Fertility Decline and Housing Boom under Geopolitical Tension: Theory and Evidence from Tax Returns

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Abstract

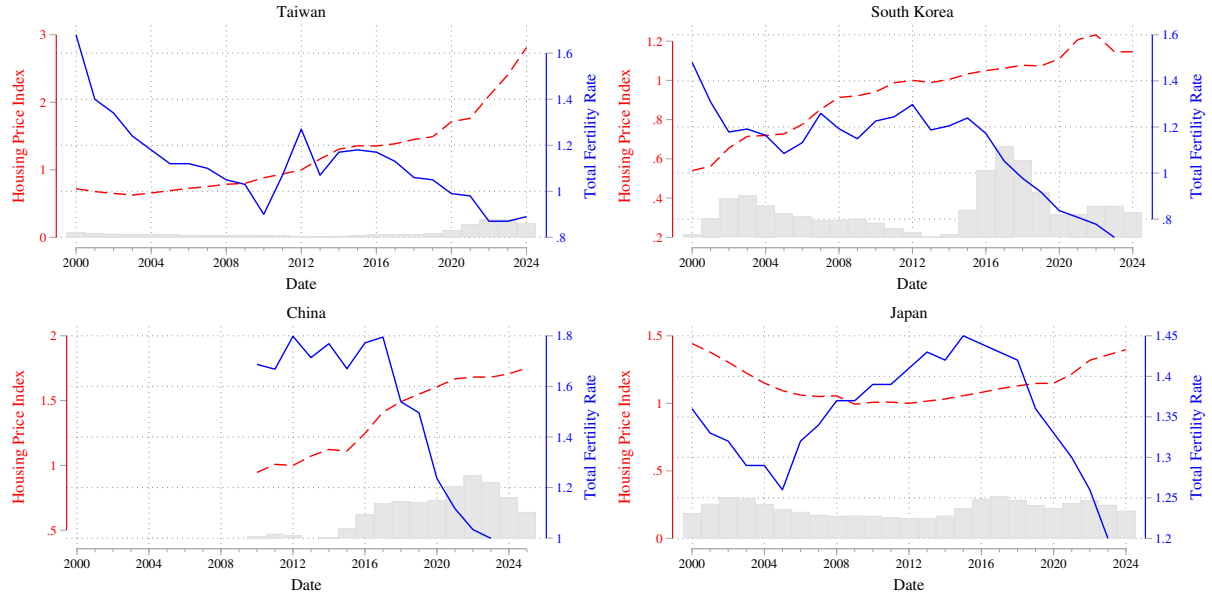
Significant declines in fertility are observed in regions such as South Korea, Taiwan, Hong Kong, Japan, and coastal China, which have been increasingly exposed to conflict risks and associated damage, despite experiencing a housing boom. We devise a novel competitive-search model of portfolio choice between housing and human capital, both of which can be damaged by wars. In our model, raising children is an equity-financed, irreversible investment, whereas housing is a defaultable debt-financed, resaleable asset. Escalation of war risk reduces demand for children but raises demand for leveraged housing in frictional markets, because defaulting on the housing debt provides implicit insurance against wars. To test our theory, we use the household-level fraction of daughters, who are exempt from compulsory military service in Taiwan, as an instrument for exposure to war risk. To measure war risk, we construct a novel sentiment index based on a textual analysis of Taiwan newspapers using a large language model. Matching Taiwan's administrative tax return data with the transaction-level data, we find evidence that households with higher exposure to war risk reduce fertility but increase mortgage leverage and housing investment. Our theory is also consistent with macro-level patterns. We conclude by discussing the policy implications of this new mechanism of fertility decline.

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Many economies facing elevated geopolitical tensions—such as Taiwan, South Korea, Japan, and parts of coastal China—have exhibited two striking trends over recent decades: a sharp decline in fertility alongside a sustained boom in housing investment. Figure 1 documents a strong negative correlation between the total fertility rate and housing prices across major economies. This negative correlation is particularly pronounced during periods of heightened geopolitical risk, denoted by the gray bars. Such episodes are evident after 2016 in Taiwan; during 2000–2004 and again since 2015 in South Korea, when the fertility rate declined sharply; and since 2015 in China. In Japan, the post-2016 trajectory has been further complicated by a macroeconomic factor where real interest rates remain persistently lower to support housing investment than those observed during the early 2000s.

Figure 1: Fertility rate, housing price, and geopolitical risk



Note: In each figure, the solid line represents the total fertility rate and corresponds to the right vertical axis, while the dashed line plots the nominal housing price index with 2012 as the base year and corresponds to the left vertical axis. For Taiwan, the total fertility rate is reported by the Taiwan National Development Council, and the housing price index is the repeat-sales hedonic unit housing price index constructed by Chi, LaPoint and Lin (2025). For the other three countries, total fertility rates are obtained from the World Bank, and the housing price index is the seasonally adjusted series from the OECD data. The gray bars indicate the year-end trend component of the monthly geopolitical risk index constructed by Caldara and Iacoviello (2022) and Caldara et al. (2025), which corresponds to the left vertical axis.

A simple geopolitical view would suggest that rising war risk discourages childbearing by increasing the expected costs of raising children. Yet this perspective leaves unresolved a central macro puzzle. Standard housing–fertility mechanisms imply that ris-

ing housing wealth should support fertility through collateral and wealth effects, while partial-equilibrium affordability arguments predict that higher housing prices should depress both fertility and housing demand. Neither logic can explain why fertility falls while housing investment and leverage rise precisely when perceived war risk increases.

This paper proposes a new explanation for this pattern based on the financial structure of household decisions. The key distinction is that fertility is an irreversible, equity-financed investment, whereas housing is predominantly financed with long-term, defaultable mortgage debt. We develop a theory of geopolitical risk-shifting in which mortgage default provides implicit insurance against war-related losses. When war risk rises, households reduce fertility but increase demand for leveraged housing because default allows them to transfer part of the downside risk to lenders. We formalize this mechanism in a general-equilibrium model with a competitive-search mortgage market and derive sharp, testable predictions that differ across mortgaged and unmortgaged households and across extensive and intensive margins of borrowing. Empirically, we bring new evidence to bear on these predictions by combining a novel, LLM-based measure of China–Taiwan war risk with administrative tax records and transaction-level housing data, and by exploiting plausibly exogenous variation in household exposure to war risk.

The theoretical framework features households that choose fertility, housing, and mortgage contracts in continuous time under the risk of a geopolitical disaster. Raising children is modeled as an irreversible investment that cannot be financed with debt, while housing is a durable asset that can be purchased using a long-term mortgage. Mortgage contracts are traded in a competitive-search market in which lenders anticipate optimal default behavior and earn zero expected profits in equilibrium. Default is endogenous and state-contingent: households optimally choose to default only in adverse states, such as when a war occurs, provided leverage lies within a feasible range. This structure generates two key debt thresholds that determine whether a mortgage is risk-free, risky but insurance-providing, or infeasible. Importantly, higher war risk increases the marginal value of default-based insurance, which raises equilibrium mortgage leverage and housing demand even as fertility declines.

To measure geopolitical risk, we construct a new news-based war-risk index using the universe of Mandarin-language articles from Taiwan's Central News Agency covering the period 1991–2025. We first identify China–Taiwan–related content using a comprehensive keyword filter and then classify each article into tension, peace, neutral, or irrelevant categories. Classification is disciplined by a manually labeled training set and implemented using a large language model with transparent, rule-based prioritization. We aggregate article-level sentiment into a continuous index that captures the relative intensity of war-

related versus peace-related language. The resulting index measures time-varying *perceived* geopolitical tail risk rather than realized conflict and exhibits clear spikes around major geopolitical events, such as the 1996 Taiwan Strait Crisis and the 2022 military exercises following high-level diplomatic visits. Importantly, the index comoves in intuitive ways with external financial indicators: increases in war risk are associated with declines in China-related stock prices, greater downside price asymmetry, and contractions in cross-border investment flows, providing independent validation of its relevance and informational content.

We combine this war-risk measure with rich administrative data from Taiwan that cover the universe of individual tax returns between 2005 and 2020. These records provide detailed demographic information, family structure (including the gender composition of children), income sources, financial assets, real-estate holdings, and outstanding mortgage balances. We further merge these data with deed-tax files and official transaction-level housing records that report prices, dates, and property characteristics for all housing transactions. This integrated dataset allows us to track fertility outcomes, mortgage borrowing, and housing investment at the household level with minimal measurement error.

Guided by the model, our empirical strategy links war risk to fertility and housing outcomes through the mortgage channel. We first document that higher war risk is associated with lower future fertility, with stronger declines for households that are more exposed to potential war-related casualties. We then examine how war risk affects mortgage participation, mortgage leverage among borrowers, and housing investment using a three-stage empirical design that separates selection into borrowing from leverage conditional on borrowing and housing investment conditional on leverage.

Household exposure to war risk is proxied by the share of sons in the household, which captures differential vulnerability to conflict through compulsory military service and therefore reflects variation in expected losses in the event of war. This measure is guided directly by the model, which predicts that households facing greater exposure adjust both fertility and financial decisions more strongly in response to increases in aggregate war risk. Consistent with these predictions, we find that higher war risk is associated with increased mortgage leverage and higher housing transaction values, but only among households that borrow. In contrast, when we restrict attention to households without mortgages, war risk and exposure have no statistically meaningful effect on housing investment.

Taken together, the results show that geopolitical risk reshapes fertility and housing outcomes primarily through financial structure—specifically, through the availability and

use of defaultable mortgage debt—rather than through direct changes in housing preferences or prices, as documented in the literature.

Related Literature The foundational economic literature treats fertility as an endogenous choice shaped by prices, income, and the opportunity cost of time. Becker and Lewis (1973) formalize the quantity–quality tradeoff, showing that an increase in desired child quality leads parents to optimally reduce fertility. Becker (1960) emphasizes the opportunity cost of maternal time as a central determinant of fertility, a mechanism that becomes more salient as female wages rise. Subsequent work embeds fertility into dynastic and growth frameworks, highlighting how household income and female labor market opportunities interact with fertility decisions (Becker, 1960; Barro and Becker, 1989; Jones, Schoonbroodt and Tertilt, 2010).

A large empirical literature identifies housing as a first-order determinant of fertility, operating through opposing affordability and housing-wealth channels with pronounced heterogeneity by tenure status. Higher house prices tend to reduce fertility among nonowners by raising entry, space, and credit constraints and by delaying marriage and first births, while increases in housing wealth can raise fertility among homeowners through collateral and wealth effects, resulting in modest average effects in the United States (Yi and Zhang, 2010; Dettling and Kearney, 2014; Clark, 2012). Quasi-experimental and household-level evidence corroborates this asymmetry: housing wealth gains increase fertility among homeowners in Australia, Denmark, and China, whereas price increases reduce fertility where affordability constraints bind for renters (Atalay, Li and Whelan, 2021; Daysal et al., 2021; Ang et al., 2024). Beyond timing responses, earlier access to adequate housing and homeownership substantially increases fertility—particularly for young and credit-constrained households—while delays in securing housing lead to meaningful reductions in completed lifetime fertility (van Doornik et al., 2025; Japaridze and Sayour, 2024). Accounting for households' endogenous choices over location and housing size, Couillard (2025) shows that higher housing costs reduce fertility rates in the United States. At longer horizons, rising house prices have been argued to play a central role in the global fertility transition since the late nineteenth century (Li, 2024).

Fertility decisions are shaped by labor market incentives and intra-household dynamics that jointly determine the costs and returns to childbearing. A large literature emphasizes the role of human-capital considerations: as returns to education and experience rise, parents substitute quality for quantity, leading to lower fertility (Becker, 1960; Becker, Murphy and Tamura, 1990; Galor and Weil, 2000). Labor market institutions further condition this trade-off by affecting flexibility, job security, and unemployment risk,

thereby altering the compatibility between career investment and family formation (Del Boca, 2002). At the same time, fertility outcomes reflect intra-household bargaining and social norms. Doepke and Kindermann (2019) show that disagreement between partners and unequal divisions of childcare reduce fertility when childbearing decisions emerge from bargaining rather than unitary household preferences. Cultural norms regarding motherhood and female employment play an independent role: traditional gender norms can interact with rising female labor market opportunities to accelerate fertility decline (Myong, Park and Yi, 2021; Goldin, 2006).

Public policy influences fertility through childcare provision, parental leave, and old-age support systems. Childcare availability raises fertility by reducing the time costs of childrearing and improving work–family compatibility (Del Boca, 2002; Olivetti and Petrongolo, 2017). Pension systems weaken incentives to rely on children for old-age security, thereby lowering fertility (Boldrin and Jones, 2002). Parental leave policies can increase higher-order births when sufficiently generous and well designed (Lalive and Zweimüller, 2009). Historical population control (like One-Child Policy) and pronatal policies further illustrate the capacity of state intervention to shape fertility trajectories.

Finally, recent work emphasizes social interactions and status concerns. Kim, Tertilt and Yum (2024) show that relative-status competition in child quality generates overinvestment in education and suppresses fertility. Building on this mechanism, Mahler, Tertilt and Yum (2025) argue that social media and intensified peer comparisons amplify comparison motives, reinforce intensive parenting norms, and contribute to persistently low fertility.

This paper develops a new mechanism linking fertility decline and housing booms to geopolitical risk through the structure of household finance.[1] We model fertility as an irreversible, equity-financed investment and housing as a debt-financed, defaultable asset traded in a competitive-search mortgage market. In this environment, mortgage default provides implicit insurance against geopolitical disasters, generating a novel risk-shifting channel through which rising war risk reduces fertility while simultaneously increasing mortgage leverage and housing investment. The model delivers sharp, testable predictions that differ between mortgaged and unmortgaged households and between extensive and intensive margins of mortgage borrowing. To evaluate these predictions, we construct a new large-scale, AI-based measure of China–Taiwan war risk and combine it with administrative tax records and transaction-level housing data. Guided by the model, we exploit variation in household exposure to war risk—proxied by the share of

[1] Bakkensen, Phan and Wong (2025) discuss the effect of defaulting mortgage to shift disaster risk but their focus is not on fertility.

sons—to identify the mortgage-insurance channel. Our results show that geopolitical risk reshapes demographic and housing outcomes through financial structure, rather than through prices or preferences alone.

Background Geopolitical tension between China and Taiwan originates in the unresolved termination of the Chinese Civil War following the Second World War. In 1949, the Communist government established control over the mainland, while the Nationalist government retreated to Taiwan. No peace treaty was ever signed. As a result, the Taiwan Strait has remained an enduring security fault line, punctuated early on by artillery crises in the 1950s and stabilized only through U.S. military involvement that prevented a forcible resolution. The conflict thus persisted in a latent form—neither resolved nor extinguished—under shifting international arrangements.

Over the past two decades, the rapid rise of China's economic and military capacity has sharply altered the cross-Strait balance. China now maintains a large standing force oriented toward Taiwan, including an estimated 1,000–1,500 short- and medium-range ballistic and cruise missiles deployed along its southeastern coast, supported by expanding naval, air, amphibious, cyber, and space capabilities. This posture has been repeatedly demonstrated through high-profile military drills, including the large-scale exercises surrounding Taiwan in 2022, encirclement-style Joint Sword exercises in 2023–2024, and frequent operations simulating blockades and precision strikes. Taiwan's response has emphasized asymmetric defense and whole-of-society mobilization. A central institutional adjustment has been the extension of compulsory military service to one year for all men, beginning in 2024. In the event of war or a declared national emergency, Taiwan's conscription system provides the legal and administrative basis for full reserve mobilization, rapid recall of former conscripts, extension of service terms, and reallocation of civilians to defense, logistics, and civil-support functions. Against this backdrop, the next section introduces a high-frequency measure of China–Taiwan war risk based on natural language processing of news and media content.

1 Model

This section develops a continuous-time model in which households jointly choose fertility and housing investment under geopolitical (war) risk. The central mechanism rests on an asymmetry in financial structure: fertility is an irreversible, equity-financed investment, while housing is financed through long-term, defaultable debt. This asymmetry allows households to use mortgage default as an implicit insurance device against war risk,

generating a decline in fertility alongside an expansion of mortgage leverage and housing investment when geopolitical risk rises.

Time is continuous, $t \in [0, \infty)$. The utility of a typical household at $t > 0$ is given by

$$U(c_t, n_t, h_t) \equiv u(n_t) + \rho(n_t - 1)\left[c_t + \phi(h_t)\right], \tag{1}$$

where c_t denotes consumption flow, and n_t and h_t the stocks of children and housing at time t. The function $u(n_t)$ captures the direct utility from having n_t children, while $\phi(h_t)$ captures the services utility from h_t units of housing. The function $\rho(b)$ is the marginal utility of consumption, capturing utility interactions with children. When $\rho'(b) > 0$, children are Edgeworth complements to consumption and housing; when $\rho'(b) < 0$, they are substitutes.

A typical household is endowed with a constant income flow y and a stock of existing children n_-. At time zero, the household chooses additional fertility $\Delta_n \geq 0$, housing investment $h \geq 0$, and, if desired, a mortgage contract to finance housing.

War. A geopolitical conflict ("war") arrives at a random time T_w, which follows a Poisson process with intensity $\bar{\lambda}$.[2] When war occurs, all housing is destroyed, all newborn Δ_n and a fraction $\sigma \in (0, 1)$ of existing children die. Households perceive the arrival rate λ, which is influented by sentiment and can be overacted to negative news such that $\lambda > \bar{\lambda}$. This wedge in belief implies that, in frictionless market, housing is not a good collateral to secure against the war risk.

Given initial children n_-, the household chooses fertility Δ_n, a mortgage contract $\mathbf{m}$ (and housing investment h_u in case of mortgage denied) to maximize expected lifetime utility net of fertility costs. The household internalizes the endogenous probability of mortgage approval ($\mathbf{1}_m = 1$), the option to default, and the stochastic arrival of war. If mortgage is denied ($\mathbf{1}_m = 0$), the household purchases housing outright using equity and enjoys housing utility until war occurs.

Before war occurs, the household's asset (human capital and housing capital) consists of

$$n_t = 1 + b, \qquad h_t = h.$$

After the war date $t \geq T_w$, housing is completely destroyed and the surviving stock of

[2]The probability that war has not yet occurred by time t is thus given by

$$\Pr(T_w \geq t) = \exp\left(-\bar{\lambda} t\right).$$

children becomes

$$n_t = 1 - \sigma, \qquad h_t = 0.$$

Mortgage contract. Housing has unit price p. Housing can be financed through a long-term mortgage contract $\mathbf{m} = \{L, i, h\}$, where L denotes the loan principal advanced at time zero, i is the contractual mortgage rate, and h is the amount of housing pledged as collateral. Mortgage contracts are offered in a competitive search market. A household that chooses contract $\mathbf{m}$ is approved with probability $\alpha(m)$. If approved, the household borrows L at time zero, purchases h units of housing, and services the mortgage by paying a continuous flow iL.

The borrower retains the option to default at any time $\tau \leq T_w$. Upon default, the borrower pays a proportional default cost fh and forfeits all housing. After default, housing services drop permanently to zero. Consumption evolves according to

$$c_t = y + w\left(1 - \mathbf{1}_{w,t}\sigma\right) - \mathbf{1}_m iL,$$

where $\mathbf{1}_m$ is an indicator for mortgage approval and w captures net transfers or support associated with adult children.

The key trade-off is that fertility is exposed to war risk without any possibility of state-contingent adjustment, while housing financed with a mortgage can be partially insured through default. This difference drives the household's portfolio choice across fertility and housing when war risk changes.

Value function. Households' expected lifetime utility is given by

$$-kb + \mathbb{E}\left[\rho(n_0)C_0 + \int_0^\infty e^{-rt}U\left(c_t, n_t, h_t\right)\mathrm{d}t - \mathbf{1}_m e^{-r\tau}\rho(n_\tau - 1)fh_0\right],$$

where $C_0 = Y_0 - (ph_0 - L)$ and we normalized $\rho\left(0\right) = 1$. Raising newborn is financed entirely through equity at cost kb at time zero. The household's preferences can be expressed recursively as

We solve the household's problem backward. Given a approved mortgage contract $\mathbf{m}$, the post-war household's non-default value is:

$$\begin{aligned} W_{\mathbf{m}}^{\text{post}}\left(b\right) &\equiv \int_0^\infty e^{-rt}\left[-\mathbf{1}_m iL + \phi\left(h_t\right)\right]\mathrm{d}t, \\ &= -\frac{i}{r}L \end{aligned}$$

where we have used the fact that the unit of housing and the number of childen after war are $h_t = 0$ and $n_t = 1 - \sigma$. The post-war household's default value is simply

$$D_{\mathbf{m}}^{\text{post}}(b) \equiv -fh.$$

Thus, the borrowing household defaults after war iff $D_{\mathbf{m}}^{\text{post}} > V_{\mathbf{m}}^{\text{post}}$, which is equivalent to

$$L > L^{\text{post}} \equiv \frac{r}{i} fh. \tag{2}$$

The pre-war household's non-default value is:

$$W_{\mathbf{m}}^{\text{pre}}(b) \equiv \mathbb{E}_{\lambda}\left\{\int_0^{T_w} e^{-rt}\left[-iL + \phi(h)\right] \mathrm{d}t + e^{-rT_w}\frac{\rho(-\sigma)}{\rho(b)} \max\left\{W_{\mathbf{m}}^{\text{post}}(b), D_{\mathbf{m}}^{\text{post}}(b)\right\}\right\},$$

Similarly, the pre-war household's default value is:

$$D_{\mathbf{m}}^{\text{pre}}(b) \equiv -fh.$$

Thus, the borrowing household defaults after war iff $D_{\mathbf{m}}^{\text{pre}} > V_{\mathbf{m}}^{\text{pre}}$. There are two relevant debt limits that determine how risky a mortgage is:

Define the risky debt limit as:

$$L^{\text{pre}} \equiv \frac{r}{i}\phi(h) + \left[1 + \lambda\left(1 - \frac{\rho(-\sigma)}{\rho(b)}\right)\right] L^{\text{post}}.$$

Given a loan contract $\mathbf{m} = (L, i, h)$, the optimal default time $\tau_{\mathbf{m}}^*$ that solves problem (**??**) is:

Lemma 1. *(a) if $L^{post} < L^{pre}$ then*

$$\tau_{\mathbf{m}}^* = \begin{cases} \infty \quad \textit{(no default)} & \textit{if } L \leq L^{post} \\ T_w \quad \textit{(default at war)} & \textit{if } L \in (L^{post}, L^{pre}] \\ 0 \quad \textit{(default immediately)} & \textit{if } L > L^{pre} \end{cases};$$

(a) if $L^{post} \geq L^{pre}$ then

$$\tau_{\mathbf{m}}^* = \begin{cases} \infty \quad \textit{(no default)} & \textit{if } L \leq L^{pre} \\ 0 \quad \textit{(default immediately)} & \textit{if } L > L^{pre} \end{cases}.$$

We provide all proofs in Appendix A.8. Intuitively, the safe debt limit is where the

borrower is indifferent between repaying and defaulting at war (at $t = T_w$). The risky debt limit is where the borrower is indifferent before the war (at any $t < T$). Any mortgage with $L \leq L^{\text{safe}}$ is safe (zero default risk) and any mortgage with $L^{\text{post}} < L \leq L^{\text{pre}}$ is risky (the loan will be defaulted at the war date). Any mortgage with $L > L^{\text{pre}}$ will incur immediate default and will not be approved by banks in equilibrium.

Competitive search. Anticipating the borrower's default strategy $\tau^*_{\mathbf{m}}$, a lender's expected revenue from a mortgage contract $\mathbf{m}$ at time zero is:

$$R_{\mathbf{m}} = \begin{cases} \mathbb{E}_{\bar{\lambda}} \int_0^{\tau^*_{\mathbf{m}}} e^{-rt} i L \mathrm{d}t, \text{ if } L \leq L^{\text{pre}}, \\ ph, \text{ if } L > L^{\text{pre}}, \end{cases} \tag{3}$$

A lender's expected profit from contract $\mathbf{m}$ is:

$$\Pi_{\mathbf{m}} \equiv R_{\mathbf{m}} - L. \tag{4}$$

The borrower is matched with a lender (out of the pool of competitive lenders) through a competitive search and matching process.[3] With search frictions, the borrower has an endogenous probability $\alpha_{\mathbf{m}}$ of finding a lender offering a mortgage contract $\mathbf{m}$. Similarly, each lender has probability $\eta_{\mathbf{m}}$ of finding a borrower. There is an exogenous matching function η that maps $\eta_{\mathbf{m}} = \eta(\alpha_{\mathbf{m}})$, where η is strictly decreasing and convex.[4]

Taking into account the search frictions as summarized by the matching function η, and given the solution $V^{\text{pre}}_{\mathbf{m}}$ to optimal default problem (**??**), the *optimal mortgage contract* $\mathbf{m}^*$ and the associated *leverage probability* $\alpha^*_{\mathbf{m}}$ maximize the borrower's value:

$$W(n) \equiv \max_{\alpha_{\mathbf{m}}, \mathbf{m}} \quad \alpha_{\mathbf{m}} \underbrace{\left[V^{\text{pre}}_{\mathbf{m}}(n) - r(ph - L)\right]}_{\text{PV of buying a house with leverage}} + (1 - \alpha_{\mathbf{m}}) \underbrace{\max_{h_c} \left\{ V^{\text{pre}}_{\{0,0,h_c\}}(n) - rph_c \right\}}_{\text{PV of buying without leverage}}, \tag{5}$$

[3] Appendix A.9.10 provides more detail. We choose the search environment not only because we think it captures the essence of credit search in practice (e.g., homebuyers searching for a mortgage lender), but also it allows us to endogenize and characterize the probability that the home purchase is financed with a mortgage—a quantity that is key in our empirical analysis. Also, the search frictions lead to a dispersion of prices and mortgage terms like maturity, loan amount and interest rate, as we see in the data.

[4] The function η is taken as given by agents but can be endogenously determined from a matching technology (see Appendix A.9.10 for details). The function maps $\alpha_{\mathbf{m}}$, which is increasing in the market thickness (the ratio of lenders to borrowers), to $\eta_{\mathbf{m}}$, which is decreasing in the market tightness. Market tightness is endogenously determined by the entry of lenders. $\eta' < 0$ implies congestion and $\eta'' > 0$ implies diminishing congestion.

subject to a free-entry condition (lenders break even):

$$\underbrace{\kappa}_{\text{entry cost}} = \underbrace{\eta(\alpha_{\mathbf{m}})\Pi_{\mathbf{m}}}_{\text{Lender's expected profit}} . \tag{6}$$

Finally, the homebuyer chooses to borrow or not by solving $\max\{U_\lambda, v_\lambda - rp\}$.

1.0.1 Optimal mortgage under the optimal default strategy

Given lenders' profit (4) and free-entry condition (6), the borrower's problem (5) becomes:

$$\max_{\alpha} \left\{ \alpha \left\{ \max_{\mathbf{m}} S_{\mathbf{m}} - \frac{\kappa}{\eta(\alpha)} \right\} \right\},$$

where $S_{\mathbf{m}}$ denotes the *joint surplus* from $\mathbf{m}$ (before the lender's service and fixed costs):

$$S_{\mathbf{m}} \equiv R_{\mathbf{m}} + V_{\mathbf{m}}^{\text{pre}} - ph - \max_{h_u} \left\{ V_{\{0,0,h_u\}}^{\text{pre}}(b) - ph_u \right\}.$$

With the block-recursive structure of the competitive search environment, we first solve for the optimal contract $\mathbf{m}^*$, which is simply the contract that maximizes $S_{\mathbf{m}}$; then we solve for the leverage probability $\alpha^* \equiv \alpha_{\mathbf{m}^*}$ that arises from the competitive search.

Under the optimal default strategy in Lemma **??**, the joint surplus $S(\mathbf{m})$ features the three cases. In the first case ($B \leq B^{\text{safe}}$), the loan balance is so low that the borrower will never default. The mortgage contract essentially redistributes the cash flow l and b between the borrower and the lender, and no joint surplus is created. Contracts in this region are dominated by no borrowing at all, due to the operational and fixed costs κ and κ_0.[5]

In the third case ($B > B^{\text{risky}}$), the loan balance is so high that the borrower finds it optimal to default right after taking on the loan at $t = 0$, which is essentially to sell the house at the liquidation price $\underline{p}$. The joint surplus is negative because it involves a dead weight loss f, and thus contracts in this region are also dominated by no borrowing at all.

In the remaining and interesting case ($B^{\text{safe}} < B \leq B^{\text{risky}}$), the loan balance is sufficiently high that the borrower will optimally default (only) when the disaster hits. In this case, the borrower uses default as *implicit insurance* against the disaster damage. Expecting the borrower's default, the break-even lender requires a higher repayment b or lends a smaller loan l. If the borrower and lender were to agree on the disaster's arrival

[5]Safe mortgages with $B \leq B^{\text{safe}}$ can occur in equilibrium for a relatively *optimistic* borrower when lenders having a lower cost of funding than the borrower(Section A.9.6).

rate (i.e., $\lambda = \bar{\lambda}$), the joint surplus from trading this implicit insurance, even excluding the deadweigh loss f from default, would be zero as the free-entry condition implies the lender is actuarial fair. With disagreement, the joint surplus becomes:

$$S(\mathbf{m}) = \underbrace{\left(\frac{1}{1+\mu+\bar{\lambda}} - \frac{1}{1+\mu+\lambda}\right) b}_{\text{net gain from the maturity channel}} \underbrace{- \left(\frac{\lambda}{1+\mu+\lambda} - \frac{\bar{\lambda}}{1+\mu+\bar{\lambda}}\right)(h-d) - \frac{\lambda f}{1+\mu+\lambda}}_{\text{net loss from the costly default channel}}. \tag{7}$$

Equation (7) highlights two opposite channels:

Maturity channel (new): The first term of (7) captures the expected net gain to the match from trading a defaultable loan with variable maturity. Both the borrower and lender know that, given the borrower's optimal default strategy, the repayment stream of b will end at date $T \wedge T_\mu$. However, given her belief of the disaster's arrival rate, the borrower's PV of this repayment stream is $\frac{b}{1+\mu+\lambda}$, while the lender's PV is $\frac{b}{1+\mu+\bar{\lambda}}$. The *maturity channel*—represented by the difference between the two valuations—is positive when the borrower is more *pessimistic* than the lender ($\lambda > \bar{\lambda}$). Here, the borrower believes that the default time will arrive sooner and hence the actual repayment will be shorter than what the lender believes. The gain due to this maturity channel is larger at *longer* maturities, i.e., the net gain term decreases in μ; for example, the term converges to zero when the contract matures immediately ($\mu \to \infty$), as the long-run risk and its disagreement become irrelevant for such a short-term mortgage. The gain is also larger at *larger* loan balance $B = b/(1+\mu)$, as the borrower can walk away a larger amount. In sum, maturity and loan balance improve and make the coverage of implicit insurance longer and larger.

Costly default channel (conventional): The remaining of (7) is standard in static models of collateralized debt with heterogeneous beliefs. It captures the expected net losses to the match due to the foreclosure cost, $-\frac{\lambda f}{1+\mu+\lambda}$, and due to the surrender of the collateral, $-\left(\frac{\lambda}{1+\mu+\lambda} - \frac{\bar{\lambda}}{1+\mu+\bar{\lambda}}\right)(h-d)$. While both the borrower and lender agree that the value of the house becomes $h-d$ once the disaster hits, they disagree on how soon it will. A relatively more pessimistic borrower ($\lambda > \bar{\lambda}$) discounts the disaster event with a higher probability than the lender does ($\frac{\lambda}{1+\mu+\lambda} > \frac{\bar{\lambda}}{1+\mu+\bar{\lambda}}$), so this default channel represents a net loss to the match. Opposite to the gain from the maturity channel, the loss due to this costly default channel is larger at *longer* maturities, as the surrender of the house becomes more likely for the pessimistic borrower under a longer maturity.

Table 1: Comparative Statics with Respect to War Risk

	Fertility		Housing			
	$\frac{\partial n}{\partial \lambda}$	$\frac{\partial^2 n}{\partial \lambda \partial \sigma}$	$\frac{\partial^2 L}{\partial \lambda \partial \sigma}$	$\frac{\partial^2 h_m}{\partial \lambda \partial \sigma}$	$\frac{\partial h_u}{\partial \lambda}$	$\frac{\partial^2 h_u}{\partial \lambda \partial \sigma}$
Complements $\rho'(n) > 0$	$-$	$-$	$+$	$+$	$-$	0
Substitutes $\rho'(n) < 0$	$-$	$-$	$-$	$-$	$-$	0

Leveraging the disagreement: Summing up, these two channels have opposite implications to the use of mortgage contract and its maturity. Whether a pessimist homebuyer will borrow depends on whether the maturity channel dominates, eventually on the strength of belief disagreement. To see this, note that whenever the borrower is more pessimistic ($\lambda > \bar{\lambda}$), the optimal mortgage contract maximizes the maturity channel in (7) by maxing out the loan balance at the risky debt limit, i.e., $B^* = B^{\text{risky}}$. In this case, the surplus $S(\mathbf{m})$ collapses to $\frac{\Delta}{1+\mu+\bar{\lambda}}$, where Δ is a *belief disagreement term* given by:

$$\Delta \equiv \frac{\lambda - \bar{\lambda}}{1+\lambda} d - \bar{\lambda} f, \tag{8}$$

where $\Delta > 0$ if and only if the borrower is sufficiently pessimistic such that $\lambda > \lambda^*$, where

$$\lambda^* \equiv \begin{cases} \bar{\lambda} \frac{d+f}{d-\bar{\lambda} f} & \text{if } d > \bar{\lambda} f \\ \infty & \text{otherwise} \end{cases}. \tag{9}$$

Note that λ^* is increasing in the foreclosure cost. If $f = 0$, then $\lambda^* = \bar{\lambda}$, implying that there would be a positive joint surplus as long as the borrower is more pessimistic than the lenders ($\lambda > \bar{\lambda}$). Otherwise, $f > 0$ implies $\lambda^* > \bar{\lambda}$.

When the borrower is sufficiently pessimistic ($\lambda > \lambda^*$), the maturity channel dominates the costly default channel, and the optimal leverage increases in the borrower's pessimism: This asymmetry yields sharp, testable predictions that discipline the empirical identification strategy.

Table 1 summarizes the model's comparative statics with respect to geopolitical risk. An increase in war risk, captured by a higher arrival rate λ, unambiguously reduces fertility. This effect reflects the higher expected loss of children in the event of conflict and holds regardless of whether children and housing are complements or substitutes in preferences. Moreover, greater exposure to war risk through either a higher arrival rate λ or a higher child mortality rate σ further depresses fertility, particularly when the marginal utility shifter $\rho(\cdot)$ exhibits strong curvature, so that households engage in precautionary

self-insurance.

The response of housing investment depends critically on the financing structure. When housing is financed through defaultable mortgages and children and housing are Edgeworth complements, that is when $\rho'(n) > 0$, higher war exposure increases the demand for mortgage-based insurance. In this case, an increase in either λ or σ raises equilibrium mortgage leverage L and mortgaged housing investment h_m. Intuitively, defaultable debt allows households to partially insure against the risk of child mortality by shifting resources toward assets whose downside risk can be state-contingently shed through default.

By contrast, when children and housing are substitutes, $\rho'(n) < 0$, the insurance motive is absent. Higher war exposure tightens the effective borrowing constraint, reducing both mortgage leverage and mortgaged housing investment. In both cases, unmortgaged housing investment h_u declines with higher war risk, reflecting the direct destruction risk of housing capital. Importantly, the cross-partial derivative of unmortgaged housing with respect to λ and σ is zero, as unmortgaged housing lacks the default option that provides insurance.

An increase in war risk λ reduces fertility because children face higher expected mortality and cannot be insured. When children and housing are complements, higher war exposure raises the value of defaultable debt as insurance, increasing both mortgage leverage and mortgaged housing investment. By contrast, unmortgaged housing does not respond to war exposure, as it lacks the default option. This asymmetry yields sharp, testable predictions that discipline the empirical identification strategy.

Taken together, the table highlights a sharp asymmetry: variation in war exposure affects housing outcomes exclusively through mortgage leverage. The interaction between war arrival risk and child mortality risk shifts the demand for defaultable debt without directly influencing unmortgaged housing investment. This asymmetry underpins the empirical identification strategy and motivates the use of mortgage leverage as the key channel linking geopolitical risk to housing investment.

2 Model extensions

The insights behind Proposition **??** hold robustly in the following series of relevant extensions.

War insurance (A.9.1). One might be concerned that the absence of war insurance in the baseline model would force the borrowers to default upon disaster as implicit insurance. Appendix A.9.1 introduces two types of war insurance: voluntary and mandatory.

We show how Proposition ?? continues to hold if an insurance market exists and its uptake is voluntary, because, given a homebuyer choosing to borrow, defaulting a long-term debt would actually crowd out the uptake of disaster insurance (even if the latter is subsidized). If war insurance is mandatory, then our main result still holds with modified λ^*, if the insurance premium is not too high, or insurance coverage is sufficiently limited, which is observed in practice (see Section ??).

War forbearance (A.9.2). In practice, borrowers might expect lenders' forbearance after disasters like war (see Section ??), which might reduce the need for default (as an implicit disaster insurance). Appendix A.9.2 shows that the anticipation of forbearance actually provides additional motive for leverage, further strengthening our theoretical prediction that pessimistic homebuyers are more likely to leverage and use a long-term mortgage.

Rental (A.9.5). One might be concerned that the model artificially assume away the option for agents to rent over homeownership. Appendix A.9.5 shows that under the presence of a rental market, Proposition ?? continues to hold, but extremely pessimistic agents now prefer renting instead. Hence, this section also formalizes the notion that renting (instead of homebuying) is a form of climate adaptation (?).

Liquidity motive of borrowing (A.9.6). So far the homebuyers borrow because of insuring the war risl. Appendix A.9.6 incorporates a liquidity motive of borrowing, where homebuyers have a higher funding cost to pay for the house purchase upfront. Our main result is modified and continues to hold.

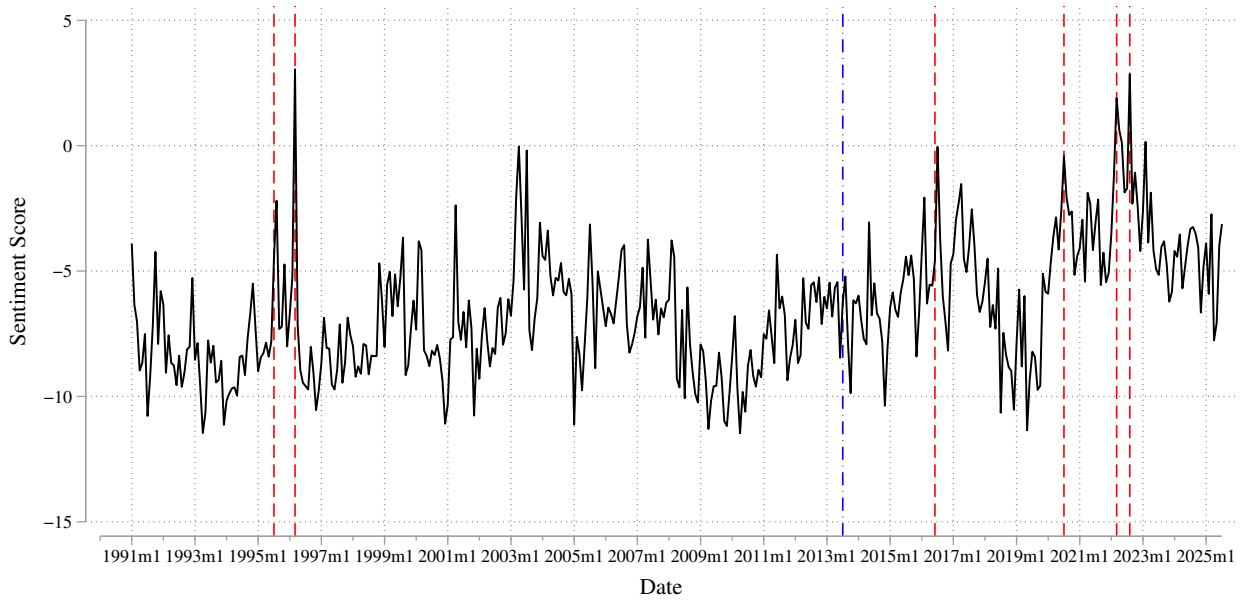
Technical extensions. Appendix A.9.9 endogenizes housing prices. Appendix A.9.10 generalizes the baseline with homebuyers with heterogeneous belief and houses with heterogeneous damage exposure, so there is also competition among borrowers for lenders. Appendix A.9.11 explicitly models deterministic maturity.

3 Measuring War Risk

We construct a news-based war (tension) index using the full archive of Mandarin-language news articles from the Central News Agency (CNA) covering the period 1991-2025. The procedure proceeds in three steps. First, we scrape all CNA news articles containing references to China and retain only those related to Taiwan and cross-Strait affairs using a comprehensive keyword filter. The resulting corpus is cleaned, segmented, and standardized using standard Mandarin text-processing tools.

Second, each article is classified into one of four sentiment categories—tension, peace, neutral, or irrelevant—based on its implications for cross-Strait conflict risk. To ensure

Figure 2: Monthly news-based war index



Note: The solid line plots the monthly news-based sentiment score. Vertical dashed lines denote key events associated with heightened cross-Strait tensions, whereas the dash-dotted line marks the event corresponding to periods of eased tension.

consistency with human judgment, we manually label a training sample and use it to guide large language model–based classification, supplemented by transparent rule-based priorities. Articles indicating increased military activity or conflict risk are classified as tension, while those reflecting cooperation or de-escalation are classified as peace.

Third, we transform the discrete sentiment classification into a continuous index capturing the intensity of war-related sentiment. For each article, we compute a normalized score based on the relative frequency of tension and peace keywords, yielding a measure bounded between 1 (maximum tension) and -1 (maximum peace). Neutral articles assigned a value of zero and irrelevant articles excluded. Appendix A provides full implementation details.

Figure 2 plots the monthly news-based war index. The index precisely highlights six major historical events that marked episodes of heightened cross-Strait tension between China and Taiwan, indicated by vertical dashed lines. These events all feature sentiment scores that are at least two standard deviations above the time-series mean. The first corresponds to July 1995, when China conducted missile tests near Taiwan following former President Teng-hui Lee's visit to the United States. The second marks March 1996, when a second round of missile launches was carried out in the run-up to Taiwan's first direct presidential election and was widely interpreted as an attempt to intimidate voters.

Table 2: Effect of war risk on the stock performance

	China-related stocks				non-China-related stocks			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	$Return_t^s$	$Return_t^s$	$Return_t^s$	$Return_t^s$	$Return_t^s$	$Return_t^s$	$Return_t^s$	$Return_t^s$
War$_{t-1}$	-0.0024	-0.0014	-0.0813**	-0.0749**	-0.0012	-0.0015	0.0341	0.0168
	(-1.23)	(-0.76)	(-3.39)	(-3.84)	(-0.05)	(-0.09)	(1.18)	(0.38)
Constant	3.954***	3.950***	5.340***	5.344***	-0.002**	-0.002**	-0.001***	-0.001***
	(25.35)	(136.21)	(438.12)	(438.05)	(-2.71)	(-2.87)	(-3.60)	(-3.67)
Other controls		✓	✓	✓		✓	✓	✓
Time FE			✓	✓			✓	✓
Stock FE				✓				✓
N	198,577	196,468	196,463	196,463	7,328,191	7,058,780	7,058,775	7,058,775
Adj R^2	0.000	0.001	0.171	0.173	0.000	0.000	0.001	0.001

Notes: $Return_t^s$ denotes the excess return of stock s on date t, measured as the daily growth rate of the close price of stock s minus the daily growth rate of the Taiwan stock market index. Other controls include daily stock-level turnover and trading volume. Standard errors are clustered at the stock–day-of-week level, as the third Wednesday of each month in Taiwan is the typical option settlement day and is associated with heightened stock price volatility. t statistics are reported in parentheses. *$p < 0.1$, ** $p < 0.05$, and *** $p < 0.01$.

In the past decade, the third event occurs in May 2016, following the inauguration of former President Ing-wen Tsai, who declined to explicitly endorse the 1992 Consensus—an informal understanding under which both sides of the Taiwan Strait acknowledge "one China" while allowing for differing interpretations. The fourth event corresponds to June 30, 2020, when China imposed the Hong Kong National Security Law, an action that significantly reinforced Taiwanese skepticism toward Beijing and coincided with a sharp increase in Chinese military pressure. The fifth event is March 2022, following Russia's invasion of Ukraine, after which China intensified military signaling toward Taiwan. The final event marks August 2022, when Nancy Pelosi visited Taiwan, triggering the largest Chinese military exercises around the island since the 1996 Taiwan Strait Crisis.

In contrast, the period spanning 2008–2016, under former President Ying-jeou Ma, was characterized by relatively lower cross-Strait tensions. During this period, the Taiwanese government announced a reduction in mandatory military service from one year to four months in 2013, a policy implemented in 2014 and denoted by the vertical dash-dotted line, after which measured war tensions declined further in the short run.

3.1 Validating the War Index

To validate the constructed war index, we first show that it is highly correlated with existing measures of geopolitical risk, including the index developed by Caldara and Iacoviello (2022) and Caldara et al. (2025). As shown in Figure A.1 in the appendix, the correlation between the two indices is 0.54. Despite this high correlation, the indices differ in both methodology and data sources. Our index combines large language models with a dictionary-based approach similar to that used in existing measures. However, while the previous index relies primarily on leading English-language newspapers, our index is constructed using news from local Taiwanese media, which are more sensitive to Taiwan-specific geopolitical risks. As a result, our index captures additional regional variation, particularly during 2011–2016, when South China Sea maritime confrontations between China and Vietnam heightened regional security tensions in June 2011, and when tensions escalated following China's deployment of an oil rig in disputed waters in May 2014. Both events are marked by vertical lines.

To further examine whether the informational content of the war index helps explain variation in asset-market outcomes, we study its comovement with financial variables that plausibly reflect geopolitical risk. Specifically, we regress daily stock returns for China-related stocks and non-China-related stocks on the lagged war index.[6] Table 2 reports the results. Across specifications, increases in the war index—corresponding to heightened cross-Strait tensions—are associated with significantly lower subsequent stock performance, measured by excess returns relative to the market index. This negative relationship is present for China-related stocks, as shown in Columns (1) to (4).Under the preferred specification in Column (4), which includes the full set of controls, a one–standard-deviation increase in war tension ($= 0.127$) is associated with a 0.95% decline in the daily excess return.

By contrast, in the placebo tests reported in Columns (5) to (8), geopolitical tension has no significant effect on the returns of non–China-related stocks, providing external validation for the relevance and informational content of the war index. Under the preferred specification in Column (8), which includes the full set of controls, we find evidence of investment spillovers from China-related stocks toward non–China-related stocks as war tensions rise, leading to a positive coefficient on the war index for non–China-related stocks. Table A.3 in the appendix presents a robustness check that focuses on the cyclical component of geopolitical risk—abstracting from the long-run trend in cross-Strait relations—and shows that the results for both groups of stocks remain qualitatively un-

[6] We identify 38 China-related stocks based on classifications from three online financial data providers, including Yahoo Finance.

Table 3: Effect of war risk on cross-border investment

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	$\frac{CaseNum_t^{TC}}{CaseNum_t^{TW}}$	$\frac{CaseNum_t^{CT}}{CaseNum_t^{WT}}$	$\frac{Invest_t^{TC}}{Invest_t^{TW}}$	$\frac{Invest_t^{CT}}{Invest_t^{WT}}$	$lnInvest_t^{TC}$	$lnInvest_t^{CT}$	$lnInvest_t^{TW}$	$lnInvest_t^{WT}$
War$_{t-1}$	-5.100*	-0.069**	-5.495**	-0.029	-2.197***	-1.211	2.509	1.215
	(-1.86)	(-2.45)	(-2.11)	(-0.39)	(-3.11)	(-0.40)	(1.55)	(0.94)
Constant	-0.623	-0.002	-1.003*	0.006	4.661***	6.873*	3.789	7.635**
	(-1.01)	(-0.26)	(-1.89)	(0.38)	(3.64)	(2.03)	(1.21)	(2.43)
Controls	✓	✓	✓	✓	✓	✓	✓	✓
N	25	15	25	15	25	15	25	25
Adj R^2	0.516	0.717	0.637	0.166	0.789	-0.027	0.652	0.263

Notes: $CaseNum_t^{ij}$ and $Invest_t^{ij}$ denote the number and value (in NTD) of annual cross-border investment from i to j, where $i, j \in \{T, C, W\}$ represent Taiwan, China, and the the total economies in the world. $lnInvest_t^{ij}$ denotes the log of cross-border investment amounts (in NTD) from i to j, where $i, j \in \{T, C, W\}$ represent Taiwan, China, and the rest of the world. Control variables include the lag dependent variable and the log of real GDP for Taiwan in percent change. t statistics are reported in parentheses. *$p < 0.1$, ** $p < 0.05$, and *** $p < 0.01$.

changed.

In addition to the stock market, the war index also predicts cross-Strait investment flows. Table 3 shows that heightened war risk is associated with a subsequent decline in cross-border investment from Taiwan to China, relative to investment from Taiwan to the rest of the world, controlling for lagged cross-border investment and Taiwan's real GDP to account for scale effects. Columns (1) and (2) measure this relative effect using the number of investment cases, while Columns (3) and (4) measure it using investment amounts. Columns (5) and (6) further show that bilateral investment amounts decline following increases in war tension. Finally, Columns (7) and (8) present placebo tests, showing no statistically significant effect on investment from Taiwan to the rest of the world.

4 Empirical Analysis

The relationship between fertility and unit housing prices is negative, as shown in Figure 1, with a correlation of -0.61. There exists a notable exception to this negative relationship in 2012, which corresponds to the Year of the Dragon—a zodiac year traditionally associated with elevated fertility in many East Asian societies.

To empirically identify the relationship between geopolitical risk and household decisions regarding fertility and housing investment, we merge the war-risk index described in Section 3 with comprehensive administrative tax return data from Taiwan. These data

provide a long-run panel that allows us to observe the timing of key household life-cycle decisions.

Guided by the model's comparative statics summarized in Section **??**, the empirical analysis is structured to map each theoretical prediction directly into a corresponding set of empirical tests. The model predicts that higher aggregate war risk reduces fertility and that this decline is amplified for households facing greater exposure, implying $\partial^2 n/\partial\lambda\partial\sigma < 0$; we test this reduced-form prediction using future birth outcomes in Tables 5 and A.2. When children and housing are complements, greater exposure to elevated war risk increases the demand for default-based insurance, raising feasible mortgage leverage on the intensive margin, $\partial^2 L/\partial\lambda\partial\sigma > 0$, which we examine in Table 6. Higher leverage, in turn, relaxes borrowing constraints only when accompanied by greater collateral, implying a positive reduced-form response of mortgaged housing investment, $\partial^2 h_m/\partial\lambda\partial\sigma > 0$, documented in Table 8.

4.1 Administrative Tax Return Data

Our analysis draws on comprehensive administrative tax return data covering the universe of individuals in Taiwan. These records contain rich demographic information—including age, gender, birth and death dates, family structure (e.g., identification of children), educational attainment, and registered filing addresses—as well as detailed measures of economic outcomes. The latter includes the level and composition of personal income, stock-level financial asset holdings, savings, ownership of durable goods (such as vehicles), and granular real-estate wealth evaluated at tax-based values, encompassing housing and land holdings together with outstanding mortgage balances.

We aggregate individual-level asset positions and housing transaction values to the household level using unique household identifiers that link multiple individuals within the same household. We exclude households in which the household head dies during the sample period. To isolate parents' fertility decisions in response to changes in their own economic conditions—without contamination from indirect income or wealth effects arising from co-residing parents—we restrict attention to two-generation households. We further exclude households recorded as divorced in a given year, as fertility decisions in Taiwan are typically made within legal marriages.

Owing to differences in the timing of household formation, the resulting dataset is an unbalanced household-year panel spanning 2008 to 2020, with housing transaction records available from 2008 to 2017. The final unbalanced panel contains 10,755,242 households, of which 7,478,068 are married households that remain married throughout the

Table 4: Summary Statistics

	N	Mean	Std.	P_1	P_{25}	P_{50}	P_{75}	P_{99}
$\overline{SS}_i$	$10,755,242$	0.304	0.386	0	0	0	0.577	1
$\overline{NumChild}_i$	$10,755,242$	0.878	0.956	0	0	0.602	1.636	3.308
$\overline{Wealth}_i$	$10,755,242$	8.712	60.3	0	0.692	3.059	8.419	80.1
$\overline{IlliquidWealth}_i$	$10,755,242$	5.691	290	0	0.090	1.948	5.318	570
$\overline{Mortgage}_i$	$10,755,242$	1.012	110	0	0	0	0.235	137
$\overline{MaleIncome}_i$	$8,846,531$	0.357	0.780	0	0	0.087	0.504	2.731
$\overline{FemaleIncome}_i$	$9,386,779$	0.200	0.390	0	0	0.013	0.282	1.535
$\overline{MaleAge}_i$	$8,846,531$	52.218	15.258	22.5	40.5	51	62	87.5
$\overline{FemaleAge}_i$	$9,386,779$	51.415	15.160	23	39.5	50.5	62	87
$HousePrice_i$	$420,144$	13.4	30.0	1.0	5.0	8.5	14.8	82.7

Note: This table reports the summary statistics of household-level time averages of annual variables, denoted by $\overline{x}_i$, where $x_{i,t}$ is observed over the period 2008–2020. P_q denotes the value at the qth percentile, where $q \in \{1, 25, 50, 75, 99\}$. $\overline{Wealth}_i$ is defined as the time average of the sum of stock holdings, bond holdings, cash, vehicles, land holdings, and housing, measured in million NTD. $\overline{IlliquidWealth}_i$ is defined as the time average of household i's land holdings and housing, measured in million NTD. $\overline{Mortgage}_i$ denotes the time average of household i's outstanding mortgage balance, measured in million NTD. $\overline{MaleIncome}_i$ and $\overline{FemaleIncome}_i$ are the time averages of annual salary income of the male and female household heads (or spouses), respectively, measured in million NTD. $\overline{MaleAge}_i$ and $\overline{FemaleAge}_i$ are the time averages of the ages of the male and female household heads (or spouses), respectively. $HousePrice_{i,t}$ denotes the housing transaction price household i purchased in year t, measured in million NTD.

sample period. The data include 2,440,215 housing units with available tax-based valuations, of which 1,811,443 are owned by married households. Table 4 reports summary statistics for the key variables used in the regression analysis.

We complement these data with transaction-level information from deed tax records, which allow us to identify precise housing transaction dates and to link buyers and sellers using unique personal identifiers. In addition, property tax files provide rich information on property characteristics, including location, size, floor level, structural attributes, usage type, assessed property holding taxes, and an indicator for owner-occupied status.

Finally, we incorporate publicly available official housing transaction records that cover the universe of housing transactions over the sample period. These data report transaction prices and dates, along with detailed property attributes—such as location, size, floor, structure, and usage—allowing us to accurately merge housing price information with the administrative microdata. In total, 420,144 transactions are successfully matched to the administrative tax return data, of which 301,187 are purchases by married households,

and 137,512 involve outstanding mortgages.

4.2 War Exposure and Future Fertility

This subsection studies how geopolitical risk affects fertility decisions at the household level. We focus on births in the subsequent period, which allows us to capture forward-looking fertility responses rather than contemporaneous timing effects. The analysis is guided by the model's prediction that fertility responds both to the aggregate level of war risk and to heterogeneity in household exposure to war-related losses.

In the model, fertility choices depend on two key objects. The first is the aggregate arrival rate of war, denoted by λ, which unambiguously reduces fertility. The second is household-level exposure to war-related losses, denoted by σ, which governs how strongly a household is affected conditional on a conflict occurring. In the data, we map λ to our time-varying war-risk index constructed from news sentiment, which captures changes in perceived geopolitical tail risk over time. We proxy for household exposure σ using the share of sons in the household. This mapping is motivated by the institutional feature that sons, but not daughters, are subject to compulsory military service in Taiwan, implying that households with more sons face greater potential losses in the event of war. The model predicts a negative interaction between war risk and son share in fertility outcomes.

To quantify the effect of geopolitical risk on fertility and its interaction with household exposure, we estimate the following baseline specification:

$$\text{Birth}_{i,t+1} = \beta_1 \,\text{SS}_{i,t} + \beta_2 \left(\text{SS}_{i,t} \times \text{War}_{i,t}\right) + \mathbf{X}'_{i,t}\boldsymbol{\gamma} + \alpha_i + \delta_t + \lambda_{l(i)} + \phi_{a^m_{i,t}} + \psi_{a^f_{i,t}} + \varepsilon_{i,t}, \quad (10)$$

where $\text{Birth}_{i,t+1}$ is an indicator equal to one if household i has a newborn in period $t+1$,[7] corresponding to fertility choice n in the model. The variable $\text{War}_{i,t}$ is our news-based war-risk index, which captures time-varying perceived geopolitical risk and maps directly to the model's disaster arrival rate λ. Household exposure to war-related losses is proxied by $\text{SS}_{i,t}$, the share of sons in the household at time t, corresponding to the exposure parameter σ in the model.

The vector $\mathbf{X}_{i,t}$ includes household-level controls such as log housing wealth, the number of existing children, and log male and female income. The specification further includes household fixed effects α_i, year fixed effects δ_t, location (township) fixed effects $\lambda_{l(i)}$, and male- and female-age fixed effects $\phi_{a^m_{i,t}}$ and $\psi_{a^f_{i,t}}$. Standard errors are clustered

[7] Note that war risk is measured in year t, while fertility and housing outcomes are measured in $t+1$.

at the location level.

Identification comes from within-household variation in exposure interacted with aggregate fluctuations in war risk. The coefficient of interest, β_2, captures how an increase in geopolitical risk differentially affects future fertility for households with greater exposure. The model predicts $\beta_2 < 0$: higher war risk reduces fertility more strongly for households facing larger expected losses in the event of conflict.

4.2.1 Result

Table 5 reports the main results. Across all specifications, the interaction between son share and war risk is negative and highly statistically significant. This implies that increases in war risk lead to a larger decline in future fertility for households with greater exposure. The result is robust to the inclusion of year fixed effects, age fixed effects, and location fixed effects, and the magnitude of the interaction remains stable across specifications.

The estimated coefficients are economically meaningful. Using the preferred specification with the full set of controls, a one-standard deviaion increase in the war-risk index ($= 0.127$) reduces the probability of a birth in the next period by approximately 0.37 percentage points for a household with all sons relative to a household with all daughters. Given the baseline annual birth probability in the sample, this represents a sizable proportional decline in fertility for highly exposed households. These findings align closely with the model's predictions. An increase in aggregate war risk (λ) reduces fertility for all households, while greater exposure (σ) strengthens this effect. The negative interaction between war risk and exposure provides direct micro-level evidence that households internalize geopolitical risk when making fertility decisions, adjusting more aggressively when the expected losses from conflict are higher.

The coefficient on son share itself is also negative and precisely estimated, reflecting lower baseline fertility among households with greater potential exposure even absent changes in war risk. An alternative interpretation is the presence of son preference, which is commonly observed in East Asian societies. The coefficient on the number of existing children is negative and stable across specifications, capturing standard parity effects in fertility: households with more existing children are less likely to have an additional birth in the subsequent period .

The effect of war tension on fertility is further highlighted in Table A.2 in the appendix, which restricts the sample to households that are more likely to have children based on the criterion that the female household head (or spouse) is below age 45. The effect of war tension on fertility decisions, conditional on the existing son share, is even stronger

Table 5: Effect of war risk on future fertility

	(1)	(2)	(3)	(4)
	$\text{Birth}_{i,t+1}$	$\text{Birth}_{i,t+1}$	$\text{Birth}_{i,t+1}$	$\text{Birth}_{i,t+1}$
$\text{SS}_{i,t} \times \text{War}_t$	-0.0744^{***}	-0.0416^{***}	-0.0286^{***}	-0.0288^{***}
	(-30.17)	(-22.13)	(-22.18)	(-22.23)
$\text{SS}_{i,t}$	-0.0196^{***}	-0.0098^{***}	-0.0122^{***}	-0.0124^{***}
	(-38.05)	(-22.83)	(-27.49)	(-28.04)
$\text{NumChild}_{i,t}$	-0.0313^{***}	-0.0330^{***}	-0.0319^{***}	-0.0321^{***}
	(-33.39)	(-35.34)	(-35.34)	(-35.56)
Constant	0.0288^{***}	0.0030^{***}	0.0082^{***}	0.0086^{***}
	(30.69)	(4.44)	(11.19)	(11.78)
Other Controls	✓	✓	✓	✓
Household FE	✓	✓	✓	✓
Year FE		✓	✓	✓
Age FEs			✓	✓
Location FE				✓
N	73,972,395	73,972,395	54,208,259	54,208,259
Adj. R^2	0.101	0.104	0.155	0.155

Note: Other controls include log household wealth, male income, and female income in year t. Age FEs denote fixed effects for the ages of the male and female household heads (or spouses). Location FE denotes city-level fixed effects based on the household's location of residence. t-statistics are in parentheses. Robust standard errors are clustered by city. $^{***}p < 0.01, ^{**}p < 0.05, ^{*}p < 0.1$.

in this subsample. Specifically, under the preferred specification with the full set of controls, a one-unit increase in the war-risk index reduces the probability of a birth in the subsequent period by approximately 6.11 percentage points for a household with all sons relative to a household with all daughters.

4.3 War Exposure, Mortgage Leverage, and Housing Investment

This subsection studies how geopolitical risk affects housing investment through mortgage borrowing. The analysis is guided by the model's prediction that war risk does not directly increase housing demand, but instead operates through defaultable mortgage leverage, which provides implicit insurance against war-related losses. As in the fertility analysis, the key forces are the aggregate level of war risk and heterogeneity in household

exposure.

Crucially, the model implies that the response of housing investment depends on the interaction between child-related losses and housing losses in adverse states. When children and housing are complements, higher war risk combined with greater exposure increases the marginal value of default-based insurance, leading households to take on more mortgage leverage and, through this channel, to expand housing investment. By contrast, if children and housing are substitutes, this insurance channel is muted and war risk does not generate a positive housing response. In either case, a key implication of the model is that war risk and exposure should not raise housing investment absent changes in mortgage borrowing.

Table 6 provides a reduced-form benchmark for this mechanism. The interaction $\text{SS}_{i,t} \times \text{War}_t$ is positive and highly significant across specifications, indicating that, when aggregate war risk rises, households with greater exposure increase housing investment. The effect is robust after adding year and location fixed effects. The interaction remains positive across financing types, but is quantitatively larger among households with positive mortgage balances.

Table 7 provides a similar reduced-form result. The interaction $\text{SS}_{i,t} \times \text{War}_t$ is positive and highly significant across specifications, indicating that, when aggregate war risk rises, households with a higher son share increase mortgage borrowing. This result remains robust after controlling for year and location fixed effects. Decomposing mortgage responses into extensive and intensive margins further shows that the effect operates along both dimensions: exposed households become more likely to carry a positive mortgage balance and, conditional on borrowing, increase the size of their housing loans. We also account for the possibility that larger households may borrow more mechanically. Table A.4 shows that the result remains robust when mortgage borrowing is measured on a per-household-member basis.

Although the reduced-form estimates in Table 6 and Table 7 are consistent with the model, they do not by themselves identify the mortgage channel. In particular, they do not show that higher mortgage uptake causes households to purchase more housing as insurance against war risk, since housing investment and borrowing decisions are jointly determined. To isolate the role of leverage, we therefore implement a three-stage empirical design that separately models (i) selection into mortgage borrowing, (ii) leverage conditional on borrowing, and (iii) housing investment conditional on leverage.

In the first stage, we model selection into mortgage borrowing:

$$D_{i,t} = \mathbb{1}\{\text{Mortgage}_{i,t} > 0\}, \qquad \mathbb{E}[D_{i,t} \mid \cdot] = F\big(\pi_1 SS_{i,t} + \pi_2 SS_{i,t} \times \text{War}_{i,t} + \mathbf{X}_{i,t-1} + \text{FE}\big),$$

Table 6: Effect of war risk on housing investment

	(1)	(2)	(3)	(4)	(5)
	$\text{HousingInv}_{i,t}$	$\text{HousingInv}_{i,t}$	$\text{HousingInv}_{i,t}$	$\text{HousingInv}_{i,t}$	$\text{HousingInv}_{i,t}$
Sample	Full	Full	Full	Mortgage $= 0$	Mortgage > 0
$\text{SS}_{i,t} \times \text{War}_t$	0.1444^{***}	0.0942^{***}	0.0941^{***}	0.0831^{***}	0.1623^{***}
	(15.71)	(13.07)	(13.09)	(13.48)	(4.58)
$\text{SS}_{i,t}$	0.0516^{***}	0.0390^{***}	0.0387^{***}	0.0359^{***}	0.0459^{***}
	(17.23)	(15.20)	(15.28)	(14.51)	(4.69)
War_t	0.3717^{***}				
	(19.19)				
$\text{NumChild}_{i,t}$	0.0270^{***}	0.0297^{***}	0.0296^{***}	0.0261^{***}	0.0665^{***}
	(21.80)	(22.57)	(22.89)	(24.05)	(13.91)
$\text{lnWealth}_{i,t-1}$	-0.0209^{***}	-0.0186^{***}	-0.0186^{***}	-0.011^{***}	-0.1087^{***}
	(-14.76)	(-13.56)	(-13.56)	(-14.28)	(-12.49)
$\text{lnMaleIncome}_{i,t-1}$	0.0002^{***}	0.0002^{**}	0.0002^{**}	0.0002^{**}	-0.0012^{***}
	(2.80)	(2.93)	(2.91)	(5.40)	(-3.34)
$\text{lnFemaleIncome}_{i,t-1}$	0.0004^{***}	0.0002^{**}	0.0002^{**}	-0.0009^{***}	0.0002^{**}
	(6.85)	(4.40)	(4.38)	(5.40)	(-2.70)
Constant	0.4331^{***}	0.3112^{***}	0.3114^{***}	0.181^{***}	1.7909^{***}
	(17.91)	(15.70)	(15.70)	(16.77)	(13.61)
Other Controls	✓	✓	✓	✓	✓
Household FE	✓	✓	✓	✓	✓
Year FE		✓	✓	✓	✓
Location FE			✓	✓	✓
N	54,705,560	54,705,560	54,705,560	47,547,121	6,489,620
Adj. R^2	0.010	0.012	0.012	0.013	0.0378

Note: $\text{HousingInv}_{i,t}$ denotes the log of the transaction price of household i's housing investment in year t. Column (4) restricts the sample to housing investments paid entirely in cash. Column (5) restricts the sample to households with positive mortgage balances, where default-based insurance is feasible. Other controls include the ages of the male and female household heads (or spouses). Location FE denotes city-level fixed effects based on the household's location of residence. War_t is ommited when the year fixed effect is included. t-statistics are in parentheses. Robust standard errors are clustered by city. $^{***}p < 0.01, ^{**}p < 0.05, ^{*}p < 0.1$.

which we estimate using Poisson pseudo-likelihood regression with multiple levels of fixed effects. From this stage, we construct a control-function residual,

$$\widehat{\mu}_{i,t} = \frac{D_{i,t} - \widehat{\mathbb{E}}[D_{i,t} \mid \cdot]}{\sqrt{\widehat{\mathbb{E}}[D_{i,t} \mid \cdot]}},$$

Table 7: Effect of war risk on mortgage borrowing

	(1) $\text{HouseLoan}_{i,t}$	(2) $\text{HouseLoan}_{i,t}$	(3) $\text{HouseLoan}_{i,t}$	(4) $\text{HouseLoan}_{i,t}$	(5) $\text{HouseLoan}_{i,t}$
Sample	Full	Full	Full	Extensive Margin	Intensive Margin
$\text{SS}_{i,t} \times \text{War}_t$	2.388^{***}	0.680^{***}	0.678^{***}	0.0526^{***}	0.199^{***}
	(24.58)	(6.76)	(6.73)	(8.09)	(8.32)
$\text{SS}_{i,t}$	0.726^{***}	0.341^{***}	0.302^{***}	0.0218^{***}	0.0333^{***}
	(28.20)	(11.08)	(10.77)	(12.26)	(5.68)
War_t	22.94^{***}				
	(45.70)				
$\text{NumChild}_{i,t}$	0.048^{***}	0.123^{***}	0.119^{***}	0.00849^{***}	-0.0193^{***}
	(7.16)	(24.57)	(13.26)	(26.08)	(-13.84)
$\text{lnWealth}_{i,t-1}$	0.362^{***}	0.399^{***}	0.398^{***}	0.0247^{***}	0.176^{***}
	(35.17)	(55.05)	(54.63)	(57.72)	(80.36)
$\text{lnMaleIncome}_{i,t-1}$	-0.00303^{***}	0.00304^{***}	0.00294^{***}	0.000220^{***}	-0.000855^{***}
	(-5.96)	(9.27)	(8.99)	(10.36)	(-6.27)
$\text{lnFemaleIncome}_{i,t-1}$	0.000589	0.00462^{***}	0.00459^{***}	0.000301^{***}	-0.000975^{***}
	(1.39)	(13.68)	(13.49)	(10.31)	(-8.33)
Constant	2.54^{***}	-3.444^{***}	-3.449^{***}	-2.02^{***}	12.441^{***}
	(18.77)	(-31.09)	(-30.41)	(-30.72)	(370.15)
Other Controls	✓	✓	✓	✓	✓
Household FE	✓	✓	✓	✓	✓
Year FE		✓	✓	✓	✓
Location FE			✓	✓	✓
N	54,705,560	54,705,560	54,705,560	54,705,560	10,167,153
Adj. R^2	0.508	0.569	0.569	0.565	0.7598

Note: $\text{HouseLoan}_{i,t}$ denotes the amount of housing loan for household i in year t. Column (4) reports the extensive margin, where the dependent variable is an indicator equal to one if the household has a positive mortgage balance in year t. Column (5) reports the intensive margin, conditional on having a positive housing loan. Other controls include the ages of the male and female household heads (or spouses). Location FE denotes city-level fixed effects based on the household's location of residence. War_t is omitted when year fixed effects are included. t-statistics are in parentheses. Robust standard errors are clustered by city. $^{***}p < 0.01, ^{**}p < 0.05, ^{*}p < 0.1$.

to account for selection into mortgage borrowing.

In the second stage, we estimate mortgage leverage among borrowers:

$$\log(\text{Mortgage})_{i,t} = \alpha_1(SS_{i,t} \times \text{War}_{i,t}) + \alpha_2\widehat{\mu}_{i,t} + \mathbf{X}_{i,t-1} + \text{FE}, \qquad \text{Mortgage}_{i,t} > 0.$$

In the third stage, we estimate housing investment using instrumented mortgage lever-

Table 8: Mortgage Leverage and Housing Investment under War Risk

	(1) PPML $\log(HV)_{i,t}$	(2) Linear $\log(HV)_{i,t}$
$\log(\widehat{Mortgage})_{i,t}$	0.0717^{***}	0.0958^{***}
	(12.58)	(7.30)
$SS_{i,t}$	-0.00965	-0.00511
	(-1.06)	(-0.51)
$\log(\text{Wealth})_{i,t-1}$	0.149^{***}	0.147^{***}
	(35.59)	(34.28)
$NumChildren_{i,t}$	-0.0374^{***}	-0.0348^{***}
	(-8.93)	(-6.90)
Other Controls	✓	✓
Age FE	✓	✓
Year FE	✓	✓
Location FE	✓	✓
N	82,882	82,882
Kleibergen–Paap	242.13	83.61

$\text{HV}_{i,t}$ denotes housing value of i's housing investment in year t. Other controls include the ages of the male and female household heads (or spouses). Location FE denotes city-level fixed effects based on the household's location of residence. War_t is ommited when the year fixed effect is included. t-statistics are in parentheses. Robust standard errors are clustered by city. $^{***}p < 0.01, ^{**}p < 0.05, ^{*}p < 0.1$.

age:

$$\log(\text{HV})_{i,t} = \beta \, \log(\widehat{\text{Mortgage}})_{i,t} + \rho \, \widehat{\mu}_{i,t} + \mathbf{X}_{i,t-1} + \text{FE},$$

where mortgage leverage is instrumented using the interaction between son share and war risk. Standard errors are clustered at the location level throughout.

4.3.1 Result

Table 8 reports the results from the final stage, which links mortgage leverage to housing investment using either PPML or linear model. Across specifications, the coefficient on instrumented mortgage leverage is positive and precisely estimated. In the preferred specification with PPML, a one-percent increase in mortgage borrowing raises housing transaction value by approximately 0.10 percent.

Across specifications, instrumented mortgage leverage has a positive and precisely estimated effect on housing investment. In the preferred PPML specification, a one-percent

increase in mortgage borrowing raises transaction values by approximately 0.10 percent. The first-stage statistics indicate a strong instrument.

To further assess the mechanism, we return to the subsample comparison in Table 6. While exposed households increase housing investment when war risk rises even among cash purchases, the response is substantially larger when mortgage borrowing is present. When we restrict the sample to households without borrowing and estimate the fertility-style specification for housing investment, the interaction between son share and war risk becomes economically small and statistically insignificant once full controls are included.

Taken together, the evidence suggests that geopolitical risk affects housing investment primarily through mortgage leverage. The reduced-form increase in housing demand among exposed households is amplified when defaultable debt is available, consistent with the mortgage–insurance channel emphasized in the model.

5 Conclusion

This paper studies a macroeconomic puzzle observed in several economies exposed to heightened geopolitical tensions: fertility declines sharply at the same time that housing investment and mortgage leverage increase. Standard explanations based on housing affordability, wealth effects, or labor-market incentives are unable to account for this joint pattern, as they typically predict that higher risk or higher housing costs should depress both fertility and housing demand.

We propose a new mechanism based on differences in the financial structure of household decisions. Fertility is an irreversible, equity-financed investment, whereas housing is typically financed with long-term, defaultable mortgage debt. In the presence of geopolitical risk, mortgage default provides implicit insurance against adverse states, allowing households to shift part of the downside risk to lenders. As a result, rising war risk reduces fertility while simultaneously increasing demand for leveraged housing, conditional on the availability of defaultable debt.

To formalize this mechanism, we develop a general-equilibrium model with a competitive-search mortgage market and endogenous default. The model delivers sharp, testable predictions that differ across mortgaged and unmortgaged households and across extensive and intensive margins of borrowing. In particular, housing investment responds to geopolitical risk only through mortgage leverage and not through direct changes in preferences for housing.

Empirically, we construct a novel, AI-based measure of China–Taiwan war risk using

textual analysis of newspaper coverage and combine it with rich administrative data on household demographics, income, mortgages, and transaction-level housing outcomes. Guided by the model, we exploit heterogeneity in household exposure to war risk, proxied by the share of sons subject to military service. We find that higher war risk reduces future fertility while increasing mortgage leverage and housing investment for exposed households, with no comparable effects among households without mortgages. These findings are consistent with the model's risk-shifting mechanism and provide micro-level evidence that households respond to geopolitical risk through financial channels.

Overall, the paper highlights the importance of financial structure for understanding how geopolitical risk shapes demographic and housing outcomes. By linking fertility decline, housing booms, and war risk through defaultable debt, our results suggest that financial markets play a central role in mediating the long-run economic consequences of geopolitical uncertainty—beyond the effects of prices or preferences alone.

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A Construction of the War Index

A.1 Data Collection and Preprocessing

We collect the universe of Mandarin-language news articles from the Central News Agency (CNA) between January 1, 1991 and July 31, 2025. Articles are scraped using the keyword "Zhongguo" (China), yielding the raw corpus.

To restrict attention to Taiwan-related and cross-Strait content, we retain an article if it contains at least one keyword referring to Taiwan, Taiwanese political institutions, major cities, political actors, cross-Strait relations, sovereignty, diplomacy, or military affairs. This filtering step yields 749,408 articles.

Each article is then cleaned by removing punctuation, URLs, HTML tags, and extraneous whitespace. We segment text into word tokens using the `jieba` Mandarin segmentation package and remove standard stopwords. The processed corpus is stored as a structured dataset containing article identifiers, timestamps, and cleaned text.

A.2 Ground Truth Labeling and Sentiment Categories

To discipline sentiment classification, we randomly sample 4,000 articles and manually assign one of four sentiment labels:

-
- 1 (**Tension**): content indicating heightened military risk, conflict, or increased probability of war;
- -1 (**Peace**): content indicating cooperation, dialogue, or de-escalation of cross-Strait tensions;
- 0 (**Neutral**): Taiwan–China–related reporting without a clear directional implication for conflict risk;
- n (**Irrelevant**): content unrelated to Taiwan–China relations.

Labeling is conducted from a Taiwan-centric perspective, with particular emphasis on military developments, strategic posturing, sovereignty-related events, and diplomatic interactions. These manually labeled articles form the ground truth used to guide subsequent automated classification.

A.3 Large Language Model Classification

Using the manually labeled sample as training data, we classify the full corpus of articles with a large language model (Gemini). To ensure consistency with human judgment and transparency, model-based classification is supplemented by a set of rule-based priorities. Articles indicating military exercises, incursions, weapons deployment, or explicit conflict threats are classified as tension, while articles emphasizing negotiations, cooperation, or de-escalation are classified as peace. Neutral articles are retained with a zero score, and irrelevant articles are excluded from index construction.

A.4 Index Construction

We transform the article-level sentiment classifications into a continuous war-risk index. Each tension article is assigned a value of $+1$, each peace article a value of -1, and neutral articles a value of 0. For each article, we then compute a normalized sentiment score based on the relative intensity of tension versus peace language, yielding an index bounded between -1 and 1. Higher values correspond to greater perceived geopolitical risk. The index can be aggregated to daily or monthly frequency depending on the application.

A.5 Interpretation and Validation

The resulting index captures time-varying *perceived* geopolitical tail risk rather than realized conflict. It exhibits clear spikes around major geopolitical events, such as the 1996 Taiwan Strait Crisis and the large-scale military exercises following high-profile diplomatic visits in 2022. In the main text, we further validate the index by showing that increases in war risk are associated with declines in China-related stock prices, greater downside price asymmetry, and contractions in cross-border investment flows, consistent with the index capturing information that is priced by financial markets. We discuss this validation in the main text.

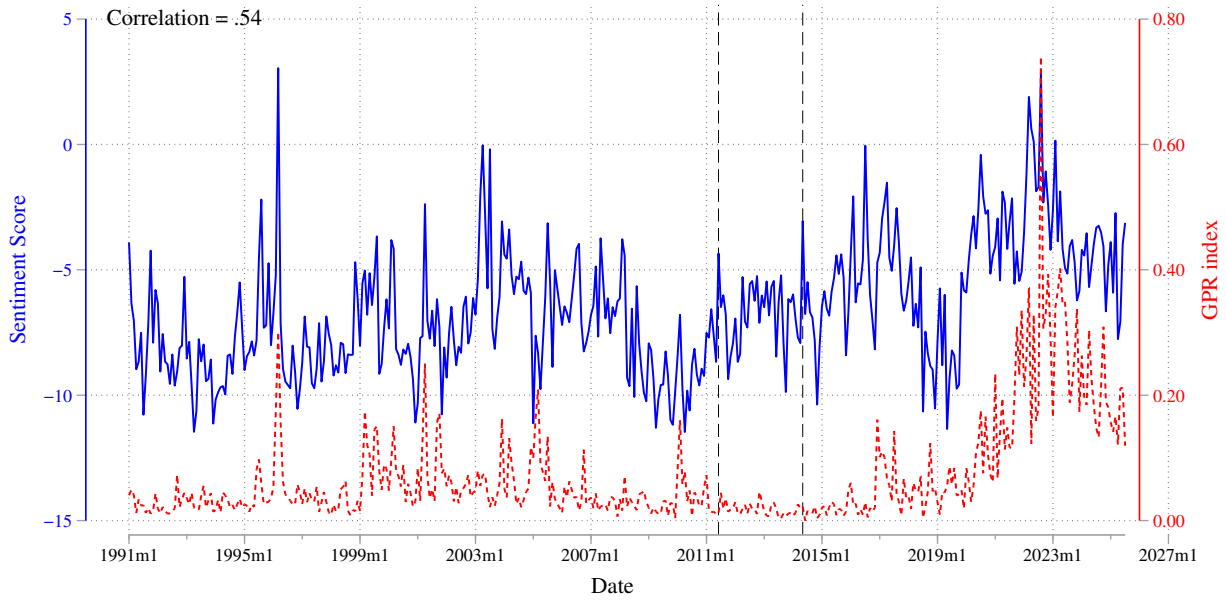
A.6 Illustrative Examples

Table A.1 provides representative examples of articles classified as peace and tension. Articles classified as peace emphasize dialogue, cooperation, and de-escalation, while articles classified as tension highlight military incursions, surveillance activity, and strategic escalation.

Table A.1: Illustrative Examples of News Articles Classified by War-Risk Sentiment

Peace Article (2002–02–20; Score = -0.79**)**	**Tension Article (2021–02–02; Score =** 0.96**)**
"…the international situation …centered on peaceful development …interaction and cooperation …made President Bush's visit possible …"	"…a Y-8 technical reconnaissance aircraft …intruded into Taiwan's southwestern ADIZ …tracked and monitored with air defense missiles …"
"…the key lies in both sides of the Strait sitting down to talk …finding solutions acceptable to both …"	"…a PLA Y-8 …harassed Taiwan in the evening …"
"…as long as both sides …do not challenge 'one China' …return to the '1992 Consensus' …a return to the negotiating table will not be far off …"	"…U.S. forces …join missions to contain China …strengthen the containment posture …"

Figure A.1: Monthly geopolitical risk in Taiwan



Note: The solid line shows the news-based war index presented in Figure 2 and corresponds to the left vertical axis. The dashed line corresponds to the right vertical axis and represents the geopolitical risk index constructed by Caldara and Iacoviello (2022) and Caldara et al. (2025), which is downloaded from https://www.matteoiacoviello.com/gpr.htm on January 20, 2026. The vertical dashed lines denote selected events that increase the sentiment score in June 2011 and May 2014.

A.7 Figures and Tables

Table A.2: Effect of war risk on future fertility at female age below 45

	(1)	(2)	(3)	(4)
	$\text{Birth}_{i,t+1}$	$\text{Birth}_{i,t+1}$	$\text{Birth}_{i,t+1}$	$\text{Birth}_{i,t+1}$
$\text{SS}_{i,t} \times \text{War}_t$	-0.1770^{***}	-0.1700^{***}	-0.0613^{***}	-0.0611^{***}
	(-47.44)	(-34.68)	(-12.35)	(-12.33)
$\text{SS}_{i,t}$	-0.0126^{***}	-0.0114^{***}	-0.0034^{***}	-0.0037^{***}
	(-10.17)	(-8.33)	(-2.19)	(-2.42)
$\text{NumChild}_{i,t}$	-0.112^{***}	-0.109^{***}	-0.111^{***}	-0.112^{***}
	(-66.40)	(-64.28)	(-57.44)	(-57.50)
Constant	0.0346^{***}	-0.0119^{***}	0.0650^{***}	0.0662^{***}
	(16.21)	(-5.78)	(21.82)	(22.41)
Other Controls	✓	✓	✓	✓
Household FE	✓	✓	✓	✓
Year FE		✓	✓	✓
Age FE			✓	✓
Location FE				✓
N	19,532,410	19,532,410	13,338,921	13,338,921
Adj. R^2	0.080	0.082	0.119	0.119

Note: Other controls include log household wealth, male income, and female income in year t. Age FE denotes fixed effects for the ages of the male and female household heads (or spouses). Location FE denotes city-level fixed effects based on the household's location of residence. t-statistics are in parentheses. Robust standard errors are clustered by city. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table A.3: Effect of cyclical component of war risk on the stock performance

	China-related stocks				non-China-related stocks			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	$Return_t^s$	$Return_t^s$	$Return_t^s$	$Return_t^s$	$Return_t^s$	$Return_t^s$	$Return_t^s$	$Return_t^s$
War_{t-1}^c	-0.0008	-0.0005	-0.0813**	-0.0749**	0.0326	0.0169	0.1147***	0.0973*
	(-0.32)	(-0.21)	(-3.39)	(-3.84)	(0.79)	(0.66)	(4.62)	(2.35)
Constant	0.0010***	-0.0053**	-0.0064**	-0.0155***	0.0080	0.0318	0.0358	0.0508
	(6.57)	(-3.45)	(-3.93)	(-6.40)	(1.70)	(1.13)	(1.31)	(1.20)
Other controls		✓	✓	✓		✓	✓	✓
Time FE			✓	✓			✓	✓
Stock FE				✓				✓
N	198,577	196,468	196,463	196,463	7,328,191	7,058,780	7,058,775	7,058,775
Adj R^2	0.000	0.001	0.171	0.173	0.000	0.000	0.001	0.001

Notes: $Return_t^s$ denotes the excess return of stock s on date t, measured as the daily growth rate of the close price of stock s minus the daily growth rate of the Taiwan stock market index. War_{t-1}^c denotes denotes the cyclical component of War_{t-1} obtained using the HP filter. Other controls include daily stock-level turnover and trading volume. Standard errors are clustered at the stock–day-of-week level, as the third Wednesday of each month in Taiwan is the typical option settlement day and is associated with heightened stock price volatility. t statistics are reported in parentheses. *$p < 0.1$, ** $p < 0.05$, and *** $p < 0.01$.

Table A.4: Effect of war risk on mortgage borrowing per household member

	(1) HouseLoanPerCap$_{i,t}$	(2) HouseLoanPerCap$_{i,t}$	(3) HouseLoanPerCap$_{i,t}$	(4) HouseLoanPerCap$_{i,t}$
Sample	Full	Full	Full	Intensive Margin
SS$_{i,t}$ × War$_t$	1.995^{***}	0.396^{***}	0.396^{***}	0.173^{***}
	(22.12)	(4.21)	(4.20)	(6.74)
SS$_{i,t}$	0.593^{***}	0.274^{***}	0.267^{***}	-0.361^{***}
	(22.08)	(9.69)	(9.24)	(-54.22)
War$_t$	21.30^{***}			
	(45.40)			
lnWealth$_{i,t-1}$	0.338^{***}	0.373^{***}	0.372^{***}	0.168^{***}
	(35.28)	(55.07)	(54.53)	(77.56)
lnMaleIncome$_{i,t-1}$	-0.00287^{***}	0.00277^{***}	0.00267^{***}	-0.00105^{***}
	(-6.07)	(9.07)	(8.78)	(-7.66)
lnFemaleIncome$_{i,t-1}$	0.00675^{*}	0.00439^{***}	0.00436^{***}	-0.000348^{***}
	(1.72)	(13.96)	(13.76)	(-3.22)
Constant	2.358^{***}	-3.135^{***}	-3.114^{***}	11.568^{***}
	(19.17)	(-30.41)	(-29.79)	(347.11)
Other Controls	✓	✓	✓	✓
Household FE	✓	✓	✓	✓
Year FE		✓	✓	✓
Location FE			✓	✓
N	54,705,560	54,705,560	54,705,560	10,167,153
Adj. R^2	0.508	0.569	0.570	0.7733

Note: HouseLoanPerCap$_{i,t}$ denotes housing loan per capita for household i in year t. Column (4) reports the intensive margin, where the sample is restricted to households with a positive housing loan. Other controls include the ages of the male and female household heads (or spouses). Location FE denotes city-level fixed effects based on the household's location of residence. War$_t$ is omitted when year fixed effects are included. t-statistics are in parentheses. Robust standard errors are clustered by city. $^{***}p < 0.01, ^{**}p < 0.05, ^{*}p < 0.1$.

A.8 Omitted derivations and proofs

A.8.1 Proof of Lemma ??

Since there are only two events, namely disaster and loan maturity, the optimal default time is contingent on the timing of the disaster and loan maturity and takes the following form:

$$\tau^* = \begin{cases} \tau_0 & \text{before the disaster and loan maturity} \\ T + \tau_d & \text{after the disaster but before loan maturity} \\ \infty & \text{after loan maturity} \end{cases},$$

where τ_0 and τ_d are constants to be determined.

Solving the buyer's default decision backward, consider the subgame after the disaster has happened at $t = T$ but before the loan matures. Denote $t_\mu = T_\mu - T$ as the distance to the maturity date, where t_μ follows the exponential distribution with parameter $r\mu$. The buyer's continuation value at $t = T$ from the strategy of defaulting at $T + \tau_d$ is given by:

$$W(\tau_d) \equiv \int r\mu e^{-r\mu t_\mu} \left\{ \begin{array}{l} \int_0^{t_\mu \wedge \tau_d} re^{-rt} (h - d - b)\, dt \\ +e^{-r\tau_d} 1_{\tau_d \leq t_\mu} \left[-f + \{h - d - B\}_+ \right] \\ +e^{-rt_\mu} 1_{\tau_d > t_\mu} (h - d) \end{array} \right\} \mathrm{d}t_\mu,$$

where we have made use of the fact that the housing price after the disaster is given by its fundamental value $(h-d)/r$. The value function satisfies the following Hamilton–Jacobi–Bellman (HJB) equation:

$$\frac{d}{d\tau} W(\tau) = r(h - d - b) + r\mu [h - d - W(\tau)] - rW(\tau). \tag{1}$$

The HJB states that the marginal value of postponing default (the left-hand side) is equal to the sum of the flow of asset return (post-disaster) net of the loan repayment, $r(h - d - b)$, and the expected gain of paying off the loan, $r\mu [h - d - W(\tau)]$, minus the cost of discounting, $rW(\tau)$. The case $\tau = 0$ represents that the buyer defaults immediately, which gives the following boundary condition to (1):

$$W(0) = -f + \{h - d - B\}_+.$$

Using the boundary condition, the HJB equation has the following solution:

$$W(\tau) = \left[1 - e^{-r(1+\mu)\tau}\right] (h - d - B) + e^{-r(1+\mu)\tau} \left[-f + \{h - d - B\}_+ \right]. \tag{2}$$

The first term is the present value of never default, and the second term is the value of immediate default. Thus, $W(\tau)$ is the average weighing the latter by the factor $\exp[-r(1+\mu)\tau]$. The optimal stopping time to default is $\tau_d = \infty$ if the first term is weakly larger and $\tau_d = 0$ if the second term is strictly larger, imposing (without loss of generality) a tie-breaking rule that a borrower chooses to repay when they are indifferent. The first term is a downward-sloping curve in B; the second term is a downward-sloping curve in B with the same slope for all $B < h - d$ but then flat at $-f$ for all $B \geq h - d$. Thus, the first term cuts the second term from above in the flat region of the second term at the single crossing point $B = B^{\text{safe}}$, where:

$$B^{\text{safe}} = h - d + f. \tag{3}$$

The optimal stopping time of default thus follows the bang-bang rule:

$$\tau_d \equiv \arg\max_{\tau} W(\tau) = \begin{cases} \infty & \text{if } B \leq B^{\text{safe}} \\ 0 & \text{if } B > B^{\text{safe}} \end{cases},$$

and the continuation value under this optimal default strategy is:

$$W^*(B) \equiv W(\tau_d) = \begin{cases} h - d - B & \text{if } B \leq B^{\text{safe}} \\ -f & \text{if } B > B^{\text{safe}} \end{cases}. \tag{4}$$

Now, consider the subgame at $t = 0$ before the disaster and loan maturity. Recall that the maturity date arrives at rate $r\mu$ while, according to the borrower's belief, the disaster date arrives at rate $r\lambda$. At $t = 0$, the buyer's continuation value of defaulting at τ_0, where $\tau_0 \leq T$, is given by:

$$V_0(\tau_0) \equiv \int r\lambda e^{-r\lambda T} \int r\mu e^{-r\mu T_\mu} \left\{ \begin{array}{l} \int_0^{\min(T_\mu, \tau_0, T)} re^{-rt}(h-b)\,dt \\ +e^{-rT} 1_{T \leq \min(T_\mu, \tau_0)} W^*(B_T) \\ +e^{-r\tau_0} 1_{\tau_0 \leq T_\mu, \tau_0 < T} \left[-f + \left\{ r\underline{p} - B_{\tau_0} \right\}_+ \right] \\ +e^{-rT} 1_{T_\mu < \tau_0, T_\mu < T} v_\lambda \end{array} \right\} dT_\mu dT \tag{5}$$

The value function satisfies the following HJB equation:

$$\frac{d}{d\tau} V_0(\tau) = r(h-b) + r\mu[v_\lambda - V_0(\tau)] - r\lambda[V_0(\tau) - W^*(B)] - rV_0(\tau). \tag{6}$$

The HJB states that the marginal value of postponing default is equal to the sum of the flow of asset return (pre-disaster) net of the loan repayment, $r(h-b)$, and the expected gain of paying off the loan, $r\mu[v_\lambda - V_0(\tau)]$, minus the expected loss from the exposure to the disaster, $r\lambda[V_0(\tau) - W^*(B)]$, and the cost of discounting, $rV_0(\tau)$. At $\tau = 0$, the borrower defaults immediately, which gives the boundary condition to (6):

$$V_0(0) = -f + \left\{ r\underline{p} - B \right\}_+.$$

Using the boundary condition, the HJB equation has the following solution:

$$V_0(\tau) = \left[1 - e^{-r(1+\lambda+\mu)\tau}\right] \underbrace{\frac{h - (1+\mu)B + \lambda W^*(B) + \mu v_\lambda}{1+\lambda+\mu}}_{V_1(B)} + e^{-r(1+\lambda+\mu)\tau} \underbrace{\left[-f + \left\{ r\underline{p} - B \right\}_+ \right]}_{V_2(B)}. \tag{7}$$

The first term is the present value of never default before the disaster, and the second term is the value of immediate default. Define the first term as $V_1(B)$ and the second term as $V_2(B)$. Thus, we have $\tau_0 = 0$ if $V_1(B) < V_2(B)$ and $\tau_0 = \infty$ if $V_1(B) \geq V_2(B)$, imposing a tie-breaking rule that a borrower chooses to never default when he is indifferent. We want to solve the region of B such that $V_1(B) < V_2(B)$.

Using $W^*(B)$ from (4), $V_1(B)$ is given by:

$$V_1(B) = \begin{cases} v_\lambda - B & \text{if } B \leq B^{\text{safe}} \\ \frac{h-\lambda f+\mu v_\lambda}{1+\lambda+\mu} - \frac{1+\mu}{1+\lambda+\mu}B & \text{if } B > B^{\text{safe}} \end{cases}.$$

$V_1(B)$ features the values for two default strategies: in the first region $B \leq B^{\text{safe}}$, the buyer never defaults; in the second region $B > B^{\text{safe}}$, the buyer defaults immediately after the disaster (which is also optimal in that subgame, shown above) but does not default beforehand. Thus, $V_1(B)$ is decreasing in B with slope equal to -1 in the first region and with slope equal to $-\frac{1+\mu}{1+1+\lambda+\mu} \in (-1, 0)$ in the second region.

On the other hand, $V_2(B)$ is decreasing in B, with the slope equal to -1 when $B \leq r\underline{p}$; otherwise the slope equals 0. Also, notice that we have $V_1(0) = v_\lambda > r\underline{p} - f = V_2(0)$. Thus, we must have $V_2(B)$ intersecting $V_1(B)$ from below at the flat region of $V_2(B)$—see the illustration in Figure A.2. Denote the intersection as $B = B^{\text{risky}}$, i.e., $V_1\left(B^{\text{risky}}\right) = V_2\left(B^{\text{risky}}\right) = -f$, where $V_1(b) > V_2(b)$ if $B < B^{\text{risky}}$ and vice versa. Notice that since $V_1\left(B^{\text{safe}}\right) = v_\lambda - (h - d + f) > -f = V_1\left(B^{\text{risky}}\right)$, we must have $B^{\text{risky}} > B^{\text{safe}}$. In sum, B^{risky} is given by setting the second region of $V_1\left(B^{\text{risky}}\right)$ to $-f$:

$$B^{\text{risky}} = h - \frac{\mu}{1+\mu}\frac{\lambda}{1+\lambda}d + f. \tag{8}$$

Summarizing the above cases, the optimal default time is as stated in Lemma **??**. □

A.8.2 Proof of Proposition ??

Given Lemma **??**, the borrower's expected gain from the mortgage contract is given by

$$V_\lambda(\mathbf{m}) - v_\lambda = \begin{cases} -B & \text{if } B \leq B^{\text{safe}} \\ -\underbrace{\frac{1+\mu}{1+\mu+\lambda}B}_{\text{PV of debt repayment}} - \underbrace{\frac{\lambda}{1+\mu+\lambda}(h-d+f)}_{\text{PV of collateral loss \& cost at default}} & \text{if } B \in (B^{\text{safe}}, B^{\text{risky}}] \\ -v_\lambda - f & \text{otherwise} \end{cases}, \tag{9}$$

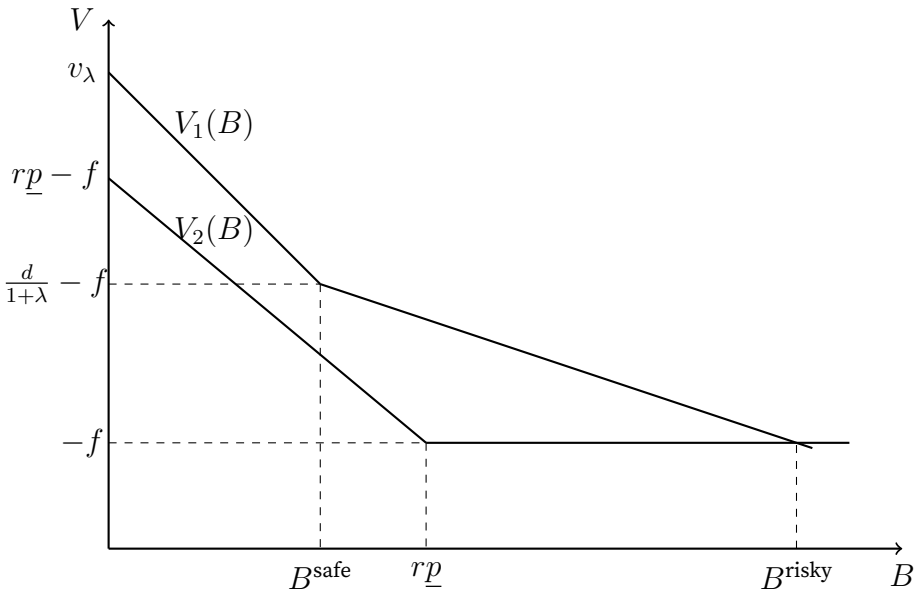


Figure A.2: Illustration of value functions $V_1(B)$ and $V_2(B)$ from equation (7) and the determination of the risky debt limit B^{risky} in (8). Note that B^{safe} can be on the left-hand side or right-hand side of $r\underline{p}$ without affecting B^{risky}.

and similarly for the lender's expected value of the mortgage's repayment stream:

$$R_{\bar{\lambda}}(\mathbf{m}) = \begin{cases} B & \text{if } B \leq B^{\text{safe}} \\ \underbrace{\frac{1+\mu}{1+\mu+\bar{\lambda}} B}_{\text{PV of repayment stream}} + \underbrace{\frac{\bar{\lambda}}{1+\mu+\bar{\lambda}}(h-d)}_{\text{PV of collateral seized at default}} & \text{if } B \in (B^{\text{safe}}, B^{\text{risky}}] \\ r\underline{p} & \text{otherwise} \end{cases}. \tag{10}$$

Equations (9) and (10) an be rewritten as:

$$V_\lambda(\mathbf{m}) - v_\lambda = \begin{cases} -B & \text{if } B \leq B^{\text{safe}} \\ -B - \frac{\lambda}{1+\mu+\lambda}(h-d+f-B) & \text{if } B \in (B^{\text{safe}}, B^{\text{risky}}] \\ -v_\lambda - f & \text{otherwise} \end{cases}$$

$$R_{\bar{\lambda}}(\mathbf{m}) = \begin{cases} B & \text{if } B \leq B^{\text{safe}} \\ B + \frac{\bar{\lambda}}{1+\mu+\bar{\lambda}}(h-d-B) & \text{if } B \in (B^{\text{safe}}, B^{\text{risky}}] \\ r\underline{p} & \text{otherwise} \end{cases}.$$

Hence, the joint surplus $S(\mathbf{m}) = V_\lambda(\mathbf{m}) - v_\lambda + R_{\bar{\lambda}}(\mathbf{m})$ from any mortgage contract $\mathbf{m}$

can be written as:

$$S(\mathbf{m}) = \begin{cases} 0 & \text{if } B \leq B^{\text{safe}} \\ \left(\frac{1+\mu}{1+\bar{\lambda}+\mu} - \frac{1+\mu}{1+\lambda+\mu}\right)(B - B^{\text{safe}}) - \frac{\bar{\lambda}}{1+\bar{\lambda}+\mu} f & \text{if } B \in (B^{\text{safe}}, B^{\text{risky}}] \\ r\underline{p} - (v_\lambda + f) < 0 & \text{otherwise} \end{cases}.$$

It is immediate that choosing a mortgage contract with the loan balance exceeding the risky limit ($B > B^{\text{risky}}$) will never occur in equilibrium, since it will yield a negative surplus of $r\underline{p} - (v_\lambda + f) < 0$. Hence, it suffices to focus on contracts with $B \leq B^{\text{risky}}$.

For a risky mortgage ($B \in (B^{\text{safe}}, B^{\text{risky}}]$), given the linearity of the joint surplus, it is immediate that the surplus is maximized by choosing $B = B^{\text{risky}}$ if $\lambda > \bar{\lambda}$, and by choosing $B = B^{\text{safe}}$ otherwise. Furthermore, when $\lambda > \bar{\lambda}$, the surplus from choosing $B = B^{\text{risky}}$ can be simplified further as:

$$\left(\frac{1+\mu}{1+\bar{\lambda}+\mu} - \frac{1+\mu}{1+\lambda+\mu}\right)(B^{\text{risky}} - B^{\text{safe}}) - \frac{\bar{\lambda}}{1+\bar{\lambda}+\mu} f = \frac{\Delta}{1+\mu+\bar{\lambda}},$$

which is positive if and only if $\Delta > 0$, which is the case if and only if $\lambda > \lambda^*$ and $d > \bar{\lambda} f$.

Hence, if $\lambda \leq \lambda^*$ or $d \leq \bar{\lambda} f$, then it is never optimal to choose a risky mortgage contract. In this case, maximal joint surplus is achieved by choosing a safe mortgage contract. However, a safe contract only yields a joint surplus of zero. Given the presence of the maintenance cost $\kappa(\mu)$ and the fixed cost of entry κ_0, borrowing with any mortgage contract is never optimal in this case.

However, if $\lambda > \lambda^*$ and $d > \bar{\lambda} f$, then it is optimal to choose a risky mortgage with the risky debt limit binding $B = B^{\text{risky}}$. The equilibrium contract will have a per-period payment of $b^* = (1+\mu)B^{\text{risky}}$, a maturity rate μ^* that maximizes:

$$\max_{\mu \geq 0} \frac{\Delta}{1+\mu+\bar{\lambda}} - \kappa(\mu),$$

and a leverage probability α^* that maximizes:

$$\max_{\alpha \in [0,1]} \alpha \cdot \left\{ \frac{\Delta}{1+\mu^*+\bar{\lambda}} - \kappa(\mu^*) - \frac{\kappa_0}{\eta(\alpha)} \right\},$$

with a loan amount given by lenders' free-entry condition (6):

$$rl^* = \frac{1+\mu}{1+\mu+\bar{\lambda}} B^{\text{risky}} + \frac{\bar{\lambda}}{1+\mu+\bar{\lambda}}(h-d) - \frac{\kappa_0}{\eta(\alpha^*)} - \kappa(\mu^*).$$

□

A.8.3 Comparative statics

Corollary 1 (Comparative statics)**.** *The following table characterizes the comparative statics (partial derivatives) with respect to λ and d for the optimal maturity rate μ^*, loan approval probability α^*, loan amount l^*, and the implied loan rate $r^* \equiv R_{\bar{\lambda}}(\mathbf{m})/l^*$:*

	Belief				*Disaster exposure*			
	$\frac{\partial \mu^*}{\partial \lambda}$	$\frac{\partial \alpha^*}{\partial \lambda}$	$\frac{\partial l^*}{\partial \lambda}$	$\frac{\partial r^*}{\partial \lambda}$	$\frac{\partial \mu^*}{\partial d}$	$\frac{\partial \alpha^*}{\partial d}$	$\frac{\partial l^*}{\partial d}$	$\frac{\partial r^*}{\partial d}$
At-risk house & pessimistic buyer ($d > \bar{\lambda} f, \lambda > \lambda^*$)	$-$	$+$	*?*	*?*	$-$	$+$	*?*	*?*
Otherwise	0	0	0	0	0	0	0	0

Note: $+$ means ≥ 0, $-$ means ≤ 0, $?$ means ambiguous.

The comparative statics for the loan amount l^* is ambiguous due to two opposite forces. On the loan demand side, more pessimism (a higher λ relative to a fixed $\bar{\lambda}$) implies more gain from trade and hence more incentive for the homebuyer to borrow. On the loan supply side, a higher λ implies a lower homebuyer's valuation of the asset v_λ, which in turn lowers their willingness to repay and lenders' willingness to lend. Similarly, a change in the disaster exposure d would have two opposite effects on the loan amount. Since the implied loan rate r^* is directly related to l^*, it follows that the comparative statics for r^* is ambiguous.

Proof. The comparative statics of μ^* and α^* are straightforward following the application of monotone comparative statics to their definition in Proposition **??**.

For the comparative statics of the loan amount l^*, notice that the free-entry (6) condition implies that:

$$rl^* = \frac{1+\mu^*}{1+\mu^*+\bar{\lambda}} B + \frac{\bar{\lambda}}{1+\mu^*+\bar{\lambda}} (h-d) - \kappa(\mu^*) - \frac{\kappa_0}{\eta(\alpha^*)}. \tag{11}$$

For $\lambda > \lambda^*$, we have $\partial \mu^* / \partial \lambda < 0$. So, via μ, an increase in λ additionally increases the first two terms of (11), but decreases the third term. The overall sign of $\partial l^* / d\lambda$ becomes ambiguous for $\lambda > \lambda^*$. The same is true for $\partial l^* / \partial d$ for $\lambda > \lambda^*$.

Finally, since the loan rate r^* is defined as:

$$r^* \equiv \frac{R_{\bar{\lambda}}(\mathbf{m}^*)}{l^*} = r + \frac{\kappa(\mu^*) + \frac{\kappa_0}{\eta(\alpha^*)}}{l^*}, \tag{12}$$

which depends on l^*, so the overall signs of $\partial r^* / \partial \lambda$ and $\partial r^* / \partial d$ are ambiguous for $\lambda > \lambda^*$.

□

A.9 Model extensions

A.9.1 Disaster insurance

This extension allows for disaster insurance. The baseline results continue to hold as long as insurance coverage is incomplete or the premium is not too high—two conditions that are realistic in the relevant empirical context of flood insurance in the U.S. (Section **??**).

Assume that at each date t before the disaster, the homebuyer has access to a disaster insurance program with an exogenous coverage c (think of the National Flood Insurance Program, or NFIP) and exogenous premium rate $r\lambda^I$. The insurance reduces the disaster damage from d to $d-c$, with c representing the pay out by the insurer when the disaster hits at T. The program charges a premium of $r\lambda^I c$ per each date $t < T$ (where λ^I can be viewed as the "effective belief" of the insurer revealed by the premium). Under this program, the homebuyer's PV of the stream of utility from an insured house is:

$$v_\lambda^I \equiv h - \frac{\lambda}{1+\lambda}(d-c) - \frac{\lambda^I}{1+\lambda}c = h^I - \frac{\lambda}{1+\lambda}d^I,$$

where $h^I \equiv h - \lambda^I c$ and $d^I \equiv d - (1+\lambda^I)c$ denote the housing utility and disaster damage with insurance.[1] To focus on the plausible case where v_λ^I is decreasing in λ (i.e., the valuation of the insured house is lower for a more pessimistic homebuyer), we assume that:

$$(1+\lambda^I)c < d. \tag{13}$$

This condition holds when insurance coverage is incomplete or the insurance premium is not too high. It is consistent with the fact that there is a maximum coverage set by the NFIP ($250,000), and that the NFIP premium rates are highly subsidized (**?**).

Mandatory insurance Let us first consider a simple scenario where insurance is mandatory (think of the flood insurance requirement in the NFIP's Special Flood Hazard Areas, see Section **??**). Then it is straightforward to show that the baseline results from Section **??** continue to hold, with parameters h and d replaced with h^I and d^I, respectively.[2]

[1] The expression for the post-insurance damage parameter d^I reflects the fact that when the disaster hits, the insurance program reduces the damage from d to $d-c$, and the homebuyer stops paying the stream of insurance premia, whose PV amounts to $\lambda^I c$.

[2] Similarly, if insurance is mandatory only when the home purchase is leveraged (but voluntary otherwise), then the results continue to hold, with a slight modification. There will be a belief threshold λ^* such that a pessimistic homebuyer with $\lambda > \lambda^*$ chooses a risky mortgage (and is hence required to purchase insurance). A homebuyer with $\lambda < \lambda^*$ but $\lambda \geq \lambda^I$ does not borrow and voluntarily purchases insurance. Finally, an optimistic homebuyer with $\lambda < \lambda^I$ does not borrow and buys no insurance.

Voluntary insurance Let us now consider the more complex scenario where insurance is voluntary. In the same way that the agent cannot commit to repaying their debt, we suppose that they cannot commit to buying insurance.[3] For each date t, there are two stages. In the first stage, the homeowner chooses whether or not to participate in the insurance program by paying the insurance premium $r\lambda^I c$. In the second stage, should the disaster arrives, the homebuyer will receive the insurance coverage if and only if she has paid the premium at the beginning of the period. In this extension, we assume $f = 0$ to simplify the analysis.

An interesting insight that arises from this extension is a *debt overhang effect*: leveraged households with risky debt have less incentive to invest in insurance, since the option to default acts as a form of implicit insurance. If there is no mortgage balance, then the homebuyer will purchase insurance if and only if the premium rate is sufficiently low such that $\lambda^I < \lambda$ (equivalently, when the homebuyer is relatively more pessimistic than the insurer). However, when the homebuyer is indebted, her demand for insurance depends on the outstanding debt, as summarized in the following lemma:

Lemma 2 (Debt overhang effect on insurance uptake)**.** *At any $t < T \wedge T_\mu$, the borrower chooses to purchase insurance if and only if* $\lambda^I < \lambda$ and $B \leq B^{safe}_{ins} \equiv B^{safe} + \left(1 - \frac{\lambda^I}{\lambda}\right) c$.

Proof. We solve the equilibrium default strategy assuming that the borrower buys the insurance. Of course, if the borrower defaults, they will stop buying insurance for his asset. Consider the subgame after the disaster has happened but the debt contract has not matured yet. Let $i \in \{0, 1\}$ denote the borrower's insurance uptake. Let τ denote their default period. Given insurance coverage c, the borrower's HJB equation is:

$$W_i(\tau) = \left[1 - e^{-r(1+\mu)\tau}\right](h - d + ic - B) + e^{-r(1+\mu)\tau}\{h - d + ic - B\}_+.$$

The optimal stopping time of default τ_i^* is given by

$$W_i^*(B) \equiv W_i(\tau_i^*) = \begin{cases} h - d + ic - B & \text{if } B \leq \overbrace{B^{\text{safe}}}^{h-d} + ic \\ 0 & \text{if } B > B^{\text{safe}} + ic \end{cases}.$$

Now, consider the subgame before the disaster and before the loan matures. The HJB

[3] In practice, most flood insurance contracts are set on an annual basis, and prescribed homebuyers can always have the option to not renew in the subsequent year.

equation has the following solution:

$$V(\tau) = \left[1 - e^{-r(1+\lambda+\mu)\tau}\right] \max_{i\in\{0,1\}} \underbrace{\frac{h - i\lambda^I c - (1+\mu)B + \lambda W_i^*(B) + \mu v_\lambda^{\text{ins}}}{1+\lambda+\mu}}_{V_1(B)} + e^{-r(1+\lambda+\mu)\tau} \underbrace{\left\{r\underline{p} - B\right\}_+}_{V_2(B)}.$$

Thus, the borrower will buy insurance ($i = 1$) if and only if the cost is smaller than the benefit:

$$\frac{\lambda^I c}{\lambda} < W_1^*(B) - W_0^*(B) = \begin{cases} c, \text{ if } B \leq B^{\text{safe}} \\ h - d + c - B, \text{ if } B \in (B^{\text{safe}}, B^{\text{safe}} + c] \\ 0, \text{ if } B > B^{\text{safe}} + c \end{cases}$$

Rearranging the above terms, we have:

$$i = 1 \Leftrightarrow \frac{\lambda^I c}{\lambda} < W_1^*(B) - W_0^*(B) \Leftrightarrow B < B_{\text{ins}}^{\text{safe}} \equiv B^{\text{safe}} + \left(1 - \frac{\lambda^I}{\lambda}\right) c.$$

Following the similar logic in the previous proof, we can summarize the borrower's value in the following regions:

$$V_\lambda(\mathbf{a}) = \begin{cases} v_\lambda^{\text{ins}} - B, \text{ if } B \leq B_{\text{ins}}^{\text{safe}} \\ \frac{1+\mu}{1+\lambda+\mu}\left(B_{\text{ins}}^{\text{risky}} - B\right), \text{ if } B \in (B_{\text{ins}}^{\text{safe}}, B_{\text{ins}}^{\text{risky}}] \\ 0, \text{ if } B > B_{\text{ins}}^{\text{risky}} \end{cases}$$

where

$$B_{\text{ins}}^{\text{risky}} \equiv h - \frac{\mu}{1+\mu}\frac{\lambda}{1+\lambda}\left[d - \left(1 - \frac{\lambda^I}{\lambda}\right)c\right].$$

In the first region, the borrower buys the insurance and never defaults. In the second region, the borrower defaults when the disaster hits and buys the insurance only after the loan has matured (and not defaulted). In the last region, the borrower defaults immediately (and does not buy insurance, of course). □

Remark 1. *Lemma 2 implies that, all else equal, home equity (price minus mortgage balance) is positively correlated with insurance uptake. This is consistent with the finding by* **?**, *which documents a strong positive correlation between home equity and insurance uptake using data aggregated across 271 MSAs between 2001 and 2015.*

Equilibrium characterization: Assume $\lambda^I \leq \bar{\lambda}$ (e.g., subsidized insurance premium). Using the same steps as in Section A.8.2, we can characterize the equilibrium mortgage

and insurance uptake as follows. There is a belief cutoff threshold:

$$\lambda^*_{\text{ins}} \equiv \bar{\lambda} + \frac{\bar{\lambda} - \lambda^I}{\frac{1}{1+\bar{\lambda}}\frac{d}{x} - 1} \geq \bar{\lambda} \geq \lambda^I,$$

such that:

1. A sufficiently pessimistic homebuyer with $\lambda > \lambda^*_{\text{ins}}$ at $t = 0$ chooses a risky mortgage contract with a risky loan amount $B = B^{\text{risky}}_{\text{ins}} \geq B^{\text{risky}}$. The buyer participates in the insurance program only after the mortgage contract has matured (i.e., at all $t \in [T_\mu, T)$). As before, the equilibrium leverage probability α^* increases and the loan maturity rate μ^* solves:

$$\max_\alpha \alpha \cdot \left\{ \max_\mu \left[\frac{\Delta_{\text{ins}}}{1 + \mu + \bar{\lambda}} - \kappa(\mu) \right] - \frac{\kappa_0}{\eta(\alpha)} \right\},$$

   where $\Delta_{\text{ins}} = \Delta - \frac{1+\bar{\lambda}}{1+\lambda}(\lambda - \lambda^I)x \leq \Delta$. As before, α^* increases and μ^* decreases in λ.

2. A sufficiently optimistic homebuyer with $\lambda \leq \lambda^*_{\text{ins}}$ chooses not to borrow at $t = 0$. A very optimistic homebuyer with $\lambda \leq \lambda^I$ will never purchase insurance. A moderately optimistic homebuyer with $\lambda^I < \lambda \leq \lambda^*_{\text{ins}}$ purchases insurance at all $t \in [0, T)$.

In summary, our baseline results continue to hold, with the cutoff threshold λ^*, risky debt limit B^{risky}, and belief disagreement term Δ replaced by λ^*_{ins}, $B^{\text{risky}}_{\text{ins}}$, and Δ_{ins}, respectively.

Moreover, since risky debt crowds out insurance uptake (Lemma 2), insurance uptake is a nonmonotone function of the homebuyer's belief. While a moderately optimistic buyer with $\lambda^I < \lambda \leq \lambda^*_{\text{ins}}$ purchases insurance at all $t \in [0, T)$, a pessimistic buyer with $\lambda > \lambda^*_{\text{ins}}$ instead chooses to leverage with a risky mortgage and purchase insurance only after the mortgage has matured, at $t \in [T_\mu, T)$.[4]

A.9.2 Disaster forbearance

This extension allows for government disaster assistance. An important source of financial assistance *specifically for mortgage borrowers* comes from disaster loan forbearance programs provided by the GSEs and federal agencies. They allow affected borrowers with

[4]Recall that both T_μ and T are random variables. If the disaster realizes before the mortgage matures ($T \leq T_\mu$), then the borrower will default at T and hence will never purchase insurance.

federally-backed mortgages to reduce or suspend their mortgage payments for a short duration, usually up to a year, without incurring late fees (**?**). In our model, a loan forbearance program can be viewed as a reduction of the loan's present value from B to $e^{-r\varepsilon}B$ at the disaster date T, where ε is the duration of forbearance (and $(1-e^{-r\varepsilon})B$ is the fiscal cost of this program). Thus, the forbearance makes the loan balance effectively contingent on the disaster state. All agents rationally anticipate the forbearance at $t = 0$.

It turns out that the forbearance will *strengthen* our main results, as it provides another reason why a pessimist might want to leverage and to use a long-term mortgage. The borrower's expected payoff $V_\lambda(\mathbf{m})$ from a mortgage $\mathbf{m}$ now becomes:

$$V_\lambda(\mathbf{m}) = v_\lambda + \begin{cases} -B + \underbrace{\dfrac{\lambda}{1+\mu+\lambda}(1-e^{-r\varepsilon})B}_{\text{borrower's PV of forbearance}} & \text{if } B \le B^{\text{safe}} \\ -\frac{1+\mu}{1+\mu+\lambda}B - \underbrace{\dfrac{\lambda}{1+\mu+\lambda}e^{-r\varepsilon}(h-d+f)}_{\text{PV of postponing the default}} & \text{if } B \in (B^{\text{safe}}, B^{\text{risky}}] \\ \text{as in the third region of (9)} & \text{if } B > B^{\text{risky}} \end{cases}. \tag{14}$$

There are two changes relative to equation (9): (i) a new term in the value of a safe mortgage, representing the PV of the borrower's gain from the forbearance, and (ii) a reduced PV of the loss of default in the value of a risky mortgage, due to the borrower postponing the default to enjoy the forbearance; both of these terms are discounted at the subjective probability $\frac{\lambda}{1+\mu+\lambda}$. Intuitively, the borrower can enjoy the forbearance only if the loan has not been defaulted, and the forbearance delays the incentive to default during the disaster period. Hence, forbearance increases the borrower's payoff via two channels: postponing the repayment to a safe mortgage and postponing the default of a risky mortgage.

Using backward induction, we can characterize the equilibrium, with the same belief threshold λ^*. Above this threshold, the borrower continues to issue a risky debt with $B = B^{\text{risky}}$ as in Proposition **??**. However, below this threshold, the borrower will issue a *safe* debt with $B = B^{\text{safe}}$, whose leverage probability and average maturity increase in the borrower's pessimism λ. Intuitively, the safe debt allows the borrowers to enjoy the forbearance. The value of a safe mortgage increases in λ, as a more pessimistic borrower expects a sooner arrival of the disaster forbearance program. The value also increases in the average loan maturity (decreases in μ), as the borrower can enjoy the forbearance only if their loan has not yet matured by the time the disaster arrives.

Formally, anticipating the forbearance program, the joint surplus at $t = 0$ from a mort-

gage $\mathbf{m}$ becomes:

$$S(\mathbf{m}) = \begin{cases} \underbrace{\frac{\lambda(1-e^{-r\varepsilon})B}{1+\mu+\lambda}}_{\text{new}} > 0 & \text{if } B \leq B^{\text{safe}} \\ \underbrace{\left(\frac{1+\mu}{1+\mu+\bar{\lambda}} - \frac{1+\mu}{1+\mu+\lambda}\right)(B-h+d-f) - \frac{\bar{\lambda}}{1+\mu+\bar{\lambda}}f}_{\text{as before}} & \\ \quad + \underbrace{\frac{\lambda\left(1-e^{-r\varepsilon}\right)}{1+\mu+\lambda}(h-d+f)}_{\text{new}} & \text{if } B \in (B^{\text{safe}}, B^{\text{risky}}] \\ \underbrace{r\underline{p} - (v_\lambda + f)}_{\text{as before}} & \text{if } B > B^{\text{risky}} \end{cases}.$$

Whenever the first option is optimal, it is optimal to set $B = B^{\text{safe}}$. The optimal maturity $\mu^{*\text{safe}}(\lambda)$ and leverage probability $\alpha^{*\text{safe}}(\lambda)$ solve:

$$\max_{\alpha\in[0,1]} \alpha \left\{ \max_{\mu\geq 0} \left[\frac{\delta}{1+\mu+\lambda} - \kappa(\mu) \right] - \frac{\kappa_0}{\eta(\alpha)} \right\},$$

where

$$\delta \equiv \lambda(1-e^{-r\varepsilon}) \overbrace{(h-d+f)}^{B^{\text{safe}}}.$$

The surplus from an optimal safe contract is given by:

$$S^{\text{safe}}(\lambda) \equiv \frac{\delta}{1+\mu^{*\text{safe}}+\lambda}.$$

Whenever the second option is optimal, it is optimal to set $B = B^{\text{risky}}$. As before, the optimal risky contract solves:

$$\max_{\alpha\in[0,1]} \alpha \left\{ \max_{\mu\geq 0} \left[\frac{\Delta}{1+\mu+\bar{\lambda}} + \frac{\delta}{1+\mu+\lambda} - \kappa(\mu) \right] - \frac{\kappa_0}{\eta(\alpha)} \right\},$$

where recall that $\Delta \equiv \frac{\lambda-\bar{\lambda}}{1+\lambda} d - \bar{\lambda} f$, and the surplus from an optimal risky contract is given by:

$$S^{\text{risky}}(\lambda) \equiv \frac{\Delta}{1+\mu^*+\bar{\lambda}} + \frac{\delta}{1+\mu^*+\lambda}.$$

Note that $S^{\text{risky}}(\lambda) > S^{\text{safe}}(\lambda)$ iff $\Delta > 0$. Thus, we have $S^{\text{risky}}(\lambda) > S^{\text{safe}}(\lambda)$ if and only if $\lambda > \lambda^*$. When $\lambda \leq \lambda^*$, the borrower prefers the optimal safe debt. When $\lambda > \lambda^*$, they prefer the optimal risky debt.

In sum, the anticipation of a forbearance program provides another motive for leverage, further strengthening our theoretical prediction that pessimistic homebuyers are more likely to leverage and use a long-term mortgage.

Remark 2 (Disaster aid)**.** *Another source of government assistance is disaster aid, although in practice, per-capita aid is often limited in both actual amounts*[5] *and amounts expected.*[6] *In our model, aid can be viewed as a reduction of the disaster damage from* d *to* $d_\epsilon \equiv (1-\epsilon)d$*. As long as* $\epsilon < 1$*, it is immediate to see that Proposition* **??** *continue to hold with* d *replaced by* d_ϵ*, and the effects of* ϵ *are summarized in the comparative statics for* d *in Corollary 1.*

A.9.3 GSEs' guarantee

This extension introduces the GSEs' guarantee program and formally shows how a guarantee subsidy will encourage lenders to optimistically evaluate the risk exposure of a loan (and especially so when the loan has a long maturity).

Suppose at $t = 0$, after entering a mortgage contract $\mathbf{m}$ with a borrower, a *GSE agent* can guarantee a fraction $1-\theta$ of the mortgage repayment against the borrower's default risk. Here, the parameter $\theta \in [0,1]$ represents the regulatory requirement that a lender retains some "skin in the game" in the loan origination and distribution process (**?**). The GSE agent charges the lender a guarantee fee (also known as "g-fee") $g \in [0,1]$ per unit of expected deficiency amount, until either the mortgage matures or the borrower defaults. The case $g < 1$ captures an underpriced g-fee (below the actuarially fair cost); our benchmark model in Section **??** is the special cases $\theta = 1$ (no guarantee) or $g = 1$ (no subsidy).

We now show how the characterization of the equilibrium mortgage in Proposition **??** continues to hold with a modified belief threshold $\lambda^*_{\text{GSE}} \leq \lambda^*$.

The lender's expected profit from obtaining GSE guaurantee for a fraction $1-\theta$ of its mortgage cash flow is now:

$$\Pi_{\bar{\lambda}}(\mathbf{m}) \equiv \theta R_{\bar{\lambda}}(\mathbf{m}) + (1-\theta)\left[B - gD_{\bar{\lambda}}(\mathbf{m})\right], \tag{15}$$

[5]For example, FEMA provides post-disaster grants to households through the Individual and Household Program, which has a cap of just over $30,000 per household, and the actual amount averages only a few thousand dollars (e.g., below $9,000 for Hurricane Harvey) (**?**).

[6]Homeowner expectations of post-disaster aid are modest. Based on calculations by the authors from the 2021 FEMA National Household Survey, among homeowners who identified flooding as one of the "types of disasters that would have the biggest impacts where you live," while 78.5% expected to receive aid from the federal government, only 32.9% expected to receive *financial assistance* from any external organization (including federal, state, local, and tribal governments). More common were expectations of in-kind donations of food, water, or temporary shelter. In addition, **?** find that residents expected government assistance to cover less than 17% of flood damages.

where $D_{\bar{\lambda}}(\mathbf{m})$ is the expected discounted deficiency amount, given by:

$$D_{\bar{\lambda}}(\mathbf{m}) = \begin{cases} 0 & \text{if } B \leq B^{\text{safe}} \\ \frac{\bar{\lambda}}{1+\mu+\bar{\lambda}}(B-h+d) & \text{if } B \in (B^{\text{safe}}, B^{\text{risky}}] \end{cases}, \tag{16}$$

where without loss of generality we rule out the excessively risky debt region $B > B^{\text{risky}}$ by assuming that the GSEs do not guarantee any loan in this region.

Focusing on the second region, the lender's expected present value of the cash flow is thus:

$$\theta R_{\bar{\lambda}}(\mathbf{m}) + (1-\theta)\left[B - gD_{\bar{\lambda}}(\mathbf{m})\right] = B - \frac{\bar{\lambda}\left[\theta + (1-\theta)g\right]}{1+\mu+\bar{\lambda}}(B-h+d).$$

Recall that the borrower's surplus is:

$$V_{\lambda}(\mathbf{m}) - v_{\lambda} = -B + \frac{\lambda}{1+\mu+\lambda}(B-h+d+f).$$

Thus, the joint surplus is:

$$S(B) \equiv V_{\lambda}(\mathbf{m}) - v_{\lambda} + \Pi_{\bar{\lambda}}(\mathbf{m}) = \left(\frac{\lambda}{1+\mu+\lambda} - \frac{\left[\theta + (1-\theta)g\right]\bar{\lambda}}{1+\mu+\bar{\lambda}}\right)(B-h+d+f) - \frac{\left[\theta+(1-\theta)g\right]\bar{\lambda}}{1+\mu+\bar{\lambda}}f.$$

So, $B > B^{\text{safe}}$ is optimal if and only if:

$$\frac{\lambda}{1+\mu+\lambda} > \frac{\left[\theta+(1-\theta)g\right]\bar{\lambda}}{1+\mu+\bar{\lambda}}. \tag{17}$$

In this case, we have:

$$S\left(B^{\text{risky}}\right) = \left[\theta+(1-\theta)g\right]\Delta\bar{T} + \frac{(1-\theta)(1-g)\lambda d}{\left(1/\bar{T}-\bar{\lambda}\right)(1+\lambda)},$$

where recall that $\bar{T} \equiv \frac{1}{1+\mu+\bar{\lambda}}$. Note that $S\left(B^{\text{risky}}\right)$ is increasing and convex in $\bar{T}$.

Thus, $\bar{T} > 0$ only if

$$\begin{aligned} 0 \quad &< \quad \left.\frac{\partial S\left(B^{\text{risky}}\right)}{\partial \bar{T}}\right|_{\bar{T}=0} = \left[\theta+(1-\theta)g\right]\Delta + \frac{(1-\theta)(1-g)\lambda d}{1+\lambda} \\ &\Leftrightarrow \quad \lambda > \lambda^{*}_{\text{GSE}} \equiv \bar{\lambda}\left[\theta+(1-\theta)g\right]\underbrace{\frac{1+f/d}{1-\bar{\lambda}\left[\theta+(1-\theta)g\right]\frac{f}{d}}}_{>1}. \end{aligned} \tag{18}$$

Note that (18) implies (17) for any $\mu \geq 0$ under the maintained assumption $d > \bar{\lambda}f$. Hence,

the homebuyer will borrow iff $\lambda > \lambda^*_{\text{GSE}}$, and when they do, they will use a risky mortgage with balance $B = B^{\text{risky}}$.

The definition of λ^*_{GSE} implies that $\lambda^*_{\text{GSE}} \leq \lambda^*$, and the inequality is strict when $(1-\theta)(1-g) > 0$. In words, the subsidized GSE guarantee leads to more risky borrowing in equilibrium.

Regarding comparative statics, from the formula of λ^*_{GSE} in (18), we have $\partial\lambda^*_{\text{GSE}}/\partial g > 0$, $\partial\lambda^*_{\text{GSE}}/\partial\theta > 0$, and $\partial\lambda^*_{\text{GSE}}/\partial\bar{\lambda} > 0$. Furthermore:

$$\begin{aligned}
\frac{\partial}{\partial g}S\left(B^{\text{risky}}\right) &= (1-\theta)\,\bar{T}\left[\Delta - \frac{\lambda d}{\left(1-\bar{\lambda}\bar{T}\right)(1+\lambda)}\right] < 0,\\
\frac{\partial}{\partial \theta}S\left(B^{\text{risky}}\right) &= (1-g)\,\bar{T}\left[\Delta - \frac{\lambda d}{\left(1-\bar{\lambda}\bar{T}\right)(1+\lambda)}\right] < 0,\\
\frac{\partial^2}{\partial\bar{T}\partial g}S\left(B^{\text{risky}}\right) &= (1-\theta)\left[\Delta - \frac{\lambda d}{\left(1-\bar{\lambda}\bar{T}\right)^2(1+\lambda)}\right] < 0,\\
\frac{\partial^2}{\partial\bar{T}\partial \theta}S\left(B^{\text{risky}}\right) &= (1-g)\left[\Delta - \frac{\lambda d}{\left(1-\bar{\lambda}\bar{T}\right)^2(1+\lambda)}\right] < 0,\\
\frac{\partial^2}{\partial\bar{T}\partial\bar{\lambda}}S\left(B^{\text{risky}}\right) &= -\left[\theta + (1-\theta)\,g\right]\left(\frac{d}{1+\lambda} + f\right) - \frac{2(1-\theta)(1-g)\lambda d}{\left(1-\bar{\lambda}\bar{T}\right)^3(1+\lambda)}\bar{T} < 0.
\end{aligned}$$

Thus, we have $\partial\alpha^*/\partial g < 0$, $\partial\alpha^*/\partial\theta < 0$, $\partial\bar{T}/\partial g < 0$, $\partial\bar{T}/\partial\theta < 0$, and $\partial\bar{T}/\partial\bar{\lambda} < 0$. Finally, note that:

$$\frac{\partial}{\partial\bar{\lambda}}S\left(B^{\text{risky}}\right) = -\left[\theta + (1-\theta)\,g\right]\bar{T}\left(\frac{d}{1+\lambda} + f\right) + \frac{(1-\theta)(1-g)\lambda d}{\left(1/\bar{T} - \bar{\lambda}\right)^2(1+\lambda)},$$

Since $\frac{\partial^2}{\partial\bar{T}\partial\bar{\lambda}}S\left(B^{\text{risky}}\right) < 0$, we have

$$\frac{\partial}{\partial\bar{\lambda}}S\left(B^{\text{risky}}\right) < \frac{\partial}{\partial\bar{\lambda}}S\left(B^{\text{risky}}\right)\Big|_{\bar{T}=0} = 0.$$

Hence, we have $\partial\alpha^*/\partial\bar{\lambda} < 0$.

As summarized in the table below, an increase in g or in θ ultimately leads to lower risky mortgage origination, lower leverage probability, and lower maturity in equilibrium, summarized in the below table. Qualitatively, the comparative statics of g and θ follow the direction of $\bar{\lambda}$, i.e., reducing the g-fee or reducing the skin in the game has a similar effect as an increase in lenders' optimism in the benchmark model.

Guarantee fee			Skin in the game			Lender's belief		
$\frac{\partial \mu^*}{\partial g}$	$\frac{\partial \alpha^*}{\partial g}$	$\frac{\partial \lambda^*_{\text{GSE}}}{\partial g}$	$\frac{\partial \mu^*}{\partial \theta}$	$\frac{\partial \alpha^*}{\partial \theta}$	$\frac{\partial \lambda^*_{\text{GSE}}}{\partial \theta}$	$\frac{\partial \mu^*}{\partial \bar{\lambda}}$	$\frac{\partial \alpha^*}{\partial \bar{\lambda}}$	$\frac{\partial \lambda^*_{\text{GSE}}}{\partial \bar{\lambda}}$
$+$	$-$	$+$	$+$	$-$	$+$	$+$	$-$	$+$

Note: $+$ *means* ≥ 0, $-$ *means* ≤ 0.

A.9.4 Securitization and MBS pool

We further extend the model to incorporate the securitization process more explicitly. Consider the generalized model with multiple types of homebuyers and houses in Section **??**. We introduce the GSE agent as in A.9.3, who guarantees and purchases the mortgages issued by the lenders to the borrowers. Following (**??**), we assume that mortgages are pooled together and shares of the pool are sold to a set of investors (think of investors in the MBS market).

Given the guarantee payment, the MBS price at the disaster date $t = T$ is simply:

$$\mathcal{P}_T = \int_{\lambda > \lambda^*, d > \bar{\lambda} f} \frac{B^{\text{risky}}(\lambda, d)}{r} \phi^*_T(\lambda, d), \tag{19}$$

where ϕ^*_T is the measure of defaulting loans defined in (**??**). At the disaster date T, all the risky debts at-risk are defaulted. For each defaulting loan, the GSE repays the MBS investors the remaining loan balance $B^{\text{risky}}(\lambda, d)/r$.

Let λ^i and r^i denote the belief parameter and the discount rate of MBS investors. We focus on the relevant case of patient investors: $r^i < r$.[7] The MBS price in any pre-disaster period $t < T$ is given by the investors' present value of the pool of mortgage repayment streams (which are guaranteed by the GSEs):

$$r^i \mathcal{P}_t = \int_{\lambda > \lambda^*, d > \bar{\lambda} f} \left(\frac{1 + \mu^*}{1 + \mu^* + \lambda^i} + \frac{r^i}{r} \frac{\lambda^i}{1 + \mu^* + \lambda^i} \right) B^{\text{risky}}(\lambda, d) \phi^*_t(\lambda, d), \tag{20}$$

where the term in the brackets takes into account the investors' belief and discount rate.

For MBS investors, the mortgage default risk is effectively transformed to a prepayment risk, due to the guaranteed provided by the GSEs (**?**). In equilibrium, when the disaster arrives, risky mortgages are defaulted, and the GSE agent makes a lump-sum payment of the unpaid balances to the MBS investors. Hence, investors receive a one-time guarantee payment, instead of the regular repayment *flows*. This prepayment of the bal-

[7] This is a reasonable assumption, as the biggest investors in agency MBSs are depository institutions and the Federal Reserve (**?**).

ances is undesirable for patient MBS investors, reflected by the fact that the "prepayment penalty" term $\frac{1+\mu^*}{1+\mu^*+\lambda^i} + \frac{r^i}{r}\frac{\lambda^i}{1+\mu^*+\lambda^i}$ in equation (20) is less than one.

Hence, even with the GSE guarantee, the disaster causes a drop in the MBS price:

$$\mathcal{P}_T < \lim_{t \to T-} \mathcal{P}_t.$$

Since a subsidized g-fee encourages risky mortgage origination, leading to a pool of mortgages with a higher default risk in equilibrium, it follows that a subsidized g-fee also has an unintended consequence of increasing the sensitivity of the MBS price to the disaster shock.

A.9.5 Rental

Our model is flexible enough to encompass the rental option. So far, we have imposed $\kappa'(0) = -\infty$ on the operational servicing cost, so that the equilibrium maturity has an interior solution ($\mu^* > 0$), and the mortgage contract has a finite maturity. We now relax this assumption and consider contracts with an infinite maturity. An infinite maturity contract can be loosely interpreted as a rental contract, where the borrower is the renter, the lender the landlord, and the payment in perpetuity b the per-period rent. When the disaster hits, instead of foreclosure, the renter now terminates the rental contract, and we interpret f as the moving cost.

Formally, consider a lease contract $\mathbf{m}^{\text{rent}} = (p - k, b, 0)$, where k is a deposit to the landlord.[8] The renter's optimization problem is a modification of the borrower's problem (5):

$$U_\lambda^{\text{rent}} \equiv \max_{\mathbf{m}^{\text{rent}}, \alpha} \alpha \left[-rk + V_\lambda(\mathbf{m}^{\text{rent}})\right] + (1-\alpha)\left(-rp + v_\lambda\right), \tag{21}$$

where V_λ is the same as before. A competitive landlord's expected profit is also a modified version of the lender's free-entry condition (4):

$$\kappa_0 + rp = \eta(\alpha)\left[R_{\bar{\lambda}}\left(\mathbf{m}^{\text{rent}}\right) + rk - \kappa(\mu)\right] + \left[1 - \eta(\alpha)\right] rp,$$

where the landlord's cost of posting a lease contract is $\kappa_0 + rp$ and $R_{\bar{\lambda}}$ is the same as before. The landlord has probability $\eta(\alpha)$ of finding a renter. Otherwise, the rental property is

[8] For simplicity, we assume the deposit is non-refundable. The model can be adjusted for a refundable deposit. Also, we assume competitive landlords with free entry (as was the case for competitive lenders). The model can be adjusted to have a fixed measure of landlords as in **?**.

sold at p for simplicity. The optimal lease contract solves:

$$\max_{\alpha\in[0,1]} \alpha \left\{ \max_{b,k} S\left(\mathbf{m}^{\text{rent}}\right) - \kappa(0) - \frac{\kappa_0}{\eta\left(\alpha\right)} \right\}.$$

The joint surplus from a given lease contract $\mathbf{m}^{\text{rent}}$ is given by:

$$S\left(\mathbf{m}^{\text{rent}}\right) = \underbrace{V_\lambda\left(\mathbf{m}^{\text{rent}}\right) - v_\lambda + r\left(p-k\right)}_{\text{renter's surplus}} + \underbrace{R_{\bar\lambda}\left(\mathbf{m}^{\text{rent}}\right) - r\left(p-k\right)}_{\text{landlord's surplus}}.$$

The main results continue to hold, except that there will be an additional belief cutoff threshold $\lambda^{**} \equiv \frac{(d+f)\bar\lambda+\kappa'(0)(\bar\lambda+1)^2}{d-f\bar\lambda-\kappa'(0)(\bar\lambda+1)^2} > \lambda^*$. In equilibrium, for an at-risk house ($d > \bar\lambda f$):

1.

2. An optimistic homebuyer ($\lambda \leq \lambda^*$) chooses to not borrow;

3. A pessimistic homebuyer ($\lambda^* < \lambda \leq \lambda^{**}$) borrows using a risky mortgage contract (with finite maturity, where the maturity rate $\mu^* > 0$ is as specified in baseline Proposition **??**);

4. A very pessimistic homebuyer ($\lambda > \lambda^{**}$) rents, using the optimal lease specified above.

A.9.6 Difference in funding costs

This section introduces another reason for trade that arises from heterogeneous funding costs. It also clarifies the conceptual difference between heterogeneous beliefs and heterogeneous discount rates. Suppose that lenders have a lower discount rate $\bar r < r$. This assumption reflects the fact that in practice, banks have access to cheaper wholesale funding, such as the Fed funds market or commercial papers, while homebuyers do not. Using the same backward induction logic, this appendix characterizes the equilibrium mortgage. There still exists a belief cutoff λ^*_ω that a relatively pessimistic homebuyer ($\lambda > \lambda^*_\omega$) will borrow with risky debt $B = B^{\text{risky}}$. The cutoff λ^* now is modified by the ratio of different discount rates, $\omega \equiv r/\bar r > 1$. The main difference is that even if the homebuyer is relatively optimistic ($\lambda \leq \lambda^*_\omega$) or the house is relatively unexposed ($d \leq \bar\lambda f$), the homebuyer will borrow using a safe mortgage contract with no default risk, $B = B^{\text{safe}}$. They do so to reap the gain from borrowing at a relatively lower funding cost. Hence, both risky and safe mortgages will be traded in equilibrium.

With funding costs, the homebuyer's optimization problem in (5) becomes:

$$U_\lambda \equiv \max_{\mathbf{m}} \quad \alpha\left[-r(p-l)+V_\lambda(\mathbf{m})\right]+(1-\alpha)\left(-rp+v_\lambda\right), \tag{22}$$

where $V_\lambda(\mathbf{m})$ is the same as before. The lender's expected profit in (4) becomes:

$$\Pi_{\bar{\lambda}}\left(\mathbf{m}\right) \equiv R_{\bar{\lambda}}\left(\mathbf{m}\right)-\bar{r}l-\kappa(\mu). \tag{23}$$

where $R_{\bar{\lambda}}(\mathbf{m})$ is now given by

$$R_{\bar{\lambda}}\left(\mathbf{m}\right)=\begin{cases} B & \text{if } B\leq B^{\text{safe}} \\ \frac{b}{1+\mu+\bar{\lambda}}+\frac{\bar{\lambda}}{1+\mu+\bar{\lambda}}\frac{h-d}{\omega} & \text{if } B\in\left(B^{\text{safe}},B^{\text{risky}}\right] \\ \bar{r}\underline{p} & \text{otherwise} \end{cases},$$

where $\omega \equiv r/\bar{r} > 1$.

The joint surplus from a given mortgage $\mathbf{m}$ is given by:

$$\begin{aligned} S\left(\mathbf{m}\right) &= V_\lambda\left(\mathbf{m}\right)-v_\lambda+\omega R_{\bar{\lambda}}\left(\mathbf{m}\right) \\ &= \begin{cases} \left(\omega-1\right)B & \text{if } B\leq B^{\text{safe}} \\ \left(1+\mu\right)\left(\frac{\omega}{1+\mu+\bar{\lambda}}-\frac{1}{1+\mu+\bar{\lambda}}\right)\left(B-B^{\text{safe}}\right) & \\ \quad+\left(\omega-1\right)B^{\text{safe}}-\frac{\bar{\lambda}}{1+\mu+\bar{\lambda}}\left[\left(\omega-1\right)B^{\text{safe}}+f\right] & \text{if } B\in\left(B^{\text{safe}},B^{\text{risky}}\right] \\ r\underline{p}-v_\lambda-f & \text{otherwise} \end{cases}. \end{aligned}$$

To avoid unrealistic mortgage contracts that mature immediately, we impose an upper bound on the maturity rate: $\mu \in [0, \mu^{\max}]$. We set $\mu^{\max}$ sufficiently large so that the constraint $\mu \leq \mu^{\max}$ does not bind in equilibrium for risky mortgages. However, as shown below, an optimistic borrower will use a safe mortgage contract with a short maturity where this constraint binds. Using the same steps as in Section A.8.2, we can characterize the equilibrium mortgage as follows: There is a belief cutoff threshold given by:

$$\lambda_\omega^* \equiv \begin{cases} \frac{\bar{\lambda}(d+f)+\Delta_2}{d-\bar{\lambda}f-\Delta_1} & \text{if } d>\bar{\lambda}f+\Delta_1 \\ \infty & \text{otherwise} \end{cases}, \tag{24}$$

where

$$\begin{aligned}\Delta_1 &\equiv \bar{\lambda}\left(1-\omega^{-1}\right)(h-d)\,,\\ \Delta_2 &\equiv \left(1-\omega^{-1}\right)\left[\bar{\lambda}h-\left(1+\mu^{\max}+2\bar{\lambda}\right)d\right].\end{aligned}$$

The equilibrium mortgage contract is now given by:

1. If the house is sufficiently exposed ($d > \bar{\lambda}f + \Delta_1$) and the homebuyer is sufficiently pessimistic with $\lambda > \lambda^*_\omega$, then:

   (a) chooses a risky mortgage contract with loan balance $B = B^{\text{risky}}$, while maturity rate μ^* and loan approval rate α^* solve:

   $$\max_{\alpha\in[0,1]} \alpha\cdot\left\{\max_{\mu\in[0,\mu^{\max}]}\left[\frac{\omega\Delta_\omega}{1+\mu+\bar{\lambda}}-\kappa(\mu)\right]-\frac{\kappa_0}{\eta(\alpha)}+(\omega-1)\left(v_\lambda+f\right)\right\}, \tag{25}$$

   where

   $$\Delta_\omega \equiv \Delta-\bar{\lambda}\left(1-\omega^{-1}\right)(h-d)\,.$$

2. Otherwise, the borrower chooses a safe mortgage contract with loan balance $B = B^{\text{safe}}$, maturity rate $\mu = \mu^{\max}$ (implying short maturity), and loan approval rate α^s solve:

   $$\max_{\alpha\in[0,1]} \alpha\cdot\left\{\omega B^{\text{safe}}-\kappa(\mu^{\max})-\frac{\kappa_0}{\eta(\alpha)}\right\}.$$

A.9.7 Heterogeneous beliefs about adaptation progress

This section extends the baseline model to have an additional layer of belief heterogeneity about the progress of future adaptation technology.[9]

Suppose that after the disaster, there may be a "build-back-better" adaptation technology that allows homeowners to rebuild their house to be more resilient to SLR inundation.[10] Specifically, at any $t \geq T_a$ (after the technology has arrived), any homeowner has the option to pay a one-time up front cost $rc_a > 0$ (e.g., construction cost). The benefit of adaptation is that it permanently reduces the house's SLR damage from d to $d_a < d$. We focus on the relevant parameter space in which, once the technology is available, the PV of adoption $a \equiv (d - d_a) - rc_a$ is positive.

[9] For a discussion of how climate change could drive innovation to meet household demand for adaptation measures, see **?**. For a related discussion on optimal capital durability in the face of disaster risk, see **?**.
[10] See **?** for a discussion of such build-back-better technologies.

Hence, with $\tau_a \geq T$ denoting the period of adaptation, equation (**??**) for the housing utility flow becomes:

$$h_t = \begin{cases} h & \text{if } t < T \\ h - d & \text{if } T \leq t < \tau_a \\ h - d_a > h - d & \text{if } t \geq \tau_a \end{cases}. \tag{26}$$

Suppose a borrower believes that this post-disaster build-back-better adaptation technology that will arrive in the future at a random date $T_a \geq T$ according to rate $r\theta$.[11] Lenders belief is $r\bar{\theta}$, where θ and $\bar{\theta}$ can be different.

Given these subjective beliefs, the borrower's and lenders' valuations of the adaptation option are $\frac{\theta}{1+\theta}a$ and $\frac{\bar{\theta}}{1+\bar{\theta}}a$, respectively. At $t = 0$, the borrower's PV of the stream of housing utility from the house with the expectation of the adaptation option is now given by:

$$v_{\lambda,\theta} \equiv h - \frac{\lambda}{1+\lambda}\left(d - \frac{\theta}{1+\theta}a\right).$$

The presence of the adaptation option will change the borrower's repayment incentive at the disaster period T. Given the positive adoption cost $rc_a > 0$, a defaulting borrower does not have an incentive to adapt. In other words, by defaulting, the borrower knows that she will effectively give up on the adaptation option. Consequently, the loan thresholds in Lemma 1 will now be adjusted to reflect the borrower's valuation of the adaptation option given her belief parameter θ:

$$B_a^{\text{safe}} = \underbrace{h - d + f}_{B^{\text{safe}}} + \frac{\theta}{1+\theta}a, \tag{27}$$

and

$$B_a^{\text{risky}} = \underbrace{h - \frac{\mu}{1+\mu}\frac{\lambda}{1+\lambda}d + f}_{B^{\text{risky}}} + \frac{\mu}{1+\mu}\frac{\lambda}{1+\lambda}\frac{\theta}{1+\theta}a. \tag{28}$$

Using the same logic as in Lemma 1, it follows that the optimal default time τ^* and the optimal adaptation time τ_a^* are given by:

Lemma 3. *There are two loan thresholds $B_a^{safe} < B_a^{risky}$ as given by (27) and (28), so that for any*

[11] That is $\mathbb{P}[T_a \leq t|T] = 1 - e^{-r\theta(t-T)}$ for $t \geq T$. This is a simplifying assumption and can be relaxed, but with additional algebra, as there will be an additional case of $T_a < T$. In this case, there is no role for adaptation belief disagreement at the disaster period T, since by that time the adaptation technology has already arrived. We assume away this case in order to focus on the role of adaptation belief disagreement.

given loan balance B,

- *When $B \leq B_a^{safe}$, the borrower will never default ($\tau^* = \infty$) and will adapt when the technology arrives ($\tau_a^* = T_a$).*
- *When $B_a^{safe} < B \leq B_a^{risky}$, borrower defaults at the disaster period ($\tau^* = T$) and will never adapt ($\tau_a^* = \infty$).*
- *When $B > B_a^{risky}$, borrower defaults immediately at $t = 0$ and never adapts ($\tau_a^* = \infty$).*

Given the optimal default and adaptation times, the borrower's expected gain from any mortgage contract $\mathbf{m}$ now becomes:

$$V_{\lambda,\theta}(\mathbf{m}) - v_{\lambda,\theta} = \begin{cases} -B & \text{if } B \leq B_a^{\text{safe}} \\ -\frac{1+\mu}{1+\mu+\lambda}B - \frac{\lambda}{1+\mu+\lambda}\left(h - d + f + \frac{\theta}{1+\theta}a\right) & \text{if } B_a^{\text{safe}} < B \leq B_a^{\text{risky}} \\ -v_{\lambda,\theta} - f & \text{otherwise} \end{cases} \tag{29}$$

and the lenders' expected value of $\mathbf{m}$ is as before

$$R_{\bar{\lambda},\bar{\theta}}(\mathbf{m}) = \begin{cases} B & \text{if } B \leq B_a^{\text{safe}} \\ \frac{(1+\mu)B + \bar{\lambda}\bar{p}_\tau}{1+\mu+\bar{\lambda}} & \text{if } B_a^{\text{safe}} < B \leq B_a^{\text{risky}} \\ r\underline{p} & \text{otherwise} \end{cases}, \tag{30}$$

except that the liquidation price of the house in the case of default $\bar{p}_\tau$ now takes into account the anticipation of adaptation technology that is believed to arrive according to parameter $\bar{\theta}$:

$$\bar{p}_\tau \equiv h - d + \frac{\bar{\theta}}{1+\bar{\theta}}a. \tag{31}$$

From (29) and (30), the surplus from a mortgage contract m in the three corresponding cases now becomes:

$$S_a(\mathbf{m}) \equiv \begin{cases} 0 \\ \underbrace{\left(\frac{1+\mu}{1+\mu+\bar{\lambda}} - \frac{1+\mu}{1+\mu+\lambda}\right)(B - (h - d + f)) - \frac{\bar{\lambda}}{1+\mu+\bar{\lambda}}f}_{\text{as before}} \\ \quad + \underbrace{\left(\frac{\bar{\lambda}}{1+\mu+\bar{\lambda}}\frac{\bar{\theta}}{1+\bar{\theta}} - \frac{\lambda}{1+\mu+\lambda}\frac{\theta}{1+\theta}\right)a}_{\text{net gain from belief disagreement about adaptation}} \\ r\bar{p} - v_{\lambda,\theta} - f < 0 \end{cases}, \tag{32}$$

where the last term in the middle case (for risky mortgage) captures the net gain from trade due to the belief disagreement about the adaptation technology (the net gain could be positive or negative, depending on belief parameters).

As in the baseline model, when $\lambda > \bar{\lambda}$ (the borrower is more SLR-pessimistic than lenders), the risky mortgage that maximizes the surplus features a binding risky loan limit ($B = B_a^{\text{risky}}$), yielding a surplus that collapses to

$$S_a(\mathbf{m}|B = B_a^{\text{risky}}) = \frac{\Delta_a}{1 + \mu + \bar{\lambda}}$$

where the baseline belief disagreement term Δ_a is given by

$$\Delta_a \equiv \frac{\lambda - \bar{\lambda}}{1 + \lambda}(d - \frac{\theta}{1 + \theta}a) - \bar{\lambda} f_a$$

except with f replaced by f_a, which the foreclosure cost plus the difference in the valuations of the adaptation option at the disaster period:

$$f_a \equiv f + \left(\frac{\theta}{1 + \theta} - \frac{\bar{\theta}}{1 + \bar{\theta}}\right) a. \tag{33}$$

The term f_a captures the deadweight loss of default. The second term in equation (33) reflects the fact that when there is a default, the house (the collateral) is transferred from a borrower with adaptation belief θ to a lender with adaptation belief $\bar{\theta}$. If the borrower is more adaptation pessimistic ($\theta < \bar{\theta}$) then the deadweight loss of default is lessened, since the foreclosed house will be transferred to a lender who will value the house more than the borrower (the reverse happens when $\theta > \bar{\theta}$).

The belief disagreement term Δ_a is positive if and only if $\lambda > \lambda_a^*$, where the belief cutoff threshold is again given by $\Delta_a = 0$, or equivalently:

$$\lambda_a^* \equiv \begin{cases} \bar{\lambda} \frac{d - \frac{\bar{\theta}}{1+\bar{\theta}} a + f}{d - \bar{\lambda} f_a - \frac{\theta}{1+\theta} a} & \text{if } d > \bar{\lambda} f_a + \frac{\theta}{1+\theta} a \\ \infty & \text{otherwise} \end{cases}, \tag{34}$$

which is the same as before except with f replaced by f_a.

Our main result (Proposition 2) continues to hold, except with λ^* replaced by λ_a^* and f by f_a:

Proposition 1. *(Optimal mortgage)*

1. *If the house is sufficiently at risk ($d > \bar{\lambda} f_a$) and the borrower is sufficiently SLR-pessimistic ($\lambda > \lambda_a^*$), then the borrower will borrow using a risky mortgage. The optimal contract*

features binding risky debt limit $B^* = B_a^{risky}$. The optimal maturity rate μ^* solves:

$$\max_{\mu} \frac{\Delta_a}{1+\mu+\bar{\lambda}} - \kappa(\mu)$$

and the leverage probability α^* solves

$$\max_{\alpha\in[0,1]} \alpha \cdot \left[\frac{\Delta_a}{1+\mu^*+\bar{\lambda}} - \kappa(\mu^*) - \frac{\kappa_0}{\eta(\alpha)}\right].$$

2. *Otherwise, the homeowner will not borrow.*

Figure **??** illustrates the result by plotting the climate belief cutoff threshold λ_a^* as a function of the borrower's adaptation belief θ. In the absence of adaptation ($a = 0$), the model collapses to the baseline, and $\lambda_a^* = \lambda^*$ is independent of θ, as illustrated by the horizontal dashed line. All borrowers with $\lambda > \lambda^*$ (the climate pessimists) will become risky borrowers in equilibrium. When there is the option to adapt ($a > 0$), λ_a^* becomes an increasing function in θ. Climate pessimistic borrowers, who are also more pessimistic about adaptation than lenders ($\theta < \bar{\theta}$), value the Dixit-Pindyck real option value to default more as there is less deadweight loss from default ($f_a < f$; as explained above: the foreclosed house will be transferred to a lender who values the house more due to the higher perceived value of the adaptation option), and hence there will be more risky leverage in equilibrium (the climate belief cutoff threshold is lower at $\lambda_a^* < \lambda^*$). The reverse happens when $\theta > \bar{\theta}$.

As in the baseline, the aggregate measure $\mathcal{A}_T^{\text{risky}}$ of risky mortgages in equilibrium at the time of disaster T will depend on the mass of borrowers with $\lambda > \lambda_a^*$ (the pink-shaded region in the figure). This mass in turns depend on the joint distribution of beliefs λ and θ. It remains an open empirical question as to how this joint distribution looks like (e.g., is adaptation belief θ positively or negatively correlated with climate belief λ), and remains open for future research.[12]

A.9.8 Belief convergence

Our results hold even when belief disagreement does not last forever. This extension

[12] To our knowledge little empirical work examines the joint distribution between climate beliefs and future climate adaptation innovation beliefs as neither the publicly available Yale Climate Opinion Survey nor the propriety Gallup survey ask respondents about both their beliefs in global warming and adaptation. The door-to-door survey that one of us conducted in **?** did not ask for these two layers of beliefs either. As noted in **?**, these beliefs may not be exogenous on the aggregate at the level as adaptation innovation can be impacted by climate skepticism in the economy. It is an interesting topic for future surveys to investigate the correlation between these two layers of beliefs.

allows beliefs to converge in a simple way by introducing an exogenous news shock about the disaster process.[13] The news shock occurs at date T_n, which arrives at a commonly known rate $r\nu$. For simplicity, we assume that upon its realization at T_n, the news shock reveals the true arrival rate of the disaster. Hence, after the news shock, agents will have common knowledge of the disaster arrival rate, and we denote this ex-post common belief by λ'. However, before the news shock, agents do not know what λ' will be, and they have different priors about the distribution of λ'. For analytical tractability, we assume that λ' can take two possible realizations: $\lambda' \in \{0, \lambda\}$. Before the news arrives, the borrower's prior places a probability weight of $\pi \in (0,1)$ on the "bad news" state $\lambda' = \lambda$, and $1-\pi$ on the "good news" state $\lambda' = 0$. Similarly, the lenders' prior places a probability weight of $\bar{\pi} \in (0,1)$ on the bad news. The borrower has a more pessimistic prior than the lenders when $\pi > \bar{\pi}$, and vice versa.

Hence, as in the baseline model, agents have heterogeneous prior beliefs at $t = 0$. However, unlike in the baseline model, *agents know that their beliefs will eventually converge at date* T_n. This leads to an interesting scenario: the realization of a *bad* news at T_n could trigger a default. In the subgame before the arrival of the news, there will be a new debt limit below which the borrower will not default immediately:

$$B^{\text{risky}}_{\text{news}} \equiv h - \frac{\mu\pi}{1+\mu+(1-\pi)(\lambda+\zeta)}\frac{\lambda}{1+\lambda}d + f > B^{\text{risky}} \equiv h - \frac{\mu}{1+\mu}\frac{\lambda}{1+\lambda}d + f.$$

The optimal default time for any given loan balance B is:

$$\tau^*_{\mathbf{m}} = \begin{cases} \infty \quad \text{(no default)} & \text{if } B \leq B^{\text{safe}} \text{ or } T > T_\mu \\ T \quad \text{(default at disaster)} & \text{if } B \in (B^{\text{safe}}, B^{\text{risky}}] \text{ and } T \leq T_\mu \\ T \wedge T_n \text{(default at disaster } \textit{or} \text{ at bad news)} & \text{if } B \in (B^{\text{risky}}, B^{\text{risky}}_{\text{news}}],\ T \wedge T_n^{\text{bad}} \leq T_\mu \\ 0 \quad \text{(default immediately)} & \text{otherwise} \end{cases}, \tag{35}$$

where T_n^{bad} denotes the date the bad news realizes.

The third region is new: if the loan balance is sufficiently large, then it is optimal for the borrower to default upon the realization of bad news, even if the disaster has not arrived. Debt in this region is riskier with a higher debt level ($B^{\text{risky}}_{\text{news}} > B^{\text{risky}}$) and a higher

[13] The news shock could represent, for example, the news of an important progress in climate science, which significantly reduces the disagreement about the speed of sea level rise. One could potentially introduce belief convergence in much more sophisticated ways, such as via a communication game as in **?**; however, interactions between agents do not necessarily lead to common beliefs when agents have different and incomplete models of learning about uncertainty (**?**). Leaving this complexity aside, we model belief convergence in a simple way via an exogenous news shock.

probability of default (which happens when either the disaster or the bad news realizes).

We now show that the main results continue to hold. A relatively optimistic homebuyer with prior $\pi \leq \bar{\pi}$ chooses not to borrow in equilibrium. A relatively pessimistic homebuyer ($\pi > \bar{\pi}$) chooses a risky mortgage that solves a slightly more complex optimization problem (36), which reflects the fact that the optimal loan balance will be either B^{risky} or $B^{\text{risky}}_{\text{news}}$, depending on parameters. Focusing on the tractable case of $f \to 0$, we can show that the latter option dominates, and hence in equilibrium, the borrower chooses a risky mortgage with $B = B^{\text{risky}}_{\text{news}}$ and will default when either the disaster or the bad news hits.

At $t = 0$, the borrower's expected gain from the mortgage is:

$$
\begin{aligned}
& V_\pi(\mathbf{m}) - v_\pi \\
= & \begin{cases}
-B & \text{if } B \leq B^{\text{safe}} \\
\pi \left[-\frac{1+\mu}{1+\mu+\lambda} B - \frac{\lambda}{1+\mu+\lambda}(h - d + f) \right] - (1-\pi) B & \text{if } B \in (B^{\text{safe}}, B^{\text{risky}}] \\
\pi \left[\begin{array}{l} -\frac{1+\mu}{1+\mu+\lambda+\nu} B - \frac{\lambda}{1+\mu+\lambda+\nu}(h - d + f) \\ -\frac{\nu}{1+\mu+\lambda+\nu}(h - \frac{\lambda}{1+\lambda} d + f) \end{array} \right] - (1-\pi) B & \text{if } B \in (B^{\text{risky}}, B^{\text{risky}}_{\text{news}}] \\
-v_\pi - f & \text{if } B > B^{\text{risky}}_{\text{news}}
\end{cases}
\end{aligned}
$$

where the expected (subjective) value of the house is:

$$
v_\pi \equiv \pi \left(h - \frac{\lambda}{1+\lambda} d \right) + (1-\pi) h = h - \frac{\pi \lambda}{1+\lambda} d.
$$

Note that the borrower is indifferent between the first and second region when $B = B^{\text{safe}}$, indifferent between the second and third region when $B = B^{\text{risky}}$, and indifferent between the third and forth region when $B = B^{\text{risky}}_{\text{news}}$, where:

$$
B^{\text{risky}}_{\text{news}} \equiv h - \frac{\mu \pi}{1 + \mu + (1-\pi)(\lambda + \zeta)} \frac{\lambda}{1+\lambda} d + f > B^{\text{risky}} = h - \frac{\mu}{1+\mu} \frac{\lambda}{1+\lambda} d + f.
$$

Similarly, the lenders' expected value of the repayment stream is:

$$
R_{\bar{\pi}}(\mathbf{m}) = \begin{cases}
B & \text{if } B \leq B^{\text{safe}} \\
-\frac{\bar{\pi}\lambda}{1+\mu+\lambda} \left(B - B^{\text{safe}} \right) + B - \frac{\bar{\pi}\lambda}{1+\mu+\lambda} f & \text{if } B \in (B^{\text{safe}}, B^{\text{risky}}] \\
-\frac{\bar{\pi}(\lambda+\nu)}{1+\mu+\lambda+\nu} \left(B - B^{\text{safe}} \right) + B + \frac{\bar{\pi}\nu}{1+\mu+\lambda+\nu} \frac{d}{1+\lambda} - \frac{\bar{\pi}(\lambda+\nu)}{1+\mu+\lambda+\nu} f & \text{if } B \in (B^{\text{risky}}, B^{\text{risky}}_{\text{news}}] \\
r\underline{p} & \text{if } B > B^{\text{risky}}_{\text{news}}
\end{cases}.
$$

The joint surplus $S(\mathbf{m}) = V_\pi(\mathbf{m}) - v_\pi + R_{\bar\pi}(\mathbf{m})$ becomes:

$$S(\mathbf{m}) = \begin{cases} 0 & \text{if } B \leq B^{\text{safe}} \\ S_1(B,\mu) \equiv \frac{(\pi-\bar\pi)\lambda}{1+\mu+\lambda}\left(B - B^{\text{safe}}\right) - \frac{\lambda\bar\pi}{1+\mu+\lambda} f & \text{if } B \in (B^{\text{safe}}, B^{\text{risky}}] \\ S_2(B,\mu) \equiv \frac{(\pi-\bar\pi)(\lambda+\nu)}{1+\mu+\lambda+\nu}\left(B - B^{\text{safe}}\right) + \frac{\frac{(\pi-\bar\pi)\nu}{1+\lambda} d - \bar\pi(\lambda+\nu) f}{1+\mu+\lambda+\nu} & \text{if } B \in (B^{\text{risky}}, B^{\text{risky}}_{\text{news}}] \\ r\underline{p} - f - v_\pi < 0 & \text{if } B > B^{\text{risky}}_{\text{news}} \end{cases}.$$

When $\pi \leq \bar\pi$ (the borrower is relatively more optimistic), both S_1 and S_2 are decreasing in B, and hence it is not optimal for the borrower to use a risky mortgage with $B > B^{\text{safe}}$. However, a safe mortgage with $B \leq B^{\text{safe}}$ yields a joint surplus of zero. Given the presence of the maintenance cost κ and the fixed cost of entry κ_0, it is thus not optimal to issue any mortgage contract in equilibrium.

When $\pi > \bar\pi$ (the borrower is relatively more pessimistic), both S_1 and S_2 are strictly increasing in B. Thus, the risky surpluses S_1 is maximized at $B = B^{\text{risky}}$ and S_2 is maximized at $B = B^{\text{risky}}_{\text{news}}$, which yields:

$$S^{\text{dis}}(\mu) \equiv S_1(B^{\text{risky}}, \mu) = \frac{1}{1+\mu}\frac{(\pi-\bar\pi)\lambda d}{1+\lambda} - \frac{\bar\pi\lambda f}{1+\mu+\lambda}.$$

$$S^{\text{news}}(\mu) \equiv S_2(B^{\text{risky}}_{\text{news}}, \mu) = \frac{1 + \left[1 - \frac{\mu\pi}{1+\mu+(1-\pi)(\lambda+\nu)}\right](\lambda+\nu)}{1+\mu+\lambda+\nu}\frac{(\pi-\bar\pi)\lambda d}{1+\lambda} - \frac{\bar\pi(\lambda+\nu) f}{1+\mu+\lambda+\nu}.$$

Putting these options together, the optimal contract (μ^*, α^*) solves:

$$\max_{\alpha\in[0,1]} \alpha \cdot \left\{ \max_{\mu\geq 0} \left[\max\left\{S^{\text{dis}}(\mu), S^{\text{news}}(\mu)\right\} - \kappa(\mu)\right] - \frac{\kappa_0}{\eta(\alpha)} \right\}. \tag{36}$$

Whether the borrower will choose a risky mortgage contract with $B = B^{\text{risky}}$ (and gets the surplus S^{dis}) or a risky mortgage contract with $B = B^{\text{risky}}_{\text{news}}$ (and gets the surplus S^{news}) depends on parameters, in particular the foreclosure cost f. However, focusing on the simple case where the foreclosure cost vanishes ($f \to 0$), it is always the case that $S^{\text{dis}} < S^{\text{news}}$ for all μ. Hence, in equilibrium the borrower will choose a risky mortgage contract in the new risky region $(B^{\text{risky}}, B^{\text{risky}}_{\text{news}}]$. In this region, the borrower will default when the disaster hits or when the bad news hits.

In summary, this extension shows that our main results continue to hold, even if agents know that beliefs will eventually converge after the realization of an exogenous news shock. However, there is an added layer of complexity, as the realization of a bad news shock could trigger equilibrium default through a repricing shock.

A.9.9 Endogenous housing price

This subsection extends the baseline model to endogenize the housing price p^*. Assume that at $t = 0$, before the loan market search takes place, the homebuyer is matched with a seller, and Nash bargaining determines the equilibrium housing price:

$$p^* = \arg\max_p (U_\lambda)^\zeta (rp - v^s)^{1-\zeta}, \tag{37}$$

where p also enters the homebuyer's utility U_λ in (5), $\zeta \in (0,1)$ is their bargaining power, and v^s is the seller's (exogenous) valuation of the house. We assume $v^s < v_\lambda$, so that the seller wants to sell the house to the homebuyer. The solution to the bargaining problem is:

$$rp^* = \underbrace{(1-\zeta)v_\lambda + \zeta v^s}_{\text{"hedonic" term}} + \underbrace{(1-\zeta)M_\lambda}_{\text{mortgage term}}, \tag{38}$$

where M_λ is the expected surplus from the optimal mortgage as derived in (**??**).

Proposition 2 (Housing price)**.** *The housing price p^* is decreasing in the homebuyer's disaster belief λ, and decreasing in the disaster exposure d.*

Proposition 2 states that, although pessimists have a higher expected mortgage surplus M_λ, the effect of a lower subjective value v_λ still dominates for asset pricing. Consequently, the asset price decreases in the degree of pessimism and disaster exposure. This is a testable implication that we also evaluate in the data.

Proof. Recall that M_λ denotes the borrower's expected value from the optimal mortgage:

$$M_\lambda \equiv \max_{\alpha\in[0,1]} \alpha \cdot \left\{ \frac{\Delta}{1+\mu^*+\bar{\lambda}} - \kappa(\mu^*) - \frac{\kappa_0}{\eta(\alpha)} \right\}.$$

The envelope theorem implies that

$$\frac{\partial M_\lambda}{\partial \lambda} = -\alpha \frac{1+\bar{\lambda}}{1+\mu+\bar{\lambda}} \frac{\partial v_\lambda}{\partial \lambda},$$

Given the bargaining solution of the house price $rp^* = (1-\zeta)v^\lambda + \zeta v^s + (1-\zeta)M_\lambda$, we have

$$r\frac{\partial p^*}{\partial \lambda} = (1-\zeta)\left[\frac{\partial v_\lambda}{\partial \lambda} + 1_{M_\lambda \neq 0}\frac{\partial M_\lambda}{\partial \lambda}\right] = (1-\zeta)\underbrace{\left[1 - 1_{M_\lambda\neq 0}\overbrace{\alpha\frac{1+\bar{\lambda}}{1+\mu+\bar{\lambda}}}^{<1}\right]}_{>0}\underbrace{\frac{\partial v_\lambda}{\partial \lambda}}_{<0} < 0.$$

Similarly,

$$r\frac{\partial p^*}{\partial d} = (1-\theta)\underbrace{\left[1 - 1_{M_\lambda \neq 0}\frac{\overbrace{\alpha\left(1-\frac{\bar{\lambda}}{\lambda}\right)}^{<1}}{1+\mu+\bar{\lambda}}\right]}_{>0}\underbrace{\frac{\partial v_\lambda}{\partial d}}_{<0} < 0.$$

□

A.9.10 General heterogeneity

Homebuyers/borrowers search for lenders in submarkets. For each type of house d, each type of borrower λ, and for each mortgage contract $\mathbf{m}$, a *submarket* consists of an (endogenous) measure $n^b_{\mathbf{m}}$ of type-λ homebuyers of a type-d house for whom contract $\mathbf{m}$ solves their optimization problem, and an (endogenous) measure $n^l_{\mathbf{m}}$ of lenders for whom approving contract $\mathbf{m}$ to a type-λ borrower satisfies their free-entry condition. Within each submarket, given $n^b_{\mathbf{m}}$ and $n^l_{\mathbf{m}}$, the number of matches produced is given by $N(n^b_{\mathbf{m}}, n^l_{\mathbf{m}})$, where N is a constant-returns-to-scale matching technology function.

The probability a borrower in the submarket finds a match (the leverage probability) is given by:

$$\alpha_{\mathbf{m}} \equiv \frac{N(n^b_{\mathbf{m}}, n^l_{\mathbf{m}})}{n^b_{\mathbf{m}}} = \mathcal{N}(n_{\mathbf{m}}) \equiv N(1, n_{\mathbf{m}}), \tag{39}$$

where $n_{\mathbf{m}} \equiv n^l_{\mathbf{m}}/n^b_{\mathbf{m}}$ is the loan market thickness and $\mathcal{N}$ is increasing and concave. Similarly, the probability that a lender finds a match is:

$$\eta_{\mathbf{m}} \equiv \frac{N(n^b_{\mathbf{m}}, n^l_{\mathbf{m}})}{n^l_{\mathbf{m}}} = N(1/n_{\mathbf{m}}, 1). \tag{40}$$

For each submarket, the lenders' free-entry condition is:

$$\eta_{\mathbf{m}}(\mathbf{m})\Pi_{\bar{\lambda}}(\mathbf{m}) = \kappa_0. \tag{41}$$

Definition 1. *A* competitive search equilibrium *consists of (i) a menu of all available loan contracts $\mathcal{M}(\lambda, d)$, (ii) measures $n^b_{\mathbf{m}}(\lambda, d)$ and $n^l_{\mathbf{m}}(\lambda, d)$ of borrowers and lenders in each submarket, and (iii) their matching probabilities $\alpha_{\mathbf{m}}(\lambda, d)$ and $\eta_{\mathbf{m}}(\lambda, d)$, for each borrower type λ, house type d, and mortgage $\mathbf{m} \in \mathcal{M}(\lambda, d)$, such that:*

1. *Given $\alpha_{\mathbf{m}}$ and $\mathcal{M}$, $n^b_{\mathbf{m}}$ is the measure of type-λ borrowers choosing loan contract $\mathbf{m}$ to finance the purchase of a type-d house in their optimization problem (5);*

2. *Given $\eta_{\mathbf{m}}$ and $\mathcal{M}$, $n^l_{\mathbf{m}}$ is the measure of lenders offering loan contract $\mathbf{m}$ to type-λ homebuyers of a type-d house, subject to (41);*

3. *Given $n^b_{\mathbf{m}}$ and $n^l_{\mathbf{m}}$, $\alpha_{\mathbf{m}}$ and $\eta_{\mathbf{m}}$ are given by (39) and (40);*

4. *$\mathcal{M}$ is the set of mortgage contracts $\mathbf{m}$ such that free-entry (41) holds with equality;*

5. *Each submarket clears:*

$$\int_{\mathbf{m}\in\mathcal{M}(\lambda,d)} n^b_{\mathbf{m}}(\lambda,d) = \phi(\lambda,d), \quad \forall \lambda, d,$$

where ϕ is the density function of the joint distribution of belief and exposure types.

It is straightforward to show that Proposition **??** continues to hold in this environment, except that now the probability that a lender is matched with a borrower $\eta_{\mathbf{m}}$ can be derived endogenously from (39) and (40) as a function of $\alpha_{\mathbf{m}}$:

$$\eta_{\mathbf{m}} = \eta(\alpha_{\mathbf{m}}) \equiv \frac{\alpha_{\mathbf{m}}}{\mathcal{N}^{-1}(\alpha_{\mathbf{m}})}, \tag{42}$$

where

$$\begin{aligned} \eta'(\alpha_{\mathbf{m}}) &= -\frac{1}{n^2_{\mathbf{m}}} < 0, \\ \eta''(\alpha_{\mathbf{m}}) &= \frac{1}{n^3_{\mathbf{m}}} \frac{1}{\mathcal{N}'(n_{\mathbf{m}})} > 0, \end{aligned}$$

as stated in the baseline model.

A.9.11 Deterministic maturity

This section provides a robustness check when mortgage maturity is deterministic instead. Suppose a mortgage is now a loan contract $\mathbf{m} = (l, b, \bar{T})$ that specifies a promised repayment flow of b per each period between $t = 0$ and a deterministic maturity period $t = \bar{T}$. At each period $0 \leq t \leq \bar{T}$, the loan balance B_t is the present value of the remaining stream of promised repayments:

$$B_t = \int_t^{\bar{T}} re^{-r(t'-t)} b dt'^{-r(\bar{T}-t)})b,$$

and follows the following law of motion:

$$\frac{dB}{dt} = -r(b - B) = -re^{-r(\bar{T}-t)}b, \tag{43}$$

which shows that the loan balance decreases over time.

The borrower's continuation value V_λ given loan balance B and repayment flow b now satisfies the following HJB equation, which takes into account law of motion (43):

$$(1+\lambda)rV_\lambda(B,b) \;=\; r(h-b) - r(b-B)\frac{\partial V_\lambda}{\partial B} + r\lambda\left[1_{B\leq B^{\text{safe}}}(h-d-B) - 1_{B>B^{\text{safe}}}f\right],$$

where $B^{\text{safe}} \equiv h - d$, and the initial condition $V_\lambda(0,b) = v_\lambda = h - \frac{\lambda}{1+\lambda}d$. The risky debt limit $B^{\text{risky}}(b)$ is the level of loan balance such that $V_\lambda(B,b) = 0$.

Similarly, the lenders' continuation value satisfies:

$$(1+\bar{\lambda})rR_{\bar{\lambda}}(B,b) \;=\; rb - r(b-B)\frac{\partial R_{\bar{\lambda}}}{\partial B} + r\bar{\lambda}\left[1_{B\leq B^{\text{safe}}}B + 1_{B>B^{\text{safe}}}(h-d)\right].$$

For tractability, for the rest of the section, we assume $f = 0$. Then, solving these equations yields:

$$B^{\text{risky}}(b) = b - (b-h)^{\frac{1}{1+\lambda}}(b - B^{\text{safe}})^{\frac{\lambda}{1+\lambda}},$$

$$V_\lambda(B,b) = \begin{cases} \frac{h-b}{1+\lambda} + \frac{b-B^{\text{safe}}}{1+\lambda}\left(\frac{b-B}{b-B^{\text{safe}}}\right)^{1+\lambda} & \text{if } B^{\text{safe}} < B \leq B^{\text{risky}}(b) \\ v_\lambda - B & \text{if } B \leq B^{\text{safe}} \end{cases},$$

and

$$R_{\bar{\lambda}}(B,b) = \begin{cases} \frac{b+\bar{\lambda}(h-d)}{1+\bar{\lambda}} - \frac{b-B^{\text{safe}}}{1+\bar{\lambda}}\left(\frac{b-B}{b-B^{\text{safe}}}\right)^{1+\bar{\lambda}} & \text{if } B^{\text{safe}} < B \leq B^{\text{risky}}(b) \\ B & \text{if } B \leq B^{\text{safe}} \end{cases}.$$

Hence, the surplus $S \equiv V_\lambda - v_\lambda + R_{\bar{\lambda}}$ is given by:

$$S(B,b) = \begin{cases} \left[\frac{\lambda-\bar{\lambda}}{(1+\lambda)(1+\bar{\lambda})} + \frac{1}{1+\lambda}\left(\frac{b-B}{b-B^{\text{safe}}}\right)^{1+\lambda} - \frac{1}{1+\bar{\lambda}}\left(\frac{b-B}{b-B^{\text{safe}}}\right)^{1+\bar{\lambda}}\right](b - B^{\text{safe}}) & \text{if } B^{\text{safe}} < B \leq B^{\text{risky}}(b) \\ 0 & \text{if } B \leq B^{\text{safe}} \end{cases}.$$

Note that for a given risky mortgage, the surplus is strictly increasing (decreasing) in B when $\lambda > \bar{\lambda}$ ($\lambda < \bar{\lambda}$). Hence, pessimistic homebuyers with $\lambda > \bar{\lambda}$ will choose the maximal risky mortgage at $B = B^{\text{risky}}(b)$, but optimistic homebuyers with $\lambda > \bar{\lambda}$ will not borrow, i.e., $B = 0$. In other words, $\lambda^* = \bar{\lambda}$ when maturity is deterministic.

Finally, as in Proposition **??**, at $t = 0$, a relative optimistic homebuyer ($\lambda \leq \lambda^*$) will

not borrow; a relatively pessimistic homebuyer ($\lambda > \lambda^*$) will borrow, and will choose a mortgage with $b^*, \bar{T}^*$ and α^* that solve:

$$\max_{\alpha\in[0,1]} \alpha \cdot \left\{ \max_{b,\bar{T}} \left[S(B^{\text{risky}}(b), b) - k(\bar{T}) \right] - \frac{\kappa_0}{\eta(\alpha)} \right\}$$

subject to $B^{\text{risky}}(b) = (1 - e^{-r\bar{T}})b$, and the loan amount l^* is given by the lender's free-entry condition:

$$\eta(\alpha) \cdot (R_{\bar{\lambda}}(B^{\text{risky}}(b^*), b^*) - rl^* - k(\bar{T}^*)) = \kappa_0,$$

where $k(\bar{T})$ and κ_0 denote the servicing cost and entry cost, with $k'(\bar{T}) > 0$ and $k''(\bar{T}) > 0$.

A.9.12 Comparison with the prediction in the literature

Our prediction contrasts with the standard prediction in models of heterogeneous beliefs that optimists leverage more (**????**). The key mechanism is the endogenous maturity channel, which is absent in the standard framework with an exogenously fixed maturity.

To see this more clearly, consider a modified case of our extended model in Section A.9.6, but assume the maturity of any mortgage contract is exogenously fixed at some T_0. The homebuyer can still choose the loan amount and repayment flow. A relatively optimistic homebuyer ($\lambda \leq \lambda^*_\omega$) will leverage with a safe loan without default, as in **?**. For a relatively pessimistic homebuyer ($\lambda > \lambda^*_\omega$), who optimally chooses a risky loan, the optimal leverage probability now solves:

$$\alpha_0 \equiv \arg\max_{\alpha\in[0,1]} \alpha \cdot \left\{ (\omega - 1)(v_\lambda + f) + \omega \Delta_\omega T_0 - \frac{\kappa_0}{\eta(\alpha)} \right\},$$

where Δ_ω denotes the belief disagreement term modified by ω. Furthermore, suppose the exogenous maturity is sufficiently short: $T_0 < \frac{1-\omega^{-1}}{1+\bar{\lambda}}$.

Then, by monotone comparative statics, the leverage probability α_0 is decreasing in λ, i.e., more pessimistic homebuyers are now *less* likely to leverage. Intuitively, when T_0 is low, the gain from maturity (captured by the term $\Delta \cdot T_0$) is relatively small, and the comparative statics is dominated by the fact that the homebuyer's PV of the flow of housing utility v_λ is decreasing in λ. In other words, the collateral channel dominates the maturity channel.

In summary, in this special case without the possibility of the borrower choosing a long-term loan contract, our model would imply that optimists would be more likely to leverage, consistent with the standard finding in the literature.

A.9.13 Time-inconsistency problem in disaster forbearance policies

First, consider a committed planner who at the beginning of $t = 0$ (before mortgage decisions are made) chooses and commits ex-ante to a disaster forbearance policy, represented by the choice of parameter ε in Section **??**. The optimal forbearance under commitment solves a cost minimization problem:

$$\varepsilon_0^* \equiv \arg\min_{\varepsilon\in[0,\bar{\varepsilon}]} \mathbb{E}\left\{ e^{-rT} \underbrace{\mathcal{A}_T^{\text{risky}}(\varepsilon)}_{\text{number of defaulting loans}} \left[\underbrace{e^{-r\varepsilon} f}_{\text{benefit of postponing default loss}} + \underbrace{\Psi(\varepsilon)}_{\text{fiscal cost}} \right] \right\},$$

where Ψ is a convex function ($\Psi > 0$, $\partial\Psi/\partial\varepsilon > 0, \partial^2\Psi/\partial\varepsilon^2 > 0$), capturing the fiscal cost of forbearing each defaulted loan, and the expectation is with respect to T according to the planner's belief. Assuming an interior solution, the first-order condition for ε_0^* is:

$$\Psi'\left(\varepsilon_0^*\right) = re^{-r\varepsilon_0^*} f - \frac{\mathbb{E}\left\{ e^{-rT} \frac{\partial \mathcal{A}_T^{\text{risky}}(\varepsilon_0^*)}{\partial\varepsilon} \right\}}{\mathbb{E}\left\{ e^{-rT} \mathcal{A}_T^{\text{risky}}(\varepsilon_0^*) \right\}} \left[e^{-r\varepsilon_0^*} f + \Psi(\varepsilon_0^*) \right]. \tag{44}$$

The key thing to notice that *the committed planner internalizes how their policy choice of ε affects the measure of risky debt in the market $\mathcal{A}_T^{risky}$*.

Second, consider a planner with limited commitment who, at date $t = T$ (after the disaster realizes), can deviate from the ex-ante policy ε_0^* and select the disaster forbearance parameter ε ex-post.[14] We will show that this planner will deviate and choose a more generous forbearance $\varepsilon_T^* > \varepsilon_0^*$. The key factor here is that at $t = T$, the mortgage choices have already been made at $t = 0$. Hence, *the limited commitment planner takes $\mathcal{A}_T^{risky}$ as given* when choosing its policy:[15]

$$\varepsilon_T^* \equiv \arg\min_{\varepsilon\in[0,\bar{\varepsilon}]} \mathcal{A}_T^{\text{risky}} \left[e^{-r\varepsilon} f + \Psi(\varepsilon) \right].$$

Again assuming interior solutions, the equilibrium ε_T^* solves the following first-order condition:

$$\Psi'(\varepsilon_T^*) = re^{-r\varepsilon_T^*} f. \tag{45}$$

The main difference between the committed and uncommitted first-order conditions is

[14] For simplicity, we assume that the limited commitment planner only has one window of opportunity to change the policy at $t = T$. One can also consider a more complex model where the limited commitment planner can instead choose at all $t \geq T$ whether to continue the forbearance.

[15] There is no expectation operator because all uncertainty will have resolved by $t = T$.

the absence of the second term of (44) in (45). This absence reflects the fact that the limited commitment planner at $t = T$ takes the mortgage choices made at $t = 0$ as given, and hence does not internalize how their policy affects the measure of risky mortgages $\mathcal{A}^{\text{risky}}$.

In a rational expectation equilibrium, private agents at $t = 0$ correctly expect the planner's choice at $t = T$, so $\mathcal{A}_T^{\text{risky}}$ in (45) is given by $\mathcal{A}_T^{\text{risky}} = \mathcal{A}_T^{\text{risky}}(\varepsilon_T^*)$. The resulting planner's choice of ε_T^* is thus time-consistent.

Since $\partial \mathcal{A}_T^{\text{risky}} / \partial \varepsilon > 0$, it follows from (44) and (45) that $\Psi'(\epsilon_T^*) > \Psi'(\epsilon_0^*)$. Furthermore, since $\partial^2 \Psi / \partial \epsilon^2 > 0$, it follows that the disaster forbearance with limited commitment is more generous:

$$\varepsilon_T^* > \varepsilon_0^*,$$

and as private agents rationally anticipate more forbearance, there will be more risky mortgages under limited commitment:

$$\mathcal{A}_T^{\text{risky}}(\varepsilon_T^*) > \mathcal{A}_T^{\text{risky}}(\varepsilon_0^*).$$

This result is very similar to how the rational expectation of more bailout from a limited commitment planner (or more inflation from a limited commitment central bank) will lead to higher equilibrium bankruptcies (or higher equilibrium inflation), as in **?** (or **?**).