Insurability and Premia in Foreign Exchange Markets*

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Abstract

We propose a tractable small-open-economy framework in which uncovered interest parity premia on foreign exchange (FX) markets are related to the endogenous insurability of exchange rates. Asymmetric information about monetary policy can make the tails of the exchange rate distribution uninsurable in FX hedging markets, which in turn generates premia in FX spot markets. There is a role for government intervention because of an information sensitivity externality: agents do not internalize that their actions can cause more exchange rates to become uninsurable. The framework reveals that premia may be amplified by weaknesses in monetary, fiscal, and financial policy frameworks. Premia can be mitigated through macroeconomic policy reforms which reduce the sensitivity of exchange rates to private information. The constrained efficient policy remedies depend on country-specific characteristics such as the composition of lenders holding the external debt. Possible remedies include delegation to an "aloof central banker", constraints on fiscal policy, more active use of macroprudential tools, and institutions designed to publish or hide information.

JEL classification codes: E52, F31, F41, G14

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1 Introduction

Many countries, especially emerging and developing ones, have foreign exchange (FX) markets characterized by significant deviations from uncovered interest parity (UIP). These deviations, i.e., UIP *premia*, mean that exchange rates become de-anchored from monetary policy rates and expectations of economic activity while capital allocation is distorted by high external financing costs.[1] In some countries, these premia emerge and dissipate abruptly, while in other countries, they persist structurally. For policymakers, it is important to understand these premia, because the appropriate policy mix to handle them depend on why the premia arise to begin with.

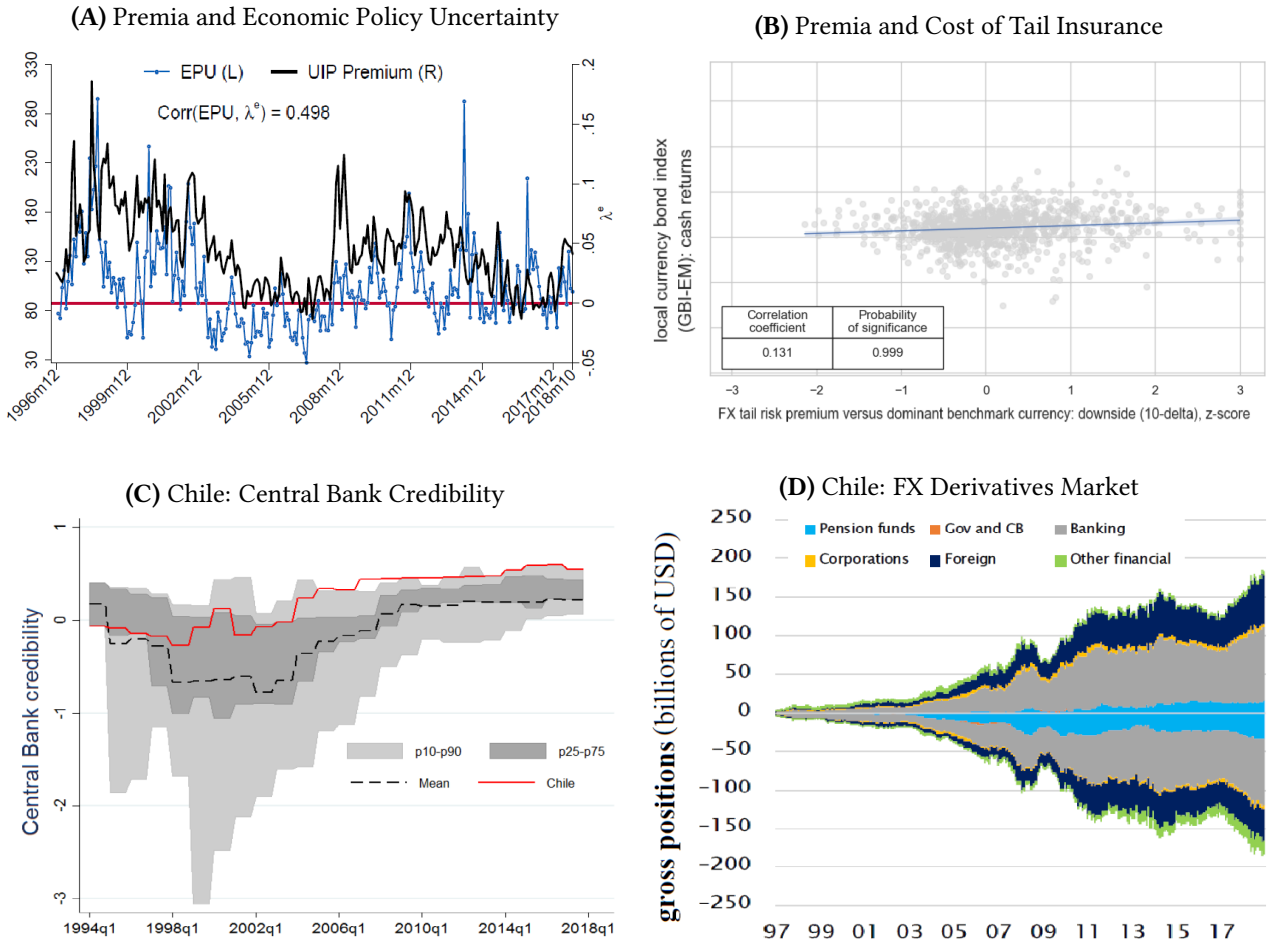
In this paper, we propose a tractable small-open-economy framework in which UIP premia are related to the *endogenous insurability* of exchange rates. In the model, the tails of the exchange rate distribution may become uninsurable because of asymmetric information in the FX hedging market, and this partial uninsurability may generate premia because of the risk aversion of external holders of local currency debt in the FX spot market. The relevant asymmetric information pertains to the country's monetary policy, because this policy affects the exchange rate distribution and its tails. Correspondingly, the constrained efficient policy mixes and institutional designs reduce premia by modifying the sensitivity of monetary policy to private information.

Our focus on insurability in the FX hedging market incorporates key elements of the practical experiences of emerging and developing countries while differentiating our work from other recent optimal policy models which do not include such a market. Recent seminal contributions by Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2021, 2023) assume that UIP premia depend on exchange rate volatility, which implies that premia can be reduced by mitigating that volatility.[2] Figure 1 plots empirical evidence with a contrasting take on what determines premia in emerging and developing countries. First, panel A illustrates the finding of Kalemli-Özcan and Varela (2021) that premia comove with policy uncertainty. Second, panel B illustrates that premia comove with a higher insurance cost for exchange rate tail risks, as suggested by the findings of Della Corte et al. (2016). Third, panels C and D show an example of an emerging market country which developed

[1] Evidence of UIP premia dates back to Fama (1984) and is summarized by Engel (2014) and Kalemli-Özcan and Varela (2021). Hassan and Zhang (2021) argue that these premia reduce capital allocation.

[2] These papers and ours follow the portfolio balance approach to FX markets, which dates back to Kouri (1976) and includes Cavallino (2019) and Fanelli and Straub (2021). Chernov et al. (2024) explore how premia can be modified by the ability to hedge using different sets of exogenously available financial assets from outside the FX market. Hassan et al. (2023) use a separate risk-based approach to propose that premia are related to the volatility of the exchange rate with respect to anchor currencies. Given the ambiguous cross-country empirical evidence, Basu et al. (2025) take a neutral stance and assume that premia are unaffected by exchange rate volatility.

Figure 1: Empirical Evidence on UIP Premia and FX Hedging Markets



Notes: **(A)** Kalemli-Özcan and Varela (2021). Economic Policy Uncertainty (EPU) index on left axis, UIP premium for 21 emerging markets on right axis. **(B)** Macrosynergy notebook using J.P. Morgan data. **(C)** Albagli et al. (2020). Central bank credibility index percentiles for 25 emerging markets and red line for Chile. **(D)** Albagli et al. (2020).

FX hedging markets over time by overhauling central bank and fiscal frameworks while increasing rather than reducing exchange rate volatility (Albagli et al., 2020). A clue to reconciling these findings comes from empirical analyses of FX market microstructure, which point to the existence of asymmetric information (e.g., Menkhoff et al., 2016; Ranaldo and Somogyi, 2021; Cespa et al., 2022). Our framework ties all this evidence together by showing that UIP premia can arise when policy-related asymmetric information causes uninsurability in exchange rate tails, and such problems can be remedied by improving policy frameworks irrespective of the degree of exchange rate volatility.

In the first part of the paper, we propose a general model which combines the portfolio balance approach with an information asymmetry inspired by the empirics of FX market microstructure. The model has two periods and two kinds of shocks. Monetary policy amounts to setting the exchange

rate in both periods. Premia are determined by the interaction of FX hedging and spot markets in the first period in anticipation of shocks. The FX hedging market comprises uninformed insurers and informed speculators who interact in a market with sticky insurance prices, while the FX spot market includes risk-averse external lenders who hold the local currency debt issued by domestic agents and insure the exchange rate volatility using FX hedges. In the second period, the shocks are realized and observable to all agents. The exchange rate can be set as a linear function of both shocks, but there is a key difference between the shocks: the *public* shock is not known to anyone in the first period, but the *private* shock is known to informed agents in that period.

Premia are generated as follows. First, the FX hedging market breaks down for a particular exchange rate if uninformed and informed insurers cannot agree on its probability. This problem occurs only if monetary policy sets exchange rates to depend on the private shock, and then only for exchange rates in the tails of the distribution. Exchange rates in the middle of the distribution can plausibly be generated by some combination of public and private shocks, so uninformed and informed agents can have different information sets but still agree on the probabilities of the exchange rates. However, exchange rates in the tails only occur when the private shock is especially small or large, so informed speculators know the probability of those exchange rates more accurately than uninformed insurers do. Uninformed insurers risk large losses when trading with informed speculators, so they will offer no contracts at such exchange rates.[3] Second, UIP premia can then arise on the FX spot market because if risk-averse external lenders cannot insure the tails of the exchange rate distribution, they charge a premium for bearing the risk of these tails themselves.

Our model suggests that premia arise structurally in countries whose monetary policy depend on shocks known beforehand only to a subset of FX hedging market participants, and may exhibit episodic surges when such shocks become more important or there is a risk of a regime shift that makes monetary policy more sensitive to such shocks. This insight may help explain the empirical connection between UIP premia and measures of policy uncertainty. It may also explain why the premia are especially large in emerging and developing countries, as they may be more susceptible to asymmetric information than advanced countries.

Since insurability is endogenous to macroeconomic policies, those policies should be altered to improve insurability and reduce premia. In our environment, there is an *information sensitivity externality*: when agents respond to the private shock in the second period, they do not internalize

[3]The above-cited FX market microstructure literature points to the existence of information asymmetry, with more informed agents making profits at the expense of less informed ones. We capture this information asymmetry in a tractable form by relating it to a specific shock and combining it with a sticky-price assumption that results in large profits and losses.

that their actions also cause the exchange rate to respond to that shock in that period, which in turn generates premia in the first period. From the planner's perspective, it may be optimal to reduce the sensitivity of the exchange rate function to the private shock.[4] One approach would be to pre-commit in the first period to a monetary policy such that the exchange rate function in the second period depends less on that shock. If pre-commitment is not possible, there may be a case for delegating monetary policy to an "aloof central banker" who responds to the public shock but down-weights the private shock in the second period.[5]

Our focus on endogenous insurability in the FX hedging market differentiates our policy recommendations from papers which assume that UIP premia depend on expected exchange rate volatility. Unlike in Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2021, 2023), the premia can be zero even with high exchange rate volatility, and the policy mix should remedy the lack of insurability in the tails of the exchange rate distribution rather than reduce exchange rate volatility per se.

In the second part of the paper, we explore four applications of the general model which each extract additional insights for the policy mix.

The first application links the magnitude of premia to the composition of external lenders and the level of external debt. We explore two options for modeling the foreign owners of the lenders. One option is that these lenders are owned by foreigners with log utility. If so, the premia on the FX market can become large if there are insurability problems and each lender's exposure to the country's external debt is large—which can arise if the set of lenders who specialize in the country's debt is small and/or the debt becomes large. The premia may also vary with how much of the debt is held by inframarginal investors (e.g., the "noise traders" in Gabaix and Maggiori, 2015). Another option is that the lenders are owned by foreigners who suffer financial constraints after large losses but not otherwise. If so, the premia on the FX market may be driven mostly by the upper tail of the exchange rate distribution (i.e., large depreciations), rather than by the lower tail.

The second application reveals that fiscal policy has a key influence on FX market premia. We tweak the general model to include public spending on imports in both periods, and we assume that the private shock in the model affects that spending in the second period. Fiscal policy affects premia via two channels. First, public spending in the first period increases external debt and thereby the

[4]This result evokes and yet contrasts with a separate literature on information sensitivity: while Dang et al. (2015, 2017) examine how private institutions can design securities to make them less sensitive to private information and thereby more liquidly tradable by uninformed agents, our framework takes as given that domestic agents issue local currency debt securities, and then it explores how the macroeconomic policy mix affects the sensitivity of the FX returns on that debt to private information: the less sensitive they are, the lower the premia.

[5]This case for monetary delegation is distinct from the inflation bias rationale in Rogoff (1985).

premia on that debt. Second, the dependence of public spending on the private shock in the second period generates macroeconomic volatility in that period, which needs to be addressed via monetary policy and thereby underlies the dependence of exchange rates on the private shock described above. To reduce premia, it may be desirable to discipline fiscal policy in the first period, and to pre-commit that fiscal policy in the second period will be less sensitive to the private shock.[6]

The third application explores how financial policies could be used in the presence of FX market premia. We consider a version of the general model in which the country's output in the second period depends on a financial shock which is known in advance by the informed speculators in the first period. We show that if macroprudential instruments are not available, the authorities should stabilize the economy by setting exchange rates to vary with the private financial shock in the second period. Next, we consider the possibility that the authorities can build an FX fund in the first period with which to offer liquidity injections in the second period. The authorities would use such a fund even with symmetric information. In the presence of asymmetric information, the fund and the injections could be larger, because exchange rates can then be made less sensitive to the private financial shock and premia can thereby be reduced.[7]

The fourth application addresses whether countries should set up institutions to publish information or keep it secret. We assume that before the two periods in the general model, the authorities can commit to one of three options: do nothing, or collect and publish all information related to the private shock, or instead hide that information from all insurers.[8] The first option achieves the outcome with partial FX market breakdown and positive premia. The other two options of publication or secrecy both achieve symmetric information in the first period, eliminating any breakdown in the FX hedging market and any premia in the FX spot market. Which of these options is preferred depends on their relative costs, which in practice depend on the kind of information. The authorities need to compare the option of doing nothing against the least costly manner of achieving zero premia.

Taken together, these four applications highlight that the development of FX markets and the

[6]This effect of fiscal policy on insurability constitutes another reason for disciplining fiscal policy beyond those in the literature on business cycle management and sovereign debt risks. In that literature, fiscal rules and institutions can help reduce public debt (e.g., Wyplosz, 2013; Halac and Yared, 2014; Hatchondo et al., 2022).

[7]This reasoning suggests that relative to recent models on the integrated use of financial policy tools alongside monetary policy (e.g., Farhi and Werning, 2016; Bianchi and Lorenzoni, 2022; Basu et al., 2025), the tools may have additional benefits (or costs) depending on whether they decrease (or increase) the sensitive of monetary policy to private information.

[8]We assume that the public and private shocks correspond to distinct underlying macroeconomic variables. This modeling is different from that in papers which consider public and private signals about the same underlying variable (e.g., global games as analyzed by Hellwig, 2002). However, in the language of these other papers, our option to collect and publish the private information can be interpreted as the production of a costly, zero-variance, and publicly-observable signal about the private shock which renders redundant the existing private signal about that shock.

behavior of premia depend on the country's use of policy instruments before and after shocks and the institutions that govern such use.

Next, we turn to the model-building and analysis. Section 2 lays out the general model. Section 3 extracts some over-arching results from this model. Section 4 then covers the four applications described above. Section 5 concludes. All the proofs of our results are in the appendix.

2 General Model

In this section, we introduce our stylized small open economy with flexible exchange rates, local currency debt, asset market segmentation, and asymmetric information in the FX market. It includes households, home goods firms, external lenders, insurers, and speculators.

2.1 Environment

Shocks and information structure. There are two periods $t \in \{1, 2\}$ and two shocks: a public shock φ and a private shock θ which follow independent uniform distributions over $[0, 1]$. At $t = 2$, both shocks are realized and observable to all agents. At $t = 1$, there are uninformed and informed agents with the respective information sets $\left\{\underline{\Omega}, \overline{\Omega}\right\}$. The value of φ is not known to anyone in this period, but the value of θ is known to the informed agents, i.e., $\underline{\Omega} = \{\varnothing\}$ and $\overline{\Omega} = \{\theta\}$. $\pi\left(\theta|\Omega_k\right)$ is the probability density function of the private shock θ given the information set Ω_k at $t = 1$, so $\pi\left(\theta|\underline{\Omega}\right) = 1$ for all θ while $\pi\left(\theta|\overline{\Omega}\right) = 1$ only for the known value of θ and $= 0$ for other values.

The lenders, insurers, and speculators have the respective information sets Ω_L, Ω_I, and Ω_S, where $\Omega_L = \Omega_I \subseteq \Omega_S$, i.e., the speculators' information set includes the same or more information than those of the others. All the other agents are uninformed, so we usually suppress the notation for their information sets. The shocks are not contractible, so no hedging contracts exist to directly insure them.

Monetary policy. The policy rate between the two periods is i_1. The exchange rates (in units of local currency per dollar) in the two periods are $\{\mathcal{E}_1, \mathcal{E}_2\left(\varphi, \theta\right)\}$, i.e., a single number at $t = 1$ and a function of both shocks at $t = 2$. We assume a linear function $\mathcal{E}_2 = \alpha\varphi + \beta\theta + \gamma$. We define Ψ as the set of exchange rates $\mathcal{E}_2$ at $t = 2$, while $\pi\left(\mathcal{E}_2|\Omega_k\right)$ is the probability density function of the exchange rate value $\mathcal{E}_2 \in \Psi$ given the information set Ω_k at $t = 1$. Correspondingly, $\pi\left(\mathcal{E}_2|\underline{\Omega}\right) = \int_{\theta=0}^{1} \pi\left(\mathcal{E}_2|\overline{\Omega}\right) d\theta$. The exchange rate is contractible, so FX hedging contracts may exist. We denote $\widetilde{\Psi}$ as the insurable set, i.e., the set of exchange rates for which such contracts exist, where $\widetilde{\Psi} \subseteq \Psi$.

Consumption and production. This part of the environment is adapted from Farhi and Werning (2016). Households maximize their welfare:

$$\max_{\{C_{Ht},C_{Ft},N_t\}} \mathbb{E}\left[\sum_{t=1}^{2} \delta^{t-1} U\left(C_{Ht}, C_{Ft}, N_t\right)\right]$$

$$\text{where } U\left(C_{Ht}, C_{Ft}, N_t\right) = \sigma_H \log C_{Ht} + \sigma_F \log C_{Ft} - N_t \text{ and } \sigma_F = 1 - \sigma_H,$$

subject to the budget constraint:

$$W_t N_t + \Pi_{Ht} + \mathcal{E}_t Y_{Ft} + T_t + D_{t+1} \geq P_H C_{Ht} + \mathcal{E}_t C_{Ft} + \left(1 + i_{t-1}\right) D_t.$$

On the income side of the constraint, W_t is the wage, N_t is labor supply, Π_{Ht} is the profit of home goods firms, Y_{Ft} is the endowment of tradable goods and is given by $\{Y_{F1}, Y_{F2}(\varphi, \theta)\}$, T_t is the lump-sum transfer from the planner, and D_{t+1} is the local currency debt at the end of period t. On the expenditure side, P_H and C_{Ht} are the local price and consumption of home goods, while C_{Ft} is the consumption of imports. We normalize the dollar prices of tradable endowments and imports to 1 and i_0 to 0. Debt can be non-zero to begin with but is extinguished at the end of time, i.e., $D_2 = 0$. Households have no access to insurance contracts.

Their first order conditions (FOCs) yield the following intratemporal and Euler conditions:

$$C_{Ht} = \frac{\sigma_H}{\sigma_F} \frac{\mathcal{E}_t}{P_H} C_{Ft}, W_t = \frac{1}{\sigma_F} \mathcal{E}_t C_{Ft} \text{ and } \frac{1}{\mathcal{E}_1 C_{F1}} = \delta\left(1 + i_1\right) \mathbb{E}\left[\frac{1}{\mathcal{E}_2 C_{F2}}\right].$$

Home goods firms are monopolistically competitive and set prices at the beginning of period $t = 1$, after which prices are fully rigid. Each firm produces a variety $j \in [0, 1]$ of the home good using the production technology $Y_{Ht}(j) = A_t N_t(j)$, where productivity is given by $\{A_1, A_2(\varphi, \theta)\}$. The home good aggregator, the variety-level demand function, and the aggregate price index are as follows:

$$Y_{Ht} = \left(\int_0^1 Y_{Ht}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}, Y_{Ht}(j) = Y_{Ht}\left(\frac{P_H(j)}{P_H}\right)^{-\varepsilon}, \text{ and } P_H = \left(\int_0^1 P_H(j)^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}},$$

where $\varepsilon > 1$. Labor is taxed at the pre-set rate ϕ. The variety is priced at $P_H(j)$ in local currency, and this price is set to maximize expected profits:

$$\max_{P_H(j)} \mathbb{E}\left[\sum_{t=1}^{2} s_t \left[P_H(j) Y_{Ht}(j) - (1 + \phi) W_t N_t(j)\right]\right],$$

where $s_t \equiv \delta^{t-1} \frac{\mathcal{E}_1 C_{F1}}{\mathcal{E}_t C_{Ft}}$ is the households' period-1 stochastic discount factor (SDF) for local currency income in period t (taken by firms as an exogenous profile across states). The FOC for price-setting

establishes the following expression:

$$P_H(j) = (1+\phi)\frac{\varepsilon}{\varepsilon-1}\frac{\mathbb{E}\left[\sum_{t=1}^{2}\delta^{t-1}\frac{1}{\mathcal{E}_t C_{Ft}}\frac{W_t}{A_t}Y_{Ht}\right]}{\mathbb{E}\left[\sum_{t=1}^{2}\delta^{t-1}\frac{1}{\mathcal{E}_t C_{Ft}}Y_{Ht}\right]}.$$

Since all firms are identical, $P_H = P_H(j)$, $Y_{Ht} = Y_{Ht}(j)$, and $N_t = N_t(j)$.

The labor market clears at the flexible wage W_t, and there is market clearing for home goods, i.e., $Y_{Ht} = C_{Ht}$.

FX markets. The FX spot and hedging markets are both open at $t = 1$. The FX spot market includes the above households who issue local currency debt and the external lenders who absorb that debt.

External lenders can undertake two kinds of financial transactions at $t = 1$. First, they can exchange currencies by taking a position of D_2 in local currency bonds financed by a position of $-\frac{D_2}{\mathcal{E}_1}$ in dollar bonds. Second, they have access to the available insurance contracts against exchange rate volatility. For each exchange rate $\mathcal{E}_2$ that lies within the insurable set $\widetilde{\Psi}$, the lenders can purchase a quantity $Q_L(\mathcal{E}_2)$ of contracts at price $p(\mathcal{E}_2)$ which each yield one dollar if that exchange rate materializes at $t = 2$. The lenders maximize the expected value of their dollar profits:

$$\max_{\{B_2, Q_L(\mathcal{E}_2)\}} \mathbb{E}\left[M_2\left(\left[\eta_2 - (1+i^*)\right]B_2 + Q_L(\mathcal{E}_2)\right)|\Omega_L\right]$$

$$\text{where } \eta_2 \equiv (1-\mu_1)(1+i_1)\frac{\mathcal{E}_1}{\mathcal{E}_2} \text{ and } B_{t+1} \equiv \frac{D_{t+1}}{\mathcal{E}_t},$$

subject to the insurance market constraint:

$$\int_{\mathcal{E}_2 \in \widetilde{\Psi}} p(\mathcal{E}_2) Q_L(\mathcal{E}_2)\, d\mathcal{E}_2 = 0.$$

M_2 is the period-1 SDF over dollar income at $t = 2$, η_2 is the dollar value of the return on local currency bonds for the external lenders, μ_1 is a tax on capital inflows set at $t = 1$, and i^* is the foreign interest rate. The SDF originates from the lenders' foreign owners. While it is not affected by the decisions of each lender, it is affected by the aggregate decisions of the set of lenders as well as other relevant equilibrium variables. Abusing notation, we assume that the SDF takes the form of a function $M_2 = M_2(B_2, Q_L(\mathcal{E}_2), \eta_2)$ defined over the lenders' aggregate decisions and the dollar return,[9] with the properties at each $\mathcal{E}_2$:

$$\frac{dM_2}{dQ_L(\mathcal{E}_2)} < 0, \lim_{Q_L(\mathcal{E}_2)\to\infty} M_2 = 0, \text{ and } \lim_{Q_L(\mathcal{E}_2)\to(Q_{\min})^+} M_2 = \infty \text{ for some } Q_{\min}.$$

[9] This assumption means that the SDF does not depend on the shocks (φ, θ) beyond their impact on the variables $\{B_2, Q_L(\mathcal{E}_2), \eta_2(\mathcal{E}_2)\}$.

Combining the SDF function with the lenders' FOCs for the quantity of insurance contracts and the bond position, we obtain the following expressions:

$$M_2 = \overline{M}\frac{(1+i^*)\,p\,(\mathcal{E}_2)}{\pi\,(\mathcal{E}_2|\Omega_L)} \text{ for all } \mathcal{E}_2 \in \widetilde{\Psi}, \text{ and } \mathbb{E}\left[M_2\left[\eta_2 - (1+i^*)\right]|\Omega_L\right] = 0,$$

where $\overline{M}$ is a positive constant.

The FX hedging markets include risk-neutral insurers and speculators alongside the external lenders described above. There is a continuum of markets, one for each exchange rate $\mathcal{E}_2 \in \Psi$. Each of these markets operates with the following structure within $t = 1$:

- Step 1. The insurer for this exchange rate first decides their participation, $d \in \{0, 1\}$. If $d = 1$, they propose a price $p\,(\mathcal{E}_2)$ and commit to sell the quantity $Q_I\,(\mathcal{E}_2)$ of contracts needed to fulfill all demands. If $d = 0$, they offer no price and do not fulfill any contracts. The insurer maximizes their expected profits:

  $$\max_d \left\{ \max_{p(\mathcal{E}_2)} \left[(1+i^*)\,p\,(\mathcal{E}_2)\,\mathbb{E}\left[Q_I\,(\mathcal{E}_2)\,|\Omega_I\right] - \mathbb{E}\left[\pi\,(\mathcal{E}_2|\Omega_S)\,Q_I\,(\mathcal{E}_2)\,|\Omega_I\right]\right], 0 \right\}.$$

  The first term of the inner maximand represents the insurer's expected revenues at $t = 1$, compounded with foreign interest, while the second term represents their expected payouts at $t = 2$. The expectation terms reflect that the insurer may be uncertain as to what the speculators will do in step 2, to the extent that the speculators have a superior information set $\Omega_S \supset \Omega_I$.

- Step 2. If $d = 1$, speculators and lenders observe $p\,(\mathcal{E}_2)$ and set their demands without observing each other's demands. Speculators purchase a quantity $Q_S\,(\mathcal{E}_2) \in \left[-\overline{Q_S}, \overline{Q_S}\right]$ of contracts to maximize their expected profits:

  $$\max_{Q_S(\mathcal{E}_2)} \left[\pi\,(\mathcal{E}_2|\Omega_S)\,Q_S\,(\mathcal{E}_2) - (1+i^*)\,p\,(\mathcal{E}_2)\,Q_S\,(\mathcal{E}_2)\right].$$

  The first term of the maximand represents speculators' expected revenues at $t = 2$, while the second term represents their expenses at $t = 1$, compounded with foreign interest. We take the limit as $\overline{Q_S} \to \infty$. Lenders purchase the quantitites $Q_L\,(\mathcal{E}_2)$ derived from the analysis above on the FX spot market. If $d = 0$, speculators and lenders cannot demand any contracts.

- Step 3. The market for insurance contracts clears, pinning down the quantity of contracts that the insurer must fulfill, i.e., $Q_I\,(\mathcal{E}_2) = Q_S\,(\mathcal{E}_2) + Q_L\,(\mathcal{E}_2)$. If $d = 1$, the exchange rate $\mathcal{E}_2$ is

insurable, i.e., $\mathcal{E}_2 \in \widetilde{\Psi}$. If $d = 0$, the exchange rate $\mathcal{E}_2$ is not insurable, so $p(\mathcal{E}_2) = \varnothing$ and $\{Q_I(\mathcal{E}_2), Q_S(\mathcal{E}_2), Q_L(\mathcal{E}_2)\} = 0$.

The outcome of this problem depends on the information sets of the insurer and speculators, and it determines the insurable set of exchange rates $\widetilde{\Psi}$. We solve it in the next subsection.

Lump sum rebates. The planner rebates to households in lump sum the revenues from taxes on labor and capital inflows:

$$T_t = \phi W_t N_t + \mu_{t-1}(1 + i_{t-1}) D_t.$$

The competitive equilibrium is defined as follows.

Definition 1 (Competitive equilibrium). *A competitive equilibrium for this economy comprises the quantities* $\left\{\{C_{Ht}, C_{Ft}, N_t, Y_{Ht}\}_{t=1}^2, D_2, \{Q_I(\mathcal{E}_2), Q_L(\mathcal{E}_2), Q_S(\mathcal{E}_2)\}_{\mathcal{E}_2 \in \Psi}\right\}$*, prices* $\left\{P_H, \{W_t\}_{t=1}^2, \{p(\mathcal{E}_2)\}_{\mathcal{E}_2 \in \Psi}\right\}$*, exchange rate sets* $\left\{\Psi, \widetilde{\Psi}\right\}$*, and probability distributions* $\left\{\pi(\mathcal{E}_2|\underline{\Omega}), \pi\left(\mathcal{E}_2|\overline{\Omega}\right)\right\}_{\mathcal{E}_2 \in \Psi}$ *that satisfy the optimization conditions and constraints of households, home goods firms, external lenders, insurers, and speculators, as well as the market clearing conditions for home goods, labor, local currency bonds, and FX hedging contracts, taking as given the SDF function* M_2 *and the values of the policy instruments* $\left\{\phi, i_1, \mu_1, \{\mathcal{E}_t\}_{t=1}^2\right\}$.

2.2 Insurability and Premia

The FX hedging market produces the following condition that must be satisfied for any exchange rate within the insurable set $\widetilde{\Psi}$. This result does not depend on the specific function $\mathcal{E}_2(\varphi, \theta)$.

Lemma 1 (Insurability condition). $\mathcal{E}_2 \in \widetilde{\Psi}$ *if and only if* $\pi(\mathcal{E}_2|\Omega_I) = \pi(\mathcal{E}_2|\Omega_S)$ *irrespective of* θ. *Within that set,* $\pi(\mathcal{E}_2|\Omega_I) = (1 + i^*) p(\mathcal{E}_2)$.

This lemma establishes that for an exchange rate to be insurable, the insurer and speculators need to agree on its probability—whether they have the same information set or not. The reason is that with the given price stickiness and market structure, the insurer makes large losses if speculators bet against them. The insurer can only avoid such betting if the speculators agree with the insurer on the probabilities.

We can establish the lemma by solving the FX hedging market backward. Here, alongside the general mathematical steps, we provide the argument assuming that $\pi(\mathcal{E}_2|\Omega_I) > 0$ and $p(\mathcal{E}_2) > 0$;

the proof in the appendix relaxes these inequalities. In step 2, speculators have the demands:

$$Q_S\left(\mathcal{E}_2\right)\begin{cases} = \overline{Q_S} & \text{if } \pi\left(\mathcal{E}_2|\Omega_S\right) > \left(1+i^*\right)p\left(\mathcal{E}_2\right) \\ \in \left(-\overline{Q_S}, \overline{Q_S}\right) & \text{if } \pi\left(\mathcal{E}_2|\Omega_S\right) = \left(1+i^*\right)p\left(\mathcal{E}_2\right) \\ = -\overline{Q_S} & \text{if } \pi\left(\mathcal{E}_2|\Omega_S\right) < \left(1+i^*\right)p\left(\mathcal{E}_2\right). \end{cases}$$

In step 1, the insurer's problem depends on the information structure. They know that each exchange rate value $\mathcal{E}_2 = \mathcal{E}_2\left(\varphi, \theta\right)$ can be the result of a combination of the public shock φ and the private shock θ. There are two cases, which we next analyze in turn.

The first case is that $\Omega_I = \Omega_S \in \left\{\underline{\Omega}, \overline{\Omega}\right\}$, i.e., the insurer and speculators are either both uninformed or both informed, irrespective of the specific value of θ. The insurer's problem is as follows:

$$\max_d \left\{ \max_{p(\mathcal{E}_2)} \left[\left(1+i^*\right)p\left(\mathcal{E}_2\right) - \pi\left(\mathcal{E}_2|\Omega_I\right)\right]\left[Q_S\left(\mathcal{E}_2\right) + Q_L\left(\mathcal{E}_2\right)\right], 0 \right\}.$$

Since the external lenders are risk averse, $Q_L\left(\mathcal{E}_2\right)$ is finite. Correspondingly, if $\overline{Q_S}$ is sufficiently large, the speculators' demand for insurance determines the sign of the square bracket on the right hand side. The insurer is willing to select $d = 1$ and earn zero profits by setting $p\left(\mathcal{E}_2\right)$ such that $\pi\left(\mathcal{E}_2|\Omega_I\right) = \pi\left(\mathcal{E}_2|\Omega_S\right) = \left(1+i^*\right)p\left(\mathcal{E}_2\right)$. At any other price, they would make losses.

The second case is that $\Omega_F = \underline{\Omega}$ and $\Omega_S = \overline{\Omega}$, i.e., the insurer is uninformed and speculators are informed. The insurer's problem can then be written as follows:

$$\max_d \left\{ \max_{p(\mathcal{E}_2)} \mathbb{E}\left[\begin{array}{c|c} \left[\left(1+i^*\right)p\left(\mathcal{E}_2\right) - \pi\left(\mathcal{E}_2|\Omega_S\right)\right] & \theta \in \{A, B, C\}, \\ \times\left[Q_S\left(\mathcal{E}_2\right) + Q_L\left(\mathcal{E}_2\right)\right] & \mathcal{E}_2 = \mathcal{E}_2\left(\varphi, \theta\right); \Omega_I \end{array} \right], 0 \right\},$$

where the regions A, B, and C partition the range of θ in the manner we explain next. If $\overline{Q_S}$ is sufficiently large, the speculators' demand for insurance again determines the sign of $Q_S\left(\mathcal{E}_2\right) + Q_L\left(\mathcal{E}_2\right)$. The insurer does not know θ when they set the price $p\left(\mathcal{E}_2\right)$ in step 1, but they know that θ can lie in one of three different regions. In region A, θ takes a value such that $\pi\left(\mathcal{E}_2|\Omega_S\right) > \left(1+i^*\right)p\left(\mathcal{E}_2\right)$, so the insurer makes losses. In region B, θ takes a value such that $\pi\left(\mathcal{E}_2|\Omega_S\right) = \left(1+i^*\right)p\left(\mathcal{E}_2\right)$, so the insurer can make zero profits. In region C, θ takes a value such that $\pi\left(\mathcal{E}_2|\Omega_S\right) < \left(1+i^*\right)p\left(\mathcal{E}_2\right)$, so the insurer again makes losses.

The upshot is that the insurer selects $d = 1$ if and only if they can set $p\left(\mathcal{E}_2\right)$ such that the condition $\pi\left(\mathcal{E}_2|\Omega_S\right) = \left(1+i^*\right)p\left(\mathcal{E}_2\right)$ holds with probability one. Since they do not know θ, they cannot set $p\left(\mathcal{E}_2\right)$ in such a manner if $\pi\left(\mathcal{E}_2|\Omega_S\right)$ varies with θ; instead, they will simply set $d = 0$. However, if $\pi\left(\mathcal{E}_2|\Omega_S\right)$ does not vary with θ, we obtain $\pi\left(\mathcal{E}_2|\Omega_I\right) = \pi\left(\mathcal{E}_2|\Omega_S\right)$ irrespective of θ, and the insurer can set $p\left(\mathcal{E}_2\right)$ to the desired value. In this case, the insurer and speculators agree on the probability

of this specific exchange rate value $\mathcal{E}_2$ despite having different information sets. In other words, the probability of the range of (φ, θ) that can jointly yield $\mathcal{E}_2$ happens to be equal to the probability of the set of φ that can yield $\mathcal{E}_2$, given any particular value of θ.

Next, we turn to the insurable set and premia associated with the exchange rate function $\mathcal{E}_2 = \alpha\varphi + \beta\theta + \gamma$. We define the UIP premium as $\rho \equiv \mathbb{E}[\eta_2] - (1 + i^*)$.

Proposition 1 (Insurability and premia). *Suppose that $\alpha > \beta \geq 0$. The external lenders' bond position satisfies:*

$$\begin{aligned} 0 = &\int_{\theta=0}^{1} \int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta} [\eta_2 - (1+i^*)]\, \pi(\theta|\Omega_I)\, d\varphi d\theta \\ &+ \int_{\theta=0}^{1} \int_{\varphi=0}^{\frac{\beta}{\alpha}(1-\theta)} m_2 [\eta_2 - (1+i^*)]\, \pi(\theta|\Omega_I)\, d\varphi d\theta \\ &+ \int_{\theta=0}^{1} \int_{\varphi=1-\frac{\beta}{\alpha}\theta}^{1} m_2 [\eta_2 - (1+i^*)]\, \pi(\theta|\Omega_I)\, d\varphi d\theta, \end{aligned}$$

where we define $m_2 \equiv \frac{M_2}{M}$. We can establish the following cases:

(a) $\Omega_I = \Omega_S \in \{\underline{\Omega}, \overline{\Omega}\}$ and/or $\beta = 0$. $\widetilde{\Psi} = \Psi$, so the probability of $\mathcal{E}_2 \in \widetilde{\Psi}$ is 1 and the premium is $\rho = 0$.

(i) If $\Omega_I = \Omega_S = \underline{\Omega}$, the insurable set is $\widetilde{\Psi} = [\gamma, \alpha + \beta + \gamma]$, while $m_2 = 1$ and $\pi(\theta|\Omega_I) = 1$.

(ii) If $\Omega_I = \Omega_S = \overline{\Omega}$, the insurable set is $\widetilde{\Psi} = [\beta\theta + \gamma, \alpha + \beta\theta + \gamma]$, while $m_2 = 1$ and the integrals over θ are removed because $\pi(\theta|\Omega_I) = 1$ only at the known value of θ and $= 0$ elsewhere.

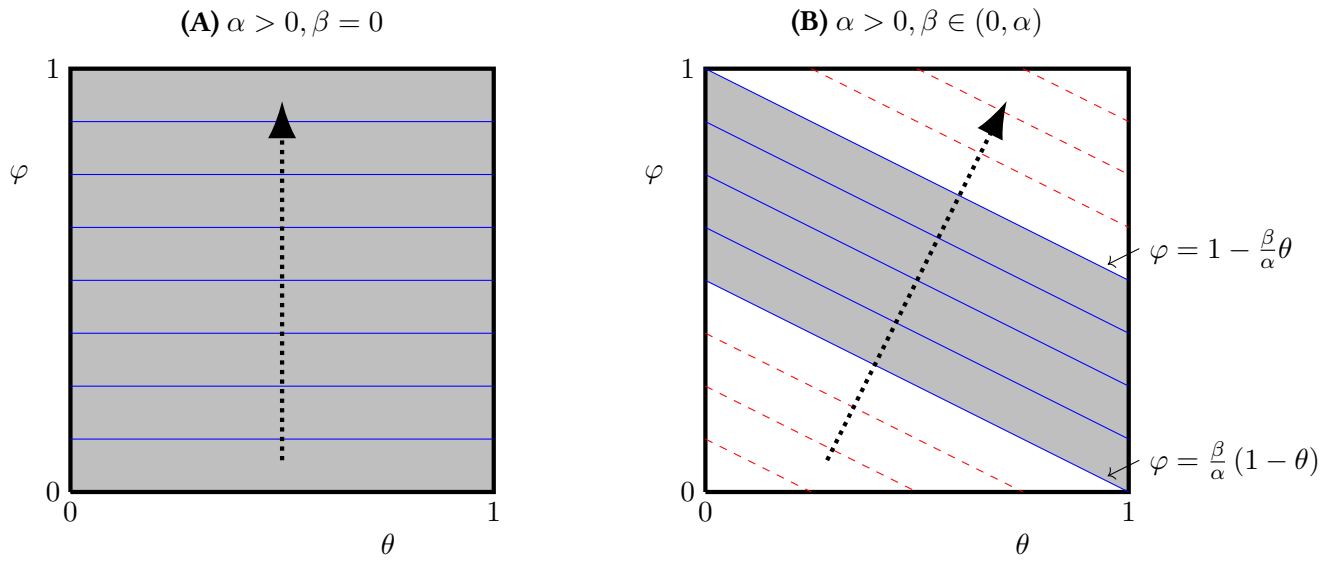
(iii) If $\Omega_I = \underline{\Omega}$, $\Omega_S = \overline{\Omega}$, and $\beta = 0$, the insurable set is $\widetilde{\Psi} = [\gamma, \alpha + \gamma]$, while $\pi(\theta|\Omega_I) = 1$ and the second and third terms of the above expression are zero.

(b) $\Omega_I = \underline{\Omega}$, $\Omega_S = \overline{\Omega}$, and $\beta > 0$. Exchange rates lie within $\Psi = [\gamma, \alpha + \beta + \gamma]$. The probability of $\mathcal{E}_2 \in \widetilde{\Psi}$ is $1 - \frac{\beta}{\alpha}$. The insurable set is $\widetilde{\Psi} = [\beta + \gamma, \alpha + \gamma]$, while $m_2 = m_2\left(B_2, \eta_2, \{\eta_2\}_{\mathcal{E}_2 \in \Psi}, \frac{\beta}{\alpha}\right)$ and $\pi(\theta|\Omega_I) = 1$. The upper and lower insurability contours are $\varphi = 1 - \frac{\beta}{\alpha}\theta$ and $\varphi = \frac{\beta}{\alpha}(1-\theta)$ respectively. The premium is $\rho = -\frac{Cov(m_2, \eta_2)}{\mathbb{E}[m_2]}$.

This proposition establishes that for premia to arise at $t = 1$, we require several ingredients: information must be asymmetric in the FX hedging market, the exchange rate must depend on the private shock at $t = 2$, and the external lenders in the FX spot market must dislike future exchange rate volatility. Exchange rate volatility alone is *not* sufficient for premia to arise.

We can illustrate the proposition using Figure 2. The figure plots the exchange rate contours as a function of φ on the vertical axis and θ on the horizontal axis, for $\alpha > \beta \geq 0$. Their slope is $-\frac{\beta}{\alpha}$. On panel A of the figure, $\beta = 0$ so the contours are horizontal and the exchange rate increases as

Figure 2: Exchange Rate Contours and Insurability



Note: The blue and red lines represent contours. The shaded region lies between the upper and lower contours $\varphi = 1 - \frac{\beta}{\alpha}\theta$ and $\varphi = \frac{\beta}{\alpha}(1 - \theta)$. The dotted arrow shows the direction in which the exchange rate is increasing.

we travel upwards. On panel B, $\beta > 0$ so the contours are downward-sloping and the exchange rate increases as we travel upwards and rightwards.

Owing to the tractability of the linear exchange rate function, we can partition the panels using the upper and lower contours given by $\varphi = 1 - \frac{\beta}{\alpha}\theta$ and $\varphi = \frac{\beta}{\alpha}(1 - \theta)$ respectively, and reflect that partition in the expression for the lenders' bond position shown in the proposition. The first term in that expression represents the shaded region between these upper and lower contours; within that region, we have plotted solid blue contours. That region covers the whole of panel A but is a parallelogram in panel B. The second and third terms represent the lower and upper triangular unshaded regions in the tails of the exchange rate distribution, and they are present only in panel B; within those regions, we have plotted dashed red contours.

How those regions give rise to insurability and premia depends on which case and subcase identified in the proposition is applicable.

Subcases (a) (i) and (ii) cover the symmetric information environment. Here, insurers and speculators agree on the probabilities of all exchange rates $\mathcal{E}_2$, whether panel A or B is relevant. From the previous lemma, it follows that there is full insurability, i.e., $\widetilde{\Psi} = \Psi$. From the lenders' FOC related to insurance, we obtain that the SDF $m_2 = 1$ for all $\mathcal{E}_2$ in all the regions of the panels. As a result, the premium ρ is zero *irrespective* of the level of exchange rate volatility given by the parameters (α, β). In subcase (a) (ii), θ is known to insurers and speculators at $t = 1$, so a smaller set of exchange rates

need to be insured.

By contrast, if there is asymmetric information, the outcome depends on whether panel A or B is relevant.

Subcase (a) (iii) applies if $\beta = 0$ as on panel A. The private information of speculators does not give them superior information about the exchange rate. Accordingly, insurers and speculators continue to agree on the probabilities of all exchange rates, and there is full insurability. We again obtain $m_2 = 1$ in the shaded region from the lenders' FOC, and there are no other regions to consider.

Case (b) applies if $\beta > 0$ as on panel B, and there is partial insurability because the tails of the exchange rate distribution are uninsurable. Insurers and speculators continue to agree on the probabilities of the exchange rates in the shaded region (i.e., this region is the insurable set $\widetilde{\Psi}$, with $m_2 = 1$ within that set coming from the lenders' FOC), but *not* in the two unshaded regions. All the exchange rates in the shaded region can occur for some value of θ, and each value of θ is equally likely. Correspondingly, while the private information of speculators is material (i.e., given θ, they can understand which values of φ need to occur for any particular exchange rate to be achieved), it does not affect their assessed probability of any exchange rate within the region. As a result, while insurers are aware that speculators are more informed, they are still willing to participate in the market. However, the exchange rates in the two unshaded regions are not insurable because insurers know that such exchange rates require the value of θ to be at one extreme and cannot occur for values of θ at the other extreme. Therefore, speculators who know that θ is at one extreme or another can bet against the insurers and make profits at their expense. As a result, insurers are not willing to participate in the market.

Partial insurability may give rise to premia. The premium ρ depends on the covariance of m_2 with the dollar return η_2. Since the lenders' FOC pins down $m_2 = 1$ within the insurable set, any nonzero covariance must be generated from the uninsurable tails of the exchange rate distribution. The value of m_2 in those regions originates from the lenders' foreign owners and is not pinned down by the lenders' FOC. On the one hand, if $m_2 = 1$ also happens to hold in those tails, the covariance and premium are both zero. On the other hand, if external lenders generally have a higher SDF m_2 during combinations of shocks where the exchange rate is more depreciated and the dollar return η_2 is lower, the covariance is negative and the premium is positive.

The function $m_2\left(B_2, \eta_2, \{\eta_2\}_{\mathcal{E}_2 \in \Psi}, \frac{\beta}{\alpha}\right)$ is derived from the definition $m_2 \equiv \frac{M_2}{M}$ and can be explained as follows. The arguments $\{B_2, \eta_2\}$ are derived from the function $M_2\left(B_2, Q_L\left(\mathcal{E}_2\right), \eta_2\right)$ described above. The arguments $\left\{\{\eta_2\}_{\mathcal{E}_2 \in \Psi}, \frac{\beta}{\alpha}\right\}$ are needed to determine $\overline{M}$, as the latter depends on

the set of dollar returns η_2 within the insurable set $\widetilde{\Psi} \subset \Psi$ whose boundaries are given by $\frac{\beta}{\alpha}$.

This proposition can be used alongside the definition of the competitive equilibrium above to characterize outcomes as a function of any given information structure and monetary policy.

3 Constrained Efficient Policies

In this section, we show how concerns related to insurability and premia alter the constrained efficient monetary policy and consumption levels.

3.1 Planner Problem

We assume that the planner's information set is identical to that of the insurers. The constrained efficient allocation is defined as follows.

Definition 2 (Constrained efficient allocation). *A constrained efficient allocation comprises the quantities* $\left\{\{C_{Ht}, C_{Ft}, N_t, Y_{Ht}\}_{t=1}^{2}, D_2, \{Q_I(\mathcal{E}_2), Q_L(\mathcal{E}_2), Q_S(\mathcal{E}_2)\}_{\mathcal{E}_2 \in \Psi}\right\}$*, prices* $\left\{P_H, \{W_t\}_{t=1}^{2}, \{p(\mathcal{E}_2)\}_{\mathcal{E}_2 \in \Psi}\right\}$*, exchange rate sets* $\left\{\Psi, \widetilde{\Psi}\right\}$*, and probability distributions* $\left\{\pi(\mathcal{E}_2|\underline{\Omega}), \pi\left(\mathcal{E}_2|\overline{\Omega}\right)\right\}_{\mathcal{E}_2 \in \Psi}$ *that maximize household welfare under full commitment, subject to the restriction that the allocation can be implemented via policy instruments* $\left\{\phi, i_1, \mu_1, \{\mathcal{E}_t\}_{t=1}^{2}\right\}$ *in a competitive equilibrium, where* $\mathcal{E}_2 = \alpha\varphi + \beta\theta + \gamma$ *for some* $\{\alpha, \beta, \gamma\}$.

Following Farhi and Werning (2016) and Basu et al. (2025), we define two wedges which summarize the distance of an allocation from the frictionless first-best frontier:

$$\tau_{Ht} \equiv 1 + \frac{1}{A_t}\frac{U_{Nt}}{U_{Ht}} = 1 - \frac{1}{A_t}\frac{C_{Ht}}{\sigma_H} \text{ for } t \in \{1, 2\}$$

$$\tau_{\Gamma 2} \equiv \mathbb{E}\left[\left[\eta_2 - (1 + i^*)\right]\frac{\sigma_F}{C_{F2}}\right].$$

The aggregate demand (AD) wedges $\{\tau_{Ht}\}_{t=1}^{2}$ compare the consumption demand for home goods against the labor disutility from the production of those goods. The UIP wedge $\tau_{\Gamma 2}$ measures the expected marginal utility of the consumption loss owing to the premia paid by domestic agents to external lenders.

The planner's period-t utility function V can be written in the following form to incorporate the households' intratemporal FOCs:

$$V\left(C_{Ft}, \frac{\mathcal{E}_t}{P_H}\right) \equiv U\left(\frac{\sigma_H}{\sigma_F}\frac{\mathcal{E}_t}{P_H}C_{Ft}, C_{Ft}, \frac{1}{A_t}\frac{\sigma_H}{\sigma_F}\frac{\mathcal{E}_t}{P_H}C_{Ft}\right),$$

It produces the following partial derivatives:

$$\frac{\partial V}{\partial C_{Ft}} = \frac{\sigma_F}{C_{Ft}}\left[1 + \frac{\sigma_H}{\sigma_F}\tau_{Ht}\right] \text{ and } \frac{\partial V}{\partial\left(\frac{\mathcal{E}_t}{P_H}\right)} = \sigma_H \frac{P_H}{\mathcal{E}_t}\tau_{Ht}.$$

We also define the auxiliary external repayment variable $\chi \equiv (1-\mu_1)(1+i_1)\mathcal{E}_1$ $(= \eta_2 \mathcal{E}_2)$, which combines several policy settings from $t=1$ and does not depend on the shocks (φ, θ).

Given these definitions, we can write the planner problem as follows:

$$V^P = \max_{\{C_{F1}, C_{F2}, \mathcal{E}_1, \alpha, \beta, \gamma, \chi, P_H\}} V\left(C_{F1}, \frac{\mathcal{E}_1}{P_H}\right) + \delta \int_{\theta=0}^{1}\int_{\varphi=0}^{1} V\left(C_{F2}, \frac{\mathcal{E}_2}{P_H}\right)\pi\left(\theta|\Omega_I\right) d\varphi d\theta \tag{1}$$

subject to the constraints:

$$B_1(1+i^*) = (Y_{F1} - C_{F1}) + (Y_{F2} - C_{F2})\frac{\mathcal{E}_2}{\chi} \text{ for all } (\varphi, \theta) \tag{2}$$

$$\begin{aligned} 0 = &\int_{\theta=0}^{1}\int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta}\left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right]\pi\left(\theta|\Omega_I\right) d\varphi d\theta \\ &+ \int_{\theta=0}^{1}\int_{\varphi=0}^{\frac{\beta}{\alpha}(1-\theta)} m_2\left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right]\pi\left(\theta|\Omega_I\right) d\varphi d\theta \\ &+ \int_{\theta=0}^{1}\int_{\varphi=1-\frac{\beta}{\alpha}\theta}^{1} m_2\left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right]\pi\left(\theta|\Omega_I\right) d\varphi d\theta \end{aligned} \tag{3}$$

$$\mathcal{E}_2 = \alpha\varphi + \beta\theta + \gamma. \tag{4}$$

The planner problem captures the public and private shocks that strike the economy and reveals the existence of three externalities. Each combination of shocks (φ, θ) constitutes a state of nature at $t=2$, and the variables C_{F2} and $\mathcal{E}_2$ may be set to depend on these shocks.

The utility function (1) incorporates the productivity level $A_2(\varphi, \theta)$, which may depend on both shocks at $t=2$. There is an *AD externality*:[10] households do not internalize the impact of their individual consumption decisions on the AD wedges $\{\tau_{Ht}\}_{t=0}^{2}$.

There is a continuum of resource constraints (2), one for each state of nature, obtained by combining the households' budget constraints with the market clearing conditions and the expression for the lump sum rebates. For $t=1$, we fix the dollar value of repayments on inherited external debt to $B_1(1+i^*)$. At $t=2$, the endowment of tradable goods $Y_{F2}(\varphi, \theta)$ may depend on both shocks. Since all external debt is in local currency, any variation in $\mathcal{E}_2$ across states alters the dollar value of repayments on external debt B_2 in those states, which in turn affects the level of imports C_{F2}.

[10] As described in Farhi and Werning (2016), such an externality naturally emerges in the New Keynesian literature on desirable monetary policies.

The premium constraint (3) aggregates the repayments on external debt across states of nature at $t = 2$ to ensure that external lenders receive the necessary premium to hold that external debt at $t = 1$. The constraint comes from the FX spot and hedging markets as summarized in Proposition 1. If case (a) of Proposition 1 applies, there is no premium. If case (b) applies, there may be a *local currency (LC) premium externality*:[11] households do not internalize that their individual borrowing decisions may affect economywide premia and thereby the UIP wedge $\tau_{\Gamma 2}$. The mechanism is that their borrowing affects B_2, which in turn affects m_2 in the uninsurable regions of exchange rates.

The exchange rate function (4) relates $\mathcal{E}_2$ to the public and private shocks. Combined with the premium constraint (3), it reveals that in our environment, there may be a novel *information sensitivity (IS) externality*: when agents respond to the shocks (φ, θ) at $t = 2$, they do not internalize that their individual actions may affect monetary policy and exchange rates in a manner that generates premia. Moreover, even an optimizing planner may not be able to avoid these premia, as they take the agents' optimization conditions as given. If the planner is induced to set β above zero, such that $\mathcal{E}_2$ responds to the private shock at $t = 2$, and if the information structure features $\Omega_I = \underline{\Omega}$ and $\Omega_S = \overline{\Omega}$, there may be a switch from case (a) to case (b) of Proposition 1. Within case (b), any decision by the planner to alter the parameters $\{\alpha, \beta\}$ affects the boundaries of the insurable set and the resulting premium. The premium also varies with any changes in the parameters $\{\alpha, \beta, \gamma\}$ affecting m_2 and/or $\frac{\chi}{\mathcal{E}_2}$ in the uninsurable regions.

The households' Euler condition and the firms' price-setting equation do not appear in the planner problem above, but remain operational in the background. Once the planner selects the variables indicated above, the households' Euler condition enables us to back out the implicit policy rate i_1 and capital inflow tax μ_1, while the price-setting equation determines the implicit labor tax ϕ.

3.2 First Order Conditions

We solve the planner problem via the first order approach and assume that the solution satisfies $\alpha > \beta \geq 0$.[12] In this subsection, we rearrange the FOCs to derive four insights for the constrained efficient allocation. We use these insights to construct some general results in subsection 3.3 and explore them further via specific applications in section 4.

[11] Such an externality naturally emerges when consider desirable policies in papers focusing on the portfolio balance approach to FX markets (e.g., Cavallino, 2019; Fanelli and Straub 2021; and Itskhoki and Mukhin, 2023).

[12] The first-order approach assumes that the problem has a unique interior solution for all variables and that the FOCs establish this welfare maximum. Appendix B contains more details on the algebraic steps, including the full set of FOCs and how to amend them when $\beta > \alpha \geq 0$. The FOC for P_H turns out to be redundant, so we normalize $P_H = 1$ throughout the remainder of the paper. For purposes of comparison, the appendix also includes a specification where there are no premia and the exchange rate function is unrestricted.

The first insight is that the AD wedge is zero at $t = 1$:

$$\tau_{H1} = 0. \tag{5}$$

This result comes directly from the FOC for $\mathcal{E}_1$. It means that irrespective of the level of import consumption C_{F1}, the planner uses exchange rate flexibility to set C_{H1} to the value that eliminates the AD externality and its associated wedge in that period.

The second insight is that the level of the external repayment variable χ is set in a manner that internalizes AD and IS externalities:

$$B_2 \int_{\theta=0}^{1} \int_{\varphi=0}^{1} \frac{1}{\mathcal{E}_2} \frac{\delta \sigma_F}{C_{F2}} \left[1 + \frac{\sigma_H}{\sigma_F} \tau_{H2}\right] \pi\left(\theta|\Omega_I\right) d\varphi d\theta = \Lambda \left[\int_{\theta=0}^{1} \int_{\varphi=0}^{1} \frac{1}{\mathcal{E}_2} \pi\left(\theta|\Omega_I\right) d\varphi d\theta + z_\chi\right] \tag{6}$$

$$\text{where } B_2 = B_1\left(1 + i^*\right) + \left(C_{F1} - Y_{F1}\right) = \left(Y_{F2} - C_{F2}\right) \frac{\mathcal{E}_2}{\chi}$$

$$z_\chi = \begin{cases} \int_{\theta=0}^{1} \int_{\varphi=0}^{\frac{\beta}{\alpha}(1-\theta)} \left\{ \left[m_2 - 1\right] \frac{1}{\mathcal{E}_2} + \frac{\partial m_2}{\partial \chi} \left[\frac{\chi}{\mathcal{E}_2} - \left(1 + i^*\right)\right] \right\} \pi\left(\theta|\Omega_I\right) d\varphi d\theta \\ + \int_{\theta=0}^{1} \int_{\varphi=1-\frac{\beta}{\alpha}\theta}^{1} \left\{ \left[m_2 - 1\right] \frac{1}{\mathcal{E}_2} + \frac{\partial m_2}{\partial \chi} \left[\frac{\chi}{\mathcal{E}_2} - \left(1 + i^*\right)\right] \right\} \pi\left(\theta|\Omega_I\right) d\varphi d\theta, \end{cases}$$

and Λ is the multiplier on the premium constraint (3). This equation comes from combining the FOCs for χ and C_{F2}. The equation reveals that Λ is non-zero only if the external debt B_2 is also non-zero. If $B_2 = 0$, changes in the external repayment variable χ do not affect C_{F2} via the resource constraints (2), so χ is set solely to make the premium constraint (3) non-binding, i.e., $\Lambda = 0$. Instead, if $B_2 \neq 0$, changes in χ affect C_{F2} and thereby welfare.

If we remove the externalities so that $\tau_{H2} = z_\chi = 0$, equation (6) is simplified. In this case, the equation shows that χ is set to balance two considerations: the left hand side reflects that an increase in χ causes a decrease in C_{F2} if the economy has positive external debt, i.e., $B_2 > 0$, while the right hand side reflects that such an increase may be needed to generate sufficient returns for external lenders to hold the debt at $t = 1$.

However, externalities generally exist and yield additional considerations. AD externalities at $t = 2$ are captured by the non-zero wedges τ_{H2} on the left hand side of equation (6), reflecting that changes in C_{F2} can induce inefficient changes in these wedges. The IS externality is captured by the non-zero term z_χ on the right hand side of the equation. This term incorporates that an increase in χ has additional effects on the SDF-weighted returns of external lenders because of the uninsurable regions of the exchange rate distribution. In the definition of the variable z_χ, the two rows capture the two uninsurable regions. Within each row, the first term reflects that there is an increase in

external repayments in regions where m_2 may differ from 1, while the second term reflects that the values of m_2 within these regions may themselves change.

The third insight is that the levels of import consumption C_{Ft} are set in a manner that internalizes AD and LC premium externalities:

$$\frac{\sigma_F}{C_{F1}} = \int_{\theta=0}^{1}\int_{\varphi=0}^{1} \frac{\chi}{\mathcal{E}_2}\frac{\delta\sigma_F}{C_{F2}}\left[1+\frac{\sigma_H}{\sigma_F}\tau_{H2}\right]\pi\left(\theta|\Omega_I\right)d\varphi d\theta + z_{F1} \tag{7}$$

$$\text{where } z_{F1} = -\Lambda\left[\begin{array}{c}\int_{\theta=0}^{1}\int_{\varphi=0}^{\frac{\beta}{\alpha}(1-\theta)}\frac{\partial m_2}{\partial B_2}\left[\frac{\chi}{\mathcal{E}_2}-(1+i^*)\right]\pi\left(\theta|\Omega_I\right)d\varphi d\theta \\ +\int_{\theta=0}^{1}\int_{\varphi=1-\frac{\beta}{\alpha}\theta}^{1}\frac{\partial m_2}{\partial B_2}\left[\frac{\chi}{\mathcal{E}_2}-(1+i^*)\right]\pi\left(\theta|\Omega_I\right)d\varphi d\theta\end{array}\right].$$

This equation comes from combining the FOCs for C_{F1} and C_{F2}.

If we remove the externalities so that $\tau_{H2} = z_{F1} = 0$, equation (7) is simplified. In this case, the equation shows that the profile of imports C_{Ft} over time is determined solely by the expected dollar value of repayments on external debt, i.e., $\frac{\chi}{\mathcal{E}_2}$.

Externalities add terms to the right hand side of the equation. AD externalities at $t = 2$ are captured by the non-zero wedges τ_{H2}. The LC premium externality is captured by the non-zero term z_{F1}. This term incorporates that an increase in C_{F1} causes an increase in B_2 and thereby changes in m_2 in the uninsurable regions of the exchange rate distribution.

The fourth insight is that the parameters $\{\alpha, \beta, \gamma\}$ in the exchange rate function are set in a manner that internalizes AD and IS externalities. For each parameter $\omega \in \{\alpha, \beta, \gamma\}$, we obtain:

$$\begin{aligned}&\int_{\theta=0}^{1}\int_{\varphi=0}^{1}\frac{\partial\mathcal{E}_2}{\partial\omega}\left\{\frac{1}{\mathcal{E}_2}\delta\sigma_H\tau_{H2} + B_2\frac{\chi}{\left(\mathcal{E}_2\right)^2}\frac{\delta\sigma_F}{C_{F2}}\left[1+\frac{\sigma_H}{\sigma_F}\tau_{H2}\right]\right\}\pi\left(\theta|\Omega_I\right)d\varphi d\theta \\ &= \Lambda\left[\int_{\theta=0}^{1}\int_{\varphi=0}^{1}\frac{\partial\mathcal{E}_2}{\partial\omega}\frac{\chi}{\left(\mathcal{E}_2\right)^2}\pi\left(\theta|\Omega_I\right)d\varphi d\theta + z_{\omega1} + z_{\omega2}\right]\end{aligned} \tag{8}$$

$$\text{where } z_{\omega1} = \left\{\begin{array}{c}\int_{\theta=0}^{1}\int_{\varphi=0}^{\frac{\beta}{\alpha}(1-\theta)}\left\{\left[m_2-1\right]\frac{\partial\mathcal{E}_2}{\partial\omega}\frac{\chi}{\left(\mathcal{E}_2\right)^2}-\frac{\partial m_2}{\partial\omega}\left[\frac{\chi}{\mathcal{E}_2}-(1+i^*)\right]\right\}\pi\left(\theta|\Omega_I\right)d\varphi d\theta \\ +\int_{\theta=0}^{1}\int_{\varphi=1-\frac{\beta}{\alpha}\theta}^{1}\left\{\left[m_2-1\right]\frac{\partial\mathcal{E}_2}{\partial\omega}\frac{\chi}{\left(\mathcal{E}_2\right)^2}-\frac{\partial m_2}{\partial\omega}\left[\frac{\chi}{\mathcal{E}_2}-(1+i^*)\right]\right\}\pi\left(\theta|\Omega_I\right)d\varphi d\theta\end{array}\right.$$

$$z_{\omega2} = \frac{\partial\left(-\frac{\beta}{\alpha}\right)}{\partial\omega}\int_{\theta=0}^{1}\left\{\begin{array}{c}(1-\theta)\left[\lim_{\varphi\to\left(\frac{\beta}{\alpha}(1-\theta)\right)^-} m_2-1\right]\left[\frac{\chi}{\beta+\gamma}-(1+i^*)\right] \\ +\theta\left[\lim_{\varphi\to\left(1-\frac{\beta}{\alpha}\theta\right)^+} m_2-1\right]\left[\frac{\chi}{\alpha+\gamma}-(1+i^*)\right]\end{array}\right\}\pi\left(\theta|\Omega_I\right)d\theta.$$

This equation comes from combining the FOCs for each parameter and C_{F2}.

If we remove the externalities so that $\tau_{H2} = z_{\omega 1} = z_{\omega 2} = 0$, equation (8) is simplified and has a similar interpretation to that of equation (6). Any change in the parameters $\{\alpha, \beta, \gamma\}$ causes movements in the exchange rates $\mathcal{E}_2$ across all states (φ, θ) at $t = 2$. The left hand side of equation (8) contains just one term, and this term reflects that these movements in $\mathcal{E}_2$ affect C_{F2} via the resource constraints (2) if $B_2 \neq 0$. The right hand side of the equation reflects that the movements in $\mathcal{E}_2$ affect whether there are sufficient returns for external lenders to hold the debt at $t = 1$.

Again, externalities generally exist and yield additional considerations.

AD externalities at $t = 2$ are captured by the non-zero wedges τ_{H2} in the two terms on the left hand side of equation (8). The first term reflects that irrespective of the level of B_2, $\mathcal{E}_2$ directly affects τ_{H2} through the households' intratemporal FOCs, while the second term reflects that if $B_2 \neq 0$, there is an additional effect of $\mathcal{E}_2$ on C_{F2}, and thereby on τ_{H2}, via the resource constraints (2).

The first term capturing the IS externality is the non-zero term $z_{\omega 1}$ on the right hand side of equation (8). This term incorporates that any change in the parameters $\{\alpha, \beta, \gamma\}$ has additional effects on the SDF-weighted returns of external lenders because of the uninsurable regions of the exchange rate distribution. In the definition of $z_{\omega 1}$, the first term in each row reflects that there are movements in exchange rates $\mathcal{E}_2$ in regions where m_2 may differ from 1, while the second term reflects that the values of m_2 within these regions may themselves change.

The second term capturing the IS externality is the term $z_{\omega 2}$ on the right hand side of the equation. This term is non-zero only for the parameters $\{\alpha, \beta\}$ because these parameters determine the boundaries of the insurable set of exchange rates $\widetilde{\Psi}$. In the definition of $z_{\omega 2}$, the term in front of the integral captures the effect of varying the parameters on the slope and size of the insurable set. On Figure 2, the slope of the upper and lower contours are $-\frac{\beta}{\alpha}$. Correspondingly, an increase in β makes the slopes steeper and shrinks the insurable set from below and above. Moreover, if $\beta > 0$ to begin with, a decrease in α has a similar effect on the insurable set. The result is an expansion of the two uninsurable regions. The terms inside the integral capture that in the premium constraint (3), the expansion of the uninsurable regions means that more of the returns to external lenders are multiplied by their uninsured SDFs. Consequently, the premium ρ needed to satisfy the constraint may change.

3.3 Policy Impact of Uninsurability

The externalities in equations (6)-(8), and therefore the constrained efficient policy mix, depend on which case and subcase from Proposition 1 applies. Different cases may apply for different countries or for different episodes within the same country.

Lemma 2 (Effective information symmetry). *For all subcases within case (a) from Proposition 1, the constrained efficient allocations satisfy constraints (2)-(4) and equations (5)-(8) with the settings* $z_\chi = z_{F1} = z_{\omega 1} = z_{\omega 2} = 0$.

This result establishes that the LC premium and IS externalities are zero in case (a) because of full insurability, i.e., $\widetilde{\Psi} = \Psi$. Subcases (a) (i) and (ii) feature full insurability arising from the symmetric information environment, irrespective of the value of β. They differ in terms of the probability density function $\pi(\theta|\Omega_I)$. Subcase (a) (iii) occurs when there is asymmetric information but the planner happens to choose $\beta = 0$. It features full insurability because the private information of speculators does not give them superior information about the exchange rate.

The next lemma identifies whether the constrained efficient allocations selected for the subcases with a symmetric information structure remain feasible when the information structure becomes asymmetric at $t = 1$. The information structure has no effect on the resource constraints (2). Accordingly, whether the constrained efficient allocations remain feasible depends on whether the premium constraint (3) continues to be satisfied.

Lemma 3 (Information sensitivity shock). *Consider a change from* $\Omega_I = \Omega_S \in \left\{\underline{\Omega}, \overline{\Omega}\right\}$ *to* $\Omega_I = \underline{\Omega}$ *and* $\Omega_S = \overline{\Omega}$. *The premium* ρ *from the constrained efficient allocation prior to the change in the information structure is too low to satisfy constraint (3) after the change in the information structure if and only if the following condition is satisfied:*

$$
\begin{aligned}
0 > & \int_{\theta=0}^{1} \int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta} \left[1-\pi\left(\theta|\Omega_I\right)\right] \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] d\varphi d\theta \\
& + \int_{\theta=0}^{1} \int_{\varphi=0}^{\frac{\beta}{\alpha}(1-\theta)} \left[m_2^{(b)} - \pi\left(\theta|\Omega_I\right)\right] \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] d\varphi d\theta \\
& + \int_{\theta=0}^{1} \int_{\varphi=1-\frac{\beta}{\alpha}\theta}^{1} \left[m_2^{(b)} - \pi\left(\theta|\Omega_I\right)\right] \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] d\varphi d\theta, \qquad (9)
\end{aligned}
$$

where the $m_2^{(b)}$ *function corresponds to case (b) but all the variable values correspond to the allocation prior to the change in the information structure.*

This lemma allows us to identify three possible scenarios arising from the change in the information structure.

The first scenario is that the country begins in subcase (a) (i) and the planner selects $\beta = 0$ in the constrained efficient allocation for that subcase. The change from symmetric to asymmetric information involves speculators becoming more informed than they were before, while external lenders

and insurers remain uninformed. The change moves the country to subcase (a) (iii). It does not violate the premium constraint (3) because the entire exchange rate distribution remains insurable at the previous variable values, i.e., $\widetilde{\Psi} = \Psi$ and $m_2 = 1$ in all states of nature (φ, θ).

The second scenario is that the country begins in subcase (a) (i) and the planner selects $\beta > 0$. The change in the information structure is now important for the constrained efficient allocation because it results in uninsurable regions of the exchange rate distribution, i.e., $\widetilde{\Psi} \subset \Psi$. The country moves to case (b). Whether the premium ρ from subcase (a) (i) is sufficient to satisfy the premium constraint (3) in case (b) depends on the values of $m_2^{(b)}$ in the uninsurable regions, as these values are no longer necessarily equal to one with partial insurability.

Condition (9) is the criterion for the premium ρ to no longer be sufficient. For this second scenario where the country starts in subcase (a) (i), we set $\pi(\theta|\Omega_I) = 1$ for all θ prior to the change in the information structure, so the first term on the right hand side of the condition is zero and only the second and third terms may be non-zero. They correspond to the uninsurable regions. The second term reflects that in the absence of insurance, external lenders gain from the higher dollar value of their returns in the lower uninsurable region, as the values of $\eta_2 \equiv \frac{\chi}{\mathcal{E}_2}$ are highest in that region and $\eta_2 > (1 + i^*)$ prevails in some of the region. The third term reflects that the lenders lose from the lower dollar value of their returns in the upper uninsurable region, as the values of η_2 are lowest in that region and $\eta_2 < (1 + i^*)$ prevails in some of the region. The inequality in the condition identifies when the external lenders gain less than they lose, e.g., if $m_2^{(b)}$ tends to be smaller in the lower insurable region and larger in the upper insurable region.

The third scenario is that the country begins in subcase (a) (ii). The change from symmetric to asymmetric information involves speculators remaining informed, while external lenders and insurers become less informed than they were before. Condition (9) is again the criterion for the previous premium ρ to no longer be sufficient. For this third scenario, we set $\pi(\theta|\Omega_I) = 1$ only for the known value of θ and $= 0$ for other values, so all three terms on the right hand side of the condition may now be non-zero. This amendment reflects that in subcase (a) (ii), external lenders must receive a premium which is appropriate given the specific known value of θ, while in case (b), they must receive a premium which is appropriate given all the possible values of θ.

Next, we turn to how exchange rates are set differently at the constrained efficient allocation with asymmetric information. Since the function $m_2^{(b)} = m_2^{(b)}\left(B_2, \eta_2, \{\eta_2\}_{\mathcal{E}_2 \in \Psi}, \frac{\beta}{\alpha}\right)$ matters when there are uninsurable regions, we condition our next result with respect to perturbations of this function. Specifically, we consider perturbed functions m_2^{ϵ} indexed by ϵ such that these functions and their

first derivatives converge uniformly on compact sets to $m_2^{(b)}$ and its first derivatives as $\epsilon \to 0$. We describe a result to be generic with respect to the function $m_2^{(b)}$ in the following sense: if the result does not hold for a specific function $m_2^{(b)}$, there exists a small perturbation $\epsilon > 0$ to the function such that the result does hold.

Proposition 2 (Modified monetary policy). *Suppose that in subcases (a) (i) and (a) (ii), the planner selects $\beta > 0$ and $B_2 > 0$. Generically with respect to the function $m_2^{(b)}$, the planner selects a different value of β in case (b). There exist functions $m_2^{(b)}$ such that the planner selects a lower value of β in case (b) than in subcase (a) (i).*

There is a case for monetary policy to be set differently when a change in the information structure causes a switch from full to partial insurability and the country has positive external debt. The planner sets monetary policy to internalize the IS externality. They recognize that the reduction in insurability pushes up the premium at $t = 1$, which in turn hurts welfare if there is external debt to be repaid. Faced with higher expected external debt repayments, the planner generically finds it optimal to alter χ and the exchange rate function $\mathcal{E}_2 = \alpha\varphi + \beta\theta + \gamma$ because they affect the external lenders' SDF (through the m_2 function) and the size of the uninsurable regions (by changing $-\frac{\beta}{\alpha}$, the slope of the upper and lower contours in Figure 2).

Specifically, the planner may reduce the information sensitivity of the exchange rate by reducing the parameter β. In the absence of insurability concerns, a positive value of β may be constrained efficient because a depreciation of the exchange rate $\mathcal{E}_2$ may be an appropriate response to the private shock θ, for example to reduce the AD wedge and/or to reduce the dollar value of external debt repayments. Once insurability concerns are added, the planner may wish to reduce the value of β to reduce the premium, even if such a policy distorts the AD wedge and external debt repayments.

We can explain the intuition for this result using the following thought experiment. Fixing all the planner's choice variables to the values corresponding to the constrained efficient allocation for subcase (a) (i), we switch to case (b) and examine whether the planner would wish to reduce β subject to the constraint that all the other choice variables must be held constant. The benefits and costs of such a change is captured by the planner's FOC for β, which we inspect by substituting $\omega = \beta$ and the expression for Λ from equation (6) into equation (8). We derive that the planner wishes to reduce β iff the following condition is satisfied:

$$\frac{\int_{\theta=0}^{1}\int_{\varphi=0}^{1}\theta\frac{\chi}{(\mathcal{E}_2)^2}d\varphi d\theta + z_{\beta 1} + z_{\beta 2}}{\int_{\theta=0}^{1}\int_{\varphi=0}^{1}\frac{1}{\mathcal{E}_2}d\varphi d\theta + z_{\chi}} > 1, \tag{10}$$

when all the planner's choice variables correspond to their constrained efficient values for subcase (a) (i) but the variables $\{z_{\beta 1}, z_{\beta 2}, z_\chi\}$ are evaluated using the function $m_2^{(b)}$ corresponding to case (b). This condition indicates that the planner would wish to reduce β when the variables $z_{\beta 1}$ and $z_{\beta 2}$ are larger. These two terms are key measures of the IS externality.

The variable $z_{\beta 1}$ assumes that the boundaries of the uninsurable regions are fixed, and captures how much an increase in β causes a decrease in the SDF-weighted returns of external lenders within these regions. The increase in β causes exchange rates to depreciate at $t = 2$, especially in states when θ is large (since $\frac{\partial \mathcal{E}_2}{\partial \beta} = \theta$). From Figure 2, such states occur with greater frequency in the upper uninsurable region than in the lower one. In the definition of $z_{\beta 1}$, the first term in each row captures the impact of this change in exchange rates taking the values of $m_2^{(b)}$ as given, with each row representing one of the uninsurable regions. The terms can sum to a large positive number if the values of $m_2^{(b)}$ are sufficiently higher in the upper insurable region than in the lower one. The second term in each row captures that the values of $m_2^{(b)}$ within these regions may themselves change. The terms can sum to a large positive number if $m_2^{(b)}$ increases more sharply in the upper uninsurable region (where the values of $\eta_2 \equiv \frac{\chi}{\mathcal{E}_2}$ are lowest and some states feature $\eta_2 < (1 + i^*)$) than in the lower insurable region (where the values of η_2 are highest and some states feature $\eta_2 > (1 + i^*)$).

The variable $z_{\beta 2}$ captures how much an increase in β causes a decrease in the SDF-weighted returns by expanding the size of the uninsurable regions. As $-\frac{\beta}{\alpha}$ decreases and the regions expand, some states of nature (φ, θ) transition from insured to uninsured, so more of the returns to the lenders are multiplied by their uninsured SDFs. In the definition of $z_{\beta 2}$, the term in front of the integral is multiplied by the negative number $-\frac{1}{\alpha}$, so $z_{\beta 2}$ can be a large positive number if the integral itself is a large negative number. Such an outcome can occur if m_2 jumps from one to a higher uninsured value and $\eta_2 < (1 + i^*)$ at the boundary of the upper insurable region, while m_2 jumps from one to a lower uninsured value and $\eta_2 < (1 + i^*)$ at the boundary of the lower insurable region.

The rationale for reducing the parameter β evokes and yet contrasts with a separate literature on information sensitivity. Dang et al. (2013, 2015) examine how private institutions can design securities to make them less sensitive to private information and thereby more liquidly tradable by uninformed agents. By contrast, our framework takes as given that domestic agents issue local currency debt securities, and then it explores how monetary policy should be modified to affect the sensitivity of the dollar returns on that debt to the private shock: the less sensitive they are, the lower the premium.

The monetary policy result in Proposition 2 differs from those in Basu et al. (2025) and Itskhoki

and Mukhin (2023) because of the different way in which we model FX market premia. In Basu et al. (2025), premia are affected by external debt but not by exchange rate volatility, so the policy mix should incorporate the LC premium externality but not otherwise modify the exchange rate movements. In Itskhoki and Mukhin (2023), premia are generated by exchange rate volatility as none of it is insurable. Correspondingly, they recommend that premia can be reduced by modifying the policy mix to reduce this volatility. By contrast, in our framework, the premia can be zero even with high exchange rate volatility, and the policy mix should remedy the lack of insurability in the tails of the exchange rate distribution rather than reduce exchange rate volatility per se. As a result, our constrained efficient monetary policy may include a lower sensitivity of the exchange rate to the private shock, but not necessarily to the public shock.

Next, we turn to how a constrained efficient allocation with modified parameters $\{\alpha, \beta, \gamma\}$ can be implemented in practice. In our framework, the planner should simply pre-commit at $t = 1$ to a monetary policy with the constrained efficient parameters $\{\alpha, \beta, \gamma\}$ at $t = 2$. Consider an alternative environment where pre-commitment is not possible, but at $t = 1$ the planner can delegate monetary policy to a central banker with preferences $\{\alpha, \beta, \gamma\}$ for the exchange rate function $\mathcal{E}_2$ at $t = 2$.

Corollary 1 (Aloof central banker). *The constrained efficient allocation can be implemented by the planner delegating monetary policy to a central banker with the constrained efficient parameters $\{\alpha, \beta, \gamma\}$ for the exchange rate function $\mathcal{E}_2$. If the constrained efficient value of β is lower in case (b) than in subcase (a) (i), the central banker's decisions should be less sensitive to the private shock θ in case (b) than in subcase (a) (i).*

This corollary states that the planner can achieve the constrained efficient allocation through delegation to a central banker with the appropriate preferences. These preferences are generically different in case (b) than in subcases (a) (i) and (ii). Moreover, in the presence of insurability concerns, there may be a case for selecting an "aloof central banker" who down-weights the private shock θ in their exchange rate function in case (b) relative to subcase (a) (i). External lenders charge a lower premium on the country's external debt at $t = 1$ if they are assured that the central banker prefers not to be sensitive to θ at $t = 2$, even if such a preference means that other economic distortions may be larger at $t = 2$. This case for monetary delegation to a central banker with specific preferences is distinct from the inflation bias rationale in Rogoff (1985), as it relies on information sensitivity rather than any fear of excessive inflation.

4 Applications

In this section, we explore four applications of the general model which each extract additional insights for the policy mix.

4.1 Composition of External Lenders

While the general model permits a broad range of possibilities for the modeling of the external lenders, here we explore two options. The first option is that the lenders are owned by foreigners with log utility. The second option is that the lenders are owned by foreigners who face binding financial constraints after large losses but not otherwise.

For our first option, we assume that the lenders are owned by foreigners who maximize the following welfare:

$$\max_{C^*_{F2}} \mathbb{E}\left[\log C^*_{F2} | \Omega_L\right],$$

subject to the budget constraint:

$$\kappa \Pi_2 + Y^*_{F2} \geq C^*_{F2},$$

where $\kappa > 0$ is the ratio of the measure of the home country relative to the measure of the external lenders, $\Pi_2 = [\eta_2 - (1 + i^*)] B_2 + Q_L(\mathcal{E}_2)$ is the dollar profit of the lenders as described in subsection 2.1, Y^*_{F2} is the foreign endowment of tradable goods, and C^*_{F2} is the foreign consumption of tradable goods. The SDF M_2 reflects the marginal value of Π_2 derived from this maximization problem.

The following result derives the corresponding function m_2 and its implication for the premium.

Proposition 3 (Log utility). *Suppose that the modeling of the external lenders follows our first option. For a country in case (b) with $\kappa > 0$, the function m_2 is as follows:*

$$m_2 = \begin{cases} 1 & \text{for } \mathcal{E}_2 \in \widetilde{\Psi} \\ \dfrac{\frac{\kappa}{1-\frac{\beta}{\alpha}} B_2 \int_{\theta=0}^{1} \int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta} \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] d\varphi d\theta + Y^*_{F2}}{\kappa B_2 \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] + Y^*_{F2}} & \text{for } \mathcal{E}_2 \notin \widetilde{\Psi}, \end{cases} \tag{11}$$

For any given parameters $\{\alpha, \beta, \gamma\}$, the premium constraint (3) yields $\frac{d\chi}{dB_2} > 0$.

This proposition relates the premium to each lender's exposure to the country's external debt. This exposure is determined as follows.

First, the values of m_2 in the uninsurable regions deviate from one only if $\kappa > 0$, i.e., if the country is large relative to the set of external lenders. Such a situation may arise for a range of countries. For example, it could arise for a small country whose external debt can only be held by an even

smaller set of lenders who specialize in that country's debt. Alternatively, it could arise for a large country whose external debt must be held by foreigners located in smaller countries. By contrast, if the country is small and the set of external lenders is large, i.e., $\kappa \to 0$, we obtain $m_2 = 1$ throughout the insurable and uninsurable regions, so there is no premium.

Second, for $\kappa > 0$, the level of the country's external debt B_2 is important for determining the premium. The second part of the proposition establishes that for any given parameters $\{\alpha, \beta, \gamma\}$ within the exchange rate function (4), the planner faces a premium schedule that is upward-sloping in external debt. Specifically, they must offer a higher external repayment χ if they choose to increase the consumption of imports C_{F1} and correspondingly issue more debt B_2 at $t = 1$.

An implication of this finding is that if the country switches from full to partial insurability without a change in the parameters $\{\alpha, \beta, \gamma\}$, the jump in the premium is increasing in the level of external debt. If the external debt is low, the jump in the premium would be small, so an episode of partial insurability may not be clearly detectable in practice. However, if the external debt is large, the jump may be large and noticeable. Information sensitivity shocks may become more macroeconomically problematic over time for countries whose external debts are increasing in magnitude.

Figures 3 and 4 show simulations for the comparative statics with respect to κ and B_1 respectively. The results in both figures are consistent with Propositions 2 and 3. We use the example functions:

$$Y_{F1} = \overline{Y}, Y_{F2}(\varphi, \theta) = \overline{Y} - \xi\left(\varphi - \frac{1}{2}\right) - \nu\left(\theta - \frac{1}{2}\right), \text{ and } A_1 = A_2(\varphi, \theta) = \overline{A}. \tag{12}$$

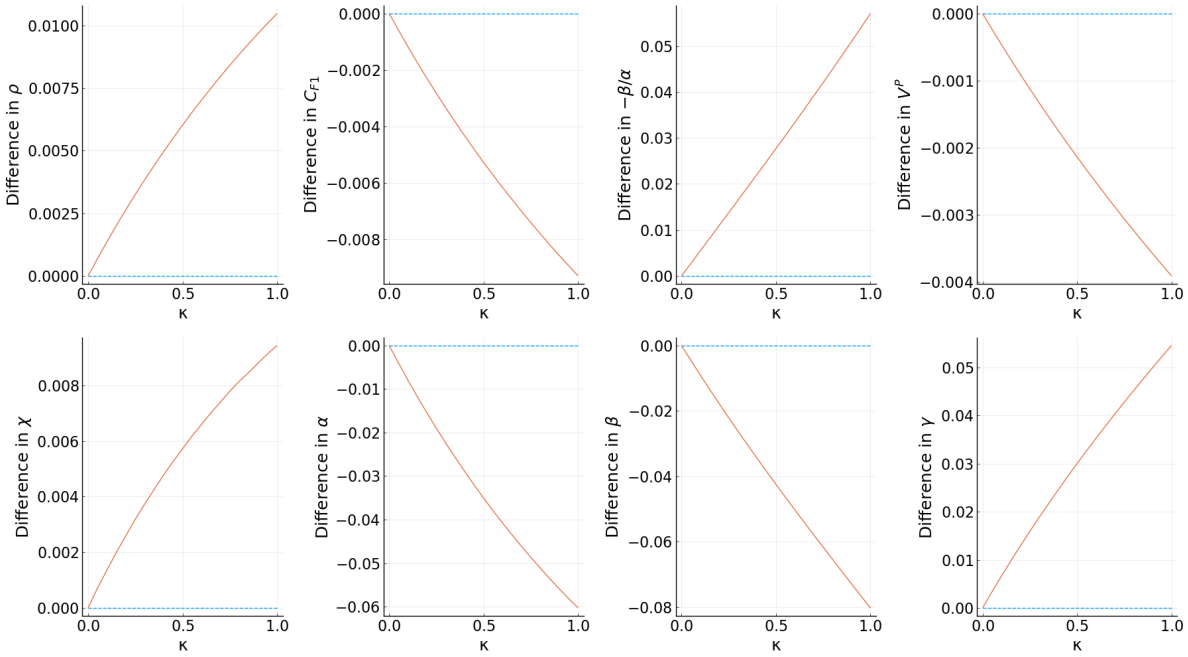
The red lines in the figures plot the difference for the constrained efficient variables going from the full insurability subcase (a) (i) to the partial insurability case (b). Both figures show that insurability problems cause a higher premium ρ, lower imports C_{F1}, a lower value of β, a higher value of $-\frac{\beta}{\alpha}$, and lower welfare V^P. Figure 3 shows that these problems become more severe as the ratio κ increases, while figure 4 shows that they become more severe as the inherited external debt B_1 increases.

Next, we consider an extension of the model with two categories of external lenders.

Corollary 2 (Inframarginal lenders). *Suppose that the marginal lenders follow our first modeling option but there also exist inframarginal lenders who hold the dollar value S_2 of local currency bonds. Proposition 3 continues to hold but with B_2 replaced by $B_2 - S_2$.*

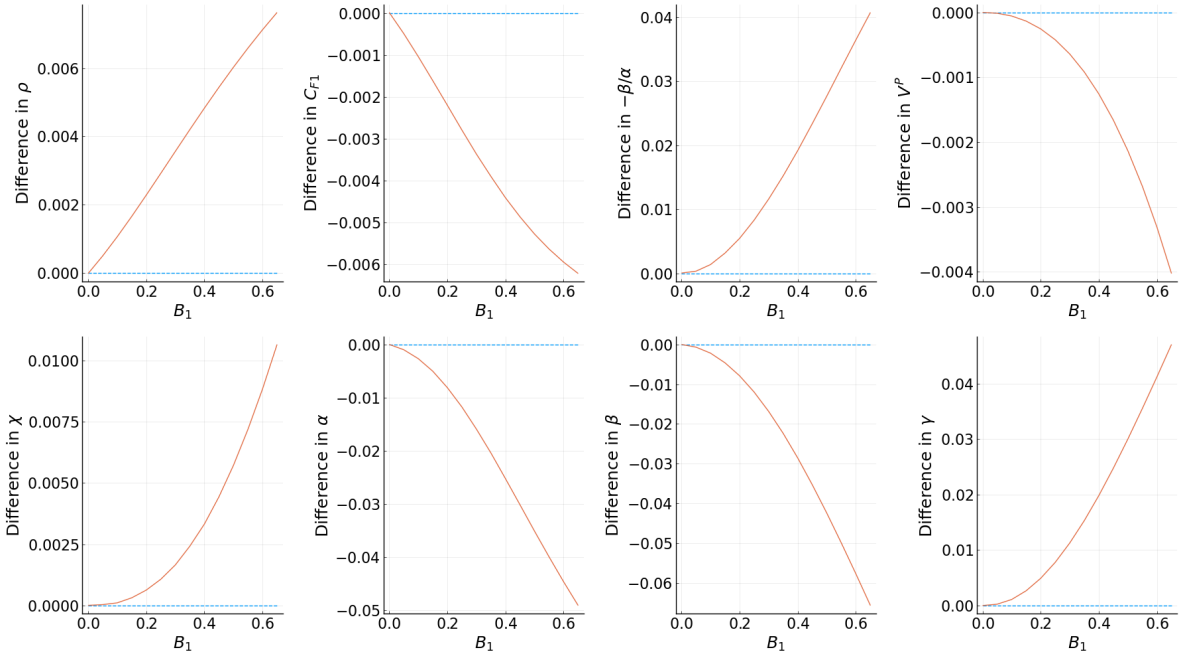
This corollary establishes a link between our papers and others which highlight the role of inframarginal return-insensitive investors in FX markets. Some papers label these investors as "noise traders" and posit that they cause high-frequency variations in FX market premia (e.g., Gabaix and Maggiori, 2015; Itskhoki and Mukhin, 2021, 2023; and Basu et al., 2025). Here, we do not explore

Figure 3: How Insurability Problems Change Allocations, Given the Value of κ



Note: Difference in variable = Constrained efficient value in case (b) minus value in subcase (a) (i). Parameter values are $\sigma_H = 0.5, \delta = 0.96, i^* = \frac{1}{\delta} - 1, B_1 = 0.5, Y^*_{F2} = 1, \xi = 0.6, \nu = 0.3,$ and $\overline{A} = 1$.

Figure 4: How Insurability Problems Change Allocations, Given the Value of B_1



Note: Difference in variable = Constrained efficient value in case (b) minus value in subcase (a) (i). Parameter values are $\sigma_H = 0.5, \delta = 0.96, i^* = \frac{1}{\delta} - 1, \kappa = 0.5, Y^*_{F2} = 1, \xi = 0.6, \nu = 0.3,$ and $\overline{A} = 1$.

the stochastic properties of such investors. Instead, we simply observe that consistent with those papers, the premium may be higher in our framework if inframarginal investors hold less of the external debt, because each marginal investor must then be more exposed to the debt. Our framework also introduces the importance of insurability: inframarginal investors affect premia only if some exchange rates are uninsurable.

For our second modeling option of the external lenders, we assume that they are owned by foreigners who maximize the following welfare:

$$\max_{C^*_{F2}} \mathbb{E}\left[\frac{C^{*1-\nu}_{F2}}{1-\nu}\middle|\,\Omega_L\right],$$

subject to the budget constraint:

$$\Pi_2 - \varsigma\Theta\left(\Pi_2\right)\frac{\left(\underline{\Pi}-\Pi_2\right)^2}{2} + Y^*_2 \geq C^*_{F2},$$

$$\text{where } \Theta\left(\Pi_2\right) = \begin{cases} 0 & \text{if } \Pi_2 \geq \underline{\Pi} \\ 1 & \text{if } \Pi_2 < \underline{\Pi} \end{cases} \quad \text{for some } \underline{\Pi} < 0.$$

The indicator function $\Theta\left(\Pi_2\right)$ captures in reduced form a one-sided financial constraint, e.g., costly margin calls when losses are high.

The corresponding function m_2 reflects the asymmetry of the financial constraint.

Proposition 4 (One-sided constraint). *Suppose that the modeling of the external lenders follows our second option and that for $\varsigma = 0$, the constrained efficient allocation for case (b) features $\{\beta, B_2\} > 0$. There exist values of $\underline{\Pi} < 0$ such that for $\varsigma > 0$, the constrained efficient allocation features $\{\beta, B_2\} > 0$, while $\Theta\left(\Pi_2\right) = 1$ only for some $\mathcal{E}_2 \in (\alpha+\gamma, \alpha+\beta+\gamma]$. We also obtain:*

$$\lim_{\nu\to 0} m_2 = \begin{cases} 1 & \text{for } \mathcal{E}_2 \in [\gamma, \alpha+\gamma] \\ 1+\varsigma\Theta\left(\Pi_2\right)\left(\underline{\Pi} - \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right]B_2\right) \geq 1 & \text{for } \mathcal{E}_2 \in (\alpha+\gamma, \alpha+\beta+\gamma]. \end{cases} \tag{13}$$

This proposition explains that the premium may be mostly determined by the upper tail of the exchange rate distribution (i.e., large depreciations). The reason is that the premium jumps if the financial constraint binds and pushes m_2 substantially above one, and such an outcome occurs only in the upper uninsurable region.

The details of the argument are as follows. In case (b), the exchange rate distribution is $\Psi = [\gamma, \alpha+\beta+\gamma]$ and the insurable set of exchange rates is $\widetilde{\Psi} = [\beta+\gamma, \alpha+\gamma]$. External lenders' losses are large enough to trigger the financial constraint only after large depreciations, and such depreciations are observed only in the upper uninsurable region. In the limit as $\nu \to 0$, we obtain $m_2 > 1$

only if the financial constraint is triggered, so the limit of m_2 is equal to one in both the insurable set and the lower uninsurable region, and above one only for a part of the upper uninsurable region.

An implication is that for this modeling of the external lenders, any monetary policy modification may also be mostly determined by the upper uninsurable region. The premium charged by external lenders is determined mostly by the largest depreciations that they could expect. Accordingly, if the planner internalizes the IS externality and modifies its monetary policy to select a lower value of β, they would do so mostly to mitigate the largest depreciations rather than to modify other parts of the exchange rate distribution.

4.2 Fiscal Policy

Here we explore two modeling options to illustrate how fiscal policy can generate insurability problems and how such problems can in turn affect fiscal policy decisions. Both modeling options relate to government spending on imports.[13] The first option is that fiscal policy is exogenous and subject to public and private shocks. The second option is that fiscal policy is endogenous and the motivation for public spending is subject to public and private shocks. In the fiscal policy context, private shocks may reflect asymmetric information about political or rent-seeking pressures.

For our first option, we consider a model of exogenous fiscal policy. Government spending on imports in the two periods are $\{G_{F1}, G_{F2}(\varphi, \theta)\}$ in dollar terms, i.e., a single number at $t = 1$ and a function of both shocks at $t = 2$. They are financed by lump sum taxation, so relative to the model in subsection 2.1, the planner's rebates to households are amended as follows:

$$T_t = \phi W_t N_t + \mu_{t-1}(1 + i_{t-1}) D_t - \mathcal{E}_t G_{Ft}.$$

There are no other shocks to tradable endowments or productivity, so we set $\{Y_{F1}, Y_{F2}(\varphi, \theta)\} = \overline{Y}_F$ and $\{A_1, A_2(\varphi, \theta)\} = \overline{A}$ in the households' and firms' maximization problems respectively.

It turns out that the planner problem associated with this model is isomorphic to the planner problem in subsection 3.1.

Proposition 5 (Exogenous fiscal policy). *Suppose that the modeling of fiscal policy follows our first option. The constrained efficient allocation is derived from the planner problem maximizing the utility function (1) subject to the constraints (2)-(4), with* $\{Y_{F1}, Y_{F2}(\varphi, \theta)\}$ *from that problem replaced by* $\left\{\overline{Y}_F - G_{F1}, \overline{Y}_F - G_{F2}(\varphi, \theta)\right\}$ *and* $\{A_1, A_2(\varphi, \theta)\} = \overline{A}$.

[13] This subsection builds on the treatment of government spending on imports in Basu and Gopinath (2026) and adds insurability problems. That paper also considers other categories of government spending and financial frictions.

This proposition reveals that fiscal policy can be the underlying source of FX market premia. It can increase the premia through two channels.

First, prior to any shocks, an increase in government spending on imports G_{F1} is identical to a reduction in the initial tradable endowment Y_{F1} and/or an increase in inherited external debt B_1. Accordingly, it tends to increase the external debt B_2 issued at $t = 1$. This debt is subject to any premium that exists on the debt. In addition, the external debt affects the value of Λ via equation (6) and the parameters $\{\alpha, \beta, \gamma\}$ in the exchange rate function via equation (8). For $B_2 > 0$, monetary policy may be useful to depreciate away the dollar value of the external debt after adverse external shocks.

Second, the shocks to government spending on imports G_{F2} may be the reason that monetary policy is sensitive to the private shock at $t = 2$ and why a premium exists at $t = 1$. Such shocks generate macroeconomic volatility in that period through the resource constraint (2). Suppose that the private shock θ causes an increase in government spending G_{F2} such that there are less resources available for private import consumption C_{F2}. If so, the planner may set $\beta > 0$ in the exchange rate function (4), because the benefit of depreciating away the dollar value of the external debt becomes higher for larger values of θ. If the country is in case (b), the sensitivity of the exchange rate to the private shock at $t = 2$ generates uninsurability in the FX hedging market and a premium on the FX spot market at $t = 1$.

An implication of this finding is that premia may arise structurally in countries whose fiscal policy depend on shocks known beforehand only to a subset of FX hedging market participants, and may exhibit episodic surges when such shocks become more important or there is a risk of a regime shift that makes fiscal policy more sensitive to such shocks.

For our second option, we consider a model of endogenous fiscal policy. Government spending on imports and the planner's lump sum rebates follow the same expressions as in our first option. However, we amend the households' welfare to the following:

$$\mathbb{E}\left[\sum_{t=1}^{2} \delta^{t-1} U\left(C_{Ht}, C_{Ft}, G_{Ft}, N_t\right)\right]$$

$$\text{where } U\left(C_{Ht}, C_{Ft}, N_t\right) = \sigma_H \log C_{Ht} + \sigma_F \log C_{Ft} + \sigma_{Gt} \log G_{Ft} - N_t,$$

and the parameter σ_{Gt} is given by $\{\sigma_{G1}, \sigma_{G2}(\varphi, \theta)\}$.

The planner problem associated with this model is amended relative to the planner problem in subsection 3.1.

Proposition 6 (Endogenous fiscal policy). *Suppose that the modeling of fiscal policy follows our second option. The constrained efficient allocation is derived from the planner problem maximizing the utility function (1) subject to the constraints (2)-(4), with* $\{Y_{F1}, Y_{F2}(\varphi,\theta)\}$ *from that problem replaced by* $\left\{\overline{Y}_F - G_{F1}, \overline{Y}_F - G_{F2}(\varphi,\theta)\right\}$, $\{A_1, A_2(\varphi,\theta)\} = \overline{A}$, *the planner's period-t utility function* V *replaced as follows:*

$$V\left(C_{Ft}, \frac{\mathcal{E}_t}{P_H}, G_{Ft}\right) \equiv U\left(\frac{\sigma_H}{\sigma_F}\frac{\mathcal{E}_t}{P_H}C_{Ft}, C_{Ft}, \frac{1}{A_t}\frac{\sigma_H}{\sigma_F}\frac{\mathcal{E}_t}{P_H}C_{Ft}, G_{Ft}\right) \text{ where } \frac{\partial V}{\partial G_{Ft}} = \frac{\sigma_{Gt}}{G_{Ft}},$$

and the planner's choice variables expanded to include $\{G_{F1}, G_{F2}\}$. *The allocation satisfies equations (5)-(8) as well as the following condition:*

$$G_{F1} = \frac{\sigma_{G1}}{\sigma_F}C_{F1} \text{ and } G_{F2} = \frac{\sigma_{G2}}{\sigma_F}\frac{C_{F2}}{1+\frac{\sigma_H}{\sigma_F}\tau_{H2}}. \tag{14}$$

This proposition reveals that if fiscal policy is endogenous, the level of government spending on imports is affected by insurability problems in FX markets. Condition (14) comes from combining the FOCs for G_{Ft} and C_{Ft} and establishes how G_{Ft} is determined in both periods.

First, prior to any shocks, the government spending on imports G_{F1} is tied to the private import consumption C_{F1}. As equation (7) indicates, the level of C_{F1} is affected by AD and LC premium externalities. Correspondingly, G_{F1} is also affected by both of these externalities. For example, if C_{F1} is small because the FX market premium is large, G_{F1} is also set to be small. In other words, fiscal discipline in normal times helps make the country ready to manage future shocks.

Second, the sensitivity of government spending on imports G_{F2} to shocks and the parameters in the exchange rate function (4) depend on each other. Equation (8) indicates that if the country is in case (b) and G_{F2} varies with the private shock θ, the parameters $\{\alpha, \beta\}$ are affected by the partial insurability problem because of the AD and IS externalities. Condition (14) indicates that after shocks, the value of $\mathcal{E}_2$ affects the value of G_{F2} by affecting the levels of C_{F2} and τ_{H2}.

The planner has the ability to eliminate the insurability problem and the associated premium by constraining fiscal policy to be insensitive to the private shock.

Corollary 3 (Constrained fiscal policy). *Suppose that the modeling of fiscal policy follows our second option but* G_{F2} *is constrained to be insensitive to the private shock* θ, *i.e.,* $G_{F2} = G_{F2}(\varphi)$. *Proposition 6 continues to hold except that condition (14) is replaced by the following condition:*

$$G_{F1} = \frac{\sigma_{G1}}{\sigma_F}C_{F1} \text{ and } \int_{\theta=0}^{1}\frac{\sigma_{G2}}{G_{F2}}\pi(\theta|\Omega_I)\,d\theta = \int_{\theta=0}^{1}\frac{\sigma_F}{C_{F2}}\left[1+\frac{\sigma_H}{\sigma_F}\tau_{H2}\right]\pi(\theta|\Omega_I)\,d\theta. \tag{15}$$

The constrained efficient allocation for subcase (a) (i) features $\beta = 0$, *and the premium* $\rho = 0$ *continues*

to satisfy the premium constraint (3) after the information sensitivity shock in Lemma 3.

Such a constraint on fiscal policy achieves full insurability in the FX hedging market and eliminates the premium in the FX spot market. In the context of our model of endogenous fiscal policy, the resource constraint (2) indicates that if G_{F2} does not vary with the private shock θ, the feasible levels of C_{F2} are not affected by that shock. As a result, there is no need for the planner to move $\mathcal{E}_2$ with θ. Accordingly, the planner sets $\beta = 0$ and the economy is in case (a) (iii). There is no premium on external debt.

Whether such a constraint on fiscal policy is constrained efficient depends on the trade-off between the volatility of σ_{G2} and the level of external debt B_2. On the one hand, welfare is hurt by the constraint on fiscal policy if σ_{G2} is a function of a shock θ that is unknown in advance, i.e., $\pi(\theta|\Omega_I) = 1$, because G_{F2} should ideally vary with θ in this case. On the other hand, welfare is improved by the removal of FX market premia, which is beneficial if B_2 is positive.

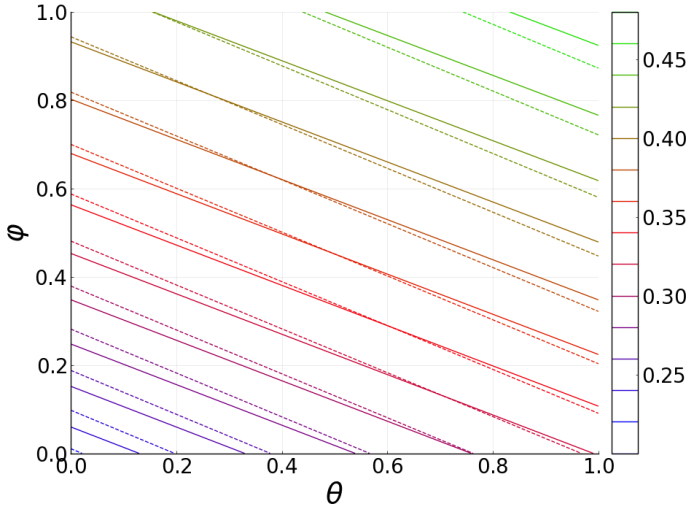
Figure 5 shows a simulation for a country where the planner reduces but does not eliminate the insurability problem. We use the m_2 function from Proposition 3, the modeling option from Proposition 6, and the example function:

$$\sigma_{G2}(\varphi, \theta) = \sigma_{G1}\left(1 + \xi\left(\varphi - \frac{1}{2}\right) + \nu\left(\theta - \frac{1}{2}\right)\right). \tag{16}$$

For the parameters listed at the bottom of the figure, we confirm that moving from the full insurability subcase (a) (i) to the partial insurability case (b) causes a higher premium ρ, lower imports C_{F1}, a lower value of β, a higher value of $-\frac{\beta}{\alpha}$, and lower welfare V^P. The figure shows how such an outcome is achieved. The dashed lines represent the contour map for the levels of G_{F2} as a function of the shocks (φ, θ) in subcase (a) (i) and the solid lines represent the contour map in case (b). The solid lines have a gentler slope than the dashed lines, revealing that the higher value of $-\frac{\beta}{\alpha}$ that is needed to reduce the insurability problem is itself facilitated by reducing the sensitivity of G_{F2} to θ.

These results connect the literature on fiscal rules and institutions with the separate international finance literature on FX market premia. Several papers have pointed to fiscal rules and institutions as being beneficial because they impose constraints that reduce public debt (e.g., Wyplosz, 2013; Halac and Yared, 2014; Hatchondo et al., 2022). Our framework identifies a novel channel through which such rules and institutions may be beneficial: if they reduce the sensitivity of fiscal policy to private shocks, they can improve the functioning of FX hedging markets and reduce FX spot market premia. Combining Corollaries 2 and 3, we also observe that by reducing these premia, such fiscal policy constraints may reduce the susceptibility of the country to inframarginal return-insensitive

Figure 5: How Insurability Problems Change Government Imports G_{F2}



Note: Dashed contours relate to subcase (a) (i) and solid contours relate to case (b). Parameter values are $\sigma_H = 0.5, \sigma_G = 0.5, \delta = 0.96, i^* = \frac{1}{\delta} - 1, B_1 = 0.5, \kappa = 0.5, Y^*_{F2} = 1, \xi = 0.6, \nu = 0.3, \overline{Y}_F = 1,$ and $\overline{A} = 1$.

investors such as noise traders in FX markets.

4.3 Financial Policy

Here we consider how the desired mix of monetary and financial policies is affected by the presence of FX market premia. We are interested in macroprudential financial policies, i.e., those which are set at $t = 1$ and generate some associated economic costs, and whose benefits depend on how severe the shocks happen to be at $t = 2$. We model one example of such a policy: we extend our general model to incorporate an FX fund that the planner must set up in advance and can be used to mitigate the output declines caused by the private shock.

Relative to the model in subsection 2.1, we reinterpret the tradable endowment Y_{F2} as the output of export firms who are subject to shocks modeled in reduced form as follows:

$$Y_{F2} = \widetilde{Y}_{F2}(\varphi) + \lambda(\theta, X),$$

where $X \in [0, X^{\max}]$ is an injection of dollar funds by the planner and $\lambda_\theta \leq 0$, $\lambda_X > 0$, $\lambda_{X\theta} > 0$, $\lambda_{XX} < 0$, $\lim_{X\to 0} \mathbb{E}[\lambda_X|\Omega_I] > (1 + i^*)$, and $\lim_{X\to X_{\max}} \lambda_X < (1 + i^*)$ for all θ. The private financial shock causes a reduction in output but this reduction can be mitigated by dollar injections that are especially valuable for large values of the private shock and have diminishing marginal benefits.

The injections must be financed by the planner borrowing X dollars from abroad and setting up an FX fund at $t = 1$, which is then used to support export firms at $t = 2$. The planner must

also repay all the funds with interest in the final period. Since the FX fund is financed by dollar borrowing, its incorporation into the framework does not alter the expression for the local currency external debt D_2 or its dollar value B_2. However, the repayments must be financed by transfers from the households, so the planner's lump sum rebates to households at $t = 2$ are amended as follows:

$$T_2 = \phi W_2 N_2 + \mu_1 (1 + i_1) D_2 - \mathcal{E}_2 X (1 + i^*).$$

The incorporation of the FX fund provides a new macroprudential policy tool to the planner. In the absence of the FX fund, the planner problem associated with this model is a special case of the planner problem in subsection 3.1. In particular, equation (8) indicates that the parameter β in the exchange rate function should not be zero. In other words, since the private financial shock destabilizes the macroeconomy at $t = 2$, the planner should attempt to remedy the problem by setting $\mathcal{E}_2$ to vary with that shock. If the planner can set up an FX fund, the planner problem associated with the extended model is amended relative to the planner problem in subsection 3.1.

Proposition 7 (Financial policy). *Suppose that the modeling of financial policy includes an FX fund. The constrained efficient allocation is derived from the planner problem maximizing the utility function (1) subject to the constraints (2)-(4), with $Y_{F2}(\varphi, \theta)$ from that problem replaced by $\widetilde{Y}_{F2}(\varphi) + \lambda(\theta, X) - X(1 + i^*)$, and the planner's choice variables expanded to include X. The allocation satisfies equations (5)-(8) as well as the following condition:*

$$(1 + i^*) - \mathbb{E}[\lambda_X | \Omega_I] = \frac{Cov\left[\lambda_X, \frac{1 + \frac{\sigma_H}{\sigma_F}\tau_{H2}}{C_{F2}} \middle| \Omega_I\right]}{\mathbb{E}\left[\frac{1 + \frac{\sigma_H}{\sigma_F}\tau_{H2}}{C_{F2}} \middle| \Omega_I\right]}. \tag{17}$$

This proposition establishes how insurability problems affect the desired size of the FX fund. We can use condition (17) to compare subcase (a) (i) and case (b).

For a country in subcase (a) (i), the planner sets $X > 0$ if the private financial shock θ causes a decrease in imports C_{F2} and an increase in the AD wedge τ_{H2}. In such a circumstance, the covariance term on the right hand side of condition (17) is positive, as the shock also increases λ_X. Accordingly, the value of X is increased until the term $\mathbb{E}[\lambda_X | \Omega_I]$ on the left hand of the condition decreases sufficiently below $(1 + i^*)$.

For a country in case (b), the planner generically selects a different value of X, and may select a higher value. Proposition 2 establishes that the planner generically selects a different value of the parameter β in case (b) than in subcase (a) (i). This change in the exchange rate function causes changes in C_{F2} and τ_{H2}, which in turn alter the desired value of X according to condition (17).

Moreover, to reduce FX market premia, the planner may select a lower value of β in case (b) than in subcase (a) (i), i.e., they may reduce the sensitivity of the exchange rate to the private shock. The side-effect of this modified monetary policy may be that the private financial shock θ causes a larger decrease in C_{F2} and increase in τ_{H2}. The covariance term on the right hand side of condition (17) may then be higher. If so, it may result in the planner selecting a larger value of X to reduce the term $\mathbb{E}\left[\lambda_X|\Omega_I\right]$ further on the left hand of the condition.

This finding has implications for the desired mix of monetary and financial policies in countries where FX market premia are salient. If insurability problems mean that the constrained efficient monetary policy features a low sensitivity of the exchange rate to the private shock, financial policies may have to play a greater role to address that shock. Indeed, if financial policies can directly mitigate the macroeconomic impact of the private shock, there is less need for the exchange rate to vary with the private shock, which means that the premia would be lower. Relative to papers which study the integrated policy mix in the context of symmetric information (e.g., Farhi and Werning, 2016; Bianchi and Lorenzoni, 2022; Basu et al., 2025), our framework in this paper suggests that there may be a greater role for the kinds of financial policies which can reduce any asymmetric information about financial conditions among FX market participants. This recommendation would be reversed for the kinds of financial policies which may exacerbate asymmetric information.

4.4 Transparency and Secrecy

Finally, we use our framework to shed light on whether countries should set up institutions to publish information or keep it secret.

We add a new period $t = 0$ which occurs before the two periods of the model in subsection 2.1. There is no discounting between $t = 0$ and $t = 1$. In the new period, no shocks are known to any agents, and the planner must commit to an institutional design $\Delta \in \{N, T, S\}$.

- $\Delta = N$ corresponds to the planner committing at $t = 0$ to do nothing at $t = 1$, so the new period has no impact on the model. We assume that if the planner does nothing, the country is in case (b) at $t = 1$, i.e., $\Omega_I = \underline{\Omega}$, $\Omega_S = \overline{\Omega}$, and $\beta > 0$.

- $\Delta = T$ corresponds to the planner committing at $t = 0$ to collect and publish all information related to the private shock at $t = 1$. We assume that the planner can collect the information at a cost $\Upsilon^T \geq 0$. If they do so, they convert the country to case (a) (ii) at $t = 1$, i.e., $\Omega_I = \Omega_S = \overline{\Omega}$.

- $\Delta = S$ corresponds to the planner committing at $t = 0$ to keep secret all information related

to the private shock at $t = 1$. We assume that the planner can hide the information at a cost $\Upsilon^S(\omega) \geq 0$, where we denote ω to be the measure of speculators who potentially have access to the private information and $\Upsilon^S_\omega > 0$. If the planner hides the information, they convert the country to case (a) (i) at $t = 1$, i.e., $\Omega_I = \Omega_S = \underline{\Omega}$.

Out of this set of options, we observe that while the options of transparency and secrecy appear diametrically opposed to each other, both of them eliminate the premium by achieving symmetric information in the first period. The authorities need to compare the option of doing nothing against the least costly manner of achieving a zero premium.

The following result relates the planner's decision at $t = 0$ to the different possible versions of the planner problem in subsection 3.1.

Proposition 8 (Institutional design). *Suppose that the planner commits to an institutional design at $t = 0$. The planner's decision at $t = 0$ solves:*

$$V^{P0} = \max_{\Delta} \left\{ V^{\Delta} \right\}$$

$$\textit{where } V^N = V^{P,(b)}, V^T = \int_{\theta=0}^{1} V^{P,(a)(ii)} d\theta - \Upsilon^T, \textit{ and } V^S = V^{P,(a)(i)} - \Upsilon^S(\omega),$$

and $V^{P,(x)}$ denotes the welfare from maximizing the utility function (1) subject to the constraints (2)-(4) for a country in case (x) at $t = 1$.

This proposition captures that the planner's choice of institutional design should weigh the benefits of reducing FX market premia against the costs of collecting or hiding information.

The planner chooses institutions at $t = 0$ to modify the information structure at $t = 1$ only if the benefits outweigh the costs. If the benefit of reducing the premium is large because the inherited external debt B_1 is large and the IS externality is severe, while the costs of either of the options of transparency or secrecy are small, the planner may commit to one of these two options instead of doing nothing. However, if the benefit of reducing the premium is small, while the costs of both of the options of transparency or secrecy are large, the planner may commit to do nothing.

The choice between transparency and secrecy generally depends on the kind of information. For "leaky" information which is easy to collect and publish but difficult to hide, i.e., Υ^T is low and ω is large, a commitment to transparency and keeping all agents informed is the least costly way of converting the country from asymmetric to symmetric information and eliminating premia. For "murky" information which is difficult to collect and publish but easy to hide, i.e., Υ^T is high and

ω is low, a commitment to secrecy and keeping all agents uninformed is the least costly way of eliminating premia.

Some kinds of information may switch between the two categories. For example, some economic data may be difficult to collect within the first week but can be inferred by a large set of private sector agents after a month. If we apply Proposition 8 to the institutional design problem within the first week, we may categorize the information as murky, so secrecy may be the selected option. However, if we apply the proposition after a month, we may categorize the information as leaky, so transparency may be the selected option. We observe that many statistical agencies around the world follow a process of keeping the collection of sensitive economic data secret from all agents to begin with, and then releasing it in a public fashion once it is collected (e.g., even the central bank may not have access to inflation or labor market data until it is formally published). Our framework suggests that this approach could help preserve an environment of symmetric information (albeit through different means at different times) and minimize the likelihood of any episodes of asymmetric information which could cause disruptions and premia in FX markets.

5 Conclusion

Policy advice for countries which face FX market premia should account for why exactly those premia exist. We propose a tractable framework that relates UIP premia to the insurability of exchange rates, and then explains how macroeconomic policies affect that insurability via a novel information sensitivity externality. Insurability problems endogenously arise from asymmetric information about the tails of the exchange rate distribution, so policies should seek to reduce the sensitivity of the exchange rate to private information. Exchange rate volatility per se is not the cause of premia, so stabilizing the exchange rate is unnecessary in addressing the externality.

In practice, the insights from our four applications of the general model may interact. One example is that if fiscal policy is the underlying source of insurability problems, the design of fiscal institutions should be related to the composition of external lenders and how it changes. If these lenders have a large exposure to the debt, it may be especially important to impose constraints on fiscal policy and/or for fiscal institutions to establish symmetric information about future fiscal projections. Another example is that the use of macroprudential policies to mitigate the most severe financial shocks may be especially beneficial if those shocks are more visible to some FX market participants than others and if the upper tail of the exchange rate distribution (i.e., large depreciations) drive most of the premia.

We leave several topics for future research. On the technical dimension, the exchange rate can be allowed to be a nonlinear function of the shocks, which would change the shape of the insurable set and the uninsurable regions. On the practical dimension, we need to better understand how premia in FX markets can be affected by additional complications such as time consistency and poor coordination across policy tools, as well as how the problem of asymmetric information may vary across countries and tools.

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APPENDIX

A Proofs of Results in Section 2

Proof of Lemma 1

As indicated in the main text, we establish the lemma by solving the FX hedging market backward. For each $\mathcal{E}_2 \in \Psi$, we suppose that the insurer for this exchange rate decides $d = 1$ such that $\mathcal{E}_2 \in \widetilde{\Psi}$, and then we establish whether there is a contradiction and the insurer prefers $d = 0$ instead.

In step 2, speculators have the demands $Q_S(\mathcal{E}_2)$ shown on page 10 of the main text. External lenders have the aggregate demands $Q_L(\mathcal{E}_2)$ derived from the expressions on page 8 of the main text. They do not observe any information that can help them update their information set $\Omega_L = \Omega_I$, and their profits do not depend on the speculators' demands. Moreover, changes in the price $p(\mathcal{E}_2)$ for a specific exchange rate $\mathcal{E}_2$ do not affect $\overline{M}$. For $\pi(\mathcal{E}_2|\Omega_L) > 0$, we obtain the following relations for the lenders' aggregate decisions:

$$\frac{dQ_L(\mathcal{E}_2)}{dp(\mathcal{E}_2)} < 0 \text{ and } \lim_{p(\mathcal{E}_2)\to 0} Q_L(\mathcal{E}_2) = \infty,$$

with $Q_L(\mathcal{E}_2)$ being finite for $p(\mathcal{E}_2) > 0$. For $\pi(\mathcal{E}_2|\Omega_L) \to 0$, we obtain that $Q_L(\mathcal{E}_2)$ is indeterminate if $p(\mathcal{E}_2) \to 0$, while $Q_L(\mathcal{E}_2) \to Q_{\min}$ if $p(\mathcal{E}_2) > 0$.

In step 1, there are two cases, which we next analyze in turn. For each of the cases, the insurer's problems are shown on page 10 of the main text; for the second case, the regions $\{A, B, C\}$ are also defined there. The insurer's profits depend on the demands of the lenders and the speculators, so the insurer must game out how these other agents respond to different prices $p(\mathcal{E}_2)$.

The first case is that $\Omega_I = \Omega_S \in \{\underline{\Omega}, \overline{\Omega}\}$.

- For $\pi(\mathcal{E}_2|\Omega_I) = \pi(\mathcal{E}_2|\Omega_S) > 0$, the insurer does not select $p(\mathcal{E}_2) = 0$ as that choice would result in losses owing to $Q_L(\mathcal{E}_2) \to \infty$ and $Q_S(\mathcal{E}_2) = \overline{Q_S} \to \infty$. If the insurer selects some $p(\mathcal{E}_2) > 0$, the argument in the main text applies, establishing that $\pi(\mathcal{E}_2|\Omega_I) = (1 + i^*)\, p(\mathcal{E}_2)$.

- For $\pi(\mathcal{E}_2|\Omega_I) = \pi(\mathcal{E}_2|\Omega_S) = 0$, the insurer can avoid losses by selecting $p(\mathcal{E}_2) = 0$, establishing that $\pi(\mathcal{E}_2|\Omega_I) = (1 + i^*)\, p(\mathcal{E}_2)$. They do not select some $p(\mathcal{E}_2) > 0$ as that choice would result in losses owing to $Q_L(\mathcal{E}_2) \to Q_{\min}$ and $Q_S(\mathcal{E}_2) = -\overline{Q_S} \to -\infty$.

The second case is that $\Omega_F = \underline{\Omega}$ and $\Omega_S = \overline{\Omega}$.

- For $\pi(\mathcal{E}_2|\Omega_I) > 0$, the insurer does not select $p(\mathcal{E}_2) = 0$ as that choice would result in losses. We would obtain $Q_L(\mathcal{E}_2) \to \infty$ and then one of regions $\{A, B\}$ may apply. There would be $Q_S(\mathcal{E}_2) = \overline{Q_S} \to \infty$ in region A, resulting in losses for the insurer, and $Q_S(\mathcal{E}_2) \in \left(-\overline{Q_S}, \overline{Q_S}\right)$ in region B, resulting in zero profits. Adapting the argument in the main text for this price $p(\mathcal{E}_2) = 0$, the insurer would prefer $d = 0$ as they cannot ensure that the condition $\pi(\mathcal{E}_2|\Omega_S) = (1 + i^*)\, p(\mathcal{E}_2)$ holds with probability one. If the insurer selects some $p(\mathcal{E}_2) > 0$, the argument in the main text applies, establishing that the insurer decides $d = 1$ only if $\pi(\mathcal{E}_2|\Omega_I) = \pi(\mathcal{E}_2|\Omega_S)$. In that case, we obtain $\pi(\mathcal{E}_2|\Omega_I) = (1 + i^*)\, p(\mathcal{E}_2)$.

- For $\pi(\mathcal{E}_2|\Omega_I) = 0$, it follows by definition that $\pi(\mathcal{E}_2|\Omega_S) = 0$ for all θ, which in turn means that $\pi(\mathcal{E}_2|\Omega_I) = \pi(\mathcal{E}_2|\Omega_S) = 0$. The insurer selects $p(\mathcal{E}_2) = 0$. For this price, we would obtain an indeterminate $Q_L(\mathcal{E}_2)$ and then region B would apply. There would be $Q_S(\mathcal{E}_2) \in \left(-\overline{Q_S}, \overline{Q_S}\right)$ in that region, resulting in zero profits for the insurer. Adapting the argument in the main text, the insurer decides $d = 1$ and we establish that $\pi(\mathcal{E}_2|\Omega_I) = (1 + i^*)\, p(\mathcal{E}_2)$. The insurer does not select some $p(\mathcal{E}_2) > 0$ as that choice would result in losses. We would obtain $Q_L(\mathcal{E}_2) \to Q_{\min}$ and then region C would apply. There would be $Q_S(\mathcal{E}_2) = -\overline{Q_S} \to -\infty$ in that region, resulting in losses for the insurer.

These arguments establish the results in the lemma. ■

Proof of Proposition 1

For $\alpha \geq \beta \geq 0$, the set of exchange rates Ψ can be partitioned into three regions as in Figure 2. Substituting $\Omega_L = \Omega_I$ and rearranging the expression for the external lenders' bond position from page 8 of the main text, we can obtain the following condition with three terms, each representing one of the regions:

$$\begin{aligned} 0 = & \int_{\theta=0}^{1} \int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta} M_2 \left[\eta_2 - (1+i^*)\right] \pi\left(\theta|\Omega_I\right) d\varphi d\theta \\ & + \int_{\theta=0}^{1} \int_{\varphi=0}^{\frac{\beta}{\alpha}(1-\theta)} M_2 \left[\eta_2 - (1+i^*)\right] \pi\left(\theta|\Omega_I\right) d\varphi d\theta \\ & + \int_{\theta=0}^{1} \int_{\varphi=1-\frac{\beta}{\alpha}\theta}^{1} M_2 \left[\eta_2 - (1+i^*)\right] \pi\left(\theta|\Omega_I\right) d\varphi d\theta, \end{aligned}$$

where $\pi\left(\theta|\Omega_I\right)$ depends on Ω_I. The probability distributions of the exchange rates $\mathcal{E}_2$ in these three respective regions depend on the information sets as follows:

$$\pi\left(\mathcal{E}_2|\overline{\Omega}\right) = \begin{cases} \frac{1}{\alpha} & \text{for } \mathcal{E}_2 \in [\beta+\gamma, \alpha+\gamma] \\ 0 \text{ if } \mathcal{E}_2 < \beta\theta+\gamma;\ \frac{1}{\alpha} \text{ if } \mathcal{E}_2 \geq \beta\theta+\gamma & \text{for } \mathcal{E}_2 \in [\gamma, \beta+\gamma] \\ \frac{1}{\alpha} \text{ if } \mathcal{E}_2 \leq \alpha+\beta\theta+\gamma;\ 0 \text{ if } \mathcal{E}_2 > \alpha+\beta\theta+\gamma & \text{for } \mathcal{E}_2 \in [\alpha+\gamma, \alpha+\beta+\gamma] \end{cases}$$

$$\pi\left(\mathcal{E}_2|\underline{\Omega}\right) = \begin{cases} \int_{\theta=0}^{1} \frac{1}{\alpha} d\theta = \frac{1}{\alpha} & \text{for } \mathcal{E}_2 \in [\beta+\gamma, \alpha+\gamma] \\ \int_{\theta=0}^{\frac{\mathcal{E}_2-\gamma}{\beta}} \frac{1}{\alpha} d\theta = \frac{\mathcal{E}_2-\gamma}{\alpha\beta} & \text{for } \mathcal{E}_2 \in [\gamma, \beta+\gamma] \\ \int_{\theta=\frac{\mathcal{E}_2-\alpha-\gamma}{\beta}}^{1} \frac{1}{\alpha} d\theta = \frac{\alpha+\beta+\gamma-\mathcal{E}_2}{\alpha\beta} & \text{for } \mathcal{E}_2 \in [\alpha+\gamma, \alpha+\beta+\gamma]. \end{cases}$$

Next, we consider the following cases. Note that in all the cases, $Q_I\left(\mathcal{E}_2\right) = Q_L\left(\mathcal{E}_2\right) + Q_S\left(\mathcal{E}_2\right)$.

Subcase (a) (i). $\Omega_I = \Omega_S = \underline{\Omega}$. Since $\Omega_I = \underline{\Omega}$, $\pi\left(\theta|\Omega_I\right) = 1$. Since $\Omega_I = \Omega_S$, $\pi\left(\mathcal{E}_2|\Omega_I\right) = \pi\left(\mathcal{E}_2|\Omega_S\right)$ for all $\mathcal{E}_2 \in \Psi$ irrespective of θ. Then Lemma 1 indicates that all exchange rates are insurable, i.e., $\widetilde{\Psi} = \Psi$. In addition, $\pi\left(\mathcal{E}_2|\Omega_I\right) = (1+i^*)\, p\left(\mathcal{E}_2\right)$ within the insurable set. Correspondingly, $Q_L\left(\mathcal{E}_2\right)$ is set for all $\mathcal{E}_2 \in \Psi$ such that $M_2 = \overline{M}$ in all the terms in the above condition for the bond position, while $Q_S\left(\mathcal{E}_2\right) \in \left(-\overline{Q_S}, \overline{Q_S}\right)$. Dividing that condition through by $\overline{M}$, we establish $\mathbb{E}\left[\eta_2\right] = (1+i^*)$ and all the results for this subcase of the proposition.

Subcase (a) (ii). $\Omega_I = \Omega_S = \overline{\Omega}$. Since $\Omega_I = \overline{\Omega}$, $\pi\left(\theta|\Omega_I\right) = 1$ only at the known value of θ and $= 0$ elsewhere. Since $\Omega_I = \Omega_S$, $\pi\left(\mathcal{E}_2|\Omega_I\right) = \pi\left(\mathcal{E}_2|\Omega_S\right)$ for all $\mathcal{E}_2 \in \Psi$ irrespective of θ. Then Lemma 1 indicates that all exchange rates are insurable, i.e., $\widetilde{\Psi} = \Psi$. In addition, $\pi\left(\mathcal{E}_2|\Omega_I\right) = (1+i^*)\, p\left(\mathcal{E}_2\right)$ within the insurable set. Correspondingly, $Q_L\left(\mathcal{E}_2\right)$ is set for all $\mathcal{E}_2 \in \Psi$ such that $M_2 = \overline{M}$ in all the terms in the above condition for the bond position, while $Q_S\left(\mathcal{E}_2\right) \in \left(-\overline{Q_S}, \overline{Q_S}\right)$. Dividing that condition through by $\overline{M}$, we establish $\mathbb{E}\left[\eta_2\right] = (1+i^*)$ and all the results for this subcase of the proposition.

Subcase (a) (iii). $\Omega_I = \underline{\Omega}$, $\Omega_S = \overline{\Omega}$, and $\beta = 0$. Since $\Omega_I = \underline{\Omega}$, $\pi\left(\theta|\Omega_I\right) = 1$. Since $\beta = 0$, $\pi\left(\mathcal{E}_2|\Omega_I\right) = \pi\left(\mathcal{E}_2|\Omega_S\right)$ for all $\mathcal{E}_2 \in \Psi$ irrespective of θ even though $\Omega_S \supset \Omega_I$. The reason is that the second and third terms in the above condition for the bond position are zero and there are no exchange rates within the regions represented by those terms. Then Lemma 1 indicates that all exchange rates are insurable, i.e., $\widetilde{\Psi} = \Psi$. In addition, $\pi\left(\mathcal{E}_2|\Omega_I\right) = (1+i^*)\, p\left(\mathcal{E}_2\right)$ within the insurable set. Correspondingly, $Q_L\left(\mathcal{E}_2\right)$ is set for all $\mathcal{E}_2 \in \Psi$ such that $M_2 = \overline{M}$ in the first and only remaining term in the above condition for the bond position, while $Q_S\left(\mathcal{E}_2\right) \in \left(-\overline{Q_S}, \overline{Q_S}\right)$. Dividing that condition through by $\overline{M}$, we establish $\mathbb{E}\left[\eta_2\right] = (1+i^*)$ and all the results for this subcase of the proposition.

Case (b). $\Omega_I = \underline{\Omega}$, $\Omega_S = \overline{\Omega}$, and $\beta > 0$. Since $\Omega_I = \underline{\Omega}$, $\pi\left(\theta|\Omega_I\right) = 1$. Then Lemma 1 indicates that only the exchange rates $\mathcal{E}_2 \in [\beta+\gamma, \alpha+\gamma]$ are insurable, i.e., $\widetilde{\Psi} = [\beta+\gamma, \alpha+\gamma]$. The probability of $\mathcal{E}_2 \in \widetilde{\Psi}$ is $1 - \frac{\beta}{\alpha}$. In addition, $\pi\left(\mathcal{E}_2|\Omega_I\right) = (1+i^*)\, p\left(\mathcal{E}_2\right)$ for all $\mathcal{E}_2 \in \widetilde{\Psi}$. Correspondingly, $Q_L\left(\mathcal{E}_2\right)$ is set for all $\mathcal{E}_2 \in \widetilde{\Psi}$

such that $M_2 = \overline{M}$ in the first term in the above condition for the bond position, while $Q_S(\mathcal{E}_2) \in \left(-\overline{Q_S}, \overline{Q_S}\right)$. Meanwhile, $p(\mathcal{E}_2) = \varnothing$ and $Q_L(\mathcal{E}_2) = Q_S(\mathcal{E}_2) = 0$ for all $\mathcal{E}_2 \notin \widetilde{\Psi}$. Dividing the condition for the bond position through by $\overline{M}$, we establish the version of the condition shown in the proposition. Applying the definition of the covariance, we obtain the expression for the premium ρ. Finally, $m_2 = m_2\left(B_2, \eta_2, \{\eta_2\}_{\mathcal{E}_2 \in \Psi}, \frac{\beta}{\alpha}\right)$ for all $\mathcal{E}_2 \notin \widetilde{\Psi}$ because M_2 depends on the variables $\{B_2, Q_L(\mathcal{E}_2), \eta_2\}$ by definition, while $\overline{M}$ depends on the variables $\{B_2, \eta_2\}$ within the insurable set $\widetilde{\Psi}$ which in turn is characterized by combining Ψ and $\frac{\beta}{\alpha}$. ■

B Planner Problem

Here we explain how to extract the details of the constrained efficient allocation. Equations (1)-(4) characterize that allocation in terms of the planner's choice variables $\{C_{F1}, C_{F2}, \mathcal{E}_1, \alpha, \beta, \gamma, \chi, P_H\}$. The remaining variables listed in Definitions 1 and 2 can be derived as follows:

$$i_1 = \frac{1}{\delta \mathcal{E}_1 C_{F1} \mathbb{E}\left[\frac{1}{\mathcal{E}_2 C_{F2}}\right]} - 1$$

$$\mu_1 = 1 - \delta \chi C_{F1} \mathbb{E}\left[\frac{1}{\mathcal{E}_2 C_{F2}}\right]$$

$$\phi = \sigma_F P_H \frac{\varepsilon - 1}{\varepsilon} \frac{1+\delta}{\frac{\mathcal{E}_1 C_{F1}}{A_1} + \delta \mathbb{E}\left[\frac{\mathcal{E}_2 C_{F2}}{A_2}\right]} - 1$$

$$\mathcal{E}_2 = \alpha\varphi + \beta\theta + \gamma$$

$$C_{Ht} = Y_{Ht} = \frac{\sigma_H}{\sigma_F} \frac{\mathcal{E}_t C_{Ft}}{P_H}, N_t = \frac{Y_{Ht}}{A_t}, \text{ and } W_t = \frac{1}{\sigma_F} \mathcal{E}_t C_{Ft} \text{ for } t \in \{1, 2\}$$

$$D_2 = \left[B_1(1+i^*) + (C_{F1} - Y_{F1})\right] \mathcal{E}_1,$$

while the quantities $\{Q_I(\mathcal{E}_2), Q_L(\mathcal{E}_2), Q_S(\mathcal{E}_2)\}_{\mathcal{E}_2 \in \Psi}$, prices $\{p(\mathcal{E}_2)\}_{\mathcal{E}_2 \in \Psi}$, exchange rate sets $\left\{\Psi, \widetilde{\Psi}\right\}$ and probability distributions $\left\{\pi(\mathcal{E}_2|\underline{\Omega}), \pi\left(\mathcal{E}_2|\overline{\Omega}\right)\right\}_{\mathcal{E}_2 \in \Psi}$ pertaining to the FX hedging market follow the expressions in the proof of Proposition 1.

Next, we complement the analysis in subsection 3.2 with the FOCs of the planner problem and additional expressions which are not provided there. We assume that the solution satisfies $\alpha > \beta \geq 0$. The FOC with respect to $\mathcal{E}_1$ is provided on page 17 of the main text. The FOC with respect to χ is as follows:

$$\frac{B_2}{\chi} \int_{\theta=0}^{1} \int_{\varphi=0}^{1} \Phi(\varphi, \theta) \pi(\theta|\Omega_I) \, d\varphi d\theta = \Lambda \left[\int_{\theta=0}^{1} \int_{\varphi=0}^{1} \frac{1}{\mathcal{E}_2} \pi(\theta|\Omega_I) \, d\varphi d\theta + z_\chi \right],$$

where $\Phi(\varphi, \theta) \pi(\theta|\Omega_I)$ is the multiplier on the resource constraint (2) for the state of nature (φ, θ). The FOCs with respect to C_{F1} and C_{F2} are as follows:

$$\frac{\sigma_F}{C_{F1}} \left[1 + \frac{\sigma_H}{\sigma_F} \tau_{H1}\right] = \int_{\theta=0}^{1} \int_{\varphi=0}^{1} \Phi(\varphi, \theta) \pi(\theta|\Omega_I) \, d\varphi d\theta + z_{F1}$$

$$\frac{\delta \sigma_F}{C_{F2}} \left[1 + \frac{\sigma_H}{\sigma_F} \tau_{H2}\right] = \Phi(\varphi, \theta) \frac{\mathcal{E}_2}{\chi}.$$

The FOCs with respect to each parameter $\omega \in \{\alpha, \beta, \gamma\}$ are as follows:

$$\int_{\theta=0}^{1} \int_{\varphi=0}^{1} \frac{\partial \mathcal{E}_2}{\partial \omega} \left\{ \frac{1}{\mathcal{E}_2} \delta \sigma_H \tau_{H2} + B_2 \frac{1}{\mathcal{E}_2} \Phi(\varphi, \theta) \right\} \pi(\theta|\Omega_I) \, d\varphi d\theta$$

$$= \Lambda \left[\int_{\theta=0}^{1} \int_{\varphi=0}^{1} \frac{\partial \mathcal{E}_2}{\partial \omega} \frac{\chi}{(\mathcal{E}_2)^2} \pi(\theta|\Omega_I)\, d\varphi d\theta + z_{\omega 1} + z_{\omega 2} \right].$$

The FOC with respect to P_H is as follows:

$$\tau_{H1} + \delta \int_{\theta=0}^{1} \int_{\varphi=0}^{1} \tau_{H2} \pi(\theta|\Omega_I)\, d\varphi d\theta = 0.$$

This FOC is redundant given the FOCs with respect to $\{\mathcal{E}_1, \alpha, \beta, \gamma, \chi\}$, provided that the following condition holds:

$$\frac{\partial m_2}{\partial \alpha}\alpha + \frac{\partial m_2}{\partial \beta}\beta + \frac{\partial m_2}{\partial \gamma}\gamma + \frac{\partial m_2}{\partial \chi}\chi = 0.$$

This condition does indeed hold because $m_2 = m_2\left(B_2, \eta_2, \{\eta_2\}_{\mathcal{E}_2 \in \Psi}, \frac{\beta}{\alpha}\right)$ and we can establish that $\frac{\partial \eta_2}{\partial \alpha}\alpha + \frac{\partial \eta_2}{\partial \beta}\beta + \frac{\partial \eta_2}{\partial \gamma}\gamma + \frac{\partial \eta_2}{\partial \chi}\chi = 0$ and $\frac{\partial\left(\frac{\beta}{\alpha}\right)}{\partial \alpha}\alpha + \frac{\partial\left(\frac{\beta}{\alpha}\right)}{\partial \beta}\beta = 0$.

Next, we turn to how to amend the FOCs when $\alpha > \beta \geq 0$ is not satisfied at an interior solution. We consider two scenarios. In the first scenario, we suppose that $\alpha = 0$ or $\beta = 0$ are binding. Then we replace the equality with the inequality $<$ in the FOC with respect to that parameter. In the second scenario, we suppose that $\beta > \alpha \geq 0$. Then we make more substantial modifications to the analysis. We amend the premium constraint (3) in Proposition 1 and the planner problem as follows:

$$0 = \int_{\theta=0}^{1} \int_{\varphi=0}^{1} m_2 \left[\frac{\chi}{\mathcal{E}_2} - (1 + i^*) \right] \pi(\theta|\Omega_I)\, d\varphi d\theta, \tag{B.1}$$

as there is no longer a shaded region in Figure 2. In subcases (a) (i) and (ii), we continue to obtain $m_2 = 1$, while subcase (a) (iii) cannot occur when $\beta > 0$. In case (b), the probability of $\mathcal{E}_2 \in \widetilde{\Psi}$ is 0, the insurable set is $\widetilde{\Psi} = \varnothing$, and there are no longer any upper or lower insurability contours. In the FOCs, we amend the following definitions:

$$z_\chi = \int_{\theta=0}^{1} \int_{\varphi=0}^{1} \left\{ [m_2 - 1] \frac{1}{\mathcal{E}_2} + \frac{\partial m_2}{\partial \chi} \left[\frac{\chi}{\mathcal{E}_2} - (1 + i^*) \right] \right\} \pi(\theta|\Omega_I)\, d\varphi d\theta$$

$$z_{F1} = -\Lambda \int_{\theta=0}^{1} \int_{\varphi=0}^{1} \frac{\partial m_2}{\partial B_2} \left[\frac{\chi}{\mathcal{E}_2} - (1 + i^*) \right] \pi(\theta|\Omega_I)\, d\varphi d\theta$$

$$z_{\omega 1} = \int_{\theta=0}^{1} \int_{\varphi=0}^{1} \left\{ [m_2 - 1] \frac{\partial \mathcal{E}_2}{\partial \omega} \frac{\chi}{(\mathcal{E}_2)^2} - \frac{\partial m_2}{\partial \omega} \left[\frac{\chi}{\mathcal{E}_2} - (1 + i^*) \right] \right\} \pi(\theta|\Omega_I)\, d\varphi d\theta$$

$$z_{\omega 2} = 0.$$

Finally, for comparison purposes, we consider how the analysis changes if the model is amended such that there are no premia and the exchange rate function is unrestricted. Specifically, we consider an amended planner problem where we set $m_2 = 1$ in all the terms of the premium constraint (3) and we remove the exchange rate function (4). There are two modifications to the analysis. First, since $m_2 = 1$ and $\frac{\partial m_2}{\partial \chi} = \frac{\partial m_2}{\partial B_2} = 0$, we obtain $z_\chi = z_{F1} = 0$ in the FOCs with respect to $\{C_{F1}, \chi\}$ and in equations (6)-(7). Second, we replace the FOCs with respect to $\omega \in \{\alpha, \beta, \gamma\}$ with new FOCs with respect to $\mathcal{E}_2$. For each state of nature (φ, θ), we obtain:

$$\frac{1}{\mathcal{E}_2} \delta \sigma_H \tau_{H2} + B_2 \frac{1}{\mathcal{E}_2} \Phi(\varphi, \theta) = \Lambda \frac{\chi}{(\mathcal{E}_2)^2}.$$

Correspondingly, we replace equation (8) with the following equation:

$$\delta \sigma_H \tau_{H2} + B_2 \frac{\chi}{\mathcal{E}_2} \frac{\delta \sigma_F}{C_{F2}} \left[1 + \frac{\sigma_H}{\sigma_F} \tau_{H2} \right] = \Lambda \frac{\chi}{\mathcal{E}_2}. \tag{B.2}$$

C Proofs of Results in Sections 3-4

Proof of Lemma 2

For all the subcases within case (a) from Proposition 1, we obtain $m_2 = 1$ for every term in the premium constraint (3) which contains m_2. Since $m_2 = 1$ and $\frac{\partial m_2}{\partial \chi} = \frac{\partial m_2}{\partial B_2} = \frac{\partial m_2}{\partial \omega} = 0$ for all $\omega \in \{\alpha, \beta, \gamma\}$, we obtain $z_\chi = z_{F1} = z_{\omega 1} = z_{\omega 2} = 0$. ■

Proof of Lemma 3

The country starts with $\Omega_I = \Omega_S \in \left\{\underline{\Omega}, \overline{\Omega}\right\}$, so it may be in subcase (a) (i) or (ii). Given Proposition 1, the constrained efficient allocation must satisfy the premium constraint (3) with $m_2 = 1$ as follows:

$$\begin{aligned} 0 = & \int_{\theta=0}^{1} \int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta} \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] \pi\left(\theta|\Omega_I\right) d\varphi d\theta \\ & + \int_{\theta=0}^{1} \int_{\varphi=0}^{\frac{\beta}{\alpha}(1-\theta)} \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] \pi\left(\theta|\Omega_I\right) d\varphi d\theta \\ & + \int_{\theta=0}^{1} \int_{\varphi=1-\frac{\beta}{\alpha}\theta}^{1} \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] \pi\left(\theta|\Omega_I\right) d\varphi d\theta. \end{aligned} \tag{C.1}$$

Next, consider a change to $\Omega_I = \underline{\Omega}$ and $\Omega_S = \overline{\Omega}$ without altering the planner's choice variables. Suppose that the premium ρ is too low to satisfy the constraint (3) after the change in the information structure. From the external lenders' FOC for the bond position, such an outcome occurs if there is an inequality $>$ instead of an equality in that constraint.

There are two cases to consider. The first case pertains to the country starting with $\beta = 0$. Then it moves to subcase (a) (iii) and the second and third terms of constraint (3) are zero:

$$0 > \int_{\theta=0}^{1} \int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta} \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] d\varphi d\theta. \tag{C.2}$$

The second case pertains to the country starting with $\beta > 0$. Then it moves to case (b) and the $m_2^{(b)}$ function applies:

$$\begin{aligned} 0 > & \int_{\theta=0}^{1} \int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta} \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] d\varphi d\theta \\ & + \int_{\theta=0}^{1} \int_{\varphi=0}^{\frac{\beta}{\alpha}(1-\theta)} m_2^{(b)} \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] d\varphi d\theta \\ & + \int_{\theta=0}^{1} \int_{\varphi=1-\frac{\beta}{\alpha}\theta}^{1} m_2^{(b)} \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] d\varphi d\theta \end{aligned} \tag{C.3}$$

Depending on which of these two cases is relevant, we subtract condition (C.1) from either condition (C.2) or condition (C.3). In both cases, we obtain the expression in the lemma. ■

Proof of Proposition 2

First, we establish that the planner generically selects a different β in case (b) than in subcases (a) (i) and (a) (ii). For each of the latter subcases, Lemma 2 indicates that the planner's choice variables including β satisfy constraints (2)-(4) and equations (5)-(8) with the settings $z_\chi = z_{F1} = z_{\omega 1} = z_{\omega 2} = 0$. Switching to case (b), $m_2^{(b)}$ is no longer always equal to 1, so those settings are no longer valid and the expressions $\{z_\chi, z_{F1}, z_{\omega 1}, z_{\omega 2}\}$ may

be non-zero. Correspondingly, equations (5)-(8) indicate that the values of all the planner's choice variables are altered, except in knife-edge cases where the expressions cancel each other out in a particular FOC. Suppose that for a specific function $m_2^{(b)}$, the value of β is not altered because of such a knife-edge configuration applying in the equation (8) for $\omega = \beta$, evaluated at the value of β from subcase (a) (i) or (a) (ii). Then there exists a small perturbation $\epsilon > 0$ to the function such that $m_2^{(b)}$ and its derivatives are altered sufficiently to take the allocation away from the knife-edge case, and the constrained efficient value of β changes accordingly.

Next, we prove that there exist functions $m_2^{(b)}$ such that the planner selects a lower value of β in case (b) than in subcase (a) (i). Taking as given that the planner selects $\beta > 0$ in subcase (a) (i), we consider a switch to case (b). In light of the discussion on page 23 of the main text, we are interested in example functions for $M_2\left(B_2, Q_L\left(\mathcal{E}_2\right), \eta_2\right)$ which may potentially generate large values of $\{z_{\beta 1}, z_{\beta 2}\}$. Specifically, we use a function as follows:

$$M_2\left(B_2, 0, \eta_2\right) = \begin{cases} M^u & \text{if } \eta_2 < (1+i^*) \\ M^m \in (0, M^u) & \text{if } \eta_2 \in [(1+i^*), (1+i^*)+h] \\ M^l \in (0, M^m) & \text{if } \eta_2 > (1+i^*)+h, \end{cases}$$

where $\left\{M^u, M^m, M^l, h\right\} > 0$ and we consider the planner problem in the limit as $M^u \to \infty$ and $M^l \to 0$.

Let us conjecture that $\alpha > \beta > 0$. The external lenders' bond position then satisfies:

$$\begin{aligned} 0 = & \int_{\theta=0}^{1} \int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta} \overline{M}\left[\eta_2 - (1+i^*)\right] d\varphi d\theta \\ & + \int_{\theta=0}^{1} \int_{\varphi=0}^{\frac{\beta}{\alpha}(1-\theta)} M_2\left[\eta_2 - (1+i^*)\right] d\varphi d\theta \\ & + \int_{\theta=0}^{1} \int_{\varphi=1-\frac{\beta}{\alpha}\theta}^{1} M_2\left[\eta_2 - (1+i^*)\right] d\varphi d\theta, \end{aligned}$$

where there are non-empty lower and upper uninsurable regions. The dollar value of the return η_2 varies with both shocks so it must take values above and below $(1+i^*)$ to be able to satisfy the above constraint. Since $\eta_2 < (1+i^*)$ for the highest values of the exchange rate $\mathcal{E}_2$ in the upper uninsurable region, there exists a non-empty set of exchange rates $\mathcal{E}_2$ where the SDF is M^u and the term in the above constraint is $M^u\left[\eta_2 - (1+i^*)\right] < 0$. In the limit as $M^u \to \infty$, we obtain that within that set, $M^u\left[\eta_2 - (1+i^*)\right] \to -\infty$. There may exist a non-empty set of exchange rates $\mathcal{E}_2$ in the lower uninsurable region where $\eta_2 > (1+i^*)+h$ and the term in the above constraint is $M^l\left[\eta_2 - (1+i^*)\right]$. In the limit as $M^l \to 0$, we obtain that within that set, $M^l\left[\eta_2 - (1+i^*)\right] \to 0$. In between those sets, there exists a set of exchange rates where $\eta_2 \in [(1+i^*), (1+i^*)+h]$ and the relevant SDFs lie within $\left[M^l, M^u\right]$, noting in particular that $\overline{M} \in \left[M^l, M^u\right]$. In the limit as $M^u \to \infty$ and $M^l \to 0$, we obtain that within that set, $M_2\left[\eta_2 - (1+i^*)\right]$ remains finite. Putting these results together, the right hand side of the above constraint tends to $-\infty$ in the limit, so the constraint cannot be satisfied for the conjectured parameter configuration.

For the constraint to be satisfied in the limit, we require that there does not exist a non-empty set of exchange rates $\mathcal{E}_2$ in the upper uninsurable region where $\eta_2 < (1+i^*)$ and the SDF is M^u. The possible solutions are that in the limit, either $\frac{\beta}{\alpha} \to 0$ (so there is no upper uninsurable region) or $\eta_2 \to (1+i^*)$ for all (φ, θ), i.e., $\{\alpha, \beta\} \to 0$ (so that $\eta_2 < (1+i^*)$ does not arise). We consider each in turn.

Possibility 1. $\frac{\beta}{\alpha} \to 0$. The planner can set $\beta \to 0$ and α to any value that they wish. This option offers superior welfare to the alternative of keeping $\beta > 0$ and setting $\alpha \to \infty$.

Possibility 2. $\{\alpha, \beta\} \to 0$. This option offers inferior welfare to Possibility 1 because α is restricted.

The upshot is that in the limit as $M^u \to \infty$ and $M^l \to 0$, the constrained efficient allocation in case (b) which satisfies the condition for the external lenders' bond position and achieves the highest welfare involves $\beta \to 0$ and a non-zero value of α. The planner problem, including the selection of β, is continuous in the values of $\left\{M^u, M^l\right\}$, so there exist finite values of those variables such that the constrained efficient allocation in case (b) features a value of β that is lower than the positive value chosen for this parameter in subcase (a) (i). Substituting these finite values into the above constraint and dividing through by $\overline{M}$, we obtain an example function $m_2^{(b)}$ which can be entered into the premium constraint (3) and satisfies the proposition.

We conclude by noting that an alternative proof would involve providing a simulation of the model with an example function $m_2^{(b)}$ and showing that the planner selects a lower value of β in case (b) than in subcase (a) (i). In subsection 4.1, we provide such a simulation. ■

Proof of Corollary 1

By inspection, as long as the exchange rate function (4) follows the constrained efficient parameters $\{\alpha, \beta, \gamma\}$, it does not matter whether it is implemented by a planner with pre-committed monetary policy at $t = 1$ or via delegation of monetary policy to a central banker with the appropriate preferences at $t = 2$. ■

Proof of Proposition 3

The foreigners' FOC with respect to C_{F2}^* is as follows:

$$\frac{1}{C_{F2}^*} = \Upsilon,$$

where $\Upsilon\pi\left(\mathcal{E}_2|\Omega_L\right)$ is the multiplier on their budget constraint for exchange rate $\mathcal{E}_2 \in \Psi$. The external lenders' SDF is equal to the marginal value of a dollar of their profits Π_2 at each exchange rate $\mathcal{E}_2$. Accordingly, we obtain:

$$M_2 = \kappa\Upsilon = \frac{\kappa}{\kappa\Pi_2 + Y_{F2}^*}. \tag{C.4}$$

A country in case (b) has the insurable set $\widetilde{\Psi} \subset \Psi$ as indicated in Proposition 1. Within that set, Lemma 1 indicates that $\pi\left(\mathcal{E}_2|\Omega_I\right) = (1 + i^*)\, p\left(\mathcal{E}_2\right)$. Since $\pi\left(\mathcal{E}_2|\Omega_L\right) = \pi\left(\mathcal{E}_2|\Omega_I\right)$, we obtain:

$$\overline{M} = \frac{\kappa}{\kappa\Pi_2 + Y_{F2}^*} \text{ for all } \mathcal{E}_2 \in \widetilde{\Psi}, \tag{C.5}$$

which means that $Q_L\left(\mathcal{E}_2\right)$ must be set such that the following condition holds:

$$\Pi_2 = \left[\eta_2 - (1 + i^*)\right] B_2 + Q_L\left(\mathcal{E}_2\right) = \overline{\Pi} \text{ for all } \mathcal{E}_2 \in \widetilde{\Psi}. \tag{C.6}$$

Integrating this condition over $\mathcal{E}_2 \in \widetilde{\Psi}$, we obtain:

$$\overline{\Pi} = \frac{1}{1 - \frac{\beta}{\alpha}} \left[\begin{array}{c} B_2 \int_{\theta=0}^{1} \int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta} \left[\eta_2 - (1 + i^*)\right] d\varphi d\theta \\ + \int_{\mathcal{E}_2 \in \widetilde{\Psi}} Q_L\left(\mathcal{E}_2\right) \pi\left(\mathcal{E}_2|\Omega_I\right) d\mathcal{E}_2 \end{array} \right], \tag{C.7}$$

where the second term in the square brackets on the right hand side can be shown to be zero by inserting $p\left(\mathcal{E}_2\right) = \frac{\pi(\mathcal{E}_2|\Omega_I)}{(1+i^*)}$ into the lenders' insurance market constraint in subsection 2.1. Combining equations (C.4)-(C.7) yields the expression (11) in the proposition.

$$\frac{\frac{\kappa}{1-\frac{\beta}{\alpha}} B_2 \int_{\theta=0}^{1} \int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta} \left[\eta_2 - (1 + i^*)\right] d\varphi d\theta + Y_{F2}^*}{\kappa\Pi_2 + Y_{F2}^*}$$

We substitute expression (11) into the premium constraint (3) to obtain:

$$0 = \frac{\int_{\theta=0}^{1} \int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta} \left[\frac{\chi}{\mathcal{E}_2} - (1 + i^*)\right] d\varphi d\theta}{\frac{\kappa}{1-\frac{\beta}{\alpha}} B_2 \int_{\theta=0}^{1} \int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta} \left[\frac{\chi}{\mathcal{E}_2} - (1 + i^*)\right] d\varphi d\theta + Y_{F2}^*}$$

$$+\int_{\theta=0}^{1}\int_{\varphi=0}^{\frac{\beta}{\alpha}(1-\theta)}\frac{\left[\frac{\chi}{\mathcal{E}_2}-(1+i^*)\right]}{\kappa B_2\left[\frac{\chi}{\mathcal{E}_2}-(1+i^*)\right]+Y^*_{F2}}d\varphi d\theta$$
$$+\int_{\theta=0}^{1}\int_{\varphi=1-\frac{\beta}{\alpha}\theta}^{1}\frac{\left[\frac{\chi}{\mathcal{E}_2}-(1+i^*)\right]}{\kappa B_2\left[\frac{\chi}{\mathcal{E}_2}-(1+i^*)\right]+Y^*_{F2}}d\varphi d\theta.$$

Fixing the parameters $\{\alpha,\beta,\gamma\}$ in the above condition and taking derivatives with respect to χ and B_2 yields:

$$\frac{d\chi}{dB_2}=\frac{\kappa}{Y^*_{F2}}\frac{Z_1}{Z_2},$$

$$\text{where } Z_1 = \frac{\frac{1}{1-\frac{\beta}{\alpha}}\left[\int_{\theta=0}^{1}\int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta}\left[\frac{\chi}{\mathcal{E}_2}-(1+i^*)\right]d\varphi d\theta\right]^2}{\left[\frac{\kappa}{1-\frac{\beta}{\alpha}}B_2\int_{\theta=0}^{1}\int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta}\left[\frac{\chi}{\mathcal{E}_2}-(1+i^*)\right]d\varphi d\theta+Y^*_{F2}\right]^2}$$
$$+\int_{\theta=0}^{1}\int_{\varphi=0}^{\frac{\beta}{\alpha}(1-\theta)}\frac{\left[\frac{\chi}{\mathcal{E}_2}-(1+i^*)\right]^2}{\left[\kappa B_2\left[\frac{\chi}{\mathcal{E}_2}-(1+i^*)\right]+Y^*_{F2}\right]^2}d\varphi d\theta$$
$$+\int_{\theta=0}^{1}\int_{\varphi=1-\frac{\beta}{\alpha}\theta}^{1}\frac{\left[\frac{\chi}{\mathcal{E}_2}-(1+i^*)\right]^2}{\left[\kappa B_2\left[\frac{\chi}{\mathcal{E}_2}-(1+i^*)\right]+Y^*_{F2}\right]^2}d\varphi d\theta$$

$$Z_2 = \frac{\int_{\theta=0}^{1}\int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta}\frac{1}{\mathcal{E}_2}d\varphi d\theta}{\left[\frac{\kappa}{1-\frac{\beta}{\alpha}}B_2\int_{\theta=0}^{1}\int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta}\left[\frac{\chi}{\mathcal{E}_2}-(1+i^*)\right]d\varphi d\theta+Y^*_{F2}\right]^2}$$
$$+\int_{\theta=0}^{1}\int_{\varphi=0}^{\frac{\beta}{\alpha}(1-\theta)}\frac{\frac{1}{\mathcal{E}_2}}{\left[\kappa B_2\left[\frac{\chi}{\mathcal{E}_2}-(1+i^*)\right]+Y^*_{F2}\right]^2}d\varphi d\theta$$
$$+\int_{\theta=0}^{1}\int_{\varphi=1-\frac{\beta}{\alpha}\theta}^{1}\frac{\frac{1}{\mathcal{E}_2}}{\left[\kappa B_2\left[\frac{\chi}{\mathcal{E}_2}-(1+i^*)\right]+Y^*_{F2}\right]^2}d\varphi d\theta.$$

Since $\{Z_1,Z_2\}>0$, this result establishes that $\frac{d\chi}{dB_2}>0$. ■

Proof of Corollary 2

If inframarginal external lenders hold the dollar value S_2 of local currency bonds, the marginal lenders now have to hold (B_2-S_2) such bonds. Correspondingly, their profit is $\Pi_2=[\eta_2-(1+i^*)](B_2-S_2)+Q_L(\mathcal{E}_2)$. By inspection of Proposition 3, the results established there continue to hold but with B_2 replaced by B_2-S_2. ■

Proof of Proposition 4

First, we establish the m_2 function. The foreigners' FOC with respect to C^*_{F2} is as follows:

$$C^{-\nu}_{F2}=\Upsilon,$$

where $\Upsilon\pi\left(\mathcal{E}_2|\Omega_L\right)$ is the multiplier on their budget constraint for exchange rate $\mathcal{E}_2 \in \Psi$. The external lenders' SDF is equal to the marginal value of a dollar of their profits Π_2 at each exchange rate $\mathcal{E}_2$. Accordingly, we obtain:

$$M_2 = \Upsilon\left[1 + \varsigma\Theta\left(\Pi_2\right)\left(\underline{\Pi} - \Pi_2\right)\right] = \frac{\left[1 + \varsigma\Theta\left(\underline{\Pi} - \Pi_2\right)\right]}{\left[\Pi_2 - \varsigma\Theta\frac{\left(\underline{\Pi}-\Pi_2\right)^2}{2} + Y_2^*\right]^\nu}. \tag{C.8}$$

A country in case (b) has the insurable set $\widetilde{\Psi} \subset \Psi$ as indicated in Proposition 1. Within that set, Lemma 1 indicates that $\pi\left(\mathcal{E}_2|\Omega_I\right) = \left(1 + i^*\right)p\left(\mathcal{E}_2\right)$. Since $\pi\left(\mathcal{E}_2|\Omega_L\right) = \pi\left(\mathcal{E}_2|\Omega_I\right)$, we obtain:

$$\overline{M} = \frac{\left[1 + \varsigma\Theta\left(\Pi_2\right)\left(\underline{\Pi} - \Pi_2\right)\right]}{\left[\Pi_2 - \varsigma\Theta\left(\Pi_2\right)\frac{\left(\underline{\Pi}-\Pi_2\right)^2}{2} + Y_2^*\right]^\nu} \text{ for all } \mathcal{E}_2 \in \widetilde{\Psi}, \tag{C.9}$$

which means that irrespective of the value of $\Theta\left(\Pi_2\right)$, $Q_L\left(\mathcal{E}_2\right)$ must be set such that the following condition holds:

$$\Pi_2 = \left[\eta_2 - \left(1 + i^*\right)\right]B_2 + Q_L\left(\mathcal{E}_2\right) = \overline{\Pi} \text{ for all } \mathcal{E}_2 \in \widetilde{\Psi}. \tag{C.10}$$

Analogously to the proof of Proposition 3, we can integrate this condition over $\mathcal{E}_2 \in \widetilde{\Psi}$, insert $p\left(\mathcal{E}_2\right) = \frac{\pi\left(\mathcal{E}_2|\Omega_I\right)}{\left(1+i^*\right)}$ into the lenders' insurance market constraint in subsection 2.1, and combine equations (C.8)-(C.10) to obtain:

$$\overline{\Pi} = \frac{1}{1 - \frac{\beta}{\alpha}}B_2\int_{\theta=0}^{1}\int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta}\left[\eta_2 - \left(1 + i^*\right)\right]d\varphi d\theta \tag{C.11}$$

$$m_2 = \frac{1 + \varsigma\Theta\left(\Pi_2\right)\left(\underline{\Pi} - \Pi_2\right)}{1 + \varsigma\Theta\left(\overline{\Pi}\right)\left(\underline{\Pi} - \overline{\Pi}\right)}\left(\frac{\overline{\Pi} - \varsigma\Theta\left(\overline{\Pi}\right)\frac{\left(\underline{\Pi}-\overline{\Pi}\right)^2}{2} + Y_2^*}{\Pi_2 - \varsigma\Theta\left(\Pi_2\right)\frac{\left(\underline{\Pi}-\Pi_2\right)^2}{2} + Y_2^*}\right)^\nu. \tag{C.12}$$

Next, we prove by contradiction that $\frac{\chi}{\alpha+\beta+\gamma} < \left(1 + i^*\right)$. Suppose instead that $\frac{\chi}{\alpha+\beta+\gamma} > \left(1 + i^*\right)$. Since $\mathcal{E}_2 = \alpha + \beta + \gamma$ is the highest value of the exchange rate, the inequality means that $\frac{\chi}{\mathcal{E}_2} > \left(1 + i^*\right)$ for all $\mathcal{E}_2 \in \Psi$. Given that $m_2 > 0$, the premium constraint (3) cannot be satisfied with equality, so there is a contradiction.

Finally, we consider the following two scenarios for the parameter ς.

Scenario 1. For $\varsigma = 0$, we assume that the constrained efficient allocation for case (b) features $\{\beta, B_2\} > 0$. The allocation does not depend on the value of $\underline{\Pi}$. Since $\frac{\chi}{\alpha+\beta+\gamma} < \left(1 + i^*\right)$, we can then set $\underline{\Pi}$ such that:

$$\underline{\Pi} = \left[\frac{\chi}{\alpha + \beta + \gamma} - \left(1 + i^*\right)\right]B_2 < 0. \tag{C.13}$$

Since $\mathcal{E}_2 = \alpha + \beta + \gamma$ is the highest value of the exchange rate, we also obtain that $\underline{\Pi} < \overline{\Pi}$.

Scenario 2. We switch to $\varsigma > 0$. For the value of $\underline{\Pi}$ in equation (C.13), the constrained efficient allocation is the same as in scenario 1. The reason is that for that value of $\underline{\Pi}$, $\Pi_2 \geq \overline{\Pi}$ for all $\mathcal{E}_2 \in \Psi$ even if we restrict $Q_L\left(\mathcal{E}_2\right) = 0$ throughout, so $\Theta\left(\Pi_2\right) = 0$ for all $\mathcal{E}_2 \in \Psi$. Now consider a marginal reduction in $\underline{\Pi}$ below the value in equation (C.13). The planner problem is continuous in the value of $\underline{\Pi}$, so there are marginal changes in $\{\beta, B_2\}$ which preserve their signs. Moreover, we end up with $\Theta\left(\Pi_2\right) = 1$ only for some $\mathcal{E}_2 \in (\alpha + \gamma, \alpha + \beta + \gamma]$, which implies that $\Theta\left(\overline{\Pi}\right) = 0$ is preserved. Finally, in the limit as $\nu \to 0$, we obtain the expression (13) in the proposition. ■

Proof of Proposition 5

Combining the households' budget constraints with the market clearing conditions and the amended expression for the lump sum rebates, we obtain the amended resource constraints:

$$B_1\left(1 + i^*\right) = \left(Y_{F1} - G_{F1} - C_{F1}\right) + \left(Y_{F2} - G_{F2} - C_{F2}\right)\frac{\mathcal{E}_2}{\chi} \text{ for all } \left(\varphi, \theta\right),$$

and we amend the definition of B_2 as follows:

$$B_2 = B_1 (1 + i^*) + (C_{F1} + G_{F1} - Y_{F1}) = (Y_{F2} - G_{F2} - C_{F2}) \frac{\mathcal{E}_2}{\chi}.$$

Setting $\{Y_{F1}, Y_{F2}(\varphi, \theta)\} = \overline{Y}_F$ and $\{A_1, A_2(\varphi, \theta)\} = \overline{A}$, we establish the proposition. ■

Proof of Proposition 6

The treatment of the resource constraints and the definition of B_2 are the same as in the proof of Proposition 5. We also amend the planner's period-t utility function V as indicated to incorporate the welfare obtained from government spending. The planner now optimizes over such spending, and the new FOCs with respect to G_{F1} and G_{F2} are as follows:

$$\frac{\sigma_{G1}}{G_{F1}} = \int_{\theta=0}^{1} \int_{\varphi=0}^{1} \Phi(\varphi, \theta) \pi(\theta|\Omega_I) \, d\varphi d\theta + z_{F1}$$

$$\frac{\delta \sigma_{G2}}{G_{F2}} = \Phi(\varphi, \theta) \frac{\mathcal{E}_2}{\chi}.$$

Combining these FOCs with those with respect to C_{F1} and C_{F2}, we obtain condition (14) in the proposition. ■

Proof of Corollary 3

Relative to the proof of Proposition 6, we restrict $G_{F2} = G_{F2}(\varphi)$. Correspondingly, we amend the FOC with respect to G_{F2} as follows:

$$\int_{\theta=0}^{1} \frac{\delta \sigma_{G2}}{G_{F2}} \pi(\theta|\Omega_I) \, d\theta = \int_{\theta=0}^{1} \Phi(\varphi, \theta) \frac{\mathcal{E}_2}{\chi} \pi(\theta|\Omega_I) \, d\varphi d\theta.$$

Combining this FOC with the one with respect to C_{F2}, we obtain the amended condition (15) in the corollary.

Next, we consider the constrained efficient allocation for subcase (a) (i) subject to the above restriction on G_{F2}. To prove that the allocation features $\beta = 0$, we first establish the solution to a relaxed version of the planner problem and then show that this solution can be achieved by the unrelaxed planner problem described in Corollary 3. Since the country is in subcase (a) (i), we set $\pi(\theta|\Omega_I) = 1$ for all θ and $m_2 = 1$ for all $\mathcal{E}_2 \in \Psi$.

Relaxed planner problem. We relax the planner problem by replacing the exchange rate function (4) as follows:

$$\mathcal{E}_2 = \alpha \varphi + f(\theta), \tag{C.14}$$

where $f(\theta)$ is a relation that can depend on θ as well as constant terms. The relaxed planner can select a relation of the form $f(\theta) = \beta\theta + \gamma$, in which case the solution to the relaxed problem can be achieved by the unrelaxed planner problem, but they can also select other relations. Replacing the exchange rate function means that we must amend the premium constraint (3). Since $m_2 = 1$ for all $\mathcal{E}_2 \in \Psi$, it reduces to the following:

$$0 = \int_{\theta=0}^{1} \int_{\varphi=0}^{1} \left[\frac{\chi}{\mathcal{E}_2} - (1 + i^*) \right] d\varphi d\theta. \tag{C.15}$$

The relaxed planner problem maximizes the utility function (1) subject to the constraints (2), (C.15), and (C.14), with $\{\beta, \gamma\}$ being replaced by $\{f(\theta)\}$ in the planner's choice variables. At the constrained efficient allocation, equations (5)-(7) hold with the settings $\pi(\theta|\Omega_I) = 1$ and $z_\chi = z_{F1} = z_{\omega 1} = z_{\omega 2} = 0$. Equation (8) also holds with those settings, but only for $\omega = \alpha$. There is a continuum of new FOCs with respect to $f(\theta)$ as follows, one for each value of θ:

$$\int_{\varphi=0}^{1} \left\{ \frac{1}{\mathcal{E}_2} \delta \sigma_H \tau_{H2} + B_2 \frac{1}{\mathcal{E}_2} \Phi(\varphi, \theta) \right\} d\varphi = \Lambda \int_{\varphi=0}^{1} \frac{\chi}{(\mathcal{E}_2)^2} d\varphi \text{ for all } \theta,$$

which we combine with the FOC for C_{F2} to obtain:

$$\int_{\varphi=0}^{1}\left\{\frac{1}{\mathcal{E}_2}\delta\sigma_H\tau_{H2}+B_2\frac{\chi}{(\mathcal{E}_2)^2}\frac{\delta\sigma_F}{C_{F2}}\left[1+\frac{\sigma_H}{\sigma_F}\tau_{H2}\right]\right\}d\varphi=\Lambda\int_{\varphi=0}^{1}\frac{\chi}{(\mathcal{E}_2)^2}d\varphi \text{ for all } \theta. \tag{C.16}$$

We can write τ_{H2} and C_{F2} as follows:

$$\tau_{H2}=1-\frac{\mathcal{E}_2 C_{F2}}{\sigma_F\overline{A}} \text{ and } C_{F2}=\overline{Y}_F-G_{F2}(\varphi)-B_2\frac{\chi}{\mathcal{E}_2},$$

so condition (C.16) is identical for each θ. As a result, for any given choice of α, the same value of $f(\theta)$ is selected for all θ. We can rewrite this solution as $f(\theta)=\beta\theta+\gamma$, where $\beta=0$.

Unrelaxed planner problem. Since the relaxed planner selects a relation of the form $f(\theta)=\beta\theta+\gamma$, the solution to the relaxed problem can be achieved by the unrelaxed planner problem described in Corollary 3. Correspondingly, the solutions of the two problems coincide. $\beta=0$ at the constrained efficient allocation for subcase (a) (i) subject to the restriction $G_{F2}=G_{F2}(\varphi)$.

Finally, we consider the impact of the information sensitivity shock in Lemma 3 on the premium constraint (3). Since $\pi(\theta|\Omega_I)=1$ and $\beta=0$ prior to the change in the information structure, condition (9) is not met. Correspondingly, the premium constraint remains satisfied after the change in the information structure, even if all the variable values correspond to the allocation prior to the change in the information structure. ■

Proof of Proposition 7

Combining the households' budget constraints with the market clearing conditions and the amended expression for the lump sum rebates, we obtain the amended resource constraints:

$$B_1(1+i^*)=(Y_{F1}-C_{F1})+\left(\widetilde{Y}_{F2}(\varphi)+\lambda(\theta,X)-X(1+i^*)-C_{F2}\right)\frac{\mathcal{E}_2}{\chi} \text{ for all } (\varphi,\theta),$$

and we amend the definition of B_2 as follows:

$$B_2=B_1(1+i^*)+(C_{F1}-Y_{F1})=\left(\widetilde{Y}_{F2}(\varphi)+\lambda(\theta,X)-X(1+i^*)-C_{F2}\right)\frac{\mathcal{E}_2}{\chi}.$$

These results establish the form of the planner problem. The planner's new FOC with respect to X is as follows:

$$\int_{\theta=0}^{1}\int_{\varphi=0}^{1}\left[\lambda_X(\theta,X)-(1+i^*)\right]\Phi(\varphi,\theta)\frac{\mathcal{E}_2}{\chi}\pi(\theta|\Omega_I)\,d\varphi d\theta=0.$$

Combining this FOC with the one with respect to C_{F2}, and then applying the covariance definition, we obtain condition (17) in the proposition. ■

Proof of Proposition 8

The planner commits to the institutional design with the highest welfare from the perspective of $t=0$.

- If the planner selects $\Delta=N$, the country is in case (b) at $t=1$, and the welfare from the perspective of $t=0$ is the same as the welfare from the perspective of $t=1$.
- If the planner selects $\Delta=T$, the country is in case (a) (ii) at $t=1$, when all agents are informed about the value of θ. From the perspective of $t=0$, the planner does not know the value of θ, so they must evaluate the welfare by integrating across all possible values of that private shock. They also need to subtract the cost of collecting the information. These considerations yield the expression for V^T in the proposition.
- If the planner selects $\Delta=S$, the country is in case (a) (i) at $t=1$. The planner needs to subtract the cost of secrecy, yielding the expression for V^S in the proposition. ■