Real Exchange Rate Fluctuations and Non-Homothetic Preferences

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First draft: March 2025

This draft: March 2026

Abstract

This study presents new evidence for procyclical markups and replicates them in a stochastic general equilibrium model with non-homothetic consumption aggregates. Theoretically, procyclical markups amplify cost responses to financial shocks but offset cost responses to productivity shocks. Consequently, a favorable financial shock leads to a larger real appreciation, whereas a favorable productivity shock creates a smaller real depreciation than the model with CES aggregates. Combining a favorable financial shock with an adverse productivity shock increases real appreciation and weakens its comovement with relative consumption. Both shocks generate greater volatility in the real exchange rate than in the terms of trade.

JEL Classifications: F31, F41, F44

Keywords: pricing to market, real exchange rate, variable markup

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1 Introduction

A limit on international risk sharing, home bias in consumption and financial shocks in dynamic and stochastic general equilibrium (DSGE) models in Itskhoki and Mukhin (2021, 2025), Kekre and Lenel (2024), and Chien et al. (2020) generate more realistic exchange rate behaviors than earlier studies.[1] These models assume constant-elasticity-of-substitution (CES) consumption aggregates. Unit income elasticity contradicts the micro evidence in Alessandria and Kaboski (2011), who found that prices of tradables are persistently higher in high- than in low-income countries. This pricing-to-market (PTM) pattern is twice as important as local nontraded inputs for differences in tradable prices.

Recent explanations for PTM rely on introducing non-homothetic preferences for consumption aggregates, whereas early studies rely on strategic complementarity and the distribution margin.[2] Auer et al. (2018) and Macedoni and Weinberger (2022) argued that variable income elasticity of demand results from differences in the quality of tradables across income levels. In contrast, Simonovska (2015) assumed the identical quality of tradables and and presented micro evidence for the variable income elasticity of demand. These studies, however, examine micro prices without investigating their aggregate effects.

We fill the gap in the literature by providing new evidence for procyclical markups, and replicating them in a DSGE model with non-homothetic preferences for consumption aggregates. Furthermore, we illustrate that procyclical markups play a crucial role in explaining real exchange rate (RER) behaviors in line with the exchange rate disconnect puzzle, as

[1] See Chari et al. (2002) for the role of nominal rigidities, Ghironi and Melitz (2005) for endogenous entries to exporting, and Naknoi (2008) for both features.

[2] See Kimball (1995) for the strategic complementarity in pricing in a closed-economy, and de Blas and Russ (2015) for this mechanism in an open-economy. See Atkeson and Burstein (2008) for the role of distribution margins in an open economy.

described in Backus and Smith (1993) and Atkeson and Burstein (2008). The functional form of our non-homothetic consumption aggregate is similar to Simonovska (2015), as we do not address quality differences. In each country, the price-setting monopolist produces a tradable good for domestic and foreign consumption, and the price of the good is flexible. Non-homothetic preferences cause the producer to choose markups that vary across markets due to differences in income, hence PTM emerges in equilibrium.

First, we derive a closed-form solution for prices, the RER and the terms of trade in response to temporary financial and productivity shocks. We supplement our analytical proofs with a numerical illustration and obtain three results. First, our PTM mechanism creates a 50% exchange rate pass-through and procyclical markups, implying that consumers become increasingly willing to pay higher prices during expansionary periods. Hence, a cross-country asymmetry in consumption booms creates asymmetric markup increases, with the degree of relative risk aversion becoming important for the magnitude of markup adjustments.

Second, combining a temporary positive shock to financial frictions and a temporary negative productivity shock in the home country creates rising inflation, and RER appreciation that is not perfectly correlated with relative consumption. Intuitively, a positive shock to financial frictions raises the relative cost of production while creating a consumption boom at home, increasing markups of both domestic and imported products in the home market. Hence, PTM amplifies inflation and RER appreciation in response to positive financial shocks. At the same time, a temporary negative shock to productivity increases the relative cost of production while having a negative effect on consumption, offsetting the expansionary effect of a positive financial shock and weakening the comovement between RER and relative consumption.

Finally, financial shocks generate greater volatility in the RER than in the terms of trade, even when the degree of relative risk aversion is as low as 1.5. This result is due to the amplifying effect of markup adjustments, which is decreasing in the degree of relative risk aversion. As for productivity shocks, they generate this volatility ranking when the degree of home bias is sufficiently high, causing the difference in volatility of these two variables to increase in the degree of relative risk aversion. To illustrate this mechanism, we compare our results with an alternative conventional model with CES aggregates without strategic complementarity. Such a model predicts lower volatility in the RER than the terms of trade, for any degree of relative risk aversion.

Closely related to Itskhoki and Mukhin (2021), our work contributes to the theoretical literature on exchange rates. they presented the first study integrating multiple shocks and low relative risk aversion in DSGE models to resolve the exchange rate disconnect puzzle. Their PTM mechanism relies on strategic complementarity, which implies countercyclical markups. Compared to their model, the strength of ours is its consistency with both micro evidence and macro evidence. Besides our study, evidence for procyclical markups is provided by Nekarda and Ramey (2020), conditioning on productivity, monetary, and government spending shocks. In theory, procyclical markups permit a positive demand shock to generate larger effects on inflation than countercyclical markups. For this reason, our model is potentially useful for studying the effects of fiscal shocks on inflation and their international spillovers.[3]

The other closely related study is the open economy model of markups by de Blas and Russ (2015). In their model, endogenous entries following trade liberalization generate a neg-

[3]For an open economy model of the recent COVID-19 pandemic inflation, see di Giovanni et al. (2023).

ative relationship between the average markup and aggregate output, similar to endogenous entries under a translog demand structure in Bergin and Feenstra (2009). However, trade liberalization is a low-frequency shock, whereas in our model the shocks are of medium frequency over business cycles. Introducing endogenous entries in our model would weaken the positive correlation between output and markups; however, favorable shocks over business cycles would likely create fewer entrants than trade liberalization.

The next section presents stylized facts about markups and RERs. We describe the model and its results in Section 3. Section 4 compares our results with those in the conventional model with CES preferences. Finally, we conclude our study in Section 5.

2 Stylized Facts

This section establishes stylized facts about the cyclicality of markups and RER volatility.

2.1 Cyclicality of Markups

To construct our measure of markups, we consider a monopolist producer with a linear technology and labor input for two reasons. First, we assume so in our model in the next section. Second, internationally comparable data of employment are available at quarterly frequency, whereas quarterly data of capital stock are available only at annual frequency.

In particular, the production function takes the follow form: $Y_t = A_t l_t$, where Y_t is output, A_t is labor productivity, and l_t is labor input. Hence, the marginal cost is W_t/A_t, where W_t is labor cost. Let the nominal price of output be P_t. Then, the country-level

markup is defined as the price level relative to the nominal marginal cost of labor:

$$\mu_t = \frac{P_t}{W_t/A_t}$$

We employ the quarterly data in various accounts of the International Monetary Fund's *International Financial Statistics* (IFS) from 2000Q1 to 2025Q4. Specifically, we use three measures of price levels: producer price index (PPI); consumer price index (CPI); and export price index (XPI). We calculate these price indices as the ratios of current-price and constant-price values of GDP, private consumption expenditures, and exports of goods and services, respectively, from the National Economic Accounts of the IFS. As for the nominal wage, it is measured by the wage rate, which is the monetary compensation for labor services in Labor Statistics in the IFS. Finally, productivity is measured by the residual in the production function or GDP per employed person from the same database.

There are 24 countries in this dataset, which is constrained by the availability of wage and employed person series. After obtaining the country-level markup, we filter its natural logarithm using the Baxter-King (BK) filter keeping cyclical components from 6 to 32 quarters, and the Hodrick-Prescott (HP) filter with a smoothing parameter 1,600. The natural logarithm of GDP is also subject to the same filters. The post-filter correlation between markups and GDP is reported in Table 1.

Evidently, the correlation between markups and GDP are positive for almost all countries, except for Greece and Portugal. Moreover, the correlation is higher than 0.50 for more than half of the sample, and the average is approximately 0.50 for all three price indices when we use the BK filter. When we use the HP filter, the correlation results in a slightly lower

Table 1: Cyclicality of Markup

	PPI Markup		CPI Markup		XPI Markup	
	BK filter	HP filter	BK filter	HP filter	BK filter	HP filter
Corr(Markup,GDP)						
Australia	0.85	0.82	0.88	0.85	0.87	0.86
Austria	0.69	0.67	0.59	0.56	0.71	0.66
Chile	0.74	0.65	0.70	0.61	0.82	0.78
Costa Rica	0.44	0.52	0.62	0.55	0.69	0.61
Croatia	0.44	0.34	0.38	0.29	0.59	0.47
Denmark	0.89	0.85	0.76	0.72	0.86	0.84
Germany	0.87	0.31	0.89	0.23	0.88	0.29
Greece	-0.23	-0.19	-0.27	-0.17	-0.27	-0.20
Hungary	0.26	0.17	0.23	0.13	0.31	0.20
Israel	0.16	0.04	0.14	0.02	0.14	0.04
Italy	0.73	0.69	0.79	0.70	0.83	0.77
Japan	0.68	0.65	0.66	0.58	0.78	0.75
Korea	0.27	0.28	0.32	0.28	0.13	0.21
Latvia	0.60	0.56	0.43	0.37	0.43	0.37
Mexico	0.49	0.31	0.34	0.17	0.36	0.26
Netherlands	0.68	0.64	0.63	0.60	0.67	0.65
New Zealand	0.67	0.72	0.76	0.82	0.53	0.52
North Macedonia	0.42	0.46	0.60	0.62	0.62	0.56
Norway	0.82	0.80	0.88	0.86	0.97	0.96
Poland	0.30	0.16	0.16	0.04	0.11	0.06
Portugal	-0.62	-0.13	-0.67	-0.09	-0.54	-0.09
Romania	0.43	0.28	0.25	0.15	0.38	0.32
Spain	0.41	0.21	0.51	0.16	0.53	0.24
Sweden	0.67	0.52	0.77	0.53	0.53	0.41
Summary						
Median	0.55	0.49	0.60	0.45	0.56	0.44
Average	0.49	0.43	0.47	0.40	0.50	0.44
Maximum	0.89	0.85	0.89	0.86	0.97	0.96
Minimum	-0.62	-0.19	-0.67	-0.17	-0.54	-0.20

Notes: Authors' calculation using quarterly data from the IMF's *International Financial Statistics*. The abbreviations PPI, CPI and XPI stand for producer price index, consumer price index, and export price index. Markup and GDP series, in natural logarithm, are filtered with the Baxter-King filter to retain cyclical components from 6 to 32 quarters, and the Hodrick-Prescott filter with smoothing parameter 1,600, respectively.

average and median than those with the BK filter, although the procyclicality of markups holds for almost all countries as well.

2.2 RER Volatility

Following Atkeson and Burstein (2008), we conceptualize an exchange rate disconnect from macro fundamentals by greater volatility of the RER than that of GDP and terms of trade. Our RER is measured by the quarterly real effective exchange rate (REER) series from the IFS. As for the terms of trade, it is equivalent to the ratio of XPI as in the previous subsection to the import price index (MPI), which is measured as the ratio of current-price and real-price values of imports of goods and services in the National Economic Accounts of the IFS. In total, there are 44 countries in this dataset, including the United States.[4]

In Figure 1, we plot the standard deviation of REER relative to that of GDP against the standard deviation of RER relative to that of terms of trade. In the top panel, all standard deviations are calculated after filtering the natural logarithm of relative price and GDP series with the BK filter which keeps cyclical components from 6 to 32 quarters. The corresponding statistics based on the HP filter with the smoothing parameter 1,600 are displayed in the bottom panel.

The vertical and horizontal lines in both panels of Figure 1 indicate the cutoff, such that the top right area captures the so-called exchange rate disconnect puzzle; highly volatile RERs do not transmit comparable degree of adjustments to GDP and terms of trade. Evi-

[4]The sample countries are Australia, Austria, Belgium, Brazil, Bulgaria, Canada, Chile, Colombia, Costa Rica, Croatia, Cyprus, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Israel, Italy, Japan, Latvia, Luxembourg, Malta, Mexico, Moldova, Morocco, the Netherlands, New Zealand, North Macedonia, Norway, Poland, Portugal, Romania, Slovak Republic, South Africa, Spain, Sweden, Switzerland, Uganda, the United Kingdom, and the United States.

dently, the majority of countries in our sample belong to this area and constitute evidence for the exchange rate disconnect puzzle.

Having established evidence for procyclical markups and the exchange rate disconnect puzzle, we describe our model and replicate these findings in the next section.

3 The Model

Consider two open economies, called Home and Foreign. In each country, a monopolist produces a tradable good for domestic consumption by representative households and exporting. We abstract from nontradables, which is critical in Chien et al. (2020), to highlight the PTM mechanism. Importantly, households select optimal demand for each product given their non-homothetic preferences, as described below.

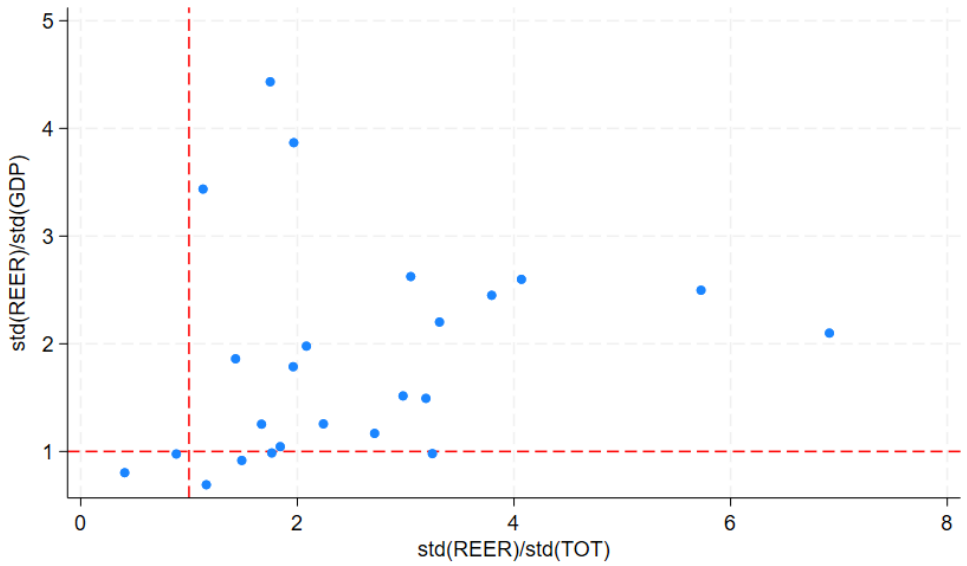
3.1 The Household's Intratemporal Problem

Let c_{it} denote consumption of product i, where $i \in \{h, f\}$ denote home and foreign products, respectively. Final consumption exhibits non-homothetic preferences over home and foreign products as follows:

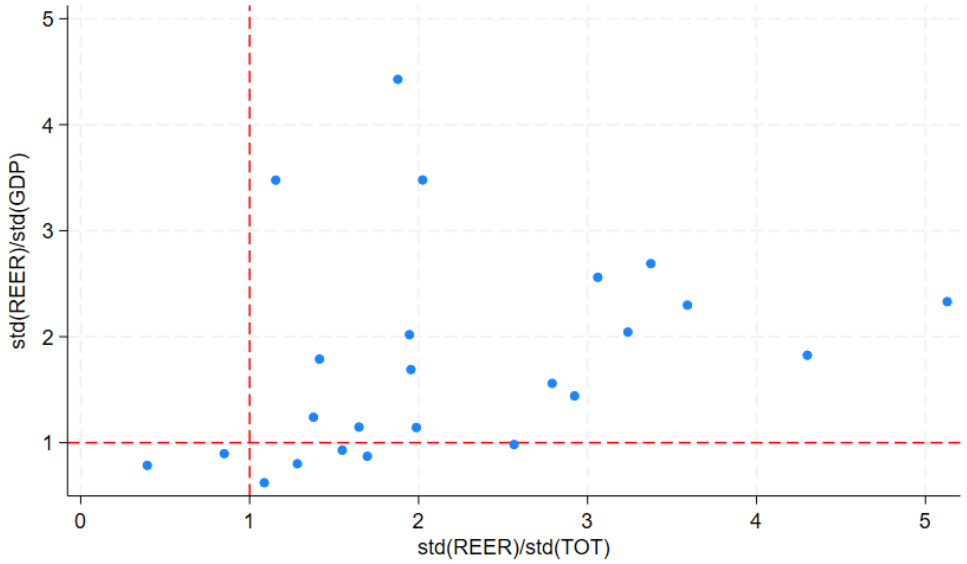
$$c_t = (c_{ht} + \gamma)^{\alpha}(c_{ft} + \gamma)^{1-\alpha}$$

where $\gamma > 0$ and $0.5 < \alpha < 1$, indicating a home bias toward the domestic product. Solving a cost minimization problem subject to unit consumption gives the optimal allocation as a

Figure 1: RER Volatility, Relative to Output Volatility and Terms of Trade Volatility



(a) BK Filter



(b) HP Filter

Notes: Authors' calculation using quarterly data from the IMF's *International Financial Statistics*. Real effective exchange rate, terms of trade and GDP series, in natural logarithm, are filtered with the Baxter-King filter to retain cyclical components from 6 to 32 quarters, and the Hodrick-Prescott filter with smoothing parameter 1,600, respectively.

function of its own price (p_{it}) and the unit cost (P_t):

$$c_{ht} = \alpha \left(\frac{P_t}{p_{ht}} \right) c_t - \gamma \tag{1}$$

$$c_{ft} = (1-\alpha) \left(\frac{P_t}{p_{ft}} \right) c_t - \gamma \tag{2}$$

$$P_t = \frac{p_{ht}^{\alpha} p_{ft}^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}. \tag{3}$$

Let the superscript * denote foreign variables, and the foreign final consumption aggregate takes a similar form:

$$c_t^* = (c_{ht}^* + \gamma)^{1-\alpha} (c_{ft}^* + \gamma)^{\alpha}$$

Likewise, we obtain the foreign optimal demand for each product and the foreign unit cost from a cost minimization problem:

$$c_{ht}^* = (1-\alpha) \left(\frac{P_t^*}{p_{ht}^*} \right) c_t^* - \gamma \tag{4}$$

$$c_{ft}^* = \alpha \left(\frac{P_t^*}{p_{ft}^*} \right) c_t^* - \gamma \tag{5}$$

$$P_t^* = \frac{p_{ht}^{*1-\alpha} p_{ft}^{*\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \tag{6}$$

3.2 The Household's Intertemporal Problem

Consider an incomplete asset market in which money and a foreign-currency bond are available. Let $\mathcal{E}_t$ denote the nominal exchange rate defined as home currency units per foreign currency. Real balances held by home residents at the end of period t are $m_t = M_t/P_t$ and

$b_t = B_t\mathcal{E}_t/P_t$, where M_t and B_t denote nominal values, respectively. Let i_t^* be the nominal interest rate on the bond between periods $t-1$ and t. Financial frictions increase the cost of borrowing above the nominal interest rate, and we denote financial frictions with $\phi_t > 1$. Let gross inflation from $t-1$ to t be $\pi = P_t/P_{t-1}$. Then, real values of asset holdings at the beginning of period t are m_{t-1}/π_t and $b_{t-1}\mathcal{E}_t/(\pi_t\mathcal{E}_{t-1})$, respectively. Let β be the discount factor, where $\beta \in (0,1)$.

The utility function exhibits separability with money and labor supply (l_t), a constant degree of relative risk aversion ($\sigma > 0$) and Frisch elasticity of labor supply ($1/\kappa$). A representative household chooses final consumption, labor supply, and the real money balance to maximize the expected lifetime utility:

$$\max_{\{c_t,l_t,m_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} c_t^{1-\sigma} - \frac{l_t^{1+\kappa}}{1+\kappa} + \ln m_t$$

subject to the intertemporal budget constraint:

$$c_t + m_t + b_t = w_t l_t + \frac{m_{t-1}}{\pi_t} + \frac{\Pi_t}{P_t} + \phi_t(1+i_t^*)\frac{b_{t-1}\mathcal{E}_t}{\pi_t\mathcal{E}_{t-1}},$$

where w_t denotes the real wage rate, and Π_t denotes dividends. The first-order conditions (FOCs) for consumption and the labor supply give labor income that is positively correlated with consumption:

$$w_t l_t^{\kappa} = c_t^{\sigma} \tag{7}$$

The FOCs with respect to consumption and bond give the Euler equation:

$$1 = \beta E_t \left[\frac{\phi_{t+1}(1+i^*_{t+1})\mathcal{E}_{t+1}}{\pi_{t+1}\mathcal{E}_t} \left(\frac{c_{t+1}}{c_t} \right)^{-\sigma} \right]$$

A similar optimization problem exists for foreign households, for which we obtain a similar Euler equation:

$$1 = \beta E_t \left[\frac{\phi^*_{t+1}(1+i^*_{t+1})}{\pi^*_{t+1}} \left(\frac{c^*_{t+1}}{c^*_t} \right)^{-\sigma} \right]$$

3.3 The Real Exchange Rate

The RER is defined as the relative price levels, $q_t = P^*_t \mathcal{E}_t / P_t$. Its movements are identical to domestic-foreign inflation differentials adjusted by the depreciation rate of the home currency. Let $\hat{x}_t$ denote a percentage deviation of variable x_t from its steady state $\bar{x}$. For analytical tractability, we assume $\bar{b} = 0$ in the steady state. The difference in the linearized versions of the two Euler equations above gives the expected rate of change in the RER:

$$E_t[\Delta \hat{q}_{t+1}] = \sigma E_t[\Delta \hat{c}_{t+1} - \Delta \hat{c}^*_{t+1}] - E_t[\hat{\phi}_{t+1} - \hat{\phi}^*_{t+1}] \tag{8}$$

The last term depends on the stochastic process of shocks to financial frictions, as in Itskhoki and Mukhin (2021, 2025).

3.4 The Monopolist Producer

The production function of the home monopolist is linear: $Y_t = A_t l_t$, where A_t is total factor productivity. The home monopolist takes its local demand in (1) and export demand in (4) as given, and sets market-specific home-currency price p_{ht} and foreign-currency price p^*_{ht} to maximize profits:

$$\max_{\{p_{ht}, p^*_{ht}\}} \Pi_t = \left(p_{ht} - \frac{W_t}{A_t}\right) c_{ht} + \left(p^*_{ht}\mathcal{E}_t - \frac{W_t}{A_t}\right) c^*_{ht}$$

The optimal prices are obtained from the FOCs as:

$$p_{ht} = \left(\frac{\alpha W_t P_t c_t}{\gamma A_t}\right)^{0.5} \tag{9}$$

$$p^*_{ht} = \left(\frac{(1-\alpha) W_t P^*_t c^*_t}{\gamma A_t \mathcal{E}_t}\right)^{0.5} \tag{10}$$

The power 0.5 indicates the elasticity of the exchange rate pass through to the local-currency price of the exported product. Additionally, these optimal prices are positively influenced by aggregate consumption in the destination market. When aggregate consumption in the domestic market and that in the foreign market are different, the firm sets different prices for each locations. Thus, PTM emerges in equilibrium. These prices also suggest that the optimal price is positively correlated with income.

To demonstrate the key mechanism, we derive the endogenous markup as the ratio of

the optimal price in each location to the marginal cost, conditioning on the optimal demand:

$$\mu_{ht} = \frac{c_{ht} + \gamma}{\gamma} \tag{11}$$

$$\mu^*_{ht} = \frac{c^*_{ht} + \gamma}{\gamma} \tag{12}$$

Proposition 1 *Variable markups are procyclical in each location.*

Proof. Let $corr(\cdot, \cdot)$ denote time series correlation of two random variables. According to (11), $corr(\hat{\mu_{ht}}, \hat{c}_{ht}) > 0$. From (1) and (7), $corr(\hat{c}_{ht}, \hat{w}_t + \hat{l}_t) > 0$. From the production function, $corr(\hat{w}_t + \hat{l}_t, \hat{Y}_t) > 0$, thus $corr(\hat{\mu}_{ht}, \hat{Y}_t) > 0$. Likewise, according to (12), $corr(\hat{\mu}^*_{ht}, \hat{c}^*_{ht}) > 0$. Additionally, (4) and the foreign counterpart of (7) imply that $corr(\hat{c}^*_{ht}, \hat{w}^*_t + \hat{l}^*_t) > 0$. From The foreign production function, $corr(\hat{w}^*_t + \hat{l}^*_t, \hat{Y}^*_t) > 0$, thus $corr(\hat{\mu}^*_{ht}, \hat{Y}^*_t) > 0$.

Proposition 1 is consistent with the effect of the price elasticity of demand, denoted by θ_{ht} in the home market and θ^*_{ht} in the foreign market. To see why, we write the markup in (11) as $\theta_{ht}/(\theta_{ht} - 1)$ where $\theta_{ht} = (c_{ht} + \gamma)/c_{ht}$. Likewise, the markup abroad in (12) is equivalent to $\theta^*_{ht}/(\theta^*_{ht} - 1)$, where $\theta^*_{ht} = (c^*_{ht} + \gamma)/c^*_{ht}$. Intuitively, increased demand reduces the price elasticity, raising the markup. In short, consumers' increasing willingness to pay during an expansionary period enables the producer to raise markups.

However, our prediction in Proposition 1 is the opposite of that of de Blas and Russ (2015). Their open economy model with Bertrand competition illustrates that bilateral trade liberalization creates new exporters and decreases the markups of domestic firms, while increasing the markups of incumbent exporters. Hence, their work predicts a negative

correlation between the average markup and aggregate output. This mechanism is similar to endogenous entries in Bergin and Feenstra (2009), who assume a translog demand structure. Compared to their models, ours is abstract from endogenous entries in the export market. Introducing endogenous entries in our model would intuitively weaken the correlation between markups and aggregate output, as new exporters would increase the price elasticity of demand. However, medium-frequency shocks over business cycles would likely generate far fewer new entrants than low-frequency trade shocks in their studies.

We obtain similar properties of markups chosen by the foreign firm. In particular, the foreign monopolist chooses market-specific home=currency price p_{ft} and foreign-currency price p^*_{ft} to maximize profits, taken as given its export demand in (2) and local demand in (5):

$$\max_{\{p_{ft},p^*_{ft}\}} \Pi^*_t = \left(\frac{p_{ft}}{\mathcal{E}_t} - \frac{W^*_t}{A^*_t}\right) c_{ft} + \left(p^*_{ft} - \frac{W^*_t}{A^*_t}\right) c^*_{ft}$$

The optimal prices for the foreign monopolist become:

$$p_{ft} = \left(\frac{(1-\alpha) W^*_t P_t c_t \mathcal{E}_t}{\gamma A^*_t}\right)^{0.5} \tag{13}$$

$$p^*_{ft} = \left(\frac{\alpha W^*_t P^*_t c^*_t}{\gamma} A^*_t\right)^{0.5} \tag{14}$$

Again, the power 0.5 indicates the degree of the exchange rate pass-through to the local-currency price of exports to the home country. Alternatively, we write endogenous markups

as:

$$\mu_{ft} = \frac{c_{ft} + \gamma}{\gamma} \tag{15}$$

$$\mu^*_{ft} = \frac{c^*_{ft} + \gamma}{\gamma} \tag{16}$$

Similar to Proposition 1, it is straightforward that price markups for foreign products in (15) and (16) are also procyclical in each location.

3.5 The Monetary Rule

The monetary rule of central banks implicitly determines the nominal exchange rate. We consider the Taylor rule:

$$\hat{i}_t = \xi \hat{\pi}_t,$$

where $\xi > 0$ and the nominal interest rate is defined as $i_t = \beta(c_{t+1}/c_t)^\sigma \pi_t - 1$. This rule and the definition of the RER determine the nominal exchange rate:

$$\hat{\mathcal{E}}_t = \hat{q}_t + \frac{\hat{\pi}_t - \hat{\pi}^*_t}{\xi}$$

As Itskhoki and Mukhin (2021, 2025) pointed out, this rule creates strong correlation between the nominal exchange rate and the RER when the central bank targets an inflation rate close to the foreign central bank. The foreign central bank is assumed to follow a similar rule.

3.6 Shock Processes

Financial frictions in each country follow the following AR(1) process:

$$\hat{\phi}_t = \rho_\phi \hat{\phi}_{t-1} + z_t, \text{ and } \hat{\phi}^*_t = \rho_\phi \hat{\phi}^*_{t-1} + z^*_t,$$

with a degree of persistence $\rho_\phi \in (0, 1)$. The random shocks z_t and z^*_t are identically and independently distributed (iid) and follow $\mathcal{N}(0, \sigma^2_z)$.

The stochastic processes for productivity at home and abroad are described by:

$$\hat{A}_t = \rho_A \hat{A}_{t-1} + v_t, \text{ and } \hat{A}^*_t = \rho_A \hat{A}^*_{t-1} + v^*_t,$$

where $\rho_A \in (0, 1)$l. The random shocks v_t and v^*_t are iid and follow $\mathcal{N}(0, \sigma^2_v)$.

To close the model, all markets must clear in every period. The goods market clears at equilibrium prices, while the labor market clears at equilibrium wages. In the asset market, there is a net zero supply of bonds in equilibrium.

3.7 The Equilibrium Real Exchange Rate

To obtain the effect of PTM on the RER, we substitute wages from (7) into optimal PTM conditions (9) and (13), and then substitute these prices into the price index (3) to obtain the equilibrium domestic price level. Likewise, the foreign counterpart of (7), optimal PTM conditions (10) and (14), and the foreign price index (6) give the equilibrium foreign price level. Assume that the population size in the two countries is identical. Then, the RER deviation from the symmetric steady state becomes:

$$\hat{q}_t = -\frac{\sigma(2\alpha - 1) + 1}{2(1 - \alpha)}(\hat{c}_t - \hat{c}^\star_t) + \frac{2\alpha - 1}{2(1 - \alpha)}(\hat{A}_t - \hat{A}^*_t - \kappa(\hat{l}_t - \hat{l}^*_t))$$

The elasticity of the relative consumption is negative, given that $\alpha \in (0.5, 1)$, illustrating the role of non-homothetic preferences in PTM. When home consumption expands and the marginal cost declines in response to positive productivity shocks at home, both foreign and home monopolists raise markups in the home market. Such a PTM pattern offsets the direct impact of productivity improvements on prices. Consequently, PTM weakens the degree of RER depreciation. As for the elasticity of the relative productivity, it is positive as long as the Frisch elasticity of the labor supply is sufficiently small.

Next, we further rewrite the expression above using the production function, $\hat{l}_t = \hat{Y}_t - \hat{A}_t$:

$$\hat{q}_t = -\frac{\sigma(2\alpha - 1) + 1}{2(1-\alpha)}(\hat{c}_t - \hat{c}_t^{\star}) + \frac{2\alpha - 1}{2(1-\alpha)}((1+\kappa)(\hat{A}_t - \hat{A}_t^*) - \kappa(\hat{Y}_t - \hat{Y}_t^*)) \tag{17}$$

where $-\eta^j \hat{\phi}_t + \eta^j \hat{A}_t = \hat{Y}_t$, and $\eta^j \in (d, s)$ denotes the elasticity of output in response to positive shocks to financial frictions and productivity, respectively. By construction, $\eta^j \in (d, s) > 0$.

To obtain the equilibrium RER, we iterate the supply side in (17) forward for one period, take the first difference and equate it with the demand side in (8). Additionally, we use the stochastic process of shocks to obtain the expectations $E_t[\Delta\hat{A}_{t+1} - \Delta\hat{A}^*_{t+1}] = -(1-\rho_A)(\hat{A}_t - \hat{A}^*_t)$ and $E_t[\hat{\phi}_{t+1} - \hat{\phi}^*_{t+1}] = \rho_\phi(\hat{\phi}_t - \hat{\phi}^*_t)$.

$$\begin{aligned} E_t[\Delta\hat{q}_{t+1}] = &-\frac{((\sigma - \kappa\eta^d)(2\alpha - 1) + 1)}{1+\sigma}\rho_\phi(\hat{\phi}_t - \hat{\phi}^*_t) \\ &- \frac{(\sigma + \kappa - \kappa\eta^s)(2\alpha - 1)}{1+\sigma}(1-\rho_A)(\Delta\hat{A}_t - \Delta\hat{A}^*_t) \end{aligned} \tag{18}$$

When $d(E_t[\Delta\hat{q}_{t+1})/d(\hat{\phi}_t))] < 0$ and $d(E_t[\Delta\hat{q}_{t+1})/d(\hat{A}_t))] < 0$), the RER is expected to

be mean-reverting. Given that $\alpha \in (0.5, 1)$, $2\alpha - 1 > 0$. This is the case when κ is sufficiently small that $\sigma > \kappa\eta^j$. In other words, this condition requires that a small increase in employment enables households to enjoy a consumption boom. Quantitative studies typically choose a small parameter value for κ to satisfy this condition; otherwise the economy will not converge to the steady state in the long run. Then, the RER follows this stochastic process: $\hat{q}_{t+1} = \delta\hat{q}_t + u_t$, where and u_t denotes white noise. Consequently, $E_t[\Delta\hat{q}_{t+1}] = -(1-\delta)\hat{q}_t$. Substituting this expectation into (18) obtains the following.

$$\hat{q}_t = \frac{((\sigma - \kappa\eta^d)(2\alpha - 1) + 1)\rho_\phi}{(1+\sigma)(1-\delta)}(\hat{\phi}_t - \hat{\phi}_t^*) + \frac{(\sigma + \kappa - \kappa\eta^s)(2\alpha - 1)(1-\rho_A)}{(1+\sigma)(1-\delta)}(\hat{A}_t - \hat{A}_t^*)$$

This expression is not exactly a closed-form solution of $\hat{q}_t$, because we know only $\eta^d > 0$ and $\eta^j > 0$ without a prediction about how other parameters determine them. For this reason, we consider a special case in which the labor supply is restricted by the size of the population; thus $\kappa = 0$, to analytically illustrate the key mechanism.

$$\hat{q}_t = \frac{(\sigma(2\alpha - 1) + 1)\rho_\phi}{(1+\sigma)(1-\delta)}(\hat{\phi}_t - \hat{\phi}_t^*) + \frac{\sigma(2\alpha - 1)(1-\rho_A)}{(1+\sigma)(1-\delta)}(\hat{A}_t - \hat{A}_t^*) \tag{19}$$

3.7.1 Persistence of the Real Exchange Rate

To solve for the first-order autocorrelation δ, we consider one shock at a time. Suppose $\hat{\phi}_t \neq 0$, and $\hat{\phi}_t^* = \hat{A}_t = \hat{A}_t^* = 0$. Then, the home demand shock generates the following first-order autocorrelation:

$$\delta = corr\left(\frac{(\sigma(2\alpha - 1) + 1)\rho_\phi}{(1+\sigma)(1-\delta)}(\rho_\phi\hat{\phi}_{t-1} + z_t), \frac{(\sigma(2\alpha - 1) + 1)\rho_\phi}{(1+\sigma)(1-\delta)}\hat{\phi}_{t-1}\right) = \rho_\phi$$

We can generalize this result for other shock variables.

Proposition 2 *RER responses to AR(1) shocks follow an AR(1) process in which persistence is identical to the persistence of underlying shocks.*

Proof. We can write (19) as $y_t = \hat{q}_t$, x_t is a shock variable, and k_x is the elasticity associated with the shock. Hence, $y_t = k_x x_t$, $k \neq 0$, and x_t is governed by the following mean-reverting process: $x_t = \rho_x x_{t-1} + n_t$, where $0 < \rho_x < 1$ and n_t is an iid white noise. Multiplying x_t with k_x gives $k_x x_t = k\rho_x x_{t-1} + k_x n_t$. This process is identical to the stochastic process of $y_t = \rho_x y_{t-1} + k n_t$. Given that n_t is an iid white noise, $k n_t$ is also an iid white noise. Hence, $corr(y_t, y_{t-1}) = \rho_x = corr(x_t, x_{t-1})$.

Unlike full-fledged international real business cycle models, our model is highly stylized and abstract from capital accumulation and investment adjustment costs; thus, the prediction in Proposition 2 is intuitive.

3.7.2 Volatility of the Real Exchange Rate

Demand-Driven Volatility. We substitute the persistence parameter, $\delta = \rho_\phi$, and shock variables, $\hat{\phi}_t \neq 0, \hat{\phi}^*_t = \hat{A}_t = \hat{A}^*_t = 0$, into (19) to obtain the closed-form solution for the RER response to a home financial shock:

$$\hat{q}^d_t = \frac{(\sigma(2\alpha - 1) + 1)\rho_\phi}{(1 + \sigma)(1 - \rho_\phi)} \hat{\phi}_t \tag{20}$$

We obtain the standard deviation of the home financial shock from its stochastic process. Consequently, the standard deviation of RER is:

$$std(\hat{q}_t^d) = \frac{(\sigma(2\alpha - 1) + 1)\rho_\phi \sigma_z}{(1 + \sigma)(1 - \rho_\phi)\sqrt{1 - \rho_\phi^2}} \tag{21}$$

Proposition 3 *Favorable financial shocks appreciate the RER, and the degree of RER appreciation is decreasing in the degree of relative risk aversion.*

Proof. From (20), $d\hat{q}_t^d / d\hat{\phi}_t > 0$ for $\alpha \in (0.5, 1)$. From (21), $d(std(\hat{q}_t^d))/d\sigma = -2(1 - \alpha)\rho_\phi \sigma_z / ((1 + \sigma)^2(1 - \rho_\phi)\sqrt{1 - \rho_\phi^2})$. Since $\alpha \in (0.5, 1)$ and $\rho_\phi \in (0, 1)$, then $d(std(\hat{q}_t^d))/d\sigma < 0$.

Understanding the intuition for Proposition 3 requires understanding how the RER adjusts to financial shocks. According to (20), a reduction in financial frictions appreciates the RER by raising the value of the home product, while increasing consumption with a home bias. Then, the PTM mechanism suggests that the home monopolist raises the local markup and lowers the export markup. In contrast, the foreign monopolist raises the export markup and lowers the local markup. For this reason, markups for both products increase in the home market and decrease in the foreign market, reinforcing the rising home-foreign wage differential. The price and RER adjustments lead home consumers to decrease exports and increase imports. Hence, this expenditure switching causes the home country to run a trade deficit and become a debtor.

Proposition 3 implies that a low degree of risk aversion amplifies RER responses to financial shocks. Intuitively, increasing risk aversion implies a declining intertemporal elasticity of

substitution and decreasing adjustments in relative consumption growth. The positive impact of consumption growth on markup adjustments declines, diminishing the amplification effect of procyclical markups. However, RER volatility in (21) will reach its lower bound if $\alpha = 0.5$, implying that the home bias is critical for our result. The importance of the home bias in consumption has been emphasized in previous studies under CES preferences including Itskhoki and Mukhin (2021, 2025) and Chien et al. (2020).

Supply-Driven Volatility. Substituting the persistence parameter, $\delta = \rho_A$, and shock variables, $\hat{A}_t \neq 0, \hat{\phi}_t = \hat{\phi}_t^* = \hat{A}_t^* = 0$, into (19) gives us the closed-form solution for the RER response to a home productivity shock:

$$\hat{q}_t^s = \frac{\sigma(2\alpha - 1)}{1 + \sigma} \hat{A}_t \tag{22}$$

Next, we obtain the standard deviation of the home productivity shock from its stochastic process to derive the standard deviation of RER responses to productivity shocks:

$$std(\hat{q}_t^s) = \frac{\sigma(2\alpha - 1)\sigma_v}{1 + \sigma\sqrt{1 - \rho_A^2}} \tag{23}$$

Proposition 4 *Favorable productivity shocks depreciate the RER, and the degree of RER depreciation increases in the degree of relative risk aversion.*

Proof. From (22), $d\hat{q}_t^s / d\hat{A}_t > 0$ for $\alpha \in (0.5.1)$. From (23), $d(std(\hat{q}_t^s))/d\sigma = (2\alpha - 1)\sigma_v / ((1 + \sigma)^2 \sqrt{1 - \rho_A^2})$. Since $\alpha \in (0.5.1)$ and $\rho_A \in (0, 1)$, then $d(std(\hat{q}_t^s))/d\sigma > 0$.

To understand Proposition 4, we must understand how the RER adjusts to a produc-

tivity shock in (22). A home productivity improvement depreciates the RER by lowering the home product's value, while expanding relative consumption. The PTM mechanism suggests that the home monopolist raises the local markup and lowers the export markup. In contrast, the foreign monopolist raises the export markup and lowers the local markup. For this reason, markups for both products increase in the home market and decrease in the foreign market, offsetting the decline in the relative cost of production. The price and RER adjustments lead home consumers to increase exports and decrease imports. Hence, this expenditure switching causes the home country to run a trade surplus and become a creditor.

According to Proposition 4, increasing risk aversion implies declining intertemporal substitution elasticity and decreasing adjustments in relative consumption growth. As a result, the offsetting effect of procyclical markups diminishes, increasing the influence of relative cost on inflation and the RER. Once again, the solution in (22) highlights the importance of the home bias. If $\alpha = 0.5$, the RER will not respond to productivity shocks.

3.8 Exchange Rate Disconnect

3.8.1 Comovement between RER and Consumption Growth

We illustrate the mechanism that generates exchange rate disconnect by deriving the correlation of the RER solution with relative consumption for each type of shock. First, we revisit (17), (20) and (22):

$$corr(\hat{q}_t^d, \hat{c}_t - \hat{c}_t^*) = -1,\ corr(\hat{q}_t^s, \hat{c}_t - \hat{c}_t^*) \in (0, 1) \tag{24}$$

These statistics imply that a positive financial shock and an adverse productivity shock both cause the RER to appreciate, while creating opposite adjustments in consumption. For this reason, the combined effect of both shocks can weaken RER comovement with relative consumption. In this case, the closed-form solution of RER responses becomes:

$$\hat{q}_t^{ds} = \hat{q}_t^d + \hat{q}_t^s = \frac{(\sigma(2\alpha-1)+1)\rho_\phi}{(1+\sigma)(1-\rho_\phi)}\hat{\phi}_t + \frac{\sigma(2\alpha-1)}{1+\sigma}\hat{A}_t \tag{25}$$

Proposition 5 *Combining a favorable financial shock with an adverse productivity shock, or an adverse financial shock with a positive productivity shock, increases RER volatility while creating an imperfect correlation between the RER and relative consumption.*

Proof. Consider $\hat{A}_t = \hat{\phi}_t \neq 0$ and $\hat{A}_t^* = \hat{\phi}_t^* = 0$. From (25), $std(\hat{q}_t^{ds}) = \sqrt{(std(\hat{q}_t^d))^2 + (std(\hat{q}_t^s))^2} > std(\hat{q}_t^d)$. From (17), $corr(\hat{q}_t^{ds}, \hat{c}_t - \hat{c}_t^*) = corr(\hat{q}_t^s, \hat{c}_t - \hat{c}_t^*)$ and thus (24) gives $corr(\hat{q}_t^{ds}, \hat{c}_t - \hat{c}_t^*) \in (-1, 0)$.

Proposition 5 and (25) suggest that a favorable financial shock and an adverse productivity shock at the same time ($\hat{q}_t^{ds} < 0$) creates RER appreciation with a small impact on relative output. The reason is that the two shocks have opposite impacts on output, while positively impacting the relative cost of production. Although this productivity shock puts downward pressure on markups, the pressure is dominated by rising costs and upward pressure on markups from the financial shock.

Combining different types of shocks to weaken the comovement between RER and real allocation is also how Itskhoki and Mukhin (2021) resolve the exchange rate disconnect puzzle. Our contribution illustrates that we can achieve a similar mechanism underlying a macro fact in an environment that is supported by micro evidence.

3.8.2 Volatility of the Terms of Trade

An alternative way to address the exchange rate disconnect puzzle is by illustrating that our model can generate greater volatility in the RER than the terms of trade, as documented in the literature (Atkeson and Burstein (2008); de Blas and Russ (2015)). We define the terms of trade as $\omega_t = p^*_{ht}\mathcal{E}_t/p_{ft}$. Thus, $\hat{\omega}_t = \hat{p}^*_{ht} + \hat{\mathcal{E}}_t - \hat{p}_{ft}$. The equilibrium prices in (10) and (13), the demand-side in (8), and the RER solution in (20) and (22) give the solution of terms of trade responses to each shock:

$$\hat{\omega}^j_t = -\tau^j \hat{q}^j_t, \tag{26}$$

where $j \in \{d, s\}$ and

$$\tau^j = \begin{cases} \frac{(1-\alpha)(\sigma-1)}{1+\sigma(2\alpha-1)} & \text{for } j = d \\ \frac{(1-\alpha)\sigma+\alpha}{\sigma(2\alpha-1)} & \text{for } j = s \end{cases}$$

Proposition 6 *Financial shocks always generate greater volatility in the RER than the terms of trade when the relative risk aversion is 3 or below. Productivity shocks will generate this result over the same range of relative risk aversion if the degree of home bias is sufficiently high.*

Proof From (40), $std(\hat{\omega}^j_t) < std(\hat{q}^j_t)$ if $\tau^j < 1$. This is the case when $\alpha > \underline{\alpha}^j$ where $\bar{\alpha}^d = 2(\sigma-1)/(2\sigma-1)$ and $\underline{\alpha}^s = 2\sigma/(2\sigma-1)$. For $1 < \sigma \leq 3$, $\underline{\alpha}^d \leq 0.5$ and $\bar{\alpha}^s > 0.75$.

The numerical illustration of Proposition (6) is provided in Table 2. Column 1 displays the degree of relative risk aversion, and the other two columns display the corresponding minimum threshold level of home bias that generates greater RER volatility than the terms

Table 2: Minimum degree of home bias required for std(RER)>std(terms of trade)

Risk aversion σ	Financial shocks $\underline{\alpha}^d$	Productivity shocks $\underline{\alpha}^s$
1.5	0.286	0.857
2	0.400	0.800
3	0.500	0.750
4	0.545	0.727
5	0.571	0.714
6	0.588	0.706

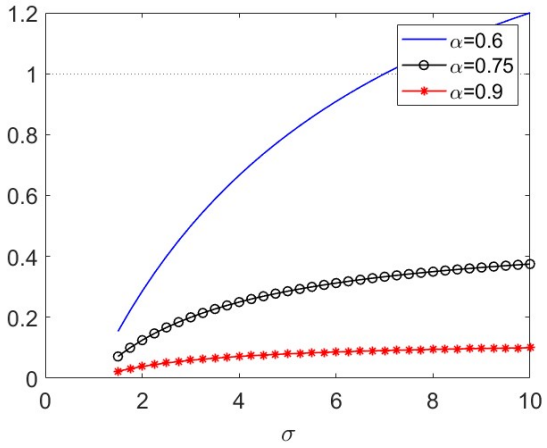
of trade for each type of shock. The threshold for financial shocks is lower than 0.5 for the risk aversion at 1.5 and 2. For any degree of relative risk aversion, the threshold level for productivity shocks is much higher than for financial shocks. This result reflects the amplification effect of markup adjustments to financial shocks and the mitigating effect of markup adjustments to productivity shocks. This difference in PTM effects across shock types decreases with the degree of risk aversion, as described in Propositions 3 and 4.

Figure 2 provides a numerical illustration of the ratio $std(\hat{\omega}_t^j)/std(\hat{q}_t^j)$, which is below 1 in the empirical literature. Panel 2a displays the result for financial shocks and Panel 2b display the result for productivity shocks. In both panels, we vary the degree of relative risk aversion, while keeping the home bias parameter constant. The solid line without markers corresponds to low degree of home bias ($\alpha = 0.6$), the solid line with circle markers corresponds to moderate degree of home bias ($\alpha = 0.75$), and the solid line with asterisk markers corresponds to high degree of home bias ($\alpha = 0.9$).

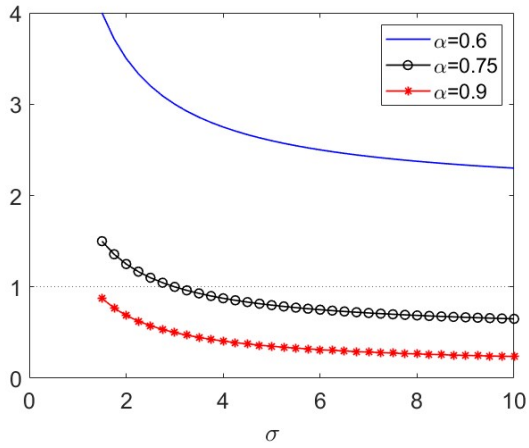
In Panel 2a, financial shocks significantly match this fact when the risk aversion is low, for all values of the home bias parameter. The lower the rise aversion, the stronger the PTM mechanism that amplifies RER movements. When $\sigma = 1.5$ and $\alpha = 0.6$, the terms of trade

Figure 2: The terms of trade volatility relative to the real exchange rate volatility

(a) Financial shocks



(b) Productivity shocks



Note: α denotes the home bias; and σ denotes the intertemporal elasticity of substitution.

volatility is only 15% of the RER volatility. When $\sigma = 2$ and $\alpha = 0.6$, the terms of trade volatility accounts for 29% of the RER volatility.

In contrast, in Panel 2b, productivity shocks do not match well when the risk aversion is low. When $\sigma = 1.5$ and $\sigma = 2$, productivity shocks replicate the empirical fact when the home bias parameter is 0.9. This extremely high level of home bias results in a lower import share in the steady state than that in Itskhoki and Mukhin (2021) and Chien et al. (2020).

In the next section, we further highlight the mechanism by comparing the prediction of our model with the alternative model with CES consumption aggregates.

4 Comparison with CES Preferences

Let θ be the elasticity of substitution between varieties and $\theta > 1$. We define the CES final consumption as:

$$c_t = \left(\alpha^{\frac{1}{\theta}} c_{ht}^{\frac{\theta-1}{\theta}} + (1-\alpha)^{\frac{1}{\theta}} c_{ft}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}$$

The household's intertemporal choice involves solving the same utility maximization problem as in our model. The difference from our model originates in the household's intratemporal choice. In particular, a cost minimization problem subject to unit consumption gives the optimal demand and the CPI as:

$$c_{ht} = \alpha \left(\frac{p_{ht}}{P_t}\right)^{-\theta} c_t \tag{27}$$

$$c_{ft} = (1-\alpha) \left(\frac{p_{ft}}{P_t}\right)^{-\theta} c_t \tag{28}$$

$$P_t = \left(\alpha p_{ht}^{1-\theta} + (1-\alpha) p_{ft}^{1-\theta}\right)^{\frac{1}{1-\theta}} \tag{29}$$

The foreign monopoly producer faces a similar cost minimization. The optimal allocation and the CPI in the foreign country are:

$$c_{ft}^* = \alpha \left(\frac{p_{ft}^*}{P_t^*}\right)^{-\theta} c_t^* \tag{30}$$

$$c_{ht}^* = (1-\alpha) \left(\frac{p_{ht}^*}{P_t^*}\right)^{-\theta} c_t^* \tag{31}$$

$$P_t^* = \left((1-\alpha) p_{ht}^{*1-\theta} + \alpha p_{ft}^{*1-\theta}\right)^{\frac{1}{1-\theta}} \tag{32}$$

Differences between the product-level demand above and that in our model lead to differences on the supply side. In particular, the home monopoly producer, facing demand in (27) and (28), chooses p_{ht} and p^*_{ht} to maximize profits. The foreign monopoly producer, facing demand in (30) and (31), chooses p_{ft} and p^*_{ft} to maximize profits. Consequently, both monopoly producers choose the same optimal markup over the marginal cost:

$$\mu_{jt} = \mu^*_{jt} = \frac{\theta}{\theta - 1} \quad j \in \{h, f\} \tag{33}$$

Clearly, aggregate consumption does not appear in the constant markup in (33), unlike optimal markups with PTM in (11), (12), (15) and (16).

Substituting the optimal markup and wage into the price indices in (29) and (32) and using the definition of the RER gives the expected future RER adjustments:

$$E_t \Delta \hat{q}_{t+1} = -\frac{\sigma(2\alpha - 1)}{2(1-\alpha)} E_t[\Delta \hat{c}_{t+1} - \Delta \hat{c}^\star_{t+1}] - \frac{2\alpha - 1}{2(1-\alpha)}(1 - \rho_A)(A_t - A^*_t) \tag{34}$$

4.1 Real Exchange Rate Volatility under CES Preferences

Equating the supply side in (34) with the demand side in (8) results in the expected RER depreciation as a function of exogenous shocks:

$$E_t \Delta \hat{q}_{t+1} = -(2\alpha - 1)\left[\rho_\phi(\hat{\phi}_t - \hat{\phi}^*_t) - (1 - \rho_A)(\hat{A}_t - \hat{A}^*_t)\right] \tag{35}$$

As in our baseline model, we solve for the RER persistence by equating $E_t \Delta \hat{q}_{t+1}$ in (35) to $-(1-\delta)\hat{q}_t$, where δ is the first-order autocorrelation of $\hat{q}_t$. Similar steps arrive at the same

result; the first-order autocorrelation of $\hat{q}_t$ is identical to the first-order autocorrelation of the underlying shocks.

Then, the closed-form solution of the RER in response to the home financial shock ($\hat{\phi}_t \neq 0, \hat{\phi}_t^* = \hat{A}_t = \hat{A}_t^* = 0$) and its standard deviation as follows:

$$\hat{q}_t^{d,ces} = (2\alpha - 1)\frac{\rho_\phi}{1 - \rho_\phi}\hat{\phi}_t \tag{36}$$

$$std(\hat{q}_t^{d,ces}) = \frac{(2\alpha - 1)\rho_\phi \sigma_z}{(1 - \rho_\phi)\sqrt{1 - \rho_\phi^2}} \tag{37}$$

Next, the closed-form solution of the RER in response to the home productivity shock ($\hat{A}_t \neq 0, \hat{\phi}_t = \hat{\phi}_t^* = \hat{A}_t^* = 0$) and its standard deviation are as follows:

$$\hat{q}_t^{s,ces} = (2\alpha - 1)\hat{A}_t \tag{38}$$

$$std(\hat{q}_t)^{s,ces} = \frac{(2\alpha - 1)\sigma_v}{\sqrt{1 - \rho_A^2}} \tag{39}$$

The solutions in both (36) and (38) reflect only adjustments of relative costs, because prices of both home and foreign products are subject to a constant markup. Thus, the degree of relative risk aversion does not matter to RER standard deviations in (37) and (39). The home bias parameter continues to play an essential role, positively affecting RER volatility generated by both types of shocks.

4.2 Terms of Trade Volatility under CES Preferences

We obtain the terms of trade from the equilibrium prices and the solution of the RER responses to shocks:

$$\hat{\omega}_t^j = -\frac{1}{2\alpha - 1}\hat{q}_t^j, \tag{40}$$

for $j \in \{d, s\}$.

Proposition 7 *The terms of trade volatility is always greater than RER volatility when the consumption aggregate exhibits a constant elasticity of substitution.*

Proof. From (40), $std(\hat{\omega}_t^j) = 1/(2\alpha - 1)std(\hat{q}_t^j)$. Since $2\alpha - 1 > 1$, $std(\hat{\omega}_t^j) > std(\hat{q}_t^j)$.

Proposition 7 suggests that the CES preference specification cannot replicate the exchange rate disconnect puzzle. This is because optimal prices under this specification lack the PTM mechanism, unlike the nested CES aggregates in Atkeson and Burstein (2008) and those with heterogeneous producers in de Blas and Russ (2015).

4.3 Comparison with the Baseline Model

The rankings of the terms of trade volatility and that of the RER in Propositions 6 and 7 clearly suggest that our model performs better than the alternative with CES preferences. However, the ranking does not necessarily imply that the absolute size of RER volatility in our model is always larger than that in the alternative model. For this reason, it is useful to compare the RER volatility across models.

Lemma 1 *Demand-driven RER volatility is greater under non-homothetic preferences than*

CES preferences, whereas supply-driven RER volatility is smaller under non-homothetic preferences than CES preferences.

Proof. From (21) and (37), $std(\hat{q}_t^d) - std(\hat{q}_t^{d,ces}) = 2(1-\alpha)\rho_\phi\sigma_z/((1+\sigma)(1-\rho_\phi)\sqrt{1-\rho_\phi^2}) > 0$ for $\alpha < 1$. From (23) and (39), $std(\hat{q}_t^s) - std(\hat{q}_t^{s,ces}) = -(2\alpha-1)\sigma_v/(1+\sigma)\sqrt{1-\rho_A^2}) < 0$ for $0.5 < \alpha < 1$.

According to Lemma 1, the ranking of RER volatility driven by the two types of shocks is reverse. As discussed earlier, PTM under non-homothetic preferences creates procyclical markup adjustments that offset the effect of productivity shocks on costs; however, such an offsetting effect does not exist under CES preferences. In contrast, the volatility of RER responses to financial shocks in our baseline model is greater than in the standard model under CES preferences, thanks to the amplifying effect of procyclical markups.

Lemma 2 *For both types of shocks, the difference in RER volatility under non-homothetic preferences and that under CES preferences decreases with the degree of relative risk aversion.*

Proof. For demand-driven RER, (21) and (37) give $d|std(\hat{q}_t^d) - std(\hat{q}_t^{d,ces})|/d\sigma = -2(1-\alpha)\rho_\phi\sigma_z/((1+\sigma)^2(1-\rho_\phi)\sqrt{1-\rho_A^2}) < 0$. For supply-driven RER, (23) and (39) give $d|std(\hat{q}_t^s) - std(\hat{q}_t^{s,ces})|/d\sigma = -(2\alpha-1)\sigma_v/((1+\sigma)^2\sqrt{1-\rho_A^2}) < 0$.

Lemma 2 further highlights the importance of our PTM mechanism. To be precise, the quantitative difference in RER volatility obtained from the two models decreases with the degree of relative risk aversion or increases with the intertemporal elasticity of substitution. Intuitively, declining intertemporal elasticity of substitution reduces relative consumption responses to shocks, making markup adjustments in our baseline model less responsive to

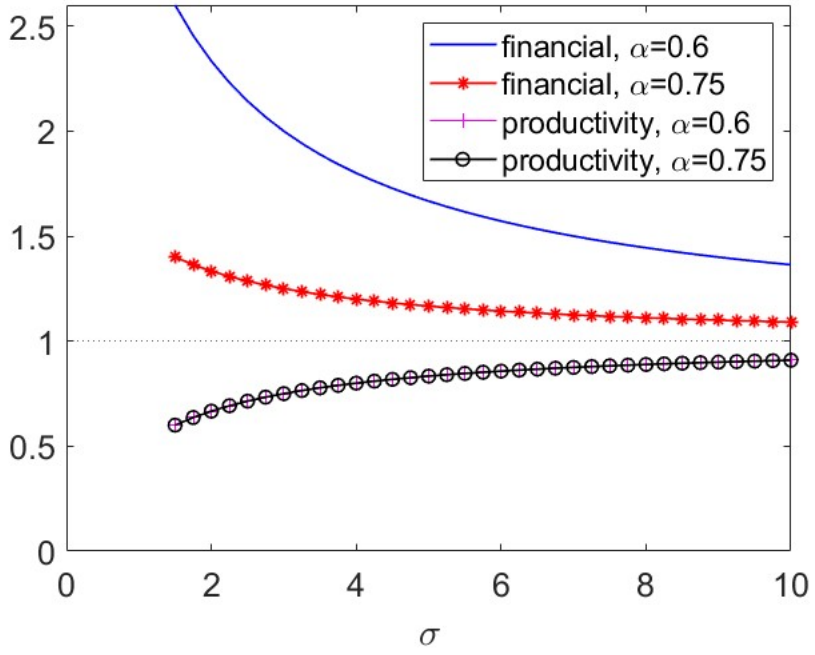
shocks. Then, both the amplification effect of markups in response to financial shocks and the offsetting effect of markups in response to productivity shocks in our model become weakened. Consequently, as the degree of relative risk aversion increases, RER volatility generated by financial shocks under non-homothetic preferences falls toward the level under CES preferences, while RER volatility generated by productivity shocks under non-homothetic preferences rises toward the level under CES preferences.

Figure 3 provides the numerical illustration for the predictions in Lemmas 1 and 2. We plot the volatility ratio $std(\hat{q}_t^d)/std(\hat{q}_t^{d,ces})$ and $std(\hat{q}_t^s)/std(\hat{q}_t^{s,ces})$ against the risk aversion, for the two home bias parameter values: 0.6 and 0.75. The higher value of home bias accentuates the PTM mechanism in response to financial shocks and yields larger differences in the two models, for any value of risk aversion. However, supply-driven volatility in the two models is equally affected by the home bias; thus, their difference is not sensitive to the degree of the home bias.

5 Concluding Remarks

We demonstrate that our DSGE model with a non-homothetic demand structure results in procyclical markups and explains the exchange rate disconnect puzzle, as in the data. Specifically, the terms of trade in our model is substantially less volatile than the RER. Furthermore, financial shocks generate this result even when the degree of relative risk aversion is as low as 1.5. For this reason, the PTM mechanism in our model is an alternative to the strategic complementarity in Itskhoki and Mukhin (2025) and de Blas and Russ (2015), and the distribution margins in Atkeson and Burstein (2008). However, we must

Figure 3: RER Volatility with non-homothetic aggregate/RER Volatility with CES aggregate



combine financial shocks with productivity shocks in the opposite direction to weaken the RER comovement with relative consumption. Thus, both types of shocks are necessary for explaining the exchange rate disconnect puzzle.

Our work also has implications for the literature on the cyclicality of markups. The novel mechanism in our model is procyclical markups, which are supported by micro evidence in the trade literature and macro evidence in our study and Nekarda and Ramey (2020). In particular, they emphasize the importance of the type of shocks, and employ US data to estimate markups conditioning on shocks, contrary to our unconditional correlation approach. They find that markups are procyclical in response to productivity, monetary and government spending shocks. However, markup adjustments in response to investment-specific technology shocks are found to be countercyclical. Their finding indicates that our model is potentially useful for studying international spillovers of fiscal shocks during and

after the COVID-19 recession. For that purpose, it is important to introduce investment into the model, and that is beyond the scope of our study.

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