Fed's Employment Mandate and Inflation Policy*

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Abstract

We formalize the maximum, optimal, and equilibrium levels of employment in a frictional model of unemployment and characterize the conditions under which it is desirable to actively pursue employment objectives via monetary policy. Maximum employment corresponds to the lowest feasible unemployment rate with voluntary participation, achieved when employers capture the entire surplus to maximize job creation, resulting in low-paying jobs. Inflationary policy can raise employment by eroding real wages in economies with restricted wage contracts, but implementing maximum employment requires prohibitively high inflation. Optimal employment target balances costs and benefits of job creation with welfare costs of inflation and differs from both maximum and equilibrium employment. Optimal monetary policy exhibits asymmetry: focusing solely on price stability when fundamentals generate low unemployment, but raising inflation targets when (natural) employment falls short of optimal levels.

1 Introduction

The Federal Reserve's congressional mandate to promote maximum employment and price stability has generated extensive research on optimal monetary policy design. While there is a vast literature on appropriate price stability goals, the employment component of the mandate has received less systematic treatment. The current approach models the employment objective indirectly through output gap stabilization or as activity levels consistent with long-run price stability, effectively subordinating employment considerations to inflation targets. This approach reflects the lack of a clear definition of the concept of "maximum" employment in existing work, where employment outcomes are primarily viewed through the lens of their inflationary consequences.

*The views expressed here are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Cleveland or the Federal Reserve Board.

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The resulting gap in the literature is particularly problematic given the Fed's explicit statutory requirement to pursue maximum employment as a co-equal objective with price stability. The theory of labor search and matching provide the analytical tools necessary to address this deficiency by offering rigorous definitions of employment concepts within frictional frameworks. However, the connection between these microeconomic foundations and macroeconomic policy targets remains underexplored. Specifically, the literature has not adequately addressed how different employment objectives – whether maximum, optimal, or equilibrium employment – translate into specific inflation rate targets that would implement these goals.

This paper fills this gap by developing a tractable framework that derives inflation targets directly from employment objectives within a frictional labor market model. We define maximum employment as the level associated with the lowest sustainable rate of unemployment that still ensures voluntary participation in the labor market by employers and households. This level is independent of monetary policy and the inflation rate in the economy. To achieve maximum employment, the planner has to incentivize employers to maximize job creation, which requires maximizing their share of the production surplus, while ensuring that the unemployed are not altogether discouraged from searching for a job. This is tantamount to a labor market allocation where employers are provided with the entirety of the surplus while workers are paid their reservation value, leaving them indifferent between unemployment and employment. The key insight is that maximum employment can only be attained by creating many low-paying jobs.

In a general equilibrium setting, workers recoup some of the surplus as dividends, because households are also the owners of firms. Nonetheless, maximum employment level is not optimal because the job creation is costly in a frictional model. The recruiting costs associated with constantly trying to fill a large number of vacant positions so as to maintain the lowest feasible level of unemployment exceeds the additional value created by those jobs. The optimal level of employment, which maximizes the welfare, better balances the costs and benefits of job creation. Equilibrium employment is distinct from the optimal and maximal levels of employment except in special cases, e.g., equilibrium aligns with maximal employment when employees have no bargaining power against employers, and, hence would have been paid their reservation wage anyways.

We then introduce trend inflation in our model and analyze its effects on equilibrium employment. To that end, we restrict the frequency of wage re-negotiations for ongoing employee-employer pairs, which puts deflationary pressure on real wages in an inflationary environment. Importantly, we allow for wage negotiations when the worker is hired and re-negotiations at random intervals thereafter, or if the real wage is deflated to a point where the workers would prefer unemployment to their job. Under these assumptions, the labor market clears at the hiring margin, since the starting salaries are flexible, and the wage rigidity never results in inefficient separations.[1] We then show that if the space of wage contracts is unrestricted, then (long-run) inflation is neutral, i.e., it does not change equilibrium employment. This is because any real effect that might arise from (anticipated) deterioration of real pay is fully negotiated away at the time of hire by frontloading wage

[1]Therefore, Barro's 1977 criticism of models where wage stickiness disallows opportunities for a mutually beneficial arrangement does not apply in our model.

payments.

At sufficiently high inflation levels, however, frontloading of pay requires the employer to compensate the worker more than their marginal product early in their tenure to balance anticipated losses in real wages. Depending on the rate of inflation, the required early compensation can be substantial, straining firms' finances. We therefore consider credit frictions that limit the extent of frontloading that can be financed, thereby, allowing inflation to raise equilibrium employment by eroding worker's share of the surplus.[2] Because such constraints do not bind at low levels of inflation, where, for instance, required frontloading in wage contracts can be handled by firms' internal sources, the model features a non-linear Phillips Curve with inflation raising employment only if it is sufficiently high.

We then ask what levels of inflation are generally consistent with maximal and optimal employment levels. We first show that trying to implement the maximal employment level is prohibitively costly. By contrast, implementing optimal employment is feasible, with the required inflation rate depending on the particular nature of the labor market and the job matching process. The inflation target that implements optimal employment is lower when the marginal value of an additional vacancy in job creation is low, or, equivalently, that of an unemployed worker is high; when the job separation rate from employment to unemployment is low; or, when the real interest rate is low. A number of policy implications emerge. For instance, when the number of job seekers and recruiters are gross complements in job creation and the market is short on unemployed workers relative to vacant positions, the model calls for a lower inflation target. Higher rates of job separation or real interest rates call for higher inflation targets.

The preceding analysis takes the employment target as given and derives the inflation rate needed to implement it. This prioritizes the employment objective over the inflation objective. A complete characterization of optimal policy requires the monetary authority to jointly determine both targets by balancing the welfare costs of inflation against the benefits of achieving an employment level that internalizes the costs and benefits of job creation.

To that end, we extend the frictional labor market framework by incorporating a money demand structure based on Sidrauski [1967]. In this environment, inflation creates a welfare cost by taxing real money balances, generating a meaningful trade-off between fully implementing the employment objective and minimizing the distortions from inflation. The optimal policy balances the marginal welfare loss from inflation against the marginal gain in output from reducing unemployment. This generally requires an inflation rate above the Friedman rule [Friedman, 1969] and an unemployment rate below what would be prescribed by the decentralized equilibrium alone. However, this interior solution may not emerge when labor market fundamentals imply a naturally low unemployment rate, or when the long-run Phillips curve is steep such that achieving meaningful employment gains requires prohibitively high inflation. In such cases, optimal policy prescribes singular focus on price stability, which in the Sidrauski framework corresponds to the Friedman rule.

A key implication of our framework is that optimal policy design and the associated targets are endogenous to labor market fundamentals. Specifically, when labor market fun-

[2]Lack of commitment to future employment by the worker could also render such supernumerary pay infeasible. Alternative models of monetary non-neutrality we considered do not change the nature of our conclusions although they might differ in the particular mechanism with which inflation affects real allocations.

damentals generate a naturally low unemployment rate – such that optimal employment is achievable at near-zero inflation – policy should focus exclusively on price stability, effectively operating as a single mandate. However, when fundamentals imply high unemployment at low inflation, optimal policy calls for moderate inflation to raise employment toward socially optimal levels, implementing a genuine dual mandate with asymmetric policy responses.

Our framework can be viewed as providing a theoretical foundation for the Fed's shortfall approach: policy should "judiciously raise inflation targets when employment falls short," but need not respond symmetrically when unemployment is low, unless inflation itself becomes problematic. Unlike the Fed's 2020 framework, however, which treats the shortfall approach primarily as a response to ZLB risks and asymmetric costs of policy errors, as articulated in Bernanke et al. [2019], our model derives this asymmetry from the fundamental structure of the labor market. As a result, whether policy should focus solely on an inflation target or jointly target both inflation and employment levels evolves with labor market conditions. Secular changes in such characteristics as matching efficiency, separation rates, or the real interest rate, for instance, necessitate corresponding adjustments to the long-run policy stance and whether policy should focus solely on an inflation target or jointly target both inflation and employment levels.

Related Literature. The classic Friedman rule, which prescribes a zero nominal interest rate to eliminate deadweight losses from economizing on money balances, provides the earliest theoretical benchmark for monetary policy targets [Friedman, 1969]. The benchmark New Keynesian models, exemplified by King and Wolman [1996] and Schmitt-Grohe and Uribe [2004], demonstrate that optimal inflation is typically zero or slightly negative when price stickiness is the primary friction, as any positive inflation creates menu costs and distortionary price dispersion. In the presence of distortionary taxation, Phelps [1973] shows that seigniorage revenue from positive inflation should be part of an optimal tax mix. Summers [1991] and, more recently Coibion et al. [2012] and Bernanke et al. [2019] among several others, have argued for higher inflation targets to provide a buffer against the zero lower bound (ZLB) during recessions. The treatment of the employment goal in this literature is typically guided by what Blanchard and Galí [2007] term the "divine coincidence": in standard New Keynesian models with only nominal rigidities, stabilizing inflation automatically stabilizes the welfare-relevant output gap. This property motivated the focus on inflation targeting, as there is no independent role for the employment mandate.[3]

Earliest engagement with long-run employment effects of inflation comes from Tobin [1972], who argued that moderate inflation "greases the wheels" of the labor market by facilitating real wage adjustments in the presence of downward nominal wage rigidity. Formalizing Tobin's argument, Akerlof et al. [1996] shows that moderate inflation can permanently reduce the natural rate of unemployment. More recently, Hobijn and Şahin [2021] and Eusepi and Şahin [2025] study the interaction between labor force participation and employment levels in a search and matching framework, and find that monetary policy focused on stabilizing jobs inherently promotes maximum employment by strengthening the long-term attachment

[3]When Blanchard and Galí [2007] introduce real wage rigidities, the divine coincidence disappears and a meaningful trade-off emerges, though their focus remains on stabilization rather than on defining employment targets.

of the workforce. The employment objective in these studies is centered around the natural unemployment rate. Michaillat and Saez [2024] propose a "full-employment" objective, where resources dedicated to job creation are minimized. Adopting a frictional model of unemployment as the recent literature, we show that maximum employment exceeds both the natural rate of unemployment and Michaillat and Saez's (2024) full-employment and jointly study both the inflation and employment objectives.

A related literature integrates the labor search framework with frictional goods markets and examine the long-run relationship between inflation and the labor market focusing on optimal inflation policy [Berentsen et al., 2011, Bethune et al., 2024, Gu et al., 2025]. Berentsen et al. [2007] examine when both optimal inflation (Friedman rule) and optimal employment (Hosios condition) can be jointly achieved, finding they coincide only under restrictive conditions on the distribution of money holdings across agents. While they characterize when both objectives align, we analyze how to implement employment targets when they do not. Furthermore, our mechanism, based on restricted wage setting, potentially allows inflation to raise employment even at higher levels, while Berentsen et al. [2007] features a negative effect of inflation on economic activity above moderate levels.

Our paper makes three key contributions relative to the literature. First, we provide the first rigorous definition of "maximum employment" within a frictional labor market model, showing it corresponds to the lowest feasible unemployment rate consistent with voluntary participation—a concept distinct from both efficient employment (the Hosios benchmark) and equilibrium employment. Second, we demonstrate that achieving maximum employment requires prohibitively high inflation, as it necessitates creating many low-paying jobs by maximizing employers' surplus. Third, we characterize the states of the economy where it is desirable to pursue an active dual-mandate policy and derive the associated inflation and employment targets.

Our paper also relates to recent work on how inflation affects labor market outcomes and worker well-being through real wage erosion. Afrouzi et al. [2026] document that the wage growth gap between job stayers and job switchers widens substantially during inflationary periods, with stayers' wages lagging behind. Exploring the behavioral responses workers undertake to mitigate wage erosion, Hajdini et al. [2023] and Pilossoph and Ryngaert [2024] use survey evidence to show that workers with higher inflation expectations increase job search effort, while Pilossoph et al. [2024] estimate the welfare costs of this intensified search activity. Guerreiro et al. [2024] find that approximately one-fifth of workers engaged in costly actions—-requesting raises, participating in union activity, or soliciting outside offers—-to restore their real wages during the recent inflation episode, documenting costs associated with wage renegotiation. These papers collectively demonstrate that wage catch-up with the same employer is costly and often incomplete, leading to persistent real wage erosion for job stayers. Our contribution is to show how this wage rigidity create profit opportunities that firms exploit through vacancy creation, linking inflation to labor market tightness.

Roadmap. In the next section, we introduce our model environment, describe the decentralized equilibrium. Section 3 discusses the interaction between inflation and economic activity. Section 4 discusses the various policy objectives and presents our key results. Section 5 conducts a (preliminary) quantitative analysis that demonstrates our results by

calibrating key model parameters. The quantitative analysis captures the essential features of the U.S. Beveridge Curve and Phillips Curve relationships. Moreover, the quantitative analysis allows us to compute optimal employment rates and illustrate how the associated inflation targets would vary across different labor market environments. Section 6 concludes.

2 Model

This section introduces a monetary economy with frictional labor markets that combines the Sidrauski [1967, 1969] model of money demand with the canonical model of random job search [Diamond, 1982, Mortensen, 1982, Pissarides, 1985], extended to include nominal wage rigidities for ongoing employment relationships.

2.1 Household Problem

Time is continuous and the horizon is infinite. The economy is populated by a representative household with a continuum of members of measure one. Following Lucas [2000] and Sidrauski [1967], households derive direct utility from holding real balances. This reduced-form specification captures in a tractable way the transaction services provided by money, which facilitate consumption purchases and economize on the time and effort required for market transactions.

The household maximizes lifetime utility derived from consumption and real money balances:

$$\max_{\{c,n\}} \int_0^\infty e^{-\rho t} U(c,n) dt, \tag{1}$$

where c denotes consumption, n denotes real money balances, and $\rho > 0$ is the subjective discount rate. We assume the utility function $U(c,n)$ is strictly increasing and concave in both arguments.

The household's budget constraint is:

$$\dot{a} = I(u,d) + T - c - \pi n, \tag{2}$$

where $a \equiv b + n$ denotes total real assets (the sum of real bond holdings b and real money balances n), T represents lump-sum government transfers, and π is the inflation rate. Real bonds earn the real interest rate r, while money is dominated in rate of return, losing value at rate π through inflation.

Household members pool their resources and perfectly insure each other against idiosyncratic employment risk [Andolfatto, 1996]. The household's real income $I(u,d)$ depends on labor market outcomes. Each representative household has a fraction $(1-u)$ of their members employed, earning wage $\bar{w}$ on average, and a fraction u unemployed, receiving flow utility z in consumption equivalent terms from home production and unemployment benefits. The household also receives dividend income d as firm owners. Thus:

$$I(u,d) = (1-u)\overline{w} + uz + d$$

Because labor market dynamics affect the household's optimization problem only through the income flow $I(u, d)$, we can decompose the household's decision problem. In particular, we can separate the analysis of the labor market equilibrium, which determines $I(u, d)$ as a function of labor market conditions, and solve the household's consumption-savings problem taking the income process as given.

2.2 Labor Market

The labor market is characterized by search and matching frictions in the spirit of the Diamond-Mortensen-Pissarides framework [Diamond, 1982, Mortensen, 1982, Pissarides, 1985]. Unemployed workers search for job opportunities while firms with vacant positions recruit employees. The bilateral nature of search creates frictions: it takes time for unemployed workers to locate suitable vacancies, and it takes time for firms to fill open positions.

We denote by u the measure of unemployed workers and by v the measure of vacant positions. The flow of new matches formed per unit of time is determined by a matching function $m(u, v)$, which depends on the number of searchers on both sides of the market. We assume $m(u, v)$ exhibits constant returns to scale and is strictly increasing and concave in both arguments: $m_u, m_v > 0$, $m_{uu}, m_{vv} < 0$, and $m_{uv} > 0$. These properties capture the congestion effects inherent in the matching process. η_u^m denotes the unemployment elasticity of the matching function, and $\eta_v^m = 1 - \eta_u^m$ is the corresponding vacancy elasticity.

Labor market conditions are summarized by ratio of vacancies per unemployed $\theta \equiv \frac{v}{u}$, which determines two essential transition rates: the job-finding rate $p(\theta) = m(u, v)/u \equiv m(1, \theta)$ for workers, and the vacancy-filling rate $q(\theta) = m(u, v)/v \equiv m(1, \theta)/\theta$ for firms. These transition rates satisfy $p'(\theta) > 0$ and $q'(\theta) < 0$: in a tighter labor market (higher θ), workers find jobs more quickly but firms find it harder to fill vacancies.

Employment relationships are subject to exogenous separation shocks that arrive at rate $\delta \in (0, 1)$. When a separation occurs, the worker returns to unemployment and the firm's position becomes vacant. In steady state, flows into and out of unemployment balance: $p(\theta)u = \delta(1 - u)$, implying the following steady-state unemployment rate:

$$u(\theta) = \frac{\delta}{\delta + p(\theta)}. \tag{3}$$

This relationship defines the Beveridge curve: a negative relationship between vacancies and unemployment, with tighter labor markets (higher θ) associated with lower unemployment.

We now characterize the value of employment and unemployment to workers and firms, taking into account how inflation affects the real value of ongoing employment relationships through nominal wage rigidity.

2.2.1 Worker Value Functions

The value function for an unemployed worker, denoted U, satisfies the Bellman equation:

$$rU = z + p(\theta)[W_0 - U], \tag{4}$$

which states that the flow return to unemployment equals the utility from being unemployed plus the expected capital gain from finding employment. An unemployed worker receives immediate utility z and transitions to employment at rate $p(\theta)$, experiencing a discrete jump in value equal to $W_0 - U$ upon successful matching, where W_0 is the value of a new job to the worker.

The value function for a newly employed worker, W_0, satisfies:

$$rW_0 = w + \delta(U - W_0) + \lambda(\pi)[W - W_0]. \tag{5}$$

The flow return to employment consists of a negotiated wage payment w minus the expected capital loss from job destruction and from inflation. With probability δ per unit time, the employment relationship dissolves and the worker returns to unemployment, experiencing a discrete drop in value equal to $W_0 - U$. At the rate $\lambda(\pi)$, where π is the inflation rate , the value of a job to the worker changes from W_0, resulting in a drop in the job's value to the worker by $W_0 - W \geq 0$. Function $\lambda(\cdot)$ captures the likelihood that inflation triggers wage depreciation, potentially allowing for the possibility of wage indexing. We assume that $\lambda(0) = 0$ and $\lambda'(\pi) > 0$.

The value of an ongoing job to a worker is given by:

$$rW = \underline{w} + \delta[U - W] + \phi[W_0 - W]. \tag{6}$$

The flow return to the worker from an ongoing employment relationship consists of a lower wage payment $\underline{w} \leq w$, which is not negotiated, minus the expected capital loss from job destruction plus the expected capital gain from renegotiating the wage rate. At rate ϕ, workers renegotiate their wage, which results in a capital gain of $W_0 - W$.

2.2.2 Firm Value Functions

The value of maintaining a vacant position, V, is characterized by:

$$rV = -\kappa + q(\theta)[J_0 - V] \tag{7}$$

A firm with a vacancy incurs the flow cost κ and successfully fills the position at rate $q(\theta)$. Upon matching with a worker, the firm experiences a discrete increase in value equal to $J_0 - V$, where J_0 represents the value of a newly filled position and satisfies the following Bellman equation:

$$rJ_0 = y - w + \delta(V - J_0) + \lambda(\pi)[J - J_0] \tag{8}$$

The firm's flow return to a newly filled position equals the worker's productivity less the negotiated wage payment, adjusted for the expected losses from job destruction and expected gains from real wage deflation. When the job is destroyed at rate δ, the firm loses $J_0 - V$. At the rate $\lambda(\pi)$, the firm's gains from the position rise from J_0 to $J \geq J_0$.

The value of an ongoing employment relationship to the firm is:

$$rJ = y - \underline{w} + \delta(V - J) + \phi[J_0 - J] \tag{9}$$

The flow return consists of a higher ongoing profit margin $y - \underline{w}$ minus the expected losses from job destruction and wage renegotiation, which happens at rate ϕ and reduces the value of the job to the firm back to J_0.

2.2.3 Wage Determination and Labor Market Equilibrium

The model has nine endogenous variables: six values in Equations 4-9, the two wage rates w and $\underline{w}$, and the labor market tightness θ. To the six value equations above, we add three conditions to define the labor market equilibrium.

First, we assume that the negotiated wage payment w is determined through Nash bargaining between the worker and firm over the total match surplus $S = (W_0 - U) + (J_0 - V)$. The Nash bargaining protocol maximizes the weighted product of the parties' gains from the match: $w = \arg\max (W_0 - U)^{\beta}(J_0 - V)^{1-\beta}$, where $\beta \in [0, 1]$ represents the worker's relative bargaining power. We restrain the set of possible wages so that at any point in time the wage rate paid may not exceed a worker's instantaneous productivity: $w \leq y$.[4] The first-order condition for this optimization problem yields the surplus-sharing rule:

$$\beta(J_0 - V) = (1 - \beta)(W_0 - U) + \mu, \tag{10}$$

where $\mu \geq 0$ is the Lagrange multiplier associated with the credit constraint. If the credit constraint does not bind ($\mu = 0$), Equation (10) states that the worker's bargaining power determines the division of the total match surplus. Otherwise, $\mu > 0$, and $w = y$.

The ongoing wage rate $\underline{w} \leq w$ is not negotiated. Without loss of generality we assume that $\underline{w}$ leaves the worker indifferent between unemployment and employment:

$$W = U. \tag{11}$$

This assumption ensures that transitions to the low-wage regime, W, from inflation does not result in a negative surplus share to the worker.

Finally, the equilibrium is characterized by free entry of firms into the vacancy posting process. This condition drives the value of maintaining a vacancy to zero in equilibrium:

$$V = 0. \tag{12}$$

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Equations (10), (11), and (12) along with the value functions 4-9 characterize the labor market equilibrium.

Equations (10), (11), and (12) along with the value functions 4-9 determine the labor market tightness in equilibrium and, thereby, household income. In equilibrium, total dividends consists of profits from filled positions net of maintaining vacant positions.

$$d = (y - \bar{w})(1 - u) - v\kappa,$$

where $\bar{w}$ is the average wage between workers that are in the two wage regimes $\underline{w}$ and w. Substituting the dividends gives household's income as a function of the unemployment rate and tightness in equilibrium:

$$I(u, \theta) = y(1 - u) + uz - u\theta\kappa.$$

At the steady state, unemployment must lie on the Beveridge Curve: $u(\theta) = \delta/(\delta + p(\theta))$, which implies that household's income is given by $I(\theta) = I(u(\theta), \theta)$.

[4]The no-credit requirement could be extended to allow for some borrowing $w \leq \gamma y$ with $\gamma > 1$. The results would be qualitatively similar.

2.3 Steady-State Consumption and Real Money Balances

Given our focus in long-run policy objectives rather than short-run policy responses to economy's deviations from those objectives, we now briefly review the household's optimality conditions in steady state.[5] The marginal rate of substitution between consumption and money balances equals the nominal interest rate:

$$\frac{U_n(c,n)}{U_c(c,n)} = i = r + \pi, \tag{13}$$

where the second equality follows from the Fisher equation. This condition equates the marginal benefit of holding an additional unit of real balances (the marginal utility U_n relative to the marginal utility of consumption) to its opportunity cost, which is the forgone nominal return i from holding interest-bearing bonds instead.

With government budget balance ($T = \pi n$) and goods market clearing, consumption equals aggregate income:

$$c = I(\theta) \tag{14}$$

Given the income process $I(\theta)$ and inflation π, equations (13) and (14) jointly determine the household's optimal choices of consumption and money balances (c^*, n^*) in steady state.

The household problem is thus characterized by two key margins. First, the consumption-money margin determines the composition of household spending between goods and liquidity services, governed by condition (13). Second, labor market equilibrium determines the level of income available to the household. By solving the labor market equilibrium first, we can treat $I(u, \theta)$ as given when analyzing the household's financial decisions, simplifying the overall characterization of equilibrium.

The complete characterization of the household's optimal choices, combined with the labor market equilibrium, defines a mapping from inflation rates to employment outcomes. This mapping, subject to credit constraints, generates the Phillips Curve relationship that is central to our policy analysis in the next section.

3 Inflation and Employment in Equilibrium

This section characterizes the relationship between inflation and economic activity in the decentralized labor market equilibrium. We establish conditions under which monetary policy affects real allocations – specifically, the level of employment and unemployment – through the labor market.

We begin with a striking neutrality result: when wage contracts are unrestricted, inflation has no effect on equilibrium employment despite the presence of nominal wage rigidities. The key insight is that forward-looking workers and firms fully internalize the effects of anticipated inflation through wage negotiations at the time of hire. Workers demand and receive compensation for future real wage erosion by negotiating higher starting wages, which exactly offsets the redistribution of surplus that would otherwise occur through inflation.

[5] We present the full set of dynamic equilibrium conditions in the Appendix where we construct the monetary authority's planning problem.

Then, we consider limitations to firms' ability to frontload wages beyond current productivity. Under these constraints, sufficiently high inflation cannot be fully neutralized through upfront compensation, allowing monetary policy to affect real allocations. The result is a nonlinear Phillips Curve relationship. At low inflation rates (below a threshold $\underline{\pi}$), credit constraints do not bind and inflation remains neutral: the equilibrium unemployment rate is independent of the inflation rate. At high inflation rates (above $\underline{\pi}$), credit constraints bind and inflation reduces unemployment by eroding workers' share of the match surplus.

We begin our analysis of the labor market equilibrium by deriving the job creation condition that determines labor market tightness as a function of model primitives.

Substituting $V = 0$ from the free-entry condition into the vacancy value function yields $\kappa = q(\theta)J_0$, which states that the flow cost of maintaining a vacancy equals the expected benefit from filling it. From equation (8), the latter is equivalent to:

$$J_0 = \frac{y - w + \lambda(\pi)[J - J_0]}{r + \delta}.$$

The job creation condition emerges from combining the free entry condition with the value function for filled positions, which yields:

$$\frac{y - w + \lambda(\pi)[J - J_0]}{r + \delta} = \frac{\kappa}{q(\theta)}, \tag{15}$$

indicating that the expected cost of hiring a worker equals the present discounted value of the profit stream from the employment relationship. When $\pi = 0$, equation (15) gives the job creation condition in the canonical random search model. When $\pi > 0$, inflation raises profits, ceteris paribus, and encourages firms to post more vacancies since $J \geq J_0$. As we show next, however, the effect of inflation on equilibrium vacancy creation, all things considered, depends on the outcome of wage negotiations.

Consider first an economy without any borrowing constraints on firms: $\mu = 0$. The equilibrium wage rate in that case is:

$$w = \beta y + (1 - \beta) rU + \lambda(\pi)(W_0 - W). \tag{16}$$

The negotiated wage rate is a convex combination of productivity and the flow reservation value of an unemployed worker, adjusted for future deflation of the real wage in an inflationary environment. Given our assumptions on $\lambda(\cdot)$, when $\pi = 0$, the model replicates the standard wage setting process in the canonical model. $\pi > 0$ implies higher wages when the worker starts their job, implying that workers recapture at least some of the anticipated loss in value from wage deflation. In fact, they capture the entirety of it if starting salaries are unbounded. This can be established by first showing that a worker's loss from deflation is equal to the firm's gain from wage deflation.

Lemma 1. $J - J_0 = W_0 - W$

Because the inflationary process effectively transfers resources from the worker to the firm, we have $J - J_0 = W_0 - W \geq 0$. Therefore the additional gains in firm's flow profits, $\lambda(\pi)(J - J_0)$ in equation (15), is paid out entirely to the worker by raising starting salaries by the same amount $\lambda(\pi)(W_0 - W)$. Substituting the wage rate in equation (16) into the job creation condition therefore gives the canonical equilibrium condition for labor market tightness in the random search model.

Proposition 1. *If wage payments are unbounded, then the equilibrium wage rate is given by:*

$$w = \beta y + (1-\beta)z + \beta\kappa\theta + \lambda(\pi)\frac{\beta}{1-\beta}\frac{\kappa}{q(\theta)}. \tag{17}$$

Considering the equilibrium valuations of jobs and unemployment to a worker, the negotiated wage reflects a combination of productivity, value of leisure, and compensation for the deflationary effect of price inflation on real wages. That compensation depends on the strength of the worker's bargaining power in wage negotiations and increases with the rate of inflation in the economy. Note that wages can easily rise above current productivity level if $\beta > 0$ and the inflation rate is high. The faster the real wage deflation, the higher the upfront negotiated wage has to be to compensate the worker.

Proposition 2. *Equilibrium level of labor market tightness is given by:*

$$\frac{y-z}{r+\delta} = \frac{\kappa}{q(\theta^*)} \times \frac{r+\delta+\beta p(\theta^*)}{(1-\beta)(r+\delta)}. \tag{18}$$

Equation (18) is the standard equilibrium condition in the canonical model and states that, in equilibrium, the discounted value of the flow surplus associated with a job equals the effective cost of creating that job to the firm. That is higher than the firm's expected cost of filling a vacancy. Because such filling costs are sunk by the firm prior to wage negotiations, workers capture some of those during negotiations as long as they have some bargaining power, $\beta > 0$, which raises the effective cost of posting vacancies in equilibrium and discourages firm entry.[6] That in turn results in lower job creation than would occur if workers had no bargaining power.

Importantly, π does not appear in the equilibrium condition above, giving our first main result on monetary neutrality.

Corollary 1. *Suppose that w is unbounded. Then the inflation rate has no real effect on equilibrium allocations.*

The model without credit constraints does not feature a long-run Phillips Curve, because the deflationary pressure on the real wage rate is negotiated away. The firm agrees to frontload pay in order to compensate the worker for future loss. Setting $w = y$ gives the inflation level above which the negotiated wage exceeds the contemporaneous productivity level:

$$\lambda(\underline{\pi}) = \frac{1-\beta}{\beta}(r+\delta). \tag{19}$$

Higher levels of inflation require the firm to borrow against future profits to pay its workers. Note that the threshold inflation rate above which this becomes necessary is lower the higher the worker's bargaining power, which determines the amount of inflation-related compensation. Furthermore, the higher the effective discount rate for the match, $r+\delta$, the lower is the required upfront pay to compensate future losses, and, hence, higher is $\underline{\pi}$.

Next we consider the scenario where the firm is limited in its ability to frontload pay due to credit constraints. Specifically, we assume the highest salary a firm can pay is equal to

[6] This is represented by the $\beta\kappa\theta$ term in the equilibrium wage function.

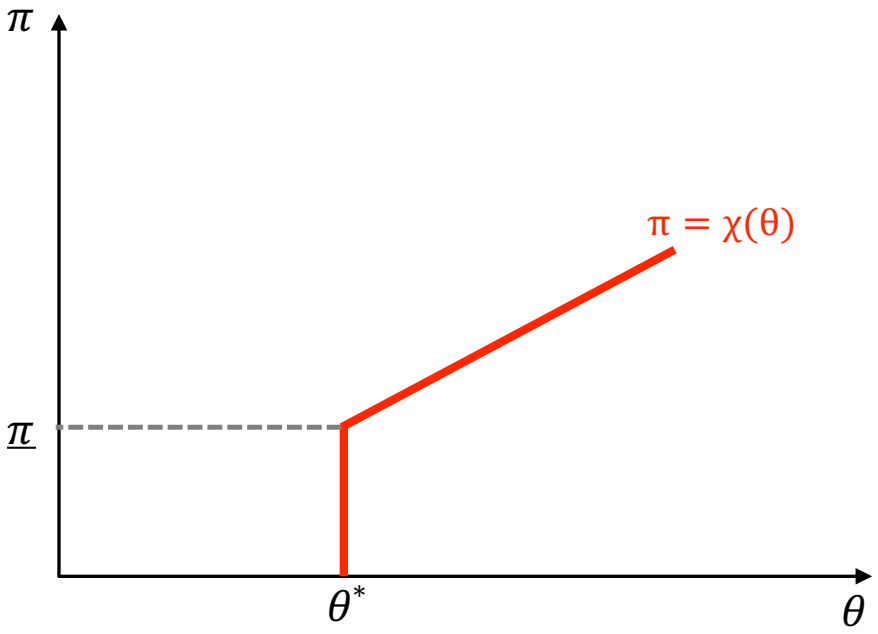


Figure 1: Inflation and Labor Market Tightness

productivity: $w \leq y$. Therefore, when $\pi \geq \underline{\pi}$, $w = y$, which gives the following job creation condition:

Proposition 3. *In an economy with credit constraints, the equilibrium level of labor market tightness solves equation* (18) *if* $\pi \leq \underline{\pi}$ *and solves the following equation if* $\pi \geq \underline{\pi}$*:*

$$\frac{\lambda(\pi)}{r+\delta+\lambda(\pi)+p(\theta)} \times \frac{(y-z)}{r+\delta} = \frac{\kappa}{q(\theta)}. \tag{20}$$

Proposition 3 describes a positive relationship between the inflation rate and equilibrium labor market tightness, and, hence, a negative relationship between inflation and the unemployment rate.

Formally, we define the long-run Phillips curve as $\pi = \chi(\theta)$, where $\chi(\theta)$ is given by:

$$\chi(\theta) = \begin{cases} [\pi_{min}, \underline{\pi}] & \text{if } \theta = \theta^* \\ \lambda^{-1}\left(\frac{\kappa[r+\delta+p(\theta)](r+\delta)}{(y-z)q(\theta)-\kappa(r+\delta)}\right) & \text{if } \theta > \theta^* \end{cases} \tag{21}$$

where θ^* is implicitly defined by (18) and π_{min} is the minimum feasible inflation the monetary authority could target (we set it at the Friedman rule). Figure 1 illustrates the non-linearity of this relationship in the model. Inflation has no effect on employment at low levels ($\pi < \underline{\pi}$), but raises employment at sufficiently high inflation rates.[7]

In what follows, we focus on the economy with credit constraints and consider different interpretations of the Fed's employment mandate and examine the levels of inflation that are consistent with each interpretation.

[7] To see this, note that the left hand side of equation (20) is increasing in π, which requires a rise in θ to rebalance since $p'(\theta) > 0$ and $q'(\theta) < 0$.

4 Policy Objectives

This section examines different policy objectives and their implementation through inflation policy. We first define two employment objectives based on the wording of the congressional mandate:

> "*The Board of Governors of the Federal Reserve System and the Federal Open Market Committee shall maintain long run growth of the monetary and credit aggregates commensurate with the* ***economy's long run potential to increase production****, so as to promote effectively the goals of* ***maximum employment****, stable prices and moderate long-term interest rates.*" (emphasis added)
>
> Section 2A
> Federal Reserve Act

We begin with a definition of "maximum" employment because it is an explicitly stated goal in the mandate. We then consider the employment level that maximizes output level. Established in [Hosios, 1990], output-maximizing unemployment balances the benefits of creating jobs with the cost of maintaining job vacancies, and is a well-known benchmark in frictional models of unemployment. It also connects, in our view, to the congressional mandate through its statement of the economy's long-run output potential as a means to achieve the listed goals.

Because equilibrium employment does not necessarily align with either of these employment objectives in frictional labor markets, we then analyze the problem of a planner that aims to achieve the employment objectives via monetary policy, weighing potential employment gains against welfare costs of inflation.

4.1 Maximum Employment

Suppose the planner wants to maximize steady-state employment, which is equivalent to minimizing unemployment in our setup. In doing so, the planner must ensure market participation by firms and workers: firms are willing to operate and post vacancies, and unemployed workers are interested in seeking employment:

$$\max_{w} n = (1-u) \quad \text{subject to} \;\; W_0 \geq U \text{ and } J_0 \geq V$$

At the steady-state equilibrium, where the unemployment rate is stable, hires equal separations: $m(u,v) = \delta(1-u)$, which gives the following unemployment rate:

$$u(\theta) = \frac{\delta}{\delta + p(\theta)}. \tag{22}$$

Equation (22) implicitly describes the Beveridge Curve relationship between vacancies and unemployment. Because $p'(\theta) > 0$, the steady-state unemployment rate is decreasing in labor market tightness. Therefore the planner's problem is equivalent to maximizing $\theta = v/u$, which requires incentivizing vacancy creation as much as possible. That entails

delivering the entire surplus from the match to the firm. The lowest value that will not deter unemployed workers from searching for a job in the first place requires $W_0 = U$, which leads to a corner solution to the planner's problem where the workers' participation constraint binds. The level of maximum attainable employment consistent with the planner's solution is, therefore, well-defined as summarized by the following proposition.

Proposition 4. *The maximum attainable labor market tightness solves*

$$\frac{y-z}{r+\delta} = \frac{\kappa}{q(\theta^{max})}, \tag{23}$$

and the corresponding maximum employment rate is:

$$e^{max} = 1 - u^{min} = \frac{p(\theta^{max})}{\delta + p(\theta^{max})} \tag{24}$$

In the planner's allocation, the firm takes all of the surplus from the match. Note that the maximum employment level only depends on the fundamental parameters of the economy. It does not depend on the inflation rate, implying that it is a stable target regardless of the adopted monetary stance.

By definition, equilibrium employment is below the maximal employment level so long as $\beta > 0$, i.e., workers capture some of the match surplus: $\theta^* < \theta^{max} \iff \beta > 0$. Recall from Proposition 3 that equilibrium employment is increasing in the inflation rate. It is natural to ask: What level of the inflation rate implements maximum employment? Comparison of equations (20) and (23) reveals, however, that equilibrium employment approaches maximum employment only as a limiting case, when the inflation rate approaches infinity $\pi \to \infty$.[8]

4.2 Maximum Output

Following Hosios [1990], our definition of the output-maximizing unemployment rate maximizes (steady-state) resources available for consumption:

$$\max_{u,v} \int_0^\infty e^{-rt}[(1-u)y + uz - v\kappa]$$

subject to

$$\dot{u} = (1-u)\delta - m(u,v),$$

where we substituted $r = \rho$ from the steady-state. The solution to this optimal control problem gives the well-known Hosios optimality condition, which is stated below:

Proposition 5 (Hosios, 1990)**.** *The optimal tightness level is given by:*

$$\frac{y-z}{r+\delta} = \frac{\kappa}{q(\theta^H)} \cdot \frac{r+\delta+\eta_u^m p(\theta^H)}{\eta_v^m (r+\delta)}, \tag{25}$$

where $\eta_u^m = m_u(u,v)u/m$ and $\eta_v^m = m_v(u,v)v/m$ denote the elasticities of the matching function with respect to the numbers of unemployed workers and vacancies.

[8] Maximum employment in equilibrium requires $\frac{\pi}{r+\delta+\pi+p(\theta)} = 1$ which can only hold if $\pi = \infty$.

The optimality of the equilibrium is paramount to understanding the recipes for monetary policy for a planner that seeks to maximize welfare. Comparing equations (25) and (18) reveals the well-known condition for equilibrium optimality in the canonical, unconstrained economy: $\beta = \eta_u^m = (1 - \eta_v^m)$. This ensures that the social marginal return to an additional unemployed worker and an additional vacant position are aligned by the private incentives of job seekers and firms.[9]

The Hosios condition is met for all values of u and v when $m(u, v)$ is given by a Cobb-Douglas function with the relevant elasticities set to β and $1-\beta$. In that case, all equilibria are efficient in the unconstrained economy. Because credit constraints do not bind at low levels of inflation, all levels of inflation $\pi < \underline{\pi}$ ensure that the equilibrium is optimal. Therefore a natural strategy for a central bank that aims to maximize welfare is to maintain a low inflation rate.

However, the requirement that the matching function be of Cobb-Douglas form with elasticities coinciding with relative bargaining powers in wage negotiations is an extremely special case. More generally, matching elasticities vary with the numbers of unemployed workers and vacancies. In that case the optimality of equilibrium employment depends on where the elasticities stand vis-à-vis the bargaining powers of workers and firms. Moreover, because inflation is non-neutral only above a certain level, the planner's ability to implement optimal employment whenever the equilibrium deviates from it depends on whether that deviation happens at high or low inflation rates. We go over these cases below.

When $\eta_u^m < \beta$, equilibrium employment is suboptimal. In that case, the planner can raise the employment level by targeting an inflation rate above $\underline{\pi}$. Comparing equations (25) and (20) gives the inflation rate that delivers maximum resources as follows.

Proposition 6. *If $\eta_u^m < \beta$, the output maximizing inflation rate is*

$$\pi^* = \lambda^{-1}\left(\frac{1 - \eta_u^m}{\eta_u^m}(r + \delta)\right) > \underline{\pi} \tag{26}$$

Resource maximization requires the planner to follow an active monetary policy by raising π above $\underline{\pi}$. The corresponding inflation target is decreasing in the elasticity of the matching function with respect to unemployed workers. If the numbers of unemployed workers and vacancies are gross complements in the matching process, then a low elasticity is associated with a higher unemployment rate relative to vacant positions. In that case the optimal inflation target rises with the unemployment rate suggesting that optimal policy is countercyclical.

When $\eta_u^m > \beta$, workers retain too low a share of the match surplus and employment is super-optimal: $\theta^H < \theta^* < \theta^{max}$. Because the inflation rate is already low in unconstrained equilibria, an active monetary policy which raises the inflation rate high enough to stimulate employment would worsen welfare by raising employment further above optimality. Therefore the optimal policy calls for a low inflation target $\pi^* < \underline{\pi}$. If inflation happens to be high

[9]The marginal value of an unemployed worker is the product of marginal matches created and the surplus each match generates: $m_u(u, v) \times S$, while the private return equals the product of the job-finding probability and the workers' share of the surplus $m(u, v)/u \times \beta S$. Equating the two gives: $\beta = \eta_u^m$. Similarly, $\beta = (1-\eta_v^m)$ follows from the social value of vacancies and firms' posting incentives: $m_v(u, v) \times S = m(u, v)/v \times (1 - \beta)S$.

enough that the economy is constrained when $\eta_u^m > \beta$, then policy could improve welfare by lowering the inflation target.

We now turn to the monetary policy and analyze the potential welfare tradeoffs between employment and price stability objectives.

4.3 Monetary Authority's Problem

The objective of the monetary authority is to maximize the representative household's welfare by choosing the growth rate of money supply. The set of constraints on policy choices include a government balanced budget and equilibrium conditions for agents' choices.[10] This methodological approach to optimal policy closely follows the primal approach [Lucas and Stokey, 1983].

We define $\tau \equiv \frac{\dot{N}}{N}$ as the growth of the nominal money supply. Hence, real balances satisfy $\dot{n} = (\tau - \pi)n$. With pure rebate of seignorage as a lump-sum transfer, the government budget balance requires:

$$T = \tau n \tag{27}$$

From workers and firms' optimal labor market choices, we obtain the long-run Phillips curve, as presented by $\pi = \chi(\theta)$ and equation (21). From the household's problem, we obtain the optimal intra- and intertemporal substitutability conditions, given by equation (13) and the Euler equation. Finally, following the methodology presented by Chari and Kehoe [1999], we construct the implementability constraint by aggregating intertemporally the household's budget constraint, while combining the household's optimal choices. Details on our derivations are presented in Appendix Section B.2.

Formally, the monetary authority problem is

$$\max_{\{\theta(t),\tau(t),c(t)\}_{t\geq 0}} \int_0^\infty e^{-\rho t} U(c,n)dt \tag{28}$$

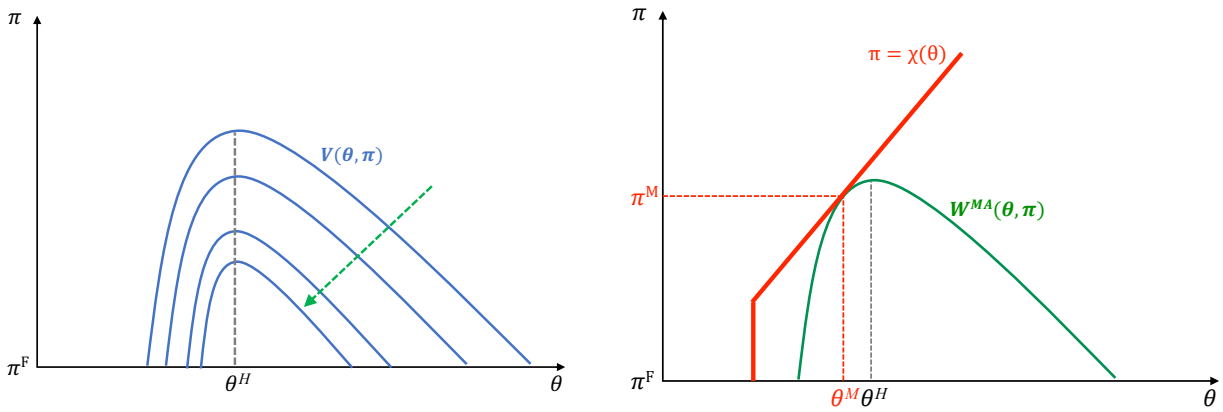
subject to:

$$\begin{cases} U_m - (r + \chi(\theta))U_c = 0 & (IT) \\ \dot{s} = (\rho - r)s & (EQ) \\ s - U_c = 0 & (SR) \\ \dot{\mathcal{A}} = \rho\mathcal{A} - \Psi & (IC) \\ \dot{u} = \delta(1-u) - up(\theta) & (LM) \\ \dot{n} = (\tau - \chi(\theta))n & (MS) \\ y(\theta,u) - c = 0 & (RC) \end{cases}$$

$$n \geq 0, \qquad \theta \geq 0$$

Constraints (IT), (EQ), (SR), and (IC) impose the household's optimality conditions. The intratemporal and intertemporal substitutions of utility are pinned down by constraints (IT), which gives money demand to consumption ratio, and (EQ), which is the Euler's equation. To simplify the problem, we augment the state space by introducing $s(t)$ as an

[10] While our formulation is a literal interpretation of the stated policy instrument, *"long run growth of the monetary and credit aggregates"*, the problem can equivalently be stated as a choice of the nominal interest rate, which is closer to Fed's current practice.



(a) Indifference Curves and Planner's Allocation

(b) Monetary Authority's Constrained Allocation

Figure 2: Monetary Authority's Policy Problem

independent state and impose the identity $s(t) = U_c(c(t), n(t))$ as a path equality constraint (SR).[11] Constraint (IC) gives us implementability, where $\mathcal{A}(t) \equiv \int_t^\infty e^{-\rho(s-t)}\Psi(t)dt$ and $\Psi(t)$ is defined as:

$$\Psi(t) \equiv U_c c + U_n n - U_c I(\theta, u) - U_c \tau n \tag{29}$$

Constraint (LM) imposes the restrictions given labor market's frictional nature and it governs transitional deviations from the steady-state unemployment rate (off the Beveridge Curve). Finally, restriction (MS) imposes the law of motion for real balances, and (RC) the resource constraint. Given that government balance constraint is always satisfied, we substitute (27) in the monetary authority's problem. Similarly, since we show in Lemma B.1 that the Phillips curve is always satisfied, we substitute it in the monetary authority's problem in order to simplify the presentation.

We start by analyzing the first-best allocation, which we dub as the social planner's problem. Differently from the monetary authority, the planner is not constrained by the households' and firms' optimal choices. In other words, the only constraints faced by the planner are (LM) and (RC). As a result, the mechanism through which inflation impacts vacancy creation – by incentivizing firms to post vacancies – is severed. As a result, the planner's optimal choice is to separately optimize π and θ. Recall that the labor market, which effectively determines the resources available for consumption, can be analyzed in isolation of the household's choices money balances. Consequently, the social planner sets $\theta = \theta^H$ to maximize available resources (Hosios condition) and $\pi = \pi_{min}$ to minimize the welfare costs of inflation (Friedman rule).

First-best allocation is illustrated in Figure 2-a. the blue curves show the planner's indifference curves between inflation and activity, as measured by labor market tightness.

[11] In the augmented Hamiltonian, s is an independent argument, so first-order conditions take partial derivatives holding s fixed; the dependence of s on (c, n) is captured solely through the constraint term $\mu_E(U_c(c(t), n(t)) - s(t))$, where μ_E is a multiplier. For more details, see Kamien and Schwartz [2012], Part II, Section 2, pages 125–127.

The preferences are satiated around (π^F, θ^H). Lemma 2 formalizes this result.

Lemma 2. *In this economy, a planner would optimally choose $\theta = \theta^H$ and $\pi = \pi_{min}$, where θ^H is given by the Hosios condition and π_{min} is given by the Friedman rule.*

Moving to the monetary authority problem, let's initially consider the case in which $\chi'(\theta) \leq 0$, where inflation is neutral due to unconstrained wage setting. In this case, because inflationary policy fails to raise economic activity, implementing the Friedman rule $\pi = \pi_{min}$ maximizes welfare. Under the Friedman rule $\pi_{min} < \underline{\pi}$, money is neutral and the monetary authority has no influence on vacancy creation. As a result, $\theta = \theta^*$. The following lemma summarizes this result.

Lemma 3. *If $\chi'(\theta) \leq 0$, then $\pi^M = \pi_{min}$ and $\theta^M = \theta^*$.*

In contrast, in our model, given equation (21), we have that, for $\theta > \theta^*$:

$$\chi'(\theta) = \frac{(r+\delta)\kappa}{\lambda'\left(\lambda^{-1}\left(\frac{\kappa[r+\delta+p(\theta)](r+\delta)}{(y-z)q(\theta)-\kappa(r+\delta)}\right)\right)} \frac{\left\{\begin{array}{c} p'(\theta)\left[(y-z)q(\theta)-\kappa(r+\delta)\right] \\ -(y-z)q'(\theta)\left[r+\delta+p(\theta)\right] \end{array}\right\}}{\left[(y-z)q(\theta)-\kappa(r+\delta)\right]^2} > 0$$

since $(y-z)q(\theta) - \kappa(r+\delta) > 0$, $p'(\theta) = m_v > 0$, $q'(\theta) < 0$, and $\lambda'(\pi) > 0$.

Solving the monetary authority's problem, we obtain θ^M through the following implicit equation:

$$\left\{\begin{array}{c} \left[\begin{array}{c} [2U_{cn} - U_{cc}\frac{U_n}{U_c}]U_n \\ +(U_{nn}U_{cc} - U_{cn}^2)rn - U_cU_{nn} \end{array}\right] u\left[\left(\frac{y-z+\theta\kappa}{r+\delta+p(\theta)}\right)q(\theta)\eta_v^m - \kappa\right] \\ -\left[U_n - rnU_{cn}\right]\chi'(\theta)U_c \end{array}\right\} = 0 \tag{30}$$

We then substitute the slope of the household's dynamic indifference curve ($\frac{d\pi}{d\theta}$), presented in equation (A.33), and rearrange to get the following optimality condition:

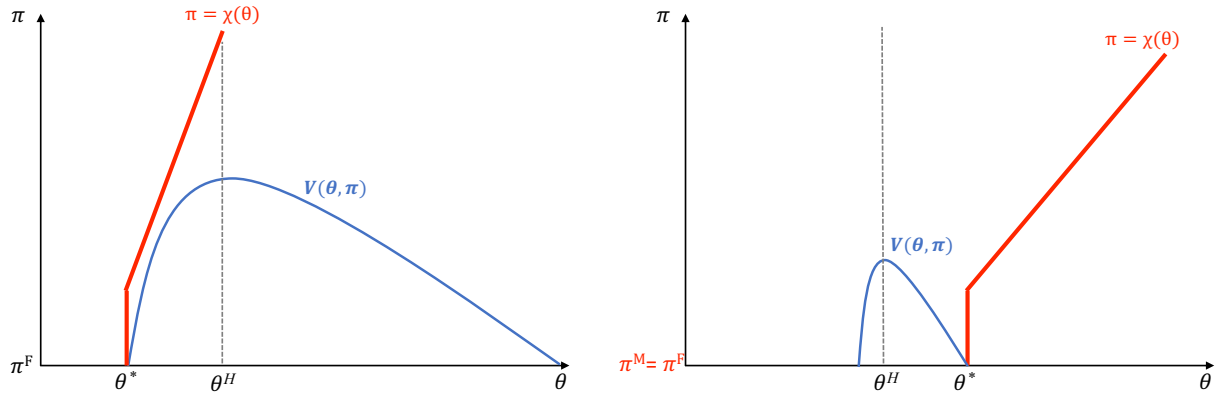
$$\chi'(\theta) = \mathcal{W} \times \frac{d\pi}{d\theta} \tag{31}$$

where $\mathcal{W}$ is the monetary authority wedge defined as:[12]

$$\mathcal{W} \equiv \frac{\left[\begin{array}{c} [2U_{cn} - U_{cc}\frac{U_n}{U_c}]U_n \\ +(U_{nn}U_{cc} - U_{cn}^2)rn - U_cU_{nn} \end{array}\right] n}{\left[U_n - rnU_{cn}\right]} \tag{32}$$

$\mathcal{W}$'s numerator summarizes how the household values a tightening-induced resource gain once both the intratemporal margin between consumption and real balances and the intertemporal shadow value of liquidity are taken into account. In contrast, $\mathcal{W}$'s denominator

[12]This wedge arises because the monetary authority solves the full dynamic Ramsey problem, not a static steady-state comparison. The steady state induced by the optimal transition generally differs from the steady state that maximizes static welfare. Equivalently, the authority optimizes over implementable (θ, π) along decentralized equilibrium paths – an implementability constraint absent from the household's problem and its (steady-state) indifference curve.



(a) Steep (or Vertical) Long-run Phillips Curve (b) Low Inflation - Low Unemployment Economy

Figure 3: Cases for Inflation-Focused Monetary Policy

Notes.— Figure displays two situations under which the optimal policy only targets a low level of inflation (single-mandate).

measures the marginal welfare cost of inflation, net of the fact that money and consumption may be complements. Hence, $\mathcal{W}$ summarizes how the full dynamic valuation of tighter labor markets differs from the household's compensation slope alone.

Taken together, (30)–(32) imply that the case for active inflation policy is inherently conditional. Even when higher inflation relaxes the labor-market distortion, the desirability of doing so depends on the sign and magnitude of $\mathcal{W}$, which captures the dynamic welfare cost of moving away from the Friedman-rule allocation. This allows the monetary authority's problem to be summarized as a trade-off between implementable employment gains and the utility cost of distorting real balances.

As a result, the monetary authority finds it desirable to use inflation actively only when the implementable employment gain from moving to a tighter labor market is large enough to offset the utility cost of higher inflation, once this trade-off is adjusted by the wedge $\mathcal{W}$. In that case, the authority chooses an interior allocation on the constrained branch, using inflation to raise θ^* above its decentralized level while still stopping short of the Hosios benchmark. By contrast, when the implementable Phillips curve is too steep or the wedge makes the dynamic welfare cost of inflation too large, the authority optimally places no independent weight on raising employment and instead focuses on the inflation objective alone. This distinction underlies the cases illustrated below.

Figure 2-b shows an interior optimum in the monetary authority problem. The green curve, $W^{MA}(\pi, \theta)$, is a level set of the authority's welfare criterion over implementable (π, θ) pairs. $W^{MA}(\pi, \theta)$ is a graphical summary of the full dynamic Ramsey problem after the remaining controls and states have been solved out. At the optimum, we have:

$$\chi'(\theta) = -\frac{W_\theta^{MA}}{W_\pi^{MA}} = \mathcal{W}\frac{d\pi}{d\theta}$$

in the case shown, $\theta^* < \theta^H$, so the authority finds it desirable to move the economy to a tighter labor market and a higher inflation allocation, while still stopping short of the Hosios

benchmark. This makes it desirable for the monetary authority to promote employment with active inflationary policy. This involves dual policy objectives $u(\theta^M)$ and $\pi^M > \pi_F$, reflecting the optimal balance between the employment gains from higher inflation and the welfare costs of distorting money holdings. The employment objective in this case lowers the unemployment rate, but remains above the resource-maximizing level: $u(\theta^*) > u(\theta^M) > u(\theta^H)$.

Figure 3 shows two situations where it is best to focus solely on the inflation objective. Panel (a) depicts a case where θ^* is sub-optimally low $< \theta^H$, but the implementable Phillips curve is too steep for the welfare gains from higher employment to justify the associated increase in inflation. Panel (b) shows a case where the $\theta^* > \theta^H$, so the low-inflation allocation already implies sufficiently strong labor-market performance. In both cases, the authority optimally places no independent weight on raising employment and instead focuses solely on the inflation objective

5 Quantitative Analysis (Preliminary)

To illustrate the quantitative implications of our findings, we calibrate the model economy to the pre-pandemic U.S. economy. We follow Lucas [2000] in setting the utility function as:

$$U(c,n) = \frac{u_0}{1-\sigma}\left[c\varphi\left(\frac{n}{c}\right)\right]^{1-\sigma}$$

The household's optimality conditions yield the money demand function $n/c = N_0(r+\pi)^{-\eta_m}$. Following Lucas [2000], we set $N_0 = 0.27$ and $\eta_m = 0.5$ to approximate the empirical money demand curve for the US.

We assume that the matching function is given by:

$$m(u,v) = A \times \frac{uv}{u+v},$$

which displays varying elasticities with respect to vacancies and unemployed: when the labor market is tight (v/u high), fewer matches are generated by additional vacancies.[13]

We calibrate the labor market at monthly frequency and set the following parameters externally. Labor productivity is normalized to $y = 1$ and the consumption equivalent value of unemployment is set to $z = 0.7$. The separation rate is $\delta = 0.036$ following Shimer [2005] and the real interest rate is calibrated to an annual value of 4 percent. We set the monthly renegotiation rate $\phi = 0.19$, which implies 90 percent of workers renegotiate their contracts with their employer at least once in a given year. Table 1 summarizes the parameters and the corresponding targets or sources.

We calibrate the remaining parameters internally. To position the economy on the empirical Beveridge curve, we target an unemployment rate of 5 percent when $v/u = 0.75$. To capture the empirical relationship between inflation and real wage depreciation, we target a ratio of 0.45. We target an annual inflation rate of 1.5 percent. Finally, we target the slope of the empirical relationship between the inflation rate and labor market tightness.

[13] We deviate from the commonly used Cobb-Douglas form, which provides a poor fit to the empirical Beveridge Curve during tight labor markets.

Parameter	Value	Target	Value
Labor productivity (y)	1.0	Normalized	—
Unemp. utility (z)	0.7	Shimer [2005]	0.7
Discount rate (%)	0.34	Real interest rate (% annual)	4.0
Money demand level (A_m)	0.27	Mean M1/GDP [Lucas, 2000]	0.27
Money demand elasticity (η)	0.5	M1/GDP by GDP[Lucas, 2000]	0.5
Renegotiation rate (ϕ)	0.19	Annual probability of ≥ 1 (%)	90.0

Table 1: Externally Calibrated Parameters

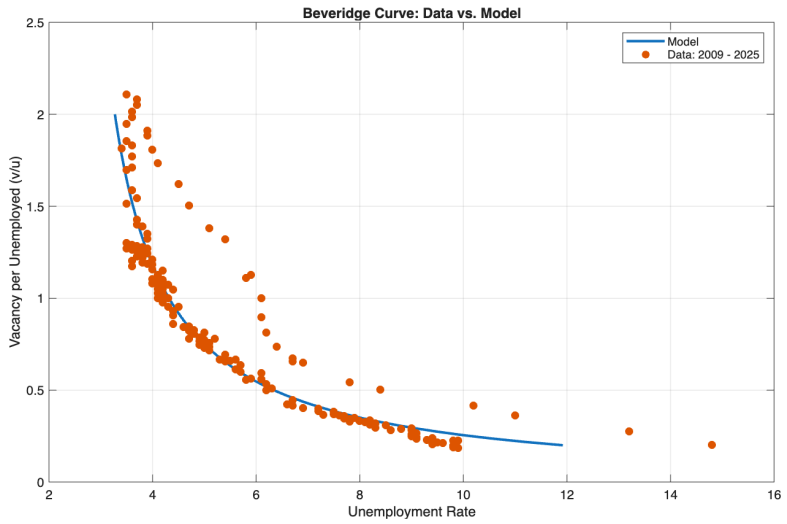


Figure 4: Beveridge Curve

Notes.– The model is calibrated to generate 5 percent unemployment at vacancy-to-unemployment ratio of 0.75. Monthly data values are taken from the BLS.

The calibrated model successfully replicates key features of the U.S. labor market and the empirical relationship between inflation and employment. The model generates a downward-sloping Beveridge curve consistent with the data, and a non-linear Phillips curve that is flat at low inflation rates and negatively sloped at higher rates.

The quantitative results indicate that maximum employment (minimum unemployment of approximately 1.9%) would require prohibitively high inflation rates approaching infinity. In contrast, optimal employment from a real perspective (Hosios condition) corresponds to an unemployment rate of approximately 3.3%, which could be implemented with an inflation rate of 4.9%. When accounting for the welfare costs of inflation through the monetary authority's problem, the optimal policy target involves a somewhat higher unemployment rate of 4.1% and a more moderate inflation rate of 2.8%.

These findings illustrate the key trade-offs in setting monetary policy under the dual mandate. While it is theoretically possible to push unemployment below its natural rate through higher inflation, the welfare costs of doing so quickly become prohibitive. The

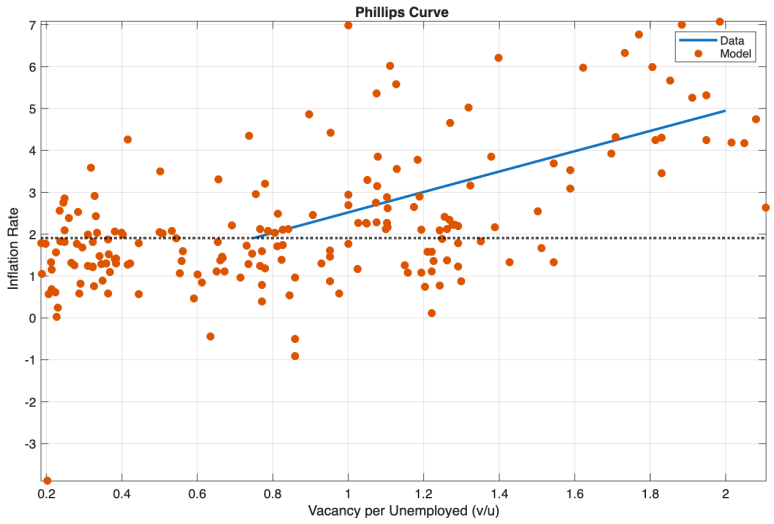


Figure 5: Inflation Rate and Labor Market Tightness

Notes.– Monthly data values for PCE inflation are taken from the FRED.

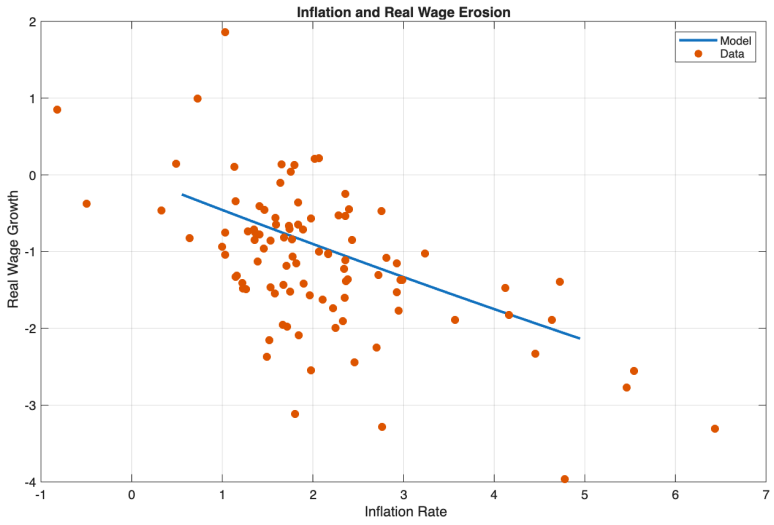


Figure 6: Inflation and Real Wage Growth

Notes.– Real wage growth reflects annualized quarterly growth rate in the employment cost index net of PCE inflation and average productivity growth of 2 percent during the sample period.

optimal policy balances the benefits of lower unemployment against the costs of higher inflation, resulting in targets that are more moderate than those that would emerge from focusing solely on either objective in isolation.

6 Conclusion

This paper develops a framework for the Federal Reserve's employment mandate by formalizing the concepts of maximum, optimal, and equilibrium employment within a frictional labor market model. Our analysis yields several key insights.

First, we show that maximum employment—defined as the lowest feasible unemployment rate consistent with voluntary labor market participation—can only be achieved by creating many low-paying jobs where employers capture the entire match surplus. Implementing this objective requires prohibitively high inflation rates, making it an impractical policy target.

Second, we demonstrate that optimal employment, which balances the costs and benefits of job creation, can be implemented with moderate inflation targets that depend systematically on labor market fundamentals. The optimal inflation rate is lower when matching efficiency is high, separation rates are low, or real interest rates are low.

Third, our framework provides a theoretical foundation for asymmetric monetary policy responses to employment conditions. When labor market fundamentals generate naturally low unemployment, policy should focus exclusively on price stability. However, when fundamentals imply high unemployment at low inflation, optimal policy calls for moderate inflation to raise employment toward socially optimal levels. This asymmetry emerges from the structure of labor market frictions rather than from concerns about the zero lower bound or asymmetric costs of policy errors.

Finally, our analysis suggests that optimal policy targets are endogenous to labor market conditions and should be updated as these fundamentals evolve over time. Secular trends in matching efficiency, separation rates, or real interest rates necessitate corresponding adjustments to long-run inflation targets.

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A Mathematical Appendix

This appendix contains the proofs for the propositions and derivations for some of the equations in the main text. We first list the equations that describe the model for convenience. The model is summarized by the following value functions:

$$\begin{aligned} rV &= -\kappa + q(\theta)(J_0 - V) \\ rJ_0 &= y - w + \delta(V - J_0) + \lambda(\pi)(J - J_0) \\ rJ &= y - \underline{\mathrm{w}} + \delta(V - J) + \phi(J_0 - J) \\ rU &= z + p(\theta)(W_0 - U) \\ rW_0 &= w + \delta(U - W_0) + \lambda(\pi)(W - W_0) \\ rW &= \underline{\mathrm{w}} + \delta(U - W) + \phi(W_0 - W) \end{aligned}$$

and the following equilibrium conditions and assumptions:

$$\begin{aligned} V &= 0 \\ W &= U \\ \beta(J_0 - V) &= (1 - \beta)(W_0 - U) + \mu, \end{aligned}$$

where $\mu \geq 0$ is the multiplier on the firm's credit constraint. Corresponding to these 9 equations, the steady state consists of 9 unknowns: 6 values, two wage rates, w and $\underline{\mathrm{w}}$, and labor market tightness, θ.

A.1 Proof of Lemma 1

Proof. The difference between equations (8) and (9) and the difference between (6) and (5) gives:

$$J - J_0 = \frac{w - \underline{\mathrm{w}}}{r + \delta + \phi + \lambda(\pi)} = W_0 - W. \tag{A.1}$$

□

A.2 Proof of Proposition 1

Proof. First note that:

$$W_0 - W = W_0 - U = \frac{\beta}{1 - \beta} \frac{\kappa}{q(\theta)}.$$

The first equality is by assumption. The second equality uses the Nash bargaining equation along with the free entry condition that implies $V = 0$ and $J_0 = \kappa / q(\theta)$:

$$W_0 - U = \frac{\beta}{1 - \beta}(J_0 - V) = \frac{\beta}{1 - \beta} J_0 = \frac{\beta}{1 - \beta} \frac{\kappa}{q(\theta)}$$

Substituting for $W_0 - U$ from the above in equation (4) gives:

$$rU = z + p(\theta) \frac{\beta}{1 - \beta} \frac{\kappa}{q(\theta)}$$

Then, substituting for rU and $W_0 - W$ from the equations above gives the following representation for the wage equation:

$$w = \beta y + (1-\beta)z + \beta\kappa\theta + \lambda(\pi)\frac{\beta}{1-\beta}\frac{\kappa}{q(\theta)}$$

□

A.3 Proof of Proposition 2

Proof. Simply substitute the solution for the wage rate from (17) and $J - J_0 = W_0 - W = \frac{\beta}{1-\beta}\frac{\kappa}{q(\theta)}$ in the job creation condition:

$$\begin{aligned}\frac{y - w + \lambda(\pi)(J - J_0)}{r+\delta} &= \frac{y - \beta y - (1-\beta)rU - \lambda(\pi)(W_0 - W) + \lambda(\pi)(J - J_0)}{r+\delta} \\ &= \frac{(1-\beta)(y - rU)}{r+\delta} \\ &= \frac{(1-\beta)(y-z)}{r+\delta} - \frac{\beta(p(\theta))}{r+\delta}\frac{\kappa}{q(\theta)} = \frac{\kappa}{q(\theta)},\end{aligned}$$

where the last equality follows from the job creation condition. Rearranging terms gives:

$$\frac{\kappa}{q(\theta)}\left[1 + \frac{\beta(p(\theta))}{r+\delta}\right] = \frac{\kappa}{q(\theta^*)} \times \frac{r+\delta+\beta p(\theta^*)}{(1-\beta)(r+\delta)} = \frac{y-z}{r+\delta}$$

□

A.4 Proof of Proposition 3

Proof. Setting $w = y$ in the job creation condition (15) gives:

$$\frac{y - y + \lambda(\pi)(J - J_0)}{r+\delta} = \frac{\kappa}{q(\theta)}$$

Subtracting J_0 from J:

$$J - J_0 = \frac{w - \underline{w}}{r + \phi + \lambda(\pi) + \delta}$$

Setting $W = U$ gives $\underline{w}$:

$$\underline{w} = z + (p(\theta) - \phi)(W_0 - U) = z + (p(\theta) - \phi)\frac{w - z}{r + \delta + \lambda(\pi) + \phi}$$

where the last equality follows from subtracting rU from rW_0. Subtracting both sides from w gives:

$$w - \underline{w} = (w - z)\frac{r + \delta + \lambda(\pi) + \phi}{r + \delta + \lambda(\pi) + p(\theta)}$$

Note, when $\phi = p(\theta)$, then $\underline{w} = z$ as expected.

Setting $w = y$ and plugging this back in for $J - J_0$ in the job creation condition gives:

$$\frac{\lambda(\pi)}{r + \delta + \lambda(\pi) + p(\theta)}\frac{(y-z)}{r+\delta} = \frac{\kappa}{q(\theta)},$$

which holds for $\pi > \underline{\pi} > 0$. □

A.5 Proof of Proposition 4

Proof. The lowest value that will not deter unemployed workers from searching for a job in the first place requires $W_0 = U$. Using equations (5) and (4), the added value to a worker from a job is:

$$W_0 - U = \frac{w - z - \lambda(\pi)(W_0 - W)}{r + \delta + p(\theta)}$$

Setting that to zero gives the lowest acceptable wage to the worker: $w = z + \lambda(\pi)(W_0 - W)$. Substituting it back in the job creation condition in (15) gives the maximum attainable labor market tightness:

$$\frac{y - z + \lambda(\pi)(J - J_0 - W_0 + W)}{r + \delta} = \frac{\kappa}{q(\theta)},$$

where $J - J_0 - W_0 + W = 0$ by Lemma 1. □

A.6 Proof of Proposition 6

Proof. First note that $\pi^* > \underline{\pi}$ whenever $\eta_u^m < \beta$. Therefore the equilibrium is constrained and $w = y$. Comparing equations (20) and (25), labor market tightness is optimal if and only if:

$$\frac{r + \delta + \lambda(\pi) + p(\theta^H)}{\lambda(\pi)} = \frac{r + \delta + \eta_u^m p(\theta^H)}{\eta_v^m (r + \delta)}$$

Solving for $\lambda(\pi)$ and considering that $\lambda(\cdot)$ is monotonic, gives the condition in the proposition. □

B Additions to the Appendix

B.1 Additional Result – Phillips Curve

Lemma B.1. *The long-run Phillips Curve is always satisfied*

Proof. Consider the case in which $\pi > \underline{p}$. Then, the out of steady state system of Bellman equations is given by:

$$\begin{cases} rV = -\kappa + q(\theta)[J_0 - V] + \dot{V} & (V) \\ rJ_0 = \delta(V - J_0) + \lambda(\pi)(J - J_0) + \dot{J}_0 & (J_0) \\ rJ = y - \underline{w} + \delta(V - J) + \phi(J_0 - J) + \dot{J} & (J) \\ rU = z + p(\theta)(W_0 - U) + \dot{U} & (U) \\ rW_0 = y + \delta(U - W_0) + \lambda(\pi)(W - W_0) + \dot{W}_0 & (W_0) \\ rW = \underline{w} + \delta(U - W) + \phi(W_0 - W) + \dot{W} & (W) \end{cases}$$

as pointed out by Pissarides [2000], since firms can open and close vacancies instantaneously, we must always have that $V = \dot{V} = 0$. Hence, from (V), we have:

$$J_0 = \frac{\kappa}{q(\theta)} \tag{A.2}$$

while from (J_0) and (J), we have:

$$\dot{J}_0 = -\lambda(\pi)J + (r + \delta + \lambda(\pi))J_0 \tag{A.3}$$

and

$$\dot{J} = -(y - \underline{w}) + (r + \delta + \phi)J - \phi J_0 \tag{A.4}$$

substituting (A.2) and (A.4), we have:

$$\dot{J} = -(y - \underline{w}) - \phi\frac{\kappa}{q(\theta)} + (r + \delta + \phi)J \tag{A.5}$$

Moreover, since $W = U$, from $(W) - (U)$, we have:

$$0 = \underline{w} - z - (p(\theta) - \phi)(W_0 - U)$$

therefore:

$$\underline{w} = z + (p(\theta) - \phi)(W_0 - U) \tag{A.6}$$

Similarly, from $(W_0) - (U)$, we have:

$$r(W_0 - U) = y - z - \delta(W_0 - U) - p(\theta)(W_0 - U) - \lambda(\pi)(W_0 - U) + \dot{W}_0 - \dot{U}$$

Rearranging it:

$$\dot{W}_0 - \dot{U} = -(y - z) + (r + \delta + \lambda(\pi) + p(\theta))(W_0 - U) \tag{A.7}$$

From (A.5) − (A.3), we have:

$$\dot{J} - \dot{J}_0 = -(y - \underline{w}) + (r + \delta + \lambda(\pi) + \phi)(J - J_0) \tag{A.8}$$

substituting (A.6), we have:

$$\dot{J} - \dot{J}_0 = -y + z + (p(\theta) - \phi)(W_0 - U) + (r + \delta + \lambda(\pi) + \phi)(J - J_0) \tag{A.9}$$

From (A.9) − (A.7), we have:

$$\dot{J} - \dot{J}_0 - \dot{W}_0 + \dot{U} = (r + \delta + \lambda(\pi) + \phi)(J - J_0 - W_0 + U) \tag{A.10}$$

Define $J = S$ and $S_0 = J_0 + W_0 - U$, we have:

$$\dot{S} - \dot{S}_0 = (r + \delta + \lambda(\pi) + \phi)(S - S_0) \tag{A.11}$$

Notice that the surpluses S and S_0 represent the match surplus in States 1 and 0. However, the only difference between the two is how the surplus is distributed and not its value. Hence $S = S_0$, $\forall t$. Therefore, $\dot{S} - \dot{S}_0 = 0$, $\forall t$.

Then, from $S - S_0 = 0$, we can show that:

$$(r + \delta)\kappa(r + \delta + \lambda(\pi) + p(\theta)) = \lambda(\pi)(y - z)q(\theta)$$

so the long-run Phillips curve is always satisfied. □

B.2 Derivations – Implementability Restrictions

In order to simplify notation and our calculations, let's define assets at time t as $a(t) \equiv b(t) + n(t)$. As a result, the household problem becomes:

$$\max_{\{\theta,c,n\}} \int_0^\infty e^{-rt} U(c,n) dt$$

subject to:

$$\begin{cases} \dot{a} = y - u(y - z + \kappa\theta) + T - c + ra - (r+\pi)n \\ \dot{u} = (1-u)\delta - up(\theta) \end{cases}$$

Then, the F.O.C.s are:

$$\begin{cases} U_c - \mu_a = 0 & (\mathcal{H}_c) \\ U_n - \mu_a(r+\pi) = 0 & (\mathcal{H}_n) \\ r\mu_a = -\dot{\mu}_a + \rho\mu_a & (\mathcal{H}_a) \\ -\mu_a(y - z + \theta\kappa) - \mu_u(\delta + p(\theta)) = -\dot{\mu}_u + \rho\mu_u & (\mathcal{H}_u) \end{cases}$$

and the transversality conditions:

$$\lim_{t\to\infty} e^{-\rho t}\mu_a(t)a(t) = 0 \tag{A.12}$$

and

$$\lim_{t\to\infty} e^{-\rho t}\mu_u(t)u(t) = 0 \tag{A.13}$$

Now, let's take the asset law of motion and rearrange:

$$c + (r+\pi)n - y + u(y - z + \kappa\theta) - T = ra - \dot{a}$$

to simplify notation, define $y(\theta, u) \equiv y - u(y - z + \kappa\theta)$. Then, substituting $y(\theta, u)$, multiplying both sides by $e^{-\rho t}\mu_a$ and integrating over $[0, \bar{T}]$, we have:

$$\int_0^{\bar{T}} e^{-\rho t}\mu_a \left[c + (r+\pi)n - y(\theta,u) - T\right] dt = \int_0^{\bar{T}} e^{-\rho t}\mu_a \left[ra - \dot{a}\right] dt$$

substituting $(\mathcal{H}_c)$ and $(\mathcal{H}_n)$ on the left hand side, we have:

$$\int_0^{\bar{T}} e^{-\rho t} \left[U_c c + U_n n - U_c y(\theta,u) - U_c T\right] dt = \int_0^{\bar{T}} e^{-\rho t}\mu_a \left[ra - \dot{a}\right] dt \tag{A.14}$$

Then, notice that:

$$\frac{d}{dt} e^{-\rho t}\mu_a a = e^{-\rho t}\left\{\dot{a}\mu_a + \dot{\mu}_a a - \rho\mu_a a\right\}$$

substituting $\dot{\mu}_a = (\rho - r)\mu_a$ and rearranging, we have:

$$\frac{d}{dt} e^{-\rho t}\mu_a a = -e^{-\rho t}\mu_a(ra - \dot{a})$$

Therefore, substituting it in the right hand side of (A.14), we have:

$$\int_0^{\bar{T}} e^{-\rho t}\mu_a \left[ra - \dot{a}\right] dt = \mu_a(0)a_0 - e^{-\rho\bar{T}}\mu_a(\bar{T})a(\bar{T})$$

from the transversality condition (A.12), we have that $\lim_{\bar{T}\to\infty} e^{-\rho\bar{T}}\mu_a(\bar{T})a(\bar{T}) = 0$. Hence, the implementability constraint becomes:

$$\int_0^{\infty} e^{-\rho t}\left[U_c c + U_n n - U_c y(\theta, u) - U_c T\right] dt = U_c(c(0), n(0))a_0. \tag{A.15}$$

Finally, let's transform the implementability constraint into an auxiliary state variable $\mathcal{A}(t)$. Define $\Psi(t)$ as:

$$\Psi(t) \equiv U_c c + U_n n - U_c y(\theta, u) - U_c T \tag{A.16}$$

then, we define $\mathcal{A}(t) \equiv \int_t^{\infty} e^{-\rho(s-t)}\Psi(t)dt$. As a result, the implementability constraint can be rewritten as:

$$\dot{\mathcal{A}} = \rho\mathcal{A} - \Psi \tag{A.17}$$

and $\lim_{t\to\infty} \mathcal{A}(t) = 0$.

Finally, from $(\mathcal{H}_c)$ and $(\mathcal{H}_a)$, we have:

$$\frac{dU_c}{dt} = (\rho - r)U_c \tag{A.18}$$

which is the Euler equation that highlights the intertemporal substitutability by households.

B.3 Derivations – Monetary Authority Problem

Let us define $\tau(t) \equiv \frac{\dot{N}}{N}$. Therefore, real balances satisfy $\dot{n} = (\tau - \pi)n$. With pure rebate of seignorage as a lump-sum transfer, we have:

$$h(t) = \tau(t)n(t) \tag{A.19}$$

Now, we must also take into account the intertemporal substitutability. We will do this by introducing a auxiliary variable $s = U_c$. Henceforth, (A.18) becomes:

$$\dot{s} = (\rho - r)s$$

In particular, we augment the state space by introducing $s(t)$ as an independent state and impose the identity $s(t) = U_c(c(t), n(t))$ as a path equality constraint. In the augmented Hamiltonian, s is an independent argument, so first-order conditions take partial derivatives holding s fixed; the dependence of s on (c, n) is captured solely through the constraint term $\mu_E(U_c(c(t), n(t)) - s(t))$, where μ_E is a multiplier.[14]

Furthermore, consider the Phillips curve $\pi(t) = \chi(\theta(t))$. As shown in lemma B.1, the Phillips curve is always satisfied. Therefore, the most compact monetary authority primal approach problem is given by:

$$\max_{\{\theta(t),\tau(t),c(t)\}_{t\geq 0}} \int_0^{\infty} e^{-\rho t}U(c, n)dt \tag{A.20}$$

[14]For more details, see Kamien and Schwartz [2012], Part II, Section 2, pages 125–127.

subject to:

$$\begin{cases} \dot{u} = \delta(1-u) - up(\theta) & (LM) \\ \dot{n} = (\tau - \chi(\theta))n & (MS) \\ \dot{\mathcal{A}} = \rho\mathcal{A} - \Psi & (IC) \\ U_n - (r + \chi(\theta))s = 0 & (IT) \\ \dot{s} = (\rho - r)s & (EQ) \\ s - U_c = 0 & (SR) \\ y(\theta, u) - c = 0 & (RC) \end{cases}$$

$$n \geq 0, \qquad \theta \geq 0$$

Then, the current-value Hamiltonian becomes:

$$\mathcal{H} = \left\{ \begin{array}{c} U(c,n) + \mu_u[\delta(1-u) - up(\theta)] + \mu_c[\rho\mathcal{A} - \Psi] + \mu_s(s - U_c) \\ +\mu_n[U_n - (r + \chi(\theta))s] + \mu_\tau[(\tau - \chi(\theta))n] + \mu_E(\rho - r)s + \mu_R(y(\theta, u) - c) \end{array} \right\}$$

where μ_u, μ_c, μ_n, μ_τ, μ_E, μ_s, and μ_R are the multipliers for the (LM), (IC), (IT) (MS), (EQ), (SR), and (RC) constraints. Moreover, keep in mind that:

$$\Psi(t) \equiv sc + U_n n - sy(\theta, u) - s\tau n$$

Then, the first order conditions are:

$$\begin{cases} U_c - \mu_c U_{nc} n - \mu_c s + \mu_n U_{nc} - \mu_s U_{cc} - \mu_R = 0 & (\mathcal{H}_c) \\ U_n - \mu_c[U_{nn} n + U_n - s\tau] + \mu_n U_{nn} + \mu_\tau(\tau - \chi(\theta)) - \mu_s U_{cn} = -\dot{\mu}_\tau + \rho\mu_\tau & (\mathcal{H}_n) \\ -\mu_u u p'(\theta) - \mu_c s u \kappa - \mu_n \chi'(\theta) s - \mu_\tau \chi'(\theta) n - \mu_R u \kappa = 0 & (\mathcal{H}_\theta) \\ -\mu_u[\delta + p(\theta)] - \mu_c(y - z + \theta\kappa)s - \mu_R(y - z + \theta\kappa) = -\dot{\mu}_u + \rho\mu_u & (\mathcal{H}_u) \\ \mu_c s n + \mu_\tau n = 0 & (\mathcal{H}_\tau) \\ -\mu_c(c - y + u(y - z + \kappa\theta) - \tau n) - \mu_n(r + \chi(\theta)) + \mu_s + \mu_E(\rho - r) = -\dot{\mu}_E + \rho\mu_E & (\mathcal{H}_s) \\ \mu_c \rho = -\dot{\mu}_c + \rho\mu_c & (\mathcal{H}_A) \end{cases}$$

From $(\mathcal{H}_\tau)$, given that $n > 0$, we have $\mu_c s = -\mu_\tau$. Replacing (SR), we have $\mu_\tau = -\mu_c U_c$. Let's then consider the steady-state equilibrium. Since $s = U_c$, $\rho = r$, $\tau = \chi(\theta)$ and $c = y - u(y - z + \kappa\theta)$, the system simplifies to:

$$\begin{cases} U_c - \mu_c U_{nc} n - \mu_c U_c + \mu_n U_{nc} - \mu_s U_{cc} - \mu_R = 0 & (\mathcal{H}_c) \\ U_n - \mu_c[U_{nn} n + U_n - U_c \tau] + \mu_n U_{nn} - \mu_s U_{cn} = \rho\mu_\tau & (\mathcal{H}_n) \\ -\mu_u u p'(\theta) - \mu_c U_c u \kappa - \mu_n \chi'(\theta) U_c - \mu_\tau \chi'(\theta) n - \mu_R u \kappa = 0 & (\mathcal{H}_\theta) \\ -\mu_u[\delta + p(\theta)] - \mu_c(y - z + \theta\kappa)U_c - \mu_R(y - z + \theta\kappa) = \rho\mu_u & (\mathcal{H}_u) \\ \mu_c U_c + \mu_\tau = 0 & (\mathcal{H}_\tau) \\ +\mu_c \chi(\theta) n - \mu_n(r + \chi(\theta)) + \mu_s = \rho\mu_E & (\mathcal{H}_s) \\ \delta(1-u) - up(\theta) = 0 & (LM) \\ U_n - (r + \chi(\theta))U_c = 0 & (IT) \end{cases}$$

Normalization of multipliers. The necessary conditions from the Pontryagin Maximum Principle (henceforth PMP) are homogeneous of degree one in the objective multiplier and

the vector of costates and multipliers: if $(\lambda_0, \{\mu_i(t)\})$ satisfies the maximum principle conditions, then so does $(k\lambda_0, \{k\mu_i(t)\})$ for any $k > 0$. Hence, for a *normal* problem (i.e. $\lambda_0 > 0$), one scalar normalization is without loss of generality. Following the standard convention, we fix the overall scale of the multiplier vector by setting $\lambda_0 = 1$.[15]

In our formulation, the implementability constraint is represented either directly as a present-value restriction with a single constant multiplier (as in the primal approach), or equivalently by the auxiliary state equation $\dot{\mathcal{A}} = \rho\mathcal{A} - \Psi$ with associated costate μ_c, which is constant along the optimum. The level of μ_c is therefore a choice of units for shadow values and is not separately identified by pointwise steady-state conditions; only ratios of multipliers enter the allocation conditions. Accordingly, after fixing the overall PMP normalization, we may (purely as a notational convenience) report other multipliers in units of μ_c (i.e. divide all multipliers by μ_c), which is equivalent to setting $\mu_c = 1$ in the normalized system.

Finally, the costate μ_E associated with the auxiliary state equation $\dot{s} = (\rho - r)s$ is attached to an auxiliary restriction that collapses in steady state when $r = \rho$. As a result, μ_E is not pinned down by steady-state allocation conditions and carries no independent economic content in the steady state; once the global normalization has been fixed, μ_E can be set by convention (e.g. $\mu_E = 0$) without affecting the allocation or the multiplier ratios used in the elimination steps.

Hence, the model further simplifies to:

$$\begin{cases} -U_{nc}n + \mu_n U_{nc} - \mu_s U_{cc} - \mu_R = 0 & (\mathcal{H}_c) \\ U_n - [U_{nn}n + U_n - U_c\chi(\theta)] + \mu_n U_{nn} - \mu_s U_{cn} = -rU_c & (\mathcal{H}_n) \\ -\mu_u up'(\theta) - U_c\left(u\kappa - \chi'(\theta)n\right) - \mu_n\chi'(\theta)U_c - \mu_R u\kappa = 0 & (\mathcal{H}_\theta) \\ -\mu_u[\delta + p(\theta)] - (y - z + \theta\kappa)U_c - \mu_R(y - z + \theta\kappa) = r\mu_u & (\mathcal{H}_u) \\ \chi(\theta)n - \mu_n(r + \chi(\theta)) + \mu_s = 0 & (\mathcal{H}_s) \\ \delta(1 - u) - up(\theta) = 0 & (LM) \\ U_n = (r + \chi(\theta))U_c & (IT) \end{cases}$$

From $(\mathcal{H}_n)$ and (IT), we have:

$$\mu_s U_{cn} = -U_{nn}n + (r + \chi(\theta))U_c + \mu_n U_{nn} \tag{A.21}$$

similarly, from $(\mathcal{H}_s)$, we have:

$$\mu_s = \mu_n(r + \chi(\theta)) - \chi(\theta)n$$

substituting into (A.21), we have:

$$\mu_n = \frac{(r + \chi(\theta))U_c + \chi(\theta)nU_{cn} - U_{nn}n}{(r + \chi(\theta))U_{cn} - U_{nn}} \tag{A.22}$$

Hence:

$$\mu_s = \frac{(r + \chi(\theta))^2 U_c - rU_{nn}n}{(r + \chi(\theta))U_{cn} - U_{nn}} \tag{A.23}$$

[15] This normalization is standard in optimal control derivations; see, e.g., the statement and use of the "normal case" normalization in Weber [2011], where λ_0 is set to one.

from $(\mathcal{H}_c)$, we have $\mu_R = -U_{nc}n + \mu_n U_{nc} - \mu_s U_{cc}$. Substituting (A.22) and (A.23) and rearranging, we have:

$$\mu_R = \frac{(r+\chi(\theta))U_c\left[U_{cn} - U_{cc}(r+\chi(\theta))\right] - rn\left[U_{cn}^2 - U_{nn}U_{cc}\right]}{(r+\chi(\theta))U_{cn} - U_{nn}} \tag{A.24}$$

while from $(\mathcal{H}_u)$, we have $\mu_u = -(U_c + \mu_R)\left(\frac{y-z+\theta\kappa}{r+\delta+p(\theta)}\right)$. Plugging in the above expression for μ_R, we have:

$$\mu_u = -\left(\frac{\left[\begin{array}{c}(r+\chi(\theta))U_c\left[2U_{cn} - U_{cc}(r+\chi(\theta))\right]\\ -rn\left[U_{cn}^2 - U_{nn}U_{cc}\right] - U_cU_{nn}\end{array}\right]}{(r+\chi(\theta))U_{cn} - U_{nn}}\right)\left(\frac{y-z+\theta\kappa}{r+\delta+p(\theta)}\right) \tag{A.25}$$

Finally, from $(\mathcal{H}_\theta)$, after substituting (A.22), (A.24), (A.25), and $p'(\theta) = q(\theta)\eta_v^m$ we have:

$$\left\{\begin{array}{c}\left[\begin{array}{c}[2U_{cn} - U_{cc}\frac{U_n}{U_c}]U_n\\ +(U_{nn}U_{cc} - U_{cn}^2)rn - U_cU_{nn}\end{array}\right]u\left[\left(\frac{y-z+\theta\kappa}{r+\delta+p(\theta)}\right)q(\theta)\eta_v^m - \kappa\right]\\ -\left[U_n - rnU_{cn}\right]\chi'(\theta)U_c\end{array}\right\} = 0 \tag{A.26}$$

Equation (A.26) pins down the optimal θ for the monetary authority. Moreover, notice that, given that $U(c,n)$ is strictly concave and $U_{cn} \geq 0$, we have that:

$$\left[\begin{array}{c}[2U_{cn} - U_{cc}\frac{U_n}{U_c}]U_n\\ +(U_{nn}U_{cc} - U_{cn}^2)rn - U_cU_{nn}\end{array}\right] > 0$$

Hence, if $\chi(\theta) = \chi'(\theta) = 0$, equation (A.26) becomes:

$$y - z = \frac{\kappa}{q(\theta)}\frac{r+\delta+\eta_u^m p(\theta)}{\eta_v^m} \tag{A.27}$$

Therefore, if the Phillips curve did not impose any restriction, the monetary authority would like to impose the Hosios condition.

B.4 Household Problem – Dynamic Indifference Curves

$$\max_{\{c,n\}} \int_0^\infty e^{-rt}U(c,n)dt \tag{A.28}$$

subject to:

$$\left\{\begin{array}{l}\dot{b} + \dot{n} = y - u(y - z + \theta\kappa) + T - c + (i-\pi)b - \pi n\\ \dot{u} = (1-u)\delta - up(\theta)\end{array}\right.$$

solving the Hamiltonian, we obtain the following optimality conditions:

$$\left\{\begin{array}{ll}U_c - \mu_b = 0 & (\mathcal{H}_c)\\ U_n - \pi\mu_b = -\dot{\mu}_b + r\mu_b & (\mathcal{H}_n)\\ (i-\pi)\mu_b = -\dot{\mu}_b + r\mu_b & (\mathcal{H}_b)\\ -\mu_b[y - z + \theta\kappa] - \mu_u(\delta + p(\theta)) = -\dot{\mu}_u + r\mu_u & (\mathcal{H}_u)\end{array}\right.$$

where μ_b and μ_u are the multipliers for the asset and unemployment laws of motion.

From government budget constraint and balanced budget, we have $h = \pi n$, while from equilibrium conditions, we have $c = (1-u)y+uz-\theta\kappa u$. Then, in steady-state, from $(\mathcal{H}_b)$, we have $r = i - \pi$, which is the Fisher equation. Similarly, combining the government balanced budget, the consumption equilibrium condition and the asset law of motion in steady state ($\dot{b} = \dot{n} = 0$), we have that $b = 0$. From $(\mathcal{H}_c)$, $(\mathcal{H}_n)$, and $(\mathcal{H}_b)$ we have:

$$\frac{U_n}{U_c} = i \tag{A.29}$$

while in steady-state, we have:

$$\frac{U_n}{U_c} = r + \pi \tag{A.30}$$

from $(\mathcal{H}_u)$ in steady state, we have:

$$\mu_u = -\frac{U_c[y - z + \theta\kappa]}{r + \delta + p(\theta)} \tag{A.31}$$

from the unemployment law of motion in steady state, we have $u^* = \frac{\delta}{\delta+p(\theta)}$. Combining u^* with the equilibrium condition for c, we have:

$$c^* = y - \frac{\delta}{\delta + p(\theta)}(y - z + \theta\kappa) \tag{A.32}$$

finally, combining (A.30) and (A.32), we are able to pin down n^*.

Let's define $\mathbf{V}(\theta, \pi) \equiv \mathbf{V}(b_0, n_0, u_0; \theta, \pi)$, i.e., the optimized lifetime utility for the household problem, given the initial state (b_0, n_0, u_0), for different values of inflation and labor market tightness. Then a "dynamic indifference curve" in the (θ, π)–plane is the level set $\{(\theta, \pi) : \mathbf{V}(\theta, \pi) = \overline{V}\}$, where $\overline{V} > 0$ is a constant. We consider this "dynamic indifference curve" evaluated at the steady-state. In other words, starting from the steady state, we consider what compensating changes in π offsets the welfare impact of changes in θ, while allowing the household to reoptimize. This reoptimization reintroduces the impact of costates. In our particular case, the shadow value of the unemployment state is discounted at rate r in the costate equation, introducing an additional factor relative to the purely steady-state comparative statics.

Since $\mathbf{V}(\theta, \pi)$ is differentiable, along the level set $d\mathbf{V} = 0$. Henceforth $\mathbf{V}_\theta d\theta + \mathbf{V}_\pi d\pi = 0$. From implicit function theorem and the envelope theorem, we have:

$$\frac{d\pi}{d\theta} = -\frac{\int_0^\infty e^{-\rho t}\left[\mu_b(-u\kappa) - \mu_u p'(\theta)u\right] dt}{\int_0^\infty e^{-\rho t}\mu_b(-n)dt}$$

considering the case of steady-state, we have:

$$\frac{d\pi}{d\theta} = -\frac{u\left[\mu_b\kappa + \mu_u p'(\theta)\right]}{\mu_b n^*}$$

substituting μ_a, μ_u, u, $p'(\theta) = q(\theta)\eta_v^m$ and $\theta p'(\theta) = p(\theta)\eta_v^m$, and simplifying we have:

$$\frac{d\pi}{d\theta} = \frac{\delta\left\{(y-z)q(\theta)\eta_v^m - \kappa\left(r + \delta + (1-\eta_v^m)p(\theta)\right)\right\}}{\left(\delta + p(\theta)\right)\left(r + \delta + p(\theta)\right)n} \tag{A.33}$$

Finally, from (A.33) notice that $\frac{d\pi}{d\theta} = 0$ if:

$$(y - z) = \frac{\kappa}{q(\theta)} \frac{r + \delta + (1 - \eta_v^m)p(\theta)}{\eta_v^m} \tag{A.34}$$

by comparing (A.34) and (25), we can see that $\frac{d\pi}{d\theta} = 0$ at $\theta = \theta^H$.

In order to pin down the dynamic indifference curve, we need first to obtain a close-form solution for n^*. From Lucas [2000], let's assume the following utility function:

$$U(c, n) = \frac{1}{1 - \sigma} \left[c\varphi \left(\frac{n}{c} \right) \right]^{1-\sigma} \tag{A.35}$$

where the function $\varphi(\cdot)$ is given by:[16]

$$\varphi \left(\frac{n}{c} \right) = \left[1 + \frac{N_0^2}{\frac{n}{c}} \right]^{-1}$$

Therefore, solving the household's problem and imposing steady-state, gives us $n^* = \frac{N_0 c^*}{\sqrt{r+\pi}}$, where c^* is given by (A.32). As a result, (A.33) delivers a separable differential equation. Define θ_0 the initial condition (initial steady-state). Hence, the solution of the differential equation gives us:

$$\pi(\theta) = \left[\sqrt{r + \pi(\theta_0)} + \frac{\delta}{2N_0} \int_0^\theta \frac{(y - z)q(s)\eta_v^m - \kappa(r + \delta + \eta_u^m)p(s)}{\left(y - \frac{\delta}{\delta + p(s)}(y - z + \kappa s) \right) (\delta + p(s))(r + \delta + p(s))} \right]^2 - r \tag{A.36}$$

Let's also consider an urn-ball matching function $m(u, v) = \frac{uv}{u+v}$. Hence, we have that $p(\theta) = \frac{\theta}{1+\theta}$, $q(\theta) = \frac{1}{1+\theta}$, $\eta_v^m = \frac{1}{1+\theta}$, and $\eta_u^m = \frac{\theta}{1+\theta}$.

Figure A.1 present the dynamic indifference curves evaluated at the steady state. As expected, they all peak at $\theta = \theta^H$. Furthermore, utility levels increase as we move southeast in the graph.

[16] Lucas [2000] picks the functional form for $\varphi(\cdot)$ in order to obtain a money demand $m(i) = N_0/\sqrt{i}$.

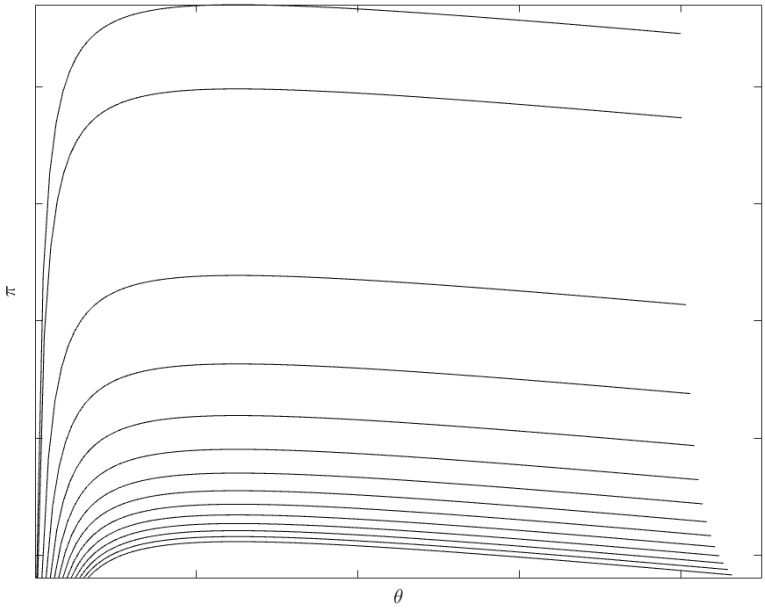


Figure A.1: Dynamic Indifference Curves

Notes.– The figure shows the indifference curves evaluated at the steady state, assuming an urn-ball matching function and the Sidrausky utility function from Lucas [2000].