Growth Accounting for the States*

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Abstract

We study United States economic growth at the state level over the period from 1971-2019. Our main contribution is the construction of new data series on capital at the state and industry level, using data on the aggregate capital stock and payments to capital at the state and industry level. We use published series and simple theory to construct this data, as there are no official data on investment or capital at the state level. Importantly, we allow for differential and time-varying profit shares by state and industry, and account for differential state taxes. We use this data to study capital accumulation and factor shares across states, regions, and industries. We find declines in labor and capital shares overall, and a growth of profits, but not uniformly. For example, the service sector accounted for an increasing share of output, but experienced an increasing labor share and decline in profits. Although there is substantial variation across states and sectors, we also find evidence of convergence across states in labor shares. We then estimate utilization-adjusted productivity at the state and industry level, decomposing productivity growth into improvements in technical efficiency and allocative efficiency. Between-sector allocative efficiency is a relatively minor factor for US as a whole, but can be important at the state level.

Keywords: Growth accounting; capital; productivity; state-level data; income share

JEL classification: E22, E25, O47, R11

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1 Introduction

The United States provides a rich environment to study economic dynamics and the impact of economic policy. The diversity of statewide and regional factors and policies allows for many interesting interactions and comparisons. In addition, the commonality of national factors, along with federal fiscal and monetary policy, can potentially allow for a clear isolation of sources of difference. A growing body of research has used this regional variation to inform estimates of policy impacts and sources of fluctuations.[1] While these papers are clever in using, developing, and compiling data sources and identification strategies, a lack of readily available data has held back the broader development of the field. This paper contributes to this research by developing a new data source for state economies.

Our main contribution is the construction of new data series on capital at the state and industry level, for the period from 1971-2019, using data on the aggregate capital stock and payments to capital at the state and industry level. While there are many macroeconomic variables on public databases, there are no official published data on capital or investment at the state level. Given the importance of capital dynamics in economic growth and growth accounting, this absence of data is critical. We use published data and simple theory to construct this data, as there are no official data on investment or capital at the state level. We use this data to study capital accumulation and convergence across states, as well as levels and trends in factor shares across states, regions, and industries.

Importantly, we allow for differential and time-varying profit shares by state and industry, and account for differential state-taxes. Earlier attempts to estimate state-level capital relied on perfect competition and thus the absence of profits, and generally made strong assumptions on capital mobility and factor shares. Instead, we build on the body of research over the past decade that has documented time variation in factor shares (particularly the labor share of income), as well as decreased competition and the growth of profits.[2]

The key to our approach is to use published data from the BEA on capital by industry at the national level, along with a separate measure of gross operating surplus (GOS) which is reported

[1] For example, Mian and Sufi (2009), Mian, Rao and Sufi (2013), Suárez Serrato and Zidar (2016), Fajgelbaum et al. (2019), Beraja, Hurst and Ospina (2019), Caliendo et al. (2018), Nakamura and Steinsson (2014), and Jones, Midrigan and Philippon (2022), among others study the impacts of economic shocks and state and federal policy across different states and regions.

[2] Elsby, Hobijn and Sahin (2013) and **?** are two key early papers on the decline of the labor share, while Barkai (2020) also showed the decline of the capital share and the growth of profits. De Loecker, Eeckhout and Unger (2020) documented the growth of market power and profits at the micro level.

by state and industry. This measure of GOS contains both measures of payments to capital income as well as profits. We decompose the GOS into its components by estimating an industry-level required rate of return, and assuming that the profit share by industry does not vary across states. While still somewhat restrictive, this represents a significant relaxation and generalization relative to previous work, and provides a convenient approach that preserves aggregation. In the end, we obtain estimates of capital by state and industry, as well as time series (by state and industry) of the shares of income going to capital, labor, and profits. Our estimates of industry-level capital by state lines up well with existing survey data on state-level manufacturing sector capital expenditures in the Annual Survey of Manufactures, better than those from the estimation approaches adopted in the existing literature with stronger assumption (e.g., Garofalo and Yamarik, 2002; Yamarik, 2013; Chirinko and Wilson, 2008).

In addition to estimating capital and income shares, we also estimate state- and industry-level TFP in the presence of time-varying markups, and decompose it into changes in true productivity and changes in allocative efficiency. Our exercise is in a similar spirit with Baqaee and Farhi (2020), but with a focus on the states.[3] We use this data to analyze the differential patterns of growth and factor accumulation across states and industries. We also study the differential patterns in the allocation of capital and productivity across states and industry.

We find substantial heterogeneity in the growth experiences of states. We show that the decline in labor's share of income was strongest in states where it was initially the highest, and while labor's share fell in manufacturing it increased in services. Thus even as manufacturing shrank and services grew as a share of total output, labor's share declined overall. While the share of output going to pure profits generally increased over time, this pattern can differ a lot by industry. For example, profit share has increased dramatically in manufacturing and finance sectors since 1980s but it declined over time in services.

For US as a whole, the impact of allocative efficiency change over time is minimal, with most of productivity growth consisting of true improvements in technical efficiency. However at a more disaggregated level, this allocative efficiency matters for some states and industries. For some states, this explains a significant portion of their TFP growth.

Related Literature. Our paper contributes to the literature estimating or using state-level capital. Garofalo and Yamarik (2002) and Yamarik (2013) apportion the national industry-level

[3]Because our data is more aggregated, we abstract from some of the additional features (such as the input-output network) in Baqaee and Farhi (2020).

capital by the relative income shares of each state in that industry. This approach has been taken by Desmet and Rossi-Hansberg (2013) as well. Similarly, Kleinman, Liu and Redding (2023) estimate state-level real capital stocks by computing state-industry level nominal gross operating surplus by subtracting labor compensation from GDP and then deflating each observation by a corresponding estimate of the national capital income deflator. Turner, Tamura and Mulholland (2013) and Khatri and Tamura (2022) apportion capital from capital-output ratio at the national industry level, which is scaled by industry value-added share in each state to construct state-industry level capital, while constructing capital for the agricultural and manufacturing sectors from surveys. Chirinko and Wilson (2008) and Wilson (2009) specifically estimate state-level capital for the manufacturing sector and R&D, respectively.

Our approach distinguishes itself from the existing literature in three key ways: 1) we avoid the strict assumptions of "equal shares" and strong capital mobility, 2) we account for taxes and we especially consider non-zero profit shares, and 3) we provide capital estimates for all major sectors in each state. As a by-product of our estimation approach, we are able to show the changes in labor, capital and profit shares by state and industry, complementing the extensive literature on income share dynamics at the country level (see Karabarbounis, 2024 for a review of this literature). In addition, our analysis of regional-level profit share is related to the recent discussion of local concentration and markups (Rossi-Hansberg, Sarte and Trachter, 2021; Smith and Ocampo, forthcoming; Becker et al., 2024).

Our growth accounting framework is related to a voluminous literature adjusting for estimation biases in Solow's classic growth accounting model, such as Basu, Fernald and Kimball (2006); Fernald (2014); Comin et al. (2025); Huo, Levchenko and Pandalai-Nayar (2023). Our productivity decomposition exercise is motivated by Baqaee and Farhi (2020), who decompose productivity changes into variations in technical and allocative efficiency. Our work is distinguished from these existing studies by our focus on state-level productivity estimation.

In previous work, Liu and Williams (2019) used a subset of (an earlier version of) our state-level data to characterize the state-level differences in response to federal fiscal shocks. They adapted the national approach of Mertens and Ravn (2013) and Romer and Romer (2010) to the state-level, finding substantial heterogeneity across states, with more than half having no significant response to the tax cuts.

The rest of this paper proceeds as follows. Section 2 summarizes the data used for estimating capital and for growth accounting. Section 3 provides results for the capital estimation together

with a descriptive analysis of factor shares and cross-state convergence. We present the growth accounting framework and results in Section 4. Section 5 presents a decomposition of US productivity to within-state and cross-state misallocation components. Section 6 concludes.

2 Data and Measurement

2.1 Data Source

The state-level data are mainly drawn from BEA's Regional Accounts, including GDP in current dollars (SAGDP2), compensation of employees (SAGDP4), taxes on production and imports less subsidies (SAGDP3), gross operating surplus (SAGDP7), real GDP in chained dollars (SAGDP9). Industry-level value added together with its components (after 1997, plus historical data 1947–1997), used for estimating industry-wide profit shares, are taken from the Industry Economic Accounts. BEA Fixed Assets Accounts include data on current cost net stock of private fixed assets by industry, chain-type quantity index for net stock of private fixed asset by industry, current cost depreciation of private fixed asset by industry, investment in private fixed asset by industry, chain-type quantity index for investment in private fixed asset by industry. Lastly, we use data on nominal gross value added (NIPA Table 1.14 line 17), compensation of employees (NIPA Table 1.14 line 20), taxes on production and imports less subsidies (NIPA Table 1.14 line 23), gross operating surplus, supplemented by Compensation of Employees by Industry (NIPA Table 6.2A B C D).

Our estimate of state-specific cost of capital R_{ijt} requires information on a proper measure of business income tax rate at the state level. We collect federal corporate tax rate from Tax Police Center for 1947–1979, Suárez Serrato and Zidar (2016) for 1980-2010, and Internal Revenue Service for 2011-2020. We hand collect state corporate income tax rates from the Book of the States for 1947–2010 and the Tax Foundation for 2011-2020. The cost of capital is estimated as a weighted average of the debt cost of capital i^D and equity cost of capital i^E, as in Barkai (2020).

We compute the series of state- and industry-level capital stocks for ten two-digit private sector industries: "Construction", "Finance and Insurance", "Farms", "Forestry and Fishing", "Manufacturing", "Mining", "Retail", "Services", "Transportation, Communication, and Utilities", and "Wholesale." To ensure consistency in the definition of sectors before and after 1997, we apply the concordance between SIC and NAICS constructed by Fort and Klimek (2018) and allocate these series in SIC-classified industries to the ten NAICS industries in proportion to the value of shipments (or employment) attributable to each SIC and NAICS pair.

For the growth accounting exercise, we collect hours per worker data from the BLS Current Employment Statistics. The BLS hours data are the closest match to hours used by establishments in the state-industry level, but they are available only for private nonfarm industries after 2007. Our benchmark productivity estimates therefore use the subsample with directly observed state-industry hours and exclude sectors with missing or extremely sparse hours coverage. To extend the productivity series backward, we impute pre-2007 state-industry hours with the corresponding national industry hours and use those longer-sample estimates as robustness checks.

The external instruments used in the productivity estimation combine aggregate shocks with sectoral or regional exposure measures. The oil instrument uses the WTI crude oil price series together with industry oil intensities constructed from the BEA input-output tables. The state-level defense instruments use the Department of Defense prime contract data from USAspending. In the productivity estimation, we deflate output and capital series using the national core CPI and deflate procurement spending with the BEA national defense deflator (Nakamura and Steinsson, 2014).

2.2 Measurement Strategy

The basic estimation problem that we face is one of separately identifying state and industry level variables from an observed composite. The BEA regional accounts provide data on output, labor income, gross operating surplus (GOS), and taxes by state and industry. Using standard notation, we can write this accounting identity as:

$$Y_{ijt} = \underbrace{W_{ijt}N_{ijt}}_{\text{Employee Compensation}} + \underbrace{R_{ijt}K_{ijt} + \Pi_{ijt}}_{\text{Gross Operating Surplus}} + \underbrace{Others_{ijt}}_{\text{Taxes on production and imports less subsidies}} . \quad (2.1)$$

The estimation problem is to decompose the observed GOS into its capital income $R_{ijt}K_{ijt}$ and profit income Π_{ijt} components, and then to separate the rate of return R_{ijt} from the capital stock K_{ijt}.

The existing approaches discussed above, such as Garofalo and Yamarik (2002), Yamarik (2013), and Kleinman, Liu and Redding (2023), impose relatively strong assumptions to make this identification. In particular, they assume competitive markets and constant returns to scale, which gives the standard zero profit condition, $\Pi_{ijt} = 0$ for all i and j. Thus they identify GOS with capital income directly. Then they assume perfect capital mobility, which enables equalization of rates of return, so either $R_{ijt} = R_{jt}$ with perfect mobility across states within a sector, or $R_{ijt} = R_t$ with

perfect capital mobility across states and sectors. These assumptions then mean that the share of national capital within a state and sector is the same as the share of national GOS within that state and sector. This is a relatively straightforward approach, which provides a way to apportion national capital by shares of GOS. However it relies on strong assumptions of the traditional neo-classical approach: constant returns, zero profits, strong capital mobility, and no taxes or frictions which prevent capital mobility and marginal factor price equalization.

Our approach weakens several of these assumptions. In particular, based on the literature estimating markups and misallocation of resources, we allow for nonzero profit shares. Thus we need a different identifying assumption to separate profits and capital income. As we discuss below, we assume that profit shares are equated across states within and industry. While restrictive, this allows us to use national data to separate $R_{ijt}K_{ijt}$ from Π_{ijt}. In particular, rather than apportioning national GOS by states, it justifies estimating profit national industry-level profit shares, then apportioning the share of non-profit income within an industry by state. Then we directly estimate required rates of return by state and industry, taking into account federal and state taxes as well as differential industry-level depreciation. These estimates of R_{ijt} then allow us to estimate the capital stocks K_{ijt}.

As discussed above, our approach allows us to weaken several of the assumptions in the previous versions. We avoid assumptions of equal factor income shares and perfect capital mobility, deriving time-varying estimates of industry- and state-level income shares of labor, capital, and profits. Importantly, we incorporate distortions by accounting for differential tax treatments and especially considering non-zero profit shares and markups. Our results provide a complete set of disaggregated capital and TFP estimates for all major sectors (which are still relatively aggregated) in each state. The main drawback of our approach is that we assume constant profit shares by industry, which conflicts with some evidence in the recent literature on spatial markups and misallocation. We are working on relaxing this assumption. A smaller practical drawback is that because we separately estimate capital by state and region, our results are no longer ensured to aggregate to the direct estimates of national capital. However the difference is small in practice.

Indirect taxes in the data are usually very small, so we focus our attention on the three income components: labor income, capital income and profit income. In order to separate capital income from profit income when only GOS is observed at the state-industry level, we assume that profit share only depends on the industry, not the state. As such, the accounting identity (2.1) can be

expressed in the following way

$$(1-\pi_{jt})Y_{ijt} = W_{ijt}N_{ijt} + R_{ijt}K_{ijt}, \tag{2.2}$$

where Y_{ijt} is the valued added at industry j for state i net of indirect taxes, that is, taxes on production and imports less subsidies.

As mentioned above, we make the assumption that profit share only depends on the industry, not the state. Therefore, we can rely on the aggregate industry-level data to estimate the profit share in each industry. For industry-level accounting

$$(1-\pi_{jt})Y_{jt} = W_{jt}N_{jt} + R_{jt}K_{jt}. \tag{2.3}$$

Combining (2.2) and (2.3), and we can express the capital income part as a function of observables

$$R_{ijt}K_{ijt} = \frac{Y_{ijt}}{Y_{jt}}\left(R_{jt}K_{jt} + W_{jt}N_{jt}\right) - W_{ijt}N_{ijt}. \tag{2.4}$$

2.3 Required Rate of Return

To estimate K_{ijt}, the key is to estimate the required rate of return on capital R_{ijt} for each industry-state pair. Instead of simply summing up the 10-year US treasury rate and a BBB bond spread, we take a more sophisticated approach following Barkai (2020), which is based on Hall and Jorgenson (1967). In particular,

$$R_{ijt} = \left(\frac{D}{D+E}i^{D}(1-\tau_{it}) + \frac{E}{D+E}i^{E} - \mathbb{E}(\pi^{k}_{jt+1}) + \delta_{jt}\right)\frac{1-z\tau_{it}}{1-\tau_{it}}, \tag{2.5}$$

where R_{ijt} is calculated as the weighted average of the debt cost of capital and equity cost of capital, with the weights given by the market value of debt and equity market, adjusted by inflation rate, depreciation rate and depreciation allowances of capital. We also consider differential tax treatments of debt and equity-financed capital, as corporate income tax rates differ significantly across states. Figure A.1 presents the estimates of required rate of return at different sectors, for both the US as a whole and states.

3 State-Level Capital and Income Shares

3.1 Capital by State and Industry

The main task in our estimation of capital is to decompose reported valued added at the state and industry level into wage payments to labor, rental payments to capital and profit income, and use them to estimate capital stock by state and industry. These estimates are then fed into a standard growth accounting framework to estimate state-level productivity. The key features of our estimation are the income approach to GDP accounting, as in Karabarbounis and Neiman (2019), an equation for the required rate of return on capital following Hall and Jorgenson (1967), and a strategy to estimate profit shares.

3.2 Income Shares

Figure 2 show markedly different factor income share paths across sectors. In the total private sector, the US-wide labor share is roughly 2/3 in the early 1970s, remains fairly stable through the 1970s and 1980s, and then drifts down from the late 1990s onward to about 0.60 by 2020. Labor income share in services trends mildly upward over time, reaches about 0.72 around 2000, dips modestly around the Great Recession, and ends close to 0.70. Manufacturing, however, looks very different: it begins much higher, around 0.75, but then declines substantially and persistently, with especially visible drops from the mid-1980s onward and again in the 2000s, ending near 1/2. Moreover, statewide series indicate much larger dispersion in manufacturing than in services. Overall, the aggregate private sector decline is modest relative to the very large fall within manufacturing, while services are broadly stable to slightly rising. Capital income share in the total private sector exhibit mildly decreasing pattern since the 1980s, while that in manufacturing has seen increase and that in service stays relatively flat. Profit shares, in contrast, increase dramatically for total private and manufacturing, yet broadly decline for the service sector.

These patterns are related to but differ from the labor share literature. Karabarbounis and Neiman (2014) document a broad decline in labor shares since the early 1980s across many countries and industries, consistent with the downward movement in the total private series. But we show that: first, these patterns can be different across sectors, and second, profit share changes largely mirror the dynamics of labor income share.

3.3 Convergence

We examine convergence across states in GDP per capita, labor (as well as capital) income share, and capital to labor ratio, following the approach by Barro and Sala-i Martin (1992). The classic U.S. regional-convergence literature focuses primarily on income and productivity levels, e.g., Barro and Sala-i Martin (1992) and Caselli and Coleman II (2001). Relative to this literature, our contribution is to shift attention from income or labor productivity to the factor structure of regional production itself. We show that interstate convergence in aggregate income can coexist with broad convergence in labor shares and more uneven convergence in capital intensity across sectors.

GDP per Capita Figure 3 plots the average annual growth rate of real per capita GDP from 1963 to 2022 against its level in 1963.[4] The fitted line is negative and statistically significant, indicating evidence for β-convergence: states that started richer tend to grow more slowly over the subsequent decades, while initially poorer states tend to catch up. This pattern is in line with Barro and Sala-i Martin (1992), who show a similar convergence relationship for per capita income across US states.

Labor Share Figure 4 shows strong convergence in labor shares across all sectors. In each sector, the fitted relationship between the 1971 labor share and its subsequent change through 2019 is negative, suggesting that states with initially high labor shares tend to experience larger subsequent declines, while states with initially low labor shares tend to see smaller declines.

Capital-to-Labor Ratio Figure 5 shows convergence in the capital-to-labor ratio in the private sector as a whole, but the pattern is much weaker for manufacturing and finance. The private sector displays a clear negative relationship between the 1971 level and the subsequent change through 2019. This pattern is much weaker in manufacturing, however, where the full cross-state sample shows little systematic tendency toward convergence.

Figure 6 focuses on manufacturing and plots 48 states, excluding Alaska, Hawaii, and the District of Columbia. The red points denote the 12 states (i.e., top quartile) with the highest value added share in manufacturing in 1971. For the full sample of states, there is little evidence of convergence in manufacturing. However, within the group of manufacturing-heavy states, the

[4]Real GDP is constructed by deflating nominal GDP by the BEA gross domestic product implicit price deflator.

relationship is significantly negative, suggesting that any convergence in manufacturing capital intensity is concentrated within a more limited subset of states rather than broadly shared across the national distribution.

4 State-Level Growth Accounting

In this section, we build a parsimonious model following Basu, Fernald and Kimball (2006) (hereafter, BFK) to present the accounting identities necessary for the estimation of capital, and to illustrate the estimation strategy for growth accounting at the state-industry level. In particular, we allow for non-negligible roles for changes in *input utilization* and *profit*.

4.1 Estimation Framework

We index states by i and industries by j. For brevity, we omit these indexes where no ambiguity occurs as the same estimation applies to all state-industry pairs. Production function takes a Cobb-Douglas form:

$$Y_t = Z_t \left(U_{k,t} K_t\right)^{\alpha_k} \left(U_{n,t} N_t H_t\right)^{\alpha_n}, \tag{4.6}$$

where Y_t denotes the gross value added at the state-industry level, Z_t the TFP; input variables K_t, N_t and H_t stand for capital, number of workers and hours per worker respectively. Following BFK, we introduce two utilization terms $U_{k,t}$ and $U_{n,t}$, which capture the effort of each worker and the capital utilization rate. Since data on intermediate inputs are not available at the state level, we study industry value added instead of gross output. Consistent with estimates in the growth accounting literature, we impose constant returns to scale: $\alpha_k + \alpha_n = 1$.

Define the growth rate of a generic variable X from $t-1$ to t as $\hat{x}_t \equiv \log(X_t) - \log(X_{t-1})$. (4.6) implies that output growth can be decomposed into three parts: technology change, input growth and the growth in input utilization:

$$\hat{y}_t = \hat{z}_t + \underbrace{\left(\alpha_k \hat{k}_t + \alpha_n \hat{n}_t + \alpha_n \hat{h}_t\right)}_{\hat{x}_t} + \underbrace{\left(\alpha_k \hat{u}_{k,t} + \alpha_n \hat{u}_{n,t}\right)}_{\hat{u}_t}. \tag{4.7}$$

As utilization changes are unobserved, we need to impose additional model structure to estimate utilization-adjusted TFP growth $\hat{z}_t$. As in BFK, we assume that firms take factor prices as given

and minimize their costs by solving the following problem

$$\min_{U_k,U_n,H,I,D} \mathbb{E}_0 \sum_{s=0}^{\infty} \left[\prod_{t=0}^{s-1} \frac{1}{1+r_s}\right] \left[W_s N_s G\left(H_s, U_{n,s}\right) V\left(U_{k,s}\right) + W_s N_s \Psi\left(\frac{D_s}{N_s}\right) + P_{I,s} K_s J_s\left(\frac{I_s}{K_s}\right)\right] \tag{4.8}$$

subject to

$$\bar{Y}_t = Z_t \left(U_{k,t} K_t\right)^{\alpha_k} \left(U_{n,t} N_t H_t\right)^{\alpha_n},$$
$$K_{t+1} = \left(1-\delta_k\right) K_t + I_t,$$
$$N_{t+1} = N_t + D_t.$$

After log linearizing the optimization conditions, we obtain $\hat{u}_t^n = \psi \hat{h}_t$ and $\hat{u}_t^k = \xi \hat{h}_t$, with ψ and ξ derived and specified in Appendix C.1. Combined with (4.7), we obtain the following equation:

$$\hat{y}_t = \hat{z}_t + \underbrace{\left(\alpha_k \hat{k}_t + \alpha_n \hat{n}_t + \alpha_n \hat{h}_t\right)}_{\hat{x}_t} + \underbrace{\left(\alpha_k \xi + \alpha_n \psi\right) \hat{h}_t}_{\hat{u}_t}. \tag{4.9}$$

Now we put back the state index i and industry index j. Equation (4.9) motivates the following estimation equation:

$$\hat{y}_{ijt} = \theta_1 \hat{x}_{ijt} + \theta_2 \hat{h}_{ijt} + \nu_{ij} + \hat{z}_{ijt}, \tag{4.10}$$

where the coefficient θ_1 is the estimate of returns to scale.[5] We seek to measure utilization-adjusted aggregate productivity change, $\hat{z}_{ijt}$. A key identification challenge is that input growth potentially correlates with the productivity shocks. To address this problem, following BFK and Huo, Levchenko and Pandalai-Nayar (2023) (hereafter, HLP), we use instruments that are plausibly uncorrelated with technology changes but have predictive power for changes in industry-level output.

4.2 Identification

In the empirical implementation, we allow the input elasticities, α_k and α_n, to vary across state-industry cells and measure them using the capital and labor shares estimated in Section 3. Because we allow for nonzero profits, these elasticities are better interpreted as factor cost shares than as simple revenue shares, in the spirit of Comin et al. (2025). Thus, $\hat{x}_{ijt}$ is constructed from the growth rates of capital, employment, and hours per worker, weighted by the corresponding state-industry

[5] In the model, we assume constant returns to scale, that is, θ_1 should be equal to 1.

shares of capital and labor income.

For the instruments, the first source of exogenous variation is the aggregate oil shock. Following HLP, we define the oil shock as the increase in log oil prices above the maximum over the previous four quarters, aggregated to the annual frequency. We then interact this common shock with industry oil intensity, computed from the BEA use tables, to obtain an instrument that varies across industries according to their direct exposure to oil as an input. The second source of exogenous variation is the state-level defense spending shock, measured as the growth in real state defense spending. We construct this measure using state military prime contracts awarded by the Department of Defense, deflate it by the price index for national defense consumption expenditures, and lag it by one year. The data sources are described in Section 2.

4.3 Estimation Results

Table 1 reports two-stage least squares estimates of (4.10). Columns (1)–(3) construct input growth using factor shares that vary at the state-industry-year level. Column (1) includes state and sector fixed effects and is our baseline specification. Column (2) uses sector fixed effects instead of the baseline fixed-effects structure, and column (3) uses state fixed effects. Column (4) returns to the baseline fixed-effects specification, but instead uses factor shares that vary across state-industry panels and are held fixed over time.

Table 1: IV Estimates of Utilization-Adjusted TFP Growth

	(1)	(2)	(3)	(4)
Returns to Scale (θ_1)	0.89***	0.90***	0.97***	0.77***
	(0.17)	(0.18)	(0.17)	(0.16)
Utilization Adjustment (θ_2)	2.36***	2.35***	1.61***	2.65***
	(0.58)	(0.59)	(0.44)	(0.56)
SW-F (θ_1)	9.69	9.41	11.37	11.85
SW-F (θ_2)	13.01	13.30	15.46	16.97
Sector FE	√	√		√
State FE	√		√	√
Time-Invariant Factor Shares				√
Observations		12,501		

Notes: Robust standard errors are in parentheses. SW-F denotes the Sanderson-Windmeijer statistic for the corresponding endogenous regressor. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Across all four columns, the main qualitative findings are stable. The estimated returns-to-scale coefficients is close to unity, which is consistent with constant returns to scale and similar to the values reported in HLP and BFK. The coefficient on hours per worker is always positive and statistically significant, suggesting that utilization adjustment is empirically important for measuring productivity. The similarity of columns (1) and (2) indicates that the results are not driven by whether cross-sectional heterogeneity is absorbed using the baseline fixed effects or sector fixed effects alone. Column (3) yields a slightly larger returns-to-scale coefficient and a smaller utilization coefficient, but the main conclusion is unchanged. Column (4) shows that the results are robust to the case where factor shares are held fixed within state-industry panels.

The first-stage results provide support for instrument relevance. Given the presence of multiple instruments and multiple endogenous regressors, we assess weak identification using the Sanderson-Windmeijer F-statistic (SW-F), which is appropriate for this environment (Huo, Levchenko and Pandalai-Nayar, 2023). The SW-F ranges from about 9 to 12 for returns to scale and from 13 to 17 for the utilization term, indicating that weak instruments are unlikely to be a concern in our specification.

Given the preferred estimates $\hat{\theta}_1$ and $\hat{\theta}_2$, utilization-adjusted productivity growth is recovered as

$$\hat{z}_{ijt} = \hat{y}_{ijt} - \hat{\theta}_1 \hat{x}_{ijt} - \hat{\theta}_2 \hat{h}_{ijt},$$

while the standard Solow residual is $\hat{z}_{ijt}^{\text{Solow}} = \hat{y}_{ijt} - \hat{x}_{ijt}$. Since that the estimated returns-to-scale coefficients are close to unity, the difference between the two reflects the correction for variable utilization. When aggregating across industries, we use cost-based Domar weights so that sectoral contributions are consistent with aggregate growth accounting (Baqaee and Farhi, 2020).

4.4 TFP by State and Industry

Figures 7 and 8 plot the year-over-year and cumulative TFP growth for each state as well as the US as a whole.

4.5 State-Level TFP and Output Growth

Figure 10 shows the output, TFP and capital growth for each state.

5 Misallocation and Productivity

We use our growth accounting results to estimate the spatial misallocation across states and its contribution to US-wide overall and industry-specific productivity changes.

5.1 Framework

Statewide production function implies:

$$\hat{y}_{it} = \hat{z}_{it} + \alpha_{ikt}\hat{k}_{it} + \alpha_{int}\hat{l}_{it} + \hat{\theta}_2\hat{h}_{it}, \tag{5.11}$$

where $\hat{l}_{it} = \hat{n}_{it} + \hat{h}_{it}$ and $\hat{z}_{it}$ captures the state-level utilization-adjusted TFP, aggregated from state-industry level productivity estimates using cost-based Domar weights. Aggregated from the N states to the US level,

$$\hat{y}_t = \sum_{i=1}^{N} b_{it}\hat{y}_{it},\ \hat{k}_t = \sum_{i=1}^{N} \omega_{it}^k \hat{k}_{it},\ \hat{l}_t = \sum_{i=1}^{N} \omega_{it}^n \hat{l}_{it},$$

where b_{it} is the output share of state i among all US states, and ω_{it} denotes the weight of state i's capital (labor) cost among the nationwide input cost. Growth accounting based on Baqaee and Farhi (2020) gives:

$$\hat{z}_t = \hat{y}_t - \lambda_t^k \hat{k}_t - \lambda_t^n \hat{l}_t.$$

A decomposition of US productivity is then given by:

$$\hat{z}_t = \sum_{i=1}^{N} b_{it}\hat{z}_{it} + \sum_{i=1}^{N} \hat{\theta}_2 b_{it}\hat{h}_{it} + \sum_{i=1}^{N} (b_{it}\alpha_{kt} - \lambda_t^k \omega_{it}^k)\hat{k}_{it} + \sum_{i=1}^{N} (b_{it}\alpha_{nt} - \lambda_t^n \omega_{it}^n)\hat{l}_{it}, \tag{5.12}$$

where

$$b_{it} = \frac{Y_{it}}{Y_t},\ \alpha_{ikt} = \frac{r_{it}k_{it}}{r_{it}k_{it} + w_{it}l_{it}},\ \alpha_{int} = \frac{w_{it}l_{it}}{r_{it}k_{it} + w_{it}l_{it}},\ \omega_{it}^k = \frac{r_{it}k_{it}}{\sum_{j=1}^{N} r_{jt}k_{jt}},\ \omega_{it}^n = \frac{w_{it}l_{it}}{\sum_{j=1}^{N} w_{jt}l_{jt}}.$$

Following the same method, we can add a subscript j on both sides of (5.12) to study the spatial misallocation for a particular industry, say, manufacturing.

$$\hat{z}_{jt} = \sum_{i=1}^{N} b_{ijt}\hat{z}_{ijt} + \sum_{i=1}^{N} \hat{\theta}_2 b_{ijt}\hat{h}_{ijt} + \sum_{i=1}^{N} (b_{ijt}\alpha_{kjt} - \lambda_t^k \omega_{ijt}^k)\hat{k}_{ijt} + \sum_{i=1}^{N} (b_{ijt}\alpha_{njt} - \lambda_t^n \omega_{ijt}^n)\hat{l}_{ijt}. \tag{5.13}$$

5.2 Explaining U.S. Productivity Change

TBA

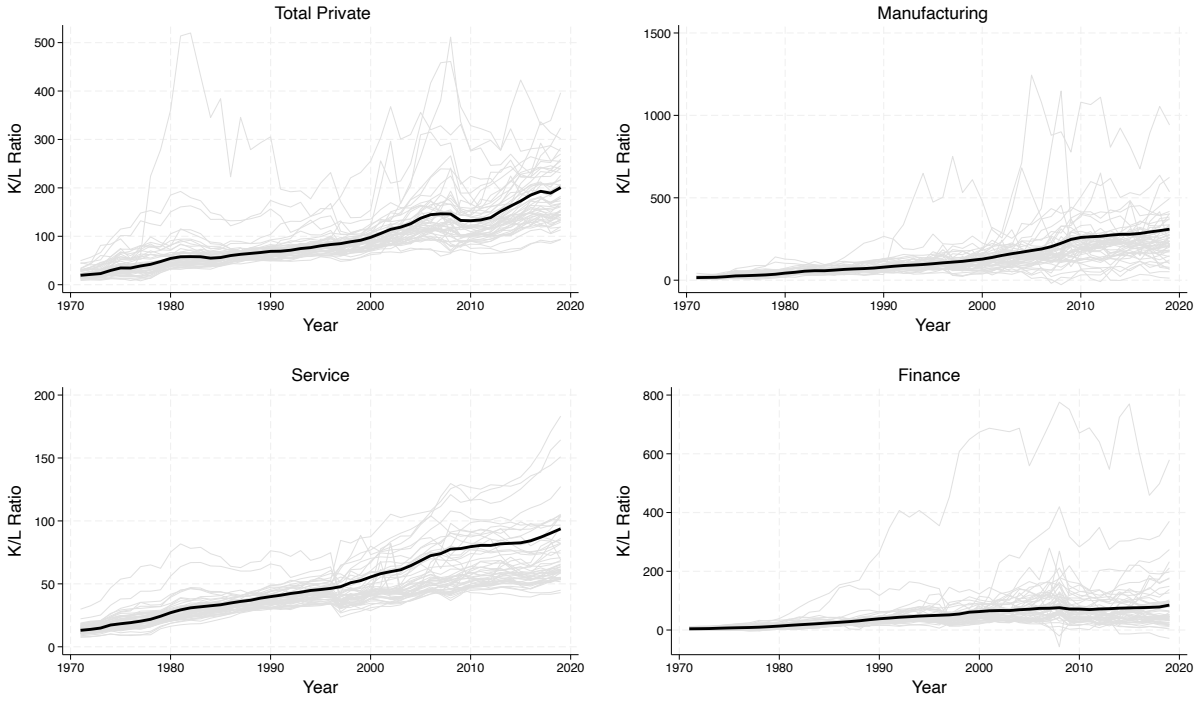
5.3 Explaining U.S. Manufacturing TFP Decline

TBA

6 Conclusion

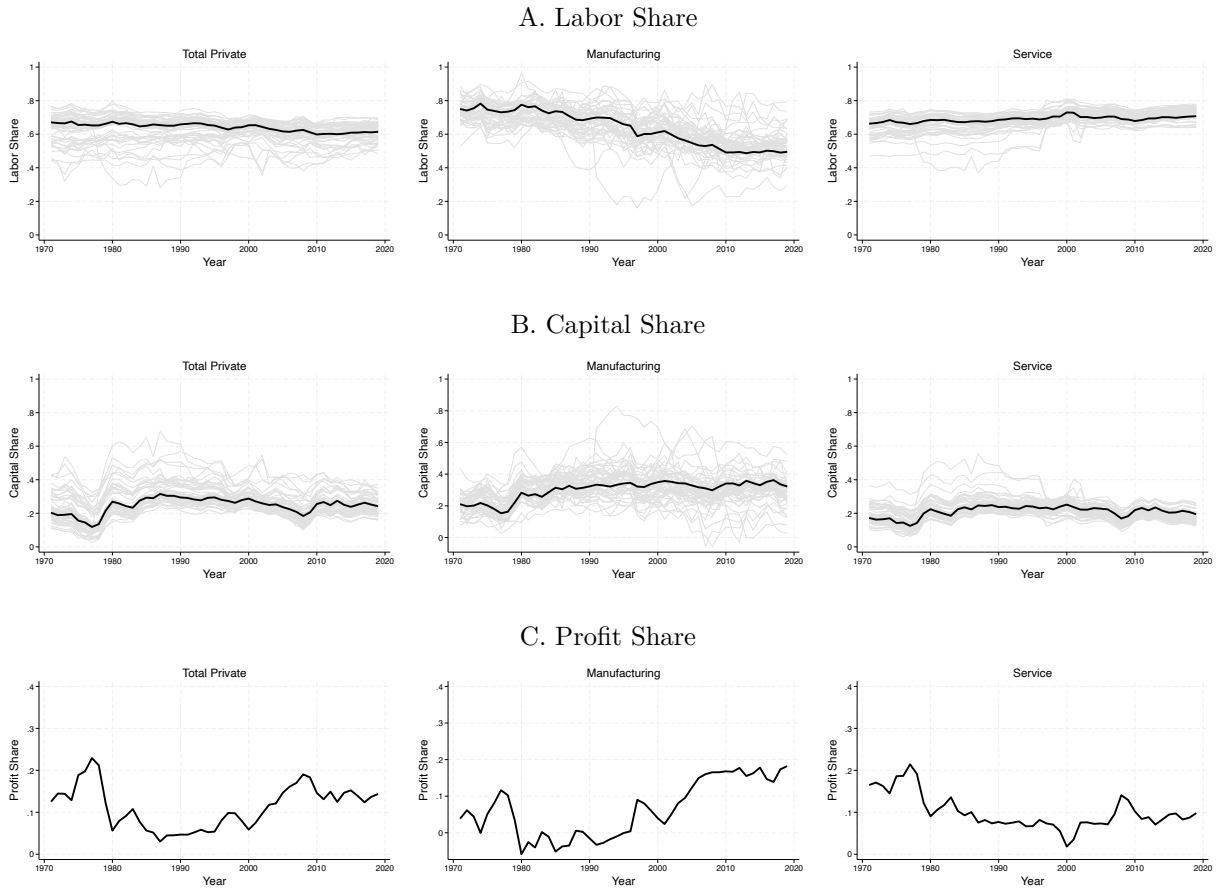
In this paper, we start by estimating a new state-level capital series for all private sectors which do not rely on strong assumptions. We then conduct growth accounting for the states. A few new facts emerge from this work. First, while, generally, we observe slight declines in labor and capital share and an increase in profit share since mid-1980s, there's substantial variation across sectors and states. Second, we find strong evidence of convergence across states in labor shares all sectors, while convergence of the capital-to-labor ratio exhibits more cross-sector heterogeneity, with the manufacturing sector seeing almost no convergence. Third, the state-level growth accounting shows that while between-sector allocative efficiency does not seem important for US as a whole, it can be a crucial driver of TFP growth at the state level.

Figure 1: Capital/Labor Ratio



Notes: These figures plot the capital per employment for each state (gray) and the United States (black).

Figure 2: Factor Shares of Income



Notes: These figures plot the factor income shares for each state (gray) and the United States (black), that is, labor, capital, and profit shares.

Figure 3: Growth Rate (1963–2022) vs. 1963 Per Capita Real GDP

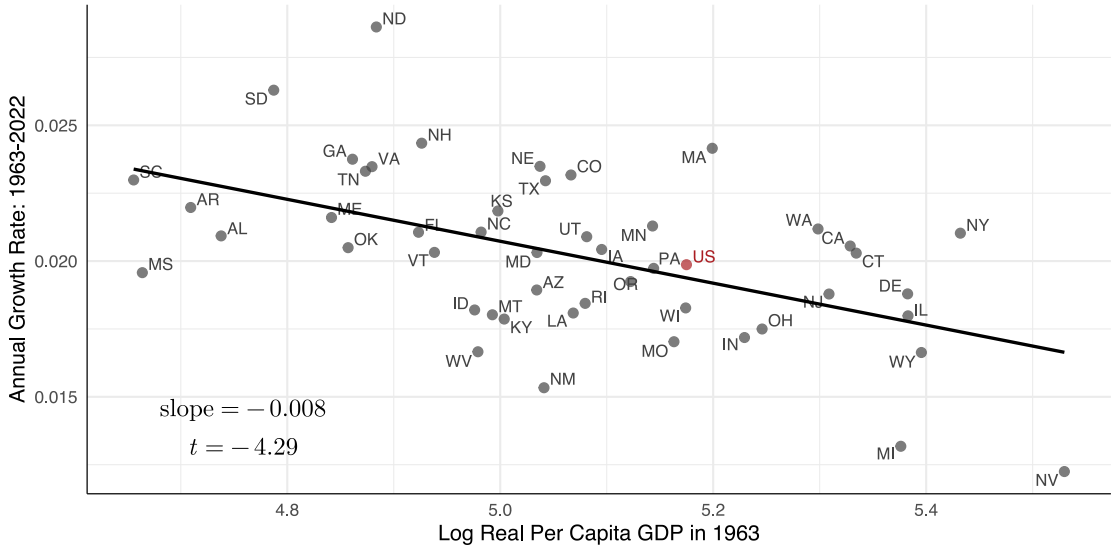


Figure 4: Sectoral Change in Labor Share (1971–2019) vs. 1971 Level

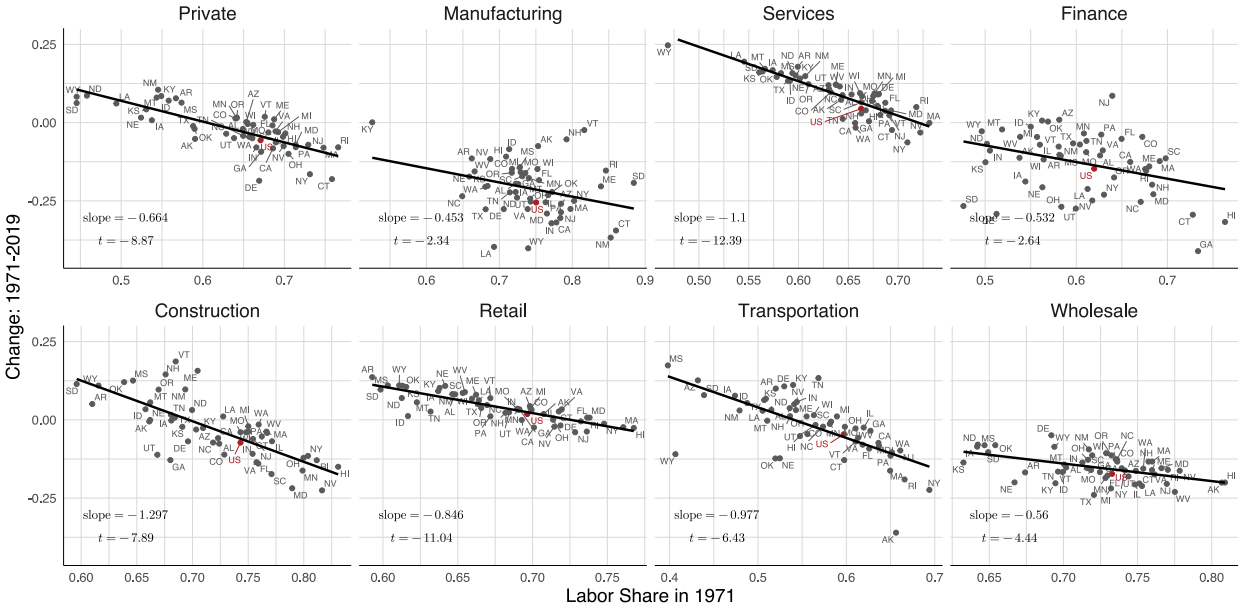


Figure 5: Sectoral Change in Capital-to-Labor Share Ratio (1971–2019) vs. 1971 Level

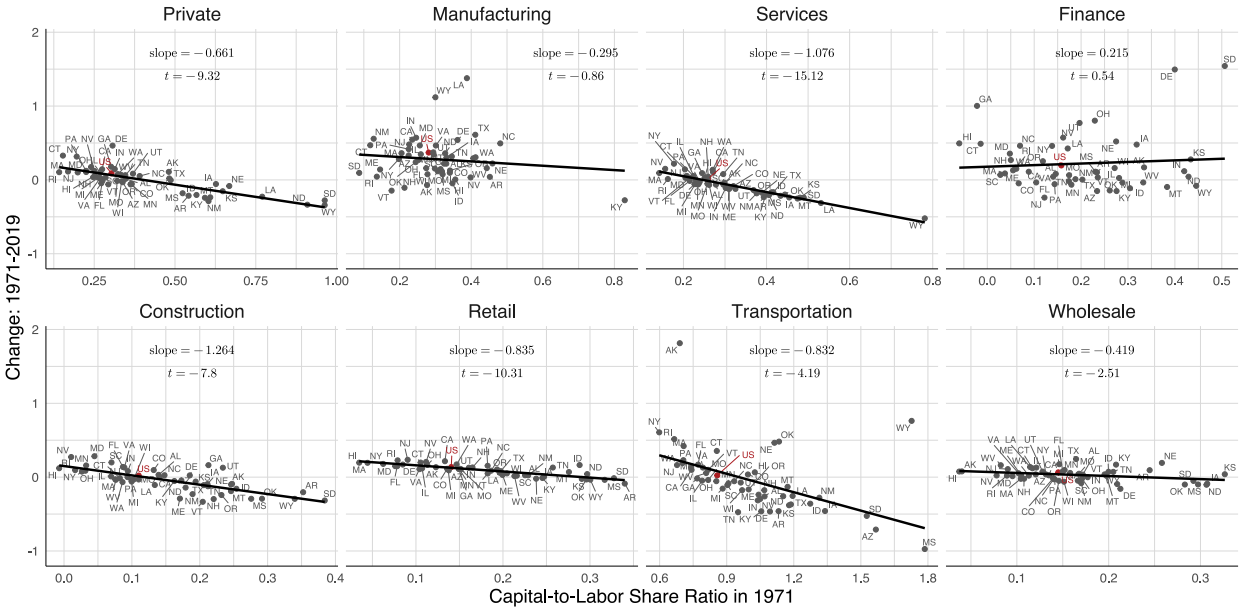


Figure 6: Manufacturing: Change in Capital-to-Labor Ratio (1971–2019) vs. 1971 Level

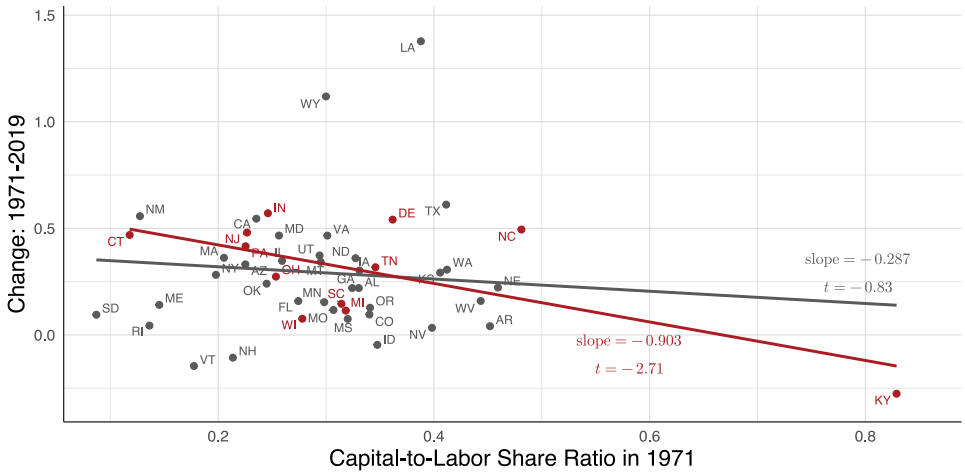
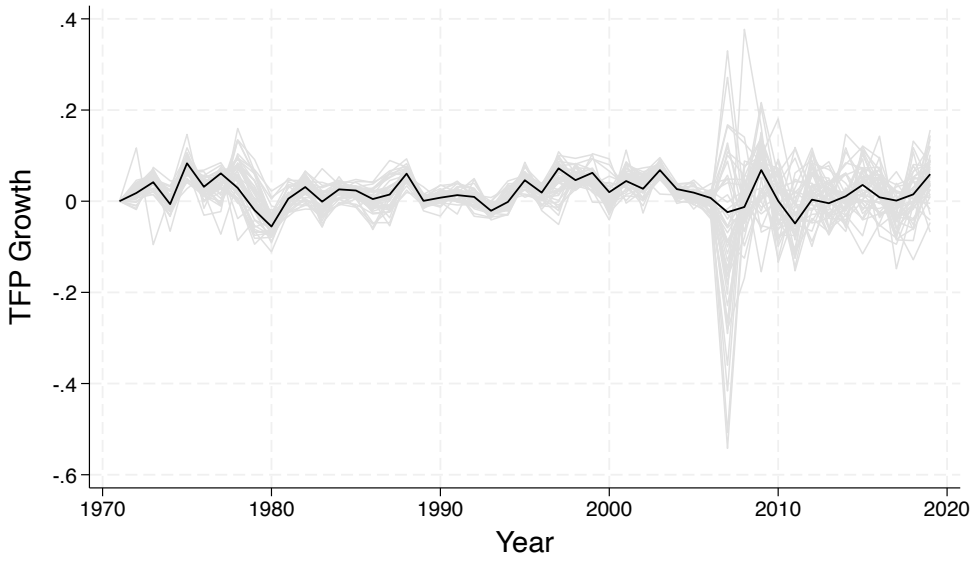
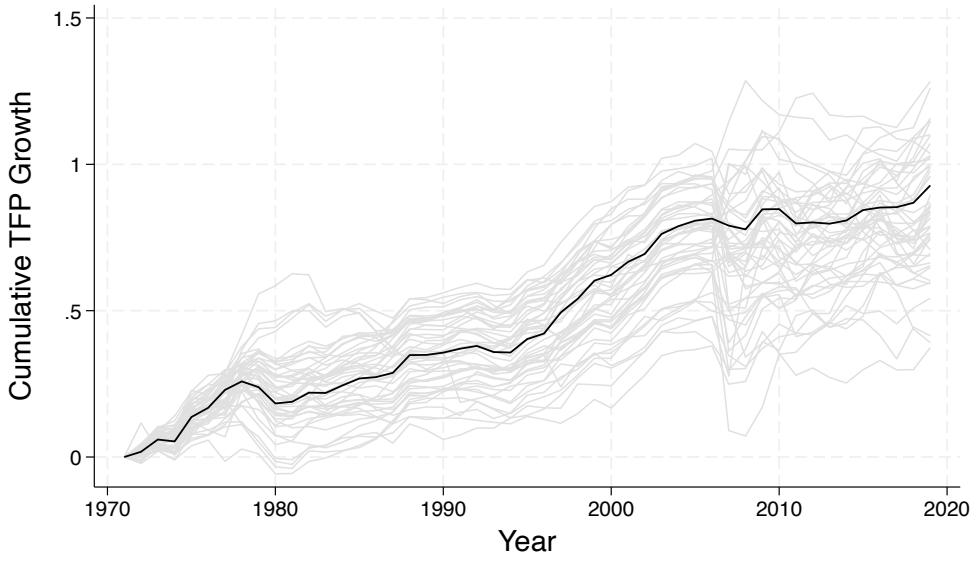


Figure 7: Utilization-Adjusted TFP Growth



Notes: Black line represents the US.

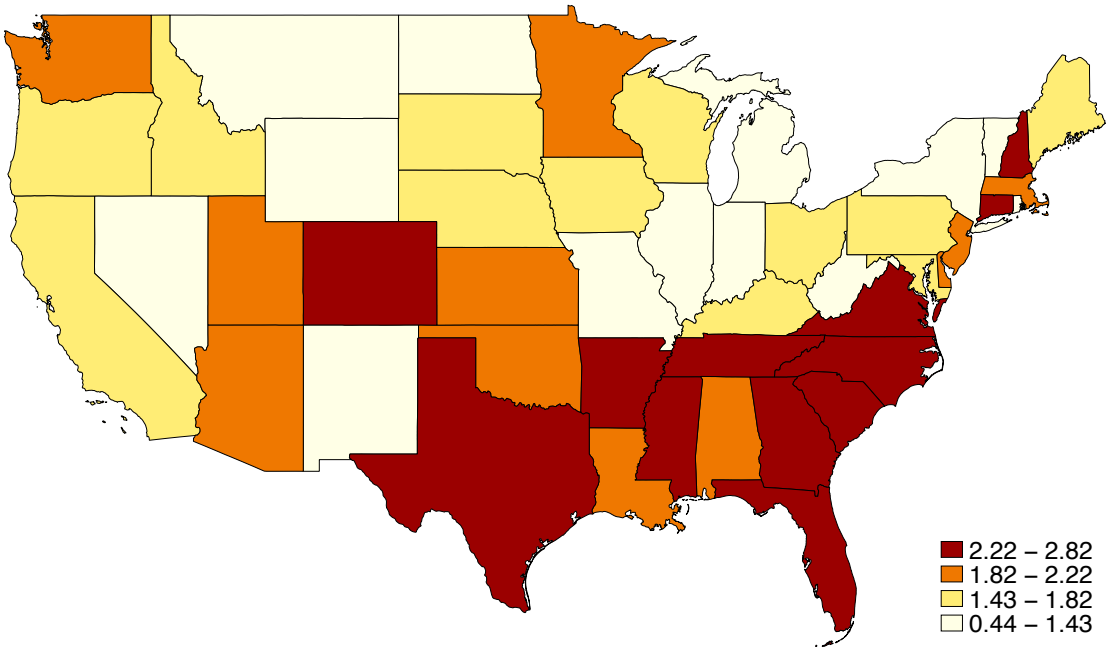
Figure 8: Cumulative Utilization-Adjusted TFP Growth



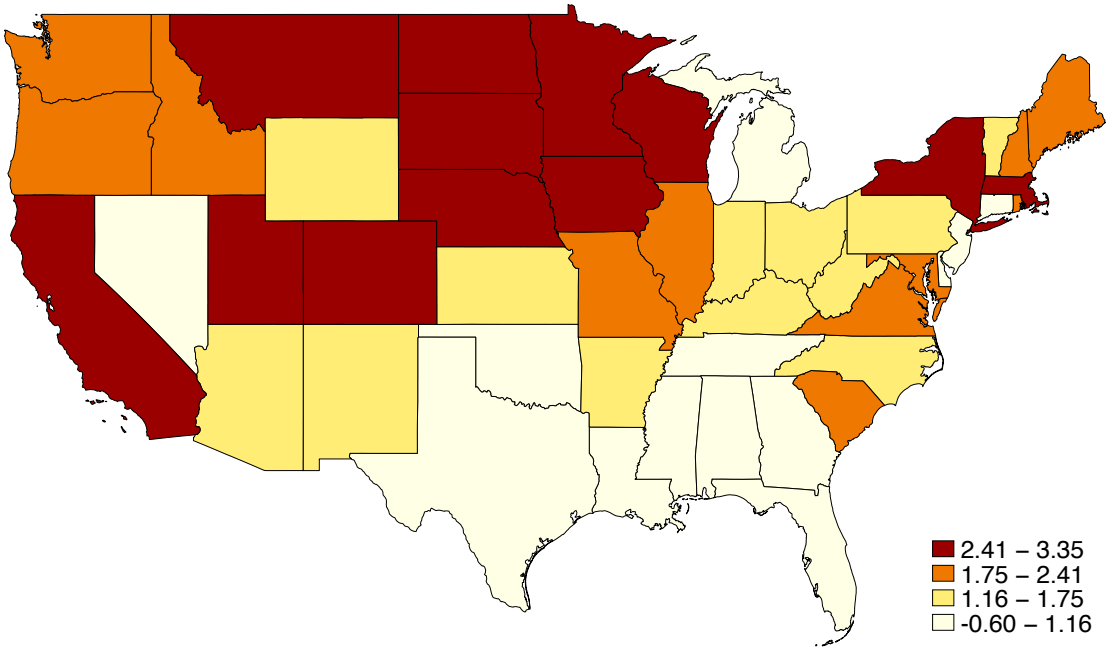
Notes: Black line represents the US.

Figure 9: Average TFP Growth

A. 1970–1997

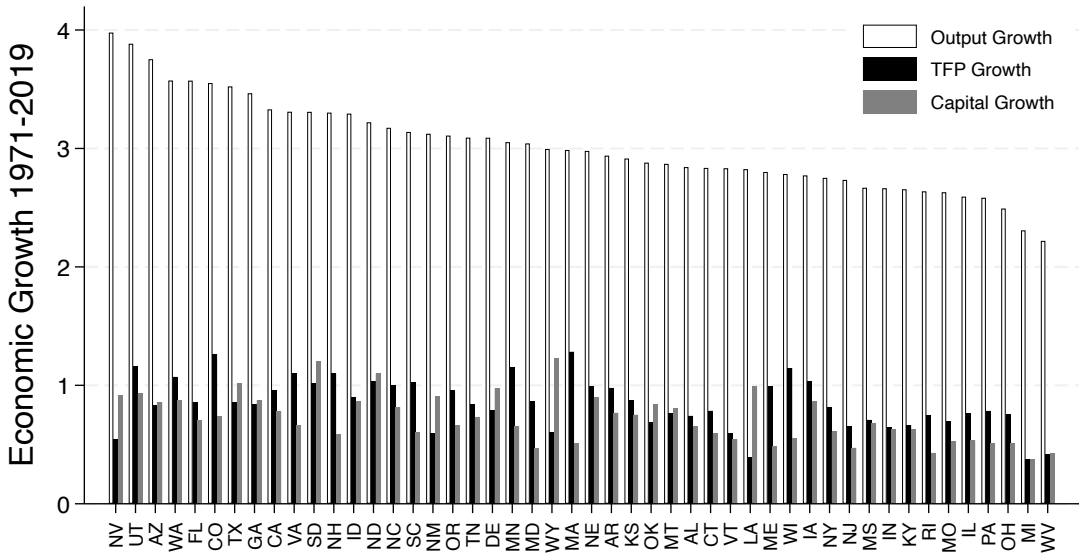


B. 1998–2019



Notes: These figures plot the average annual state TFP growth over 1970–1997 (Panel A) and 1998–2019 (Panel B).

Figure 10: State-Level Output Growth



References

Baqaee, David Rezza and Emmanuel Farhi. 2020. "Productivity and Misallocation in General Equilibrium." *The Quarterly Journal of Economics* 135(1):105–163.

Barkai, Simcha. 2020. "Declining Labor and Capital Shares." *The Journal of Finance* 75(5):2421–2463.

Barro, Robert J and Xavier Sala-i Martin. 1992. "Convergence." *Journal of Political Economy* 100(2):223–251.

Basu, Susanto, John G. Fernald and Miles S. Kimball. 2006. "Are Technology Improvements Contractionary?" *American Economic Review* 96(5):1418–1448.

Becker, Jonathan, Chris Edmond, Virgiliu Midrigan and Daniel Yi Xu. 2024. "Local Concentration, National Concentration, and the Spatial Distribution of Markups." *Working Paper* .

Beraja, Martin, Erik Hurst and Juan Ospina. 2019. "The Aggregate Implications of Regional Business Cycles." *Econometrica* 87:1789–1833.

Caliendo, Lorenzo, Fernando Parro, Esteban Rossi-Hansberg and Pierre-Daniel Sarte. 2018. "The Impact of Regional and Sectoral Productivity Changes on the U.S. Economy." *The Review of Economic Studies* 85(4):2042–2096.

Caselli, Francesco and Wilbur John Coleman II. 2001. "The US Structural Transformation and Regional Convergence: A Reinterpretation." *Journal of Political Economy* 109(3):584–616.

Chirinko, Robert S and Daniel J Wilson. 2008. "State Investment Tax Incentives: A Zero-Sum Game?" *Journal of Public Economics* 92(12):2362–2384.

Comin, Diego, Javier Quintana, Tom Schmitz and Antonella Trigari. 2025. "Revisiting Productivity Dynamics in Europe: A New Measure of Utilization-Adjusted TFP Growth." *Journal of the European Economic Association* p. jvaf003.

De Loecker, Jan, Jan Eeckhout and Gabriel Unger. 2020. "The Rise of Market Power and the Macroeconomic Implications." *The Quarterly Journal of Economics* 135(2):561–644.

Desmet, Klaus and Esteban Rossi-Hansberg. 2013. "Urban Accounting and Welfare." *American Economic Review* 103(6):2296–2327.

Elsby, Michael, Bart Hobijn and Aysegul Sahin. 2013. "The Decline of the U.S. Labor Share." *Brookings Papers on Economic Activity* Fall:1–52.

Fajgelbaum, Pablo D, Eduardo Morales, Juan Carlos Suárez Serrato and Owen Zidar. 2019. "State Taxes and Spatial Misallocation." *The Review of Economic Studies* 86:333–376.

Fernald, John. 2014. "A Quarterly, Utilization-Adjusted Series on Total Factor Productivity." *Federal Reserve Bank of San Francisco Working Paper* .

Fort, Teresa C and Shawn D Klimek. 2018. The Effects of Industry Classification Changes on US Employment Composition. Working Papers 18-28 Center for Economic Studies, U.S. Census Bureau.

Garofalo, Gasper A and Steven Yamarik. 2002. "Regional Convergence: Evidence from a New State-by-State Capital Stock Series." *Review of Economics and Statistics* 84(2):316–323.

Hall, Robert E. and Dale W. Jorgenson. 1967. "Tax Policy and Investment Behavior." *The American Economic Review* 57(3):391–414.

Huo, Zhen, Andrei A Levchenko and Nitya Pandalai-Nayar. 2023. "Utilization-adjusted TFP Across Countries: Measurement and Implications for International Comovement." *Journal of International Economics* 146:103753.

Jones, Callum, Virgiliu Midrigan and Thomas Philippon. 2022. "Household Leverage and the Recession." *Econometrica* 90(5):2471–2505.

Karabarbounis, Loukas. 2024. "Perspectives on the Labor Share." *Journal of Economic Perspectives* 38(2):107–36.

Karabarbounis, Loukas and Brent Neiman. 2014. "The Global Decline of the Labor Share." *The Quarterly Journal of Economics* 129(1):61–103.

Karabarbounis, Loukas and Brent Neiman. 2019. "Accounting for Factorless Income." *NBER Macroeconomics Annual* 33(1):167–228.

Khatri, Krishna and Robert Tamura. 2022. "Standard of Living of the States of the United States: 1800 - 2020." Working paper, Clemson University.

Kleinman, Benny, Ernest Liu and Stephen J Redding. 2023. "Dynamic Spatial General Equilibrium." *Econometrica* 91(2):385–424.

Liu, Chang and Noah Williams. 2019. "State-Level Implications of Federal Tax Policies." *Journal of Monetary Economics* 105:75–90.

Mertens, Karel and Morten O. Ravn. 2013. ""The Dynamic Effects of Personal and Corporate Income Tax Changes in the United States"." *American Economic Review* 103(4):1212–47.

Mian, Atif and Amir Sufi. 2009. "The Consequences of Mortgage Credit Expansion: Evidence from the U.S. Mortgage Default Crisis." *Quarterly Journal of Economics* 124:1449–1496.

Mian, Atif, Kamalesh Rao and Amir Sufi. 2013. "Household Balance Sheets, Consumption, and the Economic Slump." *Quarterly Journal of Economics* 128:1687–1726.

Nakamura, Emi and J. Steinsson. 2014. "Fiscal Stimulus in a Monetary Union: Evidence from U.S. Regions." *American Economic Review* 104:753–792.

Romer, Christina D. and David H. Romer. 2010. ""The Macroeconomic Effects of Tax Changes: Estimates Based on a New Measure of Fiscal Shocks"." *American Economic Review* 100(3):763–801.

Rossi-Hansberg, Esteban, Pierre-Daniel Sarte and Nicholas Trachter. 2021. "Diverging Trends in National and Local Concentration." *NBER Macroeconomics Annual* 35(1):115–150.

Smith, Dominic A. and Sergio Ocampo. forthcoming. "The Evolution of U.S. Retail Concentration." *American Economic Journal: Macroeconomics* .

Suárez Serrato, Juan Carlos and Owen Zidar. 2016. "Who Benefits from State Corporate Tax Cuts? A Local Labor Markets Approach with Heterogeneous Firms." *American Economic Review* 106(9):2582–2624.

Turner, Chad, Robert Tamura and Sean E. Mulholland. 2013. "How Important are Human capital, physical capital and total factor productivity for determining state economic growth in the United States, 1840–2000?" *Journal of Economic Growth* 18:319–371.

Wilson, Daniel J. 2009. "Beggar Thy Neighbor? The In-State, Out-of-State, and Aggregate Effects of R&D Tax Credits." *The Review of Economics and Statistics* 91(2):431–436.

Yamarik, Steven. 2013. "State-Level Capital and Investment: Updates and Implications." *Contemporary Economic Policy* 31(1):62–72.

Yuskavage, Robert E. 2007. "Converting Historical Industry Time Series Data from SIC to NAICS." *Federal Committee on Statistical Methodology* .

Online Appendix for "Growth Accounting for the States"

Chang Liu Noah Williams Weiheng Zheng

A Data Details

A.1 Industrial Classification Matching

The gross operating surplus (GOS) data are indexed by the BEA according to two different industrial classification systems. From 1963-1997, the GOS data are classified according to the Standard Industrial Classification (SIC) system. After 1997, the GOS data are classified according to the North American Classification System (NAICS). The primary difference between the two systems is that the SIC classifies establishments according to the primary economic activity in which they are engaged, while the NAICS classifies establishments according to their production process (Yuskavage, 2007). For example, a firm that supplies mining support services would be classified by the SIC as a "Mining" firm and classified by the NAICS as a "Services" firm (Fort and Klimek (2018)). As the classification paradigm changed dramatically between the two systems, merging the GOS data is not straightforward. To construct a mapping between GOS indexed by SIC and GOS indexed by NAICS, we use a concordance described in Fort and Klimek (2018) which provides cross-walks between four-digit SIC codes and six-digit NAICS codes constructed using the Economic Censuses (Esc) published by the Census Bureau. Crucially, for each SIC-NAICS pair, Fort and Klimek (2018) estimate figures for the number of establishments, value of shipments, payroll, and employment in 1997. Thus, despite the fact that each SIC code can map to multiple NAICS codes, we are able to observe what fraction of, for example, shipments associated with a given SIC code are due to each NAICS code.

Specifically, let $x_{s,n}$ denote the 1997 value of variable x for matched pair of four-digit SIC code s and six-digit NAICS code n. For example, in 1997, 32 establishments were matched to both the SIC code 1101 ("Iron ores") and the NAICS code 212210 ("Iron ore mining") (Fort and Klimek

(2018)). Therefore, we have

$$\text{establishments}_{1101,212210} = 32$$

In the Fort and Klimek (2018) concordance, each SIC code s and NAICS code n belongs to one of eight major sectors ("Construction", "Finance, insurance, and real estate", "Manufacturing", "Mining", "Retail", "Services", "Transportation, communication, and utilities", and "Wholesale"). Therefore, each SIC-NAICS pair (s, n) is accompanied by a major sector pair $(MAJOR^{SIC}, MAJOR^{NAICS})$ where

$$s \in MAJOR^{SIC}$$
$$n \in MAJOR^{NAICS}$$

For example, SIC code 1101 and NAICS code 212210 both belong to the Mining sector, so we have

$$1101 \in MINING^{SIC}$$
$$212210 \in MINING^{NAICS}$$

Given that each SIC code s and NAICS code n is associated with a major sector, we can use the estimated pairwise totals to compute major sector totals, defined either by the SIC or NAICS codes. Therefore, the total for variable x in a given major sector defined by the SIC codes is given by

$$X_{MAJOR^{SIC}} = \sum_{s \in MAJOR^{SIC}} x_{s,n}$$

and total x in the same major sector defined using the NAICS codes is given by

$$X_{MAJOR^{NAICS}} = \sum_{n \in MAJOR^{NAICS}} x_{s,n}$$

We can also calculate total x for each major sector pair $(MAJOR^{SIC}, MAJOR^{NAICS})$ as

$$X_{(MAJOR^{SIC}, MAJOR^{NAICS})} = \sum_{s \in MAJOR^{SIC}, n \in MAJOR^{NAICS}} x_{s,n}$$

The fraction of variable x in SIC-indexed major sector $MAJOR^{SIC}$ that maps to NAICS-indexed

major sector $MAJOR^{NAICS}$ is given by

$$FRACTION^{x}_{(MAJOR^{SIC},MAJOR^{NAICS})} = \frac{X_{(MAJOR^{SIC},MAJOR^{NAICS})}}{X_{MAJOR^{SIC}}}$$

We construct this mapping for two different variables x: value of shipments and employment.

A.2 Construction of Pre-1997 NAICS-Indexed Series

The BEA publishes several pre-1997 industry-level data series indexed by the SIC system. In order to convert these series to the NAICS, we assume that major sectors indexed by the SIC codes are allocated to major sectors indexed by NAICS codes in accordance with

$$FRACTION^{x}_{(MAJOR^{SIC},MAJOR^{NAICS})} = \frac{X_{(MAJOR^{SIC},MAJOR^{NAICS})}}{X_{MAJOR^{SIC}}}$$

Because this fraction is computed with 1997 data, we assume that this relationship is constant across time. To illustrate this method, consider mapping gross operating surplus (GOS) pre-1997 to NAICS-indexed major sectors. To do this, we let x be the value of shipments. So we have that year t (where $t < 1997$) GOS in NAICS-indexed major sector $MAJOR^{NAICS}$ is given by

$$GOS_{MAJOR^{NAICS},t} = \sum_{s \in MAJOR^{SIC}} FRACTION^{shipments}_{(MAJOR^{SIC},MAJOR^{NAICS})} GOS_{s,t}$$

Since the major sectors "Farms" and "Government" are omitted from Fort and Klimek (2018), we assume that the definitions of these sectors are the same in the SIC and NAICS.

A.3 Estimating Cost of Capital

Cost of capital by industry and state is estimated from the following data sources:

1. Commercial Paper: 3-Month AA Non-financial Commercial Paper Rate. (1971M4-2020M12. Monthly data. Source: Federal Reserve Economic Data.)

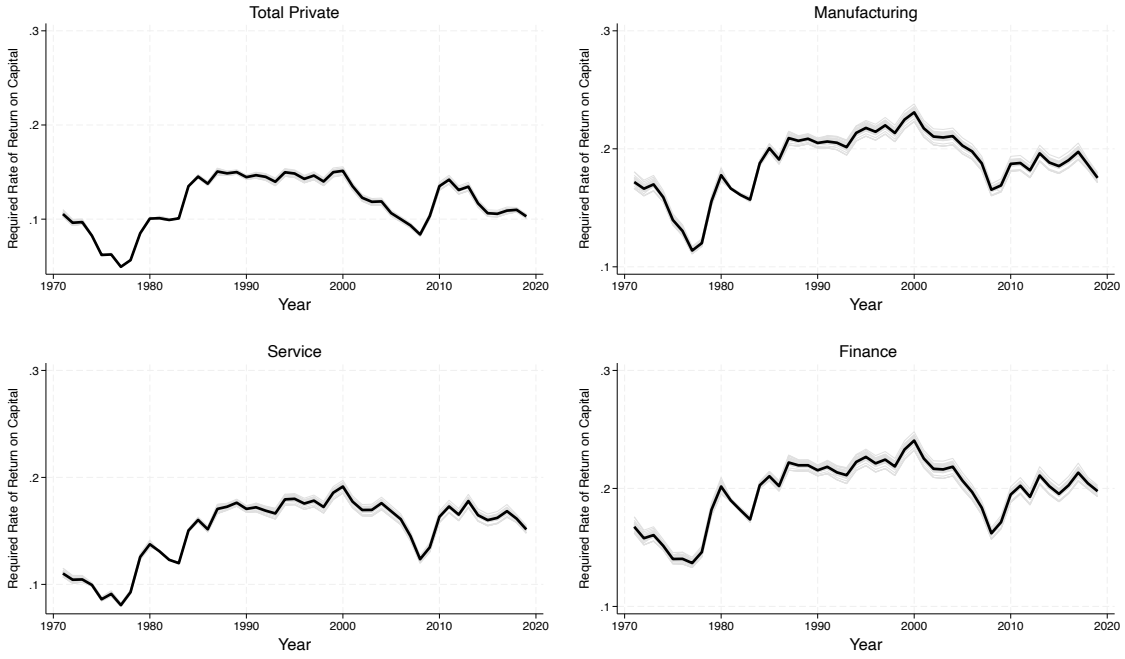
2. Integrated Macroeconomic Accounts Table S5a: Market values of Loans (line 141), Bonds (sum of lines 139 and 140) and Commercial Paper (line 138). Market values of debt (sum of lines 137 and 141) and equity (line 146). (1960-2020. Annual data. Unit: Billions of dollars. Source: Bureau of Economic Analysis, Integrated Macroeconomic Accounts Table S5a)

3. Loans: The prime lending rate (1949M1-1986M3. Monthly data. Unit: Source: Barkai (2020)) and the rate for commercial and industrial loans made by all commercial banks (1986Q2-2017Q2. Quarterly data. Source: Federal Reserve Table E.2, Survey of Terms of Business Lending)

4. Treasury Yields: The yield on the 10-year U.S. treasury (195304-202012. Monthly data. Source: Federal Reserve Economic Data)

5. Moodys Aaa: Yields on Moody's Aaa bonds (1919M1-2020M12. Monthly data. Source: Federal Reserve Economic Data)

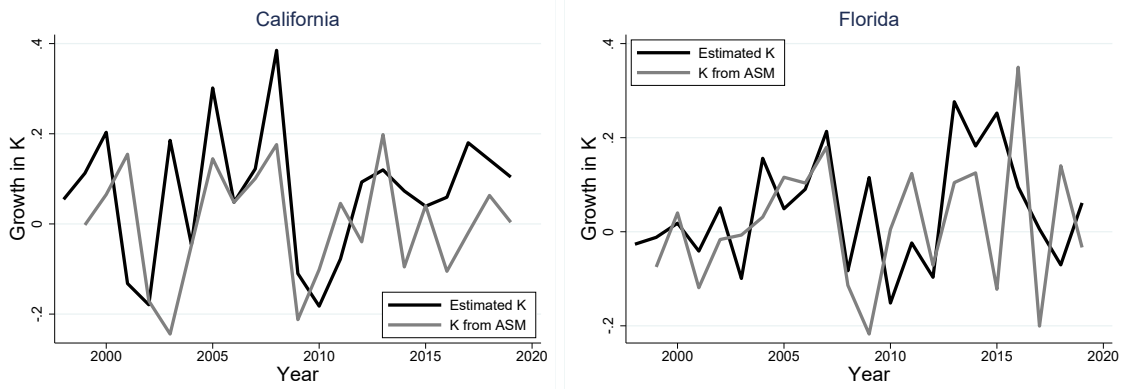
Figure A.1: Estimates of Capital Cost



Notes: Black line shows United States, gray lines show each state.

A.4 Validation of the Capital Series

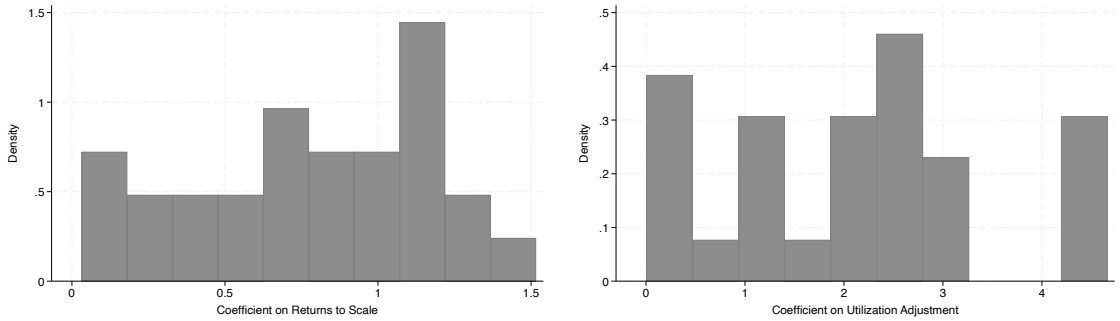
Figure A.2: Comparison with ASM



Notes: This figure shows a comparison of our state capital estimates with that from the Annual Survey of Manufactures (ASM), available from 1997.

B Additional Empirical Results

Figure B.3: State-Level Growth Accounting Coefficients



Notes: These figures present the distribution of coefficients for state-by-state estimations of Equation (4.10).

C Derivations

C.1 Optimization Conditions

$$U_t^k : \lambda K_t \alpha_k Z_t \left(U_{k,t} K_t\right)^{\alpha_k - 1} \left(U_{n,t} N_t H_t\right)^{\alpha_n} = W_t N_t G\left(H_t, U_{n,t}\right) V'\left(U_{k,t}\right) \tag{C.1}$$

$$U_t^n : \lambda N_t H_t \alpha_n Z_t \left(U_{k,t} K_t\right)^{\alpha_k} \left(U_{n,t} N_t H_t\right)^{\alpha_n - 1} = W_t N_t G_2\left(H_t, U_{n,t}\right) V\left(U_{k,t}\right) \tag{C.2}$$

$$H_t : \lambda N_t U_t^n \alpha_n Z_t \left(U_{k,t} K_t\right)^{\alpha_k} \left(U_{n,t} N_t H_t\right)^{\alpha_n - 1} = W_t N_t G_1\left(H_t, U_{n,t}\right) V\left(U_{k,t}\right) \tag{C.3}$$

It follows that:

$$U_{n,t} = \frac{G_1\left(H_t, U_{n,t}\right)}{G_2\left(H_t, U_{n,t}\right)} H_t \tag{C.4}$$

$$U_{k,t} = \frac{\alpha_k}{\alpha_n} \frac{G_1\left(H_t, U_{n,t}\right) V\left(U_{k,t}\right)}{G\left(H_t, U_{n,t}\right) V'\left(U_{k,t}\right)} H_t \tag{C.5}$$