Optimal Monetary Response to Tariffs According to HANK

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March 26, 2026*

Abstract

I characterize optimal monetary responses to tariff shocks in an open economy Heterogeneous Agent New Keynesian (HANK) model. I show that a planner has a novel motive to cut or raise nominal rates in such a model: depending on how tariff revenue is spent and how the planner values any redistribution that revenue induces, they may desire higher (lower) foreign consumption and tariff revenue, pushing them towards rate cuts (hikes). Unlike similar findings in RANK models, this fiscal externality motive does not always push towards rate cuts. When tariff revenue targets high marginal value households, such as with lump sum transfers, the planner cuts rates. If tariff revenue is spent in distributionally neutral ways, such as paying down outstanding debt, the planner expands less or raises rates. I also highlight how the initial level of redistribution and the planner's attitude towards further redistribution shapes policy: the motive to expand is weaker if the initial steady state features more redistribution or if they view further redistribution from tariff revenue negatively.

JEL codes: E52, E61, E63, F13, F41

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1 Introduction

How should an independent monetary authority respond to tariffs to maximize the welfare of its people? Due to a combination of existing inflationary pressures and the Trump administration's April 2nd, 2025 announcement of sweeping tariffs, this question is of increasing relevance and has an unclear answer.

Among Federal Reserve policymakers, there were two primary schools of thought on the optimal monetary stance. The first, a "look-through" policy argues that a tariff is a one-time mechanical increase in the price level and thus requires no change in the monetary stance[1]. The countervailing stance views views price changes as more persistent and a threat towards the fed's goal of price stability and errs towards a contractionary stance[2]

There is an additional channel under-emphasized by Fed policymakers, the *fiscal revenue channel* and the interactions between monetary and fiscal policy that a tariff shock induces. The spending of tariff revenue will have real effects on the economy and will play a key role in determining the effect of a change in interest rates, particularly when households are non-Ricardian and fiscal revenues redistribute from rich to poor agents.

I study the optimal monetary responses to a tariff shock in a heterogeneous agent New-Keynesian (HANK) environment, emphasizing the effect of the fiscal externality on optimal monetary policy. In HANK, monetary transmission (ex: Kaplan et al. (2018)) and therefore optimal policy (ex: Acharya et al. (2023)) differ, as the planner now has an additional motive to stabilize cross-sectional consumption volatility and reduce inequality. Additionally, the failure of Ricardian Equivalence in this class of models implies that the *specification of fiscal policy* and how the government's budget adjusts will affect equilibrium outcomes in a way that it cannot in RANK. Tariff revenue can be used to redistribute from rich, high foreign consumption agents to poor, low foreign consumption agents. Furthermore, the planner's *attitude towards redistribution* will determine the extent to which they desire extra fiscal revenue. A planner who has a negative attitude towards redistribution may not cut rates to

[1]From Anna Paulson: "Tariffs will increase the price level, but they won't leave a lasting imprint on inflation... Monetary policy should look through tariff effects on prices."

[2]from Adriana Kugler: "It should be a priority to make sure that inflation doesn't move up...We are very committed still to our 2% target...which should be a priority now"

increase fiscal revenue.

In a recent paper, Bianchi and Coulibaly (2025) study the optimal monetary response to tariffs in a *representative agent environment* (RANK) and find that optimal policy is always expansionary, citing this fiscal revenue channel as the primary driver of this result. In addition to stifling demand and raising prices, the tariffs create a fiscal externality. Households consume too little of the foreign good, as they do not internalize the effect of their foreign consumption on fiscal revenues and redistributed transfers[3]. Thus, the planner will prefer expansionary policy, which partially corrects this wedge by cutting rates, which increases aggregate demand, foreign consumption, and redistributed tariff revenues.

I write, calibrate, and solve a heterogeneous agent version of the model of Bianchi and Coulibaly (2025). I use the variational approach of Dávila and Schaab (2023)[4] to solve for timeless, Ramsey-optimal policy numerically and highlight some simple theoretical mechanisms. In particular, I derive the targeting rule of Dávila and Schaab (2023) in my environment and show there is a new fiscal revenue motive present.

The key insight for optimal policy is that how tariff revenue is used (redistributed), the planner's attitude towards any fiscal redistribution, and the initial amount of redistribution in the economy are all major qualitative and quantitative drivers of the optimal monetary response to a tariff shock. It is no longer the case that the fiscal externality pushes unambiguously towards expansion as in RANK. The optimal policy in HANK may involve **rate hikes** rather than rate cuts.

I show analytically that this fiscal externality is comprised of two forces in HANK: an aggregate and a distributional component. The fiscal externality pushes towards a monetary expansion when the planner perceives the aggregate effect of additional revenue as positive (the mechanism of Bianchi and Coulibaly (2025)). It further pushes towards a monetary expansion when the covariance between the planner's marginal value of each household and the income effect of fiscal revenue is positive; this is the distributional effect.

By solving the model numerically, I show that when tariff revenues are used to provide

[3]This same mechanism remains in place if fiscal revenue is spent on reducing government debt, spent on a public good, or used to reduce other tax rates

[4]This approach was initially used by Nuño and Moll (2018)

lump sum transfers, and the planner is utilitarian, rate cuts are larger. The cost of the inflation this additional expansion induces is small relative to the benefits of additional resources to (constrained) households, who the planner values highly. However, when a distributionally neutral instrument adjusts, such as the debt, the planner will no longer cut rates on impact, as there are fewer benefits to constrained households.

Next, since any HANK economy necessarily has an inefficient steady state (unlike in RANK), the monetary response to tariffs will both correct existing distortions (from incomplete markets or suboptimal long run fiscal policy) and respond to new ones. To correct this issue, I use an inverse optimal approach (like the one of McKay and Wolf (2022)) to find welfare weights that generate an efficient steady state and thus study how monetary policy should respond only to the *new* inefficiencies induced by a tariff, rather than correcting existing ones.

Again, I find that when the fiscal revenue is used to redistribute towards agents the planner values more (less) highly, they will want to cut rates more (less). Thus, if the revenue is used to reduce the debt, the optimal monetary response will be nearly identical regardless of the planner weights, as reducing the debt is roughly distributionally neutral.

However, if the revenue is not spent in a distributionally neutral way, the optimal response may be qualitatively different across different sets of planner weights. In particular, when tariff revenue is redistributed lump sum and the planner uses welfare weights that induce an initially efficient steady state, the optimal policy is contractionary and involves a small rate *hike*, as the planner views the fiscal redistribution induced by cutting rates more as an explicit bad, due to the higher welfare weight placed on the rich.

Like Bianchi and Coulibaly (2025), I then explore some additional environments to better understand how robust my results are to different model specifications. I consider a specification where the planner has an incentive to manipulate the terms of trade, showing the fiscal externality motive remains present and dominates the ToT motive. I also consider an environment with a more realistic progressive tax and transfer system, and highlight how transfer progressivity affects the aggregate and distributional components of the fiscal externality. More progressive transfers ultimately dampen the planner's incentive to inflate, as the poor are more insured pre-shock. This again highlights that initial conditions or existing

inefficiencies shape the planner's incentive to cut rates.

2 Literature

My work intersects intersects with four major strands of literature. First, I build off of the open economy (optimal) monetary policy literature. I am primarily in conversation with Bianchi and Coulibaly (2025), who emphasizes the role of a fiscal externality in shaping the optimal monetary policy response to *exogenous* tariff shocks in RANK. I extend this analysis and show how household heterogeneity changes the optimal monetary response to the same tariff shocks, highlighting that the failure of Ricardian equivalence and the planner's attitude towards redistribution can strengthen or weaken this fiscal externality. Rather than simply issuing a refund to the representative household, tariff revenue in HANK can redistribute from rich households to poor households and increase aggregate demand, strengthening incentives to inflate. Work by Alessandria et al. (2025) emphasizes the strength of this revenue channel in correcting distortions induced by other taxes. Though they primarily analyze how this revenue channel affects the optimal tariff itself, the insight that using tariff revenue as transfers to reduce the distortion from incomplete markets or suboptimal long run fiscal policy remains in my model, with this motive shaping optimal monetary policy.

Other work in this literature studies jointly optimal (endogenous) tariff and monetary policy. Jeanne (2021) studies the interaction between monetary policy instruments and a wide variety of fiscal instruments, including tariffs. Due to the difficulty of studying optimal monetary and fiscal policy jointly with heterogeneous agents, I leave questions of endogenous trade policy (or other endogenous fiscal policy) to future work. I instead use an inverse optimal approach to work around the fact that steady state fiscal policy in my model may be chosen suboptimally.

I also contribute to the HANK literature, particularly the strand of the literature concerned with optimal monetary policy. Starting from the work of Kaplan et al. (2018), the HANK literature emphasizes that monetary transmission in heterogeneous and representative agent economies differs due to the presence of high MPC households. This difference extends to the conduct of optimal monetary policy, with a planner having an incentive to

cut rates to redistribute from creditors to debtors. I primarily follow the analytical and computational framework introduced by Nuño and Moll (2018) and further developed by Nuño and Thomas (2022) and Dávila and Schaab (2023). Like them, I use continuous time and a variational approach with Gâteaux derivatives to characterize the solution to the planner's problem analytically coupled with standard computational methods to solve for optimal policy computationally. To circumvent the Ramsey planner's time 0 problem, I follow the timeless approach discussed by Dávila and Schaab (2023). My paper is the first in this particular substrand of the heterogeneous agent optimal monetary policy literature to emphasize how a realistic fiscal block affects optimal monetary policy. I show theoretically that the addition of non-trivial fiscal policy induces additional motives for the planner to inflate/deflate (expanding on the targeting rule of Dávila and Schaab (2023)) and show it is quantitatively powerful in the case of a tariff shock.

Other substrands of this literature have studied how fiscal and monetary interactions shape optimal policy in environments with limited heterogeneity. For instance, Le Grand et al. (2022) and Le Grand et al. (2025) use a truncation method to study jointly optimal fiscal and monetary policy. In an environment similar to mine, where fiscal instruments are limited, optimal monetary policy has a role to redistribute over the business cycle, though there is no such scope with sufficient rich and optimally chosen fiscal instruments. My work puts particular emphasis on how a fiscal externality (or a fiscal revenue motive) shapes optimal policy and is used to redistribute. I show that the covariance between the effects of fiscal revenue and planner values of households (a combination of planner weights and household marginal utilities) determines whether the fiscal externality pushes the planner to inflate or deflate. I use an inverse optimal approach (see Heathcote and Tsujiyama (2021) or McKay and Wolf (2022)) to find planner weights that imply an efficient steady state with fiscal policy taken as given and isolate the effect of optimal monetary policy on the new inefficiencies induced by the tariff.

Finally, in the only paper that discusses optimal monetary policy in an open economy HANK model, Acharya and Challe (2025) extend their 2023 paper to an open economy, and show that the central bank has an additional motive when conducting monetary policy: manipulation of the terms of trade. When I allow for endogenous terms of trade, this motive

remains present albeit weak. I primarily abstract from it and instead focus on the fiscal revenue effects of a tariff.

Finally, I contribute to the literature discussing the distributional consequences of trade and international shocks. Carroll and Hur (2023) studies the distributional effects of tariff shocks and emphasizes the allocation of tariff revenue is a strong determinant of the positive effects of the shock. I build on this insight, showing that the reallocation of tariff revenue plays a similarly strong role in determining the normative response of policy to a tariff shock. Borusyak and Jaravel (2021) studies the expenditure channel of trade shocks and finds that American households have roughly similar expenditure on foreign, tariffed goods across the income distribution, contrary to past work (for example, that of Carroll and Hur (2020)). Thus, I abstract from this channel in my analysis.

Outline I organize the remainder of the paper as follows. Section 3 introduces my baseline model. Section 4 describes the timeless Ramsey optimal monetary policy problem and highlights some theoretical results. Section 5 describes the quantitative results for a utilitarian planner. Section 6 quantitatively solves the same model with heterogeneous planner welfare weights that generate an ex-ante efficient steady state. Section **??** studies welfare in the baseline model and counterfactuals. Section 7 introduces additional extensions of the baseline model and discusses their properties. Finally, section 8 concludes

3 Model

I adapt the model of Bianchi and Coulibaly (2025) into continuous time and incorporate household heterogeneity in productivity.

Environment Time is continuous and infinite. The economy includes

1. A continuum of ex-ante identical households.
2. A final firm that produces the home good.
3. A continuum of intermediate firms that produce inputs for the home good.

4. A fiscal authority that levies tariffs on the foreign good and distributes subsidies and transfers to households and firms

5. A monetary authority that sets interest rates.

Households choose consumption and savings in home denominated bonds subject to a borrowing constraint, are subject to idiosyncratic labor productivity shocks and exponentially distributed life lengths with mean $1/\eta$[5], and supply labor to the monopolistically competitive intermediate good firms, who in turn sell goods to a perfectly competitive final firm. Intermediate firms demand household labor and set prices under monopolistic competition subject to Rotemberg adjustment costs. The government is subject to a period budget constraint, issuing one period non-state contingent debt, levying tariffs and an income tax, providing an employment subsidy to firms, and distributing tariff revenue to households. The monetary authority optimally chooses interest rates to maximize household welfare.

3.1 Households

There is a continuum of ex-ante identical households in the home country indexed $i \in [0, 1]$. Households receive disutility from labor $n_{i,t}$, which is divisible and indexed 0 to 1 and receive utility from consumption, aggregated according to the CES aggregator

$$(c_{i,t})^{\frac{\gamma-1}{\gamma}} = \omega(c^h_{i,t})^{\frac{\gamma-1}{\gamma}} + (1-\omega)\left(c^f_{i,t}\right)^{\frac{\gamma-1}{\gamma}}$$

where $c_{i,t}$ is aggregate consumption, $c^h_{i,t}$ is consumption of the home good, $c^f_{i,t}$ is consumption of the foreign-produced good, ω is the preference weight on home goods relative to foreign, and γ is the elasticity of substitution between home and foreign goods (the trade elasticity).

Each instant, households choose consumption, labor supply, and savings in home and foreign bonds, subject to an exogenous borrowing constraint ϕ. Household productivity z_i evolves according to the n-state Poisson process Λ with transition intensity $\lambda_{j,j'}$. Households have exponentially distributed lifetimes with mean $1/\eta$. Each period, mass η of households

[5]Random births and deaths are not needed to ensure the existence of the stationary distribution, but does ensure the generator matrix $\mathcal{A}$ has a well behaved inverse, which will be necessary when numerically solving for the monetary authority's optimal policy. The same strategy is used by Nuño and Moll (2018)

are born with no wealth and productivity drawn from the stationary distribution of Λ. These are the only sources of uncertainty; there is no aggregate risk. To simplify notation, in this subsection I avoid explicitly denoting each household's variables with i. Thus, the household's Hamilton Jacobi Bellman equation can be written as:

$$\tilde{\rho}V_t(b_t, z_{jt}) = \max_{c_t^f, c_t^h, n_t} u(c_t^f, c_t^h) - g(n_t) + \frac{\partial V_t}{\partial b_t}\dot{b}_t + \sum_{j' \neq j} \lambda_{j,j'} \left[V_t(j') - V_t(j)\right] + \frac{\partial V_t}{\partial t}$$

subject to

$$\dot{b}_t = (r_t + \eta)b_t + (1 - \tau_n)z_t w_t n_t + T_t + D_t z_t - c_t^h - p_t(1 + \tau_t)c_t^f$$
$$b_t \geq \phi$$

where $\tilde{\rho} = \rho + \eta$. c^f and c^h are consumption of the foreign and home goods. The numeraire is 1 unit of the home good; the relative price of the foreign good is given by p_t. n is labor supply, b are household positions in the risk-free home bond with real interest rate r[6], z_j is idiosyncratic productivity, w is the wage, T are lump sum transfers, D are dividends distributed proportionally to household productivity, τ^n is a labor income tax, and τ is a tariff. Following Nuño and Moll (2018), households purchase bonds from a perfectly competitive intermediary who offers a gross return η in exchange for all of the household's assets at time of death[7]. Following the assumptions of Bianchi and Coulibaly (2025), the relative price $p_t = \frac{p^f}{p^h}$ is exogenous and first best tariffs are 0[8].

Now, define the generator $\mathcal{A}$ according to

$$\mathcal{A}V = \begin{bmatrix} \frac{\partial V_1}{\partial b} s_1 + \sum_{j' \neq 1} \lambda_{1,j'} \left[V_{j'} - V_1\right] \\ \vdots \\ \frac{\partial V_J}{\partial b} s_J + \sum_{j' \neq J} \lambda_{J,j'} \left[V_{j'} - V_J\right] \end{bmatrix}$$

where V_j is shorthand for $V(b, z_j)$ and s_j similar shorthand for the savings function $s(b, z_j) = \dot{b}$. Thus, the HJB can be expressed as

$$\tilde{\rho}V_t = \max_{c_t^f, c_t^h, n_t} u(c_t^f, c_t^h) - g(n_t) + \mathcal{A}V_t + \frac{\partial V_t}{\partial t} \tag{1}$$

[6] All borrowing is done at home, my theoretical mechanisms remain if there is foreign borrowing or lending

[7] The intermediary has flow revenues ηB and pays flow costs ηb to each household;their aggregate profits are thus 0

[8] The assumption allows me to focus on the fiscal revenue channel instead of a terms of trade manipulation channel, I lift this assumption in section 7

which will be useful when deriving the solution to the planner's problem later.

When solving the model numerically, I assume CRRA utility for total consumption and power disutility of labor according to

$$u(c_{i,t}) = \frac{c_{i,t}^{1-\sigma}}{1-\sigma}$$
$$g(n_{i,t}) = \chi \frac{n_{i,t}^{1+\kappa}}{1+\kappa}$$

Household First Order Conditions The first order conditions for the household are

$$c_t^f = c_t^h \left(\frac{\omega}{1-\omega}(1+\tau_t)\frac{p_t^f}{p_t^h} \right)^{-\gamma} \tag{2}$$
$$g'(n_t) = \frac{w_t z_t}{p_t^h} u_h(c_t^h, c_t^f) \tag{3}$$
$$u_h(c_t^h, c_t^f) = \frac{\partial V(b_t, z_t)}{\partial b_t} \tag{4}$$

The first equation governs the optimal split between home and foreign goods. The second and third equations are standard labor supply and Euler equations.

Sources of Suboptimality The tariff distorts three margins, two of which are present in RANK and HANK, with the third only being present in HANK.

1. First, it directly distorts the home vs foreign consumption margin. As τ increases, c^f decreases.

2. Second, it distorts the labor/leisure margin. The price level the household faces in equilibrium can be written as $\mathcal{P}_{i,t} = P_t^h \left[\omega_i^\gamma + (1-\omega_i)^\gamma (p(1+\tau_t))^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$, which is increasing in the tariff. Thus, since consumption is more expensive, it acts as a tax on labor, distorting the labor/leisure margin indirectly.

3. Finally, unlike in a RANK economy, it distorts the intertemporal consumption savings margin, even if tariffs are constant. Since consumption is more expensive, households who are borrowing constrained can afford less consumption.[9] Thus, there is an incentive to engage in more precautionary savings.

[9]This can be shown mathematically by examining a discrete time version of the model's Euler Equation with a constant tariff rate, $\tau_t = \tau$: $(c_{i,t}^h)^{-\sigma}\mathcal{C}_{i,\tau} = \mathbb{E}\left[\beta R_{t+1}(c_{i,t+1}^h)^{-\sigma}\mathcal{C}_{i,\tau}\right] + \lambda_{i,t}$ where $\mathcal{C}_{i,\tau}$ is a time-invariant

Kolmogorov Forward Equation The distribution of households $f(b, z_j, t)$ satisfies the following Kolmogorov Forward Equation

$$\frac{\partial f_t(b_t, z_{jt})}{\partial t} = -\frac{\mathrm{d}}{\mathrm{d}b_t}[s_t(b_t, z_{jt}) f_t(b_t, z_{jt})] - f_t(b_t, z_{jt}) \sum_{j' \neq j} \lambda_{jj'} + \sum_{j' \neq j} \lambda_{j'j} f_t(b_t, z_{jt}) - \eta f_t(b_t, z_{jt}) + \eta \delta_0$$

where s is the savings functions that solve the household problem. The final two terms represent respectively the outflow due to deaths and the inflow of agents with 0 wealth and productivity drawn from the stationary distribution of Λ; δ_0 is the Dirac delta function.

The KFE can similarly be expressed in terms of the adjoint of $\mathcal{A}$, $\mathcal{A}^*$

$$\frac{\partial f}{\partial t} = \mathcal{A}^* f + \eta \Lambda \tag{5}$$

where

$$\mathcal{A}^* f = \begin{bmatrix} -\frac{\partial (s^1 f_1)}{\partial b} - f_1 \sum_{j' \neq 1} \lambda_{1j'} + \sum_{j' \neq 1} \lambda_{j'1} f_1 \\ \vdots \\ -\frac{\partial (s_J f_J)}{\partial b} - f_J \sum_{j' \neq J} \lambda_{Jj'} + \sum_{j' \neq J} \lambda_{j'J} f_J \end{bmatrix}$$

with shorthand similar to that defined in the previous section[10].

3.2 Firms

Firms are nearly identical to a standard New Keynsian production block.

Final Firm The output of the final good producer is Y_t, which combines a continuum of intermediate goods $y_{j,t}$, $j \in [0, 1]$ according to

$$Y_t = \left(\int_0^1 (y_{j,t})^{\frac{\epsilon - 1}{\epsilon}} \mathrm{d}j \right)^{\frac{\epsilon}{\epsilon - 1}},$$

where $\theta > 0$ is the elasticity of substitution across intermediate goods. The static profit maximization problem of the final firm yields the demand for intermediate goods

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t^h} \right)^{-\epsilon} Y_t$$

scalar that increases with τ. In RANK, or if $\lambda_{i,t} = 0 \forall i$, the scalar $\mathcal{C}_{i,\tau}$ cancels and the consumption savings margin is non-distorted. However, in HANK when the borrowing constraint binds, this is no longer the case and the consumption savings margin is distorted. There will be additional precautionary savings in equilibrium.

[10] See Nuño and Moll (2018) for a proof that $\mathcal{A}$ and $\mathcal{A}^*$ are adjoints.

where the price index of the home-produced good is $P_t^h = \left(\int_0^1 (p_{j,t})^{1-\epsilon} \, \mathrm{d}j\right)^{\frac{1}{1-\epsilon}}$.

Intermediate Firms The continuum of intermediate good producers each produce one variety $j \in [0,1]$ according to the linear production technology

$$y_{j,t} = A_t n_{j,t}$$

These producers are monopolistically competitive and set prices a la Rotemberg (1982). Defining producer price inflation as $\pi_t = \frac{\dot{p}_t^h}{p_t}$, the adjustment costs are

$$\Theta(\pi) = \frac{\theta}{2}\pi_t^2 Y_t$$

Given the production technology, equilibrium marginal costs are $m_t = \frac{w_t}{P_t^h A_t}$. Assuming firms discount the future at rate r_t^h and omitting j subscripts for clarity, the firm's pricing problem is cast recursively as

$$r_t J(p^h, t) = \max_{\pi} \left(\frac{p_t^h}{P_t^h}(1+S) - m_t\right)\left(\frac{p_t^h}{P_t^h}\right)^{-\epsilon} Y_t - \Theta(\pi) + J_p(p^h, t) p_t^h \pi_t + J_t(p^h, t)$$

where the production subsidy $S = \frac{1}{\epsilon - 1}$ corrects the markup distortion in steady state. Then, in a symmetric equilibrium with $n_{j,t} = N_t = A_t Y_t = A_t y_{j,t}$, $p_{j,t}^h = P_t^h$ the NKPC is

$$(r^h - \frac{\dot{N}_t}{N_t})\pi_t = \frac{\epsilon}{\theta}(m-1) + \dot{\pi} \tag{6}$$

Additionally, this implies (real) dividends are

$$\frac{D_t}{P_t^h} = (1+S)Y_t - m_t Y_t - \Theta(\pi_t) \tag{7}$$

3.3 Government

The fiscal authority issues debt, collects tariff revenue, and distributes it to households after paying for the production subsidy

$$\dot{B}_t = r_t B_t + T_t + S p_t^h Y_t - \tau_t p^f \int c_t^f \, \mathrm{d}f_t(b,z) - \tau^n w_t \int n_t z_t \, \mathrm{d}f_t(b,z) \tag{8}$$

where $\int c_f \, \mathrm{d}f(b,z)$ is aggregate consumption of the foreign good, $\int nz \, \mathrm{d}f(b,z)$ is aggregate labor, B_t are aggregate bonds, T_t are lump sum transfers, and τ^n are labor income taxes.

Since Ricardian equivalence fails, the specification of fiscal policy and which variable adjusts to satisfy the government budget will affect equilibrium outcomes. As a baseline, I assume that transfers adjust with debt and income taxes held fixed[11]. To highlight how the distributional (non-)neutrality of how tariff revenues are spent affects optimal policy, I present results with bonds adjusting in section 5.

3.4 Market Clearing

Market clearing requires

$$N_t = n_{j,t} = \int n_{i,t} z_{i,t} \, \mathrm{d}f(b^h, b^f, z) \tag{9}$$

$$B_t = \int b_t \, \mathrm{d}f_t(b, z) \tag{10}$$

$$0 = Y_t - \frac{\phi}{2}\pi_t^2 Y_t - c_t^h - p c_t^f \tag{11}$$

i.e. labor market clearing, bond market clearing, and a balance of payments condition

3.5 Equilibrium

I now definite a competitive equilibrium.

Definition 1 *Given an initial distribution over bond holdings and idiosyncratic states $f_0(b_0, z_0)$, an exogenous relative price p, sequences of government policy for S, T_t, B_t, τ^n, and τ_t, a specification for monetary policy r_t, a competitive equilibrium requires finding (sequences of) household policy functions s_t, n_t c_t^h, c_t^f and value functions V_t, solutions for the firms' problem $n_{j,t}$, $y_{j,t}$, $p_{j,t}^h$, Y_t, N_t, prices w_t, and measures $f_t(b_t, z_t)$ such that*

1. *Given prices and interest rates, the HJB holds*
2. *Given the savings function $s_t(b_t, z_t)$, f_t solves the KFE*
3. *Given wages, $n_{j,t}$ solves the firm cost minimization problem and $p_{j,t}^h$ solves the price setting problem*

[11] i.e. all tariff revenues are distributed lump sum to households

4. *Given a specification for 3 of S, T_t, B_t, and τ_t^n, the fourth is determined by the government budget constraint*

5. *Markets clear*

   (a) *Labor market:* $N_t = n_{j,t} = \int n \, \mathrm{d}f_t(b_t, z_t)$

   (b) *Bond market:* $B_t = \int b_t \, \mathrm{d}f_t(b_t, z_t)$

   (c) *Goods:* $\underbrace{Y_t - \frac{\theta}{2}\pi_t^2 Y_t - C_t^h}_{Exports} = \underbrace{pC_t^f}_{Imports}$

4 Timeless Ramsey Optimal Monetary Policy Problem

Following Dávila and Schaab (2023), I now describe the timeless Ramsey planner's optimal policy problem. The planner's only instrument is the (real) interest rate r_t. Using the timeless perspective ensures the planner has no incentive to induce inflation at time 0, a standard issue in the optimal monetary policy literature[12].

In particular, define time 0 welfare W_0 as

$$W_0[f_0(b,z)] \equiv \mathbb{E}_{f_0(b,z)} V_0(b,z) = \int_0^\infty e^{-\tilde{\rho}t} \mathbb{E}_{f_0(b,z)}[\varphi(b,z)(u(c_t^f, c_t^h) - g(n_t))] \, \mathrm{d}t \tag{12}$$

where $\tilde{\rho} = \rho + \eta$, the effective discount rate and $\varphi(b,z)$ are planner welfare weights. Informally, the Ramsey optimal policy problem requires maximizing equation 12 by choosing sequences of $r_t, \pi_t, V_t(.), c_t^f(.), c_t^h(.), n_t(.), f_t(.), w_t, N_t, C_t^f$ subject to all of the implementability conditions for the competitive equilibrium: the HJB (eq 1), all household FOC (eq 2-4),the KFE (eq 5), the NKPC (eq 6), the government budget constraint (eq 8), and market clearing (eq 9-11), taking as given an initial distribution over bonds and the idiosyncratic shock: $f_0(b,z)$.

As in Nuño and Moll (2018), the optimal value and associated polices are functionals, as they map the infinite dimension initial distribution f_0 into $\mathbb{R}$ (in the case of welfare) or other infinite dimensional distributions (in the case of, for example consumption policy functions). N

[12] See Dávila and Schaab (2023) or appendix B.4 for further details on the timeless perspective

Proposition 1 (Primal Timeless Ramsey Problem) *The timeless Ramsey planner's problem solves*

$$\min_{\{\eta_{k,t}\}_{k=1}^{9}} \max_{r_t,\pi_t,V_t(.),f_t(.),c_t^f(.),c_t^h(.),n_t(.),w_t,N_t,C_t^f} \mathcal{L}(f_0) + \mathcal{T}(\eta_{3,0},\eta_{2,0}) \tag{13}$$

with the Lagrangian functional $\mathcal{L}(f_0)$ [13] *defined as*

$$\mathcal{L}(f_0) \equiv \int_0^\infty e^{-\tilde{\rho}t}\{\int_{z_1}^{z_J}\int_\phi^\infty \varphi(b,z)\left[u(c_t^h,c_t^f)-g(n_t)\right]f_t + \eta_{1,t}(b,z)\left[\mathcal{A}^* f_t - \frac{\partial f_t}{\partial t}\right] +$$
$$\eta_{2,t}(b,z)\left[u(c_t^f,c_t^h)-g(n_t)+\mathcal{A}V_t+\frac{\partial V_t}{\partial t}-\rho V_t\right]+\eta_{3,t}\left[\pi_t(r_t^h-\frac{\dot{N}_t}{N_t})-\frac{\epsilon}{\theta}(m_t-1)-\dot{\pi}\right]+$$
$$\eta_{4,t}[b_t^h-B_t]f_t+\eta_{5,t}[n_t z_t-N_t]f_t+\eta_{6,t}[c_t^f-C_t^f]f_t+\eta_{7,t}(b,z)\left[g'(n_t)-\frac{w_t z_{j,t}(1-\tau^n)}{p_t^h}u_h(c_t^h,c_t^f)\right]$$
$$\eta_{8,t}(b,z)\left[u_h(c_t^h,c_t^f)-\frac{\partial V}{\partial b_t^h}p_t^h\right]+\eta_{9,t}(b,z)\left[u_f(c_t^h,c_t^f)-\frac{\partial V}{\partial b_t^h}p_t^h(1+\tau)\right]\mathrm{d}b\,\mathrm{d}z\}\,\mathrm{d}t$$

where $\eta_{1,t},\eta_{2,t},\eta_{7,t},\eta_{8,t},\eta_{9,t}$ *are functional Lagrange multipliers and* $\eta_{3,t}$, $\eta_{4,t}$, $\eta_{5,t}$, *and* $\eta_{6,t}$ *are scalar Lagrange multipliers and* z_1 *and* z_J *are the lower and upper bounds, respectively, on the idiosyncratic shock* z. *Additionally, I assume that transfers adjust to clear the government budget constraint; the government budget and the expression for dividends are thus included implicitly in* $\mathcal{A}$ *by substituting for transfers and dividends, respectively.*

The timeless penalty $\mathcal{T}(\eta_{3,0},\eta_{2,0})$ *is defined as in Dávila and Schaab (2023) and equals:*

$$\int_{z_1}^{z_J}\int_\phi^\infty \eta_{2,0}V_0(b,z)\,\mathrm{d}b\,\mathrm{d}z+\eta_{3,0}\pi_0$$

The corresponding Ramsey plan are the paths of multipliers $\{\eta_{k,t}\}_{k=1}^{9}$ *and choice variables* $r_t,\pi_t,V_t(.),f_t(.),c_t^f(.),c_t^h(.),n_t(.),w_t,N_t,C_t^f$ *that solve 13*

In the remainder of this section, I assume the planner is utilitarian: $\varphi(b,z)=1$. I relax this assumption in section 6. In appendix B I derive proposition 1 and all other results in more detail, as well as more mathematical details in appendix A.

[13] The Lagrangian depends on both time and the infinite dimension initial distribution f_0. To maximize with respect to the choice functions, I follow Nuño and Moll (2018) and employ Gâteaux derivatives, an extension of the derivative from $\mathbb{R}^n$ to infinite dimension function spaces. The Gâteuax derivative of $\mathcal{L}$ with respect to, for example, V_t is defined as

$$\lim_{\alpha\to 0}\frac{\mathcal{L}[v_t+\alpha h,...]-\mathcal{L}[v_t,...]}{\alpha}=\frac{\mathrm{d}}{\mathrm{d}\alpha}\mathcal{L}[v_t+\alpha h,...]|_{\alpha=0}$$

where h is any function in the same space as V_t. More mathematical details can be found in appendix A.

Now, I characterize the solution to the optimal policy problem

Proposition 2 (Ramsey Optimality Conditions) *In addition to the implementability conditions, the solution to the utilitarian Ramsey planner's optimal policy problem is characterized by the following system of equations*[14]

$$\tilde{\rho}\eta_{1,t} = u(c_t^h, c_t^f) - g(n_t) + \mathcal{A}\eta_{1,t} + \frac{\mathrm{d}\eta_{1,t}}{\mathrm{d}t} + \eta_{4,t}(b_t^h - B_t) + \eta_{5,t}(n_t - N_t) + \eta_{6,t}(c_t^f - C_t^f) \tag{14}$$

$$\frac{\partial \eta_{2,t}}{\partial t} = \mathcal{A}^*\eta_{2,t} + \frac{\partial \eta_{8,t}}{\partial b}p + \frac{\partial \eta_{9,t}}{\partial b}(1+\tau)p \tag{15}$$

$$0 = u_f(c_t^h, c_t^f) - \frac{\partial \eta_{1,t}}{\partial b}(p(1+\tau)) + \frac{\eta_{9,t}}{f_t}u_{ff}(c_t^h, c_t^f) - \frac{\eta_{7,t}}{f_t}\frac{w_t z_{j,t}(1-\tau^n)}{p_t^h}u_{hf}(c_t^h, c_t^f) + \frac{\eta_{8,t}}{f_t}u_{fh}(c_t^h, c_t^f) + \eta_{6,t} \tag{16}$$

$$0 = u_h(c_t^h, c_t^f) - \frac{\partial \eta_{1,t}}{\partial b}p + \frac{\eta_{9,t}}{f_t}u_{fh}(c_t^h, c_t^f) - \frac{\eta_{7,t}}{f_t}\frac{w_t z_{j,t}(1-\tau^n)}{p_t^h}u_{hh}(c_t^h, c_t^f) + \frac{\eta_{8,t}}{f_t}u_{hh}(c_t^h, c_t^f) \tag{17}$$

$$0 = -g'(n_t) + \frac{(1-\tau^n)w_t z_{j,t}}{p_t^h}\frac{\partial \eta_{1,t}}{\partial b} + \eta_{7,t}\frac{g''(n_t)}{f_t} + z\eta_{5,t} \tag{18}$$

$$\eta_{3,t}\pi_t = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\left(\frac{\partial \eta_{1,t}}{\partial b}f_t + \frac{\partial V_t}{\partial b}\eta_{2,t}\right)(B_t - b_t)\,\mathrm{d}b\,\mathrm{d}z \tag{19}$$

$$\eta_{3,t}\frac{\epsilon}{\theta} = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\left(\frac{\partial \eta_{1,t}}{\partial b}f_t + \frac{\partial V_t}{\partial b}\eta_{2,t}\right)(1-\tau^n)(z_t n_t - N_t) - (1-\tau^n)z u_h(c^f, c^h)\eta_{7,t}\,\mathrm{d}b\,\mathrm{d}z \tag{20}$$

$$\dot{\eta}_{3,t} = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\left(\frac{\partial \eta_{1,t}}{\partial b}f_t + \frac{\partial V_t}{\partial b}\eta_{2,t}\right)\frac{\partial s}{\partial \pi_t}\,\mathrm{d}b\,\mathrm{d}z + \eta_{3,t}(\tilde{\rho} - r + \frac{\dot{N}_t}{N_t}) \tag{21}$$

$$\eta_{5,t} = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\left(\frac{\partial \eta_{1,t}}{\partial b}f_t + \frac{\partial V_t}{\partial b}\eta_{2,t}\right)\frac{\partial s}{\partial N_t}\,\mathrm{d}b\,\mathrm{d}z + \frac{(\dot{\eta}_{3,t}\pi_t - \eta_{3,t}\tilde{\rho}\pi_t + \dot{\pi}\eta_{3,t})}{N_t} \tag{22}$$

$$\eta_{6,t} = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\left(\frac{\partial \eta_{1,t}}{\partial b}f_t + \frac{\partial V_t}{\partial b}\eta_{2,t}\right)\frac{\partial s}{\partial C_t^f}\,\mathrm{d}b\,\mathrm{d}z \tag{23}$$

with boundary conditions[15]

$$\lim_{T\to\infty} e^{-\tilde{\rho}T}\eta_{1,T} = 0$$

$$\eta_{3,0} = \eta_{3,s.s.}$$

$$\eta_{2,0} = \eta_{2,s.s.}$$

I now discuss and interpret some of the planner's first order conditions. Equation 14 can be interpreted as the planner's HJB equation, where $\eta_{1,t}(b,z)$ is the social value function.

[14] s is the household savings function or budget constraint

[15] These initial conditions are the only place the timeless penalty affects the planner's solution. All first order conditions are identical with or without a timeless penalty. Solving this problem without the timeless penalty would allow both time 0 inflation and time 0 household values to be determined freely by the planner, incentivizing the planner to induce inflation at time 0, even in the absence of shocks. Adding the timeless penalty will set the initial conditions to ensure the planner does not do this, ensuring the monetary response I report is not partially caused by time inconsistency at time 0. See appendix B.4 for a numerical experiment that highlights this fact or Dávila and Schaab (2023) for a more detailed discussion.

Understanding the difference between the social and private values is best done through the marginal value functions. The marginal social value of wealth for a household with a given state vector $\frac{\partial \eta_{1,t}(b,z)}{\partial b}$ is related to the marginal private value $\frac{\partial V_t(b,z)}{\partial b}$ through the following identity derived in Appendix B

$$\frac{\partial \eta_{1,t}(b,z)}{\partial b} = \frac{\partial V_t(b,z)}{\partial b} + \eta_4 \mathcal{M}^{-1} + \eta_5 \mathcal{M}^{-1} \frac{\partial n(b,z)}{\partial b} + \eta_6 \mathcal{M}^{-1} \frac{\partial c^f(b,z)}{\partial b}$$

defining the operator .

$$\mathcal{M} = \tilde{\rho} - s_b - \mathrm{d}t - \mathcal{A} \tag{24}$$

The 3 Lagrange multipliers $\eta_{4,t}$, $\eta_{5,t}$, and $\eta_{6,t}$ reflect the addition of pecuniary externalities. The former two terms are not unique to an open economy environment. Individual agents do not internalize how their savings and labor decisions infinitesimally affect market-clearing prices, redistributed profits, or transfers they receive. The planner internalizes these forces, which are captured by the additional terms in their HJB.

Additionally, in this open economy model with tariffs, the planner's HJB includes a fiscal externality multiplier $\eta_{6,t}$, similarly reflecting that individual agents do not internalize how their foreign consumption affects fiscal revenue and therefore the transfers they and all other agents receive. This force is unique to my model.

Thus, the difference in the marginal social and private values of wealth $\eta_4 \mathcal{M}^{-1} + \eta_5 \mathcal{M}^{-1} \frac{\partial n(b,z)}{\partial b} + \eta_6 \mathcal{M}^{-1} \frac{\partial c^f(b,z)}{\partial b}$ captures the present discounted value of future savings to aggregate excess bond demand, future labor supply to aggregate excess labor, and foreign consumption to aggregate fiscal revenue caused by an increase in time t assets. Increasing assets today will affect the marginal propensity to consume foreign goods $c^f(b,z)$, the marginal propensity to work $n(b,z)$, and directly affects asset holdings, both today and in the future (as captured by the $\mathcal{M}^{-1}$ term). When one of the Lagrange multipliers η_4, η_5, η_6 are positive (negative), the additional excess savings, labor supply, and foreign consumption are respectively beneficial (harmful). Note that relative to the closed economy model of Dávila and Schaab (2023), the η_6 term is novel and captures the planner's assessment of the value of the fiscal externality when tariffs are non-zero.

Equation 15 can be interpreted as the "planner's KFE", where $\eta_{2,t}$ is the "social density", or a distributional penalty in the language of Dávila and Schaab (2023). If the planner was

not constrained by household optimality conditions, they would prefer different individual policies and a different density than the one that prevails in equilibrium. $\eta_{2,t}$ governs this penalty/benefit on the welfare gains/losses for each type of household and works to ensure the planner satisfies the private optimality conditions.

The Fiscal Externality Equation 23 is the planner's first order conditions for optimal aggregate foreign consumption. They characterize the Lagrange multipliers in the planner's HJB equation. In the spirit of Nuño and Thomas (2022), I can rewrite the Lagrange multiplier for aggregate foreign consumption, $\eta_{6,t}$ as

$$\eta_{6,t} = \int_{z_1}^{z_J} \int_{\phi}^{\infty} \underbrace{\frac{\partial s_t}{\partial T}\frac{\partial T}{\partial C^f}}_{\text{Marginal Effect of Transfers}} \underbrace{\left(\frac{\partial \eta_{1,t}}{\partial b} + \frac{\partial V_t}{\partial b}\frac{\eta_{2,t}}{f_t}\right)}_{\text{Net Planner Marginal Value}} f_t \, \mathrm{d}b \, \mathrm{d}z \tag{25}$$

or rearranging

$$\eta_{6,t} = \underbrace{\mathrm{COV}_{f_t}\left[\left(\frac{\partial \eta_{1,t}}{\partial b} + \frac{\partial V_t}{\partial b}\frac{\eta_{2,t}}{f_t}\right), \frac{\partial s_t}{\partial T}\frac{\partial T}{\partial C^f}\right]}_{\text{Distributional Effect}} + \underbrace{\mathbb{E}_{f_t}\left[\left(\frac{\partial \eta_{1,t}}{\partial b} + \frac{\partial V_t}{\partial b}\frac{\eta_{2,t}}{f_t}\right)\right]\mathbb{E}_{f_t}\left[\frac{\partial s_t}{\partial T}\frac{\partial T}{\partial C^f}\right]}_{\text{Average Effect}}$$

which is a combination of a covariance or distributional effect term and an average effect term.

The covariance term says that the shadow value of additional foreign consumption is higher if increasing aggregate foreign consumption has a larger income effect on the those with a higher net planner marginal value (in general, the poor). The second term says the shadow value of additional foreign consumption is higher if the economy on average will receive more income as a result of increasing aggregate foreign consumption and the planner values that income positively (the average planner net marginal value is positive).

Under the baseline assumption of flat transfers $\frac{\partial s_t}{\partial T}$ is constant. However, equation 23 still holds if transfers are not flat[16]. If transfers were progressive, the covariance term would be positive indicating that the planner would prefer even more foreign consumption, holding fixed marginal values. However, even if transfers were regressive, $\eta_{6,t}$ could be positive, as the "average effect" term would remain positive even though the covariance term would be negative.

[16]See appendix G for a proof

Note that $\eta_6 > 0$ will push the planner towards cutting rates and positive inflation. Since the planner's instrument is blunt, the only way to induce additional foreign consumption is by cutting rates. This mechanism is present in RANK (see Bianchi and Coulibaly (2025)) but the additional covariance or distributional term in the definition of the fiscal externality may strength or weaken this effect.

4.1 Characterizing Optimal Policy

First, I show that steady state inflation in my model is 0, echoing a result of Dávila and Schaab (2023).

Proposition 3 (Long Run Inflation) *Long-run (or steady state) inflation is 0*

Proof. In a steady state,

$$0 = \int_{z_1}^{z_J} \int_{\phi}^{\infty} \frac{\partial \eta_{1,t}}{\partial b}(N_t \theta \pi_t) f_t + \frac{\partial V_t}{\partial b}(N_t \theta \pi_t)\eta_{2,t} \, \mathrm{d}b \, \mathrm{d}z + \eta_{3,t}(\tilde{\rho} - r)$$

The precautionary motive implies $\tilde{\rho} - r > 0$ (Sargent and Ljungqvist (2012)). To satisfy the first order condition if $\pi \neq 0$, either $\pi > 0$ and $\eta_{3,t} < 0$ or $\pi < 0$ and $\eta_{3,t} > 0$ due to properties of the social and private value functions. Additionally,

$$\eta_{3,t}\pi_t = \int_{z_1}^{z_J} \int_{\phi}^{\infty} (B_t - b_t^h)\frac{\partial \eta_{1,t}}{\partial b} f_t + (B_t - b_t^h)\frac{\partial V_t}{\partial b}\eta_{2,t} \, \mathrm{d}b \, \mathrm{d}z$$

must hold. From the previous assumption, the LHS of the above must be negative. However, the RHS of the above cannot be negative. Agents with $B_t - b_t^h >> 0$ ($B_t - b_t^h << 0$) will have the highest (lowest) net marginal value. This implies the RHS is weakly positive, a contradiction. ■

Next, I derive a targeting rule similar in spirit to the one of Dávila and Schaab (2023) that characterizes the motives and penalties faced by the central bank when choosing monetary policy.

Proposition 4 (A Targeting Rule) *Optimal interest rate policy is determined by the following equation*

$$0 = \underbrace{\mathbb{E}_{f_t}\left[(1 - m_t) z_t \frac{u_h}{p}\right]}_{\textit{Output Gap}} - \underbrace{\mathbb{E}_{f_t}\left[\frac{\theta}{2}\pi_t^2 z_t \frac{u_h}{p}\right]}_{\textit{Inflation Costs}} - \frac{\Omega_2}{\Omega_1} \underbrace{\mathbb{E}_{f_t}\left[\frac{\partial s}{\partial T}\frac{\partial T}{\partial C^f}\frac{u_h}{p}\right]}_{\textit{Tariff Fiscal Externality}} + \tag{26}$$

$$\underbrace{\mathbb{E}_{f_t}[\frac{\partial s}{\partial T}\frac{\partial IncTax}{\partial N}\frac{u_h}{p}]}_{Income\ Tax\ Fiscal\ Externality} + \frac{1}{N_t}\left(\frac{\dot{\eta}_{3,t}}{\eta_{3,t}} - \tilde{\rho} + \frac{\dot{\pi}_t}{\pi_t}\right)\underbrace{\mathbb{E}_{f_t}\left[(B-b)\frac{u_h}{p}\right]}_{Redistribution} +$$

$$\underbrace{\int\int \frac{1}{N_t}\left(\frac{\dot{\eta}_{3,t}}{\eta_{3,t}} - \tilde{\rho} + \frac{\dot{\pi}_t}{\pi_t}\right)[B-b]\frac{u_h}{p}\eta_{2,t} + \left[\frac{\partial s}{\partial N}\right]\frac{u_h}{p}\eta_{2,t}\,\mathrm{d}b\,\mathrm{d}z - \frac{\Omega_2}{\Omega_1}\int\int\left[\frac{\partial s}{\partial T}\frac{\partial T}{\partial C^f}\right]\frac{u_h}{p}\eta_{2,t}}_{Commitment\ Penalties}$$

where Ω_1 and Ω_2 are (endogenous and generally positive) constants defined in this proposition's proof in Appendix B

A change in interest rates requires balancing the "motives" (an individual term) laid out in proposition 3, all of which have the general form of marginal utility-weighted averages. Now, I describe all of these motives.

The first term and second terms are standard output gap and deadweight inflation cost terms, present in RANK and HANK. It captures the effect that a change in interest rates has on the output gap. It is scaled by z since dividends are distributed proportionally to productivity.

The third term is novel to my environment relative to the previous heterogeneous agent optimal monetary policy literature, and is present in open economy RANK and HANK models with tariffs. It captures the fiscal externality present in the model via the redistribution of tariff revenue. Cutting rates will increase expenditure on both goods, leading to more revenues to redistribute. The particular structure of transfers can strengthen or weaken this effect, as $\frac{\partial s}{\partial T}$ can be decreasing (increasing) in b, leading to high (low) marginal utility households receiving even more of the tariff revenue.

The fourth term is similar to the third in that it reflects fiscal redistribution. However, this term is not novel to an open economy environment and instead reflects the redistribution of fiscal revenue from the income tax. Note that this motive is quantitatively weak in my environment. Unlike the tariff rate, which increases substantially, any change in income tax revenue comes only from changes in labor supply.

The fifth term is exclusive to HANK environments and captures a redistributive motive. Decreasing the interest rate redistributes from high marginal utility agents ($B - b < 0$) to low marginal utility agents ($B - b > 0$), leading to potential welfare gains. This provides an additional incentive for the planner to inflate.

Following the naming convention of Dávila and Schaab (2023), the remaining terms capture commitment penalties. They penalize the welfare gains of certain households. In particular, each of the five policy motives discussed above has an associated penalty term, capturing the necessary penalty ($\eta_{2,t}(b,z) < 0$ implies a penalty on the household) assessed to each household for each motive to make the planner respect household optimality conditions on labor and consumption.

To summarize, table 1 summarizes which policy shaping motives are present in closed/open RANK/HANK economies.

Table 1: Policy Motives in RANK, HANK, Closed, and Open Economies

Motive	Closed RANK	Open RANK	Closed HANK	Open HANK
Output Gap	✓	✓	✓	✓
Inflation Costs	✓	✓	✓	✓
Tariff Fiscal Externality		✓		✓
Redistribution			✓	✓
Penalties			✓	✓

5 Numerical Results

5.1 Calibration

I calibrate the model to an annual frequency. For the baseline calibration, I assume that, as in the presentation of the theoretical results, transfers adjust to solve the government's budget constraint. Additionally, I choose to assume that government bonds are a fixed, flat number to simplify computation rather than chosen optimally. This number will be calibrated in the steady state to be a certain fraction of GDP. Since I am not concerned with (optimal) provision of public debt and how the monetary authority internalizes the effects of their policy on the provision of public debt, this assumption simplifies the fiscal block while still maintaining nontrivial monetary/fiscal interactions.

Table 2 summarizes all parameter values for the baseline calibration.

Table 2: Parameter Values

		Parameter	Value	Target/Source
Household	Effective Discount Factor	$\rho + \eta$	0.0417	4% annual RA interest rate
	Death Rate	η	.02	50 year working lifespan
	Risk Aversion	σ	1	
	Frisch Elasticity	κ	1	
	Borrowing Limit	ϕ	-0.25	One quarter of average labor income
	Labor Disutility Scaling Parameter	χ	1	Steady state output
	Persistence Idiosyncratic Shock	ρ_z	0.914	
	Std. Idiosyncratic Shock	σ_z	0.2063	
	No. of St. Dev Idiosyncratic Shock	m_z	2.0	
	Preference Weight, Home Goods	ω	0.7	Share of Foreign Consumption, US
	Trade Elasticity	γ	2	Estimate from Bianchi and Coulibaly (2025)
	Relative Price	p	1	
Firms	Intermediate Firm Elasticity	θ	10	Average markups
	Adj. Cost Parameter	ϵ	100	Implied annual slope of 0.1
Gvt.	Debt to GDP ratio	B_{perc}	1.4	FRED
	Labor Tax	τ_n	0.3	Average labor income tax , US
	Firm Subsidy	s	$\frac{1}{\epsilon-1}$	Zero markup steady state

Notes: The table reports the parameter values. The debt to GDP ratio is only for calibration, it is never internalized by the planner and is never adjusted in the transition dynamics.

I assume an effective discount factor for both the planner and the household of $\rho + \eta = \frac{1}{.96} - 1$, standard for an annually calibrated model. The death rate of $\eta = .02$ implies a (working) lifespan of 50 years. I assume that $\sigma = \kappa = 1$ to simplify the computation of the labor/lesiure choice[17]. The borrowing limit $\phi = -.25$ is calibrated to be one quarter of average labor income following Kaplan et al. (2018). The Poisson process for z is a discretization of an AR(1) process, using Rouwenhorst discretization. The transition matrix in continuous time is found by taking the (unique) matrix logarithm of the discrete time transition matrix. The preference weight on home goods ω is set so that the share of foreign consumption is 15% in the steady state, in line with US data. I set the intermediate firm demand elasticity to $\theta = 10$ and the adjustment cost parameter $\epsilon = 100$ to have an annual

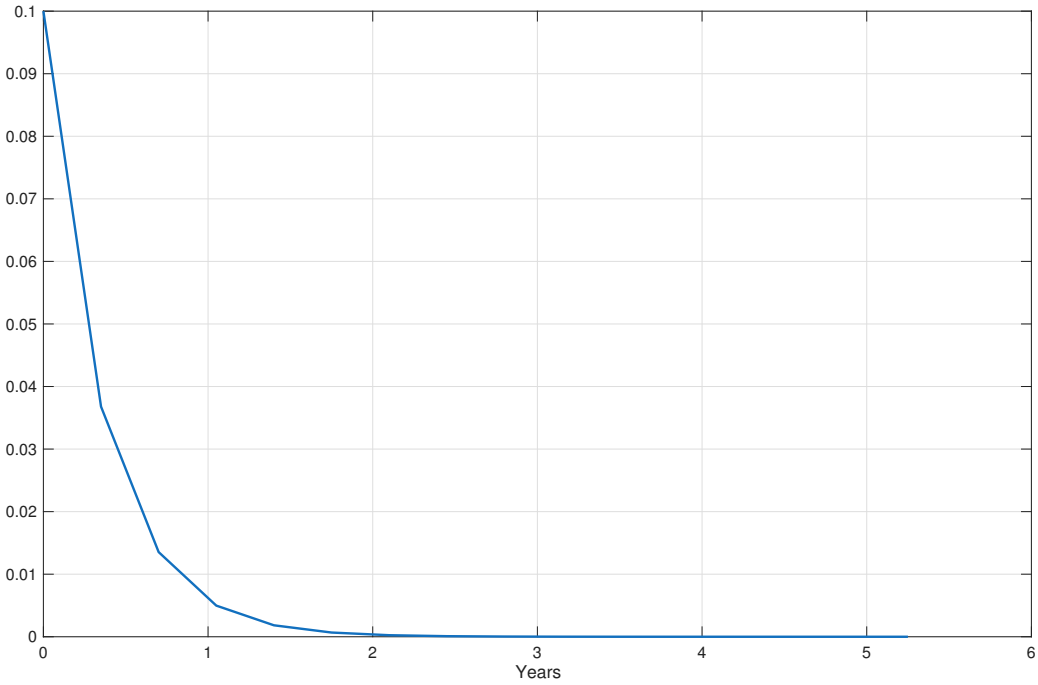
[17]This parameterization ensures that the first order condition for labor is a quadratic equation, ensuring I do not need to use a nonlinear solver to find labor choice.

slope of the Phillips curve of .1, in line with the literature. I set the constant level of debt B to equal 1.4 times GDP. Note that the debt to GDP ratio is never internalized by the monetary authority, from the perspective of the central bank, debt is constant. The labor tax is set to .3, in line with average US labor income taxes. Finally, the firm subsidy is set to eliminate firm markups in the zero inflation steady state.

5.2 Baseline Results

I investigate fully nonlinear, perfect foresight responses to a temporary MIT shock to tariffs[18]. From a zero tariff steady state[19], there is a 10 percentage point increase to the tariff rate τ which then reverts exponentially to 0. Figure 1 plots the tariff shock I use for all of my numerical experiments.

Figure 1: Path of Exogenous Tariff Shock



Notes: The path of tariffs satisfies $\tau_t = .1 \cdot e^{-\xi t}$ with $\xi = .35$.

I make the following broad assumptions about fiscal policy:

[18] I discuss the numerical algorithm in appendix C

[19] The use of the timeless approach allows the time 0 steady state to be well behaved

1. In each experiment, one fiscal variable adjusts.

2. The planner takes (rules for) the non-adjusting variables as given. Since my shocks of interest disperse after ≈ 2 years, it is unrealistic for fiscal tools to adjust broadly or to be chosen optimally over the transition.

In incomplete markets models, Ricardian equivalence fails. Thus, which variable adjusts to clear the government budget constraint affects equilibrium outcomes. Carroll and Hur (2023) highlight how the redistribution of tariff revenue affects *positively* the welfare gains and losses from tariff shocks in a heterogeneous agent neoclassical model. I build on these insights and show how the failure of Ricardian equivalence affects equilibrium outcomes in a *normative* sense.

In particular, I am interested in how the distributional (non-)neutrality of the adjusting instruments affects optimal policy. Thus, I compare Ramsey optimal policy when *transfers adjust* to when *bonds adjust*[20]. Interestingly, the Trump administration has proposed using tariff revenue to adjust both of these instruments,[21] highlighting the direct policy relevance of understanding how the failures of Ricardian Equivalance affect both equilibrium outcomes and optimal policy.

In the steady state, I calibrate the other instrument in each of the scenarios as follows

1. Transfers Adjust (Baseline): Bonds are constant as calibrated in table 2

2. Bonds Adjust: Transfers are constant and set to the same transfer to GDP level as in the steady state of the baseline model.

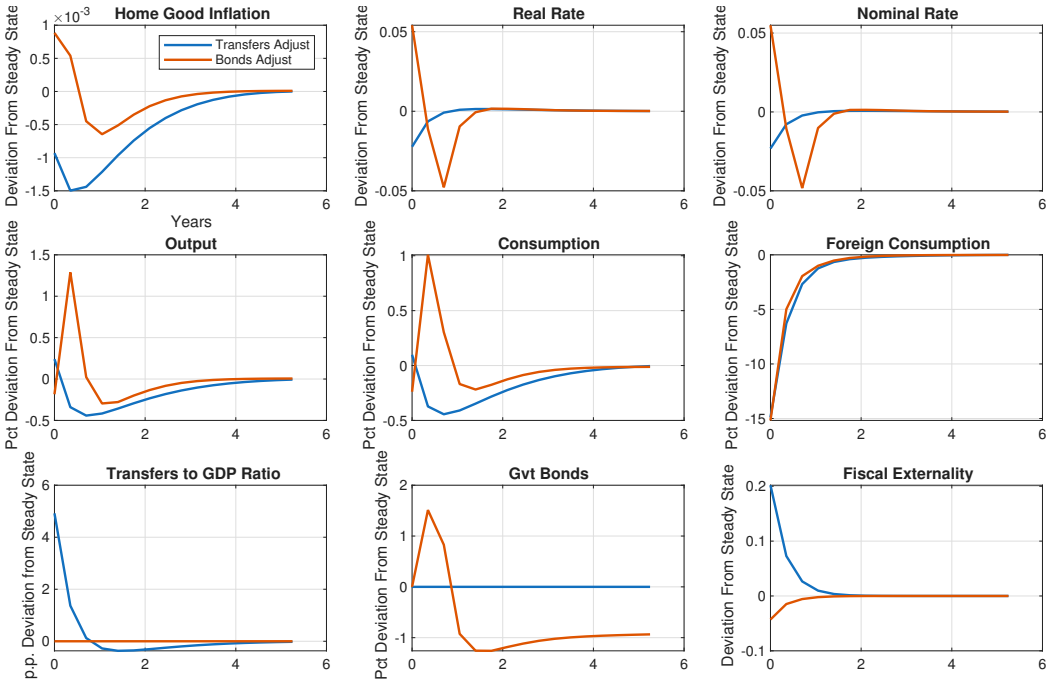
This calibration ensures the steady state outcomes are identical the two alternatives. However, the optimal policy response will not be identical since additional fiscal revenue is spent differently.

Figure 2 plots aggregate outcomes for optimal monetary policy under each of these instruments adjusting.

[20] Planner first order conditions are slightly different in each of these environments, I detail the differences in appendix D

[21] Proposal to adjust transfers, proposal to adjust debt,

Figure 2: Optimal Policy, Varying Fiscal Adjustment, Aggregate Variables



Notes: For the path of tariffs from figure 1, I plot optimal monetary responses to aggregate variables when tariff revenue is spent to increase lump sum transfers or to reduce government debt.

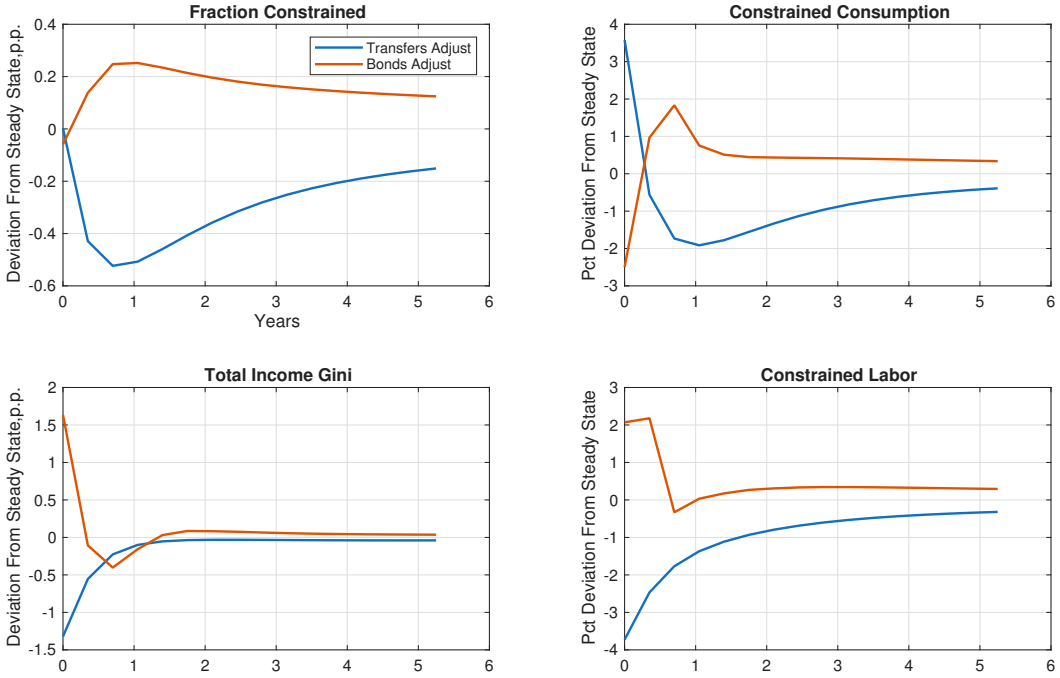
The variable that adjusts affects not only the magnitude of the optimal policy response, but also the sign. When transfers adjust, like the results of Bianchi and Coulibaly (2025), the planner cuts rates and induces a monetary expansion on impact. The transfer to GDP ratio jumps by 5 percentage points. Since (even lump sum) transfers disproportionately benefit high marginal utility poor agents, the planner places a large value on this fiscal revenue. Thus, the fiscal externality (equation 25) is positive, pushing the planner to cut rates. Doing so increases foreign consumption, which increases tariff revenue, which increases lump sum transfers. Since the rich consume more of the foreign good than the poor, they face a larger share of the tariff burden in absolute terms. Thus, in addition to the redistribution from creditors to debtors that a rate cut induces, there is additional redistribution from high foreign consumption to low foreign consumption agents.

When bonds adjust, the central bank instead chooses to raise rates on impact, causing output to fall rather than rise on impact. Reducing government debt does not have the direct impact on household income that raising transfers does. It does not directly decrease inequal-

ity in the same way that increasing transfers does. The fiscal externality becomes (slightly) negative, reflecting the fact that the planner does not value additional foreign consumption and thus lower debt positively. The other motives found in equation 26 dominate.

Figure 3 plots distributional outcomes for optimal monetary policy under each of the 3 fiscal instruments adjusting

Figure 3: Optimal Policy, Varying Fiscal Adjustment, Distributional Variables



Notes: For the path of tariffs from figure 1, I plot optimal monetary responses to distributional variables when tariff revenue is spent to increase lump sum transfers or to reduce government debt.

Similar to the aggregate outcomes, the distributional outcomes are also greatly affected by which variable adjusts. When transfers adjust, the fraction of constrained agents falls. Their consumption rises and labor supply falls. Notably, the consumption of the constrained rises by more than aggregate consumption does, further highlighting that using the tariff revenue as lump sum transfers redistributes from the rich to the poor. Income inequality thus falls.

However, when bonds adjust, the fraction of constrained agents increase since the unanticipated tariff makes them poorer. Their consumption (disproportionately) decreases and

their labor supply (disproportionately) increases. Since rates rise on impact, the increase in capital income for the rich increases income inequality. Bonds adjusting does not directly affect distributional outcomes, thus, the poor tend to suffer under this regime. This further explains why the planner does not cut rates or induce inflation to the same extent he did when transfers adjusted: he cannot use a rate cut to improve the welfare of the high marginal value, constrained agents in the same way he could when transfers adjusted. A change in the debt does not greatly affect agents on average, nor does it have particularly positive outcomes for the poor.

6 Efficient Steady State

Even with the use of the timeless penalty, the presence of market incompleteness, borrowing constraints, and potentially suboptimal fiscal policy implies the initial steady state in section 5 is inefficient. The monetary policy response to the tariff shock may then be responding to these pre-existing inefficiencies or the new ones induced by the higher tariff. The planner may be unhappy with the ex-ante amount of redistribution in the economy and uses the tariff shock as an opportunity to temporarily engage in more redistribution. Since directly solving for optimal steady state fiscal instruments is infeasible (see Auclert et al. (2024)), I follow an inverse optimal approach similar to that of McKay and Wolf (2022).

In particular, I find welfare weights for each household type $\varphi(b, z)$ such that the initial steady state from section 5 is first-best (equalizing marginal utility of consumption for each agent). In the steady state, for given parameters and fiscal policy, all allocations are invariant to the welfare weights used (since steady state inflation is 0 and all other economic variables are pinned down by implementability conditions). Thus, the welfare weights are given by

$$\varphi(b, z) = \frac{u_h(C^h_{ss}, C^f_{ss})}{u_h(c^h_{ss}, c^f_{ss})} \tag{27}$$

where C^h and C^f are aggregate home and foreign consumption. The planner then solves the timeless Ramsey problem (equation 13) with welfare weights given by equation 27. The planner first order conditions for this version of the model are described in detail in appendix E. Crucially, these welfare weights will be increasing in b. Thus, a planner will explicitly

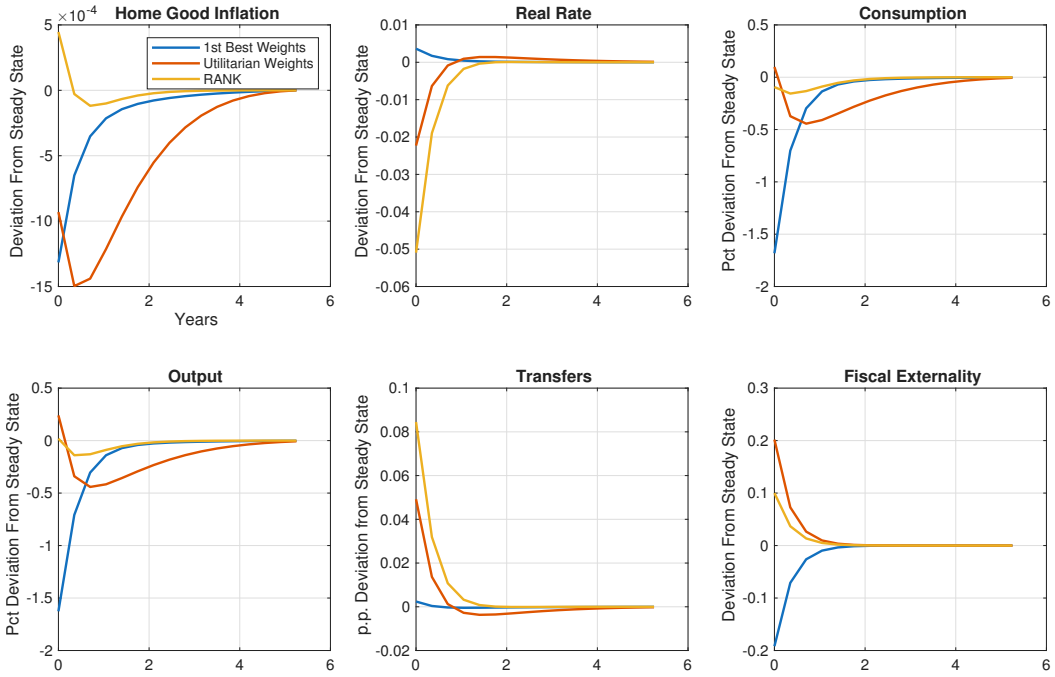
dislike redistribution from the rich to the poor, as the rich have higher welfare weights.

6.1 Numerical Results

With these welfare weights, the steady state allocation is efficient. The planner does not view the amount of redistribution in the economy's steady state as suboptimal, nor do they view fiscal variables as suboptimally chosen. Any policy response is a response to the new inefficiencies induced by the tariff. As before, I hit the economy with the MIT tariff shock given in figure 1 and calculate the fully nonlinear, perfect foresight transition paths.

First, I compare impulse responses for aggregate (figure 4) and distributional (figure 5) variables for the planner with utilitarian welfare weights, the welfare weights given by equation 27, and RANK. In all 3 cases, tariff revenue is spent on lump sum transfers T.

Figure 4: First Best vs Utilitarian Weights, Aggregate Variables, T Adjusts



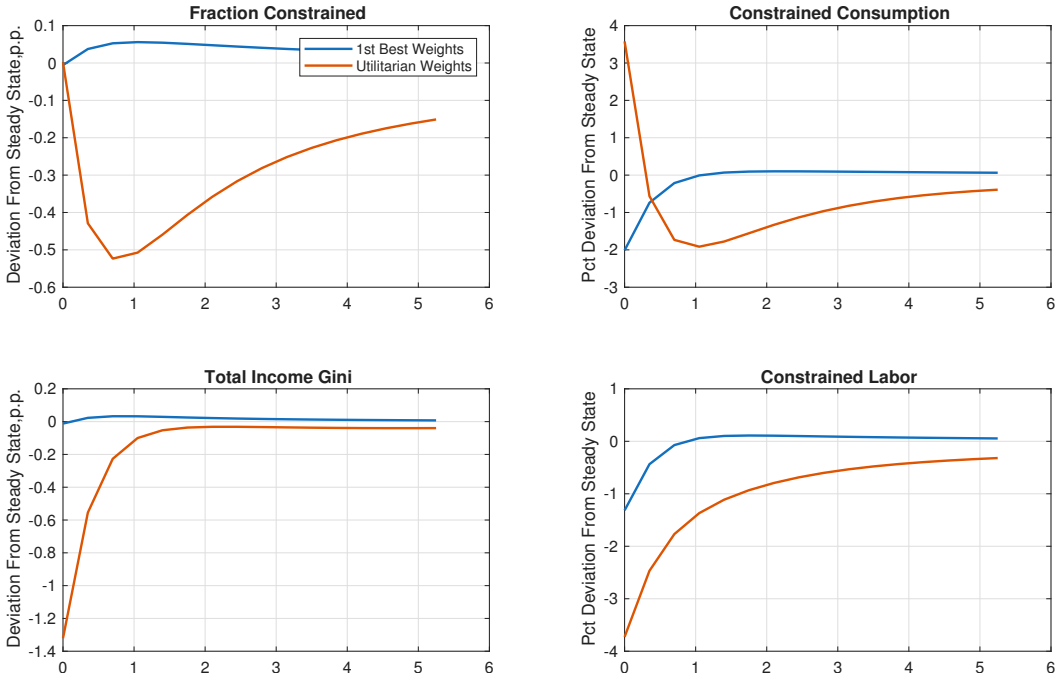
Notes: For the path of tariffs from figure 1, I plot impulse responses to aggregate variables for a planner whose has welfare weights given by equation 52 versus a planner who has a utilitarian social welfare function. In all economies, tariff revenues are spent on transfers.

In the economy with first best welfare weights, there is a very small rate hike. The optimal change in the nominal rate is close to 0; a look through policy is close to optimal.

Consumption and output both fall, while transfers remain nearly flat. This is a stark contrast from the policy with utilitarian weights, which featured inflation, sizeable rate cuts, increases in consumption, output, and transfers. The fiscal externality (equation 25) is now negative rather than positive: additional foreign consumption, tariff revenue, and transfers will redistribute from the rich to the poor. Since the planner puts higher weight on the welfare of the rich, this redistribution is now an explicit bad. The existence of this fiscal externality pushes the planner towards raising rates.

Both the RANK economy and the HANK economy with first best weights are initialized from a steady state that is efficient/first best. However, HANK prescribes a small rate hike, while RANK prescribes a rate cut. The difference comes from the planner's attitude towards redistribution in HANK. When the tariff shock is realized, there is a large increase in fiscal revenue. The HANK planner is not indifferent between where this revenue is redistributed and is not indifferent towards further redistribution. His larger welfare weight on the rich will imply he views that increasing lump sum transfers, which redistribute from high foreign consumption rich agents to low foreign consumption poor agents, is a social bad. So, he will not cut rates and obtain even more fiscal revenue through a monetary expansion. The social cost of additional foreign consumption is greater than its private cost. However, in RANK, the planner perceives the social cost of foreign consumption as lesser than its private cost. He wants to induce more foreign consumption and will thus cut rates.

Figure 5: First Best vs Utilitarian, Distributional Variables, T Adjusts

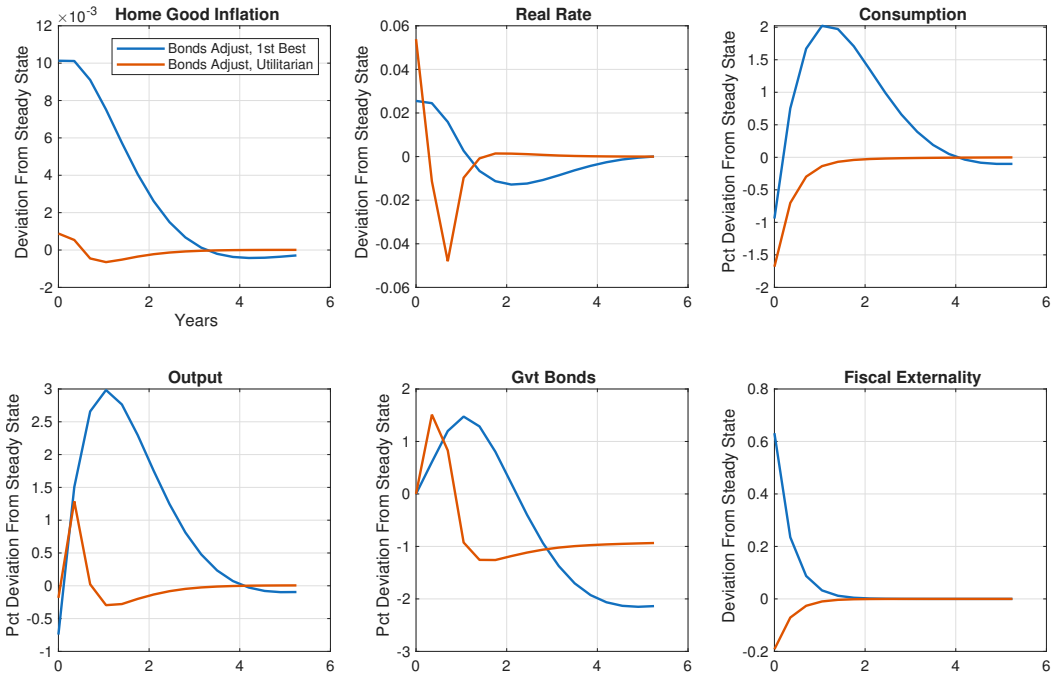


Notes: For the path of tariffs from figure 1, I plot impulse responses to for a planner whose has welfare weights given by equation 52 versus a planner who has a utilitarian social welfare function. In all economies tariff revenues are spent on transfers.

In the economy with first best welfare weights, there is nearly no change in the fraction of constrained agents, rather than the large decrease present with utilitarian weights. Their consumption falls (rises with utilitarian weights) and their labor supply contracts by less than in the utilitarian weight case.

To further emphasize how the "marginal" fiscal instrument affects the planner's optimal policy, I now allow government debt to adjust with "first-best" weights and compare to the utilitarian planner's solution. Debt adjusting, unlike transfers adjusting, is (roughly) distributionally neutral, it does not directly affect the budget of households, only indirectly through general equilibrium effects. Figures 6 and 7 plot the aggregate and distributional outcomes for these two economies.

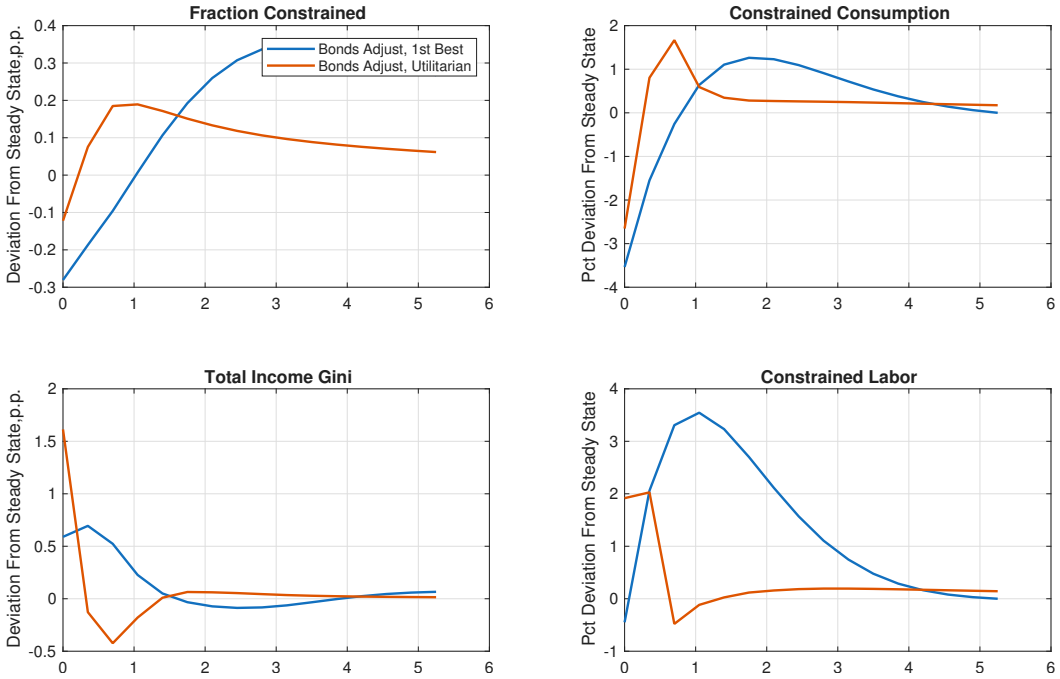
Figure 6: First Best vs Utilitarian Weights, Aggregate Variables, B Adjusts



Notes: For the path of tariffs from figure 1, I plot impulse responses to aggregate variables for a planner whose has welfare weights given by equation 52 versus a planner who has a utilitarian social welfare function. In all economies, government debt adjusts to satisfy the budget.

Now, the paths of interest rates are more similar across the first best and utilitarian weights when bonds adjust. On impact, the planner raises rates. Consumption falls, debt (eventually) falls. Again, the fiscal externality takes the opposite sign with first best welfare weights relative to the utilitarian case. Even though bonds adjusting are more distributionally neutral, they are not perfectly so. As such, it is unsurprising that a planner with a higher welfare weight on the rich would have the opposite attitude towards additional fiscal revenue than a planner with utilitarian weights.

Figure 7: First Best, Varying Fiscal Adjustment, Distributional Variables



Notes: For the path of tariffs from figure 1, I plot impulse responses to for a planner whose has welfare weights given by equation 52 versus a planner who has a utilitarian social welfare function. In all economies, government debt adjusts to satisfy the budget.

The distributional variables take (broadly) similar paths when bonds adjust, regardless of the weighting scheme used. The fraction of constrained agents (eventually) rises, constrained consumption and labor supply (eventually) rise, and income inequality rises.

The modest difference between the optimal policy with first best and utilitarian weights when bonds adjusts compared to the large difference between optimal policy with first best and utilitarian weights when transfers adjust stems from the distributional effects of the "marginal" fiscal variable. Bonds adjusting are roughly distributionally neutral. The planner's rate stance is (roughly) unchanged when rates are changed. However, transfers adjusting have a clear distributional impact. They redistribute from the rich to the poor. The planner views this as an explicit bad when they have first best welfare weights and an explicit good when they have utilitarian welfare weights. This leads to deflation and contraction in the first best case and inflation and expansion in the utilitarian case.

7 Model Extensions

I now consider some additional extensions to the model in the vein of the extensions studied by Bianchi and Coulibaly (2025). Each extension has a section in the appendix that provides more details; I write out the new planner's problem and discuss particularly important theoretical differences in the main body of the text before describing numerical results.

7.1 Progressive Fiscal Policy

I now consider a tax and transfer specification that is not flat. I use tax and transfer functions of the following form, from Ferriere et al. (2023):

$$\mathcal{T}(z_t) = \xi_t \frac{2e^{-\gamma_1(z_t-\gamma_0)}}{1+e^{-\gamma_1(z_t-\gamma_0)}}$$
$$\tau(z_t) = \exp\left(\log(\lambda_{\text{Tax}})\left(\frac{z_t}{\bar{z}}\right)^{-\theta_{\text{Tax}}}\right)$$

The transfer function is parameterized by $\gamma_1 \in \mathbb{R}^+, \gamma_0 \in \mathbb{R}$ This sigmoid is a continuous approximation of a step function with a cutoff at γ_0. Households with productivity above γ_0 receive (approximately) 0 transfers, while households with productivity below γ_0 receive (approximately) $2\xi_t$ transfers. This captures the fact that many government transfer programs have eligibility thresholds at some level of income[22]. γ_1 represents the steepness of the transfer function, if $\gamma_1 = 0$, transfers are flat, while the limiting case $\gamma_1 \to \infty$ recovers the aforementioned step function. ξ is an equilibrium object, adjusting to satisfy the government budget constraint (more generally the relationship between individual and aggregate transfers satisfies $\int \mathcal{T}(b)f(b,z)\,\mathrm{d}b\,\mathrm{d}z = T$ where T are aggregate transfers set according to an ad hoc rule or to satisfy the government budget).

I detail the different features of the model in appendix G. I highlight one particularly

[22] Indexing the same sigmoid off of wealth provides too strong of incentives to remain borrowing constrained for empirically reasonable values of γ_0 and γ_1, while indexing to total income eliminates the closed form solution to the labor supply decision, which significantly slows the solving of the model. Productivity is a reasonable proxy for total labor income, while remaining computationally tractable

relevant equation, the fiscal externality:

$$\eta_{6,t} = \underbrace{\text{COV}_{f_t}\left[\left(\frac{\partial \eta_{1,t}}{\partial b} + \frac{\partial V_t}{\partial b}\frac{\eta_{2,t}}{f_t}\right), \frac{\partial s^h}{\partial \mathcal{T}(b)}\frac{\partial \mathcal{T}(b)}{\partial T}\frac{\partial T}{\partial C^f}\right]}_{\text{Distributional Effect}} + \underbrace{\mathbb{E}_{f_t}\left[\left(\frac{\partial \eta_{1,t}}{\partial b} + \frac{\partial V_t}{\partial b}\frac{\eta_{2,t}}{f_t}\right)\right]\mathbb{E}_{f_t}\left[\frac{\partial s^h}{\partial \mathcal{T}(b)}\frac{\partial \mathcal{T}(b)}{\partial T}\frac{\partial T}{\partial C^f}\right]}_{\text{Average Effect}} \tag{28}$$

which shows the strength of the fiscal externality $\eta_{6,t}$ depends on how individual savings respond to individual transfers (one-to-one by construction), how individual transfers respond to aggregate transfers (a function of progressivity), and how aggregate transfers respond to aggregate foreign consumption (if transfers adjust in the government budget, aggregate transfers change by τp). As before, this is comprised of an average and distributional effect. When transfers are progressive, the distributional effect will be particularly strong. There will be a positive covariance between planner marginal values and the effects of transfers on household budgets. Holding fixed marginal values, this strengthens the fiscal externality.

Calibration I calibrate the parameters γ_1 and γ_0 to match the empirical transfer rate distribution, defining the transfer rate as the share of total income from transfers. I use CBO data on non-Medicare and non-SSI transfers and assume UI payments are transfers to generate the empirical transfer distribution by income percentile[23]. I follow directly the methods of Ferriere et al. (2023): using the methods and code of Habib (2018) to impute transfer levels on 2013 CPS-ASEC data, merging into IPUMS CPS-ASEC data (to obtain income), and then calculating transfer rates for each income percentile. As they do, I only incorporate transfers designed to provide income insurance to the non-elderly: SNAP (food stamps), Temporary Assistance for Needy Families, housing assistance, UI benefits, and other small means-tested programs. Since the model does not consider disability, old age, or other adverse health shocks, I exclude Medicaid and Supplemental Security Income.

Given that my function is calibrated to productivity, which takes only 5 discrete values in the numerical exercises, table 3 reports the model transfer rate for each productivity level,

[23]There is a sufficiently high correlation between income and productivity to directly compare the distribution of transfer rates by income in the data with the distribution of transfer rates by productivity in the model

its percentile in the productivity distribution, and the corresponding transfer rate for the same income percentile in the data[24].

Table 3: Transfer Rates

Percentile	6.25	31.25	68.75	93.75	100
Model	.658	.423	.115	.007	.0001
Data	.666	.056	.007	.001	.0008

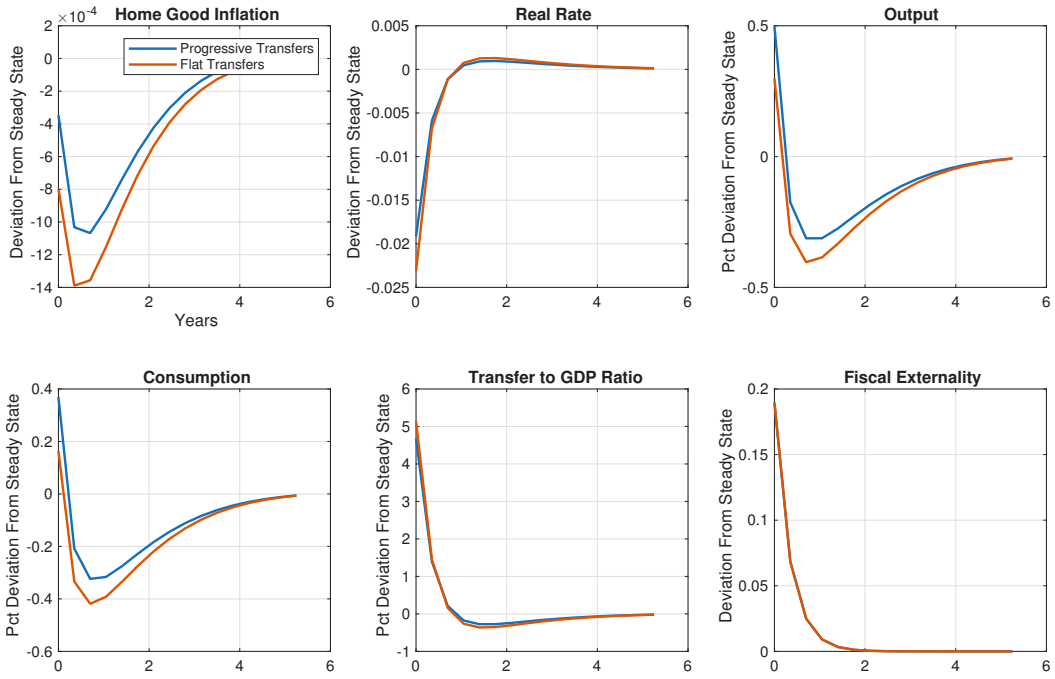
Notes: This table reports, by decile, household transfer rates in the model and in CBO data. I define a transfer rate as the fraction of total income attributed to transfers. I successfully match qualitative features of the transfer distribution and the transfer rate for the poor. However, indexing to productivity alone is too course a proxy to precisely match the middle of the transfer distribution. A function with a sharper decline will have a much higher transfer rate for the low productivity households. I prioritize matching this transfer rate for the poor rather than the steep cutoff for middle productivity households.

To calibrate the tax function, I take λ_{Tax} from Ferriere et al. (2023) and adjust θ_{Tax} so the average tax rate is the same in the flat tax and progressive tax economy.

Numerical Results Figures 8 and 9 plot results for the tariff shock in figure 1. I compare two economies, one with progressive taxes and transfers, one with progressive taxes and flat transfer. This then isolates the effect of progressive transfers on optimal policy.

[24]To do the calibration, I solve the model with flat transfers ($\gamma_1 = 0$) and obtain total income and wealth levels by decile. I then obtain transfer levels consistent with these income levels and the data. Finally, I obtain γ_1 and γ_0 by doing NLS on the corresponding levels of wealth and transfer levels (letting ξ be aggregate transfers in the model with flat transfers). I then solve the model with these new γ_1 and γ_0 coefficients, repeating the process until model transfer rates and data transfer rates are approximately equal.

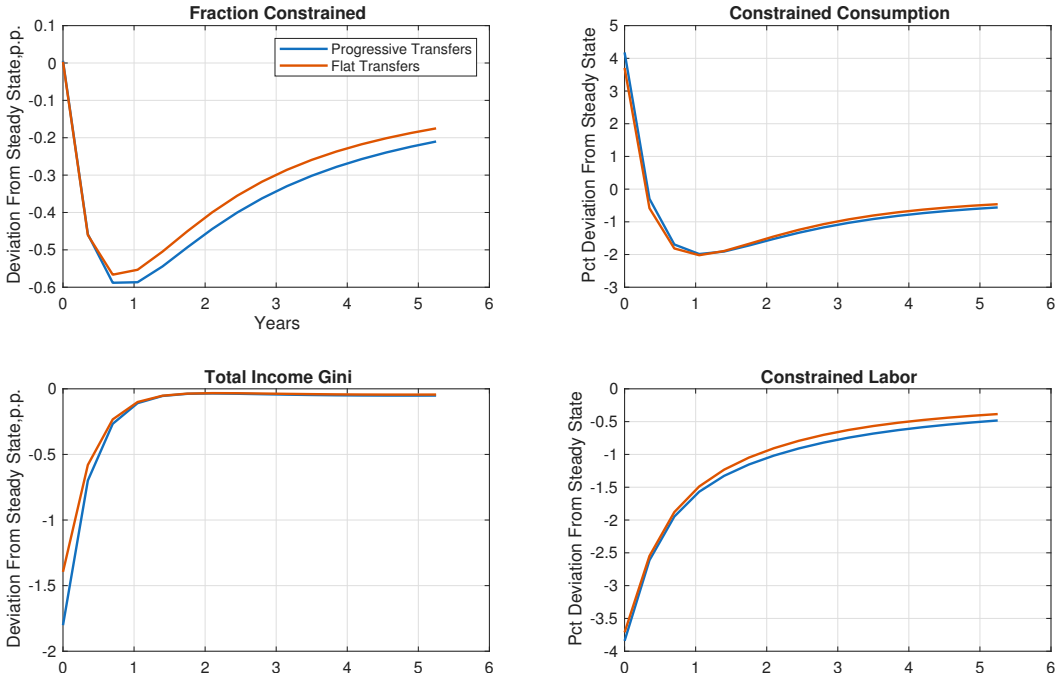
Figure 8: Tariff Shock with Progressive Transfers, Aggregate Variables



Notes: For the path of tariffs from figure 1, I plot impulse responses to aggregate variables. Home good inflation and interest rates are plotted as deviations from the steady state, while other variables are percent deviations from the steady state.

The rate cut with progressive transfers is somewhat smaller. The aggregate effect of monetary policy on output and consumption is generally similar across the two models, though the increase in output and consumption on impact is slightly dampened with flat transfers. This stems from the fact that a larger share of tariff revenues are redistributed to high MPC agents with progressive transfers. The total size of the fiscal externality and the total increase in the transfer to GDP ratio is nearly identical in the two models.

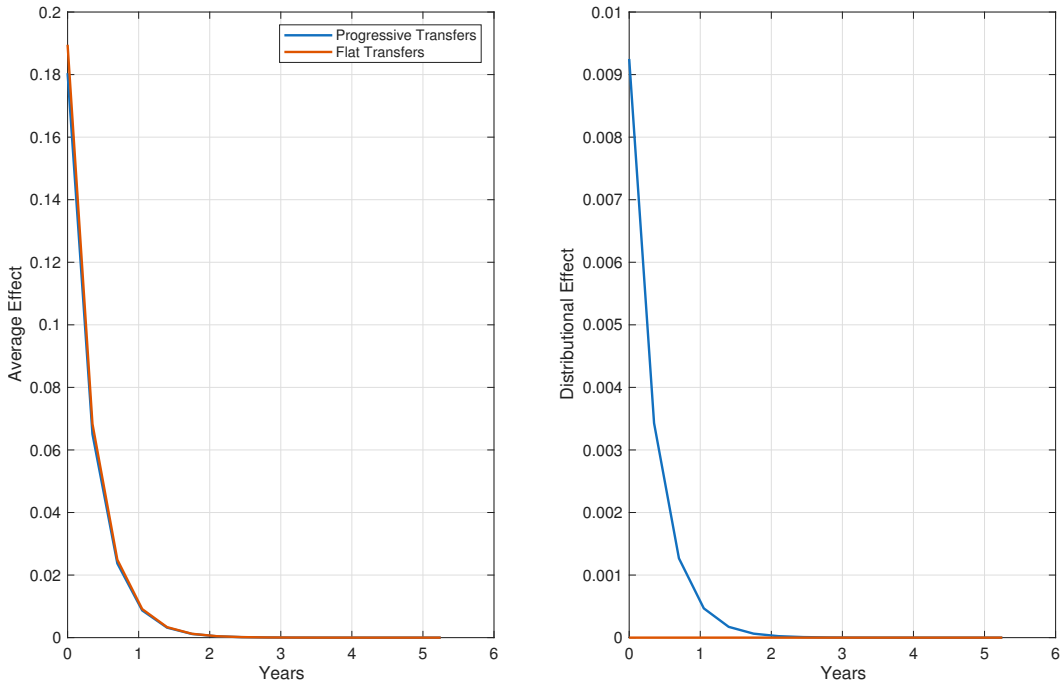
Figure 9: Tariff Shock with Progressive Transfers, Distributional Variables



Notes: For the path of tariffs from figure 1, I plot impulse responses for distributional variables.

The effect of policy on distributional variables is also similar between the models with flat and progressive transfers. The fact that a larger expansion is not induced by the planner may seem counterintuitive, as additional expansion increases resources more for low productivity agents, who, all else equal, have higher marginal values than their high productivity counterparts. Thus, the planner would want to cut rates more. However, recall that the fiscal redistribution-induced incentive to cut rates, $\eta_{6,t}$, is comprised of a distributional effect and an average effect. The distributional effect is positive, as transfers redistribute disproportionately to high marginal value agents. This pushes the planner to inflate more than in the flat transfer economy, where the distributional effect is 0 by construction. However, the average effect is smaller in the economy with progressive transfers, which drives the weakened rate cut. The poorer, low productivity agents have lower marginal values than in a flat transfer economy, since, as a baseline, they are "better off", receiving much high transfers than in the flat transfer economy. Thus, the marginal value of giving them additional wealth or consumption is smaller, and the incentive to expand, cut rates, and induce inflation is dampened.

Figure 10: Decomposition of Fiscal Externality



Notes: Using equation 28, I decompose the strength of the fiscal externality in response to the tariff shock into the distributional and average components. The fiscal externality reflects the extent to which the planner desires additional fiscal revenue, obtained through higher foreign consumption and tariff revenue. This in turn determines the strength of the rate cut.

Figure 10 highlights this result numerically. I use equation 28 to decompose the strength of the fiscal externality in the progressive transfer economy and baseline economy into the average effect and the distributional effect. Even though the distributional effect is positive in the progressive transfer economy and 0 in the baseline economy, the overall size of the fiscal externality is comparable, since the average effect is smaller. The extra progressivity in the baseline economy reduces marginal values for poor, constrained agents (they have much higher transfers, which act as insurance against being in low productivity states). This then weakens the average effect portion of the fiscal externality. The positive distributional effect compensates for this weaker average effect, but it is not strong enough to increase the total effect size above the flat transfer economy[25]. Thus, the rate cut is not stronger in the

[25]It may appear strange that the distributional effect is so small when transfers are progressive. However, indexing transfers to productivity is only a rough proxy. There are still many agents who are constrained and have higher productivity. They have high marginal values, often even higher than high wealth, low

progressive transfer economy.

7.2 Endogenous Relative Prices

In the baseline model, I assume the relative price of home and foreign goods, p, is exogenous. This implies the central bank will have no incentive or ability to use its policy instrument to affect terms of trade. Additionally, the optimal (first best) tariff in this economy is 0. Now, I extend the model to have an endogenous relative price. Following Bianchi and Coulibaly (2025), I assume the relative price takes the following functional form

$$p_t = A(N_t - C_t^h)^{\frac{1}{\vartheta}} \tag{29}$$

where A is an exogenous foreign demand shifter, N_t is aggregate output at home, C_t^h is aggregate home consumption of the home-produced good, and ϑ is the export demand elasticity.

This functional form is implied by assuming the rest of the world is comprised of a continuum of countries each with a representative household that produces one foreign variety. The foreign block is written and solved explicitly in appendix F.

Even though home households are heterogeneous, a usual static optimal tariff formula applies since foreign households are homogeneous, and the foreign export supply function is only a function of the contemporaneous price[26]. The optimal tariff then satisfies

$$\tau_{\text{opt}} = \frac{1}{\vartheta - 1} \tag{30}$$

I rewrite and solve the planner's problem in section F of the appendix.

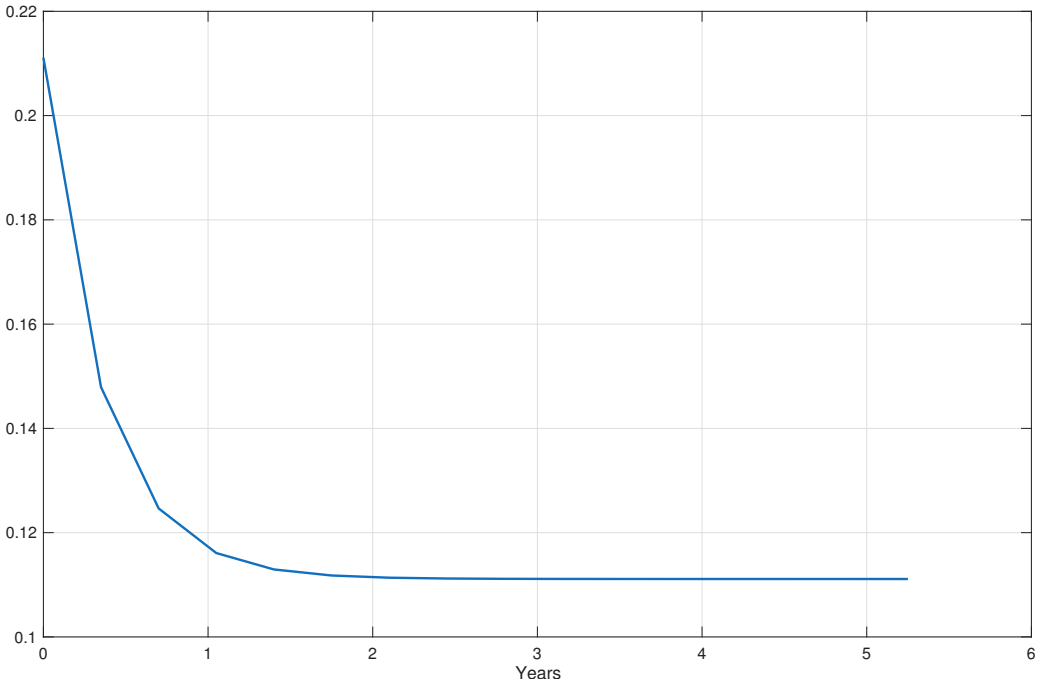
Numerical Results Similar to my previous numerical results, I consider the planner's optimal response to a one-time, temporary, MIT shock to tariffs. However, unlike before, I assume the tariff at time 0 is the optimal tariff from equation 30. There is a 10 percentage

productivity agents, who receive a large transfer. This weakens the distributional effect. Ideally, transfers would be indexed off of total income, which would avoid this problem, but doing so greatly complicates the computation, as the labor supply margin would no longer have a closed form solution.

[26] see section 2 of Dávila et al. (2025) for details

point increase to the tariff rate, which again reverts exponentially back to 0 after approximately 2 years. Figure 11 plots the tariff shock I use for the numerical experiments in this section.

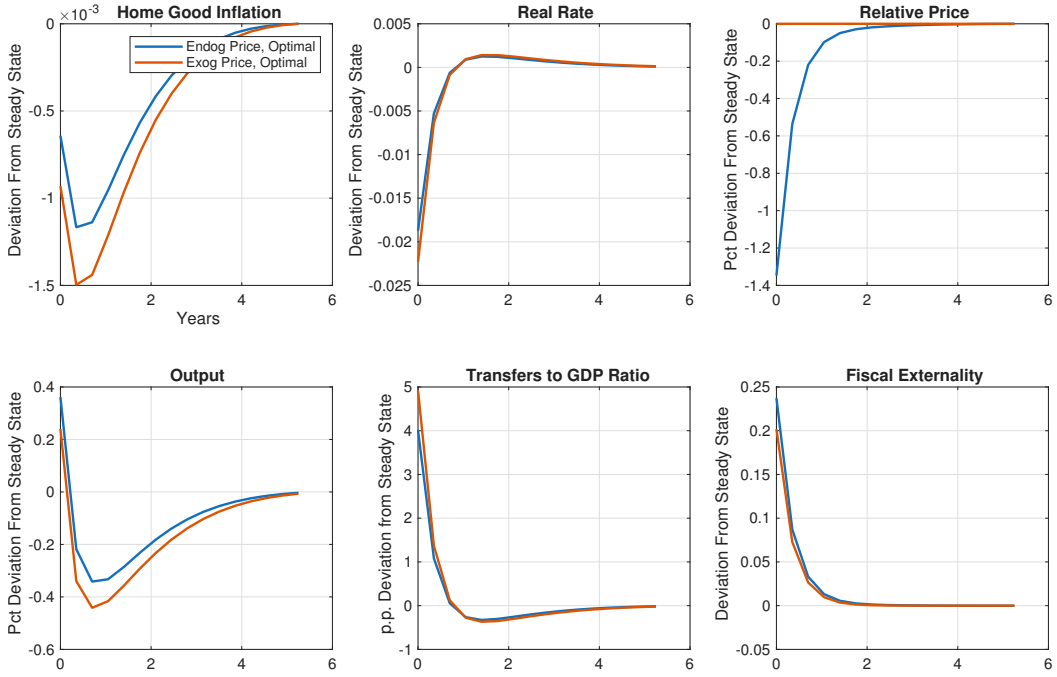
Figure 11: Path of Exogenous Tariff Shock



Notes: The path of tariffs satisfies $\tau_t = .1 \cdot e^{-\xi t}$ with $\xi = .35$.

Figures 12 and 13 compare aggregate and distributional outcomes for the Ramsey policy with endogenous and exogenous relative prices.

Figure 12: Optimal vs Look Through Policy, Aggregate Variables

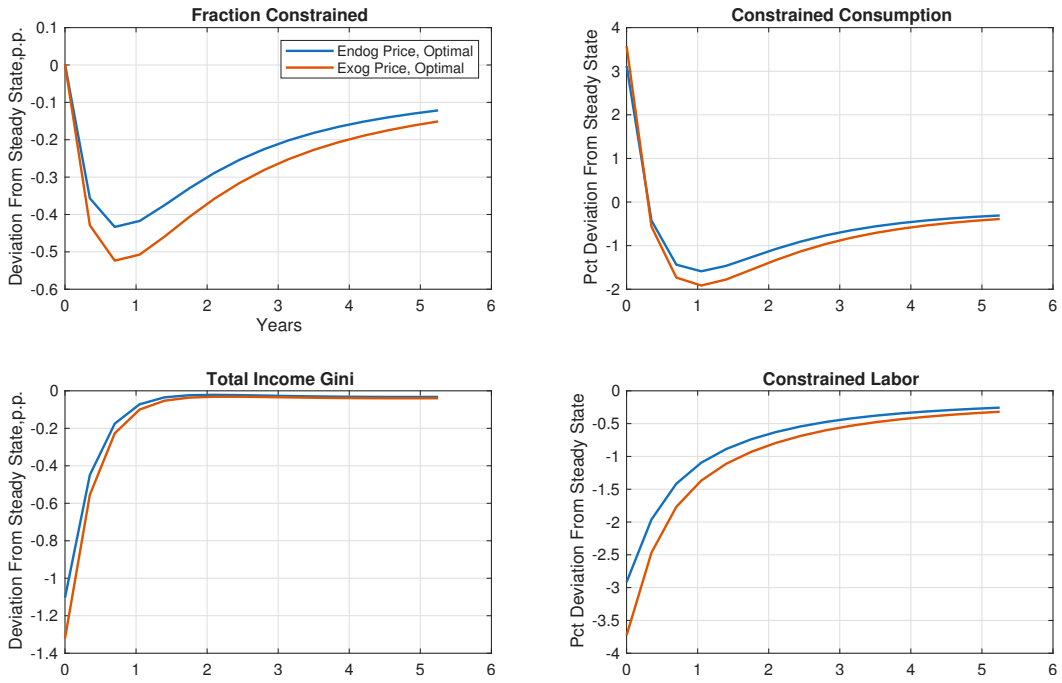


Notes: For the path of tariffs from figure 11, I plot impulse responses to aggregate variables. Home good inflation and interest rates are plotted as deviations from the steady state, while other variables are percent deviations from the steady state.

When the relative price is endogenous, the tariff shock will reduce the relative price p of the foreign good. Expansionary policy will counteract this reduction in the relative price, since it will increase N, but C^h will increase less than one for one. Since an increase in the relative price is welfare reducing for all agents in my calibration (foreign consumption is not enough of a luxury good to make $\tau C^f - (1+\tau)c^f$ positive for any agent at any time), the planner's incentive to engage in expansionary policy is dampened.

Relative to optimal policy when prices are exogenous, there is a smaller interest rate cut, inflation rises by less, consumption, output, and transfers increase by less. However, the signs of the responses to each variable are unchanged and are quantitatively not too different. This reflects the fact that the fiscal externality is still present and very powerful quantitatively. The marginal welfare costs of higher prices are weak compared to the gains from fiscal redistribution. The fiscal channel dominates the ToT manipulation channel.

Figure 13: Optimal vs Look Through Policy, Distributional Variables



Notes: For the path of tariffs from figure 11, I plot impulse responses to aggregate variables. Home good inflation and interest rates are plotted as deviations from the steady state, while other variables are percent deviations from the steady state.

Since there is less expansion and less redistribution through tariff revenue, more agents remain constrained, the consumption of the constrained rises by less, and their labor supply falls by less. Relative to the look-through policy, most of the insights from the previous sections remain; there is a much stronger rate cut, labor, consumption, and transfers all rise instead of remaining flat or falling, and there are larger improvements in distributional outcomes. Notably, the relative price falls by more in the look-through policy, since the optimal policy entails an expansion, which raises the relative price.

8 Conclusion

The presence of household heterogeneity is a key determinant of the optimal monetary response to tariff shocks. Unlike a conventional supply or demand a shock, tariff shocks are differentiated by their strong effects on fiscal revenue. They induce a fiscal externality, wherein households do not internalize that their foreign consumption increases fiscal revenue.

The central finding of my analysis is that in HANK, unlike in RANK, this fiscal externality need not always push the planner towards a monetary expansion. How fiscal revenue is spent, the initial amount of redistribution present in the economy, and the planner's attitude towards further redistribution play key roles in determining the sign and magnitude of the fiscal externality and thus the planner's preference for cutting or raising rates.

First, I show that the fiscal externality in HANK is comprised of two forces: an aggregate effect or the average effect of additional fiscal revenue on households and a distributional effect that reflects the covariance between the planner's marginal value of each household and the income effect of additional fiscal revenue on each household. Thus, when fiscal revenue targets high marginal value, poor households the fiscal externality will be stronger. When this is the case, the planner will cut rates more, as doing so will increase aggregate demand, foreign consumption, and fiscal revenue.

Next, I highlight quantitatively how the adjusting fiscal instrument affects optimal policy. Since Ricardian equivalence fails, how tariff revenue is spent will have strong effects on equilibrium outcomes in a way that it cannot in RANK. I show that when fiscal revenues are spent in more progressive ways, such as issuing transfers, the planner tends to cut rates more. When fiscal revenues are spent in distributionally neutral ways, such as reducing the debt, the planner tends to cut rates less or even raise rates. Since fiscal revenue does not target high marginal value agents, the planner no longer desires additional fiscal revenue.

I then show how the initial amount of fiscal redistribution and the planner's attitude towards redistribution shapes the optimal policy response. The initial steady state of my economy is inefficient, incomplete markets or suboptimally chosen fiscal tools imply there is too little redistribution from the perspective of a utilitarian planner. Thus, part of the monetary response may be correcting these initial inefficiencies rather than responding to the new ones induced by the tariff.

To correct this, I obtain welfare weights that make the initial steady state efficient (first best). I then solve for the monetary response to a tariff shock in this economy. When debt, which is distributionally neutral, adjusts, the planner's monetary response is qualitatively unchanged relative to a utilitarian planner. Despite having more welfare weight on the rich, additional fiscal revenues do not carry any explicitly redistributive content. However, when

transfers, which redistribute from the rich to the poor, adjust, the planner raises rates rather than cutting them. Since the planner places more welfare weight on the rich, he explicitly dislikes the redistribution that additional fiscal revenue induces. He thus raises rates to reduce the amount of fiscal redistribution.

The importance of initial conditions is further highlighted by considering an economy with a utilitarian planner and progressive taxes and transfers. Even though there is a positive distributional effect of additional fiscal revenue, the size of the fiscal externality is nearly unchanged. The ensuing rate cut is of similar magnitudes when compared to a flat transfer environment. This occurs because there is more redistribution in the initial state of the economy. The marginal values of the poor are smaller and the planner is less incentivized to induce a strong expansion to further redistribute towards the poor.

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A Mathematical Preliminaries

Following the notation of Nuño and Thomas (2022), define the following pieces of notation, letting $\Phi \equiv \{z_1, ..., z_J\} \times \mathbb{R}$, $\hat{\Phi} = [0, \infty] \times \Phi$, and $L^2(\Phi), L^2([0,\infty]), L^2(\hat{\Phi})$ be Lebesgue-integrable function spaces:

$$\langle f, g \rangle_\Phi = \int \int_\Phi f_i g_i \, \mathrm{d}b \, \mathrm{d}z = \int_\Phi f^T g \quad \forall f, g \in L^2(\Phi) \tag{31}$$

$$(f, g)_{\mathbb{R}^+} = \int_0^\infty e^{-\tilde{\rho} t} f g \, \mathrm{d}t \quad \forall f, g \in L^2[0,\infty] \tag{32}$$

$$(f, g)_\Phi = \int_0^\infty e^{-\tilde{\rho} t} \langle f, g \rangle_\Phi = \langle e^{-\tilde{\rho} t} f, g \rangle_{\hat{\Phi}} \quad \forall f, g \in L^2(\hat{\Phi}) \tag{33}$$

Then $L^2(\Phi), L^2([0,\infty])$, and $L^2(\hat{\Phi})$ are Hilbert spaces with inner products 31, 32, and 33 respectively.

To solve the central bank's problem (eq 12), I need a generalization of the derivative for function spaces. The Gâteaux derivative is this generalization. See Nuño and Moll (2018) and Nuño and Thomas (2022) for a more detailed discussion of the mathematics; here I state the necessary results for my application without proof.

Definition 2 (Gâteaux Derivative) *Let $W[f]$ be a functional in an arbitrary function space and h be any function in that function space. Then, the Gâteaux derivative of W at f with increment h is*

$$\delta W[f;h] = \lim_{\alpha \to 0} \frac{W[f+\alpha h] - W[f]}{\alpha} = \frac{\mathrm{d}}{\mathrm{d}\alpha} W[f+\alpha h]|_{\alpha=0}$$

The Gâteaux derivative can be used to generalize usual constrained optimization results to function spaces. First, I define a Lagrangian Functional

Definition 3 (Lagrangian Functional) *Suppose $W[f]$ is a functional in function space $L^2(\Phi)$ and $G[f]$ is a mapping $L^2(\Phi) \to \mathbb{R}^n$. Then the Lagrangian functional for the optimization problem*

$$\max W[f]$$
$$s.t. \quad G[f] = 0$$

is defined as $\mathcal{L}[f] = W[f] + \langle \eta, G[f] \rangle_\Phi$ where $\eta \in L^2(\Phi)$ is the Lagrange multiplier

Theorem 5 (Constrained Optimality) *Suppose $W[f]$ is a functional in function space $L^2(\Phi)$ and $G[f]$ is a mapping $L^2(\Phi) \to \mathbb{R}^n$. Then a necessary condition for W to have a maximum at f subject to the constraint $G[f] = 0$ is that there exists a Lagrange multiplier $\eta \in L^2(\Phi)$ such that the Gâteaux derivative of the Lagrangian Functional with multiplier η is 0:*

$$\delta \mathcal{L}[f;h] = 0, \quad \forall h \in L^2(\Phi)$$

B Solution to Optimal Policy Problem

B.1 Lagrangian

Using the notation from equations 32-33, the Lagrangian for the central bank's problem (eq 12) is defined in the space $L^2(\hat{\Phi})$ as

$$\mathcal{L} = \left(u(c_t^h, c_t^f) - g(n_t), f_t\right)_\Phi + \left(\eta_{1,t}, \mathcal{A}^* f_t - \frac{\partial f}{\partial t}\right)_\Phi + \left(\eta_{2,t}, u(c_t^f, c_t^h) - g(n_t) + \mathcal{A} V_t + \frac{\partial V_t}{\partial t} - \tilde{\rho} V_t\right)_\Phi$$
$$\left(\eta_{3,t}, \pi_t(r_t^h - \frac{\dot{N}_t}{N_t}) - \frac{\epsilon}{\theta}(m_t - 1) - \dot{\pi}\right)_{\mathbb{R}^+} + \left(\eta_{4,t}, f_t(b_t^h - B_t)\right)_\Phi + \left(\eta_{5,t}, f_t(n_t - N_t)\right)_\Phi + \left(\eta_{6,t}, f_t(c_t^f - C_t^f)\right)_\Phi$$

$$+\left(\eta_{7,t}, g'(n_t) - \frac{(1-\tau^n)w_t z_{j,t}}{p_t^h} u_h(c_t^h, c_t^f)\right)_\Phi + \left(\eta_{8,t}, u_h(c_t^h, c_t^f) - \frac{\partial V}{\partial b_t^h} p\right)_\Phi + \left(\eta_{9,t}, u_f(c_t^h, c_t^f) - \frac{\partial V}{\partial b_t^h} p(1+\tau)\right) \tag{34}$$

Re-defining the generator and its adjoint:

$$\mathcal{A}^* f = \begin{bmatrix} -\frac{\partial(s_1^f f_1)}{\partial b^f} - \frac{\partial(s_1^h f_1)}{\partial b^h} - f_1 \sum_{j' \neq 1} \lambda_{1j'} + \sum_{j' \neq 1} \lambda_{j'1} f_1 \\ \vdots \\ -\frac{\partial(s_J^f f_J)}{\partial b^f} - \frac{\partial(s_J^h f_J)}{\partial b^h} - f_J \sum_{j' \neq J} \lambda_{Jj'} + \sum_{j' \neq J} \lambda_{j'J} f_J \end{bmatrix}$$

and

$$\mathcal{A}V = \begin{bmatrix} \frac{\partial V_1}{\partial b^h} s_1^h + \frac{\partial V_1}{\partial b^f} s_1^f + \sum_{j' \neq 1} \lambda_{1,j'} [V_{j'} - V_1] \\ \vdots \\ \frac{\partial V_J}{\partial b^h} s_J^h + \frac{\partial V_J}{\partial b^f} s_J^f + \sum_{j' \neq J} \lambda_{J,j'} [V_{j'} - V_J] \end{bmatrix}.$$

where after substituting in for lump sum dividends and the government budget, the savings function is $s^h = rb^h + (1-\tau^n)zwn + \dot{B} - rB - sN + \tau p C_t^f + \tau^n w N_t + (1+s)N_t - m_t N_t - \frac{\theta}{2}\pi_t^2 N_t - c^h - p(1+\tau)c^f$. I define $C_t^f = \langle c_t^f, f_t \rangle_\Phi$ and add C_t^f as an additional choice of the planner. This avoids having f appear directly in the generator $\mathcal{A}$, simplifying the derivations going forward, and facilitating a more intuitive interpretation of the equilibrium conditions.

The generator and its adjoint satisfy $\langle f, \mathcal{A}v \rangle_\Phi = \langle \mathcal{A}^* f, v \rangle_\Phi$ for any $f, v \in L^2(\Phi)$. Using this result and integration by parts, it will be useful at times to rewrite the $\eta_{1,t}$ and $\eta_{2,t}$ terms of the Lagrangian as

$$\begin{aligned} &\int_0^\infty e^{-\tilde{\rho}t} \langle \eta_{1,t}, \mathcal{A}^* f_t - \frac{\partial f}{\partial t} \rangle_\Phi \, \mathrm{d}t \\ &= \int_0^\infty e^{-\tilde{\rho}t} \langle \mathcal{A}\eta_{1,t}, f_t \rangle_\Phi + e^{-\tilde{\rho}t} \langle \eta_{1,t}, -\frac{\partial f_t}{\partial t} \rangle_\Phi \\ &= \int_0^\infty e^{-\tilde{\rho}t} \langle \mathcal{A}\eta_{1,t}, f_t \rangle_\Phi + \langle \eta_{1,0}, f_0 \rangle_\Phi - \lim_{T \to \infty} \langle e^{-\tilde{\rho}T} \eta_{1,T}, f_T \rangle + e^{-\tilde{\rho}t} \langle \frac{\partial \eta_{1,t}}{\partial t} - \tilde{\rho}\eta_{1,t}, f_t \rangle_\Phi \end{aligned}$$

and

$$\begin{aligned} &\int_0^\infty e^{-\tilde{\rho}t} \langle \eta_{2,t}, u(c_t^f, c_t^h) - g(n_t) + \mathcal{A}V_t + \frac{\partial V_t}{\partial t} - \tilde{\rho}V_t \rangle_\Phi \, \mathrm{d}t \\ &= \int_0^\infty e^{-\tilde{\rho}t} \langle \eta_{2,t}, u(c_t^f, c_t^h) - g(n_t) \rangle_\Phi \, \mathrm{d}t + \int_0^\infty e^{-\tilde{\rho}t} \langle \mathcal{A}V_t + \frac{\partial V_t}{\partial t} - \tilde{\rho}V_t \rangle_\Phi \, \mathrm{d}t \\ &= \int_0^\infty e^{-\tilde{\rho}t} \langle \eta_{2,t}, u(c_t^f, c_t^h) - g(n_t) \rangle_\Phi \, \mathrm{d}t + \int_0^\infty e^{-\tilde{\rho}t} \langle \mathcal{A}^* \eta_{2,t}, V_t \rangle_\Phi \, \mathrm{d}t + \lim_{T \to \infty} \langle e^{-\tilde{\rho}T} \eta_{2,T}, V_T \rangle_\Phi - \langle \eta_{2,0}, v_0 \rangle_\Phi + \\ &\int_0^\infty e^{-\tilde{\rho}t} \langle V_t, -\frac{\partial \eta_{2,t}}{\partial t} \rangle_\Phi \, \mathrm{d}t + \sum_{j=1}^J v_{j,t}(b) s_{j,t}(b) \theta_{j,t}(b) |_{-\infty}^{\infty} \end{aligned}$$

Note that the rewritten $\eta_{1,t}$ term can be combined with the first term of the original Lagrangian to construct a "social HJB Equation"

$$\int_0^\infty e^{-\tilde{\rho}t}\langle u(c_t^h, c_t^f) - g(n_t) + \mathcal{A}\eta_{1,t} + \frac{\partial \eta_{1,t}}{\partial t} - \tilde{\rho}\eta_{1,t}, f_t\rangle_\Phi \, \mathrm{d}t + \langle \eta_{1,0}, f_0\rangle_\Phi - \lim_{T\to\infty}\langle e^{-\tilde{\rho}T}\eta_{1,T}, f_T\rangle + ...$$

where $\eta_{1,t}$ is the social value function with the latter two terms later being used to define boundary conditions.

Similarly, the $\eta_{2,t}$ term can be rewritten as a "social KFE Equation" plus a social welfare term

$$\int_0^\infty e^{-\tilde{\rho}t}\langle \eta_{2,t}, u(c_t^f, c_t^h) - g(n_t)\rangle_\Phi \, \mathrm{d}t + \int_0^\infty e^{-\tilde{\rho}t}\langle \mathcal{A}^*\eta_{2,t} - \frac{\partial \eta_{2,t}}{\partial t}, V_t\rangle_\Phi \, \mathrm{d}t + \lim_{T\to\infty}\langle e^{-\tilde{\rho}T}\eta_{2,T}, V_T\rangle_\Phi - \langle \eta_{2,0}, v_0\rangle_\Phi +$$
$$\sum_{j=1}^J v_{j,t}(b)s_{j,t}(b)\theta_{j,t}(b)|_{-\infty}^{\infty}$$

where $\eta_{2,t}$ is the planner's density (the latter three terms will be used to create usual boundary conditions).

Additionally, it will be useful to rewrite the FOC on home and foreign consumption according to

$$\begin{aligned} &-\int_0^\infty e^{-\tilde{\rho}t}\langle \eta_{8,t}, \frac{\partial V}{\partial b^h}\rangle_\Phi \\ &= \int_0^\infty e^{-\tilde{\rho}t}\langle \frac{\partial \eta_{8,t}}{\partial b^h}, V_t\rangle_\Phi \, \mathrm{d}t \end{aligned}$$

by integrating by parts.

B.2 Proof of Proposition 2

Characterizing the solution to the planner's problem and completing the proof of proposition 2 requires taking Gâteaux derivatives of 34 with respect to $\pi_t, r_t^h, V_t(.), c_t^f(.), c_t^h(.), n_t(.), f_t(.), w_t, B_t, N_t, C_t^f$.

KFE and HJB The derivative wrt f in the direction h is

$$0 = \frac{\mathrm{d}}{\mathrm{d}\alpha}\langle u(c_t^h, c_t^f) - g(n_t), f_t + \alpha h\rangle_\Phi \, \mathrm{d}t|_{\alpha=0} + \frac{\mathrm{d}}{\mathrm{d}\alpha}\langle \frac{\partial \eta_{1,t}}{\partial t} - \tilde{\rho}\eta_{1,t} + \mathcal{A}\eta_{1,t}, f_t + \alpha h\rangle_\Phi|_{\alpha=0} +$$
$$\frac{\mathrm{d}}{\mathrm{d}\alpha}\langle \eta_{1,0}, f_0 + \alpha h_0\rangle|_{\alpha=0} - \frac{\mathrm{d}}{\mathrm{d}\alpha}\lim_{T\to\infty}\langle e^{-\tilde{\rho}T}\eta_{1,T}, f_T + \alpha h_T\rangle_\Phi + \frac{\mathrm{d}}{\mathrm{d}\alpha}\langle e^{-\tilde{\rho}t}\eta_{4,t}, (b_t^h - B_t)(f + \alpha h)\rangle_\Phi|_{\alpha=0} +$$
$$\frac{\mathrm{d}}{\mathrm{d}\alpha}\langle e^{-\tilde{\rho}t}\eta_{5,t}, (z_t n_t - N_t)(f + \alpha h)\rangle_\Phi|_{\alpha=0} + \frac{\mathrm{d}}{\mathrm{d}\alpha}\langle e^{-\tilde{\rho}t}\eta_{6,t}, (c_t^f - C_t^f)(f + \alpha h)\rangle_\Phi|_{\alpha=0}$$

Which simplifies to

$$\tilde{\rho}\eta_{1,t} = u(c_t^h, c_t^f) - g(n_t) + \mathcal{A}\eta_{1,t} + \frac{\mathrm{d}\eta_{1,t}}{\mathrm{d}t} + \eta_{4,t}(b_t^h - B_t) + \eta_{5,t}(z_t n_t - N_t) + \eta_{9,t}(c_t^f - C_t^f) \tag{35}$$

$$\lim_{T\to\infty} e^{-\tilde{\rho}T}\eta_{1,T} = 0$$

This can be interpreted as a planner's HJB equation and usual transversality condition, where $\eta_{1,t}$ is the planner's value function.

The derivative wrt v in the direction h

$$0 = \frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle \mathcal{A}^*\eta_{2,t} - \frac{\partial \eta_{2,t}}{\partial t}, V_t + \alpha h\rangle_\Phi \,\mathrm{d}t|_{\alpha=0} + \frac{\mathrm{d}}{\mathrm{d}\alpha}(\lim_{T\to\infty}\langle e^{-\tilde{\rho}T}\eta_{2,T}, V_T + \alpha h_T\rangle_\Phi - \langle \eta_{2,0}, v_0 + \alpha h_0\rangle_\Phi)|_{\alpha=0}$$

$$\frac{\mathrm{d}}{\mathrm{d}\alpha}\left(\sum_{j=1}^J (v_{j,t} + \alpha h)(b)s_t^h(b,j)\eta_{2,t}(b,j)|_{b=-\infty}^{b=\infty}\right)|_{\alpha=0} + \int_0^\infty e^{-\tilde{\rho}t}\langle \frac{\partial \eta_{8,t}}{\partial b}p_t^h + \frac{\partial \eta_{9,t}}{\partial b}(1+\tau)p_t^f\rangle_\Phi$$

Which simplifies to (for $b \geq \phi$ and $t > 0$)

$$\frac{\partial \eta_{2,t}}{\partial t} = \mathcal{A}^*\eta_{2,t} + \frac{\partial \eta_{8,t}}{\partial b}p_t^h + \frac{\partial \eta_{9,t}}{\partial b}(1+\tau)p_t^f \tag{36}$$

This is a "planner's KFE" with density $\eta_{2,t}$. The boundary conditions are

$$\lim_{b\to\infty} s_t^h(b,j)\eta_{2,t}(b,j) = \lim_{b\to-\infty} s_t^h(b,j)\eta_{2,t}(b,j) = 0 \quad j = 1, ...J$$

$$\eta_{2,0} = \lim_{T\to\infty} e^{-\tilde{\rho}T}\eta_{2,T} = 0$$

Prices The derivative wrt π in the direction h

$$0 = \frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle \frac{\partial \eta_{1,t}}{\partial b}(-N_t\frac{\theta}{2}(\pi+\alpha h)^2), f_t\rangle_\Phi \,\mathrm{d}t|_{\alpha=0} + \frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle \frac{\partial v}{\partial b}(-N_t\frac{\theta}{2}(\pi+\alpha h)^2), \eta_{2,t}\rangle_\Phi \,\mathrm{d}t|_{\alpha=0} +$$

$$\frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\eta_{3,t}\left((r - \frac{\dot{N}_t}{N_t})(\pi+\alpha h) - \dot{(\pi+\alpha h)}\right)\mathrm{d}t$$

After taking derivatives wrt α, the third term requires evaluating $-\int_0^\infty e^{-\tilde{\rho}t}\eta_{3,t}\dot{h}\,\mathrm{d}t$. Using integration by parts and $\langle 1, f_t\rangle_\Phi = 1$

$$\begin{aligned}&-\int_0^\infty e^{-\tilde{\rho}t}\eta_{3,t}\dot{h}\,\mathrm{d}t \\ &= \eta_{3,0}h_0 + \int_0^\infty e^{-\tilde{\rho}t}\langle(\dot{\eta}_{3,t} - \tilde{\rho}\eta_{3,t})h_t, f_t\rangle_\Phi\end{aligned}$$

Thus, this FOC simplifies to

$$0 = \langle -\frac{\partial \eta_{1,t}}{\partial b}(N_t \theta \pi_t), f_t \rangle_\Phi + \langle -\frac{\partial V_{1,t}}{\partial b}(N_t \theta \pi_t), \eta_{2,t} \rangle_\Phi + \eta_{3,t}(r - \frac{\dot{N}_t}{N_t} - \tilde{\rho}) + \dot{\eta}_{3,t} \tag{37}$$

$$\eta_{3,0} = 0$$

The derivative wrt r in the direction h

$$0 = \frac{\mathrm{d}}{\mathrm{d}\alpha} \int_0^\infty e^{-\tilde{\rho}t} \langle (r + \alpha h)(b - B) \frac{\partial \eta_{1,t}}{\partial b}, f_t \rangle_\Phi \, \mathrm{d}t|_{\alpha=0} + \frac{\mathrm{d}}{\mathrm{d}\alpha} \int_0^\infty e^{-\tilde{\rho}t} \langle (r + \alpha h) b \frac{\partial V}{\partial b}, \eta_{2,t} \rangle_\Phi \, \mathrm{d}t|_{\alpha=0} +$$
$$\frac{\mathrm{d}}{\mathrm{d}\alpha} \int_0^\infty e^{-\tilde{\rho}t} \eta_{3,t} \left((r_t + \alpha h) - \frac{\dot{N}_t}{N_t} \right) \pi_t$$

which simplifies to

$$\eta_{3,t} \pi_t = \langle (B_t - b_t^h) \frac{\partial \eta_{1,t}}{\partial b}, f_t \rangle_\Phi + \langle (B_t - b_t^h) \frac{\partial V_{1,t}}{\partial b}, \eta_{2,t} \rangle_\Phi \tag{38}$$

The derivative wrt w in direction h

$$0 = \frac{\mathrm{d}}{\mathrm{d}\alpha} \int_0^\infty e^{-\tilde{\rho}t} \langle \left(\frac{(w + \alpha h)}{p^h} (1 - \tau^n) z n_t + \tau^n N_t \frac{(w + \alpha h)}{p^h} \right) \frac{\partial \eta_{1,t}}{\partial b}, f_t \rangle_\Phi \, \mathrm{d}t|_{\alpha=0} + \frac{\mathrm{d}}{\mathrm{d}\alpha} \int_0^\infty e^{-\tilde{\rho}t} \langle \frac{(w + \alpha h)}{p^h} (1 -$$
$$\frac{\mathrm{d}}{\mathrm{d}\alpha} \int_0^\infty e^{-\tilde{\rho}t} \eta_{3,t} \left(-\frac{\epsilon}{\theta} (\frac{w + \alpha h}{p^h} - 1) \right) \mathrm{d}t + \int_0^\infty e^{-\tilde{\rho}t} \langle \eta_{7,t}, -\frac{(w_t + \alpha h)(1 - \tau^n) z_{j,t}}{p_t^h} u_h(c_t^h, c_t^f) \rangle \, \mathrm{d}t$$

which simplifies to

$$0 = \langle (1 - \tau^n)(z_t n_t - N_t) \frac{\partial \eta_{1,t}}{\partial b}, f \rangle_\Phi + \langle (1 - \tau^n)(z_t n_t - N_t) \frac{\partial V_t}{\partial b}, \eta_{2,t} \rangle_\Phi - \eta_{3,t} \frac{\epsilon}{\theta} - \langle (1 - \tau^n) z u_h(c^f, c^h), \eta_{7,t} \rangle_\Phi \tag{39}$$

Household Choices The derivative wrt c^f in the direction h

$$0 = \frac{\mathrm{d}}{\mathrm{d}\alpha} \int_0^\infty e^{-\tilde{\rho}t} \langle u(c_t^h, c_t^f + \alpha h) - (c^f + \alpha h) p (1 + \tau) \frac{\partial \eta_{1,t}}{\partial b}, f_t \rangle_\Phi \, \mathrm{d}t|_{\alpha=0} +$$
$$\frac{\mathrm{d}}{\mathrm{d}\alpha} \int_0^\infty e^{-\tilde{\rho}t} \langle u(c_t^h, c_t^f + \alpha h) - (c^f + \alpha h) p (1 + \tau) \frac{\partial V_t}{\partial b}, \eta_{2,t} \rangle_\Phi \, \mathrm{d}t|_{\alpha=0} +$$
$$\frac{\mathrm{d}}{\mathrm{d}\alpha} \int_0^\infty e^{-\tilde{\rho}t} \langle \eta_{9,t}, u_f(c^h, c^f + \alpha h) \rangle_\Phi \, \mathrm{d}t|_{\alpha=0} + \frac{\mathrm{d}}{\mathrm{d}\alpha} \int_0^\infty e^{-\tilde{\rho}t} \langle \eta_{7,t}, -\frac{w_t z_{j,t} (1 - \tau^n)}{p_t^h} u_h(c_t^h, c_t^f + \alpha h) \rangle \, \mathrm{d}t +$$
$$\frac{\mathrm{d}}{\mathrm{d}\alpha} \int_0^\infty e^{-\tilde{\rho}t} \langle \eta_{8,t}, u_h(c_t^h, c_t^f + \alpha h) \rangle_\Phi \, \mathrm{d}t + \frac{\mathrm{d}}{\mathrm{d}\alpha} \langle e^{-\tilde{\rho}t} \eta_{6,t}, (c_t^f + \alpha h - C_t^f) f_t \rangle_\Phi |_{\alpha=0}$$

which, after noting that the private FOC hold, simplifies to

$$0 = u_f(c_t^h, c_t^f) - \frac{\partial \eta_{1,t}}{\partial b}(p(1+\tau)) + \frac{\eta_{9,t}}{f_t} u_{ff}(c_t^h, c_t^f) - \frac{\eta_{7,t}}{f_t} \frac{w_t z_{j,t}(1-\tau^n)}{p_t^h} u_{hf}(c_t^h, c_t^f) + \frac{\eta_{8,t}}{f_t} u_{fh}(c_t^h, c_t^f) + \eta_{6,t} \tag{40}$$

The derivative wrt c^h in the direction h is similar

$$\begin{aligned}
0 = &\frac{\mathrm{d}}{\mathrm{d}\alpha} \int_0^\infty e^{-\tilde{\rho} t} \langle u(c_t^h + \alpha h, c_t^f) - (c^h + \alpha h) \frac{\partial \eta_{1,t}}{\partial b}, f_t \rangle_\Phi \, \mathrm{d}t|_{\alpha=0} + \\
&\frac{\mathrm{d}}{\mathrm{d}\alpha} \int_0^\infty e^{-\tilde{\rho} t} \langle u(c_t^h + \alpha h, c_t^f) - (c^h + \alpha h) \frac{\partial V_t}{\partial b}, \eta_{2,t} \rangle_\Phi \, \mathrm{d}t|_{\alpha=0} + \\
&\frac{\mathrm{d}}{\mathrm{d}\alpha} \int_0^\infty e^{-\tilde{\rho} t} \langle \eta_{9,t}, u_f(c^h + \alpha h, c^f) \rangle_\Phi \, \mathrm{d}t|_{\alpha=0} + \frac{\mathrm{d}}{\mathrm{d}\alpha} \int_0^\infty e^{-\tilde{\rho} t} \langle \eta_{7,t}, -\frac{(1-\tau^n) w_t z_{j,t}}{p_t^h} u_h(c_t^h, c_t^f + \alpha h) \rangle \, \mathrm{d}t|_{\alpha=0} + \\
&\frac{\mathrm{d}}{\mathrm{d}\alpha} \int_0^\infty e^{-\tilde{\rho} t} \langle \eta_{8,t}, u_h(c_t^h + \alpha h, c_t^f) \rangle_\Phi \, \mathrm{d}t|_{\alpha=0}
\end{aligned}$$

and after taking into account the fact that the household FOCs must hold simplifies to

$$0 = u_h(c_t^h, c_t^f) - \frac{\partial \eta_{1,t}}{\partial b} + \frac{\eta_{9,t}}{f_t} u_{fh}(c_t^h, c_t^f) - \frac{\eta_{7,t}}{f_t} \frac{(1-\tau^n) w_t z_{j,t}}{p_t^h} u_{hh}(c_t^h, c_t^f) + \frac{\eta_{8,t}}{f_t} u_{hh}(c_t^h, c_t^f) \tag{41}$$

Note that unlike in Nuño and Moll (2018), I explicitly set the individual FOC as a constraint. Without this, the planner's FOC would simply be the partials of the planner's HJB (eqn (35)) with respect to c^h, c^f, and n respectively. However, enforcing the private FOC hold directly adds η_7, η_8, and η_9 terms (with non-zero multipliers), capturing the fact that the planner would prefer levels of individual consumption and labor not consistent with households' optimization. The values of $\eta_{7,t}(b,z)$, $\eta_{8,t}(b,z)$, $\eta_{9,t}(b,z)$ penalize the planner enough to make them choose consumption and labor consistent with household choices rather than allocation them optimally subject only to feasibility

The derivative wrt n in direction h

$$\begin{aligned}
0 = &\frac{\mathrm{d}}{\mathrm{d}\alpha} \int_0^\infty e^{-\tilde{\rho} t} \langle \frac{\partial \eta_{1,t}}{\partial b} z w(n + \alpha h) - g(n_t + \alpha h), f_t \rangle_\Phi \, \mathrm{d}t + \int_0^\infty e^{-\tilde{\rho} t} \langle \frac{\partial V_t}{\partial b} z w(n + \alpha h) - g(n_t + \alpha h), \eta_{2,t} \rangle_\Phi \, \mathrm{d}t|_{\alpha=0} \\
&+ \frac{\mathrm{d}}{\mathrm{d}\alpha} \int_0^\infty e^{-\tilde{\rho} t} \langle \eta_{5,t}, f_t(n_t + \alpha h - N_t) \rangle_\Phi \, \mathrm{d}t \int_0^\infty e^{-\tilde{\rho} t} \langle \eta_{7,t}, g'(n_t + \alpha h) \rangle \, \mathrm{d}t|_{\alpha=0}
\end{aligned}$$

This simplifies to

$$0 = -g'(n_t) + \frac{(1-\tau^n) w_t z_{j,t}}{p_t^h} \frac{\partial \eta_{1,t}}{\partial b} + \eta_{7,t} \frac{g''(n_t)}{f_t} + z \eta_{5,t} \tag{42}$$

Aggregate Choices The derivative wrt B in direction h

$$0 = \int_0^\infty e^{-\tilde{\rho}t}\langle \eta_{1,t}, \mathcal{A}^*_{B+\alpha h} f_t\rangle_\Phi \,\mathrm{d}t + \frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle \eta_{4,t}, f_t(b^h_t - (B+\alpha h))\rangle_\Phi \,\mathrm{d}t|_{\alpha=0}$$
$$+ \int_0^\infty e^{-\tilde{\rho}t}\langle V_t, \mathcal{A}^*_{B+\alpha h}\eta_{2,t}\rangle_\Phi \,\mathrm{d}t$$

which simplifies to

$$\eta_{4,t} = \langle f_t, \frac{\partial \eta_{1,t}}{\partial b}\frac{\partial s^h}{\partial B}\rangle_\Phi + \langle \eta_{2,t}, \frac{\partial V_t}{\partial b}\frac{\partial s^h}{\partial B}\rangle_\Phi \tag{43}$$

Under the assumption of flat transfers, this evaluates to (using integration by parts to evaluate the derivative of $\dot{B}$ terms)

$$\eta_{4,t} = \langle f_t, \frac{\partial \eta_{1,t}}{\partial b}\left((\rho - r) - \frac{\dot{f}_t}{f_t} - \frac{\frac{\partial \dot{\eta}_{1,t}}{\partial b}}{\frac{\partial \eta_{1,t}}{\partial b}}\right)\rangle_\Phi + \langle \eta_{2,t}, \frac{\partial V_t}{\partial b}\left((\rho - r) - \frac{\dot{\eta}_{2,t}}{\eta_{2,t}} - \frac{\frac{\partial \dot{V}_t}{\partial b}}{\frac{\partial V_t}{\partial b}}\right)\rangle_\Phi$$

The derivative wrt N in direction h

$$0 = \frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle f_t, \frac{\partial \eta_{1,t}}{\partial b}\left((1+s-s)(N_t+\alpha h) - m_t(N_t+\alpha h) - \frac{\theta}{2}\pi_t^2(N_t+\alpha h) + \tau w(N+\alpha h)\right)\rangle_\Phi \,\mathrm{d}t_{\alpha=0}+$$
$$\frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle \eta_{2,t}, \frac{\partial V}{\partial b}\left((1+s-s)(N_t+\alpha h) - m_t(N_t+\alpha h) - \frac{\theta}{2}\pi_t^2(N_t+\alpha h) + \tau w(N+\alpha h)\right)\rangle_\Phi \,\mathrm{d}t_{\alpha=0}+$$
$$\frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\eta_{3,t}\left(-\frac{N_t + \dot{\alpha h}}{N_t+\alpha h}\right)\pi_t \,\mathrm{d}t|_{\alpha=0} + \frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle \eta_{5,t}, f_t(n_t - (N+\alpha h))\rangle_\Phi \,\mathrm{d}t|_{\alpha=0}$$

which simplifies to

$$\eta_{5,t} = \langle f_t, \frac{\partial \eta_{1,t}}{\partial b}\left(1 - m - \frac{\theta}{2}\pi_t^2 + \tau w\right)\rangle_\Phi + \langle \eta_{2,t}, \frac{\partial V_t}{\partial b}\left(1 - m - \frac{\theta}{2}\pi_t^2 + \tau w\right)\rangle_\Phi +$$
$$\frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\eta_{3,t}\left(-\frac{N_t + \dot{\alpha h}}{N_t+\alpha h}\right)\pi_t \,\mathrm{d}t|_{\alpha=0}$$

The final term evaluates to

$$\int_0^\infty e^{-\tilde{\rho}t}(-\eta_{3,t})\frac{\dot{h}N_t - h\dot{N}_t}{N_t^2}\pi_t \,\mathrm{d}t$$

Using integration by parts

$$\int_0^\infty e^{-\tilde{\rho}t}(-\eta_{3,t})\frac{\dot{h}}{N_t}\,\mathrm{d}t$$
$$= -\left(\frac{e^{-\tilde{\rho}t}h_t\eta_{3,t}\pi_t}{N_t}|_0^\infty\right) + \int_0^\infty h_t\left(\frac{\pi_t\eta_{3,t}(-\tilde{\rho}e^{-\tilde{\rho}t})}{N_t} + \frac{\pi_t\dot{\eta}_{3,t}e^{-\tilde{\rho}t}}{N_t} + \frac{\dot{\pi}\eta_{3,t}e^{-\tilde{\rho}t}}{N_t} - \frac{\pi_t\eta_{3,t}e^{-\tilde{\rho}t}\dot{N}_t}{N_t^2}\right)\mathrm{d}t + \int_0^\infty h_t\frac{\pi_t\eta_{3,}}{}$$
$$= \frac{h_0\eta_{3,0}}{N_0} + \int_0^\infty \frac{h_t e^{-\tilde{\rho}t}}{N_t}\left(\dot{\eta}_{3,t}\pi_t - \eta_{3,t}\tilde{\rho}\pi_t + \dot{\pi}\eta_{3,t}\right)\mathrm{d}t$$

So finally,

$$\eta_{5,t} = \langle f_t, \frac{\partial \eta_{1,t}}{\partial b}\left(1 - m - \frac{\theta}{2}\pi_t^2 + \tau w\right)\rangle_\Phi + \langle \eta_{2,t}, \frac{\partial V_t}{\partial b}\left(1 - m - \frac{\theta}{2}\pi_t^2 + \tau w\right)\rangle_\Phi + \frac{(\dot{\eta}_{3,t}\pi_t - \eta_{3,t}\tilde{\rho}\pi_t + \dot{\pi}\eta_{3,t})}{N_t} \tag{44}$$

The derivative wrt C^f in direction h

$$0 = \frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle f_t, \frac{\partial \eta_{1,t}}{\partial b}\tau p(C_t^f + \alpha h)\rangle_\Phi \,\mathrm{d}t|_{\alpha=0} + \frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle \eta_{2,t}, \frac{\partial V_t}{\partial b}\tau p(C_t^f + \alpha h)\rangle_\Phi \,\mathrm{d}t|_{\alpha=0} +$$
$$\frac{\mathrm{d}}{\mathrm{d}\alpha}\int_0^\infty e^{-\tilde{\rho}t}\langle \eta_{9,t}, (c_t^f - C_t^f - \alpha h) f_t\rangle_\Phi \,\mathrm{d}t|_{\alpha=0}$$

which simplifies to

$$\eta_{9,t} = \langle f_t, \frac{\partial \eta_{1,t}}{\partial b}\tau p\rangle_\Phi + \langle \eta_{2,t}, \frac{\partial V_t}{\partial b}\tau p\rangle_\Phi \tag{45}$$

Combining the simplified first order conditions completes the proof of Proposition 2.

B.3 Proof of Proposition 3

First, I define some notation. Taking the derivative of the household's HJB, equation 1, and the planner's HJB, equation 35, with respect to current bonds b yields

$$\tilde{\rho}V_b = u_h \cdot c_b^h + u_f \cdot c_b^f - g' \cdot n_b + V_b s_b^h + \frac{\partial^2 V}{\partial b^2}s^h + \sum_{j' \neq j}\lambda_{j,j'}[V_b(j') - V_b(j)] + \frac{\mathrm{d}}{\mathrm{d}t}V_b$$

$$\tilde{\rho}\eta_b = \cdot c_b - g' \cdot n_b + \eta_b s_b^h + \frac{\partial^2 \eta}{\partial b^2}s^h + \sum_{j' \neq j}\lambda_{j,j'}[\eta_b(j') - \eta_b(j)] + \frac{\mathrm{d}}{\mathrm{d}t}\eta_b + \eta_4 + \eta_5 n_b + \eta_6 c_b^f$$

where variables with a subscript b denote partial derivatives with respect to bonds and u and g are the instantaneous utilities of consumption and labor. Now define the operator $\mathcal{M} = \tilde{\rho} - s_b^h - \mathrm{d}t - \mathcal{A}$. Then, noting that $\frac{\partial^2 V}{\partial b^2}s^h + \sum_{j' \neq j}\lambda_{j,j'}[V_b(j') - V_b(j)] = \mathcal{A}V_b$, the envelope conditions can be rewritten according to

$$\mathcal{M}V_b = u' \cdot c_b - g' \cdot n_b$$

$$\mathcal{M}\eta_b = u' \cdot c_b - g' \cdot n_b + \eta_4 + \eta_5 n_b + \eta_6 c_b^f$$

and following Dávila and Schaab (2023), the difference between the marginal social and public value of wealth can be written as

$$\eta_b = V_b + \eta_4 \mathcal{M}^{-1} + \eta_5 \mathcal{M}^{-1} n_b + \eta_6 \mathcal{M}^{-1} c_b^f \tag{46}$$

where the term $\eta_4\mathcal{M}^{-1} + \eta_5\mathcal{M}^{-1}n_b + \eta_6\mathcal{M}^{-1}c_b^f$ captures the present discounted value of the future savings, labor supply, and foreign consumption to aggregate excess bond demand, aggregate excess labor, and aggregate excess foreign consumption (respectively) caused by an increase in time t assets.

Next, define "promise keeping values"

$$\begin{aligned}
\mathcal{H}_t^f &\equiv -\frac{\eta_{9,t}}{f_t}u_{ff}(c_t^h, c_t^f) + \frac{\eta_{7,t}}{f_t}\frac{w_t z_{j,t}(1-\tau^n)}{p_t^h}u_{hf}(c_t^h, c_t^f) - \frac{\eta_{8,t}}{f_t}u_{fh}(c_t^h, c_t^f) \\
\mathcal{H}_t^h &\equiv -\frac{\eta_{9,t}}{f_t}u_{fh}(c_t^h, c_t^f) + \frac{\eta_{7,t}}{f_t}\frac{w_t z_{j,t}(1-\tau^n)}{p_t^h}u_{hh}(c_t^h, c_t^f) - \frac{\eta_{8,t}}{f_t}u_{hh}(c_t^h, c_t^f) \\
\mathcal{H}_t^n &\equiv -\eta_{7,t}\frac{g''(n_t)}{f_t}
\end{aligned}$$

i.e. the η_7, η_8, and η_9 terms in the planner's FOC's for individual choices (equations 40-42) which represent value/cost of the planner being forced to adhere to household optimality conditions for foreign consumption, domestic consumption, and labor rather than allocating consumption and labor only subject to feasibility.

Additionally, note that the "promise keeping values" are related to the wedge between the private and social values of wealth according to

$$\begin{aligned}
\mathcal{H}_t^f &= \eta_{6,t} - p(1+\tau)(\eta_4\mathcal{M}^{-1} + \eta_5\mathcal{M}^{-1}n_b + \eta_6\mathcal{M}^{-1}c_b^f) \\
\mathcal{H}_t^h &= -(\eta_4\mathcal{M}^{-1} + \eta_5\mathcal{M}^{-1}n_b + \eta_6\mathcal{M}^{-1}c_b^f) \\
\mathcal{H}_t^n &= z\eta_{5,t} - \frac{w_t z_{j,t}(1-\tau^n)}{p_t^h}(\eta_4\mathcal{M}^{-1} + \eta_5\mathcal{M}^{-1}n_b + \eta_6\mathcal{M}^{-1}c_b^f)
\end{aligned}$$

Now with the requisite notation defined, I am ready to prove proposition 3. The general strategy will be to substitute the planner's first order conditions for the individual choices (equations 40-42) into the first order conditions for interest rates (equation 38) and aggregates (equations 44-45)

Rewrite the first order condition for individual foreign consumption

$$\begin{aligned}
0 &= u_f(c_t^h, c_t^f) - \frac{\partial\eta_{1,t}}{\partial b}(p(1+\tau)) - \mathcal{H}_t^f + \eta_{6,t} \\
0 &= \int_{z_1}^{z_J}\int_{\phi}^{\infty}(B-b)\left[u_f(c_t^h, c_t^f) - \frac{\partial\eta_{1,t}}{\partial b}(p(1+\tau)) - \mathcal{H}_t^f + \eta_{6,t}\right] f_t\,\mathrm{d}b\,\mathrm{d}z \\
\int_{z_1}^{z_J}\int_{\phi}^{\infty}(B-b)\frac{\partial\eta_{1,t}}{\partial b}f_t\,\mathrm{d}b\,\mathrm{d}z &= \int_{z_1}^{z_J}\int_{\phi}^{\infty}(B-b)\left[\frac{u_f}{p(1+\tau)} - \frac{\mathcal{H}_t^f}{p(1+\tau)}\right] f_t\,\mathrm{d}b\,\mathrm{d}z
\end{aligned}$$

Substituting into the first order condition for interest rates and using the private consumption FOC to substitute for $\frac{\partial V_t}{\partial b}$

$$\eta_{3,t}\pi_t = \int_{z_1}^{z_J}\int_{\phi}^{\infty}(B_t - b_t^h)\left[\frac{u_f}{p(1+\tau)} - \frac{\mathcal{H}_t^f}{p(1+\tau)} + \frac{u_f}{p(1+\tau)}\frac{\eta_{2,t}}{f_t}\right] f_t \,\mathrm{d}b\,\mathrm{d}z \tag{47}$$

Next, the first order condition for aggregate foreign consumption is rewritten as

$$0 = u_f(c_t^h, c_t^f) - \frac{\partial \eta_{1,t}}{\partial b}(p(1+\tau)) - \mathcal{H}_t^f + \eta_{6,t}$$

$$0 = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\frac{\partial s}{\partial T}\frac{\partial T}{\partial C^f}\left[u_f(c_t^h, c_t^f) - \frac{\partial \eta_{1,t}}{\partial b}(p(1+\tau)) - \mathcal{H}_t^f + \eta_{6,t}\right] f_t \,\mathrm{d}b\,\mathrm{d}z$$

$$\int_{z_1}^{z_J}\int_{\phi}^{\infty}\frac{\partial s}{\partial T}\frac{\partial T}{\partial C^f}\frac{\partial \eta_{1,t}}{\partial b} f_t \,\mathrm{d}b\,\mathrm{d}z = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\frac{\partial s}{\partial T}\frac{\partial T}{\partial C^f}\left[\frac{u_f}{p(1+\tau)} - \frac{\mathcal{H}_t^f}{p(1+\tau)}\right] f_t \,\mathrm{d}b\,\mathrm{d}z + p\tau\frac{\eta_{6,t}}{p(1+\tau)}$$

paired with the first order condition for individual foreign consumption yields

$$\eta_{6,t} = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\frac{\partial s}{\partial T}\frac{\partial T}{\partial C^f}\left[\frac{u_f}{p} - \frac{\mathcal{H}_t^f}{p} + \frac{u_f}{p}\frac{\eta_{2,t}}{f_t}\right] f_t \,\mathrm{d}b\,\mathrm{d}z \tag{48}$$

Rewriting the first order condition for individual labor choice yields

$$\int_{z_1}^{z_J}\int_{\phi}^{\infty}\frac{\partial s^h}{\partial N}\frac{\partial \eta_{1,t}}{\partial b} f_t \,\mathrm{d}b\,\mathrm{d}z = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\frac{\partial s^h}{\partial N}\left[\frac{g'(n)}{w_t z_t(1-\tau^n)} + \frac{\mathcal{H}_t^n}{(1-\tau^n)w_t z_t}\right] f_t \,\mathrm{d}b\,\mathrm{d}z - \frac{\partial s^h}{\partial N}\frac{\eta_{5,t}}{w_t(1-\tau^n)}$$

Then substituting into the first order condition for aggregate labor

$$\eta_{5,t} = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\frac{\partial s^h}{\partial N}\frac{1}{(1-\tau^n)wz}\left[g'(n) + \mathcal{H}_t^n + g'(n_t)\frac{\eta_{2,t}}{f_t}\right] f_t \,\mathrm{d}b\,\mathrm{d}z - \frac{\partial s^h}{\partial N}\frac{\eta_{5,t}}{w_t(1-\tau^n)} + \frac{1}{N_t}\left(\frac{\dot{\eta}_{3,t}}{\eta_{3,t}} - \tilde{\rho} + \frac{\dot{\pi}_t}{\pi_t}\right)\pi \tag{49}$$

Next, I combine equation 47 with equation 49

$$\eta_{5,t} = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\frac{\partial s^h}{\partial N}\frac{1}{w_t z_t(1-\tau^n)}\left[g'(n) + \mathcal{H}_t^n + g'(n_t)\frac{\eta_{2,t}}{f_t}\right] f_t \,\mathrm{d}b\,\mathrm{d}z - \frac{\partial s^h}{\partial N}\frac{\eta_{5,t}}{w_t(1-\tau^n)} +$$
$$\frac{1}{N_t}\left(\frac{\dot{\eta}_{3,t}}{\eta_{3,t}} - \tilde{\rho} + \frac{\dot{\pi}_t}{\pi_t}\right)\int_{z_1}^{z_J}\int_{\phi}^{\infty}(B_t - b_t^h)\left[\frac{u_f}{p(1+\tau)} - \frac{\mathcal{H}_t^f}{p(1+\tau)} + \frac{u_f}{p(1+\tau)}\frac{\eta_{2,t}}{f_t}\right] f_t \,\mathrm{d}b\,\mathrm{d}z$$

$$\eta_{6,t} = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\frac{\partial s}{\partial T}\frac{\partial T}{\partial C^f}\left[\frac{u_f}{p} - \frac{\mathcal{H}_t^f}{p} + \frac{u_f}{p}\frac{\eta_{2,t}}{f_t}\right] f_t \,\mathrm{d}b\,\mathrm{d}z$$

I then define Ω_1 and Ω_2 according to

$$\Omega_1 = \eta_{6,t} + \int_{z_1}^{z_J}\int_{\phi}^{\infty} \frac{\partial s}{\partial T}\frac{\partial T}{\partial C^f}\frac{\mathcal{H}_t^f}{p} f_t \,\mathrm{d}b\,\mathrm{d}z$$

$$\Omega_2 = (1+\frac{\partial s^h}{\partial N})\eta_{5,t} + \int_{z_1}^{z_J}\int_{\phi}^{\infty}\left[\frac{1}{N_t}\left(\frac{\dot{\eta}_{3,t}}{\eta_{3,t}} - \tilde{\rho} + \frac{\dot{\pi}_t}{\pi_t}\right)(B-b^h)\frac{\mathcal{H}_t^f}{p(1+\tau)} - \frac{\partial s^h}{\partial N}\frac{\mathcal{H}_t^n}{w_t z_t}\right]$$

Given usual assumptions about concavity of the utility functions and standard parameter values, $\Omega_1, \Omega_2 > 0$.

So

$$1 = \frac{1}{\Omega_2}\int_{z_1}^{z_J}\int_{\phi}^{\infty}\frac{\partial s^h}{\partial N}\left[\frac{u_h}{p} + \frac{u_h}{p}\frac{\eta_{2,t}}{f_t}\right] f_t \,\mathrm{d}b\,\mathrm{d}z + \frac{1}{\Omega_2 N_t}\left(\frac{\dot{\eta}_{3,t}}{\eta_{3,t}} - \tilde{\rho} + \frac{\dot{\pi}_t}{\pi_t}\right)\int_{z_1}^{z_J}\int_{\phi}^{\infty}(B_t - b_t^h)\left[\frac{u_h}{p} + \frac{u_h}{p}\frac{\eta_{2,t}}{f_t}\right] f_t \,\mathrm{d}$$

(50)

$$1 = \frac{1}{\Omega_1}\int_{z_1}^{z_J}\int_{\phi}^{\infty}\frac{\partial s}{\partial T}\frac{\partial T}{\partial C^f}\left[\frac{u_h}{p} + \frac{u_h}{p}\frac{\eta_{2,t}}{f_t}\right] f_t \,\mathrm{d}b\,\mathrm{d}z \tag{51}$$

Combining equations 50 and 51 and rearranging completes the proof.

Note that manipulating equation 37 can allow one to obtain a more intuitive formulation of term that rescales the redistributive motive, $\left(\frac{\dot{\eta}_{3,t}}{\eta_{3,t}} - \tilde{\rho} + \frac{\dot{\pi}_t}{\pi_t}\right)$.

$$\frac{\dot{\eta}_{3,t}}{\eta_{3,t}} = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\frac{N_t\theta\pi_t}{\eta_{3,t}}\left(\frac{u_h}{p} - \frac{\mathcal{H}^h}{p} + \frac{u_h}{p}\frac{\eta_{2,t}}{f_t}\right) f_t \,\mathrm{d}b\,\mathrm{d}z + \tilde{\rho} - r + \frac{\dot{N}_t}{N_t}$$

$$\frac{\dot{\eta}_{3,t}}{\eta_{3,t}} - \tilde{\rho} + \frac{\dot{\pi}_t}{\pi_t} = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\frac{N_t\theta\pi_t}{\eta_{3,t}}\left(\frac{u_h}{p} - \frac{\mathcal{H}^h}{p} + \frac{u_h}{p}\frac{\eta_{2,t}}{f_t}\right) f_t \,\mathrm{d}b\,\mathrm{d}z + \frac{\dot{N}_t}{N_t} - r + \frac{\dot{\pi}_t}{\pi_t}$$

Finally from the NKPC

$$\frac{\dot{\eta}_{3,t}}{\eta_{3,t}} - \tilde{\rho} + \frac{\dot{\pi}_t}{\pi_t} = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\frac{N_t\theta\pi_t}{\eta_{3,t}}\left(\frac{u_h}{p} - \frac{\mathcal{H}^h}{p} + \frac{u_h}{p}\frac{\eta_{2,t}}{f_t}\right) f_t \,\mathrm{d}b\,\mathrm{d}z - \frac{\epsilon}{\theta}\frac{m_t - 1}{\pi_t}$$

This term thus implies that the redistributive motive is scaled by how the marginal costs of inflation accrue to households.

B.4 Timeless Penalty

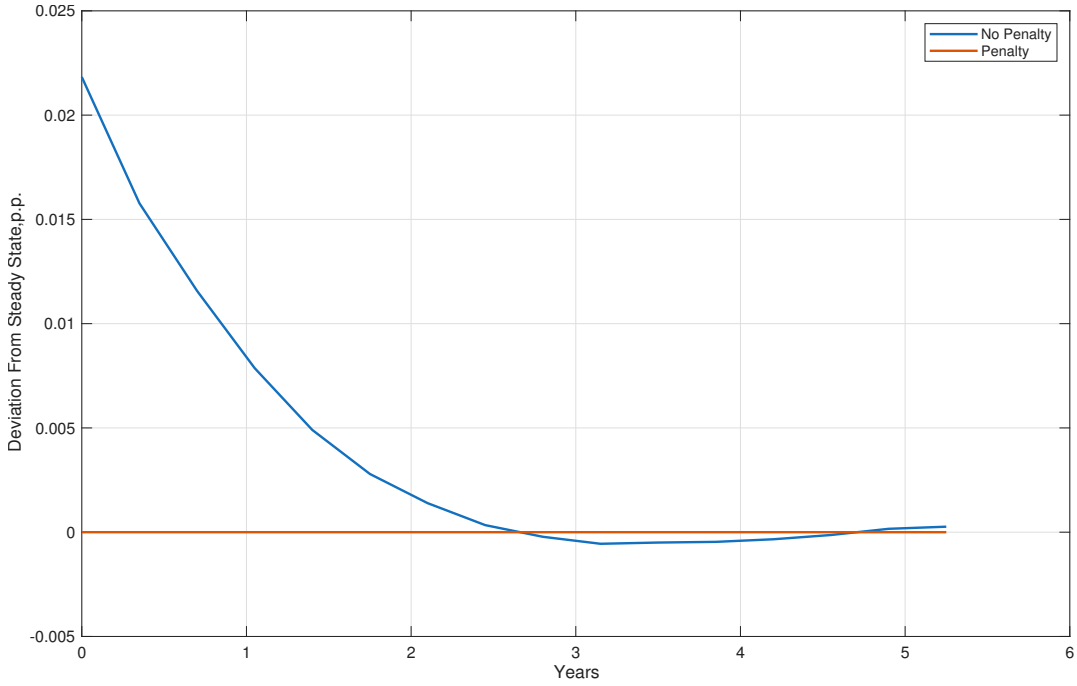
My specification of the timeless penalty is identical to the one in Dávila and Schaab (2023). In the absence of such a penalty, the initial values on the two forward looking constraints

(the HJB and Phillips curve) $\eta_{2,0}$ and $\eta_{3,0}$ are free. It is as if the planner at time 0 is not bound by any "past promises". This leads to a time 0 problem (a la Kydland and Prescott (1977)) where, even in the absence of shocks, the planner would want to raise inflation at time 0, as doing so would redistribute from creditors to debtors.

Formally, the initial conditions for such a Ramsey problem without a timeless penalty are $\eta_{2,0} = 0$ and $\eta_{3,0} = 0$. However, any stationary Ramsey plan will generically have $\eta_{2,0} \neq 0$ and $\eta_{3,0} \neq 0$. Thus, without enforcing a timeless penalty and initializing the economy at its long run steady state (i.e. setting $f_0 = f_{ss}$), the planner will not set all allocations, prices, and the instrument to their steady state values for all time t. They will instead cut rates and temporarily induce positive inflation. To counteract this problem, I define the timeless penalty so that the initial conditions for η_2 and η_3 are their steady state values.

To highlight how this time 0 problem manifests in my economy, figure 14 plots the solution to two Ramsey planning problems without any tariff shocks: one where no timeless penalty is enforced where $\eta_{2,0} = 0$ and $\eta_{3,0} = 0$ and one where the timeless penalty is enforced.

Figure 14: Inflation Without the Timeless Penalty



Notes: I solve the Ramsey planner's problem with no tariff shock for two economies: one with no timeless penalty and one with a timeless penalty, both initialized to the same long run steady state.

Despite featuring no shocks and initializing the economy at its long run steady state (i.e. setting $f_0 = f_{ss}$), the economy with no timeless penalty is not stationary. The planner chooses positive inflation since they are not constrained by "past promises". However, the economy with a timeless penalty is stationary. Thus, in all my numerical experiments, I employ this timeless penalty to ensure the policy response does not stem from time-0 time inconsistency.

C Computational Algorithm

The algorithm to calculate the optimal stationary equilibrium and optimal transitional dynamics combines insights from the algorithms of Nuño and Moll (2018) and Nuño and Thomas (2022). In particular, it is a double fixed point algorithm. In the inner loop, I solve the usual household HJB, given guesses of Lagrange multipliers η_4, η_5, η_9 and optimal inflation π_t, a standard problem with algorithms outlined in, for example, Achdou et al. (2021). In the outer loop, I solve algebraically for the planner's value function, density, and multipliers on individual choices. Then I use the planner's first order conditions on aggregates (C^f, N, B) and inflation/interest rates to verify my guesses for the unknown multipliers and inflation.

Stationary Equilibrium Algorithm First, I specify the inner loop. Entering the inner loop, I take as given values of η_4, η_5, η_9, and π. I am implicitly treating T as a residual in the baseline version of the model. So since s and τ are ex-ante specified and T is the residual, I need a specification for B. Here, I assume that it is flat, but I *calibrate* it to be equal to some constant fraction of steady state GDP: $B_t = B_{\text{perc}} Y_{ss}$, which greatly facilitates numerical implementation. Note that I am assuming bonds are flat; the monetary authority does not internalize how their choices of policy affect equilibrium supply of bonds by the fiscal authority.

So in this inner loop, I am effectively seeking the equilibrium consistent with a given level of inflation

1. Guess aggregate labor N_t, the interest rate r_t, and aggregate transfers T

2. Solve the firm's problem (eqs 6 and 7) for the wage and dividends.

3. Using standard methods, solve the household's HJB equation (eq 1)

4. Solve the KFE (eq 5) for the stationary distribution

5. Using implied aggregate bonds, use the government budget constraint to calculate the implied transfer T (eq 8)

6. Check market clearing, for bonds comparing the guessed B and the model-implied B (eqs 9 10)

7. If markets clear and the gvt budget constraint holds, stop. Otherwise, return to step 1 and update guesses until convergence

Now, I specify the outer loop, which verifies the planner's conditions holds.

1. Guess values for $\eta_4, \eta_5, \eta_6,$, and π

2. Do the inner loop, described above. Store all equilibrium policy functions, value functions, and aggregates.

3. Solve the planner's HJB (eq 35) for $\eta_{1,t}$.

4. Use equation 46 to calculate the derivative of $\eta_{1,t}$ with respect to wealth (the usual upwinding scheme is not effective since we cannot use backwinding to calculate the derivative for constrained agents. The planner's FOC for individual consumption and labor that would be used to deal with constrained agents include unknown Lagrange multiplier terms. It is less efficient to jointly solve for $\frac{\partial \eta_{1,t}}{\partial b}$ and the multipliers than it is to back out the derivative using equation 46)

5. Jointly solve the planner's first order conditions for individual choices and obtain η_7, η_8, η_9 (eqns 40-42)

6. Solve the planner's "KFE" (eq 36) for $\eta_{2,t}$

7. Using the planner's first order condition for π_t, the conjectured value of π_t, and the equilibrium objects, solve equation 37 for η_3

8. Use the planner's first order condition for r_t and w_t (equation 38 and 39) and aggregates (equations 43-45) to calculate 4 residuals

9. If the 4 residuals are 0, stop. Otherwise, return to step 1 of the outer loop and update guesses for η_4, η_5, η_9, and π.

For both loops, I use a numerical solver in matlab (the default fsolve algorithm is fine).

C.1 Transitional Dynamics Algorithm

I follow the timeless approach of, for example, Dávila and Schaab (2023). In principle, one can solve the transitional dynamics in response to an MIT shock in the same way as the stationary equilibrium, using an inner loop to guess and verify paths of N, r, and T and an outer loop to guess and verify paths of multipliers η_4, η_5, η_6, and π. However, given that most shocks I consider are relatively small (meaning initial guesses should be reasonable) and there are a large number of equations and unknowns, it is more efficient to simultaneously guess paths of N, r, T, η_4, η_5, η_6, and π and use one solver. Like in the stationary equilibrium, I assume that bond are constant ($\dot{B}_t = 0$) and calibrated to be a fixed fraction of steady state GDP.[27]

1. Solve for the initial stationary equilibria. For a temporary shock, the initial and ending equilibria are identical. For a permanent shock, the equilibria will be different

2. Discretize time. Set the ending time to be some large number $\bar{T}$ so the model converges to the steady state after $\bar{T}$. Additionally choose a number of steps ns, which will be the number of discrete time steps the model is solved on. Thus, I define $\mathrm{d}t = \frac{\bar{T}}{ns}$.

3. Specify a path for the tariff shock $\{\tau_t\}_{t=0}^{ns}$. For the temporary shock, I assume an exponentially decaying process at some rate ξ.

[27] I found that allowing bonds to be a fraction of GDP, adjusting during the transition path, and having the monetary authority internalize this made market clearing very difficult numerically. In particular, it implied that output must remain constant on impact. Household bond demand must remain constant (b is a state variable, f cannot adjust on impact due to the initial condition on the density). Thus, bond supply also needs to remain constant, so if bond supply is a fixed fraction of GDP, GDP cannot change on impact for the bond market to clear.

4. Construct initial guess of paths for $\{N\}_{t=0}^{ns}$, $\{r\}_{t=0}^{ns}$, $\{T\}_{t=0}^{ns}$, $\{\eta_4\}_{t=0}^{ns}$, $\{\eta_5\}_{t=0}^{ns}$, $\{\eta_6\}_{t=0}^{ns}$, and $\{\pi\}_{t=0}^{ns}$. I simply linearly interpolate between the initial and terminal steady state values.

5. Generate and provide initial or terminal conditions for all other endogenous equilibrium variables' paths (aggregates, household policies, the density, value function, its derivative, the generator, and all of the planner's multipliers). $\eta_{2,t}$ and f_t should have initial conditions, as they are determined by forward looking equations. All other endogenous variables should have terminal conditions.

6. Solve the firm's problem for paths of m (from the NKPC), w (from firm labor FOC), Y, D, and B (from the assumption that B is a constant fraction of GDP). Since I guess π and N, iteration over time on the NKPC is unnecessary, I can simply calculate $\dot{\pi}$ and $\dot{N}$ directly.

7. Starting from the terminal steady state, solve the HJB (eq 1). Discretize time as $\dot{V} = \frac{V_{t+1}-V_t}{\mathrm{d}t}$ and solve the HJB with the implicit method of Achdou et al. (2021).

8. Solve the KFE (equation 5) forwards. Implicit updating implies $\frac{f_{t+1}-f_t}{\mathrm{d}t} = \mathcal{A}_t' f_{t+1}$ and requires solving the aforementioned for f_{t+1}.

9. Calculate aggregate consumption, labor, and bonds implied by the path of distributions and household policies. Then solve the government budget constraint for implied transfers, discretizing $\dot{B}$ in the gvt budget as $\frac{B_{t+1}-B_t}{\mathrm{d}t}$

10. Generate 3 vectors of residuals by verifying whether the implied paths of labor, bonds, and transfers from the previous step equal the guessed paths.

11. Now I solve the social planner's FOC's. First, iterate backwards in the same way as the household problem to solve for $\eta_{1,t}$ (equation 35). Use equation 46 to find the corresponding derivative.

12. Jointly solve the planner's first order conditions for individual choices and obtain η_7, η_8, η_9 (eqns 40-42). These are linear equations in the unknown multipliers that can be solved efficiently using sparse matrices.

13. Solve the planner's KFE (eq 36) forwards in the same way as the KFE to obtain $\eta_{2,t}$.

14. Solve the π FOC backwards for a path of $\eta_{3,t}$, setting $\dot{\eta}_{3,t} = \frac{\eta_{3,t+1} - \eta_{3,t}}{\mathrm{d}t}$.

15. Use the planner's first order condition for r_t and w_t (equation 38 and 39) and aggregates (equations 43-45) to calculate 4 vectors of residuals

16. If the 7 vectors of residuals (4 from verifying social planner FOCs hold, 3 from verifying competitive equilibrium conditions hold), stop. Otherwise update the guesses and repeat until convergence

I use a numerical solver in matlab to solve this system of $7 \cdot ns$ equations in $7 \cdot ns$ unknowns. The default algorithm converges quickly and reliably. All matrix inversions should be done with sparse matrices to speed up computation; otherwise computation time will be order(s) of magnitude slower.

Boundary Conditions At the borrowing constraint,

$$
\begin{aligned}
0 \leq \dot{b} &= rb + zwn + D + T - c^h - p(1+\tau)c^f \\
c^h + p(1+\tau)c^f &\leq rb + zwn + D + T \\
c^h \left(1 + p(1+\tau)\Theta_{i,\tau}\right) &\leq rb + zwn + D + T \\
(c^h)^{-\sigma}\mathcal{C}_{i,\tau} &\geq \left(\frac{rb + zwn + D + T}{(1 + p(1+\tau)\Theta_{i,\tau})}\right)^{-\sigma} \mathcal{C}_{i,\tau} \\
u_h &\geq \left(\frac{rb + zwn + D + T}{(1 + p(1+\tau)\Theta_{i,\tau})}\right)^{-\sigma} \mathcal{C}_{i,\tau} \\
V_b &\geq \left(\frac{rb + zwn + D + T}{(1 + p(1+\tau)\Theta_{i,\tau})}\right)^{-\sigma} \mathcal{C}_{i,\tau} \\
V_b &\geq u_h \left(\frac{rb + zwn + D + T}{(1 + p(1+\tau)\Theta_{i,\tau})}\right)
\end{aligned}
$$

where $u_h(c_h) \equiv c_h^{-\sigma}\mathcal{M}_{i,\tau}$

To evaluate the above formula when the inequality binds, need a solution for n. Labor supply at the *binding* boundary constraint $b = \phi$ is determined by

$$
\begin{aligned}
n^\kappa &= \frac{wz}{p^h}(c^h)^{-\sigma}\mathcal{C}_{i,\tau} \\
c^h &= \frac{r\phi + zwn + D + T}{(1 + p(1+\tau)\Theta_{i,\tau})}
\end{aligned}
$$

which is a quadratic equation under the assumption that $\kappa = \sigma = 1$

D Adjustment of Gvt Budget, Details

The variable that adjusts in the gvt's budget constraint affects equilibrium outcomes and (some of) the conditions to determine equilibrium, as Ricardian Equivalence does not hold. As a baseline, I assume T adjusts while B is a fixed fraction of GDP and τ_n is some fixed value (i.e. tariff revenue is redistributed to households as transfers). Below, I rewrite the different equilibrium conditions if instead τ_n or B adjust (tariff revenue is used to reduce the labor income tax or reduce the deficit while other variables are fixed at a certain level or fraction of GDP) and briefly describe how the computational algorithm changes. As the production subsidy s is designed to eliminate steady state markup distortions, I will not consider a case where it adjusts and instead always set it to $s = \frac{1}{1-\epsilon}$.

D.1 Adjusting Bonds

Here I assume that τ_n is a fixed number, T is some fraction of GDP ($T = T_{\text{perc}}Y$), and B adjusts to clear the government budget constraint. Since I will be able to eliminate one guess variable in the competitive equilibrium loop, I add the government budget constraint directly as an additional planner constraint with multiplier $\eta_{10,t}$ without affecting computation speed

$$0 = rB_t + T_{\text{perc}}N_t + sN_t - \tau pC_t^f - \tau^n wN_t - \dot{B}_t$$

The new savings function is

$$s^h = (r+\eta)b^h + (1-\tau^n)zwn + T_{\text{perc}}N_t + (1+s)N_t - m_tN_t - \frac{\theta}{2}\pi_t^2 N_t - c^h - p(1+\tau)c^f$$

The first order conditions for N_t, w_t, r_t, and C_t^f are affected and there is now an FOC for B_t; all others are unchanged.

The new FOC for N_t is

$$\eta_{5,t} = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\left(\frac{\partial \eta_{1,t}}{\partial b}f_t + \frac{\partial V_t}{\partial b}\eta_{2,t}\right)\left(T_{\text{perc}} + (1+s) - m - \frac{\theta}{2}\pi_t^2\right)\mathrm{d}b\,\mathrm{d}z + \frac{(\dot{\eta}_{3,t}\pi_t - \eta_{3,t}\tilde{\rho}\pi_t + \dot{\pi}\eta_{3,t})}{N_t} + \eta_{10,t}(T_{\text{perc}} + s - \tau_n w)$$

The new FOC for C_t^f is

$$-\eta_{10,t}\tau p - \eta_{6,t} = 0$$

There is no integral containing marginal social and private values since $\frac{\partial s^h}{\partial C^f} = 0$

The new FOC for r_t is

$$0 = \int_{z_1}^{z_J} \int_{\phi}^{\infty} \left(\frac{\partial \eta_{1,t}}{\partial b} f_t + \frac{\partial V_t}{\partial b} \eta_{2,t} \right) b \, \mathrm{d}b \, \mathrm{d}z + \pi_t \eta_{3,t} + B_t \eta_{10,t}$$

The new first order condition for w_t is

$$\eta_{3,t} \frac{\epsilon}{\theta} + \eta_{10,t} \tau_n N_t = \int_{z_1}^{z_J} \int_{\phi}^{\infty} ((z_t n_t - N_t) - \tau z_t n_t) \left(\frac{\partial \eta_{1,t}}{\partial b} f_t + \frac{\partial V_t}{\partial b} \eta_{2,t} \right) - (1 - \tau^n) z u_h(c^f, c^h) \eta_{7,t} \, \mathrm{d}b \, \mathrm{d}z$$

The first order condition for B_t is

$$0 = -\eta_{4,t} + \eta_{10,t} r - (\tilde{\rho} \eta_{10,t} - \dot{\eta}_{10,t})$$

where the final term is derived by integrating by parts over the time dimension and has initial condition $\eta_{10,0} = 0$

Now, instead of guessing N, r, and T to solve the competitive equilibrium and $\eta_{4,t}$, $\eta_{5,t}$, $\eta_{6,t}$, and π_t to verify the planner FOC's, I only need to guess N and r to solve the competitive equilibrium but need to also guess $\eta_{10,t}$ to verify the planner FOC. Similarly, in the transitional dynamics, I guess a path for $\eta_{10,t}$ instead of T.

E Inverse Optimal Approach, Details

This section provides details on the inverse optimal approach I take to obtain an efficient steady state.

E.1 Model

The new Lagrangian is

$$\mathcal{L} = \Big(\varphi(b,z)(u(c_t^h, c_t^f) - g(n_t)), f_t\Big)_\Phi + \left(\eta_{1,t}, \mathcal{A}^* f_t - \frac{\partial f}{\partial t}\right)_\Phi + \left(\eta_{2,t}, u(c_t^f, c_t^h) - g(n_t) + \mathcal{A}V_t + \frac{\partial V_t}{\partial t} - \tilde{\rho}V_t\right)_\Phi$$
$$\left(\eta_{3,t}, \pi_t(r_t^h - \frac{\dot{N}_t}{N_t}) - \frac{\epsilon}{\theta}(m_t - 1) - \dot{\pi}\right)_{\mathbb{R}^+} + \Big(\eta_{4,t}, f_t(b_t^h - B_t)\Big)_\Phi + \Big(\eta_{5,t}, f_t(n_t - N_t)\Big)_\Phi + \Big(\eta_{6,t}, f_t(c_t^f - C_t^f)\Big)_\Phi$$
$$+ \left(\eta_{7,t}, g'(n_t) - \frac{(1-\tau^n)w_t z_{j,t}}{p_t^h} u_h(c_t^h, c_t^f)\right)_\Phi + \left(\eta_{8,t}, u_h(c_t^h, c_t^f) - \frac{\partial V}{\partial b_t^h} p\right)_\Phi + \left(\eta_{9,t}, u_f(c_t^h, c_t^f) - \frac{\partial V}{\partial b_t^h} p(1+\tau)\right)$$

The FOC for c^h, c^f, n, and f differ. They are

$$\tilde{\rho}\eta_{1,t} = \varphi u(c_t^h, c_t^f) - \varphi g(n_t) + \mathcal{A}\eta_{1,t} + \frac{\mathrm{d}\eta_{1,t}}{\mathrm{d}t} + \eta_{4,t}(b_t^h - B_t) + \eta_{5,t}(n_t - N_t) + \eta_{6,t}(c_t^f - C_t^f)$$
$$0 = \varphi u_f(c_t^h, c_t^f) - \frac{\partial \eta_{1,t}}{\partial b}(p(1+\tau)) + \frac{\eta_{9,t}}{f_t} u_{ff}(c_t^h, c_t^f) - \frac{\eta_{7,t}}{f_t}\frac{w_t z_{j,t}(1-\tau^n)}{p_t^h} u_{hf}(c_t^h, c_t^f) + \frac{\eta_{8,t}}{f_t} u_{fh}(c_t^h, c_t^f) + \eta_{6,t}$$
$$0 = \varphi u_h(c_t^h, c_t^f) - \frac{\partial \eta_{1,t}}{\partial b} p + \frac{\eta_{9,t}}{f_t} u_{fh}(c_t^h, c_t^f) - \frac{\eta_{7,t}}{f_t}\frac{w_t z_{j,t}(1-\tau^n)}{p_t^h} u_{hh}(c_t^h, c_t^f) + \frac{\eta_{8,t}}{f_t} u_{hh}(c_t^h, c_t^f)$$
$$0 = -\varphi g'(n_t) + \frac{(1-\tau^n)w_t z_{j,t}}{p_t^h}\frac{\partial \eta_{1,t}}{\partial b} + \eta_{7,t}\frac{g''(n_t)}{f_t} + z\eta_{5,t}$$

Computation Recall that I define the weights as

$$\varphi(b,z) = \frac{u_h(C^h, C^f)}{u_h(c_i^h, c_i^f)} \tag{52}$$

where C^h, C^f are aggregate home and foreign consumption. First, I solve for the utilitarian planner steady state and obtain the welfare weights. I solve for the stationary equilibrium again, using these welfare weights. The allocations will be identical to the baseline, as inflation will remain 0 in the steady state, but planner multipliers will be different. I use these results to initialize and solve the transitional dynamics to the same shock as before. Crucially, planner welfare weights are constant and equal to those defined in equation 52.

F Endogenous Terms of Trade, Details

This section provides details on the model with endogenous terms of trade.

Derivation of Relative Price and Optimal Tariff The rest of the world is comprised of a continuum of countries $k \in (0,1)$ each with a representative household. Each country produces a single variety a la Armington aggregated into the (composite) foreign good with elasticity of substitution ϑ, i.e.

$$C^f_{k,t} = \left(\int (C^f_{j,k,t})^{\frac{\vartheta-1}{\vartheta}} \, \mathrm{d}j \right)^{\frac{\vartheta}{\vartheta-1}}$$

where $C_{j,k,t}$ is the consumption of variety j by country k at time t. The representative foreign household in country k has a value function V_k given by

$$\rho V_{k,t}(B) = \max_{C^h_{k,t}, C_{j,k,t}, N_{k,t}} u(C^h_{k,t}, C^f_{k,t}) - g(N_{k,t}) + \frac{\partial V_{k,t}}{\partial B} \dot{B}_t + \frac{\partial V_{k,t}}{\partial t}$$

$$\dot{B}_t = W_{k,t} N_{k,t} + D_{k,t} + r_t B_{k,t} - P^h_{k,t} - \int P_{j,k,t} C_{j,k,t}$$

where home and foreign consumption are aggregated according to the CES aggregator

$$(C_{k,t})^{\frac{\gamma-1}{\gamma}} = \omega (C^h_{k,t})^{\frac{\gamma-1}{\gamma}} + (1-\omega) \left(C^f_{k,t} \right)^{\frac{\gamma-1}{\gamma}}$$

with trade elasticity γ. Solving the household's problem, invoking the market clearing condition for the home produced good $N_t = C^h_t + C^{h*}_t$ (where C^{h*}_t is aggregated foreign consumption of the home produced good) yields equation 29 with the exogenous demand shifter $A = ((1-\omega) C^*_t)^{-\frac{1}{\vartheta}}$.

In this model, the usual static optimal tariff formula applies as foreign households are homogeneous, implying the foreign export supply function is only a function of the contemporaneous price (see section 2 of Dávila et al. (2025) for details). The first best tariff for the home country is then

$$\tau_{\text{opt}} = \frac{1}{\vartheta - 1}$$

In my numerical simulations, I will consider tariff shocks that deviate from this optimal benchmark.

F.1 Model

Denoting the relative price as $p(N, C^h)$, the new planner problem is

Proposition 6 (Primal Timeless Ramsey Problem with Endogenous Prices) *The timeless Ramsey planner's problem solves*

$$\min_{\{\eta_{k,t}\}_{k=1}^{1}0} \max_{r_t,\pi_t,V_t(.),f_t(.),c_t^f(.),c_t^h(.),n_t(.),w_t,N_t,C_t^f,C_t^h} \mathcal{L}(f_0) + \mathcal{T}(\eta_{3,0},\eta_{2,0}) \tag{53}$$

with the Lagrangian functional $\mathcal{L}(f_0)$ *defined as*

$$\mathcal{L}(f_0) \equiv \int_0^\infty e^{-\tilde{\rho}t}\{\int_{z_1}^{z_J}\int_\phi^\infty \varphi(b,z)\left[u(c_t^h,c_t^f)-g(n_t)\right]f_t + \eta_{1,t}(b,z)\left[\mathcal{A}^* f_t - \frac{\partial f_t}{\partial t}\right] +$$

$$\eta_{2,t}(b,z)\left[u(c_t^f,c_t^h)-g(n_t)+\mathcal{A}V_t+\frac{\partial V_t}{\partial t}-\rho V_t\right]+\eta_{3,t}\left[\pi_t(r_t^h-\frac{\dot{N}_t}{N_t})-\frac{\epsilon}{\theta}(m_t-1)-\dot{\pi}\right]+$$

$$\eta_{4,t}[b_t^h-B_t]f_t+\eta_{5,t}[n_t z_t-N_t]f_t+\eta_{6,t}[c_t^f-C_t^f]f_t+\eta_{7,t}(b,z)\left[g'(n_t)-\frac{w_t z_{j,t}(1-\tau^n)}{p_t^h}u_h(c_t^h,c_t^f)\right]$$

$$\eta_{8,t}(b,z)\left[u_h(c_t^h,c_t^f)-\frac{\partial V}{\partial b_t^h}\right]+\eta_{9,t}(b,z)\left[u_f(c_t^h,c_t^f)-\frac{\partial V}{\partial b_t^h}p(N_t,C^h)t(1+\tau)\right]+\eta_{10,t}[c_t^h-C_t^f]f_t\,\mathrm{d}b\,\mathrm{d}z\}\,\mathrm{d}t$$

where $\eta_{1,t},\eta_{2,t},\eta_{7,t},\eta_{8,t},\eta_{9,t}$ *are functional Lagrange multipliers and* $\eta_{3,t}$, $\eta_{4,t}$, $\eta_{5,t}$, $\eta_{6,t}$, *and* $\eta_{10,t}$ *are scalar Lagrange multipliers and* z_1 *and* z_J *are the lower and upper bounds, respectively, on the idiosyncratic shock* z. *Additionally, I assume that transfers adjust to clear the government budget constraint; the government budget is thus included implicitly in* $\mathcal{A}$ *by substituting for transfers.*

The timeless penalty $\mathcal{T}(\eta_{3,0},\eta_{2,0})$ *is defined as in Dávila and Schaab (2023) and equals:*

$$\int_{z_1}^{z_J}\int_\phi^\infty \eta_{2,0}V_0(b,z)\,\mathrm{d}b\,\mathrm{d}z+\eta_{3,0}\pi_0$$

The corresponding Ramsey plan are the paths of multipliers $\{\eta_{k,t}\}_{k=1}^{1}0$ *and choice variables* $r_t,\pi_t,V_t(.),f_t(.),c_t^f(.),c_t^h(.),n_t(.),w_t,N_t,C_t^f,C_t^h$ *that solve 53*

The new multiplier $\eta_{10,t}$ reflects a pecuniary externality on the relative price caused by marginal changes in home consumption.

However, in the heterogeneous agent case, if c^f is very low relative to C^f it is possible some households will see a net resource increase. For example, if foreign goods are a luxury, low wealth households will have $c^f \approx 0$ and will gain resources from an increase in the relative price. Thus, it is ex-ante unclear how the terms of trade motive affects optimal policy. If there are households who consume very few imports and those households have high marginal utility, the planner may want to have an even greater expansion than the exogenous price

case. However, under most parameterizations (including my baseline parameterization), this will not be the case. The planner will likely prefer less expansionary policy since expansion raises the relative price, which is welfare-reducing

where $s^h = rb^h + (1-\tau^n)zwn + \dot{B} - rB - sN + \tau p(N_t, C_t^h)C_t^f + \tau^n wN_t + (1+s)N_t - m_t N_t - \frac{\theta}{2}\pi_t^2 N_t - c^h - p(N_t, C_t^h)(1+\tau)c^f$.

There are different FOC for f_t, c_t^h, N_t, and a new FOC for C_t^h.

The FOC for f_t is

$$\tilde{\rho}\eta_{1,t} = u(c_t^h, c_t^f) - g(n_t) + \mathcal{A}\eta_{1,t} + \frac{\mathrm{d}\eta_{1,t}}{\mathrm{d}t} + \eta_{4,t}(b_t^h - B_t) + \eta_{5,t}(z_t n_t - N_t) + \eta_{6,t}(c_t^f - C_t^f) + \eta_{10,t}(c_t^h - C_t^h)$$

The FOC for c_t^h is

$$0 = u_h(c_t^h, c_t^f) - \frac{\partial \eta_{1,t}}{\partial b} + \frac{\eta_{9,t}}{f_t}u_{fh}(c_t^h, c_t^f) - \frac{\eta_{7,t}}{f_t}\frac{(1-\tau^n)w_t z_{j,t}}{p_t^h}u_{hh}(c_t^h, c_t^f) + \frac{\eta_{8,t}}{f_t}u_{hh}(c_t^h, c_t^f) + \eta_{10,t}$$

The FOC for N_t is

$$\eta_{5,t} = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\left(\frac{\partial \eta_{1,t}}{\partial b}f_t + \frac{\partial V_t}{\partial b}\eta_{2,t}\right)\left(1 - m_t - \frac{\theta}{2}\pi_t^2 + \tau w + \frac{\partial p}{\partial N_t}\left(\tau C_t^f - (1+\tau)c^f\right)\right)\mathrm{d}b\,\mathrm{d}z + \frac{(\dot{\eta}_{3,t}\pi_t - \eta_{3,t}\tilde{\rho}\pi_t + \dot{\pi}\eta_{3,t})}{N_t} - \int_{z_1}^{z_J}\int_{\phi}^{\infty}\eta_{9,t}(\frac{\partial V}{\partial b_t^h}\frac{\partial p}{\partial N}(1+\tau))\,\mathrm{d}b\,\mathrm{d}z$$

The FOC for C_t^h is

$$\eta_{10,t} = \int_{z_1}^{z_J}\int_{\phi}^{\infty}\left(\frac{\partial \eta_{1,t}}{\partial b}f_t + \frac{\partial V_t}{\partial b}\eta_{2,t}\right)\frac{\partial p}{\partial C_t^h}\left(\tau C_t^f - (1+\tau)c^f\right) - \eta_{9,t}(\frac{\partial V}{\partial b_t^h}\frac{\partial p}{\partial C^h}(1+\tau))\,\mathrm{d}b\,\mathrm{d}z$$

The new multiplier $\eta_{10,t}$ reflects how marginal changes in home consumption affect the relative price. The first order condition for aggregate labor and home consumption additionally reflect how the planner optimally manipulates terms of trade, accounting for how the consumption bundle of the household changes ($\frac{\partial p}{\partial C_t^h}(1+\tau)c^f$), how the change in the relative price affects the transfer the household receives ($\frac{\partial p}{\partial C_t^h}\tau C^f$), and how the change in the relative price directly affects the household's FOC for foreign consumption ($\eta_{9,t}\frac{\partial V}{\partial b_t^h}\frac{\partial p}{\partial C^h}(1+\tau)$)

F.2 New Planner Rule

I can derive a new planner targeting rule that illustrates the terms of trade motive

Proposition 7 (An Endogenous Price Targeting Rule) *Optimal interest rate policy is determined by the following equation*

$$0 = \underbrace{\mathbb{E}_{f_t}\left[(1-m_t)\frac{u_h}{p}\right]}_{\text{Output Gap}} - \underbrace{\mathbb{E}_{f_t}\left[\frac{\theta}{2}\pi_t^2\frac{u_h}{p}\right]}_{\text{Inflation Costs}} - \frac{\Omega_2}{\Omega_1}\underbrace{\mathbb{E}_{f_t}\left[\frac{\partial s}{\partial T}\frac{\partial T}{\partial C^f}\frac{u_h}{p}\right]}_{\text{Tariff Fiscal Externality}} +$$
$$\underbrace{\mathbb{E}_{f_t}[\frac{\partial s}{\partial T}\frac{\partial IncTax}{\partial N}\frac{u_h}{p}]}_{\text{Income Tax Fiscal Externality}} + \frac{1}{N_t}\left(\frac{\dot\eta_{3,t}}{\eta_{3,t}} - \tilde\rho + \frac{\dot\pi_t}{\pi_t}\right)\underbrace{\mathbb{E}_{f_t}\left[(B-b)\frac{u_h}{p}\right]}_{\text{Redistribution}} + \underbrace{\mathbb{E}_{f_t}[\frac{\partial p}{\partial N}\left(\tau C^f - (1+\tau)c^f\right)u_h]}_{\text{Terms of Trade Manipulation}}$$
$$\underbrace{\int\int \frac{1}{N_t}\left(\frac{\dot\eta_{3,t}}{\eta_{3,t}} - \tilde\rho + \frac{\dot\pi_t}{\pi_t}\right)[B-b]\frac{u_h}{p}\eta_{2,t} + \frac{\partial s}{\partial N}\frac{u_h}{p}\eta_{2,t} - \frac{\Omega_2}{\Omega_1}\frac{\partial s}{\partial T}\frac{\partial T}{\partial C^f}\frac{u_h}{p}\eta_{2,t} + \frac{\partial p}{\partial N}\left(\tau C^f - (1+\tau)c^f\right)u_h\eta_{2,t}\,\mathrm{d}b}_{\text{Commitment Penalties}}$$

where Ω_1 and Ω_2 are (endogenous and generally positive) constants defined in this proposition's proof in Appendix F

This targeting rule features a new term that captures how the planner values terms of trade manipulation: $\mathbb{E}_{f_t}[\frac{\partial p}{\partial N}\left(\tau C^f - (1+\tau)c^f\right)u_h]$. Expansionary policy will increase the relative price p, since output will increase and home consumption will increase by less than one-to-one. Increasing the price will give all agents additional transfers equal to $\frac{\partial p}{\partial N}\tau C^f$, but make their consumption of the foreign good more expensive by a factor of $\frac{\partial p}{\partial N}(1+\tau)c^f$. In the representative agent case $c^f = C^f$, so an increase in the relative price unambiguously costs the household resources, dampening the planner's incentives to expand.

I now derive a new planner targeting rule that illustrates the terms of trade motive. Promise keeping values become

$$\mathcal{H}_t^f \equiv -\frac{\eta_{9,t}}{f_t}u_{ff}(c_t^h, c_t^f) + \frac{\eta_{7,t}}{f_t}\frac{w_t z_{j,t}(1-\tau^n)}{p_t^h}u_{hf}(c_t^h, c_t^f) - \frac{\eta_{8,t}}{f_t}u_{fh}(c_t^h, c_t^f)$$
$$\mathcal{H}_t^h \equiv -\frac{\eta_{9,t}}{f_t}u_{fh}(c_t^h, c_t^f) + \frac{\eta_{7,t}}{f_t}\frac{w_t z_{j,t}(1-\tau^n)}{p_t^h}u_{hh}(c_t^h, c_t^f) - \frac{\eta_{8,t}}{f_t}u_{hh}(c_t^h, c_t^f)$$
$$\mathcal{H}_t^n \equiv -\eta_{7,t}\frac{g''(n_t)}{f_t}$$

In the following derivations, I follow the proof of Proposition 3 in appendix B. More details can be found there. I obtain two equations, combining planner first order conditions exactly as in the baseline model

$$\eta_{6,t} = \int_{z_1}^{z_J} \int_{\phi}^{\infty} \frac{\partial s}{\partial T} \frac{\partial T}{\partial C^f} \left[\frac{u_f}{p} - \frac{\mathcal{H}_t^f}{p} + \frac{u_f}{p} \frac{\eta_{2,t}}{f_t} \right] f_t \, \mathrm{d}b \, \mathrm{d}z$$

$$\eta_{5,t} = \int_{z_1}^{z_J} \int_{\phi}^{\infty} \frac{\partial s^h}{\partial N} \left[u_h + \frac{\mathcal{H}_t^n}{(1-\tau^n)wz} + u_h \frac{\eta_{2,t}}{f_t} \right] f_t \, \mathrm{d}b \, \mathrm{d}z - \int_{z_1}^{z_J} \int_{\phi}^{\infty} \frac{\partial s^h}{\partial N} \frac{\eta_{5,t}}{z w_t (1-\tau^n)} \, \mathrm{d}b \, \mathrm{d}z$$
$$- \frac{\partial p}{\partial N} \int_{z_1}^{z_J} \int_{\phi}^{\infty} \eta_{9,t} (\frac{\partial V}{\partial b_t^h}(1+\tau)) \, \mathrm{d}b \, \mathrm{d}z + \frac{1}{N_t} \left(\frac{\dot{\eta}_{3,t}}{\eta_{3,t}} - \tilde{\rho} + \frac{\dot{\pi}_t}{\pi_t} \right) \int_{z_1}^{z_J} \int_{\phi}^{\infty} (B_t - b_t^h) \left[u_h - \frac{\mathcal{H}_t^f}{p(1+\tau)} + u_h \frac{\eta_{2,t}}{f_t} \right] f_t$$

I then define Ω_1 and Ω_2 in the same way as before.

$$\Omega_1 = \eta_{6,t} + \int_{z_1}^{z_J} \int_{\phi}^{\infty} \frac{\partial s}{\partial T} \frac{\partial T}{\partial C^f} \frac{\mathcal{H}_t^f}{p} f_t \, \mathrm{d}b \, \mathrm{d}z$$
$$\Omega_2 = \eta_{5,t} \left(1 + \int_{z_1}^{z_J} \int_{\phi}^{\infty} \frac{\partial s^h}{\partial N} \frac{1}{z w_t (1-\tau^n)} f_t \, \mathrm{d}b \, \mathrm{d}z \right) + \frac{\partial p}{\partial N} \int_{z_1}^{z_J} \int_{\phi}^{\infty} \eta_{9,t} (\frac{\partial V}{\partial b_t^h}(1+\tau)) \, \mathrm{d}b \, \mathrm{d}z$$
$$+ \int_{z_1}^{z_J} \int_{\phi}^{\infty} \left[\frac{1}{N_t} \left(\frac{\dot{\eta}_{3,t}}{\eta_{3,t}} - \tilde{\rho} + \frac{\dot{\pi}_t}{\pi_t} \right) (B - b^h) \frac{\mathcal{H}_t^f}{p(1+\tau)} - \frac{\partial s^h}{\partial N} \frac{\mathcal{H}_t^n}{w_t z_t (1-\tau^n)} \right] \mathrm{d}b \, \mathrm{d}z$$

After some algebra and simplifications, I obtain the targeting rule in the main body.

G Progressive Transfers, Details

This section provides details on the model with progressive transfers.

Model I now detail the different features of the model. In this section, I assume a general transfer function $\mathcal{T}(b, z)$. The new government budget constraint is

$$\dot{B}_t = rB_t + sp_t^h Y_t + \int \mathcal{T}(b_t, z_t) \, \mathrm{d}f(b, z) - \tau p^f \int c_f \, \mathrm{d}f(b, z) - \tau_n w_t \int nz \, \mathrm{d}f(b, z)$$

The new savings function is

$$s^h = (r+\eta)b + (1-\tau^n)zwn + \mathcal{T}(b_t, z_t) + \underbrace{z \left((1+s)N_t - m_t N_t - \frac{\theta}{2} \pi_t^2 N_t \right)}_{D(z)} - c^h - p(1+\tau)c^f$$

Proposition 8 (Primal Timeless Ramsey Problem with Progressive Transfers) *The timeless Ramsey planner's problem solves*

$$\min_{\{\eta_{k,t}\}_{k=1}^1 0} \max_{r_t, \pi_t, V_t(.), f_t(.), c_t^f(.), c_t^h(.), n_t(.), w_t, N_t, C_t^f, \mathcal{T}(.)} \mathcal{L}(f_0) + \mathcal{T}(\eta_{3,0}, \eta_{2,0}) \tag{54}$$

with the Lagrangian functional $\mathcal{L}(f_0)$ *defined as*

$$\mathcal{L}(f_0) \equiv \int_0^\infty e^{-\tilde{\rho}t}\{\int_{z_1}^{z_J}\int_\phi^\infty \left[u(c_t^h, c_t^f) - g(n_t)\right] f_t + \eta_{1,t}(b,z)\left[\mathcal{A}^* f_t - \frac{\partial f_t}{\partial t}\right] +$$

$$\eta_{2,t}(b,z)\left[u(c_t^f, c_t^h) - g(n_t) + \mathcal{A}V_t + \frac{\partial V_t}{\partial t} - \rho V_t\right] + \eta_{3,t}\left[\pi_t(r_t^h - \frac{\dot{N}_t}{N_t}) - \frac{\epsilon}{\theta}(m_t - 1) - \dot{\pi}\right] +$$

$$\eta_{4,t}[b_t^h - B_t]f_t + \eta_{5,t}[n_t z_t - N_t]f_t + \eta_{6,t}[c_t^f - C_t^f]f_t + \eta_{7,t}(b,z)\left[g'(n_t) - \frac{w_t z_{j,t}(1-\tau^n)}{p_t^h}u_h(c_t^h, c_t^f)\right]$$

$$\eta_{8,t}(b,z)\left[u_h(c_t^h, c_t^f) - \frac{\partial V}{\partial b_t^h}\right] + \eta_{9,t}(b,z)\left[u_f(c_t^h, c_t^f) - \frac{\partial V}{\partial b_t^h}p(1+\tau)\right] +$$

$$\eta_{10,t}[\mathcal{T}(b,z) - \dot{B}_t + r_t B_t + sN_t - \tau_t p C_t^f - \tau^n w_t N_t]f_t \,\mathrm{d}b\,\mathrm{d}z\}\,\mathrm{d}t$$

where $\eta_{1,t}, \eta_{2,t}, \eta_{7,t}, \eta_{8,t}, \eta_{9,t}$ *are functional Lagrange multipliers and* $\eta_{3,t}$, $\eta_{4,t}$, $\eta_{5,t}$, $\eta_{6,t}$, *and* $\eta_{10,t}$ *are scalar Lagrange multipliers and* z_1 *and* z_J *are the lower and upper bounds, respectively, on the idiosyncratic shock* z. *Additionally, I assume that transfers adjust to clear the government budget constraint; the government budget is thus included implicitly in* $\mathcal{A}$ *by substituting for transfers.*

The timeless penalty $\mathcal{T}(\eta_{3,0}, \eta_{2,0})$ *is defined as in Dávila and Schaab (2023) and equals:*

$$\int_{z_1}^{z_J}\int_\phi^\infty \eta_{2,0}V_0(b,z)\,\mathrm{d}b\,\mathrm{d}z + \eta_{3,0}\pi_0$$

The corresponding Ramsey plan are the paths of multipliers $\{\eta_{k,t}\}_{k=1}^1 0$ *and choice variables* $r_t, \pi_t, V_t(.), f_t(.), c_t^f(.), c_t^h(.), n_t(.), w_t, N_t, C_t^f, \mathcal{T}(.)$ *that solve 54*

I explicitly describe the planner first order conditions in appendix G.[28]

[28]Note that even though the problem has an additional choice variable and transfers are not flat, the only first order condition that is explicitly different is the planner HJB; each other first order condition, including the fiscal externality, is expressed exactly as in the baseline model. The planner HJB reflects a new pecuniary externality on transfers, as changes in wealth will now affect idiosyncratic and therefore aggregate transfers:

$$\tilde{\rho}\eta_{1,t} = u(c_t^h, c_t^f) - g(n_t) + \mathcal{A}\eta_{1,t} + \frac{\mathrm{d}\eta_{1,t}}{\mathrm{d}t} + \eta_{4,t}(b_t^h - B_t) + \eta_{5,t}(z_t n_t - N_t) + \eta_{9,t}(c_t^f - C_t^f) + \eta_{10,t}(\mathcal{T}(b_t, z_t) - T_t)$$

G.1 Model

The new government budget constraint is

$$\dot{B}_t = rB_t + sp_t^h Y_t + \int \mathcal{T}(b_t)\,\mathrm{d}f(b,z) - \tau p^f \int c_f\,\mathrm{d}f(b,z) - \tau_n w_t \int nz\,\mathrm{d}f(b,z)$$

The new savings function is

$$s^h = (r+\eta)b + (1-\tau^n)zwn + \mathcal{T}(b) + (1+s)N_t - m_t N_t - \frac{\theta}{2}\pi_t^2 N_t - c^h - p(1+\tau)c^f$$

I then need to add the government budget/transfer clearing as a new planner condition with multiplier $\eta_{10,t}$, so the planner Lagrangian is

$$\mathcal{L} = \left(u(c_t^h, c_t^f) - g(n_t), f_t\right)_\Phi + \left(\eta_{1,t}, \mathcal{A}^* f_t - \frac{\partial f}{\partial t}\right)_\Phi + \left(\eta_{2,t}, u(c_t^f, c_t^h) - g(n_t) + \mathcal{A}V_t + \frac{\partial V_t}{\partial t} - \tilde{\rho}V_t\right)_\Phi$$
$$\left(\eta_{3,t}, \pi_t(r_t^h - \frac{\dot{N}_t}{N_t}) - \frac{\epsilon}{\theta}(m_t - 1) - \dot{\pi}\right)_{\mathbb{R}^+} + \left(\eta_{4,t}, f_t(b_t^h - B_t)\right)_\Phi + \left(\eta_{5,t}, f_t(n_t - N_t)\right)_\Phi + \left(\eta_{6,t}, f_t(c_t^f - C_t^f)\right)_\Phi$$
$$+ \left(\eta_{7,t}, g'(n_t) - \frac{(1-\tau^n)w_t z_{j,t}}{p_t^h} u_h(c_t^h, c_t^f)\right)_\Phi + \left(\eta_{8,t}, u_h(c_t^h, c_t^f) - \frac{\partial V}{\partial b_t^h} p\right)_\Phi + \left(\eta_{9,t}, u_f(c_t^h, c_t^f) - \frac{\partial V}{\partial b_t^h} p(1+\tau)\right)_\Phi$$
$$+ \left(\eta_{10,t}, f_t(\mathcal{T}(b_t) - \dot{B}_t + rB_t + sp_t^h N_t - \tau p^f C^f - \tau_n w_t N_t)\right)_\Phi$$

adding $\mathcal{T}(b)$ as a planner choice (since it cannot be substituted for anymore in the household budget). Planner choices of r, N, C^f, w, and f are different than in the baseline model; other choices are unaffected.

The planner FOC for f is

$$\tilde{\rho}\eta_{1,t} = u(c_t^h, c_t^f) - g(n_t) + \mathcal{A}\eta_{1,t} + \frac{\mathrm{d}\eta_{1,t}}{\mathrm{d}t} + \eta_{4,t}(b_t^h - B_t) + \eta_{5,t}(z_t n_t - N_t) + \eta_{9,t}(c_t^f - C_t^f) + \eta_{10,t}(\mathcal{T}(b_t) - T_t)$$

where $T_t = \dot{B}_t - rB_t - sp_t^h N_t + \tau p^f C^f + \tau_n w_t N_t$

The planner FOC for N is

$$\eta_{5,t} = \int_{z_1}^{z_J}\int_\phi^\infty \left(\frac{\partial \eta_{1,t}}{\partial b} f_t + \frac{\partial V_t}{\partial b}\eta_{2,t}\right)\frac{\partial s^h}{\partial N}\,\mathrm{d}b\,\mathrm{d}z + \frac{(\dot{\eta}_{3,t}\pi_t - \eta_{3,t}\tilde{\rho}\pi_t + \dot{\pi}\eta_{3,t})}{N_t} + \eta_{10,t}(s - \tau_n w)$$

The planner FOC for C^f is

$$\eta_{6,t} = -\eta_{10,t}\tau p$$

The planner FOC for r_t is

$$0 = \int_{z_1}^{z_J} \int_{\phi}^{\infty} \left(\frac{\partial \eta_{1,t}}{\partial b} f_t + \frac{\partial V_t}{\partial b} \eta_{2,t} \right) b \, \mathrm{d}b \, \mathrm{d}z + \pi_t \eta_{3,t} + B_t \eta_{10,t}$$

The planner FOC for w_t is

$$\eta_{3,t} \frac{\epsilon}{\theta} + \eta_{10,t} \tau_n N_t = \int_{z_1}^{z_J} \int_{\phi}^{\infty} ((z_t n_t - N_t) - \tau z_t n_t) \left(\frac{\partial \eta_{1,t}}{\partial b} f_t + \frac{\partial V_t}{\partial b} \eta_{2,t} \right) - (1 - \tau^n) z u_h(c^f, c^h) \eta_{7,t} \, \mathrm{d}b \, \mathrm{d}z$$

The planner FOC for $\mathcal{T}(b)$ is

$$\left(\frac{\partial \eta_{1,t}}{\partial b} f_t + \frac{\partial V_t}{\partial b} \eta_{2,t} \right) \frac{\partial s^h}{\partial \mathcal{T}(b)} = -f_t \eta_{10,t} \tag{55}$$

with $\frac{\partial s^h}{\partial \mathcal{T}(b)} = 1$.

The FOC for r, N, C^f, w in this formulation can be re-expressed as exactly the FOC in the baseline formulation of the model with non-progressive transfers. I integrate eq 55 over b and z and substitute for $\eta_{10,t}$ in those 4 FOC. The planner HJB is the only FOC that has a different expression as long as transfers are not flat, i.e. $\mathcal{T}(b) = T \quad \forall b$ does not hold. Then, the social value function, $\eta_{1,t}$, and its derivative, $\frac{\partial \eta_{1,t}}{\partial b}$, are different.

H RANK, Details

Here, I explicitly characterize the first order conditions for optimal policy in the RANK limit of my economy. In particular, the RANK limit I consider is one with the death rate $\eta = 0$, the borrowing limit set to the natural borrowing limit, and the standard deviation of the idiosyncratic shock equal to zero.[29] This implies the density is a point mass, individual labor supply is equal to aggregate labor supply, individual foreign consumption is equal to aggregate foreign consumption, and individual bond holdings are equal to aggregate bond holdings. I rewrite the Lagrangian of the planner with multipliers indexed by λ and time subscripts omitted for notational simplicity:

$$\mathcal{L} = \int_0^{\infty} e^{-\rho t} \{ u(C^h, C^f) - g(N) + \lambda_1 \left[C^h \frac{1}{\sigma}(r - \rho) - \dot{C}^h \right] + \lambda_3 \left[g'(N) - u_h(C^h, C^f)(1 - \tau_n) w \right]$$
$$+ \lambda_2 \left[C^f - C^h \Theta_\tau \right] + \lambda_4 \left[\pi_t (r - \frac{\dot{N}}{N}) - \frac{\epsilon}{\theta}(w - 1) - \dot{\pi} \right] + \lambda_5 \left[N - \frac{\theta}{2} \pi^2 N - C^h - p(1 + \tau) C^f + T(C^f) \right] \}$$

[29] Since the generator $\mathcal{A}$ would be non-invertible and f would be a point mass, it is easier computationally to solve an explicit RANK representation of the model rather than the implicit representation with the idiosyncratic shock, death rate, and borrowing limit removed.

The first three planner constraints (λ_1, λ_2, λ_3) are household optimality conditions: the Euler equation w/ risk aversion coefficient σ, the distribution of home and foreign consumption with $\Theta_\tau = \left(\frac{1-\omega}{\omega(1+\tau)p}\right)^\gamma$, and the intratemporal labor/leisure choice. The final two conditions (λ_4 and λ_5) are the Phillips Curve and goods market clearing. To make the fiscal externality more explicit in the derivation that follows, I follow Bianchi and Coulibaly (2025) and define the (rebated) fiscal revenue as $T(C^f) = p\tau C^f$.

Again, this Lagrangian is a functional, so I take Gâteaux derivatives with respect to π, r, w, N, C^h, and C^f to generate planner first order conditions. The FOC are

$$\lambda_4\left(r - \frac{\dot{N}}{N} - \rho\right) + \dot{\lambda}_4 = \lambda_5\theta\pi N$$

$$\frac{\lambda_1 C^h}{\sigma} = -\lambda_4\pi$$

$$\lambda_4\frac{\epsilon}{\theta} = -\lambda_3\left(u_h(C^h, C^f)(1-\tau_n)\right)$$

$$g'(N) = \lambda_3 g''(N) + \lambda_5(1 - \frac{\theta}{2}\pi^2) + \frac{\dot{\lambda}_4\pi - \lambda_4\pi\rho + \dot{\pi}\lambda_4}{N}$$

$$0 = u_h(C^h, C^f) + \lambda_1\left[\frac{1}{\sigma}(r-\rho)\right] - \rho\lambda_1 + \dot{\lambda}_1 + \lambda_2\left[-\Theta_\tau\right] + \lambda_3\left[-u_{hh}(C^h, C^f)(1-\tau_n)w\right] - \lambda_5$$

$$0 = u_f(C^h, C^f) + \lambda_2 + \lambda_3\left[-u_{hf}(C^h, C^f)(1-\tau_n)w\right] - \lambda_5 p(1+\tau) + \lambda_5\frac{\partial T}{\partial C^f}$$

Like in HANK, multipliers λ_1, λ_2, λ_3 are promise-keeping values. The planner may prefer to not obey private first order conditions; these multipliers/penalties force him to do so. Consider the first order condition on foreign consumption. When choosing the optimal level of foreign consumption, the planner trades off the marginal utility gain u_f, the marginal cost to the household of foreign consumption $p(1+\tau)$, the marginal effect on the slackness of household first order conditions (λ_2 and λ_3 terms), and the fiscal revenue gained $\frac{\partial T}{\partial C^f}$. Since λ_5 is the multiplier on the resource constraint, I interpret λ_5 as the *social* marginal value of wealth. For the private household, the first order condition for foreign consumption is $u_f(C^h, C^f) = \tilde{\lambda}p(1+\tau)$ where $\tilde{\lambda}$ is the multiplier on the household budget constraint or the *private* marginal value of wealth. Unlike the household, the planner internalizes how changes in foreign consumption affect transfers, valuing them according to $\lambda_5\frac{\partial T}{\partial C^f} > 0$. This is the fiscal externality. Since this term is positive, the planner will want higher foreign consumption.

This intuition extends directly to HANK. First, I will rewrite the planner's FOC for foreign consumption in RANK by condensing the "promise-keeping" terms

$$0 = u_f(C_t^h, C_t^f) - \mathcal{H}_t^{f,RA} - \lambda_{5,t} p(1+\tau_t) + \lambda_{5,t} \frac{\partial T}{\partial C^f}$$

Then, in HANK, I combine the first order conditions for individual and aggregate foreign consumption, defining $\mathcal{H}_t^f$ as in appendix B.

$$0 = \int_{z_1}^{z_J} \int_{\phi}^{\infty} \left[u_f(c_t^h, c_t^f) - \mathcal{H}_t^f - \frac{\partial \eta_{1,t}}{\partial b} p(1+\tau) + \frac{\partial s^h}{\partial T} \frac{\partial T}{\partial C^f} \left(\frac{\partial \eta_{1,t}}{\partial b} + \frac{\partial V_t}{\partial b} \frac{\eta_{2,t}}{f_t} \right) \right] f_t \, \mathrm{d}b \, \mathrm{d}z$$

One can show these equations are identical when the HANK economy sets the death rate to 0, the variance of the idiosyncratic shock to 0, and the borrowing limit to its natural value (the RANK limit). When choosing the optimal level of foreign consumption in HANK, the planner still trades off the same benefits and costs as in RANK, in HANK he simply values these benefits differently. In RANK $\frac{\partial s^h}{\partial T} = 1$; in HANK this need not be the case. Similarly, $\frac{\partial \eta_{1,t}}{\partial b}$ is heterogeneous across HANK households (and the planner must account for the distributional penalties), the analogous marginal social value of wealth $\lambda_{5,t}$ is homogeneous in RANK.

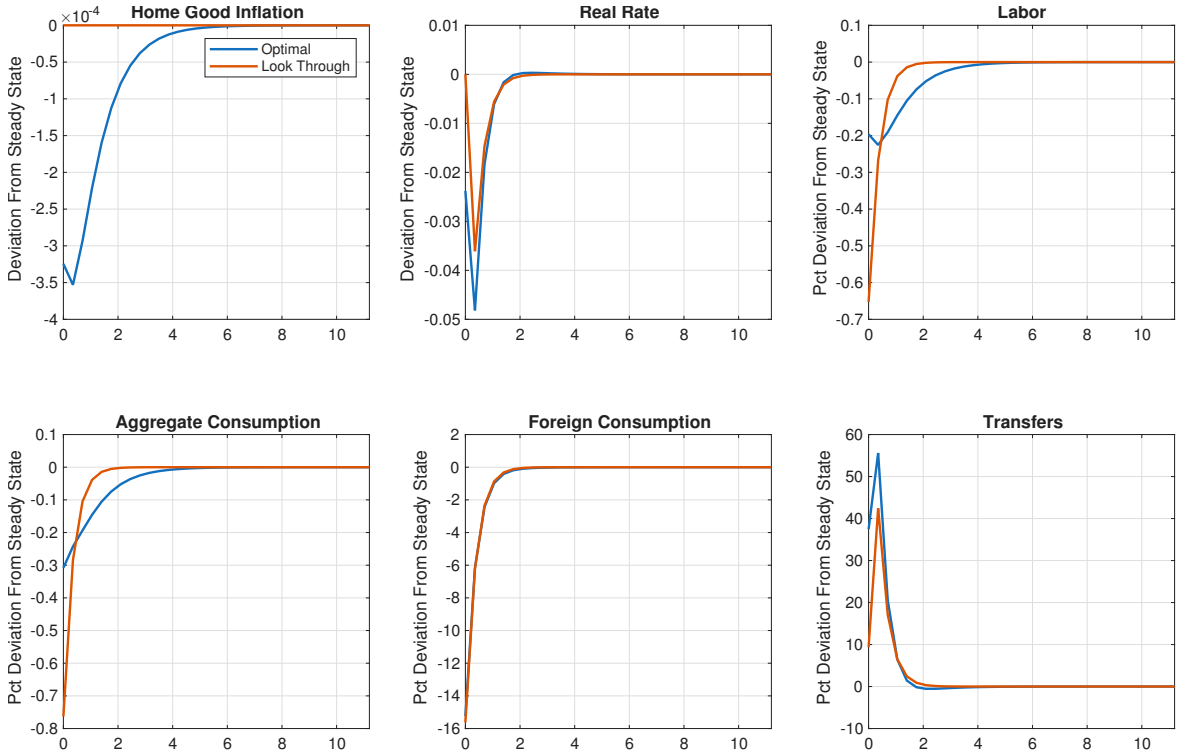
H.1 Numerical Results

The 6 planner FOC, the 3 household FOC, the Phillips curve, and market clearing jointly determine the values of all planner multipliers and the 6 economic variables. For computation, it is sufficient to guess (paths) for r, π, and λ_1. Given these guesses, it is straightforward to back out all other unknowns and then use the remaining 3 equations as residuals.

Figure 15 plots the impulse response for interest rates, inflation, aggregate and foreign consumption, labor, and transfers for the same shock as in figure 1 for the RANK model under the optimal policy response and the look-through response. Though I still see (slightly) less of a decrease in foreign consumption and an increase in transfers in line with the logic of Bianchi and Coulibaly (2025), I do not see the optimally inflationary response that they do since I assume the interest rate is endogenous, rather than set to $r^* = \rho$ as they do. However, the general intuition that the fiscal externality incentivizes the planner to have

higher output and a stronger rate cut to induce higher consumption to counteract the effects of the tariff remains in my model.

Figure 15: Optimal vs Look Through Policy, RANK



Notes: For the path of tariffs from figure 1, I plot impulse responses to aggregate variables in RANK. Home good inflation and interest rates are plotted as deviations from the steady state, while other variables are percent deviations from the steady state. 3

Figure 16: Optimal Policy, HANK vs RANK

