A Macroeconomic Model of the Cross-Section of Currencies*

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Abstract

We study the cross-section of currencies using a quantitative macroeconomic model with heterogeneous countries, segmented asset markets, and risk-averse financial intermediaries. The model admits a two-factor structure of changes in bilateral exchange rates, where the factors reflect a combination of shocks to global output and risk premia. Country exposures to these factors are heterogeneous and determined by the incidence of commodity production and dollar asset holdings. We estimate this factor structure empirically and use these estimates to discipline the model. Our results indicate substantial heterogeneity in exchange rate drivers across countries, which is primarily determined by countries' exposure to risk. Riskier currencies are those of high-commodity countries that are exposed to global business cycle risk and low-dollar countries that are exposed to US-originated fluctuations in the dollar value. Increases in risk premia depreciate all currencies relative to the dollar, with larger depreciations in risky ones. Currencies of countries with high dollar assets and low commodity exports tend to move less relative to the dollar, and their fluctuations are mostly determined by idiosyncratic shocks.

Key Words: currencies, exchange rates, risk, macroeconomy

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1 Introduction

Bilateral exchange rate movements exhibit strong common components with differential sensitivities across countries. When the US dollar's value changes, it tends to move in the same direction against most currencies but with different magnitudes. For example, during the global financial crisis, the Australian dollar depreciated 39% against the US dollar, while the euro depreciated 22% and the Peruvian sol depreciated 8%. A core question in international macroeconomics and finance is what drives these currency movements in the cross-section. This topic has been primarily analyzed in the international finance literature, which documents that most of the variation in bilateral exchange rates can be accounted for by two factors to which countries have heterogeneous exposures (Verdelhan, 2018; Lustig and Richmond, 2020).

In this paper, we develop a model of the world economy to offer a macroeconomic interpretation of the factor structure of exchange rates and their drivers in the cross-section and time series. In the model, the factors reflect a combination of aggregate shocks to output and the risk-taking capacity of global intermediaries. Countries' exposure to these factors is determined by the incidence of commodity production and dollar asset holdings. We use empirical estimates of this factor structure to discipline the model, adding a cross-sectional dimension to the standard set of macroeconomic moments. Our quantitative analysis indicates substantial heterogeneity in exchange rate drivers across countries, which is primarily determined by countries' exposure to risk. Riskier currencies are those from commodity-intensive countries exposed to global business cycle risk and non-dollarized countries exposed to US-originated fluctuations in the dollar's value. Increases in risk premia depreciate all currencies relative to the dollar, with larger depreciations occurring in risky currencies. Currencies of countries with high dollar assets and low commodity exports tend to move less relative to the dollar, and predominantly respond to idiosyncratic shocks.

We start by revisiting the empirical factor structure of exchange rates. We show that most variation in monthly changes in bilateral exchange rates can be accounted for by two factors: a "dollar factor," which measures the appreciation of the dollar against the average currency, and a "commodity factor," which measures the depreciation of commodity-intensive countries against those with low commodity exports. The loading of countries on these factors is largely explained by the share of commodities in their exports and their stock of dollar assets. Commodity exporters have high loadings on the commodity factor. Asset-dollarized countries have high loadings on

the commodity factor and low loadings on the dollar factor, meaning their currencies depreciate less in times of broad dollar appreciation. These results are largely in line with the findings of the literature. In particular, the commodity factor is similar to alternative factors that exploit cross-country heterogeneity in interest rate differentials or trade network centrality.

We then develop a quantitative macroeconomic model that maps macroeconomic fundamentals to exchange rates. Our model introduces heterogeneous countries into a world economy with segmented asset markets and financial intermediaries with limited risk-bearing capacity (as in, e.g., Gabaix and Maggiori, 2015; Itskhoki and Mukhin, 2021), along with a canonical tradable and non-tradable goods structure (as in, e.g., Schmitt-Grohé and Uribe, 2017). Aggregate shocks affect global output and the risk-taking capacity of intermediaries, broadly capturing global business and financial cycles. Motivated by our empirical results, we model countries with two dimensions of ex-ante heterogeneity: commodity endowments and the level of dollar assets.

Commodity exporters are more exposed to commodity-biased fluctuations in global tradable output. An expansion in global tradable output increases the relative price of non-tradable goods in all countries, with larger increases occurring in commodity-intensive countries. Countries are also affected by US-specific fluctuations, with their exposures determined by their dollar asset holdings. Contractions in US non-tradable output increase prices of non-tradables in the US because they reduce the supply of these goods. These shocks affect other countries' prices through two channels: an expansionary channel through which a dollar appreciation generates higher financial income through positive capital gains on their dollar holdings; and a contractionary channel because intermediaries retreat capital flows in times of dollar appreciation. In dollarized countries the first effect predominates and nontradable prices increase. In non-dollarized countries the second effect dominates and non-tradable prices decrease.

The global financial cycle in our model is generated by shocks that lead to a repricing of business cycle risk. Global financial intermediaries facilitate capital flows. In equilibrium, they absorb the global demand for dollar reserves by issuing dollar bonds and invest the proceeds in the bonds of all other currencies. From the perspective of intermediaries, currencies have different risk profiles. Commodity exporters are a poor hedge because their currencies depreciate against the dollar during periods of low global output, which is when the intermediaries realize low returns. Countries with low dollar assets are a poor hedge because their currencies depreciate more following contractions in US non-tradable output, which is also when intermediaries bear losses. Following a

shock to risk aversion, intermediaries rebalance away from risky countries, causing their currencies to depreciate through capital flight, and the dollar to appreciate due to a flight to safety.

The model generates the same factor structure we estimate in the data and allows us to map fundamental shocks into factors. The commodity factor combines shocks to global tradable output and risk premia, and (in the empirically relevant parameter space) the dollar factor combines shocks to US nontrdable output and risk premia. In special cases of the model, we derive analytical expressions for both factors and factor loadings of all countries. We show that risk premium shocks are necessary to reproduce the cross-sectional structure of factor loadings, where commodity loadings depend on the country's dollar assets due to the interaction of dollar and commodity risk on the intermediaries' balance sheets.

We use the estimated factors alongside with the standard set of macroeconomic moments to quantify our model. We argue that there is valuable information in the time series of the factors that helps to identify the relevance of the structural shocks in the model. In particular, the positive correlation between both factors estimated in the data points to the importance of risk premium shocks, because they move both factors in the same direction, and limits the importance of the two aggregate output shocks because they do not jointly affect factors. Moreover, the relative volatility of the two factors provides information about the volatility of the global tradable output shock.

We then use our model to study the drivers of currencies in the cross-section. Following prior literature, we focus on bilateral exchange rates relative to the US dollar, which in the model is given by the ratio of relative prices on nontradable goods in any given country to those in the US. We highlight two key takeaways from our quantitative analysis. First, fluctuations in risk premia stand out as a consistently relevant driver for bilateral exchange rates relative to the dollar, with the importance of this shock being much higher for bilateral exchange rates than for relative prices of nontradables in each country. This is because risk shocks affect prices of non-tradables in the US and other countries in the opposite directions. The US experiences flight-to-safety inflows in times of high risk aversion, while other countries experience capital flight. Second, we estimate substantial heterogeneity in exchange rate drivers across countries. Along the commodity dimension, fluctuations in global tradable output play a more important role explaining exchange rates in commodity-intensive countries. Along the dollar-asset dimension, the comovement induced by global shocks of country nontradable prices with those of the US is stronger for dollarized countries. This implies that the bilateral exchange rates of dollarized countries are less volatile

and more influenced by idiosyncratic shocks.

Related Literature Our paper lies at the intersection of the international finance literature that studies the cross section of currencies and the international macroeconomics literature that develops quantitative macroeconomic models of the exchange rate.

An extensive literature in international finance studies the cross-section of exchange rates. A strand of this literature has identified factors that can price currency risk and explain cross-sectional excess returns in currency markets (see e.g., Lustig and Verdelhan, 2007; Lustig et al., 2011; Menkhoff et al., 2012; Lustig et al., 2014). Other papers have documented that the variation in bilateral exchange rates exhibits strong common components with heterogeneous exposures across countries (Verdelhan, 2018; Lustig and Richmond, 2020). Motivated by this empirical literature, a body of work develops theories that explain the origins of currency risk and how countries differ in their exposure to risk (Hassan, 2013; Maggiori, 2017; Colacito et al., 2018; Richmond, 2019). Hassan and Zhang (2021) survey recent advances in this literature. Our paper relates to Ready et al. (2017), who develop a model of international trade and currency pricing in which currencies of commodity producers are more exposed to global risk. Our model builds on this idea by introducing country heterogeneity in the incidence of commodity production, which gives rise to a commodity factor, and expands on it by modeling a dollar factor that emerges because of country heterogeneity in dollar asset holdings. Closest to our work, Verdelhan (2018) documents that most of the observed monthly variation in bilateral exchange rates can be explained with a two-factor empirical model and proposes a model of the stochastic discount factors that can replicate this structure. We contribute to this literature by developing a quantitative macroeconomic model that admits a two-factor structure tightly linked to that in Verdelhan (2018), and proposes a structural interpretation of the drivers of factors and exchange rates in the cross-section.

Second, our paper is related to the literature that studies the relationship between exchange rates and international asset markets imperfections (see Maggiori, 2022, for a recent review). Following the original work of Backus and Smith (1993), a set of papers analyzes how segmentation in international asset markets shapes the equilibrium properties of exchange rates (see, e.g., Lustig and Verdelhan, 2019; Chernov et al., 2024; Jiang et al., 2023). A related body of work studies the role of imperfect financial intermediation. Gabaix and Maggiori (2015) develop a theory of exchange rate dynamics and frictional financial intermediaries that can account for various

properties of exchange rates. Lettau et al. (2014) study the role of financial intermediaries in the pricing of cross-sectional currency risk. Our paper builds on the insights from this literature by incorporating global financial intermediaries with fluctuating risk-bearing capacity that are central to explaining exchange rate movements in the cross section.

Finally, our paper builds on the growing literature that develops quantitative macroeconomic models of the exchange rate (see the recent survey by Itskhoki and Mukhin, 2025a, and references therein). A strand of this literature has studied the quantitative drivers of exchange rates (see, e.g., Stockman and Tesar, 1995; Engel and West, 2005; Itskhoki and Mukhin, 2021, 2025b; Chahrour et al., 2024; Engel and Wu, 2024; Kekre and Lenel, 2024a; Bodenstein et al., 2024). Another body of work studies the flight to safety properties of the US dollar (see, e.g., Jiang et al., 2024; Kekre and Lenel, 2024b; Bodenstein et al., 2023). We contribute to this literature by introducing country heterogeneity aimed at providing a structural explanation of the dynamics of the cross-section of exchange rates. In this sense, our paper also relates to the recent work by Kekre and Lenel (2025), who study the heterogeneous spillovers of US monetary policy across countries, and the literature that studies the heterogeneous effects of the global financial cycle, such as Morelli, Ottonello, and Perez (2022) and Oskolkov (2025).

Outline. Section 2 presents our empirical analysis. Section 3 describes the model and characterizes the equilibrium. Section 4 conducts the quantitative analysis and decomposes exchange rates into fundamental drivers in cross-section. We conclude in Section 5.

2 Empirical Analysis

In the first part of this section, we estimate a factor structure of exchange rates based on two factors: a commodity factor and a dollar factor. In the second part, we relate the estimated loadings on these factors to parameters of countries in the cross-section.

2.1 Factor Structure of Exchange Rates

We use monthly data on bilateral exchange rates vis-a-vis the US dollar for a sample of $N = 32$ countries from 2000 to 2019.[1] Exchange rates are expressed in units of local currency per US dollar, unless noted otherwise. A rising exchange rate means dollar appreciation. For each currency i, we estimate the following empirical model:

$$\Delta s_{i,t+1} = \alpha_i + \beta_{i,\text{comm}}\text{comm}_{i,t+1} + \beta_{i,\text{doll}}\text{doll}_{i,t+1} + \epsilon_{i,t+1}, \tag{1}$$

where $\Delta s_{i,t+1}$ is the monthly change in the log exchange rate of currency i per dollar in period $t+1$, $\text{doll}_{i,t+1}$ is the dollar factor, and $\text{comm}_{i,t+1}$ is the commodity factor. The factors are indexed by currency i because we exclude currency i when constructing them to avoid a mechanical relationship with exchange rates. We construct the commodity factor as the average change in the log exchange rate of high-versus-low commodity producers

$$\text{comm}_{i,t+1} \equiv \frac{2}{N-1}\sum_{j\in\mathcal{I}_{h,i}} \Delta s_{j,t+1} - \frac{2}{N-1}\sum_{j\in\mathcal{I}_{l,i}} \Delta s_{j,t+1}, \tag{2}$$

where $\mathcal{I}_{h,i}$ and $\mathcal{I}_{l,i}$ are the sets of countries in which the time-average share of commodity exports in total exports is above and below the median across countries excluding country i, respectively. We construct the dollar factor as the average change in the log exchange rates of all currencies against the dollar, excluding currency i:

$$\text{doll}_{i,t+1} \equiv \frac{1}{N-1}\sum_{j\neq i} \Delta s_{j,t+1}. \tag{3}$$

We provide further details on the construction of these variables in Appendix A.

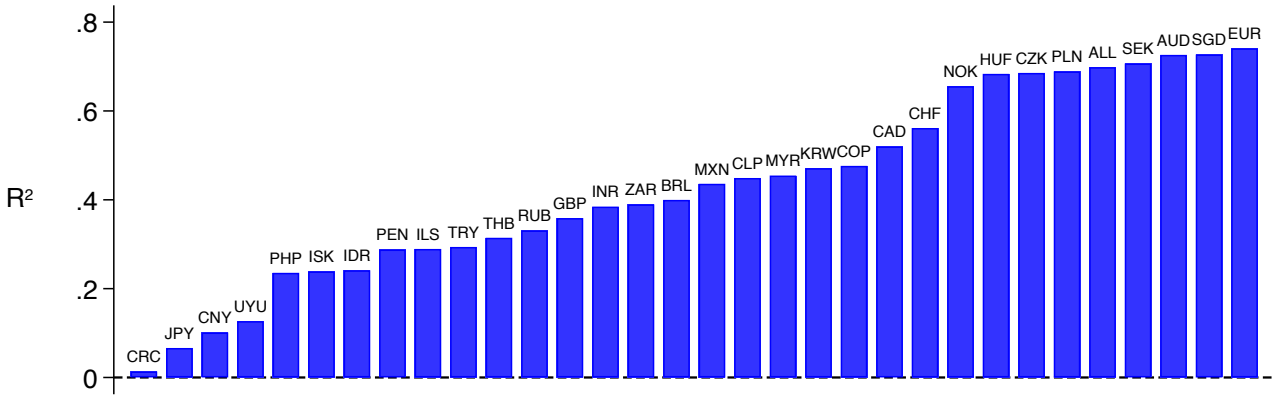
Panel (a) of Figure 1 reports the R^2 from the estimation of equation (1) for all currencies. The factor estimations have high explanatory power, with an average R^2 of 43% and as high as 74% for some countries. As a benchmark for comparison, panel (a) of Appendix Figure A.1 shows that the R^2 are close to those obtained from regressing changes in log exchange rates on their first two principal components, which is an upper bound on how much variation can be explained by any two variables. The average R^2 from the two-principal-components regressions is 51%.

Panel (b) of Figure 1 reports the loadings on the commodity factor by currency, which exhibit

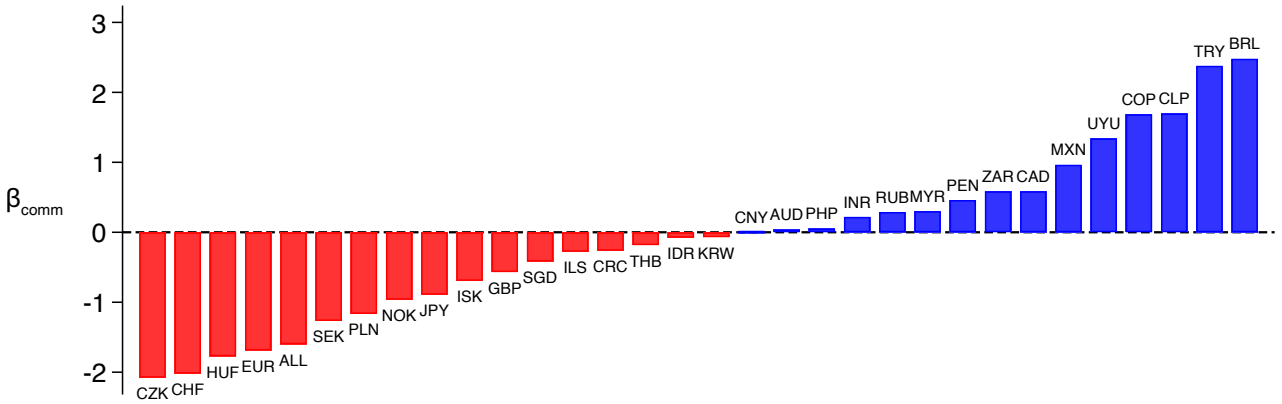
[1] The sample of countries includes Albania, Australia, Brazil, Canada, Switzerland, Chile, China, Colombia, Costa Rica, Czech Republic, Euro Area, United Kingdom, Hungary, Indonesia, Israel, India, Iceland, Japan, South Korea, Mexico, Malaysia, Norway, Peru, Philippines, Poland, Russia, Sweden, Singapore, Thailand, Turkey, Uruguay, and South Africa.

Figure 1: Factor Structure of Exchange Rates: Country Regressions

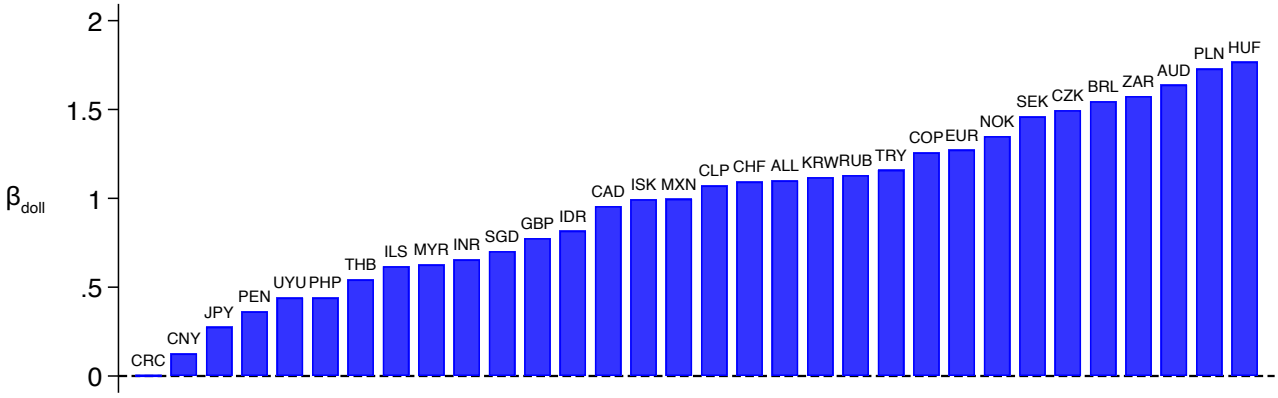
(a) Explanatory power R_i^2



(b) Commodity-factor loadings $\beta_{i,\text{comm}}$



(c) Dollar-factor loadings $\beta_{i,\text{doll}}$



Notes: Panel (a) shows the R^2, representing the fraction of variation in $\Delta s_{i,t+1}$ explained by our empirical model in equation (1) for each currency i. Panel (b) shows the estimated loading of the commodity factor $\text{comm}_{i,t+1}$ for each currency i from our empirical model in equation (1). Panel (c) shows estimated loading of the dollar factor $\text{doll}_{i,t+1}$ for each currency i from our empirical model in equation (1). .

significant cross-country heterogeneity, with roughly half of the countries loading positively and the other half negatively. Panel (c) reports the loadings on the dollar factor, which are all positive and also exhibit significant heterogeneity across countries. Table A.1 and Table A.1 report these regression estimates along with their Newly-West adjusted standard errors.

Robustness analysis. Our results are robust to the way we construct the commodity factor and the sample choice. Appendix Figure A.2 shows similar results if we consider the high-commodity set of countries as those with a share of commodity exports above the 75th percentile and the low-commodity-countries set as those below the 25th percentile. Appendix Figure A.3 shows that we obtain similar results if we consider an extended sample from 1990 to 2019.

Commodity factor and other factors. We then study how the commodity factor relates to alternative factors. We first compare it with two factors previously studied in the literature. One is the carry factor proposed by Verdelhan (2018), computed in a similar way to the commodity factor but sorting countries by their local-currency interest rates; the other is the trade-centrality factor proposed by Richmond (2019), also computed similarly to the commodity factor but sorting countries according to their trade centrality (see Appendix A for further details on the construction of these factors). Panel (a) of Appendix Figures A.4 and A.5 show that the explanatory power from our baseline two-factor specification is very similar to alternative two-factor specifications in which we replace the commodity factor with the carry factor or the trade centrality factor. Additionally, Panel (b) of Appendix Figures A.4 and A.5 shows that the country-loadings on the commodity factor are tightly correlated with the loadings on the carry and the trade-centrality factors. This analysis suggests that the commodity factor captures similar variation as the carry and trade-centrality factors, consistent with the findings of Ready et al. (2017).

Second, we document that the commodity-factor is related to a factor constructed from country variation in the sensitivity of tradable output to variations in a global commodity price index. As we clarify below, this "tradable-output-sensitivity factor" captures the mechanism through which heterogeneity in commodity incidence affects country-risk exposure. To construct this factor, we first obtain country-specific sensitivities to a global commodity price index, $\beta_{i,\text{tos}}$ by estimating the

following regression:

$$\Delta y^{T}_{i,t+1} = \alpha_i + \beta_{i,\text{tos}} \Delta p_{\text{comm},t+1} + \upsilon_{i,t}, \tag{4}$$

where $\Delta y^{T}_{i,t+1}$ is the annual log change in tradable output from country i in period $t+1$, and $\Delta p_{\text{comm},t+1}$ is the annual log change in the global commodity price index. We then construct the tradable-output-sensitivity factor as the differential average change in the log exchange rate of above-median relative to below-median tradable-output-sensitivity-countries (see Appendix A for further details).

Appendix Figure A.6 shows that the regression in which we substitute the commodity factor for the tradable-output-sensitivity factor has an explanatory power similar to the baseline regression. It also shows that the tradable-output-sensitivity factor captures similar country variation as the baseline commodity factor.

The role of dollar basis. We also argue that the dollar factor captures a global factor and not a local US factor that arises because we express currencies relative to the US dollar. We perform two related exercises to support this conclusion. First, we estimate a similar specification to equation (1) in which we measure the dependent variable (the change in log exchange rates) in all possible currency bases and include the same two factors as in equation (1). Appendix Table A.3 shows that the dollar factor remains a statistically significant factor in 732 out of 992 country-currency-base regressions. For example, when we consider the Euro as the base currency, the dollar factor is statistically significant in 22 out of 31 countries. Second, we estimate a new regression in which the dependent variable is expressed in all possible currency bases and we include three factors as independent variables: the two factors from the baseline regression equation (1), and an additional currency-base factor, constructed in a similar way as the dollar-factor. For example, when we measure exchange rates relative to the Euro, we include the commodity factor, the dollar factor, and a Euro factor as independent variables. In this other set of regressions, dollar factor remains a statistically significant factor in 786 out of 992 country-currency-base regressions (see column 2 of Appendix Table A.3).

Finally, we show that the reverse is not true. Other currency bases do not have as strong explanatory power as the dollar factor. We illustrate this by estimating a similar specification to the baseline in which we add an additional currency-base factor, for all possible currency bases.

As reported in the last column of Appendix Table A.3, all other currency bases are statistically significant in only 268 out of the 992 regressions.

2.2 Determinants of Country-Factor Loadings

We now analyze how the estimated country-loadings from equation (1) are related to certain observable country-characteristics. For this, we estimate the following specification for each factor $f = \{\text{comm}, \text{doll}\}$:

$$\beta_{i,f} = \alpha_f + \theta_{f,u} u_i + \theta_{f,r} r_i + \epsilon_{i,f} \tag{5}$$

where u_i is a measure of dollar assets incidence in country i, which in the baseline specification corresponds to the share of dollar portfolio-assets in total portfolio-assets; and r_i is a measure of commodity incidence in country i, which in the baseline specification corresponds to the share of commodity exports in total exports. Appendix A provides further details on the construction of these variables and all other variables used in the empirical analysis.

Table 1 shows the regression estimates for both factors and Appendix Figure A.7 shows a graphical representation of the variation driving the regression estimates. The commodity factor is positively related to the share of commodities in exports and negatively related to the share of dollar assets. Both coefficients in the commodity-factor regression are statistically significant and the R^2 from the regression is 57%. The dollar factor is positively related to the share of dollar assets and does not exhibit a statistically significant relationship with the share of commodity exports. The R^2 from the regression on the dollar factor is 38%.

Columns 2-6 show that the empirical relationships estimated in the regressions are robust to considering various additional controls considered in the literature (e.g., Hassan, 2013; Richmond, 2019; Lustig and Richmond, 2020): the trade centrality, trade openness, the economic size of the country, and a set of gravity variables (bilateral distance, and dummies for common borders, language, and legal origins). Finally, Appendix Figure A.8 shows that we reach similar conclusions if we replace the commodity factor with the tradable-output-sensitivity factor, suggesting that both factors capture similar cross country variation.

Overall, our results indicate that an important part of the variation in bilateral exchange rates can be explained by the commodity and dollar factors, with countries loading heterogeneously on

Table 1: Cross-Sectional Regressions of Factor Loadings on USD Assets and Commodity Exports Shares

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Commodity factor loading β_{comm}						
USD assets	2.635***	2.871***	2.765***	2.601***	2.638***	2.545***
	(0.681)	(0.716)	(0.737)	(0.716)	(0.711)	(0.899)
Comm share	2.252***	2.797***	2.649***	2.441***	2.442**	2.687**
	(0.794)	(0.887)	(0.942)	(0.878)	(0.932)	(1.211)
Constant	-2.069***	-2.603***	-2.392***	-0.334	-2.199	-5.488
	(0.329)	(0.558)	(0.689)	(1.118)	(1.896)	(4.089)
R^2	0.558	0.574	0.577	0.639	0.650	0.680
		(1)	(2)	(3)	(4)	(5)
		+	+	+	+	+
Controls		trade cent	trade open	country size	dist	gravity
Observations	32	32	32	32	32	32
Panel B: Dollar factor loading β_{doll}						
USD assets	-1.234***	-1.265***	-1.266***	-1.271***	-1.282***	-1.277***
	(0.265)	(0.289)	(0.290)	(0.290)	(0.295)	(0.328)
Comm share	0.807**	0.734*	0.733	0.727	0.727	1.025*
	(0.370)	(0.408)	(0.444)	(0.455)	(0.458)	(0.507)
Constant	1.536***	1.608***	1.609***	1.670***	2.268**	0.054
	(0.159)	(0.248)	(0.283)	(0.539)	(0.894)	(2.009)
R^2	0.378	0.380	0.380	0.380	0.387	0.461
		(1)	(2)	(3)	(4)	(5)
		+	+	+	+	+
Controls		trade cent	trade open	country size	dist	gravity
Observations	32	32	32	32	32	32

Notes: This table reports cross-sectional regressions of factor loadings $\beta_{i,\text{comm}}$ and $\beta_{i,\text{doll}}$ on the share of dollar-denominated portfolio-assets (u_i), the commodity export share (r_i), and various controls. Appendix A provides further details on the construction of these variables. Panel A presents results for the commodity factor loading $\beta_{i,\text{comm}}$. Column (1) reports coefficients for $\theta_{f,u}$, $\theta_{f,r}$, and constant α_f as specified in equation (5). Column (2)-(6) sequentially add trade centrality, trade openness, log GDP share, log harmonic geographical distance to the U.S., and a set of gravity variables as controls. Panel B repeats these specifications for the dollar factor loading $\beta_{i,\text{doll}}$.

them. The country-loadings on the commodity and dollar factors can be traced to fundamental country characteristics like the incidence of commodities in their exports and the incidence of dollar assets in their economy. These empirical patterns motivate the features of the model in the next section.

3 A Model of the Cross-Section of Exchange Rates

In this section, we develop a macroeconomic model of the cross-section of currencies. We model the global economy composed of a continuum of heterogeneous countries and a representative global financial intermediary. The model features a canonical structure with tradable and non-tradable goods, as in Backus and Smith (1993) and Schmitt-Grohé and Uribe (2017). Asset markets are segmented, following modern macroeconomic models of the exchange rate such as Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2021). We consider fluctuations in countries' economic activity, broadly capturing the global business cycle, and fluctuations in the risk-bearing capacity of intermediaries, capturing the global financial cycle.

3.1 Environment

Small open economies. There is a continuum of small open economies indexed by $i \in [0,1]$. Each economy is a representative household and a central bank. Households maximize

$$\mathbb{E}_0 \sum_{t=0}^{t=\infty} \beta^t \frac{C_{it}^{1-\rho}}{1-\rho}, \tag{6}$$

where $\beta \in (0,1)$ is the discount factor, ρ is the inverse elasticity of intertemporal substitution, and $C_{it} = \mathcal{C}(C_{it}^N, C_{it}^T)$ is a consumption aggregator that uses C_{it}^T units of tradable goods and C_{it}^N units of non-tradable goods:

$$\mathcal{C}(C_{it}^N, C_{it}^T) = \left(\frac{C_{it}^N}{\alpha}\right)^{\alpha} \left(\frac{C_{it}^T}{1-\alpha}\right)^{1-\alpha}$$

Here $\alpha \in (0,1)$ is the weight on local non-tradables. The price of tradables is normalized to one, the price of non-tradable is P_{it}^N, and $Q_{it} = (P_{it}^N)^{\alpha}$ is the ideal price index of the consumption aggregator. For brevity, we will refer to countries' consumption baskets as "local currency". In particular, the US is one of our countries with an index $i = u$, and we call the US consumption basket the US dollar. Relative prices of consumption baskets are real exchange rates between countries: Q_{it}/Q_{ut} is the bilateral real exchange rate of country i against the dollar (i.e., the price of a consumption basket in country i in terms of a consumption basket in the US).

Households are endowed with both goods. The non-traded endowment is $Y_{it}^N = N(1 + x_{it})$, where x_{it} is a mean-zero shock process that is idiosyncratic to each country. The endowment of the undifferentiated tradable goods is $Y_{it}^T = 1 + e_i z_t$, where z_t is an aggregate disturbance in

tradable endowments and e_i is country i's exposure to this shock, which is a source of permanent heterogeneity across countries. Appendix D provides a micro-foundation for this formulation of the traded endowment. It shows that this functional form of Y_{it}^T arises in a setup with two types of tradables: final goods and primary inputs (commodities). Final goods are produced using labor and commodities, and z_t is the aggregate shock that raises the relative productivity of commodities. Exposures e_i map to countries' commodity endowments: those with large e_i receive more tradable income following an increase in z_t. The micro-foundation also maps e_i into the share of commodities in country i's exports, an important moment we later use for identification.

Households can save and borrow in one-period bonds denominated in local currency. We denote their bond purchases at time t by B_{it+1}. The bonds pay an interest rate $R_{i,t+1}$, known at time t, at maturity. The household's budget constraint, expressed in tradable goods, is

$$C_{it}^T + P_{it}^N C_{it}^N = Y_{it}^T + P_{it}^N Y_{it}^N + R_{it} Q_{it} B_{it} - Q_{it} B_{i,t+1} + T_{it} + \Pi_{it}, \tag{7}$$

where T_{it} are transfers from the central bank and Π_{it} profit rebates from financial intermediaries.

Households choose sequences $\left\{C_{it}^T, C_{it}^N, B_{i,t+1}\right\}_{t\geq 0}$ to maximize equation (6) subject to equation (7). The optimality conditions are $(1-\alpha)C_{it}^T = \alpha P_{it}^N C_{it}^N$ and $C_t^{-\rho} = \beta R_{it} \mathbb{E}_t C_{t+1}^{-\rho}$.

The central bank in each small open economy accumulates foreign reserves, denominated in dollars, and transfers returns to the households. At time t, the central bank of country i buys $M_{i,t+1}$ dollar denominated bonds. Its rebate to households is

$$T_{it} = R_{ut} Q_{ut} M_{it} - Q_{ut} M_{i,t+1},$$

where u is the index of the US. We assume the following policy for reserve accumulation:

$$Q_{ut} M_{i,t+1} = \tau \cdot Q_t M_i + (1-\tau) \cdot Q_{ut} M_i,$$

where $Q_t \equiv \int Q_{it} \mathrm{d}i$ is the global average of consumption basket prices. The policy is a linear combination between targeting a constant value of reserves in the global basket of currencies and in dollars, with a weight $\tau \in (0,1)$ on the former. This weight governs the degree to which the central banks engages in exchange rate stabilization: when the USD is strong against the global basket of currencies, a high level of τ prescribes selling more USD to realize capital gains and strengthen domestic currency. The target level of dollar reserves is M_i is exogenous and is the second and last source of permanent heterogeneity across countries. A related literature studies

the determinants of asset dollarization across countries, which include varying levels of domestic policy risk (see, e.g., Drenik et al., 2022).

Global financial intermediaries. There is a representative global financial intermediary that trades bonds in all currencies. Let $A_{i,t+1}$ be the currency-i bonds purchased by the intermediary in period t, and $L_{i,t+1} \equiv Q_{it}A_{i,t+1}/Q_{ut}$ be the dollar value of the positions in currency i. The intermediary starts with no initial capital so its balance sheet is given by:

$$\int L_{i,t+1}\mathrm{d}i + A_{u,t+1} = 0, \tag{8}$$

where, as will become clearer below, we conveniently separately keep track of the position in US dollar. We assume that the intermediary has mean-variance preferences over excess returns denominated in dollars. Let $X_{i,t+1}$ be the realized excess returns, measured in dollars, on investing in i's bonds:

$$X_{i,t+1} \equiv \frac{R_{i,t+1}}{R_{u,t+1}}\frac{Q_{i,t+1}}{Q_{u,t+1}}\frac{Q_{ut}}{Q_{it}} - 1.$$

With total realized profits $\int L_{i,t+1}X_{i,t+1}\mathrm{d}i$, the intermediary chooses $\{L_{i,t+1}\}_{i\neq u}$ to maximize

$$\mathbb{E}_t\left[\int L_{i,t+1}X_{i,t+1}\mathrm{d}i\right] - \frac{\Gamma_t}{2}\mathbb{V}_t\left[\int L_{i,t+1}X_{i,t+1}\mathrm{d}i\right] - \frac{1}{2\chi}\int \left(L_{i,t+1} - L_{t+1}\right)^2 \mathrm{d}i \tag{9}$$

subject to equation (8), which determines the intermediary's dollar position once other investments are chosen. The intermediary's risk aversion is parameterized by $\Gamma_t \equiv \Gamma(1+\gamma_t)$, where $\Gamma > 0$ is the average risk aversion and γ_t is an aggregate shock. We assume that the intermediary faces portfolio management costs, the third term in equation (9). These costs capture frictions that make capital flows slow-moving and less elastic to risk-adjusted returns than predicted by frictionless models (as estimated in, e.g., Koijen and Yogo, 2020). The cost decreases in $\chi > 0$ and penalizes deviations from a portfolio in which the intermediary issues dollar bonds to satisfy the demand for reserves by all countries and invests the proceedings uniformly in all other countries. That is, the reference level of liabilities is $L_{t+1} = \int M_{i,t+1}\mathrm{d}i$. The optimal portfolio allocation is

$$L_{i,t+1} = L_{t+1} + \chi\left(\mathbb{E}_t[X_{i,t+1}] - \Gamma_t\mathbb{C}_t\left[X_{i,t+1}, \int L_{j,t+1}X_{j,t+1}\mathrm{d}j\right]\right). \tag{10}$$

The intermediary will allocate more funds to currencies that promise higher expected returns against the dollar and have lower covariance with the intermediary's total profits. Denote by

$L_{u,t+1} \equiv A_{u,t+1} + \int M_{i,t+1} \mathrm{d}i$ the intermediary's demand for dollar bonds in excess of total reserve demand. This metric indicates how much of the global demand for reserves the intermediary cannot accommodate with its own balance sheet and passes through to the US:

$$L_{u,t+1} = \chi \left(\Gamma_t \mathbb{C}_t \left[\int X_{i,t+1} \mathrm{d}i, \int L_{j,t+1} X_{j,t+1} \mathrm{d}j \right] - \mathbb{E}_t \left[\int X_{i,t+1} \mathrm{d}i \right] \right)$$

The pass-through to the US is proportional to the covariance between the average foreign currency return and the intermediary's total profits, adjusted for expected excess returns.

Finally, we assume that profits are rebated back to each country according to

$$\Pi_{it} = Q_{u,t-1} L_{it} \left(R_{it} \frac{Q_{it}}{Q_{i,t-1}} - \frac{Q_{ut}}{Q_{u,t-1}} \right) - Q_{ut} M_{it} (R_{ut} - 1).$$

Each country receives returns that the intermediary realizes on its bond, including interest income and exchange rate appreciation, net of interest payments on reserves. The US only receives the first part, since it does not maintain reserves.

Shocks and the role of the US. A feature of this model is that the US is infinitesimal in goods markets but looms large in financial markets. Since all countries accumulate reserves denominated in dollars, shocks that affect the dollar exchange rate will affect aggregate variables, even though US demand does not affect global goods markets. A consequence is that the idiosyncratic non-tradable endowment shock to the US becomes aggregate. In addition to this aggregate shock, there are two others: the global tradable endowment shock z_t and the risk aversion shock γ_t. We assume that all shocks are stationary and follow the following processes with independent increments:

$$z_{t+1} = \rho_z z_t + \sigma_z \epsilon_{z,t+1}$$

$$\gamma_{t+1} = \rho_\gamma \gamma_t + \sigma_\gamma \epsilon_{\gamma,t+1}$$

$$x_{i,t+1} = \rho_x x_{it} + \sigma_x \epsilon_{i,t+1}$$

Equilibrium. An equilibrium is a set of processes for prices $\{P_{it}^N, R_{it}, Q_{it}\}_{t \geq 0}$ and quantities $\{C_{it}^N, C_{it}^T, B_{i,t+1}, A_{i,t+1}\}_{t \geq 0}$ such that, given central banks' reserve policy $\{M_{i,t+1}\}_{t \geq 0}$, and profits and transfers policies, all quantities are chosen optimally taking prices as given, and the following

market clearing conditions hold:

$$\begin{aligned}
&\text{non-tradable good } i\text{:} && C_{it}^N = Y_{it}^N, \\
&\text{tradable good:} && \int C_{it}^T \mathrm{d}i = \int Y_{it}^T \mathrm{d}i, \\
&\text{currency } i \text{ bonds } (i \neq u)\text{:} && A_{i,t+1} + B_{i,t+1} = 0, \\
&\text{dollar bonds } (i = u)\text{:} && \int M_{i,t+1} \mathrm{d}i + A_{u,t+1} + B_{u,t+1} = 0.
\end{aligned}$$

3.2 Equilibrium Characterization

We take the linear approximation of the model around a deterministic steady state. Formally, we look for equilibrium exchange rates in the following form:

$$q_{it} = \eta_i x_{it} + \underbrace{\theta_i z_t + \xi_i x_{ut} + \mu_i \gamma_t}_{\text{exogenous global states}} + \underbrace{\omega_i l_{it} + \delta_i l_{ut} + \zeta_i m_t}_{\text{endogenous states}} \tag{11}$$

$$q_{ut} = \theta_u z_t + \xi_u x_{ut} + \mu_u \gamma_t + \omega_u l_{ut} + \zeta_i m_t \tag{12}$$

Small letters denote first-order log deviations. Individual countries' currencies are exposed to local non-traded shocks x_{it}, globally important exogenous shocks (z_t, γ_t, x_{ut}), and endogenous states (l_{it}, l_{ut}, m_t). The dollar position of intermediaries, denoted by l_{ut}, is an aggregate state because it affects the US dollar, and that propagates to all other countries through their reserve positions. Intermediarie's positions on country i, denoted by l_{it}, and its reserve holdings, denoted by m_t, are idiosyncratic states affect the country's exchange rate through it balance of payments. Reserve holdings do not have a subscript i because the policy we specify above leads to the same reaction of reserves in all countries

$$m_{t+1} = \tau(q_t - q_{ut}). \tag{13}$$

Central banks realize the same share of their capital gains on the dollar, as measured against the global aggregate of currencies.

Steady state. The portfolio of itermediaries is not well-defined in the steady state. We follow prior literature and analyze the portfolio that arises when we set the variance of shocks to zero and scale up intermedarie's risk aversion so that non-zero risk premia persists even in the linear approximation. In particular, we consider the following double limit $(\sigma_z, \sigma_\gamma, \sigma_x) \longrightarrow 0$ and $\Gamma \longrightarrow$

∞, where $(\Gamma\sigma_z^2, \Gamma\sigma_x^2, \Gamma\sigma_\gamma^2) \longrightarrow (\Gamma_z, \Gamma_x, \Gamma_\gamma)$, which is akin to one used by Borovicka and Hansen (2013) and Kekre and Lenel (2024a). It lets us keep non-zero risk premia even in the linear approximation. The intermediary prices risk associated with shocks to global tradables z_t, shocks to US non-tradables x_{ut}, and shocks to its own risk-bearing capacity γ_t. The steady-state positions are functions of $(\Gamma_z, \Gamma_x, \Gamma_\gamma)$, as well as of exposures of other variables to shocks: covariance of exchange rates with intermediary profits survives in this limit, so steady-state financial inflows into each country depend on the cyclicality of its currency, as specified by the optimal portfolio allocation in equation (10). Equilibrium exposures of exchange rates to shocks, in turn, depend on steady-state investment positions. This makes finding the steady state and the linear approximation around it a fixed point problem.

Let β_i be the vector of exposure of i's bilateral exchange rate against the dollar to global shocks:

$$\beta_i = \begin{pmatrix} \theta_i - \theta_u \\ \xi_i - \xi_u \\ \mu_i - \mu_u \end{pmatrix}$$

Let $D_\Gamma = \text{diag}\{\Gamma_z, \Gamma_x, \Gamma_\gamma\}$ be the diagonal matrix with risk weights. Define the intermediary's total exposure as $\pi = \int L_i \beta_i \mathrm{d}i$. We can now characterize steady-state investment positions.

PROPOSITION 1. *In the steady state, $L_i - L = -\chi \pi' D_\Gamma \beta_i$. The intermediary's exposure is*

$$\pi = L \cdot (1 + \chi D_\Gamma \mathbb{E}[\beta_i \beta_i'])^{-1} \mathbb{E}[\beta_i]$$

Here $\mathbb{E}[\cdot]$ denotes the cross-sectional expectation. The pass-through to the US is positive:

$$L_u = L \cdot \mathbb{E}[\beta_i']((\chi D_\Gamma)^{-1} + \mathbb{E}[\beta_i \beta_i'])^{-1} \mathbb{E}[\beta_i] \in (0, L)$$

Here L_i is the steady-state position of the intermediary in country i, and $L = \int M_i di$ is the steady-state reserve level. This proposition shows that the intermediary tilts its steady-state position away from risky countries, whose bilateral exchange rates against the dollar correlate with the intermediary's profits. The intermediary cannot fully satisfy the world's demand for dollar assets and hedges against aggregate risk by passing some demand for dollar through to US households. However, it still maintains a positive dollar exposure: $L_u < L$.

This proposition generates an insight about the case of $\chi = \infty$, which corresponds to no portfolio cost. In the limit of $\chi \longrightarrow \infty$, the intermediary's exposure π to currency risk shrinks to zero,

and bonds of different countries become perfect substitutes to each other. Interestingly, the intermediary's positions are still non-zero in this limit: they converge to $L_i - L \longrightarrow \mathbb{E}[\hat{\beta}_i](\mathbb{E}[\hat{\beta}_i\hat{\beta}_i'])^{-1}\hat{\beta}_i$, where $\hat{\beta}_i$ is the limit of β_i. These non-zero positions combine to cancel out each other's exposures and achieve a zero exposure on aggregate. At the same time, solving the model with $\chi = \infty$ instead of taking the limit $\chi \longrightarrow \infty$ also generates a trivial solution $L_i \equiv 0$ that achieves a zero aggregate exposure by fully passing through the demand for reserves to US borrowers. The limit $\chi \longrightarrow \infty$ features a discontinuity with multiple possible steady states at the point $\chi = \infty$ itself.

Exchange rate determination. Linearizing country i's resource constraint,

$$q_{it} = \alpha e_i z_t - \alpha x_{it} + \alpha L_i \Delta l_{i,t+1} - \alpha M_i \Delta m_{t+1}$$

This equation shows how i's currency price is determined. It rises when the tradable endowment increases, and a higher exposure e_i makes i's currency appreciate more in times of high z_t. The non-tradable shock x_{it} makes local goods more abundant and leads to a depreciation. Inflows $\Delta l_{i,t+1}$, scaled by the steady-state position L_i, lead to appreciation because they increase disposable income, which will in part be spent on local non-tradables. As we show below, this is the channel through which financial shocks affect exchange rates: inflows are caused by shocks to the intermediary's risk aversion, and their sign depends on the country's riskiness. Reserve accumulation Δm_{t+1}, scaled by the steady state position M_i, leads to a depreciation by the same token. This is the channel through which reserve policy stabilizes currencies: the government generates outward flows when the dollar depreciates and inward flows when the dollar is expensive, counteracting its movement.

Linearizing the intermediary's optimal portfolio in equation (10),

$$L_i l_{i,t+1} = L m_{t+1} + \chi(r_{it} - r_{ut} + \mathbb{E}_t[\Delta q_{i,t+1} - \Delta q_{u,t+1}]) + \gamma_t(L_i - L)$$

Inflows increase with the overall demand for dollar bonds m_{t+1}: the intermediary accommodates that demand, distributing its investments uniformly across countries. It invests more in currencies that promise higher expected excess returns against the dollar, the currency of its liabilities, with an elasticity of χ. This part reflects carry trades. Finally, shocks to risk-bearing capacity γ_t induce the intermediary to invest more in safe countries with $L_i > L$ and unwind its positions in risky ones with $L_i < L$. This term reflects the global risk-taking cycle: $\gamma_t < 0$ leads to a build-up of risky investment, and $\gamma_t > 0$ generates a risk-off episode, illustrating collapsing carry trades.

Lemma 1 and Lemma 2 in Appendix B provide the exact coefficients in equation (11) and equation (12). Proposition 6 additionally shows that, if the processes for exogenous shocks are stationary, exchange rates are stationary too. This property is not common to models of global economy, where net foreign asset positions, and hence exchange rates, are typically integrated. In our setup, stationarity obtains because portfolio costs and risk aversion anchor long-run levels of net foreign asset positions.

3.3 Factor structure of exchange rates

We now theoretically characterize the factor structure of currencies. Specifically, we express the dollar and commodity factors explored in Section 2 as functions of fundamental shocks and factor loadings of individual currencies as functions of cross-sectional observables. We then explain how we use this mapping to identify parameters of the model.

Let $\Delta s_{i,t+1} \equiv \Delta q_{u,t+1} - \Delta q_{i,t+1}$ be country i's bilateral exchange rate depreciation against the dollar. Define the dollar and commodity factors doll_{t+1} and comm_{t+1} as

$$\text{doll}_{t+1} \equiv \int_{\underline{e}}^{\bar{e}} \Delta s_{i,t+1} di$$
$$\text{comm}_{t+1} \equiv \int_{[\text{med}\{e\},\bar{e}]} \Delta s_{i,t+1} di - \int_{[\underline{e},\text{med}\{e\}]} \Delta s_{i,t+1} di$$

The dollar factor is simply the appreciation of the dollar against all else. The commodity factor is a high-minus-low factor that tracks the depreciation of high-commodity countries against low-commodity ones. These two factors combine all three global shocks $\{\epsilon_{z,t+1}, \epsilon_{u,t+1}, \epsilon_{\gamma,t+1}\}$.

Consider the following projection, the exact model analog of equation (1):

$$\Delta s_{i,t+1} = \alpha_i + \beta_{i,\text{comm}} \text{comm}_{t+1} + \beta_{i,\text{doll}} \text{doll}_{t+1} + \epsilon_{i,t+1}$$

with $\mathbb{E}[\epsilon_{i,t+1}] = \mathbb{E}[\epsilon_{i,t+1}\text{comm}_{i,t+1}] = \mathbb{E}[\epsilon_{i,t+1}\text{doll}_{i,t+1}] = 0$.

The mapping from three shocks to two factors is not invertible in general. However, we can fully characterize this mapping together with the estimated loadings $\beta_{i,\text{doll}}$ and $\beta_{i,\text{comm}}$ in the special cases where one of the shocks is turned off. These special cases will guide our identification and illustrate the reason to include each of the shocks in the model.

To simplify exposition, we take a limit $\chi \longrightarrow \infty$ and $\chi^{-\frac{1}{2}} M_i \longrightarrow m_i$ for all i. The latter leads to normalized financial flows $\chi^{-\frac{1}{2}} L_i \longrightarrow l_i$ and $\chi^{-\frac{1}{2}} L_u \longrightarrow l_u$. Taking this double limit allows us

to maintain non-zero exposure of the intermediary to currency risk, which is otherwise converging to zero, as established by Proposition 1. We also set all persistence coefficients to one and assume that (e_i, M_i) are uncorrelated in the cross-section for simplicity. Let e_H, e_L, and e be the averages over the segments $[\text{med}\{e\}, \overline{e}]$, $[\underline{e}, \text{med}\{e\}]$, and $[\underline{e}, \overline{e}]$. Denote $\phi = \alpha/\sqrt{\alpha + \rho(1-\alpha)}$.

We start with the simplest special case without tradable endowment shocks:

PROPOSITION 2. *Suppose $\sigma_z = 0$. Then, $comm_{t+1} \equiv 0$.*

If there is no tradable output shock, there is no commodity factor in the model. Countries are not differentially exposed to global output, and there is trivially no global business cycle risk. The shock to z_t is essential for the model.

Next, we consider the case without financial shocks:

PROPOSITION 3. *Suppose $\sigma_\gamma = 0$. Then,*

$$doll_{t+1} = (e_u - e)\alpha\sigma_z\epsilon_{z,t+1} - \alpha\sigma_x\epsilon_{x,t+1}$$
$$comm_{t+1} = (e_L - e_H)\alpha\sigma_z\epsilon_{z,t+1}$$

Their covariance is positive if and only if $e > e_u$. The estimated factor loadings are

$$\beta_{comm,i} = \frac{e_i - e}{e_H - e_L}$$
$$\beta_{doll,i} = 1 - \phi\tau(m_i - l)$$

In this case, exchange rates are driven purely by output shocks, which include global tradable and US non-tradable output. The commodity factor decreases in $\epsilon_{z,t+1}$ by construction, since it measures the depreciation of high-commodity countries, whose trade balance goes into surplus in times of high z_t, against low-commodity ones. The dollar factor is also driven by shocks to z_t if the US exposure to commodities is not exactly the world average. After a positive shock to z_t, the dollar depreciates against the average currency if the US is a low-commodity country ($e_u < e$).

Dollar factor loadings depend on the country's dollar position because of the reserve policy. Countries with high stocks of dollar assets realize more gains or losses when the dollar moves against other currencies, their non-tradable prices move closer to that of the US, and their exchange rates are more stable against the dollar. Dollar factor loadings do not depend on the country's commodity exposure, even though the dollar factor includes shocks to global tradable output z_t. This is because $\beta_{\text{doll},i}$ measures the correlation of the country's exchange rate with the dollar factor

conditional on the commodity factor, which means holding $\epsilon_{z,t+1}$ constant. Dollar factor loadings isolate exchange rate movements coming from $\epsilon_{x,t+1}$.

Commodity factor loadings, on the other hand, do not depend on the dollar position. This is because $\beta_{\text{comm},i}$ measures the correlation of the country's exchange rate with the commodity factor conditional on the dollar factor. If the dollar factor is constant, regular countries do not realize any capital gains or losses on reserves, so their reserve position is irrelevant for their exchange rate movements. Commodity loadings isolate currency movements coming from $\epsilon_{z,t+1}$ directly.

As we show in Section 2, this implication is counterfactual: the data do not support the separation of loadings into commodity intensity and dollar asset intensity. Commodity loadings strongly depend on the country's dollar position, which is why the model calls for financial shocks.

We finalize the exposition with the third special case, no US non-tradable output shocks:

PROPOSITION 4. *Suppose $\sigma_x = 0$. Then,*

$$\begin{aligned} doll_{t+1} &= (e_u - e)\alpha\sigma_z\epsilon_{z,t+1} + \sqrt{2}\cdot\phi l_u\sigma_\gamma\epsilon_{\gamma,t+1} \\ comm_{t+1} &= \frac{e_H - e_L}{e - e_u}[(e_u - e)\alpha\sigma_z\epsilon_{z,t+1} + \kappa\cdot\phi l_u\sigma_\gamma\epsilon_{\gamma,t+1}] \end{aligned}$$

Here $\kappa > 0$ is a function of model parameters. The following statements are equivalent: (i) $\kappa < \sqrt{2}$, (ii) the ratio $\mathbb{V}[e]/(e - e_u)^2$ is sufficiently high, (iii) the intermediary's profit is positively exposed to z_t, and (iv) the fixed point in Proposition 1 is stable. The covariance of the factors is positive if and only if $e > e_u$. The estimated loadings are

$$\begin{aligned} \beta_{comm,i} &= \frac{e_i - e}{e_H - e_L} + \phi\tau(m_i - l)\frac{e - e_u}{e_H - e_L} \\ \beta_{doll,i} &= 1 - 2\phi\tau(m_i - l) \end{aligned}$$

In this case, both factors reflect both operating shocks. Despite the absence of shocks to the US non-tradable output x_{ut}, the dollar factor still moves for two reasons: the US exposure to global tradable output shocks is not exactly the world average ($e_u \neq e$), which makes the broad dollar depend on $\epsilon_{z,t+1}$, and there is flight to safety after risk aversion shocks, which makes the dollar increase in $\epsilon_{\gamma,t+1}$. The multiplier $\sqrt{2}$ is due to the self-reinforcing nature of flight to safety: since it leads to a dollar appreciation, regular countries sell their dollar holdings, which compresses the intermediary's balance sheet and makes it unwind positions in regular countries even more, furthering the dollar appreciation spiral.

The commodity factor, again, decreases in $\epsilon_{z,t+1}$ by construction. Provided $e_u < e$, it increases in $\epsilon_{\gamma,t+1}$. High-commodity countries are risky: their exchange rates appreciate in times of high z_t, which is exactly when the intermediary realizes high profits, since the currency of its liabilities, the dollar, broadly depreciates if $e_u < e$. The multiplier κ is lower that $\sqrt{2}$ in a stable fixed point equilibrium. This means that the dollar factor is more strongly exposed to risk aversion shocks $\epsilon_{\gamma,t+1}$ than the commodity factor. We find this case to be relevant in Section 4. The stable fixed point equilibrium obtains when there is sufficient variation $\mathbb{V}[e]$ in commodity exposure across regular countries relative to their overall riskiness, measured by $(e - e_u)^2$. This helps the intermediary limit its exposure to commodity risk by tilting its positions toward low-exposure countries, making a positive overall exposure to global tradable output shocks tenable.

Commodity loadings now increase in the country's dollar position. The reason is that the dollar factor is more strongly exposed to risk aversion shocks than the commodity factor. This implies that, perhaps counterintuitively, the commodity factor decreases in $\epsilon_{\gamma,t+1}$ conditional on the dollar factor. Countries with higher loadings on the commodity factor then depreciate more when $\epsilon_{\gamma,t+1}$ is negative. Supposing two countries have the same commodity exposure e_i, the one that depreciates more with $\epsilon_{\gamma,t+1} < 0$ must have higher dollar assets: the only only reason it can depreciate more is that it is perceived as less risky, and since commodity risk is the same across the two countries, the safer one must be less exposed to dollar risk.

We find this case to be relevant in our empirical results in Section 2. However, this does not imply that shocks to non-tradable output x_{it} are not necessary in the model. Rather, Proposition 4 makes the case for including financial shocks. We now turn to the reason why non-tradable output shocks are essential.

3.4 Risk-sharing in the cross-section

International risk-sharing is typically quantified by $\mathbb{C}[\Delta c_{i,t+1} - \Delta c_{u,t+1}, \Delta s_{i,t+1}]$, the covariance between bilateral exchange rates and relative consumption growth. In our model, if market were complete and integrated, the following would hold state-by-state for all i:

$$\rho(\Delta c_{i,t+1} - \Delta c_{u,t+1}) = \Delta s_{i,t+1} \tag{14}$$

This implies a positive covariance, which is at odds with the data, where this unconditional covariance is zero or mildly negative, an observation due to Backus and Smith (1993). If local currency bond markets were only accessible to local households, but all households could freely trade dollar bonds without intermediation, equation (14) would hold in expectations. Since $r_{it} = \frac{1}{\rho}\mathbb{E}_t[\Delta c_{i,t+1}]$, this would capture uncovered interest parity. In our model, intermediation leads to UIP violations. Optimal choice of the intermediary implies

$$\mathbb{E}_t\left[\rho(\Delta c_{i,t+1} - \Delta c_{u,t+1}) - \Delta s_{i,t+1}\right] = \frac{1}{\chi}(L_i l_{i,t+1} - L m_{t+1} - \gamma_t(L_i - L)) \tag{15}$$

The UIP violation pays for financial inflows in country i and is additionally increased if the intermediary's risk-aversion grows and the country is risky ($L_i < L$). Equation (15) is the cross-sectional version of Lemma 1 in Itskhoki and Mukhin (2021), extended to allow countries to be endogenously more or less risky. Itskhoki and Mukhin (2021) show that this structure leaves enough space for the risk-sharing covariance to have any sign.

For this reason, we attack the covariance from the intratemporal perspective. Intratemporal optimality implies

$$c_{it} = \frac{1-\alpha}{\alpha} q_{it} + x_{it}$$

This leads to

$$\mathbb{C}[\Delta c_{i,t+1} - \Delta c_{u,t+1}, \Delta s_{i,t+1}] = \mathbb{C}[\Delta s_{i,t+1}, \Delta x_{i,t+1} - \Delta x_{u,t+1}] - \frac{1-\alpha}{\alpha}\mathbb{V}[\Delta s_{i,t+1}] \tag{16}$$

Equation (16) illustrates the importance of the shock to non-tradables. Without variation in non-traded supply, our model would imply a strongly negative covariance, overturning the Backus and Smith (1993) result but going to the opposite extreme. Non-tradability of some goods naturally induces positive co-movement between consumption and domestic price index because it is an extreme form of home bias. This is the reason the model needs a force that induces expansionary depreciations, for which shocks to non-traded supply are a good candidate. In a simple special case, we can illustrate the usefulness of the risk-sharing condition for identifying the relative sizes of the shocks analytically.

PROPOSITION 5. *Suppose* $\sigma_x = 0$. *Then,* $corr[\Delta c_{i,t+1} - \Delta c_{u,t+1}, \Delta s_{i,t+1}] = -1$. *If* $\sigma_x \neq 0$, $corr[\Delta c_{i,t+1} - \Delta c_{u,t+1}, \Delta s_{i,t+1}]$ *can take any value in* $[-1, 1]$.

Much like Proposition 4 shows that commodity factor loadings identify a role for the global

tradable endowment shocks and financial shocks, Proposition 5 makes the case for non-tradable supply shocks. If the model should match the average risk-sharing covariance close to zero, shocks to non-traded supply have to balance out the imperfectly diversified shocks to traded endowments, which lead to expansionary appreciations, and financial shocks, which directly introduce wedges in the UIP condition.

4 Quantitative Analysis

We present our quantitative results in four steps. First, we describe the calibration procedure. Second, we show impulse responses to fundamental shocks and iillustrate the origins of risk in the model. Third, we describe the drivers of the relative price of local non-tradables in individual countries. Fourth, we show the drivers of bilateral exchange rates against the dollar in the cross-section and the factor structure of exchange rates.

Calibration strategy. We solve the general case of the model, departing from the simplifying assumption on infinite portfolio management costs. The intermediaries have a finite value of portfolio cost and price risk emanating from the global tradable output shocks z_t and US non-tradable output shocks x_{ut}. We choose parameters to approximate a set of moments presented in Table 3. These include two cross-sectional moments: the ratio of standard deviations of the two factors and their correlation, estimated in the 1990-2020 sample. Other moments are computed for all countries and then averaged over the cross-section. We approximate the average persistence of the bilateral exchange rates against the USD, the correlation between depreciation against the dollar and the gap in consumption growth relative to the US, correlations between output growth, consumption growth, and depreciation, and the standard deviations of these variables.

We choose 8 parameters listed as internally calibrated in Table 2 to approximate these 11 moments. We restrict persistence of tradable and non-tradable output shocks to be the same. We set the inverse elasticity of intertemporal substitution to 2, a standard value in the literature, and set the discount rate to 0.98 for a steady-state interest rate of 2%. To compute the moments in the model, we simulate a global economy of 100 countries for 5000 months. We draw pairs (e_i, M_i) for these countries from the empirical distribution of these parameters, using the mapping from the share of commodities in exports to e_i described in Appendix D. Appendix C presents full details

Table 2: Model parameters.

Parameter	Value	Role
		Internally calibrated
α	0.81	share of non-tradables
χ	16.94	portfolio cost
Γ	0.45	risk aversion of intermediaries
ρ_γ	0.000	persistence of γ_t
ρ_y	0.997	persistence of x_{it} and z_t
σ_γ	0.5000	standard deviation of innovations to γ_t
σ_x	0.0080	standard deviation of innovations to x_{it}
σ_z	0.0428	standard deviation of innovations to z_t
		Estimated
τ	0.83	stabilization elasticity
L	3.40	total external assets
$\{e_i, M_i\}$		distribution of country types
		Set externally
ρ	2.0	inverse of EIS
β	0.98	discount factor

Table 3: Model fit on targeted moments.

Moment	Data	Model
std(doll$_t$)/std(comm$_t$)	1.50	1.49
corr(doll$_t$,comm$_t$)	0.51	0.55
$\rho(s_{it})$	0.92	0.90
corr$(\Delta s_{it}, \Delta c_{it} - \Delta c_{ut})$	-0.20	-0.18
corr$(\Delta y_{it}, \Delta y_t)$	0.62	0.28
corr$(\Delta y_{it}, \Delta c_{it})$	0.50	0.44
corr$(\Delta s_{it}, \Delta c_{it})$	0.03	-0.23
corr$(\Delta y_{it}, \Delta s_{it})$	0.22	0.34
std(Δc_{it})	1.40	1.31
std(Δs_{it})	6.80	6.79
std(Δy_{it})	1.20	1.31

of our calibration and estimation procedure.

Internally calibrated parameters jointly affect all targeted moments and all variance decompositions that we perform. However, our theoretical results provide guidance on which links are stronger than others and what moments push parameters in opposing directions. In particular, we are interested in identifying the relative strength of the shocks $\{z_t, \gamma_t, x_{ut}, x_{it}\}$ for the cross-sectional analysis of the exchange rate drivers. First, we note that in the absence of demand shocks, excess volatility of exchange rates and low, near zero, correlation between exchange rates and consumption points to risk-aversion shocks γ_t. Second, the positive correlation between local and global output suggests a non-trivial role for global tradable output shocks z_t. Third, the positive correlation between the commodity and dollar factors points to a limited role for US non-tradable output shocks x_{ut}, since they only affect the dollar. Fourth, the fact that the correlation between a country's exchange rate depreciation against the dollar and its relative consumption growth compared to the US, the Backus and Smith (1993) moment, is close to zero ensures the model attaches some importance to shocks to non-traded output x_{ut} and x_{it}, as suggested by Proposition 5.

The two cross-section moments we use, the correlation between the factors and the ratio of their volatilities, inform the relative importance of the three global shocks in cross-section. The high correlation ensures that risk originating from z_t in the model is priced. The reason is that shocks to γ_t are a strong driver of the dollar factor regardless of other parameters, so if the global business cycle risk were not priced, γ_t would not drive the commodity factor, and the correlation between the factors would be close to zero. This implies high importance of z_t and substantial variation in the drivers of exchange rates along the dimension of commodity intensity.

At the same time, the ratio of volatilities of the two factors informs the relative price of the dollar and commodity risk. The fact that the dollar factor is more volatile points to a relatively high price of the dollar risk: the relative price of commodity risk cannot be too high, since otherwise shocks to risk-taking capacity γ_t would generate too much capital flight specifically from commodity-intensive countries, inducing too much volatility in the commodity factor. This implies substantial variation in the importance of risk premium shocks stemming from fluctuations in γ_t along the dimension of dollar asset abundance.

Impulse responses. We now illustrate the impact of the three global shocks in the model at the quarterly frequency. We start with two output shocks of global importance: tradable output

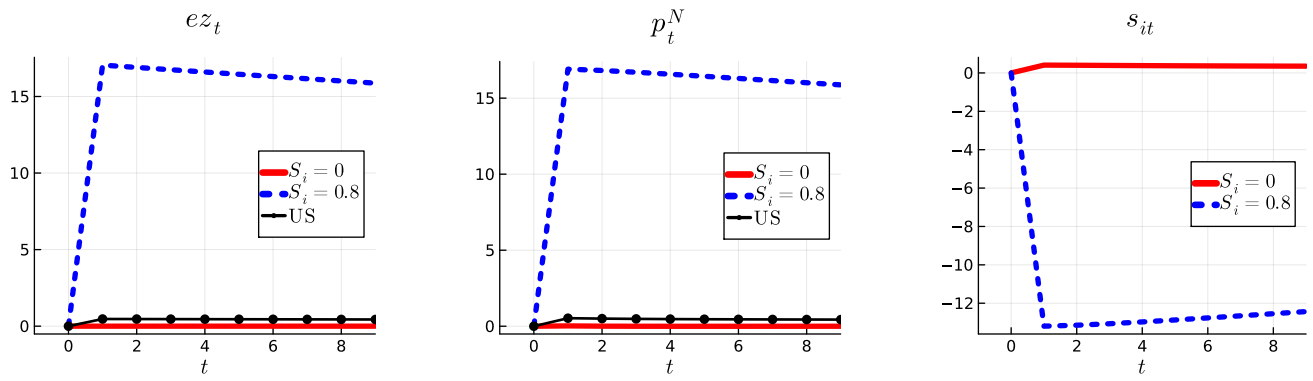
z_t and US non-tradable output x_{ut}. We explain the nature of cyclical risk they generate and then illustrate how shocks to risk-taking capacity γ_t lead to a financial cycle given the risk in the system.

Figure 2 shows the impact of a shock to z_t on tradable output, the relative price of non-tradables, and bilateral exchange rates. We show three countries: one with a high share of commodities in exports, $S_i = 0.8$, one with no commodities in exports, $S_i = 0$, and the US. Dollar assets are fixed at the average level for both regular countries. Countries with high commodity exports experience a spike in tradable income, shown on panel (a), which leads to a rise in the relative price of their non-tradable goods, shown on panel (b). The US has a non-zero share of commodities in exports, somewhere in between the two other countries. The relative price of non-tradables increases even in the country that does not export commodities at all: the tradable output shocks is aggregate, and some of the increase in tradable consumption is shared by all countries through capital flows.

Finally, panel (c) shows that currencies against the dollar appreciate in high-commodity countries and depreciate in low-commodity ones. On Figure C.1 in Appendix C, we show that the intermediary's profits are positive after positive shocks to z_t, since its assets are denominated in currencies that are on average more exposed to z_t than the dollar, which is the currency of its liabilities. This makes high-commodity currencies risky, since they generate positive returns when profits increase, and low-commodity currencies are perceived as safe.

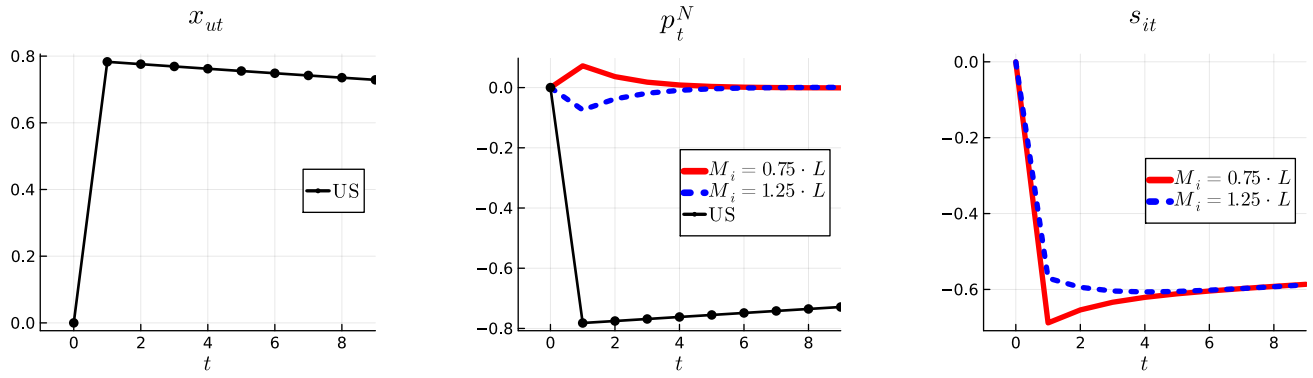
Figure 3 shows responses to a rise in US non-tradable output. In our calibration, it is close to permanent, which is why the elevation in x_{ut} does not visibly fade out on panel (a). Panel (b) shows the relative price of non-tradables in countries with high and low dollar assets and in the US. Commodity exposure is fixed at the average level for both regular countries. The dollar depreciation has two consequences. First, it leads to a rise in reserve demand from all countries, decreasing their consumption and the price of their non-tradables. Second, the increasing demand for reserves expands the intermediary's balance sheet and leads to inflows into all countries, raising their consumption and their non-tradable prices. The size of country's dollar assets relative to the average determines which force dominates: dollar-rich countries experience a decline in the prices of non-tradables. Panel (c) shows that their currency appreciates less against the dollar.

On Figure C.2 in Appendix C, we show that the intermediary's profits are positive on impact of a positive shock to x_{ut}. This means that all currencies bear dollar risk from the intermediary's perspective: they appreciate against the currency of its liabilities, the dollar, when profits are positive. Moreover, countries with large dollar assets are less risky, since their appreciation against



(a) tradable output (b) price of non-tradables (c) exchange rate

Figure 2: Responses of tradable output, relative prices of non-tradable goods, and bilateral exchange rates against USD to a standard innovation to z_t. The units of the vertical axis are percentage points. The shock happens at $t = 1$.

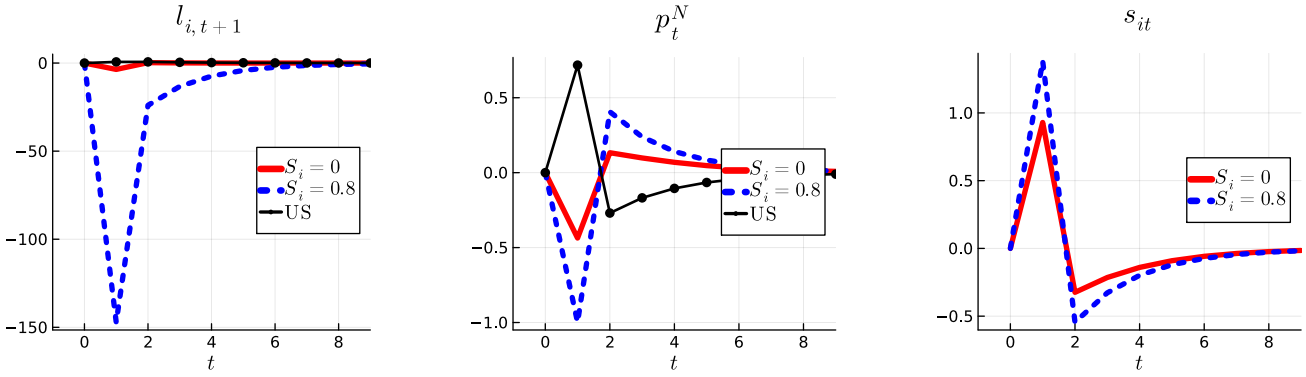


(a) US non-tradable output (b) price of non-tradables (c) exchange rate

Figure 3: Responses of non-tradable output, relative prices of non-tradable goods, and bilateral exchange rates against USD to a standard innovation to x_{ut}. The units of the vertical axis are percentage points. The shock happens at $t = 1$.

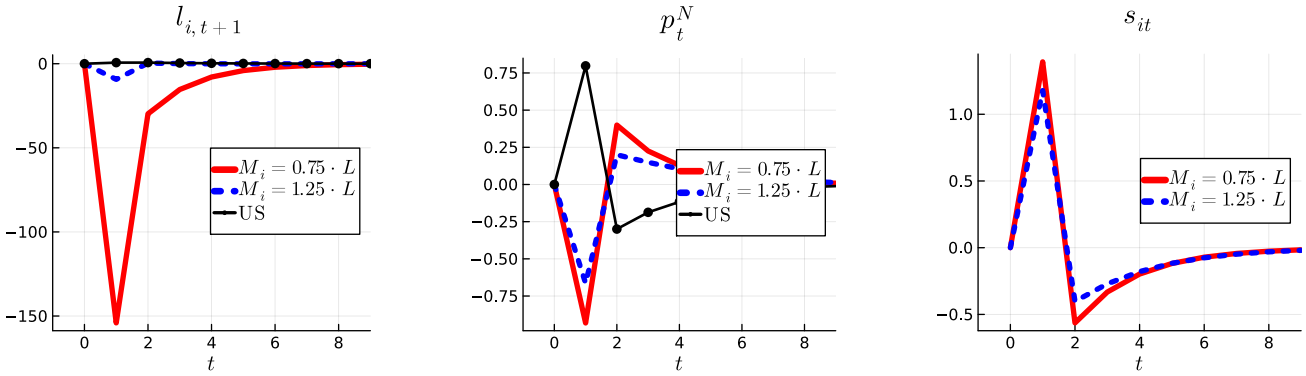
the dollar is muted. Dollar-poor countries bear more dollar risk.

Having established how commodity and dollar risk are distributed in the economy, we turn to risk-aversion shocks γ_t. Figure 4 compares the impact of this shock on countries with high and low commodity intensity. The risk-aversion shock is mediated through capital flows, which is the focus of panel (a). When the intermediary's risk-taking capacity falls after a positive shock to γ_t, it decreases its exposure to risk. Panel (a) shows capital flight from regular countries, with a larger magnitude in the high-commodity one. The low-commodity country experiences capital flight too, since it is also evaluated at the average level of dollar assets, and hence bears substantial dollar risk. Flight to safety manifests in inflows into the US. Panel (b) shows that this adjustment



(a) external debt (b) price of non-tradables (c) exchange rate

Figure 4: Responses of financial inflows, relative prices of non-tradable goods, and bilateral exchange rates against USD to a standard innovation to γ_t. The units of the vertical axis are percentage points. The shock happens at $t = 1$. Countries with different commodity shares.



(a) external debt (b) price of non-tradables (c) exchange rate

Figure 5: Responses of financial inflows, relative prices of non-tradable goods, and bilateral exchange rates against USD to a standard innovation to γ_t. The units of the vertical axis are percentage points. The shock happens at $t = 1$. Countries with different dollar assets.

in current account induces non-tradables to depreciate in regular countries and appreciate in the US. Panel (c) shows that, as a consequence, currencies of regular countries depreciate against the dollar, with a higher depreciation in the high-commodity country.

Figure 5 similarly compares the impact of a risk-aversion shock on countries with high and low dollar assets. The country with low dollar assets experiences more intense capital flight because it bears more dollar risk. The US, again, experiences flight-to-safety inflows. Non-tradable goods depreciate in regular countries, with a higher magnitude in the dollar-poor one. Panel (c) shows that both regular countries' currencies depreciate against the dollar on impact. The gap in these depreciations is narrower than the gap in the rate of capital flight on panel (a). The reason is

that what impacts the exchange rates is not inflow rate $l_{i,t+1}$ themselves but inflow levels $L_i l_{i,t+1}$, and low-risk countries have higher steady-state debt levels L_i. The fact that L_i is higher for the dollar-rich country partly offsets the fact that $l_{i,t+1}$ is more negative for the dollar-poor one.

Drivers of currencies in the cross-section. We now perform variance decompositions of the relative price of non-tradables in individual countries to gauge the relative importance of shocks. For every country i, including the US, we run the following regression in the simulated economy:

$$\Delta p_{it}^N = \beta_{i,\gamma} \Delta \gamma_t + \beta_{i,z} \Delta z_t + \beta_{i,u} \Delta x_{ut} + \epsilon_{it} \tag{17}$$

We standardize the independent variables. Since all regressors are independent, the variance share of a standardized shock $k \in \{\gamma, z, u\}$ is

$$\frac{\beta_{i,k}^2}{\sum_k \beta_{i,k}^2} \cdot R^2.$$

Table 4: Decomposition of the changes in the relative price of non-tradables in the US.

	coefficient	variance share
$\Delta\gamma_t$	2.85	0.90
Δz_t	0.47	0.02
$\Delta x_{u,t}$	-0.80	0.07
R^2	1.00	
Observations	5000	

Notes: Estimates of the coefficients from equation (17) for $i = u$. Independent variables are standardized. The units of the dependent variable are percentage points. Standard errors are 0.01 or less for all coefficients.

We first report the results for the US in Table 4. The standard deviation of changes in the relative price of US non-tradables is 3%. The dynamics of this price are mainly driven by risk aversion shocks γ_t, which account for more than 80% of the variance. Shocks to non-tradable supply turn out to contribute modestly, principally because our calibration procedure assigns them a much lower volatility.

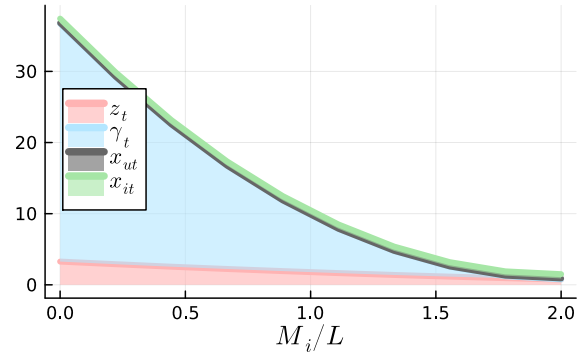
In Table 5, we report the estimates from the same regression run four four different countries in the model: combinations of high and low commodity export share and high and low dollar assets. Countries with low dollar assets are more strongly exposed to risk aversion shocks to γ_t through capital flight. Global tradable output shocks z_t increase their non-tradable prices more since the dollar depreciates in times of high global output, and these countries face lower capital losses on their dollar assets. The relative price of non-tradables in dollar-poor countries increases after

Table 5: Decomposition of the changes in the relative price of non-tradables in four countries.

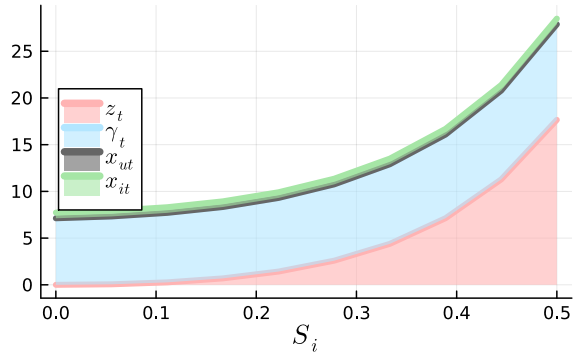
	Low M_i		High M_i	
	Low S_i	High S_i	Low S_i	High S_i
$\Delta\gamma_t$	-4.21	-4.52	-1.24	-1.56
Δz_t	0.73	3.08	0.24	2.57
$\Delta x_{u,t}$	0.19	0.19	-0.18	-0.18
$\Delta x_{i,t}$	-0.78	-0.78	-0.78	-0.78
R^2	0.99	0.99	0.99	0.99
Observations	5000	5000	5000	5000

Notes: Estimates of the coefficients from equation (17). Independent variables are standardized. The units of the dependent variable are percentage points. We use the following low-high pairs: $(0.75 \cdot L, 1.25 \cdot L)$ for M_i and $(0.1, 0.4)$ for S_i. Standard errors are 0.01 or less for all coefficients.

positive shocks to the US non-tradable supply x_{ut}: capital losses due to a depreciation of the dollar in these countries are dominated by financial inflows from the intermediaries, who expand their balance sheets in times of dollar weakness. The outcome is the opposite in dollar-rich countries, where capital losses on the dollar dominate. Countries with high commodity exports are more exposed to the tradable output shocks z_t and risk aversion shocks γ_t. Finally, local non-tradable supply shocks x_{it} are prominent in countries with low commodity exports and high dollar asset buffers. These countries are better insulated from global shocks, and local shocks are as important as risk aversion shocks γ_t and global tradable output shocks z_t.



(a) contributions of shocks to non-traded price variance, $S_i = 0.25$



(b) contributions of shocks to non-traded price variance, $M_i = L$

We next perform the same decomposition in the entire cross-section and plot contributions of the three global shocks and local non-traded output as a function of the country's permanent type (S_i, M_i), where the commodity share of exports S_i maps into e_i as $S_i = \dfrac{e_i}{1+e_i}$. We slice the cross-section two ways. First, Figure 6a shows the contributions of the four shocks $\{z_t, \gamma_t, x_{ut}, x_{it}\}$

to country i's non-tradable price volatility as a function of its dollar assets at the average level of commodity intensity. The units of the vertical axis are percentage points squared, and the layers add up to the total variance. Shocks to γ_t are responsible for most of the variation at low levels of dollar assets. The relative price of non-tradables in countries without a USD buffer is strongly exposed to risk-aversion shocks through capital flight, since these countries bear high dollar risk, and the intermediary rebalances away from them after its risk aversion rises. As the stock of dollar assets increases, exposure to risk-aversion shocks is neutralized, and the price of non-tradables becomes less volatile. Local non-tradable supply shocks increase their contribution to volatility and become an important driver.

Second, Figure 6b shows the contributions of the four shocks as functions of country i's commodity share of exports at the average level of dollar assets. The share of variation coming from tradable endowment shocks z_t increases with the commodity share of exports, although risk aversion shocks still dominate for most of the countries and increase their absolute contribution as S_i increases. At the average export share $S_i = 0.25$ and the average level of dollar assets $M_i = L$, risk aversion shocks account for around one half of the non-tradable price volatility. The reason for such consistently high importance is that all countries in the picture are evaluated at the average level of dollar assets, and hence bear substantial dollar risk.

Factor structure of bilateral exchange rates. We next turn to bilateral exchange rates against the dollar and perform a variance decomposition of the dollar and commodity factors to measure the relative importance of the three global shocks. In the simulated economy, we run the following regressions:

$$\text{doll}_t = \beta_{\text{doll},\gamma}\Delta\gamma_t + \beta_{\text{doll},z}\Delta z_{t+1} + \beta_{\text{doll},u}\Delta x_{ut} + \epsilon_{\text{doll}t} \tag{18}$$

$$\text{comm}_t = \beta_{\text{comm},\gamma}\Delta\gamma_t + \beta_{\text{comm},z}\Delta z_t + \beta_{\text{comm},u}\Delta x_{ut} + \epsilon_{\text{comm},t} \tag{19}$$

The top panel of Table 6 reports the results. We standardize the independent variables, so the coefficients in Table 6 can be interpreted as standard magnitudes of shocks. The commodity factor decreases in Δz_{t+1} due to tradable income windfalls in high-commodity countries and increases in $\Delta\gamma_{t+1}$ due to portfolio rebalancing away from the risky to safe countries. The dollar factor increases in $\Delta\gamma_{t+1}$ due to financial inflows into the US following flight to safety as well. In terms of magnitudes, tradable output shocks are responsible for roughly two thirds of the variation in

Table 6: Decomposition of factors.

	dollar factor		commodity factor	
	coefficient	variance share	coefficient	variance share
$\Delta\gamma_t$	4.60	0.94	1.25	0.16
Δz_t	-0.87	0.03	-2.90	0.84
$\Delta x_{u,t}$	-0.65	0.02	-0.11	0.00
R^2	0.99		0.97	
Observations	5000		5000	

Notes: Estimates of the coefficients from equation (18) and equation (19). Independent variables are standardized. The units of the dependent variables are percentage points. Standard errors are 0.01 or less for all coefficients.

the commodity factor, and risk-taking capacity shocks account for the rest. Notably, risk-taking capacity shocks γ_t dominate movements in the dollar factor. Flight to safety is the most important driver of the broad dollar appreciation in our model.

Drivers of bilateral exchange rates against the dollar. We now perform variance decompositions for individual exchange rates in the same way we report it above for the relative price of non-tradable goods.

Table 7: Decomposition of the dollar exchange rate depreciations in four countries.

	Low M_i		High M_i	
	Low S_i	High S_i	Low S_i	High S_i
$\Delta\gamma_t$	5.70	5.95	3.30	3.55
Δz_t	-0.21	-2.10	0.19	-1.69
$\Delta x_{u,t}$	-0.80	-0.80	-0.50	-0.50
$\Delta x_{i,t}$	0.63	0.62	0.63	0.62
R^2	0.99	0.99	1.00	0.99
Observations	5000	5000	5000	5000

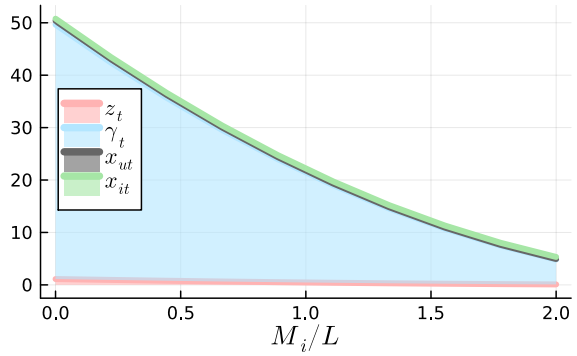
Notes: Estimates of the coefficients from equation (20). Independent variables are standardized. The units of the dependent variable are percentage points. We use the following low-high pairs: $(0.75 \cdot L, 1.25 \cdot L)$ for M_i and $(0.1, 0.4)$ for S_i. Standard errors are 0.01 or less for all coefficients.

We run the following regressions for all countries i, excluding the US:

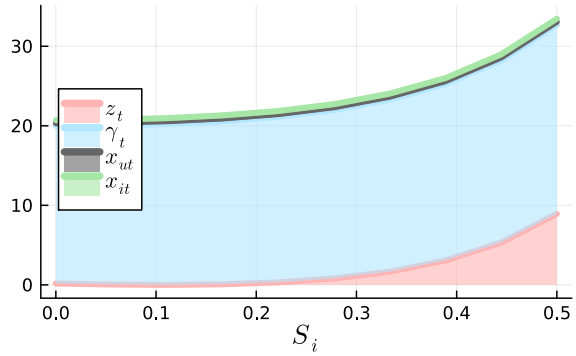
$$\Delta s_{it} = \beta_{i,\gamma}\Delta\gamma_t + \beta_{i,z}\Delta z_t + \beta_{i,u}\Delta x_{ut} + \epsilon_{it} \tag{20}$$

Here $\Delta s_{i,t+1}$ is the depreciation of currency i against the dollar. Table 7 reports regression estimates for four countries that combine high and low dollar assets and high and low commodity export shares with the same choice of countries as before. These numbers differ from the coefficients we

report for the relative price of non-tradables because bilateral exchange rate movements combine country i's non-tradable price movements with those of the US: $\Delta s_{i,t+1} = \alpha(\Delta p^N_{u,t+1} - \Delta p^N_{i,t+1})$. This manifests in the consistently high importance of risk aversion shocks γ_t, which is more prominent than for individual non-tradable prices. The reason is that risk aversion shocks systematically move non-tradable prices in the US and other countries in the opposite directions since dollar risk is strongly priced. Another difference is that the impact of US non-tradable supply shocks has the same sign for all bilateral depreciations, while its sign changes for individual non-tradable prices depending on the stock of dollar assets. The reason is that US non-tradables prices strongly and unambiguously decrease after positive supply shocks.



(a) contributions of shocks to exchange rate volatility, $S_i = 0.25$



(b) contributions of shocks to exchange rate volatility, $M_i = L$

Figure 7a shows the contributions of the four shocks $\{z_t, \gamma_t, x_{ut}, x_{it}\}$ to country i's exchange rate variance as a function of its dollar assets at the average level of commodity intensity. As before, the units of the vertical axis are percentage points squared, and the layers add up to the total variance. Countries without a USD buffer are strongly exposed to the dollar factor, making shocks to γ_t the single most important source of exchange rate volatility. As the stock of dollar assets increases, the exposure to the dollar factor and the exchange rate volatility both fall, although the contribution of shocks to γ_t does not reach zero as quickly as their contribution to the volatility of non-tradable prices.

Second, Figure 7b shows the contributions of the four shocks as functions of country i's commodity share of exports at the average level of dollar assets. Risk-taking capacity shocks still dominate for most of the countries, accounting for more than 90% of exchange rate variance at the average export share $S_i = 0.25$ and the average level of dollar assets $M_i = L$. It might seem surprising that risk-taking capacity shocks are the single dominant source of exchange rate fluc-

tuations for most countries, yet two thirds of the commodity factor are driven by the tradable output shocks. The explanation for this apparent discrepancy is that the importance of the tradable output shock varies more with S_i, so the commodity factor, which is a high-minus-low factor by construction, loads on this shock more strongly.

5 Conclusion

We develop a model of the global economy to study the drivers of exchange rate in cross-section. The model fulfills two goals. First, by virtue of reproducing the factor structure of exchange rates, the model has high explanatory power in the data. Second, the model provides an equilibrium interpretation to this factor structure, mapping fundamental shocks to factors and fundamental properties of countries to their factor loadings. We find that risk premium shocks are a consistently important driver of individual countries' price levels and a much more important driver of their dollar exchange rates, since these shocks move non-tradable good prices in the US and other countries in the opposite directions. High-commodity currencies are driven by a combination of risk premium and global output shocks. Countries with low dollar assets are affected by risk premium shocks the most. Low-dollar and high-commodity countries are the ones with most volatile price levels and dollar exchange rates. They also experience the highest dollar exchange rate volatility and the highest explanatory power of global factors overall. High-dollar and low-commodity countries, in contrast, are the ones with the lowest volatility and lowest explanatory power of global shocks: they maximize the variance share of local shocks in the local price level and the dollar exchange rate.

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A Empirical Appendix

A.1 Data description

- *Exchange rate*: The dependent variable in our empirical model equation (1) is constructed based on exchange rate contracts data from the London Stock Exchange (LSEG). We construct end-of-month bilateral exchange rates vis-a-vis the US dollar from daily to-USD spot exchange rate contracts, and then take the first difference of log exchange rate of local currency i per dollar.

- *Commodity factor*: We define high and low commodity producers based on countries' time-averaged annual share of raw materials in merchandise exports over 2015-2019, using data from the World Integrated Trade Solution (WITS). A country is categorized as a high commodity producer $i \in \mathcal{I}_{h,i}$, if its time-averaged share of raw materials in merchandise exports is above the median across countries in our sample, excluding the country itself; and as a low commodity producer $i \in \mathcal{I}_{l,i}$ if it is below the median. We then construct the country-specific commodity factor by differencing the average change in the log exchange rate of high commodity producers and that of low commodity producers as in equation (2).

- *Dollar factor*: We construct the currency-specific dollar factor as the cross-sectional-average of monthly change in the log exchange rates of all currencies against the dollar, excluding the currency itself as in equation (3).

- *Carry factor*: Following Verdelhan (2018), we first sort currencies in our sample, excluding the currency itself, into six portfolios based on their local-currency interest rates, which are approximated using the one-month forward discount rate. We then construct the currency-specific carry factor by taking the difference between the average change in the log exchange rate of the 6th portfolio (the one with the highest interest rates), and that of the first portfolio (the one with the lowest interest rates).

- *Trade-centrality factor*: We first construct the Richmond (2019) trade centrality measure as in equation (A.1), using bilateral trade data from the International Monetary Fund's (IMF) International Trade in Goods dataset (IMTS).

$$v_{it} = \sum_{j=1}^{N} \left(\frac{\widetilde{X}_{ijt} + \widetilde{X}_{jit}}{GDP_{it} + GDP_{jt}} \right) \tilde{s}_{jt} \tag{A.1}$$

  where v_{it} is the trade centrality of country i at time t; $\frac{\widetilde{X}_{ijt}+\widetilde{X}_{jit}}{GDP_{it}+GDP_{jt}}$ is the bilateral trade intensity between country i and j at time t, measured as the total bilateral exports scaled by the combined GDP; and $\tilde{s}_{jt}$ is country j's exports as a share of total trade across our sample countries at time t. We then construct the country-specific trade-centrality factor in a manner similar to the commodity factor, categorizing countries into high- or low-trade-centrality groups based on whether a country's 2015–2019 average trade centrality is above or below the median of the sample (excluding the country itself).

- *Tradable-output-sensitivity factor*: The dependent variable in the regression model equation (4) is based on data from the Organization for Economic Co-operation and Develop-

ment's (OECD) Inter-Country Input-Output tables (ICIO) (Note that the ICIO database does not include data for Albania or Uruguay from our sample). We categorize agriculture, mining, and manufacturing as the tradable sectors and deflate each country's annual tradable output (measured in USD) by U.S. CPI index. The independent variable is based on monthly commodity prices from the World Bank Commodity Price Data and we use the end-of-year total index. We estimate the empirical model in equation (4) over the 2000-2019 period to obtain $\beta_{i,\text{tos}}$ for each country i, representing its tradable output sensitivity. Finally, we construct the tradable-output-sensitivity factor in a manner similar to the commodity factor, categorizing countries into high- or low-sensitivity groups based on whether a country's β_{tos} is above or below the median of the sample (excluding the country itself).

- *Share of dollar portfolio assets*: We primarily use data from the IMF's Portfolio Investment Positions by Counterpart Economy (formerly CPIS) to construct each country's share of dollar portfolio assets. We calculate this share by dividing a country's USD-denominated portfolio assets by its total portfolio assets and using 2015-2019 average as u_i in our empirical specification equation (5). For countries not covered by the CPIS, we impute the share using the ratio of a country's holding of US Treasuries (from the Treasury International Capital (TIC) System) to its total international investment assets (from IMF's International Investment Position (IIP)).

- *Commodity incidence (or commodity share)*: We use 2015-2019 average of a country's annual share of raw materials in merchandise exports as r_i in our empirical specification equation (5).

- *Trade centrality*: Following Richmond (2019), we use 2015-2019 average of a country's trade centrality measure as an additional control variable for specification (2) of Table 1.

- *Trade openness*: We use 2015-2019 average of a country's total trade (imports and exports) scaled by its GDP as an additional control variable for specification (3) of Table 1.

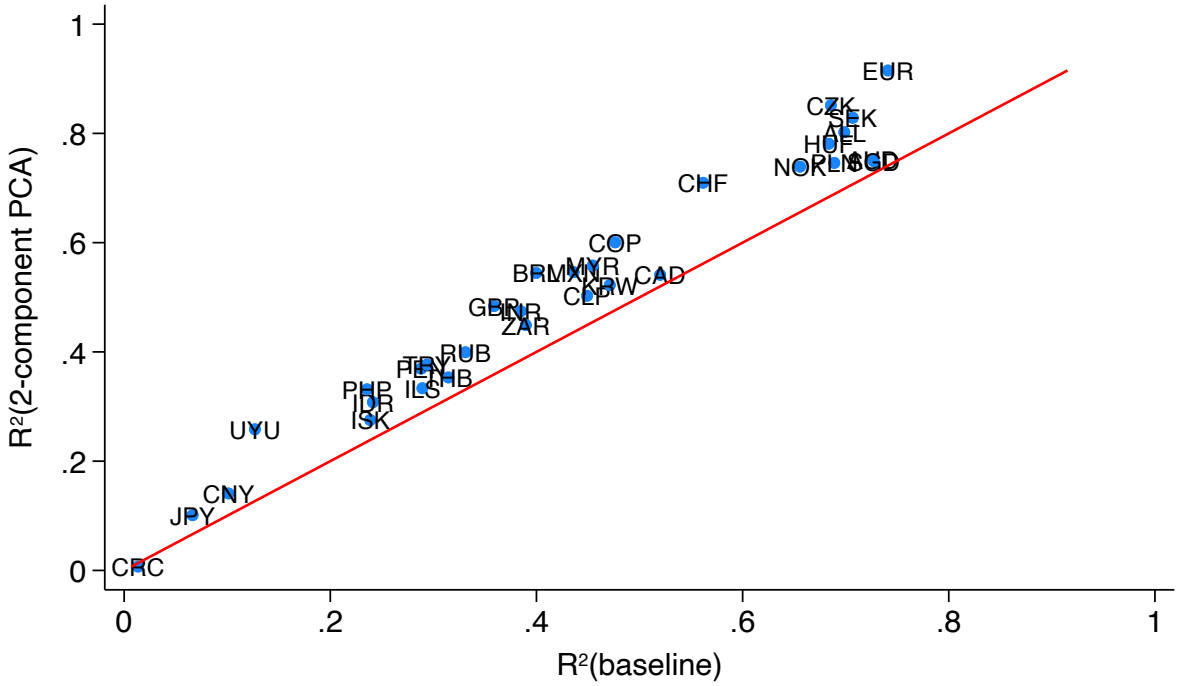
- *Country size*: Following Hassan (2013), we use 2015-2019 average of a country's log share of GDP in our sample as an additional control variable for specification (4) of Table 1.

- *Distance*: we include log harmonic geographic distance to U.S. from the World Bank's World Development Index (WDI) as an additional control variable for specification (5) of Table 1.

- *Gravity variables*: Following Lustig and Richmond (2020), we include a set of other gravity variables relative to U.S. as additional control variables in specification (6) of Table 1 using data from the Research and Expertise on the World Economy (CEPII) Gravity dataset. These include: a dummy variable indicating whether country i shares a common language with the U.S.; a dummy variable for whether it shares a common legal origin; and a dummy variable for whether country i is contiguous with the U.S.

A.2 Additional figures and tables

Figure A.1: Explanatory Power of Factor Regression and Principal Components



*Notes:*This figure plots the R^2 from a two-principal-component model (y-axis) against the R^2 from the empirical model in equation (1) (x-axis) for each currency i. Both measures represent the fraction of variation in log exchange rate changes $\Delta s_{i,t+1}$ explained by the respective models. Each scatter point represents a currency, and the red line denotes the 45-degree line.

Table A.1: Factor Structure of Exchange Rates: Regression Estimates

Country	α	β_{comm}	β_{doll}	R^2	N
Albania	-0.001	-1.601***	1.100***	0.699	240
	(0.001)	(0.183)	(0.069)		
Australia	-0.002*	0.038	1.639***	0.726	240
	(0.001)	(0.250)	(0.070)		
Brazil	0.000	2.480***	1.546***	0.400	240
	(0.002)	(0.606)	(0.136)		
Canada	-0.002*	0.586**	0.956***	0.520	240
	(0.001)	(0.293)	(0.074)		
Switzerland	-0.002*	-2.020***	1.093***	0.562	240
	(0.001)	(0.249)	(0.086)		
Chile	-0.001	1.698***	1.072***	0.449	240
	(0.002)	(0.355)	(0.109)		
China	-0.001*	0.001	0.126***	0.101	240
	(0.000)	(0.099)	(0.027)		
Colombia	-0.000	1.686***	1.258***	0.476	240
	(0.002)	(0.399)	(0.117)		
Costa Rica	0.003***	-0.264	0.003	0.014	240
	(0.001)	(0.167)	(0.041)		
Czech Republic	-0.002**	-2.079***	1.496***	0.686	240
	(0.001)	(0.299)	(0.066)		
Euro Area	-0.001	-1.688***	1.273***	0.741	240
	(0.001)	(0.206)	(0.058)		
United Kingdom	0.000	-0.570	0.776***	0.359	240
	(0.001)	(0.361)	(0.095)		
Hungary	-0.000	-1.772***	1.768***	0.684	240
	(0.001)	(0.386)	(0.110)		
Indonesia	0.002	-0.078	0.817***	0.241	240
	(0.002)	(0.312)	(0.111)		
Israel	-0.001	-0.279	0.617***	0.289	240
	(0.001)	(0.261)	(0.066)		
India	0.001	0.221	0.655***	0.385	240
	(0.001)	(0.193)	(0.063)		

Notes: This table reports regression estimates for constant α, the commodity factor loading β_{comm}, and the dollar factor loading β_{doll} for each currency i (listed in the first column), as specified in equation (1). Newly-West adjusted standard errors are reported in brackets. The table also includes the R^2, representing the fraction of variation in Δs_{t+1} explained by our empirical model for each currency's regression.

Table A.2: Factor Structure of Exchange Rates: Regression Estimates

Country	α	β_{comm}	β_{doll}	R^2	N
Iceland	0.002	-0.695	0.995***	0.239	240
	(0.002)	(0.621)	(0.170)		
Japan	0.001	-0.892**	0.276**	0.066	240
	(0.002)	(0.402)	(0.124)		
South Korea	-0.001	-0.071	1.119***	0.471	240
	(0.001)	(0.340)	(0.109)		
Mexico	0.001	0.965***	0.996***	0.436	240
	(0.001)	(0.310)	(0.102)		
Malaysia	-0.001	0.302*	0.627***	0.455	240
	(0.001)	(0.169)	(0.062)		
Norway	-0.000	-0.964***	1.350***	0.656	240
	(0.001)	(0.331)	(0.083)		
Peru	-0.001	0.461***	0.363***	0.288	240
	(0.001)	(0.152)	(0.049)		
Philippines	0.000	0.052	0.441***	0.236	240
	(0.001)	(0.204)	(0.052)		
Poland	-0.002	-1.167***	1.729***	0.689	240
	(0.001)	(0.345)	(0.097)		
Russia	0.002	0.290	1.131***	0.331	240
	(0.002)	(0.505)	(0.152)		
Sweden	-0.000	-1.268***	1.460***	0.707	240
	(0.001)	(0.242)	(0.063)		
Singapore	-0.001***	-0.417***	0.703***	0.727	240
	(0.001)	(0.118)	(0.045)		
Thailand	-0.001	-0.184	0.543***	0.315	240
	(0.001)	(0.203)	(0.049)		
Turkey	0.007**	2.380***	1.159***	0.294	240
	(0.003)	(0.546)	(0.117)		
Uruguay	0.004*	1.341***	0.441***	0.127	240
	(0.002)	(0.497)	(0.101)		
South Africa	0.001	0.585	1.575***	0.390	240
	(0.002)	(0.460)	(0.105)		

Notes: This table reports regression estimates for constant α, the commodity factor loading β_{comm}, and the dollar factor loading β_{doll} for each currency i (listed in the first column), as specified in equation (1). Newly-West adjusted standard errors are reported in brackets. The table also includes the R^2, representing the fraction of variation in Δs_{t+1} explained by our empirical model for each currency's regression.

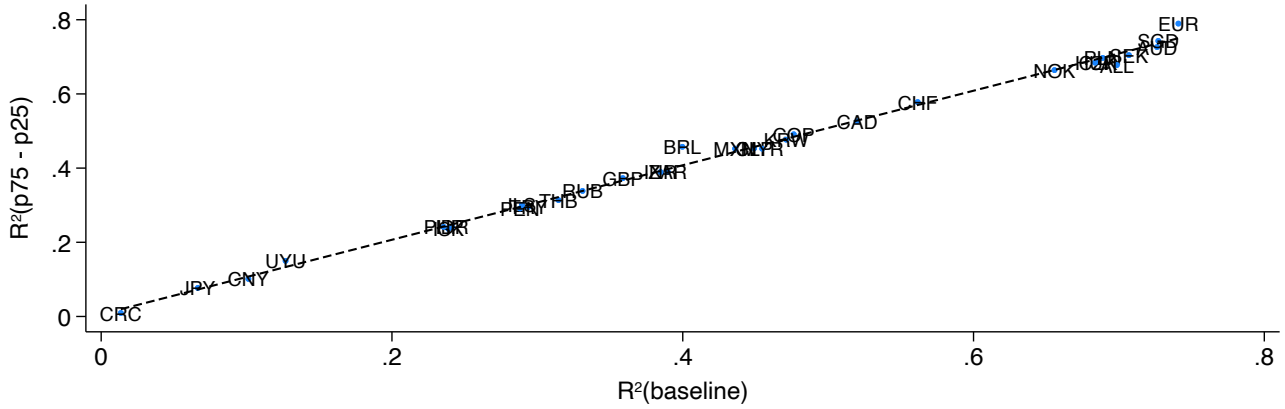
Table A.3: Factor Structure of Exchange Rates: The Role of the Dollar Basis

	# of Regressions with significant		
Currency Base	Dollar Factor	Dollar Factor	Currency-Base Factor
Albania	22	22	14
Australia	26	23	5
Brazil	22	19	4
Canada	21	22	6
Switzerland	20	23	9
Chile	22	22	1
China	30	17	6
Colombia	17	20	9
Costa Rica	31	19	3
Czech Republic	25	22	15
Euro Area	22	23	17
United Kingdom	19	22	5
Hungary	27	20	10
Indonesia	19	22	5
Israel	23	20	2
India	24	22	10
Iceland	17	22	5
Japan	25	22	9
South Korea	19	22	9
Mexico	20	22	7
Malaysia	23	22	14
Norway	22	22	12
Peru	28	21	6
Philippines	26	23	13
Poland	27	22	9
Russia	16	22	5
Sweden	24	19	14
Singapore	25	23	14
Thailand	25	22	11
Turkey	17	23	15
Uruguay	23	21	1
South Africa	25	21	3
All currency bases	732	687	268
Dependent variable relative to	currency base	currency base	USD
Independent variables	commodity factor dollar factor	commodity factor dollar factor currency-base factor	commodity factor dollar factor currency-base factor

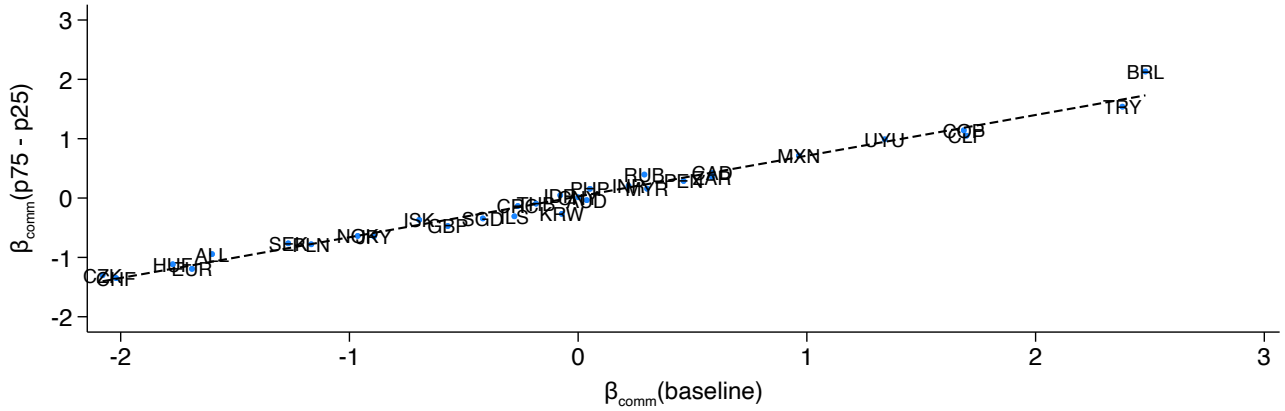
*Notes:*This table reports the counts of statistically significant dollar and currency-base factor loadings for each base currency i, evaluated at the 5% significance level. Appendix A provides details on the construction of the currency-base factor. The second column reports the number of significant dollar factor loadings obtained from two-factor regressions, where

Figure A.2: Exchange Rate Regressions: Baseline and Alternative Commodity Factor

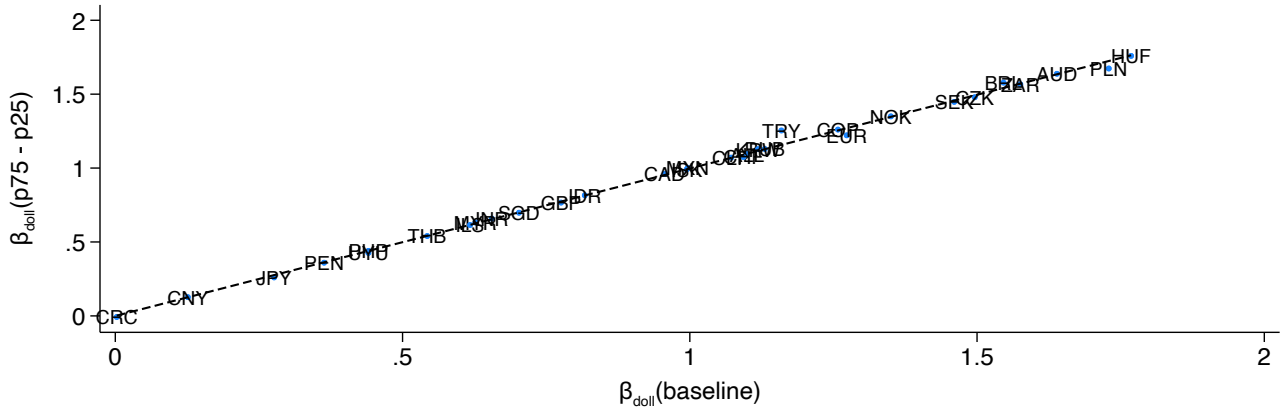
(a) Explanatory power R_i^2



(b) Commodity-factor loadings $\beta_{i,\text{comm}}$



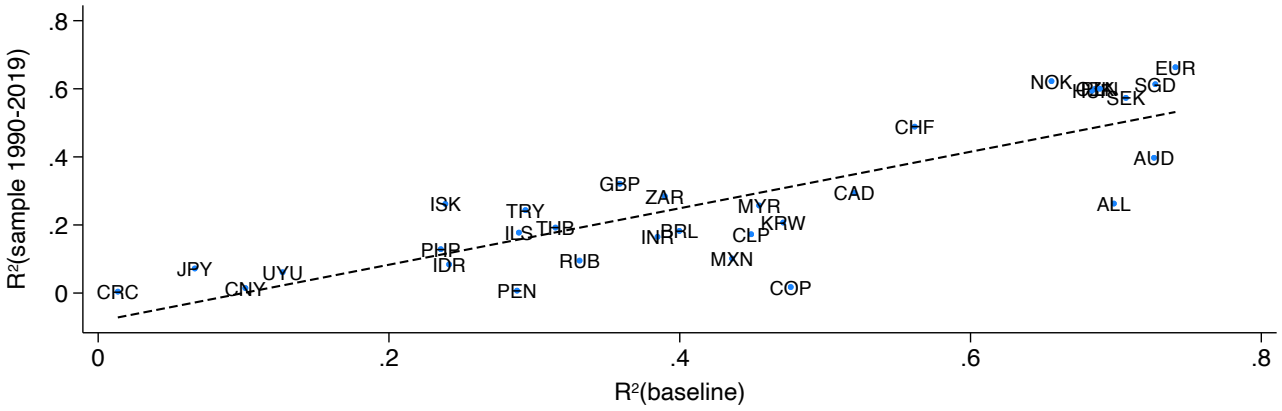
(c) Dollar-factor loadings $\beta_{i,\text{doll}}$



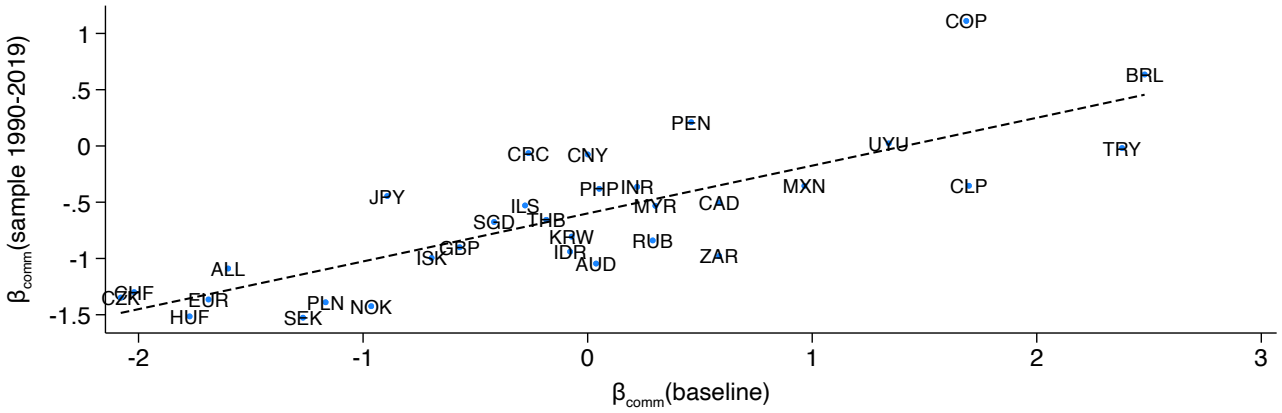
*Notes:*Panel (a) plots the R^2 for each currency i, from the modified empirical model (y-axis) - where the commodity factor is constructed using the 75th and 25 percentiles of the commodity export share - against the R^2 from the baseline model in equation (1) (x-axis). Panel (b) plots the estimated commodity factor loadings from the modified empirical model against those from the baseline model in equation (1). Panel (c) plots the estimated dollar factor loadings from the modified empirical model against those from the baseline model in equation (1). Each scatter point represents a currency, and the gray dashed line denotes the line of best fit.

Figure A.3: Exchange Rate Regressions: Baseline and 1990-2019 Sample

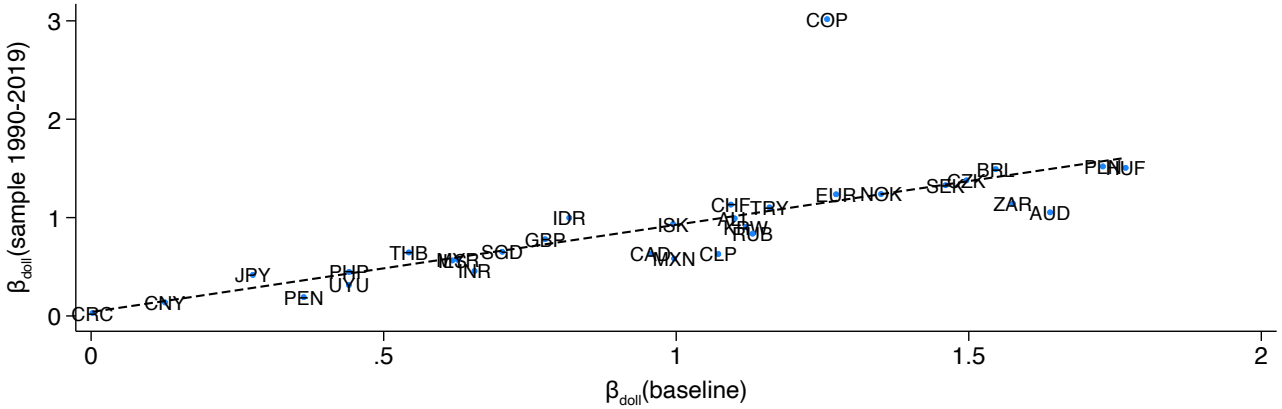
(a) Explanatory power R_i^2



(b) Commodity-factor loadings $\beta_{i,\text{comm}}$



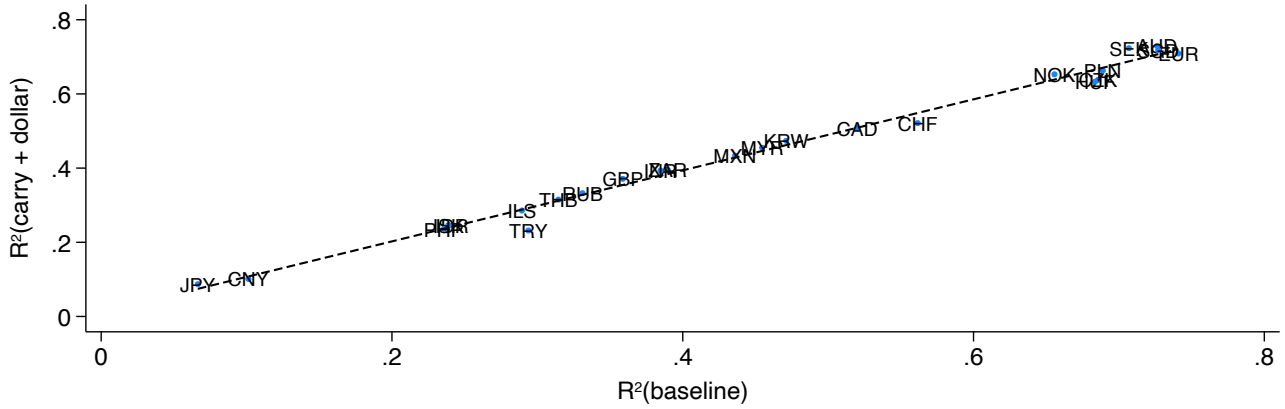
(c) Dollar-factor loadings $\beta_{i,\text{doll}}$



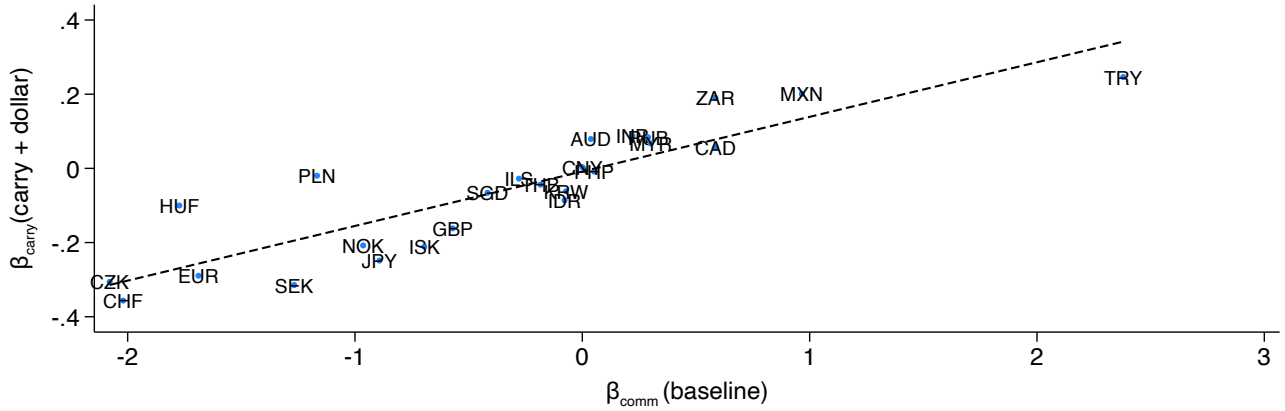
*Notes:*Panel (a) plots the R^2 for each currency i, from the 1990-2019 extended sample (y-axis), against the R^2 from the baseline model in equation (1) (x-axis), which uses the 2000-2019 sample. Panel (b) plots the estimated commodity factor loadings from the 1990-2019 extended sample against those from the baseline. Panel (c) plots the estimated dollar factor loadings from the 1990-2019 extended sample against those from the baseline. Each scatter point represents a currency, and the gray dashed line denotes the line of best fit.

Figure A.4: Exchange Rate Regressions: Baseline and Carry-Factor

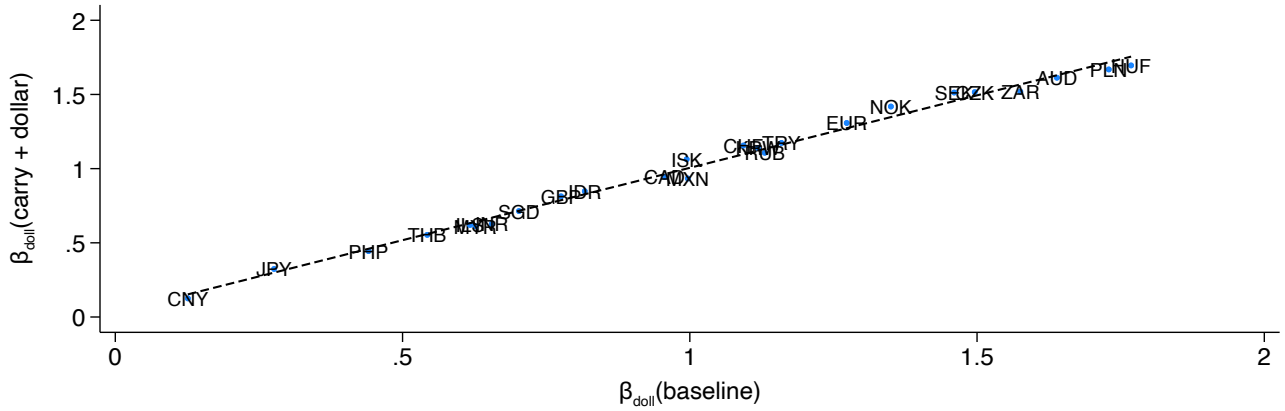
(a) Explanatory power R_i^2



(b) Commodity- and carry-factor loadings $\beta_{i,\text{comm}}$



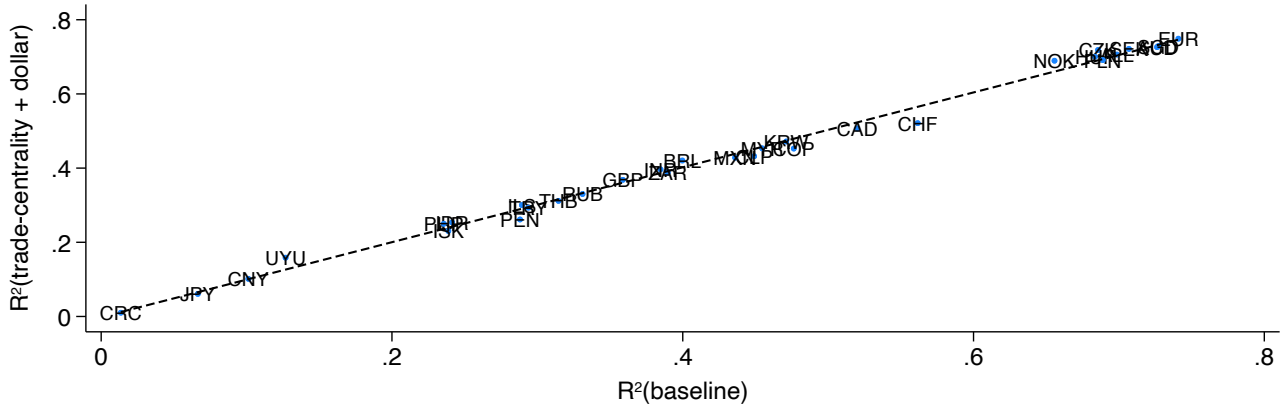
(c) Dollar-factor loadings $\beta_{i,\text{doll}}$



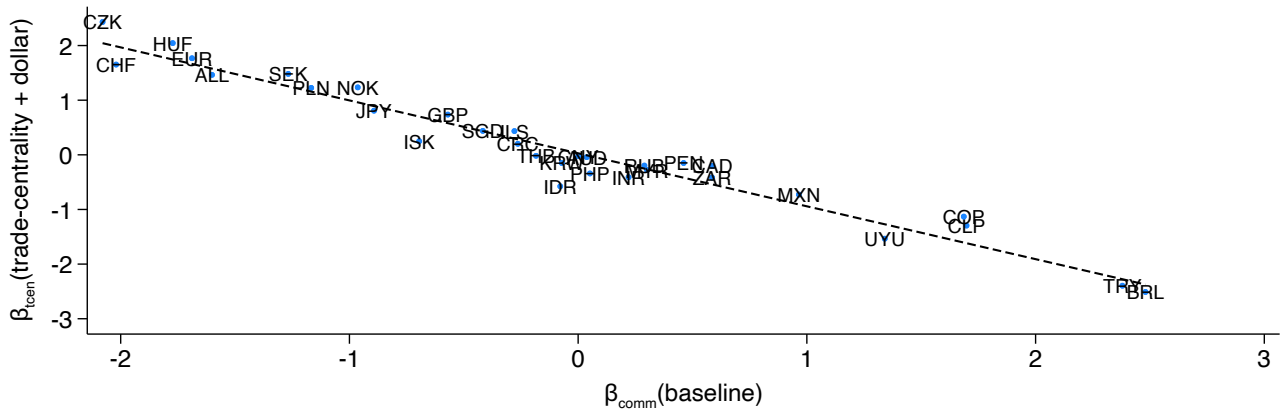
*Notes:*Appendix A provides details on the construction of carry factor proposed by Verdelhan (2018). Panel (a) plots the R^2 for each currency i, from the modified empirical model (y-axis) - where the commodity factor is replaced by the carry factor - against the R^2 from the baseline model in equation (1) (x-axis). Panel (b) plots the estimated carry factor loadings from the modified empirical model against the estimated commodity factor loadings from our baseline model. Panel (c) plots the estimated dollar factor loadings from the modified empirical model against those from the baseline model. Each scatter point represents a currency, and the gray dashed line denotes the line of best fit.

Figure A.5: Exchange Rate Regressions: Baseline and Trade-Centrality Factor

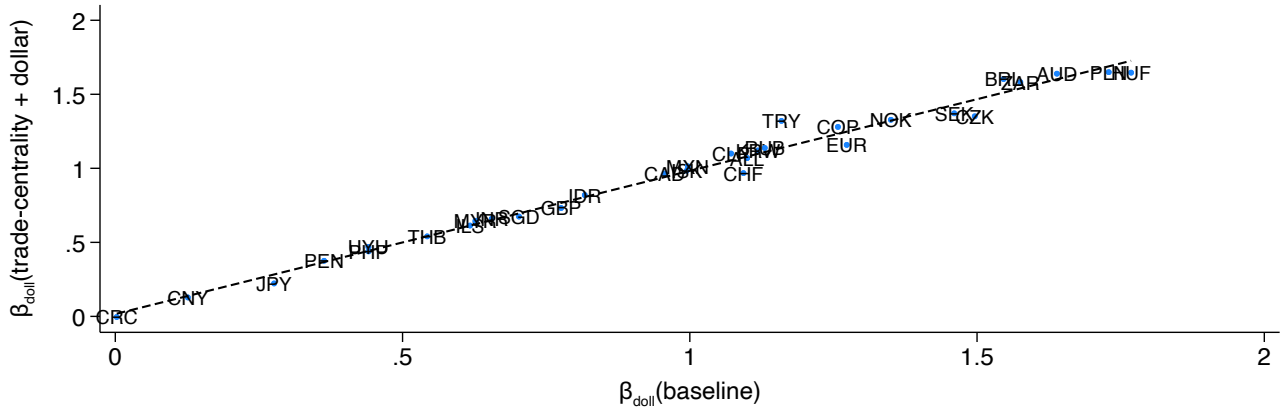
(a) Explanatory power R_i^2



(b) Commodity- and trade-centrality-factor loadings $\beta_{i,\text{comm}}$



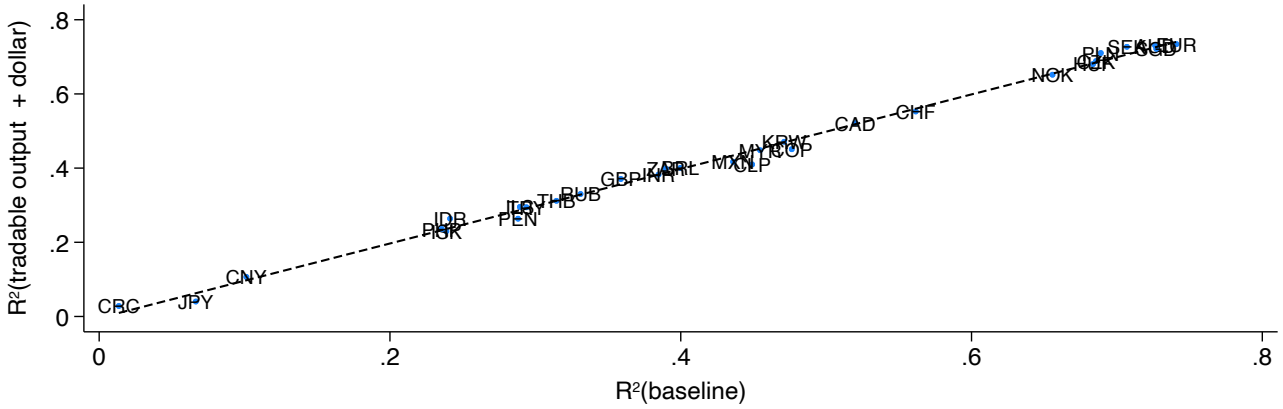
(c) Dollar-factor loadings $\beta_{i,\text{doll}}$



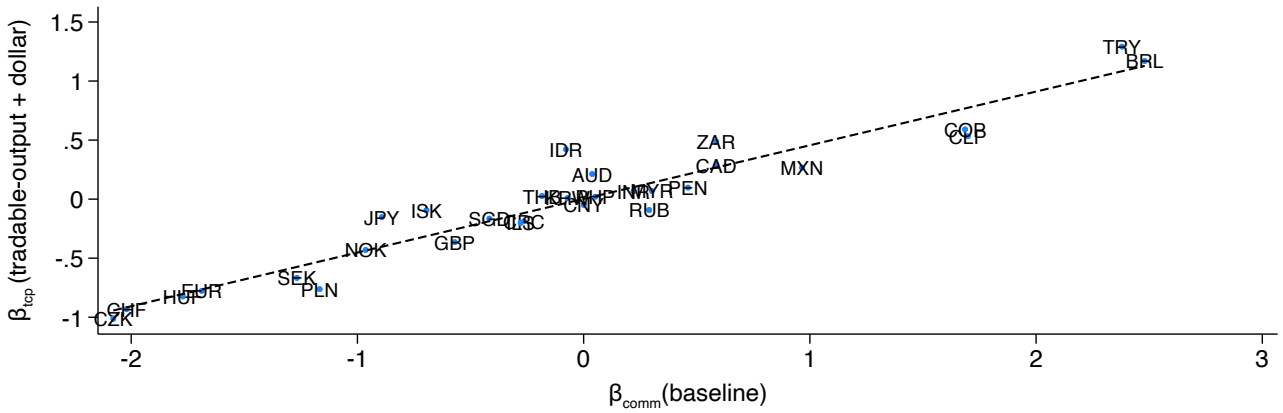
*Notes:*Appendix A provides details on the construction of the trade-centrality factor proposed by Richmond (2019). Panel (a) plots the R^2 for each currency i, from the modified empirical model (y-axis) - where the commodity factor is replaced by the trade-centrality factor - against the R^2 from the baseline model in equation (1) (x-axis). Panel (b) plots the estimated trade-centrality factor loadings from the modified empirical model against the estimated commodity factor loadings from our baseline model. Panel (c) plots the estimated dollar factor loadings from the modified empirical model against those from the baseline model. Each scatter point represents a currency, and the gray dashed line denotes the line of best fit.

Figure A.6: Exchange Rate Regressions: Baseline and Tradable-Output Sensitivity Factor

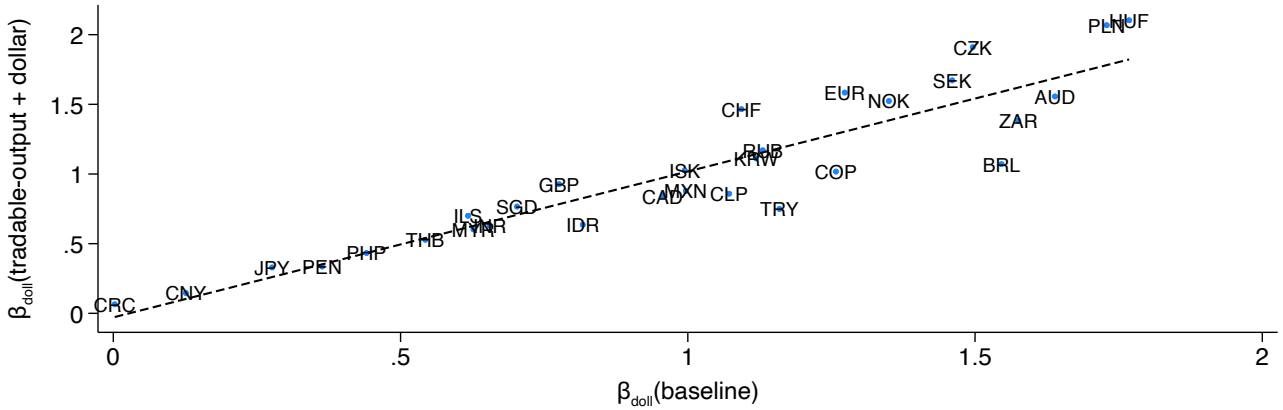
(a) Explanatory power R_i^2



(b) Commodity- and tradable-output-sensitivity-factor loadings $\beta_{i,\text{comm}}$



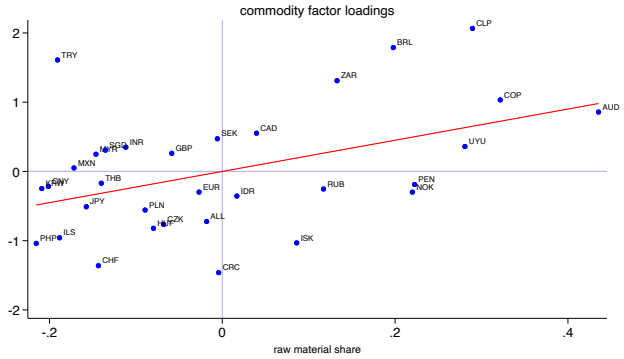
(c) Dollar-factor loadings $\beta_{i,\text{doll}}$



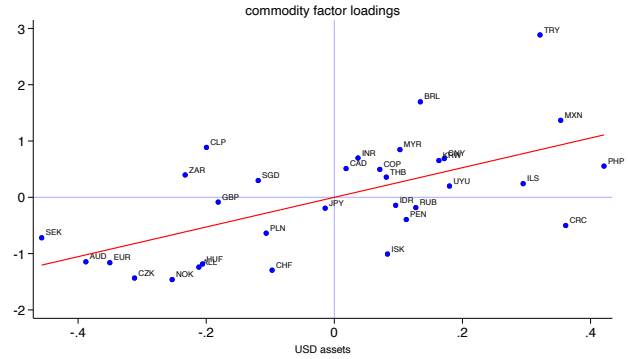
*Notes:*Appendix A provides details on the construction of the tradable-output-sensitivity factor. Panel (a) plots the R^2 for each currency i, from the modified empirical model (y-axis) - where the commodity factor is replaced by the tradable-output-sensitivity factor - against the R^2 from the baseline model in equation (1) (x-axis). Panel (b) plots the estimated tradable-output-sensitivity factor loadings from the modified empirical model against the estimated commodity factor loadings from our baseline model. Panel (c) plots the estimated dollar factor loadings from the modified empirical model against those from the baseline model. Each scatter point represents a currency, and the gray dashed line denotes the line of best fit.

Figure A.7: Determinants of Factor Loadings: Baseline Regressions

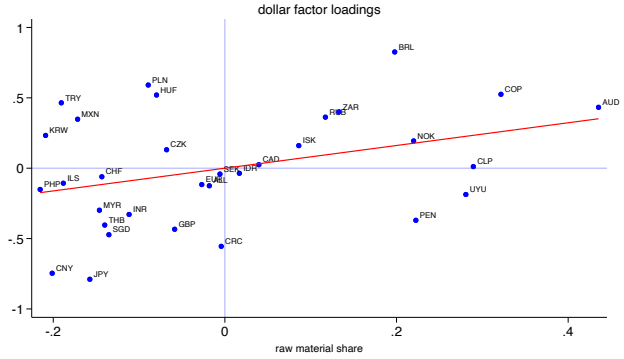
(a) Commodity-loadings on commodity incidence



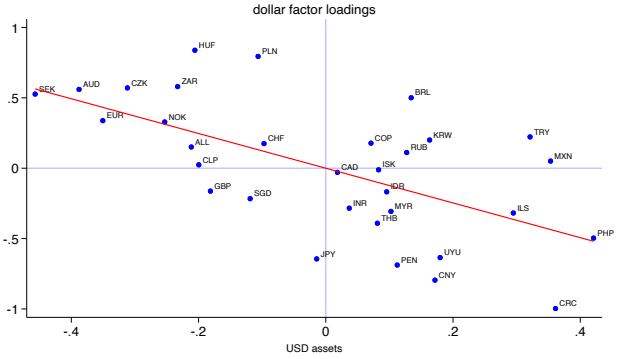
(b) Commodity-loadings on dollar assets



(c) Dollar-loadings on commodity incidence



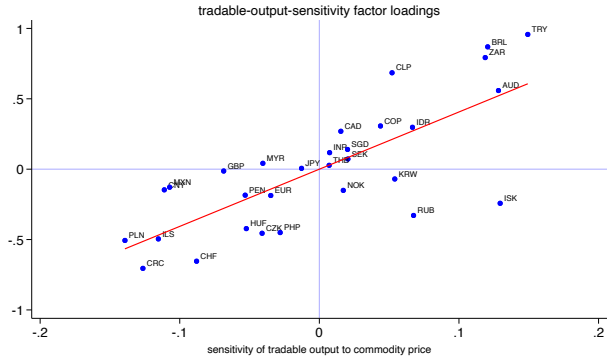
(d) Dollar-loadings on dollar assets



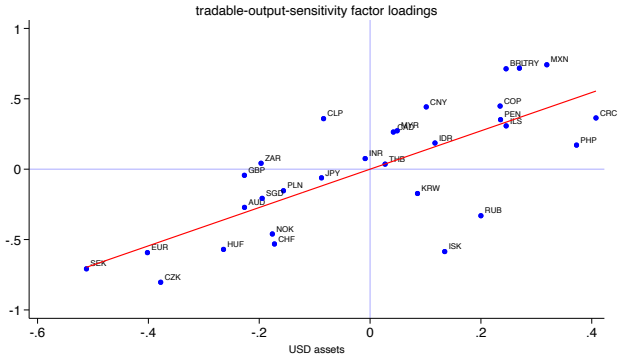
*Notes:*Panel (a) plots the residualized commodity factor loadings for each currency i (y-axis), against the residualized commodity incidence, measured by the share of commodity exports (x-axis). Both variables are residualized with respect to the share of dollar-denominated assets. Panel (b) plots the residualized commodity factor loadings for each currency i (y-axis), against the residualized share of dollar-denominated assets (x-axis). Both variables are residualized with respect to the commodity incidence. Panel (c) and (d) plot the residualized dollar factor loadings for each currency i (y-axis), against the residualized commodity incidence and the share of dollar-denominated assets (x-axes), respectively, following the same residualization logic. Each scatter point represents a currency, and the red line denotes the line of best fit.

Figure A.8: Determinants of Factor Loadings: Tradable-Output Sensitivity Specification

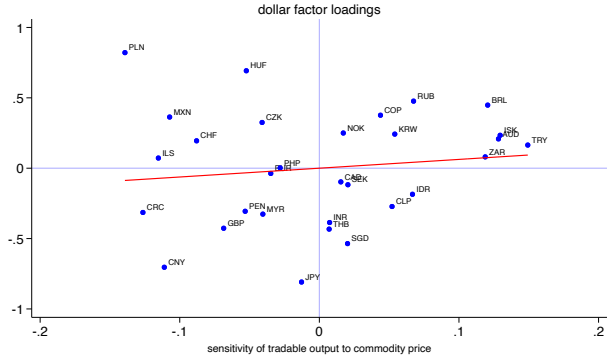
(a) TOS-loadings on tradable-output sensitivity



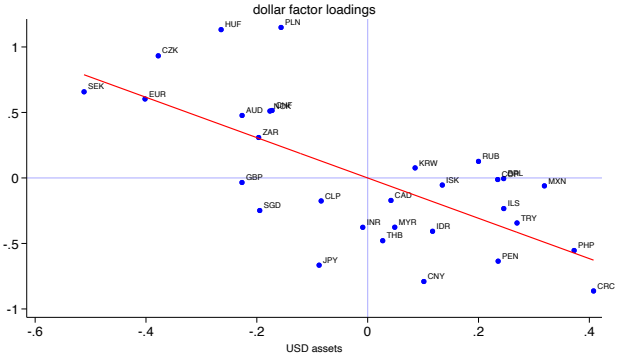
(b) TOS-loadings on dollar assets



(c) Dollar-loadings on tradable output sensitivity



(d) Dollar-loadings on dollar assets



*Notes:*Panel (a) plots the residualized tradable-output sensitivity factor loadings for each currency i (y-axis), against the residualized commodity incidence, measured by the share of commodity exports (x-axis). Both variables are residualized with respect to the share of dollar-denominated assets. Panel (b) plots the residualized tradable-output sensitivity factor loadings for each currency i (y-axis), against the residualized share of dollar-denominated assets (x-axis). Both variables are residualized with respect to the commodity incidence. Panel (c) and (d) plot the residualized dollar factor loadings for each currency i (y-axis), against the residualized commodity incidence and the share of dollar-denominated assets (x-axes), respectively, following the same residualization logic. Each scatter point represents a currency, and the red line denotes the line of best fit.

B Proofs

We start with a lemma that characterizes the linear equilibrium:

$$q_{ut} = \theta_u z_t + \xi_u x_{ut} + \mu_u \gamma_t + \omega_u l_{ut} + \zeta_u m_t \tag{B.1}$$
$$q_{it} = \theta_i z_t + \xi_i x_{ut} + \mu_i \gamma_t + \omega_i l_{it} + \zeta_i m_t + \eta_i x_{it} + \delta_i l_{ut} \tag{B.2}$$
$$l_{u,t+1} = D_{l,z} z_t + D_{l,x} x_{ut} + D_{l,l} l_{ut} + D_{l,m} m_t \tag{B.3}$$
$$m_{t+1} = D_{m,z} z_t + D_{m,x} x_{ut} + D_{m,l} l_{ut} + D_{m,m} m_t \tag{B.4}$$
$$l_{i,t+1} = D_{l,i} l_{it} + D_{l,x,i} x_{it} + D_{l,z,i} z_t + D_{l,u,i} x_{ut} + D_{l,l,i} l_{ut} + D_{l,m,i} m_t \tag{B.5}$$

We start with the globally important variables: the US price level, the evolution of the US debt, and the evolution of reserves.

LEMMA 1. *The US price level coefficients in the linear equilibrium are*

$$\theta_u = \alpha \cdot \frac{e_u - \alpha\chi\varphi\tilde{\omega}(1-\rho_z)(e+e_u)}{1 - 2\alpha\chi\varphi\tilde{\omega}(1-\rho_z)} \tag{B.6}$$
$$\xi_u = -\alpha \cdot \frac{1 - \chi\tilde{\omega}(1+\alpha\varphi)(1-\rho_x)}{1 - 2\chi\tilde{\omega}\alpha\varphi(1-\rho_x)} \tag{B.7}$$
$$\mu_u = -\frac{\alpha L_u \tilde{\omega}}{1 - 2\alpha\chi\varphi\tilde{\omega}(1-\rho_\gamma)} \tag{B.8}$$
$$\omega_u = \alpha L_u \tilde{\omega} \tag{B.9}$$
$$\zeta_u = 0 \tag{B.10}$$

The coefficients in the evolution of the US debt are

$$D_{l,z} = \frac{\alpha(e_u - e)}{\alpha L_u} \cdot \frac{\alpha\chi\varphi\tilde{\omega}(1-\rho_z)}{1 - 2\alpha\chi\varphi\tilde{\omega}(1-\rho_z)} \tag{B.11}$$
$$D_{l,x} = \frac{1}{\alpha L_u} \cdot \frac{\alpha\chi\tilde{\omega}(1-\alpha\varphi)(1-\rho_x)}{1 - 2\alpha\chi\varphi\tilde{\omega}(1-\rho_x)} \tag{B.12}$$
$$D_{l,l} = \frac{\omega_u}{\alpha L_u} + 1 \tag{B.13}$$
$$D_{l,\gamma} = \frac{\mu_u}{\alpha L_u} \tag{B.14}$$
$$D_{l,m} = 0 \tag{B.15}$$

The coefficients in the evolution of reserves are

$$D_{m,z} = \frac{\alpha\tau(e - e_u)}{1 - 2\alpha\chi\varphi\tilde{\omega}(1-\rho_z)} \tag{B.16}$$
$$D_{m,x} = \alpha\tau \cdot \frac{1 - 2\chi\tilde{\omega}(1-\rho_x)}{1 - 2\alpha\varphi\chi\tilde{\omega}(1-\rho_x)} \tag{B.17}$$
$$D_{m,l} = -2\tau\omega_u \tag{B.18}$$
$$D_{m,\gamma} = -2\tau\mu_u \tag{B.19}$$
$$D_{m,m} = 0 \tag{B.20}$$

Proof of Lemma 1. First, observe that first-order deviations of reserves are the same in all countries:

$$m_{t+1} = \tau(q_t - q_{ut}) \tag{B.21}$$

Linearizing the resource constraints,

$$q_{it} = \alpha e_i z_t - \alpha x_{it} + \alpha L_i \Delta l_{i,t+1} - \alpha M_i \Delta m_{t+1} \tag{B.22}$$
$$q_{ut} = \alpha e_u z_t - \alpha x_{ut} + \alpha L_u \Delta l_{u,t+1} \tag{B.23}$$

Integrating and adding up,

$$q_t + q_{ut} = (e + e_u)\alpha z_t - \alpha x_{ut} \tag{B.24}$$

We assume that the intermediaries use the following approximation for their profits:

$$\int X_{i,t+1} L_{i,t+1} di \approx \int (\Delta q_{i,t+1} - \Delta q_{u,t+1} + r_{it} - r_{ut}) L_i di \tag{B.25}$$

Expanding the intermediary's optimality condition and plugging this approximation for profits,

$$\begin{aligned} L_i(1 + l_{i,t+1}) \approx{}& L(1 + m_{t+1}) + \chi \mathbb{E}_t[\Delta q_{i,t+1} - \Delta q_{u,t+1} + r_{it} - r_{ut}] \\ & - \chi\Gamma(1 + \gamma_t)\mathbb{C}_t \left[\Delta q_{i,t+1} - \Delta q_{u,t+1}, \int (\Delta q_{i,t+1} - \Delta q_{u,t+1}) L_i di\right] \end{aligned} \tag{B.26}$$

In the linear equilibrium, the conditional covariance is constant and hence equal to the unconditional one. Defining β_i as in the main text,

$$\beta_i = \begin{pmatrix} \theta_i - \theta_u \\ \xi_i - \xi_u \\ \mu_i - \mu_u \end{pmatrix} \tag{B.27}$$

and defining $s_t' = (z_t \; x_{ut} \; \gamma_t)$ to be the vector of exogenous global shocks, we can rewrite this as

$$\begin{aligned} L_i(1 + l_{i,t+1}) \approx{}& L(1 + m_{t+1}) + \chi \mathbb{E}_t[\Delta q_{i,t+1} - \Delta q_{u,t+1} + r_{it} - r_{ut}] \\ & - \chi\Gamma(1 + \gamma_t)\mathbb{C}\left[\beta_i' \Delta s_{t+1}, \int \beta_i' \Delta s_{t+1} L_i di\right] \\ ={}& L(1 + m_{t+1}) + \chi \mathbb{E}_t[\Delta q_{i,t+1} - \Delta q_{u,t+1} + r_{it} - r_{ut}] - \chi\Gamma(1 + \gamma_t)\beta_i' \mathbb{V}[\Delta s_{t+1}]\pi \end{aligned} \tag{B.28}$$

Here, as in the main text, $\pi \equiv \int \beta_i L_i di$. Taking the double limit $(\sigma_z, \sigma_x, \sigma_\gamma) \longrightarrow (0, 0, 0)$ and $(\Gamma\sigma_z^2, \Gamma\sigma_x^2, \Gamma\sigma_\gamma^2) \longrightarrow (\Gamma_z, \Gamma_x, \Gamma_\gamma)$, rewrite this as

$$L_i(1 + l_{i,t+1}) \approx L(1 + m_{t+1}) + \chi \mathbb{E}_t[\Delta q_{i,t+1} - \Delta q_{u,t+1} + r_{it} - r_{ut}] - (1 + \gamma_t)\chi\beta_i' D_\Gamma \pi \tag{B.29}$$

In the zeroth order we have

$$L_i = L - \chi \pi' D_\Gamma \beta_i \tag{B.30}$$

In the first order, we have

$$L_i l_{i,t+1} = Lm_{t+1} + \chi \mathbb{E}_t[\Delta q_{i,t+1} - \Delta q_{u,t+1} + r_{it} - r_{ut}] + (L_i - L)\gamma_t \tag{B.31}$$

To eliminate the interest rates, observe that Euler equations linearize into

$$r_{it} = \rho \mathbb{E}_t[\Delta c_{i,t+1}] \tag{B.32}$$

From the condition that $\alpha Q_{it} C_{it} = P_{it} N_{it}$ and $P_{it} = Q_{it}^{\frac{1}{\alpha}}$, we get

$$c_{it} = \frac{1-\alpha}{\alpha} q_{it} + x_{it} \tag{B.33}$$

This implies

$$L_i l_{i,t+1} = Lm_{t+1} + \chi\varphi \mathbb{E}_t[\Delta q_{i,t+1} - \Delta q_{u,t+1}] + \chi \mathbb{E}_t[\Delta x_{i,t+1} - \Delta x_{u,t+1}] + (L_i - L)\gamma_t \tag{B.34}$$

Here the composite parameter φ is

$$\varphi = 1 + \frac{\rho(1-\alpha)}{\alpha} \tag{B.35}$$

Integrating equation (B.34) and using equation (B.24),

$$\begin{aligned} L_u l_{u,t+1} &= \chi\varphi \mathbb{E}_t[\Delta q_{u,t+1} - \Delta q_{t+1}] + \chi \mathbb{E}_t[\Delta x_{u,t+1}] + L_u \gamma_t \\ &= \chi\varphi \mathbb{E}_t[2\Delta q_{u,t+1} - (e+e_u)\alpha \Delta z_{t+1}] + \chi(1+\alpha\varphi)\mathbb{E}_t[\Delta x_{u,t+1}] + L_u\gamma_t \\ &= \chi\varphi(2\theta_u - \alpha(e+e_u))\mathbb{E}_t[\Delta z_{t+1}] + \chi(2\varphi\xi_u + 1 + \alpha\varphi)\mathbb{E}_t[\Delta x_{u,t+1}] + 2\chi\varphi\mu_u \mathbb{E}_t[\Delta\gamma_{t+1}] \\ &+ L_u\gamma_t + 2\chi\varphi\omega_u \Delta l_{u,t+1} + 2\chi\varphi\zeta_u \Delta m_{t+1} \end{aligned} \tag{B.36}$$

Reorganizing equation (B.36),

$$\begin{aligned} (\alpha L_u - 2\alpha\chi\varphi\omega_u)\Delta l_{u,t+1} = 2\alpha\chi\varphi\zeta_u \Delta m_{t+1} - \alpha L_u l_{ut} + (\alpha L_u + 2\chi\varphi\alpha\mu_u(\rho_\gamma - 1))\gamma_t \\ + \alpha\chi\varphi(\rho_z - 1)(2\theta_u - \alpha(e+e_u))z_t \\ + \alpha\chi(\rho_x - 1)(2\varphi\xi_u + 1 + \alpha\varphi)x_{ut} \end{aligned} \tag{B.37}$$

Plugging this into equation (B.23), we can see that $\zeta_u = 0$, and ω_u satisfies

$$\omega_u = -\frac{(\alpha L_u)^2}{\alpha L_u - 2\alpha\chi\varphi\omega_u} \tag{B.38}$$

Define $\tilde{\omega}$ by $\omega_u = \alpha L_u \tilde{\omega}$. This solves

$$2\alpha\chi\varphi\tilde{\omega}^2 - \tilde{\omega} - 1 = 0 \tag{B.39}$$

This implies

$$\alpha L_u - 2\alpha\chi\varphi\omega_u = -\frac{\alpha L_u}{\tilde{\omega}} \tag{B.40}$$

Using this to to reorganize equation (B.37),

$$\begin{aligned}\alpha L_u \Delta l_{u,t+1} = {} & \omega_u l_{ut} - (\alpha L_u + 2\chi\varphi\alpha\mu_u(\rho_\gamma - 1))\tilde{\omega}\gamma_t \\ & - \alpha\chi\varphi\tilde{\omega}(\rho_z - 1)(2\theta_u - \alpha(e + e_u))z_t - \alpha\chi\tilde{\omega}(\rho_x - 1)(2\varphi\xi_u + 1 + \alpha\varphi)x_{ut}\end{aligned} \tag{B.41}$$

This implies

$$\theta_u = \alpha \cdot \frac{e_u - \alpha\chi\varphi\tilde{\omega}(1 - \rho_z)(e + e_u)}{1 - 2\alpha\chi\varphi\tilde{\omega}(1 - \rho_z)} \tag{B.42}$$

$$\xi_u = -\alpha \cdot \frac{1 - \chi\tilde{\omega}(1 + \alpha\varphi)(1 - \rho_x)}{1 - 2\chi\tilde{\omega}\alpha\varphi(1 - \rho_x)} \tag{B.43}$$

$$\mu_u = -\frac{\omega_u}{1 - 2\alpha\chi\varphi\tilde{\omega}(1 - \rho_\gamma)} \tag{B.44}$$

Define $\tilde{\mu}$ by $\mu_u = \alpha L_u \tilde{\mu}$. For $\alpha L_u \Delta l_{u,t+1}$, we now have

$$\alpha L_u \Delta l_{u,t+1} = \omega_u l_{ut} + \mu_u \gamma_t + \frac{\alpha^2\chi\varphi\tilde{\omega}(e_u - e)(1 - \rho_z)}{1 - 2\alpha\chi\varphi\tilde{\omega}(1 - \rho_z)} z_t + \frac{\alpha\chi\tilde{\omega}(1 - \alpha\varphi)(1 - \rho_x)}{1 - 2\alpha\chi\varphi\tilde{\omega}(1 - \rho_x)} x_{ut} \tag{B.45}$$

This implies

$$D_{l,z} = \frac{\alpha(e_u - e)}{\alpha L_u} \cdot \frac{\alpha\chi\varphi\tilde{\omega}(1 - \rho_z)}{1 - 2\alpha\chi\varphi\tilde{\omega}(1 - \rho_z)} \tag{B.46}$$

$$D_{l,x} = \frac{1}{\alpha L_u} \cdot \frac{\alpha\chi\tilde{\omega}(1 - \alpha\varphi)(1 - \rho_x)}{1 - 2\alpha\chi\varphi\tilde{\omega}(1 - \rho_x)} \tag{B.47}$$

$$D_{l,l} = 1 + \tilde{\omega} \tag{B.48}$$

$$D_{l,\gamma} = \tilde{\mu} \tag{B.49}$$

Recall that

$$m_{t+1} = \tau(q_t - q_{ut}) = (\alpha(e + e_u) - 2\theta_u)\tau z_t - (\alpha + 2\xi_u)\tau x_{ut} - 2\tau\mu_u\gamma_t - 2\tau\omega_u l_{ut} \tag{B.50}$$

This uses equation (B.24). The implication is that

$$D_{m,z} = \frac{\alpha\tau(e - e_u)}{1 - 2\alpha\chi\varphi\tilde{\omega}(1 - \rho_z)} \tag{B.51}$$

$$D_{m,x} = \alpha\tau \cdot \frac{1 - 2\chi\tilde{\omega}(1 - \rho_x)}{1 - 2\alpha\varphi\chi\tilde{\omega}(1 - \rho_x)} \tag{B.52}$$

$$D_{m,l} = -2\tau\omega_u \tag{B.53}$$

$$D_{m,\gamma} = -2\tau\mu_u \tag{B.54}$$

This completes the proof. □

We next characterize the dynamics of country-specific price levels q_{it} and external debt l_{it}.

LEMMA 2. *Country-specific price level coefficients in the linear equilibrium are*

$$\theta_i = \frac{\alpha e_i - \alpha\varphi\chi\omega\theta_u(1-\rho_z) + \alpha\omega D_{m,z}(M_i - L) - \alpha\varphi\chi\omega D_{l,z}(\delta_i - \omega_u)}{1 - \alpha\varphi\chi\omega(1-\rho_z)} \tag{B.55}$$

$$\xi_i = \frac{\alpha\omega D_{m,x}(M_i - L) - \alpha\varphi\chi\omega D_{l,x}(\delta_i - \omega_u) - \alpha\chi\omega(1+\varphi\xi_u)(1-\rho_x)}{1 - \alpha\varphi\chi\omega(1-\rho_x)} \tag{B.56}$$

$$\mu_i = -\frac{\alpha\omega(L_i - L) + \mu_u\alpha\varphi\chi\omega(1-\rho_\gamma) + 2\alpha\omega\tau\mu_u(M_i - L) + \alpha\varphi\chi\omega\tilde{\mu}(\delta_i - \omega_u)}{1 - \alpha\varphi\chi\omega(1-\rho_\gamma)} \tag{B.57}$$

$$\omega_i = \alpha\omega L_i \tag{B.58}$$

$$\zeta_i = -\alpha\omega M_i \tag{B.59}$$

$$\eta_i = -\frac{\alpha}{1 - \alpha\chi\varphi\omega(1-\rho_x)} \tag{B.60}$$

$$\delta_i = \omega_u \cdot \frac{\alpha\varphi\chi\omega\tilde{\omega} - 2\alpha\omega\tau(M_i - L)}{\alpha\varphi\chi\omega\tilde{\omega} + 1} \tag{B.61}$$

The coefficients in the evolution of country-specific debt are

$$D_{l,i} = 1 + \omega \tag{B.62}$$

$$D_{l,x,i} = -\frac{\alpha\varphi\chi\omega(1-\rho_x)}{L_i(1 + \alpha\varphi\chi\omega(1-\rho_x))} \tag{B.63}$$

$$D_{l,z,i} = \frac{\theta_i - \alpha e_i + \alpha M_i D_{m,z}}{\alpha L_i} \tag{B.64}$$

$$D_{l,u,i} = \frac{\xi_i + \alpha M_i D_{m,x}}{\alpha L_i} \tag{B.65}$$

$$D_{l,\gamma,i} = \frac{\mu_i + \alpha M_i D_{m,\gamma}}{\alpha L_i} \tag{B.66}$$

$$D_{l,l,i} = \frac{\delta_i + \alpha M_i D_{m,l}}{\alpha L_i} \tag{B.67}$$

$$D_{l,m,i} = -\frac{(\omega+1)M_i}{L_i} \tag{B.68}$$

Proof of Lemma 2. Plugging the linear conjecture into equation (B.34),

$$\begin{aligned}(\alpha L_i - \alpha\chi\varphi\omega_i)\Delta l_{i,t+1} = {} & \alpha L m_{t+1} + \alpha\chi\varphi(\theta_i - \theta_u)(\rho_z - 1)z_t - \alpha\chi(1 - \varphi(\xi_i - \xi_u))(\rho_x - 1)x_{ut} \\ & + \alpha\chi\varphi\eta_i(\rho_x - 1)x_{it} + \alpha\chi\varphi\zeta_i\Delta m_{t+1} + \alpha\chi\varphi(\delta_i - \omega_u)\Delta l_{u,t+1} \\ & + (\alpha\chi\varphi(\mu_i - \mu_u)(\rho_\gamma - 1) + \alpha(L_i - L))\gamma_t - \alpha L_i l_{it}\end{aligned} \tag{B.69}$$

Plugging this back into the linear conjecture, we see that ω_i satisfies

$$\omega_i = -\frac{(\alpha L_i)^2}{\alpha L_i - \alpha\chi\varphi\omega_i} \tag{B.70}$$

Define ω by $\omega_i = \alpha L_i \omega$. This parameter solves

$$\alpha\chi\varphi\omega^2 - \omega - 1 = 0 \tag{B.71}$$

Using this, we can reorganize equation (B.69):

$$\begin{aligned}\alpha L_i \Delta l_{i,t+1} = {} & \omega_i l_{it} - \alpha\omega L m_{t+1} - \alpha\chi\varphi\omega\zeta_i \Delta m_{t+1} - \alpha\chi\varphi\omega(\delta_i - \omega_u)\Delta l_{u,t+1} \\ & + \alpha\chi\varphi\omega(\theta_i - \theta_u)(1-\rho_z)z_t + \alpha\chi\omega(\varphi(\xi_i - \xi_u) - 1)(1-\rho_x)x_{ut} + \alpha\chi\varphi\omega\eta_i(1-\rho_x)x_{it} \\ & + (\alpha\chi\varphi\omega(1-\rho_\gamma)(\mu_i - \mu_u) - \alpha\omega(L_i - L))\gamma_t \end{aligned} \tag{B.72}$$

Collecting the coefficients on m_t in the linear conjecture,

$$\zeta_i = \alpha\chi\varphi\omega\zeta_i + \alpha M_i \tag{B.73}$$

This means $\zeta_i = -\alpha\omega M_i$. Collecting the coefficients on m_{t+1} in the linear conjecture,

$$-\alpha\omega L - \alpha\chi\varphi\omega\zeta_i - \alpha M_i = \alpha\omega(M_i - L) \tag{B.74}$$

Collecting the terms on x_{it},

$$\eta_i = -\frac{\alpha}{1 - \alpha\chi\varphi\omega(1-\rho_x)} \tag{B.75}$$

This means

$$\begin{aligned} q_{it} = {} & \omega_i l_{it} + \zeta_i m_t + \eta_i x_{it} + \alpha\omega(M_i - L)m_{t+1} - \alpha\chi\varphi\omega(\delta_i - \omega_u)\Delta l_{u,t+1} \\ & + [\alpha e_i + \alpha\chi\varphi\omega(\theta_i - \theta_u)(1-\rho_z)]z_t + \alpha\chi\omega(\varphi(\xi_i - \xi_u) - 1)(1-\rho_x)x_{ut} \\ & + (\alpha\chi\varphi\omega(1-\rho_\gamma)(\mu_i - \mu_u) - \alpha\omega(L_i - L))\gamma_t \end{aligned} \tag{B.76}$$

For δ_i, we have

$$\delta_i = \omega_u \cdot \frac{\alpha\varphi\chi\omega\tilde{\omega} - 2\alpha\omega\tau(M_i - L)}{\alpha\varphi\chi\omega\tilde{\omega} + 1} \tag{B.77}$$

For θ_i and ξ_i, we have

$$\theta_i = \frac{\alpha e_i - \alpha\varphi\chi\omega\theta_u(1-\rho_z) + \alpha\omega D_{m,z}(M_i - L) - \alpha\varphi\chi\omega D_{l,z}(\delta_i - \omega_u)}{1 - \alpha\varphi\chi\omega(1-\rho_z)} \tag{B.78}$$

$$\xi_i = \frac{\alpha\omega D_{m,x}(M_i - L) - \alpha\varphi\chi\omega D_{l,x}(\delta_i - \omega_u) - \alpha\chi\omega(1+\varphi\xi_u)(1-\rho_x)}{1 - \alpha\varphi\chi\omega(1-\rho_x)} \tag{B.79}$$

Finally, for μ_i, we have

$$\mu_i = -\frac{\alpha\omega(L_i - L) + \mu_u\alpha\varphi\chi\omega(1-\rho_\gamma) + 2\alpha\omega\tau\mu_u(M_i - L) + \alpha\varphi\chi\omega\tilde{\mu}(\delta_i - \omega_u)}{1 - \alpha\varphi\chi\omega(1-\rho_\gamma)} \tag{B.80}$$

To characterize the evolution of l_{it}, use equation (B.22):

$$\alpha L_i l_{i,t+1} = \alpha L_i l_{it} + q_{it} - \alpha e_i z_t + \alpha x_{it} + \alpha M_i m_{t+1} - \alpha M_i m_t \tag{B.81}$$

This implies

$$D_{l,z,i} = \frac{\theta_i - \alpha e_i + \alpha M_i D_{m,z}}{\alpha L_i} \tag{B.82}$$

$$D_{l,u,i} = \frac{\xi_i + \alpha M_i D_{m,x}}{\alpha L_i} \tag{B.83}$$

$$D_{l,\gamma,i} = \frac{\mu_i + \alpha M_i D_{m,\gamma}}{\alpha L_i} \tag{B.84}$$

$$D_{l,i} = \omega + 1 \tag{B.85}$$

$$D_{l,m,i} = -\frac{(\omega+1)M_i}{L_i} \tag{B.86}$$

$$D_{l,l,i} = \frac{\delta_i + \alpha M_i D_{m,l}}{\alpha L_i} \tag{B.87}$$

$$D_{l,x,i} = \frac{\alpha + \eta_i}{\alpha L_i} = -\frac{\alpha\varphi\chi\omega(1-\rho_x)}{L_i(1+\alpha\varphi\chi\omega(1-\rho_x))} \tag{B.88}$$

This completes the proof. □

These lemmas imply Proposition 1 and Proposition 6.

Proof of Proposition 1. The first part of the statement of this proposition is $L_i - L = -\chi\pi' D_\Gamma \beta_i$. This is equation (B.30) in the proof of Lemma 1. Plug this into the definition of π:

$$\begin{aligned}\pi = \int L_i \beta_i di = L\mathbb{E}[\beta_i] - \chi\mathbb{E}[\beta_i\hat{\beta}_i']\pi &= L \cdot (1 + \chi\mathbb{E}[\beta_i\hat{\beta}_i'])^{-1}\mathbb{E}[\beta_i] \\ &= L \cdot D_\Gamma^{-1}(D_\Gamma^{-1} + \chi\mathbb{E}[\beta_i\beta_i'])^{-1}\mathbb{E}[\beta_i]\end{aligned} \tag{B.89}$$

Here $\mathbb{E}[\cdot]$ denotes the cross-sectional expectation. Taking the cross-sectional expectation of L_i, using $L_u = L - \mathbb{E}[L_i]$, and canceling χ,

$$L_u = L \cdot \mathbb{E}[\beta_i'](\chi^{-1}D_\Gamma^{-1} + \mathbb{E}[\beta_i\beta_i'])^{-1}\mathbb{E}[\beta_i] \tag{B.90}$$

The fact that $L_u > 0$ follows from the fact that $\mathbb{E}[\beta_i\beta_i']$ is positive definite, which makes $\chi^{-1}D_\Gamma^{-1} + \mathbb{E}[\beta_i\beta_i']$ positive definite, and hence $(\chi^{-1}D_\Gamma^{-1} + \mathbb{E}[\beta_i\beta_i'])^{-1}$ is positive definite too. It remains to prove that $L_u/L < 1$. To see why this is the case, consider a matrix M given by

$$M = \begin{pmatrix} \chi^{-1}D_\Gamma^{-1} + \mathbb{E}[\beta_i\beta_i'] & \mathbb{E}[\beta_i] \\ \mathbb{E}[\beta_i'] & 1 \end{pmatrix} \tag{B.91}$$

This matrix is positive definite, since it can be represented as

$$M = \begin{pmatrix} \chi^{-1}D_\Gamma^{-1} & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} \mathbb{E}[\beta_i\beta_i'] & \mathbb{E}[\beta_i] \\ \mathbb{E}[\beta_i'] & 1 \end{pmatrix} \tag{B.92}$$

This is a sum of two positive semidefinite matrices, one of which is generically positive definite. Now, since M is positive definite, multiplying it by a full-rank matrix A' on the left and A on the

right preserves positive-definiteness. Choose A as follows:

$$A = \begin{pmatrix} I_3 & -(\chi^{-1}D_\Gamma^{-1} + \mathbb{E}[\beta_i\beta_i'])^{-1}\mathbb{E}[\beta_i] \\ 0 & 1 \end{pmatrix} \tag{B.93}$$

Multiplying, we get

$$A'MA = \begin{pmatrix} \chi^{-1}D_\Gamma^{-1} + \mathbb{E}[\beta_i\beta_i'] & 0 \\ 0 & 1 - \mathbb{E}[\beta_i'](\chi^{-1}D_\Gamma^{-1} + \mathbb{E}[\beta_i\beta_i'])^{-1}\mathbb{E}[\beta_i] \end{pmatrix} \tag{B.94}$$

The fact that this matrix is positive definite means $1-\mathbb{E}[\beta_i'](\chi^{-1}D_\Gamma^{-1}+\mathbb{E}[\beta_i\beta_i'])^{-1}\mathbb{E}[\beta_i] > 0$, implying $1 - L_u/L > 0$. This completes the proof. □

We next show the stationarity properties of exchange rates.

PROPOSITION 6. *Endogenous states (l_{ut}, l_{it}, m_t) evolve as*

$$\begin{aligned} l_{u,t+1} &= (1+\tilde{\omega})l_{ut} + F_u(z_t, \gamma_t, x_{ut}) \\ l_{i,t+1} &= (1+\omega)l_{it} + F_i(z_t, \gamma_t, x_{ut}, x_{it}, l_{ut}, m_t) \\ m_{t+1} &= F_m(z_t, \gamma_t, x_{ut}, l_{ut}) \end{aligned}$$

Here the coefficients $\tilde{\omega}$ and ω satisfy $-1 < \omega < \tilde{\omega} < 0$ and both converge to zero if $\chi \longrightarrow \infty$.

If exogenous processes are stationary, endogenous processes, including exchange rates, are stationary too. The reason is the portfolio management cost. The intermediary's portfolio gravitates towards a fixed benchmark, which makes the economy return to a stable distribution of investment positions in the long run. The US position has a persistence coefficient $1+\tilde{\omega} \in (0,1)$, every other country's position has persistence $1+\omega \in (0,1)$, and the global reserve position does not depend on its own past values. Inflows into the US are more persistent than inflows into other countries due to their feedback: they affect the US dollar exchange rate, which further drives financial flows. Both persistence coefficients become a unit root if portfolio frictions disappear. This relates our portfolio costs to stationarity-inducing devices in the spirit of Schmitt-Grohé and Uribe (2003).

Proof of Proposition 6. Directly follows from the statements of Lemma 1 and Lemma 2. □

B.1 Small portfolio management costs

We next characterize the limit of small portfolio costs, $\chi \longrightarrow \infty$, in which asset holdings are large: $M_i = \chi^{\frac{1}{2}} m_i$, and hence $L = \chi^{\frac{1}{2}} l$, where $l \equiv \int m_i di$. For tractability, assume also $\rho_z = \rho_x = \rho_\gamma = 1$. Define the following auxiliary variables:

$$\phi \equiv \sqrt{\frac{\alpha}{\varphi}} = \alpha\sqrt{\frac{1}{\alpha + \rho(1-\alpha)}} \tag{B.95}$$

$$\psi_i \equiv \phi\tau(m_i - l) - 1 \tag{B.96}$$

With these auxiliary variables, we can characterize bilateral exchange rate depreciation against the US for all countries, our currency factors, and loadings.

LEMMA 3. *Suppose $\rho_z = \rho_x = \rho_\gamma = 1$. Then, in the limit $\chi \longrightarrow \infty$, bilateral exchange rate depreciation against the dollar is*

$$\Delta s_{i,t+1} = \bar{\theta}_i \Delta z_{t+1} + \bar{\xi}_i \Delta x_{u,t+1} + \bar{\mu}_i \Delta \gamma_{t+1} - \alpha \Delta x_{i,t+1} \tag{B.97}$$

Here the coefficients are

$$\bar{\theta}_i = \alpha(e - e_i) + \alpha\psi_i(e - e_u) \tag{B.98}$$
$$\bar{\xi}_i = \alpha\psi_i \tag{B.99}$$
$$\bar{\mu}_i = \phi l_u \cdot \frac{1 - 2(1 + \psi_i)}{1 + \sqrt{2}} - \phi(l_i - l) \tag{B.100}$$

The steady-state exposures $l_i - l$ are given recursively:

$$l_i - l = -l \cdot \mathbb{E}\left[\bar{\beta}_i'\right]\left(\mathbb{E}\left[\bar{\beta}_i \bar{\beta}_i'\right]\right)^{-1} \bar{\beta}_i, \tag{B.101}$$

where $\bar{\beta}_i = (\bar{\theta}_i \ \bar{\xi}_i \ \bar{\mu}_i)'$ and $\mathbb{E}[\cdot]$ denotes the cross-sectional expectation.

Proof of Lemma 3. Assume, for simplicity, that $\rho_z = \rho_x = \rho_\gamma = 1$. Under this condition, US coefficients are

$$\theta_u = \alpha e_u \tag{B.102}$$
$$\xi_u = -\alpha \tag{B.103}$$
$$\mu_u = -\omega_u \tag{B.104}$$

Coefficients of individual countries are

$$\theta_i = \alpha e_i + \alpha^2 \omega\tau(e - e_u)(M_i - L) \tag{B.105}$$
$$\xi_i = \alpha^2 \omega\tau(M_i - L) \tag{B.106}$$
$$\eta_i = -\alpha \tag{B.107}$$
$$\zeta_i = -\alpha\omega M_i \tag{B.108}$$
$$\delta_i = \omega_u - \omega_u \cdot \frac{1 + 2\alpha\omega\tau(M_i - L)}{1 + \alpha\varphi\chi\omega\tilde{\omega}} \tag{B.109}$$
$$\mu_i = \mu_u - \mu_u \cdot \frac{1 + 2\alpha\omega\tau(M_i - L)}{1 + \alpha\varphi\chi\omega\tilde{\omega}} - \alpha\omega(L_i - L) \tag{B.110}$$

US debt coefficients are

$$D_{l,z} = 0 \tag{B.111}$$
$$D_{l,x} = 0 \tag{B.112}$$
$$D_{l,l} = 1 + \tilde{\omega} \tag{B.113}$$
$$D_{l,\gamma} = -\tilde{\omega} \tag{B.114}$$

Total reserve demand coefficients are

$$D_{m,z} = \alpha\tau(e - e_u) \tag{B.115}$$
$$D_{m,x} = \alpha\tau \tag{B.116}$$
$$D_{m,l} = -2\tau\omega_u \tag{B.117}$$
$$D_{m,\gamma} = 2\tau\omega_u \tag{B.118}$$

Assume $\chi \longrightarrow \infty$ and $\chi^{-\frac{1}{2}} M_i \longrightarrow m_i$ for all i. Then, $\chi^{-\frac{1}{2}} L_i \longrightarrow l_i$ for all i. In this limit,

$$\tilde{\omega} \sim -\frac{1}{\sqrt{2\alpha\varphi\chi}} \tag{B.119}$$
$$\omega \sim -\frac{1}{\sqrt{\alpha\varphi\chi}} \tag{B.120}$$

The limits of the coefficients are

$$\omega_u \longrightarrow -\frac{\phi}{\sqrt{2}} l_u \tag{B.121}$$
$$\mu_u \longrightarrow \frac{\phi}{\sqrt{2}} l_u \tag{B.122}$$
$$\theta_i \longrightarrow \alpha e_i - \alpha\tau\phi(e - e_u)(m_i - l) \tag{B.123}$$
$$\xi_i \longrightarrow -\alpha\tau\phi(m_i - l) \tag{B.124}$$
$$\zeta_i \longrightarrow \phi m_i \tag{B.125}$$
$$\omega_i \longrightarrow -\phi l_i \tag{B.126}$$
$$\delta_i \longrightarrow -\frac{\phi}{\sqrt{2}} l_u \cdot \frac{1 + 2\sqrt{2}\tau\phi(m_i - l)}{1 + \sqrt{2}} \tag{B.127}$$
$$\mu_i \longrightarrow \frac{\phi}{\sqrt{2}} l_u \cdot \frac{1 + 2\sqrt{2}\tau\phi(m_i - l)}{1 + \sqrt{2}} + \phi(l_i - l) \tag{B.128}$$

Consider $\Delta l_{u,t+1}$ and m_{t+1}:

$$\Delta l_{u,t+1} = O(\chi^{-\frac{1}{2}}) \tag{B.129}$$
$$m_{t+1} \longrightarrow \alpha\tau(e - e_u) z_t + \alpha\tau x_{ut} + \sqrt{2}\phi\tau\gamma_t \tag{B.130}$$

Now consider $\alpha L_i \Delta l_{t+1}$:

$$\begin{aligned} \alpha L_i \Delta l_{i,t+1} = {} & \omega_i l_{it} - \alpha\omega L m_{t+1} + \alpha^2\varphi\chi\omega^2 M_i \Delta m_{t+1} - \alpha\chi\varphi\omega(\delta_i - \omega_u)\Delta l_{u,t+1} \\ & - \alpha\omega(L_i - L)\gamma_t \end{aligned} \tag{B.131}$$

From this, it follows that $\alpha L_i \Delta l_{i,t+1} = O(\chi^{\frac{1}{2}})$, meaning that $\Delta l_{i,t+1} = O(1)$ and hence $l_{it} = O(1)$. Now consider $\omega_i \Delta l_{i,t+1} + \zeta_i \Delta m_{t+1}$:

$$\begin{aligned} \omega_i \Delta l_{i,t+1} + \zeta_i \Delta m_{t+1} = {} & \omega(\omega_i l_{it} - \alpha\omega L m_{t+1} - \alpha\chi\varphi\omega(\delta_i - \omega_u)\Delta l_{u,t+1} - \alpha\omega(L_i - L)\gamma_t) \\ & + \alpha\omega^2 M_i \Delta m_{t+1} = O(\chi^{-\frac{1}{2}}) \end{aligned} \tag{B.132}$$

This implies that

$$\Delta q_{i,t+1} \longrightarrow \lim\ (\theta_i \Delta z_{t+1} + \xi_i \Delta x_{u,t+1} + \mu_i \Delta \gamma_{t+1} + \eta_i \Delta x_{i,t+1}) \tag{B.133}$$
$$\Delta q_{u,t+1} \longrightarrow \lim\ (\theta_u \Delta z_{t+1} + \xi_u \Delta x_{u,t+1} + \mu_u \Delta \gamma_{t+1}) \tag{B.134}$$

Denote by $\Delta s_{i,t+1}$ the depreciation of currency i against the dollar:

$$\Delta s_{i,t+1} \equiv \Delta q_{u,t+1} - \Delta q_{i,t+1} \tag{B.135}$$

This variable converges to

$$\Delta s_{i,t+1} \longrightarrow \bar{\theta}_i \Delta z_{t+1} + \bar{\xi}_i \Delta x_{u,t+1} + \bar{\mu}_i \Delta \gamma_{t+1} + \bar{\eta}_i \Delta x_{i,t+1} \tag{B.136}$$

With $\psi_i = \phi\tau(m_i - l) - 1$, the coefficients are

$$\bar{\theta}_i = \alpha(e - e_i) + \alpha\psi_i(e - e_u) \tag{B.137}$$
$$\bar{\xi}_i = \alpha\psi_i \tag{B.138}$$
$$\bar{\mu}_i = \phi l_u \cdot \frac{1 - 2(1 + \psi_i)}{1 + \sqrt{2}} - \phi(l_i - l) \tag{B.139}$$

Finally, taking the limit $\chi \longrightarrow \infty$ in Proposition 1, we arrive at

$$l_i - l = -l \cdot \mathbb{E}[\bar{\beta}_i]\big(\mathbb{E}[\bar{\beta}_i \bar{\beta}_i']\big)^{-1} \bar{\beta}_i \tag{B.140}$$

This completes the proof. □

LEMMA 4. *The factors are given by*

$$doll_{t+1} = (e_u - e)\alpha\sigma_z \epsilon_{z,t+1} - \alpha\sigma_x \epsilon_{x,t+1} + \sqrt{2}\phi l_u \sigma_\gamma \epsilon_{\gamma,t+1} \tag{B.141}$$
$$comm_{t+1} = (e_L - e_H)\alpha\sigma_z \epsilon_{z,t+1} + (l_L - l_H)\phi\sigma_\gamma \epsilon_{\gamma,t+1} \tag{B.142}$$

Proof of Lemma 4. This follows from aggregating $\Delta s_{i,t+1}$ from Lemma 3 and using the definition of (e_H, e_L, l_H, l_L). □

LEMMA 5. *Suppose $\sigma_\gamma = 0$. Then, the factors are given by*

$$doll_{t+1} = (e_u - e)\alpha\sigma_z \epsilon_{z,t+1} - \alpha\sigma_x \epsilon_{x,t+1} \tag{B.143}$$
$$comm_{t+1} = (e_L - e_H)\alpha\sigma_z \epsilon_{z,t+1} \tag{B.144}$$

Their covariance is positive if and only if $e > e_u$: $\mathbb{C}[doll_{t+1}, comm_{t+1}] = \alpha^2\sigma_z^2(e - e_u)(e_H - e_L)$. Bilateral exchange rate depreciations are

$$\Delta s_{i,t+1} = \frac{e_i - e}{e_H - e_L} \cdot comm_{t+1} - \psi_i \cdot doll_{t+1} - \alpha\sigma_x \epsilon_{i,t+1} \tag{B.145}$$

The loadings estimated in the factor regression are

$$\beta_{comm,i} = \frac{e_i - e}{e_H - e_L} \tag{B.146}$$

$$\beta_{doll,i} = 1 - \phi\tau(m_i - l) \tag{B.147}$$

Proof of Lemma 5. The first part follows directly from Lemma 4 and Lemma 3 with σ_γ set to zero. The expression for loadings follows from the fact that

$$\beta_{\text{comm},i} = \frac{\mathbb{C}[\text{comm}_{t+1}, \Delta s_{i,t+1}]\mathbb{V}[\text{doll}_{t+1}] - \mathbb{C}[\text{comm}_{t+1}, \text{doll}_{t+1}]\mathbb{C}[\text{doll}_{t+1}, \Delta s_{i,t+1}]}{\mathbb{V}[\text{comm}_{t+1}]\mathbb{V}[\text{doll}_{t+1}] - \mathbb{C}[\text{comm}_{t+1}, \text{doll}_{t+1}]^2} \tag{B.148}$$

$$\beta_{\text{doll},i} = \frac{\mathbb{C}[\text{doll}_{t+1}, \Delta s_{i,t+1}]\mathbb{V}[\text{comm}_{t+1}] - \mathbb{C}[\text{comm}_{t+1}, \text{doll}_{t+1}]\mathbb{C}[\text{comm}_{t+1}, \Delta s_{i,t+1}]}{\mathbb{V}[\text{comm}_{t+1}]\mathbb{V}[\text{doll}_{t+1}] - \mathbb{C}[\text{comm}_{t+1}, \text{doll}_{t+1}]^2} \tag{B.149}$$

Plugging these covariances and leads to the expressions for $\beta_{\text{comm},i}$ and $\beta_{\text{doll},i}$ above. □

LEMMA 6. *Suppose $\sigma_z = 0$. Then, the factors are given by*

$$doll_{t+1} = -\alpha\sigma_x\epsilon_{u,t+1} + \sqrt{2}\phi l\sigma_\gamma\epsilon_{\gamma,t+1} \tag{B.150}$$

$$comm_{t+1} = 0 \tag{B.151}$$

Their covariance is zero. Bilateral exchange rate depreciations are

$$\Delta s_{i,t+1} = -\psi_i \cdot doll_{t+1} + \sqrt{2}\phi l(\sqrt{2} - 1)(\psi_i + 1)\sigma_\gamma\epsilon_{\gamma,t+1} - \alpha\sigma_x\epsilon_{i,t+1} \tag{B.152}$$

Proof of Lemma 6. Without shocks to z_t, the vector of exposures is

$$\bar{\beta}_i = \begin{pmatrix} -\bar{\xi}_i \\ -\bar{\mu}_i \end{pmatrix} = \begin{pmatrix} -\alpha\psi_i \\ \phi l_u(\sqrt{2} - 1)(2\psi_i + 1) + \phi(l_i - l) \end{pmatrix} \tag{B.153}$$

Steady-state inflows are

$$l_i - l = -l \cdot \mathbb{E}[\bar{\beta}_i'](\mathbb{E}[\bar{\beta}_i\bar{\beta}_i'])^{-1}\bar{\beta}_i \tag{B.154}$$

Define the row vector $(a_1\ a_2) = -\mathbb{E}[\bar{\beta}_i'](\mathbb{E}[\bar{\beta}_i\bar{\beta}_i'])^{-1}$ to write

$$l_i - l = -\frac{\alpha l\psi_i a_1}{1 - \phi l a_2} + l\frac{\phi l_u a_2(\sqrt{2} - 1)(2\psi_i + 1)}{1 - \phi l a_2} \tag{B.155}$$

For l_u, we have

$$l_u = -\frac{\alpha l a_1}{1 - \sqrt{2}\phi l a_2} \tag{B.156}$$

Using this to eliminate a_1,

$$l_i - l = \frac{1 - \sqrt{2}\phi l a_2}{1 - \phi l a_2}\psi_i l_u + l\frac{\phi l_u a_2(\sqrt{2} - 1)(2\psi_i + 1)}{1 - \phi l a_2} \tag{B.157}$$

The fact that e_i is uncorrelated with m_i in cross-section implies that $l_H = l_L$. With $\sigma_z = l_L - l_H = 0$, this implies $\text{comm}_{t+1} = 0$. Plugging $l_i - l$ into the expression for $\bar{\mu}_i$,

$$\begin{aligned}\bar{\mu}_i &= -\phi l_u(\sqrt{2}-1)(2\psi_i+1) - \phi l_u \frac{(1-\sqrt{2}\phi l a_2)\psi_i}{1-\phi l a_2} - \phi l_u \frac{\phi l a_2(\sqrt{2}-1)(2\psi_i+1)}{1-\phi l a_2} \\ &= -\phi l_u \frac{(1-\sqrt{2}\phi l a_2)\psi_i + (\sqrt{2}-1)(2\psi_i+1)}{1-\phi l a_2} = \sqrt{2}\phi l_u - \phi l_u \frac{(2\sqrt{2}-1-\sqrt{2}\phi l a_2)(\psi_i+1)}{1-\phi l a_2}\end{aligned} \tag{B.158}$$

The expression for $\bar{\xi}_i$ is $\bar{\xi}_i = \alpha(\psi_i + 1) - \alpha$. Denote $V_\psi = \mathbb{E}[(\psi_i+1)^2]$. With this notation, cross-sectional second moments are

$$\mathbb{E}[\bar{\xi}_i^2] = \alpha^2(V_\psi + 1) \tag{B.159}$$

$$\mathbb{E}[\bar{\mu}_i^2] = (\phi l_u)^2 \cdot \left[2 + V_\psi \frac{(2\sqrt{2}-1-\sqrt{2}\phi l a_2)^2}{(1-\phi l a_2)^2}\right] \tag{B.160}$$

$$\mathbb{E}[\bar{\xi}_i \bar{\mu}_i] = -\alpha\phi l_u \cdot \left[\sqrt{2} + V_\psi \frac{2\sqrt{2}-1-\sqrt{2}\phi l a_2}{1-\phi l a_2}\right] \tag{B.161}$$

For the vector $(a_1\ a_2)$, we have

$$a_1 = -\frac{\alpha\mathbb{E}[\bar{\mu}_i^2] + \sqrt{2}\phi l_u \mathbb{E}[\bar{\xi}_i\bar{\mu}_i]}{\Delta} \tag{B.162}$$

$$a_2 = \frac{\sqrt{2}\phi l_u \mathbb{E}[\bar{\xi}_i^2] + \alpha\mathbb{E}[\bar{\xi}_i\bar{\mu}_i]}{\Delta} \tag{B.163}$$

Here Δ is

$$\Delta \equiv \mathbb{E}[\bar{\xi}_i^2]\mathbb{E}[\bar{\mu}_i^2] - \mathbb{E}[\bar{\xi}_i\bar{\mu}_i]^2 \tag{B.164}$$

Plugging the cross-sectional second moments,

$$\Delta = \frac{(\alpha\phi l_u)^2(\sqrt{2}-1)^2 V_\psi}{(1-\phi l a_2)^2} \tag{B.165}$$

This leads to

$$\begin{aligned}a_1 &= \frac{\alpha(\phi l_u)^2 V_\psi}{\Delta}\left[\frac{2\sqrt{2}-1-\sqrt{2}\phi l a_2}{1-\phi l a_2}\sqrt{2} - \frac{(2\sqrt{2}-1-\sqrt{2}\phi l a_2)^2}{(1-\phi l a_2)^2}\right] \\ &= \frac{\alpha(\phi l_u)^2 V_\psi}{\Delta}\frac{(1-\sqrt{2})(2\sqrt{2}-1-\sqrt{2}\phi l a_2)}{(1-\phi l a_2)^2} = \frac{2\sqrt{2}-1-\sqrt{2}\phi l a_2}{\alpha(1-\sqrt{2})} \\ &= -\frac{2+(\sqrt{2}+1)(1-\sqrt{2}\phi l a_2)}{\alpha}\end{aligned} \tag{B.166}$$

$$a_2 = \frac{\alpha^2\phi l_u V_\psi}{\Delta}\left[\sqrt{2} - \frac{2\sqrt{2}-1-\sqrt{2}\phi l a_2}{1-\phi l a_2}\right] = \frac{\alpha^2\phi l_u V_\psi}{\Delta}\frac{1-\sqrt{2}}{1-\phi l a_2} = \frac{1}{\phi l_u}\frac{1-\phi l a_2}{1-\sqrt{2}} \tag{B.167}$$

The latter implies

$$\phi a_2(l_u - (\sqrt{2}+1)l) = -(\sqrt{2}+1) \tag{B.168}$$

Plugging a_1 into l_u in equation (B.156),

$$l_u = l(\sqrt{2}+1) + \frac{2l}{1-\sqrt{2}\phi l a_2} \tag{B.169}$$

This means

$$a_2 = \frac{\sqrt{2}+1}{\sqrt{2}\phi l} \tag{B.170}$$

With this expression, we get $l_u = l$ and $\alpha a_1 = \sqrt{2}$. The fact that $l_u = l$ implies that the dollar factor is $\text{doll}_{t+1} = -\alpha\sigma_x\epsilon_{u,t+1} + \sqrt{2}\phi l\sigma_\gamma\epsilon_{\gamma,t+1}$. The expression for $\bar{\mu}_i$ becomes

$$\bar{\mu}_i = \sqrt{2}\phi l - 2\phi l(\sqrt{2}-1)(\psi_i+1) \tag{B.171}$$

Bilateral depreciations are

$$\begin{aligned}\Delta s_{i,t+1} &= \alpha\psi_i\sigma_x\epsilon_{u,t+1} - \sqrt{2}\phi l\psi_i\sigma_\gamma\epsilon_{\gamma,t+1} + \sqrt{2}\phi l(\sqrt{2}-1)(\psi_i+1)\sigma_\gamma\epsilon_{\gamma,t+1} - \alpha\sigma_x\epsilon_{i,t+1} \\ &= -\psi_i\cdot\text{doll}_{t+1} + \sqrt{2}\phi l(\sqrt{2}-1)(\psi_i+1)\sigma_\gamma\epsilon_{\gamma,t+1} - \alpha\sigma_x\epsilon_{i,t+1}\end{aligned} \tag{B.172}$$

This completes the proof. □

LEMMA 7. *Suppose $\sigma_x = 0$. Then, the factors are given by*

$$doll_{t+1} = (e_u - e)\alpha\sigma_z\epsilon_{z,t+1} + \sqrt{2}\phi l_u\sigma_\gamma\epsilon_{\gamma,t+1} \tag{B.173}$$

$$comm_{t+1} = (e_L - e_H)\alpha\sigma_z\epsilon_{z,t+1} + \frac{e_H - e_L}{e - e_u}\cdot\kappa\phi l_u\sigma_\gamma\epsilon_{\gamma,t+1} \tag{B.174}$$

Here κ and l_u are given by

$$\kappa = \frac{2\mathbb{V}[\psi](e-e_u)^2}{\mathbb{V}[\psi](e-e_u)^2 + \mathbb{V}[e]} \tag{B.175}$$

$$l_u = \frac{\mathbb{V}[\psi](e-e_u)^2 + \mathbb{V}[e]}{\mathbb{V}[\psi](e-e_u)^2 + (4\mathbb{V}[\psi]+1)\mathbb{V}[e]} \tag{B.176}$$

The coefficient κ is positive and confined within $(0,2)$. The case $\kappa \in (0,\sqrt{2})$ corresponds to a positive exposure of the intermediary to z_t and a stable fixed point in Proposition 1. The case $\kappa \in (\sqrt{2},2)$ corresponds to a negative exposure of the intermediary to z_t and an unstable fixed point in Proposition 1. The covariance of the factors is positive if and only if $e > e_u$. Their correlation reaches 1 if $e > e_u$ and $\mathbb{V}[e] = (\sqrt{2}-1)(e-e_u)^2\mathbb{V}[\psi]$. Bilateral exchange rate depreciations are

$$\Delta s_{i,t+1} = \frac{e_i - e}{e_H - e_L}\cdot comm_{t+1} - \psi_i\cdot doll_{t+1} + \phi l_u(\kappa - \sqrt{2})(\psi_i+1)\sigma_\gamma\epsilon_{\gamma,t+1} \tag{B.177}$$

Proof of Lemma 7. Without shocks to x_t, the vector of exposures is

$$\bar{\beta}_i = \begin{pmatrix} -\bar{\theta}_i \\ -\bar{\mu}_i \end{pmatrix} = \begin{pmatrix} \alpha(e_i - e) - \alpha\psi_i(e - e_u) \\ \phi l_u(\sqrt{2}-1)(2\psi_i + 1) + \phi(l_i - l) \end{pmatrix} \tag{B.178}$$

Steady-state inflows are

$$l_i - l = -l \cdot \mathbb{E}[\bar{\beta}_i'] \big(\mathbb{E}[\bar{\beta}_i \bar{\beta}_i']\big)^{-1} \bar{\beta}_i \tag{B.179}$$

Define the row vector $(a_1 \; a_2) = -\mathbb{E}[\bar{\beta}_i'] \big(\mathbb{E}[\bar{\beta}_i \bar{\beta}_i']\big)^{-1}$ to write

$$l_i - l = -\frac{\alpha l a_1 (e - e_i + \psi_i(e - e_u))}{1 - \phi l a_2} + l a_2 \frac{\phi l_u (\sqrt{2}-1)(2\psi_i + 1)}{1 - \phi l a_2} \tag{B.180}$$

For l_u, we have

$$l_u = -\frac{\alpha l a_1 (e - e_u)}{1 - \sqrt{2}\phi l a_2} \tag{B.181}$$

Using this to eliminate a_1,

$$l_i - l = \frac{1 - \sqrt{2}\phi l a_2}{1 - \phi l a_2} \frac{e - e_i}{e - e_u} l_u + 2\psi_i l_u - \frac{1 - \sqrt{2}\phi l a_2}{1 - \phi l a_2} l_u \psi_i + l_u \frac{\phi l a_2 (\sqrt{2}-1)}{1 - \phi l a_2} \tag{B.182}$$

Plugging $l_i - l$ into the expression for $\bar{\mu}_i$,

$$\bar{\mu}_i = \phi l_u \frac{1 - \sqrt{2}\phi l a_2}{1 - \phi l a_2} \frac{e_i - e}{e - e_u} + \sqrt{2}\phi l_u - \phi l_u (\psi_i + 1) \frac{2\sqrt{2} - 1 - \sqrt{2}\phi l a_2}{1 - \phi l a_2} \tag{B.183}$$

The expression for $\bar{\theta}_i$ is $\bar{\theta}_i = \alpha(e - e_i) + \alpha(\psi_i + 1)(e - e_u) + \alpha(e_u - e)$. Denote $V_\psi = \mathbb{E}[(\psi_i + 1)^2]$ and $V_e = \mathbb{E}[(e_i - e)^2]$. With this notation, cross-sectional second moments are

$$\mathbb{E}[\bar{\theta}_i^2] = \alpha^2 (V_e + (e - e_u)^2 (V_\psi + 1)) \tag{B.184}$$

$$\mathbb{E}[\bar{\mu}_i^2] = (\phi l_u)^2 \cdot \left[2 + V_\psi \frac{(2\sqrt{2} - 1 - \sqrt{2}\phi l a_2)^2}{(1 - \phi l a_2)^2} + \frac{V_e}{(e - e_u)^2} \frac{(1 - \sqrt{2}\phi l a_2)^2}{(1 - \phi l a_2)^2} \right] \tag{B.185}$$

$$\mathbb{E}[\bar{\theta}_i \bar{\mu}_i] = -\alpha\phi l_u (e - e_u) \cdot \left[\sqrt{2} + V_\psi \frac{2\sqrt{2} - 1 - \sqrt{2}\phi l a_2}{1 - \phi l a_2} + \frac{V_e}{(e - e_u)^2} \frac{1 - \sqrt{2}\phi l a_2}{1 - \phi l a_2} \right] \tag{B.186}$$

For the vector $(a_1 \; a_2)$, we have

$$a_1 = -\frac{\alpha(e - e_u)\mathbb{E}[\bar{\mu}_i^2] + \sqrt{2}\phi l_u \mathbb{E}[\bar{\theta}_i \bar{\mu}_i]}{\Delta} \tag{B.187}$$

$$a_2 = \frac{\sqrt{2}\phi l_u \mathbb{E}[\bar{\theta}_i^2] + \alpha(e - e_u)\mathbb{E}[\bar{\theta}_i \bar{\mu}_i]}{\Delta} \tag{B.188}$$

Here Δ is the determinant of $\mathbb{E}[\bar{\beta}_i \bar{\beta}_i']$:

$$\Delta \equiv \mathbb{E}[\bar{\theta}_i^2]\mathbb{E}[\bar{\mu}_i^2] - \mathbb{E}[\bar{\theta}_i \bar{\mu}_i]^2 \tag{B.189}$$

To save on space, we introduce the following notation: $\hat{\alpha} \equiv \alpha(e - e_u)$, $\hat{V}_e \equiv V_e/(e - e_u)^2$, and

$$k \equiv \frac{2\sqrt{2} - 1 - \sqrt{2}\phi l a_2}{1 - \phi l a_2} \tag{B.190}$$

$$n \equiv \frac{1 - \sqrt{2}\phi l a_2}{1 - \phi l a_2} \tag{B.191}$$

With this, we can rewrite

$$\mathbb{E}[\bar{\theta}_i^2] = \hat{\alpha}^2(\hat{V}_e + V_\psi + 1) \tag{B.192}$$

$$\mathbb{E}[\bar{\mu}_i^2] = (\phi l_u)^2 \cdot \left[2 + k^2 V_\psi + \hat{n}^2 \hat{V}_e\right] \tag{B.193}$$

$$\mathbb{E}[\bar{\theta}_i \bar{\mu}_i] = -\hat{\alpha}\phi l_u \cdot \left[\sqrt{2} + k V_\psi + n \hat{V}_e\right] \tag{B.194}$$

Plugging the cross-sectional second moments,

$$\begin{aligned} \Delta &= (\hat{\alpha}\phi l_u)^2 \cdot [(n - \sqrt{2})^2 \hat{V}_e + (k - \sqrt{2})^2 V_\psi + (n - k)^2 \hat{V}_e V_\psi] \\ &= (\hat{\alpha}\phi l_u)^2 \cdot \frac{(\sqrt{2} - 1)^2(\hat{V}_e + V_\psi + 4\hat{V}_e V_\psi)}{(1 - \phi l a_2)^2} \end{aligned} \tag{B.195}$$

With this result, we can express a_1 and a_2 as

$$\hat{\alpha} a_1 = (\sqrt{2} + 1)\frac{(1 - \sqrt{2}\phi l a_2)(\hat{V}_e - V_\psi) - (\sqrt{2} - 1)2V_\psi}{\hat{V}_e + V_\psi + 4\hat{V}_e V_\psi} \tag{B.196}$$

$$\phi l_u a_2 = (\sqrt{2} + 1)\frac{(1 - \phi l a_2)(\hat{V}_e - V_\psi)}{\hat{V}_e + V_\psi + 4\hat{V}_e V_\psi} \tag{B.197}$$

Using equation (B.181) to replace l_u in the expression for a_2,

$$\phi l a_2 \left(\hat{V}_e - V_\psi - \frac{(\sqrt{2} - 1)2V_\psi}{1 - \sqrt{2}\phi l a_2}\right) = (V_\psi - \hat{V}_e)(1 - \phi l a_2) \tag{B.198}$$

This leads to

$$\phi l a_2 = \frac{1}{\sqrt{2}} \cdot \frac{\hat{V}_e - V_\psi}{\hat{V}_e - (\sqrt{2} - 1)V_\psi} \tag{B.199}$$

If $\hat{V}_e > (\sqrt{2} - 1)V_\psi$, then either $\hat{V}_e > V_\psi$, in which case $\phi l a_2 \in (0, 1/\sqrt{2})$, or $\hat{V}_e \in ((\sqrt{2} - 1)V_\psi, V_\psi)$, in which case $\phi l a_2 < 0$. Either way, $\phi l a_2 < 1/\sqrt{2}$. If $\hat{V}_e < (\sqrt{2} - 1)V_\psi$, then $\hat{V}_e < V_\psi$ as well, and

we can rewrite

$$\phi l a_2 = \frac{1+\sqrt{2}}{\sqrt{2}} \cdot \frac{V_\psi - \hat{V}_e}{V_\psi - (\sqrt{2}+1)\hat{V}_e} > 1 + \frac{1}{\sqrt{2}} \tag{B.200}$$

Hence, $\hat{V}_e > (\sqrt{2}-1)V_\psi$ is equivalent to $\phi l a_2 < 1/\sqrt{2}$ and $\hat{V}_e < (\sqrt{2}-1)V_\psi$ is equivalent to $\phi l a_2 > 1 + 1/\sqrt{2}$. The latter corresponds to the case when the mapping $l_i - l = ... + a_2 \phi l \cdot (l_i - l)$ has an unstable fixed point since $a_2 \phi l > 1$.

Armed with this result, go back to $l_L - l_H$:

$$l_L - l_H = \kappa \cdot \frac{e_H - e_L}{e - e_u} \cdot l_u \tag{B.201}$$

Here κ is

$$\kappa \equiv \frac{1 - \sqrt{2}\phi l a_2}{1 - \phi l a_2} \tag{B.202}$$

For this parameter, we have that $\hat{V}_e > (\sqrt{2}-1)V_\psi$ is equivalent to $\kappa \in (0, \sqrt{2})$ and $\hat{V}_e < (\sqrt{2}-1)V_\psi$ is equivalent to $\kappa \in (\sqrt{2}, 2)$. Plugging a_2,

$$1 - \phi l a_2 = \frac{\sqrt{2}-1}{\sqrt{2}} \cdot \frac{\hat{V}_e + V_\psi}{\hat{V}_e - (\sqrt{2}-1)V_\psi} \tag{B.203}$$

$$1 - \sqrt{2}\phi l a_2 = \frac{V_\psi \sqrt{2}(\sqrt{2}-1)}{\hat{V}_e - (\sqrt{2}-1)V_\psi} \tag{B.204}$$

$$2\sqrt{2} - 1 - \sqrt{2}\phi l a_2 = \sqrt{2}(\sqrt{2}-1) \cdot \frac{\sqrt{2}\hat{V}_e + (\sqrt{2}-1)V_\psi}{\hat{V}_e - (\sqrt{2}-1)V_\psi} \tag{B.205}$$

This implies

$$\kappa = \frac{2V_\psi}{V_\psi + \hat{V}_e} \tag{B.206}$$

Coming back to a_1, we can express it as

$$\hat{\alpha} a_1 = \frac{\sqrt{2}(1-\sqrt{2})V_\psi}{\hat{V}_e - (\sqrt{2}-1)V_\psi} \cdot \frac{V_\psi + \hat{V}_e}{V_\psi + \hat{V}_e + 4V_\psi \hat{V}_e} \tag{B.207}$$

Hence,

$$l_u = l \cdot \frac{V_\psi + \hat{V}_e}{V_\psi + \hat{V}_e + 4V_\psi \hat{V}_e} \tag{B.208}$$

This also implies that $a_1 > 0$ if an only if $\hat{V}_e < (\sqrt{2}-1)V_\psi$, which is also equivalent to $\kappa \in (\sqrt{2}, 2)$. Since the sign of a_1 captures the inverse of the sign of the intermediary's exposure to z_t, we can conclude that $\kappa \in (0, \sqrt{2})$ is equivalent to the intermediary being positively exposed to z_t. Conversely, if $\kappa \in (\sqrt{2}, 2)$, the intermediary is negatively exposed to z_t.

For $\bar{\mu}_i$, we then have

$$\bar{\mu}_i = \kappa\phi l_u \frac{e_i - e}{e - e_u} + (\kappa - \sqrt{2})\phi l_u + (\kappa - 2\sqrt{2})\psi_i \phi l_u \tag{B.209}$$

This means that bilateral exchange rate depreciations are

$$\begin{aligned}\Delta s_{i,t+1} &= (e - e_i)\alpha\sigma_z\epsilon_{z,t+1} + \frac{e_i - e}{e - e_u}\kappa\phi l_u\sigma_\gamma\epsilon_{\gamma,t+1} + \psi_i \cdot [(e - e_u)\alpha\sigma_z\epsilon_{z,t+1} - \sqrt{2}\phi l_u\sigma_\gamma\epsilon_{\gamma,t+1}] \\ &\quad + (\kappa - \sqrt{2})(\psi_i + 1)\phi l_u\sigma_\gamma\epsilon_{\gamma,t+1} \\ &= \frac{e_i - e}{e_H - e_L} \cdot \text{comm}_{t+1} - \psi_i \cdot \text{doll}_{t+1} + (\kappa - \sqrt{2})(\psi_i + 1)\phi l_u\sigma_\gamma\epsilon_{\gamma,t+1} \end{aligned} \tag{B.210}$$

This completes the proof. □

LEMMA 8. *Suppose $\sigma_x = 0$. Then, the loadings estimated in the factor regression are*

$$\beta_{comm,i} = \frac{e_i - e}{e_H - e_L} + \phi\tau(m_i - l)\frac{e - e_u}{e_H - e_L} \tag{B.211}$$

$$\beta_{doll,i} = 1 - 2\phi\tau(m_i - l) \tag{B.212}$$

Proof of Lemma 8. Start with the regression

$$\Delta s_{i,t+1} = \beta_{\text{comm},i}\text{comm}_{t+1} + \beta_{\text{doll},i}\text{doll}_{t+1} + \varepsilon_{i,t+1} \tag{B.213}$$

Here $\mathbb{E}[\varepsilon_{i,t+1}] = \mathbb{E}[\varepsilon_{i,t+1}\text{comm}_{t+1}] = \mathbb{E}[\varepsilon_{i,t+1}\text{doll}_{t+1}] = 0$. The coefficients are

$$\beta_{i,\text{comm}} = \frac{C_{i,f}V_d - C_{i,d}C_{f,d}}{V_f V_d - C_{f,d}^2} \tag{B.214}$$

$$\beta_{i,\text{doll}} = \frac{C_{i,d}V_f - C_{i,f}C_{f,d}}{V_f V_d - C_{f,d}^2} \tag{B.215}$$

Here the notation is

$$C_{i,f} = \mathbb{C}[\Delta s_{i,t+1}, \text{comm}_{t+1}] \tag{B.216}$$

$$C_{i,d} = \mathbb{C}[\Delta s_{i,t+1}, \text{doll}_{t+1}] \tag{B.217}$$

$$C_{f,d} = \mathbb{C}[\text{doll}_{t+1}, \text{comm}_{t+1}] \tag{B.218}$$

$$V_f = \mathbb{V}[\text{comm}_{t+1}] \tag{B.219}$$

$$V_d = \mathbb{V}[\text{doll}_{t+1}] \tag{B.220}$$

$$\tag{B.221}$$

Recall equation (B.177):

$$\Delta s_{i,t+1} = \frac{e_i - e}{e_H - e_L} \cdot \text{comm}_{t+1} - \psi_i \cdot \text{doll}_{t+1} + \hat{\epsilon}_{i,t+1} \tag{B.222}$$

Here the residual is

$$\hat{\epsilon}_{i,t+1} = \phi l_u(\kappa - \sqrt{2})(\psi_i + 1) \cdot \sigma_\gamma \epsilon_{\gamma,t+1} \tag{B.223}$$

Denoting $C_{\epsilon,f,i} = \mathbb{C}[\hat{\epsilon}_{i,t+1}, \text{comm}_{t+1}]$ and $C_{\epsilon,d,i} = \mathbb{C}[\hat{\epsilon}_{i,t+1}, \text{doll}_{t+1}]$, we have

$$\beta_{i,f} = \frac{e_i - e}{e_H - e_L} + \frac{C_{\epsilon,f,i}V_d - C_{\epsilon,d,i}C_{f,d}}{V_f V_d - C_{f,d}^2} \tag{B.224}$$

$$\beta_{i,d} = -\psi_i + \frac{C_{\epsilon,d,i}V_f - C_{\epsilon,f,i}C_{f,d}}{V_f V_d - C_{f,d}^2} \tag{B.225}$$

Using Lemma 7,

$$V_f = (e_H - e_L)^2\alpha^2\sigma_z^2 + \frac{(e_H - e_L)^2}{(e - e_u)^2}\kappa^2(\phi l_u)^2\sigma_\gamma^2 \tag{B.226}$$

$$V_d = (e - e_u)^2\alpha^2\sigma_z^2 + 2(\phi l_u)^2\sigma_\gamma^2 \tag{B.227}$$

$$C_{f,d} = (e_H - e_L)(e - e_u)\alpha^2\sigma_z^2 + \frac{e_H - e_L}{e - e_u}\sqrt{2}\kappa(\phi l_u)^2\sigma_\gamma^2 \tag{B.228}$$

$$C_{\epsilon,f,i} = \frac{e_H - e_L}{e - e_u}\kappa \cdot (\psi_i + 1)(\kappa - \sqrt{2})(\phi l_u)^2\sigma_\gamma^2 \tag{B.229}$$

$$C_{\epsilon,d,i} = \sqrt{2} \cdot (\psi_i + 1)(\kappa - \sqrt{2})(\phi l_u)^2\sigma_\gamma^2 \tag{B.230}$$

The denominator in the expressions for $\beta_{i,\text{comm}}$ and $\beta_{i,\text{doll}}$ is

$$V_f V_d - C_{f,d}^2 = \sigma_\gamma^2\sigma_z^2(\alpha\phi l_u)^2(e_H - e_L)^2(\kappa - \sqrt{2})^2 \tag{B.231}$$

The numerators are

$$C_{\epsilon,f,i}V_d - C_{\epsilon,d,i}C_{f,d} = (\psi_i + 1)(\kappa - \sqrt{2})^2(\alpha\phi l_u)^2\sigma_\gamma^2\sigma_z^2(e_H - e_L)(e - e_u) \tag{B.232}$$

$$C_{\epsilon,d,i}V_f - C_{\epsilon,f,i}C_{f,d} = -(\psi_i + 1)(\kappa - \sqrt{2})^2(\alpha\phi l_u)^2\sigma_\gamma^2\sigma_z^2(e_H - e_L)^2 \tag{B.233}$$

Hence,

$$\beta_{i,\text{comm}} = \frac{e_i - e}{e_H - e_L} + (\psi_i + 1)\frac{e - e_u}{e_H - e_L} \tag{B.234}$$

$$\beta_{i,\text{doll}} = -2\psi_i - 1 \tag{B.235}$$

This completes the proof. □

Proof of Proposition 5. Consider the case $\sigma_x = 0$ first. In this case,

$$\Delta s_{i,t+1} = \frac{1 - \alpha}{\alpha}(\Delta c_{u,t+1} - \Delta c_{i,t+1}) \tag{B.236}$$

Relative consumption growth is proportional to the real depreciation, so the risk-sharing correlation $\text{corr}[\Delta s_{i,t+1}, \Delta c_{i,t+1} - \Delta c_{u,t+1}]$ is equal to -1. Now consider $\sigma_x \neq 0$. The depreciation and relative

consumption growth are

$$\Delta s_{i,t+1} = \bar{\theta}_i \Delta z_{t+1} + \alpha\psi_i \Delta x_{u,t+1} + \bar{\mu}_i \Delta\gamma_{t+1} + \alpha\Delta x_{i,t+1} \tag{B.237}$$

$$\begin{aligned}\Delta c_{i,t+1} - \Delta c_{u,t+1} = &-\frac{(1-\alpha)\bar{\theta}_i}{\alpha}\Delta z_{t+1} - (1+(1-\alpha)\psi_i)\Delta x_{u,t+1} - \frac{(1-\alpha)\bar{\mu}_i}{\alpha}\Delta\gamma_{t+1} \\ &+ \alpha\Delta x_{i,t+1}\end{aligned} \tag{B.238}$$

Here we used the fact that $\bar{\xi}_i = \alpha\psi_i$. Taking the limit $\sigma_z \longrightarrow 0$ and $\sigma_\gamma \longrightarrow 0$,

$$\text{corr}[\Delta s_{i,t+1}, \Delta c_{i,t+1} - \Delta c_{u,t+1}] = \frac{\alpha - \psi_i - (1-\alpha)\psi_i^2}{\sqrt{1+\psi_i^2}\cdot\sqrt{\alpha^2 + (1+(1-\alpha)\psi_i)^2}} \tag{B.239}$$

This correlation is equal to 1 at $\psi_i = -1$, which corresponds to $m_i = l$. By continuity, it can take any value between 1 and -1 when $(\sigma_z, \sigma_\gamma, \sigma_x)$ vary between the extreme hyperplane $\sigma_x = 0$ and the axis $(\sigma_z, \sigma_\gamma) = (0, 0)$. □

C Details of the quantitative analysis

Steady-state inflows. In our main quantitative specification, we assume that the intermediary prices risk originating in shocks to z_t and x_{ut}. Define the vector of exposures β_i as

$$\beta_i = \begin{pmatrix} \theta_i - \theta_u \\ \xi_i - \xi_u \end{pmatrix} \tag{C.1}$$

Applying the relevant version of Proposition 1,

$$L_i - L = -L \cdot \mathbb{E}[\beta_i'](D_\Gamma^{-1}\chi^{-1} + \mathbb{E}[\beta_i\beta_i'])^{-1}\beta_i \tag{C.2}$$

Here D_Γ is a diagonal matrix with $\Gamma_z = \lim\ \Gamma\sigma_z^2$ and $\Gamma_x = \lim\ \Gamma\sigma_x^2$ on the main diagonal. Denote

$$V_\theta = \int (\theta_i - \theta_u)^2 di \tag{C.3}$$

$$V_\xi = \int (\xi_i - \xi_u)^2 di \tag{C.4}$$

$$C_{\theta\xi} = \int (\theta_i - \theta_u)(\xi_i - \xi_u) di \tag{C.5}$$

$$\bar{\theta} = \mathbb{E}[\theta_i - \theta_u] \tag{C.6}$$

$$\bar{\xi} = \mathbb{E}[\xi_i - \xi_u] \tag{C.7}$$

With this notation,

$$L_i - L = -L \cdot C_\theta(\theta_i - \theta_u) - L \cdot C_\xi(\xi_i - \xi_u) \tag{C.8}$$

The coefficients C_θ and C_ξ are given by

$$C_\theta = \frac{(\chi^{-1}\Gamma_x^{-1} + V_\xi)\bar{\theta} - C_{\theta\xi}\bar{\xi}}{(\chi^{-1}\Gamma_x^{-1} + V_\xi)(\chi^{-1}\Gamma_z^{-1} + V_\theta) - C_{\theta\xi}^2} \tag{C.9}$$

$$C_\xi = \frac{(\chi^{-1}\Gamma_z^{-1} + V_\theta)\bar{\xi} - C_{\theta\xi}\bar{\theta}}{(\chi^{-1}\Gamma_x^{-1} + V_\xi)(\chi^{-1}\Gamma_z^{-1} + V_\theta) - C_{\theta\xi}^2} \tag{C.10}$$

The coefficients C_θ and C_ξ are the analogs of risk prices in the portfolio space: holdings of i's bonds are shaded by i's exposures to shocks z_t and x_{ut} scaled with these coefficients. Computing them in practice requires knowing the joint distribution of (e_i, M_i), since these permanent types determine θ_i and ξ_i. We use the empirical distribution of (e_i, M_i) for the same sample of countries on which we do our empirical analysis.

Additional impulse responses. We now show additional impulse responses. First, Figure C.1 shows responses to a shocks in z_t of financial inflows in panel (a) and consumption in panel (b) for a high-commodity country, a low-commodity country, and the US. These graphs demonstrate why there is an increase in the relative price of non-tradables at all in the country with zero commodities in exports. The reason is that the windfall of tradable income is partly shared across countries through the intermediary. Dollar appreciation leads to an expansion in the intermediary's balance sheet and inflows into all countries. This is how the low-commodity country is able to fund an initial increase in consumption, and hence an appreciation of non-tradables, even though it has no direct increase in tradable income. Panel (c) shows that the intermediary's realized profits are positive at $t = 1$, when the shock happens. This makes all currencies that appreciate in this event risky, and currencies that appreciate more are more risky than others.

Second, Figure C.2 shows responses to a shock in x_{ut} of financial inflows in panel (a) and consumption in panel (b) for a dollar-rich country, a dollar-poor country, and the US. The positive shock to x_{ut} leads to a depreciation of the dollar, which induces an expansion of the intermediary's balance sheet and rising inflows into regular countries. The fact that the inflow rate $l_{i,t+1}$ is higher in dollar-poor countries is related to the lower base level. Inflows are determined by

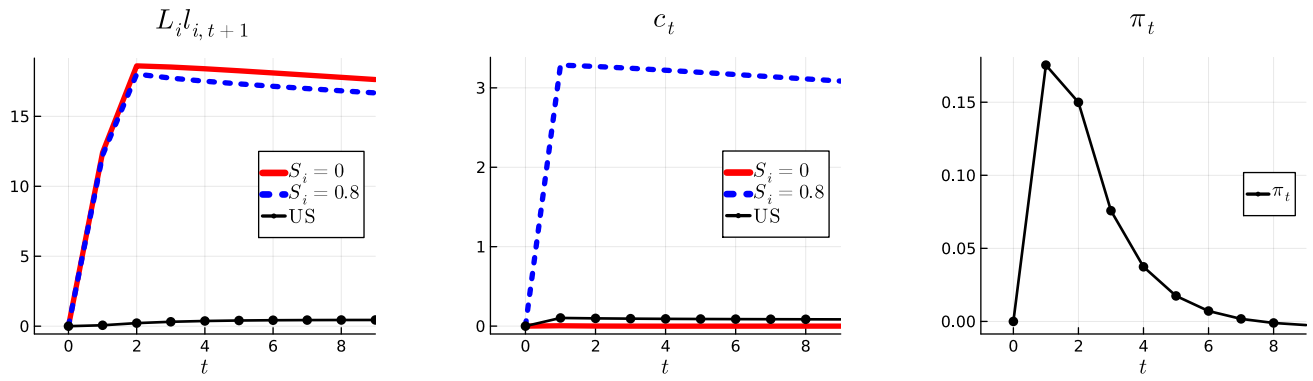
$$L_i l_{i,t+1} = Lm_{t+1} + \chi\varphi\mathbb{E}_t[\Delta q_{i,t+1} - \Delta q_{u,t+1}] \tag{C.11}$$

In dollar-poor countries, inflows dominate increasing demand for dollars, and consumption rises. In dollar-rich countries, demand for reserves expands more than inflows, and consumption falls. This is the driver of a stronger appreciation in dollar-poor countries. Panel (c) shows that the intermediary makes profits on impact, which is what makes dollar-rich countries risky, since their currencies appreciate more in times of high profits.

Estimation of τ. We estimate τ by running the following regression implied by our model:

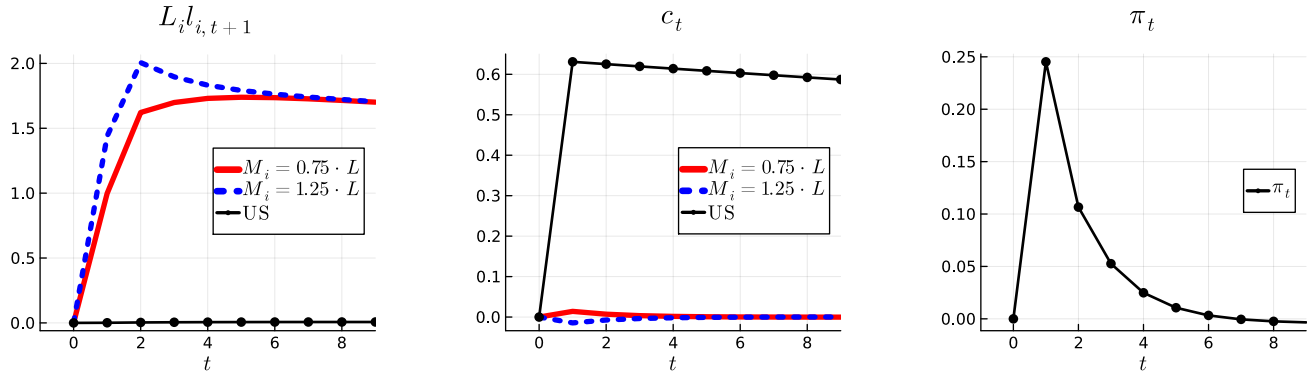
$$\Delta m_{t+1} = \alpha_{\text{reserves}} + \tau_{\text{reserves}}(\Delta q_t - \Delta q_{ut}) + \epsilon_{t+1}$$

We use the logarithm of the broad dollar index DXY as a measure of $q_{ut} - q_t$ and the logarithm of total reserves expressed in dollars as m_t. After estimating $\tau_{\text{reserves}} = 0.9$ (standard error of 0.15), we adjust it for the share of USD in reserves reported by IMF (equal to 0.6 on average). For simplicity, we assume that the non-dollar part of reserves is denominated in the broad basket of currencies



(a) external debt (b) consumption (c) intermediary profits

Figure C.1: Responses of financial inflows, consumption, and intermediary profits to a standard innovation to z_t. The units of the vertical axis are percentage points. The shock happens at $t = 1$. Countries with different commodity shares of exports. Dollar assets fixed at the average level.



(a) external debt (b) consumption (c) intermediary profits

Figure C.2: Responses of financial inflows, consumption, and intermediary profits to a standard innovation to x_{ut}. The units of the vertical axis are percentage points. The shock happens at $t = 1$. Countries with different dollar assets. Commodity shares fixed at the average level.

and is not actively managed in response to the change in the broad dollar. This means that the USD value of the non-dollar part of reserves changes one-for-one with the change in DXY. If the average share of the dollar in reserves is S_{USD}, under this assumption the relationship between the recovered τ_{reserves} and τ in our model is

$$\tau_{\text{reserves}} = (1 - S_{\text{USD}}) \cdot 1 + S_{USD} \cdot \tau$$

This leads to $\tau = 0.83$.

D A commodity-based economy

In this section, we present a simple model of a production economy that underlies the endowment economy of our main exposition. In this version of the mode, commodity intensity is well defined,

and we map it into the tradable output exposure parameter e_i in our baseline model. We also show the mapping between e_i and the steady-state export share of commodities.

Each country produces two final goods: tradable and non-tradable. It also exports non-differentiated commodities and uses labor. Non-tradable goods are simply endowed at the level Y_{it}^N. Tradable goods are produced by a representative firm using labor H_{it} and commodities X_{it}:

$$Y_{it}^T = z_i H_{it} + Z_t X_{it} \tag{D.1}$$

Here Z_t is the global productivity shock with $\mathbb{E}[Z_t] = 1$, and z_i is the country-specific labor productivity. The price of the commodity input is M_t, the wage is W_{it}. The firm solves

$$\max_{H_{it}, X_{it}} \; P_{it} Y_{it}^T - W_{it} H_{it} - M_t X_{it} \tag{D.2}$$

An interior solution implies that, for all i,

$$P_{it} Z_t = M_t \tag{D.3}$$

$$P_{it} z_i = W_{it} \tag{D.4}$$

Countries consume a Cobb-Douglas aggregator of their own non-tradable good and all tradable goods, putting a mass point on their own good:

$$\ln(C_{it}) = \theta \log(C_{it}^N) + (1-\theta)\alpha \log(C_{iit}^T) + (1-\theta)(1-\alpha) \int \log(C_{ijt}^T) dj - \nu \tag{D.5}$$

Here ν is a normalization constant:

$$\nu = \theta \log(\theta) + (1-\theta)\alpha \log((1-\theta)\alpha) + (1-\theta)(1-\alpha) \log((1-\theta)(1-\alpha)) \tag{D.6}$$

This leads to the following demand:

$$C_{it}^N = \theta \frac{Q_{it} C_{it}}{P_{it}^N} \tag{D.7}$$

$$C_{iit}^T = (1-\theta)\alpha \frac{Q_{it} C_{it}}{P_{it}^T} \tag{D.8}$$

$$C_{ijt}^T = (1-\theta)(1-\alpha) \frac{Q_{it} C_{it}}{P_{jt}^T} \tag{D.9}$$

Here the price index is

$$Q_{it} = \theta \log(P_{it}^N) + (1-\theta)\alpha \log(P_{it}^T) + (1-\theta)(1-\alpha) \int \log(P_{jt}^T) dj \tag{D.10}$$

Normalize

$$\int \log(P_{jt}^T) dj = 0 \tag{D.11}$$

Since we have $P_{it} Z_t = M_t$ for all i, which means P_{it} are the same for all i, we have $P_{it} = 1$ for all i. The individual country's price index is then $Q_{it} = (P_{it}^N)^\theta$. Moreover, $M_t = Z_t$.

Country i has a commodity endowment e_i. We parameterize it so that $e_i + z_i = 1$ for all i: natural advantage in commodities in some countries is balanced out by a natural advantage in producing final goods in others. This will allow us to arrive at a symmetric steady state. Country i's consolidated budget constraint is

$$\begin{aligned} Q_{it}C_{it} = P_{it}^N Y_{it}^N + P_{it}^T Y_{it}^T + M_t(e_i - X_{it}) \\ + R_{it}Q_{it}B_{it} - Q_{it}B_{i,t+1} + T_{it} + \Pi_{it} \end{aligned} \tag{D.12}$$

Here $B_{i,t+1}$ is borrowing in local currency bonds for the next period, B_{it} is the debt at the start of this period, T_{it} are net transfers from the government, and Π_{it} are the profit rebates from the intermediaries. Using the fact that $P_{it}^N Y_{it}^N = \theta Q_{it}C_{it}$, the fact that $Q_{it} = (P_{it}^N)^\theta$, assuming $Y_{it}^N = \theta/(1-\theta)$, the fact that $P_{it}^T Y_{it}^T = z_i H_{it} + Z_t X_{it}$, and the fact that $M_t = Z_t$,

$$(Q_{it})^\theta = z_i + Z_t e_i + R_{it}Q_{it}B_{it} - Q_{it}B_{i,t+1} + T_{it} + \Pi_{it} \tag{D.13}$$

In the steady state, $Z_t = 1$. The profit rebate rule and reserve management rules from the baseline model imply $R_{it}Q_{it}B_{it} - Q_{it}B_{i,t+1} + T_{it} + \Pi_{it} = 0$. We also have $z_i + e_i = 1$. All of this implies $Q_i = 1$. To arrive at the steady-state export shares in commodities, use the market clearing for traded goods:

$$Y_{it}^T = C_{iit}^T + \int C_{jit}^T dj \tag{D.14}$$

This leads to the following expression:

$$Y_{it}^T = \alpha(1-\theta)Q_{it}C_{it} + (1-\alpha)(1-\theta)\int Q_{jt}C_{jt}dj \tag{D.15}$$

Since $Q_i = P_i^N = 1$ in the steady state, using $P_{it}^N Y_{it}^N = \theta Q_{it}C_{it}$ leads to $C_i = 1/(1-\theta)$, so $Y_i^T = 1$ for all i. The steady-state export share of commodities is then

$$S_i = \frac{e_i}{1+e_i} \tag{D.16}$$

Denoting $z_t = Z_t - 1$ and using $e_i + z_i = 1$, we can transform the budget constraint in equation (D.12) into

$$C_{it}^T + P_{it}^N C_{it}^N = 1 + e_i z_t + R_{it}Q_{it}B_{it} - Q_{it}B_{i,t+1} + T_{it} + \Pi_{it} \tag{D.17}$$