The Global Industrial Revolution*

Robert E. Lucas Jr.
University of Chicago

Luis Felipe Sáenz
University of South Carolina

March 18, 2026

Abstract

Standards of living were roughly constant before 1800, almost everywhere. In the early nineteenth century, English-speaking countries began a process of sustained growth that continues today. This rise in living standards, first observed in Great Britain, is now a global phenomenon. Empirically, four regularities characterize the transition to modern economic growth: (i) a reallocation of labor out of agriculture, (ii) a demographic transition, (iii) rising investments in human capital, and (iv) a U-shaped pattern of female labor force participation. Motivated by these facts, we develop a unified growth model in which the industrial revolution—defined as the onset of sustained income growth—emerges from parental investments in children's quality. The model features a two-parent household that engages in specialization when choosing how to allocate time between work and child-rearing in a dual economy where traditional and modern technologies coexist. Calibrated to Great Britain (1400–2000) and tested against cross-country data, the model broadly reproduces the salient patterns of the industrial revolution.

Keywords: Industrial Revolution, Unified Growth Theory, Female Labor Force Participation
JEL codes: J11, J21, O11, O14, O40

*Bob Lucas passed away during the production of this manuscript. Completing this draft has been, in part, an effort to continue the conversation with him on a subject he encouraged all of us to think about, making it hard to think about anything else. We thank Marcelo Buchelli, Paco Buera, Bentley Coffey, Kevin Donovan, Juan Carlos Echeverry, Manuel Fernández, John McDermott, Joe Kaboski, Martí Mestieri, Rachel Ngai, Steve Parente, and particularly Nancy Stokey for encouraging and valuable comments, as well as the seminar and conference participants at EAFIT (Medellín), University of South Carolina, LACEA (Lima), UNC Greensboro, Society for Economic Dynamics (Cartagena), European Economic Asociation (Barcelona), Southern Economic Association (New Orleans), STEG Annual Conference (NYU Abu Dhabi), I-85 Macroeconomics Workshop (Athens), University of Illinois, CEER (Cartagena), Universidad del Norte (Barranquilla), Society for Computational Economics (Santiago), Universidad de los Andes (Bogotá), and the University of Nottingham.

> Suppose Galileo were here and we were to show him the world today and try to make him happy, or see what he finds out.
>
> — Richard P. Feynman, 1964.

1 Introduction

The disparities in living standards across the globe today are striking. As of 2018, an average American enjoyed a level of affluence roughly eleven times greater than that of a typical resident of the African continent. While the deep historical foundations of these differences remain the subject of intense debate, their proximate cause is clear: some societies have embarked on modern economic growth, while others have remained stagnant at low income levels, generation after generation.

The industrial revolution—the transition from stagnation to sustained growth in living standards—divides a traditional world that Ricardo and Malthus would have recognized well, one in which the incomes of ordinary people remained within a narrow range across space and over generations, from a world in which income per person grows without foreseeable limit. This revolution does not mark the beginning of technological progress. What is truly new is the fact that productivity gains materialized into rising incomes rather than expanding populations, breaking the gloomy Malthusian check in which population growth kept the level of prosperity of ordinary people around subsistence.

The industrial revolution began in Great Britain in the early nineteenth century and has since spread *unevenly* across cultures and geographies. This is an ongoing global process. At its core lie two closely connected forces: a movement out of traditional modes of production anchored in land, and a demographic transition toward smaller and better-educated families. These are not separate developments but different aspects of a single economic event.

This paper places the role of women at the center of the transition to modern economic growth. It is hard to understand this take-off without inquiring how families allocate labor and make fertility decisions—choices inherently shaped by within-household specialization. Empirically, Goldin (1994) documents a U-shaped relationship between female labor force participation and economic development, in contrast to the relative stability of male participation. We view this pattern as an integral part of a unified process that jointly determines demographic transition and human capital accumulation, triggering the escape from Malthusian stagnation and the emergence of a human-capital economy capable of sustaining growth across generations.

The purpose of this paper is twofold. First, we update the empirical knowledge of the industrial revolution, highlighting four key regularities associated with the transition to modern economic growth: (i) the reallocation of labor out of agriculture, (ii) the demographic transition, (iii) rising human capital accumulation, and (iv) a U-shaped pattern of female labor force participation. Growth is now occurring almost everywhere, including in regions of Asia and Africa that remained stagnant just three decades ago. Some of these transitions are unfolding at "catch-up" speeds, although these growth rates eventually slow down and stabilize. Miracles of growth, once confined to Japan and the Asian Tigers, have multiplied across the developing world. In the last thirty years, dozens of economies have doubled their income per capita within a decade—a pace of transformation previously unimaginable.

Second, to think about the choices that brought about the industrial revolution, we develop a unified growth model in which parents optimally allocate their time between production and child-rearing. Following Lucas (2002), who builds on Hansen and Prescott (2002) and Becker, Murphy, and Tamura (1990), our model economy produces a single good under two technologies: a traditional sector that uses land and unskilled labor, and a modern sector that is linear in human capital.[1] We extend this framework by introducing intra-household and production specialization in two-parent households. The model combines Becker (1981)'s insight on mothers' comparative advantage in child-rearing with Goldin (1990)'s observation that early industrial technologies favored physical strength. Men's "brawn", therefore, provides a temporary advantage at modest levels of human capital, but modernization erodes this gap as human capital becomes the primary engine of growth. These modeling choices are essential to account for the salient facts of the industrial revolution, including the U-shaped pattern of female labor force participation.

In our model, the transition from stagnation to modern growth is driven by households' response to rising productivity in the traditional sector. As output per person gradually increases, families can afford to invest in child "quality," with mothers reallocating time from production to child-rearing. These investments eventually turn profitable the deployment of (some) labor in a human capital-intensive production technology, triggering modernization and sustained growth. This mechanism contrasts with Hansen and Prescott (2002), where the takeoff from Malthusian stagnation is driven by exogenous technological progress in the modern sector. In our framework, productivity growth in the traditional sector is exogenous, but the escape from stagnation is endogenous, arising from household decisions over fertility and quality bequests that trigger the onset of a human-capital economy and sustained growth.

[1] In contrast to Hansen and Prescott (2002), Lucas (2002) endogenizes fertility by incorporating the Beckerian quantity–quality trade-off, as formalized in Becker (1960), Becker and Barro (1988), and Becker et al. (1990).

Modernization proceeds asymmetrically. The parent with a comparative advantage in brawn—reflecting the physical demands of early industrial production—enters the modern sector first, raising the marginal product of labor in the traditional sector and temporarily delaying his partner's transition. As human capital accumulates, the brawn advantage fades, and both parents adopt modern production. With the onset of modernization, households face a quantity–quality trade-off, opting for fewer but better-educated children. As human capital rises, the opportunity cost of child-rearing increases, leading mothers to reenter the labor force while investing more time in *fewer* children. These mechanisms reproduce the central empirical patterns of the industrial revolution: a shift out of traditional production, declining fertility, rising education, and a U-shaped path of female labor force participation.

The model is solved analytically at both ends of the development spectrum: a stagnant steady state and a balanced growth regime. To characterize the transition between these regimes, we solve the model numerically, calibrating it to Great Britain over the period 1400–2000. The parameterization is designed to match the evolution of income per capita across the *Malthusian*, *Post-Malthusian*, and *Modern Growth* regimes, in the spirit of Galor and Weil (2000). We then compare the model's predictions with historical time series for Great Britain and cross-country data. The resulting time paths broadly reproduce the key features of the industrial revolution.

Theoretically, our model belongs to the literature on Unified Growth Theory, which seeks to explain the transition from traditional modes of production to modern economic growth while accounting for the demographic transition along the path to development. Galor (2011) provides the most comprehensive overview of this literature. Closely related to our work, Galor and Weil (1996) link fertility and growth in a two-parent framework and show that the U-shaped pattern of female labor force participation can arise from the gradual closing of the gap between actual and desired fertility as biological and supply constraints—such as high infant mortality—relax with development. Our model abstracts from these important factors but offers a parsimonious account in which the U-shape emerges from optimal household behavior. It arises endogenously from intra-household specialization and the rising opportunity cost of child-rearing as human capital accumulates, rather than from non-optimization frictions or binding constraints. Moreover, we examine this mechanism quantitatively to assess its empirical relevance.

Another closely related paper is Ngai, Olivetti, and Petrongolo (2024). They reconstruct a consistent measure of male and female employment and hours in the United States from 1870 to 2019, adjusting Census data to include unpaid family work in agriculture and family businesses. Once this adjustment is made, female employment follows a clear U-shaped pattern—declining until the mid-twentieth century and rising thereafter—while male em-

ployment declines monotonically. They then develop a three-sector model of structural transformation and the marketization of home production, showing that uneven productivity growth and gender specialization can account for the U-shape. Our approach differs in both mechanism and focus: their account emphasizes the marketization of home production and is silent on fertility, whereas we connect the U-shape directly to fertility decisions and intergenerational human capital accumulation.

The empirical treatment in this paper follows the spirit of Lucas (2002), who emphasized that the industrial revolution is not a singular historical event but a global process that continues to unfold. Similar empirical analyses appear in Kremer (1993), Parente and Prescott (1993), and Galor (2011), particularly in the second chapter. More recently, Kremer, Willis, and You (2022) provide updated evidence showing that over the past 25 years, poor countries have begun to grow faster *unconditionally*, reversing centuries of divergence. They document a global shift from divergence to convergence, characterized by fewer growth "disasters" and an increasing number of "miracles," as the world's poorest regions experience sustained improvements in living standards. Our contribution builds on this evidence by extending the documentation of the global industrial revolution and linking these empirical regularities to the core forces behind this transition.

The rest of the paper is organized as follows. Section 2 illustrates some facts about the global industrial revolution. Section 3 presents a unified growth theory with two-parent households and establishes the conditions for the onset of a human capital economy. Section 4 presents the model calibration and illustrates the main quantitative results of our paper. Section 5 concludes.

2 Some Facts of the Global Industrial Revolution

Unless otherwise noted, data on income and population come from the Maddison Project Database (version 2023), constructed by Bolt and van Zanden (2025). Over the last 200 years, world production has grown steadily at about 2.4 percent annually, while global population expanded by roughly 1 percent. These figures imply that income per person increased by about 1.3 percent per year—an extraordinary contrast to the stagnation of earlier centuries. Figure 1 plots world production and population from 1820 to 2018 on a logarithmic scale, so deviations from linearity indicate accelerations. The figure shows output outpacing population since 1820 and accelerating after World War II, while population growth has slowed in recent decades. Between 1820 and 1950, world output grew at 1.6 percent annually, then accelerated to about 3.9 percent after 1950. Population growth rose from 0.7 percent in 1820–1950 to 1.6 percent in 1950–2018. Since the early twenty-first century, output growth

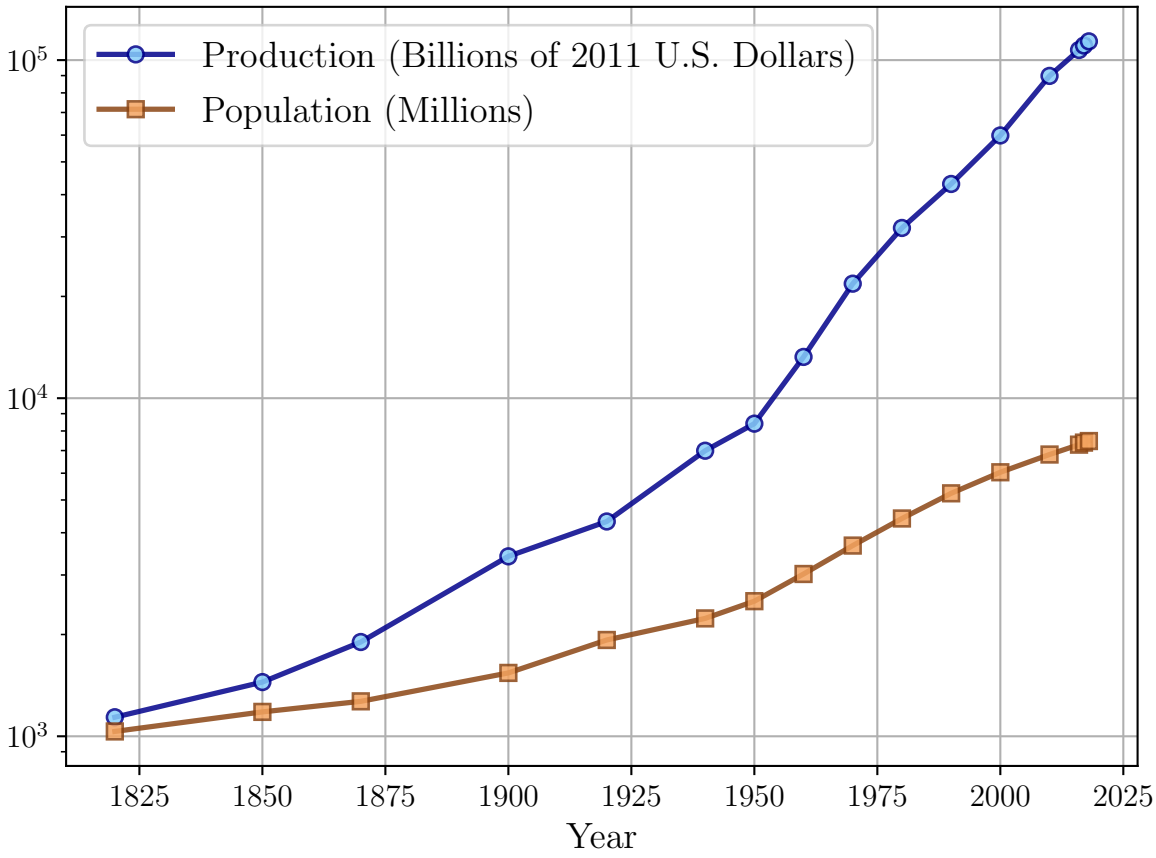


Figure 1: World production and population. 1820–2018.

has slightly decelerated to 3.6 percent, while population growth fell to 1.2 percent.

Figure 2 plots the world income per capita between 1820 and 2018. All the income numbers used in this paper are expressed in 2011 U.S. dollars. The world has steadily increased its production per person over the last 200 years, from an income per capita of about 1,100 USD in 1820 to over 15,000 USD in 2018. These numbers translate into a 13-fold increase in the standards of living worldwide. No period in history has witnessed more economic prosperity around the globe. It took 80 years, since 1820, for the world to duplicate the production per person. In contrast, income per capita multiplied about 4.5 times during the twentieth century, and between 2000 and 2018, the world witnessed an additional 50 percent increase in living standards.

2.1 The Industrial Revolution Across the World

The changes documented above have no precedent in history. Still, one must ask: Are these aggregates driven by a few successful regions or cultures, or is the industrial revolution a

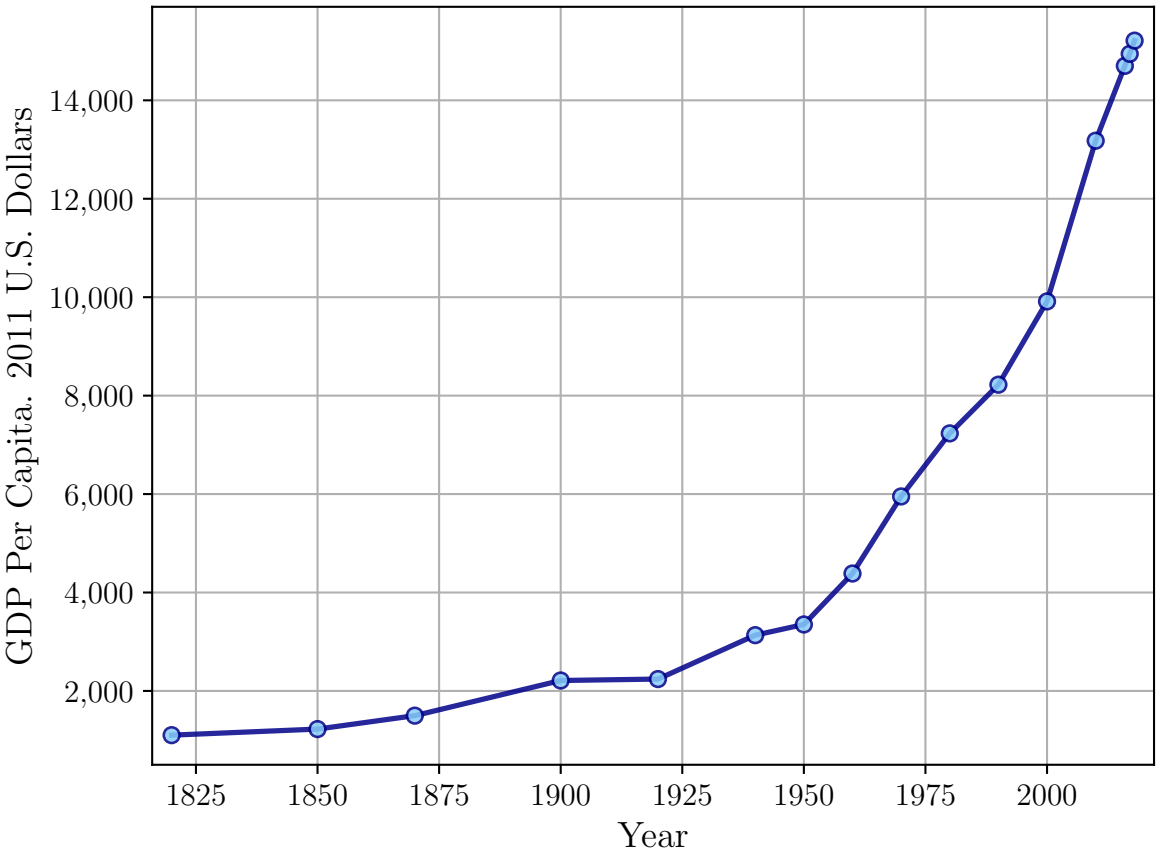
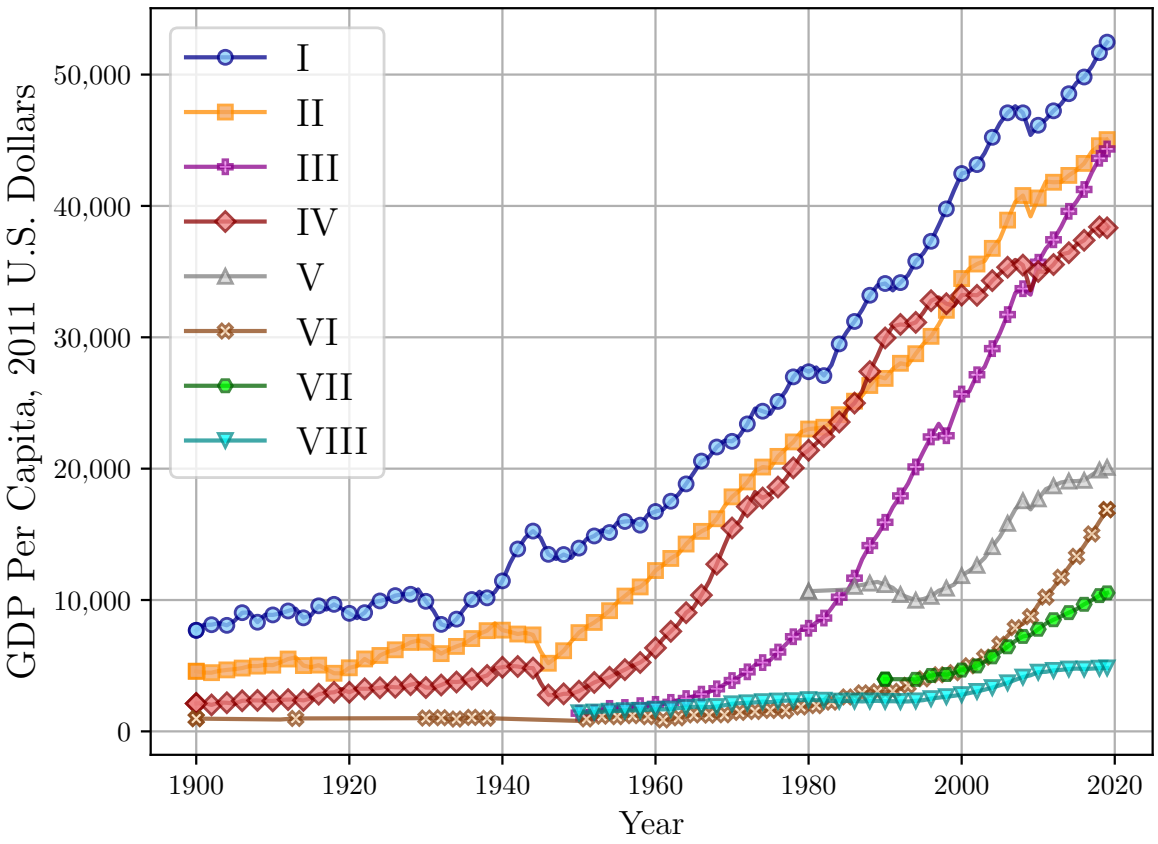


Figure 2: World income per capita (2011 U. S. Dollars). 1820–2018.

truly global phenomenon? To address this, Figure 3 plots GDP per capita from 1900 to 2018 for eight country groups, ordered by their 2018 income levels. These groupings follow Lucas (2002), with notable distinctions. In addition to Japan, we highlight two distinct development paths: the Asian Tigers (Group III)—Hong Kong, Singapore, South Korea, and Taiwan—and China (Group VI). Their extraordinary growth over recent decades warrants special attention. We also divide the poorest countries into two groups: the rest of Asia (Group VII) and Africa (Group VIII).

The English-speaking countries (Group I) were the pioneers of the industrial revolution. By 1900, they had already established a substantial lead, with living standards roughly twice as large as those of continental Europe. Since then, they have grown steadily at about 2 percent per year, maintaining their position at the productivity frontier. The major continental European economies—France, Germany, the Netherlands, and Scandinavia (Group II)—industrialized soon after the Western Offshoots. Their GDP per capita trends have run essentially parallel to Group I. Differences in level persist, but since the 1960s, Group II has consistently reached the income levels of Group I with a delay of about a decade.



I: U.K., USA, Canada, Australia, and New Zealand.
II: France, Germany, Netherlands, Scandinavia.
III: Hong Kong, Singapore, South Korea, Taiwan.
IV: Japan.
V: Rest of Western Europe, Latin America, Eastern Europe, Soviet Union.
VI: China.
VII: Rest of Asia.
VIII: Africa.

Figure 3: GDP per capita, eight groups. 1900-2018.

The Asian Tigers (Group III)—Hong Kong, Singapore, South Korea, and Taiwan—began their ascent in the second half of the twentieth century. Since the 1980s, they have grown at rates well above the frontier's 2 percent benchmark. By the late 2000s, they had already surpassed Japan in income per capita. Their trajectory exemplifies latecomer success: fast, sustained growth with no clear signs of stagnation, though growth has decelerated recently, as they join the pioneers of the industrial revolution. Japan (Group IV) traced earlier a similar path to the Asian Tigers: It lagged far behind Group I in 1900, began converging in the late 1950s, and closed much of the gap by the 1980s. Unlike the Tigers, however, Japan's growth stalled in the mid-1980s, and the gap with Group I has since widened.

The rest of Western Europe, Latin America, Eastern Europe, and the Soviet Union (Group V) also industrialized, but substantially later. By the 1980s, it was evident that their growth performance was well above stagnation. Their trajectories, however, reflect a middle-income trap—steady growth at roughly the same rate as the leaders, but at income levels well below the frontier, outside the 2:1 ratios that prevailed before the industrial revolution.

China (Group VI) represents the most remarkable latecomer at the turn of the twenty-first century, both in terms of the speed and scale of its transformation. In 1960, its income per capita was just above 1,000 USD—essentially a subsistence level. Between 1960 and 2000, China multiplied its income nearly fivefold, and between 2000 and 2018 it almost tripled again. This dramatic acceleration mirrors the earlier trajectories of Japan and the Asian Tigers, but with delays of three and five decades, respectively. The takeoff observed during the 2000s, unprecedented in China's history, is, however, a similar path already traced by the "Asian miracles" described in Groups III and IV.

Last—and perhaps the most remarkable fact of all—the poorest regions of the world, represented by Group VII (the rest of Asia) and Group VIII (Africa), have begun their own industrial revolutions. By 2018, the Asian economies in Group VII reached an average income per capita slightly above 10,000 USD. Although this still represents a development gap of about 5:1 relative to Group I, the distance was 14:1 in 1950—a striking reduction within seven decades. The poorest countries today are in Africa (Group VIII), where average income per capita remains roughly one-tenth that of Group I. While this ratio is similar to that of 1950, it follows a period of widening divergence between 1960 and 2000, when the gap reached 15:1, followed by a marked reduction in the gap up to its current levels. Africa is, therefore, the youngest newcomer to the global industrial revolution.

Figure 3 illustrates the uneven nature of the global industrial revolution. While some regions had already embarked on modern growth by the early twentieth century, others remained stagnant until the postwar decades and the end of the colonial era, and the poorest regions began to grow only toward the close of the century. The vast inequality in living standards observed today is thus a relatively recent phenomenon, largely the consequence of differences in the timing of takeoff.

To underscore how the uneven spread of the industrial revolution shaped global inequality, Figure 4 plots the share of world production and population accounted for by European-dominated economies—traced here as Groups I, V, and VI, the regions most directly influenced by European culture. The data prior to 1820 are taken from Lucas (2002). In 1750, the shares of world income and population were nearly identical. By 1820, driven mainly by Group I, the production share had risen to 41 percent, while the population share stood near

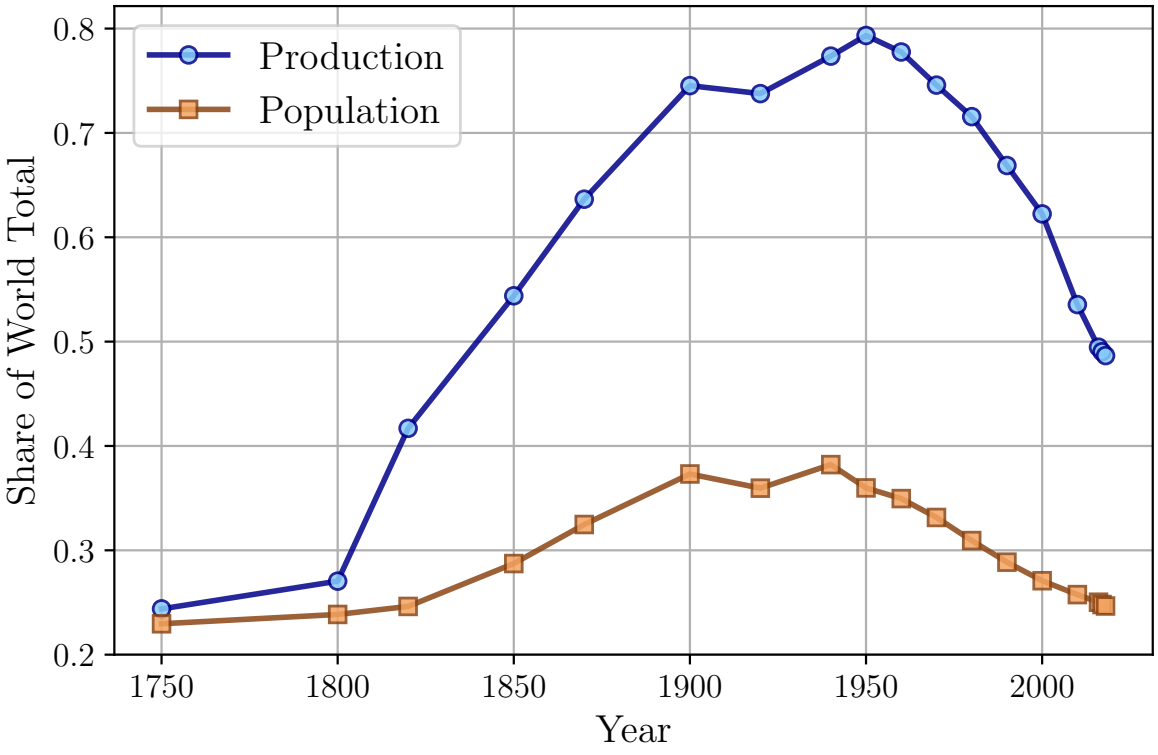


Figure 4: European-dominated economies' share of world production and population.

25 percent. The gap widened steadily, reaching its peak in 1950, when European-dominated economies produced 80 percent of the world's output but accounted for only 36 percent of its population. Since then, this imbalance has narrowed rapidly: by 2018, their population share had returned to its 1820 level, while their production share had fallen to 49 percent.

The pattern in Figure 4 shows that the extraordinary inequality created by Europe's early takeoff may be a historical transient. As the Industrial Revolution continues to unfold in the rest of the world, production shares may continue to converge toward population shares, reducing global inequality to the pre-industrial levels documented by Adam Smith. As Lucas (2002, p. 174) observed,

> At the beginning of the twentieth century, many sophisticated observers were persuaded that northern Europeans must have racial or cultural advantages that accounted for the income differentials that had emerged during the nineteenth century. Now, at the beginning of a new century, it is abundantly clear that full participation in the economic benefits of the industrial revolution is open to countries of all races and cultural backgrounds.

2.2 Growth Miracles

The regional patterns described above are further illuminated by examining episodes of rapid income growth worldwide. To highlight the speed of these transitions, Table 1 records the years in which a country's per capita income doubled within a decade—a benchmark we refer to as a *growth miracle*. This label has been used almost exclusively to describe the spectacular growth episodes in Asia, as documented in Groups III, IV, and VI.

Except for Australia in 1851, no Western Offshoot country has experienced economic growth at such a rapid pace. The first postwar miracles occurred in Europe, where France and Germany recorded sharp, reconstruction-driven expansions; these are essentially major deviations induced by wartime destruction that cannot be smoothed by five-year averages. Later, in East Asia, Japan, South Korea, Singapore, and Taiwan achieved sustained growth episodes unprecedented in earlier history. Japan's miracle lasted fifteen years, followed by South Korea's, which began in 1961 and extended for nearly a quarter of a century—the longest on record. Singapore and Taiwan also experienced rapid expansions during the 1970s. Although Hong Kong does not meet our formal criterion, its income per capita increased by more than 95 percent within a single decade.

China's miracle in the early 2000s stands out as perhaps the most remarkable growth episode of the twenty-first century. Within a single decade, more than 1.3 billion people effectively doubled their standard of living! Yet this experience is no longer unique to China or to East Asia. Groups VII and VIII in Table 1, which include the rest of Asia and Africa, show that such rapid transitions are no longer confined to the early- or mid-twentieth-century industrializers. Over the past thirty years, growth miracles have occurred in dozens of low-income countries, many of them outside the resource-exporting economies. These episodes suggest that the capacity to escape regimes of Malthusian stagnation is now a global phenomenon.

The evidence from Groups VII and VIII suggests that the "elusive quest" for growth may no longer be elusive for many poor countries. Over the past generation, the world has witnessed a proliferation of growth miracles across diverse institutional and geographic settings. Ireland, for example, experienced a sustained expansion by the end of the twentieth century, while Nigeria and Myanmar entered the new millennium with similar trajectories. Table 1 marks with an asterisk those countries classified as "oil-dependent," based on OPEC membership. While oil-exporting economies account for a significant portion of the recent growth in Groups VII and VIII, several of the most striking episodes in Table 1 occurred in economies without significant resource rents.[2]

[2]Venezuela is the only country in Group V—and the only Latin American country—to have experienced a growth miracle under our definition. Venezuela's early twentieth-century growth was certainly driven by

	Start	End		Start	End
Group I			**Group VII**		
Australia	1851	1854	Afghanistan	2007	2014
Group II			Azerbaijan	2005	2015
France	1953	1954	Bahrain	1998	2010
Germany	1957	1960	Iran*	1970	1976
Group III			Iraq*	2003	2015
Singapore	1971	1980	Kazakhstan	2006	2011
South Korea	1971	1995	Kuwait	1996	2010
Taiwan	1971	1980	Lebanon	2000	2001
Group IV			Mongolia	2003	2016
Japan	1959	1974	Myanmar	2005	2010
Group V			Oman	1966	2008
Albania	2006	2009	Qatar*	1997	2014
Austria	1956	1958	Saudi Arabia*	1967	1976
Bosnia–Herzegovina	2003	2005	Thailand	1994	1994
Ireland	1999	2004	Turkmenistan	2005	2016
Italy	1955	1956	United Arab Emirates*	2005	2005
Latvia	2006	2006	Yemen	1977	1979
Lithuania	2006	2006	**Group VIII**		
Malta	1974	1983	Angola*	2003	2015
Montenegro	1965	1966	Botswana	1971	1982
Montenegro	1968	1968	Botswana	1987	1993
Montenegro	2004	2013	Chad	2008	2013
Romania	1954	1956	Equatorial Guinea*	1966	1968
Romania	1968	1974	Equatorial Guinea*	1996	2011
Romania	2006	2012	Gabon*	1975	1977
Russian Federation	2006	2010	Libya*	1962	1974
Serbia	1964	1966	Libya*	2003	2011
Slovenia	1964	1965	Nigeria*	2008	2012
Ukraine	2007	2009	Swaziland	1968	1969
Venezuela*	1952	1953	Swaziland	1974	1979
Group VI			Zambia	2009	2012
China	2010	2012			

Table 1: Growth miracles around the globe. A country is considered to have experienced a growth miracle if its GDP per capita doubles within ten years, using a five-year rolling average to smooth out business-cycle fluctuations. The dates represent the first and last year when the five-year rolling GDP per capita was at least two times the five-year rolling GDP per capita of the previous decade.*Countries members (including former members) of the Organization of the Petroleum Exporting Countries (OPEC). The list of OPEC countries is available at https://www.opec.org/opec_web/en/about_us/25.htm.

oil, but this process was accompanied by a substantial expansion of non-oil sectors, particularly between 1936 and the 1970s. See Salazar-Carrillo (1976) for a historical account of oil and industrial development in Venezuela.

These episodes confirm that the ability to escape the Malthusian trap is no longer confined to a handful of early industrializers. Table 1 illustrates that this transition is not solely a European story, even if one is generous regarding what defines "European influence". Nor is it limited to the Asian miracles of the 1970s or the more recent Chinese takeoff. Rather, the phenomenon of rapid income doubling—once exceptional—has become increasingly common among countries that began their development process relatively late, as extensively documented by Parente and Prescott (2000). The following section examines how modern economic growth—some of it occurring at a extraordinary speed—aligns with the key structural dimensions of the global industrial revolution: the reallocation of labor out of agriculture, the demographic transition, the accumulation of human capital, and the U-shaped pattern of female labor force participation.

2.3 Out of Agriculture

The process of economic development is fundamentally intertwined with the reallocation of labor out of agriculture. Figure 5 plots the agricultural employment share for country groups I–VIII against both calendar time (left panel) and log GDP per capita in 2011 US dollars (right panel). To compute the agricultural employment share, we use data from the U.S. Department of Agriculture for the numerator, defined as the number of economically active individuals over age 15 engaged in agriculture.[3] For the denominator, we use the number of persons engaged in economic activity from the Penn World Tables 10.0.[4] These sources provide comprehensive global coverage beginning in 1961. Earlier data exist for a handful of countries, but these are concentrated in groups I–IV and offer limited insight into the transitions now underway in groups VII and VIII.

The left panel of Figure 5 shows that by 1961, Group I had already reduced its agricultural employment share to below 10 percent. This fraction continued to decline, reaching levels under 2 percent by 2018. Group II followed a similar trajectory, starting at roughly 15 percent in 1961 and converging to under 2 percent by the end of the sample. In contrast, the Asian Tigers (Group III) and Japan (Group IV) began the transition with substantially higher agricultural employment shares—45 and 28 percent, respectively. These are not negligible fractions, yet the pace of transformation in these economies was faster than in Groups I and V. By 2018, both Japan and the Asian Tigers had reduced their agricultural employment shares to below 4 percent.

Groups IV (China), VI, and VIII began with agricultural employment shares between

[3]Data available at https://www.ers.usda.gov/data-products/international-agricultural-productivity/

[4]Available at https://www.rug.nl/ggdc/productivity/pwt/?lang=en

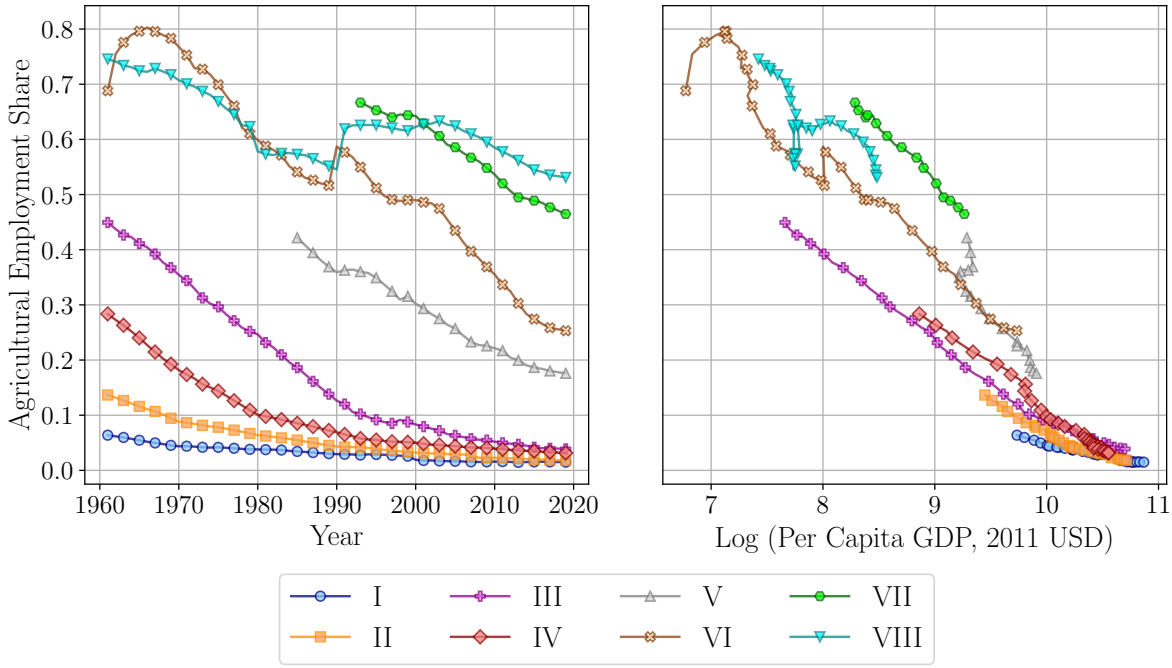


Figure 5: Employment share in agriculture, eight groups. 1961-2018.

70 and 90 percent. When plotted against calendar time, the trajectories for these groups appear roughly parallel, though the slope for Group VIII (Africa) is less steep. Even so, the decline is substantial: the share of agricultural employment in Africa has fallen markedly over the past half-century.

The right panel of Figure 5 plots agricultural employment shares against income per capita (in logs). Much of the variation observed in 1961 reflects differences in development stages across regions. When these trajectories are traced as a function of GDP per capita, a strikingly consistent pattern emerges: as income rises, the share of labor employed in agriculture declines. While level differences persist, today's poorest countries follow paths remarkably similar to those traced earlier by the now-rich economies. Historical evidence extending into the nineteenth century confirms that early industrializers underwent analogous transitions (see Kuznets (1971)). These observations reinforce a central regularity of economic development: the reallocation of labor out of traditional agriculture is not a regional anomaly but a universal feature of the industrial revolution.

From employment shares alone, one cannot determine whether the global decline in agricultural labor reflects slower growth in agrarian employment relative to the total labor force, or an absolute reduction in the number of agricultural workers. To address this, Figure 6 plots total agricultural employment for Groups I–VIII—the numerator of the shares shown in Figure 5. We use a logarithmic y-axis allows us to observe the entire world in a single plot.

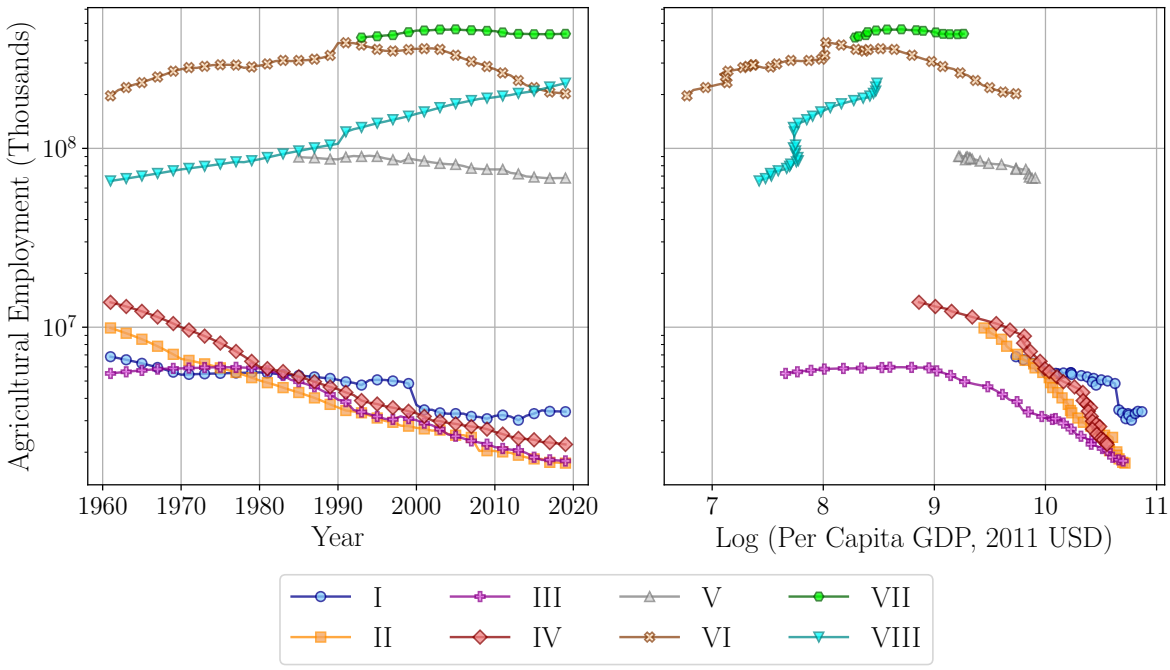


Figure 6: Employment in agriculture (thousands of workers), eight groups. 1961-2018.

The left panel displays the number of agricultural workers over time (1961–2018), while the right panel plots the same quantity against log income per capita.

The left panel reveals that Groups I through VI are not only experiencing a decline in agricultural employment shares (as shown in Figure 5), but also shedding labor from agriculture in absolute terms. Japan, for instance, employed roughly twice as many agricultural workers as all countries in Group I combined at the beginning of the sample. By 2018, Japan had lost nearly one million more agricultural workers than Group I, although the English-speaking countries shed approximately three million agricultural workers over the same period.

Notably, the bulk of agricultural workers today are outside groups I–IV. China (Group VI) continued to grow the number of workers in agriculture, similar to the poorest countries in the 1960s through the 1980s. However, by the early 1990s, China also began to employ fewer workers on farms. China still has over 200 million workers in agriculture, although the number of agricultural workers in China peaked at about 400 million in 1991, nearly twice as many as in 1961 and 2018.

Group VII displays a modest decrease in its number of farmers, albeit at a slower pace than observed in China. Africa is the only region still trending upward in terms of agricultural workers (Group VIII). Between 1961 and 2018, the number of farm workers in Africa increased by about 160 million people, from 65 million in 1961 to 225 million in 2018.

The right panel of Figure 6 plots the time series of employment in agriculture for all groups as a function of their (log) GDP per capita. There are two messages worth highlighting from this exercise. First, one observes that for income levels below exp(9) $\sim$ 8,000 USD, the number of workers in agriculture increases. In the case of Africa, this threshold has not yet been achieved. One can observe the stagnation documented in Figure 3, reflected in a vertical rise in the number of agricultural workers. Second, the logarithmic scale allows us to see a significant acceleration in the movement out of agriculture as each Group achieves an income per capita above exp(9.5) $\sim$ 13,300 USD. Importantly, this acceleration happens everywhere. Poor countries are undergoing a structural transformation, characterized by a decline in the number of people employed in agriculture, both in terms of percentage and in absolute numbers. Poor countries are beginning their industrial revolutions.

2.4 Demographic Transition

Figure 7 plots population growth across Groups I–VIII. Each point shows a group's annualized population growth rate over a decade against the log of GDP per capita at the start of that decade. The pattern is clear: population growth rises as income grows out of stagnation, just as Malthus and Ricardo would have predicted, and then declines steadily as economies become richer—a fact that would have puzzled all the classical economists. Growth rates peak for every group before income reaches about 3,000 USD. Beyond this threshold, no group in our sample sustains positive population growth alongside further gains in income per capita.

For Group I, the peak is not visible because the data begin in 1900; however, their subsequent decline aligns closely with that of other groups at similar income levels. Today, Groups I and II record population growth rates below 1 percent, while China (Group VI) has stabilized near 0.5 percent. China is unusual in several respects: its surge in population growth occurred with virtually no gains in income, and the subsequent steep decline was premature, driven mainly by Mao's famine, well before the one-child policy. Another anomaly is Japan (Group IV), where population growth fell below zero at income levels above 25,000 USD. This case reflects structural forces and restrictive migration policies rather than a temporary shock. Japan's trajectory underscores a striking contrast: a spectacular growth miracle in the mid-twentieth century followed by three decades of stagnation and demographic contraction.

Among the poorer regions, the paths for Groups VII and VIII trace the same inverted-U pattern observed in earlier industrializers. Both regions have already passed their population growth peaks and are now undergoing similar demographic transitions at comparable income levels. As these economies grow richer, their demographic trajectories are likely to converge

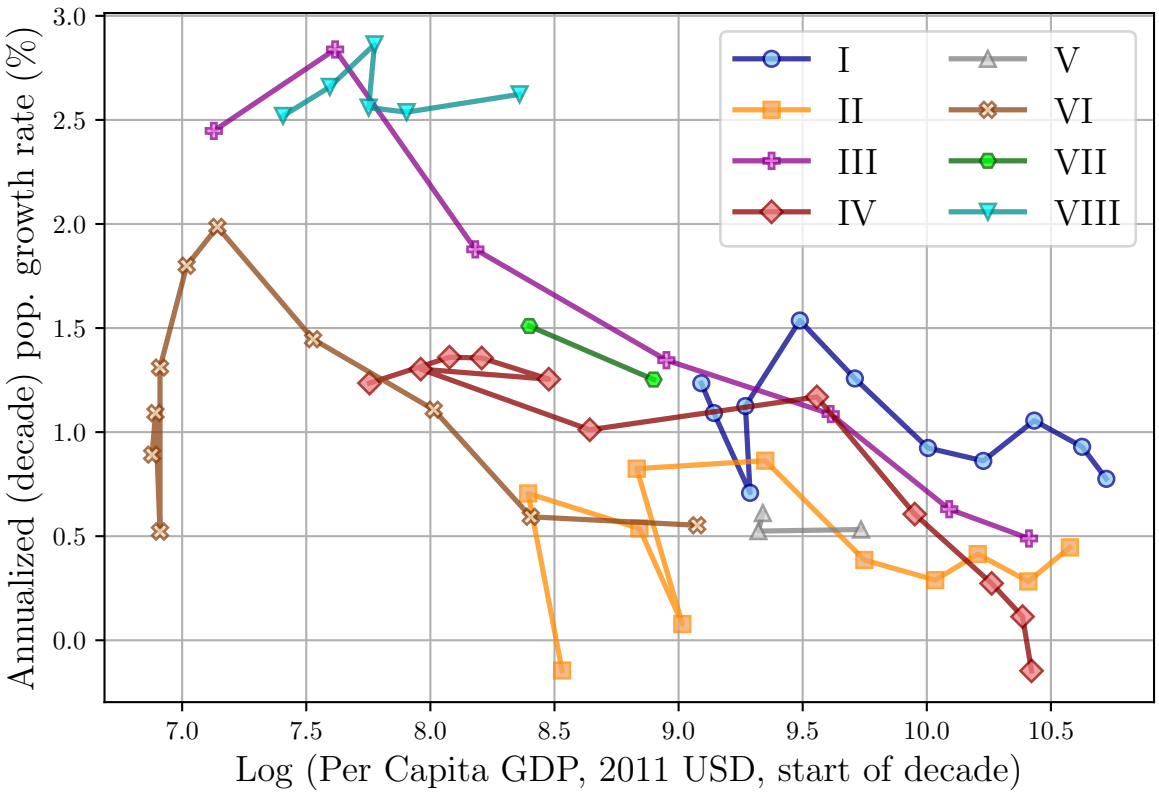


Figure 7: Demographic transitions, eight groups. 1900-2018.

with those of Groups I through III. The evidence suggests that declining population growth, once regarded as a feature of the industrial nations, is now a universal stage in the process of modern development. As the industrial revolution continues to unfold in poorer countries, one can conjecture that population growth will follow the same path traced by Groups I, II, and III.

2.4.1 Fertility

Population growth is the combined outcome of fertility, mortality, and migration. Mortality can be influenced by health and nutrition choices, and migration policies are largely societal decisions. Fertility, by contrast, is primarily a family choice. Figure 8 plots fertility—measured as the number of children a woman would bear if she lived through her childbearing years and followed the age-specific fertility rates of the given year—against income per capita (in logs).[5]

The cross-section for 1990 (left panel) shows a steep negative relationship, while the 2018 cross-section (right panel) is noticeably flatter. In 1990, Groups VII and VIII clustered in the

[5]Data source: gapminder.org.

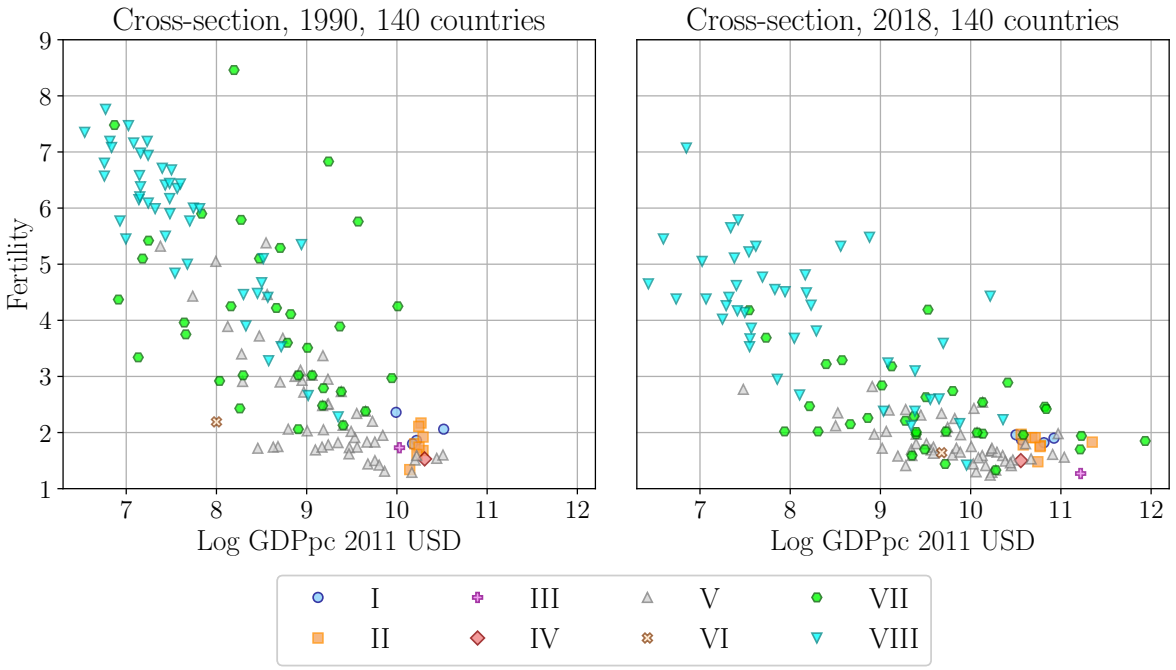


Figure 8: Fertility. Cross-section data for 140 countries in 1990 and 2018. Eight groups. Fertility is measured as the number of children a woman would bear if she lived through her childbearing years and followed the age-specific fertility rates of the given year.

low-income, large-family zone. By 2018, the demographic transition in Group VII is evident, leaving Africa (Group VIII) as the main region where high fertility persists at low income levels. It is also striking that the well-defined relation for Group VI in 1990 is less steep by 2018. For advanced economies, fertility rates were already low by 1990, so changes since then have been modest. The broader pattern, however, is clear: the link between income and fertility remains strong, with an important caveat: at any given income level, families today are smaller than they were three decades ago.

2.5 Human Capital

Human capital, as treated here, refers to investments that enhance an individual's productive abilities. These investments take many forms: reading, engaging in conversations, exploring new environments, spending time with peers and mentors, and—critically—the effort parents devote to equipping their children with skills beyond innate talent. Formal schooling is a central component of human capital, but it is not the whole of it. School attendance, however, has the advantage of being measurable and comparable across countries, making it a useful proxy for analyzing development paths, even though it captures only part of the broader process of skill accumulation.

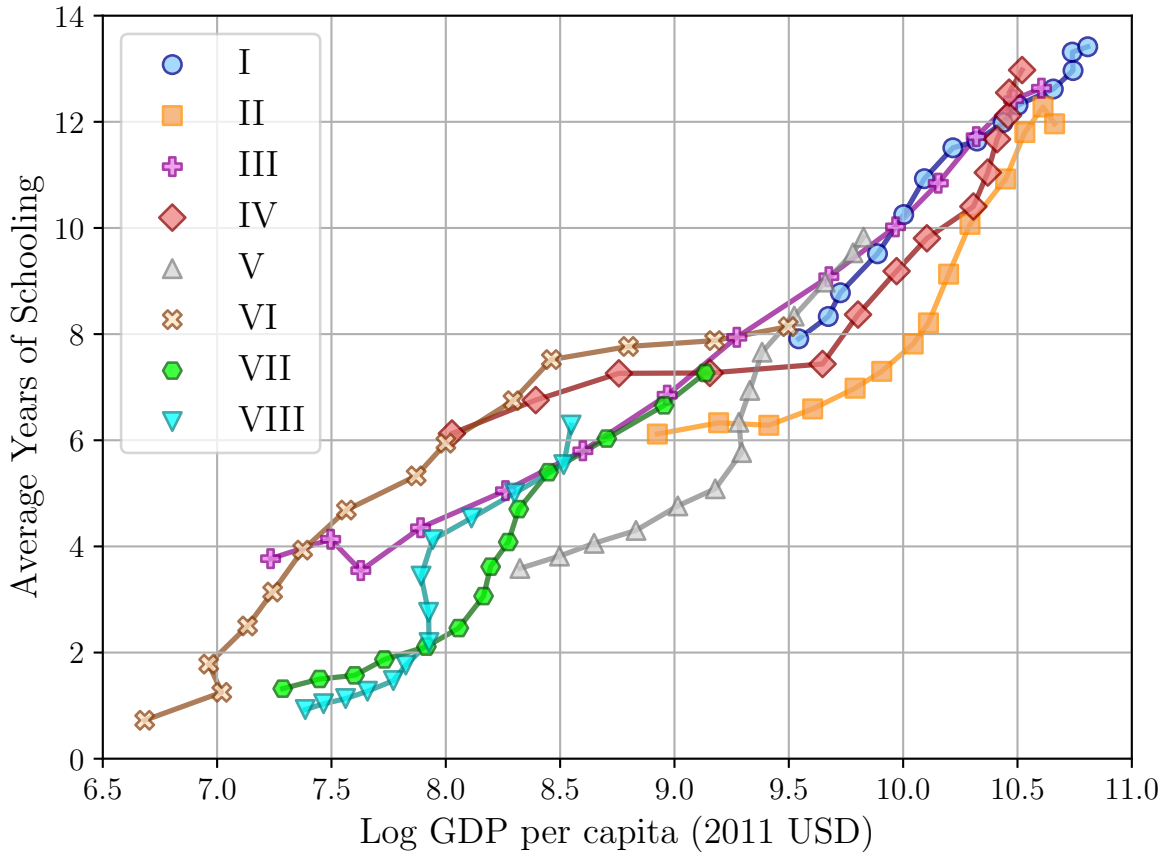


Figure 9: Average years of schooling, eight groups.

Figure 9 plots average years of schooling from the Barro and Lee dataset, which measures the average years of education for the population aged 25 and above. Despite vast differences in geography and history, all regions align along a remarkably similar path when human capital is plotted against income per capita. At low levels of development, represented by Africa (Group VIII) and the rest of Asia (Group VII), schooling averages fewer than three years. As income rises, education expands rapidly: the Asian Tigers and Japan (Groups III–IV) follow almost identical trajectories. The English-speaking offshoots and continental Europe (Groups I–II) occupy the upper end of the curve, where schooling levels exceed twelve years.

The smooth alignment across all groups suggests that the accumulation of human capital follows a broad income–education relationship, largely independent of initial conditions or timing. In this sense, Figure 9 shows that wealth and education coevolve along a broad path traced by every region since the onset of the industrial revolution.

2.6 U-Shaped Female Labor Force Participation

Goldin (1994) documents a striking empirical regularity: as economies develop, women's participation in the labor force first declines and then rises, tracing a U-shaped path. Leveraging on a cross-section of countries for 1980, she finds that participation is high at low stages of development, falls to a trough at middle stages, and rises again as economies grow richer. For the United States, Goldin argues that this pattern would be visible if data extended before 1890; historical series capture only the rising part of the U.[6]

Recently, Ngai et al. (2024) revisited the U-shaped hypothesis through the lens of structural transformation and marketization, and importantly, they contribute to our understanding of historical time series, where women's labor force participation appears to be absent. They construct a consistent measure of male and female work for the United States from 1870 to 2019, combining Census data with early sources to account for unpaid family labor in farms and non-farm businesses. This adjustment matters: while raw Census figures report female participation around 10 percent in 1870, the adjusted series places it near 56 percent. The lowest point occurs around 1940 at roughly 45 percent, followed by a steady rise above 70 percent in 2019. Second, they show that the U-shape is sharper when measured in hours rather than headcounts. In their view, the early decline reflects the erosion of unpaid agricultural work; the later rise follows the expansion of market services and the reallocation of home production to the market. Their model attributes these patterns to uneven sectoral productivity growth and gender specialization, although the model is silent regarding fertility.

Following Goldin (1994), we ask whether the U-shaped relationship between economic development and female labor force participation persists in recent cross-country data. Our data comes from the International Labour Organization (ILO), accessed through `gapminder.org`. These measures capture a broad range of work arrangements, including informal and illegal employment and, importantly, unpaid work. Such work remains prevalent in traditional agriculture and family enterprises, as emphasized by Ngai et al. (2024), and accounts for a large share of total employment in poor countries. Unfortunately, the series begins in 1989, so we cannot trace a meaningful historical evolution of the U-shape.

Figure 10 plots two cross-sections of the female labor force participation rate for women aged 15 to 65 as a function of the logarithm of GDP per capita (2011 USD). The left panel shows the relationship for 140 countries in 1990, and the right panel presents the

[6] Goldin (1994) explains the U-shape as follows: early industrialization brings a strong income effect—women withdraw from market work as household incomes rise—reinforced by a stigma against "blue-collar" manufacturing jobs. Later, education expands and white-collar employment grows; the substitution effect dominates, and women reenter the labor force in occupations where stigma is absent.

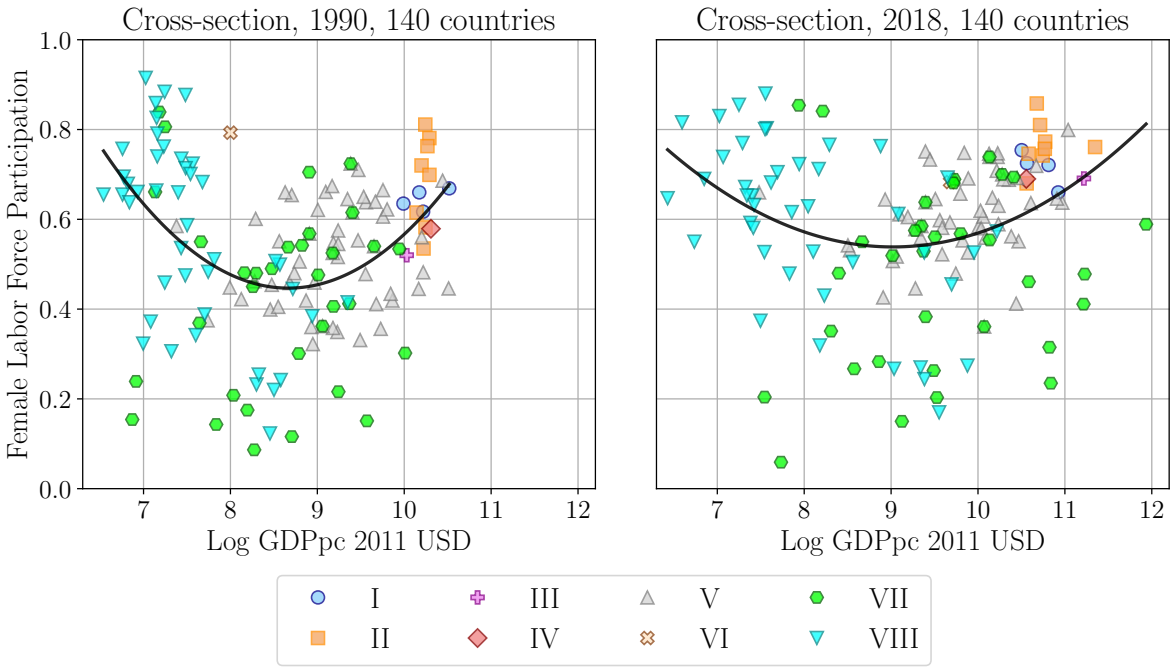


Figure 10: Female's U-shaped labor force participation over development. Cross-section data for 140 countries in 1990 and 2018. The labor force participation rate is for women aged between 15 and 65. Eight groups

same for 2018. In each panel, we fit a second-order polynomial regression of female labor force participation on income per capita. Both panels illustrate the U-shaped relationship between the female labor force and development, as documented by Goldin (1994). One observes that African countries (Group VIII) have similar participation rates to those of the advanced countries represented by Groups I and II in 1990 and 2018. One would expect lower participation rates for African countries as they embark on their industrial revolutions. This is what one observes for Group VII: More points on the northwest side of the U-shape belong to Group VII in 1990 (left panel) than in 1990 (right panel). These countries are growing wealthier *and* have lower labor participation rates for women. Goldin (1994) finds that Latin American countries in 1980 represented the bottom of the U. By 2018, the participation rates for women in Latin America had grown, and these countries are now in the rising part of the U-shape. The place at the bottom of the U now belongs predominantly to Group VII. Appendix A shows that male labor force participation, in contrast, is rather flat.

Appendix B uses panel data to address whether the relationship documented in the cross sections of Figure 10 vanishes once we control for the idiosyncratic effect that each country might have, or if these phenomena reflects general time trends rather than an inherent relation between development and female labor force participation. In other words, we aim to determine whether the within-country variation is driving the relationship. Thus, we

would expect the log income and its square coefficients to remain significant when controlling for country and time fixed effects. Table B.1 shows that the U-shaped relation between female labor force participation and development is robust to these controls, suggesting that countries that grow rich do experience a reduction in female labor force participation at early stages of development, followed by a rise roughly after about $\exp(9) \sim 8{,}000$ USD of income per capita.

This concludes our empirical examination of the industrial revolution. In what follows, we present our theoretical framework to interpret these patterns and explore the choices that drive the onset of sustained income growth.

3 A Unified Growth Theory with Two Parents

This section builds on the analysis in Lucas (2002), particularly Section 7, which integrates two defining features of the industrial revolution: the role of land as a fixed input in traditional production, and the emergence of a quality–quantity trade-off in fertility decisions that gives rise to both a demographic transition and rising human capital, trigering modern economic growth. Hansen and Prescott (2002) study a transition, from a stagnant regime—characterized by low productivity growth and decreasing returns to scale—to a modern growth regime with higher productivity and constant returns to scale. However, in their framework, the demographics are imposed, and there is no explicit role for parental investment in child quality. By contrast, Becker et al. (1990) develop a model of endogenous human capital formation during the industrial revolution, but omit the role of land in their "Malthusian" equilibrium.[7] In what follows, we attempt to unify these perspectives by embedding both features—land-based constraints in traditional production and endogenous human capital investment in a modern economy—into a unified framework.

Relative to Lucas (2002), our model incorporates two key departures. First, households are composed of two parents who jointly choose fertility and the time allocated to the formation of child quality. This setup, following Becker (1981), allows for specialization within the household. While parents are symmetric in most respects, we allow them to differ along two dimensions that define their type. We label these as Type xx and Type xy. The xx parent has a comparative advantage in child-rearing—a simplification meant to capture biological and social predispositions often associated with women. As Becker (1981, p. 37) notes, "[w]omen not only have a heavy biological commitment to the production and feeding of children, but they also are biologically committed to the care of children in other, more

[7] In their framework, the Malthusian regime corresponds to an equilibrium with zero investment in human capital.

subtle ways." Conversely, the xy possesses a comparative advantage exclusively in modern production, intended to capture men's advantage in physical strength and suitability for brawn-intensive work—what Goldin (1990, p. 7) describes as "differences among industries ... [in] their initial requirement of strength and the hazardous and dirty nature of work."

Second, we adopt the preference structure proposed by Galor and Weil (2000), departing from the dynastic framework used in Lucas (2002). In this formulation, parents value the quality of their children for its own sake, rather than as a means to maximize the children's lifetime utility. This approach allows tractability when studying the transition to modern economic growth as a sequence of static choices updated each period in response to exogenous technological progress in the traditional sector and the endogenous evolution of human capital. Crucially, the escape from Malthusian stagnation in our model is entirely driven by human capital accumulation. This contrasts with Hansen and Prescott (2002), where exogenous productivity growth in the modern sector is critical for the transition out of Malthus.

As in Lucas (2002), we work with functional forms that permit analytical solutions, allowing us to characterize both the Malthusian equilibrium—marked by low growth and high fertility—and the modern growth regime—defined by sustained gains in income and low fertility. The model captures the essential features of the industrial revolution as a shift from stagnation to growth, driven by decisions made at the household level. We now turn to a formal description of the economic environment.

3.1 Endowments

In each period, every parent is endowed with one unit of time and h_t units of human capital, inherited from their parents' investment in child quality. The economy consists of a continuum of households of measure one. Each household comprises two parents and thus has a total time endowment of two units per period. Households also have access to x_t units of land, where X denotes the fixed aggregate land supply and N_t is the total number of adults alive in period t, born in period $t-1$.

To simplify the analysis, we assume that no property rights are defined over land; thus, land is held in common and cannot be transferred across generations. This assumption reflects the communal land use typical of hunter-gatherer societies, as discussed in Lucas (2002, Section 3), and aligns with the modeling approach of Galor and Weil (2000). In this setting, land does not enter into household optimization through bequests or asset accumulation. Instead, each household receives access to a per capita share of the fixed land stock in each period, determined by the prevailing population level. This treatment

preserves Ricardo's classical insight: land is a fixed, non-reproducible factor of production. As population increases, the land-to-labor ratio declines, generating diminishing returns and constraining the possibility of sustained growth in living standards—an essential feature of the Malthusian regime.

Finally, each parent is endowed with a fixed "advantage". For each type $i \in xx, xy$, let ξ_i denote efficiency in child-rearing, and let δ_i represent a "brawn" advantage relevant when operating modern production technologies exclusively. We assume $\xi_{xx} \geq \xi_{xy}$ and $\delta_{xy} > \delta_{xx}$, so that Type xx is relatively more efficient in producing child quality, while Type xy has an advantage in operating the modern technology production. These differences generate intra-household specialization and, crucially, are fixed over time—these advantages *do not* evolve.

3.2 Preferences

Household preferences, jointly representing both parents, are given by the utility function

$$\mathcal{U}(c_t, n_t, h_{t+1}) = (1-\beta)\log(c_t) + \eta\log(n_t) + \beta\log(h_{t+1}), \tag{1}$$

where c_t denotes consumption, n_t the number of children, and h_{t+1} the human capital level of each child. This specification captures parental trade-offs over material consumption, fertility, and the quality of offspring.

The term $\beta\log(h_{t+1})$ introduces Gary Becker's "quantity-quality" trade-off, emphasizing that parents derive utility not only from the number of children but also from the human capital invested in each child, thereby requiring an optimal allocation of resources between these two dimensions. The parameter η captures the weight parents assign to fertility, while β reflects the relative importance of children's quality. We assume that $\eta > \beta$; otherwise, parents may regard the number of children as a "bad"—a point emphasized by Lucas (2002, p. 137). Finally, note that the utility function is homogeneous of degree one in (c_t, h_{t+1}) for any given n.

3.3 Human Capital Accumulation

In this model, the formation of human capital takes place entirely within the household. There is no formal schooling sector. All investments in child quality are made through the allocation of parental time, and no role is assigned to market institutions in the education process.

Let h_t denote the stock of human capital in period t. The law of motion for human

capital across generations is specified as

$$h_{t+1} = h_t \cdot \varphi(r_{xx,t}; r_{xy,t}), \tag{2}$$

where $r_{xx,t}$ and $r_{xy,t}$ denote the fractions of time devoted to child-rearing by Type xx and Type xy parents, respectively. The function $\varphi(\cdot)$ satisfies $\varphi(0;0) = 1$, and is strictly increasing in each argument. Human capital is augmented only through active investment by parents. Thus, in the absence of time spent in child-rearing, human capital remains constant.

We assume $\xi_{xx} > \xi_{xy}$, so that Type xx parents are more productive in human capital formation per unit of time invested. This assumption is in line with the empirical findings of Del Boca, Flinn, and Wiswall (2014), who show that maternal time inputs have a disproportionately large effect on child cognitive development, even after controlling for education, labor supply, and other household characteristics. The model captures this asymmetry by assigning a higher productivity in human capital production to the xx parent's time, generating efficiency gains through specialization, in the spirit of Becker (1981); ? (?).

3.4 Technology

At each point in time, two technologies are available for producing the final good: a traditional technology that combines land and unskilled labor, and a modern technology that uses skilled labor.

The traditional production function is

$$y_{Tt} = A_t x_t^{\alpha} (l_{xx,t} + l_{xy,t})^{1-\alpha}, \tag{3}$$

where x_t denotes land input, l_{it} is the labor supply of Type $i \in xx, xy$, and A_t is total factor productivity.[8]

This land-intensive technology does not employ human capital. Labor is homogeneous and equally productive by Type. The traditional sector represents the low-productivity farming systems that dominated economic life since the Neolithic Revolution—modes of production in which output was constrained by fixed land and unskilled labor, and where the returns to education were negligible. These are the "thankless" forms of cultivation that Quesnay distinguished from surplus-generating agriculture.[9]

[8] Throughout, we use "sectors" and "technologies" interchangeably.

[9] As he wrote in the Tableau Économique contrasting his agricultural kingdom to the feudal agricultural systems, "[w]e are not speaking here of small-scale cultivation carried on with the aid of oxen, in which more than a million ploughs and about two million men would be required to work 40 million arpents of land" Quesnay (1758/1965, p. vi).

The modern production technology is given by

$$y_{Mt} = B\left[(h_t + \delta_{xx})\, l_{xx,t} + (h_t + \delta_{xy})\, l_{xy,t}\right], \tag{4}$$

where B is a constant productivity parameter in the modern sector, assumed to be relatively low compared to A_t at any point in time.

Human capital plays a central role in the modern sector. Each unit of labor is augmented by h_t, reflecting productivity gains from acquired skills. In addition, raw labor productivity differs across types: $\delta_{xy} > \delta_{xx} \geq 0$, capturing an initial "brawn" advantage of Type xy in physically demanding tasks. This advantage is more relevant when human capital is scarce. As h_t rises, however, the contribution of raw strength to output diminishes, and production becomes increasingly reliant on cognitive skills. The modern economy thus transitions from one where physical attributes dominate to one where acquired knowledge drives productivity.

3.5 Household Problem

In each period, the household chooses the number of children, how each parent allocates time between child-rearing and work, and how work time is divided across production technologies. Since households do not value leisure, all non-work time is allocated to child-rearing. Letting n_t denote the number of children, the labor supply of a parent of type $i \in xx, xy$ is

$$l_{i,t} = 1 - r_{i,t} n_t. \tag{5}$$

Since output is maximized by reallocating labor across production technologies conditional on labor supply, it is convenient to treat l_{it} as given when analyzing production decisions. We then return to the fertility and time investment choice to solve the household's quantity–quality trade-off. Conditional on labor supply, households allocate work effort across sectors. Let $\theta_{it} \in [0, l_{it}]$ denote time allocated to traditional production. The remainder, $l_{it} - \theta_{it}$, is allocated to the modern sector. Figure 11 summarizes the structure of time use.

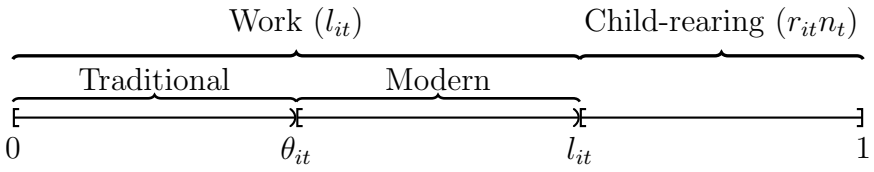


Figure 11: Time allocation for parent type $i \in xx, xy$.

3.5.1 Output Maximization

Output maximization is a static problem; thus, we omit time subscripts. Total output is given by

$$F(h, l_{xx}, l_{xy}) = \max_{\theta_{xx},\theta_{xy}} \left\{ Ax^{\alpha}(\theta_{xx}+\theta_{xy})^{1-\alpha}+B\left[(h+\delta_{xx})\,(l_{xx}-\theta_{xx})+(h+\delta_{xy})\,(l_{xy}-\theta_{xy})\right] \right\}. \quad (6)$$

The household chooses each θ_i to maximize (6). For parent type $i \in xx, xy$, the first-order condition is

$$(1-\alpha)Ax^{\alpha}(\theta_{xx}+\theta_{xy})^{-\alpha} \geq B(h+\delta_i), \text{ with equality if } \theta_i < l_i.$$

This condition governs the optimal allocation of working time across technologies by equating the marginal product of labor across sectors. Solving for θ_i, the optimal allocations to the traditional sector are

$$\theta_i = \min\left\{ \max\left\{ \left(\frac{A(1-\alpha)}{B(h+\delta_i)}\right)^{\frac{1}{\alpha}} x - \theta_{-i}, 0 \right\}, l_i \right\}, \quad i \in \{xx, xy\}. \quad (7)$$

The movement out of Malthus depends on the household's land endowment, the land intensity in traditional production, the relative productivity of the two technologies, the stock of human capital, and the partner's allocation of time. Each of these factors shapes the incentives for reallocating labor as the economy evolves.

Crucially, while A_t grows exogenously over time, B remains constant. Unlike in Hansen and Prescott (2002), where the escape from Malthus is driven by exogenous TFP growth in the modern sector, here the transition is entirely the result of endogenous human capital accumulation. Rising A_t increases traditional output, relaxing the household's resource constraint. At some point, this *may* enable parents to start investing in their children's quality, initiating a process that ultimately triggers modernization.

In light of (7), the transition to modern production unfolds asymmetrically across parent types. A parent with a comparative advantage in physical labor—reflected in a higher δ_i—exits the traditional sector earlier. This reallocation raises the marginal product of remaining labor in traditional production, thereby delaying the partner's exit. Modernization thus proceeds in stages: Type xy reallocates first, while Type xx remains in traditional production until $\theta_{xy} = 0$. Even beyond this point, diminishing returns imply that some fraction of Type xx labor remains in traditional use, though this share vanishes as h becomes large.

Given the differential timing in exit out of the traditional sector across types, there is an indirect production function during this transition, given by

$$y = \begin{cases} Ax^{\alpha}(l_{xx} + l_{xy})^{1-\alpha}, & \text{if } \theta_{xx} = l_{xx} \text{ and } \theta_{xy} = l_{xy}; \\ B(h + \delta_{xy})(l_{xx} + l_{xy}) + \alpha(1-\alpha)^{\frac{1}{\alpha}-1} A^{\frac{1}{\alpha}} [B(h + \delta_{xy})]^{1-\frac{1}{\alpha}} x, & \text{if } \theta_{xx} = l_{xx} \text{ and } \theta_{xy} < l_{xy}; \\ B(h + \delta_{xy}) l_{xy} + Ax^{\alpha} l_{xx}^{1-\alpha}, & \text{if } \theta_{xx} = l_{xx} \text{ and } \theta_{xy} = 0; \\ B[(h + \delta_{xx}) l_{xx} + (h + \delta_{xy}) l_{xy}] + \alpha(1-\alpha)^{\frac{1}{\alpha}-1} A^{\frac{1}{\alpha}} [B(h + \delta_{xx})]^{1-\frac{1}{\alpha}} x, & \text{if } \theta_{xx} < l_{xx} \text{ and } \theta_{xy} = 0. \end{cases} \tag{8}$$

3.5.2 Utility Maximization

Each period, the household faces the resource constraint

$$c_t + k n_t \leq y_t, \tag{9}$$

where k represents a fixed cost per child, in terms of units of the final good.

Preferences, as defined in (1), satisfy the following boundary conditions:

$$\lim_{c_t \to 0} \mathcal{U}_{c_t} = \infty; \quad \lim_{n_t \to 0} \mathcal{U}_{n_t} = \infty; \quad \lim_{h_{t+1} \to 1} \mathcal{U}_{h_{t+1}} = \beta. \tag{10}$$

The first two are standard Inada conditions, ensuring interior solutions for consumption and fertility. The third is a boundary condition: if no time is allocated to child-rearing, human capital remains at its baseline level, $h_{t+1} = 1$, which can be regarded as the innate ability and dexterity we are all born with. In this case, the marginal utility of quality equals β. At low levels of development, households choose quantity and consumption over quality. Quality investments begin only when the marginal returns to the former fall below β.

Once the household can to afford quality investments, who provides them? This is where Becker's insight on intra-household specialization becomes relevant. Under (2), the marginal benefit of time spent in child-rearing is always higher for the Type xx parent. The marginal cost, by contrast, is the same across parents: their time contributes equally to output at the margin. It follows that all quality investments are made by Type xx parent, with $r_{xy,t} = 0$, as long as her time suffices to meet the household's demand for quality. Otherwise, the household moves to the opposite corner: $r_{xx,t} = 1$ and $r_{xy,t} > 0$.

Given (1), the resource constraint (9) binds with equality. Substituting directly into the utility function and using the law of motion for human capital from (2), along with the corner solution $l_{xy} = 1$, the household's problem reduces to:

$$\max_{r_{xx,t},\ n_t} \mathcal{U}(n_t, r_{xx,t}) = (1-\beta)\log(y_t - k n_t) + \eta \log(n_t) + \beta \log\left[h_t \varphi(r_{xx,t})\right]. \tag{11}$$

This formulation collapses the household's choice to two variables: fertility, and the share of Type xx's time allocated to quality investment. Although parents care about the quality of their children, there are no genuine intertemporal decisions. Labor supply affects quality bequests directly, through time forgone from current production. We drop time subscripts to simplify notation. The first-order conditions for (11) are

$$\frac{1-\beta}{y-kn}\left(y_{l_{xx}} r_{xx}+k\right)=\frac{\eta}{n}, \tag{12}$$

and

$$\frac{1-\beta}{y-kn}\left(y_{l_{xx}} n\right)=\beta\frac{\varphi'(r_{xx})}{\varphi(r_{xx})}. \tag{13}$$

Solving (12) and (13) jointly yields the optimal fertility choice

$$n=\frac{y}{\left(\frac{1-\beta}{\beta}\frac{\varphi(r_{xx})}{\varphi'(r_{xx})}\right)y_{l_{xx}}+k}, \tag{14}$$

and the time allocated to each child

$$r_{xx}=\max\left(\frac{\eta}{\beta}\frac{\varphi(r_{xx})}{\varphi'(r_{xx})}-\frac{k}{y_{l_{xx}}};0\right) \tag{15}$$

To continue making progress, one needs explicit functional forms for both y and $\varphi(r_{xx})$. We already know that the indirect production function is implicitly defined in (8) when total output is produced using both traditional and modern technologies. However, from (8) one cannot solve analytically the system defined by (14) and (15) even with an explicit functional form for $\varphi(r_{xx})$ along the transition out of Malthus. Nevertheless, before proceeding to solve the equilibrium path numerically, it is insightful to characterize the solutions at both ends of the development path: a Malthusian stagnant equilibrium and an equilibrium path with large human capital levels and constant growth in living standards.

3.6 Malthusian Stagnation

The Malthusian model provides a compelling account of the long epoch spanning from the Neolithic Revolution to the dawn of the industrial revolution. As Diamond (2005) recounts, the shift from nomadic subsistence to settled agriculture furthered the ongoing domestication of plants and animals, enabling the development of transformative innovations, such as irrigation systems, crop rotation, and eventually, the foundations for more complex technologies: *guns, germs, and steel*. These advances supported larger, denser societies.

Yet from the Neolithic Revolution to the clash of civilizations at the time of the discovery of the Americas, there is little evidence of sustained gains in material living standards. How can this be? How can societies invent so much and remain so poor?

The Malthusian framework resolves this paradox, and its explanation aligns well with the facts. In a world where resources are finite and population responds to material conditions, technological progress does not raise income per capita—it raises population. Productivity gains are offset by demographic expansion, leaving average incomes near subsistence levels.

The model described above is consistent with this "date-zero" fact. The Malthusian economy is characterized by a traditional productivity level $A > B$ such that employing modern technology, while available, is not profitable. Material conditions are near subsistence, and ordinary families cannot afford quality bequests. From (10), one learns that parents will not invest in children's quality if the marginal utility of consumption or fertility is above β. In this case, $r_{xx} = 0$ and the only relevant choice is around fertility. Solving for optimal fertility yields

$$n = \frac{\eta}{k(1-\beta+\eta)} A 2^{1-\alpha}. \tag{16}$$

In line with Malthusian insights, (16) show that positive income shocks will lead parents to have more children, provided that these shocks occur within a subsistence neighborhood, where quality is still unaffordable. The cost per child, k, discourages fertility. From the first-order conditions, time investment in child-rearing remains zero as long as

$$\frac{k}{A 2^{1-\alpha}} > \frac{\beta}{1-\beta+\eta}. \tag{17}$$

To solve for the steady state, we seek a solution with $n = 1$ and c, h, y constant, where households maintain their size over time. By solving for k consistent with $n = 1$ in (16) and substituting into (17), one finds that the optimal time investment in child quality is zero in steady state if

$$\eta > \beta. \tag{18}$$

The assumption imposed on preferences—namely, that the number of children is not a "bad" relative to each child's quality—appears directly in condition (18), and is sufficient to guarantee a Malthusian steady state with no quality investments and levels of income low enough so (17) holds. While parents derive utility from bestowing quality bequests on their children, regardless of the production possibilities of their time, condition (18) implies that in low-productivity, traditional economies, they cannot afford to allocate time to child quality.

Instead, they must devote all available time to producing material goods for the subsistence of themselves and their offspring, despite their preference for children's well-being. The steady state consumption of both parents in steady state is given by

$$c = \frac{1-\beta}{1-\beta+\eta} A 2^{1-\alpha}.$$

3.7 Post-Malthusian Dynamics

The equilibria described in Section 3.6 reflect a stagnant living condition that remains near subsistence and is dynamically stable. This is what Ricardo (1817/2004, p. 93) refers to as "the natural price of labour," defined as the level "necessary to enable the labourers, one with another, to subsist and to perpetuate their race." It includes not only food and shelter but also the "conveniences required for the support of the labourer and his family." As Ricardo notes further (p. 94), in line with Malthus, this stability is maintained through "(...) the encouragement which high wages give to increase of population, the number of labourers is increased, wages fall again to their natural price, and indeed from a re-action sometimes fall below it."

Is there any path out of this trap? Hansen and Prescott (2002) suggest one: exogenous growth in the productivity of modern technology, which at some point makes its adoption inevitable. In their framework, both traditional and modern technologies grow exogenously, while population growth adjusts mechanically to maintain constant living standards until the transition occurs.

In contrast, in this model, productivity growth in the traditional sector does not, by itself, trigger a transition. It raises income per capita temporarily, but these gains are checked by endogenous population growth. Yet the improvement opens a window. If living standards rise sufficiently, parents may find it optimal to invest in their children's human capital. In turn, these investments render the modern technology profitable, triggering the transition to a regime of sustained income growth.

This mechanism gives rise to an intermediate regime that is neither fully Malthusian nor modern, marked by rising incomes and continued population growth. Galor (2011, pp. 17–18) calls this the *Post-Malthusian Regime*, noting the irony that, just as Malthus formalized his population theory, the world was already beginning to escape his trap, though fertility still responded in the very way Malthus and Ricardo had described.

Consider Section 3.6 as a description of the dynamics that governed economic life from the dawn of civilization. There are no long-run dynamics in this regime, and that is precisely the point. Shocks—famine, wars, barbarian invasions, bubonic plagues—produce only *temporary*

deviations. The system converges back to its steady-state values.

Now, treat these equilibria as the "initial state," or "date-zero," and let the level of technology, A, begin to grow at a constant, exogenous rate. Crucially, this is not a shock. Is this newfound source of growth sufficient to trigger a transition to modern economic growth? As we shall see, the answer hinges on whether parents are induced to leave quality bequest to their offspring.

First, normalize $h = 1$. As mentioned above, think of this as the basic level of dexterity and cognitive ability a child acquires in the absence of parental investment. With A growing, one can rearrange terms in (17) to express the child-rearing condition as

$$Ax^{\alpha} > \frac{\eta}{\beta}. \tag{19}$$

As A rises, the resource constraint becomes less binding. Parents can now afford to invest in child quality, but they also increase fertility, diluting the land endowment x and offsetting part of the gains from technological progress. Two possibilities then emerge: (i) a post-Malthusian convergence in which no investments in human capital occur—standards of living rise to a permanently higher level but remain stagnant thereafter; or (ii) the onset of a human-capital economy that generates sustained growth.

3.7.1 Post-Malthusian Convergence

Assume that, at some point in history, productivity in the traditional sector begins to grow at a constant exogenous rate $\gamma > 0$, but condition (19) never holds because land dilution offsets the gains from rising productivity. In this case, the economy achieves higher output per capita and population *levels*, but no sustained growth in living standards.

To see this, begin from a steady state where $n_0 = 1$ and $l_{0,i} = 1$ for $i \in \{xx, xy\}$, with normalized values $h_0 = 1$ and $A_0 = 1$. Initial output per capita is then $y_0 = 2^{1-\alpha}$. Since (19) never holds, there are no investments in child quality, so human capital remains constant at $h_t = 1$ for all t. Population evolves according to $N_{t+1} = n_t N_t$, where n_t is determined by (16), and land per household is $x_t = 1/N_t$. Solving the system recursively, one obtains

$$n_t = (1+\gamma)^{P_t}, \quad y_t = 2^{1-\alpha}(1+\gamma)^{P_t},$$

where P_t is a polynomial resembling a Pascal's triangle with alternating signs, namely

$$P_t = \sum_{k=0}^{t-1} (-1)^k \binom{t}{k+1} \alpha^k = \frac{1}{\alpha}\left[1 - (1-\alpha)^t\right].$$

As $t \to \infty$, $P_t \to 1/\alpha$, and both n_t and y_t converge to $(1+\gamma)^{\frac{1}{\alpha}}$ and $2^{1-\alpha}(1+\gamma)^{\frac{1}{\alpha}}$ respectively. This is a *level* effect. In the absence of human capital formation, sustained growth in traditional productivity alone cannot support rising living standards.

3.7.2 The Onset of a Human Capital Economy

On the other hand, if rising productivity allows condition (19) to be satisfied, the trajectory of development changes fundamentally. The gains from technology allow parents to reallocate time away from subsistence toward child quality. These choices bring about a transition from stagnation to sustained growth. With the quantity–quality trade-off now active, fertility and the time allocated to each child adjust accordingly, as given by (14) and (15), respectively.

Regarding modernization, the timing of the transition differs across parent types. Type xy begins to operate the modern technology when

$$(1-\alpha)Ax^{\alpha}(l_{xx}+l_{xy})^{-\alpha} < B(h+\delta_{xy}).$$

This first-mover characteristic arises solely from the initial "brawn" advantage in modern production, where physical effort dominates. This edge, however, is only relevant during the early stages of development, when human capital accumulation remains relatively low.

From this point onward, θ_{xy} declines steadily to equalize the marginal product of labor (of both parent types) across sectors. Here, diminishing returns in traditional production play a critical role: it is not optimal for Type xx parent to reallocate labor until $\theta_{xy} = 0$. Only then does modernization become profitable, provided that

$$\frac{(1-\alpha)A}{B}\left(\frac{x}{l_{xx}}\right)^{\alpha} < h+\delta_{xx}. \tag{20}$$

Once (20) is satisfied, Type xx parent begins reallocating time toward the modern sector. At this point, $\theta_{xx} < l_{xx}$ to equalize marginal returns across technologies. During this path, the model does not yield analytical solutions for the equilibria, but it asymptotically converges to an economy with large human capital levels and sustained growth, described in the next subsection.

3.8 Modern Economic Growth

At the other end of the development spectrum, we seek a solution in which n and c/h are constant, and living standards grow at a constant rate. In this regime, traditional technology becomes negligible as human capital accumulates without bound. From (8), one obtains the following limiting condition

$$\lim_{h\to\infty}\frac{y}{h}=B(l_{xx}+l_{xy}). \tag{21}$$

As (21) shows, "brawn" advantages in modern production vanish in a human-capital-based economy with large h. Similarly,

$$\lim_{h\to\infty}\frac{c}{h}=\lim_{h\to\infty}\frac{c+kn}{h}=B(l_{xx}+l_{xy}). \tag{22}$$

Finally,

$$\lim_{h\to\infty}\frac{\partial y/\partial l_{xx}}{h}=B. \tag{23}$$

To derive closed-form solutions, we require a functional form for $\varphi(r_{xx})$ such that $\varphi'(r_{xx})>0$, $\varphi(0)=1$, and $\frac{\varphi'(r_{xx})}{\varphi(r_{xx})}$ is independent of r_{xx}.[10] A function satisfying these properties is given by

$$\varphi(r_{xx})=e^{\xi_{xx}r_{xx}},\quad \xi_{xx}>0, \tag{24}$$

with $\frac{\varphi'(r_{xx})}{\varphi(r_{xx})}=\xi_{xx}$. Using (21) and (23) in (14) and (15), together with the labor force participation of each parent ($l_{xx}=1-r_{xx}n$ and $l_{xy}=1$ as long as $r_{xx}<1$) one obtains the optimal number of children

$$n=\frac{\beta\xi_{xx}}{1-\beta+\eta}2, \tag{25}$$

and the optimal fraction of time devoted to child-rearing

$$r_{xx}=\frac{\eta}{\beta\xi_{xx}}. \tag{26}$$

Thus, in a regime of modern growth driven by human capital accumulation, Type xx's labor force participation is given by

$$l_{xx}=\frac{1-\beta-\eta}{1-\beta+\eta}<1, \tag{27}$$

and per capita human capital grows at a constant rate given by

$$\frac{h_{t+1}}{h_t}-1=e^{\left(\frac{\eta}{\beta}\right)}-1. \tag{28}$$

Finally, combining optimal labor supplies in (22), the consumption-to-human-capital ratio is

[10] $\frac{\varphi'(r_{xx})}{\varphi(r_{xx})}$ represents the relative growth rate of φ at r_{xx}.

$$\frac{c}{h} = \left(\frac{1-\beta}{1-\beta+\eta}\right) 2B, \tag{29}$$

which is constant (yet another reason to prevent B from growing exogenously).

The allocations we just found in the Malthusian equilibrium and in the modern growth regime on their own cannot tell much about the transition that defines the industrial revolution. Similarly, Lucas (2002, p. 166) notes

> [t]hese are the long-run possibilities for this two-state-variable system: a Malthusian steady state with a constant income level, and an"Ak"-like system in which income is perpetually growing, driven by the accumulation of human capital. Little is known about the possibilities for this model away from these two steady states, and since the intended subject of this section is the transition from one to the other, this is a serious limitation.

The next section calibrates the model to explore numerically such possibilities, those governing the dynamics between these two extremes, away from Malthus' stagnant equilibria and toward sustained growth driven by human capital accumulation.

4 Calibration

Can the theory presented above generate the key features of the industrial revolution? That is, can it quantitatively account for the transition from stagnation to sustained growth, marked by labor reallocation out of agriculture, a demographic transition, rising human capital, and the U-shaped path of female labor force participation? This section addresses this by calibrating the model to historical data. We parameterize the model to account for Great Britain's development path between 1400 and 2000.

The calibration proceeds in three steps. First, we target the equilibrium allocations in Sections 3.6 and 3.8 to characterize a Malthusian steady state at one end of the development spectrum, and a human capital regime at the opposite extreme under balanced growth conditions. Second, we introduce exogenous TFP growth in the traditional sector to account for the slow gains during the post-Malthusian phase. Third, we set the TFP *level* in the modern sector to match the observed transition path.

Table 2 summarizes the parameterization. Labor intensity in the traditional sector is set to $\alpha = 0.4$, consistent the labor intensity in traditional production from Hansen and Prescott (2002). The child cost parameter, $k = 0.168$, is chosen so that $n = 1$ in the Malthusian steady state, implying that the fertility is at the replacement level: each adult produces one

Parameter	Value	Target / Comment
α	0.4	Consistent with labor intensity in Malthus (Hansen & Prescott, 2002).
η, β	0.112, 0.099	Chosen jointly to match the female labor participation in (27) and the long-run growth rate in (28) to: (i) the observed long-run income per capita growth rate (1.9%, compounded over 60 years), and (ii) the female-to-male labor force participation ratio in Great Britain in 2000 from Olivetti (2014, Appendix A.1.).
$A_{t=0}$	1	Normalization.
k	0.168	Consistent with $n = 1$ in the Malthusian steady state.
γ	0.473	Chosen to match Post-Malthusian income per capita growth net of land dilution.
B	0.164	Modern TFP *level* to match modern growth.
ξ_{xy}	1	Normalization.
ξ_{xx}	1.56	Consistent with productivity differences between mothers' and fathers' time at the mean child age of 6.32 years from Del Boca et al. (2014).
δ_{xx}	0	Consistent with normalized basic human dexterity.
δ_{xy}	0.5	"Brawn" advantage; female strength $\approx$ 67% of male, from Peolsson, Hedlund, and Öberg (2001).

Table 2: Parameterization summary.

child, so a two-parent household yields two children. The preference parameters η and β are calibrated jointly to match two targets: the modern long-run growth rate of income per capita (1.9 percent per year, compounded over 60 years), and the female-to-male labor force participation ratio in Great Britain in 2000, reported by Olivetti (2014, Appendix A.1).[11] The restriction $\eta > \beta$ is not imposed but emerges from the calibration.

TFP in the traditional sector is normalized to $A_{t=0} = 1$ and grows at a rate of $\gamma = 0.473$ from 1640 onward, corresponding to approximately 0.65 percent per year. This modest rate is sufficient to match the gradual rise in living standards during the post-Malthusian era, once land dilution "checks" the gains from technical progress. The transition from the post-Malthusian regime to the modern growth regime is dated to 1820, when a marked change in the growth rate is observed. The TFP level in the modern sector is set to $B = 0.164$, chosen to minimize the distance between the model and the observed per capita growth in the modern regime.

[11] Since the model abstracts from leisure, Type xy's labor supply is his full time endowment. The relevant data moment is therefore the female labor supply in 2000 as a fraction of the male's.

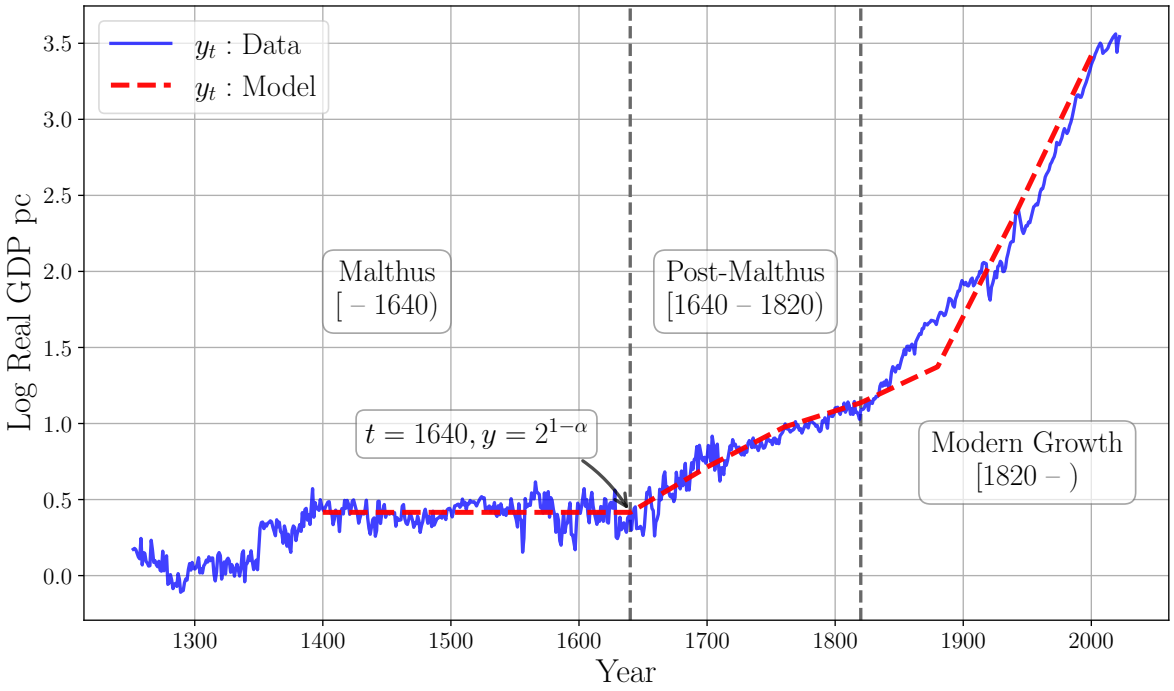


Figure 12: Development stages in Great Britain: 1252–2022.

We set $\delta_{xx} = 0$ and $\delta_{xy} = 0.5$, consistent with the normalization of basic dexterity and with estimates of relative physical strength between women and men from Peolsson et al. (2001). For child-rearing productivity, we draw on the findings of Del Boca et al. (2014), which show that mothers' active time is substantially more effective in producing child quality when children are younger, while fathers' active time becomes slightly more effective as children grow older. Specifically, we use the estimated elasticities from the one-child model evaluated at the median child age (6.32 years). This yields $\delta_{xy} = 1.56$ (consistent with the normalization of ξ_{xy}), meaning that at the mean child age, mother's active time is about 56 percent more effective than father's time in producing human capital.[12]

Figure 12 displays the path of real income per capita in Great Britain from 1252 to 2022, in logarithmic scale and normalized so that $y = 2^{1-\alpha}$ in 1640—the last period of the Malthusian regime. The log scale is useful, as it allows one to observe growth rates as slopes. There is a short-lived growth period after the Black Death, but between 1400 and 1640 income is essentially flat at about 1,500 USD, or four dollars a day. From 1640 to 1820, the slope increases slightly, corresponding to a growth rate of about 0.4 percent per year.[13]

[12]Note, however, that with the presence of B in the calibration to match the modern growth path, the precise values of these parameters are not crucial. What matters is that $\xi_{xx} > \xi_{xy}$ and $\delta_{xx} > \delta_{xy}$ to generate division of labor and specialization within the household and across production technologies.

[13]Within this period, one can locate the years that are often identified with the early stages of the Industrial Revolution. Clark (2001) cites 1770 as a conventional starting point, associated with innovations

After 1820, the slope steepens to 1.9 percent, the modern growth rate.

Figure 12 compares the model's predictions to observed data, with one model period corresponding to 60 years. This horizon is implied by equation (28), given $\eta > \beta$ and the empirical fact that modern income per capita grows at approximately 1.9 percent annually. The model replicates the evolution of income per capita in Great Britain reasonably well. This is expected, as the calibration is constructed to match this path. Prior to 1640, income per capita remains stagnant, consistent with the Malthusian dynamics described in Section 3.6. Strictly speaking, the steady-state condition requires both income and population to remain constant. Historical evidence suggests that this condition is not fully satisfied: substantial population growth occurred in the aftermath of the Black Death. One could reconcile this fact within the model by introducing a shock before 1400, corresponding to the plague, and tracing the subsequent convergence toward the Malthusian steady state. We do not pursue this extension, as it yields no additional insight beyond what is already captured by the baseline calibration.

From 1640 onward, exogenous growth in traditional-sector productivity accounts for the gradual rise in income per capita characteristic of the post-Malthusian regime. By 1820, human capital accumulation renders modern technology viable, marking the final period in which traditional production is operated exclusively. Given the model's 60-year period length, the first simulated period in which modern technology is adopted is 1880, introducing a visible lag relative to the historical record. From that point forward, the model closely tracks the observed path of British income per capita through 2000.

The take-off is generated by an exogenous source of growth in traditional productivity, interacting with endogenous household choices around human capital formation. This mechanism ultimately makes the adoption of modern production profitable, consistent with the British development experience.

Figure 13 presents the model's main predictions. Panel (i) shows the reallocation of labor away from traditional production—the Malthus sector—across types, as determined by equation (7). Modernization begins in 1820. The parent with the "brawn" advantage enters modern production first, while the other remains tied to traditional methods. Eventually, both exit Malthus and enter modern growth. For Type xy, the transition to modern production occurs within a single model period (60 years), as $\theta_{xy} = 0$ by 1880.[14]

Type xx follows a two-step path out of Malthus: first withdrawing from traditional production to invest in child quality, then gradually reallocating labor toward modern tech-

such as the spinning jenny and the water frame.

[14]In alternative simulations, a slower adjustment of θ_{xy} delays the transition for Type xx, keeping her labor confined to traditional production until $\theta_{xy} = 0$.

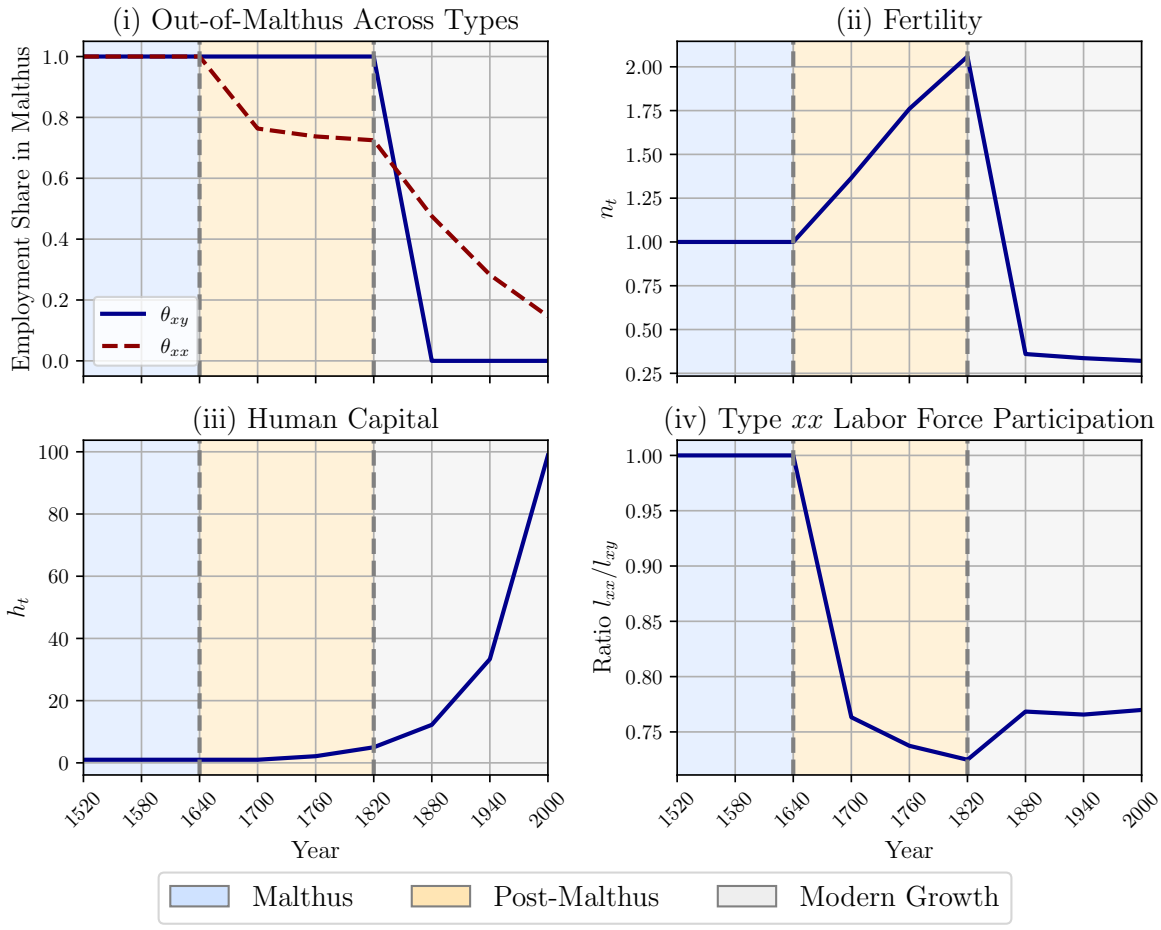


Figure 13: Main predictions.

nology. Unlike θ_{xy}, this transition is slowed by diminishing marginal returns, which continue to operate in the traditional sector.

Panel (ii) shows the evolution of fertility. In the Malthusian era, fertility remains at replacement level. During the Post-Malthusian period, productivity gains in traditional technology induce rising fertility, as shown in (14). Children are a normal goods, and families want more of them as they grow wealthier. What is new here is the persistence of technical improvements, which create new *possibilities*. These gains lead to quality investments, made exclusively by mothers. Families can now afford more and "better" children!

The Beckerian quantity–quality trade-off emerges only with modernization. In traditional technology, time spent on child-rearing is time away from traditional production, which raises the marginal product of the remaining labor due to diminishing returns. No such compensation exists in the modern sector. Thanks to generational investments in human capital, Type xx's labor becomes profitable in modern production. Modern technology exhibits *linear* returns, and so do the opportunity costs of child-rearing—the "substitution"

effect in (14). In the modern era, families choose fewer children to invest further in quality. This adjustment is sharp: within one period, fertility falls below replacement and gradually converges to its balanced-growth level, as shown in (25).

Panel (iii) illustrates the evolution of human capital per person. In the Malthusian era, h_t is normalized to 1. During the Post-Malthusian era, technical improvements enable families to bequeath human capital to their children *before* modernization occurs, as parents value child quality for its own sake—despite its irrelevance to traditional production methods. Human capital not only precedes modernization. It is the cause of it.

Panel (iv) plots the labor supply of Type xx, interpreted as female labor force participation relative to males, as the model abstracts from leisure. In the Malthusian era, men and women work side by side in subsistence farming to sustain life. Although families wish to invest in quality, they cannot afford fewer arms in the fields. Historical evidence suggests these constraints were so binding that even children were drawn into farm work—though the model abstracts from child labor. Female labor force participation declines sharply during the post-Malthusian era, driven entirely by child-rearing. This Beckerian division of labor accounts for the downward part of Goldin's "U." Panel (iv) then depicts the rising segment of the "U," associated with modern growth and falling fertility. The opportunity cost of time, augmented by human capital, does not yield a compromise in quality, but rather in quantity.

Taken together, the model reproduces the key stylized facts of the historical transition: a decline in fertility, a rise in human capital, and a reallocation of labor out of Malthus. This completes the model's core predictions. We now turn to the data to evaluate how well these mechanisms account for the salient patterns of the industrial revolution.

4.1 Model and Data

First, we compare the model's equilibrium paths with historical time series for Great Britain drawn from several sources, to contrast the model's predictions with data before and after the British Industrial Revolution. Second, we superimpose the model's predictions for Great Britain at different income per capita levels, rather than calendar time, onto a cross-section of countries in the year 2000, to assess how the predictions align with several economies at different stages of development.[15]

[15]There is nothing inherently "British" in our model calibration. The productivity path A_t and the TFP level B are chosen to replicate Britain's income per capita series, but any country's trajectory could be matched with three parameters: $\{A_0, \gamma, B\}$. If this were to be done, one would obtain a mapping between income per capita and the model's main predictions. Since the British calibration already establishes this correspondence, further country-specific calibrations would offer limited additional insight. However, it would be interesting to think of cross-country variations in B as "barriers", in the sense of Parente and Prescott

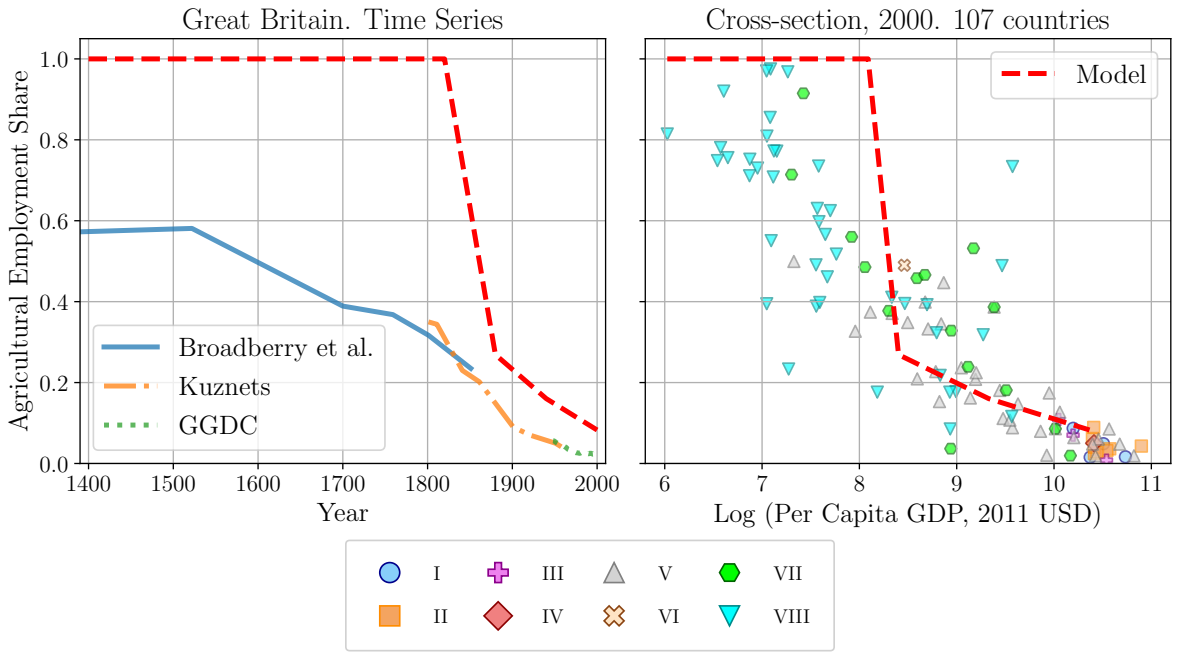


Figure 14: Out-of (traditional) agriculture.

Figure 14 compares the model's predictions for the transition out of traditional agriculture with the data.[16] The available evidence tracks employment shares in agriculture rather than in traditional modes of production, though the overlap between the two is substantial. The model, however, applies more broadly to pre-industrial or semi-feudal modes of production—still present in parts of the developing world—where land and raw labor are the primary inputs. Not all agricultural employment is Malthusian, and some non-agricultural activity may still reflect traditional production relations. Agro-industrial employment, in particular, should not be regarded as part of the Malthusian sector.

The left panel of Figure 14 compares the model with British historical data from three sources: Broadberry, Campbell, Klein, Overton, and van Leeuwen (2015), Kuznets (1966, 1971), and the GGDC 10-Sector Database of Timmer, de Vries, and de Vries (2015). Before modernization (1820), the model assigns all available labor to traditional agriculture. Historical series for Great Britain, however, show that less than 40 percent of the labor force remained in agriculture at that date. Even by the fifteenth century, the British share in agriculture was unusually low, under 60 percent. This early transformation may reflect Britain's role as the pioneer of modern growth, and the model is silent on this. Yet, from

(2000). Viewed this way, development gaps would reflect delays in modernization. These barriers could, in principle, be disciplined outside the calibration to shed light on the mechanics of growth miracles and their implications for fertility and human capital formation. We do not pursue this direction here, as our focus is on characterizing the salient patterns of the industrial revolution within a unified framework.

[16] In the model, the share of employment in traditional agriculture is given by $\frac{\theta_{xx}+\theta_{xy}}{l_{xx}+l_{xy}}$.

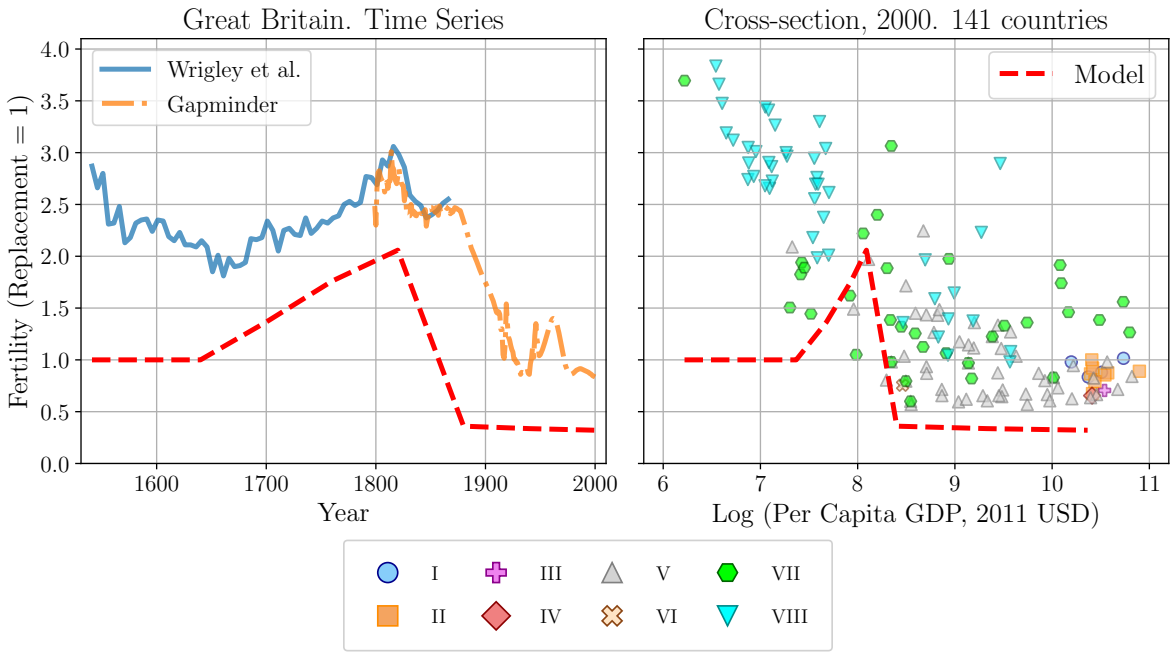


Figure 15: Fertility

1820 onward, the model predicts a sharp contraction in traditional employment, which moves nearly in parallel with the historical decline after 1880. This correspondence is decent: once modernization begins, the model and the data trace the same downward path.

The right panel of Figure 14 compares the model with agricultural employment in a cross-section of countries in 2000, using the same data sources documented in Section 2. Here, the model captures the broad development pattern: the poorest economies—mainly in Africa and Asia—show agricultural employment shares well above 60 percent (some even close to 100 percent as in the model) at low income levels, while richer economies across all regions, including Africa and Asia, exhibit low agricultural shares consistent with the model's projection.

Figure 15 compares the model's predictions for fertility with the data. The left panel plots the historical time series of the Gross Reproduction Rate (GRR) from Wrigley, Davies, Oeppen, and Schofield (1997), which measures the average number of daughters born to a woman over her childbearing years, together with fertility data from Gapminder, which reports the number of children per woman by cohort. These are the same data used in Section 2 and in the cross-country panel.

Ignoring mortality, GRR is sufficient to account for the replacement level, which is 1, while for the Gapminder series, it is 2. To make the two measures comparable and consistent with the model, we divide the Gapminder series by 2. The British historical evidence in the left panel of Figure 15 shows that the GRR during the Malthusian era remained well

above replacement, although it declined gradually until the early post-Malthusian period. As argued by Clark (2007), these high fertility rates are consistent with Malthusian dynamics: they reflect the long demographic recovery following the Black Death, a process that stabilized only as the population approached its pre-plague size.

Fertility never returned to those earlier levels, however. As the British economy entered its post-Malthusian phase, population growth accelerated once again.[17] The model calibration does not compute "plagues" or any kind of deviation from the Malthusian equilibrium. Fertility is held at replacement throughout the Malthusian era, consistent with a steady-state interpretation of stagnation. Then, as the economy enters and exits the post-Malthusian era, the timing of the fertility rise and decline in the data aligns with the model's prediction. Fertility increases until roughly 1820, marking the onset of modern growth and the demographic break from Malthus. Differences in levels are expected: the model omits mortality and other demographic feedbacks that shaped historical fertility. Even so, it captures the essential dynamics, predicting a sharp decline below replacement about one period earlier than observed, while remaining silent on later phenomena such as the post–World War II British baby boom.

The right panel of Figure 15 compares the model's predictions with cross-country data. At low income levels, the model predicts lower but rising fertility levels, a pattern not observed in the cross-country data. Poor countries—mainly in sub-Saharan Africa—display instead high but gradually declining fertility rates that the model cannot reproduce. Being poor in Africa in 2000 is not the same as being poor in pre-industrial Britain. Broad improvements in public health, nutrition, and sanitation at low income levels have allowed populations in today's poorest countries to expand far beyond the limits that once constrained Europe, where Malthusian checks on mortality and subsistence were binding. During the modern growth era, the empirical relationship between income and fertility is smoother than in the model, but the income levels at which the model predicts fertility to fall below replacement correspond roughly to several observations across regions I–VII. In all African countries, by contrast, fertility remained above replacement in 2000 at income levels where the model predicts it should already have fallen below replacement.

Figure 16 compares the model's predictions for human capital accumulation with both historical and cross-country data. The historical series are taken from Lee and Lee (2016), who report average years of schooling from 1870 onward and construct human capital stocks based on Mincerian returns. Because the model and data are not expressed in the same units,

[17] Clark (2007) also suggests that "Darwinian" selection mechanisms may have favored behavioral traits that later positioned Britain to exploit the "temporary" high wages and new opportunities emerging in the aftermath of the plague.

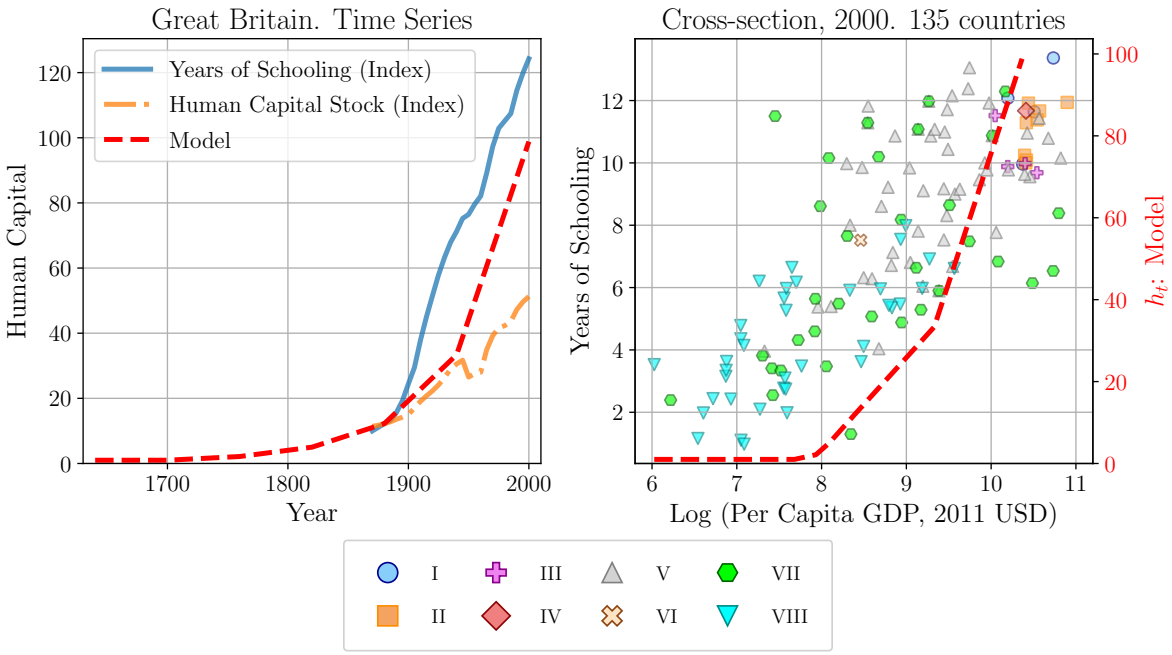


Figure 16: Human Capital.

we construct indices using observed growth rates and anchor them to the model's human capital stock in 1880. In the model, human capital is normalized to one in the Malthusian steady state; by 1880, it is twelve times higher, and by 2000, it rises by an additional factor of eight. Over the same period, the schooling index increases roughly tenfold, while the human capital index from Lee and Lee (2016) grows more than fourfold. Both model and data display a steep upward trajectory during the modern growth era, with the model falling between the two empirical series. It is important to note, however, that while the model encompasses all parental investments in child quality, the data primarily reflect formal education—a narrower dimension of human capital.

The right panel of Figure 16 extends the comparison to cross-country evidence. Here, we plot model and data on independent y-axes, sharing a common x-axis for income per capita. We also experimented with indexed versions for each country, anchoring either to a reference year or to a level of GDP per capita. These alternatives, though visually noisier, yield the same conclusion: human capital rises sharply with development. A notable feature of the figure is that even at income levels consistent with Malthusian stagnation, many African and Asian countries exhibit years of schooling well above what the model would predict for that stage. Is this a novel feature of late development? Unfortunately, comparable data for pre-industrial Britain are unavailable, leaving the question unresolved.

Figure 17 traces the U-shaped relationship between female labor force participation and economic development, originally documented by Goldin (1994), using a cross-section of

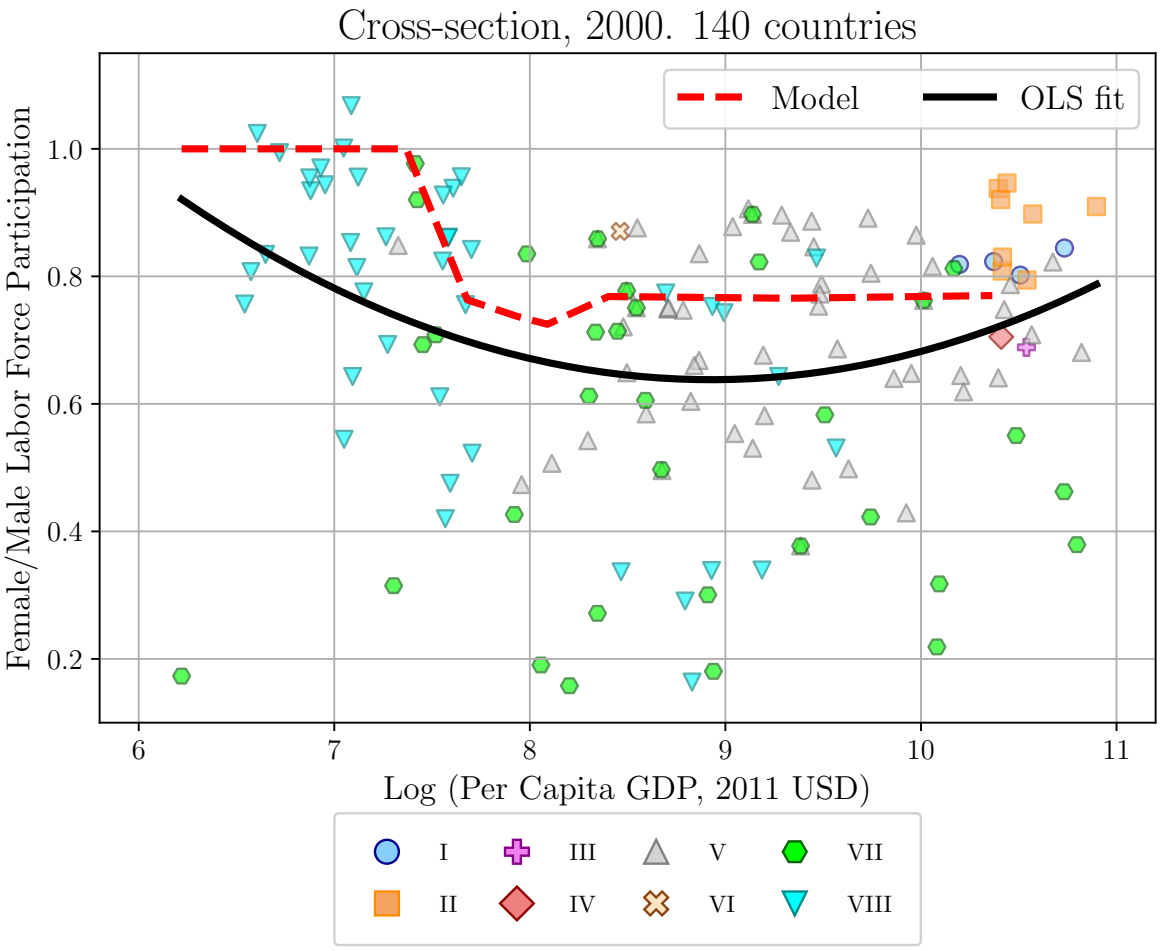


Figure 17: Female to male labor force participation ratio.

countries in the year 2000. Ideally, one would compare the model's long-run predictions with historical series for Great Britain extending back to the Malthusian era. The earliest systematic evidence we are aware of comes from Olivetti (2014), who reports data beginning in the late nineteenth century—too recent for our purposes, as these data capture only the rising portion of the "U."[18] Even for the United States, the longest available census series requires substantial adjustment. As shown by Ngai et al. (2024), when census data are corrected to include unpaid work in family farms and small enterprises, the U-shape emerges as early as the late nineteenth century.

To remain consistent with the model, which abstracts from leisure, Figure 17 plots the ratio of female to male labor force participation. Since male participation remains relatively stable across levels of development, variation in this ratio is driven almost entirely by changes

[18]Moreover, we have already used these data in calibrating the model's long-run female labor force participation.

in female participation. For comparison, the figure includes a cubic OLS fit to the cross-sectional data. Given the dispersion, this fit provides a useful benchmark. The model, despite its simplicity, roughly follows the estimated curve: it begins near one and converges to its long-run value, $\frac{1-\beta-\eta}{1-\beta+\eta}$, through a U-shaped trajectory that broadly follows the declining and rising pattern in the data.

4.1.1 Land as a Pure Public Good

So far, we have assumed that land behaves as a fixed factor that limits sustained income growth. In the traditional mode of production, this feature makes the economy Malthusian: land constrains long-run expansion because it cannot be reproduced. Unlike reproducible capital in Solow (1956), land cannot be accumulated by sacrificing current consumption to yield more acres tomorrow. This irreproducibility blocks the law of motion that drives modern growth under exogenous technological progress. To simplify the analysis, we have also abstracted from property rights over land. This assumption could be relaxed to allow for land bequests, following Lucas (2002), so that fertility decisions incorporate inheritance. Land bequests may influence fertility in early stages of development—and could motivate a discussion of differential inheritance under systems such as primogeniture—but they become secondary once the economy transitions to human-capital-based growth.

Beyond reproducibility, we have assumed that land behaves as a public good subject to congestion: population growth dilutes the gains from technological improvements in traditional production. This assumption is plausible in contexts such as the British Isles, Japan, or modern China. Yet it may not apply to economies like the United States during its westward expansion or the Russian Empire under Catherine the Great, where the capacity to expand arable land and political frontiers was virtually unconstrained. More generally, many societies have historically expanded land availability through migration, colonization, or conquest.

We are now ready to alter this assumption. To explore the implications of land abundance, we treat land as a pure public good, implying that population growth does not cause a dilution of resources. This represents the polar opposite of Malthusian congestion and provides a useful contrast for understanding how the role of land shapes the model's main predictions.

Figure 18 shows the model's main predictions when land is treated as a pure public good, effectively setting $x = X/n = 1$ for all periods. Under this assumption, exogenous growth in A guarantees a takeoff because productivity gains are never diluted. Families eventually invest in child quality, triggering modernization.

Panel (i) illustrates that mothers reallocate time away from traditional production before

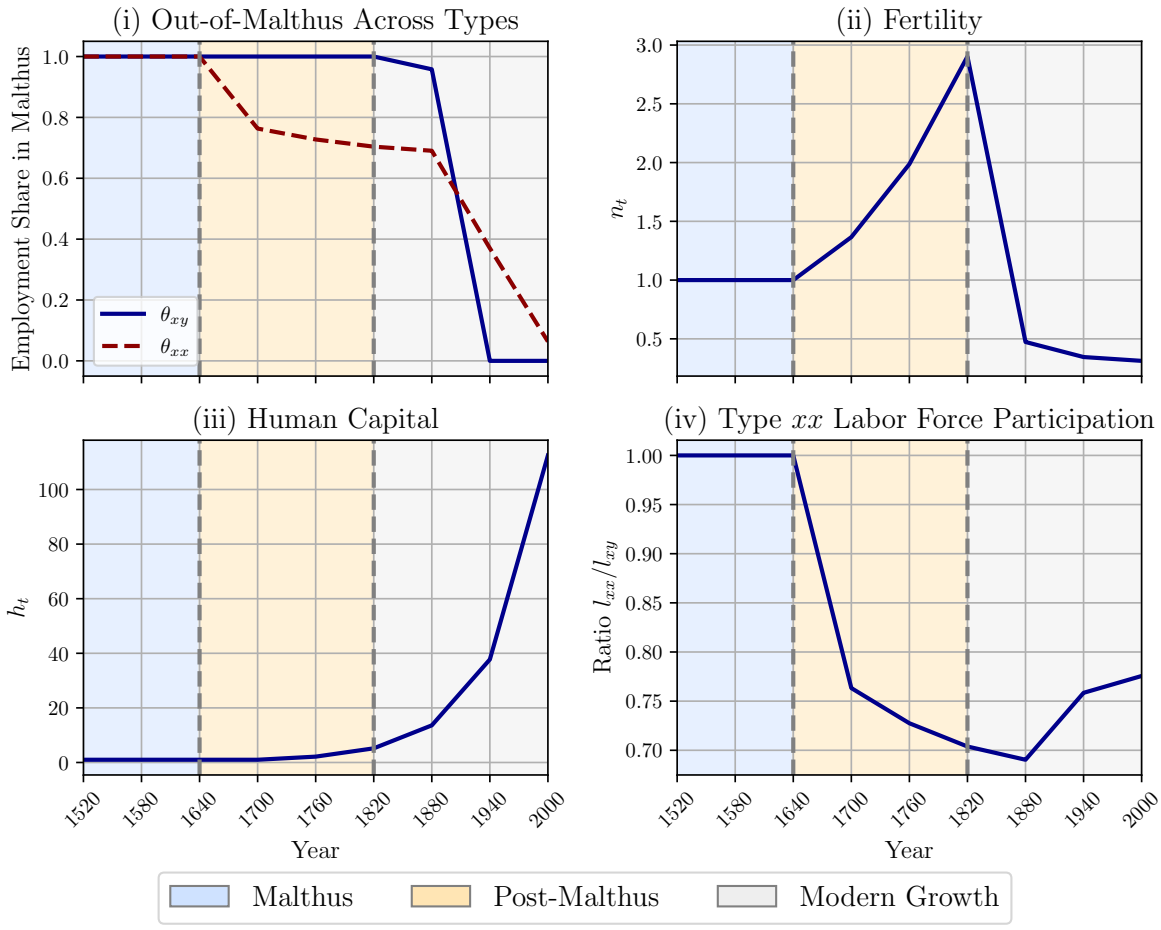


Figure 18: Main predictions with land as a pure public good. $x = X/n = 1$ for all periods.

full modernization, initiating human capital accumulation—similar to Figure 13. The key difference is timing: for Type xy, the transition now spans two periods (120 years), as labor remains profitable in agriculture when land is undiluted. Panel (ii) shows that without Malthusian checks, fertility remains substantially higher in a post-Malthusian era that is no longer Malthusian in any sense. However, the economy still enters a demographic transition once modern growth begins.

Treating land as a pure public good smooths the pace of transformation in some dimensions but does not alter the core mechanisms driving industrialization. Even when congestion is removed entirely at early stages of development, the gains in human capital—shown in Panel (iii)—are modest relative to the baseline case, where a Malthusian check offsets part of the benefits from technological progress.

In addition, because modernization for Type xx proceeds more slowly, the downward portion of the U-shaped path extends by one period, producing a smoother transition and delaying women's return to production, as shown in Panel (iv). Figure 19 compares this pre-

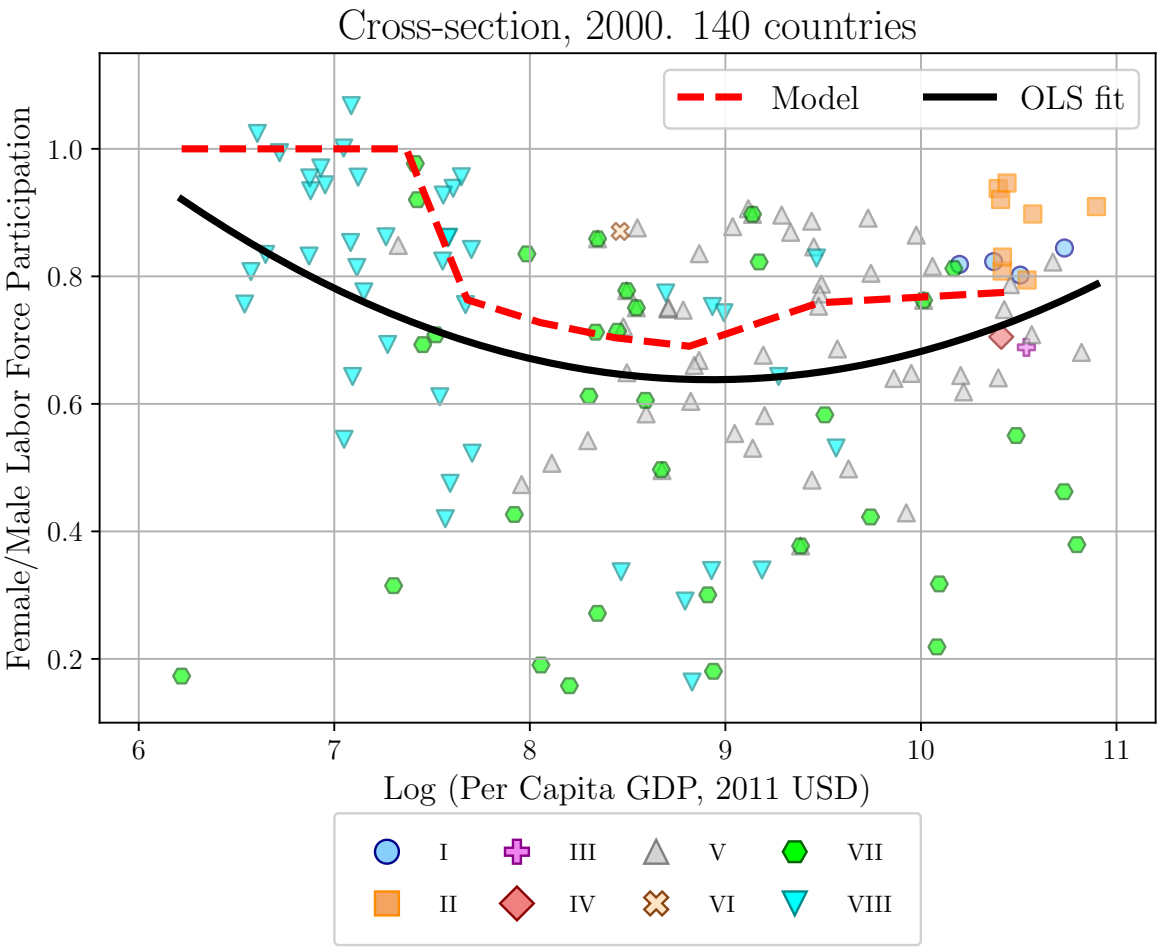


Figure 19: Female labor force participation with land as a pure public good.

diction with cross-sectional data, showing that assuming land as a pure public good yields a smoother U-shaped curve than the case with land congestion. Without congestion, modernization proceeds more gradually, affecting the timing and scale of change, but altering the role of land offers little additional insight into the fundamental forces behind the industrial revolution.

5 Conclusion

The patterns of sustained growth in income per capita, the reallocation of labor out of agriculture, rising educational attainment, and the U-shaped trajectory of female labor force participation are not isolated phenomena. They are recurring features of the development process, observed across countries and historical periods. These patterns suggest that the mechanisms underlying the industrial revolution are not specific to particular cultures or

institutions but reflect general responses to changing economic conditions.

The evidence presented here supports the view that the industrial revolution is a global process, not a historical accident confined to a few nations. As more countries embark on their industrial revolutions, the gap between poor and rich nations begins to close, and perhaps these binary labels are becoming obsolete. The forces driving this transition—technological change, human capital accumulation, and demographic choices—are universal but unfold unevenly.

The model presented here is a parsimonious fable that brings together the main components of this process in a unified way, abstracting from many important aspects that remain to be understood. Formal schooling, largely absent from the framework, may amplify human-capital accumulation through complementarities and externalities not captured by parental investment alone. Economies of scale in child-rearing—arising from siblings, grandparents, social networks, or shared caregiving—could modify the trade-off between child quantity and quality in ways our model leaves aside. Finally, many societies still face persistent barriers that delay or block the arrival of modern economic growth. Understanding how these forces interact will determine whether the global industrial revolution now underway achieves its full promise.

But it would be a mistake to end on a note of unfinished questions and lingering barriers. In the thirty years since the original articulation of these ideas, the historical record has shown that the vast inequality unleashed by the early industrial revolution is a transient phase in the development process. Hundreds of millions have escaped poverty in ways that would once have seemed miraculous. In this sense, we may now be witnessing the final stages of the global industrial revolution itself.

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Appendix

A Male Labor Force Participation Ratio over Development.

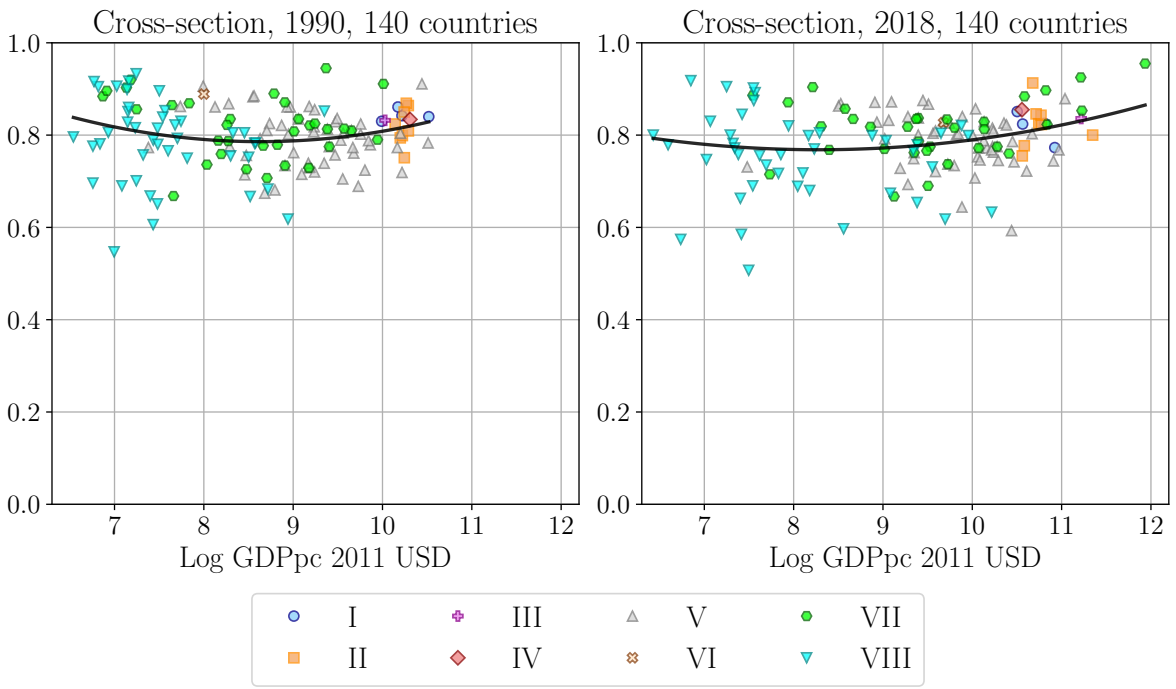


Figure A.1: Male's labor force participation over development. Cross-section data for 140 countries in 1990 and 2018. The labor force participation rate is for men aged between 15 and 65. Eight groups

B OLS Estimation: Female Labor Force Participation over Development.

	(1)	(2)	(3)	(4)
Log GDP pc (2011 USD)	-0.669***	-0.653***	-0.223***	-0.238***
	(0.033)	(0.033)	(0.056)	(0.052)
Square of Log GDP pc (2011 USD)	0.038***	0.037***	0.015***	0.014***
	(0.002)	(0.002)	(0.003)	(0.003)
Constant	3.470***	3.394***	1.309***	1.576***
	(0.145)	(0.147)	(0.254)	(0.253)
Time FE	NO	YES	NO	YES
Country FE	NO	NO	YES	YES
Obs.	4758	4758	4758	4758
Adjusted R^2	0.11	0.12	0.27	0.34

Robust standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table B.1: Female's U-shaped labor force participation over development. Panel data evidence. Dependent variable: Labor force participation rate for women aged between 15 and 65.

C OLS Estimation: Female to Male Labor Force Participation Ratio over Development.

	(1)	(2)	(3)	(4)
Log GDP pc (2011 USD)	-0.729***	-0.704***	-0.188***	-0.208***
	(0.042)	(0.042)	(0.068)	(0.056)
Square of Log GDP pc (2011 USD)	0.041***	0.039***	0.015***	0.012***
	(0.002)	(0.002)	(0.004)	(0.003)
Constant	3.906***	3.776***	1.213***	1.622***
	(0.184)	(0.185)	(0.301)	(0.273)
Time FE	NO	YES	NO	YES
Country FE	NO	NO	YES	YES
Obs.	4758	4758	4758	4758
Adjusted R^2	0.09	0.10	0.31	0.44

Robust standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table C.1: Female to male U-shaped labor force participation ratio over development. Panel data evidence. Dependent variable: Female to male labor force participation rate, for ages between 15 and 65.