Wealth Sorting and Cyclical Employment Risk

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Abstract

I present empirical evidence that US workers with low liquid wealth face significantly more cyclical employment. This finding cannot be fully explained by skills or socio-demographics; liquid wealth itself plays a role. I propose a framework that generates a negative correlation between wealth and employment risk in equilibrium, through job sorting based on wealth. Job search provides an insurance mechanism for liquidity constrained and risk-averse unemployed workers. Wealth-poor unemployed sort into relatively lower wage and lower security jobs, because these jobs offer a relatively higher job finding probability. I build a quantitative model to study how wealth sorting shapes consumption, wages and job transitions over the business cycle, and to assess its implications for the persistence of inequality. The interaction between employment risk and wealth accumulation generates a "poverty trap" which is amplified in bad times. The cost of entering unemployment during a recession amounts to 3% of lifetime consumption for the poorest, more than twice that of the wealthiest.
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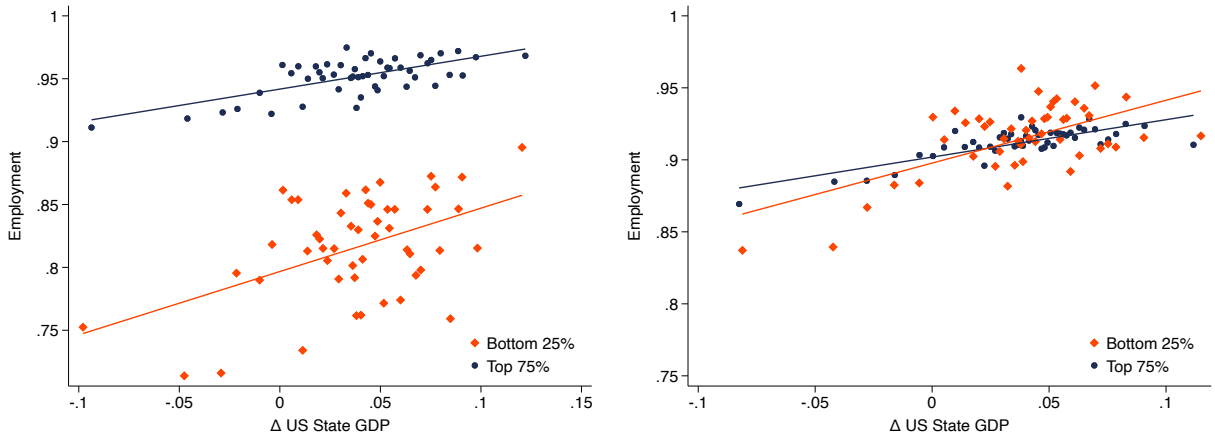
1 Introduction

Despite the high degree of turnover in the labor market, where workers frequently transition across employment states, borrowing constraints may cause financial distress to translate into persistently worse employment prospects.[1] One labor market outcome that has received relatively little attention – but is particularly relevant from the perspective of heterogeneous agent macroeconomics – is cyclical employment risk. Understanding the relationship between wealth and employment risk is important for assessing how risk is distributed across the wealth distribution, how effectively households can smooth consumption over the business cycle, and how labor market dynamics interact with inequality.
In this paper, I examine the relationship between workers' wealth and their cyclical employment risk empirically, theoretically, and through the lens of a quantitative model.

I begin by documenting that US workers at the bottom of the liquid wealth distribution experience greater sensitivity of employment to changes in output. Using the Panel Study of Income Dynamics (PSID), Figure 1 provides illustrative evidence by plotting GDP growth against employment separately for workers at the bottom and at the top of the liquid wealth distribution (diamonds and dots, respectively). Wealth-poor workers are, on average, less likely to be employed (left panel). This level difference across wealth groups disappears once observable worker characteristics and unobservable heterogeneity are controlled for (right panel) and, more importantly for this paper, the relationship between GDP growth and employment remains steeper for workers at the bottom of the liquid wealth distribution.

Figure 1: Employment and GDP fluctuations by liquid wealth

(a) No controls **(b)** With controls



Note: Observations come from the Core PSID dataset and covers years from 1999 to 2017. Dots represent mean values of x and y within 50 equally populated bins. The $x-$ axis reports log changes in US state real GDP; the $y-$axis reports the employment status (1 if employed, 0 if unemployed). The left panel displays the unconditional correlation. The right panel controls for individual fixed effects, and a quadratic in age interacted with education, gender, marital status, and race. Liquid wealth is defined as the sum of savings account balances and stock holdings. Orange diamonds represent workers in the bottom quartile of the liquid wealth distribution in a given year – roughly those with zero liquid wealth.

[1] In the United States, more than a quarter of working-age households lack sufficient savings to cover one month of expenditures after a job loss (Catherine et al., 2020).

To complement this descriptive evidence, I estimate regressions of individual employment status on output growth interacted with individuals' past liquid wealth. I find that a one standard deviation decline in US state GDP significantly reduces the employment probability of individuals with zero liquid wealth by nearly five times as much as that of individuals with median wealth. These estimates are obtained from a specification saturated with individual controls and fixed effects. As such, they capture a *residual* inequality in employment cyclicality attributable to heterogeneity in liquid wealth holdings.[2] In addition, I find that wealth-poor workers not only exhibit higher employment elasticity over the business cycle, but also face a higher probability of job loss in the cross-section. Importantly, the relevant distinction is whether a worker is *liquid* wealth-poor at a given point in time. Classifying agents based on illiquid or "permanent" wealth does not generate significant differences in employment elasticity with respect to output growth.
The fact that wealth-poor workers bear more employment risk challenges standard models of job acceptance under risk aversion, which predict that poorer workers are more likely to reject low-security jobs. A key contribution of this paper is thus to provide a theoretical rationale for the negative correlation between liquid wealth and cyclical employment risk.

Next, I develop a theoretical framework that can generate the observed empirical correlation in equilibrium. Time lasts for two periods, job search is directed, and the labor market features two-sided heterogeneity: workers differ in wealth, and jobs differ in their separation risk (security). Firms post wages, and there is free entry for vacancy posting – subject to a cost. Workers are risk-averse and can save at the risk-free interest rate, up to a borrowing constraint. Employed workers take a consumption-savings decision, while unemployed workers additionally search for a job.
In this economy, workers sort into different types of jobs based on their level of wealth. I provide sufficient conditions for the equilibrium to feature *positive wealth sorting*, i.e., for wealth-poor workers to sort into relatively riskier jobs. Positive sorting arises from a *preference* complementarity between wealth-poor risk-averse workers and jobs that offer a high job finding probability.
The mechanism works as follows. Directed search introduces a trade-off between job-finding probability and job attributes, and risk aversion induces workers with low liquid wealth to both value job finding probability relatively more and dislike separation risk relatively more. In equilibrium, these two forces compete against one another, because jobs that offer higher matching probabilities are also those characterized by higher separation risk. This result comes from the free entry condition, and is governed by the relationship between the separation and the vacancy cost function. Indeed, when the cost of having an unfilled vacancy is relatively larger for more secure jobs, firms opening secure jobs post relatively higher wages

[2]Because the model is fully saturated with individual characteristics, I interpret the residual inequality as arising from heterogeneity in job attributes – specifically, differences in cyclical job security. This rules out labor demand as a driving mechanism. Note also that the mechanism I propose can be thought to work within skill groups, or in addition to how skills may affect cyclical employment risk. While these dimensions of worker heterogeneity are surely important determinants of labor market choices, they have been the focus of some other papers already (e.g., Kramer, 2022; Griffy, 2021; Acabbi et al., 2022). Whether there are some interaction effects between wealth and skill accumulation in the current context is beyond the scope of this paper.

to attract applicants. In equilibrium, wages and job security are thus positively correlated, and "bad" jobs (low wage, high separation risk) have higher matching probability.
Thus, for the equilibrium to feature positive sorting, the force operating through the job-finding rate must outweigh that operating through separation rates. This requires that job-finding rates respond sufficiently strongly to the search for better jobs – more strongly than the change in separation risk. This depends, additionally to the shape of the vacancy cost and separation risk functions, on the elasticity of the matching function with respect to tightness, as well as on the marginal rate of transformation between vacancies and unemployed workers implied by the matching function. If this holds (which is true under reasonable parameterizations), higher job-finding rates more than compensate for the costs imposed by higher separation risk, and "bad" jobs are complements to low wealth.
Moreover, these elasticities also have implications for the cross-sectional and state-dependent strength of positive sorting. Because the elasticity of the matching function with respect to tightness is higher at low levels of tightness, the condition for positive sorting is more easily satisfied among "bad" jobs. By similar reasoning, positive sorting is also more likely to arise when aggregate tightness is low, i.e. during recessions.
More generally, any force that either increases the need for insurance or decreases the costs of searching for bad jobs strengthens the motive for positive wealth sorting. Infinite horizon and aggregate uncertainty ambiguously affect both the need for insurance and the long-term utility costs of searching for bad jobs through various channels. Understanding how wealth sorting behaves in such a framework is therefore a quantitative question.

I then integrate the sorting mechanism in a dynamic quantitative incomplete market heterogeneous agent model with aggregate risk. The model allows to study the dynamics of wages, assets, and labor market flows over the business cycle. Relative to existing research, it newly connects wealth heterogeneity with labor market sorting in an incomplete market setup. An interaction between the distributions of cyclical employment risk and wealth arises. Wealth influences job search decisions through labor market sorting, and job search outcomes influence workers' wealth through savings during the employment spell.
I show that the quantitative model is block recursive, i.e., that the individual value functions and market tightness in every submarket are independent of the distribution of workers over asset and employment states. This result allows to solve for the full stochastic equilibrium in a relatively tractable way. I calibrate the model to the US at a monthly frequency and assume firms with higher separation risk to be less productive. My calibration matches moments of the wealth distribution, as well as cross-sectional and cyclical job flows moments. In the steady state, relatively poorer workers look for relatively lower wage jobs and lower productivity jobs (with a larger separation risk). They use their labor market search as a way to smooth consumption. This mechanism is valid both across workers (low versus high assets) and also across aggregate productivity states. In bad times, wealth-poor unemployed optimally decide to apply to even worse jobs to mitigate the decrease in job finding probability. As a consequence, sorting is relatively stronger in bad aggregate states, as measured by the larger correlation between individual wealth and separation risk, wages and job productivity. Moreover, the dispersion of wages and job productivity is relatively larger in bad aggregate states.

Finally, I use the quantitative model to study the effect of recessions and targeted transfers. I simulate a 1% negative aggregate productivity shock. First, I show that following a 1% negative aggregate productivity shock, the search of unemployed workers tilts towards worse jobs – average wages and productivity decrease by around 6%. This number hides substantial heterogeneity concentrated in the bottom 1% of the wealth distribution. In fact, these workers search for wages and job productivity around 1 percentage point lower than the richest 1%. This change in job search allows to greatly mitigate the drop in their consumption. Second, I study the effect of four types of targeted transfers implemented when the aggregate productivity shock hits. The first policy is a standard unemployment benefit and targets all unemployed. I compare this to policies targeting the wealth-poor, as well as policies targeting only the employed or unemployed wealth-poor. By targeting the transfer to workers with on average lower assets, the aggregate consumption drop is mitigated. This is especially true for the transfers targeted at the unemployed. However, this distorts their job search –allowing them to look for better jobs, which depresses the following period's aggregate consumption through a relatively larger share of workers who remain unemployed (matching probabilities are lower for good jobs). This backlash effect on consumption does not happen when targeting the poorest employed workers. Lastly, I show that in the presence of aggregate risk and liquidity constraints, wealth sorting creates a "poverty trap". wealth-poor workers systematically face higher employment risk and struggle to escape their precarious state, as they are unable to save sufficiently during short and low-wage employment spells. Additionally, I show that starting an unemployment spell during a recession, as opposed to normal times, has long-lasting effects for all workers, though to a greater extent for the poorest. A job search during a recession leads to persistently lower wages and slower asset accumulation for years, especially so for the poorest workers. I quantify the lifetime cost of starting an unemployment spell during a recession (relative to a normal time) to represent 3% of lifetime consumption for the poorest, more than twice that of the wealthiest workers.

Related literature This paper relates to four strands of the literature.
First, I contribute to the empirical literature analyzing heterogeneity in earnings cyclical risk across different dimensions.[3] I add to this literature by documenting a negative relationship between liquid wealth and employment cyclicality. In terms of empirics, my work is related to Patterson (2023) who studies the correlation between marginal propensities to consume (MPCs) and earnings' elasticity to aggregate fluctuations. Differently from her, I focus on residual inequality – differences that are not explained by workers' characteristics beyond their liquid wealth.
Second, my work relates to theoretical literature on the role of wealth for various labor market outcomes, going back to Danforth (1979). Acemoglu and Shimer (1999) show that

[3]Guvenen et al. (2014) focuses on idiosyncratic labor income risk across the earning distribution; Hoynes et al. (2012) analyzes how recessions impacts labor income differently by demographic groups; Dany-Knedlik et al. (2021) document that lower and higher income individuals experience significantly larger fluctuations across the business cycle than middle-income individuals; and Cairó and Cajner (2018) find that less educated individuals in the US have less stable jobs, which is due to a more frequent job loss. Carrillo-Tudela et al. (2022) study cyclical earnings through the lens of a job ladder model.

risk-averse agents use the labor market as an insurance mechanism against consumption risk due to unemployment. Rendon (2006) and Chaumont and Shi (2022) study the relationship between job search, unemployment spells, and asset accumulation. Eeckhout and Sepahsalari (2024) introduce the notion of wealth sorting and show that workers' wealth determines their productivity through labor market sorting. The mechanism at the heart of my paper builds on this line of research in the sense that wealthier individuals are more selective and are able to wait for better job offers. The novelty I propose is to explore a job search outcome that has not been explored yet (both empirically and theoretically), namely cyclical employment risk.[4] For that, my paper builds on the directed search frameworks developed in Menzio and Shi (2010, 2011). Importantly, I also introduce aggregate risk and study the effect of wealth over business cycles.

Third, I contribute to the literature on sorting over the business cycle by integrating *wealth* sorting into a general equilibrium model with aggregate shocks. Lise and Robin (2017) study skill-sorting over the business cycles in a model with random search and alternating offers. Acabbi et al. (2022) explore how sorting of heterogeneously skilled workers varies over the business cycle in a directed search framework. These papers do not model heterogeneity in wealth, which is my focus, and have a different notion of sorting (driven by production complementarities). My mechanism complements alternative sorting theories, operating within dimensions of heterogeneity such as skills. This paper thus adds to the understanding of how aggregate shocks affect sorting in a model that relates wealth to cyclical employment risk.[5] I also prove block recursivity in my environment featuring aggregate risk, worker wealth heterogeneity, and firm productivity heterogeneity.[6]

Fourth, my paper speaks to the literature on incomplete market models with wealth heterogeneity and labor market frictions. Relative to other frameworks (e.g., Krusell et al., 2010; Shao and Silos, 2017 or Ravn and Sterk, 2021), my model implies that negative aggregate shocks lead to a selection effect, whereby workers with lower wealth face a higher probability of unemployment due to sorting into riskier jobs. While separation risk is usually modeled to be exogenous to wealth, my mechanism maps wealth into cyclical risk and cyclical risk to wealth. This gives rise to a sort of "poverty trap", or said differently, a persistent negative effect of being unemployed with low wealth. This last feature resembles Jarosch (2023), although he focuses on the risk to fall off the job ladder and how it relates to human capital accumulation.

The rest of the paper proceeds as follows. Section 2 presents empirical results. Section 3 first introduces a tractable theoretical framework of wealth-sorting into risky jobs, providing conditions for the model to be consistent with the empirical results. Second, it outlines a

[4]Other strands of the literature explored different search outcomes; see e.g., Lentz and Tranaes (2005); Lise (2013) who study how wealth affects search effort; Griffy (2021), Kaas et al. (2023) and Sepahsalari et al. (2025) who study how wealth affects workers' mobility on the job ladder.

[5]Kramer (2022) jointly studies wealth heterogeneity and income risk, but through a different mechanism and without sorting.

[6]Existing related papers have proved the existence of a block recursivity in similar environments: with skill-sorting and aggregate shocks (Acabbi et al., 2022); with a wealth distribution (Chaumont and Shi, 2022); with a wealth distribution, worker heterogeneity, and aggregate shocks (Birinci and See, 2023; Herkenhoff et al., 2024).

quantitative wealth-sorting model. Section 4 calibrates the model and quantifies sorting in the steady state. Section 5 quantifies the implications of wealth sorting for and over business cycles, and implements policy counterfactuals. Finally, Section 6 concludes.

2 Liquid Wealth and Employment Elasticity to GDP Fluctuations

This section presents empirical results on the correlation between liquid wealth and cyclical employment.

2.1 Data and Empirical Specification

The PSID is the world's longest ongoing longitudinal household panel dataset. The survey is conducted in the US, on a biennial basis since 1999. From that year onward, collected information of interest additionally includes detailed workers' financial situation, central to the exercise. Appendix I provides detailed information about the data and cleaning.

Wealth is an endogenous variable that is correlated with individual characteristics such as skills, and these characteristics may potentially explain part of the observed heterogeneity in cyclical employment by wealth. However, the focus of this paper is on whether wealth *itself* plays a role, beyond the characteristics with which it is correlated. One can thus think of the exercise as aiming to estimate a *residual* inequality, that is, the inequality that remains after controlling for these correlated variables. I estimate the employment exposure to output fluctuations by (past period) liquid wealth, controlling for individual observed and unobserved heterogeneity, as well as aggregate variables. Because the model is saturated of controls and individual FEs, the coefficients of interest are estimated using within-individual variation in wealth over time.[7] Results should be interpreted as operating *within* individual, and therefore also within skill or socio-demographic groups. The baseline specification is as follows.

$$e_{it} = \gamma_i + \gamma_{s(i)} + \gamma_t + x'_{it}\beta + \chi_1 \Delta \log Y_{s(i)t} + \chi_2 \log(a_{it-1}) + \chi_3 \log(a_{it-1}) \times \Delta \log Y_{s(i)t} + \varepsilon_{it}, \tag{1}$$

where e_{it} is a dummy of the employment status of worker i in year t (1 if employed, 0 if unemployed), $\gamma_i, \gamma_{s(i)}$ and γ_t are, respectively, worker, US State, and year fixed effects (FEs), x_{it} is a vector of time-varying worker controls, a_{it-1} is liquid wealth[8], and $Y_{s(i)t}$ is real US State GDP.

A negative value of χ_3 implies that, everything else being equal, the employment of workers with lower liquid wealth is relatively more exposed to output fluctuations. Instead, if there is no role for a residual effect of wealth on cyclical employment, the coefficient becomes insignificant as dimensions of heterogeneity underlying the correlation are controlled for.

[7] In the sample, there is significant variation of individual liquid wealth over time: the auto-correlation of liquid wealth is 0.86, while the autocorrelation of the probability to lie at the bottom 25% of the liquid wealth distribution is of 0.54.

[8] The log is taken on $a_{it-1} + 1$ to include workers with zero liquid wealth.

2.2 Empirical Results

Table 1 presents results on individual employment elasticity to GDP fluctuations by liquid wealth. In Column (1), the estimates are displayed without the inclusion of controls or fixed effects. The first coefficient indicates a positive correlation between output and individual employment probability. Specifically, a 1% increase in GDP is associated with a 0.56 percentage point (pp) increase in the probability of employment for those holding zero liquid wealth. The second coefficient shows that individuals with more liquid wealth are on average more likely to be employed. This coefficient may reflect a direct effect of wealth on the employment probability, though, without individual controls or fixed effects, it may also reflect that higher liquid wealth is generally associated with other characteristics that increase the probability to be employed, such as skills or productivity. More relevant to this paper, the estimates on the interaction term reveal that wealth-poor individuals exhibit a larger exposure of employment to GDP fluctuations, as evidenced by the significantly negative coefficient.

Column (2) displays the coefficients related to the baseline specification in Eq. (1). The set of fixed effects and controls fully explain the effect of wealth on the employment status in the absence of GDP fluctuations.[9] Instead, the interaction coefficient of interest stays significant, slightly decreasing in magnitude to -0.027. In terms of economic significance, if a US State's GDP drops by one standard deviation, i.e. roughly 3.7%, the employment probability of individuals at the 25^{th} percentile of the liquid wealth distribution (those with practically zero wealth) decreases by -0.94 pp, almost five times as much as the median wealth worker (-0.20 pp). The slope is three times as steep when regressing employment on aggregate rather than US State GDP (Appendix Table A3). I interpret this as a *residual* effect of wealth inequality on the elasticity of individual employment to output fluctuations. These findings are consistent with Patterson (2023), who shows that workers with a high MPC display relatively greater employment sensitivity to aggregate fluctuations.[10] Taken together, the evidence supports a theoretical link between liquid wealth and heterogeneous employment risk.

Next, I examine whether the heterogeneous cyclicality across the liquid wealth distribution is driven by job loss or job finding. In Columns (3) and (4), the baseline regression's dependent variable is changed to represent either a transition from employment to unemployment (job loss dummy) or a transition from unemployment to employment (job finding dummy). Results indicate that the heterogeneous employment cyclicality by liquid wealth is driven by job loss. Wealth-poor workers are more likely to lose their jobs in a recession, as indicated by the positive and significant interaction coefficient in Column (3). Moreover, liquid wealth is

[9] The loss of statistical significance for the liquid wealth coefficient is largely attributable to the inclusion of individual fixed effects (Appendix Table A2).

[10] In a standard model with imperfect insurance and idiosyncratic risk, MPCs and liquid wealth are negatively correlated. In the data however, MPCs and wealth are distinct objects: the former must generally be estimated, whereas wealth is directly measured in large-scale surveys. Replicating the MPC imputation in the PSID of Patterson (2023), I find that workers at the bottom of the liquid wealth distribution in my sample indeed have a higher MPC – see Appendix Table A1.

Table 1: Liquid wealth and employment elasticity to aggregate fluctuations

	(1) empl. status	(2) empl. status	(3) job loss	(4) job finding
$\Delta \log Y_{s(i),t}$	0.56***	0.26***	-0.25***	0.059
	(0.086)	(0.073)	(0.078)	(0.68)
$\log a_{it-1}$	0.016***	0.00047	-0.0017**	-0.018***
	(0.00088)	(0.00079)	(0.00069)	(0.0048)
$\Delta \log Y_{s(i),t} \times \log a_{it-1}$	-0.034***	-0.027***	0.027***	-0.021
	(0.0095)	(0.0085)	(0.0086)	(0.079)
Ind., US State & year FEs		✓	✓	✓
Worker controls		✓	✓	✓
R^2	0.056	0.45	0.35	0.50
N	44377	44377	38083	4078

Note: Observations come from the Core PSID dataset. The sample covers years from 1999 to 2017. Standard errors in parentheses; $^{*}p < 0.1$,$^{**}p < 0.05$,$^{***}p < 0.01$, clustered at the state level. Worker-level controls consist of a quadratic in age and its interactions with education, gender, marital status, and race. In Columns (1) and (2), the outcome variable is the employment status of the household head (0 if unemployed, 1 if employed); while in Columns (3) and (4) it is a job loss or job finding dummy (1 if the worker was employed in $t-1$ and unemployed in t, or vice versa).

significantly negatively correlated with both job loss and job finding, even after controlling for observed and unobserved worker heterogeneity. Appendix Figure A1 plots the predicted job loss and job finding probabilities as functions of liquid wealth, for both positive and negative GDP growth. In the cross-section, having low liquid wealth thus induces a higher likelihood to both find and lose jobs. Because these results pertain to *residual* inequality, I interpret the lower job-finding rate among wealthier individuals in the model as arising from a liquidity effect, and the lower job-loss rate as reflecting heterogeneity in job characteristics, specifically in separation risk.[11]

I. Sensitivity

The evidence presented above points to a role for *previous liquid* wealth in explaining heterogeneous employment exposure to output fluctuations. In the model, I interpret this pattern as arising from a liquidity-constraints channel that gets activated when a bad shock drives down a worker's liquid wealth. Table 2 further motivates this modeling choice. First, I show that liquid wealth plays a more important role than illiquid wealth. In Column (1), I augment the baseline specification by additionally controlling for illiquid wealth. The coefficients on illiquid wealth are statistically insignificant, and the coefficient of interest, χ_3, remains virtually unchanged.[12] A second result supporting a role for liquidity constraints is that being hand-to-mouth is associated with a similarly larger employment exposure to

[11]The negative relationship between wealth and job finding can also be related to the large empirical literature on the effect of liquidity on job finding rates. That literature finds a systematic negative relationship, consistent with two alternative theories: reduced search effort or an income effect that allows workers to target better jobs – see e.g., Card et al. (2007); Chetty (2008); Lammers (2014); Nekoei and Weber (2017); He and le Maire (2020); Fontaine et al. (2020); Bednarzik et al. (2021); Herkenhoff et al. (2024). This paper focuses on the liquidity channel.

[12]In principle, illiquid wealth could generate similar residual inequality and be consistent with a liquidity channel, provided that workers are able to convert it into liquid assets.

GDP fluctuations (Column (2)). Third, I rule out that the correlation is driven by a *permanent* component of wealth that would not be captured by the individual FE, but instead by differences in the histories of shocks affecting liquid wealth. The residual inequality vanishes when I use the worker's permanent position in the liquid wealth distribution, rather than their previous period's wealth as in the baseline specification (Column (3)).

Table 2: Liquidity channel

	(1)	(2)	(3)
$\Delta \log Y_{s(i),t}$	0.35***	0.039	0.040
	(0.11)	(0.047)	(0.054)
$\log a_{it-1}$	0.00019		
	(0.00070)		
$\Delta \log Y_{s(i),t} \times \log a_{it-1}$	-0.020**		
	(0.0082)		
$\log a_{it-1}^{\text{illiq}}$	0.00063		
	(0.00078)		
$\Delta \log Y_{s(i),t} \times \log a_{it-1}^{\text{illiq}}$	-0.013		
	(0.010)		
H2M_{it-1}		-0.00083	
		(0.0058)	
$\Delta \log Y_{s(i),t} \times \text{H2M}_{it-1}$		0.14**	
		(0.064)	
$\Delta \log Y_{s(i),t} \times \bar{a}_i^{p25}$			0.23
			(0.16)
R^2	0.45	0.45	0.45
N	44377	45522	45522

Note: Observations come from the Core PSID dataset. The sample covers years from 1999 to 2017. Standard errors in parentheses; $^{*}p < 0.1$,$^{**}p < 0.05$,$^{***}p < 0.01$, clustered at the state level. The outcome variable is the employment status of the household head. All specifications include worker, year, and state fixed effects. Worker-level controls consist of a quadratic in age and its interactions with education, gender, marital status, and race. $\bar{a}^{p25}$ is equal to one if the household's liquid wealth lies in the first quartile of the average liquid wealth distribution; $H2M$ is a dummy equal to 1 if the household's net worth is lower than a sixth of their labor income, following Aguiar et al. (2024).

Additional supporting evidence that the coefficient captures a residual effect of liquid wealth, rather than the effect of another variable correlated with liquid wealth, is presented in Appendix Table A4. First, if individual productivity is not fully controlled for by the individual fixed effects and education, the stronger elasticity of wealth-poor workers may come from individual productivity differences. To mitigate such concerns, Column (2) controls for workers' past labor income interacted with GDP growth. To the extent that labor income reflects idiosyncratic productivity and that the latter has persistence, this control allows cleaning the effects of wealth from productivity. The significance and magnitudes are barely affected by the addition of this control. Second, shocks to spousal income may correlate with both liquid wealth and the household head's employment. Column (3) controls for the interaction between spousal income and GDP fluctuations; with results remaining unaffected. Third, saturating the model with debt does not affect results (Column (4)). Finally, in Columns (5) and (6) I control for workers' occupation and industry interacted with GDP fluctuations. The baseline coefficient remains significantly different from zero, suggesting that neither the workers' occupations or industry can fully explain the residual inequality in employment exposure to GDP fluctuations by liquid wealth. Thus, the mechanism can be thought to operate within narrowly defined occupation, industry, and skill groups.

The next results suggest that the imposed log-linearity form of the estimated relationship is reasonable, even though approximately 25% of individuals in the sample hold no assets. Appendix Figure A2 plots the predicted employment probabilities along the liquid wealth distribution, comparing the log liquid wealth specification of Eq. (1) to a specification using bins of liquid wealth. The latter model does not seem to produce significantly different slopes, neither in booms or recessions. Consistently with models with liquidity constraints and risk aversions, effects seem to be log-linear in liquid wealth.
Finally, in the baseline specification, I use US State-level GDP to exploit greater cross-sectional variation, particularly in negative realizations. Nevertheless, residual liquid wealth inequality remains when using aggregate GDP or the unemployment rate, with larger estimated slopes (see Appendix Table A3).

2.3 Wealth and Employment Risk in Conventional Theory

The fact that workers with lower liquid wealth face more cyclical employment, as well as a higher job loss probability in the cross-section, is surprising in light of existing theories.
From the perspective of wage-compensating differentials, risk-averse agents who are in a very concave part of their utility function (holding low wealth while unemployed) should be willing to trade low wages for job security relatively more. In the data, by this reasoning we should expect low wealth to be associated with *fewer* job separations.
Consider a world where agents differ in wealth, are risk-averse, face liquidity constraints and can decide to accept job offers they draw from some distribution over wages and job loss risk (à la McCall, 1970). All else being equal, wealthier workers, who do not mind risk as much, are willing to accept riskier but better paid jobs.[13] On average, riskier jobs will thus be populated with wealthier agents. The same intuition carries over to more complex models of wage-compensating differentials. For example, in Pinheiro and Visschers (2015), the wage compensating differential for additional job loss risk is larger for those workers who have most to lose from job loss, in their case workers who are at the top of the job ladder. Applying the same logic to a model with heterogeneity in wealth rather than their job ladder, risk aversion and the presence of liquidity constraints imply that those workers who have most to lose are those with little wealth. Thus, low wealth workers should be less likely to accept highly risky jobs – the opposite of what I find in the data.
The next section reconciles the empirical evidence with a theory grounded in risk aversion. As in standard models, all else equal, agents with low liquid wealth value job safety relatively more because of risk aversion. However, I show that risk aversion can also give rise to an outweighing counteracting force working through job finding rates.

3 A Theory of Wealth-Sorting into Risky Jobs

This section develops a theoretical framework that rationalizes why wealth-poor workers may face relatively higher (cyclical or cross-sectional) separation risk, all else equal. The model is intentionally parsimonious: in line with the empirical evidence, it focuses on *residual*

[13] Another way to put this is that all else being equal (including job loss probability), wealthier unemployed workers have a lower reservation wage.

inequalities – those not explained by observable worker characteristics other than liquid wealth – and attributes the remaining heterogeneity to job characteristics rather than to workers themselves. Exposure to job risk then emerges as a by-product of workers' optimal labor market search decisions. The mechanism therefore operates through labor supply.[14]
I first develop a two-period model that isolates the trade-offs involved in worker's job search and their implications for labor market equilibrium. I then construct a quantitative model with aggregate risk that features the wealth-sorting mechanism, proving that it is block recursive.

3.1 Two-Period Model

I. Environment

Time lasts two periods, i.e., $t = 1, 2$. Aggregate productivity z in the second period is uncertain.
The economy is populated with a continuum of heterogeneous workers and firms. Workers are risk-averse and differ in their asset holdings $a_t \in \mathcal{A} = [\underline{a}, \bar{a}] \subset \mathbb{R}_+$. Their period utility function $u(c)$ is increasing and concave ($u'(c) > 0$, $u''(c) < 0$ $\forall c \in \mathbb{R}_+$) and satisfies the Inada conditions. They face a liquidity constraint such that they cannot save less than $\underline{a}$. They discount the future at rate β, and the risk-free interest rate $R = 1 + r$ is determined by the world market and taken as given. Their labor market status is either employed or unemployed. When employed, workers earn a wage w and when unemployed they benefit from home production b.
Firms have one job, are risk neutral, and face the same discount rate as workers. As is common in the search literature, I refer to a firm as a job. Jobs are indexed by their (observable) type $x \in \mathcal{X} = [\underline{x}, \bar{x}] \subset \mathbb{R}_+$, and differ in their separation probability $\delta(x, z)$. Let $\delta_x(x, z) < 0$, such that low-type jobs are those carrying relatively more separation risk. There is free entry for vacancy posting; firms opening a vacancy pay a cost that is increasing in their type, i.e. $\varphi(x)$, $\varphi'(x) > 0$.[15] The production technology is independent of job type; all jobs produce output $\bar{y}$ using labor as the only factor of production.[16] A final good competitive firm pools individual output and transforms it to the consumption good.

Labor markets Job search is directed, and the labor market is segmented in submarkets that do not compete directly with one another. Every submarket represents a subset of unemployed and firms with vacant jobs that are searching for each other. Submarkets are denoted by the firm type and the posted wage, (x, w) – different firms posting the same wage operate in different submarkets. Each entrant firm posts a wage to be paid in the production stage upon matching. Unemployed workers choose to apply to (only) one

[14] From a labor demand perspective, there is no reason to expect differential behavior based on workers' liquid wealth, all else equal, insofar as wealth is not observable.

[15] The assumption that the vacancy cost is a function of type is needed for the existence of an interior solution. Instead, the positive relationship between vacancy cost and type is needed for the possibility of positive sorting, as will become clear. Generally, one can think about it as assuming that there exist deep-pocketed entrepreneurs who can invest in the quality of their jobs, and higher type jobs require a larger investment.

[16] I will relax this assumption in the quantitative model.

submarket, and employed workers do not search on the job. The firm-to-worker ratio in each submarket is denoted by $\mathbf{v}(x,w)/\mathbf{u}(x,w) = \theta(x,w)$. For ease of notation, I hereafter drop the arguments and write θ to denote the tightness in a specific submarket. Matching is governed by a matching function that satisfies common assumptions (twice continuously differentiable, concave) and features constant returns to scale, following Menzio and Shi (2011). This function satisfies regularity conditions that required for the existence of an equilibrium. The latter allows to write matching probability in every submarket as a function of tightness, $p(\theta)$, with $p'(\theta) > 0$ and $p''(\theta) < 0$. The probability of filling a vacancy is given by $q(\theta) = p(\theta)/\theta$, with $q'(\theta) < 0$.

Timing In $t = 1$, all agents start unemployed. Firms open vacancies and post wages. Workers apply to one job offer and take their consumption-savings decision. At the end of the period, the matching function determines the matching outcome in each open submarket. In the beginning of period 2, the aggregate productivity state materializes, and a lottery destroys some matches created in the end of period 1 according to the separation function $\delta(x,z)$. If separated, workers remain unemployed and the job vacant (with zero value). Matched workers get their contract wage, and production takes place.

II. Value Functions and Equilibrium Conditions

In the second (and last) period, the values of employment E and unemployment U are, (given that $a_3 = 0$), $E_2(a_2,w) = u(Ra_2 + w)$, and $U_2(a_2) = u(Ra_2 + b)$. For simplicity of exposure and without loss of generality, in what follows I set $b = 0$. The value of a filled job, given that wages and productivity are fixed, is simply $J(x,w) = \bar{y} - w$.
In period 1, the firm decides to open a vacancy in a certain market by posting a wage, according to the following value function.

$$V(x) = \max_{w} -\varphi(x) + \beta\Delta(x)q(\theta)J(x,w),$$

where $\Delta(x) = \mathbb{E}_z(1 - \delta(x,z))$ is the expected retention rate. Given that in this model the aggregate state only affects separation probabilities and not the continuation value functions (J,E,U), what is relevant to agents is the *expected* retention, not its cyclicality.[17]
Unemployed workers' problem is to save a_2 subject to the liquidity constraint $\underline{a}$, and to choose a submarket (x,w) to which to apply. Their value function writes

$$U_1(a_1) = \max_{a_2 \geq \underline{a}, (x,w)} u(Ra_1 - a_2) + \beta[p(\theta)\Delta(x)E_2(a_2,w) + (1 - p(\theta)\Delta(x))U_2(a_2)]$$

Relevant to the unemployed' labor market search is only the *expected* retention. A search in a high-type job x affects the continuation values through the retention rate $\Delta(x)$ (endogenously), as well as through the matching probability $p(\theta)$ (endogenously).

Free entry The measure of entrant firms is determined by competitive entry. This implies that in equilibrium, the value of open vacancies is zero, i.e., the cost of opening a vacancy

[17] I discuss this assumption at the end of this section.

is equal to the expected value of the job. Formally,

$$\varphi(x) \geq \beta\Delta(x)q(\theta)J(x,w) \qquad \forall x \in \mathcal{X} \tag{2}$$

and $\theta(x,w) \geq 0$ with complementary slackness:

$$\theta(x,w) = \begin{cases} q^{-1}\left(\frac{\varphi(x)}{\beta\Delta(x)J(x,w)}\right) & \text{if } \beta\Delta(x)J(x,w) > \varphi(x) \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

For a given job type x, the free-entry condition pins down the tightness as a function of wages in any submarket that is visited by a positive number of workers. It provides firms' indifference curves in the menu of market tightness and wages; posting a high wage has to be compensated with high job filling probability.

III. Equilibrium and Positive Wealth Sorting

To solve for the equilibrium allocation, I consider the unemployed optimal search and the firms' optimal posting strategy jointly. Effectively, unemployed workers take the firms' indifference condition as given. We can then analyze the problem as a maximization with non-linear Pareto frontier $U(a,x)$, which denotes the value to the worker with assets a when matched with job type x. Conditional on a search with job type x, the maximization problem of an unemployed writes

$$\begin{aligned} U(a_1,x) = \max_{a_2,\theta} \quad & u(c_1) + \beta\left[p(\theta)\Delta(x)u\left(Ra_2+w\right) + (1-p(\theta)\Delta(x))u(Ra_2)\right] \\ \text{s.t.} \quad & c_1 + a_2 = Ra_1 \\ & w = \bar{y} - \frac{\varphi(x)}{\beta q(\theta)\Delta(x)} \end{aligned}$$

The first order conditions (FOC) with respect to a_2 and θ are, respectively:

$$-u'(c_1^*) + \beta R\left[p(\theta^*)\Delta(x)u'\left(c_{e,2}^*\right) + (1-p(\theta^*)\Delta(x))u'(c_{u,2}^*)\right] = 0 \tag{4}$$

$$\beta\Delta(x)\left[p'(\theta^*)\left(u\left(c_{e,2}^*\right) - u(c_{u,2}^*)\right) + u'\left(c_{e,2}^*\right)\frac{\varphi(x)\theta^* q'(\theta^*)}{\beta q(\theta^*)\Delta(x)}\right] = 0 \tag{5}$$

where $c_1^* = Ra_1 - a_2^*$, $c_{e,2}^* = Ra_2^* + \bar{y} - \frac{\varphi(x)}{\beta q(\theta^*)\Delta(x)}$ and $c_{u,2}^* = Ra_2^*$.

The first condition is an Euler equation where $t=2$'s consumption depends on the probability of match and match outcome; the second condition pins down market tightness for a given type x.

The worker takes the firm payoff as given – like a menu of value functions for all possible θ, and chooses the firm type that maximizes their expected utility. From the first order condition, it must hold that $\frac{\partial U(a_1,x)}{\partial x} = 0$. The optimality condition for the choice of x can

be written as

$$\beta p(\theta^*)\left[\Delta'(x)\left(u(c^*_{e,2})-u(c^*_{u,2})\right)+\Delta(x)u'(c_{e,2^*})\left(-\frac{\varphi'(x)\Delta(x)-\varphi(x)\Delta'(x)}{\beta q(\theta^*)(\Delta(x))^2}\right)\right]=0 \quad (6)$$

The equilibrium is pinned down by Eq. (4) - (6). We can now define sorting into job type x as a function of assets a.

Definition 1. *Wealth sorting is positive whenever ex-ante wealthier agents direct their search towards relatively higher type jobs, i.e. jobs with lower separation risk.*

Formally, let $\phi(a_2^*, \theta^*)$ be the maximand of $U(a_1)$ under the equilibrium constraints. Then, positive wealth sorting obtains whenever $\phi_{a_1 x} \coloneqq \frac{\partial \phi(a_2^*,\theta^*)}{\partial a_1 x} > 0$.

Proposition 1 gives the sufficient conditions for the equilibrium of the two-period model to feature positive wealth sorting.

Proposition 1. *Wealth sorting is positive if, for all x,*

$$\frac{u'(c^*_{e,2})-u'(c^*_{u,2})}{u(c^*_{e,2})-u(c^*_{u,2})} < \frac{u''(c^*_{e,2})}{u'(c^*_{e,2})}, \quad \textit{and} \quad (7)$$

$$\frac{\varphi'(x)}{\varphi(x)} > \frac{\Delta'(x)}{\Delta(x)}(\varepsilon_\theta \varepsilon_p)^{-1}, \quad (8)$$

, where $\varepsilon_\theta = (1+\gamma)^{-1}$ is the elasticity of substitution between $\mathbf{v}$ and $\mathbf{u}$ in the matching function, and $\varepsilon_p = (1+\theta^{*\gamma})^{-1}$ is the elasticity of job finding to market tightness. Moreover, the equilibrium is unique.

Proof. See Appendix B2. ■

Like in all directed search models, in this model workers' choice to apply to a certain job is the result of the optimal resolution of the trade-off between the probability to match (or, equivalently, the waiting time) and the expected value of a job. Because workers are ex-ante heterogeneous in asset holdings, cannot freely borrow against future income due to liquidity constraints, and are risk-averse, they value both the matching probability and the expected job value in a heterogeneous manner. The first inequality dictates in what way agents heterogeneously value these terms. Eq. (7) is satisfied for all utility functions featuring Decreasing Absolute Risk Aversion (DARA) (Eeckhout and Sepahsalari, 2024).

DARA utility works in two opposing forces for sorting. On the one hand, those with lower

assets value the matching probability relatively more, i.e. the surplus function ϕ is sub-modular in (θ, a_2). In fact, for workers with low assets, the unemployment state is relatively more costly in terms of utility, as they cannot rely on savings to consume. For them, there is thus a sense of "urgency" to find a job. When jobs that are easier to find are the "bad" ones (high job loss risk), this force thus works in favor of positive sorting. On the other hand, those with low assets also dislike higher job loss risk relatively more, i.e. the surplus function ϕ is super-modular in $(\delta(x), a)$. Risk aversion implies that workers with low consumption dislike separation risk relatively more. Therefore, the separation risk that is increasing in type introduces a force working against positive sorting in equilibrium. In other words, higher separation risk hurts the low wealth workers for the exact reason they like high matching probability: risk aversion. [18]

The *degree* of risk aversion affects the magnitude of the inequality (7), thus scaling the strength of the two opposing forces. Under fairly general conditions, higher risk aversion increases the gap between the two sides of the inequality.[19] To illustrate this mechanism, I solve the model numerically and parametrize it such that equations (7) and (8) hold. Figure D3 presents the equilibrium under different calibrations of risk aversion. A higher level of risk aversion scales up the benefits of matching (the dominant force in the positive sorting equilibrium) relative to the utility cost associated with separation risk. Under higher risk aversion, agents thus direct their search toward relatively worse jobs, and the effect is strongest for low-wealth workers.

For a worker deciding whether to search for a higher-type job x, the relative strength of the two opposing forces associated with DARA risk aversion depends on whether the positive utility from a higher job-finding probability outweighs the disutility induced by an increased risk of job loss.

Property 1. *From the worker's perspective, the job-finding motive dominates the job loss risk, i.e.* $|\phi_{a_2\theta}| > |\phi_{a_2x}|$*, if*

$$\frac{p'(\theta^*)}{p(\theta^*)} > \frac{\Delta'(x)}{\Delta(x)}$$

In words, the proportional gain in job-finding probability must exceed the proportional gain in job retention probability from marginally improving search to a higher job quality x. Whether this is indeed satisfied in equilibrium is determined by the second condition in Proposition 1, Eq. (8). That condition both governs which types of jobs are the ones associated with a higher effective probability of being employed, and which of the channels between job finding and job separation risk dominate in equilibrium.

First, an interior solution requires that high-type firms are those posting higher wages in

[18] Note that this counteracting force would be absent if jobs were homogeneous in their separation risk, in which case condition (7) alone is sufficient to obtain positive wealth sorting. Apart from the equilibrium concept, this model would be similar to Eeckhout and Sepahsalari (2024).

[19] See Appendix B3 for the conditions.

equilibrium, and therefore having the longest queues.[20] To understand why it is the case, it is useful to start from firms' indifference curves in the menu of wages and tightness. Free entry pins down the relationship between θ and w consistent with firms' optimal job creation: posting a high wage has to be compensated with high job filling probability; and these indifference curves are function of the job type x. Property 2, which directly follows from the free entry condition (2), characterizes the slope of these iso-profit curves as a function of x.

Property 2. *The rate of substitution between wages and tightness decreases with job type x whenever the following holds, for all x,*

$$\frac{\varphi'(x)}{\varphi(x)} > \frac{\Delta'(x)}{\Delta(x)}. \tag{9}$$

, where φ is the search cost and Δ the expected duration of a filled vacancy. When the marginal cost of opening a higher type job outweighs the marginal increase in job duration, then search costs in terms of foregone production increase with job type x. Therefore, to accept a lower job filling probability, high type jobs need to be compensated by relatively lower wages compared to low type jobs. Intuitively, high type jobs have a relatively larger incentive to offer higher wages in order to attract more workers and improve the job-filling probability. The free entry condition allows to control the slope of tightness to the job-type x through these two parameters. Ultimately, this will determine whether the endogenous force coming through market tightness can outbalance the exogenous force coming through job separation in equilibrium. From Eq. (6), it is clear that Property 2 is necessary to guarantee an interior solution to the sorting problem. Given the assumptions on φ and δ, the interior solution thus necessarily features a complementarity between wage and job safety.[21] However, it is not sufficient to guarantee positive sorting in equilibrium.

Second, the terms $\varepsilon_\theta \varepsilon_p$ capture the extent to which job-finding rates respond to changes in market tightness in equilibrium. While Eq. (9) ensures that $q(\theta)$ is increasing in type x, a positive wealth-sorting equilibrium requires that the *effective* job-finding probability, $p(\theta)\Delta(x)$, be decreasing in x. Eq. (8) guarantees this by adjusting Eq. (9) for two additional terms. The elasticity ε_θ reflects the degree of complementarity between job seekers and vacancies in the matching process, while the elasticity ε_p translates firms' indifference curves into the object relevant for workers, as it captures how changes in tightness affect the job-finding probability.

[20] The fact that the high-wage jobs are faster to fill is consistent with empirical evidence. Using Austrian data, Mueller et al. (2024) document a negative relationship between vacancy duration and wages, once conditioning on a rich set of worker and job characteristics.

[21] The lack of compensating differentials between wages and job loss risk is a common feature of the data. The coexistence of low wage and low security jobs is consistent with what Jarosch (2023) estimates in Germany. Dual labor markets are also an example of the coexistence of low-wage, low-security jobs alongside high-wage, high-security jobs.

Equilibrium allocation Positive wealth sorting implies that wealth-poor agents self-select into high matching probability jobs to solve their optimal consumption-smoothing problem. Moreover, wealth-poor workers' optimal search imply that upon matching, they get relatively lower wages and higher job separation risk. Because of risk aversion and the liquidity constraint, workers' need of consumption outweighs the utility costs of matching in low wage and low security jobs. Any force that either increases the need for insurance (e.g., higher risk aversion, tighter liquidity constraints) or decreases the utility cost of searching for bad jobs (e.g., lower separation rates) strengthens the motive for positive wealth sorting.

IV. Cyclical Sorting

Under a CES matching function, ε_p is *decreasing* in market tightness: when many vacancies are open, an additional vacancy improves matching by relatively less than when vacancies are scarce. The fact that positive sorting depends on this term has implications for both the cross-section and the cyclical behavior of sorting.
In the cross-section, the condition is more likely to be satisfied in tight submarkets (those to which low-asset workers direct their search), ceteris paribus. In fact, a relatively low θ implies a locally higher elasticity, which relaxes the inequality. A similar argument applies to exogenous fluctuations in the aggregate level of tightness. A recession associated with a decline in tightness increases the sensitivity of matching to tightness, thereby making the condition for positive sorting easier to satisfy. In this simple model, the strength of positive sorting is therefore countercyclical. The parameter ε_θ governs the slope of this relationship, scaling the magnitude of the effect.

More generally, in an infinite-horizon environment with aggregate risk, both the need for insurance and the utility costs of "bad" jobs may increase. The former because jobs are harder to find in recessions, while the latter because bad jobs entail a higher probability of unemployment, particularly when job loss is more costly (i.e., in recessions). In the two-period model, I assume that the aggregate state only affects the separation function and not second-period consumption values for employed and unemployed workers (i.e., $u\left(Ra_2 + w\right)$ and $u\left(Ra_2\right)$ are independent of z). Agents therefore internalize the *expected* separation risk, but its *cyclicality* is de facto irrelevant. By contrast, in an infinite-horizon model, sorting depends on cyclicality through the continuation values of employment and unemployment. To illustrate the impact of cyclical continuation values on sorting, Appendix G1 extends the two-period model to allow the continuation value of employment to vary with the aggregate state. In this case, agents optimize over the *expected continuation value*, which now contains a cyclical component. In particular, I assume that firm productivity is a function of the aggregate state and derive necessary conditions for positive sorting. In this environment, Eq. (7) is no longer sufficient to ensure positive wealth sorting, even when (8) holds. Nevertheless, positive sorting may still arise. The necessary conditions for positive sorting have a similar structure, but must hold in expectation. Cyclicality thus directly affects both the sign and the strength of sorting.

The quantitative model fully accounts for these additional channels and allows studying the effects of positive wealth sorting on the dynamics of wages and employment over the

business cycle.

V. Role of Directed Search

Under the assumptions on $\varphi(x)$ and $\delta(x)$, the sorting pattern in the two-period model emerges from risk-averse workers' optimization in job search through directed search. Moreover, competitive entry allows to tightly control the relative strength of the endogenous force coming through job finding rates, and the exogenous force coming through separation rates. However, the deep trade-off faced by the agents come from risk aversion, not the specificity of this equilibrium. In other words, while the trade-off emerges in my model through assumptions on the utility and matching functions, as well as a condition on φ and δ, the sorting pattern does not require this trade-off to be generated endogenously. Proposition 1 can then be viewed as a distinct way to construct a wage compensating differential, in a way that takes into account job finding rates. In models where workers can choose to accept jobs in a menu of wages and job loss risk, if the job options also differ in their matching probabilities, workers' relevant compensating differential includes that job finding probability.

VI. Testable Implications in the Data

The two-period model predicts that wealthier workers earn higher wages, ceteris paribus. Moreover, through the lens of the model, the relationship between wealth and employment status is the result of two opposing forces. On the one hand, the liquidity channel induces low-wealth workers to find jobs more quickly, reducing total unemployment duration. On the other hand, their search implies that they are more likely to lose these jobs, increasing overall unemployment duration.
Table 3 presents empirical evidence on the relationship between wealth, wages, and hours worked, as well as total months unemployed and unemployment spell duration.

Table 3: Liquid wealth and wages, hours worked, and unemployment spells

	(1) log wage (hourly)	(2) log labor income (yearly)	(3) log hours worked (yearly)	(4) months unemployed (yearly)	(5) u spell duration
$\log a_{it-1}$	0.0060***	0.023***	0.013***	-0.0013*	0.054*
	(0.0011)	(0.0054)	(0.0035)	(0.00079)	(0.032)
R^2	0.71	0.64	0.66	0.36	0.52
N	43085	50883	50883	50883	3456

Note: Observations come from the Core PSID dataset. The sample covers years from 1999 to 2017. Standard errors in parentheses; $^{*}p < 0.1$,$^{**}p < 0.05$,$^{***}p < 0.01$, clustered at the individual level. All specifications include individual and year fixed effects. Worker-level controls consist of a quadratic in age and its interactions with education, gender, marital status, and race. In Column (4), months unemployed are grouped into three categories (0; 0–6; 6–12 months). In Column (5) unemployment spell duration (conditional on experiencing at least one unemployment spell in that year) is measured using monthly information on employment status.

Higher wealth is indeed significantly and positively associated with higher hourly wages (Column (1)). Column (2) indicates that these higher wages translate into greater annual labor income, consistent with the fact that wealthier workers work more total hours (Column (3)). A related indicator is that wealthier individuals spend fewer months unemployed

within a given year (Column (4)). Despite experiencing fewer months of unemployment overall, conditional on being unemployed wealthier workers exhibit longer unemployment spell durations (in months) (Column (5)).

3.2 Quantitative Model

I. Environment

Time is discrete, and the economy is infinitely lived. As in the two-period model, the economy is populated by a continuum of risk-averse agents with asset holdings $a_t \in \mathcal{A} = [\underline{a}, \bar{a}] \subset \mathbb{R}_+$, and a continuum of risk neutral one-employee firms, with (observable) productivity $x \in \mathcal{X} = [\underline{x}, \bar{x}] \subset \mathbb{R}_+$. Let Ψ_t be the distribution of workers over employment states, and Ω_t the distribution of workers over assets. Agents have a period utility function $u(\cdot)$, which is increasing ($u'(\cdot) > 0$) and concave ($u''(\cdot) < 0$). They discount the future at rate $\beta = \tilde{\beta}(1-\omega)$, where ω is the retirement probability. They can save at the risk-free interest rate $R = 1 + r$ and can borrow up to a constraint $\underline{a}$. The interest rate is determined by the world market and taken as given. Firms have the same discount factor as workers.
When employed, agents earn a fixed, match-specific wage $w \in \mathcal{W}$. When unemployed, they receive unemployment benefits b, financed by proportional wage taxes τ collected by the government. Upon retirement, a worker's assets are seized by the government, and a newborn agent enters unemployment with zero assets.
Aggregate productivity $z \in \mathcal{Z}$ evolves according to a Markov transition probability $\pi(z_t, z_{t+1})$. Firms produce output $f(x_t, z_t)$ using labor as the only factor of production, and a final good competitive firm pools individual output and transforms it to a consumption good. The aggregate state of the economy, including the distribution of workers and the aggregate productivity state, is denoted by Γ.

Labor markets Job search is directed, and there is a separate submarket for each productivity and wage so that they do not compete directly with one another. There is free entry in every submarket and firms have to pay a vacancy cost $\varphi(x_t, z_t)$ to open a job. Each entrant firm posts wages w promised to the workers upon match formation and for the entire duration of the match. Firms have limited commitment in the sense that they can fire workers if the value of a filled job becomes negative.[22] Unemployed workers choose to apply to (only) one submarket $(x_{t+1}, w_{t+1}; \Gamma_t)$, and workers do not search on the job.[23] The firm-to-worker ratio in each submarket is denoted $\theta_t(x_{t+1}, w_{t+1}; \Gamma_t)$.[24] Matching is governed by a matching function $p(\theta_t)$ that is twice continuously differentiable, strictly increasing, strictly concave, and such that $p(0) = 0$, $p'(0) < \infty$. The probability of filling a vacancy is given by $q(\theta_t) = p(\theta_t)/\theta_t$, with $q'(\theta_t) < 0$. Matches are exogenously separated with probability $\delta(x_t, z_t)$.

[22] This assumption is not material to the results. However, it allows to bound the expected surplus function from below, to allow firms to open vacancies even if in expectations they generate negative value. In the calibrated model, job destruction coming from this assumption is immaterial.

[23] These assumptions ensure the model remains tractable, as they are essential for block recursivity. See proposition 2 and Appendix C for further details.

[24] For notational simplicity and in an abuse of notation, I hereon write θ_t to denote the tightness in a specific submarket.

Timing Within a period, there are the following four phases. (i) the aggregate shock materializes and some existing matches get endogenously destroyed, others get exogenously destroyed; (ii) firms with type x decide whether to open a vacancy, in which case they post wages; (iii) unemployed workers apply to a job offer and take their consumption-savings decision, employed workers choose optimal savings; (iv) agents potentially match on the labor market (the matching function determines the outcome) and production takes place. Matches become productive next period – if they do not get destroyed before production.

II. Value Functions and Equilibrium Conditions

Unemployed workers choose in which market to apply (summarized by the contract pair (x_t, w_t)) and solve a standard consumption saving problem

$$\begin{aligned} U(a_t;\Gamma_t) = \max_{a_{t+1},(x_t,w_t)} & u(c_{u,t}) + \beta \mathbb{E}_t \left[p(\theta_t) E(a_{t+1}, x_t, w_t; \Gamma_{t+1}) + (1 - p(\theta_t)) U(a_{t+1};\Gamma_{t+1}) \right] \\ \text{s.t.} \quad & c_{u,t} + a_{t+1} = R a_t + b_t, \qquad a_{t+1} \geq \underline{a} \end{aligned} \tag{10}$$

Employed workers with wage w_t simply choose savings

$$\begin{aligned} E(a_t, x_t, w_t;\Gamma_t) = \max_{a_{t+1}} & u(c_{e,t}) + \beta \mathbb{E}_t \left[\delta(x_t, z_t) U(a_{t+1};\Gamma_{t+1}) + (1 - \delta(x_t, z_t)) E(a_{t+1}, x_t, w_t;\Gamma_{t+1}) \right] \\ \text{s.t.} \quad & c_{e,t} + a_{t+1} = R a_t + (1-\tau) w_t, \qquad a_{t+1} \geq \underline{a} \end{aligned} \tag{11}$$

, where τ are (exogenous) taxes.
The value of posting a vacancy in a specific submarket with productivity x is

$$V(x_t,\Gamma_t) = \max_{w_t} \quad -\varphi(x_t, z_t) + \beta \mathbb{E}_t \left[q(\theta_t) J(x_t, w_t;\Gamma_{t+1}) + (1 - q(\theta_t)) V(x_{t+1};\Gamma_{t+1}) \right] \tag{12}$$

The value of a filled job in submarket (x_t, w_t) is

$$\begin{aligned} J(x_t, w_t;\Gamma_t) = \max\{0; \\ f(x_t, z_t) - w_t + \beta \mathbb{E}_t \left[\delta(x_t, z_t) V(x_{t+1};\Gamma_{t+1}) + (1 - \delta(x_t, z_t)) J(x_t, w_t;\Gamma_{t+1}) \right] \} \end{aligned} \tag{13}$$

Note that both firm productivity and wages are fixed over the employment duration.[25] The max operator comes from limited commitment by firms. At the beginning of the period, the aggregate productivity materializes, and existing matches may be subject to destruction. If firms exit the market, they send their workers to unemployment and get payoff 0. This ensures that firms do not refrain from opening vacancies even if the expected value is negative.

[25] In the absence of significant limited commitment frictions, and given that workers are risk-averse while firms are risk-neutral, I choose to model wages as fixed. Computing the fully optimal dynamic contract in this model is considerably more involved, as the contract would be a function of wealth.

Free entry Free entry drives the value of posting a vacancy to zero. The expected value of the job is then equal to the fixed cost of opening a vacancy, i.e.,

$$\beta \mathbb{E}_t \left[q(\theta_t) J(x_t, w_t; \Gamma_{t+1}) \right] = \varphi(x_t, z_t)$$

Which pins down the market tightness for markets that are visited by unemployed workers. The concept follows the notion of subgame perfect equilibrium. For unvisited markets, the market tightness is equal to zero. Given the functional assumptions stated above, it holds

$$\theta_t(x_t, w_t; \Gamma_t) = \begin{cases} q^{-1}\left(\frac{\varphi(x_t,,z_t)}{\beta \mathbb{E}_t[J(x_t, w_t; \Gamma_{t+1})]}\right) & \text{if } \beta \mathbb{E}_t \left[J(x_t, w_t; \Gamma_{t+1}) \right] > \varphi(x_t, z_t) \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

Government The government collects taxes to fund the unemployment benefits. The government pools income taxes, inherits the retired workers' assets and redistributes them to the unemployed. I do not assume a balanced budget in every state, but that in the stochastic steady state it holds that

$$\mathbb{E}_z \int^{\Psi^e} \tau w + \mathbb{E}_z \int^{\Omega} \omega a = \mathbb{E}_z \int^{\Psi^u} b \tag{15}$$

, where Ψ^e denotes the stationary distribution of employed workers over labor market states and Ψ^u that of unemployed workers. This condition implies a balanced budget in expectations. On the one hand, this avoids having a cyclical unemployment benefit or tax. On the other hand, it is also relevant for block recursivity, as will become clear in the next section.

III. Equilibrium Characterization

Definition 2. *Let $\Gamma = \Omega \times \Psi \times \mathcal{Z}$. A recursive equilibrium for the baseline economy consists of a set of value functions for workers $U : \mathcal{A} \times \Gamma \to \mathbb{R}$ and $E : \mathcal{A} \times \mathcal{W} \times \Gamma \to \mathbb{R}$; a value function for entrant firms $J : \mathcal{X} \times \mathcal{W} \times \Gamma \to \mathbb{R}$; a set of policy functions $c_u, c_e, (x, w)$; a market tightness function $\theta : \mathcal{X} \times \mathcal{W} \times \Gamma \to \mathbb{R}_+$; a destruction productivity function $\ell : \mathcal{X} \times \mathcal{W} \times \Gamma \to \{0, 1\}$; a process for aggregate shocks (z); and a law of motion for the aggregate state $\mathbb{H}$ such that:*

i) U and E satisfy, respectively, Eq. (10) *and* (11)*, taking taxes τ as given; $c_u, (x, w)$ and c_e are the associated policy functions.*

ii) V and J satisfy, respectively, Eq. (12) *and* (13)*.*

iii) There is free entry in each submarket, so that market tightness satisfies (14) *in all submarkets (x, w, Γ).*

iv) $\mathbb{H}$ is consistent with agents' policy functions and the exogenous law of motion for aggregate productivity, which evolves according to a Markov chain with transition probability $\Xi(z, z')$.

v) The government balances its budget in expectations, according to Eq. (15)*.*

Under some restrictive assumptions, the model admits a *block* recursive equilibrium.

Definition 3. *A Block Recursive Equilibrium (BRE) is a recursive equilibrium in which the functions* $\{\theta, E, U, J, \ell\}$ *only depend on the aggregate state of the economy* Γ *through the aggregate state of productivity* z*.*

In general, BRE obtains as long as workers' policy and value functions are independent of the distribution of workers across employment states (Menzio and Shi, 2010). Under some additional assumptions, this is true even when the asset distribution is a state variable of the steady state and in the presence of sorting.

Proposition 2. *The baseline quantitative model is block recursive.*

Proof. See Appendix C. ■

The combination of separated submarkets and the free entry condition is central to the existence of a BRE. To get some intuition of why the equilibrium is BR, assume that J depends on Γ solely through the aggregate component of productivity z. Importantly, I assume that firms do not observe workers' wealth when posting job contracts. Then, free entry implies that θ is also independent of Ψ (see Eq. 14). In fact, free entry determines the number of vacancies in each submarkets, independently of the distribution of workers across the employment distribution. The support of market tightness, i.e., the open markets in equilibrium, is function of the distribution of workers across employment and asset states, but is immaterial for workers' value functions. The unemployed workers' jobs search problem is thus also independent of Γ, as it merely depends on exogenous parameters and the job finding probability (function of market tightness θ). It follows that the values of search and unemployment also only depend on z. Directed search is a crucial assumption here, as if it were undirected, the worker would need to evaluate the whole distribution of open vacancies. As for the consumption-savings decision of both employed and unemployed workers, it is independent of Γ given that we assume the interest rate r to be exogenous to their savings decision. Therefore, the workers value functions and policy functions are independent of Γ. We can then prove that the mapping is a contraction, therefore that the solution is such that the equilibrium is block recursive. The assumption that the tax rate balances the stationary distribution, rather than the period-by-period distribution, is key. In particular, the tax rate is fixed and can therefore be treated as a parameter by agents.

Block recursivity implies that one can solve for the equilibrium of the model without keeping track of the distribution of agents over asset or employment states. It therefore offers a relatively tractable way to solve a fully stochastic dynamic equilibrium. I solve the model numerically with a nested algorithm and using policy and value function iteration. For

government market clearing, I start from an initial guess for τ, verify the government budget constraint, and update the guess accordingly until the constraint is satisfied. See Appendix E for more details.

4 Wealth Sorting in a Quantitative Model

4.1 Calibration

I calibrate the model at a monthly frequency for the United States. Functional forms are listed in Appendix Table D5, externally calibrated parameters in Table 4 and internally calibrated parameters in Table 5. Given my focus on *residual* inequality, the model is in nature parsimonious. My calibration exercise targets one moment of the wealth distribution, as well as the dispersion and cyclicality of matching and separation rates, consistent with the PSID evidence.

The utility and matching functions are taken from standards in the literature. I assume a CRRA utility function with an intertemporal elasticity of substitution σ equal to 2. I follow Menzio and Shi (2011) and choose a standard CES matching function. I set the interest rate to target a 2 percent annualized real rate in steady state. I set the borrowing constraint to $\underline{a} = 0$. Agents face a constant retirement probability $\omega = 0.0021$, implying an expected working life duration of 40 years. The log of aggregate productivity follows an AR(1) process: $\log z_t = \rho_z \log z_{t-1} + \sigma_z \varepsilon_t$. I estimate $\rho_z = 0.9179$ and $\sigma_z = 0.0051$ by running an AR(1) regression on real labor productivity in the nonfarm business sector, as compiled by the Bureau of Labor Statistics (BLS), over the period 1985 to 2018. Monthly estimates are reported in Table 4. I target γ to match an average elasticity of the matching function to tightness of $[0.27 - 0.5]$, in line with the literature. The discount factor β is calibrated to target an average annual MPC consistent with the literature.[26] I set unemployment benefits b to target a replacement rate of 40% for the lowest paid jobs.[27]

I follow Birinci and See (2023) in specifying the separation and matching functions, allowing them to depend on both firm type and aggregate productivity. The targets for average job finding and job loss rates are taken from average monthly CPS flows, while the targets for the cross-sectional dispersion and its cyclicality by wealth are drawn from conditional PSID evidence. In particular, I augment model (1) to allow for asymmetric effects of GDP expansions and contractions. I then compute predicted job loss and job finding rates by liquid wealth, conditional on the aggregate state – see Appendix Figure A1. From these, I recover the slopes of the relationship between labor market transitions and wealth, for both job loss and job finding, as well as the differences in average job finding and job loss rates across booms and recessions. These moments are conditional in that they control for observed and unobserved individual characteristics and capture differences across wealth levels.

The parameters $\bar{\delta}$, η^{δ_z}, and η^{δ_y} govern, respectively, the average separation rate, the cyclicality of the job destruction rate, and its cross-sectional dispersion across firms. Similarly, $(\bar{\varphi}, \eta^{\varphi_y}, \eta^{\varphi_z})$ control the average matching probability, the dispersion in matching across

[26] The corresponding annual MPC is 0.46, computed as $mpc_a = 1 - (1 - mpc_m)^{12}$.

[27] Unemployment benefits in this model are meant to capture only the pecuniary value of benefits.

assets, and the cyclicality of matching. The average vacancy cost is calibrated to match a monthly U-E flow rate of 24%, while the average separation rate is targeted to replicate a monthly E-U flow of 1.3% (CPS data, 1999–2017).

Table 4: External calibration

Parameter	Definition	Value	Parameter	Definition	Value
r	interest rate	0.0016	ρ_z	TFP, persistence	0.918
σ	risk aversion	2.0	σ_z	TFP, standard deviation	0.004
$\underline{a}$	borrowing limit	0.0	ω	retirement probability	0.0021

Table 5: Internal calibration

Parameter	Value	Target (description)	Model	Target
β	0.990	average MPC	0.050	0.03-0.056
b	3.10	replacement rate	0.349	40%
γ	0.38	matching elasticity	0.43	0.27-0.5
Separation function				
$\bar{\delta}$	0.026	avg separation	0.0163	0.0130
η^{δ_y}	-0.24	slope by asset (boom)	-0.007 $\times 10^{-2}$	-0.099 $\times 10^{-2}$
		slope by asset (recession)	-0.115 $\times 10^{-2}$	-0.283 $\times 10^{-2}$
η^{δ_z}	30.00	ratio boom/recession	0.658	0.682
Vacancy cost function				
$\bar{\varphi}$	1.00	avg matching	0.1795	0.24
η^{φ_y}	1.00	slope by asset (boom)	-0.020	-0.018
		slope by asset (recession)	-0.020	-0.017
η^{φ_z}	65.00	ratio boom/recession	1.172	1.084

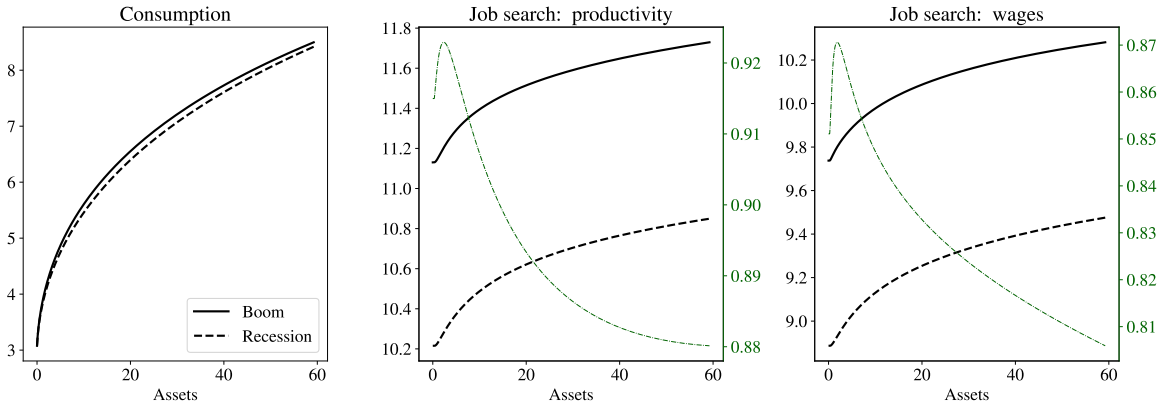
The model does a decent job at matching the average rates, the elasticity of job loss and job finding by liquid wealth. The implied unemployment rate is of 9.3% (including the newborn workers), and its standard deviation (computed from simulating the model around the stochastic steady state) is in line with the data – see Appendix Table D6.

4.2 Consumption and Job Search Policy Functions

Figure 2 illustrates the consumption and job search behavior of unemployed workers as a function of their asset holdings. The consumption policy function follows a standard pattern (left panel). The middle and right panels, respectively, plot the choice of wages and firm type as a function of assets. As shown by the monotonically increasing lines, the equilibrium exhibits positive wealth sorting, i.e., wealthier individuals apply to higher-productivity jobs that also offer higher-wage ("good" jobs), while poorer workers target lower-productivity jobs that offer lower-wage ("bad" jobs). The driving mechanism is the shorter queue length in bad jobs, which implies a higher probability of match in those submarkets. Since asset poor individuals are at risk of becoming hand-to-mouth and thereby off their Euler equation, they prefer to search in submarkets that offer a relatively higher probability of match. Instead, wealthy unemployed workers are far from the borrowing

constraint and prefer to target a relatively high wage, thereby accepting a longer waiting time. The solid and dotted lines plot the policy functions under different states of aggregate productivity. Overall market tightness declines in recessions, reflecting both weakened job creation incentives and an increased risk of separations. If unemployed were to keep their job search unchanged, the lower matching probability would result in them having to decrease their current consumption. Agents thus balance consumption smoothing against job search and future income prospects. Therefore, in bad aggregate states, unemployed workers shift their search towards "worse" jobs (lower wages and lower productivity jobs) in an effort to smooth their consumption. The difference in search policy functions between high and low aggregate productivity states is shown in the dash-dotted green lines. It decreases with asset holdings, indicating that workers with fewer assets adjust their job search relatively more over the business cycle. In the presence of incomplete markets, the labor market thus provides some insurance to unemployed workers – in the spirit of Acemoglu and Shimer (1999), at the same time implying a cyclicality of wages and employment risk. Wages decrease relatively less than productivity in bad times.
Appendix Figure D4 shows the consumption policy function for employed workers in different submarkets. Unsurprisingly, those employed in bad jobs consume less as they earn lower wages and face a higher separation probability.

Figure 2: Unemployed workers policy functions

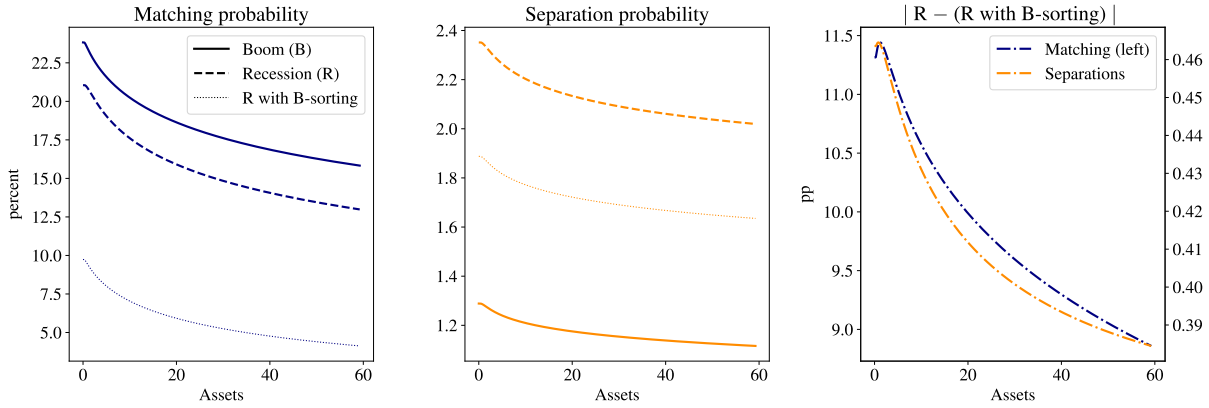


Note: The solid (dashed) lines, measured on the left axis, depict the smoothed policy functions for unemployed workers in good (bad) aggregate states. In the middle and right panels, the green dash-dotted line, measured on the right axis, represents the difference between booms and recessions.

The left panel of Figure 3 plots the matching probabilities by assets, which reflects the choice of submarkets. By directing their search towards bad jobs, poor workers have around 20% probability of exiting their precarious unemployment state in booms (versus around 15% for the wealthiest). Wealthier workers are willing to wait longer (direct their search towards markets featuring low probability of matches) to get good jobs. Although the unemployed shift their search toward relatively lower-productivity jobs in downturns, the overall matching probability remains lower than in good times, consistent with empirical evidence on the procyclicality of market tightness. The dotted line represents a counterfactual matching probability where the aggregate state is low (R), but the sorting policy functions are that

of the high aggregate state (B). If workers did not adjust their job search, their average matching probability would fall by more than half.
The middle panel shows the expected separation risk associated with unemployed workers' job search. Under the assumption that separation risk decreases with firm productivity, positive wealth sorting implies that poorer employed workers face a higher probability of separation, particularly during periods of low aggregate productivity. The fact that the dotted line lies below the recession line (dashed) indicates that workers' job search adjustments in low aggregate states come at the cost of higher separation risk once employed. In this sense, job search amplifies the volatility of separation risk over the business cycle.
The right panel shows the absolute difference in matching and separation probabilities during downturns relative to the counterfactual sorting (dashed minus dotted lines in the left and middle panels). Cyclical sorting implies that the matching and separation probabilities of poorer agents increase relatively more than those of wealthier agents. This can be interpreted as a theoretical counterpart to the empirical evidence presented in Section 2.

Figure 3: Matching and separation probabilities



Note: The solid (dashed) lines represent the smoothed matching and separation functions in good (bad) aggregate states. The dotted lines show the matching or separation in bad states but with the job search policy functions of good states. The right panel displays the difference between the dashed and dotted lines from the left and middle panels.

Appendix Figure D8 plots the value of filled vacancies ordered by the markets populated by workers with increasing assets. Jobs employing poor workers are also the ones that have the lowest value. This arises endogenously monitored by the calibration of the entry cost function of the firms.[28]

[28]This result suggests the possibility of endogenizing the separation function δ within the framework. Suppose firms face the same separation risk, and endogenous job destruction is introduced through a productivity threshold for labor market entry. Firms at the productivity margin are vulnerable to aggregate shocks, with those at the threshold being destroyed following a negative shock. Since ex-post profits increase with productivity, this mechanism implies a higher layoff risk for lower-productivity workers. Only that in that case it would come from a firm endogenous exit and not the separation function.

4.3 Stationary Distribution

The (stochastic) steady state of the model supports a stationary distribution of agents of states. Appendix Figure D5 plots the steady state distribution of agents over assets as a function of aggregate productivity z, as well as the wage distribution. While the majority of agents have substantial savings, a non-negligible share of the population holds zero assets. Bad aggregate states are associated with lower wages, and a larger share of workers at the bottom of the wage distribution. On the right panel, I plot the MPC by wealth percentiles. The lowest percentiles exhibit an MPC close to one, and the MPC is relatively high across all wealth percentiles (it is close to 0.035 even at the top of the distribution).

In equilibrium, workers with higher levels of assets are more likely to earn higher wages and have relatively more secure jobs. This dynamic could, in theory, set off a path of explosive asset accumulation, undermining the existence of a stationary equilibrium. However, stabilizing forces are sufficient to sustain a stationary equilibrium under the baseline calibration. Specifically, the asset policy function is not diverging even in the highest asset and labor market states, as illustrated in Appendix Figure D7.

First, all workers face a positive probability of job separation, with separation rates that do not exhibit large cross-sectional variation. Second, workers deplete their assets during periods of unemployment, especially if they enter unemployment with higher initial assets. Figure 4 plots the behavior of an unemployed worker holding the mean level of steady state assets, as she continues not to be selected by the matching function (she is unlucky and remains unemployed period after period). The left panel shows the worker's assets over time, while the middle and right panels illustrate their labor market search behavior. The dotted (solid) line represents the worker's choices if the aggregate state is low (high) throughout.

Figure 4: Average outcomes for cohort starting with the same level of assets



Workers with higher assets direct their search towards good jobs, which implies longer average unemployment duration due to the relatively low matching probability associated to these jobs. Consequently, these workers experience longer unemployment spells in which they deplete their asset holdings. Moreover, they do so at a faster pace the higher the initial assets, as visible from the decreasing slope in absolute terms in the left panel. Therefore, although workers with higher assets face a relatively lower probability of job separation, when

they become unemployed they deplete their assets faster and for a longer expected period. Furthermore, within individual workers, negative duration dependence in asset levels drives a shift in search behavior over time, towards lower wages and lower security jobs. In fact, as unemployment lengthens, asset holdings decline, increasing the relative preference for higher-matching probability jobs. This within-worker duration dependence contrasts with the positive duration dependence observed *across* workers, where higher-wage jobs correlate with longer unemployment spells.[29] Therefore, initially wealthier workers may ultimately accept the worst jobs if they experience extended unemployment. This outcome occurs after 12 months of unemployment for an average-wealth worker under baseline calibration.
The aggregate state influences the rate of asset depletion and the job search behavior. In low aggregate productivity states, workers exhibit a stronger precautionary savings motive, which slightly moderates the rate of asset depletion.

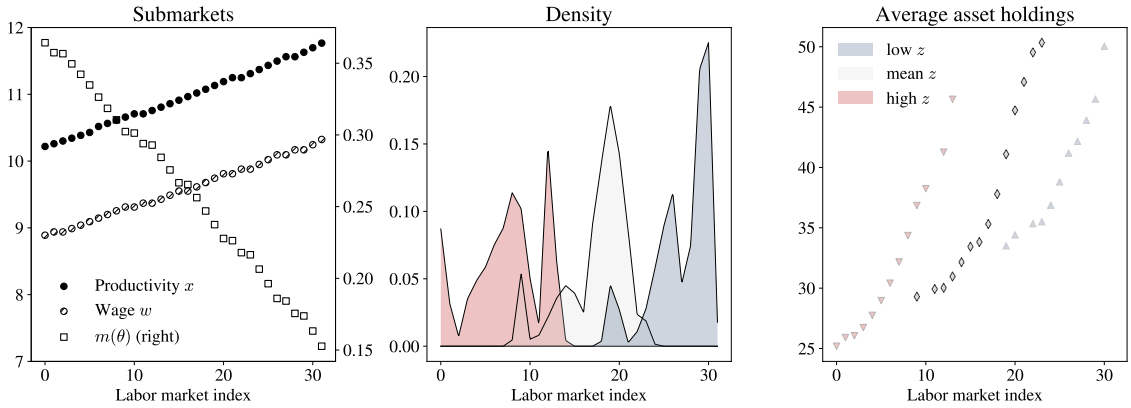
4.4 Wealth Sorting and Aggregate Productivity

Figure 5 shows the labor markets that are open in equilibrium and how they are populated in the steady state. The left panel reports the equilibrium allocation of open submarkets, ordered by increasing wages and productivity. The middle and right panels display, respectively, the density of workers across these submarkets and their average asset holdings. In both panels, distributions are computed in non-stochastic steady states. Colors distinguish aggregate conditions: red triangles (grey diamonds, blue upward triangles) denote steady states with low (mean, high) aggregate productivity.
Recessions are characterized by a shift in the distribution toward lower-quality jobs, reflecting both weaker job creation incentives and a greater need for insurance (middle panel). Moreover, within the set of open labor markets, worker mass becomes more concentrated at the lower end in bad aggregate states. In contrast, during expansions, relatively few workers occupy the lowest-quality submarkets. This is also true in the stochastic steady-state: times of low aggregate productivity are associated to a larger mass of workers in low-wage jobs (Appendix Figure D6). Average asset holdings are also lower in recessions (right panel), and the composition of workers within a given submarket varies across aggregate states: for example, labor market 10 is populated by relatively wealthy workers in low states, but by relatively poorer workers in normal times.

Moreover, the distribution of agents over labor markets also changes with aggregate productivity. Figure 6 plots the distribution of agents across different aggregate states in the stochastic steady-state. The first row shows the unemployed, and the rest show the open labor markets ordered in ascending order (the rows at the top are bad jobs, and quality improves as we read rows more to the bottom). Two features are worth mentioning. First, as expected, recessions are times with relatively more unemployed. Second, the distribution is more scattered during booms than in recessions. Looking in the top left corner, we see an increased density of agents that are poor and working in low-quality jobs. At the same time, workers at the top of the asset distribution are less likely to work in these low paid, low security jobs. The density becomes more concentrated on the diagonal, which also means

[29] This is a common feature of directed search models.

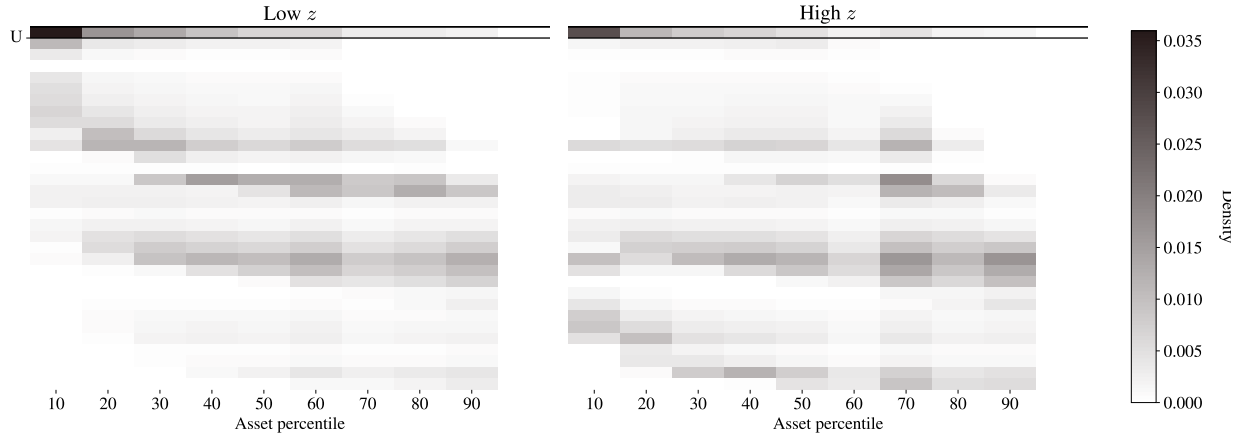
Figure 5: Distribution of open labor markets



Note: The left panel shows open labor markets bunching all aggregate states together. The middle and right panels' distributions are computed in non-stochastic steady state with different values for aggregate productivity z, corresponding to the different colors.

that sorting is stronger in recessions.

Figure 6: Stochastic steady state distribution of agents over assets



Note: The $x-$axis represents deciles of the asset distributions. The $y-$axis represents workers' labor market status; the first row is the unemployment status, while the other rows represent open labor markets (in ascending order, i.e., the bottom rows are higher productivity and wages jobs relative to the first rows). The intensity of coloring represents the density of agents in each asset bin and labor market state.

Table 6: Sorting in booms and recessions

Moment	Aggregate productivity *recessions*	*booms*	Difference
$\text{corr}(a_i, w_i)$	0.9373	0.9112	0.0261
$\text{corr}(a_i, x_i)$	0.9445	0.9298	0.0147
$\text{corr}(a_i, \Lambda_i)$	0.9382	0.9221	0.0161
$\text{sd}(w_i)$	0.1657	0.1413	0.0245
$\text{sd}(x_i)$	0.1826	0.1617	0.0210

Table 6 quantifies the dynamics of wealth sorting during booms and recessions. The strength of sorting can be evaluated by the relative correlation in good and bad aggregate states. As we can read from the positive differences, sorting is stronger in bad aggregate states. This is true by all measures of sorting (assets with wages, productivity, and retention rates). The correlation difference is relatively larger for wages, which is the margin that workers use more to insure their consumption. Moreover, the dispersion of wages and firm productivity is slightly higher in bad aggregate states.

5 Implications over the Business Cycles

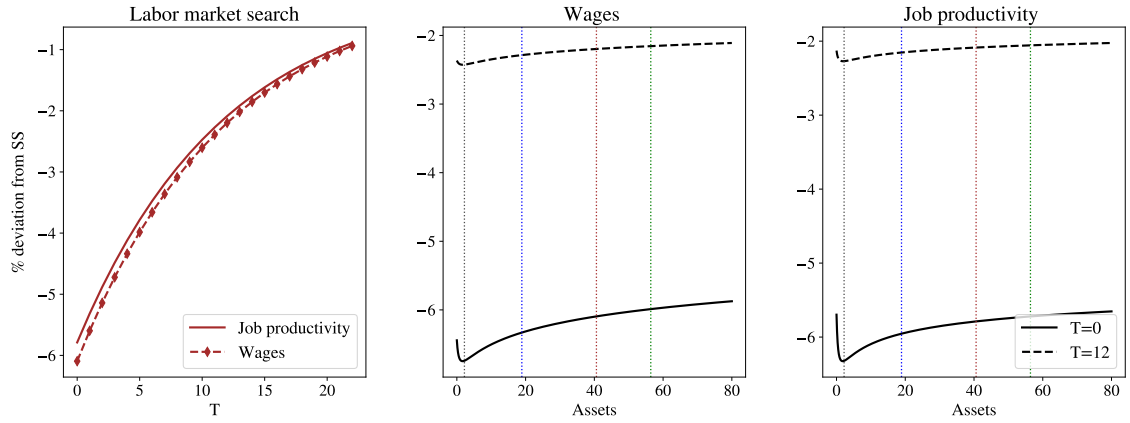
In this section, I first present the impulse response functions (IRFs) to a negative aggregate productivity shocks. I show that unemployed workers primarily adjust their job search behavior rather than their consumption, particularly among the wealth-poor. A policy that provides an unexpected one-off transfer is most effective in boosting consumption if it targets the poorest unemployed workers. I then show that wealth sorting has implications for long-run inequalities, as it generates a "poverty trap".

5.1 Short Term Responses to a Recessionary Shock

I simulate the effect of a recession on consumption and job search, both at the individual level and on aggregate. I assume that aggregate productivity follows an AR(1) process such that $z_t = \rho_z z_{t-1} + (1-\rho_z)z^{ss} + \epsilon_t$, where $\epsilon_t \sim \mathcal{N}(0,1)$; and implement a 1% negative aggregate productivity shock.

Figure 7 plots the job search response. On the left panel, we see that following a negative aggregate productivity shock, the median job search decreases both in terms of wages and job productivity. Wages and productivity decrease by around 6% on impact. This aggregate response hides important heterogeneity, as shown in the middle and right panels (for wages and productivity, respectively). The vertical lines in the middle and right panels flag percentiles of the distribution (1^{st}, 10^{th}, 50^{th} and 99^{th} percentiles, from left to right). The bottom 1% of the distribution decreases their search by relatively more, and the difference between the bottom and top 1% is of around 1% of wages, and 0.5% of productivity. After a year, the unemployed search shifts up (dashed lines) and also becomes more homogeneous across the wealth distribution. When aggregate productivity is at its lowest, its effect on the search of unemployed workers is thus the largest both in terms of magnitude and heterogeneity across workers.

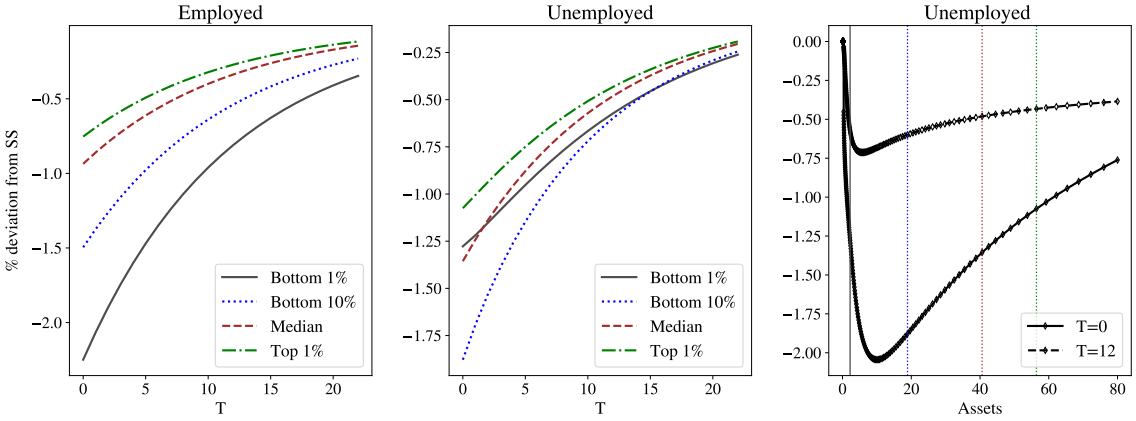
Figure 7: Unemployed workers' job search



Note: All panels plot the IRFs to a 1% negative aggregate productivity shock. On the left panel, the two lines correspond to the labor market search of the median unemployed worker. The vertical lines correspond, from left to right, to the bottom 1%, 10%, the median, and the top 1% of the wealth distribution.

The reason why the poorest unemployed adapt their search more can be seen in Figure 8 (right panel)– it allows mitigating the drop in their consumption. The left panel of Figure 8 instead plots the consumption response for employed workers. Consistently with more standard models, the poorest agents decrease their consumption by relatively more. In the middle panel, we see that besides for the poorest who are hand-to-mouth, the unemployed workers' consumption behavior mirrors that of the employed.

Figure 8: Workers' consumption response

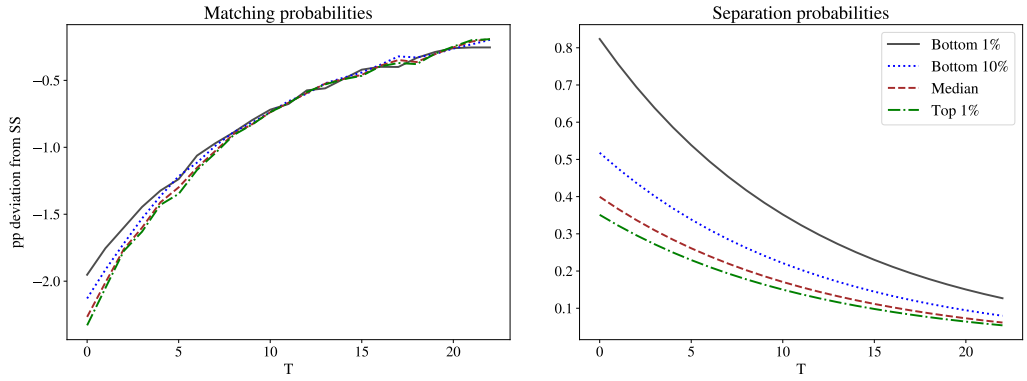


Note: All panels plot the IRFs to a 1% negative aggregate productivity shock. On the left panel, lines correspond to workers working in the same submarket but holding different wealth. The vertical lines correspond, from left to right, to the bottom 1%, 10%, the median, and the top 1% of the wealth distribution.

Positive wealth sorting implies a different matching and separation probability across the wealth distribution. As shown in Figure 9, on average, matching decreases and separations increase. Effects are heterogeneous across the wealth distribution. The change in job search

of bottom 1% unemployed allows them to mitigate the drop in their matching probability, and they experience the lowest decrease.

Figure 9: Matching and separation probability by asset

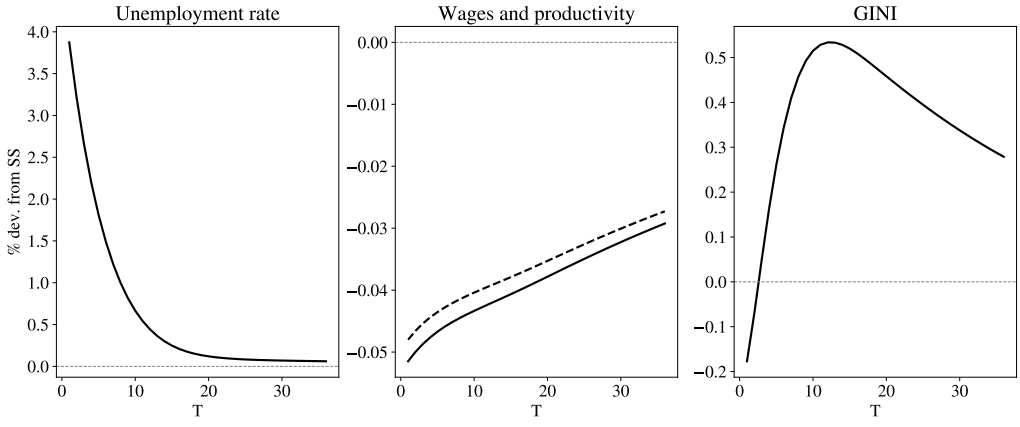


Note: All panels plot the IRFs to a 1% negative aggregate productivity shock. The different lines represent percentiles of the wealth distribution

However, the heterogeneous effects are stronger in the separation probabilities. On impact, the separation for the poorest 1% is twice as large relative to the median worker. This difference stays present throughout the shock – which is not the case for the matching probability. Therefore, recessions heterogeneously affect workers mostly through an increase in separation probability, and through the consumption response.

Figure 10 plots aggregate responses of the unemployment rate, wages and inequality following a temporary (one month) 1 standard deviation decrease in aggregate productivity. The higher incidence of separation shocks following the recessionary shock and the following higher unemployment risk faced by workers through sorting results in a persistence in the unemployment rate.

Figure 10: Hysterisis

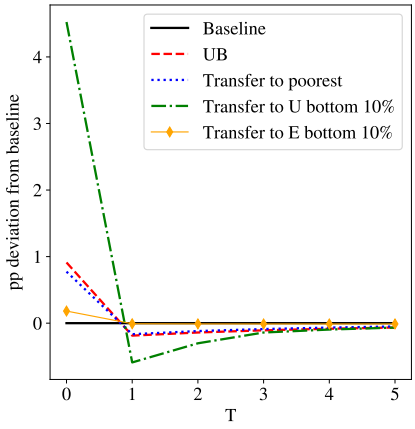


Note: All panels plot the IRFs to a temporary 1 std negative aggregate productivity shock.

5.2 Targeted Transfers and Aggregate Consumption Response

I study the effect of four types of targeted transfers implemented when the aggregate productivity shock hits for aggregate consumption response. Figure 11 plots the differential consumption response under policy counterfactuals. In the baseline, the social planner distributes an untargeted transfer to all workers in the first period of the negative aggregate productivity shock.[30] Agents do not expect it ("MIT" policy shock) and they correctly do not expect the transfer to be reiterated. I set the total value of the transfer to be equal to one-fifth of average wages (this would correspond to a transfer of roughly $1000).
The first policy is a standard unemployment benefit and targets all unemployed workers (dashed line). The second is a policy targeting the wealth-poor (dotted line), and for comparability with the first policy, I set the threshold such that the same amount of workers get the transfer. Then, I also implement policies targeting only the unemployed or employed wealth-poor (dash-dotted line and diamond line, respectively). There, I divide the total transfer to the 10% poorest. By targeting the transfer to workers with on average lower assets, the aggregate consumption drop is mitigated in the first period by more than 4 percentage points. This is especially true for the transfers targeted at the wealth-poor unemployed workers, as these are the group with the largest share of hand-to-mouth. The targeted transfer also allows them to look for better jobs, which depresses aggregate consumption in the following periods through a relatively larger share of workers who remain unemployed (matching probabilities are lower for good jobs). This backlash effect on consumption (0.5 percentage points) is strongest when targeting the unemployed poor, and does not happen when targeting the poorest employed workers.

Figure 11: Consumption effect of targeted transfers



Note: All lines correspond to the same aggregate transfer, which amounts to one tenth of average equilibrium wages. The solid line represents the aggregate consumption response when the transfer is distributed to all agents (baseline). The other lines represent differences from this baseline. The dashed line distributes the transfer equally among unemployed; the dotted line to all workers at the bottom of the wealth distribution (the mass is chosen to be equal to the mass of unemployed); the dash-dotted line to the unemployed workers at the bottom 10% of the wealth distribution; and the diamond line to the employed worker at the bottom 10% of the wealth distribution.

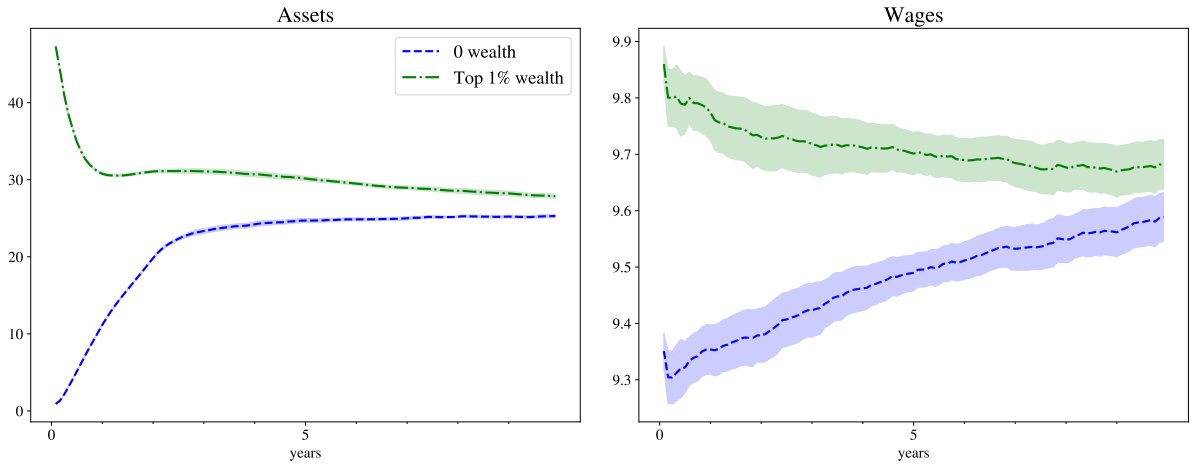
[30] Here I do not impose the government to balance its budget for simplicity. For this reason, in this section I always compare counterfactuals holding the total amount of transfer fixed.

5.3 Cyclical Shocks Have Long Term Consequences

Whether the increase in inequality following a negative aggregate productivity shock is associated with long term consequences is an important policy question. In this model, cyclical fluctuations have long-lasting effects because of workers' assets accumulation and job search.

In Figure 12, I simulate two cohorts of 10'000 individuals, each starting either with the bottom or the top 1% wealth. I track them over time as aggregate productivity evolves according to different draws of the stochastic process. The left panel plots mean asset holdings, and the right panel plots mean wages; while the shaded areas represent 90% confidence interval (across aggregate productivity states). A striking result is that the initial

Figure 12: Average outcomes for cohort starting with different levels of assets



Note: Each cohort is of 10'000 workers who share the same level of assets in $t = 0$. I then randomly simulate 20 different paths for aggregate productivity and follow each worker in the different cohorts. Lines represent the average worker, while shaded areas are the 90% confidence interval.

asset position determines different asset and wages for at least the next 10 years. Comparing the poorest cohort with the richest, the right panel reveals that after 10 years, the cohort that started with the highest wealth continues to benefit from higher wages. Wealth sorting leads wealth-poor workers to accept lower wages, limiting their ability to accumulate savings during employment and thereby increasing their probability to apply to low-wage jobs if they become unemployed. In that sense, the presence of liquidity constraints in this setting generates a "poverty trap". Heterogeneous separation risk aggravates this phenomenon, because it disproportionately affects wealth-poor workers. Bad aggregate states intensify the poverty trap for wealth-poor unemployed workers, who are then driven to seek riskier, lower-wage jobs. It is worth mentioning that the model does not embed *permanent* inequalities; all heterogeneity in asset and wages are merely due to workers' initial condition in wealth and its interaction with labor market outcomes. Given the randomness of shock processes, average effects for the different cohorts converge to the same value in the long-run.
Next, I quantify the scarring effects of recessions along the wealth distribution by comparing two simulated aggregate states: a "recession" scenario, where the aggregate state is negative for the first two quarters (following the NBER definition) and then goes back to

the unconditional mean, and a "tranquil" scenario, where the aggregate state is constant at the mean throughout. In Figure 13, I show the percentage difference in assets (left panel) and wages (right panel) for cohorts who began their unemployment spells in the "recession" scenario relative to a "tranquil" scenario. Moreover, I show how these differences vary between workers at the bottom and top of the wealth distribution (blue and green lines, respectively).

Figure 13: Long-term effects of recessions for cohort starting with different levels of assets



Note: Each cohort is of 10'000 workers who share the same level of assets in $t = 0$. Lines represent the difference between starting the unemployment spell in a recession (2 quarters negative growth) versus a tranquil period, separately for each wealth cohort. After the first 2 quarters, the aggregate state is the same across simulations.

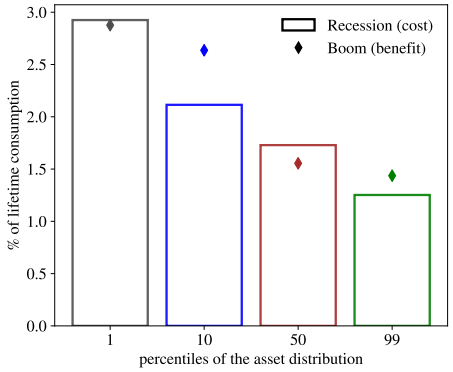
Starting the unemployment spell in a recession has long-lasting effect for all workers. Wages are around 4.5% lower on impact, and are still 1% lower after 5 years.[31] Asset accumulation is positive in the very beginning, reflecting precautionary savings motives, but then becomes and stays negative for all workers. Those suffering from larger negative effects are the wealth-poor, whose wages are between 0.5 and 1 percentage points lower compared to the wealthiest. Moreover, those with initially zero assets accumulate much less in the "recession" scenario, by as much as 8% in the first year. The wealthiest also start decumulating assets, though to a smaller extent. As a consequence, wealth inequality between these two groups increases in the first years.

Finally, I calculate the welfare cost of beginning a job spell in a recession versus a boom across the wealth distribution. The solid bars in Figure 14 represent the lifetime cost of starting a job search during a recession, computed in a similar fashion as Lucas (1987) – see Appendix F for more details. The poorest workers would sacrifice 3% of lifetime consumption to avoid the recession scenario. This is more than two times larger than the wealthiest workers, whose utility losses of starting in a recession are equivalent to 1.25% of lifetime consumption.

[31] The strong persistence is also due to the assumption of fixed wages throughout the employment duration. Wages therefore adjust to aggregate conditions only when the employed worker gets separated and finds a new job.

The diamonds show the benefits of starting in a boom relative to the tranquil scenario. The heterogeneity is slightly lower, mostly driven by the top 1%. In fact, these workers benefit more from a positive aggregate state as they suffer in a bad aggregate state – they would give up 1.5% of their lifetime consumption for the boom scenario. Instead, the benefits are symmetric to the cost of recession for the bottom 1%, while the 10^{th} percentile gains relatively more in booms than what they lose from recessions.

Figure 14: Scarring effect of recessions and welfare gains from booms



Note: Each cohort is of 10'000 workers who share the same level of assets in $t = 0$ (respectively 5^{th} and 95^{th} percentiles). I compute the welfare cost of starting in a boom or recession, compared to tranquil times. I follow each worker in the different cohorts, and show outcomes for the median worker in the cohort.

6 Conclusion

This paper explores the relationship between wealth, job search behavior, and labor income risk over the business cycle. First, I present suggestive evidence that wealth-poor workers have a relatively larger employment elasticity to aggregate output fluctuations. I then rationalize this finding with a theory based on risk aversion, liquidity constraints, and labor market search. I show that in the presence of incomplete markets, risk-aversion drives agents to sort positively on the labor market, even though bad jobs are more likely to be destroyed. This implies that wealthier individuals can afford to be more selective in their job search, thereby obtaining better job matches with higher wages and higher security. During economic downturns, wealthier workers are able to cushion a potential job loss through savings, and their resulting job search implies more stable and higher income. Conversely, poor workers seek insurance and use the labor market to smooth their consumption. Wealth sorting is relatively stronger in bad aggregate states, and so are inequalities (in terms of wages and security). Positive wealth sorting generates a higher employment elasticity to aggregate shocks for low-wealth workers, in line with the empirical evidence. Moreover, it generates a "poverty trap", in which wealth-poor workers struggle to accumulate assets due to short and low-wage employment contracts. Following a negative aggregate productivity shock, I show that a transfer targeted at the unemployed wealth-poor is the most effective way to mitigate the drop in aggregate consumption but induces a small backlash in the next period due to more workers staying unemployed.

References

E. M. Acabbi, A. Alati, and L. Mazzone. A labor market sorting model of hysteresis and scarring. *Available at SSRN 4068858*, 2022. 3, 6

D. Acemoglu and R. Shimer. Efficient unemployment insurance. *Journal of Political Economy*, 107(5):893–928, 1999. 5, 26

M. Aguiar, M. Bils, and C. Boar. Who are the hand-to-mouth? *Review of Economic Studies*, page rdae056, 2024. 10

R. Bednarzik, A. Kern, and J. Hisnanick. Displacement and debt–the role of debt in returning to work after displacement. *Journal of Financial Economic Policy*, 2021. 9

S. Birinci and K. See. Labor market responses to unemployment insurance: The role of heterogeneity. *American Economic Journal: Macroeconomics*, 15(3):388–430, 2023. 6, 24

I. Cairó and T. Cajner. Human capital and unemployment dynamics: Why more educated workers enjoy greater employment stability. *The Economic Journal*, 128(609):652–682, 2018. 5

S. Cannon and J. Mustre-del Rio. Dissecting wage dispersion. *Economic Review (Federal Reserve Bank of Kansas City)*, 102(3), 2017.

D. Card, R. Chetty, and A. Weber. Cash-on-hand and competing models of intertemporal behavior: New evidence from the labor market. *Quarterly Journal of Economics*, 122(4):1511–1560, 2007. 9

C. Carrillo-Tudela, L. Visschers, and D. Wiczer. Cyclical earnings, career and employment transitions. *CESifo Working Paper No. 9990*, 2022. 5

S. Catherine, M. Miller, and N. Sarin. Relaxing household liquidity constraints through social security. *Journal of Public Economics*, 189:104243, 2020. 2

G. Chaumont and S. Shi. Wealth accumulation, on-the-job search and inequality. *Journal of Monetary Economics*, 128:51–71, 2022. 6

R. Chetty. Moral hazard versus liquidity and optimal unemployment insurance. *Journal of political Economy*, 116(2):173–234, 2008. 9

J. P. Danforth. On the role of consumption and decreasing absolute risk aversion in the theory of job search. *Studies in the Economics of Search*, 123:109–131, 1979. 5

G. Dany-Knedlik, A. Kriwoluzky, and S. Pasch. Income business cycles. 2021. 5

W. J. Den Haan, G. Ramey, and J. Watson. Job destruction and propagation of shocks. *American economic review*, 90(3):482–498, 2000. 44

J. Eeckhout and A. Sepahsalari. The effect of wealth on worker productivity. *Review of Economic Studies*, 91(3):1584–1633, 2024. 6, 15, 16, 46

R. J. Faberman, A. I. Mueller, A. Şahin, and G. Topa. Job search behavior among the employed and non-employed. *Econometrica*, 90(4):1743–1779, 2022. 51

F. Fontaine, J. Nyborg Jensen, and R. Vejlin. Wealth, portfolios, and unemployment duration. 2020. 9

B. S. Griffy. Search and the sources of life-cycle inequality. *International Economic Review*, 62(4): 1321–1362, 2021. 3, 6

F. Guvenen, S. Ozkan, and J. Song. The nature of countercyclical income risk. *Journal of Political Economy*, 122(3):621–660, 2014. 5

A. X. He and D. le Maire. Household liquidity constraints and labor market outcomes: Evidence from a danish mortgage reform. *Available at SSRN 3481279*, 2020. 9

K. Herkenhoff, G. Phillips, and E. Cohen-Cole. How credit constraints impact job finding rates, sorting, and aggregate output. *Review of Economic Studies*, 91(5):2832–2877, 2024. 6, 9

H. Hoynes, D. L. Miller, and J. Schaller. Who suffers during recessions? *Journal of Economic Perspectives*, 26(3):27–48, 2012. 5

G. Jarosch. Searching for job security and the consequences of job loss. *Econometrica*, 91(3): 903–942, 2023. 6, 17

L. Kaas, E. Lalé, and N. Siassi. Job ladder and wealth dynamics in general equilibrium. 2023. 6

F. Karahan and S. Rhee. Geographic reallocation and unemployment during the great recession: The role of the housing bust. *Journal of Economic Dynamics and Control*, 100:47–69, 2019. 49

J. Kramer. The cyclicality of earnings growth along the distribution – causes and consequences. 2022. 3, 6

P. Krusell, T. Mukoyama, and A. Şahin. Labour-market matching with precautionary savings and aggregate fluctuations. *The Review of Economic Studies*, 77(4):1477–1507, 2010. 6

M. Lammers. The effects of savings on reservation wages and search effort. *Labour Economics*, 27: 83–98, 2014. 9

R. Lentz and T. Tranaes. Job search and savings: Wealth effects and duration dependence. *Journal of Labor Economics*, 23(3):467–489, 2005. 6

J. Lise. On-the-job search and precautionary savings. *Review of Economic Studies*, 80(3):1086–1113, 2013. 6

J. Lise and J.-M. Robin. The macrodynamics of sorting between workers and firms. *American Economic Review*, 107(4):1104–1135, 2017. 6

R. E. Lucas. Models of business cycles, 1987. 36, 56

J. J. McCall. Economics of information and job search. *Quarterly Journal of Economics*, 84(1): 113–126, 1970. 11

G. Menzio and S. Shi. Block recursive equilibria for stochastic models of search on the job. *Journal of Economic Theory*, 145(4):1453–1494, 2010. 6, 23, 49

G. Menzio and S. Shi. Efficient search on the job and the business cycle. *Journal of Political Economy*, 119(3):468–510, 2011. 6, 13, 24, 49

A. Morin. Wage dispersion over the business cycle. *Economics Letters*, 181:17–21, 2019. 51

A. I. Mueller, D. Osterwalder, J. Zweimüller, and A. Kettemann. Vacancy durations and entry wages: Evidence from linked vacancy–employer–employee data. *Review of Economic Studies*, 91 (3):1807–1841, 2024. 17

A. Nekoei and A. Weber. Does extending unemployment benefits improve job quality? *American Economic Review*, 107(2):527–61, 2017. 9

C. Patterson. The matching multiplier and the amplification of recessions. *American Economic Review*, 113(4):982–1012, 2023. 5, 8, 42

R. Pinheiro and L. Visschers. Unemployment risk and wage differentials. *Journal of Economic Theory*, 157:397–424, 2015. 11

M. O. Ravn and V. Sterk. Macroeconomic fluctuations with HANK & SAM: An analytical approach. *Journal of the European Economic Association*, 19(2):1162–1202, 2021. 6

S. Rendon. Job search and asset accumulation under borrowing constraints. *International Economic Review*, 47(1):233–263, 2006. 6

A. Sepahsalari, C. Mantovani, A. Figueiredo, and I. Baley. Self-insurance in turbulent labor markets. Technical report, 2025. 6

E. Shao and P. Silos. Wealth inequality and employment fluctuations. *Economic Modelling*, 67: 125–135, 2017. 6

Appendix

A Empirical analysis

A1 Data sources

I. Panel Study of Income Dynamics

The PSID is the world's longest ongoing longitudinal household panel dataset. This survey was conducted in the US yearly from 1968 to 1997, and biennially afterwards. In total, more than 82,000 individuals (coming from around 9,500 families) were interviewed over 50 years. The panel consists of different subsamples. The first is the so-called SRC sample, which is representative of the US population. The SEO samples add an oversample of low-income families. Together, the SRC and SEO are referred to as the Core PSID sample, which is the sample I work with. My main variables of interest are individual demographics, employment, income, and wealth.

Sample selection I restrict the PSID to the heads in the Core sample, that are aged between 25 and 62. Data on wealth is available only from 1999 onward, which restricts my sample year to 1999 to 2017. I discard individuals with inconsistent data, e.g., those having positive hourly labor income lower than half of the federal minimum wage.

Demographics Until 2017, household heads are always male unless the household does not feature a spouse. The education level is not reported consistently throughout the years. I compute a consistent variable for each respondent and year and then attribute the education to that individual that is the highest computed over all observed years.

Employment and income The employment status is reported consistently after 1999, the question I am using reads: "We would like to know about what you do–are you (HEAD) working now, looking for work, retired, keeping house, a student, or what?". I set the employment status to be $e_{i,t}$ =1 if they answer "Working now" or "Only temporarily laid off", and =0 if they answer "Looking for work, unemployed". Employment status is thus at the time of interview. For results related to income, I use the household head's individual yearly labor income, as reported in the data. It is defined as "Labor part of farm income and business income, wages, bonuses, overtime, commissions, professional practice, labor part of income from roomers and boarders or business income".

Wealth Wealth is reported at the household level since 1999. I use the total of savings and checks accounts summed to stock holdings as a measure of liquid wealth. Results are similar when using a savings and checks accounts only.

II. Aggregate data

Aggregate unemployment rates are taken from the BLS, while aggregate GDP and CPI is taken from FRED, and the US State's GDP is taken from the Bureau of Economic Analysis. Nonfarm business labor productivity is taken from FRED (OPHNFB), based on data from the BLS.

A2 Additional results

Table A1: MPCs by liquid wealth quintiles

	Q1	Q2	Q3	Q4	Q5
Mean	0.519	0.441	0.349	0.295	0.286
Median	0.409	0.373	0.341	0.300	0.194

Note: Observations come from the Core PSID dataset. The sample covers years from 1999 to 2017. MPCs are computed according to the procedure in Patterson (2023).

Table A2: Gradually saturating to the baseline model

	(1)	(2)	(3)	(4)
$\Delta \log Y_{s(i),t}$	0.56***	0.32***	0.28***	0.26***
	(0.086)	(0.070)	(0.063)	(0.073)
$\log a_{it-1}$	0.016***	0.016***	0.011***	0.00047
	(0.00088)	(0.00089)	(0.00070)	(0.00079)
$\Delta \log Y_{s(i),t} \times \log a_{it-1}$	-0.034***	-0.040***	-0.034***	-0.027***
	(0.0095)	(0.0089)	(0.0079)	(0.0085)
US State & year FEs		✓	✓	✓
Worker controls			✓	✓
Individual FE				✓
R^2	0.056	0.066	0.093	0.45
N	44377	44377	44377	44377

Note: Observations come from the Core PSID dataset. The sample covers years from 1999 to 2017. Standard errors in parentheses; $^{*}p < 0.1$,** $p < 0.05$,*** $p < 0.01$, clustered at the state level. The outcome variable is the employment status of the household head. Worker-level controls consist of a quadratic in age and its interactions with education, gender, marital status, and race.

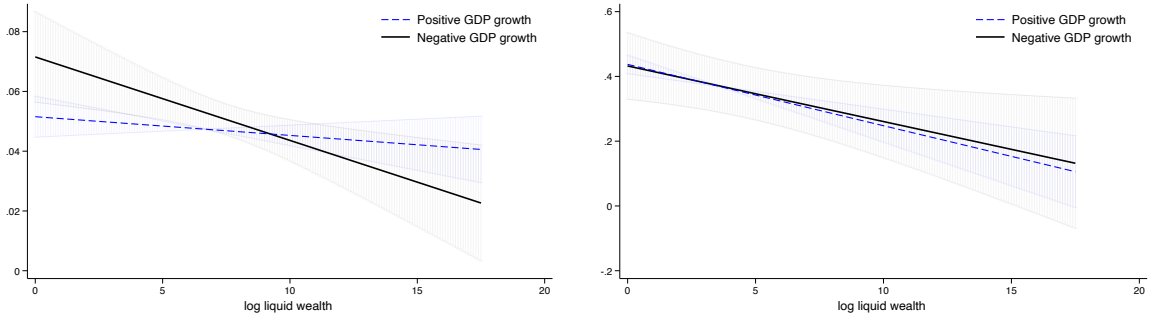
Table A3: Liquid wealth and employment exposure to aggregate fluctuations

	(1)	(2)	(3)	(4)
$\Delta \log Y_t$	1.68***			
	(0.14)			
$\log a_{it-1}$	0.020***	0.0032***	0.015***	-0.00054
	(0.00085)	(0.00079)	(0.00085)	(0.00061)
$\Delta \log Y_t \times \log a_{it-1}$	-0.12***	-0.10***		
	(0.016)	(0.015)		
Δu_t			-0.012***	
			(0.0018)	
$\Delta u_t \times \log a_{it-1}$			0.00072***	0.00098***
			(0.00024)	(0.00024)
Ind., US State & year FEs, worker controls		✓		✓
R^2	0.061	0.45	0.056	0.45
N	44377	44377	44377	44377

Note: Observations come from the Core PSID dataset. The sample covers years from 1999 to 2017. Standard errors in parentheses; $^{*}p < 0.1$,** $p < 0.05$,*** $p < 0.01$, clustered at the individual level. The outcome variable is the employment status of the household head. Worker-level controls consist of a quadratic in age and its interactions with education, gender, marital status, and race. Columns (1) to (2) use aggregate instead of State GDP in Eq. (1); and Columns (3) and (4) use the aggregate unemployment rate.

Figure A1: Predicted employment change and liquid wealth

(a) Job loss **(b)** Job finding



Note: Observations come from the Core PSID dataset. The sample covers years from 1999 to 2017. The lines plot predicted values of the job loss or finding indicator from a linear regression with individual, state, and year fixed effects. Predictions are shown as a function of lagged log liquid wealth. State-level GDP growth is set to ± one standard deviation, while all other covariates are held at their sample means. Shaded areas represent 95 percent confidence intervals for the predicted values, reflecting uncertainty in the estimated regression coefficients. Standard errors are clustered at the state level.

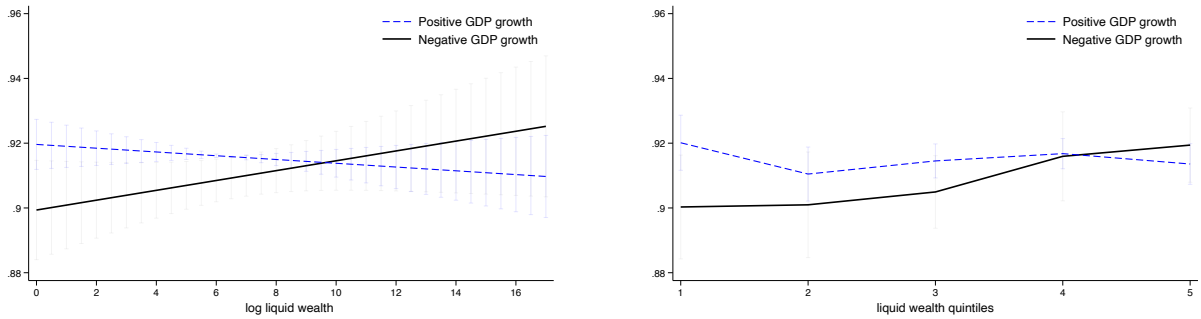
Table A4: Baseline model with additional covariates

	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta \log Y_{s(i),t}$	0.26***	0.48**	0.26***	0.23***	0.019	0.085
	(0.073)	(0.20)	(0.082)	(0.085)	(0.24)	(0.11)
$\log a_{it-1}$	0.00047	0.00024	0.00043	0.00052	0.00045	0.00050
	(0.00079)	(0.00080)	(0.00076)	(0.00078)	(0.00082)	(0.00080)
$\Delta \log Y_{s(i),t} \times \log a_{it-1}$	-0.027***	-0.023***	-0.027***	-0.028***	-0.022*	-0.020*
	(0.0085)	(0.0087)	(0.0080)	(0.0084)	(0.012)	(0.011)
$\log$ labor income$_{it-1}$		0.0013				
		(0.00099)				
$\Delta \log Y_{s(i),t} \times \log$ labor income$_{it-1}$		-0.026*				
		(0.015)				
$\log$ labor income (spouse)$_{it-1}$			0.00028			
			(0.00051)			
$\Delta \log Y_{s(i),t} \times \log$ labor income (spouse)$_{it-1}$			-0.00089			
			(0.0076)			
debt (total)$_{it-1}$				-0.00072		
				(0.00047)		
$\Delta \log Y_{s(i),t} \times$ debt (total)$_{it-1}$				0.0051		
				(0.0068)		
$\Delta \log Y_{s(i),t} \times$ Occupation$_{it-1}$					✓	
$\Delta \log Y_{s(i),t} \times$ Industry$_{it-1}$						✓
R^2	0.45	0.45	0.45	0.45	0.40	0.40
N	44377	41861	44377	44377	39061	38628

Note: Observations come from the Core PSID dataset. The sample covers years from 1999 to 2017. Standard errors in parentheses; $^{*}p < 0.1$,$^{**}p < 0.05$,$^{***}p < 0.01$, clustered at the state level. The outcome variable is the employment status of the household head. All specifications include worker, year, and state fixed effects. Worker-level controls consist of a quadratic in age and its interactions with education, gender, marital status, and race.

Figure A2: Predicted employment probability by liquid wealth in the baseline model

(a) continuous log liquid wealth **(b)** liquid wealth quintiles



Note: Observations come from the Core PSID dataset. The sample covers years from 1999 to 2017. The left panel reports predicted values of the employment status indicator from the baseline model in Eq. (1); the right panel reports predicted values from an alternative specification in which liquid wealth is discretized into bins. Predictions are shown as a function of lagged liquid wealth. State-level GDP growth is set to ± one standard deviation, while all other covariates are held at their sample means. Shaded areas represent 95 percent confidence intervals for the predicted values, reflecting uncertainty in the estimated regression coefficients. Standard errors are clustered at the state level.

B Positive sorting derivation and proof

B1 Matching function

I assume a constant returns to scale labor market matching function (Den Haan et al., 2000), specified as:

$$M(\mathbf{u},\mathbf{v}) = \mathbf{uv}(\mathbf{u}^{\gamma}+\mathbf{v}^{\gamma})^{\frac{-1}{\gamma}} \tag{16}$$

This gives rise to the following job finding and job filling probabilities, where $\theta = \mathbf{v}/\mathbf{u}$

$$\begin{aligned}
p(\theta) &= \theta(1+\theta^{\gamma})^{\frac{-1}{\gamma}}, & q(\theta) &= (1+\theta^{\gamma})^{\frac{-1}{\gamma}} \\
p'(\theta) &= (1+\theta^{\gamma})^{\frac{-1}{\gamma}-1}, & q'(\theta) &= -\theta^{\gamma-1}(1+\theta^{\gamma})^{\frac{-1}{\gamma}-1} \\
p''(\theta) &= -(1+\gamma)\theta^{\gamma-1}(1+\theta^{\gamma})^{\frac{-1}{\gamma}-2} \\
q''(\theta) &= -\theta^{\gamma-2}(1+\theta^{\gamma})^{\frac{-1}{\gamma}-2}\left[2\theta^{\gamma}-(\gamma-1)\right]
\end{aligned}$$

The elasticity of job finding to tightness is

$$\varepsilon_p = \frac{\partial \log p(\theta)}{\partial \log(\theta)} = \frac{\partial\left[\log(\theta) - \frac{1}{\gamma}\log(1+\exp(\gamma\log(\theta)))\right]}{\partial\log(\theta)} = 1 - \frac{\theta^{\gamma}}{1+\theta^{\gamma}} = (1+\theta^{\gamma})^{-1} \tag{17}$$

$$\varepsilon_q = \frac{\partial \log q(\theta)}{\partial \log(\theta)} = \frac{\partial\left[-\frac{1}{\gamma}\log(1+\exp(\gamma\log(\theta)))\right]}{\partial\log(\theta)} = -\frac{\theta^{\gamma}}{1+\theta^{\gamma}} \tag{18}$$

To compute the elasticity of substitution between $\mathbf{v}$ and $\mathbf{u}$, we start by writing the partial derivatives of the matching function as

$$\frac{\partial M}{\partial \mathbf{u}} = \mathbf{v}^{\gamma+1}\left(\mathbf{u}^{\gamma}+\mathbf{v}^{\gamma}\right)^{-1/\gamma-1} \quad , \quad \frac{\partial M}{\partial \mathbf{v}} = \mathbf{u}^{\gamma+1}\left(\mathbf{u}^{\gamma}+\mathbf{v}^{\gamma}\right)^{-1/\gamma-1}$$

The marginal rate of technical substitution of $\mathbf{v}$ for $\mathbf{u}$ is

$$\text{MRTS}_{\mathbf{uv}} \equiv \frac{\partial M/\partial \mathbf{v}}{\partial M/\partial \mathbf{u}} = \frac{\mathbf{u}^{\gamma+1}}{\mathbf{v}^{\gamma+1}} = \left(\frac{\mathbf{u}}{\mathbf{v}}\right)^{\gamma+1}. \tag{19}$$

The (Hicks) elasticity of substitution between $\mathbf{u}$ and $\mathbf{v}$ is then

$$\varepsilon_\theta \equiv \frac{d\ln(\mathbf{v}/\mathbf{u})}{d\ln \text{MRTS}_{\mathbf{uv}}} = \frac{1}{\gamma+1}. \tag{20}$$

Hence, $1+\gamma$ is the inverse elasticity of substitution between $\mathbf{u}$ and $\mathbf{v}$ in the matching technology.

B2 Proof of Proposition 1

Proof. Let $\phi(a_2^*, \theta^*)$ be the maximand of $U(a_1, x)$

$$\phi(a_2^*, \theta^*) = u(Ra_1 - a_2^*) + \beta\left[p(\theta^*)\Delta(x)u\left(Ra_2^* + \bar{y} - \frac{\varphi(x)}{\beta q(\theta^*)\Delta(x)}\right) + (1 - p(\theta^*)\Delta(x))u(Ra_2^*)\right]$$

By the implicit function theorem, we have

$$\frac{\partial a_{t+1}}{\partial x_t} = -\frac{\begin{vmatrix} \phi_{a_{t+1}x_t} & \phi_{a_{t+1}\theta_t} \\ \phi_{\theta_t x_t} & \phi_{\theta_t\theta_t} \end{vmatrix}}{|H|} = -\frac{\phi_{a_{t+1}x_t}\phi_{\theta_t\theta_t} - \phi_{\theta_t x_t}\phi_{a_{t+1}\theta_t}}{|H|}$$

$$\frac{\partial \theta_t}{\partial x_t} = -\frac{\begin{vmatrix} \phi_{\theta_t x_t} & \phi_{\theta_t a_{t+1}} \\ \phi_{a_{t+1}x_t} & \phi_{a_{t+1}a_{t+1}} \end{vmatrix}}{|H|} = -\frac{\phi_{\theta_t x_t}\phi_{a_{t+1}a_{t+1}} - \phi_{\theta_t a_{t+1}}\phi_{a_{t+1}x_t}}{|H|}$$

Where $|H| > 0$ is the determinant of the Hessian:

$$H = \begin{bmatrix} \phi_{a_{t+1}a_{t+1}} & \phi_{a_{t+1}\theta_t} \\ \phi_{\theta_t a_{t+1}} & \phi_{\theta_t\theta_t} \end{bmatrix} \tag{21}$$

We first derive the partial derivatives of the surplus function $U(a_2, x)$. Taking the envelope theorem into account, we have that

$$U_{a_1} = Ru'(Ra_1 - a_2)$$
$$U_{a_1 x} = -Ru''(Ra_1 - a_2)\frac{\partial a_2}{\partial x}$$

The condition for positive wealth sorting is

$$U_{a_1 x} > 0 \quad \Leftrightarrow \quad -Ru''(Ra_1 - a_2)\frac{\partial a_2}{\partial x} > 0$$
$$\frac{\partial a_2}{\partial x} > 0 \quad \Leftrightarrow \quad -\frac{\phi_{a_{t+1}x_t}\phi_{\theta_t\theta_t} - \phi_{\theta_t x_t}\phi_{a_{t+1}\theta_t}}{|H|} > 0$$

$$\phi_{a_2 x}\ \phi_{\theta\theta} < \phi_{\theta x}\ \phi_{a_2\theta} \tag{22}$$

Where we used the implicit function theorem in the second line. Eq. (22) is the condition for

PAM, and we now compute the relevant partial derivatives

$$\begin{aligned}\phi_{a_2,x} &= \beta R p(\theta^*)\left[\Delta'(x)(u'(c^*_{e,2}) - u'(c^*_{u,2})) + \Delta(x)u''(c^*_{e,2})\left(-\frac{\varphi'(x)\Delta(x) - \varphi(x)\Delta'(x)}{\beta q(\theta)(\Delta(x))^2}\right)\right] \\ &= \beta R p(\theta^*)\left[\Delta'(x)(u'(c^*_{e,2}) - u'(c^*_{u,2})) - \Delta'(x)u''(c^*_{e,2})\frac{u(c^*_{e,2}) - u(c^*_{u,2})}{u'(c^*_{e,2})}\right] \\ &= \beta R p(\theta^*)\Delta'(x)\left[(u'(c^*_{e,2}) - u'(c^*_{u,2})) - \frac{u''(c^*_{e,2})}{u'(c^*_{e,2})}(u(c^*_{e,2}) - u(c^*_{u,2}))\right] \end{aligned} \tag{23}$$

$$\begin{aligned}\phi_{\theta,a_2} &= \beta R\Delta(x)\left(p'(\theta)\left[u'(c^*_{e,2}) - u'(c^*_{u,2})\right] + u''(c^*_{e,2})\frac{\varphi(x)\theta q'(\theta)}{\beta\Delta(x)q(\theta)}\right) \\ &= \beta R\Delta(x)p'(\theta)\left(\left[u'(c^*_{e,2}) - u'(c^*_{u,2})\right] - u''(c^*_{e,2})\frac{(u(c^*_{e,2}) - u(c^*_{u,2}))}{u'(c^*_{e,2})}\right) \end{aligned} \tag{24}$$

With DARA utility function, it holds that $\frac{u'(c^*_{e,2}) - u'(Ra_2)}{u(c^*_{e,2}) - u(Ra_2)} < \frac{u''(c^*_{e,2})}{u'(c^*_{e,2})}$ (Eeckhout and Sepahsalari, 2024). Therefore, Eq. (23) and Eq. (24) are negative, i.e., $\phi_{a_2,x} < 0$ and $\phi_{\theta,a_2} < 0$.

$$\begin{aligned}\phi_{\theta,x} = \beta\Bigg(&\Delta'(x)\left[p'(\theta)(u(c^*_{e,2}) - u(c^*_{u,2})) + u'(c^*_{e,2})\frac{\varphi(x)\theta q'(\theta)}{\beta\Delta(x)q(\theta)}\right] \\ &+ \Delta(x)p'(\theta)u'(c^*_{e,2})\left(-\frac{\varphi'(x)\Delta(x) - \varphi(x)\Delta'(x)}{\beta q(\theta)(\Delta(x))^2}\right) \\ &+ \Delta(x)u''(c^*_{e,2})\left(-\frac{\varphi'(x)\Delta(x) - \varphi(x)\Delta'(x)}{\beta q(\theta)(\Delta(x))^2}\right)\frac{\varphi(x)\theta q'(\theta)}{\beta\Delta(x)q(\theta)} \\ &+ \Delta(x)u'(c_{e,2})\frac{\theta q'(\theta)}{\beta q(\theta)}\left(\frac{\varphi'(x)\Delta(x) - \varphi(x)\Delta'(x)}{(\Delta(x))^2}\right)\Bigg) \\ = \beta&\left(-\frac{\varphi'(x)\Delta(x) - \varphi(x)\Delta'(x)}{\beta q(\theta)(\Delta(x))^2}\right)\left(\Delta(x)p'(\theta)u'(c^*_{e,2}) + \Delta(x)u''(c^*_{e,2})\frac{\varphi(x)\theta q'(\theta)}{\beta\Delta(x)q(\theta)} - \Delta(x)u'(c_{e,2})\theta q'(\theta)\right) \\ = \beta&\left(-\frac{\varphi'(x)\Delta(x) - \varphi(x)\Delta'(x)}{\beta q(\theta)(\Delta(x))^2}\right)\left(\Delta(x)u'(c^*_{e,2})[p'(\theta) - \theta q'(\theta)] + u''(c^*_{e,2})\frac{\varphi(x)\theta q'(\theta)}{\beta q(\theta)}\right) \end{aligned} \tag{25}$$

All terms in the second bracket are positive under matching function (16), and from Eq.(6), it must be that $\frac{\varphi'(x)\Delta(x) - \varphi(x)\Delta'(x)}{\beta q(\theta)(\Delta(x))^2} > 0$. We thus find that $\phi_{\theta,x} < 0$.

$$\phi_{\theta,\theta} = \beta\Delta(x)\Bigg[p''(\theta^*)\left[u\left(c^*_{e,2}\right) - u(c^*_{u,2})\right] + p'(\theta^*)u'(c^*_{e,2})\left(\frac{\varphi(x)q'(\theta)}{\beta\Delta(x)(q(\theta))^2}\right)$$
$$+ u''(c^*_{e,2})\left(\frac{\varphi(x)q'(\theta)}{\beta\Delta(x)(q(\theta))^2}\right)\frac{\varphi'(x)\theta q'(\theta)}{\beta q(\theta)\Delta(x)} + u'(c^*_{e,2})\frac{\varphi(x)}{\beta\Delta(x)}\left(\frac{(q'(\theta)+\theta q''(\theta))q(\theta) - (q'(\theta))^2\theta}{(q(\theta))^2}\right)\Bigg]$$
$$= \beta\Delta(x)\Bigg[p''(\theta)\left[u\left(c^*_{e,2}\right) - u(c^*_{u,2})\right] \tag{26}$$
$$+ \frac{\varphi(x)q'(\theta)}{\beta\Delta(x)(q(\theta))^2}\left(p'(\theta)u'(c^*_{e,2}) + u''(c^*_{e,2})\frac{\varphi(x)\theta q'(\theta)}{\beta\Delta(x)q(\theta)} + u'(c^*_{e,2})\frac{(q'(\theta)+\theta q''(\theta))q(\theta) - (q'(\theta))^2\theta}{q'(\theta)}\right)\Bigg] \tag{27}$$

Under the matching function in Eq. (16), all terms are negative, i.e. $\phi_{\theta,\theta} < 0$.

The sufficient condition for PAM is:

$$\beta R p(\theta^*)\Delta'(x)\left[(u'(c^*_{e,2}) - u'(c^*_{u,2})) - \frac{u''(c^*_{e,2})}{u'(c^*_{e,2})}(u(c^*_{e,2}) - u(c^*_{u,2}))\right]$$
$$\beta\Delta(x)\Bigg[p''(\theta)\left[u\left(c^*_{e,2}\right) - u(c^*_{u,2})\right]$$
$$+ \frac{\varphi(x)q'(\theta)}{\beta\Delta(x)(q(\theta))^2}\left(p'(\theta)u'(c^*_{e,2}) + u''(c^*_{e,2})\frac{\varphi(x)\theta q'(\theta)}{\beta\Delta(x)q(\theta)} + u'(c^*_{e,2})\frac{(q'(\theta)+\theta q''(\theta))q(\theta) - (q'(\theta))^2\theta}{q'(\theta)}\right)\Bigg]$$
$$< \beta\frac{\varphi(x)\theta q'(\theta)}{\beta q(\theta)}\left(u'(c^*_{e,2})\frac{\varphi'(x)}{\varphi(x)} - u''(c^*_{e,2})\frac{\Delta'(x)}{\Delta(x)}\frac{u(c^*_{e,2}) - u(c^*_{u,2})}{u'(c_{e,2^*})}\right)$$
$$\beta R\Delta(x)p'(\theta)\left(\left[u'(c^*_{e,2}) - u'(c^*_{u,2})\right] - u''(c^*_{e,2})\frac{(u(c^*_{e,2}) - u(c^*_{u,2}))}{u'(c^*_{e,2})}\right)$$

With DARA utility functions, we know that the last parenthesis is negative. Simplifying on both sides yields

$$\Delta'(x)p''(\theta)\left[u\left(c^*_{e,2}\right) - u(c^*_{u,2})\right]$$
$$+ \frac{\Delta'(x)}{\Delta(x)}\frac{\varphi(x)q'(\theta)}{\beta(q(\theta))^2}\left(p'(\theta)u'(c^*_{e,2}) + u''(c^*_{e,2})\frac{\varphi(x)\theta q'(\theta)}{\beta\Delta(x)q(\theta)} + u'(c^*_{e,2})\frac{(q'(\theta)+\theta q''(\theta))q(\theta) - (q'(\theta))^2\theta}{q'(\theta)}\right)$$
$$> \frac{\varphi(x)q'(\theta)}{\beta(q(\theta))^2}\left(u'(c^*_{e,2})\frac{\varphi'(x)}{\varphi(x)} - u''(c^*_{e,2})\frac{\Delta'(x)}{\Delta(x)}\frac{u(c^*_{e,2}) - u(c^*_{u,2})}{u'(c_{e,2^*})}\right)p'(\theta)$$
$$= \frac{\Delta'(x)}{\Delta(x)}\frac{\varphi(x)q'(\theta)}{\beta(q(\theta))^2}\left(\frac{\Delta(x)}{\Delta'(x)}u'(c^*_{e,2})\frac{\varphi'(x)}{\varphi(x)} - u''(c^*_{e,2})\frac{u(c^*_{e,2}) - u(c^*_{u,2})}{u'(c_{e,2^*})}\right)p'(\theta)$$

Where in the last line we simply extended the RHS so that we can then take the difference

$$\Delta'(x)p''(\theta)\left[u\left(c^*_{e,2}\right)-u(c^*_{u,2})\right]+\frac{\Delta'(x)}{\Delta(x)}\frac{\varphi(x)q'(\theta)}{\beta(q(\theta))^2}\left(p'(\theta)u'(c^*_{e,2})\left(1-\frac{\Delta(x)}{\Delta'(x)}\frac{\varphi'(x)}{\varphi(x)}\right)\right.$$
$$\left.+u'(c^*_{e,2})\frac{(q'(\theta)+\theta q''(\theta))q(\theta)-(q'(\theta))^2\theta}{q'(\theta)}+u''(c^*_{e,2})\left(\frac{\varphi(x)\theta q'(\theta)}{\beta\Delta(x)q(\theta)}+p'(\theta)\frac{u(c^*_{e,2})-u(c^*_{u,2})}{u'(c_{e,2^*})}\right)\right)>0$$

The last term in parentheses is equal to zero. Then, using Eq. (5) we can write:

$$\Delta'(x)\left[u\left(c^*_{e,2}\right)-u(c^*_{u,2})\right]\left[p''(\theta)-\frac{(p'(\theta))^2}{p(\theta)}\left(1-\frac{\Delta(x)}{\Delta'(x)}\frac{\varphi'(x)}{\varphi(x)}+\frac{(q'(\theta)+\theta q''(\theta))q(\theta)-(q'(\theta))^2\theta}{q'(\theta)p'(\theta)}\right)\right]>0$$

The first terms are positive. From the CES matching functional form, the last term simplifies to:

$$\begin{aligned}(q'(\theta)+\theta q''(\theta))q(\theta)&=-\theta^{\gamma-1}(1+\theta^\gamma)^{\frac{-2}{\gamma}-2}\left[-\theta^\gamma+\gamma\right]\\ \theta(q'(\theta))^2&=\theta^{2\gamma-1}(1+\theta^\gamma)^{\frac{-2}{\gamma}-2}\\ p'(\theta)q'(\theta)&=-\theta^{\gamma-1}(1+\theta^\gamma)^{\frac{-2}{\gamma}-2}\\ \Leftrightarrow\frac{(q'(\theta)+\theta q''(\theta))q(\theta)-(q'(\theta))^2\theta}{q'(\theta)p'(\theta)}&=\gamma\end{aligned}$$

The sufficient condition for positive sorting is thus

$$\begin{aligned}p''(\theta)-\frac{(p'(\theta))^2}{p(\theta)}\left[1+\gamma-\frac{\Delta(x)}{\Delta'(x)}\frac{\varphi'(x)}{\varphi(x)}\right]&>0\\ \theta^{-1}(1+\theta^\gamma)^{\frac{-1}{\gamma}-2}\left[-(1+\gamma)\theta^\gamma-1+\gamma-\frac{\Delta(x)}{\Delta'(x)}\frac{\varphi'(x)}{\varphi(x)}\right]&>0\\ -(1+\gamma)\theta^\gamma&>1+\gamma-\frac{\Delta(x)}{\Delta'(x)}\frac{\varphi'(x)}{\varphi(x)}\\ \frac{\varphi'(x)}{\varphi(x)}&>\frac{\Delta'(x)}{\Delta(x)}(1+\gamma)(1+\theta^\gamma)\end{aligned}$$

Where $1+\theta^\gamma$ is the inverse elasticity of the job finding rate to tightness, ε_p, derived in Eq. (17). Uniqueness follows from the negative-semidefinite Hessian matrix (21).

■

B3 Risk aversion and strength of sorting

Under CRRA utility function, we can write the inequality as

$$\mathcal{S}\equiv\left[(c^*_{e,2})^{-\sigma}-(c^*_{u,2})^{-\sigma}+\frac{\sigma}{1-\sigma}(c^*_{e,2})^{-1}((c^*_{e,2})^{1-\sigma}-(c^*_{u,2})^{1-\sigma})\right]<0$$

, with $1-\sigma<0$.

The derivative with respect to risk aversion σ is

$$\begin{aligned}\mathcal{S}'(\sigma) = &- \ln(c^*_{e,2})(c^*_{e,2})^{-\sigma} + \ln(c^*_{u,2})(c^*_{u,2})^{-\sigma} + \frac{1}{(1-\sigma)^2}(c^*_{e,2})^{-1}\Big[(c^*_{e,2})^{1-\sigma} - (c^*_{u,2})^{1-\sigma}\Big] \\ &- \frac{\sigma}{1-\sigma}(c^*_{e,2})^{-1}\Big[\ln(c^*_{e,2})(c^*_{e,2})^{1-\sigma} - \ln(c^*_{u,2})(c^*_{u,2})^{1-\sigma}\Big]\end{aligned}$$

Using the notation $x \equiv \frac{c^*_{u,2}}{c^*_{e,2}} \in (0,1)$, we can write $\ln(c^*_{u,2}) = \ln(x) + \ln(c^*_{e,2})$. Dividing by $(c^*_{e,2})^{-\sigma} > 0$, we can write

$$\begin{aligned}\mathcal{S}'(\sigma) =& - \ln(c^*_{e,2}) + x^{-\sigma}\ln(c^*_{u,2}) + \frac{1}{(1-\sigma)^2}\Big(1 - x^{1-\sigma}\Big) - \frac{\sigma}{1-\sigma}\Big(\ln(c^*_{e,2}) - x^{1-\sigma}\ln(c^*_{u,2})\Big) \\ =&\frac{1}{(1-\sigma)^2}\Big[1 - x^{1-\sigma}\Big] + \ln(x)x^{1-\sigma}\left[\frac{\sigma}{1-\sigma} + x^{-1}\right] + \ln(c^*_{e,2})\left[x^{-\sigma} - 1 + \frac{\sigma}{1-\sigma}(x^{1-\sigma} - 1)\right] \\ =&\frac{1}{(1-\sigma)^2}\Big[1 - x^{1-\sigma}\Big] + x^{1-\sigma}\left[\frac{\sigma}{1-\sigma} + x^{-1}\right]\Big[\ln(x) + \ln(c^*_{e,2})\Big] - \ln(c^*_{e,2})\frac{1}{1-\sigma}\end{aligned}$$

In the limits we can show that

$$\lim_{x\to 0}\mathcal{S}'(\sigma) = -\infty, \qquad \lim_{x\to 1}\mathcal{S}'(\sigma) = 0$$

Moreover, the derivative with respect to x is

$$\frac{d\mathcal{S}'(\sigma, x, c^*_{e,2})}{dx} = \Big[1 - x^{-1}\Big]\Big(\sigma\ln(c^*_{u,2}) - 1\Big)$$

and the second order derivative is

$$\frac{d^2\mathcal{S}'(\sigma, x, c^*_{e,2})}{d^2x} = x^{-2}\Big(\sigma\ln(c^*_{u,2}) - 1\Big)$$

If $\sigma\ln(c^*_{u,2}) - 1 > 0$, i.e. if $c^*_{u,2} > \exp(\sigma^{-1})$, then $\frac{d\mathcal{S}'(\sigma,x,c^*_{e,2})}{dx} > 0$ and $\frac{d^2\mathcal{S}'(\sigma,x,c^*_{e,2})}{d^2x} > 0$. As a result (and together with the limits), we conclude that $S'(\sigma) < 0$. The surplus function becomes more negative, i.e. risk aversion amplifies the inequality.

C Block recursive equilibrium

The proof follows Karahan and Rhee (2019) which is an extension of Menzio and Shi (2010, 2011), and proceeds in two steps. First, we show that there exists a value function for firms as well as market tightness functions that depend on the aggregate state of the economy only through exogenous shocks, and not through any endogenous distributions. This implies that we can reduce the state space for firms to only the exogenous shocks. In the baseline model without aggregate uncertainty, this means effectively that the value functions for firms and the market tightness do not depend on any aggregate state.
Second, we collapse the employed and unemployed problems and show that the overall household problem is a contraction. If the rental rate (which is exogenous) and the tightness functions are independent of the endogenous asset distribution, we show that the functional equation maps the set of functions that do not depend on the endogenous distribution into itself. Hence, the firm

value function, market tightness functions and household problem constitute a block-recursive equilibrium.

C1 Existence

i) Let $\mathcal{J}(\mathcal{X}\times\mathcal{W}\times\mathcal{Z})$ be the set of continuous and bounded firm's value functions: $J(\mathcal{X}\times\mathcal{W}\times\mathcal{Z})\to\mathbb{R}$, and denote $\mathbb{T}_j$ as an operator associated with the firm's value function (Eq. (13)).The operator $\mathbb{T}_j$ maps $\mathcal{J}$ into itself; applying Blackwell's conditions it is straightforward to show that $\mathbb{T}_j:\mathcal{J}\to\mathcal{J}$ is a contraction.
Let $J^*\in\mathcal{J}$ be the fixed point of $\mathbb{T}_j$. J^* is a continuous, bounded function that does not depend on any endogenous aggregate state.

Then, we can rewrite the market tightness functions for open markets, from Eq. (14), as

$$\theta^*(x,w;z)=q^{-1}\left(\frac{\varphi(x)}{\mathbb{E}_t[J^*(x,w;z)]}\right)$$

ii) Denote the set of functions $\mathcal{R}:\{0,1\}\times\mathcal{A}\times\mathcal{W}\times\mathcal{Z}\to\mathbb{R}$. Let e be an indicator of the employment status ($e=1$ if the worker is employed), and let $R:\{0,1\}\times\mathcal{A}\times\mathcal{W}\times\mathcal{Z}$ denote the combined households' value functions:

$$\begin{aligned}R(e=0,a,w;\Gamma)&=U(a;\Gamma)\\R(e=1,a,w;\Gamma)&=E(a,w;\Gamma)\end{aligned}$$

If $R\in\mathcal{R}$, then the employed and unemployed value functions are also only a function of z and not Ψ. Let $\mathbb{T}_r$ be an operator such that

$$\begin{aligned}(\mathbb{T}_rR)(e,a,x,w;\Gamma)=(1-e)\Big\{&\max_{a',x,w}u(c_u)+\beta\mathbb{E}\big[p(\theta(x,w;z))R(e'=1,a',x,w;\Gamma')\\&+(1-p(\theta(x,w;z)))R(e'=0,a';\Gamma')\big]\Big\}\\&+e\left\{\max_{a'}\quad u(c_e)+\beta\mathbb{E}\left[\delta(x,z)R(e'=0,a';\Gamma')+(1-\delta(x,z))R(e'=1,a',w';\Gamma')\right]\right\}\\\text{s.t.}\quad&c_e+a'=Ra+w,\quad c_u+a'=Ra+b,\qquad a'\geq\underline{a}\end{aligned}$$

Where we used the fact that market tightness does not depend on Γ (point i)). Assuming that $u(\cdot)$ is bounded and continuous, $\mathcal{R}$ is a set of bounded and continuous functions. We can show that $\mathbb{T}_r:\mathcal{R}\to\mathcal{R}$, so that Blackwell's sufficiency conditions imply that $\mathbb{T}_r$ is a contraction. Being the space complete and non-empty, there exists a unique fixed point $R^*\in\mathcal{R}$ such that $\mathbb{T}_r(R^*\in\mathcal{R})=R^*$. R^* is the solution to the household problem, and does not depend on any aggregate state.

C2 Uniqueness

The uniqueness follows from the strict concavity of value functions. Given the assumptions on $u(\cdot)$, $R(e'=1,a',x,w;\Gamma')$ is strictly concave in w, while $R(e'=0,a';\Gamma')$ is constant in w. With

C3 Discussion

Under some different modeling assumptions, block recursivity breaks. Here are some examples.

1. If firms observe workers' wealth and post optimal contracts;

2. with on-the-job search;

3. if the interest rate is endogenous.

First, if firms would observe workers' wealth, intuitively they would post contracts to attract specific worker types; hence, the distribution of workers over asset and employment states would not be independent of their value function. However, it is in reality unlikely that the employer observes the assets of applicants. Second, on-the-job search would create differential incentives to search across the wealth distribution, and again firms would post contracts to attract targeted workers. In this case as well, firms' value functions would not only depend on the aggregate state through aggregate productivity. This assumption is a restriction on the equilibrium, as it is an established fact that workers search on the job and that job-to-job transitions account for a lot of labor market flows (see e.g., Faberman et al., 2022). Third, if the interest rate is endogenous it directly depends on the distribution of workers across assets. Interest rates influence the trade-off of labor market search – differently across the asset distribution – and therefore value functions of workers directly depend on the distribution of agents over states.

D Additional Figures and Tables

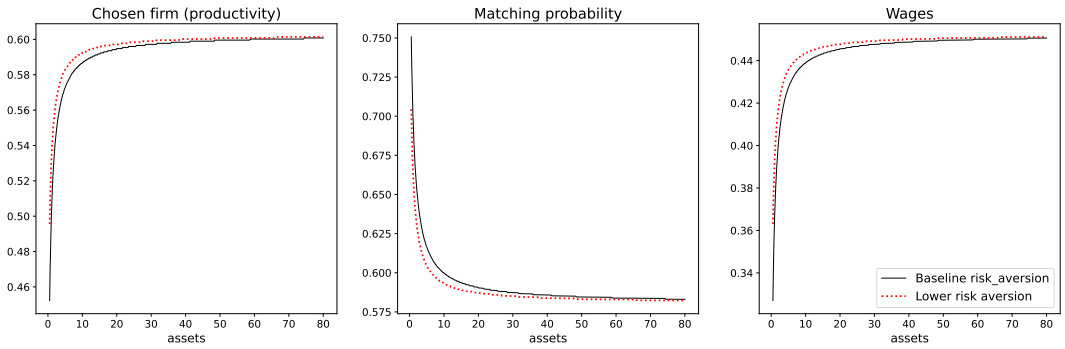
Table D5: Functional forms

Matching function (CES)	$p(\theta) = \theta(1+\theta^{\gamma})^{\frac{-1}{\gamma}}$
Utility function	$\frac{c^{1-\sigma}}{1-\sigma}$
Separation function	$\bar{\delta}\exp(\eta^{\delta_z}(z-1))\exp(\eta^{\delta_x}(x-\bar{x}))$
Vacancy cost function	$\bar{\varphi}\exp(\eta^{\varphi_z}(z-1))\exp(\eta^{\varphi_x}(x-\bar{x}))$
Production function	$x-w$

Table D6: Non targeted moments

Moment	Model	Data
unemployment rate	0.093	6% (FRED)
std. unemployment rate	0.016	0.0165
asset-to-income ratio	3.624	13.01 (2004 SIPP)
sd(w)	0.345	0.3 (residual wage dispersion, Morin 2019)

Figure D3: Equilibrium allocations with different risk aversion parameters



Notes: Baseline risk aversion: $\nu = 2$, counterfactual risk aversion: $\nu = 1.3$.

Figure D4: Consumption policy functions

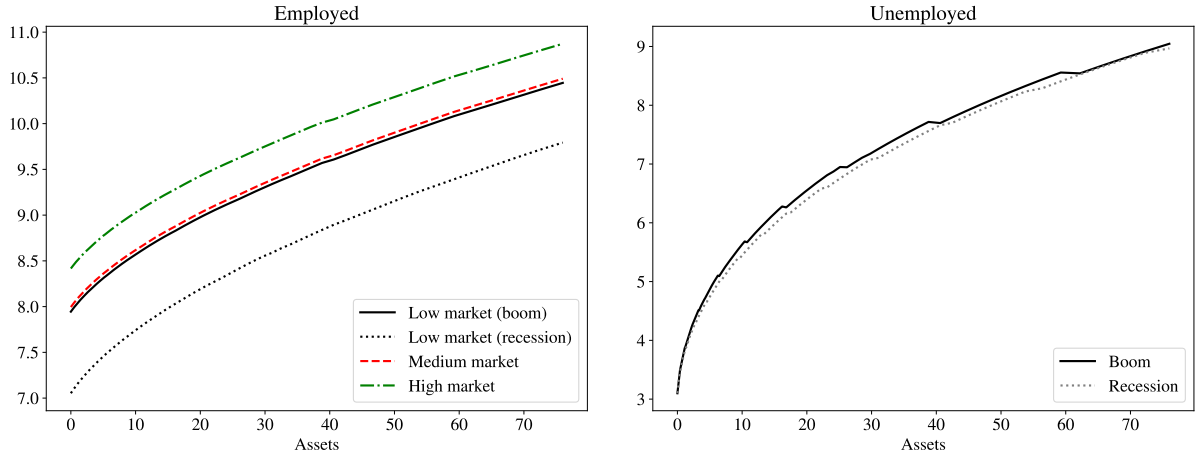
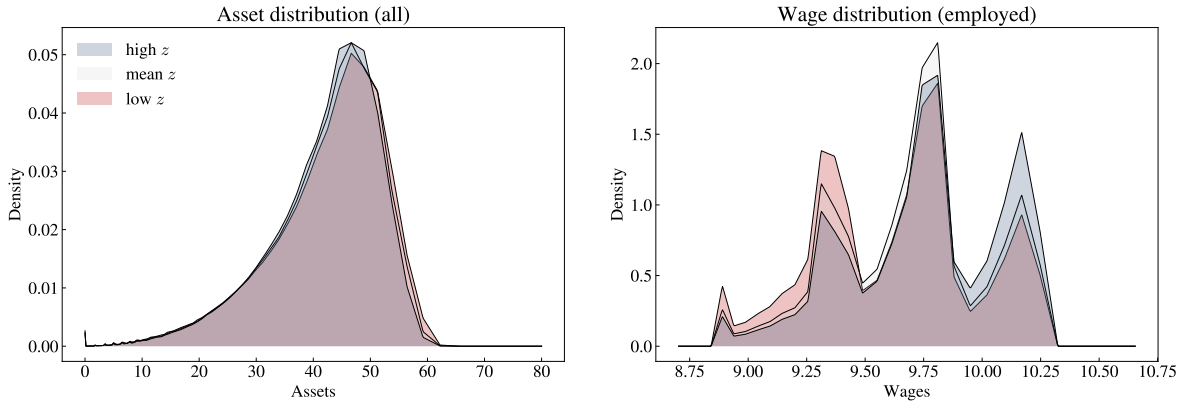


Figure D5: Steady state distribution of agents over wages and assets



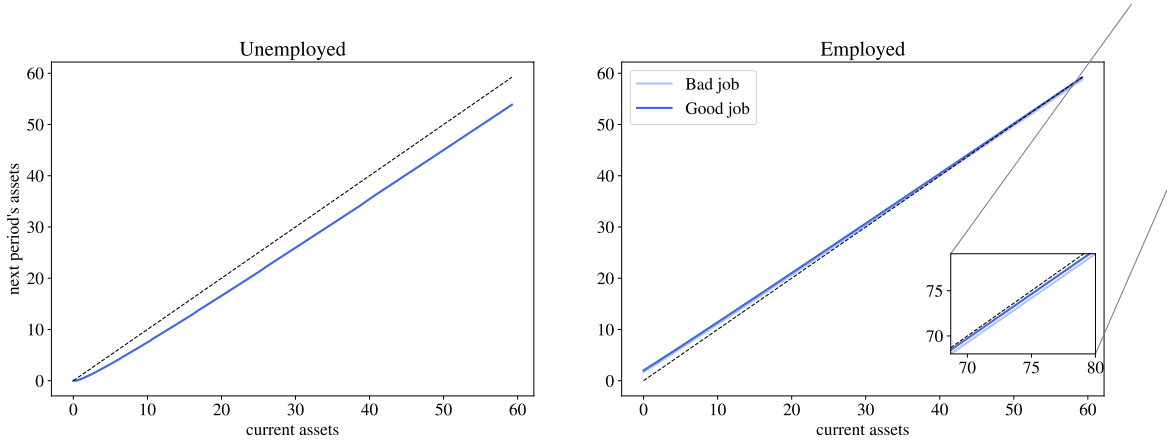
Note: The left and middle panels show the stationary asset and wage distributions, respectively, under different steady state values of aggregate productivity. On the right panel, the vertical bars show the MPC of workers in different wealth percentiles, and the blue dashed line represents the aggregate monthly MPC.

Figure D6: Stochastic steady state distribution of agents over wages and assets



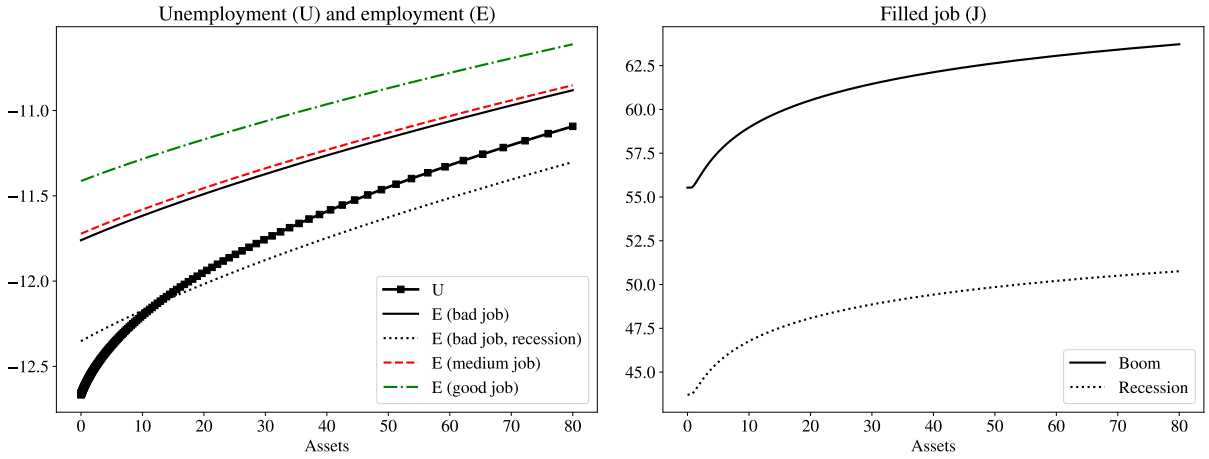
Note: The panels show the stationary asset and wage distributions, respectively, under different values of aggregate productivity in the stochastic steady-state.

Figure D7: Workers' asset accumulation



Note: The 45° line is represented in dashed black. On the right panel, the two lines represent the asset accumulation decisions of workers in the lowest and highest labor market, i.e., those with lowest (highest) wages and productivity.

Figure D8: Workers' and firms' value functions



E Solution method

With Block Recursivity (BR), the solution method is greatly simplified, as one can solve the labor market problem orthogonal to the distribution of workers across employment states. In fact, the latter is not considered by agents in their optimal decision making.
In the baseline calibration, I use 90 grid points for assets, wages, and productivity, and 5 aggregate states of productivity.
The algorithm works as follows.

1. Given $V(x) = 0$, find the fixed point of $J(x, w, z)$ using VFI on Eq. (13).
2. Compute the market tightness according to Eq. (14). Take a guess on $U^0(a, z)$, $E^0(a, x, w, z)$, $a_0^u(a, z)$, $a_0^e(a, x, w, z)$.
3. Take a guess for the tax rate τ. Solve for the workers' problem.
   (a) Apply policy function iteration (PFI) procedure on the employed workers' Euler equation.
      i. Fix a submarket (x, w). Write the employed workers' EE using the asset policy function initial guesses $a_0^u(a, z), a_0^e(a, z)$

$$u'\left(Ra + w - h^e(a,z)\right) = \beta R\, \mathbb{E}_{z'}\Big\{\delta(x,z)u'\left(Ra_0^e(a,z) + b - a_0^u\left(a_0^e(a,z), z'\right)\right) + (1-\delta(x,z))u'\left(Ra_0^e(a,z) + w - a_0^e\left((a_0^e(a,z), z'\right)\right)\Big\}$$

      Where we interpolate a_0^u and a_0^e on the value of assets given by $a_0^e(a, z)$.
      The new update for the asset policy function is constructed as

$$a_1^e(a,z) = \begin{cases} h^e(a,z) & \text{if } h^e(a,z) \geq \underline{a} \\ 0 & \text{otherwise} \end{cases}$$

      ii. Update on the policy function a^e in a next iteration k, until $a_k^e - a_{k-1}^e < \text{tol}$. Denote the fixed point policy function for a given guess on the unemployed policy function as $a^e(a, z)$.

iii. Loop over all submarkets and retrieve the consumption policy function as $c_e(a, x, w, z) = Ra + w - a^e(a, x, w, z)$.

(b) Apply PFI procedure on the unemployed workers' Euler equation.

i. Fix a submarket (x, w), construct the RHS of the EE using the asset policy function guesses $a_0^u(a, z), a^e(a, x, w, z)$

$$u'\left(Ra + b - h^u(a, z)\right) = \beta R\, \mathbb{E}_{z'}\Bigg\{p(\theta(x, w, z))u'\left(Ra_0^u(a, z) + w - a^e\left(a_0^u(a, z), x, w, z'\right)\right) + (1 - p(\theta(x, w, z)))u'\left(Ra_0^u(a, z) + b - a_0^u\left(a_0^u(a, z), z'\right)\right)\Bigg\}$$

Where we interpolate a^e and a_0^u on the value of assets given by $a_0^u(a, z)$.
The new update for the asset policy function is constructed as

$$a_1^u(a, z) = \begin{cases} h^u(a, z) & \text{if } h^u(a, z) \geq \underline{a} \\ 0 & \text{otherwise} \end{cases}$$

ii. Update on the policy function a^u in a next iteration k, until $a_k^u - a_{k-1}^u < \text{tol}$. Denote the fixed point policy function for a given guess on the unemployed policy function as $a^u(a, z)$.

iii. Loop over all submarkets and retrieve the consumption policy function as $c_u(a, x, w, z) = Ra + b - a^u(a, x, w, z)$.

(c) Apply value function iteration (VFI) to find the fixed point of the workers' VF.

i. Solve for the employed VF given the policy functions a^u and a^e.

A. Compute the RHS using policy function from above and the guesses on the workers' VFs.

$$E^1(a, x, w; z) = u(c_e(a, x, w, z)) + \beta \mathbb{E}_{z'}\Bigg[\delta(x, z)U^0\Big(a^e(a, x, w, z); z'\Big) + (1 - \delta(x, z))E^0\Big(a^e(a, x, w, z), x, w; z'\Big)\Bigg]$$

We need to interpolate U_0 and E_0 on the next value of assets given by the policy function a^e.

B. Update the VF until $E^k - E^{k-1} < \text{tol}$.

ii. Apply VFI to find the fixed point of the unemployed worker' VF.

A. Compute the RHS using the policy function evaluated for all possible labor market choices, and the new guess for the employed VF.

$$U^1(a; z) = \max_{\tilde{x}, \tilde{w}}\, u(c_u(a, \tilde{x}, \tilde{w}, z)) + \beta \mathbb{E}_{z'}\Bigg[p(\theta(\tilde{x}, \tilde{w}; z))E^1\Big(a^u(a, \tilde{x}, \tilde{w}, z), \tilde{x}, \tilde{w}; z'\Big) (1 - p(\theta(\tilde{x}, \tilde{w}; z)))U^0\Big(a^u(a, \tilde{x}, \tilde{w}, z); z'\Big)\Bigg]$$

The relationship between tightness and wages is pinned down by combining Eq. (13) and (12).

B. Update the VF until $U^k - U^{k-1} <$ tol. For every (a, z), store the submarket policy function $\vartheta(x^*, w^*)$.

(d) Verify that $c_u^1(a, z) = c_u^1(a, x^*, w^*, z)$. If not, update the guess $c_u^0(a, z) = c_u^1(a, x^*, w^*, z)$, as well as value functions and other policy functions, and go back to point 3.

4. Verify that the government budget constraint holds. If not, say taxes are too low (high) to finance unemployment benefits, increase (decrease) the tax rate and go back to 3.

F Welfare costs

Similarly to Lucas (1987), I compute the welfare gain from an alternative model, as solving the following

$$\sum_{t=0}^{T} \beta^t \frac{(c_t(1-\delta))^{1-\sigma}}{1-\sigma} = \sum_{t=0}^{T} \beta^t \frac{\tilde{c}_t^{1-\sigma}}{1-\sigma} \tag{28}$$

, where $\tilde{c}_t$ is the consumption in the alternative model. In these computations, I solve the model under a given path of aggregate productivity. Then, the welfare cost is

$$\delta = 1 - \left(\frac{\sum_{t=0}^{T} \beta^t \tilde{c}_t^{1-\sigma}}{\sum_{t=0}^{T} \beta^t (c_t)^{1-\sigma}} \right)^{\frac{1}{1-\sigma}}$$

G Alternative two-period models

G1 Model where aggregate productivity affects continuation value functions

In period 1, the firm decides to open a vacancy in a certain market by posting a wage, according to the following value function.

$$V(x) = \max_{w} -\varphi(x) + \beta \mathbb{E}_z \left[1 - \delta(x, z) q(\theta) \left[\bar{y} z - w \right] \right],$$

Unemployed workers' problem is to save a_2 subject to the liquidity constraint $\underline{a}$, and to choose a submarket (x, w) to which to apply. Their value function writes

$$U_1(a_1) = \max_{a_2 \geq \underline{a}, (x,w)} u(Ra_1 - a_2) + \beta \left[p(\theta) \mathbb{E}_z \left[(1 - \delta(x, z)) E_2(a_2, w; z) \right] + (1 - p(\theta)(1 - \delta(x, z))) U_2(a_2) \right]$$

Let $\phi(a_2^*, \theta^*)$ be the maximand of $U(a_1, x)$

$$\phi(a_2^*, \theta^*) = u(Ra_1 - a_2^*) + \beta \Bigg[p(\theta^*) \mathbb{E}_z \left[(1 - \delta(x, z)) u \left(Ra_2^* + \bar{y} z - \frac{\varphi(x)}{\beta q(\theta^*)(1 - \delta(x, z))} \right) \right] + (1 - p(\theta^*)(1 - \delta(x, z))) \, u(Ra_2^*) \Bigg]$$

The FOCs w.r.t to a_2 and θ become:

$$-u'(c_{u,1}) + \beta R \left[p(\theta^*) \mathbb{E}_z \left[(1-\delta(x,z)) u'(c_{e,2};z) \right] + (1 - p(\theta^*)(1-\delta(x,z))) u'(c_{u,2}) \right] = 0 \quad (29)$$

$$\mathbb{E}_z \left[(1-\delta(x,z)) \left[p'(\theta)(u(c_{e,2};z) - u(c_{u,2})) + u'(c_{e,2};z) \frac{\varphi(x) q'(\theta) \theta}{\beta(1-\delta(x,z)) q(\theta)} \right] \right] = 0$$

$$\mathbb{E}_z \left[(1-\delta(x,z)) p'(\theta)(u(c_{e,2};z) - u(c_{u,2})) \right] + \frac{\varphi(x) q'(\theta) \theta}{\beta q(\theta)} \mathbb{E}_z \left[u'(c_{e,2};z) \right] = 0 \quad (30)$$

Another equilibrium relationship is the derivative w.r.t. x

$$\mathbb{E}_z \left[-\delta_x(x,z) \left(u(c_{e,2};z) - u(c_{u,2}) \right) - u'(c_{e,2};z) \frac{\varphi'(x)(1-\delta(x,z)) - \varphi(x)(-\delta_x(x,z))}{\beta q(\theta)(1-\delta(x,z))} \right] = 0$$

$$\mathbb{E}_z \left[-\delta_x(x,z) \left(u(c_{e,2};z) - u(c_{u,2}) \right) \right] = \frac{\varphi'(x)}{\beta q(\theta)} \mathbb{E}_z \left[u'(c_{e,2};z) \left(1 - \frac{\varphi(x)(-\delta_x(x,z))}{\varphi'(x)(1-\delta(x,z))} \right) \right] \quad (31)$$

Recall the condition for positive sorting: $\phi_{a_2 x}\ \phi_{\theta\theta} < \phi_{\theta x}\ \phi_{a_2 \theta}$.

We start by computing $\phi_{a_2,x}$. It holds

$$\phi_{a_2,x} = \beta R p(\theta) \mathbb{E}_z \left[-\delta_x(x,z)(u'(c_{e,2};z) - u'(c_{u,2})) - u''(c_{e,2};z) \frac{\varphi'(x)(1-\delta(x,z)) + \varphi(x)\delta_x(x,z)}{\beta q(\theta)(1-\delta(x,z))} \right]$$

$$= \beta R p(\theta) \left[\mathbb{E}_z \left[-\delta_x(x,z)(u'(c_{e,2};z) - u'(c_{u,2})) \right] - \frac{\varphi'(x)}{\beta q(\theta)} \mathbb{E}_z \left[u''(c_{e,2};z) \left(1 + \frac{\varphi(x)\delta_x(x,z)}{\varphi'(x)(1-\delta(x,z))} \right) \right] \right]$$

Using Eq. (31), the FOC wrt x can be written as

$$\frac{\varphi'(x)}{\beta q(\theta)} = \frac{\mathbb{E}_z \left[-\delta_x(x,z) \left(u(c_{e,2};z) - u(c_{u,2}) \right) \right]}{\mathbb{E}_z \left[u'(c_{e,2};z) \left(1 - \frac{\varphi(x)(-\delta_x(x,z))}{\varphi'(x)(1-\delta(x,z))} \right) \right]}$$

Substituting this into the expression for $\phi_{a_2,x}$ yields

$$\phi_{a_2,x} = \beta R p(\theta) \left[\mathbb{E}_z \left[-\delta_x(x,z)(u'(c_{e,2};z) - u'(c_{u,2})) \right] - \right.$$
$$\left. \frac{\mathbb{E}_z \left[-\delta_x(x,z) \left(u(c_{e,2};z) - u(c_{u,2}) \right) \right]}{\mathbb{E}_z \left[u'(c_{e,2};z) \left(1 - \frac{\varphi(x)(-\delta_x(x,z))}{\varphi'(x)(1-\delta(x,z))} \right) \right]} \mathbb{E}_z \left[u''(c_{e,2};z) \left(1 + \frac{\varphi(x)\delta_x(x,z)}{\varphi'(x)(1-\delta(x,z))} \right) \right] \right]$$

Given that $\mathbb{E}_z \left[-\delta_x(x,z) \left(u(c_{e,2};z) - u(c_{u,2}) \right) \right] > 0$, we obtain

$$\phi_{a_2,x} < 0 \Leftrightarrow \frac{\mathbb{E}_z \left[-\delta_x(x,z)(u'(c_{e,2};z) - u'(c_{u,2})) \right]}{\mathbb{E}_z \left[-\delta_x(x,z) \left(u(c_{e,2};z) - u(c_{u,2}) \right) \right]} < \frac{\mathbb{E}_z \left[u''(c_{e,2};z) \left(1 - \frac{\varphi(x)(-\delta_x(x,z))}{\varphi'(x)(1-\delta(x,z))} \right) \right]}{\mathbb{E}_z \left[u'(c_{e,2};z) \left(1 - \frac{\varphi(x)(-\delta_x(x,z))}{\varphi'(x)(1-\delta(x,z))} \right) \right]} \quad (32)$$

We continue by computing ϕ_{θ,a_2}. It holds

$$\phi_{\theta a_2} = R\mathbb{E}_z\left[(1-\delta(x,z))\left[p'(\theta)(u'(c_{e,2};z)-u'(c_{u,2}))+u''(c_{e,2};z)\frac{\varphi(x)q'(\theta)\theta}{\beta(1-\delta(x,z))q(\theta)}\right]\right]$$

Using Eq. (30), the FOC wrt θ can be written as

$$\frac{\varphi(x)q'(\theta)\theta}{\beta q(\theta)} = -\,p'(\theta)\,\frac{\mathbb{E}_z\big[(1-\delta(x,z))\big(u(c_{e,2};z)-u(c_{u,2})\big)\big]}{\mathbb{E}_z\big[u'(c_{e,2};z)\big]}.$$

Substituting this into the expression for $\phi_{\theta a_2}$ yields

$$\begin{aligned}\phi_{\theta a_2} &= R\Bigg[p'(\theta)\,\mathbb{E}_z\big[(1-\delta(x,z))\big(u'(c_{e,2};z)-u'(c_{u,2})\big)\big]\\ &\qquad - p'(\theta)\,\frac{\mathbb{E}_z\big[(1-\delta(x,z))\big(u(c_{e,2};z)-u(c_{u,2})\big)\big]}{\mathbb{E}_z\big[u'(c_{e,2};z)\big]}\,\mathbb{E}_z\big[u''(c_{e,2};z)\big]\Bigg]\\ &= R\,p'(\theta)\left[\mathbb{E}_z\big[(1-\delta(x,z))\big(u'(c_{e,2};z)-u'(c_{u,2})\big)\big]-\frac{\mathbb{E}_z\big[(1-\delta(x,z))\big(u(c_{e,2};z)-u(c_{u,2})\big)\big]\,\mathbb{E}_z\big[u''(c_{e,2};z)\big]}{\mathbb{E}_z\big[u'(c_{e,2};z)\big]}\right]\end{aligned}$$

Given that $\mathbb{E}_z\big[(1-\delta(x,z))\big(u(c_{e,2};z)-u(c_{u,2})\big)\big]>0$, we obtain

$$\phi_{\theta a_2}<0 \Leftrightarrow \frac{\mathbb{E}_z\left[(1-\delta(x,z))\left(u'(c_{e,2};z)-u'(c_{u,2})\right)\right]}{\mathbb{E}_z\big[(1-\delta(x,z))\big(u(c_{e,2};z)-u(c_{u,2})\big)\big]} < \frac{\mathbb{E}_z\big[u''(c_{e,2};z)\big]}{\mathbb{E}_z\big[u'(c_{e,2};z)\big]} \tag{33}$$

A necessary condition for positive sorting is thus $\phi_{\theta x}<0$. It holds

$$\begin{aligned}\phi_{\theta x} &= \mathbb{E}_z\Bigg[p'(\theta)\Big((-\delta_x(x,z))\big(u(c_{e,2};z)-u(c_{u,2})\big)+(1-\delta(x,z))u'(c_{e,2};z)\,\frac{\partial c_{e,2}}{\partial x}\Big)\\ &\quad + u''(c_{e,2};z)\,\frac{\partial c_{e,2}}{\partial x}\,\frac{\varphi(x)q'(\theta)\theta}{\beta q(\theta)}+u'(c_{e,2};z)\,\frac{\varphi'(x)q'(\theta)\theta}{\beta q(\theta)}\Bigg]\\ \text{with}\quad \frac{\partial c_{e,2}}{\partial x} &= -\frac{1}{\beta q(\theta)}\frac{\varphi'(x)(1-\delta(x,z))+\varphi(x)\delta_x(x,z)}{(1-\delta(x,z))^2} = -\frac{\varphi'(x)}{\beta(1-\delta(x,z))q(\theta)}\left(1-\frac{\varphi(x)(-\delta_x(x,z))}{\varphi'(x)(1-\delta(x,z))}\right)\\ &= \mathbb{E}_z\Big[p'(\theta)(-\delta_x(x,z))\big(u(c_{e,2};z)-u(c_{u,2})\big)\Big]-p'(\theta)\frac{\varphi'(x)}{\beta q(\theta)}\mathbb{E}_z\left[u'(c_{e,2};z)\left(1-\frac{\varphi(x)(-\delta_x(x,z)}{\varphi'(x)(1-\delta(x,z))}\right)\right]\\ &\quad+\frac{\varphi'(x)}{\beta q(\theta)}\left(\mathbb{E}_z\left[-u''(c_{e,2};z)\left(1-\frac{\varphi(x)(-\delta_x(x,z)}{\varphi'(x)(1-\delta(x,z))}\right)\frac{\varphi(x)q'(\theta)\theta}{\beta(1-\delta(x,z))q(\theta)}\right]+\mathbb{E}_z\Big[u'(c_{e,2};z)\,q'(\theta)\theta\Big]\right)\end{aligned}$$

Using expression (31), the first line cancels out. We can thus write

$$\phi_{\theta x}=\frac{\varphi'(x)}{\beta q(\theta)}\left(-\mathbb{E}_z\left[u''(c_{e,2};z)\left(1-\frac{\varphi(x)(-\delta_x(x,z)}{\varphi'(x)(1-\delta(x,z))}\right)\frac{\varphi(x)q'(\theta)\theta}{\beta(1-\delta(x,z))q(\theta)}\right]+q'(\theta)\theta\;\mathbb{E}_z\Big[u'(c_{e,2};z)\Big]\right)$$

The term multiplying the outer bracket is positive, the last term is negative. If $\left(1-\frac{\varphi(x)(-\delta_x(x,z)}{\varphi'(x)(1-\delta(x,z))}\right)>$

0, then the whole expression must be negative. A sufficient condition for $\phi_{\theta x} < 0$ is thus

$$\frac{\varphi'(x)}{\varphi(x)} > \frac{(-\delta_x(x,z)}{(1-\delta(x,z))} \quad , \text{ for all } z \tag{34}$$

Moreover, as before under CES matching function, $\phi_{\theta,\theta} < 0$. We assume that it is the case.

Necessary conditions for positive sorting are thus given by Eq. (32), (33) and (34).

G2 Model with homogeneous vacancy cost

In this section, I expose why a heterogeneous vacancy cost is a necessary condition for PAM. Below, I derive the model with a homogeneous cost. The conclusion is that PAM cannot obtain under this specification.

With a homogeneous vacancy cost, the joint unemployed and firm problem becomes

$$U(a_1,x) = \max_{a_2,\theta} u(Ra_1 - a_2) + \beta\left[p(\theta)\Delta(x)u\left(Ra_2 + \bar{y} - \frac{\varphi}{\beta q(\theta)\Delta(x)}\right) + (1-p(\theta)\Delta(x))u(Ra_2)\right]$$

where $\Delta(x) = 1 - \delta(x,z)$ and $\bar{y}$ is the common production output.

The FOCs with respect to a_2 and θ are, respectively:

$$-u'(c^*_{u,1}) + \beta R\left[p(\theta^*)\Delta(x)u'\left(c^*_{e,2}\right) + (1-p(\theta^*)\Delta(x))u'(c^*_{u,2})\right] = 0 \tag{35}$$

$$\beta\Delta(x)\left[p'(\theta^*)\left(u\left(c^*_{e,2}\right) - u(c^*_{u,2})\right) + u'\left(c^*_{e,2}\right)\frac{\varphi\theta^* q'(\theta^*)}{\beta q(\theta^*)\Delta(x)}\right] = 0 \tag{36}$$

where $c^*_{u,1} = Ra_1 - a^*_2$, $c^*_{e,2} = Ra^*_2 + \bar{y} - \frac{\varphi}{\beta q(\theta^*)\Delta(x)}$ and $c^*_{u,2} = Ra^*_2$.

The first condition is an Euler equation where $t = 2$'s consumption depends on the probability of match and match outcome; the second condition pins down market tightness given productivity x.

Proposition 3. *There is no equilibrium featuring positive assortative matching.*

Proof. The first order condition for the choice of x is

$$\begin{aligned}\frac{\partial U}{\partial x} &= \beta p(\theta^*)\left[\Delta'(x)\left(u(c^*_{e,2}) - u(c^*_{u,2})\right) + \Delta(x)u'(c_{e,2^*})\left(\frac{\varphi\Delta'(x)}{\beta q(\theta^*)(\Delta(x))^2}\right)\right] \\ &= \beta p(\theta^*)\Delta'(x)\left[\left(u(c^*_{e,2}) - u(c^*_{u,2})\right) + u'(c_{e,2^*})\left(\frac{\varphi}{\beta q(\theta^*)\Delta(x)}\right)\right]\end{aligned} \tag{37}$$

The squared bracket of Eq. (37) is always positive, i.e., all workers always prefer the same type of jobs. There is no interior solution and hence no sorting. Depending on the sign of $\Delta'(x)$, all workers will apply to the maximum (if positive) or minimum (if negative) type. ■