Inflation Expectations, Liquidity Premia and Global Spillovers in Japanese Bond Markets

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&

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Abstract

Using a novel arbitrage-free dynamic term structure model of nominal and real yields, we account for liquidity premia and the deflation protection afforded by Japanese inflation-indexed bonds, known as JGBi's. Adjusting for JGBi liquidity premia lowers the estimated value of JGBi deflation protection and raises inflation risk premium estimates, while long-term inflation expectations remain relatively stable. Regression analysis using our estimated JGBi liquidity measure and bid-ask spreads of JGBs confirms the existence of significant spillovers to Japanese bond markets from global bond market illiquidity, as proxied by periods of low U.S. Treasury market depth.

JEL Classification: C32, E43, E52, G12, G17

Keywords: affine arbitrage-free term structure model, deflation risk, deflation protection, liquidity spillovers, inflation swaps

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1 Introduction

Breakeven inflation (BEI)—the difference between yields on comparable-maturity nominal and real debt—is a popular market-based indicator of inflation expectations. Long-term BEI is frequently used to measure the credibility of the central bank's inflation objective.[1] However, BEI is affected by both inflation and other risk premia. These need to be purged from estimated BEI to obtain accurate measurements of investors' inflation expectations. In this paper, we make such adjustments to prices of inflation-indexed Japanese government bonds, commonly known as JGBi.

Japanese inflation has been quite low for decades, far below the Bank of Japan's (BOJ) two-percent inflation target adopted in 2013. Japanese inflation expectations have remained around one percent in surveys such as Consensus Forecasts over this period, and appear to have been further depressed following the onset of the COVID-19 pandemic.[2] Most recently, Japanese inflation has risen above the BOJ two-percent target, in tandem with elevated inflation levels in other advanced economies. This recent movement in realized inflation elevates the need to properly gauge Japanese inflation expectations in the current environment.

This paper extends the joint model of Japanese government bond (JGB) nominal and real yields described in Christensen and Spiegel (2022, henceforth CS (2022)) to account for liquidity risk premia in Japanese indexed bonds, building on our previous model which only accounted for the value of JGBi "deflation protection."[3]

As discussed in Cardozo and Christensen (2025), inflation-indexed bonds are likely to be much less traded than nominal bonds, as they provide a natural hedge against inflation risk. Moreover, the inflation hedge applies disproportionately to domestic investors, whose consumption expenditures track the local CPI. As a consequence, the trading of inflation-indexed bonds should be concentrated among relatively "patient" domestic buy-and-hold investors, such as domestic pension funds and insurance companies.[4] The relatively modest trading in these instruments likely raises potential liquidity issues, and they should offer investors some premium for bearing the associated liquidity risk. We therefore use our extended model to examine the extent and time variation of this liquidity premium.

[1] Under credible inflation objectives, long-term inflation expectations should remain proximate to the central bank's target, as deviations from the target should be considered temporary.

[2] The pandemic, and associated shutdowns, disrupted economic activity and led to global financial market volatility. The BOJ responded to the turmoil with a number of monetary policy easings, including both policies designed to lower interest rates and policies aimed at easing financial conditions. Fiscal policy also responded aggressively. However, Christensen and Spiegel (2023) find that announcements of policy changes yielded only modest immediate responses in inflation expectations.

[3] JGBi deflation protection, offered since 2013, entails paying off the original nominal principal at maturity if cumulative deflation had occurred since issuance. These enhancements are particularly important for our sample, which contains extended periods with low and often negative Japanese inflation. CS (2022) accounts for the value of this deflation protection option using an adaptation of the approach of Christensen et al. (2012).

[4] Cardozo and Christensen (2025) confirm this conjecture for the Colombian inflation-indexed bond market, while Beauregard et al. (2024) report similar findings for the Mexican inflation-indexed bond market.

To account for any JGBi price distortions arising from such concentration among the investors in these bonds, we follow Andreasen et al. (2021, henceforth ACR (2021)) and augment the model used by CS (2022) with a bond-specific risk factor to adjust for any bond-specific risk premia in JGBi prices.

The identification of the bond-specific risk factor comes from its unique loading, which is a function of both the time since issuance and the remaining time to maturity. We posit that an increasing fraction of the outstanding value of a given security is likely to be held in buy-and-hold investors' portfolios over time. Increased time since issuance raises the sensitivity of the security to variation in the market-wide bond-specific risk factor. By observing a cross section of security prices over time, this factor can be separately identified. To test this conjecture, we run panel regressions that demonstrate that bond ages are positively correlated with bid-ask spreads of individual JGBi's at statistically significant levels. This dynamic pattern provides support for our assumed model structure.

Our analysis also accounts for the value of the deflation protection option embedded in Japanese inflation-indexed bond contracts issued since 2013. We also identify bond investors' underlying inflation expectations and inflation risk premia, as in Christensen et al. (2010). Finally, to obtain the appropriate persistence of the dynamic factors in the model, we follow Kim and Orphanides (2012) and incorporate long-term forecasts of inflation from surveys of professional forecasters.

Our results demonstrate that the average estimated JGBi liquidity premium series exhibits significant variation, and even switches sign. Furthermore, properly accounting for these liquidity premia significantly lowers the estimated value of the indexed bonds' deflation protection. After adjustment for both of these distortions in Japanese BEI rates, we find that long-term Japanese inflation expectations have remained relatively stable at levels modestly exceeding one percent during the pandemic period. In addition, the estimated inflation risk premia are more stable and less negative than they appear without accounting for the liquidity risk of JGBi's.

Our liquidity risk estimates indicate that bond illiquidity spiked towards the end of our sample, probably due to the BOJ "yield curve control" policy.[5] This policy removed a significant amount of JGBs from the open market, and lowered JGB liquidity, particularly in the 10-year segment of the market (e.g. Shiratsuka (2025)). Properly accounting for these spikes in liquidity premia during this period significantly lowers the estimated value of the bonds' deflation protection and affects estimated inflation risk premia.

As bond-specific JGBi premia can result in significant distortions in Japanese BEI rates, we also examine their implications for the pricing of Japanese inflation swap contracts.[6] We

[5]Under yield curve control, the BOJ stood ready to purchase Japanese Government Bonds (JGBs) as needed to maintain its target for the 10-year JGB yield.

[6]In an inflation swap contract, the owner of a long position pays a fixed premium in exchange for a floating payment equal to the change in the CPI. At inception, the fixed premium is set such that the contract has a value of zero.

find that, while inflation swap rates may appear to be priced consistently with BEI rates implied by JGBi yields, proper adjustments for the estimated JGBi bond-specific premia are likely to reveal that Japanese inflation swaps may be systematically mispriced by a seemingly wide margin. Thus, our estimated JGBi premia could point to the existence of significant arbitrage opportunities in the Japanese inflation swap market. However, we leave it for future research to explore that conjecture further.

We also utilize our estimated series for JGBi liquidity premia to investigate the degree of market segmentation in the Japanese inflation-indexed bond market. Japanese asset markets are commonly believed to exhibit some segmentation due to "home bias" among Japanese investors in their asset holdings (e.g. Kang and Stulz (1997)). Moreover, the guarantees afforded by inflation-indexed and deflation-protected Japanese JGBi's are likely disproportionately valuable to domestic agents, whose consumption is more exposed to Japanese inflation risk. We use weekly liquidity estimates of U.S. Treasury market depth, measured as the number of open orders from surveyed dealers for the most liquid on-the-run U.S. Treasury securities from December 2009 through December 30, 2022, as a proxy for global bond market liquidity. The U.S. market is commonly considered the most liquid global bond market, and movements in that market have been characterized as reflecting overall global financial conditions; see Rey (2015) for an example.

Using this data, we examine the implications of notable illiquidity events in U.S. Treasury markets, defined as market depth two standard deviations or more below series means, for JGBi liquidity premia. As these shocks directly measure liquidity movements, they are distinct from classic flight-to-safety events, which tend to improve both Japanese and U.S. bond market conditions.[7] We regress our generated JGBi liquidity premium series on indicators of low levels of market depth in U.S. Treasuries at various maturities, as well as other conditioning variables to account for general movements in U.S. and Japanese market conditions. Note that while our generated liquidity premium series is estimated with error, the series is the dependent variable in our specification and any estimation errors are unlikely to cause coefficient estimate bias beyond attenuation bias.

Our results indicate that exceptionally low levels of U.S. Treasury depth are associated with elevated estimates of JGBi liquidity premia. Our point estimates indicate that this relationship is economically important, as low levels of market depth in U.S. Treasuries at the 2-, 5-, or 7-year maturities are associated with over 2 standard deviation predicted increases in the estimated Japanese JGBi liquidity premia, while low market depth in the 10-year segment of the U.S. Treasury market is associated with a still notable 0.90 standard deviation increase in the estimated Japanese JGBi liquidity premia. We interpret these results as evidence of spillovers from illiquidity and market stress in the U.S. Treasury market, implying that segmentation of the Japanese JGBi market is incomplete.

[7] In Japan, this is due to Japanese investors moving funds back to Japan, while in the U.S. the improvement is driven by foreigners seeking safer assets.

As a robustness check and to examine spillovers into the conventional JGB market, we repeat the regression exercise using instead an average of on-the-run JGB bid-ask spread series as the dependent variable. The results are again statistically significant and confirm that spillovers from low U.S. Treasury market liquidity extend to JGB market liquidity as well. This makes us extra confident in concluding that Japanese bond markets are less segmented from global bond markets than widely perceived.

The remainder of this paper is structured as follows. Section 2 describes the Japanese government bond data, including our analysis of the connection between JGBi bid-ask spreads and bond age. Section 3 details our benchmark model used to decompose the nominal and real bond yields into underlying expectations components and residual risk premia, while accounting for bond-specific liquidity premia and the values of deflation protection offered by indexed bonds. It then estimates our model and summarizes its results. Section 4 examines the estimated bond-specific liquidity premia and values of deflation protection and studies their implications for the pricing of inflation swaps, while Section 5 examines the implications of our empirical BEI decomposition. Section 6 examines the evidence for global spillovers from U.S. Treasury markets to our JGBi liquidity premium estimates and JGB bid-ask spreads. Concluding comments are provided in Section 7.[8]

2 Japanese Government Bond Data

The Japanese government bond market is large and liquid by international standards. As of December 2022, the total outstanding notional amount of marketable bonds issued by the government in Japan was 1,197.6 trillion yen, of which close to 1 percent represented inflation-indexed bonds.[9] In total, Japanese government debt reached 263.9% of Japanese nominal GDP in 2022, far above the level of any other major industrialized country.[10]

2.1 Nominal Bonds

We follow CS (2022) and extend the Japanese nominal government bond yield series in Kim and Singleton (2012), which originally ended in March 2008, with Japanese nominal government zero-coupon yields through December 2022.[11] This data set contains six maturities: six-month yields and one-, two-, four-, seven-, and ten-year yields, with all yields being continuously compounded and available at daily frequency. We examine the data at

[8] An online appendix contains technical details on yield decompositions within our models along with miscellaneous estimation and robustness results.

[9] Source: https://www.mof.go.jp/english/policy/jgbs/publication/newsletter/jgb2023_04e.ps

[10] Source: tradingeconomics.com/japan/government-debt-to-gdp

[11] Extension data through the end of October 2022 is downloaded from Bloomberg, as in Christensen and Rudebusch (2015), while the data for November and December 2022 are constructed directly from nominal JGB prices downloaded from Bloomberg using the Svensson (1995) yield curve; see Andreasen et al. (2019) for details.

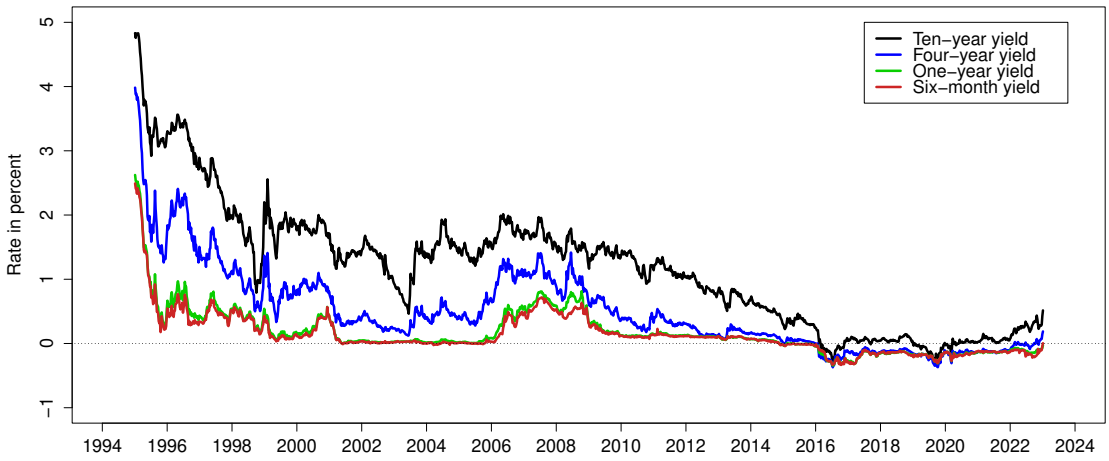


Figure 1: **Japanese Nominal Government Bond Yields**
Illustration of the Japanese nominal government zero-coupon bond yields with maturities of six months, one year, four years, and ten years. The data series are weekly covering the period from January 6, 1995, to December 30, 2022.

weekly frequency using Friday observations or the most recently available whenever Friday observations are missing.

Figure 1 shows the persistent drop in yields since the mid-1990s for four of our nominal yields.[12] We also observe a persistent decline in the yield spreads. The spread between the ten- and one-year yield was larger than 200 basis points at the start of the sample and less than 50 basis points at the end of the sample. We follow Kim and Singleton (2012), who find that a two-factor model is adequate to fit their data, and use a two-factor model for the nominal yields. Furthermore, we take the compressed term structure of the past decade as evidence that liquidity premia are likely to play only a modest role for the nominal yield series, and make no further adjustments to those yields.[13]

That choice is also supported by the available data on bid-ask spreads in the regular JGB market. Figure 2 shows the average daily bid-ask spreads of JGBs across four representative maturities downloaded from Bloomberg. With the exception of the COVID-19 pandemic period, the bid-ask spreads have been low and stable with values slightly above one basis point, which underscores the high liquidity in this market.

[12]To maintain the readability of Figure 1, we display four of the six yield series we use in the model estimation. The nominal yields not shown exhibit a similar pattern.

[13]While the BOJ's purchases of close to 45 percent of all outstanding JGBs by the end of our sample raises the possibility of illiquidity, both Kurosaki et al. (2015) and Sakiyama and Kobayashi (2018) find no evidence of market impairment during our sample period. Nishikazi et al. (2013) do find minute-by-minute spikes in JGB daily bid-ask spreads on April 5, 2013, when circuit breakers in the JGB market were triggered twice to halt transactions due to rapid price movements, but even here, the bid-ask spreads seem to have substantively stabilized that same day.

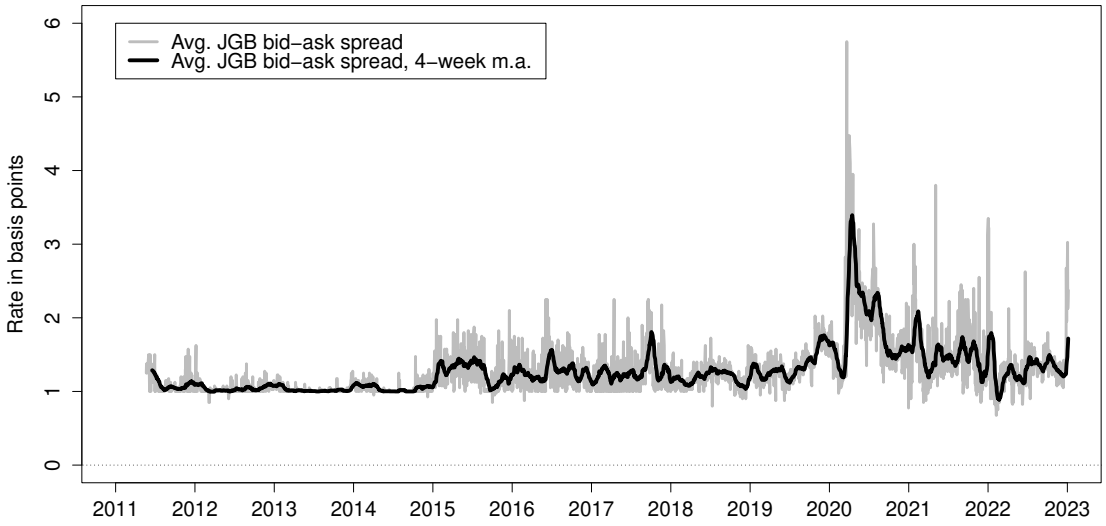


Figure 2: **Average Bid-Ask Spreads of JGBs**
Illustration of the average bid-ask spread of JGBs downloaded from Bloomberg across four constant maturities (2yr, 5yr, 7yr, and 10yr). For each day, the average of the bid-ask spreads is calculated. Also shown is the four-week moving average of the daily average bid-ask spread series. The data series is daily covering the period from May 31, 2011, to December 30, 2022.

The periodic asymmetric behavior of Japanese short-term nominal bond yields in the neighborhood of the zero lower bound is evident in Figure 1. The main approach for accounting for this unique bond yield behavior is known as shadow-rate term structure models. These are Gaussian models in which the process for the instantaneous nominal short-term interest rate, r_t^N, is instead referred to as a shadow-rate process s_t^N and the instantaneous nominal short-term interest rate used for discounting nominal cash flows is its truncated version, $r_t^N = \max\{s_t^N, 0\}$.

We encounter two primary challenges with the shadow-rate approach for Japanese bonds. First, the shadow-rate models are only supported by the data until January 29, 2016, when the Bank of Japan introduced negative interest rates. Once negative interest rates have been introduced, it is not clear what the effective lower bound is.[14]

For example, the four-year yield in our data has been as low as -0.38 percent, well below the -0.10 percent minimum of the policy overnight rate set by the Bank of Japan. Moreover, policy rates have been even lower in other countries, such as Denmark, Sweden, and Switzerland after the introduction of negative interest rates. Given that, our Gaussian approach should provide a reasonable fit to the data, at least to a first order.

Second, the results in CS (2022) and in this paper demonstrate that it is of first order

[14]See Christensen (2019) for evidence of the yield curve response to the introduction of negative interest rates in multiple jurisdictions, including Japan.

importance to account for the values of deflation protection offered by JGBi's. Unfortunately, we are unable to calculate these option values using the shadow-rate modeling approach. The need to account for JGBi deflation protection reinforces our choice to proceed with a Gaussian model.

Finally, it is clear that bond yields in Japan are subject to some variation in their volatility. However, the term structure literature has largely ignored bond yield second moment variation and instead has focused on modeling the first moment and the persistence of the state variables that constitute the primary determinants of forecast performance, yield decompositions, and the behavior of bond returns (e.g. Duffee (2002)). Our paper follows in that tradition. Moreover, it would also be challenging to extend our model to accommodate stochastic yield volatility, while also accounting for JGBi deflation protection. As a result, we leave this issue for future research.

2.2 Real Bonds

The Japanese government has issued inflation-indexed bonds—known as JGBi—since the spring of 2004. These are all ten-year bonds, which were issued in two separate periods: From March 2004 until June 2008, a total of 16 bonds were issued with a nearly quarterly frequency. The program was then temporarily halted in the aftermath of the global financial crisis, but new inflation-indexed bonds have been issued roughly once a year since the fall of 2013. These are government bonds whose principal amount fluctuates in proportion with the consumer price index (CPI) excluding fresh food.

Issuance since 2013 included the deflation protection enhancement noted in the introduction. These bonds are guaranteed to pay off at least at par at maturity, even if there was net deflation between the issuance and maturity dates.[15] Table 1 contains the contractual details of all 27 JGBi's in our sample as well as their individual number of weekly observations.

The distribution of individual JGBi's for each date in our sample is illustrated in Figure 3(a). Each bond's trajectory over time in terms of remaining years to maturity is represented by a diagonal solid black line that starts at its date of issuance with a value equal to its original maturity and ends at zero on its maturity date. The two waves of issuances of JGBi's are clearly visible.

The solid grey rectangle in Figure 3(a) indicates the sub-sample of bonds used in our empirical analysis. The sample is restricted to start on January 7, 2005, and limited to inflation-indexed bond prices with more than one year remaining to maturity.

Figure 3(b) shows the distribution of available JGBi's in the sample across time. Our sample starts with three bonds and increases to sixteen bonds by 2008. The number of bonds available then gradually began declining in 2011, as bonds from the first wave of issuances started to mature. However, since 2018, the number of outstanding bonds has

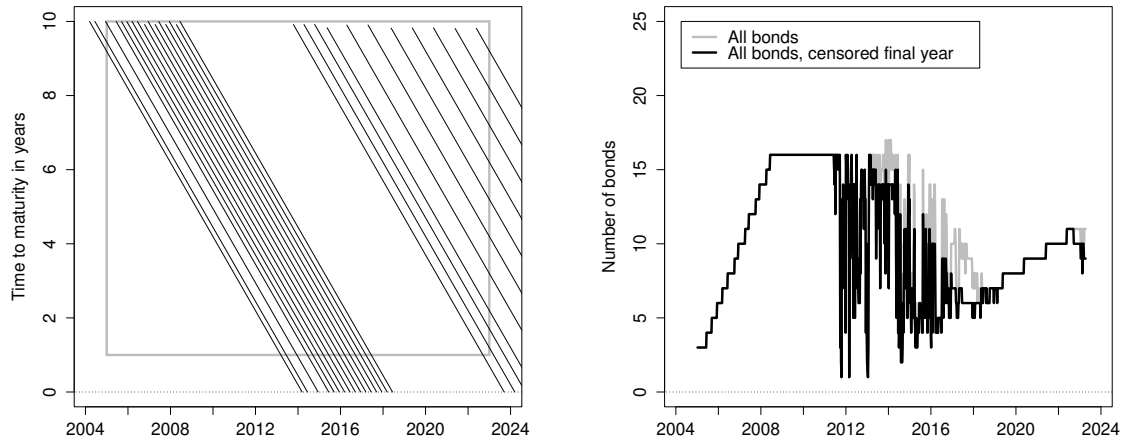
[15] See https://www.mof.go.jp/english/jgbs/topics/bond/10year_inflation/index.htm

JGBi (coupon, maturity)	No. obs.	Issuance		Number of auctions	Total notional amount
		Date	amount		
(1) 1.2% 3/10/2014	376	3/10/2004	100	1	100
(2) 1.1% 6/10/2014	389	6/10/2004	300	1	300
(3) 0.5% 12/10/2014	428	12/10/2004	500	1	500
(4) 0.5% 6/10/2015	432	6/10/2005	500	1	500
(5) 0.8% 9/10/2015	419	9/12/2005	500	1	500
(6) 0.8% 12/10/2015	402	12/12/2005	500	1	500
(7) 0.8% 3/10/2016	414	3/10/2006	500	1	500
(8) 1% 6/10/2016	394	6/12/2006	500	2	1000
(9) 1.1% 9/10/2016	395	10/11/2006	500	1	500
(10) 1.1% 12/10/2016	388	12/12/2006	500	2	1000
(11) 1.2% 3/10/2017	369	4/10/2007	500	1	500
(12) 1.2% 6/10/2017	395	6/12/2007	500	2	1000
(13) 1.3% 9/10/2017	382	10/10/2007	500	1	500
(14) 1.2% 12/10/2017	375	12/11/2007	500	2	1000
(15) 1.4% 3/10/2018	356	4/10/2008	500	1	500
(16) 1.4% 6/10/2018	358	6/10/2008	500	2	1000
(17) 0.1% 9/10/2023	463	10/10/2013	300	2	600
(18) 0.1% 3/10/2024	451	4/10/2014	400	2	800
(19) 0.1% 9/10/2024	420	10/10/2014	500	2	1000
(20) 0.1% 3/10/2025	398	5/12/2015	500	4	2000
(21) 0.1% 3/10/2026	348	4/14/2016	400	4	1600
(22) 0.1% 3/10/2027	296	4/13/2017	400	4	1600
(23) 0.1% 3/10/2028	242	5/11/2018	400	4	1600
(24) 0.1% 3/10/2029	190	5/13/2019	400	4	1600
(25) 0.2% 3/10/2030	138	5/11/2020	200	4	800
(26) 0.005% 3/10/2031	84	5/18/2021	200	4	800
(27) 0.005% 3/10/2032	33	5/17/2022	200	3	700

Table 1: **Sample of Japanese Real Government Bonds**
The table reports the characteristics, first issuance date and amount, the total number of auctions, and total amount issued in billions of Japanese yen for the sample of Japanese inflation-indexed government bonds (JGBi). Also reported are the number of weekly observation dates for each bond during the sample period from January 7, 2005, to December 30, 2022.

been gradually increasing with the second issuance wave. At the end of our sample there are nine bonds. The number of inflation-indexed bonds $n_R(t)$ combined with the time variation in the cross-sectional dispersion in the maturity dimension observed in Figure 3(a) provides the identification of the real factors in our model.

Figure 4 shows the yields to maturity for all 27 Japanese inflation-indexed bonds. We see notable changes in the level and slope of the Japanese real yield curve, which motivates our decision to model the inflation-indexed data with three real yield factors. Note also that the series for individual bonds show gaps as the bonds approach maturity. Our use of all available bond price information in combination with the Kalman filter is designed to handle



(a) Distribution of inflation-indexed bonds

(b) Number of inflation-indexed bonds

Figure 3: **Real Japanese Government Bond Sample**
Panel (a) shows the maturity distribution of available Japanese inflation-indexed government bonds (JGBi) on any given date. The solid grey rectangle indicates the sample used in our empirical analysis. The sample is restricted to start on January 7, 2005, and limited to inflation-indexed bond prices with more than one year remaining to maturity. Panel (b) reports the number of outstanding inflation-indexed bonds available at a given point in time for various samples.

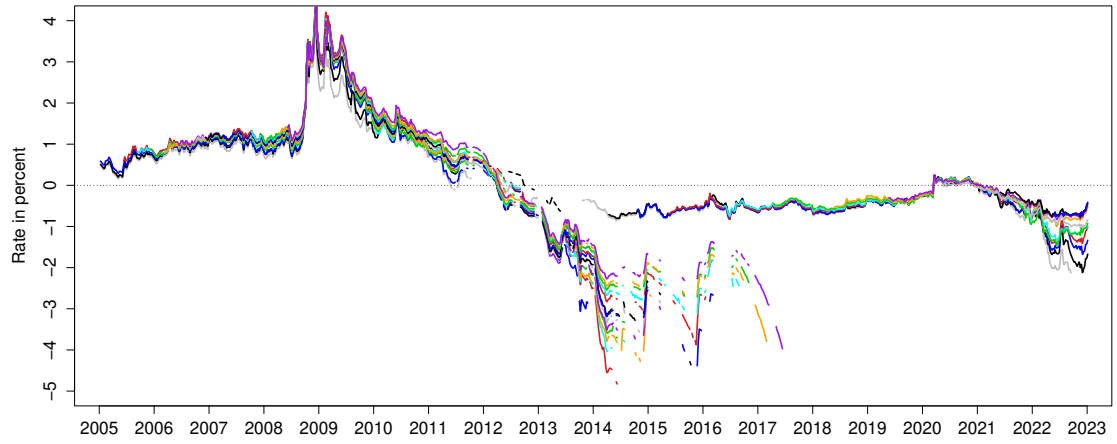


Figure 4: **Yield to Maturity of Japanese Real Government Bonds**
Illustration of the yield to maturity of the Japanese inflation-indexed bonds considered in this paper, which are subject to two sample choices: (1) sample limited to the period from January 7, 2005, to December 30, 2022; (2) censoring of a bond's price when it has less than one year to maturity.

such data gaps.[16] More importantly, the more erratic pricing patterns observed as these

[16]Finlay and Wende (2012) represent an early example of similar analysis based on prices from a limited number of Australian inflation-indexed bonds.

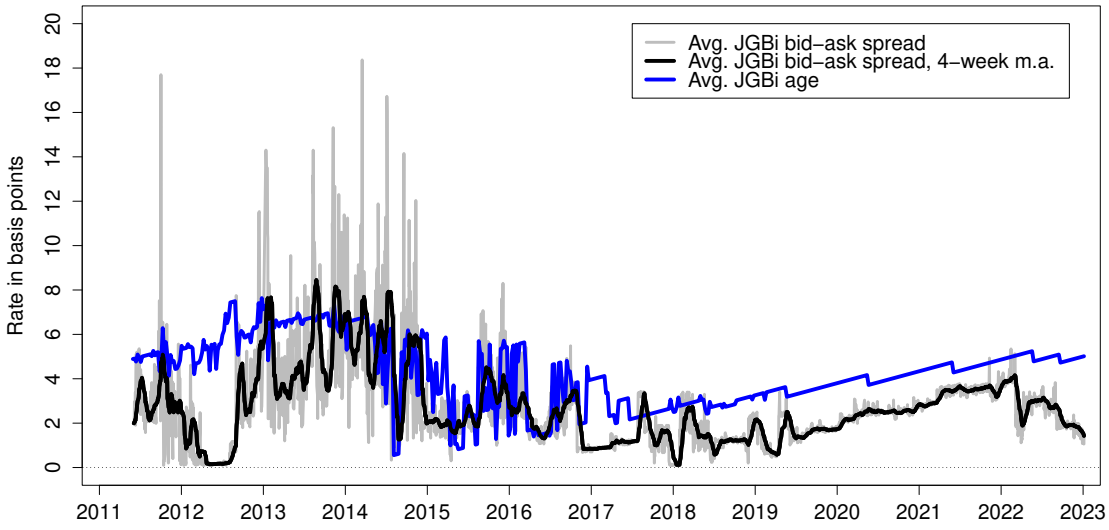


Figure 5: **Average Bid-Ask Spreads of JGBi's**
Illustration of the average bid-ask spread of JGBi's downloaded from Bloomberg. For each day, the average of the available bid-ask spreads is calculated. Also shown is the four-week moving average of the daily average bid-ask spread series as well as the average age of the JGBi's in the sample. All series are daily covering the period from May 31, 2011, to December 30, 2022, and JGBi's with less than one year remaining to maturity are censored.

bonds approach maturity suggests the presence of bond-specific liquidity premia, motivating the model structure presented in Section 3.

2.2.1 The Dynamics of JGBi Liquidity Risk

To support our adopted model structure more formally, we next examine the dynamics of JGBi bid-ask spreads, which represent frequently used measures of liquidity risk.

The data we analyze consist of daily bid-ask spreads for individual JGBi's downloaded from Bloomberg starting in May 2011. For each observation date, we calculate the average of the available bid-ask spreads, which is shown with a solid gray line in Figure 5. To smooth out the noise in the daily series, we calculate the four-week moving average, which is shown with a solid black line in Figure 5. Note the elevated bid-ask spreads in the 2013-2015 period when the JGBi market was dominated by seasoned bonds close to maturity. Finally, we include the average age of the JGBi's in our sample, which is shown with a solid blue line in Figure 5. The positive correlation between the four-week moving average of the daily average bid-ask spread series and the average JGBi age is evident.

To document that this positive connection between liquidity risk and bond age is a pervasive pattern in the data, we test for the statistical significance of this positive relationship

Regression specification	Age_t^i	Number of parameters	Adj. R^2
OLS	0.526***	2	0.107
	(0.104)		
Bond fixed effects	0.528***	28	0.107
	(0.106)		
Time fixed effects	0.394***	3,025	0.470
	(0.096)		
Bond & time fixed effects	0.410***	3,051	0.380
	(0.082)		

Table 2: **Panel Regressions of Daily Bid-Ask Spreads in the JGBi Market**
This table reports the results of separately estimating the relationship between bid-ask spreads and bond age, by OLS, with bond-specific fixed effects, time fixed effects and both fixed effects without interaction. The variable Age_t^i is in years since issuance. White's heteroscedastic standard errors (SEs) are reported in parentheses. Asterisks *, ** and *** indicate significance at the 10 percent, 5 percent, and 1 percent levels, respectively. The adjusted R^2 is computed based on the demeaned variation in the dependent variable, except for the OLS adjusted R^2. The data used are daily covering the period from May 31, 2011, to December 30, 2022, across 27 Japanese ten-year inflation-indexed bonds (JGBi), a total of 27,949 observations.

using a panel regression given by

$$BAS_t^i = \tau_t + \alpha_i + \beta Age_t^i + \varepsilon_t^i, \tag{1}$$

where BAS_t^i denotes the bid-ask spread of the ith JGBi in period t. To control for unobserved heterogeneity, we allow for both time fixed effects τ_t and bond-specific fixed effects α_i in equation (1). Finally, Age_t^i measures the time since issuance of the ith JGBi and ε_t^i is a zero-mean error term. Using all available JGBi's, we then estimate the panel regression in equation (1) by Ordinary Least Squares (OLS) with heteroscedasticity-robust standard errors.

Table 2 reports the results without fixed effects as well as with either bond or time fixed effects, or both, all based on daily data. The result that the age of individual JGBi's has a significant positive effect on their bid–ask spreads is robust to all of these alternative specifications. This relationship is also suggested by the aggregate series shown in Figure 5.[17]

These results match the patterns between bid-ask spreads and bond age in the U.S. Treasury Inflation-Protected Securities (TIPS) market reported in ACR (2021). They are also similar to the well-known finding for standard U.S. Treasuries, where newly issued securities are more liquid than existing bonds (e.g. Krishnamurthy (2002) and Fontaine and Garcia (2012)). However, the bid-ask spreads for JGBi's are much wider than those of U.S.

[17]Online Appendix A documents that these results are also robust to using weekly data.

Treasuries. It is, therefore, important to account for this dynamic pattern in JGBi liquidity to fully understand the price dynamics in the JGBi market.

More broadly, we draw two conclusions from these reduced-form regressions. First, current liquidity in the JGBi market exhibits notable variation over time, and liquidity, therefore, represents a risk factor to bond investors in this market.

Second, seasoned JGBi's are less liquid than more recently issued securities. Although equation (1) does not provide an explanation for this dynamic pattern in liquidity, anecdotal evidence suggests that it most likely arises because many securities get locked up in buy-and-hold investors' portfolios as time passes, and become unavailable for trading.

Rational and forward-looking investors should be aware of this dynamic pattern in liquidity and demand compensation for holding bonds with low future liquidity. Our model formalizes this effect and quantifies how current JGBi prices are affected by expected future JGBi liquidity.

3 Model Estimation and Results

In this section, we first detail our benchmark model, which we use to decompose the nominal and real bond yields into underlying expectations and residual risk premia, while evaluating the value of the inflation-indexed bond deflation enhancement. We then describe our identification restrictions, estimate the model, and summarize our results.

3.1 An Arbitrage-Free Model of Nominal and Real Yields with Bond-Specific Liquidity Risk

In order to precisely measure both the value of deflation protection offered by JGBi's and their individual bond-specific liquidity premia, we need an accurate model of the instantaneous nominal and real rate, r_t^N and r_t^R. We focus on a tractable affine dynamic term structure model of nominal and real yields briefly summarized below. This model can be viewed as a restricted Gaussian version of the affine term structure models introduced by Dai and Singleton (2000).

Let $X_t = (L_t^N, S_t^N, L_t^R, S_t^R, X_t^L)$ denote the state vector of our five-factor model, which we refer to as the $G^L(5)$ model using the terminology of ACR (2021). (L_t^N, S_t^N) represent level and slope factors in the nominal yield curve, while (L_t^R, S_t^R) represent separate level and slope factors in the real yield curve.[18] Finally, X_t^L represents the added liquidity risk factor in the pricing of JGBi's. Our model represents an augmented version of the four-factor $G(4)$ model used by CS (2022).

[18] Chernov and Mueller (2012) provide evidence of a hidden factor in the U.S. nominal yield curve that is observable from real yields and inflation expectations. Our joint model accommodates this stylized fact via the factors (L_t^R, S_t^R).

The instantaneous nominal and real risk-free rates are defined as

$$r_t^j = L_t^j + S_t^j, \quad j = N, R. \tag{2}$$

The risk-neutral $\mathbb{Q}$-dynamics of the state variables used for pricing are given by

$$\begin{pmatrix} dL_t^N \\ dS_t^N \\ dL_t^R \\ dS_t^R \\ dX_t^L \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda^N & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda^R & 0 \\ 0 & 0 & 0 & 0 & \kappa_L^{\mathbb{Q}} \end{pmatrix} \left[\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \theta_L^{\mathbb{Q}} \end{pmatrix} - \begin{pmatrix} L_t^N \\ S_t^N \\ L_t^R \\ S_t^R \\ X_t^L \end{pmatrix} \right] dt + \Sigma \begin{pmatrix} dW_t^{L^N,\mathbb{Q}} \\ dW_t^{S^N,\mathbb{Q}} \\ dW_t^{L^R,\mathbb{Q}} \\ dW_t^{S^R,\mathbb{Q}} \\ dW_t^{X^L,\mathbb{Q}} \end{pmatrix},$$

where Σ is a lower triangular matrix.

Based on this specification of the $\mathbb{Q}$-dynamics, *frictionless* nominal and real zero-coupon bond yields preserve a simplified Nelson and Siegel (1987) factor loading structure:

$$y_t^j(\tau) = L_t^j + \left(\frac{1 - e^{-\lambda^j \tau}}{\lambda^j \tau} \right) S_t^j - \frac{A^j(\tau)}{\tau}, \quad j = N, R, \tag{3}$$

where $\frac{A^j(\tau)}{\tau}$ represents deterministic yield-adjustment terms.

As our model allows JGBi's to be sensitive to bond-specific liquidity risks, their pricing is not performed with the standard frictionless real discount function above, but with a discount function that accounts for the bond-specific liquidity risk:

$$\overline{r}^{R,i}(t, t_0^i) = r_t^R + \beta^i (1 - e^{-\lambda^{L,i}(t - t_0^i)}) X_t^L = L_t^R + S_t^R + \beta^i (1 - e^{-\lambda^{L,i}(t - t_0^i)}) X_t^L, \tag{4}$$

where t_0^i denotes the date of issuance of the specific JGBi and β^i is its sensitivity to the variation in the bond-specific risk factor. Furthermore, the decay parameter $\lambda^{L,i}$ is assumed to vary across securities.

By Christensen and Rudebusch (2019), the net present value of one unit of consumption paid by JGBi bond i at time $t + \tau$ has the following exponential-affine form

$$\begin{aligned} P_t(t_0^i, \tau) &= E^{\mathbb{Q}} \left[e^{-\int_t^{t+\tau} \overline{r}^{R,i}(s, t_0^i) ds} \right] \\ &= \exp\Big(B_1(\tau) L_t^R + B_2(\tau) S_t^R + B_3(t, t_0^i, \tau) X_t^L + A(t, t_0^i, \tau) \Big). \end{aligned}$$

This result implies that the model belongs to the class of Gaussian affine term structure models. Note also that, by fixing $\beta^i = 0$ for all i, we recover the $G(4)$ model.

We next evaluate the value of the deflation protection enhancement in JGBi's issued since 2013. Consider an inflation-indexed bond issued at time t_0 with a reference price index value equal to Π_{t_0}. By time t, its accrued inflation compensation is $\frac{\Pi_t}{\Pi_{t_0}}$, which we define as the

"inflation index ratio." There are then two mutually exclusive scenarios: First, the net price index change from time t to maturity T could be sufficiently positive that the inflation index ratio is greater than one. Given this outcome, the bond will pay off its principal adjusted for inflation between inception and maturity ($\frac{\Pi_T}{\Pi_{t_0}}$).

Alternatively, the net price index change between t and T may be insufficient to bring the inflation index ratio for the entire period from inception to maturity to be greater than one. Given that outcome, the deflation protection option will be in the money, as the inflation-indexed bond will return its original principal. The value of the deflation protection option, DOV_t, is then given by[19]

$$DOV_t\Big(\frac{\Pi_t}{\Pi_{t_0}}\Big) = \left[E_t^{\mathbb{Q}}\Big[e^{-\int_t^T r_s^N ds}\mathbf{1}_{\{\frac{\Pi_T}{\Pi_t}\leq\frac{\Pi_{t_0}}{\Pi_t}\}}\Big] - E_t^{\mathbb{Q}}\Big[e^{-\int_t^T r_s^R ds}\mathbf{1}_{\{\frac{\Pi_T}{\Pi_t}\leq\frac{\Pi_{t_0}}{\Pi_t}\}}\Big]\right].$$

The option value will be lower when accrued inflation compensation is larger, as it is less likely that the net price index change over the bond's remaining life will be sufficiently low (or negative) to bring the option back into the money. Moreover, when accrued inflation is larger, the option value is lower the shorter is the remaining time to maturity, as the probability of bringing the option back into the money at maturity is reduced.

The "clean price" of a bond i issued at time t_0^i with maturity at $t+\tau^i$ that pays an annual coupon C^R semi-annually, i.e. that which does not account for any accrued interest and maps to our observed real bond prices, then satisfies

$$\begin{aligned}\overline{P}_t^{R,i}\Big(t_0^i,\tau^i,C^R,\frac{\Pi_t}{\Pi_0}\Big) &= C^R(t_1-t)E^{\mathbb{Q}}\Big[e^{-\int_t^{t_1}\overline{r}^{R,i}(s,t_0^i)ds}\Big] + \sum_{j=2}^{N}\frac{C^R}{2}E^{\mathbb{Q}}\Big[e^{-\int_t^{t_j}\overline{r}^{R,i}(s,t_0^i)ds}\Big] \\ &\quad + E^{\mathbb{Q}}\Big[e^{-\int_t^{t+\tau^i}\overline{r}^{R,i}(s,t_0^i)ds}\Big] + DOV_t\Big(\frac{\Pi_t}{\Pi_0}\Big).\end{aligned} \tag{5}$$

Note that we only include the option value for the inflation-indexed bonds that have this contractual feature and compute it using the four frictionless factors, $(L_t^N, S_t^N, L_t^R, S_t^R)$, within our model following an approach similar to that outlined in Christensen et al. (2012).[20]

To implement our model empirically, we need to specify the risk premia that connect these factor dynamics under the $\mathbb{Q}$-measure to the dynamics under the real-world $\mathbb{P}$-measure. It is important to note that there are no restrictions on the dynamic drift components under the empirical $\mathbb{P}$-measure beyond the requirement of constant volatility. To facilitate empirical implementation, we use the essentially affine risk premium specification introduced in Duffee

[19] For derivation, see CS (2022).

[20] See CS (2022) for details. We also do not account for the approximately 2.5 month lag in Japanese inflation indexation. Grishchenko and Huang (2013) and D'Amico et al. (2018) find that this adjustment normally is within a few basis points for the implied yield on U.S. TIPS. It is likely to be very small for our Japanese data as well.

(2002). In a Gaussian framework, this specification implies that the risk premia Γ_t depend on the state variables; that is,

$$\Gamma_t = \gamma^0 + \gamma^1 X_t,$$

where $\gamma^0 \in \mathbf{R}^5$ and $\gamma^1 \in \mathbf{R}^{5\times 5}$ contain unrestricted parameters. Thus, the resulting unrestricted five-factor joint model of nominal and real yields has $\mathbb{P}$-dynamics given by

$$dX_t = K^{\mathbb{P}}(\theta^{\mathbb{P}} - X_t) + \Sigma dW_t^{\mathbb{P}},$$

where $K^{\mathbb{P}}$ is an unrestricted 5×5 mean-reversion matrix, $\theta^{\mathbb{P}}$ is a 5×1 vector of mean levels, and Σ is a 5×5 lower triangular volatility matrix. This is the transition equation in the extended Kalman filter estimation of our model.

As shown in online Appendix B, nominal and real zero-coupon yields can be expressed as

$$y_t^N(\tau) = y_t^R(\tau) + \pi_t^e(\tau) + \phi_t(\tau),$$

where $y_t^N(\tau)$ and $y_t^R(\tau)$ are nominal and real zero-coupon yields as described in equation (3), while the market-implied average rate of inflation expected at time t for the period from t to $t+\tau$ is

$$\pi_t^e(\tau) = -\frac{1}{\tau} \ln E_t^{\mathbb{P}} \left[\frac{\Pi_t}{\Pi_{t+\tau}} \right] = -\frac{1}{\tau} \ln E_t^{\mathbb{P}} \left[e^{-\int_t^{t+\tau} (r_s^N - r_s^R) ds} \right] \tag{6}$$

and the associated inflation risk premium for the same time period is

$$\phi_t(\tau) = -\frac{1}{\tau} \ln \left(1 + \frac{cov_t^{\mathbb{P}} \left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}} \right]}{E_t^{\mathbb{P}} \left[\frac{M_{t+\tau}^R}{M_t^R} \right] \times E_t^{\mathbb{P}} \left[\frac{\Pi_t}{\Pi_{t+\tau}} \right]} \right).$$

This last equation demonstrates that the inflation risk premium can be positive or negative. It is positive if and only if

$$cov_t^{\mathbb{P}} \left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}} \right] < 0.$$

That is, the riskiness of nominal bonds relative to real bonds depends on the covariance between the real stochastic discount factor and inflation, and is ultimately determined by investor preferences, as in, for example, Rudebusch and Swanson (2012).

The BEI rate is defined as the difference between nominal and real yields of the same maturity

$$BEI_t(\tau) \equiv y_t^N(\tau) - y_t^R(\tau) = \pi_t^e(\tau) + \phi_t(\tau).$$

Note that it can be decomposed into the sum of expected inflation and the inflation risk premium.

3.2 Model Estimation and Econometric Identification

We estimate the model using a conventional likelihood-based approach, where we extract latent pricing factors from our observed data, nominal zero-coupon yields and inflation-indexed mid-market yields to maturity. The functional form for nominal yields is specified as affine and provided in equation (3), whereas the expression for the yield to maturity $\hat{y}_t^R$ of an inflation-indexed bond with maturity at T that pays an annual coupon C^R semi-annually is given by the solution to the following fixed-point problem

$$\overline{P}_t^{R,i} \quad = \quad C^R(t_1 - t)\exp\left\{-(t_1 - t)\hat{y}_t^R\right\} + \sum_{k=2}^{n} \frac{C^R}{2}\exp\left\{-(t_k - t)\hat{y}_t^R\right\} + \exp\left\{-(T - t)\hat{y}_t^R\right\}, \quad (7)$$

where $\overline{P}_t^{R,i}$ is the model-implied inflation-indexed bond price in equation (5).

Following Joslin et al. (2011), all nominal yields have independent Gaussian measurement errors $\varepsilon_t^{N,i}$ with zero mean and a common standard deviation σ_ε^N, denoted $\varepsilon_{y,t}^i \sim \mathcal{NID}\left(0, (\sigma_\varepsilon^N)^2\right)$ for $i = 1, 2, \ldots, n_N$. We also account for measurement errors in the yields to maturity of the inflation-indexed bonds through $\varepsilon_t^{R,i}$, where $\varepsilon_t^{R,i} \sim \mathcal{NID}\left(0, (\sigma_\varepsilon^R)^2\right)$ for $i = 1, 2, \ldots, n_R(t)$. We follow CS (2022) and restrict the volatility matrix Σ to a diagonal matrix. ACR (2021) show that this has at most a very small impact on the estimated liquidity premia.

Finally, as the bond-specific risk factor is a latent factor, it is not identified without additional restrictions. We let the third JGBi, which was issued right before the start of our sample period, have a unit loading on this factor, that is, the 10-year JGBi issued on December 12, 2004, and maturing on December 10, 2014, with 0.5% coupon has $\beta^i = 1$. This implies that the β^i sensitivity parameters measure bond-specific liquidity risk sensitivity relative to that of this 10-year 2014 JGBi.

3.2.1 Survey Forecasts

The inclusion of long-term survey forecasts can help the model better capture the appropriate persistence of the factors under the objective $\mathbb{P}$-dynamics, which can otherwise suffer from significant finite-sample bias.[21] We therefore also incorporate long-term forecasts of inflation from surveys of professional forecasters, using the projected ten-year CPI inflation ex fresh-food series that can be constructed semi-annually from Consensus Forecasts.[22] We only include forecasts for every other year, i.e., the April forecasts from 2005, 2007, 2009, 2011, 2013, 2015, 2017, 2019, and 2021, to minimize the impact of the survey information on our model results.

[21] See Kim and Orphanides (2012) and Bauer et al. (2012).

[22] We do not include inflation data in the model estimation. This omission is expected to, at most, have a small impact on our results due to the relatively long maturities of most of our real yield observations; see D'Amico et al. (2018).

Maturity in months	$G(4)$ model				$G^L(5)$ model			
	w.o. opt. adj.		w. opt. adj.		w.o. opt. adj.		w. opt. adj.	
	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
6	5.41	8.95	5.41	9.02	5.39	8.92	5.41	8.94
12	1.07	5.19	1.04	5.28	1.06	5.14	1.07	5.21
24	-3.99	6.70	-4.10	6.85	-4.01	6.70	-4.01	6.76
48	-5.39	10.02	-5.56	10.16	-5.41	10.05	-5.42	10.06
84	-0.56	10.81	-0.70	11.01	-0.57	10.80	-0.58	10.84
120	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
All maturities	-0.58	7.84	-0.65	7.96	-0.59	7.84	-0.59	7.87

Table 3: **Pricing Errors of Nominal Yields**
This table reports the mean pricing errors (Mean) and the root mean-squared pricing errors (RMSE) of Japanese nominal yields in the $G(4)$ and $G^L(5)$ models without and with deflation protection option adjustment. All errors are reported in basis points.

The measurement equation for the survey expectations incorporating these long-term forecasts takes the form

$$\pi_t^{CF}(10) = \pi_t^e(10) + \varepsilon_t^{CF},$$

where $\pi_t^e(10)$ is the model-implied ten-year expected inflation calculated using equation (6), which is affine in the state variables, while the measurement error is $\varepsilon_t^{CF} \sim \mathcal{NID}\left(0, (\sigma_\varepsilon^{CF})^2\right)$ with σ_ε^{CF} fixed at 0.0075, as recommended by Kim and Orphanides (2012).

We note that the model is estimated with the standard extended Kalman filter due to the nonlinear measurement equations for the inflation-indexed bond yields.[23]

Finally, we assume that the state variables are stationary, which is standard in the finance literature. As a consequence, we start the Kalman filter at the unconditional mean and covariance matrix.

3.3 Estimation Results

We next examine the results under our $G^L(5)$ model, contrasting them with the earlier CS (2022) $G(4)$ model that did not adjust for the bond-specific liquidity risk premia in JGBi prices. For both models we perform the estimation without and with adjustment for the value of the deflation protection offered by JGBi's issued since 2013.

To begin our comparison, we first examine the models' ability to fit nominal yields. Table 3 reports the summary statistics of the fitted errors of the 6 nominal yields in our sample. The table documents that our benchmark $G^L(5)$ model with option adjustment fits all of the nominal yields well, as the overall root mean-squared error (RMSE) is only 7.87 basis points. Moreover, all four specifications fit the nominal yields about equally well.

The summary statistics of the fitted errors for each JGBi calculated as described in

[23]See Andreasen et al. (2019) for evidence of the robustness of this approach.

JGBi (coupon, maturity)	$G(4)$ model				$G^L(5)$ model			
	w.o. opt. adj.		w. opt. adj.		w.o. opt. adj.		w. opt. adj.	
	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
(1) 1.2% 3/10/2014	-6.33	15.24	-6.61	15.76	-2.17	6.25	-2.13	6.37
(2) 1.1% 6/10/2014	7.39	16.11	7.27	16.39	0.50	4.41	0.56	4.54
(3) 0.5% 12/10/2014	-1.64	10.04	-1.93	10.25	3.35	5.95	2.86	5.93
(4) 0.5% 6/10/2015	6.81	11.15	6.59	11.31	1.99	5.54	1.86	5.40
(5) 0.8% 9/10/2015	2.69	8.29	2.67	8.19	-1.40	5.74	-1.22	5.71
(6) 0.8% 12/10/2015	0.02	10.97	0.12	10.78	-1.29	9.10	-0.56	9.01
(7) 0.8% 3/10/2016	-1.99	8.15	-1.73	8.32	-1.59	6.20	-1.85	6.16
(8) 1% 6/10/2016	0.84	10.51	1.11	10.70	2.36	7.81	2.38	7.83
(9) 1.1% 9/10/2016	-5.08	8.51	-4.80	8.51	-0.43	4.18	-0.23	4.08
(10) 1.1% 12/10/2016	-5.10	7.60	-4.78	7.54	-0.79	3.39	-0.34	3.46
(11) 1.2% 3/10/2017	-6.48	12.96	-6.03	12.04	-3.73	7.21	-3.80	7.20
(12) 1.2% 6/10/2017	1.03	5.33	1.37	5.67	2.29	3.98	2.74	4.44
(13) 1.3% 9/10/2017	-1.92	5.90	-1.61	5.69	-1.60	3.49	-1.35	3.46
(14) 1.2% 12/10/2017	0.84	7.79	0.97	7.69	-0.07	3.58	0.13	3.69
(15) 1.4% 3/10/2018	-2.84	11.43	-3.08	11.67	-2.81	6.06	-3.73	6.74
(16) 1.4% 6/10/2018	7.99	14.01	7.46	13.56	4.58	7.12	3.52	6.79
(17) 0.1% 9/10/2023	4.00	10.37	2.16	10.41	-0.17	4.37	0.21	4.92
(18) 0.1% 3/10/2024	2.22	6.27	-0.49	6.79	0.97	3.71	0.30	4.48
(19) 0.1% 9/10/2024	-0.71	5.25	3.60	9.07	-0.89	3.81	0.84	4.51
(20) 0.1% 3/10/2025	-0.05	6.10	2.31	7.77	0.53	3.12	0.93	3.18
(21) 0.1% 3/10/2026	-2.20	5.45	-0.83	6.45	0.13	3.46	-0.59	3.85
(22) 0.1% 3/10/2027	-5.78	10.77	-5.61	12.25	-0.48	6.61	-1.56	8.55
(23) 0.1% 3/10/2028	2.88	5.02	2.79	5.77	-0.03	2.81	-0.33	4.16
(24) 0.1% 3/10/2029	4.77	6.54	4.12	7.38	0.15	3.22	0.30	4.99
(25) 0.2% 3/10/2030	-9.13	11.18	-11.22	12.53	-0.94	3.24	-0.07	4.86
(26) 0.005% 3/10/2031	6.15	7.27	5.83	7.54	0.80	3.97	1.39	4.91
(27) 0.005% 3/10/2032	-1.43	2.85	1.68	3.14	-0.53	4.13	0.10	3.14
All yields	-0.03	9.74	0.09	10.07	-0.01	5.32	0.02	5.59
Max $\mathcal{L}^{EKF}$	97,297.21		97,132.59		101,763.2		101,514.0	

Table 4: **Pricing Errors of Japanese Real Government Bond Yields to Maturity**
This table reports the mean pricing errors (Mean) and the root mean-squared pricing errors (RMSE) of Japanese inflation-indexed bond (JGBi) yields to maturity in the $G(4)$ and $G^L(5)$ models without and with deflation protection option adjustment. The errors are computed as the difference between the observed yield to maturity from Bloomberg and the corresponding model-implied yield. All errors are reported in basis points.

equation (7) are reported in Table 4. The RMSE for all yield errors combined is 5.59 basis points in our $G^L(5)$ model that adjusts for the bond-specific liquidity premia in the JGBi prices. In contrast, the $G(4)$ model without adjustment for these premia produces a RMSE for all yield errors of 10.07 basis points. Thus, the $G^L(5)$ model achieves a markedly better fit to the JGBi data. We also find that the estimated measurement error standard deviations within our benchmark model, $\sigma_\varepsilon^N = 0.0010$ and $\sigma_\varepsilon^R = 0.0006$, are consistent with these

JGBi (coupon, maturity)	$G^L(5)$ model							
	w.o. opt. adj.				w. opt. adj.			
	β^i	SE	$\lambda^{L,i}$	SE	β^i	SE	$\lambda^{L,i}$	SE
(1) 1.2% 3/10/2014	1.6874	0.0137	1.5033	0.0001	1.6647	0.0096	1.4282	0.0114
(2) 1.1% 6/10/2014	5.6085	0.0347	0.0567	0.0004	7.6469	0.0112	0.0392	0.0004
(3) 0.5% 12/10/2014	1	n.a.	1.0144	0.0419	1	n.a.	1.0146	0.0076
(4) 0.5% 6/10/2015	17.7698	0.0660	0.0100	0.0001	18.4698	0.0244	0.0100	0.0001
(5) 0.8% 9/10/2015	3.3881	0.0617	0.0599	0.0017	3.0637	0.0149	0.0725	0.0015
(6) 0.8% 12/10/2015	1.4667	0.0480	0.1451	0.0094	1.2774	0.0107	0.2049	0.0070
(7) 0.8% 3/10/2016	0.9202	0.0359	0.2499	0.0268	1.0514	0.0109	0.2388	0.0104
(8) 1% 6/10/2016	0.7354	0.0269	0.2815	0.0309	0.8307	0.0175	0.2714	0.0122
(9) 1.1% 9/10/2016	0.4072	0.0164	2.7074	0.0488	0.4874	0.0109	2.0620	0.0097
(10) 1.1% 12/10/2016	0.3566	0.0103	2.4591	0.0450	0.4279	0.0045	1.5344	0.0161
(11) 1.2% 3/10/2017	0.5444	0.0225	0.3105	0.0475	0.6918	0.0077	0.3037	0.0148
(12) 1.2% 6/10/2017	0.4237	0.0206	0.2950	0.0481	0.5229	0.0048	0.3099	0.0059
(13) 1.3% 9/10/2017	0.4632	0.0240	0.2720	0.0524	0.5654	0.0067	0.3392	0.0152
(14) 1.2% 12/10/2017	0.4048	0.0277	0.2769	0.0523	0.5463	0.0072	0.2882	0.0115
(15) 1.4% 3/10/2018	6.2176	0.0502	0.0100	0.0003	9.0623	0.0223	0.0101	0.0000
(16) 1.4% 6/10/2018	0.4156	0.0190	0.4132	0.0660	0.7425	0.0079	0.2621	0.0001
(17) 0.1% 9/10/2023	0.0317	0.0147	10	0.0501	0.4838	0.0089	9.9902	0.0104
(18) 0.1% 3/10/2024	0.1586	0.0181	9.9999	0.0525	0.7035	0.0092	9.9900	0.0117
(19) 0.1% 9/10/2024	0.3125	0.0226	1.4286	0.0657	1.0172	0.0105	9.9778	0.0145
(20) 0.1% 3/10/2025	0.4725	0.0271	1.0786	0.0523	1.1809	0.0111	9.9994	0.0240
(21) 0.1% 3/10/2026	0.7340	0.0271	1.4177	0.0438	1.3409	0.0159	9.9999	0.0090
(22) 0.1% 3/10/2027	0.9169	0.0310	10	0.0568	1.3376	0.0101	9.9999	0.0168
(23) 0.1% 3/10/2028	1.4399	0.0519	9.9999	0.0734	1.7599	0.0138	9.9991	0.0141
(24) 0.1% 3/10/2029	1.8482	0.0982	0.9520	0.1812	1.9147	0.0131	9.9917	0.0119
(25) 0.2% 3/10/2030	1.8340	0.0897	1.6610	0.2270	1.5113	0.0145	9.9998	0.0145
(26) 0.005% 3/10/2031	2.4004	0.0888	10	0.2792	2.0686	0.0109	10	0.0311
(27) 0.005% 3/10/2032	2.6696	0.1339	10	0.3090	1.9310	0.0087	9.9999	0.0202

Table 5: **Estimated Bond-Specific Risk Parameters**
This table reports the estimated bond-specific liquidity risk parameters and their standard errors in the $G^L(5)$ model estimated without and with deflation protection option adjustment. Standard errors (SE) are not available (n.a.) for the normalized value of β^3.

results.

Second, in Table 5, we report the estimated bond-specific parameters, $(\beta^i, \lambda^{L,i})$, for each JGBi within the $G^L(5)$ model without and with deflation protection option adjustment. For the first 16 JGBi's that do not offer deflation protection we see relatively minor differences as could be expected. In contrast, for the remaining 11 JGBi's issued since 2013, which do provide the deflation protection, we see some differences across bonds. Moreover, the diverse estimated values underscore the importance of allowing the bond-specific parameters to vary across JGBi's.

Finally, we report the estimated dynamic parameters of our benchmark model in Table 6. The volatility parameters in the Σ matrix are estimated with precision. For the mean-

$K^{\mathbb{P}}$	$K^{\mathbb{P}}_{\cdot,1}$	$K^{\mathbb{P}}_{\cdot,2}$	$K^{\mathbb{P}}_{\cdot,3}$	$K^{\mathbb{P}}_{\cdot,4}$	$K^{\mathbb{P}}_{\cdot,5}$	$\theta^{\mathbb{P}}$		Σ
$K^{\mathbb{P}}_{1,\cdot}$	2.8335	2.9422	-0.3034	-0.1398	-0.0067	0.0056	σ_{11}	0.0033
	(0.0154)	(0.0163)	(0.0072)	(0.0139)	(0.0120)	(0.0040)		(0.0001)
$K^{\mathbb{P}}_{2,\cdot}$	0.0752	0.1804	0.1108	0.1041	-0.0025	-0.0085	σ_{22}	0.0034
	(0.0142)	(0.0145)	(0.0188)	(0.0237)	(0.0147)	(0.0038)		(0.0002)
$K^{\mathbb{P}}_{3,\cdot}$	-2.2618	-2.5176	0.4050	0.2966	0.0068	-0.0101	σ_{33}	0.0056
	(0.0074)	(0.0124)	(0.0070)	(0.0172)	(0.0107)	(0.0075)		(0.0000)
$K^{\mathbb{P}}_{4,\cdot}$	3.1530	3.5006	0.2192	0.4907	-0.0830	-0.0028	σ_{44}	0.0166
	(0.0178)	(0.0295)	(0.0140)	(0.0131)	(0.0124)	(0.0091)		(0.0002)
$K^{\mathbb{P}}_{5,\cdot}$	0.4554	0.3535	0.7129	-0.1509	0.5677	0.0389	σ_{55}	0.0300
	(0.0115)	(0.0079)	(0.0101)	(0.0225)	(0.0074)	(0.0113)		(0.0008)

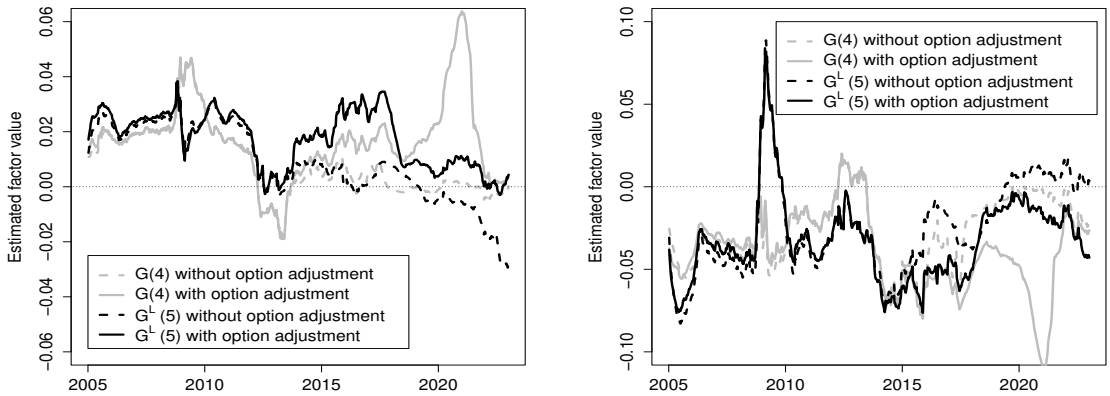
Table 6: **Estimated Dynamic Parameters in Benchmark $G^L(5)$ Model**
The estimated parameters for the mean-reversion matrix $K^{\mathbb{P}}$, the mean vector $\theta^{\mathbb{P}}$, and the volatility matrix Σ in the option-adjusted $G^L(5)$ model. The $\mathbb{Q}$-related parameters are estimated at $\lambda^N = 0.1010$ (0.0015), $\lambda^R = 0.3237$ (0.0037), $\kappa^{\mathbb{Q}}_L = 2.4086$ (0.0140), and $\theta^{\mathbb{Q}}_L = 0.0015$ (0.0001). The numbers in parentheses are the estimated standard deviations.

reversion parameters in the $K^{\mathbb{P}}$ matrix and the mean parameters in the $\theta^{\mathbb{P}}$ vector, the results are more mixed. Some of them are highly statistically significant, while others are clearly insignificant.

In Figure 6, we plot the estimated real yield factors from all four models studied so far. While the two nominal factors, L^N_t and S^N_t, are essentially identical across all four models and therefore only reported in online Appendix D, the most noticeable differences are observed for the estimated path of the real yield level and slope factors, L^R_t and S^R_t. This seems reasonable given that the liquidity premium adjustment affects the prices of JGBi most directly. As a consequence, we also see some differences in the estimated path for the frictionless instantaneous inflation rate $\pi_t = L^N_t + S^N_t - L^R_t - S^R_t$ across the four shown models (see panel (d) of Figure 6).

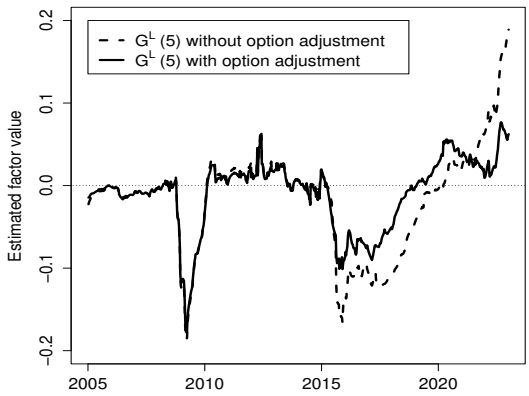
4 The JGBi Bond-Specific Premia

This section first analyzes the JGBi bond-specific liquidity premium implied by the estimated $G^L(5)$ model described in the previous section before proceeding to a detailed analysis of the estimated values of the deflation protection options. We then examine the implications of the JGBi bond-specific premia for the pricing of Japanese inflation swaps using updated data through December 2025. Finally, we briefly discuss a closely related paper by Hiraki and Hirata (2020).

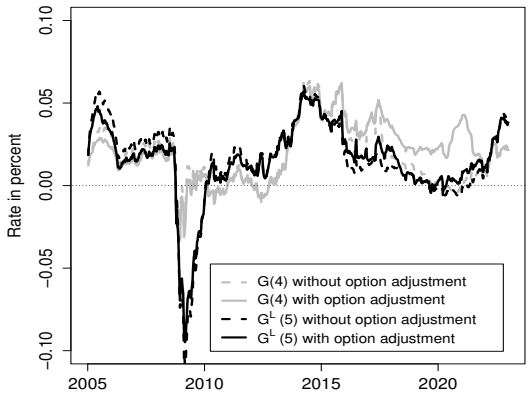


(a) L_t^R

(b) S_t^R



(c) X_t^L



(d) π_t

Figure 6: **Estimated Real Yield State Variables and Instantaneous Inflation**
Illustration of the estimated real yield state variables, (L_t^R, S_t^R, X_t^L), and the instantaneous inflation rate π_t from the $G(4)$ and $G^L(5)$ models without and with deflation protection option adjustment.

4.1 The Estimated JGBi Bond-Specific Liquidity Premia

To compute the novel bond-specific liquidity premia in the JGBi market, we first use the estimated parameters and the filtered states $\left\{X_{t|t}\right\}_{t=1}^{T}$ to calculate the fitted JGBi prices $\left\{\hat{P}_t^i\right\}_{t=1}^{T}$ for all outstanding securities in our sample. These bond prices are then converted into yields to maturity $\left\{\hat{y}_t^{c,i}\right\}_{t=1}^{T}$ by solving the fixed-point problem

$$\begin{aligned}\hat{P}_t^{R,i} &= C^R(t_1-t)\exp\left\{-(t_1-t)\hat{y}_t^{c,i}\right\} + \sum_{k=2}^{n}\frac{C^R}{2}\exp\left\{-(t_k-t)\hat{y}_t^{c,i}\right\} \\ &\quad + \exp\left\{-(T-t)\hat{y}_t^{c,i}\right\},\end{aligned} \tag{8}$$

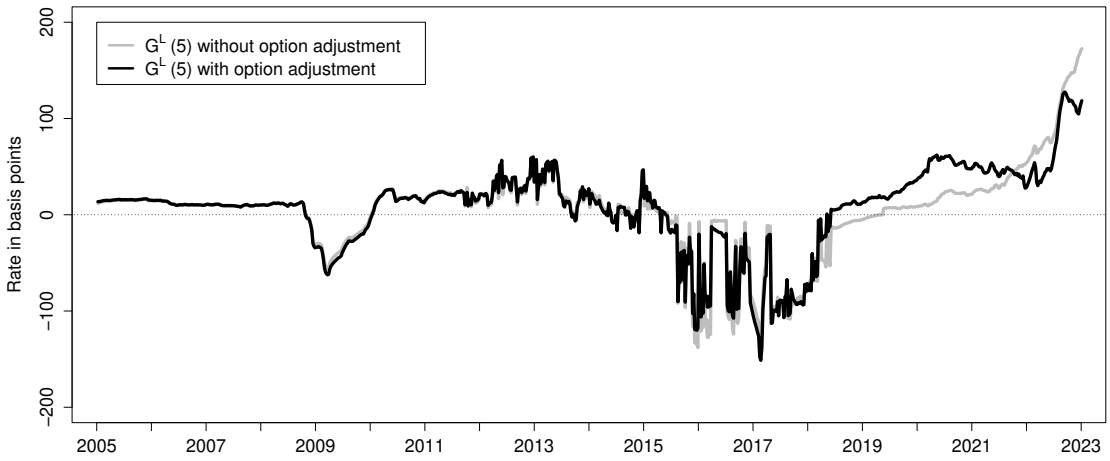


Figure 7: **Average Estimated JGBi Bond-Specific Liquidity Premium**
Illustration of the average estimated bond-specific liquidity premium of JGBi's for each observation date implied by the $G^L(5)$ model estimated without and with option adjustment. The bond-specific liquidity premia are measured as the estimated yield difference between the fitted yield to maturity and the corresponding frictionless yield to maturity of individual JGBi's with the liquidity risk factor turned off as described in the text. The data cover the period from January 7, 2005, to December 30, 2022.

for $i = 1, 2, ..., n_{R,t}$, meaning that $\left\{\hat{y}_t^{c,i}\right\}_{t=1}^{T}$ is approximately the real rate of return on the ith JGBi if held until maturity (Sack and Elsasser (2004)). To obtain the corresponding yields with correction for the bond-specific liquidity premia, a new set of model-implied bond prices are computed from the estimated $G^L(5)$ model but using only its frictionless part, i.e., using the constraints that $X_{t|t}^L = 0$ for all t as well as $\sigma_{55} = 0$ and $\theta_L^Q = 0$. These prices are denoted $\left\{\tilde{P}_t^i\right\}_{t=1}^{T}$ and converted into yields to maturity $\tilde{y}_t^{c,i}$ using equation (8). They represent estimates of the prices that would prevail in a world without any financial frictions or special demands for certain bonds.[24] The bond-specific liquidity premium for the ith JGBi is then defined as

$$\Psi_t^i \equiv \hat{y}_t^{c,i} - \tilde{y}_t^{c,i}. \tag{9}$$

Figure 7 shows the average JGBi bond-specific liquidity premium $\bar{\Psi}_t$ across the outstanding JGBi's at a given point in time without and with adjustment for the value of the JGBi deflation protection. Beyond some level difference over the 2018-2022 period, estimated average bond-specific liquidity premia are only modestly sensitive to the adjustment for the deflation protection option.

The average estimated JGBi bond-specific liquidity premium varies over time and takes

[24]For the $G^L(5)$ model estimated with option adjustment, note that DOV_t^i is included in the calculation of $\hat{P}_t^i$ and $\tilde{P}_t^i$ for JGBi's issued since 2013.

on both positive and negative values. It tends to turn negative and reflect convenience premia when uncertainty abroad is elevated, as in 2009 following the global financial crisis and in the 2015-2018 period when there was financial volatility in China associated with its credit crunch and trade disputes between it and the United States.

Both of these episodes were characterized by elevated fears of deflationary outcomes, as we document in Section 4.2. Given the level of deflation risk detected by our model, it appears that the levels of the observed real yields were surprisingly moderate. In late 2008, real yields were high, but deflation risks were severe. In contrast, in the 2015-2018 period, real yields are near all-time lows, highly negative, and well below matching nominal yields meaning BEI rates were quite positive. To rationalize the low real yields in the midst of elevated deflation risk, the model drives the bond-specific risk factor into negative territory as can be seen in Figure 6(c). Practically speaking, this means that JGBi's were trading at a convenience premium during the two deflationary spells. To support that interpretation, we note that in Figure 5 JGBi bid-ask spreads were low in the 2015-2018 period, suggesting healthy demand for these bonds.[25]

Figure 7 also features elevated JGBi liquidity risk premia towards the end of our sample. This spike in liquidity risk premia coincides with the BOJ's "yield curve control" policy, under which the BOJ stood ready to purchase Japanese Government Bonds (JGBs) as needed to maintain its target for the ten-year JGB yield. As noted in the literature, this policy was effective in reducing JGB yields to within the BOJ's announced target range (e.g. Hattori and Yoshida (2023) and Aoki and Ueda (2025)).

However, this policy removed a significant amount of JGBs from the open market, particularly in 2022 and 2023 where there were two major spikes in outright BOJ purchases; see Shiratsuka (2025). As discussed in Nakazawa and Ozawa (2024), this removal of JGBs lowered JGB liquidity, particularly in the ten-year segment of the market. This interpretation is also supported by the somewhat higher and more volatile JGB bid-ask spreads during this period as observed in Figure 2. This suggests that our finding of elevated liquidity risk premia at the end of our sample was in part a function of this yield-curve control policy, as JGB's and JGBi's are relatively close substitutes in Japanese financial markets.

4.1.1 Validation of the Estimated JGBi Liquidity Premia

While our latent-factor model is very flexible and provides an accurate fit to the bond data, it does not allow us to identify the variation in the data that is captured by the bond-specific liquidity risk factor. This section validates that the liquidity risk factor is indeed capturing the liquidity components in the prices of JGBi bonds as intended.

To do so, we run standard linear regressions with the average estimated JGBi bond-specific liquidity premium $\bar{\Psi}_t$ as the dependent variable and the four-week moving average

[25] Unfortunately, JGBi bid-ask spreads are not available prior to May 2011.

Explanatory variables	(1)	(2)	(3)	(4)	(5)
Avg. JGBi bid-ask spread	4.929**		-1.915		-1.200
	(2.096)		(2.138)		(2.091)
Avg. JBGi age		12.849***	13.947***		16.314***
		(1.981)	(2.248)		(2.445)
Avg. JGB bid-ask spread				37.137***	55.453***
				(7.255)	(9.189)
Constant	-5.813***	-47.547***	-47.064***	-39.922***	-130.032***
	(8.732)	(9.968)	(10.163)	(10.491)	(18.457)
Adjusted R^2	0.022	0.134	0.135	0.134	0.233

Table 7: **Regression Results for the Average Estimated JGBi Liquidity Premium** This table reports the results of running the regression described in equation (10) by OLS. Newey-West standard errors (SEs) with 4 lags are reported in parentheses. Asterisks *, ** and *** indicate significance at the 10 percent, 5 percent, and 1 percent levels, respectively. The data used are weekly covering the period from June 3, 2011, to December 30, 2022, a total of 605 observations.

of the average JGBi bid-ak spread, denoted $\overline{BAS}_t(JGBi)$, the average JGBi bond age, denoted $\overline{Age}_t(JGBi)$, and the four-week moving average of the average JGB bid-ak spread, denoted $\overline{BAS}_t(JGB)$, as explanatory variables, the first two of which are shown in Figure 5, while the final series is shown in Figure 2. That is, the regressions take the form

$$\bar{\Psi}_t = \alpha + \beta_1 \overline{BAS}_t(JGBi) + \beta_2 \overline{Age}_t(JGBi) + \beta_3 \overline{BAS}_t(JGB) + \varepsilon_t. \tag{10}$$

Given that JGBi bid-ask spreads and bond ages are linked and both represent bond liquidity as described in Section 2.2.1, this method allows us to establish that our latent-factor model structure produces estimated liquidity premia that are indeed capturing liquidity risk in the JGBi market. Given that the JGBi market is small relative to the standard JGB market, we include the JGB bid-ask spread series to examine whether our estimated liquidity premium series has any ties to the liquidity risk in the standard JGB market.

We run regression (10) for the period from June 3, 2011, to December 30, 2022, when the bid-ask spread series are available. The results reported in Table 7 show that both the average JGBi bid-ask spread and the average JGBi bond age are significantly positively correlated with our estimated liquidity premium series. However, the average bond age is the dominating factor and drives out the bid-ask spread series in the joint regression (3), in part due to multicollinearity between these two variables. Interestingly, our average JGBi liquidity premium is also significantly positively correlated with the JGB bid-ask spread series as shown in regression (4) pointing to some common component in the liquidity risk across the JGB and JGBi markets. Moreover, average JGBi bond age and the JGB bid-ask spreads preserve their high statistical significance in the joint regression (5) with all three variables.

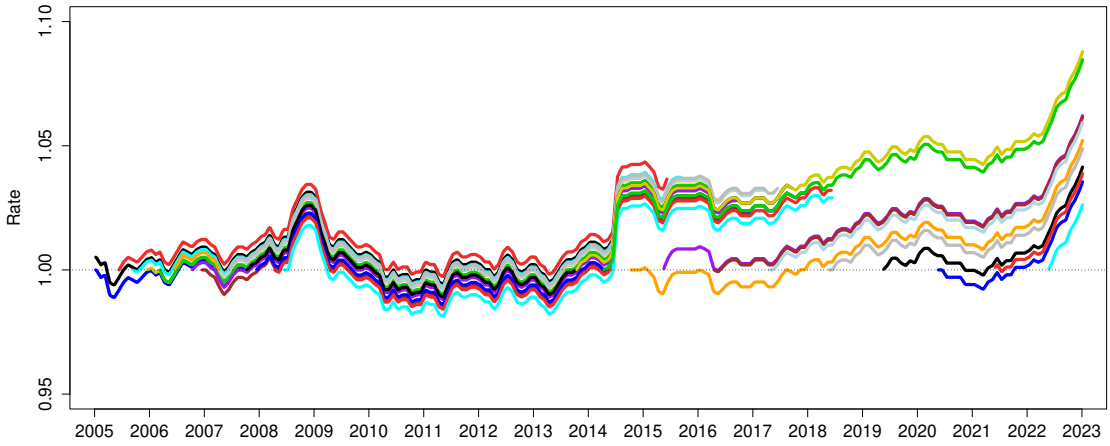


Figure 8: **Inflation Index Ratios of JGBi's**

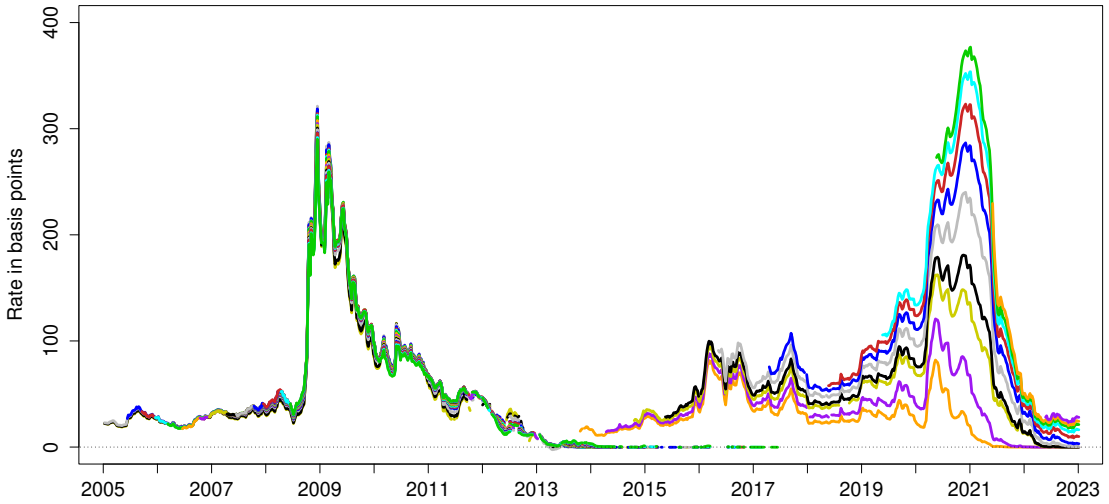
In terms of magnitudes based on regression (5), we note that when the average bond age increases by one year the average estimated JGBi liquidity risk premium tends to go up by about 16 basis points, while a 1-basis-point increase in the average JGB bid-ask spread is associated with a sizable 55-basis-point increase in the average estimated JGBi liquidity premium. Thus, both average JGBi age and JGB bid-ask spreads play an economically meaningful and statistically significant role for our estimated JGBi liquidity premia. Based on this evidence, we conclude that the added liquidity risk factor in the $G^L(5)$ model is capturing liquidity components in the JGBi prices as intended.

4.2 The Estimated JGBi Deflation Option Values

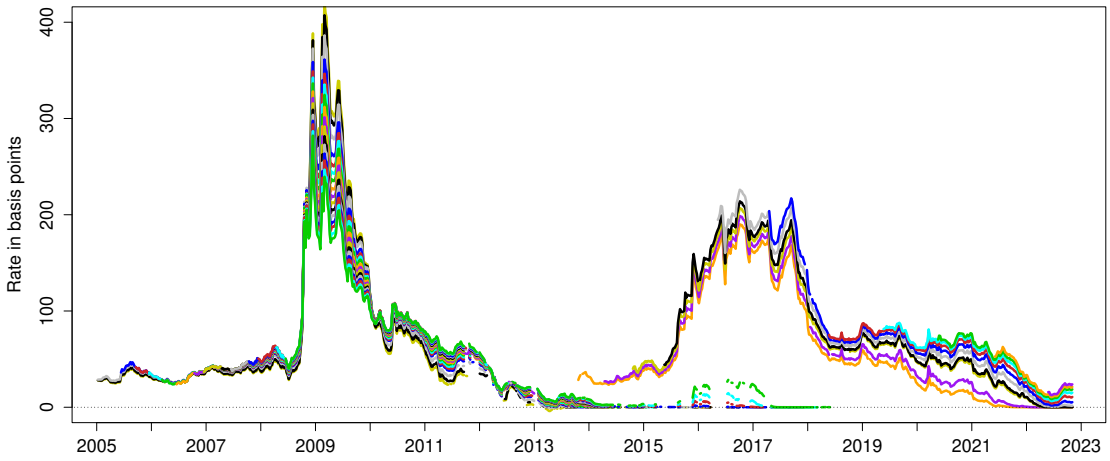
In this section, we turn our focus to the value of the deflation protection offered by the JGBi's issued since 2013.

Figure 8 shows the inflation index ratios for all 27 JGBi's in our sample. Several bonds issued since 2013 when the deflation protection clause was introduced have been exposed to periods of deflation. However, despite extended spells of deflation in Japan since 2005, so far no JGBi has reached maturity with an index ratio below one, i.e. with the deflation option in the money.

Figure 9 shows the option value for each JGBi implied by the $G(4)$ and $G^L(5)$ models, each estimated with option adjustment. We note that we measure the option value as a yield spread between the model-implied yield to maturity based on the fitted bond price *without* the option value and the model-implied yield to maturity based on the fitted bond price *with* the option value included, and we perform this calculation for all JGBi's, including the ones not offering the deflation protection.



(a) $G(4)$ model



(b) $G^L(5)$ model

Figure 9: **Value of Deflation Protection Options in JGBi's**

The results show that the liquidity premium adjustment significantly alters the estimated value of the deflation protection option starting in mid-2015. As we document in Section 5, this has dramatic implications for BEI decompositions over our sample period.

In Figure 9(b), note the two spells of deflation risk in 2009 and 2015-2018 highlighted in the discussion of our JGBi liquidity premium series in Section 4.1.

The $G(4)$ model without the liquidity risk factor at its disposal is forced to take the real yields at their face value during these two periods. This leads it to imply a notably lower level of deflation risk. Moreover, during the uptick in liquidity risk in 2020 that puts

downward pressure on measured BEI rates, the $G(4)$ model produces unrealistically high values for the individual JGBi deflation option values. In contrast, while still elevated, the $G^L(5)$ model-implied deflation option values seem more reasonable given the transitory nature of the pandemic. Overall, we take this evidence to demonstrate that the $G(4)$ model is misspecified by failing to account for the persistent changes in the liquidity premia embedded in the JGBi prices unlike our novel $G^L(5)$ model.

4.2.1 Deflation Option Values Measured as Par Yield Spreads

To have a consistent measure of deflation values across time, we follow Christensen et al. (2012) and construct synthetic ten-year real par-coupon yield spreads as described in the following. We compare prices of newly issued JGBi's without any accrued inflation compensation and similar seasoned JGBi's with sufficient accrued inflation compensation to make its deflation option value entirely negligible. First, consider a hypothetical seasoned JGBi with T years remaining to maturity that pays an annual coupon C semi-annually. Assume this bond has accrued sufficient inflation compensation so it is practically impossible to reach the deflation floor before maturity. The par-coupon bond satisfying these criteria has a coupon rate determined by the equation[26]

$$\sum_{i=1}^{2T} \frac{C}{2} E_t^Q[e^{-\int_t^{t_i} r_s^R ds}] + E_t^Q[e^{-\int_t^T r_s^R ds}] = 1. \tag{11}$$

The first term is the sum of the present value of the $2T$ coupon payments using the model's fitted real yield curve at day t. The second term is the discounted value of the principal payment. We denote the coupon payment for the seasonal bond that solves this equation as C_S.

Next, consider a new JGBi with no accrued inflation compensation and T years to maturity. Since the coupon payments are not protected against deflation, the difference is in accounting for the deflation protection on the principal payment:

$$\sum_{i=1}^{2T} \frac{C}{2} E_t^Q[e^{-\int_t^{t_i} r_s^R ds}] + E_t^Q\Big[\frac{\Pi_T}{\Pi_t} \cdot e^{-\int_t^T r_s^N ds} \mathbf{1}_{\{\frac{\Pi_T}{\Pi_t} > 1\}}\Big] + E_t^Q\Big[1 \cdot e^{-\int_t^T r_s^N ds} \mathbf{1}_{\{\frac{\Pi_T}{\Pi_t} \leq 1\}}\Big] = 1.$$

The first term is the same as before. The second term represents the present value of the principal payment conditional on a positive net change in the price index over the bond's maturity; i.e., $\frac{\Pi_T}{\Pi_t} > 1$. Under this condition, full inflation indexation applies, and the price change $\frac{\Pi_T}{\Pi_t}$ is placed within the expectations operator and weighted by the probability of accumulated inflation at time T. The third term represents the present value of the *floored* JGBi principal conditional on accumulated net deflation; i.e., when the price level change is

[26] In the $G^L(5)$ model, the relevant expectations under the risk-neutral measure are calculated using its frictionless yield curves.

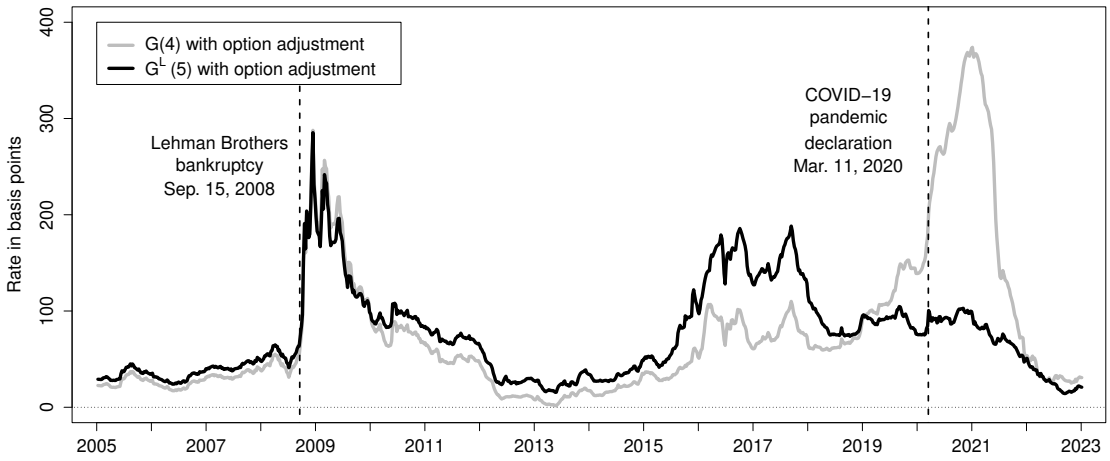


Figure 10: **Ten-Year Deflation Risk Premium**

below one, $\frac{\Pi_T}{\Pi_t}$ is replaced by a value of one to provide the promised deflation protection. Since

$$\frac{\Pi_T}{\Pi_t} = e^{\int_t^T (r_s^N - r_s^R) ds},$$

the equation can be rewritten as

$$\sum_{i=1}^{2T} \frac{C}{2} E_t^Q [e^{-\int_t^{t_i} r_s^R ds}] + E_t^Q \left[e^{-\int_t^T r_s^R ds} \right] + \left[E_t^Q \left[e^{-\int_t^T r_s^N ds} \mathbf{1}_{\{\frac{\Pi_T}{\Pi_t} \leq 1\}} \right] - E_t^Q \left[e^{-\int_t^T r_s^R ds} \mathbf{1}_{\{\frac{\Pi_T}{\Pi_t} \leq 1\}} \right] \right] = 1,$$

where the last term on the left-hand side represents the net present value of the deflation protection of the principal in the JGBi contract. The par-coupon yield of a new hypothetical JGBi that solves this equation is denoted as C_0.

The difference between C_S and C_0 is a direct measure of the advantage of being at the inflation adjustment floor for a newly issued JGBi. Figure 10 shows this par yield difference at the ten-year maturity from the same two model estimations studied above. From 2005 until 2013 there is relatively little difference between the two estimated deflation risk premia. However, in the 2015-2018 period, the average bond-specific liquidity premia turned negative and JGBi's traded with a convenience premium *even after* accounting for the value of the deflation protection. In turn, this implies that the frictionless option-adjusted BEI is *below* the fitted option-adjusted BEI. As a consequence, the estimated priced risk of deflation is notably higher according to the $G^L(5)$ model during this period than it was under our earlier $G(4)$ model without the liquidity adjustment.

In contrast, when the average bond-specific liquidity premia switched back into positive territory during 2018, the frictionless option-adjusted BEI moved back up above the fitted

option-adjusted BEI, which explains the relatively low deflation risk premium implied by the $G^L(5)$ model during the remaining years of our sample. In particular, we only obtain a modest uptick at the onset of the COVID-19 pandemic, which stands in sharp contrast to the much higher estimated values implied by our earlier $G(4)$ model.

4.3 Implications for the Pricing of Japanese Inflation Swaps

The most widely traded inflation swap (IS) derivatives in Japan and elsewhere are zero-coupon IS contracts, in which a single payment is made at maturity with one party paying a fixed amount quoted as an annual rate—the IS rate—in exchange for receiving the cumulative change in the CPI index since inception.[27] The floating leg of Japanese IS contracts is adjusted with the same price index as the cash flows of JGBi's. It therefore follows that in a frictionless world with continuous trading, BEI and IS rates of the same maturity should be identical.

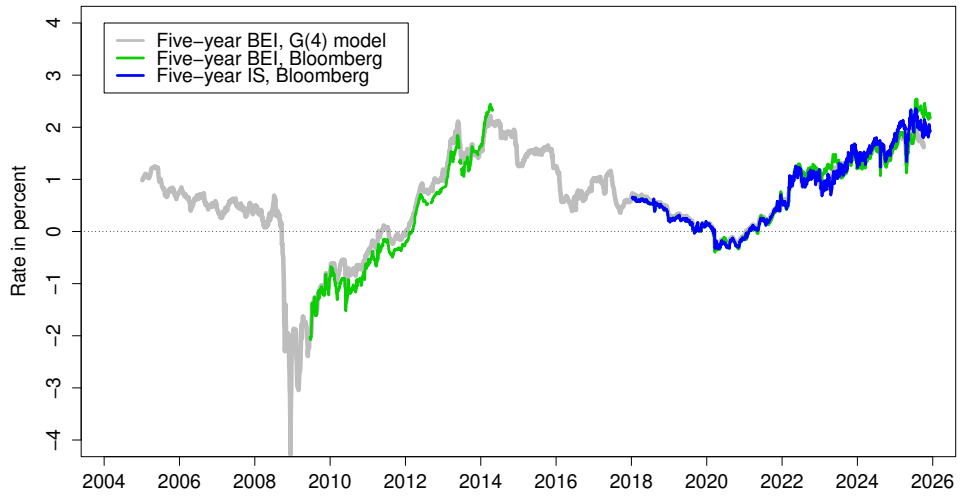
However, as documented in the previous two sections, there are large and time-varying premia in the JGBi prices tied to the liquidity risk of the bonds (i.e. the risk that an investor may have to offer a discount when selling a bond back to the market) and the JGBi deflation protection since 2013. Neither of these premia apply to Japanese IS contracts. First, as a financial derivative exchanging two cash flows, IS contracts have zero value at inception by absence of arbitrage. As a consequence, IS contracts are never sold back to the market (unlike JGBi's). Instead, either the net value of the contract is settled or an offsetting new IS contract is entered at no cost. Therefore, in principle, there should be no liquidity premia in IS rates. Second, standard IS contracts do not contain any contingent payments affording protection against deflationary outcomes as JGBi's have since 2013.

In this section, we compare JGBi prices and IS quotes to investigate whether these factors are reflected in market pricing.

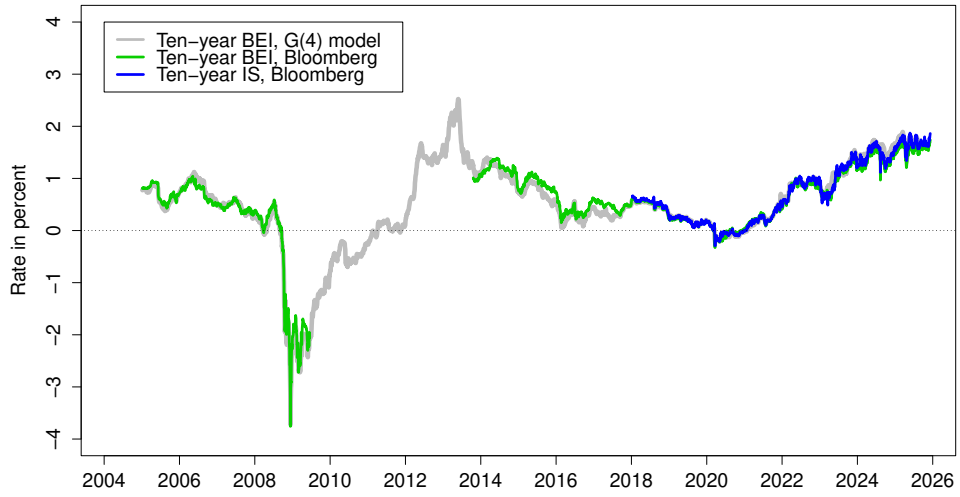
We obtain daily Japanese IS rates from Bloomberg at the one-, five-, and ten-year maturities starting in January 4, 2018, through to the present. In addition, Bloomberg provides fitted Japanese BEI rates at the five- and ten-year maturities. The ten-year BEI rates run from January 4, 2005, to June 15, 2009, and then again from October 10, 2013, through to the present, while the five-year BEI rates run from June 16, 2009, to April 22, 2014, and then again from January 11, 2018, through to the present. This uneven coverage is a function of the unusual pattern of JGBi's issued by the Japanese Finance Ministry since 2004, including the temporary halt to issuance between fall 2008 and October 2013.

For comparison with the Bloomberg data, we estimate our simple $G(4)$ model that makes no adjustments for liquidity premia or deflation protection using daily data and calculate the model-implied five- and ten-year BEI rates. Figure 11(a) compares the five-year BEI rate

[27] In Japan, the price index used is the CPI ex fresh food. This leaves cash flows in the IS market aligned with those in the JGBi market.



(a) Five-year maturity



(b) Ten-year maturity

Figure 11: **Japanese BEI and Inflation Swap Rates**
The top panel illustrates the fitted five-year BEI rate implied by the standard G(4) model estimated using daily nominal yields covering the period from January 6, 1995, to December 30, 2025, and real yields covering the period from January 4, 2005, to December 30, 2025. Also shown are the five-year BEI and IS rates downloaded from Bloomberg for the periods described in the main text. The bottom panel illustrates the fitted ten-year BEI rate implied by the G(4) model estimated using daily nominal yields covering the period from January 6, 1995, to December 30, 2025, and real yields covering the period from January 4, 2005, to December 30, 2025. Also shown are the ten-year BEI and IS rates downloaded from Bloomberg for the periods described in the main text.

implied by our $G(4)$ model with the five-year BEI and IS rates downloaded from Bloomberg,

IS-BEI, Bloomberg			
Maturity	1-yr	5-yr	10-yr
Mean	n.a.	-0.90	4.20
MAD	n.a.	9.92	5.32
RMSD	n.a.	14.95	6.90
Correlation	n.a.	0.98	1.00

IS-BEI, $G(4)$ model			
Maturity	1-yr	5-yr	10-yr
Mean	-9.05	1.30	-1.42
MAD	18.67	7.65	5.15
RMSD	24.43	10.71	7.04
Correlation	0.97	0.99	0.99

Table 8: **Summary Statistics of Differences between Inflation Swap and Breakeven Inflation Rates**

This table reports the mean difference, the mean absolute difference (MAD), and the root-mean-squared-difference (RMSD) between IS and BEI rates. In the top panel, the differences are calculated based on five- and ten-year BEI rates downloaded from Bloomberg. The data series are daily covering the period from January 4, 2018, to December 30, 2025, a total of 1,945 and 1,949 observations, respectively. In the bottom panel, the differences are calculated based on one-, five-, and ten-year BEI rates implied by the standard $G(4)$ model. The data series are daily covering the period from January 4, 2018, to December 30, 2025, a total of 1,952 observations. The last row of each panel reports the correlations between the matching IS and BEI rate series.

while Figure 11(b) shows the result of the corresponding comparison at the ten-year maturity. We note the general closeness between all three series for the overlapping periods for both maturities.

For completeness, Table 8 reports the summary statistics of the differences between IS rates, on one side, and matching JGBi-based BEI rates from Bloomberg and our standard $G(4)$ model, respectively, on the other. The results document that Japanese IS rates are systematically nearly identical to matching BEI rates, in particular at the five- and ten-year maturities. Moreover, this conclusion is not sensitive to the specific BEI measure considered.

This comparison leaves a few takeaways. First, it highlights the strength of our approach and the quality of our data. By leveraging all available bond information, the standard $G(4)$ model is able to deliver accurate estimates of BEI rates at relevant maturities for the entire period since January 2005 when our model estimation becomes feasible.

Second, and most importantly, given our earlier analysis that documents the existence of large liquidity premia and deflation protection option values in JGBi prices, Japanese BEI rates are materially distorted and *not* equal to the frictionless outcome. Combining that evidence with the findings in Figure 11 and Table 8, the strong correlation and minor differences between the IS and BEI rates suggest that Japanese IS rates are likely to be mispriced. Indeed, their close fit to the $G(4)$ model-based BEI rates suggests that they are

priced *as if* both the liquidity premia and deflation protection distortions *are* incorporated in the pricing of IS contracts.[28] In on-going research, we investigate whether this pattern can be rationalized within our novel term structure model.

Finally, despite this documented mispricing, IS rates may still serve as a useful measure of financial market participants' inflation expectations (e.g. Nakazawa and Osada (2024)).

4.4 Comparison with the Analysis in Hiraki and Hirata (2020)

In the following, we compare and contrast our analysis with the closely related paper by Hiraki and Hirata (2020, henceforth HH) that was written contemporaneously with this paper. Similar to our methodology, their paper uses the CS (2022) model augmented with a bond-specific liquidity risk factor structured as in ACR (2021). This allows them to estimate both JGBi-specific liquidity premia and deflation protection values for the JGBi's issued since 2013.

However, HH first regress IS-BEI rate spreads on bond age, obtaining a negative coefficient estimate. This leads them to impose the restriction that the $\lambda^{L,i}$ parameters should be negative. In contrast, our regression analysis based on individual daily JGBi bid-ask spread series regressed against the age of the bonds described in Section 2.2.1 clearly calls for $\lambda^{L,i}$ to be positive as liquidity deteriorates with a bond's age.

Moreover, there are a number of other disparities between the methodologies in the two papers. First, they impose the restriction that all β^i parameters are equal to one whereas we allow these parameters to be freely determined by the data. They also impose the restriction that the mean-reversion matrix $K^{\mathbb{P}}$ is diagonal, while we leave it as a flexible 5×5 matrix. These disparities in combination are likely to affect the estimated JGBi-specific liquidity premia quite materially.

Finally, in terms of the data used in the model estimation, we rely on JGB and JGBi prices and some very limited survey inflation forecasts. While HH do not use any survey information, they include a full term structure of IS rates for ten maturities with data starting in 2007. They then use IS-BEI spreads as a way to identify liquidity premia in inflation-indexed bond prices by effectively assuming that IS rates are fairly priced. However, our analysis in Section 4.3 above strongly suggests that this assumption is not satisfied by the IS data.

In short, although seemingly similar on the surface, there are numerous reasons tied to data and implementation choices why the results reported by HH differ from ours.

[28] This conclusion requires that our $G^L(5)$ model is well specified.

5 Empirical BEI Decomposition

In this section, we examine the implications for the decomposition of BEI that follow from our $G^L(5)$ model. We focus on the ten-year maturity, but our results are robust to using other maturities. We end the section with an evaluation of an estimate of the natural real rate implied by our $G^L(5)$ model.

Figure 12 shows the ten-year BEI decomposition implied by the $G^L(5)$ model and the $G(4)$ model without liquidity adjustment from CS (2022), both estimated with deflation protection option adjustment. Figure 12 also shows the semiannual ten-year inflation forecasts from the Consensus Forecasts survey that we use in the model estimation. As discussed above we only use the April-forecast every other year, highlighted in the figure with thick black dots.

The wedge between the ten-year fitted BEI and the ten-year liquidity- and option-adjusted BEI implied by our $G^L(5)$ model represents the net effect of the deflation protection and liquidity adjustments. This adjusted measure of BEI fell below fitted BEI in the 2015-2019 period, while it was above it during the last year of our sample. In contrast, by only accounting for the value of deflation protection, the adjustment produced by the $G(4)$ model leads to a distorted measure of adjusted BEI, and yields an expected inflation component that is clearly high relative to available survey evidence and a residual inflation risk premium that is low and overly volatile. In comparison, the $G^L(5)$ model yields an expected inflation estimate that is closer to the forecasts reported in the surveys of professional forecasters and a more stable inflation risk premium.

Specifically, between 2005 and 2020, the $G^L(5)$ model's ten-year expected inflation closely tracks the survey forecasts even though they are only included every other year in the model estimation and with a high measurement error standard deviation of 0.75 percent. This suggests that investors' inflation expectations were by and large in agreement with those of the professional forecasters. In contrast, a quite sizable wedge opened up in the 2021-2022 period between the survey and the model-implied ten-year expected inflation. As of October 2022, the survey forecast was 0.97 percent, while the model expects inflation to average 1.34 percent over the next ten years. Hence, long-term expected inflation in Japan may be closer to the Bank of Japan's two percent inflation target than either the surveys or unadjusted BEI may suggest.

The uptick in the liquidity- and option-adjusted BEI also implies an increase in the ten-year inflation risk premium since 2021 according to our $G^L(5)$ model. This seems reasonable in light of the significant spike in Japanese and global inflation during this period. However, by the end of our sample, the Japanese inflation risk premium is close to zero. This may be an early sign that fears about a material undershooting of the inflation target have abated.

To explore that question further, we follow Christensen and Rudebusch (2019) and define

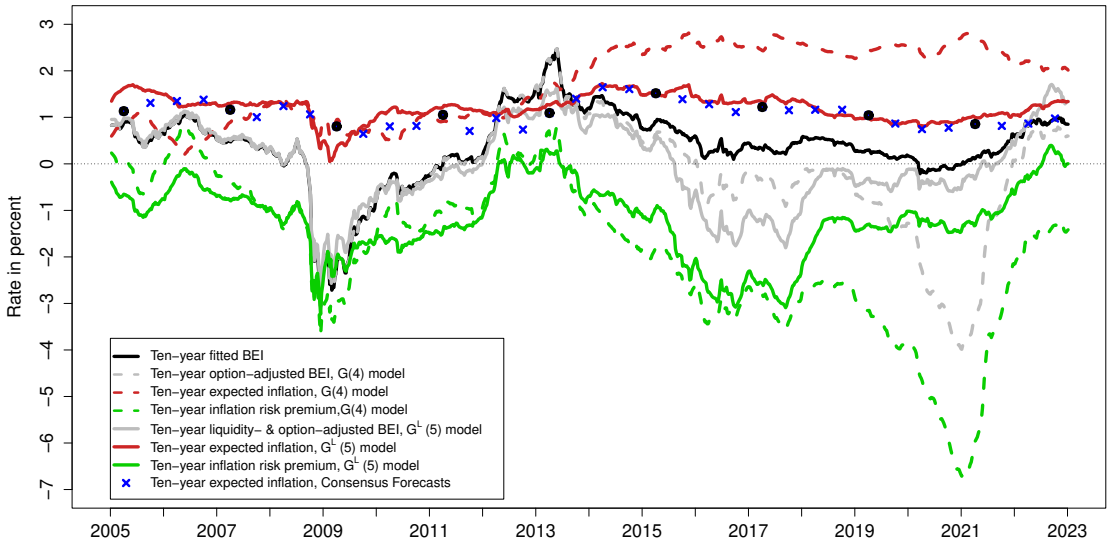


Figure 12: **Decomposition of Ten-Year BEI**
Illustration of the fitted ten-year BEI obtained by fitting the $G(4)$ model to nominal and real yields without any option adjustment or survey forecasts. Also shown are (i) the ten-year fitted option-adjusted BEI calculated as the difference between the fitted ten-year nominal yield and the fitted ten-year option-adjusted real yield from the $G(4)$ model and as the difference between the fitted ten-year nominal yield and the fitted ten-year liquidity- and option-adjusted real yield from the $G^L(5)$ model, both estimated with deflation protection option adjustment, (ii) the estimated ten-year expected inflation, and (iii) the residual ten-year inflation risk premium. Finally, the semi-annual ten-year expected inflation series from the Consensus Forecasts survey is shown with blue crosses with the biannual ones used in the model estimations highlighted in black.

the equilibrium real rate of interest r_t^* as

$$r_t^* = \frac{1}{5}\int_{t+5}^{t+10} E_t^{\mathbb{P}}[r_{t+s}^R]ds, \tag{12}$$

that is, the average expected real short rate over a five-year period starting five years ahead where the expectation is with respect to the objective $\mathbb{P}$-probability measure. This 5yr5yr forward average expected real short rate should be little affected by short-term transitory shocks.

Figure 13 shows these finance-based estimates of r_t^* from both the $G(4)$ and $G^L(5)$ models. Based on the $G(4)$ model, the equilibrium real rate has experienced a persistent decline of more than 1 percentage point on net since 2005. This would imply that monetary policy is likely to have been less stimulative than widely perceived. Furthermore, it would also imply that monetary policy has been mostly restrictive. In contrast, our novel $G^L(5)$ model produces an estimate of r_t^*, which is very stable and has remained close to -1.35 percent for

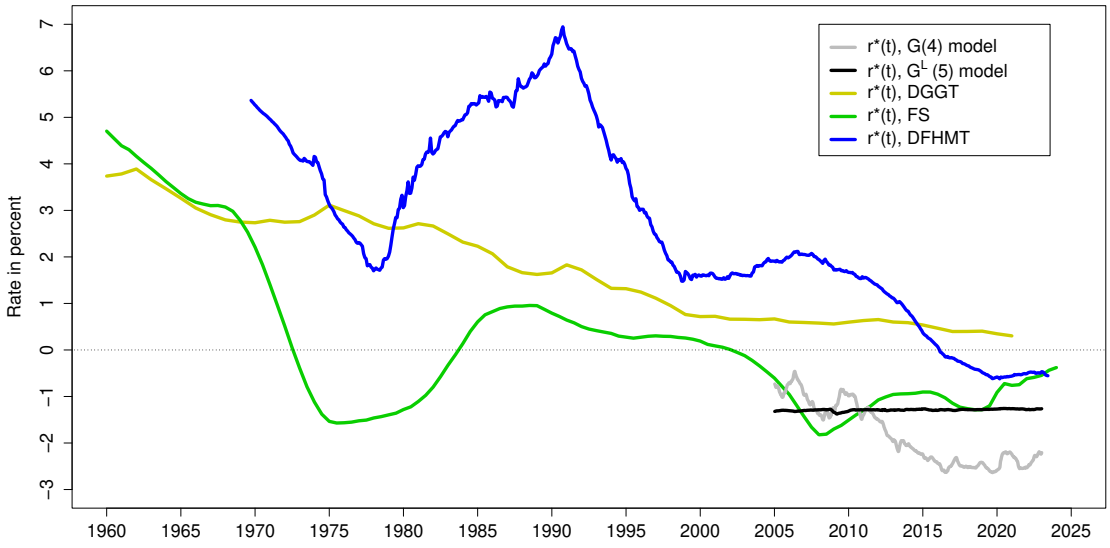


Figure 13: **Estimates of** r^*

our entire sample period.[29] Hence, the missing liquidity premium adjustment in the earlier $G(4)$ model distorts its assessment of the outlook for short-term real rates.

We also include a comparison with relevant estimates based on macroeconomic data. The first is taken from Del Negro et al. (2019, henceforth DGGT). They estimate a flexible vector autoregression model with common trends for a sample of data from 7 advanced economies, including Japan, covering the period from 1870 to 2016 and here extended through the end of 2020.[30] They use annual data on short- and long-term government bond yields, consumer prices, and real consumption per capita in addition to Moody's Baa corporate bond yields. They assume no arbitrage in the long run, which implies a factor structure for the trend of real interest rates across countries. They find that real interest rates across these 7 countries share a global common trend that has been particularly pronounced since the 1970s. Moreover, using regression analysis, they find that declining consumption growth and increasing convenience yields from the safety and liquidity offered by government bonds from these countries are the main drivers of declining real rates the past 40 years. Note that their r_t^* estimate for Japan was high back in the 1960s and 1970s before starting a pronounced secular decline in the early 1980s. Although the decline continues through the end of the shown sample, it remains positive and hence much higher than our finance-based r_t^* estimates. Given that observable real yields in Japan have remained negative since 2013,

[29] Given that the model-implied long-term expected inflation shown in Figure 12 does vary over time, the stable r_t^* estimates are *not* a result of upward bias in the estimated mean-reversion rates as could be a concern; see Bauer et al. (2012).

[30] We thank Marco Del Negro for sharing the updated data.

this positive r_t^* estimate is hard to reconcile with the available Japanese bond yield data the past decade.

The second estimate comes from Ferreira and Shousha (2023, henceforth FS). They consider a panel of 11 advanced economies and estimate the longer-run neutral real interest rates while accounting for changes in the global supply of safe assets and their convenience yields in addition to productivity and demographics and global spillovers from their developments. Their r_t^* estimate for Japan starts in 1960 and fell steadily until the mid-1970s. It reversed some of the decline in the early 1980s and remained fairly stable slightly above zero until the early 2000s. It then steadily declined until 2008 when it reached a historic low of -1.82 percent. Since then it has gradually trended higher and stood at -0.38 percent by the end of 2023. This upward trend the past 15 years with a net increase of more than 1 percentage point sets it apart from our two finance-based r_t^* estimates as well as the other considered macro-based estimates.

Given the role of safe assets for the natural real rate in the analysis of FS, we note that, according to our r_t^* estimates, productivity and demographic developments appear to have more than offset any potential boost from the large increase in Japanese government debt since 2005 and the associated added supply of safe assets.[31] In addition, according to the analysis of FS, global spillovers are likely to also have contributed negatively. Hence, we take this as early suggestive evidence that Japanese bond markets are likely to have been under the influence of global factors during our sample period, a conjecture we explore further below.

The final macro-based r_t^* estimate comes from Davis et al. (2024, henceforth DFHMT). They introduce a unified no-arbitrage macro-finance model with two trend factors used to estimate the natural rate r_t^* for 10 advanced economies, including Japan, covering a longer and wider sample than most previous studies. Using a multitude of data sources on trend growth and inflation in addition to risk premium series, DFHMT also underscore the need for a coherent model approach like ours. Importantly, the interpretation of their natural rate r_t^* is consistent with Laubach and Williams (2003) as representing a medium-run real rate anchor for monetary policy. Our finance-based definition taken from Christensen and Rudebusch (2019) is intended to capture the same concept. Hence, the DFHMT r_t^* estimate should be comparable to ours. One notable difference, though, is that by relying on a single average yield equation, their estimation does not fully exploit all the information in the yield curve, unlike our approach.

While fluctuating in the 1970s and 1980s, the DFHMT r_t^* estimate peaked in late 1989 close to 7 percent—coinciding with the peak in the Nikkei and the real estate price bubble in Japan at the time. In the subsequent more than 30 years, r_t^* fell more than 7 percentage

[31] According to the IMF, gross public debt in Japan stood at 174.59 percent of nominal GDP in 2005. That number had increased to 261.29 percent by the end of 2022; see https://www.imf.org/external/datamapper/d@FPP/USA/FRA/GBR/ESP/ITA/ZAF/JPN.

points according to their estimate and ends the sample close to -1 percent. For the overlapping period this entails a sharp contrast between their r_t^* estimate on one side and our and the FS estimates on the other. While the latter estimates have varied relatively little since 2005, the r_t^* estimate reported by DFHMT has declined almost 300 basis points. Compared to the Japanese real yields shown in Figure 4, this decline seems excessive and essentially requires a very large offset by increased real term premia or JGBi liquidity risk premia that are not supported by our analysis.[32] Moreover, using panel regressions, DFHMT relate their r_t^* estimates to economic growth and demographic variables and find that slowing growth and population aging have been significant factors in driving down the natural rate r_t^* globally, and in Japan in particular. Our finance-based r_t^* estimate from the $G^L(5)$ model suggests that these effects had already materialized by 2005 when our JGBi sample begins, as also suggested by the r_t^* estimate from FS. In contrast, DFHMT's estimate is higher and still declining by the end of the sample, suggesting that this process may have yet to run its course in Japan.

6 Global JGBi Liquidity Spillovers

We next use our estimated time series of liquidity premia in the JGBi market to evaluate whether or not liquidity conditions in Japanese bond markets are linked to disruptions in other important global bond markets. We concentrate on the U.S. Treasury market, the most liquid bond market and one that is commonly considered a determinant of global financial conditions, e.g. Rey (2015). In principle, the Japanese market might be less exposed than other national bond markets to disruptions in the U.S. Treasury market due to its own large size and the general perception that investors in Japan exhibit substantive home bias in their asset holdings; see Tesar and Werner (1995).[33] If this were the case, liquidity shocks in the U.S. Treasury market should not give rise to dislocations or illiquidity conditions in Japanese bond markets.

To measure the timing of such liquidity disruptions in the U.S. Treasury market, we obtain market depth data for the two-, five-, seven-, and ten-year on-the-run U.S. Treasury securities from December 4, 2009, through December 30, 2022. Market depth is measured as the number of open orders from surveyed dealers at the stated maturity. We take values as of the close on Fridays, if available. In a few cases this data is unavailable due to market closure,

[32] According to the analysis by DFHMT, trend inflation has changed little since 2000 across their sample of 10 advanced economies, including in Japan, and our ten-year BEI decomposition in Figure 12 supports that view. Hence, inflation trends do not appear to be driving yield declines during this period.

[33] The perception that the Japanese market is segmented appears to have changed. While earlier studies, such as Couerdacier and Rey (2013) document substantive Japanese equity home bias in 2008 relative to the United States or the euro area, more recent studies suggest less segmentation. For example, Tanaka et al. (2024) document significant responsiveness among Japanese pension fund portfolios to relative yields on domestic and foreign assets.

in which case we use the latest trading day of the week.[34] We then compare movements in these U.S. Treasury market depth measures to our estimated JGBi liquidity premium series.

6.1 Specification

To determine if liquidity spillovers from U.S. Treasury markets are disrupting the Japanese JGBi market, we start from our continuous measures of U.S. Treasury market depth, denoted $US(X)_{depth}$, where $X \in \{2, 5, 7, 10\}$ refers to the maturity in years of the considered on-the-run U.S. Treasury security. We examine time-series specifications for all four depth measures separately as well as combinations of measured disruptions in all four series.

We characterize Treasury market depth as shallow if the measured depth is two standard deviations below the mean for that series or less. We then evaluate the implications of an indicator variable, denoted $I^{US}(X)_{shallow}$, $X \in \{2, 5, 7, 10\}$, which takes a value of one during weeks exhibiting shallow market depth in the X-year on-the-run U.S. Treasury security, and 0 otherwise. We also consider two summary indicators, $I^{US}(any)_{shallow}$, which takes a value of one in weeks in which any of the individual market indicators of shallowness signals low market depth, and an in between "counting" indicator $C^{US}_{shallow}$, which is calculated as the sum of the four individual maturity indicators, and so takes values 1 through 4 based on the number of maturities indicating shallow market depth conditions, and 0 otherwise.

To isolate the impact of low market depth and control for potentially confounding factors, we include variables indicating general market conditions in both the U.S. and Japan. We include both U.S. and Japanese yields of comparable maturities, $y^{US}(X)$ with $X \in \{2, 5, 7, 10\}$, and $y^{JPN}(X)$ with $X \in \{2, 4, 7, 10\}$ measured in years for the U.S. and Japanese bond markets, respectively. Note that while we match the 2-, 7, and 10-year shallow market depth indicators with identical maturity yields, we pair the 5-year U.S. shallow market depth indicators with the 4-year Japanese maturity yield. Furthermore, we include VIX^{JGB} and $MOVE^{US}$ to proxy for volatility in Japanese and U.S. financial markets, respectively.

For example, our baseline specification for the impact of shallow market depth in the 2-year U.S. Treasury market satisfies:

$$\bar{\Psi}_t = c + \beta_1 I^{US}(2yr)_{shallow,t} + \beta_2 y^{US}(2yr)_t + \beta_3 y^{JPN}(2yr)_t + \beta_4 VIX^{JPN}_t + \beta_5 MOVE^{US}_t + \epsilon_t, \quad (13)$$

where $\bar{\Psi}_t$ is our estimate of the average JGBi liquidity premium at time t and ϵ_t is a disturbance term, with Newey-West standard errors to adjust for possible heteroskedasticity and autocorrelation. Note that while our estimates of liquidity premia likely contain errors, as they are included as the dependent variable in our estimation, they should not introduce any systematic bias in our estimates of the coefficients of interest. Indeed, if anything, the

[34] Unfortunately, this data is proprietary and cannot be released to the public.

Table 9: Summary Statistics of Regression Variables

	N	Mean	Std. Dev.	Min	Max
$\bar{\Psi}_t$	939	6.524	41.153	-101.987	89.511
VIX^{JPN}	742	2.811	1.254	1.290	6.410
$US(2yr)_{depth}$	651	1054.359	747.427	22.135	3632.355
$US(5yr)_{depth}$	651	212.767	93.727	16.162	581.166
$US(7yr)_{depth}$	651	155.749	58.473	12.478	333.293
$US(10yr)_{depth}$	651	163.160	61.653	10.868	333.778
$MOVE^{US}$	893	81.204	29.025	46.230	162.500
$y^{JPN}(2yr)$	939	0.119	0.323	-0.359	1.073
$y^{JPN}(4yr)$	939	0.244	0.428	-0.371	1.416
$y^{JPN}(7yr)$	939	0.448	0.550	-0.382	1.855
$y^{JPN}(10yr)$	939	0.720	0.658	-0.281	2.014
$y^{US}(2yr)$	893	1.607	1.492	0.102	5.230
$y^{US}(5yr)$	893	2.162	1.253	0.210	5.210
$y^{US}(7yr)$	893	2.480	1.149	0.390	5.210
$y^{US}(10yr)$	893	2.893	1.135	0.552	5.257
$JGB - basp$	596	1.263	0.254	1.000	2.159

Note: Summary statistics of the data sample for the baseline regressions.

errors may lead to attenuation bias for our estimated coefficients on those variables. Specifications for shallow market conditions in other maturities as well as the composite measures are similar.

6.2 Data

Data are weekly, and obtained from Bloomberg unless otherwise indicated, from July 1, 2009 through July 1, 2023. U.S. Treasury market depth data was obtained from Brokertec.

Summary statistics for our sample are shown in Table 9.[35] As discussed above, volatility in U.S. Treasury market depth is particularly large for the 2-year U.S. Treasury maturity.

6.3 Results

Our regression results are shown in Table 10. Weeks exhibiting shallow depth in U.S. Treasury markets are associated with higher estimates of liquidity premia in the Japanese JGBi market for all of the individual maturities studied, as well as for specifications 5 and 6, which use a measure of the existence of shallowness at any maturity in the U.S. market and the total number of maturities indicating shallow treasury market depth, respectively. All of our estimates are statistically significant, with all entering positively at a 1% confidence level except for the 10 year maturity, which enters positively at a 10% confidence level.

[35] We also show sample statistics for $JGB - basp$, the average bid ask spread for conventional JGB bonds that is used as an alternative dependent variable in Table 11

Table 10: Average JGBi Liquidity Premium Regression Results

	(1)	(2)	(3)	(4)	(5)	(6)
$I^{US}(2yr)_{shallow}$	97.195***					
	(28.013)					
$I^{US}(5yr)_{shallow}$		108.980***				
		(25.474)				
$I^{US}(7yr)_{shallow}$			104.601***			
			(23.971)			
$I^{US}(10yr)_{shallow}$				44.148*		
				(17.297)		
$I^{US}(any)_{shallow}$					82.661***	
					(23.501)	
$C^{US}_{shallow}$						33.658***
						(6.792)
VIX^{JPN}	-4.322	-7.650	-9.017	-5.821	-6.163	-7.538
	(6.119)	(6.559)	(6.578)	(7.060)	(6.435)	(6.464)
$MOVE^{US}$	0.041	0.292	0.454	0.748*	0.240	0.219
	(0.328)	(0.341)	(0.339)	(0.366)	(0.306)	(0.327)
$y^{JPN}(2yr)$	105.564*					
	(47.291)					
$y^{JPN}(4yr)$		64.554*				
		(30.250)				
$y^{JPN}(7yr)$			49.219*			
			(19.220)			
$y^{JPN}(10yr)$				31.642*	44.641**	49.279***
				(15.586)	(14.551)	(14.249)
$y^{US}(2yr)$	-2.017					
	(7.578)					
$y^{US}(5yr)$		-15.778*				
		(6.961)				
$y^{US}(7yr)$			-20.1274**			
			(7.252)			
$y^{US}(10yr)$				-19.762*	-21.297**	-23.263***
				(8.050)	(7.356)	(7.009)
Constant	16.584	24.203	23.699	-0.945	31.397	39.074
	(26.021)	(25.958)	(24.590)	(27.652)	(22.387)	(22.135)
N	606	606	606	606	606	606

Note: Results of running various specifications of the regression described in equation (13) by OLS with Newey-West standard errors (SEs) with 12 lags reported in parentheses. Asterisks *, ** and *** indicate significance at the 10 percent, 5 percent, and 1 percent levels, respectively. Data are weekly, and obtained from Bloomberg from July 1, 2009 through December 30, 2022, except U.S. Treasury market depth data, which was obtained from Brokertec.

In terms of our estimated magnitudes and the summary statistics above, our point estimates indicate that having shallow two-year U.S. Treasury markets according to our metric are associated with 97.20 basis point increases in our estimate of JGBi liquidity premia. Similarly, our estimated coefficients for shallow market conditions for 5-, 7-, and 10-year U.S. Treasuries are associated with increases of 108.98, 104.60, and 44.15 basis point deviations in the estimated JGBi liquidity premia, respectively. Our point estimate for regression model (5) indicates that having U.S. treasuries of *any* maturity exhibit shallowness is associated with a predicted increase of 82.66 basis points in the estimated liquidity premia, while our point estimate for regression model (6) indicates that having an additional U.S. Treasury market exhibiting shallowness is associated with a 33.66 basis point increase in the estimated

JGBi liquidity premia.

In terms of the conditioning variables, both the VIX^{JPN} and the $MOVE^{US}$ variables are insignificant. The Japanese yield variables, $y^{JPN}(2yr)$, $y^{JPN}(4yr)$, $y^{JPN}(7yr)$, and $y^{JPN}(10yr)$ all enter positively in their respective specifications, at least at a 10% significance level. This is the expected sign, as we would expect elevated interest rate periods to be associated with higher liquidity premia as well. In contrast, U.S. Treasury yields enter negatively, albeit only insignificantly for the 2-year yield, while the 5-, 7- and 10-year Treasury yields enter negatively at least at a 10% confidence level. Again, all of these variables enter with their expected signs.

Overall then, our results indicate significant and economically important spillovers between illiquidity disruptions in U.S. Treasury markets and estimated liquidity premia in the Japanese JGBi market. As a consequence, the Japanese JGBi market may be less segmented from global bond markets than could be expected based on the structural arguments outlined in Cardozo and Christensen (2025). All else being equal, this market may then be more liquid than other inflation-indexed bond markets, as also suggested by the mostly moderate size of our estimated JGBi liquidity premia. On the downside, the integration with global bond markets implies that the JGBi market is sensitive to liquidity shortages in other major bond markets as captured in our market depth measures for on-the-run U.S. Treasury securities.

6.4 Additional Spillover Evidence from the Conventional JGB Market

Given the strong evidence we find for JGBi-market spillovers from Treasury market disruptions, we next examine the implications for liquidity disruptions in the U.S. Treasury market for the broader conventional JGB market.

Our methodology is the same as that above. We use the market depth data for the two, five, seven, and ten year on-the-run U.S. Treasury securities from December 4, 2009, to December 30, 2022, where market depth is again measured as the number of open orders from surveyed dealers at the stated maturity.

We again characterize Treasury market depth as shallow if the measured depth is two standard deviations below its sample mean or less, using the same indicator variables: $I^{US}(X)_{shallow}$, $X \in \{2, 5, 7, 10\}$ which takes a value of one during weeks exhibiting shallow market depth in the X-year on-the-run U.S. Treasury security, and 0 otherwise; $I^{US}(any)_{shallow}$, which takes a value of one in weeks in which any of the individual market indicators of shallowness signals low market depth, and $C^{US}_{shallow}$, which is calculated as the sum of the four individual maturity indicators. We also include U.S. and Japanese yields of comparable maturities, $y^{US}(X)$ with $X \in \{2, 5, 7, 10\}$, and $y^{JPN}(X)$ with $X \in \{2, 4, 7, 10\}$ measured in years for the U.S. and Japanese bond markets, respectively. We again pair the 5-year U.S. shallow market depth indicators with the 4-year Japanese maturity yield. Finally, we

Table 11: Average JGB Bid-Ask Spread Regression Results

	(1)	(2)	(3)	(4)	(5)	(6)
$I^{US}(2yr)_{shallow}$	0.376**					
	(0.137)					
$I^{US}(5yr)_{shallow}$		0.449***				
		(0.133)				
$I^{US}(7yr)_{shallow}$			0.481**			
			(0.146)			
$I^{US}(10yr)_{shallow}$				0.300**		
				(0.107)		
$I^{US}(any)_{shallow}$					0.332***	
					(0.113)	
$C^{US}_{shallow}$						0.137***
						(0.034)
VIX^{JPN}	0.027	0.020	0.023	0.025	0.029	0.021
	(0.037)	(0.036)	(0.033)	(0.035)	(0.036)	(0.034)
$MOVE^{US}$	-0.002	-0.001	-0.001	0.000	-0.002	-0.002
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
$y^{JPN}(2yr)$	-1.279***					
	(0.310)					
$y^{JPN}(4yr)$		-0.918***				
		(0.193)				
$y^{JPN}(7yr)$			-0.553***			
			(0.108)			
$y^{JPN}(10yr)$				-0.330***	-0.300***	-0.281***
				(0.062)	(0.061)	(0.065)
$y^{US}(2yr)$	-0.085*					
	(0.041)					
$y^{US}(5yr)$		-0.141**				
		(0.048)				
$y^{US}(7yr)$			-0.165**			
			(0.052)			
$y^{US}(10yr)$				-0.198***	-0.205***	-0.213***
				(0.058)	(0.058)	(0.057)
Constant	1.336***	1.503***	1.638***	1.732***	1.829***	1.865***
	(0.120)	(0.140)	(0.157)	(0.164)	(0.178)	(0.172)
N	536	536	536	536	536	536

Note: Regression results with average JGB bid-ask spreads as dependent variable. Estimation by OLS with Newey-West standard errors (SEs) with 12 lags reported in parentheses. Asterisks *, ** and *** indicate significance at the 10 percent, 5 percent, and 1 percent levels, respectively. Data are weekly, and obtained from Bloomberg from December 4, 2009 through December 30, 2022, except U.S. Treasury market depth data, which was obtained from Brokertec.

again include VIX^{JGB} and $MOVE^{US}$ to proxy for volatility in Japanese and U.S. financial markets, respectively.

As our new dependent variable, we take the average level of JGB bid-ask spreads of 2, 5, 7, and 10 year maturities. It is likely that this variable would be positively correlated with

our JGBi liquidity premium measure, and indeed this is the case. The two variables have a positive, and statistically significant correlation coefficient of 0.21, as also documented in Section 4.1.1.

Our regression results are shown in Table 11. Similar to the results we obtained for our generated JGBi liquidity indicator, we find that weeks exhibiting shallow depth in U.S. Treasury markets are associated with higher bid-ask spreads in the broader JGB market for all of the individual maturities studied, as well as the group indicators of shallowness at any maturity in the U.S. market and the total number of maturities indicating shallow treasury market depth (Columns 5 and 6). All of these estimates are statistically significant, with all entering positively at a 1% confidence level except for the 2 year maturity, which enters positively at a 5% confidence level.

In terms of our estimated magnitudes and the summary statistics above, our point estimates indicate that having shallow two-year U.S. Treasury markets according to our metric are associated with 0.38 basis point increases in the average JGB bid-ask spread. Similarly, our estimated coefficients for shallow market conditions for 5-, 7-, and 10-year U.S. Treasuries are associated with increases of 0.45, 0.48, and 0.30 basis points in the average JGB bid-ask spread. Our point estimate for regression model (5) indicates that having U.S. Treasuries of *any* maturity exhibit shallowness is associated with a predicted increase of 0.33 basis points in the JGB bid-ask spread, while our point estimate for regression model (6) indicates that having an additional U.S. Treasury market exhibiting shallowness is associated with a 0.14 basis point increase in the average JGB bid-ask spread.

Our finding of global spillovers for the broader JGB market as well suggests that U.S. Treasury spillovers are not limited to JGBi's, but instead may have broader macroeconomic implications. Moreover, in supporting our findings for the JGBi liquidity premium estimates generated by our model, they provide a useful robustness check on those liquidity premium measures, and hence for the model itself, as also discussed in Section 4.1.1.

7 Conclusion

In this paper, we account for both bond-specific liquidity premia and the value of deflation protection in the prices of individual Japanese inflation-indexed government bonds, known as JGBi's. To do so, we extend the four-factor model of nominal and real yields introduced by CS (2022), which only accounts for the deflation protection values, by adding a bond-specific liquidity risk factor structured as in ACR (2021).

We find that both adjustments are sizable and time varying. Importantly, the liquidity adjustment has persistent trends and switches sign, meaning that it periodically transitions from being a liquidity price discount to being a convenience price premium. This adjustment is impactful, as our measure of adjusted BEI during these latter periods fall *below* observed BEI, which significantly increases the priced risk of deflation and raises the model-implied

value of the deflation protection offered by JGBi's. Similarly, when the liquidity adjustment is positive, our measure of adjusted BEI is *above* observed BEI, which leads to lower values of deflation protection compared to the standard model.

The documented distortions to Japanese BEI rates due to the large and time-varying premia in JGBi prices should leave these rates different from inflation swap rates, similar to disparities that have been found in U.S. financial markets by Fleckenstein et al. (2014) in the context of U.S. TIPS and inflation swaps. Intriguingly, we find that Japanese inflation swap rates, which are not subject to the distortions applying to matching BEI rates, are nearly identical to those rates. Thus, we provide suggestive evidence of the existence of apparent and persistent pricing anomalies in the Japanese inflation swap market.

While it is beyond the scope of this paper to speculate about the sources of these anomalies, one possibility is that pricing in the inflation swap market may reflect the perceived need among inflation swap dealers to hedge their positions in JGBi markets in some states of the world. A priori, however, the large values of these distortions seem excessive to explaining this anomaly solely through that channel. Thus, we leave it for future research to examine that question further.

Moreover, our extended model produces more stable measures of expected inflation, which are closer to the inflation forecasts reported in surveys of professional forecasters than those coming from the model without the liquidity adjustment. For that same reason the extended model also produces more stable estimates of the residual inflation risk premium. Overall, we take these results to document that the extended model represents a significant improvement relative to the existing models in the literature, including our earlier model in CS (2022).

We then utilize our estimated liquidity premium estimates to examine the degree of spillovers between U.S. and Japanese bond markets. While Japanese asset markets are commonly believed to exhibit some degree of segmentation due to "home bias" among Japanese investors, we find that exceptionally low levels of market depth for on-the-run U.S. Treasuries for a variety of maturities are associated with elevated estimated JGBi liquidity premia. Our point estimates also indicate that this relationship is economically important. We interpret these results as evidence that illiquidity and market stress in the U.S. Treasury market does spill over into the JGBi market, implying that segmentation of the Japanese JGBi market is incomplete.

We also examine the implications for liquidity disruptions in the U.S. Treasury market for the broader conventional JGB market using the same methodology. Similar to our JGBi market results, we find that weeks exhibiting shallow depth in U.S. Treasury markets are associated with higher bid-ask spreads in the broader JGB market as well. Our finding of global spillovers for the broader JGB market indicates that global spillovers to the Japanese bond market are not limited to JGBi's, and may have broader macroeconomic implications.

However, we leave it for future research to explore that further.

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Online Appendix

Inflation Expectations, Liquidity Premia and Global Spillovers in Japanese Bond Markets

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A The Dynamics of JGBi Liquidity Risk Using Weekly Data

In this appendix, to further document that the positive connection between liquidity risk and bond age is indeed a pervasive pattern of the JGBi data, we use weekly data and test for the statistical significance of this positive relationship repeating the panel regression given by

$$BAS_t^i = \tau_t + \alpha_i + \beta Age_t^i + \varepsilon_t^i, \tag{1}$$

where BAS_t^i denotes the bid-ask spread of the ith JGBi in period t. To control for unobserved heterogeneity, we allow for both time fixed effects τ_t and bond-specific fixed effects α_i in equation (1). Finally, Age_t^i measures the time since issuance of the ith JGBi and ε_t^i is a zero-mean error term. Using all available JGBi's, we then estimate the panel regression in equation (1) by Ordinary Least Squares (OLS).

Regression specification	Age_t^i	Number of parameters	Adj. R^2
OLS	0.514***	2	0.181
	(0.097)		
Bond fixed effects	0.522***	28	0.103
	(0.104)		
Time fixed effects	0.394***	605	0.465
	(0.097)		
Bond & time fixed effects	0.407***	631	0.426
	(0.083)		

Table 1: **Panel Regressions of Weekly Bid-Ask Spreads in the JGBi Market**
This table reports the results of separately estimating the relationship between bid-ask spreads and bond age, by OLS, with bond-specific fixed effects, time fixed effects and both fixed effects without interaction. The variable Age_t^i is in years since issuance. White's heteroscedastic standard errors (SEs) are reported in parentheses. Asterisks *, ** and *** indicate significance at the 10 percent, 5 percent, and 1 percent levels, respectively. The adjusted R^2 is computed based on the demeaned variation in the dependent variable, except for the OLS adjusted R^2. The data used are weekly covering the period from June 3, 2011, to December 30, 2022, across 27 Japanese ten-year inflation-indexed bonds (JGBi), a total of 5,584 observations.

Table 1 reports the results from a simple OLS regression without any fixed effects as well as those from regressions with either bond or time fixed effects or both, all based on weekly data. The results show that the age of individual JGBi's has a significant positive effect on their bid–ask spreads, as also suggested by the aggregate series shown in Figure 5 in the main text.

B Bond Yield Decomposition

In this appendix, we describe the decomposition of nominal and real bond yields into underlying expectations and residual risk premium components using arbitrage-free term structure models.

We follow Merton (1974) and assume the existence of a continuously-traded continuum of nominal and real zero-coupon bonds. This implies that inflation risk is spanned by the nominal and real yields. This allows us to decompose the nominal and real yields into the sum of the corresponding short-rate expectations and associated term premia using our arbitrage-free term structure model.

To begin, define the nominal and real stochastic discount factors as M_t^N and M_t^R, respectively. Their dynamics are standard and given by

$$\begin{aligned} dM_t^N/M_t^N &= -r_t^N dt - \Gamma_t' dW_t^{\mathbb{P}}, \\ dM_t^R/M_t^R &= -r_t^R dt - \Gamma_t' dW_t^{\mathbb{P}}, \end{aligned}$$

where Γ_t contains the risk premia.

Under our no-arbitrage condition, the price of a nominal bond that pays one unit of currency in τ years and the price of a real bond that pays one consumption unit in τ years must satisfy

$$P_t^N(\tau) = E_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^N}{M_t^N}\right] \quad \text{and} \quad P_t^R(\tau) = E_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}\right],$$

where $P_t^N(\tau)$ and $P_t^R(\tau)$ are the prices of the zero-coupon nominal and real bonds for maturity τ at time t and $E_t^{\mathbb{P}}[.]$ is the conditional expectations operator under the real-world (or $\mathbb{P}$-) probability measure.

The no-arbitrage condition also requires that the price of a consumption unit, denoted as the overall price level Π_t, is the ratio of the real and nominal stochastic discount factors:

$$\Pi_t = \frac{M_t^R}{M_t^N}.$$

By Ito's lemma, the dynamic evolution of Π_t is given by

$$d\Pi_t = (r_t^N - r_t^R)\Pi_t dt.$$

Thus, in the absence of arbitrage, the instantaneous growth rate of the price level is equal to the difference between the instantaneous nominal and real risk-free rates.[1] Correspondingly, we can express the stochastic price level at time t+τ as

$$\Pi_{t+\tau} = \Pi_t e^{\int_t^{t+\tau}(r_s^N - r_s^R)ds}.$$

The relationship between the yields and inflation expectations can be obtained by decomposing the price of the nominal bond as follows

$$\begin{aligned} P_t^N(\tau) &= E_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^N}{M_t^N}\right] = E_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R/\Pi_{t+\tau}}{M_t^R/\Pi_t}\right] = E_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}\frac{\Pi_t}{\Pi_{t+\tau}}\right] \\ &= E_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}\right] \times E_t^{\mathbb{P}}\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right] + cov_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}}\right] \\ &= P_t^R(\tau) \times E_t^{\mathbb{P}}\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right] \times \left(1 + \frac{cov_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}}\right]}{E_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}\right] \times E_t^{\mathbb{P}}\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right]}\right). \end{aligned}$$

Converting this price into yield to maturity using

$$y_t^N(\tau) = -\frac{1}{\tau}\ln P_t^N(\tau) \quad \text{and} \quad y_t^R(\tau) = -\frac{1}{\tau}\ln P_t^R(\tau),$$

we obtain

$$y_t^N(\tau) = y_t^R(\tau) + \pi_t^e(\tau) + \phi_t(\tau),$$

where the market-implied average rate of inflation expected at time t for the period from t to $t+\tau$ is

$$\pi_t^e(\tau) = -\frac{1}{\tau}\ln E_t^{\mathbb{P}}\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right] = -\frac{1}{\tau}\ln E_t^{\mathbb{P}}\left[e^{-\int_t^{t+\tau}(r_s^N - r_s^R)ds}\right]$$

and the associated inflation risk premium for the same time period is

$$\phi_t(\tau) = -\frac{1}{\tau}\ln\left(1 + \frac{cov_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}}\right]}{E_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}\right] \times E_t^{\mathbb{P}}\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right]}\right).$$

This last equation demonstrates that the inflation risk premium can be positive or nega-

[1] Note that the price level Π_t is a stochastic process as long as r_t^N and r_t^R are stochastic processes.

tive. It is positive if and only if

$$cov_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}}\right] < 0.$$

That is, the riskiness of nominal bonds relative to real bonds depends on the covariance between the real stochastic discount factor and inflation, and is ultimately determined by investor preferences.

Now, the BEI rate is defined as

$$BEI_t(\tau) \equiv y_t^N(\tau) - y_t^R(\tau) = \pi_t^e(\tau) + \phi_t(\tau),$$

that is, the difference between nominal and real yields of the same maturity. Note that it can be decomposed into the sum of expected inflation and the inflation risk premium.

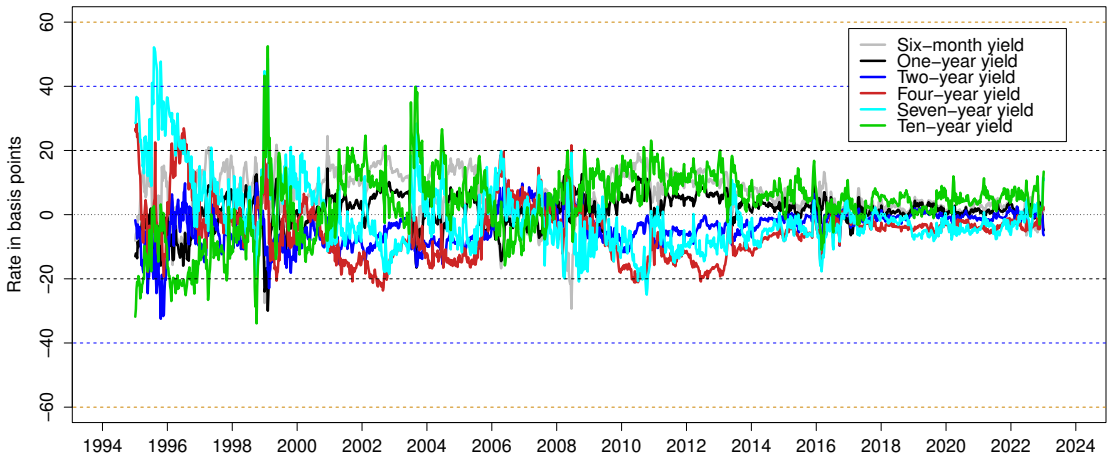
Finally, we define the nominal and real term premia as

$$\begin{aligned} TP_t^N(\tau) &= y_t^N(\tau) - \frac{1}{\tau}\int_t^{t+\tau} E_t^{\mathbb{P}}[r_s^N]ds, \\ TP_t^R(\tau) &= y_t^R(\tau) - \frac{1}{\tau}\int_t^{t+\tau} E_t^{\mathbb{P}}[r_s^R]ds. \end{aligned}$$

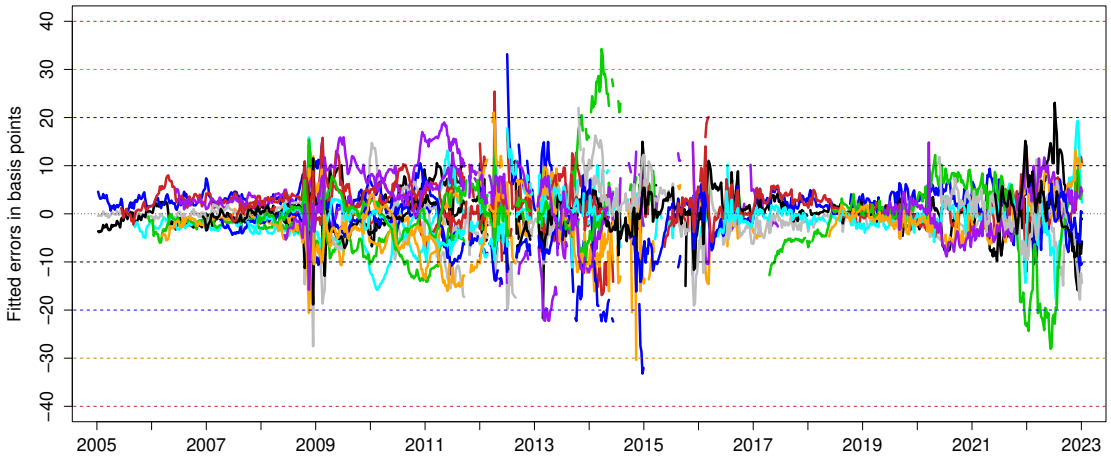
That is, the nominal term premium is the difference in expected nominal return between a buy and hold strategy for a τ-year nominal bond and an instantaneous rollover strategy at the risk-free nominal rate r_t^N. The interpretation for the real term premium is similar. The model thus allows us to decompose nominal and real yields into their respective term premia and short-rate expectations components.

C Fitted Error Series

In this appendix, we show the fitted error series of the six Japanese nominal yields in our sample in Figure 1(a), while the fitted errors of the JGBi prices measured as yield deviations are shown in Figure 1(b), all based on our benchmark $G^L(5)$ model with option adjustment.



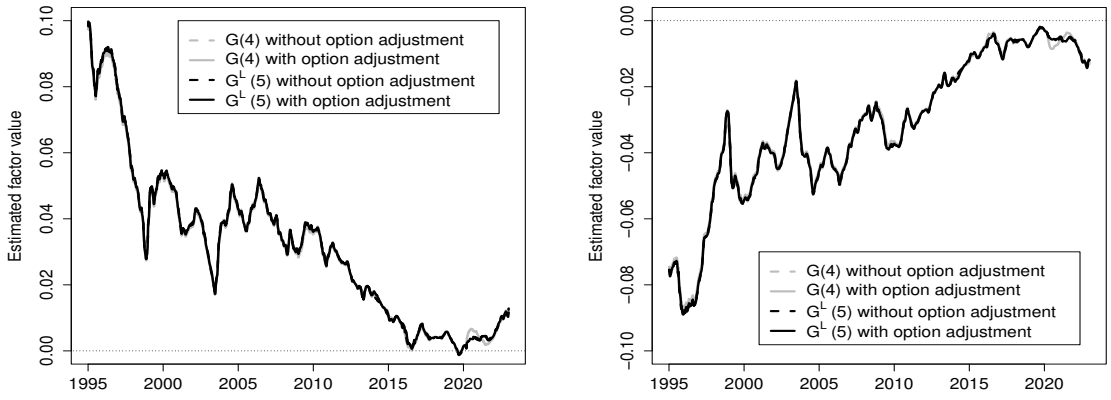
(a) JGBs



(b) JGBi's

Figure 1: **Fitted Errors of Japanese Nominal and Real Government Bonds**

D Estimated Nominal State Variables



(a) L_t^N (b) S_t^N

Figure 2: **Estimated Nominal State Variables**
Illustration of the estimated state nominal variables, L_t^N and S_t^N, from the $G(4)$ and $G^L(5)$ models without and with deflation protection option adjustment.

In this appendix, we plot the estimated nominal yield factors from all four models studied in the main text. These are shown in Figure 2. Note that the two nominal factors, L_t^N and S_t^N, are essentially identical across all four models. This also explains the very similar fit to the nominal yield data across all four models reported in Table 3 in the main text. Thus, the nominal side of our models has very little sensitivity to adjustments for either the liquidity premia of JGBi's or the values of the deflation protection they offer.

E Sensitivity of JGBi Liquidity Premia to Data Frequency

In this appendix, we assess whether the data frequency plays any role for our estimated JGBi liquidity premia. To do so, we estimate the $G^L(5)$ model with deflation protection option adjustment using daily, weekly, and monthly data.

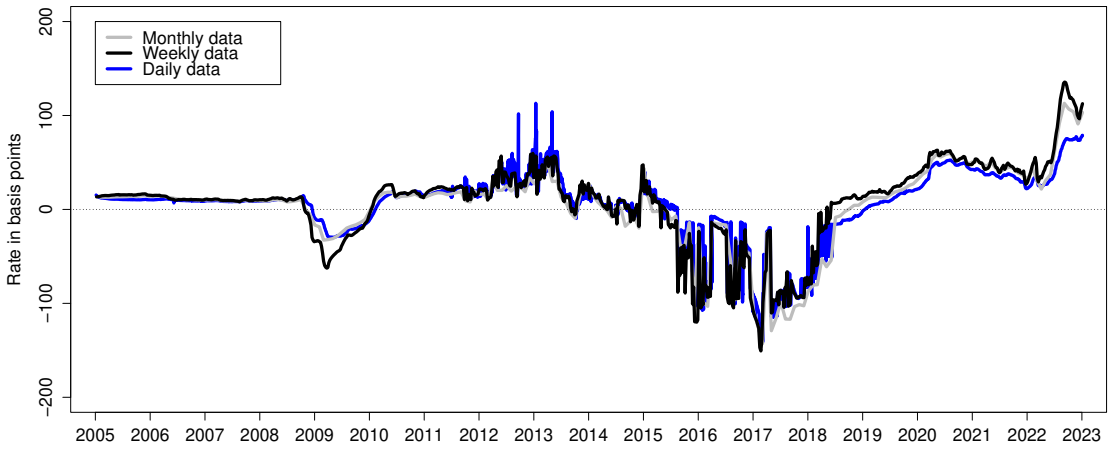


Figure 3: **Average Estimated JGBi Liquidity Premium: Data Frequency**
Illustration of the average estimated bond-specific liquidity risk premium of JGBi's for each observation date implied by the $G^L(5)$ model with deflation option adjustment when estimated using daily, weekly, and monthly data. In all three cases, the bond-specific liquidity risk premia are measured as the estimated yield difference between the fitted yield to maturity and the corresponding frictionless yield to maturity of individual JGBi's with the liquidity risk factor turned off as described in the text.

Figure 3 shows the average JGBi bond-specific liquidity risk premium from the three estimations calculated as described in Section 4.1 in the main text. Note that they are all very close to each other. Thus, we conclude that data frequency matters little for our estimated JGBi liquidity premia.

F Sensitivity of Deflation Risk Premia to Data Frequency

In this appendix, we assess whether the data frequency plays any role for our estimated ten-year deflation risk premia. To do so, we estimate the $G^L(5)$ model with deflation protection option adjustment using daily, weekly, and monthly data.



Figure 4: **Ten-Year Deflation Risk Premium**

Figure 4 shows the estimated ten-year deflation risk premia from the three estimations, each calculated as par yield spreads as described in Section 4.2.1 in the main text. Note that they are all very close to each other. Thus, we conclude that data frequency matters little for our estimated measures of deflation risk.

Overall, we take these results to suggest that our $G^L(5)$ model is well positioned to be used for real-time analysis.

References

Merton, Robert C., 1974, "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *Journal of Finance*, Vol. 29, No. 2, 449-470.