Multi-Asset Market Making

Ioanid Roșu, Andriy Shkilko, Elvira Sojli, Wing Wah Tham

Presentation: Ioanid Roșu, HEC Paris

NBER

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Motivation

> *"A market maker's success depends on . . . accurately responding to relevant market data in similar and* **correlated instruments**.*"*
>
> — Virtu Financial, S-1 Registration Statement (SEC, 2014)

- Each trade has two sides:
  - **Liquidity demand** (e.g., market order): pays the spread
    - E.g., $1 € = \$1.17$; Ask \$1.18, Bid \$1.16 $\implies$ spread = \$0.02
  - **Liquidity supply** (e.g., limit order): earns the spread; loses from **inventory**
- Standard inventory models work **one asset at a time**
  - Yet a dealer typically quotes **many correlated assets at once** (Virtu: 25,000 securities, 235 venues, 37 countries)
- **This paper:** how does a dealer manage inventory **across correlated assets**?

Modeling Liquidity Demand & Supply

- **Liquidity Demand:** Modern Portfolio Theory (MPT) (Markowitz, Sharpe, Tobin)
  - N risky assets ($+$ 1 risk-free), 1 period
  - Inputs: expected returns, standard deviations, correlations
  - Results: efficient frontier, tangency portfolio, optimal portfolio
  - If all investors agree on inputs $\Longrightarrow$ CAPM
- **Liquidity Supply:** How do market makers behave?
  - N risky assets, infinitely many periods
  - Inputs: standard deviations, correlations
  - Results: **?**

Market Makers (Dealers) and Prices

- Market makers supply liquidity by setting the bid and ask quotes at which others trade:
  - How do they set their quotes?
  - How do they manage inventory in multiple assets?
  - How much **cross-hedging** is there?
  - Which assets do they choose to make markets in?
- Do market makers affect prices?
  - Hendershott & Menkveld (2014) measure "price pressures" in **one asset**: a \$100,000 inventory shock causes a 0.98% price pressure for small-cap stocks, 0.02% for large-cap
  - If dealers cross-hedge, what does a **single-asset** inventory half-life reveal about risk aversion?
  - How does cross-stock correlation vary with frequency?
- **This paper:** dynamic model in N correlated assets + tests on TSX message-level data (45 trading days)

Model

- Discrete time $t = 0, 1, 2, \ldots$; N **risky assets**, fundamentals $v_{n,t}$ follow a random walk; Δv has covariance Σ_v
- (Monopolist) dealer posts ask a_t and bid b_t in each asset; aggregate market orders satisfy:

$$Q_t^b = \frac{k}{2}(v_t - a_t) + \ell + \varepsilon_t^b, \quad \text{with} \quad \varepsilon_t^b \overset{IID}{\sim} \mathcal{N}(0, \Sigma_L/2)$$

$$Q_t^s = \frac{k}{2}(b_t - v_t) + \ell + \varepsilon_t^s, \quad \text{with} \quad \varepsilon_t^s \overset{IID}{\sim} \mathcal{N}(0, \Sigma_L/2)$$

$\Longrightarrow$ 3 components: elastic + inelastic + random

- Inventory: $x_{t+1} = x_t + Q_t^s - Q_t^b$
- Dealer maximizes expected profit (time discount $\beta < 1$), penalized by **quadratic inventory cost** (**risk aversion** γ)

Optimal Quotes

- **Proposition 1:** In equilibrium, the dealer sets:

$$a_t \;=\; p_t + h, \qquad b_t \;=\; p_t - h, \qquad p_t \;=\; v_t - \lambda x_t$$

where $h = k^{-1}\ell$, and $\lambda = k^{-1}\Lambda$ is the **pricing matrix**, with Λ the unique symmetric positive definite solution of:

$$\Lambda^2 + \Lambda\Big(S + \frac{1-\beta}{\beta} I_N\Big) \;=\; S, \quad \text{with} \quad S \;=\; \gamma k \Sigma_v$$

- Price pressure: $p_t = v_t - \lambda x_t$
  - Diagonal λ_{ii}: own-stock price pressure
    - One-dimensional intuition (Ho–Stoll 1981): buy imbalance $\Longrightarrow$ $x_t < 0$ $\Longrightarrow$ dealer raises a_t, b_t to discourage buyers & encourage sellers $\Longrightarrow$ p_t rises
  - Off-diagonal λ_{ij}: cross-stock price pressure = **cross-hedging**

Cross-Hedging

- When $\beta \to 1$, the pricing matrix λ converges to:

$$\lambda \;=\; \frac{k^{-1}}{2}\left((S^2+4S)^{1/2}-S\right), \quad \text{with} \quad S \;=\; \gamma k \Sigma_v$$

- **Corollary 1:** asymptotics in the risk aversion γ:

  Small $\gamma \implies$ **cross-hedging**:

$$\lambda \;\approx\; \gamma^{\frac{1}{2}} k^{-\frac{1}{2}} \Sigma_v^{\frac{1}{2}}$$

  Large $\gamma \implies$ **no cross-hedging** (Σ_v drops out):

$$\lambda \;\approx\; k^{-1}$$

  - *Intuition:* dealer mean-reverts each inventory to zero so quickly that hedging across assets has no time to be valuable

Two Assets: Pricing Matrix

- **Corollary 2:** Two assets with correlation a, diagonal k:

$$\Sigma_v = \sigma_v^2 \begin{bmatrix} 1 & a \\ a & 1 \end{bmatrix}, \quad k = k_0 I_2.$$

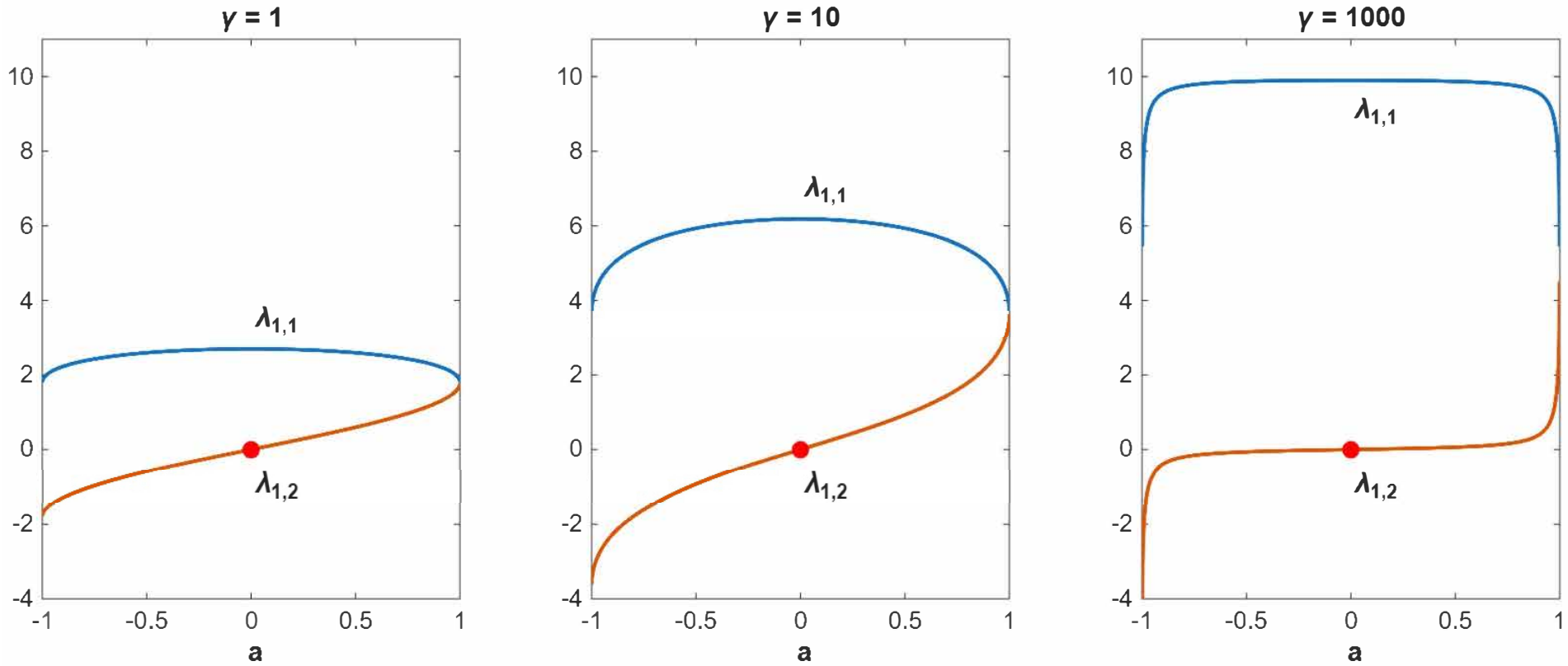
Then the pricing matrix is:

$$\lambda = \frac{1}{k_0(m_1+1)(m_2+1)} \begin{bmatrix} m_1+m_2+2 & m_1-m_2 \\ m_1-m_2 & m_1+m_2+2 \end{bmatrix}$$

$$m_1 = \left(\frac{4}{\gamma k_0 \sigma_v^2(1-a)} + 1\right)^{1/2}, \quad m_2 = \left(\frac{4}{\gamma k_0 \sigma_v^2(1+a)} + 1\right)^{1/2}$$

- Note: off-diagonal λ_{12} is proportional to $m_1 - m_2$:
  - $a = 0 \implies m_1 = m_2 \implies$ no cross-hedging
  - $a > 0 \implies m_1 > m_2 \implies$ **cross-hedging**; λ_{12} **strictly increasing in** a
- For $a \in (0,1)$: λ_{12} is **inverted U-shaped in** γ (vanishes at both extremes)

Pricing Coefficients vs Correlation & Risk Aversion



$\lambda_{1,1}$ (own) and $\lambda_{1,2}$ (cross) vs correlation $a = \mathrm{corr}(\Delta v_1, \Delta v_2)$, for risk aversion $\gamma = 1$ (left), 10 (center), 1000 (right). ($\sigma_v = 1$, $k_0 = 0.1$)

- $\lambda_{1,2}$ **rises with** a: cross-hedging strengthens with correlation
- $|\lambda_{1,2}|$ **rises, then falls with** γ (Cor 2): need risk aversion to cross-hedge, but very high γ eliminates cross-hedging

Two Assets: Numerical Example

- Set $\gamma = 1$, $\sigma_v = 1$, $k_0 = 0.1$, $a = 0.6$ $\implies$ pricing matrix:

$$\lambda = \begin{bmatrix} 2.54 & \mathbf{0.73} \\ \mathbf{0.73} & 2.54 \end{bmatrix}$$

- Suppose $v_t = [10, 10]'$. If $x_t = [-1, +1]'$ $\implies$

$$p_t = \begin{bmatrix} 10 \\ 10 \end{bmatrix} - \begin{bmatrix} 2.54 & 0.73 \\ 0.73 & 2.54 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 11.81 \\ 8.19 \end{bmatrix}$$

If instead $x_t = [-1, 0]'$ $\implies$

$$p_t = \begin{bmatrix} 10 \\ 10 \end{bmatrix} - \begin{bmatrix} 2.54 & 0.73 \\ 0.73 & 2.54 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 12.54 \\ 10.73 \end{bmatrix}$$

- Long position in (correlated) asset 2 partially hedges short position in asset 1 $\implies$ p_1 rises less (11.81 vs 12.54)
  - Substitution rate: $\lambda_{12}/\lambda_{11} = 0.73/2.54 \approx 29\%$: inventory in asset 2 absorbs about 29% of asset-1's price pressure

Two Assets: Hedging vs Correlations

- If correlation rises from $a = 0.6$ to 0.8, pricing matrix changes:

$$\lambda_{(a=0.6)} = \begin{bmatrix} 2.54 & 0.73 \\ 0.73 & 2.54 \end{bmatrix} \quad \rightarrow \quad \lambda_{(a=0.8)} = \begin{bmatrix} 2.38 & 1.06 \\ 1.06 & 2.38 \end{bmatrix}$$

- **Cross-hedging $\lambda_{1,2}$ rises**: $0.73 \rightarrow 1.06$
  - Substitution rate $\lambda_{12}/\lambda_{11}$: $29\% \rightarrow 45\%$ (reaches 100% as a converges to 1)
- **Own-hedging $\lambda_{1,1}$ falls**: $2.54 \rightarrow 2.38$
  - At $a = 1$, all entries of λ equal $1.79 \implies$ assets are perfect substitutes; less own-hedging needed as a rises

Inventory Half-Life

- **Inventory half-life** T in asset 1: starting from $x_0 = [1, 0]'$, smallest T with $\mathrm{E}[x_{1,T}] \leq 1/2$
- **Corollary 4:** For $N = 2$, diagonal k (with m_1, m_2 as before): Inventory half-life solves

$$\left(\frac{m_1-1}{m_1+1}\right)^T + \left(\frac{m_2-1}{m_2+1}\right)^T = 1$$

  - Sum inventory $(x_1 + x_2)$ decays at rate $\frac{m_2-1}{m_2+1}$ (fast)
  - Difference inventory $(x_1 - x_2)$ decays at rate $\frac{m_1-1}{m_1+1}$ (slower; cross-hedging works on the difference)
  - $x_0 = [1, 0]'$ projects equally on both $\implies$ equation for T
- Approximation when γ is small:

$$T \approx \frac{m}{\sqrt{1+a}+\sqrt{1-a}}, \qquad m = \frac{1}{\sqrt{\gamma k_0}\,\sigma_v}$$

$\implies$ T depends on correlation a

Risk-Aversion Bias: N Assets

- **Corollary 5:** N assets with same correlation a, diagonal k. When γ is small, the inventory half-life in asset 1 is:

$$T \approx \frac{m}{2\bar{r}}, \quad \text{with} \quad \bar{r} = \frac{\left(1+(N-1)a\right)^{\frac{1}{2}} + (N-1)(1-a)^{\frac{1}{2}}}{N}$$

- Intuition: correlated assets $\Longrightarrow$ **cross-hedging** $\Longrightarrow$ less urgency to unwind $\Longrightarrow$ longer half-life
- If we estimate γ by observing T (assuming $a = 0$), then $\gamma_{a=0}/\gamma = \bar{r}^2$:
  - $a = 1$: $\bar{r}^2 = \frac{1}{N}$ $\Longrightarrow$ bias $= 1 - \frac{1}{N}$:
    - 50% at $N = 2$, close to 100% for large N
  - Fixed a, large N: $\bar{r}^2 \to 1 - a$ $\Longrightarrow$ bias $\to a$

$\Longrightarrow$ **Single-asset model underestimates γ and bias grows with both N and a**

Asset Selection vs. Correlation

- Dealers also choose which assets to make markets in
- Two correlations matter:
  - $a = \text{corr}(\Delta v_1, \Delta v_2)$: **fundamental correlation**
  - $b = \text{corr}(\varepsilon_1^s - \varepsilon_1^b, \varepsilon_2^s - \varepsilon_2^b)$: **liquidity-shock correlation**
- They work in opposite directions:
  - Higher a makes assets better **substitutes** for cross-hedging
  - Higher b makes inventories move together, reducing diversification
- Risk aversion determines which force dominates:
  - High γ: dealer unwinds quickly $\Rightarrow$ **diversification wins**
  - Low γ: dealer holds inventory $\Rightarrow$ **cross-hedging can win**

Asset Selection vs. Correlation

- Focus on the empirically relevant case: $a > 0$ and $b > 0$
- With low risk aversion, compare two endpoints:
  - $a = 0$: no cross-hedging, but inventory risk is spread across two assets
  - $a = 1$: full cross-hedging, but liquidity shocks load on the same factor
- Comparison gives a simple threshold:

$$\sqrt{2}\,(1 + b^*) \;=\; 2 \qquad \Longrightarrow \qquad b^* \;=\; \sqrt{2} - 1 \;\approx\; 0.41$$

  - If $b < 0.41$: **cross-hedging wins** $\Rightarrow$ prefer highly correlated assets
  - If $b > 0.41$: **diversification wins** $\Rightarrow$ prefer less correlated assets

$\Longrightarrow$ Asset choice depends on both return correlation and liquidity commonality

Price Correlation at Short Horizons

- One-period price changes contain:

$$\Delta p_t = \underbrace{\Delta v_t}_{\text{fundamental news}} - \underbrace{\lambda \Delta x_t}_{\text{inventory pressure}}$$

- Suppose fundamentals are correlated ($a > 0$) but liquidity shocks are not ($\Sigma_L = L_0 I$). Also, risk aversion is high, so

$$\lambda \approx k_0^{-1} I$$

- If inventory pressure dominates at the one-period horizon,

$$L_0 \gg k_0 \sigma_v^2 \gg 1/\gamma, \qquad \Longrightarrow \qquad \rho_p(1) \approx 0 < a$$

- **At short horizons, observed prices are less correlated than fundamentals**

Epps Effect: Dealer-Inventory Mechanism

- Dealer quotes satisfy:

$$p_t = v_t - \lambda x_t$$

- Over a horizon of H periods:

$$p_{t+H} - p_t = \underbrace{v_{t+H} - v_t}_{\text{fundamental shocks accumulate}} - \underbrace{\lambda(x_{t+H} - x_t)}_{\text{inventory effect is transitory}}$$

- With high risk aversion and independent liquidity shocks:
  - Short horizon: dealer inventory dominates $\implies \rho_p(H) \approx 0$
  - Long horizon: fundamentals dominate $\implies \rho_p(H) \to a$

$\implies$ Observed return correlation **rises with sampling horizon**

	Pairs of Stocks					
Interval	*AMC-Chrysler*	*AMC-Ford*	*AMC-GM*	*Chrysler-Ford*	*Chrysler-GM*	*Ford-GM*
10 minutes	001	.009	– 009	– 014	007	055
20 minutes	009	018	011	017	026	118
40 minutes	.006	012	.014	041	040	197
One hour	–.043	.057	064	.023	065	.294
Two hours	029	.060	094	.112	129	383
Three hours	031	.158	111	361	518	519
One day	–.067	.170	078	342	442	571
Two days	– 020	223	186	336	449	572
Three days	– 098	203	.100	334	542	645

Empirical Design

- The theory gives a direct object to look for:

$$p_t = v_t - \lambda x_t$$

- Off-diagonal prediction:
  - If stock i and stock j are positively correlated, then $\lambda_{ij} > 0$
  - A dealer long stock i should quote stock j to **sell**
  - The response should be stronger when $\text{corr}(r_i, r_j)$ is higher
- Measurement implication:
  - Look at **quotes**, not only trades or end-of-period positions
  - Pairwise responses are small, so aggregate across the rest of the portfolio
- Empirical question:
  - Do individual market makers adjust quotes across correlated stocks in the predicted direction?

Data: TSX STAMP Internal Feed

- Message-level audit trail of all TSX orders, cancellations, and trades
  - 45 trading days in 2006
  - About 1,200 stocks per day
- Each message identifies the individual market maker and account type
- For each market maker, we reconstruct:
  - Inventory in every stock
  - Resting bid and ask quotes in every stock
  - Hence, we can relate

$$\text{inventory in stock } i \quad \longrightarrow \quad \text{quotes in stock } j, \qquad j \neq i$$

$\Longrightarrow$ Observe **off-diagonal effects** predicted by theory

Classifying Market Makers (549)

- We split market makers into three groups:
- **DMM-only** (69): designated by the exchange; trade only assigned stocks
- **EMM-only** (252): not designated; voluntarily provide two-sided liquidity
- **Both** (228): designated in some stocks; voluntarily quote in others
- Why split them?
  - DMMs have formal obligations in assigned stocks
  - EMMs choose when and where to supply liquidity
  - The theory should apply to both, but behavior may differ
- **Preview:** cross-hedging appears in all three groups

Inventory at Stock and Portfolio Levels

- Minute-level positions in each stock, summed to the **net** (signed) and **gross** (absolute) portfolio:

	Stock			Portfolio		
(means)	All	DMM	EMM	All	DMM	EMM
End-of-day |position| (000s)	459	158	544	12,729	1,556	6,923
Reversions to zero per day	0.61	1.13	0.38	1.86	3.62	0.97
Minutes between reversions	62.4	71.2	59.7	43.8	41.4	51.1
End-of-day net (000s, median)	1.1	-0.3	0.1	6.4	-2.3	0.7

- Single-stock positions are carried: the gross mean is CAD 12.7M, while the **net** is flat at the median (CAD 6,400)
- At the close, 46% of held stocks are long and 41% short: a **two-sided portfolio**

Cross-Hedging Episodes

- Goal: identify times when inventory in stock i predicts quote changes in the **rest of the portfolio**
- Quoted liquidity imbalance for market maker, stock, minute:

$$\text{imb}_j = \text{ask depth}_j - \text{bid depth}_j$$

  - $\text{imb}_j > 0$: quotes tilt toward **selling** stock j

- For each focal stock i, aggregate the rest of the book:

$$\text{imb}_{\text{not-}i} = \sum_{j \neq i} \text{imb}_j$$

- **Cross-hedging episode**: inventory in i leads ask-heavy quotes in the rest of the portfolio
  - 30-minute rolling correlation: $\text{inv}_i(t)$ with $\text{imb}_{\text{not-}i}(t+5)$
  - At least 5 consecutive minutes with positive correlation

Cross-Hedging Episodes: Results

	All	DMM	EMM	Both
Episodes	879,942	58,634	449,107	372,201
Market makers	537	68	242	227
Episode length in min. (median)	26	24	26	25
Covariance offset ratio (median)	0.130	0.096	0.131	0.133
Top-3 hedgers' cov. share (median)	0.857	1.000	0.615	0.989
Positive contributors (median)	13	4	24	6

- **Widespread**: 537 of 549 market makers have cross-hedging episodes
- **Concentrated**: the top 3 hedger stocks explain most of the response
- **Modest**: median offset is 13% of the focal inventory covariance
- Consistent with theory: cross-hedging exists, but pairwise responses are small

Which Stocks Carry the Hedge?

- Most frequent focal–hedger pairs across episodes:

Focal	Hedger	Episodes	Market makers	Mean return corr.
ECA	CNQ	4,878	70	0.446
CNQ	ECA	4,792	68	0.446
PCA	ECA	4,242	53	0.445
PCA	CNQ	4,206	59	0.423
ECA	PCA	4,143	55	0.436
TLM	CNQ	4,023	51	0.448

- The recurring pairs are mostly related energy/resource stocks
- They have high return correlations, about **0.44**
- This suggests the hedge is not random: dealers use economically related stocks

Direction Test: Correlation & Quote Direction

- Theory predicts $\lambda_{ij} > 0$ when stocks i and j are positively correlated
- If dealer is long stock i, she should quote stock j to **sell**
- Stronger prediction: tendency to sell j should $\uparrow$ with $\text{corr}(r_i, r_j)$

Return $\text{corr}(r_i, r_j)$	Share quoted to sell	Relative to low corr.
Below -0.1	0.583	–
-0.1 to 0	0.623	+4.0 %
0 to 0.1	0.647	+6.4 %
0.1 to 0.3	0.694	+11.1 %
Above 0.3	**0.728**	**+14.5 %**

- The level is not the test: episode selection makes all cells exceed 0.500
- The test is the **slope**: higher return correlation means more selling pressure in the hedger stock

Direction Test & Own-Inventory in Hedger

- Concern: maybe the dealer is also long stock j
- Then selling j may simply be own-inventory management, not cross-hedging from i
- Check: split focal–hedger pairs by the dealer's inventory correlation in i and j

	Inventory correlation between i and j			
Return correlation	Negative	Near-zero	Positive	All
Below -0.1	0.603	0.566	0.624	0.583
Above 0.3	0.743	0.699	0.730	0.728
High $-$ low	$+0.140$	$+0.133$	$+0.106$	$+0.145$

- Return-correlation slope is positive in every inventory-correlation group
- It survives even when inventories in i and j do not move together

Conclusion: Theory

- This paper develops a dynamic model of market making in N **correlated assets**
- Main object: the market maker's pricing matrix λ
  - Diagonal terms: own-inventory management
  - Off-diagonal terms: **cross-hedging** through correlated assets
- The model delivers several sharp predictions:
  - Inventory in stock i affects quotes in correlated stocks j
  - Cross-hedging is stronger when return correlation is higher
  - Pairwise cross-hedging is small when the portfolio is large
  - Single-asset estimates understate risk aversion (bias can be close to 100%)
- **Contribution:** Tractable portfolio theory of liquidity supply

Conclusion: Empirical Verification

- The empirical section asks whether the key theoretical mechanism is visible in the data
- TSX audit-trail data: observe inventories & quotes
- Main predictions are borne out:
  - Market makers manage inventory at the portfolio level
  - Cross-hedging episodes are widespread, concentrated, and modest
  - Hedger stocks are economically related and highly correlated
  - Share quoted to sell rises with return correlation
- Last test is the strongest:
  - Long stock $i \Rightarrow$ quote correlated stock j to **sell**
  - Slope survives when inventories in $i\&j$ do not move together

$\Longrightarrow$ Data support theory's off-diagonal pricing matrix