The Effect of Market Power on the Risk-free rate and the Equity Premium*

SHUBHDEEP DEB
Tilburg University†

JAN EECKHOUT
UPF Barcelona‡

JOSE LORES-DIZ
UPF Barcelona§

February 15, 2026

— PREMLIMINARY AND INCOMPLETE —
PLEASE DO NOT CITE

Abstract

In frictionless economies, the risk-free rate equals the household discount rate. With uninsurable income risk, precautionary savings depress the risk-free rate below the discount rate. We develop a model in which market power affects equilibrium asset returns through household portfolio allocation between safe assets and risky equity under financial frictions. Higher markups raise profits and increase the share of risky equity in household wealth. Because equity returns are risky, this shift increases exposure to equity risk and strengthens precautionary demand for safety. At the same time, higher markups reduce capital accumulation and contract the supply of safe assets through firms' collateral constraints. The equilibrium risk-free rate declines to reconcile stronger demand for safety with reduced safe asset supply, widening the equity premium. Quantitatively, the rise in market power and labor income risk account for most of the observed decline in the risk-free rate, while changes in markups explain the bulk of the movement in the return on equity.

Keywords. Market Power. Financial Frictions. Risk-free rate. Return on Equity.

*We are grateful for comments and feedback from numerous colleagues and seminar audiences. We acknowledge support from the ERC, Advanced grant 101198932. Jose received the support of a fellowship from Fundación Ramón Areces.

†Shubhdeep.deb@tilburguniversity.edu

‡jan.eeckhout@upf.edu – ICREA-BSE-CREI

§jose.lores@upf.edu

1 Introduction

The risk-free rate of return on bonds and the return on risky equity are determined by financial frictions the household faces. In the absence of financial frictions, households can perfectly insure income risk and the risk-free rate as well as the return on equity converges to the discount rate. Instead, when households face financial frictions, uninsurable labor and asset income risk drives the risk-free rate below the discount rate below the discount rate and drives a wedge between the return on equity and the risk-free rate, the equity premium for investing in risky assets. In this paper, we investigate how market power affects safe and risky asset returns in the presence of market power.

A priori, the effect of market power is ambiguous. Market power increases the value of equity in the household budget constraint, which changes the composition of wealth and can affect the demand for safe assets (see for example Auclert, Malmberg, Rognlie, and Straub (2025)). But market power also reduces labor income and at the same time, in the presence of uninsurable income risk from labor and risky assets, households optimally re-balance their portfolio. In this paper, we analyze financial frictions and portfolio choice and show that they are the central determinant of how markups transmit to interest rates. The framework helps reconcile seemingly conflicting results across theories and empirical narratives as the framework nests limiting cases. We quantify the model to match moments in the US economy and find that the increase in equity as a source of income and the resulting portfolio rebalancing accounts for a decline of more than 2 percentage points in the risk-free rate, which accounts for 20% of the total decline between 1984 and 2019.

We develop a tractable model of financial frictions with two assets, safe corporate debt and equity claims on (future) monopoly profits. The financial frictions restrict household asset participation and firm-side debt issuance. Households are constrained how much they can borrow to hedge uninsurable labor income and equity risk. As a result, asset prices (interest rates) are pinned by endogenous Euler conditions with precautionary motive mechanisms rather than only by time preference alone. Firms have market power and generate monopoly profits. To finance investment, they issue safe debt subject to a collateral constrained tied to capital. Firms finance the remainder of the capital issuing risky equity.

Market power affects the risk-free rate and the return on equity through the supply and demand for assets. First, there is a precautionary savings shift. As markups rise, the composition of household income shifts away from labor income toward profit-based income intermediated through equity claims. While labor income is itself risky, higher markups reduce the labor share and increase the share of income exposed to equity return risk, while simultaneously contracting the supply of safe debt. The relevant change is therefore not an increase in primitive risk, but a redistribution of consumption exposure toward the equity diffusion component. This reallocation strengthens precautionary saving motives among unconstrained households and increases the demand for safe assets, placing downward pressure on the risk-free rate.

This mechanism is formalized by the equilibrium condition that pins down the risk-free rate in our model. The safe rate is determined by three components: (i) an intertemporal substitution term governed by time preference, (ii) a precautionary component that reflects households' exposure to consumption diffusion risk, and (iii) a drift component that captures the expected consumption growth of constrained households.

Market power affects the risk-free rate primarily through the latter two channels. By shifting income

from labor toward profit-based equity income, higher markups alter the composition of consumption risk faced by unconstrained households, thereby increasing the precautionary term. At the same time, the redistribution of income changes the mass of constrained households and the relative weight of constrained consumption in aggregate demand, affecting the drift component. The combined effect determines how markups translate into movements in the equilibrium safe rate.

Second, when firms face financial frictions, the **supply** of safe versus risky assets is endogenous, and so is the return. Rising market power increases the equity value whilst at the same time depressing capital investment. The economy becomes awash in risky equity while safe debt becomes scarce relative to total wealth. This relative scarcity drives the risk-free rate down, decoupling it from the marginal product of capital. Together with the demand-side contraction, this creates a "double squeeze" that exerts further downward pressure on the risk-free rate.

In this framework we analyze the transmission mechanism from markups to the risk-free rate and the rate of return on equity. To gain a better understanding of the mechanism, we consider some limit cases. First, we consider the benchmark without financial frictions. When there is perfect insurance of income risk and the firms' collateral constraint is not binding (there is a full set of Arrow-Debreu securities) or there is no risk, then the risk-free rate and the return on equity coincide and are equal to the discount rate. Both rates are pinned down by the household's time preference and are invariant to market power. Market power both induces inefficiency through lower investment and deadweight loss, and reallocates income from labor to profits. Even if the risk-free rate remains unaffected by market power, welfare falls due to underinvestment, higher output prices and deadweight loss.

Second, when there is no uncertainty in the return on equity but uncertainty in labor income, constrained households have limited access to risk-free assets to smooth consumption driving down the risk-free rate below the discount rate. As market power rises, the redistribution of income from labor to (deterministic) equity income renders the income process less risky and hence leads to a decline in the household demand for assets, and hence an increase in the risk-free rate (which is equal to the return on equity). When there is no equity risk, the return on equity is equal to the risk-free rate, which increases the value of equity, making households richer.

Finally, financial frictions in the corporate sector play a key role in amplifying the effect of markups on the risk-free rate and the return on equity. As the borrowing constraint loosens, firms need to retain less earnings thus supplying less equity on the asset market. In the limit without a borrowing constraint, firms retain zero earnings and use bonds to finance all capital. A tighter borrowing constraint leads to inefficient production, and it increases the value of equity through retained earnings. Since equity is more risky, households again balance their portfolio towards more safe bonds and less risky assets. This leads to a further decline in the risk-free rate and an increase in the equity premium.

Related Literature Understanding the mechanisms behind the sharp decline in the risk-free rate from the 1980s onwards has grasped the attention of academics and policy makers. There are many candidate competing explanations for this phenomenon, ranging from demographics (aging population, changing fertility), technological change (productivity growth, changing capital intensity), government policy (debt, taxes and spending), the balance of payments, and market power... Several papers aim to account for the contribution of all these factors: Eggertsson, Robbins, and Wold (2021); Rachel (2025); Auclert, Malmberg, Rognlie, and Straub (2025); Peruffo and Platzer (2024) amongst others. Here we focus exclu-

sively on the role of market power in an attempt to isolate a crisp mechanism. That does not mean we believe other factors than market power are not important conceptually and quantitatively.

Eggertsson, Robbins, and Wold (2021) attributes a substantial role to market power and explain both the decline in the labor share and the decline in the risk-free rate in a model with market power. Their model is an otherwise standard neoclassical growth model where firms have market power. They explain the falling risk-free through a time-varying utility wedge in household preferences and the rise of monopoly power through rising entry costs.

In contrast to Eggertsson et al. (2021), Auclert, Malmberg, Rognlie, and Straub (2025) interpret the secular decline in the real rate as mainly demand-driven: population aging, rising inequality, foreign demand for U.S. assets, and slower productivity growth raise desired asset holdings relative to GDP, putting downward pressure on long-run returns. In their framework, higher market power does not lower the safe rate. Instead, an increase in markups raises the equilibrium return because it (i) boosts private asset *supply* through a higher capitalized value of profits and rents, and (ii) lowers the labor share – which is the only source of risk– scaling down household *demand* for insurance (see also Auclert and Rognlie (2018)). :

Our model nests this comparative static once we shut down aggregate uncertainty in equity returns, as in Auclert, Malmberg, Rognlie, and Straub (2025). In this case, the rise in markups increases the stock of effectively riskless wealth and reduces households' exposure to risky labor income, weakening the precautionary saving motive against idiosyncratic earnings shocks and therefore increasing the safe rate. When equity-return risk is reinstated, the same redistribution toward profits raises exposure to risky returns, so the precautionary wedge dominates and r_S falls in our baseline.

Instead, like Eggertsson et al. (2021), Peruffo and Platzer (2024) finds that market power has a downward effect on the risk-free rate. The main reason is, in contrast to Auclert et al. (2025), that the models in Eggertsson et al. (2021) and Peruffo and Platzer (2024) do not include the value of capitalized profits as part of the asset supply. With the capitalized profits included, profits from market power has a huge effect on the asset supply, which leads to higher interest rates as households need the incentives to hold more assets. This is indeed the case also in our framework. However, we show that including the capitalized profits in the asset supply comes with an important countervailing equilibrium force. As the supply of equity increases, so does risk. Households balance their portfolio towards bonds in order to hedge against the increased risk due to higher equity supply. We find that this insurance motive dominates and lowers the risk-free rate and increases the return on equity and the equity premium.

Our model builds on a large literature in heterogeneous agent macroeconomics. Like Kaplan, Moll, and Violante (2018) and Guvenen (2009) there are two assets, bonds and equity, with frictions that create an premium for holding equity. On top of that, we also speak to a burgeoning literature of modeling heterogeneous households in continuous time (Achdou et al., 2021; Cioffi, 2021; Fernández-Villaverde et al., 2023; Kaplan et al., 2023)

2 Model

Environment Time is continuous and measured in years. The economy consists of a unit measure of households and a unit measure of monopolistically competitive firms.

Households are heterogeneous in financial wealth and idiosyncratic labor efficiency. Labor is supplied inelastically, and income depends on the wage per efficiency unit and individual productivity. Households choose consumption and allocate their financial wealth between a safe asset and a risky equity asset, making a portfolio choice decision each period. Uncertainty is entirely idiosyncratic. Households face stochastic labor income risk and stochastic returns on equity. There is no aggregate uncertainty, so the economy admits a stationary distribution of households over wealth and productivity.

On the production side, a competitive firm aggregates inputs from intermediate goods producers who operate under monopolistic competition and issue equity and debt claims that are held by households. Prices in the economy are the wage, the risk free return, and the expected return on equity. There are a continuum of measure one intermediate goods markets i with one firm in each market.

Preferences Households supply labor inelastically in efficiency units and choose consumption and portfolio allocations. Preferences are represented by instantaneous CRRA utility

$$u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}, \qquad \gamma > 0,\ \gamma \neq 1, \tag{1}$$

and utility is discounted at rate $\rho > 0$.

Income and Portfolio Choice At each instant t, a household earns labor income equal to $w_t z_t$, where w_t denotes the wage per efficiency unit and z_t represents idiosyncratic labor productivity. Labor is supplied inelastically, so income risk arises solely from fluctuations in z_t.

Households hold total financial wealth a_t. This wealth can be allocated between a safe asset a_S and a risky equity asset a_R, so that total assets at time t are:

$$a_t = a_{St} + a_{Rt}. \tag{2}$$

The safe asset yields the risk-free return r_S. The risky asset yields a stochastic return with drift r_R and volatility σ_R. Households choose the portfolio composition (a_{St}, a_{Rt}) taking prices (w_t, r_S, r_R) as given.

Wealth Dynamics The risk-free asset yields return r_S:

$$\frac{\mathrm{d}B_t}{B_t} = r_S \,\mathrm{d}t. \tag{3}$$

The risky asset (equity) price follows

$$\frac{\mathrm{d}S_t}{S_t} = \mu_R \,\mathrm{d}t + \sigma_R \,\mathrm{d}W_{Rt}, \tag{4}$$

where μ_R is equal to the return r_R on the risky asset (see below) and W_{Rt} is a standard Wiener process, and r_R denotes the expected return and σ_R the volatility.

Financial wealth evolves according to

$$\mathrm{d}a_t = \mu_a \mathrm{d}t + \sigma_R a_R \,\mathrm{d}W_{Rt} \tag{5}$$

where $\mu_a = wz + r_S a_S + r_R a_R - c$ and $a_t = a_{St} + a_{Rt}$. The drift term μ_{at} reflects income — labor earnings, the return on total wealth in the safe asset, and the expected return earned on risky holdings — net of consumption. The diffusion term captures equity return risk, which scales proportionally with risky asset holdings a_{Rt}.

Labor Income Process Labor productivity follows a log Ornstein–Uhlenbeck process,

$$\mathrm{d}\log z_t = -\theta \log z_t \,\mathrm{d}t + \sigma_z \,\mathrm{d}W_{zt}, \qquad \theta > 0, \tag{6}$$

where θ governs the speed of mean reversion and σ_z the volatility of income shocks. Importantly, for our estimates we use the standard deviation of the stationary log income process $\nu = \sigma_z/\sqrt{2\theta}$. The processes W_{Rt} and W_{zt} are independent, so equity risk and labor income risk are uncorrelated.[1]

Financial Constraint Households face a borrowing constraint on net worth,

$$a_t \geq \underline{a}, \tag{7}$$

which limits leverage. Given this constraint, risky asset holdings must satisfy

$$0 \leq a_{Rt} \leq a_t - \underline{a}. \tag{8}$$

When $\underline{a} = 0$, households cannot borrow and therefore cannot take leveraged positions in the risky asset.

Hamilton–Jacobi–Bellman Equation Let $V(a, z)$ denote the stationary value function of a household with financial wealth a and idiosyncratic labor productivity z. Taking prices (w, r_S, r_R) as given, the household chooses consumption c and risky asset holdings a_R to maximize lifetime utility. The stationary Hamilton–Jacobi–Bellman equation is

$$\rho V(a,z) = \max_{c\geq 0,\, a_R} \left\{ u(c) + V_a \mu_a(a,z;c,a_R) + \frac{1}{2} V_{aa} \sigma_R^2 a_R^2 + V_z \mu_z(z) + \frac{1}{2} V_{zz} \sigma_z^2 z^2 \right\}. \tag{9}$$

The term $u(c)$ represents instantaneous utility from consumption. The term $V_a \mu_a$ captures the effect of expected changes in financial wealth on the continuation value. The term $\frac{1}{2} V_{aa} \sigma_R^2 a_R^2$ captures the curvature effect of equity return risk. Because equity returns are stochastic with volatility σ_R, fluctuations in wealth scale proportionally with risky holdings a_R, and the second derivative V_{aa} governs the precautionary response to this risk. The remaining terms, $V_z \mu_z(z)$ and $\frac{1}{2} V_{zz} \sigma_z^2 z^2$, arise from the stochastic evolution of labor productivity. These terms reflect the impact of expected changes and volatility in idiosyncratic labor income on the continuation value of the household.

The household chooses consumption c and the asset composition $\{a_S, a_R\}$ to maximize the discounted value $V(a, z)$ given current assets a and income shock z, subject to the borrowing constraint (7). Since total assets and consumption equal total income, a_S is determined by c and a_R.

[1] This is approximately borne out in the US data (PSID) where the correlation coefficient is 0.06.

Technology There is a representative firm producing the final good, using all goods produce in the continuum of input markets. The final good aggregates inputs with a standard Constant Elasticity of Substitution (CES) technology:

$$Y_t = \left(\int_0^1 y_{it}^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}}, \qquad \eta > 1. \tag{10}$$

The representative firm in the input market has productivity $A_{it} = A_t$ and uses a Cobb-Douglas technology with labor and capital as inputs of production:

$$y_{it} = A_{it} k_{it}^{\alpha} l_{it}^{1-\alpha}, \qquad 0 < \alpha < 1. \tag{11}$$

Each period, firms pay a sunk cost (overhead) $w_t \phi_t$, expressed in overhead labor units ϕ_t. Because firms are identical in productivity and operate in identical markets, throughout the paper we drop the i subscript whenever there is no ambiguity.

There is a spot market for labor l_t with price w_t in efficiency units. Firms need to invest to accumulate capital k_t to produce. Capital increases with investment I_t and depreciates at rate δ. We assume firms cannot divest. The law of motion of capital accumulation is:

$$\dot{k}_t = I_t - \delta k_t. \tag{12}$$

Debt, Equity and Collateral Firms have a choice to finance capital k with debt or with equity. Denote by D_t the stock of outstanding debt. The debt (or bond) is safe and interest is serviced continuously at the risk-free safe rate r_S, so the flow cost is $r_S D_t$. Denote new borrowing by b_t:

$$\dot{D}_t = b_t. \tag{13}$$

Risky assets generate a flow of dividends. The continuous-time analog of the discrete-time flow-of-funds identity is:

$$d_{Et} = \pi_t^o - I_t - r_S D_t + \dot{D}_t. \tag{14}$$

Dividends equal operating profits π_t^o (see below) minus investment minus interest payments plus net new borrowing.

Firms face financial frictions. They can borrow up to a fraction $\lambda \in [0, 1]$ of their capital stock k. With continuously serviced debt, the natural collateral restriction is on the outstanding debt stock:

$$D_t \leq \lambda k_t, \qquad \lambda > 0. \tag{15}$$

This is the continuous-time counterpart of discrete-time constraints written in terms of principals.[2]

[2]Discrete-time forms like $(1 + r_S) D_t \leq \lambda k_t$ correspond to one-period maturity with gross repayment; in continuous time, the analog would involve $e^{r_S \Delta}$ for maturity Δ.

Profits We define a firms' operating profits (accounting for the cost of labor but not the user cost of capital and overhead) as:

$$\pi_t^o = p_t y_t - w_t l_t \tag{16}$$

The firm's accounting profits also adjust for the user cost of capital and overhead

$$\pi_t^a = p_t y_t - w_t l_t - w_t \phi_t - r_{St} D_t - \delta k_t. \tag{17}$$

Economic profits also take into account the opportunity cost of holding equity:

$$\pi_t^e = p_t y_t - w_t l_t - w_t \phi_t - (r_t^S + \delta) k_t. \tag{18}$$

The firm chooses labor, investment (capital) and debt to maximize the present value of dividend payments:

$$\max_{\{l_t, I_t, D_t\}} \mathbb{E}_0 \int_0^\infty e^{-r_R t} \, d_{Et} \, dt. \tag{19}$$

3 Solution

3.1 Households solution

Optimal (Interior) Policies The household chooses consumption c, and safe and risky assets a_S, a_R. Assume an interior solution with $V_a > 0$ and $V_{aa} < 0$ (concavity in a). Then consumption solves the FOC $u'(c) = V_a$ which gives

$$c^*(a,z) = V_a(a,z)^{-1/\gamma}. \tag{20}$$

The optimal asset position satisfies the FOC in k (ignoring constraints)

$$0 = (\mu_R - r) V_a + V_{aa} \, \sigma_R^2 \, a_R \quad \Rightarrow \quad a_R^*(a,z) = -\frac{\mu_R - r_S}{\sigma_R^2} \frac{V_a(a,z)}{V_{aa}(a,z)}. \tag{21}$$

With the constraint (8), implement

$$a_R(a,z) = \min \Big\{ \max\{a_R^*(a,z), 0\}, \, a - \underline{a} \Big\}. \tag{22}$$

In the frictionless CRRA Merton problem without labor-income risk, the optimal risky *share* of wealth is constant:

$$\pi^{\text{Merton}} = \frac{\mu_R - r_S}{\gamma \sigma_R^2}. \tag{23}$$

In many heterogeneous-agent settings, as $a \to \infty$ the policies converge toward this benchmark (income becomes negligible relative to wealth), which motivates tail boundary conditions in numerical work.

Stationary Kolmogorov Forward Equation Let $g(a,z)$ be the stationary joint density on the domain $\mathcal{A} = [\underline{a}, a_{\max}] \times [z_{\min}, z_{\max}]$ (or suitable truncation). The stationary KFE is the adjoint of the infinitesimal

generator:

$$0 = -\partial_a\big(\mu_a(a,z)\,g(a,z)\big) + \tfrac{1}{2}\partial_{aa}\big(\sigma_a(a,z)^2\,g(a,z)\big) - \partial_z\big(\mu_z(z)\,g(a,z)\big) + \tfrac{1}{2}\partial_{zz}\big(\sigma_z(z)^2\,g(a,z)\big), \tag{24}$$

with normalization $\int_{\mathcal{A}} g(a,z)\,\mathrm{d}a\,\mathrm{d}z = 1$.

Boundary conditions (economic interpretation) A standard interpretation of the borrowing constraint at $a = \underline{a}$ is a state constraint: prevent probability mass from flowing out of the feasible set at the borrowing limit by ensuring the drift points weakly inward when the agent is at the boundary. Numerically, this is typically enforced by an upwind/one-sided derivative rule in the HJB and a no-outflow condition in the KFE discretization.

The Upper boundary at $a = a_{\max}$ (tail condition). Because computations truncate the state space, it is o to impose a tail condition based on asymptotic behavior of V under CRRA. One convenient condition is

$$V_{aa}(a_{\max}, z) \approx -\gamma \frac{V_a(a_{\max}, z)}{a_{\max}}, \tag{25}$$

and the implied tail risky demand at $a_{\max}$ is often close to $k(a_{\max}, z) \approx \pi^{\text{Merton}} a_{\max}$.

3.2 Firm Solution

Demand and Pricing The final goods producer aggregates the intermediate inputs produced in the monopolistically competitive market. Total output Y_t is the CES aggregat of a unit measure of varieties:

$$Y_t = \left(\int_0^1 y_{it}^{\frac{\eta-1}{\eta}}\,di\right)^{\frac{\eta}{\eta-1}}, \qquad \eta > 1, \tag{26}$$

with associated price index

$$P_t = \left(\int_0^1 p_{it}^{1-\eta}\,di\right)^{\frac{1}{1-\eta}}. \tag{27}$$

Cost minimization by the final-good producer implies variety demand y_{it} and inverse demand p_{it}

$$y_{it} = Y_t\left(\frac{p_{it}}{P_t}\right)^{-\eta} \quad \text{and} \quad p_{it} = P_t\left(\frac{y_{it}}{Y_t}\right)^{-1/\eta}. \tag{28}$$

The final goods firm's cost is the sum of each intermediate goods firm's revenue $R_{it}(y_{it}) \equiv p_{it}(y_{it})\,y_{it}$. The intermediate firm's marginal revenue is $p_{it}\,(1+\varepsilon) = p_{it}\mu_{it}^{-1}$, where we define μ_{it} as the gross markup

$$\mu \equiv \frac{\eta}{\eta - 1} > 1. \tag{29}$$

Hamiltonian The firm chooses labor l_t, investment I_t, and the path of debt D_t to maximize the present value of dividend payments, subject to capital accumulation and the collateral constraint. To characterize the optimal policy, we write the firm's problem in current-value form.

Let q_t denote the shadow value of installed capital k_t, m_t the shadow value of outstanding debt D_t, and $\gamma_t \geq 0$ the multiplier on the collateral constraint $D_t \leq \lambda k_t$. The multiplier γ_t captures the marginal

value of relaxing the borrowing constraint.

The current-value Hamiltonian is

$$\mathcal{H}_t = \pi_t - I_t - r_S D_t + b_t + q_t(I_t - \delta k_t) + m_t b_t + \gamma_t(\lambda k_t - D_t). \tag{30}$$

The first term, $\pi_t - I_t - r_S D_t + b_t$, represents the flow of dividends to equity holders. Dividends equal operating profits Π_t minus investment expenditures I_t and interest payments on outstanding debt $r_S D_t$, plus net new borrowing $b_t = \dot{D}_t$.

The term $q_t(I_t - \delta k_t)$ imposes the law of motion for capital,

$$\dot{k}_t = I_t - \delta k_t,$$

where δ is the depreciation rate. The shadow value q_t measures the marginal value of an additional unit of installed capital.

The term $m_t b_t$ imposes the law of motion for debt,

$$\dot{D}_t = b_t,$$

with m_t capturing the marginal value of relaxing the debt stock constraint through additional borrowing.

Finally, $\gamma_t(\lambda k_t - D_t)$ enforces the collateral constraint. When the constraint binds, $\gamma_t > 0$ and the firm values additional collateral capacity; when it is slack, $\gamma_t = 0$.

This Hamiltonian summarizes the firm's intertemporal trade-offs between investment, borrowing, and dividend payments, taking prices (w_t, r_S, r_R) as given.

Labor Under monopolistic competition, marginal revenue equals price divided by the gross markup $\mu = \frac{\eta}{\eta - 1}$. Since

$$\frac{\partial(p_t y_t)}{\partial y_t} = \frac{p_t}{\mu}, \qquad \frac{\partial y_t}{\partial l_t} = (1-\alpha) A k_t^{\alpha} l_t^{-\alpha},$$

the first-order condition for labor is

$$\frac{p_t}{\mu}(1-\alpha) A k_t^{\alpha} l_t^{-\alpha} = w_t. \tag{31}$$

The wage equals the marginal revenue product of labor. Relative to perfect competition, the markup μ drives a wedge between marginal product and marginal revenue.

Investment The first-order condition with respect to investment is

$$-1 + q_t = 0 \quad \Rightarrow \quad q_t = 1. \tag{32}$$

Debt The first-order condition with respect to new borrowing b_t is

$$1 + m_t = 0 \quad \Rightarrow \quad m_t = -1. \tag{33}$$

Issuing an additional unit of debt raises contemporaneous dividends one-for-one, implying the current-value shadow price of debt equals minus one. The dynamic implications of leverage are governed by

the debt costate equation:

Debt Costate Equation The intertemporal condition for debt is given by the costate equation

$$\dot{m}_t = r^r m_t - \frac{\partial \mathcal{H}_t}{\partial D_t}. \tag{34}$$

Since

$$\frac{\partial \mathcal{H}_t}{\partial D_t} = -r_S - \gamma_t,$$

it follows that

$$\dot{m}_t = r^r m_t + r_S + \gamma_t. \tag{35}$$

Using the first-order condition $m_t = -1$ and imposing stationarity ($\dot{m}_t = 0$), we obtain

$$\gamma_t = r^r - r_S. \tag{36}$$

The multiplier on the collateral constraint therefore equals the spread between the required return on equity and the risk-free rate. Complementary slackness requires

$$\gamma_t \geq 0, \qquad \lambda K_t - D_t \geq 0, \qquad \gamma_t(\lambda K_t - D_t) = 0. \tag{37}$$

Hence, if $r^r > r_S$, the constraint binds and the firm leverages up to the collateral limit, $D_t = \lambda K_t$. If $r^r = r_S$, the constraint is slack. If $r^r < r_S$, debt is strictly dominated and the firm sets $D_t = 0$.

Capital Euler Equation The intertemporal optimality condition for capital is given by the costate equation

$$\dot{q}_t = r_R q_t - \frac{\partial \mathcal{H}_t}{\partial k_t}. \tag{38}$$

This condition states that the return required by equity holders on the shadow value of installed capital, $r_R q_t$, must equal the marginal contribution of capital to firm value. The term $\frac{\partial \mathcal{H}_t}{\partial k_t}$ captures how a marginal increase in the capital stock affects the firm's flow value through production, depreciation, and collateral capacity.

Since

$$\frac{\partial \mathcal{H}_t}{\partial k_t} = \frac{p_t}{\mu} \alpha A k_t^{\alpha-1} L_t^{1-\alpha} - q_t \delta + \lambda \gamma_t,$$

the marginal effect of capital has three components.

First,

$$\frac{p_t}{\mu} \alpha A k_t^{\alpha-1} L_t^{1-\alpha}$$

is the marginal revenue product of capital. Under monopolistic competition, marginal revenue equals price divided by the markup $\mu = \frac{\eta}{\eta-1}$. Hence markups reduce the productive return to capital relative to perfect competition.

Second, the term $-q_t \delta$ reflects depreciation. An additional unit of capital loses value at rate δ, and since q_t measures the shadow value of capital, depreciation reduces firm value proportionally.

Third, the term $\lambda\gamma_t$ captures the collateral benefit of capital. Because firms face the borrowing constraint $D_t \leq \lambda k_t$, a marginal increase in capital expands borrowing capacity by λ. The shadow value of relaxing this constraint is γ_t, so capital generates additional value through improved financing conditions whenever the constraint binds.

In steady state, $\dot{q}_t = 0$, and from the investment first-order condition without adjustment costs we have $q_t = 1$. Substituting into the Euler equation yields

$$r_R + \delta = \frac{p_t}{\mu}\alpha A k_t^{\alpha-1} L_t^{1-\alpha} + \lambda\gamma_t. \tag{39}$$

This equation equates the full user cost of capital, $r_R + \delta$, to the total marginal benefit of capital. The left-hand side represents the return required by equity holders including depreciation. The right-hand side consists of the productive return and the collateral value of capital.

Substituting $\gamma_t = r_R - r_S$,

$$r_R + \delta = \frac{p_t}{\mu}\alpha A k_t^{\alpha-1} l_t^{1-\alpha} + \lambda(r_R - r_S). \tag{40}$$

The collateral channel therefore depends directly on the equity premium $r_R - r_S$. When $r_R > r_S$, capital provides financing services in addition to productive services. The strength of this channel is governed by λ, which measures the tightness of the borrowing constraint.

Steady State under Symmetry Under $p = 1$ (normalization), $L = 1$ (fixed labour supply), and $r_R = \mu_R$, the Euler condition simplifies to

$$\mu_R + \delta = \frac{1}{\mu}\alpha A k^{\alpha-1} + \lambda(\mu_R - r_S). \tag{41}$$

The left-hand side represents the required return on capital. The first term on the right-hand side is the marginal revenue product of capital under monopolistic competition, while the second term captures the collateral benefit arising from the spread between the required equity return and the risk-free rate.

Solving for capital:

$$k = \left[\frac{\frac{1}{\mu}\alpha A}{\mu_R + \delta - \lambda(\mu_R - r_S)}\right]^{\frac{1}{1-\alpha}}. \tag{42}$$

The denominator

$$\mu_R + \delta - \lambda(\mu_R - r_S)$$

can be interpreted as the effective user cost of capital. Financial frictions reduce this effective cost when $r_R > r_S$, because capital relaxes borrowing constraints. At the same time, market power reduces the numerator through the factor $\frac{1}{\mu}$, lowering the productive return to capital.

Wages:

$$w = \frac{1}{\mu}(1-\alpha)Ak^{\alpha}. \tag{43}$$

Balance sheet:

$$D = \lambda k, \qquad E = k - D = (1 - \lambda)k. \tag{44}$$

3.3 Market Clearing

We assume a unit measure of identical firms (symmetric equilibrium), and households solve the portfolio choice problem given prices (r, μ_R, σ_R, w), generating policies $c(a,z)$, $a_R(a,z)$, and a stationary density $g(a,z)$.

Asset market clearing We interpret the unique risky asset held by households as the *market portfolio of equity*. Then aggregate household demand for risky dollars equals the market value of equity:

$$\iint a_R(a,z)\, g(a,z)\, da\, dz \;=\; V.$$

Risk-free asset clearing equates aggregate household safe holdings to the supply of risk-free claims. If corporate debt is the only risk-free claim held by households, then

$$\iint \big(a - a_R(a,z)\big)\, g(a,z)\, da\, dz \;=\; \iint a_S(a,z)\, g(a,z)\, da\, dz \;=\; D.$$

Labor market cleaing. Since labor is supplied inelastically, our aggregate labor supply is given by $L^S = \iint z\, g(a,z)\, da\, dz$. We normalize units so that $L^S = 1$, which pins down the scale of z (or equivalently the normalization of the stationary distribution).[3] On the demand side, each intermediate producer chooses labor l_{it} given wages and its capital stock. Aggregate labor demand is the integral over varieties,

$$L^D \;=\; \int_0^1 l_i\, di,$$

which equals l in a symmetric equilibrium, with homogeneous firms. Labor market clearing requires

$$L^D \;=\; L^S \;=\; 1.$$

Goods market clearing. The resource constraint is

$$C_t + I_t = Y_t, \qquad I_t = \dot{k}_t + \delta k_t,$$

so in a steady state ($\dot{K} = 0$) it becomes

$$C + \delta K = Y,$$

matching your discrete-time steady-state condition.

3.4 Market Power and Risk Free Rate

Once we have solved the model for the policy functions on consumption $c(a,z)$ and equity $a_R(a,z)$; and obtained the stationary distribution $g(a,z)$ from the Kolmogorov Forward Equation, we can characterize

[3] With this normalization, the wage share coincides with the labor share.

r_S as the rate at which households can bring consumption into the future that satisfies stationarity of aggregate consumption $\frac{dC}{dt} = 0$. To do so, in the Appendix we derive the expected consumption drift for each individual household $\frac{E[dc|a,z]}{dt}$, and aggregate them to get the stationary aggregate consumption drift:

$$0 = \frac{dC}{dt} = \iint \frac{E[dc \mid a,z]}{dt} g(a,z)\, da\, dz = \iint_U \frac{E[dc \mid a,z]}{dt} g(a,z)\, da\, dz \; + \; \iint_C \frac{E[dc \mid a,z]}{dt} g(a,z)\, da\, dz$$

where in the last equality we divide households into constrained and unconstrained groups. Next, we plug the derived expected consumption drifts and define $\widetilde{E}^S$ as the consumption weighted average over group S. Constrained households' consumption depends exclusively on their efficiency units drift and real wages [4], while unconstrained households choose their optimal policies subject to labor and equity risks. Together:

$$0 = \underbrace{\frac{C^U}{C}\frac{1}{\gamma}(r_S - \rho)}_{\text{Intertemporal substitution}} + \underbrace{\frac{C^U}{C}\frac{\gamma+1}{2}\widetilde{E}^U\left[\sigma_z(z)^2\left(\frac{c_z}{c}\right)^2 + (\sigma_R a_R)^2\left(\frac{c_a}{c}\right)^2\right]}_{\text{precautionary motive}} + \underbrace{\frac{C^C}{C}\widetilde{E}^C[\eta(z)]}_{\text{expected drift of constrained}} \tag{45}$$

Equation (45) decomposes the stationary aggregate consumption drift, which must be zero in steady state, in three terms that reflect how each type of household's optimal consumption expected drift: (i) unconstrained households' intertemporal substitution weighted by its share in aggregate consumption; (ii) the precautionary savings motive generated by idiosyncratic risk arising from either labor efficiency units risk or equity returns risk, with the same weight (iii) the expected drift in consumption of constrained households, weighted by they share of aggregate consumption, this drift is generally positive as constrained households are ''Hand-to-mouth" and their income is expected to revert to the mean. When constrained households (who are tied more closely to current income) tend to sit in states with higher expected mean reversion, their average drift exceeds that of the unconstrained, so the economy requires the unconstrained sector to absorb a larger predictable drift imbalance in order to keep aggregate consumption stationary. That is, in the stationary equilibrium, the risk-free rate r must be such that these three forces must offset each other. Solving for the risk-free rate we get the following equation:

$$r_S \approx \rho - \gamma\,\frac{\gamma+1}{2}\,\widetilde{E}^U\left[\sigma_z(z)^2\left(\frac{c_z}{c}\right)^2 + (\sigma_R a_R)^2\left(\frac{c_a}{c}\right)^2\right] - \gamma\,\frac{C^C}{C^U}\,\widetilde{E}^C[\eta(z)] \tag{46}$$

Equation (46) makes clear that departures of the safe rate from the time-preference benchmark arise only through two channels: (i) household-level consumption risk (the precautionary term) and (ii) the presence of households whose consumption is pinned to current labor income (the drift term). In the knife-edge case with no idiosyncratic risk and no constrained households (so $\sigma_z = \sigma_R = 0$ and $C^C = 0$), the wedges vanish and the stationary rate collapses to $r_S = \rho$.

[4] We use $\eta(z) \equiv \frac{\mu_z(z)}{z}$. If constrained households consume wage income $c = wz$ (hand-to-mouth), then $dc = w\,dz$ so

$$\frac{E[dc \mid a,z]}{dt} = w\mu_z(z) = (wz)\frac{\mu_z(z)}{z} = c(a,z)\,\eta(z) \qquad \text{for } (a,z) \in C.$$

Risk and the precautionary wedge. The first wedge is a consumption-weighted average of the instantaneous variance of consumption growth faced by unconstrained households. It combines labor-income risk and equity-return risk,

$$\sigma_z(z)^2\left(\tfrac{c_z}{c}\right)^2 \quad \text{and} \quad (\sigma_R a_R)^2\left(\tfrac{c_a}{c}\right)^2,$$

where $\frac{c_z}{c}$ and $\frac{c_a}{c}$ measure how strongly optimal consumption responds to shocks in productivity and wealth at a given state (a, z). The factor $(\gamma + 1)$ captures prudence: higher curvature raises the strength of precautionary saving for a given amount of diffusion risk. Since this wedge enters with a negative sign, greater exposure to either source of risk lowers the equilibrium risk-free rate required to keep aggregate consumption stationary.

Constraints and the drift wedge. The second wedge reflects predictable movements in constrained households' consumption induced by the drift in their labor income process. Under the hand-to-mouth boundary rule $c = wz$, constrained consumption inherits the conditional mean reversion of z through $\eta(z) = \mu_z(z)/z$. When constrained households are disproportionately located in states with positive expected income growth (e.g. low z states under mean reversion), the consumption-weighted average $\widetilde{E}^C[\eta(z)]$ is positive. The scaling factor $\frac{C^C}{C^U}$ then maps this predictable drift in the constrained block into the amount of offsetting adjustment required from the unconstrained block; in equilibrium, this adjustment operates through a lower r_S.

Link to market power. Market power (markups) affects r_S through the same two wedges. By shifting the split of aggregate income between wages and profits, changing the equilibrium portfolio position $a_R(a, z)$, and reshaping the stationary distribution $g(a, z)$, markups modify (i) the relative contribution of labor versus equity risk inside the precautionary term and (ii) the consumption mass of constrained households and the income-drift states they occupy. This decomposition is therefore useful for interpreting comparative statics: any change in the safe rate across markup regimes can be traced to changes in risk exposure/sensitivities $\left(\frac{c_z}{c}, \frac{c_a}{c}, a_R\right)$ and/or to changes in the constrained consumption share and drift statistics $\left(\frac{C^C}{C^U}, \widetilde{E}^C[\eta]\right)$.

4 Limit Cases and Comparative Statics

4.1 Limit Cases

Full insurance. When agents can trade a full set of Arrow-Debreu securities, or idiosyncratic risk is shut down, consumption becomes a deterministic object, and it holds that:

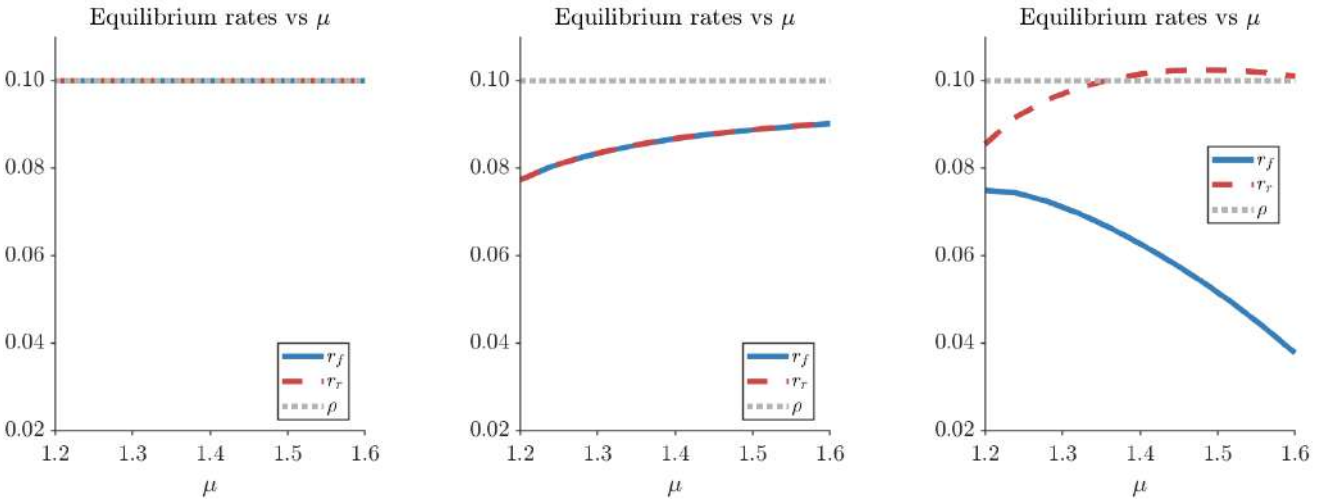
$$\frac{\mathrm{d}c_t}{c_t} = \frac{1}{\gamma}(r_S - \rho) \tag{47}$$

That is, the risk-free rate is completely determined by **intertemporal substitution** motives and constant in the steady state $\rho = r_S$.

One asset. Now we focus on the standard incomplete markets model with just one risk-free asset. Following Kaplan et al. (2023), r_S can be written as:

$$r_S \approx \rho - \gamma \frac{\gamma+1}{2} \widetilde{E}^U \left[\sigma_z(z)^2 \left(\frac{c_z}{c} \right)^2 \right] - \gamma \frac{C^C}{C^U} \widetilde{E}^C[\eta(z)] \tag{48}$$

which yields the standard incomplete-markets intuition: precautionary savings arising from idiosyncratic labor-income risk shift the risk-free rate below the time discount rate ρ.



(a) Deterministic Ricardian economy

(b) One risk-free asset economy ($\sigma_R = 0, r = r_S = r_R$)

(c) Two-asset economy with safe debt and risky equity

Figure 1: Comparative statics of equilibrium interest rates with respect to markups μ. Panel (a) shows the deterministic Ricardian benchmark where $\sigma_z = \sigma_R = 0$. In Panel (b) $\sigma_R = 0$ so equity is risk-free and $r = r_S = r_R$. Panel (c) features both safe debt and risky equity ($\sigma_R > 0$), generating an endogenous spread $r_R - r_S$.

Limit cases and comparative statics over markups. Figure 1 illustrates how equilibrium interest rates respond to changes in markups μ across three limit cases of the model.

Panel 1a sets $\sigma_z = \sigma_R = 0$, shutting down all idiosyncratic risk. In this limit case, households perfectly smooth consumption and the precautionary motive vanishes. The stationary equilibrium condition reduces to the standard Euler equation, implying $r = r_S = r_R = \rho$. Because there is no risk and no borrowing constraint distortion affecting intertemporal trade, markups only affect levels of wages and profits, but not the intertemporal price of consumption. As a result, equilibrium interest rates are invariant to μ.

Panel 1b sets $\sigma_R = 0$, so equity becomes risk-free. In this case, $r = r_S = r_R$ and there is no equity premium. Although labor-income risk may still be present, the asset structure collapses to a single risk-free store of value.

An increase in markups raises monopoly profits and therefore increases the market value of claims to those profits. When these claims are risk-free, this expansion effectively increases the supply of safe stores of value in the economy. To clear the asset market, the equilibrium interest rate must rise. Consequently, both r_S and r_R increase with μ in this environment.

This case corresponds to a fully capitalized profit channel operating through riskless wealth.

Panel 1c restores risky equity ($\sigma_R > 0$) and introduces a distinction between safe debt and risky equity. In this environment, r_S and r_R are determined separately in equilibrium.

Two forces operate when markups increase.

First, markups reduce the marginal revenue product of capital through the factor $1/\mu$, which tends to reduce equilibrium capital. Since safe debt is constrained by collateral,$D \leq \lambda K$, a lower capital stock tightens the supply of safe debt. This contraction in safe asset supply puts downward pressure on the risk-free rate r_S.

Second, equity remains risky and must compensate households for bearing return risk. As the safe rate falls and portfolios rebalance, the required return on equity need not fall one-for-one. Instead, the equilibrium spread $r_R - r_S$ widens. In the figure, r_R increases modestly while r_S declines, generating a rising equity premium.

This limit case illustrates the central mechanism of the model: markups alter the composition of asset supply. When safe debt is collateral-constrained and equity is risky, higher markups can simultaneously depress the risk-free rate and sustain or increase the required return on equity.

4.2 Model's Comparative Statics

Comparative statics on markups An increase in markups μ generates a sharp decline in the safe rate r_S and a modest increase in the return on equity r_R, implying a widening equity premium $r_R - r_S$. This separation between safe and risky returns reflects the interaction between redistribution, capital accumulation, and asset supply.

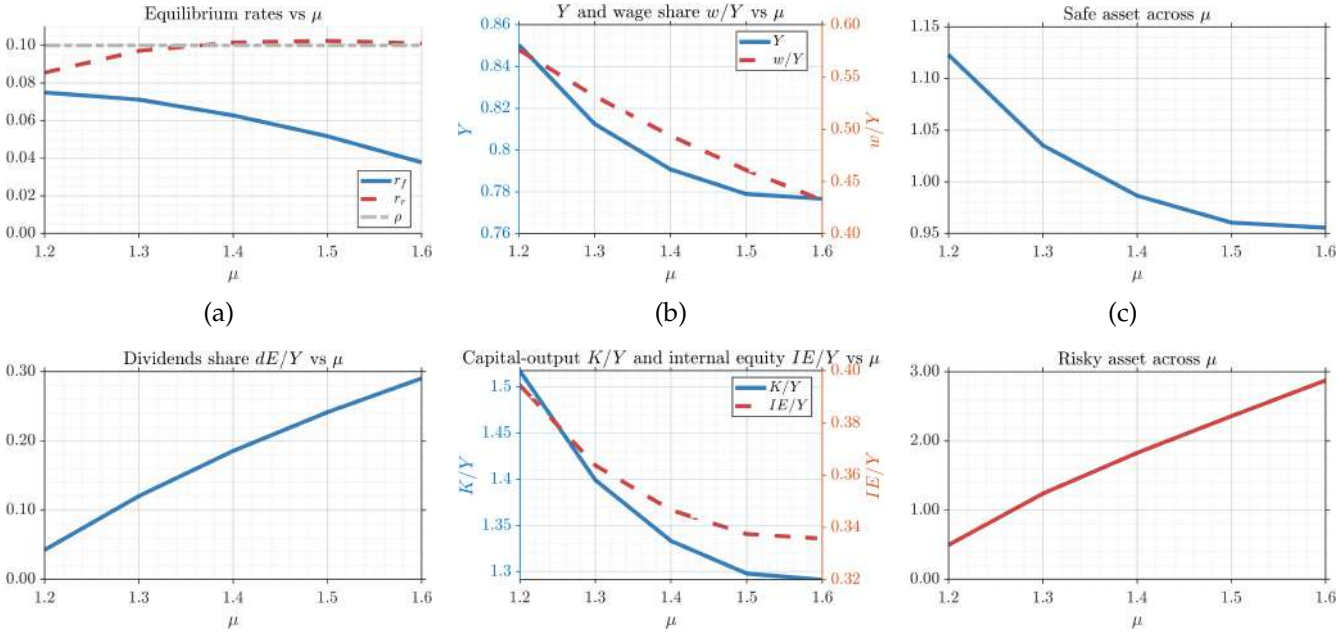


Figure 2: Comparative statics markups μ

Higher markups distort production by reducing the marginal revenue product of capital through the factor $1/\mu$, which lowers equilibrium capital and output. Since labor supply is normalized to $L = 1$, the

wage share w/Y equals the labor share. Its decline represents a redistribution of income from labor to profits. Lower labor income increases households' effective exposure to idiosyncratic risk and strengthens precautionary saving motives, which depresses the equilibrium safe rate through the aggregate Euler condition.

Because safe debt is constrained by $D \leq \lambda K$, the decline in capital mechanically contracts the supply of safe assets. With precautionary demand for safety remaining strong, the reduction in collateralizable capital intensifies safe asset scarcity and further lowers r_S.

At the same time, the shift toward profits raises dividend payments relative to output and increases the scale of equity claims. The market value of risky equity expands as profit income becomes more concentrated. Since equity remains risky with volatility $\sigma_R > 0$, greater exposure to profit risk does not reduce the required return. Instead, the fall in the safe rate combined with higher dividend exposure sustains or increases r_R.

Capital deepening weakens and internal equity declines alongside the contraction in safe debt, reinforcing the link between market power, lower collateralizable capital, and reduced safe asset supply. The expansion in risky asset valuation occurs simultaneously with the contraction in safe assets, altering the composition of aggregate wealth toward riskier claims.

Rising markups therefore reallocate income from labor to profits, compress capital accumulation, reduce the supply of safe assets through the collateral channel, and expand the scale of risky equity. The equilibrium consequence is a lower risk-free rate and a wider equity premium.

Comparative statics on collateral constraint Figure 3 illustrates how relaxing the collateral constraint, through an increase in λ in $D \leq \lambda K$, reshapes equilibrium allocations and asset prices. Because λ governs the supply capacity of safe debt.

Beginning with interest rates in Panel (a), we observe that a higher λ raises the safe rate r_S while slightly lowering the return on equity r_R, thereby compressing the spread $r_R - r_S$. Intuitively, a looser collateral constraint expands the supply of safe assets. With precautionary demand for safe claims largely unchanged, equilibrium requires a higher safe rate to induce households to absorb the larger stock of safe debt. At the same time, the reduced scarcity of safe assets diminishes the compensation required for holding risky equity, leading to a modest decline in r_R.

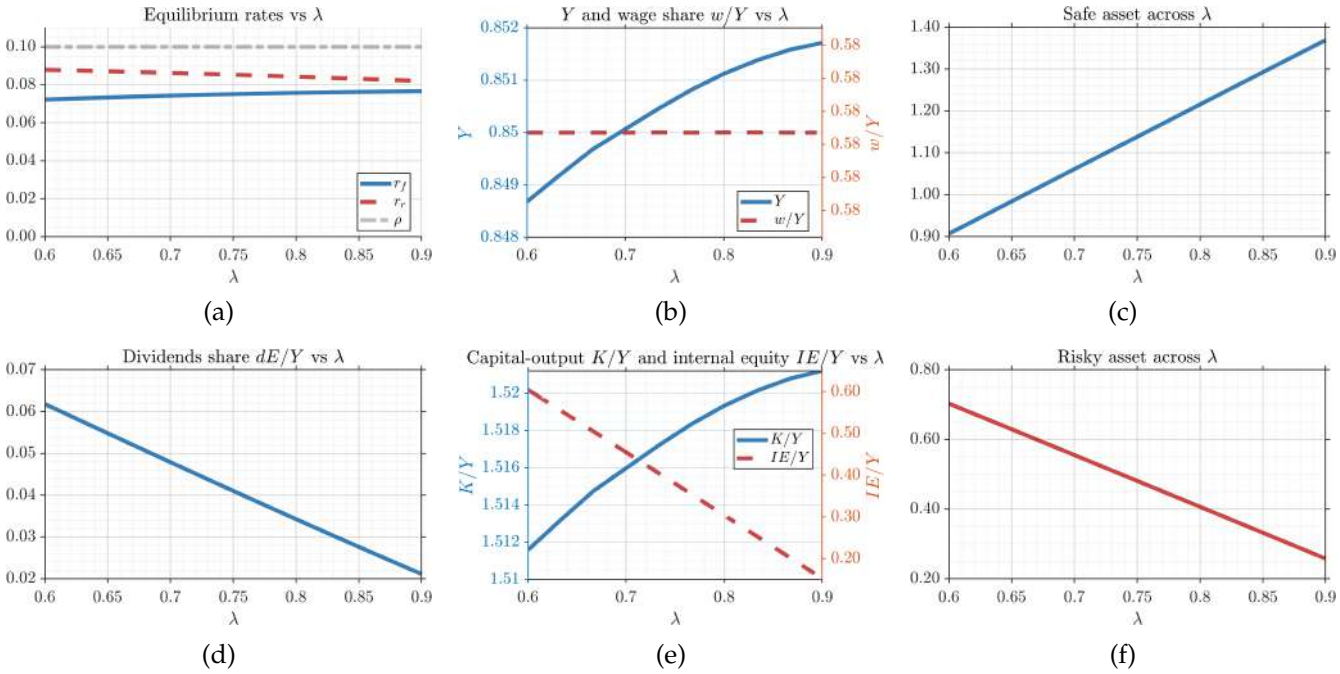


Figure 3: Comparative statics collateral constraint λ

In Panel (b), output Y increases monotonically with λ. Improved access to debt financing alleviates distortions in capital accumulation, raising the capital stock and therefore production. In contrast to the markup experiment, the wage share w/Y remains essentially unchanged. Since markups are fixed in this exercise, there is no redistribution between labor and profits; the effects operate purely through financing conditions rather than income shares.

Panel (c) makes explicit the central mechanism: safe debt expands strongly as λ rises. This follows directly from the collateral constraint. A larger λ allows firms to issue more safe claims against a given capital stock. The expansion of safe asset supply is the primary force behind the increase in r_S documented earlier.

The implications for income distribution within firms are reflected in Panel (d), where the dividend share d_E/Y declines as λ increases. As firms rely more heavily on debt financing, a greater portion of operating income is directed toward interest payments, reducing the residual flow accruing to equity holders.

These financing shifts are further clarified in Panel (e). The capital-output ratio K/Y rises with λ, reflecting more efficient capital accumulation under looser borrowing constraints. Simultaneously, internal equity IE/Y declines as firms substitute toward debt. The overall capital structure tilts away from equity and toward safe debt.

Finally, Panel (f) shows that the aggregate risky asset position declines as λ increases. With safe debt more abundant, households reallocate portfolios toward safe assets and away from risky equity. The reduction in aggregate exposure to profit risk contributes to the observed decline in the required return on equity.

Taken together, the panels reveal a coherent financing mechanism. Relaxing collateral constraints expands the supply of safe assets, increases capital and output, shifts firm financing toward debt, and reduces aggregate exposure to risk. As a consequence, the safe rate rises and the equity premium com-

presses. This exercise provides the natural counterpart to the markup comparative statics: whereas rising market power contracts safe asset supply and depresses the risk-free rate, loosening collateral expands safe asset supply and raises it.

Comparative statics on fixed costs An increase in fixed costs ϕ, paid in labor units, affects equilibrium allocations primarily through redistribution and profitability effects. Because fixed costs are financed by wage payments $w\phi$, a higher ϕ directly increases overhead labor demand and shifts income toward workers.

As shown in panel (a), the safe rate r_S increases modestly while the return on equity r_R declines, leading to a compression of the equity premium. The decline in profit income reduces aggregate exposure to risky dividend flows, weakening precautionary demand for safe assets and placing upward pressure on the safe rate.

Output declines with higher ϕ, as seen in panel (b), reflecting lower net profitability and weaker investment incentives. At the same time, the wage share w/Y rises. Since overhead labor is compensated at the wage rate, higher fixed costs mechanically raise total wage payments while output contracts, increasing the labor share.

The response of safe debt is non-monotonic, as illustrated in panel (c). For moderate increases in ϕ, lower profitability reduces capital accumulation and contracts collateralizable capital under the constraint $D \leq \lambda K$, limiting safe asset supply. At higher values of ϕ, this contraction attenuates, generating a U-shaped pattern.

Dividend payments relative to output decline monotonically with ϕ, as reported in panel (d), consistent with lower net profits. The aggregate risky asset correspondingly declines.

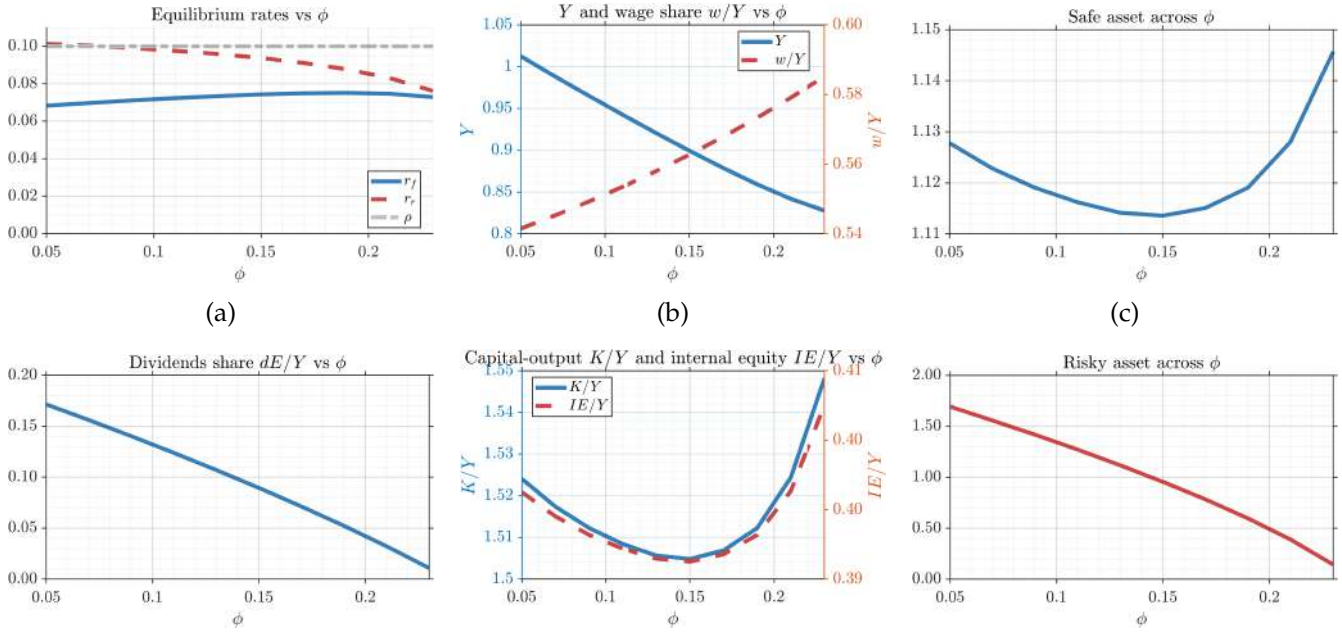


Figure 4: Comparative statics fixed cost ϕ

A similar non-monotonic pattern appears in the capital-output ratio K/Y and internal equity IE/Y,

shown in panel (e). Initial increases in ϕ reduce capital through weaker profitability; at higher levels, the decline moderates.

Panel (f) confirms that the aggregate risky asset falls steadily as fixed costs rise. The redistribution from profits to labor reduces the scale of risky equity claims in the economy.

Higher fixed costs therefore operate primarily through redistribution from profits to labor. The compression of the equity premium and the increase in the safe rate reflect reduced aggregate exposure to profit risk rather than changes in marginal production incentives.

Comparative statics on volatility of equity return Higher equity return volatility σ_R lowers the safe rate r_S and raises the return on equity r_R, thereby widening the equity premium $r_R - r_S$, as illustrated in panel (a). Greater financial risk increases the compensation required for holding equity and strengthens precautionary demand for safe claims, which places downward pressure on the equilibrium safe rate.

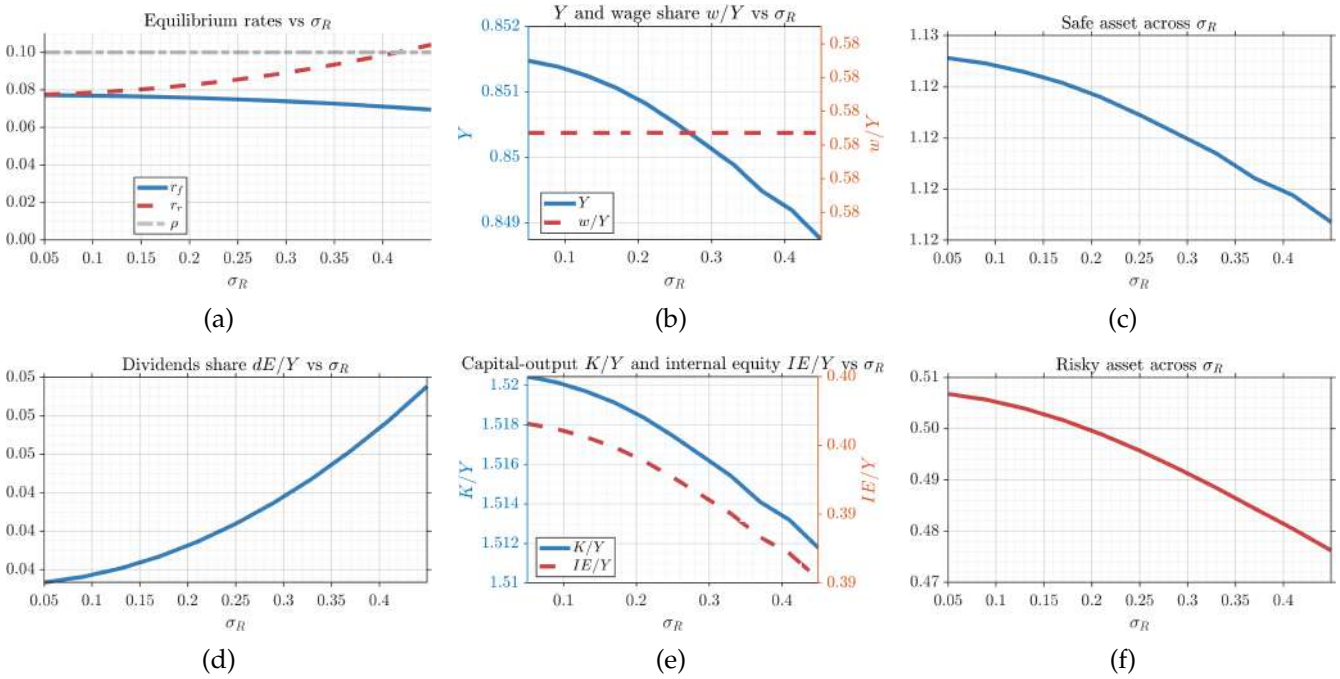


Figure 5: Comparative statistics volatility returns on equity σ_R

As seen in panel (b), output declines modestly with σ_R, while the wage share w/Y remains essentially unchanged. Since production parameters and markups are fixed, income distribution between labor and profits does not adjust. The decline in output reflects weaker capital accumulation induced by a higher risk-adjusted user cost of capital.

Safe debt contracts alongside capital, a pattern visible in panel (c). Under the collateral constraint $D \leq \lambda K$, lower investment reduces collateralizable capacity and shrinks the supply of safe assets, reinforcing the decline in r_S.

Dividend payments relative to output rise slightly with higher volatility, as shown in panel (d), consistent with the higher required return on equity. The capital-output ratio K/Y and internal equity IE/Y both decline, as panel (e) indicates, reflecting reduced investment and a smaller steady-state capital stock under elevated risk.

Panel (f) shows that the aggregate risky asset declines as σ_R increases. Households reduce exposure to risky equity in response to higher volatility, consistent with the portfolio condition $a_R^* \propto (r_R - r_S)/(\gamma\sigma_R^2)$. The contraction in risky holdings is therefore a direct portfolio reallocation effect.

Higher equity volatility thus operates through precautionary and portfolio channels: it increases the equity premium, depresses the safe rate, and reduces capital accumulation through the risk-adjusted user cost of capital.

Comparative statics on standard deviation of log income As shown in panel (a), higher labor income risk, measured by the stationary dispersion ν, lowers both the safe rate r_S and the return on equity r_R, with a larger decline in r_S and therefore a widening spread $r_R - r_S$. Stronger uninsurable background risk increases precautionary saving demand and depresses the equilibrium safe rate.

Panel (b) indicates that output Y rises with ν while the wage share w/Y remains essentially unchanged. With labor supply normalized to $L = 1$, the stability of w/Y implies that income distribution between labor and profits does not adjust materially. The expansion in output is consistent with higher capital accumulation induced by stronger saving motives.

Safe debt increases sharply as ν rises, as seen in panel (c). Given the collateral constraint $D \leq \lambda K$, the expansion in capital raises collateralizable capacity and supports a larger stock of safe claims in equilibrium.

Dividend payouts relative to output decline with higher ν, as shown in panel (d), indicating that profit flows represent a smaller share of aggregate production even as the economy expands. The capital-output ratio K/Y and internal equity IE/Y both rise, as seen in panel (e), reflecting capital deepening and an expansion in firms' balance sheets.

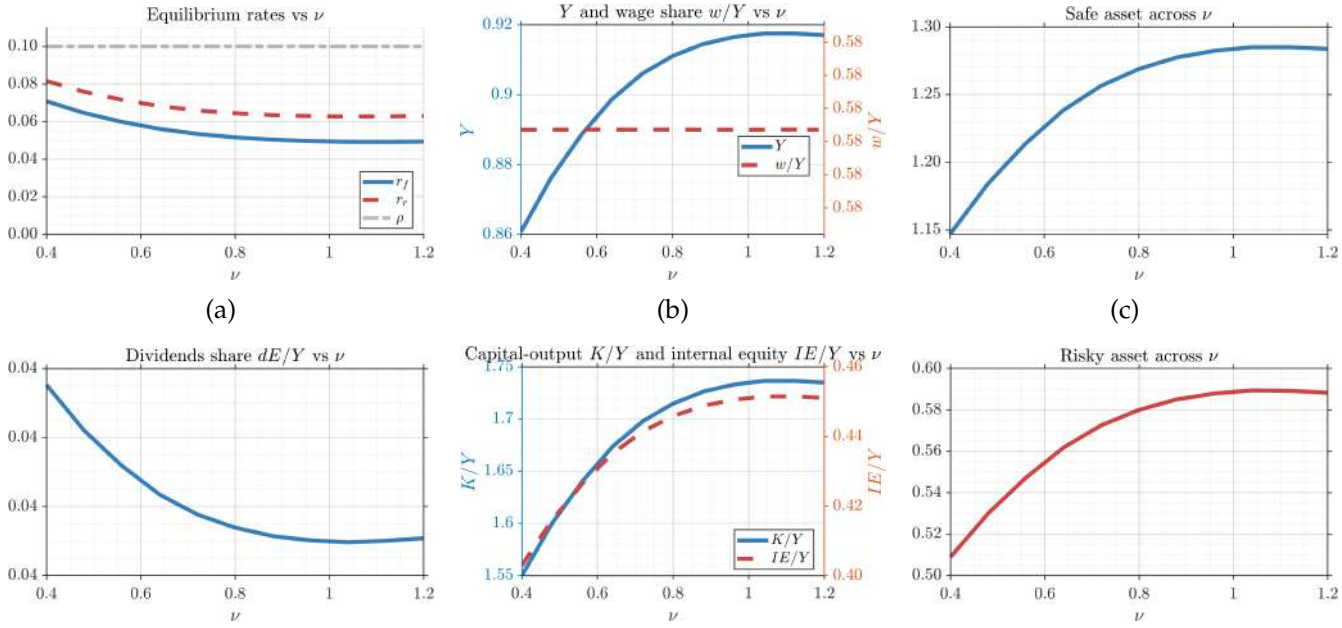


Figure 6: Comparative statics std log income ν

An increase in ν is also associated with a larger aggregate risky asset, shown in panel (f). Although

dividend shares fall, the decline in r_R raises equity valuations, and the larger scale of capital accumulation increases the total quantity of risky claims held in equilibrium.

5 Quantification

5.1 Data

We summarize the data sources and definitions in Table 1 together with the externally calibrated parameters. Our goal is to match the model-generated moments with data. For the firm block, our primary source is Compustat, from which we retrieve annual data on total costs, total assets, and debt over the period 1984–2019. We decompose total costs into administrative expenses (SGA), labor costs, cost of goods sold, and capital costs. To produce markup estimates, we follow De Loecker et al. (2020). More specifically, each firm aggregate moment is computed as follows:

1. **Markups.** We aggregate markup estimates as: $\mu_t = \sum_i \mu_{it} \times cost_{it} / \sum_i cost_{it}$.
2. **Fixed costs.** For our measure of fixed to total costs we use the data to compute the ratio of total: $\Phi_t = \frac{\sum_i SGA_{it}}{\sum_i Tot.Cost_{it}}$.
3. **Debt to total assets.** In the model, we interpret Total Assets as Debt plus Internal Equity $K = D + E$. Since $D = \lambda K$, the collateral constraint can be matched by computing $\frac{\sum_i Debt_{it}}{\sum_i Tot.Assets_{it}}$.

For the interest rate data, we retrieve the following macroeconomic aggregates from FRED (Federal Reserve Bank of St. Louis, 2026): the five-year Treasury constant maturity rate, the return on equity of non-financial corporations, and the CPI inflation rate. We use the latter to convert nominal rates into real terms using: $r_t = (1 + i_t)/(1 + \pi_t^{cpi}) - 1$, where π_t^{cpi} denotes the yearly inflation rate. To remove excessive noise from the data and focus on structural trends, we take the three-year moving average (MA) of each series.

5.2 Moments and Estimates

Model fit Figure 7 compares the time series implied by the estimated model with their empirical counterparts for the five targeted moments. The model is estimated by matching steady states period by period, allowing the structural parameters governing markups, fixed costs, collateral constraints, equity risk, and labor income risk to vary so as to reproduce the joint evolution of the data moments.

Panel (a) reports cost-weighted markups. The model tracks both the upward trend and medium-run fluctuations in markups, capturing the persistent increase over the sample as well as the temporary plateau in the early 2000s. The close co-movement between the data and the model indicates that the estimated markup parameter successfully absorbs the secular rise in market power.

The share of fixed costs in total costs, shown in panel (b), is also well reproduced. The model captures the rise during the 1990s, the subsequent stabilization, and the renewed increase toward the end of the sample. This alignment suggests that the estimated fixed cost parameter effectively accounts for the changing composition of firm costs.

Panel (c) compares the debt-to-asset ratio. The model replicates the gradual increase in leverage and its subsequent stabilization, although short-run fluctuations are somewhat smoother in the model. The

Table 1: Calibration and estimation targets

Parameter	Name	Value / estimates	Target / source
Exogenously set parameters			
ρ	Discount rate	0.1	Match initial high r_S
γ	CRRA parameter	2.0	standard
θ	OU mean reversion	0.1054	standard ($\rho_z = 0.9$)
α	Cobb Douglas capital share	0.36	Match average capital to GDP
δ	Depreciation	0.12	(De Loecker et al., 2021)
Calibrated / estimated parameters (time-varying)			
μ_t	Markups cost weighted	$\widehat{\mu}_t$	Target: (De Loecker et al., 2020)
Φ_t	Fixed costs	$\widehat{\Phi}_t$	Target: SG&A over total cost
λ_t	Firms collateral constraint	$\widehat{\lambda}_t$	Target: Debt to total assets ratio
$\sigma_{R,t}$	Risk (risky return volatility)	$\widehat{\sigma}_{R,t}$	Target: ROE
ν_t	log std. deviation wages	$\widehat{\nu}_t$	Target: Risk-Free rate

Notes: The estimated parameter paths $\{\widehat{\mu}_t, \widehat{\phi}_t, \widehat{\lambda}_t, \widehat{\sigma}_{R,t}, \widehat{\nu}_t\}$ are obtained by matching model-implied moments to data moments year-by-year using a Sobol grid of precomputed equilibria (nearest-neighbor criterion).

estimated collateral parameter therefore reproduces the medium-run leverage dynamics observed in the data.

Turning to asset prices, panel (d) shows the risk-free rate. The model closely tracks the long-run decline, including the pronounced peak in the early part of the sample and the sustained downward movement thereafter. The joint calibration of risk and collateral parameters allows the model to reproduce both the level and the trend in safe rates.

Panel (e) presents the return on equity. The model captures the broad cyclical movements and the average level of equity returns, while smoothing some high-frequency volatility. Importantly, it reproduces the persistent wedge between the return on equity and the risk-free rate.

Overall, the model simultaneously matches the evolution of market power, cost structure, leverage, and asset returns within a unified framework. The joint fit across real and financial moments indicates that the estimated structural parameters generate internally consistent movements in firm balance sheets and equilibrium interest rates.

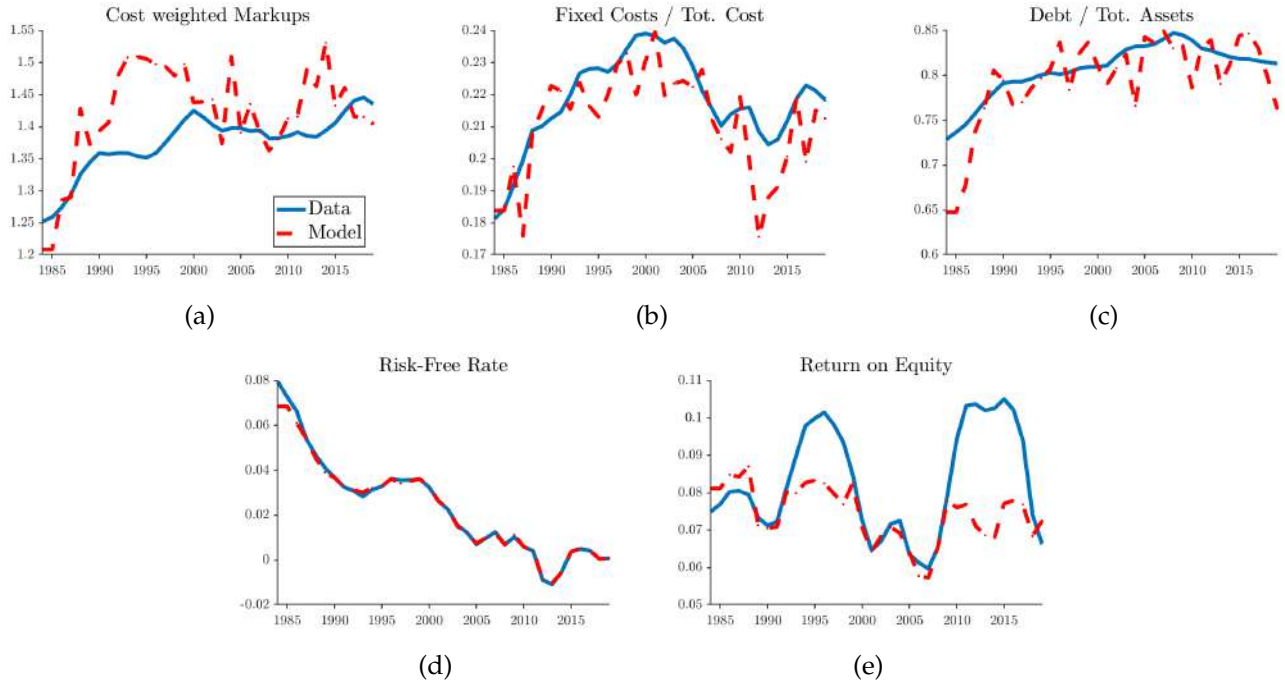


Figure 7: Model fit.

Estimated structural parameters Figure 8 reports the time series of the estimated structural parameters obtained from the period-by-period steady-state matching. The estimates reveal distinct medium-run movements across parameters, with differing degrees of volatility.

The markup parameter μ, shown in panel (a), exhibits a clear upward trend over the sample. It begins at a level slightly above 1.2 in the early 1980s and rises toward values around 1.45–1.55 in later decades. Although there are cyclical fluctuations, the dominant pattern is a persistent increase in market power. The magnitude of this change implies a substantial rise in price-cost margins over time.

The fixed cost parameter ϕ, reported in panel (b), increases from roughly 0.17–0.20 in the early part of the sample to levels near 0.28–0.30 at its peak. The series displays moderate variation around this higher plateau, with some temporary declines during downturns. Overall, the estimates indicate a gradual increase in the importance of overhead labor costs relative to production scale.

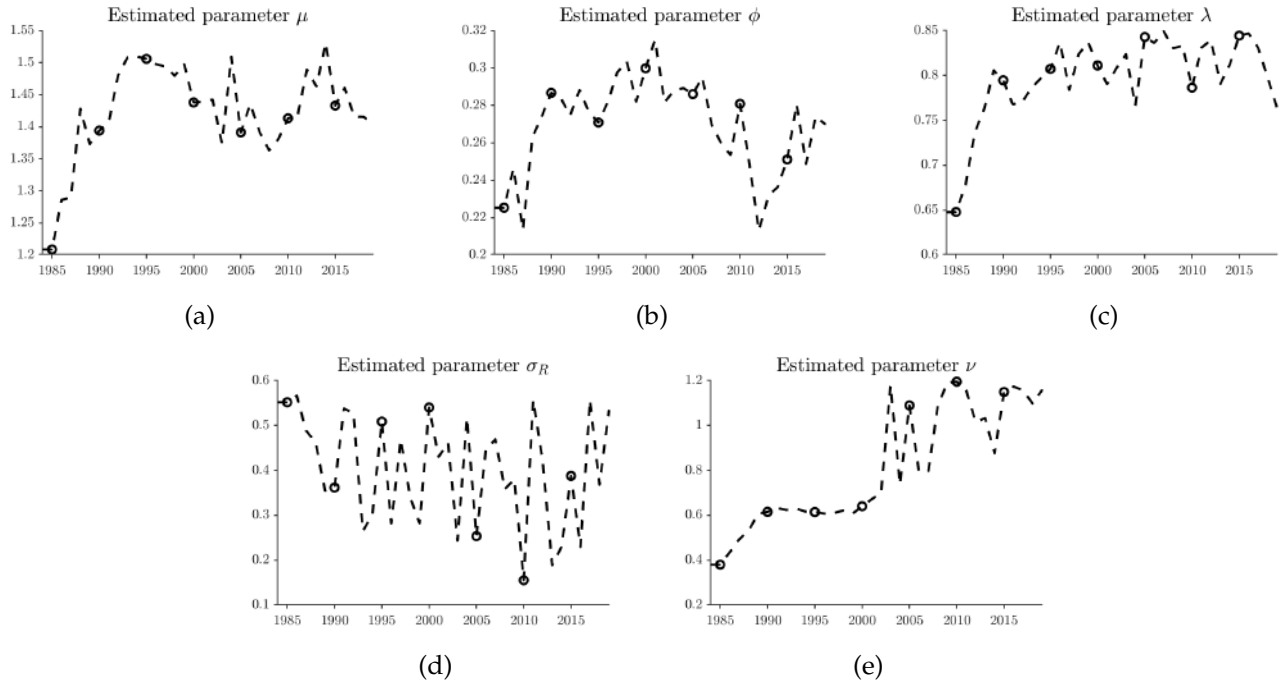


Figure 8: Estimated parameters.

The collateral parameter λ, shown in panel (c), rises from values close to 0.65 in the early 1980s to levels around 0.80–0.85 in subsequent decades. While the series is somewhat noisy, the long run movement points to a loosening of borrowing constraints, consistent with higher leverage capacity over time.

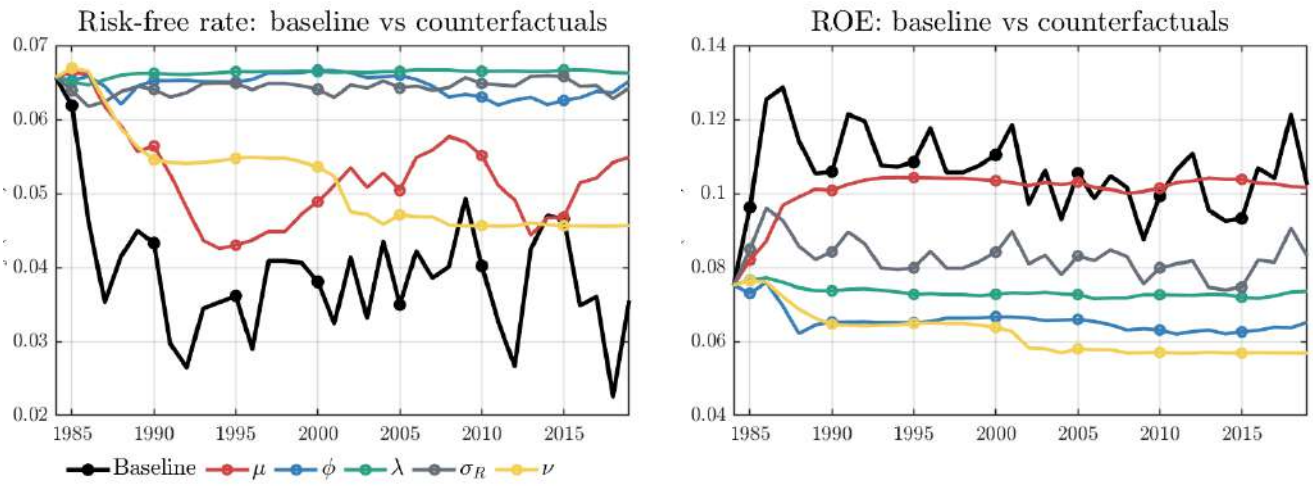
The volatility of equity returns σ_R, displayed in panel (d), is comparatively volatile. The estimates fluctuate within a broad range, roughly between 0.2 and 0.55, without a strong deterministic trend. This variability reflects the sensitivity of the model's asset pricing block to changes in observed return dispersion.

Finally, the stationary dispersion of log labor income ν, shown in panel (e), increases markedly over the sample. It declines initially from a value near 1.0 to below 0.5 in the early years, before rising steadily to levels above 1.0 in later decades. Despite short-run fluctuations, the long run pattern indicates a substantial increase in labor income risk.

Taken together, the estimates suggest a structural environment characterized by rising market power, higher fixed operating costs, and greater labor income dispersion, accompanied by a gradual relaxation of collateral constraints. The risk parameter governing equity returns remains comparatively volatile, reflecting time variation in financial conditions.

5.3 Counterfactual of the Risk-free Rate

To assess the quantitative contribution of each structural force, we conduct a sequence of counterfactual experiments. Starting from a common initial steady state, we vary one estimated parameter at a time along its estimated path while holding all other parameters fixed at their baseline values. This procedure isolates the partial-equilibrium effect of each force on the risk-free rate and the return on equity within the general equilibrium of the model. By comparing these counterfactual paths, we quantify the



(a) (b)

Figure 9: Decomposing the change in rates r_S and r_R.

relative importance of changes in markups, labor income risk, equity return volatility, fixed costs, and the collateral constraint for the observed movements in asset returns.

Counterfactual decomposition of the risk-free rate Figure 9(a) decomposes the evolution of the risk-free rate by allowing one parameter at a time to follow its estimated time series while holding all others fixed at their initial level. Two forces account for almost the entire decline in r_S: rising markups μ and increasing labor income dispersion ν. When only μ varies, the safe rate declines persistently. Higher markups reduce the labor share and compress capital accumulation through the marginal revenue distortion, thereby contracting collateralizable capital and shrinking the supply of safe debt. In equilibrium, the reduction in safe asset supply lowers r_S in order to clear the asset market.

Allowing only labor income risk ν to vary generates a similarly strong decline in the safe rate. Higher income risk strengthens precautionary saving motives among unconstrained households, increasing demand for safe claims and bidding down r_S through the aggregate Euler condition. In contrast, varying fixed costs ϕ, the collateral parameter λ, or equity volatility σ_R in isolation produces very little movement in the safe rate. This reflects both weak structural sensitivity of r_S to these parameters and the fact that their estimated paths exhibit comparatively limited long-run change. The counterfactual contribution therefore combines model elasticities with the magnitude of the estimated parameter shifts, and only μ and ν change sufficiently to generate sizable effects.

We find that, between 1984 and 2019, markups directly contribute to 20% of the total decline in the risk-free rate, and labor income risk contributes to 35% of the total decline. However, these effects are only direct, and the contribution of their interaction with other parameters exceed the direct effect.

Counterfactual decomposition of the return on equity Figure 9(b) reports the analogous decomposition for the return on equity. The counterfactual paths reveal that different parameters operate in opposite directions. Allowing markups μ to rise in isolation increases r_R. Higher markups raise the

profit share and increase the exposure of aggregate income to risky dividend flows, requiring a higher expected return to compensate households for bearing equity risk. Similarly, higher equity volatility σ_R raises r_R directly through the portfolio condition, increasing the risk premium embedded in the user cost of capital. We find that the increase in markups can rationalize the observed increase in the return to equity between 1984 and 2019.

By contrast, increases in labor income dispersion ν generate a substantial decline in r_R, accounting for most of the downward movement in the baseline path. Greater income risk increases aggregate precautionary saving lowering required returns even on risky claims. Rising fixed costs ϕ contribute a smaller negative effect through reduced profitability and lower dividend flows, while changes in the collateral parameter λ have minimal impact on r_R. As in the case of the safe rate, the quantitative importance of each counterfactual reflects not only the structural mapping from parameters to returns but also the extent to which each estimated parameter moves in the data. The large estimated increases in μ and ν therefore dominate the decomposition, whereas the relatively flat paths of λ and σ_R imply limited aggregate effects.

5.4 Mechanisms for the decline in the risk free rate

The counterfactuals highlight that the key drivers of the decline in the risk-free rate r_S are the rising-markup μ and an increase in labor income risk ν. To understand why these two parameters play a central role, we revisit the key equation of our paper in equation (46) and analyze the mechanisms and tradeoffs behind the decline in the risk free rate.

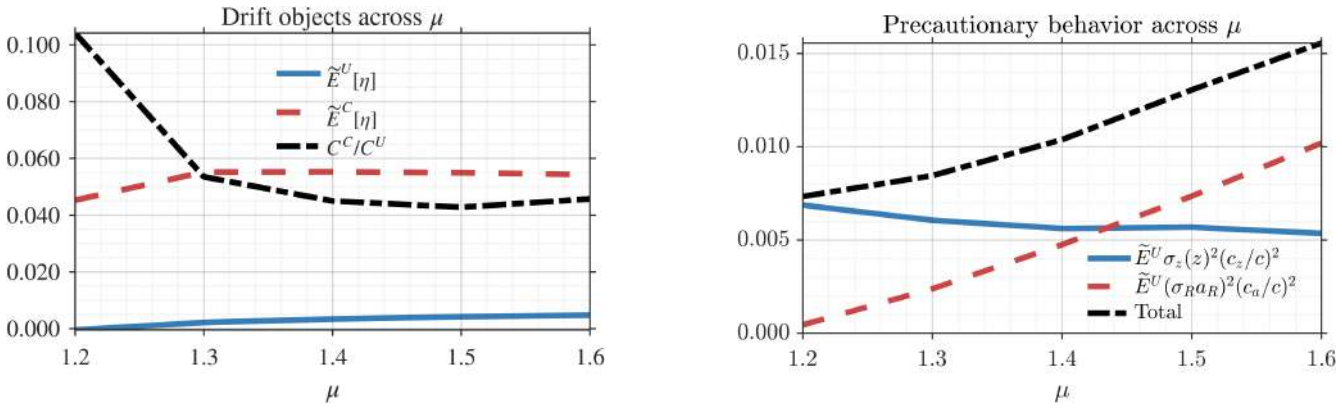
In steady-state the risk free rate satisfies:

$$r_S \approx \underbrace{\rho}_{\text{Time Preference Benchmark}} \underbrace{-\gamma \frac{\gamma+1}{2} \widetilde{E}^U \left[\sigma_z(z)^2 \left(\frac{c_z}{c}\right)^2 + (\sigma_R a_R)^2 \left(\frac{c_a}{c}\right)^2\right]}_{\text{Precautionary wedge among unconstrained agents}} - \underbrace{\gamma \frac{C^C}{C^U} \widetilde{E}^C[\eta(z)]}_{\text{Constrained income-drift wedge}} \tag{49}$$

The equation decomposes the risk free rate into three components: 1. the time preference benchmark ρ; 2. a precautionary wedge arising from consumption risk among unconstrained households; and 3. a drift wedge reflecting income dynamics of constrained households. The operators $\widetilde{E}^U$ and $\widetilde{E}^C$ denote the consumption weighted expectations over the unconstrained and constrained agents, respectively. Whereas, the ratio $\frac{C^C}{C^U}$ measures the relative consumption mass of unconstrained agents.

Finally, the precautionary wedge contains two distinct diffusion terms: $\sigma_z(z)^2 \left(\frac{c_z}{c}\right)^2$ which captures the exposure to idiosyncratic labor income risk and $(\sigma_R a_R)^2 \left(\frac{c_a}{c}\right)^2$ which captures the exposure to risky equity returns. The terms c_Z/c and c_a/c measure the sensitivity of consumption to income and wealth respectively and the term a_R is the equilibrium portfolio position. The term $\gamma+1$ reflects prudence; higher curvature amplifies the impact of diffusion risk on precautionary saving.

Mechanism for markups First, an increase in markups increases the profit share and reduces the labor share. This redistribution affects both the diffusion components of the precautionary wedge. On the labor side, the decline in wage income reduces the relative importance of labor risk in household consumption. The term $\sigma_z(z)^2 \left(\frac{c_z}{c}\right)^2$ declines modestly. On the equity side, higher markups increase the



(a) Drift components.

(b) Precautionary term components

Figure 10: Decomposing the change in r_S due to markups.

share of income derived from profits. Because equity returns are risky, ($\sigma_R > 0$), households become more exposed to aggregate profit fluctuations. In equilibrium, the risky position a_R rises and consumption becomes more sensitive to wealth fluctuations, therefore increasing $(\sigma_R a_R)^2 \left(\frac{c_a}{c}\right)^2$. This equity diffusion component rises strongly and dominates the modest decline in the labor component. As a result, the total precautionary wedge increases.

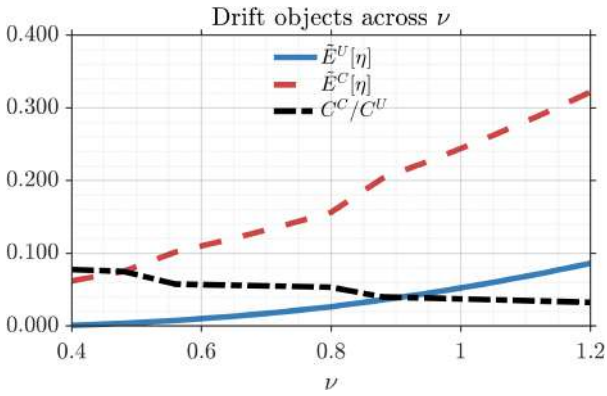
Since the precautionary wedge enters negatively in the expression for r_S, the equilibrium safe rate falls. At the same time, the constrained drift wedge moves in the opposite direction early on. As μ rises, e mass of constrained households declines, reducing $\frac{C^C}{C^U}$. Because this ratio scales the drift term, the constrained wedge shrinks in magnitude and partially offsets the precautionary force. This explains the relatively modest decline in the safe rate at lower levels of μ.

Once $\frac{C^C}{C^U}$ stabilizes, the continued rise in the precautionary component of equity dominates, generating a sharp decline in r_S. This nonlinear response in the safe rate thus reflects the interaction between the declining mas of constrained agents and rising equity exposure.

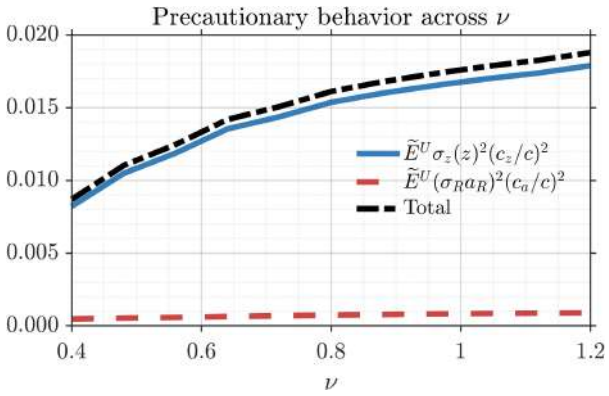
This mechanism differs fundamentally from environments without priced equity risk. In models where households face only labor income risk, the second diffusion term is absent. In that case, higher markups reduce labor exposure and lower the precautionary wedge, implying an increase in the risk-free rate (see for example Auclert et al. (2025)). The presence of the equity risk and hence the diffusion term in equation (49) reverses this logic. Markups increase exposure to risky profit income, raising precautionary demand for safe assets and lowering the equilibrium safe rate.

Mechanism for labor income risk We now consider increases in labor income dispersion ν. An increase in ν raises the variance of the income process $\sigma_z(z)^2$ directly. This mechanically increases the labor diffusion component $\sigma_z(z)^2 \left(\frac{c_z}{c}\right)^2$. Because households cannot fully insure idiosyncratic income shocks, higher background risk increases precautionary saving among unconstrained agents. The precautionary wedge therefore rises unambiguously.

Unlike the markup case, the equity diffusion component $(\sigma_R a_R)^2 \left(\frac{c_a}{c}\right)^2$ remains largely unchanged when ν increases. The dominant adjustment occurs through the labor risk channel. The constrained drift wedge also changes as the stationary distribution shifts, but its quantitative contribution remains small relative to the precautionary component. Thus, the increase in ν lowers the risk-free rate primarily



(a) Drift components.



(b) Precautionary term components

Figure 11: Decomposing the change in r_S due to labor risk.

through stronger precautionary motives arising from labor income risk.

As we have seen in the preceding discussion of the mechanisms, the equilibrium condition makes clear that the risk-free rate declines when aggregate exposure to consumption diffusion risk increases. Markups affect the safe rate not directly, but through their effect on income composition and portfolio exposure. Labor income risk affects the safe rate through background income uncertainty. The central theoretical contribution is the presence of the equity diffusion term $(\sigma_R a_R)^2 \left(\frac{c_a}{c}\right)^2$. This term is absent in labor-only incomplete markets models. It introduces a new channel through which market power alters equilibrium interest rates: by increasing exposure to risky profit income, higher markups raise precautionary demand for safety and depress the risk-free rate.

Appendix

Derivations

Step 1: HJB. Let $V(a,z)$ be the stationary value function. The stationary HJB is

$$\rho V(a,z) = \max_{c\geq 0,\, a_R\in[0,a-\underline{a}]} \Big\{u(c) + V_a(a,z)\mu_a(a,z;a_R,c) + \tfrac{1}{2}V_{aa}(a,z)\,\sigma_a(a,z;a_R,c)^2 + V_z(a,z)\mu_z(z) + \tfrac{1}{2}V_{zz}(a,z)\,\sigma_z(z)^2\Big\}. \tag{50}$$

where recall:

$$\begin{aligned}
\mu_a(a,z;k,c) &= (wz + ra + (\mu_R - r)a_R - c)\\
\sigma_a(a,z;k,c)^2 &= (\sigma_R a_R)^2\\
\mu_z(z) &= (-\theta\log z + \frac{1}{2}\sigma_z^2 z)\\
\sigma_z(z)^2 &= (\sigma_z z)^2
\end{aligned}$$

during the derivations I will use LHS or RHS of the above equations interchangable.

Step 2: FOCs (unconstrained/interior). For an unconstrained household, the first-order conditions are:

$$u'(c) = V_a(a,z), \tag{51}$$

$$0 = (\mu_R - r)V_a(a,z) + V_{aa}(a,z)\sigma_R^2 a_R \tag{52}$$

With CRRA, $u'(c) = c^{-\gamma}$, hence

$$V_a(a,z) = c(a,z)^{-\gamma} \qquad \Longleftrightarrow \qquad c(a,z) = V_a(a,z)^{-1/\gamma}.$$

From (52),

$$a_R(a,z) = -\frac{\mu_R - r}{\sigma_R^2}\frac{V_a(a,z)}{V_{aa}(a,z)}.$$

Importantly, there is an extra constraint: households cannot get into debt in the risk-free asset to invest in equity. This generates an additional term in our derivation, which we will ignore for now.

Step 3: Differentiate the HJB w.r.t. a. Differentiate (50) with respect to a. Use the envelope cancellations implied by (51) and (52): terms proportional to c_a cancel because $u'(c) = V_a$, and terms proportional to $a_{R,a}$ cancel because $(\mu_R - r)V_a + V_{aa}\sigma_R^2 a_R = 0$.

The derivative is

$$\rho V_a = V_{aa}\Big(wz + ra + (\mu_R - r)a_R - c\Big) + rV_a + \tfrac{1}{2}V_{aaa}\,\sigma_R^2 k^2 + V_{az}\mu_z(z) + \tfrac{1}{2}V_{azz}\,\sigma_z(z)^2.$$

$$(\rho - r)V_a = V_{aa}\Big(wz + ra + (\mu_R - r)a_R - c\Big) + \tfrac{1}{2}V_{aaa}\,\sigma_R^2 a_R^2 + V_{az}\mu_z(z) + \tfrac{1}{2}V_{azz}\,\sigma_z(z)^2. \tag{53}$$

Step 4: Apply Itô's Lemma for $V_a(a,z)$ along the state process. Itô to $V_a(a,z)$ using (**??**) and $dz = \mu_z(z)\,dt + \sigma_z(z)\,dW^z$. Because covariance $dW^R\,dW^z = 0$, the cross-variation term is zero.

$$dV_a = V_{aa}\,da + V_{az}\,dz + \tfrac{1}{2}V_{aaa}\,(da)^2 + \tfrac{1}{2}V_{azz}\,(dz)^2.$$

Since $(da)^2 = (\sigma_R a_R)^2 dt$ and $(dz)^2 = \sigma_z(z)^2 dt$,

$$\begin{aligned} dV_a = &\Big[V_{aa}\Big(wz + ra + (\mu_R - r)a_R - c\Big) + \tfrac{1}{2}V_{aaa}\sigma_R^2 a_R^2 + V_{az}\mu_z(z) + \tfrac{1}{2}V_{azz}\sigma_z(z)^2\Big]dt \\ &+ V_{aa}\sigma_R a_R\,dW^R + V_{az}\sigma_z(z)\,dW^z. \end{aligned} \tag{54}$$

Use (53) to replace the drift bracket:

$$dV_a = (\rho - r)V_a\,dt + V_{aa}\sigma_R a_R\,dW^R + V_{az}\sigma_z(z)\,dW^z. \tag{55}$$

Step 5: Apply Itô's Lemma for $c(a,z)$ and its quadratic variation. Apply Itô to the stationary policy $c(a,z)$:

$$dc = c_a\,da + c_z\,dz + \tfrac{1}{2}c_{aa}(da)^2 + \tfrac{1}{2}c_{zz}(dz)^2.$$

Hence the diffusion part of dc is

$$dc = \Big[c_a\Big(wz + ra + (\mu_R - r)a_R - c\Big) + \tfrac{1}{2}c_{aa}\sigma_R^2 a_R^2 + c_z\mu_z(z) + \tfrac{1}{2}c_{zz}\sigma_z(z)^2\Big]dt + c_a\sigma_R a_R\,dW^R + c_z\sigma_z(z)\,dW^z,$$

and therefore the quadratic variation is

$$(dc)^2 = \Big(c_a\sigma_R a_R\Big)^2 dt + \Big(c_z\sigma_z(z)\Big)^2 dt. \tag{56}$$

Step 6: Use $V_a = c^{-\gamma}$ to obtain $E[dc \mid a,z]$. Since $V_a = c^{-\gamma}$, apply Itô to $c^{-\gamma}$:

$$d(c^{-\gamma}) = -\gamma c^{-\gamma-1}dc + \tfrac{1}{2}\gamma(\gamma+1)c^{-\gamma-2}(dc)^2.$$

But $d(c^{-\gamma}) = dV_a$.

Brief detour: taking conditional expectations. Conditioning on (a,z), the increments satisfy

$$E[dW^R \mid a,z] = 0, \qquad E[dW^z \mid a,z] = 0,$$

$$E[(dW^R)^2 \mid a,z] = dt, \qquad E[(dW^z)^2 \mid a,z] = dt,$$

and, since W^R and W^z are independent,

$$E[dW^R\,dW^z \mid a,z] = 0.$$

Therefore

$$E[dV_a \mid a,z] = (\rho - r)V_a\,dt \tag{57}$$

$$E[(dc)^2 \mid a,z] = \left[(c_a\sigma_R a_R)^2(c_z\sigma_z(z))^2\right]dt \tag{58}$$

$$E[d(c^{-\gamma}) \mid a,z] = -\gamma c^{-\gamma-1}E[dc \mid a,z] + \frac{1}{2}\gamma(\gamma+1)c^{-\gamma-2}E[(dc)^2 \mid a,z] \tag{59}$$

Putting all together:

$$E[d(c^{-\gamma}) \mid a,z] = -\gamma c^{-\gamma-1}E[dc \mid a,z] + \frac{1}{2}\gamma(\gamma+1)c^{-\gamma-2}\left[(c_a\sigma_R a_R)^2 + (c_z\sigma_z(z))^2\right]dt \tag{60}$$

Because $dV_a = d(c^{-\gamma})$, taking conditional expectations gives

$$E[dV_a \mid a,z] = E[d(c^{-\gamma}) \mid a,z].$$

Use (57) on the left and (60) on the right, and replace V_a by $c^{-\gamma}$:

$$(\rho - r)c^{-\gamma}dt = -\gamma c^{-\gamma-1}E[dc \mid a,z] + \frac{1}{2}\gamma(\gamma+1)c^{-\gamma-2}\left[(c_a\sigma_R k)^2 + (c_z\sigma_z(z))^2\right]dt$$

Divide both sides by dt:

$$(\rho - r)c^{-\gamma} = -\gamma c^{-\gamma-1}\frac{E[dc \mid a,z]}{dt} + \frac{1}{2}\gamma(\gamma+1)c^{-\gamma-2}\left[(c_a\sigma_R a_R)^2 + (c_z\sigma_z(z))^2\right]$$

Now solve for $E[dc \mid a,z]/dt$:

$$\frac{E[dc \mid a,z]}{dt} = \frac{r-\rho}{\gamma}\,c + \frac{\gamma+1}{2}\,\frac{(c_a\sigma_R a_R)^2 + (c_z\sigma_z(z))^2}{c}$$

Finally, reorder and make explicit that c is policy function $c(a,z)$ and k is policy function $a_R(a,z)$:

$$\frac{E[dc \mid a,z]}{dt} = \frac{1}{\gamma}(r-\rho)\,c(a,z) + \frac{\gamma+1}{2}\left[(\sigma_R a_R(a,z))^2\left(\frac{c_a(a,z)}{c(a,z)}\right)^2 + \sigma_z(z)^2\left(\frac{c_z(a,z)}{c(a,z)}\right)^2\right]c(a,z) \tag{61}$$

Step 7: Aggregation Let $g(a,z)$ denote the stationary density and split the state space into

$$U = \{(a,z) : \text{unconstrained (Euler holds)}\}, \qquad C = \{(a,z) : \text{constrained}\}.$$

Define aggregate consumption levels

$$C \equiv \iint c(a,z)\,g(a,z)\,da\,dz, \qquad C^U \equiv \iint_U c(a,z)\,g(a,z)\,da\,dz, \qquad C^C \equiv \iint_C c(a,z)\,g(a,z)\,da\,dz,$$

so $C = C^U + C^C$. Define, for $S \in \{U, C\}$,

$$C^S \equiv \iint_S c(a,z)\,g(a,z)\,da\,dz, \qquad \widetilde{E}^S[X] \equiv \frac{1}{C^S}\iint_S c(a,z)\,X(a,z)\,g(a,z)\,da\,dz,$$

and the overall consumption-weighted average

$$\widetilde{E}[X] \equiv \frac{1}{C} \iint c(a,z)\, X(a,z)\, g(a,z)\, da\, dz, \qquad C = C^U + C^C.$$

Income drift object for the constrained rule. Let z follow

$$dz = \mu_z(z)\, dt + \sigma_z(z)\, dW^z,$$

We use

$$\eta(z) \equiv \frac{\mu_z(z)}{z}.$$

If constrained households consume wage income $c = wz$ on C (hand-to-mouth), then $dc = w\, dz$ so

$$\frac{E[dc \mid a,z]}{dt} = w\mu_z(z) = (wz)\frac{\mu_z(z)}{z} = c(a,z)\,\eta(z) \qquad \text{for } (a,z) \in C.$$

Stationarity of aggregate consumption. With no aggregate shocks, aggregate consumption is constant in steady state, hence

$$0 = \frac{dC}{dt} = \iint \frac{E[dc \mid a,z]}{dt}\, g(a,z)\, da\, dz = \iint_U \frac{E[dc \mid a,z]}{dt} g(a,z)\, da\, dz \;+\; \iint_C \frac{E[dc \mid a,z]}{dt} g(a,z)\, da\, dz.$$

Now we just need to plug in, the equation for the unconstrained (61) and the equation for the constrained (5.4). Focus first in the unconstrained case:

$$\begin{aligned}
\iint_U \frac{E[dc \mid a,z]}{dt} g(a,z)\, da\, dz &= \frac{1}{\gamma}(r-\rho) \iint_U c(a,z)\, g(a,z)\, da\, dz \\
&\quad + \frac{\gamma+1}{2} \iint_U c(a,z) \left[\sigma_z(z)^2 \left(\frac{c_z(a,z)}{c(a,z)} \right)^2 + (\sigma_R k)^2 \left(\frac{c_a(a,z)}{c(a,z)} \right)^2 \right] g(a,z)\, da\, dz \\
&= \frac{1}{\gamma}(r-\rho) C^U + \frac{\gamma+1}{2} \iint_U c(a,z) \left[\sigma_z(z)^2 \left(\frac{c_z(a,z)}{c(a,z)} \right)^2 + (\sigma_R a_R)^2 \left(\frac{c_a(a,z)}{c(a,z)} \right)^2 \right] g(a,z)\, da\, dz
\end{aligned} \tag{62}$$

now the constrained drift:

$$\iint_C \frac{E[dc \mid a,z]}{dt} g(a,z)\, da\, dz = \iint_C c(a,z)\eta(z)\, g(a,z)\, da\, dz \tag{63}$$

Add them together to get the stationary aggregate consumption drift:

$$\begin{aligned}
0 = \frac{1}{\gamma}(r-\rho) C^U + \frac{\gamma+1}{2} \iint_U c(a,z) \left[\sigma_z(z)^2 \left(\frac{c_z(a,z)}{c(a,z)} \right)^2 + (\sigma_R a_R(a,z))^2 \left(\frac{c_a(a,z)}{c(a,z)} \right)^2 \right] g(a,z)\, da\, dz \\
+ \iint_C c(a,z)\, \eta(z)\, g(a,z)\, da\, dz.
\end{aligned} \tag{64}$$

Divide the above equation by C and take expectations weighted by consumption:

$$0 = \frac{C^U}{C}\frac{1}{\gamma}(r-\rho) + \frac{C^U}{C}\widetilde{E}^U\left[\frac{\gamma+1}{2}\left\{\sigma_z(z)^2\left(\frac{c_z}{c}\right)^2 + (\sigma_R a_R)^2\left(\frac{c_a}{c}\right)^2\right\}\right] + \frac{C^C}{C}\widetilde{E}^C[\eta(z)] \tag{65}$$

Solution Algorithm

Numerical solution of the stationary HJB We solve for the value function on a 2D grid in (a, z) using finite differences, following the standard continuous-time implicit method presented in Achdou et al. (2021). Starting from an initial guess V^0, iterate:

1. compute V_a with forward/backward differences and select the derivative by an *upwind scheme* (use the forward derivative where the implied drift $\mu_a > 0$ and the backward derivative where $\mu_a < 0$), while imposing a state-constraint (no-outflow) at $a = \underline{a}$ and a CRRA tail condition at $a_{\max}$;

2. update policies from the FOCs, $c^n = (V_a^n)^{-1/\gamma}$ and $a_R^{*,n} = -(\mu_R - r)\, V_a^n / (\sigma_R^2 V_{aa}^n)$, and ensure k^n is inside the feasible set;

3. with (c^n, k^n) fixed, assemble the discrete operator $A^n = A^{a,n} + A^z$, where $A^{a,n}$ uses upwinded drift and centered diffusion; the upwind construction ensures A^n is an *intensity matrix* (nonnegative off-diagonals and rows summing to zero);

4. update the value function using the *false-transient / implicit* step

$$\left(\tfrac{1}{\Delta} + \rho\right)V^{n+1} - A^n V^{n+1} \;=\; u(c^n) + \tfrac{1}{\Delta}V^n,$$

   where $\Delta = 1000$ and stop when $\|V^{n+1} - V^n\|_\infty$ is below tolerance. It imitates VFI in a tiny step (continuous time).

Extra Figures

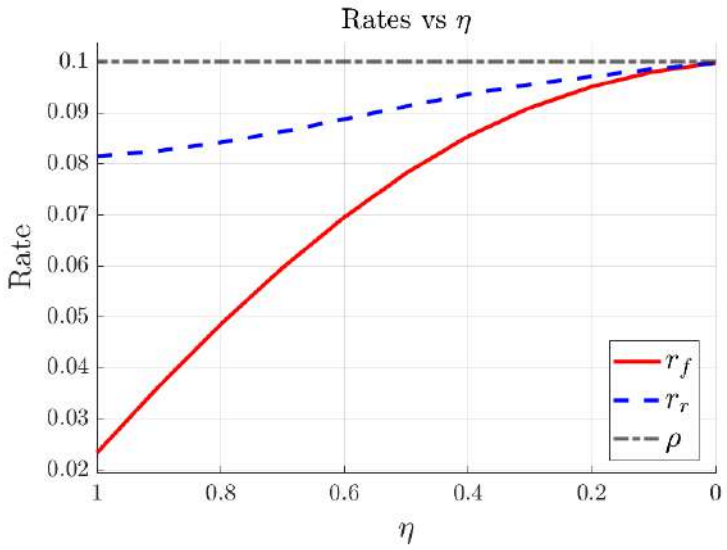
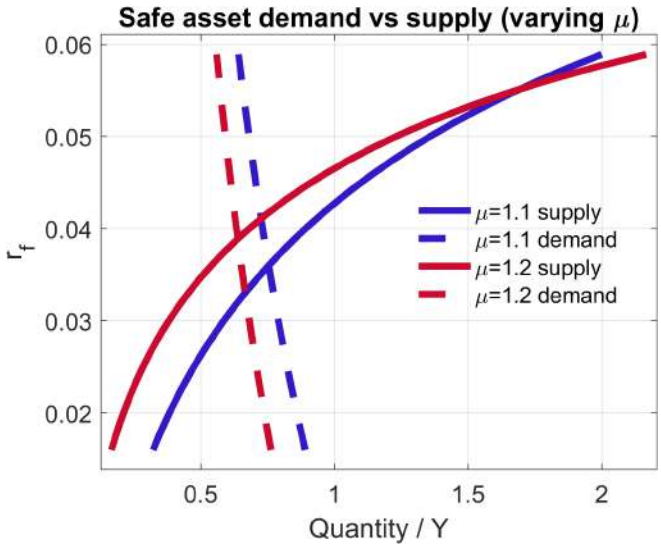
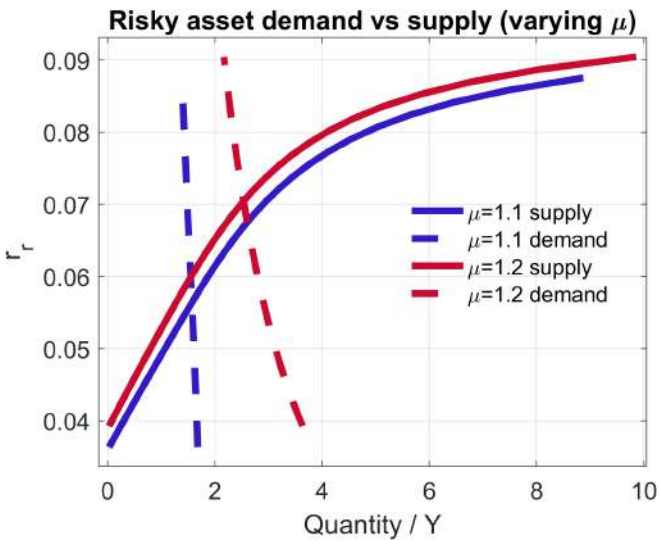


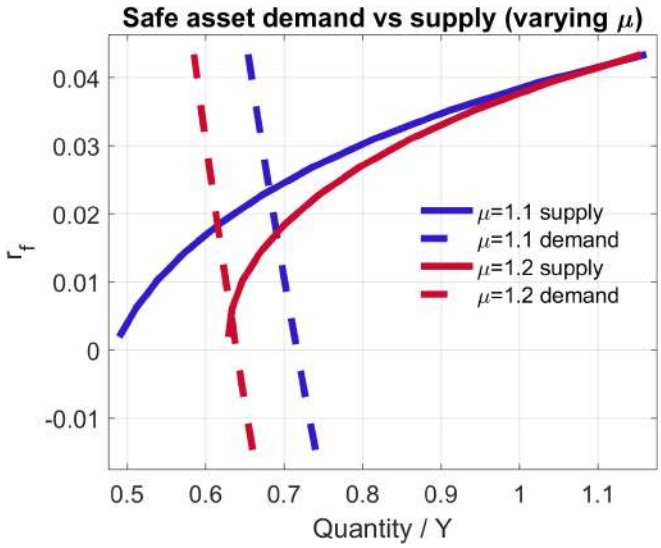
Figure 12: Comparative statics $\eta \in \{1, 0.9, 0.8, ..., 0.0001\}$ setting $\sigma_z = \eta \times \sigma_z^{\text{baseline}}$ and $\sigma_R = \eta \times \sigma_R^{\text{baseline}}$



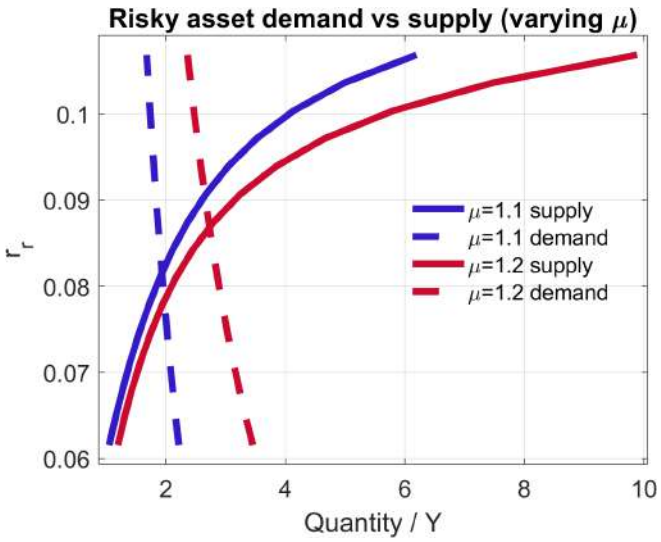
(a) Safe asset market (r_f up case)



(b) Risky asset market (r_f up case)



(c) Safe asset market (r_f down case)



(d) Risky asset market (r_f down case)

Figure 13: Supply and demand in safe and risky asset markets under changes in μ. Top row: r_f up case. Bottom row: r_f down case.

References

ACHDOU, Y., J. HAN, J.-M. LASRY, P.-L. LIONS, AND B. MOLL (2021): "Income and Wealth Distribution in Macroeconomics: A Continuous-Time Approach," *The Review of Economic Studies*, 89, 45–86.

AUCLERT, A., H. MALMBERG, M. ROGNLIE, AND L. STRAUB (2025): "The race between asset supply and asset demand," Tech. rep., National Bureau of Economic Research.

AUCLERT, A. AND M. ROGNLIE (2018): "Inequality and aggregate demand," Tech. rep., National Bureau of Economic Research.

CIOFFI, R. (2021): "Heterogeneous Risk Exposure and the Dynamics of Wealth Inequality," Tech. rep., Princeton University.

DE LOECKER, J., J. EECKHOUT, AND S. MONGEY (2021): "Quantifying Market Power and Business Dynamism in the Macroeconomy," Working Paper 28761, National Bureau of Economic Research.

DE LOECKER, J., J. EECKHOUT, AND G. UNGER (2020): "The Rise of Market Power and the Macroeconomic Implications," *Quarterly Journal of Economics*, 135, 561–644.

EGGERTSSON, G. B., J. A. ROBBINS, AND E. G. WOLD (2021): "Kaldor and Piketty's facts: The rise of monopoly power in the United States," *Journal of Monetary Economics*, 124, S19–S38.

FEDERAL RESERVE BANK OF ST. LOUIS (2026): "FRED Economic Data," https://fred.stlouisfed.org, accessed on February 8, 2026.

FERNÁNDEZ-VILLAVERDE, J., S. HURTADO, AND G. NUÑO (2023): "Financial Frictions and the Wealth Distribution," *Econometrica*, 91, 869–901.

GUVENEN, F. (2009): "A parsimonious macroeconomic model for asset pricing," *Econometrica*, 77, 1711–1750.

KAPLAN, G., B. MOLL, AND G. L. VIOLANTE (2018): "Monetary policy according to HANK," *American Economic Review*, 108, 697–743.

KAPLAN, G., G. NIKOLAKOUDIS, AND G. L. VIOLANTE (2023): "Price Level and Inflation Dynamics in Heterogeneous Agent Economies," Working Paper 31433, National Bureau of Economic Research.

PERUFFO, M. AND J. PLATZER (2024): "Secular Drivers of the Natural Rate of Interest in the United States: A Quantitative Evaluation," *Manuscript, August*.

RACHEL, L. (2025): "What Next for r*? A Capital Market Equilibrium Perspective on the Natural Rate of Interest," Bpea conference draft, Brookings Papers on Economic Activity (Brookings Institution).