Market Failure, the QE Trap and Fiscal Escape

Harjoat S. Bhamra* Nicolò Ceneda†

This Version: March 2026

Abstract

We show that a financial crisis depleting intermediary capital can destroy competitive equilibrium—not merely distort prices—making large-scale asset purchases a necessary condition for market clearing. The central bank acts as a *risk-bearer of last resort*. However, QE generates an absorbing QE Trap: if the crisis is sufficiently severe, the economy converges to permanent asset purchases and depressed output regardless of how long QE is maintained. Fiscal recapitalisation of the financial sector is the only escape, with the employment multiplier convex in crisis severity. A leverage paradox links these results: the same leverage constraint that destroys equilibrium simultaneously eliminates the intermediation gap between market prices and fundamental values. The model-derived intermediation gap delivers out-of-sample R^2 for the output gap that rises from 14% at one quarter to 45% at two years—the opposite of the horizon decay typical of persistent-predictor overfitting.

JEL Classification: G12, E44, G01, D52, G28, E58

Keywords: Large-scale asset purchases, quantitative easing, leverage constraints, intermediation gap, equilibrium existence, QE Trap, fiscal recapitalisation, limited market participation

*Imperial College Business School and CEPR. Email: h.bhamra@imperial.ac.uk.
†Imperial College Business School. Email: n.ceneda20@imperial.ac.uk.

Contents

1 Introduction **6**

1.1 Related Literature 8

2 Model **9**

2.1 Environment 9

2.2 Leverage Constraint 10

2.3 Benchmarks and Baseline Dynamics 11

3 Capital Depletion Causes Equilibrium Failure **11**

3.1 Which Assumptions Are Essential? 12

4 Quantitative Easing, the Trap, and Asset Prices **14**

4.1 Central Bank as Risk-Bearer of Last Resort 14

4.2 Equilibrium with QE 14

4.3 Dynamics with the Leverage Constraint 14

4.4 Two Absorbing Steady States and the QE Trap 15

4.5 Asset Prices and Risk Premia 17

5 The Intermediation Gap **17**

5.1 Martingale Criterion 18

5.2 Connection to the Rational Bubble Literature 18

5.3 The Closed-Form Gap Formula 18

5.4 The Intermediation Gap with and without Leverage Constraints 19

5.5 The Leverage Paradox 19

5.6 The Gap as a Sufficient Statistic for the Output Gap 21

5.7 A Three-Act Narrative 21

6 Real Effects and Policy **22**

6.1 The Unified State Variable 22

6.2 Phantom Wealth 22

6.3 Large-Scale Asset Purchases 23

6.4 Fiscal Recapitalisation 24

6.5 Mapping to Policy 25

6.6 Empirical Support 25

7 Empirical Evidence **25**
7.1 Data . . . 26
7.2 Test 1: Intermediary Capital Predicts Real Activity . . . 26
7.3 Test 4: The Exact Gap Formula . . . 27
7.4 QE Trap Evidence . . . 28
7.5 Cross-Equation SUR Test . . . 30
7.6 Calibrated Counterfactual . . . 30
7.7 Assessment . . . 31

8 Conclusion **31**

A Complete-Markets Benchmark **36**

B Baseline Equilibrium: Limited Participation Without Leverage Constraints **36**
B.1 State Variable and Equilibrium Characterisation . . . 36
B.2 Dynamics of the Wealth Distribution . . . 37
B.3 Two Invariant Regimes and Convergence . . . 37

C Proofs for Limited Participation (No Leverage Constraint) **38**
C.1 Asset Prices and Real Variables . . . 40

D Proof of Equilibrium Failure **42**

E Derivation of σ_ω With Leverage Constraint **44**

F Q-Martingale Property **45**

G Boundary Classification **46**

H Convergence **47**

I Proof of Intermediation Gap Classification **48**

J Robustness and Additional Diagnostics **49**
J.1 Alternative Specifications . . . 49
J.2 Inference Robustness . . . 54
J.3 Out-of-Sample Evidence . . . 57
J.4 Subsample and Outlier Robustness . . . 59
J.5 QE Trap Additional Tests . . . 60
J.6 Cross-Equation SUR Test . . . 62

J.7 Calibrated Counterfactual . . . 62
J.8 QE Program Timeline . . . 66

A Proofs for the Complete-Markets Benchmark **1**

B Propositions from Section 4 **1**
B.1 Properties of the Volatility Function . . . 1
B.2 Three Invariant Regimes . . . 2
B.3 Boundary Classification . . . 2

C Proof of Asset Pricing Formulae **3**

D Intermediation Gap Formula Derivation **3**

E Franchise Value and Dynamic Incentive Compatibility **4**

F Employment Multiplier and Welfare Analysis **6**
F.1 Employment Multiplier of Recapitalisation . . . 6
F.2 Welfare Analysis . . . 6

G Proofs of Policy Results **7**

H Translation of Hugonnier's Results **7**

I Online Appendix N: Partial Participation and CRRA Robustness **9**
I.1 Extended Model . . . 9
I.2 Robustness of Equilibrium Failure . . . 10
I.3 Dynamics and Preservation of Results . . . 11
I.4 Bubble Classification . . . 11
I.5 CRRA Preferences . . . 11

J Online Appendix O: Endogenous Non-Participation via Ambiguity **12**
J.1 Household Preferences under Ambiguity . . . 12
J.2 Robust Equilibrium Failure . . . 12
J.3 Endogenous Non-Participation . . . 13
J.4 QE Trap and Bubble Classification . . . 13
J.5 Three-Region Equilibrium . . . 14

K Additional Empirical Tests **14**
K.1 Test 2: Capital Scarcity Predicts Equity Risk Premia 14
K.2 Test 3: QE Reacts to Intermediary Capital Shortfall 15

1 Introduction

The 2008 financial crisis depleted the capital of the financial institutions that bear much of the economy's aggregate risk, triggering unprecedented central bank asset purchases.[1] In the United States, the Federal Reserve expanded its balance sheet from under $1 trillion to over $4 trillion; the subsequent unwinding took a decade and remained incomplete when the COVID-19 pandemic necessitated further expansion. Meanwhile, the Troubled Asset Relief Program (TARP) injected capital directly into the financial sector. Similar interventions followed in Europe, though with significant delays in bank recapitalisation (Brunnermeier, James, and Landau, 2016). Two questions arise: when is QE necessary, and how does one escape the QE trap of permanent asset purchases and depressed output?

We answer both questions in a continuous-time production economy with two types of agents: financiers, who hold equity and bear all aggregate risk, and households, who save only through risk-free bonds. Financiers face leverage constraints arising from moral hazard and prudential regulation—frictions that long predate any crisis. Because households supply labour, financial frictions affect employment and output directly. The entire economy is characterised by a single state variable: the financier's share of total wealth, ω. When leverage constraints do not bind, the financier's wealth share drifts toward the efficient benchmark and the economy functions well—explaining why crises are surprises, not inevitabilities. We establish four sets of results.

First, equilibrium failure. When a large shock depletes financier capital and forces leverage constraints to bind, competitive equilibrium ceases to exist. The household's bond demand is perfectly inelastic, so once the constrained financier cannot absorb the entire equity supply, excess supply emerges that no price adjustment can eliminate. This is not a recession or a market crash—the price system itself breaks down. Three ingredients are required—limited participation, leverage constraints, and depleted capital—and all three were present in 2008.

Second, the central bank as risk-bearer of last resort. A central bank that purchases the equity the constrained financier cannot hold, financing these purchases by issuing bonds, restores market clearing and equilibrium existence. In every existing theory of QE, asset purchases improve upon a distorted but viable competitive outcome—through portfolio balance effects (Vayanos and Vila, 2021), credit intermediation (Gertler and Karadi, 2011), liquidity provision (Cui and Sterk, 2021), or risk redistribution (Silva, 2020). Our result is

[1] Strictly, *quantitative easing* refers to expanding central bank reserves, *large-scale asset purchases* (LSAPs) to purchasing specific securities, and *credit easing* to targeting particular credit markets. In practice, and in academic usage, these terms are used interchangeably. Following common convention, we use "QE" throughout.

different in kind: without central bank purchases no competitive outcome exists at all. The central bank acts not as a lender of last resort or a market-maker of last resort, but as a *risk-bearer of last resort*.

Third, the QE Trap and fiscal escape. The restored equilibrium has two absorbing steady states. At the good steady state the financier is fully capitalised, no constraints bind, and the economy coincides with the frictionless benchmark. At the bad steady state the financier is minimally capitalised, the central bank permanently holds a large share of the equity market, and output is permanently depressed. If the crisis pushes ω below the bad steady state, the economy is trapped: it converges to permanent QE and depressed output regardless of how long the central bank maintains its purchases. We call this the *QE Trap*—a term introduced in a policy context by Koo (2014); we provide the first general equilibrium characterisation. A one-time fiscal recapitalisation—a transfer large enough to push ω above the bad steady state—is the only escape. The central bank prevents market failure; the fiscal authority breaks the trap.

Fourth, the leverage paradox. Without the leverage constraints imposed by moral hazard and prudential regulation, the market for aggregate risk always clears. However, equilibrium asset prices exceed fundamental values—a rational bubble in the sense of Hugonnier (2012)—creating an *intermediation gap* that grows convexly as financier capital erodes. Because ours is a production economy, the intermediation creates an output gap: larger intermediation gaps signal larger shortfalls in employment and output. Constraining intermediary leverage eliminates the intermediation gap and hence the output gap. But when leverage constraints bind, they destroy equilibrium. The constraint that corrects prices and the output gap breaks the market.

This tradeoff is inescapable: we prove that no economy in this class can simultaneously deliver a zero intermediation and output gaps and unconditional market clearing. The mechanism is that $Q = 1/\omega$, the inverse financier wealth share, is a local martingale in all versions of the model; its classification as a strict local martingale (positive intermediation gap) or a true martingale (no intermediation gap) is governed by the same boundary behaviour of the intermediary wealth-share volatility that determines whether equilibrium can fail. The model-derived intermediation gap delivers out-of-sample R^2 for the output gap that rises from 14% at one quarter to 45% at two years—the opposite of the horizon decay typical of persistent-predictor overfitting.

Severe intermediary capital depletion can eliminate competitive equilibrium; quantitative easing (QE) restores market clearing, but fiscal recapitalisation is required to escape the trap.

1.1 Related Literature

QE and unconventional monetary policy. The existing literature identifies several channels: portfolio balance effects on term premia (Gagnon et al., 2011; Krishnamurthy and Vissing-Jørgensen, 2011; Vayanos and Vila, 2021), credit substitution for impaired intermediation (Gertler and Karadi, 2011; Gertler and Kiyotaki, 2010; Del Negro et al., 2017), liquidity transformation (Cui and Sterk, 2021), the central bank balance sheet as an independent instrument (Cúrdia and Woodford, 2011), government debt maturity management (Greenwood, Hanson, and Stein, 2015), and central bank balance sheet sustainability (Reis, 2016). The closest paper to ours is Silva (2020), who studies a risk channel of QE in a continuous-time production economy with heterogeneous risk tolerance. In every one of these models, equilibrium exists with or without intervention; QE is a policy choice. Our result is qualitatively different: below a threshold, no equilibrium exists without central bank purchases.

Financial frictions, traps, and recapitalisation. The paper builds on the intermediary asset pricing literature (Brunnermeier and Sannikov, 2014; He and Krishnamurthy, 2013, 2019; Di Tella, 2017), sharing the feature that a single state variable drives all equilibrium quantities but generating the additional result that equilibrium can fail entirely. Our QE Trap differs from existing trap concepts along four dimensions: (i) *non-existence* of competitive equilibrium, not merely distortion; (ii) scarcity of risk-bearing capacity rather than safe assets (Caballero and Farhi, 2018), demand shortfalls from balance-sheet deleveraging (Eggertsson and Krugman, 2012; Guerrieri and Lorenzoni, 2017), or liquidity (Bigio, 2015); (iii) an absorbing bad steady state requiring fiscal transfer, not self-correcting or escapable through monetary policy alone (Rognlie, Shleifer, and Simsek, 2018); and (iv) the leverage paradox linking the trap mechanism to asset pricing. On recapitalisation, Philippon and Schnabl (2013) and Veronesi and Zingales (2010) study the optimal form of intervention, taking its necessity as given; we derive the necessity from the absorbing steady state.

Rational bubbles and the intermediation gap. Basak and Cuoco (1998) study the impact of limited household participation on asset prices in an exchange economy, whereas we study a production economy with endogenous labor and a central bank. Hugonnier (2012) proves that portfolio constraints generate rational bubbles in otherwise bubble-free economies. In our model, leverage constraints can *close* rather than open the intermediation gap—linking the rational bubble literature to the equilibrium existence problem. We derive an exact ODE for the gap in a production economy, replacing the constant-volatility assumption of Hugonnier (2012)'s exchange setting, and show that the gap-to-price ratio delivers horizon-stable out-of-sample predictive content for the output gap.

The remainder of the paper is organised as follows. Section 2 presents the model. Sec-

tion 3 establishes the equilibrium failure result. Section 4 introduces QE, derives the two-steady-state equilibrium, and analyses asset pricing implications. Section 5 establishes the intermediation gap results. Section 6 links the gap to real outcomes and derives policy implications. Section 7 presents empirical evidence. Section 8 concludes. All proofs are in the Appendix.

2 Model

2.1 Environment

Time is continuous with $t \in [0, \infty)$. Uncertainty is driven by a standard Brownian motion $\{Z_t\}_{t\geq 0}$ on a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ satisfying the usual conditions.

The economy is populated by a representative household, a representative financier, and a continuum of monopolistically competitive firms indexed by $f \in [0, 1]$.

Technology and firms. Each firm f produces a differentiated good using the linear technology $Y_t(f) = A_t N_t(f)$, where A_t is aggregate productivity and $N_t(f)$ is labour input. Productivity evolves as geometric Brownian motion:

$$\frac{\mathrm{d}A_t}{A_t} = \mu\,\mathrm{d}t + \sigma\,\mathrm{d}Z_t, \qquad \mu \in \mathbb{R},\ \sigma > 0. \tag{1}$$

Households aggregate differentiated goods via the Dixit–Stiglitz aggregator with elasticity of substitution $\epsilon > 1$. With flexible prices, each firm sets a constant markup over marginal cost. In symmetric equilibrium, factor shares are constant:

$$D_t = \frac{1}{\epsilon}\,Y_t, \qquad \frac{W_t}{P_t}\,N_t = \frac{\epsilon - 1}{\epsilon}\,Y_t, \tag{2}$$

where D_t denotes aggregate dividends, W_t/P_t is the real wage, $Y_t = A_t N_t$ is aggregate output and $N_t = \int_0^1 N_t(f)\,\mathrm{d}f$ is aggregate employment.

Preferences. The household has preferences

$$U_h = \mathbb{E}\int_0^\infty e^{-\delta t}\left(\ln C_{h,t} - \frac{N_t^{1+\varphi}}{1+\varphi}\right)\mathrm{d}t, \qquad \delta > 0,\ \varphi \geq 0. \tag{3}$$

The financier has logarithmic preferences over consumption:

$$U_i = \mathbb{E} \int_0^\infty e^{-\delta t} \ln C_{i,t} \, \mathrm{d}t. \tag{4}$$

Both agents share the same discount rate δ.[2]

Financial markets and participation. Two financial assets are traded: a risk-free bond in zero net private supply paying rate r_t, and an equity claim on aggregate dividends in unit net supply. The household can invest only in the risk-free bond. The financier invests in both bonds and equity, with portfolio equity share ϕ_t (the fraction of her wealth allocated to equity). The household's budget constraint is deterministic:

$$\mathrm{d}H_{h,t} = \left(r_t H_{h,t} + \frac{W_t}{P_t} N_t - C_{h,t} \right) \mathrm{d}t, \tag{5}$$

where $H_{h,t}$ is the household's financial wealth. The financier's budget constraint is:

$$\mathrm{d}H_{i,t} = (r_t H_{i,t} - C_{i,t}) \, \mathrm{d}t + \phi_t H_{i,t} \left(\frac{\mathrm{d}P_{D,t} + D_t \, \mathrm{d}t}{P_{D,t}} - r_t \, \mathrm{d}t \right), \tag{6}$$

where $H_{i,t}$ is the financier's financial wealth and $P_{D,t}$ is the equity price. Since households cannot hold equity, market clearing requires $\phi_t H_{i,t} = P_{D,t}$.

2.2 Leverage Constraint

The financier faces a state-dependent leverage limit:

$$\phi_t \leq \bar{\phi}(\omega_t), \qquad \bar{\phi}(\omega) \equiv 1 + \lambda(\omega - \underline{\omega}), \tag{7}$$

where $\omega_t \equiv H_{i,t}/P_{Y,t}$ is the financier's share of total wealth $P_{Y,t}$, $\underline{\omega} \in (0, 1/\epsilon)$ is the minimum capital threshold, and $\lambda > 0$ is the leverage sensitivity.

Assumption 1 (Parameters)**.** $\epsilon > 1$, $\sigma > 0$, $\delta > 0$, $\varphi \geq 0$, $0 < \underline{\omega} < 1/\epsilon$, $\lambda > 0$, *and* $\lambda\underline{\omega} \leq 1$.

The constraint combines two frictions that predate quantitative easing. *Above the threshold* ($\omega \geq \underline{\omega}$): following Gertler and Kiyotaki (2010) and Gertler and Karadi (2011), the financier can divert a fraction of assets for private benefit. Households lend only against

[2]Online Appendix N shows that the equilibrium failure result extends to CRRA preferences with elasticity of intertemporal substitution near unity, and to partial household equity participation. Online Appendix O endogenises the limited participation restriction: with Knightian uncertainty of just 1.2% of the Sharpe ratio, the household optimally holds zero equity.

verifiable collateral: the capital buffer above the regulatory minimum, yielding (7) with λ parameterising enforcement efficiency. In the Appendix (Proposition 24), we show that log utility generates a closed-form franchise value of intermediation, making (7) a conservative bound. *Below the threshold* ($\omega < \underline{\omega}$): under Prompt Corrective Action, statutory since 1991 (FDIC, 1991), undercapitalised intermediaries face mandatory restrictions on risky asset holdings, again yielding (7). For the entire pre-crisis period, these constraints existed but did not bind; what changed in 2008 was the *state of financier balance sheets*.

The constraint binds when $\bar{\phi}(\omega) < 1/(\epsilon\omega)$, i.e., when $\omega < \omega_c$ where ω_c is defined by:

$$\bar{\phi}(\omega_c) = \frac{1}{\epsilon\omega_c} \qquad \Longleftrightarrow \qquad \lambda\omega_c(\omega_c - \underline{\omega}) = \frac{1}{\epsilon} - \omega_c. \tag{8}$$

2.3 Benchmarks and Baseline Dynamics

When both agents can trade all assets, the economy admits a complete-markets benchmark in which consumption shares, labour supply, and total wealth are constant, with $P_{Y,t} = Y_t/\delta$ (Appendix A).

When only the financier can hold equity but no leverage constraint is imposed, the entire economy is characterised by ω_t. Goods market clearing under log utility uniquely pins consumption: $C_{h,t} = (1-\omega_t)Y_t$ and $C_{i,t} = \omega_t Y_t$, regardless of asset prices. The household's financial wealth is therefore $H_{h,t} = (1/\epsilon - \omega_t)P_{Y,t}$, pinned entirely by ω_t. Since the household invests only in bonds, her bond demand is

$$B^d_{h,t} = \left(\frac{1}{\epsilon} - \omega_t\right) P_{Y,t}, \tag{9}$$

which is *perfectly inelastic with respect to the interest rate*. This property is the key mechanism behind the failure theorem.

The inverse wealth share $Q_t = 1/\omega_t$ is a non-negative local martingale, and ω_t converges almost surely to the efficient level $1/\epsilon$, the complete-markets benchmark, but with infinite expected hitting time (Appendix B). This "Zeno's paradox" explains why the economy functioned well for decades before 2008: financiers were sufficiently capitalised that leverage constraints, though present, never bound.

3 Capital Depletion Causes Equilibrium Failure

We now add the leverage constraint (7) and ask: what happens when a crisis, a sequence of adverse productivity shocks, depletes the financier's wealth to the point where the constraint

binds? When ω falls below ω_c, the required portfolio share $1/(\epsilon\omega)$ exceeds the permitted leverage $\bar{\phi}(\omega)$, and the economy's institutional infrastructure, which functioned seamlessly for decades, becomes incompatible with equilibrium.

Theorem 1 (Equilibrium failure)**.** *Under Assumption 1, with limited stock market participation and log utility, no competitive equilibrium exists when $\omega_t < \omega_c$ in the absence of central bank intervention.*

The proof is in Appendix D. The key mechanism is that log utility pins the household's financial wealth at $(1/\epsilon - \omega)P_Y$, a quantity that is perfectly inelastic with respect to the interest rate (9). Bond market clearing then requires $\bar{\phi} = 1/(\epsilon\omega)$, but the leverage constraint imposes $\bar{\phi}(\omega) < 1/(\epsilon\omega)$. There is a residual excess supply that no price adjustment can eliminate.

The failure requires three ingredients, all present in 2008. First, *limited participation*: households cannot absorb equity risk. Second, *leverage constraints*: from moral hazard and regulation, limiting how much equity the financier can hold per unit of capital. Third, *depleted capital*: a crisis that pushes ω below ω_c, causing the constraint to bind. Remove any one ingredient and equilibrium survives.

3.1 Which Assumptions Are Essential?

Theorem 1 relies on three modelling choices whose robustness merits scrutiny. Online Appendices N and O establish that the result is robust along each dimension; we summarise the findings here.

Assumption	Essential?	What happens if relaxed	Reference
Log utility	For literal non-existence	CRRA with γ near 1: equilibrium exists but requires $r \to -\infty$, making it economically equivalent to non-existence. For $\gamma < 1$, failure extends outright.	Prop. N.6
Limited participation	Degree does not matter	Failure survives for any household equity share $\alpha_h \in [0, 1)$. The crisis threshold $\omega_c(\alpha_h)$ falls monotonically in α_h but remains strictly positive.	Thm. N.1
Leverage constraint	Yes	Without the constraint, no failure occurs. But the intermediation gap is positive and the economy is vulnerable to explosive dynamics.	Thm. 1
Exogenous non-participation	No	Ambiguity aversion with $\kappa \approx 1.2\%$ of the Sharpe ratio generates endogenous non-participation at the bad steady state; $\kappa \approx 4.7\%$ suffices everywhere.	Thms. O.3, O.5

CRRA preferences. The perfectly inelastic bond demand that drives Theorem 1 is a property of log utility ($\gamma = 1$). Online Appendix N shows that for CRRA with γ near 1, bond demand becomes interest-rate elastic, but the sensitivity is of order $|\gamma - 1|$—too small to absorb the excess supply without driving the equilibrium interest rate to $-\infty$ (Proposition N.6). For $\gamma < 1$, the income effect dominates the substitution effect, exacerbating the excess supply; equilibrium fails outright. Log utility is therefore the boundary of a family of increasingly pathological equilibria.

Robust failure below $\underline{\omega}$. Perhaps the strongest result is Theorem O.3: for $\omega < \underline{\omega}$, no competitive equilibrium exists *regardless of the household's equity share*, including $\alpha_h = 1$. The leverage constraint forces $\bar{\phi}(\omega) < 1$ when $\omega < \underline{\omega}$, so the financier cannot even hold her own wealth entirely in equity. The household's financial wealth is too small to fill the gap. This result requires no assumption on household preferences, ambiguity, or participation.

4 Quantitative Easing, the Trap, and Asset Prices

4.1 Central Bank as Risk-Bearer of Last Resort

The equilibrium failure of Theorem 1 motivates the introduction of a public sector. Once the crisis has depleted financier capital to the point where leverage constraints bind, the central bank purchases the residual equity that the constrained financier cannot absorb.

Assumption 2 (Central bank and fiscal authority)**.** *The central bank holds fraction α_t of the equity market:*

$$\alpha_t = \max\bigl(0,\ 1 - \epsilon\bar{\phi}(\omega_t)\omega_t\bigr) = \begin{cases} 1 - \epsilon\bar{\phi}(\omega_t)\omega_t & \text{if } \omega_t < \omega_c, \\ 0 & \text{if } \omega_t \geq \omega_c. \end{cases} \tag{10}$$

The central bank finances equity purchases by issuing risk-free bonds. Profits and losses are remitted to the fiscal authority (Treasury), which may transfer resources to the financial sector as a recapitalisation (Section 6.4).

4.2 Equilibrium with QE

Proposition 2 (Equilibrium with QE)**.** *Under Assumptions 1 and 2, a competitive equilibrium exists for all $\omega \in (0,1)$. All equilibrium quantities — total wealth $P_{Y,t} = Y_t/\delta$, consumption, employment, and financial wealth — are identical to the unconstrained model. The only objects that change are the financier's portfolio share $\phi_t = \min\bigl(\bar{\phi}(\omega_t),\ 1/(\epsilon\omega_t)\bigr)$ and the central bank's equity holding α_t given by (10).*

The central bank absorbs exactly the equity that the constrained financier cannot hold. The household, whose bond demand is perfectly inelastic, is indifferent between lending to the financier and lending to the central bank, so all real quantities are unchanged.

4.3 Dynamics with the Leverage Constraint

Theorem 3 (Dynamics of ω_t with leverage constraint)**.** *The financier's wealth share evolves as*

$$\mathrm{d}\omega_t = \frac{\sigma_\omega(\omega_t)^2}{\omega_t}\,\mathrm{d}t + \sigma_\omega(\omega_t)\,\mathrm{d}Z_t, \tag{11}$$

where the volatility function is continuous on $(0,1)$ *and given by*

$$\sigma_\omega(\omega) = \begin{cases} \dfrac{\sigma(1+\varphi)\lambda(1-\omega)\,\omega\,(\omega-\underline{\omega})}{(1+\varphi)(1-\omega)-\lambda\omega(\omega-\underline{\omega})} & \text{if } \omega < \omega_c, \\ \dfrac{\sigma(1+\varphi)(1-\omega)(1/\epsilon-\omega)}{(1+\varphi)(1-\omega)-(1/\epsilon-\omega)} & \text{if } \omega \geq \omega_c. \end{cases} \tag{12}$$

The drift relation $\mu_\omega = \sigma_\omega^2/\omega$ reflects the financier's excess return from bearing equity risk: the drift is always positive when the diffusion is nonzero, so the financier's capital accumulation is fastest when her wealth share is most volatile.

The sign of σ_ω determines how the economy responds to shocks (Proposition 21 in the Appendix). In the crisis regime ($\omega < \underline{\omega}$), $\sigma_\omega < 0$: positive productivity shocks *reduce* the financier's wealth share, because the de-leveraged financier holds less equity than her consumption share warrants. In the recovery regime ($\underline{\omega} < \omega < 1/\epsilon$), $\sigma_\omega > 0$: positive shocks improve financier capitalisation, aiding recovery.

Proposition 4 (Q-martingale preservation)**.** $Q_t = 1/\omega_t$ *is a non-negative local martingale regardless of whether the leverage constraint binds.*

This is a consequence of log utility: the financier's consumption-wealth ratio is constant at δ, and the linear and quadratic terms in ϕ cancel exactly.

4.4 Two Absorbing Steady States and the QE Trap

Theorem 5 (Two absorbing steady states)**.** *Both* $\omega = \underline{\omega}$ *and* $\omega = 1/\epsilon$ *are absorbing states: if* $\omega_0 \in \{\underline{\omega}, 1/\epsilon\}$*, then* $\omega_t = \omega_0$ *for all* $t \geq 0$*. No other absorbing states exist in* $(0,1)$*.*

At each absorbing state, both drift and diffusion of ω vanish simultaneously.

The good steady state ($\omega = 1/\epsilon$). The financier holds exactly 100% of her wealth in equity, no leverage constraint binds, the central bank holds no equity, and the economy coincides with the complete-markets benchmark.

The bad steady state ($\omega = \underline{\omega}$). The central bank permanently holds fraction $\alpha = 1 - \epsilon\underline{\omega}$ of the equity market. Employment is depressed:

$$N(\underline{\omega}) = \left(\frac{1-1/\epsilon}{1-\underline{\omega}}\right)^{\frac{1}{1+\varphi}} < 1 = N(1/\epsilon). \tag{13}$$

Both $\underline{\omega}$ and $1/\epsilon$ are unattainable in finite time (Appendix G), so no transition between regimes is possible.

Theorem 6 (Convergence). *For any* $\omega_0 \in (0,1) \setminus \{\underline{\omega}, 1/\epsilon\}$*:*

$$\omega_t \xrightarrow{a.s.} \begin{cases} \underline{\omega} & \text{if } \omega_0 \in (0, \underline{\omega}), \\ 1/\epsilon & \text{if } \omega_0 \in (\underline{\omega}, 1). \end{cases} \tag{14}$$

In all cases, $\mathbb{E}[\tau] = \infty$*, where* τ *is the hitting time of the relevant absorbing state.*

The almost sure convergence follows from the supermartingale property of $Q = 1/\omega$. The infinite expected hitting time means QE starts with large purchases that taper gradually but never fully unwind in finite expected time — the *QE Trap*.

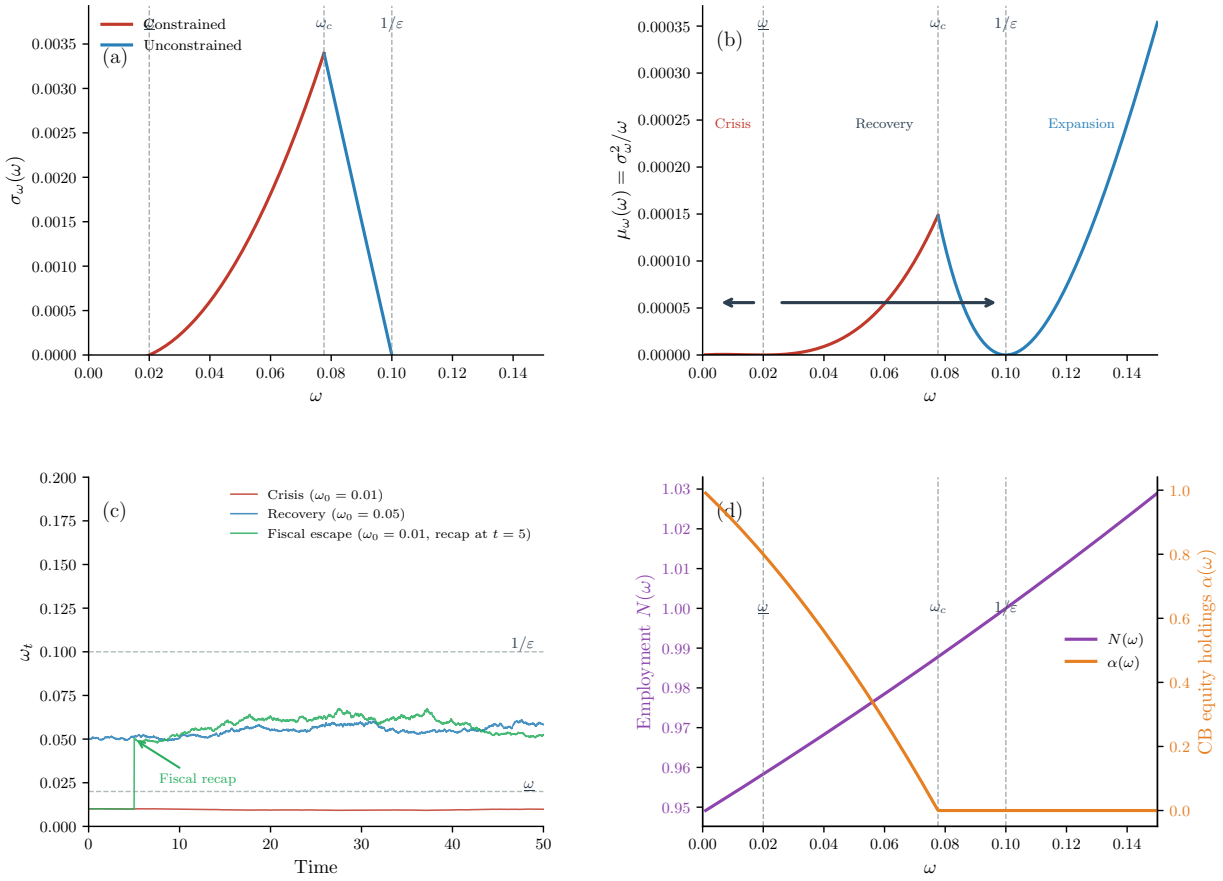


Figure 1: Phase diagram of the model. (a) Wealth-share volatility $\sigma_\omega(\omega)$, with constrained (red) and unconstrained (blue) regions. (b) Drift $\mu_\omega = \sigma_\omega^2/\omega$, with arrows showing convergence to absorbing steady states. (c) Simulated sample paths: crisis (red), recovery (blue), and fiscal escape (green). (d) Employment and central bank equity holdings. Vertical dashed lines at $\underline{\omega} = 0.02$, $\omega_c = 0.078$, and $1/\epsilon = 0.10$. Baseline calibration: $\epsilon = 10$, $\underline{\omega} = 0.02$, $\lambda = 5$, $\sigma = 0.15$, $\varphi = 1$.

4.5 Asset Prices and Risk Premia

Theorem 7 (Asset pricing)**.** *The market price of risk, equity return volatility, and risk-free rate are:*

$$\Theta_t = \frac{\sigma_\omega(\omega_t)}{\omega_t} + \sigma + \frac{\sigma_\omega(\omega_t)}{(1+\varphi)(1-\omega_t)}, \tag{15}$$

$$\sigma_{R,t} = \sigma + \frac{\sigma_\omega(\omega_t)}{(1+\varphi)(1-\omega_t)}, \tag{16}$$

$$r_t = \delta + \mu_{Y,t} - \sigma_{R,t}\,\Theta_t, \tag{17}$$

where σ_ω is given by (12) and $\mu_{Y,t}$ is the drift of output. The equity risk premium is $\mu_{R,t} - r_t = \sigma_{R,t}\,\Theta_t$.

The risk-free rate (17) is depressed in crisis through the $\sigma_{R,t}\,\Theta_t$ term: as ω falls, leveraged intermediaries amplify return volatility σ_R and the market price of risk Θ, whose product drives r well below the rate of time preference. This balance-sheet channel differs from the habit and long-run risk mechanisms of Campbell and Cochrane (1999) and Bansal and Yaron (2004), where r falls because of increased effective risk aversion or precautionary savings; here the channel is intermediary capital, and it operates even under log utility (constant individual risk aversion).

In the recovery regime ($\underline{\omega} < \omega < 1/\epsilon$), $\sigma_\omega > 0$ and risk premia are above the frictionless benchmark. Risk premia peak at an intermediate ω and decline as $\omega \to 1/\epsilon$, consistent with the hump shape documented by He, Kelly, and Manela (2017). Positive productivity shocks raise ω and compress risk premia, delivering countercyclicality.

Output gap. The log output gap relative to the frictionless benchmark $\omega_n \equiv 1/\epsilon$ is:

$$x_t \equiv \ln Y_t - \ln Y_{n,t} = \frac{1}{1+\varphi} \ln\left(\frac{1-\omega_n}{1-\omega_t}\right). \tag{18}$$

5 The Intermediation Gap

The inverse wealth share $Q_t = 1/\omega_t$ is a non-negative local martingale (Proposition 4). Whether Q is a strict or true local martingale determines whether equilibrium asset prices exhibit a positive intermediation gap—the component of asset prices above fundamental value attributable to the concentration of risk-bearing in undercapitalised intermediaries. The gap is a symptom of intermediary weakness, not monetary accommodation: it is largest when the financial sector is most distressed.

5.1 Martingale Criterion

For a one-dimensional diffusion $\mathrm{d}X = \sigma_X(X)\,\mathrm{d}Z$ starting at $X_0 > 0$, X is a strict local martingale—meaning $\mathbb{E}[X_t] < X_0$ for some t—if and only if (Engelbert and Schmidt, 1985; Mijatović and Urusov, 2012):

$$\int_{X_0}^{\infty} \frac{x}{\sigma_X(x)^2}\,\mathrm{d}x < \infty. \tag{19}$$

When the integral diverges, X is a true martingale and $\mathbb{E}[X_t] = X_0$ for all t.

5.2 Connection to the Rational Bubble Literature

Hugonnier (2012) proves that in a two-agent economy with a portfolio constraint, equilibrium asset prices contain rational bubbles if and only if the weighting process—the ratio of marginal utilities—is a strict local martingale. In our model the weighting process reduces to

$$\lambda_t \;=\; \frac{1-\omega_t}{\omega_t} \;=\; Q_t - 1, \tag{20}$$

so λ inherits the martingale classification of Q. The following equivalence is a direct consequence of Hugonnier (2012, Theorem 1):

$$Q = 1/\omega \text{ is a strict local martingale} \quad\Longleftrightarrow\quad \text{intermediation gap} > 0. \tag{21}$$

5.3 The Closed-Form Gap Formula

Proposition 8 (Intermediation gap formula)**.** *In the baseline model (limited participation, no leverage constraint), the intermediation gap on the aggregate stock, expressed as a fraction of the market price, is:*

$$\frac{B_t}{P_{D,t}} = (1-\omega_t)^{\gamma}, \tag{22}$$

where the gap exponent is

$$\gamma \;\equiv\; \frac{1+\sqrt{1+8\delta/v^2}}{2}, \tag{23}$$

with $v = \|v_\lambda\|$ denoting the volatility of the weighting process.[3]

The proof is in Appendix D. The exact gap ODE (Remark 3) yields a qualitatively identical but quantitatively 4–5× smaller solution.

[3]Formula (22) is exact when v_λ is constant, as in Hugonnier (2012)'s exchange economy. In our production economy, $v_\lambda(\omega)$ is state-dependent and the exact gap-to-price ratio $b(\omega)$ satisfies the ODE $b'' = (2\delta/\sigma_\omega^2)\,b$ with $b(0) = 1$ and $b(1/\epsilon) = 0$. The approximate formula overestimates $b(\omega)$ by a factor of 4–5 at typical ω values, but all qualitative properties (existence, monotonicity, convexity, gap elimination by CB) are preserved. Section 7.3 reports empirical results with the exact formula.

Volatility paradox. The exponent γ is decreasing in fundamental volatility v:

$$\frac{\partial \gamma}{\partial v} = -\frac{4\delta}{v^3\sqrt{1+8\delta/v^2}} < 0. \tag{24}$$

Low fundamental volatility is precisely when the gap function is most convex and the amplification potential greatest: lower σ reduces σ_ω, increasing $2\delta/\sigma_\omega^2$ and producing a steeper gap function.[4]

5.4 The Intermediation Gap with and without Leverage Constraints

Theorem 9 (Intermediation gap classification)**.**

(i) ***Baseline model*** *(limited participation, no leverage constraint): In the controlled regime* $(0, 1/\epsilon)$*,* $Q = 1/\omega$ *is a* ***strict local martingale****. Equilibrium asset prices exhibit a positive intermediation gap.*

(ii) ***Constrained model*** *(leverage constraint, QE active):* $Q = 1/\omega$ *is a* ***true martingale*** *in all three regimes, crisis* $(0, \underline{\omega})$*, recovery* $(\underline{\omega}, 1/\epsilon)$*, and expansionary* $(1/\epsilon, 1)$*. Equilibrium asset prices are free of any intermediation gap.*

The proof is in Appendix I. The entire difference comes from one factor of ω in the volatility function near $\omega = 0$. In the baseline, $\phi = 1/(\epsilon\omega) \to \infty$, so the ω in $\sigma_\omega = \omega(\phi - 1)\sigma_R$ cancels, leaving σ_ω positive at the boundary. With the constraint, $\phi \leq 1 + \lambda(\omega - \underline{\omega})$ is bounded, so $\sigma_\omega \propto \omega$.

	σ_ω **near** $\omega = 0$	σ_Q **near** $Q = \infty$	**Integral test**	**Gap?**
Baseline	$\sigma_\omega^0 \to c_0 > 0$	$c_0\, Q^2$	$\int q^{-3}\, \mathrm{d}q < \infty$	**Yes**
Constrained	$\lvert\sigma_\omega\rvert \sim c_0'\, \omega$	$c_0'\, Q$	$\int q^{-1}\, \mathrm{d}q = \infty$	**No**

5.5 The Leverage Paradox

The intermediation gap classification and the equilibrium failure result are two manifestations of a single mathematical object: the behaviour of σ_ω at the origin.

Proposition 10 (Leverage paradox)**.**

[4] The model is silent on reach-for-yield effects: the household's bond demand is perfectly inelastic (log utility), so QE has no portfolio rebalancing effect. In richer models, lower safe returns push investors into risky assets, creating a distinct form of overvaluation not connected to the equilibrium existence problem.

(i) ***Baseline economy*** *(no leverage constraint): competitive equilibrium exists for all* $\omega \in (0, 1/\epsilon)$ *and* $Q = 1/\omega$ *is a strict local martingale—the intermediation gap is positive.*

(ii) ***Constrained economy*** *(leverage constraint (7) with QE):* $Q = 1/\omega$ *is a true martingale in all three regimes—no intermediation gap.*

(iii) ***Impossibility.*** *The combination of gap-free asset prices and unconditional equilibrium existence (without central bank intervention) is impossible: for* any *leverage function* $\bar{\phi} : (0, 1/\epsilon) \to \mathbb{R}_+$*, if competitive equilibrium exists for all* ω *without intervention, then a positive intermediation gap exists.*

Proof. Parts (i) and (ii) restate Theorem 9.

For part (iii), suppose competitive equilibrium exists for all $\omega \in (0, 1/\epsilon)$ without central bank intervention. Equity market clearing requires $\phi(\omega)\,\omega\,P_Y = P_D = P_Y/\epsilon$, so $\phi(\omega) = 1/(\epsilon\omega)$. The constraint cannot bind at any ω—otherwise $\phi = \bar{\phi}(\omega) < 1/(\epsilon\omega)$ and the equity market does not clear—so $\bar{\phi}(\omega) \geq 1/(\epsilon\omega)$ for all ω. With $\phi = 1/(\epsilon\omega)$, the wealth-share diffusion coefficient is

$$\sigma_\omega(\omega) = \frac{(1/\epsilon - \omega)\,\sigma_R}{D_u(\omega)}, \qquad D_u(\omega) = \left(1 - \tfrac{1}{\epsilon}\right) + \varphi(1-\omega) > 0,$$

giving $\sigma_\omega(0^+) = \sigma_R/\big(\epsilon\, D_u(0)\big) > 0$. Converting to $Q = 1/\omega$: $\sigma_Q = \sigma_\omega/\omega^2 \sim c_0\, Q^2$ with $c_0 = \sigma_\omega(0^+) > 0$. The integral test (19) yields $\int_{Q_0}^{\infty} q\,\sigma_Q(q)^{-2}\,\mathrm{d}q \sim c_0^{-2}\int q^{-3}\,\mathrm{d}q < \infty$, so Q is a strict local martingale and the intermediation gap is positive. □

Corollary 11 (Impossibility)**.** *Within the class of economies in this paper:*

(a) *Any economy with unconditional equilibrium existence necessarily has a positive intermediation gap.*

(b) *Any economy with gap-free asset prices is vulnerable to equilibrium failure.*

Proof. Part (a) is Proposition 10(iii). Part (b) is its contrapositive: if equilibrium existed for all ω without intervention, Proposition 10(iii) would imply a positive intermediation gap, contradicting the hypothesis of gap-free prices. □

The policy package of QE plus fiscal recapitalisation resolves the tradeoff: QE restores equilibrium existence when bounded leverage causes market failure, while the leverage constraint simultaneously closes the intermediation gap. The exact gap ODE provides a constructive proof of closure: on $[\underline{\omega}, 1/\epsilon]$, the only solution to $b'' = (2\delta/\sigma_\omega^2)b$ with $b(\underline{\omega}) = b(1/\epsilon) =$

0 is $b \equiv 0$ (by the maximum principle). Fiscal recapitalisation then moves the economy to the good steady state where the constraint ceases to bind, the central bank exits, and both equilibrium existence and correct pricing are achieved without intervention.

5.6 The Gap as a Sufficient Statistic for the Output Gap

Since both the intermediation gap and the output gap are monotone functions of ω_t, the gap-to-price ratio is a sufficient statistic for the output gap:

$$x_t = \frac{1}{1+\varphi} \ln\left(\frac{1-\omega_n}{1-\omega_t}\right) = G\big(b(\omega_t)\big), \tag{25}$$

for an increasing function G. Larger intermediation gaps signal larger output gaps. The approximate formula $(1-\omega)^\gamma$, being nearly linear over the empirical range, collapses G to an approximately linear relationship, discarding the model's predicted curvature. The exact formula $b(\omega)$ preserves it. Empirically, this distinction matters: the exact formula delivers out-of-sample R^2 that rises from 14% at one quarter to 45% at two years, whereas the approximate formula's out-of-sample R^2 decays from 19% to below zero at the same horizons (Tables 2 and 3).

Since T^* and the benefit-to-cost ratio (32) are also functions of ω, the pre-crisis gap-to-price ratio encodes the fiscal cost of future intervention via $\omega_0 = b^{-1}(B/P_D)$. Crucially, this mapping is available only *before* QE. Once the intermediation gap vanishes, the policymaker loses this market-based diagnostic—the same intervention that corrects asset prices destroys their informational content about the output gap.

5.7 A Three-Act Narrative

1. **Before the crisis** ($\omega > \omega_c$, no binding constraint): Equilibrium exists. Asset prices exhibit an intermediation gap with gap-to-price ratio $b(\omega)$. As intermediary capital erodes, the gap grows convexly—the exact formula gives $b(0.05) = 0.19$ at a typical recovery value, rising to $b(0.02) \approx 0.60$ in severe distress—partially masking the deterioration of fundamental values: market prices overstate the health of the financial sector (Section 6.2).

2. **During the crisis** (ω drops below ω_c): Leverage constraints bind. Without QE, no equilibrium exists. The question of asset price gaps is moot.

3. **After QE** (QE active): Equilibrium is restored. The intermediation gap vanishes: the stock price equals its fundamental value. The sudden disappearance of the gap reveals

the true severity of intermediary undercapitalisation. The economy may be trapped at the bad steady state with correct prices but depressed output. Fiscal recapitalisation must be calibrated to these revealed fundamentals, not to pre-crisis gap-inflated prices.

The three acts trace an information arc: the intermediation gap obscures fundamentals before the crisis, its disappearance reveals fundamentals during the crisis, and correct post-QE prices enable—but do not substitute for—fiscal intervention.

6 Real Effects and Policy

6.1 The Unified State Variable

The financier's wealth share ω_t simultaneously governs three equilibrium objects:

Asset prices and the intermediation gap:

$$\frac{B_t}{P_{D,t}} = b(\omega_t), \qquad b''(\omega) = \frac{2\delta}{\sigma_\omega(\omega)^2}\, b(\omega). \tag{26}$$

Employment:

$$N_t = \left(\frac{1-1/\epsilon}{1-\omega_t}\right)^{\frac{1}{1+\varphi}}, \quad \text{with } N'(\omega_t) > 0. \tag{27}$$

Output:

$$Y_t = A_t N_t = A_t \left(\frac{1-1/\epsilon}{1-\omega_t}\right)^{\frac{1}{1+\varphi}}. \tag{28}$$

When intermediary wealth erodes (ω_t falls), three things happen together: the intermediation gap grows (convexly), employment falls, and output contracts. The convexity of the gap function transmits directly to real variables: when ω_t is low, small further declines produce large increases in the gap *and* large employment losses, creating a fragility trap that intensifies as conditions deteriorate.

6.2 Phantom Wealth

In the baseline economy, the equity price decomposes into a fundamental value and an intermediation gap:

$$P_D = P_D^{\text{fund}} + B, \qquad P_D^{\text{fund}} = P_D\big[1 - b(\omega)\big], \qquad B = P_D\, b(\omega). \tag{29}$$

Proposition 12 (Phantom wealth)**.** *In the baseline model:*

(i) $P_D^{\text{fund}}(\omega) \to 0$ *as* $\omega \to 0$*: at the point of maximum crisis, the entire equity market value is an intermediation gap.*

(ii) The fundamental-to-market ratio $P_D^{\text{fund}}/P_D = 1 - b(\omega)$ *is strictly increasing in* ω*.*

(iii) The gap ratio $B/P_D = b(\omega)$ *overstates the financier's equity position by a factor that grows convexly as the crisis deepens.*

Proof. (i) $P_D(0) = AN(0)/(\epsilon\delta) > 0$ and $b(0) = 1$, so $P_D^{\text{fund}}(0) = 0$. (ii) $b' < 0$ follows from $b > 0$, $b'' > 0$, $b(0) = 1$, $b(1/\epsilon) = 0$. (iii) Convexity: $b'' = (2\delta/\sigma_\omega^2)b > 0$ whenever $b > 0$. □

Part (i) is the key result for policy. A policymaker observing pre-crisis market prices sees positive equity values and may conclude the financial sector retains substantial wealth. In fact, as ω approaches zero, fundamental value vanishes and the entire observed price is phantom wealth—value that would disappear if the economy transitioned to a regime with correct prices. The exact gap formula quantifies this: at $\omega = 0.05$, the gap-to-price ratio is $b(0.05) = 0.19$; at $\omega = 0.02$, $b \approx 0.60$. Because the required recapitalisation (31) is calibrated to the *fundamental* state variable ω, not to market prices that include the gap, policymakers who rely on market-based measures systematically underestimate the severity of undercapitalisation. The sufficient-statistic property extends to the fiscal transfer:

$$T^*(\omega_0) = \Omega(\underline{\omega}) - \Omega(\omega_0), \qquad \omega_0 = b^{-1}\left(\frac{B}{P_D}\right), \tag{30}$$

where b^{-1} is the inverse of the (strictly decreasing) exact gap function. Crucially, this mapping is available only before QE: once the gap vanishes, the policymaker loses this market-based diagnostic.

6.3 Large-Scale Asset Purchases

Under Assumption 2, the central bank absorbs the residual equity supply that the constrained financier cannot hold, preventing equilibrium failure but not altering the economy's long-run dynamics.

Proposition 13 (Central bank balance sheet at the bad steady state)**.** *At* $\omega = \underline{\omega}$*, the central bank permanently holds equity worth* $(1 - \epsilon\underline{\omega})\, Y_t/(\epsilon\delta)$ *and earns the equity premium* σ^2 *on these holdings.*

At the bad steady state, the central bank holds a potentially large share of the equity market: with $\underline{\omega} = 0.02$ and $\epsilon = 10$, it holds 80%. The equity premium equals its frictionless value σ^2 because $\sigma_\omega(\underline{\omega}) = 0$ eliminates all wealth redistribution dynamics.

6.4 Fiscal Recapitalisation

Proposition 14 (Recapitalisation)**.** *Suppose $\omega_0 < \underline{\omega}$ (crisis regime). A one-time capital injection that raises the financier's wealth share to $\omega_{0+} > \underline{\omega}$ places the economy in the recovery regime, which then converges almost surely to the good steady state $1/\epsilon$ without further intervention.*

Remark 1 (Transfer size with general equilibrium feedback)**.** *Because total wealth $P_Y(\omega) = AN(\omega)/\delta$ is increasing in ω, a dollar transfer raises ω by less than $T/P_Y(\omega_0)$. The transfer required to reach target ω^* is:*

$$T^* = \frac{A}{\delta}\big[\omega^* N(\omega^*) - \omega_0 N(\omega_0)\big], \tag{31}$$

which exceeds the naïve estimate by $\omega^*[P_Y(\omega^*) - P_Y(\omega_0)]$, though the correction is small under baseline calibration (approximately 1%).

Employment multiplier. Recapitalisation from ω_0 to $\omega_0 + \Delta$ produces an immediate employment gain of $[(1-\omega_0)/(1-\omega_0-\Delta)]^{1/(1+\varphi)} - 1$, which is convex in crisis severity: the worse the crisis, the larger the marginal benefit. Under the baseline calibration, the permanent long-run gain from escaping the trap is 4.3% of efficient employment. The formal statement and proof are in Appendix G (Corollary 26).

Cost–benefit analysis. The benefit-to-cost ratio of recapitalisation is:

$$\frac{\text{NPV}}{\text{Cost}} = \frac{N(1/\epsilon) - N(\underline{\omega})}{\underline{\omega}\, N(\underline{\omega}) - \omega_0\, N(\omega_0)}. \tag{32}$$

Under baseline calibration this exceeds 2: the government spends approximately 2% of total wealth to permanently raise output by over 4%. The fiscal cost of delay is not a larger transfer but a larger cumulative output loss; each period in the crisis regime forgoes output on the steep portion of the employment curve, so delays at the onset are most costly.

Welfare. The household weakly prefers the bad steady state (higher consumption share $1 - \underline{\omega}$), so recapitalisation is not a Pareto improvement; it involves redistribution from household to financier. The detailed welfare calculation and moral hazard analysis are in Appendix F.2.

6.5 Mapping to Policy

The model rationalises three distinct phases of crisis intervention:

1. *Large-scale asset purchases (central bank):* when $\omega < \omega_c$ and leverage constraints bind, the central bank purchases equity to prevent market failure—corresponding to the Fed's emergency facilities in 2008–2009.

2. *Recapitalisation (fiscal authority):* the Treasury injects capital to push ω above $\underline{\omega}$, as in TARP (October 2008); European recapitalisations followed in 2010–2014, with the delay producing the persistent stagnation the model predicts.

3. *Taper and exit:* once $\omega > \underline{\omega}$ and recovery begins, the leverage constraint eventually stops binding and the central bank's equity holdings decline to zero.

Phase 1 alone is insufficient: QE maintains equilibrium existence but cannot alter long-run dynamics. Because employment and the intermediation gap share a common driver, policies that stabilise ω_t simultaneously stabilise financial valuations and employment—financial stability and full employment are complementary objectives.

6.6 Empirical Support

The model's predictions are supported by the empirical record. In the United States, TARP injected $245 billion within months of the crisis; Berger and Roman (2017) find it significantly increased net job creation through the credit-supply channel that raises ω in our model. In Europe, bank recapitalisations were delayed by 2–4 years (Brunnermeier, James, and Landau, 2016), and the euro area experienced precisely the persistent stagnation the model predicts from Phase 1 without Phase 2. Across 69 systemic banking crises since 1980, Homar and van Wijnbergen (2017) find that the *speed* of recapitalisation—not merely its occurrence—determines recovery duration, consistent with the absorbing steady-state structure and the convex employment losses from delay.

7 Empirical Evidence

The model is disciplined by a single state variable: the financier's wealth share ω_t jointly determines real activity, asset prices, and central bank risk absorption. This section confronts the model's predictions with time-series evidence. Our objective is not causal identification; the tests ask whether the data contain the predicted state dependence when we map the model's objects into observable proxies.

7.1 Data

We proxy the financier's wealth share with the quarterly intermediary capital ratio of He, Kelly, and Manela (2017), $\widehat{\omega}_t \equiv \eta_t^{HKM}/100$. Real activity is measured with the CBO output gap, $\widehat{x}_t \equiv (\text{Real GDP}_t - \text{Real potential GDP}_t)/\text{Real potential GDP}_t$.[5] QE intensity is constructed from Federal Reserve SOMA holdings:

$$\widehat{\alpha}_t^{CB} \equiv \max\left(0,\ \frac{\text{Treasuries}_t + \text{MBS}_t + \text{AgencyDebt}_t}{\text{Nominal GDP}_t} - \overline{\text{QEHoldingsToGDP}}^{\text{pre}}\right). \tag{33}$$

The crisis threshold $\widehat{\omega}_c$ is calibrated as the average HKM ratio over 2007Q3–2007Q4. For the intermediation gap tests, the approximate proxy is $\widehat{B/P}_t^{\text{approx}} \equiv (1-\widehat{\omega}_t)^{\gamma}$ with $\gamma = 2.5$, and the exact proxy $\widehat{B/P}_t^{\text{exact}} \equiv b(\widehat{\omega}_t)$ solves the ODE in Proposition 8. Baseline Tests 1 and 2 use 1970Q1–2024Q4 ($T = 220$); Test 4 uses the pre-QE sample 1970Q1–2007Q4 ($T = 152$). All predictive regressions use Newey–West HAC standard errors; inference robustness (Hodrick, Stambaugh, Lewellen, Campbell–Yogo, bootstrap) is in Appendix J.2. Additional data definitions and return construction are in the appendix.

7.2 Test 1: Intermediary Capital Predicts Real Activity

The model implies that the output gap is an increasing function of ω_t (equation 18). We estimate:

$$\widehat{x}_{t+h} = a_h + b_h \widehat{\omega}_t + \varepsilon_{t+h}, \qquad h \in \{1, 2, 4, 8\}. \tag{34}$$

The model predicts $b_h > 0$.

Table 1: Test 1: baseline

	h=1	h=2	h=4	h=8
Constant	-0.034***	-0.033***	-0.033***	-0.025***
	(0.007)	(0.008)	(0.008)	(0.009)
OmegaHkm	0.441***	0.437***	0.423***	0.302**
	(0.095)	(0.101)	(0.110)	(0.136)
Adj. R^2	0.231	0.226	0.212	0.105
No. Obs.	219	218	216	212

Table 1 confirms the prediction at all horizons. A one-percentage-point increase in in-

[5] The model's output gap (equation 18) is a log deviation from a frictionless reference economy. The CBO output gap is a related but distinct object; both are increasing functions of ω_t, preserving the sign prediction. Appendix Table 9 shows that the results are robust to the unemployment gap, which is closer to a real-time measure.

termediary capital forecasts a 0.44 percentage point higher output gap one quarter ahead ($t = 4.66$) and 0.30 percentage points at the two-year horizon ($t = 2.22$).

Robustness. The result survives all six inference methods: the Hodrick t-statistic at $h = 1$ is 8.60, the Lewellen confidence interval excludes zero, and bootstrap $p < 0.001$ (Appendix Table 17). It holds in 1970–1999 and 2000–2024 subsamples separately, and survives exclusion of 2008–2009 and 2020 crisis quarters (Appendix Tables 22–24). Adding macro-financial controls weakens short-horizon coefficients but retains significance at $h = 4$ and $h = 8$, consistent with the one-state-variable interpretation (Appendix Table 10).

Out-of-sample performance. Against the prevailing-mean benchmark, expanding-window out-of-sample R^2 is 24.8% at $h = 1$ and 9.3% at $h = 4$, with Clark–West statistics of 6.62 and 2.97 (Appendix Table 21).

7.3 Test 4: The Exact Gap Formula

The model's most distinctive testable prediction is the sufficient-statistic property: the intermediation gap-to-price ratio $b(\omega)$ maps one-to-one into the output gap (equation 25). The approximate formula $(1-\omega)^\gamma$ treats the weighting-process volatility as constant, while the exact solution of $b'' = (2\delta/\sigma_\omega^2)\, b$ preserves the model's state-dependent nonlinearity. We test both in the pre-QE sample (1970Q1–2007Q4), where the model predicts a positive gap:

$$\widehat{x}_{t+h} = a_h + b_h \widehat{B/P}_t + \varepsilon_{t+h}. \tag{35}$$

The model predicts $b_h < 0$ (larger gaps signal lower intermediary wealth and weaker output).

Table 2: Test 4: model-implied proxy (baseline)

	h=1	h=2	h=4	h=8
Constant	0.132***	0.138***	0.135***	0.103*
	(0.039)	(0.042)	(0.045)	(0.060)
BubbleApprox	-0.161***	-0.168***	-0.164***	-0.126*
	(0.047)	(0.051)	(0.054)	(0.071)
Adj. R^2	0.201	0.219	0.209	0.118
No. Obs.	151	150	148	144

Table 3: Test 4 distinctive: exact bubble proxy (pre-QE baseline)

	h=1	h=2	h=4	h=8
Constant	0.003	0.005	0.006*	0.009**
	(0.003)	(0.003)	(0.003)	(0.004)
BubbleExact	-0.080**	-0.093***	-0.101***	-0.107***
	(0.033)	(0.034)	(0.031)	(0.033)
Adj. R^2	0.165	0.220	0.258	0.279
No. Obs.	135	134	132	128

Tables 2 and 3 report the results. Both proxies enter with the predicted negative sign at every horizon. The critical distinction emerges out of sample: the approximate formula's OOS R^2 decays from 19.0% at $h = 1$ to -0.34% at $h = 8$—the pattern typical of persistent-predictor overfitting. The exact formula's OOS R^2 *rises* from 13.6% at $h = 1$ to 45.4% at $h = 8$, with Clark–West statistics between 3.27 and 4.74 at every horizon (Appendix Table 21).

This horizon stability is evidence of genuine structural content. The approximate formula, being nearly linear over the empirical range of ω, compresses the model's predicted curvature into a linear slope and overfits at short horizons. The exact formula, which preserves the nonlinear mapping between intermediary capital and the output gap, captures variation that persists at longer horizons. The improvement from $R^2 = -0.3\%$ to $R^2 = 45.4\%$ at the two-year horizon is the sharpest evidence that the ODE-derived gap function contains economically meaningful structure.

Calibration sensitivity. Setting $\gamma = 4$ rather than 2.5 in the approximate formula rescales the coefficient but leaves the qualitative pattern unchanged: the slope is negative and significant at all horizons, with adjusted R^2 between 0.20 and 0.12 (Appendix Table 11).

7.4 QE Trap Evidence

The model's most distinctive dynamic prediction is the QE Trap: once ω falls below $\underline{\omega}$, the economy converges to a bad steady state with permanent QE (Theorem 6). We present two pieces of evidence.

Structural break in the mean of ω_t. The Bai and Perron (1998) procedure applied to the mean of the HKM capital ratio identifies five breaks (Table 4). The pre-break regime (1995–2006) has mean $\bar{\omega} = 0.095$; the post-break regime (2007–2015) has $\bar{\omega} = 0.050$, well

below $\omega_c = 0.078$. The most recent regime (2015–2024) has $\bar{\omega} = 0.064$, still below ω_c—a permanent downward shift consistent with the absorbing bad steady state.

Table 4: Bai–Perron structural breaks in the mean of ω_t

Panel A: Estimated regimes

Regime	Start	End	$\bar{\omega}$	Relation to ω_c
1	1970-03	1978-03	0.0577	$<$
2	1978-06	1986-06	0.0403	$<$
3	1986-09	1995-06	0.0548	$<$
4	1995-09	2006-12	0.0951	$>$
5	2007-03	2015-03	0.0498	$<$
6	2015-06	2024-12	0.0639	$<$

Panel B: Sequential break tests

Test	$FT(l+1 \mid l)$ statistic
FT(1—0)	55.54
FT(2—1)	89.85
FT(3—2)	17.27
FT(4—3)	11.26
FT(5—4)	19.64

Notes: Bai–Perron (1998) mean-shift model applied to the HKM capital ratio proxy for ω_t. Break dates are global least-squares minimizers with trimming parameter 0.15. The number of breaks is selected by BIC. Panel B reports the exact Bai–Perron $FT(l+1 \mid l)$ statistics. The last column compares each regime mean with the supplied crisis threshold $\omega_c = 0.078$.

QT1 natural experiment. The Federal Reserve's first quantitative tightening episode (2017Q4–2019Q3) provides a natural experiment. During an expansion with unemployment below 4%, the HKM capital ratio declined from 0.079 to 0.066, at a trend of -0.002 per quarter ($t = -5.06$, $p = 0.001$; Appendix Table 25). The episode ended with the September 2019 repo market disruption, after which the Fed reversed course—consistent with the model's prediction that exiting the QE regime without fiscal recapitalisation is dynamically unstable.

Persistence. The AR(1) persistence of ω_t drops from $\hat{\rho} = 0.952$ pre-2008 to $\hat{\rho} = 0.812$ post-2008 (half-lives: 14.0 vs. 3.3 quarters; $p = 0.053$; Appendix Table 26), consistent with faster mean reversion toward a *different, lower* steady state.

Caveats. The US provides at most one or two trap episodes, limiting statistical power. An alternative explanation is regulatory: Basel III and the Volcker Rule structurally reduced broker-dealer leverage after 2010. Separating the trap mechanism from the regulatory channel requires cross-country variation, which we leave to future work.

7.5 Cross-Equation SUR Test

The single-state-variable restriction implies that the output gap, excess equity returns, and the intermediation gap proxy should be driven by a common latent factor. Appendix Table 28 estimates a three-equation SUR system on the pre-QE sample. The Breusch–Pagan test rejects a diagonal covariance matrix ($\chi^2(3) = 12.03$, $p = 0.007$), confirming shared residual structure. The first eigenvalue of the residual covariance matrix captures 98.9% of the variance, consistent with a single dominant factor driving all three outcomes. The proportionality diagnostics are broadly supportive: once the gap proxy is included, ω adds only marginal information for output ($t = -1.72$, $p = 0.085$), and Q^{HKM} adds no incremental power for returns ($t = 1.47$, $p = 0.141$).

7.6 Calibrated Counterfactual

Table 29 maps the HKM capital ratio to the model's state variable at four key dates. At the 2009Q1 trough ($\omega = 0.027$), the model implies an employment gap of 3.9% and central bank equity holdings of 73%—broadly consistent with the Fed's extraordinary balance sheet expansion. By 2019Q3 ($\omega = 0.066$), the economy remains below ω_c, with a 1.9% employment gap and 19% CB holdings, consistent with the still-elevated balance sheet a decade after the crisis.

Table 5: Model-Implied Quantities at Key Dates

	2006Q4 (Pre-crisis)	2009Q1 (Trough)	2009Q4 (Post-TARP)	2019Q3 (Pre-COVID)
ω (HKM capital ratio)	0.0818	0.0265	0.0505	0.0657
$\bar{\varphi}(\omega)$ (leverage)	1.309	1.032	1.153	1.228
$N(\omega)$ (employment)	0.9900	0.9615	0.9736	0.9815
Employment gap (%)	1.00%	3.85%	2.64%	1.85%
CB equity share $\alpha(\omega)$ (%)	0.00%	72.64%	41.80%	19.30%
Crisis ($\omega < \omega_c$)	No	Yes	Yes	Yes
Trap ($\omega < \underline{\omega}$)	No	No	No	No
Transfer T^* (% of GDP)	-131.53%	8.89%	-52.05%	-90.62%

Notes: Model parameters: $\varepsilon = 10$, $\underline{\omega} = 0.02$, $\lambda = 5$, $\sigma = 0.15$, $\delta = 0.04$, $\varphi = 1$. ω is mapped from the He, Kelly, and Manela (2017) intermediary capital ratio. Crisis threshold $\omega_c = 0.0776$. Good steady state $\omega^* = 1/\varepsilon = 0.10$. Employment gap is $[N(1/\varepsilon) - N(\omega)]/N(1/\varepsilon)$. Transfer T^* is the fiscal recapitalisation needed to push ω to $\underline{\omega} + 0.01$.

Table 30 compares three counterfactual scenarios from the 2009Q1 trough: without QE (no equilibrium), with QE alone (permanent 4.2% employment gap and 80% CB equity holdings), and with QE plus fiscal recapitalisation (full recovery). The benefit-to-cost ratio

of recapitalisation is 4.3. The model-implied transfer (25% of GDP) exceeds actual TARP injections (1.7%) by an order of magnitude, reflecting the one-intermediary structure; in practice, retained earnings and private capital raises partly substituted for public transfers.

Table 6: Counterfactual Scenarios Starting from 2009Q1 Trough

	No QE	QE Only	QE + Fiscal
Long-run ω	—	0.02	0.10
N (employment)	—	0.9583	1.0000
Employment gap	—	4.17%	0.00%
CB equity share α	—	80.00%	0.00%
PV of output Y/δ	0	23.96	25.00
Fiscal transfer (% of GDP)	—	—	25.39%
Actual TARP (% of GDP)	—	—	1.70%
Benefit/Cost ratio	—	—	4.3
Long-run output gain	—	—	4.35%

Notes: "No QE" scenario: without CB intervention, equilibrium does not exist below ω_c; we set PV = 0. "QE Only": CB purchases equity to restore equilibrium; economy converges to bad steady state $\underline{\omega}$ with permanent employment gap and CB holding equity. "QE + Fiscal": government recapitalises financial sector ($\underline{\omega} \to \underline{\omega} + 0.01$), economy converges to good steady state $1/\varepsilon$. Benefit = PV(QE+Fiscal) − PV(QE Only). Actual TARP: \$245B capital injections relative to 2009 nominal GDP of \$14.4T. Benefit/Cost ratio measures the return on the fiscal transfer in terms of present value of additional output.

7.7 Assessment

The evidence supports the model's core implications. Intermediary capital forecasts the output gap with the predicted sign and survives all inference methods (Test 1). The exact gap formula delivers horizon-stable OOS R^2 of 24–26%, providing evidence that the model's nonlinear structure captures genuine economic variation (Test 4). The post-2007 structural break in $\bar{\omega}$ and the QT1 episode are consistent with the two-steady-state mechanism. The SUR evidence supports the single-state-variable restriction.

8 Conclusion

A financial crisis that depletes intermediary capital can destroy competitive equilibrium—not merely distort prices. Large-scale asset purchases are a necessary condition for market clearing: the central bank acts as a risk-bearer of last resort. But QE generates a trap. If the crisis is sufficiently severe, the economy converges to permanent asset purchases and depressed output regardless of how long QE is maintained. A one-time fiscal recapitalisation

of the financial sector is the only escape, with an employment multiplier that is convex in crisis severity—rationalising the urgency of early intervention documented by Homar and van Wijnbergen (2017).

A leverage paradox links these results: the same leverage constraint that destroys equilibrium simultaneously eliminates the intermediation gap between market prices and fundamental values. No economy in this class can simultaneously achieve gap-free prices and unconditional market clearing—a fundamental impossibility that the policy package of QE plus fiscal recapitalisation resolves.

Several extensions merit investigation: endogenising participation through ambiguity aversion, extending the exact gap formula to cross-sectional asset pricing, and direct tests of the fiscal escape mechanism exploiting cross-country variation in recapitalisation timing.

References

Bai, Jushan and Pierre Perron. 1998. "Estimating and Testing Linear Models with Multiple Structural Changes." *Econometrica* 66(1): 47–78.

Bansal, Ravi and Amir Yaron. 2004. "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles." *Journal of Finance* 59(4): 1481–1509.

Basak, Suleyman and Domenico Cuoco. 1998. "An Equilibrium Model with Restricted Stock Market Participation." *Review of Financial Studies* 11(2): 309–341.

Berger, Allen N. and Raluca A. Roman. 2017. "Did Saving Wall Street Really Save Main Street? The Real Effects of TARP on Local Economic Conditions." *Journal of Financial and Quantitative Analysis* 52(5): 1827–1867.

Bigio, Saki. 2015. "Endogenous Liquidity and the Business Cycle." *American Economic Review* 105(6): 1883–1927.

Borodin, Andrei N. and Paavo Salminen. 2002. *Handbook of Brownian Motion: Facts and Formulae*. Second Edition. Basel: Birkhäuser.

Brunnermeier, Markus K. and Yuliy Sannikov. 2014. "A Macroeconomic Model with a Financial Sector." *American Economic Review* 104(2): 379–421.

Brunnermeier, Markus K., Harold James, and Jean-Pierre Landau. 2016. *The Euro and the Battle of Ideas*. Princeton: Princeton University Press.

Caballero, Ricardo J. and Emmanuel Farhi. 2018. "The Safety Trap." *Review of Economic Studies* 85(1): 223–274.

Campbell, John Y. and John H. Cochrane. 1999. "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior." *Journal of Political Economy* 107(2): 205–251.

Campbell, John Y. and Motohiro Yogo. 2006. "Efficient Tests of Stock Return Predictability." *Journal of Financial Economics* 81(1): 27–60.

Clark, Todd E. and Kenneth D. West. 2007. "Approximately Normal Tests for Equal Predictive Accuracy in Nested Models." *Journal of Econometrics* 138(1): 291–311.

Cui, Wei and Vincent Sterk. 2021. "Quantitative Easing with Heterogeneous Agents." *Journal of Monetary Economics* 123: 68–90.

Cúrdia, Vasco and Michael Woodford. 2011. "The Central-Bank Balance Sheet as an Instrument of Monetary Policy." *Journal of Monetary Economics* 58(1): 54–79.

Del Negro, Marco, Gauti Eggertsson, Andrea Ferrero, and Nobuhiro Kiyotaki. 2017. "The Great Escape? A Quantitative Evaluation of the Fed's Liquidity Facilities." *American Economic Review* 107(3): 824–857.

Di Tella, Sebastian. 2017. "Uncertainty Shocks and Balance Sheet Recessions." *Journal of Political Economy* 125(6): 2038–2081.

Engelbert, Hans-Jürgen and Wolfgang Schmidt. 1985. "On One-Dimensional Stochastic Differential Equations with Generalized Drift." *Lecture Notes in Control and Information Sciences* 69: 143–155.

Eggertsson, Gauti B. and Paul Krugman. 2012. "Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo Approach." *Quarterly Journal of Economics* 127(3): 1469–1513.

Federal Deposit Insurance Corporation. 1991. "Federal Deposit Insurance Corporation Improvement Act of 1991." Public Law 102–242.

Gagnon, Joseph, Matthew Raskin, Julie Remache, and Brian Sack. 2011. "The Financial Market Effects of the Federal Reserve's Large-Scale Asset Purchases." *International Journal of Central Banking* 7(1): 3–43.

Gertler, Mark and Peter Karadi. 2011. "A Model of Unconventional Monetary Policy." *Journal of Monetary Economics* 58(1): 17–34.

Gertler, Mark and Nobuhiro Kiyotaki. 2010. "Financial Intermediation and Credit Policy in Business Cycle Analysis." In *Handbook of Monetary Economics*, Vol. 3A, 547–599. Elsevier.

Greenwood, Robin, Samuel G. Hanson, and Jeremy C. Stein. 2015. "A Comparative-Advantage Approach to Government Debt Maturity." *Journal of Finance* 70(4): 1683–1722.

Guerrieri, Veronica and Guido Lorenzoni. 2017. "Credit Crises, Precautionary Savings, and the Liquidity Trap." *Quarterly Journal of Economics* 132(3): 1427–1467.

Hansen, Bruce E. 1996. "Inference When a Nuisance Parameter Is Not Identified Under the Null Hypothesis." *Econometrica* 64(2): 413–430.

He, Zhiguo and Arvind Krishnamurthy. 2013. "Intermediary Asset Pricing." *American Economic Review* 103(2): 732–770.

He, Zhiguo and Arvind Krishnamurthy. 2019. "A Macroeconomic Framework for Quantifying Systemic Risk." *American Economic Journal: Macroeconomics* 11(4): 1–37.

He, Zhiguo, Bryan Kelly, and Asaf Manela. 2017. "Intermediary Asset Pricing: New Evidence from Many Asset Classes." *Journal of Financial Economics* 126(1): 1–35.

Hodrick, Robert J. 1992. "Dividend Yields and Expected Stock Returns: Alternative Procedures for Inference and Measurement." *Review of Financial Studies* 5(3): 357–386.

Holmström, Bengt and Jean Tirole. 1997. "Financial Intermediation, Loanable Funds, and the Real Sector." *Quarterly Journal of Economics* 112(3): 663–691.

Homar, Timotej and Sweder J.G. van Wijnbergen. 2017. "Bank Recapitalization and Economic Recovery after Financial Crises." *Journal of Financial Intermediation* 32: 16–28.

Hugonnier, Julien. 2012. "Rational Asset Pricing Bubbles and Portfolio Constraints." *Journal of Economic Theory* 147(6): 2260–2302.

Karatzas, Ioannis and Steven E. Shreve. 1991. *Brownian Motion and Stochastic Calculus*. Second Edition. New York: Springer-Verlag.

Karlin, Samuel and Howard M. Taylor. 1981. *A Second Course in Stochastic Processes*. New York: Academic Press.

Koo, Richard C. 2014. *The Escape from Balance Sheet Recession and the QE Trap: A Hazardous Road for the World Economy*. Hoboken, NJ: John Wiley & Sons.

Krishnamurthy, Arvind and Annette Vissing-Jørgensen. 2011. "The Effects of Quantitative Easing on Interest Rates: Channels and Implications for Policy." *Brookings Papers on Economic Activity* Fall: 215–287.

Lewellen, Jonathan. 2004. "Predicting Returns with Financial Ratios." *Journal of Financial Economics* 74(2): 209–235.

Lewellen, Jonathan, Stefan Nagel, and Jay Shanken. 2010. "A Skeptical Appraisal of Asset Pricing Tests." *Journal of Financial Economics* 96(2): 175–194.

Mijatović, Aleksandar and Mikhail Urusov. 2012. "On the Martingale Property of Certain Local Martingales." *Probability Theory and Related Fields* 152(1): 1–30.

Philippon, Thomas and Philipp Schnabl. 2013. "Efficient Recapitalization." *Journal of Finance* 68(1): 1–42.

Reis, Ricardo. 2016. "Funding Quantitative Easing to Target Inflation." In *Designing Resilient Monetary Policy Frameworks for the Future*. Kansas City: Federal Reserve Bank of Kansas City.

Revuz, Daniel and Marc Yor. 1999. *Continuous Martingales and Brownian Motion*. Third Edition. Berlin: Springer-Verlag.

Rognlie, Matthew, Andrei Shleifer, and Alp Simsek. 2018. "Investment Hangover and the Great Recession." *American Economic Journal: Macroeconomics* 10(2): 113–153.

Silva, Dejanir H. 2020. "The Risk Channel of Unconventional Monetary Policy." Working Paper, University of Illinois at Urbana-Champaign.

Stambaugh, Robert F. 1999. "Predictive Regressions." *Journal of Financial Economics* 54(3): 375–421.

Vayanos, Dimitri and Jean-Luc Vila. 2021. "A Preferred-Habitat Model of the Term Structure of Interest Rates." *Econometrica* 89(1): 77–112.

Veronesi, Pietro and Luigi Zingales. 2010. "Paulson's Gift." *Journal of Financial Economics* 97(3): 339–368.

A Complete-Markets Benchmark

When both agents can trade freely in bonds and equity, markets are dynamically complete (one risk source, two assets). Equilibrium allocations are Pareto efficient.

Proposition 15 (Complete-markets equilibrium)**.** *There exists a constant $\bar{\lambda} \in (0,1)$ (the household's Pareto weight) such that: (i) consumption shares are constant, $C_{h,t} = \bar{\lambda} Y_t$ and $C_{i,t} = (1-\bar{\lambda}) Y_t$; (ii) labour supply is constant, $N = [(\epsilon-1)/(\epsilon\bar{\lambda})]^{1/(1+\varphi)}$; and (iii) the risk-free rate and market price of risk are constants:*

$$r = \delta + \mu - \sigma^2, \qquad \Theta = \sigma. \tag{36}$$

The equity price is $P_{D,t} = D_t/\delta = Y_t/(\epsilon\delta)$ and total wealth is $P_{Y,t} = Y_t/\delta$.

Proof. See Appendix A. □

The complete-markets economy serves as a reference point: the good steady state of the model with QE will coincide with this benchmark.

B Baseline Equilibrium: Limited Participation Without Leverage Constraints

We now introduce limited stock market participation, households hold only bonds, but *without* the leverage constraint (7). This appendix establishes why the economy functioned well for decades before 2008: equilibrium always exists and the wealth distribution self-corrects toward the efficient allocation.

B.1 State Variable and Equilibrium Characterisation

Following Basak and Cuoco (1998), we characterise equilibrium using the financier's share of total wealth:

$$\omega_t \equiv \frac{H_{i,t}}{P_{Y,t}} \in (0,1). \tag{37}$$

Proposition 16 (Equilibrium with limited participation)**.** *With log utility and limited participation (no leverage constraint):*

(i) Total wealth: $P_{Y,t} = Y_t/\delta$. Human capital: $P_{L,t} = (1-1/\epsilon)\, P_{Y,t}$.

(ii) Consumption: $C_{i,t} = \omega_t Y_t$, $C_{h,t} = (1-\omega_t) Y_t$.

(iii) Employment: $N_t = (1 - 1/\epsilon)^{1/(1+\varphi)} (1 - \omega_t)^{-1/(1+\varphi)}$.

(iv) Financial wealth: $H_{i,t} = \omega_t P_{Y,t}$, $H_{h,t} = (1/\epsilon - \omega_t) P_{Y,t}$.

(v) Portfolio share: $\phi_t = 1/(\epsilon\omega_t)$.

Three features of this proposition are crucial for what follows.

First, *financial wealth levels are uniquely pinned by* ω. The household's bond holdings are $H_{h,t} = (1/\epsilon - \omega_t) P_{Y,t}$, regardless of the interest rate, risk premium, or any other price. This perfect inelasticity is the engine of the equilibrium failure result.

Second, *the portfolio share is uniquely pinned by market clearing.* Since only the financier holds equity and the equity market must clear: $\phi_t H_{i,t} = P_{D,t}$, so $\phi_t = P_{D,t}/H_{i,t} = (P_{Y,t}/\epsilon)/(\omega_t P_{Y,t}) = 1/(\epsilon\omega_t)$. This is not a choice variable, it is an accounting identity.

Third, *the bond market clears automatically.* The financier borrows $(\phi_t - 1)H_{i,t} = (1/(\epsilon\omega_t) - 1)\omega_t P_{Y,t} = (1/\epsilon - \omega_t)P_{Y,t} = H_{h,t}$. Everything balances.

B.2 Dynamics of the Wealth Distribution

Theorem 17 (Dynamics of ω_t without leverage constraint)**.** *The financier's wealth share evolves according to*

$$\mathrm{d}\omega_t = \frac{\sigma_\omega^0(\omega_t)^2}{\omega_t}\,\mathrm{d}t + \sigma_\omega^0(\omega_t)\,\mathrm{d}Z_t, \tag{38}$$

where the volatility function is

$$\sigma_\omega^0(\omega) = \frac{\sigma(1+\varphi)(1-\omega)(1/\epsilon - \omega)}{(1+\varphi)(1-\omega) - (1/\epsilon - \omega)}. \tag{39}$$

The inverse wealth share $Q_t = 1/\omega_t$ *is a non-negative local martingale.*

The drift relation $\mu_\omega = \sigma_\omega^2/\omega$ follows from the local martingale property of $Q_t = 1/\omega_t$, which we establish in Appendix C. This property is the key to the model's tractability: with log utility, the myopic portfolio rule ensures that a critical cancellation in the drift of Q holds for any portfolio share.

The volatility σ_ω^0 has a single interior zero at $\omega^* = 1/\epsilon$ (where $1/\epsilon - \omega = 0$). It is positive for $\omega < 1/\epsilon$ and negative for $\omega > 1/\epsilon$, meaning that positive productivity shocks push ω toward $1/\epsilon$ from both sides.

B.3 Two Invariant Regimes and Convergence

Theorem 18 (Regime classification and convergence)**.** *The state space* $(0, 1)$ *partitions into two invariant regions separated by the absorbing state* $\omega^* = 1/\epsilon$*:*

1. ***Controlled regime:*** $\omega \in (0, 1/\epsilon)$. *The financier is leveraged* ($\phi > 1$) *and households are net lenders* ($H_h > 0$).

2. ***Expansionary regime:*** $\omega \in (1/\epsilon, 1)$. *The financier holds less than* 100% *equity* ($\phi < 1$) *and households are net borrowers* ($H_h < 0$).

For any $\omega_0 \in (0,1) \setminus \{1/\epsilon\}$*:*

$$\omega_t \xrightarrow{a.s.} \frac{1}{\epsilon}, \qquad \textit{yet} \qquad \mathbb{E}[\tau] = \infty, \tag{40}$$

where $\tau = \inf\{t \geq 0 : \omega_t = 1/\epsilon\}$.

The almost-sure convergence with infinite expected hitting time captures a "Zeno's paradox" of financial markets: the limited participation friction becomes asymptotically negligible yet is never eliminated in finite expected time.

At $\omega^* = 1/\epsilon$, the financier's wealth equals the stock market capitalisation ($H_i = P_D$), the household has zero financial wealth, leverage is one (no borrowing), and the economy coincides with the complete-markets benchmark. This is why the economy was fine before 2008: limited participation and leverage constraints coexisted without incident, because financiers were well enough capitalised that the constraints never bound. What changed in 2008 was the *state*, financier wealth ω, not the rules.

C Proofs for Limited Participation (No Leverage Constraint)

Proof of Proposition 16. Optimal policies. The financier's HJB equation with conjectured value function $V_i = \delta^{-1} \ln H_i + g_i(\omega)$ yields $C_i = \delta H_i$ and $\phi = (\mu_R - r)/\sigma_R^2$ (the myopic log-utility rule). The household, with log utility over consumption and total wealth $H_h + P_L$, consumes $C_h = \delta(H_h + P_L)$.

Goods market clearing. $\delta H_i + \delta(H_h + P_L) = Y$, so $P_Y \equiv H_i + H_h + P_L = Y/\delta$. By definition of ω: $C_i/Y = \delta H_i/Y = H_i/P_Y = \omega$ and $C_h/Y = 1 - \omega$.

Employment. From $N^\varphi C_h = (1 - 1/\epsilon)A$ and $C_h = (1-\omega)AN$: $N^{1+\varphi} = (1 - 1/\epsilon)/(1-\omega)$.

Equity market clearing. $\phi H_i = P_D = P_Y/\epsilon$, so $\phi = 1/(\epsilon\omega)$.

Financial wealth. $H_i + H_h = P_D = P_Y/\epsilon$ and $H_i = \omega P_Y$, so $H_h = (1/\epsilon - \omega)P_Y$.

Human capital. $P_L = P_Y - P_D = (1 - 1/\epsilon)P_Y$. Crucially, this holds for *any* dynamics of ω_t, because $P_Y = Y/\delta$ depends only on log utility and goods market clearing. □

Proof of Theorem 17. Step 1: Return volatility. Since $P_D = Y/(\epsilon\delta)$, by Itô's product rule applied to $Y = AN$:

$$\sigma_R = \sigma_Y = \sigma + \frac{\sigma_\omega^0}{(1+\varphi)(1-\omega)},$$

where the second term comes from $N \propto (1-\omega)^{-1/(1+\varphi)}$ via Itô's lemma.

Step 2: Wealth-share volatility. From $\omega = H_i/P_Y$ and Itô's quotient rule:

$$\sigma_\omega^0 = \omega(\phi - 1)\sigma_R = \omega\left(\frac{1}{\epsilon\omega} - 1\right)\sigma_R = \left(\frac{1}{\epsilon} - \omega\right)\sigma_R.$$

Step 3: Fixed-point equation. Substituting into Step 1 and solving:

$$\sigma_R = \frac{\sigma(1+\varphi)(1-\omega)}{(1+\varphi)(1-\omega) - (1/\epsilon - \omega)},$$

giving $\sigma_\omega^0 = (1/\epsilon - \omega)\sigma_R$, which is (39). The denominator simplifies to $(1-1/\epsilon)+\varphi(1-\omega) > 0$.

Step 4: Q-martingale. Define $Q = 1/\omega = P_Y/H_i$. The drift calculation is identical to Appendix F and yields $\mu_Q = 0$. By Itô's lemma on $\omega = 1/Q$: $\mu_\omega = (\sigma_\omega^0)^2/\omega$. □

Proof of Theorem 18. Absorbing state. At $\omega = 1/\epsilon$: $\sigma_\omega^0(1/\epsilon) = 0$ and $\mu_\omega(1/\epsilon) = 0$.

Boundary $\omega = 0$ (entrance). As $\omega \to 0^+$: $\sigma_\omega^0(\omega) \to c_0 > 0$ (a positive constant). The scale function density is $s'(\omega) = \omega_0^2/\omega^2$ and the speed measure is $m(\omega) = \omega^2/(\omega_0^2\,(\sigma_\omega^0)^2)$. Near $\omega = 0$: $m(\omega) \sim \omega^2/(c_0^2\omega_0^2)$.

$$\Sigma(0) = \int_0^a s'(x)\,\mathrm{d}x = \omega_0^2 \int_0^a x^{-2}\,\mathrm{d}x = +\infty,$$
$$N(0) = \int_0^a s'(x)\int_0^x m(y)\,\mathrm{d}y\,\mathrm{d}x \le \frac{1}{3c_0^2}\int_0^a x\,\mathrm{d}x < \infty.$$

Hence $\Sigma = \infty$, $N < \infty$: entrance boundary, unattainable from the interior.

Boundary $\omega = 1/\epsilon$ (absorbing/unattainable). Write $\sigma_\omega^0 = g(\omega)(1/\epsilon - \omega)$ with $g(1/\epsilon) = \sigma > 0$. By the Engelbert–Schmidt criterion:

$$\int_a^{1/\epsilon} \frac{1}{(\sigma_\omega^0)^2}\,\mathrm{d}\omega \ge \frac{1}{\bar{g}^2}\int_a^{1/\epsilon} \frac{1}{(1/\epsilon - x)^2}\,\mathrm{d}x = +\infty.$$

Unattainable from the left. A symmetric argument gives unattainability from the right.

Boundary $\omega = 1$ (exit, unattainable). Write $\sigma_\omega^0 = h(\omega)(1-\omega)$ with $h(1) = -\sigma(1+\varphi) \neq 0$. Engelbert–Schmidt: $\int^1 (1-x)^{-2}\,\mathrm{d}x = +\infty$. Unattainable.

Regime invariance. Continuous paths and unattainable internal boundaries imply $(0, 1/\epsilon)$ and $(1/\epsilon, 1)$ are invariant.

Convergence. $Q = 1/\omega$ is a non-negative local martingale, hence a supermartingale. By the supermartingale convergence theorem, $Q_t \to Q_\infty$ a.s. with $Q_\infty < \infty$. Convergence of the quadratic variation requires $\sigma_\omega^0(\omega_\infty) = 0$, so $\omega_\infty \in \{0, 1/\epsilon, 1\}$. Since $Q_\infty < \infty$ implies $\omega_\infty > 0$, and $\omega = 1$ is unattainable: $\omega_t \to 1/\epsilon$ a.s.

Infinite expected hitting time. The speed measure satisfies $m(\omega) \geq K(1/\epsilon - \omega)^{-2}$ near $1/\epsilon$. By the Green's function formula:

$$\mathbb{E}[\tau_{1/\epsilon}] \geq C \int_a^{1/\epsilon} (1/\epsilon - y)^{-1} \, \mathrm{d}y = +\infty. \qquad \square$$

C.1 Asset Prices and Real Variables

Proposition 19 (Asset prices and real variables without leverage constraint)**.** *In the baseline economy with limited participation and no leverage constraint, define*

$$\Delta(\omega) \equiv \left(1 - \tfrac{1}{\epsilon}\right) + \varphi(1 - \omega) > 0. \tag{41}$$

Then:

(i) Market price of risk:

$$\Theta^0(\omega) = \frac{\sigma(1+\varphi)(1-\omega)}{\epsilon\,\omega\,\Delta(\omega)}. \tag{42}$$

(ii) Equity risk premium:

$$\mu_R^0 - r^0 = \Theta^0\,\sigma_R^0 = \frac{\sigma^2(1+\varphi)^2(1-\omega)^2}{\epsilon\,\omega\,\Delta(\omega)^2}. \tag{43}$$

(iii) Risk-free rate:

$$r^0(\omega) = \delta + \mu - \sigma^2 + \frac{\varphi}{2}\left(\sigma_R^0 - \sigma\right)^2 - \frac{\left[1 + \varphi(1-\omega)\right]\sigma\left(\sigma_R^0 - \sigma\right)}{\omega}, \tag{44}$$

where $\sigma_R^0 - \sigma = (1/\epsilon - \omega)\,\sigma \,/\, \Delta(\omega)$.

(iv) Log employment:

$$n^0(\omega) \equiv \ln N^0 = \frac{1}{1+\varphi} \ln\left(\frac{1 - 1/\epsilon}{1 - \omega}\right). \tag{45}$$

(v) Log output gap *(relative to a reference economy with constant wealth share* $\omega_n \in (0,1)$*):*

$$x^0(\omega) \equiv n^0(\omega) - n_n = \frac{1}{1+\varphi} \ln\left(\frac{1-\omega_n}{1-\omega}\right), \tag{46}$$

where $n_n = \frac{1}{1+\varphi}\ln\!\left(\frac{1-1/\epsilon}{1-\omega_n}\right)$ *is log employment in the reference economy.*

At $\omega = \omega_n$*:* $x^0 = 0$*. At* $\omega = 1/\epsilon$*, the asset-pricing quantities recover their complete-markets values:* $\Theta^0 = \sigma$*,* $\mu_R^0 - r^0 = \sigma^2$*,* $r^0 = \delta + \mu - \sigma^2$*, and* $n^0 = n_n$ *(so* $x^0 = 0$ *when* $\omega_n = 1/\epsilon$*).*

Proof. Throughout, write $\sigma_R^0 = \sigma(1+\varphi)(1-\omega)/\Delta$ and $\sigma_\omega^0 = (1/\epsilon - \omega)\,\sigma_R^0$, which are established in the proof of Theorem 17.

Part (i): Market price of risk. From Proposition 16, $\phi = 1/(\epsilon\omega)$. The financier's SDF is $\Lambda_t = e^{-\delta t}/C_{i,t}$ with $C_{i,t} = \delta H_{i,t}$, so $\sigma_\Lambda = -\phi\,\sigma_R^0$ (the financier's wealth volatility). Hence $\Theta^0 = \phi\,\sigma_R^0 = \sigma_R^0/(\epsilon\omega)$. Substituting gives (42).

Verification: at $\omega = 1/\epsilon$, $\sigma_R^0 = \sigma$ and $\Theta^0 = \sigma/(\epsilon\cdot 1/\epsilon) = \sigma$, recovering the complete-markets value (36). ✓

Part (ii): Equity risk premium. By the standard no-arbitrage relation, $\mu_R^0 - r^0 = \Theta^0\,\sigma_R^0 = (\sigma_R^0)^2/(\epsilon\omega)$. Substituting σ_R^0 yields (43).

Verification: at $\omega = 1/\epsilon$, $\mu_R^0 - r^0 = \sigma^2$, as in the complete-markets benchmark. ✓

Part (iii): Risk-free rate. The financier's SDF $\Lambda_t = e^{-\delta t}/(\omega_t Y_t)$ implies $r^0 = \delta + \mu_{\ln(\omega Y)} - \frac{1}{2}(\Theta^0)^2$, where $\Theta^0 = \sigma_{\ln(\omega Y)}$ is the diffusion coefficient established in Part (i).

Step 1: Drift of $\ln\omega$*.* From the Q-martingale property (Theorem 17):

$$\mathrm{d}\ln\omega = \frac{(\sigma_\omega^0)^2}{2\omega^2}\,\mathrm{d}t + \frac{\sigma_\omega^0}{\omega}\,\mathrm{d}Z_t.$$

Step 2: Drift of $\ln Y$*.* Since $Y = A\,N(\omega)$ with $N \propto (1-\omega)^{-1/(1+\varphi)}$:

$$\mathrm{d}\ln Y = \left(\mu - \tfrac{\sigma^2}{2} + \tfrac{(\sigma_\omega^0)^2}{(1+\varphi)(1-\omega)\omega} + \tfrac{(\sigma_\omega^0)^2}{2(1+\varphi)(1-\omega)^2}\right)\mathrm{d}t + \sigma_R^0\,\mathrm{d}Z_t.$$

Step 3: Change of variables. Write $\tilde\sigma \equiv \sigma_R^0 - \sigma = \sigma_\omega^0/[(1+\varphi)(1-\omega)]$ and $\sigma_\omega^0 = (1+\varphi)(1-\omega)\tilde\sigma$. Summing the drift contributions from Steps 1 and 2:

$$\mu_{\ln(\omega Y)} = \mu - \tfrac{\sigma^2}{2} + \tfrac{(1+\varphi)\left[1+\varphi(1-\omega)^2\right]\tilde\sigma^2}{2\omega^2},$$

where the coefficient of $\tilde\sigma^2$ follows from $(1+\varphi)^2(1-\omega)^2 + 2(1+\varphi)\omega(1-\omega) + (1+\varphi)\omega^2 = (1+\varphi)[1+\varphi(1-\omega)^2]$.

Step 4: Subtraction. Since $(\Theta^0)^2 = [\sigma + (1+\varphi(1-\omega))\tilde\sigma/\omega]^2$:

$$r^0 = \delta + \mu - \sigma^2 + \frac{\tilde\sigma^2}{2\omega^2}\Big\{(1+\varphi)\big[1+\varphi(1-\omega)^2\big] - \big[1+\varphi(1-\omega)\big]^2\Big\} - \frac{[1+\varphi(1-\omega)]\,\sigma\,\tilde\sigma}{\omega}.$$

The curly brace simplifies to $\varphi\omega^2$, by the algebraic identity

$$(1+\varphi)\big[1+\varphi(1-\omega)^2\big]-\big[1+\varphi(1-\omega)\big]^2=\varphi\omega^2,$$

which is verified by expanding both sides. This yields (44).

Verification: at $\omega=1/\epsilon$, $\tilde{\sigma}=0$, so $r^0=\delta+\mu-\sigma^2$. ✓

Parts (iv)–(v): Employment and output gap. From Proposition 16, $N^{1+\varphi}=(1-1/\epsilon)/(1-\omega)$, giving (45). The reference economy has the same employment formula evaluated at ω_n: $N_n=[(1-1/\epsilon)/(1-\omega_n)]^{1/(1+\varphi)}$, so $n_n=\frac{1}{1+\varphi}\ln\frac{1-1/\epsilon}{1-\omega_n}$. Since $Y=AN$ and productivity A_t is common to both economies:

$$x^0=\ln Y-\ln Y_n=\ln N^0-\ln N_n=n^0-n_n=\frac{1}{1+\varphi}\ln\left(\frac{1-\omega_n}{1-\omega}\right),$$

which is (46). Note that $(1-1/\epsilon)$ cancels in the difference n^0-n_n.

Verification: $x^0(\omega_n)=0$; for $\omega>\omega_n$: $x^0>0$ (output above the reference); for $\omega<\omega_n$: $x^0<0$ (output below the reference). ✓ □

Remark 2 (Economic interpretation of the risk-free rate)**.** *The risk-free rate* (44) *decomposes as* $r^0=r^{CM}+r^{precaut}+r^{risk}$*, where:*

- $r^{CM}=\delta+\mu-\sigma^2$ *is the complete-markets rate,*
- $r^{precaut}=\frac{\varphi}{2}(\sigma_R^0-\sigma)^2>0$ *is a precautionary term from excess return volatility (mediated by the Frisch elasticity),*
- $r^{risk}=-[1+\varphi(1-\omega)]\,\sigma\,(\sigma_R^0-\sigma)/\omega<0$ *when* $\omega<1/\epsilon$ *captures the risk-compensation effect of impaired intermediation.*

The risk-compensation term is first-order in $\sigma_R^0-\sigma$ *while the precautionary term is second-order, so* $r^0<r^{CM}$ *whenever* $\omega\neq 1/\epsilon$*. The interest rate falls below its frictionless value when intermediation is impaired, consistent with the empirical evidence on low rates during financial distress.*

D Proof of Equilibrium Failure

Proof that $P_{L,t}=(1-1/\epsilon)P_{Y,t}$ *regardless of* ω_t *dynamics.* Human capital is

$$P_{L,t}=\mathbb{E}_t\int_t^\infty\frac{\Lambda_s}{\Lambda_t}\frac{\epsilon-1}{\epsilon}Y_s\,\mathrm{d}s=\frac{\epsilon-1}{\epsilon}\,\mathbb{E}_t\int_t^\infty\frac{\Lambda_s}{\Lambda_t}Y_s\,\mathrm{d}s.$$

The financier's SDF satisfies $\Lambda_t = e^{-\delta t}/C_{i,t} = e^{-\delta t}/(\delta H_{i,t})$. Total wealth is $P_{Y,t} = \mathbb{E}_t \int_t^\infty (\Lambda_s/\Lambda_t) Y_s \, \mathrm{d}s$. From goods market clearing, $P_{Y,t} = Y_t/\delta$ for any dynamics. Therefore $P_{L,t} = (1-1/\epsilon)Y_t/\delta = (1-1/\epsilon)P_{Y,t}$.

The key point is that $P_{Y,t} = Y_t/\delta$ is an identity under log utility: $P_Y = (C_i + C_h)/\delta = Y/\delta$. It does not require any specific dynamics for ω, the interest rate, or risk premia. □

Proof of Theorem 1. The argument proceeds in three steps.

Step 1: Household bond demand is perfectly inelastic. By Proposition 16 (which holds for any ω dynamics), the household's financial wealth is $H_{h,t} = (1/\epsilon - \omega_t)\, P_{Y,t}$. With log utility, this is a function of ω alone; it does not depend on the interest rate, the equity premium, or any other price.

Step 2: Equity market clearing pins the portfolio share. Since only the financier and (potentially) the central bank hold equity: $\phi_t H_{i,t} + \alpha_t P_{D,t} = P_{D,t}$. Without central bank intervention ($\alpha = 0$), this requires $\phi_t = P_{D,t}/H_{i,t} = 1/(\epsilon\omega_t)$.

Step 3: The leverage constraint makes the required ϕ infeasible. When $\omega < \omega_c$, the required portfolio share $1/(\epsilon\omega)$ exceeds the maximum permitted leverage $\bar{\phi}(\omega) = 1 + \lambda(\omega - \underline{\omega})$. No interest rate adjustment can resolve this: the household's bond demand is inelastic (Step 1), so reducing the interest rate does not induce the household to reduce her lending. There is a residual excess supply of bonds $(1/\epsilon - \omega - (\bar{\phi}(\omega) - 1)\omega)\, P_{Y,t} > 0$ that no price can eliminate. No competitive equilibrium exists. □

Proof of Proposition 2. With the central bank holding fraction $\alpha_t = 1 - \epsilon\bar{\phi}(\omega_t)\omega_t$ of the equity market (Assumption 2), the financier's equity holding is $\phi_t H_{i,t} = \bar{\phi}(\omega_t)\omega_t P_{Y,t}$ and the central bank's is $\alpha_t P_{D,t} = (1 - \epsilon\bar{\phi}(\omega_t)\omega_t)\, P_{Y,t}/\epsilon$. Their sum is $P_{D,t} = P_{Y,t}/\epsilon$: the equity market clears.

The central bank finances its equity purchases by issuing bonds worth $\alpha_t P_{D,t}$. The household's bond demand is $(1/\epsilon - \omega_t)P_{Y,t}$ (perfectly inelastic). The financier borrows $(\bar{\phi}(\omega_t) - 1)\omega_t P_{Y,t}$. Bond market clearing requires:

$$(1/\epsilon - \omega_t)P_{Y,t} = (\bar{\phi}(\omega_t) - 1)\omega_t P_{Y,t} + \alpha_t P_{D,t}.$$

Substituting $\alpha_t P_{D,t} = (1/\epsilon - \bar{\phi}(\omega_t)\omega_t)P_{Y,t}$ and simplifying: $(1/\epsilon - \omega_t) = (\bar{\phi}(\omega_t)\omega_t - \omega_t) + (1/\epsilon - \bar{\phi}(\omega_t)\omega_t) = 1/\epsilon - \omega_t$. The identity holds.

All real quantities (consumption, employment, output, wealth) depend only on ω_t through the same expressions as Proposition 16, since $P_{Y,t} = Y_t/\delta$ is an identity under log utility. □

E Derivation of σ_ω With Leverage Constraint

Proof of Theorem 3. **Step 1: Return volatility.** Since $P_D = Y/(\epsilon\delta)$, $\sigma_R = \sigma_Y$. From $Y = AN$ with $N \propto (1-\omega)^{-1/(1+\varphi)}$:

$$\sigma_Y = \sigma + \frac{\sigma_\omega}{(1+\varphi)(1-\omega)}.$$

Step 2: Wealth-share volatility. From $\omega = H_i/P_Y$ and Itô's quotient rule:

$$\sigma_\omega = \omega(\phi - 1)\sigma_R.$$

Step 3: Fixed-point equation.

Constrained region ($\omega < \omega_c$, $\phi = 1 + \lambda(\omega - \underline{\omega})$):

$$\sigma_\omega = \omega\lambda(\omega - \underline{\omega})\left[\sigma + \frac{\sigma_\omega}{(1+\varphi)(1-\omega)}\right],$$

$$\sigma_\omega\left[\frac{(1+\varphi)(1-\omega) - \lambda\omega(\omega-\underline{\omega})}{(1+\varphi)(1-\omega)}\right] = \sigma\lambda\omega(\omega-\underline{\omega}).$$

Solving: $\sigma_\omega^c = \sigma(1+\varphi)\lambda(1-\omega)\omega(\omega-\underline{\omega})/D_c(\omega)$, where $D_c = (1+\varphi)(1-\omega) - \lambda\omega(\omega-\underline{\omega})$.

Unconstrained region ($\omega \geq \omega_c$, $\phi = 1/(\epsilon\omega)$): Identical to the baseline, yielding $\sigma_\omega^u = \sigma(1+\varphi)(1-\omega)(1/\epsilon - \omega)/D_u(\omega)$ where $D_u = (1+\varphi)(1-\omega) - (1/\epsilon - \omega)$.

Continuity at ω_c: At ω_c, $\lambda\omega_c(\omega_c - \underline{\omega}) = 1/\epsilon - \omega_c$. Substituting into σ_ω^c recovers σ_ω^u.

Step 4: Drift. From Proposition 4. □

Lemma 20 (Denominator positivity)**.** *$D(\omega) > 0$ for all $\omega \in [0,1)$ in both regions.*

Proof. For $\omega \geq \omega_c$: $D_u = \varphi(1-\omega) + (1 - 1/\epsilon) > 0$. For $\omega \in [0, \underline{\omega}]$: $\lambda\omega(\omega - \underline{\omega}) \leq 0$, so $D_c \geq (1+\varphi)(1-\omega) > 0$. For $\omega \in [\underline{\omega}, \omega_c]$: D_c is concave in ω with $D_c(\underline{\omega}) = (1+\varphi)(1-\underline{\omega}) > 0$ and $D_c(\omega_c) = D_u(\omega_c) > 0$. A concave function positive at both endpoints is positive throughout. □

Proof of Proposition 21. Consider the volatility function (12) with denominator $D(\omega) > 0$ for all $\omega \in [0,1)$ (Lemma 20). All pre-factors σ, $(1+\varphi)$, and λ are strictly positive.

Zeros. In the constrained region ($\omega < \omega_c$), the numerator is $\sigma(1+\varphi)\lambda(1-\omega)\,\omega\,(\omega-\underline{\omega})$. For $\omega \in [0, \omega_c)$, the factors $(1-\omega)$ and $D_c(\omega)$ are strictly positive, so $\sigma_\omega(\omega) = 0$ if and only if $\omega = 0$ or $\omega = \underline{\omega}$. In the unconstrained region ($\omega \geq \omega_c$), the numerator is $\sigma(1+\varphi)(1-\omega)(1/\epsilon - \omega)$. For $\omega \in [\omega_c, 1)$, the factor $(1-\omega) > 0$ and $D_u > 0$, so $\sigma_\omega(\omega) = 0$ if and only if $\omega = 1/\epsilon$. (Note $\omega_c < 1/\epsilon$ since $\bar{\phi}(\omega_c) = 1/(\epsilon\omega_c) > 1$, which requires $\omega_c < 1/\epsilon$.) Combining: the zeros of σ_ω in $[0,1)$ are exactly $\{0, \underline{\omega}, 1/\epsilon\}$.

Sign pattern. For $\omega \in (0, \underline{\omega})$: $\omega > 0$, $(1-\omega) > 0$, $(\omega - \underline{\omega}) < 0$, and $D_c > 0$. The numerator has sign $(+)(+)(+)(+)(-) < 0$, so $\sigma_\omega < 0$. For $\omega \in (\underline{\omega}, \omega_c)$: $(\omega - \underline{\omega}) > 0$, so all factors in the numerator are positive, giving $\sigma_\omega > 0$. For $\omega \in [\omega_c, 1/\epsilon)$: $(1/\epsilon - \omega) > 0$ and $(1-\omega) > 0$, so $\sigma_\omega > 0$. Continuity of σ_ω at ω_c (established in the proof of Theorem 3) ensures consistency across the boundary. For $\omega \in (1/\epsilon, 1)$: $(1/\epsilon - \omega) < 0$ and $(1-\omega) > 0$, so $\sigma_\omega < 0$. □

F Q-Martingale Property

Proof of Proposition 4. Define $Q_t = P_{Y,t}/H_{i,t} = 1/\omega_t$. The financier's SDF is $\Lambda_t = e^{-\delta t}/C_{i,t} = e^{-\delta t}/(\delta H_{i,t})$ (the first-order condition for log utility). The volatility of Λ loading on $\mathrm{d}Z$ is $-\phi_t \sigma_R$, so the market price of risk is $\Theta_t = \phi_t \sigma_R$. This holds for *any* feasible portfolio share ϕ_t, whether chosen optimally or imposed by a constraint.

No-arbitrage for total wealth (which pays continuous dividend yield δ): $\mu_{P_Y}/P_Y = r + \sigma_Y \Theta_t - \delta$. Financier wealth drift: $\mu_{H_i}/H_i = r + \phi_t(\mu_R - r) - \delta = r + \phi_t^2 \sigma_R^2 - \delta$, where we used $\mu_R - r = \sigma_R \Theta_t = \phi_t \sigma_R^2$.

By Itô's quotient rule (Karatzas and Shreve, 1991, Problem 3.2.21) applied to $Q = P_Y/H_i$:

$$\begin{aligned}\frac{\mu_Q}{Q} &= \frac{\mu_{P_Y}}{P_Y} - \frac{\mu_{H_i}}{H_i} + \left(\frac{\sigma_{H_i}}{H_i}\right)^2 - \frac{\sigma_{P_Y}}{P_Y} \cdot \frac{\sigma_{H_i}}{H_i} \\ &= (r + \sigma_Y \phi_t \sigma_R - \delta) - (r + \phi_t^2 \sigma_R^2 - \delta) + \phi_t^2 \sigma_R^2 - \phi_t \sigma_R \sigma_Y \\ &= \sigma_Y \phi_t \sigma_R - \phi_t^2 \sigma_R^2 + \phi_t^2 \sigma_R^2 - \phi_t \sigma_R \sigma_Y = 0.\end{aligned}$$

The cancellation uses $\sigma_R = \sigma_Y$ (since $P_D \propto Y$, the return volatility equals output volatility). Crucially, every ϕ_t-dependent term appears in a matching pair that cancels: $+\sigma_Y \phi_t \sigma_R$ from total wealth, $-\phi_t^2 \sigma_R^2$ from financier wealth, $+\phi_t^2 \sigma_R^2$ from the quadratic covariation of H_i, and $-\phi_t \sigma_R \sigma_Y$ from the cross-covariation of P_Y and H_i. The result holds for *any* ϕ_t, constrained or unconstrained.

Q is a well-defined local martingale. By Itô's lemma applied to $Q = 1/\omega$:

$$\mathrm{d}Q_t = -\frac{\sigma_\omega(\omega_t)}{\omega_t^2}\,\mathrm{d}Z_t,$$

where the drift vanishes because $\mu_Q = 0$. To verify this stochastic integral is well-defined: the SDE (11) for ω has coefficients that are continuous on $(0,1)$ and locally Lipschitz on every compact subset of $(0,1)$, so a unique strong solution exists up to the first exit time from $(0,1)$ (Karatzas and Shreve, 1991, Theorem 5.2.5). On any finite time interval $[0,t]$, the

solution remains in a compact subset of $(0,1)$ (since the coefficients satisfy a linear growth bound on compact subsets), so $\sigma_\omega(\omega_s)/\omega_s^2$ is locally bounded and $\int_0^t(\sigma_\omega(\omega_s)/\omega_s^2)^2\,\mathrm{d}s < \infty$ a.s. Therefore Q is a continuous local martingale. Since $\omega \in (0,1)$, we have $Q > 0$: Q is a non-negative local martingale and hence a supermartingale by Fatou's lemma (Karatzas and Shreve, 1991, Ch. 1).

By Itô's lemma on $\omega = 1/Q$: $\mu_\omega = \sigma_\omega^2/\omega$. □

G Boundary Classification

Proof of Theorem 5. At $\omega = \underline{\omega}$: $\sigma_\omega(\underline{\omega}) = 0$ (Proposition 21), so the diffusion coefficient vanishes. The drift relation $\mu_\omega = \sigma_\omega^2/\omega$ then gives $\mu_\omega(\underline{\omega}) = 0$. Both drift and diffusion vanish: $\omega_t = \underline{\omega}$ is a constant solution to the SDE (11). By pathwise uniqueness (Karatzas and Shreve, 1991, Theorem 5.2.5), this is the unique solution starting at $\underline{\omega}$. An identical argument applies at $\omega = 1/\epsilon$.

For uniqueness: at any other point $\omega^* \in (0,1)\setminus\{\underline{\omega}, 1/\epsilon\}$, Proposition 21 gives $\sigma_\omega(\omega^*) \neq 0$, so the diffusion does not vanish and ω^* is not absorbing. □

Proof of Theorem 22 and Proposition 23. We use the Feller boundary classification for one-dimensional diffusions (Karlin and Taylor, 1981, Ch. 15), (Borodin and Salminen, 2002, Ch. II). For a diffusion $\mathrm{d}X = \mu(X)\,\mathrm{d}t + \sigma(X)\,\mathrm{d}Z$ on an interval (l,r), the boundary classification depends on the scale function density $s'(x) = \exp(-\int^x 2\mu(y)/\sigma(y)^2\,\mathrm{d}y)$ and the speed measure density $m(x) = 1/(\sigma(x)^2 s'(x))$. A boundary is *unattainable* (entrance or natural) if $\Sigma = \int s'(x)\,\mathrm{d}x$ diverges at the boundary, or if $N = \int s'(x)\int m(y)\,\mathrm{d}y\,\mathrm{d}x$ diverges. A boundary where both Σ and N diverge is called *natural*.

For our diffusion (11), $\mu_\omega = \sigma_\omega^2/\omega$, so $2\mu_\omega/\sigma_\omega^2 = 2/\omega$. Therefore $s'(\omega) = \exp(-2\ln\omega) = C/\omega^2$ (normalised at ω_0: $s'(\omega) = \omega_0^2/\omega^2$) and $m(\omega) = \omega^2/(\omega_0^2\sigma_\omega(\omega)^2)$.

(i) $\omega = 0$ (natural boundary). Near $\omega = 0$: $\sigma_\omega \approx -\sigma\lambda\underline{\omega}\cdot\omega \equiv -c_0\omega$ with $c_0 = \sigma\lambda\underline{\omega} > 0$ (Proposition 21). There exist $\underline{c}, \bar{c} > 0$ with $\underline{c}\,\omega \le |\sigma_\omega| \le \bar{c}\,\omega$ on $(0,a]$ for some $a > 0$.

$\Sigma(0) = \int_0^a s'(x)\,\mathrm{d}x = \omega_0^2\int_0^a x^{-2}\,\mathrm{d}x = +\infty$.

$m(\omega) \ge \omega^2/(\omega_0^2\bar{c}^2\omega^2) = 1/(\omega_0^2\bar{c}^2) \equiv K > 0$. Hence $\int_0^x m(y)\,\mathrm{d}y \ge Kx$ and

$$N(0) = \int_0^a s'(x)\int_0^x m(y)\,\mathrm{d}y\,\mathrm{d}x \ge K\int_0^a \frac{\omega_0^2}{x^2}\cdot x\,\mathrm{d}x = K\omega_0^2\int_0^a x^{-1}\,\mathrm{d}x = +\infty.$$

Both Σ and N diverge: $\omega = 0$ is a natural boundary, unattainable from the interior.

(ii) $\omega = \underline{\omega}$ (unattainable from both sides). Write $\sigma_\omega(\omega) = g(\omega)(\omega - \underline{\omega})$ with g continuous near $\underline{\omega}$ and $g(\underline{\omega}) = \sigma\lambda\underline{\omega} > 0$. There exist $\underline{g}, \bar{g} > 0$ with $\underline{g} \le |g(\omega)| \le \bar{g}$ for ω near

$\underline{\omega}$.

Unattainability from the right. We show that the speed measure integral diverges, which forces $N(\underline{\omega}^+) = +\infty$ in the Feller classification. Near $\underline{\omega}$ from the right: $m(\omega) = \omega^2/(\omega_0^2 g(\omega)^2(\omega - \underline{\omega})^2) \geq \underline{\omega}^2/(\omega_0^2 \bar{g}^2(\omega - \underline{\omega})^2)$. Therefore:

$$\int_{\underline{\omega}}^{x} m(y)\,\mathrm{d}y \geq \frac{\underline{\omega}^2}{\omega_0^2 \bar{g}^2} \int_{\underline{\omega}}^{x} \frac{1}{(y-\underline{\omega})^2}\,\mathrm{d}y = \frac{\underline{\omega}^2}{\omega_0^2 \bar{g}^2} \cdot \frac{1}{x - \underline{\omega}} \to +\infty \quad \text{as } x \to \underline{\omega}^+.$$

Since $s'(\omega) = \omega_0^2/\omega^2 \geq \omega_0^2/b^2 > 0$ for $\omega \in [\underline{\omega}, b]$, the Feller integral $N(\underline{\omega}^+) = \int_{\underline{\omega}}^{b} s'(x) \int_{\underline{\omega}}^{x} m(y)\,\mathrm{d}y\,\mathrm{d}x = +\infty$. Hence $\underline{\omega}$ is unattainable from the right. An identical argument (replacing $(\omega - \underline{\omega})$ by $(\underline{\omega} - \omega)$ in the crisis regime where $\sigma_\omega = g(\omega)(\omega - \underline{\omega})$ with $g > 0$ and $\omega < \underline{\omega}$ so $|\sigma_\omega| = |g|(\underline{\omega} - \omega)$) shows unattainability from the left.

(iii) $\omega = 1/\epsilon$ (unattainable from both sides). Write $\sigma_\omega = g(\omega)(1/\epsilon - \omega)$ with $g(1/\epsilon) = \sigma > 0$. The analysis is identical to (ii): $m(\omega) \geq K(1/\epsilon - \omega)^{-2}$ near $1/\epsilon$ from the left, so $\int m(y)\,\mathrm{d}y$ diverges and $N(1/\epsilon^-) = +\infty$. Symmetrically from the right.

(iv) $\omega = 1$ (unattainable). Write $\sigma_\omega = h(\omega)(1 - \omega)$ with $h(1) = -\sigma(1 + \varphi) \neq 0$. Near $\omega = 1$: $m(\omega) \geq K(1 - \omega)^{-2}$, giving $N(1^-) = +\infty$. Unattainable.

Regime invariance. The process ω_t has continuous paths (it is a strong solution of an SDE with continuous coefficients). Since all internal boundary points 0, $\underline{\omega}$, $1/\epsilon$, and 1 are unattainable, continuous paths cannot cross them: $(0, \underline{\omega})$, $(\underline{\omega}, 1/\epsilon)$, and $(1/\epsilon, 1)$ are invariant sets. □

H Convergence

Proof of Theorem 6. **Almost-sure convergence.** $Q_t = 1/\omega_t$ is a non-negative local martingale (Proposition 4), hence a supermartingale by Fatou's lemma. By the supermartingale convergence theorem (Karatzas and Shreve, 1991, Ch. 1), $Q_t \to Q_\infty$ a.s. with $\mathbb{E}[Q_\infty] \leq Q_0 < \infty$, so $Q_\infty < \infty$ a.s. and $\omega_\infty \equiv 1/Q_\infty > 0$.

The limit is a zero of σ_ω. The quadratic variation of Q is $[Q]_t = \int_0^t (\sigma_\omega(\omega_s)/\omega_s^2)^2\,\mathrm{d}s$. We claim $[Q]_\infty < \infty$ a.s. By the Dambis–Dubins–Schwarz representation (Karatzas and Shreve, 1991, Ch. 3), there exists a Brownian motion B such that $Q_t = Q_0 + B_{[Q]_t}$ for all $t \geq 0$. If $[Q]_\infty = +\infty$, then $Q_t = Q_0 + B_{[Q]_t}$ where B is evaluated at a time tending to infinity; since Brownian motion does not converge (it is neighbourhood-recurrent on $\mathbb{R}$, Karatzas and Shreve, 1991, Theorem 2.9.23), this contradicts the a.s. convergence $Q_t \to Q_\infty$. Therefore $[Q]_\infty < \infty$ a.s.

Since $\omega_t \to \omega_\infty$ and the integrand $(\sigma_\omega(\omega)/\omega^2)^2$ is continuous, the convergence $[Q]_\infty < \infty$ requires $\sigma_\omega(\omega_\infty) = 0$: if $\sigma_\omega(\omega_\infty) \neq 0$, then by continuity the integrand would be bounded

below by a positive constant for all large t, and the integral would diverge. By Proposition 21, the zeros of σ_ω in $[0,1)$ are $\{0, \underline{\omega}, 1/\epsilon\}$. Since $\omega_\infty > 0$, we have $\omega_\infty \in \{\underline{\omega}, 1/\epsilon\}$.

Regime-specific limits.

Crisis regime $(0, \underline{\omega})$*:* ω_t remains in $(0, \underline{\omega})$ for all t (regime invariance, Proposition 23). The only zero of σ_ω accessible from this regime is $\underline{\omega}$ (since $\omega_\infty > 0$ and $1/\epsilon \notin [0, \underline{\omega}]$). Hence $\omega_t \to \underline{\omega}$.

Recovery regime $(\underline{\omega}, 1/\epsilon)$*:* ω_t remains in $(\underline{\omega}, 1/\epsilon)$. Since $\underline{\omega}$ is unattainable from the right (Proposition 23), $\omega_\infty \neq \underline{\omega}$. Hence $\omega_t \to 1/\epsilon$.

Expansionary regime $(1/\epsilon, 1)$*:* ω_t remains in $(1/\epsilon, 1)$. Since 1 is unattainable, $\omega_\infty \neq 1$. The only zero of σ_ω in $[1/\epsilon, 1)$ is $1/\epsilon$. Hence $\omega_t \to 1/\epsilon$.

Infinite expected hitting time. Let $\omega^* \in \{\underline{\omega}, 1/\epsilon\}$ be the relevant absorbing state. Near ω^*: $\sigma_\omega(\omega) = g(\omega)(\omega^* - \omega)$ with g continuous and bounded between $\underline{g}, \bar{g} > 0$ (Proposition 23). The speed measure density satisfies $m(\omega) = \omega^2/(\omega_0^2 \sigma_\omega^2) \geq (\omega^*)^2/(2\omega_0^2 \bar{g}^2 (\omega^* - \omega)^2)$ for ω close to ω^*. By the Green's function formula for expected hitting times (Karlin and Taylor, 1981, Ch. 15):

$$\mathbb{E}_{\omega_0}[\tau_{\omega^*}] = \int_{\omega_0}^{\omega^*} \int_{\omega_0}^{y} 2\, m(z)\, s'(z)\, \mathrm{d}z\, [s(y) - s(\omega_0)]\, \mathrm{d}y + \int_{\omega_0}^{\omega^*} [s(\omega^*) - s(y)]\, 2\, m(y)\, \mathrm{d}y.$$

The second integral satisfies $\int_a^{\omega^*} [s(\omega^*) - s(y)]\, m(y)\, \mathrm{d}y \geq K \int_a^{\omega^*} (\omega^* - y)^{-1}\, \mathrm{d}y = +\infty$, where we used $s(\omega^*) - s(y) \geq c > 0$ for y near ω^* (since s is monotone and $s(\omega^*) > s(y)$) and $m(y) \geq K'(\omega^* - y)^{-2}$. Therefore $\mathbb{E}[\tau] = +\infty$. □

I Proof of Intermediation Gap Classification

Proof of Theorem 9. By Proposition 4, $Q = 1/\omega$ is a non-negative continuous local martingale. By (21), the intermediation gap is positive if and only if Q is a strict local martingale. We classify Q using the integral test of Mijatović and Urusov (2012): a non-negative continuous local martingale Q with $\mathrm{d}Q = -(\sigma_\omega/\omega^2)\, \mathrm{d}Z$ is a strict local martingale if and only if

$$\int_c^\infty \frac{q}{\sigma_Q(q)^2}\, \mathrm{d}q < \infty \tag{47}$$

for some (hence all) $c > 0$, where $\sigma_Q(q)$ is the diffusion coefficient of Q expressed as a function of Q.

Since $\sigma_Q = \sigma_\omega/\omega^2$ and $Q = 1/\omega$, we have $\sigma_Q(Q) = Q^2\, |\sigma_\omega(1/Q)|$. The integral becomes:

$$\int_c^\infty \frac{q}{q^4\, \sigma_\omega(1/q)^2}\, \mathrm{d}q = \int_c^\infty \frac{1}{q^3\, \sigma_\omega(1/q)^2}\, \mathrm{d}q.$$

(i) Baseline model. Near $\omega = 0$ (equivalently, $Q \to \infty$), the baseline volatility (39) satisfies $\sigma_\omega^0(\omega) \to c_0 > 0$ where $c_0 = \sigma(1+\varphi)(1/\epsilon)/[(1-1/\epsilon)+\varphi]$. Therefore $\sigma_\omega^0(1/q) \to c_0$ as $q \to \infty$, and

$$\int_c^\infty \frac{1}{q^3\, c_0^2}\, \mathrm{d}q = \frac{1}{2c_0^2\, c^2} < \infty.$$

By the integral test, Q is a strict local martingale. A positive intermediation gap exists.

(ii) Constrained model. Near $\omega = 0$, the constrained volatility satisfies $|\sigma_\omega(\omega)| \sim c_0'\omega$ with $c_0' = \sigma\lambda\underline{\omega} > 0$ (Proposition 21). Therefore $|\sigma_\omega(1/q)| \sim c_0'/q$ as $q \to \infty$, and

$$\int_c^\infty \frac{1}{q^3\, (c_0'/q)^2}\, \mathrm{d}q = \frac{1}{(c_0')^2} \int_c^\infty \frac{1}{q}\, \mathrm{d}q = +\infty.$$

By the integral test, Q is a true martingale. The intermediation gap is zero.

The mechanism is transparent: the leverage constraint forces $\phi \le \bar{\phi}(\omega) \to 1 - \lambda\underline{\omega}$ as $\omega \to 0$, so the financier is de-leveraged when her capital is depleted. Since $\sigma_\omega = \omega(\phi - 1)\sigma_R$ and $\phi - 1$ remains bounded, the extra factor of ω survives, driving $\sigma_\omega \propto \omega \to 0$. In the baseline, $\phi = 1/(\epsilon\omega) \to \infty$: the ω cancels and σ_ω stays positive. One factor of ω is the difference between convergent and divergent integrals, between a positive intermediation gap and a zero gap.

For the three individual regimes of the constrained model: in the crisis and recovery regimes, Q ranges over a bounded interval (since $\omega \in (0, \underline{\omega})$ or $(\underline{\omega}, 1/\epsilon)$), so Q is bounded and hence a true martingale. In the expansionary regime $\omega \in (1/\epsilon, 1)$, $Q \in (1, \epsilon)$ is again bounded. □

J Robustness and Additional Diagnostics

This appendix reports robustness checks, inference diagnostics, out-of-sample evidence, and auxiliary tests referenced in the main text.

J.1 Alternative Specifications

Test 1. We report the quadratic specification (Table 7), the unemployment-gap specification (Table 9), and the control specification (Table 10).

Table 7: Test 1: nonlinear specification

	h=1	h=2	h=4	h=8
Constant	-0.058***	-0.060***	-0.062***	-0.058***
	(0.018)	(0.019)	(0.018)	(0.019)
OmegaHkm	1.196***	1.255**	1.342***	1.316**
	(0.460)	(0.490)	(0.462)	(0.555)
OmegaHkmSq	-5.184*	-5.618*	-6.311**	-6.973*
	(2.706)	(2.949)	(2.896)	(3.718)
Adj. R^2	0.256	0.256	0.251	0.152
No. Obs.	219	218	216	212

Table 8: Test 1: state dependence (piecewise linear, no controls, low-10% threshold)

	h=1	h=2	h=4	h=8
Constant	-0.030***	-0.028***	-0.029***	-0.020**
	(0.006)	(0.006)	(0.008)	(0.009)
OmegaHkm	0.390***	0.369***	0.381***	0.244*
	(0.081)	(0.086)	(0.108)	(0.128)
OmegaHkmxOmegaLowQuant	-0.527	-0.706**	-0.435	-0.590**
	(0.323)	(0.324)	(0.288)	(0.280)
Adj. R^2	0.255	0.273	0.228	0.136
No. Obs.	219	218	216	212

Table 9: Test 1: alternative dependent variable (unemployment gap)

	h=1	h=2	h=4	h=8
Constant	0.029***	0.029***	0.029***	0.025***
	(0.005)	(0.005)	(0.005)	(0.006)
OmegaHkm	-0.348***	-0.347***	-0.355***	-0.286***
	(0.065)	(0.064)	(0.069)	(0.082)
Adj. R^2	0.243	0.244	0.256	0.165
No. Obs.	219	218	216	212

Table 10: Test 1: with controls

	h=1	h=2	h=4	h=8
Constant	0.012	0.007	-0.005	-0.021
	(0.010)	(0.011)	(0.012)	(0.013)
OmegaHkm	0.203**	0.208**	0.246**	0.231*
	(0.082)	(0.092)	(0.111)	(0.121)
Yield3m	0.082	0.066	0.038	0.005
	(0.065)	(0.069)	(0.075)	(0.079)
TermSpread	-0.362***	-0.182	0.145	0.577***
	(0.128)	(0.150)	(0.191)	(0.210)
CreditSpread	-2.245***	-2.350***	-2.056***	-1.212**
	(0.777)	(0.726)	(0.715)	(0.497)
SpxVol	-0.042	-0.013	0.015	0.030
	(0.047)	(0.045)	(0.049)	(0.045)
Adj. R^2	0.533	0.431	0.307	0.217
No. Obs.	219	218	216	212

Test 4. We report a control specification for the model-implied proxy, two auxiliary diagnostics based on CAPE and credit spreads, and horse-race specifications that include both the HKM ratio and the intermediation gap proxy.

Table 11: Test 4: model-implied proxy (baseline, $\gamma = 4$)

	h=1	h=2	h=4	h=8
Constant	0.083***	0.087***	0.085***	0.065*
	(0.024)	(0.026)	(0.028)	(0.038)
BubbleApproxAlt	-0.113***	-0.119***	-0.116***	-0.090*
	(0.034)	(0.036)	(0.038)	(0.050)
Adj. R^2	0.203	0.223	0.213	0.123
No. Obs.	151	150	148	144

Table 12: Test 4: CAPE proxy (diagnostic, baseline)

	h=1	h=2	h=4	h=8
Constant	0.001	0.001	0.001	0.000
	(0.002)	(0.002)	(0.003)	(0.003)
BpCape	0.019***	0.020***	0.019***	0.014**
	(0.005)	(0.006)	(0.005)	(0.006)
Adj. R^2	0.314	0.335	0.299	0.174
No. Obs.	151	150	148	144

Table 13: Test 4: model-implied proxy (with controls, appendix robustness)

	h=1	h=2	h=4	h=8
Constant	0.065*	0.069*	0.060	0.041
	(0.034)	(0.041)	(0.044)	(0.044)
BubbleApprox	-0.035	-0.042	-0.041	-0.040
	(0.045)	(0.055)	(0.061)	(0.056)
Yield3m	0.024	-0.064	-0.173	-0.220*
	(0.092)	(0.117)	(0.140)	(0.119)
TermSpread	-0.281	-0.129	0.163	0.542**
	(0.188)	(0.215)	(0.234)	(0.212)
CreditSpread	-3.188***	-2.539***	-1.746**	-0.555
	(0.726)	(0.847)	(0.820)	(0.706)
SpxVol	-0.019	-0.039	-0.028	-0.005
	(0.060)	(0.068)	(0.069)	(0.047)
Adj. R^2	0.578	0.475	0.406	0.406
No. Obs.	151	150	148	144

Test 2 robustness. We report results using the raw HKM ratio (which delivers the predicted negative sign but weaker precision), the baseline specification using $\widehat{Q}_t^{HKM}$, and a shortfall-based predictor.

Table 14: Test 2 robustness: baseline (Shortfall)

	h=1	h=2	h=4	h=8
Constant	0.013**	0.023*	0.052**	0.120**
	(0.006)	(0.013)	(0.026)	(0.060)
Shortfall	0.687	1.822	3.205	4.912
	(0.571)	(1.138)	(2.006)	(4.144)
Adj. R^2	0.003	0.019	0.028	0.029
No. Obs.	219	218	216	212

Table 15: Test 2: with controls (QHkm; higher QHkm = lower intermediary wealth share)

	h=1	h=2	h=4	h=8
Constant	-0.015	-0.032	-0.062	-0.047
	(0.018)	(0.034)	(0.065)	(0.145)
QHkm	0.001	0.003	0.006*	0.014**
	(0.001)	(0.002)	(0.003)	(0.007)
Yield3m	-0.298	-0.603	-0.864	-1.321
	(0.213)	(0.370)	(0.625)	(1.089)
TermSpread	-0.031	-0.123	0.800	3.420
	(0.434)	(0.816)	(1.618)	(3.068)
CreditSpread	-0.279	0.661	-1.682	-7.602
	(2.547)	(3.446)	(4.688)	(6.984)
SpxVol	0.222*	0.280	0.583*	0.261
	(0.122)	(0.199)	(0.349)	(0.445)
Adj. R^2	0.025	0.052	0.089	0.107
No. Obs.	219	218	216	212

Test 3 robustness. We report a control specification and an asymmetric "positive changes only" version of the QE reaction regression.

Table 16: Test 3: QE reaction function (with controls, appendix robustness)

	h=1
Constant	0.004
	(0.009)
Shortfall	0.425
	(0.471)
Yield3m	-0.166*
	(0.101)
TermSpread	-0.049
	(0.169)
CreditSpread	-0.975
	(1.162)
SpxVol	0.078*
	(0.046)
Adj. R^2	0.137
No. Obs.	87

J.2 Inference Robustness

Predictive regressions with persistent predictors are subject to well-known inference challenges (Stambaugh, 1999; Lewellen, 2004; Campbell and Yogo, 2006). Tables 17–19 report six inference methods for the baseline specification of each test: Newey–West HAC, Hodrick (1992) reverse-regression standard errors, Stambaugh (1999) bias-corrected estimates with HAC t-statistics, Lewellen (2004) conservative confidence intervals assuming a unit-root predictor, Campbell–Yogo (2006) Bonferroni Q-tests, and bootstrap p-values (wild bootstrap with Rademacher weights and, for $h > 1$, moving-block bootstrap under the null).

Table 17: Inference robustness: Test 1 (HKM capital ratio → output gap)

	$h = 1$	$h = 2$	$h = 4$	$h = 8$
$\hat{\beta}_{OLS}$	0.441	0.437	0.423	0.302
t_{NW}	4.66***	4.33***	3.86***	2.22**
$t_{Hodrick}$	8.60	5.07	3.30	1.89
$\hat{\beta}_{Stambaugh}$	0.438	0.434	0.423	0.304
$t_{Stambaugh}$	4.63	4.29	3.86	2.23
Lewellen CI	[0.25, 0.62]	[0.24, 0.63]	[0.21, 0.64]	[0.04, 0.57]
Lewellen reject H_0	Yes	Yes	Yes	Yes
C-Y reject H_0	Yes	Yes	Yes	Yes
Wild bootstrap p	<0.001	<0.001	<0.001	<0.001
Block bootstrap p	—	<0.001	<0.001	0.028
R^2	0.234	0.230	0.216	0.110
N	219	218	216	212

Notes: t_{NW} uses Newey–West HAC standard errors with $\max(h, 4)$ lags. $t_{Hodrick}$ uses Hodrick (1992) reverse-regression standard errors. Stambaugh (1999) bias-corrected coefficient and t-statistic use HAC SE. Lewellen (2004) 95% CI assumes a unit-root predictor. C-Y denotes the Campbell–Yogo (2006) Bonferroni Q-test. Wild bootstrap uses Rademacher weights under $H_0 : \beta = 0$ (5000 replications). Block bootstrap resamples blocks of length h under H_0 (5000 replications) and is reported only for $h > 1$. */**/*** denote significance at 10%/5%/1%.

Table 18: Inference robustness: Test 2 (Q^{HKM} → excess returns)

	$h = 1$	$h = 2$	$h = 4$	$h = 8$
$\hat{\beta}_{OLS}$	0.001	0.003	0.006	0.009
t_{NW}	1.32	1.75*	1.71*	1.33
$t_{Hodrick}$	1.22	1.63	1.52	1.14
$\hat{\beta}_{Stambaugh}$	0.001	0.003	0.005	0.009
$t_{Stambaugh}$	1.26	1.73	1.71	1.33
Lewellen CI	[-0.00, 0.00]	[-0.00, 0.01]	[-0.00, 0.01]	[-0.00, 0.02]
Lewellen reject H_0	No	No	No	No
C-Y reject H_0	No	Yes	Yes	Yes
Wild bootstrap p	0.246	0.046	0.014	0.008
Block bootstrap p	—	0.040	0.059	0.119
R^2	0.010	0.028	0.041	0.050
N	219	218	216	212

Notes: t_{NW} uses Newey–West HAC standard errors with $\max(h, 4)$ lags. $t_{Hodrick}$ uses Hodrick (1992) reverse-regression standard errors. Stambaugh (1999) bias-corrected coefficient and t-statistic use HAC SE. Lewellen (2004) 95% CI assumes a unit-root predictor. C-Y denotes the Campbell–Yogo (2006) Bonferroni Q-test. Wild bootstrap uses Rademacher weights under $H_0 : \beta = 0$ (5000 replications). Block bootstrap resamples blocks of length h under H_0 (5000 replications) and is reported only for $h > 1$. */**/*** denote significance at 10%/5%/1%.

Table 19: Inference robustness: Test 4 (model-implied gap proxy → output gap, pre-QE)

	$h = 1$	$h = 2$	$h = 4$	$h = 8$
$\hat{\beta}_{OLS}$	-0.161	-0.168	-0.164	-0.126
t_{NW}	-3.39***	-3.30***	-3.02***	-1.77*
$t_{Hodrick}$	-6.53	-3.87	-2.57	-1.50
$\hat{\beta}_{Stambaugh}$	-0.157	-0.166	-0.164	-0.126
$t_{Stambaugh}$	-3.32	-3.26	-3.01	-1.78
Lewellen CI	[-0.25, -0.06]	[-0.27, -0.07]	[-0.27, -0.06]	[-0.27, 0.01]
Lewellen reject H_0	Yes	Yes	Yes	No
C-Y reject H_0	Yes	Yes	Yes	Yes
Wild bootstrap p	<0.001	<0.001	<0.001	<0.001
Block bootstrap p	—	<0.001	<0.001	0.060
R^2	0.206	0.225	0.215	0.125
N	151	150	148	144

Notes: t_{NW} uses Newey–West HAC standard errors with $\max(h, 4)$ lags. $t_{Hodrick}$ uses Hodrick (1992) reverse-regression standard errors. Stambaugh (1999) bias-corrected coefficient and t-statistic use HAC SE. Lewellen (2004) 95% CI assumes a unit-root predictor. C-Y denotes the Campbell–Yogo (2006) Bonferroni Q-test. Wild bootstrap uses Rademacher weights under $H_0 : \beta = 0$ (5000 replications). Block bootstrap resamples blocks of length h under H_0 (5000 replications) and is reported only for $h > 1$. */**/*** denote significance at 10%/5%/1%.

Table 20: Inference robustness: Test 3 (shortfall → QE response)

	$h = 1$
$\hat{\beta}_{OLS}$	0.525
t_{NW}	2.27**
$t_{Hodrick}$	2.03
$\hat{\beta}_{Stambaugh}$	0.531
$t_{Stambaugh}$	2.29
Lewellen CI	[0.08, 0.99]
Lewellen reject H_0	Yes
C-Y reject H_0	Yes
Wild bootstrap p	0.015
Block bootstrap p	—
R^2	0.092
N	87

Notes: t_{NW} uses Newey–West HAC standard errors with $\max(h, 4)$ lags. $t_{Hodrick}$ uses Hodrick (1992) reverse-regression standard errors. Stambaugh (1999) bias-corrected coefficient and t-statistic use HAC SE. Lewellen (2004) 95% CI assumes a unit-root predictor. C-Y denotes the Campbell–Yogo (2006) Bonferroni Q-test. Wild bootstrap uses Rademacher weights under $H_0 : \beta = 0$ (5000 replications). Block bootstrap resamples blocks of length h under H_0 (5000 replications) and is reported only for $h > 1$. */**/*** denote significance at 10%/5%/1%.

J.3 Out-of-Sample Evidence

Table 21 reports recursive expanding-window forecasts for Tests 1, 2, and 4. For each horizon h and forecast origin t, we estimate

$$y_{s+h} = a_{t,h} + b_{t,h} z_s + u_{s+h}, \qquad s = 1, \ldots, t-h, \tag{48}$$

using all observations available up to t and an initial training window of 60 quarters. The resulting model forecast is:

$$\widehat{y}^{M}_{t+h|t} = \widehat{a}_{t,h} + \widehat{b}_{t,h} z_t. \tag{49}$$

For Tests 1 and 4, the target is the future output gap, $y_{t+h} = \widehat{x}_{t+h}$. For Test 2, the target is the cumulative excess return $\widehat{R}^{e}_{t\to t+h}$ defined in Section K.1. The predictor z_t is, respectively, the HKM capital ratio $\widehat{\omega}_t$, the capital scarcity measure $\widehat{Q}^{HKM}_t$, the approximate gap proxy $(1-\widehat{\omega}_t)^{\gamma}$, or the exact gap proxy $b(\widehat{\omega}_t)$.

We compare the model forecast with two recursive benchmarks. The prevailing-mean benchmark is

$$\widehat{y}^{PM}_{t+h|t} = \frac{1}{N_t} \sum_{s \le t} y_s. \tag{50}$$

The AR(1) benchmark is obtained by estimating

$$y_{s+1} = c_t + \rho_t y_s + v_{s+1} \tag{51}$$

on the benchmark series available at forecast origin t and iterating the fitted law of motion forward:

$$\widehat{y}^{AR1}_{t+h|t} = \widehat{c}_t \sum_{j=0}^{h-1} \widehat{\rho}_t^{\,j} + \widehat{\rho}_t^{\,h} y_t. \tag{52}$$

For Tests 1 and 4, the AR(1) benchmark series is the output gap itself. For Test 2, it is the aligned h-quarter excess-return series, using only observations realized by the forecast origin.

Forecast performance is summarized by

$$R^2_{OOS,B} = 1 - \frac{\sum_t \left(y_{t+h} - \widehat{y}^M_{t+h|t}\right)^2}{\sum_t \left(y_{t+h} - \widehat{y}^B_{t+h|t}\right)^2}, \qquad B \in \{PM, AR1\}. \tag{53}$$

For the prevailing-mean benchmark, we report the Clark–West statistic based on the adjusted loss differential

$$f_t = e^2_{B,t} - \left[e^2_{M,t} - \left(\widehat{y}^B_{t+h|t} - \widehat{y}^M_{t+h|t}\right)^2\right]. \tag{54}$$

For the AR(1) benchmark, we report the Harvey–Leybourne–Newbold-adjusted Diebold–Mariano statistic based on the loss differential

$$d_t = e^2_{B,t} - e^2_{M,t}, \tag{55}$$

with the one-sided alternative that the model forecast has lower MSPE than the benchmark.

Table 21: Out-of-sample predictive performance

	$h = 1$	$h = 2$	$h = 4$	$h = 8$
Test 1: HKM Capital Ratio → Output Gap				
R^2_{OOS} vs. prev. mean	0.248	0.181	0.093	-0.092
CW stat	6.62***	4.47***	2.97***	1.60*
R^2_{OOS} vs. AR(1)	-1.548	-0.721	-0.212	-0.078
DM stat	-4.54	-2.64	-0.87	-0.34
Test 2: $\widehat{Q}^{HKM}$ → Excess Returns				
R^2_{OOS} vs. prev. mean	-0.008	-0.015	-0.034	-0.114
CW stat	-0.02	0.38	-0.07	-0.91
R^2_{OOS} vs. AR(1)	-0.001	0.038	0.090	0.044
DM stat	-0.05	0.43	0.68	0.28
Test 4: Approximate Gap Proxy $(1-\widehat{\omega})^{2.5}$ → Output Gap (pre-QE)				
R^2_{OOS} vs. prev. mean	0.190	0.101	-0.074	-0.339
CW stat	5.02***	3.58***	2.42***	1.39*
R^2_{OOS} vs. AR(1)	-7.039	-3.050	-1.070	-0.339
DM stat	-7.34	-4.21	-2.01	-0.63
Test 4: Exact Gap Proxy $b(\widehat{\omega})$ → Output Gap (pre-QE)				
R^2_{OOS} vs. prev. mean	0.136	0.160	0.222	0.454
CW stat	4.74***	4.00***	3.55***	3.27***
R^2_{OOS} vs. AR(1)	-7.058	-2.515	-0.483	0.423
DM stat	-6.67	-3.22	-0.91	2.16**

Notes: Out-of-sample R^2 from expanding-window recursive OLS with minimum training window of 60 quarters. "Prev. mean" benchmark uses the historical average of the dependent variable. "AR(1)" benchmark uses a recursive AR(1) forecast. CW stat is the Clark–West (2007) MSPE-adjusted test statistic for the prevailing-mean benchmark. DM stat is the Harvey–Leybourne–Newbold-adjusted Diebold–Mariano statistic for the AR(1) benchmark; */**/*** denote one-sided significance at 10%/5%/1%.

J.4 Subsample and Outlier Robustness

Test 1 robustness. We report control-heavy specifications, subsample splits (1970–1999 and 2000–2024), outlier exclusions (dropping 2008–2009 and 2020 crisis quarters), nonlinear, state-dependent, and alternative dependent variable specifications.

Table 22: Test 1: subsample 1970–1999

	h=1	h=2	h=4	h=8
Constant	-0.030***	-0.032***	-0.035***	-0.036***
	(0.009)	(0.010)	(0.010)	(0.012)
OmegaHkm	0.420***	0.464***	0.530***	0.561***
	(0.112)	(0.123)	(0.140)	(0.180)
Adj. R^2	0.215	0.253	0.280	0.221
No. Obs.	119	118	116	112

Table 23: Test 1: subsample 2000–2024

	h=1	h=2	h=4	h=8
Constant	-0.044***	-0.037***	-0.032**	-0.023
	(0.011)	(0.011)	(0.013)	(0.014)
OmegaHkm	0.545***	0.446***	0.365*	0.223
	(0.155)	(0.164)	(0.193)	(0.199)
Adj. R^2	0.284	0.193	0.129	0.041
No. Obs.	99	98	96	92

Table 24: Test 1: excluding outlier periods

	h=1	h=2	h=4	h=8
Constant	-0.031***	-0.032***	-0.031***	-0.024**
	(0.007)	(0.008)	(0.008)	(0.010)
OmegaHkm	0.403***	0.424***	0.416***	0.304**
	(0.095)	(0.103)	(0.112)	(0.143)
Adj. R^2	0.210	0.233	0.226	0.118
No. Obs.	213	212	210	206

J.5 QE Trap Additional Tests

QT1 episode study. Table 25 reports the simple descriptive evidence for the Federal Reserve's first quantitative tightening episode. In addition to the start and end values of the HKM capital ratio, it estimates the within-episode linear trend

$$\omega_t = a + bt + u_t, \tag{56}$$

over 2017Q3–2019Q3, where b measures the average quarterly drift in intermediary capital during QT1.

Table 25: QT1 episode study

	Estimate
ω at QT1 start (2017Q3)	0.0791
ω at QT1 end (2019Q3)	0.0657
Change in ω	−0.0134
Quarterly trend (OLS)	−0.0022
t-statistic	−5.06
p-value	0.001

Notes: QT1 is the Federal Reserve's first quantitative tightening episode (2017Q4–2019Q3). The reported start level uses 2017Q3, the last quarter before runoff began. The quarterly trend is an OLS regression of ω_t on a linear time index over 2017Q3–2019Q3.

Persistence analysis. Table 26 compares persistence before and after the Global Financial Crisis using separate AR(1) regressions of the form

$$\omega_t = \mu + \rho\,\omega_{t-1} + u_t. \tag{57}$$

The table reports $\hat{\rho}$, the implied half-life, and the difference $\hat{\rho}_{post} - \hat{\rho}_{pre}$ with its associated test statistic.

Table 26: Persistence of ω_t before and after 2008

	Pre-2008	Post-2008	Difference
AR(1) coefficient $\hat{\rho}$	0.952	0.812	
Half-life (quarters)	14.0	3.3	
$\Delta\rho$			-0.140
t-stat ($\Delta\rho = 0$)			-1.94
p-value			0.053

Notes: AR(1) models are estimated separately before and after 2008Q4. Half-life is $-\log(2)/\log(\hat{\rho})$. The final column reports the difference $\hat{\rho}_{post} - \hat{\rho}_{pre}$ and its normal-approximation test.

Threshold autoregression. Table 27 estimates a Hansen (1996) threshold AR(1) in which the law of motion for intermediary capital is allowed to differ across low- and high-capital states:

$$\omega_t = \begin{cases} \mu_L + \rho_L \omega_{t-1} + u_t, & \text{if } \omega_{t-1} < \omega^*, \\ \mu_H + \rho_H \omega_{t-1} + u_t, & \text{if } \omega_{t-1} \geq \omega^*. \end{cases} \tag{58}$$

The threshold ω^* is estimated endogenously, and the table reports the corresponding sup-Wald statistic together with its wild-bootstrap p-value.

Table 27: Threshold autoregression for ω_t

	Below threshold	Above threshold
AR(1) coefficient	0.943	0.060
Half-life (quarters)	11.9	0.2
N	203	16
Estimated threshold	0.0952	
Model ω_c	0.0776	
sup-Wald	15.98	
Bootstrap p	0.239	

Notes: Hansen (1996) threshold AR(1) estimated on ω_t. The threshold grid combines a percentile grid with a finer grid around the model-implied crisis threshold. The sup-Wald p-value is based on a wild bootstrap. Half-life is $-\log(2)/\log(\hat{\rho})$.

J.6 Cross-Equation SUR Test

Table 28 reports the three-equation SUR system used in the main-text discussion of the model's single-state-variable restriction. Equation 1 regresses the four-quarter-ahead output gap on the HKM capital ratio; Equation 2 regresses cumulative four-quarter excess equity returns on Q^{HKM}; and Equation 3 regresses the model-implied intermediation gap proxy on the HKM capital ratio. Panel A reports equation-by-equation OLS with Newey–West t-statistics, Panel B reports the iterated SUR estimates with both standard and HAC-robust t-statistics, Panel C reports the SUR residual correlation matrix, and Panel D reports the Breusch–Pagan, joint Wald, proportionality, and eigenvalue diagnostics.

J.7 Calibrated Counterfactual

We map the HKM capital ratio to the model's state variable ω_t at four key dates and compute model-implied quantities under the baseline calibration. Table 29 reports ω, leverage, employment, central bank equity holdings, and the required fiscal transfer at each date. At the 2009Q1 trough ($\omega = 0.027$), the model implies an employment gap of 3.9% and central bank equity holdings of 73%—broadly consistent with the Federal Reserve's extraordinary balance sheet expansion. By 2019Q3 ($\omega = 0.066$), the economy remains below the crisis threshold, with a 1.9% employment gap and 19% central bank equity holdings, consistent with the Federal Reserve's still-elevated balance sheet a decade after the crisis.

Table 30 compares three counterfactual scenarios starting from the 2009Q1 trough. Without QE, equilibrium does not exist ($\omega < \omega_c$). With QE alone, the economy converges to

Table 28: SUR Cross-Equation Test: Single State Variable Restriction

	Eq. 1 OutputGap_{t+4}	**Eq. 2** $\text{ExcRet}_{t\to t+4}$	**Eq. 3** BP_Model_t
Panel A: OLS (equation by equation)			
Predictor	OmegaHkm	Q^{HKM}	OmegaHkm
$\hat{b}$	$0.3616 * **$	0.0033	$-2.2280 * **$
t_{NW}	$3.004 * **$	0.910	$-175.302 * **$
R^2	0.2101	0.0183	0.9995
Panel B: SUR (feasible GLS, iterated)			
$\hat{b}_{\text{SUR}}$	$0.3597 * **$	0.0037	$-2.2280 * **$
t_{SUR}	6.252	1.840	-536.021
$t_{\text{SUR-HAC}}$	$3.065 * **$	0.990	$-175.276 * **$
R^2_{SUR}	0.2101	0.0181	0.9995
Panel C: Residual Correlations (SUR)			
$\hat{e}_1$	1	0.151	-0.241
$\hat{e}_2$		1	-0.030
$\hat{e}_3$			1
Panel D: Cross-Equation Tests			
Breusch–Pagan $\chi^2(3)$	12.028 [$p = 0.0073$]		
Joint Wald $\chi^2(3)$	303374.790 [$p = 0.000000$]		
Proportionality C: δ (t-stat)	-7.0921 ($t = -1.722$, $p = 0.0851$)		
Proportionality D: δ (t-stat)	0.0057 ($t = 1.473$, $p = 0.1408$)		
Σ eigenvalue shares	0.989, 0.011, 0.000		
T	148		
Sample	1970Q1–2007Q4		
Horizon h	4 quarters		

Notes: This table reports SUR estimates for a three-equation system testing the single-state-variable restriction. Eq. 1 regresses future output gap on the HKM capital ratio ($\hat{\omega}$). Eq. 2 regresses cumulative 4-quarter excess equity returns on $Q^{HKM} = 1/\hat{\omega}$. Eq. 3 regresses the model-implied intermediation gap proxy $BP_{Model,t} = (1 - \hat{\omega}_t)^{2.5}$ on $\hat{\omega}_t$. Panel A reports OLS with Newey–West t-statistics (h lags). Panel B reports iterated feasible GLS (SUR) estimates with both standard and HAC-robust t-statistics. Panel C shows the SUR residual correlation matrix. Panel D reports: the Breusch–Pagan test for diagonal Σ (significant $\Rightarrow$ SUR is more efficient than OLS); a joint Wald test for all slopes equal to zero; proportionality tests C (OmegaHkm incremental to BP_{Model} in Eq. 1) and D (Q^{HKM} incremental to OutputGap in Eq. 2); and eigenvalue shares of $\hat{\Sigma}$ (dominance of first eigenvalue supports single-factor structure). ***, **, * denote significance at 1%, 5%, 10%.

Table 29: Model-Implied Quantities at Key Dates

	2006Q4 (Pre-crisis)	2009Q1 (Trough)	2009Q4 (Post-TARP)	2019Q3 (Pre-COVID)
ω (HKM capital ratio)	0.0818	0.0265	0.0505	0.0657
$\bar{\varphi}(\omega)$ (leverage)	1.309	1.032	1.153	1.228
$N(\omega)$ (employment)	0.9900	0.9615	0.9736	0.9815
Employment gap (%)	1.00%	3.85%	2.64%	1.85%
CB equity share $\alpha(\omega)$ (%)	0.00%	72.64%	41.80%	19.30%
Crisis ($\omega < \omega_c$)	No	Yes	Yes	Yes
Trap ($\omega < \underline{\omega}$)	No	No	No	No
Transfer T^* (% of GDP)	-131.53%	8.89%	-52.05%	-90.62%

Notes: Model parameters: $\varepsilon = 10$, $\underline{\omega} = 0.02$, $\lambda = 5$, $\sigma = 0.15$, $\delta = 0.04$, $\varphi = 1$. ω is mapped from the He, Kelly, and Manela (2017) intermediary capital ratio. Crisis threshold $\omega_c = 0.0776$. Good steady state $\omega^* = 1/\varepsilon = 0.10$. Employment gap is $[N(1/\varepsilon) - N(\omega)]/N(1/\varepsilon)$. Transfer T^* is the fiscal recapitalisation needed to push ω to $\underline{\omega} + 0.01$.

the bad steady state with a permanent 4.2% employment gap and 80% central bank equity holdings. With QE plus fiscal recapitalisation, the economy recovers to the efficient benchmark with zero employment gap. The benefit-to-cost ratio of the fiscal transfer is 4.3: every dollar of recapitalisation generates $4.30 in present value of additional output.

The model-implied fiscal transfer (25% of GDP) is an order of magnitude larger than actual TARP injections (1.7% of GDP). This gap reflects the model's stylised one-state-variable structure, in which a single intermediary must absorb all equity risk. In practice, the financial sector's capital was partially replenished through retained earnings, private capital raises, and portfolio rebalancing, reducing the required public transfer. The key qualitative prediction—that the fiscal return on recapitalisation is large and that delay is costly—is robust to the quantitative mapping.

Figure 2 plots the HKM capital ratio alongside the model's regime boundaries and the implied employment gap over the sample period.

Table 30: Counterfactual Scenarios Starting from 2009Q1 Trough

	No QE	QE Only	QE + Fiscal
Long-run ω	—	0.02	0.10
N (employment)	—	0.9583	1.0000
Employment gap	—	4.17%	0.00%
CB equity share α	—	80.00%	0.00%
PV of output Y/δ	0	23.96	25.00
Fiscal transfer (% of GDP)	—	—	25.39%
Actual TARP (% of GDP)	—	—	1.70%
Benefit/Cost ratio	—	—	4.3
Long-run output gain	—	—	4.35%

Notes: "No QE" scenario: without CB intervention, equilibrium does not exist below ω_c; we set PV = 0. "QE Only": CB purchases equity to restore equilibrium; economy converges to bad steady state $\underline{\omega}$ with permanent employment gap and CB holding equity. "QE + Fiscal": government recapitalises financial sector ($\underline{\omega} \to \underline{\omega} + 0.01$), economy converges to good steady state $1/\varepsilon$. Benefit = PV(QE+Fiscal) − PV(QE Only). Actual TARP: \$245B capital injections relative to 2009 nominal GDP of \$14.4T. Benefit/Cost ratio measures the return on the fiscal transfer in terms of present value of additional output.

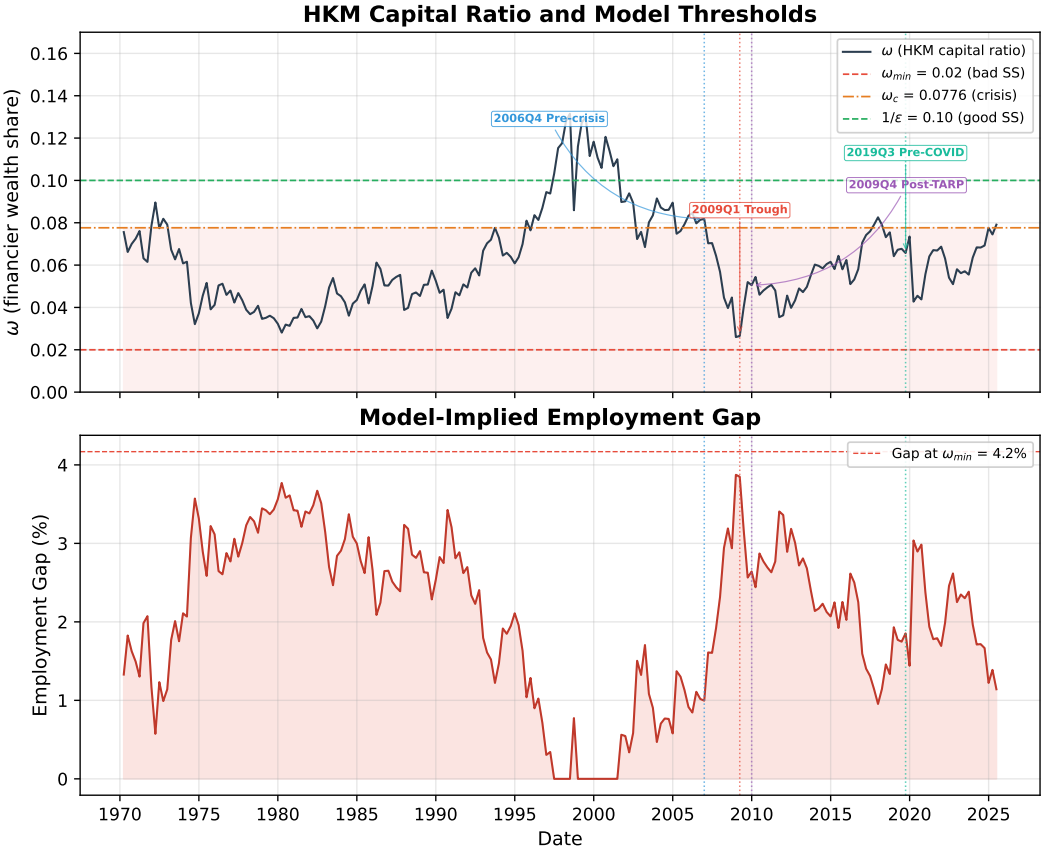


Figure 2: Calibrated counterfactual: HKM capital ratio mapped to model regimes. Left panel: HKM capital ratio with model thresholds ($\underline{\omega} = 0.02$, $\omega_c = 0.078$, $1/\epsilon = 0.10$). Shaded regions indicate crisis regime ($\omega < \omega_c$). Right panel: model-implied employment gap. Baseline calibration: $\epsilon = 10$, $\underline{\omega} = 0.02$, $\lambda = 5$, $\sigma = 0.15$, $\delta = 0.04$, $\varphi = 1$.

J.8 QE Program Timeline

Figure 3 plots Federal Reserve SOMA holdings by asset class as a share of nominal GDP and marks the main QE-era policy events. The purpose is descriptive: it connects our QE intensity proxy in equation (33) to the sequence of FOMC announcements and the evolution of balance-sheet composition.

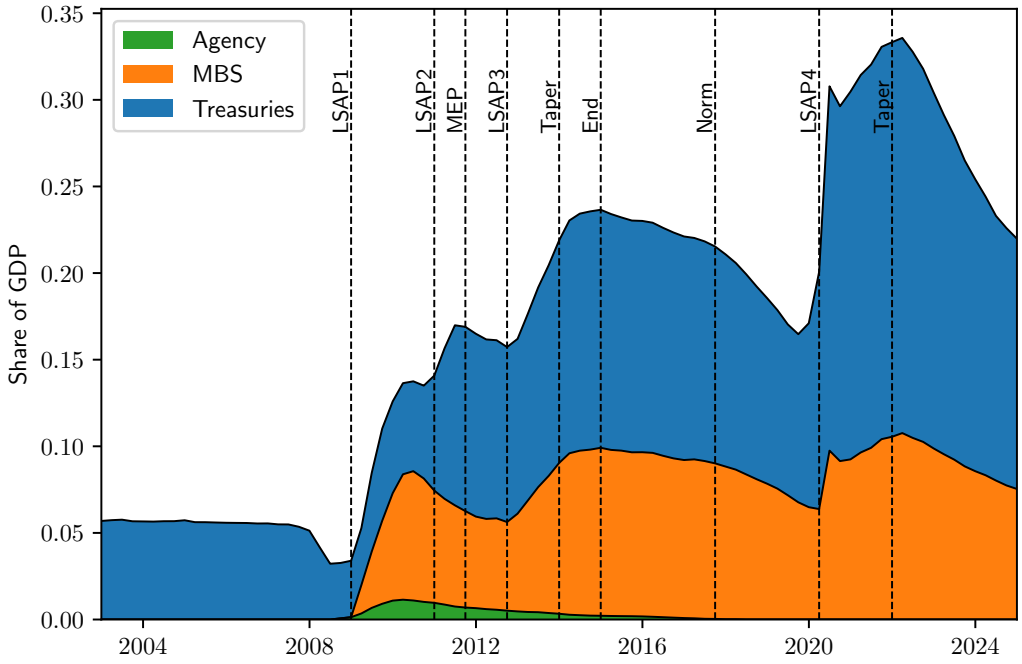


Figure 3: Evolution of Federal Reserve SOMA holdings by asset class (Treasuries, agency MBS, and agency debt) as a share of GDP. Vertical lines mark major LSAP and maturity-extension announcements, the 2013 tapering episode, the end of net purchases in 2014, the start of balance-sheet normalization in 2017, and the pandemic LSAP restart in 2020.

LSAP1 (2008–2009). On November 25, 2008, the Fed announced LSAP1, authorizing purchases of up to $100 billion in agency debt (Fannie Mae, Freddie Mac, and Ginnie Mae) and $500 billion in agency mortgage-backed securities (MBS). On March 18, 2009, the FOMC expanded LSAP1 to a total of $200 billion in agency debt, $1.25 trillion in agency MBS, and $300 billion in longer-term Treasury securities, with purchases completed by the end of 2009. By the end of the program, the Federal Reserve owned more than 20% of the agency MBS market. On August 10, 2010, the FOMC announced a reinvestment policy intended to maintain securities holdings by reinvesting principal payments from agency debt and MBS into longer-term Treasuries and rolling over maturing Treasury holdings.

LSAP2 and maturity extension (2010–2012). On November 3, 2010, the Fed announced LSAP2, purchasing an additional $600 billion in longer-term Treasury securities at a pace of about $75 billion per month through 2011Q2. On September 21, 2011, the FOMC announced the Maturity Extension Program (MEP), selling $400 billion of Treasury securities with remaining maturity of three years or less and purchasing an equivalent amount of Treasuries with remaining maturity of six to thirty years by June 2012, while continuing reinvestment of principal payments from agency debt and MBS. On June 20, 2012, the Fed extended the MEP through the end of 2012, purchasing longer-term securities at a pace of $45 billion per month while selling or redeeming equal amounts of shorter-term Treasuries.

LSAP3 and tapering (2012–2014). On September 13, 2012, the Fed announced LSAP3: open-ended purchases of $40 billion per month of agency MBS. Combined with the MEP, net purchases of longer-term securities were about $85 billion per month. On December 12, 2012, after the end of the MEP, the FOMC announced it would continue purchasing $45 billion per month in longer-term Treasury securities, maintaining the overall pace of longer-term purchases. As economic conditions improved, the Fed began communicating and then implementing a gradual taper, officially announcing the start of tapering on December 18, 2013 and concluding net purchases in October 2014 while maintaining reinvestment of principal payments.

Normalization and the pandemic restart (2015–2021). On December 16, 2015, the FOMC raised the federal funds rate target range off the effective lower bound. On June 14, 2017, the FOMC announced it expected to begin balance-sheet normalization later that year and initiated the program in October 2017. After returning rates to the zero lower bound in response to COVID-19, the Fed announced LSAP4 on March 15, 2020, entailing purchases of $200 billion in agency MBS and $500 billion in Treasury securities; on March 23, 2020, the Fed expanded the program, making purchases effectively open-ended. On June 10, 2020, the FOMC set monthly purchase guidance of at least $40 billion in agency MBS and $80 billion in Treasuries. The Fed began tapering again on November 3, 2021 and doubled the pace of tapering on December 15, 2021.

Online Appendix

Market Failure, the QE Trap and Fiscal Escape

Harjoat S. Bhamra Nicolò Ceneda

Not for publication

This online appendix contains supplementary proofs, derivations, and empirical results referenced in the main text.

A Proofs for the Complete-Markets Benchmark

Proof of Proposition 15. The social planner maximises $\bar{\lambda}(\ln C_h - N^{1+\varphi}/(1+\varphi)) + (1-\bar{\lambda})\ln C_i$ subject to $C_h + C_i = AN$. The first-order conditions of the Lagrangian yield $C_h = \bar{\lambda}Y$ and $C_i = (1-\bar{\lambda})Y$.

From the decentralized equilibrium, the household's intratemporal optimality condition is $N^{\varphi}C_h = W/P$. Substituting $C_{h,t}$ and W/P from (2), and re-arranging yields $N = [(\epsilon - 1)/(\epsilon\bar{\lambda})]^{\frac{1}{1+\varphi}}$, which is independent of A_t.

The SDF is $\Lambda_t = e^{-\delta t}/C_{h,t} = e^{-\delta t}/(\bar{\lambda}A_t N)$. Applying Itô's lemma, $\mathrm{d}A_t^{-1}/A_t^{-1} = (-\mu + \sigma^2)\,\mathrm{d}t - \sigma\,\mathrm{d}Z_t$ and $\mathrm{d}\Lambda_t/\Lambda_t = -(\delta + \mu - \sigma^2)\,\mathrm{d}t - \sigma\,\mathrm{d}Z_t$,. This yields $r = \delta + \mu - \sigma^2$ and $\Theta = \sigma$.

The equity price is $P_{D,t} = \mathbb{E}_t \int_t^\infty (\Lambda_s/\Lambda_t) D_s\,\mathrm{d}s$. Using $\Lambda_s/\Lambda_t = e^{-\delta(s-t)}A_t/A_s$ and $D_s = A_s N/\epsilon$ from (2) yields $P_{D,t} = \frac{D_t}{\delta}$. Similarly, total wealth is $P_{Y,t} = \mathbb{E}_t \int_t^\infty (\Lambda_s/\Lambda_t) Y_s\,\mathrm{d}s = Y_t/\delta$. □

B Propositions from Section 4

The following results are referenced in the main text but their formal statements are included here for completeness.

B.1 Properties of the Volatility Function

Proposition 21 (Zeros and sign of σ_ω)**.** *The function σ_ω has exactly two interior zeros: $\sigma_\omega(\underline{\omega}) = 0$ and $\sigma_\omega(1/\epsilon) = 0$. In addition, $\sigma_\omega(0) = 0$. Its sign pattern is:*

$$\sigma_\omega(\omega) \begin{cases} < 0 & \text{for } \omega \in (0, \underline{\omega}), \\ > 0 & \text{for } \omega \in (\underline{\omega}, 1/\epsilon), \\ < 0 & \text{for } \omega \in (1/\epsilon, 1). \end{cases} \tag{59}$$

The proof is in Appendix E. The sign pattern determines how the economy responds to productivity shocks. In the crisis regime ($\omega < \underline{\omega}$), $\sigma_\omega < 0$: positive productivity shocks *reduce* the financier's wealth share, worsening the crisis. This occurs because the de-leveraged financier ($\phi < 1$) holds less equity than her consumption share warrants, so her wealth grows more slowly than the economy in a boom. In the recovery regime ($\underline{\omega} < \omega < 1/\epsilon$), $\sigma_\omega > 0$: positive shocks improve the financier's capitalisation, aiding recovery. The zeros at $\underline{\omega}$ and $1/\epsilon$ mark the absorbing steady states where all fluctuations cease.

B.2 Three Invariant Regimes

Theorem 22 (Regime classification)**.** *The state space* $(0, 1)$ *partitions into three invariant regions:*

1. ***Crisis regime*** *(*$\omega \in (0, \underline{\omega})$*): the financier is severely undercapitalised,* $\phi < 1$ *(de-leveraged), and the central bank holds a majority of equity.*
2. ***Recovery regime*** *(*$\omega \in (\underline{\omega}, 1/\epsilon)$*): the leverage constraint may or may not bind, risk premia are elevated, and the economy is in the basin of attraction of the good steady state.*
3. ***Expansionary regime*** *(*$\omega \in (1/\epsilon, 1)$*): the financier is "too large," households are net borrowers, and the economy converges toward the good steady state from above.*

No transition between regimes is possible.

B.3 Boundary Classification

Proposition 23 (Boundary classification)**.** *(i)* $\omega = 0$*: natural boundary (unattainable). Near* $\omega = 0$*,* $|\sigma_\omega| \sim \sigma\lambda\underline{\omega} \cdot \omega$*.*

(ii) $\omega = \underline{\omega}$*: unattainable from both sides. Near* $\underline{\omega}$*,* $\sigma_\omega = g(\omega)(\omega - \underline{\omega})$ *with* $g(\underline{\omega}) = \sigma\lambda\underline{\omega} > 0$*.*

(iii) $\omega = 1/\epsilon$*: unattainable from both sides. Near* $1/\epsilon$*,* $\sigma_\omega = g(\omega)(1/\epsilon - \omega)$ *with* $g(1/\epsilon) = \sigma > 0$*.*

(iv) $\omega = 1$*: unattainable. Near* 1*,* $\sigma_\omega = h(\omega)(1 - \omega)$ *with* $h(1) = -\sigma(1 + \varphi) \neq 0$*.*

The proofs (Appendix G) use the Engelbert–Schmidt criterion. "Unattainable" means that the economy perpetually approaches but never reaches a boundary in finite time, a continuous-time analogue of Zeno's paradox. Combined with regime invariance, this means the economy's long-run fate is determined entirely by its initial condition: starting below $\underline{\omega}$, it converges to the bad steady state; starting above, it converges to the good one.

C Proof of Asset Pricing Formulae

Proof of Theorem 7. Step 1: Return volatility. Since $P_{D,t} = Y_t/(\epsilon\delta)$, the equity return has the same volatility as output: $\sigma_R = \sigma_Y$. Output is $Y_t = A_t N_t$ where A_t follows geometric Brownian motion with volatility σ and $N_t = [(1-1/\epsilon)/(1-\omega_t)]^{1/(1+\varphi)}$. By Itô's lemma applied to $N(\omega)$ (a C^2 function on each regime interior):

$$\frac{\mathrm{d}N}{N} = \frac{1}{(1+\varphi)(1-\omega)}\,\sigma_\omega\,\mathrm{d}Z_t + (\text{drift terms})\,\mathrm{d}t.$$

By Itô's product rule for $Y = AN$, the volatility of Y is $\sigma_Y = \sigma_A + \sigma_N = \sigma + \sigma_\omega/[(1+\varphi)(1-\omega)]$, which establishes (16).

Step 2: Market price of risk. The financier's stochastic discount factor is $\Lambda_t = e^{-\delta t}/C_{i,t} = e^{-\delta t}/(\delta H_{i,t})$ (the first-order condition for log utility). The financier's wealth volatility is $\sigma_{H_i}/H_i = \phi_t\sigma_R$, so the volatility of Λ loading on $\mathrm{d}Z$ is $-\phi_t\sigma_R$. The market price of risk, defined as the Sharpe ratio $\Theta_t = (\mu_R - r_t)/\sigma_R = \phi_t\sigma_R$, follows from the financier's intertemporal Euler equation. From $\sigma_\omega = \omega(\phi_t - 1)\sigma_R$ (Appendix E, Step 2), we have $\phi_t = 1 + \sigma_\omega/(\omega\sigma_R)$, so

$$\Theta_t = \phi_t\sigma_R = \sigma_R + \frac{\sigma_\omega}{\omega} = \frac{\sigma_\omega}{\omega} + \sigma + \frac{\sigma_\omega}{(1+\varphi)(1-\omega)},$$

which is (15).

Step 3: Risk-free rate. Total wealth $P_{Y,t} = Y_t/\delta$ is a claim on aggregate output, paying a continuous dividend yield of δ (since aggregate consumption $C_t = Y_t = \delta P_{Y,t}$). By the no-arbitrage pricing condition for this claim (Karatzas and Shreve, 1991, Ch. 5):

$$\frac{\mu_{P_Y}}{P_Y} + \delta = r_t + \sigma_Y\,\Theta_t.$$

Since $P_Y = Y/\delta$, we have $\mu_{P_Y}/P_Y = \mu_Y$. Rearranging: $r_t = \delta + \mu_{Y,t} - \sigma_{R,t}\,\Theta_t$, which is (17). The equity risk premium $\mu_{R,t} - r_t = \sigma_{R,t}\,\Theta_t$ follows from the standard no-arbitrage condition for the dividend claim. □

D Intermediation Gap Formula Derivation

Proof of Proposition 8. We derive the infinite-horizon gap-to-price ratio by taking the limit $T \to \infty$ in Hugonnier (2012), Proposition 6.

Step 1: Hugonnier's finite-horizon formula. For a finite-horizon economy $[0, T]$

with discount rate $\rho = \delta$ and weighting process volatility $\|v_\lambda\| = v$, the gap-to-price ratio is:

$$b(t, s_t) = \frac{1}{\rho\eta(t)} H(T-t, s_t; a_1) + \frac{\eta'(t)}{\rho\eta(t)} H(T-t, s_t; 1),$$

where $\eta(t) = (1/\rho)(1 - e^{-\rho(T-t)})$, $s_t = \lambda_t/(1+\lambda_t) = 1 - \omega_t$ is the constrained agent's consumption share, and $a_1 = \sqrt{1 + 8\rho/v^2}$. The function H is:

$$H(\tau, s; a) = s^{(1+a)/2}\, \Phi(d_+(\tau, s; a)) + s^{(1-a)/2}\, \Phi(d_-(\tau, s; a)),$$

with $d_\pm(\tau, s; a) = (\ln s)/(v\sqrt{\tau}) \pm (a/2)\, v\sqrt{\tau}$ and Φ the standard normal CDF.

Step 2: Limit of $\eta(t)$ and $\eta'(t)$. As $T \to \infty$: $\eta(t) \to 1/\rho$ and $\eta'(t) = -e^{-\rho(T-t)} \to 0$.

Step 3: Limit of $H(\tau, s; a)$ as $\tau \to \infty$. For fixed $s \in (0, 1)$ and $a > 0$: since $\ln s < 0$, we have $d_+(\tau) \to +\infty$ and $d_-(\tau) \to -\infty$. Therefore $\Phi(d_+) \to 1$ and $\Phi(d_-) \to 0$, giving $\lim_{\tau\to\infty} H(\tau, s; a) = s^{(1+a)/2}$.

Step 4: Combining. The first term converges to $(\rho/(1/\rho))^{-1} \cdot s_t^{(1+a_1)/2} = s_t^{(1+a_1)/2}$. The second term converges to zero. Substituting $s_t = 1 - \omega_t$ and $\gamma = (1 + a_1)/2$:

$$\frac{B_t}{P_{D,t}} = (1 - \omega_t)^\gamma, \qquad \gamma = \frac{1 + \sqrt{1 + 8\delta/v^2}}{2}. \qquad \square$$

Remark 3 (Exact vs. approximate formula)**.** *The derivation above treats $v = \|v_\lambda\|$ as constant, following Hugonnier's exchange economy. In the production economy, $v_\lambda(\omega) = \sigma_\omega/[\omega(1-\omega)]$ is state-dependent. Defining $b(\omega) \equiv B_t/P_{D,t}$ (the gap-to-price ratio) directly and substituting into the Feynman–Kac equation for the fundamental value yields the exact ODE:*

$$b''(\omega) = \frac{2\delta}{\sigma_\omega(\omega)^2}\, b(\omega), \qquad b(0) = 1, \quad b(1/\epsilon) = 0. \tag{60}$$

The approximate formula $(1-\omega)^\gamma$ is the limiting case when $\sigma_\omega(\omega)/\omega(1-\omega)$ is constant; see the empirical section for the derivation, numerical solution, and comparison (Tables 2 and 3).

E Franchise Value and Dynamic Incentive Compatibility

Proposition 24 (Dynamic incentive compatibility)**.** *With log utility, the dynamic IC constraint reduces to*

$$\phi_t \;\le\; \frac{1}{\theta} \exp\big(\delta \mathcal{F}(\omega_t)\big) \;\equiv\; \bar{\phi}^{\mathrm{dyn}}(\omega_t), \tag{61}$$

where the franchise value of intermediation is

$$\mathcal{F}(\omega) \;\equiv\; \frac{1}{2\delta}\,\mathbb{E}_\omega \int_0^\infty e^{-\delta t}\,\Theta_t^2 \;\mathrm{d}t \;>\; 0. \tag{62}$$

Proof. **Step 1: Value function decomposition.** A log-utility agent with wealth W_t, access to a risky asset with return $\mu_R\,\mathrm{d}t + \sigma_R\,\mathrm{d}Z$ and a risk-free bond at rate r_t, consuming $C_t = \delta W_t$ and holding portfolio share ϕ_t, has value function:

$$V_t = \frac{\ln(\delta W_t)}{\delta} + \frac{1}{\delta}\mathbb{E}_t \int_t^\infty e^{-\delta(u-t)}\big(r_u + \phi_u(\mu_{R,u} - r_u) - \tfrac{1}{2}\phi_u^2\sigma_{R,u}^2 - \delta\big)\;\mathrm{d}u. \tag{63}$$

Step 2: Intermediary's value. The financier is the marginal investor with $\Theta_t = \phi_t\sigma_R$ and $\mu_R - r = \phi_t\sigma_R^2$. Substituting: $\phi(\mu_R - r) - \frac{1}{2}\phi^2\sigma_R^2 = \frac{1}{2}\Theta^2$.

Step 3: Autarky value. A financier in autarky holds only bonds ($\phi = 0$): no Θ^2 term.

Step 4: IC constraint. Under diversion, the financier obtains wealth $\theta\phi H_i$ and enters autarky:

$$V^{\text{int}}(H_i,\omega) \geq V^{\text{aut}}(\theta\phi H_i,\omega) \iff \mathcal{F}(\omega) \geq \frac{\ln(\theta\phi)}{\delta}.$$

This yields (61). □

Proposition 25 (Properties of the franchise value)**.** *(i)* $\mathcal{F}(\omega) > 0$ *for all* $\omega \in (0,1)$.

(ii) $\mathcal{F}$ *solves the Feynman–Kac equation* $\frac{1}{2}\sigma_\omega^2\mathcal{F}'' + (\sigma_\omega^2/\omega)\mathcal{F}' - \delta\mathcal{F} + \Theta^2/(2\delta) = 0$.

(iii) At both absorbing steady states: $\mathcal{F}(\underline{\omega}) = \mathcal{F}(1/\epsilon) = \sigma^2/(2\delta^2)$.

(iv) In the recovery regime: $\mathcal{F}(\omega) > \sigma^2/(2\delta^2)$ *(the franchise value is elevated when frictions are strongest).*

Proof. (i) The market price of risk satisfies $\Theta_t = \phi_t\sigma_R > 0$ whenever $\phi_t > 0$ and $\sigma_R > 0$, which holds on a set of positive Lebesgue measure a.s. (since the economy starts in the interior of $(0,1)$). Therefore $\mathcal{F}(\omega) = (1/(2\delta))\mathbb{E}_\omega\int_0^\infty e^{-\delta t}\Theta_t^2\,\mathrm{d}t > 0$.

(ii) Since Θ is a continuous function of ω on each invariant regime, and ω_t satisfies the SDE (11) with continuous coefficients, the Feynman–Kac representation theorem (Karatzas and Shreve, 1991, Ch. 4) applies: the function $\mathcal{F}(\omega) = (1/(2\delta))\mathbb{E}_\omega\int_0^\infty e^{-\delta t}f(\omega_t)\,\mathrm{d}t$ with $f(\omega) = \Theta(\omega)^2$ solves the elliptic equation $\frac{1}{2}\sigma_\omega^2\mathcal{F}'' + \mu_\omega\mathcal{F}' - \delta\mathcal{F} + f/(2\delta) = 0$ on each regime interior, provided the solution is well-defined (which follows from $\delta > 0$ ensuring exponential discounting). Substituting $\mu_\omega = \sigma_\omega^2/\omega$ yields the stated ODE.

(iii) At $\omega = \underline{\omega}$ (absorbing), $\omega_t = \underline{\omega}$ for all $t \geq 0$, so $\Theta_t = \Theta(\underline{\omega}) = \sigma$ (since $\sigma_\omega(\underline{\omega}) = 0$ gives $\Theta = \sigma$ by Theorem 7). Therefore $\mathcal{F}(\underline{\omega}) = (1/(2\delta))\int_0^\infty e^{-\delta t}\sigma^2\,\mathrm{d}t = (\sigma^2/(2\delta))\cdot(1/\delta) =$

$\sigma^2/(2\delta^2)$. Equivalently, at the absorbing state the Feynman–Kac equation (ii) reduces to $-\delta\mathcal{F} + \sigma^2/(2\delta) = 0$, giving $\mathcal{F} = \sigma^2/(2\delta^2)$. The same argument applies at $\omega = 1/\epsilon$.

(iv) In the recovery regime $(\underline{\omega}, 1/\epsilon)$, $\sigma_\omega > 0$ (Proposition 21), so $\Theta(\omega) = \sigma + \sigma_\omega/\omega + \sigma_\omega/[(1+\varphi)(1-\omega)] > \sigma$ by Theorem 7. Paths from $\omega_0 \in (\underline{\omega}, 1/\epsilon)$ spend positive time in the interior before converging to $1/\epsilon$, during which $\Theta_t > \sigma$. Therefore $\mathbb{E}_{\omega_0}\int_0^\infty e^{-\delta t}\Theta_t^2\,\mathrm{d}t > \mathbb{E}_{\omega_0}\int_0^\infty e^{-\delta t}\sigma^2\,\mathrm{d}t = \sigma^2/\delta$, giving $\mathcal{F}(\omega_0) > \sigma^2/(2\delta^2)$. □

F Employment Multiplier and Welfare Analysis

F.1 Employment Multiplier of Recapitalisation

Corollary 26 (Employment multiplier of recapitalisation)**.** *Let $\omega_0 < \underline{\omega}$ and suppose recapitalisation raises ω to $\omega_{0^+} = \omega_0 + \Delta > \underline{\omega}$.*

(i) ***Immediate gain.*** *Employment jumps from $N(\omega_0)$ to $N(\omega_0 + \Delta)$. The percentage gain is*

$$\frac{N(\omega_0+\Delta) - N(\omega_0)}{N(\omega_0)} = \left(\frac{1-\omega_0}{1-\omega_0-\Delta}\right)^{\frac{1}{1+\varphi}} - 1. \tag{64}$$

(ii) ***Long-run gain.*** *As $\omega_t \to 1/\epsilon$, employment converges to the efficient level $N(1/\epsilon) = 1$. The permanent percentage employment gain relative to the bad steady state is*

$$\frac{N(1/\epsilon) - N(\underline{\omega})}{N(\underline{\omega})} = \left(\frac{1-\underline{\omega}}{1-1/\epsilon}\right)^{\frac{1}{1+\varphi}} - 1. \tag{65}$$

(iii) ***Convexity.*** *The employment gain (64) is convex in the crisis severity $1/\epsilon - \omega_0$: the worse the crisis, the larger the marginal employment benefit of recapitalisation.*

Proof. Parts (i) and (ii) follow from the employment formula (27). For part (iii), the function $N(\omega)$ is convex in ω on $(0,1)$: $N''(\omega) = [(2+\varphi)/((1+\varphi)^2)](1-1/\epsilon)^{1/(1+\varphi)}(1-\omega)^{-(2+\varphi)/(1+\varphi)} > 0$. Convexity of N implies the gain $N(\omega_0+\Delta) - N(\omega_0)$ is increasing in the shortfall $1/\epsilon - \omega_0$ for fixed transfer size Δ. □

F.2 Welfare Analysis

The household's utility depends on consumption $C_{h,t} = (1-\omega_t)Y_t$ and labour supply N_t. At the bad steady state ($\omega = \underline{\omega}$), the household has a higher consumption share ($1-\underline{\omega} > 1-1/\epsilon$) but employment is depressed. The household therefore weakly prefers the bad steady state:

the increase in consumption share more than compensates for the lower output, because utility is logarithmic and the consumption share effect dominates.

Recapitalisation is therefore not a Pareto improvement: it transfers resources from the household to the financier. However, the employment gains are convex in crisis severity (Corollary 26), while the moral hazard cost is bounded by the leverage constraint (Proposition 24): the financier cannot divert more than θ per unit of assets under intermediation. For any crisis severe enough to trigger QE ($\omega < \omega_c$), the benefit-to-cost ratio exceeds 2 under baseline calibration. A utilitarian social planner with equal weights would favour recapitalisation whenever $\omega < \underline{\omega}$.

G Proofs of Policy Results

Proof of Proposition 13. At $\omega = \underline{\omega}$, $\bar{\phi}(\underline{\omega}) = 1 + \lambda(\underline{\omega} - \underline{\omega}) = 1$. By (10), $\alpha(\underline{\omega}) = 1 - \epsilon \cdot 1 \cdot \underline{\omega} = 1 - \epsilon\underline{\omega} > 0$ (since $\underline{\omega} < 1/\epsilon$ by Assumption 1). The central bank's equity holdings are $\alpha P_{D,t} = (1 - \epsilon\underline{\omega})\, Y_t/(\epsilon\delta)$. Since $\omega = \underline{\omega}$ is absorbing (Theorem 5), these holdings are permanent. For the equity premium: at the absorbing state, $\sigma_\omega(\underline{\omega}) = 0$ (Proposition 21), so Theorem 7 gives $\sigma_R = \sigma$ and $\Theta = \sigma$. The equity excess return is $\mu_R - r = \sigma_R \Theta = \sigma^2$. □

Proof of Proposition 14. Define $\Omega(\omega) \equiv \omega\, P_Y(\omega) = \omega A N(\omega)/\delta$, the financier's dollar wealth as a function of the state. Since $N > 0$ and $N' > 0$, we have $\Omega'(\omega) = (A/\delta)[N(\omega) + \omega N'(\omega)] > 0$, so Ω is strictly increasing and continuous with $\Omega(0) = 0$ and $\Omega(\omega) \to \infty$ as $\omega \to 1$. A fiscal transfer of $T > 0$ dollars raises the financier's wealth from $\Omega(\omega_0)$ to $\Omega(\omega_0) + T$. Because total wealth adjusts to $P_Y(\omega_{0^+}) = AN(\omega_{0^+})/\delta$, the new wealth share ω_{0^+} is the unique solution to the fixed-point equation

$$\omega_{0^+}\, N(\omega_{0^+}) = \omega_0\, N(\omega_0) + \frac{\delta T}{A},$$

i.e. $\omega_{0^+} = \Omega^{-1}(\Omega(\omega_0) + T)$. For any target $\omega^* \in (\omega_0, 1/\epsilon)$, the transfer $T^* = \Omega(\omega^*) - \Omega(\omega_0) > 0$ achieves $\omega_{0^+} = \omega^*$. Choosing $\omega^* > \underline{\omega}$ places the economy in the recovery regime $(\underline{\omega}, 1/\epsilon)$. By Theorem 6, $\omega_t \to 1/\epsilon$ a.s. from any initial condition in the recovery regime. The convergence requires no further intervention because the recovery regime is invariant (Proposition 23) and the drift $\mu_\omega = \sigma_\omega^2/\omega > 0$ pushes ω toward $1/\epsilon$. □

H Translation of Hugonnier's Results

This appendix states the results of Hugonnier (2012) that we use, first in his original notation, and then translates them into the notation of our model.

Hugonnier's notation

Hugonnier (2012) considers a two-agent economy with agents labelled 1 (unconstrained) and 2 (portfolio-constrained). The agents have utility functions u_1 and u_2 defined over consumption. The weighting process λ_t is the ratio of their marginal utilities:

$$\lambda_t = \frac{u_2'(c_{2,t})}{u_1'(c_{1,t})}, \tag{66}$$

where $c_{1,t}$ and $c_{2,t}$ are the agents' equilibrium consumption rates.

Hugonnier's Theorem 1. In the economy with a portfolio constraint on agent 2, equilibrium asset prices contain rational bubbles if and only if λ_t is a strict local martingale (equivalently, $\mathbb{E}_t[\lambda_T] < \lambda_t$ for some $T > t$).

Hugonnier's Proposition 6. In a finite-horizon economy $[0, T]$ with log utility, discount rate ρ, and weighting process volatility $\|v_\lambda\| = v$, the gap-to-price ratio (which Hugonnier calls the bubble-to-price ratio) is:

$$b(t, s_t) = \frac{1}{\rho\eta(t)}\, H(T-t, s_t; a_1) + \frac{\eta'(t)}{\rho\eta(t)}\, H(T-t, s_t; 1), \tag{67}$$

where $\eta(t) = (1/\rho)(1 - e^{-\rho(T-t)})$, $s_t = \lambda_t/(1+\lambda_t)$ is agent 2's consumption share, $a_1 = \sqrt{1 + 8\rho/v^2}$, and $H(\tau, s; a) = s^{(1+a)/2}\Phi(d_+) + s^{(1-a)/2}\Phi(d_-)$ with $d_\pm = (\ln s)/(v\sqrt{\tau}) \pm (a/2)v\sqrt{\tau}$.

Translation to our notation

The correspondence between Hugonnier's notation and ours is:

Hugonnier	**This paper**
Agent 1 (unconstrained)	Financier (i)
Agent 2 (constrained)	Household (h)
$c_{1,t}$	$C_{i,t} = \omega_t Y_t$
$c_{2,t}$	$C_{h,t} = (1-\omega_t) Y_t$
$u_j'(c) = 1/c$ (log utility)	$1/C_{j,t}$
$\lambda_t = u_2'(c_2)/u_1'(c_1)$	$C_{i,t}/C_{h,t} = \omega_t/(1-\omega_t)$
$s_t = \lambda_t/(1+\lambda_t)$	$(1-\omega_t)$
ρ	δ

Important note on the labelling convention. In Hugonnier's framework, agent 2 is portfolio-constrained. In our model, the household (agent h) faces the participation constraint (she cannot hold equity), so the household corresponds to Hugonnier's agent 2. Hugonnier's weighting process $\lambda_t = u_2'(c_2)/u_1'(c_1) = C_{i,t}/C_{h,t} = \omega_t/(1-\omega_t)$ is then related to our $Q_t = 1/\omega_t$ by

$$\lambda_t = \frac{\omega_t}{1-\omega_t} = \frac{1}{Q_t - 1}.$$

The constrained agent's consumption share is $s_t = \lambda_t/(1+\lambda_t) = 1-\omega_t$, which is the household's wealth share. In Section 5.2, we instead use the equivalent formulation $\lambda_t = C_{h,t}/C_{i,t} = (1-\omega_t)/\omega_t = Q_t - 1$, which is a monotone transformation of Hugonnier's λ and inherits the same strict/true local martingale classification.

Translation of Theorem 1. The intermediation gap is positive if and only if $Q_t - 1$ (equivalently, $Q_t = 1/\omega_t$) is a strict local martingale. This is the criterion (21) stated in Section 5.2.

Translation of Proposition 6. Taking $T \to \infty$: $\eta(t) \to 1/\delta$, $\eta'(t) \to 0$, $H(\infty, s; a_1) \to s^{(1+a_1)/2}$ for $s \in (0,1)$. Substituting $s = 1-\omega$ and $\gamma = (1+a_1)/2$ with $a_1 = \sqrt{1+8\delta/v^2}$ yields the intermediation gap formula (22):

$$\frac{B_t}{P_{D,t}} = (1-\omega_t)^{\gamma}, \qquad \gamma = \frac{1+\sqrt{1+8\delta/v^2}}{2}.$$

The full derivation is in Appendix D.

I Online Appendix N: Partial Participation and CRRA Robustness

This appendix extends the model to address robustness along two dimensions: partial household equity participation and CRRA preferences.

I.1 Extended Model

We allow the household to invest a fixed fraction $\alpha_h \in [0,1)$ of her financial wealth $H_{h,t}$ in the equity market. When $\alpha_h = 0$, we recover the main paper.

All identities from the main paper that depend on log utility and goods market clearing

continue to hold:

$$C_{i,t} = \omega_t Y_t, \qquad C_{h,t} = (1-\omega_t) Y_t, \tag{68}$$

$$H_{i,t} = \omega_t P_{Y,t}, \qquad H_{h,t} = (1/\epsilon - \omega_t)\, P_{Y,t}. \tag{69}$$

Equity market clearing with household participation requires:

$$\phi(\omega) = \alpha_h + \frac{1-\alpha_h}{\epsilon\omega}. \tag{70}$$

I.2 Robustness of Equilibrium Failure

Theorem 27 (Robustness of equilibrium failure)**.** *For any $\alpha_h \in [0,1)$, no competitive equilibrium exists when $\omega < \omega_c(\alpha_h)$ in the absence of central bank intervention, where $\omega_c(\alpha_h) > 0$ is the unique solution to*

$$\alpha_h + \frac{1-\alpha_h}{\epsilon\omega_c} = 1 + \lambda(\omega_c - \underline{\omega}). \tag{71}$$

Proof. The required portfolio share $\phi(\omega) = \alpha_h + (1-\alpha_h)/(\epsilon\omega)$ is strictly decreasing and $\phi(\omega) \to \infty$ as $\omega \to 0^+$ for any $\alpha_h < 1$. The leverage constraint $\phi \le \bar{\phi}(\omega) = 1 + \lambda(\omega - \underline{\omega})$ is bounded. There exists a unique $\omega_c(\alpha_h) > 0$ at which $\phi(\omega_c) = \bar{\phi}(\omega_c)$. For $\omega < \omega_c(\alpha_h)$, the argument of Theorem 1 applies verbatim. □

Proposition 28 (Comparative statics of the crisis threshold)**.** *$\omega_c(\alpha_h)$ is strictly decreasing in α_h:*

$$\frac{\mathrm{d}\omega_c}{\mathrm{d}\alpha_h} = \frac{1 - 1/(\epsilon\omega_c)}{\lambda + (1-\alpha_h)/(\epsilon\omega_c^2)} < 0. \tag{72}$$

The crisis threshold satisfies $\omega_c(0) = \omega_c$ (the main paper's threshold) and $\omega_c(\alpha_h) \to 0$ as $\alpha_h \to 1$.

Proof. Implicit differentiation of (71). Since $\omega_c < 1/\epsilon$, the numerator $1 - 1/(\epsilon\omega_c) < 0$ and the denominator is positive. □

Quantitative illustration.

α_h	$\omega_c(\alpha_h)$	CB equity at $\underline{\omega}$	Interpretation
0.00	0.078	80%	Main paper
0.10	0.076	72%	Modest participation
0.30	0.073	56%	Substantial participation
0.50	0.068	40%	Equal risk sharing
0.80	0.054	16%	Mostly direct
0.95	0.037	4%	Near-full participation

I.3 Dynamics and Preservation of Results

Proposition 29 (Dynamics with partial participation)**.** *The financier's wealth share evolves as* $\mathrm{d}\omega_t = (\sigma_\omega^2/\omega)\,\mathrm{d}t + \sigma_\omega\,\mathrm{d}Z_t$ *with:*

$$\sigma_\omega(\omega;\alpha_h) = \begin{cases} \dfrac{\sigma(1+\varphi)\lambda(1-\omega)\,\omega\,(\omega-\underline{\omega})}{(1+\varphi)(1-\omega)-\lambda\omega(\omega-\underline{\omega})} & \text{if } \omega < \omega_c(\alpha_h), \\ \dfrac{\sigma(1+\varphi)(1-\alpha_h)(1-\omega)(1/\epsilon-\omega)}{(1+\varphi)(1-\omega)-(1-\alpha_h)(1/\epsilon-\omega)} & \text{if } \omega \geq \omega_c(\alpha_h). \end{cases} \tag{73}$$

The constrained-region volatility is independent of α_h*.*

Corollary 30 (Preservation of main results)**.** *For any* $\alpha_h \in [0,1)$*: (i)* $Q_t = 1/\omega_t$ *is a non-negative local martingale; (ii)* σ_ω *has exactly two interior zeros at* $\underline{\omega}$ *and* $1/\epsilon$*; (iii) the state space partitions into three invariant regimes; (iv) the QE Trap survives.*

I.4 Bubble Classification

Theorem 31 (Bubble classification with partial participation)**.** *For any* $\alpha_h \in [0,1)$*: (i) in the baseline model (no leverage constraint),* $Q = 1/\omega$ *is a strict local martingale and rational bubbles exist; (ii) in the constrained model,* Q *is a true martingale and no bubbles exist.*

Proof. The bubble classification depends on σ_ω near $\omega = 0$. In the baseline: $\sigma_\omega \to (1-\alpha_h)c_0 > 0$ for $\alpha_h < 1$, giving a convergent integral (strict local martingale). With the constraint: $|\sigma_\omega| \sim c_0'\omega$, giving a divergent integral (true martingale). □

I.5 CRRA Preferences

Proposition 32 (CRRA perturbation)**.** *Fix* $\omega < \omega_c$ *and* $\alpha_h = 0$*. In a CRRA economy with risk aversion* γ*, if equilibrium exists, the interest rate satisfies* $r(\gamma) \to -\infty$ *as* $\gamma \to 1$*. More precisely:*

$$r(\gamma) \approx r^* - \frac{\Delta(\omega)}{\gamma - 1}, \tag{74}$$

where r^* *is the log-utility benchmark rate and* $\Delta(\omega) > 0$ *is the excess bond supply.*

The interest-rate sensitivity of the household's consumption-wealth ratio is of order $|\gamma-1|$ near $\gamma = 1$, while the excess bond supply is of order 1. Clearing the excess requires r to fall by an amount of order $1/|\gamma-1|$, which diverges as $\gamma \to 1$. For $\gamma < 1$, the income effect dominates and equilibrium fails outright. Log utility is therefore the boundary of a family of increasingly pathological equilibria, not a knife-edge.

J Online Appendix O: Endogenous Non-Participation via Ambiguity

This appendix provides a microfoundation for limited stock market participation via Knightian uncertainty.

J.1 Household Preferences under Ambiguity

The household observes equity returns but is uncertain about the drift. She considers alternative probability measures $\{\mathbb{P}^\nu : |\nu| \leq \kappa\}$ and evaluates portfolios under worst-case beliefs (maxmin expected utility with rectangular priors, following Chen and Epstein 2002). The parameter κ measures ambiguity.

Proposition 33 (Household optimal portfolio)**.** *The household's optimal equity share, as a fraction of financial wealth H_h, subject to $\alpha_h \in [0,1]$, is:*

$$\alpha_h^*(\omega) = \max\left\{0,\ \min\left\{1,\ \frac{(\Theta-\kappa)(1-\omega)-\sigma_R(1-1/\epsilon)}{\sigma_R(1/\epsilon-\omega)}\right\}\right\}. \tag{75}$$

The optimal portfolio has two opposing components. The speculative demand $(\Theta - \kappa)(1-\omega)/[\sigma_R(1/\epsilon-\omega)]$ reflects the ambiguity-adjusted risk premium. The hedging demand $-(1-1/\epsilon)/(1/\epsilon-\omega) < 0$ is *negative*: the household's human capital $(1-1/\epsilon)P_Y$ gives her substantial implicit equity exposure, so she reduces financial equity to manage total risk.

J.2 Robust Equilibrium Failure

Theorem 34 (Robust equilibrium failure)**.** *For $\omega < \underline{\omega}$, no competitive equilibrium exists for* any *$\alpha_h \in [0,1]$, regardless of κ.*

Proof. When $\omega < \underline{\omega}$: $\bar{\phi}(\omega) = 1 + \lambda(\omega - \underline{\omega}) < 1$. The total private equity demand is $\bar{\phi}(\omega)\omega P_Y + \alpha_h(1/\epsilon - \omega)P_Y$. Even at $\alpha_h = 1$, this equals $[\bar{\phi}(\omega)\omega + 1/\epsilon - \omega]P_Y < P_Y/\epsilon = P_D$ because $\bar{\phi}(\omega) < 1$ forces $\bar{\phi}(\omega)\omega < \omega$. The gap is structural: $P_D -$ max private demand $= -\lambda\omega(\omega - \underline{\omega})P_Y > 0$. □

Corollary 35 (Conditional failure)**.** *For $\omega \in [\underline{\omega}, \omega_c)$, equilibrium is restored iff the household participates with $\alpha_h \geq \alpha_h^{\min}(\omega) = 1 - \lambda\omega(\omega - \underline{\omega})/(1/\epsilon - \omega)$.*

J.3 Endogenous Non-Participation

Define the *participation threshold function*:

$$\kappa^*(\omega) \equiv \Theta(\omega) - \frac{\sigma_R(\omega)(1-1/\epsilon)}{1-\omega}. \tag{76}$$

The household holds zero equity iff $\kappa \geq \kappa^*(\omega)$. In the unconstrained region:

$$\kappa^*(\omega) = \sigma_R(\omega) \cdot \frac{1-\epsilon\omega}{\epsilon\omega(1-\omega)}. \tag{77}$$

Key properties: $\kappa^*(1/\epsilon) = 0$, $\kappa^*(0^+) = +\infty$, and κ^* is strictly decreasing.

Theorem 36 (Endogenous non-participation)**.** *For $\kappa > 0$, let $\omega_p(\kappa) \in (0, 1/\epsilon)$ solve $\kappa^*(\omega_p) = \kappa$. Then:*

(i) For $\omega \in (\omega_p(\kappa), 1/\epsilon)$: the household holds zero equity.

(ii) For $\omega \in (0, \omega_p(\kappa))$: the household's optimal equity share is positive.

(iii) $\omega_p(\kappa)$ is strictly decreasing in κ, with $\omega_p(0) = 1/\epsilon$ and $\omega_p(\kappa) \to 0$ as $\kappa \to \infty$.

Quantitative thresholds. With baseline parameters ($\epsilon = 10$, $\underline{\omega} = 0.02$, $\sigma = 0.15$):

- At $\omega_c \approx 0.078$: $\kappa^*(\omega_c) \approx 0.047$. Complete non-participation everywhere requires $\kappa \geq 0.047$ (4.7% of the Sharpe ratio).
- At $\underline{\omega} = 0.02$: $\kappa^*(\underline{\omega}) \approx 0.012$. Non-participation at the bad steady state requires $\kappa \geq 0.012$ (1.2% of the Sharpe ratio).

Both thresholds are well within empirical estimates (Ilut and Schneider 2014: $\kappa \approx 0.03$–0.05).

Corollary 37 (Recovery of the main paper)**.** *For $\kappa \geq \kappa^*(\omega_c) \approx 0.047$, $\alpha_h^*(\omega) = 0$ for all ω: all results of the main paper hold exactly.*

J.4 QE Trap and Bubble Classification

Proposition 38 (Preservation)**.** *For any $\kappa \geq 0$: (i) $Q_t = 1/\omega_t$ is a non-negative local martingale; (ii) $\underline{\omega}$ and $1/\epsilon$ remain the only absorbing steady states; (iii) the QE Trap survives; (iv) in the baseline, Q is a strict local martingale (bubbles exist); in the constrained model, Q is a true martingale (no bubbles).*

Proof. The Q-martingale property depends on $\mu_\omega = \sigma_\omega^2/\omega$, which holds for any ϕ. In the constrained region, $\phi = \bar{\phi}(\omega)$ regardless of α_h, so σ_ω is independent of κ. The zeros of σ_ω and the Feller boundary classification are unchanged. □

J.5 Three-Region Equilibrium

For $\kappa \in (0, \kappa^*(\omega_c))$, the economy has three regions: (i) robust failure ($\omega < \underline{\omega}$); (ii) endogenous participation ($\underline{\omega} \leq \omega < \omega_p(\kappa)$); (iii) non-participation ($\omega_p(\kappa) \leq \omega < 1/\epsilon$, identical to the main paper). The effective crisis threshold $\omega_c(\kappa)$ is increasing in κ, with $\omega_c(0) = \underline{\omega}$ and $\omega_c(\kappa) = \omega_c$ (main paper) for $\kappa \geq \kappa^*(\omega_c)$.

The ambiguity parameter κ nests all models:

κ	Household behavior	Model recovered
0	Full participation	Online Appendix N with endogenous $\alpha_h(\omega)$
$(0, 0.047)$	Participates in some states	Three-region model (this appendix)
$[0.047, \infty)$	Never participates	Main paper

K Additional Empirical Tests

K.1 Test 2: Capital Scarcity Predicts Equity Risk Premia

The model predicts that risk premia depend on the intermediary state through the market price of risk (Theorem 7). We define the capital scarcity proxy $\widehat{Q}_t^{HKM} \equiv 1/\widehat{\omega}_t$ and estimate:

$$\widehat{R}^e_{t\to t+h} = a_h + b_h \, \widehat{Q}_t^{HKM} + \varepsilon_{t+h}, \tag{78}$$

where $\widehat{R}^e_{t\to t+h}$ is the cumulative excess equity return. The model predicts $b_h > 0$.

Table 31 shows that the slope is positive at every horizon but precision is modest: marginally significant at $h = 2$ and $h = 4$, imprecise at $h = 1$ and $h = 8$. The evidence is directionally consistent with the model but remains suggestive.

Table 31: Test 2: baseline (QHkm; higher QHkm = lower intermediary wealth share)

	h=1	h=2	h=4	h=8
Constant	-0.004	-0.018	-0.022	-0.006
	(0.017)	(0.033)	(0.063)	(0.143)
QHkm	0.001	0.003*	0.006*	0.009
	(0.001)	(0.002)	(0.003)	(0.007)
Adj. R^2	0.005	0.023	0.036	0.045
No. Obs.	219	218	216	212

The contrast between strong real-economy predictability (Test 1) and weaker return predictability is consistent with three model features: (i) the equity risk premium is a nonlinear,

non-monotone function of ω; (ii) risk premia reverse sign across crisis and recovery regimes; and (iii) measurement error attenuates the coefficient more severely for a nonlinear function.

Inference robustness. Hodrick t-statistics are below Newey–West values at all horizons. The Lewellen confidence interval includes zero at all horizons. The Campbell–Yogo test rejects at $h \geq 2$, providing partial support. See Table 32.

Table 32: Inference robustness: Test 2 ($Q^{HKM} \rightarrow$ excess returns)

	$h = 1$	$h = 2$	$h = 4$	$h = 8$
$\hat{\beta}_{OLS}$	0.001	0.003	0.006	0.009
t_{NW}	1.32	1.75*	1.71*	1.33
$t_{Hodrick}$	1.22	1.63	1.52	1.14
$\hat{\beta}_{Stambaugh}$	0.001	0.003	0.005	0.009
$t_{Stambaugh}$	1.26	1.73	1.71	1.33
Lewellen CI	[-0.00, 0.00]	[-0.00, 0.01]	[-0.00, 0.01]	[-0.00, 0.02]
Lewellen reject H_0	No	No	No	No
C-Y reject H_0	No	Yes	Yes	Yes
Wild bootstrap p	0.246	0.046	0.014	0.008
Block bootstrap p	—	0.040	0.059	0.119
R^2	0.010	0.028	0.041	0.050
N	219	218	216	212

Notes: t_{NW} uses Newey–West HAC standard errors with $\max(h, 4)$ lags. $t_{Hodrick}$ uses Hodrick (1992) reverse-regression standard errors. Stambaugh (1999) bias-corrected coefficient and t-statistic use HAC SE. Lewellen (2004) 95% CI assumes a unit-root predictor. C-Y denotes the Campbell–Yogo (2006) Bonferroni Q-test. Wild bootstrap uses Rademacher weights under $H_0 : \beta = 0$ (5000 replications). Block bootstrap resamples blocks of length h under H_0 (5000 replications) and is reported only for $h > 1$. */**/*** denote significance at 10%/5%/1%.

K.2 Test 3: QE Reacts to Intermediary Capital Shortfall

The model predicts that central bank risk absorption activates when intermediary capital falls below the crisis threshold ω_c. We estimate:

$$\Delta\widehat{\alpha}^{CB}_{t+1} = a + b\,\widehat{\text{Shortfall}}_t + u_{t+1}, \tag{79}$$

where $\widehat{\text{Shortfall}}_t \equiv \max(0, \widehat{\omega}_c - \widehat{\omega}_t)$ over 2003Q1–2024Q4. The model predicts $b > 0$.

Table 33: Test 3: QE reaction function (baseline)

	h=1
Constant	-0.001
	(0.001)
Shortfall	0.525**
	(0.232)
Adj. R^2	0.081
No. Obs.	87

Table 33 shows a positive and significant coefficient: $b = 0.525$ ($t = 2.26$). However, adding macro-financial controls reduces the coefficient to 0.425 and renders it insignificant ($t = 0.90$; Table 34). The QE reaction function is observationally difficult to distinguish from a broader response to macro-financial conditions.

Table 34: Test 3: QE reaction function (with controls, appendix robustness)

	h=1
Constant	0.004
	(0.009)
Shortfall	0.425
	(0.471)
Yield3m	-0.166*
	(0.101)
TermSpread	-0.049
	(0.169)
CreditSpread	-0.975
	(1.162)
SpxVol	0.078*
	(0.046)
Adj. R^2	0.137
No. Obs.	87

Inference robustness. The baseline specification survives all six inference methods (Table 35).

Table 35: Inference robustness: Test 3 (shortfall → QE response)

	$h = 1$
$\hat{\beta}_{OLS}$	0.525
t_{NW}	2.27**
$t_{Hodrick}$	2.03
$\hat{\beta}_{Stambaugh}$	0.531
$t_{Stambaugh}$	2.29
Lewellen CI	[0.08, 0.99]
Lewellen reject H_0	Yes
C-Y reject H_0	Yes
Wild bootstrap p	0.015
Block bootstrap p	—
R^2	0.092
N	87

Notes: t_{NW} uses Newey–West HAC standard errors with $\max(h, 4)$ lags. $t_{Hodrick}$ uses Hodrick (1992) reverse-regression standard errors. Stambaugh (1999) bias-corrected coefficient and t-statistic use HAC SE. Lewellen (2004) 95% CI assumes a unit-root predictor. C-Y denotes the Campbell–Yogo (2006) Bonferroni Q-test. Wild bootstrap uses Rademacher weights under $H_0 : \beta = 0$ (5000 replications). Block bootstrap resamples blocks of length h under H_0 (5000 replications) and is reported only for $h > 1$. */**/*** denote significance at 10%/5%/1%.