Estimating the Aggregate Implications of Incomplete Markets using an Incomplete Model*

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March 27, 2026

Abstract

We show how to use an incomplete model—a representative-agent benchmark that omits household heterogeneity—to estimate the aggregate implications of incomplete markets. Our main theorem establishes that, for any complete-markets benchmark path, there exists a household-specific transfer sequence that makes an incomplete-markets economy replicate the benchmark allocation in aggregates. These transfers are identified from household budget constraints and therefore provide a direct measure of redistribution without requiring specification of a full heterogeneous-agent model. Combined with marginal propensities to consume, they deliver the aggregate consumption correction induced by market incompleteness. Under empirically relevant nominal rigidities, the same correction provides a first-order approximation to the heterogeneous-agent general-equilibrium consumption response. We use these results to construct heterogeneity-robust counterfactuals from representative-agent impulse responses and to estimate HANK through a latent RANK benchmark. In model-generated data from benchmark and alternative HANK economies, the procedure recovers aggregate responses and structural parameters.

*We thank Gianluca Violante for constructive suggestions that led to a change in the direction to take the paper, and our discussants, Vincent Sterk, Pengfei Wang, Markus Sihvonen, Wan-Jung Cheng, and Byoungchan Lee. Marcus died as we were finishing the first draft of this paper. We will remember Marcus as a dear friend and cherished co-author and for the incredibly high standard he set for all of us with his intellectual clarity, his principled stance, and his relentless pursuit of understanding. We benefited from interactions with seminar and conference participants at MacCaLM Workshop 2017, SED Edinburgh, AEA Annual Meeting 2018, Bank of Finland and CEPR Conference on Monetary Economics and Reality 2019, SITE 2019, Stanford, ECB Money Macro Workshop 2019, 2nd European Midwest Micro/Macro Conference, HKMA-FRBNY Conference on "Monetary Policy and Heterogeneity" 2019, Boston College, Princeton, 2nd Copenhagen Macro Days, AEA Annual Meeting 2021, SED Madison, SED Barcelona, 11th Annual CIGS conference on Macroeconomic Theory and Policy, 3D-In-Macro Workshop BI Oslo, Minnesota Macro 2024, Midwest Macro 2024, Mizzou, UC Santa Cruz, University of Virginia, Universidad Autonoma de Madrid. This paper has been previously presented under the titles "Monetary Policy in Incomplete Markets Models: Theory and Evidence" and "Does Market Incompleteness Matter for Monetary Policy? Theory and Evidence." Support from the National Science Foundation Grant No. SES-1357903, FRIPRO Grant No. 250617, the European Research Council Starting Grant 759482, Consolidator Grant 101171856, and the Ragnar Söderbergs stiftelse is gratefully acknowledged.

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1 Introduction

Over the last decade(s), macroeconomists have made substantial progress in incorporating household heterogeneity into workhorse models. This progress reflects the view that shocks and policies redistribute resources across households and may therefore change aggregate dynamics. The open macro question is whether that redistribution materially changes aggregate dynamics or whether representatitive-agents benchmarks remain a good guide. HANK models show that either outcome is possible, but rely on fully specified heterogeneous-agent models, whose conclusions can be sensitive to assumptions that are hard to verify or quantify in the data. This paper addresses that gap. We develop a methodology for measuring the aggregate implications of incomplete markets without specifying a full heterogeneous-agent model. Put differently, we use an incomplete model to measure the effects of incomplete markets. Our contribution is methodological: we establish the theoretical result, develop the measurement framework, and evaluate its quantitative performance.

Our starting point is a transfer-equivalence theorem. Consider the response of a complete markets economy to an aggregate shock, e.g., a monetary shock. We establish that there exists a household-specific transfer sequence that makes the incomplete-markets economy replicate the complete-markets allocation in terms of observable aggregates (prices and quantities). Intuitively, under complete markets, household income, consumption, and assets move in proportion to the aggregates.[1] The transfers we construct are exactly the household-specific wedges needed to restore that proportional scaling in an incomplete-markets economy. These transfers are not only an ingredient for recovering aggregates. Because they are identified from household budget constraints, they provide a direct measure of redistribution due to market incompleteness, and are therefore of independent interest as a welfare-relevant object.

In order to go from compensating transfers to the macro implications of incomplete-markets we need to exploit data on marginal propensities to consume (MPCs). Combining MPCs with the transfers and integrating across the distribution, they yield $\Delta C = \text{transfers} \times$

[1] Note that, in dollar terms, budget sets of different individuals change by different amounts, but this redistribution does not imply that markets are incomplete.

MPCs, the aggregate consumption correction induced by market incompleteness. At RANK prices, this correction is exact. Under empirically relevant nominal rigidities, it also provides a first-order approximation to the HANK general-equilibrium consumption response. This delivers heterogeneity-robust counterfactuals from RANK impulse responses. This allows us to develop a novel method for estimating HANK via impulse-response matching using a latent-RANK model.

So far, the complete-markets response to a shock has been taken as known. In practice, it need not be. Indeed, to the extent that the observed data are generated by an incomplete-markets model, directly estimating a RANK model on these data would yield biased parameter estimates as the model is misspecified. Instead, to construct the latent complete markets benchmark (the complete markets version of the HANK model that generated the data) one needs to take the misspecification into account. Leveraging the fact that the HANK and RANK models differ only in terms of the consumption block, our main theoretical results provide a natural way of using the ΔC correction to do so. We begin by specifying the RANK model and an initial guess for its parameter vector. This yields impulse responses to, e.g., monetary shocks. Using data on household budget constraints, we then compute transfers and combine them with MPCs to compute the aggregate consumption correction ΔC due to market incompleteness, as described above. Having obtained the total consumption response (the one from the latent RANK model plus ΔC), we solve for the remaining equilibrium variables, evaluate likelihood, and update the parameters of the latent RANK model, if necessary. This procedure not only provides the relevant benchmark to assess the consequences of market incompleteness but also allows us to estimate the parameters relevant to the RANK model without biases induced by heterogeneity and market incompleteness in the data.

We validate these ideas in model-generated data from benchmark and alternative HANK economies. In our benchmark model, we recover the aggregate consumption correction ΔC, show when RANK impulse responses plus ΔC recover HANK general-equilibrium responses, and demonstrate in Monte Carlo exercises that the latent-RANK estimator recovers the true structural parameters. We then show that the same procedure performs well in leading alternative HANK environments. In a companion paper, Hagedorn et al. (2026), we develop the empirical measurement of iMPCs and apply the method to U.S. monetary

shocks.

The rest of the paper is organized as follows. After reviewing related literature, in Section 2 we state and prove our main theorem. In Section 3 we describe the model framework which incorporates several standard features of heterogeneous agents incomplete markets models with price and wage rigidities. While the method does not require specifying the complete model, the model provides a controlled benchmark that enables us to study the method's quantitative performance. In Section 4, we show how to go from transfers to aggregates. We present the results from our latent-RANK estimator in Section 5, and show that the methodology works well on the data generated from key quantitative HANK models in the literature. Section 6 concludes.

1.1 Related Literature

This paper relates first to the burgeoning literature on heterogeneous-agent macroeconomics with nominal rigidities. At one end, Kaplan et al. (2018) show that HANK and RANK can imply sharply different responses to monetary policy in a quantitative model. More broadly, Kaplan and Violante (2018) review the emerging literature that combines nominal rigidities, idiosyncratic risk, and aggregate shocks, building on early contributions such as Gornemann et al. (2012), Oh and Reis (2012), and Guerrieri and Lorenzoni (2017). At the other end of the spectrum, Werning (2015) derives an aggregate Euler representation under conditions that make incomplete-markets economies much closer to representative-agent benchmarks. Our paper is complementary to this literature. Rather than taking a stand on a fully specified HANK micro block, we ask how much of the aggregate effect of incomplete markets can be recovered from household budget sets, MPC objects, and a representative-agent benchmark. This perspective is also complementary to recent work estimating richer HANK models directly, such as Bayer et al. (2024).

Our first-order results are also related to the literature seeking low-dimensional or sufficient-statistic summaries of heterogeneity. Auclert (2019) shows that redistribution matters for aggregate demand when households' exposures to aggregate shocks covary with their MPCs. Auclert et al. (2024) show that the iMPC matrix is sufficient to characterize the dynamic response of output to fiscal policy. Wolf (2025) establishes an equivalence

between interest-rate cuts and uniform stimulus payments in standard demand-determined models, while Bilbiie et al. (2025) propose sufficient statistics to assess whether heterogeneity amplifies demand shocks in Norwegian micro data. Our approach is close in spirit but differs in object. The central object in our analysis is a household-specific compensating-transfer sequence, identified directly from the budget constraint and defined relative to a complete-markets benchmark. Combined with iMPCs, these transfers yield the aggregate consumption correction due to incomplete markets and can be used for heterogeneity-robust counterfactuals and latent-RANK estimation.

Most closely related is Berger et al. (2023). They show that a broad class of HANK models admits an equivalent representative-agent economy with a discount-factor wedge and a labor wedge, both measurable from micro data on consumption shares and relative wages. In their empirical implementation, the discount-factor wedge is identified using grouped household data, with the high-income/low-net-worth group proxying for financially unconstrained households. As they emphasize, because these wedges are endogenous to the policy regime, their method is best suited to ex post evaluations. Our approach differs in both object and interpretation. Rather than backing out representative-agent wedges, we construct household-specific compensating transfers directly from the budget constraint, relative to a complete-markets benchmark. This lets us work at the household level, delivers a welfare-relevant transfer object, and connects naturally to aggregate recovery through MPCs.

Finally, our paper is related to work measuring the distributional incidence and welfare effects of aggregate shocks. Del Canto et al. (2025) develop a feasible-set approach in which the first-order welfare effect of a shock is summarized by changes in households' budget and borrowing constraints, and they use it to study the incidence of inflationary shocks and monetary expansions. That paper is close in spirit because it also builds a money-metric household object from budget-set information. The key difference is that their object measures the total first-order welfare incidence of a shock, whereas our transfers isolate the redistribution due specifically to incomplete markets relative to a complete-markets benchmark. For that reason, our transfers are of independent interest as welfare-relevant compensating transfers, but they are also the input into our aggregate correction and counterfactual exercises.

In the companion paper, Hagedorn et al. (2026), we develop an empirical implementation strategy for the theoretical methods proposed in this paper and conduct a substantive empirical analysis on the effects of monetary policy. Specifically, using data from the Panel Survey of Income Dynamics (PSID), we observe the components of each household's budget constraint and calculate the transfers identified by the theory. We then combine the transfers with the estimated dynamic MPCs to derive the aggregate consumption responses. We find that monetary policy leads to an initial consumption response of approximately 50% larger than in the incomplete market model. Thus, the data reveal that heterogeneity and market incompleteness have an important role in shaping aggregate implications and the distributional consequences of monetary policy.

In Hagedorn et al. (2026) we also exploit the fact that the method proposed in this aper measures transfers at the level of individual households, allowing for a detailed and nuanced analysis of the distributional effects of economic shocks and policies. Focusing on measuring the redistributive effects of monetary policy, for each household in the PSID we compute the transfer needed to compensate it for the redistribution induced by monetary policy. This compensating transfer is our measure of redistribution for each household. We then aggregate the household measures to obtain measures of redistribution between groups of people, including low/high labor or asset income, young and old, men and women, and black and white. We find that the redistribution across these groups induced by monetary policy is large in the data.

2 Measurement: A Model-Light Transfer Framework

This section develops the theoretical core of the paper in a deliberately model-light environment. We do not begin with the full New Keynesian structure used later for quantitative analysis. Instead, we start from the minimal ingredients needed to define redistribution relative to a complete-markets benchmark: a steady-state distribution of households, a household budget constraint, and a benchmark aggregate response path. These objects are sufficient to construct household-specific compensating transfers directly from budget sets and to state the paper's main transfer-equivalence theorem.

Model-light here does not mean assumption-free. The theorem still relies on the minimal

household structure needed for proportional scaling to be well defined and optimal. But by postponing the full aggregate block to Section 3, we can cleanly separate what is intrinsic to the measurement idea from what is specific to the benchmark quantitative laboratory used later for validation, general-equilibrium recovery, and estimation. Because the theorem is an exact nonlinear statement, we use multiplicative γ-notation in this section; Section **??** will switch to first-order d-notation.

2.1 Minimal household environment

We begin with the minimal household structure needed for the transfer-equivalence result. Households face idiosyncratic risk, save in a non-contingent asset, receive labor income and transfers, and choose consumption and savings subject to a budget constraint. At this stage we do not specify firms, price setting, monetary policy, or the rest of the aggregate block. Those objects matter later for quantitative implementation, but not for the construction of the transfer object itself.

Let Ω_{ss} denote the invariant distribution of household states in the incomplete-markets steady state, and let

$$\bar{C},\ \bar{H},\ \bar{Y},\ \bar{A},\ \bar{w},\ \bar{\Gamma},\ \bar{r}^a,\ \bar{\tau}_w,\ \bar{\tau}_k$$

denote the corresponding steady-state aggregates and policies. For household i at date t, let

$$c_{i,t}^{IM,ss}, \qquad h_{i,t}^{IM,ss}, \qquad a_{i,t}^{IM,ss}$$

denote the choices that household would make at date t if aggregate prices and policies were forever fixed at their steady-state values. These objects remain indexed by calendar time: although aggregate variables are constant in steady state, households continue to move across idiosyncratic states within the invariant distribution. The benchmark for our transfer construction is therefore the household-specific stationary path, not a static cross section.

Define steady-state after-tax labor income by

$$\bar{\ell}_{i,t} \equiv (1-\bar{\tau}_w)\bar{w}\, s_{i,t} h_{i,t}^{IM,ss},$$

and the steady-state gross after-tax return by

$$\bar{R}^a \equiv 1 + (1 - \bar{\tau}_k)\bar{r}^a.$$

Then the household budget constraint in the stationary incomplete-markets equilibrium is

$$c_{i,t}^{IM,ss} + a_{i,t+1}^{IM,ss} = \lambda_{i,t}^r \bar{R}^a a_{i,t}^{IM,ss} + \bar{\ell}_{i,t} + \lambda_{i,t}\bar{\Gamma}. \tag{1}$$

Here $\lambda_{i,t}$ governs the household-specific incidence of aggregate transfers and $\lambda_{i,t}^r$ allows for heterogeneity in returns. When returns are common across households, $\lambda_{i,t}^r \equiv 1$.

Equation (1) contains all the objects needed for the transfer construction below. Nothing in this subsection requires the full quantitative model introduced in Section 3.

2.2 Benchmark response path and proportional scaling

With the household resource constraint in hand, the only additional ingredient needed for the theorem is a benchmark aggregate response path. Take as given a benchmark path, which we interpret as the complete-markets counterpart of the economy under study, and define the corresponding gross responses

$$\gamma_t^C \equiv \frac{C_t^{CM}}{\bar{C}}, \qquad \gamma_t^H \equiv \frac{H_t^{CM}}{\bar{H}}, \qquad \gamma_t^A \equiv \frac{A_t^{CM}}{\bar{A}}, \qquad \gamma_t^Y \equiv \frac{Y_t^{CM}}{\bar{Y}}, \tag{2}$$

$$\gamma_t^w \equiv \frac{w_t^{CM}}{\bar{w}}, \qquad \gamma_t^\Gamma \equiv \frac{\Gamma_t^{CM}}{\bar{\Gamma}}, \qquad \gamma_t^{\tau_w} \equiv \frac{1 - \tau_{w,t}^{CM}}{1 - \bar{\tau}_w}, \tag{3}$$

and let

$$R_t^a \equiv 1 + (1 - \tau_{k,t}^{CM}) r_t^{a,CM} \tag{4}$$

denote the benchmark gross after-tax return on assets.

We assume throughout that the complete-markets and incomplete-markets economies are aligned at the same aggregate steady state. As is standard, this may require different discount-factor parameters in the two environments, but it ensures that the comparison is about responses to shocks rather than differences in steady states.

Our benchmark notion of *no redistribution due to incomplete markets* is that household variables scale relative to their own stationary paths by the same gross factors as their

aggregate counterparts. That is, at date t,

$$c_{i,t}^{IM} = \gamma_t^C c_{i,t}^{IM,ss}, \qquad h_{i,t}^{IM} = \gamma_t^H h_{i,t}^{IM,ss}, \qquad a_{i,t+1}^{IM} = \gamma_{t+1}^A a_{i,t+1}^{IM,ss}. \tag{5}$$

This proportional-scaling benchmark is defined relative to the movements households would have made inside the invariant distribution absent the aggregate shock. If (5) holds, incomplete markets do not alter the aggregate transmission of the shock relative to the complete-markets benchmark. When it fails, we interpret the deviation as redistribution due to incomplete markets and ask what household-specific transfers would restore it.

2.3 Constructing compensating transfers

Proportional scaling defines our benchmark notion of no redistribution. The transfers are then the household-specific residuals needed to restore that benchmark.

Given the benchmark path

$$\left\{\gamma_t^C, \gamma_t^H, \gamma_t^A, \gamma_t^w, \gamma_t^\Gamma, \gamma_t^{\tau_w}, R_t^a\right\}_{t\geq 0},$$

define the transfer to household i at date t by

$$\Delta_{i,t} = (\gamma_t^C - 1)c_{i,t}^{IM,ss} - (\gamma_t^H \gamma_t^w \gamma_t^{\tau_w} - 1)\bar{\ell}_{i,t} - \lambda_{i,t}(\gamma_t^\Gamma - 1)\bar{\Gamma} + \lambda_{i,t}^r a_{i,t}^{IM,ss}\left(\bar{R}^a - \gamma_t^A R_t^a\right) + a_{i,t+1}^{IM,ss}(\gamma_{t+1}^A - 1). \tag{6}$$

We adopt the convention that a positive $\Delta_{i,t}$ is a payment to the household.

Equation (6) has a simple interpretation. The first term captures the consumption scaling required by the benchmark path. The second and third terms account for deviations in after-tax labor income and transfers from proportional scaling. The last two terms capture asset-income and savings effects. Thus $\Delta_{i,t}$ is exactly the household-specific residual needed to make the proportional-scaling allocation feasible at the household budget constraint.

This transfer object is the central microeconomic output of the paper. It measures, at the household level, the redistribution due to incomplete markets relative to the complete-markets benchmark. For that reason it is of independent interest even before we aggregate it using MPCs: it is the compensating transfer required to undo the redistributive effect of incomplete markets.

2.4 Transfer-equivalence theorem

The next theorem shows that these residuals are not merely accounting devices: when the benchmark path is itself generated by a complete-markets equilibrium, they exactly implement that path in the incomplete-markets economy.

Theorem 2.1 (Transfer equivalence) *Consider a benchmark path*

$$\left\{\gamma_t^C, \gamma_t^H, \gamma_t^A, \gamma_t^Y, \gamma_t^w, \gamma_t^\Gamma, \gamma_t^{\tau_w}, R_t^a\right\}_{t\geq 0}$$

generated by a complete-markets equilibrium, and suppose the incomplete-markets and complete-markets economies share the same aggregate steady state. Let the household transfers $\Delta_{i,t}$ be given by (6)*. Then there exists an equilibrium of the incomplete-markets economy with these transfers such that*

$$c_{i,t}^{IM} = \gamma_t^C c_{i,t}^{IM,ss}, \qquad h_{i,t}^{IM} = \gamma_t^H h_{i,t}^{IM,ss}, \qquad a_{i,t+1}^{IM} = \gamma_{t+1}^A a_{i,t+1}^{IM,ss}. \tag{7}$$

Consequently, aggregate quantities in the incomplete-markets economy coincide with the benchmark complete-markets path:

$$C_t^{IM} = C_t^{CM}, \qquad H_t^{IM} = H_t^{CM}, \qquad A_t^{IM} = A_t^{CM}, \qquad Y_t^{IM} = Y_t^{CM}, \tag{8}$$

and all prices and policies inherited from the common non-household block also coincide with the benchmark path.

Proof sketch. The argument has three steps. First, by construction, substituting (6) into the household budget constraint makes the scaled allocation in (7) feasible household by household. Second, under the maintained household structure, the same scaled allocation satisfies the Euler and intratemporal optimality conditions: for unconstrained households, the homotheticity implied by CRRA preferences preserves the Euler equation, while for constrained households the Kuhn-Tucker conditions scale in the same way. Third, once the household block is aligned with the benchmark path, all remaining equilibrium conditions are inherited from the complete-markets equilibrium. A full proof is given in the appendix.

The theorem immediately implies that the transfers sum to zero in every period:

$$\int \Delta_{i,t}\, d\Omega_{ss}(i) = 0 \qquad \forall t. \tag{9}$$

The transfer object therefore measures redistribution across households rather than a net resource injection. This zero-sum property will be central below when we take the household-

level transfers to the aggregate data. In the appendix, we show how the theorem can be extended to allow for return heterogeneity.

2.5 Using transfers Δ to measure redistribution

Theorem 2.1 allows for a straightforward measurement of redistribution induced by monetary policy. The transfers $\Delta_{i,t}$ are defined to ensure, once taken into account, that all individual variables evolve as if there was no redistribution, i.e. the ratio of individual to aggregate variables is constant. We therefore find that the

$$\text{redistributional effect on idividual } i \text{ in Period } t: \quad -\Delta_{i,t}.$$

This definition makes sense since according to theorem 2.1 the transfer $\Delta_{i,t}$ exactly undoes the redistribution of monetary policy. The transfer $\Delta_{i,t}$ are measuring redistribution since we show that they integrate to zero in all periods:

$$\int_{\text{All households i}} \Delta_{i,t} di = 0.$$

To get a better idea of the welfare significance of this redistribution measure, we can divide $\Delta_{i,t}$ by disposable income of individual i in Period t in the absence of the monetary policy change.

Our redistribution measure for a group $\mathcal{G}$ of households at time t is

$$\int_{i \in \mathcal{G}} \Delta_{i,t} di.$$

This is again a proper measure of redistribution between households in group $\mathcal{G}$ and households not in group $\mathcal{G}$, since the measure for the two groups adds up to zero,

$$\int_{i \in \mathcal{G}} \Delta_{i,t} di + \int_{i \notin \mathcal{G}} \Delta_{i,t} di = \int_{\text{All households i}} \Delta_{i,t} di = 0.$$

In our empirical implementation the group $\mathcal{G}$ will be low income households, low asset households, young, women and blacks, respectively.

3 Benchmark Quantitative HANK/RANK Economy

This section introduces the quantitative environment used to operationalize and validate the transfer-equivalence theorem. The model is a benchmark HANK economy with sticky prices and wages, capital accumulation, government debt, and a mutual fund that implies a common asset return. We also define its complete-markets/RANK counterpart. The role of this section is not to provide the foundation for the theorem in Section 2, but to provide a disciplined laboratory in which the transfer object, the aggregate correction, and the latent-RANK estimator can be evaluated quantitatively.

Two features of the benchmark model are central for the rest of the paper. First, the household budget constraint nests exactly the objects entering the transfer formula from Section 2. Second, outside the consumption-savings block, the equilibrium system is common across RANK and HANK. This common block is what later allows us to move from the exact transfer theorem at benchmark prices to general-equilibrium recovery, heterogeneity-robust counterfactuals, and latent-RANK estimation.

3.1 Household block

We begin with the incomplete-markets household block in the Bewley-Huggett-Aiyagari-Imrohoroglu tradition. Households face idiosyncratic labor risk, save in a non-contingent asset, receive labor income and transfers, and choose consumption and savings subject to a borrowing constraint. The key output of this subsection is the mapping from aggregate prices and policies into household budget sets, together with the invariant distribution and the iMPC objects used later in the paper.

We keep the household block separate because it is the only part of the model that differs between HANK and RANK. Everything outside this consumption-savings problem will be collected in the next subsection as a common non-household block.

The economy consists of a continuum of agents normalized to measure 1 who are ex-ante heterogenous with respect to their subjective discount factors and who have CRRA preferences over consumption and additively separable preferences for leisure:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t),$$

where

$$u(c,h) = \begin{cases} \frac{c^{1-\sigma}-1}{1-\sigma} - g(h) & \text{if } \sigma \neq 1 \\ \log(c) - g(h) & \text{if } \sigma = 1, \end{cases}$$

$\beta \in (0,1)$ is the household-specific subjective discount factor, and $g(h)$ is the disutility of labor. Agents' labor productivity $\{s_t\}_{t=0}^{\infty}$ is stochastic and is characterized by an N-state Markov chain that can take on values $s_t \in \mathcal{S} = \{s_1, \cdots, s_N\}$ with transition probability characterized by $p(s_{t+1}|s_t)$ and $\int s = 1$. Agents rent their labor services, hs, to firms for a real wage w_t and their real assets a to the capital market for a real return $(1 + r_t^a)$. The real return on bonds is $1 + r_t = \frac{1+i_t}{1+\pi_t}$, where i is the nominal return and $1 + \pi_t = \frac{P_t}{P_{t-1}}$ is the inflation rate (the equilibrium final good price P_t is derived below). There are two classes of assets, bonds and capital with potentially different returns, but households can invest in one asset A, which the mutual fund (described below) collects and allocates to bonds and capital.

Thus, at time t an agent faces the following budget (written in real terms) constraint:

$$c_t + a_{t+1} = (1 + r_t^a(1 - \tau_k))a_t + (1 - \tau_t)w_t h_t s_t + \lambda_t T_t,$$

where τ_t and τ_k are a proportional labor and capital income taxes and T_t is a government transfer. We allow for deviations from T_t being lump-sum and allow time-varying individual-specific share λ_t, that a household receives a transfer $\lambda_t T_t$ in period t, which can be different for different individuals if λ_t is different.

Agents are price takers. In addition, households take wages and hours from the middleman's wage-setting problem (described below) as given. Thus, we can rewrite the agent's problem recursively:

$$\begin{aligned} V(a,s,\beta;\Omega) = \max_{c\geq 0, a'\geq 0} & u(c,h) + \beta \sum_{s\in\mathcal{S}} p(s'|s)V(a',s',\beta;\Omega') \\ \text{subj. to} \quad & c + a' = (1 + r^a(1-\tau_k))a + (1-\tau)whs + \lambda T/P \\ & \Omega' = \Upsilon(\Omega), \end{aligned} \tag{10}$$

where $\Omega(a,s,\beta)$ is the distribution on the space $X = A \times S \times \mathcal{B}$, agents asset holdings $a \in A$, labor productivity $s \in S$ and discount factor $\beta \in \mathcal{B}$, across the population, which will

together with the policy variables determine the equilibrium prices. Let $\mathbb{B}(X) = \mathbb{A} \times \mathcal{P}(S) \times \mathcal{P}(\mathcal{B})$ be the σ-algebra over X, defined as the cartesian product over the Borel σ-algebra on A and the power sets of S and $\mathcal{B}$. Define our space $M = (X, \mathbb{B}(X))$, and let $\mathcal{M}$ be the set of probability measures over M. Υ is an equilibrium object that specifies the evolution of the distribution Ω.

Having defined the household side, we now collect the equilibrium conditions outside the consumption-savings problem. These equations will be common across HANK and RANK and will later be the key object behind the general-equilibrium recovery results.

3.2 Common non-household block

This subsection collects the production, pricing, wage-setting, capital-accumulation, fiscal, monetary, and mutual-fund blocks of the model. We present these equations together because, once a consumption path is specified, they form the common propagation mechanism shared by HANK and its complete-markets/RANK counterpart.

3.2.1 Final Good Producer

A competitive representative final goods producer aggregates a continuum of intermediate goods indexed by $l \in [0, 1]$ and with prices p_l:

$$Y_t = \left(\int_0^1 y_{lt}^{\frac{\epsilon-1}{\epsilon}} dl \right)^{\frac{\epsilon}{\epsilon-1}},$$

where ϵ is the elasticity of substitution across goods. Given a level of aggregate demand Y, cost minimization for the final goods producer implies that the demand for the intermediate good l is given by

$$y_{lt} = y(p_{lt}; P_t, Y_t) = \left(\frac{p_{lt}}{P_t} \right)^{-\epsilon} Y_t, \tag{11}$$

where P is the (equilibrium) price of the final good and can be expressed as

$$P_t = \left(\int_0^1 p_{lt}^{1-\epsilon} dl \right)^{\frac{1}{1-\epsilon}}.$$

3.2.2 Intermediate-Goods Firms

A monopolist produces intermediate good $l \in [0,1]$ using the following technology:

$$Y_{lt} = \begin{cases} Z_t K_{lt}^{\alpha} H_{lt}^{1-\alpha} - Z_t F & \text{if} \geq 0 \\ 0 & \text{otherwise} \end{cases}, \tag{12}$$

where $0 < \alpha < 1$, K_{lt} is capital services rented, H_{lt} is labor services rented and the fixed cost of production are denoted $F > 0$.

Intermediate-goods firms rent capital and labor in perfectly competitive factor markets. Profits are fully taxed by the government. A firm's real marginal cost is $mc_{lt} = \partial S_t(Y_{lt})/\partial Y_{lt}$, where

$$S_t(Y_{lt}) = \min_{K_{lt}, H_{lt}} r_t^k K_{lt} + w_t H_{lt}, \text{ and } Y_{lt} \text{ is given by (12).} \tag{13}$$

Given our functional forms, we have

$$mc_{lt} = \left(\frac{1}{\alpha}\right)^{\alpha} \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \frac{(r_t^k)^{\alpha}(w_t)^{1-\alpha}}{Z_t} \tag{14}$$

and

$$\frac{K_{lt}}{H_{lt}} = \frac{\alpha w_t}{(1-\alpha) r_t^k}. \tag{15}$$

Prices are sticky as intermediate-goods firms face Rotemberg (1982) price adjustment costs. Given last period's individual price p_{jt-1} and the aggregate state $(P_t, Y_t, Z_t, w_t, r_t)$, the firm chooses this period's price p_{jt} to maximize the present discounted value of future profits, satisfying all demand. The firm's pricing problem is

$$V_t\left(p_{lt-1}\right) \equiv \max_{p_{lt}} \frac{p_{lt}}{P_t} y\left(p_{lt}; P_t, Y_t\right) - S(y(p_{lt}; P_t, Y_t)) - \frac{\theta}{2}\left(\frac{p_{lt}}{p_{lt-1}} - \overline{\Pi}\right)^2 Y_t - Z_t F + \frac{1}{1+r_t} V_{t+1}\left(p_{lt}\right),$$

where F are fixed operating costs.

Some algebra (in the appendix) yields the New Keynesian Phillips Curve

$$(1-\epsilon) + \epsilon mc_t - \theta\left(\pi_t - \overline{\Pi}\right)\pi_t + \frac{1}{1+r_t}\theta\left(\pi_{t+1} - \overline{\Pi}\right)\pi_{t+1}\frac{Y_{t+1}}{Y_t} = 0.$$

The equilibrium real profit of each intermediate goods firm is then

$$d_t^{IG} = Y_t - Z_t F - S(Y_t).$$

The claims to profits of the intermediate goods sector are paid to the holders of shares v_t^{IG}, where we normalize the mass of shares to one. The price of the intermediate goods shares is given by p_t^{IG}.

3.2.3 Recruiting Firms and Unions

To allow for sticky wages we follow the literature and assume that each household j provides differentiated labor services, h_{jt}. These differentiated labor services are transformed by a representative, competitive labor packer firm into an aggregate effective labor input, H_t, using the following technology:

$$H_t = \left(\int_0^1 s_{jt} (h_{jt})^{\frac{\epsilon_w - 1}{\epsilon_w}} dj \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}, \tag{16}$$

where ϵ_w is the elasticity of substitution across labor services.

A middleman firm (e.g. a union) sells households labor services to the labor packer, which given aggregate labor demand H_t by the intermediate goods sector, minimizes costs

$$\int_0^1 W_{jt} s_{jt} h_{jt} dj, \tag{17}$$

implying a demand for the labor services of household j:

$$h_{jt} = h(W_{jt}; W_t, H_t) = \left(\frac{W_{jt}}{W_t} \right)^{-\epsilon_w} H_t, \tag{18}$$

where W_t is the (equilibrium) nominal wage which can be expressed as

$$W_t = \left(\int_0^1 s_{jt} W_{jt}^{1-\epsilon_w} dj \right)^{\frac{1}{1-\epsilon_w}}.$$

The middleman sets a nominal wage $\hat{W}_t$ for an effective unit of labor (so that $W_{jt} = \hat{W}_t$) to maximize profits subject to wage adjustment costs modeled similarly to the price adjustment costs in Rotemberg (1982). These adjustment costs are proportional to idiosyncratic productivity s, are measured in units of aggregate output, and are given by a quadratic function of the change in wages above and beyond steady state wage inflation $\overline{\Pi}^w$,

$$\Theta \left(s_{jt}, W_{jt} = \hat{W}_t, W_{jt-1} = \hat{W}_{t-1}; H_t \right) = s_{jt} \frac{\theta_w}{2} \left(\frac{W_{jt}}{W_{jt-1}} - \overline{\Pi}^w \right)^2 H_t = s_{jt} \frac{\theta_w}{2} \left(\frac{\hat{W}_t}{\hat{W}_{t-1}} - \overline{\Pi}^w \right)^2 H_t.$$

The middleman's wage setting problem is to maximize[2]

$$V_t^w\left(\hat{W}_{t-1}\right) \equiv \max_{\hat{W}_t} \int \left(\frac{s_{jt}(1-\tau_t)\hat{W}_t}{P_t} h(\hat{W}_t; W_t, H_t) - \frac{g(h(\hat{W}_t; W_t, H_t))}{u'(C_t)}\right) dj - \int s_{jt} \frac{\theta_w}{2} \left(\frac{\hat{W}_t}{\hat{W}_{t-1}} - \overline{\Pi}^w\right)^2 H_t dj + \frac{1}{1+r_t} V_{t+1}^w\left(\hat{W}_t\right), \tag{19}$$

where C_t is aggregate consumption.

Some algebra (see the appendix) yields, using $h_{jt} = H_t$ and $\hat{W}_t = W_t$ and defining the real wage $w_t = \frac{W_t}{P_t}$, the wage inflation equation

$$\theta_w\left(\pi_t^w - \overline{\Pi}^w\right)\pi_t^w = (1-\tau_t)(1-\epsilon_w)w_t + \epsilon_w \frac{g'(h(\hat{W}_t; W_t, H_t))}{u'(C_t)} + \frac{1}{1+r_t}\theta_w\left(\pi_{t+1}^w - \overline{\Pi}^w\right)\pi_{t+1}^w \frac{H_{t+1}}{H_t}. \tag{20}$$

The wage adjustment process does not involve actual costs but is as-if those costs were actually present. We make this assumption to avoid significant movements of these adjustment costs in response to e.g. a fiscal stimulus or in a liquidity trap. Such swings would matter in our incomplete markets model and might yield quite different implications from price setting à la Calvo.

3.2.4 Capital-producing Firms

The capital-producing firms own the physical capital in the economy, rent it out to intermediate-goods firms, and face adjustment costs in investment. The objective function of a capital-goods producing firm is:

$$\max_{\{I_t, K_{t+1}\}_{t=0}^{\infty}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left[r_t^k K_t - I_t - \Phi(I_t, I_{t-1})\right] \tag{21}$$

$$\text{subject to} \quad K_{t+1} = (1-\delta)K_t + I_t \tag{22}$$

where $\Phi(I_t, I_{t-1}) = \Phi(K_{t+1} - (1-\delta)K_t, K_t - (1-\delta)K_{t-1})$ is convex and increasing in both arguments. Letting q_t^k be the Lagrange multiplier on the accumulation equation, some algebra (in the appendix) yields the optimal investment and price of capital:

[2]Equivalently one can think of a continuum of middlemen each setting the wage for a representative part of the population with $\int s = 1$ at all times.

$$q_t^k = 1 + \Phi_1(I_t, I_{t-1}) + \frac{1}{1+r_t}\Phi_2(I_{t+1}, I_t) \tag{23}$$

$$q_t^k = \frac{1}{1+r_t}\left(r_{t+1}^k + (1-\delta)q_{t+1}^k\right) \tag{24}$$

The dividends paid out by the capital-producing firm are given by $d_t^k = r_t^k K_t - I_t - \Phi(I_t, I_{t-1})$. Dividends are paid out to shares in the capital-producing firms, v_t^k (normalized to 1) that trade at price p_t^k.

3.3 Government

The government obtains revenue from taxing labor income, capital income, and profits as well as issuing bonds. Household labor income wsh is taxed progressively with a lump-sum transfer T_t and a proportional tax τ:

$$\tilde{T}(wsh) = -T + \tau wsh.$$

The government issues long-term bonds B_{t+1}^L to finance its expenditures. Bonds mature stochastically at rate δ^B. Let $\widetilde{i_{t+1}}$ be in nominal interest rate paid on long-term debt. We want government bonds to sell at "par", meaning that the price is constant (and equal to 1). Then, we have $q_t^{B^L} = q_{t+1}^{B^L} = 1 \quad \forall t$. Then, we can exploit the above equation:

$$\begin{aligned} 1 &= \frac{\delta^B + \widetilde{i^{ss}} + 1 - \delta^B}{1 + i^{ss}} \\ \Leftrightarrow \quad \widetilde{i^{ss}} &= i^{ss} \end{aligned}$$

In steady state the government budget constraint reads:

$$q_t^{B^L}\left(B_{t+1}^L - (1-\delta^B)B_t^L\right) = \left(\delta^B + i^{ss}\right)B_t^L + G_t - \tau_k i_t^a A_t - \int \tilde{T}_t(w_t s_t h_t) d\Omega_t.$$

So in a steady state with constant inflation and real variables we essentially have (now, we have $q_t^{B^L} = 1, B_{t+1}^L / B_t^L = 1 + \pi$):

Ok, now let's consider what happens outside of steady state when a shock occurs in period 0. When the government has a stock of legacy debt B_0^L, duration δ^B and pays nominal

interest i^{ss}. During the transition, the government finances itself by issuing one period debt (B_{t+1}) that pays a nominal interest ratet i_{t+1} as set by the monetary authority (discussed below). Note that as $t \to \infty$ if we converge back to the same steady state, it doesn't matter that along the transition we will have effectively replaced all long-term debt with short-term debt. Thus, the government budget constraint becomes:

$$B_{t+1} = (1 + i_t)B_t + (\delta^B + i^{ss})B_t^L + G_t - \tau_k i_t^a A_t - \int \tilde{T}_t(w_t s_t h_t) d\Omega_t. \tag{25}$$

where $B_0 = 0$ and B_0^L is given from steady state, such that:

$$B_{t+1} = (1 + i_t)B_t + (\delta^B + i^{ss})(1 - \delta^B)^t B_0^L + G_t - \tau_k i_t^a A_t - \int \tilde{T}_t(w_t s_t h_t) d\Omega_t. \tag{26}$$

Monetary Policy The monetary authority follows a Taylor rule, which sets the nominal interest rate according to:

$$\begin{aligned} i_{t+1} &= \max(\tilde{i}_{t+1}, 0), \text{ where} \\ \tilde{i}_{t+1} &= \left(\frac{1}{\zeta}\right)\left(\frac{1 + \pi_t}{1 + \pi_{ss}}\right)^{\phi^\pi(1-\rho_R)} [\zeta(1 + i_t)]^{\rho_R} - 1, \end{aligned} \tag{27}$$

for parameters ϕ^π and ρ_R.

3.3.1 Mutual Fund

The mutual fund collects households' real assets A_{t+1} and pays a real return $\tilde{r}_t^a$, and invests them in real bonds b_{t+1}, and shares of the capital-producing firms' v_t^k and intermediate goods firms v_t^f. It maximizes

$$\begin{aligned} \max_{b_{t+1}, b_{t+1}^L, v_t^k, v_t^f} \quad & \left(p_{t+1}^f + d_{t+1}^f\right) v_{t+1}^f + \left(p_{t+1}^k + d_{t+1}^k\right) v_{t+1}^k + (1 + r_{t+1})\, b_{t+1} \\ & + \left(\delta^B + \widetilde{i_{ss}} + (1 - \delta^B)\right) q_{t+1}^{B^L} b_{t+1}^L - \left(1 + r_{t+1}^a\right) A_{t+1} \\ \text{subj. to} \quad & A_{t+1} = b_{t+1} + q_t^B b_{t+1}^L + p_t^f v_{t+1}^f + p_t^k v_{t+1}^f. \end{aligned}$$

Note that we drop expectation operators for simplicity since we consider perfect foresight

experiments only. The equilibrium first-order conditions are

$$\begin{aligned} r_{t+1} &= r^a_{t+1}, \\ p^f_t &= \frac{1}{1+r_t}\left(p^f_{t+1} + d^f_{t+1}\right), \\ p^k_t &= \frac{1}{1+r_t}\left(p^k_{t+1} + d^k_{t+1}\right) \\ q^{B^L}_t &= \frac{\delta^B + \widetilde{r_{ss}} + (1-\delta^B) q^{B^L}_{t+1}}{1+r_{t+1}} \\ A_{t+1} &= b_{t+1} + q^B_t b^L_{t+1} + p^f_t v^f_{t+1} + p^k_t v^f_{t+1}. \end{aligned}$$

The corresponding nominal return is

$$1 + i^a_{t+1} = (1 + r^a_{t+1})\frac{P_{t+1}}{P_t}.$$

3.4 Equilibrium

Market clearing requires that the labor demanded by the firm is equal to the aggregate labor supplied by households, that the demand for long and short bonds issued by the government and capital equal their supplies, that the markets for shares in capital-producing and intermediate goods firms clear, and that the amount of assets provided by households equals their demand by the mutual fund:

$$b_{t+1} + q^B_t b^L_{t+1} + p^f_t + p^k_t = A_{t+1} = \int_{a_t} \sum_{s_t \in \mathcal{S}} \sum_{b_t \in \mathcal{B}} a_{t+1}(a_t, s_t, \beta_t) d\Omega_t(a_t, s_t, \beta_t) \tag{28}$$

$$\int b_{t+1} = B^g_{t+1} \tag{29}$$

$$\int b^L_{t+1} = B^L_{t+1} \tag{30}$$

$$K_t = \int K_{lt} dl \tag{31}$$

$$H_t = \int H_{lt} dl = H_{lt} = \int h_{jt} dj = h_{jt} \tag{32}$$

where we have abused notation slightly here, $a_{t+1}(a_t, s_t, \beta_t)$ is the asset choice of an agent with asset level a_t and productivity s_t and discount factor β_t.

Definition: A monetary competitive equilibrium is a sequence of prices P_t, tax rates τ_t and τ_k, nominal transfers T_t, nominal government spending G_t, bonds B^g_t, a value functions

$v_t : M \to \mathcal{R}$ with policy functions $a_t : M \to \mathcal{R}_+$ and $c_t : M \to \mathcal{R}_+$, hours choices $H_t, H_{lt}, h_{jt} : M \to \mathcal{R}_+$, capital decisions $K_t, K_{lt} : M \to \mathcal{R}_+$, pricing functions $r_t, r_t^k, r_t^a, p_t^f, p_t^k, q_t^k, q_t^{B^L} : M \to \mathcal{R}$ and $w_t : M \to \mathcal{R}_+$, and a law of motion $\Upsilon : \mathcal{M} \to \mathcal{M}$, such that:

1. v_t satisfies the Bellman equation with corresponding policy functions a_t and c_t given price sequences $r_t^a(), w_t()$ and hours h_t.

2. Firms maximize profits taking prices P_t, r_t^k, w_t as given.

3. Wages are set optimally by middlemen.

4. The mutual fund maximizes profits taking prices as given.

5. For all $\Omega \in \mathcal{M}$:

$$\begin{aligned}
b_{t+1} + q_t^B b_{t+1}^L + p_t^f + p_t^k &= \int \frac{a_{t+1}(a_t, s_t, \beta_t)}{P_{t+1}} d\Omega_t, \\
\int b_{t+1} &= B_{t+1}^g \\
\int b_{t+1}^L &= B_{t+1}^L \\
K_t &= \int K_{lt} dl \\
H_t &= \int H_{lt} dl = H_{lt} = \int h_{jt} dj = h_{jt} \\
Y_t = Z_t K_t^\alpha H_t^{1-\alpha} &= \int c_t(a_t, s_t, \beta_t) d\Omega_t + \frac{G_t}{P_t} + Z_t F + K_{t+1} \\
&\quad -(1-\delta)K_t + \Phi(I_t, I_{t-1}).
\end{aligned}$$

6. Aggregate law of motion Υ generated by a' and p.

3.5 Complete-markets/RANK counterpart

We next define the complete-markets/RANK counterpart to the benchmark economy. It shares the same non-household block as HANK but replaces the incomplete-markets household sector with its representative-agent, complete-markets analog. This counterpart

therefore provides the benchmark aggregate response path used in Section 2 and, later, the latent-RANK benchmark used for counterfactuals and estimation.

This subsection plays a narrower role than the abstract benchmark path in Section 2. There the benchmark path was taken as given. Here it is pinned down by a specific quantitative model.

The complete markets model arises as a special case without idiosyncratic uncertainty, $s \equiv 1$. The variables in the complete markets model are denoted with a superscript CM: Inflation rate π_t^{CM}, wage inflation rate $\pi_t^{CM,w}$, real interest rate on bonds $(1 + r_t^{CM}) = \frac{1+i_t}{1+\pi_t^{CM}}$, real interest rates $1 + r_t^{CM,a}$, $1 + r_t^{CM,k}$, output Y_t^{CM}, physical capital stock K_t^{CM}, total hours H_t^{CM}, consumption C_t^{CM} and wages w_t^{CM}. Transfers and bonds are indeterminate by Ricardian equivalence in the complete markets/representative agent economy. We define total assets below. The proportional tax rate is unchanged at its steady state level τ_{ss}. Real government expenditure is allowed to respond and now equals g_t and is equal to g_{ss} in a steady state and we define $\gamma_t^g = \frac{g_t}{g_{ss}}$ as the change in government expenditure.

These variables, capital stock K_t^{CM}, consumption C_t^{CM}, output Y_t^{CM}, hours H_t^{CM} wages w_t^{CM}, wage inflation $\pi_t^{CM,w}$, inflation π_t^{CM} and the interest rates i_t, r_t^{CM}, $r_t^{CM,a}$, $\tilde{r}_t^{CM,a}$, $r_t^{CM,k}$ satisfy

$$
\begin{aligned}
Y_t^{CM} = Z_t (K_t^{CM})^{\alpha} (H_t^{CM})^{1-\alpha} &= C_t^{CM} + g_t + Z_t F + K_{t+1}^{CM} - (1-\delta) K_t^{CM} + \Phi(K_{t+1}^{CM}, K_t^{CM}) \\
\theta_w \left(\pi_t^{CM,w} - \overline{\Pi}^w \right) \pi_t^{CM,w} &= (1-\tau_t)(1-\epsilon_w) w_t^{CM} + \epsilon_w \frac{g'(H_t^{CM}))}{u'(C_t^{CM})} + \frac{1}{1+r_t^{CM}} \theta_w \left(\pi_{t+1}^w - \overline{\Pi}^w \right) \pi_{t+1}^w \frac{H_{t+1}^{CM}}{H_t^{CM}}. \\
u_c(C_t^{CM}) = (C_t^{CM})^{-\sigma} &= \beta (1 + r_{t+1}^{CM,a} (1-\tau_k)) (C_{t+1}^{CM})^{-\sigma} \\
(1-\epsilon) + \epsilon m c_t^{CM} &= \theta \left(\pi_t^{CM} - \overline{\Pi} \right) \pi_t^{CM} - \frac{1}{1+r_t^{CM}} \theta \left(\pi_{t+1}^{CM} - \overline{\Pi} \right) \pi_{t+1} \frac{Y_{t+1}^{CM}}{Y_t^{CM}}. \\
m c_t^{CM} &= \left(\frac{1}{\alpha} \right)^{\alpha} \left(\frac{1}{1-\alpha} \right)^{1-\alpha} \frac{(r_t^{CM,k})^{\alpha} (w_t^{CM})^{1-\alpha}}{Z_t} \\
\frac{K_t^{CM}}{H_t^{CM}} &= \frac{\alpha w_t^{CM}}{(1-\alpha) r_t^{CM,k}} \\
1 + r_{t+1}^{CM} &= 1 + r_{t+1}^{CM,a} \\
1 + \Phi_1(I_t, I_{t-1}) + \frac{1}{1+r_t} \Phi_2(I_{t+1}, I_t) &= \frac{1}{1+r_t} \left(r_{t+1}^k + (1-\delta) q_{t+1}^k \right)
\end{aligned}
$$

where the real return on bonds satisfies

$$1 + r_t^{CM} = \frac{1 + i_t^{CM}}{1 + \pi_t^{CM}}$$

and the government budget constraint (in real terms) holds

$$b^g_{t+1} = (1 + r_t)b^g_t + (\delta^B + i^{ss})(1 - \delta^B)^t B^L_0/P_t + g_t - \tau_k r^a_t A_t - \int \tilde{T}_t(w_t s_t h_t) d\Omega_t. \tag{33}$$

for real government bonds b^g_{t+1}, real government expenditure g_t and real tax revenue $\hat{T}$ and fully taxing intermediate goods firms profits d. The level of bonds does not matter for the CM economy but it does for the IM economy and the construction of the Δ. We define the real value of bonds below. For every sequence of real bonds b^{CM}_t define the aggregate real transfers paid to households

$$\Gamma^{CM}_t = \tau w^{CM}_t H^{CM}_t + \tau_k r^a_t A^{CM}_t + b^{CM}_{t+1} - (1 + r^{CM}_t)b^{CM}_t - (\delta^B + i^{ss})(1 - \delta^B)^t B^L_0/P_t - g_t,$$

which satisfies the government budget constraint. And Ricardian equivalence implies that any combination of bonds and transfers constitutes an equilibrium.

4 From Transfers to Aggregate Counterfactuals

This section shows how the objects constructed in Section 2 can be used to recover the aggregate implications of incomplete markets using information from impulse responses, household budget sets, and MPCs. We proceed in increasing order of ambition. We first construct the latent transfers on model-generated data and verify that, at RANK prices, the transfer correction exactly reconciles the HANK household block with the RANK consumption response. We then characterize when the same first-round correction also recovers the full HANK general-equilibrium response. Finally, we translate these results into a practical procedure for computing heterogeneity-robust counterfactuals from RANK impulse responses. Throughout this section, fiscal policy is taken as given and common across RANK and HANK; the empirical measurement of fiscal responses is left to the companion paper, Hagedorn et al. (2026).

4.1 Constructing latent transfers on model-generated data

We begin by calibrating the complete markets version of the economy described in Section 3.5. The calibration of preferences, technology and nominal rigidities largely follows that

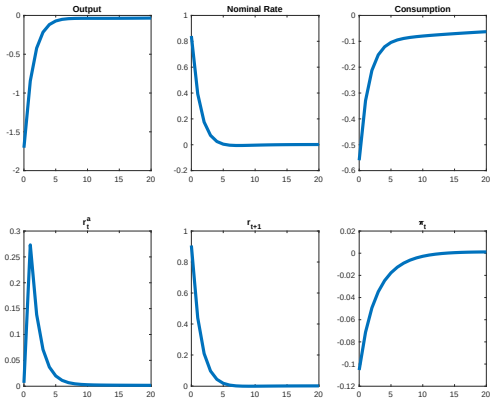


Figure 1: 25 BP innovation to the Taylor Rule in RANK

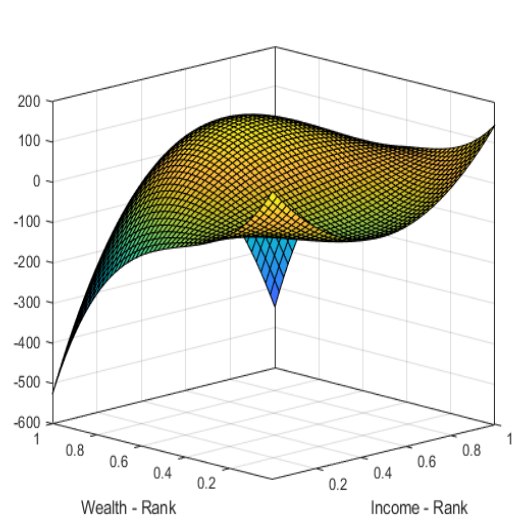
outlined in Hagedorn et al. (2019). The full list of calibrated parameters is included in the Appendix. We linearize the model in the sequence space following Boppart et al. (2018) and solve for the impulse response to a one-time innovation to the Taylor rule of 25BP with a quarterly persistence of 0.5. The result is plotted in Figure 1. We use these deviations from steady state to compute the γs from Section **??** as the input for computing the latent transfers $\Delta_t(a, y)$.

In order to validate our method, we want to compute the transfers in a "controlled environment" where we can verify the theory. To do so we use the heterogeneous-agent version of our model to generate a cross-section of households, for whom we can compute the Δs. Our calibration for the income process and preferences follows Krueger et al. (2016), which enables us to match the joint distribution of income, consumption, and wealth in the PSID data, the key ingredients needed for computing the transfers. The full details of the calibration are provided in the Appendix. After solving for the cross-section distribution, we can use the γs from the previous section to compute the distribution of transfers $\Delta_k(a, y)$ over time following our methodology. The distribution of $\Delta_k(a, y)$ is plotted for $k = 1, 2, 3, 4$ in figure 2.

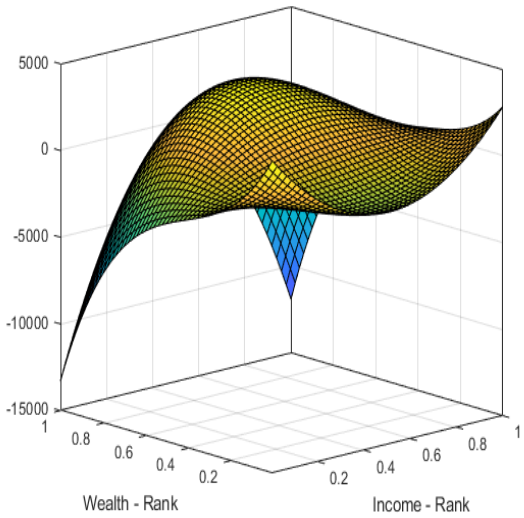
4.2 Validation in partial equilibrium

Now that we've constructed $\Delta_k(a, y)$ in the model, we validate that the aggregate consumption response to those sequences of transfers replicated the consumption response from the complete markets model. To do so requires computing the aggregate MPCs out of those

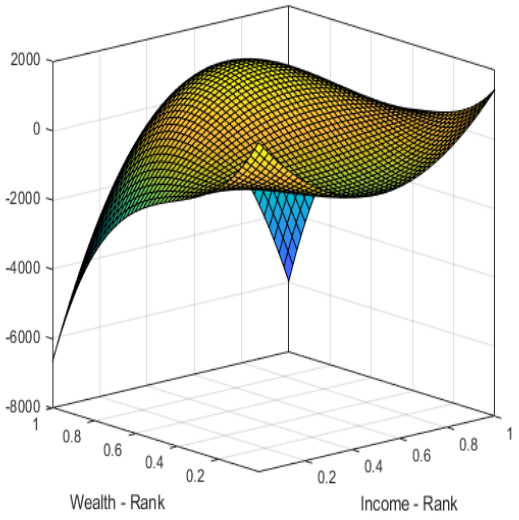
Figure 2: Model $\Delta_k(a, y)$ at different time horizons



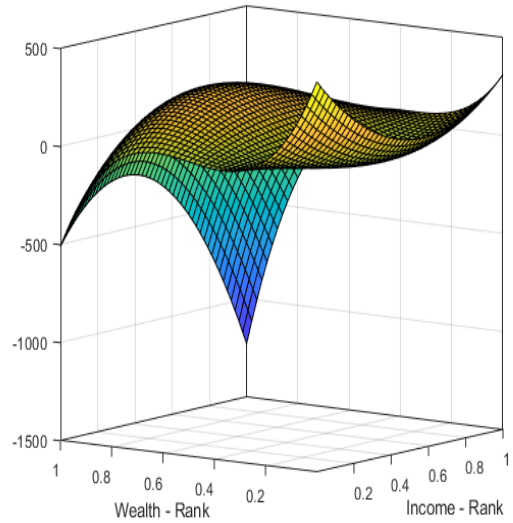
(a) Δ_i at $t = 0$



(b) Δ_i at $t = 1$



(c) Δ_i at $t = 2$



(d) Δ_i at $t = 4$

transfers. That is, we need $MPC_{t,k}(a, y)$, the *aggregate consumption* response in time t to a transfer in time k to a household with (a, y) in period k. In the subsequent section, we will use these same MPCs for computing aggregate consumption responses based on empirical impulse responses to monetary policy shocks and transfers constructed based on the PSID data. We resort to the model-generated MPCs (only for our aggregate counterfactual analysis) because we can't yet reliably measure these MPCs directly in the data.

We then combine the MPCs and the transfers to compute the aggregate consumption response:

$$\Delta C_t = \sum_{k=0}^{\infty} \int_{a \in \mathcal{A}, y \in \mathcal{Y}} \Delta_k(a, y) MPC_{t,k}(a, y) da dy$$

The aggregate consumption response is plotted as the blue solid line in figure 3. To validate our methodology, we next solve for the impulse response of the HANK model using the *prices* from the RANK model in response to the monetary policy shock. The aggregate consumption response to the RANK MP shock and prices is plotted as the red dot-dashed line in figure 3. Finally, we add the aggregate consumption response to the transfers to the consumption response from the HANK model with RANK prices and plot that as the purple dashed line in the figure. This exactly coincides with the aggregate consumption response from the RANK model, thus validating that our methodology indeed constructs transfers that makes the aggregate response to a monetary policy shock in HANK identical to that in RANK.

The remainder of the section asks how far this first-round object can take us once we leave the RANK price sequence and return to HANK general equilibrium.

4.3 Recovering HANK in General Equilibrium

Section 4.2 showed that, when HANK households are evaluated at the RANK prices, the transfer correction exactly reconciles the HANK household block with the RANK consumption response. The natural next question is whether the same object is informative about the full HANK general-equilibrium response. The answer is yes at first order: the transfer-implied correction ΔC is the first-round gap between RANK and HANK, while the remaining difference is governed by how the common non-household block feeds the

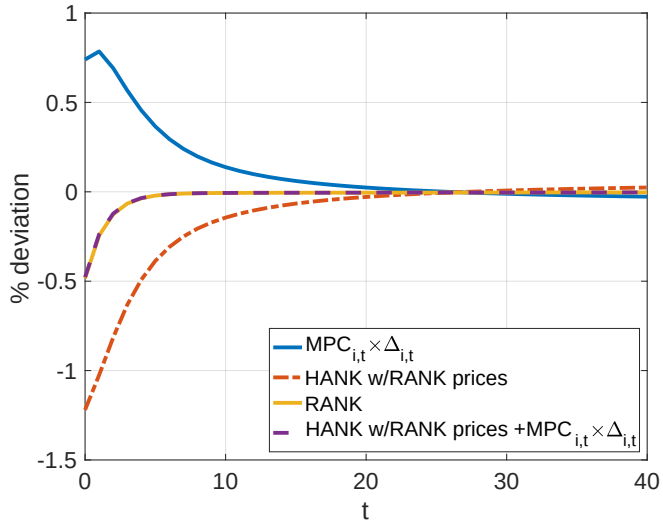


Figure 3: Validation Using Model Generated Data

consumption gap back into household prices and incomes. Proposition 1 gives the exact first-order statement.

To connect the nonlinear theorem in Section 2 to a linearized general-equilibrium comparison, we switch from the gross-response notation γ_t to additive d-notation. Define

$$\bar{\ell} \equiv (1-\bar{\tau}_w)\bar{w}\bar{H}, \qquad \widetilde{r}_t \equiv (1-\tau_{k,t})r_t^a,$$

and map the objects from Theorem 2.1 into first-order changes via

$$dC_t = (\gamma_t^C - 1)\bar{C}, \qquad dA_t = (\gamma_t^A - 1)\bar{A}, \qquad d\Gamma_t = (\gamma_t^\Gamma - 1)\bar{\Gamma}, \qquad d\ell_t = (\gamma_t^\ell - 1)\bar{\ell}, \tag{34}$$

where $\gamma_t^\ell \equiv \gamma_t^w \gamma_t^H \gamma_t^{\tau_w}$. Let

$$dz^R \equiv (dC^R, d\ell^R, d\Gamma^R, dA^R, d\widetilde{r}^R)$$

collect the RANK responses of the aggregate objects entering the transfer formula, and let B denote the Jacobian of the transfer map in Theorem 2.1, so that

$$d\Delta^R = B\, dz^R. \tag{35}$$

Let M denote the iMPC operator mapping a transfer sequence into aggregate consumption,

$$[Md\Delta]_t = \sum_{k=0}^{\infty} \int_{\mathcal{A}\times\mathcal{Y}} MPC_{t,k}(a,y)\, d\Delta_k(a,y)\, da\, dy, \tag{36}$$

and define the associated aggregate consumption correction by

$$\Delta C \equiv Md\Delta^R = MB\, dz^R. \tag{37}$$

This is exactly the object constructed in Section 4.2.

Proposition 1 (First-order recovery of HANK consumption from RANK and Δ's) *Suppose the RANK and HANK economies share the same aggregate steady state and consider a first-order approximation around that steady state. Let*

$$X \equiv (\widetilde{r}, \ell, \Gamma)$$

denote the common household environment, and let the linearized HANK household block be

$$dC = J_X dX + Md\Delta = J_{\widetilde{r}} d\widetilde{r} + J_\ell d\ell + J_\Gamma d\Gamma + Md\Delta, \tag{38}$$

where $J_X = (J_{\widetilde{r}}, J_\ell, J_\Gamma)$ is the Jacobian of aggregate consumption with respect to the common household environment, holding transfers fixed. Suppose further that, outside the household block, the linearized equilibrium system implies

$$dX = G_C dC + G_\varepsilon d\varepsilon. \tag{39}$$

Then:

(i) Holding the RANK price-income system fixed,

$$dC^{H|R} = dC^R - \Delta C. \tag{40}$$

(ii) In HANK general equilibrium,

$$(I - J_X G_C)(dC^R - dC^H) = \Delta C, \tag{41}$$

so that

$$dC^H = dC^R - (I - J_X G_C)^{-1} \Delta C. \tag{42}$$

(iii) If $\rho(J_X G_C) < 1$, then

$$dC^R - dC^H = \Delta C + J_X G_C\, \Delta C + (J_X G_C)^2 \Delta C + \cdots. \tag{43}$$

Hence

$$dC^H \approx dC^R - \Delta C \tag{44}$$

whenever the higher-round feedback matrix $J_X G_C$ is small.

Proof. See Appendix A.

Proposition 1 has a simple interpretation. At RANK prices, ΔC is the exact first-order correction that maps the HANK household block into the RANK consumption response. In general equilibrium, the one-step approximation $dC^H \approx dC^R - \Delta C$ fails only because the RANK and HANK economies induce different paths for the household environment $(\widetilde{r}, \ell, \Gamma)$. In particular,

$$dC^R - dC^H - \Delta C = J_X(dX^R - dX^H) = J_{\widetilde{r}}\,\delta\widetilde{r} + J_\ell\,\delta\ell + J_\Gamma\,\delta\Gamma, \tag{45}$$

where $\delta x \equiv dx^R - dx^H$ denotes the RANK–HANK gap in any aggregate object. The remainder of this subsection characterizes the structure of these higher-round terms.

Corollary 4.1 (Common-block structure of the higher-round term) *Under the conditions of Proposition 1, let s_t collect the core non-household variables in the linearized system, for example*

$$s_t \equiv (R^n_t, \pi_t, w_t, H_t, r^k_t, q^k_t, K_t, I_t)'.$$

Assume that government spending is common across RANK and HANK. Then there exist bounded linear operators $\mathcal{G}^W$ and $\mathcal{G}^{RC}$, and a linear map $\mathcal{H}_X$ from the core path δs into the household-relevant environment gap $\delta X \equiv (\delta\widetilde{r}, \delta\ell, \delta\Gamma)$, such that

$$\delta s = (\mathcal{G}^W + \mathcal{G}^{RC})\delta C, \qquad \delta X = \mathcal{H}_X(\mathcal{G}^W + \mathcal{G}^{RC})\delta C,$$

where:

- *$\mathcal{G}^W$ is associated with the forcing term $-\eta_w\sigma\,\delta C_t$ in the differenced wage Phillips curve;*
- *$\mathcal{G}^{RC}$ is associated with the forcing term $(\bar{C}/\bar{Y})\,\delta C_t$ in the differenced resource constraint.*

Consequently,

$$(I - \mathcal{K}^W - \mathcal{K}^{RC})\delta C = \Delta C, \qquad \mathcal{K}^W \equiv J_X\mathcal{H}_X\mathcal{G}^W, \qquad \mathcal{K}^{RC} \equiv J_X\mathcal{H}_X\mathcal{G}^{RC}. \tag{46}$$

Moreover, if the slope of the NKPC is zero, $\eta_p = 0$, then

$$\delta\pi_t = 0, \qquad \delta R^n_t = 0, \qquad \delta\widetilde{r}_t = 0 \qquad \forall t. \tag{47}$$

Finally, under the benchmark fiscal closure with a fixed debt path and common government spending, the transfer gap satisfies

$$d\Gamma_t \equiv \bar{\Gamma}\,\delta\hat{\Gamma}_t = \tau_\ell\bar{w}\bar{H}(\delta\hat{w}_t + \delta\hat{H}_t) + \tau_k\bar{r}^a\bar{A}(\delta\hat{r}^a_t + \delta\hat{A}_t) - \bar{B}\bar{R}\left[1 - (1-\delta_b)^t\right]\delta\hat{R}_t. \tag{48}$$

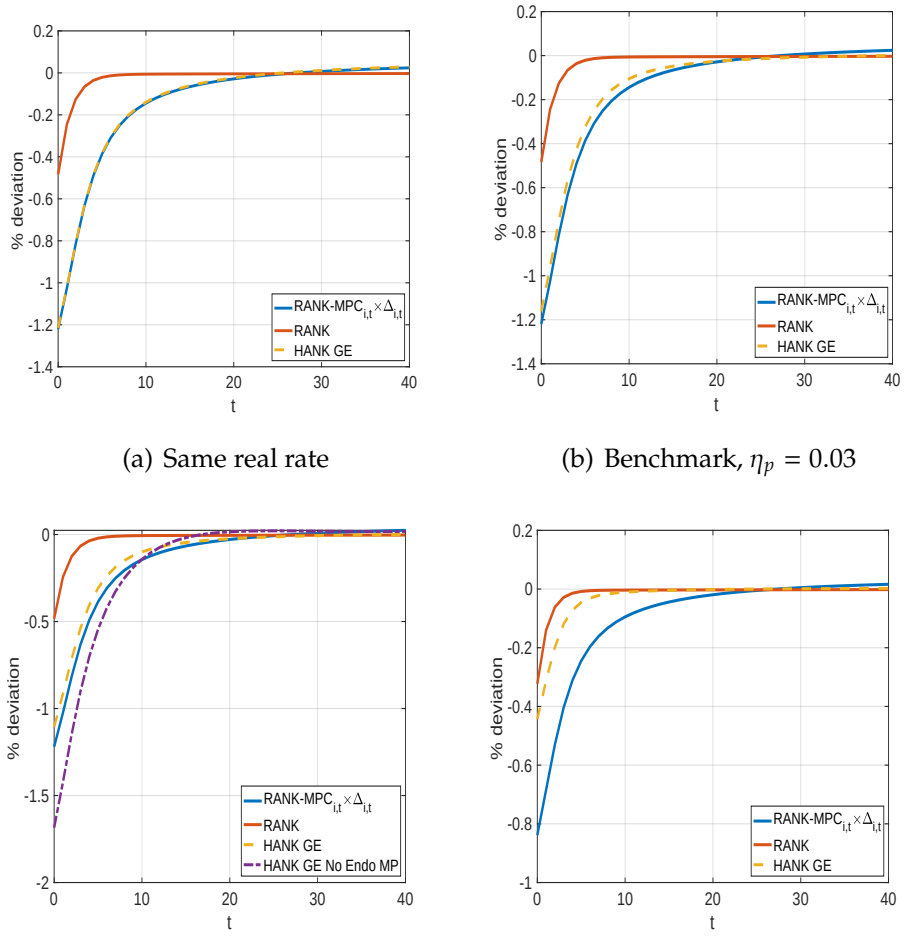
Proof. See Appendix A.

Corollary 4.1 sharpens the proposition in two ways. First, it shows that the higher-round term $J_X G_C$ is not a generic black box. In our model it is generated by only two direct wedges in the common non-household block: a wage-setting wedge and a resource-constraint wedge. Second, it delivers a sharp nominal-rigidity result. When $\eta_p = 0$, the inflation gap satisfies a homogeneous forward equation, so boundedness implies $\delta\pi_t = 0$; the Taylor rule then implies $\delta R_t^n = 0$, and hence the real-rate gap $\delta\widetilde{r}_t$ disappears exactly. More generally, lower values of η_p weaken the pass-through from the common-block wedges into inflation and real rates. Lower values of η_w work in the same direction by shrinking the wage-setting wedge directly and by reducing the wage component of marginal cost.

The fixed-debt government budget constraint further shows that the transfer gap is not an independent source of error. What matters is not that transfers move in levels, but whether transfers move differently in RANK and HANK. Under the benchmark fiscal closure, $\delta\Gamma_t$ is pinned down by labor-tax-base differences, capital-tax-base differences, and the debt-service consequences of the real-rate gap. Thus the benchmark error (45) can be interpreted as the combined effect of three household-relevant wedges—the after-tax return wedge, the disposable-income wedge, and the transfer wedge—all induced by the propagation of the common-block wedges through prices, incomes, and fiscal accounting.

Figure 4 quantifies these mechanisms. Panel (a) imposes the same real-rate path in RANK and HANK, so that $\delta\widetilde{r}_t = 0$ by construction. Consistent with Proposition 1, the path $C_t^R - \Delta C_t$ then coincides almost exactly with the full HANK general-equilibrium response. Panel (b) returns to the benchmark Taylor-rule economy with a low NKPC slope, $\eta_p = 0.03$. The approximation remains close in this empirically relevant case, even though the equality from panel (a) no longer holds exactly. Panel (c) keeps the same benchmark calibration but shuts down endogenous monetary-policy feedback by feeding the nominal-rate path from RANK directly into HANK. By construction the nominal-rate gap is zero in this experiment, but the real-rate gap can still differ through the inflation response. The resulting HANK response is more contractionary than in panel (b). This shows that the benchmark closeness of C_t^H and $C_t^R - \Delta C_t$ is not due solely to negligible higher-round feedback. Rather, HANK features stronger quantity-side amplification, and the endogenous Taylor-rule response partly offsets that amplification in the benchmark. Panel (d) increases the slope of the

Figure 4: Recovering HANK GE from our methodology



(a) Same real rate

(b) Benchmark, $\eta_p = 0.03$

(c) Benchmark, no endogenous MP response

(d) Higher NKPC slope

NKPC. As prices become less sticky, the mapping from the common-block wedges into inflation and real rates becomes stronger, the induced gaps between RANK and HANK widen, and the approximation deteriorates.

Taken together, Proposition 1, Corollary 4.1, and Figure 4 show that ΔC is the correct first-round object and that the remaining gap has a transparent general-equilibrium interpretation. This is the basis for the heterogeneity-robust counterfactual procedure in the next subsection.

4.4 Heterogeneity-robust counterfactuals from RANK impulse responses

1. Solve the counterfactual policy experiment in a RANK model.
2. Use the implied aggregate paths to construct latent Δs from household budget sets.
3. Combine Δs with the MPC matrix to obtain ΔC.
4. Recover the HANK consumption path using $C^H \approx C^R - \Delta C$.
5. Recover the remaining GE variables from the common block conditional on C^H.

5 Estimating HANK Using a Latent RANK Model

Section 4 showed how to recover the HANK consumption response from a representative-agent benchmark and a transfer-based correction for market incompleteness. We begin with one basic observation, outside the household block, the linearized equilibrium conditions are common across RANK and HANK. Once a HANK consumption path is specified, the remaining equilibrium objects can therefore be recovered from the common representative block. This observation suggests a natural estimation strategy. Rather than estimating a misspecified RANK model directly on HANK-generated data, we estimate a latent RANK benchmark and correct its consumption block using the redistribution objects identified by our theory. In this way, we recover the complete-markets benchmark relevant for the data while explicitly accounting for the effects of heterogeneity and incomplete markets on aggregates outcomes.

5.1 Estimation algorithm

Our measurement results imply a simple impulse-response matching procedure for estimating HANK through a latent RANK model. We treat the aggregate data as if they were generated by a HANK economy, so that the empirical impulse responses are the HANK impulse responses. We then exploit the fact that the difference between RANK and HANK is concentrated in the household consumption block. For any candidate parameter vector, we first solve the latent RANK model, then use household budget sets together with iMPCs

to recover the correction to aggregate consumption implied by market incompleteness, and finally solve for the remaining equilibrium variables conditional on the corrected consumption path. The resulting impulse responses can then be compared to the data in a standard likelihood-based estimation routine.

Algorithm 1 Estimating HANK using a latent RANK model

1: **Input:** HANK impulse responses, household budget-set objects, iMPC matrices, and a parameter vector Θ
2: Specify the RANK counterpart of the HANK model to be estimated
3: Guess a parameter vector Θ
4: Solve the latent RANK model and compute its impulse responses, denoted $\gamma_t^R(\Theta)$
5: Use the household budget constraints implied by $\gamma_t^R(\Theta)$ to construct the latent transfers $\Delta_{i,t}(\Theta)$
6: Combine $\Delta_{i,t}(\Theta)$ with the iMPC matrix to obtain the aggregate consumption correction $\Delta C_t(\Theta)$
7: Form the corrected HANK consumption path

$$C_t^H(\Theta) \approx C_t^R(\Theta) - \Delta C_t(\Theta)$$

8: Solve the remaining general-equilibrium block conditional on $C_t^H(\Theta)$
9: Stack the model-implied impulse responses into a moment vector $m(\Theta)$
10: Evaluate the likelihood by comparing $m(\Theta)$ to the target HANK impulse responses
11: Update Θ and iterate until convergence of the posterior sampler

The algorithm is deliberately high-level it could, for example, be implemented as a maximum likelihood routine or through a Bayesian sampler after assigning priors (our preferred method). Its purpose is to summarize the economic logic rather than the exact implementation. In Appendix D.2 we detail how the model-implied HANK objects can be constructed in exactly this way: a RANK transition path is computed first, the associated transfers are computed from household budget constraints and are combined with the iMPC and cross-sectional distribution to obtain a correction to aggregate consumption, and the remaining variables are then recovered from the common non-household block conditional on that corrected consumption path.

5.2 Validation in the benchmark model

We begin with a benchmark validation exercise on model-generated data. The pseudo-data are impulse responses from the HANK model following a monetary policy shock. We

estimate the parameter vector

$$\Theta = \left(\eta_w, \eta_p, \rho_R, \phi_k, \phi_\pi, \sigma_R\right)',$$

where η_w is the slope of the wage Phillips curve, η_p is the slope of the price Phillips curve, ρ_R is Taylor-rule inertia, ϕ_k is the investment adjustment-cost parameter, ϕ_π is the Taylor-rule response to inflation, and σ_R is the monetary policy innovation.

The target moments are constructed from a subset of HANK impulse responses. For consumption, hours, investment, inflation, and the real wage, we use cumulative responses and retain horizons $h = 12, \ldots, 16$. For the nominal interest rate, we use the level response over horizons $h = 1, \ldots, 16$. Stacking these objects yields a 41-dimensional target vector. In the current implementation, the weighting matrix is diagonal with a common variance normalization, so the likelihood is a Gaussian discrepancy criterion over impulse-response moments. This criterion can be written as

$$\log L(\Theta) = -\frac{1}{2}\left(m(\Theta) - \mu\right)' V^{-1}\left(m(\Theta) - \mu\right) + \text{const},$$

where μ is the target HANK moment vector and $m(\Theta)$ is the model-implied counterpart.

The model-implied moment map is hybrid rather than fully nonlinear. For each candidate Θ, we first solve the linearized RANK system. We then use the resulting paths for wages, hours, transfers, and returns together with the steady-state household distribution, policy rules, to construct the redistribution terms $\Delta_{i,t}$. Aggregating these terms through the iMPC object yields the consumption correction ΔC_t. We then set

$$C_t^H(\Theta) \approx C_t^R(\Theta) - \Delta C_t(\Theta)$$

and solve the reduced non-household equilibrium block conditional on that path. Thus, the estimation routine does not resolve the full HANK model at each draw. Instead, it estimates a latent RANK benchmark and uses the theory developed above to map that benchmark into a HANK-consistent aggregate response.

We estimate the model in a Bayesian framework using a Metropolis–Hastings sampler initialized at the posterior mode. Priors are informative for the investment-adjustment and Taylor-rule inflation parameters and impose admissible support restrictions on the

remaining coefficients. Posterior computation is standard, and implementation details are reported in the Appendix.

The posterior summaries show that the procedure recovers some parameters more accurately than others. The two Phillips-curve slope parameters are estimated close to their true values, and the inverse adjustment-cost parameter is also recovered reasonably well. Recovery is weaker for the monetary-policy block. In particular, the posterior mean of ρ_R lies below its true value, and the posterior for σ_R remains somewhat dispersed. The largest discrepancy concerns ϕ_π, whose posterior mean is substantially below the true value used to generate the pseudo-data.

Parameter	Mean	5%	95%
η_w	0.022	0.019	0.025
η_p	0.012	0.011	0.014
ρ_R	0.656	0.641	0.672
ϕ_k	0.105	0.100	0.115
ϕ_π	1.056	1.048	1.062
σ_R	0.196	0.173	0.218

Table I: Posterior Means and 90% Credible Intervals

This pattern is economically informative. The selected impulse-response moments appear to pin down the slopes of the wage and price blocks and the investment margin relatively well, but they provide less independent information about the monetary-policy block. That is plausible in the present setup, because the shock-size parameter governs the overall amplitude of the impulse responses, while ρ_R and ϕ_π govern closely related aspects of monetary-policy propagation. The validation exercise therefore supports a qualified conclusion: the latent-RANK procedure is able to recover several important structural parameters from HANK-generated data.

5.3 Robustness in alternative HANK models

5.4 Kaplan et al. (2018)

TBC

5.5 Auclert et al. (2024)

TBC

5.6 McKay et al. (2016)

TBC

6 Conclusion

TBC

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A Proofs

A.1 Proof of Theorem 2.1

The proof proceeds in three steps. First, the transfers are constructed so that the proposed scaled allocation satisfies each household's budget constraint. Second, under the assumed household structure, the proportional-scaling allocation satisfies the household optimality conditions. Third, once the household block is aligned with the benchmark path, the remaining equilibrium conditions are inherited from the complete-markets allocation.

Defining Δ therefore requires the individual decision in a steady state in the incomplete markets economy and otherwise we only need complete markets variables:

$$\begin{aligned}
\Delta_{i,t} &= (\gamma_t^C - 1)c_{i,t}^{IM,ss} - \left(\gamma_t^H \gamma_t^w \gamma_t^{\tau_w} - 1\right) w_{ss}^{CM}(1 - \tau_{w,ss}^{CM}) s_{it} h_{i,t}^{IM,ss} \\
&- \lambda_{i,t}(\gamma_t^\Gamma - 1)\Gamma_{ss}^{CM} \\
&+ a_{i,t}^{IM,ss}(1 + (1 - \tau_{k,ss}^{CM}) r_{ss}^{CM,a}) \\
&- a_{i,t}^{IM,ss}\gamma_t^A(1 + (1 - \tau_{k,t}^{CM}) r_t^{CM,a}) + a_{i,t+1}^{IM,ss}(\gamma_{t+1}^A - 1).
\end{aligned}$$

The allocations and prices constitute a CM equilibrium implying that all equilibrium conditions in the IM economy unrelated to the household sector, government budget constraint, MF conditions, ... are automatically satisfied. We therefore consider first whether the household budget constraint for the allocation in (**??**) which reads now

$$\begin{aligned}
& c_{i,t}^{IM} + a_{i,t+1}^{IM} \\
= \;& \Delta_{i,t} + (1 + (1 - \tau_{k,t}^{IM}) r_t^{a,IM}) a_{i,t}^{IM} + (1 - \tau_{w,t}^{IM}) s_{i,t} w_t^{IM} h_{i,t}^{IM} + \lambda_{i,t} \Gamma_t^{IM}
\end{aligned}$$

holds.

Note that the budget constraint for household i at time t holds in the steady state:

$$c_{i,t}^{IM,ss} + a_{i,t+1}^{IM,ss} = (1 + (1 - \tau_{k,ss}^{IM}) r_{ss}^a) a_{i,t}^{IM,ss} + (1 - \tau_{w,ss}^{IM}) w_{ss}^{IM} h_{i,t}^{IM,ss} s_{i,t} + \lambda_{i,t} \Gamma_{ss}^{IM}.$$

For the same household i, the period t budget constraint is now

$$\begin{aligned}
& c_{i,t}^{IM} + a_{i,t+1}^{IM} \\
= \; & \gamma_t^C c_{i,t}^{IM,ss} + a_{i,t+1}^{IM,ss}\gamma_{t+1}^A \\
= \; & (\gamma_t^C - 1)c_{i,t}^{IM,ss} + a_{i,t+1}^{IM,ss}(\gamma_{t+1}^A - 1) + c_{i,t}^{IM,ss} + a_{i,t+1}^{IM,ss} \\
= \; & (\gamma_t^C - 1)c_{i,t}^{IM,ss} + a_{i,t+1}^{IM,ss}(\gamma_{t+1}^A - 1) + (1 + (1 - \tau_{k,ss}^{IM})r_{ss}^a)a_{i,t}^{IM,ss} + (1 - \tau_{w,ss}^{IM})w_{ss}^{IM}h_{i,t}^{IM,ss}s_{i,t} + \lambda_{i,t}\Gamma_{ss}^{IM} \\
= \; & \Delta_{i,t} + \gamma_t^H \gamma_t^w \gamma_t^{\tau_w}(1 - \tau_{w,ss}^{IM})s_{i,t}w_{ss}^{IM}h_{i,t}^{IM,ss} + (1 + (1 - \tau_k)r_t^{a,IM})a_{i,t}^{IM,ss}\gamma_t^A + \lambda_{i,t}\Gamma_t^{IM} \\
= \; & \Delta_{i,t} + (1 - \tau_{w,t}^{IM})s_{i,t}w_t^{IM}h_{i,t}^{IM} + (1 + (1 - \tau_{k,t}^{IM})r_t^{a,IM})a_{i,t}^{IM} + \lambda_{i,t}\Gamma_t^{IM}
\end{aligned}$$

that is the household budget constraint holds.

The conjectured allocation (**??**) also satisfies the consumption Euler equation since the CM economy implies that

$$(\frac{\gamma_{t+1}^C}{\gamma_t^C})^{-\sigma} = \frac{(\frac{C_{t+1}^{CM}}{C_{ss}^{CM}})^{-\sigma}}{(\frac{C_t^{CM}}{C_{ss}^{CM}})^{-\sigma}} = \frac{1 + r_{ss}^{a,CM}(1 - \tau_{k,ss}^{CM})}{1 + r_{t+1}^{a,CM}(1 - \tau_{k,t+1}^{CM})} = \frac{1 + r_{ss}^{a,IM}(1 - \tau_{k,ss}^{IM})}{1 + r_{t+1}^{a,IM}(1 - \tau_{k,t+1}^{IM})}, \tag{A.1}$$

so that

$$(\gamma_t^C)^{-\sigma}(1 + r_{ss}^{a,IM}(1 - \tau_{k,ss}^{IM})) \;\; = \;\; (\gamma_{t+1}^C)^{-\sigma}(1 + r_{t+1}^{a,IM}(1 - \tau_{k,t+1}^{IM})). \tag{A.2}$$

Households who are credit constrained in the steady state at period t are still constrained so that their consumption Euler equation holds with inequality. For non-constrained households

$$(c_{i,t}^{IM})^{-\sigma} = (\gamma_t^C c_{i,t}^{IM,ss})^{-\sigma} = (\gamma_t^C)^{-\sigma}\beta(1 + r_{ss}^{a,IM}(1 - \tau_{k,ss}^{IM}))E_t(c_{i,t+1}^{IM,ss})^{-\sigma} \tag{A.3}$$

$$= \; (\gamma_{t+1}^C)^{-\sigma}\beta(1 + r_{t+1}^{a,IM}(1 - \tau_{k,t+1}^{IM}))E_t(c_{i,t+1}^{IM,ss})^{-\sigma} = \beta(1 + r_{t+1}^{a,IM}(1 - \tau_{k,t+1}^{IM}))E_t(c_{i,t+1}^{IM})^{-\sigma} \tag{A.4}$$

where we have used (A.2). Since all other equations characterizing the equilibrium are identical in the complete and incomplete markets economy (only the consumption block is different), we have an equlibrium.

Aggregate consumption, hours and assets at time t in the incomplete markets "shock"

equilibrium equal

$$H_t^{IM} = \int s h_{i,t}^{IM} di = \int s \gamma_t^H h_{i,t}^{IM,ss} di = \gamma_t^H \int s h_{i,t}^{IM,ss} di = \gamma_t^H H^{IM,ss} = H_t^{CM} \quad \text{(A.5)}$$

$$C_t^{IM} = \int c_{i,t}^{IM} di = \int s \gamma_t^C c_{i,t}^{IM,ss} di = \gamma_t^C \int s c_{i,t}^{IM,ss} di = \gamma_t^C C^{IM,ss} = C_t^{CM} \quad \text{(A.6)}$$

$$A_{t+1}^{IM} = \int a_{i,t+1}^{IM} di = \int s \gamma_t^A a_{i,t}^{IM,ss} di = \gamma_t^A \int s a_{i,t}^{IM,ss} di = \gamma_t^A A^{IM,ss} = A_t^{CM} \quad \text{(A.7)}$$

We also have that the average transfer is zero:

$$\begin{aligned}
& \int \Delta_{i,t} di \\
= \; & (\gamma_t^C - 1) \int c_{i,t}^{IM,ss} di - \left(\gamma_t^H \gamma_t^w - 1\right) w_{ss}^{CM} (1 - \tau_{ss}) \int s_{it} h_{i,t}^{IM,ss} di \\
- \; & (\gamma_t^\Gamma - 1) \Gamma_{ss}^{CM} \underbrace{\int \lambda_{i,t} di}_{=1} \\
+ \; & \left((1 + (1 - \tau_k) r_{ss}^{a,CM}) - \gamma_t^A (1 + (1 - \tau_k) r_t^{a,CM}) \right) \int a_{i,t}^{IM,ss} di \\
+ \; & (\gamma_{t+1}^A - 1) \int a_{i,t+1}^{IM,ss} di \\
= \; & C_t^{CM} - C_{ss} - (1 - \tau_{ss})(H_t^{CM} w_t^{CM} - H_{ss}^{CM} w_{ss}^{CM}) - (\Gamma_t^{CM} - \Gamma_{ss}^{CM}) \\
+ \; & A_{ss}^{CM} (1 + (1 - \tau_k) r_{ss}^{a,CM}) - A_t^{CM} (1 + (1 - \tau_k) r_t^{a,CM}) \\
+ \; & A_{t+1}^{CM} - A_{ss}^{CM} \\
= \; & 0 - 0
\end{aligned}$$

since the CM household budget constraint:

$$A_{t+1}^{CM} = (1 - \tau_{ss}) H_t^{CM} w_t^{CM} + A_t^{CM} (1 + (1 - \tau_k) r_t^{a,CM}) - C_t^{CM} + \Gamma_t^{CM}$$

B Return Heterogeneity

B.1 Return heterogeneity

We now allow that households earn different returns $\lambda^r_{i,t}(1+(1-\tau_k)r^a_t)$ on their assets a. To ensure that the average gross return equals $(1+(1-\tau_k)r^{a,IM}_t)$ in the steady state

$$\frac{\int \lambda^r_{i,t}(1+(1-\tau_k)r^{a,IM}_{ss})a^{IM,ss}_{i,t}}{\int a^{IM,ss}_{i,t}} = 1+(1-\tau_k)r^{a,IM}_{ss}.$$

and outside steady states:

$$\frac{\int \lambda^r_{i,t}(1+(1-\tau_k)r^{a,IM}_{t})\gamma^A_t a^{IM,ss}_{i,t}}{\int \gamma^A_t a^{IM,ss}_{i,t}} = 1+(1-\tau_k)r^{a,IM}_{t}$$

which both are equivalent to

$$\frac{\int \lambda^r_{i,t} a^{IM,ss}_{i,t}}{\int a^{IM,ss}_{i,t}} = 1.$$

In the data this condition is easy to satisfy if one sets the interest rate income (after taxes) equal to $\lambda^r_{i,t}(1+(1-\tau_k)r^{a,IM}_t a^{IM}_{i,t}$ for individual i at time t.

We have to adjust $\Delta_{i,t}$:

$$\begin{aligned}
\Delta_{i,t} &= (\gamma^C_t-1)c^{IM,ss}_{i,t} - \left(\gamma^H_t\gamma^w_t-1\right)w^{CM,ss}_{i,t}(1-\tau_{ss})s_{it}h^{IM,ss}_{i,t} \\
&- \lambda_{i,t}(\gamma^\Gamma_t-1)\Gamma^{CM}_{ss} \\
&+ a^{IM,ss}_{i,t}\lambda^r_{i,t}(1+(1-\tau_k)r^{a,CM}_{ss}) \\
&- a^{IM,ss}_{i,t}\gamma^A_t\lambda^r_{i,t}(1+(1-\tau_k)r^{a,CM}_t) + a^{IM,ss}_{i,t+1}(\gamma^A_{t+1}-1).
\end{aligned}$$

The Euler equations are satisfied

$$\begin{aligned}
&(c^{IM}_{i,t})^{-\sigma} = (\gamma^C_t c^{IM,ss}_{i,t})^{-\sigma} = (\gamma^C_t)^{-\sigma}E_t[\beta\lambda^r_{i,t+1}(1+r^{a,IM}_{ss}(1-\tau_k))(c^{IM,ss}_{i,t+1})^{-\sigma}] \\
= \;& (\gamma^C_{t+1})^{-\sigma}E_t[\beta\lambda^r_{i,t+1}(1+r^{a,IM}_{t+1}(1-\tau_k))E_t(c^{IM,ss}_{i,t+1})^{-\sigma}] = E_t[\beta\lambda^r_{i,t+1}(1+r^{a,IM}_{t+1}(1-\tau_k))(c^{IM}_{i,t+1})^{-\sigma}]
\end{aligned}$$

B.2 Proof of Proposition

Proof Theorem 2.1 implies that, when HANK households face the RANK price-income system together with the compensating transfers, the aggregate consumption path coincides

with the RANK path. Linearizing that exact identity around the common steady state gives

$$dC^R = J_X dX^R + M d\Delta^R.$$

Using $d\Delta^R = B\, dz^R$ yields

$$dC^R = J_X dX^R + \Delta C,$$

which rearranges to

$$dC^{H|R} = J_X dX^R = dC^R - \Delta C.$$

This proves part (i).

For part (ii), in HANK general equilibrium there is no compensating transfer, so

$$dC^H = J_X dX^H.$$

Because the non-household block is common across RANK and HANK, Appendix C implies that once a consumption path is specified, the remaining equilibrium objects are pinned down linearly, i.e.

$$dX^j = G_C dC^j + G_\varepsilon d\varepsilon, \qquad j \in \{R, H\}.$$

Subtracting the HANK equation from the RANK equation yields

$$dC^R - dC^H = \Delta C + J_X(dX^R - dX^H) = \Delta C + J_X G_C(dC^R - dC^H),$$

which is equivalent to

$$(I - J_X G_C)(dC^R - dC^H) = \Delta C.$$

This proves part (ii). Part (iii) follows from the Neumann expansion of

$$(I - J_X G_C)^{-1}.$$

C RANK Model

In this section, we present the equilibrium conditions of the RANK model, the log-linear perfect-foresight response to a monetary shock, and the specific functional forms used in the estimation.

C.1 Households

Preferences. Households consist of a continuum of members $j \in [0,1]$, each deriving utility from consumption c_{jt} and disutility from supplying labor h_{jt}. Preferences are additively separable in consumption and leisure, with expected lifetime utility

$$U_j = E_0 \sum_{t=0}^{\infty} \beta^t u(c_{jt}, h_{jt}).$$

The period utility function is

$$u(c_{jt}, h_{jt}) = \begin{cases} \dfrac{c_{jt}^{1-\sigma} - 1}{1-\sigma} - \dfrac{\chi}{1+\gamma} h_{jt}^{1+\gamma}, & \text{if } \sigma \neq 1, \\ \log(c_{jt}) - \dfrac{\chi}{1+\gamma} h_{jt}^{1+\gamma}, & \text{if } \sigma = 1, \end{cases}$$

where σ is the coefficient of relative risk aversion (the inverse of the intertemporal elasticity of substitution), γ governs the Frisch elasticity of labor supply, and χ scales the disutility of labor.

Wages. As described in the main text, the representative wage setter (or "middleman") chooses nominal wages to maximize the discounted value of supplying differentiated labor services. The problem is

$$V_t^w\left(\hat{W}_{t-1}\right) = \max_{\hat{W}_t} \int \left[\frac{s_{jt}(1-\tau_l)\hat{W}_t}{P_t} h\left(\hat{W}_t; W_t, H_t\right) - \frac{g\left(h(\hat{W}_t; W_t, H_t)\right)}{u'(C_t)} \right] dj$$
$$- \int s_{jt} \frac{\theta_w}{2} \left(\frac{\hat{W}_t}{\hat{W}_{t-1}} - \overline{\Pi}^w \right)^2 H_t \, dj + \frac{1}{1+r_t} V_{t+1}^w\left(\hat{W}_t\right),$$

where C_t denotes aggregate consumption. Using $\int s_{jt}\, dj = 1$ and substituting the demand for differentiated labor services, the recursive formulation becomes

$$V_t^w(W_{jt-1}) = \max_{W_{jt}} \left\{ \frac{W_{jt}}{P_t} \left(\frac{W_{jt}}{W_t} \right)^{-\epsilon_w} (1-\tau_l) H_t - \frac{\chi}{1+\gamma} \frac{\left[\left(\frac{W_{jt}}{W_t} \right)^{-\epsilon_w} H_t \right]^{1+\gamma}}{C_t^{-\sigma}} \right.$$
$$\left. - \frac{\theta_w}{2} \left(\frac{W_{jt}}{W_{jt-1}} - \bar{\Pi}^w \right)^2 H_t + \frac{1}{1+r_{t+1}} V_{t+1}^w(W_{jt}) \right\}.$$

The first-order condition with respect to W_{jt} is

$$0 = (1-\epsilon_w)\left(\frac{W_{jt}}{W_t}\right)^{-\epsilon_w}\frac{(1-\tau_l)H_t}{P_t} + \frac{\epsilon_w\chi H_t}{W_t}\frac{\left[\left(\frac{W_{jt}}{W_t}\right)^{-\epsilon_w}H_t\right]^{\gamma}}{C_t^{-\sigma}}\left(\frac{W_{jt}}{W_t}\right)^{-\epsilon_w-1}$$
$$-\theta_w\left(\frac{W_{jt}}{W_{jt-1}}-\bar{\Pi}^w\right)\frac{H_t}{W_{jt-1}} + \frac{1}{1+r_{t+1}}\frac{\partial V_{t+1}^w(W_{jt})}{\partial W_{jt}},$$

and the envelope condition is

$$\frac{\partial V_{t+1}^w(W_{jt})}{\partial W_{jt}} = \theta_w\left(\frac{W_{jt+1}}{W_{jt}}-\bar{\Pi}^w\right)\frac{H_{t+1}W_{jt+1}}{W_{jt}^2}.$$

In a symmetric equilibrium where $W_{jt} = W_t$ for all j, the wage Phillips curve simplifies to

$$\theta_w(\Pi_t^w-\bar{\Pi}^w)\Pi_t^w = (1-\epsilon_w)(1-\tau_l)w_t + \epsilon_w\frac{\chi H_t^{\gamma}}{C_t^{-\sigma}} + \frac{\theta_w}{1+r_{t+1}}(\Pi_{t+1}^w-\bar{\Pi}^w)\Pi_{t+1}^w\frac{H_{t+1}}{H_t},$$

where $w_t = W_t/P_t$ is the real wage. In the steady state,

$$0 = (1-\epsilon_w)(1-\tau_l)w + \epsilon_w\chi H^{\gamma}C^{\sigma}.$$

A log-linear approximation around the steady state yields

$$\begin{aligned}\theta_w\hat{\Pi}_t^w &= (1-\epsilon_w)(1-\tau_l)w\hat{w}_t + \epsilon_w\chi H^{\gamma}C^{\sigma}(\gamma\hat{H}_t+\sigma\hat{C}_t) + \frac{\theta_w}{1+r}\hat{\Pi}_{t+1}^w\\ &= (1-\epsilon_w)(1-\tau_l)w\left(\hat{w}_t-\gamma\hat{H}_t-\sigma\hat{C}_t\right) + \frac{\theta_w}{1+r}\hat{\Pi}_{t+1}^w,\end{aligned}$$

where $\Pi_t^w = (w_t/w_{t-1})\Pi_t$ implies

$$\hat{\Pi}_t^w = \hat{w}_t - \hat{w}_{t-1} + \hat{\Pi}_t.$$

Substituting and rearranging gives the compact form

$$\hat{w}_t - \hat{w}_{t-1} + \hat{\Pi}_t = \eta_w\left(\hat{w}_t-\gamma\hat{H}_t-\sigma\hat{C}_t\right) + \frac{\hat{w}_{t+1}-\hat{w}_t+\hat{\Pi}_{t+1}}{1+r},$$

where

$$\eta_w = \frac{(1-\epsilon_w)(1-\tau_l)w}{\theta_w}$$

is the slope of the wage Phillips curve.

Consumption and Saving. Under complete asset markets and the assumptions on labor supply, the household problem aggregates to a representative agent choosing an aggregate consumption stream $\{C_t\}_{t=0}^{\infty}$. The representative household maximizes

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\frac{C_t^{1-\sigma}-1}{1-\sigma}\right],$$

subject to the real budget constraint

$$C_t + A_{t+1} = (1-\tau_l)w_t H_t + \left[1 + r_t^a(1-\tau_k)\right]A_t + \Gamma_t,$$

where $\beta \in (0,1)$ is the discount factor, $1/\sigma$ the intertemporal elasticity of substitution, A_t denotes one-period real assets with gross return $1 + r_t^a$, and Γ_t denotes real lump-sum transfers from the government. The first-order conditions imply the standard Euler equation

$$1 = \beta E_t\left[\left(1 + r_{t+1}^a(1-\tau_k)\right)\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\right].$$

In the steady state, where $r^a = r$,

$$1 = \beta\left(1 + r(1-\tau_k)\right).$$

Log-linearizing around the steady state yields

$$\hat{c}_{t+1} - \hat{c}_t = \frac{1}{\sigma}\frac{(1-\tau_k)r}{1+(1-\tau_k)r}\hat{r}_t^a,$$

which links consumption growth to the after-tax real interest rate.

C.2 Firms

Intermediate-Good Firms. Following the main text, each intermediate-good producer $l \in [0,1]$ hires capital K_{lt} and labor H_{lt} to produce differentiated output using technology $Y_{lt} = Z_t K_{lt}^{\alpha} H_{lt}^{1-\alpha}$, where Z_t denotes aggregate productivity. Cost minimization implies the marginal-cost and factor-demand relations

$$mc_{lt} = \left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\frac{(r_t^k)^{\alpha}(w_t)^{1-\alpha}}{Z_t}, \qquad \frac{K_{lt}}{H_{lt}} = \frac{\alpha w_t}{(1-\alpha)r_t^k}.$$

Each firm sets its nominal price p_{lt} to maximize the present discounted value of profits subject to quadratic price-adjustment costs:

$$V_t(p_{lt-1}) \equiv \max_{p_{lt}} \left\{ \frac{p_{lt}}{P_t} y(p_{lt}; P_t, Y_t) - S\big(y(p_{lt}; P_t, Y_t)\big) - \frac{\theta}{2}\left(\frac{p_{lt}}{p_{lt-1}} - \overline{\Pi}\right)^2 Y_t - Z_t F + \frac{1}{1+r_t} V_{t+1}(p_{lt}) \right\},$$

where $S(\cdot)$ denotes total production cost, F fixed operating costs, and $\theta > 0$ the degree of nominal rigidity. Substituting the demand from the final-good producer, $Y_{lt} = (p_{lt}/P_t)^{-\epsilon} Y_t$, yields

$$V_t(p_{lt-1}) = \max_{p_{lt}} \left\{ \frac{p_{lt}}{P_t}\left(\frac{p_{lt}}{P_t}\right)^{-\epsilon} Y_t - S\left[\left(\frac{p_{lt}}{P_t}\right)^{-\epsilon} Y_t\right] - \frac{\theta}{2}\left(\frac{p_{lt}}{p_{lt-1}} - \overline{\Pi}\right)^2 Y_t + \frac{1}{1+r_{t+1}} V_{t+1}(p_{lt}) \right\}.$$

The first-order condition is

$$0 = (1-\epsilon)\left(\frac{p_{lt}}{P_t}\right)^{-\epsilon} \frac{Y_t}{P_t} + \epsilon\, mc_t \left(\frac{p_{lt}}{P_t}\right)^{-\epsilon-1} \frac{Y_t}{P_t} - \theta\left(\frac{p_{lt}}{p_{lt-1}} - \overline{\Pi}\right)\frac{Y_t}{p_{lt-1}} + \frac{1}{1+r_{t+1}} \frac{\partial V_{t+1}(p_{lt})}{\partial p_{lt}},$$

with envelope condition

$$\frac{\partial V_{t+1}(p_{lt})}{\partial p_{lt}} = \theta\left(\frac{p_{lt+1}}{p_{lt}} - \overline{\Pi}\right)\frac{Y_{t+1} p_{lt+1}}{p_{lt}^2}.$$

In a symmetric equilibrium where $p_{lt} = P_t$, the New Keynesian Phillips curve (NKPC) is

$$\theta(\Pi_t - \overline{\Pi})\Pi_t = (1-\epsilon) + \epsilon\, mc_t + \frac{\theta}{1+r_{t+1}}(\Pi_{t+1} - \overline{\Pi})\Pi_{t+1}\frac{Y_{t+1}}{Y_t}.$$

Log-linearizing around the steady state gives

$$\hat{\Pi}_t = \eta_p\left[\alpha\, \hat{r}_t^k + (1-\alpha)\hat{w}_t\right] + \frac{1}{1+r}\hat{\Pi}_{t+1},$$

where $\eta_p = \epsilon - 1$, and $\hat{r}_t^k$ and $\hat{w}_t$ are log-deviations of the real rental rate of capital and the real wage. The log-linearized labor-demand equation implied by cost minimization is

$$\hat{H}_t = -\hat{w}_t + \hat{r}_t^k + \hat{K}_t.$$

Dividends paid by intermediate-good firms are

$$d_t^{IG} = Y_t - S(Y_t),$$

so in steady state

$$d^{IG} = Y - S(Y).$$

In response to an MP shock,

$$\begin{aligned}
\partial d_t^{IG} &= \partial Y_t - S'(Y)\,\partial Y_t, \\
\Rightarrow \quad d^{IG}\hat{d}^{IG} &= \left(1 - \left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}(r^k)^{\alpha} w^{1-\alpha}\right) Y\,\hat{Y}_t, \\
\Rightarrow \quad \hat{d}^{IG} &= \left(1 - \left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}(r^k)^{\alpha} w^{1-\alpha}\right)\left(\frac{C}{d^{IG}}\hat{C}_t + \frac{\delta K}{d^{IG}}\hat{I}_t + \frac{G}{d^{IG}}\hat{G}_t\right).
\end{aligned}$$

Capital-Producing Firms. The recursive problem is

$$\begin{aligned}
V_t(I_{t-1}, K_t) = \max_{I_t, K_{t+1}} &\left\{ r_t^k K_t - I_t - \Phi(I_t, I_{t-1}) + \frac{1}{1+r_{t+1}} V_{t+1}(I_t, K_{t+1}) \right\} \\
\text{s.t.} \quad & K_{t+1} = (1-\delta)K_t + I_t,
\end{aligned}$$

where $\Phi(\cdot)$ denotes investment adjustment costs and δ is the depreciation rate. Let q_t^k be the multiplier on the accumulation constraint. Equivalently,

$$\begin{aligned}
V_t(I_{t-1}, K_t) = \max_{I_t, K_{t+1}, q_t^k} &\left\{ r_t^k K_t - I_t - \Phi(I_t, I_{t-1}) + \frac{1}{1+r_{t+1}} V_{t+1}(I_t, K_{t+1}) \right. \\
&\left. + q_t^k \left[(1-\delta)K_t + I_t - K_{t+1}\right] \right\}.
\end{aligned}$$

The first-order and envelope conditions are

$$\begin{aligned}
\partial I_t: \quad & 0 = -1 - \Phi_{1,t} + \frac{1}{1+r_{t+1}} \frac{\partial V_{t+1}}{\partial I_t} + q_t^k, \\
\partial K_{t+1}: \quad & 0 = \frac{1}{1+r_{t+1}} \frac{\partial V_{t+1}}{\partial K_{t+1}} - q_t^k, \\
& \frac{\partial V_{t+1}}{\partial I_t} = -\Phi_{2,t+1}, \qquad \frac{\partial V_{t+1}}{\partial K_{t+1}} = r_{t+1}^k + (1-\delta) q_{t+1}^k.
\end{aligned}$$

Combining these yields

$$\begin{aligned}
q_t^k &= 1 + \Phi_{1,t} + \frac{1}{1+r_{t+1}} \Phi_{2,t+1}, \\
q_t^k &= \frac{r_{t+1}^k + (1-\delta) q_{t+1}^k}{1+r_{t+1}}.
\end{aligned}$$

The adjustment cost is quadratic:

$$\Phi(I_t, I_{t-1}) = \frac{\phi_k}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 I_{t-1},$$

which implies

$$\Phi_{1,t} = \phi_k\left(\frac{I_t}{I_{t-1}} - 1\right), \qquad \Phi_{2,t} = -\frac{\phi_k}{2}\left(\frac{I_t^2}{I_{t-1}^2} - 1\right),$$

$$\Phi_{11,t} = \frac{\phi_k}{I_{t-1}}, \qquad \Phi_{22,t} = \phi_k\frac{I_t^2}{I_{t-1}^3}, \qquad \Phi_{12,t} = \Phi_{21,t} = -\phi_k\frac{I_t}{I_{t-1}^2}.$$

In steady state,

$$\Phi_{ss} = \Phi_{1,ss} = \Phi_{2,ss} = 0, \qquad \Phi_{11,ss} = \Phi_{22,ss} = \frac{\phi_k}{I}, \qquad \Phi_{12,ss} = -\frac{\phi_k}{I}.$$

Evaluating at steady state gives

$$q^k = 1, \qquad 1 + r = 1 + r^k - \delta.$$

Linearizing around steady state yields

$$\hat{I}_t - \hat{I}_{t-1} = \frac{1}{\phi_k}\hat{q}_t^k + \frac{1}{1+r}(\hat{I}_{t+1} - \hat{I}_t),$$

$$\hat{q}_t^k = \frac{r^k\hat{r}_{t+1}^k + (1-\delta)\hat{q}_{t+1}^k - r\hat{r}_{t+1}}{1+r},$$

which shows that investment growth depends on the discounted value of the marginal return to capital. Dividends paid by the capital-producing firm are

$$d_t^k = r_t^k K_t - I_t - \Phi(I_t, I_{t-1}),$$

so in steady state

$$d^k = (r^k - \delta)K = rK.$$

Log-linearizing dividends yields

$$\hat{d}_t^k = \left(\frac{r+\delta}{r}\right)\left(\hat{r}_t^k + \hat{K}_t\right) - \left(\frac{\delta}{r}\right)\hat{I}_t.$$

C.3 Mutual Fund

In a complete-markets setting, the mutual fund is a pass-through for the representative household, consolidating all asset holdings. The household holds government debt (b_t, b_t^L), equity in intermediate-goods firms v_t^{IG}, and equity in capital-goods firms v_t^k, and solves

$$\begin{aligned} V_t^{MF}(b_t, b_t^L, v_t^{IG}, v_t^k) = \max_{d_t^{MF}, b_{t+1}, b_{t+1}^L, v_{t+1}^{IG}, v_{t+1}^k} & \left\{ d_t^{MF} + \beta_t^{MF} V_{t+1}(b_{t+1}, b_{t+1}^L, v_{t+1}^{IG}, v_{t+1}^k) \right\} \\ \text{s.t.} \quad d_t^{MF} + \underbrace{b_{t+1} + q_t^b b_{t+1}^L + p_t^{IG} v_{t+1}^{IG} + p_t^k v_{t+1}^k}_{A_{t+1}} & \\ = (1 + r_t) b_t + \left[r_t^L + \delta^b + (1 - \delta^b) q_t^b \right] b_t^L & \\ + v_t^{IG} \left(p_t^{IG} + d_t^{IG} \right) + v_t^k (p_t^k + d_t^k). & \end{aligned}$$

Let ξ_t denote the Lagrange multiplier on the budget constraint. The first-order conditions are

$$\begin{aligned} \partial d_t^{MF}: \quad & 1 = \xi_t, & \partial b_{t+1}: \quad & \beta_t^{MF} \frac{\partial V_{t+1}}{\partial b_{t+1}} = \xi_t, \\ \partial b_{t+1}^L: \quad & \beta_t^{MF} \frac{\partial V_{t+1}}{\partial b_{t+1}^L} = q_t^b \xi_t, & \partial v_{t+1}^{IG}: \quad & \beta_t^{MF} \frac{\partial V_{t+1}}{\partial v_{t+1}^{IG}} = p_t^{IG} \xi_t, \\ \partial v_{t+1}^k: \quad & \beta_t^{MF} \frac{\partial V_{t+1}}{\partial v_{t+1}^k} = p_t^k \xi_t. & & \end{aligned}$$

The envelope conditions are

$$\begin{aligned} \frac{\partial V_{t+1}}{\partial b_{t+1}} &= \xi_{t+1} \left(1 + r_{t+1} \right), \\ \frac{\partial V_{t+1}}{\partial b_{t+1}^L} &= \xi_{t+1} \left[r_{t+1}^L + \delta^b + (1 - \delta^b) q_{t+1}^b \right], \\ \frac{\partial V_{t+1}}{\partial v_{t+1}^{IG}} &= \xi_{t+1} \left[p_{t+1}^{IG} + d_{t+1}^{IG} \right], \\ \frac{\partial V_{t+1}}{\partial v_{t+1}^k} &= \xi_{t+1} \left[p_{t+1}^k + d_{t+1}^k \right]. \end{aligned}$$

Combining yields the intertemporal optimality conditions:

$$\begin{aligned}
\beta_t^{MF} &= \frac{1}{1+r_{t+1}}, \\
1+r_{t+1} &= \frac{r_{t+1}^L + \delta^b + (1-\delta^b)q_{t+1}^b}{q_t^b}, \\
1+r_{t+1} &= \frac{p_{t+1}^{IG} + d_{t+1}^{IG}}{p_t^{IG}}, \\
1+r_{t+1} &= \frac{p_{t+1}^{k} + d_{t+1}^{k}}{p_t^{k}}.
\end{aligned}$$

The first equation implies that to rationalize risk-free debt holdings, the mutual fund must discount at the risk-free rate. The remaining equations are standard asset-pricing conditions equating all asset returns to the risk-free rate. Hence, the return on aggregate assets satisfies

$$1 + r_t^a = \frac{d_t^{MF} + A_{t+1}}{A_t} = 1 + r_t.$$

Steady state. Assuming long-term debt is issued at par ($q^b = 1$), or equivalently $r^L = r$, so that

$$r = r^L = \frac{d^{IG}}{p^{IG}} = \frac{d^k}{p^k}.$$

Log-linearization. For long-term debt pricing and equity claims (intermediate- and capital-goods),

$$\begin{aligned}
\hat{q}_t^b &= (1-\delta^b)(1-\rho)\hat{q}_{t+1}^b + \rho\,(\hat{r}_{t+1}^L - \hat{r}_{t+1}), \\
\hat{p}_t^{IG} &= (1-\rho)\,\hat{p}_{t+1}^{IG} + \rho\,(\hat{d}_{t+1}^{IG} - \hat{r}_{t+1}), \\
\hat{p}_t^{k} &= (1-\rho)\,\hat{p}_{t+1}^{k} + \rho\,(\hat{d}_{t+1}^{k} - \hat{r}_{t+1}),
\end{aligned}$$

where $\rho = \frac{r}{1+r}$. In what follows we will assume the real rate on long-term bonds is constant out of steady state so $\hat{r}_t^L = 0$.

Aggregation of financial wealth. Define total financial assets

$$\begin{aligned} A_{t+1} &= b_{t+1} + q_t^b b_{t+1}^L + p_t^{IG} v_{t+1}^{IG} + p_t^k v_{t+1}^k \\ &= \underbrace{b_{t+1} + b_{t+1}^L}_{B_{t+1}} + (q_t^b - 1) b_{t+1}^L + p_t^{IG} v_{t+1}^{IG} + p_t^k v_{t+1}^k. \end{aligned}$$

In the initial steady state, $b_{-1} = 0$ and $b_{-1}^L = B$; after the shock, $b_t \to B$ and $b_t^L = (1-\delta^b)^t B \to 0$. With $q^b = 1$ and equity shares summing to one, the steady-state level is

$$A = B + p^{IG} + p^k = B + p^{IG} + K,$$

using $p^k = K$ in steady state. Log-linearizing gives

$$\hat{A}_{t+1} = \frac{B}{A}\left(\hat{B}_{t+1} + (1-\delta^b)^{t+1}\hat{q}_t^b\right) + \frac{p^{IG}}{A}\hat{p}_t^{IG} + \frac{K}{A}\hat{p}_t^k.$$

C.4 Government

Fiscal Policy. Prior to the initial steady state, the government issues long-term debt at par, $q_{ss}^L = 1$. Its real budget constraint is

$$\underbrace{b_{t+1}^L - \left(1-\delta^b\right) b_t^L}_{\text{Net issuance}} = \left(\delta^b + r\right) b_t^L + G + \Gamma - \tau_l w H - \tau_k r^a A.$$

Revenue from net issuance thus finances the difference between expenditures on debt service $(\delta^b + r) b_t^L$, government purchases G, and transfers Γ, and revenues from labor and capital income taxes, $\tau_l w H$ and $\tau_k r^a A$.

Let B denote the steady-state level of debt. Out of steady state, the government ceases long-term issuance and instead rolls over one-period bonds, implying the constraint

$$b_{t+1} = (1 + r_t) b_t + (\delta^b + r) b_t^L + G_t + \Gamma_t - \tau_l w_t H_t - \tau_k r_t^a A_t.$$

Hence $b_t \to B$ and $b_t^L = (1-\delta^b)^t B \to 0$ as $t \to \infty$. To ensure a consistent log-linearization,

define total government debt $B_t = b_t + b_t^L$. Then

$$\begin{aligned} B_{t+1} &= (1 + r_t)B_t + (\delta^b + r - r_t)b_t^L + G_t + \Gamma_t - \tau_l w_t H_t - \tau_k r_t^a A_t \\ &= (1 + r_t)B_t + (\delta^b + r - r_t)(1 - \delta^b)^t B + G_t + \Gamma_t - \tau_l w_t H_t - \tau_k r_t^a A_t. \end{aligned}$$

This representation satisfies $B_{-1} = B_\infty = B$, ensuring the initial and terminal steady states coincide. The linearized government budget constraint is

$$\hat{B}_{t+1}B = \left[\hat{R}_t\left(1 - (1 - \delta^b)^t\right) + \hat{B}_t\right]BR + \hat{G}_t G + \hat{\Gamma}_t\Gamma - (\hat{w}_t + \hat{H}_t)\tau_l wH - (\hat{r}_t^a + \hat{A}_t)\tau_k rA.$$

The effect of a change in the gross real interest rate $\hat{R}_t$ depends on the maturity composition of debt through $1 - (1 - \delta^b)^t$. At shorter horizons, rate movements have smaller effects on expenditures because a larger share of debt remains in legacy long-term bonds. Here $\hat{A}_t$ denotes the deviation of real assets from steady state.

Monetary Policy. Given the Taylor rule in the main text, the log-linearized response of the nominal interest rate $R_t^i = 1 + i_t$ is

$$\hat{R}_{t+1}^i = \phi^\pi(1 - \rho_R)\hat{\Pi}_t + \rho_R\hat{R}_t^i.$$

C.5 Log-Linear Perfect-Foresight Response

Here we outline the equations describing the response of the log-linear system to a monetary shock. We interpret the shock to be a sequence of exogenous disturbances to the nominal rate $\left\{\epsilon_t^R\right\}_{t=0}^\infty$, as well as the fiscal response of the treasury $\left\{\hat{B}_t, \hat{G}_t, \hat{\Gamma}_t\right\}_{t=0}^\infty$ which are all assumed to die out in the limit.

Core Equations. The system can be separated into a set of nine core variables $\left\{\hat{C}_t, \hat{R}_t^n, \hat{w}_t, \hat{\Pi}_t, \hat{H}_t, \hat{r}_t^k, \hat{q}_t^k, \hat{K}_t, \hat{I}_t\right\}_{t=0}^\infty$ that are jointly determined in response to the shocks

according to:

$$
\begin{aligned}
&\text{Household} && \begin{cases} \hat{C}_{t+1} - \hat{C}_t = \dfrac{1}{\sigma}\dfrac{(1-\tau_k)(1+r)}{1+(1-\tau_k)r}\left(\hat{R}^n_t - \hat{\Pi}_{t+1}\right) \\ \hat{w}_t - \hat{w}_{t-1} + \hat{\Pi}_t = \eta_w\left(\hat{w}_t - \gamma\hat{H}_t - \sigma\hat{C}_t\right) + \dfrac{\hat{w}_{t+1} - \hat{w}_t + \hat{\Pi}_{t+1}}{1+r} \end{cases} \\
&\text{Firms} && \begin{cases} \hat{\Pi}_t = \eta_p\left[\alpha\,\hat{r}^k_t + (1-\alpha)\hat{w}_t\right] + \dfrac{1}{1+r}\hat{\Pi}_{t+1} \\ \hat{H}_t = -\hat{w}_t + \hat{r}^k_t + \hat{K}_t \\ \hat{I}_t - \hat{I}_{t-1} = \dfrac{1}{\phi_k}\hat{q}^k_t + \dfrac{1}{1+r}(\hat{I}_{t+1} - \hat{I}_t), \\ \hat{q}^k_t = \dfrac{r^k\hat{r}^k_{t+1} + (1-\delta)\hat{q}^k_{t+1} - (1+r)\left(\hat{R}^n_t - \hat{\Pi}_{t+1}\right)}{1+r} \end{cases} \\
&\text{Government} && \left\{\hat{R}^n_t = \phi^\pi(1-\rho_R)\hat{\Pi}_t + \rho_R\hat{R}^n_{t-1} + \epsilon^R_t\right. \\
&\text{Agg. Resource} && \begin{cases} \alpha\hat{K}_t + (1-\alpha)\hat{H}_t = \dfrac{C}{Y}\hat{C}_t + \dfrac{I}{Y}\hat{I}_t + \dfrac{G}{Y}\hat{G}_t \\ \hat{K}_{t+1} = (1-\delta)\hat{K}_t + \delta\hat{I}_t \end{cases}
\end{aligned}
$$

Auxiliary Equations

$$
\begin{aligned}
&\text{Firms} && \begin{cases} \hat{d}^{IG} = \left(1 - \left(\dfrac{1}{\alpha}\right)^\alpha\left(\dfrac{1}{1-\alpha}\right)^{1-\alpha}(r^k)^\alpha(w)^{1-\alpha}\right)\left(\dfrac{C}{d^{IG}}\hat{C}_t + \dfrac{\delta K}{d^{IG}}\hat{I}_t + \dfrac{G}{d^{IG}}\hat{G}_t\right) \\ \hat{d}^k_t = \left(\dfrac{r+\delta}{r}\right)\left(\hat{r}^k_t + \hat{K}_t\right) - \left(\dfrac{\delta}{r}\right)\hat{I}_t \end{cases} \\
&\text{Mutual Fund} && \begin{cases} \hat{q}^b_t = (1-\delta^b)(1-\rho)\hat{q}^b_{t+1} - \rho\,\hat{r}_{t+1} \\ \hat{p}^{IG}_t = (1-\rho)\,\hat{p}^{IG}_{t+1} + \rho\,(\hat{d}^{IG}_{t+1} - \hat{r}_{t+1}) \\ \hat{p}^k_t = (1-\rho)\,\hat{p}^k_{t+1} + \rho\,(\hat{d}^k_{t+1} - \hat{r}_{t+1}) \\ \hat{A}_{t+1} = \dfrac{B}{A}\left(\hat{B}_{t+1} + (1-\delta^b)^{t+1}\hat{q}^b_t\right) + \dfrac{p^{IG}}{A}\hat{p}^{IG}_t + \dfrac{K}{A}\hat{p}^k_t. \end{cases} \\
&\text{Government} && \left\{\hat{B}_{t+1}B = \left[\hat{R}_t\left(1-(1-\delta^b)^t\right) + \hat{B}_t\right]BR + \hat{G}_tG + \hat{\Gamma}_t\Gamma - (\hat{w}_t + \hat{H}_t)\tau_l wH - (\hat{r}_t + \hat{A}_t)\tau_k rA\right.
\end{aligned}
$$

D Estimation

In this section, we provide additional details on the impulse-response-matching estimation procedure described in the main text. We use the limited information approach of Bayesian impulse response matching (Christiano et al., 2010). There are two sets of parameters that determine the dynamics of the system in the previous section (i) parameters set externally or using steady state relationships, and (ii) the remaining parameters $\theta = \left(\eta_w, \eta_p, \phi_k, \phi_\pi, \rho_R\right)$ that are estimated to match the moments of interest.

D.1 Impulse Responses

The perfect-foresight equilibrium path can be written as

$$A_t x_{t+1} + B_t x_t + C_t x_{t-1} = D_t z_t,$$

where (A_t, B_t, C_t, D_t) are known (possibly time-varying) coefficient matrices determined by parameters θ. We impose $x_{-1} = z_{-1} = 0$ and approximate $x_\infty = 0$ by setting $x_{T+1} = 0$ for a sufficiently large terminal horizon T. This leads to

$$\begin{aligned} A_1 x_2 + B_1 x_1 &= D_1 z_1, \\ A_t x_{t+1} + B_t x_t + C_t x_{t-1} &= D_t z_t, \qquad 1 < t < T, \\ B_T x_T + C_T x_{T-1} &= D_T z_T. \end{aligned}$$

Stacking yields the block-tridiagonal system

$$\underbrace{\begin{bmatrix} B_1 & A_1 & 0 & \cdots & 0 \\ C_2 & B_2 & A_2 & \ddots & \vdots \\ 0 & C_3 & B_3 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & A_{T-1} \\ 0 & \cdots & 0 & C_T & B_T \end{bmatrix}}_{\mathcal{M}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_T \end{bmatrix}}_{X} = \underbrace{\operatorname{diag}(D_1, \ldots, D_T)}_{\mathcal{D}} \underbrace{\begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_T \end{bmatrix}}_{Z}.$$

Thus,

$$X = \mathcal{M}^{-1} \mathcal{D} Z.$$

D.2 Counterfactual HANK

Let $\tilde{x}_t$ denote the vector of endogenous variables *excluding* consumption, and suppose the sequence $\{c_t\}$ is imposed exogenously. Eliminating the Euler equation yields the reduced system

$$\tilde{A}\tilde{x}_{t+1} = \tilde{B}\tilde{x}_t + \tilde{C}\tilde{x}_{t-1} + \tilde{D}e_t, \tag{D.1}$$

where $\tilde{A}$ may be singular and $\tilde{B}$ is nonsingular. Define the companion state $\tilde{s}_t = (\tilde{x}_t', \tilde{x}_{t-1}')'$, so (D.1) is equivalent to

$$\widetilde{\mathcal{S}}\tilde{s}_{t+1} = \widetilde{\mathcal{N}}\tilde{s}_t + \widetilde{\mathcal{D}}\, e_t, \tag{D.2}$$

where

$$\widetilde{\mathcal{S}} = \begin{pmatrix} 0 & -\tilde{A} \\ I & 0 \end{pmatrix}, \quad \widetilde{\mathcal{N}} = \begin{pmatrix} \tilde{B} & \tilde{C} \\ 0 & I \end{pmatrix}, \quad \widetilde{\mathcal{D}} = \begin{pmatrix} \tilde{D} \\ 0 \end{pmatrix}.$$

Let (λ_i, v_i) be the generalized eigenpairs of $(\widetilde{\mathcal{N}}, \widetilde{\mathcal{S}})$:

$$\widetilde{\mathcal{N}}v_i = \lambda_i \widetilde{\mathcal{S}}v_i.$$

Under standard saddle-path stability,

$$k \equiv \#\{|\lambda_i| > 1\} < m.$$

Let $\widetilde{Z}^u$ collect the right eigenvectors with $|\lambda_i| > 1$ and partition

$$\widetilde{Z}^u = \begin{pmatrix} \widetilde{U}_1 \\ \widetilde{U}_2 \end{pmatrix}, \qquad \widetilde{U}_1, \widetilde{U}_2 \in \mathbb{R}^{m \times k},$$

conformably with $\tilde{s}_t = (\tilde{x}_t', \tilde{x}_{t-1}')'$. In the full model (with endogenous c_t), boundedness of equilibrium dynamics imposes the stable-manifold condition

$$\widetilde{U}_1^\top \tilde{x}_t + \widetilde{U}_2^\top \tilde{x}_{t-1} = 0. \tag{D.3}$$

When c_t is imposed exogenously, (D.3) is no longer enforced automatically, and generic solutions to (D.1) contain explosive components. To restore equilibrium selection, we impose

(D.3) only at the terminal horizon T (where c_t has returned arbitrarily close to steady state):

$$\widetilde{U}_1^\top \tilde{x}_T + \widetilde{U}_2^\top \tilde{x}_{T-1} = 0. \tag{D.4}$$

Stacking (D.1) for $t = 1, \ldots, T$ yields

$$\widetilde{\mathscr{M}}\widetilde{X} = \widetilde{\mathscr{D}}\, E,$$

where

$$\widetilde{\mathscr{M}} = \begin{bmatrix} \tilde{B} & \tilde{A} & 0 & \cdots & 0 \\ \tilde{C} & \tilde{B} & \tilde{A} & \ddots & \vdots \\ 0 & \tilde{C} & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \tilde{B} & \tilde{A} \\ 0 & \cdots & 0 & \tilde{C} & \tilde{B} \end{bmatrix}, \quad \widetilde{\mathscr{D}} = \operatorname{diag}(\tilde{D}, \ldots, \tilde{D}),$$

and $\widetilde{X} = (\tilde{x}_1', \ldots, \tilde{x}_T')'$. Appending the terminal condition (D.4) gives

$$\underbrace{\begin{bmatrix} \widetilde{\mathscr{M}} \\ (\cdots\ 0\ \ \widetilde{U}_2^\top\ \ \widetilde{U}_1^\top) \end{bmatrix}}_{\widetilde{\mathscr{M}}_T} \widetilde{X} = \underbrace{\begin{bmatrix} \widetilde{\mathscr{D}}E \\ 0 \end{bmatrix}}_{\widetilde{\mathscr{R}}_T}.$$

Lemma 1 (Uniqueness of the Bounded Counterfactual Path) *The terminal condition* (D.4) *selects a unique bounded solution* $\{\tilde{x}_t\}_{t=0}^T$ *to* (D.1).

Proof A QZ decomposition of $(\widetilde{\mathcal{N}}, \widetilde{\mathcal{S}})$ block-diagonalizes the dynamics into stable ($|\lambda| < 1$) and unstable ($|\lambda| > 1$) blocks. Boundedness requires the unstable component to be zero at T, which is exactly (D.4). This pins down $\widetilde{X}$ uniquely by backward recursion.

Interpretation as a Stable-Manifold Projection. The terminal restriction (D.4) is equivalent to projecting the terminal state $(\tilde{x}_T, \tilde{x}_{T-1})$ onto the stable manifold of the dynamical system. In the full model, the Euler equation selects exactly the linear combination of states that rules out explosive motion in the unstable eigenspace. When c_t is imposed exogenously, this selection mechanism is removed: the solution space to the reduced system (D.1) contains both stable and unstable components. Imposing $\widetilde{U}_1^\top \tilde{x}_T + \widetilde{U}_2^\top \tilde{x}_{T-1} = 0$ explicitly eliminates the unstable component at the terminal horizon and therefore enforces the

same equilibrium selection principle as in the full forward-looking system. Operationally, this is a projection onto the $m - k$ dimensional invariant stable subspace of the generalized eigenvalue system $(\widetilde{\mathcal{N}}, \widetilde{\mathcal{S}})$.