Learning Segmentation and Economic Development*

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Abstract

How does learning from others shape economic development? We answer this question with a quantitative model of multiple labor markets and modern and traditional sectors, capturing key features of dual economies in developing countries. Workers sort across sectors and labor markets and accumulate human capital by learning from peers. Learning opportunities are sector-specific and depend on peer composition within the labor market. The model makes one main prediction: more skilled workers sort into the modern sector and, as a consequence, workers in this sector experience faster wage growth because they are exposed to higher-skilled peers. We test this prediction using longitudinal worker data from Chile and find empirical support for it. Finally, we estimate the model and use it to quantify how income per capita and welfare would change if traditional-sector workers had access to the same learning opportunities as modern-sector workers.

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1 Introduction

Human interactions are a crucial source of learning. People learn from others in the workplace, in the places they live, or from individuals in similar occupations who perform similar tasks. Lower-income economies tend to be characterized by dual economies, in which workers are split between modern and traditional sectors that coexist despite large differences in scale. This idea, central to the study of economic development, goes back to Lewis (1954) and is often reflected in agricultural versus non-agricultural productivity gaps, rural versus urban income differences, and the coexistence of many small firms alongside relatively few large firms within the same industries.[1] In turn, such dual economies likely shape who interacts with whom, segmenting the learning opportunities available to workers. In this paper, we study how these segmented learning markets influence workers' skill formation and, through this channel, economic development. How do segmented learning markets affect workers' learning? How does workers' human capital influence their labor market choices? What are the aggregate implications for income and welfare?

We answer these questions using a quantitative model of learning from peers in a dual economy with segmented interactions, together with panel data on workers from Chile. The model features multiple labor markets and include modern and traditional sectors. Workers differ in skill, choose where to work, and accumulate human capital by learning from peers in their labor market. Importantly, learning is segmented within labor markets, with a worker's learning depending on the skill composition of peers in their own sector. The model predicts that higher-skill workers select into the modern sector. It also predicts a dynamic modern-sector wage premium, with modern-sector workers experiencing faster wage growth when learning is sufficiently segmented. We then test these predictions using the Chilean panel, which allows us to track workers across labor markets and across the modern and traditional sectors. Consistent with the model, modern-sector workers earn higher wages and experience faster wage growth. We provide suggestive evidence that this dynamic premium reflects learning from peers: exposure to higher-paid, more skilled peers predicts higher future wage growth, and modern-sector workers are more exposed to such peers. Finally, we estimate the model and use it to quantify how much of the low income levels in developing countries stem from lost learning opportunities for workers in the traditional sector.

The model is designed to capture key features of dual economies in developing countries. Labor markets differ in productivity and include a modern and a traditional sector. Modern-sector firms operate a scalable constant-returns technology but face hiring frictions in the form of payroll taxes and worker registration costs. Traditional-sector firms avoid these regulations, but operate a decreasing-returns technology, capturing small-scale production or subsistence self-employment. Workers differ in skill, choose where to work, and accumulate human capital over time. In this sense, our framework relates to recent quantitative dual-economy models with human capital accumulation, such as Ma et al. (2024) and Amaral and Rivera-Padilla (2024). Our main departure is that we incorporate a local learning externality: workers learn from peers in their sector, and they do not internalize that their location decision affects the learning opportunities of others by changing the peer skill composition within each sector and labor market. This mechanism

[1] Gollin (2014) offers a recent review of the Lewis model and the dual economy concept.

connects our framework to classic work on knowledge spillovers and social learning (Lucas, 2004; Lucas and Moll, 2014) and to empirical and quantitative studies of learning from coworkers and local interactions (Jarosch et al., 2021; Herkenhoff et al., 2024; Crews, 2023; Lhuillier, 2024). This externality drives the different patterns of skill accumulation across sectors.

We characterize the steady-state equilibrium of our heterogeneous-worker framework and highlight two key predictions. In equilibrium, workers sort both across labor markets and within markets across sectors. Across markets, the invariant distribution exhibits positive assortative matching, as higher-skill workers concentrate in more productive labor markets due to the complementarity between worker skill and market productivity in production. Within each labor market, higher-skill workers select into the modern sector, where the scalable constant-returns technology makes high productivity particularly valuable despite regulatory costs. In contrast, lower-skill workers find it optimal to operate in the traditional sector, where decreasing returns are less binding at small scale and regulations are avoided. In addition to this sorting pattern, the model generates a dynamic modern-sector wage premium: when learning is sufficiently segmented, modern-sector workers are more exposed to higher-skill peers, accumulate human capital faster, and therefore experience steeper wage growth than traditional-sector workers, even within the same labor market.

We then examine the model's main predictions using panel data on workers from Chile. As an important developing economy, Chile features large and segmented sectors that map naturally onto the model's dual-economy structure. We use the Encuesta de Protección Social (EPS), a nationally representative longitudinal survey that tracks about 17,000 workers over 2002–2016. We classify workers into modern and traditional sectors based on their formality status. Informal workers lack formal labor contracts and are not subject to payroll taxes, consistent with our modeling of the traditional sector. In addition, they face informality costs that rise with output, such as expected fines or avoidance expenses (Perry et al., 2007; Ulyssea, 2020b), which imply diminishing returns to production in the traditional sector. Accordingly, we classify modern-sector workers as those affiliated with pension funds and holding a labor contract. The EPS also provides complete formal and informal job histories dating back to 1980, which allows us to follow workers across sectors over their careers. With these data in hand, we test the model's three main predictions.

First, we show that higher-skilled workers select into the modern sector. Since skill is not directly observed in the data, we use workers' wage outcomes as a proxy for skill, and interpret persistent wage differences across workers as reflecting differences in ability. Controlling for observable characteristics, modern-sector workers in Chile earn 17% more than their traditional-sector counterparts. However, once we account for worker fixed effects, we find that more than 40% of this wage premium is explained by unobserved worker heterogeneity. Worker fixed effects are positively correlated with modern-sector participation, indicating that modern-sector workers have higher unobserved average skill.

Second, we provide evidence for the model's dynamic modern-sector wage premium. We analyze individual wage trajectories and examine how wage growth relates to modern-sector participation, controlling for worker demographics, industry, occupation, and firm size. We find that modern-sector workers experience larger wage increases over time than traditional-sector workers. After

one year, modern-sector workers earn 2.7% more than their traditional-sector counterparts, with the effect peaking at 8% after four years. Using wage trajectories of switchers, workers who change sectors, we show that this dynamic modern-sector premium persists even after accounting for worker fixed effects. Workers who transition from a traditional-sector to a modern-sector job experience a wage increase that is 4% higher over four years than that of workers who remain in the traditional sector. This dynamic wage premium helps explain why modern-sector workers' wages rise over the life cycle while traditional-sector workers' wages stagnate.[2]

Third, we find that learning from better-paid workers in the same labor market contributes to the dynamic modern-sector wage premium. Guided by our framework, we use information on workers' residence, industry, and occupation to define the labor markets in which workers interact. Then, in the spirit of Jarosch et al. (2021) and Herkenhoff et al. (2024), we assess whether exposure to higher-earning peers predicts higher future wage growth. We find that such exposure in the same labor market significantly increases future wage growth, and that this effect holds across industries, occupations, and firm-size brackets. Moreover, consistent with our sorting results, modern-sector workers tend to be more exposed to these higher-earning peers. Taken together, these findings suggest that peer learning, and the concentration of high-skilled workers in the modern sector, are key drivers of the dynamic modern-sector wage premium.

In the final section, we estimate the model and use it to quantify the aggregate importance of learning for explaining low income levels in developing countries. We estimate the key parameters of the model using a Simulated Method of Moments approach. We infer the parameters governing workers' learning by matching transition probabilities across wage-distribution quartiles and across the modern and traditional sectors. We also pin down parameters of the production side using the share of modern-sector workers and the static modern-sector wage premium. We then use the estimated model to assess the importance of learning for wage trajectories, aggregate income, and welfare. In our main exercise, we ask how much income and welfare would change if traditional-sector workers had the same learning opportunities as modern-sector workers. In a second exercise, we evaluate the role of learning by doing relative to learning from peers for aggregate outcomes.

The first exercise focuses on learning segmentation across sectors. We compare the estimated economy to counterfactual economies in which workers' learn only from workers in their sector, no segmentation, and another in which workers learn from all other workers in their labor market, modern and traditional workers. This comparison allows us to quantify how differences in learning opportunities across the modern and traditional sectors translate into differences in aggregate income and welfare. Importantly, because workers impose a learning externality on others, it is not obvious that eliminating segmentation is always beneficial. While exposing traditional-sector workers to modern-sector workers improves learning opportunities in the traditional sector, it can also reduce learning opportunities for modern-sector workers, who now interact with lower-skill peers, creating a crowding-out effect. The net aggregate effect of eliminating learning segmentation is therefore a quantitative question, and the underlying tradeoff is similar to the idea-diffusion mechanism in semi-endogenous growth models such as Buera and Oberfield (2016).

[2]We classify modern-sector jobs as formal and traditional-sector jobs as informal. To the best of our knowledge, these patterns have not been documented previously in the informality literature.

The second complementary exercise disentangles learning from peers from learning by doing. We vary the relative importance of peer-based learning versus learning that depends on the productivity of the labor market but is independent of peer composition. Then, we compare the implied steady states to the one inferred from the data. This exercise is important because it clarifies the quantitative strength of the learning externality. It also helps us think about the effects of policies that change how work is organized and who interacts with whom. This distinction is especially important given the learning externality at the core of our paper. If skills grow mostly through peer effects, then changing interactions can have large consequences. If skills grow mostly through learning by doing, then peer interactions matter less for development outcomes.

Related Literature. Our work contributes to four strands of the literature. First, our work is most closely related to the literature on learning from peers. We generalize the ideas in Lucas (2004) by allowing endogenous learning environments in both the modern and traditional sectors, with interactions both within and across sectors, and we provide empirical evidence for this peer-learning mechanism. We are related to the subsequent endogenous growth frameworks of Lucas and Moll (2014), Perla and Tonetti (2014), and Akcigit et al. (2018), but we depart from them by emphasizing how worker sorting across sectors shapes aggregate income in developing economies. We also highlight the idea that workers learn from better peers, as in Jarosch et al. (2021) and Herkenhoff et al. (2024), with the distinction that learning in our framework is not limited to the firm. In this sense, we are closer to Crews (2023) and Lhuillier (2024), who study worker sorting across cities with local learning externalities. Finally, labor regulations associated with operating in the modern sector generate an endogenous productivity cutoff that separates workers across sectors, akin to the exporting-related sorting mechanism in Perla et al. (2021).

Second, we relate to papers that study how human capital accumulation shapes wage dynamics in developing economies. Building on Lagakos et al. (2018), Ma et al. (2024) show that firm-provided training in developing economies accelerates workers' skill formation, helping explain steeper age-wage profiles relative to developing economies. We differ from their framework by emphasizing learning from peers, our local externality, as the fundamental driver of differences in human capital accumulation between the modern and traditional sectors, which can also help rationalize the observed differences in wage-age profiles across developed and developing economies. Along these lines, Engbom (2022) finds that countries with higher rates of job-to-job transitions exhibit steeper wage profiles, which contrasts with Donovan et al. (2023), who document that workers in developing economies transition more across jobs without moving into better-paying ones. Abstracting from labor market frictions, we offer a reconciling explanation for these findings by showing how segmented labor markets in developing countries hinder human capital accumulation, generating flatter wage profiles in the aggregate.

Third, our work also connects to the literature on how technological differences across sectors affect aggregate productivity in the context of economic development.[3] Lagakos (2016) shows how retail firms in low-income countries optimally adopt traditional technologies with lower measured

[3]Herrendorf et al. (2014) provides a detailed overview of this literature in the broader context of structural transformation, and Gollin and Rogerson (2025) highlights the more recent use of heterogeneous-agent macro models in this literature, as we do.

productivity when ownership of durable goods is low. Duarte and Restuccia (2019) and Schwartzman (2025) study structural transformation from traditional to modern services, showing that the reallocation of demand and workers across these segments is quantitatively important for aggregate income differences. We complement their results by showing that worker reallocation across traditional and modern sectors can also lead to higher aggregate income by improving the learning opportunities of lower-skill workers in the traditional sector. Closer to our work are Herrendorf and Schoellman (2018) and Amaral and Rivera-Padilla (2024), who study how complementarities between workers' human capital investments and firms' technology amplify cross-country income differences. Engbom et al. (2025) shows how low education levels in developing countries give rise to an endogenous dual economy with a prevalent traditional-sector organization of production. We depart from these studies by allowing for continuous skill accumulation over the life cycle, rather than a one-time human capital investment, and by emphasizing how peer composition within modern and traditional sectors shapes learning and wage dynamics.

Finally, by linking the modern and traditional sectors to formality status, we contribute to the literature on informal labor markets and their impact on aggregate outcomes. Ulyssea (2020b) provides a comprehensive review of recent studies analyzing firms and workers. Ulyssea (2018) develops a firm dynamics model with intensive and extensive margins of informality, while Dix-Carneiro et al. (2021) evaluates trade gains in economies with large informal labor markets. Both papers use static frameworks that abstract from worker human capital dynamics. In contrast, Meghir et al. (2015) considers a dynamic wage-posting setting but assumes homogeneous and time-invariant worker skills. We depart from these studies by emphasizing wage dynamics arising from ongoing skill accumulation through peer learning. More closely related to our approach are Lopez Garcia (2015) and Bobba et al. (2022), who link informality to schooling investments, and Akcigit et al. (2024) and Lopez-Martin (2019), who study informality's implications for growth. In contrast, we abstract from growth and one-time schooling decisions, and instead emphasize continuous skill accumulation and sectoral mobility over the life cycle, which shape workers' choices between the modern-formal and traditional-informal sectors.[4]

The rest of the paper is organized as follows. Section 2 outlines our framework and its main predictions. Section 3 describes the data and empirical tests of the model's predictions. Section 4 details our quantification strategy. Finally, Section 5 presents our counterfactual exercises and concluding remarks.

2 A Dynamic Model of Peer Learning in a Dual Economy

We develop a model of worker allocation across labor markets and sectors in which human capital accumulation is shaped by local learning externalities. Workers differ in their human capital and choose where to work and whether to be employed in the formal or informal sector. Labor markets are heterogeneous in productivity, and firms demand labor in efficiency units under perfect competition, but face sector-specific hiring costs. These costs generate differences in wages across

[4] Ulyssea (2020a) emphasizes that we still have limited understanding of how workers make forward-looking decisions about formality status.

sectors, which in a static environment would induce sorting based solely on current income.

In the dynamic environment, however, sectoral choices also affect the evolution of human capital. Learning depends on the distribution of skills within each labor market and sector, so that workers in more vibrant environments experience faster skill growth. As a result, workers face a trade-off between current consumption and future earnings: sectors that offer lower wages today may provide better learning opportunities and higher wage growth over time. This trade-off shapes both occupational choices within labor markets and mobility decisions across labor markets, as workers seek environments that maximize the value of their future human capital.

2.1 Environment

Time is continuous and indexed by $t \geq 0$. The economy consists of a unit mass of workers distributed across a finite number of labor markets indexed by $n \in \{1, \ldots, N\}$. Within each labor market, workers are employed in one of two sectors, modern (f) or traditional (i). Each worker is endowed with a level of human capital $z \in \mathbb{R}_+$. Human capital determines a worker's efficiency units of labor. At each point in time, workers are characterized by their labor market n, sector $s \in \{f, i\}$, and human capital z. Moreover, workers are infinitely lived but face a Poisson death shock with intensity $\delta > 0$. Upon death, a worker is replaced by a newborn worker with productivity z drawn from an exogenous distribution $\psi(z)$ and can freely chose the labor market n based on their productivity and preferences. Finally, workers discount future utility at rate $\rho > 0$.

Let $N_{n,s}(z,t)$ denote the mass of workers in labor market n and sector s with human capital z at time t. The total mass of workers with human capital z in labor market n is given by

$$N_n(z,t) = N_{n,f}(z,t) + N_{n,i}(z,t),$$

and the total population in labor market n is

$$L_n(t) = \int_0^\infty N_n(z,t)\, dz.$$

Workers derive utility from consumption according to a utility function $u(c)$. We assume that there are no savings, so workers are hand-to-mouth and consumption is equal to labor income in each period plus government transfers $b(t)$.

2.2 Production and Labor Demand

Each labor market n is populated by a representative firm that produces a homogeneous consumption good using labor as the only input. Production is linear in efficiency units of labor and given by

$$Y_n = A_n \int_0^\infty \{z\ell_{n,f}(z) + \phi_n(z)\ell_{n,i}(z)\}\mathrm{d}z,$$

where $A_n > 0$ denotes the productivity of labor market n, and $\ell_{n,s}(z)$ is the measure of workers with human capital z employed in sector $s \in \{f, i\}$. This specification implies that, conditional on

human capital z, differences in production in the modern and traditional sector arise from access to different technologies. Specifically we assume $\phi_n(z)$ to be a strictly increasing and concave function satisfying $\phi_n(z) = 0$ with Inhada conditions

$$\lim_{z\to 0} \phi_n(z) = \infty, \qquad \lim_{z\to\infty} \phi_n(z) = 0$$

Labor markets are perfectly competitive, and firms take wages as given. Given wage schedules $\{w_{n,f}(z), w_{n,i}(z)\}$, the representative firm in labor market n chooses $\boldsymbol{\ell}_n = \{\ell_{n,f}(z), \ell_{n,i}(z)\}_{z\geq 0}$ to maximize profits:

$$\pi_n(\boldsymbol{\ell}_n) = \max_{\boldsymbol{\ell}_n} A_n \int_0^\infty \Big\{ z\ell_{n,f}(z) + \ell_{n,i}(z)\phi_n(z) - \ell_{n,f}(z)(1+\tau)w_{n,f}(z) - \ell_{n,i}(z)w_{n,i}(z) \Big\} \mathrm{d}z.$$

Profit maximization implies that, for each z, firms hire workers in sector s up to the point where the marginal product equals the marginal cost. In equilibrium, this yields the following wage schedules:

$$w_{n,f}(z) = \frac{A_n z}{1+\tau}, \qquad w_{n,i}(z) = A_n \phi_n(z). \tag{1}$$

These wage schedules have a single-crossing property as a function of human capital. In a static framework, this would imply perfect sorting of workers across sectors. Low skill workers will benefit from the Inhada conditions in their marginal productivity of labor but as they become more skilled the marginal diminishing returns of the production technology of the traditional sector will make them sort into the modern sector. Moreover, this implies that the modern sector wage premium will depend both on the level of human capital z of the worker, and the curvature of the traditional sector technology. Hence, the mean modern sector premium depends on the curvature of the production technology and on the distribution of of human capital in the respective labor market.

2.3 Mobility Across Labor Markets

Workers receive opportunities to move across labor markets according to a Poisson process with intensity $\lambda > 0$. In the absence of such an opportunity, a worker remains in their current labor market. When a mobility opportunity arrives, a worker currently employed in labor market n can move to any labor market $m \in \{1, \ldots, N\}$ by paying a mobility cost ξ_{nm}, with $\xi_{nn} = 0$.

In addition to mobility costs, workers draw idiosyncratic utility shocks over labor markets. Let $\{\varepsilon_m\}_{m=1}^N$ denote the vector of taste shocks associated with the different labor markets. We assume that these shocks are independently distributed Type-I extreme value with scale parameter $1/\nu$. In this setting, ν controls the dispersion of preferences for different labor markets. In the absence of preferences shocks all workers would sort into the most productive labor market as it will pay the highest wage given their level of human capital z. Letting $V_{n,s}(z,t)$ denote the value of being employed in sector $s \in \{f, i\}$ in labor market n at time t for a worker with human capital z, the

value of labor market n is then given by the outer envelope

$$V_n(z,t) = \max\{V_{n,f}(z,t), V_{n,i}(z,t)\}.$$

Conditional on receiving a mobility opportunity, a worker in labor market n chooses the destination labor market that maximizes continuation value net of mobility costs and taste shocks. We assume that when changing labor markets, the worker can freely sort into their previous sector or change sector upon arriving to their new sector. The arrival of mobility opportunities therefore contributes the following term to the worker's continuation value:

$$\mathcal{M}_n[V](z,t) = \lambda \left[\mathbb{E}_\varepsilon \left(\max_m \{V_m(z,t) - \xi_{nm} + \varepsilon_m\} \right) - V_{n,s}(z,t) \right].$$

Under the Type-I extreme value assumption, this operator can be written in closed form as

$$\mathcal{M}_n[V](z,t) = \lambda \left[\frac{1}{\nu} \log \left(\sum_{m=1}^{N} e^{\nu(V_m(z,t) - \xi_{nm})} \right) - V_n(z,t) \right].$$

The same structure implies standard logit migration shares. Conditional on a mobility opportunity, the probability that a worker currently in labor market n moves to labor market m is

$$m_{nm}(z,t) = \frac{e^{\nu(V_m(z,t) - \xi_{nm})}}{\sum_{k=1}^{N} e^{\nu(V_k(z,t) - \xi_{nk})}}.$$

These migration shares govern the reallocation of workers across labor markets in the population dynamics below.

2.4 Learning and Skill Dynamics

Workers accumulate human capital through a learning process that depends on the distribution of human capital in their labor market and sector. Learning is shaped by local externalities that arise from interactions with other workers.

For each labor market n and sector $s \in \{f, i\}$, let $g_{n,s}(z,t)$ denote the conditional density of human capital. We define sectoral vibrancy as

$$Z_{n,s}(t) = \left[\int_0^\infty z^\theta g_{n,s}(z,t)\, \mathrm{d}z \right]^{1/\theta},$$

where $\theta > 0$ controls the sensitivity of vibrancy to the upper tail of the human capital distribution. In this context, θ acts as substitution parameter in terms of learning. When $\theta = 1$ then learning from high skills is a perfect substitute from learning from learning from low skilled. The edge case where $\theta = 0$ is the case where the vibrancy is a Cobb-Douglas production function on the skills of workers. Finally, for $\theta < 0$ learning from different workers become complements with the limit case of perfect complement as $\theta \to -\infty$. Consequently, different levels of θ will imply different levels

of modularity of the vibrancy and will determine the strength of agglomeration effects withing a labor market.

Learning opportunities within a labor market depend on both traditional and modern vibrancy. For each labor market n and sector $s \in \{f, i\}$, the deterministic component of the learning process is defined as

$$\mu_{n,s}(t) = (1-\gamma)A_n + \gamma \left[\pi_s Z_{n,s}(t)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\pi_s) Z_{n,-s}(t)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2}$$

where $\gamma \in [0,1]$ governs the importance of learning from peers relative to learning by doing, $\pi_s \in [0,1]$ captures the degree of within-sector learning segmentation, $\varepsilon > 0$ is the elasticity of substitution between traditional and modern vibrancy in the learning process, and $-s$ denotes the opposite sector.

A worker employed in labor market n and sector s accumulates human capital according to a geometric Brownian motion:

$$\mathrm{d} \log z_{n,s}(t) = \mu_{n,s}(t)\, \mathrm{d}t + \sigma_s\, \mathrm{d}W(t),$$

where $W(t)$ is a standard Wiener process and $\sigma_s > 0$ is a sector-specific volatility parameter. This specification implies that learning is scale invariant in human capital and that the growth of skills depends on both labor-market productivity and the composition of human capital within and across sectors in the same labor market. Labor markets with a higher composition of high skilled workers will experience higher growth on their human capital as their vibrancy will be higher. However, learning segmentation will also play a major role on determining the learning opportunities for different sectors. Particularly, if the learning segmentation is high (i.e π_s close to 1) then most of the learning comes from within the sector. Hence, if high skilled individuals sort into the modern sector and segmentation is high, traditional sector workers will experience lower wage growth.

2.5 Workers' Dynamic Problem

Now we characterize the value function $V_{n,s}(z,t)$. Workers derive flow utility from consumption, which equals labor income plus government transfers, so that instantaneous utility is given by $u\big(w_{n,s}(z,t) + b(t)\big)$. In addition, workers face three sources of dynamics: mobility across labor markets, accumulation of human capital, and a Poisson death shock. The value function therefore satisfies the Hamilton-Jacobi-Bellman equation

$$\begin{aligned} \rho V_{n,s}(z,t) = {} & u\big(w_{n,s}(z,t)\big) + \mathcal{M}_n[V](z,t) + \mu_{n,s}(t)\, z\, \partial_z V_{n,s}(z,t) + \frac{1}{2}\sigma_s^2 z^2\, \partial_{zz} V_{n,s}(z,t) \\ & + \partial_t V_{n,s}(z,t) - \delta V_{n,s}(z,t). \end{aligned} \tag{3}$$

The second term captures the expected gains from mobility across labor markets, as defined in the previous subsection. The third and fourth terms reflect the evolution of human capital, where the drift $\mu_{n,s}(t)$ depends on local vibrancy and σ_s governs the volatility of skill accumulation. The final term captures the Poisson death shock, under which workers exit the economy and are replaced

by newborns.

2.5.1 Workers' Occupational Choice

Workers choose between formal and informal employment within a labor market by comparing the associated value functions. For a worker with human capital z in labor market n at time t, the optimal sector choice is given by

$$s^*(n, z, t) = \arg\max_{s \in \{f,i\}} V_{n,s}(z, t).$$

The structure of wages and learning implies that the value functions satisfy a single-crossing property in human capital. As a result, there exists a cutoff $\bar{z}_n(t)$ such that workers with human capital above the cutoff choose one sector, while those below choose the other.

The cutoff $\bar{z}_n(t)$ is defined implicitly by the indifference condition

$$V_{n,f}(\bar{z}_n(t), t) = V_{n,i}(\bar{z}_n(t), t).$$

At this cutoff, both value matching and smooth pasting conditions are satisfied.

The cutoff reflects both the static differences in wages across sectors and the dynamic differences in learning opportunities. In particular, while the static wage schedules imply sorting based solely on contemporaneous labor income, the dynamic problem incorporates the effect of sectoral learning on future human capital accumulation. As a result, workers may be willing to accept lower current wages in exchange for higher expected growth in skills.

This implies that the cutoff in the dynamic economy is generally lower than the cutoff that would arise in a static framework, as forward-looking workers internalize the value of learning when making occupational choices.

2.6 Distribution Dynamics

We now characterize the evolution of the cross-sectional distribution of workers across labor markets, sectors, and human capital. Let $N_{n,s}(z, t)$ denote the mass of workers with human capital z employed in labor market n and sector $s \in \{f, i\}$ at time t. The occupational cutoff $\bar{z}_n(t)$ partitions workers within each labor market according to

$$z \leq \bar{z}_n(t) \Rightarrow s = i, \qquad z > \bar{z}_n(t) \Rightarrow s = f.$$

It is useful to define the probability flux in labor market n and sector s as

$$J_{n,s}(z, t) = \left(\mu_{n,s}(t) - \frac{\sigma_s^2}{2}\right) z N_{n,s}(z, t) - \frac{\sigma_s^2}{2} z^2 \partial_z N_{n,s}(z, t).$$

The evolution of the sectoral masses is governed by a system of Kolmogorov-Forward equations.

For workers in the modern sector, that is, for $z > \bar{z}_n(t)$, the KFE is

$$\begin{aligned}\partial_t N_{n,f}(z,t) = -\partial_z J_{n,f}(z,t) + \lambda \left(\sum_{k=1}^{N} m_{kn}(z,t) N_k(z,t) - N_{n,f}(z,t) \right) \\ + \delta \left[\psi(z) \Pi_n(z,t) - N_{n,f}(z,t) \right].\end{aligned} \tag{4}$$

Similarly, for informal workers, that is, for $z \leq \bar{z}_n(t)$,

$$\begin{aligned}\partial_t N_{n,i}(z,t) = -\partial_z J_{n,i}(z,t) + \lambda \left(\sum_{k=1}^{N} m_{kn}(z,t) N_k(z,t) - N_{n,i}(z,t) \right) \\ + \delta \left[\psi(z) \Pi_n(z,t) - N_{n,i}(z,t) \right],\end{aligned} \tag{5}$$

The first term in each equation captures the endogenous evolution of human capital through learning. The second term reflects reallocation across labor markets induced by mobility opportunities. The final term captures worker turnover: incumbents exit at rate δ, while newborn workers enter with initial human capital distributed according to $\psi(z)$ and choose labor market n with probability

$$\Pi_n(z,t) = \frac{e^{\nu V_n(z,t)}}{\sum_{k=1}^{N} e^{\nu V_k(z,t)}}. \tag{6}$$

Equation (6) leverages the fact upon birth workers draw idiosyncratic preference shocks and is able to sort The law of motion is completed by standard boundary and matching conditions. At the upper bound, the sectoral masses vanish:

$$\lim_{z \to \infty} N_{n,s}(z,t) = 0 \qquad \text{for } s \in \{f, i\}.$$

Finally, at the occupational cutoff $z = \bar{z}_n(t)$, both the mass of workers and the probability flux are continuous:

$$\begin{aligned}\lim_{z \uparrow \bar{z}_n(t)} N_{n,i}(z,t) &= \lim_{z \downarrow \bar{z}_n(t)} N_{n,f}(z,t), \\ \lim_{z \uparrow \bar{z}_n(t)} J_{n,i}(z,t) &= \lim_{z \downarrow \bar{z}_n(t)} J_{n,f}(z,t).\end{aligned}$$

These conditions ensure that worker mass is conserved within each labor market and that the aggregate population remains constant over time.

2.7 Equilibrium

An equilibrium is a collection of time paths

$$\{w_{n,f}(z),\, w_{n,i}(z),\, V_{n,f}(z,t),\, V_{n,i}(z,t),\, V_n(z,t),\, N_{n,f}(z,t),\, N_{n,i}(z,t),\, \bar{z}_n(t),\, b(t)\}_{n=1}^{N}$$

such that, for all labor markets n, human capital levels z, and times t, the following conditions hold.

(i) Production and labor demand. Firms optimize in each labor market, and wages satisfy (1)

(ii) Workers' dynamic problem. For each sector $s \in \{f, i\}$, the value function $V_{n,s}(z,t)$ satisfies (3) and the labor-market value is given by

$$V_n(z,t) = \max\{V_{n,f}(z,t), V_{n,i}(z,t)\}.$$

(iii) Occupational choice. The sectoral cutoff $\bar{z}_n(t)$ is consistent with workers' optimal sector choice and satisfies

$$V_{n,f}(\bar{z}_n(t),t) = V_{n,i}(\bar{z}_n(t),t),$$

together with the associated smooth-pasting condition at $\bar{z}_n(t)$.

(v) Distribution dynamics. The sectoral masses evolve according to the Kolmogorov-Forward equations. For $z > \bar{z}_n(t)$, the distribution satisfies equation (4) and for $z < \bar{z}_n(t)$, the distribution satisfies equation (5)

(vi) Government budget constraint. The lump-sum transfer $b(t)$ satisfies

$$b(t) = \sum_{n=1}^{N} \int_{\bar{z}_n(t)}^{\infty} \tau \, N_{n,f}(z,t) \, w_{n,f}(z,t) \, dz.$$

(vii) Labor Markets. Within labor markets clear $\ell_{n,s}(z) = N_{n,s}(z)$ and resources constrains are satisfied:

$$\sum_{n=1}^{N} \int_{0}^{\infty} L_n(t) = 1.$$

Our main object of interest is a stationary equilibrium. In such equilibrium the distributions are ergodic and hence

$$\partial_t N_{n,s}(z,t) = 0$$

and $\dot{V}_{n,s}(z,t) = 0$. Hence we drop the time index from our equilibrium objects.

We solve for the stationary equilibrium using finite differences, following Achdou et al. (2022). Appendix A provides a detailed description of the algorithm.

2.8 Predictions of the Model

We now turn our attention to some of the predictions of our model. First, our model implies that more skilled individuals will sort into the modern sector. To that end, the data should reflect a differences in levels in favor of workers in the modern sector due to the sorting. The magnitude however of the modern wage premium will be entirely dependent on the distribution of human

capital across labor markets. More specifically, the average wage premium observed in the data is

$$\sum_{n=1}^{N} \int_0^{\infty} \frac{z}{(1+\tau)\phi_n(z)} N_n(z) \mathrm{d}z$$

Second, we study the differences in wages trajectories over the life cycle for workers. Particularly, we are interested in understanding the dynamic effects of sorting into the traditional vs the modern sector over the life cycle. To do so, notice that the wage growth in a given labor market and sector can be written as

$$\begin{aligned} \mathrm{d}w_{n,i}(x(t)) &= A_n \phi_n'(x(t)) \left[\mu_{n,i}\mathrm{d}t + \sigma_i \mathrm{d}W(t)\right], \\ \mathrm{d}w_{n,f}(x(t)) &= \frac{A_n}{1+\tau} \left[\mu_{n,f}\mathrm{d}t + \sigma_f \mathrm{d}W(t)\right]. \end{aligned}$$

Here we can note several things. For workers in the modern sector, the wage growth only depends on the process on their local labor market but there are no scale effects from their current level of human capital. On the contrary, wage growth for workers in the traditional sector does depend on their current level of human capital will determine their level of wage growth. Specifically, as they become more skilled, the marginal diminishing returns of the technology of the traditional sector reduce the marginal productivity of labor and hence dampens the wage growth. Moreover, the value functions will inherit the sorting property of the static block and hence if $\pi_s > 1/2$ the drift for workers in the modern sector will be higher leading to higher wage growth.[5] Our model will imply that workers in the modern sector not only experience higher wage growth but show a premium attributed to sorting and differences in technologies.

3 Empirical Evidence

This section first describes our definitions of modern and traditional jobs and the data we use to test the model's predictions. Our main analysis focuses on Chile, where we can exploit rich worker survey data in an important developing economy in which a large fraction of workers still operate in the traditional sector.

3.1 Definitions and Data

We define modern and traditional-sector jobs in the data in a way that is consistent with our framework. In the model, modern-sector workers are those who comply with labor regulations. Accordingly, we define a modern-sector job as one with a formal labor contract and pension contributions, which correspond to payroll taxation in the model. We define a traditional-sector

[5] While we have yet to formalize this result, the edge case where $\sigma_i = \sigma_f$ is a natural case where to build intuition. Consider two workers with the same initial level of human capital, one sorting into the traditional sector vs. the other in the modern sector. The distribution of potential human capital levels for the worker in the modern sector will first order dominate the distribution of the worker in the traditional sector as the deterministic component is higher and the idiosyncratic component is i.i.d for both of them.

job as any job that does not comply with these labor regulations. In doing so, our definitions of modern and traditional employment coincide with the standard distinction between formal and informal work.[6]

Our main data source is the Chilean *Encuesta de Protección Social* (EPS). The EPS is a comprehensive longitudinal survey designed to gather data on various aspects of social protection. The survey spans five waves conducted in 2002, 2004, 2006, 2009, and 2015.[7] We utilize two blocks of the survey. First, we use the block containing information on each individual's demographics, such as age and education. Second, we use the block containing the individual's job history dating back to 1980.[8] The worker's job history includes information on job types, occupation, economic sector, region, weekly hours worked, starting and ending dates, type of contract (if any), pension fund contributions, job category (e.g., employed by a firm or self-employed), and the number of workers in the firm. Since 2004, individuals also provide information on monthly disposable income for each job. The first wave conducted in 2002 only considered workers' affiliation with a pension fund, making it representative only of formal workers. We restrict our sample starting from the 2004 wave, the year in which individuals without pension affiliation were included, making the survey representative of all workers.

The EPS offers four advantages for analyzing employment dynamics for modern and traditional workers. First, the detailed information for each job allows us to classify workers as modern or traditional based on their contract type and pension fund contributions. Second, the data on disposable monthly income and the number of hours worked allows us to compute the hourly wage rate for jobs starting from 2002 onwards.[9] Third, despite the survey being conducted over scattered years, the job history block enables us to reconstruct the employment history for each worker since 1980. We aggregate different jobs for each individual to construct our baseline sample, which is an unbalanced year-level panel with 18,650 workers from 2002 to 2016.[10] The length and coverage of our sample allow us to overcome the shortcomings of previous studies by tracking modern and traditional workers over long periods. For example, we can follow 50% of the workers in our sample for at least five years and 20% for at least ten years.[11] Finally, although we only use data for jobs active from 2002 onwards due to wage availability, we use workers' job histories since 1980 to measure experience, which we can further disaggregate into modern and traditional experiences.[12]

We exclude certain types of workers from our analysis. First, in accordance with Chilean labor regulations, we include male workers aged 15 to 65 and female workers aged 15 to 60, aligning

[6] Chile's statutory labor laws state that at the start of any labor relation, any non-affiliated worker generates automatic affiliation to the pension system and the obligation to contribute 10% of their salary to the fund.

[7] We exclude the 2012 and 2019 waves from our analysis. The 2012 wave is not representative at the national level, and the questions for 2019 changed because of the COVID-19 pandemic.

[8] When an individual enters the survey, they are asked for their job history since 1980. If, in a specific wave, the individual was already part of the survey, they are asked about their job history since the survey's last wave.

[9] We construct wages for 2002 and 2003 using job history information.

[10] For workers with multiple jobs in a given year, we take the job with the highest total hours worked.

[11] In contrast, Meghir et al. (2015) use a rotating panel in Brazil that follows individuals for five consecutive months and then for another four months one year after their entry into the sample. Samaniego De La Parra and Fernández Bujanda (2020) use a rotating panel in Mexico that follows workers for only five quarters.

[12] We compute experience by dividing the cumulative hours worked up to a particular year by the number of hours corresponding to a full work schedule of 48 hours per week.

Table 1: Summary Statistics

	(1) Traditional	(2) Modern	(3) All
Fraction of workers	0.31	0.69	···
Mean log wage	2.01	2.40	2.28
Std. deviation log wage	0.73	0.67	0.71
Mean weekly working hours	42.10	46.00	44.81
Fraction of male workers	0.58	0.63	0.61
Mean experience (years)	15.32	15.24	15.27
Transitions (by initial status)			
Frac. of traditional workers next year	0.92	0.03	···
Frac. of modern workers next year	0.08	0.97	···
Number of observations	35,767	86,617	122,384
Number of workers	···	···	17,308

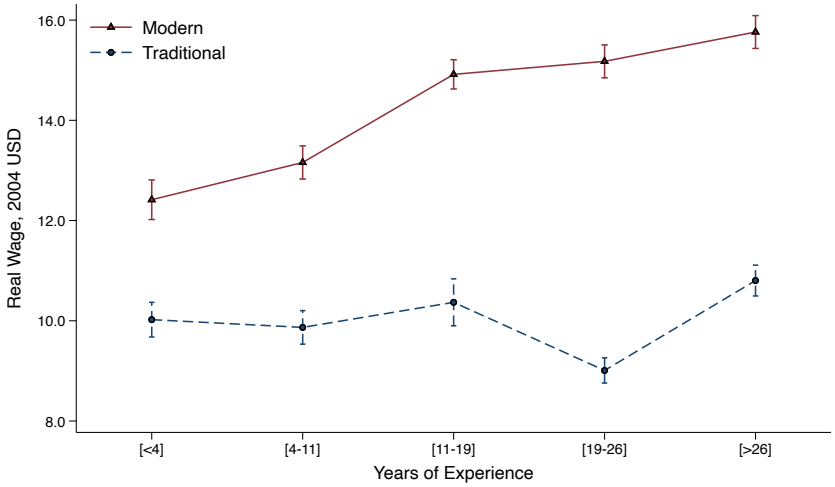
Notes: Table 1 displays summary statistics for the baseline sample 2002-2016. We include male workers aged 15 to 65 and female workers aged 14 to 60. We exclude workers without remuneration, public-sector and armed forces employees. Wage is the real hourly wage. The variable experience corresponds to the equivalent in years of a full-time work schedule of 48 hours per week.

with the country's legal starting working age and retirement age for men and women, respectively. Second, in our baseline specifications, we omit workers without remuneration, as well as public-sector and armed forces employees. Importantly, our data allows us to include self-employed workers and entrepreneurs which represent a large fraction of traditional sector workers.[13]

Table 1 displays summary statistics for our baseline sample. Three key points are worth highlighting. First, on average, modern workers are paid more than traditional workers. Nevertheless, wages for traditional workers are more dispersed than those for modern workers, consistent with previous findings. Second, modern and traditional workers work similar hours per week and have similar years of experience. This similarity supports our modeling choice of abstracting from labor supply decisions and from an explicit time allocation between working and learning. Third, transition probabilities between the modern and traditional sectors are highly asymmetric. On average, 8% of traditional workers move to the modern sector the following year, whereas only 3% of modern workers become traditional the next year. Importantly, these transition probabilities are computed by restricting the sample to active workers, which excludes transitions into and out of unemployment. Appendix B provides additional descriptive statistics, including modern and traditional sector affiliation by age, education, firm size, occupation, and industry.

[13]This inclusion is also consistent with previous informality studies, such as Meghir et al. (2015)

Figure 1: Wages Over the Life Cycle for modern and Traditional Workers



Notes: Figure 1 displays wage paths over the life cycle for modern and traditional workers. We compute the average wage for each experience bin for modern and traditional workers. The figure shows the average for all workers, not only salaried workers. Vertical dashed lines denote 90 percent confidence intervals.

3.2 Stylized Facts

This section presents empirical evidence on wages for modern and traditional workers. We begin by documenting wage-experience profiles that highlight the differential wage growth patterns for workers in each sector. We then test the model predictions in two steps. We first present evidence of high-skill workers sorting into the modern sector and then we test the presence of the modern dynamic wage premium.

Figure 1 presents experience-wage profiles for modern and traditional workers. These profiles reveal stark differences in life cycle wage growth between the two groups. We construct these profiles by first classifying workers as modern or traditional according to our definition. We then divide workers into five experience bins and compute the average wage for each bin.

Figure 1 conveys two important messages. First, there is a significant difference in wage levels and growth between modern and traditional workers throughout the life cycle. Modern workers experience a wage increase of approximately 30% by the end of their careers relative to their early career wages. In contrast, traditional workers' wages remain mostly flat over time. Second, a larger fraction of the wage divergence between modern and traditional workers occurs early in the life cycle. modern workers experience substantial wage growth during the first two decades of their careers, while traditional workers see modest growth that plateaus or even declines later.

These life cycle wage patterns for modern and traditional workers align with recent findings on wage growth across countries. Lagakos et al. (2018) document that wages in developed countries increase relatively more over the life cycle than wages in developing economies. Our findings suggest that the modern sector behaves like a "developed economy labor market," whereas the traditional sector displays patterns typical of a "developing economy labor market." In Appendix B, we report similar wage trajectories using age bins instead of experience bins and for salaried workers only. The patterns remain consistent across these alternative specifications.

Table 2: Modern-Traditional Wage Gap

	Dependent variable: $\log w_{i,t}$			
	(1)	(2)	(3)	(4)
Modern	0.396***	0.161***	0.0950***	0.0805***
	(0.0151)	(0.0132)	(0.0167)	(0.0176)
$\text{asinh}\exp_{i,t}^{M}$				0.0322**
				(0.0149)
$\text{asinh}\exp_{i,t}^{T}$				-0.0292**
				(0.0138)
Corr(Modern$_{it}$, δ_i)			0.168	0.104
			(0.004)	(0.004)
Observations	58,926	58,926	58,926	58,926
Adj R-squared	0.0746	0.460	0.839	0.839
Controls		✓	✓	✓
Worker Fixed Effects			✓	✓

Notes: Table 2 reports estimates for β in equation (7). Column (1) displays the results without including any controls. Column (2) shows the results when controlling for worker characteristics but without worker fixed effects. Column (3) reports the results when controlling for worker characteristics and worker fixed effects. Column (4) additionally controls separately for experience in the modern and traditional sectors, rather than for total experience. Controls include age, education, experience, occupation, industry, region, firm size, and time fixed effects. All columns include time fixed effects. Standard errors in parentheses are clustered at the individual level. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

3.2.1 Worker Sorting

We now examine the modern-traditional wage gap and investigate the role of worker sorting in explaining this gap. We estimate the following regression equation:

$$\log w_{i,t} = \beta \text{Modern}_{i,t} + \gamma X_{i,t} + \delta_t + \delta_i + \varepsilon_{i,t}, \tag{7}$$

where $\log w_{i,t}$ is the log wage of worker i at time t, Moderni, t is an indicator equal to one if worker i is in the modern sector in year t and zero if they are in the traditional sector, Xi, t is a vector of worker characteristics including age, education, gender, occupation, industry, region, firm size, and experience, and δ_t and δ_i denote time and worker fixed effects.

Table 2 presents the results from estimating equation (7). Column 1 shows that, without any controls, modern workers earn wages approximately 49% higher than traditional workers. Once we control for worker characteristics in Column 2, the modern-traditional wage gap falls to 17.5%, indicating that differences in observable characteristics account for a substantial share of the raw wage gap. Most importantly, when we include worker fixed effects in Column 3, the modern-traditional wage gap decreases further to 10%, suggesting that approximately 60% of the raw modern wage premium is explained by worker sorting on observable and unobservable characteristics. In addition, we find a positive correlation of 0.168 between formality status and worker fixed

effects, providing direct evidence that more skilled workers, as captured by the fixed effects, tend to sort into the modern sector. Hence, consistent with our model, we find empirical support for the idea that more skilled workers sort into the modern sector. Importantly, this sorting reflects both observable characteristics, such as education, and unobserved skill, as captured by worker fixed effects. In our model, these two components are captured by the initial skill level and the subsequent lifetime path of worker skills.

Table 2 also informs the technological differences between the modern and traditional sectors. The modern wage premium in the specification with controls and worker fixed effects shows that workers with the same skill level earn higher wages in the modern sector. Through the lens of our model, this provides empirical evidence of decreasing returns to production in the traditional sector. Given these technological differences, the same worker has a higher marginal product in the modern sector. Importantly, because we measure wages net of taxes, this modern wage premium cleanly identifies differences in technology across the two sectors rather than differences in regulatory costs. We target this estimate in our quantitative analysis to discipline the parameter governing production in the traditional sector.

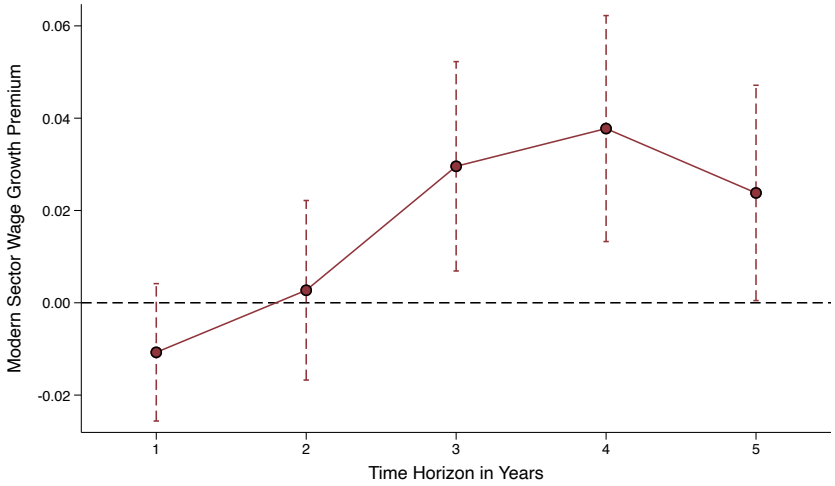
3.2.2 Dynamic Modern-Sector Premium

We now turn to our model's main novel prediction: the dynamic premium of the modern sector. We test this prediction in three steps. First, we examine the cumulative effects of experience in the modern and traditional sectors on wages. Second, we show the short- and medium-run effects of working in the modern sector on future wages. Finally, we provide evidence that learning from better-paid peers in the modern sector generates persistent wage differences between workers in the modern and traditional sectors.

We study the cumulative effects of working in the modern and traditional sectors by revisiting a variant of equation (7). Specifically, we examine how accumulated experience in each sector affects current wages by controlling separately for experience in the modern and traditional sectors. To increase statistical power and include workers with zero experience in a given sector, we use the inverse hyperbolic sine, asinh, rather than the natural logarithm. Accordingly, $\text{asinh}, \exp_{i,t}^{M}$ and $\text{asinh}, \exp_{i,t}^{T}$ denote the inverse hyperbolic sine of years of experience in the modern and traditional sectors, respectively. Column 4 of Table 2 shows that modern-sector experience is positively associated with wages, with an estimated coefficient of 0.032. In contrast, traditional-sector experience is negatively associated with wages, with an estimated coefficient of -0.029. These results indicate that experience accumulated in the modern sector raises wages, whereas experience accumulated in the traditional sector has nearly the opposite effect. These results support the idea that human capital accumulation is higher in the modern sector than in the traditional sector.

We then now formally study the short and medium run effect of being in the modern sector on future wages. To do so, we relate workers' future wage dynamics to their sectorial status and various observable characteristics. Specifically, we estimate the following relationship separately for different time horizons h:

Figure 2: Modern Sector Dynamic Wage Premium



Notes: The figure displays the modern future wage premium based on the estimates of β_h from equation (8) including worker fixed effects. Each dot represents the estimated wage premium at horizon $t+h$: $\exp(\hat{\beta}_h)-1$, for $h = 1, \ldots, 5$. Controls include age, education, gender, occupation, industry, region, experience, wage decile, and time and worker fixed effects. Vertical dotted lines denote 90 percent confidence intervals.

$$\Delta \log w_{i,t+h} = \beta_h \text{Modern}_{i,t} + \gamma X_{i,t} + \delta_t + \delta_i + \varepsilon_{i,t}, \tag{8}$$

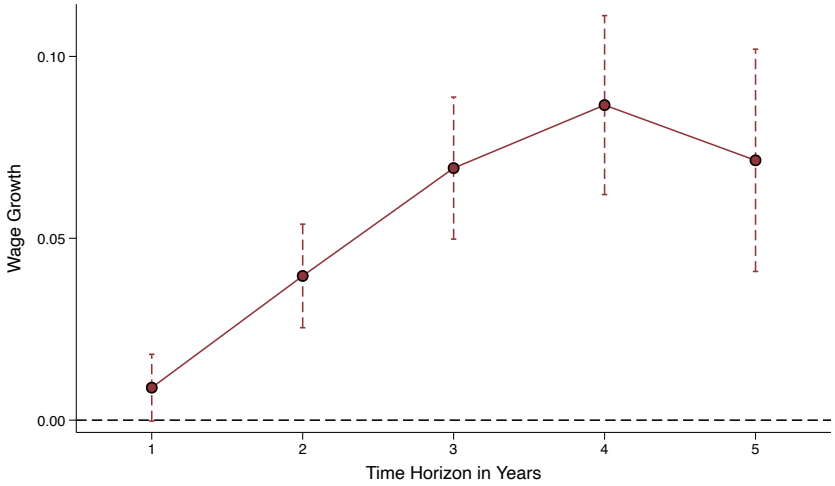
where $\Delta \log w_{i,t+h}$ is the change in worker i's log wage from year t to year $t+h$, and $X_{i,t}$ includes worker characteristics, and wage decile fixed effects, and δ_t and δ_i being time and worker fixed effects, respectively. The parameter of interest is β_h, which captures the average difference in wage growth over h years between modern and traditional workers, conditional on observable characteristics and fixed effects.

Figure 2 presents the estimated future wage premium associated with working in the modern sector across different time horizons. This premium is initially statistically insignificant at short horizons. However, it increases with the horizon, reaching approximately 4% after four years. Crucially, because we include worker fixed effects, identification comes from workers who change their formality status, allowing us to compare the wage growth trajectories of the same worker during periods of modern and traditional employment. Hence, this exercise suggests that workers who switch from the traditional to the modern sector experience cumulative wage gains in subsequent years.

Importantly, we do not condition on workers remaining in the modern sector in subsequent years. This is consistent with our model, in which human capital is portable across sectors. As a result, our estimates should be interpreted as a lower bound on the dynamic modern-sector premium that would arise if we conditioned on workers who continue in the modern sector. In addition, the estimated effects become statistically insignificant after seven years. This is natural in our setting, since workers are not required to remain in the modern sector and may switch sectors over time in response to idiosyncratic skill shocks.

Which forces drive the modern-sector dynamic wage premium? Following the approach of Jarosch et al. (2021) and Herkenhoff et al. (2024), we examine whether interactions with better-paid peers

Figure 3: Peer Effects on Wage Growth



Notes: Figure displays the effect of peer wages on workers' future wage growth based on the estimates of β_h from equation (9). Each dot represents the estimated effect at the time horizon $t + h$. Controls include age, education, gender, occupation, industry, region, experience, wage decile, and time fixed effects. Vertical dotted lines denote 90 percent confidence intervals.

contribute to workers' wage growth. According to our framework, workers interact with others in the same labor market. Hence, we define a labor market in our data to be a region, industry, and firm size bracket in a given year. Then, for each worker i, we compute the average wage of the other workers in the same labor market, whcih we denote by $w_{-i,t}$ and estimate the following reduced form equation:

$$\Delta \log w_{i,t+h} = \beta_h \log w_{-i,t} + \gamma X_{i,t} + \varepsilon_{i,t}, \tag{9}$$

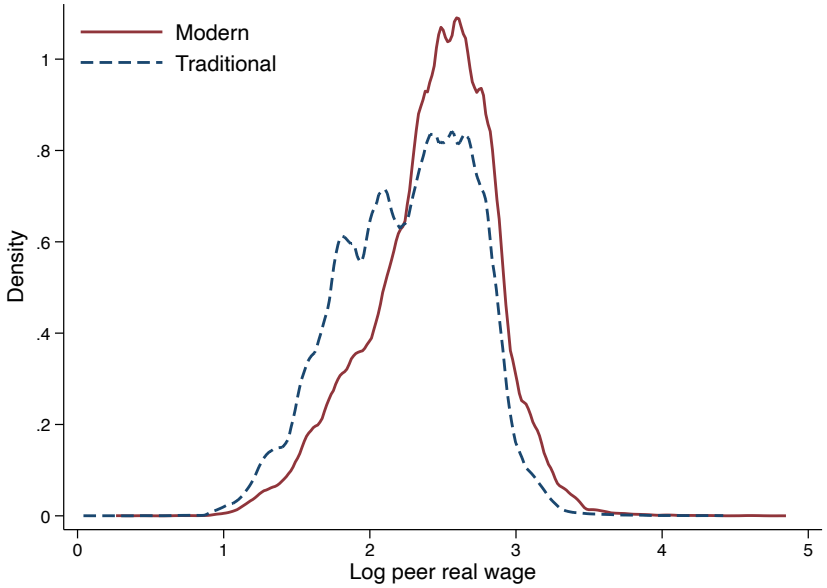
where the parameter β_h captures how the quality of a worker's peers (measured by their average wage) affects the worker's future wage growth.

Figure 3 presents the estimated peer effects on wage growth across different time horizons. We find strong evidence that workers with better-paid peers experience significantly higher wage growth. This effect increases with the time horizon, suggesting that peer quality has a cumulative impact on skill acquisition. These findings align with the literature on learning from coworkers and provide direct evidence that the quality of interactions plays a crucial role in workers' wage trajectories.

Having established that peers matter for wage growth, we now examine whether modern sector workers systematically have access to better peers than traditional sector workers. Figure 4 compares the distribution of peer wages ($w_{-i,t}$) for modern and traditional workers.

Figure 4 reveals a striking pattern: the distribution of peer wages for modern workers first-order stochastically dominates that of traditional workers. Modern workers have peers with substantially higher wages compared to traditional workers. The distribution of peer wages for modern workers is shifted to the right and displays a higher variance, indicating both higher average peer quality and greater dispersion in peer quality. This finding provides the missing link in our narrative: modern workers experience higher wage growth partly because they interact with better-paid peers, and these interactions contribute to faster skill accumulation.

Figure 4: Distribution of Peer Wages for Modern and Traditional Workers



Notes: Figure displays the density of log real wages for peers of modern and traditional workers, estimated using kernel density method. Peers are defined as workers in the same Region × Year × Industry × Occupation.

3.2.3 Taking Stock

In this section, we provided evidence that modern-sector workers earn higher wages throughout their careers than traditional-sector workers. We also showed that a large share of the wage differences across sectors reflects sorting of higher-skilled workers into the modern sector, consistent with our model. Moreover, we provide evidence consistent with the model's main dynamic prediction: modern-sector workers experience a wage premium in the form of higher future wage growth. Finally, we presented evidence that an important force behind this dynamic premium is learning from peers, as modern-sector workers are more exposed to better-paid peers and such exposure predicts higher future wage growth.

Taken together, these facts support our model's narrative. Higher-skilled workers select into the modern sector, where they are exposed to other skilled workers. In contrast, lower-skilled workers select into the traditional sector, where they are exposed to less skilled peers. These sorting patterns generate better learning opportunities for modern-sector workers, which translate into steeper wage trajectories over time. Consistent with this mechanism, modern-sector workers experience substantially faster wage growth over their careers than traditional-sector workers, whose wages remain relatively flat.

Importantly, our evidence suggests that peer effects operate both within and outside the workplace. While a portion of the modern dynamic wage premium is explained by firm characteristics (as shown in our analysis controlling for firm size), a significant part remains unexplained, indicating that learning likely occurs through various channels. Face-to-face interactions outside the workplace, such as those studied by Atkin et al. (2022), Catalini et al. (2020), and Andrews (2023), could be particularly relevant. To the extent that high-skilled modern workers tend to live

or work in proximity, attend the same schools, or frequent the same social venues, the quality of interactions for modern workers would be higher than for traditional workers.

It is important to recognize, however, that learning from peers is not the only channel through which workers accumulate human capital. Two other forces can shape skill accumulation in our framework: learning by doing and sector-specific idiosyncratic human capital shocks. Our reduced-form evidence does not rule out that modern-sector workers learn faster because of technological differences across sectors that raise learning by doing, independent of peer composition. Similarly, consistent with the arguments in Donovan et al. (2023), traditional-sector workers may face more frequent idiosyncratic productivity shocks, either because of the technologies they operate or because they are less likely to be insured through social security. Our data do not allow us to cleanly measure the relative importance of peer learning versus these other forces. Fortunately, our model provides a lens to disentangle them. We do so in the next section, where we quantify the model and use counterfactual exercises to isolate the contributions of peer learning, learning by doing, and idiosyncratic shocks.

4 Model Quantification

This section describes the estimation of the model parameters. We summarize the parameters described in our theory and discuss the various strategies employed for their estimation. Next, we examine the variation in the data that helps identify these parameters.

4.1 Set Up

We define a labor market in our data as a Region–Industry–Occupation triplet, consistent with the empirical exercises in Section 3. The EPS records information for 13 regions, 10 industries, and 10 occupations, yielding a total of $N = 1,300$ labor markets.

To take the model to the data, we specify two functional forms. First, we assume the initial skill distribution, $\psi(z)$, is Pareto, with the location parameter normalized to one and shape parameter ϑ. Second, we parameterize the production technology for traditional-sector workers as $\phi_n(z) = \phi(z)z^{\eta}$, with $\eta < 1$.

Since time is continuous in our model, we interpret an interval $[t, t+1)$ as one year. Furthermore, we average wages and transition probabilities across modern and traditional sectors over 2002–2016 and interpret these averages as the steady state of the economy.

4.2 Estimation

With our functional form assumptions, we have 14 parameters in addition to the labor-market productivity parameters and bilateral migration costs:

$$\left\{\rho, \tau, \nu, \theta, \mu, \delta, \{\xi_{nm}\}_{n,m=1}^{N}, \{A_n\}_{n=1}^{N}, \gamma, \epsilon, \pi_i, \pi_f, \sigma_i, \sigma_f, \eta, \vartheta\right\}$$

We divide these parameters into three groups, according to the estimation strategy we follow to estimate each of them.

External Calibration (4 parameters): ρ, τ, ν, θ

We set workers' discount rate, ρ, to 0.05, a standard value in the literature (Akcigit et al., 2021). We set payroll taxes in the modern sector to $\tau = 0.18$, consistent with Chilean statutory regulations. These costs include pension and social security contributions. We set the inverse dispersion parameter of labor-market idiosyncratic preference shocks to $\nu = 1/1.5$, following Kleinman et al. (2025). Finally, we set the power-mean parameter governing labor market vibrancy to $\theta = 6.22$, following Crews (2023), who estimates it in a similar environment using U.S. metropolitan statistical areas. We recognize that the last two parameters, governing the dispersion of idiosyncratic preference shocks and the aggregation of labor market vibrancy, are estimated using U.S. data and may differ in the Chilean context. In ongoing work, we plan to incorporate them into our internal estimation strategy.

Minimum Distance Estimated Parameters ($2 + N^2$ parameters): δ, μ, $\{\xi_{nm}\}_{n,m=1}^{N}$

We estimate the death shock, δ, using the implied age distribution. Letting $H(a)$ denote the age CDF, the model implies the log-linear relation

$$\log\left(1 - H(a)\right) = -\delta a.$$

We therefore estimate δ by regressing $\log\left(1 - H^e(a)\right)$ on age a without a constant, where $H^e(a)$ is the empirical age CDF.

For the migration rate, μ, we use the worker panel data and set it equal to the overall fraction of workers who change labor markets in a given year. In the model, migration opportunities arrive according to a Poisson process with intensity μ, implying that a fraction μ of workers receive an opportunity to move each period. Using the EPS, we identify movers and non-movers and compute μ as the fraction of workers who move across labor markets between consecutive survey waves.

Given our value for ν, we estimate bilateral migration costs, $\{\xi_{nm}\}_{n,m=1}^{N}$, nonparametrically using the bilateral migration flows observed in the EPS. To conduct this nonparametric estimation, we impose symmetric migration costs, as we explain in Appendix C.

Our estimation delivers $\delta = 0.037$ and $\mu = 0.08$. The estimated value of δ implies that around 3.7% of workers in our model exit employment each year and that, at any point in time, active workers have a remaining expected work life of about 26 years. This number is higher than Chile's overall mortality rate, which is natural since δ captures not only death but also exits from employment into unemployment or out of the labor force. Moreover, our estimated value of μ lies in the lower range of the values estimated by Crews (2023) for the United States.

Simulated Method of Moments (8 + N parameters): $A_{n\,n=1}^{N}, \gamma, \epsilon, \pi_i, \pi_f, \sigma_i, \sigma_f, \eta, \vartheta$

We estimate the remaining $8 + N$ parameters using a Simulated Method of Moments approach. While the moments identify the parameters jointly, it is useful to describe the estimation in three blocks: labor-market productivities $A_{n\,n=1}^{N}$, learning parameters $(\gamma, \epsilon, \pi_i, \pi_f, \sigma_i, \sigma_f)$, and the remaining technology and initial-condition parameters (η, ϑ).

We estimate labor-market productivity by matching the average wage of traditional workers in each labor market. Equation (1) implies

$$\mathbb{E}[w_{nf}(z)] = \frac{A_n \mathbb{E}_n[z|\text{modern}]}{1+\tau}, \tag{10}$$

where $\mathbb{E}n[z|\text{modern}]$ is the average skill of formal workers in labor market n. We therefore construct a moment condition based on (10) and choose the vector $A_n n = 1^N$ so that, market by market, the model-implied average wage of modern-sector workers matches its empirical counterpart.

We next turn to the learning parameters $\gamma, \epsilon, \pi_i, \pi_f, \sigma_i, \sigma_f$. To identify them, we target the observed transition patterns of workers over time. Our first set of moments matches transition probabilities across quartiles of the wage distribution. These transitions are informative about the parameters governing the drift and variance of the law of motion for workers' skill. Perla et al. (2021) adopt a similar strategy to estimate parameters governing the law of motion of firm productivity when it follows a geometric Brownian motion, as in our setting. We construct these moments as follows. For each year, we sort workers into wage quartiles and compute the probabilities that they move across quartiles in the following year, conditional on being employed. We then average these transition probabilities across all years in the EPS (2002–2016). Appendix C provides further details on how we compute the corresponding transition probabilities in the model from the equilibrium invariant skill distribution. Our second set of moments matches the transition probabilities between modern and traditional employment reported in Table 1.

We then identify η, the parameter governing technology in the traditional sector, using the static modern-sector wage premium. Under our parametrization, equation (1) implies that the modern-sector static wage premium in the model is given by $z^{1-\eta}/(1+\tau) - 1$. We therefore target the estimated modern-sector wage premium reported in Column 4 of Table 2 to identify η.

Finally, we choose the Pareto shape parameter of the newborn skill distribution, ϑ, to match the observed variance of real wages among young workers, following Lhuillier (2024). Intuitively, a smaller ϑ implies a heavier tail and greater dispersion in initial skills, which translates into greater wage dispersion early in workers' careers.[14]

Overall, we have a total of 18 moments that we use to identify 9 parameters. As the model is over-identified, in current work we conduct robustness exercises by varying the moment set and assessing the sensitivity of the estimates to alternative weighting matrices.

[14] In the data, we define young workers as those with 2 years of experience or less.

5 The Role of Learning in Economic Development

We are currently refining our simulated method of moments estimation. Although we already have preliminary results, we are tuning the estimation to improve the model's fit. While we do not expect the estimates to change substantially, we decided not to report them in this version of the draft. Once we finalize the estimates, we will conduct a series of counterfactual exercises to quantify the role of learning in economic development.

Our main counterfactual exercise quantifies the role of peer learning for aggregate income. In the model, learning segmentation is governed by the weights π_f and π_i, which determine how learning depends on sector-specific peer composition for modern- and traditional-sector workers. In the extreme case $\pi_f = \pi_i = 1$, workers in each sector learn only from other workers in the same sector within their labor market. This corresponds to complete learning segmentation. At the other extreme, when $\pi_f = \pi_i = 1/2$, learning is not segmented: the learning technology collapses to one in which workers learn from all workers in their labor market, regardless of sector. We therefore vary these weights and compare aggregate income and welfare across steady states in the estimated economy and in counterfactual economies with complete segmentation and with no segmentation. Comparing these steady states provides a measure of the importance of learning segmentation in dual economies for aggregate income and welfare.

Do we expect learning segmentation to always be a barrier to economic development? The answer, perhaps surprisingly, is no. Workers impose a learning externality on others. When they choose labor markets and sectors, they do not internalize that these decisions affect other workers' learning opportunities. Eliminating learning segmentation altogether can actually reduce the learning opportunities of the most skilled workers, because they are now exposed to less skilled peers, creating a crowding-out effect. At the same time, eliminating segmentation increases the learning opportunities of traditional-sector workers, who become more exposed to higher-skilled counterparts. Whether, in the aggregate, removing learning segmentation raises or lowers income and welfare is therefore a quantitative question. Importantly, this crowding-out effect also appears in semi-endogenous growth models such as Buera and Oberfield (2016), who show that in a world with free trade, idea diffusion can reduce growth in some countries.

In our final and complementary counterfactual exercise, we study the role of learning by doing relative to learning from peers for aggregate income. In the model, learning by doing captures the idea that workers in more productive labor markets accumulate skills faster, independently of peer composition. The relative importance of learning by doing and peer learning is governed by γ. When $\gamma = 1$, all learning comes from peers, while when $\gamma = 0$, learning is independent of peer composition. We vary γ and compare the resulting steady states to the one inferred from the data to assess the strength of these two learning channels. This exercise is informative about the quantitative importance of the learning externality. If learning by doing is the main driver of skill accumulation, concerns about policies that operate through peer interactions are less central. If, instead, peer learning is the dominant channel, then policies that affect interactions and idea diffusion in developing economies may have larger aggregate consequences.

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A Computational Appendix

This appendix is a self-contained summary of the equations and algorithms used to solve the model.

The numerical procedure consists of three components. First, conditional on an aggregate stationary distribution, we solve the worker's partial-equilibrium problem using a finite-difference approximation to the Hamilton–Jacobi–Bellman (HJB) equation, which yields a market-specific sector cutoff policy rule. Second, given these policy rules, we solve the Kolmogorov forward equation (KFE) for the stationary distribution, obtaining the equilibrium distribution of modern and traditional workers across markets. Third, we compute the general equilibrium by iterating jointly on the value function, the stationary distribution, and the learning drifts until the sector cutoffs converge.

A.1 HJB Numerical Solution (Partial Equilibrium)

A.1.1 Summary of Equations

Fix market productivities $\{A_n\}_{n=1}^N$, migration costs $\{\xi_{nm}\}_{n,m=1}^N$, model parameters, a transfer b, and a candidate aggregate stationary density

$$N_n(z), \qquad n = 1, \ldots, N.$$

In partial equilibrium the distribution is taken as given when solving the worker problem. Given sector policy weights $(\omega_{n,\text{mod}}(z), \omega_{n,\text{trad}}(z))$ with $\omega_{n,\text{mod}}(z) + \omega_{n,\text{trad}}(z) = 1$, sectoral densities are

$$N_{n,\text{mod}}(z) = \omega_{n,\text{mod}}(z) N_n(z), \qquad N_{n,\text{trad}}(z) = \omega_{n,\text{trad}}(z) N_n(z). \tag{11}$$

Once the aggregate density is split across sectors, the within-sector normalized densities are

$$g_{n,s}(z) = \frac{N_{n,s}(z)}{\int N_{n,s}(x)\, dx}, \qquad s \in \{\text{mod}, \text{trad}\}, \tag{12}$$

whenever the denominator is positive.

The sectoral vibrancy indexes, the core learning terms, are

$$\widetilde{Z}_{n,s} = \left(\int z^\theta g_{n,s}(z)\, dz \right)^{1/\theta}, \qquad s \in \{\text{mod}, \text{trad}\}, \tag{13}$$

and the corresponding drifts are

$$\mu_{n,s} = (1-\gamma) A_n + \gamma \left[\pi_s \widetilde{Z}_{n,s}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\pi_s) \widetilde{Z}_{n,-s}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{14}$$

The wage schedules are

$$w_{n,\text{mod}}(z) = \frac{A_n z - F}{1+\tau}, \qquad w_{n,\text{trad}}(z) = A_n z - \varphi_n(z), \tag{15}$$

with traditional-sector cost

$$\varphi_n(z) = \lambda_\phi A_n z + \frac{e^{-\eta A_n z} - 1}{\eta}. \tag{16}$$

Given the transfer b, flow utility is

$$u_{n,s}(z) = \log\big(\max\{w_{n,s}(z) + b, \underline{c}\}\big), \qquad \underline{c} > 0 \text{ small}. \tag{17}$$

The positive constant $\underline{c}$ is a numerical safeguard that avoids evaluating the logarithm at non-positive consumption levels.

Let $V_{n,s}(z)$ denote the sector-specific continuation value and let

$$V_n(z) = \max\{V_{n,\mathrm{mod}}(z), V_{n,\mathrm{trad}}(z)\} \tag{18}$$

be the free-switching envelope. The continuation value associated with a relocation opportunity is

$$\mathcal{C}_n[V](z) = \frac{1}{\nu} \log\left(\sum_{m=1}^{N} e^{\nu(V_m(z) - \xi_{nm})}\right). \tag{19}$$

For each market n, the envelope Bellman equation is

$$(\rho + \delta + \lambda_M) V_n(z) = \lambda_M \mathcal{C}_n[V](z) + \max_{s \in \{\mathrm{mod},\mathrm{trad}\}} \{u_{n,s}(z) + \mathcal{B}_{n,s} V_n(z)\}, \tag{20}$$

where the drift–diffusion operator associated with learning in sector s, for a generic value function $\phi(z)$ defined on the productivity grid, is

$$\mathcal{B}_{n,s}\phi(z) = \left(\mu_{n,s} + \frac{\sigma_s^2}{2}\right) z\, \partial_z \phi(z) + \frac{\sigma_s^2}{2} z^2\, \partial_{zz}\phi(z). \tag{21}$$

After solving for the envelope, the sector-specific values are recovered from

$$V_{n,s}(z) = \frac{u_{n,s}(z) + \mathcal{B}_{n,s} V_n(z) + \lambda_M \mathcal{C}_n[V](z)}{\rho + \delta + \lambda_M}, \qquad s \in \{\mathrm{mod}, \mathrm{trad}\}. \tag{22}$$

The market-level sector cutoff is then defined by the indifference condition

$$V_{n,\mathrm{mod}}(\bar{z}_n) = V_{n,\mathrm{trad}}(\bar{z}_n). \tag{23}$$

A.1.2 Summary of Equations For Numerical Solution

To solve the Bellman problem numerically, we bound the productivity state space by a finite interval $[z_{\min}, z_{\max}]$. Let

$$z^{\mathrm{ex}} = \{z_j\}_{j=1}^{J} \subset [z_{\min}, z_{\max}], \qquad z_j = z_{\min} + (j-1)\Delta z, \tag{24}$$

be the computational grid, where $\Delta z = (z_{\max} - z_{\min})/(J-1)$. In our calculations the grid is uniform, but the formulas below also clarify how the problem is written once the continuous state has been replaced by a finite collection of nodes.

The Bellman equation is approximated only on the bounded interval. The lower boundary is reflecting and the upper boundary is treated as a vanishing tail. These conditions are imposed directly in the boundary rows of the sparse matrices that approximate the drift–diffusion operator. Hence the numerical problem becomes: find values $\{V_n(z_j)\}_{n,j}$ such that, at each grid point,

$$(\rho + \delta + \lambda_M)V_n(z_j) = \lambda_M \mathcal{C}_n[V](z_j) + \max_{s\in\{\text{mod},\text{trad}\}} \{u_{n,s}(z_j) + [\mathcal{B}_{n,s}V_n](z_j)\}, \tag{25}$$

subject to the boundary conditions embedded in the discrete operator.

Once the grid has been fixed, the remaining numerical tasks are threefold. First, one must approximate the differential operator $\mathcal{B}_{n,s}$ with a sparse matrix. Second, one must determine, node by node, which sector delivers the larger Bellman right-hand side. Third, given that sector assignment, one must solve the resulting nonlinear fixed-point problem for the envelope values.

A.1.3 HJB Discretization with Upwind Finite-Differences

Grid and discretized functions

For each market n, let

$$V_n = (V_n(z_1), \ldots, V_n(z_J))' \in \mathbb{R}^J$$

denote the vector of envelope values on the grid, and analogously define the vectors of flow utilities

$$u_{n,s} = (u_{n,s}(z_1), \ldots, u_{n,s}(z_J))', \qquad s \in \{\text{mod}, \text{trad}\}.$$

The numerical problem is therefore to solve for the finite-dimensional object $\{V_n\}_{n=1}^N$.

Discretizing the drift–diffusion operator

For each market–sector pair (n, s), consider a generic value function $\phi(z)$ defined on the productivity grid. The drift–diffusion operator entering the HJB is

$$\mathcal{B}_{n,s}\phi(z) = \left(\mu_{n,s} + \frac{\sigma_s^2}{2}\right) z\, \partial_z\phi(z) + \frac{\sigma_s^2}{2} z^2\, \partial_{zz}\phi(z). \tag{26}$$

To discretize this operator on the productivity grid, we approximate the first derivative with an upwind finite difference and the second derivative with a centered second difference. This choice is standard in continuous-time problems because it preserves stability when drift dominates diffusion.

For an interior node z_j, it is convenient to define

$$a_{n,s}(z_j) \equiv \frac{\sigma_s^2}{2} z_j^2, \qquad b_{n,s}(z_j) \equiv \left(\mu_{n,s} + \frac{\sigma_s^2}{2}\right) z_j,$$

so that

$$\mathcal{B}_{n,s}\phi(z_j) = a_{n,s}(z_j)\,\partial_{zz}\phi(z_j) + b_{n,s}(z_j)\,\partial_z\phi(z_j).$$

This decomposition separates the diffusion and drift components and makes clear how each enters the discrete operator.

Discrete HJB at an interior grid point

We begin with the diffusion term. Using the centered second difference,

$$\partial_{zz}\phi(z_j) \approx \frac{\phi_{j+1} - 2\phi_j + \phi_{j-1}}{\Delta z^2},$$

we obtain

$$a_{n,s}(z_j)\,\partial_{zz}\phi(z_j) \approx d_{n,s}(z_j)\phi_{j-1} - 2d_{n,s}(z_j)\phi_j + d_{n,s}(z_j)\phi_{j+1},$$

where

$$d_{n,s}(z_j) \equiv \frac{a_{n,s}(z_j)}{\Delta z^2} = \frac{\sigma_s^2 z_j^2}{2\Delta z^2}. \tag{27}$$

Next consider the drift term. The sign of the local drift determines the upwind direction. Define

$$q_{n,s}(z_j) \equiv \frac{b_{n,s}(z_j)}{\Delta z} = \left(\mu_{n,s} + \frac{\sigma_s^2}{2}\right)\frac{z_j}{\Delta z}, \tag{28}$$

and write

$$q_{n,s}^+(z_j) = \max\{q_{n,s}(z_j), 0\}, \qquad q_{n,s}^-(z_j) = \max\{-q_{n,s}(z_j), 0\}.$$

If $q_{n,s}(z_j) \geq 0$, we use the forward difference

$$\partial_z\phi(z_j) \approx \frac{\phi_{j+1} - \phi_j}{\Delta z},$$

while if $q_{n,s}(z_j) < 0$, we use the backward difference

$$\partial_z\phi(z_j) \approx \frac{\phi_j - \phi_{j-1}}{\Delta z}.$$

These two cases can be written compactly as

$$b_{n,s}(z_j)\,\partial_z\phi(z_j) \approx q_{n,s}^-(z_j)\phi_{j-1} - \big(q_{n,s}^+(z_j) + q_{n,s}^-(z_j)\big)\phi_j + q_{n,s}^+(z_j)\phi_{j+1}.$$

Combining the diffusion and drift approximations yields the discrete HJB operator at the interior node z_j:

$$[\mathcal{B}_{n,s}\phi](z_j) \approx \big(d_{n,s}(z_j) + q_{n,s}^-(z_j)\big)\phi_{j-1} - \big(2d_{n,s}(z_j) + q_{n,s}^+(z_j) + q_{n,s}^-(z_j)\big)\phi_j + \big(d_{n,s}(z_j) + q_{n,s}^+(z_j)\big)\phi_{j+1}. \tag{29}$$

Equation (29) makes clear that the coefficient on ϕ_j is the negative of the total local outflow, while the off-diagonal terms govern movement toward adjacent productivity nodes.

Constructing the sparse generator $A_{n,s}^{\mathrm{HJB}}$

Equation (29) defines the interior rows of the sparse matrix $A_{n,s}^{\mathrm{HJB}}$ that approximates $\mathcal{B}_{n,s}$ on the productivity grid. Specifically, for an interior row j,

$$\begin{aligned} A_{n,s}^{\mathrm{HJB}}(j, j-1) &= d_{n,s}(z_j) + q_{n,s}^{-}(z_j), \\ A_{n,s}^{\mathrm{HJB}}(j, j) &= -\Big(2d_{n,s}(z_j) + q_{n,s}^{+}(z_j) + q_{n,s}^{-}(z_j)\Big), \\ A_{n,s}^{\mathrm{HJB}}(j, j+1) &= d_{n,s}(z_j) + q_{n,s}^{+}(z_j), \end{aligned} \tag{30}$$

with all other entries in row j equal to zero. Hence $A_{n,s}^{\mathrm{HJB}}$ is tridiagonal in the interior and satisfies

$$\mathcal{B}_{n,s}\phi \approx A_{n,s}^{\mathrm{HJB}}\phi.$$

This representation is useful because it translates the continuous drift–diffusion operator into a sparse linear operator that can be used directly in the HJB iteration. Once the matrices $\{A_{n,s}^{\mathrm{HJB}}\}_{n,s}$ are assembled, they can be combined with the active-sector selection and continuation values to form the semi-implicit system solved at each iteration.

Boundary rows

The first and last rows of $A_{n,s}^{\mathrm{HJB}}$ are modified to impose the boundary conditions described above. At the lower boundary, we impose a reflecting condition, so mass is not allowed to drift below the minimum productivity point. At the upper boundary, we impose a vanishing-tail condition, which approximates the fact that the value function becomes locally flat sufficiently far in the upper tail.

These boundary adjustments preserve the same logic as in the interior. Off-diagonal terms describe local movement across adjacent grid points, while the diagonal entry captures the total outflow implied by the local drift–diffusion approximation and the boundary condition. In this way, the full matrix $A_{n,s}^{\mathrm{HJB}}$ provides a discrete approximation to the continuous operator on the entire bounded grid.

Lower boundary. At the first node z_1, the process cannot move below the grid. Define

$$d_{n,s}(z_1) = \frac{\sigma_s^2 z_1^2}{2\Delta z^2}, \qquad q_{n,s}^{+}(z_1) = \max\left\{\left(\mu_{n,s} + \frac{\sigma_s^2}{2}\right)\frac{z_1}{\Delta z}, 0\right\}.$$

The lower-boundary row is

$$A_{n,s}^{\mathrm{HJB}}(1,1) = -\big(d_{n,s}(z_1) + q_{n,s}^{+}(z_1)\big), \qquad A_{n,s}^{\mathrm{HJB}}(1,2) = d_{n,s}(z_1) + q_{n,s}^{+}(z_1), \tag{31}$$

with all remaining entries in row 1 equal to zero. If the drift points downward at the lower bound, that outward motion is suppressed by the reflecting condition; if it points upward, the drift contribution is added to the upper-diagonal entry.

Upper boundary. At the last node z_J, define

$$d_{n,s}(z_J) = \frac{\sigma_s^2 z_J^2}{2\Delta z^2}, \qquad q_{n,s}^-(z_J) = \max\left\{-\left(\mu_{n,s} + \frac{\sigma_s^2}{2}\right)\frac{z_J}{\Delta z}, 0\right\}.$$

With the vanishing-upper-tail condition used in the code, the upper-boundary row is

$$A_{n,s}^{\mathrm{HJB}}(J, J-1) = 2d_{n,s}(z_J) + q_{n,s}^-(z_J), \qquad A_{n,s}^{\mathrm{HJB}}(J, J) = -\big(2d_{n,s}(z_J) + q_{n,s}^-(z_J)\big), \tag{32}$$

with all remaining entries in row J equal to zero. The factor of $2d_{n,s}(z_J)$ is the discrete counterpart of imposing a locally flat upper tail rather than reflecting mass back from an image point above the grid.

Sector comparison and cutoff recovery

Given a candidate envelope V, sectoral choice is determined by comparing the two Bellman right-hand sides evaluated on the same envelope. Define

$$D_n(z_j) \equiv \left[u_{n,\mathrm{mod}}(z_j) + [A_{n,\mathrm{mod}}^{\mathrm{HJB}} V_n]_j\right] - \left[u_{n,\mathrm{trad}}(z_j) + [A_{n,\mathrm{trad}}^{\mathrm{HJB}} V_n]_j\right]. \tag{33}$$

If $D_n(z_j) > 0$, the modern sector is preferred at (n, z_j); if $D_n(z_j) < 0$, the traditional sector is preferred. The cutoff is recovered from the first sign change of $D_n(z_j)$ along the productivity grid. If $D_n(z_j)$ and $D_n(z_{j+1})$ have opposite signs, the cutoff is obtained by linear interpolation:

$$\alpha_n = -\frac{D_n(z_j)}{D_n(z_{j+1}) - D_n(z_j)}, \qquad \bar{z}_n = z_j + \alpha_n(z_{j+1} - z_j). \tag{34}$$

If no sign change is found, the cutoff is assigned to the relevant boundary, which corresponds to a corner solution in that market.

Semi-implicit fixed-point iteration

The Bellman equation is solved by pseudo-time iteration. Let V_n^k denote the envelope at iteration k, and let $\Delta_\tau > 0$ be the pseudo-time step. The migration continuation term is evaluated at V^k, and the sector assignment at each node is chosen from the sign of (33). Let $W_{n,\mathrm{mod}}^k$ and $W_{n,\mathrm{trad}}^k$ be diagonal matrices collecting these node-by-node sector weights. In the present application they are effectively indicator matrices, so that each node is assigned to the sector that delivers the larger Bellman right-hand side.

Define the selected operator and flow vector by

$$A_n^{\mathrm{sel},k} = W_{n,\mathrm{mod}}^k A_{n,\mathrm{mod}}^{\mathrm{HJB}} + W_{n,\mathrm{trad}}^k A_{n,\mathrm{trad}}^{\mathrm{HJB}}, \qquad u_n^{\mathrm{sel},k} = W_{n,\mathrm{mod}}^k u_{n,\mathrm{mod}} + W_{n,\mathrm{trad}}^k u_{n,\mathrm{trad}}. \tag{35}$$

Then the update for the envelope solves, market by market,

$$\left[(\rho + \delta + \lambda_M) + \Delta_\tau^{-1}\right] I_J V_n^{k+1} - A_n^{\mathrm{sel},k} V_n^{k+1} = u_n^{\mathrm{sel},k} + \lambda_M \mathcal{C}_n[V^k] + \Delta_\tau^{-1} V_n^k. \tag{36}$$

This step treats the selected drift–diffusion operator implicitly and the nonlinear terms explicitly. The resulting linear system is sparse and can therefore be solved efficiently for each market.

The iteration is initialized either from the previous outer-loop value function or from a static approximation,

$$V_n^0(z_j) = \frac{\max\{u_{n,\mathrm{mod}}(z_j), u_{n,\mathrm{trad}}(z_j)\}}{\rho + \delta + \max\{\lambda_M, \varepsilon_0\}}, \qquad \varepsilon_0 > 0 \text{ small.} \tag{37}$$

Iteration continues until the sup norm of successive envelope values falls below the prescribed tolerance. Once the envelope has converged, sector-specific values are recovered from (22), and the cutoff is obtained from (34).

A.2 KFE Numerical Solution (Partial Equilibrium)

A.2.1 Summary of Equations

Once the value function has been computed, workers switch freely between sector states according to the policy implied by the Bellman comparison. It is therefore convenient to solve first for the aggregate stationary density $N_n(z)$ and then recover sectoral densities from the policy. Let

$$\pi_n^0(z) = \frac{e^{\nu V_n(z)}}{\sum_{m=1}^N e^{\nu V_m(z)}} \tag{38}$$

denote the probability that a newborn worker selects market n at productivity z, and let $\psi(z)$ denote the exogenous newborn skill density.

Given a sector assignment, define the active forward operator

$$\mathcal{A}_n^{\mathrm{act},*} = W_{n,\mathrm{mod}} A_{n,\mathrm{mod}}^{\mathrm{KFE}} + W_{n,\mathrm{trad}} A_{n,\mathrm{trad}}^{\mathrm{KFE}}, \qquad A_{n,s}^{\mathrm{KFE}} = \left(A_{n,s}^{\mathrm{HJB}}\right)', \tag{39}$$

where the diagonal matrices $W_{n,s}$ include the sectoral policy weights:

$$W_{n,s} = \mathrm{diag}\left(\omega_{n,s}\left(z_1\right), ..., \omega_{n,s}\left(z_J\right)\right). \tag{40}$$

Thus the forward operator is the discrete adjoint of the HJB generator. At each productivity level z, migration across markets is governed by the multinomial logit shares

$$m_{nm}(V(z)) = \frac{e^{\nu(V_m(z) - \xi_{nm})}}{\sum_{\ell=1}^N e^{\nu(V_\ell(z) - \xi_{n\ell})}}. \tag{41}$$

The stationary KFE for the aggregate density is

$$0 = \mathcal{A}_n^{\mathrm{act},*} N_n(z) + \lambda_M \left(\sum_{k=1}^N m_{kn}(V(z)) N_k(z) - N_n(z) \right) - \delta N_n(z) + \delta \psi(z) \pi_n^0(z). \tag{42}$$

Equation (42) makes clear that the density evolves through four channels: learning-induced drift and diffusion in productivity, migration across markets, death, and newborn entry.

After solving for the aggregate density, sectoral densities are recovered from the policy weights:

$$N_{n,\text{mod}}(z) = \omega_{n,\text{mod}}(z)N_n(z), \qquad N_{n,\text{trad}}(z) = \omega_{n,\text{trad}}(z)N_n(z). \tag{43}$$

A.2.2 Summary of Equations For Numerical Solution

As in the HJB problem, the KFE is solved on the bounded grid $\{z_j\}_{j=1}^J$. Let

$$N_n = (N_n(z_1), \ldots, N_n(z_J))' \in \mathbb{R}^J$$

denote the vector of aggregate density values in market n. The numerical problem is therefore to find $\{N_n\}_{n=1}^N$ satisfying the discretized counterpart of (42) on the finite grid.

The forward equation inherits the same treatment of boundaries used in the Bellman problem. Because the drift–diffusion operator in the KFE is the adjoint of the HJB operator, the corresponding boundary conditions are built into the first and last rows of the transposed sparse matrices. In addition, after solving the linear system, the resulting density is renormalized to satisfy the unit-mass condition

$$\sum_{n=1}^{N} \int N_n(z)\, dz = 1. \tag{44}$$

Hence the numerical KFE problem becomes a finite-dimensional sparse linear system coupled across markets through migration and newborn inflows.

A.2.3 KFE Discretization

Discrete forward operator

The discretization starts from the same sparse operators used in the HJB problem. For each market and sector, the forward operator is the transpose of the corresponding HJB matrix:

$$A_{n,s}^{\text{KFE}} = \left(A_{n,s}^{\text{HJB}}\right)'.$$

This is the standard relationship between backward and forward equations, given the uniform productivity grid. Once the sector assignment has been determined, the selected forward operator is assembled according to (39). As a result, within each market the density evolves under the law of motion implied by the sector chosen at each productivity node.

Stacked system across markets

Let $\boldsymbol{n} \in \mathbb{R}^{NJ}$ stack the densities $\{N_n(z_j)\}_{n=1,\ldots,N}^{j=1,\ldots,J}$. The stationary KFE (42) can then be written as a global sparse linear system

$$G\boldsymbol{n} + \boldsymbol{b} = 0, \tag{45}$$

where the operator G can be decomposed into two components. First, each market block contains the within-market forward operator $\mathcal{A}_n^{\text{act},*}$ together with the death term $-\delta I_J$. Second, for each

productivity node, markets are coupled through the migration matrix

$$\lambda_M\big(m(V(z_j))' - I_N\big), \tag{46}$$

where $m(V(z_j))$ collects the shares in (41). The vector $\boldsymbol{b}$ collects newborn inflows of the form $\delta\psi(z_j)\pi_n^0(z_j)$.

Writing the problem in this stacked form is useful because all interactions—drift, diffusion, migration, death, and entry—are represented in a single sparse linear system. Equivalently, one may write

$$(-G)\boldsymbol{n} = \boldsymbol{b}. \tag{47}$$

This system is solved directly using sparse linear-algebra methods.

Post-processing and recovery of sectoral densities

After solving (47), the vector solution is reshaped into an $N \times J$ array. Small negative entries generated by numerical error are set to zero, and the distribution is renormalized so that (44) holds exactly on the grid. The sectoral distributions are then reconstructed from (43). These distributions are the objects that enter the updating formulas for vibrancy, drifts, and the transfer in general equilibrium.

A.3 General-Equilibrium Solution Algorithm

A.3.1 Summary of Equations

General equilibrium requires solving jointly for the aggregate stationary density $\{N_n(z)\}_{n=1}^N$, the free-switching value function $\{V_n(z)\}_{n=1}^N$, and the cutoff vector $\{\bar{z}_n\}_{n=1}^N$. It also requires the learning drifts $\{\mu_{n,\text{mod}}, \mu_{n,\text{trad}}\}_{n=1}^N$ and the transfer b. The fixed-point problem combines the Bellman equation (20), the stationary forward equation (42), the learning rule (14), and the balanced-budget transfer condition

$$b = \tau \sum_{n=1}^{N} \int N_{n,\text{mod}}(z) w_{n,\text{mod}}(z)\, dz. \tag{48}$$

An equilibrium is therefore a collection of objects such that: (i) workers optimize given the stationary distribution and transfer; (ii) the stationary distribution is consistent with optimal behavior; (iii) learning drifts are induced by the resulting sectoral distributions; and (iv) the transfer is financed by modern-sector labor tax revenues.

A.3.2 Summary of Equations For Numerical Solution

The numerical equilibrium problem is a fixed point in distributions, values, cutoffs, drifts, and the transfer. It is convenient to use the cutoff vector as the main convergence object because the sectoral allocation enters both the Bellman and the forward equations through the node-by-node sector assignment.

The outer iteration starts from an initial density that is Pareto in productivity and assigns positive mass to every market. Specifically, let

$$g_0(z) = \frac{\theta_p \kappa^{\theta_p}}{z^{\theta_p+1}}, \qquad z \geq \kappa, \tag{49}$$

be the initial productivity density, projected onto the computational grid and renormalized. Combining this with market weights $\{\omega_n^0\}_{n=1}^N$ yields the initial market distributions

$$N_n^0(z) = \omega_n^0 g_0(z), \qquad \sum_{n=1}^{N} \int N_n^0(z)\, dz = 1. \tag{50}$$

The initial cutoff in each market is set equal to the mean productivity under N_n^0, and the transfer starts from an initial scalar guess.

Given a current iterate $(N^r, \bar{z}^r, b^r)$, one computes the implied sectoral distributions, updates the learning drifts from (14), solves the HJB, solves the stationary KFE, updates the transfer from (48), and checks whether the cutoff vector has converged. This yields a finite-dimensional fixed-point iteration over the equilibrium objects.

A.3.3 General Equilibrium Solution Algorithm

Let superscript r denote the general-equilibrium iteration.

Step 1. Split the current distribution across sectors. Given $(N^r, \bar{z}^r)$, construct the policy weights $(\omega_{n,\text{mod}}^r, \omega_{n,\text{trad}}^r)$ and the sectoral densities using (11).

Step 2. Update learning objects. Compute the within-sector densities (12), the vibrancy indexes (13), and the drift terms (14).

Step 3. Update wages and the transfer. Compute the wage schedules from (15) and update the transfer using the balanced-budget rule (48).

Step 4. Solve the worker problem. Using the current drifts and transfer, solve the envelope Bellman equation by the finite-difference procedure described in Section A. This yields updated value functions and a new cutoff vector $\bar{z}^{r+1}$.

Step 5. Solve for the stationary distribution. Holding the updated policy fixed, solve the stationary KFE by the method described in Section B. This yields an updated aggregate distribution N^{r+1} and, through (43), updated sectoral densities.

Step 6. Check convergence. Compute

$$\max_n |\bar{z}_n^{r+1} - \bar{z}_n^r|.$$

If this distance is below the prescribed tolerance, stop. Otherwise set $r \leftarrow r+1$ and return to Step 1.

The output of this outer loop is a stationary equilibrium in which the value function, the sectoral

allocation, the stationary distribution, the drift terms, and the transfer are mutually consistent.

B Empirical Appendix

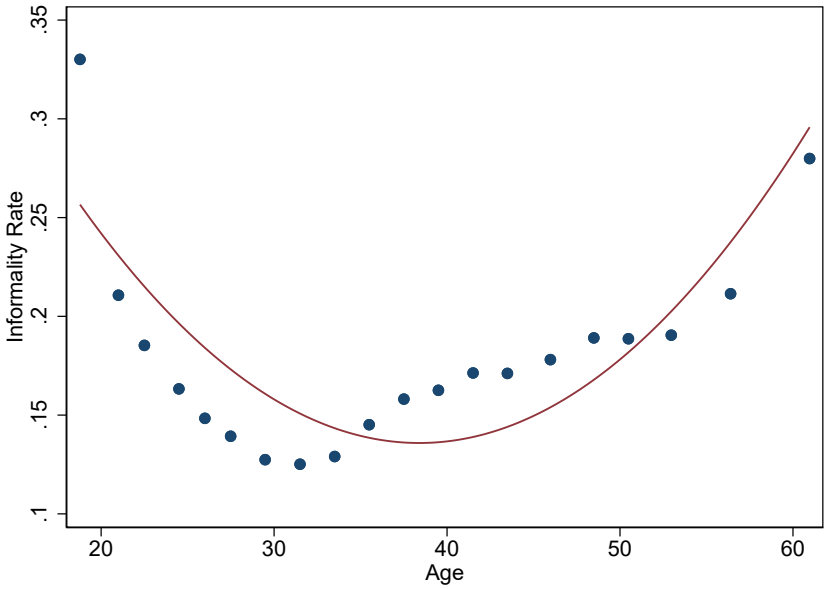
B.1 Additional Descriptive Statistics

This section reports additional descriptive statistics for informal workers in Chile. Although slightly inconsistent with the notation in the main text, we refer to modern workers as formal and traditional workers as informal, following the definitions we use in the data. We confirm previously well-established informality patterns for the Chilean economy:[15]

1. Informality and wage follow a U-shaped pattern.
2. Informality decreases with educational attainment.
3. Informality decreases with firm size.

Then, we report informality rates by occupation and economic sector.

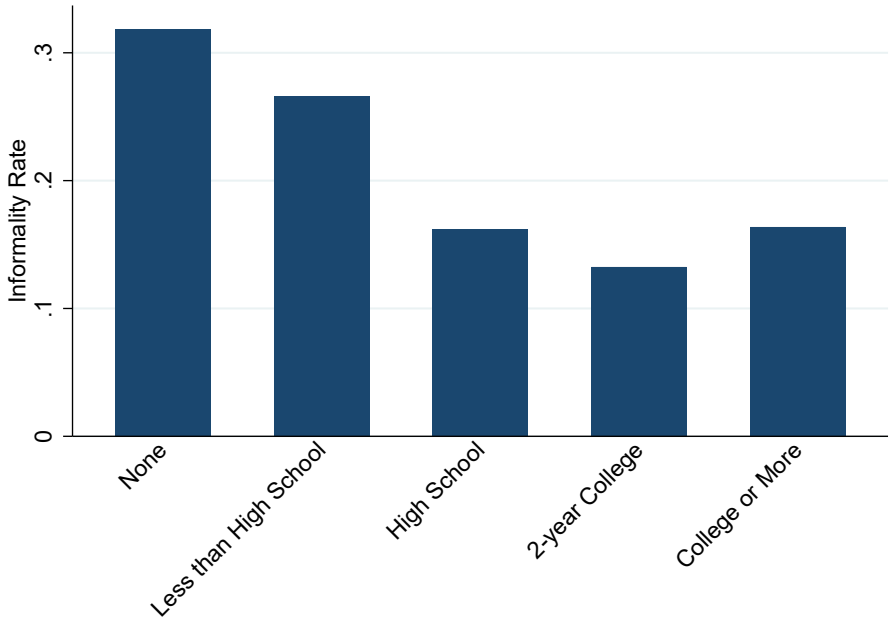
Figure 5: Informality Rate by Age



Notes: Figure 5 displays a binscatter plot of informality rates and age. We consider 20 equally-sized bins according to worker's age. Moreover, we restrict our sample to salaried workers, excluding self-employed workers, entrepreneurs, workers without remuneration, and public-sector and armed forces employees.

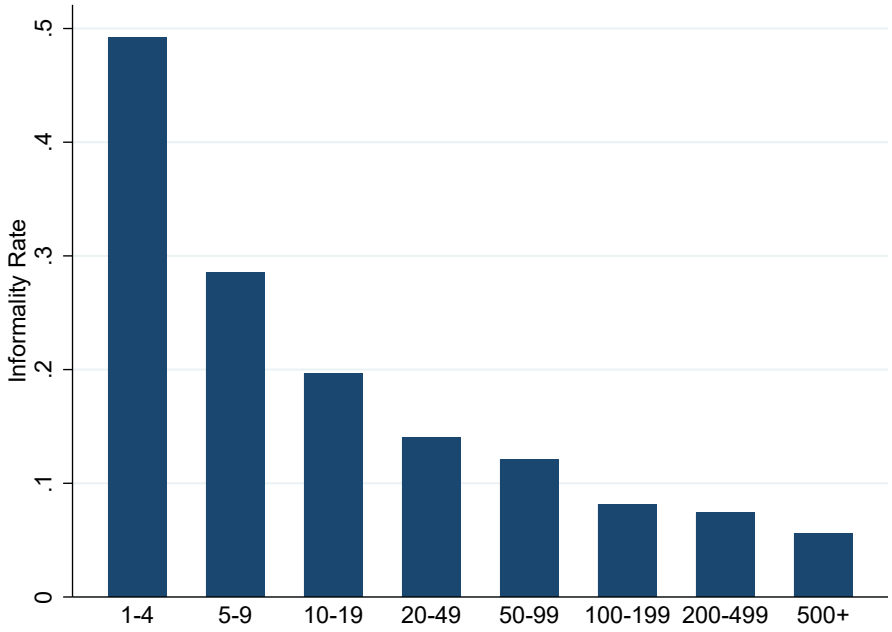
[15] See Ulyssea (2020a) for a comprehensive review of informal workers empirical regularities.

Figure 6: Informality Rate by Educational Attainment



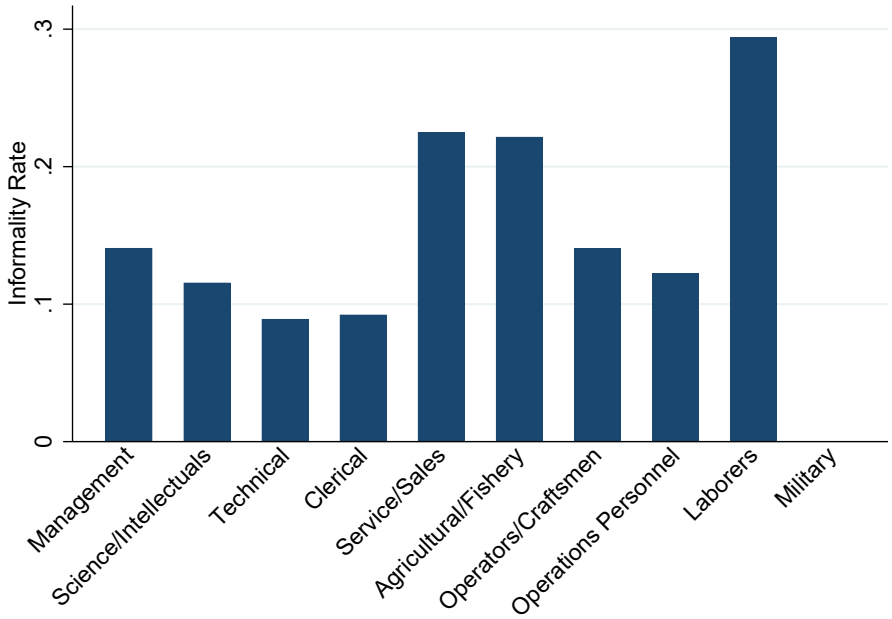
Notes: Figure 6 displays informality rates by education groups. We restrict our sample to salaried workers, excluding self-employed workers, entrepreneurs, workers without remuneration, and public-sector and armed forces employees.

Figure 7: Informality Rate by Firm Size



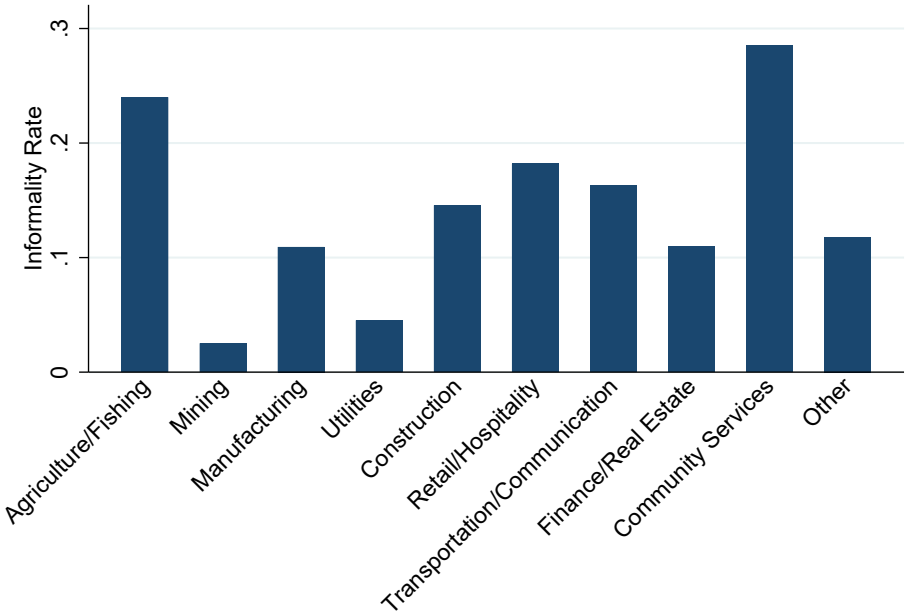
Notes: Figure 7 displays informality rates by firm size. Bars denote firm size groups, and labels denote the range of employees in each group. We restrict our sample to salaried workers, excluding self-employed workers, entrepreneurs, workers without remuneration, and public-sector and armed forces employees.

Figure 8: Informality Rate by Occupation



Notes: Figure 8 displays informality rates by occupations. We restrict our sample to salaried workers, excluding self-employed workers, entrepreneurs, workers without remuneration, and public-sector and armed forces employees.

Figure 9: Informality Rate by Sector

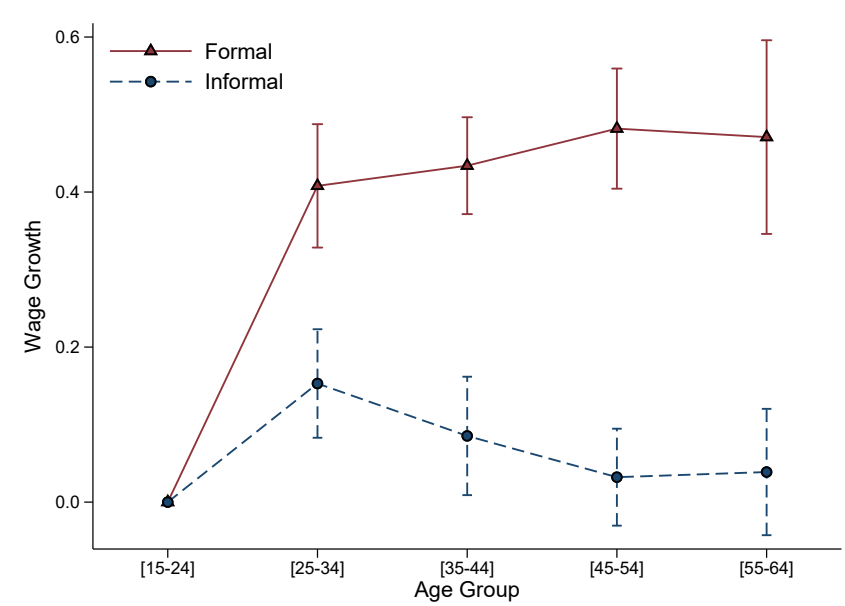


Notes: Figure 9 displays informality rates by economic sector. We restrict our sample to salaried workers, excluding self-employed workers, entrepreneurs, workers without remuneration, and public-sector and armed forces employees.

B.2 Additional Figures on Wages Over the Life Cycle

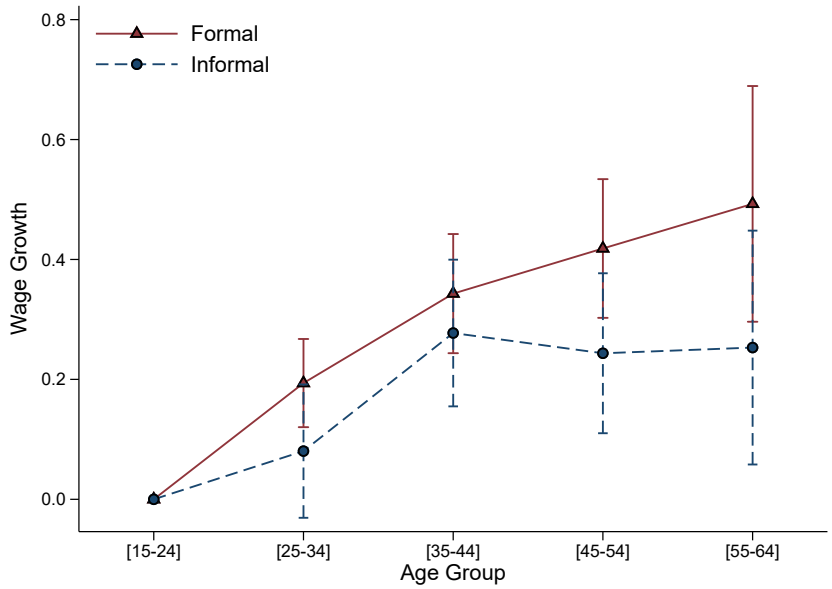
In this section, we report additional figures on wages over the life cycle for Chile. We begin by presenting wage profiles over the life cycle for all workers in the economy, including entrepreneurs, self-employed individuals, and public sector employees. Then, focusing exclusively on salaried workers, we present wage-age profiles by controlling for observable worker characteristics, as well as wage-experience profiles, both in the raw data and after controlling for observables.

Figure 10: Wages Over the Life Cycle for Formal and Informal Workers, All Workers



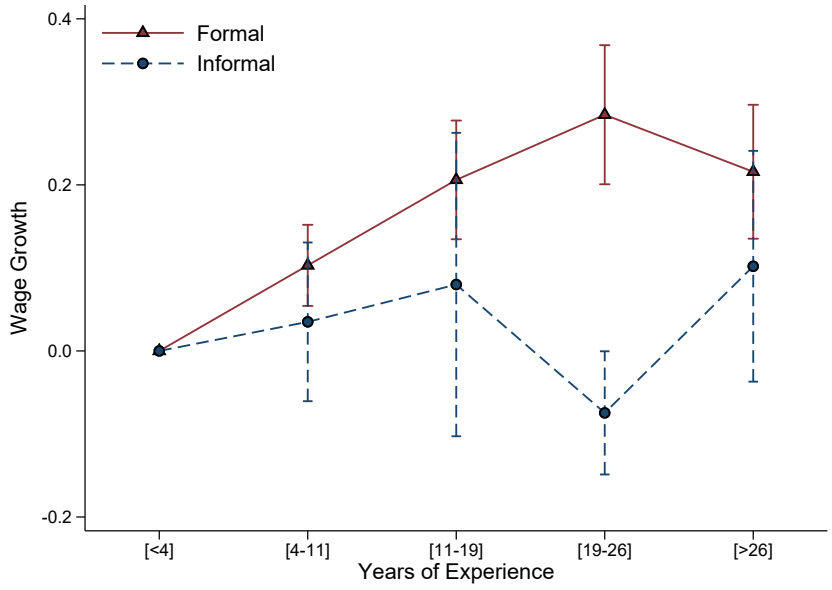
Notes: Figure 10 displays wage paths over the life cycle for formal and informal workers. We consider all workers in the economy, including self-employed workers, entrepreneurs, workers without remuneration, and public-sector and armed forces employees. We compute the average wage for each 10-year age bin relative to the average wage for workers less than 24 years. Vertical dashed lines denote 90 percent confidence intervals.

Figure 11: Wages Over the Life Cycle for Formal and Informal Workers, Controls



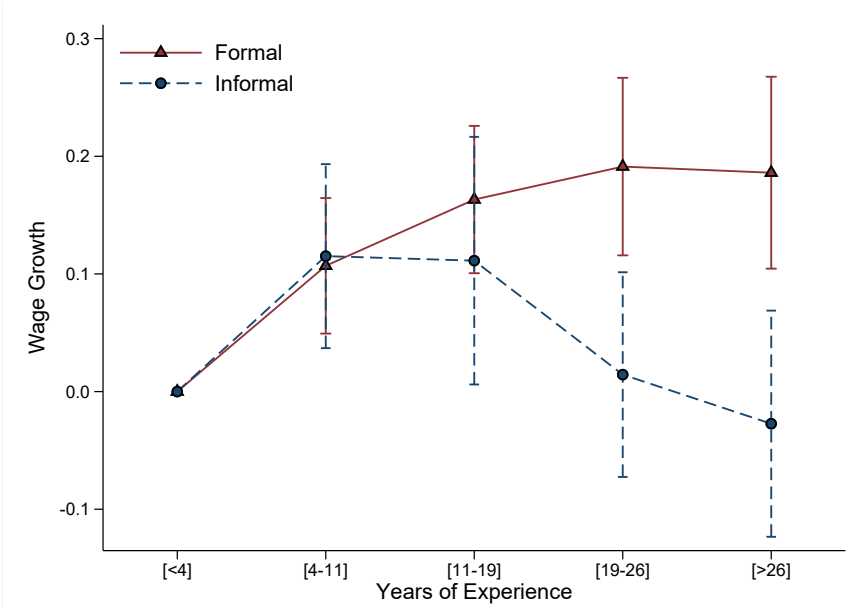
Notes: Figure 11 displays wage paths over the life cycle for formal and informal workers while residualizing wages by: education, gender, occupation, sector, region, firm size, and year fixed effects, respectively. We restrict our sample to salaried workers, excluding self-employed workers, entrepreneurs, workers without remuneration, and public-sector and armed forces employees. We compute the average wage for each 10-year age bin relative to the average wage for workers less than 24 years. Vertical dashed lines denote 90 percent confidence intervals.

Figure 12: Wages-Experience Profiles



Notes: Figure 12 displays wage-experience paths for formal and informal workers. We restrict our sample to salaried workers, excluding self-employed workers, entrepreneurs, workers without remuneration, and public-sector and armed forces employees. We compute the average wage for each experience bin relative to the average wage for workers less than 4 years of experience. Vertical dashed lines denote 90 percent confidence intervals.

Figure 13: Wages-Experience Profiles, Controls



Notes: Figure 13 displays wage-experience paths for formal and informal workers, while residualizing wages by: education, gender, occupation, sector, region, firm size, and year fixed effects, respectively. We restrict our sample to salaried workers, excluding self-employed workers, entrepreneurs, workers without remuneration, and public-sector and armed forces employees. We compute the average wage for each experience bin relative to the average wage for workers less than 4 years of experience. Vertical dashed lines denote 90 percent confidence intervals.

C Quantitative Appendix

This appendix describes how the solved stationary equilibrium is converted into a synthetic worker panel for estimation. The purpose of the simulation is to translate the equilibrium objects from the computational appendix into annual worker histories that can be summarized in the same moments observed in the data.

C.1 Simulation of Worker Panel

The simulation starts from the stationary equilibrium characterized in the computational appendix. In particular, it uses four objects from the solved model:

1. The stationary distribution over market and productivity states, denoted by $N_n(z)$ in continuous time and by the stacked vector $\boldsymbol{n} \in \mathbb{R}^{NJ}$ on the discretized grid.
2. The policy rule $(\omega_{n,\text{mod}}(z), \omega_{n,\text{trad}}(z))$, which determines the probability of modern and traditional work at each state.
3. The associated wage schedules $(w_{n,\text{mod}}(z), w_{n,\text{trad}}(z))$.
4. The continuous-time law of motion for the joint market–productivity process, summarized by the generator matrix Q.

The last object is the key input for the simulation. Let Q denote the continuous-time generator for the discretized joint state (n, z), where n indexes markets and z indexes productivity on the grid. Thus, Q is an $NJ \times NJ$ matrix that summarizes the same state transitions used in the stationary KFE. Within each market it contains productivity drift and diffusion, and across markets it contains migration flows governed by the value-based migration rule.[16] In this sense, the simulation uses exactly the same law of motion as the stationary distribution problem rather than introducing a separate transition approximation.

To simulate the model at a monthly frequency, we convert this continuous-time generator into a one-step transition matrix,

$$\Pi(\Delta t) = \exp\big(\Delta t\, Q\big), \qquad \Delta t = 1/12.$$

This matrix gives the probabilities of moving from one discretized (n, z) state to another over one month.

A synthetic cross section of workers is then initialized by drawing states from the stationary distribution implied by the solved equilibrium. The code also assigns each worker an initial level of

[16] Formally, for each market n, Q reproduces the first two terms of the stationary KFE in the computational appendix,

$$\mathcal{A}_n^{\text{act},*} N_n(z) + \lambda_M \left(\sum_{k=1}^{N} m_{kn}(V(z)) N_k(z) - N_n(z) \right).$$

Deaths and newborn inflows are added separately in the simulation.

accumulated experience so that the starting cross section is close to the stationary age profile generated by the death process. Conditional on a worker's current (n, z) state, modern or traditional sector status is drawn using the model-implied policy weights. Wages are then assigned from the corresponding equilibrium wage schedule.

Each simulated month has three steps. First, given the worker's current state, the simulation draws whether the worker is in the modern or traditional sector during that month and updates total, modern, and traditional experience accordingly. Second, the worker moves across market–productivity states according to the transition matrix $\Pi(1/12)$. Third, the worker faces the exogenous death hazard from the model. When a death occurs, that worker is replaced by a newborn entrant drawn from the model-implied entry distribution over markets and productivity. Newborn workers start with zero experience.

It is useful to separate the roles of these two mechanisms. The matrix Q governs movement across the discretized market–productivity state space, while death and replacement are imposed separately in the simulation because the code tracks individual worker histories. This is why turnover is handled outside the matrix transition step even though the stationary distribution itself is consistent with the same birth–death structure.

At the end of each simulated year, the code records one observation for every worker. The resulting annual panel contains worker identifiers, calendar year, market, sector status, wage, log wage, and the worker's accumulated total, modern, and traditional experience. In the baseline implementation, the panel contains 2,500 workers simulated for 15 years.

C.2 Construction of Model Moments

The annual simulated panel is then used to construct the moments matched in estimation.

The first set of moments is the wage-transition matrix. In each simulated year, workers are assigned to wage quartiles using that year's cross-sectional wage distribution. The code then links observations in year t and year $t + 1$ by worker identifier and computes the implied 4×4 transition matrix across quartiles. Averaging these matrices across adjacent years yields the model counterpart to the empirical wage-transition moments.

The second set of moments is the modern–traditional transition matrix. Using the same year-to-year matching procedure, the code computes the 2×2 transition matrix between modern and traditional employment and averages it across adjacent years. Because deceased workers are replaced with new identifiers, these transitions are constructed only from workers who are observed in consecutive annual snapshots.

The remaining moments are cross-sectional summaries of the simulated equilibrium. The traditional-sector employment share is the average share of worker-year observations in traditional employment. The modern-sector wage premium is computed directly from the solved wage schedules as the stationary-mass-weighted average of the ratio of modern to traditional wages across states. Finally, the model's young-worker wage-dispersion moment is the variance of log wages among workers with at most two years of total experience in the simulated annual panel.