A Lost Decade of Fiscal Misallocation

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Abstract

This paper examines how fiscal expansion influenced resource allocation during Japan's Lost Decade (1993-2002). We document two key empirical patterns: a deterioration in the allocation efficiency of government investment, and a substantial increase in government borrowing driven by interest rate declines. We develop a politico-economic theory of the fiscal curse of easy borrowing: relaxing government borrowing constraint induces allocation to become more politically driven than efficiency-oriented. This mechanism is incorporated into a quantitative model calibrated to the Japanese economy, and we conduct counterfactual exercises to assess its quantitative effects. If the government had been forced to spend the extra borrowing revenue on consumption while keeping the shadow price of government investment intact, the allocation of government capital would have improved substantially, raising aggregate TFP by 0.43% and welfare by 0.31% during the Lost Decade.

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1 Introduction

Governments worldwide have accumulated substantial public debt since the 1980s. The scope for fiscal adjustment has been far greater than previously recognized, and such adjustments have become increasingly consequential for resource allocation and aggregate productivity. Japan's experience during the "Lost Decade", which refers to the period 1993-2002 in this paper, is particularly revealing for two reasons.[1] First, the Lost Decade features a significant fiscal expansion: the total government expenditure-to-GDP ratio increased by 2.5 percentage points relative the previous decade. This expansion was primarily driven by substantially more aggressive government borrowing and massive interest rate cut. Second, we document a novel fact that fiscal resource allocation efficency deteriorated significantly during the Lost Decade. While low aggregate productivity growth in the Lost Decade has been extensively studied since Hayashi and Prescott (2002), it remains unclear how fiscal expansion affects fiscal resource allocation efficiency and to what extent a deterioration in such efficiency contributes to low aggregate productivity.

This paper begins with a comprehensive documentation of Japan's fiscal resource allocation. We find that the Lost Decade features a higher government-to-private investment ratio than either the previous or subsequent decade. Importantly, this investment shift is associated with a widening gap between the returns to government and private-sector capital during the Lost Decade. Moreover, there is a significant deterioration in the spatial allocation of government investment. Prefecture-level government investments exhibited a strong negative correlation with economic returns, a pattern unique to the Lost Decade. We also find Japanese government debt dynamics follow a fiscal rule: a borrowing-constrained government always borrows up to the maximum debt level, which is determined by its fiscal capacity to service interest payments and repay maturing government bonds. This rule implies that the fiscal expansion in the Lost Decade was primarily driven by low-interest-rate-induced increases in government borrowing, which we refer to as "easy borrowing" for brevity.

We next propose a politico-economic theory of the fiscal curse of easy borrowing to explain the worsening of fiscal misallocation during the Lost Decade. The government balances the tradeoff between maximizing household welfare and pursuing political objectives by responding to local demands for government investment. Political incentives are modeled through local lobbying capacity.[2] Compared to the first-best allocation, this political economy exhibits government capital

[1] Our definition of the Lost Decade aligns with the conventional timeframe (early 1990s to early 2000s) commonly adopted in the literature. Hayashi and Prescott (2002), for instance, focuses on 1991-2000 as the Lost Decade, while Peek and Rosengren (2005) and Caballero, Hoshi and Kashyap (2008) examine the periods 1993-1999 and 1990-2002, respectively. We deliberately exclude the initial years of the 1990s to create a clear distinction between the Lost Decade and the preceding asset price bubble burst in 1990-1992.

[2] Local-interest groups exert considerable influence in obtaining substantial transfers from the central government, predominantly in the form of public works (Ihori et al. (2009)). Moreover, these funds are often directed to politically favoured but unproductive uses (Sato (2002)).

wedges that distort both the consumption-investment decision and the spatial investment allocation in the government sector. These distorted allocations represent a compromise between economic efficiency and political influence. In this political economy framework, a looser government borrowing constraint reduces the shadow price of the fiscal budget, not only increasing government investment, but also inducing its spatial allocation to be more politically driven than efficiency-oriented. These model predictions align consistently with the empirical patterns observed during Japan's Lost Decade.

We generalize the simple theory to a small open economy with overlapping generations of households. While interest rate declines led to fiscal expansion during the Lost Decade, our model treats them as exogenous, since the mechanism of the fiscal curse of easy borrowing does not depend on any particular source of the loosening of government borrowing constraints. Unlike households, the government administration is long-lived, though it operates with a lower discount factor, reflecting the probability of being replaced by another administration in subsequent periods. The government optimally chooses sequences of consumption, investments, subject to its budget constraint, private-sector decisions, general equilibrium conditions, as well as an endogenous debt limit determined by XXX.[3]

The model incorporates multiple regions, each characterized by a time-varying institutional parameter that captures local lobbying capacity to influence government capital allocation, as well as time-varying private-sector capital and labor wedges that distort local factor prices despite perfect factor mobility across regions. We calibrate these wedges to match private-sector capital and labor distributions in each prefecture. Following the "institutional accounting" framework developed by Song and Xiong (2024), we recover local institutional parameters to match cross-prefecture government capital allocation. Despite the model's rich variation in institutions and frictions across prefectures and over time, our counterfactual analysis shows that these factors play a surprisingly small role in explaining both the deterioration in the allocation of investment between the private and government sectors and the spatial misallocation of government investment during the Lost Decade.

We design a specific counterfactual to assess the quantitative effects of the fiscal curse of easy borrowing. First, we estimate the magnitude of the loosening of government budget constraint in each period based on changes in expected yield-to-maturity on newly issued government bonds. In our counterfactual, the government is forced to spend the extra bugdet on consumption, making the shadow price of the fiscal budget intact for government investment. By deliberately isolating the effect of interest rate cuts on the fiscal budget, we avoid confounding our analysis with the broader impact of interest rate cuts on private consumption and investment – an important but

[3]The welfare weights assigned to different household generations in the government objective function ensure time consistency in the dynamic optimization problem. Our main findings remain robust when we assume commitment technology and solve the corresponding Ramsey problem under alternative welfare weights.

contentious issue on which the literature has yet to reach consensus.

This counterfactual tightens the government budget, representing a reversal of the fiscal curse of easy borrowing. The gap between the returns to government and private-sector capital narrows from 3.9% to 2.4%. The strong negative cross-prefecture correlation between government investment and its returns disappears. The improved cross-sector and spatial allocation of government investments increases the aggregate TFP in the Lost Decade by 0.43%. Welfare also increases by 0.31% using the equivalent consumption variation. Unlike counterfactuals that vary local institutional parameters and frictions, this exercise demonstrates that the loosening of the government borrowing constraint alone played a decisive role in shaping the allocation of resources throughout the Lost Decade.

While the Lost Decade has been widely regarded as a period characterized by low productivity growth since Hayashi and Prescott (2002), the fiscal dimensions of this era remain understudied. The fiscal curse of easy borrowing is related to the misallocation of fiscal windfalls in the literature. Caselli and Michaels (2013) demonstrate how fiscal windfalls from oil output affect disproportionately government expenditures in Brazil. Similarly, Brollo et al. (2013) find that transfers from central to local governments in the same country significantly increase corruption levels. Our study extends this literature by analyzing the production allocation efficiency of "fiscal windfalls" due to a loosening of government borrowing constraint and quantifying their effects on aggregate TFP and welfare.

Our work is also complementary to the literature on low interest rate and stagnation: Caballero, Hoshi and Kashyap (2008) on zombie lending, Kiyotaki, Moore and Zhang (2021) on overvalued (intangible) assets; and Asriyan et al. (2024) on misallocation via financial frictions.

Finally, it is worth emphasizing that Japan operates under a highly centralized fiscal system (e.g., Sato (2002); Ihori et al. (2009)). The central government actively influenced resource allocation through large government investments around the Lost Decade. This centralization minimizes the confounding influences of local government policies, thereby providing analytical clarity that sharpens our understanding of the underlying politico-economic mechanisms.

The rest of the paper is organized as follows. Section 2 presents the basic facts. We construct a simple model in Section 3 to illustrate the politico-economic mechanism behind fiscal windfall-induced misallocation. We extend the simple model into a dynamic framework and calibrate it in Section 4. Section 5 conducts the counterfactual analysis. Section 6 concludes.

2 Facts

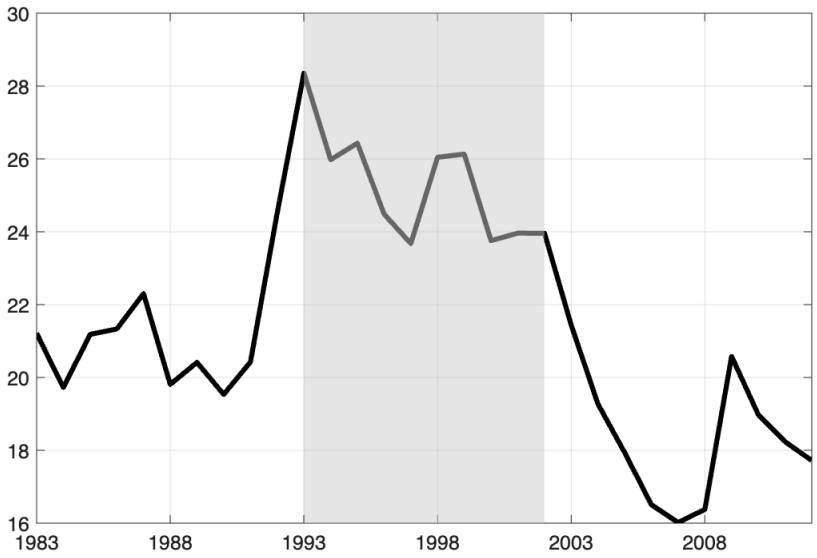
In this section, we present a set of facts regarding Japanese government investment and borrowing during the Lost Decade and the decades before and after. All data are obtained from official sources, including Japan's Ministry of Finance, the National Accounts of Japan, and the Cabinet Office.

Appendix A provides detailed information on data construction procedures.

2.1 Government Investment

Figure 1 plots the government-to-private investment ratio over the period 1983–2012. The shaded area represents the Lost Decade. Government investment (I_t^G) is measured by gross fixed capital formation (GFCF) by the central and local governments.[4] Private investment (I_t^K) is calculated as total GFCF minus government investment. The government-to-private investment ratio increased dramatically from 20.4% in 1991 to 28.4% in 1993, remained at a high level throughout most of the Lost Decade and then rapidly declined afterward, stabilizing around 18% after 2003.

Figure 1: Government-to-Private Investment Ratio (%)

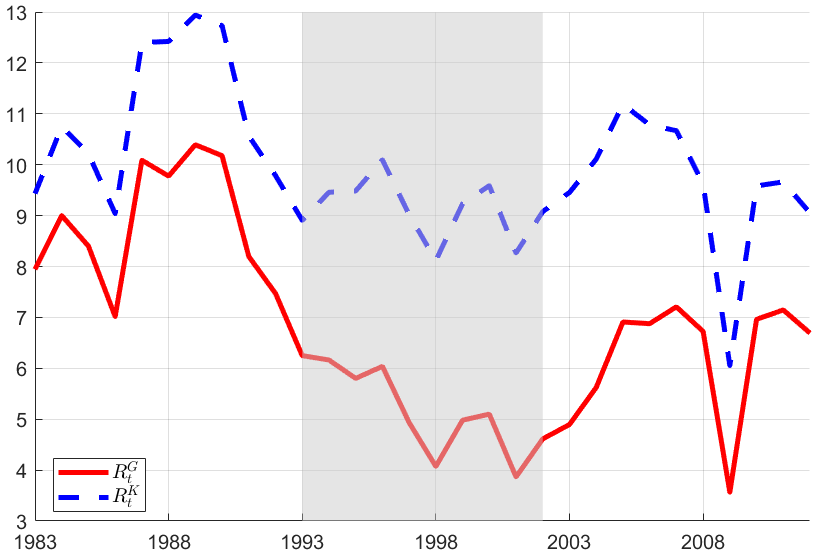


We calculate the aggregate returns to government capital (R_t^G) and private-sector capital (R_t^K) based on a Cobb-Douglas aggregate production function estimated using prefecture-level data.[5] Figure 2 illustrates these returns, with R_t^G represented by the solid red line and R_t^K by the dashed blue line. While both returns reached their lowest levels during the Lost Decades, the gap between them widened to its maximum. The average difference ($R_t^K - R_t^G$) measured 3.9% during the Lost Decades, exceeding the averages of 2.2% and 3.4% observed in the preceding and subsequent decades, respectively.

[4] We exclude GFCF provided by social security funds and public enterprises. Our finding is robust to the inclusion of these entities.

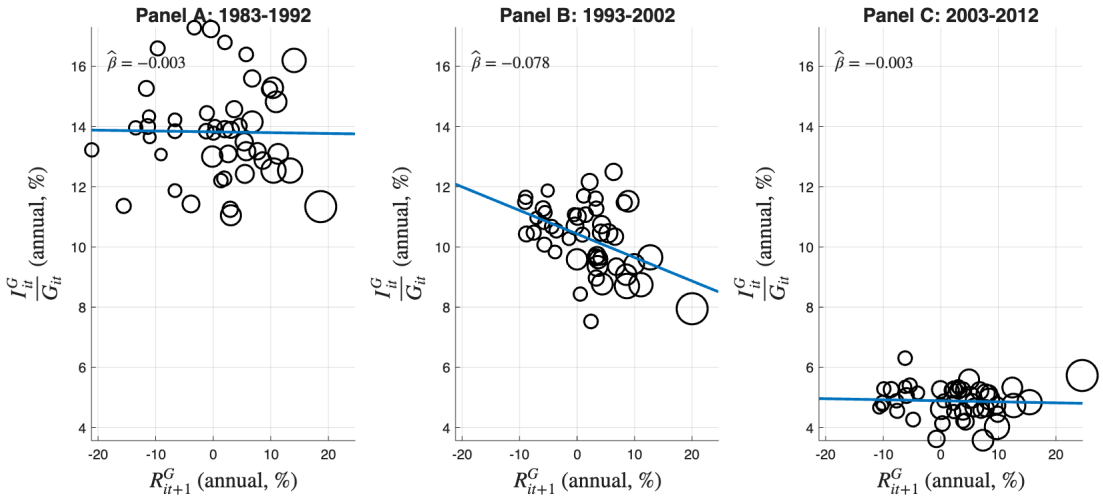
[5] See Section 4.6.2 for details of the estimation.

Figure 2: Aggregate Returns to Government and Private-sector Capital (%)



We next compile prefecture-level data on government investment and capital stock, denoted by I_{it}^G and G_{it}, respectively. We next calculate the aggregate return to prefecture government capital, R_{it}^G, based on a spatial equilibrium model with Cobb-Douglas production technology in each prefecture.[6] Figure 3 plots the relationship between R_{it+1}^G and I_{it}^G/G_{it} across the three time periods.

Figure 3: $\frac{I_{it}^G}{G_{it}} \sim R_{it+1}^G$



[6]See Section 4.6.2 for the definition and estimation of R_{it}^G. Note that the aggregate return differs from the local returns, which is essentially determined by the ratio of local output to local government capital in each prefecture and therefore does not capture general equilibrium effects in the spatial model. However, the two returns are highly correlated, and all results presented below are robust to using local returns instead.

The first notable observation is the large but relatively stable cross-prefecture dispersion of R^G_{it+1}, with standard deviations of 8.7%, 6.3%, and 7.6% in the respective time periods. However, the correlation between R^G_{it+1} and I^G_{it}/G_{it} varies substantially across periods. The local government investment rate is uncorrelated with returns both before and after the Lost Decade. This pattern contradicts the prediction of standard investment models, which suggest that investment rates should be higher when returns are higher, thus indicating spatial misallocation of government capital during these periods. More strikingly, there is a strong negative correlation during the Lost Decade, signaling a significant deterioration in spatial allocation efficiency during that period. These observations are confirmed by simple OLS regression results presented in Table 1.

Table 1: $\frac{I^G_{it}}{G_{it}} \sim R^G_{it+1}$

$\frac{I^G_{it}}{G_{it}}$	1983-1992	1993-2002	2003-2012
R^G_{it+1}	-0.003	-0.078***	-0.003
	(0.028)	(0.024)	(0.010)
Observations	47	47	47
R^2	0.000	0.190	0.002

These observations indicate substantial variation in prefecture-level government investments over time. The average government investment rate during the Lost Decade exhibits zero correlation with the corresponding averages in either the preceding or subsequent decades (correlation of 0.04 and 0.06).[7]

The time-varying spatial allocation of government investment cannot be explained by changes in local fiscal conditions. Rather, changes in prefecture-level government investment are highly correlated with fluctuations in fiscal transfers from the central government to prefecture governments, which is supported by simple OLS regression presented in Table 2. This finding aligns with the highly centralized structure of the Japanese fiscal system, where central authorities maintain substantial control over regional investment patterns.

[7]The cross-prefecture correlation of government investment growth, I^G_{it}/I^G_{it-1}, between the Lost Decade and the following decade is also weak (correlation of -0.18).

Table 2: $\frac{I^G_{it}}{G_{it}} \sim \frac{\text{Transfer}_{it}}{\text{Revenue}_{it}}$

$\frac{I^G_{it}}{G_{it}}$	1983-1992	1993-2002	2003-2012
$\frac{\text{Transfer}_{it}}{\text{Revenue}_{it}}$	0.009	0.056***	0.004
	(0.019)	(0.014)	(0.008)
Observations	47	47	47
R^2	0.004	0.255	0.007

We also compile prefecture-level data on government consumption, denoted by C^G_{it}, where i represents each prefecture, from the national accounts (Appendix A.1.6). The spatial allocation of government consumption demonstrates considerably greater persistence. The correlation of government consumption growth $\frac{C^G_{it}}{C^G_{it-1}}$ between the Lost Decade and the subsequent decade reaches 0.61, which stands in stark contrast to the zero correlation observed in government investment growth. In the following analysis, we will abstract away from the spatial allocation of government consumption and focus on the spatial allocation of government investment.

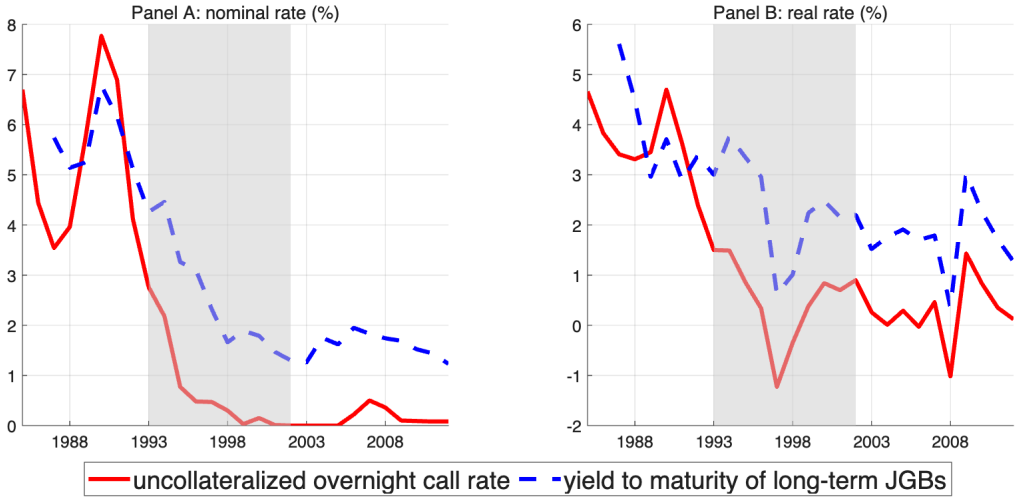
In summary, the Lost Decade features a higher government-to-private investment ratio than both the preceding and subsequent decades. The widening dispersion of aggregate returns to government and private-sector capital indicates a deterioration of capital allocation efficiency during the Lost Decade. The cross-prefecture correlation between government investment rates and returns to government capital further suggests a significant decline in spatial allocation efficiency during the Lost Decade.

2.2 Interest Rate Cuts

Japan's monetary policy during the Lost Decade has been intensively studied in the literature. Following Hayashi and Koeda (2013), we plot the uncollateralized overnight call rate in Figure 4 (solid line) to illustrate the rapid adoption of the near-zero interest rate policy – a pivotal policy shift that underlies the mechanism central to our study. A similarly pronounced decline is observable in Japanese Government Bond (JGB) interest rates, as depicted by the dashed line representing the yield to maturity of long-term JGBs.[8]

[8]These interest rate cuts were largely unanticipated and initially perceived as temporary, primarily due to the BOJ's optimistic assessments of economic conditions (as evidenced in the *Monthly Bulletin* publications of 1991-92 analyzed by Jinushi, Kuroki and Miyao (2000)). Similarly, the implementation of the Zero Interest Rate Policy (ZIRP) in 1999 and subsequent Quantitative Easing Policy (QEP) in 2001 represented unexpected monetary interventions that significantly influenced market expectations. As Nakazawa and Osada (2024) demonstrates, these policies had significant effects on the expected short-term rate component of long-term JGB yields.

Figure 4: Interest Rate

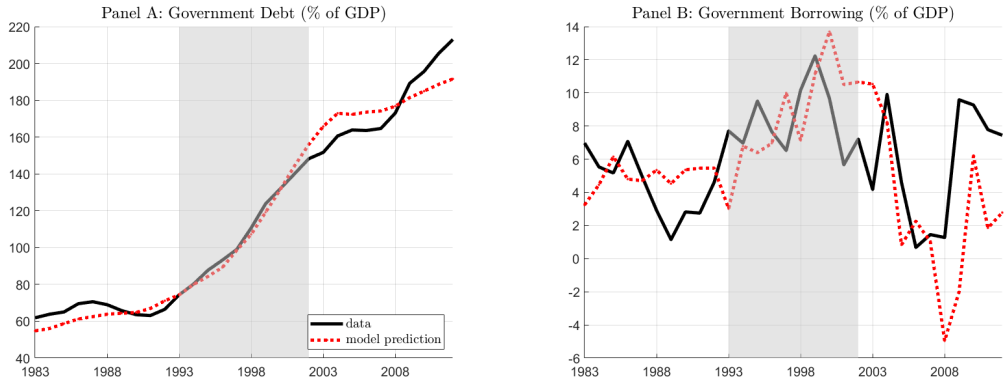


The right panel presents real interest rates – nominal rates adjusted for inflation using the CPI. While the decline in real interest rates appears less dramatic, it remains quantitatively significant. The real yield to maturity of long-term JGBs fell from an average of 3.84% during 1987-1992 to 2.38% during the Lost Decade. As demonstrated in Section 4, this substantial reduction in government borrowing costs significantly expanded fiscal capacity and can account for all the empirical patterns documented above.

2.3 A Simple Debt Rule

The dramatic interest rate cut in the Lost Decade was associated with a substantial increase in government borrowing. We let D_t denote total government debt, which includes both central and local government debt but excludes Fiscal Investment and Loan Program (FILP) bonds from central government debt (see Appendix A.2). The solid line in the left panel of Figure 5 shows that the government debt GDP ratio started to take off since the Lost Decade. Denote by $B_t \equiv D_{t+1} - D_t$ government borrowing. The solid line in the right panel plots the government borrowing GDP ratio increased from an average of 4.4% during 1983-1992 to 8.3% during the Lost Decade.

Figure 5: Government Debt and Borrowing



Furman and Summers (2020) documented that despite the rapidly rising government debt GDP ratios in the G7, their interest payment-to-GDP ratios had been stable or even declining since the mid-1980s. They argue that the interest payments as a share of GDP, a flow-to-flow measure, provides a more appropriate metric of fiscal sustainability than the government debt-to-GDP ratio, a stock-to-flow measure. The flow-flow perspective "looks at how much that debt costs today, in interest payments, as compared to income today, providing a sense of how affordable that debt is."

A constant interest payment-to-GDP ratio has a straightforward interpretation when all debt is financed by issuing perpetual bonds. Without loss of generality, we assume that taxes are collected at the end of each period and interest is paid at the beginning of each period. The maximum debt level for which interests can be serviced, denoted by $\bar{D}$, is then given by $\bar{D} = \frac{bY_t}{r_{t+1}}$, where b denotes the fraction of output that can be collected as taxes to cover interest payments. A borrowing-constrained government will therefore borrow up to the limit, conforming to the constant interest payment-to-GDP ratio $\frac{r_{t+1}D_{t+1}}{Y_t} = b$.

We extend the debt limit by accommodating non-perpetual bonds. Denote by a the fraction of outstanding debt that cannot be rolled over. The debt limit is generalized to

$$\bar{D} = \frac{bY_t}{a + r_{t+1}}. \tag{1}$$

This simple debt rule fits the dynamics of Japanese government borrowing well, not only during the lost decade but also in the decades before and after. We jointly estimate a and b by minimizing the sum of squared errors between the debt in the data and that implied by the debt rule. The estimated values of a and b are 0.014 and 0.048, with bootstrapped standard errors of 0.0016 and 0.0021, respectively, and R^2 is 0.98. The high R^2 value suggests that the interest rate decline accounts for essentially all of the increase in government borrowing during the lost decade. The dotted lines in Figure 5 plot the predicted D_{t+1} and B_{t+1} as a percent of Y_t. XXX footnote: the importance of a XXX

2.4 Fiscal Expansion

The decline in interest rates and the associated increase in government borrowing led to a significant fiscal expansion during the Lost Decade. The government's nominal budget can be decomposed into three components:

$$E_t = NB_t + B_t - r_t D_t, \tag{2}$$

where E_t denotes total government expenditure. The right-hand side captures government net revenue, including non-borrowing income NB_t (mainly tax revenue) and borrowing revenue B_t net of interest payment $r_t D_t$, where r_t denotes the interest rate for government debt. Table 3 reports fiscal revenue components for the periods 1983-1992, 1993-2002, and 2003-2012.

Table 3: Fiscal Revenue (% of GDP)

	NB_t	B_t	$r_t D_t$
1983-1992	23.3	4.2	4.0
1993-2002	21.3	8.3	3.5
2003-2012	21.2	5.6	2.4

The non-borrowing income-to-GDP ratio declined by two percentage points during the lost decade. However, this decline was more than offset by the large increase in government borrowing depicted in Figure 5. The borrowing revenue-to-GDP ratio increased by four percentage points, leading to a sizable fiscal expansion during the lost decade, a 2.6 percentage-point increase in the total government expenditure-to-GDP ratio. The low interest rates shown in Figure 4 also directly contributed to fiscal expansion through reduced $r_t D_t$ during the lost decade and subsequent periods, though this effect is much smaller than the borrowing channel during the lost decade.

3 A Simple Model

This section develops a simple model that identifies efficiency wedges by contrasting a multi-region political economy with the first-best allocation. Government capital wedges emerge as the central mechanism through which more government wealth from easy borrowing may lead to overinvestment and spatial misallocation of government capital.

3.1 The Economy

We consider a two-period economy consisting of N regions indexed by i. Production takes place in the first period. Each region i has a production function given by:

$$Y_i = A_i G_i^{\alpha}, \tag{3}$$

where Y_i denotes regional output, A_i represents regional total factor productivity, G_i is regional government capital, and $\alpha \in (0,1)$ is the output elasticity of government capital. Aggregate government capital and output are defined as $G \equiv \sum_i G_i$ and $Y \equiv \sum_i Y_i$, respectively.

The representative household lives for two periods. In each period, they derive utility $u(C_t)$ from a Cobb-Douglas aggregation of household consumption C_t^H and government consumption C_t^G, where

$$C_t = \left(C_t^H\right)^{\rho} \left(C_t^G\right)^{1-\rho}, \tag{4}$$

where $\rho \in [0,1]$ captures the intensity of preference for household consumption. $u' > 0$, $u'' < 0$, $\lim_{C\to 0} u' = \infty$, $\lim_{C\to\infty} u' = 0$. Unlike G_i, which varies across region i, we do not distinguish region-specific government consumption.[9] The household life-time utility $U^H = u(C_1) + \beta u(C_2)$, where β is the discount factor.

We assume a small open economy facing an exogenous world interest rate r with no initial wealth[10]. The consolidated budget constraint is given by:

$$C_1^G + C_1^H + G + \frac{C_2^G + C_2^H}{1+r} = Y + W^G + W^H \tag{5}$$

, where W^G and W^H captures the exogeneously determined initial wealth for government and household, respectively.

3.2 First-Best Allocation

A benevolent planner maximizes U subject to the resource constraint (5). The first-order condition with respect to G_i is

$$R_i^G = 1, \tag{6}$$

where $R_i^G \equiv \frac{\partial Y}{\partial G_i} = \alpha \frac{Y_i}{G_i}$ denotes the aggregate return to G_i. Hence, R_i^G is equalized across regions. Define aggregate TFP as

$$A \equiv \frac{Y}{G^{\alpha}}. \tag{7}$$

[9] This simplification reflects the empirical finding that cross-prefecture allocation of government consumption has exhibited remarkable persistence over time (see Section 2).

[10] A more general case considering the initial government and initial household wealth doesn't influence any mechanism mentioned in the section.

It follows that aggregate TFP is maximized when R_i^G is equalized across regions, yielding

$$\bar{A} = \left(\sum_i A_i^{\frac{1}{1-\alpha}} \right)^{1-\alpha}, \tag{8}$$

where $\bar{A}$ denotes the maximum level of aggregate TFP.

The first-order conditions with respect to C^G and C^H imply the optimal consumption allocation rule: $\frac{C^G}{C^H} = \frac{1-\rho}{\rho}$ and the optimal intertemporal condition is: $\frac{u'(C_1)}{u'(C_2)} = \beta(1+r)$.

3.3 Political Economy

We now introduce a political economy framework wherein resource allocation decisions are made by the central government, which balances the tradeoff between maximizing household welfare and pursuing political objectives. Specifically, we assume the following objective function for the first-period government:

$$U^G = u(C_1) + \beta u(C_2) + \sum_i \kappa_i v(G_i), \tag{9}$$

where $\kappa_i > 0$ is an institutional parameter that captures local lobbying capacity and $\kappa \equiv \sum_i \kappa_i$; $v' > 0$ and $v'' < 0$.

The first-period government collects tax revenue and borrows to finance government consumption and regional government capital. The government budget constraint at the first period is

$$C_1^G + \sum_i G_i = W^G + T_1 + \tau Y + D, \tag{10}$$

where τ denotes a flat-rate tax on output, and T_1 is a lump-sum tax on, or transfer to, the household. The first-period government chooses the debt level D which should be repaid at the begining of second period. Crucially, the government is constrained by the maximum debt level $D \leq \bar{D}$. For simplicity, we assume that ρ is sufficiently high for this borrowing constraint to bind in equilibrium. For illustrative purposes, we first treat $\bar{D}$ as an exogenous debt limit. In the last part of this section, we will impose the same endogenous debt limit as in (1).

The second-period government collects lump-sum taxes T_2 from the household to repay the debt D_2 and to provide government consumption C_2^G. For simplicity, we assume exogenous T_2 so that the second-period government passively chooses

$$C_2^G = T_2 - (1+r)D \tag{11}$$

to satisfy the budget constraint.

The households collect after-tax output and lump-sum transfers from the government. Their

budget constraints are given by

$$C_1^H + \frac{C_2^H}{1+r} = W^H + (1-\tau)Y - T_1 - \frac{T_2}{1+r}. \tag{12}$$

The central government optimally selects aggregate government consumption C_1^G, region-specific government capital allocations G_i to maximize (9), subject to its budget constraint (10) and (11). We take τ, T_1, and T_2 as exogenously given, and assume in addition that τ is sufficiently high.

To guarantee the non-emptyness of the feasibility set, we assume the debt limit $\bar{D}$ is larger than the lower bound $\underline{D} \equiv -\left(\tau \bar{A}\right)^{\frac{1}{1-\alpha}} (\alpha)^{\frac{\alpha}{1-\alpha}} (1-\alpha) - W^G - T_1$ making sure $C_1^G > 0$. And we also assume T_2 and W^H are sufficiently high making sure $C_2^G, C_1^H, C_2^H > 0$

The allocation of regional government capital follows:

$$\left(1 + \underbrace{\frac{\kappa_i C_1^G}{(1-\rho)\,\alpha Y_i} \frac{v'(G_i)\,G_i}{u'(C_1)C_1}}_{\text{Government Capital Wedge}}\right) R_i^G = 1, \tag{13}$$

Comparing equation (13) with equation (6) reveals capital wedges arising from lobbying incentives, which direct greater government capital toward regions with higher political weights κ_i.

The capital wedges lead to two sources of misallocation. First, it distorts the allocation between C_1^G and $\{G_i\}_i$ by overinvesting $\{G_i\}_i$ relative to the first-best allocation. Moreover, the capital wedge creates misallocation of government capital across regions. To see this, we rewrite equation (13) as

$$\kappa_i v'(G_i) = \underbrace{\frac{(1-\rho)\,C_1}{C_1^G} u'(C_1)}_{\text{Shadow Price}} \times \left(1 - R_i^G\right) \tag{14}$$

The left-hand side of equation (14) represents the marginal benefit the government derives from increasing G_i. On the right-hand side, the first component captures the shadow price of G_i, while the second component reflects the marginal cost of providing G_i through its effect on aggregate output Y in the budget constraint (10). Unless local lobby capacity precisely aligns with local productivity – where $\kappa_i v'(G_i)$ is function of R_i^G – government capital returns R_i^G will not be equalized across regions, resulting in aggregate TFP losses.

3.4 Government Borrowing Limit and Misallocation

In this subsection, we first distinguish two special types of cross-region government capital allocation in our political economy framework, in contrast to the first-best government capital allocation $G_i = (\alpha A_i)^{\frac{1}{1-\alpha}}$ implied by $R_i^G = 1$. The first type, $G_i \propto A_i^{\frac{1}{1-\alpha}}$, features identical marginal returns

R_i^G and thereby maximizes aggregate TFP. We term this the efficiency-driven spatial allocation of government capital.

Minimal Borrowing and Efficiency-Driven Spatial Allocation One can easily see that as $\bar{D}$ approaches its lower bound $\underline{D}$, the allocation G_i always converges to the efficient spatial allocation. An impoverished government faces an infinitely high shadow price for G_i and, consequently, prohibitive marginal costs for any inefficient spatial allocation. This compels the government to prioritize efficiency, which overrides political considerations.

The second type is the politically-driven spatial allocation, where $v'(G_i) \propto \frac{1}{\kappa_i}$ is determined entirely by political considerations captured by the left-hand side of equation (14).

Abundant Borrowing and Politically-Driven Spatial Allocation As $\bar{D} \to \infty$, $\{G_i\}$ converges to the politically-driven spatial distribution. The intuition is straightforward: abundant government borrowing reduces the shadow price to zero, allowing allocation decisions to be driven entirely by political considerations.

We now endogenize the debt limit using the same specification as in equation (1). Combining the first-order conditions with respect to C_1^G and G_i yields

$$\kappa_i v'(G_i) = \underbrace{\frac{(1-\rho)\,C_1}{C_1^G} u'(C_1)}_{\text{Shadow Price}} \times \left(1 - \left(1 + \underbrace{\frac{b}{a+r}}_{\text{Easy-Borrowing Effect}}\right) R_i^G\right). \tag{15}$$

Relative to equation (14), the first term on the right-hand side plays the same role as before: it is the shadow price of G_i. A lower interest rate relaxes the government's borrowing constraint, increases its first-period budget, and therefore lowers this shadow price. As a result, political motives become more influential in the allocation of government capital.

The endogenous debt limit introduces an additional channel. Because $\bar{D}$ is increasing in aggregate output Y, a marginal increase in G_i not only raises contemporaneous output, but also expands the government's borrowing limit. This effect is captured by the term $\frac{b}{a+r}$, which reduces the effective opportunity cost of investing in government capital. When the interest rate falls, this easy-borrowing effect becomes stronger, further encouraging government capital accumulation and making the allocation of $\{G_i\}_i$ more responsive to political consideration than to regional productivity. Hence, a decline in r exacerbates both the aggregate overinvestment in government capital and the spatial misallocation of G_i.

Under additional assumptions, the following proposition formalizes these comparative statics.

Proposition 1 (Government Borrowing Limit and Misallocation) *Suppose that* $u(\cdot) = v(\cdot) = \log(\cdot)$, $\kappa_i \geq 0$ *for all* i, *and the tax rate* τ *is sufficiently high. Then the following statements hold:*

1. ***Level of Government Capital.*** *For each region* i, *government capital* G_i *exceeds its first-best level. Moreover, the gap between* G_i *and its first-best level increases monotonically with the borrowing limit* $\bar{D}$ *and lump-sum tax (transfer)* T_1. *When* $\bar{D}$ *is endogenously determined, the same gap also decreases monotonically with the interest rate* r.

2. ***Spatial Allocation of Government Capital.*** *Spatial allocation efficiency, as measured by aggregate TFP* A, *decreases monotonically with the borrowing limit* $\bar{D}$ *and lump-sum tax (transfer)* T_1. *When* $\bar{D}$ *is endogenously determined, aggregate TFP increases monotonically with the interest rate* r.

The proposition shows that a decline in the interest rate expands the government's borrowing limit and thereby exacerbates both the overaccumulation and the spatial misallocation of government capital. A lowerer T_1—that is, a larger lump-sum transfer—mitigates these distortions.

4 A Quantitative Model

4.1 Household

The economy is inhabited by overlapping generations of households. Each household chooses a region i at birth and resides there for two periods. The preferences of a household born in period t and residing in region i are given by:

$$U_{it}^H = \log\left(\left(\left(c_{it,t}^H\right)^\gamma \left(h_{it}\right)^{1-\gamma}\right)^\rho \left(C_{it}^G\right)^{1-\rho}\right) + \beta \log\left(\left(c_{it,t+1}^H\right)^\rho \left(C_{it+1}^G\right)^{1-\rho}\right) + \log \bar{Z}_{it}. \tag{16}$$

Here, $c_{it,t}^H$ and $c_{it,t+1}^H$ represent goods consumption in periods t and $t+1$, respectively; h_{it} denotes residential land use when young; C_{it}^G and C_{it+1}^G are total government consumption in region i in the two periods; and $\bar{Z}_{it}$ captures exogenous region-specific amenities. Households take government consumption as given. Housing services enter utility only in the first period of life. Accordingly, households pay land rents only when young. The parameter $\rho \in (0, 1]$ captures the weight on private consumption relative to government consumption, $\gamma \in [0, 1]$ captures the weight on goods consumption relative to land use, and β is the discount factor.

The household budget constraints are $c_{it,t}^H + r_{it}^H h_{it} + s_{it}^H = y_{it}^H$ and $c_{it,t+1}^H = (1 + r_{t+1})\, s_{it}^H$, where s_{it}^H denotes household saving, y_{it}^H denotes the income of the young household (to be specified later), r_{it}^H denotes the land rent in region i, and r_{t+1} denotes the interest rate. Under logarithmic preferences, the household saving decision is independent of government consumption. The resulting

closed-form solution is:

$$c_{it,t}^{H} = \frac{\gamma}{1+\beta} y_{it}^{H}, \quad h_{it} = \frac{1-\gamma}{1+\beta} \frac{y_{it}^{H}}{r_{it}^{H}}, \quad c_{it,t+1}^{H} = \frac{\beta\left(1+r_{t+1}\right)}{1+\beta} y_{it}^{H}. \tag{17}$$

Young households are free to move across regions. Therefore, in equilibrium, utility must be equalized across regions so that young households are indifferent over where to reside and supply labor. Let $\bar{U}_t^H$ denote this common equilibrium utility level.

4.2 Local Production and Aggregate Output

We consider a small open economy consisting of N regions. Regional output in period t is given by:

$$Y_{it} = A_{it} G_{it}^{\alpha_G} K_{it}^{\alpha_K} L_{it}^{\alpha_L}, \tag{18}$$

where A_{it}, G_{it}, K_{it} and L_{it} represent total factor productivity, government capital, private-sector capital, and labor in region i in period t, respectively.

Each region has a representative firm. The output of the firm is subject to a proportional tax rate τ_t that is identical across regions. Private capital and labor are mobile across regions but government capital is immobile.

Denote by r_t^K and w_{it} the rental rate of private capital and local wage rate, respectively. In this small open economy, the rental rate of private capital r_t^K is exogenously determined by the world interest rate, denoted r_t^w. From the bank's zero-profit condition, r_t^K satisfies

$$1 + r_t^K = \frac{\left(1+r_t^w\right) q_{t-1} - \left(1-\delta\right) q_t}{1-\xi_t^K}, \tag{19}$$

where q_t is the relative price of investment in terms of consumption, δ is the capital depreciation rate, and ξ_t^K is an inverse measure of financial intermediation efficiency in the private sector (e.g., Song, Storesletten and Zilibotti (2011)).

In contrast, the local wage rate w_{it} is determined endogenously through regional labor market clearing. We allow for region-specific distortions in private capital markets through the wedge (τ_{it}^K). In contrast to private inputs, government capital is provided to firms within each region without direct user charges.

The firm's profit-maximization problem is given by:

$$\max_{K_{it}, L_{it}} \Pi_{it} \equiv \left(1-\tau_t\right) Y_{it} - \left(1+\tau_{it}^K\right)\left(1+r_t^K\right) K_{it} - w_{it} L_{it}. \tag{20}$$

The first-order conditions imply that local labor demand is given by

$$L_{it} = \left((1-\tau_t) \left(\frac{\alpha_K}{\left(1+\tau_{it}^K\right)\left(1+r_t^K\right)} \right)^{\alpha_K} \left(\frac{\alpha_L}{w_{it}} \right)^{1-\alpha_K} A_{it} G_{it}^{\alpha_G} \right)^{\frac{1}{1-\alpha_K-\alpha_L}}, \tag{21}$$

while local private capital demand and local output are given by

$$K_{it} = \left(\frac{\alpha_K (1-\tau_t)}{\left(1+\tau_{it}^K\right)\left(1+r_t^K\right)} \right)^{\frac{1}{1-\alpha_K}} A_{it}^{\frac{1}{1-\alpha_K}} G_{it}^{\frac{\alpha_G}{1-\alpha_K}} L_{it}^{\frac{\alpha_L}{1-\alpha_K}}, \tag{22}$$

$$Y_{it} = \left(\frac{\alpha_K (1-\tau_t)}{\left(1+\tau_{it}^K\right)\left(1+r_t^K\right)} \right)^{\frac{\alpha_K}{1-\alpha_K}} A_{it}^{\frac{1}{1-\alpha_K}} G_{it}^{\frac{\alpha_G}{1-\alpha_K}} L_{it}^{\frac{\alpha_L}{1-\alpha_K}}. \tag{23}$$

4.3 Spatial Equilibrium

Household income includes wage income (w_{it}), local firm profit ($\frac{\Pi_{it}}{L_{it}}$) and central government transfers (ϕ_{it}^T). In addition, land income is redistributed lump-sum to young residents in the same region. In equilibrium, the land income received by each young resident equals $\frac{1-\gamma}{1+\beta} y_{it}^H$. Household income satisfies

$$y_{it}^H = \frac{\Pi_{it}}{L_{it}} + w_{it} + \phi_{it}^T + \frac{1-\gamma}{1+\beta} y_{it}^H = \frac{1+\beta}{\gamma+\beta} \left(\frac{1-\alpha_K}{\alpha_L} w_{it} + \phi_{it}^T \right). \tag{24}$$

Land supply is fixed at $\bar{H}_{it}$. Land market clearing then implies that the equilibrium land rent is:

$$r_{it}^H = \frac{1-\gamma}{1+\beta} \frac{y_{it}^H}{\bar{H}_{it}} L_{it}. \tag{25}$$

The household's indirect utility can be written as:

$$v_{it}^H = \log \left(\bar{Z}_{it} \left(C_{it}^G \left(C_{it+1}^G \right)^{\beta} \right)^{(1-\rho)} \frac{\left(y_{it}^H \right)^{\rho(1+\beta)}}{\left(r_{it}^H \right)^{\rho(1-\gamma)}} \right) + \beta\rho \log\left(1+r_{t+1}\right) + \log \Upsilon, \tag{26}$$

where $\Upsilon \equiv \frac{(\gamma)^{\rho\gamma}(1-\gamma)^{\rho(1-\gamma)}(\beta)^{\rho\beta}}{(1+\beta)^{\rho(1+\beta)}}$ is a constant. Combining (26) with (25), equalization of v_{it}^H across regions implies

$$y_{it}^H \propto \left(\bar{Z}_{it} \left(C_{it}^G \left(C_{it+1}^G \right)^{\beta} \right)^{(1-\rho)} \right)^{-\frac{1}{\rho(\beta+\gamma)}} \left(\frac{\bar{H}_{it}}{L_{it}} \right)^{-\frac{1-\gamma}{\beta+\gamma}}, \tag{27}$$

thus, equilibrium household income must be higher in regions with lower amenities, lower government consumption, or less residential land per capita.

Combining (27) with labor demand in (21), we obtain the equilibrium labor allocation. For

simplicity, we assume $\phi_{it}^T \propto w_{it}$, so that $y_{it}^H \propto w_{it}$, and the equilibrium labor allocation is given by:

$$L_{it} \propto \left((Z_{it})^{\frac{1}{\rho(\beta+\gamma)}} \left(\frac{A_{it}}{\left(1+\tau_{it}^K\right)^{\alpha_K}} \right)^{\frac{1}{1-\alpha_K}} \right)^{\frac{(1-\alpha_K)\eta}{\alpha_G}} \left(C_{it}^G \left(C_{it+1}^G \right)^{\beta} \right)^{\frac{(1-\rho)(1-\alpha_K)\eta}{\rho(\beta+\gamma)\alpha_G}} (G_{it})^{\eta}, \tag{28}$$

where $Z_{it} \equiv \bar{Z}_{it} \left(\bar{H}_{it}\right)^{\rho(1-\gamma)}$, $\eta \equiv \frac{\alpha_G}{1-\alpha_K-\alpha_L+\frac{1-\gamma}{\beta+\gamma}(1-\alpha_K)}$.

We assume fixed aggregate labor supply $L_t \equiv \sum_{i=1}^N L_{it}$. Local output Y_{it} is obtained by substituting (28) into (23). Its allocation across regions is

$$Y_{it} \propto (Z_{it})^{\frac{\alpha_L \eta}{\rho(\beta+\gamma)\alpha_G}} \left(\frac{A_{it}}{\left(1+\tau_{it}^K\right)^{\alpha_K}} \right)^{\frac{1+\frac{\alpha_L\eta}{\alpha_G}}{1-\alpha_K}} \left(C_{it}^G \left(C_{it+1}^G \right)^{\beta} \right)^{\frac{(1-\rho)\alpha_L\eta}{\rho(\beta+\gamma)\alpha_G}} (G_{it})^{\frac{\alpha_G-\frac{1-\gamma}{\beta+\gamma}\alpha_L\eta}{1-\alpha_K-\alpha_L}}. \tag{29}$$

In a partial equilibrium environment with exogenous wage rate w_{it} and land rent r_{it}^H, the elasticity of local output with respect to G_{it} is $\frac{\alpha_G}{1-\alpha_K-\alpha_L}$. In general equilibrium, however, changes in G_{it} trigger a reallocation of labor L_{it} across regions, which generates additional impacts through the endogenously determined w_{it} and r_{it}^H.

The aggregate marginal product of government capital G_{it} is

$$\frac{\partial Y_t}{\partial G_{it}} = \frac{\alpha_G}{1-\alpha_K-\alpha_L}\frac{Y_{it}}{G_{it}} - \left(\frac{(1-\gamma)\,\alpha_L\eta}{(\beta+\gamma)\,(1-\alpha_K-\alpha_L)}\frac{Y_{it}}{G_{it}} + \frac{\alpha_L\eta}{1-\alpha_K}\frac{L_{it}}{L_t}\frac{Y_t}{G_{it}} \right), \tag{30}$$

where $Y_t \equiv \sum_{i=1}^N Y_{it}$. The first term on the right-hand side of equation (30) captures the direct effect of G_{it} on production in region i, while the second and third terms capture general equilibrium effects through land rents and wages, respectively.

In equilibrium, the common utility level $\bar{U}_t^H$ is given by

$$\bar{U}_t^H = \rho\,(\beta+\gamma)\log\left(\frac{Y_t^H}{\sum_{i=1}^N \left(Z_{it} \left(C_{it}^G \left(C_{it+1}^G \right)^{\beta} \right)^{(1-\rho)} \right)^{-\frac{1}{\rho(\beta+\gamma)}} L_{it}^{\frac{1+\beta}{\beta+\gamma}}} \right) + \log \tilde{\Upsilon}_t. \tag{31}$$

where $Y_t^H \equiv \sum_{i=1}^N y_{it}^H L_{it}$ is aggregate household income, the denominator is an effective labor aggregate adjusted for regional differences and congestion, and $\tilde{\Upsilon}_t \equiv \Upsilon \left(\frac{1-\gamma}{1+\beta}\right)^{-\rho(1-\gamma)} (1+r_{t+1})^{\beta\rho}$ is an exogenous term. Equation (24) implies $Y_t^H = \frac{1+\beta}{\beta+\gamma}\left((1-\tau_t)\,(1-\alpha_K)\,Y_t + T_t\right)$, where $T_t \equiv \sum_{i=1}^N \phi_{it}^T L_{it}$ is the aggregate lump-sum government transfer or tax.

The marginal effect of G_{it} on household welfare is

$$\frac{\partial \bar{U}_t^H}{\partial G_{it}} = \rho\,(1+\beta)\,\frac{(1-\tau_t)\,(1-\alpha_K)}{Y_t^H}\frac{\partial Y_t}{\partial G_{it}} - \rho\,(1+\beta)\,\eta\left(\frac{y_{it}^H L_{it}}{Y_t^H} - \frac{L_{it}}{L_t}\right)\frac{1}{G_{it}}, \tag{32}$$

Government investment affects household welfare through aggregate output (the first term) and labor reallocation across regions (the second term).

4.4 Central Government

The economy has a centralized fiscal system, where the central government chooses government consumption, investment, and borrowing at the beginning of period t. We denote local government investment as I_{it}^G, with local government capital evolving according to $G_{it+1} = I_{it}^G + (1-\delta)G_{it}$. Although the central government could allocate government consumption across regions, Section 2 demonstrates that the cross-regional allocation of government consumption has remained remarkably stable over time. We therefore take the regional distribution of government consumption as given and assume $C_{it}^G = \psi_i C_t^G$. Moreover, because local governments must obtain prior permission from the central government to issue bonds (Ihori and Sato (2002)), we abstract from local government borrowing and consider only aggregate government borrowing.

The central government's objective function is

$$U_t^C = \omega^o \bar{U}_{t-1}^H + \sum_{j=0}^{\infty} (\beta_C)^j \left(\bar{U}_{t+j}^H + \sum_{i=1}^{N} \kappa_{it+j} \log G_{it+j+1} \right), \tag{33}$$

where κ_{it} is the same institutional parameter as in the simple model that indicates the local government's lobbying capacity for government investment in that region. The household utility follows equation (31). The lifetime utility of the generation born in $t-1$ is affected by government consumption in period t, so the central government values the welfare of the current old generation as well as that of current and future young generations. It discounts future generations' utilities geometrically with a discount factor $\beta_C \in (0,1)$, while the weight on the current old generation is ω^o. We assume $\omega^o = \frac{\beta}{\beta_C}$ such that the central government's preference is time-consistent.[11] The central government's budget constraint is:

$$C_t^G + \sum_{i=1}^{N} q_t I_{it}^G + T_t + \Phi_t = \tau_t Y_t + D_{t+1} - \left(1 + r_t^D\right) D_t, \tag{34}$$

Here, D_t denotes the total government debt at the beginning of period t, r_t^D is the real effective

[11] Abstracting from terms that are independent of the central government's choices, the objective can be simplified to the following time-consistent preference:

$$\hat{U}_t^C = \sum_{j=0}^{\infty} \beta_C^j \left(\hat{\bar{U}}_{t+j}^H + \left(1 + \frac{\beta}{\beta_C}\right)(1-\rho) \log C_{t+j}^G + \sum_{i=1}^{N} \kappa_{it+j} \log G_{it+j+1} \right),$$

where

$$\hat{\bar{U}}_t^H = \rho\,(\beta+\gamma) \log Y_t^H - \rho\,(\beta+\gamma) \log \left(\sum_{i=1}^{N} \left(Z_{it}^{-\frac{1}{\rho(\beta+\gamma)}} \psi_i^{-\frac{(1-\rho)(1+\beta)}{\rho(\beta+\gamma)}} L_{it}^{\frac{1+\beta}{\beta+\gamma}} \right) \right).$$

interest rate of government debt, and Φ_t is a residual component capturing government spendings that cannot be easily classified as government consumption, investment, or transfer. Government borrowing is subject to the fiscal rule

$$\frac{\left(a_t + r^D_{t+1}\right) D_{t+1}}{Y_t} = b, \tag{35}$$

where we allow a_t to be time-varying.

The central government chooses $\{C^G_{t+j}, G_{it+j+1}, D_{t+j+1}\}_{i=1,j=0}^{N,\infty}$ to maximize the objective function (33), subject to the budget constraint (34) and fiscal rule (35).

4.5 Equilibrium Conditions

We derive the optimality conditions for the central government's problem in Appendix B.4.3. The first-order conditions w.r.t. G_{it+j+1} are

$$\begin{aligned}\frac{C^G_{t+j+1}}{C^G_{t+j}} = \frac{\beta_C}{q_{t+j}} \Bigg(\left(\tau_{t+j+1} + \frac{\partial D_{t+j+2}}{\partial Y_{t+j+1}} \right) \frac{\partial Y_{t+j+1}}{\partial G_{it+j+1}} + q_{t+j+1}\left(1-\delta\right) \Bigg) \\ - \frac{\beta_C^2}{q_{t+j}} \frac{C^G_{t+j+1}}{C^G_{t+j+2}} \left(1 + r^D_{t+j+2}\right) \frac{\partial D_{t+j+2}}{\partial Y_{t+j+1}} \frac{\partial Y_{t+j+1}}{\partial G_{it+j+1}} \\ + \frac{C^G_{t+j+1}}{\left(1+\beta/\beta_C\right)\left(1-\rho\right) q_{t+j}} \left(\beta_C \frac{\partial \bar{U}^H_{t+j+1}}{\partial G_{it+j+1}} + \kappa_{it+j} \frac{1}{G_{it+j+1}} \right)\end{aligned} \tag{36}$$

where $\frac{\partial Y_t}{\partial G_{it}}$ follows equation (30), and $\frac{\partial \bar{U}^H_{t+j}}{\partial G_{it+j}}$ follows equation (32). The right-hand side of equation (36) reveals the multifaceted nature of government capital allocation decisions. The first and second lines capture the economic return to government capital, including its effects on tax revenue and debt capacity. The term $\beta_C \frac{\partial \bar{U}^H_{t+j+1}}{\partial G_{it+j+1}}$ represents the marginal benefit of government investment in region i through positive spillover effects on household consumption. The final term $\kappa_{it+j} \frac{1}{G_{it+j+1}}$ reflects the political gain from allocating government capital to region i.

4.6 Calibration

4.6.1 Production Elasticities

We calibrate the output elasticities of private capital and labor to $\alpha_K = 0.406$ and $\alpha_L = 0.559$, respectively, by matching their average income shares in the national accounts over 1994–2012 (see Appendix A.1.4 for details). The same approach cannot be applied to government capital, because the observed income share of government capital does not capture its positive externalities. We therefore estimate α_G using a proxy-variable framework inspired by Olley and Pakes (1996), Levinsohn and Petrin (2003), and Ackerberg, Caves and Frazer (2015).

To do so, we first subtract the contributions of private capital and labor from output and get the Solow residual term as

$$\text{Solow Residual}_{it} = \log Y_{it} - \alpha_K \log K_{it} - \alpha_L \log L_{it}.$$

This residual contains the contributions of government capital, productivity, and measurement error:

$$\text{Solow Residual}_{it} = \alpha_G \log G_{it} + \log A_{it} + \varepsilon_{it}, \tag{37}$$

where the unobserved productivity term A_{it} may be correlated with G_{it}. Following the proxy-variable approach, we use private investment as a proxy for productivity and assume that investment satisfies

$$\log I_{it}^K = f_t\left(\log G_{it}, \log K_{it}, \log I_{it}^G, \log A_{it}\right). \tag{38}$$

Under invertibility, this implies $\log A_{it} = f_t^{-1}(\log G_{it}, \log K_{it}, \log I_{it}^G, \log I_{it}^K)$.

The estimation proceeds in two stages. In the first stage, we estimate

$$\text{Solow Residual}_{it} = \underbrace{\alpha_G \log G_{it} + f_t^{-1}\left(\log G_{it}, \log K_{it}, \log I_{it}^G, \log I_{it}^K\right)}_{\mathcal{F}_t\left(\log G_{it}, \log K_{it}, \log I_{it}^G, \log I_{it}^K\right)} + \varepsilon_{it}, \tag{39}$$

and obtain $\hat{\mathcal{F}}_{it} \equiv \hat{\mathcal{F}}_t(\log G_{it}, \log K_{it}, \log I_{it}^G, \log I_{it}^K)$. In the second stage, we assume that log productivity follows an AR(1) process:

$$\log A_{it} = \rho_0 + \rho_1 \log A_{it-1} + \xi_{it}, \tag{40}$$

where the innovation ξ_{it} is independent of G_{it}. Substituting this expression into equation (37) yields

$$\text{Solow Residual}_{it} = \alpha_G \log G_{it} + \rho_0 + \rho_1\left(\hat{\mathcal{F}}_{it-1} - \alpha_G \log G_{it-1}\right) + \xi_{it} + \varepsilon_{it}. \tag{41}$$

For each candidate value of α_G, we first recover $\log A_{it}(\alpha_G) = \hat{\mathcal{F}}_{it} - \alpha_G \log G_{it}$, then estimate $\hat{\rho}_0(\alpha_G)$ and $\hat{\rho}_1(\alpha_G)$ from equation (40), and finally compute the composite residual from equation (41):

$$\widehat{\xi_{it} + \varepsilon_{it}}\left(\alpha_G\right) = \text{Solow Residual}_{it} - \alpha_G \log G_{it} - \hat{\rho}_0\left(\alpha_G\right) - \hat{\rho}_1\left(\alpha_G\right)\left(\hat{\mathcal{F}}_{it-1} - \alpha_G \log G_{it-1}\right). \tag{42}$$

We consider three estimators for α_G based on equation (42). First, we choose α_G to minimize $\sum_{i,t}(\widehat{\xi_{it} + \varepsilon_{it}}(\alpha_G))^2$, which yields $\alpha_G = 0.082$ with a bootstrap standard error of 0.021. Second, we impose the orthogonality conditions between $\widehat{\xi_{it} + \varepsilon_{it}}(\alpha_G)$ and $(\log G_{it}, \log G_{it-1})$, which gives a GMM estimate of $\alpha_G = 0.050$ with a bootstrap standard error of 0.021; the Hansen J-statistic is close to zero. Third, we use Bayesian estimation and obtain a posterior mean of $\alpha_G = 0.072$, with a 95% credible interval of $[0.01, 0.14]$. In the benchmark model, we set $\alpha_G = 0.072$. In the

robustness analysis, we also consider $\alpha_G = 0.050$ and $\alpha_G = 0.082$.

4.6.2 External Calibration

Each period in the model spans ten years. We set $\delta = 1 - (1 - 0.089)^{10} = 0.606$, following the calibrated annual capital depreciation rate of 0.089 in Hayashi and Prescott (2002).

We set the household annual discount factor to 0.98, yielding a decadal discount factor of $\beta = 0.98^{10} = 0.82$. For the central government, we assume a discount factor $\beta_C = \beta p_C$, where $p_C \in (0, 1)$ captures political turnover. We calibrate the annualized political survival probability to $p_C = 0.82$, implying that an administration has a 13% probability of remaining in office for a full decade. This parameterization yields a government discount factor of $\beta_C = 0.8^{10} = 0.11$. We calibrate γ to match the aggregate housing expenditure share over the period 1983-2012, which is 21.7%. In our model, housing expenditure is paid only by young households, which implies $\gamma = 0.628$. The preference parameter $\rho = 0.965$ is calibrated so that the observed government-to-household consumption ratio during the 1983-1992 period matches the first-best allocation.

All the variables for output, capital, consumption, and debt are detrended by the long-run growth rate $g = (1 + g_A)^{\frac{1}{1-\alpha_G-\alpha_K}} - 1$, where g_A denotes the long-run TFP growth rate, which is set to an annual rate of 0.5%.

We set the initial household wealth W_0^H to match household financial assets at the end of 1982, which is 160% of GDP.

Table 4 summarizes all time-invariant externally calibrated parameter values.

Table 4: Externally Calibrated Time-Invariant Parameters

Parameters	Value	Target
α_K	0.406	private capital share
α_L	0.559	labor share
α_G	0.072	production elasticity estimation
γ	0.628	share of housing expenditure in private consumption = 21.7%
annualized δ	0.089	capital depreciate rate in Hayashi and Prescott (2002)
annualized β	0.980	
annualized β_C	0.800	13% probability of staying in office in a full decade
ρ	0.965	government to household consumption ratio in 1983-1992
annualized g_A	0.005	0.5% annual TFP growth
b	0.048	fiscal rule estimation using annual data in Section 2

In the calibrated model, $t = 0, 1, 2$ represents the period 1983-1992, 1993-2002, and 2003-2012, respectively. We use the relative price of fixed capital formation to GDP from the national accounts

for q_t, and assume both government and private investment share this same price. The parameter τ_t is chosen to match the ratio of total government revenue to GDP. The results are reported in the first two rows of Table 5).

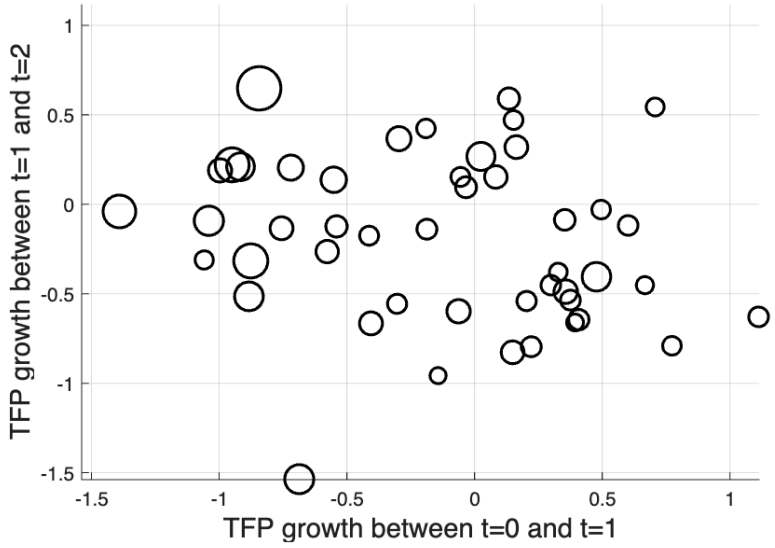
Table 5: Externally Calibrated Time-Varying Parameters

Parameters	$t=0$ (83-92)	$t=1$ (93-02)	$t=2$ (03-12)	Target
q_t	1.41	1.41	1.45	relative price of fixed capital formation
τ_t (%)	23.26	21.26	21.17	total government revenue to GDP ratio
r_t^K(annualized, %)	6.91	4.44	3.45	rental rate of private capital

We derive local productivity, private capital, and labor wedges from the above externally calibrate parameters. Using the perpetual inventory method, we construct prefecture-level government and private capital stocks G_{it} and K_{it} (see Appendix A.1.7 for details). Prefecture TFP is computed as the Solow residual: $\log A_{it} = \log Y_{it} - \alpha_G \log G_{it} - \alpha_K \log K_{it} - \alpha_L \log L_{it}$, where L_{it} denotes employment in the prefecture.

The annualized aggregate TFP growth is -0.46% between 1983-1992 and 1993-2002, and recovers to -0.04% in the subsequent decade. Figure 6 illustrates prefecture-level TFP growth across consecutive decades. The horizontal axis shows growth between 1983-1992 and 1993-2002, while the vertical axis represents growth between 1993-2002 and 2003-2012. The correlation is -0.19, suggesting no persistence in prefecture-level TFP growth.

Figure 6: Prefecture-Level TFP Growth (annual, %)

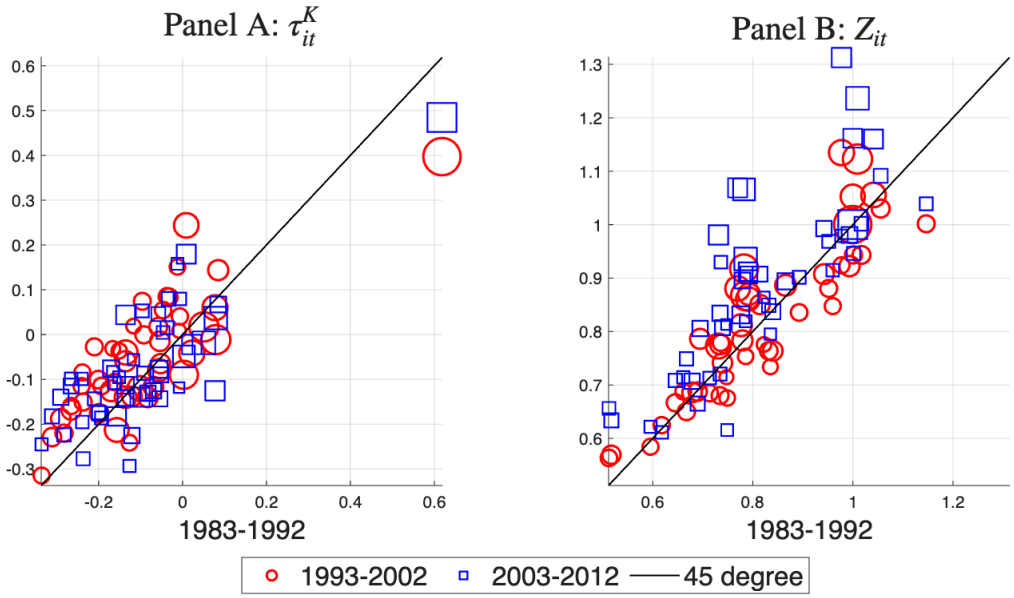


Firms' first-order conditions imply the following prefecture-specific private capital and labor wedges: $1+\tau_{it}^K \propto \frac{Y_{it}}{K_{it}}$. We pin down the level of wedges by imposing $\sum_{i=1}^N \tau_{it}^K K_{it} = 0$ – i.e., these

wedges do not affect aggregate private capital and labor income. The exogenous rental rate of private capital is then given by $r_t^K = \alpha_K(1-\tau_t)\frac{Y_t}{K_t} - 1$.[12] Amenities $\bar{Z}_{it}$ and residential land supply $\bar{H}_{it}$ are not separately identified in the model. We therefore calibrate their composite term, Z_{it}, using the labor-allocation equation (28), and normalize Tokyo's Z_{it} to one.

The values of r_t^K are reported in the last row of Table 5. The left panel of Figure 7 plots τ_{it}^K for three time periods: 1983-1992 (x-axis) against 1993-2002 (y-axis, red circles) and 2003-2012 (y-axis, blue squares). The right panel presents the corresponding comparison for Z_{it}. The visualization shows that both the prefecture-level private capital wedges and the composite amenity term exhibit strong persistence across time periods.

Figure 7: Prefecture-Level Private Capital Wedges and Composite Amenity Term



We generalize the aggregate returns to G_{it} in the simple model to

$$R_{it}^G \equiv \frac{\partial Y_t}{q_{t-1}\partial G_{it}} + \frac{q_t}{q_{t-1}}(1-\delta) - 1. \tag{43}$$

Figure (3) plots the correlation between I_{it}^G/G_{it} and R_{it+1}^G. Moreover, we define the average aggregate returns to government and private capital as

$$R_t^{\mathcal{Z}} = \alpha_{\mathcal{Z}}\frac{Y_t}{q_t\mathcal{Z}_t} + \frac{q_t}{q_{t-1}}(1-\delta) - 1, \quad \mathcal{Z} \in \{G, K\}, \tag{44}$$

and plot them in Figure (2).

[12] Since r_t^K is the only price that influences the demand for private capital, we need not separately specify the world interest rate r_t^w and the capital friction parameter ξ_t^K in equation (19).

4.6.3 Internal Calibration

The remaining parameters are interally calibrated to match the observed data: Φ_t to government consumption C_t^G, T_t to household consumption C_t^H, $\{\kappa_{it}\}_{i=1}^N$ to prefecture-level government capital $\{G_{it+1}\}_{i=1}^N$, and a_t to government debt D_{t+1}.[13]

To simulate the dynamic model, we make two key assumptions. First, for all time-varying parameters (including those in Table 5, prefecture-level $\{\tau_{it}^K, Z_{it}\}$, and the internally calibrated $\{\Phi_t, T_t, \{\kappa_{it}\}, a_t\}$) with $t > 2$, we set their values equal to their respective values in period $t = 2$. Additionally, we decompose prefecture-level TFP into two components $A_{it} = \hat{A}_{it} A_t$, and assume A_t grows at the constant rate g_A for $t > 2$ while $\hat{A}_{it}$ equals to their values in period $t = 2$.

Second, regarding expectations, we assume perfect foresight for aggregate TFP A_t, but MIT-shock expectation for all the other parameters. Specifically, for the set $X_t = \{i_{t+1}, \pi_t, r_t^K, q_t, \tau_t, \Phi_t, T_t, L_t, a_t, \{\tau_{it}^K\}, \{Z_{it}\}, \{\kappa_{it}\}, \{\hat{A}_{it}\}\}$, we assume

$$\mathbb{E}_t \left[X_{t+j+1} \right] = X_t, \quad \forall j \geq 0. \tag{45}$$

We use the uncollateralized overnight call rate for i_t and the CPI for π_t. The expected real interest rate follows $\mathbb{E}_t[r_{t+j+1}] = \mathbb{E}_t[i_{t+j+1} - \pi_{t+j+1}] = i_{t+1} - \pi_t$, $\forall j \geq 0$. The results are reported in Table (6).

Determining expected real effective rate of government debt, $\mathbb{E}_t[r_{t+j+1}^D]$ $\forall j \geq 0$, is more complex, as it depends on both the historical issuance rates and the maturity composition of outstanding debt. Let $i_t(m)$ denote the yield-to-maturity on newly issued m-year bonds at year t, and let $D_{s,t+1}(m)$ represent the outstanding stock of m-year debt issued at year $t - s$. The nominal effective rate of government debt, i_{t+1}^D, is determined by

$$i_{t+1}^D = \frac{1}{D_{t+1}} \left(\sum_{s=0}^{m-1} i_{t-s}(m) D_{s,t+1}(m) \right). \tag{46}$$

Due to data constraints, we group maturities into three categories (shorter than or equal to one year; between one and ten years; and longer than or equal to ten years) and assume that $D_{t+1}(m) \equiv \sum_{s=0}^{m-1} D_{s,t+1}(m)$ was issued uniformly over the past m years (see Appendix A.2.2 for detailed data descriptions). Despite these limitations, the constructed i_{t+1}^D closely tracks the effective interest rate in the data, as illustrated in Figure 8.

To form expectation for $\mathbb{E}_t[i_{t+j+1}^D]$ for $j \geq 0$, we similarly adopt MIT-shock expectations for

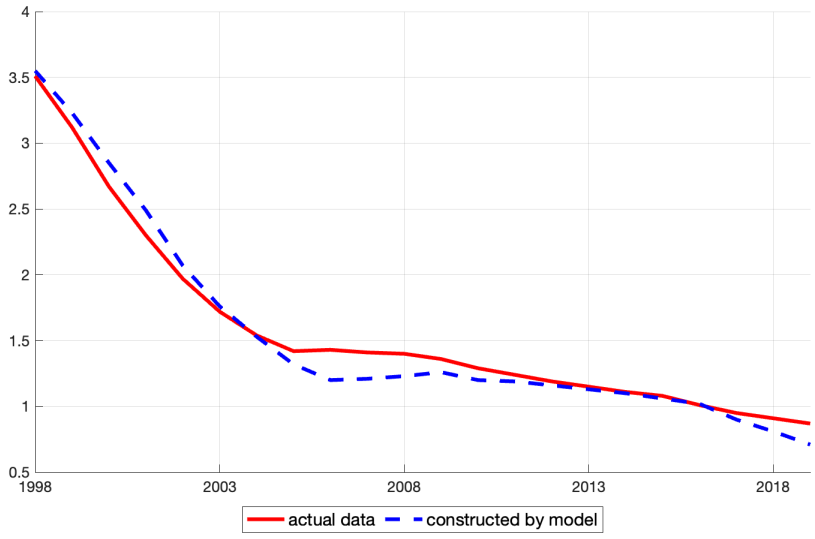
[13] Government consumption, C_t^G, is measured by "government final consumption" provided by the central and local governments as reported in the National Accounts, while private consumption, C_t^H, is measured by private final consumption expenditure. We exclude "government final consumption" provided by social security funds. However, using more inclusive definitions of government consumption would lead to the same finding or even strengthen it.

future issuance rates and maturity shares:

$$\mathbb{E}_t\left[i_{t+j+1}(m)\right] = i_t(m), \quad \mathbb{E}_t\left[\frac{D_{t+j+1}(m)}{D_{t+j+1}}\right] = \frac{D_t(m)}{D_t}, \quad \forall j \geq 0,$$

We then apply equation (46) to generate $\mathbb{E}_t[i^D_{t+j+1}]$, and obtain $\mathbb{E}_t[r^D_{t+j+1}] = \mathbb{E}_t[i^D_{t+j+1} - \pi_{t+1}]$.[14] Both $\mathbb{E}_t[i^D_{t+j+1}]$ and $\mathbb{E}_t[r^D_{t+j+1}]$ are reported in Table (6).

Figure 8: Effective interest rate i^D_t (%)



[14] We first construct annual expectations using yearly data, and then convert these into ten-year expectations, which are computed as the product of the annual forecasts for $j = 1, \ldots, 10$.

Table 6: Expected Interest Rates (%)

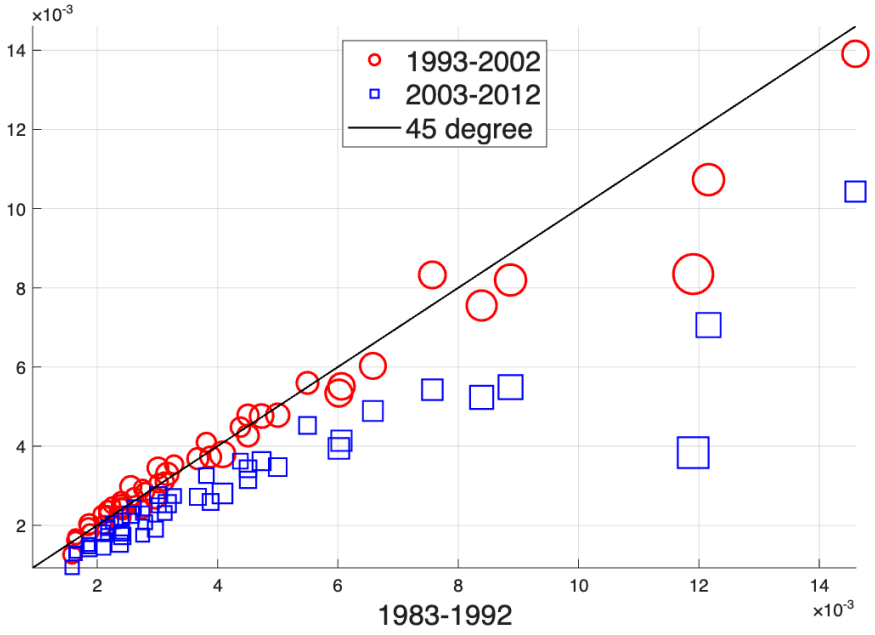
	$t=1$ (93-02)	$t=2$ (03-12)	$t=3$ (13-22)
r_t	0.96	0.26	-0.70
$\mathbb{E}_0[r_t]$	-0.66	-0.66	-0.66
$\mathbb{E}_1[r_t]$	-	-0.04	-0.04
$\mathbb{E}_2[r_t]$	-	-	0.13
i_t^D	3.71	1.40	0.95
$\mathbb{E}_0[i_t^D]$	5.03	4.60	4.60
$\mathbb{E}_1[i_t^D]$	-	1.12	0.85
$\mathbb{E}_2[i_t^D]$	-	-	0.96
r_t^D	3.53	1.53	0.94
$\mathbb{E}_0[r_t^D]$	3.24	2.80	2.80
$\mathbb{E}_1[r_t^D]$	-	0.95	0.67
$\mathbb{E}_2[r_t^D]$	-	-	1.09

The calibration solves for 150 unknows: 4 aggregate variables (Φ_t, T_t, a_t), and 47 prefecture-level variables (κ_{it}) across three periods. The results are summarized in Table 7. Figure 9 plots κ_{it} for three time periods: 1983-1992 (x-axis) against 1993-2002 (y-axis, red circles) and 2003-2012 (y-axis, blue squares). Like prefecture-level wedges, κ_{it} is also highly persist across time periods.

Table 7: Internally Calibrated Parameters

Parameters	$t=0$ (83-92)	$t=1$ (93-02)	$t=2$ (03-12)	Target
$\frac{\Phi_t}{Y_t}$(%)	9.84	11.03	11.59	government consumption
$\frac{T_t}{Y_t}$(%)	-1.91	-1.39	-0.94	household consumption
a_t (annualized)	-0.005	0.022	0.014	government debt
mean of κ_{it} (10^{-3})	4.12	3.99	2.92	prefecture government capital
std of κ_{it} (10^{-3})	2.92	2.54	1.73	prefecture government capital

Figure 9: Prefecture-level κ_{it}



4.7 Fiscal Windfall

We now quantify the magnitude of the fiscal windfall during the Lost Decade. For notational ease, we define changes in expectations as $\Delta\mathbb{E}_t[X_{t+j}] \equiv \mathbb{E}_t[X_{t+j}] - \mathbb{E}_{t-1}[X_{t+j}]$. We can now formally write down the definition of fiscal windfalls as an extension of equation (2). The government in period t perceives the following fiscal windfall in period $t+j$:

$$\mathbb{E}_t\left[\text{Windfall}_{t+j}\right] = \Delta\mathbb{E}_t\left[NB_{t+j}\right] + \Delta\mathbb{E}_t\left[B_{t+j}\right] - \Delta\mathbb{E}_t[r^D_{t+j}D_{t+j}] \tag{47}$$

Here, the government non-borrowing income NB_t is

$$NB_t = \tau_t Y_t - \Phi_t - T_t.$$

In the Lost Decade, the expected real interest rate declined from $\mathbb{E}_0[r^D_2] = 2.80\%$ to $\mathbb{E}_1[r^D_2] = 0.95\%$, which saved interest payments. A lower expected real rate also increases borrowing by $\Delta\mathbb{E}_t[B_t]$. Finally, there are other forces that change government non-borrowing net income, $\Delta\mathbb{E}_t[NB_t]$. We report each component as a percentage of GDP in Table 8.

Table 8: Fiscal windfalls perceived at 1993-2002 ($t = 1$, in percent of GDP)

	$t+j=1$ (93-02)	$t+j=2$ (03-12)	$t+j=3$ (13-22)	steady state
$\Delta\mathbb{E}_1[NB_{t+j}]$	-0.59	-0.11	-0.26	-0.49
$\Delta\mathbb{E}_1[B_{t+j}]$	7.22	1.92	0.44	0.79
$\Delta\mathbb{E}_1[-r^D_{t+j}D_{t+j}]$	-0.27	0.91	1.10	1.13
$\mathbb{E}_1[\text{Windfall}_{t+j}]$	6.36	2.72	1.28	1.44

5 Counterfactuals

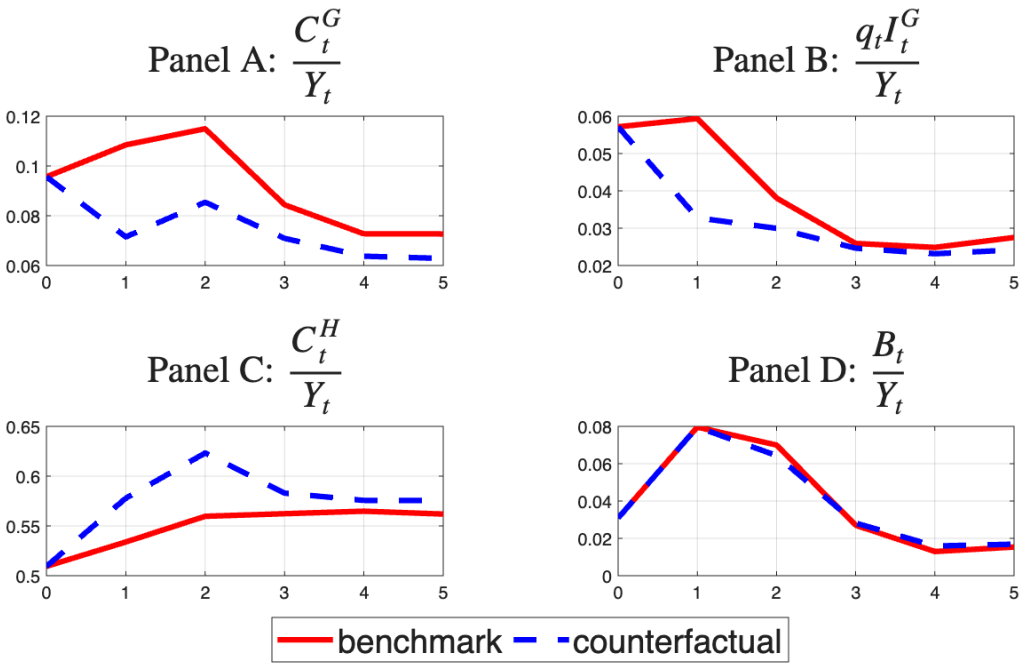
As demonstrated in the simple model, fiscal windfalls may generate several inefficiencies in the economy. In the data, we observe a higher government investment-to-GDP ratio and a worsened spatial allocation of government investment, both consistent with our model predictions. Moreover, we observe a higher government-to-household consumption ratio, aligning with insufficient adjustments in household transfers in response to fiscal windfalls.

To quantitatively examine the effects of these fiscal windfalls, we conduct a counterfactual analysis in which the central government commits to transferring the fiscal windfall $\mathbb{E}_t[\text{Windfall}_{t+j}]$ perceived during the 1993-2002 period ($t = 1$). Consistent with our model, $\mathbb{E}_t[\text{Windfall}_{t+j}]$ is directed exclusively to the young generation in period $t+j$ and is financed from the government budget. The government optimizes its choices of C^G_{t+j}, $\{G_{it+1+j}\}$ and D_{t+j+1} for $j \geq 0$ in each period $t \geq 1$.

The transitional dynamics of aggregate variables are shown in Figure 10. In the counterfactual, government consumption and investment GDP ratios decrease substantially in the two periods when the government hands out the additional transfers (Panel A and B). In contrast, the household consumption GDP ratio increases (Panel C).[15] Government borrowing is unchanged in period 1 because of the fiscal rule, but deviates slightly from the benchmark in subsequent periods (Panel D).

[15] Household consumption is defined as $C^H_t = C^H_{t,t} + C^H_{t-1,t}$, where $C^H_{t,t} = \sum_{i=1}^N (c^H_{t,t} + r^H_{it}h_{it})L_{it}$, $C^H_{t,t+1} = \sum_{i=1}^N c^H_{t,t+1}L_{it}$.

Figure 10: Transitional Dynamics



The additional transfers tighten the government budget. Proposition 1 predicts an increase in the spatial allocation efficiency of government capital in the one-period model. This also happens in the quantitative model as shown by Figure 11. The strong negative correlation between I_{it}^G/G_{it} and R_{it+1}^G in the data is now replaced with zero correlation in the counterfactual (see also the first two columns of Table 9. As a result, the counterfactual aggregate TFP at $t = 2$ is 0.43% higher than the calibrated economy. Aggregate output declines by 3.99%, primarily due to substantially reduced government investment. However, household income increases by 10.11%.

To measure the welfare gain, we assume social planner's objective function follows equation (33), but with $\kappa_{it} = 0$ and $\beta_C = \beta$. The counterfactual welfare gain or loss is measured using the equivalent consumption variation (φ) relative to the calibrated economy. Specifically, φ is calculated to make the social planner indifferent between two scenarios: a lifetime consumption stream in the calibrated economy, $\{C_t^G, c_{it-1,t}^H, c_{it,t}^H\}_{t=t_0}^{\infty}$, each scaled by $(1+\varphi)$, and a lifetime consumption stream in the counterfactual allocation. t_0 is the period when we start to conduct the counterfactual exercise. The welfare gain in the counterfactual is $\varphi = 0.31\%$. These results are summarized in the first column of Table 10.

Figure 11: $\frac{I^G_{it}}{G_{it}} \sim r^G_{it+1}$ at 1993-2002

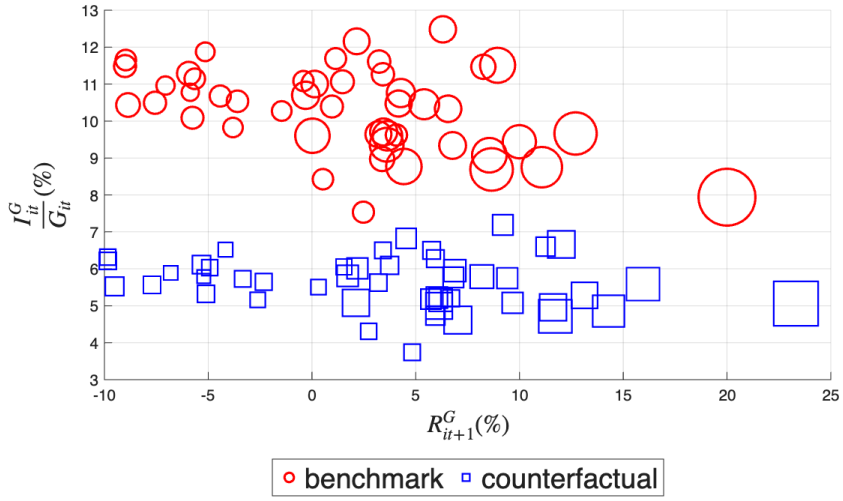


Table 9: $\frac{I^G_{it}}{G_{it}} \sim R^G_{it+1}$ at 1993-2002

$\frac{I^G_{it}}{G_{it}}$	Benchmark	Counterfactuals			
		transfers	$\kappa_{it} = \kappa_i$	$\tau^K_{it} = \tau^K_i$	$Z_{it} = Z_i$
R^G_{it+1}	-0.078***	-0.020	-0.051***	-0.113***	-0.086***
	(0.024)	(0.014)	(0.013)	(0.040)	(0.025)
Obs.	47	47	47	47	47
R^2	0.190	0.044	0.258	0.152	0.203

Table 10: Aggregate TFP, output and welfare changes

	Counterfactuals			
	transfers	$\kappa_{it} = \kappa_i$	$\tau^K_{it} = \tau^K_i$	$Z_{it} = Z_i$
Aggregate TFP change at 2003-2012 (%)	0.43	0.34	-0.59	-0.10
Aggregate output change at 2003-2012 (%)	-3.99	-0.93	0.10	-0.24
Household income change in 1993-2002 (%)	10.11	0	-0.92	-0.40
Welfare change (φ, %)	0.31	-0.08	-0.02	-0.16

5.1 Lobby Incentives, Private Capital Wedges and Amenities

We next conduct the following counterfactuals to examine the effects of changes in κ_{it}, τ_{it}^K, Z_{it} in the Lost Decade.

1. Set the local lobby capacity parameter in the lost decade to its average level: $\kappa_{i1} = \frac{1}{3}\sum_{t=0}^{2}\kappa_{it}$.
2. Set the local private capital wedge or composite amenity term in the lost decade to its average level: $\tau_{i1}^K = \frac{1}{3}\sum_{t=0}^{2}\tau_{it}^K$ or $Z_{i1} = \frac{1}{3}\sum_{t=0}^{2}Z_{it}$.

The κ counterfactual modestly weakens the negative correlation between I_{it}^G/G_{it} and R_{it+1}^G (the third column of Table 9). The resulting counterfactual aggregate TFP in 1993-2002 is 0.34% higher than in the calibrated economy, much smaller than the efficiency gain observed in the counterfactual where fiscal windfalls are transferred to households. This limited improvement is expected given the high persistence in κ_{it} (Figure 9). The welfare gain is $\varphi = -0.08\%$.

The counterfactuals for prefecture-level wedges prove inconsequential for the spatial allocation of government capital (as shown in the last two columns of Table 9). This outcome is unsurprising given the high persistence of these wedges demonstrated in Figure 7. The counterfactual aggregate TFP and welfare are both lower than in the calibrated economy. These results are summarized in the last three columns of Table 10.

These results confirm that insufficient adjustments to transfers in response to low-interest-rate-generated fiscal windfalls can largely account for the deterioration in spatial allocation efficiency of government capital. Transferring these fiscal windfalls to households can significantly increase aggregate TFP and welfare. In contrast, changes in local lobbying capacity and local private capital and labor wedges have, at best, marginal effects on aggregate TFP and welfare.

5.2 Benevolent Planner

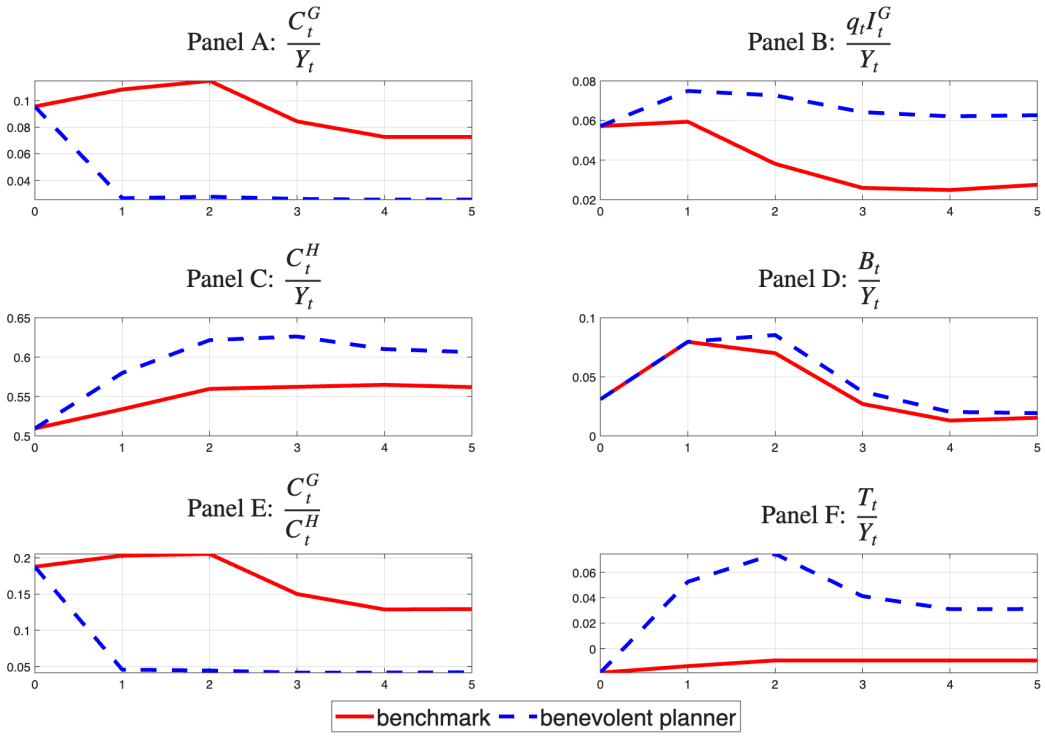
We solve the benevolent planner problem, whose objective function follows equation (33), but with $\kappa_{it} = 0$ and $\beta_C = \beta$. The planner operates under the same budget constraint (34) and possesses the same policy instruments as the central government, with one critical addition: the ability to determine the sequence of lump-sum transfers to young households ($\{T_{t+j}\}_{j\geq 0}$). The first-order condition w.r.t T_{t+j} is:

$$\frac{C_{t+j,t+j}^H}{C_{t+j}^G} = \frac{\rho}{2\left(1-\rho\right)}, \tag{48}$$

and the first-order conditions with respect to G_{it+j+1} (36) remain applicable.

Figure 12 plots the transitional dynamics. The bottom right panel shows an increase in the optimal transfers. In this case, the aggregate TFP and output increase by 9.22% and 12.82% in 2003-2012, respectively. The welfare increases by 11.93%.

Figure 12: Transitional Dynamics



5.3 Institutional Perspective

Through the lens of our model, the Lost Decade of fiscal misallocation concluded with the stabilization of interest rates near zero, which terminated the fiscal windfall created by unanticipated interest rate cuts. However, fiscal restructuring typically necessitates substantial institutional reforms. From an institutional perspective, the end of this decade-long period of fiscal misallocation was achieved through a series of comprehensive fiscal policy reforms. These institutional changes complemented the macroeconomic stabilization and were instrumental in redirecting Japan's fiscal trajectory.

- FILP Reform
- "Trinity Reforms" on central government transfers to local governments
  - Sticks: Fiscal restructuring (Yubari city)
  - Carrots: "Great Heisei Mergers"

6 Conclusion

To be written

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A Data

A.1 National-level Data from National Accounts

A.1.1 Output and Employment

We collect data on nominal and real gross domestic product (GDP) for the period 1955-2022, and employment data for period 1970-2022.

Japan's national accounts data employ different accounting standards over time. For GDP data, the Cabinet Office publishes 2008SNA standard data covering 1980-2022 and 1968SNA standard data covering 1955-1998. To construct a consistent time series using the 2008SNA standard for 1955-1979, we first calculate growth rates for each GDP component (private final consumption, government final consumption, gross domestic capital formation, exports, and imports) between 1955 and 1980 using the 1968SNA standard. We then apply these growth rates to the 1980 values from the 2008SNA data to reconstruct the historical series. This method yields nominal and real GDP for the entire 1955-2022 period, from which we derive the GDP deflator.

For employment data, the Cabinet Office publishes three series: 2008SNA standard data for 1994-2022, 1993SNA standard data for 1980-2009, and 1968SNA standard data for 1970-1998. We employ a similar methodology, using growth rate information from the 1993SNA and 1968SNA series to construct a consistent time series spanning 1970-2022.

The consumption and investment data discussed in subsequent sections face similar challenges with changing accounting standards over time. We apply analogous methods to construct consistent time series for these variables, though we omit detailed descriptions of this process in the following sections.

A.1.2 Consumption

We collect private final consumption expenditure and government final consumption expenditure for 1955-2022. Private final consumption expenditure serves as our measure of household consumption C_t^H.

In the National Accounts, government final consumption is provided by the central government, local governments, and social security funds. For our measure of government consumption C_t^G, we use government final consumption by central and local governments only. This decomposed data is available for 1980-2022.

A.1.3 Investment

We collect nominal and real gross fixed capital formation (GFCF) for both private and public sectors covering 1955-2022. Public sector GFCF can be decomposed into investments by the central government, local governments, social security funds, and public enterprises. For our measure of

government investment I_t^G, we use GFCF by central and local governments only. This decomposed data is available for 1970-2022.

To extend the series back to 1955, we assume that the share of central and local government GFCF relative to total public sector GFCF during 1955-1969 equals the 1970 level. This assumption allows us to construct a complete government investment series for 1955-2022. Total GFCF excluding government investment serves as our measure of private investment I_t^K. The investment deflator is calculated by dividing nominal GFCF by real GFCF.

A.1.4 Production Elasticity of Private Capital and Labor

This appendix describes how we construct the factor income shares used to calibrate α_K and α_L for Japan. Unless otherwise noted, all series are current-price data from the Cabinet Office of Japan compiled under SNA 2008. Throughout this appendix, y denotes calendar year and $\widetilde{y}$ denotes fiscal year. When needed, the superscripts CY and FY indicate calendar-year and fiscal-year series, respectively; GG denotes general government and SSF denotes social security funds.

Annual factor shares. Our main source is "GDP and Factor Income by Economic Activities." This table is available only on a calendar-year basis and begins in 1994, which is also the first year for which factor-income items for general government are reported. We therefore construct annual factor shares for $y = 1994, \ldots, 2012$ and calibrate α_K and α_L as the simple averages of these annual shares:

$$\alpha_K = \frac{1}{19} \sum_{y=1994}^{2012} \widetilde{\alpha}_{K,y}, \qquad \alpha_L = \frac{1}{19} \sum_{y=1994}^{2012} \widetilde{\alpha}_{L,y}.$$

The annual shares are defined by

$$\widetilde{\alpha}_{K,y} = \frac{\mathrm{KI}_y}{\mathrm{NGDP}_y - \tau_y^p - \mathrm{MI}_y}, \qquad \widetilde{\alpha}_{L,y} = \frac{\mathrm{LC}_y}{\mathrm{NGDP}_y - \tau_y^p - \mathrm{MI}_y}.$$

Here, NGDP_y denotes nominal GDP, τ_y^p denotes taxes on production and imports less subsidies, LC_y denotes compensation of employees, MI_y denotes mixed income, and KI_y denotes private capital income.

Private capital income. We construct private capital income as

$$\mathrm{KI}_y = \mathrm{OP}_y + \mathrm{CFC}_y^K,$$

where OP_y is private operating surplus and CFC_y^K is private consumption of fixed capital. The source table reports "operating surplus and mixed income" rather than operating surplus alone. Let OSMI_y denote that combined series. Assigning all of mixed income to capital would mechanically

overstate the capital share. Following the convention used in the Feenstra, Inklaar and Timmer (2015), we instead assume that mixed income is split between labor and capital in the same proportions as in the rest of the economy. Under this assumption, allocating mixed income is algebraically equivalent to excluding MI_y from total factor income and recovering operating surplus as

$$\mathrm{OP}_y = \mathrm{OSMI}_y - \mathrm{MI}_y,$$

where MI_y is taken from "Generation of Income Account." Accordingly, labor income in $\widetilde{\alpha}_{L,y}$ is measured by LC_y alone. Because "operating surplus and mixed income" is recorded as zero for general government in "GDP and Factor Income by Economic Activities," OP_y corresponds to operating surplus outside general government, which we treat as private operating surplus in the factor-share calculation.

Private consumption of fixed capital. We obtain private consumption of fixed capital by subtracting the government component from total consumption of fixed capital:

$$\mathrm{CFC}_y^K = \mathrm{CFC}_y - \mathrm{CFC}_y^G,$$

where CFC_y is total consumption of fixed capital and CFC_y^G is government consumption of fixed capital under our definition of the government sector. The calendar-year table reports consumption of fixed capital for general government. Denote this series by $\mathrm{CFC}_y^{\mathrm{GG,CY}}$. Under SNA 2008, general government includes central government, local government, and social security funds. Our measure of government capital excludes social security funds, so $\mathrm{CFC}_y^{\mathrm{GG,CY}}$ must be adjusted accordingly.

The necessary breakdown is available in "Account Classified by the Sub-sectors of General Government," which is reported on a fiscal-year basis. For each calendar year y, we use the share from the fiscal year with the same label, denoted $\widetilde{y}$, and define

$$s_{\widetilde{y}}^{\mathrm{SSF}} = \frac{\mathrm{CFC}_{\widetilde{y}}^{\mathrm{SSF,FY}}}{\mathrm{CFC}_{\widetilde{y}}^{\mathrm{GG,FY}}},$$

where $\mathrm{CFC}_{\widetilde{y}}^{\mathrm{SSF,FY}}$ is consumption of fixed capital of social security funds and $\mathrm{CFC}_{\widetilde{y}}^{\mathrm{GG,FY}}$ is consumption of fixed capital of general government in fiscal year $\widetilde{y}$. We assume that this share is the same in fiscal-year and calendar-year data. Government consumption of fixed capital is then computed as

$$\mathrm{CFC}_y^G = \left(1 - s_{\widetilde{y}}^{\mathrm{SSF}}\right) \mathrm{CFC}_y^{\mathrm{GG,CY}}.$$

This procedure yields the annual series $\{\widetilde{\alpha}_{K,y}, \widetilde{\alpha}_{L,y}\}_{y=1994}^{2012}$.

A.1.5 Output and Employment

We collect nominal GDP for each prefecture for the period 1955-2020 and employment for each prefecture for the period 1975-2020. Since aggregated prefecture-level data do not perfectly match national totals, we use national-level data for aggregate figures and allocate these to each prefecture proportionally based on the corresponding prefecture-level values.

A.1.6 Consumption

We obtain private and government final consumption expenditures for each prefecture over 1955-2020. We derive household consumption C_{it}^H using the same allocation method as for output and employment.

From Section A.1.2, we have central government consumption C_{0t}^G and the aggregate local government consumption $\sum_{i=1}^N C_{it}^G$. Unlike national-level data, government final consumption expenditure data at the prefecture level do not provide further decomposed information, leaving us without a direct measure for C_{it}^G. We therefore assume that each prefecture's share of C_{it}^G in aggregate local government consumption $\sum_{i=1}^N C_{it}^G$ equals its share of government final consumption in aggregate local government final consumption. This assumption yields our measure of prefecture-level local government consumption C_{it}^G.

A.1.7 Investment and Capital

We collect nominal GFCF for both private and public sectors for each prefecture for 1955-2020. Denote by $I_{it}^{G,\text{ local}}$ the investment provided by the local government in prefecture i. Since decomposed data within public sector GFCF are not available at the prefecture level, we assume that each prefecture's share of $I_{it}^{G,\text{ local}}$ in $I_t^{G,\text{ local}}$ equals its share of public sector GFCF in aggregate public sector GFCF. This yields prefecture-level local government investment $I_{it}^{G,\text{ local}}$. We further assume that central government investment $I_t^{G,\text{ central}}$ is allocated across prefectures in proportion to their local government investment. Thus, total government investment in prefecture i is

$$I_{it}^G = I_{it}^{G,\text{ local}} + I_t^{G,\text{ central}} \times \frac{I_{it}^{G,\text{ local}}}{\sum_{i=1}^N I_{it}^{G,\text{ local}}}. \tag{49}$$

Total GFCF data for each prefecture provide I_{it}, from which we derive private investment as $I_{it}^K = I_{it} - I_{it}^G$.

We estimate initial government and private capital stocks at the beginning of 1955 using

$$Z_{i,1955} = \frac{I_{i,1955}^Z}{\log\left(I_{i,1960}^Z / I_{i,1955}^Z\right)/5 + \delta}, \tag{50}$$

where $Z \in \{G, K\}$. We then construct capital series using the perpetual inventory method:

$$Z_{it+1} = I_{it}^{Z} + (1 - \delta) Z_{it}. \tag{51}$$

A.2 Government Debt and Government Bonds

A.2.1 Government Debt and Effective Interest Rate

Government debt comprises two components: central government debt (CGD) and local government debt (LGD). Our CGD calculation includes general bonds, borrowings, financial bills, and government-guaranteed debt, collected from the Ministry of Finance for 1973-2023. We exclude Fiscal Investment and Loan Program (FILP) bonds from the CGD calculation for two reasons: (1) the 2001 FILP reform makes the data before and after that year incomparable, (2) to avoid double-counting with LGD.

Our LGD calculation encompasses both prefecture-level and municipal-level debt, obtained from the "Local Government Finance Settlement" for 1974-2022.

We calculate the effective interest rate by dividing interest payments by the outstanding debt amount. Since interest payment data are available only for general bonds, which constitute more than half of total government debt (56% during 1983-2012), we use the effective interest rate on general bonds as a proxy for the rate on total government debt.

A.2.2 Issuance Rate and Maturity Composition of Government Debt

We collect Japanese government bond auction results from the Ministry of Finance (link). Each auction record contains information on maturity, yield to maturity (YTM), and issuance amount. We classify bonds into three maturity categories: long-term (40, 30, 20, 15, and 10 years), medium-term (6, 5, 3, and 2 years), and short-term (1 year and 6 months). For each category, we calculate the annual issuance-weighted average YTM. The earliest observations begin in 1987, 1979, and 1985 for long-term, medium-term, and short-term bonds, respectively.

The Ministry of Finance publishes outstanding amounts of long-term, medium-term, and short-term government debt for 2005-2023 (link). For years prior to 2005, we assume that each category's share of total government debt remains constant at its 2005 level.

When we apply equation (46) to estimate the future effective interest rate, we set $m = 10, 5, 1$ for long-term, medium-term, and short-term bonds, respectively.

A.3 Others

A.3.1 Interest Rate

The uncollateralized overnight call rate is available from 1985 onward. For earlier years, we follow Hayashi and Koeda (2013) and use the collateralized call rate as a proxy.

A.3.2 Government Revenue

Total government revenue comprises tax and non-tax revenues from both central and local governments. We collect these data from the Ministry of Finance for 1975–2022.

A.4 Constructing Ten-Year Variables

This appendix details the conversion of annual variables to ten-year variables. For time-invariant annual parameters $X \in \{\beta, \beta_C, 1-\delta\}$, the conversion is straightforward: the ten-year counterpart is X^{10}. Time-variant variables require a more involved process. For expositional ease, we denote X_{yt} as the annual variable X in year yt, and X_t as the corresponding ten-year variable at period t in the model.

For capital stocks and nominal government debt, we set them to their corresponding annual variable at the beginning of the ten year at period t. For interest rates $X_{yt} \in \{1 + r^D_{yt}, 1 + r_{yt}\}$, $X_t = \prod_{yt} X_{yt}$. For nominal variables NX_t where $X_t \in \{I^G_{it}, I^K_{it}, Y_{it}, C^G_{it}, C^H_{it}\}$, ten-year variables are obtained by simple summation: $NX_t = \sum_{yt} NX_{yt}$.

The ten-year price index for output follows $P^Y_t = \frac{NY_t}{\sum_{yt} Y_{yt}}$, and we apply this to obtain ten-year real output and consumption: $X_t = \frac{NX_t}{P^Y_t}$, $X_t \in \{Y_{it}, C^G_{it}, C^H_{it}\}$. Ten-year price index for investment, P^{FCF}_t, is inferred from $P^{\text{FCF}}_t = \frac{NI^G_t + NI^K_t}{(G_{t+1}+K_{t+1}) - (1-\delta)(G_t+K_t)}$, which ensures the consistency with ten-year total capital stocks and depreciation rate. We apply this to obtain ten-year real investment: $X_t = \frac{NX_t}{P^{\text{FCF}}_t}$, $X_t \in \{I^G_{it}, I^K_{it}\}$.

Finally, real government debt is given by $D_t = \frac{ND_t}{P^Y_{t-1}}$, where ND_t denotes nominal government debt at the beginning of period t. This allows us to express the budget constraint in real terms as in equation (34).

B Proofs

To guarantee the non-emptyness of the feasibility set, we assume borrowing limit $\bar{D}$ is larger than the lower bound $\underline{D} \equiv -\left(\tau \bar{A}\right)^{\frac{1}{1-\alpha}} (\alpha)^{\frac{\alpha}{1-\alpha}} (1-\alpha) - W^G - T_1$.

B.1 Proof for Minimal Government Wealth

As government wealth $\bar{D}$ approaches its lower bound $\underline{D}$, we have $C^G \to 0$, so the shadow price $\frac{(1-\rho)C}{C^G} u'(C)$ diverges to infinity.

equation (14) requires R_i^G to converge to 1, that is $G_i \propto A_i^{\frac{1}{1-\alpha}}$.

B.2 Proof for Abundant Government Wealth

When $\bar{D} \to \infty$, G_i will converge to ∞. We prove this by contradiction. Suppose not, then the right-hand side of equation (14) will converge to 0 because the shadow price will converge to 0, while the left-hand side remains positive. This yields a contradiction.

Since G_i converges to ∞, R_i^G converges to zero. We have $\lim_{\bar{D}\to\infty} \kappa_i v'(G_i) = \frac{(1-\rho)C}{C^G} u'(C)$.

B.3 Proof of Proposition 1

Notice that when τ is large enough (for simplicity, we consider $\tau \to 1$) under exogenous transfer, and we recall the condition here:

$$\left[1 + \frac{\kappa_i C^G}{(1-\rho)\,\alpha Y_i}\right] R_i^G = 1 \tag{52}$$

B.3.1 Level of Government Capital

Since $\kappa_i > 0$, $(1-\rho) > 0$, $C^G > 0$, and $G_i > 0$, we have $R_i^G < 1$.

Taking the total differential of equations (10), and (52) yields:

$$\frac{dG_i}{dC^G} = \frac{\frac{\kappa_i}{1-\rho}}{1 - \alpha^2 A_i G_i^{\alpha-1}} > 0$$

$$\frac{dC^G}{d\bar{D}} = \frac{1}{1 + \sum_i \frac{\kappa_i (1-R_i^G)}{(1-\rho)(1-\alpha R_i^G)}} > 0$$

Therefore:

$$\frac{dG_i}{d\bar{D}} = \frac{dG_i}{dC^G} \frac{dC^G}{d\bar{D}} > 0$$

Hence, the discrepancy between G_i and its first-best level increases monotonically with borrowing $\bar{D}$.

B.3.2 Spatial Allocation of Government Capital

Notice that:

$$\begin{aligned}\frac{d\log(A)}{d\bar{D}} &= \alpha\sum_i \frac{G_i}{G}\left(\frac{\sum_i G_i}{\sum_i Y_i}A_i G_i^{\alpha-1}-1\right)\frac{d\log G_i}{d\bar{D}}\\ &= \alpha \mathrm{Cov}_{\frac{G_i}{G}}\left(\frac{\sum_i G_i}{\sum_i Y_i}A_i G_i^{\alpha-1}, \frac{d\log G_i}{d\bar{D}}\right)\end{aligned}$$

This can be interpreted as a covariance between $\frac{\sum_i G_i}{\sum_i Y_i}A_i G_i^{\alpha-1}$ and $\frac{d\log G_i}{d\bar{D}}$ with weights $\frac{G_i}{G}$. Notice that:

meaning that $A_i G_i^{\alpha-1}$ and $\frac{d\log G_i}{d\bar{D}}$ are monotonically negatively correlated. So, we have:

$$\frac{d\log(A)}{d\bar{D}} = \alpha \mathrm{Cov}_{\frac{G_i}{G}}\left(\frac{\sum_i G_i}{\sum_i Y_i}A_i G_i^{\alpha-1}, \frac{d\log G_i}{d\bar{D}}\right) < 0$$

B.4 Quantitative Model Details

B.4.1 Local Production and Aggregate Output

Firm's profit-maximization problem is

$$\max_{K_{it},L_{it}} \Pi_{it} = (1-\tau_t)Y_{it} - \left(1+\tau_{it}^K\right)\left(1+r_t^K\right)K_{it} - w_{it}L_{it}, \tag{53}$$

where $Y_{it} = A_{it}G_{it}^{\alpha_G}K_{it}^{\alpha_K}L_{it}^{\alpha_L}$. The first-order conditions for firms are

$$(1-\tau_t)\alpha_K\frac{Y_{it}}{K_{it}} = \left(1+\tau_{it}^K\right)\left(1+r_t^K\right), \tag{54}$$

$$(1-\tau_t)\alpha_L\frac{Y_{it}}{L_{it}} = w_{it}. \tag{55}$$

Thus firm profit equals to

$$\Pi_{it} = (1-\tau_t)(1-\alpha_K-\alpha_L)Y_{it}. \tag{56}$$

Combine Equation (54) and (55), we derive local labor demand as

$$L_{it} = \left((1-\tau_t)\left(\frac{\alpha_K}{\left(1+\tau_{it}^K\right)\left(1+r_t^K\right)}\right)^{\alpha_K}\left(\frac{\alpha_L}{w_{it}}\right)^{1-\alpha_K}A_{it}G_{it}^{\alpha_G}\right)^{\frac{1}{1-\alpha_K-\alpha_L}}. \tag{57}$$

Then, local private capital demand and local output are given by

$$K_{it} = \left(\frac{\alpha_K (1-\tau_t)}{\left(1+\tau_{it}^K\right)\left(1+r_t^K\right)} \right)^{\frac{1}{1-\alpha_K}} A_{it}^{\frac{1}{1-\alpha_K}} G_{it}^{\frac{\alpha_G}{1-\alpha_K}} L_{it}^{\frac{\alpha_L}{1-\alpha_K}}, \tag{58}$$

$$Y_{it} = \left(\frac{\alpha_K (1-\tau_t)}{\left(1+\tau_{it}^K\right)\left(1+r_t^K\right)} \right)^{\frac{\alpha_K}{1-\alpha_K}} A_{it}^{\frac{1}{1-\alpha_K}} G_{it}^{\frac{\alpha_G}{1-\alpha_K}} L_{it}^{\frac{\alpha_L}{1-\alpha_K}}. \tag{59}$$

B.4.2 Spatial Equilibrium

Household optimal choice (17) implies total land income in region i is $\frac{1-\gamma}{1+\beta} y_{it}^H L_{it}$. Therefore, each young resident receives $\frac{1-\gamma}{1+\beta} y_{it}^H$. Additionally, land market clearing requires total land income equals to $r_{it}^H \bar{H}_{it}$, which implies

$$r_{it}^H = \frac{1-\gamma}{1+\beta} \frac{y_{it}^H}{\bar{H}_{it}} L_{it}. \tag{60}$$

Household indirect utility is derived by plugging household optimal choice (17) into preference (16), which is

$$v_{it}^H = \log \left(\bar{Z}_{it} \left(C_{it}^G \left(C_{it+1}^G \right)^{\beta} \right)^{(1-\rho)} \frac{\left(y_{it}^H\right)^{\rho(1+\beta)}}{\left(r_{it}^H\right)^{\rho(1-\gamma)}} \right) + \beta\rho \log\left(1+r_{t+1}\right) + \log \Upsilon, \tag{61}$$

Combine with equation (60), the equalization of v_{it}^H across regions implies

$$y_{it}^H \propto \left(\bar{Z}_{it} \left(C_{it}^G \left(C_{it+1}^G \right)^{\beta} \right)^{(1-\rho)} \right)^{-\frac{1}{\rho(\beta+\gamma)}} \left(\frac{\bar{H}_{it}}{L_{it}} \right)^{-\frac{1-\gamma}{\beta+\gamma}}, \tag{62}$$

Labor demand (57) implies that

$$w_{it} \propto \left(\frac{A_{it} G_{it}^{\alpha_G} L_{it}^{-(1-\alpha_K-\alpha_L)}}{\left(1+\tau_{it}^K\right)^{\alpha_K}} \right)^{\frac{1}{1-\alpha_K}}, \tag{63}$$

To simplify the analysis, we assume $\phi_{it}^T \propto w_{it}$, so that $y_{it}^H \propto w_{it}$. Then, combining (63) with (62) gives

$$L_{it} \propto \left((Z_{it})^{\frac{1}{\rho(\beta+\gamma)}} \left(\frac{A_{it}}{\left(1+\tau_{it}^K\right)^{\alpha_K}} \right)^{\frac{1}{1-\alpha_K}} \right)^{\frac{(1-\alpha_K)\eta}{\alpha_G}} \left(C_{it}^G \left(C_{it+1}^G \right)^{\beta} \right)^{\frac{(1-\rho)(1-\alpha_K)\eta}{\rho(\beta+\gamma)\alpha_G}} (G_{it})^{\eta}, \tag{64}$$

where $Z_{it} \equiv \bar{Z}_{it} \left(\bar{H}_{it}\right)^{\rho(1-\gamma)}$, $\eta \equiv \frac{\alpha_G}{1-\alpha_K-\alpha_L+\frac{1-\gamma}{\beta+\gamma}(1-\alpha_K)}$.

We assume fixed aggregate labor supply $L_t \equiv \sum_{i=1}^N L_{it}$. Local output is obtained by substituting

(64) into (59)

$$
\begin{aligned}
Y_{it} &= \left(\frac{\alpha_K\left(1-\tau_t\right)}{\left(1+\tau_{it}^K\right)\left(1+r_t^K\right)}\right)^{\frac{\alpha_K}{1-\alpha_K}} A_{it}^{\frac{1}{1-\alpha_K}} G_{it}^{\frac{\alpha_G}{1-\alpha_K}} L_{it}^{\frac{\alpha_L}{1-\alpha_K}} \\
&= \left(\frac{\alpha_K\left(1-\tau_t\right)}{\left(1+\tau_{it}^K\right)\left(1+r_t^K\right)}\right)^{\frac{\alpha_K}{1-\alpha_K}} A_{it}^{\frac{1}{1-\alpha_K}} G_{it}^{\frac{\alpha_G}{1-\alpha_K}} \times \\
&\quad \left(\frac{\left(Z_{it}^{\frac{1}{\rho(\beta+\gamma)}}\left(\frac{A_{it}}{\left(1+\tau_{it}^K\right)^{\alpha_K}}\right)^{\frac{1}{1-\alpha_K}}\right)^{\frac{(1-\alpha_K)\eta}{\alpha_G}}\left(C_{it}^G\left(C_{it+1}^G\right)^{\beta}\right)^{\frac{(1-\rho)(1-\alpha_K)\eta}{\rho(\beta+\gamma)\alpha_G}} G_{it}^{\eta}}{\sum_i\left(Z_{it}^{\frac{1}{\rho(\beta+\gamma)}}\left(\frac{A_{it}}{\left(1+\tau_{it}^K\right)^{\alpha_K}}\right)^{\frac{1}{1-\alpha_K}}\right)^{\frac{(1-\alpha_K)\eta}{\alpha_G}}\left(C_{it}^G\left(C_{it+1}^G\right)^{\beta}\right)^{\frac{(1-\rho)(1-\alpha_K)\eta}{\rho(\beta+\gamma)\alpha_G}} G_{it}^{\eta}} L_t\right)^{\frac{\alpha_L}{1-\alpha_K}} \\
&= \frac{\left(\frac{\alpha_K(1-\tau_t)}{1+r_t^K}\right)^{\frac{\alpha_K}{1-\alpha_K}} L_t^{\frac{\alpha_L}{1-\alpha_K}}}{\left(\sum_i\left(Z_{it}^{\frac{1}{\rho(\beta+\gamma)}}\left(\frac{A_{it}}{\left(1+\tau_{it}^K\right)^{\alpha_K}}\right)^{\frac{1}{1-\alpha_K}}\right)^{\frac{(1-\alpha_K)\eta}{\alpha_G}}\left(C_{it}^G\left(C_{it+1}^G\right)^{\beta}\right)^{\frac{(1-\rho)(1-\alpha_K)\eta}{\rho(\beta+\gamma)\alpha_G}} G_{it}^{\eta}\right)^{\frac{\alpha_L}{1-\alpha_K}}} \times \\
&\quad Z_{it}^{\frac{\alpha_L\eta}{\rho(\beta+\gamma)\alpha_G}}\left(\frac{A_{it}}{\left(1+\tau_{it}^K\right)^{\alpha_K}}\right)^{\frac{1+\frac{\alpha_L\eta}{\alpha_G}}{1-\alpha_K}}\left(C_{it}^G\left(C_{it+1}^G\right)^{\beta}\right)^{\frac{(1-\rho)\alpha_L\eta}{\rho(\beta+\gamma)\alpha_G}} G_{it}^{\frac{\alpha_G+\alpha_L\eta}{1-\alpha_K}}.
\end{aligned} \tag{65}
$$

Therefore, aggregate output $Y_t \equiv \sum_{i=1}^N Y_{it}$ is given by

$$
Y_t = \left(\frac{\alpha_K\left(1-\tau_t\right)}{1+r_t^K}\right)^{\frac{\alpha_K}{1-\alpha_K}} L_t^{\frac{\alpha_L}{1-\alpha_K}} \frac{\sum_{i=1}^N Z_{it}^{\frac{\alpha_L\eta}{\rho(\beta+\gamma)\alpha_G}}\left(\frac{A_{it}}{\left(1+\tau_{it}^K\right)^{\alpha_K}}\right)^{\frac{1+\frac{\alpha_L\eta}{\alpha_G}}{1-\alpha_K}}\left(C_{it}^G\left(C_{it+1}^G\right)^{\beta}\right)^{\frac{(1-\rho)\alpha_L\eta}{\rho(\beta+\gamma)\alpha_G}} G_{it}^{\frac{\alpha_G-\frac{1-\gamma}{\beta+\gamma}\alpha_L\eta}{1-\alpha_K-\alpha_L}}}{\left(\sum_{i=1}^N\left(Z_{it}^{\frac{1}{\rho(\beta+\gamma)}}\left(\frac{A_{it}}{\left(1+\tau_{it}^K\right)^{\alpha_K}}\right)^{\frac{1}{1-\alpha_K}}\right)^{\frac{(1-\alpha_K)\eta}{\alpha_G}}\left(C_{it}^G\left(C_{it+1}^G\right)^{\beta}\right)^{\frac{(1-\rho)(1-\alpha_K)\eta}{\rho(\beta+\gamma)\alpha_G}} G_{it}^{\eta}\right)^{\frac{\alpha_L}{1-\alpha_K}}}.
$$

Now we look at the effect of regional government investment G_i. First of all, labor allocation rule (64) implies

$$
\frac{\partial L_{nt}}{\partial G_{it}} = \eta\frac{L_{nt}}{G_{it}}\cdot 1_{\{i=n\}} - \eta\frac{L_{it}}{L_t}\frac{L_{nt}}{G_{it}}. \tag{66}
$$

Then, local production function (59) implies

$$
\begin{aligned}
\frac{\partial Y_{nt}}{\partial G_{it}} &= \frac{\alpha_G}{1-\alpha_K}\frac{Y_{nt}}{G_{it}}\cdot 1_{\{i=n\}} + \frac{\alpha_L}{1-\alpha_K}\frac{Y_{nt}}{L_{nt}}\frac{\partial L_{nt}}{\partial G_{it}} \\
&= \left(\frac{\alpha_G}{1-\alpha_K} + \frac{\alpha_L}{1-\alpha_K}\eta\right)\frac{Y_{nt}}{G_{it}}\cdot 1_{\{i=n\}} - \frac{\alpha_L}{1-\alpha_K}\eta\frac{L_{it}}{L_t}\frac{Y_{nt}}{G_{it}}.
\end{aligned} \tag{67}
$$

Therefore,

$$\begin{aligned}\frac{\partial Y_t}{\partial G_{it}} &= \sum_{n=1}^{N} \frac{\partial Y_{nt}}{\partial G_{it}} \\ &= \sum_{n=1}^{N} \left(\left(\frac{\alpha_G}{1-\alpha_K} + \frac{\alpha_L}{1-\alpha_K}\eta \right) \frac{Y_{nt}}{G_{it}} \cdot 1_{\{i=n\}} - \frac{\alpha_L}{1-\alpha_K}\eta \frac{L_{it}}{L_t}\frac{Y_{nt}}{G_{it}} \right) \\ &= \frac{\alpha_G}{1-\alpha_K-\alpha_L}\frac{Y_{it}}{G_{it}} - \left(\frac{(1-\gamma)\,\alpha_L\eta}{(\beta+\gamma)\,(1-\alpha_K-\alpha_L)}\frac{Y_{it}}{G_{it}} + \frac{\alpha_L\eta}{1-\alpha_K}\frac{L_{it}}{L_t}\frac{Y_t}{G_{it}} \right).\end{aligned} \tag{68}$$

Next, we look at the equilibrium household utility. Equation (62) implies

$$\begin{aligned}y_{it}^H L_{it} &\propto \left(Z_{it} \left(C_{it}^G \left(C_{it+1}^G \right)^{\beta} \right)^{(1-\rho)} \right)^{-\frac{1}{\rho(\beta+\gamma)}} L_{it}^{\frac{1+\beta}{\beta+\gamma}} \\ &= \frac{\left(Z_{it} \left(C_{it}^G \left(C_{it+1}^G \right)^{\beta} \right)^{(1-\rho)} \right)^{-\frac{1}{\rho(\beta+\gamma)}} L_{it}^{\frac{1+\beta}{\beta+\gamma}}}{\sum_i \left(Z_{it} \left(C_{it}^G \left(C_{it+1}^G \right)^{\beta} \right)^{(1-\rho)} \right)^{-\frac{1}{\rho(\beta+\gamma)}} L_{it}^{\frac{1+\beta}{\beta+\gamma}}} Y_t^H.\end{aligned} \tag{69}$$

Then, the indirect utility (61) can be rewritten as a term common across regions:

$$\begin{aligned}\bar{U}_t^H &= \log\left(Z_{it} \left(C_{it}^G \left(C_{it+1}^G \right)^{\beta} \right)^{(1-\rho)} L_{it}^{-\rho(1+\beta)} \left(y_{it}^H L_{it} \right)^{\rho(\beta+\gamma)} \right) + \log \tilde{\Upsilon}_t \\ &= \rho\,(\beta+\gamma) \log\left(\left(Z_{it} \left(C_{it}^G \left(C_{it+1}^G \right)^{\beta} \right)^{(1-\rho)} \right)^{\frac{1}{\rho(\beta+\gamma)}} L_{it}^{-\frac{1+\beta}{\beta+\gamma}} y_{it}^H L_{it} \right) + \log \tilde{\Upsilon}_t \\ &= \rho\,(\beta+\gamma) \log\left(\frac{Y_t^H}{\sum_{i=1}^N \left(Z_{it} \left(C_{it}^G \left(C_{it+1}^G \right)^{\beta} \right)^{(1-\rho)} \right)^{-\frac{1}{\rho(\beta+\gamma)}} L_{it}^{\frac{1+\beta}{\beta+\gamma}}} \right) + \log \tilde{\Upsilon}_t\end{aligned} \tag{70}$$

where $Y_t^H \equiv \sum_{i=1}^N y_{it}^H L_{it}$ is aggregate household income, and $\tilde{\Upsilon}_t \equiv \Upsilon \left(\frac{1-\gamma}{1+\beta} \right)^{-\rho(1-\gamma)} (1+r_{t+1})^{\beta\rho}$ is an exogenous term. Equation (24) implies $Y_t^H = \frac{1+\beta}{\beta+\gamma}\left((1-\tau_t)\,(1-\alpha_K)\,Y_t + T_t \right)$, where $T_t \equiv \sum_{i=1}^N \phi_{it}^T L_{it}$ is the aggregate lump-sum government transfer or tax. Take derivative w.r.t. G_{it}, we obtain

$$\frac{\partial \bar{U}_t^H}{\partial G_{it}} = \rho\,(1+\beta)\,\frac{(1-\tau_t)\,(1-\alpha_K)}{Y_t^H}\frac{\partial Y_t}{\partial G_{it}} - \rho\,(1+\beta)\,\eta \left(\frac{y_{it}^H L_{it}}{Y_t^H} - \frac{L_{it}}{L_t} \right) \frac{1}{G_{it}}. \tag{71}$$

B.4.3 Central Planner Problem

Central planner chooses $\left\{C^G_{t+j}, G_{it+j+1}, D_{t+j+1}\right\}^{N,\infty}_{i=1,j=0}$ to maximize

$$U^C_t = \omega^o \bar{U}^H_{t-1} + \sum_{j=0}^{\infty} (\beta_C)^j \left(\bar{U}^H_{t+j} + \sum_{i=1}^{N} \kappa_{it+j} \log G_{it+j+1} \right), \tag{72}$$

where $\bar{U}^H_{t+j}$ follows (70). We assume $\omega = \frac{\beta}{\beta_C}$ and $C^G_{it} = \psi_i C^G_t$, abstracting from terms that are independent of the central government's choices, the objective can be simplified to the following time-consistent preference:

$$\hat{U}^C_t = \sum_{j=0}^{\infty} \beta^j_C \left(\hat{\bar{U}}^H_{t+j} + \left(1 + \frac{\beta}{\beta_C}\right)(1-\rho) \log C^G_{t+j} + \sum_{i=1}^{N} \kappa_{it+j} \log G_{it+j+1} \right),$$

where

$$\hat{\bar{U}}^H_t = \rho\,(\beta+\gamma) \log Y^H_t - \rho\,(\beta+\gamma) \log \left(\sum_{i=1}^{N} \left(Z_{it}^{-\frac{1}{\rho(\beta+\gamma)}} \psi_i^{-\frac{(1-\rho)(1+\beta)}{\rho(\beta+\gamma)}} L_{it}^{\frac{1+\beta}{\beta+\gamma}} \right) \right).$$

Government choices are subject to the following budget constraint:

$$C^G_{t+j} + \sum_{i=1}^{N} q^G_{t+j} I^G_{it+j} + T_{t+j} + \Phi_{t+j} = \tau_{t+j} Y_{t+j} + D_{t+j+1} - \left(1 + r^D_{t+j}\right) D_{t+j}, \tag{73}$$

and fiscal rule

$$\frac{\left(a_{t+j} + r^D_{t+j+1}\right) D_{t+j+1}}{Y_{t+j}} = b. \tag{74}$$

The Lagrangian equation is

$$\begin{aligned}\mathcal{L} = \sum_{j=0}^{\infty} \beta^j_C \Bigg[& (1-\rho) \left(1 + \frac{\beta}{\beta_C}\right) \log C^G_{t+j} + \hat{\bar{U}}^H_{t+j} + \sum_{i=1}^{N} \kappa_{it+j} \log G_{it+j+1} \\ & + \lambda_{t+j} \left(\tau_{t+j} \sum_{i=1}^{N} Y_{it+j} + D_{t+j+1} - \left(1 + r^D_{t+j}\right) D_{t+j} - C^G_{t+j} - \sum_{i=1}^{N} q^G_{t+j} I^G_{it+j} - T_{t+j} - \Phi_{t+j} \right) \Bigg]\end{aligned}$$

The first-order condition w.r.t C^G_{t+j} is

$$0 = (1-\rho) \left(1 + \frac{\beta}{\beta_C}\right) \frac{1}{C^G_{t+j}} - \lambda_{t+j}. \tag{75}$$

Using this, the first-order conditions w.r.t. G_{it+j+1} for $i = 1, \ldots, N$ are

$$\begin{aligned}\frac{C^G_{t+j+1}}{C^G_{t+j}} = \frac{\beta_C}{q_{t+j}} \Bigg(\left(\tau_{t+j+1} + \frac{\partial D_{t+j+2}}{\partial Y_{t+j+1}} \right) \frac{\partial Y_{t+j+1}}{\partial G_{it+j+1}} + q_{t+j+1} \left(1 - \delta\right) \Bigg) \\ - \frac{\beta_C^2}{q_{t+j}} \frac{C^G_{t+j+1}}{C^G_{t+j+2}} \left(1 + r^D_{t+j+2}\right) \frac{\partial D_{t+j+2}}{\partial Y_{t+j+1}} \frac{\partial Y_{t+j+1}}{\partial G_{it+j+1}} \\ + \frac{C^G_{t+j+1}}{\left(1 + \beta/\beta_C\right)\left(1 - \rho\right) q_{t+j}} \left(\beta_C \frac{\partial \bar{U}^H_{t+j+1}}{\partial G_{it+j+1}} + \kappa_{it+j} \frac{1}{G_{it+j+1}} \right)\end{aligned} \tag{76}$$

where $\frac{\partial Y_t}{\partial G_{it}}$ follows (68) and $\frac{\partial \bar{U}_t}{\partial G_{it}}$ follows (71).