

Aggregating across investors with $\bar{\eta} = \frac{1}{2}(\eta_G + \eta_B)$ and multiplying both sides by $\gamma\Sigma_r$:

$$\gamma\bar{y}\Sigma_r\bar{\theta} = (\bar{r} - r_f\mathbf{1}) + \bar{\eta}\Sigma_{rE} + \gamma\lambda_R\bar{E}\,\sigma_\tau^2\,D\bar{P}.$$

The endogenous quantities are affine in $\bar{\theta}$: $\bar{r} = \bar{R} - \frac{\bar{y}}{2}\Phi\bar{\theta}$ (from Theorem 1(i) and $K_s = \bar{y}\bar{\theta}_s$), $\Sigma_{rE} = \bar{y}\text{diag}(D_s^2\sigma_{AP,s})\bar{\theta}$ (from Lemma 3), $\bar{E}\,D\bar{P} = \bar{y}(D\bar{P})(D\bar{P})'\bar{\theta}$. Substituting and collecting terms:

$$\big[\gamma\bar{y}\Sigma_r + \tfrac{\bar{y}}{2}\Phi - \bar{y}\bar{\eta}\text{diag}(D_s^2\sigma_{AP,s}) - \gamma\lambda_R\bar{y}\sigma_\tau^2(D\bar{P})(D\bar{P})'\big]\bar{\theta} = \bar{R} - r_f\mathbf{1}.$$

Identifying the bracket as $\widetilde{M}$ gives (2.4). Bond market clearing $\mathbf{1}'\bar{\theta} = 1$ yields (2.5). □

Proof of Corollary 1

Proof. From (B.2):

$$\text{Var}(Z_i) = \gamma^2 y_i^2\theta_i'\Sigma_r\theta_i + \eta_i^2\text{Var}(E) - 2\gamma y_i\eta_i\theta_i'\Sigma_{rE} + \gamma^2\lambda_R^2\bar{E}^2\sigma_\tau^2 + 2\gamma^2\lambda_R y_i\bar{E}\,\theta_i'\Sigma_{r\tau},$$

where we used $\text{Cov}(E, \epsilon_\tau) = 0$. The FOC $\partial[\mathbb{E}[Z_i] + \frac{1}{2}\text{Var}(Z_i)]/\partial\theta_i = 0$ gives:

$$-\gamma y_i(\bar{r} - r_f\mathbf{1}) + \gamma^2 y_i^2\Sigma_r\theta_i - \gamma y_i\eta_i\Sigma_{rE} + \gamma^2\lambda_R y_i\bar{E}\,\Sigma_{r\tau} = 0.$$

Solving for θ_i and substituting $\Sigma_{r\tau} = -\sigma_\tau^2 D\bar{P}$ yields (2.6). The portfolio difference (2.7) follows since only η_i is type-specific. □

Proof of Proposition 1

Proof. From the aggregate FOC (proof of Theorem 1):

$$\bar{r} - r_f\mathbf{1} = \gamma\bar{y}\Sigma_r\bar{\theta} - \bar{\eta}\Sigma_{rE} + \gamma\lambda_R\bar{E}\,\Sigma_{r\tau}.$$

Element-by-element, $(\Sigma_r\bar{\theta})_s = \mathrm{Cov}(r_s, r_M)$, $(\Sigma_{rE})_s = \mathrm{Cov}(r_s, E)$, $(\Sigma_{r\tau})_s = \mathrm{Cov}(r_s, \epsilon_\tau)$. This gives (2.8). The beta form follows by dividing each covariance by the corresponding factor variance. □

Corollary 2 (Comparative Statics: Policy)**.**

(a) Tax level τ^*.

(i) Expected returns $\bar{R}_s$ are decreasing in τ^*, with the brown sector declining faster: $|\partial\bar{R}_b/\partial\tau^*| > |\partial\bar{R}_g/\partial\tau^*|$ when $D_b\bar{P}_b > D_g\bar{P}_g$.

(ii) Expected aggregate emissions $\bar{E}$ are decreasing in τ^*.

(b) Tax uncertainty σ_τ^2.

(i) The political risk premium $\lambda_\tau = \gamma\lambda_R\bar{E}\sigma_\tau^2$ is increasing in σ_τ^2.

(ii) Return variance $Var(r_s)$ is increasing in σ_τ^2, with the brown sector more affected when $D_b\bar{P}_b > D_g\bar{P}_g$.

(iii) When $\lambda_R < 1$, higher σ_τ^2 shifts capital from brown to green ($d\bar{\theta}_g/d\sigma_\tau^2 > 0$) and widens the expected return spread $\bar{r}_b - \bar{r}_g$. When $\lambda_R = 1$, political uncertainty has no effect on capital allocation.

Proof of Corollary 2

Proof. Part (a). (i) $\bar{R}_s = \alpha\mu_s D_s$ where $\mu_s := \bar{A}_s - \tau^*\bar{P}_s$. Logarithmic differentiation of $D_s = [(1-\alpha)\mu_s/w_s]^{(1-\alpha)/\alpha}$ gives $\partial D_s/\partial\tau^* = -\frac{(1-\alpha)\bar{P}_s}{\alpha\mu_s}D_s$, so $\partial\bar{R}_s/\partial\tau^* = \alpha(-\bar{P}_s D_s + \mu_s\,\partial D_s/\partial\tau^*) = -\bar{P}_s D_s < 0$. The comparison follows from $D_b\bar{P}_b > D_g\bar{P}_g$. (ii) $\bar{E} = \sum_s \bar{P}_s D_s K_s$, so at fixed S: $\partial\bar{E}/\partial\tau^* = \sum_s \bar{P}_s(\partial D_s/\partial\tau^*)K_s < 0$ since $\partial D_s/\partial\tau^* < 0$.

Part (b). (i) $\lambda_\tau = \gamma\lambda_R\bar{E}\sigma_\tau^2$, so $\partial\lambda_\tau/\partial\sigma_\tau^2 = \gamma\lambda_R\bar{E}$. (ii) $\mathrm{Var}(r_s) = D_s^2(v_s + \bar{P}_s^2\sigma_\tau^2)$, so $\partial\mathrm{Var}(r_s)/\partial\sigma_\tau^2 = D_s^2\bar{P}_s^2$, with $D_b^2\bar{P}_b^2 > D_g^2\bar{P}_g^2$ when $D_b\bar{P}_b > D_g\bar{P}_g$.

(iii) Differentiate the equilibrium condition $\widetilde{M}\bar{\theta} = \bar{R} - r_f\mathbf{1}$ with respect to σ_τ^2, treating $\bar{E} = \bar{y}(D\bar{P})'\bar{\theta}$ as endogenous (evaluated at the current equilibrium). Since $\bar{R}$ is independent of σ_τ^2, and $\partial\widetilde{M}/\partial\sigma_\tau^2 = \gamma(1-\lambda_R)\bar{y}(D\bar{P})(D\bar{P})'$ (the variance contribution $\gamma\bar{y}(D\bar{P})(D\bar{P})'$ from Σ_r minus the hedging correction $\gamma\lambda_R\bar{y}(D\bar{P})(D\bar{P})'$):

$$\widetilde{M}\frac{d\bar{\theta}}{d\sigma_\tau^2} = -\gamma(1-\lambda_R)\bar{E}\,D\bar{P} - \frac{dr_f}{d\sigma_\tau^2}\mathbf{1}.$$

Since $\mathbf{1}'\frac{d\bar{\theta}}{d\sigma_\tau^2} = 0$, any zero-sum two-vector is proportional to $(1,-1)'$, so write $\frac{d\bar{\theta}}{d\sigma_\tau^2} = \beta_\sigma\binom{1}{-1}$. Premultiplying by $\binom{1}{-1}'$:

$$\beta_\sigma = \gamma(1-\lambda_R)\bar{E}\cdot\frac{D_b\bar{P}_b - D_g\bar{P}_g}{\binom{1}{-1}'\widetilde{M}\binom{1}{-1}}.$$

The denominator $\binom{1}{-1}'\widetilde{M}\binom{1}{-1} > 0$ by positive definiteness. When $\lambda_R < 1$ and $D_b\bar{P}_b > D_g\bar{P}_g$, the numerator is positive, so $\beta_\sigma > 0$, giving $d\bar{\theta}_g/d\sigma_\tau^2 > 0 > d\bar{\theta}_b/d\sigma_\tau^2$. When $\lambda_R = 1$, $\beta_\sigma = 0$.

For the return spread: $\bar{r}_b - \bar{r}_g = (\bar{R}_b - \bar{R}_g) - \frac{\bar{y}}{2}(\phi_b\bar{\theta}_b - \phi_g\bar{\theta}_g)$. Differentiating and using $d\bar{\theta}_b/d\sigma_\tau^2 = -\beta_\sigma$:

$$\frac{d(\bar{r}_b - \bar{r}_g)}{d\sigma_\tau^2} = -\frac{\bar{y}}{2}(-\phi_b\beta_\sigma - \phi_g\beta_\sigma) = \frac{\bar{y}(\phi_g+\phi_b)}{2}\beta_\sigma > 0.$$

□

Corollary 3 (Comparative Statics: Emission Risks)**.**

(a) Emission–productivity covariance $\sigma_{AP,s}$. *Higher $\sigma_{AP,s}$ increases $Cov(r_s, E)$ and amplifies portfolio heterogeneity between green and brown investors without affecting return variance v_s.*

(b) Emission intensity variance $\sigma_{P_s}^2$. *Since the tax is on expected emission intensity, $\sigma_{P_s}^2$ does not enter v_s or Σ_r. However, higher $\sigma_{P_s}^2$ increases $Var(E)$ and the magnitude of the emission risk premium $|\lambda_E|$.*

(c) Aggregate productivity variance σ_A^2. *Higher σ_A^2 increases return covariance across sectors without affecting emission hedging or political risk hedging channels.*

Proof of Corollary 3

Proof. Part (a). $\mathrm{Cov}(r_s, E) = D_s^2 K_s\,\sigma_{AP,s}$, so higher $\sigma_{AP,s}$ directly increases this covariance. At fixed capital allocation, portfolio heterogeneity $\theta_G - \theta_B = \frac{\eta_G - \eta_B}{\gamma y}\Sigma_r^{-1}\Sigma_{rE}$ increases in the component corresponding to sector s when $\sigma_{AP,s} > 0$. Since $v_s = \sigma_A^2 + \sigma_{A,s}^2$ does not depend on $\sigma_{AP,s}$, there is no indirect effect through Σ_r.

Part (b). Since the tax is on expected emission intensity, $\sigma_{P_s}^2$ does not enter v_s or Σ_r. However, $\mathrm{Var}(E) = \sum_s D_s^2 K_s^2 \sigma_{P_s}^2$, so higher $\sigma_{P_s}^2$ increases $|\lambda_E| = \bar{\eta}\,\mathrm{Var}(E)$.

Part (c). $\partial V/\partial\sigma_A^2 = \mathbf{1}\mathbf{1}'$, so $\partial\Sigma_r/\partial\sigma_A^2 = D(\mathbf{1}\mathbf{1}')D$, giving $\partial(\Sigma_r)_{ss}/\partial\sigma_A^2 = D_s^2$ and $\partial(\Sigma_r)_{gb}/\partial\sigma_A^2 = D_g D_b$. Since $\sigma_{AP,s}$, Σ_{rE}, and $\Sigma_{r\tau}$ do not depend on σ_A^2 (the aggregate shock ϵ_A is independent of ϵ_{P_s} and ϵ_τ), these channels are unaffected. □

Proof of Proposition 2

Proof. Write the equilibrium as $\widetilde{M}\bar{\theta} = \bar{R} - r_f\mathbf{1}$ with $\mathbf{1}'\bar{\theta} = 1$. Since $\partial\widetilde{M}/\partial\phi_g = \frac{\bar{y}}{2}e_g e_g'$ (where $e_g = (1,0)'$):

$$\widetilde{M}\frac{d\bar{\theta}}{d\phi_g} = -\frac{\bar{y}\bar{\theta}_g}{2}e_g - \frac{dr_f}{d\phi_g}\mathbf{1}.$$

Since $\mathbf{1}'\frac{d\bar{\theta}}{d\phi_g} = 0$, write $\frac{d\bar{\theta}}{d\phi_g} = \xi\binom{1}{-1}$. Premultiplying by $\binom{1}{-1}'$:

$$\xi = -\frac{\bar{y}\bar{\theta}_g/2}{\binom{1}{-1}'\widetilde{M}\binom{1}{-1}} < 0.$$

Hence $d\bar{\theta}_g/d\phi_g = \xi < 0$ and $d\bar{\theta}_b/d\phi_g = -\xi > 0$, establishing part (1).

For parts (2)–(3): $\bar{r}_s = \bar{R}_s - \frac{\phi_s\bar{y}}{2}\bar{\theta}_s$, where $\bar{R}_s$ is independent of ϕ_g. Let $\mu := \binom{1}{-1}'\widetilde{M}\binom{1}{-1} > 0$. Expanding μ:

$$\mu = \gamma\bar{y}\begin{pmatrix}1\\-1\end{pmatrix}'\Sigma_r\begin{pmatrix}1\\-1\end{pmatrix} + \tfrac{\bar{y}}{2}(\phi_g+\phi_b) - \bar{y}\bar{\eta}(D_g^2\sigma_{AP,g} + D_b^2\sigma_{AP,b}) - \gamma\lambda_R\bar{y}\sigma_\tau^2(D_g\bar{P}_g - D_b\bar{P}_b)^2.$$

For the green sector:

$$\frac{d\bar{r}_g}{d\phi_g} = -\frac{\bar{y}}{2}\bar{\theta}_g - \frac{\phi_g\bar{y}}{2}\xi = -\frac{\bar{y}\bar{\theta}_g}{2}\Big(1 - \frac{\phi_g\bar{y}}{2\mu}\Big).$$

Under condition (2.9), the risk compensation term exceeds the (possibly negative) hedging corrections, ensuring $\mu > \frac{\bar{y}}{2}(\phi_g+\phi_b) > \frac{\bar{y}}{2}\phi_g$. Hence the factor in parentheses is positive, so $d\bar{r}_g/d\phi_g < 0$: a decrease in ϕ_g increases $\bar{r}_g$. For the brown sector: $d\bar{r}_b/d\phi_g = \frac{\phi_b\bar{y}}{2}\xi < 0$, so a decrease in ϕ_g also increases $\bar{r}_b$.

To compare magnitudes, note $|d\bar{r}_g/d\phi_g| = \frac{\bar{y}\bar{\theta}_g}{2}(1 - \frac{\phi_g\bar{y}}{2\mu})$ and $|d\bar{r}_b/d\phi_g| = \frac{\phi_b\bar{y}^2\bar{\theta}_g}{4\mu}$. The former exceeds the latter if and only if $\mu > \frac{(\phi_g+\phi_b)\bar{y}}{2}$, which holds under (2.9). Hence $\bar{r}_g$ increases more than $\bar{r}_b$ when ϕ_g decreases.

For part (3): $\frac{d(\bar{r}_b-\bar{r}_g)}{d\phi_g} = \frac{\bar{y}\bar{\theta}_g}{2}(1 - \frac{(\phi_g+\phi_b)\bar{y}}{2\mu}) > 0$ under (2.9), so a decrease in ϕ_g narrows the spread. □

Proof of Lemma 1 (Welfare Decomposition) We derive the common financial welfare $\Psi(\tau)$ from the CARA-normal certainty equivalent.

Under $\sigma_{P_s} = 0$ and $\sigma_{AP,s} = 0$, emissions $E = \bar{E} = \sum_s \bar{P}_s D_s K_s$ are deterministic. All investors hold the same portfolio $\bar{\theta}$, and consumption is

$$c = \bar{y}\big[r_f + \bar{\theta}'(r - r_f\mathbf{1})\big] + \lambda_R \tilde{\tau}\,\bar{E},$$

where $\tilde{\tau} = \tau + \epsilon_\tau$. Using $\tilde{\tau}\bar{E} = \tau\bar{E} + \bar{E}\epsilon_\tau$:

$$c - \mathbb{E}[c] = \bar{y}\,\bar{\theta}'(r - \bar{r}) + \lambda_R\,\bar{E}\,\epsilon_\tau.$$

The variance is

$$\mathrm{Var}(c) = \bar{y}^2\bar{\theta}'\Sigma_r\bar{\theta} + \lambda_R^2\bar{E}^2\sigma_\tau^2 + 2\bar{y}\lambda_R\bar{E}\,\bar{\theta}'\mathrm{Cov}(r, \epsilon_\tau),$$

where $\mathrm{Cov}(r_s, \epsilon_\tau) = -D_s\bar{P}_s\sigma_\tau^2$ (from the return covariance structure in Section 2). Since $\bar{y}\bar{\theta}' D\bar{P} = \bar{E}$:

$$\mathrm{Var}(c) = \bar{y}^2\bar{\theta}'\Sigma_r\bar{\theta} - \lambda_R(2 - \lambda_R)\sigma_\tau^2\bar{E}^2.$$

Expected consumption: using the equilibrium relation $\bar{r} - r_f\mathbf{1} = \gamma\bar{y}(\Sigma_r - \lambda_R\sigma_\tau^2(D\bar{P})(D\bar{P})')\bar{\theta}$:

$$\begin{aligned}\mathbb{E}[c] &= \bar{y}r_f + \bar{y}\bar{\theta}'(\bar{r} - r_f\mathbf{1}) + \lambda_R\tau\bar{E} \\ &= \bar{y}r_f + \gamma\bar{y}^2\bar{\theta}'\Sigma_r\bar{\theta} - \gamma\lambda_R\sigma_\tau^2\bar{E}^2 + \lambda_R\tau\bar{E}.\end{aligned}$$

The certainty equivalent $\Psi = \mathbb{E}[c] - \frac{\gamma}{2}\mathrm{Var}(c)$ then gives

$$\begin{aligned}\Psi &= \bar{y}r_f + \frac{\gamma\bar{y}^2}{2}\bar{\theta}'\Sigma_r\bar{\theta} + \lambda_R\tau\bar{E} + \gamma\sigma_\tau^2\bar{E}^2\big[-\lambda_R + \tfrac{1}{2}\lambda_R(2 - \lambda_R)\big] \\ &= \bar{y}r_f + \frac{\gamma\bar{y}^2}{2}\bar{\theta}'\Sigma_r\bar{\theta} + \lambda_R\tau\bar{E} - \frac{\gamma}{2}\lambda_R^2\sigma_\tau^2\bar{E}^2,\end{aligned}$$

which is (3.3). □

Lemma (Emissions Decreasing in Tax)

Lemma 4. *Under the maintained condition $D_b(\tau)\bar{P}_b > D_g(\tau)\bar{P}_g$, the marginal tax reduces aggregate benchmark emissions: $\bar{E}'(\tau) < 0$ for all $\tau \in [0, \bar{\tau})$, where $\bar{\tau} := \min_s \bar{A}_s/\bar{P}_s$.*

Proof. Differentiating $\bar{E}(\tau) = \bar{y}\sum_s \bar{P}_s D_s(\tau)\bar{\theta}_s(\tau)$ and using $\sum_s \bar{\theta}'_s = 0$:

$$\bar{E}'(\tau) = \underbrace{\bar{y}\sum_s \bar{P}_s D'_s(\tau)\bar{\theta}_s(\tau)}_{\text{intensive } (<0)} + \underbrace{\bar{y}(D_g\bar{P}_g - D_b\bar{P}_b)\bar{\theta}'_g(\tau)}_{\text{extensive } (\leq 0)}.$$

The intensive margin is negative: $D'_s(\tau) = -\frac{(1-\alpha)\bar{P}_s}{\alpha(\bar{A}_s - \tau\bar{P}_s)} D_s(\tau) < 0$ for $\tau < \bar{\tau}$. For the extensive margin, the cross-sectional portfolio response solves

$$\bar{\theta}'_g(\tau) = \frac{g_g(\tau) - g_b(\tau)}{\binom{1}{-1}' \widetilde{M} \binom{1}{-1}},$$

where $\widetilde{M}$ is the climate pricing matrix of Section 2, $g_s = \bar{R}'_s(\tau) - (\dot{M}\bar{\theta})_s$, and $\bar{R}'_s(\tau) = -D_s(\tau)\bar{P}_s$. The denominator is positive. The direct return effect gives $g_g - g_b = (D_b\bar{P}_b - D_g\bar{P}_g) + O(\rho_s)$ with $\rho_s := (1-\alpha)\bar{P}_s/(\alpha\bar{A}_s)$, where the $O(\rho_s)$ correction arises from general-equilibrium repricing through $\dot{M}\bar{\theta}$. Under the maintained condition, $\bar{\theta}'_g > 0$: capital flows from brown to green, and the extensive margin reinforces the intensive margin. □

Proof of Proposition 3 (Ordering of Ideal Tax Rates) From (3.5) and Lemma 4: $W'_G(\tau) - W'_B(\tau) = -\frac{\eta_G - \eta_B}{\gamma}\bar{E}'(\tau) > 0$ for all $\tau \in [0, \bar{\tau})$.

Case 1: τ_B^* is interior. Then $W'_B(\tau_B^*) = 0$, so $W'_G(\tau_B^*) > 0$. Quasi-concavity of W_G implies $\tau_G^* > \tau_B^*$.

Case 2: $\tau_B^* = 0$ (corner). Here $W'_B(0) \leq 0$. From single-crossing, $W'_G(0) > W'_B(0)$, but $W'_G(0)$ could be positive or non-positive. If $W'_G(0) > 0$, then $\tau_G^* > 0 = \tau_B^*$. If $W'_G(0) \leq 0$, then $\tau_G^* = 0 = \tau_B^*$. Using (3.6), $W'_G(0) > 0$ iff $\Psi'(0) > \frac{\eta_G}{\gamma}\bar{E}'(0)$, i.e., $\Psi'(0) + \frac{\eta_G}{\gamma}(-\bar{E}'(0)) > 0$, which is the stated sufficient condition. □

Computation of Equilibrium Derivatives We compute $\Psi'(0)$, $\bar{E}'(0)$, $\Psi''(0)$, $\bar{E}''(0)$ at the no-policy benchmark ($\tau = 0, \sigma_\tau = 0$).

Notation. A superscript ^{0} denotes evaluation at $\tau = 0$:

$$D_s^0 = \left[\frac{(1-\alpha)\bar{A}_s}{w_s}\right]^{(1-\alpha)/\alpha}, \quad \bar{R}_s^0 = \alpha\bar{A}_s D_s^0, \quad \Sigma_r^0 = D^0 V D^0,$$

where $D^0 = \text{diag}(D_g^0, D_b^0)$ and $V = \begin{pmatrix} v_g & \sigma_A^2 \\ \sigma_A^2 & v_b \end{pmatrix}$ with $v_s = \sigma_A^2 + \sigma_{A,s}^2$. The no-policy pricing matrix is $M^0 = \gamma\bar{y}\Sigma_r^0 + \frac{\bar{y}}{2}\Phi$ (with $\Phi = \text{diag}(\phi_g, \phi_b)$); at $\sigma_\tau^2 = 0$, this coincides

with $\widetilde{M}^0$ from the main text. The baseline equilibrium satisfies $M^0\bar{\theta}^0 = \bar{R}^0 - r_f^0\mathbf{1}$, $\mathbf{1}'\bar{\theta}^0 = 1$.

Key derivatives of equilibrium objects at $\tau = 0$. Define the emission-productivity ratio $\rho_s := (1-\alpha)\bar{P}_s/(\alpha\bar{A}_s)$. Then:

$$D_s'(0) = -\rho_s D_s^0,$$
$$\bar{R}_s'(0) = -D_s^0\bar{P}_s.$$

Differentiating the equilibrium system $M^0\bar{\theta}^0 = \bar{R}^0 - r_f^0\mathbf{1}$:

$$M^0\bar{\theta}'(0) = \mathbf{g} - r_f'(0)\mathbf{1}, \qquad \mathbf{1}'\bar{\theta}'(0) = 0, \tag{B.3}$$

where $\mathbf{g} := \bar{R}'(0) - \dot{M}^0\bar{\theta}^0$ with $\dot{M}^0 = -\gamma\bar{y}D^0(\mathrm{diag}(\rho_s)V + V\mathrm{diag}(\rho_s))D^0$. Solving:

$$r_f'(0) = \frac{\mathbf{1}'(M^0)^{-1}\mathbf{g}}{\mathbf{1}'(M^0)^{-1}\mathbf{1}}, \qquad \bar{\theta}'(0) = (M^0)^{-1}\left[\mathbf{g} - r_f'(0)\mathbf{1}\right].$$

Marginal emission reduction $-\bar{E}'(0)$.

$$-\bar{E}'(0) = \underbrace{\bar{y}\sum_s \bar{P}_s\rho_s D_s^0\bar{\theta}_s^0}_{\text{intensive margin}} - \underbrace{\bar{y}\sum_s \bar{P}_s D_s^0\bar{\theta}_s'(0)}_{\text{extensive margin}},$$

with both terms positive when $D_b^0\bar{P}_b > D_g^0\bar{P}_g$ (Lemma 4).

Marginal financial benefit $\Psi'(0)$. At $\sigma_\tau = 0$: $\Psi(\tau) = \bar{y}r_f + \frac{\gamma\bar{y}^2}{2}\bar{\theta}'\Sigma_r\bar{\theta} + \lambda_R\tau\bar{E}$. Differentiating and using $\mathbf{1}'\bar{\theta}'(0) = 0$:

$$\Psi'(0) = \bar{y}\,r_f'(0) + \gamma\bar{y}^2\,\bar{\theta}'(0)'\Sigma_r^0\bar{\theta}^0 + \frac{\gamma\bar{y}^2}{2}\bar{\theta}^{0\prime}\dot{\Sigma}_r^0\bar{\theta}^0 + \lambda_R\bar{E}^0,$$

where $\dot{\Sigma}_r^0 = \frac{d\Sigma_r}{d\tau}\big|_0 = -D^0(\mathrm{diag}(\rho_s)V + V\mathrm{diag}(\rho_s))D^0$. The term $\lambda_R\bar{E}^0$ is the marginal rebate benefit; the remaining terms constitute the portfolio distortion cost $\Psi'_{\text{fin}}(0) < 0$.

Second-order derivatives. The financial curvature $-\Psi''(0)$ and emission curvature $\bar{E}''(0)$ require second derivatives of the equilibrium, obtained by differentiating (B.4) once more. The leading contribution to $-\Psi''(0)$ is the deadweight loss from tax

distortion:

$$-\Psi''(0) \geq \frac{(1-\alpha)}{\alpha}\,\bar{y}\sum_s \frac{\bar{P}_s^2}{\bar{A}_s} D_s^0 \bar{\theta}_s^0 + (\text{GE corrections}) > 0.$$

The emission curvature has three components:

$$\bar{E}''(0) = \bar{y}\sum_s \bar{P}_s\Big[\underbrace{\rho_s^2 D_s^0 \tfrac{1-2\alpha}{1-\alpha}\bar{\theta}_s^0}_{\text{intensive curvature}} \underbrace{-2\rho_s D_s^0 \bar{\theta}_s'(0)}_{\text{cross term}} + \underbrace{D_s^0 \bar{\theta}_s''(0)}_{\text{extensive curvature}}\Big],$$

where $\bar{\theta}''(0)$ is obtained from the second differentiation of the equilibrium system.

B.1 Proof of Proposition 4

Define:

- $D := -\Psi''(0) + \frac{\bar{\eta}_\beta}{\gamma}\bar{E}''(0) > 0$: welfare curvature at the politically-weighted preference;
- $D_i := -\Psi''(0) + \frac{\eta_i}{\gamma}\bar{E}''(0) > 0$ for $i \in \{G, B\}$: individual welfare curvatures;
- $\Xi := \bar{E}'(0)\Psi''(0) - \bar{E}''(0)\Psi'(0) > 0$: positive by the ordering $\tau_G^* > \tau_B^*$.

Positivity of Ξ. From (3.9), the ideal-point gap satisfies $\tau_G^* - \tau_B^* = (\eta_G - \eta_B)\Xi/(\gamma D_G D_B)$. Since $\eta_G > \eta_B$, $D_G, D_B > 0$ (the regularity assumption), and $\tau_G^* > \tau_B^*$ (Proposition 3), we conclude $\Xi > 0$.

Part (a). Write $\tau^* = N/D$ with $N := \Psi'(0) - \frac{\bar{\eta}_\beta}{\gamma}\bar{E}'(0)$. Only $\bar{\eta}_\beta = \beta\eta_G + (1-\beta)\eta_B$ depends on β, with $\partial\bar{\eta}_\beta/\partial\beta = \eta_G - \eta_B$. By the quotient rule:

$$\frac{\partial\tau^*}{\partial\beta} = \frac{(\eta_G - \eta_B)}{\gamma}\cdot\frac{\big(-\bar{E}'(0)\big)D - N\,\bar{E}''(0)}{D^2}.$$

Expanding $(-\bar{E}'(0))D - N\bar{E}''(0) = (-\bar{E}'(0))(-\Psi''(0)) + \frac{\bar{\eta}_\beta}{\gamma}(-\bar{E}'(0))\bar{E}''(0) - \Psi'(0)\bar{E}''(0) + \frac{\bar{\eta}_\beta}{\gamma}\bar{E}'(0)\bar{E}''(0)$; the $\frac{\bar{\eta}_\beta}{\gamma}$ terms cancel, leaving $\bar{E}'(0)\Psi''(0) - \Psi'(0)\bar{E}''(0) = \Xi$.

Part (b). With $\bar{\eta}$ fixed, $\bar{\eta}_\beta = \bar{\eta} + (\beta - 1/2)(\eta_G - \eta_B)$. So $\partial\tau^*/\partial(\eta_G - \eta_B) = (\beta - 1/2)\,\partial\tau^*/\partial\bar{\eta}_\beta$, and $\partial\tau^*/\partial\bar{\eta}_\beta = \Xi/(\gamma D^2)$ by the same quotient-rule argument as in part (a). For the gap: parameterize $\eta_G = \bar{\eta} + \Delta\eta/2$, $\eta_B = \bar{\eta} - \Delta\eta/2$, so $D_G D_B = D_{\bar{\eta}}^2 - (\frac{\Delta\eta\bar{E}''(0)}{2\gamma})^2$. The gap is $\frac{\Delta\eta\,\Xi}{\gamma(D_{\bar{\eta}}^2 - b^2\Delta\eta^2)}$ with $b = \bar{E}''(0)/(2\gamma)$. Differentiating in $\Delta\eta$ yields a numerator $\Xi(D_{\bar{\eta}}^2 + b^2\Delta\eta^2)/\gamma > 0$.

Part (c). Same quotient-rule structure as part (a), differentiating in γ with $\bar{\eta}_\beta/\gamma$ as the moving part. The numerator becomes $-\frac{\bar{\eta}_\beta}{\gamma^2}[\cdots]$, and the identical cancellation

yields $(-\bar{E}'(0))D + N\bar{E}''(0) = -\Xi$. For the gap: the direct effect on $\tau_G^* - \tau_B^*$ through $1/\gamma$ gives the stated formula.

□

B.2 Computation of Equilibrium Derivatives

We compute $\Psi'(0)$, $\bar{E}'(0)$, $\Psi''(0)$, $\bar{E}''(0)$ at the no-policy benchmark ($\tau = 0, \sigma_\tau = 0$).

Notation. A superscript ^{0} denotes evaluation at $\tau = 0$:

$$D_s^0 = \left[\frac{(1-\alpha)\bar{A}_s}{w_s}\right]^{(1-\alpha)/\alpha}, \quad \bar{R}_s^0 = \alpha \bar{A}_s D_s^0, \quad \Sigma_r^0 = D^0 V D^0,$$

where $D^0 = \mathrm{diag}(D_g^0, D_b^0)$ and $V = \begin{pmatrix} v_g & \sigma_A^2 \\ \sigma_A^2 & v_b \end{pmatrix}$ with $v_s = \sigma_A^2 + \sigma_{A,s}^2$. The no-policy pricing matrix is $M^0 = \gamma\bar{y}\Sigma_r^0 + \frac{\bar{y}}{2}\Phi$ (with $\Phi = \mathrm{diag}(\phi_g, \phi_b)$); at $\sigma_\tau^2 = 0$, this coincides with $\widetilde{M}^0$ from the main text. The baseline equilibrium satisfies $M^0\bar{\theta}^0 = \bar{R}^0 - r_f^0\mathbf{1}$, $\mathbf{1}'\bar{\theta}^0 = 1$.

Key derivatives of equilibrium objects at $\tau = 0$. Define the emission-productivity ratio $\rho_s := (1-\alpha)\bar{P}_s/(\alpha\bar{A}_s)$. Then:

$$D_s'(0) = -\rho_s D_s^0,$$
$$\bar{R}_s'(0) = -D_s^0 \bar{P}_s.$$

Differentiating the equilibrium system $M^0\bar{\theta}^0 = \bar{R}^0 - r_f^0\mathbf{1}$:

$$M^0\bar{\theta}'(0) = \mathbf{g} - r_f'(0)\mathbf{1}, \qquad \mathbf{1}'\bar{\theta}'(0) = 0, \tag{B.4}$$

where $\mathbf{g} := \bar{R}'(0) - \dot{M}^0\bar{\theta}^0$ with $\dot{M}^0 = -\gamma\bar{y}D^0(\mathrm{diag}(\rho_s)V + V\mathrm{diag}(\rho_s))D^0$. Solving:

$$r_f'(0) = \frac{\mathbf{1}'(M^0)^{-1}\mathbf{g}}{\mathbf{1}'(M^0)^{-1}\mathbf{1}}, \qquad \bar{\theta}'(0) = (M^0)^{-1}\left[\mathbf{g} - r_f'(0)\mathbf{1}\right].$$

Marginal emission reduction $-\bar{E}'(0)$.

$$-\bar{E}'(0) = \underbrace{\bar{y}\sum_s \bar{P}_s\rho_s D_s^0\bar{\theta}_s^0}_{\text{intensive margin}} + \underbrace{\bar{y}(D_b^0\bar{P}_b - D_g^0\bar{P}_g)\bar{\theta}_g'(0)}_{\text{extensive margin}},$$

with both terms positive when $D_b^0\bar{P}_b > D_g^0\bar{P}_g$ (Lemma 4).

Marginal financial benefit $\Psi'(0)$. At $\sigma_\tau = 0$: $\Psi(\tau) = \bar{y} r_f + \frac{\gamma \bar{y}^2}{2} \bar{\theta}' \Sigma_r \bar{\theta} + \lambda_R \tau \bar{E}$. Differentiating and using $\mathbf{1}' \bar{\theta}'(0) = 0$:

$$\Psi'(0) = \bar{y}\, r_f'(0) + \gamma \bar{y}^2\, \bar{\theta}'(0)' \Sigma_r^0 \bar{\theta}^0 + \frac{\gamma \bar{y}^2}{2} \bar{\theta}^{0\prime} \dot{\Sigma}_r^0 \bar{\theta}^0 + \lambda_R \bar{E}^0,$$

where $\dot{\Sigma}_r^0 = \frac{d\Sigma_r}{d\tau}\big|_0 = -D^0(\mathrm{diag}(\rho_s)V + V\mathrm{diag}(\rho_s))D^0$. The term $\lambda_R \bar{E}^0$ is the marginal rebate benefit; the remaining terms constitute the portfolio distortion cost $\Psi'_{\text{fin}}(0) < 0$.

Second-order derivatives. The financial curvature $-\Psi''(0)$ and emission curvature $\bar{E}''(0)$ require second derivatives of the equilibrium, obtained by differentiating (B.4) once more. The leading contribution to $-\Psi''(0)$ is the deadweight loss from tax distortion:

$$\text{(1-}\alpha\text{)}\alpha\, \bar{y} \sum_s \tfrac{\bar{P}_s^2}{\bar{A}_s} D_s^0 \bar{\theta}_s^0 + \text{(GE corrections)} > 0. \quad -\Psi''(0) \geq$$

The emission curvature has three components:

$$\bar{E}''(0) = \bar{y} \sum_s \bar{P}_s [\ \underbrace{\rho_s^2 D_s^0 \tfrac{1-2\alpha}{1-\alpha} \bar{\theta}_s^0}_{\text{intensive curvature}} \ \underbrace{- 2\rho_s D_s^0 \bar{\theta}_s'(0)}_{\text{cross term}} + \ \underbrace{D_s^0 \bar{\theta}_s''(0)}_{\text{extensive curvature}} \ ],$$

where $\bar{\theta}''(0)$ is obtained from the second differentiation of the equilibrium system. The intensive curvature is proportional to $(1-2\alpha)/(1-\alpha)$: it vanishes at $\alpha = 1/2$, is positive for $\alpha < 1/2$, and negative for $\alpha > 1/2$. The cross and extensive curvatures are typically small relative to $(-\Psi''(0))\gamma/\eta_G$, so the concavity condition $D_i > 0$ and the positivity $\Xi > 0$ are robust across parameter configurations.

B.3 Derivation of the Coordinated Green FOC

We derive the first-order condition (4.3). Consumption is $c_G = \bar{y}[r_f + (r - r_f \mathbf{1})' \theta_G] + \lambda_R \tau \bar{E}$, so the certainty equivalent is:

$$W_G^{\text{coord}} = \bar{y}[r_f + (\bar{r} - r_f \mathbf{1})' \theta_G] + \lambda_R \tau \bar{E} - \frac{\gamma}{2} \bar{y}^2 \theta_G' \Sigma_r \theta_G - \frac{\eta_G}{\gamma} \bar{E}.$$

From (4.2): $\frac{\partial r_f}{\partial \theta_G} = \frac{\gamma \bar{y}}{\mathbf{1}' \Sigma_r^{-1} \mathbf{1}} \mathbf{1}$. Brown investors respond via (4.1): $\frac{\partial \theta_B}{\partial \theta_G} = -\frac{\Sigma_r^{-1} \mathbf{1}\mathbf{1}'}{\mathbf{1}' \Sigma_r^{-1} \mathbf{1}}$.

Capital allocation $K = \frac{\bar{y}}{2}(\theta_G + \theta_B)$ satisfies:

$$\frac{\partial K}{\partial \theta_G} = \frac{\bar{y}}{2}\left(I - \frac{\Sigma_r^{-1}\mathbf{1}\mathbf{1}'}{\mathbf{1}'\Sigma_r^{-1}\mathbf{1}}\right),$$

so $\frac{\partial \bar{E}}{\partial \theta_G} = v^{P\prime}\frac{\partial K}{\partial \theta_G} = \frac{\bar{y}}{2}\Delta v^{P\prime}$, where $\Delta v^P := v^P - \bar{v}^P\mathbf{1}$ with $\bar{v}^P := \frac{\mathbf{1}'\Sigma_r^{-1}v^P}{\mathbf{1}'\Sigma_r^{-1}\mathbf{1}}$.

Differentiating W_G^{coord} with respect to θ_G:

$$\bar{y}(\bar{r} - r_f\mathbf{1}) + \frac{\gamma\bar{y}^2(1 - \mathbf{1}'\theta_G)}{\mathbf{1}'\Sigma_r^{-1}\mathbf{1}}\mathbf{1} - \gamma\bar{y}^2\Sigma_r\theta_G + \left(\lambda_R\tau - \frac{\eta_G}{\gamma}\right)\frac{\bar{y}}{2}\Delta v^P = 0.$$

This is (4.3). □

B.4 Proof of Theorem 3

(i). Part (i): Portfolio scale. Multiply the FOC (4.3) by $\mathbf{1}'\Sigma_r^{-1}$. The emission term vanishes: $\mathbf{1}'\Sigma_r^{-1}\Delta v^P = \mathbf{1}'\Sigma_r^{-1}(v^P - \bar{v}^P\mathbf{1}) = 0$ by definition of $\bar{v}^P$. Using $r_f(\theta_G)$ from (4.2):

$$\bar{y}\mathbf{1}'\Sigma_r^{-1}(\bar{r} - r_f\mathbf{1}) = \gamma\bar{y}^2(2 - \mathbf{1}'\theta_G).$$

Combined with the remaining terms:

$$\gamma\bar{y}^2(2 - \mathbf{1}'\theta_G) - \gamma\bar{y}^2\mathbf{1}'\theta_G + \gamma\bar{y}^2(1 - \mathbf{1}'\theta_G) = \gamma\bar{y}^2(3 - 3\mathbf{1}'\theta_G) = 0,$$

which gives $\mathbf{1}'\theta_G^* = 1$. □

(ii). Part (ii): Corner solution. When $\theta_{G,b} = 0$ binds, the FOC in $\theta_{G,g}$ contains the emission term $(\lambda_R\tau - \frac{\eta_G}{\gamma})\frac{\bar{y}}{2}\Delta v_g^P$. Since $\Delta v_g^P < 0$ (green has lower emission intensity than the precision-weighted average), this term is positive when $\eta_G > \gamma\lambda_R\tau$, pushing toward higher $\theta_{G,g}$. As $\eta_G \to \infty$, satisfying the FOC requires $\theta_{G,g} \to \infty$, implying $\mathbf{1}'\theta_G > 1$: green investors borrow to finance green investment. The threshold $\bar{\eta}_G$ is determined by the parameter values at which the interior solution first yields $\theta_{G,b}^* = 0$. □

B.5 Derivation of the Risk Wedge

From the green FOC (4.3), dividing by $\bar{y}$:

$$(\bar{r} - r_f \mathbf{1}) - \gamma \bar{y} \Sigma_r \theta_G + \frac{\gamma \bar{y}(1 - \mathbf{1}'\theta_G)}{\mathbf{1}'\Sigma_r^{-1}\mathbf{1}} \mathbf{1} + \frac{1}{2}\left(\lambda_R \tau - \frac{\eta_G}{\gamma}\right) \Delta v^P = 0.$$

The brown FOC gives $(\bar{r} - r_f \mathbf{1}) = \gamma \bar{y} \Sigma_r \theta_B$. Subtracting, and using $\mathbf{1}'\theta_G^* = 1$ (so the interest rate term vanishes):

$$-\gamma \bar{y} \Sigma_r (\theta_G - \theta_B) + \frac{1}{2}\left(\lambda_R \tau - \frac{\eta_G}{\gamma}\right) \Delta v^P = 0.$$

Solving for $\xi_G - \xi_B = \bar{y}(\theta_G - \theta_B)$:

$$\xi_G - \xi_B = \frac{1}{2\gamma} \Sigma_r^{-1} \left(\lambda_R \tau - \frac{\eta_G}{\gamma}\right) \Delta v^P,$$

which is (4.5). □

B.6 Proof of Proposition 5 (Inefficiency)

Efficient risk sharing requires equal exposure to aggregate consumption risk: $\xi_G = \xi_B$ (equation (4.4)). Under the stated conditions, equation (4.5) implies $\xi_G \neq \xi_B$: the term $(\lambda_R \tau - \frac{\eta_G}{\gamma})$ is nonzero (since $\eta_G \neq \gamma \lambda_R \tau$), and $\Sigma_r^{-1} \Delta v^P \neq 0$ (since $D_g P_g \neq D_b P_b$ implies $\Delta v^P \neq 0$, and Σ_r is positive definite). The planner could improve welfare by maintaining the same capital allocation while redistributing consumption risk to equalize exposures. □

B.7 Proof of Efficient Risk Sharing (Equation 4.4)

With CARA utility, the planner optimizes state by state. For each realization of aggregate consumption C and emissions $\bar{E}$ (deterministic), the planner solves:

$$\max_{c_G, c_B} \omega_G(-e^{-\gamma c_G + \eta_G \bar{E}}) + \omega_B(-e^{-\gamma c_B}) \quad \text{s.t.} \quad c_G + c_B = C.$$

The first-order condition is $\omega_G \gamma e^{-\gamma c_G + \eta_G \bar{E}} = \omega_B \gamma e^{-\gamma c_B}$. Taking logs: $\ln(\omega_G/\omega_B) - \gamma c_G + \eta_G \bar{E} = -\gamma c_B$. Using the constraint $c_B = C - c_G$:

$$c_G = \frac{\ln(\omega_G/\omega_B) + \eta_G \bar{E}}{2\gamma} + \frac{C}{2}, \qquad c_B = C - c_G.$$

Hence c_G is linear in C with slope $1/2$. Since $\bar{E}$ is deterministic, the environmental term enters the intercept, not the risk loading. Defining $a_i := \mathbb{E}[c_i]$, we have $c_i = a_i + \frac{1}{2}(C - \bar{C})$ for $i \in \{G, B\}$. Both groups bear equal consumption risk. □

B.8 Proof of Theorem 4 (Efficiency with Swaps)

Augment the market with a return swap paying $(r_g - r_b)$ per dollar of position. Let ϕ_i denote investor i's swap position and p_s the swap price. Each investor's risk exposure is $\xi_i = \bar{y}\theta_i + \phi_i e$, where $e := (1, -1)'$.

The brown atomistic investor's FOC for ϕ_B:

$$e'\gamma\Sigma_r\xi_B = \frac{p_s}{2}.$$

The coordinated green bloc's FOC for ϕ_G (noting ϕ_G does not affect capital allocation or emissions):

$$e'\gamma\Sigma_r\xi_G = \frac{p_s}{2}.$$

Both have identical right-hand sides. Subtracting: $e'\Sigma_r(\xi_G - \xi_B) = 0$. Combined with bond market clearing ($\mathbf{1}'(\xi_G - \xi_B) = 0$, since $\mathbf{1}'\theta_G + \mathbf{1}'\theta_B = 2$ and $\phi_G + \phi_B = 0$), we have two independent linear equations. Since Σ_r is positive definite and $e, \mathbf{1}$ are linearly independent, the unique solution is $\xi_G = \xi_B$. Consumption therefore takes the form $c_i = a_i + \frac{1}{2}(C - \bar{C})$, matching the planner's optimal risk sharing (4.4).

With equal risk exposure, capital allocation is determined by the green bloc's equity choice, which solves the same problem as the social planner (for welfare weights determined by equilibrium prices), establishing Pareto efficiency. □