On Markups, Borrowing Constraints and Firm Size*

A. Andrés Bellofatto† Begoña Domínguez‡ Patrick Elkington§
Alicia N. Rambaldi¶

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Abstract

This paper studies the relationship between firm-level borrowing constraints and markups. Using comprehensive administrative data on businesses operating in Australia, we proxy borrowing-constraint tightness by combining managers' survey responses on financial decision-making, and firms' balance sheet data from business income tax returns. Within narrowly defined industries, we find that firms facing tighter borrowing constraints tend to charge considerably higher markups. This relationship is negatively related to firm size: at the median of the size distribution, constrained firms set markups about 3% higher than unconstrained firms, but such difference can exceed 10% among the smallest firms. To interpret these findings, we develop a model in which working capital requirements generate a positive link between borrowing constraints and markups. Consistent with our theory, we further show empirically that higher working capital requirements substantially amplify the pass-through from borrowing constraints to markups.

Keywords: Markups; Borrowing Constrained Firms; Administrative Firm Level Data.
JEL Codes: D24, D33, D42, D43, D5, D50, D57, D61, E01, E1, E25, E52, O4, O41.

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†University of Queensland, School of Economics, Brisbane, Australia. E-mail: a.bellofatto@uq.edu.au.
‡University of Queensland, School of Economics, Brisbane, Australia. E-mail: b.dominguez@uq.edu.au.
§Reserve Bank of Australia, Sydney, Australia. E-mail: p.elkington@uq.edu.au.
¶University of Queensland, School of Economics, Brisbane, Australia. E-mail: a.rambaldi@uq.edu.au.

1 Introduction

Over recent decades, a large and influential literature has documented a substantial increase in markups—defined as the wedge between prices and marginal costs—as well as significant dispersion in markups across firms (e.g., De Loecker et al. (2020)). The determinants of this dispersion matter for macroeconomic outcomes, including resource misallocation and policy effectiveness.

This paper aims to advance our understanding of cross-firm variation in markups by focusing on a largely overlooked determinant: borrowing constraints. The paper's main contribution is empirical. To guide the analysis, we first develop a model of monopolistic competition with heterogeneous firms in which borrowing constraints arise from working capital requirements. We then analytically characterize the implications for firms' pricing behavior and markups. Our main data comes from the Business Longitudinal Analysis Data Environment (BLADE), which is constructed by the Australian Bureau of Statistics (ABS). BLADE is the most exhaustive data source on Australian businesses, as it covers all active Australian businesses paying Gross Sales Tax. From these data, we construct firm-level measures of borrowing-constraint tightness by combining managers' survey responses on financial decision-making with balance sheet information from income tax returns. Specifically, we construct two binary indicators: one based on responses of managers/owners to a sequence of questions in the Business Characteristics Survey (BCS) and a second one based on predicted probabilities on being borrowing constrained. Using these measures, we examine the empirical relationship between markups and borrowing-constraint tightness using comprehensive Australian administrative data covering firms operating in Australia.

Our findings are as follows. The theoretical model shows that, all else equal, borrowing constraints increase markups. The difference in markups between constrained and unconstrained firms reflects the balance between two opposing forces. The first is a "pass-through" effect, which operates through working capital requirements and tends to increase markups. The second is a "demand-elasticity" effect, arising from changes in the elasticity of demand as firms reduce production, which tends to reduce markups. We further examine how these effects vary with firm size. The pass-through effect weakens with firm size, while the demand-elasticity effect becomes stronger. Both patterns imply that the impact of borrowing constraints on markups is larger for smaller firms. Our simulation exercises confirm these results.

For our empirical analysis, we adopt a cost-share approach (following Bond et al.

(2021)) to study markup variation across firms within narrowly defined manufacturing industries. We find that, on average, firms facing tighter borrowing constraints (across all measures) charge substantially higher markups. Consistent with the model, this relationship weakens with firm size. At the median of the size distribution, constrained firms set markups approximately 3 % higher than unconstrained firms, while the differential exceeds 10 % among the smallest firms. Consistent with our theory, we further show empirically that higher working capital requirements substantially amplify the pass-through from borrowing constraints to markups.

We next provide a summary of the related literature. Section 2 presents the model and the theoretical results. Section 3 describes the data. Section 4 introduces the empirical framework and the measures of borrowing constraints. Section 5 discusses our empirical results. Section 6 concludes. Lastly, the Appendix includes all proofs.

Related Literature. Building on Hall (1988), De Loecker and Warzynski (2012) developed a methodology to estimate markups using firm-level data. However, several papers argue that such methodology does not accurately capture the "true" markup set by firms. Basu (2019), for example, argues that the methodology can produce unrealistic results. Bond et al. (2021) suggest that there are important identification and estimation issues that present problems when interpreting markups from revenue-based data and production function estimation.[1] Given these concerns, we rely on a "cost-share approach" and study markup variation through variations in the cost share of individual firms. This approach abstracts from issues associated with identifying markup levels and estimating output elasticities using firm-level data.

A growing literature suggests that markups vary substantially across different firm characteristics. Previous studies show that firms alter their markups in response to shocks related to foreign competition (Loecker et al., 2016), exporting decisions (De Loecker and Warzynski, 2012; Garcia-Marin and Voigtländer, 2019) or exchange rates (Berman et al., 2021). Markups also tend to rise as firms increase in size (Amiti et al., 2019; Autor et al., 2020; Gamber, 2023; Burya and Mishra, 2023) and as they grow older (Peters, 2020).

Market characteristics also play a central role in determining firm-level markups. For example, Campbell and Hopenhayn (2005) show that larger markets tend to be associated with greater competition and lower markups. Dhingra and Morrow (2019) and Syverson (2011) show that more productive firms in less competitive markets charge higher

[1] De Ridder et al. (2024) argue that markups estimated from revenue-based data estimate the dispersion and change in markups reasonably well.

markups, leading to inefficient resource allocation and lower overall economic welfare. De Loecker et al. (2021) expand upon these papers and show that market structure determines how markups change in response to technological innovations. In the context of international trade, Atkenson and Burstein (2008) show that markups adjust across countries in response to trade costs and de Blas and Russ (2015) find that the market structure matters for pass-through and gains from trade. More recently, Döpper et al. (2024) reveal that declining consumer price sensitivity allowed firms to raise markups, which highlights the important role of shifting demand-side preferences.

Most closely related to our paper is a branch of the literature that explores the relationship between markups and financial constraints. Chevalier and Scharfstein (1996) represents a cornerstone of this literature, highlighting that liquidity constraints and capital market imperfections lead to countercyclical markups. During recessions, liquidity constrained firms boost short-run profits (at the expense of long run market share) by raising prices relative to marginal cost. They suggest this occurs because consumers are 'sticky' and face switching costs, demand may be less elastic during downturns, collusion is harder in booms and that firms are more likely to have low cash flow and difficulties raising external funds during recessions.

More recently, Gilchrist et al. (2017) show that firms with limited internal liquidity and budget constraints significantly increase their prices. Incorporating financial frictions into a New Keynesian model, they show that working capital constraints affect firms' marginal costs, producing a feedback mechanism and leading to higher prices even in weak demand conditions. Dou and Ji (2021) highlights a similar mechanisms.

Further empirical evidence highlights how financial constraints drive firms to raise output prices Renkin and Zullig (2024) show that credit supply shocks lead firms to raise output prices, while Meinen and Soares (2022) find financially constrained firms increase markups. Duca et al. (2017) shows a similar relationship using survey data and suggest that persistence of demand shocks amplifies the countercyclicality of markups.

These papers mainly discuss the "liquidity channel", which argues that firms can raise liquidity in the short run by increasing their markups when external credit becomes scarce. An alternative channel is the "working capital channel" which suggests that firms have to prefinance part of their variable production costs in each period. Higher interest rates or tighter credit constraints increase the cost of working capital for firms. Christiano and Eichenbaum (1992) introduced working capital requirements into business cycle models and revealed that monetary policy is more potent when firms must borrow to finance working capital. Christiano et al. (2015) extend on this work by building a

richer DSGE model that embeds borrowing constraints, credit shocks and banking sector dynamics.

These papers are complemented by Bigio (2015) and Balleer et al. (2020). Bigio (2015) develops a micro-founded model with working capital constraints and shows that shocks increasing uncertainty tighten these constraints, reducing investment and employment. Balleer et al. (2020) present a partial equilibrium menu cost model, which suggests that tighter financial constraints are associated with lower nominal rigidities, higher prices and lower output. Taken together, these papers highlight the important role of working capital in determining aggregate outcomes and the business cycle. Our paper extends upon this mechanism and shows that working capital requirements are an important channel which leads to higher markups.

Other papers in the literature draw different conclusions on the relationship between pricing and financial constraints. Brander and Lewis (1986), for example, suggest that high debt levels increase firm output and lower prices by incentivizing more aggressive production decisions due to limited liability effects. Showalter (1995) and Hendel (1996) build on this foundation, incorporating asymmetric information and managerial incentives, and similarly find that financial constraints influence competitive behaviour and market outcomes. More recently, Kim (2021) argues that firms will cut prices to liquidate their inventory and boost cash flow in response to a negative credit supply shock. This effect is particularly strong for smaller firms.

Finally, Li et al. (2025) is most closely related to our paper and links borrowing constraints to firm markups in the long run, highlighting that earnings-based borrowing constraints can affect markups. They argue that less financially constrained firms tend to have higher markups, especially in sectors where borrowing against physical assets is difficult and firms rely more on earnings to finance borrowing. Markup dispersion across firms is also found to be greater in industries that are more reliant on earnings-based borrowing. Using a Kimball demand framework, they present a two-way feedback mechanism where looser borrowing constraints allow firms to charge higher markups, and higher markups lead to higher earnings, further loosening a firm's borrowing constraint. Our paper shows that this relationship varies across the distribution of firm size, which can be rationalised through working capital requirements.

2 The Model

This section presents a theoretical model linking firms' borrowing constraints and markups. All proofs and a more detailed description of the model are relegated to the Appendix.

Consider an industry featuring a continuum of firms indexed by $i \in [0,1]$, who compete monopolistically by selling differentiated varieties. Time is discrete and indexed by $t \in \{0,1,...,T\}$ with $T \leq \infty$. Firms live in all periods, and there is no entry of new firms. At each period t, firm i produces output according to the technology

$$y_{it} = F(k_{it}, v_{it}^1, ..., v_{it}^J),$$

where y_{it} is output, k_{it} is capital, and $\{v_{it}^j\}_{j=1}^J$ are variable inputs which firms can adjust within a period.

The production function F satisfies the following properties: (i) F is strictly increasing in k_{it} and $\{v_{it}^j\}_{j=1}^J$; (ii) $F_{kk} \equiv \partial^2 F/\partial k_{it}^2 < 0$; (iii) the Hessian of F with respect to $\{v_{it}^j\}_{j=1}^J$, denoted $\boldsymbol{F_{vv}}$, is negative definite; and (iv) for all k_{it} and all $\{v_{it}^j\}_{j=1}^J$, F satisfies:

$$F_k - \boldsymbol{F_v}^T (\boldsymbol{F_{vv}})^{-1} \boldsymbol{F_{vk}} > 0, \tag{1}$$

where $F_k \equiv \partial F/\partial k$, $\boldsymbol{F_v}$ is the gradient of F with respect to $\{v_{it}^j\}_{j=1}^J$, and $\boldsymbol{F_{vk}}$ is a $J \times 1$ vector of cross-partials with generic element $\partial^2 F/\partial v_{it}^j \partial k_{it}$. This feature on the production function is the least standard and deserves some further clarification. Condition (1) requires that the marginal product of capital be sufficiently large, and is key to ensure that the marginal cost decreases with capital. It is straightforward to show that (1) is satisfied under standard assumptions, e.g., if F is Cobb-Douglas and conditions (i)-(ii) also hold.[2]

Intermediate inputs $[y_{it}]_{i=0}^1$ are combined to produce the final good Y_t according to the Kimball (1995) aggregator

$$\int_0^1 \Phi\left(\frac{y_{it}}{Y_t}\right) = 1,$$

where Φ is a strictly increasing and strictly convex function. This implies that the inverse demand function for intermediate good i in period t is defined by final producer's optimality condition

$$p_{it} = \Phi'\left(q_{it}\right) P_t D_t, \tag{2}$$

[2]Suppose that $F(\cdot) = A k_{it}^{\rho} \prod_{j=1}^J (v_{it}^j)^{\alpha^j}$. Then $\boldsymbol{F_v}^T (\boldsymbol{F_{vv}})^{-1} \boldsymbol{F_{vk}} = \boldsymbol{F_v}^T (\boldsymbol{F_{vv}})^{-1} \boldsymbol{F_v} \frac{\rho}{k_{it}} < 0$, since $\boldsymbol{F_{vv}}$ is negative definite.

where p_{it} is the price set by firm i, $q_{it} \equiv y_{it}/Y_t$ is the relative firm size of firm i, $P_t \equiv \int_0^1 p_{it} q_{it} d_i$ is the ideal price index, and $D_t \equiv \left(\int_0^1 \Phi' \left(q_{it} \right) q_{it} di \right)^{-1}$ is the Kimball demand index in period t.

We denote the demand elasticity faced by firm i at t by $\sigma(q_{it}) \equiv -\Phi'(q_{it})/(\Phi''(q_{it})q_{it})$. As standard in the literature, we assume throughout the analysis is that Φ is such that larger firms face more inelastic demands, i.e.,

$$\sigma'(q_{it}) \leq 0. \tag{3}$$

In other words, the *superelasticity* of demand $\varepsilon_\sigma(q) \equiv -\sigma'(q_{it})q_{it}/\sigma(q_{it})$ is *positive*. Edmond et al. (2023), among others, provide empirical support validating this assumption.

Firms rent the variable input j in a spot market at the price w_t^j, which firms take as given. Capital is owned and accumulated by the firms, and evolves according to $k_{it+1} = (1-\delta)k_{it} + x_{it}$, where $\delta \in (0,1)$ is the depreciation rate, and x_{it} is investment.

Firms can only obtain external financing by issuing debt. We assume that there are two types of firm-level debt, namely, intertemporal and intratemporal borrowing. The former consists of one-period risk-free bonds b_{it} issued by the firms at the net interest rate r_t. In addition, within each period t, firms must pay a fraction $\psi \in [0,1]$ of variable input costs $\sum_{i=1}^J w_t^j v_{it}$ before revenues are realized. To cover this cashflow mismatch, firms issue intraperiod "working capital" loans at zero interest (see, e.g., Jermann and Quadrini (2012)).

Importantly, total liabilities are limited by the borrowing constraint

$$b_{it+1} + \psi \sum_{j=1}^{J} w_t^j v_{it}^j \leq (1-\theta_{it})k_{it+1}, \tag{4}$$

at each point in time, where $\theta_{it} \in [0,1]$ is an idiosyncratic credit shock which follows a Markov process. According to (4), the maximum level of debt is a fraction of the resale value of a firm's assets. In the presence of limited enforcement, this constraint prevents that firms default in equilibrium (see, e.g., Gopinath et al. (2017)). The coefficient $(1-\theta_{it})$ represents the fraction of next period's capital that lenders can claim as collateral in case of default, and thus higher values θ_{it} are associated with tighter borrowing constraints.[3]

The goal of firm i is to maximize the expected discounted utility of its owner/manager $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_{it})$, where c is consumption, $\beta \in (0,1)$, and U is an increasing and concave

[3]We implicitly assume that next period's capital is priced at replacement cost, but this assumption is not essential for our results.

function.[4] Firm i chooses $\{c_{it}, \{v_{it}^j\}_{j=1}^J, x_{it}, y_{it}, p_{it}, b_{it+1}, k_{it+1}\}$ at each period t subject to (2), (4), and the sequence of budget constraints

$$c_{it} + x_{it} + (1 + r_{it})b_{it} = p_{it}y_{it} - \sum_{j=1}^{J} w_t^j v_{it}^j + b_{it+1}, \tag{5}$$

for all t and idiosyncratic shock realizations.

In what follows, it will be useful to define the variable cost function:

$$C^v(y_{it}, k_{it}) \equiv \min_{\{v_{it}^j\}_{j=1}^J} \sum_{j=1}^{J} w_t^j v_{it}^j \quad \text{s.t.} \quad F(k_{it}, v_{it}^1, ..., v_{it}^J) \geq y_{it}. \tag{6}$$

where we omit the dependency on the input costs $\{w_t^j\}_{j=1}^J$ to simplify the notation. Note that the function C^v is a variable cost function which abstracts from the effect of borrowing constraints.[5]

For future reference, we establish some important properties of the variable cost function next.

Lemma 1. *The variable cost function satisfies*

$$C_y^v(y_{it}, k_{it}) > 0, \quad C_k^v(y_{it}, k_{it}) < 0,$$

$$C_{yy}^v(y_{it}, k_{it}) > 0, \quad C_{yk}^v(y_{it}, k_{it}) < 0 \quad \textit{and} \quad C_{kk}^v(y_{it}, k_{it}) > 0,$$

where $C_y^v \equiv \partial C^v / \partial y_{it}$, $C_k^v \equiv \partial C^v / \partial k_{it}$, *and similar notation applies to second derivatives.*

2.1 Markups and Borrowing Constraints

At the optimum, firms price varieties by charging a markup over marginal costs. More specifically, we define the markup for firm i at time t as the distortion μ_{it} such that

$$p_{it} = \mu_{it} C_y^v(y_{it}, k_{it}). \tag{7}$$

[4] Our main results do not require strict concavity of the utility function U, so we contemplate the possibility of risk neutral firm owners/managers.

[5] An alternative characterization of the model solution would incorporate the borrowing limit enters as a constraint in the definition of the variable cost function (as in, e.g., Duval et al. (2023)). The results on the effect of borrowing constraints on markups are robust to this alternative formulation. However, our baseline approach—where the borrowing limit is a constraint in the profit maximization problem—permits isolating the effect of borrowing constraints more clearly through a markup formula.

By definition, the markup μ_{it} encapsulates the effect of borrowing constraints in distorting the equality between price and marginal cost. Proposition 1 disentangles that effect:

Proposition 1. *Suppose that firms face asset-based borrowing constraints as in* (4)*. Then firm i's markup* $\mu_{it} \geq 1$ *satisfies*

$$\mu_{it} = \left(1 + \gamma_{it}\frac{\psi}{\lambda_{it}}\right)\frac{\sigma(q_{it})}{\sigma(q_{it}) - 1}, \tag{8}$$

where $\gamma_{it} \geq 0$ *and* $\lambda_{it} > 0$ *are, respectively, the Lagrange multipliers on the borrowing constraint and on the budget constraint.*

Equation (8) shows that markups are affected by two key determinants.[6] The last term on the right hand side is standard, and reflects that markups are inversely related to the elasticity of demand. The first term, on the other hand, is novel: it emerges whenever borrowing constraints are binding, and firms have working capital needs (i.e., $\psi > 0$). By this term, binding borrowing constraints create a positive wedge which pushes up μ_{it}, all else equal.

To build intuition around Proposition 1, consider two firms which share the same level of capital k_t, but differ in their borrowing-constraint status (e.g., due to differences in θ_{it}). Let μ^c_{it} and μ^u_{it} denote the markups of the constrained and an unconstrained firm, respectively. Equation (8) implies:[7]

$$\log\left(\frac{\mu^c_{it}}{\mu^u_{it}}\right) = \underbrace{\log\left(1 + \gamma^c_{it}\frac{\psi}{\lambda^c_{it}}\right)}_{\text{"pass-through" effect}} - \underbrace{\left[\log\left(\frac{\sigma(q^u_{it})}{\sigma(q^u_{it}) - 1}\right) - \log\left(\frac{\sigma(q^c_{it})}{\sigma(q^c_{it}) - 1}\right)\right]}_{\text{"demand elasticity" effect}}, \tag{9}$$

where we use c- (u-) superscripts to label constrained- (unconstrained-) firm's variables. According to the decomposition in (9), the difference in markups between borrowing-constrained and unconstrained firms reflects the balance between two forces. The first is a direct "pass-through" effect, which arises from the binding constraint multiplier and is amplified by working capital requirements ψ. The second is an indirect "demand-elasticity" effect, which operates through changes in σ, as firms reduce production due to the borrowing constraints.

[6] It is straightforward to show that $\mu_{it} \geq 1$, as firms produce on the relatively elastic section of the demand curve where $\sigma(q) \geq 1$. More precisely, note that the first order condition with respect to output can be written as

$$\left(1 - \frac{1}{\sigma(q_{it})}\right) p_{it} = C^v_y(y_{it}, k_{it})\left(1 + \gamma_{it}\frac{\psi}{\lambda_{it}}\right).$$

As the right-hand side is positive, it must be that $\sigma(q_{it}) \geq 1$.

[7] Throughout the paper we use $\log(x)$ to denote the *natural* logarithm of x.

Intuitively, when its borrowing constraint becomes binding, the firm reduces short-term working-capital borrowing and, as a result, contemporaneous production is scaled down. Lowering output, in turn, triggers two opposing forces. First, the marginal cost C_y^v decreases (see Lemma 1), and prices increase since firms face downward slopping demands. This force is manifested as a higher markup, and is encapsulated by the "pass-through" effect in (9).[8] On the other hand, by lowering output, the constrained firm faces a more elastic demand (see (3)). This impact, which we refer to as the "demand-elasticity" effect, pushes for lower markups. The following lemma formalizes the preceding discussion:[9]

Lemma 2. *All else equal, $y_{it}^c \leq y_{it}^u$. This implies that the effect of borrowing constraints on markups is the result of two opposing forces defined in (9): a positive "pass-through" effect which pushes for higher markups, and a positive "demand-elasticity" effect which pushes for lower markups.*

The effect of a binding borrowing constraint on markups is illustrated in Figure 1. We draw the diagram in log terms, which helps clarify the exposition. As shown in the figure, a binding borrowing constraint creates a positive wedge between the marginal cost curve $\log(C_y^v)$, and the "shadow" marginal cost that the firm equalizes to marginal revenue.[10] The constrained firm reduces output—relative to an unconstrained firm—and the new markup equals the original markup plus the difference between the pass-through effect and the demand elasticity effect.[11] To isolate the latter, we draw the tangent at the original production level $\log(y^u)$. This dash-dotted line has slope $\sigma(y^u)^{-1}$ in absolute terms, and it is the hypothetical demand curve that the firm would face (e.g., under a CES aggregator) if the demand elasticity did not change as the market share of the firm shrinks.

An implication from Lemma 2 is that the sign of the effect of borrowing constraints on markups is in principle undetermined, as the demand elasticity effect at least partially offsets the pass-through effect in formula (9). However, the overall effect is positive, unless the marginal cost function is locally decreasing (i.e., increasing returns) which are

[8] Duval et al. (2023) analyzes a similar mechanism to study the transmission of monetary policy when firms have market power.

[9] A few observations are in order. Clearly, the presence of working capital loans is essential for the mechanism to hold, because such loans link financial frictions to output within a time period. In addition, note that the level of the recovery rate $(1 - \theta_{it})$ is only affecting markups implicitly through the Lagrange multipliers and the production levels.

[10] Such wedge might decrease with firm size, as we argue in detail below.

[11] Geometrically, the length of the pass-through effect labeled in the diagram is $\log(1 + \gamma\psi/\lambda)$, i.e., the distance between the "shadow" marginal cost curve and $\log(C_y^v)$.

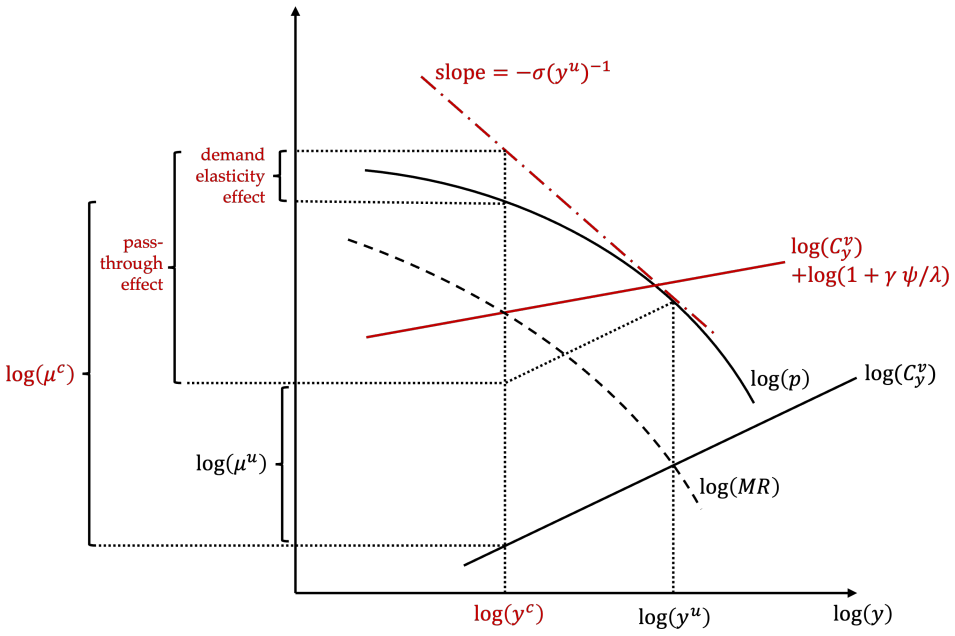


Figure 1: Effect of Borrowing Constraints on Markups.

currently being ruled out (see Lemma 1). This result is readily verified graphically using Figure 1.

Remarks on Markup Definition. Consistent with the theory, we define variable costs as the costs arising from variable inputs that are not subject to adjustment costs, such as borrowing constraints. Accordingly, in our model, marginal costs C_y^v exclude financial costs arising from borrowing constraints, in line with the assumption that intraperiod working capital loans carry a zero interest rate.

2.2 Firm Size Effects

Using (9), we can analyze how the effect of borrowing constraints on markups changes across firm sizes. Throughout this section, we index firm size using capital k_{it}. Proxying firm size with k_{it} is convenient because, for being a state variable, capital is not affected by the borrowing-constrained status of the firm (unlike, e.g., production or variable input demand). However, other dimensions of firm size are positively correlated with k_{it}.[12]

[12]For example, $C_{yk}^v(y_{it}, k_{it}) < 0$ ensures that y_{it} and k_{it} are positively related.

Clearly:

$$\frac{\partial}{\partial k_{it}} \log\left(\frac{\mu^c_{it}}{\mu^u_{it}}\right) = \frac{\partial}{\partial k_{it}} \log\left(1 + \gamma^c_{it}\frac{\psi}{\lambda^c_{it}}\right) - \frac{\partial}{\partial k_{it}}\left[\log\left(\frac{\sigma(q^u_{it})}{\sigma(q^u_{it})-1}\right) - \log\left(\frac{\sigma(q^c_{it})}{\sigma(q^c_{it})-1}\right)\right]. \quad (10)$$

It is generally hard to determine the sign of each of the terms on the right hand side of (10). In what follows, though, we present a simple example in which the sign of those terms can be pinned down via a few sufficient conditions. This will be useful to interpret our empirical findings.

2.2.1 A Streamlined Example

Consider a special case of the model presented in this section. The economy lasts for only two periods, $t \in \{0, 1\}$. We also concentrate on the problem of a single firm and, hence, drop the subindex i. In the first period, the firm faces constraints analogous to those in (4) and (5). In the second period, the firm does not face a borrowing constraint on neither its intertemporal debt nor on its working capital.[13] Without loss of generality, we normalize aggregate output to $Y = 1$. The constraints that the firm faces are therefore:

$$c_0 + k_1 + (1+r)b_0 = p(y_0)y_0 - C^v(y_0, k_0) + (1-\delta)k_0 + b_1, \quad (11)$$

$$b_1 + \psi C^v(y_0, k_0) \leq (1-\theta)k_1, \quad (12)$$

$$c_1 + (1+r)b_1 = p(y_1)y_1 - C^v(y_1, k_1) + (1-\delta)k_1, \quad (13)$$

and nonnegativity constraints on consumption, bonds, and capital holdings.

In this example, we also assume that the firm operates a Cobb-Douglas production function using only one variable input v under constant returns to scale: $F(k_t, v_t) = k_t^{\alpha} v_t^{1-\alpha}$. This implies that the variable cost function is

$$C^v(y_t, k_t) = w_t\, y_t^{\frac{1}{1-\alpha}}\, k_t^{-\frac{\alpha}{1-\alpha}}. \quad (14)$$

where w_t is the variable input price. Moreover, following Klenow and Willis (2016), we parameterize the Kimball demand system using a constant superelasticity of demand, so

[13] Evidently, this is equivalent to assuming that the borrowing constraint is never binding in the second period.

that

$$\sigma(y_t) = \bar{\sigma} y_t^{-\frac{\varepsilon}{\bar{\sigma}}}, \tag{15}$$

where $\bar{\sigma} > 0$ and $\frac{\varepsilon}{\bar{\sigma}} > 0$ is the superelasticity of demand.

Next we establish two results. First, we characterize how the pass-through effect changes with firm size. Second, we present sufficient conditions such that the demand elasticity effect increases with firm size.

Proposition 2. *Consider interior solutions to the simple example in which intertemporal debt and investment in physical capital are both strictly positive. Then:*

1. *The pass-through effect of the borrowing constraint on the markup decreases with firm size k_0 if $\frac{\partial k_1^c}{\partial k_0} > 0$, and decreases otherwise, i.e.,*

$$sign\left[\frac{\partial}{\partial k_0}\left(\frac{\gamma_0^c}{\lambda_0^c}\right)\right] = sign\left[-\frac{\partial k_1^c}{\partial k_0}\right], \tag{16}$$

   where k_1^c is the level of capital in $t = 1$, and λ_0^c and γ_0^c denote the Lagrange multipliers on the constraints (11) and (12), respectively, when the borrowing constraint (12) binds.

2. *Up to a first order approximation around y_0^u, the demand elasticity effect increases with firm size k_0 if*

$$\frac{\partial(y_0^u - y_0^c)}{\partial k_0} > 0, \quad \text{and} \quad \frac{\varepsilon}{\bar{\sigma}} > 1 - \frac{1}{\sigma(y_0^u)}, \tag{17}$$

   where y_0^c and y_0^u denote the levels of production at $t = 0$ of a borrowing-constrained and an unconstrained firm, respectively, which share the same levels of k_0.

The algebra underlying Part 1 is lengthy, but the intuition behind the result is straightforward. Next period's capital serves as collateral and relaxes borrowing constraints. Accordingly, if investment in physical capital increases with firm size, the shadow value of their borrowing constraints (and thus the pass-through effect) should be smaller for larger firms, all else equal. This is essentially what the sufficient condition (16) conveys.

We turn to the interpretation of Part 2. The demand elasticity effect, which pushes for lower markups, can increase with firm size through to two channels. The first channel—embodied in the first inequality of (17)—operates if large firms reduce their production by more than small firms when becoming borrowing constrained. The second channel, on the other hand, encapsulates that even if the above drop in production did not change with firm size, large firms could still face a larger increase in their demand elasticities than

small firms when borrowing constraints bind. This would be possible if the demand elasticity reacted more to changes in production for large firm sizes, i.e., if the superelasticity of demand is large enough. Such requirement is manifested in the second inequality of (17) which, due to Kimball demand, is more likely to hold when production is sufficiently large.

Simulations. In this section, we take the two-period setup and empirically simulate these results under a set of standard parameters (Table 1). We set the discount rate β to 0.95 and α to 0.30, which are standard values used. We use a constant relative risk aversion utility function with risk aversion parameter ρ set to 2.00. Firms start with an initial stock of debt b_0 of 0.7 units of the consumption good, which gives us a realistic range of leverage across the distribution of initial capital levels. In our simple model, we assume that all firms have the same working capital share of variable inputs. We set ψ to 0.8, which is consistent with firm-level estimates from Gil (2025). Following the Kimball demand structure of our theoretical model, we also set parameter values for s_0 and ϵ_K. Hambur and Freestone (2025) calibrate the general equilibrium model from (Edmond et al., 2023) to Australian microdata and estimate a superelasticity of 0.13. As such, we set s_0 to 6.00 and ϵ_K tp 1.20 to produce a similar superelasticity.

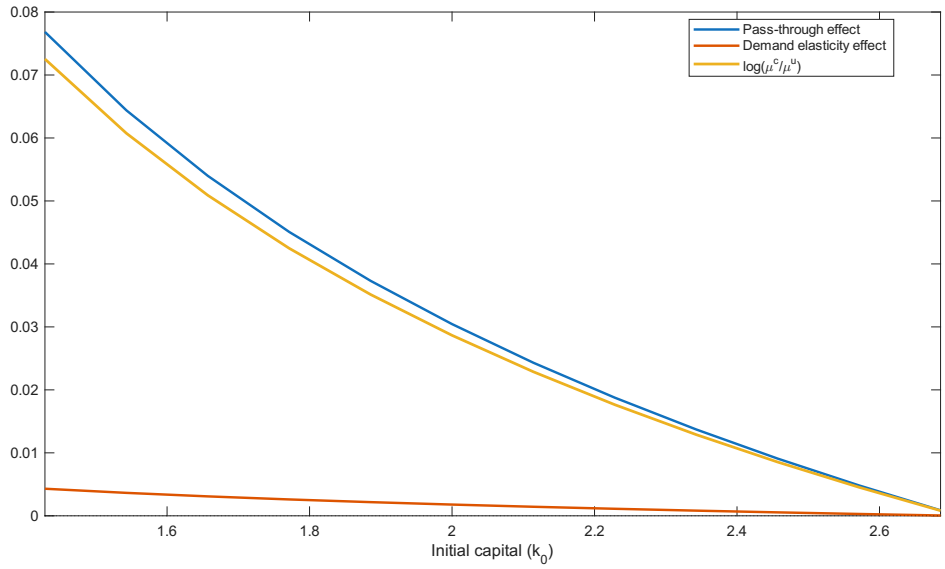
Table 1: Model Parameters

Parameter	Description	Value
α	Capital share of income	0.30
β	Discount rate	0.96
ρ	Risk aversion parameter	2.00
δ	Depreciation rate	0.10
r	Real interest rate	0.04
v	Variable input cost	1.00
ψ	Working capital share of variable cost	0.8
b_0	Initial stock of debt	0.7
s_0	Baseline elasticity of substitution	6.00
ϵ_K	Kimball curvature parameter	1.20

There are two types of firms in our calibration: firms with tight asset-based borrowing constraints (reflected by a high value of θ, set to 0.8) and firms with looser asset-based borrowing constraints (reflected by a low value of θ, set to 0.1). For a given level of initial capital, firms with a high value of θ are generally more likely to face binding borrowing constraints.

Our quantitative results align with the theoretical results discussed above. First, our results suggest that borrowing constrained firms will charge higher markups than non-borrowing constrained firms, and that effect decreases with size. Second, our model confirms Lemma 2. Decomposing the logarithmic difference in markups between the two types of firms reveals that a positive "pass-through" effect and a positive "demand-elasticity" effect exist over a range of initial capital where the chosen values of θ produce firms that can be either constrained or unconstrained. In Figure 2, we plot the decomposition for a range of initial capital values. Under a standard set of parameter values, the "demand-elasticity" effect is quantitatively small, implying that the pass-through effect is the primary channel through which higher markups arise for borrowing constrained firms.

Figure 2: Markup Decomposition for Low- and High- θ Firms



3 Data

The main data source for estimating the empirical relationship between borrowing constraints and markups is the Business Longitudinal Analysis Data Environment (BLADE) constructed by the Australian Bureau of Statistics (ABS). BLADE is a collection of linked firm-level panel data sets, which integrates information from government administrative tax data, business surveys, intellectual property and trade data, among other sources. BLADE is the most exhaustive micro-level data source on Australian firms, as it includes

all active businesses operating in Australia paying Gross Sales Tax (GST).[14]

We collect data from various datasets under the heading of the "BLADE Standard" product. Certain administrative datasets of interest within BLADE Standard—namely, Business Income Tax Returns (BIT), Business Activity Statements (BAS), and Pay as You Go Statements (PAYG)—are available in either *cross-sectional* or *longitudinal* formats. The former is updated more frequently, but the latter is curated by the ABS in order to reduce longitudinal inconsistencies (such as changes in business structures). We use the June 2023 version of the longitudinal files, which is the latest version available of the longitudinal data at the time of writing.

Within BLADE Standard, we also draw survey data from the ABS Business Characteristics Survey (BCS). This annual survey incorporates information on information technology usage, innovation, business practices and, particularly, financial decisions, which are critical for constructing borrowing constraint measures in our empirical analysis. In its latest releases, the BCS collects data from a random sample of approximately 7,000 firms.[15]

Longitudinal firm-level data in BLADE Standard is available at an annual frequency between fiscal years 2001-02 and 2022-23. In Australia, a fiscal year starts on 1 July and finishes on the 30 June the following year. To simplify the exposition, in what follows we refer the "year" of a particular observation as the *year end* of its corresponding fiscal year. To adjust for inflation, we construct implicit price deflators based on ABS data, and use separate price indices for each division's output, intermediate inputs, and capital (see, e.g., Hambur (2023)).

Firms carrying on a business in Australia are identified by the public and the government—for tax and other purposes—through a unique Australian Business Number (ABN). However, our unit of observation (i.e., what we refer to as a "firm" or a "business") is not an ABN, but an anonymised ID number created by the ABS known as a Type of Activity Unit (TAU). Each TAU represents an economic production unit with a predominant activity within an industry, and which can report a minimum set of productive and employment data.[16]

Notably, the mapping between and ABN and a TAU depends on whether a firm be-

[14]BLADE includes both local and foreign firms carrying on an enterprise in Australia. The GST payment threshold applies to firms with annual revenues above AUD 75,000. See Hansell and Rafi (2018) for an overview of BLADE.

[15]All firms with 1,000 employees or more in a particular year are included in the sample. For details, refer to https://www.abs.gov.au/methodologies/characteristics-australian-business-methodology.

[16]Specifically, each TAU reports at least total capital expenditure, income from the sale of goods and services, wages and salaries, total inventories, total purchases, and selected expenses.

longs to the "non-profiled" or "profiled" populations in BLADE. The first group, which comprises the vast majority of firms in BLADE, includes businesses with relatively simple organizational structures for which the relationship between a TAU and an ABN is one-to-one. The profiled population, on the other hand, consists of firms with more complex organization structures, for which the mapping between ABNs and TAUs is either one-to-many or many-to-one.[17] The ABS curates businesses in this way in order to distinguish *legal* entities (ABNs) from *economic* entities (TAUs).[18]

3.1 Sample Selection

Our baseline sample spans years 2006 through 2018, inclusive. We start in 2006 because this is the first year available of BCS data. The sample is truncated in 2018 for two reasons. First, firm observations drop significantly after 2018, potentially due to a change in reporting requirements. Second, truncating the sample in 2018 eliminates confounding effects that emerged due to the COVID-19 pandemic.

We only keep non-profiled businesses, i.e., firms for which the mapping between an ABN and a TAU is one-to-one. This sample restriction yields a clearer interpretation of our unit of observation, and improves comparability across observations. Non-profiled firms are also highly representative of the universe of firms operating in Australia: they constitute about 97% of the firms in BLADE Standard.[19]

Importantly, we only keep firms within the manufacturing industry division. This sample restriction is standard in the literature estimating markups since inputs, outputs, and price deflators are, in general, more reliably measured in manufacturing than in other divisions (De Ridder et al. (2026)). We remove industries with few competitors to avoid dealing with highly regulated markets. Specifically, we include all industries within the manufacturing division, except 3-digit Australian and New Zealand Standard Industrial Classification (ANZIC) industries with less than 20 observations per year.

As for legal forms of organization, we keep companies,[20] partnerships and trusts, but drop sole proprietors. We remove the latter because sole proprietors do not report as-

[17] An example of multiple TAUs to one ABN is an enterprise group with subsidiaries across multiple industries. Conversely, a large firm—such as a supermarket chain—could operate within one industry under a single TAU, but still keep multiple ABNs for tax purposes.

[18] Anonymised ABNs and concordances with TAUs are also available in the BLADE Standard product. However, datasets in longitudinal format are only available at the TAU level.

[19] The profiled population essentially includes a handful of very large, and organizationally complex firms.

[20] Companies in Australia are akin to US corporations which pay corporate taxes. See Australian Securities and Investments Commission website for details https://www.asic.gov.au.

Table 2: Firm Counts

Year	BCS Sample	BLADE Sample
2006	714	25,773
2007	675	23,252
2008	738	24,651
2009	603	23,902
2010	543	24,587
2011	343	23,793
2012	281	23,444
2013	407	23,542
2014	346	24,062
2015	354	23,987
2016	330	22,923
2017	442	23,353
2018	412	22,370
Total	6,188	309,639

sets, and this data is needed to construct our measures of borrowing-constrained status. We should highlight that our sample includes both publicly-traded and privately-held businesses, and that the latter are close to 99% of the sample.

We drop other observations by applying standard criteria. In particular, we drop public sector firms and not-for profits, firms with negative income, capital, wages or input costs, and those firms reporting full-time equivalent employees (FTE) below 1. Finally, we restrict the sample to employing firms which submit PAYG statements to the Australian Taxation Office (ATO). This is because our estimation method requires a measure of FTE, and this is only available in PAYG data.

Table 2 provides the firm count across years in our sample. The last column ("BLADE Sample") shows the number of firms within BLADE Standard which satisfy our sample criteria. This is the largest sample we work with. The first column ("BCS Sample") displays the number of firms meeting our sample criteria within BCS. This is a subsample used to construct our baseline measures of borrowing-constraint status, as discussed next.

4 Empirical Framework

In this section we discuss the empirical framework for estimating the relationship between markups and borrowing constraints.

Let μ_{idt} denote the markup of firm i in industry d at time t. As a baseline, we specify

the linear regression model

$$\log(\mu_{idt}) = \beta_1 BC_{idt} + \beta_2 BC_{idt} \log(FTE_{idt}) + \beta_3 \log(FTE_{idt}) + \delta_d + \delta_t + \varepsilon_{idt}. \quad (18)$$

Our main coefficients of interest are β_1 and β_2. The variable BC_{idt} is a binary measure of borrowing constraint status, whose construction is discussed in detail in Section 4.2. FTE_{idt} denotes the number of full-time equivalent employees, and it is our proxy of firm size. δ_d and δ_t are industry and time fixed effects.

Throughout the empirical analysis, industries (indexed by d) are defined at the 3-digit ANZSIC level within the manufacturing division. This is a fairly fine classification, with a total of 55 industries. In particular, the 3-digit ANZSIC classification is considerably more granular than the 3-digit North American Industry Classification System (NAICS), which only contains 21 industries.[21]

4.1 Measuring Markups Variation

A key obstacle for measuring markup *levels*—and, thus, the dependent variable in (18)—is that we do not observe firm-level *prices* in BLADE data. Accordingly, standard production function approaches for estimating markup levels entail identification issues. Bond et al. (2021), in particular, show that if *output* elasticities are replaced by *revenue* elasticities for estimating markups (due to lack of firm-level output prices), markup estimators can actually be uninformative about true markups.[22] To circumvent this problem, we study markup variation across firms through variations in the cost share of variable inputs in total revenues. This "cost share approach" builds on a methodology proposed by Bond et al. (2021).

In our environment, the cost share approach works as follows. As shown by Hall (1988), it is known that the first order condition of the cost minimization problem (6) with

[21] For example, industries within the 3-digit ANZSIC classification include *Meat and Meat Product Manufacturing*, or *Seafood Processing*, while 3-digit NAICS subsectors comprise broader categories like *Food Manufacturing*.

[22] The authors specifically study the "ratio estimator" of markups, which is defined as the ratio of the output elasticity and the cost share of a variable input. They show that in order to be consistent with profit maximization and cost minimization first order conditions, the ratio estimator should be identically equal to one. De Ridder et al. (2026) show that markup trends and cross-firm dispersion can still be measured accurately even without firm-level output price data.

respect to any variable input v^j_{idt} can be written as

$$\mu_{idt} = \alpha^j_{idt} \frac{p_{idt}\, y_{idt}}{w^j_t\, v^j_{idt}},$$

where $\alpha^j_{idt} \equiv \partial \log(y_{idt}) / \partial \log(v^j_{idt})$ is the output elasticity with respect to variable input v^j_{idt}. Equivalently:

$$\log(\mu_{idt}) = \log\left(\alpha^j_{idt}\right) - \log\left(s^j_{idt}\right), \tag{19}$$

where s^j_{idt} is the cost share of the variable input v^j_{idt} in total revenues. In order to identify markup variation from variations in the cost share, we need the following assumption:

Assumption 1. *Firms within each industry d operate a Cobb-Douglas production technology, so that*

$$\alpha^j_{idt} = \alpha^j_d, \tag{20}$$

for all i and t.

Applying (19) and (20) into (18) leads to

$$-\log\left(s^j_{idt}\right) = \beta_1\, BC_{idt} + \beta_2\, BC_{idt}\, \log(FTE_{idt}) + \beta_3\, \log(FTE_{idt}) + \tilde{\delta}^j_d + \delta_t + \varepsilon_{idt}, \tag{21}$$

where $\tilde{\delta}^j_d \equiv \delta_d - \log(\alpha^j_d)$ is an industry fixed effect.

In what follows, we use "labor" measured in full-time equivalent units as the variable input j for estimating β_1 and β_2 through (21).

4.2 Measuring Borrowing-Constraint Tightness

We construct two binary indicators to proxy borrowing-constraint tightness. We denote these as BC_survey_{idt}, and BC_logit_{idt}. The former is based on responses of managers and firm owners to a sequence of survey questions in BCS. The latter is based on predicted probablitities of being borrowing constrained under BC_survey_{idt}. A key difference between these two indicators is their coverage: BC_survey_{idt} is only available for a subsample of firms in BCS data, while BC_logit_{idt} is available for all firms that meet our sample selection criteria. Further details are provided next.

***BC_survey*$_{idt}$:** The indicator BC_survey_{idt} equals one if firm i within industry d meets at least one of the following conditions in year t:

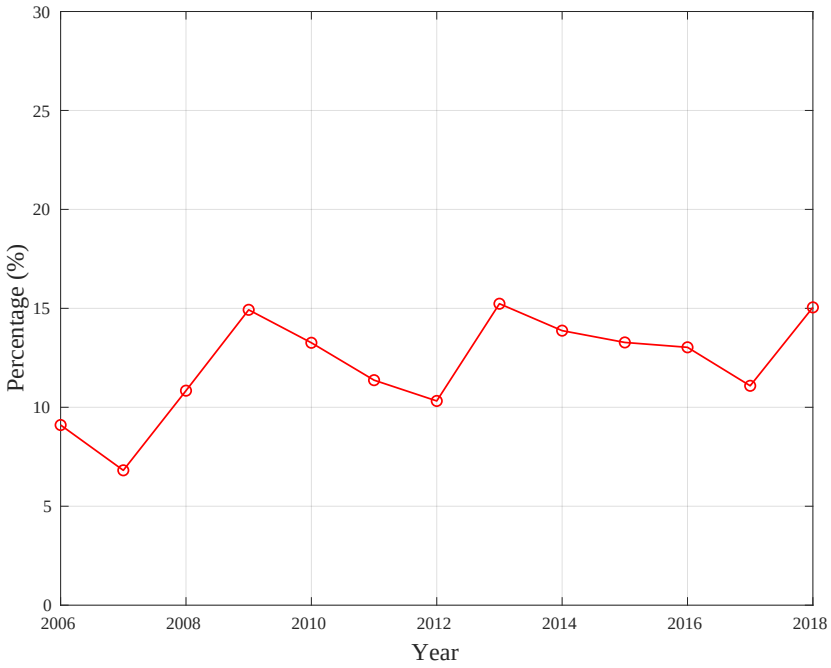


Figure 3: Percentage of firms within BCS with $BC_survey = 1$ over the sample period.

1. *The firm sought debt or equity finance, and was unable to obtain either.*

2. *The firm did not attempt to obtain debt or equity finance, but cited lack of access to additional funds as a factor significantly hampering business activities or performance.*

Otherwise, BC_survey_{it} is set to zero, or missing, accordingly. Figure 3 shows the percentage of firms within the BCS sample being borrowing-constrained across years under the measure BC_survey_{it}. This percentage ranges between 10–15% for most years.

Figures 4 and 5 illustrate the relationship between BC_survey_{idt} and other proxies of borrowing constraint tightness which are widely used in the literature, namely, *cash-to-assets* and *debt-to-assets* ratios. These are constructed using financial information from business tax returns in BLADE data.[23] The box plot of Figure 4 shows that being borrowing constrained under BC_survey_{idt} tends to be associated with lower levels of cash-to-assets, and higher levels of debt-to-assets across firm size quartiles.[24] By Figure 5, such

[23]Specifically, the denominator in each of these ratios is total assets, which comprises both current assets (expected to be depleted within 12 months) and non-current assets (longer term investments). Debt corresponds to total liabilities, and cash is defined as current assets less inventories and trade debtors. A direct measure of cash is not available in BLADE, so we follow other works which construct proxies for cash using BLADE (see e.g., Reserve Bank of Australia (2020)).

[24]For all box plots displayed in this paper, the center horizontal line in each box represents the 50th percentile (median) within a particular category. The bottom (top) of each box represents the 25th (75th) percentile. The bottom (top) whisker is equal to the 25th percentile minus (plus) 1.5 times the interquartile

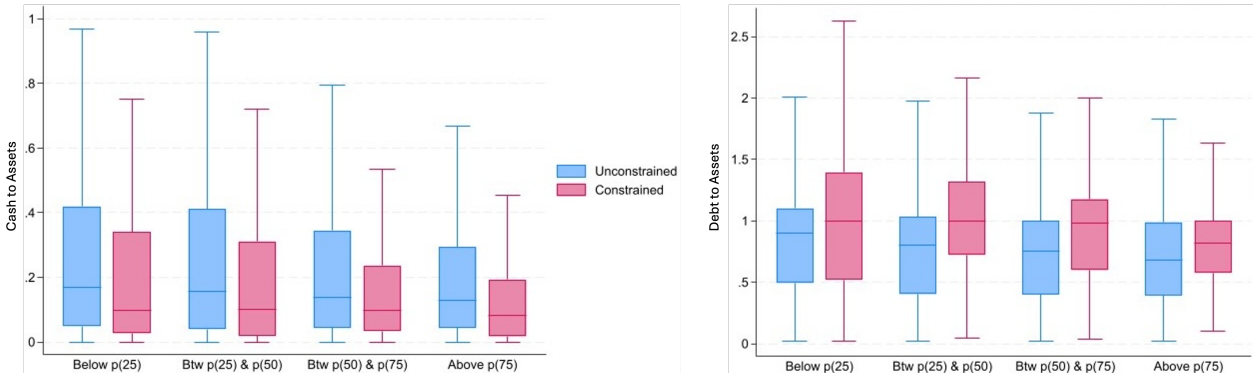


Figure 4: Box plots of *cash-to-assets* and *debt-to-assets* across FTE and BC_survey status.

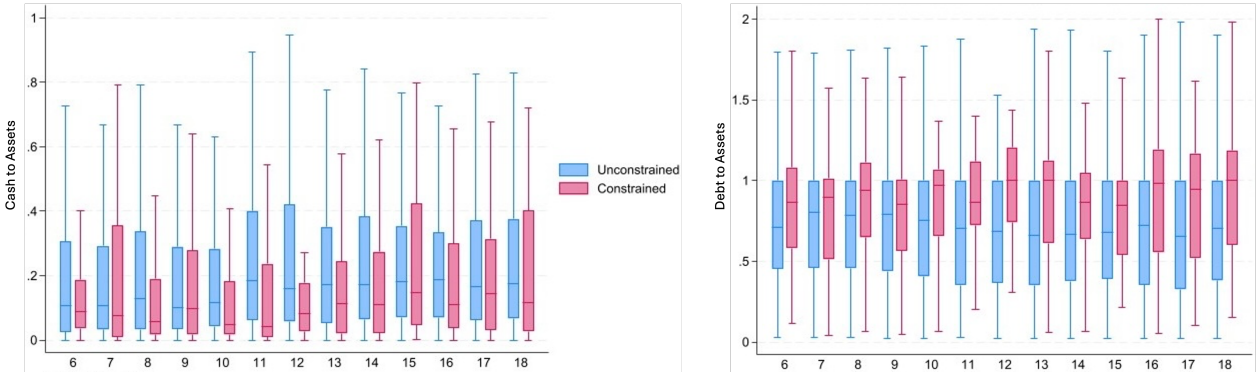


Figure 5: Box plots of *cash-to-assets* and *debt-to-assets* across years and BC_survey status.

relationship also holds across years.

BC_logit $_{idt}$**:** The indicator BC_logit_{idt} is defined based on predicted probabilities of being borrowing constrained under BC_survey_{idt}. Specifically, we start by fitting the *logit* model

$$BC_survey_{idt} = b_0 + b_1 \log(FTE_{idt}) + b_2\, cash\text{-}to\text{-}assets_{idt} + b_3\, debt\text{-}to\text{-}assets_{idt} + d_d + d_t + \varepsilon_{idt}, \quad (22)$$

to estimate the probability of $BC_survey_{idt} = 1$ within BCS data. Our key predictors are firm size, cash-to-assets, and debt-to-assets ratios. We also control for industry and time

range.

fixed effects.

Upon estimating (22) for the firms within the BCS sample, we compute the predicted probability that $BC_survey_{idt} = 1$ for each of the firms in our entire sample. We then define $BC_logit_{idt} = 1$ if the predicted probability lies above a probability threshold, which is chosen by maximizing Youden's J statistic.[25]

4.3 Sample Characteristics

Table 3 present summary statistics for the variables in the BCS and BLADE samples. To mitigate the effects of extreme values and measurement error, all continuous variables derived from other variables are winsorized at the 1st and 99th percentiles. Accordingly, we winsorize the labor share (our dependent variable), *cash-to-assets*, and *debt-to-assets*.

Most variables display similar distributions across samples, as illustrated by the moments shown in Tables 3. The exception is FTE: compared to the more representative BLADE sample, BCS oversamples large firms. This is a standard feature of firm-data surveys.

Table 3: Summary Statistics.

Sample	Variable	Mean	SD	p5	p25	p50	p75	p90	p95
BCS	s (labor share)	0.284	0.160	0.068	0.171	0.262	0.366	0.474	0.565
	FTE	28.677	70.913	1.668	4.634	10.926	26.728	59.328	94.725
	BC_survey	0.118	0.323	0	0	0	0	1	1
	BC_logit	0.346	0.476	0	0	0	1	1	1
	cash-to-assets	0.218	0.230	0.001	0.041	0.133	0.329	0.572	0.712
	debt-to-assets	0.815	0.640	0.114	0.424	0.754	1.000	1.293	1.703
BLADE	s (labor share)	0.303	0.162	0.085	0.188	0.282	0.387	0.500	0.600
	FTE	12.194	23.479	1.292	2.733	5.785	12.715	26.463	43.144
	BC_logit	0.396	0.489	0	0	0	1	1	1
	cash-to-assets	0.244	0.244	0.001	0.049	0.161	0.377	0.628	0.771
	debt-to-assets	0.882	0.712	0.114	0.460	0.838	1.004	1.415	1.980

Notes: The table reports summary statistics for the variables involved in regression (21) under the BCS and BLADE samples. Continuous variables derived from other variables—namely, the labor share, *cash-to-assets*, and *debt-to-assets*—are winsorized at the 1st and 99th percentiles.

[25] Younden's J statistic is defined as $J = sensitivity + specificity - 1$, where $sensitivity$ and $specificity$ denote the the true positive and true negative rates, respectively, at a given probability threshold. Youden's is a standard criterion to optimize the performance of dichotomous diagnostic test.

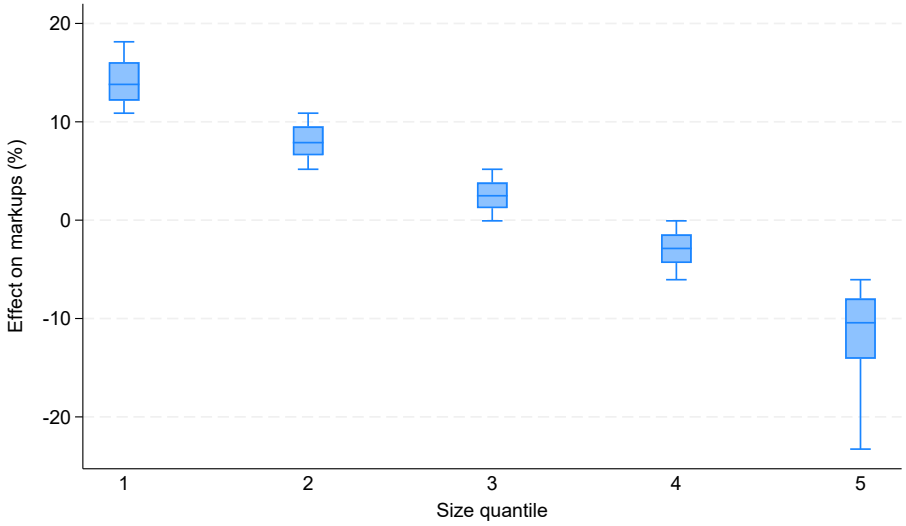


Figure 6: Box plot of the percentage effect of $BC_logit = 1$ on markups.

5 Empirical Results

5.1 Baseline Regressions

In Table 4 we show the results of our baseline regressions. Columns (1) and (2) correspond to the regressions with the BCS sample using BC_survey_{idt} and BC_logit_{idt}, respectively, as the measure of borrowing constraint status. Column (3) displays the regression results for the BLADE sample. In this case we only work with BC_logit_{idt}.

Consider column (1). The estimated coefficient on the indicator BC_survey_{idt} is significant, and sits at 0.233. This means that being borrowing constrained according to the survey indicator is associated with a positive and significant effect among small firms. But our estimates also uncover a negative interaction with firm size: the coefficient -0.094 on $BC_survey_{idt} \times \log(FTE_{idt})$ is negative and statistically significant. This relationship between borrowing-constraint tightness and markups also emerges in the models of columns (2) and (3), i.e., when using BC_logit_{idt} as the borrowing-constraint indicator on either sample. It is also worth noting that the effect of firm size on markups is positive and significant, in line with the Kimball demand structure of our theoretical model from previous sections.

How should the estimates of β_1 and β_2—the key coefficients of interest—be interpreted in economic terms? Given a borrowing constraint measure, (18)-(20) imply that the estimated percentage difference in the markup of a constrained vs. an unconstrained

firm is given by

$$\left(\frac{\widehat{\mu}^{c}_{idt}}{\widehat{\mu}^{u}_{idt}} - 1\right) \times 100 = \left[\exp\left(\widehat{\beta}_1 + \widehat{\beta}_2 \log(FTE_{idt})\right) - 1\right] \times 100, \tag{23}$$

where $\widehat{\beta}_1$ and $\widehat{\beta}_2$ are the estimates of β_1 and β_2, respectively. The box plot of Figure 6 illustrates the magnitudes of (23) across firm size quintiles (fixing the values of $\widehat{\beta}_1$ and $\widehat{\beta}_2$). In this plot we use BC_logit_{idt} as the measure of borrowing-constraint tightness, i.e., the proxy of borrowing-constraint tightness which is available for our most comprehensive sample.

According to Figure 6, being borrowing constrained has a *positive* effect on firm-level markups across most of the firm-size distribution. In particular, the percentage effect is about 3% for "representative" median-sized firms. This is consistent with the positive "pass-through" effect we isolated in Lemma 2 within our theoretical model.

However, the magnitude of the effect for the median firm masks significant heterogeneity across firm sizes. Within lower size quintiles, the percentage effect can be well above 10%. On the opposite side of the spectrum, binding borrowing constraints are associated with *lower* markups among large firms.

The negative effect of borrowing constraints on markups is seemingly at odds with the positive effect of borrowing constraint tightness on markups shown within the model of Section 2. Two comments are in order, though. First, the positive effect isolated in our theoretical model relies on the assumption of decreasing returns (i.e., an increasing marginal cost curve). If marginal cost was locally decreasing in output, markups could potentially rise when borrowing constraints bind.[26] Second, the size interaction is linear in our regression. This specification may bias marginal effect estimates at the tails of the size distribution.

[26] Banerjee and Duflo (2014) provide empirical support of locally increasing returns among Indian firms.

Table 4: Baseline Empirical Model (Regression Estimates).

Dependent var.: -log(s^j)	BCS Sample		BLADE Sample
	(1)	(2)	(3)
BC_survey	0.233***		
	(0.068)		
$BC_survey \times \log(FTE)$	-0.094***		
	(0.025)		
BC_logit		0.401***	0.167***
		(0.060)	(0.010)
$BC_logit \times \log(FTE)$		-0.146***	-0.094***
		(0.021)	(0.004)
$\log(FTE)$	0.143***	0.176***	0.052***
	(0.013)	(0.014)	(0.004)
Fixed Effects	Year and Industry (3-digit ANZSIC)		
Obs.	6,188	6,188	309,639
R^2	0.828	0.831	0.834

Notes: Clustered standard errors are in parentheses. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively. The dependent variable is the log of the labor share.

5.2 The Role of Working Capital Requirements

In this section, we quantify the role of working capital requirements in affecting the relationship between markups and borrowing constraints.

Recall that, according to our theoretical model, working capital requirements moderate the impact of borrowing constraint tightness on markups. This is evident from equation (8), which can be written as

$$\mu_{it} = \left(1 + \gamma_{it}\, \overline{WCR}_{it}\right) \frac{\sigma(q_{it})}{\sigma(q_{it}) - 1}. \tag{24}$$

where $\overline{WCR}_{it} \equiv \psi/\lambda_{it}$ is a *normalised* working capital requirement. By (24), higher values of $\overline{WCR}_{it}$ amplify the effect of binding borrowing constraints on markups, all else equal. In what follows, we quantify this channel by iteracting our measures of borrowing-constraint tightness with a proxy of $\overline{WCR}_{it}$.

Turning to empirical measures, let WCR_{idt} denote the working capital requirement of

firm i within industry d at time t.[27] We set

$$WCR_{idt} = \frac{\textit{short-term liabilities}_{idt}}{\textit{cost of variable inputs}_{idt}}, \tag{25}$$

The measure of WCR_{idt} above exploits that, in our theory, working capital maps to short-term liabilities, and working capital requirements are computed as a proportion of variable costs (see equation (4)).[28] This approach is in line with Bianchi and Mendoza (2018), who measure working capital requirements in aggregate data.

The *normalised* working capital requirement $\overline{WCR}_{idt}$ is proxied as

$$\overline{WCR}_{idt} \approx \frac{WCR_{idt}}{(expenses)_{idt}/(income)_{idt}} \tag{26}$$

The approximation in (26) captures that the borrowing-constraint "pass-through" effect decreases with the shadow value of income.[29]

Table 5 shows the regression results. Since $\overline{WCR}_{idt}$ includes income and expenses measures which can potentially lead to endogeneity, our new measures of borrowing constraint status are lagged by one period. Note that now we lose significance in model (1), but our BC_logit_{idt} measures still give significant results with the expected signs, as in the baseline regressions.

A more concrete interpretation of the main estimated coefficients is provided in Figure 7. These box plots show the percentage effect of BC_logit_{idt} on markups—which now depend on not only size, but also on the magnitude of the working capital requirements. In line with our theory predictions, higher working capital requirements substantially amplify the pass-through effect of borrowing constraint tightness. This feature holds across the entire distribution of firm sizes.

[27] To streamline the presentation, the theoretical model of Section 2 assumes that the working capital requirement fraction does not vary neither across firms nor across time. However, that assumption is irrelevant for our theoretical results.

[28] All of the variables used to compute (25) are available in the BIT databases within BLADE. More precisely, we map $\textit{short-term liabilities}_{idt}$ to current liabilities. In the denominator, $\textit{cost of variable inputs}_{idt}$ is the sum of labor costs and intermediate inputs. The latter, in turn, is the difference between total expenses and the sum of labor costs, depreciation, and fixed costs.

[29] To compute the denominator of (26) we use total expenses for $(expenses)_{idt}$, and total business income for $(income)_{idt}$.

Table 5: The Effect of Working Capital Requirements (Regression Estimates).

Dependent var.: -ln(s^j)	BCS Sample		BLADE Sample
	(1)	(2)	(3)
$(BC_survey \times \overline{WCR})_{t-1}$	0.225		
	(0.262)		
$(BC_survey \times \overline{WCR})_{t-1} \times \ln(FTE)$	-0.163		
	(0.105)		
$(BC_logit \times \overline{WCR})_{t-1}$		0.410***	0.208***
		(0.142)	(0.016)
$(BC_logit \times \overline{WCR})_{t-1} \times \ln(FTE)$		-0.155***	-0.092***
		(0.057)	(0.009)
$\ln(FTE)$	0.147***	0.158***	0.047***
	(0.015)	(0.016)	(0.004)
Fixed Effects	Year and Industry (3-digit ANZSIC)		
Obs.	3,119	3,119	220,641
R^2	0.834	0.835	0.840

Notes: Clustered standard errors are in parentheses. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively. The dependent variable is the log of the labor share.

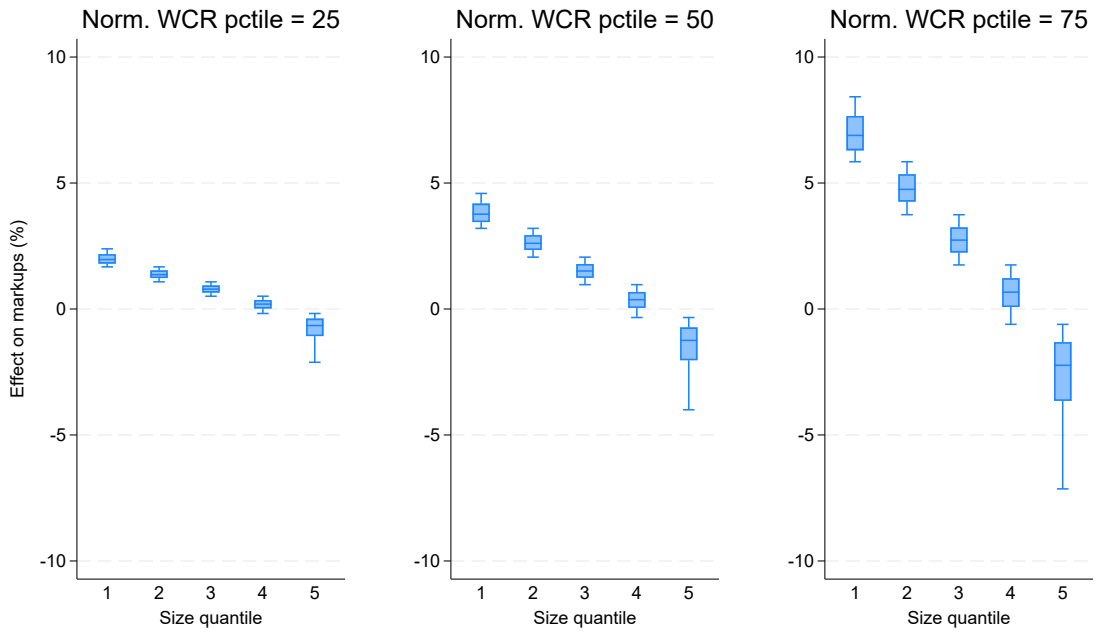


Figure 7: Box plot of the percentage effect of $BC_logit = 1$ on markups across working capital requirements.

6 Conclusions

We examine the empirical relationship between markups and borrowing constraints using administrative data covering both privately held and publicly listed firms. Our results show that the effect of borrowing constraints on markups is substantial for small and medium-sized firms but declines with firm size. We show that this pattern is consistent with the mechanism implied by our theoretical model, in which borrowing constraints operate through working capital requirements.

An important avenue for future research is to explore the implications of this channel for resource misallocation, in particular by disentangling markup dispersion arising from productivity differences from that associated with borrowing constraints. Another promising direction is to study the implications of this mechanism for monetary policy transmission and financial regulation.

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Appendix

A Details of the Model

A.1 Proof of Lemma 1

The Lagrangian associated with (6) is

$$\mathcal{L} = \sum_{j=1}^{J} w^j v^j + \eta[y - F(k, v^1, ..., v^J)].$$

First order conditions are

$$w^j - \eta F_j(k, v^1, ..., v^J) = 0, \quad \forall j, \tag{A.1}$$

and

$$F(k, v^1, ..., v^J) - y = 0, \tag{A.2}$$

where $F_j \equiv \frac{\partial F}{\partial v^j}$. Note that (A.1) and the fact that F is strictly increasing in v^j prove that $\eta > 0$. This implies that $C_y^v > 0$, since

$$C_y^v = \frac{\partial \mathcal{L}}{\partial y} = \eta, \tag{A.3}$$

thanks to the envelope theorem. Applying this theorem once again we get

$$C_k^v = \frac{\partial \mathcal{L}}{\partial k} = -\eta F_k, \tag{A.4}$$

which is strictly negative, since $F_k > 0$ by assumption.

We turn to show that $C_{yy}^v > 0$. First note that by (A.3), $C_{yy}^v = \frac{\partial \eta}{\partial y}$, so we need to prove that $\frac{\partial \eta}{\partial y} > 0$. Deriving (A.1)-(A.2) with respect to y leads to

$$-F_j \frac{\partial \eta}{\partial y} - \eta \sum_{m=1}^{J} F_{jm} \frac{\partial v^m}{\partial y} = 0, \quad \forall j,$$

and

$$\sum_{m=1}^{J} F_j \frac{\partial v^j}{\partial y} - 1 = 0,$$

or in matrix form

$$-\boldsymbol{F_v} \frac{\partial \eta}{\partial y} - \eta \boldsymbol{F_{vv}} \frac{\boldsymbol{\partial v}}{\boldsymbol{\partial y}} = \boldsymbol{0}, \tag{A.5}$$

$$\boldsymbol{F_v}^T \frac{\boldsymbol{\partial v}}{\boldsymbol{\partial y}} - 1 = 0, \tag{A.6}$$

where $\frac{\boldsymbol{\partial v}}{\boldsymbol{\partial y}} \equiv [\frac{\partial v^1}{\partial y}, \ldots, \frac{\partial v^J}{\partial y}]^T$. From equation (A.5) we obtain

$$-\boldsymbol{F_v}^T (\boldsymbol{F_{vv}})^{-1} \boldsymbol{F_v} \frac{\partial \eta}{\partial y} = \eta \boldsymbol{F_v}^T \frac{\boldsymbol{\partial v}}{\boldsymbol{\partial y}}.$$

Applying (A.6) and rearranging leads to

$$C_{yy}^v = \frac{\partial \eta}{\partial y} \equiv -\frac{\eta}{\boldsymbol{F_v}^T (\boldsymbol{F_{vv}})^{-1} \boldsymbol{F_v}} > 0. \tag{A.7}$$

The inequality follows because $\eta > 0$, as shown earlier, and because $\boldsymbol{F_{vv}}$ is a negative definite matrix.

Now we establish that $C_{yk}^v < 0$. Deriving (A.1)-(A.2) with respect to k gives (in matrix form):

$$-\boldsymbol{F_v} \frac{\partial \eta}{\partial k} - \eta \left(\boldsymbol{F_{vv}} \frac{\boldsymbol{\partial v}}{\boldsymbol{\partial k}} + \boldsymbol{F_{vk}} \right) = \boldsymbol{0}, \tag{A.8}$$

$$F_k + \boldsymbol{F_v}^T \frac{\boldsymbol{\partial v}}{\boldsymbol{\partial k}} = 0, \tag{A.9}$$

where $\frac{\partial \boldsymbol{v}}{\partial k} \equiv [\frac{\partial v^1}{\partial k}, \ldots, \frac{\partial v^J}{\partial k}]^T$. Operating on (A.8) we get

$$-\boldsymbol{F_v}^T(\boldsymbol{F_{vv}})^{-1}\boldsymbol{F_v}\frac{\partial \eta}{\partial k} = \eta\left(\boldsymbol{F_v}^T\frac{\partial \boldsymbol{v}}{\partial k} + \boldsymbol{F_v}^T(\boldsymbol{F_{vv}})^{-1}\boldsymbol{F_{vk}}\right).$$

So applying (A.9) on the right-hand side, and rearranging gives

$$\frac{\partial \eta}{\partial k} = \frac{\eta}{\boldsymbol{F_v}^T(\boldsymbol{F_{vv}})^{-1}\boldsymbol{F_v}}\left(F_k - \boldsymbol{F_v}^T(\boldsymbol{F_{vv}})^{-1}\boldsymbol{F_{vk}}\right),$$

or

$$C^v_{yk} = \frac{\partial \eta}{\partial k} = -C^v_{yy}\left(F_k - \boldsymbol{F_v}^T(\boldsymbol{F_{vv}})^{-1}\boldsymbol{F_{vk}}\right),$$

which follows from (A.3)-(A.4). Since $C^v_{yy} > 0$ and $\left(F_k - \boldsymbol{F_v}^T(\boldsymbol{F_{vv}})^{-1}\boldsymbol{F_{vk}}\right) > 0$, it follows that $C^v_{yk} < 0$.

Finally, consider C^v_{kk}. Differentiating (A.4) gives

$$C^v_{kk} = -\frac{\partial \eta}{\partial k}F_k - \eta F_{kk}.$$

Since $\frac{\partial \eta}{\partial k} < 0$, $F_k > 0$, $\eta > 0$ and $F_{kk} < 0$ we get $C^v_{kk} > 0$. □

A.2 Proof of Lemma 2

The value function of an intermediate-good firm is

$$V(k, b, \theta) = \max_{c,y,b',k'} \left\{U(c) + \beta \mathbb{E}\left[V(k', b', \theta')|\theta\right]\right\}$$

subject to

$$c + k' + (1+r)b = p(y)y - C^v(y,k) + (1-\delta)k + b', \tag{A.10}$$

$$b' + \psi C^v(y,k) \leq (1-\theta)k', \tag{A.11}$$

$$c \geq 0, \quad b' \geq 0, \quad k' \geq 0. \tag{A.12}$$

Let $\lambda > 0$, $\gamma \geq 0$ and $\xi \geq 0$ denote the Lagrange multipliers, respectively, on the budget constraint, the borrowing constraint, and the nonnegativity constraint on consumption.

First order conditions are

$$U'(c) + \xi - \lambda = 0, \tag{A.13}$$

$$\lambda \left[p'(y)y + p(y) - C_y^v(y,k) \right] - \gamma\psi C_y^v(y,k) = 0, \tag{A.14}$$

$$\beta \frac{\partial}{\partial b'} \mathbb{E}[V(k',b',\theta')|\theta] + \lambda - \gamma = 0, \tag{A.15}$$

$$\beta \frac{\partial}{\partial k'} \mathbb{E}[V(k',b',\theta')|\theta] - \lambda + \gamma(1-\theta) = 0, \tag{A.16}$$

$$c + k' + (1+r)b - p(y)y + C^v(y,k) - (1-\delta)k - b' = 0, \tag{A.17}$$

together with the Khun-Tucker conditions on (A.11) and (A.12).

Assume towards a contradiction that $y^c > y^u$. Since the marginal cost is strictly increasing in output, it follows that

$$C_y^v(y^u,k) \leq C_y^v(y^c,k) < C_y^v(y^c,k) \left(1 + \gamma^c \frac{\psi}{\lambda^c} \right), \tag{A.18}$$

where we used that $\lambda^c > 0$ and $\gamma^c > 0$ when the borrowing constraint is binding. Normalize $Y = 1$ for simplicity. Then combining (A.18) with the first order condition (A.14), we obtain

$$p'(y^u)y^u + p(y^u) < p'(y^c)y^c + p(y^c),$$

which contradicts that marginal revenue is decreasing in output under Kimball demand.[30]

□

A.3 Proof of Proposition 2 - Part 1

We start by deriving the expression for $\frac{\gamma_0^c}{\lambda_0^c}$. Consider an interior solution in which b_1 and k_1 are both strictly positive. If the firm is borrowing constrained, first order conditions

[30] Just note that marginal revenue with Kimball demand can be written as

$$p'\left(\frac{y}{Y}\right)\frac{y}{Y} + p\left(\frac{y}{Y}\right) = \left(1 - \frac{1}{\sigma(y/Y)}\right) p\left(\frac{y}{Y}\right),$$

Under a Kimball aggregator, the demand is globally downward slopping and $\sigma(y/Y)$ decreases with y. Hence, both terms on the right hand side decrease with y.

include:

$$\lambda_0 - \gamma_0 - \lambda_1(1+r) = 0, \tag{A.19}$$

$$-\lambda_0 + (1-\theta)\gamma_0 + \left[(1-\delta) - C_k^v(y_1,k_1)\right]\lambda_1 = 0, \tag{A.20}$$

$$\lambda_0\left[p'(y_0)y_0 + p(y_0) - C_y^v(y_0,k_0)\right] - \gamma_0\psi C_y^v(y_0,k_0) = 0, \tag{A.21}$$

$$p'(y_1)y_1 + p(y_1) - C_y^v(y_1,k_1) = 0. \tag{A.22}$$

where we omitted the c-superscript to avoid notational clutter. Solving (A.19) for λ_1, plugging into (A.20), and rearranging gives:

$$\frac{\gamma_0}{\lambda_0} = \left[1 - \theta(1+r)X(k_1)\right]^{-1},$$

where

$$X(k_1) \equiv r + \delta + C_k^v(y_1(k_1),k_1).$$

Note that we applied that (A.22) defines a one-to-one mapping between y_1 and k_1. Applying the chain rule:

$$\frac{\partial}{\partial k_0}\left(\frac{\gamma_0}{\lambda_0}\right) = \frac{\partial}{\partial k_1}\left(\frac{\gamma_0}{\lambda_0}\right)\frac{\partial k_1}{\partial k_0}. \tag{A.23}$$

To prove the result, we should show that $\frac{\partial}{\partial k_1}\left(\frac{\gamma_0}{\lambda_0}\right) < 0$. Note that

$$\frac{\partial}{\partial k_1}\left(\frac{\gamma_0}{\lambda_0}\right) = -\frac{\theta(1+r)}{\left[1-\theta(1+r)X(k_1)\right]^2 X(k_1)^2}\frac{\partial X(k_1)}{\partial k_1}. \tag{A.24}$$

Hence

$$sign\left[\frac{\partial}{\partial k_1}\left(\frac{\gamma_0}{\lambda_0}\right)\right] = sign\left[-\frac{\partial X(k_1)}{\partial k_1}\right]. \tag{A.25}$$

Clearly:

$$\frac{\partial X(k_1)}{\partial k_1} = C_{yk}^v(y_1,k_1)\frac{\partial y_1}{\partial k_1} + C_{kk}^v(y_1,k_1). \tag{A.26}$$

To evaluate this expression, we need the following result:

Claim 1.

$$\frac{\partial y_1}{\partial k_1} = -\frac{C_{yk}^v}{C_{yy}^v\left(1 + \frac{1-\alpha}{\alpha}\frac{1-1/\sigma+\varepsilon_\sigma}{\sigma-1}\right)} \tag{A.27}$$

Proof. Equation (A.22) can be written as

$$\underbrace{\left(1-\frac{1}{\sigma}\right)}_{\equiv r(y_1)} p(y_1) - C_y^v = 0.$$

Totally differentiating with respect to k_1:

$$\left[r(y_1)p'(y_1) + r'(y_1)p(y_1) - C_{yy}^v\right]\frac{\partial y_1}{\partial k_1} - C_{yk}^v = 0. \tag{A.28}$$

Note:

$$\begin{aligned} r(y_1)p'(y_1) &= p'(y_1)\left(1-\frac{1}{\sigma}\right) \\ &= -\frac{p(y_1)}{y\sigma}\left(1-\frac{1}{\sigma}\right), \end{aligned} \tag{A.29}$$

since $\sigma^{-1} \equiv -yp'(y)/p(y)$, and

$$\begin{aligned} r'(y_1)p(y_1) &= \sigma^{-2}\sigma' p(y_1) \\ &= \frac{1}{\sigma}\left(-\frac{\varepsilon_\sigma}{y}\right) p(y_1), \end{aligned} \tag{A.30}$$

where we applied the definition of ε_σ. Using (A.29)-(A.30) into (A.28) we get

$$\left[-\frac{p(y_1)}{y\sigma}\left(1-\frac{1}{\sigma}+\varepsilon_\sigma\right) - C_{yy}^v\right]\frac{\partial y_1}{\partial k_1} - C_{yk}^v = 0. \tag{A.31}$$

Using (A.22) we can write the inverse demand as $p(y_1) = C_y^v\left(1-1/\sigma\right)^{-1}$. Plugging into (A.31) and rearranging leads to

$$\frac{\partial y_1}{\partial k_1} = \frac{-C_{yk}^v}{C_{yy}^v + \frac{C_y^v}{y}\frac{1-1/\sigma+\varepsilon_\sigma}{\sigma-1}}. \tag{A.32}$$

Combining (A.3) and (A.7) we get

$$\frac{C_y^v}{y} = -C_{yy}^v \frac{\boldsymbol{F_v}^T(\boldsymbol{F_{vv}})^{-1}\boldsymbol{F_v}}{y},$$

and using our Cobb-Douglas specification, the right hand side boils down to

$$\frac{C_y^v}{y} = -C_{yy}^v \frac{\alpha - 1}{\alpha}.$$

Plugging into (A.32) and rearranging gives (A.27). □

Using (A.27) into (A.26), it follows that

$$\frac{\partial X(k_1)}{\partial k_1} > 0$$

if and only if

$$1 + \frac{1-\alpha}{\alpha} \frac{1 - 1/\sigma + \varepsilon_\sigma}{\sigma - 1} > \frac{\left(C_{yk}^v\right)^2}{C_{kk}^v C_{yy}^v} = 1, \tag{A.33}$$

where the last equality follows by applying (14). Hence, by (A.25),

$$\frac{\partial}{\partial k_0}\left(\frac{\gamma_0}{\lambda_0}\right) < 0 \iff \frac{1 - 1/\sigma + \varepsilon_\sigma}{\sigma - 1} > 0,$$

which always holds since $\sigma > 1$ and $\varepsilon_\sigma > 0$. □

A.4 Proof of Proposition 2 - Part 2

Let

$$Z(y^u, y^c) \equiv \log\left(\frac{1 - 1/\sigma(y^u)}{1 - 1/\sigma(y^c)}\right) = g(y^u) - g(y^c), \tag{A.34}$$

where $g(y) \equiv \log\left(1 - \sigma(y)^{-1}\right)$. Note that the demand elasticity effect equals $-Z(y^u, y^c)$. We are thus looking for sufficient conditions such that $Z(y^u, y^c)$ falls with k_0. Linearizing $g(y)$ around y^u gives

$$g(y) \approx g(y^u) + g'(y^u)(y - y^u).$$

Applying this approximation into (A.34) we obtain

$$\log\left(\frac{1 - 1/\sigma(y^u)}{1 - 1/\sigma(y^c)}\right) \approx g'(y^u)(y^u - y^c).$$

It follows that last expression decreases with k_0 if and only if

$$g'(y_0^u)\frac{\partial(y_0^u - y_0^c)}{\partial k_0} + (y_0^u - y_0^c)\frac{\partial g'(y_0^u)}{\partial k_0} < 0. \tag{A.35}$$

Clearly, a sufficient condition for (A.35) to hold is that both terms be negative. The first term is negative when $\frac{\partial(y_0^u - y_0^c)}{\partial k_0} > 0$ since $g'(y_0^u) < 0$ under Kimball demand. To derive a condition for the second term to be negative, first note that we can write

$$g'(y) = \frac{\sigma'(y)}{\sigma(y)(\sigma(y)-1)} = -\frac{s}{y(\sigma(y)-1)},$$

where $s \equiv \frac{\varepsilon}{\bar{\sigma}}$. Also, using that $(y_0^u - y_0^c) < 0$ and $\frac{\partial y_0^u}{\partial k_0} > 0$, it follows that the second term in (A.35) is negative if

$$(y_0^u)^{-2}(\sigma(y_0^u)-1)^{-1} + (y_0^u)^{-1}(\sigma(y_0^u)-1)^{-2}\sigma'(y_0^u) < 0,$$

which boils down to

$$\sigma(y_0^u) < \frac{1}{1-s}$$

as was to be shown. □