# INTERMEDIARY CONSTRAINTS AND INTERNATIONAL SAFE ASSET PREMIA *

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(This paper is constantly updated through the link here)

**Abstract**

We develop a theory of international asset returns in a two-country Lucas-tree model with a leverage constraint on financial intermediaries. Our model explains why U.S. Treasuries earn a convenience yield that rises in crises precisely when equity markets fall and international risk sharing breaks down. In normal times, the larger country (the U.S.) absorbs more global consumption risks and earns lower Treasury returns. In crisis times, the leverage constraint tightens and Treasuries receive preferential balance sheet treatment, making safe assets scarce and generating a convenience yield. These frictions force sequential portfolio adjustment—equity retrenchment followed by bond retrenchment—segmenting bond markets and giving U.S. Treasuries a unique role that endogenously generates the convenience yield. The model matches observed empirical facts that highlight how intermediary constraints and country size shape safe-asset premia in crisis times.

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# 1 Introduction

In times of global market stress, U.S. Treasury yields fall sharply relative to yields on other safe government bonds. During the 2008 global financial crisis and the 2020 Covid crisis, investors flocked to U.S. Treasuries, driving their yields far below what standard risk factors or exchange-rate movements would predict. These Treasury convenience yields[1], measured as the spread between G10 government bond returns and Treasury returns, or as deviations from covered interest parity (CIP) – rise systematically in crisis episodes.[2] What is striking about these episodes is not just the magnitude but the precise timing: Treasury convenience yields spike exactly when equity markets crash and international risk sharing breaks down (Acharya and Laarits (2025)). Moreover, these spikes occur only during high-stress periods. This state-dependent pattern raises some fundamental questions: What generates U.S. Treasury convenience yields in crisis times? And why do they exhibit regime-dependent behavior tied to equity market stress?

This paper tackles these questions by developing a tractable two-country Lucas-Tree model in which U.S. Treasuries endogenously earn convenience yields in crisis times due to a single leverage constraint on U.S. financial intermediaries that operates differently across asset classes and tightens in crisis times. The key insight is that when balance sheet constraints bind, different assets take up different amounts of scarce balance sheet space: U.S. Treasuries receive preferential treatment (relaxing constraints more per dollar held), while FX derivatives become prohibitively expensive to intermediate in crisis times. This asymmetry in constraint treatment, combined with country size effects, generates the empirical patterns we document: equity retrenchment exhibits asymmetry in magnitude, while bond retrenchment exhibits asymmetry in direction for U.S. and G10 investors. These distinct patterns arise from the interaction of country size effects and U.S. Treasuries' unique role as the preferred safe asset for satisfying binding constraints.

We embed a leverage constraint into an otherwise standard two-country Lucas-tree framework from Cochrane, Longstaff, and Santa-Clara (2007). U.S. financial intermediaries face a

[1]We use government bond-implied CIP deviations as our primary measure of return differentials between U.S. Treasuries and G10 government bonds. Following the literature (Du, Im, and Schreger 2018; Jiang, Krishnamurthy, and Lustig 2021), we refer to these return differentials as reflecting "Treasury convenience yields," though we recognize that CIP deviations capture *both* convenience yield differentials and international arbitrage frictions as shown in recent work by Diamond and Van Tassel (2024).

[2]For example, during the 2008 global financial crisis, CIP spiked from roughly 20 basis points to over 120 basis points within weeks; during the 2020 Covid crisis, CIP surged to 100 basis points in March 2020.

He-Krishnamurthy style constraint where their "efficient assets" (weighted by balance sheet space consumed) cannot exceed a multiple of net worth. In normal times, constraints are slack, cross-border equity trading is unconstrained and international risk sharing is complete. The larger country – the United States – absorbs more global risk and earns lower bond returns, consistent with the U.S. "exorbitant privilege" (Gourinchas and Rey (2007); Maggiori (2017); Jiang, Krishnamurthy, and Lustig (2021)). U.S. Treasuries earn a modest convenience yield from their collateral role. In crisis times, the constraint tightens and sequential portfolio adjustment begins. First, both U.S. and G10 investors increase domestic equity holdings (equity retrenchment), but larger U.S. market size means U.S. intermediaries absorb far more domestic equity risk in absolute terms, generating the magnitude asymmetry. Second, as constraints tighten further, only U.S. Treasuries receive preferential balance sheet treatment, incentivizing fire-sales of domestic equity for safe bonds. This generates the directional asymmetry in bond retrenchment, with concentrated demand for U.S. Treasuries from both countries driving the convenience yield spike.

In crisis times, the leverage constraint tightens through two channels: intermediaries' maximum allowable leverage falls, and FX derivatives become dramatically more expensive in terms of balance sheet space. This creates a sequential portfolio adjustment mechanism. When constraints first bind, U.S. intermediaries cannot provide market-making for FX swaps because these derivatives would consume prohibitive amounts of scarce balance sheet capacity. This breaks covered interest parity, generating CIP deviations as the arbitrage mechanism fails. As constraints tighten further, intermediaries sequentially adjust their portfolios: first dumping foreign equities to buy back domestic equity claims from foreign investors (equity retrenchment), then fire-selling domestic equity to households in exchange for U.S. Treasuries (bond retrenchment).

The equity retrenchment stage reduces intermediary liabilities without triggering costly fire-sales, as intermediaries use their foreign equity holdings as a buffer. But when this buffer is exhausted and constraints continue to tighten, the costlier bond retrenchment stage begins. Intermediaries fire-sell domestic equity to less-efficient households, acquiring Treasuries that receive preferential treatment in the leverage constraint – Treasuries relax constraints more per dollar held than other assets. This fire-sale depresses equity prices while simultaneously driving up Treasury prices, amplifying the convenience yield. The shadow price of scarce balance sheet capacity, the Lagrangian multiplier on the binding constraint, manifests as the crisis-time convenience yield.

This single constraint with time-varying tightness and asset-specific weights generates the observed crisis patterns: leverage constraints force portfolio retrenchment, but cannot alone explain why demand concentrates specifically on U.S. Treasuries; the preferential balance sheet treatment of Treasuries explains this concentration. Thus, it is the interaction of constraint tightness with asymmetric asset treatment, combined with country-size asymmetry, that produces the observed crisis-time convenience yield spikes and the negative comovement of equity and bond returns.

We highlight four empirical facts that motivate our theoretical framework. First, we establish that Treasury return differentials and government bond-implied CIP deviations exhibit state-dependent sensitivity to global market stress. During normal times with low market stress, US Treasury return differentials relative to other safe government bonds (foreign minus US) remain small (or negative) but spike up in crisis times. The relationship between government bond-implied CIP deviations and VIX are essentially flat in normal times, while CIP deviations increase sharply with VIX during high-global stress periods. This regime-switching behavior is confirmed by rolling window regressions where the sensitivity of CIP deviations to VIX rises as average VIX rises, and falls near zero when VIX is low.

Second, we document evidence of equity retrenchment during global market stress and when capital constraints tighten. Using the He-Kelly-Manela (HKM) intermediary capital ratio as our measure of constraint tightness, we show that when HKM falls (equity capital constraints tighten), both U.S. and G10 investors increase domestic equity holdings[3]. While the direction of the response is symmetric, the U.S. response is tens of folds larger than that of G10. This asymmetric magnitude validates that leverage constraints bind more severely for U.S. investors, consistent with our model's prediction that equity retrench more severely for U.S. investors in crisis regimes, forcing them to absorb substantially more domestic equity risk. This pattern echoes findings on equity home bias intensification during crises (Coeurdacier and Rey (2013); Forbes and Warnock (2012); Broner, Didier, Erce, and Schmukler (2013)).

Third, we document asymmetric direction of bond retrenchment—the key empirical signature of spillover. When HKM falls (equity constraints tighten), U.S. investors sharply increase Treasury bond holdings while G10 investors show no significant response in their own government

[3]As introduced in detail in section 2 of the paper, we define investors holdings of domestic assets as the holdings of assets divided by the total supply of assets (either bonds or equities).

bond holdings. Unlike equity markets where both countries retrench with different magnitudes, bond markets exhibit directional asymmetry: only the U.S. retrenches to U.S. Treasuries. This pattern suggests G10 investors are not fleeing to safety generally; they specifically seek U.S. Treasuries rather than their own government bonds. Our finding on asset retrenchments as constraints tighten is consistent with intermediary-based asset pricing theories (He and Krishnamurthy (2013); Adrian, Etula, and Muir (2014)) but novel in documenting asset-class-specific transmission from equity to bond markets.

Fourth, we run horse-race regressions to identify which portfolio rebalancing drives CIP deviations. We find that U.S. portfolio rebalancing drives massive CIP deviations, while G10 bond rebalancing has little or no effect. CIP rises when either U.S. or G10 equity home retrenchment increases, but the increase from the latter is ten times smaller. Including all four portfolio measures simultaneously, we find the coefficient of U.S. home bond retrenchment dominates while U.S. home equity retrenchment becomes insignificant once bond measures are included. This validates the spillover mechanism that equity retrenchment leads to bond retrenchment, and that it is the bond-market stage that directly drives CIP deviations.

**Related Literature.** Our paper contributes to several strands of literature in macro-finance and international asset pricing, including U.S. Treasury convenience yields in international markets and safe-asset determination, continuous-time asset pricing, and international portfolio rebalancing.

First, we develop a tractable two-country continuous-time model where balance sheet constraints on financial intermediaries endogenously generate Treasury convenience yields through asymmetric treatment of different assets. Previous work has documented that intermediary constraints affect asset prices (He and Krishnamurthy (2013); He, Kelly, and Manela (2017); Adrian, Etula, and Muir (2014)) and that safe assets carry convenience yields (Krishnamurthy and Vissing-Jorgensen (2012); Jiang, Krishnamurthy, and Lustig (2021)), but these mechanisms have been studied separately. We show theoretically how a single leverage constraint with time-varying tightness and asset-specific weights generates both effects: when constraints tighten, the preferential balance sheet treatment of Treasuries creates concentrated demand from both domestic and foreign investors. This spillover generates convenience yields endogenously rather than positing them as exogenous preference wedges, providing microfoundations for the "safe asset demand" driving

exchange rates in recent work (Jiang, Krishnamurthy, Lustig, and Sun (2024); Kekre and Lenel (2024)).

Second, we provide a unified framework where bonds, equities, and exchange rates are jointly determined through time-varying constraints. Existing international finance models typically study bond markets (Du, Im, and Schreger (2018a); Diamond and Van Tassel (2024)), equity markets (Hau and Rey (2006); Camanho, Hau, and Rey (2022)), or exchange rates (Gabaix and Maggiori (2015); Itskhoki and Mukhin (2021); Kekre and Lenel (2024); Beltran and He (2025)) in isolation. We incorporate both equity and bond markets in a continuous-time Lucas-tree framework where a leverage constraint activates in crisis states. As the constraint tightens and asset weights shift, this forces sequential portfolio adjustment from equities to bonds, generating endogenous convenience yields and return differentials.

Third, we characterize crisis and normal regimes where asset prices respond differently to constraints. This contributes to the intermediary asset pricing literature on occasionally binding constraints (He and Krishnamurthy (2013); Brunnermeier and Sannikov (2014)) by showing how our leverage constraint activates sequentially as financial stress intensifies. In normal times when constraints are slack, return differentials reflect only country-size effects (Hassan (2013); Maggiori (2017)) and markets achieve perfect risk-sharing. When the leverage ratio tightens, foreign equity positions become too expensive to maintain, creating equity retrenchment and SDF wedges that impair international arbitrage. If constraints tighten further, the FX derivative weight becomes prohibitive and domestic equity fire-sales begin, generating the convenience yield component. This sequential activation mechanism rationalizes empirical findings that international finance relationships appear weak in pooled data but strong during crises (Lilley, Maggiori, Neiman, and Schreger (2022); Jiang, Krishnamurthy, Lustig, and Sun (2024)) and provides theoretical foundations for the post-crisis emergence of persistent CIP deviations (Du, Tepper, and Verdelhan (2018b)).

Fourth, we show that country size interacts with financial constraints to generate asymmetric safe asset premia, departing from standard two-country Lucas tree models (Cochrane, Longstaff, and Santa-Clara (2007); Pavlova and Rigobon (2007); Martin (2011);Colacito and Croce (2011)). While country size alone generates level differences in bond returns (Hassan (2013); Maggiori (2017)), it cannot explain the state-dependent relationship between return differentials and financial stress that we document empirically. We introduce intermediary frictions into an otherwise

standard continuous-time framework and show that only when the leverage constraint on the intermediaries bind do convenience yields spike in response to financial stress. This mechanism complements recent work on global safe asset demand (Caballero, Farhi, and Gourinchas (2017); He, Krishnamurthy, and Milbradt (2019)) by showing how market segmentation from binding constraints, rather than preference-based safe asset scarcity alone, drives crisis-time convenience yield spikes. Our evidence on bond retrenchment also speaks to the flight-to-safety literature (Forbes and Warnock (2012); Broner et al. (2013)) by demonstrating that retrenchment patterns differ systematically across asset classes depending on which specific constraint binds.

# 2 Motivating Empirical Facts

We present four empirical facts documenting the relationship between U.S. Treasury return differentials, intermediary constraints, and international portfolio rebalancing during crisis episodes. These facts motivate and discipline our theoretical framework. Throughout the empirical analysis, we treat the United States as the larger home country (denoted H) and G10 currencies as the smaller foreign country (denoted F), consistent with our model notation.

## 2.1 Definitions and Terminologies

### CIP and Convenience Yields

We use government bond-implied *covered interest parity (CIP) deviations* as our primary empirical measure of return differentials between U.S. Treasuries and G10 government bonds. A CIP deviation is defined as the return differential between investing in a foreign government bond versus a U.S. Treasury after hedging currency risk using forward contracts; positive deviations indicate U.S. Treasuries earn lower returns.

Following Jiang, Krishnamurthy, and Lustig (2021), we refer to these return differentials as *Treasury convenience yields*, though we recognize this terminology conflates both (i) international arbitrage frictions arising from financial intermediaries' balance sheet constraints and (ii) pure convenience yield differentials – differences in domestic liquidity and safety premia between countries' government bonds (Diamond and Van Tassel (2024)). Our theoretical framework en-

compasses both components: the leverage constraint impairs international arbitrage by creating wedges in stochastic discount factors and segmenting markets, while generating a pure convenience yield as the shadow price of scarce Treasury collateral capacity at the same time.

We also define *crisis times* operationally as periods when the VIX index exceeds 20, following the portfolio choice literature's convention of using implied volatility to measure financial stress (Coeurdacier and Rey (2013)), rather than relying on ex-post consumption growth data which is available only at quarterly frequency. In the model, high VIX episodes also coincide with leverage constraint binding and international market segmentation.

**Portfolio Share Measures**

We construct four portfolio share measures $\chi$ to track home bias in equity and bond markets for both countries. Both equities and bonds has its respective $\chi^{H,H}$, $\chi^{F,H}$, $\chi^{F,F}$ and $\chi^{H,F}$. For equities, we define $\chi^{H,H}$ for equities as the share of home investors' holdings of home equities over the total supply of home equities; $\chi^{F,H}$ for equities is foreign investors' holdings of home equities over the total supply of home equities; $\chi^{H,F}$ is home investors' holdings of foreign equities over the total supply of foreign equities; Finally, $\chi^{F,F}$ for equities is foreign investors' holdings of foreign equities over the total supply of foreign equities. By construction, $\chi^{H,H}$ and $\chi^{F,H}$ add up to 1 as they share the total supply of home (USD) equities as the same denominator; $\chi^{H,F}$ and $\chi^{F,F}$ also add up to 1 and both use the total supply of foreign (G10) equities as the denominator. The definitions are analogous for bond holdings except that we use the total issuance of bonds (rather than equities) in the denominator.

**HKM Intermediary Capital Ratio**

Our primary measure of constraint tightness is the HKM intermediary capital ratio (He, Kelly, and Manela (2017)), defined as the aggregate market equity of New York Fed primary dealers divided by their market equity plus book debt. This measure was explicitly constructed to operationalize the intermediary capital ratio that serves as the key state variable in intermediary asset pricing models (He and Krishnamurthy (2013); Brunnermeier and Sannikov (2014)).

We use the HKM measure for three reasons. First, it directly captures balance sheet constraints. When intermediaries' equity cushion falls (HKM declines), leverage rises and equity financing

constraints bind more tightly, forcing intermediaries to reduce risk-taking and scale back cross-border arbitrage. This is precisely the mechanism our model emphasizes. Second, the measure was explicitly constructed to operationalize the intermediary capital ratio that serves as the key state variable in intermediary asset pricing models (He and Krishnamurthy (2013); Brunnermeier and Sannikov (2014)), providing theoretical grounding for our empirical proxy. Third, using holding-company market equity rather than book values or subsidiary balance sheets ensures the measure responds quickly to financial stress[4] and is particularly well-suited for identifying crisis episodes. Taken together, the HKM measure is thus both theoretically grounded and empirically powerful for identifying when intermediary constraints bind.[5]

## 2.2 Details on Constructing Portfolio Measures

We construct portfolio share measures ($\chi$) that describe how equity and bond wealth is allocated across countries. The world in the model consists of two regions: the United States and a composite foreign region comprising the G10 countries. Portfolio shares are defined based on the residence of the holder rather than the issuer of the asset. As a result, asset demand (holdings) and asset supply (issuance) are treated as distinct objects, and the construction of some portfolio shares relies on accounting identities.

Equity supply is measured using the market capitalization of listed domestic companies from the World Bank's World Development Indicators, following the methodology in Koijen and Yogo (2020). This variable captures the total market value of domestically listed firms at end-of-year prices and serves as our proxy for issuer-side equity supply. For euro-area countries, we use the World Bank's euro-area aggregate series, which is constructed consistently at the area level. Because the theoretical model includes only the United States and the G10, equity supply is adjusted so that it reflects issuance held within this two-region world.

[4]HKM exhibits pronounced drops during the 1998 LTCM crisis, 2008 financial crisis, and 2020 COVID crisis precisely when intermediary leverage constraints bind and balance sheet capacity becomes scarce.

[5]While our model features a leverage constraint as a theoretical parameter limiting intermediaries' efficient assets relative to net worth, the HKM capital ratio measures their actual equity capitalization at each point in time. These are not identical as the the model's constraint is a fixed boundary while HKM varies continuously, but HKM serves as the natural empirical proxy for constraint tightness. When HKM falls toward low levels (as in 2008 when it reached 0.03), intermediaries approach the boundary where constraints bind; when HKM is high, constraints are slack. Thus, declines in HKM empirically indicate periods when the model's leverage constraint would be binding. This alignment between theory and measurement is by design: HKM constructed their measure specifically to capture the intermediary capital dynamics emphasized in He and Krishnamurthy (2013).

Cross-border equity holdings are obtained from the IMF's Coordinated Portfolio Investment Survey (CPIS), which reports market-value portfolio equity holdings by residence of the reporting country. CPIS captures foreign holdings only and excludes domestic residents by construction. We therefore directly observe U.S. holdings of foreign equities and foreign (G10) holdings of U.S. equities from CPIS. These bilateral foreign holdings are used to construct the cross-border portfolio shares ($\chi^{HF}$) and ($\chi^{FH}$).

Domestic equity portfolio shares, ($\chi^{HH}$) and ($\chi^{FF}$), are not directly observed in the data. Instead, they are inferred residually using the accounting identities implied by the portfolio shares. Specifically, domestic equity shares are constructed as one minus the corresponding foreign share. This approach is necessary because no dataset provides consistent measures of domestic residents' equity holdings at the country level across our sample. As discussed extensively in the literature, CPIS data are not suitable for directly measuring domestic equity ownership, but they can be used reliably to measure foreign holdings, from which domestic shares can be inferred.

Bond supply is measured using the BIS Debt Securities Statistics, which report outstanding debt securities by issuer residence across all markets and currencies. This measure captures total debt issuance held by both residents and non-residents. Market-value series are used when available; when market values are not reported, we rely on nominal outstanding amounts, which is standard practice in the literature. As with equities, bond supply is adjusted so that it reflects issuance held within the two-region world consisting of the United States and the G10.

Bilateral bond holdings are constructed by combining U.S. Treasury International Capital (TIC) data with CPIS. TIC provides bilateral positions between the United States and foreign countries, while CPIS is used to measure bond holdings among G10 countries, including intra-euro-area positions that are not captured by TIC. This combined approach allows us to construct consistent bilateral bond holdings while avoiding double counting, particularly within the euro area.

Portfolio shares for bonds are then computed analogously to equities. Cross-border bond shares are constructed directly from observed bilateral holdings, while domestic bond shares are inferred residually using accounting identities. Together, these constructions yield internally consistent measures of international portfolio allocation that align closely with the structure of the theoretical model.

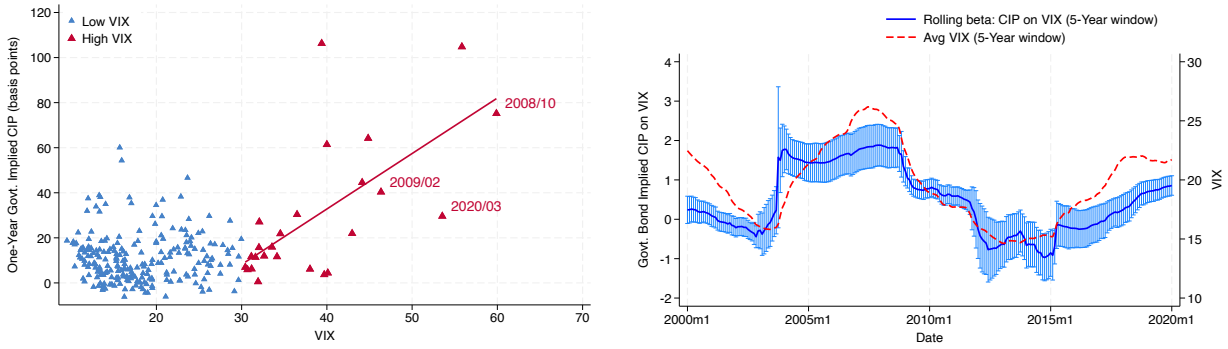
## 2.3 Four Stylized Empirical Facts

***Fact I**: Treasury Return Differentials Exhibit State-Dependent Sensitivity to Market Stress.*

We show that treasury return differentials, measured via government bond-implied covered interest parity (CIP) deviations, exhibit regime-switching sensitivity to financial stress rather than a stable linear relationship. During low-stress periods (VIX $<$ 20), CIP deviations remain small (0-40 basis points) and show essentially zero sensitivity to VIX movements. During high-stress periods (VIX $\geq$ 20), CIP deviations spike sharply with VIX, reaching 75-120 basis points during the 2008 and 2020 crises. This state-dependent pattern is well-documented in recent work on post-crisis CIP violations and challenges simple linear models of international asset pricing.

Figure 1: RETURN DIFFERENTIALS IS STATE-DEPENDENT TO MARKET STRESS

(a) CIP WITH HIGH VS. LOW VIX (b) ROLLING BETA OF CIP ON VIX




**Note**: This panel of figures presents the relation between the one-year government bond implied CIP deviations for G10 currencies and the VIX index. The left figure (a) gives scatter plot of average CIP across G10 currencies and VIX for the subsample of low VIX ($<$ 20) and subsample of high VIX ($>$ 20). The right figure (b) reports the coefficient of rolling window regression of CIP deviations in G10 currencies on VIX with a window of 5 years (60 months); the regressions are run with country fixed effects and the 95% confidence interval reported as the shaded blue area. The samples are in monthly observations for figure (a) and month-country observations for figure (b) from 2000 to 2020 .

We present two pieces of evidence for this regime-switching behavior, confirming patterns documented in the broader literature. First, Figure 1 (a) shows the scatter plot of government bond-implied CIP deviations against VIX, with observations color-coded by regime. Blue triangles (VIX $<$ 20) cluster in the lower left with no visible slope—the relationship is essentially flat in normal times. Red triangles (VIX $\geq$ 20) show a steep positive relationship, with the three major crisis episodes (2008/10, 2009/02, 2020/03) labeled. The fitted line has a positive slope driven entirely by high-VIX observations, confirming the relationship exists only in crisis states. This

visual pattern mirrors the structural break documented by Lilley, Maggiori, Neiman, and Schreger (2022)[6] and the regime-switching dynamics formalized in Cerutti, Obstfeld, and Zhou (2021).

Second, Figure 1 (b) formalizes this regime-switching using rolling window regressions. We regress CIP deviations on VIX using a 5-year (60-month) rolling window with country fixed effects, plotting the estimated coefficient (the "VIX beta") over time. The rolling beta rises above 2 during 2004-2009 when average VIX (red dashed line) exceeded 25, falls to near zero during 2010-2016 when VIX averaged below 20, and rises again during 2018-2020. The 95% confidence bands confirm these movements are statistically significant. The rolling beta tracks average VIX almost perfectly, confirming the sensitivity is state-dependent. This time-varying relationship is consistent with the mechanism emphasized by Du and Schreger (2021).[7]

This regime-switching pattern has important implications for our theoretical framework. When intermediary constraints are slack (low VIX or normal times), return differentials reflect only country-size effects and remain small, as in our model's normal regime. When the intermediary constraint binds (high VIX or crisis times), both the arbitrage friction component and the pure convenience yield component activate, producing large spikes in measured return differentials.[8] Our model provides microfoundations for this occasionally-binding-constraint by explicitly modeling how the leverage constraint activate in crisis times, generating the observed nonlinear relationship between VIX and return differentials.

***Stylized Fact II**: Intermediary constraints drive retrenchments to domestic equities for both U.S. and G10 currencies.*

Figure 2 presents visual and regression evidence that when U.S. intermediary equity capital constraints tighten (measured by HKM declines), both U.S. and G10 investors increase their domestic equity holdings. Panels (a) and (b) show dual-axis time series plots of domestic equity

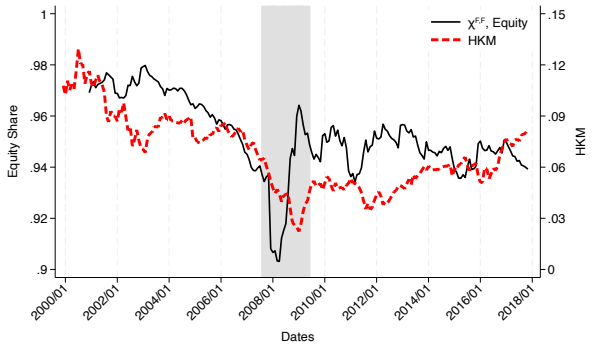
[6]Lilley et al. (2022) show that the explanatory power of risk measures for exchange rates jumped from near-zero pre-2007 to above 50% during crisis periods.

[7]As Du and Schreger (2021) wrote, "the supply curve for dollar funding became upward sloping" after 2008, creating the structural foundation for state-dependent sensitivity. Pre-crisis, dollar funding supply was perfectly elastic and CIP held; post-crisis, intermediary balance sheet constraints made supply upward-sloping, allowing demand shocks to move CIP deviations.

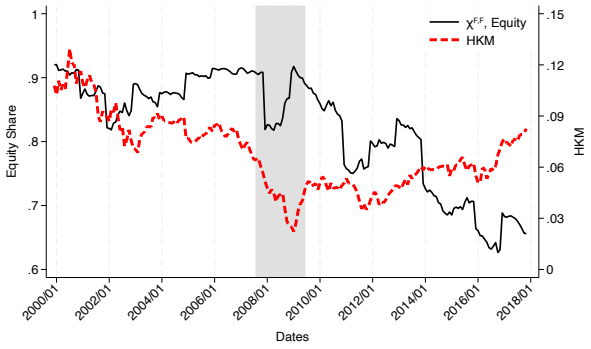
[8]The mechanism operates through intermediary balance sheets: Avdjiev et al. (2019) document that "a stronger dollar goes hand in hand with larger deviations from covered interest parity and contractions of cross-border bank lending in dollars," with the dollar-credit sensitivity coefficient rising from -0.2 pre-crisis to -0.5 post-crisis—itself evidence of a regime shift in how financial stress transmits to international markets.

Figure 2: EQUITY RETRENCHMENTS AND INTERMEDIARY CAPITAL RATIO

(a) $\chi^{HH}$ and HKM




(b)$\chi^{FF}$ and HKM




DEPENDENT VARIABLE: INTERMEDIARY CAPITAL RATIO

| | (1) | (2) | (3) |
|---|---|---|---|
| $\chi^{HH,\text{Equity}}$ | -0.252*** | | -0.253*** |
| | (0.024) | | (0.025) |
| $\chi^{FF,\text{Equity}}$ | | -0.006** | -0.006*** |
| | | (0.002) | (0.002) |
| Constant | 0.309*** | 0.073*** | 0.316*** |
| | (0.024) | (0.002) | (0.024) |
| Country FE | Y | Y | Y |
| Year FE | Y | Y | Y |
| Obs. | 1020 | 1044 | 984 |
| Adj. $R^2$ | 0.919 | 0.928 | 0.921 |

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

**Note**: Figure (a) and (b) slots the time series of $\chi^{HH}$ equity for US and average $\chi^{FF}$ for G10 currencies against the He-Kelly-Manela (HKM) measure of of intermediary capital ratio. The table reports the panel regression results with country and year fixed:

$$\text{HKM}_t = \alpha_i + \delta_t + \beta \cdot \chi^{i,i,\text{Equity}} + \epsilon_{i,t} \qquad i \in \{H, F\}$$

where column (1) and (2) include $\chi^{H,H}$ and $\chi^{F,F}$ for equities, and column (3) includes both measures.

holdings[9] ($\chi$ measures, left axis) and HKM (right axis) for the U.S. and G10 respectively. U.S. domestic equity holdings (panel a) remain in a tight range around 0.95 but move sharply in the opposite direction of HKM, particularly during the 2008 crisis (shaded region) when HKM collapsed and $\chi^{H,H}$ spiked from approximately 0.94 to 0.98—a small absolute movement that is large relative to its historical variation. In contrast, G10 domestic equity holdings (panel b) exhibit substantially more volatility overall, fluctuating between 0.65 and 0.95 over the sample period, but show much weaker co-movement with HKM during crisis episodes.

The weak co-movement between HKM and G10 domestic equity holdings can be formalized through regression in the table of Figure 2. When U.S. intermediary equity capital constraints tighten (measured by HKM declines), both U.S. and G10 investors increase their domestic equity holdings—the direction of response is symmetric. But the magnitudes differ dramatically: Column (1) shows that U.S. domestic equity holdings predict HKM with a coefficient of -0.252: when $\chi^{H,H}$ increases by 0.01 (for example, from 0.94 to 0.95), HKM falls by 0.252 percentage points; this coefficient is 42 times larger than the G10 coefficient. This asymmetry is particularly striking because U.S. domestic equity holdings exhibit far less variation than G10 holdings overall, yet respond much more strongly to HKM. This pattern suggests intermediary constraints are a primary driver of U.S. portfolio adjustment but play only a minor role for G10 investors.

***Stylized Fact III****: Bond markets show stark asymmetric patterns in retrenchment during crises and only U.S. investors retrench to home bonds.*

When intermediary constraints tighten, U.S. investors sharply increase Treasury holdings, but G10 investors show no significant response in their own government bond holdings. Unlike equity markets (where both countries retrench with different magnitudes), bond markets exhibit directional asymmetry—only the U.S. retrenches to its own bonds. The panel regression results are reported in Figure 3.

[9] We define $\chi^{H,H}$ as U.S. investors' holdings of U.S. equities divided by total U.S. equity supply, and $\chi^{F,F}$ as G10 investors' holdings of G10 equities divided by total G10 equity supply. These measures differ from the "equity home bias" typically studied in the international portfolio choice literature (e.g., French and Poterba 1991; Coeurdacier and Rey 2013), which compares actual portfolio shares to CAPM-implied optimal shares. Our $\chi$ measures capture absolute holdings as fractions of supply rather than deviations from a benchmark, avoiding the term "home bias" to prevent confusion.

Figure 3: BOND MARKETS RETRENCHMENTS AND INTERMEDIARY CAPITAL RATIO

DEPENDENT VARIABLE: INTERMEDIARY CAPITAL RATIO

| | (1) | (2) | (3) |
|---|---|---|---|
| $\chi^{HH,\text{Bond}}$ | -0.232** | | -0.235** |
| | (0.110) | | (0.110) |
| $\chi^{FF,\text{Bond}}$ | | -0.006 | -0.006 |
| | | (0.005) | (0.005) |
| Constant | 0.298*** | 0.073*** | 0.306*** |
| | (0.109) | (0.005) | (0.109) |
| Country FE | Y | Y | Y |
| Year FE | Y | Y | Y |
| Obs. | 1080 | 1080 | 1080 |
| Adj. $R^2$ | 0.927 | 0.927 | 0.927 |

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

**Note**: The table reports the panel regression results with country and year fixed:

$$\text{HKM}_t = \alpha_i + \delta_t + \beta \cdot \chi^{i,i,\text{Bond}} + \epsilon_{i,t} \qquad i \in \{H, F\}$$

where column (1) and (2) include $\chi^{H,H}$ and $\chi^{F,F}$ for bonds, and column (3) includes both measures.

This is the key empirical signature of asymmetric spillover. G10 investors are not engaging in generic "flight to safety" where they retreat to their own government bonds during crises. Instead, they specifically seek U.S. Treasuries and the U.S. investors are buying the same. This creates one-sided demand pressure on U.S. Treasuries. In our model, this occurs because U.S. Treasuries are the only asset that both hedges equity risk (via negative stock-bond correlation in crises) and satisfies collateral requirements (via the global risk-free bond construction). G10 government bonds do not share these properties in our framework, so G10 investors join U.S. investors in demanding Treasuries rather than their own bonds. The directional asymmetry in bond retrenchment—absent in equity retrenchment—provides strong evidence for this mechanism.

***Stylized Fact IV**: U.S. portfolio rebalancing drives large CIP deviations; G10 bond rebalancing does not.*

When all four portfolio measures are included simultaneously in explaining CIP deviations, U.S. bond home bias dominates with a coefficient nearly 100 times larger than G10 equity home bias and infinitely larger than G10 bond home bias (which is insignificant). This validates that concentrated demand for U.S. Treasuries—not generic portfolio retrenchment—drives crisis-time CIP spikes.

Figure 4: CIP DEVIATIONS AND RETRENCHMENTS

DEPENDENT VARIABLE: CIP DEVIATIONS

| | (1) | (2) | (3) |
|---|---|---|---|
| $\chi^{HH,\text{Equity}}$ (%) | 4.137*** | | 1.126 |
| | (0.725) | | (0.839) |
| $\chi^{FF,\text{Equity}}$ (%) | 0.462*** | | 0.301*** |
| | (0.068) | | (0.070) |
| $\chi^{HH,\text{Bond}}$ (%) | | 32.124*** | 27.772*** |
| | | (3.051) | (3.990) |
| $\chi^{FF,\text{Bond}}$ (%) | | -0.172 | -0.237 |
| | | (0.145) | (0.173) |
| Constant | -424*** | -3154*** | -2850*** |
| | (70.471) | (302.631) | (357.579) |
| Country FE | yes | yes | yes |
| Year FE | yes | yes | yes |
| Obs. | 965 | 1036 | 965 |
| Adj. $R^2$ | 0.577 | 0.589 | 0.598 |

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

**Note**: The table reports the regression results for the panel regression of CIP deviations on equity and bond shares ($\chi$ measures). Note that all the $\chi$ measures in this regression are in percentage points (and only in this regression) while CIP are still in basis points.

Figure 4 presents horse-race regressions where we regress CIP deviations on all four portfolio measures simultaneously. Columns (1)-(2) show that in univariate specifications, both equity measures are significant, as are bond measures separately. But once all are included together, only U.S. bonds and (weakly) G10 equities survive. The fact that U.S. equity becomes insignificant validates the spillover mechanism: equity retrenchment matters only insofar as it leads to bond retrenchment, and it is the bond-market stage that directly affects CIP.

Column (3) shows the full specification. When U.S. domestic bond home retrenchment increases by one percentage point, CIP deviations rise by 27.8 basis points. This is the dominant driver. G10 equity home bias shows a significant but tiny effect (coefficient 0.301). U.S. equity home bias becomes insignificant once bond measures are included, while G10 bond home bias remains insignificant.

Taken together, our four empirical facts trace the complete mechanism. Fact 1 establishes that convenience yields exhibit regime-switching behavior consistent with occasionally binding constraints. Fact 2 shows equity retrenchment is asymmetric in magnitude (U.S. responses are more than ten folds larger), validating country-size effects. Fact 3 shows bond retrenchment is asym-

metric in direction (only U.S. retrenches), providing the empirical signature of spillover. Fact 4 validates that U.S. bond demand dominates CIP. Together, these facts discipline our theoretical framework and provide quantitative targets for calibration.

# 3 Model Set-up

Time is continuous and infinite horizon, $t \in [0, +\infty)$. There are two countries in the world, home country (denoted by $H$, U.S.) and foreign country (denoted by $F$, G-10). For ease of illustration, we will call home country the US and foreign country the UK (one of G10-currency countries).

There are four classes of agents. The first two classes are made up of locals in each country, which are households who consume a fraction of their own country's output and do not eat the foreign country's output. Locals do not participate in the international financial market. They are also less good at managing domestic risky claims (Brunnermeire-Sannikov style assumption). The third and fourth class comprises the financial investors in each country: these individuals eat goods from both countries, consuming the remaining fraction $1 - \psi$ of each country's output. The Home (u.s.) financial investors are global dealers/intermediaries, who hold both domestic and foreign risky claims and have direct access to U.S Treasury and repo market. They are also the market maker for FX swaps/forward. The foreign financial investors can also hold both domestic and foreign risky claims, they represent end users hedger (e.g., insurers, pension funds, asset managers, corporates, reserve managers, banks hedging structural FX exposures, not a market maker/arbitrage desk). Foreign end users hedge currency risk using FX forwards/swaps. They do not have direct access to U.S. Treasury/repo markets and therefore cannot implement cash-and-carry trades.

**Technology** Each country is endowed with a tree producing domestic good. The two trees evolve as follows,

$$\frac{dY_{H,t}}{Y_{H,t}} = \mu_{H,t}\, dt + \sigma_{H,t}\, dZ_{H,t}$$
$$\frac{dY_{F,t}}{Y_{F,t}} = \mu_{F,t}\, dt + \sigma_{F,t}\, dZ_{F,t}$$

where $\{\mu_{H,t}, \mu_{F,t}, \sigma_{H,t}, \sigma_{F,t}\}$ are exogenous parameters (or processes). For simplicity, we assume throughout the paper that $\mu_{H,t} = \mu_{F,t} = \mu$ and $\sigma_{H,t} = \sigma_{F,t} = \sigma$. When managed by financial institutions, trees produce full output. When managed by domestic households, trees produce less output because household are less good at managing them.

**Preferences** In order to highlight our mechanism, we assume homogeneous preference, logarithmic utility, and no consumption home bias for the financial investors of the two countries. The final consumption is a CES aggregate of the two goods produced by the two countries. The expected utility of the financial investors in country $i$, takes the form

$$\mathbb{E}\int_0^\infty e^{-\rho t}\log C_{H,t}\,dt$$

where

$$C_{H,t} = \left[a^{\frac{1}{\eta}} C_{HH,t}^{\frac{\eta-1}{\eta}} + (1-a)^{\frac{1}{\eta}} C_{HF,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

$a$ is the share parameter [10]. $\eta$ is the elasticity of substitution between the two goods, assumed to be smaller than 1.[11]

**Numeraire** Define 1 unit of the CES basket of total output $\overline{Y}_t$ as numeraire throughout the paper,

$$\overline{Y}_t \equiv \left[a^{\frac{1}{\eta}} Y_{H,t}^{\frac{\eta-1}{\eta}} + (1-a)^{\frac{1}{\eta}} Y_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

Denote the process of total output $\overline{Y}_t$ as

$$\frac{d\overline{Y}_t}{\overline{Y}_t} = \overline{\mu}_t dt + \overline{\sigma}_t dZ_t$$

where $dZ_t = [dZ_{H,t} \;\; dZ_{F,t}]^T$.

[10] Unlike Pavlova and Rigobon (2008) where $a$ represents the country size, here in our model $a$ is not a key parameter of interest.

[11] We assume $\eta < 1$ (complementarity between goods) to generate home bias in consumption and ensure portfolio rebalancing has real effects on equilibrium prices. This assumption is standard in international macro models and can be microfounded through habit formation or adjustment costs in consumption.

**International trade market and exchange rate** The financial investors can trade home and foreign goods frictionlessly. Denote $p_t^H$ as the price of home good and $p_t^F$ as the price of foreign good. The real exchange rate is given by the relative price of home and foreign good[12],

$$e_t \equiv \frac{p_t^H}{p_t^F} \tag{1}$$

and $e_t$ is also the terms of trade in this model. And denote the endogenous process of real exchange rate, $e_t$, as

$$\frac{de_t}{e_t} = \mu_t^e dt + (\sigma_t^e)^T dZ_t$$

**Equity** Each country can issue domestic equity shares in unit supply. The equities are risky claims to domestic trees. Denote $S_t^H$ and $S_t^F$ as the total value of domestic equity and foreign equity respectively. Define $\chi_t^{H,S^H}$ as the the share of home stock market (apple tree) held by home investor, $\chi_t^{H,S^F}$ as the share of foreign stock market (orange tree) held by home investor. And similarly define $\chi_t^{F,S^H}$ as the share of home stock market (apple tree) held by foreign investor and $\chi_t^{F,S^F}$ as the share of foreign stock market (orange tree) held by foreign investor.

**U.S. Treasury and FX market** The U.S issues domestic bonds (U.S Treasury), priced by domestic households. U.S financial intermediary also has direct access to the Treasury and repo market. Treasury is special in the sense that it provides collateral for secured borrowing (which generates convenience yield in normal times). There also exists a FX market where U.S. intermediary is a market maker, and can provide FX swaps for foreign end users, when they have the capacity. The home domestic bond (U.S. Treasury), denoted as $B_t^H$, is instantaneously *risk-free in domestic good* but *not* risk-free in terms of numeraire. The FX swap contract has a present value of $0$ at initiation, but suffers from FX forward price movements at delivery. Foreign financial investors hedge currency risk with FX forwards/swaps.

The idea of this set-up is to separate the funding market (U.S. Treasury) and the FX hedging market (FX swaps/forwards). The specialty of the U.S. Treasury in funding market (as a collateral and liquid/safe asset) is what generates convenience yield. And the friction on the FX market is what generates the CIP deviations.

[12] An increase of $e_t$ corresponds to an appreciation of home currency relative to foreign currency.

**Asset returns** We introduce notations for asset returns which are *endogenous* processes. Recall that $B_t^H$ is instantaneously risk-less bond in home good and denote the return process of home bond (in terms of numeraire) as:

$$dr_t^{B^H} = (\mu_{p^H,t}t + r_t^f)\,dt + \sigma_{p^H,t}\,dZ_t$$

where $\mu_{p^H,t}t$ and $\sigma_{p^H,t}$ are given by the endogenous process:

$$\frac{dp_t^H}{p_t^H} = \mu_{p^H,t}\,dt + \sigma_{p^H,t}\,dZ_t$$

Denote the return process of FX forward (in terms of numeraire) as $dr_t^{FX}$.

Recall that $S_t^H$ is the total value of home equity and define $q_t^H$ as the per unit price of home equity in terms of numeraire $\overline{Y}_t$, that is, $S_t^H = q_t^H \overline{Y}_t$. And postulate the endogenous process of $q_t^H$ as follows

$$\frac{dq_t^H}{q_t^H} = \mu_{q^H,t}dt + \sigma_{q^H,t}dZ \tag{2}$$

The return of home equity in terms of numeraire is given by

$$dr_t^{S^H} = \underbrace{\frac{p_t^H Y_{H,t}}{q_t^H \overline{Y}_t}dt}_{\text{dividend yield}} + \underbrace{\frac{d(q_t^H \overline{Y}_t)}{q_t^H \overline{Y}_t}}_{\text{capital gain}}$$

and similarly

$$dr_t^{S^F} = \underbrace{\frac{p_t^F Y_{F,t}}{q_t^F \overline{Y}_t}dt}_{\text{dividend yield}} + \underbrace{\frac{d(q_t^F \overline{Y}_t)}{q_t^F \overline{Y}_t}}_{\text{capital gain}}$$

**Wealth and wealth share** We introduce notations for wealth and wealth shares, which will be determined in equilibrium. Denote the aggregate wealth of home country as $W_t^H$ and the aggregate wealth of foreign country as $W_t^F$. And define the wealth share of home country as $n_t = \frac{W_t^H}{W_t^H + W_t^F}$.

**Portfolio shares** Denote $\theta_t^{H,S^H} = \frac{\chi_t^{H,H} S_t^H}{W_t^H}$ as the portfolio share of home equity for home country, $\theta_t^{H,S^F} = \frac{\chi_t^{H,F} S_t^F}{W_t^H}$ as the portfolio share of foreign equity for home country. And similarly, denote $\theta_t^{F,S^H} = \frac{\chi_t^{F,H} S_t^H}{W_t^F}$ as the portfolio share of home equity for foreign country and $\theta_t^{F,S^F} = \frac{\chi_t^{F,F} S_t^F}{W_t^F}$ as

the portfolio share of foreign equity for foreign country.

We can similarly define portfolio shares of bonds in home and foreign country. Denote $\theta_t^{H,B^H}$ as the portfolio share of home bond (U.S. Treasury) for home country, $\theta_t^{H,FX}$ as the portfolio share of FX forwards for home country, And similarly, denote $\theta_t^{F,FX}$ as the portfolio share of FX forwards for foreign country.

**Leverage constraint** There is a leverage constraint (in the spirit of He-Krishnamurthy) facing home financial intermediaries. We first define the asset, efficient asset, liability and net worth of the home financial intermediary:

$$A_t^{eff} = S_t^H + \chi_t^{H,S^F} S_t^F + B_t^H + \omega_{fx}|FX_t| \tag{3}$$

where $S_t^H$ is total value of domestic risky capital/equity, $\chi_t^{H,S^F} S_t^F$ is home intermediary's total holding of foreign risky capital/equity, $B_t^H$ is home intermediary's total holding of Treasury, and $FX_t = \theta_t^{FX} W_t^H$ is the total value of FX forward held by home financial intermediary. If $\omega_{fx} = 0$, the FX swaps do not take any space asset positions (off-balance sheet), this is the standard definition of asset $A_t$; if $\omega_{fx} > 0$, the off-balance sheet FX positions (even if they have 0 present value) also take scarce space on asset books, this is what we define as efficient asset $A_t^{eff}$.

The liability is given

$$L_t = (1 - \chi_t^{H,S^H}) S_t^H + B_t^{repo} \tag{4}$$

where $(1 - \chi_t^{H,H}) S_t^H$ represents the outside claims of home capital issued to foreign investors, and $B_t^{repo}$ is collateralized borrowing backed by Treasury, and we have

$$B_t^{repo} \leq B_t^H \tag{5}$$

We do not allow direct shorting of the Treasury,

$$B_t^H \geq 0 \tag{6}$$

Now we have the net worth of home financial intermediary as

$$W_t^H = A_t - L_t \tag{7}$$

And the HK-style leverage constraint

$$\frac{A_t^{eff}}{W_t^H} \leq \phi_t \tag{8}$$

This $\phi_t$ is the leverage ratio of home financial intermediary. We will have leverage ratio $\phi_t$ goes down (constraint tightens) in crisis and $\omega_{fx}$, the coefficient associated with the FX forward position (off-balance sheet asset), goes up in crisis. [13]

**Country Size** Define relative production size of home country as follows.

$$s_t = \frac{a^{\frac{1}{\eta}} Y_{H,t}^{\frac{\eta-1}{\eta}}}{a^{\frac{1}{\eta}} Y_{H,t}^{\frac{\eta-1}{\eta}} + (1-a)^{\frac{1}{\eta}} Y_{F,t}^{\frac{\eta-1}{\eta}}} = a^{\frac{1}{\eta}} \left( \frac{Y_{H,t}}{\overline{Y_t}} \right)^{\frac{\eta-1}{\eta}} \tag{9}$$

**Optimization problems** The optimization problem for home financial investors is as follows:

$$\begin{aligned}
\max_{\{C_{HH,t}, C_{HF,t}, \chi_t^{H,S^H}, \chi_t^{H,S^F}, \theta_t^{H,B^H}, \theta_t^{H,FX}\}_{t=0}^{\infty}} \quad & \mathbb{E}\left[ \int_0^{\infty} e^{-\rho t} \log \left( \left[ a^{\frac{1}{\eta}} C_{HH,t}^{\frac{\eta-1}{\eta}} + (1-a)^{\frac{1}{\eta}} C_{HF,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \right) dt \right] \\
\text{s.t.} \quad & \frac{dW_t^H}{W_t^H} = \frac{\chi_t^{H,S^H} S_t^H}{W_t^H} dr_t^{S^H} + \frac{\chi_t^{H,S^F} S_t^F}{W_t^H} dr_t^{S^F} + \theta_t^{H,B^H} dr_t^{B^H} + \theta_t^{H,FX} dr_t^{FX} \\
& \qquad - \frac{p_t^H C_{HH,t} + p_t^F C_{FF,t}}{W_t^H} dt \\
& 1 = \frac{\chi_t^{H,S^H} S_t^H}{W_t^H} + \frac{\chi_t^{H,S^F} S_t^F}{W_t^H} + \theta_t^{H,B^H} \\
& S_t^H + \chi_t^{H,S^F} S_t^F + B_t^H + \omega_{fx,t} |FX|_t \leq \phi_t (\chi_t^{H,S^H} S_t^H + \chi_t^{H,S^F} S_t^F - B_t^{repo})
\end{aligned} \tag{10}$$

The optimization problem for foreign financial investors is analogous, with the key difference being that foreign investors do not face the leverage constraint and do not have direct access to U.S. Treasury or repo markets. The complete characterization of the foreign investor's problem is

[13] We take the tightening of leverage ratio $\phi_t$ as exogenous. Our assumption that $\omega_{fx}$ goes up in crisis is an extremely simplified assumption to generate CIP deviations. This can be relaxed in extensions while preserving the core mechanism.

provided in the appendix.

**Market clearing conditions** Home equity market clears,

$$\chi_t^{H,S^H} + \chi_t^{F,S^H} = 1 \tag{11}$$

Foreign equity market clears,

$$\chi_t^{H,S^F} + \chi_t^{F,S^F} = 1 \tag{12}$$

FX market clears,

$$\theta_t^{H,FX} W_t^H + \theta_t^{F,FX} W_t^F = 0 \tag{13}$$

Financial investors' total consumption of home (foreign) good equals $\psi$ share of total production of home (foreign) good, where $\psi \in (0,1)$ represents the fraction of output allocated to financial investors. The remaining fraction $1-\psi$ is consumed by local households who do not participate in international financial markets.

$$C_{HH,t} + C_{FH,t} = \psi Y_{H,t} \tag{14}$$

$$C_{HF,t} + C_{FF,t} = \psi Y_{F,t} \tag{15}$$

# 4 Summary of Theoretical Results

We characterize equilibrium asset prices and portfolio allocations in both normal and crisis regimes. The model generates Treasury convenience yields through the interaction between country size and binding financial constraints, producing testable predictions about cross-asset spillovers and portfolio rebalancing.

## 4.1 Asset Prices and Returns

**Normal Times.** When leverage constraints do not bind, the international financial market is complete and agents from both countries can perfectly share consumption risks.

*UIP deviations.* Sovereign bonds exhibit return differentials reflecting country-size effects

(Proposition 2). The larger country's bond earns lower expected returns, serving as a global safe asset. This safety premium arises because the larger country constitutes most of world consumption risk, making its bond a better hedge for aggregate consumption fluctuations.

*Convenience yield.* U.S. Treasury provides positive convenience yield in normal times (Proposition 3) because it serves as collateral for secured borrowing. The collateral value arises from the constraints $B_t^H \geq 0$ and $B_t^{repo} \leq B_t^H$, which allow Treasuries to back repo borrowing while preventing direct shorting.

**Crisis Times.** When the leverage ratio $\phi_t$ tightens and the weight of FX swaps on the efficient asset book $\omega_{fx,t}$ increases, financial market segmentation occurs.

*Convenience yield increases.* U.S. financial intermediaries face tightened leverage constraints and engage in fire-sales of domestic risky capital to domestic households in exchange for U.S. Treasury (Proposition 6). Holding U.S. Treasury relaxes the binding leverage constraint because Treasuries receive preferential treatment in the constraint ($\omega_{Treasury} < 1$). This generates higher convenience yields in crisis times relative to normal times, with the shadow price of the binding collateral constraint manifesting as the crisis-time convenience yield component.

*CIP deviations.* In crisis, we model extreme balance sheet scarcity by assuming $\omega_{fx,t} \to +\infty$, implying FX swaps take prohibitively large space on the efficient asset book (Proposition 6). As a result, U.S. intermediaries cannot provide market-making for FX swaps, leading to $|FX|_t = 0$. The cost of providing even minimal FX swap services becomes infinite, preventing intermediaries from arbitraging away CIP deviations. This reduced-form approach captures the empirical observation that intermediaries' balance sheets become so scarce during crises that covered interest parity fails. While this assumption is extreme, the core mechanism—balance sheet costs preventing arbitrage—would persist under more general specifications.

*Negative comovement of equity and bond returns.* During crises, U.S. financial intermediaries fire-sell domestic risky capital to households, causing equity prices to collapse (Proposition 8). Households are less efficient at managing risky capital than financial institutions, generating downward price pressure. Simultaneously, U.S. Treasury prices rise as intermediaries purchase and hold Treasuries to satisfy the tightened leverage constraint. The convenience yield of U.S. Treasury increases in crisis, implying even lower returns and higher prices. This generates the

characteristic negative correlation between equity and bond returns observed empirically during crisis periods, with Treasury prices rising precisely when equity markets experience stress.

### 4.2 Portfolio Holdings and Quantities

The model generates rich predictions about how financial intermediaries sequentially adjust their portfolios in response to tightening constraints.

*Equity retrenchment.* When the leverage ratio first tightens ($\phi_t \downarrow$), U.S. financial intermediaries' initial response is to rebalance from foreign risky equity to domestic risky equity (Proposition 4). This portfolio shift allows them to buy back outside equity claims previously issued to foreign investors, thereby reducing their liabilities without triggering costly fire-sales of domestic capital. The equity rebalancing continues until intermediaries exhaust their foreign equity holdings, reaching the corner solution $\chi_t^{H,S^F} = 0$. This first-stage adjustment avoids the price collapse associated with fire-sales by using holdings of foreign risky assets as a buffer.

*Bond retrenchment.* If the leverage ratio tightens severely such that equity rebalancing alone cannot satisfy the constraint ($\phi_t \downarrow\downarrow$), U.S. intermediaries must engage in the costlier second-stage adjustment: fire-sales of domestic risky capital to domestic households (Proposition 5). In exchange for domestic equity, they acquire U.S. Treasury holdings, which receive favorable treatment in the leverage constraint. The fire-sale depresses domestic equity prices (as less-efficient households now manage more capital) while elevating Treasury prices, amplifying the convenience yield. Bond retrenchment continues until the leverage constraint is satisfied, with the fraction $\psi_t$ of domestic capital remaining with intermediaries and fraction $1 - \psi_t$ transferred to households.

This sequential adjustment—equity rebalancing followed by bond retrenchment—generates testable predictions about the timing and magnitude of portfolio flows during crisis episodes, which we validate using international portfolio data.

## 5 Normal times: UIP deviations and convenience yield

When the leverage ratio constraint does not bind, the international financial market for risky claims and FX swaps are complete.

Postulate two stochastic discount factor processes for the two countries, $\xi_{H,t} = e^{-\rho t}\frac{1}{C_{H,t}}$ and $\xi_{F,t} = e^{-\rho t}\frac{1}{C_{F,t}}$, as

$$\frac{d\xi_{H,t}}{\xi_{H,t}} = -r^f_{H,t}\,dt - m^T_{H,t}\,dZ_t$$

$$\frac{d\xi_{F,t}}{\xi_{F,t}} = -r^f_{F,t}\,dt - m^T_{F,t}\,dZ_t$$

respectively. $m_{H,t}$ is the vector of risk prices in home country and also the consumption risk[14] in the logarithmic utility case.

In the complete market case, there exists a unique stochastic discount factor $\xi_t$ such that $\frac{d\xi_{H,t}}{\xi_{H,t}} = \frac{d\xi_{F,t}}{\xi_{F,t}} = \frac{d\xi_t}{\xi_t}$.

**Proposition 1** (UIP deviations)**.** *Expected return difference between home bond and foreign bond is given by*

$$\begin{aligned}\frac{\mathbb{E}_t\left[dr^{FX}_t\right]}{dt} = m^T_{F,t}\sigma^e_t &= \frac{\mathbb{E}_t\left[dr^{B^H}_t - dr^{B^F}_t\right]}{dt} + \lambda_t \\ &= \mu_{p^H,t}t + r^f_t - (\mu_{p^F,t}t + r^f_{F,t}) + \lambda_t \\ &= \mu_{p^H,t}t + r^f_{H,t} - (\mu_{p^F,t}t + r^f_{F,t}) \\ &= m^T_{H,t}\sigma^e_t\end{aligned} \tag{16}$$

*where $dr^{B^H}_t$ is the return process for home bond, $dr^{B^F}_t$ is the implied return process for foreign bond, $\sigma^e_t$ is the exchange rate risk.*

$$dr^{B^F}_t = (\mu_{p^F,t}t + r^f_{F,t})\,dt + \sigma_{p^F,t}\,dZ_t \tag{17}$$

*where $r^f_{F,t}$ is the drift of foreign investors' SDF process.*

*If home country's consumption risk is positively correlated with exchange rate risk (domestic consumption is low when domestic currency depreciates), then home bond earns a positive risk premium.*

*If home country's consumption risk is negatively correlated with its exchange rate (domestic consumption is high when domestic currency depreciates), then home bond earns a negative safety premium.*

[14] Consumption risk of a country is defined as the volatility vector of consumption process of that country, $\frac{dC_{H,t}}{C_{H,t}}$.

*Proof.* see appendix □

The intuition is as follows: A bond is considered safe if it has high value when consumption is low, because the bond insures investors against bad times. In our example, US treasury pays lower expected return than UK government bond if GBP depreciates against USD when consumption is low. Because in this case, US treasury is a good hedge for consumption risk and is viewed as safe, while UK government bond does not hedge consumption risk and is viewed as risky. Lustig and Verdelhan (2007) provides empirical evidence for proposition 1.

In our model, uncertainty comes from production fluctuations of the trees. When UK production declines due to a negative shock, the supply of UK good declines, and the relative price of UK good should go up, implying a higher expected return of UK bond. However, this is not the whole story for bond safety. Because the final consumption is an aggregate of both countries' goods, another competing force emerges: the demand for US good increases because of consumption smoothing motive. The final consumption shifts more towards US good than before due to a supply drop of UK good. This positive demand shock for US good will put upward pressure on the expected return of US bond. The next proposition shows that the relative magnitude of the supply force and demand force is determined by the relative country size.

**Proposition 2** (Country size spillover effect)**.** *Solving for* (16)*, we have*

$$\frac{\mathbb{E}_t\left[dr_t^{FX}\right]}{dt} = \frac{\mathbb{E}_t[dr_t^{B^H} - dr_t^{B^F}]}{dt} = \overline{\sigma}_t^T \sigma_t^e - \lambda_t = \begin{bmatrix} s_t\sigma_H & (1-s_t)\sigma_F \end{bmatrix} \begin{bmatrix} -\frac{1}{\eta}\sigma_H \\ \frac{1}{\eta}\sigma_F \end{bmatrix} \tag{18}$$

$$= \frac{1}{\eta}(-s_t\sigma_H^2 + (1-s_t)\sigma_F^2) - \lambda_t \tag{19}$$

*and the safety threshold*

$$s^C = \frac{\sigma_F^2}{\sigma_H^2 + \sigma_F^2} \tag{20}$$

*If $s_t < s^C$, home country is a relatively small country and home bond is riskier than foreign bond.*

*If $s_t > s^C$, home country is a relatively large country, country bond is safer than foreign bond.*

*Proof.* see appendix □

Country size spillover effect states that larger country's bond is safer. US treasury is safer than UK government bond because the size of US economy is a larger than the size of UK economy. Since US contributes a larger share to world consumption, the world consumption risk also consists largely US risk. US treasury becomes a safe asset and pays lower expected return because it is a better hedge for world consumption risk. This is often referred to as the exorbitant privilege of the US Dollar. Going back to the supply force and demand force discussed earlier: the larger the country's share in the world consumption, the larger the magnitude of supply or demand change of its good. When the small country becomes smaller, the large country's production becomes more dominant in world consumption, strengthening the hedging benefit of the large country's bond. On the other hand, when the small country grows larger, world consumption depends less on the large country's production, reducing the hedging benefit of the large country's bond. Country size spillover effect is stronger when there is more asymmetry in $s_t$ and $1 - s_t$. Expected return differences of sovereign bonds are sizable and persistent across countries as discussed in Hassan (2013), which also provides empirical evidence for country size spillover effect.

**Proposition 3** (U.S. Treasury convenience yield in normal times)**.**

$$\frac{\mathbb{E}_t\left[dr_t^H - r_{H,t}^f\, dt\right]}{dt} = \mu_{p^H,t}t + r_t^f - r_{H,t}^f = m_{H,t}^T \sigma_t^e - \underbrace{\lambda_t}_{\textit{Convenience yield, normal times}} \tag{21}$$

*where $\lambda_t$ is the Lagrangian multiplier associated with the $\theta_t^{H,B^H} \geq 0$, and we have:*

$$\lambda_t = (1-s_t)(\mu_{H,t} - \mu_{F,t}) + \frac{1}{2\eta} s_t(1-s_t)(\sigma_H^2 + \sigma_F^2) - s_t^2\sigma_H^2 - (1-s_t)^2\sigma_F^2 \tag{22}$$

*Under symmetric assumption that $\mu_H = \mu_F = \mu$ and $\sigma_H = \sigma_F = \sigma$, we have:*

$$\lambda_t = (2 - \frac{1}{\eta}) s_t(1-s_t)\sigma^2 \tag{23}$$

*The U.S. Treasury has a positive convenience yield in normal times if $\eta > \frac{1}{2}$.*

# 6 Crisis Regime

We model the crisis time as an exogenous tightening of the leverage constraint: In crisis, the leverage ratio $\phi_t$ goes down, and the weight of FX swap on the efficient asset book $\omega_{fx,t}$ goes up.

## 6.1 Retrenchment

**A relatively moderate tightening:** $\phi_t \downarrow$, but no fire-sale of capital, and thus net holding of US Treasury $\theta_t^{H,B^H} = 0$. In this case, we have efficient asset book:

$$A_t^{eff} = S_t^H + \chi_t^{H,S^F} S_t^f + \omega_{fx,t}|FX|_t \tag{24}$$

and net worth of the intermediary:

$$W_t^H = \chi_t^{H,S^H} S_t^H + \chi_t^{H,S^F} S_t^F \tag{25}$$

The leverage constraint:

$$\frac{A_t^{eff}}{W_t^H} = \frac{S_t^H + \chi_t^{H,S^F} S_t^f + \omega_{fx,t}|FX|_t}{\chi_t^{H,S^H} S_t^H + \chi_t^{H,S^F} S_t^F} \leq \phi_t \tag{26}$$

Rearranging, we have

$$(\phi_t \chi_t^{H,S^H} - 1) S_t^H + (\phi_t - 1)\chi_t^{H,S^F} S_t^F - \omega_{fx,t}|FX|_t \geq 0 \tag{27}$$

Now if $\phi_t \downarrow$, such that $\phi_t \chi_t^{H,S^H} - 1 < 0$, and this constraint binds, the U.S. financial intermediary can rebalance between domestic risky claims and foreign risky claims. Because this reduce the outside equity issuance (liability) of the intermediary and thus relax the leverage constraint. The maximum they can rebalance is given by

$$\Delta\chi_t \, S_t^H = \chi_t^{H,S^F} S_t^F \tag{28}$$

and after re-balancing, we have that

$$\overline{\chi}_t^{H,S^H} = \chi_t^{H,S^H} + \Delta\chi_t = \chi_t^{H,S^H} + \frac{\chi_t^{H,S^F} S_t^F}{S_t^H} \tag{29}$$

and

$$\underline{\chi}_t^{H,S^F} = 0, \quad \underline{\theta}_t^{H,S^F} = 0 \tag{30}$$

**Proposition 4** (Equity retrenchment)**.** *When the leverage constraint tightens moderately such that* $\phi_t \chi_t^{H,S^H} - 1 < 0$ *but equity rebalancing can still satisfy the constraint, U.S. financial intermediaries optimally rebalance from foreign equity to domestic equity. The intermediary sells all foreign equity holdings (*$\chi_t^{H,S^F} \to 0$*) and increases domestic equity holdings by* $\Delta\chi_t = \chi_t^{H,S^F} S_t^F / S_t^H$*, reaching post-rebalancing domestic equity share:*

$$\overline{\chi}_t^{H,S^H} = \chi_t^{H,S^H} + \frac{\chi_t^{H,S^F} S_t^F}{S_t^H} \tag{31}$$

*This rebalancing reduces outside equity issuance (liabilities) without requiring fire-sales, thereby avoiding price collapse of domestic capital.*

**A severe tightening:** $\phi_t \downarrow\downarrow$, such that $\phi_t \overline{\chi}_t^{H,S^H} - 1 < 0$, and that the equity rebalancing is not enough to relax the leverage constraint. The fire-sale of domestic risky capital starts, U.S. financial intermediary sell $S_t^H$ to domestic households to scale down and hold positive amount of US Treasury. In this case, we have efficient asset book:

$$A_t^{eff} = \psi_t S_t^H + \omega_{Treasury} B_t^H + \omega_{fx,t} |FX|_t \tag{32}$$

where $\psi_t$ is the fraction of domestic risky capital $S_t^H$ still held by U.S. financial intertmediary and thus $1 - \psi_t$ is the fraction sold to the U.S. households. And $\omega_{Treasury}$ is the weight of U.S. Treasury on the efficient asset book. If $\omega_{Treasury} = 1$, the U.S. Treasury takes the same space on efficient asset book as risky assets. If $\omega_{Treasury} < 1$, the U.S. Treasury has specialty over risky assets in terms of relaxing the leverage constraint. And the net worth of the intermediary:

$$W_t^H = \overline{\chi}_t^{H,S^H} \psi_t S_t^H + B_t^H \tag{33}$$

where

$$B_t^H = (1 - \psi_t) S_t^H \tag{34}$$

The leverage constraint:

$$\frac{A_t^{eff}}{W_t^H} = \frac{\psi_t S_t^H + \omega_{Treasury} B_t^H + \omega_{fx,t} |FX|_t}{\overline{\chi}_t^{H,S^H} \psi_t S_t^H + B_t^H} \leq \phi_t \tag{35}$$

Rearranging, we have

$$\psi_t (\phi_t \overline{\chi}_t^{H,S^H} - 1) S_t^H + (\phi_t - \omega_{Treasury}) B_t^H - \omega_{fx,t} |FX|_t \geq 0 \tag{36}$$

Since the U.S. financial intermediary is the market maker for FX market, when the leverage constraint binds, we can express their positions of FX swaps/forward as:

$$|FX|_t = \frac{(\phi_t - \omega_{Treasury}) B_t^H - \psi_t (1 - \phi_t \overline{\chi}_t^{H,S^H}) S_t^H}{\omega_{fx,t}} \tag{37}$$

From equation (37), we can see that the cost of accommodating/arbitraging FX swaps is to fire sell more domestic risky capital and hold more U.S. Treasury.

To make our life easier, in this basic version, we also make the assumption that $\omega_{fx,t} \to +\infty$ in crisis. As a result, the U.S. financial intermediary stops making the FX market $|FX|_t = 0$ since the cost is infinity, which generates CIP deviations in crisis. We will relax this constraint in later version of the model, but the mechanism and intuition stays the same.

**Proposition 5** (Bond retrenchment)**.** *When the leverage constraint tightens severely such that* $\phi_t \overline{\chi}_t^{H,S^H} - 1 < 0$ *and equity rebalancing alone cannot satisfy the constraint, U.S. financial intermediaries engage in fire-sales of domestic capital to households. The intermediary reduces its domestic equity holdings from* $S_t^H$ *to* $\psi_t S_t^H$ *and acquires U.S. Treasury holdings* $B_t^H = (1 - \psi_t) S_t^H$*. The fraction* $\psi_t$ *is determined by the binding leverage constraint:*

$$\psi_t (\phi_t \overline{\chi}_t^{H,S^H} - 1) S_t^H + (\phi_t - \omega_{Treasury}) B_t^H - \omega_{fx,t} |FX|_t = 0 \tag{38}$$

*U.S. Treasury receives preferential treatment in the constraint (*$\omega_{Treasury} < 1$*), incentivizing inter-*

*mediaries to hold Treasuries despite lower returns. This fire-sale depresses domestic equity prices (as less-efficient households now manage capital) while elevating Treasury prices, amplifying the convenience yield.*

## 6.2 Convenience yield in crisis times

**Proposition 6** (U.S. Treasury convenience yield in crisis time)**.**

$$\frac{\mathbb{E}_t\left[dr_t^H - r_{H,t}^f\, dt\right]}{dt} = \mu_{p^H,t}t + r_t^f - r_{H,t}^f = m_{H,t}^T\sigma_t^e - \underbrace{\mu_t}_{\textit{Convenience yield, crisis}} \tag{39}$$

*where $\mu_t$ is the Lagrangian multiplier associated with the normalized leverage constraint (the coefficient of U.S. Treasury holding is nomalized to* 1*: $\psi_t \frac{\phi_t \overline{\chi}_t^{H,S^H} - 1}{\phi_t - \omega_{Treasury}} S_t^H + B_t^H - \frac{\omega_{fx,t}}{\phi_t - \omega_{Treasury}} |FX|_t \geq 0$), and we have:*

$$\mu_t = \frac{\overline{\chi}_t^{H,S^H}(\phi_t - \omega_{Treasury})}{1 - \phi_t\omega_{Treasury}}(1 - \theta_t^{H,S^H})\sigma_H^2 \tag{40}$$

*where $\theta_t^{H,S^H}$ is U.S. financial intermediary's portfolio share of domestic risky capital $S_t^H$, and $\sigma_H$ is the volatility of domestic risky capital.*

## 6.3 CIP deviations

Moving on to prices and returns in crisis, we have both rising convenience yield and CIP deviations.

**Proposition 7** (CIP deviations)**.**

$$\begin{aligned}\frac{\mathbb{E}_t\left[dr_t^{FX}\right]}{dt} &= \frac{\mathbb{E}_t\left[dr_t^{B^H} - dr_t^{B^F}\right]}{dt} + \mu_t + (m_{H,t}^T - m_{F,t}^T)\sigma_{p^H,t} \\ &= \mu_{p^H,t}t + r_{H,t}^f - (\mu_{p^F,t}t + r_{F,t}^f) + (m_{H,t}^T - m_{F,t}^T)\sigma_{p^H,t} \\ &= m_{F,t}^T\sigma_t^e = m_{H,t}^T\sigma_t^e - \kappa_t\end{aligned} \tag{41}$$

*Proof.* see appendix. □

### 6.4 Equity risk-premium increases in crisis

**Proposition 8** (Negative comovement of equity and bond returns in crisis)**.** *In crisis regime when U.S. financial intermediaries engage in fire-sales of domestic capital (Proposition 5), domestic equity returns and U.S. Treasury returns exhibit negative comovement. Specifically:*

- ***Equity returns fall:*** *Fire-sales transfer domestic capital from efficient financial intermediaries to less-efficient households, causing equity prices $q_t^H$ to decline and expected equity returns $\mathbb{E}_t[dr_t^{S^H}]/dt$ to rise.*
- ***Treasury returns fall (prices rise):*** *Increased intermediary demand for U.S. Treasury as a constraint-relaxing asset causes Treasury prices to rise. The convenience yield component $\mu_t$ (Proposition 6) increases, lowering expected Treasury returns $\mathbb{E}_t[dr_t^{B^H}]/dt$.*

*The correlation between domestic equity returns and Treasury returns becomes negative:*

$$Corr_t\left(dr_t^{S^H}, dr_t^{B^H}\right) < 0 \quad \textit{in crisis regime} \tag{42}$$

*This negative correlation arises endogenously from the binding leverage constraint forcing simultaneous equity fire-sales and Treasury accumulation, creating the flight-to-safety pattern observed empirically.*

## 7 Conclusion

This paper develops a two-country continuous-time model where a leverage constraint on U.S. financial intermediaries generates endogenous Treasury convenience yields through asymmetric asset treatment. When constraints tighten in crisis times, different assets consume different amounts of scarce balance sheet space: U.S. Treasuries receive preferential treatment while FX derivatives become prohibitively expensive. This asymmetry, combined with country size effects, creates the observed spillover from equity market stress to bond market convenience yields.

Our model delivers three main insights. First, convenience yields arise from Treasuries' collateral value in normal times and from the shadow price of binding constraints in crisis times, generating state-dependent sensitivity to financial stress. Second, sequential portfolio adjustment occurs

as constraints tighten: equity retrenchment (both countries increase domestic equity holdings, but U.S. magnitude is larger due to country size), then bond retrenchment (only U.S. retrenches to Treasuries due to their preferential balance sheet treatment). This explains both magnitude and directional asymmetries in retrenchment patterns. Third, negative stock-bond comovement emerges endogenously as binding constraints force simultaneous equity fire-sales and Treasury accumulation.

The model provides theoretical foundations for empirical regularities we document: state-dependent CIP sensitivity to stress, asymmetric equity retrenchment magnitudes, asymmetric bond retrenchment directions, and negative crisis-time stock-bond correlation. Our mechanism—a single leverage constraint with time-varying tightness and asset-specific weights—distinguishes this framework from alternatives based on multiple separate constraints or exogenous safe-asset preferences.

# References


Viral V. Acharya and Toomas Laarits. When do treasuries earn the convenience yield? — a hedging perspective. *Working paper*, (31863), 2025.

Tobias Adrian, Erkko Etula, and Tyler Muir. Financial intermediaries and the cross-section of asset returns. *Journal of Finance*, 69(6):2557–2596, 2014.

Paula Beltran and Chang He. Inelastic financial markets and implications for foreign exchange interventions. *Working paper*, 2025.

Fernando Broner, Tatiana Didier, Aitor Erce, and Sergio L. Schmukler. Gross capital flows: Dynamics and crises. *Journal of Monetary Economics*, 60(1):113–133, 2013.

Markus K Brunnermeier and Yuliy Sannikov. A macroeconomic model with a financial sector. *American Economic Review*, 104(2):379–421, 2014.

Ricardo J. Caballero, Emmanuel Farhi, and Pierre-Olivier Gourinchas. The safe assets shortage conundrum. *Journal of Economic Perspectives*, 31(3):29–46, 2017. 10.1257/jep.31.3.29.

Nelson Camanho, Harald Hau, and Hélène Rey. Global portfolio rebalancing and exchange rates. *Review of Financial Studies*, 35(11):5228–5274, 2022.

Eugenio M. Cerutti, Maurice Obstfeld, and Haonan Zhou. Covered interest parity deviations: Macrofinancial determinants. *Journal of International Economics*, 130:103447, 2021.

John H. Cochrane, Francis A. Longstaff, and Pedro Santa-Clara. Two Trees. *The Review of Financial Studies*, 21(1):347–385, 11 2007. ISSN 0893-9454.

Nicolas Coeurdacier and Helene Rey. Home bias in open economy financial macroeconomics. *Journal of Economic Literature*, 51(1):63–115, 2013.

Riccardo Colacito and Marc F. Croce. Purchases or sales: What do international investors really exchange? *American Economic Review*, 101(7):2797–2824, 2011. 10.1257/aer.101.7.2797.

Douglas W. Diamond and Peter Van Tassel. Bond market segmentation and safe asset pricing. Working paper, 2024.

Wenxin Du and Jesse Schreger. Cip deviations, the dollar, and frictions in international capital markets. Technical report, National Bureau of Economic Research, 2021.

Wenxin Du, Joanne Im, and Jesse Schreger. The u.s. treasury premium. *Journal of International Economics*, 112:167–181, 2018a.

Wenxin Du, Alexander Tepper, and Adrien Verdelhan. Deviations from covered interest rate parity. *The Journal of Finance*, 73(3):915–957, 2018b.

Kristin J. Forbes and Francis E. Warnock. Capital flow waves: Surges, stops, flight, and retrenchment. *Journal of International Economics*, 88(2):235–251, 2012.

Xavier Gabaix and Matteo Maggiori. International Liquidity and Exchange Rate Dynamics *. *The Quarterly Journal of Economics*, 130(3):1369–1420, 03 2015. ISSN 0033-5533. 10.1093/qje/qjv016.

Pierre-Olivier Gourinchas and Helene Rey. International financial adjustment. *Journal of political economy*, 115(4):665–703, 2007.

Tarek Hassan. Country size, currency unions, and international asset returns. *The Journal of Finance*, 68(6):2269–2308, 2013.

Harald Hau and Hélène Rey. Exchange rates, equity prices, and capital flows. *Review of Financial Studies*, 19(1):273–317, 2006.

Zhiguo He and Arvind Krishnamurthy. Intermediary asset pricing. *American Economic Review*, 103(2):732–770, 2013.

Zhiguo He, Bryan Kelly, and Asaf Manela. Intermediary asset pricing: New evidence from many asset classes. *Journal of Financial Economics*, 126(1):1–35, 2017.

Zhiguo He, Arvind Krishnamurthy, and Konstantin Milbradt. A model of safe asset determination. *American Economic Review*, 109(4):1230–1262, 2019. 10.1257/aer.20160216.

Oleg Itskhoki and Dmitry Mukhin. Exchange rate disconnect in general equilibrium. *Journal of Political Economy*, 129(8):000–000, 2021.

Zhengyang Jiang, Arvind Krishnamurthy, and Hanno Lustig. Dollar safety and the global financial cycle. *Quarterly Journal of Economics*, 136(4):2013–2061, 2021.

Zhengyang Jiang, Arvind Krishnamurthy, Hanno Lustig, and Meng Sun. Safe asset demand and exchange rates. Working paper, 2024.

Rohan Kekre and Moritz Lenel. The flight to safety and international risk sharing. *American Economic Review*, 114(6):1650–1691, 2024.

Ralph SJ Koijen and Motohiro Yogo. Exchange rates and asset prices in a global demand system. Technical report, National Bureau of Economic Research, 2020.

Arvind Krishnamurthy and Annette Vissing-Jorgensen. The aggregate demand for treasury debt. *Journal of Political Economy*, 120(2):233–267, 2012.

Andrew Lilley, Matteo Maggiori, Brent Neiman, and Jesse Schreger. Exchange rate reconnect. *Review of Economics and Statistics*, 104(4):845–855, 2022. 10.1162/rest_a_00978.

Hanno Lustig and Adrien Verdelhan. The cross section of foreign currency risk premia and consumption growth risk. *American Economic Review*, 97(1):89–117, 2007.

Matteo Maggiori. Financial intermediation, international risk sharing, and reserve currencies. *American Economic Review*, 107(10):3038–71, 2017.

Ian Martin. The forward premium puzzle in a two-country world. Working Paper 17564, National Bureau of Economic Research, November 2011.

Anna Pavlova and Roberto Rigobon. Asset Prices and Exchange Rates. *The Review of Financial Studies*, 20(4):1139–1180, 01 2007. ISSN 0893-9454.

Anna Pavlova and Roberto Rigobon. The role of portfolio constraints in the international propagation of shocks. *The Review of Economic Studies*, 75(4):1215–1256, 2008.

# Appendix

## A Additional Proofs and Derivations

Trade market: Notice that international trade is a static problem for both countries. Since there is no friction in the international trade market and homogeneous preferences, we have that

$$\frac{C_{HH,t}}{C_{HF,t}} = \frac{C_{FH,t}}{C_{FF,t}} = \frac{Y_{H,t}}{Y_{F,t}} \tag{43}$$

Using market clearing condition for home good and foreign good,

$$C_{HH,t} + C_{FH,t} = Y_{H,t}$$
$$C_{HF,t} + C_{FF,t} = Y_{F,t}$$

we have

$$C_{H,t} + C_{F,t} = \overline{Y}_t$$

where

$$C_{H,t} = \left[ a^{\frac{1}{\eta}} C_{HH,t}^{\frac{\eta-1}{\eta}} + (1-a)^{\frac{1}{\eta}} C_{HF,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$
$$C_{F,t} = \left[ a^{\frac{1}{\eta}} C_{FH,t}^{\frac{\eta-1}{\eta}} + (1-a)^{\frac{1}{\eta}} C_{FF,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$
$$\overline{Y}_t = \left[ a^{\frac{1}{\eta}} Y_{H,t}^{\frac{\eta-1}{\eta}} + (1-a)^{\frac{1}{\eta}} Y_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

As a result, the prices of the two goods produced by the two trees are given by

$$p_t^H = \left( \frac{Y_{H,t}}{a\overline{Y}_t} \right)^{-\frac{1}{\eta}} \quad \text{and} \quad p_t^F = \left( \frac{Y_{F,t}}{(1-a)\overline{Y}_t} \right)^{-\frac{1}{\eta}} \tag{44}$$

We have that

$$C_{H,t} = p_t^H C_{HH,t} + p_t^F C_{HF,t}$$
$$C_{F,t} = p_t^H C_{FH,t} + p_t^F C_{FF,t}$$

Recall that country size if defined as follows

$$s_t = \frac{a^{\frac{1}{\eta}} Y_{H,t}^{\frac{\eta-1}{\eta}}}{a^{\frac{1}{\eta}} Y_{H,t}^{\frac{\eta-1}{\eta}} + (1-a)^{\frac{1}{\eta}} Y_{F,t}^{\frac{\eta-1}{\eta}}} = a^{\frac{1}{\eta}} \left( \frac{Y_{H,t}}{\overline{Y_t}} \right)^{\frac{\eta-1}{\eta}}$$

as home country's share of the world total output, i.e. the *country size* of home country, which will turn out to be an important state variable. We have that

$$p_t^H Y_{H,t} = s_t \overline{Y_t} \quad \text{and} \quad p_t^F Y_{F,t} = (1-s_t)\overline{Y_t}$$

The aggregate wealth of home country is

$$W_t^H = \chi_t^{H,H} S_t^H + \chi_t^{H,F} S_t^F + p_t^H B_t^{H,H} + p_t^F B_t^{H,F}$$

The aggregate wealth of foreign country is

$$W_t^F = \chi_t^{F,H} S_t^H + \chi_t^{F,F} S_t^F + p_t^H B_t^{F,H} + p_t^F B_t^{F,F}$$

The optimization problem for home country is as follows:

$$\begin{aligned}
\max_{\{C_{HH,t}, C_{HF,t}, \chi_t^{H,H}, \chi_t^{H,f}, \theta_t^{H,B^H}, \theta_t^{H,B^F}\}_{t=0}^{\infty}} \quad & \mathbb{E}\left[ \int_0^\infty e^{-\rho t} \log\left( \left[ a^{\frac{1}{\eta}} C_{HH,t}^{\frac{\eta-1}{\eta}} + (1-a)^{\frac{1}{\eta}} C_{HF,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \right) dt \right] \\
\text{s.t.} \quad & \frac{dW_t^H}{W_t^H} = \frac{\chi_t^{H,H} S_t^H}{W_t^H} dr_t^{S^H} + \frac{\chi_t^{H,F} S_t^F}{W_t^H} dr_t^{S^F} + \theta_t^{H,B^H} dr_t^{B^H} + \theta_t^{H,B^F} dr_t^{B^F} \\
& \qquad - \frac{p_t^H C_{HH,t} + p_t^F C_{FF,t}}{W_t^H} dt \\
& 1 = \frac{\chi_t^{H,H} S_t^H}{W_t^H} + \frac{\chi_t^{H,F} S_t^F}{W_t^H} + \theta_t^{H,B^H} + \theta_t^{H,B^F} \\
& 0 \le \chi_t^{H,H} \le 1 \\
& 0 \le \chi_t^{H,F} \le \overline{\chi}^{H,F} \\
& 0 = p_t^H B_t^{H,H} + p_t^F B_t^{H,F}
\end{aligned} \tag{45}$$

Define home country's wealth share as

$$n_t = \frac{W_t^H}{W_t^H + W_t^F} \tag{46}$$

**Lemma 1.** *Home country's wealth share* $n_t = \frac{W_t^H}{W_t^H + W_t^F}$ *is a function of country size* $s_t$*. And we always have that*

$$p_t^H B_t^{H,H} + p_t^F B_t^{H,F} = 0 \tag{47}$$

*Proof.* From optimization problems, we have that

$$C_{H,t} = \rho W_t^H \quad \text{and} \quad C_{F,t} = \rho W_t^H$$

In the aggregate, we have that total consumption equals total output

$$C_{HH,t} + C_{FH,t} = Y_{H,t}$$
$$C_{HF,t} + C_{FF,t} = Y_{F,t}$$

And using the result from trade market optimization, we have

$$C_{H,t} + C_{F,t} = \overline{Y_t}$$

That is,

$$\rho W_t^H + \rho W_t^F = \overline{Y_t}$$

The total wealth in the world is given by

$$W_t^H + W_t^F = \frac{\overline{Y_t}}{\rho}$$

There are two state variables, wealth share $n_t$ and country size $s_t$. In equilibrium, all the prices and quantities must be functions of state variables $n_t$ and $s_t$. We can rewrite home country's wealth as

$$p_t^H B_t^{H,H} + p_t^F B_t^{H,F} = [n_t - (\chi_t^{H,H} q_t^H + \chi_t^{H,F} q_t^F)]\overline{Y_t}$$

where $q_t^H = \frac{S_t^H}{\overline{Y}_t}$ and $q_t^F = \frac{S_t^F}{\overline{Y}_t}$ are per unit price of home and foreign equity, respectively. Recall that

$$p_t^H Y_{H,t} = s_t \overline{Y}_t \quad \text{and} \quad p_t^F Y_{F,t} = (1 - s_t)\overline{Y}_t$$

we have

$$p_t^H = a^{\frac{1}{\eta-1}} s_t^{-\frac{1}{\eta-1}} \quad \text{and} \quad p_t^F = (1-a)^{\frac{1}{\eta-1}} (1 - s_t)^{-\frac{1}{\eta-1}}$$

And in equilibrium $\theta_t^{H,B^H} = \frac{p_t^H B_t^{H,H}}{W_t^H}$, $\theta_t^{H,B^F} = \frac{p_t^F B_t^{H,F}}{W_t^H}$, $q_t^H$, $q_t^F$, $\chi_t^{H,H}$ and $\chi_t^{H,F}$ must be functions of state variables $n_t$ and $s_t$. We have that

$$n_t = \frac{\chi_t^{H,H} q_t^H + \chi_t^{H,F} q_t^F}{1 - (\theta_t^{H,B^H} + \theta_t^{H,B^F})} \equiv f(s_t, n_t) \tag{48}$$

Equation (48) is an implicit function and we can solve for $n_t$ as a function of $s_t$.

Now we have that in equilibrium $\theta_t^{H,B^H} = \frac{p_t^H B_t^{H,H}}{W_t^H}$, $\theta_t^{H,B^F} = \frac{p_t^F B_t^{H,F}}{W_t^H}$, $q_t^H$, $q_t^F$, $\chi_t^{H,H}$ and $\chi_t^{H,F}$ must be functions of the *only* state variable $s_t$.

Recall the dynamic budget constraint of home country,

$$\begin{aligned}\frac{dW_t^H}{W_t^H} = &\frac{\chi_t^{H,H} S_t^H}{W_t^H} dr_t^{S^H} + \frac{\chi_t^{H,F} S_t^F}{W_t^H} dr_t^{S^F} + \theta_t^{H,B^H} dr_t^{B^H} + \theta_t^{H,B^F} dr_t^{B^F} \\ &- \frac{p_t^H C_{HH,t} + p_t^F C_{FF,t}}{W_t^H} dt\end{aligned}$$

And the asset return processes,

$$dr_t^{S^H} = \underbrace{\frac{p_t^H Y_{H,t}}{q_t^H \overline{Y}_t} dt}_{\text{dividend yield}} + \underbrace{\frac{d(q_t^H \overline{Y}_t)}{q_t^H \overline{Y}_t}}_{\text{capital gain}} = \mu_t^{S^H} dt + (\frac{(q^H)'(s_t) s_t}{q_t^H} \sigma_{s_t} + \overline{\sigma}_t) dZ_t$$

and similarly

$$dr_t^{S^F} = \underbrace{\frac{p_t^F Y_{F,t}}{q_t^F \overline{Y}_t} dt}_{\text{dividend yield}} + \underbrace{\frac{d(q_t^F \overline{Y}_t)}{q_t^F \overline{Y}_t}}_{\text{capital gain}} = \mu_t^{S^F} dt + (\frac{(q^F)'(s_t) s_t}{q_t^F} \sigma_{s_t} + \overline{\sigma}_t) dZ_t$$

$$dr_t^{B^H} = \frac{d(p_t^H B_t^H)}{p_{H,t} B_t^H} = (\mu_{p^H,t} + r_t^H)\, dt + \sigma_{p^H,t}\, dZ_t$$

$$dr_t^{B^F} = \frac{d(p_t^F B_t^F)}{p_{F,t} B_t^F} = (\mu_{p^F,t} + r_t^F)\, dt + \sigma_{p^F,t}\, dZ_t$$

And we have

$$\sigma_{p_t^H} = -\frac{1}{\eta - 1}\sigma_{s_t}$$

$$\sigma_{p_t^F} = \frac{s_t}{(\eta - 1)(1 - s_t)}\sigma_{s_t}$$

Since $\rho W_t^H = n_t \overline{Y_t}$, we have

$$\frac{dW_t^H}{W_t^H} = \mu_t^{W^H} dt + (\frac{n'(s_t)s_t}{n_t}\sigma_{s_t} + \overline{\sigma}_t)\, dZ_t$$

Note that

$$\overline{\sigma}_t = [s_t \sigma^H, (1 - s_t)\sigma^F]$$

$$\sigma_{s_t} = \frac{\eta - 1}{\eta}(1 - s_t)[\sigma^H, -\sigma^F]$$

are *linearly independent* for non-degenerate $s_t$. Matching terms for $\overline{\sigma}_t$, we must have that

$$\frac{\chi_t^{H,H} S_t^H}{W_t^H} + \frac{\chi_t^{H,F} S_t^F}{W_t^H} = 1 \tag{49}$$

That is

$$p_t^H B_t^{H,H} + p_t^F B_t^{H,F} = 0 \tag{50}$$

□

**Corollary 2.** *The total capital flow of home country induced by equity trade is given by*

$$\begin{aligned} dQ_t^H = {} & S_t^H d\chi_t^{H,H} - \chi_t^{H,H}(p_t^H Y_{H,t})dt + S_t^F d\chi_t^{H,F} - \chi_t^{H,F}(p_t^F Y_{F,t})dt \\ & + d\chi_t^{H,H} dS^H + d\chi_t^{H,F} dS_t^F \end{aligned} \tag{51}$$

*And such capital flow must be financed and absorbed by trading in bonds and consumption goods*

$$dQ_t^H = (\theta_t^{B^{H,H}} dr_t^{B^H} + \theta_t^{B^{H,F}} dr_t^{B^F}) W_t^H - (p_t^H C_{HH,t} + p_t^F C_{HF,t})dt \tag{52}$$

*Similarly the total capital flow of foreign country induced by equity trade is given by*

$$\begin{aligned} dQ_t^F = & S_t^H d\chi_t^{F,H} - \chi_t^{F,H}(p_t^H Y_{H,t})dt + S_t^F d\chi_t^{F,F} - \chi_t^{F,F}(p_t^F Y_{F,t})dt \\ & + d\chi_t^{F,H} dS^H + d\chi_t^{F,F} dS_t^F \end{aligned} \tag{53}$$

*and such capital flow must be financed and absorbed by trading in bonds and consumption goods*

$$dQ_t^F = (\theta_t^{B^{F,H}} dr_t^{B^H} + \theta_t^{B^{F,F}} dr_t^{B^F}) W_t^F - (p_t^H C_{FH,t} + p_t^F C_{FF,t})dt \tag{54}$$

*Proof.* This follows from Lemma 1. Since we have

$$W_t^H = \chi_t^{H,H} S_t^H + \chi_t^{H,F} S_t^F$$

Taking total differentiation on both sides, we have that

$$dW_t^H = \chi_t^{H,H} S_t^H \, dr_t^{S^H} + \chi_t^{H,F} S_t^F \, dr_t^{S^F} + dQ_t^H \tag{55}$$

Combining with dynamic budget constraint

$$\begin{aligned} \frac{dW_t^H}{W_t^H} = & \frac{\chi_t^{H,H} S_t^H}{W_t^H} dr_t^{S^H} + \frac{\chi_t^{H,F} S_t^F}{W_t^H} dr_t^{S^F} + \theta_t^{H,B^H} dr_t^{B^H} + \theta_t^{H,B^F} dr_t^{B^F} \\ & - \frac{p_t^H C_{HH,t} + p_t^F C_{FF,t}}{W_t^H} dt \end{aligned}$$

We have that

$$dQ_t^H = (\theta_t^{B^{H,H}} dr_t^{B^H} + \theta_t^{B^{H,F}} dr_t^{B^F}) W_t^H - (p_t^H C_{HH,t} + p_t^F C_{HF,t})dt$$

Similar proof for foreign country. □

**Lemma 3.** *In crisis region $s_t \in [0, s^u]$, we have that*

$$\chi_t^{H,H} = 1, \quad \chi_t^{H,F} = \overline{\chi}^{H,F}$$

$$p_t^H B_t^{H,H} = p_t^F B_t^{H,F} = 0$$

*Proof.* At the crisis region boundary $s_t = s^u$, we have $\chi_t^{H,H} = 1$ and $\chi_t^{H,F} = \overline{\chi}^{H,F}$,

$$dQ_t^H = S_t^H d\chi_t^{H,H} - (p_t^H Y_{H,t})dt + S_t^F d\chi_t^{H,F} - \overline{\chi}^{H,F}(p_t^F Y_{F,t})dt + d\chi_t^{H,H} dS^H + d\chi_t^{H,F} dS_t^F$$

For *any* realization of $ds_t < 0$ at $s_t = s^u$, it must be that $d\chi_t^{H,H} \leq 0$ and $d\chi_t^{H,F} \leq 0$. To satisfy this, we must have $d\chi_t^{H,H}$ and $d\chi_t^{H,F}$ are deterministic for any realization of $ds_t < 0$ (thus, any $s < s^u$) at $s_t = s^u$ .

Collecting terms for $\sigma_{s_t}$ and $\overline{\sigma}_t$, we have

$$dQ_t^H - \mathbb{E}[dQ_t^H] = [(S_t^H d\chi_t^{H,H} + S_t^F d\chi_t^{H,F})\overline{\sigma}_t + (S_t^H d\chi_t^{H,H} \frac{(q^H)'(s_t)s_t}{q_t^H} + S_t^F d\chi_t^{H,F} \frac{(q^F)'(s_t)s_t}{q_t^F})\sigma_{s_t})]\, dZ_t$$

On the other side, we have

$$dQ_t^H = (\theta_t^{B^{H,H}} dr_t^{B^H} + \theta_t^{B^{H,F}} dr_t^{B^F})W_t^H - (p_t^H C_{HH,t} + p_t^F C_{HF,t})dt$$

which only consists of $\sigma_{s_t}$ risk. As a result of matching terms for $\overline{\sigma}_t$, we have

$$S_t^H d\chi_t^{H,H} + S_t^F d\chi_t^{H,F} = 0$$

That is, $d\chi_t^{H,H} = d\chi_t^{H,F} = 0$. It follows that matching terms for $\sigma_{s_t}$ on both sides should also be $0$, and we have

$$-\theta_t^{B^{H,H}} + \frac{s_t}{1 - s_t}\theta_t^{B^{H,F}} = 0$$

Combining with

$$\theta_t^{B^{H,H}} + \theta_t^{B^{H,F}} = 0$$

We have that in crisis region,

$$\theta_t^{B^{H,H}} = \theta_t^{B^{H,F}} = 0$$

□

While in crisis when $s_t \in [0, s^U]$, $P_t^H = (1 - \overline{\chi}^{H,F})s_t + \overline{\chi}^{H,F}$ and $P_t^F = (1 - \overline{\chi}^{H,F})(1 - s_t)$.

Because the two countries have different consumption prices which are non-degenerate stochastic processes, the same (real) bond corresponds to different return processes in the two countries.

That is, real bond returns bear consumption price risks and real bonds can not help overcome consumption price deviations. As a result, any real bond will not be traded in the constrained equilibrium [15]. The equity holding constraints creates financial friction that can not be overcome by sovereign bonds.

**With one equity constraint** Recall the wealth of home country and its evolution

$$W_t^H = \chi_t^{H,H} S_t^H + \chi_t^{H,F} S_t^F + p_t^H B_t^{H,H} + p_t^F B_t^{F,F} \tag{56}$$

$$\frac{dW_t^H}{W_t^H} = \frac{\rho \chi_t^{H,H} q_t^H}{n_t} dr_t^{S^H} + \frac{\rho \chi_t^{H,F} q_t^F}{n_t} dr_t^{S^F} + \frac{p_t^H B_{1t}}{W_t^H} dr_t^{B_1} + \frac{p_t^F B_{2t}}{W_t^H} dr_t^{B_2} - \rho dt \tag{57}$$

Denote

$$\frac{dW_t^H}{W_t^H} = \mu_{W_t^H}\, dt + \sigma_{W_t^H}\, dZ_t \tag{58}$$

$$\frac{dW_t^F}{W_t^F} = \mu_{W_t^F}\, dt + \sigma_{W_t^F}\, dZ_t \tag{59}$$

$$\frac{dn_t}{n_t} = \mu_{n_t} dt + \sigma_{n_t} dZ_t \tag{60}$$

There are two risks in this world: the aggregate consumption risk, $\overline{\sigma}_t$, and the distribution risk, $\sigma_{s_t}$. Since there are four financial assets, there is some redundancy. With only one equity constraint, the two countries can still perfectly share consumption risk. So we have

$$\sigma_{n_t} = \sigma_{1-n_t} = 0 \tag{61}$$

and

$$\sigma_{W_t^H} = \sigma_{W_t^F} = \overline{\sigma}_t \tag{62}$$

To find the portfolio weights on each asset, we have

$$\overline{\sigma}_t = \frac{\rho \chi_t^{H,H} q_t^H}{n_t} (\sigma_{q_t^H} + \overline{\sigma}_t) + \frac{\rho \chi_t^{H,F} q_t^F}{n_t} (\sigma_{q_t^F} + \overline{\sigma}_t) + \frac{p_t^H B_t^{H,H}}{W_t^H} \sigma_{p_t^H} + \frac{p_t^F B_t^{H,F}}{W_t^H} \sigma_{p_t^F} \tag{63}$$

[15] The symmetric setting in discount rate and preferences matters. (conjecture)

Now we need to find $\sigma_{q_t^H}$. In the complete market case, we have

$$q_t^H = \mathbb{E}_t\Big[\int_0^\infty e^{-\rho\tau} s_\tau d\tau\Big] \tag{64}$$

Using Ito's lemma we have

$$\sigma_{q_t^H} = \frac{(q^H)'(s_t)s_t}{q_t^H}\sigma_{s_t} \tag{65}$$

and since

$$q_t^F = \frac{1}{\rho} - q_t^H \tag{66}$$

we have

$$\sigma_{q_t^F} = -\frac{q_t^H}{q_t^F}\sigma_{q_t^H} = -\frac{(q^H)'(s_t)s_t}{q_t^F}\sigma_{s_t} \tag{67}$$

And we have

$$\sigma_{p_t^H} = -\frac{1}{\eta - 1}\sigma_{s_t} \tag{68}$$

$$\sigma_{p_t^F} = \frac{s_t}{(\eta-1)(1-s_t)}\sigma_{s_t} \tag{69}$$

Note that

$$\overline{\sigma}_t^T = [s_t\sigma_1, (1-s_t)\sigma_2] \tag{70}$$

$$\sigma_{s_t}^T = \frac{\eta-1}{\eta}(1-s_t)[\sigma_1, -\sigma_2] \tag{71}$$

are *linearly independent*. Now coming back to the risk of home country's wealth (63) and matching $\overline{\sigma}_t$ term, we have

$$\frac{\rho\chi_t^{H,H}q_t^H}{n_t} + \frac{\rho\chi_t^{H,F}q_t^F}{n_t} = 1 \tag{72}$$

That is

$$W_t^H = \chi_t^{H,F}S_t^H + \chi_t^{H,F}S_t^F \tag{73}$$

and thus

$$p_t^H B_t^{H,H} + p_t^F B_t^{H,F} = 0 \tag{74}$$

To determine portfolio weights on bonds, we match $\sigma_{s_t}$ terms in home country's wealth

$$\frac{\rho\chi_t^{H,H}q_t^H}{n_t}\frac{(q^H)'(\frac{s_t}{n_t})s_t}{q_t^H}+\frac{\rho\chi_t^{H,F}q_t^F}{n_t}(-\frac{(q^H)'(s_t)s_t}{q_t^F})+\frac{p_t^H B_t^{H,H}}{W_t^H}(-\frac{1}{\eta-1}-\frac{s_t}{(\eta-1)(1-s_t)})=0 \tag{75}$$

Simplified to

$$\frac{\rho(q^H)'(s_t)s_t(\chi_t^{H,H}-\chi_t^{H,F})}{n_t}-\frac{p_t^H B_t^{H,H}}{W_t^H}\frac{1}{(1-s_t)(\eta-1)}=0 \tag{76}$$

So we have

$$\frac{p_t^H B_t^{H,H}}{W_t^H}=\frac{\rho\chi_t^{H,H}(q^H)'(s_t)s_t(1-s_t)(\chi_t^{H,H}-\chi_t^{H,F})(\eta-1)}{n_t}>0 \tag{77}$$

and

$$\frac{p_t^F B_t^{H,F}}{W_t^H}=-\frac{p_t^H B_t^{H,H}}{W_t^H}<0 \tag{78}$$

We can also find the drift of home country wealth share $n_t$ by looking at the drift term of the wealth. Using market clearing condition $B_t^{H,H}=-B_t^{F,H}$ and $B_t^{H,F}=-B_t^{F,F}$,

$$\mu_{W_t^F}=-\frac{n_t}{1-n_t}\mu_{n_t}+\overline{\mu}_t \tag{79}$$

$$=\frac{\mathbb{E}_t[dr_t^{S^F}]}{dt}-\frac{n_t}{1-n_t}\frac{p_t^H B_t^{H,H}}{W_t^H}m_t\sigma_{e_t}^T-\rho \tag{80}$$

and we have that

$$\mu_{q_t^H}=\rho-\frac{s_t}{q_t^H}+\mu_{n_t}-\sigma_{n_t}^2+\sigma_{q_t^H}\sigma_{n_t} \tag{81}$$

and $\sigma_{n_t}=0$,

$$\mu_{q_t^H}=\rho-\frac{s_t}{q_t^H}+\mu_{n_t} \tag{82}$$

and thus

$$\frac{\mathbb{E}_t[dr_t^{S^F}]}{dt}=\frac{1-s_t}{q_t^F}+(-\frac{q_t^H}{q_t^F}\mu_{q_t^H}+\overline{\mu}_t+\sigma_{q_t^F}\overline{\sigma}_t^T) \tag{83}$$

Substituting into (79), and solving for $\mu_{n_t}$, we have

$$\mu_{n_t}=\frac{\frac{q_t^H}{q_t^F}\sigma_{q_t^H}\overline{\sigma}_t^T+\frac{n_t\theta_t^{H,B^H}}{1-n_t}\sigma_{e_t}\overline{\sigma}_t^T}{\frac{n_t}{1-n_t}-\frac{q_t^H}{q_t^F}}=0 \tag{84}$$

as

$$\frac{q_t^H}{q_t^F}\sigma_{q_t^H}\overline{\sigma}_t^T = -\frac{(q^H)'(\frac{s_t}{n_t})s_t(1-s_t)(\eta-1)}{q_t^F}\sigma_{e_t}\overline{\sigma}_t^T = \frac{n_t\theta_t^{H,B^H}}{1-n_t}\sigma_{e_t}\overline{\sigma}_t^T \tag{85}$$

**With two equity holding constraints** The second step is to explore what will happen with two equity holding constraints. Now the two countries can not always perfectly share consumption risk and the wealth shares are not always constant.

Full model special case: with two equity holding constraints $0 \leq \chi_t^{H,F} \leq \overline{\chi}^{H,F}$, $0 \leq \chi_t^{F,H} \leq \overline{\chi}^{F,H}$, and symmetric parameters $\sigma_1 = \sigma_2 = \sigma$ there are three safety thresholds $s^c$, $s^u$ and $s^a$, $n_0 = \frac{1}{2}$:

$$s^c = \frac{1}{2} \tag{86}$$

$$q_1(s^u) = \frac{1-2\overline{\chi}^{H,F}}{2\rho(1-\overline{\chi}^{H,F})} \tag{87}$$

and

$$0 < s^a < s^u \tag{88}$$

with parameter restrictions on $(\overline{\chi}^{H,F}, \eta)$. The equity shares are

$$\chi_{1t} = 1 \tag{89}$$

and

$$\chi_t^{H,F} = \begin{cases} \overline{\chi}^{H,F} & if \quad s_t < s^U(\overline{\chi}^{H,F}) \\ \frac{1-2\rho q_t^H}{2(1-\rho q_t^H)} & if \quad s^U(\overline{\chi}^{H,F}) < s_t < \frac{1}{2} \\ 0 & if \quad s_t > \frac{1}{2} \end{cases} \tag{90}$$

bond holdings are (in equilibrium)

$$\theta_t^{H,B^H} = \theta_t^{H,B^F} = \theta_t^{F,B^H} = \theta_t^{F,B^F} = 0 \tag{91}$$

where

$$\frac{q_t^{H,H}}{n_t} = \mathbb{E}_t\Big[\int_0^\infty e^{-\rho\tau}\frac{s_\tau}{n_\tau}d\tau\Big] \tag{92}$$

*Proof.* The equity holding constraint binds when $s_t < s^u$. There are no cross-border bond trading. The implied bond returns (within domestic country) are given as follows

$$\frac{\mathbb{E}_t[dr_t^{B^{H,H}}]}{dt} - r_t^{H,f} = m_{1t}^T \sigma_{p_t^H} = (\sigma_{n_t} + \overline{\sigma}_t)\sigma_{p_t^H} \tag{93}$$

$$\frac{\mathbb{E}_t[dr_t^{B^{H,F}}]}{dt} - r_t^{H,f} = m_{1t}^T \sigma_{p_t^F} = (\sigma_{n_t} + \overline{\sigma}_t)\sigma_{p_t^F} \tag{94}$$

$$\frac{\mathbb{E}_t[dr_t^{B^{F,H}}]}{dt} - r_t^{F,f} = m_{2t}^T \sigma_{p_t^H} = (-\frac{n_t}{1-n_t}\sigma_{n_t} + \overline{\sigma}_t)\sigma_{p_t^H} \tag{95}$$

$$\frac{\mathbb{E}_t[dr_t^{B^{F,F}}]}{dt} - r_t^{F,f} = m_{2t}^T \sigma_{p_t^F} = (-\frac{n_t}{1-n_t}\sigma_{n_t} + \overline{\sigma}_t)\sigma_{p_t^F} \tag{96}$$

where $-r_t^{H,f}$ and $-m_{1t}$ are the drift and volatility of home country's SDF and similarly $-r_t^{F,f}$ and $-m_{2t}$ are for foreign country.

$$r_t^{H,f} = \rho + \mu_{n_t} + \overline{\mu}_t + \sigma_{n_t}\overline{\sigma}_t - (\sigma_{n_t} + \overline{\sigma}_t)^2 \tag{97}$$

$$r_t^{F,f} = \rho - \frac{n_t}{1-n_t}\mu_{n_t} + \overline{\mu}_t - \frac{n_t}{1-n_t}\sigma_{n_t}\overline{\sigma}_t - (-\frac{n_t}{1-n_t}\sigma_{n_t} + \overline{\sigma}_t)^2 \tag{98}$$

$$m_{1t} = \sigma_{n_t} + \overline{\sigma}_t \tag{99}$$

$$m_{2t} = \sigma_{1-n_t} + \overline{\sigma}_t \tag{100}$$

When $s_t < s(\overline{\chi}^{H,F})$, we can write out Country 1 and Country 2's wealth as

$$W_t^H = S_t^{H,F} + \overline{\chi}^{H,F} S_t^{H,F} \tag{101}$$

$$W_t^F = (1 - \overline{\chi}^{H,F}) S_t^{F,F} \tag{102}$$

And from the optimization of logarithmic utility, we have

$$\rho W_t^H = C_{H,t} = n_t \overline{Y}_t \tag{103}$$

$$\rho W_t^F = C_{F,t} = (1 - n_t)\overline{Y}_t \tag{104}$$

Looking at the volatility of $W_t^H$,

$$\sigma_{W_t^H} = \sigma_{n_t} + \overline{\sigma}_t \tag{105}$$

$$= \frac{\rho q_t^{H,H}}{n_t}(\sigma_{q_t^{H,H}} + \overline{\sigma}_t) + \frac{\rho \overline{\chi}^{H,F} q_t^{H,F}}{n_t}(\sigma_{q^{H,F}} + \overline{\sigma}_t) \tag{106}$$

and the volatility of $W_t^F$,

$$\sigma_{W_t^F} = -\frac{n_t}{1-n_t}\sigma_{n_t} + \overline{\sigma}_t \tag{107}$$

$$= \frac{\rho(1-\overline{\chi}^{H,F}) q_t^{F,F}}{1-n_t}(\sigma_{q_t^{F,F}} + \overline{\sigma}_t) \tag{108}$$

For both countries, portfolio weights add up to 1

$$\frac{\rho q_t^{H,H}}{n_t} + \frac{\rho \overline{\chi}^{H,F} q_t^{H,F}}{n_t} = 1 \tag{109}$$

$$\frac{\rho(1-\overline{\chi}^{H,F}) q_t^{F,F}}{1-n_t} = 1 \tag{110}$$

so we have

$$\frac{\rho q_t^{H}}{n_t} + \frac{\rho \overline{\chi}^{H,F} q_t^{H,F}}{n_t} + \frac{\rho(1-\overline{\chi}^{H,F}) q_t^{F,F}}{1-n_t} = 2 \tag{111}$$

where

$$\frac{q_t^{H,H}}{n_t} = \mathbb{E}_t\Big[\int_0^\infty e^{-\rho\tau}\frac{s_\tau}{n_\tau}d\tau\Big] \tag{112}$$

$$\frac{q_t^{H,F}}{n_t} = \mathbb{E}_t\Big[\int_0^\infty e^{-\rho\tau}\frac{1-s_\tau}{n_\tau}d\tau\Big] \tag{113}$$

$$\frac{q_t^{F,F}}{1-n_t} = \mathbb{E}_t\Big[\int_0^\infty e^{-\rho\tau}\frac{1-s_\tau}{1-n_\tau}d\tau\Big] \tag{114}$$

That is,

$$\mathbb{E}_t\Big[\int_0^\infty e^{-\rho\tau}(\frac{s_\tau + \overline{\chi}^{H,F}(1-s_\tau)}{n_\tau} + \frac{(1-\overline{\chi}^{H,F})(1-s_\tau)}{1-n_\tau})d\tau\Big] = \frac{2}{\rho} \tag{115}$$

Using Feynman-Kac formula, we have

$$\frac{s_t + \overline{\chi}^{H,F}(1-s_t)}{n_t} + \frac{(1-\overline{\chi}^{H,F})(1-s_t)}{1-n_t} = 2 \tag{116}$$

So $n_t$ is a function of the only state variable $s_t$ (the other solution $n_t = \frac{1}{2}$ is not achievable with equity constraint binding).

$$n_t = (1 - \overline{\chi}^{H,F})s_t + \overline{\chi}^{H,F} \tag{117}$$

$$1 - n_t = (1 - \overline{\chi}^{H,F})(1 - s_t) \tag{118}$$

$$\theta_t^{H,B^H} = \theta_t^{H,B^F} = 0 \tag{119}$$

And also we have

$$\frac{C_{H,t}}{C_{F,t}} = \frac{n_t}{1 - n_t} \tag{120}$$

Now we can solve for prices of risks,

$$m_{1t} = \sigma_{n_t} + \overline{\sigma}_t = \frac{(1 - \overline{\chi}^{H,F})s_t}{(1 - \overline{\chi}^{H,F})s_t + \overline{\chi}^{H,F}}\sigma_{s_t} + \overline{\sigma}_t \tag{121}$$

and solve for $s^a$ using

$$\frac{\mathbb{E}_t[dr_t^{B^{H,H}} - dr_t^{B^{H,F}}]}{dt} = m_{1t}(\sigma_{p_t^H} - \sigma_{p_t^F}) < 0 \tag{122}$$

we have

$$\frac{2(1 - \overline{\chi}^{H,F})s^2 + (2\overline{\chi}^{H,F}\eta + (1 - \overline{\chi}^{H,F})(\eta - 2))s - \eta\overline{\chi}^{H,F}}{(1 - \overline{\chi}^{H,F})s_t + \overline{\chi}^{H,F}} > 0 \tag{123}$$

For $0 < s^a < s^u$, we need the right range for parameter pair $(\overline{\chi}^{H,F}, \eta)$. For example, if $\overline{\chi}^{H,F} = 0$, we have $s^u = \frac{1}{2}$ and $s^a = \frac{2-\eta}{2}$. And if $\eta = \infty$, we have $s^a = \frac{\overline{\chi}^{H,F}}{1+\overline{\chi}^{H,F}}$.

With a special parameter case, $\rho = (\frac{\eta-1}{\eta}\sigma)^2$, we can solve everything we need analytically. The price of equity 1 in unconstrained case is

$$q_t^H(s) = \frac{1}{2\rho}\Big(1 + \frac{1-s}{s}\ln(1-s) - \frac{s}{1-s}\ln(s)\Big) \tag{124}$$

$$q_{1t}'(s) = -\frac{1}{2\rho}\frac{1}{s(1-s)}\Big(1 + \frac{1-s}{s}\ln(1-s) + \frac{s}{1-s}\ln(s)\Big) \tag{125}$$

$$q_{1t}''(s) = -\frac{1}{2\rho}\frac{1}{s^2(1-s)^2}\Big((2s-1) - \frac{(1-s)^2}{s}\ln(1-s) + \frac{s^2}{1-s}\ln(s)\Big) \tag{126}$$

□