A Quantitative Theory of Trade in Services

Preliminary and Incomplete

Benny Kleinman Tamar Yerushalmi

Stanford University and NBER Stanford University

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Abstract

Trade in services is a large and growing share of global trade. Which countries specialize in services and why, how large are the gains from trade in services, and how does the global economy adjust to rising trade in services? To study these questions, we develop a dynamic multi-sector model of trade, enriched with three key features to capture the unique characteristics of traded services: (1) firms produce and accumulate non-rival assets, with trade in services being particularly intensive in such assets; (2) preferences are non-homothetic, with services featuring high income elasticities; and (3) trade takes place through both cross-border transactions and commercial presence, capturing the key modes of service delivery. We find that heterogeneous income elasticities and non-rivalry are both quantitatively important for explaining the magnitude and cross-country variation of services specialization. Gains from trade in services are comparable to those from goods. Trade shocks lead to gradual adjustment of comparative advantage as countries accumulate non-rival assets. Along the transition path following a reduction in trade barriers, countries specializing in goods experience higher short-run gains and run current account surpluses, while those specializing in services see higher long-run gains and run current account deficits.

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Kleinman: Stanford University and NBER, bennyk@stanford.edu. Yerushalmi: Stanford University, tamarye@stanford.e
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1 Introduction

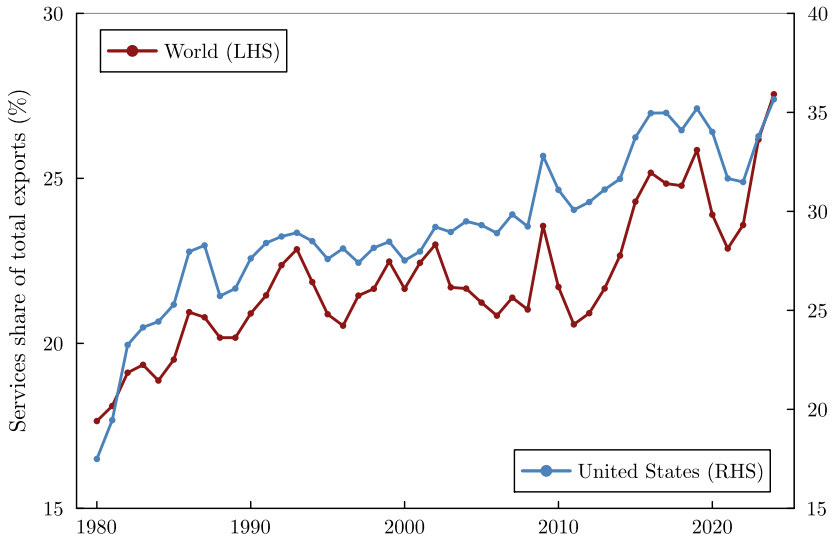
> *"Services are often largely overlooked in discussions on global trade, yet they account for the majority of trade in many developed economies and are growing rapidly in many developing economies as well."*
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> — Roberto Azevêdo, WTO Director-General, in WTO (2019)

Trade in services accounts for a large and growing share of global trade. Figure 1 shows the services share of total exports for the world and the United States since 1980, as recorded in IMF Balance of Payments data. The global share has risen from 18 percent in 1980 to over 27 percent in 2024, with the United States consistently above the world average at around 35 percent, and is projected to exceed 40 percent by 2040 (WTO 2019).[1] Furthermore, this increase is driven by "modern services" – intellectual property, ICT, business, and financial services (Baldwin et al. 2024). The rising importance of services in global trade raises multiple questions for trade theory: how different is trade in services from trade in goods? Which countries specialize in services and why? And how large are the gains from reducing trade frictions in services?

In this paper, we develop a model of the global economy with trade in goods and services, which highlights two key distinguishing features of modern services. The first is their strong reliance on the accumulation of non-rival assets – software, data, and other intangibles (see Hill 1999, for a taxonomy) – which for brevity we label *intellectual property* (IP). While wine and cloth shipped to one country cannot be shipped to another, the consumption of Netflix services by one country does not diminish their availability to other countries. The second is the observation that as countries get richer, their demand for services increases (Comin et al. 2021), particularly for modern services. We show that these two features have important implications for classic questions in international trade, including the gains from trade, the patterns of specialization, and the dynamics of trade imbalances. Richer countries specialize in services due to home market effects arising from the combination of higher demand for services and high scale effects in production. Globalization generates income effects that improve terms of trade for countries that specialize in services. Comparative advantage evolves along the transition path as countries accumulate

[1] This share is even higher once broader definitions of trade in services are used, particularly when including sales by foreign affiliates (Mode 3 of trade in services per the GATS classification).

Figure 1: Services Share of World Exports, 1980–2024



Source: IMF Balance of Payments Statistics. Services share is computed as total services credit (exports) divided by total goods plus services credit. Red: world aggregate. Blue: United States.

non-rival assets across sectors, and countries that specialize in high-increasing-returns activities are more likely to run current account deficits.

Our framework features a world economy with multiple countries and sectors. As in standard trade models, countries differ in size, productivity, and bilateral trade frictions. Three features distinguish our setting. First, on the demand side, preferences are *non-homothetic* with heterogeneous income elasticities across sectors, so that richer countries allocate a larger share of expenditure to services. Second, on the production side, firms dynamically accumulate *non-rival assets* – software, data, and other intangibles – and combine them with rival labor, with the intensity of such assets in production varying across sectors. Third, sectors differ in their *mode of delivery*: production may rely on workers in the origin country (cross-border supply, or Mode 1 in trade-policy parlance) or on workers hired in the destination country (commercial presence, or Mode 3). When delivery requires local labor, the only income flowing back to the origin country is the return on the firm's non-rival assets. Notably, a sector's trade can be intensive in non-rival assets even when its production is not, if all rival labor must be hired near consumers. The model allows for dynamic adjustment of IP stocks over time and for international saving and borrowing by households.

We analytically characterize the model's steady state and provide ex-ante sufficient statistics for long-run comparative statics, as well as ex-post sufficient statistics to evaluate welfare gains relative to autarky. As in standard trade models, much of the gains from trade can be summarized by sectoral trade openness and trade elasticities. In addition, welfare effects depend on the reallocation of labor across sectors with varying income elasticities and with varying scale effects in production, which can either dampen or amplify the gains from trade.

A notable special case of the model is the class of multi-sector trade models with external economies of scale, such as Kucheryavyy et al. (2023) and Bartelme et al. (2025). This class of models is isomorphic to our steady-state equilibrium when income elasticities are unitary and all trade is cross-border. However, unlike in these models, there are no external returns to scale in our framework. While the equilibrium is mathematically equivalent, there is no misallocation of factors across sectors with different degrees of returns to scale, and the decentralized equilibrium is constrained-efficient.

We quantify the model by combining standard data on trade in goods, newly available data on bilateral trade in services from the IMF Bilateral Trade in Services (BiTS) database (Li et al. 2025), data on sales by foreign affiliates from the OECD AMNE database, and standard national accounts data. We recover the sectoral intensity of non-rival assets from cost shares of labor dedicated to IP creation, which we construct from occupational task-based data. Disciplining scale elasticities from cost shares is feasible because investments in IP are internalized by firms: while IP is non-rival, it is excludable from other firms. Our sectoral scale elasticities closely match other existing metrics of non-rival asset intensity, such as intangible intensity.

We use our model to investigate three key questions. First, what determines the patterns of trade in services? We show that both the income effects in demand and the scale effects in production are consequential in determining which countries specialize in services. For instance, shutting down both margins leads the USA from having a very strong revealed comparative advantage (henceforth RCA) in services – second only to the United Kingdom – to a negative RCA in services. The USA's high demand for services reflects its high income, and its large size and high labor productivity magnify its comparative advantage in sectors with high scale effects. Together, these two channels make the USA a strong services exporter.

Second, how large are the welfare gains from trade in services? To this end, we revisit the

classic hypothetical exercise of measuring the gains from moving from autarky to contemporary levels of trade. We find that the gains from services trade are comparable to those from trade in goods – and larger for some countries, notably in Europe. This is largely driven by the large weight of services in consumption (especially for rich countries), despite the low levels of trade openness in services. Perhaps surprisingly, the ability to accumulate non-rival assets does not amplify these gains. This lack of amplification differs from the multiplier effect one would typically obtain when allowing for standard capital accumulation. The reason is that non-rival assets are produced with labor, so their accumulation is limited by labor endowments, and any effect on the gains from trade operates solely through the reallocation of labor across sectors with varying scale elasticities. This contrasts with physical capital accumulation, which is typically assumed to use final goods as inputs, so that its supply can increase following reductions in trade frictions.

Third, what are the dynamic effects of global trade liberalization? We study this question by tracing the model's transition path following a uniform reduction in trade frictions across all sectors. Globalization deepens specialization patterns, and this deepening is amplified over time as countries accumulate non-rival assets in their comparative-advantage sectors and draw them down in other sectors. Countries that specialize in sectors with low increasing returns – such as China – experience larger consumption gains in the short run and run current account surpluses, while countries that specialize in sectors with high increasing returns – such as the United Kingdom – experience larger consumption gains in the long run and run current account deficits.

Relation to the literature. This paper contributes to several strands of the literature. First, we contribute to the quantitative trade literature (Eaton and Kortum 2002; Arkolakis et al. 2012; Costinot and Rodríguez-Clare 2014), and in particular to the small literature that incorporates trade in services into this analysis (Eaton and Kortum 2018). Beyond tailoring quantitative trade models to the case of trade in services, our framework allows for more general analysis of trade with dynamically accumulated non-rival assets. A related literature studies trade in services within countries, including Eckert (2019), Hsieh and Rossi-Hansberg (2023), Eckert et al. (2025), and Kleinman (2025). We also relate to a small literature on the theory of trade policy in services (Staiger and Sykes 2021).

Second, we relate to the growing empirical literature on trade in services (Francois and Hoek-

man 2010; Barattieri 2014; Baldwin 2016; Gervais and Jensen 2019; Baldwin et al. 2024; Li et al. 2025). Our theory captures many of the features highlighted in this literature, including the dominance of services trade by high-income countries; the centrality of trade in intangible assets; and the relationship between comparative advantage in services and the current account.

Third, we relate to multi-sector models of trade with economies of scale, such as Kucheryavyy et al. (2023), Lashkaripour and Lugovskyy (2023), and Bartelme et al. (2025). As noted above, we differ by assuming that non-rival assets are excludable, so that all scale economies are internal to the firm. As a result, multiplicity of equilibria is ruled out, the decentralized equilibrium is constrained-efficient, and sectoral scale elasticities can be measured from cost shares.

Fourth, we relate to models of non-homothetic demand and trade, including Markusen (2013), Caron et al. (2014), Kehoe et al. (2018), and Matsuyama (2019). Our NH-CES demand structure follows Comin et al. (2021); we combine differences in sectoral income elasticities with sectoral accumulation of non-rival assets and show how these two margins shape the patterns of trade and the gains from trade in services.

Fifth, we relate to models of multinational production that highlight the role of multinationals in facilitating trade in knowledge, such as Markusen (1984), Ramondo and Rodríguez-Clare (2013), Tintelnot (2017), Arkolakis et al. (2018), Alviarez et al. (2023), and Santacreu (2025). Relative to these models, our framework features dynamic accumulation of non-rival assets in a multi-sector environment where sectors differ in the importance of such assets for production.

Finally, we relate to the literature on intangible capital in macroeconomics, including Corrado et al. (2009), McGrattan and Prescott (2009), McGrattan and Prescott (2010), and Crouzet et al. (2022). We embed the dynamic accumulation of such assets in a multi-sector model of trade.

2 Model

There are N countries, indexed by i (origin) and n (destination), and K sectors, indexed by k. Time is discrete, $t = 0, 1, 2, \ldots$. Country n is endowed with L_{nt} units of labor.

2.1 Economic Environment

Preferences. The representative household in country n has CRRA intertemporal preferences:

$$U_{n0} = \sum_{t=0}^{\infty} \beta^t \frac{C_{nt}^{1-1/\psi}}{1-1/\psi}, \tag{1}$$

where $\beta \in (0,1)$ is the discount factor and $\psi > 0$ is the elasticity of intertemporal substitution. The index of real consumption C_{nt} is defined implicitly by the nonhomothetic CES (NH-CES) aggregator of Sato (1977) and Comin et al. (2021):

$$\sum_{k=1}^{K} \bar{\chi}_{nk}^{1/\eta} \left(\frac{C_{nkt}}{C_{nt}^{\varepsilon_k}} \right)^{(\eta-1)/\eta} = 1, \tag{2}$$

where $\eta \in (0,1) \cup (1,\infty)$ is the elasticity of substitution across sectors, $\bar{\chi}_{nk} > 0$ are country-sector demand shifters, and $\varepsilon_k > 0$ are nonhomotheticity parameters that control relative income elasticities across sectors. When all ε_k are equal, the aggregator reduces to homothetic CES; when additionally $\eta \to 1$, it reduces to Cobb–Douglas. Each sectoral good C_{nkt} is itself a CES aggregate over varieties from all origins:

$$C_{nkt} = \left[\sum_{i=1}^{N} \int_{\omega \in \Omega_{inkt}} a_{inkt}^{1/\sigma_k} \, c_{inkt}(\omega)^{(\sigma_k-1)/\sigma_k} \, d\omega \right]^{\sigma_k/(\sigma_k-1)}, \tag{3}$$

with $\sigma_k > 1$ the elasticity of substitution across varieties and $a_{inkt} \geq 0$ a bilateral demand shifter that captures the appeal of origin-i goods for consumers in n. The set Ω_{inkt} denotes the mass of varieties supplied by origin i to destination n in sector k at time t.

Technology. Each sector k has an exogenous mass M_{ik} of symmetric firms in country i. A firm from i selling to n in sector k produces output using three inputs: a non-rival intellectual property (IP) stock (x_{ikt}), origin labor (l_{inkt}), and destination labor (ι_{inkt}):

$$y_{inkt} = x_{ikt}^{\gamma_k} (z_{ikt} \, l_{inkt})^{\alpha_k} (z_{nkt} \, \iota_{inkt})^{1-\alpha_k}, \tag{4}$$

where z_{ikt} and z_{nkt} are exogenous labor-augmenting productivity shifters; $\gamma_k \in [0, 1)$ is the IP intensity of sector k; and $\alpha_k \in (0, 1)$ is the origin-labor share of rival inputs. The key feature of the production function is that the IP stock x_{ikt} is *non-rival* across destinations: the same stock of intellectual property – product designs, software, patents, brand equity – enters production for every market n without being depleted. IP is, however, *excludable*: it is proprietary to the firm. The labor inputs l_{inkt} and ι_{inkt}, by contrast, are rival and must be hired separately for each destination.

The parameter α_k captures the mode of delivery. When $\alpha_k = 1$, all rival labor is hired in the origin country, corresponding to *Mode 1* (cross-border supply) under the GATS classification. When $\alpha_k = 0$, all rival labor is hired in the destination country through a local affiliate, corresponding to *Mode 3* (commercial presence), and the only income flowing from destination to origin is the return to x_{ikt}. Intermediate values of α_k capture blends of these modes.

IP accumulation. IP stocks evolve according to

$$x_{ikt+1} = (1 - \delta_k)\, x_{ikt} + \xi_k\, x_{ikt}^{1-\upsilon} \left(z_{ikt}^{\mu_k}\, h_{ikt} \right)^{\upsilon}, \tag{5}$$

where $h_{ikt} \geq 0$ is innovation labor hired in origin i; $\delta_k \in (0, 1)$ is the depreciation rate; $\xi_k > 0$ is investment efficiency; $\upsilon \in (0, 1]$ governs adjustment frictions; and $\mu_k > 0$ captures the skill-intensity of IP production. The gross increment is Cobb–Douglas in investment inputs $(z_{ikt}^{\mu_k} h_{ikt})$ and the existing stock x_{ikt}, following Lucas and Prescott (1971) and Eaton et al. (2016). Setting $\upsilon < 1$ means that investment is less efficient when done in large amounts relative to the existing stock, capturing absorptive capacity constraints. When $\upsilon = 1$, $\mu_k = 1$, and $\xi_k = 1$, the law of motion reduces to $x_{ikt+1} = (1 - \delta_k)\, x_{ikt} + z_{ikt}\, h_{ikt}$.

Table 1 collects the primitives of the economic environment.

2.2 Equilibrium

We now characterize the equilibrium of the model. We first describe the household's intratemporal and intertemporal problems, then turn to the firm's static and dynamic decisions, and finally state the market clearing conditions and equilibrium definitions.

Table 1: Summary of the Economic Environment

Preferences	
Utility	$U_{n0} = \sum_{t=0}^{\infty} \beta^t \, \dfrac{C_{nt}^{1-1/\psi}}{1-1/\psi}$
NH-CES aggregator	$\sum_{k=1}^{K} \bar{\chi}_{nk}^{1/\eta} \left(\dfrac{C_{nkt}}{C_{nt}^{\varepsilon_k}} \right)^{(\eta-1)/\eta} = 1$
Sectoral consumption	$C_{nkt} = \left[\sum_{i=1}^{N} \int a_{inkt}^{1/\sigma_k} \, c_{inkt}(\omega)^{(\sigma_k-1)/\sigma_k} \, d\omega \right]^{\sigma_k/(\sigma_k-1)}$
Technology	
Production	$y_{inkt} = x_{ikt}^{\gamma_k} (z_{ikt} \, l_{inkt})^{\alpha_k} (z_{nkt} \, \iota_{inkt})^{1-\alpha_k}$
IP accumulation	$x_{ikt+1} = (1-\delta_k) \, x_{ikt} + \xi_k \, x_{ikt}^{1-\upsilon} \left(z_{ikt}^{\mu_k} \, h_{ikt} \right)^{\upsilon}$
Resource constraints	
Goods	$y_{inkt}(\omega) = c_{inkt}(\omega) \quad$ for all i, n, k, ω, t
Labor	$L_{it} = \sum_k M_{ik} \left(h_{ikt} + \sum_n l_{inkt} \right) + \sum_k \sum_n M_{nk} \, \iota_{nikt} \quad$ for all i, t

Households. Let P_{nkt} denote the unit cost of the sectoral consumption bundle C_{nkt} (derived from the firm's pricing problem below), and define consumption expenditure $E_{nt} \equiv \sum_k P_{nkt} \, C_{nkt}$ and the NH-CES cost-of-living index $P_{nt} \equiv E_{nt}/C_{nt}$. Unlike in the homothetic case, P_{nt} depends on the level of real consumption C_{nt} because expenditure shares vary with income. The implicit Marshallian demand for each sector is (Comin et al. 2021, Lemma 1):

$$C_{nkt} = \bar{\chi}_{nk} \left(\frac{P_{nkt}}{E_{nt}} \right)^{-\eta} C_{nt}^{(1-\eta)\varepsilon_k} . \tag{6}$$

Sectoral expenditure shares are

$$\chi_{nkt} \equiv \frac{P_{nkt} \, C_{nkt}}{E_{nt}} = \bar{\chi}_{nk} \left(\frac{P_{nkt}}{E_{nt}} \right)^{1-\eta} C_{nt}^{(1-\eta)\varepsilon_k} , \tag{7}$$

with $\sum_k \chi_{nkt} = 1$ implicitly determining C_{nt} given prices and total expenditure. The expenditure elasticity of demand for sector k is

$$\frac{\partial \log C_{nkt}}{\partial \log E_{nt}} = \eta + (1-\eta)\,\frac{\varepsilon_k}{\bar{\varepsilon}_{nt}}, \qquad \bar{\varepsilon}_{nt} \equiv \sum_j \chi_{njt}\,\varepsilon_j\,. \tag{8}$$

When $\eta < 1$ (gross complements), sectors with $\varepsilon_k > \bar{\varepsilon}_{nt}$ have expenditure elasticity above one (luxuries) and those with $\varepsilon_k < \bar{\varepsilon}_{nt}$ below one (necessities). As real consumption grows, expenditure shifts toward high-ε_k sectors, generating structural change in the composition of demand.

Households can trade an international bond B_{nt} denominated in a common numeraire that pays gross return $1 + \rho_t$. The budget constraint is

$$E_{nt} + B_{n,t+1} + \frac{\zeta}{2}\,P_{nt}\left(\frac{B_{n,t+1}}{P_{nt}}\right)^2 = (1+\rho_t)\,B_{nt} + Y_{nt}\,, \tag{9}$$

where Y_{nt} is household income (wages plus firm profits) and the quadratic term is a portfolio adjustment cost following Schmitt-Grohé and Uribe (2003), with $\zeta > 0$, that ensures determinacy of the international asset distribution.[2] The intratemporal allocation across sectors follows (7), while the intertemporal Euler equation determines bond positions:

$$B_{n,t+1} = \frac{P_{nt}^2}{\zeta}\left(\beta(1+\rho_{t+1})\,\frac{P_{nt}}{P_{n,t+1}}\,\frac{\bar{\varepsilon}_{nt}}{\bar{\varepsilon}_{n,t+1}}\left(\frac{C_{nt}}{C_{n,t+1}}\right)^{1/\psi} - 1\right). \tag{10}$$

The term $\bar{\varepsilon}_{nt}/\bar{\varepsilon}_{n,t+1}$ arises from the NH-CES aggregator: because the marginal utility of expenditure depends on the composition of consumption, shifts in expenditure shares as income grows affect intertemporal incentives. When all ε_k are equal, $\bar{\varepsilon}_{nt}$ is constant and the standard Euler equation obtains. Under financial autarky, $B_{nt} = 0$ for all n, t, so $E_{nt} = Y_{nt}$

Firms. We now turn to the firm's problem. Taking the IP stock x_{ikt} as given, the firm chooses labor inputs to maximize profits. CES demand yields bilateral revenue

$$r_{inkt} = a_{inkt}^{1/\sigma_k}\left(z_{ikt}^{\alpha_k}\,z_{nkt}^{1-\alpha_k}\right)^{(\sigma_k-1)/\sigma_k} E_{nkt}^{1/\sigma_k}\,P_{nkt}^{(\sigma_k-1)/\sigma_k}\left(x_{ikt}^{\gamma_k}\,l_{inkt}^{\alpha_k}\,\iota_{inkt}^{1-\alpha_k}\right)^{(\sigma_k-1)/\sigma_k}. \tag{11}$$

[2] In the quantitative implementation, we normalize the adjustment cost by steady-state income $\bar{Y}_n$ to ensure comparability across countries of different sizes: the cost becomes $(\zeta/2\bar{Y}_n^2)(B_{n,t+1}/P_{nt})^2$.

The first-order conditions for labor imply that origin and destination labor each absorb a constant fraction of bilateral revenue:

$$w_{it}\, l_{inkt} = \alpha_k \, \frac{\sigma_k - 1}{\sigma_k} \, r_{inkt}, \tag{12}$$

$$w_{nt}\, \iota_{inkt} = (1 - \alpha_k) \, \frac{\sigma_k - 1}{\sigma_k} \, r_{inkt}. \tag{13}$$

Substituting back yields revenue as a function of the IP stock alone:

$$r_{inkt} = a_{inkt} \left(z_{ikt}^{\alpha_k} z_{nkt}^{1-\alpha_k} \right)^{\sigma_k - 1} E_{nkt}\, P_{nkt}^{\sigma_k - 1}\, x_{ikt}^{\gamma_k(\sigma_k - 1)} \left(\tfrac{\sigma_k - 1}{\sigma_k}\, \alpha_k^{\alpha_k} (1-\alpha_k)^{1-\alpha_k}\, w_{it}^{-\alpha_k} w_{nt}^{-(1-\alpha_k)} \right)^{\sigma_k - 1}. \tag{14}$$

CES price aggregation yields the sectoral price index:

$$P_{nkt} = \left(\frac{\sigma_k - 1}{\sigma_k}\, \alpha_k^{\alpha_k} (1 - \alpha_k)^{1-\alpha_k} \right)^{-1} \times \left(\sum_i M_{ik}\, a_{inkt} \left(z_{ikt}^{\alpha_k}\, z_{nkt}^{1-\alpha_k} \right)^{\sigma_k - 1} x_{ikt}^{\gamma_k(\sigma_k - 1)} \left(w_{it}^{-\alpha_k} w_{nt}^{-(1-\alpha_k)} \right)^{\sigma_k - 1} \right)^{1/(1-\sigma_k)}. \tag{15}$$

Summing over destinations gives total firm revenue $r_{ikt} \equiv \sum_n r_{inkt}$. The share of country n's sector-k spending on goods from origin i is

$$S_{nikt} = \frac{M_{ik}\, a_{inkt}\, z_{ikt}^{\alpha_k(\sigma_k - 1)}\, x_{ikt}^{\gamma_k(\sigma_k - 1)}\, w_{it}^{-\alpha_k(\sigma_k - 1)}}{\sum_{i'} M_{i'k}\, a_{i'nkt}\, z_{i'kt}^{\alpha_k(\sigma_k - 1)}\, x_{i'kt}^{\gamma_k(\sigma_k - 1)}\, w_{i't}^{-\alpha_k(\sigma_k - 1)}}, \tag{16}$$

where the destination productivity z_{nkt} cancels between numerator and denominator. The non-rival IP stock x_{ikt} enters with exponent $\gamma_k(\sigma_k - 1)$: increases in IP raise the firm's competitiveness across *all* destination markets simultaneously.

Dynamic investment. The firm chooses a sequence of innovation labor $\{h_{ikt}\}$ to maximize the present value of profits:

$$\max_{\{h_{ikt}\}} \sum_{t=0}^{\infty} \left(\prod_{s=1}^{t} \frac{1}{1 + \rho_s} \right) \left[\frac{1}{\sigma_k}\, r_{ikt} - w_{it}\, h_{ikt} \right] \qquad \text{s.t.} \quad x_{ikt+1} = (1 - \delta_k)\, x_{ikt} + \xi_k\, x_{ikt}^{1-\upsilon} \left(z_{ikt}^{\mu_k} h_{ikt} \right)^{\upsilon}, \tag{17}$$

where we have used the fact that variable labor costs absorb a fraction $\frac{\sigma_k - 1}{\sigma_k}$ of revenue, so per-period operating profit is $\frac{1}{\sigma_k} r_{ikt}$. It is convenient to define the *investment share of revenue* $\phi_{ikt} \equiv w_{it} h_{ikt} / r_{ikt}$. Eliminating the shadow value of IP from the first-order conditions yields the investment-share Euler equation:

$$\phi_{ikt-1} = \left[\phi_{ikt} \frac{(1-\delta_k) x_{ikt}}{x_{ikt+1} - (1-\delta_k) x_{ikt}} + (1-\upsilon) \phi_{ikt} + \upsilon \gamma_k \frac{\sigma_k - 1}{\sigma_k} \right] \times \frac{1}{1+\rho_t} \frac{r_{ikt}}{r_{ikt-1}} \left[1 - (1-\delta_k) \frac{x_{ikt-1}}{x_{ikt}} \right], \quad (18)$$

and the accumulation equation in terms of the investment share:

$$x_{ikt+1} = (1-\delta_k) x_{ikt} + \xi_k x_{ikt}^{1-\upsilon} \left(z_{ikt}^{\mu_k} \frac{\phi_{ikt} r_{ikt}}{w_{it}} \right)^{\upsilon}. \quad (19)$$

Together, equations (18) and (19) are the two key dynamic equations of the model. In steady state ($1 + \rho = 1/\beta$, constant revenues), the investment share reduces to

$$\phi_k^{SS} = \frac{\delta_k \upsilon \gamma_k}{\rho + \delta_k \upsilon} \frac{\sigma_k - 1}{\sigma_k}, \quad (20)$$

and the IP stock is

$$x_{ik} = \left(\frac{\xi_k}{\delta_k} \right)^{1/\upsilon} z_{ik}^{\mu_k} \frac{\phi_k^{SS} r_{ik}}{w_i}. \quad (21)$$

The steady-state IP stock is proportional to total firm revenue, so that countries and sectors with larger markets accumulate more IP.

Market clearing. Aggregate bilateral revenue is $R_{inkt} = M_{ik} r_{inkt}$. The goods market clears when

$$R_{inkt} = \chi_{nkt} E_{nt} S_{nikt}. \quad (22)$$

Country i's income is the sum of wages and net profits. Substituting the first-order conditions:

$$Y_{it} = \sum_k \sum_n \left[\left(\frac{\sigma_k - 1}{\sigma_k} \alpha_k + \frac{1}{\sigma_k} \right) R_{inkt} + \frac{\sigma_k - 1}{\sigma_k} (1 - \alpha_k) R_{nikt} \right]. \quad (23)$$

The first term collects export earnings – production labor costs plus operating profits from sales by country i's firms abroad. The second term is income earned by country i's workers who are employed in local affiliates of foreign firms. The wage bill is:

$$w_{it} L_{it} = \sum_k \left[\frac{\sigma_k - 1}{\sigma_k} \alpha_k \sum_n R_{inkt} + \frac{\sigma_k - 1}{\sigma_k} (1 - \alpha_k) \sum_n R_{nikt} + \phi_{ikt} R_{ikt} \right], \tag{24}$$

where $R_{ikt} \equiv \sum_n R_{inkt}$ is total revenue of country i's firms in sector k. The labor market clears in each country:

$$L_{it} = \sum_k M_{ik} \left(h_{ikt} + \sum_n l_{inkt} \right) + \sum_k \sum_n M_{nk} \, \iota_{nikt} \qquad \forall \; i, t, \tag{25}$$

and the bond market clears:

$$\sum_{n=1}^{N} B_{nt} = 0 \qquad \forall \; t. \tag{26}$$

Equilibrium. We can now define the equilibrium of the model.

Definition 1 (Sequential equilibrium)**.** *Given initial conditions* $\{x_{ik0}, B_{n0}\}$*, a* sequential equilibrium *is a sequence of wages* $\{w_{it}\}$*, sectoral prices* $\{P_{nkt}\}$*, expenditure shares* $\{\chi_{nkt}\}$*, revenues* $\{R_{inkt}\}$*, bond positions* $\{B_{nt}\}$*, real consumption* $\{C_{nt}\}$*, IP stocks* $\{x_{ikt}\}$*, investment shares* $\{\phi_{ikt}\}$*, and an interest rate* $\{\rho_t\}$ *such that, at every* t*:*

(i) expenditure shares satisfy (7) *with* $\sum_k \chi_{nkt} = 1$*;*

(ii) households optimize intertemporally, satisfying (10) *and* (9)*;*

(iii) firms choose production labor optimally, satisfying (12)–(13)*, with prices* (15)*;*

(iv) firms choose innovation labor optimally, satisfying (18)*;*

(v) IP stocks evolve according to (19)*;*

(vi) trade shares are given by (16)*;*

(vii) the goods market clears: (22)*;*

(viii) the labor market clears: (25)*;*

(ix) the bond market clears: (26)*.*

Definition 2 (Steady-state equilibrium)**.** *A* steady-state equilibrium *is a sequential equilibrium in which all variables are constant over time,* $B_n = 0$*, and* $1 + \rho = 1/\beta$*, with* ϕ_k^{SS} *and* x_{ik} *given by* (20)–(21)*. The equilibrium is a vector* $\{w_i, \chi_{nk}\}$ *(with one wage normalized) satisfying:*

(i) *expenditure shares* χ_{nk} *from* (7) *with* $\sum_k \chi_{nk} = 1$ *(using* $E_n = Y_n$*);*

(ii) *trade shares* S_{nik} *from* (16)*, given wages and IP stocks;*

(iii) *IP stocks* x_{ik} *from* (21)*;*

(iv) *revenues* $R_{ink} = \chi_{nk}\, Y_n\, S_{nik}$*;*

(v) *income from* (23)*;*

(vi) *labor market clearing from* (24)*.*

2.3 Steady-State Characterization

We characterize the steady-state equilibrium through two analytical results.

Proposition 3 (Sufficient statistics for gains from trade)**.** *Consider two steady states that differ in bilateral appeals* $\{a_{ink}\}$ *but share the same domestic fundamentals, and let* $\hat{x} \equiv x'/x$ *denote ratios between them. The change in real consumption relative to autarky is*

$$\hat{C}_n = \hat{\ell}_n^{-(1+\bar{\gamma}_n)/\bar{\varepsilon}_n} \left(\frac{\sum_k \Phi_{nk} \hat{\varrho}_{nk}}{\sum_k \Lambda_{nk} \hat{\chi}_{nk}} \right)^{1/\bar{\varepsilon}_n} \prod_k \hat{\varrho}_{nk}^{\chi_{nk}\gamma_k/\bar{\varepsilon}_n} \prod_k \hat{\chi}_{nk}^{\chi_{nk}/((1-\eta)\bar{\varepsilon}_n)} \prod_k \hat{S}_{nnk}^{-\chi_{nk}/[\bar{\varepsilon}_n(\sigma_k-1)]}, \tag{27}$$

where $\bar{\varepsilon}_n \equiv \sum_k \chi_{nk}\varepsilon_k$*;* $\bar{\gamma}_n \equiv \sum_k \chi_{nk}\gamma_k$*;* $\varrho_{nk} \equiv R_{nk}/\sum_{k'} R_{nk'}$*;* $\ell_n \equiv w_n L_n / \sum_k R_{nk}$ *is the labor share of total revenues; and* Φ_{nk}*,* Λ_{nk} *are income- and expenditure-side weights defined in Appendix C.*

The formula nests Arkolakis, Costinot and Rodríguez-Clare (2012) as a special case. When $\gamma_k = 0$ and $\varepsilon_k = 1$ for all k, revenue composition and expenditure shares are unchanged and the expression reduces to $\hat{C}_n = \prod_k \hat{S}_{nnk}^{-\chi_k/(\sigma_k-1)}$. In the general case, welfare depends on three additional sufficient statistics beyond the domestic trade shares $\hat{S}_{nnk}$: changes in revenue composition $\hat{\varrho}_{nk}$, which matter because sectors differ in their degree of increasing returns; changes in the labor share of revenues $\hat{\ell}_n$, which reflect how the sectoral mix of production alters the division of income between labor and IP rents; and changes in expenditure shares $\hat{\chi}_{nk}$, which arise from nonhomothetic demand.

Proposition 4 (Steady-state comparative statics)**.** *To a first order, the log-change in real consumption following small changes in productivities* $\{d \ln z_{ik}\}$ *and bilateral appeals* $\{d \ln a_{ink}\}$ *is*

$$d \ln C_n = \frac{1}{\bar{\varepsilon}_n} \left[\boldsymbol{\lambda}_n' \left(\mathbf{I} - \boldsymbol{\Gamma} \right)^{-1} \mathbf{b} \; + \; d_n \right], \tag{28}$$

where $\mathbf{b}$ *is the direct (partial-equilibrium) effect of the shocks on sectoral revenues,* $\mathbf{\Gamma}$ *is the general-equilibrium feedback matrix capturing how revenue changes in one country-sector propagate to all others,* $\boldsymbol{\lambda}_n$ *is the consumption exposure to revenue changes, and* d_n *is the direct effect of the shocks on consumption holding revenues fixed. The direct effects take the form*

$$b_{ik} = (\alpha_k+\mu_k\gamma_k)(\sigma_k-1)\sum_n T_{ink}\sum_{i'} S_{ni'k}\, d\ln z_{i'k} + \sum_n T_{ink}\sum_{i'} S_{ni'k}\, d\ln a_{i'nk}\,, \tag{29}$$

$$d_n = \sum_k \chi_{nk}\left[(\alpha_k+\mu_k\gamma_k)\sum_i S_{nik}\, d\ln z_{ik} + \tfrac{1}{\sigma_k-1}\sum_i S_{nik}\, d\ln a_{ink}\right], \tag{30}$$

where $T_{ink} \equiv \chi_{nk}Y_nS_{nik}/R_{ik}$ *is the share of sales to country* n*'s consumers in country-sector* (i,k)*'s total revenues. The equilibrium response of revenues is* $d\ln\mathbf{R} = (\mathbf{I}-\mathbf{\Gamma})^{-1}\mathbf{b}$*, with a unique solution when* $\phi_k^{SS} < 1/\sigma_k$ *for all* k*. The objects* $\mathbf{\Gamma}$*,* $\boldsymbol{\lambda}_n$*,* $\mathbf{b}$*, and* d_n *are microeconomic sufficient statistics: they depend only on the elasticities of substitution* (σ_k,η)*, the production elasticities* (α_k,γ_k)*, the sectoral income elasticities* (ε_k)*, and observable shares in the initial equilibrium – bilateral trade shares* S_{nik}*, expenditure shares* χ_{nk}*, revenue composition* ϱ_{nk}*, and the labor share* ℓ_n*.*

Appendix B provides the explicit expressions for each object in terms of these primitives.

3 Data and Calibration

In this section, we describe the data and the calibration of the model's parameters. We begin in Section 3.1 by describing our data sources. In Section 3.2, we construct model-consistent bilateral trade flows that combine cross-border exports and multinational affiliate sales. In Section 3.3, we develop a novel measure of the non-rival content of production in each sector, which disciplines the key parameter γ_k. In Section 3.4, we describe the inversion procedure that recovers the remaining unobserved fundamentals from the data. In Section 3.5, we describe the calibration of all remaining parameters. Our sample covers $N = 10$ country groups and $K = 19$ ISIC Rev. 4 sectors (12 manufacturing, 7 services), with 2022 as the reference year.[3] Appendix G provides further detail on each step.

[3]The underlying data draw on 80 economies from the OECD IOT tables, aggregated into 10 groups: Brazil, Canada, China, the EU, Great Britain, India, Japan, Russia, the United States, and a rest-of-world aggregate. Appendix G.5 lists the country groups and Appendix G.4 lists the sectors.

3.1 Data Sources

We combine data from several sources on trade flows, multinational activity, occupational tasks, and macroeconomic aggregates.

Trade flows. We use the BACI dataset (Gaulier and Zignago 2010) for bilateral goods trade. BACI reconciles exporter- and importer-reported flows from UN COMTRADE at the HS92 6-digit level for over 200 countries. For bilateral services trade, we use the IMF Bilateral Trade in Services (BiTS) dataset (Loungani et al. 2017; Li et al. 2025), which covers up to 245 economies in 12 EBOPS 2010 service categories. We obtain unilateral export totals and domestic output by sector from the OECD National Input-Output Tables (IOT), which provide a balanced representation of production and trade linkages for 80 economies over 1995–2020.

Multinational activity. We measure foreign affiliate sales using the OECD Analytical Activity of Multinational Enterprises (Analytical AMNE) database, which reports bilateral sales by the country of the ultimate controlling owner and the host country. For the United States, we supplement AMNE with the Bureau of Economic Analysis (BEA) tables on majority-owned foreign affiliates (MOFA).

Tasks and occupations. We obtain occupation-level task descriptions from version 29.1 (August 2024) of the O*NET database, restricting to base occupations (excluding detailed .XX variants). This yields 15,638 task statements across 774 occupations. We combine these with employment and wage data by occupation and 3-digit NAICS sector from the BLS Occupational Employment and Wage Statistics (OEWS) program (2024 release).

Other. We measure country-level population and GDP per capita using the Penn World Tables (PWT 10.01, Feenstra et al. 2015).

3.2 Constructing Model-Consistent Trade Flows

Bilateral revenues R_{ink} in the model combine cross-border exports (Mode 1) and affiliate sales (Mode 3). The bilateral datasets (BACI for goods, BiTS for services) and the balanced OECD IOT tables differ in their aggregate levels. To reconcile the two, we use the bilateral data only for the *distribution* of trade across partners, not for levels. For each exporter and sector, we compute the share of each destination in total exports from the bilateral data, and then distribute the

OECD IOT unilateral export total across destinations according to these shares. This ensures that bilateral flows are consistent with the balanced IOT totals while preserving the bilateral patterns from the underlying micro trade data. We compute domestic absorption as the IOT residual. Appendix G.2 provides the full step-by-step procedure.

The model also distinguishes between cross-border trade and sales through foreign affiliates. The parameter α_k in the production function (4) governs this split: from the first-order conditions (12)–(13), the ratio of cross-border export revenue to affiliate sales revenue equals $\alpha_k/(1-\alpha_k)$. We calibrate α_k using the AMNE and BEA data on the ratio of affiliate sales to cross-border exports in each sector. For country pairs where AMNE data are missing, we impute affiliate sales as $\mathcal{A}_{ij,k} = [(1-\alpha_k)/\alpha_k] \times X_{ij,k}$. Total bilateral revenue is then $R_{ij,k} = X_{ij,k} + \mathcal{A}_{ij,k} = X_{ij,k}/\alpha_k$.

3.3 Measuring Sectoral Non-Rival Intensity

A distinctive feature of the model is the parameter γ_k, which controls the extent to which production in sector k relies on non-rival assets. In the model, each firm in sector k allocates labor between two types of tasks: rival production tasks (serving specific customers in specific markets) and non-rival innovation tasks (building IP that benefits all markets simultaneously). In steady state, the share of sector k's total labor compensation directed toward non-rival tasks is a monotone function of γ_k and other calibrated parameters:

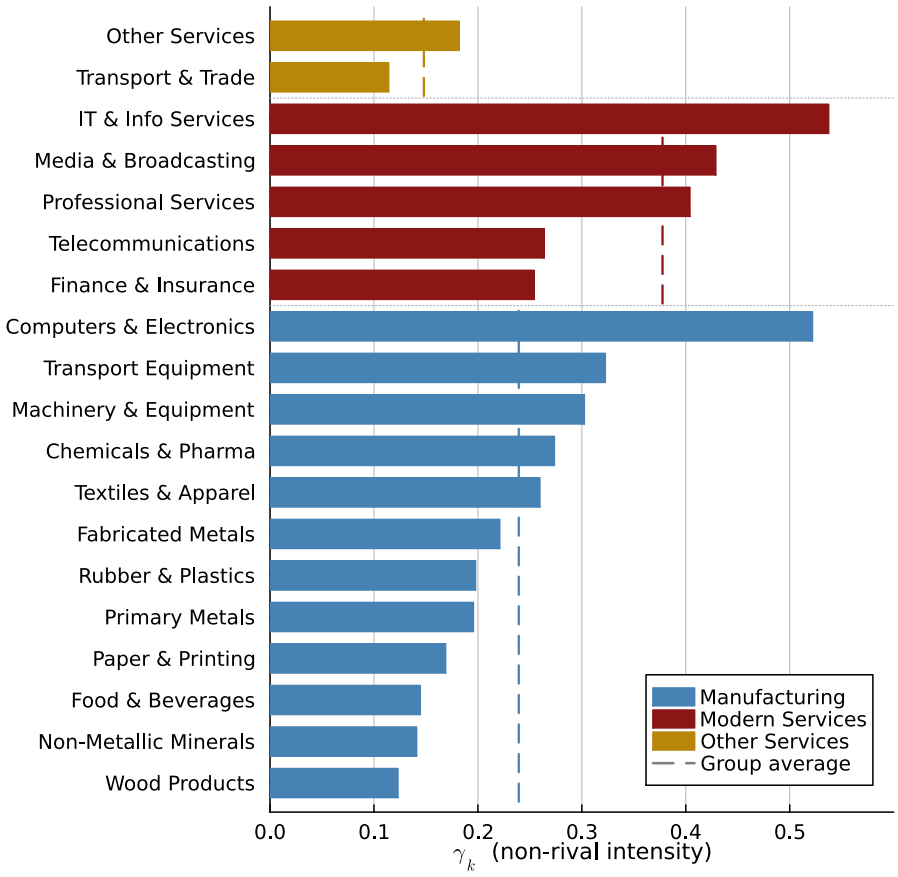
$$\gamma_k = \frac{H_k}{1-H_k} \frac{\alpha_k + (1-\alpha_k)\,\chi_{ik} Y_i/R_{ik}}{\delta^*}, \tag{31}$$

where H_k is the non-rival payroll share, $\chi_{ik} Y_i/R_{ik}$ is the ratio of expenditure to revenue in sector k, and δ^* is the discount-adjusted depreciation rate (see Appendix H.3 for the derivation). This means that if we can measure the non-rival share of labor compensation in the data, we can recover γ_k.

Figure 2 reports the resulting γ_k for each of the 19 sectors, grouped into manufacturing, knowledge-intensive services, and other services. Services sectors are substantially more non-rival intensive than manufacturing: IT and information services ($\gamma = 0.54$), computers and electronics ($\gamma = 0.52$), media and broadcasting ($\gamma = 0.43$), and professional services ($\gamma = 0.40$) all exhibit γ_k above 0.4, while most manufacturing sectors have γ_k below 0.3. Within manufactur-

ing, the most non-rival intensive sectors are those with high R&D expenditure: transport equipment ($\gamma = 0.32$), machinery ($\gamma = 0.30$), and chemicals and pharmaceuticals ($\gamma = 0.27$). The least non-rival intensive sectors are wood products ($\gamma = 0.12$) and transport and trade ($\gamma = 0.11$).

Figure 2: Non-Rival Intensity by Sector



Note: Each bar shows γ_k, the non-rival intensity parameter, for one of the 19 sectors. γ_k is computed from the non-rival payroll share H_k using equation (31). H_k is constructed by classifying O*NET 29.1 tasks as non-rival, ambiguous, or rival, dropping ambiguous tasks, and aggregating to the sector level using BLS OEWS payroll weights at the 3-digit NAICS level. Sectors grouped into manufacturing (blue), knowledge-intensive services (red), and other services (gold), sorted by γ_k within each group.

We construct H_k using the O*NET database, which provides detailed task descriptions for each occupation, combined with employment and wage data from the BLS. We classify each task into one of three categories – *non-rival*, *ambiguous*, or *rival* – using a large language model (Claude Opus 4, Anthropic). A task is classified as non-rival if performing it contributes to the creation of a firm-specific intangible asset – such as software, product designs, patents, or brand equity –

that is (i) proprietary (owned exclusively by the firm), (ii) non-depletable (usable simultaneously across all customers and markets), and (iii) durable (persists beyond the period in which it was created). A task is classified as rival if its output clearly fails any of these three conditions – for example, because it is consumed in delivery, must be repeated for each customer, or benefits only a specific market. A task is classified as ambiguous if it plausibly contributes to a non-rival asset under some circumstances but not others, or if the task description is too vague to determine (e.g., management tasks that may or may not produce reusable organizational knowledge). Across the 15,638 tasks, 8.6% are classified as non-rival, 15.0% as ambiguous, and 76.4% as rival. Appendix G.3 reports the full classification prompt and additional details. For each occupation, we compute the non-rival share as $H_o = \mathrm{NR}_o/(\mathrm{NR}_o + \mathrm{R}_o)$, excluding ambiguous tasks from both the numerator and denominator. We then aggregate to the sector level by weighting each occupation by its payroll share in the sector, using BLS OEWS data at the 3-digit NAICS level. Table 2 illustrates the classification with examples drawn from the O*NET data.

Table 2: Examples of Rival and Non-Rival Task Classifications

Task description	Classification
Design, develop, and modify software systems, using scientific analysis and mathematical models to predict and measure outcomes and consequences of design	Non-rival
Develop new or improved varieties of field crops, focusing on characteristics such as yield, quality, and disease resistance	Non-rival
Develop and organize training manuals, multimedia visual aids, and other educational materials	Non-rival
Design electronic components, software, products, or systems for commercial, industrial, medical, military, or scientific applications	Non-rival
Develop new food items for production, based on consumer feedback	Non-rival
Resolve customer complaints regarding sales and service	Rival
Inspect guest rooms, public areas, and grounds for cleanliness and appearance	Rival
Oversee the maintenance and repair of machinery, equipment, and electrical and mechanical systems	Rival
Schedule and receive food and beverage deliveries, checking delivery contents to verify product quality and quantity	Rival
Monitor customer preferences to determine focus of sales efforts	Rival

Note: Selected examples from the O*NET 29.1 task database. Each task is classified as non-rival, ambiguous, or rival. A task is non-rival if it produces a firm-specific intangible asset that is proprietary, non-depletable, and durable; rival if it clearly fails any condition; ambiguous if the answer depends on context. Ambiguous tasks are excluded when computing the non-rival share H_k. See Appendix G.3 for the full classification prompt.

3.4 Model Inversion

Given the observed trade shares $\{S_{nik}\}$, expenditure shares $\{\chi_{nk}\}$, and the calibrated parameters described above, a number of fundamentals remain unobserved: country-sector productivities $\{z_{ik}\}$, bilateral demand shifters $\{a_{ink}\}$, and NH-CES demand shifters $\{\bar{\chi}_{nk}\}$. We recover these by inverting the model's steady-state equilibrium conditions to match the data exactly.

The bilateral demand shifters a_{ink} are recovered from the trade share equation (16) using the Head-Ries index (Head and Ries 2001), which takes the geometric mean of bilateral trade share ratios to cancel all country-sector fundamentals and isolate the bilateral component. The resulting matrix is symmetric ($a_{ink} = a_{nik}$) with diagonal entries equal to one ($a_{iik} = 1$). We recover the country-sector productivities $\{z_{ik}\}$ by solving for the vector that rationalizes the observed home trade shares $\{S_{iik}\}$. Given a candidate set of productivities, we compute wages, revenues, and IP stocks from the steady-state system, then analytically update z_{ik} by inverting the home trade share expression. We iterate on this mapping using a damped fixed point until the model-implied home trade shares match the data to within a tolerance of 10^{-6}. Productivities are normalized so that the US value equals one in each sector, and then scaled so that model-implied relative wages match observed relative GDP per capita from PWT.

With the converged productivities, wages, and prices in hand, we recover the NH-CES demand shifters $\bar{\chi}_{nk}$ by inverting the expenditure share equation (7). Given η, ε_k, equilibrium prices P_{nk}, and income Y_n, the demand shifters $\bar{\chi}_{nk}$ are the unique values such that the NH-CES system reproduces the observed expenditure shares χ_{nk}. Appendix H.5 provides further detail on the algorithm and reports the model fit (Figure 7).

3.5 Additional Parameters

We now describe the calibration of the remaining model parameters. Some parameters are set directly from external estimates or data; others are recovered by the inversion procedure described above. Table 3 provides a summary.

The NH-CES demand system requires the across-sector elasticity of substitution η and the nonhomotheticity parameters ε_k. We draw on the estimates of Comin et al. (2021), who estimate the NH-CES system using cross-country panel data on sectoral expenditure shares and income.

We set $\eta = 0.5$, so that sectors are gross complements. For the nonhomotheticity parameters, CLM report estimates at a coarser sector classification than ours. We assign all manufacturing sectors a common ε_k corresponding to CLM's manufacturing estimate, and map each services sector to the closest CLM category (or to the average of two adjacent categories when the mapping is not one-to-one).[4] The resulting ε_k values rank services as luxuries ($\varepsilon_k > \bar{\varepsilon}$) and manufacturing as necessities ($\varepsilon_k < \bar{\varepsilon}$), consistent with Engel's law. We set the within-sector elasticity of substitution σ_k to equalize net profit margins at 10% of revenue across all sectors, yielding σ_k ranging from 2.9 (IT services, highest γ_k) to 6.0 (transport and trade, lowest γ_k). This ensures that high-IP sectors, which devote a larger share of revenue to innovation, earn the same residual profit rate as low-IP sectors. It also generates the empirically plausible pattern that IP-intensive sectors command higher markups (Appendix H.3). The IP depreciation rate is $\delta = 0.2$, common across sectors, the discount factor is $\beta = 0.96$ (corresponding to an annual interest rate of approximately 4%), and the adjustment friction parameter is $\upsilon = 0.5$. Country labor endowments L_i are set equal to population from the Penn World Tables (2019 values).

4 Quantitative Analysis

In this section, we use the calibrated model to study three questions. First, in Section 4.1, we ask which features of the model are responsible for the observed pattern of comparative advantage in services. We decompose each country's revealed comparative advantage in services into contributions from nonhomothetic demand and increasing returns to scale, and show that both forces are quantitatively important. Second, in Section 4.2, we compute the welfare gains from trade in goods and services, and compare them across countries and model specifications. Third, in Section 4.3, we study the dynamic response of the global economy to a reduction in trade barriers, tracing out the transition path as countries accumulate non-rival assets and adjust their patterns of specialization.

4.1 The Determinants of Comparative Advantage in Services

We begin by asking: why do some countries specialize in services while others specialize in goods? In the model, two forces shape comparative advantage in services beyond the exogenous

[4] Appendix G.1 provides the detailed mapping from CLM categories to our sector classification.

Table 3: Calibration Summary

Parameter	Description	Value	Source / Target
Set externally			
β	Discount factor	0.96	
η	Across-sector elasticity	0.5	Based on Comin et al. (2021)
ε_k	Nonhomotheticity	Sector-specific (0.81–1.75)	Based on Comin et al. (2021)
δ	IP depreciation rate	0.2	Avg. IPP depreciation, BEA Fixed Assets
υ	Adjustment friction	0.5	Quadratic adjustment cost
L_i	Labor endowment	Country-specific	PWT population
Computed directly from data			
α_k	Origin labor share	Sector-specific (0.14–0.53)	AMNE/BEA affiliate data
γ_k	Non-rival intensity	Sector-specific (0.11–0.54)	O*NET/BLS task measure H_k
σ_k	Within-sector elasticity	Sector-specific (2.9–6.0)	Constant net profit margin (10%)
Recovered from model inversion			
a_{ink}	Bilateral demand shifters	Bilateral×sector	Trade shares
z_{ik}	Productivity	Country×sector	Home trade shares, GDP p.c.
$\bar{\chi}_{nk}$	NH-CES demand shifters	Country×sector	Expenditure shares

Note: Parameters in the top panel are set from external data or prior estimates. Parameters in the middle panel are computed directly from data sources described in the text. Parameters in the bottom panel are recovered by inverting the model's steady-state equilibrium conditions to match observed trade shares, home trade shares, GDP per capita, and expenditure shares.

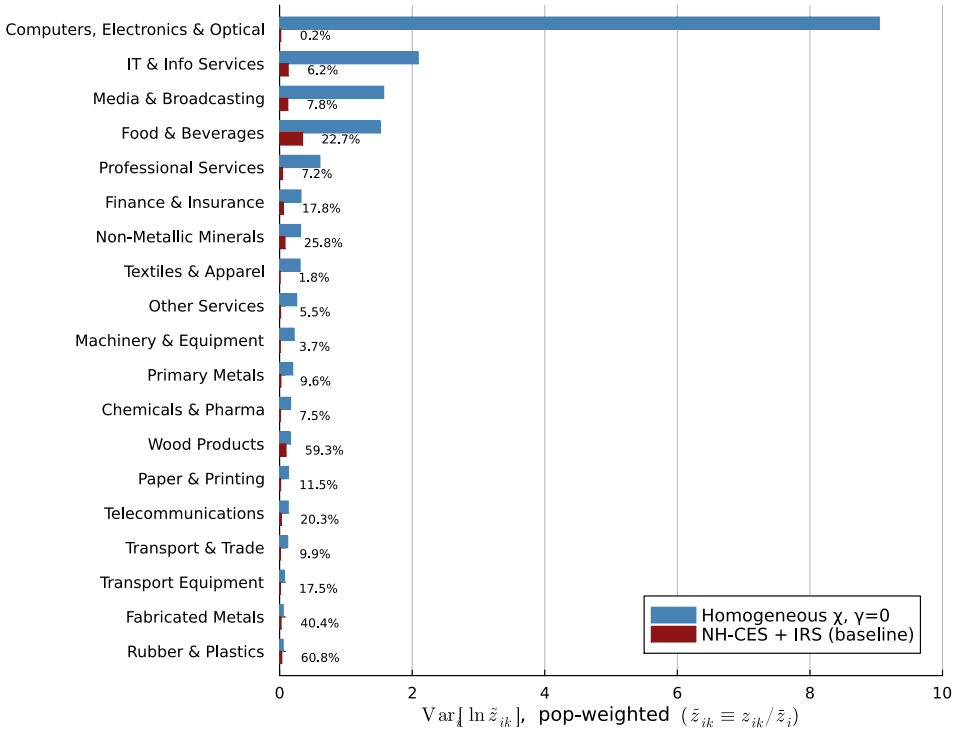
fundamentals (z_{ik}, a_{ink}). First, nonhomothetic demand implies that richer countries allocate a larger share of expenditure to services, which are luxuries ($\varepsilon_k > \bar{\varepsilon}$). Second, increasing returns to scale through non-rival IP ($\gamma_k > 0$) imply that countries with larger market access in IP-intensive sectors accumulate more IP and become more competitive in those sectors. Since services are more IP-intensive than manufacturing, this force amplifies services comparative advantage for countries with large or well-connected markets.

A first way to assess the role of nonhomothetic demand and increasing returns is to ask: how much exogenous comparative advantage variation is needed to rationalize the data? We perform two model inversions that match the same observed trade shares and bilateral appeal parameters, but differ in the forces allowed to shape trade patterns. The baseline inversion uses the full model with NH-CES demand and $\gamma_k > 0$. The counterfactual inversion shuts down both channels, imposing homogeneous expenditure shares and $\gamma_k = 0$. For each inversion, we recover the sector-level productivities z_{ik} and compute the population-weighted cross-country variance of demeaned log productivity, $\mathrm{Var}_i[\ln \tilde{z}_{ik}]$, where $\tilde{z}_{ik} \equiv z_{ik}/\bar{z}_i$ and $\bar{z}_i$ is the geometric mean of z_{ik} across sectors. This demeaning isolates comparative advantage – the relative productivity of country i in sector k – from absolute advantage.

Figure 3 reports the results. The baseline model requires substantially less exogenous comparative advantage dispersion than the counterfactual to rationalize the same trade data. Across all sectors, the baseline variance is on average 18% of the counterfactual, with the reduction being more pronounced for services (11%) than for manufacturing (22%). The sectors with the largest reductions are those with high non-rival intensity: computers, electronics and optical equipment ($\gamma = 0.52$), IT and information services ($\gamma = 0.54$), and media and broadcasting ($\gamma = 0.43$). This pattern reflects two mechanisms. First, with increasing returns, the scale effect through accumulated IP (the trade share depends on x_{ik}, which in steady state is proportional to revenue) endogenously generates cross-country variation in competitiveness, reducing the role that exogenous productivity must play. Second, nonhomothetic demand channels income differences into heterogeneous expenditure compositions. Together, these forces absorb a large share of the variation that a standard model would attribute to exogenous technology differences.

To decompose the contribution of each force, we compute the log Balassa RCA in services under four specifications that progressively shut down the two channels, holding exogenous funda-

Figure 3: How Large Is Sectoral Productivity Dispersion Across Countries?



Note: Each pair of bars shows the population-weighted cross-country variance of demeaned log productivity $\mathrm{Var}_i[\ln \tilde{z}_{ik}]$ for sector k, where $\tilde{z}_{ik} \equiv z_{ik}/\bar{z}_i$ and $\bar{z}_i$ is the geometric mean across sectors. Blue bars: counterfactual inversion with homogeneous expenditure shares and $\gamma_k = 0$. Red bars: baseline inversion with NH-CES demand and $\gamma_k > 0$. Percentages indicate the baseline variance as a share of the counterfactual. Sectors sorted by counterfactual variance.

mentals (z_{ik}, a_{ink}, L_i) fixed: (1) the baseline with NH-CES demand and $\gamma_k > 0$; (2) NH-CES demand but $\gamma_k = 0$; (3) common expenditure shares but $\gamma_k > 0$; and (4) common expenditure shares and $\gamma_k = 0$.

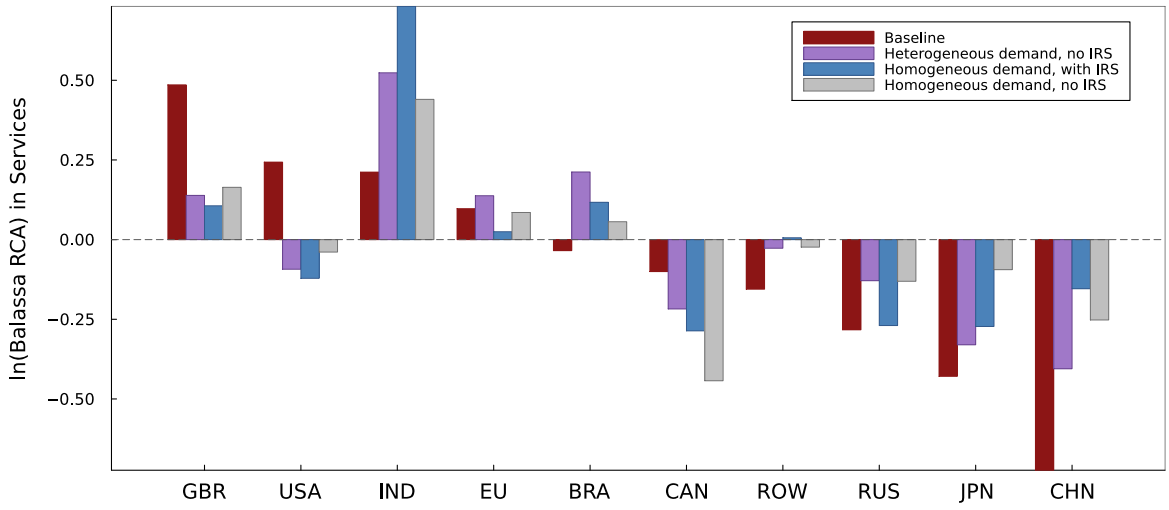
Figure 4 reports the results. In the baseline, Britain leads in services specialization with log RCA of 0.49, followed by the United States (0.24), India (0.21), and Europe (0.10). China has the lowest services RCA (−0.72), followed by Japan (−0.43) and Russia (−0.28). Several patterns stand out.

First, IRS plays an important role in determining services specialization, and its role differs across countries. For the United States and Britain, IRS drives services specialization. Shutting it down (specification 2) reduces the US services RCA from 0.24 to −0.09 – a sign reversal – indicating that the scale effect from non-rival IP is the dominant source of US services comparative advantage. For Britain, IRS accounts for roughly three-fifths of its services RCA. India, however, exhibits the opposite pattern: shutting down IRS raises its services RCA from 0.21 to 0.52. This is because IRS disproportionately benefits countries with large market access in γ-intensive sectors – like the United States and Britain – holding down India's relative position.

Second, nonhomothetic demand affects specialization in the same direction as IRS for most countries. For rich countries, NH-CES demand tilts expenditure toward services, which are luxuries, amplifying their services specialization. For poor countries such as India and China, it does the opposite, tilting expenditure toward manufacturing and suppressing their services RCA. Shutting down demand heterogeneity while keeping IRS (specification 3) therefore reduces the US services RCA from 0.24 to −0.12, while raising India's from 0.21 to 0.73. An outlier to this pattern is Europe, for which IRS slightly reduces services specialization while NH demand increases it.

Third, the two channels interact and their interaction matters. For example, in India, shutting down both simultaneously (specification 4) raises its services RCA by 0.23 – less than the 0.31 from removing IRS alone or the 0.52 from removing demand heterogeneity alone. IRS amplifies the advantage of richer countries partly through the market access that comes with income, so removing demand heterogeneity already dissipates some of the distortion that IRS generates.

Figure 4: Decomposition of Revealed Comparative Advantage in Services



Note: Each bar shows the log Balassa RCA in services for a given country under one of four model specifications. Baseline: full model with NH-CES demand and increasing returns to scale (IRS). Heterogeneous demand, no IRS: NH-CES demand with $\gamma_k = 0$. Homogeneous demand, with IRS: common expenditure shares with $\gamma_k > 0$. Homogeneous demand, no IRS: common expenditure shares with $\gamma_k = 0$. Countries ordered by baseline RCA. Exogenous fundamentals (z_{ik}, a_{ink}, L_i) held fixed across specifications.

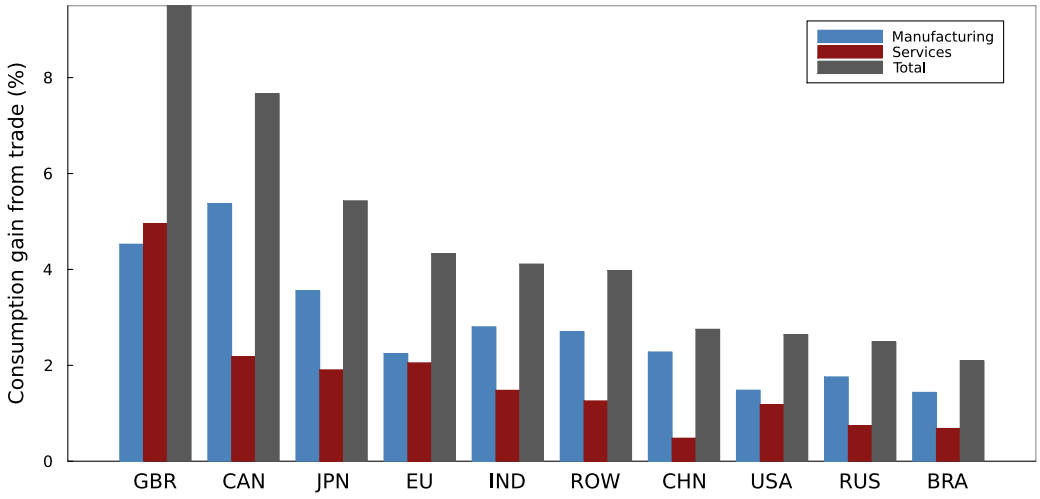
4.2 Welfare Gains From Trade in Goods and Services

We now turn to the welfare gains from trade. For each country, we compute the percentage loss in real consumption from moving to autarky. We report three measures: total gains from trade (shutting down all international trade), gains from manufacturing trade (shutting down trade in the 12 manufacturing sectors only), and gains from services trade (shutting down trade in the 7 services sectors only). The sector-specific measures hold trade in other sectors at baseline levels, so they do not sum to the total due to general-equilibrium interactions.

Figure 5 reports the manufacturing and services components. Total gains from trade range from 2.1% (Brazil) to 9.5% (Britain). Two findings stand out. First, the gains from trade in services are large and economically significant for all countries. For Britain, the gains from services trade (5.0%) exceed those from manufacturing trade (4.5%), making it the country for which services trade matters most in relative terms. For the EU, the gains from services (2.1%) are close to those from manufacturing (2.2%), reflecting Europe's deep integration in cross-border services and financial markets. For the United States, the gains from services trade (1.2%) are comparable to those from manufacturing (1.5%), with total gains of 2.7%.

Second, there is considerable heterogeneity across countries in the relative importance of goods versus services trade. For most countries, manufacturing trade generates larger gains than services trade – particularly for Canada (5.4% vs. 2.2%, total 7.7%), China (2.3% vs. 0.5%, total 2.8%), and India (2.8% vs. 1.5%, total 4.3%). These are countries that are either deeply integrated into manufacturing supply chains (Canada) or that have comparative advantage concentrated in goods sectors (China). At the other end, Britain is the only economy where services trade generates gains exceeding manufacturing, reflecting its specialization in financial services, professional services, and other IP-intensive sectors.

Figure 5: Gains from Trade in Manufacturing and Services



Note: Each bar shows the percentage consumption loss from moving to autarky in the indicated sector, holding trade in all other sectors at baseline levels. Manufacturing (blue) aggregates the 12 manufacturing sectors; services (red) aggregates the 7 services sectors. Countries sorted by total gains from trade (manufacturing plus services).

4.3 The Dynamics of Globalization

We now use the full dynamic model to study the transition path following a permanent reduction in trade barriers. A key prediction of the model is that trade liberalization does not merely reallocate production across countries – it triggers a gradual accumulation and depletion of non-rival assets, financed by international capital flows. Because non-rival IP accumulates slowly, comparative advantage adjusts over time rather than shifting instantaneously, and the direction of adjustment is shaped by each country's initial specialization and income level. We consider a 20% decline in global trade frictions, modeled as a uniform increase in bilateral appeal a_{ink} for

all country pairs $i \neq n$ and all sectors k, implemented as an unanticipated permanent shock. We solve for the perfect-foresight transition path from the initial steady state to the new steady state, allowing expenditure shares to adjust endogenously along the path through the NH-CES demand system.

Figure 6 reports the impulse response functions for four countries – China, India, the United States, and Great Britain – along three dimensions: consumption, revealed comparative advantage in manufacturing and modern services, and net foreign asset positions.

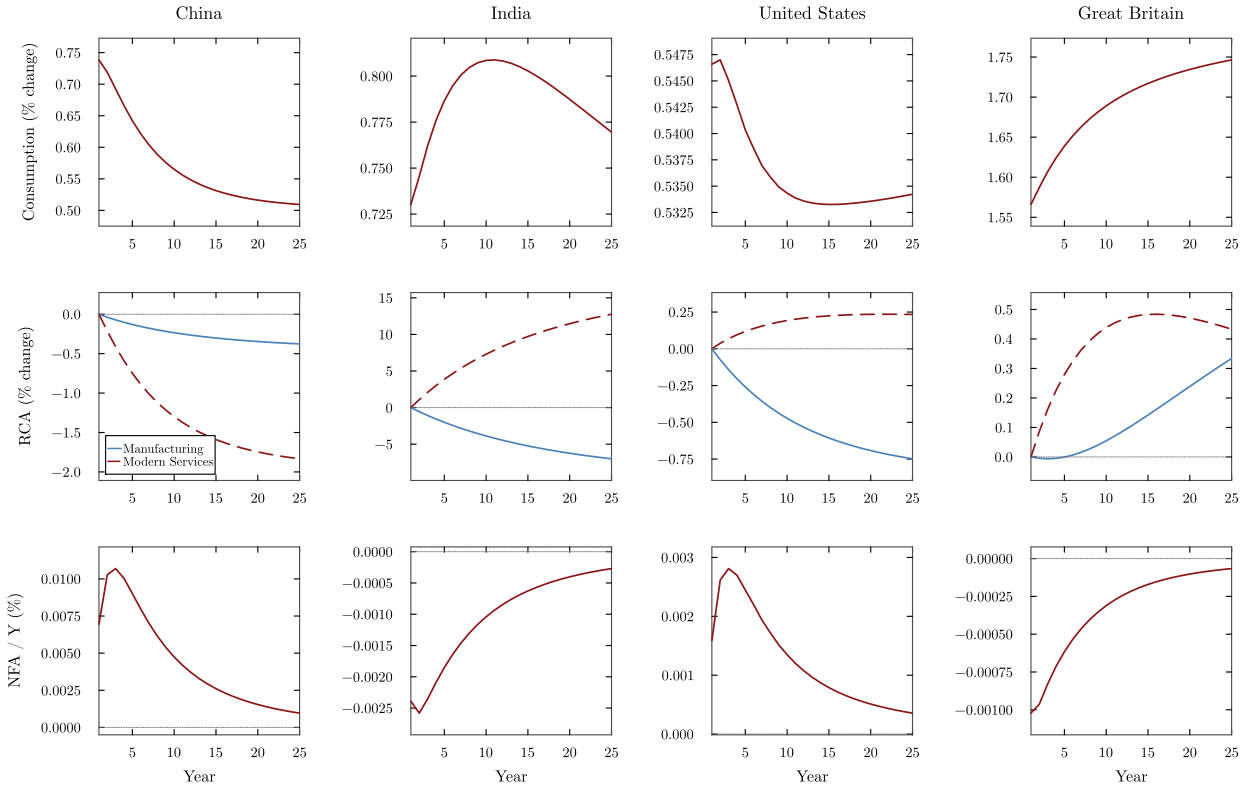
The top row shows the consumption response. All countries experience immediate welfare gains from reduced trade barriers, but the magnitudes differ substantially. Britain gains the most (nearly 1.8%), reflecting its high trade openness and specialization in IP-intensive services sectors where the scale effect amplifies the gains from market access. India gains roughly 0.9%, while the United States and China gain around 0.5–0.7%. For most countries, consumption overshoots its new steady-state level on impact and then gradually converges as IP stocks and expenditure shares adjust; Britain is an exception, with consumption rising monotonically as its services IP accumulates.

The middle row reveals dynamic changes in comparative advantage following changes in non-rival asset stocks. India exhibits the most pronounced shift: its revealed comparative advantage in modern services rises substantially over 25 years, while its manufacturing RCA falls. This reflects the endogenous accumulation of non-rival IP in services sectors: trade liberalization expands market access, raising revenues in IP-intensive sectors, which in turn increases the return to IP investment and triggers a virtuous cycle of IP accumulation and cost reduction. Because India starts with relatively low income, the NH-CES demand system initially dampens services demand, but as India grows richer along the transition, expenditure tilts toward services, reinforcing the shift in comparative advantage. The United States experiences a modest decline in services RCA and a rise in manufacturing RCA, consistent with increased competitive pressure from countries like India that are rapidly building services IP. For Britain, the services RCA rises gradually, deepening its existing specialization. China's comparative advantage adjusts more in manufacturing than in services, consistent with its initial specialization structure.

The bottom row shows net foreign asset positions, which reveal how the global economy finances the accumulation and depletion of non-rival assets. India runs a current account deficit

during the transition, borrowing from abroad to finance the rapid accumulation of services IP. China and the United States run modest surpluses, lending to countries that are investing heavily in IP. Britain's NFA position exhibits a hump-shaped surplus that gradually declines as returns from new IP stocks materialize. These capital flow patterns are a distinctive prediction of the model: unlike standard trade models where adjustment is instantaneous, the presence of non-rival assets that must be accumulated over time generates persistent current account imbalances along the transition path. Countries that are building comparative advantage in IP-intensive sectors run deficits today to finance investments whose returns accrue over many years, while countries lending to them earn returns in the short run but face increased competition in the long run.

Figure 6: Transition Dynamics Following a 20% Decline in Global Trade Frictions



Note: Each column shows the impulse response for one country following a permanent 20% decline in global trade frictions (modeled as a uniform increase in bilateral appeal a_{ink} for all $i \neq n$ and all k). Top row: consumption (percent change from initial steady state). Middle row: change in log Balassa RCA for manufacturing (solid blue) and modern services (dashed red), in percentage points. Bottom row: net foreign assets as a share of initial income (percent). Expenditure shares adjust endogenously along the transition via the NH-CES demand system. Horizontal axis: years after the shock.

5 Conclusion

In this paper, we have developed a dynamic multi-sector model of trade that captures three distinguishing features of trade in services: the central role of non-rival assets in production, the nonhomothetic nature of demand for services, and the coexistence of cross-border trade and commercial presence as modes of delivery. We find that both nonhomothetic demand and increasing returns through non-rival assets are quantitatively important for understanding which countries specialize in services; that the welfare gains from trade in services are large and comparable to those from manufacturing; and that trade liberalization generates a gradual accumulation and depletion of non-rival assets across sectors, with implications for the dynamics of comparative advantage and the current account.

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A Compact Steady-State System

Let L_{ik} denote total labor of country i in sector k, encompassing innovation labor, origin production labor, and destination labor hired by foreign firms:

$$L_{ik} \equiv M_{ik}\left(h_{ik} + \sum_n l_{ink}\right) + \sum_n M_{nk}\, \iota_{nik}\,, \qquad \sum_k L_{ik} = L_i\,. \tag{32}$$

Steady-state prices. Substituting the SS IP stock (21) into the sectoral price index (15), the home trade share S_{nnk} absorbs the ratio of the home numerator to the full sum. Up to a positive constant that depends only on exogenous fundamentals and structural parameters:

$$P_{nk} \propto S_{nnk}^{1/(\sigma_k-1)}\, w_n^{1+\gamma_k}\, R_{nk}^{-\gamma_k}\,. \tag{33}$$

Expenditure shares. In steady state, $E_n = Y_n$ (balanced trade). Substituting (33) into the NH-CES expenditure shares (7):

$$\chi_{nk} \propto \bar{\chi}_{nk}\left(\frac{S_{nnk}^{1/(\sigma_k-1)}\, w_n^{1+\gamma_k}\, R_{nk}^{-\gamma_k}}{Y_n}\right)^{1-\eta} C_n^{(1-\eta)\varepsilon_k}\,, \qquad \sum_k \chi_{nk} = 1\,, \tag{34}$$

where C_n is implicitly determined by the adding-up constraint and the proportionality constant from (33) is absorbed by C_n.

Sectoral wage bill. Using the labor FOCs and the SS investment share (20):

$$w_i\, L_{ik} = \left(\alpha_k\, \frac{\sigma_k - 1}{\sigma_k} + \phi_k^{SS}\right) R_{ik} + v_{ik}\, Y_i\,, \tag{35}$$

where the first term reflects origin production labor and innovation labor (both proportional to R_{ik}), and the second reflects destination labor hired by foreign firms (proportional to country i's spending on sector k, which equals $\chi_{ik}\, Y_i$).

Destination labor absorption. Define the *destination labor absorption rate*

$$\nu_{ik} \equiv (1 - \alpha_k) \frac{\sigma_k - 1}{\sigma_k} \chi_{ik}\,, \tag{36}$$

so the second term in (35) is $\nu_{ik}\, Y_i$.

Income. From (23):

$$Y_i = \sum_k \left(\alpha_k \frac{\sigma_k - 1}{\sigma_k} + \frac{1}{\sigma_k} \right) R_{ik} + Y_i \sum_k \nu_{ik}\,. \tag{37}$$

Goods market clearing.

$$R_{ik} = \sum_n \chi_{nk}\, Y_n\, S_{nik}\,. \tag{38}$$

Trade shares. Substituting the SS IP stock (21) into (16) (sector-common parameters cancel in the ratio):

$$S_{nik} = \frac{a_{ink}\, z_{ik}^{(\alpha_k+\mu_k\gamma_k)(\sigma_k-1)}\, M_{ik}^{1-\gamma_k(\sigma_k-1)}\, R_{ik}^{\gamma_k(\sigma_k-1)}\, w_i^{-(\alpha_k+\gamma_k)(\sigma_k-1)}}{\sum_{i'} a_{i'nk}\, z_{i'k}^{(\alpha_k+\mu_k\gamma_k)(\sigma_k-1)}\, M_{i'k}^{1-\gamma_k(\sigma_k-1)}\, R_{i'k}^{\gamma_k(\sigma_k-1)}\, w_{i'}^{-(\alpha_k+\gamma_k)(\sigma_k-1)}}. \tag{39}$$

Structure of the system. Equations (34)–(39) together with $\sum_k L_{ik} = L_i$ and a wage normalization form a self-contained system in the unknowns $\{w_i,\, L_{ik}\}$. Given a candidate $(w_i,\, L_{ik})$, the auxiliary variables $(R_{ik},\, Y_i,\, \chi_{nk},\, C_n,\, S_{nik})$ are jointly determined as a nonlinear fixed point: expenditure shares (34) depend on home trade shares and revenues, trade shares (39) depend on revenues and wages, and revenues and income (35)–(37) depend on expenditure shares through the destination-labor channel. Equation (38) provides the equilibrium conditions. When all ε_k are equal and $\eta \to 1$, expenditure shares reduce to the constants $\bar{\chi}_{nk}$ and decouple from the rest of the system.

B Log-Linearized Steady State

This appendix totally differentiates the steady-state system of Appendix A. Throughout, $d\ln x \equiv dx/x$ denotes the log-change of x. All shares and coefficients are evaluated at the initial steady state.

Expenditure shares. Log-differentiating (7) (equivalently (34), since the proportionality constant is fixed):

$$d\ln \chi_{nk} = (1-\eta)\, d\ln P_{nk} - (1-\eta)\, d\ln Y_n + (1-\eta)\, \varepsilon_k\, d\ln C_n\,, \tag{40}$$

where $d\ln P_{nk}$ is given by (47) below. Using $\sum_k \chi_{nk} = 1$, the weighted average gives

$$0 = \textstyle\sum_k \chi_{nk}\, d\ln \chi_{nk} = (1-\eta)\left[\sum_k \chi_{nk}\, d\ln P_{nk} - d\ln Y_n + \bar{\varepsilon}_n\, d\ln C_n\right],$$

so

$$d\ln C_n = \frac{1}{\bar{\varepsilon}_n}\left(d\ln Y_n - \sum_k \chi_{nk}\, d\ln P_{nk}\right), \tag{41}$$

where $\bar{\varepsilon}_n \equiv \sum_k \chi_{nk}\, \varepsilon_k$. Substituting (41) back into (40) and simplifying:

$$d\ln \chi_{nk} = (1-\eta)\left[d\ln P_{nk} - \frac{\varepsilon_k}{\bar{\varepsilon}_n}\sum_{k'} \chi_{nk'}\, d\ln P_{nk'}\right] + (1-\eta)\left(\frac{\varepsilon_k}{\bar{\varepsilon}_n} - 1\right) d\ln Y_n\,. \tag{42}$$

The first bracket is $d\ln P_{nk}$ minus $(\varepsilon_k/\bar{\varepsilon}_n)$ times the expenditure-weighted average price change; the second term is the income effect on expenditure composition. Combining with $d\ln Y_n$:

$$d\ln \chi_{nk} + d\ln Y_n = \left(\eta + (1-\eta)\,\frac{\varepsilon_k}{\bar{\varepsilon}_n}\right) d\ln Y_n \;+ (1-\eta)\left[d\ln P_{nk} - \frac{\varepsilon_k}{\bar{\varepsilon}_n}\sum_{k'} \chi_{nk'}\, d\ln P_{nk'}\right]. \tag{43}$$

The coefficient on $d\ln Y_n$ is the expenditure elasticity from (8).

Trade shares. Log-differentiating (39), each term enters as the deviation of origin i from the trade-share-weighted average across origins:

$$\begin{aligned} d\ln S_{nik} = {} & \gamma_k(\sigma_k-1)\left[d\ln R_{ik} - \textstyle\sum_{i'} S_{ni'k}\, d\ln R_{i'k}\right] \\ & - (\alpha_k+\gamma_k)(\sigma_k-1)\left[d\ln w_i - \textstyle\sum_{i'} S_{ni'k}\, d\ln w_{i'}\right] \\ & + \left[d\ln a_{ink} - \textstyle\sum_{i'} S_{ni'k}\, d\ln a_{i'nk}\right] \\ & + (\alpha_k+\mu_k\gamma_k)(\sigma_k-1)\left[d\ln z_{ik} - \textstyle\sum_{i'} S_{ni'k}\, d\ln z_{i'k}\right] \\ & + \left[1-\gamma_k(\sigma_k-1)\right]\left[d\ln M_{ik} - \textstyle\sum_{i'} S_{ni'k}\, d\ln M_{i'k}\right]. \end{aligned} \tag{44}$$

Goods market. Log-differentiating (38):

$$d\ln R_{ik} = \sum_n T_{ink}\left(d\ln\chi_{nk} + d\ln Y_n + d\ln S_{nik}\right), \tag{45}$$

where the revenue source share T_{ink} is the fraction of country i's sector-k revenue sourced from market n:

$$T_{ink} \equiv \frac{\chi_{nk}\, Y_n\, S_{nik}}{R_{ik}}, \qquad \sum_n T_{ink} = 1. \tag{46}$$

Prices. Log-differentiating (15) and substituting $d\ln x_{ik}$ from (21):

$$\begin{aligned} d\ln P_{nk} = {} & (\alpha_k+\gamma_k)\sum_i S_{nik}\, d\ln w_i \;+\; (1-\alpha_k)\, d\ln w_n \;-\; \gamma_k \sum_i S_{nik}\, d\ln R_{ik} \\ & - (\alpha_k+\mu_k\gamma_k)\sum_i S_{nik}\, d\ln z_{ik} \;-\; (1-\alpha_k)\, d\ln z_{nk} \;-\; \frac{\gamma_k}{\upsilon}\, d\ln\xi_k \\ & + \frac{1}{1-\sigma_k}\sum_i S_{nik}\, d\ln a_{ink} \;+\; \frac{1-\gamma_k(\sigma_k-1)}{1-\sigma_k}\sum_i S_{nik}\, d\ln M_{ik}. \end{aligned} \tag{47}$$

Wage bill and income. Log-differentiating (35):

$$\begin{aligned} d\ln w_i + d\ln L_{ik} = {} & \frac{\left(\alpha_k\,\frac{\sigma_k-1}{\sigma_k} + \phi_k^{SS}\right) R_{ik}}{w_i\, L_{ik}}\, d\ln R_{ik} \\ & + \frac{v_{ik}\, Y_i}{w_i\, L_{ik}}\left(d\ln\chi_{ik} + d\ln Y_i\right). \end{aligned} \tag{48}$$

Log-differentiating (37):

$$d \ln Y_i = \sum_k \Phi_{ik} \, d \ln R_{ik} + \sum_k \Psi_{ik} \, d \ln \chi_{ik} \,, \tag{49}$$

where

$$\Phi_{ik} \equiv \frac{\left(\alpha_k \, \frac{\sigma_k - 1}{\sigma_k} + \frac{1}{\sigma_k} \right) R_{ik}}{\sum_{k'} \left(\alpha_{k'} \, \frac{\sigma_{k'} - 1}{\sigma_{k'}} + \frac{1}{\sigma_{k'}} \right) R_{ik'}} \,, \qquad \sum_k \Phi_{ik} = 1 \,, \tag{50}$$

and

$$\Psi_{ik} \equiv \frac{v_{ik} \, Y_i}{\sum_{k'} \left(\alpha_{k'} \, \frac{\sigma_{k'} - 1}{\sigma_{k'}} + \frac{1}{\sigma_{k'}} \right) R_{ik'}} \,. \tag{51}$$

The term Ψ_{ik} captures how shifts in expenditure composition affect income through the destination-labor channel.

Labor market. Differentiating $\sum_k L_{ik} = L_i$:

$$\sum_k \frac{L_{ik}}{L_i} \, d \ln L_{ik} = d \ln L_i \,. \tag{52}$$

Wage weights. Combining (48)–(49) with (52):

$$d \ln w_i = \sum_k \Xi_{ik} \, d \ln R_{ik} - d \ln L_i + \sum_k \left[\frac{v_{ik} \, Y_i}{w_i \, L_i} + \frac{Y_i \sum_{k'} v_{ik'}}{w_i \, L_i} \, \Psi_{ik} \right] d \ln \chi_{ik} \,. \tag{53}$$

where

$$\Xi_{ik} \equiv \frac{\left(\alpha_k \, \frac{\sigma_k - 1}{\sigma_k} + \phi_k^{SS} \right) R_{ik}}{w_i \, L_i} + \frac{Y_i \sum_{k'} v_{ik'}}{w_i \, L_i} \, \Phi_{ik} \,, \qquad \sum_k \Xi_{ik} = 1 \,. \tag{54}$$

The first term of Ξ_{ik} is the direct effect of sector-k revenue on wages through export-related labor; the second is the indirect effect through income and destination labor. The coefficient on $d \ln \chi_{ik}$ captures the effect of expenditure-share changes on wages.

Eliminating expenditure shares from income. Substitute (42) into (49) to eliminate $d \ln \chi_{jk}$ from $d \ln Y_j$. Since (42) contains $d \ln Y_j$ on the right-hand side, the result involves $d \ln Y_j$ on both sides. Collecting, define the scalar

$$\Omega_j \equiv 1 - (1 - \eta) \sum_k \Psi_{jk} \left(\frac{\varepsilon_k}{\bar{\varepsilon}_j} - 1 \right), \tag{55}$$

The resolved income equation is

$$d \ln Y_j = \frac{1}{\Omega_j} \left[\sum_k \Phi_{jk} \, d \ln R_{jk} + (1 - \eta) \sum_k \Psi_{jk} \left(d \ln P_{jk} - \frac{\varepsilon_k}{\bar{\varepsilon}_j} \sum_{k'} \chi_{jk'} \, d \ln P_{jk'} \right) \right]. \tag{56}$$

Substituting into (43), the combined object entering the goods-market equation (45) becomes

$$\begin{aligned} d \ln \chi_{nk} + d \ln Y_n = {} & \frac{e_{nk}}{\Omega_n} \sum_{k'} \Phi_{nk'} \, d \ln R_{nk'} \\ & + \frac{e_{nk}(1-\eta)}{\Omega_n} \sum_{k'} \Psi_{nk'} \left(d \ln P_{nk'} - \frac{\varepsilon_{k'}}{\bar{\varepsilon}_n} \sum_{k''} \chi_{nk''} \, d \ln P_{nk''} \right) \\ & + (1-\eta) \left(d \ln P_{nk} - \frac{\varepsilon_k}{\bar{\varepsilon}_n} \sum_{k'} \chi_{nk'} \, d \ln P_{nk'} \right), \end{aligned} \tag{57}$$

where $e_{nk} \equiv \eta + (1 - \eta) \, \varepsilon_k / \bar{\varepsilon}_n$ is the expenditure elasticity (8). The first line is the income channel (revenue raises Y_n, scaled by the expenditure elasticity). The second line is the income-composition interaction: revenue-induced price changes shift expenditure shares, which change income through the destination-labor channel $\Psi_{nk'}$, which in turn shifts spending. The third line is the direct demand-composition effect: price changes reallocate spending across sectors.

Eliminating expenditure shares from wages. The same substitution of (42) into (53), using (56) to replace $d \ln Y_j$, gives

$$d \ln w_j = \sum_k \left[\Xi_{jk} + \frac{(1-\eta)\,\Phi_{jk}}{\Omega_j} \sum_{k'} \left(\frac{\nu_{jk'}\, Y_j}{w_j\, L_j} + \frac{Y_j \sum_{k''} \nu_{jk''}}{w_j\, L_j}\, \Psi_{jk'} \right) \left(\frac{\varepsilon_{k'}}{\bar{\varepsilon}_j} - 1 \right) \right] d \ln R_{jk}$$
$$- \; d \ln L_j \; + \; (1-\eta) \sum_k \left[\frac{\nu_{jk}\, Y_j}{w_j\, L_j} + \frac{Y_j \sum_{k'} \nu_{jk'}}{w_j\, L_j}\, \Psi_{jk} \right.$$
$$\left. + \frac{1}{\Omega_j} \left(\frac{Y_j \sum_{k'} \nu_{jk'}}{w_j\, L_j} \right) \sum_{k'} \left(\frac{\nu_{jk'}\, Y_j}{w_j\, L_j} + \frac{Y_j \sum_{k''} \nu_{jk''}}{w_j\, L_j}\, \Psi_{jk'} \right) \left(\frac{\varepsilon_{k'}}{\bar{\varepsilon}_j} - 1 \right) \Psi_{jk} \right]$$
$$\times \left(d \ln P_{jk} - \frac{\varepsilon_k}{\bar{\varepsilon}_j} \sum_{k'} \chi_{jk'}\, d \ln P_{jk'} \right). \quad (58)$$

The first line augments the revenue-to-wage elasticity Ξ_{jk} with a correction proportional to $(1-\eta)$ from the expenditure-share feedback through destination labor. The remaining lines give the sensitivity of wages to relative price changes, also proportional to $(1-\eta)$.

Prices as a function of revenues and wages. From (47), sectoral prices are linear in wages, revenues, and exogenous shocks:

$$d \ln P_{nk} = -\gamma_k \sum_i S_{nik}\, d \ln R_{ik} + (\alpha_k + \gamma_k) \sum_i S_{nik}\, d \ln w_i + (1-\alpha_k)\, d \ln w_n$$
$$+ \; \pi^{\text{exog}}_{nk}, \quad (59)$$

where π^{exog}_{nk} collects exogenous shock terms from (47):

$$\pi^{\text{exog}}_{nk} \equiv -(\alpha_k + \mu_k \gamma_k) \sum_i S_{nik}\, d \ln z_{ik} - (1-\alpha_k)\, d \ln z_{nk} - \frac{\gamma_k}{\upsilon}\, d \ln \xi_k$$
$$+ \frac{1}{1-\sigma_k} \sum_i S_{nik}\, d \ln a_{ink} + \frac{1-\gamma_k(\sigma_k - 1)}{1-\sigma_k} \sum_i S_{nik}\, d \ln M_{ik}\,. \quad (60)$$

Substituting (58) into (59) yields $d \ln P_{nk}$ as a linear function of $\{d \ln R_{jk'}\}$, exogenous shocks, and the relative price terms $d \ln P_{jk} - (\varepsilon_k / \bar{\varepsilon}_j) \sum_{k'} \chi_{jk'}\, d \ln P_{jk'}$. The last dependence creates a self-referential system: prices depend on wages, which depend on expenditure shares, which depend

on prices. This feedback is proportional to $(1-\eta)$ and to the destination-labor parameters $(1-\alpha_k)$. Collecting price terms on the left yields a linear system that can be solved by matrix inversion to express $d\ln P_{nk}$ as a function of $\{d\ln R_{jk'}\}$ and exogenous shocks alone. Write the resolved mapping as

$$d\ln P_{nk} = \sum_{j,k'} \mathcal{P}_{nk,jk'}\, d\ln R_{jk'} + \bar{\pi}_{nk}\,, \tag{61}$$

where $\mathcal{P}_{nk,jk'}$ is the total (general-equilibrium) elasticity of P_{nk} with respect to $R_{jk'}$ and $\bar{\pi}_{nk}$ collects all exogenous terms after the price resolution.

Revenue system. Substituting (57), (44), and the resolved prices (61) into (45) yields a self-contained linear system:

$$(\mathbf{I}-\boldsymbol{\Gamma})\, d\ln \mathbf{R} = \mathbf{b}\,, \tag{62}$$

where $\mathbf{b}$ is the direct effect of exogenous shocks on sectoral revenues and $\boldsymbol{\Gamma}$ is a feedback matrix that captures how changes in sectoral revenues in one country-sector affect sectoral revenues in all other country-sectors. The (ik, jk') entry of $\boldsymbol{\Gamma}$ is

$$\begin{aligned}
\Gamma_{ik,jk'} = {} & \sum_n T_{ink}\,\frac{e_{nk}}{\Omega_n}\,\Phi_{nk'}\,\mathbf{1}_{j=n} \\
& + \gamma_k(\sigma_k-1)\,\mathbf{1}_{k=k'}\left[\mathbf{1}_{i=j} - \sum_n T_{ink}\,S_{njk}\right] \\
& - (\alpha_k+\gamma_k)(\sigma_k-1)\left[\mathbf{1}_{i=j} - \sum_n T_{ink}\,S_{njk}\right] \\
& \times \left[\Xi_{jk'} + \frac{(1-\eta)}{\Omega_j}\,\Phi_{jk'}\sum_{k''}\left(\frac{\nu_{jk''}\,Y_j}{w_j\,L_j} + \frac{Y_j\sum_\ell \nu_{j\ell}}{w_j\,L_j}\,\Psi_{jk''}\right)\left(\frac{\varepsilon_{k''}}{\bar{\varepsilon}_j}-1\right)\right] \\
& + (\alpha_k+\gamma_k)(\sigma_k-1)\sum_n T_{ink}\,S_{njk}\,(1-\alpha_k)\,\frac{\partial\ln w_n}{\partial\ln R_{jk'}} \\
& + (1-\eta)\sum_n T_{ink}\left[\mathcal{P}_{nk,jk'} - \frac{\varepsilon_k}{\bar{\varepsilon}_n}\sum_{k''}\chi_{nk''}\,\mathcal{P}_{nk'',jk'}\right] \\
& + \frac{(1-\eta)\,e_{nk}}{\Omega_n}\sum_n T_{ink}\sum_{k''}\Psi_{nk''}\left[\mathcal{P}_{nk'',jk'} - \frac{\varepsilon_{k''}}{\bar{\varepsilon}_n}\sum_{k'''}\chi_{nk'''}\,\mathcal{P}_{nk''',jk'}\right],
\end{aligned} \tag{63}$$

where $\partial \ln w_n / \partial \ln R_{jk'}$ is the coefficient on $d \ln R_{jk'}$ in (58) for country n, and $\mathcal{P}_{nk,jk'}$ is the resolved price elasticity from (61).

Row sums. Each row of $\mathbf{\Gamma}$ sums to one, reflecting numeraire indeterminacy. After fixing a numeraire, the system has the unique solution

$$d \ln \mathbf{R} = (\mathbf{I} - \mathbf{\Gamma})^{-1} \mathbf{b} \tag{64}$$

under $\gamma_k(\sigma_k - 1) < 1$ for all k.

Consumption response. From (41):

$$d \ln C_n = \frac{1}{\bar{\varepsilon}_n} \left[\boldsymbol{\lambda}_n' \, (\mathbf{I} - \mathbf{\Gamma})^{-1} \mathbf{b} \; + \; d_n \right], \tag{65}$$

where $\boldsymbol{\lambda}_n$ is the consumption exposure to changes in sectoral revenues and d_n is the direct effect of shocks on consumption holding sectoral revenues constant. The entries of $\boldsymbol{\lambda}_n$ are

$$\lambda_{n,jk'} = \frac{1}{\Omega_n} \left[\Phi_{nk'} \, \mathbf{1}_{j=n} + (1-\eta) \sum_k \Psi_{nk} \left(\mathcal{P}_{nk,jk'} - \frac{\varepsilon_k}{\bar{\varepsilon}_n} \sum_{k''} \chi_{nk''} \, \mathcal{P}_{nk'',jk'} \right) \right] - \sum_k \chi_{nk} \, \mathcal{P}_{nk,jk'} \, . \tag{66}$$

C Ex-Post Welfare Sufficient Statistics

Compare two steady-state equilibria that differ in bilateral appeals $\{a_{ink}\}$ but share the same home fundamentals (a_{nnk}, z_{nk}, M_{nk}, ξ_k unchanged). Write $\hat{x} \equiv x'/x$ for ratios.

Price index. From (15) and (21), the sectoral price index satisfies

$$P_{nk} \propto S_{nnk}^{1/(\sigma_k-1)} \, w_n^{1+\gamma_k} \, R_{nk}^{-\gamma_k}$$

(up to exogenous fundamentals that cancel across equilibria).

Consumption index. In steady state, $E_n = Y_n$, and each summand of the adding-up constraint $\sum_k \chi_{nk} = 1$ satisfies $\chi_{nk} = \bar{\chi}_{nk}(P_{nk}/Y_n)^{1-\eta} \, C_n^{(1-\eta)\varepsilon_k}$. Raising to the power $1/((1-\eta)\varepsilon_k)$ and solving for C_n:

$$C_n = \frac{\chi_{nk}^{1/((1-\eta)\varepsilon_k)}}{\bar{\chi}_{nk}^{1/((1-\eta)\varepsilon_k)}} \left(\frac{Y_n}{P_{nk}}\right)^{1/\varepsilon_k} \qquad \forall \; k. \tag{67}$$

This holds for every sector k simultaneously. Taking the ratio across equilibria:

$$\hat{C}_n = \hat{\chi}_{nk}^{1/((1-\eta)\varepsilon_k)} \left(\frac{\hat{Y}_n}{\hat{P}_{nk}}\right)^{1/\varepsilon_k} \qquad \forall \; k. \tag{68}$$

Since $\hat{C}_n$ is the same for all k, raising each expression to the power $\chi_{nk} \, \varepsilon_k$ and taking the product over k:

$$\hat{C}_n^{\bar{\varepsilon}_n} = \hat{Y}_n \prod_k \hat{P}_{nk}^{-\chi_{nk}} \prod_k \hat{\chi}_{nk}^{\chi_{nk}/(1-\eta)}, \tag{69}$$

where $\bar{\varepsilon}_n \equiv \sum_k \chi_{nk} \, \varepsilon_k$ as before. Equivalently:

$$\hat{C}_n = \hat{Y}_n^{1/\bar{\varepsilon}_n} \prod_k \hat{P}_{nk}^{-\chi_{nk}/\bar{\varepsilon}_n} \prod_k \hat{\chi}_{nk}^{\chi_{nk}/((1-\eta)\bar{\varepsilon}_n)}. \tag{70}$$

The three factors are: an income effect (with exponent $1/\bar{\varepsilon}_n$); a price-index effect (with sector weights $\chi_{nk}/\bar{\varepsilon}_n$ proportional to expenditure shares); and an expenditure-reallocation effect from $\hat{\chi}_{nk}$, which is one when expenditure shares do not change. The weights $\chi_{nk} \, \varepsilon_k$ used in the aggregation ensure that the ε_k-specific exponents in (68) collapse to a common denominator $\bar{\varepsilon}_n$,

yielding a cleaner expression than the alternative of raising to χ_{nk} alone.

Sufficient statistics. Define revenue composition $\varrho_{nk} \equiv R_{nk}/\sum_{k'} R_{nk'}$, the *labor share of total revenues*

$$\ell_n \equiv \frac{w_n L_n}{\sum_k R_{nk}}, \tag{71}$$

and the *expenditure-side income weight*

$$\Lambda_{nk} \equiv \frac{\left(\alpha_k \frac{\sigma_k - 1}{\sigma_k} + \frac{1}{\sigma_k}\right) \chi_{nk}}{\sum_{k'} \left(\alpha_{k'} \frac{\sigma_{k'} - 1}{\sigma_{k'}} + \frac{1}{\sigma_{k'}}\right) \chi_{nk'}}, \qquad \sum_k \Lambda_{nk} = 1\,. \tag{72}$$

From (15) and (21), the sectoral price ratio is

$$\hat{P}_{nk} = \hat{S}_{nnk}^{1/(\sigma_k - 1)}\, \hat{w}_n^{1+\gamma_k}\, \hat{\varrho}_{nk}^{-\gamma_k}\, \widehat{\textstyle\sum R_{nk'}}^{-\gamma_k}\,. \tag{73}$$

From (37) and (35), both $\hat{Y}_n$ and $\hat{w}_n$ are proportional to $\widehat{\sum R_{nk'}}$:

$$\hat{Y}_n = \frac{\sum_k \Phi_{nk}\, \hat{\varrho}_{nk}}{\sum_k \Lambda_{nk}\, \hat{\chi}_{nk}}\, \widehat{\textstyle\sum R_{nk'}}\,, \tag{74}$$

$$\hat{w}_n = \frac{\sum_k \left[\alpha_k \frac{\sigma_k - 1}{\sigma_k} + \phi_k^{SS} + \frac{\sum_j \nu'_{nj}}{1 - \sum_j \nu'_{nj}} \left(\alpha_k \frac{\sigma_k - 1}{\sigma_k} + \frac{1}{\sigma_k}\right)\right] \varrho_{nk}\, \hat{\varrho}_{nk}}{\sum_k \left[\alpha_k \frac{\sigma_k - 1}{\sigma_k} + \phi_k^{SS} + \frac{\sum_j \nu_{nj}}{1 - \sum_j \nu_{nj}} \left(\alpha_k \frac{\sigma_k - 1}{\sigma_k} + \frac{1}{\sigma_k}\right)\right] \varrho_{nk}}\, \widehat{\textstyle\sum R_{nk'}}\,. \tag{75}$$

Substituting (73)–(75) into (70), the total exponent on $\widehat{\sum R_{nk'}}$ is $1/\bar{\varepsilon}_n - (1 + \bar{\gamma}_n)/\bar{\varepsilon}_n + \bar{\gamma}_n/\bar{\varepsilon}_n = 0$, so total revenue cancels:

$$\hat{C}_n = \left(\frac{\sum_k \Phi_{nk}\, \hat{\varrho}_{nk}}{\sum_k \Lambda_{nk}\, \hat{\chi}_{nk}}\right)^{1/\bar{\varepsilon}_n} \hat{\ell}_n^{-(1+\bar{\gamma}_n)/\bar{\varepsilon}_n} \times \prod_k \hat{S}_{nnk}^{-\chi_{nk}/[\bar{\varepsilon}_n(\sigma_k - 1)]} \prod_k \hat{\varrho}_{nk}^{\chi_{nk}\gamma_k/\bar{\varepsilon}_n} \prod_k \hat{\chi}_{nk}^{\chi_{nk}/((1-\eta)\bar{\varepsilon}_n)}, \tag{76}$$

where $\bar{\gamma}_n \equiv \sum_k \chi_{nk}\, \gamma_k$ and $\hat{\ell}_n = \hat{w}_n/\widehat{\sum R_{nk'}}$ is the change in the labor share (71), using $\hat{L}_n = 1$. The ratio $\hat{w}_n/\widehat{\sum R_{nk'}}$ is given by (75) without the trailing $\widehat{\sum R_{nk'}}$ factor. All exponents share the common denominator $\bar{\varepsilon}_n$, and all terms depend only on baseline shares $(\chi_{nk}, \varrho_{nk})$, changes

$(\hat{S}_{nnk}, \hat{\varrho}_{nk}, \hat{\chi}_{nk})$, and structural parameters.

Special cases. When α_k and σ_k are common across sectors, $\Lambda_{nk} = \chi_{nk}$ and the wage factor simplifies to $(\sum_k \Xi_{nk}\, \hat{\varrho}_{nk})^{-(1+\bar{\gamma}_n)/\bar{\varepsilon}_n}$, so (76) involves only weighted means of $\hat{\varrho}_{nk}$, $\hat{\chi}_{nk}$, and products of $\hat{S}_{nnk}$.

When all $\varepsilon_k = 1$ and $\eta \to 1$ (Cobb–Douglas), $\hat{\chi}_{nk} = 1$, $\bar{\varepsilon}_n = 1$, and (76) reduces to

$$\hat{C}_n = \left(\sum_k \Phi_{nk}\, \hat{\varrho}_{nk}\right)\left(\sum_k \Xi_{nk}\, \hat{\varrho}_{nk}\right)^{-(1+\bar{\gamma})} \prod_k \hat{\varrho}_{nk}^{\gamma_k \chi_k} \prod_k \hat{S}_{nnk}^{-\chi_k/(\sigma_k - 1)}, \tag{77}$$

where $\bar{\gamma} \equiv \sum_k \chi_k \gamma_k$. When further $\gamma_k = 0$ for all k: $\hat{C}_n = \prod_k \hat{S}_{nnk}^{-\chi_k/(\sigma_k-1)}$ (the ACR formula).

D Innovation Labor Share of the Sectoral Wage Bill

In steady state, total innovation expenditure in country-sector (i,k) is $\phi_k^{SS} R_{ik}$. Dividing (35) by R_{ik} and using $T_{iik} = \chi_{ik} Y_i S_{iik}/R_{ik}$ from (46):

$$\frac{w_i L_{ik}}{R_{ik}} = \alpha_k \frac{\sigma_k - 1}{\sigma_k} + \phi_k^{SS} + (1-\alpha_k) \frac{\sigma_k - 1}{\sigma_k} \frac{T_{iik}}{S_{iik}}.$$

Substituting $\phi_k^{SS} = \frac{\delta_k \upsilon \gamma_k}{\rho + \delta_k \upsilon} \frac{\sigma_k - 1}{\sigma_k}$ from (20), the factor $\frac{\sigma_k - 1}{\sigma_k}$ is common:

$$\frac{\phi_k^{SS} R_{ik}}{w_i L_{ik}} = \frac{\dfrac{\delta_k \upsilon \gamma_k}{\rho + \delta_k \upsilon}}{\alpha_k + \dfrac{\delta_k \upsilon \gamma_k}{\rho + \delta_k \upsilon} + (1-\alpha_k) \dfrac{T_{iik}}{S_{iik}}} \tag{78}$$

The ratio $T_{iik}/S_{iik} = \chi_{ik} Y_i / R_{ik}$ measures how much country i's own spending contributes to sector-k revenue relative to the home trade share; it exceeds one when domestic demand is large relative to export revenue.

E Q Theory of IP Investment

This appendix restates the firm's IP investment problem (Section 2.2) in terms of Tobin's Q, the shadow value of installed IP.

Shadow value. Attach Lagrange multiplier Q_{ikt} (in units of the numeraire) to the IP accumulation constraint (5) at date t, so Q_{ikt} is the shadow value of a unit of IP available at $t+1$. Writing $\mathcal{R}_{0t} \equiv \prod_{s=1}^{t}(1+\rho_s)^{-1}$ for the cumulative discount factor ($\mathcal{R}_{00}=1$), the firm's Lagrangian is

$$\mathcal{L} = \sum_{t=0}^{\infty} \mathcal{R}_{0t}\left[\frac{1}{\sigma_k} r_{ikt} - w_{it}\, h_{ikt} + Q_{ikt}\left((1-\delta_k)\, x_{ikt} + \xi_k\, x_{ikt}^{1-\upsilon}(z_{ikt}^{\mu_k}\, h_{ikt})^{\upsilon} - x_{ikt+1}\right)\right], \tag{79}$$

subject to the transversality condition $\lim_{t\to\infty} \mathcal{R}_{0t}\, Q_{ikt}\, x_{ikt+1} = 0$.

First-order conditions. Differentiating (79) with respect to h_{ikt}, multiplying both sides by h_{ikt}, and using (5):

$$\phi_{ikt}\, r_{ikt} = Q_{ikt}\, \upsilon[x_{ikt+1} - (1-\delta_k)\, x_{ikt}]\,. \tag{80}$$

Differentiating with respect to x_{ikt+1}, collecting terms from periods t and $t+1$, and substituting (5):

$$Q_{ikt} = \frac{1}{1+\rho_{t+1}}\left[\gamma_k\, \frac{\sigma_k - 1}{\sigma_k}\, \frac{r_{ikt+1}}{x_{ikt+1}} + Q_{ikt+1}\left(1 - \delta_k + (1-\upsilon)\, \frac{x_{ikt+2} - (1-\delta_k)\, x_{ikt+1}}{x_{ikt+1}}\right)\right]. \tag{81}$$

Equation (80) equates innovation expenditure to $Q_{ikt}\,\upsilon$ times the gross IP increment. Equation (81) is the asset-pricing equation for IP: the shadow value today equals the discounted marginal operating profit plus the value of the undepreciated stock plus the reduction in future innovation costs from a larger base (weighted by $1-\upsilon$).

Equivalence with (18). Using (80) to write $Q_{ikt} = \phi_{ikt}\, r_{ikt}/\{\upsilon[x_{ikt+1} - (1-\delta_k)x_{ikt}]\}$ and substituting into (81) recovers the investment-share Euler equation (18).

Steady state. In steady state, gross investment replaces depreciation: $\xi_k \, x_{ik}^{1-\upsilon} (z_{ik}^{\mu_k} h_{ik})^{\upsilon} = \delta_k \, x_{ik}$. The asset-pricing equation (81) becomes

$$Q_{ik}^{SS}(1+\rho) = \gamma_k \, \frac{\sigma_k - 1}{\sigma_k} \, \frac{r_{ik}}{x_{ik}} + Q_{ik}^{SS}(1 - \upsilon \, \delta_k) \,,$$

giving the steady-state shadow value:

$$Q_{ik}^{SS} = \frac{\gamma_k \, (\sigma_k - 1)}{\sigma_k \, (\rho + \upsilon \, \delta_k)} \, \frac{r_{ik}}{x_{ik}} \,. \tag{82}$$

Marginal vs. average Q. The per-firm market value is $V_{ik} \equiv \frac{1}{\rho}\big(\frac{1}{\sigma_k} - \phi_k^{SS}\big) r_{ik}$. Since the investment technology (5) is CRS in $(x_{ikt}, \, h_{ikt})$, the replacement cost of the IP stock is $Q_{ik}^{SS} \, x_{ik}$. Average Q is

$$\bar{Q}_k \equiv \frac{V_{ik}}{Q_{ik}^{SS} \, x_{ik}} = \frac{\rho + \delta_k \, \upsilon [1 - \gamma_k (\sigma_k - 1)]}{\rho \, \gamma_k \, (\sigma_k - 1)} \,, \tag{83}$$

which depends only on sector parameters.

F Mode 1 and Mode 3 Revenue Decomposition

This appendix maps bilateral revenue to the GATS modes of supply. Under the GATS, Mode 1 (cross-border supply) covers services delivered from i to n without commercial presence; Mode 3 (commercial presence) covers services supplied through affiliates in n.

Decomposition. The production function (4) can be rewritten as Cobb–Douglas in an origin composite and a destination composite:

$$y_{inkt} = \left(x_{ikt}^{\gamma_k} z_{ikt} l_{inkt}\right)^{\alpha_k} \left(x_{ikt}^{\gamma_k} z_{nkt} \iota_{inkt}\right)^{1-\alpha_k}. \tag{84}$$

The non-rival IP stock enters both composites simultaneously. Because IP is non-rival, the same stock x_{ik} serves both composites, so the rental cost is shared: a fraction α_k is attributed to the origin composite and $(1-\alpha_k)$ to the destination composite, matching the Cobb–Douglas output elasticities. Since the CES markup scales total cost uniformly, the revenue split inherits the same proportions:

$$R_{ink}^{M1} = \alpha_k R_{ink}, \qquad R_{ink}^{M3} = (1-\alpha_k) R_{ink}. \tag{85}$$

When $\alpha_k = 1$: $R_{ink}^{M1} = R_{ink}$, $R_{ink}^{M3} = 0$ (pure cross-border). When $\alpha_k = 0$: $R_{ink}^{M1} = 0$, $R_{ink}^{M3} = R_{ink}$ (pure commercial presence).

G Data Appendix

This appendix describes the data sources and construction of the key variables used in the quantitative analysis. Section G.1 lists the raw data sources. Section G.2 describes the construction of bilateral trade flows. Section G.3 describes the measurement of non-rival task content. Section G.4 lists the sector classification, and Section G.5 lists the country groups.

G.1 Data Sources

Table 4 summarizes the data sources used in the quantitative analysis.

Bilateral trade in services. We use the IMF Bilateral Trade in Services (BiTS) dataset, which builds on the OECD-WTO Balanced Trade in Services (BaTiS) database and supplements it with data from UNCTAD-WTO, Eurostat, and UN Comtrade historical records (Loungani et al. 2017; Li et al. 2025). The dataset covers bilateral trade flows for up to 245 reporter and partner economies. We restrict attention to years 2005 and later, when bilateral coverage is reliable. Services are classified according to the Extended Balance of Payments Services Classification (EBOPS 2010), consistent with BPM6, distinguishing 12 broad service categories which we map to ISIC Rev. 4 sectors. The key variable is $ex^{BiTS}_{ij,s,t}$ – exports of services from country i to country j in sector s and year t, denominated in current USD.

Bilateral trade in goods. We use the BACI dataset maintained by CEPII (Gaulier and Zignago 2010), which reconciles exporter- and importer-reported bilateral trade flows from UN COMTRADE. BACI covers over 200 countries and approximately 5,000 products classified at the 6-digit level of HS92 (Harmonized System, 1992 revision), spanning 1994–2022. Values are reported in current USD. We restrict the sample to 2005 onward to align with the BiTS coverage. The key variable is $ex^{BACI}_{ij,p,t}$ – exports of goods from country i to country j in HS92 product p and year t. Because HS92 is considerably finer than ISIC Rev. 4, each 6-digit HS code can be uniquely assigned to a single ISIC 4-digit industry using the standard UN concordance table. Aggregating to the OECD ISIC classification yields sector-level bilateral goods flows $ex^{BACI}_{ij,s,t}$.

Unilateral exports by sector. Total exports by country and ISIC Rev. 4 sector come from the OECD National Input-Output Tables (IOT), which provide a symmetric, industry-by-industry representation of production and total exports and imports for 80 economies. The tables are fully balanced and cover the period 1995–2020 at annual frequency.

Multinational affiliate sales. Data on sales of majority-owned foreign affiliates (MOFA) come from two sources. For the United States, we use the Bureau of Economic Analysis (BEA) tables on majority-owned foreign affiliates (i.e., affiliates where the US parent holds $> 50\%$ equity), which report affiliate sales $\mathcal{A}_{s,t}^{BEA}$ by industry. For the broader cross-country sample, we use the OECD Analytical Activity of Multinational Enterprises (Analytical AMNE) database, which reports bilateral sales $\mathcal{A}_{i,j,s,t}^{OECD}$ by the country of the ultimate controlling owner (UCO) and the host country, covering the same 80 countries and ISIC Rev. 4 sectors as the IOT tables.

Intellectual property stocks. Industry-level data on the stock and depreciation of intellectual property (IP) come from the BEA Fixed Assets tables, at the 2-digit NAICS industry level:

- Current-Cost Net Stock of Intellectual Property Products (IPP) by Industry, denoted $K_{s,t}^{IPP}$ – the perpetual-inventory estimate of the productive stock of IPP assets (software, R&D, entertainment originals) at current replacement cost.
- Current-Cost Depreciation of Private Intellectual Property Products by Industry, denoted $D_{s,t}^{IPP}$ – the annual flow of economic depreciation on the same asset stock.

Non-rival task content. Occupation-level task descriptions come from version 29.1 (August 2024) of the O*NET database (Occupational Information Network), maintained by the U.S. Department of Labor, National Center for O*NET Development. We restrict to base occupations only (excluding detailed .XX variants), yielding 15,638 task statements across 774 occupations. For each occupation o, O*NET reports a list of tasks, each accompanied by a brief natural-language description.

Occupational employment and wages. The Bureau of Labor Statistics (BLS) Occupational Employment and Wage Statistics (OEWS) program (2024 release) provides annual cross-sectional

estimates of employment $n_{o,s,t}$ (the number of workers in occupation o in 3-digit NAICS sector s) and mean annual wage $w_{o,s,t}$, enabling construction of an occupation-sector payroll measure $\pi_{o,s,t} = n_{o,s,t} \times w_{o,s,t}$. We use sector-specific payroll weights at the 3-digit NAICS level (not pooled manufacturing), mapped to the 19 OECD ISIC sectors via a NAICS-to-ISIC concordance with occupation group-code fallback matching, yielding 1,173 matched BLS occupations.

Population. Country-level population data come from the Penn World Tables (PWT 10.01), using 2019 values (Feenstra et al. 2015).

GDP benchmarks. Nominal GDP per capita (in current U.S. dollars, normalized so that USA = 1) come from the World Bank's World Development Indicators, for the year 2022.

Table 4: Data Sources

Source	Description	Coverage
Trade flows		
IMF BiTS[a]	Bilateral trade in services (EBOPS 2010, 12 categories)	245 economies, 2005–
CEPII BACI[b]	Bilateral trade in goods (HS92, 6-digit)	200+ countries, 1994–2022
OECD IOT[c]	Exports by country and ISIC Rev. 4 sector	80 countries, 1995–2020
Multinational activity		
BEA MOFA[d]	US foreign affiliate sales by industry ($\mathcal{A}^{BEA}_{s,t}$)	US, annual
OECD Analytical AMNE[c]	Affiliate sales by UCO and host ($\mathcal{A}^{OECD}_{i,j,s,t}$)	80 countries
IP stocks and tasks		
BEA Fixed Assets[d]	IPP stock ($K^{IPP}_{s,t}$) and depreciation ($D^{IPP}_{s,t}$) by 2-digit NAICS	US, annual
O*NET 29.1[e]	Task descriptions, 15,638 tasks across 774 base occupations	US, Aug 2024
BLS OEWS 2024[f]	Employment and wages by occupation × sector (3-digit NAICS)	US, 2024
Macro aggregates		
PWT 10.01[g]	Population	Cross-country, 2019
World Bank WDI[h]	Nominal GDP per capita	Cross-country, 2022

[a] https://data.imf.org/bts. See Loungani et al. (2017) and Li et al. (2025).
[b] https://www.cepii.fr/CEPII/en/bdd_modele/bdd_modele_item.asp?id=37. See Gaulier and Zignago (2010).
[c] https://stats.oecd.org.
[d] https://www.bea.gov.
[e] https://www.onetcenter.org/database.html. U.S. Department of Labor.
[f] https://www.bls.gov/oes/.
[g] https://www.rug.nl/ggdc/productivity/pwt/. See Feenstra et al. (2015).
[h] https://data.worldbank.org.

G.2 Bilateral Trade Matrix

We construct a square matrix of bilateral trade flows across N countries and K sectors by combining goods and services trade data and distributing unilateral export totals from the OECD IOT tables. We use the year 2022 as the reference year for the baseline analysis.

Step 1: Sector mapping for BACI (goods). Each HS92 6-digit product code from BACI is mapped to a unique ISIC Rev. 4 industry using the standard UN HS–ISIC concordance table. Flows are then aggregated to the OECD ISIC sector classification:

$$ex^{BACI}_{ij,s,t} = \sum_{p \in \mathcal{P}(s)} ex^{BACI}_{ij,p,t},$$

where $\mathcal{P}(s)$ is the set of HS92 products mapping to sector s.

Step 2: Bilateral export shares. For each dataset $d \in \{\text{BACI}, \text{BiTS}\}$, sector s, origin i, and year t, we restrict to the OECD sample of 80 countries and to years 2005 and later, and compute the share of each destination j in origin i's exports:

$$\hat{\sigma}^{d}_{ij,s,t} = \frac{ex^{d}_{ij,s,t}}{\sum_{j'} ex^{d}_{ij',s,t}} = \frac{ex^{d}_{ij,s,t}}{ex^{d}_{i,s,t}}.$$

Missing country-pair–sector–year observations are assigned a share of zero, ensuring the panel is balanced:

$$\sigma^{d}_{ij,s,t} = \begin{cases} \hat{\sigma}^{d}_{ij,s,t} & \text{if } (i,j,s,t) \text{ observed} \\ 0 & \text{otherwise} \end{cases}, \qquad \text{with } \sum_{j} \sigma^{d}_{ij,s,t} = 1 \;\; \forall\, i,s,t\,.$$

Step 3: Sector mapping for BiTS (services). We map EBOPS 2010 service categories from BiTS to ISIC Rev. 4 sectors. Because ISIC is finer than EBOPS in some areas, certain EBOPS categories map to multiple ISIC sectors; in these cases we apply the same bilateral shares to each mapped ISIC sector.

Step 4: Distribute OECD export totals. The bilateral trade flow is obtained by applying the share matrix to the OECD IOT unilateral export total:

$$ex_{ij,s,t}^{OECD} = E_{i,s,t}^{OECD} \times \sigma_{ij,s,t}^{d}$$

where d = BACI for goods sectors and d = BiTS for services sectors. The result is a square 80×80 matrix for each sector and year. This procedure ensures that bilateral flows are consistent with the OECD's unilateral export totals while preserving the bilateral distribution from the micro trade data. Domestic absorption (the $i = n$ diagonal) is computed as the residual from the IOT tables.

G.3 Non-Rival Task Classification

We construct a measure of the non-rival task content of each sector using occupation-level task data from O*NET 29.1 and payroll data from BLS OEWS 2024.

Step 1: LLM-based task classification. Each of the 15,638 O*NET task descriptions (774 base occupations) is submitted to a large language model (Claude Opus 4, Anthropic) with a structured classification prompt. The prompt assigns each task to one of three categories: *non-rival*, *ambiguous*, or *rival*. The core definition is as follows:

> *A task is non-rival if spending one hour on it contributes to the creation of a firm-specific intangible asset — such as software, product designs, patents, or brand equity — that is: (1) Proprietary: owned or controlled by the firm; (2) Non-depletable: can be used simultaneously across all customers, locations, and markets without being used up; (3) Durable: persists beyond the period in which it was created. A task is rival if its output clearly fails any of these three conditions — because it is consumed in delivery, must be repeated for each customer, or benefits only a specific customer or market. A task is ambiguous if it plausibly contributes to a non-rival asset under some circumstances but not others, or if the task description is too vague to determine.*

The prompt instructs the model to work through a decision procedure for each task: (i) does the task produce an asset the firm retains after delivery? If clearly not, the task is rival. (ii) Can that asset be used for all customers simultaneously without depletion? If clearly not, rival. (iii) Does

the asset persist beyond the creation period? If clearly not, rival. If yes to all three, non-rival. If the answer to any question is uncertain or context-dependent, the task is ambiguous. The prompt also provides guidance on common misclassifications (e.g., a task that benefits many customers is not automatically non-rival unless it creates a durable asset; deploying an existing non-rival asset is not itself non-rival; customization for a specific client is rival) and worked examples with reasoning for all three categories. The full prompt text is available on request.

Across the 15,638 tasks, the classifier assigns 8.6% as non-rival, 15.0% as ambiguous, and 76.4% as rival. We validate the LLM classifications against a hand-coded sample using a rule-based classifier that applies the same decision procedure; the two agree on 83.5% of tasks.

Step 2: Occupation-level non-rival share. For each occupation o, we compute the non-rival task share excluding ambiguous tasks from both the numerator and denominator:

$$x_o = \frac{\mathrm{NR}_o}{\mathrm{NR}_o + \mathrm{R}_o},$$

where NR_o and R_o are the number of tasks classified as non-rival and rival, respectively, in occupation o. Ambiguous tasks are excluded so that occupations with many borderline tasks are not mechanically pushed toward either extreme.

Step 3: Sector-level non-rival payroll share. The occupation-level shares x_o are merged with BLS OEWS 2024 payroll data at the 3-digit NAICS level. Each 3-digit NAICS industry is mapped to one of the 19 OECD ISIC sectors; occupation matching uses group-code fallback when exact SOC codes are unavailable, yielding 1,173 matched BLS occupations. The payroll of occupation o in sector s is $\pi_{o,s} = n_{o,s} \times w_{o,s}$. The payroll-weighted non-rival share in sector k is

$$H_k = \sum_o \frac{\pi_{o,k}}{\Pi_k} x_o, \tag{86}$$

where $\Pi_k = \sum_o \pi_{o,k}$ is total payroll in sector k. The rival payroll share is accordingly $1 - H_k$.

G.4 Sector Classification

Our baseline specification uses $K = 19$ ISIC Rev. 4 broad sectors, excluding Agriculture (A), Mining (B), and Real Estate (L). Of the 19 sectors, 12 are in manufacturing and 7 are in services. Table 5 lists the sectors, their ISIC codes, and the constituent ISIC divisions.

Table 5: Sector Classification

Code	Sector	ISIC Divisions
Manufacturing		
C10T12	Food products, beverages, tobacco	C10–C12
C13T15	Textiles, wearing apparel, leather	C13–C15
C16	Wood and cork products	C16
C17T18	Paper products; printing	C17–C18
C19T21	Chemicals, pharmaceuticals, petroleum	C19–C21
C22	Rubber and plastic products	C22
C23	Other non-metallic minerals	C23
C24	Primary metals	C24
C25	Fabricated metal products	C25
C26T27	Computer, electronic, electrical equipment	C26–C27
C28	Machinery and equipment n.e.c.	C28
C29T30	Transport equipment	C29–C30
Services		
GH	Transport, warehousing, and trade	G, H49–H53
J58T60	Publishing, audiovisual, broadcasting	J58–J60
J61	Telecommunications	J61
J62T63	IT and other information services	J62–J63
K	Financial and insurance activities	K
M	Professional, scientific, technical activities	M
OS	Other services[a]	I, N–S, T

[a] Includes accommodation and food services (I), administrative and support services (N), public administration (O), education (P), health and social work (Q), arts and recreation (R), and other service activities (S, T).

G.5 Country Groups

The underlying bilateral trade data draw on 80 economies from the OECD IOT tables (38 OECD members and 42 partner countries). For the quantitative analysis, we aggregate these into $N = 10$ country groups listed in Table 6. The EU group aggregates all EU member states in the OECD IOT sample. The rest-of-world (ROW) group aggregates all remaining countries not assigned to one of the other nine groups.

Table 6: Country Groups

Index	Code	Country / Group
1	BRA	Brazil
2	CAN	Canada
3	CHN	China
4	EU	European Union
5	GBR	United Kingdom
6	IND	India
7	JPN	Japan
8	ROW	Rest of World
9	RUS	Russia
10	USA	United States

H Calibration Appendix

This appendix describes how the structural parameters are calibrated from data and how country-sector productivities are recovered via model inversion. Section H.1 lists the externally set parameters. Sections H.2–H.4 detail the measurement of α_k, γ_k, and δ_k. Section H.5 describes the inversion procedure that recovers productivities $\{z_{ik}\}$ and bilateral appeals $\{a_{ink}\}$ from data.

H.1 Externally Set Parameters

Table 7 lists the externally set parameters, the parameters computed directly from data, and the parameters recovered from model inversion.

We set σ_k to equalize the net profit margin across sectors at 10% of revenue. In steady state, the net profit share is $1/\sigma_k - \phi_k^{SS}$, where $\phi_k^{SS} = \delta_k \upsilon \gamma_k (\sigma_k - 1)/[\sigma_k(\rho + \delta_k \upsilon)]$ is the innovation share of revenue. Setting $1/\sigma_k - \phi_k^{SS} = \bar{\pi} = 0.10$ and solving yields

$$\sigma_k = \frac{1 + \gamma_k \delta^*}{\bar{\pi} + \gamma_k \delta^*},$$

where $\delta^* = \delta\upsilon/(\rho + \delta\upsilon)$. This gives σ_k ranging from 2.9 (IT services, highest γ_k) to 6.0 (transport and trade, lowest γ_k). High-IP sectors have lower σ_k (higher markups), consistent with the empirical observation that IP-intensive industries enjoy greater pricing power. All sectors satisfy the uniqueness bound $\sigma_k < \sigma_k^{\max}$.

Table 7: Parameter Summary

Parameter	Symbol	Value	Source / Rationale
Set externally			
Discount factor	β	0.96	Standard annual calibration
Across-sector elasticity	η	0.5	Based on Comin et al. (2021)
Nonhomotheticity	ε_k	Sector-specific (0.81–1.75)	Based on Comin et al. (2021)
IP depreciation	δ	0.20	Avg. IPP depreciation, BEA Fixed Assets
Adjustment friction	υ	0.5	Quadratic adjustment cost
EIS	ψ	0.5	Baseline
Bond adjustment cost	ζ	0.01	Standard
Population	L_i	Country-specific	PWT 10.01 (2019 values)
Computed directly from data			
Origin labor share	α_k	Sector-specific (0.14–0.53)	Analytical AMNE/BEA affiliate data
Non-rival intensity	γ_k	Sector-specific (0.11–0.54)	O*NET/BLS task measure H_k
Within-sector elasticity	σ_k	Sector-specific (2.9–6.0)	Constant net profit margin (10%)
Recovered from model inversion			
Bilateral demand shifters	a_{ink}	Bilateral×sector	Trade shares
Productivity	z_{ik}	Country×sector	Home trade shares, GDP p.c.
NH-CES demand shifters	$\bar{\chi}_{nk}$	Country×sector	Expenditure shares

H.2 Origin-Labor Share α_k

The parameter α_k governs the split of rival labor input between origin labor l_{inkt} and destination labor ι_{inkt} in sector k. As shown in Appendix F, α_k also equals the Mode 1 (cross-border) share of bilateral revenue. We measure α_k using the ratio of direct exports to total foreign sales (exports plus affiliate sales):

$$\alpha_k = \frac{X_k}{X_k + \mathcal{A}_k}, \tag{87}$$

where the two components are defined as follows:

- X_k – *Direct exports* in sector k, from the bilateral trade matrix described in Appendix G.2. In the baseline (US-only) analysis, this is the sum of all US bilateral export flows $\sum_j X_{US,j,k}$. In the full-sample analysis, we sum across all origin countries $\sum_i \sum_j X_{ij,k}$.
- $\mathcal{A}_k$ – *Affiliate sales* in sector k. In the US-only analysis, we use $\mathcal{A}_k^{BEA}$ from the BEA MOFA tables, or equivalently $\sum_j \mathcal{A}_{US,j,k}^{OECD}$ from OECD Analytical AMNE. In the full-sample analysis, we use $\sum_i \sum_j \mathcal{A}_{ij,k}^{OECD}$ from OECD Analytical AMNE.

Note that $\alpha_k \in [0,1]$, with $\alpha_k \to 1$ indicating that the sector relies almost exclusively on direct (cross-border) exports, and $\alpha_k \to 0$ indicating near-complete reliance on multinational affiliate sales.

H.3 Non-Rival Intensity γ_k

Given the non-rival payroll share H_k from the task classification (Appendix G.3), the structural parameters α_k and δ_k, and the investment friction υ, the steady-state relationship between H_k and the IP intensity parameter γ_k is

$$\gamma_k = \frac{H_k}{1 - H_k} \frac{\alpha_k + (1 - \alpha_k)\, \chi_{ik} Y_i / R_{ik}}{\delta_k^*}, \tag{88}$$

where $\delta_k^* \equiv \delta_k \upsilon / (\rho + \delta_k \upsilon)$ is the discount-adjusted depreciation rate, $\rho = 1/\beta - 1$ is the discount rate, and $\chi_{ik} Y_i / R_{ik}$ is the ratio of country i's expenditure on sector k to total revenue of country i's firms in sector k (set to the US value in the baseline).

Uniqueness condition. The firm's problem is well-defined when the steady-state innovation share is less than the markup: $\phi_k^{SS} < 1/\sigma_k$, which is equivalent to $\gamma_k \delta^* (\sigma_k - 1) < 1$. Our calibration targets $1/\sigma_k - \phi_k^{SS} = 0.10$ for all sectors, so this condition is satisfied by construction. A stronger condition – $\gamma_k(\sigma_k - 1) < 1$ – ensures uniqueness of the log-linearized system (Proposition 4). With sector-specific σ_k, the binding sectors are IT services ($\gamma(\sigma - 1) = 1.01$) and electronics ($\gamma(\sigma - 1) = 1.00$), which marginally exceed this bound. In practice, the full nonlinear equilibrium converges uniquely for all specifications we consider, and the log-linearized system remains well-behaved because $\delta^* \gamma_k (\sigma_k - 1) < 1$ for all sectors.

H.4 IP Depreciation Rate δ_k

We measure the sector-level IP depreciation rate as the ratio of current-cost depreciation to the current-cost net stock of intellectual property products:

$$\delta_k = \frac{D_k^{IPP}}{K_k^{IPP}}, \tag{89}$$

where K_k^{IPP} is the current-cost net stock of IPP (software, R&D, entertainment originals) in sector k and D_k^{IPP} is the corresponding current-cost depreciation flow, both from the BEA Fixed Assets tables. Both series are at the 2-digit NAICS industry level and are crosswalked to ISIC Rev. 4 sectors; because NAICS is coarser than ISIC in some areas, certain NAICS industries map to multiple ISIC sectors. Across the 19 sectors, δ_k ranges from approximately 0.12 (chemicals, C19T21) to 0.42 (financial services, K), reflecting the rapid obsolescence of software-intensive IP. In the baseline specification, we set δ_k to a common value of 0.20 across sectors.

The country groups used in the quantitative analysis are listed in Appendix G.5, Table 6.

Brazil, China, India, Japan, Russia, the United Kingdom, and the United States each form their own group. Canada is kept separate from the US despite geographic proximity because of its distinct trade structure. The European Union is aggregated because intra-EU trade barriers are low relative to external trade costs. All remaining countries – including Mexico, South Korea, and all other OECD ICIO economies – are absorbed into the rest-of-world (ROW) group.

H.5 Model Inversion

Given the calibrated parameters $\{\alpha_k, \gamma_k, \delta_k, \sigma_k, v\}$ and the observed bilateral trade shares $\{S_{nik}\}$ and expenditure shares $\{\chi_{nk}\}$ for the year 2022, we recover country-sector productivities $\{z_{ik}\}$ and bilateral appeals $\{a_{ink}\}$ by inverting the steady-state equilibrium conditions.

Bilateral appeals. From the trade share equation (39), bilateral appeal is recovered as a geometric mean of bilateral trade share ratios:

$$a_{ink} = \sqrt{\frac{S_{nik}\, S_{ink}}{S_{iik}\, S_{nnk}}}, \tag{90}$$

which cancels all country-sector fundamentals (z_{ik}, w_i, R_{ik}) and isolates the bilateral component. The resulting matrix is symmetric ($a_{ink} = a_{nik}$) with home diagonal entries equal to one ($a_{iik} = 1$).

Productivities. We solve for $\{z_{ik}\}$ using a damped fixed-point iteration. Given a candidate vector of productivities, we compute wages $\{w_i\}$, revenues $\{R_{ik}\}$, and IP stocks $\{x_{ik}\}$ from the steady-state system (Appendix A), then analytically update z_{ik} to match the observed home trade shares $\{S_{iik}\}$. The iteration converges when the implied home trade shares match the data to

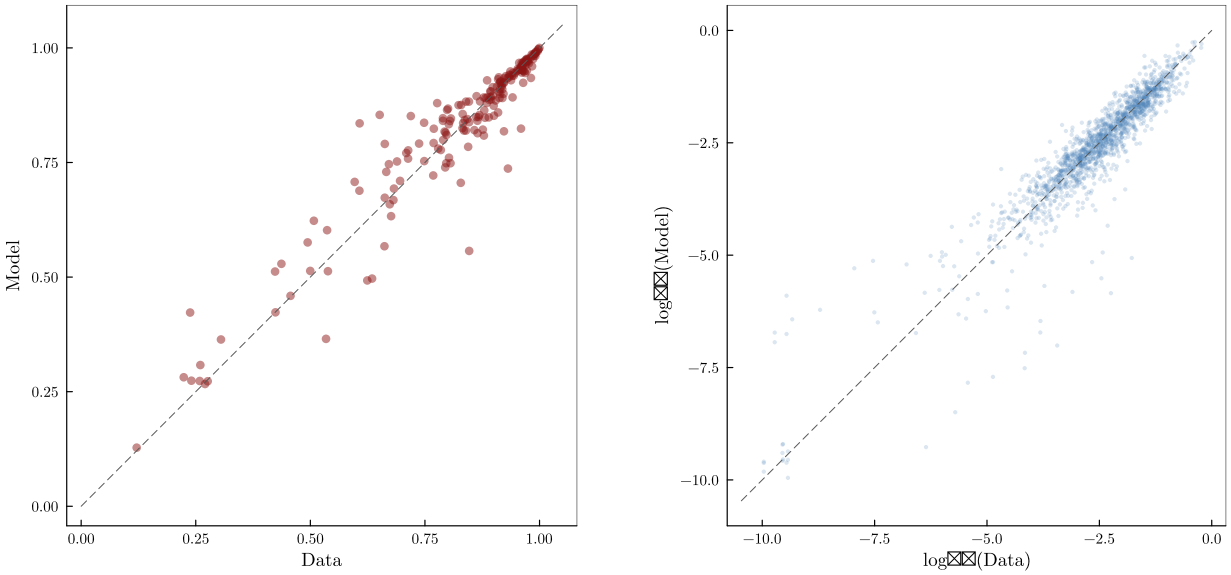
within a tolerance of 10^{-6}.

Productivities are normalized so that the US wage equals one. They are then scaled by nominal GDP per capita (from the World Bank, USA = 1) so that model-implied relative wages match observed relative income levels.

Figure 7 reports the model fit. The left panel compares model-implied home trade shares S_{iik} with their data counterparts; the right panel does the same for off-diagonal trade shares S_{nik} ($n \neq i$) in logs.

Figure 7: Model Fit: Trade Shares

(a) Diagonal trade shares S_{iik} (b) Off-diagonal trade shares S_{nik}



Note: Each dot represents a country-sector pair. The left panel plots model-implied home trade shares S_{iik} against their data counterparts in levels. The right panel plots model-implied off-diagonal trade shares S_{nik} ($n \neq i$) against data in $\log_{10}$. The dashed line is the 45-degree line.