A Quantitative Theory of Trade in Services

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Abstract

Trade in services accounts for a large and growing share of global trade, with recent growth driven entirely by "modern services" (information, communication, financial, and professional services). We revisit classic questions in trade theory in an environment that captures a defining feature of these sectors: their intensive use of non-rival assets (e.g., software, data, and intellectual property). We develop a dynamic multi-sector model in which sectors differ in their intensity of non-rival assets, and measure this intensity by computing sectoral expenditure on non-rival asset production. We show that non-rivalry is central to understanding trade patterns and the gains from trade. First, it helps rationalize the observed specialization of rich countries in modern services through its interaction with heterogeneous demand patterns. Second, non-rivalry increases the gains from integration through an amplified capital multiplier, while tilting those gains toward high-income countries. Third, it gives rise to heterogeneous timing of consumption gains following global shocks, rationalizing the observed relationship between sectoral specialization and current account imbalances. Finally, we illustrate the importance of non-rival assets for evaluating barriers to services trade by applying our framework to a leading example: China's Great Firewall.

Keywords: trade in services, non-rival assets, intangibles, structural transformation, gains from trade
JEL Codes: F11, F12, F13, O34, O41

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1 Introduction

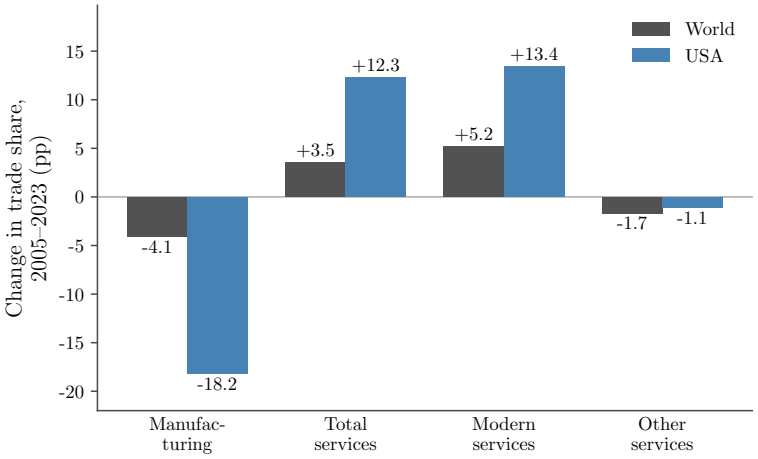
> *"Services are often largely overlooked in discussions on global trade, yet they account for the majority of trade in many developed economies and are growing rapidly in many developing economies as well."*
>
> — Roberto Azevêdo, WTO Director-General, in WTO (2019)

Services are a large and growing component of global trade, accounting for close to 30% of total direct exports globally and close to 40% in the United States, and they are expected to continue growing as a share of total trade (WTO 2019; Li et al. 2025). Virtually all of the growth in services trade over the past two decades has been driven by "modern services" (Loungani et al. 2017; Baldwin et al. 2024) – comprising information, communication, financial, and professional services. As can be seen in Figure 1, modern services have experienced a strong increase in their share of total trade, while manufacturing and other services have declined. How does the growing importance of these sectors change the answers to classic questions in trade theory, such as the patterns of trade, the gains from trade, and the dynamic adjustment to trade shocks? What are the global implications of trade frictions in these sectors, such as barriers to digital-services trade (e.g. China's "Great Firewall")?

To study these questions, we develop a dynamic multi-sector model of the global economy that captures a defining feature of modern services: their heavy reliance on *non-rival assets*, such as software, data, and other intangibles (Hill 1999).[1] Unlike merchandise such as agricultural goods, whose consumption by one country precludes its consumption by another, the consumption of services such as Netflix by one country does not diminish their availability to others. We show that modeling such non-rivalry – and the heterogeneity in its importance across sectors – is central to understanding these questions. We analyze how non-rival assets shape patterns of trade, demonstrating that their interaction with non-homothetic demand leads richer countries to specialize in modern services, consistent with the data. We show that non-rival assets amplify the gains from trade while tilting them toward richer countries. We also show that the timing of the consumption gains from integration differs systematically across countries, generating a

[1] In the EUKLEMS-INTANProd database, intangible capital accounts for 52% of the total capital stock in modern-services sectors, compared with 30% in manufacturing and 11% in other services.

Figure 1: Change in share of total trade for selected sectors, 2005-2023.



Note: Change in each group's share of total cross-border trade between 2005 and 2023, for the world and the United States: manufacturing, total services, and the split of total services into modern and other services. Trade flows combine BACI for goods and the IMF Bilateral Trade in Services database for services; modern services comprise information, communication, financial, and professional services.

link between specialization and current account balances following global shocks. Finally, we apply our framework to a key barrier to services trade – China's Great Firewall – and show that its global effects depend critically on the accumulation of non-rival assets.

Our framework features a global economy with multiple countries and sectors. As in standard trade models, countries and sectors differ in size, productivity, and trade frictions. Three features distinguish our setting. First, firms dynamically accumulate *non-rival assets*– software, data, and other intangibles – and combine them with rival factors (labor and physical capital), with sectors differing in their intensity of non-rival asset use. Second, demand is *non-homothetic*, with heterogeneous income elasticities across sectors, so that richer countries allocate a larger share of expenditure to services. Finally, to capture the importance of foreign affiliates in services trade, we allow for heterogeneity in the *mode of delivery* across sectors, encompassing both cross-border exports and sales through commercial presence.[2]

We characterize the model's steady state analytically through a series of comparative-statics exercises, highlighting three main insights. First, non-rival assets and non-homothetic demand

[2]These two modes of supply (Modes 1 and 3 in the terminology of the General Agreement on Trade in Services (GATS)) account for around 90% of global services trade. We abstract from the remaining two modes of supply – Mode 2 (consumption abroad) and Mode 4 (presence of natural persons) – which rely on the international movement of people.

jointly shape specialization. Through the interaction of income and scale effects, richer countries specialize in sectors characterized by high income elasticity and high non-rival intensity – both of which are a strong feature of modern services. Second, we show how non-rivalry shapes the gains from trade by extending standard sufficient-statistic representations, highlighting two channels. With physical capital, trade increases the amount of rival inputs available for the production of non-rival assets, generating a larger multiplier effect on consumption from reductions in trade frictions. In addition, trade reallocates production toward sectors of comparative advantage, leading to relatively larger gains for countries that specialize in sectors with high non-rival intensity. Since these tend to be richer countries through the forces emphasized above, the gains from trade are tilted toward them. Finally, we highlight that, despite the presence of sectoral scale effects, the decentralized equilibrium is efficient (up to inefficiencies generated by exogenous differences in sectoral markups), because the scale economies are internal to firms.

We quantify the model for 73 countries and 23 sectors spanning goods and services. Our central measurement task is the sectoral intensity of non-rival assets, which we recover from the allocation of sectoral expenditure to the production of such assets. We rely on the assumption that these scale effects are internal to firms and can therefore be measured from firms' optimization problems.[3] To operationalize this approach, we compute the share of labor costs in each sector devoted to the production of non-rival assets using the sector's occupational composition and the task content of each occupation. The recovered intensities vary plausibly across sectors: they are highest in information technology and other information services, professional services, computer and electronic equipment, and pharmaceuticals, and lowest in real estate, food manufacturing, and wood products. On average, the non-rival intensity in modern services is twice as large as in manufacturing, and much higher than in other services. We show that this model-consistent measure aligns closely with independent proxies for sectoral intangible intensity, such as the intellectual-property share of capital in the BEA fixed-asset accounts. We further show that our estimates are robust to alternative measurement procedures, including ones that allow the economy to be either in or out of its steady state.

We use our quantified model to investigate three key questions. First, what determines the

[3]We do not deny the existence of external economies of scale – which are typically very difficult to measure – but instead focus on analyzing and measuring the internal scale effects that arise from the accumulation of non-rival assets.

pattern of trade in services? We find that both the scale effects generated by non-rival assets and heterogeneity in demand across countries are important in explaining countries' specialization in services. While our model exactly rationalizes the observed pattern of specialization through a set of residuals capturing exogenous comparative advantage, we show that much of the observed specialization remains even after shutting down these exogenous comparative advantages by eliminating idiosyncratic country-sector productivity differences. Importantly, the same exercise yields a substantially poorer fit to the observed pattern of specialization in alternative models that abstract from non-rival assets and/or realistic cross-country differences in demand.

Second, how large are the welfare gains from trade in services, and how do the gains from trade more generally change with the accumulation of non-rival assets? We find that the gains from trade in services – particularly modern services – are comparable to those from trade in goods, and are even larger for many high-income economies. This reflects the large share of services in consumption, relatively low elasticities of substitution, and stronger scale effects. We also show, in line with the model's predictions, that the gains from trade are tilted toward rich countries and countries that specialize in non-rival-intensive sectors relative to what one would infer from a model that abstracts from these mechanisms.

Third, how does the accumulation of non-rival assets change the dynamic effects of global shocks? Because countries differ in their sectoral composition, shocks that amplify specialization generate relatively larger long-run responses in countries that specialize in high-non-rival-intensity sectors, which must accumulate non-rival assets over time, and relatively larger short-run responses in countries that specialize in low-non-rival-intensity activities. We demonstrate this by tracing the response of the global economy to two common shocks that were arguably among the most consequential of recent decades: a uniform reduction in global trade frictions, and a global improvement in the productivity of modern services such as information technology. In both cases, countries that specialize in non-rival-intensive sectors show a larger gap between their short-run and long-run consumption responses and, when we allow for trade in financial assets, run current account deficits along the transition. These patterns are consistent with empirical evidence documenting a link between current account imbalances and specialization in services in recent decades (Barattieri 2014), and with our own findings documenting such a pattern. They are also consistent with the idea that current account imbalances are connected to the rise of

intangibles (McGrattan and Prescott 2010), but our framework provides an economic rationale for this link based on heterogeneous specialization rather than mismeasurement.

Finally, we illustrate the applicability of our framework to a concrete barrier to services trade. We study China's "Great Firewall" as an example, treating it as a non-tariff barrier to digital-services trade. We show that this barrier is reflected in dramatically higher model-implied costs of importing information-technology and telecommunications services into China than into comparable East Asian economies. Reducing China's inward trade costs in these sectors to the levels of comparable neighboring economies raises Chinese real consumption while shifting production away from digital services and toward manufacturing. The effects abroad hinge on the accumulation of non-rival assets: nearby exporters, such as Japan and the Southeast Asian economies, expand their sales to China and, as they accumulate non-rival assets, become more productive and more specialized in these services. By contrast, more distant exporters specializing in IT services, such as India and Ireland, lose through a deterioration in their terms of trade as scale effects improve the competitiveness of Southeast Asian countries in third markets. This last effect disappears when non-rival assets are shut down, underscoring the central role of non-rival asset accumulation in shaping the effects of barriers to trade in services.

Relation to the literature. This paper contributes to several strands of the literature. First, we contribute to the quantitative trade literature (Eaton and Kortum 2002; Arkolakis et al. 2012; Costinot and Rodríguez-Clare 2014; Caliendo and Parro 2015), and in particular to the small literature that incorporates trade in services into this analysis (Eaton and Kortum 2018).[4] We extend these models to allow for the dynamic accumulation of non-rival assets, and analyze its implications for the patterns of trade and the gains from trade, including its interaction with non-homothetic demand. We also relate to the literature on dynamic general-equilibrium trade and spatial models (Eaton et al. 2016; Ravikumar et al. 2019; Caliendo et al. 2019; Boehm et al. 2024; Kleinman et al. 2025), extending the analysis beyond labor mobility and physical-capital accumulation to the accumulation of non-rival assets. Another related literature studies trade in services within countries using static spatial equilibrium models, including Eckert (2019), Hsieh and Rossi-Hansberg (2023), Eckert et al. (2025), and Kleinman (2025).

Second, we relate to the growing empirical literature on trade in services (Francois and

[4](Staiger and Sykes 2021) study trade policy in the context of services.

Hoekman 2010; Barattieri 2014; Baldwin 2016; Gervais and Jensen 2019; Baldwin et al. 2024; Li et al. 2025). Our theory captures many of the features highlighted in this literature, including the dominance of services trade by high-income countries; the centrality of trade in intangible assets; and the relationship between comparative advantage in services and the current account.

Third, we relate to multi-sector models of trade with external economies of scale, tracing back to Ethier (1982) and Helpman and Krugman (1985), and more recently Kucheryavyy et al. (2023), Lashkaripour and Lugovskyy (2023), and Bartelme et al. (2025). In our setting, sectoral scale elasticities arise from the endogenous accumulation of non-rival assets. The resulting sectoral economies of scale do not distort the decentralized equilibrium, and can be measured from firms' optimization problems.

Fourth, we relate to models of non-homothetic demand and trade, including Markusen (2013), Caron et al. (2014), Kehoe et al. (2018), and Matsuyama (2019). Our non-homothetic demand structure follows Comin et al. (2021); we combine differences in sectoral income elasticities with sectoral accumulation of non-rival assets and show how these two margins shape the patterns of trade and the gains from trade in services.

Fifth, we relate to models of multinational production that highlight the role of multinationals in facilitating trade in knowledge, such as Helpman (1984), Markusen (1984), Antràs et al. (2006), Ramondo and Rodríguez-Clare (2013), Tintelnot (2017), Arkolakis et al. (2018), Alviarez et al. (2023), and Santacreu (2025). Relative to these models, our framework features dynamic accumulation of non-rival assets in a multi-sector environment where sectors differ in the importance of such assets for production. We show how these features matter particularly for the analysis of trade in services, both through cross-border transactions and through affiliate sales.

Finally, we relate to the literature on intangible capital in macroeconomics, including Corrado et al. (2009), McGrattan and Prescott (2009), McGrattan and Prescott (2010), Crouzet et al. (2022), and De Ridder (2024). We embed the dynamic accumulation of such assets in a multi-sector model of trade.

Roadmap. The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 characterizes the steady state analytically and derives comparative statics on the patterns of trade and the gains from trade. Section 4 describes the data and our measurement of the sectoral intensity of non-rival assets. Section 5 presents the quantitative analysis, covering the

determinants of specialization, the gains from trade, the dynamic response to global shocks, and the application to China's Great Firewall. Section 6 concludes.

2 Model

2.1 Economic Environment

We begin with a description of the physical environment, which is also summarized in Table 1. We consider a world economy with N countries, indexed by i and n, and S sectors, indexed by s. Time is discrete, $t = 0, 1, 2, \ldots$. Each country n is endowed with L_n units of labor.

Table 1: Summary of the Economic Environment

Preferences	
Lifetime utility	$U_{n0} = \sum_{t=0}^{\infty} \beta^t \frac{C_{n,t}^{1-1/\psi}}{1-1/\psi}$
Output aggregators	
Final good aggregator	$\begin{cases} \sum_s \bar{\chi}_{ns}^{\frac{1}{\eta}} \left(\frac{\mathcal{Y}_{n,t}}{L_n}\right)^{\frac{1-\eta}{\eta}(\varepsilon_s-1)} \left(\frac{\mathcal{Y}_{ns,t}}{\mathcal{Y}_{n,t}}\right)^{\frac{\eta-1}{\eta}} = 1, & \eta \neq 1, \\ \mathcal{Y}_{n,t} = \prod_s \mathcal{Y}_{ns,t}^{\bar{\chi}_{ns}}, & \eta = 1. \end{cases}$
Sectoral aggregator	$\mathcal{Y}_{ns,t} = \left[\sum_i \int_{\omega \in \Omega_{is}} a_{ins,t}^{\frac{1}{\sigma_s}} y_{ins,t}(\omega)^{\frac{\sigma_s-1}{\sigma_s}} d\omega\right]^{\frac{\sigma_s}{\sigma_s-1}}$
Production of varieties	
Production function	$y_{ins,t} = x_{is,t}^{\gamma_s} \tau_{ins,t}^{-1} \left[g_{is,t}(l_{ins,t}^o, k_{ins,t}^o)\right]^{\alpha_s} \left[g_{ns,t}(l_{ins,t}^d, k_{ins,t}^d)\right]^{1-\alpha_s}$
Rival-input composite	$g_{js,t}(l,k) = (z_{js,t}\, l)^{\mu_s} k^{1-\mu_s}$
Factor accumulation	
Non-rival asset stock	$x_{is,t+1} = (1-\delta_x)\, x_{is,t} + g_{is,t}(l_{is,t}^x, k_{is,t}^x)$
Physical capital stock	$K_{n,t+1} = (1-\delta_K)\, K_{n,t} + I_{n,t}$
Resource constraints	
Final output	$\mathcal{Y}_{n,t} = L_n\, C_{n,t} + I_{n,t}$
Labor	$L_i \geq \sum_s \left[l_{is,t}^x + \sum_n l_{ins,t}^o\right] + \sum_{j,s} l_{jis,t}^d$
Capital	$K_{i,t} \geq \sum_s \left[k_{is,t}^x + \sum_n k_{ins,t}^o\right] + \sum_{j,s} k_{jis,t}^d$

Preferences. The representative household in country n has CRRA intertemporal preferences:

$$U_{n0} = \sum_{t=0}^{\infty} \beta^t \frac{C_{n,t}^{1-1/\psi}}{1-1/\psi}, \tag{1}$$

where $\beta \in (0,1)$ is the discount factor, $\psi > 0$ is the elasticity of intertemporal substitution, and $C_{n,t}$ is per-capita consumption of the local final good.

Final good. Households consume a local final good $\mathcal{Y}_{n,t}$, which is used for both consumption and investment $I_{n,t}$ in physical capital. The final good is produced from sectoral input bundles $\mathcal{Y}_{ns,t}$ using the non-homothetic CES aggregator (Sato 1977; Comin et al. 2021)

$$\begin{cases} \displaystyle\sum_{s=1}^{S} \bar{\chi}_{ns}^{\frac{1}{\eta}} \left(\frac{\mathcal{Y}_{n,t}}{L_n}\right)^{\frac{1-\eta}{\eta}(\varepsilon_s-1)} \left(\frac{\mathcal{Y}_{ns,t}}{\mathcal{Y}_{n,t}}\right)^{\frac{\eta-1}{\eta}} = 1, & \eta \neq 1, \\ \mathcal{Y}_{n,t} = \displaystyle\prod_{s=1}^{S} \mathcal{Y}_{ns,t}^{\bar{\chi}_{ns}}, & \eta = 1, \end{cases} \tag{2}$$

where $\eta > 0$ is the elasticity of substitution across sectors, $\bar{\chi}_{ns} > 0$ are country-sector demand shifters, and $\varepsilon_s > 0$ are sectoral expenditure elasticities that govern how the allocation of demand across sectors changes with per-capita usage of the final good. When $\varepsilon_s = 1$ for all s, the aggregator reduces to a standard CES across sectors. When $\eta = 1$, the aggregator reduces to a Cobb–Douglas across sectors. When $\eta < 1$ (gross complements), a higher ε_s makes sector s relatively more of a luxury: as per-capita consumption and investment grow, expenditure shifts toward high-ε_s sectors and away from low-ε_s ones.[5]

The sectoral input bundle $\mathcal{Y}_{ns,t}$ is a CES aggregator across all the locally supplied varieties, such that

$$\mathcal{Y}_{ns,t} = \left[\sum_{i=1}^{N} \int_{\omega \in \Omega_{is}} a_{ins,t}^{\frac{1}{\sigma_s}} y_{ins,t}(\omega)^{\frac{\sigma_s-1}{\sigma_s}} d\omega\right]^{\frac{\sigma_s}{\sigma_s-1}}, \tag{3}$$

with $\sigma_s > 1$ the elasticity of substitution across varieties in sector s, $y_{ins,t}(\omega)$ the quantity of variety ω from origin i used in country n, and $a_{ins,t} \geq 0$ a bilateral demand shifter that captures the

[5] Relative to Comin et al. (2021), whose non-homothetic aggregator operates over per-capita consumption, we let the sectoral shifters vary with per-capita usage of the final good $\mathcal{Y}_{n,t}/L_n$. Because the final good is used for both consumption and investment, this maintains the same sectoral composition in the consumption and investment bundles, which we adopt for simplicity.

appeal of origin-i goods in destination n, reflecting, for instance, language barriers and cultural differences.

Production of varieties. We associate each variety with a single firm. We assume that each country n has a measure one of symmetric firms in each sector, which supply their varieties both locally and abroad. Due to the symmetry of firm decisions, we index firms by their sector and origin country, omitting the variety index.

A firm from country i selling to country n in sector s produces output by combining its stock of non-rival assets, $x_{is,t}$, with rival inputs (labor and capital) hired in both the origin and destination countries. For convenience, we first define a local rival-input composite that combines chosen quantities of labor and capital into a single bundle:

$$g_{js,t}(l,k) \equiv \left(z_{js,t}\, l\right)^{\mu_s} k^{1-\mu_s}, \tag{4}$$

where l and k denote some chosen quantities of labor and capital in country j and sector s; $z_{js,t}$ is an exogenous labor-augmenting productivity shifter; and $\mu_s \in (0,1]$ is the labor share in the rival-input composite.

The output supplied by an i-based firm to market n is given by

$$y_{ins,t} = x_{is,t}^{\gamma_s}\, \tau_{ins,t}^{-1} \left[g_{is,t}\left(l^o_{ins,t},\, k^o_{ins,t}\right)\right]^{\alpha_s} \left[g_{ns,t}\left(l^d_{ins,t},\, k^d_{ins,t}\right)\right]^{1-\alpha_s}, \tag{5}$$

where $l^o_{ins,t}$, $k^o_{ins,t}$ are labor and capital hired in the origin country i; $l^d_{ins,t}$, $k^d_{ins,t}$ are labor and capital hired in the destination country n; γ_s is the non-rival asset intensity of sector s; $\alpha_s \in [0,1]$ is the intensity of rival inputs from the origin country in production relative to rival inputs from the destination country; and $\tau_{ins,t}$ is a bilateral "iceberg" trade friction that reflects diminished efficiency of supplying output from country i to country n. The asset stock $x_{is,t}$ is *non-rival* across destinations: the same stock – product designs, software, patents, brand equity – enters production for every market n without being depleted. Non-rival assets are, however, *excludable*: they are proprietary to the firm. The labor and capital inputs, by contrast, are rival and must be hired separately for each destination.

This production structure allows us to encompass a wide range of international trade modes

and types of services. For intuition, consider a few examples. The output of Netflix – a U.S.-based firm – to the Canadian market is highly intensive in its stock of non-rival assets (platform, software, content). At the same time, it depends also on rival inputs in the U.S. (e.g. customer support), and to a lesser degree, on rival inputs in Canada. Therefore, through the lens of our model, it would be characterized by high γ_s and high α_s. Starbucks, on the other hand, is arguably less intensive in non-rival assets and more intensive in rival inputs that are hired locally, near consumers, corresponding to low γ_s and low α_s. In the extreme case of $\alpha_s = 1$, international transactions are entirely cross-border, corresponding to Mode 1 of trade in services in the General Agreement on Trade in Services (GATS), with no local presence in the destination country. At the other extreme of $\alpha_s = 0$, international transactions are entirely through local affiliates, corresponding to Mode 3 of trade in services in GATS. In this latter case, international transactions reflect compensation for the usage of non-rival assets.

Notably, non-rival assets give rise to increasing returns to scale at the firm level, with returns to scale in sector s equal to $1 + \gamma_s$. This follows the standard specification in the growth literature, based on Romer's replication argument (Romer 1990; Jones 2022): if $x_{is,t}$ is truly non-rival, the firm should be able to double output by replicating its existing operation, hiring twice as many rival inputs while reusing the same stock of non-rival assets $x_{is,t}$. We will refer to γ_s as both the intensity of non-rival assets in production and the "scale elasticity".

Accumulation of non-rival assets. Firms can change the stock of non-rival assets by investing in new assets. The stock of non-rival assets evolves according to

$$x_{is,t+1} = (1 - \delta_x)\, x_{is,t} + g_{is,t}\left(l^x_{is,t},\, k^x_{is,t}\right), \tag{6}$$

where $l^x_{is,t}$ and $k^x_{is,t}$ are the rival factors – labor and capital – used to produce non-rival assets in origin i, and $\delta_x \in (0, 1)$ is the depreciation rate of non-rival assets.[6] For simplicity, we assume that new assets are produced from the same origin rival composite $g_{is,t}$ as in production. In the above examples of Netflix and Starbucks, $l^x_{is,t}$ and $k^x_{is,t}$ would capture the labor and capital used to create new content, software, designs, and recipes in the USA.

[6] In the quantitative model below, we add convex adjustment costs to the accumulation of non-rival assets (Section 5.3.1).

Capital accumulation. The physical capital stock (structures and machines) in country n follows a standard law of motion

$$K_{n,t+1} = (1 - \delta_K) K_{n,t} + I_{n,t} , \tag{7}$$

where $I_{n,t} \geq 0$ is real gross investment (in units of the final good) and $\delta_K \in (0, 1)$ is the depreciation rate. Capital is a country-level aggregate and all sectors rent from the same stock.

Resource constraints. We complete the description of the physical environment with the economy's resource constraints. The final good is allocated between aggregate consumption and gross investment in physical capital,

$$\mathcal{Y}_{n,t} = L_n C_{n,t} + I_{n,t} . \tag{8}$$

Total usage of labor and capital (for the production of local and foreign varieties, and for the creation of non-rival assets) cannot exceed the available amounts in each country:

$$L_i \geq \sum_s \left[l^x_{is,t} + \sum_n l^o_{ins,t} \right] + \sum_{j,s} l^d_{jis,t} , \tag{9}$$

$$K_{i,t} \geq \sum_s \left[k^x_{is,t} + \sum_n k^o_{ins,t} \right] + \sum_{j,s} k^d_{jis,t} . \tag{10}$$

2.2 Equilibrium

We now lay out the equilibrium solution of the model. We report the resulting equilibrium conditions here and provide their derivation in the appendix. To this end, we define the following prices: wages $w_{i,t}$, capital rental rates $r_{i,t}$, variety-level prices $p_{ins,t}$, sectoral price indices $P_{ns,t}$, and the price index of the final good $P_{n,t}$.

Households' problem. Taking prices as given, the representative household chooses consumption and investment, as well as the composition of sectors and varieties that make up the final good, to maximize lifetime utility (1) subject to the final-good technology (2)–(3), the capital law

of motion (7), and the budget constraint,

$$Y_{n,t} = E_{n,t} = P_{n,t}\left(L_n\, C_{n,t} + I_{n,t}\right)\,, \tag{11}$$

where we use $E_{n,t}$ for total expenditure and $Y_{n,t}$ for total national income. At this point we assume households are in financial autarky, such that total income equals total expenditure. We relax this assumption and explore implications for international financial flows in the quantitative section below (Section 5.3.1).

Expenditure shares. Cost minimization yields the following expenditure share of country n on sector s,

$$\chi_{ns,t} = \bar{\chi}_{ns}\left(\frac{P_{ns,t}}{P_{n,t}}\right)^{1-\eta}\left(\frac{\mathcal{Y}_{n,t}}{L_n}\right)^{(1-\eta)(\varepsilon_s-1)}, \qquad \sum_s \chi_{ns,t} = 1, \tag{12}$$

where $P_{n,t}$ is the price index of the final-good aggregator, defined implicitly by the adding-up condition $\sum_s \chi_{ns,t} = 1$ given the sectoral prices $\{P_{ns,t}\}$ and per-capita final-good consumption $\mathcal{Y}_{n,t}/L_n$. Total expenditure is $E_{n,t} = P_{n,t}\mathcal{Y}_{n,t}$. Notably, because expenditure shares vary with $\mathcal{Y}_{n,t}$, $P_{n,t}$ equals the average, but not marginal, expenditure per unit of the final good. Holding sectoral prices fixed,

$$\frac{\partial E_{n,t}}{\partial \mathcal{Y}_{n,t}} = \bar{\varepsilon}_{n,t} P_{n,t},$$

where $\bar{\varepsilon}_{n,t} \equiv \sum_s \chi_{ns,t}\varepsilon_s$ is the expenditure-share-weighted average expenditure elasticity. Under the empirically relevant case $\eta < 1$, the share of spending on sector s rises with its relative price index $P_{ns,t}/P_{n,t}$ and increases with per-capita usage of the final good $\mathcal{Y}_{n,t}/L_n$ when $\varepsilon_s > 1$.

Within the sectoral bundle, the share of spending on varieties from origin i – the bilateral trade share – takes the standard gravity structure:

$$S_{nis,t} = \frac{a_{ins,t}\, p_{ins,t}^{1-\sigma_s}}{\sum_{i'} a_{i'ns,t}\, p_{i'ns,t}^{1-\sigma_s}}\,, \tag{13}$$

and the sectoral price index is the CES aggregate of variety prices,

$$P_{ns,t} = \left(\sum_i a_{ins,t}\, p_{ins,t}^{1-\sigma_s}\right)^{\frac{1}{1-\sigma_s}}. \tag{14}$$

These shares and indices depend on the variety prices $p_{ins,t}$, which firms set in their optimization below.

Consumption and saving. The household's consumption-saving problem yields an Euler equation for capital accumulation, by which the marginal cost of installing one more unit of capital – equal to $\bar{\varepsilon}_{n,t}P_{n,t}$, since capital is denominated in units of the final good – equals its discounted return, the rental income plus the value of the undepreciated stock:

$$\bar{\varepsilon}_{n,t}\, P_{n,t} = \Lambda_{n,t,t+1}\left[r_{n,t+1} + (1-\delta_K)\,\bar{\varepsilon}_{n,t+1}\, P_{n,t+1}\right], \tag{15}$$

with $\Lambda_{n,t,t+1}$ denoting the household's stochastic discount factor,

$$\Lambda_{n,t,t+1} \equiv \beta\, \frac{P_{n,t}}{P_{n,t+1}}\, \frac{\bar{\varepsilon}_{n,t}}{\bar{\varepsilon}_{n,t+1}} \left(\frac{C_{n,t}}{C_{n,t+1}}\right)^{1/\psi}. \tag{16}$$

Firms' problem. The problem of a firm based in country i, sector s is to maximize the present value of profits from selling its variety in all destination markets n,

$$\max \sum_{t=0}^{\infty} \left(\prod_{\tau=0}^{t-1} \Lambda_{i,\tau,\tau+1}\right) \Bigg[\sum_n p_{ins,t}\, y_{ins,t} - w_{i,t}\left(\sum_n l^o_{ins,t} + l^x_{is,t}\right) - r_{i,t}\left(\sum_n k^o_{ins,t} + k^x_{is,t}\right) - \sum_n \left(w_{n,t}\, l^d_{ins,t} + r_{n,t}\, k^d_{ins,t}\right) \Bigg], \tag{17}$$

taking as given the path of factor prices it pays – $w_{i,t}, r_{i,t}$ at home and $w_{n,t}, r_{n,t}$ in each destination – the downward-sloping demand curve it faces in each market, $y_{ins,t} = a_{ins,t}\left(\frac{p_{ins,t}}{P_{ns,t}}\right)^{-\sigma_s} \mathcal{Y}_{ns,t}$, the law of motion for non-rival assets (6), and the stochastic discount factor (16) of its owner household in country i.

Firm pricing. The optimization problem yields a standard pricing rule of a constant markup over marginal cost,

$$p_{ins,t} = \frac{\sigma_s}{\sigma_s - 1}\, \frac{c_{ins,t}}{x^{\gamma_s}_{is,t}}, \tag{18}$$

where we define $c_{ins,t}$ as the unit cost of the required rival inputs to deliver one unit of output to

market n, and $c_{is,t}$ as the unit cost of the local rival-input composite in country i, given by

$$c_{ins,t} \equiv \tau_{ins,t} \frac{c_{is,t}^{\alpha_s} c_{ns,t}^{1-\alpha_s}}{\alpha_s^{\alpha_s}(1-\alpha_s)^{1-\alpha_s}}, \qquad c_{is,t} \equiv \frac{w_{i,t}^{\mu_s} r_{i,t}^{1-\mu_s}}{\mu_s^{\mu_s}(1-\mu_s)^{1-\mu_s} z_{is,t}^{\mu_s}}.$$

Firm input choices. Define $R_{ins,t} \equiv p_{ins,t}\, y_{ins,t}$ as the revenue of firm i in market n, and $R_{is,t} \equiv \sum_n R_{ins,t}$ as its total revenue. The first-order conditions for the rival inputs hired in the origin and destination countries then yield

$$\begin{aligned} l_{ins,t}^{o} &= \mu_s\, \alpha_s\, \tfrac{\sigma_s-1}{\sigma_s} \frac{R_{ins,t}}{w_{i,t}}, & k_{ins,t}^{o} &= (1-\mu_s)\, \alpha_s\, \tfrac{\sigma_s-1}{\sigma_s} \frac{R_{ins,t}}{r_{i,t}}, \\ l_{ins,t}^{d} &= \mu_s\, (1-\alpha_s)\, \tfrac{\sigma_s-1}{\sigma_s} \frac{R_{ins,t}}{w_{n,t}}, & k_{ins,t}^{d} &= (1-\mu_s)\, (1-\alpha_s)\, \tfrac{\sigma_s-1}{\sigma_s} \frac{R_{ins,t}}{r_{n,t}}. \end{aligned} \tag{19}$$

Investment in non-rival assets. The firm's remaining choice is how much to invest in its non-rival stock. The optimal accumulation rule equates the marginal cost of one more unit of the asset, $c_{is,t}$, to its discounted return,

$$c_{is,t} = \Lambda_{i,t,t+1} \left[\frac{\gamma_s(\sigma_s-1)}{\sigma_s} \frac{R_{is,t+1}}{x_{is,t+1}} + (1-\delta_x)\, c_{is,t+1} \right], \tag{20}$$

where $\frac{\gamma_s(\sigma_s-1)}{\sigma_s} \frac{R_{is,t+1}}{x_{is,t+1}}$ is the marginal operating profit of the firm in period $t+1$.

Concavity of the firm's problem. The increasing returns in the production function imply that firms have a downward sloping long-run supply schedule. Therefore, a high level of γ_s relative to demand curvature could make operating profits locally convex in the asset stock. We rule this out by imposing $\gamma_s(\sigma_s - 1) < 1$ for every sector – the slope of the demand curve that firms face is steeper than the slope of its long-run supply curve – which makes the firm's problem concave, and the first-order conditions above sufficient for optimality.[7]

[7] To see this, recall the residual inverse demand $p_{ins,t}(y) = P_{ns,t}(a_{ins,t} \mathcal{Y}_{ns,t}/y)^{1/\sigma_s}$, so $d\log p / d\log y = -1/\sigma_s$. Now scale the firm's destination quantities in fixed proportions by q. Producing this scale with stock x requires rival inputs proportional to q/x^{γ_s}, while the flow cost of building or maintaining the stock is proportional to x. Minimizing $Aq/x^{\gamma_s} + Bx$ gives $x \propto q^{1/(1+\gamma_s)}$, so long-run marginal cost falls with log slope $-\gamma_s/(1+\gamma_s)$. The demand curve is steeper exactly when $1/\sigma_s > \gamma_s/(1+\gamma_s)$, which is equivalent to $\gamma_s(\sigma_s - 1) < 1$.

National income. Summing operating profits and factor payments across all markets, national income in country i is

$$Y_{i,t} = \sum_s \sum_n \left[\left(\frac{1}{\sigma_s} + \alpha_s \frac{\sigma_s - 1}{\sigma_s} \right) R_{ins,t} + (1 - \alpha_s) \frac{\sigma_s - 1}{\sigma_s} R_{nis,t} \right], \tag{21}$$

where the first term collects the profits and origin factor payments of i's firms across all markets, and the second the destination factor payments its labor and capital earn from foreign firms operating in i.

Equilibrium. Given initial stocks $\{x_{is,0}, K_{i,0}\}$, labor endowments $\{L_n\}$, sequences of fundamentals $\{z_{is,t}, a_{ins,t}, \tau_{ins,t}\}$, and world GDP $\sum_i Y_{i,t}$ as the numeraire, a competitive equilibrium is a sequence of prices $\{w_{i,t}, r_{i,t}, p_{ins,t}, P_{ns,t}, P_{n,t}\}$ and allocations $\{C_{n,t}, I_{n,t}, \mathcal{Y}_{n,t}, \mathcal{Y}_{ns,t}, y_{ins,t}, l^o_{ins,t}, k^o_{ins,t}, l^d_{ins,t}, k^d_{ins,t}, l^x_{is,t}, k^x_{is,t}, x_{is,t}, K_{n,t}\}$ such that, given prices: (i) households maximize utility (1) subject to their budget constraint (11); (ii) firms maximize the present value of profits (17) subject to the production technology (5) and the demand they face; (iii) the capital and non-rival stocks evolve according to (7) and (6); (iv) the final-good, labor, and capital markets clear, so that (8) and the feasibility constraints (9)–(10) hold with equality; (v) income equals expenditure, $Y_{n,t} = E_{n,t}$ (11); and (vi) the transversality conditions hold.[8] A *steady state* is an equilibrium in which all quantities and prices are constant and the stochastic discount factor equals β in every country.

Efficiency. Despite the increasing returns generated by non-rival assets, the decentralized equilibrium is efficient up to the standard distortions that markups create:

Proposition 1 (Efficiency). *If $\sigma_s = \sigma$ and $\mu_s = 1$ for every sector, the steady-state equilibrium is Pareto efficient. Otherwise it is inefficient, and efficiency is restored by subsidies that offset the markups $\sigma_s/(\sigma_s - 1)$, at rates that depend on the elasticities σ_s but not on the scale elasticities γ_s.*

Proof. See Appendix A.6. □

When either condition fails, the allocation of resources between sectors, and between consumption and investment, is distorted, but it can be corrected using subsidies that depend on σ_s.

[8] The terminal conditions on the two stocks are $\lim_{T\to\infty} \left(\prod_{t=0}^{T-1} \Lambda_{n,t,t+1} \right) \bar{\varepsilon}_{n,T} P_{n,T} K_{n,T+1} = 0$ for capital and $\lim_{T\to\infty} \left(\prod_{t=0}^{T-1} \Lambda_{i,t,t+1} \right) c_{is,T} x_{is,T+1} = 0$ for non-rival assets.

These corrections do not depend on the scale elasticities γ_s, which do not introduce a distortion, since firms internalize the decisions on non-rival asset accumulation.

Remark: relation to models of external economies of scale. In the special case $\eta = 1$, $\mu_s = \alpha_s = 1$ – Cobb–Douglas demand across sectors, no capital, and purely cross-border supply – the steady state of the model reduces to a multi-sector static trade model with sectoral economies of scale, and it admits the same mathematical structure as the gravity models with external scale economies studied by Kucheryavyy et al. (2023), Lashkaripour and Lugovskyy (2023), and Bartelme et al. (2025). However, unlike in those models, where the scale economies are external to firms and left unmodeled, here returns to scale arise from firms' accumulation of non-rival assets and are fully internalized. This has three implications. First, the scale elasticities γ_s can be measured from the firm's optimization problem, as we do in Section 4. Second, the heterogeneity in scale elasticities does not by itself create inefficiency, as discussed above. Third, the condition that ensures a concave firm problem, $\gamma_s(\sigma_s - 1) < 1$ for all s, coincides with the condition from Kucheryavyy et al. (2023) for the uniqueness of the equilibrium.

3 Comparative Statics

This section studies the patterns of trade and the gains from trade in our model: which forces might generate the specialization patterns observed in the data, and how the presence of non-rival assets reshapes the gains from trade. As an intermediate step, we first present a log-linearized version of the steady-state system that supports the comparative-statics exercises below, in the tradition of Jones (1965).

3.1 Linearized steady-state system

To trace how small shocks to fundamentals move the steady-state equilibrium, we linearize the equilibrium conditions of Section 2.2 around the steady state. The resulting system appears in compact form below, and the full linearization is in Appendix B.1. Throughout, $\tilde{v} \equiv d \log v$ denotes the log-deviation of any positive variable v from its baseline value, and we consider shocks to sectoral productivities $\tilde{z}_{is}$, bilateral trade frictions $\tilde{\tau}_{ins}$, demand shifters $\tilde{\tilde{\chi}}_{ns}$, and population $\tilde{L}_n$.

For convenience, we define the following steady-state shares, which appear as coefficients in the linearized system. Let $T_{ins} \equiv R_{ins}/R_{is}$ be the share of market n in the revenue of firm (i,s), and $\bar{\varepsilon}_n \equiv \sum_s \chi_{ns}\varepsilon_s$ the average expenditure elasticity in country n. $\phi_s \equiv \frac{\delta_x \gamma_s(\sigma_s-1)}{(\rho+\delta_x)\sigma_s}$ is the steady-state share of revenue that a firm spends on the creation of non-rival assets, with $\rho \equiv 1/\beta - 1$. Three shares of national income collect the payments generated by sales: $\Phi^o_{is} \equiv (\alpha_s \frac{\sigma_s-1}{\sigma_s} + \phi_s)\frac{R_{is}}{Y_i}$ (origin-country factor payments and spending on non-rival assets out of home firms' revenue), $\Phi^\pi_{is} \equiv (\frac{1}{\sigma_s} - \phi_s)\frac{R_{is}}{Y_i}$ (home firms' profits net of that spending), and $\Phi^d_{is} \equiv (1-\alpha_s)\frac{\sigma_s-1}{\sigma_s}\chi_{is}$ (destination-country factor payments on total sales in the home market). Finally, $\lambda^L_i \equiv w_i L_i/Y_i$ and $\lambda^K_i \equiv r_i K_i/Y_i$ are the labor and capital shares of income, and $s^C_n \equiv L_n C_n/\mathcal{Y}_n$ is the consumption share of final-good use.

Proposition 2 (Log-linearized equilibrium). *Around the steady state, the response of equilibrium prices and quantities to small shocks $(\tilde{z}_{is}, \tilde{\tau}_{ins}, \tilde{\tilde{\chi}}_{ns}, \tilde{L}_n)$ solves the linear system*

$$\tilde{p}_{ins} = \mu_s\big[\alpha_s(\tilde{w}_i - \tilde{z}_{is}) + (1-\alpha_s)(\tilde{w}_n - \tilde{z}_{ns})\big] + (1-\mu_s)\big[\alpha_s\tilde{r}_i + (1-\alpha_s)\tilde{r}_n\big] - \gamma_s\tilde{x}_{is} + \tilde{\tau}_{ins}, \tag{22}$$

$$\tilde{x}_{is} = \tilde{R}_{is} - \mu_s\tilde{w}_i - (1-\mu_s)\tilde{r}_i + \mu_s\tilde{z}_{is}, \tag{23}$$

$$\tilde{\chi}_{ns} = \tilde{\tilde{\chi}}_{ns} + (1-\eta)\big(\textstyle\sum_i S_{nis}\tilde{p}_{ins} - \tilde{P}_n\big) + (1-\eta)(\varepsilon_s - 1)\big(\tilde{Y}_n - \tilde{P}_n - \tilde{L}_n\big), \tag{24}$$

$$\bar{\varepsilon}_n\tilde{P}_n = \tfrac{1}{1-\eta}\textstyle\sum_s \chi_{ns}\tilde{\tilde{\chi}}_{ns} + \sum_s\sum_i \chi_{ns}S_{nis}\tilde{p}_{ins} + (\bar{\varepsilon}_n - 1)\big(\tilde{Y}_n - \tilde{L}_n\big), \tag{25}$$

$$\tilde{R}_{is} = \textstyle\sum_n T_{ins}\big[\tilde{\chi}_{ns} + \tilde{Y}_n + (1-\sigma_s)\big(\tilde{p}_{ins} - \sum_j S_{njs}\tilde{p}_{jns}\big)\big], \tag{26}$$

$$\tilde{w}_i = \tfrac{1}{\lambda^L_i}\textstyle\sum_s \mu_s\big[\Phi^o_{is}\tilde{R}_{is} + \Phi^d_{is}\big(\tilde{\chi}_{is} + \tilde{Y}_i\big)\big] - \tilde{L}_i, \tag{27}$$

$$\tilde{r}_i = \tilde{P}_i + \tfrac{1}{\bar{\varepsilon}_i}\textstyle\sum_s \chi_{is}\varepsilon_s\tilde{\chi}_{is}, \tag{28}$$

$$\tilde{Y}_i = \textstyle\sum_s \big(\Phi^o_{is} + \Phi^\pi_{is}\big)\tilde{R}_{is} + \sum_s \Phi^d_{is}\big(\tilde{\chi}_{is} + \tilde{Y}_i\big), \tag{29}$$

together with a numeraire $\sum_i Y_i\tilde{Y}_i = 0$. After solving it, the capital stock and consumption follow recursively from $\tilde{K}_i = \frac{1}{\lambda^K_i}\sum_s(1-\mu_s)[\Phi^o_{is}\tilde{R}_{is} + \Phi^d_{is}(\tilde{\chi}_{is}+\tilde{Y}_i)] - \tilde{r}_i$ and $\tilde{C}_n = (\tilde{Y}_n - \tilde{P}_n - (1-s^C_n)\tilde{K}_n)/s^C_n - \tilde{L}_n$.[9]

Proof. See Appendix B.1. □

[9] When $\eta = 1$, the aggregator (2) is Cobb–Douglas and the parameters ε_s play no role. The system then applies with $\varepsilon_s = 1$, so that (24) reads $\tilde{\chi}_{ns} = \tilde{\tilde{\chi}}_{ns}$, (25) reads $\tilde{P}_n = \sum_s \chi_{ns}\tilde{P}_{ns}$ with share shocks satisfying $\sum_s \chi_{ns}\tilde{\tilde{\chi}}_{ns} = 0$, and (28) reads $\tilde{r}_n = \tilde{P}_n$. All results below that impose $\eta = 1$ use the system in this form.

3.2 Patterns of trade

A robust feature of the data is that revealed comparative advantage in modern services correlates strongly with income per capita, but only weakly with total population. To assess which forces in the model can generate this pattern, we ask how a country's growth – a uniform rise in labor productivity across all its sectors, $\tilde{z}_n$ – reshapes its specialization, and contrast it with the effect of a rise in population $\tilde{L}_n$. Specialization is measured by the Balassa index of revealed comparative advantage (RCA), the share of sector s in country n's exports relative to that sector's share in world exports,

$$RCA_{ns} \equiv \frac{X_{ns} \big/ \sum_k X_{nk}}{\sum_i X_{is} \big/ \sum_{i,k} X_{ik}}, \qquad X_{ns} \equiv \sum_{m \neq n} R_{nms},$$

where X_{ns} is country n's exports in sector s. A full characterization of the response of specialization to the above shocks involves inverting the system of Proposition 2 and results in a complex set of expressions. However, key insights can be obtained by examining a series of special cases. We start by studying how a country's growth reshapes its specialization in an environment with homothetic demand, $\eta = 1$.

Proposition 3 (Specialization under homothetic demand)**.** *Consider a country n that experiences a uniform rise in labor productivity, $\tilde{z}_{ns} = \tilde{z}_n$ for all s, or a rise in population $\tilde{L}_n$, with all other fundamentals unchanged, and suppose $\eta = 1$. Then:*

(a) *The response of specialization to a uniform labor productivity shock is equivalent to the response to an equal population shock: $\widetilde{RCA}_{ns}$ depends on $\tilde{z}_n$ and $\tilde{L}_n$ only through their sum $\tilde{z}_n + \tilde{L}_n$.*

(b) *Scale effects: Suppose $\mu_k = \alpha_k = 1$ and $\sigma_k = \sigma > 1$ for all k, baseline sector expenditure shares are common across countries, $\chi_{ik} = \chi_k$, and baseline trade shares are common across sectors and symmetric across countries, $S_{iik} = H \in (0,1)$ and $S_{ijk} = \frac{1-H}{N-1}$ for $j \neq i$. Then a positive productivity or population shock raises $\widetilde{RCA}_{ns}$ relative to $\widetilde{RCA}_{ns'}$ if and only if $\gamma_s > \gamma_{s'}$, for every pair of sectors. It raises $\widetilde{RCA}_{ns}$ itself if and only if γ_s exceeds a weighted average of the other sectors' scale elasticities,*

$$\sum_{k \neq s} \frac{\chi_k (\gamma_s - \gamma_k)}{1 - \Xi \gamma_k} > 0, \tag{30}$$

where $\Xi \equiv (\sigma-1)\left[1-\left(\frac{NH-1}{N-1}\right)^2\right]$. *In particular, the shock raises the RCA of the sector with the highest scale elasticity and lowers that of the sector with the lowest.*

(c) *Home market effects: Under the assumptions of part* (b)*, consider an exogenous expenditure-share shock in country* n *toward sector* s*,* $\tilde{\tilde{\chi}}_{ns} = d > 0$*. Let* $\bar{\gamma}_{-s}$ *be the weighted average of the other sectors' scale elasticities,*

$$\bar{\gamma}_{-s} \equiv \sum_{k\neq s} \frac{\chi_k \gamma_k}{1-\Xi\gamma_k} \Big/ \sum_{k\neq s} \frac{\chi_k}{1-\Xi\gamma_k}. \tag{31}$$

Then the sign of $\widetilde{RCA}_{ns}$ *equals the sign of*

$$(1-\chi_s)\left(1+\tfrac{\delta_x}{\rho+\delta_x}\bar{\gamma}_{-s}\right)\left[NH(\sigma-1)\gamma_s-1\right] + \chi_s\left(1+\tfrac{\delta_x}{\rho+\delta_x}\gamma_s\right)\left[NH(\sigma-1)\bar{\gamma}_{-s}-1\right]. \tag{32}$$

A sufficient condition for the demand shock to raise $\widetilde{RCA}_{ns}$ *is that* $\gamma_k > 1/[NH(\sigma-1)]$ *for every sector.*[10]

Proof. See Appendix B.2. □

Part (a) of Proposition 3 shows that under homothetic demand a productivity rise and a population rise have identical effects on specialization, as measured by revealed comparative advantage. Part (b) shows that non-rival assets generate scale effects: a rise in a country's size or labor productivity raises its RCA in sectors with higher scale elasticities (Graham 1923; Ethier 1982). This is one rationale for why richer countries might specialize in high-γ_s sectors such as modern services. But because size and productivity act identically here, part (b) equally predicts that *larger* countries specialize in these sectors, a pattern absent from the data, where specialization tracks income per capita rather than country size.

Part (c) highlights a different way in which non-rival assets could shape specialization, through home-market effects (Krugman 1980): with increasing returns, even when the scale elasticity is common across sectors, a rise in a sector's home demand increases the country's RCA in that sector, provided home bias is sufficiently strong and/or the scale elasticity is sufficiently high.

[10] Parts (b) and (c) extend to common $\mu_k = \mu \in (0,1]$ and $\alpha_k = \alpha \in (0,1]$. With $\mu = 1$ all formulas hold verbatim for any α. With $\mu < 1$, the weights $1 + \frac{\delta_x}{\rho+\delta_x}\gamma_k$ in Proposition 7 and in (32) are replaced by the capital-augmented weights given in Appendix B.2. The pairwise rule, condition (30), and the threshold $1/[NH(\sigma-1)]$ are unchanged.

Expression (32) further shows that when scale elasticities differ across sectors, the home-market effect is stronger when the targeted sector is initially small (low baseline χ_s) and characterized by greater non-rival intensity (higher γ_s). This points to a demand-based mechanism through which rich countries might specialize in modern services, but it also requires demand for such services to vary systematically with income per capita. A natural explanation is the non-homotheticity of demand, as highlighted in the literature on structural transformation.

Proposition 4 (Specialization under non-homothetic demand)**.** *Let $\eta < 1$ and take two countries $\{n, n'\}$ and two sectors $\{s, s'\}$ with $\mu_k = \alpha_k = 1$, common $\sigma > 1$, common baseline sector expenditure shares $\chi_{nk} = \chi_{n'k} = \chi_k$, and common own trade shares $S_{nnk} = S_{n'n'k} = H \in (\frac{1}{2}, 1)$. Let $\varepsilon_s > \varepsilon_{s'}$, so that s is the high-income-elasticity sector, and consider shocks to productivity and population in country n.*

(a) *If $\gamma_s = \gamma_{s'} = 0$, then*

$$\widetilde{RCA}_{ns} = \frac{\chi_{s'}(1-\eta)(\varepsilon_s - \varepsilon_{s'})}{2\bar{\varepsilon}_n\big[1 + 2H(\sigma-1)\big]}\Big[-(2H\sigma - 1)\,\tilde{z}_n + 2(1-H)\,\tilde{L}_n\Big].$$

(b) *If $\gamma_s = \gamma_{s'} = \gamma \geq 0$, then*

$$\widetilde{RCA}_{ns} = \frac{\chi_{s'}(1-\eta)(\varepsilon_s - \varepsilon_{s'})[2H(\sigma-1)\gamma - 1]}{2\bar{\varepsilon}_n[1 + 2H(\sigma-1)]\big\{1 + \gamma[(1-\eta) - 4H(1-H)(\sigma-\eta)]\big\}} \times \Big[(1+\gamma)(2H\sigma - 1)\,\tilde{z}_n + \big(\gamma(2H\sigma - 1) - 2(1-H)\big)\tilde{L}_n\Big].$$

Hence the effect following a productivity shock is positive if and only if $\gamma > 1/[2H(\sigma-1)]$, and it is negative below this threshold.

Proof. See Appendix B.2. □

Part (a) of Proposition 4 shows that non-homothetic demand breaks the equivalence between labor productivity and population, and the two now move specialization in opposite directions. On its own, however, this force is an unlikely account of why rich countries specialize in modern services, because it pushes the richer country to specialize in *low* income-elasticity sectors. A country that grows richer per capita consumes and imports more of the high-ε good at home, so its RCA there falls. A more populous, poorer country consumes less and exports relatively more.

Non-homotheticity alone therefore makes rich countries relative importers of high-ε sectors such as services.

Part (b) shows that this reverses once non-homotheticity is combined with strong enough non-rival intensity. When γ exceeds the threshold $1/[2H(\sigma-1)]$, a rise in labor productivity specializes the country *into* its high-income-elasticity sectors. Here the home-market force of Proposition 3 meets the rise in per-capita income: the extra demand that a richer country directs to high-ε sectors is met at home, leading to more accumulation of non-rival assets and a larger advantage in foreign markets. In the data, we find that high-ε_s sectors such as modern services are on average also higher-γ_s sectors, which further amplifies this effect.

It is worth contrasting these results with the structural transformation literature, which focuses on why the consumption and production of services are more prevalent in rich countries. To additionally generate the *specialization* of rich countries in services, non-homothetic demand alone is insufficient (and, by itself, would imply the opposite pattern). Instead, specialization requires the interaction of non-homothetic demand with scale effects.

3.3 Gains from trade

We now turn to how non-rival assets shape the gains from trade, beginning with the gains relative to autarky, in the spirit of Arkolakis et al. (2012). We again begin by focusing on the case of homothetic demand ($\eta = 1$), which isolates the independent role of non-rival assets and delivers a sufficient-statistic formula for the gains from trade relative to autarky. Appendix B.4 (Proposition 8) presents the more general expression under non-homothetic demand, which, while highlighting the key forces at play, cannot be evaluated from observed trade shares alone because it requires counterfactual autarky expenditure shares.

We denote $R_{ns} = \sum_m R_{nms}$ to be the total revenue of country n's firms in sector s across destinations, $R_n = \sum_s R_{ns}$, and $\vartheta_{ns} \equiv R_{ns}/R_n$ the sectoral revenue composition, and $\ell_n \equiv w_n L_n/R_n$ the labor share of revenue. Write $\hat{X} \equiv X/X^A$ for the ratio of the trade steady-state value to its autarky counterpart.

Proposition 5 (Gains from trade relative to autarky under homothetic demand)**.** *Suppose* $\eta = 1$ *and* $\sum_s \chi_{ns}(1-\mu_s)(1+\gamma_s) < 1$*, and let the autarky steady state be the one obtained by sending*

$\tau_{ins} \to \infty$ *for all* $i \neq n$. *Define the capital multiplier*

$$\mathcal{M}_n \equiv \frac{1}{1 - \sum_s \chi_{ns}(1-\mu_s)(1+\gamma_s)}, \tag{33}$$

which exceeds one whenever production uses capital ($\mu_s < 1$). *Then the steady-state gains from trade relative to autarky are a function of model elasticities and observed trade and sectoral shares,*

$$\begin{aligned} \ln \hat{C}_n = & \underbrace{-\mathcal{M}_n \sum_s \frac{\chi_{ns}}{\sigma_s - 1} \ln S_{nns}}_{\text{trade openness}} \\ & + \underbrace{\mathcal{M}_n \sum_s \chi_{ns}\gamma_s \ln \frac{\vartheta_{ns}}{\chi_{ns}}}_{\text{reallocation to high-}\gamma_s\text{ sectors}} \\ & + \underbrace{\ln \frac{s_n^C}{s_n^{C,A}} + \ln \frac{Y_n}{R_n} - \mathcal{M}_n\Big(\sum_s \chi_{ns}\mu_s(1+\gamma_s)\Big) \ln \frac{\ell_n}{\ell_n^A}}_{\text{transmission from real revenue to consumption}}. \end{aligned} \tag{34}$$

The ratios Y_n/R_n, ℓ_n, *and* s_n^C *are explicit functions of* $\{\vartheta_{ns}, \chi_{ns}\}$ *and parameters, given in Appendix B.3, and autarky values follow by setting* $\vartheta_{ns} = \chi_{ns}$.

Proof. See Appendix B.3. □

The first line in Equation (34) is the standard multi-sector version of the trade-openness formula from Arkolakis et al. (2012) (ACR), now scaled by the *capital multiplier* $\mathcal{M}_n$. The multiplier arises because capital is an accumulable factor whose long-run supply is elastic (Baqaee and Malmberg 2025; Kleinman et al. 2026). Even without non-rival assets ($\gamma_s = 0$), the static gain from lower trade costs raises real income, and capital keeps accumulating until its return is driven back to the required rate. The larger capital stock raises income again, and this capital-deepening loop scales the gain by $1/[1 - \sum_s \chi_{ns}(1-\mu_s)]$, with $1-\mu_s$ the capital share in sector s. This factor is the multi-sector counterpart of the long-run capital amplification in Baqaee and Malmberg (2025). Non-rival assets make the multiplier unambiguously larger: because they are produced with capital, each round of capital deepening also expands the resources available for producing them, which raises productivity and real income further, leading to more accumulation of capital

and non-rival assets. The per-round feedback becomes $(1-\mu_s)(1+\gamma_s)$ and the multiplier grows to $\mathcal{M}_n$. Absent capital ($\mu_s = 1$), the multiplier disappears, $\mathcal{M}_n = 1$: with labor as the only input and in fixed supply, trade cannot expand the resources available for producing non-rival assets, and can only reallocate them across sectors.

The middle line captures such reallocation. In autarky a country's production mix coincides with its spending mix, $\vartheta_{ns}^A = \chi_{ns}$. Trade allows the two to part ways, and the ratio ϑ_{ns}/χ_{ns} measures specialization, exceeding one in sectors whose revenue share runs ahead of the country's spending share. Because non-rival assets make a sector's productivity increasing in its scale, this specialization contributes $\gamma_s \ln(\vartheta_{ns}/\chi_{ns})$ to consumption for each sector, valued at its expenditure weight χ_{ns}. This term increases with the expenditure-weighted covariance between the log specialization ratios and the scale elasticities, and is positive when trade tilts production toward high-γ sectors. If high-income countries specialize in high-γ sectors, as suggested by the analysis above and as we find in the data, the gains from trade are tilted toward them. Finally, the capital multiplier scales this term up as well, since the productivity gains from reallocation feed the same capital-deepening loop.

The last line converts the real-sales gains of the first two terms into consumption gains. Real sales and consumption move together in a standard trade model, so this conversion is trivial there. They diverge here for two reasons: physical capital absorbs part of any real-sales gain as reinvestment to sustain a larger stock, and commercial presence separates a country's sales from its national income, since affiliate sales pay factors abroad while domestic affiliates of foreign firms pay factors at home. Like the reallocation term, this line varies with trade only through the specialization ratios ϑ_{ns}/χ_{ns}. The consumption share $s_n^C = 1 - \frac{\delta_K}{\rho+\delta_K}\lambda_n^K$ nets out capital maintenance: a permanently larger stock must be permanently replaced, so a fraction $\delta_K/(\rho+\delta_K)$ of each unit of capital income goes to replacement investment and consumption keeps only the net return. The income-to-revenue ratio Y_n/R_n reflects the mode of delivery, since a unit of revenue in sector s generates domestic income $\alpha_s + (1-\alpha_s)/\sigma_s$: it rises when specialization tilts toward sectors supplied cross-border or with high margins, and falls when revenue shifts toward affiliate-served sectors, whose factor payments accrue abroad. Finally, the labor share of revenue ℓ_n prices the economy's fixed input: when specialization tilts factor payments toward labor, the wage rises relative to revenue and enters each sector's costs with weight $\mu_s(1+\gamma_s)$ – rival production plus

the labor embodied in non-rival assets – compounded by the multiplier $\mathcal{M}_n$.

The formula nests familiar benchmarks. Without non-rival assets, capital, or commercial presence ($\gamma_s = 0$, $\mu_s = 1$, $\alpha_s = 1$) and common σ, it collapses to the multi-sector variant of Arkolakis et al. (2012), $\hat{C}_n = \prod_s S_{nns}^{-\chi_{ns}/(\sigma-1)}$. Without capital or commercial presence ($\mu_s = 1$, $\alpha_s = 1$) but with heterogeneous scale elasticities, the multiplier is one and the formula is similar to the one obtained from multi-sector models with sectoral external economies of scale.

This comparison speaks only to the gains relative to autarky, not to the welfare effect of a small change in trade frictions in the current equilibrium. For the latter we turn to the linearized system of Proposition 2. While the general solution does not admit a comparably simple expression, similar insights can be obtained from a uniform global reduction in trade frictions, $\tilde{\tau}_{ins} = -d_s$ for $n \neq i$.

Proposition 6 (Reduction of trade frictions with symmetric countries)**.** *Suppose the baseline steady state is symmetric, with common fundamentals across countries, and consider a uniform reduction in trade frictions, $\tilde{\tau}_{ins} = -d_s$ for $i \neq n$.*

(a) *Homothetic demand with non-rival assets: If $\eta = 1$, then, provided $\sum_s \chi_s(1+\gamma_s)(1-\mu_s) < 1$,*

$$\tilde{C}_n = \mathcal{M}_n \sum_s \chi_s(1 - S_{nns})d_s.$$

(b) *Non-homothetic demand without non-rival assets: If $\gamma_s = 0$ for all s, $\mu_s = 1$, and σ is common across sectors, then*

$$\tilde{C}_n = \frac{\sum_s \chi_s(1 - S_{nns})d_s}{\bar{\varepsilon}_n}.$$

Proof. See Appendix B.5. □

Part (a) of the proposition shows that the amplification through the capital multiplier is not specific to the autarky comparison: the same denominator scales the gains from a small trade shock around an observed trade steady state. Part (b) instead turns off non-rival assets and capital to isolate the role of non-homothetic preferences in the symmetric case. The only difference relative to homothetic demand is that the gains are deflated by the average expenditure elasticity $\bar{\varepsilon}_n$, which exceeds one when spending tilts toward high-ε_s sectors.

In sum, the analysis in this section highlights two margins through which the accumulation of non-rival assets shapes the gains from trade. First, it raises the average gains, through a larger capital multiplier. Second, it raises the relative gains of countries that specialize in high-γ sectors in the trade equilibrium. In the next Section, we explore quantiatively how important are these forces.

4 Data and Quantification

We now take the model to the data, to quantify how the accumulation of non-rival assets shapes the patterns of trade, the gains from trade, and the dynamic adjustment to global shocks. We map the model to 72 countries and a rest-of-the-world aggregate, for a total of $N = 73$, and to $S = 23$ broad sectors spanning goods and services. Section 4.1 describes the data. Section 4.2 constructs the bilateral sales matrix and the mode-of-delivery parameters α_s. Section 4.3 measures the non-rival intensities γ_s, our central parameters. Section 4.4 completes the calibration.

4.1 Data

We briefly describe our data sources, leaving details to Appendix C.

Trade flows. We use bilateral data on goods trade from BACI (Gaulier and Zignago 2010) and on services trade from the IMF Bilateral Trade in Services database (Loungani et al. 2017; Li et al. 2025).

Multinational activity. We use data on the output, exports, and value added of foreign-owned affiliates and domestically owned firms, by country and sector, from the OECD Analytical AMNE tables.

Occupations and tasks. To measure the intensity of non-rival assets, we use data on the task content of occupations from the O*NET database and on employment and wages by occupation and industry from the BLS Occupational Employment and Wage Statistics.

Other sources. We use population from the Penn World Table (Feenstra et al. 2015), sectoral capital shares from input-output accounts, and data on intellectual-property investment in the U.S. from the BEA fixed-asset accounts.

4.2 Trade Flows and Modes of Delivery

A key empirical object in the quantification is the matrix of bilateral sales in each sector, R_{ins}: the sales in destination n of firms from origin i in sector s. In the model, this object captures both direct cross-border exports and sales through affiliates in the destination country. In particular, when $\alpha_s = 1$, all of R_{ins} is direct exports (or mode 1 sales in the context of trade in services), and when $\alpha_s = 0$, all of R_{ins} is sales through affiliates (or mode 3 sales). In the data, we measure R_{ins} as the sum of direct exports and affiliate sales.[11] We then compute α_s, the intensity of output in origin rival factors, as the ratio of direct exports to total sales in each sector.[12]

We measure each origin-sector's total direct exports from the OECD Analytical AMNE tables, and distribute them across destinations using bilateral export shares, from BACI for goods and from the IMF Bilateral Trade in Services database for services. We measure affiliate sales by owner and host country from the OECD Analytical AMNE bilateral affiliate output data, netting out affiliate exports so that they record sales into the host market. Domestic sales on the diagonal are the output of domestically owned firms net of their exports. Finally, because we do not explicitly model intermediate inputs, we translate bilateral sales to a value-added basis, scaling each flow by the value-added-to-output ratio of the relevant sector. Appendix C.2 reports the construction of the sales matrices in full.

[11] Modes 1 and 3 capture around 90% of the global sales of services. The remaining modes of delivery – mode 2 (consumption abroad) and mode 4 (presence of natural persons) – require modeling the movement of people across borders, from which we abstract.

[12] To see how the intermediate case $0 < \alpha_s < 1$ maps to the data, think of the firm as a chain of three units. An origin unit in i produces an intermediate from origin rival factors and these assets, $x_{is,t}^{\gamma_s} g_{is,t}(l_{ins,t}^o, k_{ins,t}^o)$, and ships it to n; a destination unit in n produces a second intermediate from local rival factors and the same assets, $x_{is,t}^{\gamma_s} g_{ns,t}(l_{ins,t}^d, k_{ins,t}^d)$; and a final assembler in n combines the two to deliver the variety, $y_{ins,t} = \tau_{ins,t}^{-1}\left[x_{is,t}^{\gamma_s} g_{is,t}(l_{ins,t}^o, k_{ins,t}^o)\right]^{\alpha_s}\left[x_{is,t}^{\gamma_s} g_{ns,t}(l_{ins,t}^d, k_{ins,t}^d)\right]^{1-\alpha_s}$. Because the firm's assets are non-rival, the stock $x_{is,t}^{\gamma_s}$ is available to both intermediate units at once. In the data, the origin unit's sales to the final assembler would be recorded as direct cross-border exports and the destination unit's as sales through a foreign affiliate. The total sales of the final assembler, R_{ins}, would equal the sum of payments to the two intermediate units, that is, the sum of the two flows; and the ratio of direct exports to total sales would equal α_s.

4.3 Measuring the Intensity of Non-Rival Assets (γ_s)

The central parameters for our analysis are the sectoral non-rival intensities γ_s, which govern how much firms in each sector produce and accumulate non-rival assets. We recover them from a simple observation: a sector's non-rival intensity is reflected in the share of its costs that firms devote to producing non-rival assets.

To see this, let $H_{is,t}$ denote the share of the sector-s wage bill in country i paid to workers who produce non-rival assets,

$$H_{is,t} \equiv \frac{w_{i,t}\, l^x_{is,t}}{w_{i,t}\left(l^x_{is,t} + \sum_n l^o_{ins,t} + \sum_j l^d_{jis,t}\right)},$$

where the denominator is the total sector-s labor compensation in country i. This payroll share mixes country i's own firms with the affiliates of foreign firms operating in i, so we define a companion object $\mathcal{H}_{is,t}$ that re-bases the non-rival payroll onto the revenue of i's own firms,

$$\mathcal{H}_{is,t} \equiv \frac{H_{is,t}}{1 - H_{is,t}} \left(\alpha_s + (1-\alpha_s)\frac{\sum_j R_{jis,t}}{R_{is,t}}\right),$$

where $R_{is,t} = \sum_n R_{ins,t}$ is the revenue of country i's firms in sector s and $\sum_j R_{jis,t}$ is total spending on sector s in i.

We can then rewrite the firm's accumulation rule for non-rival assets (20) to express the non-rival intensity γ_s as a function of the affiliate-adjusted non-rival share of payroll, $\mathcal{H}_{is,t}$, sectoral revenue, and the growth of the non-rival asset stock:

$$\gamma_s = (1+\rho_{t+1})\,\frac{R_{is,t}}{R_{is,t+1}}\,\frac{x_{is,t+1}}{x_{is,t+1} - (1-\delta_x)x_{is,t}}\,\mathcal{H}_{is,t} - \frac{(1-\delta_x)\,x_{is,t+1}}{x_{is,t+2} - (1-\delta_x)x_{is,t+1}}\,\mathcal{H}_{is,t+1}, \tag{35}$$

where ρ_{t+1} is the rate of return between periods t and $t+1$.

A simpler expression obtains under the further assumption that the economy is in steady state: γ_s is then proportional to $\mathcal{H}_{is,t}$, and its computation requires no data on the firm's non-rival asset stock, an object that is typically hard to measure,

$$\gamma_s = \frac{\rho + \delta_x}{\delta_x}\,\mathcal{H}_{is,t}. \tag{36}$$

As shown below, both approaches – the dynamic approach that requires data on the non-rival asset stock and the steady-state approach that does not – yield similar estimates of γ_s.

We implement this approach using data from the United States, assuming that sectoral non-rival intensities are common across countries, so that cross-country variation in aggregate non-rival intensity stems from sectoral composition. The key empirical challenge is measuring the share of the sectoral wage bill accruing to workers who engage in the production of non-rival assets, $H_{is,t}$. We then translate this into the measure that adjusts for the presence of multinationals, $\mathcal{H}_{is,t}$, using the sales data from Section 4.2. The dynamic version also requires the growth of the non-rival asset stock, $x_{is,t+1}/x_{is,t}$, which we measure using sectoral intellectual-property stocks from the BEA fixed-asset accounts; the steady-state version applies equation (36) directly. In both cases, we set $\delta_x = 0.20$, consistent with the depreciation of intellectual property in the BEA fixed-asset accounts, and an interest rate of $\rho = 0.04$.

To measure $H_{is,t}$, we rely on the occupational distribution of a sector's payroll. Intuitively, a programmer who develops software is likely contributing to the production of non-rival assets, while a waiter who serves food is likely part of the rival-input composite. In practice, we allow each occupation to engage in both non-rival and rival activities. We use a large language model to classify each task the occupation performs – drawn from the O*NET task database – as non-rival, rival, or ambiguous.[13] A task is non-rival if it builds a firm-specific asset, such as software, a product design, a patent, training material, or organizational knowledge, that is (a) proprietary, (b) durable, and (c) usable across customers or markets without being depleted; it is rival if it must be repeated for each customer or is consumed in delivery. For each occupation we drop ambiguous tasks and take the non-rival share of the remainder; the sectoral non-rival payroll share $H_{is,t}$ is then the payroll-weighted average of these occupation shares, using BLS occupation-by-sector payroll weights. Additional details, including the full LLM prompt, are provided in Appendix C.3.

Table 2 illustrates the classification with example tasks. Tasks such as developing software systems, developing improved crop varieties, and drawing product blueprints are classified as contributing to the creation of non-rival assets, while tasks such as transporting patients, serving drinks, and performing minor vehicle repairs are classified as rival.

[13] We also create a classification with a smooth rivalness score between 0 and 1 for each task, one minus the score being the task's non-rival share, and average the results. For details see Appendix C.3.

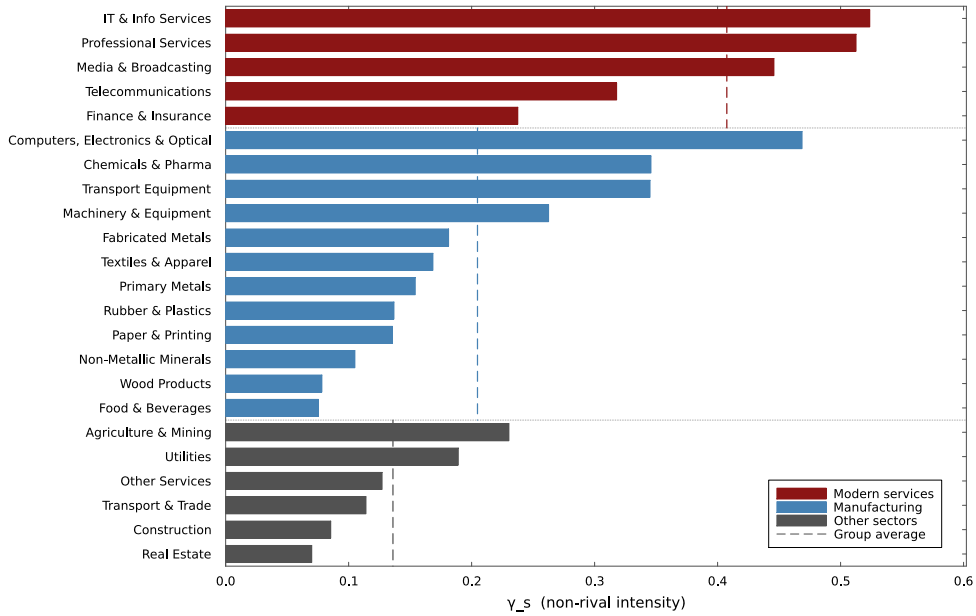
Table 2: Examples of Rival and Non-Rival Task Classifications

Task description	Classification
Design, develop and modify software systems, using scientific analysis and mathematical models to predict and measure outcome and consequences of design	Non-rival
Conduct experiments to develop new or improved varieties of field crops, focusing on characteristics such as yield, quality, disease resistance, nutritional value, or adaptation to specific soils or climates	Non-rival
Obtain, organize, or develop training procedure manuals, guides, or course materials, such as handouts or visual materials	Non-rival
Design scale or full-size blueprints of specialty items such as furniture and automobile body or chassis components	Non-rival
Create specialty dishes and develop recipes to be used in dining facilities	Non-rival
Transport patients to treatment units, testing units, operating rooms, or other areas, using wheelchairs, stretchers, or moveable beds	Rival
Drive vehicles with capacities under three tons to transport materials to and from specified destinations, such as railroad stations, plants, residences, offices, or within industrial yards	Rival
Prepare and serve cold drinks, or frozen milk drinks or desserts, using drink-dispensing, milkshake, or frozen custard machines	Rival
Clean, shape, and polish fingernails and toenails, using files and nail polish	Rival
Perform minor repairs, such as adjusting brakes, replacing spark plugs, or changing engine oil or filters	Rival

Note: Selected examples from the O*NET task database, on which all of our classification variants agree. A task is classified as non-rival if it contributes to the creation of a firm-specific asset that is proprietary, durable, and usable across customers or markets without being depleted.

The recovered values of γ_s range from 0.07 to 0.52, and are plotted by sector in Figure 2. The most non-rival-intensive sectors are information technology and other information services, professional services, computer and electronic equipment, publishing and audiovisual services, chemicals and pharmaceuticals, and transport equipment. The least non-rival-intensive sectors are real estate, food manufacturing, and wood products.

Figure 2: Non-Rival Intensity by Sector

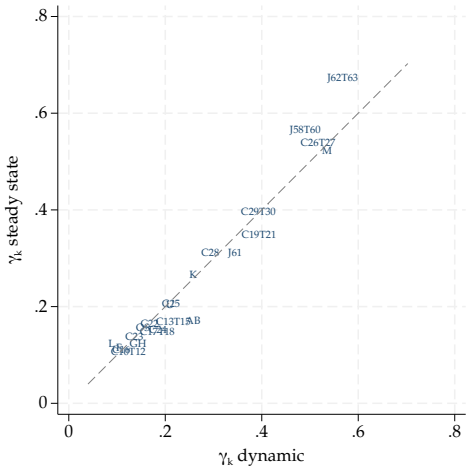


Note: Each bar shows the baseline non-rival intensity γ_s, recovered from U.S. non-rival payroll shares, sectoral revenue and expenditure, and intellectual-property-stock growth through the non-rival accumulation relationship in equation (35), as implemented in Appendix D.4.

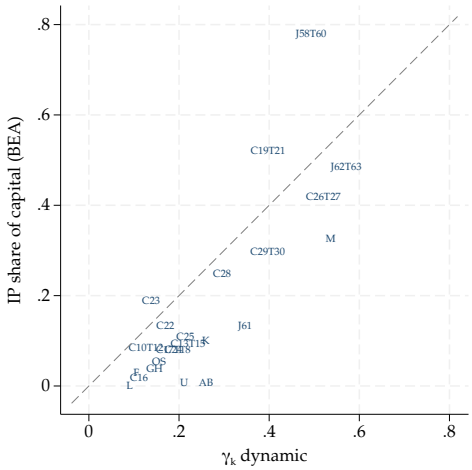
We find that the dynamic and steady-state approaches indeed yield similar estimates of γ_s, as shown in Figure 3. The left panel plots the two sets of estimates against each other: the correlation is 0.97 and the average absolute difference is 0.04. The right panel plots the dynamic estimates against the share of intellectual property in total fixed assets from the BEA, an external measure of intangible intensity that we do not use in constructing γ_s. While this measure does not map into the model as cleanly as our payroll-based measure, the similar ranking of sectors across the two is reassuring that our approach captures the intensity of intangible assets. We use the dynamic approach in our baseline calibration.

Figure 3: Robustness of the Non-Rival Intensity Estimates

(a) Steady-state versus dynamic estimates



(b) Comparison to BEA IP share of assets



Note: Each point is a sector, labeled by its code (Table 6); dashed lines are 45-degree lines. Panel (a) compares our two approaches to measuring the non-rival intensity γ_s: the horizontal axis is the baseline estimate from the dynamic accumulation equation (35), which uses data on the growth of the non-rival asset stock, and the vertical axis is the estimate from the steady-state relationship (36), which does not. Panel (b) compares the baseline estimates (horizontal axis) with an external measure of intangible intensity that plays no role in their construction: the share of intellectual-property capital in total fixed assets from the BEA fixed-asset accounts (vertical axis). Sectors that our procedure identifies as non-rival-intensive also hold more of their capital in intellectual property.

4.4 Model Inversion and Additional Calibration Details

We now briefly describe the rest of the calibration, which is summarized in Table 7 and described in detail in Appendix D.

External parameters. We set the discount factor and the intertemporal elasticity of substitution to standard values in the literature, $\beta = 0.96$ and $\psi = 0.5$. We set the depreciation rate of physical capital to $\delta_K = 0.05$, and the depreciation rate of non-rival assets to $\delta_x = 0.20$, consistent with the depreciation rate of IP capital in the BEA fixed-asset accounts.

Elasticity of substitution across sectors. We follow Comin et al. (2021) and set the expenditure elasticities ε_s to their estimates, mapped to our sectors, which range from 0.35 to 2.17. We set the across-sector elasticity of substitution to $\eta = 0.5$.

Elasticities of substitution across varieties. Given the limited available evidence on the elasticity of substitution outside manufacturing, we adopt the following approach. In our model, the steady-state net operating profit share of total sales – the operating margin net of the share of revenue spent on the creation of non-rival assets – is given by

$$\frac{1}{\sigma_s} - \frac{\delta_x}{\rho + \delta_x} \gamma_s \frac{\sigma_s - 1}{\sigma_s},$$

where ρ is the steady-state interest rate.[14] This share is decreasing in σ_s given γ_s, so we set the within-sector elasticity of substitution σ_s to target a net operating profit share of ten percent in every sector, which is consistent with standard estimates for the aggregate net profit share in the U.S. economy. This approach yields values of σ_s consistent with the trade literature's estimates of the elasticity of substitution across foreign varieties, ranging from 2.8 to 6.8, with lower values in more differentiated sectors such as IT and professional services, and higher values in less differentiated sectors such as real estate and food manufacturing. Importantly, we obtain similar values if, instead of imposing a 10 percent profit margin, we use average sectoral accounting profit

[14] Appendix D.5 reports the corresponding expression under the quantitative model of Section 5.3.1, which we use in the implementation.

margins from corporations' income statements – see Figure 8 in Appendix D.5 for a comparison of the two approaches.

Inversion of model fundamentals. Finally, we choose the model fundamentals – country-sector productivities z_{is}, bilateral iceberg trade costs τ_{ins}, and across-sector taste shifters $\bar{\chi}_{ns}$ – to exactly match the observed pattern of expenditure across origins and sectors in 2019, constructed from the bilateral sales matrix R_{ins} of Section 4.2.[15] Productivities are pinned by domestic sourcing shares together with factor-market clearing, bilateral trade costs by the off-diagonal sourcing shares, and taste shifters by the sectoral expenditure shares. The inversion is exact and reproduces the 2019 sales matrix for each sector by construction. The inversion is described in detail in Appendix D.7.

5 Quantitative Analysis

In this section, we use the calibrated model to study how the presence of non-rival assets shapes the patterns of trade, the gains from trade, and the dynamic response of the global economy to trade shocks. Section 5.1 asks which forces generate the observed pattern of specialization. Section 5.2 quantifies the gains from trade and the contribution of non-rival assets to them. Section 5.3 traces the dynamic adjustment to global shocks. Section 5.4 applies the framework to one particular trade barrier in the real world, China's "Great Firewall."

5.1 The Determinants of the Patterns of Trade

We first ask what forces generate the observed pattern of specialization. Section 3.2 suggests that the combination of heterogeneous demand through income effects and the scale effects arising from the accumulation of non-rival assets can qualitatively generate the observed direction of specialization. Whether these forces actually account for the specialization we see in the data is not something that our baseline model inversion can answer on its own: by construction, we invert the model residuals – country-sector productivities z_{is}, bilateral trade frictions τ_{ins}, and demand shifters $\bar{\chi}_{ns}$ – to exactly match the observed patterns of exports and specialization.

[15] The bilateral trade costs τ_{ins} and the bilateral demand shifters a_{ins} enter the model isomorphically – both scale the relative sales of origin i in market n – so recovering one is equivalent to recovering the other. We work with τ_{ins} throughout.

To study the role of the model's endogenous forces relative to exogenous comparative advantage, we ask how much of the observed specialization would remain if we eliminated exogenous variation in comparative advantage from the fundamentals, both in our baseline model and in simpler models that lack our features: one that removes scale effects ($\gamma_s = 0$), one that removes observed demand heterogeneity (common χ across countries), and one that removes both. We invert each variant separately, so that each has its own fundamentals and matches the same bilateral sales matrix under its own structure. We then remove sector-specific comparative advantage from those fundamentals across all variants. To this end, we decompose each inverted log productivity into a country and a sector component,

$$\log z_{is} = z_i^{\text{country}} + z_s^{\text{sector}} + \tilde{z}_{is},$$

and retain only the fitted part $z_i^{\text{country}} + z_s^{\text{sector}}$, discarding the country-by-sector residual $\tilde{z}_{is}$ – the term that encodes "country i is idiosyncratically productive in sector s." We treat bilateral trade frictions the same way, decomposing the off-diagonal iceberg costs into a country-pair and a sector component,

$$\log \tau_{ins} = \tau_{in}^{\text{pair}} + \tau_s^{\text{sector}} + \tilde{\tau}_{ins},$$

retaining $\tau_{in}^{\text{pair}} + \tau_s^{\text{sector}}$ and discarding the residual $\tilde{\tau}_{ins}$, with domestic frictions left unchanged. What remains of each fundamental varies only across countries, across sectors, or across country pairs, never idiosyncratically across country-sector cells. Finally, we solve each variant under these counterfactual fundamentals and compare the specialization patterns that emerge – as measured by revealed comparative advantage (RCA) – with the data.

Table 3 reports how well each variant reproduces the data once exogenous comparative advantage is removed. Each row is a model variant, and the columns report different metrics to capture the distance of the comparative-advantage-free model from the data: the root mean squared error between log RCA in the model and in the data, their correlation, the regression slope between them and its 95% confidence interval, and the R^2 from that regression. The first row is the baseline model. Its RCA remains closely tied to the data: the correlation is 0.521, the RMSE is 1.137, and the regression slope is 0.716, with a confidence interval $[0.581, 0.852]$. The remaining rows report the restricted variants. Removing scale effects ($\gamma_s = 0$) is the most damaging, cutting

the correlation to 0.042 and the slope to 0.182, indistinguishable from zero. Removing demand heterogeneity (common χ) is milder, leaving a correlation of 0.233 and a slope of 0.237, suggesting that scale effects without heterogeneous demand can still generate part of the observed pattern, though less well than when combined with realistic demand heterogeneity. Removing both forces reverses the relationship, giving a correlation of -0.352 and a slope of -0.955. The exercise thus shows that the baseline model with realistic demand heterogeneity and scale effects is much better at capturing observed patterns of specialization than models without these forces.

Table 3: How Close Is the Model to the Data Without Exogenous Comparative Advantage?

			Regression of model on data log RCA		
Model variant	RMSE	Correlation	Slope	95% CI	R^2
Baseline (both forces on)	1.137	0.521	0.716	[0.581, 0.852]	0.272
No scale effects ($\gamma_s = 0$)	4.321	0.042	0.182	[-0.322, 0.686]	0.002
No non-homothetic demand (common χ)	1.154	0.233	0.237	[0.123, 0.351]	0.054
Neither force (common χ, $\gamma_s = 0$)	3.141	-0.352	-0.955	[-1.248, -0.662]	0.124

Note: Each row is a model variant, inverted separately to match the same bilateral sales data. We then remove exogenous, sector-specific comparative advantage from the inverted fundamentals and re-solve: for productivity we replace $\log z_{is}$ by its country and sector fixed effects, and for bilateral trade frictions we replace the off-diagonal $\log \tau_{ins}$ by its origin-destination-pair and sector fixed effects, leaving domestic frictions unchanged. The columns report the fit of the re-solved model's log RCA to the data across the 73×4 country-by-broad-sector cells: the root mean squared error and the correlation, together with the slope, 95% confidence interval, and R^2 from a regression of model on data log RCA. Higher correlation and R^2, lower RMSE, and a slope nearer one indicate that the model reproduces the observed specialization without exogenous comparative advantage. "Common χ" fixes sectoral expenditure shares to common global values; $\gamma_s = 0$ removes non-rival assets and the scale effects they generate.

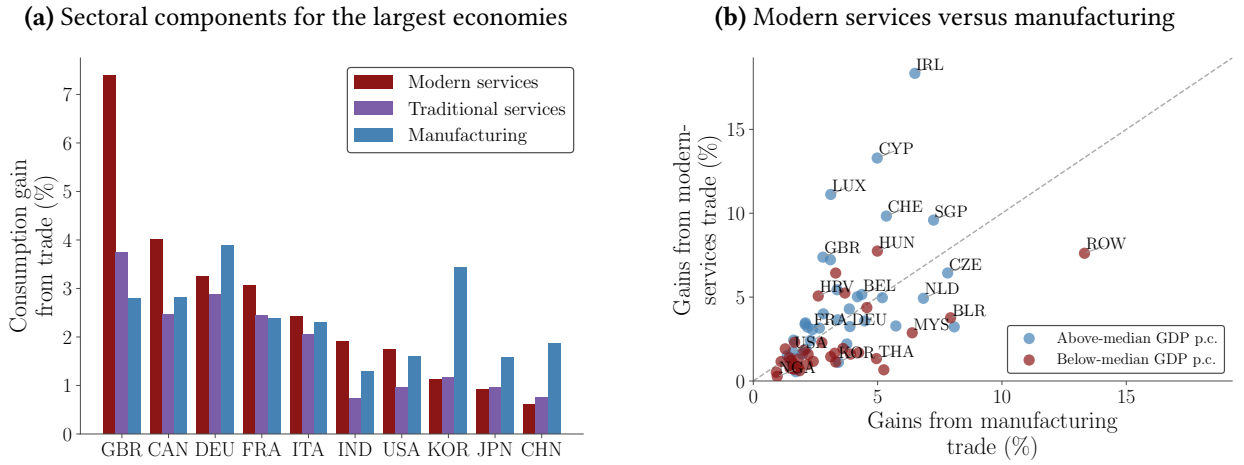
5.2 Welfare Gains From Trade

We now ask how large the welfare gains from trade are, and how the presence of non-rival assets shapes them. We pose the question in two parts: first, how large are the gains from trade in modern services relative to those in goods, and in manufacturing in particular; and second, how large are the gains in our framework relative to a framework that does not model non-rival assets. Throughout, we measure the gains from trade as the percentage gain in per-capita real consumption, either relative to autarky, or following a decline in trade frictions τ_{ins}.

We begin by comparing the gains from trade across broad sectors. For each country we compute the gains from trade in each of three broad sector groups – manufacturing, modern services, and

traditional services – as the loss in per-capita real consumption from shutting down international trade in that group of sectors alone, holding trade frictions in all other sectors fixed. Figure 4 reports the results for the ten largest economies (panel a) and compares the modern-services and manufacturing gains across all 73 countries (panel b).

Figure 4: How Large Are the Gains from Trade?



Note: Panel (a) reports the gains from trade for the ten largest economies by GDP, measured as the percentage gain in per-capita real consumption relative to autarky. For each country, the bars show the modern-services, traditional-services, and manufacturing (12 sectors) components; each measure shuts down trade in that sector group only, holding trade in all other sectors at baseline levels. Countries are sorted by their gains from modern-services trade. Panel (b) plots the gains from trade in modern services against the gains from trade in manufacturing, measured in the same units. The dashed 45-degree line indicates equal gains; points above it are countries for which modern-services trade contributes more to welfare than manufacturing trade. Marker colors indicate whether a country's GDP per capita is above or below the sample median. All 73 countries are shown.

Two features of Figure 4 stand out. First, the gains from trade in modern services are large, and for many countries they are comparable to or larger than the gains from trade in manufacturing. In panel (a), the modern-services contribution exceeds the manufacturing contribution for the United Kingdom, Canada, France, Italy, the United States, and India, and it is largest of all for the United Kingdom, at around 7 percent of consumption. For the more manufacturing-oriented economies of South Korea, Japan, Germany, and China, manufacturing trade remains the dominant source of gains. Second, panel (b) shows that this pattern is widespread: a large share of countries lie above the 45-degree line, so that trade in modern services contributes more to their welfare than trade in manufacturing. The countries furthest above the line are high-income and small open economies – Ireland, Cyprus, Luxembourg, Switzerland, Singapore, and the United Kingdom

– while those below it are manufacturing exporters such as Vietnam, China, South Korea, and Thailand. Trade in modern services is thus a first-order source of the gains from trade, of the same order of magnitude as trade in manufacturing, and disproportionately so for richer economies. Importantly, these results stem not just from the existence of non-rival assets, which we turn to analyze next, but also from the simple fact that expenditure shares on these sectors are relatively large in advanced economies, and that they are characterized by low elasticities of substitution (σ_s) in our measurement.

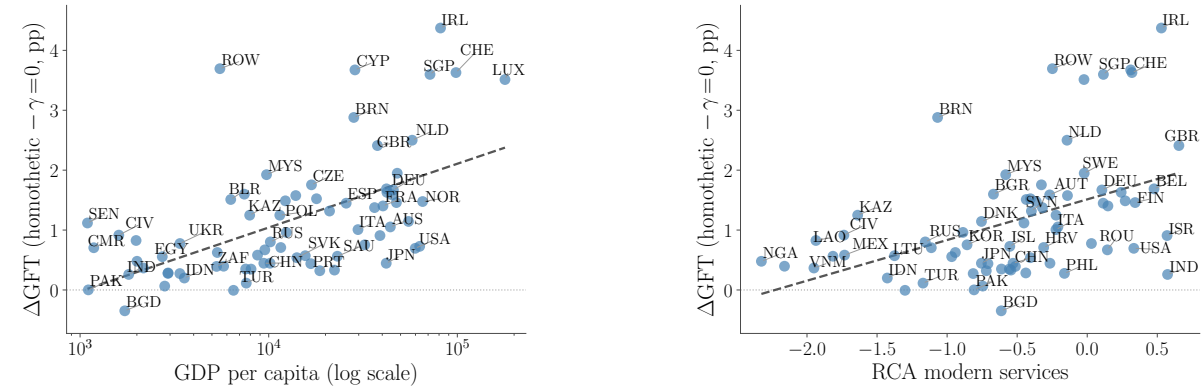
We next ask how important the accumulation of non-rival assets is for the gains from trade. Concretely, we compare the welfare gain from a unilateral trade liberalization – a 10 percent reduction in each country's iceberg trade costs on all of its bilateral trade – between a model with non-rival assets ($\gamma_s > 0$) and an otherwise-identical model without them ($\gamma_s = 0$). We invert the two models separately, so that each reproduces the same observed bilateral trade shares and expenditure shares before the liberalization; the two economies are therefore observationally equivalent in the cross section and differ only in whether the observed trade is generated in the presence of non-rival assets. We denote by ΔGFT the difference between the two welfare gains, in percentage points of per-capita real consumption.

We perform this comparison in two versions of the model – one with homothetic demand and one with our baseline non-homothetic demand – in order to isolate the role of non-rivalry from that of non-homothetic demand. The first row of Figure 5 reports the homothetic case. First, the model with non-rival assets attributes a higher gain from the liberalization to all but two countries, with an average difference of 1.15 percentage points. Second, this difference rises steeply both with income per capita (panel a) and with specialization in modern services (panel b); it is largest for high-income, small open economies such as Ireland, Switzerland, and Singapore, and is close to zero for low-income economies specialized in goods. These two findings are the quantitative counterparts of the two roles of non-rivalry highlighted in Section 3.3: non-rival assets raise the average gains from trade, through the larger capital multiplier $\mathcal{M}_n$ in equation (34), and its sectoral-reallocation term tilts the gains toward countries that specialize in high-γ sectors. The upshot is that, even under a standard homothetic demand system, a framework that abstracts from non-rival assets understates the gains for high-income countries and for countries that specialize in highly non-rival sectors such as modern services and, relative to them, overstates the gains for

Figure 5: Non-Rival Assets and the Aggregate Gains from Trade Liberalization

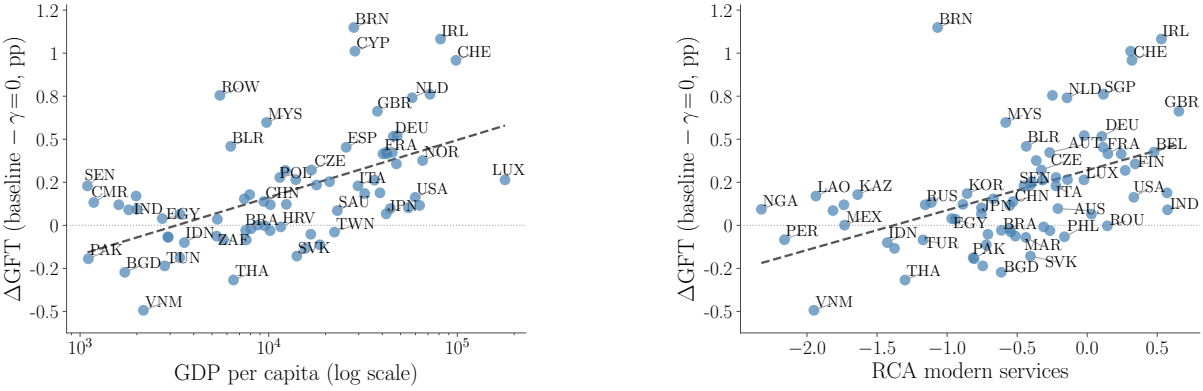
(a) Homothetic: by GDP per capita

(b) Homothetic: by RCA in modern services



(c) Non-homothetic: by GDP per capita

(d) Non-homothetic: by RCA in modern services



Note: ΔGFT is the contribution of non-rival assets to a country's welfare gain from a 10% reduction in the iceberg trade cost (τ) on all of its bilateral trade: the difference between the gains from trade with non-rival assets ($\gamma_s > 0$) and the gains without them ($\gamma_s = 0$), holding the trade elasticities σ_s fixed. The top row (panels a, b) computes it in a homothetic economy; the bottom row (panels c, d) in the baseline non-homothetic one. Both economies are re-solved to match the same observed trade before the liberalization. ΔGFT is expressed in percentage points of per-capita real consumption. Panels (a) and (c) plot ΔGFT against GDP per capita (log scale); panels (b) and (d) against the country's revealed comparative advantage in modern services (log Balassa index). Dashed lines are ordinary least squares fits. All 73 economies are shown. For reference, the mean gain from trade with non-rival assets is 7.79 percentage points under homothetic demand and 4.07 percentage points under non-homothetic demand.

manufacturing exporters.

This pattern is preserved when we add non-homothetic demand, in our baseline model (second row of Figure 5): ΔGFT again rises with income per capita (panel c) and with specialization in modern services (panel d), with an almost unchanged cross-country ordering. The overall level of the difference is lower, however – the largest contributions, again in economies such as Ireland and Switzerland, are around one percentage point rather than the three to four of the homothetic case – because non-homotheticity dampens the gains from trade as a whole.[16] Indeed, with non-homothetic demand the difference turns negative for nearly a third of countries – typically goods-exporting economies – so that a model that omits non-rival assets over-predicts their gains from the liberalization.

5.3 Gradual Adjustment to Global Shocks

Because non-rival assets are durable and accumulate only gradually, they generate heterogeneous timing across countries in the response to even a common global shock. A country specialized in non-rival-intensive sectors responds to a favorable shock by specializing further in them and accumulating more non-rival assets, a process that plays out over time. Two consequences follow. First, the country's gains accrue later: its consumption rises more in the long run, relative to poorer and less non-rival-intensive countries whose gains arrive quickly. Second, with access to international borrowing, the country borrows against these deferred gains, running a current-account deficit along the transition.

5.3.1 Adding Richer Dynamics

For the analysis of transitional dynamics, we enrich the model of Section 2 with two features standard in dynamic trade and international macroeconomics. First, we add convex adjustment costs to the accumulation of physical capital following the specification in Lucas and Prescott (1971) and Eaton et al. (2016). We add a similar friction to the accumulation of non-rival assets. The laws of motion become

$$x_{is,t+1} = (1 - \delta_x)\, x_{is,t} + x_{is,t}^{1-\upsilon_x} \left[g_{is,t}\left(l_{is,t}^x,\, k_{is,t}^x\right) \right]^{\upsilon_x},$$

[16] Non-homothetic demand reallocates expenditure toward less tradable sectors, lowering the overall gains from trade.

$$K_{n,t+1} = (1 - \delta_K) K_{n,t} + K_{n,t}^{1-\upsilon_K} I_{n,t}^{\upsilon_K},$$

where $\upsilon_x, \upsilon_K \in (0, 1]$ govern the convexity of the adjustment friction, with the frictionless case nested at $\upsilon_x = \upsilon_K = 1$. In our calibration, we set both υ_x and υ_K to 0.75, midway between the two common choices in the literature: frictionless adjustment ($\upsilon = 1$) and a quadratic adjustment cost.

Second, we relax financial autarky by letting countries trade a riskless bond denominated in the numeraire, so that the budget constraint of country n becomes

$$E_{n,t} + B_{n,t+1} = (1 + \rho_t) B_{n,t} + Y_{n,t}, \tag{37}$$

where $E_{n,t}$ is total expenditure, $B_{n,t}$ is the bond holding, paying gross return $1 + \rho_t$, and $Y_{n,t}$ is national income. Following Schmitt-Grohé and Uribe (2003), we also impose a small quadratic cost of bond holdings, which induces stationarity in net foreign asset positions and a unique steady state; Appendix A.3 states the cost and the resulting bond Euler equation. Bonds are in zero net supply, $\sum_n B_{n,t} = 0$, with zero net foreign assets in the steady state, $B_n = 0$.

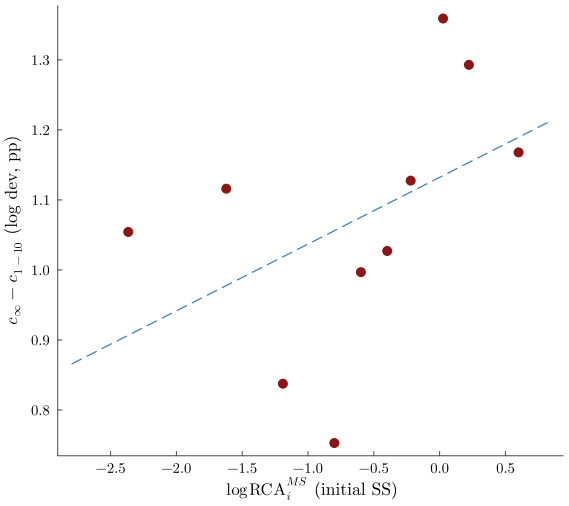
5.3.2 Specialization and the Dynamic Response to Global Shocks

We illustrate these dynamics with two global shocks motivated by the experience of recent decades. The first is a trade expansion that lowers trade frictions in all sectors and countries. The second is a global improvement in the productivity of modern services. In each case we solve for the full transition path of the world economy from the initial steady state to the new one. For each country, we measure how gradually its consumption gains accrue by $c_\infty - \bar{c}_{1-10}$, the difference between the long-run gain and the average gain over the first ten years (log-deviation, percentage points); a larger value means more of the gain arrives later. We measure its external position by $\overline{B/Y}_{1-10}$, the average net foreign asset position over the first ten years (percent of GDP); a negative value indicates net borrowing. Figure 6 relates both objects to a country's baseline specialization in modern services, $\log \mathrm{RCA}^{MS}$.

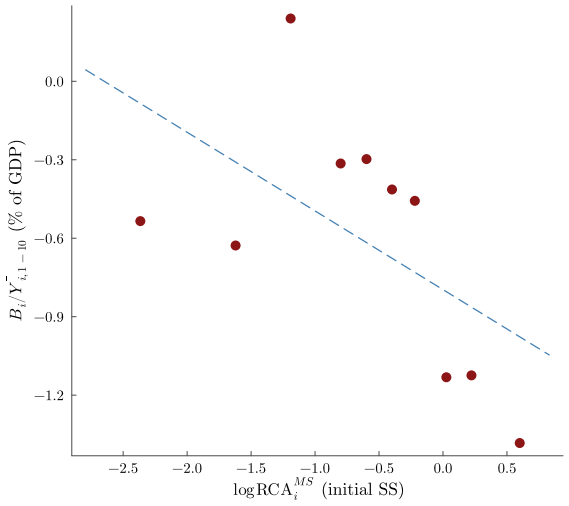
The two experiments deliver the same message: although each shock is global and symmetric across countries, its effects are highly heterogeneous and sort by baseline specialization. In the left panels, consumption gains accrue more gradually the more a country specializes in modern services; the most specialized economies realize the smallest share of their long-run gains in

Figure 6: Heterogeneous Dynamic Responses to Global Shocks

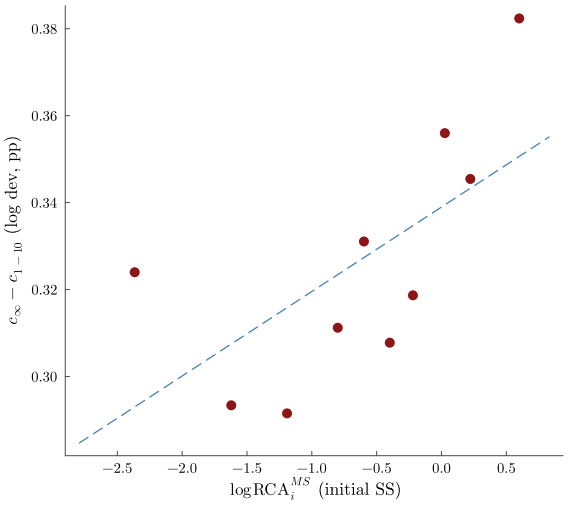
(a) Trade expansion: deferred consumption gains



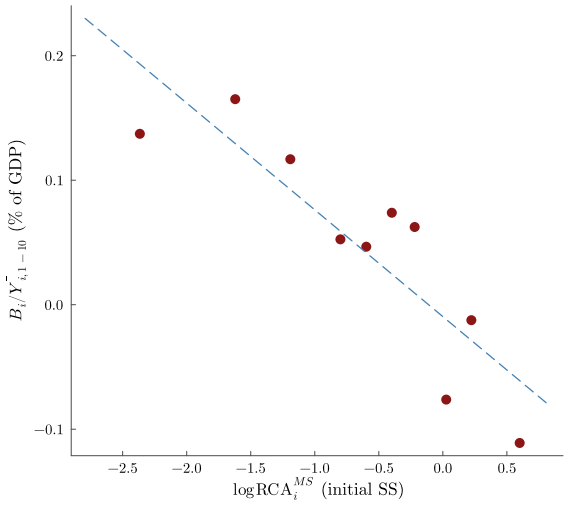
(b) Trade expansion: net foreign assets



(c) Modern-services productivity: deferred gains



(d) Modern-services productivity: net foreign assets



Note: Binned scatter plots. Countries are grouped into equal-sized bins by their baseline log Balassa index of revealed comparative advantage in modern services (horizontal axis), and each point is a bin mean. The top row reports a global trade expansion that lowers trade frictions in all sectors and countries; the bottom row a global improvement in the productivity of modern services. Left panels plot $c_\infty - \bar{c}_{1-10}$, the gap between the long-run consumption gain and the average gain over the first ten years (log-deviation, percentage points); larger values mean the gains are more deferred to the long run. Right panels plot $\overline{B/Y}_{1-10}$, the average net foreign asset position over the first ten years (percent of GDP); negative values indicate net borrowing over the transition. Dashed lines are ordinary least squares fits on the underlying country-level data.

the first decade. In the right panels, these same economies run current-account deficits over the transition, borrowing against gains that arrive later, while economies specialized in fast-adjusting sectors run surpluses.

A natural question is whether this timing pattern is visible in the data. Barattieri (2014) documents a systematic link between specialization in services and current-account deficits over recent decades, consistent with the adjustment our model describes. We provide complementary evidence in Appendix E, focusing on modern services: in a panel of countries, a rise in a country's revealed comparative advantage in modern services is associated with a deterioration of its cumulative trade balance.

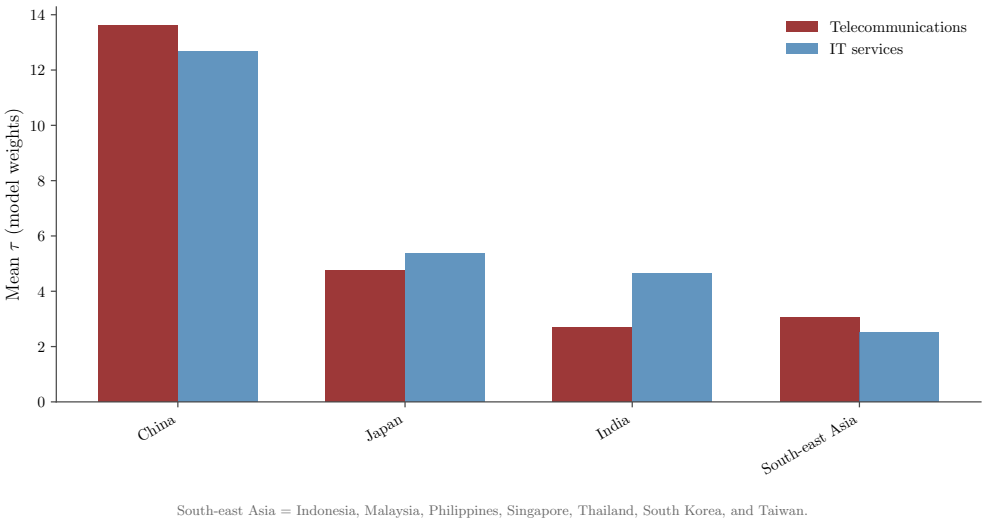
5.4 An Application to China's Great Firewall

As a final exercise, we demonstrate how our framework can be used to study barriers to trade in services, with the example of China's "Great Firewall." The Great Firewall is China's system of internet censorship and cross-border network control, which combines technical filtering, legal regulation, and platform governance to restrict access to many foreign websites and digital services while promoting domestic alternatives (Zhou 2024; Bernard et al. 2025). As such, it acts as a non-tariff barrier to digital trade in China. We use our model to assess the global consequences of removing this barrier and granting foreign digital services wider access to the Chinese market.

We first verify that the Great Firewall indeed appears in our inversion as a large barrier to inflows of digital services into China. Figure 7 reports the model-implied iceberg trade cost τ_{ins} on services flowing into China in the telecommunications (J61) and IT-services (J62T63) sectors, averaged across origin countries, and compares it with the analogous costs faced by other regional economies: Japan, India, and a South-east Asia composite (Indonesia, Malaysia, the Philippines, Singapore, Thailand, South Korea, and Taiwan). The implied cost of importing these services into China is roughly 410 percent higher in IT services and 345 percent higher in telecommunications than into the South-east Asia composite, our comparison group.

We then consider a counterfactual that lowers the inward frictions into China in these two sectors to the South-east Asia benchmark, and we solve for the new steady state. Table 4 reports the implications for China and for a set of other economies, both in the baseline model and in a version with non-rival assets shut down ($\gamma_s = 0$). Removing the barrier raises Chinese real

Figure 7: Model-Implied Trade Costs on Digital Services Into China



Note: Bars report the trade-weighted average iceberg trade cost τ_{ins} on services imported by each destination in the telecommunications (J61) and IT-services (J62T63) sectors. South-east Asia comprises Indonesia, Malaysia, the Philippines, Singapore, Thailand, South Korea, and Taiwan. Bilateral model trade flows are used as weights.

consumption by 1.10 percent: domestic users gain access to cheaper and more varied foreign digital services. At the same time, China's own digital-services sector contracts, with its revealed comparative advantage in IT and telecommunications falling by roughly 93 percent and resources reallocating toward manufacturing, whose RCA rises modestly.

The effects on other economies follow their position relative to China. Countries with good access to the Chinese market and strong IT and telecommunications sectors, such as Japan and the Southeast Asian economies, gain and become increasingly specialized in these services, with their comparative advantage in IT and telecommunications rising. Countries that specialize in the same sectors but are economically and geographically distant from China, such as India and Ireland, instead lose and reduce their comparative advantage in IT and telecommunications. One might expect these exporters to benefit from China's wider market access, but in practice they suffer a terms-of-trade deterioration. As Japan and the Southeast Asian economies expand their sales to China, the accumulation of non-rival assets makes them increasingly productive in these sectors, which erodes the competitiveness of more distant exporters in third markets where they compete. Indeed, when we consider the same counterfactual in the model without non-rival assets ($\gamma_s = 0$), India and Ireland no longer lose. Modeling the accumulation of non-rival assets is

therefore essential for studying the effects of changes in trade barriers such as the Great Firewall.

Table 4: Steady-State Effects of Removing China's Great Firewall

	Baseline ($\gamma_s > 0$)			No non-rival assets ($\gamma_s = 0$)		
Country	%Δ Cons.	%Δ RCA (IT & Tel.)	%Δ RCA (Manuf.)	%Δ Cons.	%Δ RCA (IT & Tel.)	%Δ RCA (Manuf.)
China	+1.10	−92.84	+5.15	+0.38	−16.51	+0.27
Southeast Asia	+0.38	+102.68	−10.82	+0.08	+34.74	−1.36
Japan	+0.01	+24.70	+0.11	0.00	+40.30	+0.05
India	−0.01	−20.16	+2.53	0.00	−3.07	+0.12
Ireland	−0.08	−32.17	+1.83	0.00	−14.56	+0.56
United States	+0.06	−9.43	−0.71	+0.01	+6.12	−0.01

Note: Steady-state percent changes relative to baseline, from a counterfactual that lowers the inward iceberg trade cost on China's telecommunications (J61) and IT-services (J62T63) imports to the South-east Asia composite level (Indonesia, Malaysia, the Philippines, Singapore, Thailand, South Korea, and Taiwan). The left panel uses the baseline model with non-rival assets ($\gamma_s > 0$); the right panel re-solves the same counterfactual with non-rival assets shut down ($\gamma_s = 0$). "Cons." is per-capita real consumption; "RCA" is the log Balassa revealed comparative advantage index in IT-and-telecommunications services and in manufacturing. Southeast Asia is a value-added-weighted average of Indonesia, Malaysia, the Philippines, Singapore, and Thailand.

6 Conclusion

In this paper, we revisited classic questions in trade theory – the patterns of trade, the gains from trade, and the dynamic adjustment to global shocks – in a dynamic multi-sector model that captures a defining feature of modern services: their heavy reliance on non-rival assets. The interaction of non-rival assets with non-homothetic demand leads richer countries to specialize in modern services, as observed in the data. Non-rival assets amplify the gains from trade and tilt them toward richer countries. Because they accumulate only gradually, the timing of the gains from integration differs across countries, linking specialization to current account balances following global shocks. Finally, applying the framework to China's Great Firewall, we find that the global effects of barriers to services trade depend critically on the accumulation of non-rival assets.

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A Derivation of Equilibrium Conditions

This appendix derives the equations used in Section 2.2, together with the quantitative-model extensions (bonds and adjustment costs) introduced in Section 5.3.1. We write

$$D_s \equiv \frac{\sigma_s - 1}{\sigma_s}, \qquad R_{is,t} \equiv \sum_n R_{ins,t}, \qquad \bar{\varepsilon}_{n,t} \equiv \sum_s \chi_{ns,t}\varepsilon_s.$$

A.1 Demand

Fix $\mathcal{Y}_{n,t}$, L_n, and sectoral prices $\{P_{ns,t}\}_s$. The final-good producer solves

$$\min_{\{\mathcal{Y}_{ns,t}\}_s} \sum_s P_{ns,t}\mathcal{Y}_{ns,t} \quad \text{s.t.} \quad \sum_s \bar{\chi}_{ns}^{1/\eta} \left(\frac{\mathcal{Y}_{n,t}}{L_n}\right)^{\frac{1-\eta}{\eta}(\varepsilon_s-1)} \left(\frac{\mathcal{Y}_{ns,t}}{\mathcal{Y}_{n,t}}\right)^{\frac{\eta-1}{\eta}} = 1. \tag{38}$$

The first-order conditions imply

$$\mathcal{Y}_{ns,t} = \bar{\chi}_{ns} \left(\frac{P_{ns,t}}{P_{n,t}}\right)^{-\eta} \mathcal{Y}_{n,t} \left(\frac{\mathcal{Y}_{n,t}}{L_n}\right)^{(1-\eta)(\varepsilon_s-1)}. \tag{39}$$

Using $E_{n,t} = P_{n,t}\mathcal{Y}_{n,t} = \sum_s P_{ns,t}\mathcal{Y}_{ns,t}$ gives

$$\chi_{ns,t} \equiv \frac{P_{ns,t}\mathcal{Y}_{ns,t}}{E_{n,t}} = \bar{\chi}_{ns} \left(\frac{P_{ns,t}}{P_{n,t}}\right)^{1-\eta} \left(\frac{\mathcal{Y}_{n,t}}{L_n}\right)^{(1-\eta)(\varepsilon_s-1)}, \qquad \sum_s \chi_{ns,t} = 1, \tag{40}$$

which is equation (12).

To get the marginal cost of the final good, hold sectoral prices and population fixed and differentiate $\sum_s \chi_{ns,t} = 1$:

$$0 = \sum_s \chi_{ns,t}(1-\eta)\left[-d\log P_{n,t} + (\varepsilon_s - 1)d\log \mathcal{Y}_{n,t}\right]. \tag{41}$$

Hence

$$\frac{\partial \log P_{n,t}}{\partial \log \mathcal{Y}_{n,t}} = \bar{\varepsilon}_{n,t} - 1, \qquad \frac{\partial E_{n,t}}{\partial \mathcal{Y}_{n,t}} = \bar{\varepsilon}_{n,t} P_{n,t}. \tag{42}$$

For the sectoral CES bundle, cost minimization gives

$$y_{ins,t} = a_{ins,t}\left(\frac{p_{ins,t}}{P_{ns,t}}\right)^{-\sigma_s} \mathcal{Y}_{ns,t}, \qquad P_{ns,t} = \left(\sum_i a_{ins,t} p_{ins,t}^{1-\sigma_s}\right)^{1/(1-\sigma_s)}. \tag{43}$$

Therefore the bilateral expenditure share is

$$S_{nis,t} \equiv \frac{p_{ins,t} y_{ins,t}}{P_{ns,t}\mathcal{Y}_{ns,t}} = \frac{a_{ins,t} p_{ins,t}^{1-\sigma_s}}{\sum_j a_{jns,t} p_{jns,t}^{1-\sigma_s}}, \tag{44}$$

which is equation (13).

A.2 Static Firm Choices

The unit cost of one local rival-input bundle $(z_{is,t}l)^{\mu_s}k^{1-\mu_s}$ is

$$c_{is,t} = \min_{l,k}\{w_{i,t}l + r_{i,t}k : (z_{is,t}l)^{\mu_s}k^{1-\mu_s} \geq 1\} = \frac{w_{i,t}^{\mu_s} r_{i,t}^{1-\mu_s}}{\mu_s^{\mu_s}(1-\mu_s)^{1-\mu_s} z_{is,t}^{\mu_s}}. \tag{45}$$

The cost of the rival-input bundle used by an i firm serving n is

$$c_{ins,t} = \tau_{ins,t} \frac{c_{is,t}^{\alpha_s} c_{ns,t}^{1-\alpha_s}}{\alpha_s^{\alpha_s}(1-\alpha_s)^{1-\alpha_s}}. \tag{46}$$

Given $x_{is,t}$, marginal cost is $c_{ins,t}/x_{is,t}^{\gamma_s}$. CES demand then gives the markup rule

$$p_{ins,t} = \frac{\sigma_s}{\sigma_s - 1}\frac{c_{ins,t}}{x_{is,t}^{\gamma_s}}, \tag{47}$$

which is equation (18).

Substituting (47) into (43),

$$R_{ins,t} = a_{ins,t}\chi_{ns,t}E_{n,t}P_{ns,t}^{\sigma_s-1}\left(\frac{\sigma_s-1}{\sigma_s}\right)^{\sigma_s-1} x_{is,t}^{\gamma_s(\sigma_s-1)} c_{ins,t}^{1-\sigma_s}. \tag{48}$$

The price index can be written as

$$P_{ns,t}^{1-\sigma_s} = \left(\frac{\sigma_s - 1}{\sigma_s}\right)^{\sigma_s - 1} \sum_i a_{ins,t} x_{is,t}^{\gamma_s(\sigma_s-1)} c_{ins,t}^{1-\sigma_s}. \tag{49}$$

Using (46), the destination cost term $c_{ns,t}^{1-\alpha_s}$ is common across origins and cancels from bilateral shares:

$$S_{nis,t} = \frac{a_{ins,t}\tau_{ins,t}^{1-\sigma_s} x_{is,t}^{\gamma_s(\sigma_s-1)} c_{is,t}^{-\alpha_s(\sigma_s-1)}}{\sum_j a_{jns,t}\tau_{jns,t}^{1-\sigma_s} x_{js,t}^{\gamma_s(\sigma_s-1)} c_{js,t}^{-\alpha_s(\sigma_s-1)}}. \tag{50}$$

Variable rival-input payments equal marginal cost times quantity, hence equal $D_s R_{ins,t}$. Cost shares imply

$$\begin{aligned} w_{i,t} l_{ins,t}^o &= \mu_s \alpha_s D_s R_{ins,t}, & r_{i,t} k_{ins,t}^o &= (1-\mu_s)\alpha_s D_s R_{ins,t}, \\ w_{n,t} l_{ins,t}^d &= \mu_s (1-\alpha_s) D_s R_{ins,t}, & r_{n,t} k_{ins,t}^d &= (1-\mu_s)(1-\alpha_s) D_s R_{ins,t}. \end{aligned} \tag{51}$$

Dividing by factor prices gives equation (19).

A.3 Bonds, Capital, and Non-Rival Asset Accumulation

The general household budget constraint is

$$E_{n,t} + B_{n,t+1} + \frac{\zeta}{2} P_{n,t} \left(\frac{B_{n,t+1}}{P_{n,t}}\right)^2 = (1+\rho_t) B_{n,t} + Y_{n,t}. \tag{52}$$

The marginal utility value of one unit of expenditure is $C_{n,t}^{-1/\psi}/(L_n\, \bar{\varepsilon}_{n,t} P_{n,t})$; the constant L_n cancels from the discount factor below. Hence

$$\Lambda_{n,t,t+1} \equiv \beta \frac{P_{n,t}}{P_{n,t+1}} \frac{\bar{\varepsilon}_{n,t}}{\bar{\varepsilon}_{n,t+1}} \left(\frac{C_{n,t}}{C_{n,t+1}}\right)^{1/\psi}, \tag{53}$$

and the bond first-order condition is

$$1 + \zeta \frac{B_{n,t+1}}{P_{n,t}} = \Lambda_{n,t,t+1}(1+\rho_{t+1}). \tag{54}$$

The main theory section imposes financial autarky, $B_{n,t} = 0$, so $E_{n,t} = Y_{n,t}$. The transition exercises keep (52)–(54) together with bond-market clearing, $\sum_n B_{n,t} = 0$.

For physical capital, the general law of motion is

$$K_{n,t+1} = (1 - \delta_K)K_{n,t} + K_{n,t}^{1-\upsilon_K} I_{n,t}^{\upsilon_K}. \tag{55}$$

Let $Q_{n,t}^K$ be the value of one unit of $K_{n,t+1}$. The first-order conditions are

$$\bar{\varepsilon}_{n,t} P_{n,t} = Q_{n,t}^K \upsilon_K K_{n,t}^{1-\upsilon_K} I_{n,t}^{\upsilon_K - 1}, \tag{56}$$

$$Q_{n,t}^K = \Lambda_{n,t,t+1} \left[r_{n,t+1} + Q_{n,t+1}^K \left(1 - \delta_K + (1 - \upsilon_K) K_{n,t+1}^{-\upsilon_K} I_{n,t+1}^{\upsilon_K} \right) \right]. \tag{57}$$

In steady state,

$$I_n = \delta_K^{1/\upsilon_K} K_n, \qquad \frac{r_n}{P_n} = \bar{\varepsilon}_n \frac{\rho + \upsilon_K \delta_K}{\upsilon_K \delta_K^{1-1/\upsilon_K}}. \tag{58}$$

If $\upsilon_K = 1$, then $K_{n,t+1} = (1 - \delta_K)K_{n,t} + I_{n,t}$ and (57) reduces to the Euler equation in the main theory section,

$$\bar{\varepsilon}_{n,t} P_{n,t} = \Lambda_{n,t,t+1} \left[r_{n,t+1} + (1 - \delta_K) \bar{\varepsilon}_{n,t+1} P_{n,t+1} \right]. \tag{59}$$

Let the innovation bundle be $h_{is,t}^x \equiv (z_{is,t} l_{is,t}^x)^{\mu_s} (k_{is,t}^x)^{1-\mu_s}$. The general non-rival asset law of motion is

$$x_{is,t+1} = (1 - \delta_x) x_{is,t} + x_{is,t}^{1-\upsilon_x} (h_{is,t}^x)^{\upsilon_x}. \tag{60}$$

Since one unit of $h_{is,t}^x$ costs $c_{is,t}$, define

$$\phi_{is,t} \equiv \frac{c_{is,t} h_{is,t}^x}{R_{is,t}}. \tag{61}$$

Then

$$x_{is,t+1} = (1 - \delta_x) x_{is,t} + x_{is,t}^{1-\upsilon_x} \left(\frac{\phi_{is,t} R_{is,t}}{c_{is,t}} \right)^{\upsilon_x}. \tag{62}$$

Let $Q_{is,t}^x$ denote the value of one unit of $x_{is,t+1}$. The first-order condition for innovation input is

$$c_{is,t} = Q_{is,t}^x \upsilon_x x_{is,t}^{1-\upsilon_x} (h_{is,t}^x)^{\upsilon_x - 1}. \tag{63}$$

Because $R_{is,t} \propto x_{is,t}^{\gamma_s(\sigma_s-1)}$, the marginal operating-profit gain from $x_{is,t}$ is

$$\frac{\partial(R_{is,t}/\sigma_s)}{\partial x_{is,t}} = \gamma_s D_s \frac{R_{is,t}}{x_{is,t}}. \tag{64}$$

The asset Euler equation is

$$Q^x_{is,t} = \Lambda_{i,t,t+1}\left[\gamma_s D_s \frac{R_{is,t+1}}{x_{is,t+1}} + Q^x_{is,t+1}\left(1-\delta_x + (1-\upsilon_x)x_{is,t+1}^{-\upsilon_x}(h^x_{is,t+1})^{\upsilon_x}\right)\right]. \tag{65}$$

Equivalently, in terms of the investment share,

$$\phi_{is,t-1} = \left[\phi_{is,t}\frac{(1-\delta_x)x_{is,t}}{x_{is,t+1}-(1-\delta_x)x_{is,t}} + (1-\upsilon_x)\phi_{is,t} + \upsilon_x\gamma_s D_s\right] \times \Lambda_{i,t-1,t}\frac{R_{is,t}}{R_{is,t-1}}\left[1-(1-\delta_x)\frac{x_{is,t-1}}{x_{is,t}}\right]. \tag{66}$$

In steady state,

$$\phi_s = \frac{\delta_x \upsilon_x \gamma_s}{\rho + \delta_x \upsilon_x} D_s, \qquad x_{is} = \delta_x^{-1/\upsilon_x}\frac{\phi_s R_{is}}{c_{is}}. \tag{67}$$

If $\upsilon_x = 1$, then $x_{is,t+1} = (1-\delta_x)x_{is,t} + h^x_{is,t}$ and (65) becomes the main-text Euler equation,

$$c_{is,t} = \Lambda_{i,t,t+1}\left[\gamma_s D_s \frac{R_{is,t+1}}{x_{is,t+1}} + (1-\delta_x)c_{is,t+1}\right]. \tag{68}$$

A.4 Income and Market Clearing

Per-period operating profit before innovation spending is $(1-D_s)R_{ins,t} = R_{ins,t}/\sigma_s$. Innovation spending is paid to domestic labor and capital and subtracted from firm profits, so it cancels from national income. Country i's income is

$$Y_{i,t} = \sum_{s,n}\left(\alpha_s D_s + \frac{1}{\sigma_s}\right)R_{ins,t} + \sum_{s,n}(1-\alpha_s)D_s R_{nis,t}, \tag{69}$$

which is equation (21).

Labor and capital clearing use the production payments in (51) and the innovation payments

$c_{is,t}h^x_{is,t} = \phi_{is,t}R_{is,t}$:

$$w_{i,t}L_i = \sum_s \mu_s \left[(\alpha_s D_s + \phi_{is,t})R_{is,t} + (1-\alpha_s)D_s \sum_n R_{nis,t} \right], \tag{70}$$

$$r_{i,t}K_{i,t} = \sum_s (1-\mu_s) \left[(\alpha_s D_s + \phi_{is,t})R_{is,t} + (1-\alpha_s)D_s \sum_n R_{nis,t} \right]. \tag{71}$$

Goods-market clearing is

$$R_{ins,t} = \chi_{ns,t}E_{n,t}S_{nis,t}. \tag{72}$$

Under financial autarky, $E_{n,t} = Y_{n,t}$.

A.5 Steady-State Equilibrium Conditions

A steady state has $1 + \rho = 1/\beta$, $B_n = 0$, and $E_n = Y_n$. Given fundamentals $\{L_n, z_{is}, a_{ins}, \tau_{ins}, \bar{\chi}_{ns}\}$ and one numeraire, an equilibrium is a vector of wages, rental rates, prices, expenditure shares,

revenues, capital stocks, and asset stocks satisfying:

$$\chi_{ns} = \bar{\chi}_{ns}\left(\frac{P_{ns}}{P_n}\right)^{1-\eta}\left(\frac{Y_n}{P_n L_n}\right)^{(1-\eta)(\varepsilon_s-1)}, \qquad \sum_s \chi_{ns} = 1, \tag{73}$$

$$S_{nis} = \frac{a_{ins}\tau_{ins}^{1-\sigma_s} x_{is}^{\gamma_s(\sigma_s-1)} c_{is}^{-\alpha_s(\sigma_s-1)}}{\sum_j a_{jns}\tau_{jns}^{1-\sigma_s} x_{js}^{\gamma_s(\sigma_s-1)} c_{js}^{-\alpha_s(\sigma_s-1)}}, \tag{74}$$

$$P_{ns}^{1-\sigma_s} = \left(\frac{\sigma_s-1}{\sigma_s}\right)^{\sigma_s-1} \sum_i a_{ins} x_{is}^{\gamma_s(\sigma_s-1)} c_{ins}^{1-\sigma_s}, \tag{75}$$

$$R_{ins} = \chi_{ns} Y_n S_{nis}, \qquad R_{is} = \sum_n R_{ins}, \tag{76}$$

$$Y_i = \sum_{s,n}\left(\alpha_s D_s + \frac{1}{\sigma_s}\right) R_{ins} + \sum_{s,n}(1-\alpha_s) D_s R_{nis}, \tag{77}$$

$$w_i L_i = \sum_s \mu_s \left[(\alpha_s D_s + \phi_s) R_{is} + (1-\alpha_s) D_s \sum_n R_{nis}\right], \tag{78}$$

$$r_i K_i = \sum_s (1-\mu_s) \left[(\alpha_s D_s + \phi_s) R_{is} + (1-\alpha_s) D_s \sum_n R_{nis}\right], \tag{79}$$

$$\phi_s = \frac{\delta_x \upsilon_x \gamma_s}{\rho + \delta_x \upsilon_x} D_s, \qquad x_{is} = \delta_x^{-1/\upsilon_x} \frac{\phi_s R_{is}}{c_{is}}, \tag{80}$$

$$\frac{r_i}{P_i} = \bar{\varepsilon}_i \frac{\rho + \upsilon_K \delta_K}{\upsilon_K \delta_K^{1-1/\upsilon_K}}, \qquad I_i = \delta_K^{1/\upsilon_K} K_i, \tag{81}$$

$$\frac{Y_i}{P_i} = L_i C_i + I_i. \tag{82}$$

A.6 Proof of Proposition 1

Given initial stocks, a planner in the environment of Section 2.1 chooses all quantities directly, subject only to the technologies and resource constraints, to maximize

$$\sum_n \omega_n \sum_{t=0}^{\infty} \beta^t u(C_{n,t}), \qquad \omega_n > 0, \qquad u(C) = \frac{C^{1-1/\psi}}{1-1/\psi} \ \ (u = \ln C \text{ at } \psi = 1), \tag{83}$$

subject to the aggregators (2)–(3), the production technology (5), the accumulation laws (6) and (7), and the resource constraints (8) and (9)–(10), with no prices, budget constraints, or balanced-trade requirement. An allocation that attains the maximum in (83) for some strictly positive weights is Pareto efficient, since a feasible allocation that raised one country's lifetime utility without

lowering another's would raise the weighted sum. Under the maintained restriction $\gamma_s(\sigma_s - 1) < 1$, each firm's contribution to (3), $a_{ins}^{1/\sigma_s} y_{ins}^{(\sigma_s-1)/\sigma_s}$, is by (5) Cobb–Douglas in the firm's own stock and inputs with exponents summing to $(1+\gamma_s)(\sigma_s-1)/\sigma_s < 1$, and is hence strictly concave. The planner therefore allocates identically across the firms of a given origin and sector, and we index a firm by its origin and sector (i, s). Attach a multiplier to each constraint, all discounted by β^t: $\lambda_{n,t}$ to the final-good use (8), $\Pi_{ns,t}$ to the sector bundle (3), $\Psi_{ins,t}$ to the production constraint (5), $Q^x_{is,t}$ and $Q^K_{n,t}$ to the accumulation laws (6) and (7), $w^P_{i,t}$, $r^P_{i,t}$ to labor and capital feasibility (9)–(10), and one multiplier to the aggregator (2), which the share form in (86) eliminates. Differentiating the Lagrangian and using the Cobb–Douglas form of (4) and (5),

$$\text{(consumption)} \qquad \lambda_{n,t} = \omega_n \, u'(C_{n,t})/L_n, \tag{84}$$

$$\text{(investment)} \qquad Q^K_{n,t} = \lambda_{n,t}, \tag{85}$$

$$\text{(sectors)} \qquad \lambda_{n,t} = \bar{\varepsilon}^P_{n,t} \, P^P_{n,t}, \qquad \chi^P_{ns,t} = \bar{\chi}_{ns} \left(\frac{\Pi_{ns,t}}{P^P_{n,t}}\right)^{1-\eta} \left(\frac{\mathcal{Y}_{n,t}}{L_n}\right)^{(1-\eta)(\varepsilon_s-1)}, \tag{86}$$

$$\text{(varieties)} \qquad \Psi_{ins,t} = \Pi_{ns,t} \, a_{ins,t}^{1/\sigma_s} \left(\mathcal{Y}_{ns,t}/y_{ins,t}\right)^{1/\sigma_s}, \tag{87}$$

$$\text{(rival inputs)} \qquad w^P_{i,t} \, l^o_{ins,t} = \mu_s \alpha_s \, \Psi_{ins,t} \, y_{ins,t}, \quad \text{and likewise for } k^o, l^d, k^d, \tag{88}$$

$$\text{(asset inputs)} \qquad w^P_{i,t} \, l^x_{is,t} = \mu_s \, Q^x_{is,t} \, g^x_{is,t}, \qquad r^P_{i,t} \, k^x_{is,t} = (1-\mu_s) \, Q^x_{is,t} \, g^x_{is,t}, \tag{89}$$

$$\text{(assets)} \qquad Q^x_{is,t} = \beta \left[\gamma_s \frac{\sum_n \Psi_{ins,t+1} \, y_{ins,t+1}}{x_{is,t+1}} + (1-\delta_x) \, Q^x_{is,t+1} \right], \tag{90}$$

$$\text{(capital)} \qquad Q^K_{n,t} = \beta \left[r^P_{n,t+1} + (1-\delta_K) \, Q^K_{n,t+1} \right], \tag{91}$$

where $g^x_{is,t} = g_{is,t}(l^x_{is,t}, k^x_{is,t})$, and $\chi^P_{ns,t} \equiv \Pi_{ns,t}\mathcal{Y}_{ns,t} / \sum_{s'} \Pi_{ns',t}\mathcal{Y}_{ns',t}$, $P^P_{n,t} \equiv \sum_s \Pi_{ns,t}\mathcal{Y}_{ns,t}/\mathcal{Y}_{n,t}$, and $\bar{\varepsilon}^P_{n,t} \equiv \sum_s \chi^P_{ns,t}\varepsilon_s$ are the shadow expenditure shares, the shadow price of the final good, and the shadow average expenditure elasticity. Conditions (86) restate the first-order conditions for $\mathcal{Y}_{ns,t}$ and $\mathcal{Y}_{n,t}$ by the algebra of (39)–(42), with $\Pi_{ns,t}$ in place of $P_{ns,t}$. The sum over destinations in (90) collects the production constraints (5) of every market in which $x_{is,t+1}$ appears.

Matching under a common elasticity. Take a competitive-equilibrium steady state with $\sigma_s = \sigma$, and write $\rho \equiv 1/\beta - 1$. Evaluate the planner conditions at its allocation with multipliers built from equilibrium prices,

$$w^P_i = w_i, \quad r^P_i = r_i, \quad \Psi_{ins} = \tfrac{\sigma-1}{\sigma} \, p_{ins}, \quad \Pi_{ns} = \tfrac{\sigma-1}{\sigma} \, P_{ns}, \quad Q^x_{is} = c_{is}, \quad \lambda_n = Q^K_n = \tfrac{\sigma-1}{\sigma} \, \bar{\varepsilon}_n P_n, \tag{92}$$

and welfare weights chosen so that (84) holds,

$$\omega_n = \tfrac{\sigma-1}{\sigma}\, L_n\, C_n^{1/\psi}\, \bar{\varepsilon}_n P_n, \tag{93}$$

which are strictly positive. We now check the remaining planner conditions one by one at this allocation. Dividing (87) by $\frac{\sigma-1}{\sigma}$ gives $p_{ins} = P_{ns}\, a_{ins}^{1/\sigma} (\mathcal{Y}_{ns}/y_{ins})^{1/\sigma}$, which is the inverse demand curve (43). Since $\Psi_{ins} y_{ins} = \frac{\sigma-1}{\sigma} R_{ins}$, condition (88) becomes $w_i l^o_{ins} = \mu_s \alpha_s \frac{\sigma-1}{\sigma} R_{ins}$, which is the firm's input choice (19), and the same holds for k^o, l^d, k^d; with $Q^x_{is} = c_{is}$, conditions (89) likewise reproduce the firm's cost-minimizing choice of asset-creation inputs from (45). Summing over destinations gives $\sum_n \Psi_{ins} y_{ins} = \frac{\sigma-1}{\sigma} R_{is}$, so with $Q^x_{is} = c_{is}$ condition (90) reads $c_{is} = \beta\big[\frac{\gamma_s(\sigma-1)}{\sigma}\frac{R_{is}}{x_{is}} + (1-\delta_x)c_{is}\big]$, which is the firm's accumulation rule (20) at steady state. Because σ is common across sectors, $\Pi_{ns}/\Pi_{ns'} = P_{ns}/P_{ns'}$, so the shadow expenditure shares χ^P_{ns} coincide with the market shares χ_{ns}, the shadow price satisfies $P^P_n = \frac{\sigma-1}{\sigma} P_n$, and the shadow elasticity satisfies $\bar{\varepsilon}^P_n = \bar{\varepsilon}_n$; the two parts of (86) then reduce to the equilibrium share equation (12) and to $\lambda_n = \frac{\sigma-1}{\sigma}\bar{\varepsilon}_n P_n$, which holds by (92). Only the capital condition (91) does not match: at steady state it requires $r_n = (\rho+\delta_K)\lambda_n = (\rho+\delta_K)\frac{\sigma-1}{\sigma}\bar{\varepsilon}_n P_n$, whereas the equilibrium Euler equation (15) gives $r_n = (\rho+\delta_K)\bar{\varepsilon}_n P_n$. These two differ at any finite σ, but when $\mu_s = 1$ in every sector no capital is used and the condition is vacuous, so every planner condition other than (91) holds at the equilibrium allocation.

Efficiency without capital. Set $\mu_s = 1$ in every sector. The rival composite is then $g_{js,t} = z_{js,t}\, l$, new assets are linear in asset-creation labor, and capital enters no technology, so investment is wasteful and $I_{n,t} = 0$ at any optimum, leaving $\mathcal{Y}_{n,t} = L_n C_{n,t}$. The planner's only choices are the labor allocations, and these are subject to the linear feasibility constraints (9), so the feasible set is convex and every quantity on it is bounded. Iterating the accumulation law (6), each stock is affine in the history of asset-creation labor, $x_{is,t} = (1-\delta_x)^t x_{is,0} + \sum_{\tau<t}(1-\delta_x)^{t-1-\tau} z_{is,\tau} l^x_{is,\tau}$. We show that $\ln C_{n,t}$ is a concave function of these labor allocations. By the production technology (5), each bundle term $a_{ins}^{1/\sigma} y_{ins,t}^{(\sigma-1)/\sigma}$ is Cobb–Douglas in $(x_{is,t}, l^o_{ins,t}, l^d_{ins,t})$ with exponents summing to $(1+\gamma_s)(\sigma-1)/\sigma$, which is below one exactly when $\gamma_s(\sigma-1) < 1$; the term is therefore concave in those arguments, and concave in the labor allocations once the affine stock map is substituted in. The sector bundle $\mathcal{Y}_{ns,t}^{(\sigma-1)/\sigma}$ is a sum of such terms over origins, so it too is concave, and $\ln \mathcal{Y}_{ns,t} = \frac{\sigma}{\sigma-1}\ln\big(\mathcal{Y}_{ns,t}^{(\sigma-1)/\sigma}\big)$ is concave as a positive multiple of the logarithm of a positive concave function. It remains to

pass from the sector bundles to the final good. Writing $V = \ln \mathcal{Y}_n$ and $v_s = \ln \mathcal{Y}_{ns}$, we claim that the aggregator (2) makes V a concave and nondecreasing function of $(v_1, \ldots, v_S)$. At $\eta = 1$ this map is linear, $V = \sum_s \bar{\chi}_{ns} v_s$ up to a constant. For $\eta < 1$, V solves $G(V, v) = 1$, where $G(V, v) \equiv \sum_s \bar{\chi}_{ns}^{1/\eta} L_n^{-(1-\eta)(\varepsilon_s - 1)/\eta} \exp\left[\frac{(1-\eta)\varepsilon_s}{\eta} V + \frac{\eta - 1}{\eta} v_s\right]$; each term is the exponential of an affine function of (V, v), so G is convex, increasing in V, and decreasing in each v_s. The sublevel set $\{G \leq 1\}$ is therefore convex, and its upper boundary $V(v)$ is concave and nondecreasing, which proves the claim. Since $\ln C_{n,t} = \ln \mathcal{Y}_{n,t} - \ln L_n$, it is concave in the labor allocations, and jointly concave in the labor allocations and the initial stocks $\{x_{is,0}\}$ because the stock map is linear in both.

With this concavity in hand, we verify that the constant equilibrium path solves the planner's problem. Write the objective as $\sum_n \omega_n \sum_t \beta^t f(\ln C_{n,t})$, where $f(v) = e^{(1-1/\psi)v}/(1 - 1/\psi)$ (and $f(v) = v$ at $\psi = 1$) is increasing and, for $\psi \leq 1$, concave; the objective is then concave in the labor allocations. The constant equilibrium path satisfies (84)–(90) at the multipliers (92) and weights (93), with the capital condition (91) vacuous, and it is interior because $a_{ins} > 0$ and $\tau_{ins} < \infty$. For a concave objective on a convex set, first-order conditions at an interior point are sufficient for optimality over any finite horizon: the concavity inequality against any competing feasible path leaves only the boundary term $\beta^T \sum_{i,s} Q_{is}^x (x_{is,T+1} - x'_{is,T+1})$, where x' are the competing path's stocks, and this term vanishes as $T \to \infty$ because $Q_{is}^x = c_{is}$ is constant and the stocks are bounded. The constant path is therefore a global maximum of (83), and because the weights are strictly positive, the allocation is Pareto efficient. For $\psi > 1$ the utility curvature works against concavity, and the conclusion instead requires $\eta = 1$ together with $(1 - 1/\psi)(1 + \sum_s \bar{\chi}_{ns} \gamma_s) \leq 1$ for all n. Scaling the labor allocations and the initial stocks by a common factor scales each $x_{is,t}$ by that factor, hence each $\mathcal{Y}_{ns,t}$ by $1 + \gamma_s$ and, through the Cobb–Douglas case of (2), $C_{n,t}$ by $1 + \sum_s \bar{\chi}_{ns} \gamma_s$. Thus $C_{n,t}^{1-1/\psi}$ is positive, homogeneous of degree $(1 - 1/\psi)(1 + \sum_s \bar{\chi}_{ns} \gamma_s) \in (0, 1]$, and log-concave, hence quasi-concave, by the concavity of $\ln C_{n,t}$. A positive, quasi-concave function that is homogeneous of degree $\theta \in (0, 1]$ on a convex cone is concave, so, restricted to the planner's fixed initial stocks, the objective is again concave in the labor allocations. The role of $\eta = 1$ is exactly this scaling step: for $\eta < 1$ the solution of (2) is homogeneous along the ray only if $(1 + \gamma_s)/\varepsilon_s$ is common across sectors.

Inefficiency with capital or heterogeneous elasticities, and its correction. We now treat the

two remaining cases, showing that each is inefficient and constructing the subsidy that restores efficiency. Consider first a sector with $\mu_s < 1$ and positive activity, and perturb country n's allocation alone. Investing ϵ more of the final good at date t lowers per-capita consumption by ϵ/L_n through (8), a change of $-\lambda_n\epsilon$ in the objective by (84) and (93). At $t+1$ the extra capital relaxes the constraint (10) by ϵ; assign these services to a home-supplied variety in the sector and lower $I_{n,t+1}$ by $(1-\delta_K)\epsilon$, which leaves $K_{n,t+2}$ and every later quantity unchanged. One unit of capital services raises the final good by $(\partial \mathcal{Y}_n/\partial y)(\partial y/\partial k)$. Here $\partial \mathcal{Y}_n/\partial y = p_{ins}/(\bar{\varepsilon}_n P_n)$ by the inverse demand (43) and the marginal cost of the final good (42), and $r_n = \frac{\sigma-1}{\sigma} p_{ins}\, \partial y/\partial k$ by the firm's rental choice (19), so the extra final good per unit of services is $\frac{\sigma}{\sigma-1} r_n/(\bar{\varepsilon}_n P_n) = \frac{\sigma}{\sigma-1}(\rho+\delta_K)$, using the steady-state rental from (15). Consuming this together with the undepreciated $(1-\delta_K)\epsilon$ raises country n's discounted utility, and no other country's, by

$$\Delta = \lambda_n \epsilon \big[-1 + \beta(1-\delta_K) + \beta \tfrac{\sigma}{\sigma-1}(\rho+\delta_K) \big] = \lambda_n \epsilon\, \beta\, \tfrac{\rho+\delta_K}{\sigma-1} > 0,$$

using $1-\beta(1-\delta_K) = \beta(\rho+\delta_K)$. This increment is strictly positive, so the equilibrium steady state is not Pareto efficient. Efficiency is restored by a capital-income subsidy at rate $1/(\sigma-1)$, financed by a domestic lump-sum tax: the subsidy replaces $r_{n,t+1}$ by $\frac{\sigma}{\sigma-1} r_{n,t+1}$ in the Euler equation (15), so the steady-state rental becomes $r_n = \frac{\sigma-1}{\sigma}(\rho+\delta_K)\bar{\varepsilon}_n P_n$ and (91) now holds. The subsidized steady state then satisfies every planner condition (84)–(91) at the multipliers (92)–(93). Because the final good is converted one for one into capital, the concavity argument of the previous part does not apply, and efficiency in this case is established at the level of the first-order conditions.

Consider next sector-specific σ_s, again with $\mu_s = 1$. The matching of the first part now fails only at the sector condition: conditions (88) and (87) force $\Pi_{ns}/P_{ns} = \frac{\sigma_s-1}{\sigma_s}$, which varies across sectors, so (86) cannot hold. To see the inefficiency directly, move one unit of labor in country n from a home-supplied variety in sector s' to one in sector s. By the firm's labor choice (19) and $\partial \mathcal{Y}_n/\partial y = p_{nns}/(\bar{\varepsilon}_n P_n)$ as above, this changes the final good by $\frac{w_n}{\bar{\varepsilon}_n P_n}\big(\frac{\sigma_s}{\sigma_s-1} - \frac{\sigma_{s'}}{\sigma_{s'}-1}\big)$ per unit of labor moved. Whenever $\sigma_s \neq \sigma_{s'}$ this change is nonzero, so one direction of the move raises country n's consumption alone and the steady state is not Pareto efficient. Efficiency is restored by sector-specific production subsidies that equalize the effective markups across sectors, which reinstate the matching with the common effective markup in place of $\frac{\sigma-1}{\sigma}$ in (92)–(93). The concavity argument

is unaffected, since its exponent sum $(1+\gamma_s)(\sigma_s-1)/\sigma_s < 1$ and the aggregator step each use only the sector's own elasticity, so it continues to hold sector by sector under $\gamma_s(\sigma_s-1) < 1$. Under $\mu_s = 1$ the subsidized steady state is therefore Pareto efficient, and when $\mu_s < 1$ the capital subsidy above is also required, with the same first-order caveat.

Finally, the scale elasticities enter the planner conditions only through the asset condition (90), and that condition matches its equilibrium counterpart for arbitrary $\{\gamma_s\}$ and $\{\sigma_s\}$: with $\Psi_{ins} = \frac{\sigma_s-1}{\sigma_s} p_{ins}$ it reads $c_{is} = \beta\big[\frac{\gamma_s(\sigma_s-1)}{\sigma_s}\frac{R_{is}}{x_{is}} + (1-\delta_x)c_{is}\big]$, which is the firm's accumulation rule (20) at steady state. No distortion therefore involves γ_s, and the two corrective subsidies, the capital subsidy $1/(\sigma-1)$ and the sector subsidies that equalize markups, depend on the elasticities σ_s but not on the scale elasticities γ_s. □

B Log-Linearization and Comparative-Statics Proofs

B.1 The log-linearized system

This subsection proves Proposition 2 by totally differentiating the steady-state system (73)–(82), written in terms of the prices p_{ins} through the pricing rule (47). Let $\tilde{v} \equiv d\log v$, with all shares evaluated at the initial steady state. The shocks are $\{\tilde{z}_{is}, \tilde{\tau}_{ins}, \tilde{\tilde{\chi}}_{ns}, \tilde{L}_n\}$, while the shifters a_{ins} and all parameters are held fixed, so parameter composites such as D_s and ϕ_s are constants. The concavity condition $\gamma_s(\sigma_s-1) < 1$ is not used below: it only ensures that the baseline steady state satisfies the conditions being differentiated.

Prices and assets. Log-differentiating (45) and (46),

$$\tilde{c}_{ins} = \tilde{\tau}_{ins} + \mu_s[\alpha_s\tilde{w}_i + (1-\alpha_s)\tilde{w}_n] + (1-\mu_s)[\alpha_s\tilde{r}_i + (1-\alpha_s)\tilde{r}_n] - \mu_s[\alpha_s\tilde{z}_{is} + (1-\alpha_s)\tilde{z}_{ns}]. \quad (94)$$

Log-differentiating the pricing rule (47),

$$\tilde{p}_{ins} = \tilde{c}_{ins} - \gamma_s\tilde{x}_{is}, \quad (95)$$

and substituting (94) gives equation (22). In (80), x_{is} equals R_{is}/c_{is} times the constant $\delta_x^{-1/\upsilon_x}\phi_s$, so, using (45) for $\tilde{c}_{is}$,

$$\tilde{x}_{is} = \tilde{R}_{is} - \tilde{c}_{is} = \tilde{R}_{is} - \mu_s\tilde{w}_i - (1-\mu_s)\tilde{r}_i + \mu_s\tilde{z}_{is}, \tag{96}$$

which is equation (23).

Price indices and expenditure shares. Log-differentiating the price index (14) at fixed a_{ins} puts weight $a_{ins}(p_{ins}/P_{ns})^{1-\sigma_s} = S_{nis}$ on each $\tilde{p}_{ins}$, by (13):

$$\tilde{P}_{ns} = \sum_i S_{nis}\tilde{p}_{ins}. \tag{97}$$

The system of Proposition 2 substitutes (97) for $\tilde{P}_{ns}$ throughout. Log-differentiating (73),

$$\tilde{\chi}_{ns} = \tilde{\bar{\chi}}_{ns} + (1-\eta)(\tilde{P}_{ns} - \tilde{P}_n) + (1-\eta)(\varepsilon_s - 1)(\tilde{Y}_n - \tilde{P}_n - \tilde{L}_n), \tag{98}$$

which is equation (24). Differentiating the adding-up condition in (73) gives $\sum_s \chi_{ns}\tilde{\chi}_{ns} = 0$. Substituting (98) into it, dividing by $1-\eta$ (here $\eta \neq 1$, with the case $\eta = 1$ treated at the end of this section), and collecting the $\tilde{P}_n$ terms using $\sum_s \chi_{ns}(\varepsilon_s - 1) = \bar{\varepsilon}_n - 1$,

$$\bar{\varepsilon}_n\tilde{P}_n = \frac{1}{1-\eta}\sum_s \chi_{ns}\tilde{\bar{\chi}}_{ns} + \sum_s \chi_{ns}\tilde{P}_{ns} + (\bar{\varepsilon}_n - 1)(\tilde{Y}_n - \tilde{L}_n), \tag{99}$$

which is equation (25).

Revenue. By (13) and (14), $S_{nis} = a_{ins}(p_{ins}/P_{ns})^{1-\sigma_s}$. Log-differentiating at fixed a_{ins},

$$\tilde{S}_{nis} = (1-\sigma_s)(\tilde{p}_{ins} - \tilde{P}_{ns}). \tag{100}$$

Log-differentiating $R_{is} = \sum_n \chi_{ns}Y_nS_{nis}$ from (76), with the weights $T_{ins} = R_{ins}/R_{is}$ defined in Section 3, and substituting (100),

$$\tilde{R}_{is} = \sum_n T_{ins}\left[\tilde{\chi}_{ns} + \tilde{Y}_n + (1-\sigma_s)(\tilde{p}_{ins} - \tilde{P}_{ns})\right], \tag{101}$$

which is equation (26).

Labor, rental rates, and income. Recall the income weights of Section 3,

$$\Phi^o_{is} \equiv (\alpha_s D_s + \phi_s)\frac{R_{is}}{Y_i}, \qquad \Phi^\pi_{is} \equiv \left(\frac{1}{\sigma_s} - \phi_s\right)\frac{R_{is}}{Y_i}, \qquad \Phi^d_{is} \equiv (1-\alpha_s)D_s\chi_{is},$$

with $\lambda^L_i \equiv w_i L_i / Y_i$ and $\lambda^K_i \equiv r_i K_i / Y_i$. In (78), $\sum_n R_{nis} = \chi_{is} Y_i$ by (76), since the shares S_{ins} sum to one over origins. Log-differentiating (78) and dividing by Y_i,

$$\lambda^L_i(\tilde{w}_i + \tilde{L}_i) = \sum_s \mu_s \left[\Phi^o_{is}\tilde{R}_{is} + \Phi^d_{is}(\tilde{\chi}_{is} + \tilde{Y}_i)\right], \tag{102}$$

and dividing by λ^L_i gives equation (27).

In (81), r_i/P_i equals $\bar{\varepsilon}_i$ times a constant. Log-differentiating, with $\tilde{\bar{\varepsilon}}_i$ obtained by differentiating $\bar{\varepsilon}_i = \sum_s \chi_{is}\varepsilon_s$,

$$\tilde{r}_i = \tilde{P}_i + \tilde{\bar{\varepsilon}}_i, \qquad \tilde{\bar{\varepsilon}}_i = \frac{1}{\bar{\varepsilon}_i}\sum_s \chi_{is}\varepsilon_s\tilde{\chi}_{is}, \tag{103}$$

which is equation (28).

Splitting the coefficient $\alpha_s D_s + \frac{1}{\sigma_s}$ in (77) as $(\alpha_s D_s + \phi_s) + (\frac{1}{\sigma_s} - \phi_s)$ and using $\sum_n R_{nis} = \chi_{is} Y_i$,

$$Y_i = \sum_s (\alpha_s D_s + \phi_s) R_{is} + \sum_s \left(\frac{1}{\sigma_s} - \phi_s\right) R_{is} + \sum_s (1-\alpha_s) D_s \chi_{is} Y_i. \tag{104}$$

Differentiating and dividing by Y_i,

$$\tilde{Y}_i = \sum_s (\Phi^o_{is} + \Phi^\pi_{is})\tilde{R}_{is} + \sum_s \Phi^d_{is}(\tilde{\chi}_{is} + \tilde{Y}_i), \tag{105}$$

which is equation (29). Differentiating the numeraire normalization, world GDP $\sum_i Y_i$ constant,

$$0 = \sum_i Y_i \tilde{Y}_i, \tag{106}$$

the numeraire condition stated after the system of Proposition 2. Equations (22)–(29) provide one equation per unknown in $(\tilde{p}_{ins}, \tilde{x}_{is}, \tilde{\chi}_{ns}, \tilde{P}_n, \tilde{R}_{is}, \tilde{w}_i, \tilde{r}_i, \tilde{Y}_i)$, but the income rows are redundant. Multiplying (105) by Y_i, summing over i, substituting (101), and relabeling the destination index

in the Φ^d_{is} term using $\chi_{ns}Y_n = \sum_i R_{ins}$,

$$\sum_i Y_i\tilde{Y}_i = \sum_{s,i,n} R_{ins}\left[\left(\alpha_s D_s + \tfrac{1}{\sigma_s}\right)\left(\tilde{\chi}_{ns} + \tilde{Y}_n + \tilde{S}_{nis}\right) + (1-\alpha_s)D_s\left(\tilde{\chi}_{ns} + \tilde{Y}_n\right)\right].$$

The $\tilde{S}_{nis}$ terms drop because $\sum_i S_{nis}\tilde{S}_{nis} = 0$ by (100) and (97). The remaining coefficient is $\alpha_s D_s + \frac{1}{\sigma_s} + (1-\alpha_s)D_s = 1$, so the right side equals $\sum_{n,s}\chi_{ns}Y_n(\tilde{\chi}_{ns} + \tilde{Y}_n) = \sum_n Y_n\tilde{Y}_n$, using $\sum_s \chi_{ns}\tilde{\chi}_{ns} = 0$, which (98) and (99) together imply. The Y_i-weighted sum of the income rows is therefore an identity given (97)–(101), so any one income row is implied by the remaining equations. Replacing it with (106) yields a square system, whose solution is unique whenever its coefficient matrix is nonsingular.

Capital and consumption. Log-differentiating capital clearing (79), by the same steps as (102),

$$\lambda^K_i(\tilde{r}_i + \tilde{K}_i) = \sum_s (1-\mu_s)\left[\Phi^o_{is}\tilde{R}_{is} + \Phi^d_{is}(\tilde{\chi}_{is} + \tilde{Y}_i)\right], \tag{107}$$

and dividing by λ^K_i gives the expression for $\tilde{K}_i$ in Proposition 2, which determines $\tilde{K}_i$ from the solution of the square system. Since $I_i = \delta_K^{1/\upsilon_K}K_i$ by (81), $\tilde{I}_i = \tilde{K}_i$. Log-differentiating the resource constraint (82) and solving for $\tilde{C}_i$,

$$\tilde{C}_i = \frac{\tilde{Y}_i - \tilde{P}_i - (1-s^C_i)\tilde{I}_i}{s^C_i} - \tilde{L}_i, \qquad s^C_i \equiv \frac{L_iC_i}{Y_i/P_i}, \tag{108}$$

the expression for $\tilde{C}_n$ in Proposition 2, whose s^C_n coincides with s^C_i here because $\mathcal{Y}_i = Y_i/P_i$ in steady state. This proves the proposition for $\eta \neq 1$.

The case $\eta = 1$. It remains to verify the footnote to Proposition 2. At $\eta = 1$ the aggregator (2) is Cobb–Douglas with $\sum_s \bar{\chi}_{ns} = 1$, and ε_s does not enter. Expenditure minimization gives $\chi_{ns} = \bar{\chi}_{ns}$ exactly, so (98) reads $\tilde{\chi}_{ns} = \tilde{\bar{\chi}}_{ns}$, and differentiating $\sum_s \bar{\chi}_{ns} = 1$ forces $\sum_s \chi_{ns}\tilde{\bar{\chi}}_{ns} = 0$. Expenditure is linear in $\mathcal{Y}_n$ at fixed sectoral prices, so marginal and average expenditure per unit of the final good coincide. This sets $\bar{\varepsilon}_i = 1$ in (81), and (103) reads $\tilde{r}_i = \tilde{P}_i$. Finally, P_n is the Cobb–Douglas unit cost $\prod_s (P_{ns}/\bar{\chi}_{ns})^{\bar{\chi}_{ns}}$, and log-differentiating it at fixed $\bar{\chi}_{ns}$ gives $\tilde{P}_n = \sum_s \chi_{ns}\tilde{P}_{ns}$, the $\varepsilon_s = 1$ form of (99). Under the share shocks the unit cost moves in addition by $\sum_s \bar{\chi}_{ns}\tilde{\bar{\chi}}_{ns}\log(P_{ns}/\bar{\chi}_{ns})$, a change of the

aggregator itself. This term feeds back into no other equation when $\mu_s = 1$ for all s, since $\tilde{P}_n$ and $\tilde{r}_n$ then appear only in (99) and (103). All remaining displays are unchanged, which yields the system of the footnote.

B.2 Closed-form specialization responses and proofs of Propositions 3 and 4

Throughout, write ∂_z and ∂_L for the derivatives of a response with respect to a uniform productivity shock $\tilde{z}_{ns} = \tilde{z}_n$ (all s) and a population shock $\tilde{L}_n$ in country n, all other fundamentals held fixed. Let $\mathbf{A}$ be the coefficient matrix of the system of Proposition 2, with one income row – redundant by Walras's law – replaced by the numeraire; assume $\mathbf{A}$ is nonsingular, so $\mathbf{u} = \mathbf{A}^{-1}\mathbf{b}$ is the unique response. Any permutation of countries that leaves the baseline and the shock unchanged permutes the equations and unknowns of $\mathbf{Au} = \mathbf{b}$ while leaving $\mathbf{A}$ and $\mathbf{b}$ unchanged; by uniqueness the permuted solution equals $\mathbf{u}$, so countries interchanged by such a permutation have identical responses. On a symmetric baseline this applies to the unshocked countries and, when the shock is common, to all countries. This symmetry argument is used repeatedly below.

The following proposition records the closed-form responses behind Proposition 3(b) and (c), with $\Xi \equiv (\sigma-1)\left[1-\left(\frac{NH-1}{N-1}\right)^2\right]$ as defined with (30).

Proposition 7 (Closed-form specialization responses)**.** *Under the conditions of Proposition 3(b),*

$$\widetilde{RCA}_{ns} = \frac{\frac{N-2+H}{N-1}(\sigma-1)\left[1+\frac{\delta_x}{\rho+\delta_x}\sum_k \chi_k\gamma_k\right]\sum_k \chi_k(\gamma_s-\gamma_k)\prod_{j\notin\{s,k\}}\left(1-\Xi\gamma_j\right)}{D_H}(\tilde{z}_n+\tilde{L}_n), \tag{109}$$

where

$$D_H \equiv \sum_k \chi_k\left(1+\tfrac{\delta_x}{\rho+\delta_x}\gamma_k\right)\prod_{j\neq k}\left(1-\Xi\gamma_j\right) > 0. \tag{110}$$

Under the conditions of Proposition 3(c), with $\bar{\gamma}_{-s}$ as in (31),

$$\widetilde{RCA}_{ns} = \frac{(1-\chi_s)\left(1+\frac{\delta_x}{\rho+\delta_x}\bar{\gamma}_{-s}\right)\left[NH(\sigma-1)\gamma_s-1\right]+\chi_s\left(1+\frac{\delta_x}{\rho+\delta_x}\gamma_s\right)\left[NH(\sigma-1)\bar{\gamma}_{-s}-1\right]}{N\,D_H}d, \tag{111}$$

where D_H is (110) evaluated for the pair (χ_s,γ_s) and $(1-\chi_s,\bar{\gamma}_{-s})$; the numerator is expression (32).

The RCA response. The Balassa index of Section 3.2 is

$$RCA_{ns} \equiv \frac{X_{ns} \big/ \sum_k X_{nk}}{\sum_i X_{is} \big/ \sum_{i,k} X_{ik}}, \qquad X_{ns} \equiv \sum_{m \neq n} R_{nms}.$$

Log-differentiating it gives

$$\widetilde{RCA}_{ns} = \tilde{X}_{ns} - \sum_k \frac{X_{nk}}{\sum_{k'} X_{nk'}} \tilde{X}_{nk} - \sum_i \frac{X_{is}}{\sum_j X_{js}} \tilde{X}_{is} + \sum_{i,k} \frac{X_{ik}}{\sum_{j,k'} X_{jk'}} \tilde{X}_{ik}, \qquad \tilde{X}_{ns} = \sum_{m \neq n} \frac{R_{nms}}{X_{ns}} \tilde{R}_{nms}, \tag{112}$$

whose only endogenous inputs are the export responses $\tilde{X}_{ns}$. Each bilateral revenue response is $\tilde{R}_{nms} = \tilde{\chi}_{ms} + \tilde{Y}_m + \tilde{S}_{mns}$, from the bilateral relation $R_{nms} = \chi_{ms} Y_m S_{mns}$ underlying (101), with $\tilde{S}_{mns}$ given by (100); hence it is a fixed linear combination of $\mathbf{u}$. Collecting the export weights of (112) in a vector $\mathbf{q}_{ns}$ gives $\widetilde{RCA}_{ns} = \mathbf{q}'_{ns}\mathbf{u} = \mathbf{q}'_{ns}\mathbf{A}^{-1}\mathbf{b}$.

Productivity versus population (Proposition 3(a)). Change the unknown $\tilde{w}_n$ to the *effective wage* $\check{w}_n \equiv \tilde{w}_n - \tilde{z}_n$; this relabels one unknown, so $\mathbf{A}$ and $\mathbf{q}_{ns}$ are unchanged. With $\tilde{z}_{ns} = \tilde{z}_n$, every appearance of $\tilde{w}_n$ and $\tilde{z}_n$ in the cost, price, asset, and trade-share rows (94)–(101) occurs in the combination $\tilde{w}_n - \tilde{z}_n = \check{w}_n$ (each productivity term $-\mu_s\alpha_s\tilde{z}_{ns}$ or $-\mu_s(1-\alpha_s)\tilde{z}_{ns}$ pairs with the corresponding wage term), and the labor-clearing row (102) contains $\tilde{w}_n + \tilde{L}_n = \check{w}_n + (\tilde{z}_n + \tilde{L}_n)$. Writing $\check{L}_n \equiv \tilde{z}_n + \tilde{L}_n$ for the effective labor supply, the two shocks therefore enter the system only as

$$\mathbf{A}\,\mathbf{u} = \mathbf{b}_{\check{L}}\,\check{L}_n + \mathbf{b}_L\,\tilde{L}_n, \tag{113}$$

where $\mathbf{b}_{\check{L}}$ enters only the labor-clearing row and $\mathbf{b}_L$ collects the residual $\tilde{L}_n$ terms, which appear *only* through the per-capita arguments of the expenditure-share row (98) and the price-index row (99), with coefficients $-(1-\eta)(\varepsilon_s - 1)$ and $-(\bar{\varepsilon}_n - 1)$. Hence $\partial_z\mathbf{u} = \mathbf{A}^{-1}\mathbf{b}_{\check{L}}$, $\partial_L\mathbf{u} = \mathbf{A}^{-1}(\mathbf{b}_{\check{L}} + \mathbf{b}_L)$, and

$$\partial_L\widetilde{RCA}_{ns} - \partial_z\widetilde{RCA}_{ns} = \mathbf{q}'_{ns}\mathbf{A}^{-1}\mathbf{b}_L. \tag{114}$$

If $\varepsilon_s = 1$ for all s, then $\varepsilon_s - 1 = 0$ and $\bar{\varepsilon}_n = \sum_s \chi_{ns}\varepsilon_s = 1$, so $\mathbf{b}_L = \mathbf{0}$. If $\eta = 1$, the system applies with $\varepsilon_s = 1$ (footnote to Proposition 2), so again $\mathbf{b}_L = \mathbf{0}$. In either case $\partial_z\widetilde{RCA}_{ns} = \partial_L\widetilde{RCA}_{ns}$ by (114), and since the response is linear in the shocks, $\widetilde{RCA}_{ns}$ depends on $(\tilde{z}_n, \tilde{L}_n)$ only through $\tilde{z}_n + \tilde{L}_n$, which

is Proposition 3(a).

The scale channel (Proposition 3(b)). Set $\eta = 1$, $\mu_k = \alpha_k = 1$, and $\sigma_k = \sigma$, with expenditure shares $\chi_{ik} = \chi_k$ common across countries and symmetric baseline trade shares $S_{iik} = H$, $S_{ijk} = \frac{1-H}{N-1}$ for $j \neq i$, so that baseline revenues are $R_{ik} = \sum_n \chi_k Y_n S_{nik} = \chi_k$ and incomes $Y_i = 1$ under the world-GDP numeraire. Since $\Xi \leq \sigma - 1$, the maintained condition $\gamma_k(\sigma - 1) < 1$ implies $\Xi\gamma_k < 1$ for every sector, which is used throughout. With $\eta = 1$ and no expenditure-share shocks, $\tilde{\chi}_{ik} = 0$ by (98), and $\partial_z = \partial_L$ by part (a); the shock enters only as $\check{L}_n = \tilde{z}_n + \tilde{L}_n$.

From the full system to two equations per sector. The unshocked countries are interchangeable, so their responses coincide; and summing (101) over origins at baseline weights gives $\sum_i \tilde{R}_{ik} = \sum_m \tilde{Y}_m = 0$ by the numeraire, so $\tilde{R}_{ik} = -\tilde{R}_{nk}/(N-1)$ and $\tilde{Y}_i = -\tilde{Y}_n/(N-1)$ for $i \neq n$. With $\mu_k = \alpha_k = 1$, equations (94)–(96) make the delivered price destination-independent, $\tilde{p}_{ijk} = (1+\gamma_k)(\tilde{w}_i - \tilde{z}_{ik}) - \gamma_k \tilde{R}_{ik}$ for every destination j; write $\tilde{p}_{ik}$ for this common value. Substituting (100) into (101) and using the baseline shares – the price-index gaps evaluate to $\tilde{p}_{nk} - \tilde{P}_{nk} = (1-H)(\tilde{p}_{nk} - \tilde{p}_{ik})$ in the home market and $\tilde{p}_{nk} - \tilde{P}_{mk} = \frac{N-2+H}{N-1}(\tilde{p}_{nk} - \tilde{p}_{ik})$ in every foreign market $m \neq n$ – the shocked country's revenue deviations satisfy, for each sector k and any $i \neq n$,

$$\tilde{R}_{nk} = \frac{NH-1}{N-1}\,\tilde{Y}_n - (\sigma-1)\frac{(1-H)(N-2+NH)}{N-1}\,(\tilde{p}_{nk} - \tilde{p}_{ik}), \tag{115}$$

$$\tilde{p}_{nk} - \tilde{p}_{ik} = (1+\gamma_k)(\check{w}_n - \tilde{w}_i) - \gamma_k \frac{N}{N-1}\,\tilde{R}_{nk}. \tag{116}$$

Subtracting the labor row (102) of any $i \neq n$ from that of n – here $\Phi^d_{ik} = 0$ and $\Phi^o_{ik} = (D_k + \phi_k)\chi_k$ with $D_k + \phi_k = D_k\big(1 + \frac{\delta_x}{\rho+\delta_x}\gamma_k\big)$, and the common $D_k = (\sigma-1)/\sigma$ cancels against λ^L_i – gives

$$\check{w}_n - \tilde{w}_i = \frac{N}{N-1}\,\frac{\sum_k \chi_k\big(1 + \frac{\delta_x}{\rho+\delta_x}\gamma_k\big)\tilde{R}_{nk}}{\sum_k \chi_k\big(1 + \frac{\delta_x}{\rho+\delta_x}\gamma_k\big)} - \check{L}_n. \tag{117}$$

Eliminating $\tilde{R}_{nk}$ between (115) and (116), and noting $\frac{N}{N-1}(\sigma-1)\frac{(1-H)(N-2+NH)}{N-1} = \Xi$,

$$(1 - \Xi\gamma_k)\,(\tilde{p}_{nk} - \tilde{p}_{ik}) = (1+\gamma_k)(\check{w}_n - \tilde{w}_i) - \gamma_k \frac{N(NH-1)}{(N-1)^2}\,\tilde{Y}_n. \tag{118}$$

For any two sectors s, s', the identities $(1+\gamma_s)(1-\Xi\gamma_{s'}) - (1+\gamma_{s'})(1-\Xi\gamma_s) = (1+\Xi)(\gamma_s - \gamma_{s'})$

and $\gamma_s(1-\Xi\gamma_{s'})-\gamma_{s'}(1-\Xi\gamma_s)=\gamma_s-\gamma_{s'}$ give

$$(\tilde{p}_{ns}-\tilde{p}_{is})-(\tilde{p}_{ns'}-\tilde{p}_{is'})=\frac{(\gamma_s-\gamma_{s'})\left[(1+\Xi)(\check{w}_n-\tilde{w}_i)-\frac{N(NH-1)}{(N-1)^2}\,\tilde{Y}_n\right]}{(1-\Xi\gamma_s)(1-\Xi\gamma_{s'})}, \tag{119}$$

whose bracket is common to all sectors. To determine its sign, solve (116) and (118) for $\tilde{R}_{nk}$ sector by sector, substitute into the wage row (117), and simplify using $(1+\gamma_k)=\left[(1+\Xi)-(1-\Xi\gamma_k)\right]/\Xi$; the terms $\sum_k \chi_k\left(1+\frac{\delta_x}{\rho+\delta_x}\gamma_k\right)(\check{w}_n-\tilde{w}_i)$ cancel between the two sides, leaving

$$(1+\Xi)(\check{w}_n-\tilde{w}_i)-\frac{N(NH-1)}{(N-1)^2}\,\tilde{Y}_n=-\frac{\sum_k \chi_k\left(1+\frac{\delta_x}{\rho+\delta_x}\gamma_k\right)}{\sum_k \frac{\chi_k\left(1+\frac{\delta_x}{\rho+\delta_x}\gamma_k\right)}{1-\Xi\gamma_k}}\,\check{L}_n, \tag{120}$$

whose right-hand side is negative for a positive shock whenever $\Xi\gamma_k<1$ for all k.

From prices to RCA. On the symmetric baseline the export weights in (112) equal χ_k across sectors and $1/N$ across origins, and each origin's exports are split equally across its $N-1$ destinations. Evaluating the bilateral revenue responses $\tilde{R}_{ijk}=\tilde{Y}_j+\tilde{S}_{jik}$ term by term, the export responses are

$$\begin{aligned}\tilde{X}_{nk}&=-\frac{\tilde{Y}_n}{N-1}-\frac{(\sigma-1)(N-2+H)}{N-1}(\tilde{p}_{nk}-\tilde{p}_{ik}),\\ \tilde{X}_{ik}&=\frac{\tilde{Y}_n}{(N-1)^2}+\frac{(\sigma-1)(N-2+H)}{(N-1)^2}(\tilde{p}_{nk}-\tilde{p}_{ik}),\qquad i\neq n,\end{aligned}$$

so the world-sector average $\frac{1}{N}\tilde{X}_{nk}+\frac{N-1}{N}\tilde{X}_{ik}$ is exactly zero for every sector, and differencing the first display across sectors gives

$$\widetilde{RCA}_{ns}-\widetilde{RCA}_{ns'}=-\frac{(\sigma-1)(N-2+H)}{N-1}\left[(\tilde{p}_{ns}-\tilde{p}_{is})-(\tilde{p}_{ns'}-\tilde{p}_{is'})\right]. \tag{121}$$

Combining (119), (120), and (121): whenever $\Xi\gamma_k<1$ for every sector k, every factor multiplying $(\gamma_s-\gamma_{s'})\check{L}_n$ is positive, so $\operatorname{sign}\left(\widetilde{RCA}_{ns}-\widetilde{RCA}_{ns'}\right)=\operatorname{sign}(\gamma_s-\gamma_{s'})$ for a positive shock – the pairwise claim of part (b).

Because the world-sector averages vanish, the third and fourth terms of (112) drop out, so

$\widetilde{RCA}_{ns} = \tilde{X}_{ns} - \sum_k \chi_k \tilde{X}_{nk}$, and the export responses above give

$$\widetilde{RCA}_{ns} = -\frac{(\sigma-1)(N-2+H)}{N-1} \sum_k \chi_k \Big[(\tilde{p}_{ns} - \tilde{p}_{is}) - (\tilde{p}_{nk} - \tilde{p}_{ik}) \Big].$$

Substituting (119) for each pair (s,k) and then (120),

$$\widetilde{RCA}_{ns} = \frac{(\sigma-1)(N-2+H)}{N-1} \, \frac{\sum_k \chi_k \left(1 + \frac{\delta_x}{\rho+\delta_x}\gamma_k\right)}{\sum_k \frac{\chi_k \left(1+\frac{\delta_x}{\rho+\delta_x}\gamma_k\right)}{1-\Xi\gamma_k}} \sum_k \frac{\chi_k(\gamma_s - \gamma_k)}{(1-\Xi\gamma_s)(1-\Xi\gamma_k)} \, \check{L}_n,$$

and multiplying the numerator and the denominator by $\prod_j (1-\Xi\gamma_j)$ yields (109) with D_H as in (110); $D_H > 0$ because $\Xi\gamma_j < 1$ makes each of its terms positive. In the display above, every factor multiplying the final sum is positive, and $1-\Xi\gamma_s > 0$, so $\operatorname{sign} \widetilde{RCA}_{ns} = \operatorname{sign} \sum_k \chi_k(\gamma_s - \gamma_k)/(1-\Xi\gamma_k)$ for a positive shock, which is the sign characterization (30); when the γ_k are not all equal, this sum is positive for the sector with the highest γ_k and negative for the sector with the lowest, so their RCA responses are positive and negative, respectively. Finally, since the third and fourth terms of (112) vanish, $\widetilde{RCA}_{nk} = \tilde{X}_{nk} - \sum_{k'} \chi_{k'} \tilde{X}_{nk'}$ implies $\sum_k \chi_k \widetilde{RCA}_{nk} = 0$; for two sectors this gives $\widetilde{RCA}_{ns} = \chi_{s'} \big(\widetilde{RCA}_{ns} - \widetilde{RCA}_{ns'} \big)$, which part (c) uses.

The taste-shock channel (Proposition 3(c)). Keep the restrictions and baseline of part (b), including the abbreviation Ξ. With homothetic demand, sector expenditure shares are exogenous, $\tilde{\chi}_{ik} = \tilde{\tilde{\chi}}_{ik}$ by (98), and the shock is $\tilde{\tilde{\chi}}_{ns} = d$, $\tilde{\tilde{\chi}}_{nk} = -\frac{\chi_s}{1-\chi_s} d$ for $k \neq s$, in country n only; it satisfies $\sum_k \chi_k \tilde{\tilde{\chi}}_{nk} = 0$, as the $\eta = 1$ system requires.

Split the shock into a component common to all countries, $\tilde{\tilde{\chi}}_{mk} = \tilde{\tilde{\chi}}_{nk}/N$ in every country m, and a remainder that sums to zero across countries. Under the common component all countries' responses coincide (the symmetry argument of the preamble), so producer prices are common across origins and (100) moves no trade shares; with $\tilde{Y}_m = 0$ by the numeraire, all export responses within a sector coincide, and (112) gives $\widetilde{RCA} = 0$. The RCA response is therefore governed by the remainder, to which all response variables refer from here on. Under the remainder the unshocked countries are again interchangeable and, because its share shocks sum to zero across countries, the cross-country relations of part (b) continue to hold. Carrying the demand terms $\tilde{\chi}_{mk}$ through (101),

the revenue row (115) becomes

$$\tilde{R}_{nk} = \frac{NH-1}{N-1}\left(\tfrac{N-1}{N}\tilde{\tilde{X}}_{nk} + \tilde{Y}_n\right) - (\sigma-1)\frac{(1-H)(N-2+NH)}{N-1}\,(\tilde{p}_{nk} - \tilde{p}_{ik}), \tag{122}$$

while (116) and the wage row (117) are unchanged (with $\check{w}_n = \tilde{w}_n$ and $\check{L}_n = 0$, as there is no productivity or population shock). Repeating the steps of part (b), equation (120) becomes

$$(1+\Xi)(\tilde{w}_n - \tilde{w}_i) - \frac{N(NH-1)}{(N-1)^2}\,\tilde{Y}_n = \frac{NH-1}{N-1}\,\frac{\sum_k \frac{\chi_k\left(1+\frac{\delta_x}{\rho+\delta_x}\gamma_k\right)}{1-\Xi\gamma_k}\,\tilde{\tilde{X}}_{nk}}{\sum_k \frac{\chi_k\left(1+\frac{\delta_x}{\rho+\delta_x}\gamma_k\right)}{1-\Xi\gamma_k}}. \tag{123}$$

Evaluating the export responses as in part (b), the demand terms add $-\tilde{\tilde{X}}_{nk}/N$ to $\tilde{X}_{nk}$ and $\tilde{\tilde{X}}_{nk}/[N(N-1)]$ to each $\tilde{X}_{ik}$, so each world-sector average remains zero and the third and fourth terms of (112) again drop out; using $\sum_k \chi_k \tilde{\tilde{X}}_{nk} = 0$,

$$\widetilde{RCA}_{ns} = -\frac{\tilde{\tilde{X}}_{ns}}{N} - \frac{(\sigma-1)(N-2+H)}{N-1}\sum_k \chi_k\Big[(\tilde{p}_{ns} - \tilde{p}_{is}) - (\tilde{p}_{nk} - \tilde{p}_{ik})\Big]. \tag{124}$$

Eliminating $\tilde{R}_{nk}$ between (122) and the price row, as in part (b), gives

$$(1-\Xi\gamma_k)(\tilde{p}_{nk} - \tilde{p}_{ik}) = (1+\gamma_k)(\tilde{w}_n - \tilde{w}_i) - \gamma_k\Big[\frac{NH-1}{N-1}\,\tilde{\tilde{X}}_{nk} + \frac{N(NH-1)}{(N-1)^2}\,\tilde{Y}_n\Big];$$

taking differences across sectors with the identities of part (b),

$$\begin{aligned}&(\tilde{p}_{ns} - \tilde{p}_{is}) - (\tilde{p}_{nk} - \tilde{p}_{ik})\\ &\quad= \frac{(\gamma_s - \gamma_k)\Big[(1+\Xi)(\tilde{w}_n - \tilde{w}_i) - \frac{N(NH-1)}{(N-1)^2}\,\tilde{Y}_n\Big] - \frac{NH-1}{N-1}\Big[\gamma_s\tilde{\tilde{X}}_{ns}(1-\Xi\gamma_k) - \gamma_k\tilde{\tilde{X}}_{nk}(1-\Xi\gamma_s)\Big]}{(1-\Xi\gamma_s)(1-\Xi\gamma_k)}.\end{aligned}$$

Summing over k with weights χ_k and substituting the bracket of (123) – the shocks $\tilde{\tilde{X}}_{nk} = -\frac{\chi_s}{1-\chi_s}d$ being common across $k \neq s$ – the other sectors enter (124) only through the two sums $\sum_{k\neq s}\chi_k/(1-\Xi\gamma_k)$ and $\sum_{k\neq s}\chi_k\gamma_k/(1-\Xi\gamma_k)$. A two-sector economy in which the second sector has expenditure share $1-\chi_s$, the same shock, and scale elasticity $\bar{\gamma}_{-s}$ of (31) generates the same

two sums: since $\sum_{k\neq s}\chi_k\frac{1-\Xi\gamma_k}{1-\Xi\gamma_k}=1-\chi_s$,

$$\frac{1-\chi_s}{1-\Xi\bar\gamma_{-s}}=\sum_{k\neq s}\frac{\chi_k}{1-\Xi\gamma_k},\qquad \frac{(1-\chi_s)\,\bar\gamma_{-s}}{1-\Xi\bar\gamma_{-s}}=\sum_{k\neq s}\frac{\chi_k\gamma_k}{1-\Xi\gamma_k}.$$

Since the weights $\chi_k/(1-\Xi\gamma_k)$ are positive, $\bar\gamma_{-s}$ is a convex combination of $\{\gamma_k\}_{k\neq s}$ and $\Xi\bar\gamma_{-s}<1$. The response therefore equals its value in that two-sector economy. There, $\widetilde{RCA}_{ns}=(1-\chi_s)\big(\widetilde{RCA}_{ns}-\widetilde{RCA}_{ns'}\big)$ by the two-sector identity of part (b), with s' the composite sector; differencing (124) across the two sectors and substituting the sector contrast above with the bracket of (123) gives (111), with D_H of (110) evaluated for the pair (χ_s,γ_s), $(1-\chi_s,\bar\gamma_{-s})$. Since $D_H>0$ as in part (b), the sign of $\widetilde{RCA}_{ns}$ is that of the numerator of (111), which is expression (32).

If $\gamma_k>1/[NH(\sigma-1)]$ for every sector, then $\bar\gamma_{-s}>1/[NH(\sigma-1)]$ as well, because $\bar\gamma_{-s}$ is a convex combination of $\{\gamma_k\}_{k\neq s}$, so both brackets of (32) are positive and the demand shock raises $\widetilde{RCA}_{ns}$ – the sufficient condition of the proposition. For a common $\gamma_k=\gamma$, (31) gives $\bar\gamma_{-s}=\gamma$, and the numerator of (111) factors as $[NH(\sigma-1)\gamma-1]\big[1+\frac{\delta_x}{\rho+\delta_x}\gamma\big]$, so the shock raises $\widetilde{RCA}_{ns}$ if and only if $\gamma>1/[NH(\sigma-1)]$.

Capital and commercial presence (extension of parts (b) and (c)). Let $\mu_k=\mu\in(0,1]$ and $\alpha_k=\alpha\in(0,1]$, keeping the remaining restrictions and baseline of part (b); with $\eta=1$ the rental row reads $\tilde r_i=\tilde P_i$ with $\tilde P_i=\sum_k\chi_k\tilde P_{ik}$ (footnote to Proposition 2). For this extension abbreviate, for any x,

$$\Upsilon(x)\equiv 1+\tfrac{\delta_x}{\rho+\delta_x}\,x+\frac{1-\mu}{\mu}\Big(1+\tfrac{\delta_x}{\rho+\delta_x}\textstyle\sum_j\chi_j\gamma_j\Big)\Big[\bar\alpha\Big(\frac{N(1-H)}{N-1}-\Xi x\Big)-\frac{NH-1}{N-1}\,x\Big],\qquad \bar\alpha\equiv\frac{\alpha}{1+\alpha(\sigma-1)\frac{N-2+NH}{N-1}},$$

in which $\sum_j\chi_j\gamma_j$ is always the average over all S sectors. Then all displays of parts (b) and (c) hold once the weight $1+\frac{\delta_x}{\rho+\delta_x}\gamma_k$ is replaced by $\Upsilon(\gamma_k)$ in (110), and by $\Upsilon(\bar\gamma_{-s})$ and $\Upsilon(\gamma_s)$ in the two numerator weights of (111), while the factor $1+\frac{\delta_x}{\rho+\delta_x}\sum_k\chi_k\gamma_k$ in the numerator of (109) is kept; the conditions become $\Xi\gamma_k<1$ and $\Upsilon(\gamma_k)>0$ for every k. In particular the pairwise rule, the sign condition (30), the composite (31), the sufficient condition of part (c), and the common-γ threshold $1/[NH(\sigma-1)]$ are unchanged; and at $\mu=1$, $\Upsilon(x)=1+\frac{\delta_x}{\rho+\delta_x}x$ for every α, so parts (b) and (c) hold verbatim under commercial presence alone.

The proof is modified as follows. With $\alpha<1$ the delivered price is no longer destination-

independent: by (94)–(96), $\tilde{p}_{ink} = (\alpha+\gamma_k)\tilde{c}_{ik} - \gamma_k\tilde{R}_{ik} + (1-\alpha)\tilde{c}_{nk}$, where $\tilde{c}_{ik} = \mu(\tilde{w}_i - \tilde{z}_{ik}) + (1-\mu)\tilde{r}_i$, and the destination component $(1-\alpha)\tilde{c}_{nk}$ is common across origins and drops out of the trade shares (100), which respond to the origin component $(\alpha+\gamma_k)\tilde{c}_{ik} - \gamma_k\tilde{R}_{ik}$ alone. The revenue and price-gap rows keep the form of (115)–(116), with this component in place of $\tilde{p}_{ik}$, with the cost gap $\mu(\check{w}_n - \tilde{w}_i) + (1-\mu)(\tilde{r}_n - \tilde{r}_i)$ – common to all sectors – in place of the wage gap, and with $(\alpha+\gamma_k)$ in place of $(1+\gamma_k)$. The income rows reduce to $\tilde{Y}_i = \sum_k \chi_k\tilde{R}_{ik}$, because $\alpha_k D_k + 1/\sigma_k = 1-(1-\alpha)D$ is common across sectors under a common σ and $\sum_k \chi_k\tilde{\chi}_{ik} = 0$; substituting this into the Φ^d terms of (102) restores the wage row (117) verbatim. Eliminating $\tilde{R}_{nk}$ sector by sector, the sector contrasts factor exactly as in (119) – with the identity $(\alpha+\gamma_s)(1-\Xi\gamma_{s'}) - (\alpha+\gamma_{s'})(1-\Xi\gamma_s) = (1+\alpha\Xi)(\gamma_s-\gamma_{s'})$ in place of its counterpart – and with the bracket, common to all sectors, now

$$G \equiv (1+\alpha\Xi)\Big[\mu(\check{w}_n - \tilde{w}_i) + (1-\mu)(\tilde{r}_n - \tilde{r}_i)\Big] - \frac{N(NH-1)}{(N-1)^2}\,\tilde{Y}_n.$$

Using $(\alpha+\gamma_k) = \big[(1+\alpha\Xi) - (1-\Xi\gamma_k)\big]/\Xi$, each sector's revenue satisfies

$$\frac{N}{N-1}\,\tilde{R}_{nk} = \Big[\mu(\check{w}_n - \tilde{w}_i) + (1-\mu)(\tilde{r}_n - \tilde{r}_i)\Big] - \frac{G}{1-\Xi\gamma_k},$$

and subtracting the rental rows $\tilde{r}_i = \tilde{P}_i$ of countries n and $i \neq n$ – on the baseline shares each price-index gap equals $\frac{NH-1}{N-1}$ times the origin-component gap plus $(1-\alpha)$ times the cost gap – and substituting the price-gap rows gives

$$\begin{aligned}(\tilde{r}_n - \tilde{r}_i)\Big[1 - (1-\mu)\Big((1-\alpha) + \tfrac{NH-1}{N-1}\big(\alpha + \textstyle\sum_k \chi_k\gamma_k\big)\Big)\Big] \\ = \mu\Big((1-\alpha) + \tfrac{NH-1}{N-1}\big(\alpha + \textstyle\sum_k \chi_k\gamma_k\big)\Big)(\check{w}_n - \tilde{w}_i) - \tfrac{N(NH-1)}{(N-1)^2}\textstyle\sum_k \chi_k\gamma_k\tilde{R}_{nk}.\end{aligned}$$

Substituting the revenue display into the wage row (117), into the rental display, and – through $\tilde{Y}_n = \sum_k \chi_k\tilde{R}_{nk}$ – into the definition of G leaves three linear equations in G, $\check{w}_n - \tilde{w}_i$, and $\tilde{r}_n - \tilde{r}_i$; eliminating the two gaps, using $\sum_k \frac{\chi_k}{1-\Xi\gamma_k} - \Xi\sum_k \frac{\chi_k\gamma_k}{1-\Xi\gamma_k} = 1$ and $1+\alpha\Xi - \frac{NH-1}{N-1} = \big(1-\frac{NH-1}{N-1}\big)\big[1+\alpha(\sigma-1)\big(1+\frac{NH-1}{N-1}\big)\big]$

– the origin of $\bar{\alpha}$ – yields the counterpart of (120),

$$G = -\frac{\sum_k \chi_k \left(1 + \frac{\delta_x}{\rho+\delta_x}\gamma_k\right)}{\sum_k \frac{\chi_k \Upsilon(\gamma_k)}{1-\Xi\gamma_k}}\,\check{L}_n.$$

The RCA responses follow as in part (b): the export responses depend on producer prices only through the origin components, the export weights and the coefficient of (121) are unchanged, and the world-sector averages still vanish, so substituting G into the sector contrasts and repeating the assembly delivers (109) with $\Upsilon(\gamma_k)$ in place of $1 + \frac{\delta_x}{\rho+\delta_x}\gamma_k$ in (110); every factor multiplying $(\gamma_s - \gamma_{s'})\check{L}_n$ is positive under the stated conditions, which gives the pairwise rule and (30). For the taste channel, the decomposition into a common component and a remainder applies word for word – with $\mu < 1$ the common component also moves the rental rate, but equally in every country, so trade shares and RCA are still unmoved. Repeating the elimination with the demand terms carried through shows that the counterpart of (123) holds with the weights $1 + \frac{\delta_x}{\rho+\delta_x}\gamma_k$ replaced by $\Upsilon(\gamma_k)$ in its numerator and denominator, where the numerator replacement uses $\sum_k \chi_k \tilde{\tilde{\chi}}_{nk} = 0$ once more; the composite-sector argument then goes through with (31) unchanged, provided $\sum_j \chi_j\gamma_j$ inside Υ is kept at its full-economy value, and yields (111) with the stated weights. Since Υ is affine, $\Upsilon(\bar{\gamma}_{-s}) > 0$ whenever $\Upsilon(\gamma_k) > 0$ for all k, so the sign of the response is again that of the numerator and the sufficient condition of part (c) is unchanged; at a common γ the factor $\Upsilon(\gamma)$ cancels between the numerator and the denominator, so the threshold $1/[NH(\sigma-1)]$ is independent of μ and α.

Non-homothetic demand without scale (Proposition 4(a)). Take two countries $\{n, n'\}$ and two sectors $\{s, s'\}$ with $\eta < 1$, $\mu_k = \alpha_k = 1$, $\sigma_k = \sigma$, expenditure shares $\chi_{ik} = \chi_k$ common across countries, own trade shares $S_{nnk} = S_{n'n'k} = H \in (\frac{1}{2}, 1)$, a uniform productivity shock $\tilde{z}_{ns} = \tilde{z}_{ns'} = \tilde{z}_n$ and a population shock $\tilde{L}_n$ in country n only, and no other shocks. Summing the goods-clearing conditions (76) over sectors gives $Y_n = HY_n + (1-H)Y_{n'}$, so baseline incomes are equal and the baseline weights are $R_{ik}/Y_i = \chi_k$, $T_{iik} = H$, $T_{ijk} = 1 - H$ for $j \neq i$, and $\bar{\varepsilon}_{n'} = \bar{\varepsilon}_n$. Both parts of the proposition impose a common scale elasticity $\gamma_s = \gamma_{s'} = \gamma$ – equal to zero in part (a) – so we reduce the system once for common $\gamma \geq 0$. With $\alpha_k = 1$, $\Phi^d_{ik} = 0$ and $\Phi^o_{ik} + \Phi^\pi_{ik} = \chi_k$, so the income rows (105) read $\tilde{Y}_i = \sum_k \chi_k \tilde{R}_{ik}$ (the row replaced by the numeraire is implied

by the remaining equations, by Walras's law); the labor rows (102) carry weights $(D_k+\phi_k)\chi_k$, proportional to χ_k because σ and γ are common across sectors, so $\tilde{w}_i+\tilde{L}_i=\sum_k \chi_k\tilde{R}_{ik}=\tilde{Y}_i$ and hence $\tilde{w}_n-\tilde{w}_{n'}=(\tilde{Y}_n-\tilde{Y}_{n'})-\tilde{L}_n$.

From (112) to sector contrasts. With $\mu_k=\alpha_k=1$, delivered prices are destination-independent, $\tilde{p}_{ik}=(1+\gamma)(\tilde{w}_i-\tilde{z}_{ik})-\gamma\tilde{R}_{ik}$, as in the proof of Proposition 3(b). On this baseline the export weights in (112) equal χ_k across sectors and $\frac{1}{2}$ across origins, and each country's exports are its sales to the other, $\tilde{X}_{nk}=\tilde{\chi}_{n'k}+\tilde{Y}_{n'}+(1-\sigma)(\tilde{p}_{nk}-\tilde{P}_{n'k})$, so (112) reduces to $\widetilde{RCA}_{ns}=\frac{\chi_{s'}}{2}\big[(\tilde{X}_{ns}-\tilde{X}_{n's})-(\tilde{X}_{ns'}-\tilde{X}_{n's'})\big]$. Using $\tilde{P}_{nk}-\tilde{P}_{n'k}=(2H-1)(\tilde{p}_{nk}-\tilde{p}_{n'k})$ from (97), the export gaps are $\tilde{X}_{nk}-\tilde{X}_{n'k}=-(\tilde{\chi}_{nk}-\tilde{\chi}_{n'k})-(\tilde{Y}_n-\tilde{Y}_{n'})-2H(\sigma-1)(\tilde{p}_{nk}-\tilde{p}_{n'k})$, and the income terms – common to both sectors – drop out of the sector contrast:

$$\widetilde{RCA}_{ns}=-\frac{\chi_{s'}}{2}\Big[(\tilde{\chi}_{ns}-\tilde{\chi}_{n's})-(\tilde{\chi}_{ns'}-\tilde{\chi}_{n's'})+2H(\sigma-1)\big[(\tilde{p}_{ns}-\tilde{p}_{n's})-(\tilde{p}_{ns'}-\tilde{p}_{n's'})\big]\Big]. \tag{125}$$

Country differences. Because the baseline coefficients are common to the two countries, differencing the rows of Proposition 2 across countries gives, for $k\in\{s,s'\}$,

$$\tilde{p}_{nk}-\tilde{p}_{n'k}=(1+\gamma)\big(\tilde{w}_n-\tilde{w}_{n'}-\tilde{z}_n\big)-\gamma\big(\tilde{R}_{nk}-\tilde{R}_{n'k}\big), \tag{126}$$

$$\tilde{R}_{nk}-\tilde{R}_{n'k}=(2H-1)\big[(\tilde{\chi}_{nk}-\tilde{\chi}_{n'k})+(\tilde{Y}_n-\tilde{Y}_{n'})\big]-4H(1-H)(\sigma-1)\big(\tilde{p}_{nk}-\tilde{p}_{n'k}\big), \tag{127}$$

$$\begin{aligned}\tilde{\chi}_{nk}-\tilde{\chi}_{n'k}=&(1-\eta)\big[(2H-1)(\tilde{p}_{nk}-\tilde{p}_{n'k})-(\tilde{P}_n-\tilde{P}_{n'})\big]\\&+(1-\eta)(\varepsilon_k-1)\big[(\tilde{Y}_n-\tilde{Y}_{n'})-(\tilde{P}_n-\tilde{P}_{n'})-\tilde{L}_n\big],\end{aligned} \tag{128}$$

the first from the price and asset rows (22)–(23) at $\mu_k=\alpha_k=1$, the second from the revenue rows (101) with (100) and $1-(2H-1)^2=4H(1-H)$, the third from the expenditure-share rows (98). Multiplying (127) by χ_k and summing over sectors, the adding-up condition $\sum_k\chi_k\tilde{\chi}_{ik}=0$ and the income rows leave $(\tilde{Y}_n-\tilde{Y}_{n'})\big[1-(2H-1)\big]=-4H(1-H)(\sigma-1)\sum_k\chi_k(\tilde{p}_{nk}-\tilde{p}_{n'k})$; the same weighting of (126), with the income rows and $\tilde{w}_n-\tilde{w}_{n'}=(\tilde{Y}_n-\tilde{Y}_{n'})-\tilde{L}_n$, gives $\sum_k\chi_k(\tilde{p}_{nk}-\tilde{p}_{n'k})=(\tilde{Y}_n-\tilde{Y}_{n'})-(1+\gamma)(\tilde{z}_n+\tilde{L}_n)$. Solving the pair,

$$\tilde{Y}_n-\tilde{Y}_{n'}=-2H(\sigma-1)\sum_k\chi_k\big(\tilde{p}_{nk}-\tilde{p}_{n'k}\big),\qquad\sum_k\chi_k\big(\tilde{p}_{nk}-\tilde{p}_{n'k}\big)=-\frac{(1+\gamma)\big(\tilde{z}_n+\tilde{L}_n\big)}{1+2H(\sigma-1)}. \tag{129}$$

Differencing the price-index rows (99) across countries, $\bar{\varepsilon}_n(\tilde{P}_n - \tilde{P}_{n'}) = (2H-1)\sum_k \chi_k(\tilde{p}_{nk} - \tilde{p}_{n'k}) + (\bar{\varepsilon}_n - 1)(\tilde{Y}_n - \tilde{Y}_{n'} - \tilde{L}_n)$, and substituting both parts of (129),

$$(\tilde{Y}_n - \tilde{Y}_{n'}) - (\tilde{P}_n - \tilde{P}_{n'}) - \tilde{L}_n = \frac{(1+\gamma)(2H\sigma - 1)\,\tilde{z}_n + \big[\gamma(2H\sigma-1) - 2(1-H)\big]\tilde{L}_n}{\bar{\varepsilon}_n\big[1 + 2H(\sigma-1)\big]}. \tag{130}$$

Part (a). Set $\gamma = 0$. Then (126) makes the price gap common across sectors, $\tilde{p}_{nk} - \tilde{p}_{n'k} = \tilde{w}_n - \tilde{w}_{n'} - \tilde{z}_n$, so the price contrast in (125) vanishes and the sector contrast of (128) retains only the ε_k terms:

$$\widetilde{RCA}_{ns} = -\frac{\chi_{s'}}{2}(1-\eta)(\varepsilon_s - \varepsilon_{s'})\big[(\tilde{Y}_n - \tilde{Y}_{n'}) - (\tilde{P}_n - \tilde{P}_{n'}) - \tilde{L}_n\big].$$

Substituting (130) at $\gamma = 0$ gives

$$\widetilde{RCA}_{ns} = \frac{\chi_{s'}(1-\eta)(\varepsilon_s - \varepsilon_{s'})}{2\bar{\varepsilon}_n\big[1 + 2H(\sigma-1)\big]}\Big[-(2H\sigma - 1)\,\tilde{z}_n + 2(1-H)\,\tilde{L}_n\Big],$$

the expression in part (a) of the proposition.

Non-homothetic demand with common scale (Proposition 4(b)). Keep the setup and displays (125)–(130) above, which hold for any common $\gamma \geq 0$. Substituting (127) into (126) eliminates the revenue gaps; differencing the result across sectors removes the wage and income terms, which are common to both sectors, leaving

$$\big[1 - 4H(1-H)(\sigma-1)\gamma\big]\big[(\tilde{p}_{ns} - \tilde{p}_{n's}) - (\tilde{p}_{ns'} - \tilde{p}_{n's'})\big] = -\gamma(2H-1)\big[(\tilde{\chi}_{ns} - \tilde{\chi}_{n's}) - (\tilde{\chi}_{ns'} - \tilde{\chi}_{n's'})\big]. \tag{131}$$

The sector contrast of (128) is

$$\begin{aligned}(\tilde{\chi}_{ns} - \tilde{\chi}_{n's}) - (\tilde{\chi}_{ns'} - \tilde{\chi}_{n's'}) = {} & (1-\eta)(2H-1)\big[(\tilde{p}_{ns} - \tilde{p}_{n's}) - (\tilde{p}_{ns'} - \tilde{p}_{n's'})\big] \\ & + (1-\eta)(\varepsilon_s - \varepsilon_{s'})\big[(\tilde{Y}_n - \tilde{Y}_{n'}) - (\tilde{P}_n - \tilde{P}_{n'}) - \tilde{L}_n\big]. \end{aligned} \tag{132}$$

Substituting (132) into (131) and collecting the price contrast, using $(2H-1)^2 = 1-4H(1-H)$ to write $(1-\eta)(2H-1)^2 - 4H(1-H)(\sigma-1) = (1-\eta) - 4H(1-H)(\sigma-\eta)$,

$$\Big\{1+\gamma\big[(1-\eta)-4H(1-H)(\sigma-\eta)\big]\Big\}\big[(\tilde{p}_{ns}-\tilde{p}_{n's})-(\tilde{p}_{ns'}-\tilde{p}_{n's'})\big] \\ = -\gamma(2H-1)(1-\eta)(\varepsilon_s-\varepsilon_{s'})\big[(\tilde{Y}_n-\tilde{Y}_{n'})-(\tilde{P}_n-\tilde{P}_{n'})-\tilde{L}_n\big]. \quad (133)$$

The brace is positive: $4H(1-H) \le 1$ and $\sigma-\eta = (\sigma-1)+(1-\eta) > 0$ imply $(1-\eta)-4H(1-H)(\sigma-\eta) \ge -(\sigma-1)$, so with $\gamma \ge 0$ the brace is at least $1-\gamma(\sigma-1)$, positive under the maintained condition $\gamma(\sigma-1) < 1$. Dividing (133) by the brace expresses the price contrast as a multiple of the left side of (130); substituting it into (132) does the same for the share contrast, with factor $(1-\eta)(\varepsilon_s-\varepsilon_{s'})\big[1-4H(1-H)(\sigma-1)\gamma\big]$ over the brace, by the same identity. Inserting the two contrasts into (125) and combining their coefficients with $4H(1-H)+2H(2H-1) = 2H$,

$$\widetilde{RCA}_{ns} = \frac{\chi_{s'}}{2}\,\frac{(1-\eta)(\varepsilon_s-\varepsilon_{s'})\big[2H(\sigma-1)\gamma-1\big]}{1+\gamma\big[(1-\eta)-4H(1-H)(\sigma-\eta)\big]}\,\big[(\tilde{Y}_n-\tilde{Y}_{n'})-(\tilde{P}_n-\tilde{P}_{n'})-\tilde{L}_n\big];$$

substituting (130),

$$\widetilde{RCA}_{ns} = \frac{\chi_{s'}(1-\eta)(\varepsilon_s-\varepsilon_{s'})[2H(\sigma-1)\gamma-1]}{2\bar{\varepsilon}_n[1+2H(\sigma-1)]\big\{1+\gamma[(1-\eta)-4H(1-H)(\sigma-\eta)]\big\}} \\ \times\Big[(1+\gamma)(2H\sigma-1)\,\tilde{z}_n + \big(\gamma(2H\sigma-1)-2(1-H)\big)\tilde{L}_n\Big], \quad (134)$$

the expression in part (b) of the proposition. For the sign of the productivity response, every factor multiplying $\tilde{z}_n$ in (134) except $2H(\sigma-1)\gamma-1$ is positive: $\chi_{s'} > 0$, $1-\eta > 0$ and $\varepsilon_s > \varepsilon_{s'}$ by hypothesis, $\bar{\varepsilon}_n > 0$ by $\varepsilon_k > 0$, $1+2H(\sigma-1) > 0$, the denominator is positive as shown above, and $(1+\gamma)(2H\sigma-1) > 0$ by $\gamma \ge 0$, $H > \frac{1}{2}$, and $\sigma > 1$. Hence $\partial_z\widetilde{RCA}_{ns} > 0$ if and only if $\gamma > 1/[2H(\sigma-1)]$, and it is negative below this threshold. □

B.3 Proof of Proposition 5

The trade and autarky steady states both solve (73)–(82) at $v_x = v_K = 1$, with the same $\{L_n, z_{ns}, \bar{\chi}_{ns}\}$ and the same home fundamentals (a_{nns}, τ_{nns}); every relation below is exact at both. Since $\eta = 1$, (73) gives $\chi_{ns} = \bar{\chi}_{ns}$ – the same in both steady states – and $\bar{\varepsilon}_n = 1$. Recall $D_s \equiv$

$(\sigma_s - 1)/\sigma_s$, $\rho = 1/\beta - 1$, and, from (80) at $\upsilon_x = 1$, $\phi_s = \delta_x \gamma_s D_s/(\rho + \delta_x)$ and $x_{ns} = \delta_x^{-1}\phi_s R_{ns}/c_{ns}$. The proof has four steps: the ratios Y_n/R_n, ℓ_n, s_n^C as functions of $\{\vartheta_{ns}, \chi_{ns}\}$ (Step 1); the autarky benchmark (Step 2); an exact relation between P_{ns}, S_{nns}, c_{ns}, and R_{ns} (Step 3); and the solution of the system in hats (Step 4).

Step 1. Summing goods clearing (76) over origins, country n's purchases in sector s are $\sum_m R_{mns} = \chi_{ns} Y_n \sum_m S_{nms} = \chi_{ns} Y_n$ – the sales base of the destination factor payments in (77)–(79). Substituting this into the income identity (77), $Y_n = \sum_s (\alpha_s D_s + 1/\sigma_s) R_{ns} + \sum_s (1 - \alpha_s) D_s \, \chi_{ns} Y_n$; dividing by R_n and solving for Y_n/R_n,

$$\frac{Y_n}{R_n} = \frac{\sum_s (\alpha_s D_s + 1/\sigma_s)\vartheta_{ns}}{1 - \sum_s (1 - \alpha_s) D_s \, \chi_{ns}}. \tag{135}$$

If $\alpha_s = 1$ for every s, the numerator is $\sum_s \vartheta_{ns}(D_s + 1/\sigma_s) = 1$ and the denominator is 1, so $Y_n/R_n = 1$: the last claim of the proposition. The same substitution in labor clearing (78), divided by R_n, gives

$$\ell_n = \sum_s \mu_s (\alpha_s D_s + \phi_s)\vartheta_{ns} + \frac{Y_n}{R_n} \sum_s \mu_s (1 - \alpha_s) D_s \, \chi_{ns}, \tag{136}$$

and capital clearing (79) gives $r_n K_n/R_n$ as the same expression with μ_s replaced by $1 - \mu_s$. At $\upsilon_K = 1$ and $\bar{\varepsilon}_n = 1$, (81) reads $r_n = (\rho + \delta_K) P_n$ and $I_n = \delta_K K_n$. Substituting both into the resource constraint (82) and using $s_n^C \equiv L_n C_n / \mathcal{Y}_n$ with $P_n \mathcal{Y}_n = Y_n$ (11),

$$s_n^C = 1 - \frac{\delta_K K_n P_n}{Y_n} = 1 - \frac{\delta_K}{\rho + \delta_K} \frac{r_n K_n}{Y_n}, \qquad \frac{r_n K_n}{Y_n} = \frac{r_n K_n/R_n}{Y_n/R_n}. \tag{137}$$

Equations (135)–(137) are the explicit functions of $\{\vartheta_{ns}, \chi_{ns}\}$ promised in the proposition.

Step 2. With $\tau_{ins}^{1-\sigma_s} = 0$ for $i \neq n$, (74) gives $S_{nns}^A = 1$, so (76) gives $R_{ns}^A = \chi_{ns} Y_n^A$. Summing over s and using $\sum_s \chi_{ns} = 1$ (73) yields $R_n^A = Y_n^A$, hence

$$\vartheta_{ns}^A = \chi_{ns} \qquad \text{and} \qquad Y_n^A/R_n^A = 1,$$

as claimed in the proposition. (Consistently, at $\vartheta_{ns} = \chi_{ns}$ the numerator of (135) is $\sum_s \chi_{ns}[1 - (1 - \alpha_s) D_s]$, its denominator.) Every autarky ratio of Step 1 follows by replacing ϑ_{ns} with χ_{ns}.

Step 3. Substituting the bilateral cost (46) into the price index (75) and factoring out the destina-

tion component $c_{ns}^{(1-\alpha_s)(1-\sigma_s)}$ leaves, up to the factor $[\alpha_s^{\alpha_s}(1-\alpha_s)^{1-\alpha_s}]^{\sigma_s-1}$, the sum in the denominator of (74), which equals the numerator of S_{nns} divided by S_{nns}. Combining $c_{ns}^{-\alpha_s(\sigma_s-1)} c_{ns}^{(1-\alpha_s)(1-\sigma_s)} = c_{ns}^{1-\sigma_s}$,

$$P_{ns}^{1-\sigma_s} = \left(\frac{\sigma_s-1}{\sigma_s}\right)^{\sigma_s-1} \left[\alpha_s^{\alpha_s}(1-\alpha_s)^{1-\alpha_s}\right]^{\sigma_s-1} \frac{a_{nns}\,\tau_{nns}^{1-\sigma_s}\,x_{ns}^{\gamma_s(\sigma_s-1)}\,c_{ns}^{1-\sigma_s}}{S_{nns}}.$$

Raising to the power $1/(1-\sigma_s)$ and substituting $x_{ns} = \delta_x^{-1}\phi_s R_{ns}/c_{ns}$,

$$P_{ns} = \frac{\sigma_s}{\sigma_s-1} \frac{\tau_{nns}\,a_{nns}^{-1/(\sigma_s-1)}}{\alpha_s^{\alpha_s}(1-\alpha_s)^{1-\alpha_s}} \left(\frac{\delta_x}{\phi_s}\right)^{\gamma_s} S_{nns}^{1/(\sigma_s-1)}\, c_{ns}^{1+\gamma_s}\, R_{ns}^{-\gamma_s}. \tag{138}$$

The prefactor collects parameters and (a_{nns}, τ_{nns}) only, so it is the same in both steady states and cancels from the hat of (138).

Step 4. Taking the ratio of each relation across the two steady states,

$$\begin{aligned}
\hat{w}_n &= (\ell_n/\ell_n^A)\,\hat{R}_n && \text{from } w_n L_n = \ell_n R_n \text{ with } L_n \text{ fixed},\\
\hat{r}_n &= \hat{P}_n && \text{from } r_n = (\rho+\delta_K)P_n \text{ (Step 1)},\\
\hat{c}_{ns} &= \hat{w}_n^{\mu_s}\hat{r}_n^{1-\mu_s} && \text{from (45) with } z_{ns} \text{ fixed},\\
\hat{R}_{ns} &= (\vartheta_{ns}/\chi_{ns})\,\hat{R}_n && \text{from } R_{ns} = \vartheta_{ns}R_n \text{ and } \vartheta_{ns}^A = \chi_{ns} \text{ (Step 2)},\\
\hat{P}_{ns} &= S_{nns}^{1/(\sigma_s-1)}\,\hat{c}_{ns}^{1+\gamma_s}\,\hat{R}_{ns}^{-\gamma_s} && \text{from (138) and } S_{nns}^A = 1 \text{ (Step 2)},\\
\hat{P}_n &= \textstyle\prod_s \hat{P}_{ns}^{\chi_{ns}} && \text{from } P_n = Y_n/\mathcal{Y}_n = \textstyle\prod_s (P_{ns}/\chi_{ns})^{\chi_{ns}},
\end{aligned}$$

where the last line uses $\mathcal{Y}_{ns} = \chi_{ns}Y_n/P_{ns}$, the Cobb–Douglas aggregator (2) at $\eta = 1$, and the invariance of χ_{ns}. Substituting the first four lines into the fifth, taking logs, and aggregating with weights χ_{ns} as in the sixth,

$$\begin{aligned}
\ln\hat{P}_n = \sum_s \frac{\chi_{ns}}{\sigma_s-1}\ln S_{nns} + \Big(\sum_s \chi_{ns}\mu_s(1+\gamma_s)\Big)\Big(\ln\frac{\ell_n}{\ell_n^A} + \ln\hat{R}_n\Big)\\
+ \Big(\sum_s \chi_{ns}(1-\mu_s)(1+\gamma_s)\Big)\ln\hat{P}_n - \sum_s \chi_{ns}\gamma_s\Big(\ln\frac{\vartheta_{ns}}{\chi_{ns}} + \ln\hat{R}_n\Big).
\end{aligned}$$

Because $\sum_s \chi_{ns} = 1$, the net coefficient on $\ln\hat{R}_n$, $\sum_s \chi_{ns}\mu_s(1+\gamma_s) - \sum_s \chi_{ns}\gamma_s$, equals $1 - \sum_s \chi_{ns}(1-\mu_s)(1+\gamma_s) = 1/\mathcal{M}_n$ by the definition (33) of $\mathcal{M}_n$, and this is also the coefficient of $\ln\hat{P}_n$ after collecting its terms on the left; the display therefore determines only $\ln\hat{R}_n - \ln\hat{P}_n$. Rearranging

and exponentiating,

$$\left(\frac{\hat{R}_n}{\hat{P}_n}\right)^{1/\mathcal{M}_n} = \left(\frac{\ell_n}{\ell_n^A}\right)^{-\sum_s \chi_{ns}\mu_s(1+\gamma_s)} \prod_s S_{nns}^{-\chi_{ns}/(\sigma_s-1)} \prod_s \left(\frac{\vartheta_{ns}}{\chi_{ns}}\right)^{\chi_{ns}\gamma_s}. \tag{139}$$

The definition of s_n^C and $P_n\mathcal{Y}_n = Y_n$ give $P_nC_nL_n = s_n^C Y_n$; dividing by R_n and taking the ratio across the two steady states at fixed L_n,

$$\hat{C}_n = \frac{s_n^C}{s_n^{C,A}} \frac{Y_n/R_n}{Y_n^A/R_n^A} \frac{\hat{R}_n}{\hat{P}_n}.$$

The hypothesis $\sum_s \chi_{ns}(1-\mu_s)(1+\gamma_s) < 1$ makes $1/\mathcal{M}_n$ positive, so $\mathcal{M}_n$ is finite and (139) may be raised to the power $\mathcal{M}_n$; substituting the result into the logarithm of the last display, with $Y_n^A/R_n^A = 1$ from Step 2, and grouping terms gives (34).

For the nesting stated after the proposition, set $\gamma_s = 0$, $\mu_s = 1$, $\alpha_s = 1$, and $\sigma_s = \sigma$. Then $\mathcal{M}_n = 1$; $Y_n/R_n = Y_n^A/R_n^A = 1$ by Step 1; $\phi_s = 0$, so (136) gives $\ell_n = \frac{\sigma-1}{\sigma}\sum_s \vartheta_{ns} = \frac{\sigma-1}{\sigma} = \ell_n^A$; $r_nK_n/R_n = 0$, so $s_n^C = s_n^{C,A} = 1$ by (137); and the reallocation term of (34) vanishes at $\gamma_s = 0$. What remains is $\hat{C}_n = \prod_s S_{nns}^{-\chi_{ns}/(\sigma-1)}$. □

B.4 Non-homothetic gains from trade relative to autarky

Proposition 8 (Non-homothetic autarky gains). *Suppose $\eta < 1$, $\sum_s \chi_{ns}(1-\mu_s)(1+\gamma_s) < 1$, and $\bar{\varepsilon}_n - 1 + 1/\mathcal{M}_n \neq 0$. Write $\hat{\bar{\varepsilon}}_n \equiv \bar{\varepsilon}_n/\bar{\varepsilon}_n^A$, and $\hat{\vartheta}_{ns} = \vartheta_{ns}/\chi_{ns}^A$, $\hat{\chi}_{ns} = \chi_{ns}/\chi_{ns}^A$ (closed-economy clearing gives $\vartheta_{ns}^A = \chi_{ns}^A$ and $Y_n^A = R_n^A$). Then*

$$\ln \hat{C}_n = \ln \frac{s_n^C}{s_n^{C,A}} + \frac{1}{\bar{\varepsilon}_n - 1 + 1/\mathcal{M}_n}\left[\frac{1}{\mathcal{M}_n}\ln\frac{Y_n}{R_n} - \left(\frac{1}{\mathcal{M}_n} + \sum_s \chi_{ns}\gamma_s\right)\ln \hat{\ell}_n \right.$$
$$\left. - \left(1 - \frac{1}{\mathcal{M}_n}\right)\ln \hat{\bar{\varepsilon}}_n - \sum_s \frac{\chi_{ns}}{\sigma_s - 1}\ln S_{nns} + \sum_s \chi_{ns}\gamma_s \ln \hat{\vartheta}_{ns} + \sum_s \frac{\chi_{ns}}{1-\eta}\ln \hat{\chi}_{ns}\right]. \tag{140}$$

As $\varepsilon_s \to 1$ and $\eta \to 1$, $\hat{\chi}_{ns} \to 1$, $\bar{\varepsilon}_n \to 1$, $\hat{\bar{\varepsilon}}_n \to 1$, and the denominator $\to 1/\mathcal{M}_n$, so (140) reduces to the logarithm of (34); the term $\sum_s \chi_{ns}\ln\hat{\chi}_{ns}/(1-\eta)$ has a finite limit because $\ln\hat{\chi}_{ns}/(1-\eta) \to (\ln\hat{P}_{ns} - \ln\hat{P}_n) + (\varepsilon_s - 1)\ln\hat{\mathcal{Y}}_n$, whose χ-weighted sum vanishes under the Cobb–Douglas index. It is not an ex post statistic: χ_{ns}^A – hence $\hat{\vartheta}_{ns}$, $\hat{\chi}_{ns}$, $\hat{\bar{\varepsilon}}_n$ – is not observed under non-homothetic demand.

Proof. Hats are ratios of the trade steady state to the autarky steady state of Proposition 5, obtained

by sending $\tau_{ins} \to \infty$ for $i \neq n$ and holding all other fundamentals fixed; shares without hats are trade steady-state values.

Step 1: exact relations between the two steady states. In autarky, goods clearing (76) with $S^A_{nns} = 1$ gives $R^A_{ns} = \chi^A_{ns} Y^A_n$; summing over s gives $R^A_n = Y^A_n$, hence $\vartheta^A_{ns} = \chi^A_{ns}$, as claimed. Taking the ratio of the expenditure-share equation (73) across the two steady states, with $\bar{\chi}_{ns}$ and L_n fixed and $\mathcal{Y}_n = Y_n/P_n$ by financial autarky (Appendix A.5), gives, for every s,

$$\ln \hat{\chi}_{ns} = (1-\eta)\big(\ln \hat{P}_{ns} - \ln \hat{P}_n\big) + (1-\eta)(\varepsilon_s - 1)\ln\big(\hat{Y}_n/\hat{P}_n\big), \tag{141}$$

and taking the ratio of $C_n = s^C_n\, Y_n/(P_n L_n)$, the definition of s^C_n, gives

$$\ln \hat{C}_n = \ln \tfrac{s^C_n}{s^{C,A}_n} + \ln \hat{Y}_n - \ln \hat{P}_n. \tag{142}$$

The steady-state capital Euler equation, (59) at $\Lambda_n = \beta$, is $r_n = (\rho + \delta_K)\bar{\varepsilon}_n P_n$, so $\hat{r}_n = \hat{P}_n \hat{\bar{\varepsilon}}_n$. Combining the sectoral price index with the domestic trade share, (43) and (44) at $i = n$, gives $P_{ns} = S^{1/(\sigma_s-1)}_{nns} a^{-1/(\sigma_s-1)}_{nns} p_{nns}$; substituting the pricing rule (47), $c_{nns} \propto c_{ns}$ from (46) at $i = n$, and $x_{ns} = \delta_x^{-1}\phi_s R_{ns}/c_{ns}$, the $\upsilon_x = 1$ case of (80) with ϕ_s a parameter, gives $P_{ns} \propto S^{1/(\sigma_s-1)}_{nns} c^{1+\gamma_s}_{ns} R^{-\gamma_s}_{ns}$, with a factor of proportionality that involves only a_{nns}, τ_{nns}, and parameters, all unchanged across the two steady states. Taking the ratio, with $S^A_{nns} = 1$, $\hat{c}_{ns} = \hat{w}^{\mu_s}_n \hat{r}^{1-\mu_s}_n$ from (45) with z_{ns} fixed, and $\hat{R}_{ns} = \hat{\vartheta}_{ns}\hat{R}_n$,

$$\hat{P}_{ns} = S^{1/(\sigma_s-1)}_{nns}\big(\hat{w}^{\mu_s}_n (\hat{P}_n \hat{\bar{\varepsilon}}_n)^{1-\mu_s}\big)^{1+\gamma_s}\big(\hat{\vartheta}_{ns}\hat{R}_n\big)^{-\gamma_s}, \tag{143}$$

in which $\hat{w}_n = \hat{\ell}_n \hat{R}_n$ and $\hat{Y}_n = \frac{Y_n}{R_n}\hat{R}_n$, by the definitions of ℓ_n and Y_n/R_n, with L_n fixed and $Y^A_n = R^A_n$ from the opening of this step.

Step 2: the demand envelope. Multiply (141) by $\chi_{ns}/(1-\eta)$ – the hypothesis $\eta < 1$ ensures $\eta \neq 1$ – and sum over s, using $\sum_s \chi_{ns} = 1$ and $\bar{\varepsilon}_n = \sum_s \chi_{ns}\varepsilon_s$:

$$\tfrac{1}{1-\eta}\textstyle\sum_s \chi_{ns} \ln \hat{\chi}_{ns} = \sum_s \chi_{ns} \ln \hat{P}_{ns} - \ln \hat{P}_n + (\bar{\varepsilon}_n - 1)\ln(\hat{Y}_n/\hat{P}_n).$$

Substituting $\ln \hat{P}_n = \ln \frac{s^C_n}{s^{C,A}_n} + \ln \hat{Y}_n - \ln \hat{C}_n$ and $\ln(\hat{Y}_n/\hat{P}_n) = \ln \hat{C}_n - \ln \frac{s^C_n}{s^{C,A}_n}$ from (142) and rearranging

gives

$$\bar{\varepsilon}_n \ln \hat{C}_n = \bar{\varepsilon}_n \ln \tfrac{s_n^C}{s_n^{C,A}} + \ln \hat{Y}_n - \textstyle\sum_s \chi_{ns} \ln \hat{P}_{ns} + \frac{1}{1-\eta} \sum_s \chi_{ns} \ln \hat{\chi}_{ns}. \tag{144}$$

Step 3: elimination of $\ln \hat{R}_n$. Substitute (143), $\hat{w}_n = \hat{\ell}_n \hat{R}_n$, and $\hat{Y}_n = \frac{Y_n}{R_n} \hat{R}_n$ into (144), then eliminate $\ln \hat{P}_n = \ln \frac{s_n^C}{s_n^{C,A}} + \ln \hat{Y}_n - \ln \hat{C}_n$ using (142). The term $\ln \hat{R}_n$ enters through $\ln \hat{Y}_n$, through $\hat{w}_n^{\mu_s(1+\gamma_s)}$, through $(\hat{\vartheta}_{ns} \hat{R}_n)^{-\gamma_s}$, and through the eliminated $\ln \hat{P}_n$; its coefficient is

$$1 - \textstyle\sum_s \chi_{ns} \mu_s (1+\gamma_s) + \sum_s \chi_{ns} \gamma_s - \sum_s \chi_{ns} (1-\mu_s)(1+\gamma_s) = 1 + \sum_s \chi_{ns} \gamma_s - \sum_s \chi_{ns} (1+\gamma_s) = 0,$$

so $\ln \hat{R}_n$ drops out. Collecting the remaining terms, with $1/\mathcal{M}_n = 1 - \sum_s \chi_{ns}(1-\mu_s)(1+\gamma_s)$, which is positive by hypothesis,

$$\begin{aligned}\left(\bar{\varepsilon}_n - 1 + \tfrac{1}{\mathcal{M}_n}\right)\left(\ln \hat{C}_n - \ln \tfrac{s_n^C}{s_n^{C,A}}\right) = \tfrac{1}{\mathcal{M}_n} \ln \tfrac{Y_n}{R_n} - \sum_s \chi_{ns} \mu_s (1+\gamma_s) \ln \hat{\ell}_n - \left(1 - \tfrac{1}{\mathcal{M}_n}\right) \ln \hat{\bar{\varepsilon}}_n \\ - \sum_s \frac{\chi_{ns}}{\sigma_s - 1} \ln S_{nns} + \sum_s \chi_{ns} \gamma_s \ln \hat{\vartheta}_{ns} + \frac{1}{1-\eta} \sum_s \chi_{ns} \ln \hat{\chi}_{ns}.\end{aligned}$$

Since $\sum_s \chi_{ns} \mu_s (1+\gamma_s) = \frac{1}{\mathcal{M}_n} + \sum_s \chi_{ns} \gamma_s$, dividing by $\bar{\varepsilon}_n - 1 + 1/\mathcal{M}_n$, nonzero by hypothesis, gives (140).

Step 4: the homothetic limit. Let $\varepsilon_s \to 1$ for all s and $\eta \to 1$. At $\eta = 1$, equation (73) gives $\chi_{ns} = \bar{\chi}_{ns}$ in both steady states, so $\hat{\chi}_{ns} \to 1$; at $\varepsilon_s = 1$, $\bar{\varepsilon}_n = \bar{\varepsilon}_n^A = 1$, so $\hat{\bar{\varepsilon}}_n \to 1$, the term $(1 - 1/\mathcal{M}_n) \ln \hat{\bar{\varepsilon}}_n$ vanishes, and the denominator tends to $1/\mathcal{M}_n$. For the last term of (140), dividing (141) by $1 - \eta$ gives, for every $\eta \neq 1$,

$$\frac{\ln \hat{\chi}_{ns}}{1-\eta} = \ln \hat{P}_{ns} - \ln \hat{P}_n + (\varepsilon_s - 1) \ln \hat{\mathcal{Y}}_n,$$

the limit expression in the statement; its χ_{ns}-weighted sum equals $\sum_s \chi_{ns} \ln \hat{P}_{ns} - \ln \hat{P}_n + (\bar{\varepsilon}_n - 1) \ln \hat{\mathcal{Y}}_n$, which vanishes in the limit because $\bar{\varepsilon}_n \to 1$ and at $\eta = 1$ the final-good price index is Cobb–Douglas, $\ln \hat{P}_n = \sum_s \chi_{ns} \ln \hat{P}_{ns}$. Equation (140) therefore tends to

$$\ln \hat{C}_n = \ln \tfrac{s_n^C}{s_n^{C,A}} + \mathcal{M}_n \left[\tfrac{1}{\mathcal{M}_n} \ln \tfrac{Y_n}{R_n} - \left(\tfrac{1}{\mathcal{M}_n} + \sum_s \chi_{ns} \gamma_s \right) \ln \hat{\ell}_n - \sum_s \tfrac{\chi_{ns}}{\sigma_s - 1} \ln S_{nns} + \sum_s \chi_{ns} \gamma_s \ln \hat{\vartheta}_{ns} \right],$$

which is the logarithm of (34), since $\frac{1}{\mathcal{M}_n} + \sum_s \chi_{ns} \gamma_s = \sum_s \chi_{ns} \mu_s (1+\gamma_s)$ and $\hat{\vartheta}_{ns} = \vartheta_{ns} / \chi_{ns}$ at $\eta = 1$. □

B.5 Proof of Proposition 6

The symmetric baseline and the shock $\tilde{\tau}_{ins} = -d_s$ for $i \neq n$ are invariant under every permutation of countries, so by the uniqueness argument of Appendix B.2 all responses are common across countries, and the numeraire (106) gives $\tilde{Y}_i = 0$. On the symmetric baseline, $\chi_{ns} = \chi_s$ and S_{nns} are common across countries and $S_{nis} = \frac{1-S_{nns}}{N-1}$ for $i \neq n$, so $\sum_n S_{nis} = 1$ and (76) gives $R_{is} = \chi_s Y_i$, hence $T_{ins} = S_{nis}$ and $R_{is}/Y_i = \chi_s$.

Trade shares and revenues. The wage, rental, and asset responses in (94)–(95) are common across countries – for $\alpha_s < 1$ this includes the destination terms $\tilde{w}_n$ and $\tilde{r}_n$ – so within each sector $\tilde{p}_{ins} - \tilde{\tau}_{ins}$ is the same for every pair (i, n). Subtracting (97), with $\tilde{\tau}_{nns} = 0$ and $\sum_{j\neq n} S_{njs} = 1 - S_{nns}$, gives $\tilde{p}_{ins} - \tilde{P}_{ns} = \tilde{\tau}_{ins} + (1 - S_{nns})d_s$, so (100) yields

$$\tilde{S}_{nns} = (1-\sigma_s)(1-S_{nns})d_s, \qquad \tilde{S}_{nis} = (\sigma_s - 1)S_{nns}d_s \quad (i \neq n).$$

Substituting these into (101), with $\tilde{Y}_n = 0$, $T_{ins} = S_{nis}$, and $\tilde{\chi}_{ns}$ common across countries,

$$\tilde{R}_{is} = \tilde{\chi}_{is} + \left[S_{nns}(1-\sigma_s)(1-S_{nns}) + (1-S_{nns})(\sigma_s - 1)S_{nns}\right]d_s = \tilde{\chi}_{is}.$$

Part (a). At $\eta = 1$ the system applies with $\varepsilon_s = 1$ (footnote to Proposition 2), so with no demand shocks (98) gives $\tilde{\chi}_{ns} = 0$, hence $\tilde{R}_{is} = 0$, and (103) gives $\tilde{r}_i = \tilde{P}_i$. The right-hand side of the labor row (102) vanishes term by term, so with $\tilde{L}_i = 0$ and $\lambda_i^L > 0$, $\tilde{w}_i = 0$. Then (96) gives $\tilde{x}_{is} = -(1-\mu_s)\tilde{P}_i$; substituting this, $\tilde{w}_i = 0$, and $\tilde{r}_i = \tilde{P}_i$ into (94)–(95), with $\tilde{P}_i$ common across countries, gives $\tilde{p}_{ins} = \tilde{\tau}_{ins} + (1+\gamma_s)(1-\mu_s)\tilde{P}_n$, and (97) becomes

$$\tilde{P}_{ns} = -(1-S_{nns})d_s + (1+\gamma_s)(1-\mu_s)\tilde{P}_n.$$

Summing with weights χ_s, the price-index row at $\eta = 1$, $\tilde{P}_n = \sum_s \chi_s \tilde{P}_{ns}$, gives

$$\tilde{P}_n\left[1 - \sum_s \chi_s(1+\gamma_s)(1-\mu_s)\right] = -\sum_s \chi_s(1-S_{nns})d_s.$$

The hypothesis $\sum_s \chi_s(1+\gamma_s)(1-\mu_s) < 1$ makes the bracket positive, so $\tilde{P}_n = -\mathcal{M}_n \sum_s \chi_s(1-S_{nns})d_s$.

The right-hand side of the capital row (107) vanishes for the same reason as the labor row. If $\lambda_n^K > 0$, it gives $\tilde{K}_n = -\tilde{r}_n = -\tilde{P}_n$, and (108) with $\tilde{Y}_n = \tilde{L}_n = 0$ and $\tilde{I}_n = \tilde{K}_n$ gives $s_n^C \tilde{C}_n = -\tilde{P}_n + (1 - s_n^C)\tilde{P}_n$, so $\tilde{C}_n = -\tilde{P}_n$. If instead $\mu_s = 1$ for every s, then $r_n K_n = 0$ by (79), so $s_n^C = 1$ by (82) and (108) again gives $\tilde{C}_n = -\tilde{P}_n$. In both cases $\tilde{C}_n = \mathcal{M}_n \sum_s \chi_s (1 - S_{nns}) d_s$, the expression in part (a) of the proposition.

Part (b). With $\gamma_s = 0$, (80) gives $\phi_s = 0$, so with $R_{is}/Y_i = \chi_s$ and the common $D = (\sigma - 1)/\sigma$ the income weights are $\Phi_{is}^o = \alpha_s D \chi_s$ and $\Phi_{is}^d = (1 - \alpha_s) D \chi_s$. Substituting $\tilde{R}_{is} = \tilde{\chi}_{is}$, $\tilde{Y}_i = 0$, and $\tilde{L}_i = 0$ into the labor row (102), with $\mu_s = 1$,

$$\lambda_i^L \tilde{w}_i = \sum_s \left[\alpha_s D + (1 - \alpha_s) D\right] \chi_s \tilde{\chi}_{is} = D \sum_s \chi_s \tilde{\chi}_{is} = 0$$

by the adding-up condition $\sum_s \chi_{is} \tilde{\chi}_{is} = 0$, so $\tilde{w}_i = 0$ ($\lambda_i^L = D > 0$). This is the only step that uses the common σ: with sector-varying σ_s the weights D_s differ across sectors and the sum need not vanish. With $\mu_s = 1$ the rental rate drops from (94), and with $\gamma_s = 0$ the asset term drops from (95), so $\tilde{p}_{ins} = \tilde{\tau}_{ins}$ and (97) gives $\tilde{P}_{ns} = -(1 - S_{nns}) d_s$. The price-index row (99) with no demand shocks and $\tilde{Y}_n = \tilde{L}_n = 0$ reads $\bar{\varepsilon}_n \tilde{P}_n = \sum_s \chi_s \tilde{P}_{ns}$. Since $\mu_s = 1$ for every s, $s_n^C = 1$ as in part (a), and (108) gives $\tilde{C}_n = \tilde{Y}_n - \tilde{P}_n = -\tilde{P}_n = \left[\sum_s \chi_s (1 - S_{nns}) d_s\right] / \bar{\varepsilon}_n$, the expression in part (b) of the proposition. □

C Data Appendix

This appendix documents the construction of the data used in the quantitative analysis. The central input is the bilateral sales matrix, built from cross-border trade and multinational affiliate activity and measured on a value-added basis.

C.1 Data Sources

Table 5 summarizes the sources.

Table 5: Data Sources

Source	Use	Coverage
Bilateral sales		
OECD Analytical AMNE, domestic MNE module	Domestic and foreign-owned output, exports, and value added by country-sector-year	80 countries, 2008–2020, ISIC sectors
OECD Analytical AMNE, bilateral output of foreign affiliates	Bilateral affiliate output by owner and host country	80 countries, 2008–2020, ISIC sectors
CEPII BACI (Gaulier and Zignago 2010)	Bilateral distribution of direct goods exports	200+ countries, 1994–2022, HS92 6-digit goods, mapped to ISIC sectors
IMF BiTS (Loungani et al. 2017; Li et al. 2025)	Bilateral distribution of direct services exports	200+ countries, 1985–2023, EBOPS 2010, 12 categories mapped to ISIC sectors
Sectoral parameters		
O*NET task database	Occupation-level task descriptions used to classify non-rival work	13,902 tasks with descriptions
BLS OEWS	Occupation-by-sector payroll weights for non-rival shares	US, 2012–2024, NAICS sectors mapped to ISIC sectors
BEA Fixed Assets, Table 3.1I	Intellectual-property stock growth	US, 1995–2022, BEA sectors mapped to ISIC sectors
OECD Input-Output tables	Rival capital shares used to construct μ_s	80 countries, 1995–2022, ISIC sectors
Comin et al. (2021)	Expenditure-elasticity parameters ε_s	Mapped to ISIC sectors
Penn World Table (Feenstra et al. 2015)	Population, used for labor endowments L_n	2019 population

C.2 Bilateral Sales Matrix

The bilateral sales matrix has one entry for each source country, destination country, year, and sector, decomposed into direct exports and affiliate sales.

Output and exports by ownership. The OECD Analytical AMNE tables report output, exports, and value added separately for foreign-owned affiliates, domestically owned multinationals, and other domestic firms. For country i, sector s, and year t, we form domestically owned output and exports as the sum across domestic firm types, and total value added across all three,

$$\begin{aligned} \mathrm{GO}^{D}_{is,t} &= \mathrm{GO}^{D_MNE}_{is,t} + \mathrm{GO}^{D_OTH}_{is,t}, \\ X^{D}_{is,t} &= X^{D_MNE}_{is,t} + X^{D_OTH}_{is,t}, \\ \mathrm{VA}_{is,t} &= \mathrm{VA}^{D_MNE}_{is,t} + \mathrm{VA}^{D_OTH}_{is,t} + \mathrm{VA}^{F}_{is,t}. \end{aligned}$$

Foreign-owned output and exports are kept separate because they are needed to remove affiliate exports from affiliate sales.

Direct exports. For each origin-sector-year, BACI and BiTS provide bilateral destination shares. Let $b_{ins,t}$ be the share of origin i's direct exports in sector s that go to destination n. Then

$$X_{ins,t} = X^{D}_{is,t}\, b_{ins,t},$$

using BACI for goods sectors and BiTS for services sectors, with missing bilateral observations treated as zero after the origin-sector-year total is formed.

Affiliate sales. Bilateral affiliate output is converted into destination-market sales by removing affiliate exports in the host market. For owner $i \neq n$, host n, sector s, and year t,

$$A_{ins,t} = \mathrm{FAOutput}_{ins,t} - X^{F}_{ns,t} \frac{\mathrm{FAOutput}_{ins,t}}{\sum_{i' \neq n} \mathrm{FAOutput}_{i'ns,t}}.$$

The diagonal affiliate component is zero; domestic sales are handled by the diagonal formula below.

Domestic diagonal. The diagonal entry is domestic sales to the home market, equal to output of domestic firms net of domestic-firm exports,

$$R_{iis,t} = \mathrm{GO}^{D}_{is,t} - X^{D}_{is,t}.$$

Off the diagonal, $R_{ins,t} = X_{ins,t} + A_{ins,t}$.

Value-added basis. We measure each component on a value-added basis. Let

$$v_{is,t} = \frac{\mathrm{VA}_{is,t}}{\mathrm{GO}_{is,t}}$$

be the sectoral value-added-to-output ratio, where $\mathrm{GO}_{is,t} = \mathrm{GO}^{D}_{is,t} + \mathrm{GO}^{F}_{is,t}$ is total sector output across ownership types. Direct exports are scaled by the source ratio, affiliate sales by the host ratio, and the diagonal by the home ratio,

$$X^{VA}_{ins,t} = v_{is,t} X_{ins,t}, \qquad A^{VA}_{ins,t} = v_{ns,t} A_{ins,t}, \qquad R^{VA}_{iis,t} = v_{is,t} R_{iis,t},$$

with $R^{VA}_{ins,t} = X^{VA}_{ins,t} + A^{VA}_{ins,t}$ off the diagonal.

Sample and aggregation. We use the years 2008–2020 and take 2019 as the baseline cross-section. Economies outside the main sample are collapsed into a single rest-of-world region, and sectors A and B are combined into a single agriculture-and-mining sector. For the combined sector, static parameters are 2019 global-sales-weighted averages, and non-rival payroll levels are summed before the share is recomputed.

C.3 Non-Rival Task Classification

The non-rival payroll share is built from O*NET occupation-level task descriptions. We create two classifications independently: a binary one and a continuous one.

Binary classification. Each task is assigned to one of three classes: non-rival, rival, or ambiguous. A task is non-rival if it contributes to a firm-specific asset, such as software, a product design, a patent, training material, brand capital, or organizational knowledge, that is proprietary, durable, and usable across customers or markets without being depleted. A task is rival if it must be repeated for each customer or is consumed in delivery. Ambiguous tasks are excluded when computing occupation-level shares. The following prompt is given to large language models:

> You are classifying job tasks for an economics research project. For each task, you will determine whether it is non-rival, rival, or ambiguous according to the definitions below.

Core definition. A task is non-rival if spending one hour on it contributes to the creation or improvement of a firm-specific intangible asset that is:

- **Proprietary:** owned or controlled by the firm
- **Non-depletable:** can be used simultaneously across all customers, locations, and markets without being used up
- **Durable:** persists beyond the period in which it was created

Examples of non-rival assets include (but are not limited to): software, code, algorithms, databases, system architectures; product designs, blueprints, engineering specifications, prototypes, schematics; patents, trade secrets, proprietary formulas, new chemical compounds; brand identity, marketing strategies, pricing strategies; training programs, curricula, and educational materials deployable across all firm locations; research findings, methodologies, reusable analytical frameworks, mathematical models; new products, services, processes, or organizational systems; creative works (books, films, music, visual designs, illustrations, animations); standard operating procedures, compliance frameworks, quality systems; flow charts, diagrams, schemas, network architectures, logical designs; new scientific principles, theories, or mathematical methods.

A task is rival if its output is consumed in delivery, tied to a specific customer or transaction, or must be repeated for each instance. Rival tasks do not create or improve a durable intangible asset.

A task is ambiguous if it plausibly contributes to a non-rival asset under some circumstances but not others, or if the task description is too vague to determine. Typical ambiguous cases include management and coordination tasks that may or may not produce reusable organizational knowledge; analysis or planning that could result in either a one-off deliverable or a reusable framework; teaching or training that could be developing new curricula (non-rival) or delivering existing material (rival); supervision tasks where it is unclear whether the supervisor is designing systems or performing routine oversight; attending meetings or conferences; and reviewing applications or proposals.

Decision procedure. For each task, work through these questions in order:

1. Does this task produce or improve an intangible asset (software, design, strategy, training program, creative work, organizational system, research output, etc.) that the firm retains? If clearly no → rival.
2. Can that asset be used across all of the firm's customers, locations, or markets simultaneously without being used up? If clearly no → rival.
3. Does the asset persist and retain value beyond the period it was created? If clearly no → rival.
4. If yes to all three → non-rival.
5. If the answer to any question is uncertain or depends on context → ambiguous.

Important: a task that uses an existing non-rival asset (e.g., deploying software, operating a system) is not itself non-rival unless it adds to or improves the asset. Using is rival; creating or improving is non-rival.

Common classifications.

Non-rival: developing software, writing code, building algorithms, designing system architectures; designing new products, devices, circuits, prototypes, or engineering specifications; creating training programs, curricula, lesson plans, or educational materials deployable across locations; developing pricing strategies, marketing campaigns, brand identities; writing patents, filing IP, developing trade secrets; creating standard operating procedures, compliance frameworks, quality systems; research that produces reusable methodologies, models, datasets, or scientific knowledge; devising flow charts, logical diagrams, schemas, or workflow designs; creating films, books, music, illustrations, visual designs, or other creative works; developing new mathematical principles, theories, or analytical methods; designing conceptual, logical, or physical network/system designs; establishing employee selection programs, organizational standards, or operational procedures; configuring security systems, access controls, or network infrastructure.

Rival: serving individual customers (sales calls, support calls, help desk tickets); physical labor (assembly, installation, welding, cleaning, loading, repair); routine administration tied to specific instances (filing documents, scheduling appointments, processing invoices); transporting goods or people; performing medical/dental/personal care procedures on individual patients; inspecting, testing, or

checking specific items for defects or compliance; troubleshooting specific system malfunctions or user problems; backing up data, performing routine system maintenance; operating or monitoring existing equipment or machinery; training users on how to use existing systems (delivering, not developing, training); providing technical support to individual users; testing specific software installations or hardware configurations (executing test plans, not designing them); collecting samples, counting inventory, taking measurements on specific items; negotiating or processing individual contracts, claims, or transactions; cooking, preparing food, or serving meals; patrolling, guarding, or responding to specific emergencies; entering data, copying records, or performing routine clerical work.

Ambiguous: "plan and coordinate activities" (could be creating reusable systems or one-off coordination); "analyze data to evaluate performance" (could produce reusable methodology or a one-off report); "develop and implement policies" (development is non-rival but implementation may be rival); "oversee department operations"; "confer with staff to discuss issues"; "attend and participate in meetings"; "review applications for mergers or acquisitions"; "recommend interventions or strategies to clients"; "supervise or train other technologists"; "prepare budgets or proposals"; "keep informed of regulations".

Examples with reasoning.

Non-rival examples:

- Developing a new software feature → durable proprietary code reusable across all markets.
- Creating a company-wide training program on safety procedures → reusable curriculum deployable at all firm locations.
- Designing a new product line → proprietary product design usable across all markets.
- Developing a pricing strategy for new markets → reusable strategic framework applicable across markets.
- Writing a screenplay or producing a film → durable creative work exploitable across all markets.
- Shade or color drawings to clarify and emphasize details and dimensions → creates or refines visual design assets the firm retains.
- Develop and implement employee selection and placement programs → creates a reusable organizational system deployable firm-wide.
- Devise flow charts and diagrams describing logical operational steps of programs → creates durable design documentation reusable across projects.
- Conduct research to extend mathematical knowledge → creates new mathematical knowledge, a durable, non-depletable intellectual asset.
- Develop conceptual, logical, or physical network designs → creates proprietary network architecture documents reusable across deployments.
- Establish and implement operational standards and procedures → creates durable organizational standards applicable across all departments.
- Prepare research results for publication in journals, books, manuals, and film → creates durable published research assets reusable indefinitely.
- Select, schedule, or conduct orientation or in-service education programs → develops reusable training programs deployable to all new employees.

Rival examples:

- Live customer support call → service consumed in delivery, not reusable.
- Installing electrical wiring at a construction site → physical labor tied to a specific site.
- Processing an individual insurance claim → tied to a specific transaction, must be repeated.
- Troubleshoot program and system malfunctions → fixes a specific instance; the fix itself is consumed in delivery.
- Provide staff and users with assistance solving computer-related problems → help desk support consumed per interaction.
- Back up network data → routine maintenance tied to a specific point in time; must be repeated.
- Test, maintain, and monitor computer programs and systems → consumed in delivery, tied to specific system states.
- Train users to use new or modified equipment → delivers existing knowledge to specific users.
- Perform cesarean sections or other surgical procedures → medical procedure on a specific patient;

must be repeated per patient.
- Count currency, coins, and checks received for deposit → routine transaction processing.
- Weigh, measure, and mix ingredients according to recipes → physical food preparation tied to a specific batch.
- Walk aisles of planes to verify regulatory compliance → inspection of a specific flight instance.
- Negotiate contracts with farmers for purchase of farm products → tied to a specific transaction with a specific counterparty.

Ambiguous examples:

- Analyze operations to evaluate performance of a company → could produce a reusable analytical framework or a one-off report.
- Confer with board members to discuss issues and coordinate activities → could lead to organizational innovation or be routine coordination.
- Develop and implement policies for organizational operations → policy development is non-rival but implementation is rival; net effect unclear.
- Supervise or train other cardiology technologists or students → could involve developing new training methods or routine supervision.
- Review applications for mergers, acquisitions, or registration of securities → could develop reusable evaluation criteria or be case-by-case assessment.
- Read and analyze specifications to determine how to set equipment controls → interpreting existing documents vs. developing know-how.
- Prepare budgets for approval, including funding or implementation of programs → could create reusable budgeting frameworks or be a one-off budget document.

Output instructions. For each task:

1. In one sentence, identify the primary output of the task.
2. State whether the output meets the three conditions (proprietary, non-depletable, durable), fails them, or is uncertain.
3. Assign the classification: non-rival, rival, or ambiguous.

Then produce a CSV with two columns: `task_id` and `classification`, where `classification` is "non-rival", "rival", or "ambiguous".

For occupation o, let NR_o and R_o be the numbers of non-rival and rival tasks. The occupation-level non-rival share is

$$h_o = \frac{NR_o}{NR_o + R_o}.$$

Continuous classification. Each task is assigned a rivalness score between 0 and 1 reflecting the expected share of the task's output that is rival, so that one minus the score is the task's non-rival share. We define non-rival output as contributing to a firm-specific asset, such as software, a product design, a patent, training material, brand capital, or organizational knowledge, that is proprietary, durable, and usable across customers or markets without being depleted. The following prompt is given to large language models:

You are classifying job tasks for an economics research project. For each task, you will assign a rivalness score between 0 and 1, where:

- 0.00 = purely non-rival (the task's output is entirely a durable, proprietary, non-depletable intangible asset)

- 1.00 = purely rival (the task's output is entirely consumed in delivery, tied to a specific instance, or must be repeated for each occurrence)
- Values between 0 and 1 reflect the expected share of the task's output that is rival (equivalently: 1 minus the probability-weighted share of the hour that contributes to a durable, firm-retained intangible asset).

Core definitions. A task contributes to a non-rival output to the extent that one hour spent on it produces or improves a firm-specific intangible asset that is:

- **Proprietary** — owned or controlled by the firm
- **Non-depletable** — usable simultaneously across all customers, locations, and markets without being used up
- **Durable** — persists beyond the period in which it was created

Examples of non-rival assets: software, code, algorithms, databases, system architectures; product designs, blueprints, engineering specifications, prototypes, schematics; patents, trade secrets, proprietary formulas, new chemical compounds; brand identity, marketing strategies, pricing strategies; training programs, curricula, and educational materials deployable across firm locations; research findings, methodologies, reusable analytical frameworks, mathematical models; new products, services, processes, or organizational systems; creative works (books, films, music, visual designs, illustrations, animations); standard operating procedures, compliance frameworks, quality systems; flow charts, diagrams, schemas, network architectures, logical designs; new scientific principles, theories, or mathematical methods.

A task contributes to a rival output to the extent that its output is consumed in delivery, tied to a specific customer, transaction, or instance, or must be repeated for each occurrence. Rival tasks do not create or improve durable intangible assets.

Important: A task that uses an existing non-rival asset (deploying software, operating a system, delivering an existing curriculum) is rival. Only creating or improving the asset is non-rival.

Scoring rubric. Use the following anchors. Choose the value that best fits; you may use any value in $[0, 1]$, not only the anchors.

- **0.00–0.10** — Almost purely non-rival. Output is essentially all durable intangible asset (writing code, designing a product, creating a curriculum, conducting fundamental research, drafting a patent).
- **0.15–0.30** — Predominantly non-rival, minor rival component. Asset creation is the main output, but some portion is consumed in delivery, or the asset has somewhat narrower reuse than firm-wide (e.g., creates an SOP but also walks one team through it).
- **0.35–0.50** — Mixed, leans non-rival or balanced. Task plausibly produces a reusable framework or document but might instead produce only a one-off deliverable; or the task genuinely splits roughly evenly between asset creation and delivery (e.g., "develop and implement policies").
- **0.50–0.65** — Mixed, leans rival. Task occasionally yields reusable output but is usually one-off or delivery-focused; or supervision/coordination work where reusable system-design is possible but routine oversight is the norm.
- **0.70–0.85** — Predominantly rival, minor non-rival component. Mostly delivery, instance-specific, or repeated work, with a small chance of producing incidental reusable knowledge (e.g., a complex one-off analysis that occasionally seeds a methodology).
- **0.90–1.00** — Almost purely rival. Output is consumed in delivery and must be repeated per instance: live customer support, physical labor, individual medical procedures, routine clerical work, monitoring or operating equipment, troubleshooting specific malfunctions.

When in doubt between two adjacent bands, prefer the value that reflects the modal hour spent on the task across firms and contexts.

Decision procedure. For each task, work through these questions:

1. Asset question. Does an hour on this task plausibly produce or improve an intangible asset (software, design, strategy, curriculum, creative work, organizational system, research output)? If clearly no → score 0.90–1.00.
2. Reuse question. If an asset is produced, can it be used across the firm's customers, locations, or

markets simultaneously without being depleted? If clearly no → score toward the rival end.
3. Durability question. Does the asset persist and retain value beyond the period of creation? If clearly no → score toward the rival end.
4. All three yes, unambiguously → score 0.00–0.10.
5. Mixed or context-dependent → estimate the share of a typical hour spent on durable-asset creation versus delivery/instance-specific work, and place the score accordingly using the rubric.

Bias toward the middle bands (0.35–0.65) when the task description is genuinely vague or the rival/non-rival mix depends heavily on unstated context. Reserve scores ≤ 0.10 and ≥ 0.90 for cases where the task description itself makes the classification near-certain.

Calibration examples.

Score 0.00 — pure asset creation:

- Developing a new software feature → durable proprietary code, reusable across all markets.
- Writing a screenplay or producing a film → durable creative work exploitable across all markets.
- Conducting research to extend mathematical knowledge → durable, non-depletable intellectual asset.
- Designing a new product line → proprietary design usable across markets.

Score 0.05–0.15 — predominantly non-rival:

- Devise flow charts and diagrams describing logical operational steps of programs → durable design documentation, with minor instance-specific framing.
- Develop conceptual, logical, or physical network designs → reusable architecture documents, occasionally tailored per deployment.
- Establish and implement operational standards and procedures → SOP creation dominates; some implementation effort is consumed in delivery.
- Prepare research results for publication in journals, books, manuals → durable published assets; some editing effort is one-off.

Score 0.25–0.40 — leans non-rival, meaningful delivery component:

- Develop and implement employee selection and placement programs → reusable system, but rollout has rival elements.
- Select, schedule, or conduct orientation or in-service education programs → developing programs is non-rival; conducting them is rival.
- Shade or color drawings to clarify details and dimensions → refines a durable visual asset, though portions may be one-off.

Score 0.45–0.55 — genuinely ambiguous:

- Develop and implement policies for organizational operations → development non-rival, implementation rival; net split roughly even.
- Analyze operations to evaluate performance of a company → may produce a reusable framework or a one-off report.
- Confer with board members to discuss issues and coordinate activities → may yield organizational innovation or be routine.
- Prepare budgets for approval, including for funding or implementation of programs → may create reusable templates or be one-off.
- Review applications for mergers, acquisitions, or registration of securities → may develop reusable criteria or be case-by-case.

Score 0.60–0.75 — leans rival:

- Supervise or train other cardiology technologists or students → may develop new training methods, but routine supervision is more typical.
- Read and analyze specifications to determine how to set equipment controls → mostly applies existing know-how to a specific run, with occasional reusable insight.
- Recommend interventions or strategies to clients → mostly one-off advice; sometimes seeds reusable frameworks.

Score 0.85–0.95 — predominantly rival:

- Test, maintain, and monitor computer programs and systems → tied to specific system states; minor chance of producing reusable diagnostics.
- Train users to use new or modified equipment → delivers existing knowledge; rarely creates new training content.
- Troubleshoot program and system malfunctions → instance-specific fix; occasional pattern may inform reusable knowledge.

Score 1.00 — pure delivery / rival:

- Live customer support call → consumed in delivery.
- Installing electrical wiring at a construction site → physical labor tied to a specific site.
- Processing an individual insurance claim → tied to a specific transaction.
- Perform cesarean sections or other surgical procedures → procedure on a specific patient.
- Count currency, coins, and checks received to prepare for deposit → routine transaction processing.
- Weigh, measure, and mix ingredients according to recipes → physical preparation tied to a specific batch.
- Walk aisles of planes to verify passenger compliance with regulations → inspection of a specific flight.
- Negotiate contracts with farmers for purchase of farm products → tied to a specific transaction and counterparty.
- Back up network data → routine, point-in-time, must be repeated.
- Provide staff and users with assistance solving computer-related problems → consumed per interaction.

Output instructions. For each task, internally:

1. In one sentence, identify the primary output of the task.
2. State whether that output is proprietary, non-depletable, and durable; clearly fails one or more of these; or is uncertain.
3. Assign a numeric `rivalness_score` in $[0, 1]$ using the rubric above. Use two decimal places.

Then produce a CSV with two columns: `task_id` and `rivalness_score`.

- `rivalness_score` must be a number between 0.00 and 1.00 inclusive, rounded to two decimal places.
- Do not include the textual rationale in the CSV; the CSV should contain only the two columns.

For occupation o, let N_o be the total number of tasks and let $s_{o,\tau}$ be the rivalness score of task τ performed by occupation o. The occupation-level non-rival share is

$$h_o = 1 - \frac{1}{N_o} \sum_{\tau} s_{o,\tau}.$$

We merge these with BLS payroll weights for the United States, dropping the country subscript in what follows. Letting $\pi_{os,t}$ denote the payroll of occupation o in sector s and year t, the sectoral non-rival payroll share is

$$H_{s,t} = \sum_{o} \frac{\pi_{os,t}}{\sum_{o'} \pi_{o's,t}} h_o.$$

We compute annual sectoral measures for 2012–2024 under both the binary and the continuous classifications, each produced by two large language models – Anthropic Claude Opus 4.6 and OpenAI GPT 5.4. We then take the sector-year average of the resulting four measures before

applying the non-rival-intensity relationship of equation (35).

C.4 Sector Classification

The model has $S = 23$ sectors, listed in Table 6.

Table 6: Sector Classification

Code	Sector	ISIC coverage
AB	Agriculture, forestry, fishing, and mining	A, B
C10T12	Food products, beverages, tobacco	C10–C12
C13T15	Textiles, wearing apparel, leather	C13–C15
C16	Wood and cork products	C16
C17T18	Paper products and printing	C17–C18
C19T21	Petroleum, chemicals, pharmaceuticals	C19–C21
C22	Rubber and plastic products	C22
C23	Other non-metallic minerals	C23
C24	Primary metals	C24
C25	Fabricated metal products	C25
C26T27	Computer, electronic, and electrical equipment	C26–C27
C28	Machinery and equipment n.e.c.	C28
C29T30	Motor vehicles and other transport equipment	C29–C30
F	Construction	F
GH	Wholesale, retail, transport, and storage	G, H
J58T60	Publishing, audiovisual, and broadcasting	J58–J60
J61	Telecommunications	J61
J62T63	IT and other information services	J62–J63
K	Financial and insurance activities	K
L	Real estate activities	L
M	Professional, scientific, and technical activities	M
OS	Other services	I, N–T
U	Utilities	D–E

C.5 Country Sample

The sample comprises 73 economies: 72 entering individually and one rest-of-world aggregate. The individual economies are

ARG, AUS, AUT, BEL, BGD, BGR, BLR, BRA, BRN, CAN, CHE, CHL, CHN, CIV, CMR, COL, CRI, CYP, CZE, DEU, DNK, EGY, ESP, EST, FIN, FRA, GBR, GRC, HRV, HUN, IDN, IND, IRL, ISL, ISR, ITA, JOR, JPN, KAZ, KOR, LAO, LTU, LUX, LVA, MAR, MEX, MYS, NGA, NLD, NOR, NZL, PAK, PER, PHL, POL, PRT, ROU, RUS, SAU, SEN, SGP, SVK, SVN, SWE, THA, TUN, TUR, TWN, UKR, USA, VNM, ZAF.

The rest-of-world aggregate contains HKG, KHM, MLT, and MMR. Population is summed across the same mapping and normalized to sum to one.

D Calibration Appendix

This appendix details the parameterization. Section D.1 lists the externally set parameters and the directly measured sectoral objects. Sections D.2–D.5 document the construction of α_s, μ_s, ε_s, γ_s, σ_s, and the investment parameters. Section D.6 describes the balancing of the bilateral sales matrix, and Section D.7 the inversion of productivities, bilateral trade costs, and taste shifters.

D.1 Parameter Summary

Table 7 reports the baseline parameter values and ranges. The baseline uses non-homothetic demand and data-driven γ_s. We also consider robustness variants that set $\gamma_s = 0$, impose homothetic demand, impose common sectoral expenditure shares, or set $\mu_s = \alpha_s = 1$ to recover a labor-only, purely cross-border benchmark.

D.2 Direct-Export Share α_s

The production parameter α_s is the share of rival inputs hired in the origin country rather than the destination country. A higher α_s corresponds to more cross-border supply, and a lower α_s to more commercial presence through affiliates. We measure α_s as the direct-export share of foreign sales in the 2019 sample,

$$\alpha_s = \frac{\sum_{i,n} X_{ins,2019}}{\sum_{i,n} X_{ins,2019} + \sum_{i,n} A_{ins,2019}}. \tag{145}$$

The sums use the value-added direct-export and affiliate-sales components, computed before balancing, so that α_s reflects the empirical mode mix rather than the balanced matrix used for inversion.

D.3 Rival Labor Share μ_s and Demand Parameters

The parameter μ_s is the labor share in the rival labor-capital composite. We measure the rival capital share of each sector from input-output accounts and set

$$\mu_s = 1 - (\text{rival capital share})_s,$$

Table 7: Parameterization

Parameter	Description	Baseline value	Source / target
Set externally			
β	Discount factor	0.96	Annual calibration
ψ	Elasticity of intertemporal substitution	0.5	Baseline
η	Across-sector elasticity	0.5	Based on Comin et al. (2021)
δ_x	Non-rival asset depreciation	0.20	IP depreciation
υ_x	Non-rival adjustment exponent	0.75	Baseline
δ_K	Physical-capital depreciation	0.05	Baseline
υ_K	Physical-capital adjustment exponent	0.75	Baseline
ζ	Bond-adjustment parameter	0.00	Baseline steady state
L_n	Labor endowment	Country-specific	PWT population, normalized to sum to one
Measured from data			
ε_s	Expenditure elasticity	0.355–2.170	Comin et al. (2021) mapping
μ_s	Rival labor share	0.101–0.716	One minus rival capital share
α_s	Direct-export share	0.000–0.859	Direct exports and affiliate sales, 2019
γ_s	Non-rival intensity	0.070–0.524	O*NET/BLS payroll shares and IP growth
σ_s	Within-sector elasticity	2.765–6.815	Ten percent net operating margin
ϕ_s	Non-rival investment share	0.047–0.262	Steady-state accumulation
Recovered by inversion			
z_{is}	Productivity	Country×sector	Home shares and factor-market clearing
τ_{ins}	Bilateral trade costs	Bilateral-sector	Bilateral sourcing shares
$\bar{\chi}_{ns}$	Taste shifters	Country×sector	Sectoral expenditure shares

Note: The sample has 73 economies and 23 sectors, with 2019 as the baseline year.

using the world capital-share measure. For the combined agriculture-and-mining sector, the underlying shares are averaged using 2019 global sales weights.

The expenditure-elasticity parameters ε_s are the estimates of Comin et al. (2021), mapped to the model sectors and combined for agriculture and mining using the same 2019 sales weights. Manufacturing sectors receive the common manufacturing value, and service and nonmanufacturing sectors the corresponding mapped values.

D.4 Non-Rival Intensity γ_s

We recover γ_s from the firm's investment-share Euler equation (66),

$$\phi_{is,t-1} = \left[\phi_{is,t}\frac{(1-\delta_x)x_{is,t}}{x_{is,t+1}-(1-\delta_x)x_{is,t}} + (1-\upsilon_x)\phi_{is,t} + \upsilon_x\gamma_s D_s\right] \times \Lambda_{i,t-1,t}\,\frac{R_{is,t}}{R_{is,t-1}}\left[1-(1-\delta_x)\frac{x_{is,t-1}}{x_{is,t}}\right].$$

In the calibration with $\zeta = 0$ the bond Euler equation (54) reduces the discount factor (16) to $\Lambda_{i,t-1,t} = 1/(1+\rho_t)$, where ρ_t is the net return between $t-1$ and t.

The investment share maps to the non-rival payroll share of Section 4.3,

$$H_{is,t} = \frac{w_{i,t}\, l^x_{is,t}}{w_{i,t}\left(l^x_{is,t} + \sum_n l^o_{ins,t} + \sum_j l^d_{jis,t}\right)}. \tag{146}$$

The factor demands (19) give the production components of its denominator, $w_{i,t}\sum_n l^o_{ins,t} = \mu_s\alpha_s D_s R_{is,t}$ and $w_{i,t}\sum_j l^d_{jis,t} = \mu_s(1-\alpha_s)D_s\sum_j R_{jis,t}$, while $w_{i,t}l^x_{is,t} = \mu_s c_{is,t}h^x_{is,t} = \mu_s\phi_{is,t}R_{is,t}$. Inserting these into (146), the labor share μ_s cancels and

$$\phi_{is,t} = D_s\,\mathcal{H}_{is,t}, \tag{147}$$

where $\mathcal{H}_{is,t} = \frac{H_{is,t}}{1-H_{is,t}}\left(\alpha_s + (1-\alpha_s)\frac{\sum_j R_{jis,t}}{R_{is,t}}\right)$ is the affiliate-adjusted payroll share of Section 4.3.

Substituting (147) into both sides of (66), the factor D_s is common to every term, the two investment-share terms and the marginal-profit term $\upsilon_x\gamma_s D_s$, and cancels:

$$\mathcal{H}_{is,t-1} = \left[\mathcal{H}_{is,t}\frac{(1-\delta_x)x_{is,t}}{x_{is,t+1}-(1-\delta_x)x_{is,t}} + (1-\upsilon_x)\mathcal{H}_{is,t} + \upsilon_x\gamma_s\right]\frac{1}{1+\rho_t}\,\frac{R_{is,t}}{R_{is,t-1}}\left[1-(1-\delta_x)\frac{x_{is,t-1}}{x_{is,t}}\right].$$

This is linear in γ_s. Solving gives the moment in levels,

$$\gamma_{s,t} = \frac{1}{\upsilon_x}\left[(1+\rho_t)\,\frac{R_{is,t-1}}{R_{is,t}}\,\frac{x_{is,t}}{x_{is,t}-(1-\delta_x)x_{is,t-1}}\,\mathcal{H}_{is,t-1} - \left(\frac{(1-\delta_x)x_{is,t}}{x_{is,t+1}-(1-\delta_x)x_{is,t}} + 1 - \upsilon_x\right)\mathcal{H}_{is,t}\right]. \tag{148}$$

At $\upsilon_x = 1$ this reduces to equation (35) in the main text; the implementation evaluates it with U.S. data and $\upsilon_x = 0.75$. The baseline γ_s is the average of $\gamma_{s,t}$ across the available years.

D.5 IP Depreciation, IP Growth, and Markups

The depreciation rate of non-rival assets is $\delta_x = 0.20$ for all sectors. The IP growth factors and their leads enter only the recovery of γ_s in equation (148). Physical capital uses $\delta_K = 0.05$ and $\upsilon_K = 0.75$.

We target a net profit share after IP investment of ten percent in every sector. With a common net margin $m = 0.10$, the within-sector elasticity is

$$\sigma_s = \frac{1+\gamma_s\delta^*}{m+\gamma_s\delta^*}, \qquad \delta^* = \frac{\delta_x\upsilon_x}{\rho+\delta_x\upsilon_x}. \tag{149}$$

The relation between the σ_s recovered using the common margin and the σ_s recovered using raw sectoral margins from Compustat is shown in Figure 8. The implied non-rival investment share is

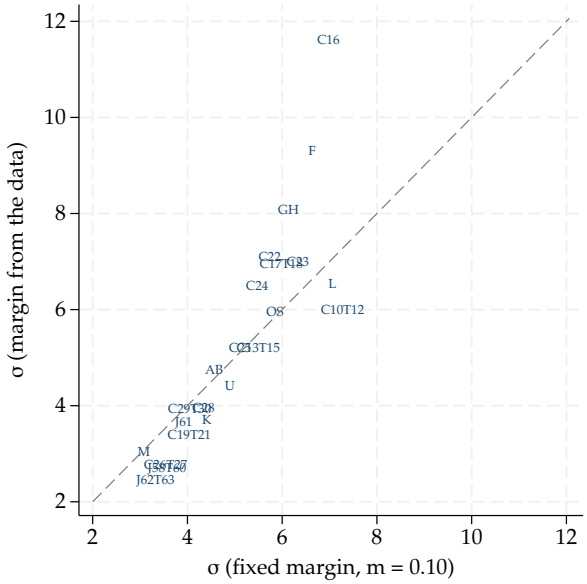
$$\phi_s = \frac{\delta_x\upsilon_x\gamma_s}{\rho+\delta_x\upsilon_x}\,\frac{\sigma_s-1}{\sigma_s}. \tag{150}$$

D.6 Model-Consistent Balancing

Before inversion, we apply a small floor to off-diagonal flows and rebuild the 2019 bilateral matrix to satisfy the model's income–expenditure balance. The floor raises any positive destination-sector off-diagonal flow below 10^{-8} of destination-sector expenditure to that threshold, which prevents exact-zero trade shares from creating home shares of one. The floor applies only to the total bilateral matrix; the direct-export and affiliate-sales components used for α_s are left unscaled.

The balancing step preserves two empirical objects: destination-sector expenditure shares χ_{ns}

Figure 8: Robustness of σ_s



Note: Each point shows the relation between the σ_s recovered using a common net margin $m = 0.10$ (horizontal axis) and the σ_s recovered using the net margin from Compustat data (vertical axis).

and sourcing shares S_{nis}. It chooses country expenditure E_n and rebuilds bilateral sales as

$$R_{ins} = S_{nis}\chi_{ns}E_n. \tag{151}$$

The vector E is chosen so that model income equals expenditure. With $D_s = (\sigma_s - 1)/\sigma_s$, income is

$$Y_i = \sum_s \left(\alpha_s D_s + \frac{1}{\sigma_s}\right) R_{is} + \sum_s (1 - \alpha_s) D_s \chi_{is} E_i, \tag{152}$$

where $R_{is} = \sum_n R_{ins}$. Solving the Perron problem implied by $E_i = Y_i$ yields a maximum relative income imbalance near machine precision (below 10^{-14}).

D.7 Model Inversion

The inversion starts from the balanced matrix R_{ins} and constructs

$$R_{is} = \sum_n R_{ins}, \qquad E_{ns} = \sum_i R_{ins}, \qquad S_{nis} = \frac{R_{ins}}{E_{ns}}, \qquad \chi_{ns} = \frac{E_{ns}}{\sum_{s'} E_{ns'}}.$$

All nominal quantities are normalized so that total revenue equals one. Because balancing imposes $E_n = Y_n$, the inversion uses E_n as both expenditure and income.

We first solve for wages from the wage-bill equation,

$$w_i L_i = \sum_s \mu_s \left(\alpha_s D_s + \phi_s\right) R_{is} + \sum_s \mu_s (1 - \alpha_s) D_s \chi_{is} E_i. \tag{153}$$

We then iterate on the capital rental vector r_i. For a candidate r_i, each sector has an inner fixed point over productivities z_{is} and bilateral trade costs τ_{ins}: τ_{ins} updates from off-diagonal sourcing-share ratios, and z_{is} from home sourcing shares, with one country's productivity normalized in each sector. Given the resulting prices, the NH-CES dual price index is computed in per-capita terms, and the rental vector is updated using

$$r_i = \bar{\varepsilon}_i \frac{r}{P} P_i, \qquad \bar{\varepsilon}_i = \sum_s \chi_{is} \varepsilon_s, \tag{154}$$

where $r/P = (\rho + \upsilon_K \delta_K) \big/ \left(\upsilon_K \delta_K^{1-1/\upsilon_K}\right)$ is the common real rental pinned by the physical-capital Euler equation (58), so the rental update embeds the non-homothetic wedge $\bar{\varepsilon}_i$. After convergence, we recover the taste shifters $\bar{\chi}_{ns}$ so that the model reproduces the observed sectoral expenditure shares χ_{ns}, using per-capita income Y_n/L_n and scaling back by population. The resulting fundamentals satisfy the model's equations and reproduce the balanced 2019 bilateral sales matrix by construction.

D.8 Sectoral Parameter Values

Table 8 reports the baseline values of the measured sectoral parameters – the direct-export share α_s, the within-sector elasticity σ_s, the expenditure elasticity ε_s, the rival labor share μ_s, and the non-rival intensity γ_s – for each of the 23 sectors, grouped by broad sector category and ordered by γ_s within each group.

Table 8: Sectoral Parameters

Sector	α_s	σ_s	ε_s	μ_s	γ_s
Modern services					
IT and other information services	0.47	2.77	1.81	0.60	0.52
Professional, scientific, and technical activities	0.52	2.80	1.68	0.64	0.51
Publishing, audiovisual, and broadcasting activities	0.55	3.00	1.31	0.72	0.45
Telecommunications	0.28	3.58	1.44	0.28	0.32
Financial and insurance activities	0.23	4.15	2.17	0.47	0.24
Traditional services					
Other services	0.54	5.51	1.18	0.70	0.13
Transport, warehousing, and trade	0.60	5.75	1.53	0.51	0.11
Real estate activities	0.31	6.82	2.17	0.10	0.07
Manufacturing					
Manufacture of computer, electronic and optical equipment	0.56	2.93	1.00	0.53	0.47
Manufacture of chemicals, pharmaceuticals and petroleum products	0.39	3.43	1.00	0.39	0.35
Manufacture of transport equipment	0.36	3.43	1.00	0.50	0.35
Manufacture of machinery and equipment	0.61	3.95	1.00	0.58	0.26
Manufacture of fabricated metals	0.59	4.72	1.00	0.55	0.18
Manufacture of textiles, wearing apparel, leather and related products	0.86	4.88	1.00	0.56	0.17
Manufacture of primary metals	0.62	5.08	1.00	0.36	0.15
Manufacture of rubber and plastic products	0.49	5.35	1.00	0.56	0.14
Manufacture of paper and paper products; printing and reproduction of recorded media	0.44	5.37	1.00	0.50	0.14
Manufacture of other non-metallic mineral products	0.30	5.94	1.00	0.45	0.11
Manufacture of wood and of products of wood and cork	0.67	6.58	1.00	0.51	0.08
Manufacture of food products; beverages and tobacco products	0.38	6.66	1.00	0.42	0.08
Other					
Agriculture, hunting, forestry & mining and extraction	0.81	4.21	0.35	0.36	0.23
Utilities	0.00	4.63	1.59	0.29	0.19
Construction	0.04	6.39	1.03	0.57	0.09

Note: Baseline calibration, 23 sectors, 2019. α_s direct-export share, σ_s within-sector elasticity, ε_s expenditure elasticity, μ_s rival labor share, γ_s non-rival intensity. Sectors are grouped by broad category and ordered by γ_s within each group.

E Modern-Services Specialization and the Trade Balance

The transition experiments in Section 5 imply that economies specialized in modern services borrow against gains that arrive later, running current-account deficits and accumulating negative net external positions along the way. This appendix documents the cross-country counterpart of that prediction: countries more specialized in modern services record smaller cumulative trade balances over the same period. A related pattern, linking comparative advantage in services to global imbalances, is documented by Barattieri (2014).

We build a country-by-period panel over 2005–2019, using two window lengths (five-year and ten-year windows). The outcome is a country's cumulative goods-and-services balance over the window, scaled by window-mean GDP, constructed from the IMF Balance of Payments statistics. The regressor of interest is a country's revealed comparative advantage in modern services, measured as the log Balassa index of its modern-services exports (cross-border plus affiliate sales, in value-added terms). Modern services comprise information and communication, finance, and professional and business services. We include the analogous log Balassa indices for manufacturing, traditional services, and mining and agriculture, so that the modern-services coefficient reflects specialization relative to the remaining sectors, together with log GDP per capita and demographic controls (the working-age share, life expectancy, and population growth). The panel specifications include country and period fixed effects, so identification comes from within-country changes in specialization, and standard errors are clustered by country.

Table 9 reports the results. A higher revealed comparative advantage in modern services is associated with a lower cumulative goods-and-services balance. In the ten-year panel, the modern-services coefficient is negative and significant at the one-percent level across specifications, ranging from -0.384 to -0.519. The relationship is stable across the three five-year windows, where the coefficient lies between -0.097 and -0.113 and is significant in each. The estimates are consistent with the model's mechanism: greater specialization in slow-adjusting, non-rival-intensive sectors coincides with net borrowing rather than net lending over the horizon.

Table 9: Modern-Services Specialization and the Cumulative Trade Balance

	Ten-year panel			Five-year window
	(1)	(2)	(3)	(4)
RCA: Modern services	-0.384^{***}	-0.404^{***}	-0.519^{***}	-0.113^{**}
	(0.133)	(0.148)	(0.185)	(0.052)
RCA: Manufacturing	−0.555	−0.582	−0.509	−0.102
	(0.404)	(0.395)	(0.343)	(0.077)
RCA: Traditional services	−0.373	−0.405	−0.001	-0.426^{***}
	(0.301)	(0.323)	(0.334)	(0.107)
RCA: Mining and agriculture	0.045	0.049	−0.059	-0.101^{**}
	(0.185)	(0.185)	(0.172)	(0.040)
Log GDP per capita	No	Yes	Yes	Yes
Demographic controls	No	No	Yes	Yes
Country FE	Yes	Yes	Yes	No
Period FE	Yes	Yes	Yes	No
Num. obs.	150	150	150	75
R^2	0.913	0.914	0.929	0.663

Note: The outcome is the cumulative goods-and-services balance over the window, scaled by window-mean GDP. RCA is the log Balassa index of value-added exports (cross-border plus affiliate sales) by sector group. Columns (1)–(3) are the ten-year panel with country and period fixed effects; standard errors clustered by country are in parentheses. Column (4) is the middle five-year window (2010–2014), estimated by ordinary least squares with heteroskedasticity-robust standard errors; the modern-services coefficient is negative and significant in all three five-year windows. Stars denote significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.