Who Built the Service Economy?

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Abstract

We document a new fact about U.S. structural transformation: the reallocation of employment from manufacturing to services is not primarily driven by entry or young firms. Instead, it is dominated by the expansion of large service incumbents. Decomposing sectoral employment changes by firm age, size, and establishment growth reveals that incumbent scaling along the extensive margin accounts for the bulk of service-sector expansion. Standard firm dynamics models can replicate aggregate sectoral employment shares, but they fail on the underlying micro margins, overstating the role of entry. We develop a quantitative firm dynamics model with customer capital in order to replicate the joint behavior of sectoral reallocation, firm-size and establishment dynamics. We find that policies that operate through entry (e.g., startup subsidies) have limited aggregate impact, whereas policies that affect incumbent expansion operate on the quantitatively dominant margin.

Keywords: Firm dynamics, structural transformation, labor reallocation, entry

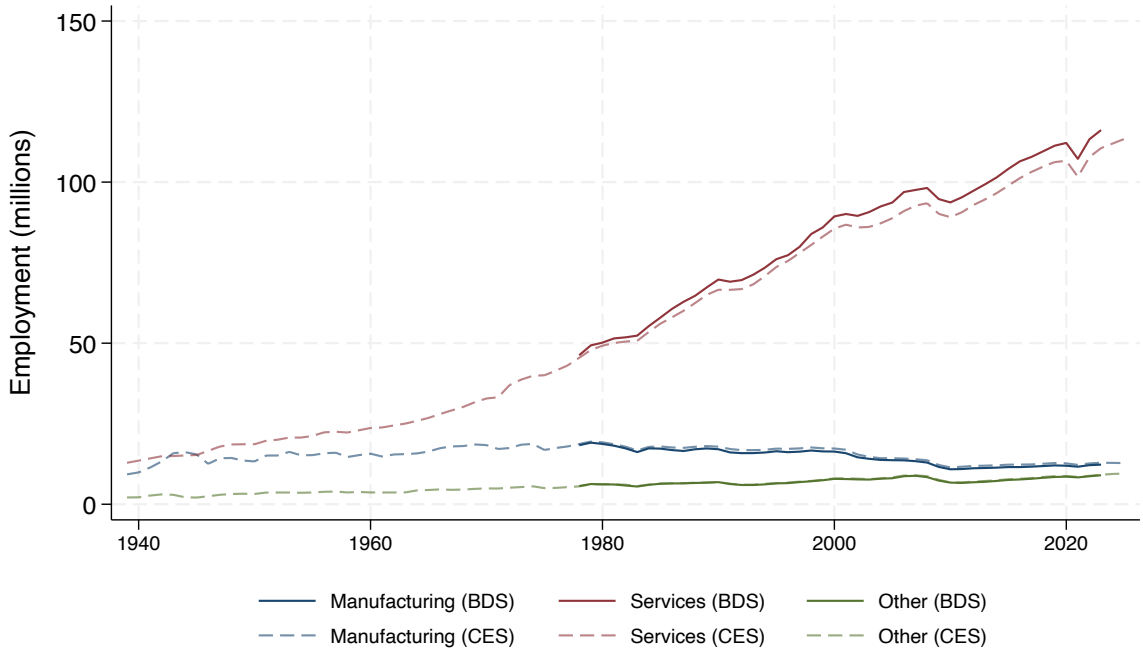
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1 Introduction

Structural transformation is a defining feature of modern growth. As shown in Figure 1, there has been a steady reallocation of labor away from manufacturing and toward services, which accelerated around the 1980s. Relative to the beginning of the sample period, service employment has increased by a factor nine, while manufacturing employment has only increased by roughly 50 percent. This implies that the service sector's share of total employment has increased from 50 to over 80 percent, while that of manufacturing has fallen from 40 to 10 percent.

FIGURE 1: SECTORAL EMPLOYMENT



Notes: Figure 1 shows the evolution of manufacturing and service employment. The definitions are shown in Appendix A. The BLS-CES data begins coverage in 1939. The Census BDS data provides measures of business dynamics beginning in 1978. Data source: BDS, BLS-CES.

How did this large reallocation of labor come about? Was it driven by entry of new service-sector firms, by the decline and exit of manufacturing incumbents, or by differential growth patterns among surviving firms? Did structural transformation primarily reflect the rise of small, young service firms, or the expansion of large, established incumbents? To what extent did changes in firm size distributions, rather than within-firm employment growth, account for the observed shift in aggregate employment? More broadly, what role do firm entry, exit, and scaling play in

shaping long-run sectoral employment dynamics? In this paper, we take a firm-dynamics perspective to answer these questions, focusing on the relative contributions of entry, exit, and incumbent growth to structural transformation.

The trends shown in Figure 1 are well documented. However, much of the structural transformation literature treats sectors as aggregates—often implicitly as collections of representative firms—and therefore abstracts from the underlying firm dynamics. As a result, it typically does not distinguish whether sectoral reallocation reflects net entry and exit, differential growth among incumbents, or changes in the size distribution of firms within sectors. This omission is surprising in light of a large empirical literature showing that business dynamism—entry, exit, and reallocation across firms—is central to both gross and net job creation. In particular, entrepreneurship and startup activity are key objects, and a substantial body of evidence documents that firm turnover and heterogeneous firm growth account for a large share of employment dynamics in the aggregate economy (Davis et al., 1998; Haltiwanger et al., 2013).

We find that structural transformation is primarily an incumbent scaling phenomenon: service employment growth is driven mainly by large, mature firms expanding through additional establishments, while manufacturing employment declines largely through contraction and reallocation among large incumbents rather than through a collapse in the number of firms.

The main database we use in this paper is the Census Business Dynamics Statistics (BDS). It provides annual measures of business dynamics, including job creation and destruction, firm startups and shutdowns, and is broken down by establishment and firm characteristics. The BDS data begins coverage in 1978. The aggregate employment trends mirror very closely the data by the Bureau of Labor Statistics (BLS), as is shown in Figure 1, where the BLS data is plotted as the dashed lines that extend back to the late 1930s.

We proceed in four steps. First, we document that entry and exit rates differ across sectors but do not map cleanly into employment reallocation. Second, we decompose sectoral employment changes into net entry, net incumbent expansion, and establishment churn within incumbent firms, and show that service-sector growth is dominated by incumbent expansion rather than net entry. Third, we show that these patterns are concentrated among large firms. Finally, we distinguish extensive-margin scaling (more establishments per firm) from intensive-margin scaling (employment per establishment), and show that service-sector incumbents expand primarily through the

extensive margin.

Related literature The reallocation of aggregate employment labor toward services has been well documented, important examples include Ngai and Pissarides (2007), Buera and Kaboski (2012), and Herrendorf et al. (2014). These papers provide alternative explanations for the large employment reallocation toward services, ranging from uneven sectoral productivity growth (Ngai and Pissarides, 2007; Herrendorf et al., 2015), to shifts in demand toward skill-intensive services as a result of increased gains from specialization (Buera and Kaboski, 2012). We differ from these papers in that we account for firm heterogeneity, entry, and exit.

Two closely related papers are Dent et al. (2016) and Hopenhayn et al. (2025), who quantitatively evaluate the ability of population growth, sectoral technology, and international trade to replicate sectoral firm dynamics along with the observed reallocation away from manufacturing.

Our paper also relates to Hsieh and Rossi-Hansberg (2023) who show that service firms operate in growing numbers of local markets and to Autor et al. (2020) who document an increase in concentration. Jarmin et al. (2009) document a rise in employment in large retail establishments.

Fort et al. (2018) and Hyun et al. (2025) find that within firms that have at least one manufacturing establishment there has been an increase in service employment and a decrease in manufacturing employment. This implies that firms might create more service establishments and that is where employment is growing.

2 Data and Measurement

Our empirical analysis is based on the Business Dynamics Statistics (BDS) housed by the U.S. Census Bureau. The BDS are constructed from the Longitudinal Business Database (LBD), the most comprehensive longitudinally linked dataset covering the universe of U.S. establishments with at least one paid employee. The BDS span the period 1978–2023 and report a wide range of establishment- and firm-level statistics across industries, sectors, and geographic regions.

Throughout the paper, we adopt the definitions of establishments and firms used in the BDS. An establishment is a single physical location at which business is conducted, whereas a firm is a collection of establishments under common ownership. In the BDS, a firm entry is recorded when

a new firm is created consisting entirely of age-0 establishments, and a firm exit is recorded when all of a firm's establishments—regardless of changes in ownership over time—cease operations and exit.

Definition of manufacturing and services sectors. The manufacturing sector spans NAICS industries 31–33. The service sector is comprised of multiple industries, including wholesale and retail trade, transportation, information, FIRE (finance, insurance, and real estate), professional services, and educational services among others. The remaining industries—such as agriculture, mining, and construction—are classified as "other". Table 3 in the Appendix provides the mapping between NAICS two-digit industries and these broad sector categories.

3 Empirical results

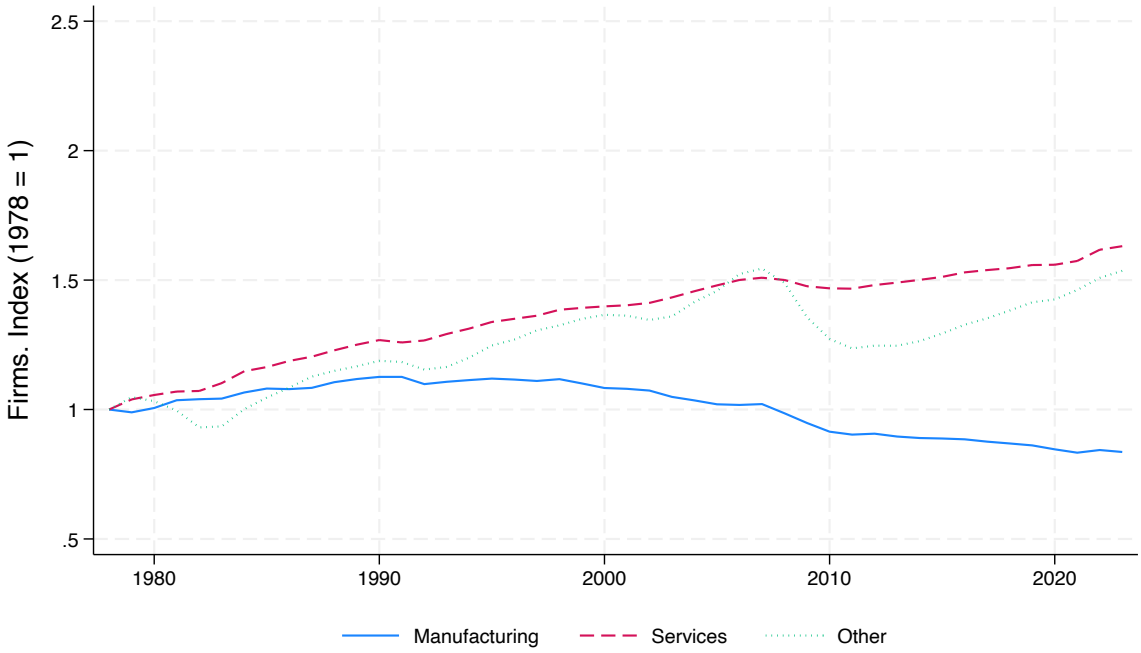
The central motivation for this paper is to uncover the firm dynamics that lie behind the shift toward services that was shown in Figure 1. In this section we use the BDS data to show that, to our surprise, net entry contributes very little to overall employment growth, particularly for the service sector. The key factor behind the employment growth patterns is the behavior of large firms. While in services, the key mechanism is extensive-margin scaling (the birth of new, large establishments by incumbent firms), the experience in the manufacturing sector is characterized by intensive-margin shrinkage, i.e., the decline of large, incumbent manufacturing firms. The remainder of this section shows the details.

3.1 Number of firms

Given that employment in services has grown by a factor of 2.5 since the late 1970s, one might expect to see a similar increase in the number of service firms. Figure 2 shows that this is not the case. The number of service firms 'only' increased by a total of 63%, suggesting that the trend in the extensive margin of firms is much less extreme than that of total employment. Turning to manufacturing, total employment declined by over 30% since the late 1970s, yet the number

of manufacturing firms shows a hump-shaped pattern, with an eventual decline of 16%.[1] These results suggest important changes in average firm size in both the manufacturing and the service sector, a point we come back to further below.

FIGURE 2: NUMBER OF FIRMS



Notes: Figure 2 shows the evolution of the number of firms in manufacturing and services. Series are normalized to one in 1978. Data source: BDS.

The patterns shown in Figure 2 indicate that viewing sectors as collections of representative firms is misleading. Aggregate employment reallocation may reflect firm selection through entry and exit, changes in the size distribution of firms, or both. To understand which of these margins is quantitatively relevant, it is necessary to examine firm dynamics in more detail.

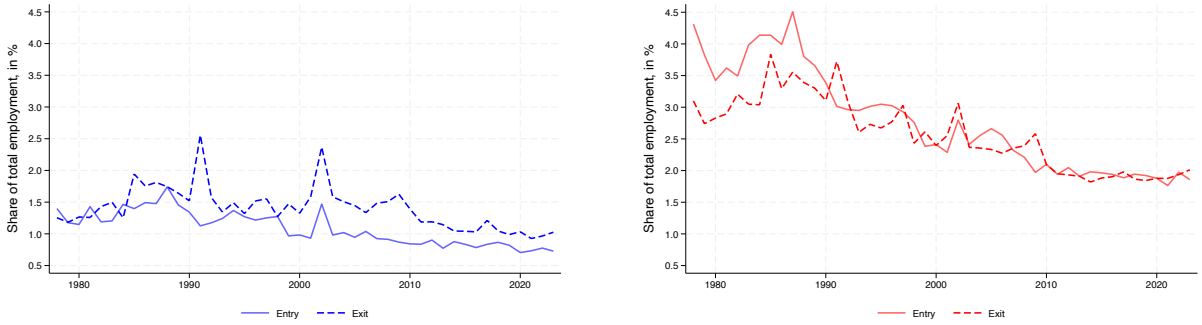
3.2 Entry and exit

A natural starting point is the employment contribution of entry and exit. Firm counts may understate the role of entry and exit if employment is generated by many small firms entering each

[1] In absolute terms, the number of service firms increased from 2.9 million to 4.7 million, while the number of manufacturing firms decreased from .26 million to .22 million. The results for the number of establishments are very similar. Figure 12 in the Appendix shows the evolution of average firm sizes. Figure 11 in the Appendix shows sector level employment relative to the first year of the BDS data.

period, even though a similar number of firms exit so that the total number of firms remains relatively stable or only increases modestly. In particular, even if firms enter small, a high volume of entry could generate substantial employment at the time of birth. To assess the quantitative importance of this margin, we examine the employment share of entering and exiting firms directly. This is shown in Figure 3, which shows the share of total sectoral employment accounted for by entering and exiting firms.

FIGURE 3: THE ROLE OF ENTRY AND EXIT



(A) Manufacturing (B) Services

Notes: Notes: Figure 3 plots the employment share of entrants (defined as total employment of entering firms as a fraction of total sectoral employment) and the employment share of exiting firms (defined as total employment of exiting firms as a fraction of total lagged sectoral employment). Data source: BDS.

Panel a) shows the employment share of entering and exiting firms for the manufacturing. Panel b) shows results for the service sector. There are three takeaways: 1) The share of total sector employment that is accounted for by entering and exiting firms has declined throughout the sample period. 2) The importance of gross entry and exit for employment growth is significantly smaller in manufacturing. 3) Manufacturing has seen an employment loss from net entry to the order of about .3% of sector employment on average per year. No similar pattern is discernible for the service sector.[2]

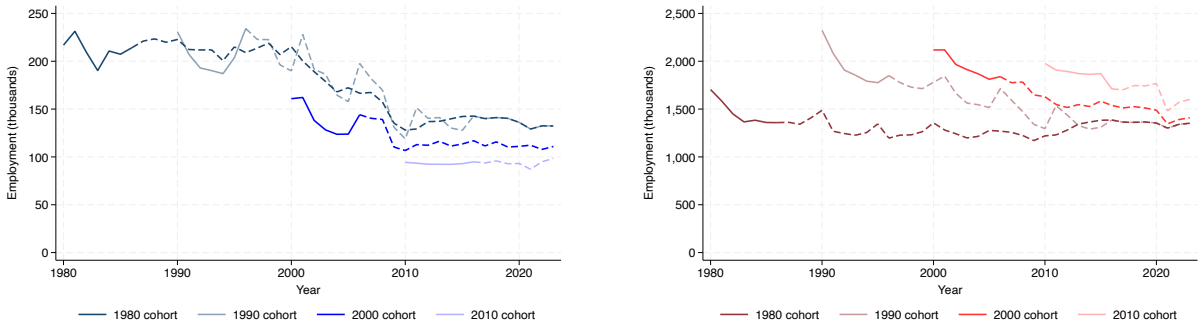
While a large literature documents a secular decline in firm entry rates, the evidence here shows that service-sector employment has expanded substantially despite a steadily diminishing contemporaneous contribution of employment at entry. By contrast, net firm entry has played

[2]The sectoral entry and exit rates (as fractions of all firms) are shown in Figure 13 in the Appendix. Both in manufacturing and in services, firm entry rates have fallen over most of the sample period, while starting to increase again since around 2010. Exit rates show no discernible pattern. The service sector exhibits higher gross turnover than manufacturing.

a more visible role for manufacturing, where exit contributed meaningfully to the employment decline. Taken together, these patterns indicate that, by itself, contemporaneous net entry cannot account for the expansion of service employment.

Post-entry dynamics A natural objection to our conclusion regarding entry and exit rates could be be that the analysis focused on employment at the time of firm entry, which may understate the role of entry if firms enter small but subsequently expand. In that view, employment reallocation occurs through post-entry growth rather than contemporaneous job creation at birth. To address this concern, we adopt a cohort-based perspective that tracks firms after entry and measures a cohort's employment over time. The results are shown in Figure 4. For each sector, we measure employment at time t accounted for by firms that entered in year $c \leq t$. The figure shows results for $c = \{1980, 1990, 2000, 2010\}$, but the results generalize to other cohorts as well.

FIGURE 4: COHORT EMPLOYMENT OVER TIME



(A) Manufacturing (B) Services

Notes: Figure 4 shows total employment for selected cohorts in the manufacturing and service sector. The solid line segments are based on data on ages 0–5. The dashed line segments are created using average employment in age bins. Data source: BDS.

In the manufacturing sector, cohort sizes have declined over time. The post-entry dynamics show that cohort employment tends to decline as the cohort ages, driven by the exit of firms. In the service sector, while there is no clear trend in the size of entering cohorts, cohort-level employment remains relatively stable over time. Given that the employment share of entrants is around 3% (*cf.* Figure 3), we conclude that even by taking post-entry dynamics into account, entrants only contribute marginally to aggregate employment growth, even at longer horizons.

The limited role of entry cohorts shifts attention toward the behavior of continuing firms, sug-

gesting that service-sector growth must primarily reflect scaling by incumbents rather than the cumulative impact of successive waves of entrants. This is what we turn to next.

3.3 Incumbent firms

Given the limited role of firm entry for service-sector employment growth, understanding structural transformation requires a closer examination of employment dynamics among continuing firms. Employment reallocation among these firms can occur along several distinct dimensions. First, employment may shift across firms of different sizes. Second, size dynamics may reflect differences across firms at different stages of the life cycle, making it important to distinguish size effects from age effects. Third, employment at continuing firms may adjust through changes at existing establishments or through the opening and closing of establishments by multi-unit firms. The analysis below documents the quantitative importance of these dimensions for understanding sectoral employment reallocation.

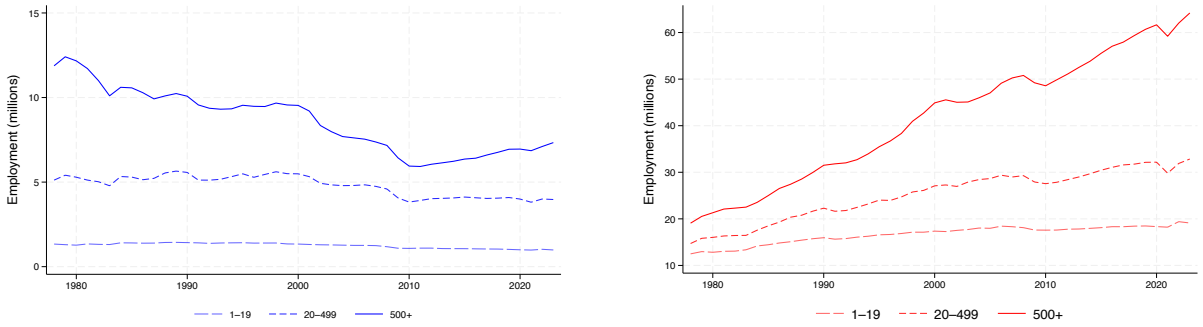
The growing role of large firms We define small firms as those with 1–19 employees, medium firms as those with 20–499 employees, and large firms as those with 500 or more employees. Figure 5 shows each sector's employment by firm size group.[3] In manufacturing, employment in large firms accounts for a vast majority of the secular decline, having fallen by about five million employees, or about 40%. Employment in small and medium firms is comparatively stable. In services, the expansion of sectoral employment is primarily the result of growth in employment at large firms, whose total employment has more than tripled. There is more muted growth among medium firms and the lowest growth among small firms. Today, more than 55% of all service sector employment is concentrated in the largest firms. This share was close to 40% at the beginning of the sample.

Taken together, Figure 5 implies that the key firm-dynamics margin behind structural transformation is not a shift toward more small firms, but the changing scale of large incumbents.

This fact is also reflected in entry rates: Figure 6 shows that the number of new firms has decreased nearly uniformly across firm sizes in the manufacturing sector. In services, however,

[3] In Figure 14 in the Appendix we show that using initial firm size, rather than current firm size, leads to virtually unchanged results.

FIGURE 5: SECTORAL EMPLOYMENT BY SIZE GROUP

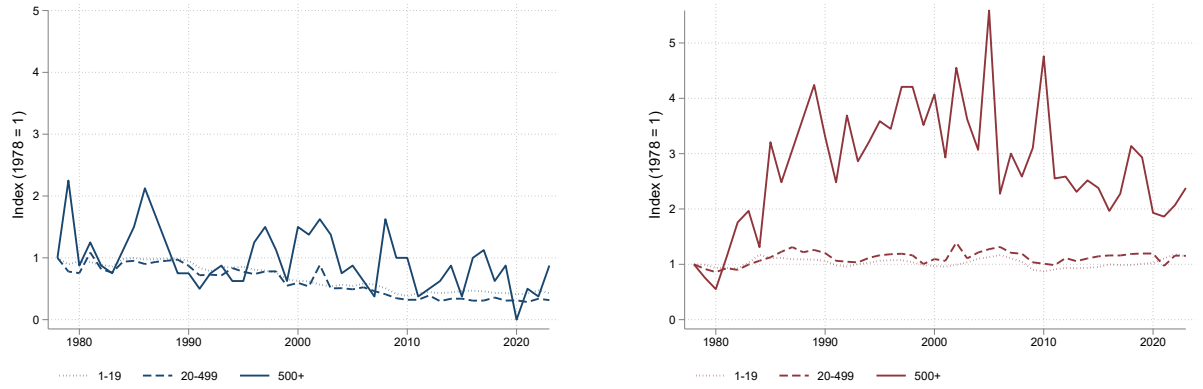


(A) Manufacturing (B) Services

Notes: Figure 5 shows the evolution of the employment by size group in manufacturing and services. Every group's employment level is normalized to one at the beginning of the sample. Data source: BDS.

the number of small (1–19 employees) and medium-sized (20–499 employees) entrants has been roughly stable over time, while the number of new large firms (500+) has been about three times higher since the late 1980s (relative to 1978).

FIGURE 6: COMPOSITION OF ENTRANTS BY FIRM SIZE (1978=1)



(A) Manufacturing (B) Services

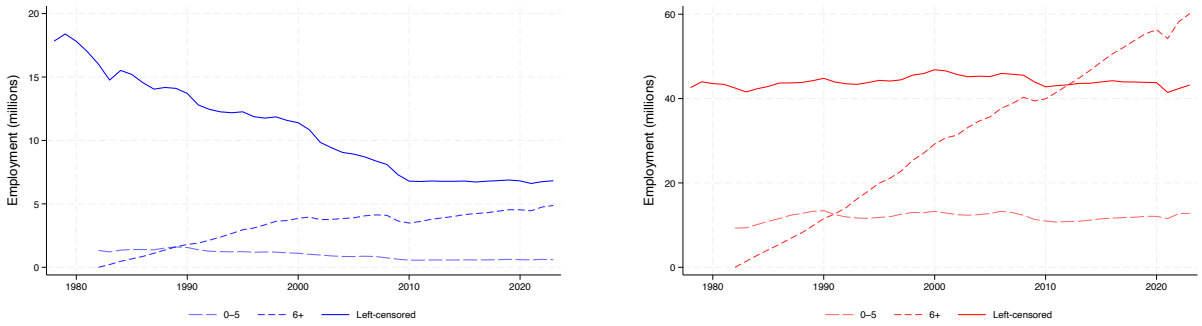
Notes: Figure 6 plots the number of entrants by initial firm size for manufacturing (panel (a)) and services (panel (b)) sectors. All time series are normalized to be 1 in 1978. Within the largest size category, data points for years 1987 and 1988 were linearly interpolated due to outliers. Data source: BDS.

The role of firm age The size-based patterns documented above raise a natural question about the role of firm age. Firm size and firm age are closely related but conceptually distinct dimensions of heterogeneity: large firms tend to be older on average, yet firms of similar size can be at very

different stages of the life cycle. As a result, employment reallocation attributed to firm size may reflect differences in life-cycle dynamics rather than size per se. Moreover, firm age is central for assessing whether employment expansion occurs through the maturation of younger firms or through the continued growth of mature incumbents. Examining employment dynamics by firm age therefore helps disentangle size effects from life-cycle effects and provides a more complete picture of the mechanisms underlying sectoral employment reallocation.

To analyze firm-age dynamics, we classify firms into three groups. Young firms are defined as firms aged 0–5 years. Old firms are defined as firms aged 6 years or older that enter after the beginning of the sample. Firms that were already present prior to the start of the sample are left-censored and cannot be assigned an exact age.[4] We report results for young and old firms starting in 1982, the first year in which firms aged 0–5 can be observed in the data. The results are shown in Figure 7.

FIGURE 7: SECTORAL EMPLOYMENT BY FIRM AGE



(A) Manufacturing (B) Services

Notes: Figure 7 shows the evolution of the employment by age group in manufacturing and services. Data source: BDS.

In manufacturing, young firms play a minor role, employing fewer than one million workers in a typical year, with little change over time. Employment among old firms—those aged six years or older—rises as cohorts age into this category, but stabilizes around the year 2000 at approximately five million workers. By contrast, the secular decline in manufacturing employment is concentrated among left-censored firms. Employment in this group falls by more than 11 million workers over the sample period, with the pace of decline moderating somewhat after 2010.

[4] At the end of the sample, 13.4% of manufacturing firms are classified as left-censored, compared to 6% in the service sector.

In the service sector, employment among young firms also plays a relatively minor role and plateaus at around twelve million workers. However, the age profiles of old and left-censored firms differ sharply from those in manufacturing. Employment among old service firms continues to grow throughout the sample period, reaching roughly 60 million workers by the end of the sample. At the same time, employment in left-censored firms – a group of firms that shrinks from initially 2.2 million firms to 280.000 firms at the end of the sample – is stable over time.

Taken together, the age patterns reinforce the conclusions drawn from the firm-size analysis. Employment among young firms remains quantitatively small and largely stable in both manufacturing and services, indicating that the expansion of large firms documented above does not primarily reflect the maturation of initially small entrants. Instead, the evidence points to employment reallocation occurring predominantly among mature firms. In manufacturing, the secular employment decline is concentrated among firms that were already operating prior to the start of the sample, consistent with contraction among long-established, presumably larger, incumbents. In services, by contrast, employment growth is driven by continued expansion among firms that are already mature, including left-censored firms. These results suggest that structural transformation operates mainly through the expansion and contraction of mature firms, rather than through life-cycle dynamics associated with entry and early growth.

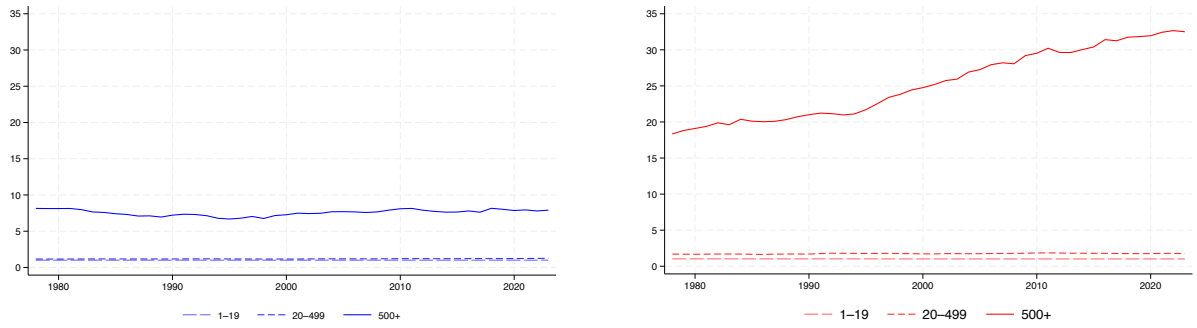
Intensive vs. extensive margin within incumbents One important margin for the expansion of incumbent firms is the number of establishments. Figure 8 shows the average number of establishments for firms of different size classes in manufacturing and services. The figure shows that in both sectors, firms with less than 250 employees essentially consist of a single establishment. The average number of establishments among large manufacturing is eight and has been stable over time. In contrast, the number of establishments per large service firm has seen a secular increase. It stands at 33 establishments per firm, on average, at the end of the sample period. Coupled with the fact that employment per establishment is stable, this suggests that extensive margin growth – the opening of new establishments by incumbent firms – is a key margin for growth in the service sector.[5] In manufacturing, where the number of establishments per firm has stayed constant, employment declines mainly through falling employment per establishment.

[5] See also Figure 15 in the Appendix.

Extensive vs. intensive growth of firms. Is manufacturing (services) employment declining (increasing) due to fewer (more) establishments per firm, or due to lower (higher) employment per establishment? Figure 8 shows that the underlying patterns differ markedly across sectors. In the manufacturing sector, the number of establishments per firm has been very stable over time, and the employment decline is primarily driven by a reduction in employment per establishment among the largest firms. On average, establishments in very large firms employed over 300 workers in the late 1970s; this figure has declined by roughly one-third, to about 200 employees. At the same time, there is no comparable trend among medium- and small-sized firms.

In the services sector, the patterns are quite different. Average employment per establishment has remained very stable across firm-size groups, while the largest firms have come to operate a substantially larger number of establishments—rising from fewer than 20 per firm in 1978 to nearly 35 per firm in 2023. Medium- and small-sized firms show no corresponding growth in the number of establishments.

FIGURE 8: ESTABLISHMENTS PER FIRM



(A) Manufacturing (B) Services

Notes: Figure 8 the number of average establishments per firm in manufacturing and services, broken down by firm size. Data source: BDS.

3.4 Employment accounting decomposition

To summarize our results, we decompose the sectoral employment change from year $t-1$ to t into three components, net entry, net expansion of continuing establishments, and net establishment

expansion of incumbent firms. We do so using the identity

$$\Delta E_{s,t} = \underbrace{\left(JC_{s,t}^{\text{birth},f=0} - \text{FirmDeathEmp}_{s,t}\right)}_{\textbf{Net entry of firms}} + \underbrace{\left(JC_{s,t}^{\text{cont}} - JD_{s,t}^{\text{cont}}\right)}_{\textbf{Net expansion/contraction of incumbents}} + \underbrace{\left(JC_{s,t}^{\text{birth},f\geq 1} - \left(JD_{s,t}^{\text{death}} - \text{FirmDeathEmp}_{s,t}\right)\right)}_{\textbf{Net establishment expansion of incumbent firms}}, \quad (1)$$

where $JC_{s,t}^{\text{birth},f=0}$ denotes job creation by age-0 firms in sector s at time t, $\text{FirmDeathEmp}_{s,t}$ denotes job destruction due to firm exit, $JC_{s,t}^{\text{cont}}$ and $JD_{s,t}^{\text{cont}}$ denote job creation and job destruction by continuing establishments, respectively. $JC_{s,t}^{\text{birth},f\geq 1}$ denotes job creation due to establishment entry within incumbent firms, and $JD_{s,t}^{\text{death}}$ denotes job destruction by exiting establishments.

Table 1 reports sectoral decomposition results for the period 1978–2023. We find that net entry accounts for about 40 percent of the decline in manufacturing employment. In contrast, its role in the services and other sectors is much smaller, at about 7 percent. Moreover, although net entry plays a relatively minor role in services and other sectors, it masks substantial entry and exit margins that largely offset each other in these sectors.

4 Model

4.1 Environment

Time is discrete, $t = 0, 1, 2, \ldots$. There are two final–goods sectors $j \in \{M, S\}$: manufacturing (M) and services (S). A representative household supplies $\bar{L}$ units of labor inelastically and owns all firms. Each sector is populated by a measure of incumbent firms and by potential entrants. Incumbent firms are heterogeneous in idiosyncratic productivity z and customer base n. Sectoral aggregate productivity is denoted by A_{jt}.

The customer base n summarizes the firm's stock of clients or relationships and imposes a capacity constraint on how much the firm can sell in a given period. Potential entrants must search for business opportunities in order to enter, and search is costly. Successful entrants become incumbents with initial productivity of z_0 and customer capital n_0.

TABLE 1: DECOMPOSITION OF EMPLOYMENT CHANGES BY SECTOR, 1978–2023

Component	Δ Employment (millions)	Share of Total Change (%)
Panel A: Manufacturing		
Net entry	−2.065	37.7
Entry (startups)	7.642	
Exit (firm deaths)	−9.709	
Net expansion	−0.603	11.0
Job creation (continuers)	52.666	
Job destruction (continuers)	−53.269	
Net establishment expansion of incumbent firms	−2.815	51.3
Total change	−5.483	**100.0**
Panel B: Services		
Net entry	4.883	6.8
Entry (startups)	98.407	
Exit (firm deaths)	−93.524	
Net expansion	37.600	52.6
Job creation (continuers)	381.400	
Job destruction (continuers)	−343.900	
Net establishment expansion of incumbent firms	29.000	40.6
Total change	71.483	**100.0**
Panel C: Other		
Net entry	0.306	7.1
Entry (startups)	12.455	
Exit (firm deaths)	−12.148	
Net expansion	3.387	78.7
Job creation (continuers)	46.834	
Job destruction (continuers)	−43.447	
Net establishment expansion of incumbent firms	0.456	10.6
Total change	4.149	**100.0**

Notes: Table 1 reports decomposition of employment changes into net entry, net expansion and net establishment expansion of incumbent firms margins for manufacturing, services and other sectors (see Equation 1). Data source: BDS.

4.2 Household

The representative household has homothetic preferences over a composite consumption good:

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ln(C_t), \tag{2}$$

where the consumption composite is given by a constant elasticity of substitution (CES) aggregator,

$$C_t = \left[\alpha C_{Mt}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\alpha) C_{St}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{3}$$

with $\varepsilon > 0$ denoting the elasticity of substitution between manufacturing and services consumption, and $\alpha \in (0,1)$ governing their relative importance in preferences.

Manufacturing is the numeraire, so we normalize its price P_{Mt} to one. The household budget

constraint is therefore

$$C_{Mt} + P_{St} C_{St} = W_t (\bar{L} - L_t^e) + \Pi_t, \tag{4}$$

where P_{St} is the relative price of services, W_t is the wage, and Π_t denotes aggregate firm profits rebated to the household. The household pays for aspiring entrepreneurs' search efforts, which are denominated in labor; L_t^e denotes the total amount of labor allocated to these efforts. The household inelastically supplies $\bar{L}$ units of labor each period.

Cost minimization implies the following price index for the consumption composite:

$$P_t = \left[\alpha + (1-\alpha) P_{St}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \tag{5}$$

so that the total consumption expenditure satisfies:

$$P_t C_t = C_{Mt} + P_{St} C_{St} = W_t (\bar{L} - L_t^e) + \Pi_t. \tag{6}$$

Intratemporal optimality implies the following sectoral consumption demands:

$$C_{Mt} = \alpha \left(\frac{1}{P_t}\right)^{-\varepsilon} C_t = \alpha P_t^{\varepsilon} C_t, \tag{7}$$

$$C_{St} = (1-\alpha) \left(\frac{P_{St}}{P_t}\right)^{-\varepsilon} C_t. \tag{8}$$

4.3 Incumbent Firms

An incumbent in sector j of type k with state (z,n) uses labor l to produce and sell its good. The production technology is:

$$q_{jk} = A_{jt} e^z l^{\theta_{jk}}, \quad \theta_{jk} \in (0,1). \tag{9}$$

We assume that each sector j is populated by two types of firms $k \in \{H,L\}$; these firm types differ only in terms of returns to scale: $\theta_{jL} < \theta_{jH}$.

In order to produce, incumbent firms must pay a fixed operating cost f, denominated in units of labor. Let $V_{jk,t}(z,n)$ denote the value of an incumbent firm. Anticipating a transition dynamics exercise—in which firms' decisions depend on paths of aggregate productivity, prices, and cross-

sectional distributions—we index value functions by time t. The incumbent firm's problem is then:

$$V_{jk,t}(z,n) = \max_{y,l,n' \geq 0} \Big\{ P_{jt} y - W_t l - f W_t + \beta (1 - p_{exit}) \max\{0, \mathbb{E}_t \left[V_{jk,t+1}(z',n') \right] \} \Big\}, \tag{10}$$

whereby output y is constrained by the production technology (9) and customer capital constraint:

$$y \leq \kappa_j n. \tag{11}$$

The tightness of the sector-specific customer capital constraint (11) is governed by the parameters $\{\kappa_j\}$. In Section 5, we normalize κ_M to one and calibrate κ_S to a value strictly smaller than one. The rationale for this asymmetry stems from a customer-facing time constraint. In the manufacturing sector, the production process decouples production from customer interaction: output produced in a given period can be delivered to customers over subsequent periods. In contrast, in the services sector customer-facing time becomes binding, as the firm's time is rival across customers. Serving customers requires allocating scarce customer-facing resources—such as the time of managers, professionals, and other frontline workers—which limits the amount of output that can be delivered to the existing clientele.

The law of motion for customer capital is:

$$n' = (1 - \delta_n) y. \tag{12}$$

That is, we assume that firms may be producing less than what is feasible given the production technology (9) in case they have not accumulated enough customer capital n. The amount of customer capital in period $t+1$ depends on the amount of realized output y at time t. We allow for customer attrition at a rate δ_n.

We allow for endogenous as well as exogenous firm exit. Incumbent firms may choose to exit in case their expected continuation value falls below the value of exit which we normalize to 0: $\mathbb{E}_t[V_{jk,t+1}(z',n')] < 0$. Let $\{ex_{jk,t}(z,n)\}$ capture the exit decision of a firm of type k in sector j with idiosyncratic states (z,n). Exogenous firm exit occurs with probability p_{exit}, and exiting firms leave the economy in the end of the period after production takes place.

Finally, idiosyncratic productivity evolves according to an AR(1) process:

$$z' = \rho z + \sigma_z \varepsilon', \quad \varepsilon' \sim \mathcal{N}(0,1), \tag{13}$$

where ε is i.i.d. across firms and time.

4.4 Entrants and Entrepreneurial Search

Potential entrants can direct their startup efforts toward a specific type of business. To operationalize this, we introduce a coordination friction which is reminiscent of the literature on directed search and is modeled similar to Sedláček and Sterk (2017) and Smirnyagin (2023).

Let $\{\psi_{jk}\}$ denote time-invariant masses of available to potential entrants business opportunities (indexed by firm type k and sector j). In order to enter, an aspiring entrepreneur has to pay a cost χ denominated in units of labor. This entrance costs is designed to capture expenditures associated with market research, development of a business plan, and alike. Upon paying the cost, a potential entrant has a chance to seize one of the available project ideas ψ_{jk}. Thus, parameters $\{\psi_{jk}\}$ govern the relative success probability of starting different project types.

Given the coordination friction, not all business opportunities are seized, while others are seized by several aspiring entrepreneurs. This friction is modeled by a matching function which returns the mass of successful entrants of type k in sector j, $m_{jk,t}$:

$$m_{jk,t} = (e_{jk,t})^{\phi} (\psi_{jk})^{1-\phi}, \tag{14}$$

where $e_{jk,t}$ is the mass of potential entrants who chose a type-j project in sector k, and $\phi \in (0,1)$ is an elasticity of successful entrants with respect to startup attempts.

Potential entrants are indifferent with respect to project types and sectors in equilibrium; this consideration gives rise to a set of free-entry conditions. Specifically, these conditions state that the cost of starting a business has to match the expected benefit:

$$\chi W_t = \frac{m_{jk,t}}{e_{jk,t}} V_{jk,t}(z_0, n_0), \tag{15}$$

where the ratio on the right-hand side is a success probability of starting a project of type k in

sector j.

By way of combining the matching technology (14) with free-entry conditions (15), one can derive the following closed-form expression for the mass of entrants of each type in each sector:

$$m_{jk,t} = (\chi W_t)^{\frac{\phi}{\phi-1}} (V_{jk,t}[z_0, n_0])^{\frac{\phi}{1-\phi}} \psi_{jk}. \tag{16}$$

We assume that successful at time t entrants become incumbents with the idiosyncratic state (z_0, n_0) in the next time period $t+1$.

4.5 Stationary Recursive Competitive Equilibrium

The recursive (stationary) competitive equilibrium in this environment is comprised of the set of value functions $\{V_{jk}\}$, firms' decision rules $\{y_{jk}(z,n), l_{jk}(z,n), n'_{jk}(z,n), ex_{jk}(z,n)\}$, cross-sectional distributions $\{\mu_{jk}\}$, aggregate profits Π, prices $\{W, P_M, P_S\}$, household consumption choice $\{C_M, C_S\}$, and an influx of entrants $\{m_{jk}\}$, such that:

1. given prices $\{W, P_M, P_S\}$ and aggregate firms' profits Π, household consumption choice $\{C_M, C_S\}$ maximizes utility (2) subject to the budget constraint (4);

2. given prices $\{W, P_M, P_S\}$, production technology (9) and stochastic process for idiosyncratic productivity (13), firms' value functions $\{V_{jk}\}$ and decision rules $\{y_{jk}(z,n), l_{jk}(z,n), n'_{jk}(z,n), ex_{jk}(z,n)\}$ solve (10)-(12);

3. aggregate firms' profits Π are given by

$$\Pi = \sum_{j\in\{M,S\}} \sum_{k\in\{H.L\}} \int \left(P_j y_{jk}(z,n) - W l(z,n) - fW\right) \mathrm{d}\mu_{jk};$$

4. the influx of successful entrants $\{m_{jk}\}$ satisfies (16);

5. goods markets (M and S) clear:

$$\sum_{k\in\{H,L\}} y_{Mk}(z,n)\mathrm{d}\mu_{Mk} = C_M, \qquad \sum_{k\in\{H,L\}} y_{Sk}(z,n)\mathrm{d}\mu_{Sk} = C_S;$$

6. labor market clears:

$$\sum_{j\in\{M,S\}}\sum_{k\in\{H.L\}}\int\left(l_{jk}(z,n)+f\right)\mathrm{d}\mu_{jk}+L^e=\bar{L},$$

where $L^e=\sum_{j\in\{M,S\}}\sum_{k\in\{H.L\}}\chi e_{jk}$;

7. stationary distributions $\{\mu_{jk}\}$ are induced by the stochastic process (13), incumbent firms' decision rules $\{l_{jk}(z,n),n'_{jk}(z,n),ex_{jk}(z,n)\}$, and masses of successful entrants $\{m_{jk}\}$.

We reserve computational details to Appendix C.1.

5 (Preliminary) Parameterization

In this section, we describe our parameterization strategy. We provide a heuristic identification argument that motivates the choice of targeted moments. Although each targeted moment is jointly affected by all parameters, we discuss them individually in relation to the parameter for which, intuitively, the moment provides the greatest identifying power.

The model period is one year, which aligns with the frequency of the data. We partition the parameter set into two groups: fixed and fitted. Parameters in the first group are assigned exogenously, outside the model, while parameters in the second group are chosen so that the model matches particular empirical moments.

Fixed parameters. Given that the model period is one year, we set $\beta=0.96$. The household inelastically supplies one unit of labor each period. Customer capital depreciates at a rate of $\delta_n=0.15$, consistent with estimates in Gourio and Rudanko (2014). The persistence of idiosyncratic productivity shocks is set to $\rho_z=0.90$, a value in line with those commonly used in the literature. We set σ_z at 0.20; this value will be later set to match the cross-sectional dispersion in employment changes. A key mechanism in our model is that as productivity in manufacturing increases, relative to services, the relative price of output in that sector decreases and so does the manufacturing share. Thus, the elasticity of substitution between manufacturing good and services must be less than one. We set it to $\varepsilon=0.4$ which is in line with the literature on structural transformation (e.g., Desmet

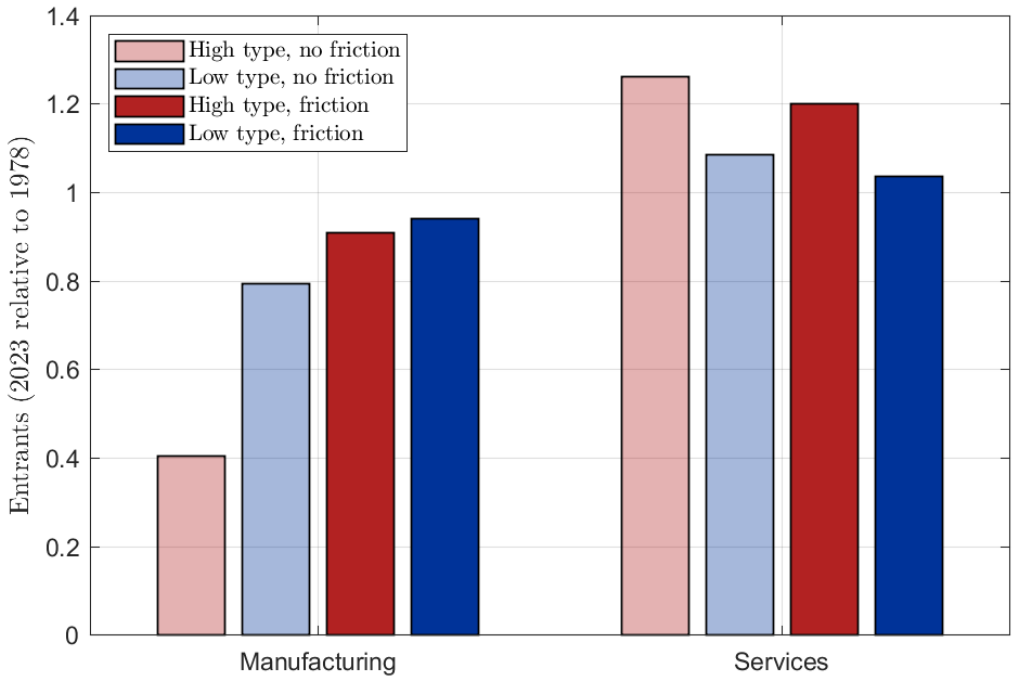
TABLE 2: MODEL PARAMETERS

Parameter	Description	Value	Target / Source	Data	Model
β	Discount factor	0.96			
ρ_z	Prod. persistence	0.90			
σ_z	Prod. volatility	0.20			
ϕ	Entry matching elasticity	0.60			
$\bar{L}$	Labor endowment	1		Normalization	
ε	Substitution elasticity	0.40		Desmet and Rossi-Hansberg (2014)	
δ_n	Cust. capital depreciation	0.15		Gourio and Rudanko (2014)	
α	Mnf cons. weight	0.30	Mnf expenditure share	0.35	0.31
θ_{MH}	RTS, mnf large	0.72	Emp. share 500+ firms in mnf	0.59	0.85
θ_{ML}	RTS, mnf small	0.62	Emp. share 0-100 firms in mnf	0.35	0.15
θ_{SH}	RTS, serv large	0.69	Emp. share 500+ firms in serv	0.46	0.78
θ_{SL}	RTS, serv large	0.65	Emp. share 0-100 firms in serv	0.32	0.20
f	Fixed operating cost	0.02	Average exit rate	0.10	0.11
p_{exit}	Exog. exit probability	0.05	Exit rate mature firms	0.05	0.05
χ	Entry cost	0.03	Entry cost to income ratio	0.07	0.10
ψ_{MH}	Mnf, large opp. mass	0.10	Share 500+ firms in mnf	0.02	0.05
ψ_{ML}	Mnf, small opp. mass	0.20	Share 0-100 firms in mnf	0.93	0.90
ψ_{SH}	Serv, large opp. mass	0.30	Share 500+ firms in serv	0.01	0.03
ψ_{SL}	Serv, small opp. mass	0.40	Share 0-100 firms in serv	0.96	0.88
z_0	Entry productivity	0.10	Relative entr. productivity	0.95	0.98

and Rossi-Hansberg, 2014). Finally, we assume that successful entrants start with a small amount of customer capital, which is set to the lowest grid point in our quantitative implementation.

Fitted parameters. Parameter α is chosen to match the manufacturing expenditure share of 0.35 observed in the data. The fixed operating cost parameter f and the probability of exogenous exit p_{exit} govern firm exit; accordingly, we target an average annual exit rate of 10 percent, as well as the exit rate of mature firms at 5 percent. The relative success probability parameters $\{\psi_{jk}\}$ are jointly chosen to match the share of manufacturing entrants in total entry, as well as the shares of large and small firms—defined as firms with above 500 and less than 100 employees, respectively—within the manufacturing and services sectors. One of the parameters in $\{\psi_{jk}\}$ is determined residually to ensure that their sum equals one. Returns to scale parameters $\{\theta_{jk}\}$ are chosen to match employment shares of large and small firms within sectors. Starting productivity z_0 is calibrated to match the relative entrants' productivity of 0.95 (Lee and Mukoyama, 2015). Table 2 summarizes model parameterization.

FIGURE 9: CHANGE IN ENTRANTS' COMPOSITION (1978 VS. 2023)



Notes: Figure 9 plots the change in the number of entrants between 1978 and 2023 for manufacturing and services sectors. "No friction" case corresponds to the version of the model without customer capital constraint (11).

6 (Preliminary) Quantitative Results

Implications for the composition of entrants. We consider two versions of the model laid out in Section 4: a baseline version and a frictionless version in which we relax firms' problems by dropping the customer capital constraint (11). Our main quantitative experiment is a transition-dynamics exercise, in which we trace out the evolution of sectoral employment and the entry of different firm types over the period 1978–2023. The driving force in the model is the change in sectoral productivities $\{A_j\}$ in manufacturing and services. In particular, we choose the initial and terminal values of sectoral productivities so that the model matches relative employment across the two sectors in 1978 and 2023. We further assume that sectoral productivities are linearly interpolated over the transition horizon.[6]

We illustrate the workings of the model by considering a steady-state-to-steady-state comparison. Figure 9 shows how the number of high- and low-type entrants in 2023 compares to that in

[6]See Appendix C.2 for computational details.

1978. The version of the model without the customer capital constraint predicts that the number of high-type firms declines more rapidly in manufacturing, while increasing disproportionately in services. This prediction stands in stark contrast to the data, where entry has declined nearly uniformly across size groups in manufacturing. By contrast, the version of the model with customer frictions can qualitatively account for the data, delivering a uniform decline in manufacturing and a more rapid growth in the entry of high-type firms in services. Below, we elaborate on why the baseline model can capture this pattern in the data.

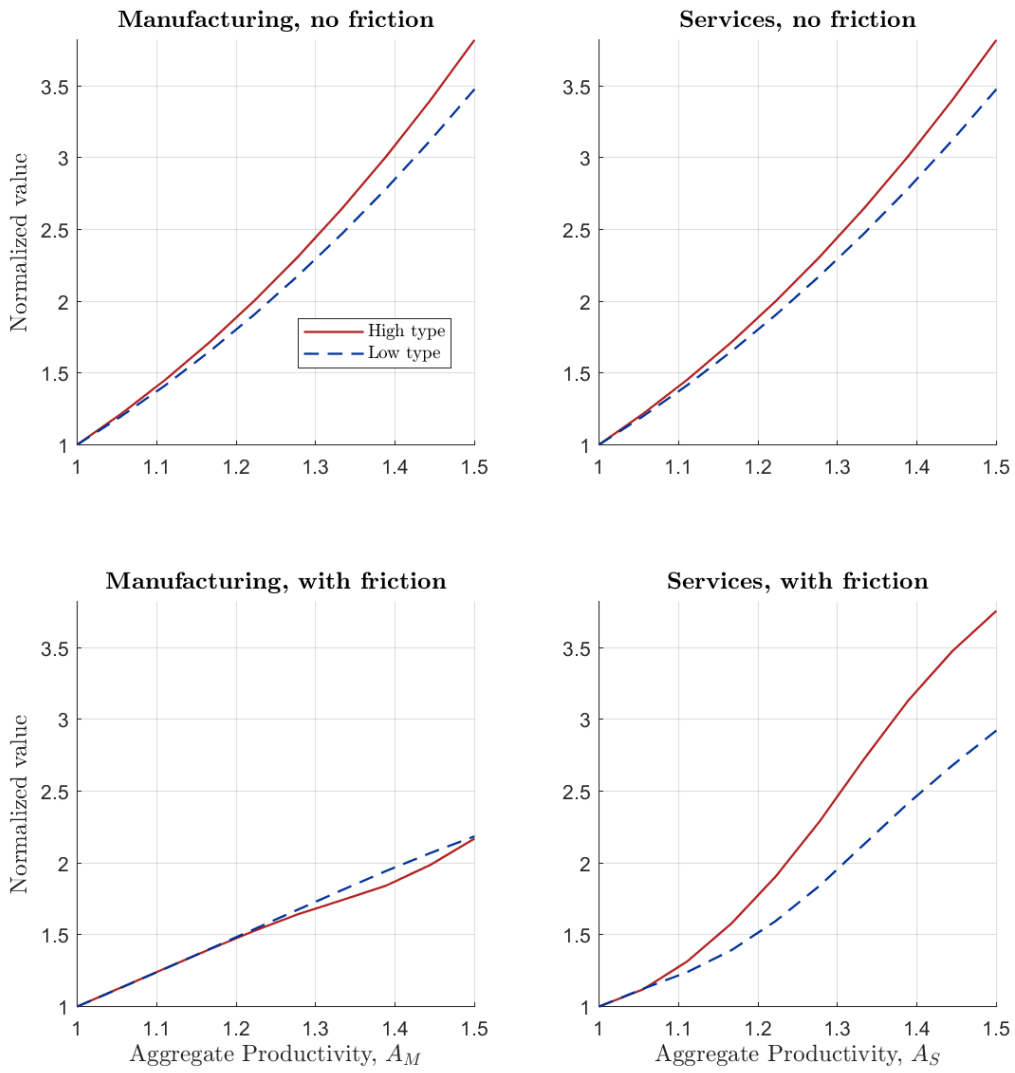
Customer capital constraint, aggregate productivity, and entry. In the data, the number of large entrants in the services sector has increased more rapidly relative to medium- and small-sized entrants, while in the manufacturing sector the number of entrants across firm sizes has declined at a similar rate (Figure 6). How does a change in sectoral productivity affect the entry of different firm types in the model?

In order to visualize the impact of changing sectoral productivity on firm entry, and to highlight the role of the customer capital constraint, we plot the value of entrants $V_{jk}(z_0, n_0)$ in both sectors as sectoral aggregate productivity varies. To isolate the role of sectoral productivity, we compute entrants' values in partial equilibrium, holding prices fixed at one.

The top row of Figure 10 shows that, in the frictionless setup, the values of high- and low-type firms increase by similar amounts as sectoral productivity rises. Given the expression for the mass of successful entrants (16), this implies a proportionate change in entry across firm types. This prediction is at odds with the data for the services sector, where the number of large entrants has increased by more relative to smaller entrants.

The bottom row shows that a version of the model with customer capital constraint can qualitatively account for the data. In manufacturing, where the constraint is less tight, the values of high- and low-type entrants increase by similar amounts. In contrast, in the services sector—where the constraint is tighter—the value of high-type firms rises more rapidly. Intuitively, high-type firms take longer to reach scale due to the constraint; as a result, an increase in sectoral productivity allows them to accumulate clientele more quickly, leading to a steeper increase in firm value. Thus, incorporating a customer capital constraint enables the model to account for features of the data that the frictionless framework fails to capture.

FIGURE 10: ENTRANT VALUE (NORMALIZED)



Notes: Figure 10 reports the value of entrants as sectoral productivity varies, holding prices (price of services and wages) fixed at one.

7 Conclusion

There is a longstanding policy intuition that small (or young) firms are the engine of job creation. This paper shows that even if startups matter for gross job flows, they may be second-order for long-run sectoral reallocation in employment.

In fact, structural transformation is not just a shift in sectoral employment; it is a shift in the

organizational form of production. If services grow primarily via scaling of multi-unit incumbents (rather than net entry), then policies built around "more startups" are mis-targeted.

Our contribution is to connect that insight specifically to structural transformation: if services growth was driven by incumbent-scaling, then policies that mainly affect entry will not affect the observed sectoral dynamics (and may not be the welfare-relevant margin). Many relevant policies are spatial / regulatory / market-access policies (zoning, licensing, regulations, occupational licensing, etc.).

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Appendix

Table of Contents for Appendix

A Data **29**

B Additional empirical results **30**

B.1 More details about the service sector 30

C Model **33**

C.1 Computational details 33

C.2 Transition dynamics exercise 35

A Data

Table **??** shows our baseline definition of service, manufacturing, and other sectors.

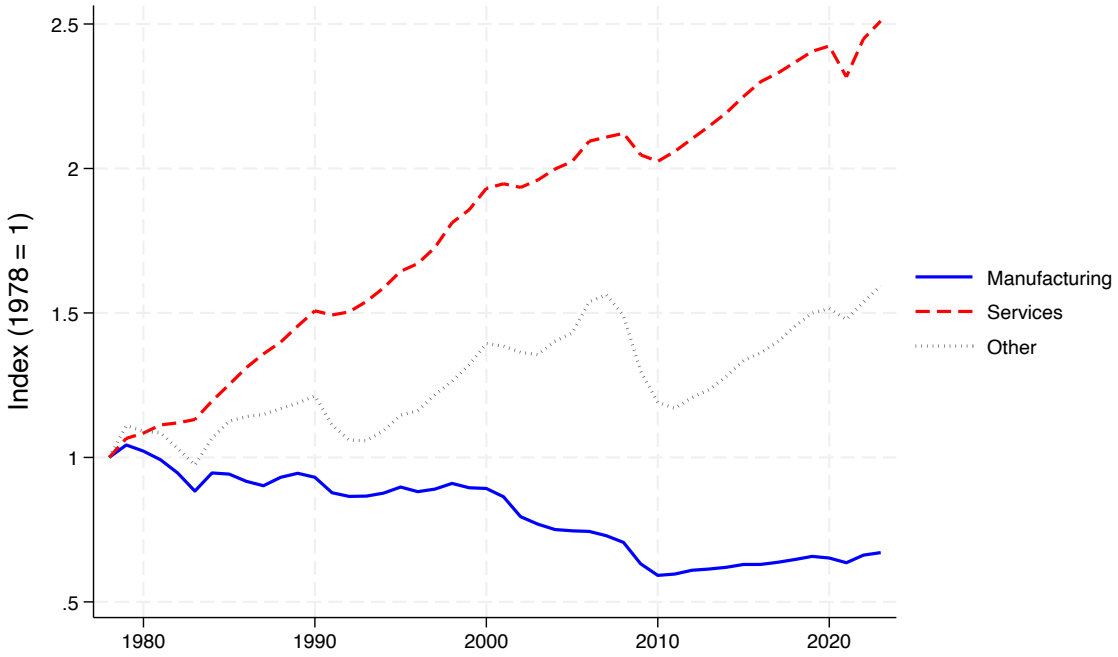
TABLE 3: SECTOR CLASSIFICATION

Code	Sector Name	Category
11	Agriculture, Forestry, Fishing and Hunting	Other
21	Mining, Quarrying, and Oil and Gas Extraction	Other
22	Utilities	Other
23	Construction	Other
31–33	Manufacturing	Manufacturing
42	Wholesale Trade	Services
44–45	Retail Trade	Services
48–49	Transportation and Warehousing	Services
51	Information	Services
52	Finance and Insurance	Services
53	Real Estate and Rental and Leasing	Services
54	Professional, Scientific, and Technical Services	Services
55	Management of Companies and Enterprises	Services
56	Administrative and Support and Waste Management Services	Services
61	Educational Services	Services
62	Health Care and Social Assistance	Services
71	Arts, Entertainment, and Recreation	Services
72	Accommodation and Food Services	Services
81	Other Services (except Public Administration)	Services

Notes: Table **??** shows how sectors in BDS data map into manufacturing, services and other groups.

B Additional empirical results

FIGURE 11: SECTORAL EMPLOYMENT RELATIVE TO 1978

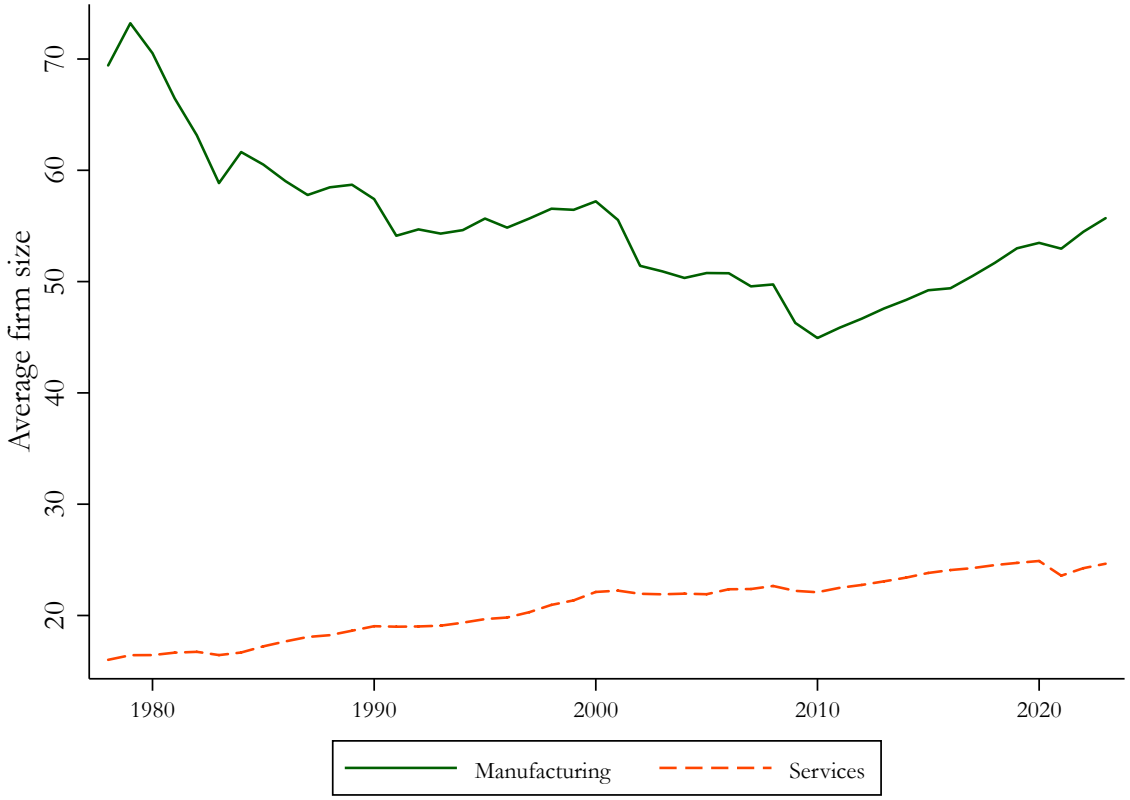


Notes: Figure 11 shows the evolution of manufacturing and service employment relative to 1978. Data source: BDS.

B.1 More details about the service sector

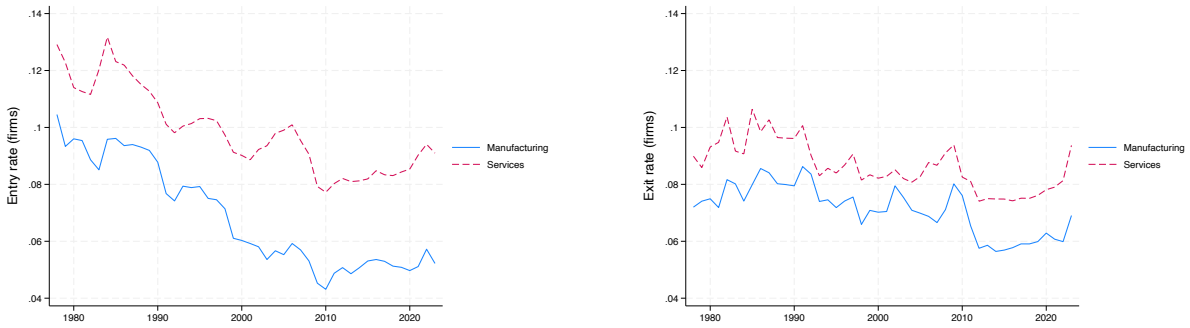
Put here some breakdowns. They might not be super consequential for our results, but they are good to know and collect somewhere.

FIGURE 12: AVERAGE FIRM SIZE



Notes: Figure 12 shows the evolution of the average firm size in manufacturing and services. Data source: BDS.

FIGURE 13: ENTRY AND EXIT RATES

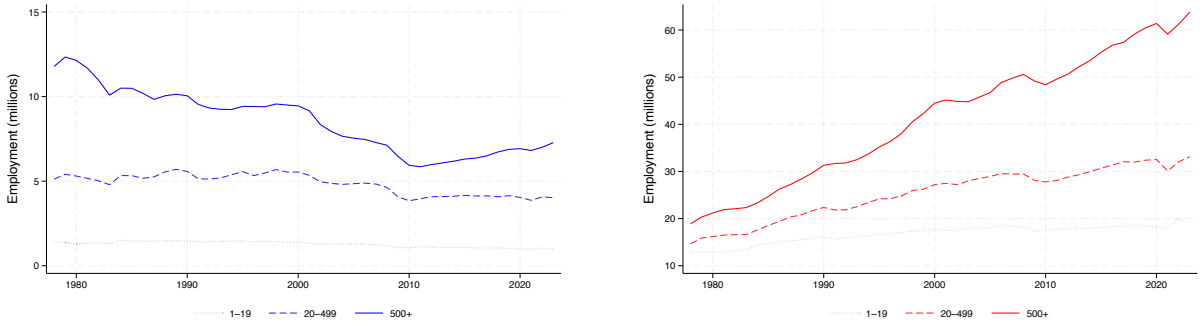


(A) Sectoral entry rates

(B) Sectoral exit rates

Notes: Notes: Figure 13 plots entry and exit rates for manufacturing and services sectors. The entry rate is defined as the number of entrants in a sector as a fraction of all firms in a sector. The exit rate is defined as the number of exiting firms as a fraction of all firms that were active in the previous year. Data source: BDS.

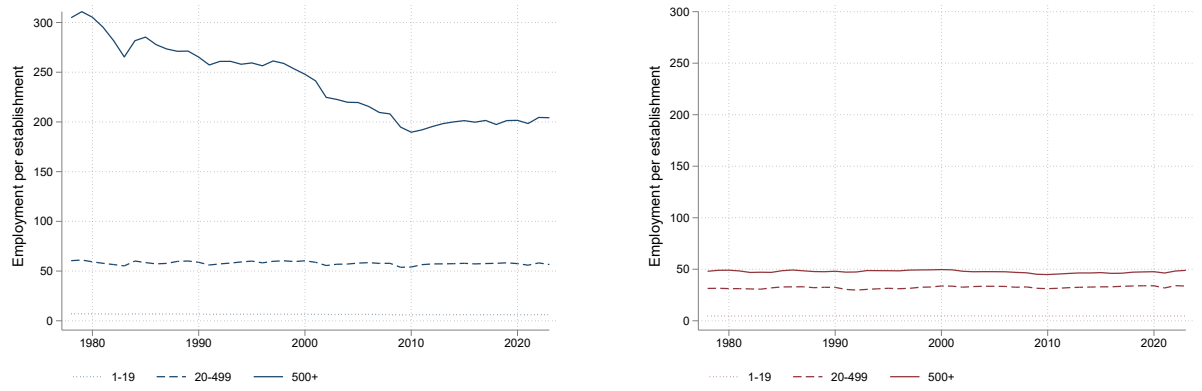
FIGURE 14: SECTORAL EMPLOYMENT BY INITIAL SIZE GROUP



(A) Manufacturing (B) Services

Notes: Figure 14 shows the evolution of the employment by initial size group in manufacturing and services. Data source: BDS.

FIGURE 15: EMPLOYMENT PER ESTABLISHMENT



(A) Emp. per establishment, manufacturing (B) Emp. per establishment, services

Notes: Figure 15 plots the average number of employees per establishment by firm size group. Data source: BDS.

C Model

C.1 Computational details

C.1.1 Outline of the Algorithm

We use collocation methods to solve the firm's functional equations. In practice, we use Chebyshev polynomials to approximate value functions. To streamline the exposition, we present the algorithm assuming one industry and one firm type (hence no j and k subscripts).

We set up a grid of collocation nodes $\mathscr{N} \times \mathscr{Z}$, with N_i nodes in each dimension, $i \in \{\mathscr{N}, \mathscr{Z}\}$. The computation of the stationary state of the model proceeds in the following 5 steps:

1. guess the equilibrium wage rate, W, and price of the service sector good P_S;
2. solve for individual decision rules $\{n', l, ex\}$;
3. compute the influx of entrants (16);
4. given the decision rules, compute stationary histogram (distribution of firms over the idiosyncratic state space);
5. compute the excess demand on the labor and service good markets. If it exceeds some prespecified tolerance, adjust the price vector guess correspondingly and go back to Step 2. Otherwise, terminate.

The price of the manufacturing good is normalized to one; we check ex post that the manufacturing good market clears due to Walras law.

C.1.2 Approximation of Value Functions

We represent firm's value function (10) as a weighted sum of orthogonal polynomials:

$$V(n,z) = \sum_{a,b=1,1}^{N_{\mathscr{N}},N_{\mathscr{Z}}} \theta^{ab} T^a(n) T^b(z),$$

where $\{\theta^{a,b}\}$ are approximation coefficients, and $T^i(\cdot)$ denotes the Chebyshev polynomial of order i.

We use a collocation method to simultaneously solve for $\{\theta^{a,b}\}$. Collocation method requires setting the residual equation to hold exactly at $N = N_{\mathscr{N}} \times N_{\mathscr{Z}}$ points ; therefore, we essentially solve for N unknown coefficients. We compute the basis matrices for Chebyshev polynomials using Miranda and Fackler (2002) Compecon toolbox. Subsequently, we solve for a vector of unknown coefficients by iterating on the value function. Given the current guess of coefficients, we solve for the optimal policy $n'(n,z)$ using vectorized golden search. After we solve for the policy function, we recompute decision rules on a finer grid, and, subsequently, compute the stationary distribution.

C.1.3 Stationary Distribution

When we solve for a stationary distribution, we iterate on a mapping using firms' decisions rules:

$$L' = \mathbf{Q}'((1 - p_{exit}) \cdot (1 - ex(n,z))L) + L^e,$$

where L is the current distribution of firms across the state space, and L^e is the distribution of successful entrants in the (n,z)-space. Matrix $\mathbf{Q}$ is a transition matrix, which determines how mass of firms shifts in the (n,z)-space. It is a direct product of two transition matrices $\mathbf{Q}_n$ and $\mathbf{Q}_z$:

$$\mathbf{Q} = \mathbf{Q}_n \odot \mathbf{Q}_z,$$

which govern the shift of mass along n- and z-dimensions, respectively. While $\mathbf{Q}_z$ is completely determined by the exogenous stochastic process (13), matrix $\mathbf{Q}_n$ is constructed so that the model generates an unbiased distribution in terms of aggregates.[7] More precisely, element (i,j) of the transition matrix $\mathbf{Q}_n$ informs which fraction of firms with the current idiosyncratic state n_i will end up having n_j tomorrow. Therefore, this entry of the matrix is computed as:

$$\mathbf{Q}_n(i,j) = \left[\mathbf{1}_{n' \in [n_{j-1}, n_j]} \frac{n' - n_j}{n_j - n_{j-1}} + \mathbf{1}_{n' \in [n_j, n_{j+1}]} \frac{n_{j+1} - n'}{n_{j+1} - n_j} \right].$$

Tensor product of matrices $\mathbf{Q}_n$ and $\mathbf{Q}_z$ is computed using the `dprod` function from the Miranda and Fackler (2002) toolkit.

[7] See Young (2010) for more details.

C.2 Transition dynamics exercise

In this section, we outline an algorithm for computing transition dynamics. Specifically, we study the perfect foresight transition of the economy driven by changes in aggregate productivity in manufacturing and services sectors.

1. Compute the steady-state for year 1977 (last pre-BDS year) normalizing manufacturing and services sector productivities $\{A_j\}$ to one; that is, firms solve their problems believing that $\{A_j\}$ will stay at the steady-state level indefinitely;

2. Consider a transition horizon T. The horizon should be large enough to ensure that the economy converges back to the steady-state by time $T + 1977$. In practice, we set $T = 100$. Solve for the steady-state corresponding to terminal values of $\{A_{j,T+1977}\}$;

3. We assume that firms learn the series of aggregate productivities $\{A_{jt}\}_{t=1978}^{T+1977-1}$ at time $t = 1978$. We construct the sequence of productivities by linearly interpolating between the initial and terminal levels for the time period 1978-2023, and keeping $\{A_{jt}\}_{t=2024}^{T+1977-1}$ at 2023 level until the end of the simulation horizon $T + 1977$;

4. Guess a sequence of wages $\{\widehat{W}_t\}_{t=1978}^{T+1977-1}$, prices of the services good $\{\widehat{p_{St}}\}_{t=1978}^{T+1977-1}$, and marginal utilities $\{\widehat{MU}_t\}_{t=1978}^{T+1977-1}$;

5. Given that we know the value function in the terminal period $T + 1977$, $\widetilde{V}_{jk,T+1977}$, we can solve for the optimal intertemporal decisions at $t = T + 1977 - 1$:

$$\{\widehat{n';l;y}\}_{jk,t}(z,n) = \arg\max_{n',l,y\geq 0} \Big(\widehat{MU}_t \times (\widehat{P_{jt}}y - \widehat{W}_t l - \widehat{W}_t f) + \\ + \beta(1-p_{exit})\max\{0, \mathbb{E}_t \widetilde{V}_{jkt+1}(z',n')\}\Big).$$

   Note that we are using value functions scaled by the marginal utility: $\widetilde{V}_{jkt} = \widehat{MU}_t \times V_{jkt}$. We also recover the value function $\widetilde{V}_t$ that corresponds to the obtained decision rules;

6. Solving backwards (i.e., by repeatedly executing the previous step), we can recover the entire path of decision rules for $t = 1978, \ldots, T + 1977 - 1$;

7. Take the steady-state distribution for period $t = 1977$ (which is the same in period $t = 1978$ since distribution is backward-looking). Apply the recovered sequence of decision rules, $\{\widehat{n'}_{jkt}(z,n)\}_{t=1978}^{T+1977-1}$, to compute the evolution of the cross-sectional distribution over the entire transition horizon;

8. Compute excess demands on the labor and service good markets (in each year along the transition path), as well as the deviation of the implied sequence of marginal utilities from the guessed one;

9. If the norm of deviations taken across time is sufficiently small, terminate. Otherwise, update the guess of prices and marginal utilities and go back to step (5).