Prices versus Quantities Revisited: The Case of a Transboundary Stock Pollutant*

Tao Wang† Tianhao Wang‡

December 2025

Abstract

The climate policy debate has emphasized the importance of the relative slopes of marginal damages and abatement costs for ranking taxes versus cap and trade under technological uncertainty. It finds that taxes dominate quotas as the social cost of carbon curve is extremely flat. We study the relative merits of price versus quantity instruments in a dynamic model with two economies facing a transboundary stock pollutant like CO_2. Our model features persistent cost shocks that generate perfectly correlated shifts of these two curves; these are as important to the ranking as is the slope ratio. Gradual diffusion of innovations generally favors quotas by changing the relative shifts. A larger relative economic size also tends to favor quotas, although its effect differs depending on whether policies are chosen unilaterally or strategically. Overall, our findings suggest that in the presence of multiple regulators, strategic policy commitments endogenously redistribute technological uncertainty across countries.

JEL Classification: Q50, H20, D80.

Keywords: prices versus quantities; tax vs quota; uncertainty; carbon price; international environmental agreement.

---

*We thank Larry Karp, Zhengyuan Yang, and Victor Barranca for helpful advice. We thank Guy Berreby for excellent research assistance. Tianhao Wang thanks Stergios Skaperdas and workshop participants at the Center for Global Peace and Conflict Studies. We are indebted to the generous support of Swarthmore College for this project.

†Swarthmore College. Email: twang1@swarthmore.edu

‡University of California, Irvine. Email: tianhw15@uci.edu

1 Introduction

Global warming, driven primarily by anthropogenic greenhouse gas (GHG) emissions, threatens to roll back decades of global development progress and puts lives, livelihoods, and economic growth at risk.[1] Setting a price for the emissions, commonly referred to as carbon pricing, is an essential part of the solution. Explicit carbon pricing policies are enacted by a government mandate and impose a price based on greenhouse gas content, typically in the form of a carbon tax or an emission trading system (ETS). According to a World Bank (2023) report, in 2023, there are 73 carbon pricing instruments (CPIs) in operation, covering 23% of global GHG emissions. These instruments differ greatly in their carbon prices and their coverage of emissions. A wide range of countries and subnational governments continue to move towards carbon pricing, in particular ETS's. Some of these have already scheduled start dates, while others are beginning with pilots. Other jurisdictions are in early stages of considering their policy options. One key decision for the various governments is whether to adopt a carbon tax, a price instrument, or an ETS, a quantity instrument.

We revisit the classic question of "prices versus quantities," asked in Weitzman's (1974) seminal paper, in the case of carbon pricing in an international context. With full information, a price instrument and a quantity instrument to regulate pollution could be equivalent. When uncertainty is present, the ranking of the two market-based policies will depend on a number of factors. In this paper, we explore the conditions under which a tax or a quota is preferred by a regulator of a global stock pollutant facing technological uncertainty in a two-economy world. Our model features a non-cooperative dynamic game between two economies, each strategically pursuing its carbon pricing policy in response to the other. In particular, in our settings, each economy adopts a price or quantity instrument to maximize its expectation of the present discounted stream of net benefit flows, conditional on the instrument choice by the other economy.

To build intuition, we start with the case where only one economy actively implements a carbon pricing policy. Our findings suggest that in a unilateral setting, the ranking criterion depends on ratios of slopes of marginal abatement costs and damage. In particular, gradual technology diffusion

[1] Recent reports from the Intergovernmental Panel on Climate Change (IPCC) (2014; 2018; 2023) renew calls to limit temperature increase to below 2°C, or even 1.5°C, relative to pre-industrial levels.

in the regulating economy and a higher responsiveness of the SCC to cumulative emissions favor quotas, while a larger relative size of the non-regulating economy also favors quotas. In strategic settings, as the relative size of an economy increases, its regulator becomes more responsive to cumulative emissions and the observed technological level. A larger size of an economy relative to the other tends to favor quotas. The role of relative size differs across these settings because in the unilateral case it governs exposure to stock uncertainty generated abroad, whereas in the strategic setting, it governs an economy's ability to influence the volatility of the shared pollution stock. The resulting equilibrium depends on both the relative sizes of the economies and the speed of technology diffusion.

Our main contributions to the literature are (i) we show that a simple and intuitive ranking criterion similar to that in Karp and Traeger (2024a) extends to the case in which an economy unilaterally pursues carbon pricing policy, (ii) we show that when multiple economies regulate simultaneously, instrument choice becomes interdependent across countries: relative economic size determines an economy's ability to influence the volatility of the shared pollution stock through its policy commitment, and (iii) while slower technology diffusion can favor quotas in both settings, we show that the effect of relative size diverges between the unilateral and strategic cases. Strategic policy commitments endogenously reallocate technological uncertainty across regulators.

This paper seek to relate two features that lie at the core of carbon pricing policies. On the one hand, regulators face uncertainty about abatement costs and damages arising from a persistent pollution stock. On the other hand, the pollution stock is regulated by multiple sovereign jurisdictions, giving rise to strategic interaction and free riding at the international level.

Weitzman's original analysis establishes that a tax is preferred to a quota if and only if the marginal abatement cost curve is steeper than the marginal damage curve. Following this reasoning, it is often inferred that society would be better off using taxes to address climate change since the per-period marginal damage curve of GHG emissions tends to be very flat. However, Weitzman's work models a local flow pollutant, i.e., the pollutant does not have persistence or cumulative impact, and there is only one relevant policy maker. This is not the case with climate change. The most persistent GHG in the atmosphere, carbon dioxide (CO_2) emissions, can affect the climate for hundreds or thousands of years. Additionally, GHGs are global pollutants and its abatement suffers

from the free riding problem at the international level. Therefore, many scholars have attempted to extend the original analysis to consider stock pollutants, technology shocks, and/or strategic policy with multiple economies. Under certain scenarios, a quota can be better than a tax.

Much of this research focuses on the stock versus flow characteristic of the pollutant. Surveys on this strand of literature include Hepburn (2006); Aldy et al. (2010) and Goulder and Schien (2013). Under a tax, technological uncertainty creates uncertainty about emissions, and consequently about damages. Under a binding quota, such uncertainty does not alter emissions, but it creates cost uncertainty for firms. If the (aggregate) marginal abatement cost curve is steeper than the marginal damage curve, then uncertainty about abatement costs harms society more than uncertainty about damages. Hence, in the case of a flow pollutant, taxes are preferred if and only if the marginal abatement cost curve is steeper than the marginal damage curve. In a dynamic setting with stock pollution, in each period we have to compare current abatement costs against the discounted stream of damages occurring over an extended time horizon. Hereafter, in discussing stock pollutants, we refer to the discounted stream of marginal damages simply as "marginal damages." In the climate context, the relevant marginal damage is referred to as the social cost of carbon (SCC). Applying the logic described above to the climate context, one might be tempted to rank taxes versus quotas by comparing the slopes the marginal abatement cost curve and the SCC. Indeed, the literature often makes this leap (e.g. Nordhaus 2008; Aldy et al. 2010; Goulder and Schien 2013; Weitzman 2020).

However, as pointed out by Karp and Traeger (2024a), this leap can be incorrect. They model the policy making process as a dynamic programming problem, where the regulator of an economy needs to balance the gains from current emissions and potential future damages due to the added stock. The model features serially correlated technological shocks and gradual diffusion of new technologies, under asymmetric information. In this dynamic setting, the abatement technology is a stochastic process. Every period gives rise to an innovation unknown to the policy maker at the onset of the regulation period. Because technology is highly persistent, the innovation affects not only the abatement cost in the present period, but also those in the future periods. Consequently, the innovation also changes future emissions, thus changing the future cumulative stock, or the future baseline concentration, of the pollutant. With convex damages, the marginal

damage caused by additional emissions in a period depends on the pollutant's concurrent baseline concentration. As a result, the technology shock not only shifts the marginal abatement cost curve, but it simultaneously shifts the marginal damage curve. Hence, the policy ranking depends as much on this shift as it does on relative slopes. In some cases, this additional shift implies that a quota is the preferred instrument even if the marginal damage curve is much flatter than the abatement cost curve.

This mechanism was either not present in earlier models or was not recognized clearly. Hoel and Karp (2002) analyze prices versus quantities for a stock pollutant in a setting where current innovations have no impact on future technology, thereby ruling out the aforementioned result. Newell and Pizer (2003) rank the two policies when cost shocks are serially correlated in an open-loop setting, where the regulator chooses current and future policy levels at the initial time. They show that positively correlated cost shocks increase stock volatility under taxes, favoring quotas, although not by enough to tip the balance. Their intuition depends crucially on the open loop assumption that future policy makers do not respond when the realized technology and emission levels drift away from anticipated levels. In contrast, Karp and Traeger's result depends on the ability of the regulator to update the policy instruments after observing past technological innovations. Karp and Zhang (2005) compare the policy ranking across the open loop and feedback settings, with correlated shocks. They observe that the ranking of tax versus quota changes, but do not recognize the different mechanism in the feedback setting and its ability to make quota dominate tax in the climate context.

A related line of literature studies international environmental agreements (IEAs) [2]. Aldy and Stavins (2007); Aldy et al. (2010) and Marrouch and Chaudhuri (2016) provide recent surveys on this research that focuses on international policy design for transboundary environmental problems.

[2]Game-theoretic frameworks have also been developed to assess IEAs and the relative performance of price- versus quantity-based instruments. Kornek and Marschinski (2018) underscore the strategic implications of instrument choice under uncertainty, endogenizing coalition size and showing that larger coalitions have a greater incentive to adopt quantity instruments, with tradable permits yielding higher global welfare. Similarly, Harstad et al. (2022) highlight the importance of technology regulation, finding that in self-enforcing IEAs, price instruments tend to dominate quotas due to stricter penalties for defection - especially when firms can adjust investment levels following governmental non-compliance.

The cross-border externality creates a strategic interdependence between involved countries. Since GHG emissions are a global pollutant, its regulation suffers from the free rider problem at the global level. To successfully address the issue, there needs to widespread international cooperation, at least among all major emitters of GHGs. Proposed architectures for international emissions control regimes can be loosely classified into those based on bottom-up versus top-down (i.e., internationally negotiated) approaches and cap-and-trade systems versus systems of emissions taxes. Mideksa and Weitzman (2019) extend the prices-versus-quantities framework to multiple heterogeneous jurisdictions interacting strategically, but in a static regulatory game. In their static framework, uncertainty is independent across jurisdictions, so each country's optimal instrument choice is independent of the policies adopted by others.

Jørgensen et al. (2010), Van Long (2012) and Calvo and Rubio (2012) offer recent surveys of dynamic models of IEAs focusing on stock pollutants. A limitation revealed in these surveys is that, in the analysis of transboundary pollution problems, almost none of the models consider the uncertainty or asymmetric information issue discussed previously. Wirl (2007) considers uncertainty in the sense that global warming follows a stochastic process, but considers only the price instrument.

This paper combines the intuitions of previous research and build on the model of Karp and Traeger (2024a) to incorporate the international aspect with a transboundary stock pollutant, and extend Mideksa and Weitzman (2019) by introducing a dynamic framework in which heterogeneous countries differ in their shares of global emissions. In contrast to the existing literature, we focus on the strategic spillover of uncertainty. We show that the choice of the pricing mechanisms, i.e., tax versus quota, depends crucially on the sizes of the economies. More importantly, the decision also depends on whether global collaboration occurs through a leader-follower arrangement among economies or through multiple economies engaging in a dynamic, non-cooperative game.

Section 2 presents a two-economy model in which only one is actively setting a carbon pricing policy. Section 3 presents the full strategic interaction model where both economies pursue policies in response to each other. Section 4 calibrates the model based on widely-used estimates of abatement costs and climate-related damages, and presents numerical results on how the ranking of tax versus quota depends on key parameters. Section 5 concludes.

2 Dynamic Model with Unilateral Policy

We present a two-economy model in which only one economy, denoted by *Home*, is actively setting a carbon pricing policy. In contrast, the other economy, *Foreign*, effectively adopts a 'zero carbon tax' by not setting any carbon pricing mechanism. In our analysis, each economy may represent a single nation or a bloc of countries with unified policies and integrated economies. Under this framework, firms in Foreign maximize their emission benefits each period.

Our analysis centers on two key asymmetries that drive the non-equivalence between taxes and quotas for Home: the relative size of the two economies and technology-related costs. The difference in the relative sizes of Home and Foreign affect each economy's share of global emissions. The asymmetry is amplified when firms treat technology-related costs as private information during their emissions decision-making.

2.1 Description of the Model

The model is characterized by two state variables: a stock pollutant S_t and a technology level θ_t. The law of motion for the stock pollutant is given by $S_{t+1} = \delta S_t + E_t$, where E_t is the annual flow of total emission of the two blocs, and $1 - \delta \in (0, 1)$ is the decay factor of the harmful pollution. The annual damage caused by the stock pollutant is $\frac{b}{2} S_t^2$.

The abatement technology θ_t is assumed to be symmetric for each economy, and is described by a deterministic trend and a stochastic deviation from this trend. The equation of motion for θ_t is $\theta_t = \rho\theta_{t-1} + \epsilon_t$, with $\rho > 0, \epsilon_t \sim iid(0, \sigma^2)$, and $\mathbb{E}_t(\epsilon_t) = 0$. The regulator knows θ_{t-1} but not ϵ_t. Firms adopt only a fraction $\alpha \in (0, 1]$ of the technological innovations in the current period, such that $\hat{\theta}_t = \rho\theta_{t-1} + \alpha\epsilon_t$.

The baseline environment corresponds to the single-regulator framework of Karp and Traeger (2024a), in which a regulator chooses between taxes and quotas under persistent technology shocks. To capture differences in relative sizes of the economies, we introduce a scalar $k > 0$, so that the size of Foreign is k times that of Home, where size is measured by relative emissions contributing to the global pollution stock. Home and Foreign have different speeds of technological diffusion

$\alpha_H \in (0,1]$ and $\alpha_F \in (0,1]$, respectively. Firms in Home and Foreign also have different slopes of the marginal abatement cost curve f_H and f_F.

Foreign does not actively implement any carbon pricing policies. The firms in Foreign have emission benefits $\hat{\theta}_t E_{F,t} - \frac{f_F}{2} E_{F,t}^2$, implying the first-order condition $E_{F,t} = \frac{\hat{\theta}_t}{f_F}$. Therefore, the Foreign firms' emission decision becomes $k(\frac{\phi}{f_F}(\rho\theta_{t-1}) + \frac{\alpha_F}{f_F}\epsilon_t)$. The parameter ϕ represents the time step; if the unit of time is one year, $\phi = 1$.

For Home, the firm's payoff under a tax τ_t is $\hat{\theta}_t E_{H,t} - \frac{f_H}{2} E_{H,t}^2 - \tau_t E_{H,t}$. Firms maximize their payoff, implying the decision rule

$$E_t^T = e_t^T + \alpha_H \frac{\epsilon_t}{f_H} \text{ with } e_t^T \equiv \frac{\rho\theta_{t-1} - \tau_t}{f_H}. \tag{1}$$

Taking expectations,

$$\mathbb{E}_t\left[\hat{\theta}_t E_{H,t} - \frac{f_H}{2} E_{H,t}^2\right] = \rho\theta_{t-1} e_t - \frac{f_H}{2} e_t^2 + \frac{\alpha_H^2}{2f_H}\sigma^2.$$

Hence, the expected flow payoff under a tax is

$$\rho\theta_{t-1} e_t - \frac{f_H}{2} e_t^2 + \frac{\alpha_H^2}{2f_H}\sigma^2 - \frac{b}{2} S_t^2. \tag{2}$$

Under a binding quota, the regulator chooses the actual emissions level E_T^Q with expected flow net benefit being

$$\rho\theta_{t-1} E_t^Q - \frac{f_H}{2}(E_t^Q)^2 - \frac{b}{2} S_t^2. \tag{3}$$

To formulate the regulator's problem for Home, we adopt a linear-quadratic structure to address the case when the regulator employs either a tax or a quota. We introduce an indicator function, Φ, defined as

$$\Phi = \begin{cases} 1 & \text{if tax,} \\ 0 & \text{if quota.} \end{cases}$$

Let $x^i_{H,t}$ denote Home emissions under instrument $i \in \{T, Q\}$. Home's regulator's problem is

$$\begin{aligned}
&\max_{x^i_{H,t}} \mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^\tau \left[\rho\theta_{t+\tau-1} x^i_{H,t+\tau} - \frac{f_H}{2}(x^i_{H,t+\tau})^2 + \Phi \frac{\alpha_H^2}{2f_H}\sigma^2 - \frac{b}{2} S^2_{t+\tau} \right] \phi \\
&\text{subject to } S_{t+\tau+1} = \delta S_{t+\tau} + \phi \frac{k}{f_F}(\rho\theta_{t+\tau-1}) + \phi x^i_{H,t+\tau} + \Phi\phi\alpha_H \frac{\epsilon_{t+\tau}}{f_H} + k\phi\alpha_F \frac{\epsilon_{t+\tau}}{f_F}, \qquad (4) \\
&\qquad \theta_t = \rho\theta_{t-1} + \epsilon_t.
\end{aligned}$$

This formulation explicitly incorporates the relative size of Foreign and its emissions into the update rule for the stock pollutant.

We define the state vector as $Y_t = (S_t, \theta_{t-1})'$ and the following matrices:

$$Q = \begin{bmatrix} -b & 0 \\ 0 & 0 \end{bmatrix}, A = \begin{bmatrix} \delta & \phi\frac{k\rho}{f_F} \\ 0 & \rho \end{bmatrix}, W = \begin{bmatrix} 0 & \rho \end{bmatrix}, B = \begin{bmatrix} \phi \\ 0 \end{bmatrix}, C = \begin{bmatrix} k\phi\frac{\alpha_F}{f_F} + \Phi\phi\frac{\alpha_H}{f} \\ 1 \end{bmatrix}.$$

Then the net flow payoff and equation of motion for the generic problem for Home are:

$$\left[-\frac{f_H}{2} x^2_{H,t} + \frac{1}{2} Y_t' Q Y_t + W Y_t x_{H,t} + \Phi \frac{\alpha_H^2}{2f}\sigma^2 \right] \phi \qquad (5)$$

$$Y_{t+1} = AY_t + Bx_{H,t} + C\epsilon_t. \qquad (6)$$

2.2 Policy ranking

The Social Cost of Carbon (SCC) measures the change in the expected value of all future climate-related damages resulting from an incremental increase in the current pollution flow. We assume that the marginal damage of the stock pollutant is given by

$$SCC_t = \lambda S_t + \mu\theta_{t-1},$$

where λ is the marginal impact of cumulative emissions S_t on the SCC, and μ is the responsiveness of the SCC to the technology realization. Both coefficients are positive constants that are endogenously determined and are key determinants of policy choices in our model.

The ranking of policy instruments depends on two sources of uncertainty. Cost uncertainty arises because technology shocks directly affect current-period abatement costs, making marginal abatement costs uncertain at the time policy instruments are set. Stock uncertainty arises because emissions accumulate over time, so technology shocks also affect the stream of future pollution stock and damages. Therefore, the regulator faces a trade-off between regulatory flexibility in responding to cost-saving technological shocks and control over the future pollution stock.

We define $R \equiv \frac{\phi\lambda}{f_H}$, which relates the convexity of stock damages to that of abatement costs. We arrive at the following lemma.

Lemma 1. *Under both taxes and quotas, the slope of the SCC of Home with respect to the stock of carbon, relative to the slope of marginal abatement cost is*

$$R \equiv \frac{\lambda}{f_H}\phi = \frac{1}{2\beta}(-(1-\beta\delta^2-\beta\frac{b}{f_H}\phi^2)+\sqrt{(1-\beta\delta^2-\beta\frac{b}{f_H}\phi^2)^2+\frac{4\beta\phi^2 b}{f_H}}). \tag{7}$$

The parameter $\frac{b}{f_H}$ represents the ratio of the slopes of the marginal flow damage and the marginal abatement cost. The link between this flow ratio and the stock ratio R is determined by the persistence of the pollutant δ and the discount factor β. The following proposition offers a straightforward and intuitive criterion for ranking taxes and quotas based on the relative sizes of the economies and other parameters.

Proposition 1. *Suppose that* $1-\beta\delta\rho > 0$ *hold, taxes dominate quotas if and only if*

$$R < \frac{1}{\beta} - \frac{2\mu}{\alpha_H} - 2\frac{\alpha_F\lambda\phi}{f_F\alpha_H} \iff R < R^{crit} \equiv -\frac{1}{2}\kappa_1 + \frac{1}{2}\sqrt{\kappa_1^2+4\kappa_0} \tag{8}$$

with

$$\kappa_1 = \frac{(1-\beta\delta\rho)\beta(\alpha_H f_F + 2k\alpha_F f_H) + 2\beta^2\delta\rho(f_F + f_H k) - \alpha_H f_F\beta}{(\alpha_H f_F + 2k\alpha_F f_H)\beta^2},$$
$$\kappa_0 = \frac{f_F\alpha_H(1-\beta\delta\rho)}{(\alpha_H f_F + 2k\alpha_F f_H)\beta^2}.$$

Proposition 1 shows that an increased μ, the derivative of the SCC with respect to the technology realization, distinctly favors quotas for Home. In addition, slower technology adoption in Home – relative to the rapid diffusion in Foreign – reinforces this preference for quotas. In addition, a higher responsiveness λ of the SCC to the cumulative emissions favors quotas for Home. The roles of the relative sizes of the two economies and the speed of technology diffusion are summarized in the following proposition.

Proposition 2. *Assume that* $f_H, f_F, b, \beta > 0$ *and* $1 - \beta\delta\rho > 0$ *hold.*

(a) A increase in k*, which corresponds to a relatively smaller Home economy, lowers* R^{crit} *and thereby favors quotas in Home.*

(b) An increase in α_H*, that is, a faster technology diffusion for Home, favors taxes in Home.*

These results are similar to what occurs in the single-economy setting as in Karp and Traeger (2024a), where quantity regulation typically exhibits an excessive responsiveness of the emissions price, and as a result slower diffusion of technology reduces the welfare loss under a quota. The effect of the relative size of Foreign, k, is primarily transmitted through μ, the marginal impact of technology level on the SCC. An increase in k leads to a higher μ, and a greater responsiveness of the SCC to technology shocks favors quotas for Home.

Equations A14, A15 in Appendix A.1 provide formulae for λ and μ, respectively. Changes in relative size affect how strongly technological innovations translate into future emissions and the stock, but do not change the sensitivity of marginal damages to the stock S_t. Since Foreign emissions are exogenous, technological innovations affect the social cost of carbon only through their effect on the future stock of emissions. As Home becomes larger (smaller k), its emissions account for a larger share of total emissions. The variability of the exogenous emissions from Foreign as a result of technological innovation is smaller. Technological innovation leads to a smaller change in the future pollution stock. Therefore, the corresponding shift in future marginal damages is smaller, which lowers the sensitivity of SCC to technology (lower μ).

This mechanism can be interpreted in terms of the uncertainty faced by Home. When Home's economy is smaller (larger k), a larger share of variation in the pollution stock is driven by Foreign's unregulated emissions. Any Home's emissions will amplify the variability of the pollution stock in

the future. As a result, it tends to overemphasize the uncertainty from future damages arising from fluctuations in the pollution stock to its own economy, instead of efficiency gains from allowing emissions to adjust flexibly to current shocks. Hence, Home minimizes the variation in its emissions to reduce uncertainty about future emissions. As shown in Section 3, however, this preference can shift when both economies actively implement carbon pricing policies.

3 Strategic Interaction Model

When both economies regulate emissions, each internalizes how its policy choice affects the volatility of the shared pollution stock and, in turn, the uncertainty faced by the other economy. As an economy becomes larger, its policy choice has a stronger effect on future stock dynamics and therefore on the volatility of future damages.

We extend the non-strategic dynamic model to a strategic framework in which each economy takes the other's instrument choice as given when solving its dynamic optimization problem. In this setting, Foreign adopts an active carbon pricing policy and responds to the policy instrument chosen by Home.

Let $C_H = (c_{1H}, c_{2H})$ and $C_F = (c_{1F}, c_{2F})$ denote the strategies chosen by Home and Foreign, respectively. The state vector Y_t is given by $Y_t = (S_t, \theta_{t-1})'$. Let k_H and k_F represent the relative sizes of the Home and Foreign, respectively, such that $k_H + k_F = 1$.

Home treats Foreign's strategy as fixed, so that Foreign's control is given by $C_F Y_t = c_{1F} S_t + c_{2F} \theta_{t-1}$. Symmetrically, Foreign takes Home's control $C_H Y_t = c_{1H} S_t + c_{2H} \theta_{t-1}$ as given. A Nash equilibrium for this non-cooperative game is a pair of policy rules (C_H, C_F) such that each is a best response to the other.

Given the structure of each economy's optimization problem, each regulator faces a linear–quadratic control problem under the assumption that the other economy employs a linear strategy (i.e., its optimal control is linear in state variables). The equilibrium of the two-economy game is characterized by a Markov-perfect Nash equilibrium in linear strategies.

To formulate the regulator's problem for both Home and Foreign, we again introduce indicator functions Φ_1 and Φ_2 for Home and Foreign, respectively:

$$\Phi_1 = \begin{cases} 1 & \text{if tax} \\ 0 & \text{if quota} \end{cases} \quad \text{and } \Phi_2 = \begin{cases} 1 & \text{if tax} \\ 0 & \text{if quota} \end{cases}.$$

Then Home's regulator's problem (for $i \in \{T, Q\}$) is :

$$\begin{aligned} &\max_{x^i_{H,t}} \mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^\tau \left[\rho\theta_{t+\tau-1} x^i_{H,t+\tau} - \frac{f_H}{2}(x^i_{H,t+\tau})^2 + \Phi_1 \frac{\alpha_H^2}{2f_H}\sigma^2 - \frac{k_H b}{2} S^2_{t+\tau} \right] \phi \\ &\text{subject to } S_{t+\tau+1} = \delta S_{t+\tau} + k_H \phi x^i_{H,t+\tau} + \phi k_F C_F Y_{t+\tau} + \Phi_1 k_H \phi \alpha_H \frac{\epsilon_{t+\tau}}{f_H} + \Phi_2 k_F \phi \alpha_F \frac{\epsilon_{t+\tau}}{f_F}, \\ &\qquad \theta_t = \rho\theta_{t-1} + \epsilon_t. \end{aligned} \tag{9}$$

Similarly, Foreign's regulator's problem (for $i \in \{T, Q\}$) is :

$$\begin{aligned} &\max_{x^i_{F,t}} \mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^\tau \left[\rho\theta_{t+\tau-1} x^i_{F,t+\tau} - \frac{f_F}{2}(x^i_{F,t+\tau})^2 + \Phi_2 \frac{\alpha_F^2}{2f_F}\sigma^2 - \frac{k_F b}{2} S^2_{t+\tau} \right] \phi \\ &\text{subject to } S_{t+\tau+1} = \delta S_{t+\tau} + k_F \phi x^i_{F,t+\tau} + \phi k_H C_H Y_{t+\tau} + \Phi_1 k_H \phi \alpha_H \frac{\epsilon_{t+\tau}}{f_H} + \Phi_2 k_F \phi \alpha_F \frac{\epsilon_{t+\tau}}{f_F}, \\ &\qquad \theta_t = \rho\theta_{t-1} + \epsilon_t. \end{aligned} \tag{10}$$

3.1 Policy Ranking

The Social Cost of Carbon for Home, which represents the marginal damage of the stock pollutant, is given by

$$SCC_{H,t} = \lambda_H S_t + \mu_H \theta_{t-1}.$$

Similarly, Foreign's Social Cost of Carbon is

$$SCC_{F,t} = \lambda_F S_t + \mu_F \theta_{t-1}.$$

We use the same procedure as outlined in Appendix A.1 to derive the expressions for control variables by writing the right side of these dynamic programming problems in terms of the control variables. Appendix A.2 provides details for solving for λ, the marginal impact of cumulative emissions S_t on the SCC, and μ, the derivative of the SCC with respect to the technology realization, in each economy. Equation A44 provides expressions for λ and μ. In contrast to the non-strategic setting, both parameters are endogenous functions of model primitives as well as equilibrium strategy coefficients.

By our assumption, the control variables take the forms of

$$\begin{aligned} x_{H,t} &= \psi_{1H} S_t + \psi_{2H} \theta_{t-1}, \\ x_{F,t} &= \psi_{1F} S_t + \psi_{2F} \theta_{t-1}. \end{aligned} \tag{11}$$

Substituting the expressions for λ and μ into the control variables, we obtain a system of nonlinear equations that can be solved for the strategy profiles C_F and C_H.

$$\begin{aligned} c_{1H} &= \psi_{1H}(\lambda_H, \beta, \delta, \phi, k_H, k_F, f_H, c_{1F}), \\ c_{2H} &= \psi_{2H}(\lambda_H, \beta, \mu_H, \rho, \phi, k_H, k_F, f_H, c_{2F}), \\ c_{1F} &= \psi_{1F}(\lambda_H, \beta, \delta, \phi, k_H, k_F, f_F, c_{1H}), \\ c_{2F} &= \psi_{2F}(\lambda_H, \beta, \mu_H, \rho, \phi, k_H, k_F, f_F, c_{2H}). \end{aligned} \tag{12}$$

As in the Unilateral policy model in Section 2, we define Δ_H and Δ_F to be the differences in payoffs under taxes and quotas for Home and Foreign, respectively. The following proposition characterizes Home's preference given Foreign's choice of policy instrument.

Proposition 3. *Given Foreign's choice of policy instruments, Home prefers taxes over quotas if and only if*

$$\Delta_H = \begin{cases} \dfrac{\alpha_H \sigma^2 (\alpha_H - 2\beta\mu_H)}{2 f_H (1-\beta)} - \dfrac{\alpha_H^2 \beta \lambda_H \phi^2 \sigma^2}{2 f_H^2 (1-\beta)} > 0 & \text{for } \Phi_2 = 0, \\ -\dfrac{\alpha_H^2 \beta \lambda_H \phi^2 \sigma^2}{2 f_H^2 (1-\beta)} - \dfrac{\alpha_H \phi \sigma^2 (2\beta f_F \mu_H - \alpha_H f_F + 2\alpha_F \beta \lambda_H \phi)}{2 f_F f_H (1-\beta)} > 0 & \text{for } \Phi_2 = 1. \end{cases} \tag{13}$$

As shown in Proposition 3, conditional on Foreign's policy choice, λ_H and μ_H have the same directional effects on Home's policy ranking. When λ_H or μ_H increase, any given technology shock shifts not only its current and future marginal abatement cost curve but also its future marginal damage curve by a greater amount, favoring quotas. Intuitively, when a large economy adopts a quantity instrument, its current period emissions no longer respond to short-run technology shocks, reducing the stock uncertainty. This shifts the remaining uncertainty faced by the other economy toward abatement costs, increasing the relative attractiveness of a tax. Corollary 1 formally captures the strategic substitution in policy instruments, showing that Foreign's adoption of a tax shifts Home's policy ranking toward quotas. This dynamic gives rise to asymmetric equilibrium instrument choices. The mechanism differs from the unilateral setting, where relative size primarily governs exposure to stock volatility generated abroad.

Corollary 1. *Suppose $\phi = 1$. Home's payoff advantage of taxes is strictly lower when Foreign adopts a tax than when Foreign adopts a quota. In particular,*

$$\Delta_{H,\Phi_2=0} - \Delta_{H,\Phi_2=1} = \frac{\alpha_H \alpha_F \beta \lambda_H}{f_H f_F (1-\beta)} \sigma^2 \; > \; 0.$$

The complexity of this dynamic programming problem, particularly with the inclusion of strategic parameters, makes it difficult to obtain analytical characterization for such equilibrium instrument choices. Instead, we solve the model numerically to offer an accurate empirical description. To ensure the model's economic relevance, we carefully select parameter values that reflect a meaningful context.

4 Calibration and Numerical Results

The solutions for the strategic model described in section 3 is many times more complicated than that of the unilateral model described in section 2. Therefore, we calibrate the dynamic model and present the numerical results.

4.1 Baseline Parameter Values

We adopt similar baseline parameter values used in Karp and Traeger (2024a), which calibrates their model to reasonable climate change scenarios using DICE 2013 (Nordhaus and Sztorc 2013) and IPCC (2014). Table 1 summarizes the baseline values for the key parameters in the model, the way they are calibrated and the range that they vary in our numerical calculation.

Marginal abatement cost f and marginal damage b are calibrated using DICE 2013. Setting abatement at 75% of the optimal level in DICE, we obtain values for marginal abatement costs during the period 2015 - 2050. Fitting the average of these values to the linear marginal abatement cost function we obtain f = 1.846 in units of billion dollars per billion tonnes of CO_2 (G\$/Gt CO_2).

To calculate the damage parameter b we set annual Gross World Product (GWP) to the IMF's 2016 estimate of 120 trillion dollars using purchasing power parity. In DICE's climate model, an increase of atmospheric carbon dioxide by 1270 Gt CO_2 over the preindustrial level implies a medium to long-run temperature increase of 2°C. DICE assumes that this temperature increase lowers output by approximately 1%. This calibration assumption implies $b \approx 0.0015$ (G\$/Gt CO_2).

For α, we follow Karp and Traeger (2024b), who estimate technological diffusion by regressing US CO_2 emissions in 1995-2010 on (stocks and flows of) green patents. Their preferred estimate is slightly above .25. We use $\alpha = .25$ as our baseline. We use the annual discount factor $\beta = 0.98$, consistent with the median 2% discount rate in the recent expert survey by Drupp et al. (2018). We assume that technological innovation is highly persistent and use $\rho = .99$. We calibrate the persistence of atmospheric carbon to Joos et al.'s (2013) model for carbon removal from the atmosphere, adopted in the 5th Assessment Report of the IPCC (2014). A least square fit over 1000 years delivers an annual removal rate of 0.3%, or $\delta = 0.997$. In the results presented below, we use $\phi = 1$[3] and $\sigma^2 = 1$[4].

[3]Using a different value of ϕ would not change the results qualitatively.

[4]For the set of results presented below, σ^2 only affects Δ. We can read the scale for Δ as in units of σ^2.

4.2 Numerical Results

To investigate how changes in the key parameters influence the equilibrium of the dynamic model, we solve the two-economy problem numerically by varying each parameter while holding others at the baseline values.

The first set of results, presented in Figure 1, concerns the size parameter k_H, with the other parameters at baseline values and identical for Home and Foreign. Panel (a) shows c_1's, the coefficients for Home and Foreign regulator's optimal response to cumulative emissions; Panel (b) shows c_2's, the response to technology level observed by the regulators at the onset of a regulatory period; Panel (c) shows the λ's, the marginal impact of cumulative emissions S_t on the SCC; and Panel (d) shows the μ's, the marginal impact of technology level θ_t on the SCC. Since the two economies have the same parameters except for their relative sizes, each set of coefficients are symmetric about $k_H = .50$, at which point the two economies are identical. As the relative size of an economy increases, its regulator becomes more responsive to cumulative emissions (i.e., c_1 becomes more negative) and less responsive to the observed technological level (i.e., c_2 becomes less positive). Consistently, λ becomes more positive, as a larger economy suffers more from future damage of cumulative emissions. Meanwhile, μ initially becomes more positive, but then peaks and decreases.

Panels (e) and (f) show the value of Δ_H for $\alpha = .25$ and $.50$ respectively. The blue curves represent Δ_H when Foreign chooses quota, and the red curves represent Δ_H when Foreign chooses tax. When Foreign chooses tax, it introduces additional uncertainty in the cumulative emissions in future periods associated with the technology innovation, thereby favoring quota for Home[5]. We observe that when the economy is relatively small, tax is the preferred instrument. As the relative size becomes larger, Δ_H decreases and becomes negative, leaving quota the preferred instrument. With some high α values, tax will be preferred by Home with small and large k_Hs, with quota the preferred instrument in an intermediate range of k_H.

[5]Equation A54 in Appendix A.2 shows the difference between $\Delta_{H,\Phi_F=0}$ and $\Delta_{H,\Phi_F=1}$. This difference increases with λ, which is increasing in k_H

The instrument preference for Foreign is not depicted directly in the figures. However, Δ_F and Δ_H must be symmetric about $k_H = .50$ in a similar fashion to the series in Panels (a) through (d). We can then use the information provided in Panel (e) to find the Nash equilibrium of the non-cooperative game between Home and Foreign when $\alpha = .25$. When k_H is small, Home prefers tax ($\Delta_H > 0$) and Foreign prefers quota ($\Delta_F < 0$) regardless of the instrument of choice of the other economy; therefore, Home choosing tax and Foreign choosing quota ($\Phi_H = 1$ and $\Phi_F = 0$, we denote it by T-Q) is a Nash equilibrium for the non-cooperative game. As k_H increases, Δ_Hs decrease and intersect with the zero line. Since Foreign still prefers quota ($\Delta_F < 0$ regardless of Φ_H) at the relevant values of k_H, the relevant series is the blue curve ($\Delta_{H,\Phi_F=0}$). After it hits zero, quota-quota (Q-Q) becomes the equilibrium. By symmetry, after ($\Delta_{F,\Phi_H=0}$) turns positive, Q-T becomes the equilibrium. To summarize, for small values of k_H, T-Q is the equilibrium; for intermediate values of k_H, Q-Q is the equilibrium; and for large values of k_H, Q-T is the equilibrium. This pattern remains for $\alpha = .50$, but the intermediate range with Q-Q being the equilibrium is now smaller. Larger values of α will lead to T-T being the intermediate equilibrium. α close to 1 will have both economies preferred tax at all times.

The previous discussion suggests that a small α, or slow diffusion of technology, tends to favor quota. The next set of results concern in more detail with the effect of α on instrument preference. As shown in Appendix A.2, the α's do not factor into the solution to the dynamic problem, i.e., they do not affect c_1, c_2, λ or μ. Rather, they appear in the value function, and therefore the expressions for Δs. Figure 2 presents the numerical results when α_H and α_F vary together. Panels (a), (b) and (c) show the value of Δ_H when $k_H = .25, .50$ and $.75$ respectively. Same as previously, the blue curves represent Δ_H when Foreign chooses quota, and the red curves represent Δ_H when Foreign chooses tax. In each case, $\Delta_H = 0$ when $\alpha = 0$; decreases as α increases from 0; reaches a minimum and then increases as α increases further. The value of α where the minimum of Δ_H is attained seems to be larger with a larger k_H[6]. When $\alpha = 0$, there is in fact no uncertain adoption of technological innovation that is not observed by the regulator, therefore the price and quantity instruments are equivalent. Small values of α favor quota while larger values favor tax.

[6]This is not always the case when k_H is over a certain value. As shown in Panels (e) and (e) of Figure 1, Δ_H is not monotonic with respect to k_H

We can then characterize the equilibrium of the non-cooperative game between Home and Foreign in choosing pricing instruments. Using the same logic for symmetry discussed previously, we can view Panels (a), (b) and (c) as depicting Δ_F when $k_H = .75, .50$ and $.25$ respectively. When $k_H = .25$, Q-Q is the equilibrium with low values of α, as both Δ_H (Panel (a)) and Δ_F (Panel (c)) are negative; T-T is the equilibrium with high values of α. In the intermediate values of α between where the blue curve in Panel (a) crosses zero and where the red curve in Panel (c) crosses zero, T-Q is the equilibrium. This pattern applies to low values of k_H. With high values of k_H, the intermediate equilibrium is Q-T. When $k_H = .50$, the intermediate range is between the points at which the blue and red curves intersect the zero line in Panel (b). In this small range, there are multiple equilibria in Q-T and T-Q. Similar multiple equilibria exist when the segments of α between the crossing of the blue and red curves and the zero line for Home and Foreign overlap each other.

Figure 3 partitions (k_H, α) and provides a graphical representation of the equilibrium characterization discussed above. At extremes when $k_H = 0$ or $k_H = 1$, the setting effectively reduces to the case of a global planner as in Karp and Traeger (2024a), where the speed of diffusion affects the ranking of the two carbon pricing policies. When α is close to 1, taxes always dominate quotas. For a larger Home economy, α does not need to be particularly low for quotas to remain advantageous. Conversely, for a smaller economy (lower k_H), taxes will dominate unless diffusion is sufficiently slow. The same logic applies symmetrically to Foreign. Along the boundaries of these regions, multiple equilibria emerge.

4.3 Discussion

We find both similarities and important differences between the strategic interaction model and the unilateral policy setting. Notably, the speed of technology diffusion plays a similar role in the ranking of taxes versus quotas. Slower technology diffusion (smaller α_H) tends to favor quotas, as shown in Figure 2.

Where the strategic setting differs from the unilateral case is in the role of relative economic size. Importantly, while μ_H and λ_H are independent of α_H, their dependence on Home's relative size differs across policy environments. In a noncooperative strategic environment, an increase in

k_H raises both λ_H and μ_H. As established in Proposition 3, higher values of either μ_H or λ_H favor quantity regulation. Proposition 2(a) establishes the same ranking logic in the unilateral setting. However, Equation A15 shows that, in that case, an increase in Home's relative size reduces μ_H, thus favoring taxes.

In the strategic setting, the intuition must therefore be adjusted to account for the spillover of uncertainty through global abatement. When Home becomes larger (larger k_H), it accounts for a larger share of total emissions. Because technology persists, the shock-induced changes in its future emissions become larger as Home is larger. Home's adoption of a quantity instrument reduces the stock uncertainty faced by Foreign, leaving its remaining uncertainty primarily about marginal abatement costs. Similarly, when Foreign commits to a quota, its emissions no longer respond to the common technology shock, and the stock variation driven by Foreign disappears. The remaining uncertainty faced by Home is then primarily about its own current abatement costs, favoring taxes. When Foreign instead uses a tax, Foreign's firms also adjust emissions in response to the shock. This additional emission response amplifies variation in the future pollution stock, increasing the shift in Home's SCC relative to its marginal abatement cost shift, and tilting Home's ranking toward quotas. As a result, Home's social cost of marginal emissions responds more strongly to technology realization (higher μ_H).

In addition, a larger Home economy bears a larger share of future damages from the stock and its SCC becomes more sensitive to the current stock S_t (higher λ_H). Because a larger Home internalizes a larger share of future damages from changes in the pollution stock, the marginal impact of Home's current emissions on the future stock trajectory matters more for Home. Home thus has a stronger incentive to stabilize the stock, thereby strengthening the case for quotas.

This intuition also helps clarify what happens as Home becomes sufficiently large. As $k_H \to 1$, Foreign's emissions become negligible, our model converges to the single-regulator benchmark of Karp and Traeger (2024b). Foreign's choice of policy instruments has a vanishing effect on pollution stock and effectively behaves as if setting a "zero tax". Uncertainty spillovers through Foreign's policy choice thus become negligible, and the dependence of the responsiveness of marginal damages

on relative economic size vanishes [7]. Home's instrument choice then follows the familiar ranking criterion, under which persistent shocks and slower technology diffusion favor quantity regulation.

5 Conclusion

We revisit the classic question of "prices versus quantities," and extend Karp and Traeger (2024a) and Mideksa and Weitzman (2019) to a transboundary stock pollutant in an international setting with heterogeneous countries that differ in their shares of global emissions. We present a dynamic, game-theoretic model to illustrate the effects of technology diffusion and relative sizes of economies on the ranking criterion of carbon pricing instruments. In addition, we contrast a unilateral policy environment, in which only one economy regulates emissions, with a strategic setting where both economies simultaneously pursue regulation of a transboundary stock pollutant featuring correlated technology shocks.

Our theoretical model and numerical results suggest that once the analysis is extended to an international setting, the ranking of taxes versus quotas also depends crucially on the relative size of economies and the nature of policy interactions across countries. With a stock pollutant, instrument choice becomes interdependent across economies. Relative size enters through differences in exposure to, and influence over, fluctuations in the pollution stock, affecting how uncertainty about abatement costs and uncertainty about emissions are weighed in instrument choice. In a unilateral setting, size primarily determines an economy's exposure to uncertainty about emissions generated abroad: a small regulating economy faces substantial volatility in the stock pollutant driven by foreign emissions and may prefer quotas to limit the variability of its own emissions. In a strategic setting, size instead determines an economy's ability to influence the volatility of the pollution stock: a large economy internalizes a larger share of future damages from changes in the pollution stock and can reduce stock uncertainty by committing to a quantity instrument. This results in asymmetric equilibrium instrument choices across economies of different sizes.

[7] When $k_H \to 1$, the policy response coefficients satisfy $c_{1H}, c_{2H} \to 0$ and $c_{1F}, c_{2F} \to 1$, as shown in Panels (a) and (b) of Figure 1. Substituting these values into the expressions for λ_H and μ_H implies that they converge to the corresponding closed-form solutions in the single-regulator problem.

These results are policy relevant for the effectiveness of international climate agreements. As countries around the world move to design their domestic policies to achieve their commitment under the Paris Agreement, policymakers may adopt different instruments for carbon pricing. It will be interesting to compare the instruments of choice of countries of varying sizes. Additionally, the Technology Mechanism in the Paris Agreement may help innovation and transfer of climate-friendly technology, and it may tip the scale of quantity versus price instruments under uncertainty. These insights carry significant policy implications in light of the United States' withdrawal from the Paris Agreement for a second time and the Environmental Protection Agency's move to rescind the 2009 endangerment finding under the Clean Air Act. With the U.S. no longer participating at the federal level, the global climate regime could shift toward a different equilibrium in the absence of a large economy's active participation in international climate policy. When a large emitter exits climate regulation, its emissions become an exogenous and potentially more volatile component of the global pollution stock, which increases the shadow value of the stock faced by remaining regulators. Relative to the strategic equilibrium, remaining countries may therefore "over-regulate," in the sense of choosing a higher effective domestic carbon price, as a partial offset to higher expected future marginal damages.

In our analysis, we have made a number of simplifying assumptions. In particular, we work within a linear–quadratic framework, which yields linear policy rules in equilibrium. These assumptions are made for tractability. Relaxing these assumptions would generally lead to nonlinear equilibrium policy rules. However, the core mechanism should remain intact: uncertainty spills over across countries through global abatement, so that one country's instrument choice affects not only current-period abatement but also the volatility and persistence of future damages faced by other countries. We also recognize that the model is missing some important features of the world economy. In particular, there is no trade between countries, with the only connection through the global bad of emissions. Such issues need to be addressed in future research.

References

Aldy, Joseph E. and Robert N. Stavins eds. (2007) *Architectures for Agreement: Addressing Global Climate Change in the Post-Kyoto World*: Cambridge University Press.

Aldy, Joseph, Alan Krupnick, Richard Newell, Ian Parry, and William Pizeer (2010) "Designing Climate Mitigation Policy," *Journal of Economic Literature*, Vol. 48, No. 4, pp. 903–934.

André, Francisco J. and Luis M. de Castro (2017) "Scarcity Climate Rents under a Carbon Price with Oligopoly Competition," *Oxford Economic Papers*, Vol. 69, No. 2, pp. 470–495.

Barrett, Scott (2005) "The Theory of International Environmental Agreements," *Handbook of Environmental Economics*, Vol. 3, pp. 1457–1516.

Calvo, Emilio and Santiago Rubio (2012) "Dynamic Models of International Environmental Agreements: A Differential Game Approach," *International Review of Environmental and Resource Economics*, Vol. 6, pp. 289–339.

Coulomb, Renaud and Fanny Henriet (2011) "Carbon Price and Optimal Extraction of a Polluting Fossil Fuel with Restricted Carbon Capture," Working Paper 322, Banque de France.

Drupp, Moritz A., Mark C. Freeman, Ben Groom, and Frikk Nesje (2018) "Discounting Disentangled," *American Economic Journal: Economic Policy*, Vol. 10, No. 4, pp. 109–34.

Eichner, Thomas and Rüdiger Pethig (2015) "Self-enforcing international environmental agreements and trade: taxes versus caps," *Oxford Economic Papers*, Vol. 67, No. 4, pp. 897–917.

Gerlagh, Reyer and Roweno Heijmans (2021) "Regulating Stock Externalities," Working Paper 7383, CESifo.

Gerlagh, Reyer and Roweno Wan (2018) "Optimal Stabilization in an Emission Permits Market," Working Paper 6950, CESifo.

Goulder, Lawrence and Andrew Schien (2013) "Carbon Taxes Versus Cap and Trade: A Critical Review," *Climate Change Economics*, Vol. 4, No. 3, pp. 1–28.

Gries, Thomas (2000) "Climate and Industrial Policy in an Asymmetric World," Working Paper 2011/079, UNU-WIDER.

Hansen, Lars and William Brock (2018) "Wrestling with Uncertainty in Climate Economic Models," No. 2019-71.

Harstad, Bård, Francesco Lancia, and Alessia Russo (2022) "Prices vs. quantities for self-enforcing agreements," *Journal of Environmental Economics and Management*, Vol. 111 (102595).

Hepburn, Cameron (2006) "Regulation by Prices, Quantities, or Both: A Review of Instrument Choice," *Oxford Review of Economic Policy*, Vol. 22, No. 2, pp. 226–247.

Hoel, Michael and Larry Karp (2000) "Taxes and Quotas for a Stock Pollutant with Multiplicative Uncertainty," Working Paper 25030, CUDARE.

——— (2002) "Taxes versus quotas for a stock pollutant," *Resource and Energy Economics*, Vol. 24, pp. 367–384.

IPCC (2014) *Climate Change 2014: Synthesis Report. Contribution of Working Groups I, II and III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change*, IPCC, Geneva, Switzerland.

——— (2018) *Summary for Policymakers. In: Global Warming of 1.5°C. An IPCC Special Report on the impacts of global warming of 1.5°C above pre-industrial levels and related global greenhouse gas emission pathways, in the context of strengthening the global response to the threat of climate change, sustainable development, and efforts to eradicate poverty*, IPCC, Geneva, Switzerland.

——— (2023) *Climate Change 2023 Synthesis Report*, IPCC, Geneva, Switzerland.

Joos, F. et al. (2013) "Carbon dioxide and climate impulse response functions for the computation of greenhouse gas metrics: a multi-model analysis," *Atmospheric Chemistry and Physics*, Vol. 13, No. 5, pp. 2793–2825.

Jørgensen, Steffen, Guiomar Martín-Herrán, and Georges Zaccour (2010) "Dynamic Games in the Economics and Management of Pollution," *Environmental Modeling & Assessment*, Vol. 15, pp. 433–467.

Kaplow, Louis (2010) "Taxes, Permits, and Climate Change," Working Paper 16268, National Bureau of Economic Research.

Karp, Larry, Sauleh Siddiqui, and Jon Strand (2016) "Dynamic Climate Policy with Both Strategic and Non-strategic Agents: Taxes Versus Quantities," *Environmental & Resource Economics,*, Vol. 65, No. 1, pp. 135–158.

Karp, Larry and Christian Traeger (2024a) "Prices Versus Quantities Reassessed," *Journal of Environmental Economics and Management*, Vol. 125, p. 102951.

——— (2024b) "Smart Cap," *Journal of the European Economic Association*, Advance online publication.

Karp, Larry and Jiangfeng Zhang (2005) "Regulation of Stock Externalities with Correlated Abatement Costs," *Environmental & Resource Economics*, Vol. 32, pp. 273–299.

——— (2012) "Taxes versus quantities for a stock pollutant with endogenous abatement costs and asymmetric information," *Economic Theory*, Vol. 49, No. 2, pp. 371–409.

Kornek, Ulrike and Robert Marschinski (2018) "Prices vs quantities for international environmental agreements," *Oxford Economic Papers*, Vol. 70, No. 4, pp. 1084–1107.

Lapan, Harvey E. and Shiva Sikdar (2022) "Strategic Environmental Policy and International Market Share Rivalry under Differentiated Bertrand Oligopoly," *Oxford Economic Papers*, Vol. 74, No. 1, pp. 215–235.

Lopez, Ramon and Sang Yoon (2013) "On optimal emission control – Taxes, substitution and business cycles," Article 24, The Bank of Finland.

——— (2014) "Environmental Sustainability with a Pollution Tax."

Marrouch, Walid and Amrita Ray Chaudhuri (2016) "International Environmental Agreements: Doomed to Fail or Destined to Succeed? A Review of the Literature," *International Review of Environmental and Resource Economics*, Vol. 9, No. 3–4, pp. 245–319.

Mideksa, Torben and Martin Weitzman (2019) "Prices versus Quantities across Jurisdictions," *Journal of the Association of Environmental and Resource Economists*, Vol. 6, No. 5, pp. 883–891.

Mohajan, Haradhan (2012) "Certainty and Uncertainty in Cap and Trade System or in Carbon Tax for Green Accounting to Decrease Greenhouse gas Emissions," *Indus Journal of Management & Social Sciences*, Vol. 6, No. 2, pp. 108–122.

Montero, Juan-Pablo (2002) "Prices versus quantities with incomplete enforcement," *Journal of Public Economics*, Vol. 85, pp. 435–454.

Newell, Richard and William Pizer (2003) "Regulating stock externalities under uncertainty," *Journal of Environmental Economics and Management*, Vol. 45, pp. 416–432.

Nordhaus, William (2008) *A Question of Balance: Weighing the Options on Global Warming Policies.*, New Haven; London: Yale University Press.

Nordhaus, William and Paul Sztorc (2013) *DICE 2013R: Introduction and User's Manual.*

Pizer, William and Brian Prest, "Prices versus Quantities with Policy Updating," *Journal of the Association of Environmental and Resource Economists*, Vol. 7, No. 3, pp. 483–518.

Strand, Jon (2010) "Taxes versus Cap-and-Trade in Climate Policy when only some Fuel Importers Abate," Working Paper 3233, CESifo.

Van Long, Ngo (2012) "Applications of Dynamic Games to Global and Transboundary Environmental Issues: A Review of the Literature," *Strategic Behavior and the Environment*, Vol. 2, No. 1, pp. 1–59.

Weitzman, Martin (1974) "Prices vs. Quantities," *The Review of Economic Studies*, Vol. 41, No. 4, pp. 477–491.

Weitzman, Martin L. (2020) "Prices or Quantities Can Dominate Banking and Borrowing," *The Scandinavian Journal of Economics*, Vol. 122, No. 2, pp. 437–463.

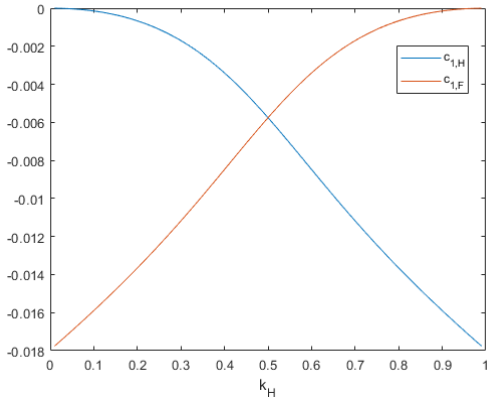
Wirl, Franz (2007) "Energy Prices and Carbon Taxes under Uncertainty about Global Warming," *Environmental & Resource Economics*, Vol. 36, No. 3, pp. 313–340.

World Bank (2023) *State and Trends of Carbon Pricing 2023*, Washington, DC: World Bank.

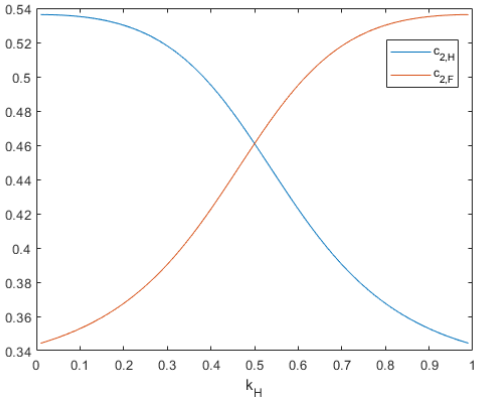
Figures and Tables

Table 1: Baseline Parameter Values and Range in Simulation

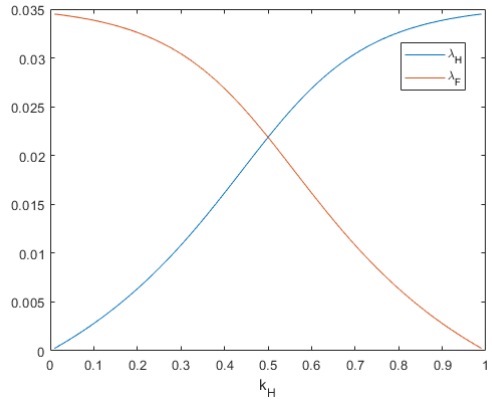
Parameter	Method/Source	Baseline Value	Range
f	DICE model	1.846	0.01-3
b	DICE model + IMF	0.0015	0.0003-0.0099
α	Karp and Traeger (2024b)	0.25	0.01-0.99
β	Drupp et al. (2018)	0.98	0.90-0.99
δ	Joos et al. (2013)	0.997	0.01-0.99
ρ	by assumption	0.99	0.01-0.99
k_H	by assumption	0.5	0.01-0.99



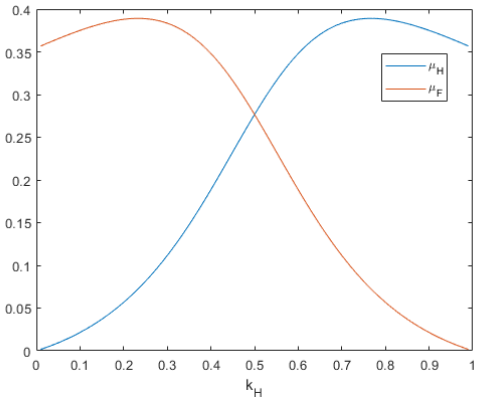
(a) $c_{1,H}$ and $c_{1,F}$



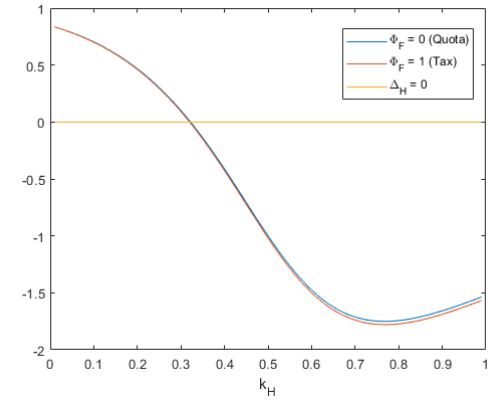
(b) $c_{2,H}$ and $c_{2,F}$



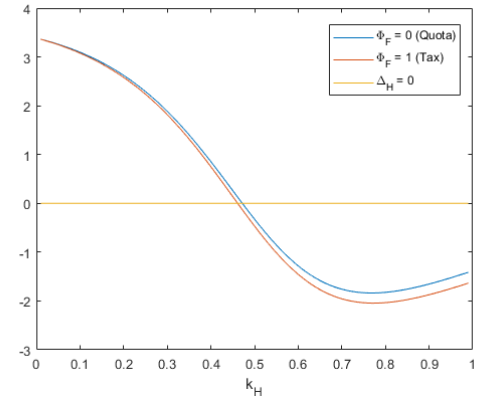
(c) λ_H and λ_F



(d) μ_H and μ_F



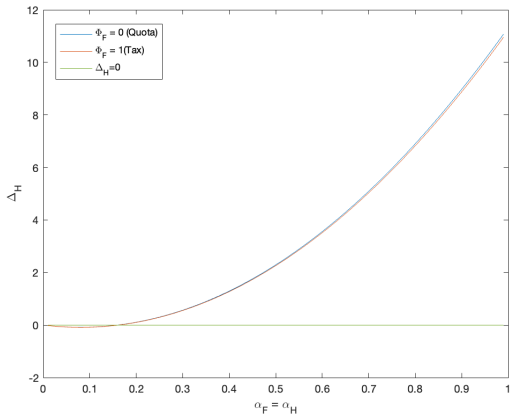
(e) $\Delta_H(\alpha = .25)$



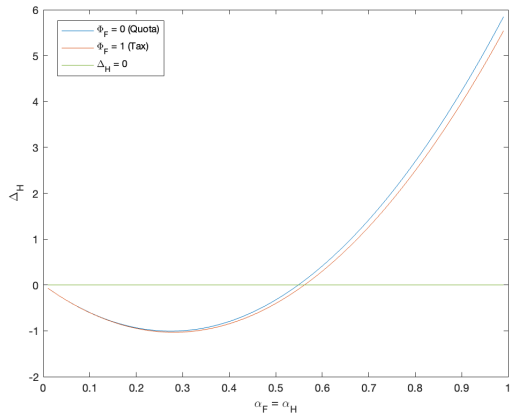
(f) $\Delta_H(\alpha = .5)$

Figure 1: Numerical Results as k_H Varies.

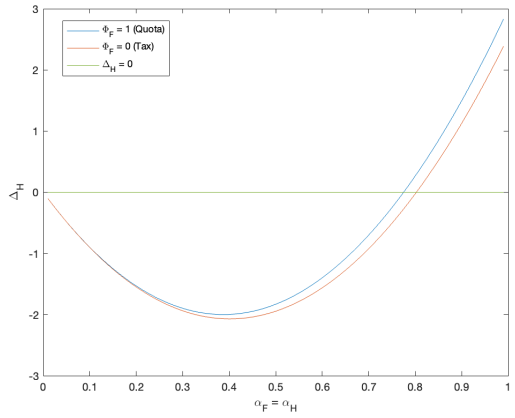
Note: Unless otherwise specified, all parameters take baseline values as listed in Table 1.
Source: Authors' calculation.



(a) Δ_H ($k_H = .25$)



(b) Δ_H ($k_H = .5$)



(c) Δ_H ($k_H = .75$)

Figure 2: Numerical Results as α Varies.

Note: Unless otherwise specified, all parameters take baseline values as listed in Table 1. α_H and α_F assume the same value in this set of calculation.
Source: Authors' calculation.

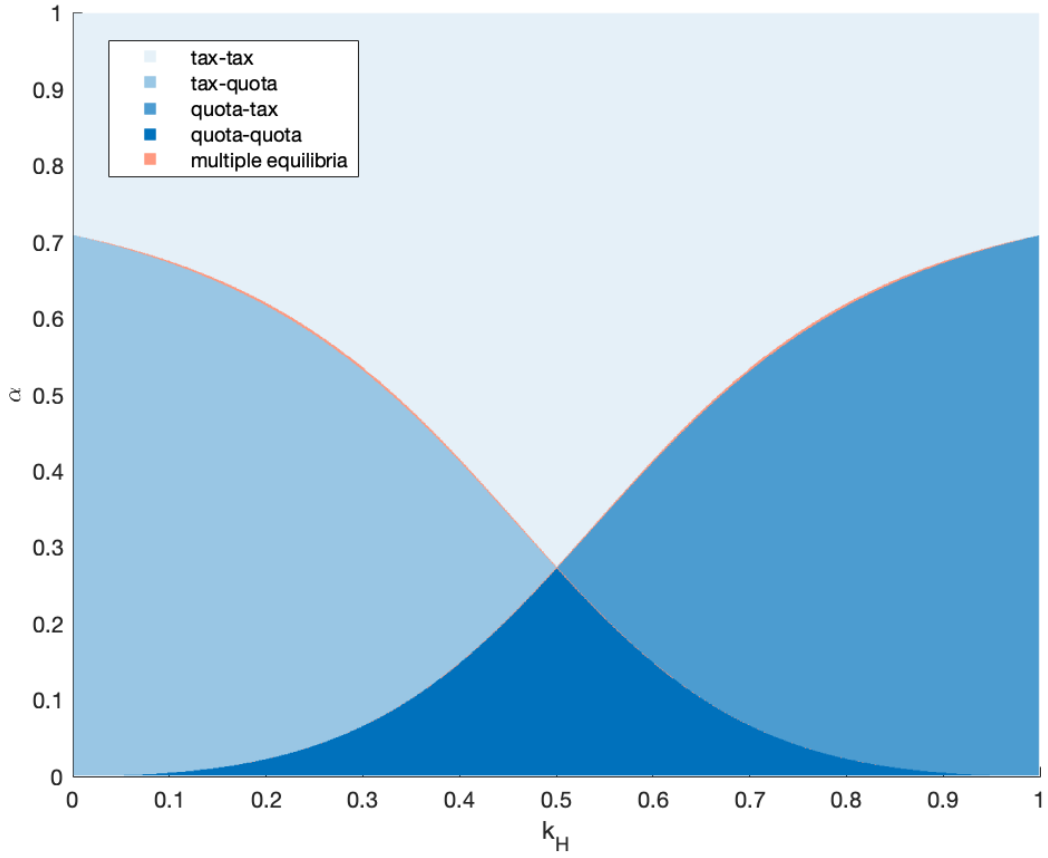


Figure 3: Equilibrium Results as α and k_H Varies.

Note: Unless otherwise specified, all parameters take baseline values as listed in Table 1. α_H and α_F assume the same value in this set of calculation.
Source: Authors' calculation.

A Appendix

A.1 Proofs for Section 2

Proof for Lemma 1. The dynamic programming equation for the generic problem is:

$$J_t(Y_t) = \max_{x_{H,t}}[-\frac{f_H}{2}x_{H,t}^2 + \frac{1}{2}Y_t'QY_t + WY_tx_{H,t} + \Phi\frac{\alpha_H^2}{2f_H}\sigma^2]\phi + \beta\mathbb{E}_tJ_{t+1}^i(Y_{t+1}) \tag{A1}$$

The value function for the LQ problem is

$$J_t^i(Y_t) = V_0^i + \frac{1}{2} Y_t' V_2 Y_t \tag{A2}$$

where

$$V_2 = -\begin{bmatrix} \lambda & \mu \\ \mu & \nu \end{bmatrix}. \tag{A3}$$

The right side of the DPE is

$$[-\frac{f_H}{2} x_{H,t}^2 + \frac{1}{2} Y_t' Q Y_t + W Y_t x_{H,t} + \Phi \frac{\alpha_H^2}{2 f_H} \sigma^2]\phi + \beta \mathbb{E}_t (V_{0,t+1} + \frac{1}{2} Y_{t+1}' V_2 Y_{t+1}) \tag{A4}$$

Substituting the equations of motion and take expectation, we have:

$$\begin{aligned} (-\frac{f_H}{2} x_{H,t}^2 + \frac{1}{2} Y_t' Q Y_t + W Y_t x_{H,t} + \Phi \frac{\alpha_H^2}{2 f_H} \sigma^2)\phi + \beta [V_{0,t+1} + \\ \frac{1}{2} (A Y_t + B x_{H,t})' V_2 (A Y_t + B x_{H,t}) + \frac{1}{2} \sigma^2 C' V_2 C] \end{aligned} \tag{A5}$$

The FOC is:

$$\phi(-f_H x_{1,t} + W Y_t) + \beta (B' V_2 B x_{H,t} + B' V_2 A Y_t) \tag{A6}$$

$$\implies \text{(control rule) } x_t = \frac{1}{f_H \phi - \beta (B' V_2 B)} (W\phi + \beta B' V_2 A) Y_t \tag{A7}$$

Rewrite x_t in terms of Z and Y_t such that

$$Z = \frac{W\phi + \beta B' V_2 A}{f_H \phi - \beta (B' V_2 B)} \tag{A8}$$

Then, we have $x_t = Z Y_t$.

Substituting the control rule into the expectation of the r.h.s of the DPE, we have

$$[-\frac{1}{2}f_H Y_t' Z' Z Y_t + \frac{1}{2}Y_t' Q Y_t + W Y_t Z Y_t + \Phi\frac{\alpha_H^2}{2f_H}\sigma^2]\phi + \tag{A9}$$

$$\beta(V_{0,t+1}^i + \frac{1}{2}(AY_t + BZY_t)'V2(AY_t + BZY_t)) + \frac{1}{2}C'V_2C\sigma^2) \tag{A10}$$

The terms quadratic in Y_t are:

$$\frac{1}{2}Y_t'[(Q - f_H Z'Z + W'Z + Z'W)\phi + \beta(A + BZ)'V_2(A + BZ)]Y_t \tag{A11}$$

So we have

$$V_2 = [(Q - f_H Z'Z + W'Z + Z'W)\phi + \beta(A + BZ)'V_2(A + BZ)] \tag{A12}$$

Then, solving for λ, we obtain

$$\lambda = \frac{(b^2\beta^2\phi^4 + 2b\beta^2\delta^2 f_H\phi^2 + 2b\beta f_H\phi^2 + \beta^2\delta^4 f_H^2 - 2\beta\delta^2 f_H^2 + f_H^2)^{1/2} - f_H + \beta\delta^2 f_H + b\beta\phi^2}{(2\beta\phi)} \tag{A13}$$

Let $\overline{\omega} \equiv f_H(1 - \beta\delta^2 - \beta\frac{b}{f_H}\phi^2)$, the positive root of is

$$\lambda = \frac{1}{2\beta\phi}(-\overline{\omega} + \sqrt{\overline{\omega^2} + 4\beta\phi^2 b f_H}). \tag{A14}$$

□

Similarly we can write μ as follows:

$$\mu = \beta\delta\frac{\rho}{f_H + \beta\lambda\phi}(f_H\mu + \lambda\phi + \frac{f_H}{f_F}k\lambda\phi) \implies \mu = \frac{(\beta\delta\lambda\phi\rho(f_F + f_H k))}{(f_F(f_H + \beta\lambda\phi - \beta\delta f_H\rho))} \tag{A15}$$

Hence we have

$$\mu = \beta\delta\rho\frac{\lambda}{f_H}\frac{\phi}{1-\beta\delta\rho+\beta\frac{\lambda}{f_H}\phi}(1+\frac{f_H}{f_F}k) \tag{A16}$$

Proof for Proposition 1. We know that

$$V_0^i = (\Phi\frac{\alpha_H^2}{2f_H}\sigma^2)\phi + \beta[\frac{1}{2}\sigma^2 C' V_2 C] \tag{A17}$$

Then we have

$$\Delta = V_0^T - V_0^Q \tag{A18}$$

$$= \frac{\alpha_H^2}{2f_H}\sigma^2\phi + \beta\Delta - \frac{1}{2}\beta\sigma^2(\begin{bmatrix} k\phi\frac{\alpha_F}{f_F}+\phi\frac{\alpha_H}{f} & 1\end{bmatrix}\begin{bmatrix}\lambda & \mu\\ \mu & \nu\end{bmatrix}\begin{bmatrix} k\phi\frac{\alpha_F}{f_F}+\phi\frac{\alpha_H}{f}\\ 1\end{bmatrix} - \begin{bmatrix} k\phi\frac{\alpha_F}{f_F} & 1\end{bmatrix}\begin{bmatrix}\lambda & \mu\\ \mu & \nu\end{bmatrix}\begin{bmatrix} k\phi\frac{\alpha_F}{f_F}\\ 1\end{bmatrix}) \tag{A19}$$

$$= \frac{\alpha_H^2}{2f_H}\sigma^2\phi + \beta\Delta - \frac{1}{2}\beta\sigma^2\phi\alpha_H\frac{2f_F f_H\mu + 2\alpha_F f_H k\lambda\phi + \alpha_H f_F\lambda\phi}{f_F f_H^2} \tag{A20}$$

Therefore, the difference between taxes and quotas is

$$\Delta = \frac{1}{1-\beta}\frac{\alpha_H\phi}{2f_H}\sigma^2(\alpha_H - \beta\frac{2f_F f_H\mu + 2\alpha_F f_H k\lambda\phi + \alpha_H f_F\lambda\phi}{f_F f_H}). \tag{A21}$$

Using the definition $R \equiv \frac{\lambda}{f_H}\phi$, we obtain

$$\Delta = \frac{1}{1-\beta}\frac{\alpha_H\phi}{2f_H}\sigma^2(\alpha_H - \beta(2\mu + 2\frac{\alpha_F k\lambda\phi}{f_F} + \alpha_H R)). \tag{A22}$$

This equation implies taxes dominate quotas if and only if

$$\alpha_H - \beta(2\mu + 2\frac{\alpha_F k\lambda\phi}{f_F} + \alpha_H R) > 0. \tag{A23}$$

So

$$R < \frac{1}{\beta} - \frac{2\mu}{\alpha_H} - 2\frac{\alpha_F k\lambda\phi}{f_F\alpha_H}. \tag{A24}$$

Using the definition $R \equiv \frac{\lambda}{f_H}\phi$ and $\mu = \frac{\beta\delta\rho R}{(1-\beta\delta\rho)+\beta R}(1 + \frac{f_H}{f_F}k)$, we have

$$\alpha_H - \beta(2\frac{\beta\delta\rho R}{(1-\beta\delta\rho)+\beta R}(1 + \frac{f_H}{f_F}k) + 2\frac{\alpha_F k\lambda\phi}{f_F} + \alpha_H R) > 0. \tag{A25}$$

Note that $\frac{\alpha_F\lambda\phi}{f_F} = \frac{\alpha_F R f_H}{f_F}$.

Then,

$$\begin{aligned}
&\alpha_H - \beta(2\frac{\beta\delta\rho R}{(1-\beta\delta\rho)+\beta R}(1 + \frac{f_H}{f_F}k) + 2k\frac{\alpha_F f_H}{f_F}R + \alpha_H R) > 0 \\
\equiv R^2 + &\frac{(1-\beta\delta\rho)\beta(\alpha_H f_F + 2k\alpha_F f_H) + 2\beta^2\delta\rho(f_F + f_H k) - \alpha_H f_F\beta}{(\alpha_H f_F + 2k\alpha_F f_H)\beta^2}R - \frac{f_F\alpha_H(1-\beta\delta\rho)}{(\alpha_H f_F + 2k\alpha_F f_H)\beta^2} < 0 \\
\equiv R^2 + &\kappa_1 R - \kappa_0 < 0
\end{aligned} \tag{A26}$$

with

$$\begin{aligned}
\kappa_1 &= \frac{(1-\beta\delta\rho)\beta(\alpha_H f_F + 2k\alpha_F f_H) + 2\beta^2\delta\rho(f_F + f_H k) - \alpha_H f_F\beta}{(\alpha_H f_F + 2k\alpha_F f_H)\beta^2}, \text{ and} \\
\kappa_0 &= \frac{f_F\alpha_H(1-\beta\delta\rho)}{(\alpha_H f_F + 2k\alpha_F f_H)\beta^2}.
\end{aligned}$$

It is apparent from the above expressions that κ_0 is positive. We can rearrange the terms to show that κ_1 is also positive. In particular, note that

$$\kappa_1 = \frac{\beta\delta\rho f_F(2-\alpha_H) + 2f_H k[\alpha_F(1-\beta\delta\rho) + \beta\delta\rho])}{\beta^2(\alpha_H f_F + 2k\alpha_F f_H)}.$$

Since $\alpha_H \leq 1$ and $\beta\delta\rho \leq 1$, both terms in the numerator are positive. □

Proof for Proposition 2a. To show that a reduction in k favors taxes, we note that R^{crit} is a differentiable function of k. Using the chain rule and the definitions of κ_0 and κ_1, we obtain

$$\frac{\partial R^{crit}}{\partial k} = \frac{1}{2}\frac{1}{\sqrt{\kappa_1^2 + 4\kappa_0}}[\frac{\partial \kappa_1}{\partial k}(\kappa_1 - \sqrt{\kappa_1^2 + 4\kappa_0}) + 2\frac{\partial \kappa_0}{\partial k}]. \tag{A27}$$

with

$$\frac{\partial \kappa_1}{\partial k} = 2\frac{f_F f_H[\alpha_H\alpha_F + \beta\delta\rho(\alpha_H - 2\alpha_F)]}{\beta(\alpha_H f_F + 2k\alpha_F f_H)},$$
$$\frac{\partial \kappa_0}{\partial k} = -2\frac{\alpha_F\alpha_H f_F f_H(1-\beta\delta\rho)}{\beta^2(\alpha_H f_F + 2k\alpha_F f_H)} < 0.$$

To determine the sign of $\frac{\partial R^{crit}}{\partial k}$, we consider the term $\frac{\partial \kappa_1}{\partial k}$. Let $N = \alpha_H\alpha_F + \beta\delta\rho(\alpha_H - 2\alpha_F)$, and $D = \alpha_H f_F + 2\alpha_F f_H k$.

If $N \geq 0$, then $\frac{\partial \kappa_1}{\partial k} \geq 0$, so the first term in the square brackets, $\frac{\partial \kappa_1}{\partial k}(\kappa_1 - \sqrt{\kappa_1^2 + 4\kappa_0})$, becomes non-negative. Given $\frac{\partial \kappa_0}{\partial k} < 0$, it follows that $\frac{\partial R^{crit}}{\partial k}$ is strictly negative.

Now suppose $N < 0$. In this case, we want to show that $|\frac{\partial \kappa_1}{\partial k}(\kappa_1 - \sqrt{\kappa_1^2 + 4\kappa_0})| < |2\frac{\partial \kappa_0}{\partial k}|$. We can rewrite the inequality to be

$$\frac{2f_H f_F}{\beta D^2}|N|[\sqrt{\kappa_1^2 + 4\kappa_0} - \kappa_1] < \frac{4\alpha_F\alpha_H f_H f_F(1-\beta\delta\rho)}{\beta^2 D^2}.$$

This can be simplified to

$$-N[\sqrt{\kappa_1^2 + 4\kappa_0} - \kappa_1] < \frac{2\alpha_F\alpha_H(1-\beta\delta\rho)}{\beta}.$$

Using the algebraic identity $\sqrt{\kappa_1^2+4\kappa_0}-\kappa_1=\frac{4\kappa_0}{\sqrt{\kappa_1^2+4\kappa_0}+\kappa_1}$ and substituting the expression for κ_0 in the numerator, we have

$$\sqrt{\kappa_1^2+4\kappa_0}-\kappa_1=\frac{4f_F\alpha_H(1-\beta\delta\rho)}{\beta^2D\sqrt{\kappa_1^2+4\kappa_0}+\kappa_1}$$

Thus we can rearrange the above inequality to be

$$-2Nf_F<\beta\alpha_FD[\sqrt{\kappa_1^2+4\kappa_0}+\kappa_1]$$

A lower bound for $\sqrt{\kappa_1^2+4\kappa_0}+\kappa_1$ is $2\kappa_1$. Hence, it suffices to show that $-2Nf_F<\beta\alpha_FD\cdot 2\kappa_1$. Substituting in the definition of κ_1 dividing both sides by α_Ff_F, we have

$$-\alpha_H-\beta\delta\rho\frac{\alpha_H-2\alpha_F}{\alpha_F}<\delta\rho(2-\alpha_H)+\frac{2f_Hk}{\beta f_F}[\alpha_F(1-\beta\delta\rho)+\beta\delta\rho]$$

After some algebra, we arrive at

$$\alpha_H(-1-\frac{\beta\delta\rho}{\alpha_F}+\delta\rho)<\frac{2f_Hk}{\beta f_F}[\alpha_F(1-\beta\delta\rho)+\beta\delta\rho]+(1-\beta)2\delta\rho.$$

The left-hand side is negative while the right-hand side is positive, so the inequality holds.

□

Proof for Proposition 2b. R^{crit} is a differentiable function of α_H. Using the chain rule and the definitions of κ_0 and κ_1, we obtain

$$\frac{\partial R^{crit}}{\partial\alpha_H}=-\frac{1}{2}\frac{\partial\kappa_1}{\partial\alpha_H}+\frac{1}{4}\frac{1}{\sqrt{\kappa_1^2+4\kappa_0}}\frac{\partial(\kappa_1^2+4\kappa_0)}{\partial\alpha_H}$$

This simplifies to

$$\frac{\partial R^{crit}}{\partial\alpha_H}=\frac{1}{2}\frac{1}{\sqrt{\kappa_1^2+4\kappa_0}}\frac{\partial\kappa_1}{\partial\alpha_H}(\kappa_1-\sqrt{\kappa_1^2+4\kappa_0})+\frac{1}{\sqrt{\kappa_1^2+4\kappa_0}}\frac{\partial\kappa_0}{\partial\alpha_H} \tag{A28}$$

with

$$\frac{\partial \kappa_1}{\partial \alpha_H} = -2\frac{f_F}{\beta(\alpha_H f_F + 2\alpha_F f_H k)^2}[k\alpha_F f_H + \beta\delta\rho(f_F + f_H k)] < 0$$
$$\frac{\partial \kappa_0}{\partial \alpha_H} = \frac{2k\alpha_F f_H f_F(1-\beta\delta\rho)}{\beta^2(\alpha_H f_F + 2\alpha_F f_H k)^2} > 0.$$

Since $\kappa_0 > 0$, we have $\kappa_1 - \sqrt{\kappa_1^2 + 4\kappa_0} < 0$. Therefore, the first term in the derivative in Equation A28 is straightforwardly positive.

□

A.2 Materials for Section 3

Under the structure of the model, each country's optimization problem features a quadratic flow payoff and a linear law of motion for the state vector. As a result, the optimal control is linear in the state variables. Moreover, best responses to linear policy rules are themselves linear, so equilibrium policy choices can be characterized by linear feedback rules of the form

$$x_{i,t} = c_{1i} S_t + c_{2i}\theta_{t-1}, \quad i \in \{H, F\}.$$

We now proceed to define the following matrices:

$$\begin{aligned} Q_H &= \begin{bmatrix} -k_H b_H & 0 \\ 0 & 0 \end{bmatrix}, A_H = \begin{bmatrix} \delta + \phi c_{1F} k_F & \phi c_{2F} k_F \\ 0 & \rho \end{bmatrix}, \\ W_H &= \begin{bmatrix} 0 \\ \rho \end{bmatrix}', B_H = \begin{bmatrix} \phi k_H \\ 0 \end{bmatrix}, C_H = \begin{bmatrix} \Phi_1 k_H \phi \frac{\alpha_H}{f_H} + \Phi_2 k_F \phi \frac{\alpha_F}{f_F} \\ 1 \end{bmatrix}. \end{aligned} \tag{A29}$$

With this notation, the net flow payoff and equation of motion for the linear quadratic problem of Home are:

$$[-\frac{f_H}{2}x_{H,t}^2 + \frac{1}{2}Y_t' Q_H Y_t + W_H Y_t x_{H,t} + \Phi_2 \frac{\alpha_H^2}{2f_H}\sigma^2]\phi \tag{A30}$$

$$Y_{t+1} = A_H Y_t + B_H x_{H,t} + C_H \epsilon_t \tag{A31}$$

Since the linear quadratic problems for Home and Foreign are symmetric, we define the following matrices for the generic problem for Foreign:

$$\begin{aligned} Q_F &= \begin{bmatrix} -k_F b_F & 0 \\ 0 & 0 \end{bmatrix}, A_F = \begin{bmatrix} \delta + \phi c_{1H} k_H & \phi c_{2H} k_H \\ 0 & \rho \end{bmatrix}, \\ W_F &= \begin{bmatrix} 0 \\ \rho \end{bmatrix}', B_F = \begin{bmatrix} \phi k_F \\ 0 \end{bmatrix}, C_F = \begin{bmatrix} \Phi_1 k_H \phi \frac{\alpha_H}{f_H} + \Phi_2 k_F \phi \frac{\alpha_F}{f_F} \\ 1 \end{bmatrix}. \end{aligned} \tag{A32}$$

Then the net flow payoff and equation of motion for the linear quadratic problem of Foreign are:

$$[-\frac{f_F}{2}x_{F,t}^2 + \frac{1}{2}Y_t' Q_F Y_t + W_F Y_t x_{F,t} + \Phi_1 \frac{\alpha_F^2}{2f_F}\sigma^2]\phi \tag{A33}$$

$$Y_{t+1} = A_F Y_t + B_F x_{F,t} + C_F \epsilon_t \tag{A34}$$

We use the same procedure as in appendix A.1 to derive the expressions for control variables by writing the right side of these dynamic programming problems in terms of the control variables. Throughout, we solve Home's dynamic programming problem, taking Foreign's strategy as given. For notational simplicity, we suppress the subscript on the matrices Q_H, A_H, B_H, C_H, W_H, as well as λ_H and μ_H.

The dynamic programming equation for Home's generic problem is:

$$J_t(Y_t) = \max_{x_{H,t}}[-\frac{f_H}{2}x_{H,t}^2 + \frac{1}{2}Y_t'QY_t + WY_t x_{H,t} + \Phi_1\frac{\alpha_H^2}{2f_H}\sigma^2]\phi + \beta\mathbf{E}_t J_{t+1}^i(Y_{t+1}) \tag{A35}$$

The value function for the linear quadratic problem is

$$J_t^i(Y_t) = V_0^i + \frac{1}{2}Y_t'V_2Y_t \tag{A36}$$

where

$$V_2 = -\begin{bmatrix} \lambda & \mu \\ \mu & \nu \end{bmatrix}. \tag{A37}$$

The right side of the DPE is

$$[-\frac{f_H}{2}x_{H,t}^2 + \frac{1}{2}Y_t'QY_t + WY_t x_{H,t} + \Phi_1\frac{\alpha_H^2}{2f_H}\sigma^2]\phi + \beta\mathbf{E}_t(V_{0,t+1} + \frac{1}{2}Y_{t+1}'V_2Y_{t+1}) \tag{A38}$$

Substituting the equations of motion and taking expectation, we have:

$$\begin{aligned}(-\frac{f_H}{2}x_{H,t}^2 + \frac{1}{2}Y_t'QY_t + WY_t x_{H,t} + \Phi\frac{\alpha_H^2}{2f_H}\sigma^2)\phi + \beta[V_{0,t+1} \\ +\frac{1}{2}(AY_t + Bx_{H,t})'V_2(AY_t + Bx_{H,t}) + \frac{1}{2}\sigma^2 C'V_2C]\end{aligned} \tag{A39}$$

The FOC is:

$$\phi(-f_H x_{1,t} + WY_t) + \beta(V_1'B + B'V_2Bx_{H,t} + B'V_2AY_t) \tag{A40}$$

$$\implies x_{H,t} = \frac{1}{f_H\phi - \beta(B'V_2B)}(\beta V_1'B + (W\phi + \beta B'V_2A)Y_t) \tag{A41}$$

Rewrite x_t in terms of Z_0, Z and Y_t such that

$$Z_0 = \frac{\beta V_1' B}{f_H \phi - \beta(B' V_2 B)} \text{ and } Z = \frac{W\phi + \beta B' V_2 A}{f_H \phi - \beta(B' V_2 B)}. \tag{A42}$$

Using this result, the quadratic part of the right side of the maximized DPE is:

$$\frac{1}{2} Y_t' [(Q - f_H Z'Z + W'Z + Z'W)\phi + \beta(A + BZ)' V_2 (A + BZ)] Y_t \tag{A43}$$

To calculate C_F, we solve for λ, μ, and C_H for Home's linear quadratic problem. There are 2 initial values for λ, as the equation for it is normally a quadratic form. We then solve for those same parameters in the Foreign system. This gives us 4 different values of Foreign's λ_F, which results in 4 different systems of equations for C_F. Solving these systems, we found the values for C_F, λ, and μ that satisfy our constraints.

The expressions for λ and μ are as follows:

$$\begin{aligned} \lambda &= b\phi + \frac{f_H \lambda \beta c_{1F}^2 k_F^2 \phi^2 + 2 f_H \lambda \beta c_{1F} \delta k_F \phi + f_H \lambda \beta \delta^2}{\lambda \phi \beta k_H^2 + f_H}, \\ \mu &= \frac{\beta(\delta + c_{1F} k_F \phi)(f_H \mu \rho + k_H \lambda \phi \rho + c_{2F} f_H k_F \lambda \phi)}{\lambda \phi \beta k_H^2 + f_H}. \end{aligned} \tag{A44}$$

And ν_{1t} equals the following:

$$\nu_{1t} = \frac{\beta f_H \nu_{1,t+1}(\delta + c_{1F} k_F \phi)}{\lambda \phi \beta k_H^2 + f_H} \tag{A45}$$

When $\Phi_2 = 0$, and we vary Φ_1 between 0 and 1, these are the two different V_0 values:

With $\Phi_1 = 0$

$$V_{0,t} = \begin{bmatrix} v_{0,00,t} & v_{0,01,t} \\ v_{0,10,t} & v_{0,11,t} \end{bmatrix}. \tag{A46}$$

Where each component is:

$$v_{0,ij,t} = \beta \left(v_{0,ij,t+1} - \frac{1}{2}\nu\sigma^2 + \frac{\beta k_H^2 v_{1,t+1}^2 \phi^2}{\lambda\beta k_H^2\phi^2 + f_H\phi} - \frac{\beta^2 k_H^4 \lambda v_{1,t+1}^2 \phi^4}{2(\lambda\beta k_H^2\phi^2 + f_H\phi)^2} \right) - \frac{\beta^2 f_F k_H^2 v_{1,t+1}^2 \phi^3}{2(\lambda\beta k_H^2\phi^2 + f_H\phi)^2} \tag{A47}$$

such that i and j takes the value of 0 or 1.

With $\Phi_1 = 1$,

$$V_{0,t} = \begin{bmatrix} v_{0,00,t} & v_{0,01,t} \\ v_{0,10,t} & v_{0,11,t} \end{bmatrix}. \tag{A48}$$

Where each component is:

$$\begin{aligned} v_{0,ij,t} = \phi \left(\frac{\alpha_H^2\sigma^2}{2f_H} - \frac{\beta^2 f_F k_H^2 v_{1,t+1}^2 \phi^2}{2(\lambda\beta k_H^2\phi^2 + f_H\phi)^2} \right) - \beta(\frac{1}{2}\nu\sigma^2 - v_{0,ij,t+1} + \\ \frac{\alpha_H\phi\left(\frac{1}{2}\mu\sigma^2 + \frac{\alpha_H\lambda\phi\sigma^2}{2f_H}\right)}{f_H} + \frac{\alpha_H\mu\phi\sigma^2}{2f_H} - \frac{\beta k_H^2 v_{1,t+1}^2\phi^2}{\lambda\beta k_H^2\phi^2 + f_H\phi} + \frac{\beta^2 k_H^4\lambda v_{1,t+1}^2\phi^4}{2(\lambda\beta k_H^2\phi^2 + f_H\phi)^2}) \end{aligned} \tag{A49}$$

The difference $V_{0,T} - V_{0,Q} = \Delta_t$ when $\Phi_2 = 0$ is:

$$\Delta_t = \frac{\alpha_H\sigma^2(\alpha_H - 2\beta\mu)}{2f_H} + \beta\Delta_{t+1} - \frac{\alpha_H^2\beta\lambda\phi^2\sigma^2}{2f_H^2} \tag{A50}$$

The steady state solution is:

$$\Delta_{H,\Phi_2=0} = \frac{\alpha_H\sigma^2(\alpha_H - 2\beta\mu)}{2f_H(1-\beta)} - \frac{\alpha_H^2\beta\lambda\phi^2\sigma^2}{2f_H^2(1-\beta)} \tag{A51}$$

The difference $V_{0,T} - V_{0,Q} = \Delta_t$ when $\Phi_2 = 1$ is:

$$\Delta_t = \beta\Delta_t - \frac{\alpha_H^2\beta\lambda\phi^2\sigma^2}{2f_H^2} - \frac{\alpha_H\phi\sigma^2(2\beta f_F\mu - \alpha_H f_F + 2\alpha_F\beta\lambda\phi)}{2f_F f_H} \tag{A52}$$

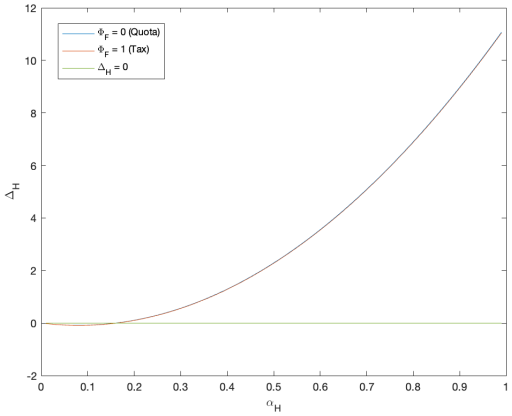
The steady state solution is:

$$\Delta_{H,\Phi_2=1} = -\frac{\alpha_H^2 \beta \lambda \phi^2 \sigma^2}{2 f_H^2 (1-\beta)} - \frac{\alpha_H \phi \sigma^2 (2\beta f_F \mu - \alpha_H f_F + 2\alpha_F \beta \lambda \phi)}{2 f_F f_H (1-\beta)} \tag{A53}$$

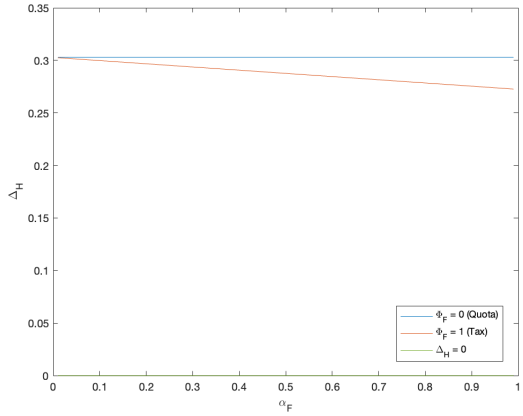
The difference between $\Delta_{H,\Phi=0}$ and $\Delta_{H,\Phi=1}$ is given by

$$\Delta_{H,\Phi_2=0} - \Delta_{H,\Phi=1} = \frac{\alpha_H \sigma^2 (\alpha_H - 2\beta\mu)}{2 f_H (1-\beta)} + \frac{\alpha_H \phi \sigma^2 (2\beta f_F \mu - \alpha_H f_F + 2\alpha_F \beta \lambda \phi)}{2 f_F f_H (1-\beta)} \tag{A54}$$

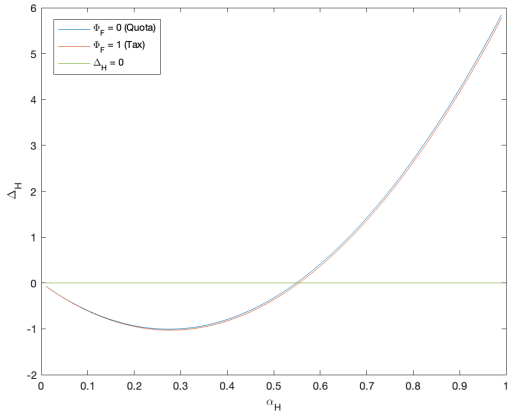
B Additional Figures



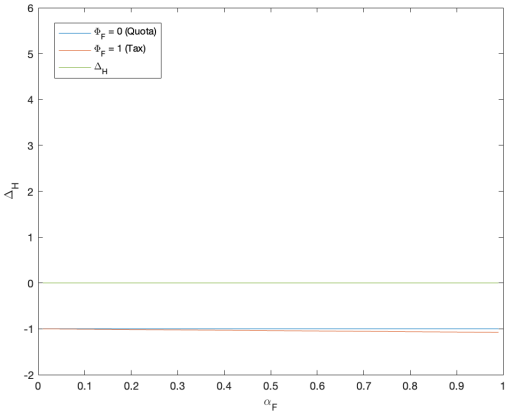
(a) Δ_H ($k_H = .25$) as α_H varies



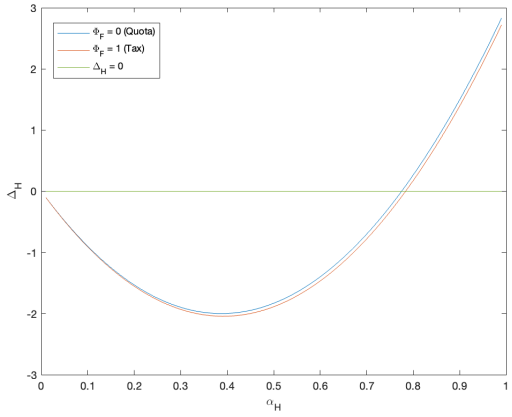
(b) Δ_H ($k_H = .25$) as α_F varies



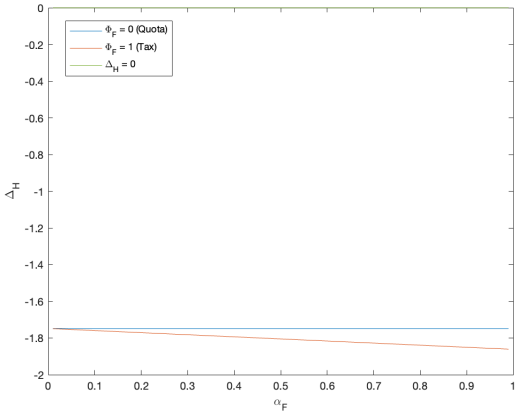
(c) Δ_H ($k_H = .5$) as α_H varies



(d) Δ_H ($k_H = .5$) as α_F varies



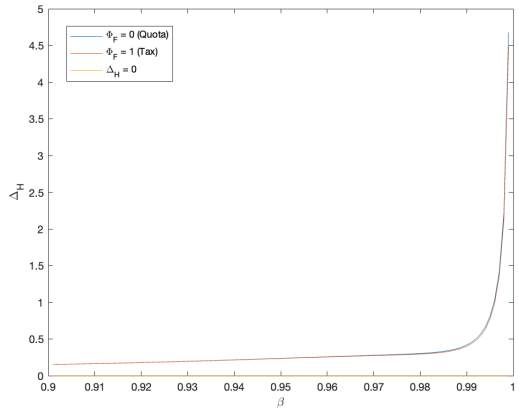
(e) Δ_H ($k_H = .75$) as α_H varies



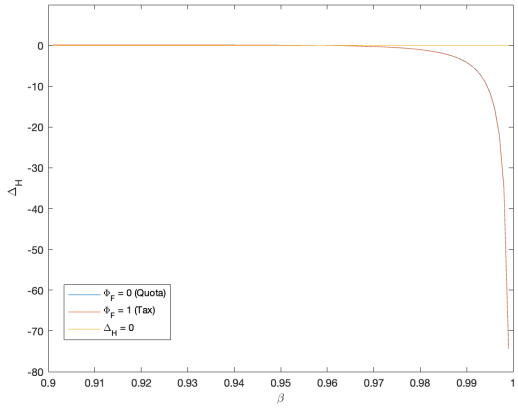
(f) Δ_H ($k_H = .75$) as α_F varies

Figure B1: Numerical Results as α_H or α_F Varies.

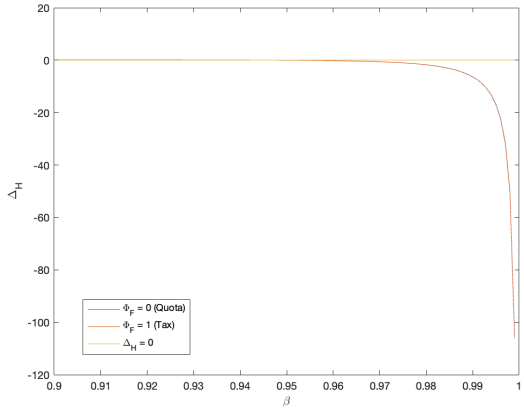
Note: Unless otherwise specified, all parameters take baseline values as listed in Table 1. One α is fixed at .25 while the other varies.
Source: Authors' calculation.



(a) Δ_H ($k_H = .25$)



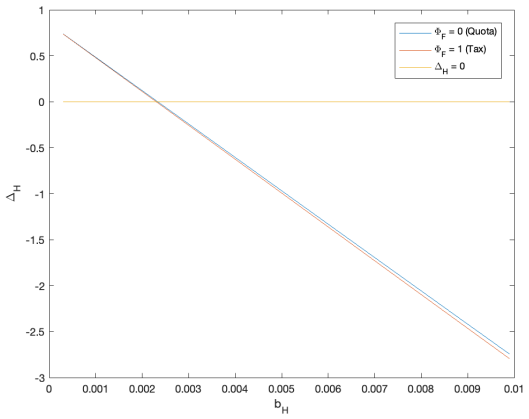
(b) Δ_H ($k_H = .5$)



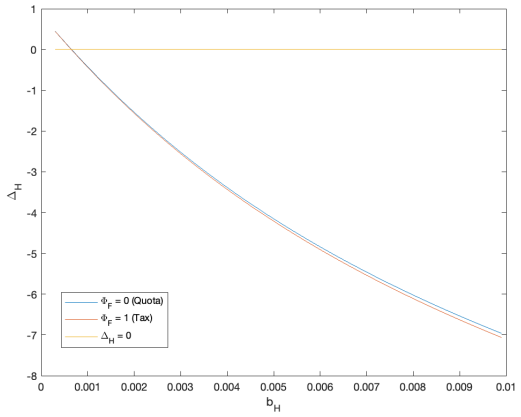
(c) Δ_H ($k_H = .75$)

Figure B2: Numerical Results as β Varies.

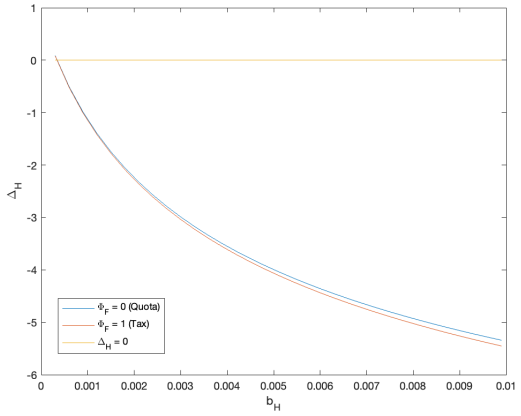
Note: Unless otherwise specified, all parameters take baseline values as listed in Table 1. Contingent on other parameter values, in panel B2b, β^{crit} ranges from 0.959 to 0.960, irrespective of whether Foreign chooses taxes or quotas. In panel B2c, β^{crit} is bounded between 0.947 and 0.948 when Foreign chooses taxes, and between 0.948 and 0.949 when Foreign chooses quotas.
Source: Authors' calculation.



(a) Δ_H ($k_H = .25$)



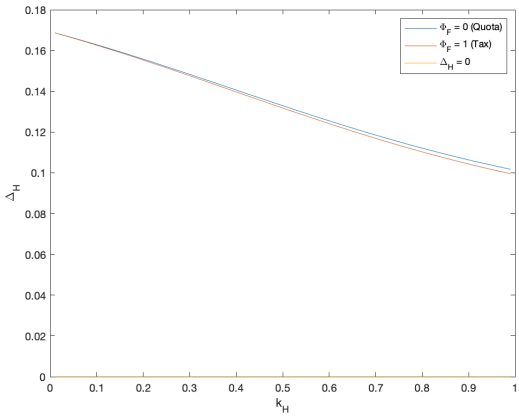
(b) Δ_H ($k_H = .5$)



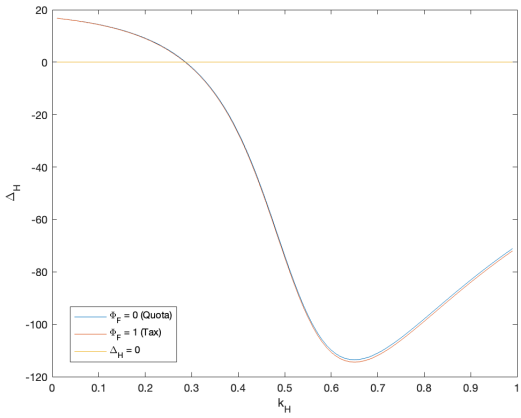
(c) Δ_H ($k_H = .75$)

Figure B3: Numerical Results as b_H Varies.

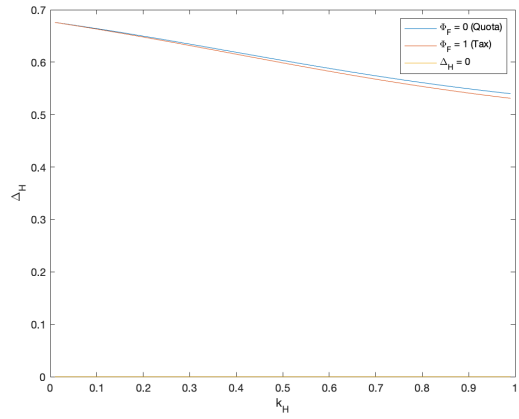
Note: Unless otherwise specified, all parameters take baseline values as listed in Table 1.
Source: Authors' calculation.



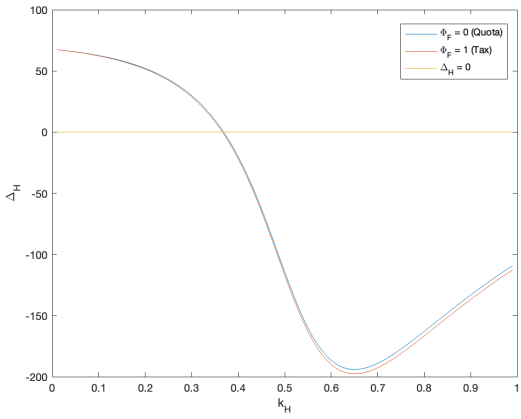
(a) Δ_H ($\beta = .90$)



(b) Δ_H ($\beta = .999$)



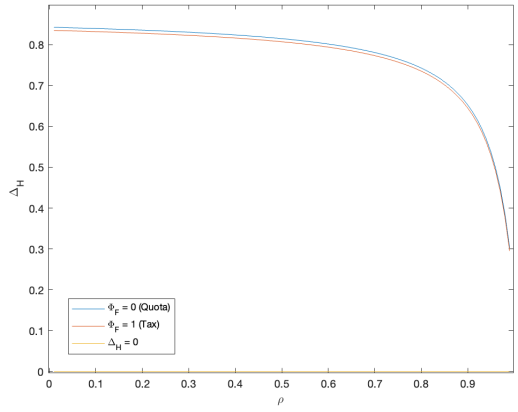
(c) Δ_H ($\beta = .90, \alpha = .50$)



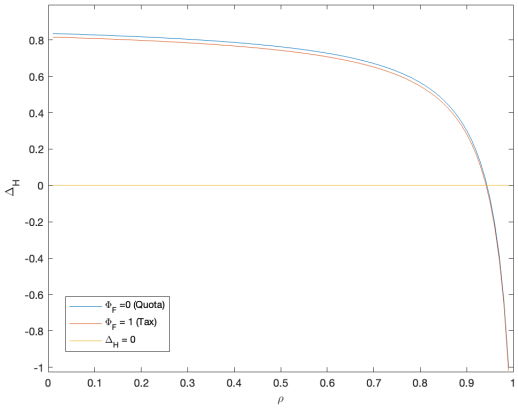
(d) Δ_H ($\beta = .999, \alpha = .50$)

Figure B4: Numerical Results as k_H Varies.

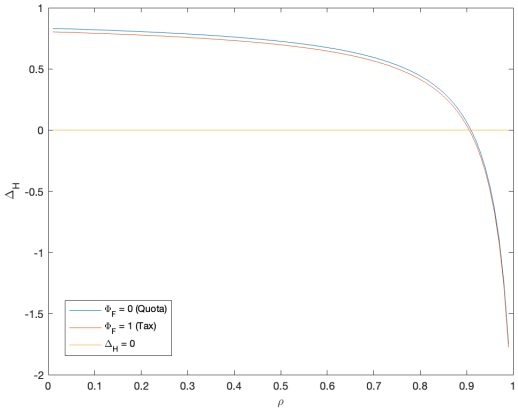
Note: Unless otherwise specified, all parameters take baseline values as listed in Table 1.
Source: Authors' calculation.



(a) Δ_H ($k_H = .25$)



(b) Δ_H ($k_H = .5$)



(c) Δ_H ($k_H = .75$)

Figure B5: Numerical Results as ρ Varies.

Note: Unless otherwise specified, all parameters take baseline values as listed in Table 1.
Source: Authors' calculation.