Optimal Subsidies for Renewable Electricity*

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Abstract

I examine the optimal subsidy for renewable electricity in the absence of an optimal carbon tax. The current most common form of renewable subsidization, the fixed feed-in premium, provides a constant additive markup over wholesale prices. In contrast, the optimal subsidy encourages investment in the most socially valuable renewable production through a multiplicative markup. In the baseline model, this optimal ad valorem subsidy is easily calculable by policymakers, and the optimality of subsidies which increase with wholesale electricity price is robust to a wide range of extensions of the baseline model. Furthermore, I show that the optimal subsidy is preferred by risk-averse investors to a fixed feed-in premium.

1 Introduction

Climate change due to greenhouse gas emissions is one of the biggest global challenges facing the world. Energy production is both a major contributor to this issue, producing roughly one third of global emissions in 2019 (IPCC, 2023) and is also critical for decarbonizing the economy through the electrification of previously fossil-fueled processes. One way to encourage the necessary decarbonization in the electricity sector, favored by many economists, is the implementation of carbon taxes, which would align the public and private costs of emissions. However, full carbon taxation is often not politically feasible[1], and so policymakers have turned to subsidies for renewable electricity production as a second-best alternative. Between 2015 and 2023, the EU alone spent over

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[1]The largest carbon pricing scheme currently implemented is the European Union's Emission Trading Scheme (ETS). As of the writing of this paper, it prices carbon emissions at around 80 €/ton. The EU as a whole has not published an estimated social cost of carbon (SCC), but in 2024, the German Environment Agency, in charge of environmental regulation in the largest European economy, adopted a social cost of carbon between 300 and 880 €/ton (Matthey et al., 2024)

€700 billion on subsidies for renewables (European Commission, 2025).

The most common form of renewable subsidy is the fixed feed-in premium (FFiP).[2] Under such a system, renewable producers sell their electricity on the wholesale market and additionally receive a fixed subsidy per unit of electricity that they generate. When carbon is taxed below its social marginal cost, renewable energy creates social value by displacing fossil-fuel based electricity production and the emissions that it generates (Andor, Voss, 2016). FFiPs would therefore be the optimal subsidy structure if the marginal unit of electricity generation being abated had a constant emissions intensity. However, the marginal emissions rate varies significantly across time and space, and so, to encourage the most socially efficient investments, subsidies should as well.[3]

An example of the heterogeneity in the emissions intensity of the marginal producer across both time and space can clearly be seen in a market like CAISO (the California Independent System Operator, California's primary electrical grid operator). For over 30% of May 2024, real-time wholesale energy prices in CAISO were negative. Renewable producers made more energy than demanded much of the time and could profitably submit negative energy-price bids because of the FFiPs and other abatement-independent subsidies that they received. In the same month, real-time wholesale prices hit a peak of over $1000/MWh when a period of high demand and low renewable supply meant that highly inefficient power plants were called on to provide the marginal units of energy (California Independent System Operator, 2024). In the same period, there were large price differentials between CAISO and its neighbors, as transmission congestion meant that prices couldn't equilibrate across markets. An extra unit of renewable production in the negatively-priced periods and places would not have reduced carbon emissions, while in the high-priced periods and places it would have significantly reduced carbon emissions. An optimal subsidy for renewables should take into account this heterogeneity in abatement value in order to generate the correct investment incentives for investors in renewables.

The key insight of this paper is that only significant variable input to fossil-fuel-based electricity generation is the fuel itself. And in a competitive market, the variable cost of the marginal producer sets the market price. This means that that there is a linear relationship between the price of electricity and the amount of fuel used to generate the marginal unit, and therefore the amount of emissions the marginal unit of generation produces. Therefore, a subsidy proportional

[2]See Kröger et al. (2022) for an overview of the existing renewable energy subsidization landscape.
[3]For empirical analyses of the extent of this heterogeneity see Callaway et al. (2018)

to the electricity price, an ad valorem subsidy, is able to track the externality value of generating more renewable electricity and abating the marginal unit of fossil-fuel-based electricity production.

To formalize this insight, I develop a theoretical model with an arbitrarily large space of potential renewable technologies and locations and show that the welfare-maximizing subsidy takes the form

$$\sigma_t^* \approx p_t \cdot \frac{d - \tau_c}{\gamma_t + \tau_c}$$

where p_t is the real-time wholesale electricity price, d is the social cost of carbon, τ_c is the existing carbon tax, and γ_t is the price of natural gas. This subsidy is independent of any technology or location, and is calculable using only data already routinely collected by governments, making it feasible to implement.

The ad valorem structure also possesses a desirable property with respect to investment risk. Revenue uncertainty, particularly from policy and geopolitical shocks to gas prices and carbon tax levels, drives a significant fraction of the risk premium demanded by investors in renewable electricity projects (Srivastav, 2024; Leblanc, 2023). I show that for any form of risk aversion, an investor would prefer the optimal ad valorem subsidy to an expectationally equivalent FFiP, as it attenuates the change in revenue driven by gas price and carbon tax shocks, while a FFiP does not. This means that a given level of investment incentive can be provided at lower expected fiscal cost under the optimal subsidy than under current FFiP designs.

I examine the robustness of these results across a range of extensions to the baseline model: endogenous fossil capacity investment, costly public funds, multiple fossil fuel types, unobserved heterogeneity in gas contract prices, fixed consumer retail prices, market power in either the renewable or fossil sector, the availability of an electricity consumption tax, and the inclusion of storage and dispatchable clean generation. While most extensions introduce additional terms, my main qualitative result, that optimal renewable subsidies should be increasing in wholesale electricity prices, is preserved in all cases.

This paper builds on three strands of previous literature, one empirical, and two theoretical, which have considered multiple renewable producers and the value of subsidies that might encourage more investment by one producer than another. The empirical strand has shown that subsidies that align revenues and socially-optimal production decisions tend to lead to preferable outcomes.

For example, Aldy et al. (2023) studies the question of how to structure subsidies by comparing capacity and feed-in subsidies for wind installations in the US and found that the feed-in subsidies resulted in significantly higher production than the capacity subsidies. However, they are only able to study the production subsidies as implemented (FFiPs) and not the optimal form that such subsidies should take in a hypothetical world. Similarly, Reguant (2019) compares the welfare effects and incidence of carbon taxation, renewable subsidies, and renewable portfolio standards, but again takes as given that subsidies are fixed. I consider the problem of optimal subsidization theoretically and so am unconstrained by what policymakers have already implemented. This allows me to extend the insight of these papers and show that the optimal subsidy for renewables results in a stronger relationship between market prices and revenues than any subsidy that has yet been implemented.

The first theoretical strand, which includes papers such as Ambec, Crampes (2019) and Helm, Mier (2021), abstracts away from the heterogenity in abatement value in the renewable sector by using a single representative renewable technology. This abstraction means that the optimal subsidies derived in these papers can be thought of as optimal levels of total subsidization of the renewable sector. They do not provide guidance about the structure which an optimal subsidy should take to correctly incentivize investment across a wide portfolio of renewable technologies. Allowing an arbitrarily large space of potential renewable projects enables me to consider how different structures of renewable subsidies incentivize different investment decisions.

The second theoretical strand does consider the optimal subsidy for renewables in the presence of a large and heterogeneous space of potential renewable energy projects. Three papers, Andor, Voss (2016), Abrell et al. (2019), and Tangerås, Wolak (2019), show that the optimal subsidy for renewables is equal to the social value generated by abating the carbon emissions of the marginal unit of fossil-fuel powered generation. Andor and Voss consider the joint problems of emissions and learning externalities, Abrell et al. consider the problem of a revenue-neutral pair of subsidies and consumption taxes, and Tangerås and Wolak consider the joint problems of emissions and transmission externalities. Antweiler (2017) condsiders the joint problems of emissions externalities and capacity requirements and calculates an optimal two part tariff - a uniform contract for differences (CfD) plus a site-specific capacity subsidy. None of the papers, however, thoroughly consider the implementation of these theoretically optimal subsidies. Determining the optimal subsidies proposed in these papers would require governments to have accurate long-term electricity system models for both supply and demand and to use such models to provide individualized subsidies to

each renewable project that is proposed. While such models are potentially feasible, they are likely to be imprecise and their recalculation for every new installation could impose a significant cost on the government. In this paper, I limit my consideration to only emissions externalities. With this simplification, I show that ad valorem optimal subsidies are uniform across projects and do not require any modeling or new information to be collected. Therefore, the subsidy that I calculate in this paper substantially simplifies implementation by policymakers.

2 Model

I develop a simple model to examine the structure of optimal subsidies for renewables. For the baseline model I make several strong assumptions, many of which will be relaxed in a later section.

The model is broken into three periods. First, there is a pre-period when investment decisions are made, then a state of nature is drawn and observed by all agents, and finally there is a realization period when producers produce electricity and consumers consume it. The state of nature is drawn from a given distribution and is denoted t, which encompasses all of the relevant variables for both consumers and producers, such as weather, season, day of the week, etc. This allows the model to accurately represent the possibility that production from various sources might have differing levels of correlation with other production sources as well as with demand.

During the realization period, a representative consumer purchases electricity on the wholesale market at real-time prices and receives utility $V_t(Q_t)$ for consuming Q_t units of electricity in state t. $V_t(Q_t)$ is assumed to be increasing in Q_t, concave, and twice differentiable.

This electricity is produced by two sectors, renewables and fossil fuels. Renewable electricity can be produced with many technologies, in many locations, and with many project-specific parameters, the combination of which describe an individual project. For each project i, installing n_i units of renewable generation capacity costs $I_i(n_i)$, with each $I_i(\cdot)$ assumed to be convex. The convexity represents the fact that the easiest building sites for a given project are used first. Each project produces $n_i e_{it}$ units of electricity in state t with zero marginal cost, for total renewable production $r_t = \sum_i n_i e_{it}$. e_{it} is determined exogenously by the state t and is conceptualized as the weather-dependent real-time capacity factor in a given location.[4]

[4]For simplicity, only the projects that have investment are considered in this model, that is $n_i > 0$ for all i, since including unbuilt projects ($n_i = 0$) does not change the model conclusions but increases notational complexity.

Fossil fuel-based electricity is produced by an existing fleet of plants (conceptualized as natural gas plants) which produce q_t units of electricity for a cost of $C(q_t, \gamma_t)$ for a given exogenous unit cost of gas, γ_t. For simplicity, this model assumes that the only cost of operating these plants is the gas necessary to run them, as the cost of natural gas accounts for approximately 97% of the variable costs (Oh et al., 2021). Thus, $C(q_t, \gamma_t) = \gamma_t G(q_t)$. Additionally, $G(q(t))$ is assumed to be increasing and convex, reflecting the fact that the most efficient fossil plants are used first. Each unit of gas burned produces d units of money-valued climate externality for a total externality cost at a given level of production of $d \cdot G(q_t)$. Consistent with decarbonization pledges, there is no new investment in gas plants in the base model.[5]

Electricity is traded in a single wholesale market with a single representative consumer, a single representative gas-based electricity producer, and a variety of renewable electricity producers. In the base model, all agents are assumed to be price takers. Additionally, electricity is assumed to be a homogeneous good with price p_t, abstracting away from issues of transmission and congestion. More generally, all of the results in this paper apply whenever transmission is separably priced at its social marginal cost and p_t is the locational marginal price of energy.

3 Baseline analysis

3.1 First-best allocation

Total welfare is the expected value of total electricity consumption minus the resource cost of gas and the externality cost of gas, minus the total investment cost of renewable installation. The planner is risk-neutral and places equal weight on all sources of welfare, so he or she maximizes expected total welfare with respect to q_t and n_i, taking the expectation over all possible states t.

$$\mathbb{E}\left[V_t\left(q_t + \sum_i n_i e_{it}\right) - G(q_t)(\gamma_t + d)\right] - \sum_i I_i(n_i) \tag{1}$$

[5]The decision to suppress investment in fossil fuel based generation capacity was made because that is what is needed to meet the climate pledges made by leaders at all levels of government. In a policy broadly representative of those being implemented and discussed across the developed world, 10 EU governments, representing more then 60% of the blocs energy generation, have pledged to decarbonize their electricity sectors by 2035. (Euronews, 2024) In 2018, the average age of retirement for natural gas power plants in the US was over 30 years. (Duquiatan, 2019) Thus, investment in fossil fuel powered capacity is incompatible with the current clean energy goals, and so can be assumed to be negligible under preferred government policies in many of the regions currently implementing significant renewable energy subsidies.

This results in a first-best solution (q_t^*, n_i^*) defined such that

$$p_t^* = V_t' \left(q_t^* + \sum_i n_i^* e_{it} \right) = G'(q_t^*)(\gamma_t + d) \tag{2}$$

and for every i

$$I_i'(n_i^*) = \mathbb{E}[p_t e_{it}] \tag{3}$$

This allocation can be decentralized with competitive investment in renewables, competitive production of electricity, a Pigouvian carbon tax set at d, and no subsidies.

3.2 Politically feasible second-best allocation

Now assume that there is an existing carbon tax fixed at $\tau_c \neq d$. If τ_c is negative this represents a carbon subsidy, if $0 < \tau_c < d$ this represents an inadequate carbon tax, and if $\tau_c > d$ this represents the overtaxation of carbon. In order to correct distortions resulting from a suboptimal carbon tax, the government may introduce a subsidy for renewable energy production (or tax in the case of $\tau_c > d$). A priori, this subsidy, σ_{it}, can vary with state, t, and project, i, and is funded by nondistortionary lump-sum taxation. This model assumes frictionless entry of renewable energy investment, so the zero-profit condition must hold for the marginal unit of investment at every i. Thus:

$$I_i'(n_i) = \mathbb{E}[(p_t + \sigma_{it})e_{it}] \tag{4}$$

Gas-based electricity is assumed to be produced competitively, up to the point where marginal revenue is equal to marginal cost, $p_t = G'(q_t)(\gamma_t + \tau_c)$. This can be rewritten in terms of $V_t'(Q_t)$, the marginal value that consumers get for consuming an extra unit of electricity, and so the price that they are willing to pay to do so.

$$V_t' \left(q_t + \sum_i n_i e_{it} \right) = G'(q_t)(\gamma_t + \tau_c) \tag{5}$$

To solve for the σ_{it}^* which maximizes total welfare a Lagrangian can be used, with the planner maximizing total welfare as in (1) through the choice of σ_{it}, subject to the constraints (5) that arises from gas generators maximizing revenue with respect to q_t. Because the model does not yet include a cost of public funds, there is no cost to the planner for transferring lump-sum payments to or from renewable producers, meaning that (4), the zero-entry condition for renewable investors, does not bind in this step, but will be used to pin down the structure of the optimal subsidy in a

later step. We have

$$\begin{aligned}\mathcal{L} = \mathbb{E}\left[V_t\left(q_t + \sum_i n_i e_{it}\right) - G(q_t)(\gamma_t + d)\right] - \sum_i I_i(n_i) \\ + \lambda_t\left(G'(q_t)(\gamma_t + \tau_c) - V_t'\left(q_t + \sum_i n_i e_{it}\right)\right)\end{aligned} \tag{6}$$

This gives two first derivatives

$$\frac{\partial \mathcal{L}}{\partial q_t} = V_t'(Q_t) - G'(q_t)(\gamma_t + d) + \lambda_t\left(G''(q_t)(\gamma_t + \tau_c) - V''(Q_t)\right) \tag{7}$$

$$\frac{\partial \mathcal{L}}{\partial n_i} = \mathbb{E}[V_t'(Q_t)e_i] - I_i'(n_i) - \mathbb{E}[\lambda_t V_t''(Q_t)e_{it}] \tag{8}$$

Solving (7) for λ_t and substituting the solution into (8) gives

$$\lambda_t = \frac{G'(q_t)(d - \tau_c)}{G''(q_t)(\gamma_t + \tau_c) - V_t''(Q_t)} \tag{9}$$

$$I_i'(n_i) = \mathbb{E}\left[e_i\left(V_t'(Q_t) - \frac{G'(q_t)(d - \tau_c)}{G''(q_t)(\gamma_t + \tau_c) - V_t''(Q_t)}V_t''(Q_t)\right)\right] \tag{10}$$

Substituting in equation (5) and the definitions $\varepsilon_t^D = -\frac{p_t}{Q_t} \cdot \frac{1}{V''(q_t)}$ and $\varepsilon_t^S = \frac{p_t}{Q_t} \cdot \frac{1}{G''(q_t)(\gamma_t + \tau_c)}$, the state varying elasticities of demand and suppy, respectively, then gives

$$I_i'(n_i) = \mathbb{E}\left[\left(p_t + (d - \tau_c)\frac{p_t}{\gamma_t + \tau_c}\frac{\varepsilon_t^S}{\varepsilon_t^S + \varepsilon_t^D}\right)e_{it}\right] \tag{11}$$

Comparing (11) to (4), the renewable investors zero marginal profit constraint, gives the main result of this paper.

Proposition 1. *There exists a unique optimal subsidy which is independent of i. It is an ad valorem subsidy.*

$$\sigma_{it}^* = \sigma_t^* = \underbrace{p_t \cdot \frac{d - \tau_c}{\gamma_t + \tau_c}}_{(a)} \cdot \underbrace{\frac{\varepsilon_t^S}{\varepsilon_t^S + \varepsilon_t^D}}_{(b)}$$

This optimal subsidy has two parts. (a) is the abatement externality generated by producing one extra unit of carbon-free energy in state t. The unpriced carbon externality per unit of energy is $(d - \tau_c)G'(q_t)$. Due to perfect competition, the price of electricity is equal to the cost of gas used to produce it, $p_t = (\gamma_t - \tau_c)G'(q_t)$. Combining these, the unpriced carbon externality is $p_t \cdot \frac{d - \tau_c}{\gamma_t + \tau_c}$. (b) corrects for the fact that when demand is elastic, an additional unit of renewable production does not displace an entire unit of fossil fuel production.

Renewable production decisions are fixed conditional on investment, so subsidies impact the production by a given renewable project only through their impact on investment. This means that there is a whole space of optimal subsidies pinned down by equivalent $\mathbb{E}[\sigma_{it}e_{it}]$, or expected total subsidy for each renewable project. This optimal project-specific subsidy level can be achieved in an infinite continuum of ways, but they all result in the same uniquely optimal set of investments, and policymakers can chose the structure of subsidy that minimizes their implementation costs. Choosing the σ_{it} which is independent of i, i.e. which can be announced once, and then does not need to be recalculated whenever a new project is proposed, therefore seems the most natural. The σ_t^* found in Proposition 1, which does not depend on i, shows that enacting an ad valorem subsidy does this.

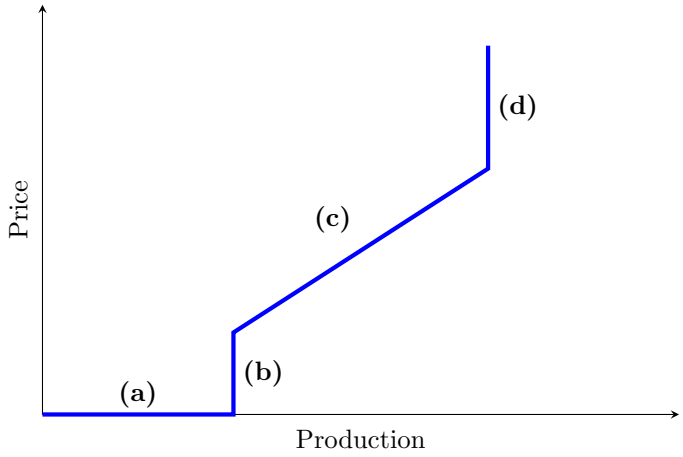


Figure 1: A conceptual merit-order curve

Most governments already collect electricity and gas price data, have estimates of the social cost of carbon, and know what carbon taxes they levy. This means that the first term, $p_t \cdot \frac{d-\tau_c}{\gamma_t+\tau_c}$, can easily be calculated without the need to collect any new data. The elasticity term requires a little more consideration, but can easily be very closely approximated without any new data being collected. To show this, I will break the electricity supply curve into four sections (see Figure 1). (a) is the region in which renewables produce more electricity than is demanded. Because the price of electricity in this region is zero, $\sigma_t^* = 0$ for all ε_t^S. (b) is the region in which there is positive residual demand beyond that supplied by renewable production, but the equilibrium price is below the cost of the most efficient natural gas plant. In this region, $\varepsilon_t^S = 0$, and so $\sigma_t^* = 0$. (c) is the region in which gas producers elastically provide the marginal unit of energy. In this region $\sigma_t^* \approx p_t \cdot \frac{d-\tau_c}{\gamma_t+\tau_c}$. This is because the elasticity of demand for electricity is in general much lower

than the elasticity of supply.[6] Finally, (d) is the region above the the maximum total production capacity of the grid. In this region, $\varepsilon_t^S = 0$, and so $\sigma_t^* = 0$. Summing up,

$$\sigma_t^* \approx \begin{cases} p_t \cdot \frac{d-\tau_c}{\gamma_t+\tau_c} & \frac{p_t}{\gamma_t+\tau_c} \in [G'(0), G'(q_{max})] \\ 0 & \frac{p_t}{\gamma_t+\tau_c} \notin [G'(0), G'(q_{max})] \end{cases}$$

In the regions with zero subsidy, an additional unit of electricity does not generate any abatement, either because all demand is already met by renewables or because the gas-based electricity supply is inelastic. Using this formula, governments can approximate the optimal subsidy very closely without needing to collect any new information about state-varying elasticities. The only variable parameters which need to continuously collected are electricity and gas prices, which governments tend to already collect. The other parameters (the carbon tax rate, social cost of carbon (SCC), and efficiencies of the least- and most-efficient gas-fired power plants) only need to be decided or collected once. See Figure 2 for a sketch of what the optimal markup looks like over time in New England. In 2024, under the optimal subsidy, a SCC of \$141/tonne, and given the existing electricity production system in New England, wind producers would have received an average subsidy of 4.9c/kWh, solar producers would have received an average of 5.2c/kWh, and nuclear producers would have received an average of 5.3c/kWh.

4 Extensions

Having developed a simple base model which provides intuitive and implementable results at the cost of requiring some strong assumptions, in this section I examine the impact of loosening many of those assumptions in order to demonstrate the robustness of the base model's qualitative, if not quantitative, results. For all of the extensions, except the one considering risk aversion, I will treat τ_c as zero in order to simplify the notation. This has no impact on the results, and τ_c can simply be subtracted from each d and added to each γ_t in order to recover the full results in all cases.

[6] The largest use of real-time pricing for consumers to-date is in Spain, which has switched to an opt-out system of real-time pricing nationwide, with roughly 50% of households using real-time pricing. In a study of Spanish residential consumers exploiting exogenous variation in wind production, Fabra et al. (2021) find a short-term elasticity of demand indistinguishable from zero. Taylor et al. (2005) find slightly higher short-term elasticities (0.27) for industrial users in North Carolina facing real-time pricing. Deryugina et al. (2020) study residential consumers in Illinois over the longer term, exploiting variation in community-wide electricity provider changes, and find an average elasticity of 0.29 after 27 months. But these all are insignificant compared to elasticities of supply. While elasticity of supply can vary significantly across regions and levels of demand, in ISO-NE (the grid in the North Eastern United States), for example, the average elasticity of supply on day-ahead markets within $\pm 10\%$ of median load was approximately 6, and longer-term elasticities are presumably much higher. (Hinkel et al., 2010) Thus, even accounting for long-term demand elasticities and more elastic industrial consumers, $\frac{\varepsilon_t^S}{\varepsilon_t^S+\varepsilon_t^D} \approx 1$.

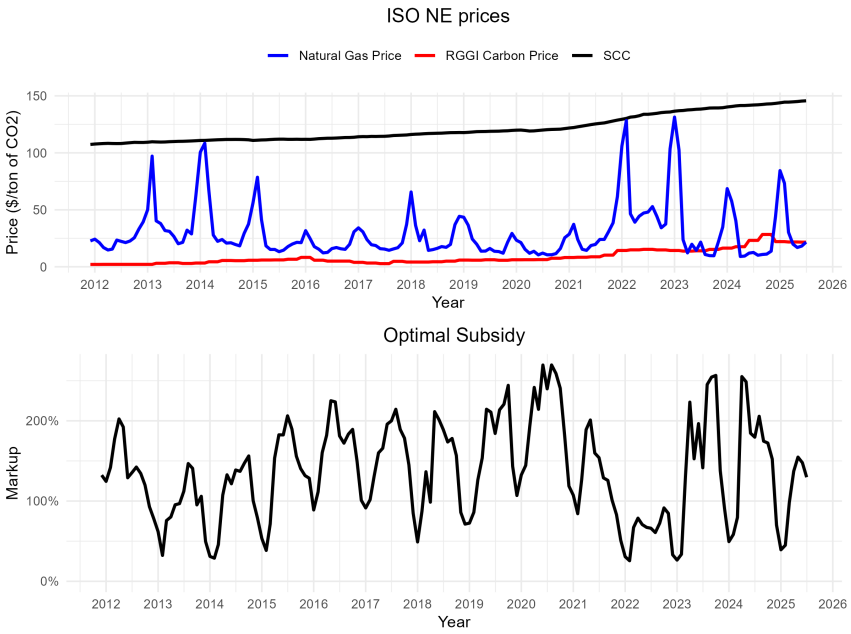


Figure 2: The optimal subsidy in the New England Independent System Operator (ISO NE) given the price of natural gas, the cost of carbon permits from the Regional Greenhouse Gas Initiative (RGGI), and assuming a social cost of carbon of $120 in 2020 dollars. Note that the fluctuations in the percentage markup are largely driven by gas price fluctuations, as RGGI carbon prices have be relatively stable. As discussed in Section 4.7 these fluctuations in subsidy lead to relatively more stable revenues for renewable investors than constant subsidies, as high gas prices drive up the price of electricity.

4.1 Investment in gas-fired generation

Consider the possibility of gas investments. The baseline model assumed that there is an exogenous existing fossil generation base. While this is a reasonable assumption in much of the world today, there are also many regions which still see significant new gas investment, and even more where routine upkeep is necessary to maintain the trajectory of gas production. To extend the applicability of the model to those regions, endogenize gas investments by assuming that the gas supply curve can be broken into many homogeneous gas portfolios, such that for a given quantity of gas investment, n_f, the new fossil production cost function $C(q_t, \gamma_t, n_f) = \gamma_t n_f G\left(\frac{q_t}{n_f}\right)$ and has investment cost $n_f p_f$. This means that each unit of gas investment stretches the ex-post supply curve to the right. This gives an updated social welfare function which accounts for the investment cost in gas-based generation.

$$W = \mathbb{E}\left[V_t\left(q_t + \sum_i n_i e_{it}\right) - n_f(d+\gamma_t)G\left(\frac{q_t}{n_f}\right)\right] - \sum_i I_i(n_i) - n_f p_f \tag{12}$$

This is subject to the same market clearing constraint as in the base model (5), as well as a zero-profit constraint for gas investment

$$\gamma_t G'\left(\frac{q_t}{n_f}\right) = V_t'\left(q_t + \sum_i n_i e_{it}\right) \tag{13}$$

$$p_f = \mathbb{E}\left[\gamma_t\left(G'\left(\frac{q_t}{n_f}\right)\frac{q_t}{n_f} - G\left(\frac{q_t}{n_f}\right)\right)\right] \tag{14}$$

Setting up the Lagrangian and taking derivatives with respect to the choice variables, n_i, q_t, and n_f, then gives the following three equations, omitting variables within functions for notational simplicity.

$$\frac{\partial \mathcal{L}}{\partial n_i} = \mathbb{E}[V_t' e_i] - I_i' - \mathbb{E}[\lambda_t V_t'' e_i] \tag{15}$$

$$\frac{\partial \mathcal{L}}{\partial q_t} = V_t' - (d+\gamma_t)G' - \lambda_t\left(V_t'' - \frac{\gamma_t}{n_f}G''\right) + \mu\left(\frac{\gamma_t}{n_f}G''\frac{q_t}{n_f}\right) \tag{16}$$

$$\frac{\partial \mathcal{L}}{\partial n_f} = d\mathbb{E}\left[G'\frac{q_t}{n_f} - G\right] - \mathbb{E}\left[\lambda_t\frac{\gamma_t}{n_f}G''\frac{q_t}{n_f}\right] - \frac{\mu}{n_f^3}\mathbb{E}\left[\gamma_t G'' q_t^2\right] \tag{17}$$

(16) and (17) can then be combined to solve for λ_t and μ.

$$\lambda_t = -\frac{dG' + \mu\frac{\gamma_t q_t}{n_f^2}G''}{V'' - \frac{\gamma_t}{nf}G''} \tag{18}$$

$$\mu = d\frac{\mathbb{E}\left[\frac{q_t}{n_f}G'\frac{\varepsilon_S}{\varepsilon_S+\varepsilon_D} - G\right]}{\mathbb{E}\left[\frac{\gamma_t q_t^2}{n_f^3}G''\frac{\varepsilon_S}{\varepsilon_S+\varepsilon_D}\right]} \tag{19}$$

Plugging these results for into (15), and comparing it to the zero-marginal profit constraint for renewable producers (4) then allows solving for σ_{it}^*. As in the baseline model, there is a continuum of solutions, but only one which does not depend on i, which I present here, as that is the one that would likely be implemented by policy-makers for feasibility reasons.

$$\sigma_t^* = \underbrace{p_t\frac{d}{\gamma_t}\frac{\varepsilon_S}{\varepsilon_S+\varepsilon_D}}_{(a)} + \underbrace{\frac{\frac{\gamma_t q_t}{n_f^2}G''\frac{\varepsilon_S}{\varepsilon_S+\varepsilon_D}}{\mathbb{E}\left[\frac{\gamma_t q_t^2}{n_f^3}G''\frac{\varepsilon_S}{\varepsilon_S+\varepsilon_D}\right]}}_{(b)}\underbrace{\mathbb{E}\left[G - \frac{q_t}{n_f}G'\frac{\varepsilon_S}{\varepsilon_S+\varepsilon_D}\right]}_{(c)} \tag{20}$$

This subsidy can be broken into five pieces, labeled (a) through (c) above. (a) is the ad valorem optimal subsidy from the baseline model which captures the externality benefit of an extra unit of renewable production. (b) is a scale factor which captures the reduction in gas investment from an extra unit of renewable production. (c) captures the tradeoff inherent in reducing fossil investment in this model. Less investment means that prices are higher, driving down electricity consumption

and so decreasing emissions. But at the same time less investment means that at any given level of demand, electricity is produced less efficiently, increasing emissions. Which effect dominates depends on the shape of the supply curve.

4.2 Subsidization of more-efficient gas plants

In the baseline model, subsidies are restricted to fully renewable power plants. However, it might be beneficial to also subsidize more-efficient gas plants, as they can also reduce emissions when less-efficient plants are the marginal producer. To generalize the analysis, assume that for every unit of electricity produced, new generators need to burn h_i units of gas. For fully renewable plants, $h_i = 0$, while for gas plants $h_i > 0$. For gas plants, let $e_{it} \in \{0, 1\}$ represent whether it will be used to produce electricity in state t. A gas plant will only be used if electricity prices cover its marginal fuel costs, i.e. $e_{it} = 1$ if $p_t > h_i \gamma_t$ and 0 otherwise.

This gives an updated social welfare function, which now includes the fuel and emissions costs of the new gas plants:

$$W = \mathbb{E}\left[V_t\left(q_t + \sum_i n_i e_{it}\right) - G(q_t)(\gamma_t + d) - \sum_i n_i e_{it} h_i(\gamma_t + d)\right] - \sum_i I_i(n_i) \tag{21}$$

This is subject to the market-clearing condition as in the baseline model (5). Setting up the Lagrangian:

$$\begin{aligned}\mathcal{L} =& \mathbb{E}\left[V_t\left(q_t + \sum_i n_i e_{it}\right) - G(q_t)(\gamma_t + d) - \sum_i n_i e_{it} h_i(\gamma_t + d)\right] - \sum_i I_i(n_i) \\ &+ \mathbb{E}\left[\lambda_t\left(G'(q_t)\gamma_t - V_t'\left(q_t + \sum_i n_i e_{it}\right)\right)\right]\end{aligned}$$

Taking the first-order derivatives with respect to the choice variables q_t and n_i gives:

$$\frac{\partial \mathcal{L}}{\partial q_t} = V_t' - G'(q_t)(\gamma_t + d) + \lambda_t \left(G''(q_t)\gamma_t - V_t''\right) \tag{22}$$

$$\frac{\partial \mathcal{L}}{\partial n_i} = \mathbb{E}\left[V_t' e_{it} - h_i(\gamma_t + d)e_{it} - \lambda_t V_t'' e_{it}\right] - I_i'(n_i) \tag{23}$$

Solving the first equation for λ_t and using the market clearing condition $V_t' = G'(q_t)\gamma_t = p_t$ gives:

$$\lambda_t = \frac{G'(q_t)d}{G''(q_t)\gamma_t - V_t''} \tag{24}$$

Substituting λ_t into the derivative for n_i provides the socially optimal marginal investment condition:

$$I_i'(n_i) = \mathbb{E}\left[e_{it}\left(p_t - h_i(\gamma_t + d) - \frac{G'(q_t)d}{G''(q_t)\gamma_t - V_t''}V_t''\right)\right] \tag{25}$$

Applying the definitions of the elasticities of supply and demand and simplifying the final term with $\psi_t = \frac{d}{\gamma_t}\frac{\varepsilon_t^S}{\varepsilon_t^S + \varepsilon_t^D}$ gives the investment condition for the optimal allocation:

$$I_i'(n_i) = \mathbb{E}\left[e_{it}\left(p_t - h_i(\gamma_t + d) + p_t\psi_t\right)\right] \tag{26}$$

To decentralize this allocation, the policymaker sets a subsidy σ_{it} equal to the net unpriced social abatement value in expectation:

$$\mathbb{E}[\sigma_{it}^*] = \mathbb{E}[p_t\psi_t - h_i d] \tag{27}$$

The policymaker cannot offer $\sigma_{it}^* = p_t\psi_t - h_i d$ because this might sometimes be negative, meaning that more efficient new plants might be dispatched after less efficient existing plants not in the subsidy scheme. To ensure that the most efficient collection of available plants is always dispatched first, it is necessary to impose a non-negativity constraint. To achieve this, the policymaker truncates the subsidy so that $\sigma_{it}^* = \max(0, p_t\psi_t - h_i d - K_i)$, for some individual K_i chosen to ensure that the truncation does not cause deviation from the optimal level of investment found in (26).

Let Ω be the set of all states t, with associated probability measure $dF(t)$. Let be Ω_{p_i} the subset of states where plant i produces energy, i.e. $e_{it} = 1$, and Ω_{s_i} the subset of states where plant i is subsidized, i.e. $\sigma_{it}^* > 0$. Then from (27):

$$\int_{\Omega_{s_i}} \left(p_t\psi_t - h_i d - K_i\right) dF(t) = \int_{\Omega_{p_i}} \left(p_t\psi_t - h_i d\right) dF(t) \tag{28}$$

$$= \int_{\Omega_{p_i}\setminus\Omega_{s_i}} \left(p_t\psi_t - h_i d\right) dF(t) + \int_{\Omega_{s_i}} \left(p_t\psi_t - h_i d\right) dF(t) \tag{29}$$

$$K_i = \frac{\int_{\Omega_{p_i}\setminus\Omega_{s_i}} \left(h_i d - p_t\psi_t\right) dF(t)}{Pr\left(\Omega_{s_i}\right)} \tag{30}$$

This gives the final optimal subsidy for a generalized set of new renewable and gas plants:

$$\sigma_{it}^* = \max\left(0, p_t\frac{d}{\gamma_t}\frac{\varepsilon_S}{\varepsilon_S + \varepsilon_D} - h_i d - \frac{\int_{\Omega_{p_i}\setminus\Omega_{s_i}} \left(h_i d - p_t\frac{d}{\gamma_t}\frac{\varepsilon_S}{\varepsilon_S+\varepsilon_D}\right) dF(t)}{Pr\left(\Omega_{s_i}\right)}\right) \tag{31}$$

While this K_i is difficult for policymakers to calculate, in practice it is likely to not be that significant. Because it is correcting for truncating the region where $\frac{p_t}{\gamma_t}\frac{\varepsilon_S}{\varepsilon_S+\varepsilon_D} < h_i$ but $\frac{p_t}{\gamma_t} > h_i$, as

$\varepsilon_D \to 0$, $K_i \to 0$. This means that in most cases, where $\varepsilon_D << \varepsilon_S$, failing to exactly calculate K_i, or even not including it entirely, is likely to lead to only small distortions from the second-best. The optimal subsidy for new gas plants is very close to simply being equal to the baseline subsidy plus a carbon tax.

4.3 Costly public funds

The baseline model assumes that the cost of public funds is 1, that subsidies have no inherent deadweight cost from the taxes necessary to raise to fund them. In reality, however, taxation is costly. To account for this, assume that each unit of subsidy generates a welfare loss of ϕ. Given that it is now valuable to raise public funds, government will attempt to design a transfer scheme which extracts all inframarginal rents from the renewable sector, $\mathbb{E}[p_t r_t] - \sum I_i$. This gives the new welfare equation with the same constraints as the baseline model:

$$W = \mathbb{E}\left[V_t\left(q_t + \sum_i n_i e_{it}\right) - (d+\gamma_t)G(q_t)\right] - \sum_i I_i(n_i) - \phi\left(I_i(n_i) - \mathbb{E}\left[p_t \sum_i n_i e_{it}\right]\right) \quad (32)$$

Setting up the Lagrangian and taking derivatives with respect to the choice variables n_i and q_t, then gives the following two equations, omitting variables within functions for notational simplicity

$$\frac{\partial \mathcal{L}}{\partial n_i} = \mathbb{E}[V_t' e_i] - (1+\phi)I_i' + \phi\mathbb{E}[V_t'' r_t e_{it} + V_t' e_{it}] - \mathbb{E}[\lambda_t V_t'' e_{it}] \quad (33)$$

$$\frac{\partial \mathcal{L}}{\partial q_t} = V_t' - (d+\gamma_t)G' + \phi V_t'' r_t + \lambda_t(\gamma_t G'' - V_t'') \quad (34)$$

Solving these for $I'(n_i)$ then gives

$$I_i'(n_i^*) = \mathbb{E}\left[e_{it}\left(p_t + \frac{1}{1+\phi}\left(p_t \frac{d}{\gamma_t}\frac{\varepsilon_S}{\varepsilon_S+\varepsilon_D} - \phi\frac{p_t \frac{r_t}{Q_t}}{\varepsilon_S+\varepsilon_D}\right)\right)\right] \quad (35)$$

Plugging this into the renewable investment constraint, (4), and choosing the form independent of i:

$$s_t^* = \underbrace{\frac{1}{1+\phi}}_{(a)} p_t \left(\underbrace{\frac{d}{\gamma_t}\frac{\varepsilon_S}{\varepsilon_S+\varepsilon_D}}_{(b)} - \underbrace{\phi\frac{\frac{r_t}{Q_t}}{\varepsilon_S+\varepsilon_D}}_{(c)}\right) \quad (36)$$

This marginal subsidy can be broken into three parts. (b) is simply the subsidy from the baseline model. It is deflated by (a) to account for the fact that each unit of subsidy generates a loss ϕ, meaning that the total social value of a subsidized unit of renewable is reduced, and so the subsidy is reduced commensurately. Finally, (c) accounts for the fact that each additional unit of renewables reduces the profits of all other renewables. Because the government is now collecting

all inframarginal rents, this reduces government revenue and so is costly.

The subsidy calculated above is only the optimal marginal subsidy. As discussed previously, the full optimal subsidy in the case of costly public funds needs to collect all inframarginal rents. The simplest way to do this is to pay the optimal marginal subsidy s_t^* on all production and levy a lump-sum tax on each project equal to their expected rent.

$$F_i^* = \mathbb{E}[e_{it}(p_t + s_t^*)] - I_i(n_i^*) \tag{37}$$

However, this scheme requires a lot of money to flow from investors to the government and back to investors again. If there are capital constraints, or a lack of trust in the government to follow through with the subsidies after lump-sum taxes have been collected, this could inefficiently increase costs. If a project would be unprofitable without subsidies, $\mathbb{E}[e_{it}p_t] < I_i(n_i^*)$, then the optimal subsidy with costly public funds can be implemented by subsidizing production at a rate s_t^* on for those units installed in at given project beyond a threshold, $\bar{n}_i = \frac{F_i}{\mathbb{E}[\sigma_{it}^* e_{it}]}$. This threshold is chosen so that $\mathbb{E}[e_{it}(p_t + \sigma_t^*)] = I_i(n_i^*)$, allowing the government to capture all rents.

In the case of costly public funds, the optimal marginal subsidy is still approximately ad valorem and does not require project-specific information to calculate. However, the collection of all inframarginal rents requires the government to know the investment costs and expected revenues of a given project in order to calculate those rents. This means that the optimal total subsidy scheme in the case of costly public funds will require the collection of project-specific information.

4.4 Multiple fuel types

Assume now that there are multiple types of fuel used to produce electricity. The fuels are indexed by j and each have their own price p_j and externality d_j. Each fuel is turned into electricity by representative plants, each requiring $F_j(q_t^j)$ units of fuel to produce q_t^j units of electricity. Each F_j is increasing and convex, reflecting the fact that the most efficient production units are used first. Each representative plant is a price taker, meaning that for all fuels which are marginal, i.e. $\frac{\partial q_t^j}{\partial p_t} \neq 0$, $p_t = p_j F_j'(q_t^j)$. Further define a fuel-specific supply function as the inverse of F_j' such that $S_j\left(\frac{p_t}{p_j}\right) = q_t^j$. Each type of fuel is "fractionally marginal" in proportion to its contribution to the slope of the aggregate supply curve at the market equilibrium with $\frac{\partial q_t^j}{\partial q_t} = \frac{\frac{\partial q_t^j}{\partial p_t}}{\sum_k \frac{\partial q_t^k}{\partial p_t}} = \frac{\frac{1}{p_j} S_j'\left(\frac{p_t}{p_j}\right)}{\sum_k \frac{1}{p_k} S_k'\left(\frac{p_t}{p_k}\right)}$.

As in the baseline model, total expected welfare is equal to the value of electricity to consumers

minus the cost of generating it, except that now the cost of generating fossil-fuel-based electricity is the sum of the costs associated with each fuel

$$W = \mathbb{E}\left[V_t(Q_t) - \sum_j (p_j + d_j)F_j(q_t^j)\right] - \sum_i I_i(n_i) \tag{38}$$

Setting up the Lagrangian and taking its derivatives with respect to q_t^j and n_i then gives

$$\frac{\partial \mathcal{L}}{\partial q_t^j} = V_t' - (p_t + d_j)F_j' + \lambda_{tj} p_j F_j'' - V_t'' \sum \lambda_{tj} \tag{39}$$

$$\frac{\partial \mathcal{L}}{\partial n_i} = \mathbb{E}[V_t' e_{it}] - I_i' - \mathbb{E}\left[\lambda_{tj} V_t'' e_{it}\right] \tag{40}$$

These can then be solved for $I_i' = \mathbb{E}[(p_t + \sigma_t^*)e_{it}]$ and the unique optimal subsidy independent of i, σ_t^*, can be chosen. As in the baseline model, the expected production-weighted subsidy is equal to the expected production weighted externality abated by generating one extra unit of renewables in state t:

$$\sigma_t^* = -\sum_j d_j F_j' \frac{\partial q_t^j}{\partial r_t} \tag{41}$$

$$= -\sum_j d_j F_j' \frac{\partial q_t^j}{\partial q_t} \frac{\partial q_t}{\partial r_t} \tag{42}$$

Because $\frac{\partial q_t^j}{\partial q_t} = 0$ for all j which are not marginal, F_j' can be replaced with $\frac{p_t}{p_j}$ without issue. $\frac{\partial q_t^j}{\partial q_t}$ can be replaced with $\frac{\frac{1}{p_j} S_j'\left(\frac{p_t}{p_j}\right)}{\sum_k \frac{1}{p_k} S_k'\left(\frac{p_t}{p_k}\right)} = w_j$, as defined above. And $\frac{\partial q_t}{\partial r_t}$ can be replaced with $-\frac{\varepsilon_S}{\varepsilon_S + \varepsilon_D}$ as in the baseline. This gives the final formula for the optimal subsidy in the case of multiple fuels:

$$\sigma_t^* = p_t \cdot \sum_j \left(\frac{d_j}{p_j} \cdot w_j\right) \cdot \frac{\varepsilon_S}{\varepsilon_S + \varepsilon_D} \tag{43}$$

This formula is almost identical to the baseline result, except that now the contribution of each fuel type is weighted by its share of the marginal unit of non-renewable production. These weights are not necessarily trivial for policymakers to compute, as they likely vary across the supply curve and with fuel prices. However, in the case the fuels are "well ordered" such that at any point in the supply curve only one fuel is marginal, e.g. coal-generated electricity is always cheaper than gas-generated, then $w_j \in \{0, 1\}$ and policymakers can simply set a threshold, q^*, between the least efficient coal plant and the most efficient gas plant. Above this threshold $\sigma_t^* = p_t \cdot \frac{d_{gas}}{p_{gas}} \cdot \frac{\varepsilon_S}{\varepsilon_S + \varepsilon_D}$ and below it $\sigma_t^* = p_t \cdot \frac{d_{coal}}{p_{coal}} \cdot \frac{\varepsilon_S}{\varepsilon_S + \varepsilon_D}$.

4.5 Unobserved heterogeneity in gas prices

Spot prices for natural gas are typically publicly available with very high temporal granularity. However, most natural gas used by power plants is supplied via private fixed contracts[7], and so the marginal fuel cost of the marginal plant is generally not easily observed. In the baseline model, the elasticity of the optimal subsidy with respect to electricity prices, $\varepsilon_\sigma = \frac{\partial \mathcal{L}n(\sigma_t^*)}{\partial \mathcal{L}n(p_t)} = 1$, i.e. the optimal subsidy is ad valorem. However, as will be proven in this section, when there is unobserved heterogeneity in the price of gas the relationship between electricity price and the optimal subsidy is weakened, with $0 < \varepsilon_\sigma < 1$, and this elasticity is monotonically decreasing in the scale of the heterogeneity. For this section, assume that $\frac{\varepsilon_S}{\varepsilon_S+\varepsilon_D} = 1$ for simplicity, although the results apply when $\frac{\varepsilon_S}{\varepsilon_S+\varepsilon_D} = C$ for any constant C.

Assume that there is a continuum of infinitesimal price-taking natural gas plants indexed by j, each drawing an individual heat rates, h_j, and gas prices, γ_j, from common, known distributions of both. Denote the log of an individual plant's heat rate as $ln(h_j) = x_j$, the log of an individual plant's gas price as $ln(\gamma_j) = y_j$, and the log of the electricity price as $ln(p_t) = z_t$. x_j and y_j are assumed to be distributed independently and to have log-concave densities. Analogously to the baseline model, σ_{it}^* is defined such that $\mathbb{E}[\sigma_{it}^* \cdot e_{it}] = d \cdot \mathbb{E}\left[h_j^{marginal} \cdot e_{it}\right]$, and we will select the σ_t^* independent of i such that $\sigma_t^* = d \cdot \mathbb{E}\left[h_j^{marginal}\right]$. A given plant is marginal when $p_t = h_j \cdot \gamma_j$, or in logs when $z_t = x_j + y_j$. This means that the optimal subsidy can be rewritten as

$$\sigma_t^* = d \cdot \mathbb{E}\left[e^{x_j} | z_t = x_j + y_j\right] \tag{44}$$

To prove the lower bound, $\varepsilon_\sigma > 0$, first expand the derivative of the log of σ_t^* with respect to z_t.

$$\frac{\partial \mathcal{L}n(\sigma_t^*)}{\partial z_t} = \frac{\partial}{\partial z_t} ln\left(d \cdot \mathbb{E}\left[e^{x_j} | z_t = x_j + y_j\right]\right) \tag{45}$$

$$= \underbrace{\frac{1}{\mathbb{E}\left[e^{x_j} | z_t = x_j + y_j\right]}}_{(a)} \cdot \underbrace{\frac{\partial}{\partial z_t} \mathbb{E}\left[e^{x_j} | z_t = x_j + y_j\right]}_{(b)} \tag{46}$$

The first term, (a), is clearly positive. The second term, (b), is positive by Efron's monotonicity property (Efron, 1965) because e^u is a monotonically increasing function. This means that the overall expression is positive, $\varepsilon_\sigma > 0$.

To prove the upper bound, follow the same procedure, replacing x_j with $z_t - y_j$ in the expo-

[7] In 2024 79% of natural gas used by power plants in the US was supplied by firm contracts (U.S. Energy Information Administration, 2025).

nent.

$$\frac{\partial \mathcal{L}n(\sigma_t^*)}{\partial z_t} = \frac{\partial}{\partial z_t} ln\left(d \cdot \mathbb{E}\left[e^{z_t - y_j} | z_t = x_j + y_j\right]\right) \tag{47}$$

e^{z_t} is constant conditional on z_t and so can be removed from the expectation leaving

$$\begin{aligned}
\frac{\partial \mathcal{L}n(\sigma_t^*)}{\partial z_t} &= \frac{\partial}{\partial z_t} ln\left(d \cdot e^{z_t} \cdot \mathbb{E}\left[e^{-y_j} | z_t = x_j + y_j\right]\right) && (48)\\
&= 1 + \frac{\partial}{\partial z_t} ln\left(\mathbb{E}\left[e^{-y_j} | z_t = x_j + y_j\right]\right) && (49)\\
&= 1 + \underbrace{\frac{1}{\mathbb{E}\left[e^{-y_j} | z_t = x_j + y_j\right]} \cdot \frac{\partial}{\partial z_t} \mathbb{E}\left[e^{-y_j} | z_t = x_j + y_j\right]}_{(a)} && (50)
\end{aligned}$$

Using Efron's monotonicity property as above, and noting that e^{-u} is a monotonically decreasing positive function, it is clear that $(a) < 0$ and therefore that $\varepsilon_\sigma < 1$

To show that ε_σ monotonically decreases in the scale of the heterogeneity, rewrite y_j as $y_j = \bar{y} + \lambda \epsilon_j$, with a shared constant, $\bar{y}$, a plant-specific heterogeneity term, ϵ_j, and a scale parameter on the heterogeneity, λ. Assume further that both ϵ_j and x_j are distributed symmetrically with quasi-concave and log-concave densities, respectively. Then, Vaeth (2025) shows that increasing λ will monotonically compress the conditional expectation, $\mathbb{E}\left[e^{x_j} | z_t = x_j + \bar{y} + \lambda \epsilon_j\right]$, towards the unconditional expectation, $\mathbb{E}\left[e^{x_j}\right]$, reducing ε_σ.

4.6 Fixed consumer prices

In the baseline model, consumers are assumed to directly face wholesale market prices. In reality, most electricity consumers buy their electricity at a fixed price from utilities. To model this, assume that $V_t' = \bar{p}$, that is consumers always face a price of electricity $\bar{p}$ and make their consumption decisions based on that. I will assume that this $\bar{p}$ is set competitively by profit-maximizing utilities which additionally charge a fixed connection fee. Following Joskow, Tirole (2006), the fixed price will be equal to the long-term average of the wholesale price weighted by the derivative of the demand function $\bar{p} = \frac{\mathbb{E}\left[\frac{p_t}{V_t''}\right]}{\mathbb{E}\left[\frac{1}{V_t''}\right]}$. The welfare function is the same as in the baseline (1).
Setting up the Lagrangian and taking derivatives with respect to the choice variables n_i and q_t,

then gives the following two equations, omitting variables within functions for notational simplicity

$$\frac{\partial \mathcal{L}}{\partial n_i} = E[V' e_{it}] - I'(n_i) + \mathbb{E}[\lambda_t V_t'' e_{it}] - \mu \mathbb{E}\left[\frac{(\gamma_t G' - \bar{p}) V_t''' e_{it}}{(V_t'')^2}\right] \tag{51}$$

$$\frac{\partial \mathcal{L}}{\partial q_t} = V_t' - (d + \gamma_t) G' + \lambda_t V_t'' + \mu \left(\frac{\gamma_t G'' V'' - (\gamma G' - \bar{p}) V_t'''}{(V_t'')^2}\right) \tag{52}$$

(51) and (52) can be solved for λ_t and μ, which can then be plugged into (51) and compared to the renewable investment zero-profit condition (4), and a σ_t^* can be selected.

$$\sigma_t^* = \underbrace{p_t \frac{d}{\gamma_t}}_{(a)} - \underbrace{\frac{\mathbb{E}\left[\frac{p_t}{V_t''}\frac{d}{\gamma}\right]}{\mathbb{E}\left[\frac{\gamma_t G''}{(V_t'')^2} - \frac{(p_t - \bar{p}) V_t'''}{(V_t'')^3} - \frac{1}{V_t''}\right]}}_{(b)} \underbrace{\frac{\varepsilon_D}{\varepsilon_S}}_{(c)}$$

The optimal subsidy is made up of three parts. (a) is the baseline subsidy. With fixed retail prices, the short-run elasticity of demand is now zero, with one extra unit of renewable generation fully displacing a unit of gas generation. (b) is the shadow cost of fixed prices, and (c) is the value of price smoothing. It is increasing in ε_D because as elasticity falls, the fixed consumer price becomes a more accurate reflection of true demand inelasticity, and so the fixed prices generate less dead-weight loss. It is decreasing in ε_S because when supply becomes more elastic it is less costly to smooth prices on the supply side.

Introducing fixed consumer prices to the model does add an additional price-smoothing term to the optimal subsidy equation, but there are two key features of it for the robustness of the main results. Firstly, like in the baseline model the optimal subsidy with fixed consumer prices can be formulated such that it is independent of i. This means that the government does not need to collect project-specific information to implement the optimal subsidy. Secondly, like in the baseline model, this happens when the subsidy has an ad valorem component. With fixed consumer prices, there now is also an additively separable component that accounts for the value of price smoothing. However, this smoothing value is not unique to the renewable sector, and would optimally be applied across the electricity sector. If such a sector-wide optimal smoothing subsidy were already in place, the optimal renewable-specific subsidy would simply be the subsidy found in the baseline model.

4.7 Monopoly

The baseline model assumes perfect competition. Electricity markets are often highly concentrated, however, and so firms might be expected to use their market power. First consider the case of market power in the renewable sector. Assume that a firm owns a portfolio of renewable production facilities j, and makes its investment decisions to maximize total profits across its portfolio. This means that the marginal profit constraint (4) for each investment i is replaced with

$$I_i'(n_i) = \mathbb{E}\left[\left(p_t(t) + \sigma_{it}\right) e_{it} - \frac{\partial p_t(t)}{\partial n_i} \sum_j e_j(t) n_j\right] \tag{53}$$

With $\frac{\partial p_t(t)}{\partial n_i} = V_t'' e_{it} = \frac{\varepsilon_t^D e_{it}}{q_t}$, using the same methodology as in the baseline model, it is easy to see that the monopoly subsidy is a simple additive correction to the optimal subsidy, following traditional monopoly subsidization results.

$$\sigma_t^* = p_t(t) \frac{d - \tau_c}{\gamma_t(t) + \tau_c} \cdot \frac{\varepsilon_t^S}{\varepsilon_t^S + \varepsilon_t^D} + \varepsilon_t^D \sum_j \frac{e_j(t) n_j}{q_t} \tag{54}$$

The case where fossil producers have market power can be modeled similarly, by adding a markup m within the market-clearing condition.

$$V_t'\left(q_t + \sum_i n_i e_{it}\right) = G'(q_t)(\gamma_t(t) + \tau_c + m) \tag{55}$$

Because there is no incentive for the planner to redistribute, and there is no cost of public funds, in the base model this is indistinguishable from a carbon tax. Therefore, the optimal subsidy simply changes to:

$$\sigma_t^* = p_t(t) \frac{d - \tau_c - m}{\gamma_t(t) + \tau_c + m} \cdot \frac{\varepsilon_t^S}{\varepsilon_t^S + \varepsilon_t^D} \tag{56}$$

Generally, these results demonstrate that the presence of market power in either sector does not significantly alter the main results of this paper. Market power in the renewable sector can be dealt with separably from the optimal subsidy which is motivated by emissions concerns, and market power in the fossil sector is indistinguishable from a carbon tax and so fits easily in the optimal subsidy.

4.8 Renewable subsidy and electricity consumption tax

Now expand the potential instruments available to the government to include a time-variable consumption tax on electricity consumption as well as a subsidy for renewable energy production.

This slightly adjusts the relationship between price and the value function, so $V_t'(Q_t) = p_t(t)+\tau_e(t)$ for some consumption tax $\tau_e(t)$. Following Ambec, Crampes (2019), this tax then allows first-best welfare to be achieved and for the removal of the elasticity correction term in the subsidy by eliminating the overproduction of electricity relative to optimal due to the subsidy. At optimal subsidy and tax levels, the price faced by consumers of gas-based electricity is $G'(q_t)\gamma_t(t) + \tau_e = (\gamma_t+d)G'(q_t)$, the full social cost including externalities, and the price for renewables is undistorted. This results in an optimum of

$$\sigma_{it}^* = \tau_e^*(t) = p_t(t)\frac{d}{\gamma_t(t)} \tag{57}$$

There is a distributional difference between this renewable-subsidy-and-consumption-tax system and a carbon tax, however. This system taxes all gas-based electricity at $dG'(q_t)$, even electricity that is produced by more efficient facilities. Because $G(q_t)$ is convex, this means that this system will collect more revenue than an optimal carbon tax. If there were gas-based capacity investment in this model, that would distort investment away from more efficient facilities, decreasing welfare relative to the first-best. However, because there is no such investment, this distortion does not occur and the first-best can be achieved even without a carbon tax.

Not only is the first-best achieved without a carbon tax, but it is achieved with very little additional information gathering required by the government. All that the government needs to collect to implement this optimum is electricity and gas prices, along with a single calculation of the social cost of carbon. This is no more costly than implementing a carbon tax, as gas sales need to be monitored in each case, and electricity prices are already collected by system operators.

4.9 Storage and dispatchable renewables

The baseline model is presented as a static model in which there is just a pre-period and a realization period, with investors maximizing profits in expectation. That profit maximization in expectation can equally well be thought of as the steady-state response to an infinite horizon of random draws with replacement from the state space, with zero intertemporal elasticity of demand or supply. In this interpretation of the model, then, there is the possibility to store electricity across states. Assuming optimal storage and discharge behavior, storage, or any other dispatchable emissions-free generation, fits directly into the model like any other potential renewable energy source, just one which might sometimes have negative production. Investors will select a "project" that specifies their storage and discharge decisions in all states of the world, making their net production ex-post exogenous as for other renewables. This does not change the formula for the optimal subsidy, but

it does require that the storage operators and investors have perfect foresight about future states, as optimal storage decisions under uncertainty is currently an unsolved problem.

4.10 Variability of profits and risk aversion

Investors have so far been assumed to be risk-neutral, however there is strong evidence that there is actually a significant risk premium for renewable electricity projects with uncertain revenues and reducing this risk premium has been one of the key justifications for implementing fixed feed-in premiums. (Srivastav, 2024) This risk premium is in large part driven by policy and technology uncertainty, which is particularly hard for investors to accurately model compared to demand and weather uncertainty. (Leblanc, 2023) Thus, when comparing possible subsidy mechanisms, policy-makers may want to reduce the variability of potential investors' revenue streams with respect to unanticipated political and geopolitical shocks in order to reduce the cost at which projects can be built.

In this section I show that the optimal subsidy is less sensitive to unanticipated shocks to gas prices and carbon taxes than fixed feed-in premiums. This means that a risk averse investor with any form of risk aversion over those two prices, will prefer the baseline subsidy to an additively equivalent fixed subsidy. This means that when investors are risk averse, the same investment incentive generated by a fixed subsidy can be generated by the baseline subsidy at a lower cost to the government.

To make this comparison I calculate the derivatives of revenue with respect to shocks under two different subsidy systems for the same investment: the revenue under the optimal subsidy defined in Proposition 1, omitting the elasticity correction for notational simplicity, and an expectationally equivalent fixed feed-in premium. For the response of revenue to gas price shocks, I am able to bound the change in revenue under the optimal subsidy between zero and the change in revenue under the fixed subsidy. By doing so, I show that an investor facing any distribution of shocks with any form of risk aversion will prefer the optimal subsidy to the fixed feed-in premium.[8] The revenue responses to changes in carbon taxes have opposite signs under the optimal and fixed subsidies, so I assume that shocks are symmetric and bound the magnitude of the response under the optimal subsidy between zero and the response under a fixed subsidy. This imposes an assumption

[8]It is hard to imagine that this method for comparing the preferences of arbitrarily risk-averse agents is novel, but I have been unable to find another example of it in the literature, so it is at least not widely used. I hope that in addition to contributing to the study of optimal subsidization, this paper might also help to popularize this method of risk-aversion analysis, as it allows one to consider risk aversion without needing to specify a particular functional form, either for the agent's preferences or the distribution of shocks.

of symmetry on the distribution of potential carbon tax shocks, but maintains the agnosticism to the form of risk aversion.

Define the revenue per unit of investment under an optimal subsidy as

$$\pi_i^* = e_i \left(p_t + p_t \frac{(d - \tau_c)}{\gamma_t + \tau_c} \right) \tag{58}$$

and revenue per unit of investment under a fixed feed-in premium with the same expected value as

$$\pi_i^F = e_{it} \left(p_t + \frac{\mathbb{E}\left[e_{it}(d - \tau_c) \frac{p_t}{\gamma_t + \tau_c} \right]}{\mathbb{E}[e_{it}]} \right) \tag{59}$$

Taking the derivatives of both revenues with respect to a change in gas prices and noting that $\frac{p_t}{\gamma_t + \tau_c} = G'$ and $\frac{\partial p_t}{\partial \gamma_t} = G' \frac{\varepsilon_t^S}{\varepsilon_t^S + \varepsilon_t^D}$ results in

$$\frac{\partial \pi_i^*}{\partial \gamma_t} = e_i \left(\frac{\varepsilon_t^S}{\varepsilon_t^S + \varepsilon_t^D} G' - \frac{d - \tau_c}{\gamma_t + \tau_c} \left(\frac{\varepsilon_t^D}{\varepsilon_t^S + \varepsilon_t^D} G' \right) \right) \tag{60}$$

$$\frac{\partial \pi_i^F}{\partial \gamma_t} = e_i \frac{\varepsilon_t^S}{\varepsilon_t^S + \varepsilon_t^D} G' \tag{61}$$

Assuming that $\frac{\varepsilon_t^S}{\varepsilon_t^D} \geq \frac{d - \tau_c}{g + \tau_c}$, as is the case in virtually all electricity markets[9], then

$$0 \leq \frac{\partial \pi_i^*}{\partial g} \leq \frac{\partial \pi_i^F}{\partial g}, \tag{62}$$

meaning that any risk averse investor facing any distribution of potential gas price shocks would prefer the optimal subsidy (13) to an equivalent fixed feed-in premium. This happens because at any point along the gas generation curve, the baseline subsidy is constant. But when an increase in gas prices drives up electricity price, elastic consumers consume less electricity, moving down the gas generation curve and pushing down the subsidy. This has the effect of dampening changes in revenue. Note also that subsidy expenditures fall during positive gas price shocks, generating fiscal space for other policy responses.

[9] Using a carbon price of €180 per ton, a carbon tax of €65 per ton, and a cost of natural gas of €45 per MWh, $\frac{d - \tau_c}{g + \tau_c}$ is approximately 0.04, as in Europe, an example of an economy with both high carbon taxes and gas prices, and so a low value of this quantity. In the US where the carbon tax is approximately zero, and natural gas is much cheaper (€7.50 per MWh), $\frac{d - \tau_c}{g + \tau_c}$ is approximately 0.37, providing a rough measure of the opposite end of the range of these quantities around the world. $\frac{\varepsilon_t^S}{\varepsilon_t^D}$ is much higher than either 0.04 or 0.37, however, as discussed previously. So for risk-averse investors in both Europe and the US, moving to the optimal subsidy from an equivalent fixed subsidy would be preferred.

Following the same procedure for carbon tax shocks,

$$\frac{\partial \pi_i^*}{\partial \tau_c} = e_i \left(\frac{\partial p_t}{\partial \gamma_t} - G' \right) \left(\frac{d - \tau_c}{\gamma_t + \tau_c} + 1 \right) \tag{63}$$

$$= e_i G' \left(\frac{\varepsilon_t^S}{\varepsilon_t^S + \varepsilon_t^D} - 1 \right) \left(\frac{d - \tau_c}{g + \tau_c} + 1 \right) \tag{64}$$

$$\frac{\partial \pi_i^F}{\partial \tau_c} = e_i G' \frac{\varepsilon_t^S}{\varepsilon_t^S + \varepsilon_t^D} \tag{65}$$

Comparing the two derivatives and assuming $\frac{1+\frac{d-\tau_c}{g+\tau_c}}{2+\frac{d-\tau_c}{g+\tau_c}} \leq \frac{\varepsilon_t^S}{\varepsilon_t^S+\varepsilon_t^D}$ then gives[10]

$$\left| \frac{\partial \pi_i^*}{\partial \tau_c} \right| \leq \left| \frac{\partial \pi_i^F}{\partial \tau_c} \right| \tag{66}$$

This means that for any symmetrically distributed carbon taxation shocks, any risk-averse investor would prefer the optimal subsidy to an expectationally equivalent fixed subsidy, and so could be encouraged to make the same investment at a lower expected cost to the government.

4.11 Discussion

These extensions have shown that even when the strong assumptions of the baseline model are weakened, the main result that optimal renewable electricity subsidies should be proportional to prices still generally holds as long as other market frictions are addressed optimally. While renewable subsidization can be used to, at least partially, address a wide range range of frictions other than inadequate carbon taxation, it generally does so separably, and when those frictions are addressed with a different instrument, their existence doesn't change the underlying result that renewables should be subsidized roughly proportionally to electricity prices.

5 Conclusion

The model in this paper allows for a fully general space of renewable technologies and siting decisions as well as a fully general subsidy function, a significant generalization from previous papers which assumed either a single representative renewable producer or fixed per-unit production subsidies. Not only does this allow the model to more accurately represent the real world, unlike previous papers this allows the optimal structure of renewable subsidies to be examined.

[10] $\frac{1+\frac{d-\tau_c}{g+\tau_c}}{2+\frac{d-\tau_c}{g+\tau_c}} \leq \frac{\varepsilon_t^S}{\varepsilon_t^S+\varepsilon_t^D}$ is again true for all realistic markets because as discussed above $\frac{d-\tau_c}{g+\tau_c}$ ranges from approximately 0 to approximately 0.4, while $\frac{\varepsilon_t^S}{\varepsilon_t^S+\varepsilon_t^D} \approx 1$

This paper shows that governments can implement optimal renewable subsidies without needing any site-specific information by utilizing a subsidy that varies with wholesale electricity price. In fact, a very close approximation of the optimal subsidy can be implemented without any knowledge of the market structure - only real-time electricity prices, along with long-run gas prices and an estimated social cost of carbon, are needed, all data already collected by governments. Additionally, such a subsidy would likely command a lower risk premium from risk-averse investors than current fixed feed-in premiums, further increasing the welfare gains from switching to such a subsidy.

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