Sectoral Labor Reallocation, Inflation and Monetary Policy

Marc de la Barrera[*] IESE
Masao Fukui[†] Boston University
Bumsoo Kim[‡] Williams College

March 26, 2026

Click here for most updated version.

Abstract

We study how frictional labor mobility across sectors shapes inflation dynamics and monetary policy in a multi-sector New Keynesian economy, where workers' dynamic discrete-choice decisions govern sectoral mobility. We derive analytical expressions for welfare and a generalized Phillips curve and show that imperfect labor reallocation influences sectoral inflation through two sufficient statistics: a mobility elasticity and the steady-state labor transition matrix. Using a spectral decomposition, we show that the persistence of sectoral inflation is governed not by average price rigidity or aggregate labor mobility alone, but by the price rigidity of the sectors that workers move to and from. When labor reallocation across sticky sectors is imperfect, the effects of sectoral shocks on these sectors become highly persistent. In response, monetary policy balances stabilization of standard aggregate output gaps with persistent labor and price misallocation. When these sticky sectors face a negative productivity shock, optimal monetary policy has an extra incentive to tighten to decrease relative demand and thereby incentivize labor outflows.

[*]IESE Business School. Email: mdelabarrera@iese.edu
[†]Boston University Department of Economics. Email: mfukui@bu.edu
[‡]Williams College Department of Economics. Email: bk12@williams.edu

1 Introduction

Sectoral shocks are pervasive: energy prices spike, supply chains break, demand rotates across goods and services, and technologies reshape which tasks are valuable. These events call for relative adjustment—prices realign and workers move toward expanding activities. In a frictionless benchmark, both margins operate efficiently. In practice, both margins are sluggish. On the nominal side, sticky prices delay relative price adjustments, generating distortions that misallocate expenditure. On the real side, worker reallocation unfolds gradually through a complex network of bilateral flows shaped by skills, geography, and institutional barriers. These two frictions are individually well established, serving as the foundations of the New Keynesian framework and the trade and labor literatures respectively.

In reality, the two margins interact: price distortions contaminate the signals that guide reallocation, while labor frictions feed back into the cost pressures that drive inflation. Moreover, because sectors differ in price rigidity, aggregate demand affects prices unevenly. As a result, monetary policy does more than influence the aggregate price level: it changes the relative prices that guide labor reallocation. Aggregate stabilization may impede the relative price adjustments required for labor reallocation, while facilitating reallocation could generate sectoral inflation. Which force dominates depends on how sectoral price rigidities map onto the labor flow network. This interaction shapes the transmission of sectoral shocks, the persistence of inflation, and optimal monetary policy.

We study these interactions in a multi-sector New Keynesian model that incorporates frictional labor reallocation through an Artuç et al. (2010) dynamic discrete choice block. On the nominal side, sectors differ in Calvo price rigidity; on the real side, workers choose sectors based on forward-looking continuation values, subject to bilateral mobility costs and idiosyncratic preference shocks. The model nests the standard multi-sector New Keynesian economy with integrated labor markets (e.g., Aoki, 2001) when mobility is frictionless, and the multi-sector economy with segmented labor markets when mobility costs are prohibitive. Despite the richness of the underlying micro structure, the linearized equilibrium depends on the reallocation block only through two sufficient statistics: a mobility elasticity ϕ that governs the responsiveness of worker flows to sectoral value differentials, and the steady-state transition matrix F that summarizes the pattern of gross worker flows across sector pairs.

In Section 3, we derive a generalized Phillips curve: a recursive mapping from aggregate demand and sectoral productivity to sectoral inflation, with a state tracking inherited price misalignment and employment misallocation. The eigenvalues of the associated transition matrix govern the persistence of sectoral inflation. Under homogeneous price stickiness, aggregate demand shifts marginal costs uniformly, so the system decouples into an aggregate block that monetary policy controls and a relative block that it cannot. At the polar cases, impact and persistence move in opposite directions: segmented markets produce a larger initial inflation

response but faster subsequent convergence, because firms that cannot anticipate cost relief from worker inflows reprice more aggressively.

In general, the eigenvalues of the transition matrix are implicit functions of the full parameter vector and have no closed-form expressions. We obtain analytical results by imposing a mild symmetry restriction on reallocation costs. Under uniform price stickiness, the system decomposes into independent components, each pairing a Phillips curve with a labor reallocation equation. The persistence of each component is governed by the interaction between nominal rigidity and the speed of labor reallocation along that dimension, with the slower-adjusting margin acting as the bottleneck.

Allowing price stickiness to vary across sectors breaks this independence but yields two key insights. First, when sectors that adjust prices slowly also sit along slow-moving reallocation dimensions, nominal and real frictions compound and adjustment stalls. Second, monetary policy has leverage over reallocation only when sticky- and flexible-price sectors load on opposite sides of a component, so that aggregate demand can tilt relative prices along it.

We formalize the optimal policy response as a dynamic Ramsey problem (Section 5). The second-order welfare loss penalizes aggregate output gaps, sectoral inflation dispersion, relative price distortions, and cross-sector employment misallocation—the last of which is new to our setting and compounds over time when labor reallocation is slow. As in any multi-sector model, the central bank faces a fundamental constraint: a single instrument can move sectoral prices only along the direction of Phillips curve slopes, which generically differs from the direction of efficient relative price adjustment.

The optimal targeting criterion nests the textbook rule when price stickiness is uniform but introduces a reallocation wedge when it is not. The wedge is governed by the interaction between the two diagnostic statistics: effective rigidity determines whether misalignment along a reallocation component is persistent enough to warrant intervention, and the demand spillover determines whether aggregate demand has leverage to correct it. When both are large along the same component, the planner actively tilts demand to redirect worker flows—tightening when efficient reallocation requires outflows from sticky sectors, loosening in the reverse case.

We bring the model to U.S. sectoral data (Section 6), estimating the worker transition matrix from industry-to-industry worker flows in the CPS and measuring sectoral price rigidity from micro price data. We evaluate the alignment between the labor flow network and the distribution of nominal rigidities to assess the quantitative relevance of these channels.

Related Literature

Our paper is most closely related to work on monetary policy during structural reallocation. Guerrieri et al. (2021) show in a two-sector model with downward wage rigidity that sectoral reallocation can shift optimal policy in either direction: contractionary when easing slows

reallocation, expansionary when easing accelerates it. Ferrante et al. (2023) study the interaction of inflation and labor reallocation in a calibrated 66-sector model, showing that labor reallocation frictions combined with heterogeneous price stickiness account for a large share of post-pandemic U.S. inflation. We generalize the policy insight of the former and provide analytical foundations for the mechanism in the latter, deriving sufficient statistics computable from worker flow data and sectoral price adjustment frequencies that determine both when reallocation frictions amplify the persistence of inflation and when they affect optimal policy.

We also contribute to the study of optimal monetary policy in multi-sector New Keynesian economies. Optimal stabilization puts more weight on sticky-price sectors (Aoki, 2001; Benigno, 2004), a principle that extends to heterogeneous rigidities and production networks (Pasten et al., 2020; La'O and Tahbaz-Salehi, 2022; Rubbo, 2023; Afrouzi and Bhattarai, 2025). In much of this literature, labor markets are either perfectly integrated or exogenously segmented.[1] We study the intermediate case of imperfect sectoral mobility and show that it generates propagation channels and policy wedges that are absent at either polar benchmark.

In our model, imperfect sectoral labor mobility is a source of real rigidity, connecting our framework to the literature on inflation dynamics and the Phillips curve. Nominal rigidities generate large macroeconomic effects when accompanied by real rigidities (Ball and Romer, 1990; Woodford, 2003), with wage rigidity serving as a prominent microfoundation (Blanchard and Galí, 2007); the empirical Phillips curve is quite flat (Hazell et al., 2022), consistent with substantial real rigidity. In our setting, a different labor market friction—imperfect sectoral reallocation—generates persistent endogenous cost-push dynamics: inherited employment misallocation shifts sectoral marginal costs, and these shifts decay slowly because the same frictions that created them also delay their correction.

The magnitude of this mechanism depends on the distribution of nominal frictions across sectors. Micro evidence documents substantial heterogeneity in price adjustment frequencies (Bils and Klenow, 2004; Nakamura and Steinsson, 2008), and this heterogeneity has become central to understanding inflation persistence (Altissimo et al., 2009; Pasten et al., 2020; Carvalho et al., 2021; Pastén et al., 2024). We show that the interaction between this nominal heterogeneity and the topology of worker flows—which sectors are connected, and how quickly workers move between them—determines which dimensions of misalignment are persistent and which matter for aggregate inflation.

Our micro-foundation for labor reallocation builds on the trade and labor literature on dynamic worker reallocation, originating from Lilien (1982) and formalized via dynamic discrete choice by Artuç et al. (2010) with general equilibrium applications in Caliendo et al. (2019); Allen et al. (2020); Seo and Oh (2025); Kim et al. (2026). We embed this micro-foundation into a New

[1] Recent work incorporates limited labor adjustment via imperfect two-sector mobility (Cantelmo and Melina, 2023) or convex hiring costs (Ferrante et al., 2023). Instead, we model worker reallocation as an explicit dynamic transition system that shapes the Phillips curve and welfare.

Keynesian model and show that the reallocation block enters the Phillips curve and welfare loss through our sufficient statistics. Methodologically, we adapt spectral decomposition techniques used in spatial migration (Kleinman et al., 2023) and input-output production networks (Liu and Tsyvinski, 2024) to characterize the dynamics of cross-sector worker flows.

2 Model

We build a multi-sector New Keynesian model in which sectors differ in price rigidity and workers reallocate across sectors through a dynamic discrete choice problem. We describe the production side, household preferences, and the labor reallocation problem, then state the equilibrium and its log-linear approximation.

Time is discrete, $t = 0, 1, \ldots$ The economy consists of N sectors indexed by $i, j \in \mathcal{N} = \{1, \ldots, N\}$.

2.1 Production and Pricing

Final and sectoral aggregation. The economy produces a final good Y_t which is a Cobb-Douglas aggregate of sectoral bundles $\{Y_{it}\}_{i \in \mathcal{N}}$:

$$Y_t = \prod_{i \in \mathcal{N}} (Y_{it}/\alpha_i)^{\alpha_i}, \qquad \sum_{i \in \mathcal{N}} \alpha_i = 1, \tag{1}$$

with associated consumer price index $P_t^C = \prod_i P_{it}^{\alpha_i}$, where P_{it} is the price index of sector i. Each sectoral bundle is a CES aggregate of differentiated varieties $f \in [0, 1]$ with elasticity of substitution $\epsilon_i > 1$:

$$Y_{it} = \left(\int_0^1 Y_{it}(f)^{\frac{\epsilon_i - 1}{\epsilon_i}} \, df \right)^{\frac{\epsilon_i}{\epsilon_i - 1}}, \tag{2}$$

yielding demand for variety f and the sectoral price index:

$$Y_{it}(f) = \left(\frac{P_{it}(f)}{P_{it}} \right)^{-\epsilon_i} Y_{it}, \qquad P_{it} = \left(\int_0^1 P_{it}(f)^{1-\epsilon_i} \, df \right)^{\frac{1}{1-\epsilon_i}}. \tag{3}$$

Technology. Each firm f in sector i operates a linear production technology:

$$Y_{it}(f) = A_{it} L_{it}(f), \tag{4}$$

where A_{it} is sector-specific productivity and $L_{it}(f)$ is labor input. Nominal marginal cost is $MC_{it} = W_{it}/A_{it}$, where W_{it} is the nominal wage in sector i.

Pricing. Firms set prices subject to Calvo frictions. In each period, a fraction $1-\delta_i \in (0,1)$ of firms in sector i can reset their prices. The government provides a revenue subsidy of rate $\frac{1}{\epsilon_i - 1}$ to each sector, financed by lump-sum taxes, that offsets the monopolistic markup and ensures the flexible-price steady state is efficient. Under this subsidy, a re-optimizing firm chooses $P_{it}^*(f)$ to maximize:

$$\max_{P_{it}^*(f)} \mathbb{E}_t \sum_{s=0}^{\infty} \delta_i^s \, Q_{t,t+s} \left(\frac{P_{it}^*(f)}{P_{it+s}} \right)^{-\epsilon_i} Y_{it+s} \left[\frac{\epsilon_i}{\epsilon_i - 1} P_{it}^*(f) - MC_{it+s} \right], \tag{5}$$

where $Q_{t,t+s} = \beta^s (C_{t+s}/C_t)^{-\gamma} (P_t^C / P_{t+s}^C)$ is the stochastic discount factor.

2.2 Households

A representative family consists of a unit mass of workers distributed across sectors. Let N_{it} denote the fraction of workers in sector i at the start of period t, with $\sum_{i \in \mathcal{N}} N_{it} = 1$. Each worker in sector i supplies L_{it} hours during period t. The distribution $\{N_{it}\}$ evolves through a reallocation process described in Section 2.3.

Preferences. The family maximizes:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\gamma}}{1-\gamma} - \sum_{i \in \mathcal{N}} N_{it} \, \zeta_i \frac{L_{it}^{1+\varphi}}{1+\varphi} - \Psi_t \right], \tag{6}$$

where $\beta \in (0,1)$ is the discount factor, $\gamma > 0$ is the inverse of the elasticity of intertemporal substitution, $\varphi > 0$ is the inverse Frisch elasticity, $\zeta_i > 0$ are sector-specific disutility shifters, and Ψ_t is the cost of labor reallocation incurred at period t (defined in Section 2.3).

Budget constraint. The family faces the nominal budget constraint:

$$P_t^C C_t + B_t \leq (1 + i_{t-1}) B_{t-1} + \sum_{i \in \mathcal{N}} W_{it} L_{it} N_{it} + \Pi_t + T_t, \tag{7}$$

where B_t are one-period nominal bonds paying gross interest $1 + i_t$ set by the central bank, Π_t are aggregate profits, and T_t are net lump-sum transfers from the government.

Optimality conditions. Maximizing (6) subject to (7), taking $\{N_{it}\}$ as given, yields the intratemporal labor supply condition for each sector i:

$$\frac{W_{it}}{P_t^C} = \zeta_i L_{it}^{\varphi} C_t^{\gamma}, \tag{8}$$

and the intertemporal Euler equation:

$$1 = \beta(1 + i_t)\,\mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\frac{P_t^C}{P_{t+1}^C}\right]. \tag{9}$$

2.3 Labor Reallocation

We now describe how workers move across sectors between periods. At the end of period t, after production and consumption decisions have been made, the representative family coordinates the reallocation of its workers for the start of period $t+1$.[2]

Reallocation cost. The per-period reallocation cost in the family's objective (6) takes the form:

$$\Psi_t = \sum_{i\in\mathcal{N}} N_{it}\,\psi_i\left(\{\mu_t^{ij}\}_{j\in\mathcal{N}}\right), \tag{10}$$

where $\psi_i(\cdot)$ is a sector-specific cost function that is strictly convex in the transition probabilities $\{\mu_t^{ij}\}$ satisfying $\mu_t^{ij} \geq 0, \sum_{j\in\mathcal{N}} \mu_t^{ij} = 1$. We adopt the following parametric specification, standard in the dynamic discrete choice literature (Anderson et al., 1988; Artuç et al., 2010; Caliendo et al., 2019):

$$\psi_i\left(\{\mu_t^{ij}\}_j\right) = \sum_{j\in\mathcal{N}} \mu_t^{ij}\tau^{ij} + \frac{1}{\theta}\sum_{j\in\mathcal{N}} \mu_t^{ij}\log\mu_t^{ij}, \tag{11}$$

where $\tau^{ij} \geq 0$ are bilateral reallocation costs (we normalize $\tau^{ii} = 0$). This aggregate penalty is equivalent to a decentralized setting in which individual workers draw i.i.d. type-I extreme value preference shocks with dispersion $1/\theta$ (Anderson et al., 1992). The parameter $\theta > 0$ governs the elasticity of labor reallocation: as θ falls, flows become less responsive to continuation-value differentials. In the limit $\theta \to 0$, sectoral employment shares do not respond to shocks at first order. Conversely, as $\theta \to \infty$, the family sorts workers deterministically toward high-value destinations, approaching a frictionless benchmark.

Value functions and optimal flows. The family chooses the flows $\{\mu_t^{ij}\}$ to maximize welfare. Let V_{it} denote the marginal continuation value to the family of having an additional worker in sector i at time t. Under the functional form for ψ_i, optimal transition probabilities take the logit

[2] The timing is: (1) period t begins with employment distribution $\{N_{it}\}$ and last period's prices $\{P_{it-1}\}$ given; (2) shocks are realized, firms set prices under Calvo frictions, and workers supply hours L_{it}; (3) at the end of period t, workers choose which sector to enter for period $t+1$; (4) this determines $\{N_{it+1}\}$.

form:

$$\mu_t^{ij} = \frac{\exp\left(\theta[\beta\,\mathbb{E}_t V_{jt+1} - \tau^{ij}]\right)}{\sum_{j'\in\mathcal{N}} \exp\left(\theta[\beta\,\mathbb{E}_t V_{j't+1} - \tau^{ij'}]\right)}. \tag{12}$$

Substituting these optimal flows back into the family's objective yields a recursive expression for the marginal value of a worker:

$$V_{it} = U_{it}^{\text{flow}} + \frac{1}{\theta}\log\left(\sum_{j\in\mathcal{N}} \exp\left(\theta[\beta\mathbb{E}_t V_{jt+1} - \tau^{ij}]\right)\right), \tag{13}$$

where the flow utility from working in sector j at time t is:

$$U_{jt}^{\text{flow}} \equiv \frac{W_{jt}}{P_t^C} C_t^{-\gamma} L_{jt} - \zeta_j \frac{L_{jt}^{1+\varphi}}{1+\varphi}. \tag{14}$$

Using the intratemporal labor supply condition (8), this simplifies to:

$$U_{jt}^{\text{flow}} = \zeta_j \frac{\varphi}{1+\varphi} L_{jt}^{1+\varphi}. \tag{15}$$

Steady-state worker flows. Let $F \equiv \{F_{ij}\} = \{\bar{\mu}^{ij}\}$ denote the *forward* transition matrix, so that F_{ij} is the steady-state probability that a worker in sector i moves to sector j. The stationary distribution satisfies $\sum_i \bar{N}_i F_{ij} = \bar{N}_j$. Define the *backward* transition matrix B by

$$B_{ji} \equiv \frac{\bar{N}_i}{\bar{N}_j} F_{ij}. \tag{16}$$

The entry B_{ji} gives the fraction of workers currently in sector j who arrived from sector i: F describes where workers go, B describes where they came from. Both are row-stochastic with the same stationary distribution $\bar{\mathbf{N}}$. Since $\bar{\mathbf{N}}$ is itself determined by F, the backward matrix B is fully pinned down by F. We show in Section 2.5 that the linearized equilibrium depends on the micro structure of reallocation only through F, B, and a scalar mobility elasticity ϕ.

2.4 Equilibrium

Market clearing. In equilibrium, the goods market for each variety and the aggregate economy must clear:

$$Y_{it}(f) = C_{it}(f), \quad Y_t = C_t, \tag{17}$$

where C_t is household consumption. Combined with Cobb-Douglas demand, this implies that nominal sectoral output equals the expenditure share of total nominal consumption:

$$P_{it}Y_{it} = \alpha_i P_t^C C_t. \tag{18}$$

Labor market clearing in each sector requires:

$$\int_0^1 L_{it}(f)\, df = N_{it}L_{it}. \tag{19}$$

The government finances the output subsidies through lump-sum taxes, balancing its budget each period:

$$T_t + \sum_{i \in \mathcal{N}} \frac{1}{\epsilon_i - 1} P_{it}Y_{it} = 0. \tag{20}$$

The central bank sets the nominal interest rate i_t according to a policy rule specified below.

Definition 1 (Equilibrium)**.** *Given initial conditions* $(\{P_{i,-1}\}, \{N_{i0}\})$ *and a sequence of productivity shocks* $\{A_{it}\}$*, an equilibrium is a collection of prices* $\{P_{it}(f), P_{it}, P_t^C, W_{it}, i_t\}$*, quantities* $\{C_t, Y_{it}, L_{it}(f), L_{it}, B_t\}$*, employment shares* $\{N_{it}\}$*, transition probabilities* $\{\mu_t^{ij}\}$*, and value functions* $\{V_{it}\}$ *such that: (i) households optimize consumption, savings, labor supply, and reallocation; (ii) firms optimize pricing subject to Calvo constraints; (iii) all markets clear and the government budget constraint* (20) *holds; and (iv) the central bank follows its policy rule.*

Steady state. We impose that steady-state flow surplus is equalized across sectors: $\zeta_i \bar{L}_i^{1+\varphi}$ is constant across i. This ensures an interior steady state: without it, the frictionless limit ($\theta \to \infty$) would concentrate all workers in the highest-surplus sector. We achieve this by normalizing $\bar{L}_i = \bar{L}$ and $\zeta_i = \zeta$ for all i.[3] Combining steady-state labor supply (8) with goods market clearing (18) gives $\alpha_i \bar{C}^{1-\gamma} = \zeta \bar{N}_i \bar{L}^{1+\varphi}$, so $\bar{N}_i = \alpha_i$: steady-state employment shares equal expenditure shares.

2.5 Linearized Equilibrium

Log-linearization of the equilibrium conditions yields the following system; derivations are in Appendix A. The system organizes into three groups: cost determination, pricing and demand. The latter two blocks are standard; the cost determination block is where labor reallocation enters.

[3]The normalization $\bar{L}_i = \bar{L}$ is without loss of generality: N_{it} measures the mass of workers in sector i, and L_{it} can be interpreted as either per-worker hours or the fraction of workers who supply labor (Galí, 2011). The condition $\zeta_i = \zeta$ then ensures equal surplus.

Notation. For any variable X_t, we write $x_t \equiv \log X_t - \log \bar{X}$ for its log-deviation from steady state. We use the following conventions throughout:

- *Natural (flexible-price) levels*: x_t^{nat} denotes the value under flexible prices.
- *Gaps*: $\tilde{x}_t \equiv x_t - x_t^{\text{nat}}$ denotes the deviation from the natural level.
- *Relative variables*: $\hat{p}_{it} \equiv p_{it} - p_t^C$ is the log relative price, $\hat{w}_{it} \equiv w_{it} - p_t^C$ is the log real wage, $\hat{\pi}_{it} \equiv \pi_{it} - \pi_t^C$ is relative inflation, and $\hat{a}_{it} \equiv a_{it} - \sum_j \alpha_j a_{jt}$ is relative productivity.

We make the standard exception for interest rates: i_t and r_t^{nat} denote levels, not log-deviations from steady state. In this subsection, we write $\overset{(1)}{=}$ for equalities that hold up to a first-order approximation, and $\overset{(2)}{=}$ for second-order. Plain $=$ denotes exact relationships.

2.5.1 Real Marginal Cost and Labor Reallocation

We begin with the supply side, where labor reallocation enters. Log-linearizing nominal marginal cost $MC_{it} = W_{it}/A_{it}$ and substituting the labor supply condition (8), production function (4), and Cobb–Douglas sectoral demand yields:

$$mc_{it} - p_{it} \overset{(1)}{=} (\gamma + \varphi)\, y_t - (1 + \varphi)(\hat{p}_{it} + a_{it}) - \varphi\, n_{it}. \tag{21}$$

Three terms drive sectoral cost pressures: aggregate demand y_t, the relative price–productivity gap $\hat{p}_{it} + a_{it}$, and inherited employment n_{it}. The first two are standard; the third is the novel channel. An overstaffed sector ($n_{it} > 0$) has lower hours per worker and hence lower marginal cost, while an understaffed sector faces elevated cost pressure.

Relative prices evolve as:

$$\hat{p}_{it} = \hat{p}_{it-1} + \pi_{it} - \pi_t^C. \tag{22}$$

The employment term n_{it} in (21) is governed by the labor reallocation block. Using bold for N-vectors and normalizing value-function and flow-utility deviations by $\varphi\zeta\bar{L}^{1+\varphi}$:

$$\mathbf{n}_{t+1} \overset{(1)}{=} B\,\mathbf{n}_t + \phi\beta(I - BF)\,\mathbb{E}_t[\mathbf{v}_{t+1}], \tag{23}$$

$$\mathbf{v}_t \overset{(1)}{=} \mathbf{u}_t + \beta\, F\, \mathbb{E}_t[\mathbf{v}_{t+1}], \tag{24}$$

$$u_{it} \overset{(1)}{=} y_t - \hat{p}_{it} - a_{it} - n_{it}, \tag{25}$$

where $\phi \equiv \varphi\theta\zeta\bar{L}^{1+\varphi}$ is the *mobility elasticity*, F and B are the forward and backward transition

matrices defined in (16), and the aggregate constraint $\sum_i \alpha_i n_{it} = 0$ holds throughout.[4] The mobility elasticity has a direct interpretation: $\phi\beta/\varphi$ is the elasticity of relative transition probabilities to destination wages.[5]

Equation (23) is the law of motion for employment: absent value differentials, the distribution reverts toward steady state through B; continuation-value gaps tilt flows through the operator $(I - BF)$ scaled by ϕ. Equation (24) is the Bellman recursion: iterating forward,

$$\mathbf{v}_t \overset{(1)}{=} \sum_{s=0}^{\infty} \beta^s F^s \, \mathbb{E}_t[\mathbf{u}_{t+s}], \tag{26}$$

so the continuation value in each sector reflects the entire expected path of flow utilities, discounted by both β and F.

2.5.2 Pricing

Given real marginal costs, sectoral inflation is determined by the New Keynesian Phillips curve:

$$\pi_{it} \overset{(1)}{=} \kappa_i \left[mc_{it} - p_{it} \right] + \beta \, \mathbb{E}_t \pi_{it+1}, \tag{27}$$

where $\kappa_i \equiv (1 - \delta_i)(1 - \beta\delta_i)/\delta_i$.

2.5.3 Demand

The IS curve is:

$$\tilde{y}_t \overset{(1)}{=} \mathbb{E}_t \tilde{y}_{t+1} - \frac{1}{\gamma}\left(i_t - \mathbb{E}_t \pi^C_{t+1} - r^{\text{nat}}_t \right), \tag{28}$$

where $\pi^C_t = \sum_i \alpha_i \pi_{it}$ is CPI inflation and $r^{\text{nat}}_t = -\log\beta + \gamma(\mathbb{E}_t y^{\text{nat}}_{t+1} - y^{\text{nat}}_t)$ is the natural real rate. The central bank sets the nominal interest rate according to:

$$i_t = \psi_\pi \, \pi_t(\omega) + \psi_y \, \tilde{y}_t, \tag{29}$$

for some inflation index $\pi_t(\omega) = \sum_i \omega_i \pi_{it}$ with weights summing to one. Sectoral demand follows from Cobb–Douglas preferences: $y_{it} - y_t = -\hat{p}_{it}$.

The demand block is identical to a textbook New Keynesian model: labor reallocation enters the system entirely through the supply side, via real marginal cost in equations (21)–(27).

[4] Here $\mathbf{v}_t$ and $\mathbf{u}_t$ are normalized deviations from steady state, with $v_{it} \equiv (V_{it} - \bar{V}_i)/(\varphi\zeta\bar{L}^{1+\varphi})$ and u_{it} defined analogously. Combining steady-state labor supply (8) with goods market clearing (18) gives $\zeta\bar{L}^{1+\varphi} = \bar{C}^{1-\gamma}$, so $\phi = \varphi\theta\bar{C}^{1-\gamma}$.

[5] Appendix D.2 derives this result and maps ϕ to the dispersion parameter ν in Artuç et al. (2010); Caliendo et al. (2019), giving $\phi = \varphi/\nu$.

Because the demand side is standard and separable, any alternative demand structure (hand-to-mouth consumers, heterogeneous agents) would replace only the IS curve (28), leaving our core mechanism unchanged. Analogously, because reallocation enters only through real marginal cost, the mechanism is agnostic to the specific micro-foundation of price setting.

3 Dynamics of Inflation and Labor Reallocation

We now characterize the dynamics of the supply side. We establish two general results that hold for any labor transition matrix F. First, the system admits a recursive representation in the state $\mathbf{s}_t = (\chi_{t-1}, \mathbf{n}_t)$, driven by the expected paths of aggregate demand and sectoral productivity. Second, the eigenvalues of the associated transition matrix $\mathbf{P}$ govern the persistence of sectoral inflation and labor misallocation.

Throughout this section, we analyze how the economy absorbs shocks conditional on a given path of aggregate demand; Section 5 studies the optimal choice of that path.

3.1 Sufficient Statistics and the State

Despite the rich micro structure of the reallocation block (23)–(25), the linearized system depends on it only through two objects:

Proposition 1 (Sufficient Statistics)**.** *The linearized system depends on the labor reallocation structure only through:*

(i) the mobility elasticity $\phi = \varphi\theta\zeta\bar{L}^{1+\varphi}$*, and*

(ii) the forward transition matrix F *(and the backward matrix* B *derived from it via* (16)*).*

The limiting cases are: $\phi \to 0$ *(segmented markets), in which* $\mathbf{n}_t = \mathbf{0}$ *for all* t*; and* $\phi \to \infty$ *(free mobility), in which expected continuation values equalize across sectors (*$\mathbb{E}_t[\mathbf{v}_{t+1}] \propto \mathbf{1}$*) and, for all* $t \geq 1$*,* n_{it} *offsets the relative price-productivity gap (*$n_{it} = -(\hat{p}_{it} + \hat{a}_{it})$ *for* $t \geq 1$*).*

Proof. See Appendix C.1. □

This parallels findings in dynamic spatial models, where the linearized system similarly reduces to a transition matrix and a mobility elasticity (Kleinman et al., 2023; Seo and Oh, 2025). Our contribution is embedding this structure into a New Keynesian model, where it interacts with nominal rigidities through sectoral marginal cost.

To identify the state variables of the system, we first characterize the flexible-price benchmark. Because employment is predetermined, the planner can adjust hours within the period but must wait for the reallocation block to correct the distribution of workers across sectors:

Lemma 1 (Natural Allocation). *The flexible-price allocation satisfies:*

(i) Natural output $y_t^{\text{nat}} = \frac{1+\varphi}{\gamma+\varphi} \sum_i \alpha_i a_{it}$ *and the natural real rate* $r_t^{\text{nat}} = -\log\beta + \gamma\, \mathbb{E}_t(y_{t+1}^{\text{nat}} - y_t^{\text{nat}})$ *are independent of* $\mathbf{n}_t$.

(ii) Sectoral hours and natural relative prices depend on inherited employment:

$$l_{it}^{\text{nat}} = \frac{1-\gamma}{1+\varphi}\, y_t^{\text{nat}} - \frac{1}{1+\varphi}\, n_{it}, \tag{30}$$

$$\hat{p}_{it}^{\text{nat}} = \hat{p}_{it}^{\text{nat},0} - \frac{\varphi}{1+\varphi}\, n_{it}, \tag{31}$$

where $\hat{p}_{it}^{\text{nat},0} = \sum_j \alpha_j a_{jt} - a_{it}$ *is the natural relative price when employment is at its steady-state distribution. An overstaffed sector* ($n_{it} > 0$) *has lower hours and a lower natural relative price.*

Proof. See Appendix C.1. □

Part (i) means that the IS curve and natural real rate are standard: labor reallocation affects only the supply side. Part (ii) motivates the state variables. Define the *relative price distortion*

$$\chi_{it} \equiv \hat{p}_{it} + \hat{a}_{it}, \tag{32}$$

the deviation of relative prices from the steady-state benchmark $\hat{p}_{it}^{\text{nat},0}$. We collect $\hat{p}_{it}$ and $\hat{a}_{it}$ into χ_{it} because they always appear together in the marginal cost (21), the flow utility (25), and the natural relative price (31).[6] In this notation, the marginal cost equation (21) becomes:

$$mc_{it} - p_{it} = (\gamma + \varphi)\, \tilde{y}_t - (1+\varphi)\, \chi_{it} - \varphi\, n_{it}. \tag{33}$$

Two variables summarize the supply side: χ_{it} measures *price misalignment* and n_{it} measures *employment misallocation*. The relative price distortion evolves as

$$\boldsymbol{\chi}_t = \boldsymbol{\chi}_{t-1} + \hat{\boldsymbol{\pi}}_t + \Delta\hat{\mathbf{a}}_t, \tag{34}$$

which follows from (22). Since relative inflation $\hat{\boldsymbol{\pi}}_t$ is determined by period-t pricing decisions, $\boldsymbol{\chi}_t$ is endogenous within the period; only $\boldsymbol{\chi}_{t-1}$ is inherited. Employment $\mathbf{n}_t$, by contrast, is predetermined. The inherited state entering period t is therefore $\mathbf{s}_t = (\boldsymbol{\chi}_{t-1}, \mathbf{n}_t)$. Sticky prices create a feedback loop between the two margins: price misalignment contaminates the flow-utility signals that drive worker transitions, and the resulting misallocation feeds back into marginal cost. The next subsection characterizes the dynamics of this feedback loop.

[6] The natural relative price is $\hat{p}_{it}^{\text{nat}} = \hat{p}_{it}^{\text{nat},0} - \frac{\varphi}{1+\varphi}\, n_{it}$ (part (ii)), so the deviation from the contemporaneous flexible-price benchmark is $\hat{p}_{it} - \hat{p}_{it}^{\text{nat}} = \chi_{it} + \frac{\varphi}{1+\varphi}\, n_{it}$. Defining χ_{it} relative to the steady-state benchmark ensures that χ_{it} and n_{it} enter additively, making the two margins of adjustment transparent.

3.2 Transition Dynamics and the Generalized Phillips Curve

The marginal cost equation (33), the sectoral pricing rule (27), and the reallocation block (23)–(25) jointly define a *generalized Phillips curve*: a mapping from the paths of aggregate demand and productivity into sectoral inflation,

$$\{\pi_{it}\}_{i,t} = \mathcal{K}(\{y_t\}_t,\ \{a_{it}\}_{i,t})\,. \tag{35}$$

We now show that this mapping admits a recursive representation. Solving out the forward-looking variables (sectoral inflation $\boldsymbol{\pi}_t$ and continuation values $\hat{\mathbf{v}}_t$) yields a first-order law of motion for the predetermined state $\mathbf{s}_t = (\boldsymbol{\chi}_{t-1}, \mathbf{n}_t)$, driven by the expected paths of aggregate demand and productivity.[7]

Proposition 2 (State-Space Representation)**.** *Let $\mathcal{R} \equiv \{x \in \mathbb{R}^N : \boldsymbol{\alpha}^\top x = 0\}$ denote the relative subspace. Under determinacy, the supply-side equilibrium admits a first-order linear state-space representation with transition equation*

$$\mathbf{s}_{t+1} = \mathbf{P}\,\mathbf{s}_t + \mathbf{b}_t^y + \mathbf{b}_t^a, \qquad \mathbf{s}_t \equiv (\boldsymbol{\chi}_{t-1},\, \mathbf{n}_t) \in \mathcal{R} \times \mathcal{R}, \tag{36}$$

with

$$\mathbf{b}_t^y = \sum_{j=0}^{\infty} \mathbf{H}_j^y\, \mathbb{E}_t[\tilde{\mathbf{y}}_{t+j}], \qquad \mathbf{b}_t^a = \sum_{j=0}^{\infty} \mathbf{H}_j^a\, \mathbb{E}_t[\Delta\hat{\mathbf{a}}_{t+j}],$$

and the observation equation

$$\hat{\boldsymbol{\pi}}_t = \Theta_{11}\,\boldsymbol{\chi}_{t-1} + \Theta_{12}\,\mathbf{n}_t + \sum_{j=0}^{\infty} \Pi_j^y\, \mathbb{E}_t[\tilde{\mathbf{y}}_{t+j}] + \sum_{j=0}^{\infty} \Pi_j^a\, \mathbb{E}_t[\Delta\hat{\mathbf{a}}_{t+j}], \tag{37}$$

with all coefficient sequences decaying exponentially. The $2(N-1) \times 2(N-1)$ transition matrix $\mathbf{P}$ depends only on $(\varphi, \beta, \phi, F, B, K)$ and satisfies $\rho(\mathbf{P}) < 1$ when F is primitive. The impact matrices Θ_{11} and Θ_{12} map inherited price misalignment and employment misallocation into current relative inflation.

Proof. See Appendix C.1. □

Proposition 2 pins down relative inflation $\hat{\pi}_{it} = \pi_{it} - \pi_t^C$ but not the aggregate component $\pi_t^C = \sum_i \alpha_i \pi_{it}$. Since both χ_{it} and n_{it} have zero expenditure-weighted means, aggregate inflation depends on the state only through its covariance with sectoral Phillips curve slopes. Define

$$\mathrm{Cov}_\alpha(\kappa_i,\, x_{it}) \equiv \sum_i \alpha_i \kappa_i\, x_{it} = \boldsymbol{\alpha}^\top \mathbf{K}\, \mathbf{x}_t$$

for any $\mathbf{x}_t \in \mathcal{R}$. We have:

[7] Both $\boldsymbol{\chi}_{t-1}$ and $\mathbf{n}_t$ satisfy $\sum_i \alpha_i \chi_{it} = 0$ and $\sum_i \alpha_i n_{it} = 0$, so the state is $2(N-1)$-dimensional.

Proposition 3 (Generalized Phillips Curve). *The mapping* $\{\pi_{it}\}_{i,t} = \mathcal{K}(\{y_t\}_t,\ \{a_{it}\}_{i,t})$ *is given by*

$$\boldsymbol{\pi}_t = \hat{\boldsymbol{\pi}}_t + \pi_t^C \mathbf{1}, \tag{38}$$

where $\hat{\boldsymbol{\pi}}_t$ *satisfies Equation* (37) *and aggregate inflation satisfies*

$$\pi_t^C = \bar{\kappa}(\gamma + \varphi)\,\tilde{y}_t - (1+\varphi)\ \mathrm{Cov}_\alpha(\kappa_i,\, \chi_{it}) - \varphi\ \mathrm{Cov}_\alpha(\kappa_i,\, n_{it}) + \beta\,\mathbb{E}_t \pi_{t+1}^C, \tag{39}$$

with $\bar{\kappa} \equiv \sum_i \alpha_i \kappa_i$. *Under uniform stickiness* ($\kappa_i = \kappa$), *both covariance terms vanish and aggregate inflation follows the standard one-sector Phillips curve.*

Proof. Weighting the sectoral Phillips curve (27) by α_i and summing gives $\pi_t^C = \sum_i \alpha_i \kappa_i (mc_{it} - p_{it}) + \beta\,\mathbb{E}_t \pi_{t+1}^C$. Substituting (33) yields (39). Under uniform κ, $\boldsymbol{\alpha}^\top \mathbf{K}\, \boldsymbol{\chi}_t = \kappa \sum_i \alpha_i \chi_{it} = 0$ and likewise for $\mathbf{n}_t$. □

The generalized Phillips curve is the supply-side constraint: given the inherited state $\mathbf{s}_t$ and the expected paths of aggregate demand and productivity, equations (36)–(39) recursively determine current relative inflation, aggregate inflation, and, via (34), the next period's state. The mapping $\mathcal{K}$ therefore is the object for the monetary policy problem in Section 5.

Equation (39) makes precise the channel through which inherited sectoral distortions feed into aggregate inflation. The two covariance terms act as endogenous cost-push shifters. The first, $\mathrm{Cov}_\alpha(\kappa_i, \chi_{it})$, is inflationary when sticky-price sectors (low κ) have relative prices that are too high (high χ). The second, $\mathrm{Cov}_\alpha(\kappa_i, n_{it})$, is inflationary when sticky-price sectors are overstaffed: excess employment lowers their marginal cost, while flexible-price sectors have high cost, and those sectors transmit into inflation quickly.

Under uniform stickiness, both covariances vanish and the relative and aggregate blocks decouple entirely:

Corollary 1 (Block Separation). *If* $\kappa_i = \kappa$ *for all* i, *then* $\mathbf{H}_j^y = \mathbf{0}$ *and* $\Pi_j^y = \mathbf{0}$ *for all* j: *aggregate demand drops out of both the transition equation* (36) *and the observation equation* (37), *and the state drops out of the aggregate Phillips curve* (39). *The relative block evolves independently of monetary policy, and the aggregate economy reduces to a standard one-sector New Keynesian model in* $(\pi_t^C, \tilde{y}_t)$.

Proof. See Appendix C.1. □

When $\kappa_i = \bar{\kappa}$, monetary policy can stabilize the aggregate economy but cannot influence sectoral reallocation: aggregate demand shifts all sectoral marginal costs equally, leaving relative prices and reallocation incentives unchanged.[8]

[8] When κ_i varies across sectors, flexible sectors respond more to aggregate demand than sticky ones, generating relative price movements that tilt reallocation incentives. Section 4 characterizes when this channel is quantitatively important.

Under heterogeneous stickiness, the covariances are generically nonzero, and the relative block spills over to the aggregate Phillips curve whenever misalignment is correlated with the distribution of nominal rigidities.

Persistence and the transition matrix. The persistence of these cost-push dynamics is governed by the eigenvalues of $\mathbf{P}$: slow-decaying eigenvalues sustain the covariance terms in (39) long after the initial shock. The next subsection characterizes these eigenvalues through a spectral decomposition of $\mathbf{P}$.

3.3 Spectral Analysis

To isolate which patterns of misalignment are most persistent and how shocks map into them, we eigendecompose the transition matrix $\mathbf{P}$.[9] Write $\mathbf{P} = \mathbf{U}\,\mathbf{\Lambda}\,\mathbf{V}$, where $\mathbf{\Lambda} = \mathrm{diag}(\lambda_1, \ldots, \lambda_{2(N-1)})$ contains the eigenvalues in decreasing order by absolute value, $\mathbf{U}$ collects the right eigenvectors $\mathbf{u}_h$ as columns, and $\mathbf{V} = \mathbf{U}^{-1}$ collects the left eigenvectors $\mathbf{v}_h^\top$ as rows:

$$\lambda_h\,\mathbf{u}_h = \mathbf{P}\,\mathbf{u}_h, \qquad \lambda_h\,\mathbf{v}_h^\top = \mathbf{v}_h^\top\,\mathbf{P}, \qquad \mathbf{v}_j^\top\,\mathbf{u}_h = \mathbf{1}_{j=h}. \tag{40}$$

Since $\rho(\mathbf{P}) < 1$ (Proposition 2), all eigenvalues satisfy $|\lambda_h| < 1$. Each right eigenvector $\mathbf{u}_h = (\mathbf{u}_h^\chi, \mathbf{u}_h^n)$ identifies a *corridor*: a self-contained joint pattern of price misalignment and employment misallocation across sectors that shrinks by the factor λ_h each period through $\mathbf{P}$ without coupling to other corridors.

To see how the corridors shape impulse responses, start from the steady-state $(\boldsymbol{\chi}_{t-1}, \boldsymbol{n}_t) = 0$ and consider a one-time permanent productivity shock $\Delta\hat{\mathbf{a}}_0 = \hat{\mathbf{a}}$ with $\tilde{y}_t = 0$. The shock generates a post-impact state $\mathbf{s}_1 = \mathbf{H}_0^a\,\hat{\mathbf{a}}$ that subsequently evolves as $\mathbf{s}_t = \mathbf{P}^{t-1}\mathbf{s}_1$. Expanding in the eigenbasis:

Lemma 2 (Corridor Expansion). *The state at $t \geq 1$ satisfies*

$$\mathbf{s}_t = \sum_{h=1}^{2(N-1)} \lambda_h^{t-1}\,(\mathbf{v}_h^\top \mathbf{s}_1)\,\mathbf{u}_h, \tag{41}$$

with half-life $\lceil -\ln 2 / \ln|\lambda_h| \rceil$ along corridor h and cumulative state $\sum_{t=1}^{\infty} \mathbf{s}_t = \sum_h \frac{\mathbf{v}_h^\top \mathbf{s}_1}{1-\lambda_h}\,\mathbf{u}_h$. Slow corridors are amplified by $1/(1-\lambda_h)$: their contribution to cumulative misalignment exceeds their share of the post-impact state.

Proof. The expansion is immediate from the eigenbasis. The cumulative formula sums the geometric series $\sum_{t=1}^{\infty} \lambda_h^{t-1} = 1/(1-\lambda_h)$. □

[9] This adapts the spectral approach to spatial transition matrices in Kleinman et al. (2023) and to input-output networks in Liu and Tsyvinski (2024) to the joint dynamics of prices and employment in a New Keynesian setting.

The lemma characterizes how any shock decomposes into corridors and how those corridors accumulate. To give this decomposition economic content, consider a shock in a single sector j and separate the inflation response into its price-misalignment and employment-misallocation components. For corridor h, define the inflation response pattern

$$\mathbf{q}_h \equiv \Theta \mathbf{u}_h = \underbrace{\Theta_{11}\mathbf{u}_h^{\chi}}_{\mathbf{q}_h^{\chi}} + \underbrace{\Theta_{12}\mathbf{u}_h^{n}}_{\mathbf{q}_h^{n}}, \tag{42}$$

so that q_{ih} gives sector i's relative inflation when corridor h is active. The decomposition $\mathbf{q}_h = \mathbf{q}_h^{\chi} + \mathbf{q}_h^{n}$ separates two channels: $\mathbf{q}_h^{\chi}$ is the inflation generated by inherited relative-price misalignment along corridor h, while $\mathbf{q}_h^{n}$ is the inflation generated by inherited employment misallocation. The second term is the labor-reallocation cost-push channel: employment misallocation enters marginal cost directly through (33), and $\mathbf{q}_h^{n}$ captures how this cost pressure maps into inflation along corridor h.

For a unit productivity shock to sector j ($\Delta\hat{a}_{i0} = \delta_{ij} - \alpha_j$), define the *shock exposure* of corridor h as

$$c_h^{(j)} \equiv \mathbf{v}_h^{\top} \mathbf{H}_0^{a} (\mathbf{e}_j - \alpha_j \mathbf{1}), \tag{43}$$

where $\mathbf{e}_j$ is the j-th standard basis vector. The exposure $c_h^{(j)}$ measures how much the shock activates corridor h, and is determined by sector j's position in the labor flow network.

Proposition 4 (Shock-Corridor Decomposition)**.** *Consider a one-time permanent unit productivity shock to sector j with $\tilde{y}_t = 0$ for all t. Impact relative inflation is $\hat{\boldsymbol{\pi}}_0^{(j)} = \Pi_0^{a} (\mathbf{e}_j - \alpha_j \mathbf{1})$. For $t \geq 1$:*

$$\hat{\boldsymbol{\pi}}_t^{(j)} = \sum_{h=1}^{2(N-1)} \lambda_h^{t-1} \, c_h^{(j)} \, \mathbf{q}_h. \tag{44}$$

Cumulative relative inflation in sector i up to horizon T is

$$\sum_{t=0}^{T} \hat{\pi}_{it}^{(j)} = \left[\Pi_0^{a} (\mathbf{e}_j - \alpha_j \mathbf{1})\right]_i + \sum_{h=1}^{2(N-1)} \frac{1 - \lambda_h^{T}}{1 - \lambda_h} \, q_{ih} \, c_h^{(j)}. \tag{45}$$

The exposures $c_h^{(j)}$ are computable from the worker transition matrix F and sectoral price rigidities K. As $T \to \infty$, this converges to the long-run identity $\sum_{t=0}^{\infty} \hat{\pi}_{it}^{(j)} = \alpha_j - \delta_{ij}$.

Proof. The post-impact state is $\mathbf{s}_1 = \mathbf{H}_0^{a} (\mathbf{e}_j - \alpha_j \mathbf{1})$. For $t \geq 1$, $\Delta\hat{\mathbf{a}}_t = \mathbf{0}$ and $\tilde{y}_t = 0$, so Lemma 2 gives $\mathbf{s}_t = \sum_h \lambda_h^{t-1} c_h^{(j)} \mathbf{u}_h$. Applying $\hat{\boldsymbol{\pi}}_t = \Theta \mathbf{s}_t$ yields (44). Summing from $t = 1$ to T and adding impact inflation gives (45). The long-run identity follows because relative prices must eventually fully reflect the permanent productivity shock: the shocked sector j experiences cumulative

relative deflation of $1 - \alpha_j$ while every other sector experiences cumulative relative inflation of α_j. □

Equation (45) decomposes the adjustment path into three ingredients. The shock exposure $c_h^{(j)}$ captures which corridors a shock to sector j activates, determined by the sector's position in the labor flow network. The amplification factor $(1 - \lambda_h^T)/(1 - \lambda_h)$ governs how much each corridor contributes through horizon T; persistent corridors accumulate because they decay slowly. The inflation incidence q_{ih} determines which sectors bear the resulting inflation along each corridor; its decomposition $q_{ih} = q_{ih}^{\chi} + q_{ih}^{n}$ distinguishes inflation from inherited price misalignment from inflation driven by employment misallocation through marginal cost (33).

The persistence of inflation following a shock to sector j is therefore not determined by sector j's own price rigidity. It is determined by the stickiness of the sectors that j exchanges workers with—those sitting on the other side of the corridors the shock activates. A shock to a flexible-price sector tightly connected to sticky-price sectors through labor flows activates slow corridors and generates persistent misalignment, even though the shocked sector's own prices adjust quickly.

For aggregate inflation, persistence further requires that the activated corridors align with sectoral Phillips curve slopes. For example, a negative productivity shock to a flexible sector that primarily exchanges workers with sticky sectors activates slow corridors whose misalignment concentrates where κ_i is low, generating a persistent positive cost-push shifter in (39).

More generally, projecting the transition equation (36) onto corridor h shows that each corridor evolves as an AR(1):

$$s_{ht+1}^{*} = \lambda_h \, s_{ht}^{*} + \mathbf{v}_h^{\top} (\mathbf{b}_t^{y} + \mathbf{b}_t^{a}), \qquad s_{ht}^{*} \equiv \mathbf{v}_h^{\top} \, \mathbf{s}_t. \tag{46}$$

A corridor that decays slowly ($|\lambda_h|$ close to one) generates persistent misalignment, and the term $\mathbf{v}_h^{\top} \mathbf{b}_t^{y}$ determines whether aggregate demand can steer that corridor. Which corridors decay slowly and whether aggregate demand can steer them depends on the interaction of price stickiness and the labor flow network. Section 3.4 builds intuition at the polar cases of ϕ; Section 4 provides a full characterization under a symmetry condition on F.

3.4 Segmented versus Integrated Markets

To clarify how labor reallocation affects the persistence of the system, we examine the eigenvalues of $\mathbf{P}$ at two polar extremes of labor mobility. At $\phi = 0$, sectoral labor markets are fully segmented; at $\phi \to \infty$, labor is perfectly mobile, recovering the baseline multi-sector New Keynesian economy (e.g., Rubbo, 2023).

When labor markets are segmented ($\phi = 0$), employment never moves ($\mathbf{n}_t = \mathbf{0}$ for all t). The shock-generated dynamics lie in the subspace $\{(\chi, \mathbf{0})\}$, and firms face the full intensive-margin

slope $\kappa_i(1+\varphi)$ in marginal cost. Under uniform stickiness, the restriction of $\mathbf{P}$ to this subspace has stable root $\lambda_{\text{seg}} \in (0,1)$ solving

$$\beta\lambda^2 - [\kappa(1+\varphi) + \beta + 1]\lambda + 1 = 0. \tag{47}$$

When labor is perfectly mobile, employment offsets price distortions ($n_{it} = -\chi_{it}$ for $t \geq 1$). The shock-generated dynamics lie in the subspace $\{(\chi, -\chi)\}$, and the effective slope drops to κ_i: reallocation absorbs the intensive-margin cost pressure. Under uniform stickiness, the restriction of $\mathbf{P}$ to this subspace has stable root $\lambda_{\text{int}} \in (0,1)$ solving

$$\beta\lambda^2 - (\kappa + \beta + 1)\lambda + 1 = 0. \tag{48}$$

The two characteristic equations differ only in the coefficient on λ: $\kappa(1+\varphi) + \beta + 1$ versus $\kappa + \beta + 1$. Since $\kappa\varphi > 0$, the segmented-market equation has a larger middle coefficient, which pushes the stable root down: $\lambda_{\text{seg}} < \lambda_{\text{int}}$. Thus we have:

Proposition 5 (Segmented vs. Integrated Markets)**.** *Under uniform stickiness ($\kappa_i = \kappa$), consider a one-time permanent productivity shock with $\tilde{y}_t = 0$:*

(i) Segmented labor markets ($\phi = 0$) produce a larger initial inflation response and faster subsequent decay at rate λ_{seg}.

(ii) Integrated labor markets ($\phi \to \infty$) produce a smaller initial inflation response and slower subsequent decay at rate $\lambda_{\text{int}} > \lambda_{\text{seg}}$.

(iii) Cumulative relative inflation is $\sum_{t=0}^{\infty} \hat{\pi}_{it} = -\hat{a}_i$ in both cases.

Proof. See Appendix C.1. □

The mechanism is forward-looking price setting. Under integration, firms anticipate that worker inflows will offset cost pressures. This anticipated relief reduces the price adjustment firms choose today, dampening impact inflation. But the same anticipation weakens the cost pressure that drives convergence, raising persistence. Under segmentation, cost pressures are unmitigated and firms reprice aggressively: the initial response is larger, but convergence is faster.
Part (iii) is an accounting identity: long-run relative prices must fully reflect the productivity shock regardless of mobility. Mobility reshapes the path but not the destination.

For intermediate ϕ, both margins are active and the eigenvalues of $\mathbf{P}$ depend on their interaction. In general, these eigenvalues can only be computed numerically, may be complex, and offer no direct interpretation of how price adjustment and labor reallocation jointly determine persistence. The next section obtains closed-form expressions under an additional mild restriction on the reallocation cost structure.

4 Closed-Form Results under Quasi-symmetry

The eigenvalues of $\mathbf{P}$ are implicit functions of the full parameter vector $(\varphi, \beta, \phi, F, B, \mathbf{K})$ and have no closed-form expressions in general. In this section, we analytically characterize them by restricting bilateral reallocation costs to be quasi-symmetric:

Assumption 1 (Quasi-Symmetric Reallocation Costs)**.** *The bilateral reallocation costs satisfy* $\tau^{ii} = 0$ *and* $\tau^{ij} = \tau_i^{out} + \tau_j^{in} + \hat{\tau}^{ij}$ *for* $i \neq j$*, where* $\hat{\tau}^{ij} = \hat{\tau}^{ji}$ *for all* i, j*.*

This is standard in the dynamic discrete choice literature (Artuç et al., 2010; Allen et al., 2020; Seo and Oh, 2025), nesting symmetric costs and additive origin-destination costs. Its key implication is that F is reversible ($B = F$), with real eigenvalues and an orthogonal eigenbasis:

Lemma 3 (Diagonalization of F)**.** *Under Assumption 1:*

(i) The steady-state transition matrix satisfies $\bar{N}_i F_{ij} = \bar{N}_j F_{ji}$*, so* $B = F$*.*

(ii) The matrix $Q \equiv A^{1/2} F A^{-1/2}$*, where* $A = \operatorname{diag}(\alpha_i)$*, is real symmetric with eigenvalues* $1 = \rho_1 > \rho_2 \geq \cdots \geq \rho_N > -1$ *and orthonormal eigenvectors* $\{s_k\}$*.*

Proof. See Appendix C.2. □

Reversibility is a close approximation in U.S. data: the asymmetry measure $\|Q - Q^\top\| / \|Q\|$ is 8×10^{-4} (Appendix D.1). Under uniform stickiness (Section 4.1), the diagonalization of F block-diagonalizes the full supply-side system into $N-1$ independent components, each pairing a Phillips curve with a reallocation equation. Section 4.2 relaxes uniform stickiness and derives two diagnostic statistics that characterize when reallocation frictions amplify persistence and when monetary policy has leverage.[10]

Table 1 collects the key objects from Sections 3.3–4.2 for reference.

4.1 Independent Components under Uniform Stickiness

Under uniform stickiness ($\kappa_i = \kappa$), quasi-symmetry (Lemma 3) and block separation (Corollary 1) jointly diagonalize the relative block into $N - 1$ independent systems.

Lemma 4 (Spectral Decoupling)**.** *Under Assumption 1 and uniform stickiness* ($\kappa_i = \kappa$)*, the coordinate transformation* $x_{kt}^* = s_k^\top A^{1/2} \hat{\mathbf{x}}_t$ *for* $x \in \{\chi, n, \hat{v}, \hat{a}\}$ *decomposes the relative block into* $N-1$ *independent*

[10] We reserve *corridor* (index h) for the eigenvectors of the full transition matrix $\mathbf{P}$, which are joint price-employment patterns in $\mathbb{R}^{2(N-1)}$ (Section 3.3), and *component* (index k) for the eigenvectors of F, which are pure labor-flow patterns in $\mathbb{R}^N$. Under uniform stickiness, each component generates a pair of corridors through the quartic (52).

Object	Space	Role
λ_h	**P**-eigenvalue	persistence of corridor h
$c_h = \mathbf{v}_h^\top \mathbf{s}_1$	scalar	how much a shock projects onto corridor h
$\Theta \mathbf{u}_h$	$\mathbb{R}^{N-1}$	relative inflation along corridor h
ρ_k	F-eigenvalue	labor mixing speed along component k
$\Delta\kappa_k^{\text{eff}}$	scalar	whether component k connects sticky sectors
d_k	scalar	whether demand tilts relative prices along component k

Table 1: Key objects. The first three are defined in Section 3.3 for any F; the last three require quasi-symmetry (Assumption 1).

three-variable systems, one for each spectral component $k \geq 2$:

$$\beta\, \mathbb{E}_t[\chi^*_{kt+1}] - [\kappa(1+\varphi) + \beta + 1]\, \chi^*_{kt} + \chi^*_{kt-1} = \kappa\varphi\, n^*_{kt} - \Delta\hat{a}^*_{kt} + \beta\, \mathbb{E}_t[\Delta\hat{a}^*_{kt+1}], \tag{49}$$

$$n^*_{kt+1} = \rho_k\, n^*_{kt} + \phi\beta(1-\rho_k^2)\, \mathbb{E}_t[\hat{v}^*_{kt+1}], \tag{50}$$

$$\hat{v}^*_{kt} = -\chi^*_{kt} - n^*_{kt} + \beta\rho_k\, \mathbb{E}_t[\hat{v}^*_{kt+1}]. \tag{51}$$

Proof. See Appendix C.2. □

Each component k pairs a Phillips curve (49) with a reallocation block (50)–(51). The Phillips curve is a second-order difference equation in χ^*_{kt}, with employment misallocation n^*_{kt} entering as an endogenous cost-push shifter. The reallocation block drives workers to close the gap $-\chi^*_{kt} - n^*_{kt}$ at a speed governed by ρ_k and ϕ. The dynamics along component k depend on F only through its eigenvalue ρ_k.

Eliminating the forward-looking variables from the component-k system yields a quartic characteristic polynomial whose stable roots are the eigenvalues of **P** associated with component k.

Lemma 5 (Characteristic Polynomial)**.** *For each component $k \geq 2$, the eigenvalues of* **P** *associated with component k solve:*

$$\Big[(\lambda - \rho_k)(1 - \beta\rho_k\lambda) + \phi\beta(1-\rho_k^2)\lambda\Big]\Big[\beta\lambda^2 - (\kappa(1+\varphi) + \beta + 1)\lambda + 1\Big] + \kappa\varphi\phi\beta(1-\rho_k^2)\lambda^2 = 0. \tag{52}$$

Exactly two roots satisfy $|\lambda| < 1$. Let λ_k^* denote the largest in absolute value. Then:

$$\lim_{\phi\to 0} \lambda_k^* = \max\{\lambda_{\text{seg}}, |\rho_k|\},$$

$$\lim_{\phi\to\infty} \lambda_k^* = \lambda_{\text{int}},$$

where λ_{seg} and λ_{int} are the stable roots of (47) and (48), and λ_k^* is continuous in ϕ.

Proof. See Appendix C.2. □

When $\phi = 0$, the coupling term vanishes and the polynomial factors: persistence is the slower of the two margins, $\max\{\lambda_{\text{seg}}, |\rho_k|\}$. Positive ϕ couples them, and the limiting cases formalize the tradeoff from Section 3.4: moving from segmented to integrated markets raises persistence when prices are the bottleneck ($\lambda_{\text{seg}} > |\rho_k|$) and lowers it when labor is ($|\rho_k| > \lambda_{\text{int}}$).

4.2 Heterogeneous Stickiness

When κ_i varies across sectors, the independent component structure of Section 4.1 breaks down: eigenvalues shift from their uniform-κ baselines, and the demand channel $\mathbf{b}_t^y$ becomes active (breaking Corollary 1). Using a perturbation method, we characterize the first-order effect of heterogeneous stickiness on persistence.

Proposition 6 (Effective Rigidity)**.** *Write $\kappa_i = \bar{\kappa} + \varepsilon\,\Delta\kappa_i$ with $\sum_i \alpha_i \Delta\kappa_i = 0$ and $|\varepsilon|$ small. For each component $k \geq 2$, define the effective rigidity*

$$\Delta\kappa_k^{eff} \equiv \sum_i \alpha_i\,\Delta\kappa_i\,v_{k,i}^2, \tag{53}$$

where $v_{k,i} = \alpha_i^{-1/2} s_{k,i}$ is sector i's loading on component k. To first order in ε, the dynamics along component k behave as if governed by a uniform slope $\bar{\kappa} + \varepsilon\,\Delta\kappa_k^{eff}$, and the dominant eigenvalue of $\mathbf{P}$ along component k shifts to

$$\lambda_k^*(\varepsilon) = \lambda_k^* + \varepsilon\,\Delta\kappa_k^{eff} \cdot \Gamma_k + O(\varepsilon^2), \tag{54}$$

where λ_k^* is the baseline eigenvalue under uniform $\bar{\kappa}$ (Lemma 5) and $\Gamma_k < 0$ is a constant. A negative $\Delta\kappa_k^{eff}$ strictly increases persistence.

Proof. See Appendix C.2. □

The effective rigidity $\Delta\kappa_k^{\text{eff}}$ is a weighted average of sectoral stickiness deviations $\Delta\kappa_i$, with weights $v_{k,i}^2$ proportional to each sector's squared loading on component k. It is negative when the sectors that load heavily on component k also tend to have stickier prices. When $\Delta\kappa_k^{\text{eff}} < 0$, nominal and real frictions compound: sluggish price adjustment mutes the value differentials that induce reallocation, while slow labor flows sustain the marginal cost gaps that drive inflation. The effect is strongest for slow-mixing components (high ρ_k), where labor frictions are already the bottleneck.

Heterogeneous κ_i also breaks the block separation of Corollary 1: aggregate demand spills into relative dynamics.

Proposition 7 (Demand Spillover). *A unit change in $\tilde{y}_t$ shifts relative inflation in component $k \geq 2$ by $(\gamma + \varphi)\, d_k$, where*

$$d_k \;\equiv\; \sum_i \alpha_i \, \Delta\kappa_i \, v_{k,i} = \mathrm{Cov}_\alpha(\Delta\kappa_i,\, v_{k,i}). \tag{55}$$

Under uniform κ, $d_k = 0$ for all k, recovering block separation.

Proof. Project the demand term $(\gamma + \varphi)\, M\mathbf{K1}\, \tilde{y}_t$ from the demeaned Phillips curve (C.2) onto the Q-eigenbasis. The k-th component receives $(\gamma + \varphi)\, s_k^\top A^{1/2} M \mathbf{K1}\, \tilde{y}_t = (\gamma + \varphi)\, d_k\, \tilde{y}_t$, since $s_k^\top A^{1/2}\mathbf{1} = 0$ for $k \geq 2$ implies $s_k^\top A^{1/2} M = s_k^\top A^{1/2}$. □

The spillover d_k is exact for arbitrary dispersion in κ_i. It is large when sticky and flexible sectors sit on opposite sides of a reallocation component: aggregate demand moves their prices at different speeds, tilting reallocation incentives along that component. For example, if component k separates a flexible sector (positive loading) from a sticky sector (negative loading), then $d_k > 0$: expansionary policy raises the flexible sector's relative price, drawing workers toward the sticky sector.

The two statistics capture distinct channels. The effective rigidity $\Delta\kappa_k^{\text{eff}}$ weights stickiness by the squared loading $v_{k,i}^2$, measuring whether sectors involved in a component tend to be sticky; it governs persistence of misallocation and inflation. The spillover d_k weights stickiness by the signed loading $v_{k,i}$, measuring whether sticky and flexible sectors are separated by the component; it governs monetary policy's leverage over reallocation. The two are independent: in a symmetric two-sector economy with $\kappa_1 \neq \kappa_2$, the spillover is maximal but the effective rigidity is exactly zero.

Both statistics are computable from the labor transition matrix F and sectoral price adjustment frequencies. The effective rigidity is a first-order approximation in ε; whether it correctly identifies the most persistent corridors when dispersion in κ_i is large is a quantitative question addressed in Section 6. The spillover is exact. Together, they determine when and how monetary policy interacts with sectoral reallocation (Section 5).

4.3 Example: A Three-Sector Illustration

We illustrate how the alignment of price rigidity with the labor flow network governs inflation persistence and monetary policy leverage. The example compares two economies with identical average mobility, identical labor-flow eigenvalues, and the same shock. They differ only in which sectors are connected by the slowest labor-flow component.

Setup. Consider three equal-sized sectors ($\alpha_i = 1/3$). Sectors A and B are sticky ($\kappa_A = \kappa_B < \bar{\kappa}$), while sector C is flexible ($\kappa_C > \bar{\kappa}$). A permanent negative productivity shock hits sector A.

We compare two labor networks with transition probabilities m and ℓ, where $0 < m < \ell \ll \frac{1}{2}$. In the first, workers move slowly between A and B:

$$F^{AB} = \begin{bmatrix} 1-m-\ell & m & \ell \\ m & 1-m-\ell & \ell \\ \ell & \ell & 1-2\ell \end{bmatrix}, \tag{56}$$

In the second, workers move slowly between A and C:

$$F^{AC} = \begin{bmatrix} 1-m-\ell & \ell & m \\ \ell & 1-2\ell & \ell \\ m & \ell & 1-m-\ell \end{bmatrix}. \tag{57}$$

One matrix is a permutation of the other, so they share the same eigenvalues:

$$\rho_1 = 1, \qquad \rho_2 = 1-2m-\ell, \qquad \rho_3 = 1-3\ell. \tag{58}$$

Since $m < \ell$, component 2 is the slow bottleneck ($\rho_2 > \rho_3$) in both economies. If persistence were determined by average mobility or sectoral price stickiness per se, the two economies would behave identically.

Different bottlenecks. What differs is which sectors the slow component connects. The eigenvectors of the slow mode are:

$$s_2^{AB} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \qquad s_2^{AC} = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}.$$

Under F^{AB}, the slow mode separates the two sticky sectors A and B. Under F^{AC}, it separates the sticky sector A from the flexible sector C.

Sufficient statistics. We evaluate $\Delta\kappa_k^{\text{eff}}$ and d_k for the slow component. With equal shares, sector i's loading simplifies to $v_{k,i} = \sqrt{3}\, s_{k,i}$. Write $\tilde{\kappa}_i \equiv \kappa_i - \bar{\kappa}$, so $\tilde{\kappa}_A = \tilde{\kappa}_B = -\varepsilon$ and $\tilde{\kappa}_C = 2\varepsilon$.

Network AB. The slow mode loads on A and B: $v_{2,A} = \sqrt{3/2}$, $v_{2,B} = -\sqrt{3/2}$, $v_{2,C} = 0$. Because A and B have equal stickiness, the signed terms cancel and the squared terms reinforce:

$$\Delta\kappa_2^{\text{eff},AB} = \textstyle\sum_i \alpha_i\, \tilde{\kappa}_i\, v_{2,i}^2 = \frac{1}{3}\Big[(-\varepsilon)\cdot\frac{3}{2} \;+\; (-\varepsilon)\cdot\frac{3}{2} \;+\; 2\varepsilon\cdot 0\Big] = -\varepsilon, \tag{59}$$

$$d_2^{AB} = \textstyle\sum_i \alpha_i\, \tilde{\kappa}_i\, v_{2,i} \;= \frac{1}{3}\Big[(-\varepsilon)\cdot\sqrt{\frac{3}{2}} \;+\; (-\varepsilon)\cdot\big(-\sqrt{\frac{3}{2}}\big) \;+\; 2\varepsilon\cdot 0\Big] = 0. \tag{60}$$

Network AC. The slow mode now loads on A (sticky) and C (flexible): $v_{2,A} = -\sqrt{3/2}$, $v_{2,B} = 0$, $v_{2,C} = \sqrt{3/2}$. Because A and C differ in stickiness, both statistics are nonzero:

$$\Delta\kappa_2^{\text{eff},AC} = \textstyle\sum_i \alpha_i \tilde{\kappa}_i v_{2,i}^2 = \frac{1}{3}\left[(-\varepsilon)\cdot\frac{3}{2} \;+\; (-\varepsilon)\cdot 0 \;+\; 2\varepsilon\cdot\frac{3}{2}\right] = +\tfrac{\varepsilon}{2}, \tag{61}$$

$$d_2^{AC} = \textstyle\sum_i \alpha_i \tilde{\kappa}_i v_{2,i} \;= \frac{1}{3}\left[(-\varepsilon)\cdot\left(-\sqrt{\tfrac{3}{2}}\right) \;+\; (-\varepsilon)\cdot 0 \;+\; 2\varepsilon\cdot\sqrt{\tfrac{3}{2}}\right] = \sqrt{\tfrac{3}{2}}\,\varepsilon. \tag{62}$$

	Network AB	Network AC
Effective rigidity $\Delta\kappa_2^{\text{eff}}$	$-\varepsilon$	$+\varepsilon/2$
Demand spillover d_2	0	$\sqrt{3/2}\,\varepsilon$

Interpretation. Under F^{AB}, the slow component connects two equally sticky sectors. The effective rigidity is negative ($\Delta\kappa_2^{\text{eff}} = -\varepsilon$): neither sector adjusts prices quickly, so the relative price signal needed to redirect workers out of A remains muted. Nominal and real frictions compound, and by Proposition 6 the persistence eigenvalue rises above the uniform-κ baseline ($\lambda_{2,AB}^* > \lambda_2^*$). The demand spillover is zero: because A and B have equal stickiness, aggregate demand moves their prices symmetrically, giving the central bank no leverage over reallocation along this component.

Under F^{AC}, the slow component connects a sticky sector to a flexible one. The effective rigidity is positive ($\Delta\kappa_2^{\text{eff}} = +\varepsilon/2$): sector C's rapid price adjustment generates a sharp wage differential that accelerates reallocation out of A, and persistence falls below the uniform baseline ($\lambda_{2,AC}^* < \lambda_2^*$). The demand spillover is positive ($d_2 = \sqrt{3/2}\,\varepsilon$): expansionary policy raises C's price faster than A's, widening the relative wage gap and tilting reallocation incentives along the slow component. Monetary policy now has direct leverage over the economy's reallocation bottleneck.

Sector A is equally sticky and faces the same shock in both economies. What differs is its network position: which sectors it exchanges workers with along the slow-mixing dimension. This network topology determines both inflation persistence and the central bank's ability to influence reallocation.

5 Optimal Monetary Policy

Sections 3–4 characterized the generalized Phillips curve $\mathcal{K}$ (35) conditional on the path of aggregate demand. We now choose that path optimally. This section derives the welfare loss function and the Ramsey problem (Section 5.1), characterizes the optimal targeting criterion (Section 5.2), and identifies conditions under which optimal policy deviates from the standard rule (Section 5.3).

5.1 The Ramsey Problem

We adopt the timeless perspective of Woodford (2003). The efficient steady-state subsidy eliminates first-order terms, so the problem admits a linear-quadratic approximation.

Proposition 8 (Ramsey Problem under Commitment)**.** *Under commitment in the timeless perspective, the central bank chooses a sequence* $\{\tilde{y}_t, \boldsymbol{\pi}_t, \mathbf{n}_{t+1}, \boldsymbol{\chi}_t\}_{t=0}^{\infty}$ *to minimize the intertemporal welfare loss*

$$\min \quad \mathcal{W} = \sum_{t=0}^{\infty} \beta^t \, \mathcal{L}_t, \tag{63}$$

where the per-period loss is

$$\mathcal{L}_t \overset{(2)}{=} \frac{\gamma + \varphi}{2} \tilde{y}_t^2 + \sum_i \frac{\alpha_i \epsilon_i}{2\kappa_i} \pi_{it}^2 + \sum_i \alpha_i \left[\frac{1+\varphi}{2} \chi_{it}^2 + \varphi \, \chi_{it} \, n_{it} + \frac{\varphi}{2} n_{it}^2 \right], \tag{64}$$

subject to, for each sector i *and date* t*:*

$$\pi_{it} = \kappa_i [(\gamma + \varphi) \, \tilde{y}_t - (1+\varphi) \, \chi_{it} - \varphi \, n_{it}] + \beta \, \mathbb{E}_t \pi_{it+1}, \tag{65}$$

$$\chi_{it} = \chi_{it-1} + \pi_{it} - \pi_t^C + \Delta \hat{a}_{it}, \tag{66}$$

$$\mathbf{n}_{t+1} = B \, \mathbf{n}_t + \phi \beta (I - BF) \, \mathbb{E}_t[\hat{\mathbf{v}}_{t+1}], \tag{67}$$

$$\hat{\mathbf{v}}_t = -\boldsymbol{\chi}_t - \mathbf{n}_t + \beta F \, \mathbb{E}_t[\hat{\mathbf{v}}_{t+1}], \tag{68}$$

taking as given initial conditions $(\hat{\mathbf{p}}_{-1}, \mathbf{n}_0)$ *and the exogenous productivity process* $\{\mathbf{a}_t\}$*.*

Proof. The per-period loss (64) follows from a second-order expansion of household welfare around the efficient steady state (Appendix B). The constraints restate the marginal cost (33) and NKPC (27), relative price identity, and reallocation block from Section 2.5. □

Equivalently, the Ramsey problem is an LQ control problem in which the planner chooses $\tilde{y}_t$ to steer the state $\mathbf{s}_t = (\boldsymbol{\chi}_{t-1}, \mathbf{n}_t)$ through the generalized Phillips curve. The per-period loss (64) penalizes the state directly: persistent misalignment in $\boldsymbol{\chi}$ and $\mathbf{n}$ accumulates welfare costs over time, making slow-decaying corridors of the transition matrix $\mathbf{P}$ disproportionately costly.

5.2 The Targeting Criterion

The first-order conditions of the Ramsey problem involve multipliers $\boldsymbol{\lambda}_t$ (Phillips curves), $\boldsymbol{\eta}_t$ (relative price identities), $\boldsymbol{\psi}_t$ (employment law of motion), and $\boldsymbol{\xi}_t$ (continuation values); the full system is in Appendix C.3. These collapse into a targeting criterion:

Proposition 9 (Monetary Policy Target). *Under commitment, optimal monetary policy is characterized by the targeting rule*

$$\tilde{y}_t - \tilde{y}_{t-1} + \sum_i \alpha_i \epsilon_i \, \pi_{it} \;=\; (MK\mathbf{1})^\top \boldsymbol{\eta}_t = \sum_i (\kappa_i - \bar{\kappa}) \eta_{it} \tag{69}$$

where $\boldsymbol{\eta}_t$ is determined by the system of first-order conditions in Appendix C.3.

Proof. See Appendix C.3. □

The left-hand side is the standard targeting criterion. The right-hand side is a *reallocation wedge*: it is nonzero when reallocation needs (η_{it}) are systematically correlated with Phillips curve slopes (κ_i). When all sectors share the same slope, the wedge vanishes and the criterion reduces to

$$\tilde{y}_t - \tilde{y}_{t-1} + \sum_i \alpha_i \epsilon_i \, \pi_{it} = 0. \tag{70}$$

which, if additionally $\epsilon_i = \epsilon$, becomes the textbook rule $\tilde{y}_t - \tilde{y}_{t-1} + \epsilon \, \pi_t^C = 0$.

The reallocation wedge. The multiplier η_{it} is the shadow value of raising sector i's relative price. Rearranging the relative-price FOC (equation (F_χ) in Appendix C.3) and iterating forward:

$$\boldsymbol{\eta}_t \;=\; -\sum_{s=0}^{\infty} \beta^s \, \mathbb{E}_t \, \mathbf{g}_{t+s}, \tag{71}$$

where $\mathbf{g}_t \equiv A[(1+\varphi)\,\boldsymbol{\chi}_t + \varphi\,\mathbf{n}_t] + (1+\varphi)\,K\,\boldsymbol{\lambda}_t + \boldsymbol{\xi}_t$ collects the marginal welfare costs of the current relative-price configuration: direct misallocation ($A[\cdot]$), cost-push pressure in sectoral Phillips curves ($(1+\varphi)K\boldsymbol{\lambda}_t$), and distortion of reallocation incentives ($\boldsymbol{\xi}_t$). Thus η_{it} is large when sector i's relative price is persistently too low. The more slowly the underlying misalignment decays, the larger is $|\boldsymbol{\eta}_t|$ and the more policy deviates from (70). Slow-decaying corridors of $\mathbf{P}$ generate large cumulative distortions, raising the shadow value and enlarging the wedge. Section 5.3 makes this link precise.

The sign of the wedge. Aggregate demand moves flexible-sector prices more than sticky-sector prices. The planner exploits this asymmetry to tilt relative prices toward the side of the economy that needs to shed workers. Consider a negative productivity shock in a sticky sector. Workers must reallocate outward, so the planner wants to raise that sector's relative price ($\eta_{it} > 0$). Because the sector has $\kappa_i < \bar{\kappa}$, the wedge is negative, and optimal policy tightens: contraction lowers flexible-sector prices more than sticky-sector prices, raising the stuck sector's relative price and incentivizing outflows. The planner accepts a negative output gap to accelerate

reallocation that sticky prices would otherwise delay. The logic reverses when workers must leave flexible sectors.

Benchmark: closing the output gap. An aggregation result following Rubbo (2023) provides a useful benchmark. Summing the NKPC (21) with weights α_i/κ_i and using $\sum_i \alpha_i n_{it} = 0$:

$$\pi_t^R = (\gamma + \varphi)\,\tilde{y}_t + \beta\,\mathbb{E}_t \pi_{t+1}^R, \tag{72}$$

independent of $\mathbf{n}_t$, $\hat{\mathbf{p}}_t$, or F. Stabilizing $\pi_t^R = 0$ therefore closes the output gap at all dates. Gap-closing policy is nonetheless suboptimal: the targeting criterion (69) shows that the planner accepts output gap costs whenever the reallocation wedge is nonzero.

5.3 When Does the Wedge Matter?

A pattern of misalignment matters for monetary policy under two conditions: the pattern must be slow to self-correct, so that cumulative welfare losses are large; and aggregate demand must be able to tilt relative prices along that pattern, so that the central bank has a lever. When either condition fails, the reallocation wedge in (69) is small and the standard targeting rule (70) is a good approximation.

The corridor decomposition from Section 3.3 makes both conditions concrete. Equation (46) shows that each corridor evolves as a scalar AR(1) with persistence λ_h, driven by aggregate demand through the term $\mathbf{v}_h^\top \mathbf{b}_t^y$. A slow corridor generates persistent misalignment in $\boldsymbol{\chi}$ and $\mathbf{n}$, which the per-period loss (64) penalizes directly. Through the present-value representation (71), this persistence raises the shadow value $|\boldsymbol{\eta}_t|$, enlarging the reallocation wedge. But the wedge is large only if $\mathbf{v}_h^\top \mathbf{b}_t^y$ is also nonzero, so that the planner can steer the corridor by choosing $\tilde{y}_t$.

Under quasi-symmetric costs (Assumption 1), the statistics from Section 4 make both conditions precise. Persistence along component k is governed by the eigenvalue λ_k^* (Lemma 5) and its shift under heterogeneous stickiness, $\Delta\kappa_k^{\text{eff}}$ (Proposition 6): a negative $\Delta\kappa_k^{\text{eff}}$ raises λ_k^*, slowing self-correction and increasing the shadow value $|\eta_{kt}^*|$ through the present-value representation (71). Demand leverage along component k is governed by the spillover coefficient d_k (Proposition 7): when $|d_k|$ is large, a unit change in $\tilde{y}_t$ materially tilts relative prices along that component.[11]

Projecting the targeting criterion (69) onto the F-eigenbasis (Lemma 3) decomposes the wedge component by component. Since $MK\mathbf{1} = K\mathbf{1} - \bar{\kappa}\mathbf{1}$ is in the relative subspace, it can be

[11] Both statistics are entries of the matrix $\mathcal{G}_{kj} \equiv \sum_i \kappa_i \alpha_i v_{k,i} v_{j,i}$: $d_k = \mathcal{G}_{k1}$ and $\bar{\kappa} + \Delta\kappa_k^{\text{eff}} = \mathcal{G}_{kk}$.

expanded as

$$MK\mathbf{1} \;=\; \sum_{k=2}^{N} A^{-1/2} s_k \, d_k, \qquad d_k \;\equiv\; s_k^\top A^{1/2} \, MK\mathbf{1} \;=\; \sum_i \alpha_i^{1/2} \, s_{k,i} \, (\kappa_i - \bar{\kappa}). \tag{73}$$

Substituting into (69) decomposes the reallocation wedge into a sum of component-level contributions, each determined by the demand spillover and the shadow value of misalignment along that component:

Proposition 10 (Spectral Targeting Rule)**.** *Under Assumption 1, the targeting criterion (69) decomposes as*

$$\tilde{y}_t - \tilde{y}_{t-1} + \sum_i \alpha_i \epsilon_i \, \pi_{it} \;=\; \sum_{k=2}^{N} d_k \, \eta_{kt}^*, \qquad \eta_{kt}^* \equiv s_k^\top A^{1/2} \, \boldsymbol{\eta}_t, \tag{74}$$

where η_{kt}^* is the shadow value of tilting relative-price gaps in component k. The reallocation wedge along component k is large when $|d_k|$ is large, so that aggregate demand tilts relative prices along k, and $|\eta_{kt}^*|$ is large, which by (71) requires persistent misalignment along k (large $|\Delta\kappa_k^{\text{eff}}|$). When $d_k \approx 0$ for all k, the wedge vanishes and optimal policy approximates the standard rule (70).

Proof. Expand $MK\mathbf{1}$ in the F-eigenbasis using (73) and substitute into $(MK\mathbf{1})^\top \boldsymbol{\eta}_t$. □

The sign of each component's contribution is $d_k \, \eta_{kt}^*$, which is invariant to the sign normalization of the eigenvectors. The planner expands when $d_k \, \eta_{kt}^* > 0$ and contracts when $d_k \, \eta_{kt}^* < 0$. The logic is the same as in the concrete case of Section 5.2: aggregate demand moves flexible-sector prices more than sticky-sector prices, so the planner chooses the sign of the output gap that tilts relative prices toward the side that should shed workers.

When $d_k \approx 0$ for all k, the one-sector Ramsey problem

$$\min_{\{\tilde{y}_t\}} \sum_{t=0}^{\infty} \beta^t \left[\frac{\gamma + \varphi}{2} \, \tilde{y}_t^2 + \frac{\epsilon}{2\kappa} \, (\pi_t^C)^2 \right] \quad \text{s.t.} \quad \pi_t^C = \kappa(\gamma + \varphi) \, \tilde{y}_t + \beta \, \mathbb{E}_t \pi_{t+1}^C + u_t \tag{75}$$

serves as a close approximation, even when the welfare costs of misallocation $\{\chi, \mathbf{n}\}$ are large. The planner cannot do better because aggregate demand has no leverage over the relative prices that drive reallocation.

6 Quantitative Evidence

We bring the model to U.S. sectoral data to assess whether the analytical channels identified in Section 3 are quantitatively relevant. We estimate the transition matrix F, B from worker flows, measure sectoral price rigidity from micro price data, and characterize the generalized Phillips

curve and assess the quantitative importance of reallocation frictions for inflation persistence and monetary policy.

6.1 Data, Calibration, and Flow Facts

We set the model period to one quarter. We set $\gamma = 1$ (log utility over consumption), $\varphi = 2$ (Frisch elasticity of 0.5), and $\beta = 0.99$ (quarterly discount factor), following standard calibrations in the New Keynesian literature (e.g., Rubbo, 2023).

We measure sector-level price adjustment frequencies using the estimates of Pasten et al. (2020), who estimate monthly price adjustment probabilities at the industry level. We convert these to quarterly Calvo parameters: for a sector with monthly adjustment probability f_i, the quarterly probability of non-adjustment is $\delta_i = (1 - f_i)^3$, yielding the Phillips curve slope $\kappa_i = (1 - \delta_i)(1 - \beta\delta_i)/\delta_i$.

We estimate the sectoral transition matrix F from industry-to-industry worker flows in the March CPS, converting to quarterly frequency following Artuç et al. (2010).[12] The backward matrix B is constructed from F and the stationary employment shares via (16), and the expenditure shares $\{\alpha_i\}$ are set equal to the stationary distribution of F.

The mobility elasticity ϕ is pinned down by the elasticity of worker flows to sectoral wages. In Appendix D.2, we show that linearizing the logit transition probabilities (12) around steady state yields a semi-elasticity of log relative transition probabilities to log destination wages of $\beta\phi/\varphi$. In Caliendo et al. (2019), where the flow payoff is $\log w_j$ and preference shocks have dispersion ν, the corresponding semi-elasticity is β/ν; thus $\phi = \varphi/\nu$, and Caliendo et al. (2019) estimate $\nu = 5.34$ at quarterly frequency. With $\varphi = 2$, this yields $\phi \approx 0.37$.

Sectoral classification. We aggregate industries into 14 broad sectors, chosen to balance granularity with reliable cell sizes in the CPS worker-flow data.[13]

Table 2 reports the expenditure share, Phillips curve slope, and implied price duration for each sector, sorted from stickiest to most flexible. Price rigidity varies by nearly two orders of magnitude across sectors, from $\kappa = 0.008$ (Professional & Business Services) to $\kappa = 0.758$ (Agriculture, Mining & Extraction), with an expenditure-weighted mean of $\bar{\kappa} = 0.112$. The correlation between expenditure share and Phillips curve slope is -0.27: sticky sectors are not systematically larger or smaller.

[12] Respondents report their current industry and the industry of their longest-held job in the previous year. Recollection bias implies an effective horizon of approximately 5 months (Artuç et al., 2010); we treat the raw matrix as F_{5m} and convert via $F = F_{5m}^{3/5}$.

[13] The 14 sectors are: Agriculture, Mining & Extraction (AGM); Construction (CON); Nondurable Manufacturing (NDM); Durable & Advanced Manufacturing (DUR); Utilities (UTL); Trade, Wholesale & Retail (TRD); Transportation & Warehousing (TRN); Leisure, Hospitality & Other Services (LSR); Information & Communications (INF); Finance, Insurance & Real Estate (FIR); Professional & Business Services (PBS); Government (GOV); Education (EDU); and Healthcare (HLT).

Table 2: Sectoral price rigidity and expenditure shares

Code	Sector	α_i	κ_i	Duration (q)
PBS	Professional & Business Services	0.091	0.008	12.4
HLT	Healthcare	0.118	0.011	10.7
INF	Information & Communications	0.041	0.012	9.9
DUR	Durable & Advanced Manufacturing	0.085	0.030	6.4
LSR	Leisure, Hospitality & Other Services	0.061	0.039	5.7
EDU	Education	0.096	0.041	5.6
GOV	Government	0.053	0.056	4.8
TRN	Transportation & Warehousing	0.048	0.059	4.7
FIR	Finance, Insurance & Real Estate	0.101	0.093	3.9
NDM	Nondurable Manufacturing	0.025	0.157	3.1
UTL	Utilities	0.012	0.194	2.8
TRD	Trade (Wholesale & Retail)	0.180	0.235	2.6
CON	Construction	0.067	0.304	2.4
AGM	Agriculture, Mining & Extraction	0.021	0.758	1.8
Expenditure-weighted mean		—	0.112	4.8

Notes. Phillips curve slopes $\kappa_i = (1-\delta_i)(1-\beta\delta_i)/\delta_i$ computed from quarterly Calvo parameters. Duration is median price duration in quarters. Sectors sorted by κ_i ascending (stickiest first).

Worker flows. The estimated transition matrix implies that 6.75% of workers change their sector each quarter. The asymmetry ratio (the maximum deviation from the detailed-balance condition $\alpha_i F_{ij} = \alpha_j F_{ji}$) is 0.08%, confirming that quasi-symmetry (Assumption 1) is a reasonable empirical approximation. The slowest-mixing component of the transition matrix has eigenvalue $\rho_2 = 0.955$, corresponding to a half-life of 15 quarters; the fastest has eigenvalue $\rho_{13} = 0.898$ with a half-life of 6 quarters. Whether this pace of labor reallocation is fast enough to matter for quarterly inflation dynamics is the question we turn to next.

We replicate all results at a 63-industry BEA classification; qualitative findings are unchanged and are reported in the Appendix.

6.2 Reallocation Frictions and the Spectrum of Persistence

We construct the transition matrix $\mathbf{P}$ from Section 3.2 and characterize its eigenvalues under four configurations: *Data* (calibrated F, ϕ, heterogeneous κ_i), *Uniform* κ (all $\kappa_i = \bar{\kappa}$), *Segmented* ($\phi = 0$), and *Integrated* ($\phi \to \infty$).

The labor-flow eigenbasis. The labor transition matrix F has 13 non-trivial eigenvalues ranging from $\rho_2 = 0.955$ (half-life 3.8 years) to $\rho_{13} = 0.898$ (half-life 1.6 years). The slowest component separates Construction and Agriculture (flexible) from Education and Healthcare (sticky), with $\Delta\kappa_2^{\text{eff}} = +0.031$: nominal and real frictions partially offset. The second-slowest ($\rho_3 = 0.948$) separates Education and Construction from Healthcare and Government, with $\Delta\kappa_3^{\text{eff}} = -0.078$:

Table 3: Corridor thickening under alternative configurations

	Dominant $\lvert\lambda\rvert$	Half-life (q)	Corridors HL $>$ 4q	Corridors HL $>$ 8q
Segmented	0.853	4.4	1	0
Uniform κ	0.897	6.4	11	0
Data	0.918	8.1	12	1
Integrated	0.914	7.7	3	0

Notes. Each configuration has $2(N-1) = 26$ eigenvalues. Data: calibrated F, $\phi = 0.37$, heterogeneous κ_i. Uniform κ: same F and ϕ, all $\kappa_i = \bar{\kappa}$. Segmented: $\phi = 0$. Integrated: $\phi \to \infty$.

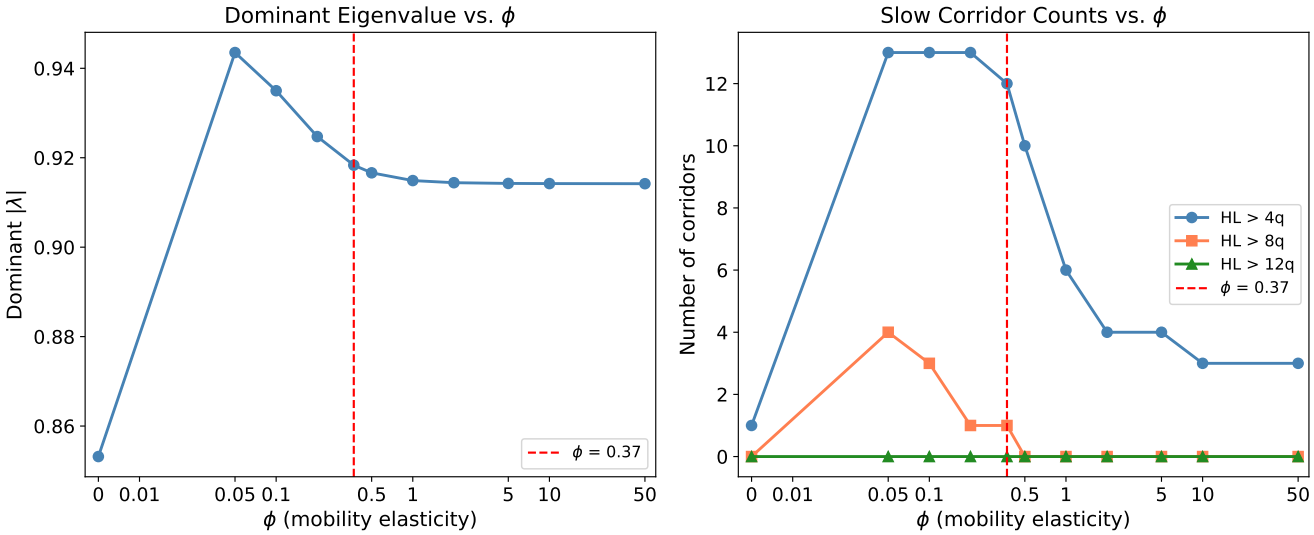


Figure 1: Dominant eigenvalue and corridor counts as a function of ϕ.

Notes. Vertical dashed line: calibrated $\phi = 0.37$. The x-axis is log-scale ϕ, with $\phi = 0$ segmented labor market.

here sticky sectors dominate the slow-mixing side, and the two frictions compound. Overall, 6 of 13 components have $\Delta\kappa_k^{\text{eff}} < 0$. The full table and details are in the Appendix.

Corridor thickening. Table 3 reports the eigenvalues of $\mathbf{P}$ under each configuration.

Under segmentation, only one corridor exceeds a four-quarter half-life. Introducing calibrated mobility raises the count to 12: finite mobility populates the spectrum with slow-decaying corridors of price-employment misalignment. Heterogeneous κ_i further raises the dominant half-life from 6.4 to 8.1 quarters, consistent with Proposition 6.

As a robustness test, we eigendecompose $\mathbf{P}$ for various levels of ϕ. The activation is sharp in ϕ: at $\phi = 0.05$, already 13 corridors exceed the four-quarter threshold (Figure 1).

Diagnostic checks. We can check that the perturbation-based results from Proposition 6 hold well: the predicted relationship between $\Delta\kappa_k^{\text{eff}}$ and actual eigenvalue shifts of $\mathbf{P}$ holds with a correlation of -0.51 when eigenvectors are matched by overlap. To assess whether the observed

Table 4: Cumulative price level response S_H

	S_0	S_4	S_8	S_{20}	S_{40}
Data	0.217	0.110	0.094	0.084	0.082
Uniform κ	0.335	0.335	0.335	0.335	0.335
Segmented	0.215	0.103	0.088	0.082	0.082
Integrated	0.254	0.132	0.102	0.085	0.082

Notes. $S_H = \sum_{h=0}^{H} \partial \pi^C_{t+h} / \partial \tilde{y}_t$. Under uniform κ, block separation (Corollary 1) implies $S_H = \bar{\kappa}(\gamma + \varphi) = 0.335$ for all H.

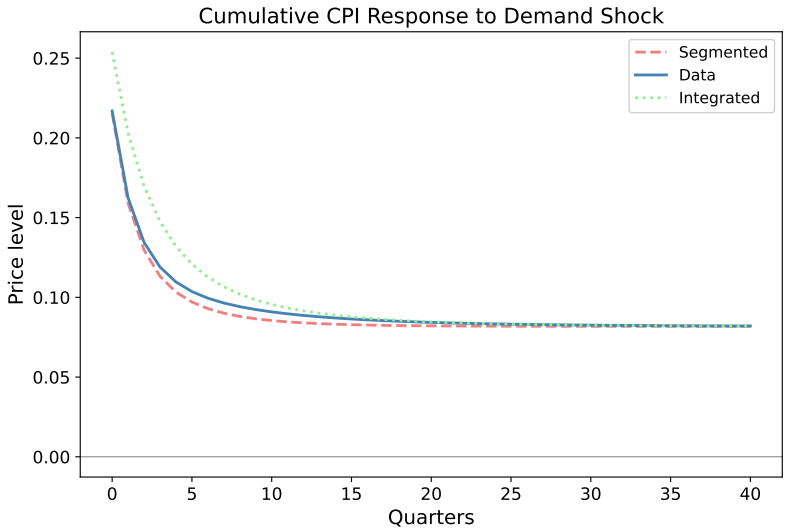


Figure 2: Cumulative price level response S_H under four configurations.

alignment of price rigidity to the labor network is generic, we permute κ across sectors while holding F fixed (1000 draws). In the U.S. data, the observed price-rigidity assignment produces a dominant eigenvalue near the median of random reassignments, but a slow-corridor count exceeding 93% of them.

6.3 The Demand Jacobian of the Phillips Curve

The generalized Phillips curve (35) maps aggregate demand and sectoral productivity into inflation. We characterize each channel in turn, starting with demand. Define the cumulative price level response

$$S_H \equiv \sum_{h=0}^{H} \frac{\partial \pi^C_{t+h}}{\partial \tilde{y}_t}, \tag{76}$$

the total CPI price level change through horizon H induced by a one-time unit demand impulse. Table 4 reports S_H at key horizons; Figure 2 plots the full paths.

Under uniform price rigidity, S_H is perfectly flat: a unit output gap raises the price level

by $\bar{\kappa}(\gamma + \varphi) = 0.335$ at every horizon. Aggregate demand shifts all sectoral costs equally, so relative prices and reallocation incentives are unaffected (Corollary 1).

Heterogeneous κ_i breaks this decoupling. At impact, flexible-price sectors transmit the demand shock disproportionately into the price level. As the economy adjusts, these sectors correct: their high κ_i means they track current marginal cost closely, so prices that jumped at impact revert once demand has passed, generating mild deflation at medium horizons. The remaining misalignment concentrates in sticky sectors, and the marginal price level response decays: by $H = 20$, S_H has lost more than half its impact value. This decay is driven primarily by the distribution of κ_i, not by labor mobility—the Data and Segmented paths are close throughout.

Where mobility matters is in the comparison with the Integrated benchmark. Under frictionless labor markets, the price level response is uniformly higher: $S_0 = 0.254$ versus 0.217 at impact, $S_8 = 0.102$ versus 0.094 at two years. When workers can freely offset cost pressures, firms anticipate relief and reprice less aggressively (Proposition 5), so the pass-through of demand into prices is larger at every horizon. In the calibrated economy, sluggish reallocation dampens the initial response but sustains it for longer.

Despite these differences in the path, the long-run price level effect S_∞ is identical across Data, Segmented, and Integrated (0.082). Mobility reshapes *when* demand passes through into prices, but not the long-run price level effect.

Sectoral transmission. Figure 3 disaggregates the demand Jacobian to the sector level, plotting the sectoral price level response $\sum_{h=0}^{H} \partial \pi_{it+h} / \partial \tilde{y}_t$ under three mobility configurations. Flexible sectors (AGM, $\kappa = 0.758$, red) pass through the demand impulse almost entirely on impact, then revert within a few quarters as their prices track the fading cost pressure. Sticky sectors (PBS, $\kappa = 0.008$, blue) barely respond on impact but accumulate price adjustment gradually over years. Despite these sharply different paths, all 14 sectors converge to the same long-run value (≈ 0.082) by $H = 40$: the permanent price level effect of a demand shock is uniform across sectors.

The three panels show how mobility reshapes the fan of sectoral paths. Under segmentation, the fan is narrow and closes by $H = 10$. Under calibrated mobility, anticipated worker flows dampen price adjustment in both flexible and sticky sectors, widening the fan and extending convergence to $H \approx 20$. Under integration, the fan is widest at medium horizons, reflecting the larger initial pass-through that frictionless mobility permits. The aggregate price level path (dashed) is the α-weighted average of these sectoral paths, and its shape mirrors the S_H curves in Table 4.

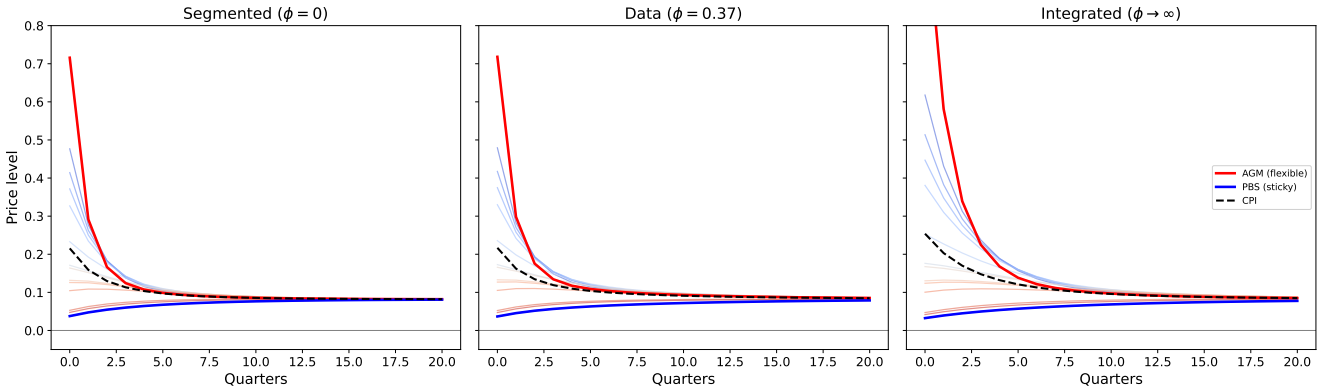


Figure 3: Sectoral demand Jacobian: cumulative inflation $\sum_{h=0}^{H} \partial \pi_{it+h} / \partial \tilde{y}_t$ for each sector. Left: Segmented ($\phi = 0$). Center: Data ($\phi = 0.37$). Right: Integrated ($\phi \to \infty$). Colors range from blue (stickiest, PBS) to red (most flexible, AGM). Dashed black line: CPI (α-weighted average). All sectors converge to the same long-run value.

6.4 The Supply Jacobian: Sectoral Productivity Shocks

We now turn to the supply side of the generalized Phillips curve. The state-space representation (36) is driven by two forcing variables: the output gap $\tilde{y}_t$ and relative productivity growth $\Delta \hat{\mathbf{a}}_t$. Section 6.3 characterized the price level response to the former; we now characterize the response to the latter. We feed a unit impulse in $\Delta \hat{a}_{it}$—equivalently, a permanent unit productivity increase in sector i—holding $\tilde{y}_t = 0$, and trace the CPI price level response $\sum_{h=0}^{H} \pi^C_{t+h}$. This isolates the supply-side propagation channel: how sectoral shocks interact with the labor flow network and heterogeneous price rigidity.

Sectoral transmission. Figure 4 plots the CPI inflation response $\partial \pi^C_{t+h} / \partial \Delta \hat{a}_{it}$ to a permanent unit productivity increase in each sector, under three mobility configurations. The sign of the response is governed by own-price rigidity. A positive productivity shock to a flexible sector (TRD, red; AGM, purple) lowers CPI on impact: the shocked sector passes through its cost reduction quickly, pulling the price level down. A positive productivity shock to a sticky sector (PBS, blue) raises CPI: the shocked sector barely adjusts its own price, while the demeaned productivity decline in other sectors raises their marginal costs, and the flexible ones among them pass this through into higher prices. The result is persistent positive CPI inflation that decays only as the sticky sector's price gradually adjusts downward. Intermediate sectors (DUR, EDU) exhibit sign reversals, with initial inflation giving way to deflation as the cross-sectoral adjustment unfolds.

The three panels reveal how mobility reshapes these dynamics. Under frictionless integration, the inflation response to any sectoral shock is small but persistent: workers reallocate to offset productivity differences, compressing the initial impact, but the gradual reallocation sustains price misalignment over a longer horizon. Calibrated mobility produces larger impact

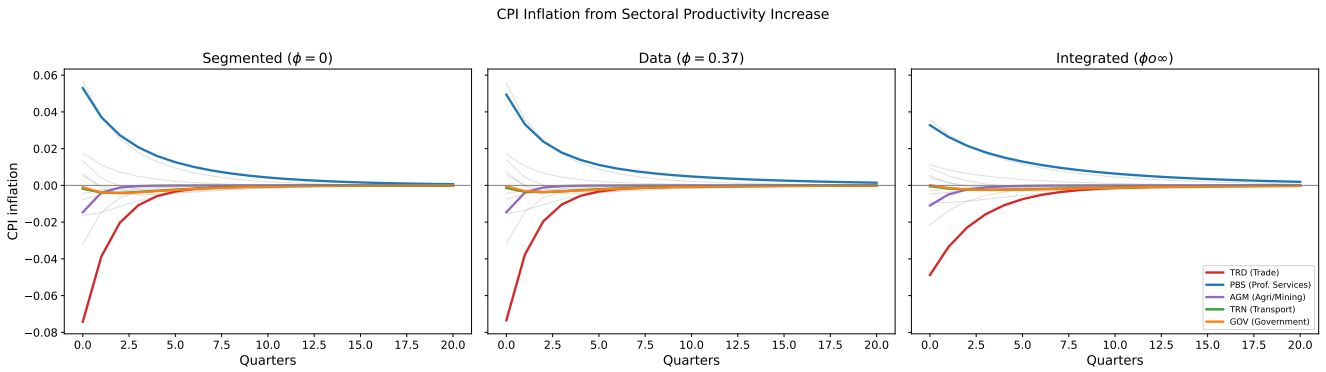


Figure 4: CPI inflation response to sectoral supply shocks (positive productivity increase) under three mobility configurations. Each line is one sector's unit shock. Highlighted: TRD (red), PBS (blue), AGM (purple), TRN (green), GOV (orange). Gray: remaining sectors.

responses—reallocation frictions prevent workers from immediately offsetting the shock—but smaller persistence. Under full segmentation, the fan is widest at impact but closes quickly: without worker flows to sustain misalignment, inflation responses die out within 10 quarters.

The pricing-relevant reallocation share. A natural question is how much of this persistence comes from labor reallocation rather than from price misalignment alone. We decompose the marginal-cost path relevant for firms' pricing decisions. Under Calvo pricing, a resetting firm in sector i sets its price as a function of the discounted path of future marginal costs, with weights $(\beta\delta_i)^s$. The marginal cost equation (33) has two state-dependent terms: the relative-price distortion $-(1+\varphi)\chi_{it}$ and the employment-misallocation term $-\varphi n_{it}$. We define the pricing-relevant reallocation share (PRS) as

$$\text{PRS}_{ij} \equiv \frac{\sum_{s\geq 0}(\beta\delta_i)^s \left|\varphi\, n^{(j)}_{i,t+s}\right|}{\sum_{s\geq 0}(\beta\delta_i)^s \left[\left|(1+\varphi)\,\chi^{(j)}_{i,t+s}\right| + \left|\varphi\, n^{(j)}_{i,t+s}\right|\right]}, \tag{77}$$

the share of the pricing-relevant marginal-cost path for sector i, following a shock to sector j, that is attributable to inherited employment misallocation rather than relative-price distortions. Under segmented labor markets, this must be zero; under integrated labor markets, Proposition 1 implies $n_{it} = -\chi_{it}$, so the upper bound is $\varphi/(1+2\varphi) = 0.40$. Across most sector pairs, the PRS lies in the range 35–40%, with a mean of 37.8%. Thus, more than one-third of the pricing-relevant marginal-cost path reflects employment reallocation.

The implication is straightforward: labor reallocation is quantitatively important for price-setting. Even though worker flows are gradual, they accumulate over the effective horizon of a sticky price, so employment misallocation accounts for a large share of the marginal-cost variation firms internalize when resetting prices.

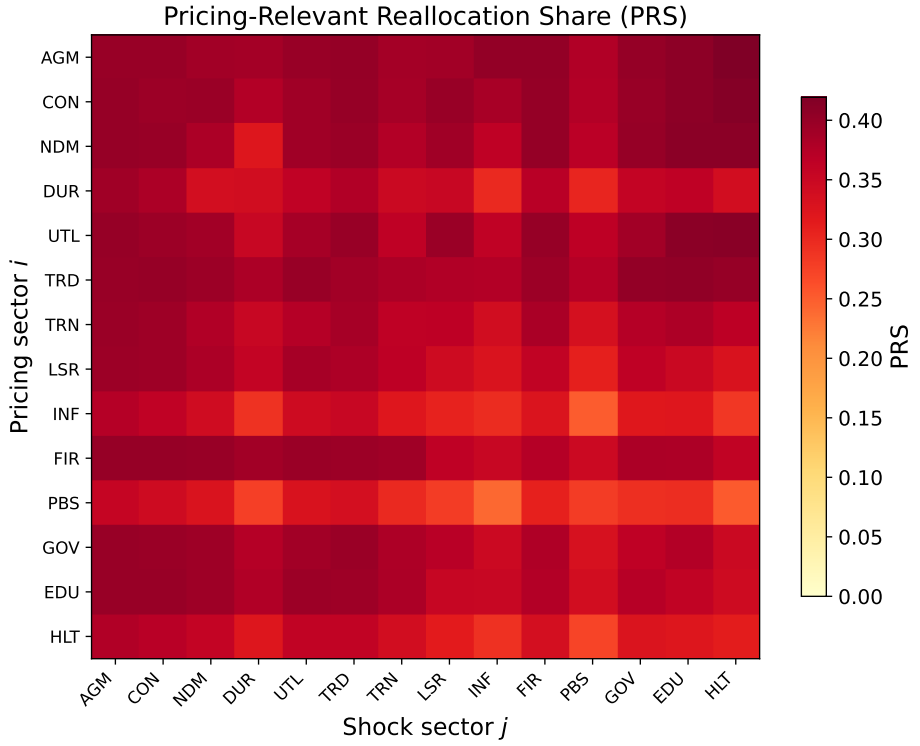


Figure 5: Pricing-relevant reallocation share (PRS) across sector pairs. Entry (i, j) is the share of sector i's pricing-relevant marginal-cost path attributable to employment misallocation following a shock to sector j.

Sectoral versus aggregate effects. The contrast between Figures 4 and 5 is instructive. At the sectoral level, labor reallocation accounts for 37% of the pricing-relevant marginal-cost path—close to the frictionless benchmark of 40%. Yet the aggregate CPI paths under calibrated mobility are nearly indistinguishable from those under segmentation. The aggregate Phillips curve (39) provides an explanation: since $\sum_i \alpha_i n_{it} = 0$, employment misallocation enters CPI inflation only through its covariance with sectoral Phillips curve slopes. This covariance is modest in our U.S. data exercises—the diagnostic checks in Section 6.2 confirm that the alignment of κ_i to the labor network is not strongly non-generic. Reallocation frictions are quantitatively important for understanding *which* sectors face persistent cost pressure, but their aggregate CPI footprint is limited by the degree to which sticky and flexible sectors sit on opposite sides of the reallocation network.

6.5 Monetary Policy and the Reallocation Wedge

Sections 6.3–6.4 characterized the generalized Phillips curve implied by sectoral reallocation frictions. We now close the model with the IS curve (28) and compare three monetary-policy closures: *zero-gap* ($\tilde{y}_t = 0$), a *Taylor rule* ($\psi_\pi = 1.5$, $\psi_y = 0.5$), and *strict inflation targeting* ($\pi_t^C = 0$).

Table 5: Welfare comparison across policy closures (TRD shock)

	$\mathcal{W}_{\text{total}}$	$\mathcal{W}_\pi$	$\mathcal{W}_y$	$\mathcal{W}_\chi$	$\mathcal{W}_r$
Zero-gap	1.101	1.038	0	0.022	0.041
Taylor	1.138	1.063	0.005	0.024	0.045
Strict IT	1.788	1.665	0.042	0.029	0.053

Notes. Each column corresponds to a welfare loss term, with (i) $\mathcal{W}_\pi$: welfare loss from Calvo dispersion; (ii) $\mathcal{W}_y$: Output gap welfare loss; (iii) $\mathcal{W}_\chi$: relative-price distortion; (iv) $\mathcal{W}_r$: employment misallocation. Welfare loss normalized by shock variance.

Table 5 reports welfare for a supply shock on the tradable sector.[14]

Among the simple rules considered, zero-gap delivers the lowest welfare loss among the simple rules. Strict inflation targeting performs substantially worse: although it stabilizes CPI by construction, it does so by inducing output movements that increase sectoral distortions. For this class of shocks, it is cheaper to accommodate the inflation generated by heterogeneous adjustment than to offset it with aggregate demand.

7 Conclusion

We have developed a multi-sector New Keynesian model in which frictional labor reallocation, summarized by a mobility elasticity and a steady-state transition matrix, reshapes the transmission of sectoral shocks and the design of optimal monetary policy. Three results stand out. First, labor mobility creates countervailing effects on sectoral inflation: it dampens impact but slows convergence. Second, the persistence of shocks depends not on average mobility or average stickiness alone, but on whether displaced workers flow into flexible or sticky sectors. Third, optimal policy under commitment deviates from standard sticky-price targeting precisely when price rigidity aligns with the slowest-mixing component of the reallocation network, as the planner exploits flexible sectors' price responsiveness to redirect worker flows.

Several extensions are natural. Our model assumes that aggregate demand affects demand for all sectors symmetrically: in practice, interest rate changes have heterogeneous incidence across sectors—durables respond more than services, for instance—which would change the incidence of aggregate demand onto sectoral demand. Incorporating production networks alongside the labor network would allow both input-output and worker-flow linkages to shape persistence and policy simultaneously.

[14] Numerical results for the Ramsey problem are in progress.

References

AFROUZI, H. AND S. BHATTARAI (2025): "Inflation and GDP Dynamics in Production Networks: A Sufficient Statistics Approach," *Working Paper*.

ALLEN, T., C. ARKOLAKIS, AND Y. TAKAHASHI (2020): "Universal Gravity," *Journal of Political Economy*, 128, 393–433.

ALTISSIMO, F., B. MOJON, AND P. ZAFFARONI (2009): "Can aggregation explain the persistence of inflation?" *Journal of Monetary Economics*, 56, 231–241.

ANDERSON, S. P., A. DE PALMA, AND J.-F. THISSE (1988): "A Representative Consumer Theory of the Logit Model," *International Economic Review*, 461–466.

——— (1992): *Discrete Choice Theory of Product Differentiation*, MIT Press.

AOKI, K. (2001): "Optimal monetary policy responses to relative-price changes," *Journal of Monetary Economics*, 48, 55–80.

ARTUÇ, E., S. CHAUDHURI, AND J. MCLAREN (2010): "Trade Shocks and Labor Adjustment: A Structural Empirical Approach," *American Economic Review*, 100, 1008–1045.

BALL, L. AND D. ROMER (1990): "Real Rigidities and the Non-Neutrality of Money," *The Review of Economic Studies*, 57, 183–203.

BENIGNO, P. (2004): "Optimal monetary policy in a currency area," *Journal of International Economics*, 63, 293–320.

BILS, M. AND P. KLENOW (2004): "Some Evidence on the Importance of Sticky Prices," *Journal of Political Economy*, 112, 947–985.

BLANCHARD, O. AND J. GALÍ (2007): "Real Wage Rigidities and the New Keynesian Model," *Journal of Money, Credit and Banking*, 39, 35–65.

CALIENDO, L., M. DVORKIN, AND F. PARRO (2019): "Trade and Labor Market Dynamics: General Equilibrium Analysis of the China Trade Shock," *Econometrica*, 87, 741–835.

CANTELMO, A. AND G. MELINA (2023): "Sectoral labor mobility and optimal monetary policy," *Macroeconomic Dynamics*, 27, 1–26.

CARVALHO, C., J. W. LEE, AND W. Y. PARK (2021): "Sectoral Price Facts in a Sticky-Price Model," *American Economic Journal: Macroeconomics*, 13, 216–256.

FERRANTE, F., S. GRAVES, AND M. IACOVIELLO (2023): "The inflationary effects of sectoral reallocation," *Journal of Monetary Economics*, 140, S64–S81.

GALÍ, J. (2011): "The Return of the Wage Phillips Curve," *Journal of the European Economic Association*, 9, 436–461, _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.1542-4774.2011.01023.x.

GUERRIERI, V., G. LORENZONI, L. STRAUB, AND I. WERNING (2021): "Monetary Policy in Times of Structural Reallocation," *Working Paper*.

HAZELL, J., J. HERREÑO, E. NAKAMURA, AND J. STEINSSON (2022): "The Slope of the Phillips Curve: Evidence from U.S. States," *The Quarterly Journal of Economics*, 137, 1299–1344.

KIM, B., M. DE LA BARRERA, AND M. FUKUI (2026): "Currency Pegs, Trade Deficits and Unemployment: A Reevaluation of the China Shock," *Working Paper*.

KLEINMAN, B., E. LIU, AND S. J. REDDING (2023): "Dynamic Spatial General Equilibrium," *Econometrica*, 91, 385–424, _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.3982/ECTA20273.

LA'O, J. AND A. TAHBAZ-SALEHI (2022): "Optimal Monetary Policy in Production Networks," *Econometrica*, 90, 1295–1336, _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.3982/ECTA18627.

LILIEN, D. M. (1982): "Sectoral Shifts and Cyclical Unemployment," *Journal of Political Economy*, 90, 777–793.

LIU, E. AND A. TSYVINSKI (2024): "A Dynamic Model of Input–Output Networks," *The Review of Economic Studies*, 91, 3608–3644.

NAKAMURA, E. AND J. STEINSSON (2008): "Five Facts about Prices: A Reevaluation of Menu Cost Models," *The Quarterly Journal of Economics*, 123, 1415–1464.

PASTEN, E., R. SCHOENLE, AND M. WEBER (2020): "The propagation of monetary policy shocks in a heterogeneous production economy," *Journal of Monetary Economics*, 116, 1–22.

PASTÉN, E., R. SCHOENLE, AND M. WEBER (2024): "Sectoral Heterogeneity in Nominal Price Rigidity and the Origin of Aggregate Fluctuations," *American Economic Journal: Macroeconomics*, 16, 318–352.

RUBBO, E. (2023): "Networks, Phillips curves, and monetary policy," *Econometrica*, 91, 1417–1455.

SEO, J. AND R. OH (2025): "Sectoral Shocks and Labor Market Dynamics:," *Working Paper*.

WOODFORD, M. (2003): *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.

Appendix

A Log-Linearization Derivations

This appendix details the log-linearization of the model around the steady state. Because all output is consumed, we impose goods market clearing $y_t = c_t$ and $y_{it} = c_{it}$ throughout, using y to denote both output and consumption.

Notation. For any variable X_t, we write $x_t \equiv \log X_t - \log \bar{X}$ for its log-deviation from steady state. We use the following conventions throughout:

- *Natural (flexible-price) levels*: x_t^{nat} denotes the value under flexible prices.
- *Gaps*: $\tilde{x}_t \equiv x_t - x_t^{\text{nat}}$ denotes the deviation from the natural level.
- *Relative prices*: $\hat{p}_{it} \equiv p_{it} - p_t^C$ is the log sectoral price relative to the CPI.
- *Relative price distortion*: $\chi_{it} \equiv \hat{p}_{it} + \hat{a}_{it}$, the deviation of relative prices from the natural benchmark under steady-state employment. When employment is away from steady state, the contemporaneous natural relative price also depends on n_{it}; see the discussion after Lemma 1.

We write $\overset{(1)}{=}$ for equalities that hold up to a first-order approximation, and $\overset{(2)}{=}$ for second-order. Plain $=$ denotes exact relationships or definitions.

A.1 The Macro Block

A.1.1 Sectoral Marginal Cost

Real marginal cost in sector i (in log-deviations) is given by $mc_{it} - p_{it} = w_{it} - a_{it} - p_{it}$. Using the intratemporal labor supply condition (8) and substituting aggregate consumption with output ($c_t = y_t$), the sectoral wage satisfies:

$$w_{it} - p_t^C = \varphi l_{it} + \gamma y_t. \tag{A.1}$$

Substituting this into the marginal cost expression yields:

$$mc_{it} - p_{it} = \gamma y_t + \varphi l_{it} - a_{it} - \hat{p}_{it},$$

where $\hat{p}_{it} \equiv p_{it} - p_t^C$ is the relative sectoral price. To eliminate l_{it}, we use the log-linearized production function ($l_{it} = y_{it} - a_{it} - n_{it}$) and substitute out sectoral output y_{it} using the demand

curve ($y_{it} = y_t - \hat{p}_{it}$). Grouping terms directly yields Equation (21) from the main text:

$$mc_{it} - p_{it} \overset{(1)}{=} (\gamma + \varphi) y_t - (1 + \varphi)(\hat{p}_{it} + a_{it}) - \varphi n_{it}. \tag{A.2}$$

A.1.2 Phillips Curve

Standard log-linearization of the Calvo pricing problem (5) yields the sectoral New Keynesian Phillips curve. A re-optimizing firm in sector i sets its price as a markup over a weighted average of current and expected future marginal costs. Log-linearizing the optimal reset price around the zero-inflation steady state, and combining with the Calvo price index evolution $P_{it} = [\delta_i P_{it-1}^{1-\epsilon_i} + (1 - \delta_i)(P_{it}^*)^{1-\epsilon_i}]^{1/(1-\epsilon_i)}$, yields:

$$\pi_{it} \overset{(1)}{=} \kappa_i (mc_{it} - p_{it}) + \beta \, \mathbb{E}_t \pi_{it+1}, \tag{A.3}$$

where $\kappa_i \equiv (1 - \delta_i)(1 - \beta\delta_i)/\delta_i$. This is Equation (27).

A.1.3 The IS Curve and Natural Real Rate

Note on convention: As is standard, we make an exception to the log-deviation convention for interest rates. We let i_t and r_t^{nat} denote the levels of the net nominal and natural real interest rates, respectively.

The Euler equation (9) is

$$1 = \beta(1 + i_t) \, \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \frac{P_t^C}{P_{t+1}^C} \right].$$

Taking logs and using the first-order approximation $\log(1 + i_t) \overset{(1)}{=} i_t$:

$$0 \overset{(1)}{=} \log \beta + i_t - \gamma \, \mathbb{E}_t (y_{t+1} - y_t) - \mathbb{E}_t \pi_{t+1}^C,$$

where we have substituted $c_t = y_t$ (goods market clearing) and $\pi_{t+1}^C = p_{t+1}^C - p_t^C$. Rearranging yields:

$$y_t \overset{(1)}{=} \mathbb{E}_t y_{t+1} - \frac{1}{\gamma}(i_t - \mathbb{E}_t \pi_{t+1}^C + \log \beta). \tag{A.4}$$

The same equation holds under flexible prices with y_t^{nat} in place of y_t and i_t^{nat} in place of i_t (and $\pi_t^C = 0$ under flexible prices). We define the natural real rate (in levels) as:

$$r_t^{\text{nat}} \equiv -\log \beta + \gamma(\mathbb{E}_t y_{t+1}^{\text{nat}} - y_t^{\text{nat}}).$$

Subtracting the flexible-price Euler equation from (A.4) gives the IS curve, Equation (28):

$$\tilde{y}_t \overset{(1)}{=} \mathbb{E}_t \tilde{y}_{t+1} - \frac{1}{\gamma}\left(i_t - \mathbb{E}_t \pi^C_{t+1} - r^{\text{nat}}_t\right). \tag{A.5}$$

A.1.4 Goods Market Clearing and Relative Prices.

Cobb-Douglas demand gives $P_{it}C_{it} = \alpha_i P^C_t C_t$, so sectoral output satisfies $P_{it}Y_{it} = \alpha_i P^C_t C_t$. In logs:

$$y_{it} - y_t = -\hat{p}_{it}. \tag{A.6}$$

Log aggregate output is $y_t = \sum_i \alpha_i a_{it} + \sum_i \alpha_i (n_{it} + l_{it})$, which uses $y_{it} = a_{it} + n_{it} + l_{it}$ (from the production function and labor market clearing) weighted by consumption shares.

The CPI definition $P^C_t = \prod_i P^{\alpha_i}_{it}$ implies $p^C_t = \sum_i \alpha_i p_{it}$, so CPI inflation is $\pi^C_t = \sum_i \alpha_i \pi_{it}$. Relative prices evolve as:

$$\hat{p}_{it} - \hat{p}_{it-1} = (p_{it} - p_{it-1}) - (p^C_t - p^C_{t-1}) = \pi_{it} - \pi^C_t, \tag{A.7}$$

which is Equation (22).

A.2 Dynamic Labor Reallocation Block

The dynamic reallocation block consists of the flow utility, the Bellman equation, and the employment evolution equation.

A.2.1 Flow Utility

Using the intratemporal labor supply condition (8), flow utility from (14) simplifies to $U^{\text{flow}}_{it} = \zeta \frac{\varphi}{1+\varphi} L^{1+\varphi}_{it}$ (Equation (15)). Define the normalized deviation $u_{it} \equiv (U^{\text{flow}}_{it} - \bar{U}^{\text{flow}})/(\varphi \zeta \bar{L}^{1+\varphi})$. Since $U^{\text{flow}}_{it} = \zeta \frac{\varphi}{1+\varphi} L^{1+\varphi}_{it}$, we compute:

$$\frac{U^{\text{flow}}_{it} - \bar{U}^{\text{flow}}}{\varphi \zeta \bar{L}^{1+\varphi}} = \frac{\zeta \frac{\varphi}{1+\varphi}(L^{1+\varphi}_{it} - \bar{L}^{1+\varphi})}{\varphi \zeta \bar{L}^{1+\varphi}} = \frac{L^{1+\varphi}_{it} - \bar{L}^{1+\varphi}}{(1+\varphi)\bar{L}^{1+\varphi}}.$$

A first-order expansion of $L^{1+\varphi}_{it}$ around $\bar{L}$ gives $L^{1+\varphi}_{it} \approx \bar{L}^{1+\varphi}(1 + (1+\varphi) l_{it})$, so:

$$u_{it} \overset{(1)}{=} l_{it}.$$

Substituting $l_{it} = y_t - \hat{p}_{it} - a_{it} - n_{it}$ (from goods market clearing and the production function) yields Equation (25):

$$u_{it} = y_t - \hat{p}_{it} - a_{it} - n_{it}. \tag{A.8}$$

A.2.2 Value Function Recursion

The Bellman equation (13) is:

$$V_{it} = U_{it}^{\text{flow}} + \frac{1}{\theta} \log \left(\sum_{j \in \mathcal{N}} \exp\left(\theta[\beta \, \mathbb{E}_t V_{jt+1} - \tau^{ij}]\right) \right).$$

In steady state, $\bar{V}_i = \bar{U}^{\text{flow}} + \frac{1}{\theta} \log\Big(\sum_j \exp(\theta[\beta \bar{V}_j - \tau^{ij}])\Big)$. Define the log-sum-exp term as $\Lambda_i(\mathbf{z}) \equiv \frac{1}{\theta} \log\Big(\sum_j \exp(\theta z_j)\Big)$ evaluated at $z_j = \beta \, \mathbb{E}_t V_{jt+1} - \tau^{ij}$. The first-order Taylor expansion of Λ_i around the steady-state values $\bar{z}_j = \beta \bar{V}_j - \tau^{ij}$ uses the standard logit property:

$$\left. \frac{\partial \Lambda_i}{\partial z_j} \right|_{\bar{z}} = \frac{\exp(\theta \bar{z}_j)}{\sum_{j'} \exp(\theta \bar{z}_{j'})} = F_{ij},$$

where $F_{ij} = \bar{\mu}^{ij}$ is the steady-state transition probability from sector i to sector j (Equation (12) evaluated at steady state). Therefore:

$$\Lambda_i(\mathbf{z}_t) - \Lambda_i(\bar{\mathbf{z}}) \overset{(1)}{=} \sum_j F_{ij} \, \beta (\mathbb{E}_t V_{jt+1} - \bar{V}_j).$$

Subtracting the steady-state Bellman from the time-t Bellman and normalizing by $\varphi \zeta \bar{L}^{1+\varphi}$:

$$v_{it} = u_{it} + \beta \sum_j F_{ij} \, \mathbb{E}_t[v_{jt+1}].$$

In matrix notation, this is Equation (24):

$$\mathbf{v}_t = \mathbf{u}_t + \beta F \, \mathbb{E}_t[\mathbf{v}_{t+1}]. \tag{A.9}$$

A.2.3 Employment Evolution

Employment evolves according to $N_{jt+1} = \sum_i \mu_t^{ij} N_{it}$, where the transition probabilities are given by the logit form (12):

$$\mu_t^{ij} = \frac{\exp\left(\theta[\beta \, \mathbb{E}_t V_{jt+1} - \tau^{ij}]\right)}{\sum_{j'} \exp\left(\theta[\beta \, \mathbb{E}_t V_{j't+1} - \tau^{ij'}]\right)}.$$

Linearizing μ_t^{ij}. The transition probability has the logit form, so its first-order expansion around the steady state $(F_{ij}, \bar{V}_j)$ is:

$$\begin{aligned}\mu_t^{ij} - F_{ij} &\overset{(1)}{=} F_{ij}\,\theta\beta\left(\mathbb{E}_t[V_{jt+1} - \bar{V}_j] - \sum_{j'} F_{ij'}\,\mathbb{E}_t[V_{j't+1} - \bar{V}_{j'}]\right)\\ &= F_{ij}\,\phi\beta\left(\mathbb{E}_t[v_{jt+1}] - \sum_{j'} F_{ij'}\,\mathbb{E}_t[v_{j't+1}]\right),\end{aligned}$$

where the second line uses the normalization $v_{jt} = (V_{jt} - \bar{V}_j)/(\varphi\zeta\bar{L}^{1+\varphi})$ and $\phi \equiv \varphi\theta\zeta\bar{L}^{1+\varphi}$. That is:

$$\mu_t^{ij} - F_{ij} \overset{(1)}{=} \phi\beta\,F_{ij}\big(\mathbb{E}_t[v_{jt+1}] - [F\,\mathbb{E}_t\mathbf{v}_{t+1}]_i\big)\,. \tag{A.10}$$

Deriving the employment law of motion. Log-linearize $N_{jt+1} = \sum_i \mu_t^{ij} N_{it}$ around $\bar{N}_j = \alpha_j$. Since $\bar{N}_j = \sum_i F_{ij}\bar{N}_i$, taking deviations:

$$\bar{N}_j\,n_{jt+1} \overset{(1)}{=} \sum_i F_{ij}\bar{N}_i\,n_{it} + \sum_i \bar{N}_i(\mu_t^{ij} - F_{ij}).$$

We handle the two terms in turn. For the first term, the definition of the backward matrix $B_{ji} \equiv \bar{N}_i F_{ij}/\bar{N}_j$ (Equation (16)) gives $\bar{N}_i F_{ij} = \bar{N}_j B_{ji}$, so:

$$\frac{1}{\bar{N}_j}\sum_i F_{ij}\bar{N}_i\,n_{it} = \sum_i B_{ji}\,n_{it} = [B\,\mathbf{n}_t]_j.$$

For the second term, substituting (A.10):

$$\sum_i \bar{N}_i(\mu_t^{ij} - F_{ij}) = \phi\beta\sum_i \bar{N}_i F_{ij}\big(\mathbb{E}_t[v_{jt+1}] - [F\,\mathbb{E}_t\mathbf{v}_{t+1}]_i\big)\,.$$

Applying $\bar{N}_i F_{ij} = \bar{N}_j B_{ji}$ and dividing by $\bar{N}_j$:

$$\begin{aligned}\text{(second term)}/\bar{N}_j &= \phi\beta\sum_i B_{ji}\big(\mathbb{E}_t[v_{jt+1}] - [F\,\mathbb{E}_t\mathbf{v}_{t+1}]_i\big)\\ &= \phi\beta\big(\mathbb{E}_t[v_{jt+1}] - [BF\,\mathbb{E}_t\mathbf{v}_{t+1}]_j\big)\,,\end{aligned}$$

where the second line uses $\sum_i B_{ji} = 1$ (row-stochasticity of B).

Combining both terms:

$$n_{jt+1} = [B\,\mathbf{n}_t]_j + \phi\beta[(I - BF)\,\mathbb{E}_t\mathbf{v}_{t+1}]_j\,. \tag{A.11}$$

In vector form, this is Equation (23):

$$\mathbf{n}_{t+1} = B\,\mathbf{n}_t + \phi\beta(I - BF)\,\mathbb{E}_t[\mathbf{v}_{t+1}]. \tag{A.12}$$

Limiting cases. When $\phi = 0$ (segmented markets), the second term vanishes and $\mathbf{n}_{t+1} = B\,\mathbf{n}_t$. Starting from $\mathbf{n}_0 = \mathbf{0}$, we have $\mathbf{n}_t = \mathbf{0}$ for all t: no reallocation occurs. When $\phi \to \infty$, boundedness of $\mathbf{n}_{t+1}$ requires $(I - BF)\,\mathbb{E}_t[\mathbf{v}_{t+1}] \to \mathbf{0}$. Since BF is row-stochastic, $\mathbf{1}$ is a fixed point; irreducibility of F and B ensures it is the unique fixed point (up to scale). Thus $\mathbb{E}_t[\mathbf{v}_{t+1}] \to c_t\mathbf{1}$: continuation values equalize across sectors. Iterating the value recursion (24) then gives $\mathbf{u}_t \propto \mathbf{1}$, and since $\sum_i \alpha_i u_{it} = 0$, we have $\mathbf{u}_t = \mathbf{0}$. By (25), $n_{it} = -(y_t - \hat{p}_{it} - a_{it}) = -\chi_{it}$: employment fully offsets relative price distortions.

B Second-Order Expansion Details

This appendix derives the quadratic welfare loss function in 8. We expand household utility around the efficient (natural) allocation to second order, term by term.

B.1 Proposition 8: Dynamic LQ

We derive the per-period welfare loss under the timeless perspective:

$$\mathcal{L}_t \overset{(2)}{=} \frac{\gamma+\varphi}{2}\,\tilde{y}_t^2 + \sum_i \frac{\alpha_i\epsilon_i}{2\kappa_i}\,\pi_{it}^2 + \sum_i \alpha_i\left[\frac{1+\varphi}{2}\,\chi_{it}^2 + \varphi\,\chi_{it}\,n_{it} + \frac{\varphi}{2}\,n_{it}^2\right].$$

The derivation proceeds in two steps. Step 1 expands the consumption-minus-labor-disutility component $u(C_t) - \sum_i N_{it}\,v(L_{it})$ to second order in steady-state deviations. Step 2 shows that the mobility cost Ψ_t is a term whose discounted sum telescopes into a boundary term that enters the Ramsey Lagrangian only through the date-0 first-order conditions. Under the timeless perspective, the per-period loss is therefore the second-order expansion of $u - \sum Nv$ alone.

Notation. All variables denote log-deviations from the efficient steady state, as defined in Appendix A. We write $h_{it} \equiv l_{it} - l_t^{\alpha}$ for cross-sector hours and normalize by $U_C\bar{C} = \bar{C}^{1-\gamma} = \zeta\bar{L}^{1+\varphi}$. Three structural identities are used throughout:

(i) *Production:* $y_t = \bar{a}_t + l_t^{\alpha} + \sum_i \alpha_i\,n_{it} + \sum_i \alpha_i\,\xi_{it}$, where $\xi_{it} \equiv -\log\Delta_{it} \le 0$ is the within-sector price-dispersion wedge. This is exact.

(ii) *Cross-section:* $h_{it} \overset{(1)}{=} -(\chi_{it} + n_{it})$, from goods market clearing and the production function.

(iii) *Adding-up:* $\sum_i \alpha_i\,n_{it} \overset{(2)}{=} -\frac{1}{2}\sum_i \alpha_i\,n_{it}^2$, from $\sum_i \alpha_i\,e^{n_{it}} = 1$.

B.1.1 Second-Order Expansion of $u(C_t) - \sum_i N_{it}\, v(L_{it})$

Consumption. A standard expansion gives:

$$\frac{u(C_t) - u(\bar{C})}{U_C \bar{C}} \overset{(2)}{=} y_t + \frac{1-\gamma}{2} y_t^2. \tag{B.1}$$

Labor disutility. Expanding $N_{it}\, v(L_{it}) = \frac{\alpha_i \zeta \bar{L}^{1+\varphi}}{1+\varphi} \exp\big(n_{it} + (1+\varphi) l_{it}\big)$ to second order, summing over i, dividing by $U_C \bar{C}$, and using identity (iii) to cancel the n_{it} and n_{it}^2 terms:

$$\frac{\sum_i N_{it}\, v(L_{it}) - \sum_i \alpha_i\, v(\bar{L})}{U_C \bar{C}} \overset{(2)}{=} l_t^{\alpha} + \sum_i \alpha_i\, n_{it}\, h_{it} + \frac{1+\varphi}{2}\Big[(l_t^{\alpha})^2 + \mathcal{M}_t\Big], \tag{B.2}$$

where $\mathcal{M}_t \equiv \sum_i \alpha_i\, h_{it}^2$.

Period return. Subtracting (B.2) from (B.1):

$$\frac{\big[u(C_t) - \sum_i N_{it}\, v(L_{it})\big] - \text{SS}}{U_C \bar{C}} \overset{(2)}{=} \underbrace{(y_t - l_t^{\alpha})}_{\text{first-order}} + \underbrace{\frac{1-\gamma}{2} y_t^2 - \frac{1+\varphi}{2} (l_t^{\alpha})^2}_{\text{aggregate quadratic}} - \underbrace{\frac{1+\varphi}{2} \mathcal{M}_t + \sum_i \alpha_i\, n_{it}\, h_{it}}_{\text{cross-sector}}. \tag{B.3}$$

We evaluate each group of terms in turn.

Aggregate quadratic. Since $l_t^{\alpha} \overset{(1)}{=} y_t - \bar{a}_t$ (from identity (i)), the aggregate quadratic in (B.3) is:

$$\frac{1-\gamma}{2} y_t^2 - \frac{1+\varphi}{2} (y_t - \bar{a}_t)^2 \;=\; -\frac{\gamma+\varphi}{2} y_t^2 + (1+\varphi)\, y_t\, \bar{a}_t - \frac{1+\varphi}{2} \bar{a}_t^2. \tag{B.4}$$

Substituting $y_t = y_t^{\text{nat}} + \tilde{y}_t$ with $y_t^{\text{nat}} = \frac{1+\varphi}{\gamma+\varphi} \bar{a}_t$, the coefficient of $\tilde{y}_t$ is $-(\gamma+\varphi)\, y_t^{\text{nat}} + (1+\varphi)\, \bar{a}_t = 0$. Hence:

$$\text{(B.4)} \;=\; -\frac{\gamma+\varphi}{2} \tilde{y}_t^2 + (\text{t.i.p}), \tag{B.5}$$

with t.i.p denoting terms independent of policy.[15]

[15] The vanishing of the cross-term is the multi-sector counterpart of the standard result: the efficient-subsidy condition ensures that the first variation of $u - \sum N v$ with respect to aggregate output vanishes at the natural level.

Cross-sector terms. Using identity (ii), the cross-sector contribution in (B.3) is, for each sector i (with the minus sign from the subtraction):

$$-\frac{1+\varphi}{2}\left(\chi_{it}+n_{it}\right)^2+n_{it}\left(\chi_{it}+n_{it}\right) \;=\; -\frac{1+\varphi}{2}\,\chi_{it}^2-\varphi\,\chi_{it}\,n_{it}-\frac{\varphi-1}{2}\,n_{it}^2. \tag{B.6}$$

Output–labor gap. The first-order term $y_t - l_t^{\alpha}$ in (B.3) has a nonzero second-order component. From the exact production identity (i) and the exogeneity of $\bar{a}_t$:

$$y_t-l_t^{\alpha} \;=\; \bar{a}_t+\sum_i \alpha_i\,n_{it}+\sum_i \alpha_i\,\xi_{it} \;\overset{(2)}{=}\; \text{t.i.p.}-\frac{1}{2}\sum_i \alpha_i\,n_{it}^2+\sum_i \alpha_i\,\xi_{it}, \tag{B.7}$$

using identity (iii). The Calvo price-dispersion recursion implies the present-value equivalence (assuming $\xi_{i,-1}=0$):

$$\sum_{t=0}^{\infty}\beta^t\sum_i \alpha_i\,\xi_{it} \;\overset{(2)}{=}\; -\sum_{t=0}^{\infty}\beta^t\sum_i \frac{\alpha_i\epsilon_i}{2\kappa_i}\,\pi_{it}^2. \tag{B.8}$$

Assembly. Substituting (B.5), (B.6), and (B.7) into (B.3), the period return relative to steady state is, in discounted-sum equivalence:

$$\begin{aligned}\frac{\left[u-\sum Nv\right]-\text{SS}}{U_C\bar{C}} \;\overset{(2)}{=}\; \text{t.i.p.}&-\frac{\gamma+\varphi}{2}\,\tilde{y}_t^2-\sum_i \frac{\alpha_i\epsilon_i}{2\kappa_i}\,\pi_{it}^2\\ &-\sum_i \alpha_i\left[\frac{1+\varphi}{2}\,\chi_{it}^2+\varphi\,\chi_{it}\,n_{it}+\frac{\varphi}{2}\,n_{it}^2\right],\end{aligned} \tag{B.9}$$

where the coefficient $\frac{\varphi}{2}$ on n_{it}^2 combines $\frac{\varphi-1}{2}$ from the cross-sector terms (B.6) and $\frac{1}{2}$ from the output–labor gap (B.7). The former reflects the convexity of the disutility of hours; the latter reflects the concavity of the Cobb–Douglas aggregator, which penalizes uneven employment even holding total labor input fixed.

Negating (B.9) and dropping terms independent of policy:

$$\mathcal{L}_t^{u-Nv} \;\overset{(2)}{=}\; \frac{\gamma+\varphi}{2}\,\tilde{y}_t^2+\sum_i \frac{\alpha_i\epsilon_i}{2\kappa_i}\,\pi_{it}^2+\sum_i \alpha_i\left[\frac{1+\varphi}{2}\,\chi_{it}^2+\varphi\,\chi_{it}\,n_{it}+\frac{\varphi}{2}\,n_{it}^2\right], \tag{B.10}$$

in discounted-sum equivalence.[16]

[16] Because the π_{it}^2 term arises from the present-value equivalence (B.8), equation (B.10) is the per-period term whose discounted sum equals the normalized welfare loss to second order.

B.1.2 Mobility Cost Telescopes into a Startup Term

Step 1 expanded $u(C_t) - \sum_i N_{it}\, v(L_{it})$. The full per-period utility also includes the mobility cost $-\Psi_t$. We now construct a sector-specific net continuation value that strips out future flow utility, yielding a boundary term that enters the Ramsey Lagrangian only through the initial values ignored in the timeless perspective.

Sector-specific net continuation value. For each sector i and date t, define the flow-utility continuation

$$F_{it} \;\equiv\; U_{it}^{\text{flow}} + \beta \sum_{j \in \mathcal{N}} \mu_t^{ij}\, \mathbb{E}_t\, F_{jt+1}, \tag{B.11}$$

and the net continuation value

$$\check{V}_{it} \;\equiv\; V_{it} - F_{it}. \tag{B.12}$$

Intuitively, F_{it} captures the expected discounted flow utility a worker in sector i would earn along the realized transition path, and $\check{V}_{it}$ isolates the remaining component of continuation value—the part attributable to the option value of reallocation net of mobility costs.

Lemma B.1. *Let $\check{\mathcal{B}}_t \equiv \sum_i N_{it}\, \check{V}_{it}$. Then:*

$$-\Psi_t \;=\; \check{\mathcal{B}}_t - \beta\, \mathbb{E}_t\, \check{\mathcal{B}}_{t+1}. \tag{B.13}$$

Consequently, if $\lim_{T\to\infty} \beta^T\, \mathbb{E}_0 |\check{\mathcal{B}}_T| = 0$, then

$$\mathcal{W}_0 \;=\; \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big[u(C_t) - \sum_i N_{it}\, v(L_{it}) \Big] \;+\; \check{\mathcal{B}}_0. \tag{B.14}$$

Proof. The Bellman equation (13), rewritten using the logit probabilities (12) and the reallocation cost (11), takes the envelope form:

$$V_{it} \;=\; U_{it}^{\text{flow}} + \beta \sum_j \mu_t^{ij}\, \mathbb{E}_t\, V_{jt+1} - \psi_i\Big(\{\mu_t^{ij}\}_j\Big). \tag{B.15}$$

Subtracting the F_{it} recursion (B.11) from (B.15), the flow-utility terms cancel:

$$\check{V}_{it} \;=\; \beta \sum_j \mu_t^{ij}\, \mathbb{E}_t\, \check{V}_{jt+1} - \psi_i\Big(\{\mu_t^{ij}\}_j\Big). \tag{B.16}$$

Multiplying by N_{it}, summing over i, and using $N_{jt+1} = \sum_i N_{it}\,\mu_t^{ij}$:

$$\sum_i N_{it}\,\check{V}_{it} \;=\; \beta\,\mathbb{E}_t \sum_j N_{jt+1}\,\check{V}_{jt+1} - \sum_i N_{it}\,\psi_i\Big(\{\mu_t^{ij}\}_j\Big)\,,$$

which is $\check{\mathcal{B}}_t = \beta\,\mathbb{E}_t\,\check{\mathcal{B}}_{t+1} - \Psi_t$, giving (B.13). Multiplying by β^t, summing, and applying the transversality condition telescopes the mobility cost into $-\check{\mathcal{B}}_0$. Substituting into $\mathcal{W}_0 = \mathbb{E}_0 \sum_{t\geq 0} \beta^t [u - \sum Nv - \Psi_t]$ yields (B.14). □

Equation (B.14) is exact: the entire contribution of the mobility block to lifetime welfare is captured by $\check{\mathcal{B}}_0 = \sum_i N_{i0}\,\check{V}_{i0}$. The key property of this construction is that $\check{\mathcal{B}}_0$ is a *pure startup term*: it depends on the future allocation only through the initial net continuation values $\{\check{V}_{i0}\}$, which are themselves determined by the recursion (B.16). In the Ramsey problem with $\{\check{V}_{it}\}$ treated as auxiliary variables subject to this recursion, $\check{\mathcal{B}}_0$ affects only the date-0 first-order conditions—it is the source of the well-known tension between the optimal long-run policy rule and the incentive to deviate at the initial date.

Under the timeless perspective (Woodford, 2003), the planner commits to a policy rule that would have been optimal had it been adopted far in the past. This amounts to requiring that the same targeting criterion hold at every date, including $t = 0$, rather than exploiting the special initial conditions. $\check{\mathcal{B}}_0$ affects only the exceptional startup conditions of the date-0 Ramsey problem. Under the timeless perspective, these startup conditions are replaced by the same self-consistent targeting conditions that apply for $t \geq 1$, and the per-period loss for the Ramsey problem under the timeless perspective is therefore $\mathcal{L}_t^{u-Nv}$ from (B.10).

Combining Steps 1 and 2 and dropping terms independent of policy, the per-period loss for the Ramsey problem is:

$$\mathcal{L}_t \overset{(2)}{=} \frac{\gamma+\varphi}{2}\,\tilde{y}_t^2 + \sum_i \frac{\alpha_i \epsilon_i}{2\kappa_i}\,\pi_{it}^2 + \sum_i \alpha_i \left[\frac{1+\varphi}{2}\,\chi_{it}^2 + \varphi\,\chi_{it}\,n_{it} + \frac{\varphi}{2}\,n_{it}^2\right], \tag{B.17}$$

in discounted-sum equivalence, which is equation (64). □

C Proofs of Propositions and Lemmas

C.1 Proofs for Section 3

Proof of Proposition 1. Sufficient statistics: inspection of (23)–(25): the transition rates $\{\mu^{ij}\}$ enter only through F, B, and ϕ.

When $\phi = 0$, (23) gives $\mathbf{n}_{t+1} = B\,\mathbf{n}_t$ with $\mathbf{n}_0 = \mathbf{0}$, so $\mathbf{n}_t = \mathbf{0}$.

When $\phi \to \infty$, bounded $\mathbf{n}_{t+1}$ in (23) requires $(I - BF)\,\mathbb{E}_t[\mathbf{v}_{t+1}] \to \mathbf{0}$. Since BF is row-stochastic and irreducible, $\mathbf{1}$ is its unique fixed point, so $\mathbb{E}_t[\mathbf{v}_{t+1}] = c_t\mathbf{1}$. Under perfect foresight,

$\mathbf{v}_t = c_{t-1}\mathbf{1}$ for $t \geq 1$, so (24) gives $\mathbf{u}_t = (c_{t-1} - \beta c_t)\mathbf{1}$. Working in relative deviations (where cross-sector means are zero), $\mathbf{u}_t = \mathbf{0}$ for $t \geq 1$, and (25) gives $\mathbf{n}_t = -\boldsymbol{\chi}_t$. At impact ($t = 0$), $\mathbf{n}_0$ is predetermined and cannot jump. □

Proof of Lemma 1. Under flexible prices with the efficient subsidy, $mc_{it}^{\text{nat}} = p_{it}^{\text{nat}}$, so the log-linearized marginal cost (21) at the natural allocation gives

$$(1+\varphi)(\hat{p}_{it}^{\text{nat}} + a_{it}) = (\gamma+\varphi)\, y_t^{\text{nat}} - \varphi\, n_{it}. \tag{C.1}$$

Weighting by α_i and summing, the left-hand side becomes $(1+\varphi)\sum_i \alpha_i a_{it}$ (since $\sum_i \alpha_i \hat{p}_{it}^{\text{nat}} = 0$) and the right-hand side becomes $(\gamma+\varphi)\, y_t^{\text{nat}}$ (since $\sum_i \alpha_i n_{it} = 0$). Solving gives part (i):

$$y_t^{\text{nat}} = \frac{1+\varphi}{\gamma+\varphi} \sum_i \alpha_i a_{it},$$

which is independent of $\mathbf{n}_t$. The natural real rate follows from the Euler equation (9) evaluated at the natural allocation.

For part (ii), solving (C.1) for $\hat{p}_{it}^{\text{nat}}$ and substituting y_t^{nat}:

$$\hat{p}_{it}^{\text{nat}} = \sum_j \alpha_j a_{jt} - a_{it} - \frac{\varphi}{1+\varphi}\, n_{it} = \hat{p}_{it}^{\text{nat},0} - \frac{\varphi}{1+\varphi}\, n_{it}.$$

Sectoral hours follow from the production function and goods market clearing ($l_{it} = y_t - \hat{p}_{it} - a_{it} - n_{it}$):

$$l_{it}^{\text{nat}} = \frac{1-\gamma}{1+\varphi}\, y_t^{\text{nat}} - \frac{1}{1+\varphi}\, n_{it}.$$ □

Proof of Proposition 2. We write the conditional supply block as a linear rational-expectations system and apply the Blanchard–Kahn theorem.

All variables are N-vectors constrained to the relative subspace $\mathcal{R} \equiv \{x \in \mathbb{R}^N : \boldsymbol{\alpha}^\top x = 0\}$; equivalently, after choosing a basis for $\mathcal{R}$, all matrices become $4(N-1) \times 4(N-1)$. Let $M \equiv I - \mathbf{1}\boldsymbol{\alpha}^\top$ and $\mathbf{K} \equiv \text{diag}(\kappa_i)$.

Using $\boldsymbol{\chi}_t = \boldsymbol{\chi}_{t-1} + \hat{\boldsymbol{\pi}}_t + \Delta\hat{\mathbf{a}}_t$ to eliminate $\boldsymbol{\chi}_t$ from the right-hand side, the demeaned Phillips curve becomes

$$\begin{aligned}(I + (1+\varphi)M\mathbf{K})\, \hat{\boldsymbol{\pi}}_t - \beta\, \mathbb{E}_t \hat{\boldsymbol{\pi}}_{t+1} = &-(1+\varphi)\, M\mathbf{K}\, \boldsymbol{\chi}_{t-1} - \varphi\, M\mathbf{K}\, \mathbf{n}_t \\ &+ (\gamma+\varphi)\, M\mathbf{K}\mathbf{1}\, \tilde{y}_t - (1+\varphi)\, M\mathbf{K}\, \Delta\hat{\mathbf{a}}_t.\end{aligned} \tag{C.2}$$

Since $\hat{\mathbf{u}}_t = -\chi_t - \mathbf{n}_t$, the demeaned value recursion is

$$\hat{\mathbf{v}}_t - \beta F \, \mathbb{E}_t[\hat{\mathbf{v}}_{t+1}] = -\chi_{t-1} - \hat{\boldsymbol{\pi}}_t - \Delta\hat{\mathbf{a}}_t - \mathbf{n}_t. \tag{C.3}$$

Together with the state equations $\chi_t = \chi_{t-1} + \hat{\boldsymbol{\pi}}_t + \Delta\hat{\mathbf{a}}_t$ and $\mathbf{n}_{t+1} = B\,\mathbf{n}_t + \phi\beta(I - BF)\,\mathbb{E}_t[\hat{\mathbf{v}}_{t+1}]$, the system takes the form

$$\mathcal{A}\,\mathbb{E}_t \begin{bmatrix} \mathbf{x}_{t+1} \\ \mathbf{z}_{t+1} \end{bmatrix} = \mathcal{B} \begin{bmatrix} \mathbf{x}_t \\ \mathbf{z}_t \end{bmatrix} + \mathcal{C}_y\,\tilde{y}_t + \mathcal{C}_a\,\Delta\hat{\mathbf{a}}_t, \tag{C.4}$$

where $\mathbf{x}_t = (\chi_{t-1}, \mathbf{n}_t)$ collects the predetermined variables, $\mathbf{z}_t = (\hat{\boldsymbol{\pi}}_t, \hat{\mathbf{v}}_t)$ the forward-looking variables, and

$$\mathcal{A} = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & -\phi\beta(I - BF) \\ 0 & 0 & \beta I & 0 \\ 0 & 0 & 0 & \beta F \end{bmatrix}, \qquad \mathcal{B} = \begin{bmatrix} I & 0 & I & 0 \\ 0 & B & 0 & 0 \\ (1+\varphi)M\mathbf{K} & \varphi\,M\mathbf{K} & I + (1+\varphi)M\mathbf{K} & 0 \\ I & I & I & I \end{bmatrix},$$

$$\mathcal{C}_y = \begin{bmatrix} 0 \\ 0 \\ -(\gamma+\varphi)\,M\mathbf{K}\mathbf{1} \\ 0 \end{bmatrix}, \qquad \mathcal{C}_a = \begin{bmatrix} I \\ 0 \\ (1+\varphi)\,M\mathbf{K} \\ I \end{bmatrix}.$$

Under determinacy, the pencil $\mathcal{B} - \lambda\mathcal{A}$ restricted to $\mathcal{R} \times \mathcal{R}$ has exactly $2(N-1)$ stable generalized eigenvalues. The Blanchard–Kahn theorem then delivers a unique bounded solution

$$\mathbf{z}_t = \boldsymbol{\Theta}\,\mathbf{x}_t + \sum_{j=0}^{\infty} \boldsymbol{\Phi}^y_j\,\mathbb{E}_t[\tilde{y}_{t+j}] + \sum_{j=0}^{\infty} \boldsymbol{\Phi}^a_j\,\mathbb{E}_t[\Delta\hat{\mathbf{a}}_{t+j}], \tag{C.5}$$

for a fixed matrix $\boldsymbol{\Theta}$ and absolutely summable sequences $\{\boldsymbol{\Phi}^y_j\}$, $\{\boldsymbol{\Phi}^a_j\}$. Partition $\boldsymbol{\Theta}$ conformably with $(\hat{\boldsymbol{\pi}}_t, \hat{\mathbf{v}}_t)$ and $(\chi_{t-1}, \mathbf{n}_t)$ into blocks $\Theta_{11}, \Theta_{12}, \Theta_{21}, \Theta_{22}$.

Substituting (C.5) into $\chi_t = \chi_{t-1} + \hat{\boldsymbol{\pi}}_t + \Delta\hat{\mathbf{a}}_t$ and into the employment law of motion (the latter requires evaluating $\mathbb{E}_t[\hat{\mathbf{v}}_{t+1}]$ via (C.5) at $t+1$ and solving for $\mathbf{n}_{t+1}$), then collecting terms in $(\chi_{t-1}, \mathbf{n}_t)$ yields $\mathbf{x}_{t+1} = \mathbf{P}\,\mathbf{x}_t + \mathbf{b}^y_t + \mathbf{b}^a_t$ with

$$\mathbf{P} = \begin{bmatrix} I + \Theta_{11} & \Theta_{12} \\ \widetilde{B}\,\phi\beta(I - BF)\,\Theta_{21}(I + \Theta_{11}) & \widetilde{B}\big[B + \phi\beta(I - BF)\,\Theta_{21}\,\Theta_{12}\big] \end{bmatrix},$$

where $\widetilde{B} \equiv [I - \phi\beta(I - BF)\,\Theta_{22}]^{-1}$, which is nonsingular under determinacy.

Properties. The eigenvalues of $\mathbf{P}$ are the $2(N-1)$ stable generalized eigenvalues of $\mathcal{B} - \lambda\mathcal{A}$, so $\rho(\mathbf{P}) < 1$. The matrices $\mathcal{A}$ and $\mathcal{B}$ depend on $(\mathbf{K}, \varphi, \beta, \phi, F, B)$ but not on γ; since $\mathbf{\Theta}$ is determined by the stable eigenspace of $(\mathcal{B}, \mathcal{A})$, $\mathbf{P}$ inherits the same dependence. The parameter γ enters only through $\mathcal{C}_y$, affecting $\{\mathbf{H}_j^y\}$ but not $\mathbf{P}$. □

Proof of Corollary 1. If $\kappa_i = \kappa$ for all i, then $M\mathbf{K}\mathbf{1} = \kappa\, M\mathbf{1} = \mathbf{0}$, so $\tilde{y}_t$ drops out of the demeaned Phillips curve (C.2). The demeaned value recursion (C.3) contains no $\tilde{y}_t$ either. Therefore none of the expectational equations for the relative block involve $\tilde{y}_{t+j}$, and $\mathbf{H}_j^y = \mathbf{0}$ for all $j \geq 0$. □

Proof of Proposition 5. We first derive the characteristic equations, then evaluate persistence, impact, and cumulative inflation.

Characteristic equations. Substituting $\tilde{y}_t = 0$ into the marginal cost equation (33) and the NKPC (27) gives $\pi_{it} = \kappa[-(1+\varphi)\chi_{it} - \varphi\, n_{it}] + \beta\, \mathbb{E}_t \pi_{it+1}$. Using $\chi_{it} = \chi_{it-1} + \hat{\pi}_{it} + \Delta\hat{a}_{it}$ to eliminate $\hat{\pi}_{it}$, and noting that $\Delta\hat{a}_{it} = 0$ for $t \geq 1$:

$$\beta\, \mathbb{E}_t[\chi_{it+1}] - [1 + \beta + \kappa(1+\varphi)]\, \chi_{it} + \chi_{it-1} = \kappa\varphi\, n_{it}. \tag{C.6}$$

Under segmentation ($\phi = 0$), $n_{it} = 0$ for all t. Conjecturing $\chi_{it} = \chi_{i0}\lambda^t$ for $t \geq 1$ yields the characteristic equation (47). Under perfect mobility ($\phi \to \infty$), $n_{it} = -\chi_{it}$ for $t \geq 1$, so the right-hand side of (C.6) becomes $-\kappa\varphi\, \chi_{it}$, reducing the coefficient on χ_{it} from $1 + \beta + \kappa(1+\varphi)$ to $1 + \beta + \kappa$. The same conjecture yields (48).

Persistence. Both characteristic equations take the form $f(\lambda; \Gamma) = \beta\lambda^2 - \Gamma\lambda + 1 = 0$. Since $f(0;\Gamma) = 1 > 0$ and $f(1;\Gamma) = \beta + 1 - \Gamma < 0$ for $\Gamma > 1 + \beta$ (which holds in both cases), there is a unique stable root $\lambda \in (0,1)$. The implicit function theorem gives $d\lambda/d\Gamma = \lambda/(2\beta\lambda - \Gamma) < 0$ (since $2\beta\lambda < \Gamma$ at the stable root), so λ is strictly decreasing in Γ. The segmented coefficient $\Gamma_{\text{seg}} = 1 + \beta + \kappa(1+\varphi)$ exceeds the integrated coefficient $\Gamma_{\text{int}} = 1 + \beta + \kappa$ by $\kappa\varphi > 0$, so $\lambda_{\text{seg}} < \lambda_{\text{int}}$.

Impact. At $t = 0$, employment is predetermined: $n_{i0} = 0$ regardless of ϕ. The difference equation (C.6) at $t = 0$ therefore uses the full slope $\Gamma_{\text{seg}} = 1 + \beta + \kappa(1+\varphi)$ in both cases, with $\chi_{i,-1} = 0$ and $\Delta\hat{a}_{i0} = \hat{a}_i$:

$$\beta\, \mathbb{E}_0[\chi_{i1}] - \Gamma_{\text{seg}}\, \chi_{i0} = -\hat{a}_i.$$

The two regimes differ only in the continuation dynamics. Under segmentation, $\chi_{it} = \chi_{i0}\,\lambda_{\text{seg}}^t$ for all $t \geq 0$, so $\mathbb{E}_0[\chi_{i1}] = \chi_{i0}\,\lambda_{\text{seg}}$, giving

$$\chi_{i0}^{\text{seg}} = \frac{\hat{a}_i}{\Gamma_{\text{seg}} - \beta\lambda_{\text{seg}}}.$$

Under perfect mobility, $n_{it} = -\chi_{it}$ for $t \geq 1$, so the state decays at rate λ_{int} from $t \geq 1$ onward: $\chi_{it} = \chi_{i0}\,\lambda_{\text{int}}^t$ for $t \geq 1$, and $\mathbb{E}_0[\chi_{i1}] = \chi_{i0}\,\lambda_{\text{int}}$. Substituting:

$$\chi_{i0}^{\text{int}} = \frac{\hat{a}_i}{\Gamma_{\text{seg}} - \beta\lambda_{\text{int}}}.$$

Since $\lambda_{\text{int}} > \lambda_{\text{seg}}$, the denominator is strictly smaller in the integrated case: $\Gamma_{\text{seg}} - \beta\lambda_{\text{int}} < \Gamma_{\text{seg}} - \beta\lambda_{\text{seg}}$, so $|\chi_{i0}^{\text{int}}| > |\chi_{i0}^{\text{seg}}|$. Impact relative inflation is $\hat{\pi}_{i0} = \chi_{i0} - \hat{a}_i$, so $|\hat{\pi}_{i0}| = |\hat{a}_i - \chi_{i0}|$. A larger $|\chi_{i0}|$ closes more of the gap at impact, leaving a smaller inflation response. Segmented markets therefore produce a strictly larger initial inflation response.[17]

Cumulative. Relative inflation satisfies $\hat{\pi}_{it} = \Delta\chi_{it} - \Delta\hat{a}_{it}$. Summing from $t = 0$ to T and telescoping:

$$\sum_{t=0}^{T} \hat{\pi}_{it} = \chi_{iT} - \chi_{i,-1} - (\hat{a}_{iT} - \hat{a}_{i,-1}).$$

Starting from steady state ($\chi_{i,-1} = 0$, $\hat{a}_{i,-1} = 0$) with a permanent shock ($\hat{a}_{it} = \hat{a}_i$ for all $t \geq 0$), and using $\chi_{iT} \to 0$ as $T \to \infty$ (since $|\lambda| < 1$):

$$\sum_{t=0}^{\infty} \hat{\pi}_{it} = -\hat{a}_i.$$

This holds for any ϕ, since it depends only on the initial and terminal values of χ_{it} and $\hat{a}_{it}$. □

C.2 Proofs for Section 4

Proof of Lemma 3. We prove each part in turn.

[17] The result does not depend on the predetermined-employment timing. If labor could adjust within the period so that $n_{i0} = -\chi_{i0}$ at impact, the comparison simplifies: using $\Gamma - \beta\lambda = 1/\lambda$ from each characteristic equation, impact inflation becomes $\hat{a}_i(1 - \lambda_{\text{seg}})$ under segmentation and $\hat{a}_i(1 - \lambda_{\text{int}})$ under integration. Since $\lambda_{\text{seg}} < \lambda_{\text{int}}$, the ordering is unchanged. The predetermined-n_0 timing creates a hybrid—impact uses Γ_{seg} while continuation uses λ_{int}—but the qualitative tradeoff between impact and persistence is driven by the effective marginal cost slope, not by the timing convention.

Part (i): Detailed balance. Evaluating the logit transition probabilities (12) at steady state:

$$F_{ij} = \frac{\exp\big(\theta[\beta \bar{V}_j - \tau^{ij}]\big)}{\sum_k \exp\big(\theta[\beta \bar{V}_k - \tau^{ik}]\big)}.$$

Under Assumption 1, $\tau^{ij} = \tau_i^{\text{out}} + \tau_j^{\text{in}} + \hat{\tau}^{ij}$ for $i \neq j$. The origin-specific component τ_i^{out} cancels between numerator and denominator, giving

$$F_{ij} = \frac{\tilde{\theta}_j \,/\, e^{\theta \hat{\tau}^{ij}}}{\varphi_i}, \qquad \tilde{\theta}_j \equiv e^{\theta(\beta \bar{V}_j - \tau_j^{\text{in}})}, \qquad \varphi_i \equiv \sum_k \frac{\tilde{\theta}_k}{e^{\theta \hat{\tau}^{ik}}}.$$

We claim $\bar{N}_i \propto \tilde{\theta}_i \, \varphi_i$. To verify, compute $\sum_i \bar{N}_i F_{ij}$:

$$\sum_i \bar{N}_i F_{ij} = c \sum_i \tilde{\theta}_i \, \varphi_i \cdot \frac{\tilde{\theta}_j \,/\, e^{\theta \hat{\tau}^{ij}}}{\varphi_i} = c \, \tilde{\theta}_j \sum_i \tilde{\theta}_i \, e^{-\theta \hat{\tau}^{ij}} = c \, \tilde{\theta}_j \, \varphi_j = \bar{N}_j,$$

where the third equality uses $\hat{\tau}^{ij} = \hat{\tau}^{ji}$, so $\sum_i \tilde{\theta}_i \, e^{-\theta \hat{\tau}^{ij}} = \sum_i \tilde{\theta}_i \, e^{-\theta \hat{\tau}^{ji}} = \varphi_j$. Therefore $\bar{N}_i = \tilde{\theta}_i \, \varphi_i / \sum_k \tilde{\theta}_k \, \varphi_k$ is the stationary distribution, and detailed balance holds:

$$\frac{\bar{N}_i \, F_{ij}}{\bar{N}_j \, F_{ji}} = \frac{\tilde{\theta}_i \, \varphi_i}{\tilde{\theta}_j \, \varphi_j} \cdot \frac{\tilde{\theta}_j \, e^{-\theta \hat{\tau}^{ij}} \,/\, \varphi_i}{\tilde{\theta}_i \, e^{-\theta \hat{\tau}^{ji}} \,/\, \varphi_j} = \frac{e^{-\theta \hat{\tau}^{ij}}}{e^{-\theta \hat{\tau}^{ji}}} = 1,$$

since $\hat{\tau}^{ij} = \hat{\tau}^{ji}$. By (16), $B_{ji} = (\bar{N}_i / \bar{N}_j) \, F_{ij} = F_{ji}$, so $B = F$.

Part (ii): Spectral properties. Define $Q \equiv A^{1/2} F A^{-1/2}$ where $A = \text{diag}(\alpha_i)$. Detailed balance in matrix form is $AF = F^\top A$, so

$$Q^\top = A^{-1/2} F^\top A^{1/2} = A^{-1/2} (AFA^{-1}) A^{1/2} = A^{1/2} F A^{-1/2} = Q,$$

and Q is real symmetric with real eigenvalues $\rho_1 \geq \cdots \geq \rho_N$ and orthonormal eigenvectors $\{s_k\}$. Since Q is similar to F, they share the same eigenvalues. The row-stochastic property $F\mathbf{1} = \mathbf{1}$ gives $Q(A^{1/2}\mathbf{1}) = A^{1/2}\mathbf{1}$, so $\rho_1 = 1$ with eigenvector $s_1 = A^{1/2}\mathbf{1} / \|A^{1/2}\mathbf{1}\|$. Under the logit specification (12), $F_{ij} > 0$ for all i, j, so by the Perron–Frobenius theorem $\rho_1 = 1$ is simple and $|\rho_k| < 1$ for $k \geq 2$, giving $1 = \rho_1 > \rho_2 \geq \cdots \geq \rho_N > -1$. □

Proof of Lemma 4. Define the coordinate transformation $x_{kt}^* \equiv s_k^\top A^{1/2} \hat{\mathbf{x}}_t$ for each variable $x \in \{\chi, n, \hat{v}, \hat{a}\}$, where $\hat{\mathbf{x}}_t$ denotes the demeaned vector (lying in the relative subspace $\mathcal{R}$). Since $s_1 \propto A^{1/2}\mathbf{1}$ and $\boldsymbol{\alpha}^\top \hat{\mathbf{x}}_t = 0$, we have $x_{1t}^* = s_1^\top A^{1/2} \hat{\mathbf{x}}_t \propto \mathbf{1}^\top A \, \hat{\mathbf{x}}_t = \boldsymbol{\alpha}^\top \hat{\mathbf{x}}_t = 0$, so only components $k \geq 2$ are nontrivial.

The key identity is that for any polynomial p, the transformation diagonalizes $p(F)$:

$$s_k^\top A^{1/2}\, p(F)\, \hat{\mathbf{x}}_t = s_k^\top\, p(Q)\, A^{1/2}\hat{\mathbf{x}}_t = p(\rho_k)\, x_{kt}^*,$$

using $Q = A^{1/2}FA^{-1/2}$ and $Qs_k = \rho_k s_k$. In particular, F maps to ρ_k, F^2 to ρ_k^2, and $I - F^2$ to $1 - \rho_k^2$. We apply $s_k^\top A^{1/2}$ to each equation in the supply-side system, using $B = F$ throughout and $\kappa_i = \kappa$.

Phillips curve. Apply $s_k^\top A^{1/2}$ to the demeaned NKPC (C.2) with $\tilde{y}_t = 0$ (by block separation, Corollary 1). Since $\kappa_i = \kappa$, the operator $M\mathbf{K} = \kappa M$ acts as a scalar on the relative subspace, so $s_k^\top A^{1/2} M\mathbf{K}\hat{\mathbf{x}}_t = \kappa\, x_{kt}^*$. Combined with the relative price identity $\chi_{kt}^* = \chi_{k,t-1}^* + \hat{\pi}_{kt}^* + \Delta\hat{a}_{kt}^*$ (which is immediate since the projection commutes with the identity and M), eliminating $\hat{\pi}_{kt}^*$ yields (49).

Employment evolution. Apply $s_k^\top A^{1/2}$ to $\mathbf{n}_{t+1} = F\,\mathbf{n}_t + \phi\beta(I - F^2)\,\mathbb{E}_t[\hat{\mathbf{v}}_{t+1}]$ (using $B = F$). The diagonalization identities give $n_{k,t+1}^* = \rho_k\, n_{kt}^* + \phi\beta(1 - \rho_k^2)\,\mathbb{E}_t[\hat{v}_{k,t+1}^*]$, which is (50).

Value recursion. Apply $s_k^\top A^{1/2}$ to $\hat{\mathbf{v}}_t = -\boldsymbol{\chi}_t - \mathbf{n}_t + \beta F\,\mathbb{E}_t[\hat{\mathbf{v}}_{t+1}]$. The diagonalization identity on the last term gives $\hat{v}_{kt}^* = -\chi_{kt}^* - n_{kt}^* + \beta\rho_k\,\mathbb{E}_t[\hat{v}_{k,t+1}^*]$, which is (51).

Each projected system for component k involves only $(\chi_{kt}^*, n_{kt}^*, \hat{v}_{kt}^*)$ and the forcing $\hat{a}_{kt}^*$, with no coupling to $k' \neq k$. The uniform-κ assumption is essential: with heterogeneous κ_i, the NKPC projection would involve $s_k^\top A^{1/2}\mathbf{K}A^{-1/2}s_{k'} \neq 0$ for $k' \neq k$, coupling the components. □

Proof of Lemma 5. We derive the characteristic polynomial and consider the limiting cases.

Characteristic polynomial. After shocks have died out, conjecture geometric decay: $\chi_{kt}^* = \lambda^t C$, $n_{kt}^* = \lambda^t N$, $\hat{v}_{kt}^* = \lambda^t V$. Substituting into the component-k system (49)–(51):

$$\Big[\beta\lambda^2 - (\kappa(1+\varphi) + \beta + 1)\lambda + 1\Big]\, C = \kappa\varphi\lambda\, N, \tag{C.7}$$

$$(\lambda - \rho_k)\, N = \phi\beta(1 - \rho_k^2)\lambda\, V, \tag{C.8}$$

$$(1 - \beta\rho_k\lambda)\, V = -(C + N). \tag{C.9}$$

Substituting (C.9) into (C.8) to eliminate V, then using the result to express N in terms of C and substituting into (C.7), yields (52) after cancelling $C \neq 0$. The polynomial is degree four: the product of the two brackets (each quadratic) is degree four, and the additive term is degree two.

The component-k system has two predetermined variables ($\chi^*_{k,t-1}$ and n^*_{kt}) and two forward-looking variables (χ^*_{kt} and $\hat{v}^*_{kt}$), giving a state dimension of four. By the Blanchard–Kahn theorem, a unique bounded solution requires exactly two stable roots ($|\lambda| < 1$) and two unstable roots.

Limiting case $\phi \to 0$. Setting $\phi = 0$ in (52) gives:

$$(\lambda - \rho_k)(1 - \beta\rho_k\lambda)\left[\beta\lambda^2 - (\kappa(1+\varphi) + \beta + 1)\lambda + 1\right] = 0.$$

The first factor gives $\lambda = \rho_k$ (stable). The second gives $\lambda = 1/(\beta\rho_k)$ (unstable). The quadratic is the segmented-market equation (47), whose roots have product $1/\beta > 1$: one stable root λ_{seg} and one unstable. The two stable roots are $\{\lambda_{\text{seg}}, \rho_k\}$, so $\lambda^*_k(0) = \max\{\lambda_{\text{seg}}, |\rho_k|\}$.

Limiting case $\phi \to \infty$. Divide (52) by ϕ and let $\phi \to \infty$. The terms not proportional to ϕ vanish, leaving:

$$\beta(1 - \rho_k^2)\lambda\left[\beta\lambda^2 - (\kappa(1+\varphi) + \beta + 1)\lambda + 1 + \kappa\varphi\lambda\right] = 0.$$

The factor in brackets simplifies to $\beta\lambda^2 - (\kappa + \beta + 1)\lambda + 1 = 0$, which is the integrated-market equation (48) with stable root λ_{int}. The root $\lambda = 0$ reflects the collapse of one state variable: as $\phi \to \infty$, employment fully tracks relative prices ($n^*_{kt} \to -\chi^*_{kt}$), eliminating one independent degree of freedom.

Continuity. The roots of a polynomial depend continuously on its coefficients. Since the coefficients of (52) are continuous in ϕ and the stable roots remain bounded away from the unit circle for all finite $\phi > 0$, λ^*_k is continuous on $[0, \infty)$. □

Proof of Proposition 6. We prove each part in turn.

Effective slope. Write $\mathbf{K} = \text{diag}(\kappa_i) = \bar{\kappa}\, I + \varepsilon\, \text{diag}(\Delta\kappa_i)$. Under uniform $\bar{\kappa}$, the NKPC projection onto component k (Lemma 4) involves the diagonal entry $s_k^\top A^{1/2}\mathbf{K}A^{-1/2}s_k = \bar{\kappa}$. Under the perturbation:

$$s_k^\top A^{1/2}\mathbf{K}A^{-1/2}s_{k'} = \bar{\kappa}\,\mathbf{1}_{k=k'} + \varepsilon \sum_i \Delta\kappa_i\, s_{k,i}\, s_{k',i}.$$

Since $s_{k,i} = \alpha_i^{1/2}\, v_{k,i}$, the diagonal entry becomes $\bar{\kappa} + \varepsilon\, \Delta\kappa_k^{\text{eff}}$. Off-diagonal entries are $O(\varepsilon)$ but affect the eigenvalue only at $O(\varepsilon^2)$ by standard non-degenerate perturbation theory. To first order, the component-k characteristic polynomial (52) is therefore obtained by replacing κ with $\bar{\kappa} + \varepsilon\, \Delta\kappa_k^{\text{eff}}$.

Eigenvalue shift. The baseline dominant stable root λ_k^* satisfies $p(\lambda_k^*;\, \bar{\kappa},\, \rho_k) = 0$, where p denotes (52). By the effective slope result, the perturbed polynomial is $p(\lambda;\, \bar{\kappa} + \varepsilon\, \Delta\kappa_k^{\text{eff}},\, \rho_k) + O(\varepsilon^2) = 0$. The implicit function theorem at the simple root λ_k^* gives (54) with

$$\Gamma_k = -\left.\frac{\partial_\kappa\, p}{\partial_\lambda\, p}\right|_{(\lambda_k^*,\, \bar{\kappa},\, \rho_k)}.$$

Sign of Γ_k. Decompose $p = R \cdot S + T$ where

$$\begin{aligned} R(\lambda) &\equiv (\lambda - \rho_k)(1 - \beta\rho_k\lambda) + \phi\beta(1 - \rho_k^2)\lambda, \\ S(\lambda;\kappa) &\equiv \beta\lambda^2 - [\kappa(1+\varphi) + \beta + 1]\lambda + 1, \\ T(\lambda;\kappa) &\equiv \kappa\varphi\phi\beta(1 - \rho_k^2)\lambda^2. \end{aligned}$$

Since T is linear in κ, $\partial_\kappa T = T/\kappa$. Using $p(\lambda_k^*) = 0$, i.e. $T(\lambda_k^*) = -R(\lambda_k^*)S(\lambda_k^*)$:

$$\partial_\kappa\, p\big|_{\lambda_k^*} = -(1+\varphi)\lambda_k^*\, R(\lambda_k^*) + \frac{T(\lambda_k^*)}{\kappa} = -\frac{R(\lambda_k^*)}{\kappa}\Big[\kappa(1+\varphi)\lambda_k^* + S(\lambda_k^*)\Big].$$

Substituting the definition of S collapses the bracket:

$$\kappa(1+\varphi)\lambda_k^* + S(\lambda_k^*) = \beta(\lambda_k^*)^2 - (\beta+1)\lambda_k^* + 1 = (\lambda_k^* - 1)(\beta\lambda_k^* - 1) > 0,$$

where the inequality uses $\lambda_k^* \in (0,1)$ and $\beta < 1$. For the remaining signs: $T(\lambda_k^*) > 0$ and $RS = -T$ imply $R(\lambda_k^*)$ and $S(\lambda_k^*)$ have opposite signs. Since $S(\lambda_k^*) = \beta(\lambda_k^*)^2 - [\kappa(1+\varphi) + \beta + 1]\lambda_k^* + 1 < 0$ (the segmented-market polynomial evaluates negatively at the coupled root $\lambda_k^* > \lambda_{\text{seg}}$), we have $R(\lambda_k^*) > 0$. Together, $\partial_\kappa\, p|_{\lambda_k^*} < 0$.

For $\partial_\lambda\, p$: the polynomial satisfies $p(0) > 0$ and crosses zero from above at λ_k^* (since p is positive for small λ and has exactly two roots inside the unit circle), so $\partial_\lambda\, p|_{\lambda_k^*} < 0$. Therefore $\Gamma_k = -(\partial_\kappa\, p)/(\partial_\lambda\, p) < 0$: a higher effective slope strictly accelerates convergence. □

C.3 Proof of Proposition 9

This appendix derives the first-order conditions of the Ramsey problem (Proposition 8) and shows how they imply the targeting criterion (69).

Notation. Collect all N sectors into vectors $\boldsymbol{\pi}_t,\, \boldsymbol{\chi}_t,\, \mathbf{n}_t,\, \hat{\mathbf{v}}_t \in \mathbb{R}^N$ and define the diagonal matrices

$$A \equiv \operatorname{diag}(\alpha_i), \qquad K \equiv \operatorname{diag}(\kappa_i), \qquad D_\pi \equiv \operatorname{diag}\!\left(\tfrac{\alpha_i \epsilon_i}{\kappa_i}\right),$$

together with $M \equiv I - \mathbf{1}\boldsymbol{\alpha}^\top$, which removes the aggregate component: $M\boldsymbol{\pi}_t = \boldsymbol{\pi}_t - \pi_t^C \mathbf{1}$. Denote by $\boldsymbol{\lambda}_t$, $\boldsymbol{\eta}_t$, $\boldsymbol{\psi}_t$, and $\boldsymbol{\xi}_t$ the multipliers on the Phillips curve, relative price identity, employment law of motion, and continuation value equation, respectively.

Constraints in vector form. The constraints (65)–(68) can be written as

$$\boldsymbol{\pi}_t - \beta\,\mathbb{E}_t\boldsymbol{\pi}_{t+1} - K[(\gamma+\varphi)\,\mathbf{1}\,\tilde{y}_t - (1+\varphi)\,\boldsymbol{\chi}_t - \varphi\,\mathbf{n}_t] = \mathbf{0}, \qquad [\boldsymbol{\lambda}_t] \tag{C.10}$$

$$\boldsymbol{\chi}_t - \boldsymbol{\chi}_{t-1} - M\boldsymbol{\pi}_t - \Delta\hat{\mathbf{a}}_t = \mathbf{0}, \qquad [\boldsymbol{\eta}_t] \tag{C.11}$$

$$\mathbf{n}_{t+1} - B\,\mathbf{n}_t - \phi\beta(I - BF)\,\mathbb{E}_t\hat{\mathbf{v}}_{t+1} = \mathbf{0}, \qquad [\boldsymbol{\psi}_t] \tag{C.12}$$

$$\hat{\mathbf{v}}_t + \boldsymbol{\chi}_t + \mathbf{n}_t - \beta F\,\mathbb{E}_t\hat{\mathbf{v}}_{t+1} = \mathbf{0}. \qquad [\boldsymbol{\xi}_t] \tag{C.13}$$

Lagrangian. The planner minimizes (63) subject to (C.10)–(C.13). The date-t contribution to the Lagrangian is

$$\begin{aligned}
\mathcal{L}_t \;=\; & \frac{\gamma+\varphi}{2}\,\tilde{y}_t^2 + \tfrac{1}{2}\,\boldsymbol{\pi}_t^\top D_\pi\,\boldsymbol{\pi}_t + \tfrac{1}{2}(1+\varphi)\,\boldsymbol{\chi}_t^\top A\,\boldsymbol{\chi}_t + \varphi\,\boldsymbol{\chi}_t^\top A\,\mathbf{n}_t + \tfrac{1}{2}\varphi\,\mathbf{n}_t^\top A\,\mathbf{n}_t \\
& + \boldsymbol{\lambda}_t^\top\big[\boldsymbol{\pi}_t - \beta\,\mathbb{E}_t\boldsymbol{\pi}_{t+1} - K\big((\gamma+\varphi)\,\mathbf{1}\,\tilde{y}_t - (1+\varphi)\,\boldsymbol{\chi}_t - \varphi\,\mathbf{n}_t\big)\big] \\
& + \boldsymbol{\eta}_t^\top\big[\boldsymbol{\chi}_t - \boldsymbol{\chi}_{t-1} - M\boldsymbol{\pi}_t - \Delta\hat{\mathbf{a}}_t\big] \\
& + \boldsymbol{\psi}_t^\top\big[\mathbf{n}_{t+1} - B\,\mathbf{n}_t - \phi\beta(I - BF)\,\mathbb{E}_t\hat{\mathbf{v}}_{t+1}\big] \\
& + \boldsymbol{\xi}_t^\top\big[\hat{\mathbf{v}}_t + \boldsymbol{\chi}_t + \mathbf{n}_t - \beta F\,\mathbb{E}_t\hat{\mathbf{v}}_{t+1}\big]\,.
\end{aligned}$$

First-order conditions. Differentiating $\sum_{t=0}^{\infty}\beta^t\,\mathcal{L}_t$ with respect to each choice variable and collecting terms across dates yields:

Output gap $\tilde{y}_t$. The output gap appears in the quadratic loss and in the Phillips curve constraint. Differentiating:

$$(\gamma+\varphi)\,\tilde{y}_t - (\gamma+\varphi)\,\mathbf{1}^\top K\,\boldsymbol{\lambda}_t = 0 \quad\Longrightarrow\quad \tilde{y}_t = \mathbf{1}^\top K\,\boldsymbol{\lambda}_t. \tag{F$_{\tilde{y}}$}$$

Sectoral inflation $\boldsymbol{\pi}_t$. Inflation at date t appears in the date-t loss ($D_\pi\,\boldsymbol{\pi}_t$), in the date-$t$ Phillips curve multiplier ($\boldsymbol{\lambda}_t$), in the date-$(t-1)$ Phillips curve through the $\beta\,\mathbb{E}_{t-1}\boldsymbol{\pi}_t$ term (contributing $-\boldsymbol{\lambda}_{t-1}$ after discounting), and in the date-t relative price identity through $M\boldsymbol{\pi}_t$. Collecting:

$$D_\pi\,\boldsymbol{\pi}_t + \boldsymbol{\lambda}_t - \boldsymbol{\lambda}_{t-1} - M^\top\boldsymbol{\eta}_t = \mathbf{0}. \tag{F$_\pi$}$$

The term $-\boldsymbol{\lambda}_{t-1}$ is the commitment term: current inflation appears in last period's Phillips curve, and the planner internalizes this dependence.

Relative prices $\boldsymbol{\chi}_t$. The relative price state enters the date-t loss, the date-t Phillips curve (through marginal cost), the relative price identities at dates t and $t+1$, and the continuation value equation. Collecting:

$$A[(1+\varphi)\,\boldsymbol{\chi}_t + \varphi\,\mathbf{n}_t] + (1+\varphi)\,K\,\boldsymbol{\lambda}_t + \boldsymbol{\eta}_t - \beta\,\mathbb{E}_t\boldsymbol{\eta}_{t+1} + \boldsymbol{\xi}_t = \mathbf{0}. \tag{F$_\chi$}$$

The first term is the direct marginal welfare cost of relative price misalignment. The second captures the cost-push effect on sectoral Phillips curves. The difference $\boldsymbol{\eta}_t - \beta\,\mathbb{E}_t\boldsymbol{\eta}_{t+1}$ is the net shadow value of adjusting the relative price state, and $\boldsymbol{\xi}_t$ reflects the effect on workers' reallocation incentives through continuation values.

Employment $\mathbf{n}_{t+1}$. The employment state at $t+1$ enters the date-$(t+1)$ loss and Phillips curve (after discounting by β), the employment law of motion at dates t and $t+1$, and the continuation value equation at $t+1$. Differentiating with respect to $\mathbf{n}_{t+1}$ and simplifying:

$$A\varphi\big(\boldsymbol{\chi}_{t+1} + \mathbf{n}_{t+1}\big) + \varphi\,K\,\boldsymbol{\lambda}_{t+1} + \boldsymbol{\xi}_{t+1} - B^\top\boldsymbol{\psi}_{t+1} + \beta^{-1}\boldsymbol{\psi}_t = \mathbf{0}, \tag{F$_n$}$$

or equivalently, shifting the time index back by one period to express in terms of date-t variables:

$$A\varphi(\boldsymbol{\chi}_t + \mathbf{n}_t) + \varphi\,K\,\boldsymbol{\lambda}_t + \boldsymbol{\xi}_t - B^\top\boldsymbol{\psi}_t + \beta^{-1}\boldsymbol{\psi}_{t-1} = \mathbf{0}.$$

The term $\varphi K\boldsymbol{\lambda}_t$ captures the shadow value of the employment distribution through the sectoral Phillips curves: because $\mathbf{n}_t$ enters marginal costs, employment misallocation matters through pricing distortions as well as directly through the welfare loss.

Continuation values $\hat{\mathbf{v}}_t$. The continuation value appears in the employment law of motion at date $t-1$ (through $\mathbb{E}_{t-1}\hat{\mathbf{v}}_t$) and in the continuation value equations at dates t and $t-1$ (through $\mathbb{E}_{t-1}\hat{\mathbf{v}}_t$). Collecting:

$$\boldsymbol{\xi}_t - F^\top\boldsymbol{\xi}_{t-1} - \phi\,(I - BF)^\top\boldsymbol{\psi}_{t-1} = \mathbf{0}. \tag{F$_v$}$$

The lagged multiplier $\bar{\mu}^\top\boldsymbol{\xi}_{t-1}$ introduces history dependence: under commitment, current policy internalizes that promised continuation values and past employment-transition incentives affect today's reallocation margin.

Timeless perspective. Under the timeless perspective, the planner commits to the policy rule that would have been optimal had it been adopted in the infinite past. The lagged multipliers $(\boldsymbol{\lambda}_{-1}, \boldsymbol{\psi}_{-1}, \boldsymbol{\xi}_{-1})$ are initialized at their unconditional steady-state values, so that the targeting rule holds at every date including $t = 0$.

Derivation of the targeting criterion. We now derive equation (69). Pre-multiply (F_π) by $\mathbf{1}^\top K$:

$$\mathbf{1}^\top K D_\pi \boldsymbol{\pi}_t + \mathbf{1}^\top K \boldsymbol{\lambda}_t - \mathbf{1}^\top K \boldsymbol{\lambda}_{t-1} - \mathbf{1}^\top K M^\top \boldsymbol{\eta}_t = 0. \tag{C.14}$$

We evaluate each term in turn.

First term. Since K and D_π are both diagonal, $\mathbf{1}^\top K D_\pi = \mathbf{1}^\top \operatorname{diag}(\kappa_i \cdot \alpha_i \epsilon_i / \kappa_i) = (\alpha_1 \epsilon_1, \ldots, \alpha_N \epsilon_N)$. Therefore

$$\mathbf{1}^\top K D_\pi \boldsymbol{\pi}_t = \sum_i \alpha_i \epsilon_i \, \pi_{it}.$$

Second and third terms. From ($F_{\tilde{y}}$), $\mathbf{1}^\top K \boldsymbol{\lambda}_t = \tilde{y}_t$ and $\mathbf{1}^\top K \boldsymbol{\lambda}_{t-1} = \tilde{y}_{t-1}$.

Fourth term. We have $\mathbf{1}^\top K M^\top = (MK\mathbf{1})^\top$. Since $M = I - \mathbf{1}\boldsymbol{\alpha}^\top$,

$$MK\mathbf{1} = K\mathbf{1} - \mathbf{1}\boldsymbol{\alpha}^\top K\mathbf{1} = \begin{pmatrix} \kappa_1 \\ \vdots \\ \kappa_N \end{pmatrix} - \bar{\kappa}\,\mathbf{1},$$

where $\bar{\kappa} \equiv \boldsymbol{\alpha}^\top K\mathbf{1} = \sum_i \alpha_i \kappa_i$. Therefore

$$\mathbf{1}^\top K M^\top \boldsymbol{\eta}_t = (MK\mathbf{1})^\top \boldsymbol{\eta}_t = \sum_i (\kappa_i - \bar{\kappa})\, \eta_{it}.$$

Substituting into (C.14):

$$\sum_i \alpha_i \epsilon_i \, \pi_{it} + \tilde{y}_t - \tilde{y}_{t-1} - \sum_i (\kappa_i - \bar{\kappa})\, \eta_{it} = 0,$$

which rearranges to (69). □

D Data Appendix

D.1 Empirical fit of reversibility

Assumption 1 is imposed for tractability. Its observable implication, established in Lemma 3, is that the steady-state transition matrix is reversible:

$$\alpha_i F_{ij} = \alpha_j F_{ji}, \qquad \text{equivalently} \qquad Q \equiv A^{1/2} F A^{-1/2} \text{ is symmetric,}$$

where $A = \operatorname{diag}(\alpha_i)$.

We assess this implication using CPS sector-to-sector transition flows. Figure D.1 shows that

reversibility is a close approximation in the data. The asymmetry measure is small,

$$\frac{\|Q - Q^\top\|}{\|Q\|} = 8 \times 10^{-4},$$

and the correlation between $\alpha_i F_{ij}$ and $\alpha_j F_{ji}$ across sector pairs is 0.9962. Thus, while Assumption 1 is not itself directly testable from observed flows, its implied symmetry restriction on labor transitions fits the data closely.

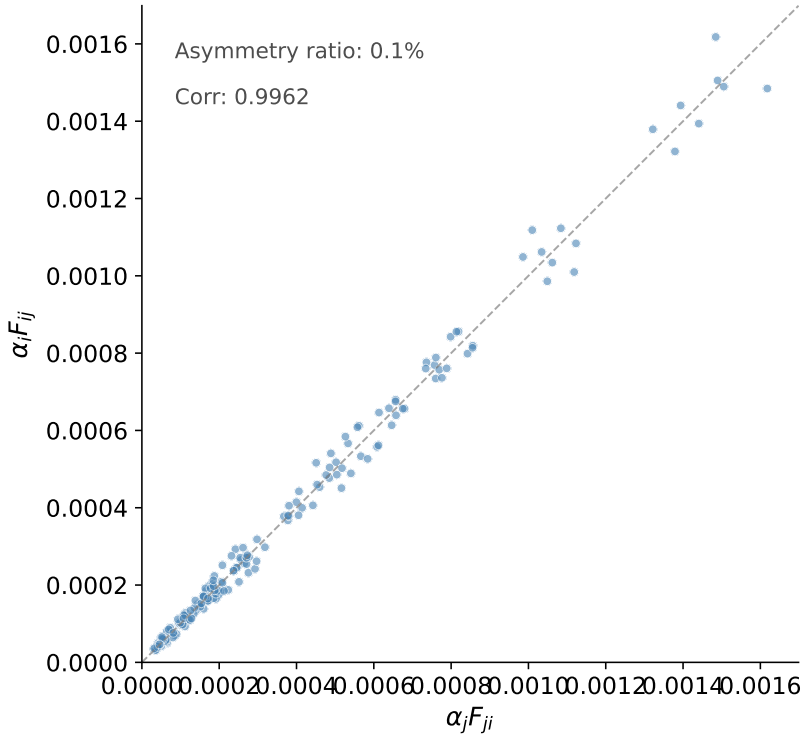


Figure D.1: Empirical fit of reversibility in sectoral labor transitions. The figure plots $\alpha_i F_{ij}$ against $\alpha_j F_{ji}$ using CPS sector-to-sector transition flows. Under reversibility, all observations lie on the 45-degree line. In the data, the asymmetry measure $\|Q - Q^\top\|/\|Q\|$ is 8×10^{-4} and the correlation between $\alpha_i F_{ij}$ and $\alpha_j F_{ji}$ is 0.9962, indicating that the symmetry implication of Lemma 3 is a close approximation empirically.

D.2 Calibrating the Mobility Elasticity

We derive the mapping between the mobility elasticity ϕ in our model and the migration dispersion parameter ν estimated by Caliendo et al. (2019).

Our model. The logit (12) gives, for $i \neq j$:

$$\log(\mu_t^{ij}/\mu_t^{ii}) = \theta\,\beta\,(E_t V_{jt+1} - E_t V_{it+1}) - \theta(\tau^{ij} - \tau^{ii}). \tag{D.1}$$

For a temporary change in destination wages, $\Delta V_j \approx \Delta U_j^{\text{flow}}$. From the flow utility (15) and the labor supply condition (8):

$$\frac{\partial U_j^{\text{flow}}}{\partial \log w_j} = \zeta \, \varphi \, \bar{L}^{\varphi} \cdot \frac{\bar{L}}{\varphi} = \zeta \, \bar{L}^{1+\varphi}, \tag{D.2}$$

so the semi-elasticity of relative log flows to log destination wages is

$$\frac{\partial \log(\mu_t^{ij} / \mu_t^{ii})}{\partial \log w_{j,t+1}} = \theta \, \beta \, \zeta \, \bar{L}^{1+\varphi} = \frac{\phi \, \beta}{\varphi}, \tag{D.3}$$

using $\phi = \varphi \, \theta \, \zeta \, \bar{L}^{1+\varphi}$.

Caliendo et al. (2019) model. In their formulation, the flow payoff is $\log(w_j / p_j)$ and idiosyncratic shocks have dispersion ν, so the logit gives:

$$\log(\mu_t^{ij} / \mu_t^{ii}) = \frac{1}{\nu} \left[\beta (E_t V_{jt+1} - E_t V_{it+1}) - (\kappa^{ij} - \kappa^{ii}) \right]. \tag{D.4}$$

Since $\partial V_j / \partial \log w_j = 1$ for a temporary wage change, the elasticity is

$$\frac{\partial \log(\mu_t^{ij} / \mu_t^{ii})}{\partial \log w_{j,t+1}} = \frac{\beta}{\nu}. \tag{D.5}$$

Equating (D.3) and (D.5) gives:

$$\frac{\phi \, \beta}{\varphi} = \frac{\beta}{\nu} \qquad \Longrightarrow \qquad \phi = \frac{\varphi}{\nu}. \tag{D.6}$$

Caliendo et al. (2019) estimate $\nu = 5.34$ at quarterly frequency. With $\varphi = 2$, this implies $\phi = 2/5.34 \approx 0.37$.