International Equity Flows, Monetary Policy, and Time Consistency*

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Abstract

This paper studies optimal monetary policy in an open economy with international equity flows. Foreign ownership creates a time-consistency problem: once equity is held abroad, the central bank has an ex post incentive to deviate from the natural allocation, which raises real wages and compresses profits paid to foreign shareholders. Anticipating this discretionary policy response, foreign investors require a discount when purchasing domestic equity, reducing domestic wealth. Because households do not internalize that their collective equity sales exacerbate the central bank's future incentive to compress profits, they over-divest, generating multiple Pareto-ranked equilibria. In the bad equilibrium, equity valuations are depressed, and employment is lower. Unlike existing theories of capital-flow management, our model provides a rationale for capital controls on equity inflows.

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1 Introduction

Over the past three decades, international equity flows have expanded markedly. This development reflects the increasing depth and integration of global financial markets. As Figure 1 shows, the combined value of foreign direct investment (FDI) and foreign ownership of equity (FOE) has increased from roughly 20% of GDP in 1995 to nearly 80% in 2024. While a common view holds that cross-border equity positions promote risk-sharing and improve welfare, their growing scale raises important questions about their implications for macroeconomic policy and aggregate welfare.

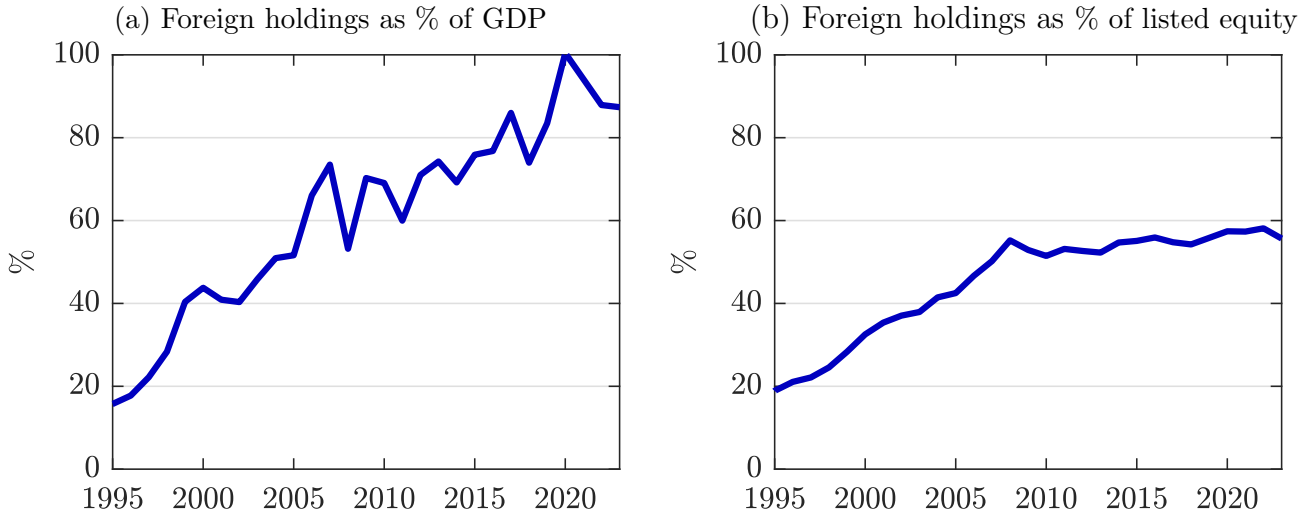


Figure 1: Foreign ownership of domestic equity

Notes: Panel (a) shows the stock of domestic equity owned by foreigners as a share of GDP, and panel (b) shows it as a share of the domestic equity market. Foreign ownership of domestic equity is measured as the sum of foreign direct investment (FDI) and foreign ownership of non-financial and bank domestic equity stocks (FOE). The series represent a simple average across the following countries: Austria, Belgium, Brazil, Bulgaria, Canada, Chile, Colombia, Croatia, the Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Israel, Italy, Japan, South Korea, Mexico, Norway, Poland, Portugal, the Slovak Republic, Slovenia, Spain, Sweden, Turkey, the United Kingdom, and the United States. Data source: External Wealth of Nations (EWN) database (Lane and Milesi-Ferretti, 2018) and OECD National Accounts Statistics.

This paper explores the implications of international equity flows for monetary policy and welfare. We show that foreign ownership of domestic equity introduces a time inconsistency problem for monetary policy, as the central bank has an incentive to depart ex post from the natural allocation in order to reduce firms' profits and erode the value of equity claims held by foreign investors. Moreover, we show that opening to international equity flows exposes the economy to a Pareto-inferior equilibrium in which households sell excessive amounts of domestic equity ex ante. This arises because households do not internalize that greater

foreign ownership exacerbates the central bank's ex post temptation to reduce profits, which depresses the equity price at which they sell the equity claims.

We formalize these results in a tractable open-economy New Keynesian framework in which households trade domestic equity with foreign investors, in addition to foreign bonds. To transparently illustrate our results, the baseline model abstracts from uncertainty, imperfections in capital markets, and domestic heterogeneity. In the case where households own the entire share of domestic equity, optimal monetary policy attains the natural allocation—equivalently, a zero labor wedge or zero output gap—by targeting a level of employment such that the marginal rate of substitution between consumption and leisure equals the marginal product of labor.

When households own the entire domestic equity stock, an increase in the real wage—holding employment fixed—has no effect on aggregate resources: the induced changes in labor income and profits exactly offset in the representative household's budget constraint. Hence, the split between labor income and profits is irrelevant for allocations. This irrelevance no longer holds when foreign investors own domestic equity. In that case, a higher real wage reduces the profits accruing to foreigners, while increasing households' income. Although domestic households also receive lower profits per equity claim, their benefit from higher wages exceeds the loss in equity payouts since they hold only a fraction of the total equity. The split between labor income and profits now matters.

With nominal rigidities, monetary policy can affect real wages; thus, a central bank has an incentive to raise real wages when foreign investors own domestic equity. In our baseline model, which features nominal wage rigidities, we show that the central bank finds it optimal to over- appreciate the exchange rate relative to the full-employment allocation. The logic is that starting from an allocation with a zero labor wedge, there is a first-order gain from reducing employment because the disutility from working exceeds the marginal utility value of the additional resources, as a share of the output revenue is, in effect, leaked abroad. Accordingly, foreign ownership leads the central bank to appreciate the exchange rate to induce a positive labor wedge, and reallocate resources from foreign investors toward domestic households.

Our key mechanism can be summarized by the central bank's intratemporal optimality condition,

$$-U_h \;=\; \lambda\big(F_h - \theta_1^* \Pi_h\big),$$

where U_h is the marginal disutility of labor, F_h is the marginal product of labor, Π_h is the marginal effect of employment on equilibrium profits, λ is the shadow value of relaxing the country's intertemporal resource constraint, and θ_1^* is the foreign-owned share of domestic

equity at the time of the nominal rigidity. With full domestic ownership ($\theta_1^* = 0$), the condition reduces to the standard efficiency rule that equates the marginal rate of substitution between consumption and leisure to the marginal product of labor. With foreign ownership ($\theta_1^* > 0$), the central bank internalizes that part of the marginal revenue created by higher employment accrues as profits paid abroad. In effect, the domestic marginal benefit of production is reduced by a "leakage" term proportional to foreign equity ownership.

The time-inconsistency problem arises because foreign investors anticipate the central bank's ex post incentive to erode their profits and therefore purchase equity only at a discount. This lack of commitment reduces the wealth of domestic households. When households sell equity, they fail to internalize that greater foreign ownership intensifies the central bank's incentive to compress future profits, which depresses equity valuations today and lowers welfare. As a result, households collectively sell too much equity, generating an ex post distortion in output without delivering ex ante benefits.

Moreover, we show that multiple equilibria arise quite naturally. When households sell more assets, this lowers future dividends and reduces the price of equity shares, to the extent that deep-pocketed foreign investors take the other side in the market. While individual investors are indifferent between holding or not holding the equity, when they collectively sell more equity, these sales reduce domestic wealth. Interestingly, the presence of multiple equilibria is not a result of fire sales and amplification through financial constraints, as in previous literature.

A key policy implication is that restricting foreign purchases of domestic equity is optimal. When domestic households initially own the entire equity stock, the optimal policy is to shut down all equity flows. This is because any sales of equity result in lower allocative efficiency and financial losses, as investors buy the equity at discounted prices. When foreign investors already hold some equity, there is a benefit from buying back the equity claims from foreign investors, as this leads to a less distorted economy in the future. However, as households increase their holdings of domestic equity, they purchase back the equity at a higher price and suffer a financial loss. The government thus faces a tradeoff between closing the output gap against capital gains accruing to foreigners. At the optimum, the government induces households to repurchase a limited share of the equity held by foreigners.

In one extension of the model, we incorporate non-tradables. For the incentives of monetary policy to deviate from the natural allocation, not only the overall share of equity owned by foreigners matters, but also the composition of the ownership between firms in the tradable and non-tradable sectors. This is because the price of non-tradables is determined domestically in equilibrium, which can generate opposite responses of firm profits to exchange

rate movements across sectors. As in the baseline model, profits in the tradable sector always fall following an exchange rate appreciation, because of the resulting increase in real wages. By contrast, profits in the non-tradable sector can increase—provided that tradable and non-tradable goods are complements, which appears to be the empirically relevant case —because the induced contraction in aggregate consumption requires an increase in the relative price of non-tradables for domestic markets to clear. The central bank can therefore have incentives to over-appreciate or over-depreciate the exchange rate relative to the full-employment allocation depending on whether foreigners predominantly hold shares in the tradable or the non-tradable sector. Moreover, there can exist a composition of foreign ownership across sectors such that the nominal exchange rate does not affect the aggregate profits accruing to foreign investors.

These results have important implications for the management of capital controls on equity inflows. In particular, by curbing equity inflows so as to achieve such a composition—which, in a calibrated version of the model, we estimate to be approximately 60% in tradable-sector firms and 40% in non-tradable-sector firms—capital controls can eliminate the ex post incentives of the central bank to deviate from the natural allocation when foreigners hold equity shares.

Related literature. Our paper belongs to the literature on monetary policy in open economies. A central theme in this literature, as in that with closed-economy models, is macro-stabilization: when prices are rigid, monetary policy can help align relative prices toward the full-employment or natural allocation (e.g., Friedman, 1953; Mundell, 1960; Schmitt-Grohé and Uribe, 2001). There are also studies that examine channels by which a departure from the natural allocation may be optimal in environments where trade in financial assets is restricted to bonds. One strand of this literature is concerned with terms of trade manipulation. When countries have market power in the tradable goods they produce, central banks can use monetary policy to tilt the terms of trade on their favor (e.g., Obstfeld and Rogoff, 1995 and Corsetti and Pesenti, 2001). Another strand of this literature considers the case when bonds are in domestic currency and analyzes how variations in exchange rates can generate valuation effects on the net foreign asset position. For instance, Fanelli (2023) shows how the optimal monetary policy under commitment balances macro-stabilization and insurance while Ottonello and Perez (2019), Du, Pflueger and Schreger (2020) and Engel and Park (2022) study the resulting inflationary bias when the government cannot commit. In contrast to these contributions, we consider a model with trade in equity claims and show how this gives rise to a novel source of time inconsistency with distinct implications for policy.

Our paper is also related to the literature on international portfolios.[1] A central theme in this literature is understanding the diversification of equity portfolios (e.g., Engel and Matsumoto, 2009; Coeurdacier, Kollmann and Martin, 2010; Kollmann, 2006; Baxter and Jermann, 1997; Devereux and Sutherland, 2006, Tille and Van Wincoop, 2010; Heathcote and Perri, 2013). In contrast to the existing literature, our paper takes a normative perspective and uncovers an aggregate welfare cost from such diversification. In addition, most of these studies are in the context of real models, thus abstracting from the role of monetary policy. A notable exception is Engel and Matsumoto (2009), who show how a portfolio of home and foreign equities and a forward position in foreign exchange can achieve to a first-order the complete market allocations. However, they do not consider optimal monetary policy.[2]

Our work also relates to recent studies examining the implications of the rise in cross-country asset ownership. Atkeson, Heathcote and Perri (2022) document that the U.S. "exorbitant privilege" from valuation effects largely disappeared after 2007. They show the deterioration is due to the surge in U.S. corporate equity values, driven by a rise in markups. Quadrini and Ríos-Rull (2024) analyze optimal taxation in a setting where multinationals engage in profit shifting. In contrast, our focus is on studying the implications of the rise in cross-country asset ownership for monetary policy.[3]

Our paper is also related to the literature on monetary unions—in particular, those emphasizing how joining a monetary union can deal with time inconsistency problems in monetary policy. In Alesina and Barro (2002), joining a monetary union reduces the inflationary bias generated by the time inconsistency problem of monetary policy, as stressed by Barro and Gordon (1983). Fornaro (2022) presents an environment where the central bank has incentives to depreciate the exchange rate to reduce the real value of non-tradable collateral that foreign investors can seize from defaulting firms ex post. Because of this time inconsistency problem, he shows that joining a monetary union can help increase bond inflows. One feature of our paper that differentiates it from these studies is that the time inconsistency problem of monetary policy is related to international equity flows.

Our paper is also related to the literature on monetary policy with heterogeneous agents in closed economies. Much of the HANK literature assumes an equal share of profits between

[1] See, for example, Engel and Matsumoto (2009), Coeurdacier, Kollmann and Martin (2010), Heathcote and Perri (2013); Kollmann (2006); Baxter and Jermann (1997) Devereux and Sutherland (2006), and Tille and Van Wincoop (2010).

[2] Of course, models of optimal monetary policy with complete markets implicitly allow trade in equity claims. However, much of this literature (e.g., Clarida et al., 2002; Devereux and Engel, 2003; Obstfeld and Rogoff, 1995; Corsetti et al., 2010) studies optimal policy under commitment and/or limited international cooperation, settings in which the mechanism we highlight is absent.

[3] Empirical contributions include Lane and Milesi-Ferretti (2005); Coppola et al. (2021); Beck et al. (2024).

households or considers setups where equilibrium profits are zero, muting redistributive motives through equity holdings (e.g., Dávila and Schaab, 2023). Broer et al. (2020) analyze the role of the distribution of profits for the transmission mechanism (see also Bilbiie, 2008). Two papers that share a normative focus are Bhandari, Evans, Golosov and Sargent (2021) and Acharya, Challe and Dogra (2020), but these examine optimal policy under commitment. Moreover, they assume that fiscal policies that effectively equalize marginal utilities of consumption at the steady state are in place. Thus, while their focus is on how monetary policy can provide insurance against aggregate shocks, we examine a redistribution motive that emerges naturally because the government is unable to tax foreign investors and does not place any weight on their welfare.

Our paper also relates to the literature on capital controls and capital flow management policies. This literature has developed a rationale for taxes on debt flows, based on pecuniary externalities (e.g., Bianchi, 2011) and aggregate demand externalities (e.g., Schmitt-Grohé and Uribe, 2016a; Farhi and Werning, 2016). See Bianchi and Lorenzoni (2021) for a review of the literature. In contrast, we provide a rationale for capital controls on equity flows.

Outline. The remainder of the paper is organized as follows. Section 2 introduces the environment. Section 3 studies the optimal exchange rate policy. Section 4 examines the role for capital controls. Section 5 analyzes extensions of the baseline framework. Section 6 concludes.

2 The Model

Consider a small open economy (SOE) with time indexed by $t \in \{0, 1, 2, \ldots\}$. The SOE is composed of a continuum of identical households and firms, and a monetary authority. There is an international financial market with a risk-free bond that pays an exogenous real return. Our key departure from the standard framework is that foreign investors may also trade equity claims on domestic firms.

We restrict attention to a deterministic environment where nominal rigidities will be present at $t = 1$. In addition, our baseline model assumes that households do not face any borrowing limits. This will allow us to highlight the key mechanism at play and characterize analytically the equilibrium. We next describe the decision problems of households, firms, and foreign investors, before analyzing the competitive equilibrium and optimal policies.

2.1 Firms

There is a unit mass of identical firms, which produce a single final good using labor, according to a strictly increasing and strictly concave production function $F(h)$. We assume that the production function satisfies $F(0) = 0, \lim_{h\to 0} F'(h) = \infty$, and $\lim_{h\to\infty} F'(h) = 0$.

We assume that the final good is tradable in the international market and the law of one price holds. Normalizing the foreign-currency price to one implies that the domestic-currency price of the good is $P_t = E_t$, where E_t denotes the nominal exchange rate, defined as the price of foreign currency in terms of domestic currency.

The firms' problem consists of choosing employment to maximize profits:

$$\max_{h_t \geq 0} \{E_t F(h_t) - W_t h_t\}, \tag{1}$$

where W_t is the nominal wage in domestic currency.

Profit maximization implies that labor demand satisfies

$$F'(h_t) = w_t \ , \ \forall t \geq 0, \tag{2}$$

where $w_t \equiv W_t/E_t$ is the real wage. We denote by π_t the firms' profits in units of foreign currency.

2.2 Households

There is a continuum of identical households of measure one. Their preferences are represented by

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_t) - v(\ell_t)\right],$$

where $\beta \in (0,1)$ is the discount factor, c_t denotes consumption, and ℓ_t denotes hours worked. The utility function $u(\cdot)$ is strictly increasing and strictly concave, and the disutility from working is strictly increasing and convex. We assume the usual Inada conditions: $\lim_{c\to 0} u'(c) = \infty$, $\lim_{c\to\infty} u'(c) = 0$, $\lim_{\ell\to 0} v'(\ell) = 0$, and $\lim_{\ell\to\infty} v'(\ell) = \infty$.

Households receive labor income, consume, and trade domestic firms' shares and bonds in international financial markets.[4] We let $\theta_t \geq 0$ and b_t respectively denote the holdings of firms' shares and bonds carried to period t (chosen at $t-1$). Bonds are assumed to be

[4]Our results would be unchanged if households held foreign equities in addition to bonds. The reason is that a small open economy's policy does not affect outcomes in foreign markets.

one-period and denominated in foreign currency or, equivalently, units of consumption. Firms' shares entitle their owner to a fraction of the firm's profits. Given a constant amount of firms' shares, which we normalize to one, a household that owns θ_t units of equity receives $\pi_t\theta_t$ in period t.

Households' budget constraint is given by

$$c_t = w_t\ell_t + (\pi_t + q_t)\theta_t - q_t\theta_{t+1} + (1+r)b_t - b_{t+1}\,, \tag{3}$$

where q_t is the price of equity (in units of foreign currency) and r is the constant interest rate on bonds. We highlight that while r is exogenously given, the price of equity q_t is endogenously determined in equilibrium. Moreover, we index the price by t to capture that aggregate states will fluctuate over time.

For our baseline model, we assume households do not face borrowing limits, except for a no-Ponzi-game condition:

$$\lim_{t\to\infty} \frac{b_{t+1} + q_t\theta_{t+1}}{(1+r)^t} \geq 0\,.$$

Wage rigidity. In period $t = 1$, the nominal wage W_1 is rigid. We assume that at that period, labor is demand-determined, which means that households must supply the hours of labor that firms demand. We focus attention on the case where hours are evenly distributed across households. Using (2), we then have that the number of hours that each household works in equilibrium must be given by

$$\ell_1 = F'^{-1}\left(\frac{\overline{W}_1}{E_1}\right), \tag{4}$$

where $\overline{W}_1$ denotes the exogenous rigid wage.[5]

Household problem. At every period, households choose consumption, asset portfolios, and labor supply (except at period 1, when the number of labor hours is determined by (4)). We let $V_t(b_t, \theta_t)$ denote the value at the beginning of the period for a household with an initial individual asset portfolio (b_t, θ_t), where we index the value function by t because the aggregate portfolio and thus prices vary over time. The households' optimization problem

[5]Our modeling of period-1 wage rigidity is motivated by expositional simplicity and can be extended to allow for wage setting in period 0 with differentiated labor, subject to pricing frictions.

can be represented recursively as follows:

$$V_t\left(b_t,\theta_t\right)=\max_{c_t,\ell_t,b_{t+1},\theta_{t+1}\geq 0}\left\{u\left(c_t\right)-v\left(\ell_t\right)+\beta V_{t+1}\left(b_{t+1},\theta_{t+1}\right)\right\}\,, \tag{5}$$

subject to the budget constraint

$$c_t=w_t\ell_t+(\pi_t+q_t)\theta_t-q_t\theta_{t+1}+(1+r)b_t-b_{t+1}\,,$$

the labor-supply schedule (4) for $t=1$, and the no-Ponzi game.

Household optimality requires that

$$u'(c_t)w_t=v'(\ell_t)\,,$$

for all $t\neq 1$. It also requires the Euler equations

$$u'(c_t)=\beta(1+r)u'(c_{t+1})\,, \tag{6}$$

$$q_t\,u'(c_t)\geq\beta(\pi_{t+1}+q_{t+1})u'(c_{t+1})\,, \tag{7}$$

for all $t\geq 0$, where the second condition holds with equality if $\theta_{t+1}>0$.

2.3 Foreign investors

The key departure from a standard framework is that foreign investors can buy and sell equity of domestic firms. We assume that foreign investors have deep pockets—that is, they face no binding wealth or borrowing constraints—and that they face a no-short-selling constraint in domestic equity, so their holdings must be non-negative, $\theta^*_{t+1}\geq 0$. It follows from investors' optimality and (6)-(7) that for the market of equity to clear, it must be that the return on bonds equals the return on firms' shares.[6] That is,

$$1+r=\frac{\pi_{t+1}+q_{t+1}}{q_t}. \tag{8}$$

Equation (8) implies that both households and foreign investors are indifferent about the relative composition of equity to bonds in their portfolios.

[6]Foreign investors' portfolio condition is $(1+r)q_t\geq(\pi_{t+1}+q_{t+1})$, with equality if $\theta^*_{t+1}>0$. Likewise, (6)-(7) yield $(1+r)q_t\geq(\pi_{t+1}+q_{t+1})$ with equality if $\theta_{t+1}>0$. For $\theta_{t+1}+\theta^*_{t+1}=1$, (8) must hold.

2.4 Competitive equilibrium

We can now define a competitive equilibrium for a given exchange rate policy. We restrict attention to a symmetric equilibrium, in which all of the households choose the same portfolios.

Definition 1. Given an initial portfolio (b_0, θ_0), a world real interest rate r, a rigid wage $\overline{W}_1$, and an exchange rate policy, $\{E_t\}_{t=0}^{\infty}$, a *competitive equilibrium* is a sequence of allocations $\{c_t, h_t\}_{t=0}^{\infty}$, portfolios $\{b_{t+1}, \theta_{t+1}, \theta^*_{t+1}\}_{t=0}^{\infty}$, profits π_t, and prices $\{W_t, q_t\}_{t=0}^{\infty}$ such that:

(i) Households optimize. That is, consumption, labor, and the portfolios solve problem (5).

(ii) Firms choose employment optimally. That is, (2) holds.

(iii) The price for firms' shares satisfies condition (8) and $\theta_{t+1} + \theta^*_{t+1} = 1$.

Profits, GNI, and balance of payment accounting. A key equilibrium variable in our analysis is firms' profits. Combining (1) and (2), we get that equilibrium profits are a function of aggregate employment only:

$$\pi_t = F(h_t) - F'(h_t)h_t \equiv \Pi(h_t), \tag{9}$$

where the profit function $\Pi(\cdot)$ satisfies $\Pi(0) = 0$ and $\Pi'(h) = -F''(h)h > 0$ for $h > 0$.

Following the standard balance of payment accounting, we define gross national income (GNI), the current account (CA), and the net foreign asset position (NFA) as

$$\begin{aligned} GNI_t &\equiv F(h_t) + rb_t - \theta^*_t \Pi(h_t), \\ CA_t &\equiv GNI_t - c_t, \\ NFA_t &\equiv b_t - q_t \theta^*_t. \end{aligned}$$

GNI is the income accruing to domestic residents: $F(h_t)$ is domestic output, $\theta^*_t \Pi(h_t)$ is profit income paid out to foreign shareholders, and rb_t is interest received on the country's net foreign bond position.[7] The CA measures the economy's net saving vis-a-vis the rest of the

[7] Notice that we can write the GNI equivalently as

$$GNI_t = \theta_t F(h_t) + (1 - \theta_t)F'(h_t)h_t + rb_t,$$

an average of total output and labor income (weighted by the fraction of equity owned by domestic households)

world: income accruing to residents minus domestic absorption. The NFA position is the value of foreign assets b_t net of the market value $q_t\theta_t^*$ of domestic equity held by foreigners, which represents a liability for the small open economy.

Using the household budget constraint, market clearing for equity claims $\theta_t^* = 1 - \theta_t$, and the expression for GNI yields the balance of payment condition: a current-account deficit must be financed either by reducing bond holdings b_t or by issuing additional domestic equity to foreign investors (raising θ_t^*):

$$CA_t = (b_{t+1} - b_t) - q_t(\theta_{t+1}^* - \theta_t^*).$$

Given this expression, our definition of the NFA, we can express its evolution as

$$NFA_t - NFA_{t-1} = CA_t - \theta_{t-1}^*(q_t - q_{t-1}).$$

That is, the change in the NFA position equals the current account plus a valuation term $-\theta_t^*(q_t - q_{t-1})$, which captures capital gains and losses on the outstanding stock of equity claims held by foreign investors.

3 International Equity Flows and Exchange Rate Policy

In this section, we characterize equilibria and analyze the implications of international equity flows for the optimal exchange rate policy. To streamline the analysis, we assume $\beta(1+r) = 1$, so the real interest rate equals the households' discount rate and consumption is constant.

Road map. We proceed with our analysis as follows: First, we characterize the steady-state equilibrium for $t \geq 2$. Second, we analyze the competitive equilibrium at $t = 1$ and the optimal exchange rate policy. Finally, we study the scope for capital controls in period $t = 0$.

3.1 Steady-state equilibrium for $t \geq 2$

The assumption that nominal wages are flexible after $t \geq 2$ ensures monetary neutrality in this continuation equilibrium. Moreover, given that $\beta(1 + r) = 1$, we have that all allocations are constant. We use the variables c and h to denote steady-state values. Lemma 1 characterizes this continuation equilibrium for any given exchange rate policy and for any initial portfolio of assets and liabilities that characterizes the economy's NFA position (b_2, θ_2^*).

plus interest on foreign bond holdings.

Lemma 1 (Steady-state equilibrium)**.** *Given an initial portfolio* (b_2, θ_2^*)*, the continuation equilibrium for* $t \geq 2$ *features constant consumption and labor. In particular, the steady-state values,* h *and* c*, are given by*

$$\frac{v'(h)}{u'(c)} = F'(h)\,, \tag{10}$$

$$c = F(h) + rb_2 - \Pi(h)\theta_2^*\,, \tag{11}$$

and the sequence of portfolios $\left\{b_{t+1}, \theta_{t+1}^*\right\}_{t\geq 2}$ *and the steady-state price of equity,* q*, satisfy*

$$b_{t+1} - q\,\theta_{t+1}^* = b_2 - q\,\theta_2^* \tag{12}$$

and

$$q = \frac{F(h) - F'(h)h}{r}. \tag{13}$$

Proof. In Appendix A.1. □

Condition (10) is the standard condition equating the marginal rate of substitution between consumption and labor to the marginal product. For the analysis that follows, it will be useful to define $h = \mathcal{H}(c)$, as the level of employment consistent with (10), where $\mathcal{H}'(\cdot) < 0$, which reflects the standard wealth effect on labor supply.[8]

Condition (11) says that in steady state—when the current account is balanced—consumption equals gross national income (GNI). Condition (12) states that any portfolio of bonds and domestic equity that keeps the NFA constant is consistent with equilibrium, reflecting that households are indifferent across portfolios that deliver the same level of consumption. Condition (13) states the asset price equals the present discounted value of profits.

3.2 Continuation equilibrium at period $t = 1$

In this section, we characterize the equilibrium starting from $t = 1$, given an initial portfolio (b_1, θ_1^*). We use the employment function $\mathcal{H}(\cdot)$, defined above, in the following lemma, which characterizes the equilibrium outcome in period 1 for any given exchange rate E_1 and initial

[8] To see that $\mathcal{H}'(c) < 0$, we differentiate $v'(\mathcal{H}(c)) = F'(\mathcal{H}(c))u'(c)$ and obtain

$$\mathcal{H}'(c) = \frac{u''(c)F'(\mathcal{H}(c))}{v''(\mathcal{H}(c)) - u'(c)F''(\mathcal{H}(c))} < 0,$$

where the inequality follows from $u'' < 0, F'' < 0, v'' > 0$.

portfolio (b_1, θ_1^*).

Lemma 2 (Equilibrium for $t \geq 1$)**.** *Given an initial portfolio (b_1, θ_1^*), a rigid wage $\overline{W}_1$, an exchange rate E_1, the equilibrium for $t \geq 1$ is such that*

i) h_1 is given by

$$\frac{\overline{W}_1}{E_1} = F'(h_1); \tag{14}$$

ii) $h_t = \mathcal{H}(c)$ for $t > 1$, and $c_t = c$ for $t \geq 1$, with c given by

$$c = \frac{r}{1+r}\left[F(h_1) + rb_1 - \Pi(h_1)\theta_1^*\right] + \frac{1}{1+r}\left[F\big(\mathcal{H}(c)\big) + rb_1 - \Pi(\mathcal{H}(c))\theta_1^*\right]; \tag{15}$$

iii) $q_t = q$, with q given by (13), and the sequence of portfolios is such that

$$(b_2 - q\,\theta_2^*) \;=\; (b_1 - q\,\theta_1^*) + \frac{F(h_1) - \Pi(h_1)\theta_1^* - \left[F\left(\mathcal{H}(c)\right) - \Pi\left(\mathcal{H}(c)\right)\theta_1^*\right]}{1+r} \tag{16}$$

and $\left\{b_{t+1}, \theta_{t+1}^*\right\}_{t\geq 2}$ satisfy (12).

Proof. In Appendix A.4. □

Condition (14) captures that when the nominal wage is preset, the exchange rate pins down the real wage, and employment is determined by firms' demand at that real wage. Condition (15) reflects that consumption must be constant over time, given that $\beta(1+r) = 1$ and that preferences between consumption and labor are separable. The expression for consumption follows from the intertemporal budget constraint: the two terms are the annuity value of period-1 GNI and of future GNI, respectively. Recall that returns on bonds and equity are equal in equilibrium, which implies that households are indifferent across any portfolio of bonds and domestic equity that supports the equilibrium level of consumption.[9] Condition (16) describes the evolution of the NFA. Notice that because in general, period-1 GNI differs from the future GNI, the current account in period 1 is not necessarily balanced; hence, the NFA can vary between periods 1 and 2.

[9]Moreover, this also implies that the future GNI can also be expressed as $F(\mathcal{H}(c)) + rNFA_1$.

3.3 Optimal exchange rate policy

We now study the problem of a central bank that chooses monetary policy in period 1 to maximize households' welfare in the competitive equilibrium. Notice that by setting the period-1 exchange rate, the central bank can effectively choose labor in period 1. Using Lemma 2, we can formulate this problem as choosing a period-1 level of employment h_1, a constant level of consumption c, and an associated steady-state level of employment $\mathcal{H}(c)$, subject to a country budget constraint. That is, the optimal policy problem is

$$V_1(b_1, \theta_1^*) = \max_{c, h_1} \left\{ u(c) - v(h_1) + \frac{u(c) - v\left(\mathcal{H}(c)\right)}{r} \right\}, \tag{17}$$

subject to:

$$\frac{1+r}{r} c = F(h_1) + r b_1 - \Pi(h_1)\theta_1^* + \frac{F\left(\mathcal{H}(c)\right) + r b_1 - \Pi\left(\mathcal{H}(c)\right)\theta_1^*}{r}. \tag{18}$$

Denoting by $\lambda > 0$ the Lagrange multiplier on the intertemporal country budget constraint (15), we have the following optimality condition with respect to period-1 employment h_1:

$$v'(h_1) = \lambda \left[F'(h_1) - \Pi'(h_1)\theta_1^* \right]. \tag{19}$$

Condition (19) equates the marginal disutility of working an additional unit of time in the period to the marginal benefits, which are given by the additional period-1 resources times the marginal benefit of relaxing the country budget constraint. Crucially, the change in period-1 resources from a marginal increase in employment is the increase in *output net of the rise in profits*—that is, $F'(h_1) - \theta_1^* \Pi'(h_1)$. The latter term captures the additional profits accruing to foreign shareholders, and their effect on domestic resources is

$$-\theta_1^* \Pi'(h_1) = \theta_1^* F''(h_1) h_1 \leq 0,$$

where the inequality is strict whenever $\theta_1^* > 0$ because the production function is strictly concave: $F''(h) < 0$. That is, higher employment raises output but lowers wages—as the central bank needs to depreciate the exchange rate to stimulate labor demand—and thus increases firms' profits. Because a share θ_1^* of firms' profits accrue abroad, a more stimulative policy generates an increase in the effective liabilities that the SOE faces. This is why stimulating employment carries a higher marginal cost than in the case without foreign ownership.

The optimality condition with respect to consumption c yields

$$u'(c) + \frac{1}{1+r}\Big[- v'\big(\mathcal{H}(c)\big) + \lambda\big(F'\big(\mathcal{H}(c)\big) - \Pi'\big(\mathcal{H}(c)\big)\theta_1^*\big)\Big]\mathcal{H}'(c) = \lambda. \tag{20}$$

Equation (20) states that the marginal benefit of increasing consumption must equal the shadow cost of the additional resources required to finance it. The first term on the left-hand side is the direct marginal utility of an additional permanent unit of consumption. The second term captures the indirect effect of higher consumption on welfare through its impact on steady-state employment: because $\mathcal{H}'(c) < 0$, an increase in c reduces future hours, which in turn lowers both the marginal disutility from working $v'\big(\mathcal{H}(c)\big)$ and the marginal annuity value of future output and profits $F'\left(\mathcal{H}(c)\right) - \Pi'\left(\mathcal{H}(c)\right)\theta_1^*$.

Combining optimality conditions (19) and (20), we obtain

$$\underbrace{F'(h_1)u'(c) - v'(h_1)}_{\text{Labor wedge}} = \frac{\Pi'(h_1)u'(c) - \frac{1}{1+r}\Big[\Pi'(h_1)\, v'\big(\mathcal{H}(c)\big) - \Pi'(\mathcal{H}(c))\, v'(h_1)\Big]\mathcal{H}'(c)}{1 - \frac{1}{1+r}F'(\mathcal{H}(c))\mathcal{H}'(c)}\theta_1^*\,. \tag{21}$$

The left-hand side of (21) represents the labor wedge. When foreign investors do not own equity claims on domestic firms, $\theta_1^* = 0$, the right-hand side of (21) is zero, and the central bank sets the wedge to zero, which replicates the flexible-wage (natural) allocation. By contrast, when $\theta_1^* > 0$, the right-hand side of (21) is generally different from zero, so the optimal policy features a non-zero labor wedge, and the central bank finds it optimal to depart from the natural allocation. The intuition is that the central bank uses the exchange rate to compress profits accruing to foreign shareholders, thereby reducing the value of the economy's foreign liabilities. As discussed above, a reduction in period-1 employment h_1 reduces contemporaneous profits, which induces the central bank to generate a positive labor wedge, as shown by the first term of the numerator on the right side. However, at the same time, a lower h_1 reduces period-1 total income and the net foreign asset position, which generates a negative wealth effect that induces households to supply more labor in steady state, raising long-run profits. Under the conditions stated in the proposition below, this indirect effect is dominated by the contemporaneous benefit of higher wages in period 1. This implies that the right-hand side of (21) is positive and that the optimal labor wedge is strictly positive whenever $\theta_1^* > 0$.

Let the labor wedge induced under optimal exchange rate policy be given by

$$\zeta_1(b_1, \theta_1^*) \equiv u'(c(b_1, \theta_1^*))F'(h_1(b_1, \theta_1^*)) - v'(h_1(b_1, \theta_1^*))\,,$$

where consumption $c(b_1, \theta_1^*)$ and period-1 employment $h_1(b_1, \theta_1^*)$ are jointly determined by (18) and (21), given international portfolio (b_1, θ_1^*). The proposition below establishes conditions under which $\zeta_1(b_1, \theta_1^*) > 0$ whenever $\theta_1^* > 0$.

Assumption 1. *Functions $u(\cdot)$, $v(\cdot)$, and $F(\cdot)$ satisfy either one of the following two properties:*

(i) $|\mathcal{H}'(\cdot)| < \varepsilon$, where $\varepsilon > 0$ is positive, but arbitrarily small.

(ii) $u(\cdot)$, $v(\cdot)$, and $F(\cdot)$ are isoelastic.

Proposition 1 (Positive labor wedge)**.** *Suppose that Assumption 1 holds. Then, $\zeta_1(b_1, \theta_1^*) \geq 0$, with strict inequality if $\theta_1^* > 0$.*

Proof. In Appendix A.2 □

Under property (i) in Assumption 1, wealth effects on labor supply are arbitrarily small. This implies that the change in the net foreign asset position in period 1 does not almost affect future wages, and hence (21) can be approximated by

$$F'(h_1)u'(c) - v'(h_1) = \theta_1^* \Pi'(h_1)u'(c) \geq 0\,.$$

That is, the optimal labor wedge is determined almost solely by the contemporaneous effect of period-1 employment on the share of firm profits accrued by foreigners. Given that $\Pi' > 0$, it follows that the labor wedge is strictly positive for any $\theta_1^* > 0$.

Under property (ii), starting from an allocation with a zero labor wedge, the future effects of wealth on future wages are second-order for the determination of the wedge. It then follows that the central bank chooses to implement an allocation with a strictly positive labor wedge as long as $\theta_1^* > 0$.

Comparative statics. Next, we study how the labor wedge responds to marginal changes in the share of equity owned by foreigners. Naturally, everything else fixed, changes in foreign equity ownership affect the wealth of the SOE and, through a resulting wealth effect, the labor wedge. To control for these wealth effects, we restrict attention to perturbations to the overall international portfolio that in the absence of changes in the price of domestic equities, keep the NFA fixed. Formally, let $q_0(b_1, \theta_1^*)$ denote the equilibrium price of domestic equities

in period 0 given international portfolio (b_1, θ_1^*). This price is given by

$$q_0(b_1, \theta_1^*) = \frac{1}{1+r}\left[\Pi(h_1(b_1, \theta_1^*)) + \frac{1}{r}\,\Pi(\mathcal{H}(c(b_1, \theta_1^*)))\right]. \tag{22}$$

We consider perturbations of the portfolios such that

$$\hat{\theta}_1^* = \theta_1^* + \hat{\varepsilon} \quad \text{and} \quad \hat{b}_1 = b_1 + q_0\left(b_1, \theta_1^*\right)\hat{\varepsilon}\,,$$

where $\hat{\varepsilon} > 0$ is positive, but arbitrarily small.[10] The following proposition holds.

Proposition 2 (Increasing labor wedge)**.** *Suppose that either property (i) in Assumption 1 holds and $F(\cdot)$ is isoelastic, or property (ii) holds. Then,*

$$\left.\frac{dh_1(\hat{b}_1, \hat{\theta}_1^*)}{d\hat{\varepsilon}}\right|_{\hat{\varepsilon}=0} < 0, \quad \left.\frac{dc(\hat{b}_1, \hat{\theta}_1^*)}{d\hat{\varepsilon}}\right|_{\hat{\varepsilon}=0} < 0, \quad \left.\frac{d\zeta_1(\hat{b}_1, \hat{\theta}_1^*)}{d\hat{\varepsilon}}\right|_{\hat{\varepsilon}=0} > 0\,.$$

Proof. In Appendix A.3 □

The proposition shows that an increase in foreign equity leads the central bank to induce lower employment and consumption and a higher labor wedge. That is, if wealth is held constant, higher holdings of domestic equity claims by foreign investors lead the central bank to follow a tighter monetary policy to reduce payments to foreign investors. We note that the assumed properties are sufficient to guarantee the result, but not necessary.

Foreign equity as a negative productivity shock. To further illustrate the comparative statics, consider the special case where the production function is given by $F(h) = h^\alpha$, with $\alpha \in (0, 1)$. In this case, GNI_t can be expressed as

$$GNI_t = \left[1 - (1-\alpha)\theta_t^*\right]F\left(h_t\right) + rb_t\,.$$

The factor $(1-\alpha)\theta_t^*$ captures the wedge between domestic production and domestic income induced by foreign equity ownership. Because the monetary authority internalizes that a portion of firms' profits go to foreign equity owners, when choosing employment, it behaves as if the economy's effective productivity were lower. If the bond position b_t adjusts to keep aggregate wealth constant, this mechanism leads the authority to select a lower level of employment.

[10] Note that $NFA_1 = \hat{b}_1 - q_0\left(b_1, \theta_1^*\right)\hat{\theta}_1^* = b_1 - q_0\left(b_1, \theta_1^*\right)\theta_1^*$ for any $\hat{\varepsilon} > 0$, which implies that the NFA would remain constant if equity was evaluated at the prevailing price before the perturbation.

3.4 Quantitative results

In this section, we provide a simple quantification to assess the role of international equity flows in shaping the optimal exchange rate policy and the welfare implications. We parametrize $\{u(\cdot), v(\cdot), F(\cdot)\}$ using isoelastic functional forms:

$$u(c) = \frac{1}{1-\frac{1}{\sigma}} c^{1-\frac{1}{\sigma}}, \quad v(\ell) = \chi \frac{1}{1+\psi} \ell^{1+\psi}, \quad F(h) = h^{\alpha},$$

with $\sigma > 0$, $\chi > 0$, $\psi > 0$ and $\alpha \in (0,1)$.

The time period is a year. We set the intertemporal elasticity of substitution to $\sigma = 1$, the inverse labor supply elasticity to $\psi = 1$, the labor share to $\alpha = 0.65$, and the interest rate to $r = 0.04$. These values are standard in the literature. We also normalize the disutility from labor χ so that output equals 1 under flexible prices.

We set the initial NFA to -60% of output, roughly the U.S. value in 2025. A reference value for θ, following Figure 1, is $\theta = 0.5$. In the simulations below, we consider values for $\theta \in [0,1]$ and adjust the initial bond holdings so that the net foreign asset position remains constant in the flexible price allocation. In particular, we vary b_1 so that $b_1 - \tilde{q}\,\theta_1^* = -60\%$, where $\tilde{q}$ denotes the price of equity under flexible wages.

Figure 2 presents the results for the optimal exchange rate policy at $t = 1$. The top panels plot employment, the labor wedge, and profits over the natural allocation; the bottom panels display the exchange rate, consumption, and welfare gains. All of these variables are expressed in percentages, measuring deviation from an economy without foreign equity ownership (or, equivalently, flexible prices), except for the labor wedge, which is expressed in levels.[11]

When $\theta_1^* = 0$, the optimal policy sets the labor wedge to zero (panel b), reproducing the flexible-wage allocation. As foreign holdings increase, the central bank appreciates the exchange rate (panel [e]), and this leads to lower employment (panel [a]), profits (panel [c]), and consumption (panel [d]). We can see that the deviations from the flexible-price allocation are sizable. For $\theta_1^* = 0.5$, the monetary authority appreciates the exchange rate by 4%, employment falls by about 10%, and consumption declines by 0.15% relative to the case without foreign equity ownership.

Panel (f) reports the consumption-equivalent welfare gains from adopting the optimal exchange rate policy relative to a benchmark that maintains a zero labor wedge (i.e., the natural allocation). The figure indicates substantial gains from departing from the natural

[11] We normalize the labor wedge by $u'(c)F'(h)$ so that it can be interpreted as a labor income tax.

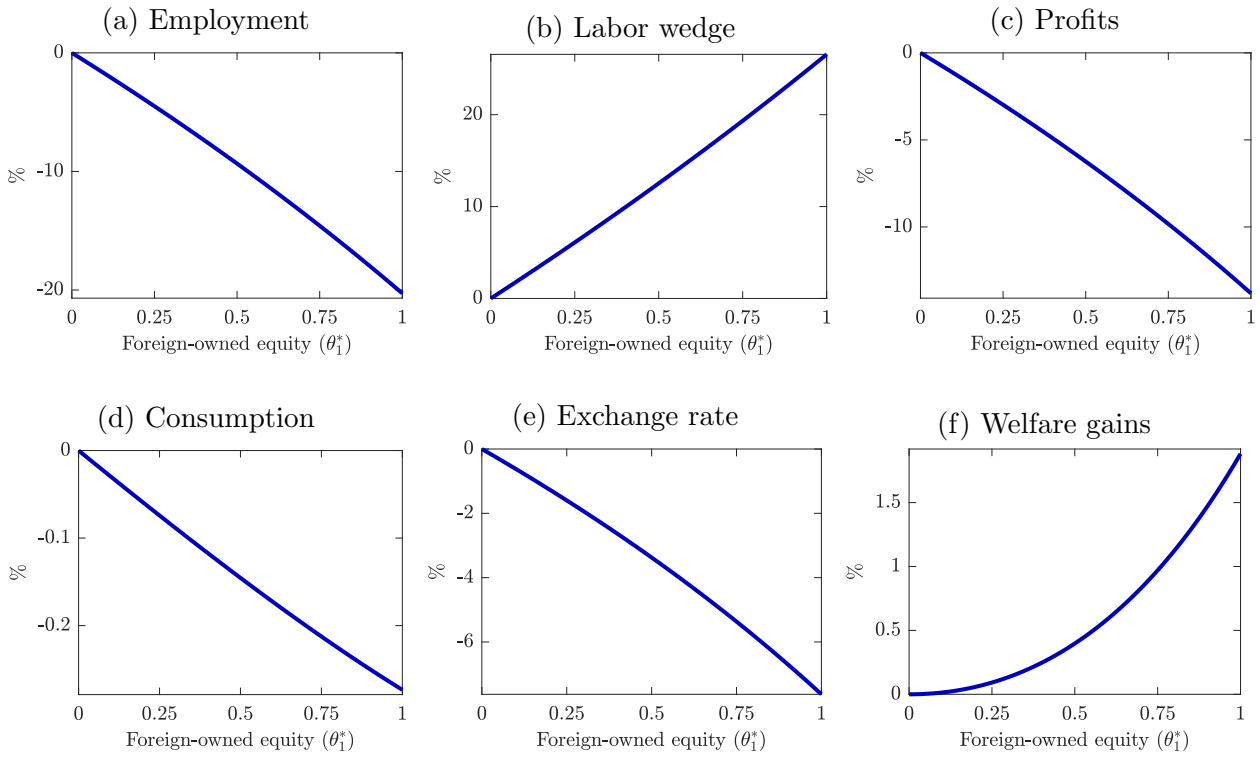


Figure 2: Equilibrium under optimal exchange rate policy

Notes: The figure plots the continuation equilibrium at period $t = 1$ under the optimal exchange rate as a function of the foreign-owned fraction of domestic firms' shares.

allocation. In the limiting case in which foreign investors own 100% of firms, households would be willing to forgo 1.75% of period-1 consumption to switch to the optimal policy.

Takeaway. This section highlighted the implications of foreign equity ownership for the determination of the optimal exchange rate. The key insight is that when foreigners own equity, the central bank has incentives to appreciate the exchange rate beyond the natural allocation to boost real wages and reduce firm profits.

4 International Equity Flows and Capital Controls

We now turn to the determination of the initial portfolio $\{b_1, \theta_1^*\}$ and explore the implications of foreign equity ownership for the desirability of capital controls.

4.1 Equilibria for $t \geq 0$

We study the equilibrium for $t \geq 0$ when it is expected that the central bank will follow at period 1 the optimal exchange rate policy $E_1(b_1, \theta_1^*)$. The lemma below shows that there is a continuum of equilibria with different allocations and welfare implications.

Lemma 3 (Equilibrium)**.** *Given (b_0, θ_0^*) and the period-1 optimal exchange rate policy $E_1(b_1, \theta_1^*)$, there exists a continuum of Pareto-ranked competitive equilibria indexed by (b_1, θ_1^*), where the set of possible equilibrium values for (b_1, θ_1^*) is characterized by*

$$c(b_1, \theta_1^*) = F(\mathcal{H}(c(b_1, \theta_1^*))) - \Pi(\mathcal{H}(c(b_1, \theta_1^*)))\theta_0^* + q_0(b_1, \theta_1^*)\,(\theta_1^* - \theta_0^*) + (1+r)b_0 - b_1\,, \quad (23)$$

where q_0 is given by (22).

Proof. In Appendix A.5. □

As in the continuation equilibria in Section 3, households are indifferent across any portfolio of bonds and equity shares that implement the same level of permanent consumption. However, while the indeterminacy in the portfolio was inconsequential in the continuation equilibria, the indeterminacy at period 0 gives rise to multiple equilibria with different prices and allocations. This happens because the portfolio chosen in period 0 influences the period-1 optimal exchange rate, which in turn influences the period-0 equity price and the country's budget constraint (23). In particular, following Proposition 2, when households choose to sell more of their equity claims, the central bank chooses in period 1 a more appreciated exchange rate, and firm profits are lowered as a result. Since investors perfectly anticipate the central bank decision, they are willing to buy the equity claims only at a lower price, which ultimately erodes the wealth of the SOE and tightens the country's budget constraint.

Figure 3 illustrates the multiplicity of equilibria for the isoelastic case, where we use the same parameter values as in subsection 3.4, and assume for simplicity that $\theta_0^* = 0$. Panels (a) and (b) respectively present the equity price and consumption in period 0 as a function of the equity claims that households choose to sell in equilibrium. Panel (c) presents the welfare losses—in terms of period-0 consumption—relative to the equilibrium in which households continue to hold all the equity. The worst possible equilibrium is the one in which households sell all equity, where the welfare losses are about -2%. In that case, as the figure illustrates, the equity price falls by about 0.4%, and permanent consumption falls by roughly 0.3%.

Time inconsistency and multiple equilibria. The multiplicity of Pareto-ranked equi-

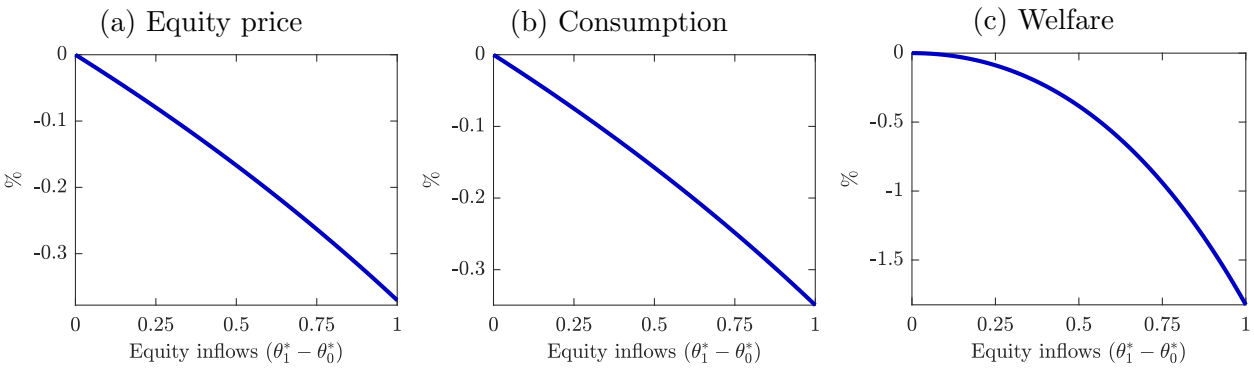


Figure 3: Equilibrium as a function of equity inflows

Notes: The figure plots the equilibrium outcome as a function of the possible equity inflows for a case in which the SOE initially owns all equity shares (i.e., $\theta_0^* = 0$).

libria reflects a coordination problem among domestic households. Individually, households are indifferent between holding equity and bonds, because both assets deliver the same return in equilibrium. Collectively, however, larger equity sales raise foreign ownership, which exacerbates the central bank's *ex post* incentive to compress profits. Anticipating lower future profits, foreign investors are willing to purchase domestic equity only at a lower price in period 0. This price decline reduces domestic wealth and lowers welfare. At the heart of the coordination failure is limited commitment: once domestic equity is held abroad, the central bank finds it optimal *ex post* to reduce profits, and this validates the lower equity valuation *ex ante*.

By contrast, if the central bank were instead able to commit, the competitive equilibrium would feature unique values of consumption and labor in all periods, regardless of the portfolio composition chosen by households in period 0. The proposition below formalizes this result, and shows additionally that if $\theta_0^* = 0$, the equilibrium corresponds to the natural allocation.

Proposition 3 (Equilibrium under commitment)**.** *Suppose that the central bank at $t = 0$ can commit to a period-1 exchange rate E_1. Then, given international portfolio (b_0, θ_0^*), the competitive equilibrium under the optimal exchange rate features unique values of consumption and labor in all periods. Moreover, if $\theta_0^* = 0$, the optimal period-1 exchange rate policy implements the natural allocation in period 1.*

Proof. In Appendix A.6. □

The coordination failure suggests that limiting foreign equity ownership could be desirable, a point we turn next.

4.2 Capital controls on international equity flows

We consider a benevolent domestic financial regulatory authority, which can set capital controls on international equity flows in period 0 to maximize the welfare of the households in the SOE. Crucially, the regulatory authority takes into account that monetary policy, in period 1, is set in a time-consistent manner. We assume that in period 0, the regulatory authority chooses the portfolio position directly on behalf of households and lets households choose consumption, labor, and bonds.[12]

The problem of the regulatory authority can be expressed as

$$V_0(b_0, \theta_0^*) = \max_{c_0, b_1, \theta_1^* \in [0,1]} \left\{ u(c_0) - v(\mathcal{H}(c_0)) + \frac{1}{1+r} V_1(b_1, \theta_1^*) \right\}, \tag{24}$$

subject to:

$$c_0 = F(\mathcal{H}(c_0)) - \Pi(\mathcal{H}(c_0))\theta_0^* + q_0(b_1, \theta_1^*)\,(\theta_1^* - \theta_0^*) + (1+r)b_0 - b_1\,, \tag{25}$$

$$c_0 = c(b_1, \theta_1^*)\,, \tag{26}$$

where the country's budget constraint (25) already incorporates that the price at which households sell depends on the endogenous portfolio. Equilibrium constraint (26) reflects that because households can choose their bond holdings b_1 in the initial period, for any policy $\theta_1^* = 1 - \theta_1$, perfect consumption smoothing must hold.

The proposition below characterizes the optimal capital controls on foreign purchases of domestic equity.

Proposition 4 (Positive capital controls)**.** *Suppose that the properties assumed in Proposition 2 hold. Then, the solution to problem (24) is such that*

$$\theta_1^* = \begin{cases} 0 & \text{if } \theta_0^* = 0, \\ \in (0, \theta_0^*) & \text{if } \theta_0^* > 0. \end{cases}$$

Proof. In Appendix A.7. □

Consider first the case with $\theta_0^* = 0$. When households initially own all of the equity shares, the regulatory authority restricts households from selling any equity claims. The reason is as follows. Keeping the same portfolio induces the monetary authority to implement the

[12] Without loss of generality, we assume that the regulatory authority does not intervene in future periods.

natural allocation. Moreover, any sales of equity claims would lead the monetary authority to depart from the natural allocation and reduce profits. However, because the reduction in profits is priced ex ante, this implies a financial loss for households. Any sales of equity claims, therefore, result in a distorted allocation and a financial loss.

When $\theta_0^* > 0$, the regulatory authority finds it optimal to induce *equity outflows*. As in the case with $\theta_0^* = 0$, allowing households to sell domestic equity claims is detrimental because the economy ends up with more distorted allocations and with financial losses. Moreover, now there is a benefit from buying back the equity claims from foreign investors, as this leads to a higher allocative efficiency in period 1. However, as households buy more equity, this raises the price of their purchases, leading again to a financial loss. In other words, as households buy back more equity from foreign investors, the foreign investors appropriate in effect part of the higher allocative efficiency gains. At the optimum, the regulatory authority therefore induces a limited number of buybacks, with $\theta_1^* \in (0, \theta_0^*)$.[13]

5 Extensions

In this section, we extend the theory in two directions. First, we allow for binding financial constraints. Second, we incorporate a non-tradable good in the model.

5.1 The case with a binding debt limit

In our baseline model, the SOE is not subject to borrowing constraints. This implies that independently of any restriction imposed on equity inflows by the regulatory authority, households are able to smooth consumption by increasing external debt. Next, we consider a situation in which households may face limits to external borrowing.

We assume that in period 0 bond holdings b_1 must satisfy

$$b_1 \geq -\underline{b}\,,$$

[13] In our analysis, we have assumed that the regulatory authority controls only equity flows. If the regulatory authority also chooses households' bond position b_1 in addition to equity holdings, the consumption-smoothing constraint $c_0 = c(b_1, \theta_1^*)$ no longer applies. We can show that the optimal policy then leaves foreign equity holdings unchanged, $\theta_1^* = \theta_0^*$ (no equity repurchases or sales), and sets higher initial consumption, $c_0 > c(b_1, \theta_1^*)$. This higher c_0 generates a positive wealth effect that shifts period-0 labor supply inward, raising real wages $w_0 = F'(\mathcal{H}(c_0))$ and reducing firm profits $\Pi(\mathcal{H}(c_0))$. In other words, control of bond and equity flows allows the SOE to avoid any financial losses, but the period-1 allocation remains distorted as this allows the regulatory authority to extract resources from the initial investors.

where $\underline{b} > 0$ is an exogenous limit to external debt. We interpret this constraint as capturing the limit enforceability of international debt contracts. For simplicity, we assume that the borrowing constraint applies only at period 0.

The regulatory authority now faces a problem that can be expressed as

$$V_0(b_0, \theta_0^*) = \max_{c_0, b_1, \theta_1^* \in [0,1]} \left\{ u(c_0) - v(\mathcal{H}(c_0)) + \frac{1}{1+r} V_1(b_1, \theta_1^*) \right\}, \tag{27}$$

subject to:

$$c_0 = F(\mathcal{H}(c_0)) - \Pi(\mathcal{H}(c_0))\theta_0^* + q_0(b_1, \theta_1^*)\,(\theta_1^* - \theta_0^*) + (1+r)b_0 - b_1\,, \tag{28}$$

$$c_0 \leq c(b_1, \theta_1^*)\,, \tag{29}$$

$$b_1 \geq -\underline{b}\,, \tag{30}$$

$$0 = (c_0 - c(b_1, \theta_1^*))\,(b_1 + \underline{b})\,, \tag{31}$$

where condition (29) allows consumption in period 0 to be below consumption in period 1, and where equilibrium constraint (31) specifies that if that is the case, the debt limit (i.e., condition (30)) must be binding. Constraint (31) is also needed because if the solution were such that $c_0 < c(b_1, \theta_1^*)$, households would rather deviate from it, by reducing b_1 to narrow the gap between c_0 and $c(b_1, \theta_1^*)$.

We restrict attention to a situation in which $b_0 < -\underline{b}$. This implies that foreign investors are not willing to hold the initial debt anymore. Thus, the SOE cannot roll it over. We interpret this situation as a sudden stop.

Proposition 5 (Optimal control with binding debt limit)**.** *Suppose that the properties assumed in Proposition 2 hold. If $\theta_0^* = 0$, then $\theta_1^* > \theta_0^*$ is optimal. More generally, if the solution to (27) is such that $c_0 < c\,(b_1, \theta_1^*)$, then it must satisfy $\theta_1^* > \theta_0^*$.*

Proof. In Appendix A.8 □

If the SOE initially owns all the equity (i.e., $\theta_0^* = 0$), a sudden stop does not allow it to simultaneously retain the equity and achieve the consumption path of the natural allocation. This is because, in this case, reducing debt without selling equity shares forces the SOE to reduce consumption in period 0 below its level in period 1. The regulatory authority then finds it optimal to resolve this trade-off by selling a fraction of the equity shares to foreign investors. The reason is that not selling shares would require a relatively large contraction in period-0 consumption, which would significantly reduce welfare, as shown in Figure 4. By

contrast, if $\theta_0^* > 0$, a sudden stop may not necessarily induce the SOE to sell equity shares. This is because, as shown in Proposition 4, absent the sudden stop, the optimal policy would actually be to repurchase equity shares. The regulatory authority therefore finds it optimal to sell equity shares only when the sudden stop is sufficiently severe. For instance, if the sudden stop is such that smoothing consumption perfectly between period 0 and period 1 is not feasible, then selling equity shares is desirable. In general, selling all the equity shares to foreign investors is never optimal.

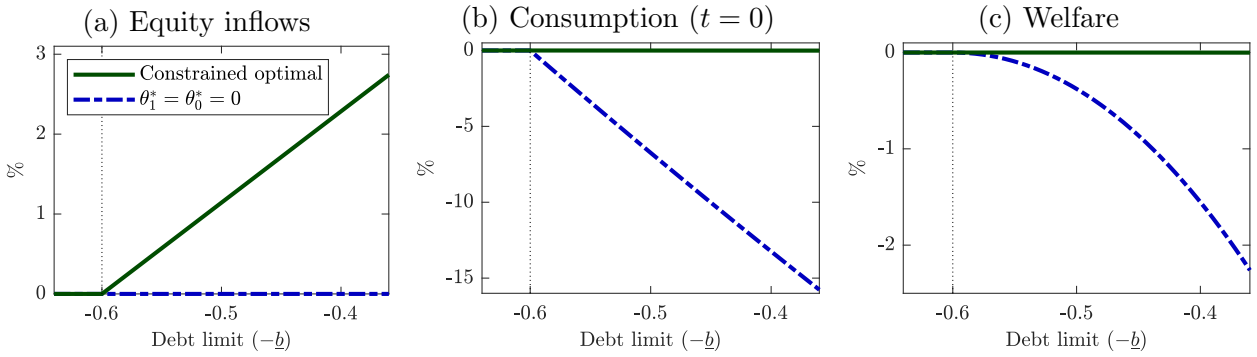


Figure 4: Equilibrium under constrained optimal capital control

Notes: The figure plots the equilibrium outcome under the constrained optimal capital control for a case in which the SOE initially owns all equity shares. The initial debt is $b_0 = -0.6$ (i.e., $NFA = -60\%$ of output). Values represent deviations from the natural allocation.

The key takeaway from this extension is that even in a situation where equity inflows are socially desirable, the regulatory authority still has incentives to reduce the amount of inflows relative to the unregulated competitive equilibrium. Notice that the optimal equity inflows restriction helps support equity prices, which indeed reduces the SOE's need to sell domestic equity as well as mitigates a vicious cycle—usually referred to as "fire sales"—between drops in equity prices and the need for further equity sales. Importantly, in this environment, equity prices fall not because of inherent differences in valuations between domestic and foreign investors, but because of distortionary incentives induced by equity inflows in future exchange rate policy.

5.2 Model with tradable and non-tradable goods

Until now, we have considered an environment with a single final good, assumed to be tradable internationally. In this section, we extend the baseline model by introducing a non-tradable good, and analyze the implications for monetary policy of international equity flows to firms in both the tradable and non-tradable sectors.

Households. Preferences are given again by

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_t) - v\left(\ell_t\right) \right],$$

but now $c_t = \tilde{c}(c_t^T, c_t^N)$ is a linearly homogeneous aggregator of the tradable good, c_t^T, and the non-tradable good, c_t^N, and $\ell_t = \ell_t^T + \ell_t^N$ is the total number of hours households work in the tradable sector, ℓ_t^T, and in the non-tradable sector, ℓ_t^N. For convenience, we will use $U(c_t^T, c_t^N) \equiv u(\tilde{c}(c_t^T, c_t^N))$ to denote utility as a function of the two goods.

The households' budget constraint, again in units of foreign currency (or the tradable good), is given by

$$c_t^T + p_t^N c_t^N = w_t \ell_t + (\pi_t^T + q_t^T)\theta_t^T + (\pi_t^N + q_t^N)\theta_t^N - q_t^T \theta_{t+1}^T - q_t^N \theta_{t+1}^N + (1+r)b_t - b_{t+1},$$

where q_t^T and q_t^N are the equity prices of firms in the tradable and the non-tradable sector, respectively, and p_t^N is the relative price of the non-tradable good.

In addition to the optimality conditions presented in Section 2, we have now a condition equating the marginal rate of substitution between the goods to their relative price,

$$\frac{U_N(c_t^T, c_t^N)}{U_T(c_t^T, c_t^N)} = p_t^N, \tag{32}$$

where U_T and U_N respectively denote the marginal utility with respect to the tradable and the non-tradable goods.

As in our baseline model, the nominal wage in units of domestic currency is fixed at a level $\overline{W}_1$ in period 1.[14] Thus, hours worked in each sector are determined by firms' labor demand.

Firms. The profits of firms in the tradable sector are now given by π_t^T, with

$$\pi_t^T \equiv \max_{h_t^T \geq 0} \left\{ F_T\left(h_t^T\right) - w_t h_t^T \right\}, \tag{33}$$

whereas those of firms in the non-tradable sector are given by π_t^N, with

$$\pi_t^N \equiv \max_{h_t^N \geq 0} \left\{ p_t^N F_N\left(h_t^N\right) - w_t h_t^N \right\}. \tag{34}$$

[14] An analysis with sticky prices for non-tradables is available upon request.

Note that the price of the non-tradable good, p_t^N, can differ from 1 since in equilibrium it is domestically determined.

Foreign investors. Lastly, foreign investors can hold equity shares of firms in either sector, but they cannot short-sell the shares. Put formally, $\theta_{t+1}^{*T} \geq 0$ and $\theta_{t+1}^{*N} \geq 0$. As in the model presented in Section 2, for markets of equity shares to clear, it must be that the return on the shares equals the return on bonds. That is,

$$1 + r = \frac{\pi_{t+1}^j + q_{t+1}^j}{q_t^j} \text{ for } j \in \{N, T\}. \tag{35}$$

Definition 2 (Competitive equilibrium with non-tradables)**.** Given initial conditions $(b_0, \theta_0^{*T}, \theta_0^{*N})$, a rigid wage $\overline{W}_1$, and an exchange rate policy, $\{E_t\}_{t=0}^{\infty}$, a *competitive equilibrium* is a sequence of allocations $\{c_t^T, c_t^N, h_t^T, h_t^N\}_{t=0}^{\infty}$, portfolios $\{b_{t+1}, \theta_{t+1}^T, \theta_{t+1}^{*T}, \theta_{t+1}^N, \theta_{t+1}^{*N}\}_{t=0}^{\infty}$, profits $\{\pi_t^T, \pi_t^N\}_{t=0}^{\infty}$, and prices $\left\{w_t, p_t^N, q_t^T, q_t^N\right\}_{t=0}^{\infty}$ such that:

(i) Households and firms optimize.

(ii) The market for the non-tradable good domestically clears. That is, $c_t^N = F_N\left(h_t^N\right)$.

(iii) The prices of firms' shares satisfy (35) and $\theta_{t+1}^j + \theta_{t+1}^{*j} = 1$ for both $j \in \{N, T\}$.

As in Section 2, we assume $\beta(1+r) = 1$ to ensure stationary in the continuation equilibria from period 2 onward.

Employment and Profits. Consider for simplicity the case with isoelastic functions. Formally:

$$U\left(c^T, c^N\right) = \frac{1}{1-\frac{1}{\sigma}}[\omega(c^T)^{1-\frac{1}{\gamma}} + (1-\omega)(c^N)^{1-\frac{1}{\gamma}}]^{\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\gamma}}}, \quad v(\ell) = \chi\frac{1}{1+\psi}\ell^{1+\psi}, \quad F_j(h) = h^{\alpha_j},$$

where $\omega \in (0,1)$ is the utility weight of the tradable good relative to that of the non-tradable good, $\gamma > 0$ is the elasticity of substitution between the two goods, and the rest of the parameters are interpreted in the same manner as described in Section 3.4. In this case, from combining (32) and (34) together with $c_t^N = F_N\left(h_t^N\right)$, one gets that in equilibrium

employment and profits in the non-tradable sector are related according to

$$\pi_t^N = \underbrace{\frac{1-\omega}{\omega}\left(\frac{F_N\left(h_t^N\right)}{c_t^T}\right)^{-1/\gamma}}_{=p_t^N}(1-\alpha_N)\,F_N\left(h_t^N\right).$$

Moreover, from (33), one obtains that employment and profits in the tradable sector are related as follows:

$$\pi_t^T = (1-\alpha_T)F_T\left(h_t^T\right).$$

These expressions show that in the tradable sector, employment and profits are always positively related, while in the non-tradable sector—all else equal—they are positively or inversely related depending on whether the goods are substitutes or complements. Specifically, if the goods are substitutes (i.e., $\gamma > 1$), the relationship is positive, whereas if they are complements (i.e., $\gamma < 1$) it is negative. This happens because the elasticity of substitution modulates the strength of the relationship between price and quantity consumed, making it weaker or stronger than inversely proportionally depending on whether γ is above or below one. An opposite sign in the employment-profits relationship between the sectors critically matters for monetary policy. Notably, if foreign equity ownership is such that foreign-owned profits in the tradable and in the non-tradable sector offset, then the central bank finds it optimal not to distort the allocation, despite the foreign ownership.

Optimal labor wedge. The proposition below formalizes this discussion by characterizing the set of international portfolios $\{b_1, \theta_1^{*T}, \theta_1^{*N}\}$ that eliminates the incentives of the central bank to distort the allocation when foreigners own equity shares in period 1. International portfolios not belonging to this set naturally incentivize the central bank to induce either a positive or a negative labor wedge. The set is particularly important from the perspective of period 0, because it specifies the controls needed on the *composition* of international equity inflows—between the tradable and the non-tradable sector—to preclude distortionary incentives to future monetary policy.

Proposition 6 (Zero optimal labor wedge)**.** *The set of international portfolios* $\{b_1, \theta_1^{*T}, \theta_1^{*N}\}$ *that eliminates the incentives of the central bank to distort the allocation when foreigners own equity shares in period 1 is characterized by*

$$\theta_1^{*N} = -\frac{\Pi_T'\left(\mathcal{H}_T\left(c^T\right)\right)\mathcal{H}_T'\left(c^T\right)}{\mathcal{P}_N'\left(c^T\right)\Pi_N(\mathcal{H}_N\left(c^T\right)) + \mathcal{P}_N\left(c^T\right)\Pi_N'(\mathcal{H}_N\left(c^T\right))\mathcal{H}_N'\left(c^T\right)}\theta_1^{*T}, \tag{36}$$

where mappings $\Pi_j(h)$*, with* $j \in \{N, T\}$*, are given by*

$$\Pi_j(h) \equiv F_j(h) - F_j'(h)h\,,$$

mappings $\mathcal{H}_T(\cdot)$*,* $\mathcal{H}_N(\cdot)$ *and* $\mathcal{P}_N(\cdot)$ *are jointly determined by*

$$\frac{v'(\mathcal{H}_T\left(c^T\right) + \mathcal{H}_N\left(c^T\right))}{U_T(c^T, F_N(\mathcal{H}_N\left(c^T\right)))} = F_T'(\mathcal{H}_T\left(c^T\right))\,,$$

$$\underbrace{\frac{U_N(c^T, F_N(\mathcal{H}_N\left(c^T\right)))}{U_T(c^T, F_N(\mathcal{H}_N\left(c^T\right)))}}_{\equiv \mathcal{P}_N(c^T)} = \frac{F_T'(\mathcal{H}_T\left(c^T\right))}{F_N'(\mathcal{H}_N\left(c^T\right))}\,,$$

and tradable consumption is constant and given by

$$c^T = F_T(\mathcal{H}_T\left(c^T\right)) + rb_1 - \left[\Pi_T(\mathcal{H}_T\left(c^T\right))\theta_1^{*T} + \mathcal{P}_N\left(c^T\right)\Pi_N(\mathcal{H}_N\left(c^T\right))\theta_1^{*N}\right]\,.$$

Proof. In Appendix A.9. □

Figure 5 illustrates the set for the isoelastic case. In our calibration, the elasticity of substitution between the goods equals $\gamma = 0.8$, which is consistent with empirical estimates by Ostry and Reinhart (1992) and Mendoza (1995). The range of empirical estimates in the literature for the elasticity in general lies between 0.45 and 0.85 (Stockman and Tesar, 1995; Gonzalez-Rozada et al., 2003; Lorenzo et al., 2005). All these values suggest that the goods are complements. The intensity of labor in the non-tradable sector is set equal to $\alpha^N = 0.75$ and that in the tradable sector is set equal to $\alpha^T = 0.37$. The former value is consistent with Schmitt-Grohé and Uribe (2016b) while the latter is chosen to that the aggregate labor share of output equals 0.65. The weight of the tradable good in the CES aggregator, i.e. $\omega = 0.18$, is chosen so that under flexible nominal wages, the share of tradable output in total output equals 75%, in line with data. Lastly, the rest of the parameters are determined as described in Section 3.4. Their values are given by $\sigma = 1$, $\chi = 0.015$, $\psi = 1$, and $r = 4\%$. The NFA is set equal to -60% of output. This value is kept fixed for all the international portfolios displayed in the figure.

The set of efficiency-inducing portfolios is plotted with the solid blue line. In the region below the line, the central bank finds it optimal to induce a positive labor wedge, whereas in the region above, it finds it optimal to induce a negative labor wedge. The slope of the line is approximately $\theta_1^{*N}/\theta_1^{*T} \simeq 0.45$. This implies that foreign investors must hold approximately half a unit of firms' shares in the non-tradable sector per unit of firms'

shares held in the tradable sector not to tempt the central bank to deviate from a zero labor wedge. Put equivalently, because the ratio of equity prices between the sectors is approximately $q_1^N/q_1^T \simeq 1.48$, the worth of the former shares in their equity portfolio must be approximately $2/3 \simeq 1.48 * 0.45$ the worth of the latter, or in terms of portfolio weights, the former must have approximately a weight of 40% and the latter a weight of 60%. Overall, starting from a situation in which initially the SOE owns all the equity, the set of efficiency-inducing portfolios allows equity inflows to the SOE of up to approximately $\left(q_1^T + 0.45q_1^N\right) / \left(F_T\left(h_1^T\right) + p_1^N F_N\left(h_1^N\right)\right) \simeq 6.85$ times GDP without inducing distortionary incentives for the central bank.

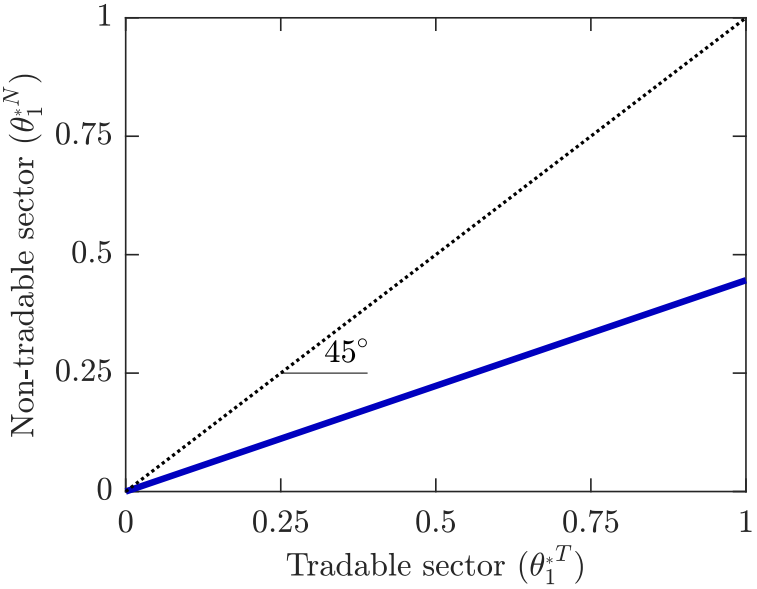


Figure 5: Set of efficiency-inducing international portfolios

Notes: The figure plots the set of international equity portfolios that eliminates the incentives of the central bank to distort the allocation when in period 1 foreign investors own equity shares.

An important takeaway from this extension is that the composition of equity flows across sectors critically matters for monetary policy. In particular, a central bank may want to depreciate or appreciate the exchange rate depending on whether foreign investors predominantly hold firms' shares in sectors with a negative or a positive relationship between firm profits and employment. Moreover, a regulatory authority that wants to reduce the temptation of the central bank to erode investors' rents in the future does not necessarily must restrict equity inflows. Rather, it must curb the inflows appropriately across sectors with opposite signs in the sensitivity of profits to employment.

6 Conclusion

In this paper, we have examined the implications of international equity flows for monetary policy in a small open economy. We develop a tractable New Keynesian framework in which foreign investors can hold domestic equity claims and the central bank sets the exchange rate under nominal wage rigidity. We show that when foreign investors own domestic equity, the central bank has an incentive to deviate from the natural allocation by appreciating the exchange rate, raising real wages, and compressing the profits that accrue to foreign investors. In effect, foreign ownership acts like a reduction in productivity: from the perspective of the domestic planner, a share of the output leaks in the form of dividends paid to foreigners, and the central bank responds by choosing lower employment than in the flexible-wage allocation.

Foreign ownership creates a novel source of time inconsistency for monetary policy. Anticipating lower returns, foreign investors purchase equity at discounted prices, which reduces domestic households' wealth and gives rise to multiple Pareto-ranked equilibria in which the bad equilibrium features larger equity inflows, depressed asset prices, and higher unemployment. We show that to mitigate this time inconsistency problem, it is optimal to manage equity flows. In particular, it is optimal to limit equity inflows ex ante, to reduce the temptation of the central bank to over-appreciate the exchange rate relative to the natural allocation ex post.

Taken together, our results highlight that the globalization of equity portfolios has important implications for monetary policy and for capital flow management. To highlight the key mechanisms at play, we have presented a parsimonious model without risk, frictions on financial intermediaries, or heterogeneity, and with a simple, temporary nominal rigidity. For future work, it would be valuable to extend the framework along these dimensions and to confront it with data, in order to further analyze the normative implications of foreign equity ownership.

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A Proofs

A.1 Proof of Lemma 1

Proof. Consider the problem (5). Together with the envelope conditions, the first-order conditions with respect to $c_t, l_t, b_{t+1}, \theta_{t+1}$ yield

$$w_t = \frac{v'(l_t)}{u'(c_t)}, \tag{A.1}$$

$$u'(c_t) = \beta(1+r)\,u'(c_{t+1}) \tag{A.2}$$

$$u'(c_t)q_t = \beta(\pi_{t+1} + q_{t+1})u'(c_{t+1}) \tag{A.3}$$

where the latter condition already incorporates that any $\theta_{t+1} \geq 0$ is optimal, a result that follows directly from (A.2) and (8). Using that $\beta(1+r) = 1$ we obtain from (A.2) that $c_t = c_{t+1} = c$ for all t. From (A.2),(A.3) and (8), it is also immediate that the portfolio between bonds and equity is undetermined.

Given $c_t = c$, equations (A.1) and (2), imply that $\ell_t = h_t = h$ and $w_t = w$ for all $t \geq 2$. We thus have that profits are constant as well and given by $\pi = F(h) - F'(h)h$. The expression (13) in the lemma then follows by iterating forward on (8) and using a no-bubble condition.

Iterating forward on the budget constraint (3) and using No-Ponzi, we obtain

$$c = w\ell + \pi\theta_2 + rb_2,$$

Combining this expression with (2) and market clearing conditions $h_t = \ell_t$ and $\theta_t + \theta_t^* = 1$ we arrive to (11). Finally, replacing c in the budget constraint (3), we obtain (12).

□

A.2 Proof of Proposition 1

Proof. Consider first a case in which property (i) holds. Then, substituting the property into (21) one gets:

$$\zeta_1(b_1, \theta_1^*) \geq \frac{\Pi'(h_1)u'(c) - \frac{1}{1+r}\left|\Pi'(h_1)\,v'\big(\mathcal{H}(c)\big) - \Pi'(\mathcal{H}(c))\,v'(h_1)\right|\varepsilon}{1 + \frac{1}{1+r}F'(\mathcal{H}(c))\,\varepsilon}\theta_1^*\,.$$

Taking the limit as $\varepsilon \to 0$ approaches zero, one obtains:

$$\zeta_1(b_1, \theta_1^*) \geq \Pi'(h_1) u'(c) \theta_1^* .$$

This implies that for any $\varepsilon > 0$ positive, but arbitrarily small, both $\zeta_1(b_1, \theta_1^*) \geq 0$ and $\zeta_1(b_1, \theta_1^*) > 0$ if $\theta_1^* > 0$ hold.

Consider now a case in which property (ii) holds. Specifically, without loss of generality, assume

$$u(c) = \frac{1}{1 - \frac{1}{\sigma}} c^{1 - \frac{1}{\sigma}}, \quad v(\ell) = \chi \frac{1}{1+\psi} \ell^{1+\psi}, \quad F(h) = h^{\alpha},$$

with $\sigma > 0$, $\chi > 0$, $\psi > 0$ and $\alpha \in (0, 1)$.

Then, from (10) it follows that

$$\mathcal{H}(c) = \left(\frac{\alpha}{\chi} c^{-1/\sigma} \right)^{\frac{1}{1+\psi-\alpha}}, \tag{A.4}$$

and from (21) one gets that

$$h_1 = z(c) \mathcal{H}(c), \tag{A.5}$$

with

$$z(c) \equiv \left[1 - (1-\alpha) \theta_1^* \frac{(1+\psi-\alpha) c}{(1+\psi-\alpha) c + \frac{1}{\sigma} \frac{1}{1+r} [1 - (1-\alpha)\theta_1^*] \alpha (\mathcal{H}(c))^{\alpha}} \right]^{\frac{1}{1+\psi-\alpha}}.$$

Note that $z(c) = 1 \, \forall c > 0$ if $\theta_1^* = 0$ and that $z(c) < 1 \, \forall c > 0$ if $\theta_1^* > 0$. This implies that $h_1 = \mathcal{H}(c)$ if $\theta_1^* = 0$ and $h_1 < \mathcal{H}(c)$ if $\theta_1^* > 0$, which in turn implies that both $\zeta_1(b_1, \theta_1^*) \geq 0$ and $\zeta_1(b_1, \theta_1^*) > 0$ if $\theta_1^* > 0$ hold.

□

A.3 Proof of Proposition 2

Proof. Applying the Implicit Function Theorem to {(18),(21)} to derive the partial derivatives of $\{h_1(b_1, \theta_1^*), c(b_1, \theta_1^*)\}$ with respect to b_1 and θ_1^*, one obtains:

$$\frac{\partial h_1}{\partial b_1} q_0 + \frac{\partial h_1}{\partial \theta_1^*} = -\frac{\mathcal{A}_{\theta_1^*}}{\Delta} \mathcal{Z}_c, \tag{A.6}$$

$$\frac{\partial c}{\partial b_1} q_0 + \frac{\partial c}{\partial \theta_1^*} = \frac{\mathcal{A}_{\theta_1^*}}{\Delta} \mathcal{Z}_{h_1}, \tag{A.7}$$

with

$$\mathcal{A}_{\theta_1^*} = -\Pi'(h_1)u'(c) - \frac{1}{1+r}\Big(- \Pi'(h_1)v'(\mathcal{H}(c)) + \Pi'(\mathcal{H}(c))v'(h_1)\Big)\mathcal{H}'(c)\,,$$

$$\mathcal{Z}_c = \frac{1}{1+r}\Big[F'(\mathcal{H}(c)) - \Pi'(\mathcal{H}(c))\theta_1^*\Big]\,\mathcal{H}'(c) - 1\,,$$

$$\mathcal{Z}_{h_1} = \frac{r}{1+r}\Big[F'(h_1) - \Pi'(h_1)\theta_1^*\Big]\,,$$

$$\begin{aligned}\mathcal{A}_c = {} & [F'(h_1) - \Pi'(h_1)\theta_1^*]u''(c) + \\ & - \frac{1}{1+r}\Bigg[[F'(h_1) - \Pi'(h_1)\theta_1^*]\,v''(\mathcal{H}(c))[\mathcal{H}'(c)]^2 - [F''(\mathcal{H}(c)) - \Pi''(\mathcal{H}(c))\theta_1^*]\,v'(h_1)[\mathcal{H}'(c)]^2 + \\ & + \Big[[F'(h_1) - \Pi'(h_1)\theta_1^*]v'(\mathcal{H}(c)) - [F'(\mathcal{H}(c)) - \Pi'(\mathcal{H}(c))\theta_1^*]v'(h_1)\Big]\,\mathcal{H}''(c)\Bigg]\,,\end{aligned}$$

$$\begin{aligned}\mathcal{A}_{h_1} = {} & [F''(h_1) - \Pi''(h_1)\theta_1^*]u'(c) - v''(h_1) + \\ & - \frac{1}{1+r}\Bigg[[F''(h_1) - \Pi''(h_1)\theta_1^*]\,v'(\mathcal{H}(c)) - [F'(\mathcal{H}(c)) - \Pi'(\mathcal{H}(c))\theta_1^*]\,v''(h_1)\Bigg]\mathcal{H}'(c)\,,\end{aligned}$$

$$\Delta = \mathcal{A}_{h_1}\mathcal{Z}_c - \mathcal{A}_c\mathcal{Z}_{h_1}\,.$$

Note that

$$\frac{\partial\zeta_1}{\partial b_1}q_0 + \frac{\partial\zeta_1}{\partial\theta_1^*} = [F''(h_1)u'(c) - v''(h_1)]\left(\frac{\partial h_1}{\partial b_1}q_0 + \frac{\partial h_1}{\partial\theta_1^*}\right) + F'(h_1)u''(c)\left(\frac{\partial c}{\partial b_1}q_0 + \frac{\partial c}{\partial\theta_1^*}\right)\,.$$

Note also that $F(h) = h^\alpha$ implies that $\Pi(h) = (1-\alpha)F(h)$.

Assume property (i) in Assumption 1 holds. Then,

$$\mathcal{A}_{\theta_1^*} < -\Pi'(h_1)u'(c) + \frac{1}{1+r}\Big| - \Pi'(h_1)v'(\mathcal{H}(c)) + \Pi'(\mathcal{H}(c))v'(h_1)\Big|\,\varepsilon\,,$$

$$\begin{aligned}\mathcal{A}_c < {} & [F'(h_1) - \Pi'(h_1)\theta_1^*]u''(c) + \\ & - \frac{1}{1+r}\Bigg[[F'(h_1) - \Pi'(h_1)\theta_1^*]\,v''(\mathcal{H}(c))[\mathcal{H}'(c)]^2 - [F''(\mathcal{H}(c)) - \Pi''(\mathcal{H}(c))\theta_1^*]\,v'(h_1)[\mathcal{H}'(c)]^2 + \\ & + \Big|F'(h_1) - \Pi'(h_1)\theta_1^*]v'(\mathcal{H}(c)) - [F'(\mathcal{H}(c)) - \Pi'(\mathcal{H}(c))\theta_1^*]v'(h_1)\Big|\,\varepsilon\Bigg]\,.\end{aligned}$$

Taking the limit as $\varepsilon \to 0$ and using that $F(\cdot)$ is isoelastic, one gets $\mathcal{A}_{\theta_1^*} < 0$ and $\mathcal{A}_c < 0$. Additionally, $F(\cdot)$ isoelastic implies that $\mathcal{Z}_c < 0$, $\mathcal{Z}_{h_1} > 0$, and $\mathcal{A}_{h_1} < 0$. Substituting these

results into (A.6) and (A.7), one obtains:

$$\frac{\partial h_1}{\partial b_1} q_0 + \frac{\partial h_1}{\partial \theta_1^*} < 0\,, \quad \frac{\partial c}{\partial b_1} q_0 + \frac{\partial c}{\partial \theta_1^*} < 0\,, \quad \frac{\partial \zeta_1}{\partial b_1} q_0 + \frac{\partial \zeta_1}{\partial \theta_1^*} > 0\,. \tag{A.8}$$

Assume now property (ii) holds. Then, (A.4) implies that $\mathcal{H}''(c) > 0$ and (A.5) implies that $-\Pi'(h_1)v'(\mathcal{H}(c)) + \Pi'(\mathcal{H}(c))v'(h_1) < 0$ and $[F'(h_1) - \Pi'(h_1)\theta_1^*]v'(\mathcal{H}(c)) - [F'(\mathcal{H}(c)) - \Pi'(\mathcal{H}(c))\theta_1^*]v'(h_1) > 0$. These results guarantee that $\mathcal{A}_{\theta_1^*} < 0$ and $\mathcal{A}_c < 0$. $F(\cdot)$ isoelastic implies that $\mathcal{Z}_c < 0$, $\mathcal{Z}_{h_1} > 0$, and $\mathcal{A}_{h_1} < 0$. Thus, $\Delta > 0$, and

$$\frac{\partial h_1}{\partial b_1} q_0 + \frac{\partial h_1}{\partial \theta_1^*} < 0\,, \quad \frac{\partial c}{\partial b_1} q_0 + \frac{\partial c}{\partial \theta_1^*} < 0\,, \quad \frac{\partial \zeta_1}{\partial b_1} q_0 + \frac{\partial \zeta_1}{\partial \theta_1^*} > 0\,. \tag{A.9}$$

□

A.4 Proof of Lemma 2

Proof. First, note that for $t \geq 2$, the economy is in a steady-state equilibrium. Therefore, we have labor, wages, profits, and prices for $t \geq 2$, as characterized in Lemma 1.

At $t = 1$, households solve

$$V_1(b_1, \theta_1) = \max_{c_1, b_2, \theta_2 \geq 0} \left\{ u(c_1) - v(\ell_1) + \frac{1}{1+r} V(b_2, \theta_2) \right\} \tag{A.10}$$

subject to:

$$c_1 + b_2 + q\,\theta_2 = \frac{\bar{W}_1}{E_1} \ell_1 + (1+r) b_1 + (\pi_1 + q)\,\theta_1\,, \tag{A.11}$$

where we have used that $q_1 = q$ and that the value function is constant for $t \geq 2$, a result that follows because in a steady-state equilibrium, wages, profits, and equity prices are constant.

The value for $t \geq 2$ is given by

$$V(b_2, \theta_2) = \frac{1+r}{r} \left[u(c) - v(\ell) \right]\,,$$

with $c = w\ell + \pi\theta_2 + r b_2$ and $w = v'(\ell)/u'(c)$.

From optimality with respect to b_2 and using the envelope condition, we obtain $c_1 = c$. Using that consumption, wages, and hours are constant and that q satisfies (8), we obtain

by iterating forward on the budget constraint that

$$c = \frac{r}{1+r}\left(\frac{\bar{W}_1}{E_1}\ell_1 + \pi_1\theta_1\right) + \frac{1}{1+r}\left(w\ell + \pi\theta_1\right) + rb_1 \ .$$

Combining this expression with (2), market clearing conditions $\ell_t = h_t$ and $\theta_t + \theta_t^* = 1$, and mapping $h = \mathcal{H}(c)$, we arrive to (15). Finally, replacing c_1 together with (2), $\ell_t = h_t$, $\theta_t + \theta_t^* = 1$, and $h = \mathcal{H}(c)$ in the budget constraint (A.11), we obtain (16).

□

A.5 Proof of Lemma 3

Proof. At $t = 0$, households solve

$$\begin{aligned} V_0\left(b_0, \theta_0\right) = \max_{c_0, b_1, \theta_1 \geq 0} & \left\{ u\left(c_0\right) - v\left(\ell_0\right) + \frac{1}{1+r} V_1\left(b_1, \theta_1\right) \right\} \\ & \text{subject to:} \\ & c_0 + b_1 + q_0\theta_1 = w_0\ell_0 + (1+r)b_0 + \left(\pi_0 + q_0\right)\theta_0 \ , \end{aligned} \tag{A.12}$$

where $q_0 = \frac{1}{1+r}(\pi_1 + q)$ already uses $q_1 = q$ and where recall that for $t \geq 2$ the economy is in a steady-state equilibrium. From problem (A.10) it follows that

$$\frac{\partial V_1}{\partial b_1} = (1+r)\,\lambda_1\left(b_1, \theta_1\right) \ \text{ and } \ \frac{\partial V_1}{\partial \theta_1} = (1+r)\,\lambda_1\left(b_1, \theta_1\right) q_0 \, .$$

These conditions together with optimality at $t = 0$ imply that $c_0 = c$. Substituting this result, optimality condition (2), market clearing conditions $\ell_t = h_t$ and $\theta_t + \theta_t^* = 1$, and mapping $h = \mathcal{H}(c)$ into (A.12), we arrive to (23).

□

A.6 Proof of Proposition 3

Proof. The proof has two parts. For the first part, observe first that (6) implies that consumption is constant, and in addition, we have $h_t = h = \mathcal{H}(c)$ for all $t \neq 1$. Using that

$$q_0 = \frac{1}{1+r}\left[\Pi\left(h_1\right) + \frac{1}{r}\Pi\left(\mathcal{H}(c_0)\right)\right].$$

and iterating forward on the budget constraint, and using the transversality condition we

have that c solves

$$\left(\frac{1+r}{r}\right)c = F(\mathcal{H}(c)) + (1+r)b_0 - q_0\theta_0^* + \frac{F(h_1)}{1+r} + \frac{F\big(\mathcal{H}(c)\big)}{(1+r)^2} \tag{A.13}$$

where h_1 is given by

$$\frac{\overline{W}}{E_1} = F'(h_1)\,,$$

The fact that $\mathcal{H}'(\cdot) < 0$ implies the right-hand side of (A.13) is decreasing in c. Because the left-hand side of (A.13) is increasing in c, the level of c must be unique, and so is h.

For the second part, we use that c satisfies (A.13) for any period-1 exchange rate policy and using that $\theta_0^* = 0$, we can express the central bank problem as

$$V_0(b_0, 0) = \max_{c, b_1, \theta_1^*, h_1} \left\{ u(c) - v(\mathcal{H}(c)) + \frac{u(c) - v\left(h_1\right)}{1+r} + \frac{1}{(1+r)^2}\frac{u(c) - v\left(\mathcal{H}(c)\right)}{r} \right\},$$

subject to:

$$\left(\frac{1+r}{r}\right)c \leq F(\mathcal{H}(c)) + (1+r)b_0 + \frac{F(h_1)}{1+r} + \frac{F\big(\mathcal{H}(c)\big)}{(1+r)^2}$$

Optimality with respect to h_1 and c result in

$$v'(h_1) = u'(c)F'(h_1)$$

which implies a zero labor wedge in period 1. □

A.7 Proof of Proposition 4

Proof. Let $\mathcal{L}_0$ denote the Lagrangian associated with problem (24), and let λ_0 and μ_0 respectively denote the Lagrange multipliers associated with the two constraints.

$$\begin{aligned}\mathcal{L}_0 =& u(c_0) - v(\mathcal{H}(c_0)) + \frac{1}{1+r}V_1(b_1, \theta_1^*) + \\ &+ \lambda_0\Big[F(\mathcal{H}(c_0)) - \Pi(\mathcal{H}(c_0))\theta_0^* + q_0(b_1, \theta_1^*)\left(\theta_1^* - \theta_0^*\right) + (1+r)b_0 - b_1 - c_0\Big] + \\ &+ \mu_0\Big[c(b_1, \theta_1^*) - c_0\Big]\,.\end{aligned}$$

The first-order derivatives of the Lagrangian are:

$$\frac{\partial \mathcal{L}_0}{\partial c_0} = u'(c_0) - v'\left(\mathcal{H}(c_0)\right)\mathcal{H}'(c_0) + \lambda_0\left[[F'(\mathcal{H}(c_0)) - \Pi'(\mathcal{H}(c_0))\theta_0^*]\mathcal{H}'(c_0) - 1\right] - \mu_0\,,$$

$$\frac{\partial \mathcal{L}_0}{\partial b_1} = \frac{1}{1+r}\frac{\partial V_1}{\partial b_1} + \lambda_0\left[\frac{\partial q_0}{\partial b_1}\left(\theta_1^* - \theta_0^*\right) - 1\right] + \mu_0\frac{\partial c}{\partial b_1}\,,$$

$$\frac{\partial \mathcal{L}_0}{\partial \theta_1^*} = \frac{1}{1+r}\frac{\partial V_1}{\partial \theta_1^*} + \lambda_0\left[\frac{\partial q_0}{\partial \theta_1^*}\left(\theta_1^* - \theta_0^*\right) + q_0(b_1,\theta_1)\right] + \mu_0\frac{\partial c}{\partial \theta_1^*}\,,$$

$$\frac{\partial \mathcal{L}_0}{\partial \lambda_0} = F(\mathcal{H}(c_0)) - \Pi(\mathcal{H}(c_0))\theta_0^* + q_0(b_1,\theta_1^*)\left(\theta_1^* - \theta_0^*\right) + (1+r)b_0 - b_1 - c_0\,,$$

$$\frac{\partial \mathcal{L}_0}{\partial \mu_0} = c(b_1,\theta_1^*) - c_0\,,$$

where note that from problem (17):

$$\frac{\partial V_1}{\partial b_1} = (1+r)\,\lambda_1(b_1,\theta_1^*)\,, \tag{A.14}$$

$$\frac{\partial V_1}{\partial \theta_1^*} = -(1+r)\,\lambda_1(b_1,\theta_1^*)\,\frac{1}{1+r}\left[\Pi(h_1(b_1,\theta_1^*)) + \frac{1}{r}\Pi(\mathcal{H}(c(b_1,\theta_1^*)))\right]. \tag{A.15}$$

Using that q_0 is determined by (22), and substituting (A.14) and (A.15) into first-order conditions $\partial\mathcal{L}_0/\partial b_1 = 0$ and $\partial\mathcal{L}_0/\partial\theta_1^* = 0$, we obtain

$$0 = \lambda_1(b_1,\theta_1^*) + \lambda_0\left[\frac{\partial q_0}{\partial b_1}\left(\theta_1^* - \theta_0^*\right) - 1\right] + \mu_0\frac{\partial c}{\partial b_1}\,, \tag{A.16}$$

$$0 = -\lambda_1(b_1,\theta_1^*)q_0(b_1,\theta_1^*) + \lambda_0\left[\frac{\partial q_0}{\partial \theta_1^*}\left(\theta_1^* - \theta_0^*\right) + q_0(b_1,\theta_1^*)\right] + \mu_0\frac{\partial c}{\partial \theta_1^*}\,,$$

and summing up these two conditions—after multiplying both sides of the first one by $q_0(b_1,\theta_1^*)$—one obtains:

$$0 = \lambda_0\left[\frac{\partial q_0}{\partial b_1}q_0(b_1,\theta_1^*) + \frac{\partial q_0}{\partial \theta_1^*}\right]\left(\theta_1^* - \theta_0^*\right) + \mu_0\left[\frac{\partial c}{\partial b_1}q_0(b_1,\theta_1^*) + \frac{\partial c}{\partial \theta_1^*}\right]. \tag{A.17}$$

Note that

$$\lambda_0 > 0\,. \tag{A.18}$$

This holds because following an increase in b_0, it is possible to increase both c_0 and b_1 in a manner that $c_0 = c(b_1,\theta_1^*)$ holds and θ_1^* remains unchanged.

Results (A.8) and (A.9) derived in Proof A.3 to Proposition 2 imply that under either stated condition in the proposition,

$$\frac{\partial c}{\partial b_1} q_0(b_1, \theta_1^*) + \frac{\partial c}{\partial \theta_1^*} < 0\,. \tag{A.19}$$

From pricing condition (22) it follows that

$$\begin{aligned}\frac{\partial q_0}{\partial b_1} q_0(b_1, \theta_1^*) + \frac{\partial q_0}{\partial \theta_1^*} =& \frac{1}{1+r}\left[\Pi'(h_1(b_1, \theta_1^*))\left[\frac{\partial h_1}{\partial b_1} q_0(b_1, \theta_1^*) + \frac{\partial h_1}{\partial \theta_1^*}\right] + \right. \\ &\left. + \frac{1}{r}\,\Pi'(\mathcal{H}(c(b_1, \theta_1^*)))\,\mathcal{H}'(c(b_1, \theta_1^*))\left[\frac{\partial c}{\partial b_1} q_0(b_1, \theta_1^*) + \frac{\partial c}{\partial \theta_1^*}\right]\right] = \\ =& \frac{1}{1+r}\frac{\mathcal{A}_{\theta_1^*}}{\Delta}\left[-\Pi'(h_1(b_1, \theta_1^*))\,\mathcal{Z}_c + \frac{1}{r}\,\Pi'(\mathcal{H}(c(b_1, \theta_1^*)))\,\mathcal{H}'(c(b_1, \theta_1^*))\mathcal{Z}_{h_1}\right]\end{aligned}$$

where the second equality follows from substituting (A.6) and (A.7) into the first one. As shown in Proof A.3 to Proposition 2, under either stated condition in the proposition, $\mathcal{A}_{\theta_1^*} < 0$ and $\Delta > 0$. Moreover, as corollary from Proof A.2 and Proof A.3, the bracketed term is positive. These results then imply that

$$\frac{\partial q_0}{\partial b_1} q_0(b_1, \theta_1^*) + \frac{\partial q_0}{\partial \theta_1^*} < 0\,. \tag{A.20}$$

Consider first the case with $\theta_0^* > 0$ and conjecture that $\mu_0 \leq 0$. Then, inequalities (A.18), (A.19) and (A.20) together with condition (A.17) imply that $\theta_1^* \geq \theta_0^*$. Moreover, first-order condition $\partial \mathcal{L}_0 / \partial c_0 = 0$ implies that

$$\lambda_0 \geq u'(c_0) - v'\left(\mathcal{H}(c_0)\right)\mathcal{H}'(c_0) + \lambda_0[F'(\mathcal{H}(c_0)) - \Pi'(\mathcal{H}(c_0))\theta_0^*]\mathcal{H}'(c_0)\,, \tag{A.21}$$

and condition (A.16) together with $\theta_1^* \geq \theta_0^*$ and partial derivatives $\partial c / \partial b_1 > 0$ and $\partial q_0 / \partial b_1 < 0$—these derivatives directly follow from applying the Implicit Function Theorem to {(18),(21)}—imply that

$$\lambda_1(b_1, \theta_1^*) \geq \lambda_0\,. \tag{A.22}$$

Inequalities (A.21) and (A.22) together with conditions (20) and $v'(\mathcal{H}(c)) = F'(\mathcal{H}(c))u'(c)$

and functional form $F(\cdot) = h^{\alpha}$ imply that

$$\frac{u'(c_0)}{u'(c(b_1,\theta_1^*))} \leq \frac{1-[F'(\mathcal{H}(c_0))-\Pi'(\mathcal{H}(c_0))\theta_0^*]\mathcal{H}'(c_0)}{1-\frac{1}{1+r}\Big[F'\big(\mathcal{H}(c(b_1,\theta_1^*))\big)-\Pi'\big(\mathcal{H}(c(b_1,\theta_1^*))\big)\theta_1^*\Big]\mathcal{H}'(c(b_1,\theta_1^*))}\times$$
$$\times \frac{1-\frac{1}{1+r}F'\big(\mathcal{H}(c(b_1,\theta_1^*))\big)\mathcal{H}'(c(b_1,\theta_1^*))}{1-F'\left(\mathcal{H}(c_0)\right)\mathcal{H}'(c_0)}.$$

Under either stated condition in the proposition, the right-hand side on the above inequality is below 1. Thus, $c_0 > c(b_1,\theta_1^*)$, but this result is not consistent with the restriction that $c_0 = c(b_1,\theta_1^*)$. This contradiction then implies that $\mu_0 > 0$, which together with inequalities (A.18), (A.19) and (A.20) and condition (A.17) in turn imply that $\theta_1^* < \theta_0^*$ is optimal when $\theta_0^* > 0$.

Consider now the case with $\theta_0^* = 0$. Note that $\theta_1^* = \theta_0^* = 0$, $b_1 = b_0$, $c_0 = c(b_0,\theta_0^*)$ and $\mu_0 = 0$ are consistent with the first-order conditions.

□

A.8 Proof of Proposition 5

Proof. Let express the Lagrangian of the problem of the regulatory authority as follows

$$\begin{aligned}\mathcal{L}_0 =& u(c_0) - v(\mathcal{H}(c_0)) + \frac{1}{1+r}V_1(b_1,\theta_1^*) + \\ &+ \lambda_0\Big[F(\mathcal{H}(c_0)) - \Pi(\mathcal{H}(c_0))\theta_0^* + q_0(b_1,\theta_1^*)\left(\theta_1^*-\theta_0^*\right) + (1+r)b_0 - b_1 - c_0\Big] + \\ &+ \mu_0\Big[c(b_1,\theta_1^*) - c_0\Big] \\ &+ \kappa_0\left(\underline{b}+b_1\right) \\ &+ \xi_0\left(c_0 - c(b_1,\theta_1^*)\right)\left(\underline{b}+b_1\right).\end{aligned}$$

Note that Lagrange multiplier $\xi_0 = 0$ equals zero. This is because if both $c_0 < c(b_1,\theta_1^*)$ and $b_1 > -\underline{b}$ hold, then it is possible to marginally reduce b_1 in a manner that restrictions {(28),(29),(30)} hold, the gap between the consumption falls, and the value attained by objective (27) increases. Setting $\xi_0 = 0$ and proceeding in the same manner as in the proof to Proposition 4, one gets:

$$0 = \lambda_0 \underbrace{\left[\frac{\partial q_0}{\partial b_1}q_0(b_1,\theta_1^*) + \frac{\partial q_0}{\partial \theta_1^*}\right]}_{<0}\left(\theta_1^*-\theta_0^*\right) + \mu_0 \underbrace{\left[\frac{\partial c}{\partial b_1}q_0(b_1,\theta_1^*) + \frac{\partial c}{\partial \theta_1^*}\right]}_{<0} + q_0(b_1,\theta_1^*)\,\kappa_0\,, \quad \text{(A.23)}$$

where $\lambda_0 > 0$ and $\mu_0 \geq 0$ hold and where the inequalities on the right-hand side follow from (A.19) and (A.20). Note that $\kappa_0 \geq 0$ because a higher $\underline{b}$ relaxes the borrowing constraint. If $c_0 = c(b_1, \theta_1^*)$ is not optimal, then $\mu_0 = 0$ must hold. Moreover, $\kappa_0 > 0$ must hold as well. Substituting these results into (A.23), one gets that $\theta_1^* > \theta_0^*$ must be optimal.

□

A.9 Proof of Proposition 6

Proof. The proof proceeds in three steps as follows. First, we derive the continuation equilibrium for any period $t \geq 2$. Then, we solve the continuation equilibrium at period 1. Lastly, we lay out the problem of the central bank in period 1, and derive its FOCs.

Step 1. Given an initial portfolio $\left(b_2, \theta_2^{*T}, \theta_2^{*N}\right)$, the continuation equilibrium for any $t \geq 2$ features constant quantities and prices over time. Quantities $\{h^T, h^N, c^T\}$ are jointly determined by

$$\frac{F_T'(h^T)}{F_N'(h^N)} = \frac{U_N(c^T, F_N(h^N))}{U_T(c^T, F_N(h^N))}\,, \tag{A.24}$$

$$F_T'(h^T) = \frac{v'(h^T + h^N)}{U_T(c^T, F_N(h^N))}\,, \tag{A.25}$$

$$c^T = F_T(h^T) + rb_2 - \left[\Pi_T(h^T)\theta_2^{*T} + p^N \Pi_N(h^N)\theta_2^{*N}\right]\,, \tag{A.26}$$

the sequence of portfolios $\left\{b_{t+1}, \theta_{t+1}^{*T}, \theta_{t+1}^{*N}\right\}_{t\geq 2}$ satisfies

$$b_{t+1} - \left(q^T\,\theta_{t+1}^{*T} + q^N\,\theta_{t+1}^{*N}\right) = b_2 - \left(q^T\,\theta_2^{*T} + q^N\,\theta_2^{*N}\right) \tag{A.27}$$

equity prices $\{q^T, q^N\}$ are determined by

$$q^T = \frac{1}{r}\,\underbrace{\left[F_T(h^T) - F_T'(h^T)h^T\right]}_{\equiv \Pi_T(h^T)}\,, \tag{A.28}$$

$$q^N = \frac{1}{r}\,p^N\,\underbrace{\left[F_N(h^N) - F_N'(h^N)h^N\right]}_{\equiv \Pi_N(h^N)}\,, \tag{A.29}$$

and price of non-tradable good p^N is given by

$$p^N = \frac{U_N(c^T, F_N(h^N))}{U_T(c^T, F_N(h^N))}\,. \tag{A.30}$$

Note that condition (A.24) follows from combining household optimality with firm optimality. Define mappings $\mathcal{H}_T(\cdot)$ and $\mathcal{H}_N(\cdot)$ as the solution to $h^T = \mathcal{H}_T\left(c^T\right)$ and $h^N = \mathcal{H}_N\left(c^T\right)$ in $\{$(A.26),(A.25)$\}$ for any given c^T. Additionally define mapping $\mathcal{P}_N\left(c^T\right)$ as the solution to $p^N = \mathcal{P}_N\left(c^T\right)$ in (A.30) as function of c^T. These mappings will be useful for characterizing the continuation equilibrium at period 1.

Step 2. Given an initial portfolio $\left(b_1, \theta_1^{*T}, \theta_1^{*N}\right)$, a rigid wage $\overline{W}_1$, an exchange rate E_1, the equilibrium for $t \geq 1$ is such that

i) h_1^T and h_1^N are respectively given by

$$\frac{\overline{W}_1}{E_1} = F_T'\left(h_1^T\right) \text{ and } \frac{\overline{W}_1}{E_1} = \underbrace{\frac{U_N(c^T, F_N(h_1^N))}{U_T(c^T, F_N(h_1^N))}}_{=p_1^N} F_N'\left(h_1^N\right) \tag{A.31}$$

ii) $h_t^j = \mathcal{H}_j(c^T)$ for $t > 1$, and $c_t^T = c^T$ for $t \geq 1$, with c^T given by

$$\begin{aligned} c^T = &\frac{r}{1+r}\left[F_T(h_1^T) + rb_1 - \left[\Pi_T\left(h_1^T\right)\theta_1^{*T} + p_1^N \Pi_N(h_1^N)\theta_1^{*N}\right]\right] + \\ &\frac{1}{1+r}\left[F_T\left(\mathcal{H}_T\left(c^T\right)\right) + rb_1 - \left[\Pi_T\left(\mathcal{H}_T\left(c^T\right)\right)\theta_1^{*T} + \mathcal{P}_N\left(c^T\right)\Pi_N\left(\mathcal{H}_N\left(c^T\right)\right)\theta_1^{*N}\right]\right] \end{aligned}$$

iii) $q_t^j = q^j$ for $j \in \{N, T\}$, with q^T and q^N respectively given by (A.28) and (A.29), and the sequence of portfolios are such that

$$\begin{aligned} b_2 - \left(q^T\theta_2^{*T} + q^N\theta_2^{*N}\right) = b_1 - \left(q^T\theta_1^{*T} + q^N\theta_1^{*N}\right) + \\ \frac{F_T(h_1^T) - \left[\Pi_T\left(h_1^T\right)\theta_1^{*T} + p_1^N\Pi_N(h_1^N)\theta_1^{*N}\right]}{1+r} - \\ \frac{F_T\left(\mathcal{H}_T\left(c^T\right)\right) - \left[\Pi_T\left(\mathcal{H}_T\left(c^T\right)\right)\theta_1^{*T} + \mathcal{P}_N\left(c^T\right)\Pi_N\left(\mathcal{H}_N\left(c^T\right)\right)\theta_1^{*N}\right]}{1+r} \end{aligned}$$

and $\left\{b_{t+1}, \theta_{t+1}^*\right\}_{t\geq 2}$ satisfy (A.27).

Combining the two equations in (A.31) one gets

$$\frac{F_T'(h_1^T)}{F_N'(h_1^N)} = \frac{U_N(c^T, F_N(h_1^N))}{U_T(c^T, F_N(h_1^N))}$$

Define mapping $\tilde{\mathcal{H}}(\cdot)$ as the solution to $h_1^N = \tilde{\mathcal{H}}\left(h_1^T, c^T\right)$ in the above equation as a

function of $\left(h_1^T, c^T\right)$. Additionally define

$$\tilde{\mathcal{P}}\left(h_1^T, c^T\right) \equiv \frac{U_N\left(c^T, F_N\left(\tilde{\mathcal{H}}\left(h_1^T, c^T\right)\right)\right)}{U_T\left(c^T, F_N\left(\tilde{\mathcal{H}}\left(h_1^T, c^T\right)\right)\right)}.$$

Note that $\tilde{\mathcal{H}}\left(\mathcal{H}_T\left(c^T\right), c^T\right) = \mathcal{H}_N\left(c^T\right)$ and $\tilde{\mathcal{P}}\left(\mathcal{H}_T\left(c^T\right), c^T\right) = \mathcal{P}_N\left(c^T\right)$. These mappings will be useful for laying out the problem of the central bank in period 1.

Step 3. The problem of the central bank in period 1 can be expressed as

$$\max_{c^T, h_1^T}\Bigg\{U\left(c^T, F_N\left(\tilde{\mathcal{H}}\left(h_1^T, c^T\right)\right)\right) - v\left(h_1^T + \tilde{\mathcal{H}}\left(h_1^T, c^T\right)\right) + \\ + \frac{1}{r}\left[U\left(c^T, F_N\left(\mathcal{H}_N\left(c^T\right)\right)\right) - v\left(\mathcal{H}_T\left(c^T\right) + \mathcal{H}_N\left(c^T\right)\right)\right]\Bigg\},$$

subject to

$$\frac{1+r}{r}c^T = F_T(h_1^T) + rb_1 - \left[\Pi_T\left(h_1^T\right)\theta_1^{*T} + \tilde{\mathcal{P}}\left(h_1^T, c^T\right)\,\Pi_N(\tilde{\mathcal{H}}\left(h_1^T, c^T\right))\theta_1^{*N}\right] + \\ \frac{1}{r}\Big[F_T\big(\mathcal{H}_T\left(c^T\right)\big) + rb_1 - \left[\Pi_T\left(\mathcal{H}_T\left(c^T\right)\right)\theta_1^{*T} + \mathcal{P}_N\left(c^T\right)\,\Pi_N\left(\mathcal{H}_N\left(c^T\right)\right)\theta_1^{*N}\right]\Big]$$

Let λ denote the Lagrange multiplier associated with the constraint (written as the sum of the terms in the right-hand side minus the term in the left-hand side equal to zero). The FOC with respect to h_1^T is

$$0 = \left[U_N F_N' - v'\right]\tilde{\mathcal{H}}_1 - v' + \lambda\left[F_T' - \Pi_T'\,\theta_1^{*T} - \left[\tilde{\mathcal{P}}_1\,\Pi_N + \tilde{\mathcal{P}}\,\Pi_N'\,\tilde{\mathcal{H}}_1\right]\theta_1^{*N}\right] \quad \text{(A.32)}$$

and the FOC with respect to c^T is

$$0 = U_T + \left[U_N\,F_N' - v'\right]\tilde{\mathcal{H}}_1 + \frac{1}{r}\left[U_T - v'\,\mathcal{H}_T'\right] + \quad \text{(A.33)} \\ + \lambda\Bigg[-\frac{1+r}{r} - \left[\tilde{\mathcal{P}}_2\,\Pi_N + \tilde{\mathcal{P}}\,\Pi_N'\,\tilde{\mathcal{H}}_2\right]\theta_1^{*N} + \\ + \frac{1}{r}\left[F_T'\,\mathcal{H}_T' - \Pi_T'\,\mathcal{H}_T'\,\theta_1^{*T} - \left[\mathcal{P}_N'\,\Pi_N + \mathcal{P}_N\,\Pi_N'\,\mathcal{H}_N'\right]\theta_1^{*N}\right]\Bigg]$$

where we omit the arguments in the functions to reduce clutter and where note that in the latter FOC result $\frac{1}{r}\left[U_N F_N' - v'\right]\mathcal{H}_N' = 0$ has already been substituted away. This result

follows from (A.24) and (A.25).

A zero labor wedge is equivalent to $h_1^T = \mathcal{H}_T$. In conditions (A.32) and (A.33), under a zero labor wedge, the following holds:

$$
\begin{aligned}
&i)\, \tilde{\mathcal{H}} = \mathcal{H}_N \,; \\
&ii)\, \left[U_N\, F'_N - v'\right] \tilde{\mathcal{H}}_1 = 0 \,; \\
&iii)\, U_T\, F'_T - v' = 0 \,; \\
&iv)\, \tilde{\mathcal{P}} = \mathcal{P}_N \,; \\
&v)\, \tilde{\mathcal{P}}_1 \mathcal{H}'_T + \tilde{\mathcal{P}}_2 = \mathcal{P}'_N \,; \\
&vi)\, \tilde{\mathcal{H}}_1 \mathcal{H}'_T + \tilde{\mathcal{H}}_2 = \mathcal{H}'_N \,.
\end{aligned}
$$

Substituting i)-vi) into (A.32) and (A.33), one obtains that the following must hold for the conditions to be consistent with a zero labor wedge:

$$
\begin{aligned}
\lambda &= U_T \,, \\
0 &= \Pi'_T \mathcal{H}'_T {\theta_1^*}^T + \left[\mathcal{P}'_N\, \Pi_N + \mathcal{P}_N\, \Pi'_N\, \mathcal{H}'_N\right] {\theta_1^*}^N \,.
\end{aligned}
$$

where the last condition is equivalent to condition (36) in the paper. □