Technological Change and Career Ladders

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First Draft: February 2026
This Draft: March 27, 2026

Abstract

In this paper, we analyze the effects of a technological innovation like AI on the career dynamics of workers *within* an occupation over their lifetime. We consider a technology that is substitutable with the human capital of workers in entry level positions who execute tasks but complements their human capital on higher rungs of the job ladder when they are creating new tasks. The key focus of our analysis is endogenizing the human capital process that occurs from on-the-job learning when there is a decline in the price of the technology. We formalize the conditions under which the new technology can reduce the economy's stock of human capital which accrues from on-the-job learning. We highlight conditions where the economy can enter a low-human capital trap where human capital falls and the task frontier is constrained by the lack of skilled workers. However, the model illustrates that if using the new technology expands the task frontier sufficiently, a high human capital equilibrium can be sustained. Finally, we develop the planners problem and show that the planner may want to tax or subsidize the new technology depending on whether the technology automate jobs faster or slower than it expands the task frontier. Welfare gains from the planners optimal tax or subsidy is largest when the economy is closest to the threshold of entering the low human capital trap.

JEL Codes: TBC
Key Words: TBC

"Learning is the product of experience. Learning can only take place through the attempt to solve a problem and therefore only takes place during activity."... *Kenneth Arrow (1962)*

1 Introduction

The recent rapid advance of artificial intelligence (AI) has prompted a variety of academics, business leaders and politicians to speculate about this new technology's potential labor market impact. On the one hand, there is speculation that AI will dramatically reduce the number of entry level positions in white collar occupations.[1] The argument is that AI is rapidly mastering abstract tasks such as coding, data gathering, document review, introductory financial research, and summarizing information; these are the types of tasks that are done in many entry level positions across a variety of white collar occupations. On the other hand, there is a belief that AI will dramatically increase the productivity of individuals at higher levels of the organization.[2] The argument here is that AI is also a tool that facilitates the creation of new ideas, products, and management practices. These two beliefs highlight that AI may be distinct from other types of technology in that *for a given worker* it may act as a substitute for their skills on early rungs of their career ladder when they are executing entry level tasks but may complement their skills as they advance through their chosen career when the creation of new ideas is more important.

In this paper, we analyze the effects of a technological innovation like AI on the career dynamics of workers *within* an occupation over their lifetime. Similar to the hypothesized effects of AI, we model the new technology as a substitute for worker labor on lower rungs of the career ladder (entry-level positions) and as a complement on higher rungs (manager positions). Entry-level positions execute existing tasks; managers create new ones. The key focus of our analysis is endogenizing the human capital accumulation process that occurs from on-the-job learning. By working on lower rungs of the career ladder, workers accumulate human capital that makes them more productive on higher rungs of the career ladder. Over a career, the same worker travels up the ladder creating a dynamic interaction between automation and human

[1] For example, Antropic's CEO Dario Amodei has claimed that AI could wipe out half of all entry-level white collar jobs over the next few years (Cooper (2025)). See Roose (2025) and Telford (2026) for discussions in the *New York Times* and *Washington Post*, respectively, about how AI is reducing entry level jobs. Brynjolfsson, Chandar, and Chen (2025) finds preliminary evidence that AI may, in fact, be reducing employment in entry level white collar jobs.

[2] For example, OpenAI CEO Sam Altman has stated that continued AI advances will make "figuring out what questions to ask even more important than figuring out the answer" (Grant (2025)). Similarly, Fossen, McLemore, and Sorgner (2024) surveys the growing literature show that AI could increase entrepreneurial activity.

capital accumulation.[3]

We formalize how the elimination of entry level positions can negatively affect the evolution of worker productivity throughout their life cycle. However, at the same time, we also formalize how AI may generate new entry-level tasks that accelerate on-the-job learning by expanding the task frontier. The latter is a countervailing force whose strength determines whether automation ultimately helps or harms the career pipeline. Our set up allows us to specify the conditions under which the new technology will increase the aggregate human capital stock and expand the number of entry level positions despite large amounts of automation of existing tasks. We further specify the conditions under which firms may eliminate too many entry level positions in response to the technological change relative to the planner's problem. Finally, we highlight the conditions under which the economy enters a low human capital trap where the lack of human capital development severely limits the productivity gains associated with the declining price of the new technology. Throughout, we formalize that in many cases there are multiple equilibrium for the same set of parameter values.

Similar to Arrow (1962), Stokey (1988) and Young (1991), the on-the-job learning process in our model is a function of the number of tasks done by workers. Consistent with the quote from Arrow (1962) above, we assume that learning takes place through the experience of performing tasks.[4] This nature of learning is visible across many white-collar occupations. For example, new lawyers learn how to advise clients and generate new business by reviewing contracts and researching precedents. Junior data scientists develop intuitions about data quality and model reliability by doing entry-level coding. Beginning management consultants build the judgment to advise senior clients by spending time doing the analytical legwork through which they learn not just institutional details but how organizations actually respond to potentially

[3]There is a large literature discussing how technological shifts can reallocate workers *across* occupations. See, for example, Autor (2015), Mokyr, Vickers, and Ziebarth (2015), and Acemoglu and Johnson (2024) for how the power loom displaced handloom weavers; Acemoglu and Restrepo (2020) for how robots displaced workers in manufacturing occupations; and Autor, Levy, and Murnane (2003) and Goos, Manning, and Salomons (2014) for how computerization hollowed out routine clerical and production occupations. Our paper differs from these in that we are focusing on how a technology such as AI can affect career dynamics *within* a given occupation. As we highlight throughout, such a distinction is important when human capital is endogenous as workers climb their career ladder.

[4]The view that learning comes from experience is pervasive in both the fields of education and cognitive psychology. See Dewey (1938), Anderson (1982), and Ericsson, Krampe, and Tesch-Römer (1993) for canonical references.

difficult recommendations. In each case, the tasks that AI substitute for are not merely low-value work; they are also the curriculum through which workers acquire the human capital that makes them more productive in senior level positions. By learning these skills, the workers who in the future will become managers are better able to use technologies like AI to develop new tasks that are valuable to the firm. These workers will have the expertise to know "what questions to ask" given the experience they accumulated early on in their career.

Our model has two agents: owners of firms and individuals who start their career as workers and eventually become managers as they move up their career ladder.[5] The owners of firms maximize profits by hiring workers, hiring managers, purchasing technology that can automate worker tasks, and purchasing technology that can make managers better at generating new tasks. The owners of the firms consume their profits. We assume that the price of both technologies is the same and is exogenously set; a technology like AI can both automate tasks and enhance the productivity of managers in generating new tasks.

Managers determine the task frontier in the economy by generating new tasks. As the price of the technology declines, it is cheaper for firms to purchase the technology that helps them expand the task frontier. However, we assume it takes a high human capital stock of managers to unlock the technology's potential to expand the task frontier. As a result, the quality of managers, as measured by their level of human capital, acts as a constraint in the technology's ability to expand the task frontier. It is in this sense that technology and manager human capital are complements on higher rungs of the job ladder.

We model firm production similar to Acemoglu and Restrepo (2022) in that workers and technology are perfect substitutes at the task level. There are a set of tasks to be produced in the economy at any given point that are endogenously determined by the quality of managers and the availability of technology as described above. Firms choose how many tasks to automate depending on the price of workers (which is endogenously determined) relative to the price of the technology scaled by the ease at which additional tasks can be automated. As in Acemoglu and

[5]There is a large literature thinking about hierarchical structures of workers and managers within a firm where workers generate problems and managers help them solve the problems. See, for example, Garicano (2000), Garicano and Rossi-Hansberg (2006), and Caicedo, Lucas, and Rossi-Hansberg (2019). Our framework has similarities but we have endogenous human capital accumulation due to learning by doing, managers who determine the task frontier (i.e., generate the problems to be solved), and a role for technology that both complements manager human capital and substitutes with worker human capital.

Restrepo (2022), this generates a cutoff rule such that tasks below some threshold get automated while the rest are done by workers. All else equal, a decline in the price of the technology results in firms automating more tasks.

In our baseline model, we assume no worker heterogeneity, all individuals are born with the minimum level of human capital, and there are no other sectors where the worker could allocate their labor. A worker accumulates human capital based on the total number of tasks done by workers in the economy which is a function of the task frontier and the task automation threshold, both of which are endogenous objects in the model. Productivity as a worker is linear in their level of human capital. As a manager, an individual's productivity is a convex function of their human capital level. Given the model setup, there is a cutoff level of human capital such that when workers hit this level of human capital they switch to becoming a manager. We assume all individuals die at a constant rate and are immediately replaced by a worker with the minimum level of human capital.

An equilibrium in the model is described by (i) a cutoff level of tasks below which firms choose to automate, (ii) a cutoff level of human capital above which an individual becomes an manager, (iii) the speed of human capital adjustment, (iv) the total number of managers in the economy, and (v) the wages of workers and managers per unit of human capital. In the decentralized model, we assume that both firms and workers are sufficiently small that they do not internalized the effects of their choices on the human capital accumulation process. This is the key externality in our model.

Our model has three possible steady-state outcomes depending on the price of the new technology and the structural features of the economy. When the price of the technology is high: firms automate relatively few tasks, workers continue to perform a wide range of entry-level work, and the economy settles into a unique equilibrium with robust human capital accumulation. As the price of the technology falls into an intermediate range, the equilibrium structure changes fundamentally. Two very different long-run outcomes become simultaneously possible: a good equilibrium in which workers still perform many tasks, accumulate human capital rapidly, and become highly skilled managers who enable the technology to expand the frontier of new products and ideas; and a bad equilibrium in which automation has reduced entry-level task opportunities enough that workers accumulate little human capital, transition

to management with limited skills, and generate a narrow frontier that in turn creates few new entry-level tasks for the next generation. Both outcomes are internally consistent — in each case, firms' automation decisions, workers' career choices, and the supply of managerial talent are all mutually compatible. Finally, when the price of the technology falls far enough, the good equilibrium can disappear and the economy can get stuck in a human capital trap. AI becomes so cheap that firms always find it profitable to automate all entry-level tasks, the learning pipeline collapses entirely, managers arrive with minimal skills, and the technology's potential to generate new products and ideas goes unrealized.

Whether an economy can tip into the bad equilibrium as the price of technology declines depends on four key structural parameters. The first is how effectively AI helps managers generate new products and tasks. When this complementarity between AI and senior workers is strong, the good equilibrium is resilient and survives to lower technology prices before disappearing. The logic is intuitive. If AI can help managers dramatically expand the task frontier, workers can learn from the new tasks and the learning equilibrium is preserved. The second key parameter is how quickly workers learn from the tasks they perform. Fast learning means that even a modest supply of entry-level tasks generates substantial human capital, making the good equilibrium harder to dislodge even as automation rises. The third parameter is how broadly AI substitutes for workers across the task spectrum. When AI is only productive at a narrow range of the most routine tasks, automation displaces relatively few learning opportunities, the good equilibrium is more resilient. However, when the new technology results in a large amount of tasks to be automated, the learning process could slow and the economy is more likely to switch to the low human capital equilibrium. The final parameter is how quickly the workforce turns over. High turnover means the economy constantly needs to rebuilds its stock of experienced workers making the good equilibrium harder to sustain. Taken together, these four features determine, for any given technology price, whether the economy remains in the region where only the good equilibrium exists, in the zone where both equilibrium coexist and history determines outcomes, or where the economy is stuck in the low human capital trap.

We next formalize the planner's problem. Firms in the decentralized equilibrium fail to internalize two distinct externalities. First, when they automate an entry-level task they reduce workers' learning opportunities, which is a negative externality on human capital accumulation.

Second, when they invest in technology that expands the frontier of new products, they create new entry-level tasks, which is a positive externality on human capital accumulation. Both externalities operate through the same technology, and the planner's optimal policy depends on which one dominates. When AI is highly productive at automating a wide range of entry-level tasks and only modestly effective at helping managers generate new products, the negative externality dominates: firms automate too aggressively relative to what the planner would choose, and the optimal policy is to tax entry-level automation. This slows the destruction of learning opportunities and nudges the economy back toward the good equilibrium. Separately, when AI is only modestly productive at automating entry-level tasks while also being highly effective at helping managers expand the task frontier, the positive externality dominates. In this case, firms under-invest in the technology relative to the social optimum, because they do not internalize that expanding the frontier creates new learning opportunities for workers throughout the economy. Here the planner wants to subsidize technology adoption, pushing firms to invest more in AI than they would choose on their own. The corrective policy therefore depends on the structural features of the occupation. In both cases the magnitude of the intervention is largest when the economy is near the boundary of the low human capital trap equilibrium, where the social cost of a small shift in automation is amplified by the risk of tipping into the bad equilibrium.

Collectively, our results speak to the current debate about how AI is going to effect the career pipeline of workers. The question is not simply whether AI will eliminate entry-level positions; existing evidence increasingly suggests it will. The deeper questions are whether it will create enough new positions to replace them and whether the workers who perform those tasks will learn faster or slower with AI. Our model formalizes all of these questions and identifies the conditions under which each matters. For economies with strong managerial complementarity, fast workplace learning, and low workforce turnover — conditions that may describe many high-skill professional occupations — the model predicts that only extreme declines in AI prices would push the economy into a low human capital trap. For sectors where the complementarity is weaker, the model warns of a potential for a low human capital trap. In that instance, there is a role for policy makers to tax the price of AI to preserve the human capital pipeline.

Related Literature: In addition to the broader literature referenced above, there are two contemporaneous papers, Garicano and Rayo (2025) and Ide (2025), that also directly examine how AI affects human capital development over a worker's career. Both do so through the lens of the Becker training problem.[6] We view the papers as complementary: they explore different aspects of how AI can affect the evolution of human capital throughout a worker's career. Garicano and Rayo (2025) build on the dynamic contracting model of Garicano and Rayo (2017) to derive conditions under which the bilateral apprenticeship relationship between an "expert" and a "novice" survives the introduction of AI. AI simultaneously reduces the value of novice labor and increases the productivity of the expert, and the survival of the apprentice relationship depends on whether the resulting surplus from training remains sufficiently large. Critically, there is no externality in their framework: the bilateral surplus efficiently determines whether training takes place, leaving no role for a social planner to improve outcomes.

Ide (2025) embeds a similar training mechanism as Garicano and Rayo (2025) in an overlapping-generations framework and shows that a wage floor on novice compensation creates a friction: even when the surplus from training is positive, novices cannot compensate experts sufficiently to sustain efficient training when the wage floor binds, generating socially excessive automation and a potential role for a corrective tax on automation. Three features distinguish our paper from Ide (2025). First, in Ide (2025)'s baseline model AI reduces the value of novice output but does not expand the task frontier available to experts. In our model, this is a countervailing force that can offset the decline in learning caused from entry-level automation and its strength determines whether the planner wants more or less automation relative to the decentralized equilibrium. Second, our mechanism requires no wage floor. Because our model endogenizes the timing at which workers transition to managers, workers never face a binding wage floor. If worker wages fall sufficiently, workers optimally become managers earlier, an equilibrium adjustment that Ide (2025)'s overlapping generation structure cannot accommodate. Finally, and most importantly, our externality is not a market incompleteness in the wage-setting process

[6] It is worth noting that the learning mechanism in our model is distinct from the classic training problem formalized in Becker (1962). In the Beckerian tradition, workers can compensate firms for providing general human capital training by accepting wages below their marginal product. While workers in our model would collectively benefit from accepting lower wages to discourage automation, no individual atomistic worker has an incentive to do so unilaterally, since their individual wage concession would not meaningfully affect the firm's automation decision. This free-rider problem prevents the Becker wage adjustment from restoring efficiency in the provision of general human capital by firms.

but a failure of atomistic firms to internalize the effect of their automation decisions on the economy's aggregate stock of human capital. This is a distinct and complementary source of inefficiency that can generate low human capital traps and potentially provide a role for the planner to tax (or subsidize) the technology.

2 Model

Time is continuous and indexed by $t \geq 0$. The economy is populated by a unit measure of households who each supply one unit of labor inelastically and accumulate human capital, denoted x, through a process described below. Individuals transition from being workers early in their career to managers later in their career. Workers execute "tasks" while managers develop new "tasks". Tasks can be thought of as activities done by the firm and can represent new ideas, new product lines, or new varieties of an existing product. All agents discount the future at rate $\rho > 0$ and face a constant death rate $\chi > 0$; upon death, each household is immediately replaced by a new entrant who starts with the minimum level of human capital. A representative firm produces a final good from a continuum of differentiated products. The firm is owned by a measure-zero set of capitalists who consume all profits.

2.1. Firm's Problem

Firms decide how many workers to hire, how many managers to hire and their use of technology in production. There is one common technology with price q but the firm can use the technology in two ways. First, the technology can help managers create new tasks. In this way, the technology is a complement with managerial ability. Second, the technology can by used to execute existing tasks. Here, the technology is a substitute with worker human capital. The key externality in our model is our assumption that firms are atomistic and, as a result, do not internalize how their decisions to use technology – either their decision to create new tasks or their decision to automate existing tasks – affect individual human capital accumulation.

Product frontier. The firm produces a set of differentiated products indexed by $\tau \in [0, f(t)]$, where the *product frontier* $f(t)$ is determined jointly by managerial input and technology: $f(t) \equiv \min\left\{\bar{\tau}\, a_m(t)^{\omega}, s(t)\right\}$, where $\bar{\tau}$ is a constant, $a_m(t)$ is the firm's demand for managerial technology, and ω controls how the product frontier expands with $a_m(t)$. The managerial input $s(t) \equiv$

$\int h(x)dN_m^d(x,t)$ aggregates the human capital of managers, with $N_m^d(x,t)$ denoting the CDF of human capital demanded for management at time t. We assume $h(x) = x^\xi$ with $\xi > 1$, so that a manager's productivity in maintaining the frontier is convex in their human capital x.

Three things are worth noting about our specification of the product frontier. First, $a_m(t)$ is how much of the common technology that a firm uses to complement managerial talent in expanding the task frontier. The price per unit of technology is q. This is the technology that is complements managerial ability. Second, we assume an extreme form of complementarity given our Leontief specification. We adopt this specification for tractability; our key results extend to any production technology in which managerial human capital and technology are complements in expanding the frontier. Finally, ω is a key parameter in our analysis. It governs how fast the technological frontier expands as the price of the technology falls. A higher value of ω implies that the task frontier will increase more for any given decline in q.

Production technology. Total output of the firm aggregates across all products within the frontier:

$$y(t) = \int_0^{f(t)} y(\tau,t)^{1-\eta^{-1}} d\tau, \quad \eta > 1 \tag{1}$$

Similar to Acemoglu and Restrepo (2022), we assume each product τ can be produced either by workers or by technology:

$$y(\tau,t) = \max\left\{\int x dN_e(x,t;\tau), z(\tau)a(\tau,t)\right\} \tag{2}$$

Here, $N_e(x,t;\tau)$ is the CDF of the firm's demand for human capital at time t for production of product τ, $a(\tau,t)$ is its demand for technology for the same product at price q, and $z(\tau)$ is the productivity of technology in producing product τ. We order products so that $z(\tau)$ is decreasing—technology has a comparative advantage in lower-indexed products—and assume the functional form $z(\tau) = \tau^{-\varepsilon}$. In other words, tasks are ordered so that lower-index tasks are more amenable to automation.

Two things are worth noting about our specification of the production technology. First, $a(\tau,t)$ is distinct from $a_m(t)$ described above; $a(\tau,t)$ is the technology used to produce existing tasks (as opposed to generating new tasks) and is a substitute for worker human capital. Second, ε is another key parameter in our analysis. It governs how many existing tasks will be automated as q falls; a lower value of ε implies that task automation is more elastic when the price of

technology falls.

To summarize, a technology in our framework can be described by a combination of ω and ε where, in response to a decline in the technology's price, ω describes how that technology will help generate new ideas (tasks) and ε describes how that same technology will automate existing tasks.

Labor markets and wages. Labor markets are competitive and indexed by position (worker or manager). We let $w_e(t)$ denote the equilibrium wage per unit of human capital for a worker and $w_m(t)$ the wage per unit of $h(x) = x^{\xi}$ for a manager.

Firm's optimization. Therefore, the firm chooses an allocation:

$$\mathscr{A}_f := \{N_e(x,t;\tau), a(\tau,t), N_m(x,t), s(t), a_m(t)\}_{\tau\in[0,f(t)],x\in\mathbb{R}_+,t\geq 0}$$

and its problem is

$$\begin{aligned}
\max_{\mathscr{A}_f}\Big\{ & \int_0^{f(t)} \max\Big\{x(\tau,t)^{1-\eta^{-1}} - w_e(t)x(\tau,t), (z(\tau)a(\tau,t))^{1-\eta^{-1}} - qa(\tau,t)\Big\}d\tau \\
& - w_m(t)\int h(x)dN_m(x,t) - qa_m(t)\Big\} \\
s.t. \quad & x(\tau,t) \equiv \int x dN_e(x,t;\tau) \\
& s(t) = \int h(x)dN_m(x,t), \quad f(t) = \min\{\bar{\tau}a_m(t)^{\omega}, s(t)\}
\end{aligned} \tag{3}$$

where the firm takes wages $w_e(t)$ and $w_m(t)$ along with the price of technology, q, as given.

A natural implication of the production technology is that each product is produced entirely by workers or entirely by technology. We let $\mathscr{T}(t) \in \mathscr{B}\left([0,f(t)]\right)$ denote the Borel set of products at time t that are produced by workers and write $|\mathscr{T}(t)|$ for its measure. Formally, defining $N_e(x,t) \equiv \int_0^{f(t)} N_e(x,t;\tau)d\tau$ as the firm's total demand for workers with human capital of x or less, we have $\mathscr{T}(t) = \text{supp}(N_e)$ in equilibrium.

2.2. Households

Households are born with one unit of human capital ($x = 1$) and have linear utility. At each instant, a household with human capital x at time t chooses whether to supply its labor for production at wage $w_e(t)x$ or manage at wage $w_m(t)h(x)$. We denote this decision as $l(x,t) : [1,\infty) \to \{e,m\}$.

The key feature of the model is that human capital accumulates through learning by doing, where a worker's human capital grows at rate $\varphi(t)$, proportional to the measure of products

produced by workers:

$$\dot{x}(t) = \varphi(t)x(t), \quad \varphi(t) \equiv \delta|\mathcal{T}(t)| \tag{4}$$

This learning-by-doing function is the heart of our model. A few comments are worth noting. First, we are assuming that learning is proportional to the range of tasks performed by workers in the economy. As workers, collectively, perform more tasks they learn faster. The rate of learning is governed by δ; δ will be another crucial parameter in our analysis because it will govern the strength of learning in the model. Second, learning is proportional to a workers current level of human capital such that a more skilled worker learns faster for a given level of tasks. Finally, φ is an economy-wide object. Because firms are atomistic, they take φ as given when optimizing.

A manager's human capital, by contrast, remains constant:[7]

$$\dot{x}(t) = 0$$

The value of a household with human capital x at time t therefore satisfies

$$(\rho + \chi)V(x,t) = \max\left\{\underbrace{w_e(t)x + \varphi(t)x\partial_x V(x,t)}_{l(x,t)=e}, \underbrace{w_m(t)h(x)}_{l(x,t)=m}\right\} + \partial_t V(x,t) \tag{5}$$

Thus, working pays the flow wage $w_e(t)x$ plus a capital gain $\varphi(t)x\partial_x V(x,t)$ from rising human capital; management pays $w_m(t)h(x)$ with no further accumulation.

2.3. Distribution of Human Capital

Let $N(x,t)$ denote the CDF of human capital in the economy at time t. Given the occupational choice of households, let

$$N_e(x,t) = \int_1^x \mathbf{1}_{\{l(x',t)=e\}} N(dx',t) \tag{6}$$

denote the CDF of those choosing production. The distribution of human capital, $N(x,t)$, thus evolves according to

$$\partial_t N(x,t) = \chi\,(1 - N(x,t)) - \varphi(t)x\partial_x N_e(x,t) \tag{7}$$

with no mass below $x = 1$.

[7] This is a simplifying assumption. If managers continued to accumulate human capital, the distribution at $x*$ (formalized below) would not be a mass point, but the threshold policy and the key externality would remain.

2.4. Equilibrium

A *competitive equilibrium* is a household allocation $\mathscr{A}_h \equiv \{V(x,t), l(x,t) : [1,\infty) \to \{e,m\}\}_{t\geq 0}$; a firm allocation $\mathscr{A}_f \equiv \{N_e(x,t;\tau), a(\tau,t), N_m(x,t), s(t), a_m(t)\}_{t\geq 0, \tau\in[0,f(t)], x\in\mathbb{R}_+}$; and prices with a rate of human capital growth $\mathscr{P} \equiv \{w_e(t), w_m(t), \varphi(t)\}$ such that: *(i) Optimality:* given $\mathscr{P}$, $\mathscr{A}_h$ and $\mathscr{A}_f$ solve the household and firm problems. *(ii) Market clearing:* labor markets for both positions clear and the rate of human capital growth is consistent with equilibrium automation:

$$\int_0^{f(t)} N_e(x,t;\tau)\,d\tau = N_e(x,t), \quad \forall x \geq 1, \forall t \geq 0$$

$$N_m(x,t) = N(x,t) - N_e(x,t) \quad \forall x \geq 1, \forall t \geq 0$$

$$\varphi(t) = \delta|\mathscr{T}(t)|, \quad \mathscr{T}(t) = \text{supp}(N_e(x,t))$$

where the distribution of human capital evolves according to the equilibrium decisions of households in Equation (7).

Stationary equilibrium. An equilibrium is *stationary* if all allocations, prices, and distributions are time-invariant.

3 Characterization of Equilibrium

3.1. Firm's Optimality Conditions

Automation threshold. Consider a product $\tau \in [0, f(t)]$. If the firm produces it with technology, the first-order condition is

$$(1-\eta^{-1})\,y(\tau,t)^{-\eta^{-1}} = \frac{q}{z(\tau)} = \tau^{\varepsilon} q \tag{8}$$

If instead the firm produces it with workers, the objective depends only on the total human capital allocated to the product, $\int x\,dN_e(x,t;\tau)$, and the composition across workers is irrelevant. The first-order condition is

$$(1-\eta^{-1})\,y(\tau,t)^{-\eta^{-1}} = w_e(t) \tag{9}$$

Comparing (8) and (9), the firm automates product τ whenever $\tau^{\varepsilon} q < w_e(t)$. Thus, for $w_e(t) > 0$, there is a threshold

$$\tau^*(t) \equiv \left(\frac{w_e(t)}{q}\right)^{\frac{1}{\varepsilon}} \tag{10}$$

below which all products are automated. If $\tau^*(t) \geq f(t)$, all products within the frontier are automated.

Output and profits. Inverting the first-order conditions, equilibrium output per product is

$$y(\tau, t) = \left(\frac{\min\{\tau^{\varepsilon} q,\, w_e(t)\}}{1 - \eta^{-1}} \right)^{-\eta} \tag{11}$$

and profits per product are

$$\pi(\tau, t) = \eta^{-1} \left(\frac{\min\{\tau^{\varepsilon} q,\, w_e(t)\}}{1 - \eta^{-1}} \right)^{1-\eta} \tag{12}$$

All worker-produced products share the same output level since they face a common marginal cost $w_e(t)$, and output is continuous at $\tau^*(t)$.

Product frontier. Profits per product depend only on aggregate managerial human capital $s(t)$, not on its composition across managers. The Leontief structure of the frontier technology implies that at any interior optimum both constraints bind: $f(t) = \bar{\tau}\, a_m(t)^{\omega} = s(t)$. The total cost of sustaining a frontier $f(t)$ is $w_m(t) f(t) + q(f(t)/\bar{\tau})^{\omega^{-1}}$, so the optimal frontier equates the marginal value of an additional product to its marginal cost:

$$\underbrace{\eta^{-1} \left(\frac{\min\left\{ f(t)^{\varepsilon} q, w_e(t) \right\}}{1 - \eta^{-1}} \right)^{1-\eta}}_{\text{marginal value}} = \underbrace{w_m(t) + q\omega^{-1} \bar{\tau}^{-\omega^{-1}} f(t)^{\omega^{-1}-1}}_{\text{marginal cost}} \tag{13}$$

A lower technology price q affects the frontier through two channels: it directly reduces the cost of expanding the frontier, and it raises the automation threshold $\tau^*(t)$ in (10), which feeds back into the marginal value of frontier products through the worker wage in general equilibrium.

3.2. Household's Problem

The household's problem (5) requires each household to choose, at every instant, whether to work or manage. Since households can freely switch between occupations, the optimal policy is not *a priori* a stopping time: a household could manage for a while, return to working to accumulate additional human capital, and then manage again. However, the structure of the model—in particular, that human capital is frozen during management—restricts the set of policies that can be optimal. We establish this formally for stationary equilibria.

Lemma 1 ***(Threshold Policy).*** *Consider a stationary equilibrium and suppose $\xi\varphi < \rho + \chi$. Then:*

(i) The optimal policy is a stopping time: once a household begins managing, it never returns

to working.

(ii) There exists a threshold $x^* \in (1,\infty)$ *such that* $l(x) = e$ *for* $x \in [1, x^*)$ *and* $l(x) = m$ *for* $x \geq x^*$*, with*

$$\frac{w_e}{w_m} = x^{*\xi-1}\left(1 - \frac{\xi\varphi}{\rho+\chi}\right) \tag{14}$$

Proof. See Section A. ■

When $\xi\varphi \geq \rho + \chi$, human capital grows so fast that the option value of remaining a worker always dominates the flow payoff from management. No household ever transitions, and the economy cannot sustain a stationary equilibrium with active managers. The bound $\xi\varphi < \rho + \chi$ is therefore necessary for both occupations to be active.

The stationary distribution of human capital, implied by the Kolmogorov forward equation (7) with $\partial_t N = 0$, is

$$N(x) = \begin{cases} 1 - x^{-\chi/\varphi} & x < x^* \\ x^{*-\chi/\varphi} & x = x^* \end{cases} \tag{15}$$

The value function is given in (29). The factor supplies and market clearing conditions are derived in Section B.

3.3. Stationary Equilibria

We now characterize all stationary equilibria, dropping time subscripts throughout. For analytical tractability, we set $\omega = 1$ so that $f = \min\{\bar{\tau} a_m, s\}$; since ω and $\bar{\tau}$ enter only through $\bar{\tau} a_m^{\omega}$, this is without loss up to a renormalization of $\bar{\tau}$.

3.3.1. Equilibria with no human capital accumulation. We first consider stationary equilibria with $\varphi = 0$. In such an equilibrium, the firm automates all products ($\tau^* \geq f$), the stationary distribution is a mass point at $x = 1$, and all households optimally choose to manage: $V(1) = w_m > w_e$. Since every agent is a manager with $h(1) = 1$, the frontier is $f = s = 1$.

Proposition 1 ***(No-Accumulation Equilibria).*** *A stationary equilibrium with* $\varphi = 0$ *exists if and only if*

$$q < \bar{q}(\bar{\tau}, \eta) \equiv \left(\frac{1}{1+\bar{\tau}^{-1}}\right)^{\eta^{-1}} \left(\eta^{-1}\right)^{\eta^{-1}} \left(1-\eta^{-1}\right)^{1-\eta^{-1}} \tag{16}$$

When this condition holds, there is a continuum of equilibria indexed by the worker wage w_e, with w_e taking any value in the interval

$$q < w_e < \eta^{-1}\left(\frac{q}{1-\eta^{-1}}\right)^{1-\eta} - q\bar{\tau}^{-1} \tag{17}$$

In each such equilibrium, $f = s = 1$, all products are automated, and all households are managers.

Proof. Full automation requires $q < w_e$ (from (10)). The frontier FOC (13) with $\omega = 1$ and $f = 1$ gives $w_m = \eta^{-1}(q/(1-\eta^{-1}))^{1-\eta} - q\bar{\tau}^{-1}$. The household condition $w_e < w_m$ delivers the upper bound. The interval is non-empty precisely when (16) holds. ■

The threshold $\bar{q}$ is increasing in $\bar{\tau}$: the more productive technology is in expanding the frontier, the more the price must fall before no-accumulation equilibria become possible. The indeterminacy in w_e reflects the fact that when no products are worker-produced, the worker wage is not pinned by any market clearing condition.

3.3.2. Equilibria with positive human capital accumulation. We now characterize equilibria with $\tau^* < f$ and $\varphi = \delta(f - \tau^*) > 0$. Combining the automation threshold (10), frontier FOC (13), household cutoff (14), factor supplies (30)–(31), and market clearing (32) (derived in Section B), we can eliminate w_e, w_m, and f to obtain a system of three equations in (x^*, φ, τ^*).

Proposition 2 ***(Positive-Accumulation Equilibria).*** *A stationary equilibrium with $\varphi > 0$ is characterized by (x^*, φ, τ^*) solving*

$$\eta^{-1}\left(\frac{q\tau^{*\varepsilon}}{1-\eta^{-1}}\right)^{1-\eta} = \frac{q\tau^{*\varepsilon}}{x^{*\xi-1}\left(1 - \frac{\varphi\xi}{\rho+\chi}\right)} + q\bar{\tau}^{-1} \tag{18}$$

$$\left(\frac{q\tau^{*\varepsilon}}{1-\eta^{-1}}\right)^{-\eta} = \frac{\delta}{\varphi}\frac{\chi/\varphi}{1-\chi/\varphi}\left(x^{*1-\chi/\varphi} - 1\right) \tag{19}$$

$$\varphi = \delta(x^{*\xi-\chi/\varphi} - \tau^*) \tag{20}$$

subject to $\varphi > 0$ and $\xi\varphi < \rho + \chi$.

Equation (18) is the frontier FOC after substituting the automation threshold and household cutoff to eliminate wages. Equation (19) is worker market clearing after the same substitutions. Equation (20) links the rate of human capital accumulation to the measure of worker-produced products. As discussed in the introduction, this system can admit multiple solutions for the same technology price q, corresponding to the high and low human capital equilibria described there.

4 A Decline in the Price of the Technology: A Numerical Example

We illustrate the results of Section 3 by solving the system in Proposition 2 numerically for a range of technology prices q. The parameter values are reported in Table 1. Figure 1 plots the key equilibrium objects as functions of q. For each value of q in an intermediate range, the system in (18)–(20) admits two solutions, corresponding to what we refer to as the *high human capital* equilibrium (dashed red) and the *low human capital* equilibrium (solid blue). The two branches coexist over a wide interval of q: at high q only one equilibrium exists; as q falls a second branch appears and the gap between the two widens.

Table 1: Assigned parameter values

Parameter	Value	Description
ρ	0.05	Discount rate
χ	0.01	Death rate
η	4	Demand elasticity
ξ	1.05	Managerial human capital elasticity
ε	1	Automation cost elasticity
$\bar{\tau}$	20	Frontier technology parameter
δ	0.02	Learning rate

The career ladder. Panels (a)–(c) display the objects that characterize the internal structure of the career ladder. In the high human capital equilibrium, workers accumulate substantially more human capital before transitioning to management: the cutoff x^* in panel (a) rises steeply as q falls, exceeding 10^2 at the lowest prices. The rate of human capital accumulation φ in panel (b) mirrors this pattern, rising from near zero to roughly 0.05. The automation cutoff τ^* in panel (c) also increases with cheaper technology, reflecting the fact that although more products are automated, the frontier expands by enough that workers still perform a large share of tasks. In the low human capital equilibrium, by contrast, $x^* \approx 1$ and $\varphi \approx 0$ across most of the range: workers barely accumulate any human capital before switching to management, and the rate of learning is negligible.

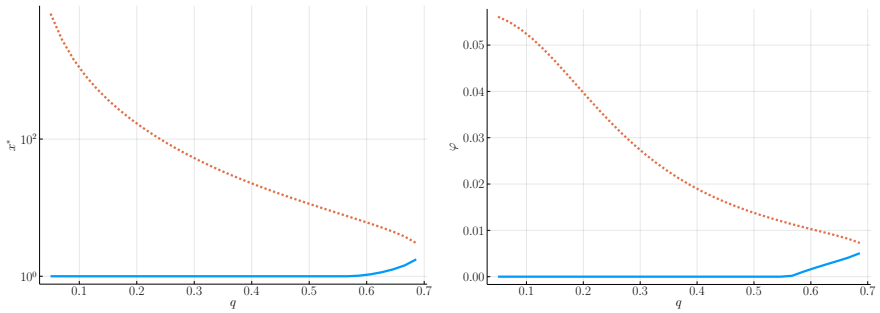
Wages. Panels (d) and (e) show the worker and manager wages. A striking feature is that the worker wage w_e is *higher* in the low human capital equilibrium than in the high human capital equilibrium. This reflects a composition effect: in the low human capital equilibrium, workers

transition to management almost immediately, leaving very few workers in the economy. The scarcity of worker labor drives up w_e despite the low level of human capital. In the high human capital equilibrium, workers remain in the labor force longer, generating a larger effective supply of worker labor that keeps w_e lower. The household cutoff condition in Lemma 1 links w_e to the manager wage w_m through the threshold x^*: a higher x^* requires a lower ratio w_e/w_m, consistent with the patterns in panels (d) and (e). Manager wages are higher in the high human capital equilibrium because the expanded frontier raises the marginal value of managerial human capital.

Aggregate outcomes. Panels (f)–(h) display the product frontier, the value of an entering worker, and total firm profits. The product frontier f in panel (f) captures the core mechanism behind the multiplicity. In the high human capital equilibrium, highly skilled managers combined with cheap technology expand the frontier by an order of magnitude as q falls. In the low human capital equilibrium, $f \approx 1$: managers arrive with minimal human capital and cannot leverage the technology to create new products, so the frontier barely expands regardless of how cheap the technology becomes. The value of an entering worker $V(1)$ in panel (g) is higher in the low human capital equilibrium despite the collapsed frontier. This reflects the same scarcity mechanism that drives wages: since workers transition to management almost immediately when $x^* \approx 1$, they quickly earn the high manager wage w_m. In the high human capital equilibrium, workers spend longer accumulating human capital at the lower wage w_e before transitioning, which reduces $V(1)$ even though the career path ultimately yields higher managerial productivity. Total firm profits Π in panel (h) are higher in the high human capital equilibrium, as the expanded frontier generates more products and greater surplus.

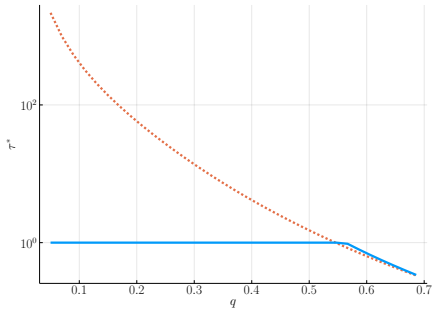
The multiplicity visible in Figure 1 arises from a complementarity between human capital accumulation and the task frontier. In the high human capital equilibrium, workers perform enough tasks to accumulate substantial human capital, become effective managers, and expand the frontier—which in turn creates more tasks for the next generation of workers. In the low human capital equilibrium, this virtuous cycle is absent: few tasks are available for learning, managers arrive with low human capital, and the frontier stagnates. As q falls further, the low human capital branch can become the only equilibrium, corresponding to the human capital trap discussed in the introduction.

Figure 1: Equilibrium objects as functions of the AI price q

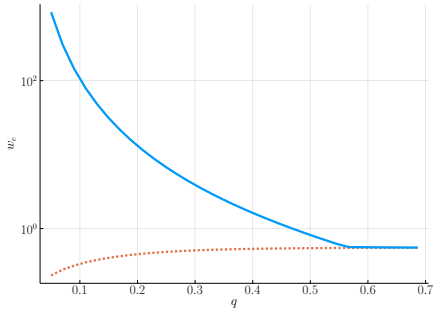


(a) x^*

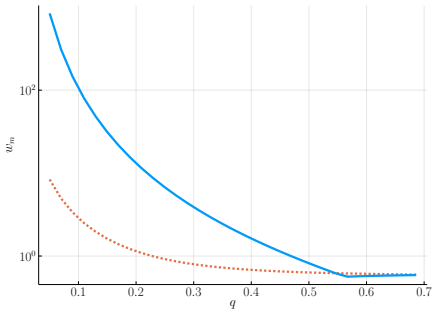
(b) φ



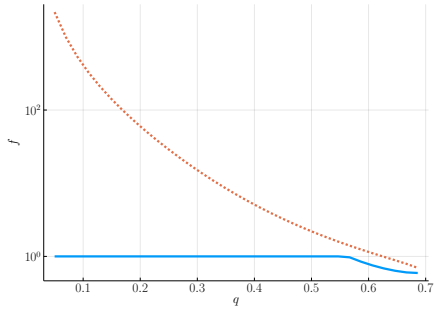
(c) τ^*



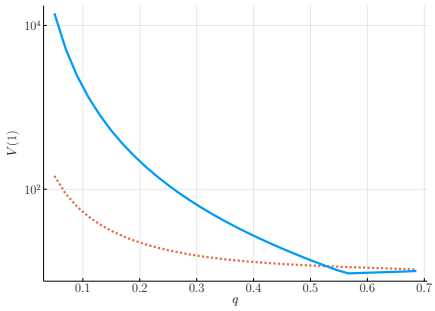
(d) w_e



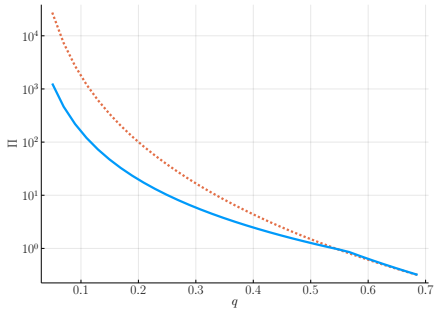
(e) w_m



(f) f



(g) $V(1)$



(h) Π

Notes: Each panel plots a stationary equilibrium object against the technology price q. The solid blue line is the low human capital equilibrium; the dashed red line is the high human capital equilibrium. Panels show: (a) worker-manager cutoff x^*, (b) human capital growth rate φ, (c) automation cutoff τ^*, (d) worker wage w_e, (e) manager wage w_m, (f) product frontier f, (g) value of an entering worker $V(1)$, (h) total firm profits Π.

5 Conclusion

This paper develops a model of how a new technology that substitutes for workers at the bottom of the career ladder while complementing workers at the top can affect the pipeline of human capital that flows through an economy over time. The central mechanism is learning by doing: workers accumulate human capital by performing tasks, that human capital determines the quality of future managers, and the quality of managers in turn determines how many new tasks the technology can help create. When a firm automates an entry-level task, it captures the full productivity gain but ignores the effect of that decision on the learning process for every worker in the economy. Likewise, when it uses the technology to expand the task frontier, it fails to internalize its use of the technology on worker learning. This externality — invisible to any individual firm but consequential in the aggregate — is the source of the model's main results. We show that the decentralized economy can sustain a high-human-capital equilibrium in which declining technology prices generate more automation and more new tasks simultaneously, and that when the technology is sufficiently good at complementing managers, this good equilibrium is the unique long-run outcome. But when the technology's complementarity with managers is weaker relative to its substitutability for workers, the economy enters an environment in which two equilibria coexist: a good equilibrium with robust human capital accumulation and a bad equilibrium with a depleted learning pipeline. When the price of the technology falls be a sufficient large level, a low human capital trap economy can emerge where the economy's level of human capital can remain low. We will have lots of "managers" using technology to come up with new ideas but the ideas are not very good.

We also characterize the optimal policy response. When the extent to which the technology automates existing tasks is sufficiently large relative to how the technology expands the task frontier,fFirms in the decentralized economy automate too much relative to the social optimum because they do not internalize the effect of automation on the aggregate learning rate. Overall, the direction of the corrective policy depends on which externality dominates. When automation is the dominant force — when the technology is broadly productive at substituting for entry-level tasks — the planner may want to tax automation to preserve learning opportunities and steer the economy away from the trap. When frontier expansion is the dominant force — when

the technology is highly productive at helping managers generate new tasks but only weakly productive at automating existing ones — firms under-invest in the technology and the planner may want to subsidize it. The corrective wedge is largest precisely when the economy is closest to the boundary of of falling into a low human capital trap, reflecting that the social cost of a small shift in automation is highest when the economy is most fragile.

Our quantitative analysis illustrates these mechanisms for empirically motivated parameterizations and traces how the equilibrium objects — the automation share, the task frontier, the learning rate, the career ladder threshold, and the wage gap between workers and managers — respond as the price of the technology falls. The numerical results confirm the three-regime structure predicted by the theory, identify the parameter configurations that place an economy at risk of tipping into the trap, and provide examples of the welfare gains from the planner's optimal corrective policy. A natural next step is to bring the model to data: occupations differ substantially in how broadly AI substitutes for entry-level tasks, how rapidly junior workers learn, and how effectively AI tools help senior workers generate new ideas and products, and these differences should predict which occupations are most at risk of a disrupted human capital pipeline as AI prices continue to fall. We leave that quantitative extension for future work.

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A Proof of Lemma 1

Part (i): Irreversibility. In a stationary equilibrium, the HJB (5) reduces to $(\rho+\chi)V(x) = \max\{w_e x + \varphi x V'(x),\ w_m x^{\xi}\}$, which depends only on x. Since human capital is frozen during management, a managing household remains at x and faces the identical problem at every future instant. Therefore, the decision never reverses and the problem is a stopping time.

Part (ii): Threshold in x. By Part (i), the problem is a stopping time: the household works while $x < x^*$ and manages for $x \geq x^*$. In the worker region, the HJB (5) reduces to

$$(\rho+\chi)V(x) = w_e\, x + \varphi\, x\, V'(x), \qquad x \in [1, x^*) \tag{21}$$

In the manager region, $V(x) = w_m x^{\xi}/(\rho+\chi)$. At the boundary x^*, value matching and smooth pasting require

$$V(x^*) = \frac{w_m (x^*)^{\xi}}{\rho+\chi} \tag{22}$$

$$V'(x^*) = \frac{w_m\, \xi\, (x^*)^{\xi-1}}{\rho+\chi} \tag{23}$$

The general solution to (21) is

$$V(x) = C x^{\frac{\rho+\chi}{\varphi}} + \frac{w_e\, x}{\rho+\chi-\varphi} \tag{24}$$

where C is a constant of integration. Value matching (22) gives

$$C = \left[\frac{w_m (x^*)^{\xi}}{\rho+\chi} - \frac{w_e\, x^*}{\rho+\chi-\varphi}\right] (x^*)^{-\frac{\rho+\chi}{\varphi}} \tag{25}$$

Differentiating (24):

$$V'(x) = C\frac{\rho+\chi}{\varphi}\, x^{\frac{\rho+\chi}{\varphi}-1} + \frac{w_e}{\rho+\chi-\varphi} \tag{26}$$

Evaluating at $x = x^*$ and substituting (25):

$$V'(x^*) = \left[\frac{w_m (x^*)^{\xi-1}}{\rho+\chi} - \frac{w_e}{\rho+\chi-\varphi}\right]\frac{\rho+\chi}{\varphi} + \frac{w_e}{\rho+\chi-\varphi} \tag{27}$$

Setting (27) equal to the smooth-pasting condition (23):

$$\left[\frac{w_m (x^*)^{\xi-1}}{\rho+\chi} - \frac{w_e}{\rho+\chi-\varphi}\right]\frac{\rho+\chi}{\varphi} + \frac{w_e}{\rho+\chi-\varphi} = \frac{w_m\, \xi\, (x^*)^{\xi-1}}{\rho+\chi}$$

The second and third terms on the left combine to $w_e/\varphi \cdot [\varphi/(\rho+\chi-\varphi) - (\rho+\chi)/(\rho+\chi-\varphi) + 1] = 0$, leaving $w_m (x^*)^{\xi-1}/\varphi$ on the left. Rearranging:

$$w_m (x^*)^{\xi-1}\left(1 - \frac{\xi\varphi}{\rho+\chi}\right) = w_e \tag{28}$$

which is (14). Substituting (25) back into (24):

$$V(x) = \left[\frac{w_m (x^*)^{\xi}}{\rho+\chi} - \frac{w_e\, x^*}{\rho+\chi-\varphi}\right]\left(\frac{x}{x^*}\right)^{\frac{\rho+\chi}{\varphi}} + \frac{w_e\, x}{\rho+\chi-\varphi} \tag{29}$$

■

B Stationary Equilibrium Derivations

This appendix collects the factor supply and market clearing expressions used in Section 3.2. All derivations assume the stationary threshold policy of Lemma 1.

From the stationary distribution (15), the aggregate supply of managerial human capital is

$$s = x^{*\xi-\chi/\varphi} \tag{30}$$

and the aggregate supply of worker human capital is

$$X^s = \frac{\chi}{\varphi-\chi}\left(x^{*1-\chi/\varphi} - 1\right) \tag{31}$$

Each worker-produced product demands the same human capital input (from (11)), so the firm's total demand is $(f-\tau^*)(w_e/(1-\eta^{-1}))^{-\eta}$. Using $\varphi = \delta(f-\tau^*)$, market clearing for worker human capital requires

$$\varphi\left(\frac{w_e}{1-\eta^{-1}}\right)^{-\eta} = \frac{\delta\chi}{\varphi-\chi}\left(x^{*1-\chi/\varphi} - 1\right) \tag{32}$$