Spatial Returns to Matching*

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Abstract

The properties of the job finding rate are crucial for unemployment and labor market policies. We estimate the Matching Function for job search using variation in vacancies and unemployment across geographical locations. We find strong and robust evidence for *decreasing returns to scale*, with elasticities on vacancies and unemployment adding up to 0.9. We propose a model of unemployment and job search across local labor markets that allows for decreasing returns to matching. We derive a modified Hosios condition for efficiency and show that it does not hold under directed search or in the random search matching model. We quantify the model to match data on the US labor market.

Keywords: *Matching Function. Unemployment. Vacancies. Job Search. Returns to Scale. Spatial Sorting. Spatial Unemployment*

JEL: *E24. J61. J64. D24*

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1 Introduction

The speed with which workers find jobs is crucial for the unemployment rate and unemployment duration. The job finding rate depends on the number of vacancies and the number of workers searching. More vacancies increases job finding because there is more demand for workers. More workers searching decreases job finding because there is more supply and hence more competition. The matching technology transforms inputs, vacancies and searching workers, into outputs, the probability of finding a job or of filling a vacancy, and depends on matching efficiency and the empirical and theoretical properties of the matching function.

Despite its central importance in theory and applications, remarkably little is known about the properties of the matching technology. In the absence of estimation results of matching functions nearly all papers in the vast literature on unemployment and search assume the constant returns to scale Cobb-Douglas technology. Yet, these properties are critically important for the features of equilibrium search behavior, for unemployment duration and the unemployment rate, for efficiency and welfare, and for optimal unemployment insurance and labor market policies. Open questions are whether models should incorporate returns to scale, what the elasticity of substitution is, whether the data fit a Cobb-Douglas matching technology,...

In this paper, we estimate the matching technology using variation across locations in the number of vacancies and job searchers. Labor markets in large cities have more job searchers and firms post more vacancies. We also observe lower job finding rates. We use tools from the literature on production function estimation.[1] A production technology transforms inputs, capital, labor and intermediate inputs, into outputs, the number of goods and services produced and sold. Similarly, a matching technology transforms the number of vacancies and job searchers in labor market into the number of matches.

The estimation of the matching technology faces a technical and conceptual challenge. One input, vacancies, is a flow (or jump) variable that adjusts quickly, either immediately (see for example Pissarides (2000)) or with a minor delay only. The other input, the number of job searchers, is a stock variable and adjusts slowly. When projecting vacancies and job searchers on the job finding rate, the sluggish adjustment of the stock of job searchers creates simultaneity bias, conflating the role of the vacancies with that of matching efficiency. Changes in the matching efficiency simultaneously determine the vacancies and hence the Ordinary Least Square (OLS) estimate is biased. To address this bias, we use a control function approach by assuming that productivity follows a statistical process, akin to the solution proposed by Olley and Pakes (1996) for production function estimation and the estimation of dynamic panel data (see Arellano and Bond (1991)). We then exploit valid moment conditions to estimate the matching technology with Generalized Method of Moments (GMM).

We use monthly data at the Metropolitan Statistical Area (MSA) level on employment, unemployment, vacancies, as well as hires, fires and quits for the United States labor market. We find that the

[1] See Cobb and Douglas (1928); Solow (1957) for the very early estimations of production technologies in the aggregate, and Griliches and Mairesse (1995); Olley and Pakes (1996); Lange and Papageorgiou (2020); Ackerberg et al. (2015); Hsieh and Klenow (2009); De Loecker and Warzynski (2012) using micro data. For review articles, see Syverson (2011); Eberhardt (2019); Ackerberg et al. (2007); De Loecker and Goldberg (2014)

matching technology exhibits decreasing returns to scale, with an elasticity of 0.7 on vacancies and 0.2 on the number of job searchers. The sum of both coefficients at 0.9 is significantly below one, hence decreasing returns to scale.[2] This finding is robust and consistent with different specifications of the model. When we estimate a Translog technology – which is an approximation of the Constant Elasticity of Substitution (CES) technology – we find that the elasticity of substitution is 0.7, indicating that vacancies and job searchers are gross complements.

The properties of the Beveridge Curve allow us to confirm these findings from a different angle. The Beveridge Curve derives from the law of motion in steady state of the number of job searchers and establishes the negative relationship between the unemployment rate and the vacancy rate (Beveridge (1944)). This is a particularly useful tool to study labor market dynamics over the business cycle. In a boom when unemployment is low, vacancies are high. The converse is true in recessions. Shifts in the Beveridge Curve indicate structural change in the labor market, changing matching efficiency, mismatch or varying search intensity (see Eeckhout and Lindenlaub (2019)). The presence of decreasing returns to matching adds a new insight to the interpretation of the Beveridge Curve.

One prediction of the Beveridge Curve regarding constant returns to scale is that the unemployment rate is independent of the number of job searchers, while the unemployment rate increases in the number of job searchers if there are decreasing returns to matching (decreases there are increasing returns to matching). When we estimate the Beveridge curve, we find that the estimated coefficient on the number of job searchers is significantly larger than zero, which corresponds to deceasing returns to matching. Moreover, the estimated coefficient is robust to different specifications, and its magnitude is consistent with the values we obtain in our benchmark estimation. This validates our main finding that the matching technology exhibits decreasing returns to scale with coefficients that sum up to 0.9.

To analyze the implications of the estimated matching technology, we build a Diamond-Mortensen-Pissarides model of job search (as in Pissarides (2000)), augmented with location choice across different local labor markets (MSAs). There is free entry of vacancies in local labor markets and worker mobility equates the value of unemployment across local labor markets, taking into account the local cost of living, as in Rosen (1979); Roback (1982).

In this economy with local labor markets and decreasing returns in the matching technology, the efficient allocation imposes conditions on the split of the bargaining surplus between the firm and the worker, in the spirit of Hosios (1990). We find that this condition for second best efficiency depends on the returns to matching in the technology. However, this efficiency condition can now no longer be implemented with directed or competitive search as is the case in the model with constant returns to scale and one location (see Moen (1997)). With decreasing returns to matching, the planner's requirement that also prevails from directed search is a necessary condition, but it is not sufficient. The planner also wants to steer the allocation of workers and jobs across locations, which introduces a novel wedge that directed search does not internalize. We find that with non-constant returns to matching, there is not a unique bargaining share that implements the second-best allocation within each local labor market. That is, effi-

[2]The output elasticity on vacancies of 0.7 is in line with Shimer (2005, 2010); Mengano (2022a).

ciency requires a location-specific bargaining share, yet there is a unique bargaining share that achieves second-best efficiency between locations. Therefore, there is no bargaining share that simultaneously achieves second-best efficiency within and between locations. Instead, under constant returns, there is a unique bargaining share that achieves the second-best planner's allocation across locations, and directed search implements it.

We document that the job finding rate is decreasing in the employment size of the local labor market, while the unemployment rate is constant across locations. We use the structure of the model to decompose the job finding and unemployment rates into their sources. We find that decreasing returns to scale lowers, varying matching efficiency raises, market tightness lowers the job finding rate. We also account for observable difference such as demographic composition (age, skill, gender) across cities of different size.

We quantify our model calibrating it to key labor market moments from the US economy and analyze the implications of decreasing returns to scale for the optimal spatial allocation, for spatial agglomeration externalities, unemployment duration, and for optimal Unemployment Insurance.

Related Literature The seminal paper on the topic of the matching technology is Petrongolo and Pissarides (2001). They investigate a Cobb-Douglas, constant-returns-to-scale matching function at the aggregate, country-wide level. They find it is empirically robust across countries. Our approach uses the variation across metropolitan areas and exploits the panel dimension of the data to address the econometric challenges of simultaneity of vacancies and unemployment and the resulting endogeneity bias.

There are several papers analyzing and estimating the matching function. An early paper Anderson and Burgess (2000) considers variation across states and industries, in a hybrid model of search and job competition. Their estimates are for the ordinary least squares regression. Lange and Papageorgiou (2020) estimates a matching technology under the assumption of constant returns to scale (or non-constant returns to scale if the returns to scale are known). Brancaccio, Kalouptsidi, and Papageorgiou (2020) focus on job search in the spatial dimension, as we do, for any form of decentralized trade between such as passengers searching for taxis, ships meeting cargo, exporters meeting importers,... They focus on settings with missing data and methods to estimate matching technology in the absence of data on one side of the market and recover the unobserved side of the market with limited assumptions. Borowczyk-Martins, Jolivet, and Postel-Vinay (2013) tackle the endogeneity bias when estimating matching function elasticities. They show that under structural assumptions about shock process to matching efficiency, their estimation method addresses that bias. Applied to aggregate U.S. data from JOLTS, they show that the bias is quantitatively important. Kangasharju, Pehkonen, and Pekkala (2005) focus on the returns to matching when search effort is endogenous which creates positive externalities and hence multiple equilibria, similar to Eeckhout and Lindenlaub (2019). Using Finnish labor market data, they find this endogenous search effort model exhibits constant to increasing returns to matching depending on the specification.

Manning and Petrongolo (2017) and Amior and Manning (2018) analyze a job search model with location decisions where they allow the search cost to depend on the distance between the worker's home

and job location. They find that matching rates drop fast in distance, and that this drop is substantially more pronounced for low education workers compared to high education workers.

There is a growing literature on spatial agglomeration externalities and worker sorting. Variation across locations in the labor market outcomes can inform us about the determinants of agglomeration externalities. The length and steepness of the job ladder varies across locations, and so does learning and human capital accumulation (De La Roca and Puga (2017); Davis and Dingel (2019); Eckert, Ganapati, and Walsh (2022); Lhuillier (2025); Crews (2023); Martellini (2022); Dauth, Findeisen, Moretti, and Suedekum (2022)). Wages tend to be more backloaded in large cities (Zanella (2025)), and credit constraints are tighter in larger cities (Aguilar (2023)) which leads to mismatch. Variation in agglomeration externalities across cities also affects inequality across locations (see Card, Rothstein, and Yi (2025); Nanos and Schluter (2018); Heise and Porzio (2022); Bilal (2023); Eeckhout, Pinheiro, and Schmidheiny (2014); Eeckhout, Hedtrich, and Pinheiro (2026); Kuhn, Manovskii, and Qiu (2026)).

In one of the influential theoretical papers on assortative matching and search, returns to matching matter for the proof of the main results. Shimer and Smith (2000) assume Increasing Returns to Scale (quadratic, with coefficients on vacancies and job searchers adding up to two) to establish their main result. Subsequently, Lauermann et al. (2020) have generalized the proof to allow for any returns to scale in the matching technology.

Organization As is common in the literature on production functions, the estimation of a production technology requires a formal framework and theoretical underpinnings, but it does not require a full general equilibrium model. This is also the case for the estimation of the matching function. In Section 2, we therefore first outline the minimal requirements to estimate the matching function. We need assumptions on the matching technology and ..., but we do not require a full general equilibrium theory to estimate the properties of the matching function. To evaluate the equilibrium consequences of the properties of the matching function, most notably efficiency and welfare, we do need a general equilibrium model. In section 3 we lay out such a GE model, in which we derive the augmented Hosios (1990) condition. We then investigate the quantitative importance of returns to scale in the matching function in section 4.

2 Empirics: Estimating the Matching Function

Define an aggregate matching function for location ℓ at time t of the form:

$$M_{\ell t} = m(U_{\ell t}, V_{\ell t}), \tag{1}$$

where $M_{\ell t}$ is number of matches, $U_{\ell t}$ is the number of searching workers, $V_{\ell t}$ is the number of job openings (vacancies). The matching function can take any functional form about which we are agnostic. Our estimation strategy starts from a parametric functional form, and in what follows we use different functional forms and methods to estimate the matching technology.

In our baseline specification, we estimate the Cobb-Douglas technology:

$$M_{\ell t} = A_{\ell t} U_{\ell t}^{\alpha_u} V_{\ell t}^{\alpha_v}, \tag{2}$$

where $A_{\ell t}$ is the matching efficiency. Crucially here, we depart from the common assumption (see also Petrongolo and Pissarides (2001)) that $\alpha_u + \alpha_v \neq 1$, i.e., we allow for non-constant returns to scale (RTS). Below, we estimate other functional forms of the matching function, among which is the translog functional form, which is an approximation to the Constant Elasticity of Substitution (CES) functional form.

Estimation Strategy Define the job finding rate $f_{\ell t} := M_{\ell t}/U_{\ell t}$, labor market tightness $\theta_{\ell t} := V_{\ell t}/U_{\ell t}$. Taking logs, we obtain a relation between the job finding rate f, the labor market tightness θ and the matching efficiency A. When returns to scale are non-constant, the *level* of unemployment also determines the job finding rate.

$$\log f_{\ell t} = \log A_{\ell t} + \alpha_v \log \theta_{\ell t} + (\alpha_u + \alpha_v - 1) \log U_{\ell t}. \tag{3}$$

This relationship is inherits the dynamic properties of the the labor market. In particular, unemployment is a stock variable that adjusts slowly, whereas vacancies are a flow (or jump) variable that adjusts quickly. As laid out in the introduction, this leads to classic simultaneity bias: the matching efficiency $A_{\ell t}$ is not observed and yet it determines $\theta_{\ell t}$ because unemployment adjusts sluggishly. As a result, matching efficiency enters the residual and the Ordinary Least Squares (OLS) estimate is biased.

To addresss this simultaneity bias, we resort to the solution methods from the production function literature. In particular, we impose structure on the process of $A_{\ell t}$ akin to the production function, as in the control function approach in Olley and Pakes (1996) and the dynamic panel data literature Arellano and Bond (1991).

We assume $\log A_{\ell t}$ follows a known statistical process, e.g., ARMA(p, q) and exploit valid moment conditions to estimate via Generalized Method of Moments (GMM). If $\log A_{\ell t}$ follows an ARMA(p, q) process, then we arrive at the following empirical specification:

$$\begin{aligned} \log f_{\ell t} = \sum_{i=1}^{p} \rho_i \log f_{\ell t-i} + (\alpha_u + \alpha_v - 1) \left(\log U_{\ell t} - \sum_{i=1}^{p} \rho_i \log U_{\ell t-i} \right) \\ + \alpha_v \left(\log \theta_{\ell t} - \sum_{i=1}^{p} \rho_i \log \theta_{\ell t-i} \right) + \varepsilon_{\ell t} + \sum_{j=1}^{q} \psi_j \varepsilon_{\ell t-j}. \end{aligned} \tag{4}$$

Thus, the following set of moment conditions can be used for estimation:

$$\mathbb{E}\left[\xi_{\ell t} \times z_{\ell t} \right] = 0$$

where

$$\xi_{\ell t} := \varepsilon_{\ell t} + \sum_{j=1}^{q} \psi_j \varepsilon_{\ell t-j} \quad \text{and} \quad z_{\ell t} := \{ \log U_{\ell t-\tau}, \log \theta_{\ell t-\tau-1}, \log f_{\ell t-\tau-1}; \tau \geq q \} \tag{5}$$

Data We use local labor market data for geographically different locations. Our definition of a local labor market is the Metropolitan Statistical Area (MSA). We have data from the Bureau of Labor Statistics (BLS)[3] on employment, unemployment and civilian labor force at MSA-level at a monthly rate.

From the Job Openings and Labor Turnover Survey (JOLTS) administered by the BLS, we obtain monthly MSA-level information on vacancies, hires, quits and layoffs for the period 2001-2019. The MSA level JOLTS data is available only for the 18 largest MSAs.

We further use the Current Population Survey (CPS) to construct estimates of search effort at the MSA-level. We follow the procedure in Barnichon and Figura (2015). And we use the Job Search Supplement (JSS) of the Survey of Consumer Expectations (SCE) to construct measures of acceptance rate at MSA-level (see Faberman, Mueller, Sahin, and Topa (2022)).

Estimation Results We estimate the matching function for different specifications and for different estimation methods. In Table 1 we report the estimates of Cobb-Douglas matching function in equation (3).

	1 OLS	**2** GMM	**3** GMM	**4** GMM	**5** GMM	**6** GMM	**7** GMM	**8** GMM	**9** Sys. GMM	**10** GMM
α_v	0.671^{***}	0.722^{***}	0.710^{***}	0.707^{***}	0.616^{***}	0.555^{***}	0.896^{***}	0.769^{***}	0.539^{***}	0.691^{***}
	(0.025)	(0.026)	(0.026)	(0.026)	(0.033)	(0.039)	(0.027)	(0.019)	(0.021)	(0.023)
$\alpha_u + \alpha_v - 1$	-0.117^{***}	-0.094^{***}	-0.100^{***}	-0.101^{***}	-0.140^{***}	-0.069^{***}	-0.071^{***}	-0.061^{***}	-0.088^{***}	-0.128^{***}
	(0.023)	(0.018)	(0.020)	(0.021)	(0.036)	(0.014)	(0.018)	(0.015)	(0.003)	(0.029)
σ_{uv}										0.740^{***}
										(0.143)
Shock process										
ARMA(p,q)	(0,0)	(1,0)	(1,1)	(2,2)	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)
Controls										
Search intensity					✓					
All effective searchers						✓				
Recall hiring							✓			
Location characteristics								✓		
Matching function										
Cobb-Douglas	✓	✓	✓	✓	✓	✓	✓	✓	✓	
Translog										✓
Fixed effects										
Month	✓	✓	✓	✓	✓	✓	✓	✓	✓	
Location and time									✓	
Observations	4,081	4,063	4,045	4,009	3,343	4,063	2,310	3,804	4,063	4,063

Table 1: Estimation results for Cobb-Douglas Matching Function

Notes: Column 10 reports the estimates for the translog matching technology.

Across all estimations, we observe a robust and consistent output elasticity on vacancies α_v in the range $[0.53, 0.89]$ with the preferred estimate of 0.72 for the GMM estimate in column 2. Most importantly, we find that the sum of the output elasticities is consistently below one, in the range $[0.86, 0.94]$, with a midpoint of 0.9. This is strong evidence of deceasing returns to scale in the matching technology.

[3]The data on labor market stocks is taken from the Local Area Unemployment Statistics (LAUS) program administered by the BLS, which uses data from various sources (the CPS, CES, state unemployment insurance systems, and the ACS) to construct local level estimates

Translog Matching Function The translog technology is a Taylor approximation of the CES technology. Write the CES matching technology as:

$$M(U,V) \;=\; A\Big[\tilde{\alpha}_u\, U^{\frac{\sigma-1}{\sigma}} + \tilde{\alpha}_v\, V^{\frac{\sigma-1}{\sigma}}\Big]^{\frac{\sigma}{\sigma-1}\,\varphi}, \tag{6}$$

where $A > 0$ is matching efficiency; $\tilde{\alpha}_u, \tilde{\alpha}_v > 0$ are input weights with $\tilde{\alpha}_u + \tilde{\alpha}_v = 1$; $\sigma > 0$ is the elasticity of substitution between unemployment and vacancies; and $\varphi > 0$ governs overall returns to scale. Three special cases are immediate: (i) $\varphi = 1$ yields constant returns; (ii) $\sigma \to 1$ collapses (6) to the Cobb–Douglas form $M = A\, U^{\varphi\tilde{\alpha}_u} V^{\varphi\tilde{\alpha}_v}$; and (iii) $\sigma \to \infty$ gives a linear matching technology $M = A(\tilde{\alpha}_u U + \tilde{\alpha}_v V)^{\varphi}$.

Taking logarithms of (6) yields

$$\ln M \;=\; \ln A \;+\; \frac{\sigma\varphi}{\sigma-1} \ln\Big[\tilde{\alpha}_u\, e^{\frac{\sigma-1}{\sigma}\ln U} + \tilde{\alpha}_v\, e^{\frac{\sigma-1}{\sigma}\ln V}\Big]. \tag{7}$$

Because $\ln M$ is not log-linear in $(\ln U, \ln V)$ for $\sigma \neq 1$, the CES matching function cannot be estimated by standard log-linear methods. The translog provides a flexible second-order approximation that is log-linear in parameters.

In Appendix Appendix A we derive in detail the derivation of thte translog matching technology, mapping the coefficients:

$$m \;=\; \ln A_0 \;+\; \alpha_u^*\,\tilde{u} \;+\; \alpha_v^*\,\tilde{v} \;+\; \tfrac{1}{2}\beta_{uu}\,\tilde{u}^2 \;+\; \tfrac{1}{2}\beta_{vv}\,\tilde{v}^2 \;+\; \beta_{uv}\,\tilde{u}\,\tilde{v}, \tag{8}$$

where the coefficients are defined in the Appendix and how they map into the equivalent Cobb-Douglas coefficients and the elasticity of substitution.

Column 10 of Table 1 reports the estimates from the Translog matching technology with the output elasticities that are similar in magnitude to those for the Cobb-Douglas matching function: the matching elasticity of vacancies is 0.69 and the sum of the matching elasticities is 0.87. The translog estimates also show that the output elasticity is 0.74, implying that vacancies and unemployment are gross complements.

Beveridge Curve An alternative approach that can confirm the robustness of our findings is to estimate the Beveridge Curve (BC). As we outlined in the introduction, the BC specifies a relation between the unemployment rate and the vacancy rate.

We can derive this relation from the law of motion of unemployment

$$U_{\ell t+1} = (1 - f_{\ell t})U_{\ell t} + \delta(N_{\ell t} - U_{\ell t}), \tag{9}$$

where $N_{\ell t}$ is employment, $f_{\ell t}$ is the job-finding rate and δ is the job-separation rate. Since the unemployment rate is equal to the ratio of the number of unemployed job searchers U to the number of people in

the labor force N, in steady state from equation (9) we can write the unemployment rate as:

$$u_\ell = \frac{\delta}{\delta + f_\ell}, \tag{10}$$

where the BC depends on θ_ℓ *and* U_ℓ through f_ℓ.

Under constant RTS, the BC is independent of the level of unemployment in a location. To see this, take a first-order Taylor expansion of the BC of the Cobb-Douglas matching technology around (u, θ, U):

$$\log u_\ell \approx \beta_o + \beta_\theta \log \theta_\ell + \beta_U \log U_\ell$$

where $\beta_o := \log u + (u-1)(\alpha_v \log \theta + (\alpha_u + \alpha_v - 1)\log U)$, $\beta_\theta := -(1-u)\alpha_v < 0$, and $\beta_U := -(1-u)(\alpha_v + \alpha_u - 1)$. Note that $\beta_U \neq 0$ if and only if $\alpha_v + \alpha_u \neq 1$ and $\beta_U > 0$ if and only if there decreasing returns to scale, i.e., $\alpha_u + \alpha_v < 1$.

We use the empirical steady state BC relationship for the Cobb-Douglas technology to validate our empirical estimate of the returns to scale, that is, we test whether $\beta_U > 0$.

	1 OLS	**2** TSLS	**3** OLS	**4** OLS	**5** OLS	**6** OLS	**7** OLS	**8** OLS	**9** OLS
β_θ	-0.499^{***}	-0.482^{***}	-0.494^{***}	-0.436^{***}	-0.786^{***}	-0.540^{***}	-0.578^{***}	-0.205^{***}	-0.117^{***}
	(0.021)	(0.036)	(0.021)	(0.026)	(0.029)	(0.021)	(0.038)	(0.020)	(0.023)
β_U	0.085^{**}	0.094^{**}	0.082^{**}	0.240^{***}	0.106^{**}	0.047^{*}	0.050^{*}	0.662^{***}	0.788^{***}
	(0.034)	(0.043)	(0.034)	(0.041)	(0.046)	(0.026)	(0.025)	(0.037)	(0.046)
$\alpha_u + \alpha_v - 1$	-0.090	-0.099	-0.086	-0.253	-0.112	-0.050	-0.053	-0.698	-0.830
Controls									
Separation rate			✓						
Search intensity				✓					
All effective searchers					✓				
Location characteristics						✓			
Instrument									
Labor productivity		✓							
Fixed effects									
Month	✓	✓	✓	✓	✓	✓		✓	
Location								✓	✓
Time							✓		✓
Observations	4,081	4,081	4,081	3,361	4,081	3,821	4,081	4,081	4,081

Table 2: Estimation results for Beveridge Curve

Notes: The last two columns estimate the variation in job finding rates from the cyclical variation in the Beveridge Curve because the location fixed effects absorb the variation of the job finding rate in the locations.

Table 2 reports the estimation results for the BC and confirms the results from our baseline estimation. For different specifications, we find that $\beta_U > 0$ which is consistent with decreasing returns to scale in the matching technology. Moreover, the magnitude of the estimates for most specifications is in the order of those that we find for the Baseline model in Table 1, with the implied sum of the matching elasticities around 0.9.

Decomposition We can now decompose the variation of the job finding rate into the different determinants: decreasing returns to matching, matching efficiency, market tightness, and observable character-

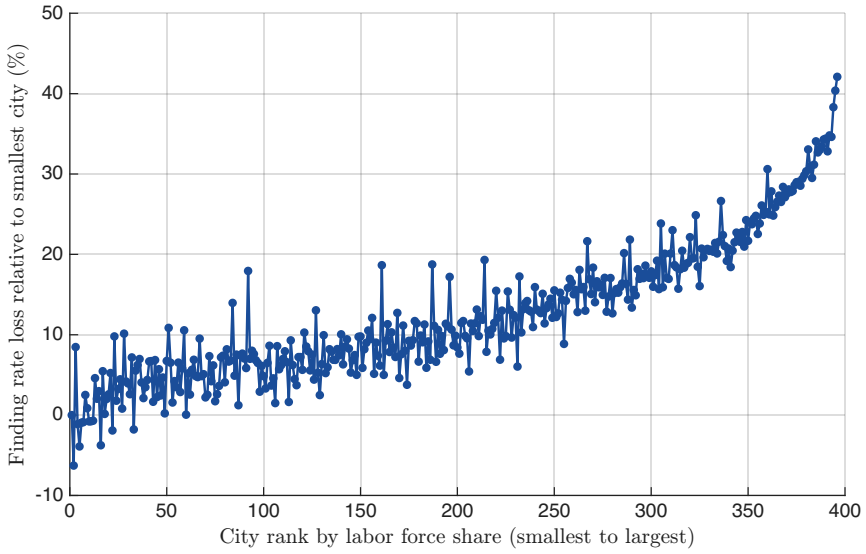


Figure 1: Finding Rate Drag due to Decreasing Returns to Scale

istics of the worker composition in each local labor market.

We have an estimate of the returns to scale from our empirical analysis $\phi = 0.9$, as well as estimates for the matching efficiency in each local labor market. We further have data on the tightness as well as the demographics of the worker composition (age, skill, gender, sector, occupation).

Using the finding rate definition in Equation (3), we can express differences in job-finding rates across locations as

$$\frac{f_{\ell t}}{f_{\ell_0 t}} = \left(\frac{\mathcal{A}_{\ell t}}{\mathcal{A}_{\ell_0 t}}\right)\left(\frac{\theta_{\ell t}}{\theta_{\ell_0 t}}\right)^{\alpha_v}\left(\frac{U_{\ell t}}{U_{\ell_0 t}}\right)^{\alpha_u+\alpha_v-1}, \tag{11}$$

where ℓ_0 denotes a reference location, taken to be the smallest MSA.

Holding matching efficiency and market tightness fixed, variation in finding rates is governed by unemployment levels. We define the unemployment-driven drag from decreasing returns to scale as

$$\Delta_{\ell t} \equiv 1 - \left(\frac{U_{\ell t}}{U_{\ell_0 t}}\right)^{\alpha_u+\alpha_v-1}, \tag{12}$$

which measures the fraction by which the finding rate in location ℓ is lower relative to the reference location.

Figure 1 plots this object against city size. The drag is modest for small and medium-sized MSAs, with a median value of around 10%, but increases sharply in the upper tail, reaching up to 40% for the largest cities. This implies that, through the returns to scale channel alone, job-finding rates in large cities are substantially lower than in smaller locations. Note that we can calculate this even for cities for which we don't have data on vacancies, since we only need the data on unemployed searchers and the estimated returns to scale.

Consistent with this pattern, the implied doubling effect is quantitatively meaningful: a doubling of city size reduces the finding rate by approximately 6.31% relative to an otherwise identical location.

3 A Model of Spatial Job Search with Non-Constant Returns to Matching

Here we lay out the basic model, which is the standard frictional Diamond-Mortensen-Pissarides (DMP) job search model augmented with heterogeneous local labor markets. In the Quantification in Section 4 we further extend to include quantitatively relevant features such as amenities for example.

3.1 Model Setup

Environment The economy is infinite-horizon and time is discrete. Local labor markets, indexed by $\ell \in \{1, 2, \ldots, \mathcal{L}\}$. All agents discount future payoffs with factor $\beta = \frac{1}{1+r} \in (0, 1)$ where r is the discount rate.

Workers There is a continuum of measure one of identical workers, who are either unemployed or employed. When unemployed, workers search for a job and earn unemployment benefit b_t. When employed, they earn a wage $w_{\ell t}$, specific to the local labor market. All workers inelastically demand one unit of housing for which they pay cost of living $c_{\ell t}$. The unemployed can freely and costlessly move across locations as in Rosen (1979) and Roback (1982).

Firms There is a continuum of firms of a large measure who can freely enter any market. When a firm enters a local market ℓ, it posts a vacancy at per-period cost κ. When a firm and a worker match, they bargain over the surplus that the match generates. Due to the search frictions there is a temporary bilateral monopoly, and they split the surplus using the Nash Bargaining rule where the firm receives a share $\gamma \in (0, 1)$ of the surplus from an active match and the worker receives $1 - \gamma$.

We assume that the cost of producing housing in location ℓ depends on the number of workers in a given market and is given by $g(N_\ell)$, where $g(\cdot)$ is twice differentiable and convex.[4] The housing market is perfectly competitive.

Search Technology Job search is random and the search frictions depend on the degree of competition for jobs, as captured by the ratio of vacancies to unemployed workers. As above, denote the number of job searchers by $U_{\ell t}$ and the number of posted vacancies by $V_{\ell t}$, and let $\theta_{\ell t} = \frac{V_{\ell t}}{U_{\ell t}}$ be the market tightness.

The matching function as specified in equation (1) relates the number of matches $M_{\ell t}$ formed in a local labor market as a function of the number of job searchers $U_{\ell t}$ and the number of vacancies posted $V_{\ell t}$. Matches separate with exogenous probability δ.

To derive our analytical results, we use a Cobb-Douglas matching function with non-constant RTS:

$$M_\ell = \left(U_\ell^{1-\alpha} V_\ell^{\alpha}\right)^{\phi} = U_\ell^{\phi-1} U_\ell^{1-\alpha\phi} V_\ell^{\alpha\phi} = \left(U_\ell \theta_\ell^{\alpha}\right)^{\phi} \tag{13}$$

[4] This is a reduced form way to obtain that the cost of living is increasing in city size as observed in the data. For a micro foundation with land and construction see Eeckhout (2004)

where $\alpha \in (0,1)$ and $\phi > 0$ is the extent of the returns to scale of the matching technology with respect to search activity. Denote the job finding rate for a searching worker p and the filling rate q for a vacancy as:

$$p(\theta_\ell, U_\ell) := M_\ell / U_\ell = \theta_\ell^{\alpha\phi} U_\ell^{\phi-1} \tag{14}$$

$$q(\theta_\ell, U_\ell) := M_\ell / V_\ell = V_\ell^{\phi-1} \theta_\ell^{\phi(\alpha-1)} = \theta_\ell^{\alpha\phi-1} U_\ell^{\phi-1} \tag{15}$$

where crucially, the contact rates also depend on labor market market size U_ℓ, not just tightness, when $\phi \neq 1$.

Value Functions Denote the value of and unemployed worker by $\mathbb{U}_\ell$, the value of an employed worker by $\mathbb{W}_\ell$, the value of a vacancy by $\mathbb{V}_\ell$, and the value of a filled job by $\mathbb{J}_\ell$. Then we can write the system of value functions as:

$$\mathbb{U}_\ell = b - c_\ell + \beta\Big\{ p(\theta_\ell, U_\ell)\mathbb{W}'_\ell + (1 - p(\theta_\ell, U_\ell))\mathbb{U}'_\ell \Big\} \tag{16}$$

$$\mathbb{W}_\ell = w_\ell - c_\ell + \beta\Big\{ (1-\delta)\mathbb{W}'_\ell + \delta\mathbb{U}'_\ell \Big\} \tag{17}$$

$$\mathbb{V}_\ell = -\kappa + \beta\Big\{ q(\theta_\ell, U_\ell)\mathbb{J}'_\ell + (1 - q(\theta_\ell, U_\ell))\mathbb{V}'_\ell \Big\} \tag{18}$$

$$\mathbb{J}_\ell = z_\ell - w_\ell + \beta\Big\{ (1-\delta)\mathbb{J}'_\ell + \delta\mathbb{V}'_\ell \Big\}, \tag{19}$$

where $\mathbb{U}'_\ell, \mathbb{W}'_\ell, \mathbb{V}'_\ell, \mathbb{J}'_\ell$ denote the continuation value in the next period $t+1$.

Nash bargaining with share γ to the firm over the match surplus pins down the wage w_ℓ in each local labor market:

$$w_\ell = \text{argmax}_{w_\ell} \ (\mathbb{J}_\ell - \mathbb{V}_\ell)^\gamma \, (\mathbb{U}_\ell - \mathbb{W}_\ell)^{1-\gamma}. \tag{20}$$

Equilibrium We are solving for the steady state equilibrium allocation where firms enter and post vacancies until the value of a vacancy is zero, and workers choose locations to equate the value of unemployment across locations, and where the value functions satisfy (16), (17), (18), (19).

3.2 Solution and Results

We first solve for the decentralized equilibrium allocation, then we solve the planner's problem, and then we solve for the equilibrium allocation under directed search. Our main results characterize the efficiency properties of the equilibrium allocation under random and directed search.

Decentralized Equilibrium In the steady state, $\mathbb{U}_\ell = \mathbb{U}'_\ell$. Likewise for $\mathbb{W}_\ell, \mathbb{V}_\ell, \mathbb{J}_\ell$. Imposing the free entry condition $\mathbb{V}_\ell = 0$ and solving for the Nash bargaining rule $\mathbb{J}_\ell = \gamma \mathbb{S}_\ell$ yields

$$\frac{\kappa}{q(\theta_\ell, U_\ell)} = \gamma\beta\mathbb{S}_\ell \tag{21}$$

where

$$\mathbb{S}_\ell = z_\ell - b + \beta(1 - \delta - (1-\gamma)p(\theta_\ell, U_\ell))\mathbb{S}'_\ell. \tag{22}$$

Combining these two conditions yields the decentralized job creation condition:

$$\gamma \frac{z_\ell - b}{r + \delta + (1-\gamma)\theta_\ell q(\theta_\ell, U_\ell)} = \frac{\kappa}{q(\theta_\ell, U_\ell)} \tag{23}$$

which determines the level of job creation V_ℓ for a given measure of unemployed workers U_ℓ.

Unemployment in each location is determined by the law of motion:

$$U'_\ell = \delta(N_\ell - U_\ell) + (1 - p(\theta_\ell, U_\ell))U_\ell \tag{24}$$

where N_ℓ is the size of the labor force in location ℓ and the measure of workers in the economy is one: $\sum_{\ell \in \mathcal{L}} N_\ell = 1$.

In the steady state this determines the Beveridge Curve:

$$U_\ell = \frac{\delta}{\delta + p(\theta_\ell, U_\ell)} N_\ell. \tag{25}$$

The spatial allocation of workers is governed by the equalization of the value of the unemployed as in Rosen (1979) and Roback (1982): $\mathbb{U}_\ell = \mathbb{U}\ \forall\ \ell$. By assumption, all workers consume one unit of housing. The cost of producing housing depends on the population in location ℓ. Equilibrium in the perfectly competitive housing market implies: $c_\ell = g'(N_\ell)$.

Combining the value of unemployment, the Nash bargaining rule and the job creation condition yields:

$$(1-\beta)\mathbb{U} = b - g'(N_\ell) + \frac{1-\gamma}{\gamma}\kappa\theta_\ell. \tag{26}$$

Directed Search Consider a directed search variation of the model where firms post wages in each local labor market. Let the matching function be Cobb-Douglas with non-CRT from equation (13) and corresponding job finding and job filling rates, and where $U_\ell \equiv \int U_\ell(w)dw$ and $V_\ell \equiv \int V_\ell(w)dw$.

In steady state, under directed search firms post wages to maximize:

$$\max_w \ q(\theta_\ell(w), U_\ell)\mathbb{J}_\ell(w) \tag{27}$$

$$\text{s.t. } p(\theta_\ell(w), U_\ell)[\mathbb{W}_\ell(w) - \mathbb{U}_\ell] = [(1-\beta)\mathbb{U}_\ell - b]\beta^{-1}, \tag{28}$$

where the values functions are specified as under random search with Nash bargaining.

The FOC and the total derivative of (IC) constraint:

$$q'(\theta_\ell(w), U_\ell)\theta'_\ell(w)\mathbb{J}_\ell(w) + q(\theta_\ell(w), U_\ell)\mathbb{J}'_\ell(w) = 0 \tag{29}$$

$$p'(\theta_\ell(w), U_\ell)\theta'_\ell(w)[\mathbb{W}_\ell(w) - \mathbb{U}_\ell] + p(\theta_\ell(w), U_\ell)\mathbb{W}'_\ell(w) = 0 \tag{30}$$

Combining these two equations and using the fact that $\mathbb{J}'_\ell(w) = -\mathbb{W}'_\ell(w)$ yields

$$\alpha\phi[\mathbb{W}_\ell(w) - \mathbb{U}_\ell] = (1 - \alpha\phi)\mathbb{J}_\ell(w). \tag{31}$$

In equilibrium $\theta_\ell(w) = \theta_\ell$, $\forall w$. Inspection of the equilibrium conditions under random search and comparing to equation (31) then leads to the following result.

Lemma 1 *The equilibrium allocation of the random search economy coincides with that of the directed search economy provided* $\gamma = \alpha\phi$.

When the bargaining share γ is equal to the elasticity of vacancies $\alpha\phi$, directed search is isomorphic to random search. This is reminiscent of the constant returns to scale matching technology with $\phi = 1$, where the directed and random search economies coincide when $\gamma = \alpha$.

Planner's Solution We solve for the constrained planner's solution. The planner is constrained by the search frictions, which is a technological constraint that the planner has no instruments to alter. Instead, we do assume the planner can affect the free entry of firms and free mobility of workers. The zero profit condition on vacancy creation for firms need not hold and unemployed workers' utility does not necessarily equate across locations.

The planner chooses vacancies V_ℓ and workers N_ℓ in each local labor market ℓ to solve:

$$\begin{aligned} \mathcal{W}(\{E_\ell\}_{\ell\in\mathcal{L}}) = \max_{\{V_\ell, N_\ell\}_{\ell\in\mathcal{L}}} \sum_{\ell\in\mathcal{L}} \{z_\ell E_\ell + b(N_\ell - E_\ell) - \kappa V_\ell - g(N_\ell)\} + \beta\mathcal{W}(\{E'_\ell\}_{\ell\in\mathcal{L}}) \\ \text{s.t. } E'_\ell = (1-\delta)E_\ell + \left((N_\ell - E_\ell)^{1-\alpha} V_\ell^\alpha\right)^\phi \quad \text{and} \quad \sum_{\ell\in\mathcal{L}} N_\ell = 1, \end{aligned} \tag{32}$$

where $E_\ell = N_\ell - U_\ell$ is employment in location ℓ. The planner's problem yields the following first-order conditions:

$$\text{FOC I:} \qquad \kappa = \beta \frac{\partial \mathcal{W}'}{\partial E'_\ell} \phi\alpha q(\theta_\ell, U_\ell) \tag{33}$$

$$\text{FOC II:} \qquad \lambda = b - g'(N_\ell) + \phi(1-\alpha)p(\theta_\ell, U_\ell)\beta \frac{\partial \mathcal{W}'}{\partial E'_\ell} \tag{34}$$

$$\text{Envelope condition:} \qquad \frac{\partial \mathcal{W}}{\partial E_\ell} = z_\ell - b + \left(1 - \delta - \phi(1-\alpha)p(\theta_\ell, U_\ell)\right)\beta \frac{\partial \mathcal{W}'}{\partial E'_\ell}, \tag{35}$$

where λ is the Lagrange multiplier on the condition that clears the population market across locations. Observe that the choice of N_ℓ is equivalent to the choice of E_ℓ or U_ℓ since by definition $N_\ell = E_\ell + U_\ell$ and the two constraints in the maximization problem (33) pin down E_ℓ, U_ℓ given N_ℓ.

Efficiency within locations Combining the FOC for vacancies (33) and the envelope condition (35) yields the following:

$$\phi\alpha \frac{z_\ell - b}{r + \delta + \phi(1-\alpha)\theta_\ell q(\theta_\ell, U_\ell)} = \frac{\kappa}{q(\theta_\ell, U_\ell)}, \tag{36}$$

which can also be expressed as:

$$\underbrace{\underbrace{\gamma \frac{z_\ell - b}{r+\delta+(1-\gamma)\theta_\ell q(\theta_\ell, U_\ell)}}_{\text{Decentralized eq.}} \times \underbrace{\left(1 - \frac{(\gamma-\phi\alpha)}{\gamma}\frac{r+\delta+\theta_\ell q(\theta_\ell, U_\ell)}{r+\delta+(1-\phi\alpha)\theta_\ell q(\theta_\ell, U_\ell)}\right)}_{\text{Hosios}}}_{\text{Directed search}} \times \underbrace{\left(1-(\phi-1)\frac{\theta_\ell q(\theta_\ell, U_\ell)}{r+\delta+\phi(1-\alpha)\theta_\ell q(\theta_\ell, U_\ell)}\right)}_{\text{Non-constant returns wedge}} = \frac{\kappa}{q(\theta_\ell, U_\ell)} \tag{37}$$

Efficiency between locations Combining the two FOCs yields the following:

$$\lambda = b - g'(N_\ell) + \frac{1-\alpha}{\alpha}\kappa\theta_\ell, \tag{38}$$

which can be re-written as follows:

$$\lambda = b - g'(N_\ell) + \underbrace{\underbrace{\frac{1-\gamma}{\gamma}\kappa\theta_\ell}_{\text{Decentralized eq.}} \times \underbrace{\left(1+\frac{(\gamma-\phi\alpha)}{(1-\gamma)\phi\alpha}\right)}_{\text{Hosios}}}_{\text{Directed search}} \times \underbrace{\left(1+\frac{(\phi-1)}{1-\phi\alpha}\right)}_{\text{Non-constant returns wedge}} \tag{39}$$

Inspection of the efficiency conditions (37) and (39) shows the usual wedge between the decentralized random search equilibrium allocation and the planner's problem, which is now a modified Hosios condition $\gamma = \phi\alpha$ (see Lemma 1). It is a modified Hosios condition because the elasticity of matches with respect to vacancies is now $\phi\alpha$ instead of α under constant returns. The directed search equilibrium allocation internalizes this wedge. However, there is now an additional wedge that is exclusively due to non-constant returns to scale. This second wedge is driving our efficiency results next.

Main Results First, we establish that the directed search equilibrium allocation is not efficient. Then we show that the decentralized is never efficient.

Proposition 1 *When $\phi \neq 1$, the directed search equilibrium allocation does not coincide with the planner's problem and is not efficient.*

This follows immediately from observing the wedge non-constant returns wedge in equations (37) and (39). Only when $\phi = 1$ does the wedge disappear. A Corollary of this result is that the modified Hosios condition $\gamma = \phi\alpha$, is not sufficient to achieve efficiency. The Hosios condition implements the optimal split between the firm and the worker, but it does not implement the optimal allocation across matches in different locations.

Novel wedges are also needed because planner wants to control the size of the pie across the locations. instead calls for policies

Proposition 2 *When $\phi \neq 1$, the decentralized equilibrium is never efficient.*

There exists *no* common economy-wide γ^{within} such that the within-location allocation is efficient, but there exists a place-based $\gamma_\ell^{\text{within}}$:

$$(23) = (36) \quad \Leftrightarrow \quad \gamma_\ell^{\text{within}} := \phi\alpha \frac{r+\delta+\theta_\ell q(\theta_\ell, U_\ell)}{r+\delta+\phi\theta_\ell q(\theta_\ell, U_\ell)} \neq \gamma^{\text{within}} \tag{40}$$

There exists a common, economy-wide γ^{between} such that the between-location allocation is efficient:

$$(26) = (38) \quad \Leftrightarrow \quad \gamma^{\text{between}} := \alpha \tag{41}$$

However, there is no $\gamma^\star$ such that *both* the within- and between-location allocation are efficient:

$$\gamma^\star = \gamma^{\text{within}} = \gamma^{\text{between}} \quad \text{but} \quad \gamma^{\text{within}} \neq \gamma_\ell^{\text{within}} \neq \gamma^{\text{between}} \tag{42}$$

Implementation The planner's allocation can be implemented through a system of place-based transfers that decentralizes both the job creation and the spatial allocation margins. The key insight is that, with decreasing returns to matching, efficiency requires correcting two distinct wedges: a within-location wedge in vacancy creation and a between-location wedge in worker allocation. In the decentralized equilibrium with location specific unemployment benefits and housing subsidies, job creation satisfies

$$\gamma \frac{z_\ell - (\tau_\ell^b + b)}{r+\delta+(1-\gamma)\theta_\ell q(\theta_\ell, U_\ell)} = \frac{\kappa}{q(\theta_\ell, U_\ell)}, \tag{43}$$

while the value of unemployment is given by

$$(1-\beta)\mathbb{V}^u = (\tau_\ell^b + b) - (1-\tau_\ell^h) g'(N_\ell) + \frac{1-\gamma}{\gamma}\kappa\theta_\ell. \tag{44}$$

These two conditions highlight that vacancy creation and worker location decisions are distorted by the same pair of wedges induced by bargaining and non-constant returns.

The optimal policy uses two instruments to target these distortions directly. The unemployment benefit is set to

$$\tau_\ell^b = (z_\ell - b)\left[1 - \left(1 - \frac{(\gamma-\phi\alpha)}{\gamma}\frac{r+\delta+\theta_\ell q(\theta_\ell, U_\ell)}{r+\delta+(1-\phi\alpha)\theta_\ell q(\theta_\ell, U_\ell)}\right)\left(1-(\phi-1)\frac{\theta_\ell q(\theta_\ell, U_\ell)}{r+\delta+\phi(1-\alpha)\theta_\ell q(\theta_\ell, U_\ell)}\right)\right], \tag{45}$$

which replicates the planner's job creation condition by correcting both the standard Hosios distortion and the additional wedge from decreasing returns. Conditional on this correction, the housing subsidy is chosen as

$$\tau_\ell^h = \frac{1}{g'(N_\ell)}\left\{\frac{1-\gamma}{\gamma}\kappa\theta_\ell\left[\left(1+\frac{(\gamma-\phi\alpha)}{(1-\gamma)\phi\alpha}\right)\left(1+\frac{(\phi-1)}{1-\phi\alpha}\right)-1\right]-\tau_\ell^b\right\}, \tag{46}$$

so as to align the private return to location choice with the planner's shadow value. Together, these instruments decentralize the planner's allocation by simultaneously restoring efficiency in vacancy creation within locations and in worker allocation across locations.

4 Quantification

4.1 Quantitative Model

To capture aspects of the economy that may be quantitatively relevant, we augment the quantitative model with additional features: heterogenous amenities across locations (this implies that the effective house price is $g'(N_\ell) - A_\ell$), on-the-job search (OJS) which happens at rate η. The details of the model are in Appendix B.

4.2 Calibration

We calibrate the model to a monthly frequency to match key cross-sectional moments of local labor markets summarized in Table 3. A subset of parameters is set exogenously using standard values in the literature or our own calculations and estimates. These include the discount factor β, separation rates δ, the matching function elasticity α, returns to scale ϕ, the bargaining weight γ, the housing supply elasticity ε_{N,p^h}, and the OJS rate η. We also normalize productivity in the median location to one.

Table 3: Model Calibration

Parameter	Description	Value	Source
	Exogenous Parameters		
β	Discount factor	0.997	Annual interest rate $r = 0.04$
δ	Job separation rate	0.013	JOLTS, monthly
γ	Firm bargaining power	0.830	Mengano, 2022b
α	Matching elasticity (vacancies)	0.7	Estimated
ϕ	Returns to scale in matching	0.9	Estimated
ε_{N,p^h}	Housing supply elasticity	0.67	Literature
η	On-the-job search rate	0.08	Literature
	Endogenous Parameters		
b	Unemployment benefit	0.938	Calibrated to unemployment rate
κ	Vacancy posting cost	0.545	Calibrated to vacancy rate
μ	Matching efficiency	0.205	Calibrated to finding rate
A_ℓ	Amenities		Match city sizes
z_ℓ	Productivity		Match local unemployment rates

The remaining parameters $\{b, \kappa, \mu, \{z_\ell\}_\ell, \{A_\ell\}_\ell\}$ are calibrated to match the median finding rate, filling rate, and the joint distribution of unemployment rates and location sizes across metropolitan areas. The amenities can be thought of as the residual as they pick up any differences between the data and the model. The model therefore fits the data exactly and replicates both the dispersion of unemployment rates and the skewed distribution of location sizes (see Figure C.1 in Appendix C).

4.3 Quantitative Findings

Figures 2 and 3 summarize the model's quantitative implications for cross-sectional labor market outcomes. Figure shows how the key distributional outcomes vary with the unemployment rate. Consistent with the data, the model generates a strong negative relationship between unemployment and labor market tightness, as well as between unemployment and vacancy filling rates. Low-unemployment locations are characterized by higher productivity, which induces greater vacancy creation and higher tightness. As a result, firms face a thicker market with more vacancies per worker, reducing vacancy filling rates while increasing job finding rates.

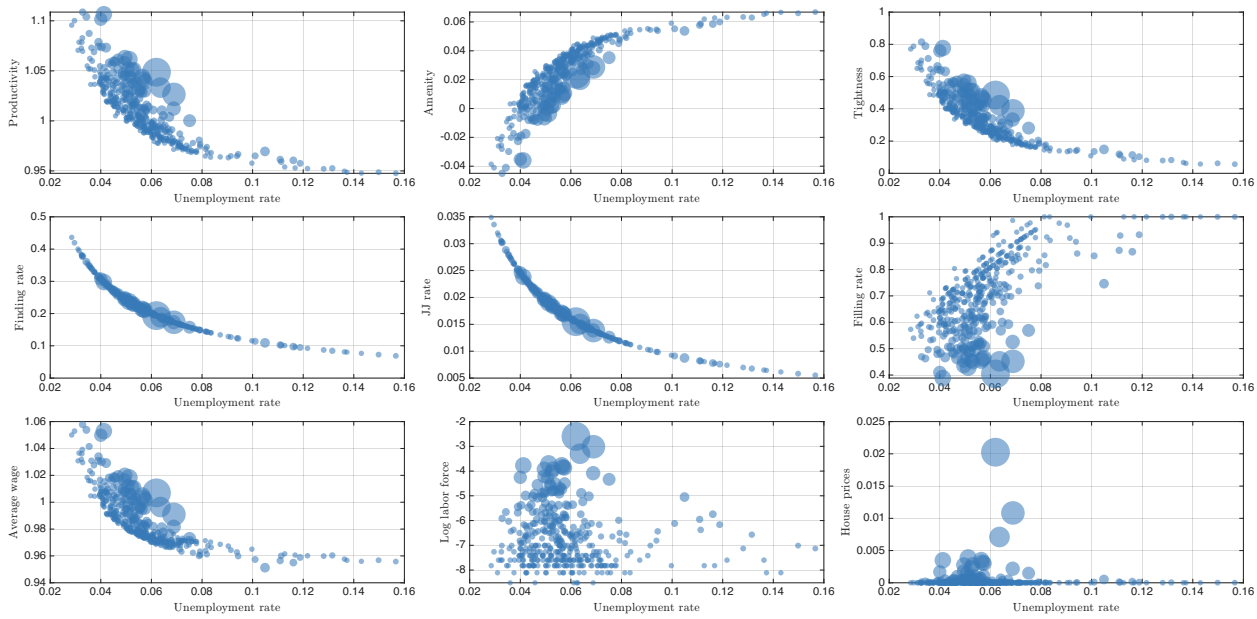


Figure 2: Heterogeneity by Across Location by Unemployment rate

Figure 3 plots the relation of outcomes with the emplo[yment size of the local labor market. The model generates a flat relationship between unemployment rates and location size, while simultaneously producing increasing wages and productivity with city size. As productivity rises in larger locations, firms create more vacancies and workers sort into these markets, yet equilibrium unemployment remains stable due to offsetting adjustments in labor market tightness and mobility. Moreover, because productivity increases with city size, the model does not require amenities to be strongly correlated with location size to rationalize the spatial distribution of workers. Instead, higher housing costs in larger cities are sufficient to offset productivity advantages and sustain spatial equilibrium.

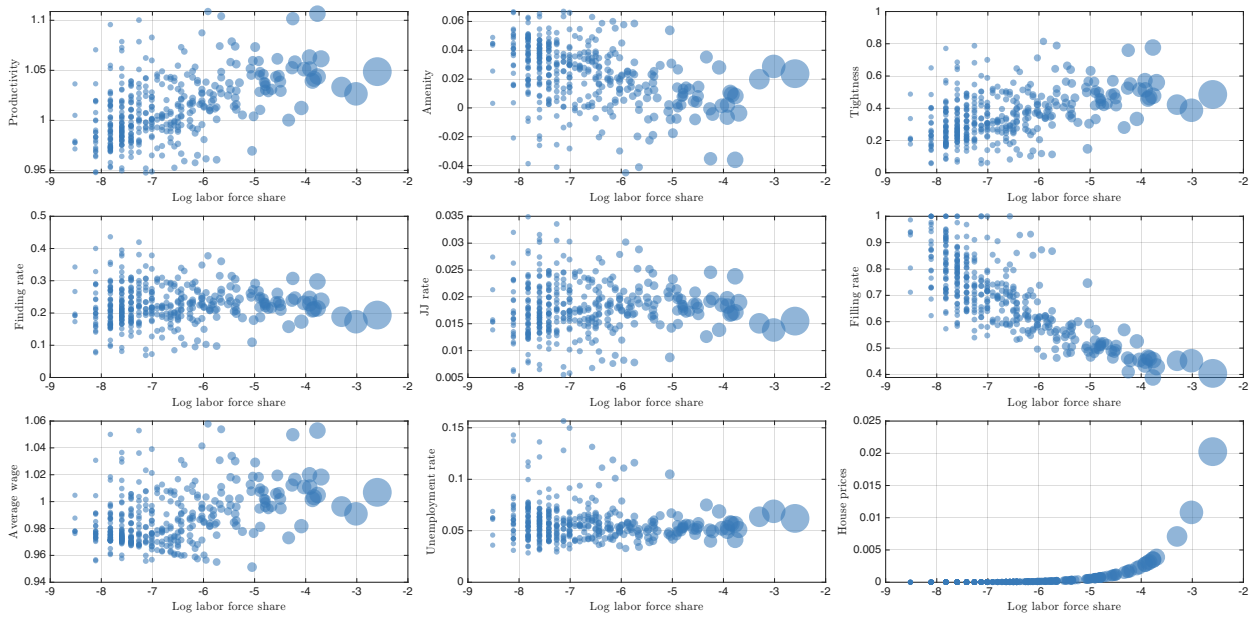


Figure 3: Heterogeneity by Across Location by Labor Force

5 Conclusion

We estimate the matching technology using variation across local labor markets and find robust evidence of decreasing returns to scale. We validate this finding using an alternative estimatioin method based on the Beveridge curve. This is consistent with falling job finding rates in larger cities that we observe in the data.

We develop a model of spatial job search and show that decreasing returns has profound implications for the efficiency of the equilibrium allocation of workers across space. The main driver of the inefficiency is the impact of decreasing returns on the job creation margin. The worker mobility margin is not directly affected.

We quantify the model to match key moments of the US labor market. Decreasing returns lead to too many vacancies in high productivity locations. This leads to disparities in welfare outcomes across locations.

Appendix

Appendix A The Translog Approximation

Appendix A.1 Second-order Taylor expansion

Let $u \equiv \ln U$, $v \equiv \ln V$, and $m \equiv \ln M$. Define the expansion point $(\bar{u}, \bar{v})$, to be specified below. A second-order Taylor expansion of $m(u, v)$ around $(\bar{u}, \bar{v})$ gives:

$$\begin{aligned} m \;\approx\; & m(\bar{u},\bar{v}) \;+\; m_u(\bar{u},\bar{v})\,(u-\bar{u}) \;+\; m_v(\bar{u},\bar{v})\,(v-\bar{v}) \\ & +\; \tfrac{1}{2}\,m_{uu}(\bar{u},\bar{v})\,(u-\bar{u})^2 \;+\; \tfrac{1}{2}\,m_{vv}(\bar{u},\bar{v})\,(v-\bar{v})^2 \\ & +\; m_{uv}(\bar{u},\bar{v})\,(u-\bar{u})(v-\bar{v}), \end{aligned} \tag{A.1}$$

where subscripts denote partial derivatives. Defining demeaned log inputs $\tilde{u} \equiv u - \bar{u}$ and $\tilde{v} \equiv v - \bar{v}$, and collecting the constant into the intercept, the translog matching function is:

$$m \;=\; \ln A_0 \;+\; \alpha_u^*\,\tilde{u} \;+\; \alpha_v^*\,\tilde{v} \;+\; \tfrac{1}{2}\beta_{uu}\,\tilde{u}^2 \;+\; \tfrac{1}{2}\beta_{vv}\,\tilde{v}^2 \;+\; \beta_{uv}\,\tilde{u}\,\tilde{v}, \tag{A.2}$$

where $\ln A_0 \equiv m(\bar{u}, \bar{v})$, and we define:

$$\alpha_u^* \equiv m_u(\bar{u},\bar{v}), \qquad \alpha_v^* \equiv m_v(\bar{u},\bar{v}), \qquad \beta_{uu} \equiv m_{uu}(\bar{u},\bar{v}), \qquad \beta_{vv} \equiv m_{vv}(\bar{u},\bar{v}), \qquad \beta_{uv} \equiv m_{uv}(\bar{u},\bar{v}). \tag{A.3}$$

By symmetry of second derivatives, $\beta_{uv} = \beta_{vu}$, so the translog has five free parameters in addition to matching efficiency.

Appendix A.2 Why demeaning is necessary

The expansion point $(\bar{u}, \bar{v})$ is the point at which the translog coincides exactly with the CES. The interpretation of the linear coefficients α_u^* and α_v^* as input elasticities of the matching function evaluated *at the expansion point* is exact only when evaluated at $(\tilde{u}, \tilde{v}) = (0, 0)$, i.e., at $(u, v) = (\bar{u}, \bar{v})$.

If one estimates (8) without demeaning (i.e., setting $\bar{u} = \bar{v} = 0$), the linear coefficients recover derivatives at $U = V = 1$, a point that has no economic content and that typically lies far outside the support of the data. In that case, the estimated α_u^* and α_v^* would not recover the elasticities at any economically meaningful point, and the second-order terms would partially absorb the linear effects.

Choice of expansion point. We set $(\bar{u}, \bar{v})$ equal to the sample means $(\mathbb{E}[\ln U], \mathbb{E}[\ln V])$. This choice ensures that (i) the linear coefficients recover elasticities at the average market, (ii) the demeaned regressors $\tilde{u}$ and $\tilde{v}$ are mean-zero so that the second-order terms are orthogonal to the linear ones in expectation, and (iii) the approximation is most accurate in the region of highest data density.

Why demeaning is innocuous. Demeaning is a reparametrization, not a restriction on the matching technology. Given estimates of the translog coefficients, the CES is recovered exactly through the map-

pings in Section Appendix A.3. The structural parameters—returns to scale, input elasticities, and the elasticity of substitution—are invariant to the choice of expansion point; only the interpretation of the estimated intercept changes. The Cobb–Douglas and constant-returns-to-scale restrictions remain testable in the same way regardless of whether demeaning is applied.

Appendix A.3 Mapping Between CES and Translog Parameters

We now compute the derivatives of $m(u, v)$ implied by the CES (7) and evaluate them at $(\bar{u}, \bar{v})$. Define the inner CES aggregate:

$$\mathcal{C} \equiv \tilde{\alpha}_u U^{\frac{\sigma-1}{\sigma}} + \tilde{\alpha}_v V^{\frac{\sigma-1}{\sigma}}, \tag{A.4}$$

so that $m = \ln A + \frac{\sigma\varphi}{\sigma-1} \ln \mathcal{C}$. At the expansion point, let $s_u \equiv \tilde{\alpha}_u \bar{U}^{(\sigma-1)/\sigma}/\bar{\mathcal{C}}$ and $s_v \equiv \tilde{\alpha}_v \bar{V}^{(\sigma-1)/\sigma}/\bar{\mathcal{C}}$ denote the CES expenditure shares, with $s_u + s_v = 1$.

First derivatives: input elasticities Direct computation yields:

$$\alpha_u^* \equiv m_u(\bar{u}, \bar{v}) = \varphi s_u, \qquad \alpha_v^* \equiv m_v(\bar{u}, \bar{v}) = \varphi s_v. \tag{A.5}$$

These are the *output elasticities of matches* with respect to unemployment and vacancies, evaluated at the sample means. Their sum recovers the overall returns to scale:

$$\varphi = \alpha_u^* + \alpha_v^*. \tag{A.6}$$

Second derivatives: elasticity of substitution The second-order coefficients are:

$$\beta_{uu} = m_{uu}(\bar{u}, \bar{v}) = -\varphi s_u s_v \frac{\sigma - 1}{\sigma}, \tag{A.7}$$

$$\beta_{vv} = m_{vv}(\bar{u}, \bar{v}) = -\varphi s_u s_v \frac{\sigma - 1}{\sigma}, \tag{A.8}$$

$$\beta_{uv} = m_{uv}(\bar{u}, \bar{v}) = \varphi s_u s_v \frac{\sigma - 1}{\sigma}. \tag{A.9}$$

Three observations follow immediately. First, the CES imposes the *symmetry restriction* $\beta_{uu} = \beta_{vv} = -\beta_{uv}$, which is testable. Second, adding (A.7)–(A.9):

$$\beta_{uu} + \beta_{vv} - 2\beta_{uv} = -4\varphi s_u s_v \frac{\sigma - 1}{\sigma}. \tag{A.10}$$

Third, using (A.5) and $s_u + s_v = 1$:

$$\frac{\sigma - 1}{\sigma} = \frac{-\beta_{uu}}{\varphi s_u s_v} = \frac{-\beta_{uu}}{\alpha_u^* \alpha_v^*/\varphi}. \tag{A.11}$$

Solving (A.11) for σ yields the formula for the elasticity of substitution in terms of translog coefficients. Because the translog is estimated in $(\tilde{u}, \tilde{\theta})$ where $\theta \equiv V/U$ denotes tightness and $\tilde{\theta} \equiv \ln \theta - \overline{\ln \theta}$, the regressors are $(\tilde{u}, \tilde{\theta})$ rather than $(\tilde{u}, \tilde{v})$. The change of variables $\tilde{v} = \tilde{\theta} + \tilde{u}$ implies the following

Jacobian transformation for the second-order coefficients:

$$B_{UU} \equiv \beta_{uu} - 2\beta_{uv} + \beta_{vv}, \qquad B_{VV} \equiv \beta_{vv}, \qquad B_{UV} \equiv \beta_{uv} - \beta_{vv}, \tag{A.12}$$

where B_{UU}, B_{VV}, B_{UV} are the second derivatives of m with respect to $(\ln U, \ln V)$ expressed in terms of the $(\tilde{u}, \tilde{\theta})$ coefficients estimated in the data. The full formula for the elasticity of substitution is then:

$$\sigma \;=\; \frac{-\,\alpha_u^*\,\alpha_v^*\,\varphi}{(\alpha_v^*)^2 B_{UU} + (\alpha_u^*)^2 B_{VV} - 2\,\alpha_u^*\,\alpha_v^*\,B_{UV}}, \tag{A.13}$$

with α_u^*, α_v^*, and φ as defined in (A.5)–(A.6).

Complementarity and substitutability The sign of $\sigma - 1$ determines whether unemployment and vacancies are gross substitutes or complements in the matching technology:

- $\sigma > 1$: unemployment and vacancies are *gross substitutes*. Increasing one input raises the marginal product of the other less than proportionally; thick-market effects are dampened. From (A.7), this corresponds to $\beta_{uu} < 0$ and $\beta_{uv} > 0$.
- $\sigma = 1$: Cobb–Douglas. The elasticity of substitution is unity, the marginal rate of technical substitution is proportional to the input ratio, and $\beta_{uu} = \beta_{vv} = \beta_{uv} = 0$.
- $\sigma < 1$: unemployment and vacancies are *gross complements*. Matching becomes harder when the input mix is unbalanced, generating a thick-market externality that is stronger than in the Cobb–Douglas case. This corresponds to $\beta_{uu} > 0$ and $\beta_{uv} < 0$.

The threshold is thus $\sigma = 1$: below unity the inputs are complements, above unity they are substitutes. This boundary corresponds to the Cobb–Douglas case in which neither complementarity nor substitutability is present.

Appendix A.4 The Estimating Equation

Job-finding rate representation Let $f_{\ell t} \equiv M_{\ell t}/U_{\ell t}$ denote the job-finding rate in location ℓ at date t, and let $\theta_{\ell t} \equiv V_{\ell t}/U_{\ell t}$ denote tightness. The translog matching function (8) implies, after subtracting $\ln U$ from both sides and using $\ln V = \ln \theta + \ln U$:

$$\ln f_{\ell t} \;=\; \ln A_{\ell t} \;+\; \underbrace{(\alpha_u^* + \alpha_v^* - 1)}_{\varphi - 1}\widetilde{\ln U}_{\ell t} \;+\; \alpha_v^*\,\widetilde{\ln \theta}_{\ell t} \;+\; \tfrac{1}{2}\beta_{uu}\,\widetilde{\ln U}_{\ell t}^{\,2} \;+\; \tfrac{1}{2}\beta_{vv}\,\widetilde{\ln \theta}_{\ell t}^{\,2} \;+\; \beta_{uv}\,\widetilde{\ln U}_{\ell t}\,\widetilde{\ln \theta}_{\ell t}, \tag{A.14}$$

where $\widetilde{\ln U}_{\ell t} \equiv \ln U_{\ell t} - \overline{\ln U}$ and $\widetilde{\ln \theta}_{\ell t} \equiv \ln \theta_{\ell t} - \overline{\ln \theta}$ are demeaned log unemployment and log tightness, respectively, and overbars denote sample means. The coefficient on $\widetilde{\ln U}$ identifies $\varphi - 1$: it is zero under constant returns, negative under decreasing returns, and positive under increasing returns. The coefficient on $\widetilde{\ln \theta}$ identifies the vacancy elasticity α_v^* directly.

Notation. For compactness define:

$$b_u \;\equiv\; \varphi - 1 \;=\; \alpha_u^* + \alpha_v^* - 1, \qquad b_v \;\equiv\; \alpha_v^*, \tag{A.15}$$

so that

$$\alpha_v^* = b_v, \qquad \alpha_u^* = b_u - b_v + 1, \qquad \varphi = b_u + 1. \tag{A.16}$$

Matching efficiency and the ARMA structure Matching efficiency $\ln A_{\ell t}$ is unobserved and correlated with inputs, generating simultaneity bias in OLS. Following Arellano and Bond (1991) and the matching function literature (Petrongolo and Pissarides, 2001), we model $\ln A_{\ell t}$ as an ARMA$(1,0)$ process:

$$\ln A_{\ell t} \;=\; \rho \, \ln A_{\ell,t-1} \;+\; \varepsilon_{\ell t}, \qquad |\rho| < 1, \tag{A.17}$$

where $\varepsilon_{\ell t}$ is a white-noise innovation. Substituting (A.17) into (A.14) and quasi-differencing—i.e., subtracting ρ times the period-$t-1$ equation—eliminates $\ln A_{\ell t}$ and yields the estimating equation:

$$\begin{aligned} \ln f_{\ell t} \;=\; & \rho \, \ln f_{\ell,t-1} \;+\; b_u\Big(\widetilde{\ln U}_{\ell t} - \rho\, \widetilde{\ln U}_{\ell,t-1}\Big) \;+\; b_v\Big(\widetilde{\ln \theta}_{\ell t} - \rho\, \widetilde{\ln \theta}_{\ell,t-1}\Big) \\ & +\; \tfrac{1}{2}\beta_{uu}\Big(\widetilde{\ln U}_{\ell t}^{\,2} - \rho\, \widetilde{\ln U}_{\ell,t-1}^{\,2}\Big) \;+\; \tfrac{1}{2}\beta_{vv}\Big(\widetilde{\ln \theta}_{\ell t}^{\,2} - \rho\, \widetilde{\ln \theta}_{\ell,t-1}^{\,2}\Big) \\ & +\; \beta_{uv}\Big(\widetilde{\ln U}_{\ell t}\,\widetilde{\ln \theta}_{\ell t} - \rho\, \widetilde{\ln U}_{\ell,t-1}\,\widetilde{\ln \theta}_{\ell,t-1}\Big) \;+\; \varepsilon_{\ell t}. \end{aligned} \tag{A.18}$$

Equation (A.18) is the structural counterpart to the moment condition implemented in the GMM estimation.

Month fixed effects and the Frisch–Waugh–Lovell theorem To control for aggregate time variation in matching efficiency common to all markets—seasonal patterns, business-cycle fluctuations, aggregate shocks to search technology—we include month fixed effects μ_t. Incorporating μ_t additively into $\ln A_{\ell t}$ preserves the quasi-differencing argument as long as the fixed effects are absorbed prior to estimation.

By the Frisch–Waugh–Lovell (FWL) theorem, including μ_t in the estimating equation is numerically equivalent to first regressing each variable in (A.18) on a full set of month dummies and then running GMM on the residuals. Denoting by $\tilde{x}_{\ell t}$ the residual from projecting any variable $x_{\ell t}$ onto month dummies, the GMM moment condition becomes:

$$\mathbb{E}[\xi_{\ell t} \cdot z_{\ell t}] = 0, \tag{A.19}$$

where

$$\xi_{\ell t} \;\equiv\; \widetilde{\ln f}_{\ell t} - b_u \widetilde{\ln U}_{\ell t} - b_v \widetilde{\ln \theta}_{\ell t} - \tfrac{1}{2}\beta_{uu} \widetilde{\ln U}_{\ell t}^{\,2} - \tfrac{1}{2}\beta_{vv} \widetilde{\ln \theta}_{\ell t}^{\,2} - \beta_{uv} \widetilde{\ln U}_{\ell t} \widetilde{\ln \theta}_{\ell t} - \rho\,[\text{same at } t-1], \tag{A.20}$$

double tildes denote variables residualized on month dummies, and the instrument vector is:

$$z_{\ell t} \;\equiv\; \Big\{\widetilde{\ln U}_{\ell t},\, \widetilde{\ln U}_{\ell t}^{\,2},\, \widetilde{\ln U}_{\ell,t-1} \cdot \widetilde{\ln \theta}_{\ell,t-1},\, \widetilde{\ln \theta}_{\ell,t-1},\, \widetilde{\ln f}_{\ell,t-1},\, \widetilde{\ln \theta}_{\ell,t-1}^{\,2}\Big\}. \tag{A.21}$$

The instruments exploit the predetermined nature of unemployment (valid at t) and the lagged values of tightness, job-finding rate, and the second-order interaction terms. The latter are required for identification of the additional parameters β_{uu}, β_{vv}, and β_{uv} beyond those in the Cobb–Douglas case.

Parameter recovery and testable restrictions Given GMM estimates $(\hat{b}_u, \hat{b}_v, \hat{\beta}_{uu}, \hat{\beta}_{vv}, \hat{\beta}_{uv}, \hat{\rho})$, the structural parameters are recovered as:

$$\hat{\alpha}_v^* = \hat{b}_v, \tag{A.22}$$

$$\hat{\alpha}_u^* = \hat{b}_u - \hat{b}_v + 1, \tag{A.23}$$

$$\hat{\varphi} = \hat{b}_u + 1, \tag{A.24}$$

$$\hat{\sigma} = \frac{-\hat{\alpha}_u^* \hat{\alpha}_v^* \hat{\varphi}}{(\hat{\alpha}_v^*)^2 \hat{B}_{UU} + (\hat{\alpha}_u^*)^2 \hat{B}_{VV} - 2\hat{\alpha}_u^* \hat{\alpha}_v^* \hat{B}_{UV}}, \tag{A.25}$$

where $\hat{B}_{UU}$, $\hat{B}_{VV}$, $\hat{B}_{UV}$ are obtained from the Jacobian in (A.12). Standard errors for all four quantities are obtained by the delta method via `nlcom` in Stata.

The following restrictions are directly testable:

1. **Constant returns to scale**: $H_0 \colon \varphi = 1 \Leftrightarrow b_u = 0$.
2. **Cobb–Douglas**: $H_0 \colon \beta_{uu} = \beta_{vv} = \beta_{uv} = 0$ (joint F-test).
3. **CES symmetry**: $H_0 \colon \beta_{uu} = \beta_{vv}$, which is implied by the CES structure.
4. **Decreasing returns**: $H_0 \colon \varphi < 1 \Leftrightarrow b_u < 0$ (one-sided).

Finally, two observations. First, under the Cobb–Douglas restriction ($\beta_{uu} = \beta_{vv} = \beta_{uv} = 0$), the denominator of (A.25) is zero and σ is not identified from the second-order terms. This is the familiar result that the elasticity of substitution cannot be distinguished from unity within the Cobb–Douglas family. Rejection of the joint Cobb–Douglas restriction is therefore a prerequisite for estimating σ.

Second, the translog is a *local* approximation. Its accuracy declines as observations move away from the expansion point $(\bar{u}, \bar{v})$. Demeaning ensures the approximation is centered on the region of highest data density, but the recovered σ should be interpreted as the elasticity of substitution evaluated at the average local labor market.

Appendix B The Quantitative Model

Appendix B.1 Decentralized Equilibrium

Value Functions

Value function of unemployed workers:

$$\mathcal{U}_\ell = b - p_\ell + \frac{1}{1+r}\Big\{ (1 - f(\theta_\ell, S_\ell))\, \mathcal{U}'_\ell + f(\theta_\ell, S_\ell) \left[\delta \mathcal{U}'_\ell + (1-\delta)\mathbb{E}_{\varepsilon'} \max\{\mathcal{W}'_\ell(\varepsilon'), \mathcal{U}'_\ell\}\right] \Big\} \tag{B.26}$$

$$= b - p_\ell + \frac{1}{1+r}\Big\{ \mathcal{U}'_\ell + f(\theta_\ell, S_\ell)(1-\delta)\mathbb{E}_{\varepsilon'} \left[\mathcal{W}'_\ell(\varepsilon') - \mathcal{U}'_\ell\right]^+ \Big\} \tag{B.27}$$

Value function of employed workers:

$$\mathcal{W}_\ell(\varepsilon) = w_\ell(\varepsilon) - p_\ell + \frac{1}{1+r}\Big\{ (1-\phi f(\theta_\ell, S_\ell)) \left[\delta \mathcal{U}'_\ell + (1-\delta)\mathbb{E}_{\varepsilon'} \max\{\mathcal{W}'_\ell(\varepsilon'), \mathcal{U}'_\ell\}\right] + \phi f(\theta_\ell, S_\ell)(1-\chi_\ell) \left[\delta \mathcal{U}'_\ell + (1-\delta)\mathbb{E}_{\varepsilon'} \max\{\mathcal{W}'_\ell(\varepsilon'), \mathcal{U}'_\ell\}\right] + \phi f(\theta_\ell, S_\ell)\chi_\ell \left[\delta \mathcal{U}'_\ell + (1-\delta)\mathbb{E}_{\varepsilon'} \max\{\mathcal{W}'_\ell(\varepsilon'), \mathcal{U}'_\ell\}\right] \Big\} \tag{B.28}$$

$$= w_\ell(\varepsilon) - p_\ell + \frac{1}{1+r}\Big\{ \mathcal{U}'_\ell + (1-\delta)\mathbb{E}_{\varepsilon'} \left[\mathcal{W}'_\ell(\varepsilon') - \mathcal{U}'_\ell\right]^+ \Big\} \tag{B.29}$$

Value function of vacancy posting firms:

$$\mathcal{V}_\ell = -\kappa + \frac{1}{1+r}\Big\{ (1-q(\theta_\ell, S_\ell))\, \mathcal{V}'_\ell + q(\theta_\ell, S_\ell)\frac{U_\ell}{S_\ell} \left[\delta \mathcal{V}'_\ell + (1-\delta)\mathbb{E}_{\varepsilon'} \max\{\mathcal{J}'_\ell(\varepsilon'), \mathcal{V}'_\ell\}\right] + q(\theta_\ell, S_\ell)\frac{\phi(N_\ell - U_\ell)}{S_\ell} \left[(1-\chi_\ell)\mathcal{V}'_\ell + \chi_\ell \left(\delta \mathcal{V}'_\ell + (1-\delta)\mathbb{E}_{\varepsilon'} \max\{\mathcal{J}'_\ell(\varepsilon'), \mathcal{V}'_\ell\}\right)\right] \Big\} \tag{B.30}$$

$$= -\kappa + \frac{1-\delta}{1+r} q(\theta_\ell, S_\ell) \left(\frac{U_\ell + \chi_\ell \phi(N_\ell - U_\ell)}{U_\ell + \phi(N_\ell - U_\ell)}\right) \mathbb{E}_{\varepsilon'} \left[\mathcal{J}'_\ell(\varepsilon')\right]^+ \tag{B.31}$$

Value function of producing firms:

$$\mathcal{J}_\ell(\varepsilon) = z_\ell \varepsilon - w_\ell(\varepsilon) + \frac{1}{1+r}\Big\{ (1-\phi f(\theta_\ell, S_\ell)) \left[\delta \mathcal{V}'_\ell + (1-\delta)\mathbb{E}_{\varepsilon'} \max\left\{\mathcal{J}'_\ell(\varepsilon'), \mathcal{V}'_\ell\right\}\right] + \phi f(\theta_\ell, S_\ell)(1-\chi_\ell) \left[\delta \mathcal{V}'_\ell + (1-\delta)\mathbb{E}_{\varepsilon'} \max\left\{\mathcal{J}'_\ell(\varepsilon'), \mathcal{V}'_\ell\right\}\right] + \phi f(\theta_\ell, S_\ell)\chi_\ell \mathcal{V}'_\ell \Big\} \tag{B.32}$$

$$= z_\ell \varepsilon - w_\ell(\varepsilon) + \frac{1-\delta}{1+r} (1 - \phi\chi_\ell f(\theta_\ell, S_\ell))\, \mathbb{E}_{\varepsilon'} \left[\mathcal{J}'_\ell(\varepsilon')\right]^+ \tag{B.33}$$

Equilibrium Characterization

Within-Location Equilibrium Surplus of a match:

$$\mathcal{S}_\ell(\varepsilon) \equiv \mathcal{W}_\ell(\varepsilon) - \mathcal{U}_\ell + \mathcal{J}_\ell(\varepsilon) - \mathcal{V}_\ell \tag{B.34}$$

$$= z_\ell \varepsilon - b + \frac{1-\delta}{1+r} (1 - (1 - \gamma + \gamma\phi\chi_\ell) f(\theta_\ell, S_\ell))\, \mathbb{E}_{\varepsilon'} [\mathcal{S}'_\ell(\varepsilon')]^+ \tag{B.35}$$

Job creation:

$$\frac{\kappa}{q(\theta_\ell, S_\ell)\left(\frac{U_\ell+\chi_\ell\phi(N_\ell-U_\ell)}{U_\ell+\phi(N_\ell-U_\ell)}\right)} = \frac{1-\delta}{1+r}\mathbb{E}_{\varepsilon'}\left[\gamma(z_\ell\varepsilon'-b)+\left(\frac{U'_\ell+\phi(N'_\ell-U'_\ell)}{U'_\ell+\chi_\ell\phi(N'_\ell-U'_\ell)}\right)\frac{(1-(1-\gamma+\gamma\phi\chi_\ell)f(\theta'_\ell,S'_\ell))\,\kappa}{q(\theta'_\ell,S'_\ell)}\right]^+ \tag{B.36}$$

$$= \frac{1-\delta}{1+r}(1-\rho)(1-F(\bar{\varepsilon}'_\ell))\left[\gamma(z_\ell\mathbb{E}_{\varepsilon'\geq\bar{\varepsilon}'_\ell}\varepsilon'-b)+\left(\frac{U'_\ell+\phi(N'_\ell-U'_\ell)}{U'_\ell+\chi_\ell\phi(N'_\ell-U'_\ell)}\right)\frac{(1-(1-\gamma+\gamma\phi\chi_\ell)f(\theta'_\ell,S'_\ell))\,\kappa}{q(\theta'_\ell,S'_\ell)}\right]$$
$$+\frac{1-\delta}{1+r}\rho\left[\gamma(z_\ell\mathbb{E}\varepsilon'-b)+\left(\frac{U'_\ell+\phi(N'_\ell-U'_\ell)}{U'_\ell+\chi_\ell\phi(N'_\ell-U'_\ell)}\right)\frac{(1-(1-\gamma+\gamma\phi\chi_\ell)f(\theta'_\ell,S'_\ell))\,\kappa}{q(\theta'_\ell,S'_\ell)}\right] \tag{B.37}$$

$$= \frac{1-\delta}{1+r}\left((1-\rho)(1-F(\bar{\varepsilon}'_\ell))+\rho\right)\left[\gamma\left(z_\ell\left(\frac{(1-\rho)\int_{\varepsilon'\geq\bar{\varepsilon}'_\ell}\varepsilon' dF(\varepsilon')+\rho\mathbb{E}\varepsilon'}{((1-\rho)(1-F(\bar{\varepsilon}'_\ell))+\rho)}\right)-b\right)\right.$$
$$\left.+\left(\frac{U'_\ell+\phi(N'_\ell-U'_\ell)}{U'_\ell+\chi_\ell\phi(N'_\ell-U'_\ell)}\right)\frac{(1-(1-\gamma+\gamma\phi\chi_\ell)f(\theta'_\ell,S'_\ell))\,\kappa}{q(\theta'_\ell,S'_\ell)}\right] \tag{B.38}$$

Separation threshold:

$$\mathcal{S}_\ell(\bar{\varepsilon}_\ell) = 0 \tag{B.39}$$

$$= z_\ell\bar{\varepsilon}_\ell - b + \frac{(1-(1-\gamma+\gamma\phi\chi_\ell)f(\theta_\ell,S_\ell))\,\kappa}{\gamma q(\theta_\ell,S_\ell)}\left(\frac{U_\ell+\phi(N_\ell-U_\ell)}{U_\ell+\chi_\ell\phi(N_\ell-U_\ell)}\right) \tag{B.40}$$

Unemployment level:

$$U'_\ell = \left(1-f(\theta_\ell,S_\ell)(1-\delta)(1-F(\bar{\varepsilon}'_\ell))\right)U_\ell + (1-(1-\delta)(1-F(\bar{\varepsilon}'_\ell)))(N_\ell-\mathcal{U}_\ell) \tag{B.41}$$

Housing:

$$\max_{N_\ell} p_\ell N_\ell - g(N_\ell) \quad \rightarrow \quad p_\ell = g'(N_\ell) \tag{B.42}$$

Across-Location Equilibrium No-arbitrage across location among unemployed workers:

$$\mathcal{U}_\ell = \frac{r}{1+r}\mathcal{U} \tag{B.43}$$

$$= b - g'(N_\ell) + \frac{1-\gamma}{\gamma}\kappa\theta_\ell\left(\frac{U_\ell+\phi(N_\ell-U_\ell)}{U_\ell+\chi_\ell\phi(N_\ell-U_\ell)}\right) \tag{B.44}$$

Overall Equilibrium in Steady State Steady state job creation equation:

$$\frac{\kappa}{\widetilde{q}(\theta_\ell,S_\ell)} = \frac{(1-\delta)\gamma\left((1-\rho)(1-F(\bar{\varepsilon}_\ell))(z_\ell\mathbb{E}_{\varepsilon\geq\bar{\varepsilon}_\ell}\varepsilon-b)+\rho(z_\ell\mathbb{E}\varepsilon-b)\right)}{1+r-(1-\delta)\left((1-\rho)(1-F(\bar{\varepsilon}_\ell))+\rho\right)(1-(1-\gamma+\gamma\phi\chi_\ell)f(\theta_\ell,S_\ell))} \tag{B.45}$$

where $\widetilde{q}(\theta_\ell,S_\ell) = \left(\frac{U_\ell+\chi_\ell\phi(N_\ell-U_\ell)}{U_\ell+\phi(N_\ell-U_\ell)}\right)q(\theta_\ell,S_\ell)$. Separation threshold in steady state:

$$\bar{\varepsilon}_\ell = \frac{1}{z_\ell}\left(b - \frac{(1-(1-\gamma+\gamma\phi\chi_\ell)f(\theta_\ell,S_\ell))\,\kappa}{\gamma\widetilde{q}(\theta_\ell,S_\ell)}\right) \tag{B.46}$$

Steady state level of unemployment:

$$U_\ell = \frac{1-(1-\delta)(1-F(\bar{\varepsilon}_\ell))}{f(\theta_\ell,S_\ell)(1-\delta)(1-F(\bar{\varepsilon}_\ell))+1-(1-\delta)(1-F(\bar{\varepsilon}_\ell))}N_\ell \tag{B.47}$$

Steady state level of population:

$$\frac{r}{1+r}\mathcal{U} = b - g'(N_\ell) + \frac{1-\gamma}{\gamma}\left(\frac{U_\ell + \phi(N_\ell - U_\ell)}{U_\ell + \chi_\ell\phi(N_\ell - U_\ell)}\right)\kappa\theta_\ell \tag{B.48}$$

Thus, a steady state in this economy is a set $\{\theta_\ell, \bar{\varepsilon}, U_\ell, N_\ell\}$ such that Equations (B.45)-(B.52) are satisfied.

Overall Equilibrium in Steady State with Full Acceptance Rate ($\chi_\ell = 1$) Steady state job creation equation:

$$\frac{\kappa}{q(\theta_\ell, S_\ell)} = \frac{(1-\delta)\gamma\left((1-\rho)(1-F(\bar{\varepsilon}_\ell))(z_\ell\mathbb{E}_{\varepsilon\geq\bar{\varepsilon}_\ell}\varepsilon - b) + \rho(z_\ell\mathbb{E}\varepsilon - b)\right)}{1 + r - (1-\delta)\left((1-\rho)(1-F(\bar{\varepsilon}_\ell)) + \rho\right)\left(1 - (1-\gamma+\gamma\phi)f(\theta_\ell, S_\ell)\right)} \tag{B.49}$$

where $\widetilde{q}(\theta_\ell, S_\ell) = \left(\frac{U_\ell + \chi_\ell\phi(N_\ell - U_\ell)}{U_\ell + \phi(N_\ell - U_\ell)}\right) q(\theta_\ell, S_\ell)$. Separation threshold in steady state:

$$\bar{\varepsilon}_\ell = \frac{1}{z_\ell}\left(b - \frac{\left(1 - (1-\gamma+\gamma\phi)f(\theta_\ell, S_\ell)\right)\kappa}{\gamma q(\theta_\ell, S_\ell)}\right) \tag{B.50}$$

Steady state level of unemployment:

$$U_\ell = \frac{1 - (1-\delta)(1-F(\bar{\varepsilon}))}{f(\theta_\ell, S_\ell)(1-\delta)(1-F(\bar{\varepsilon})) + 1 - (1-\delta)(1-F(\bar{\varepsilon}))}N_\ell \tag{B.51}$$

Steady state level of population:

$$\frac{r}{1+r}\mathcal{U} = b - g'(N_\ell) + \frac{1-\gamma}{\gamma}\kappa\theta_\ell \tag{B.52}$$

Thus, a steady state in this economy is a set $\{\theta_\ell, \bar{\varepsilon}, U_\ell, N_\ell\}$ such that Equations (B.45)-(B.52) are satisfied.

Appendix B.2 Planner Economy

Planner Problem

Planner value function:

$$\begin{aligned}\boldsymbol{\mathcal{W}}(E_\ell, \bar{\varepsilon}_\ell) = \max_{V_\ell, \bar{\varepsilon}'_\ell, N_\ell} z_\ell &\left(\frac{(1-\rho)\int_{\varepsilon\geq\bar{\varepsilon}_\ell}\varepsilon dF(\varepsilon) + \rho\mathbb{E}\varepsilon}{\left((1-\rho)(1-F(\bar{\varepsilon}_\ell)) + \rho\right)}\right)E_\ell \\ &+ b(N_\ell - E_\ell) - \kappa V_\ell - g(N_\ell) + \frac{1}{1+r}\boldsymbol{\mathcal{W}}(E'_\ell, \bar{\varepsilon}'_\ell),\end{aligned} \tag{B.53}$$

subject to:

$$E'_\ell = (1-\delta)\left((1-\rho)(1-F(\bar{\varepsilon}'_\ell)) + \rho\right)\left(E_\ell + \left((N_\ell - E_\ell + \phi E_\ell)^{1-\alpha}(V_\ell)^\alpha\right)^\nu \frac{N_\ell - E_\ell}{N_\ell - E_\ell + \phi E_\ell}\right) \tag{B.54}$$

$$N = \sum_\ell N_\ell \tag{B.55}$$

Planner Equilibrium

First order conditions:

$$V_\ell: \quad -\kappa + \left((1-\rho)(1-F(\bar{\varepsilon}'_\ell))+\rho\right)\alpha\nu q(\theta_\ell, S_\ell)\frac{U_\ell}{S_\ell}\frac{1-\delta}{1+r}\frac{\partial \boldsymbol{\mathcal{W}}(E'_\ell, \bar{\varepsilon}'_\ell)}{\partial E'_\ell} = 0 \tag{B.56}$$

$$\bar{\varepsilon}'_\ell: \quad \frac{\partial(1-F(\bar{\varepsilon}'_\ell))}{\partial \bar{\varepsilon}'_\ell}\frac{(1-\rho)E'_\ell}{(1-\rho)(1-F(\bar{\varepsilon}'_\ell))+\rho}\frac{\partial \boldsymbol{\mathcal{W}}(E'_\ell, \bar{\varepsilon}'_\ell)}{\partial E'_\ell} + \frac{\partial \boldsymbol{\mathcal{W}}(E'_\ell, \bar{\varepsilon}'_\ell)}{\partial \bar{\varepsilon}'_\ell} = 0 \tag{B.57}$$

$$N_\ell: \quad b - g'(N_\ell) - \mu + \left((1-\rho)(1-F(\bar{\varepsilon}'_\ell))+\rho\right) f(\theta_\ell, S_\ell)\left(\frac{\nu(1-\alpha)U_\ell + \phi E_\ell}{S_\ell}\right)\frac{1-\delta}{1+r}\frac{\partial \boldsymbol{\mathcal{W}}(E'_\ell, \bar{\varepsilon}'_\ell)}{\partial E'_\ell} = 0 \tag{B.58}$$

Envelope conditions:

$$\begin{aligned}\frac{\partial \boldsymbol{\mathcal{W}}(E_\ell, \bar{\varepsilon}_\ell)}{\partial E_\ell} &= z_\ell\left(\frac{(1-\rho)\int_{\varepsilon\geq\bar{\varepsilon}_\ell}\varepsilon dF(\varepsilon) + \rho\mathbb{E}\varepsilon}{((1-\rho)(1-F(\bar{\varepsilon}_\ell))+\rho)}\right) - b \\ &\quad + \left((1-\rho)(1-F(\bar{\varepsilon}'_\ell))+\rho\right)\left(1 - \left(\frac{\nu(1-\alpha)(1-\phi)U_\ell + \phi N_\ell}{S_\ell}\right) f(\theta_\ell, S_\ell)\right)\frac{1-\delta}{1+r}\frac{\partial \boldsymbol{\mathcal{W}}(E'_\ell, \bar{\varepsilon}'_\ell)}{\partial E'_\ell}\end{aligned} \tag{B.59}$$

$$\begin{aligned}\frac{\partial \boldsymbol{\mathcal{W}}(E_\ell, \bar{\varepsilon}_\ell)}{\partial \bar{\varepsilon}_\ell} &= -\frac{1-\rho}{(1-\rho)(1-F(\bar{\varepsilon}_\ell))+\rho}\frac{\partial 1-F(\bar{\varepsilon}_\ell)}{\partial \bar{\varepsilon}_\ell} z_\ell\left(\frac{(1-\rho)\int_{\varepsilon\geq\bar{\varepsilon}_\ell}\varepsilon dF(\varepsilon) + \rho\mathbb{E}\varepsilon}{((1-\rho)(1-F(\bar{\varepsilon}_\ell))+\rho)}\right)E_\ell \\ &\quad + \frac{1-\rho}{(1-\rho)(1-F(\bar{\varepsilon}_\ell))+\rho}\frac{\partial 1-F(\bar{\varepsilon}_\ell)}{\partial \bar{\varepsilon}_\ell} z_\ell \bar{\varepsilon}_\ell E_\ell\end{aligned} \tag{B.60}$$

Combining Equations (B.56) and (B.59) yields:

$$\begin{aligned}\frac{\partial \boldsymbol{\mathcal{W}}(E_\ell, \bar{\varepsilon}_\ell)}{\partial E_\ell} &= z_\ell\left(\frac{(1-\rho)\int_{\varepsilon\geq\bar{\varepsilon}_\ell}\varepsilon dF(\varepsilon) + \rho\mathbb{E}\varepsilon}{((1-\rho)(1-F(\bar{\varepsilon}_\ell))+\rho)}\right) - b \\ &\quad + \left(\frac{S_\ell - (\nu(1-\alpha)(1-\phi)U_\ell + \phi N_\ell) f(\theta_\ell, S_\ell)}{U_\ell}\right)\frac{\kappa}{\nu\alpha q(\theta_\ell, S_\ell)}\end{aligned} \tag{B.61}$$

Combining Equations (B.56) and (B.61) yields the planner job creation equation:

$$\frac{\kappa}{q(\theta_\ell, S_\ell)}\frac{S_\ell}{U_\ell} = \frac{1-\delta}{1+r}\left((1-\rho)(1-F(\bar{\varepsilon}'_\ell))+\rho\right)\left[\nu\alpha\left(z_\ell\left(\frac{(1-\rho)\int_{\varepsilon'\geq\bar{\varepsilon}'_\ell}\varepsilon' dF(\varepsilon') + \rho\mathbb{E}\varepsilon'}{((1-\rho)(1-F(\bar{\varepsilon}'_\ell))+\rho)}\right) - b\right) + \left(\frac{S_\ell - (\nu(1-\alpha)(1-\phi)U_\ell + \phi N_\ell) f(\theta_\ell, S_\ell)}{U_\ell}\right)\frac{\kappa}{q(\theta'_\ell, S'_\ell)}\right] \tag{B.62}$$

Combining Equations (B.57), (B.60) and (B.61) yields the planner job separation equation:

$$z_\ell\bar{\varepsilon}_\ell - b + \left(\frac{S_\ell - (\nu(1-\alpha)(1-\phi)U_\ell + \phi N_\ell) f(\theta_\ell, S_\ell)}{U_\ell}\right)\frac{\kappa}{\nu\alpha q(\theta_\ell, S_\ell)} = 0 \tag{B.63}$$

Finally, combining (B.56) and (B.58), yields the following equation that determines the allocation of people in space:

$$\mu = b - g'(N_\ell) + \left(\frac{1-\alpha}{\alpha} + \frac{\phi}{\alpha\nu}\frac{E_\ell}{U_\ell}\right)\kappa\theta_\ell \tag{B.64}$$

Planner Steady State

Planner steady state job creation equation:

$$\frac{\kappa}{\widetilde{q}(\theta_\ell, S_\ell)} = \frac{(1-\delta)\nu\alpha\left((1-\rho)(1-F(\bar{\varepsilon}_\ell))(z_\ell \mathbb{E}_{\varepsilon \geq \bar{\varepsilon}_\ell}\varepsilon - b) + \rho(\mathbb{E}\varepsilon - b)\right)}{1 + r - (1-\delta)((1-\rho)(1-F(\bar{\varepsilon}_\ell)) + \rho)\left(1 - \frac{\nu(1-\alpha)(1-\phi)U_\ell + \phi N_\ell}{S_\ell} f(\theta_\ell, S_\ell)\right)}, \tag{B.65}$$

where $\widetilde{q}(\theta_\ell, S_\ell) \equiv q(\theta_\ell, S_\ell)\frac{U_\ell}{S_\ell}$. Planner job destruction threshold:

$$\bar{\varepsilon}_\ell = \frac{1}{z_\ell}\left(b - \frac{\left(1 - \frac{\nu(1-\alpha)(1-\phi)U_\ell + \phi N_\ell}{S_\ell} f(\theta_\ell, S_\ell)\right)\kappa}{\nu\alpha\widetilde{q}(\theta_\ell, S_\ell)}\right) \tag{B.66}$$

Planner steady state level of population:

$$\mu = b - g'(N_\ell) + \left(\frac{1-\alpha}{\alpha} + \frac{\phi}{\alpha\nu}\frac{E_\ell}{U_\ell}\right)\kappa\theta_\ell \tag{B.67}$$

System Reduction

Planner steady state level of population:

$$\mu = b - g'(N_\ell) + \left(\frac{1-\alpha}{\alpha} + \frac{\phi}{\alpha\nu}\frac{E_\ell}{U_\ell}\right)\kappa\theta_\ell \tag{B.68}$$

$$= b - g'(N_\ell) + \left(\frac{1-\alpha}{\alpha} + \frac{\phi}{\alpha\nu}\frac{1-u_\ell}{u_\ell}\right)\kappa\theta_\ell, \tag{B.69}$$

thus $N_\ell = N(\theta_\ell, u_\ell, \mu)$. Planner job destruction threshold:

$$\bar{\varepsilon}_\ell = \frac{1}{z_\ell}\left(b - \frac{\left(1 - \frac{\nu(1-\alpha)(1-\phi)U_\ell + \phi N_\ell}{S_\ell} f(\theta_\ell, S_\ell)\right)\kappa}{\nu\alpha\widetilde{q}(\theta_\ell, S_\ell)}\right) \tag{B.70}$$

$$= \frac{1}{z_\ell}\left(b - \frac{\kappa}{\nu\alpha q(\theta_\ell, S(\theta_\ell, u_\ell, \mu))\left(\frac{u_\ell}{u_\ell + \phi(1-u_\ell)}\right)} + \frac{\nu(1-\alpha)(1-\phi)u_\ell + \phi}{\nu\alpha u_\ell}\kappa\theta_\ell\right), \tag{B.71}$$

where $S_\ell = N(\theta_\ell, u_\ell, \mu)(u_\ell + \phi(1-u_\ell)) \equiv S(\theta_\ell, u_\ell, \mu)$ and thus $\bar{\varepsilon}_\ell \equiv \bar{\varepsilon}(\theta_\ell, u_\ell, \mu)$. Social planner unemployment rate:

$$u_\ell = \frac{1 - (1-\delta)(1 - F(\bar{\varepsilon}(\theta_\ell, u_\ell, \mu)))}{f(\theta_\ell, S(\theta_\ell, u_\ell, \mu))(1-\delta)(1 - F(\bar{\varepsilon}(\theta_\ell, u_\ell, \mu))) + 1 - (1-\delta)(1 - F(\bar{\varepsilon}(\theta_\ell, u_\ell, \mu)))}. \tag{B.72}$$

Planner job creation equation:

$$\frac{\kappa}{q(\theta_\ell, S(\theta_\ell, u_\ell, \mu))\left(\frac{u_\ell}{u_\ell + \phi(1-u_\ell)}\right)} = \tag{B.73}$$

$$\frac{(1-\delta)\nu\alpha\left((1-\rho)(1-F(\bar{\varepsilon}(\theta_\ell, u_\ell, \mu)))(z_\ell \mathbb{E}_{\varepsilon \geq \bar{\varepsilon}(\theta_\ell, u_\ell, \mu)}\varepsilon - b) + \rho(\mathbb{E}\varepsilon - b)\right)}{1 + r - (1-\delta)((1-\rho)(1-F(\bar{\varepsilon}(\theta_\ell, u_\ell, \mu))) + \rho)\left(1 - \frac{\nu(1-\alpha)(1-\phi)u_\ell + \phi}{u_\ell + \phi(1-u_\ell)} f(\theta_\ell, S(\theta_\ell, u_\ell, \mu))\right)}, \tag{B.74}$$

Appendix C Additional Figures

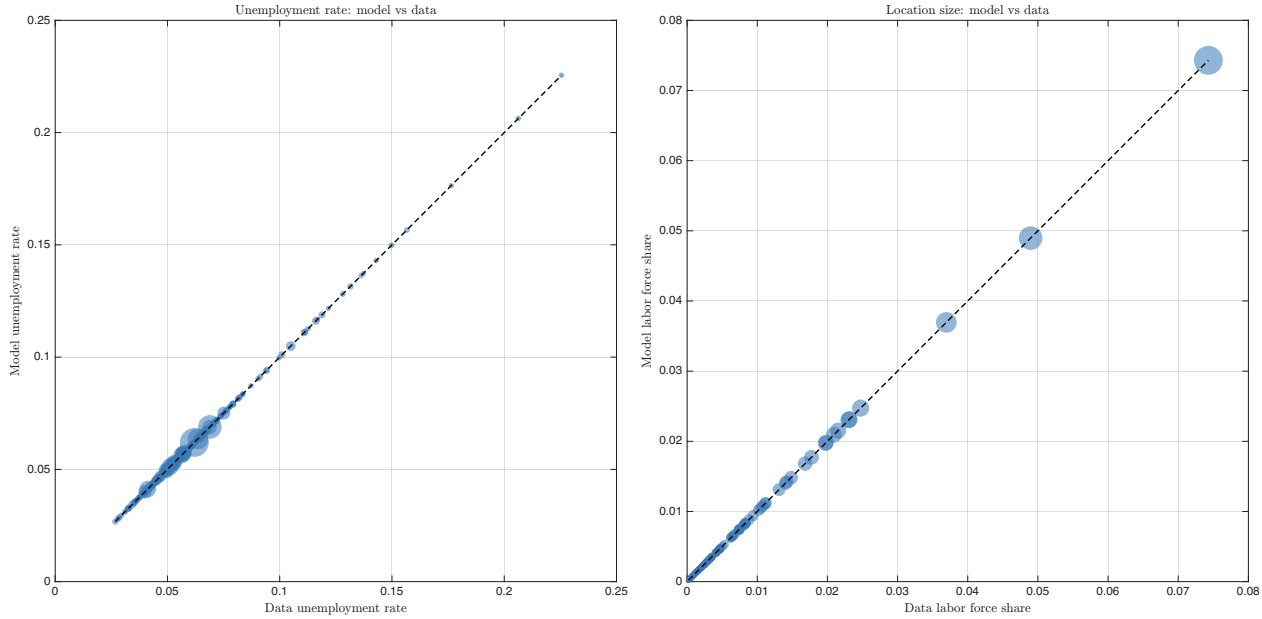


Figure C.1: Caption

References

ACKERBERG, D. A., C. L. BENKARD, S. BERRY, AND A. PAKES (2007): "Econometric Tools for Analyzing Market Outcomes," in *Handbook of Econometrics*, ed. by J. J. Heckman and E. E. Leamer, Elsevier, vol. 6A, 4171–4276.

ACKERBERG, D. A., K. CAVES, AND G. FRAZER (2015): "Identification Properties of Recent Production Function Estimators," *Econometrica*, 83, 2411–2451.

AGUILAR, E. (2023): "Credit Constraints, Learning, and Spatial Misallocation," Working paper.

AMIOR, M. AND A. MANNING (2018): "The Persistence of Local Joblessness," *American Economic Review*, 108, 1942–1970.

ANDERSON, P. M. AND S. M. BURGESS (2000): "Empirical matching functions: estimation and interpretation using state-level data," *Review of Economics and Statistics*, 82, 93–102.

ARELLANO, M. AND S. BOND (1991): "Some tests of specification for panel data: Monte Carlo evidence and an application to employment equations," *The review of economic studies*, 58, 277–297.

BARNICHON, R. AND A. FIGURA (2015): "Labor market heterogeneity and the aggregate matching function," *American Economic Journal: Macroeconomics*, 7, 222–249.

BEVERIDGE, W. H. (1944): *Full Employment in a Free Society*, London: George Allen & Unwin.

BILAL, A. (2023): "Geography, Inequality, and Aggregate Demand," *Working paper*.

BOROWCZYK-MARTINS, D., G. JOLIVET, AND F. POSTEL-VINAY (2013): "Accounting for endogeneity in matching function estimation," *Review of Economic Dynamics*, 16, 440–451.

BRANCACCIO, G., M. KALOUPTSIDI, AND T. PAPAGEORGIOU (2020): "A guide to estimating matching functions in spatial models," *International Journal of Industrial Organization*, 70, 102533.

CARD, D., J. ROTHSTEIN, AND M. YI (2025): "Location, Location, Location," *American Economic Journal: Applied Economics*, 17, 297–336.

COBB, C. W. AND P. H. DOUGLAS (1928): "A Theory of Production," *American Economic Review*, 18, 139–165.

CREWS, J. (2023): "Firm Dynamics and Agglomeration in Cities," Working paper.

DAUTH, W., S. FINDEISEN, E. MORETTI, AND J. SUEDEKUM (2022): "Matching in Cities," *Journal of the European Economic Association*, 20, 1478–1521.

DAVIS, D. R. AND J. I. DINGEL (2019): "A Spatial Knowledge Economy," *American Economic Review*, 109, 153–193.

DE LA ROCA, J. AND D. PUGA (2017): "Learning by Working in Big Cities," *Review of Economic Studies*, 84, 106–142.

DE LOECKER, J. AND P. K. GOLDBERG (2014): "Firm Performance in a Global Market," *Annual Review of Economics*, 6, 201–227.

DE LOECKER, J. AND F. WARZYNSKI (2012): "Markups and Firm-Level Export Status," *American Economic Review*, 102, 2437–2471.

EBERHARDT, M. (2019): "A Critical Review of Firm-Level Production Function Estimators," Working paper.

ECKERT, F., S. GANAPATI, AND C. WALSH (2022): "The Geography of Remote Work," *Regional Science and Urban Economics*, 93, 103770.

EECKHOUT, J. (2004): "Gibrat's law for (all) cities," *American Economic Review*, 94, 1429–1451.

EECKHOUT, J., C. HEDTRICH, AND R. PINHEIRO (2026): "IT and urban polarization," *American Economic Journal: Macroeconomics*, 18, 223–259.

EECKHOUT, J. AND I. LINDENLAUB (2019): "Unemployment cycles," *American Economic Journal: Macroeconomics*, 11, 175–234.

EECKHOUT, J., R. PINHEIRO, AND K. SCHMIDHEINY (2014): "Spatial Sorting," *Journal of Political Economy*, 122, 554–620.

FABERMAN, R. J., A. I. MUELLER, A. SAHIN, AND G. TOPA (2022): "Job Search Behavior among the Employed and Non-Employed," *Econometrica*, 90, 1743–1779.

GRILICHES, Z. AND J. MAIRESSE (1995): "Production Functions: The Search for Identification," in *Econometrics and Economic Theory in the 20th Century*, ed. by S. Strom, Cambridge University Press, 169–203.

HEISE, S. AND T. PORZIO (2022): "Spatial Frictions and Inequality in Labor Markets," *Working paper*.

HOSIOS, A. J. (1990): "On the Efficiency of Matching and Related Models of Search and Unemployment," *Review of Economic Studies*, 57, 279–298.

HSIEH, C.-T. AND P. J. KLENOW (2009): "Misallocation and Manufacturing TFP in China and India," *Quarterly Journal of Economics*, 124, 1403–1448.

KANGASHARJU, A., J. PEHKONEN, AND S. PEKKALA (2005): "Returns to scale in a matching model: evidence from disaggregated panel data," *Applied Economics*, 37, 115–118.

KUHN, M., I. MANOVSKII, AND X. QIU (2026): "The geography of job creation and job destruction," *Journal of Monetary Economics*, 103898.

LANGE, F. AND T. PAPAGEORGIOU (2020): "Beyond Cobb-Douglas: flexibly estimating matching functions with unobserved matching efficiency," Tech. rep., National Bureau of Economic Research.

LAUERMANN, S., G. NÖLDEKE, AND T. TRÖGER (2020): "The Balance Condition in Search-and-Matching Models," *Econometrica*, 88, 595–618.

LHUILLIER, J.-P. (2025): "Search Frictions and Inequality Across Local Labor Markets," Working paper.

MANNING, A. AND B. PETRONGOLO (2017): "How Local Are Labor Markets? Evidence from a Spatial Job Search Model," *American Economic Review*, 107, 2877–2907.

MARTELLINI, P. (2022): "Agglomeration Economies with History and Expectations," *Working paper*.

MENGANO, P. (2022a): "Trends in worker bargaining power," *Available at SSRN 4466868*.

——— (2022b): "Trends in worker bargaining power," *Available at SSRN 4466868*.

MOEN, E. R. (1997): "Competitive Search Equilibrium," *Journal of Political Economy*, 105, 385–411.

NANOS, P. AND C. SCHLUTER (2018): "Search Across Local Labour Markets," Working paper.

OLLEY, G. S. AND A. PAKES (1996): "The Dynamics of Productivity in the Telecommunications Equipment Industry," *Econometrica*, 1263–1297.

PETRONGOLO, B. AND C. A. PISSARIDES (2001): "Looking into the black box: A survey of the matching function," *Journal of Economic literature*, 39, 390–431.

PISSARIDES, C. A. (2000): *Equilibrium unemployment theory*, MIT press.

ROBACK, J. (1982): "Wages, Rents, and the Quality of Life," *Journal of Political Economy*, 90, 1257–1278.

ROSEN, S. (1979): "Wage-Based Indexes of Urban Quality of Life," in *Current Issues in Urban Economics*, ed. by P. Mieszkowski and M. Straszheim, Baltimore: Johns Hopkins University Press, 74–104.

SHIMER, R. (2005): "The cyclical behavior of equilibrium unemployment and vacancies," *American economic review*, 95, 25–49.

——— (2010): *Labor Markets and Business Cycles*, Princeton, NJ: Princeton University Press.

SHIMER, R. AND L. SMITH (2000): "Assortative matching and search," *Econometrica*, 68, 343–369.

SOLOW, R. M. (1957): "Technical Change and the Aggregate Production Function," *Review of Economics and Statistics*, 39, 312–320.

SYVERSON, C. (2011): "What Determines Productivity?" *Journal of Economic Literature*, 49, 326–365.

ZANELLA, S. (2025): "Wealth and the Geography of Job Ladders," Job Market Paper.