Filtering and Affordability Along the Housing Ladder

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Abstract

How do changes in building costs propagate to rental prices of old housing units at the bottom of the housing ladder? We document three empirical facts. First, housing markets with higher building costs feature disproportionately higher rental prices for old units. Second, they feature old units occupied by relatively higher income households. Third, they feature less new construction and more maintenance of the housing stock. We rationalize all three facts in a housing equilibrium model with filtering. The facts and the model raise the possibility of *amplification*: changes in building costs can have larger effects on the prices of old housing units at the bottom of the housing ladder than on the prices of new housing units. We study welfare implications and transition dynamics in a quantitative version of the model. Changes in building costs disproportionately affect the welfare of low-income households, especially in the long run.

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1 Introduction

Housing affordability has considerably declined in the US since 2000. While households across all income groups have been affected, the largest impact has been on households at the bottom of the income distribution. We document this pattern in Table 1. Low-income households have been particularly affected because they spend a higher share of their income on housing. In 2000, the rent-to-income ratio was 0.27 for working-age households in the bottom 25% of the income distribution, compared to 0.12 for households in the top 50%. Low-income households have been particularly affected also because the housing prices they face have increased by more relative to their income levels. Indeed, the rent-to-income ratio has increased by 34.9% for households in the bottom 25% of the income distribution, compared to an increase of 27.2% for households in the top 50%. Table 1 shows how a similar pattern holds when considering owner-occupied prices instead of rental prices.

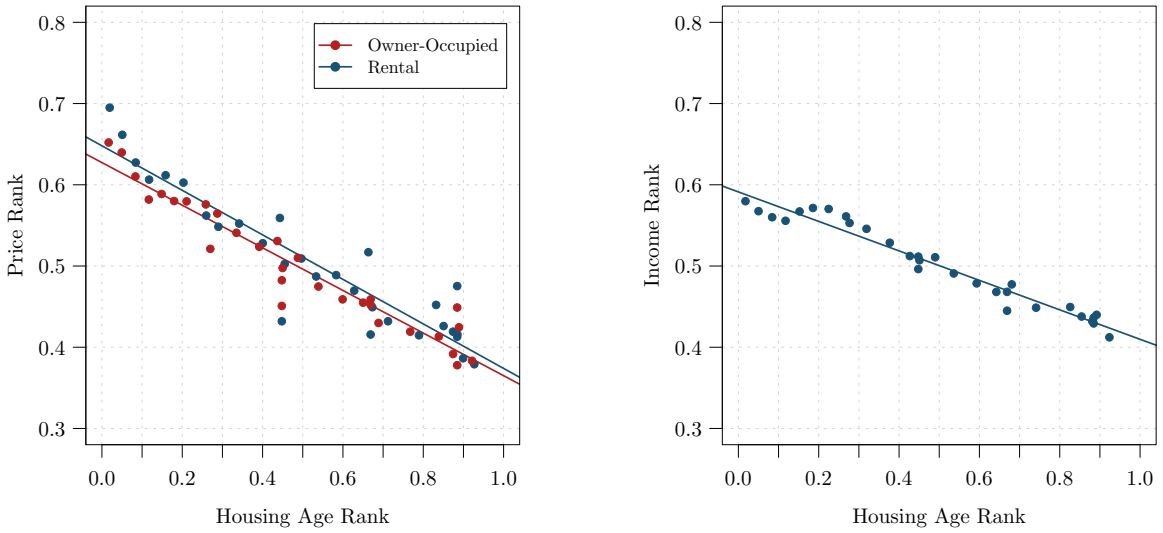
Table 1: Changes in Rent-to-Income Ratios

	Rent-to-Income		Price-to-Income	
	Bottom 25 %	Top 50 %	Bottom 25 %	Top 50 %
2000	0.27	0.12	3.29	1.73
2024	0.36	0.15	5.55	2.41
% Increase	34.9%	27.2%	68.6%	39.4%

Note: Tabulations based on the American Community Survey. Sample restricted to working-age households. Rent-to-income ratios are measured for rental housing and price-to-income ratios are measured for owner-occupied housing.

Economists have long promoted supply-side intervention to improve housing affordability. Recent policy initiatives such as California's AB 130 or New York's Let Them Build program have embraced this view in attempting to lower effective housing building costs by accelerating permitting processes. While there is broad consensus over supply-side policies' ability to make new housing more affordable, there is debate over the extent to which they can make housing more affordable for low-income households (Been et al. 2025). Indeed, a key feature of housing markets is that new housing units tend to be built at the top of the housing ladder. In 2019, approximately 82% of newly built rental housing units were above median rental prices. The equivalent fraction was 75% for owner-occupied units. It follows that low-income households sort into old housing units. Figure 1a shows the negative relationship between within-county housing age rank and price rank of housing units. Figure 1b shows the negative relationship between within-county housing age rank and household income rank, indicative of low-income households sorting into old housing units.

Figure 1: Within-County Relationship Between Housing Age, Prices, and Income.



(a) Housing Age and Price. **(b)** Housing Age and Household income.

Note: Tabulations based on the 2019 American Community Survey. Sample restricted to working-age households and to housing units of less than 45 years of age.

Key to tackling housing affordability for low-income households is therefore understanding the extent to which supply-side policies that target the cost of building new housing units can improve affordability of older units at the bottom of the housing ladder. In this paper, we contribute to answering this question in two ways. First, we document three empirical facts connecting building costs to prices of new and old housing units, household sorting, and supply-side outcomes. Second, we consider an equilibrium model of the housing market where housing units *filter* down the housing ladder as they age. We use the model to study how changes in building costs propagate to rental prices of old units at the bottom of the housing ladder. The model rationalizes our three empirical facts and provides further insights into housing affordability along the housing ladder.

In most of what follows, we focus on rental prices. Focusing on rental prices simplifies accounting for land value in interpreting our empirical facts and is consistent with a user-cost view of housing. Furthermore, we note that low-income households are disproportionately renters. 57.6% of households in the bottom 25% of the income distribution were renters in 2019, compared to 22.9% for the top 50% of the income distribution. Our model features rental prices but also housing values, which can be interpreted as owner-occupied prices. We postpone a discussion of how our results on rental prices extend to owner-occupied prices until later in the paper.

Three Empirical Facts. We study the cross-section of US counties and document three empirical facts connecting building costs to outcomes along the housing ladder. First, we show that counties with higher building costs feature disproportionately higher rental prices for old units relative to new units. We refer to this result as *amplification*: the effect of higher building costs on rental prices are amplified toward the bottom of the housing ladder. Second, we show that counties with higher building costs feature old units occupied by relatively higher income households. Third, we show that counties with higher building costs feature less new construction and more maintenance of the housing stock. Recognizing that building costs may be endogenous to housing market outcomes, we discuss how our empirical facts are robust to exploiting plausibly exogenous variation in building costs induced by terrain ruggedness.

Equilibrium Model with Filtering. In our model, housing units are differentiated along a single quality dimension. The model features *filtering*: units lose quality as they age (Sweeney 1974). As a result, the rental price of any given unit falls over time and that unit is occupied by progressively lower income households (Rosenthal 2014). Filtering can be thought of as the modeling assumption that a unit of quality q becomes a unit of quality $(q - \delta)$ after one unit of time. In appearance, the filtering assumption is therefore similar to the more standard assumption of quantity depreciation, where a unit of quality q becomes $(1 - \delta)$ units of quality q after one period of time (e.g. Favilukis et al. 2023). However, filtering has important implications for outcomes along the housing ladder. Consider a housing market with construction at a single quality. Absent filtering, all households face a single rental price per unit of housing and do not sort based on housing age. In the presence of filtering, households instead face a rental price schedule $p(q)$ for units of different qualities based on their age, leading to sorting consistent with the data. Filtering is therefore key to understanding how supply-side policies propagate down the housing ladder to older units and the households that occupy them.

The demand side of housing in our model consists of households with heterogeneous income. All households are renters and sort into housing units based on the rental price schedule. The supply side of housing in our model consists of landlords who make both construction and maintenance decisions. Consistent with our empirical facts, we treat building new housing units and maintaining existing housing units as two different ways in which the economy can supply housing. Landlords choose to build new housing units when present discounted future rent net of maintenance spending outweighs building costs and land value. Landlords choose to maintain existing housing units so long as flow rent net of maintenance spending outweighs an opportunity cost which depends on land value.

In response to an increase in building costs, the magnitude of the increase in rental prices of old, low quality housing units depends on the size of two effects. First, the increase in building costs worsens affordability in the economy, increasing *relative* demand for old units. In an equilibrium with filtering, relative housing supply is fixed, and rental prices of old units must increase by *more* than rental prices of new units to clear markets. We call this effect the affordability effect. Second, the increase in prices of old units leads to an increase in maintenance, increasing the stock of old housing units in the economy. This lowers the price pressure on old units. We call this effect the maintenance effect. When the affordability effect dominates the maintenance effect, the changes in rental prices induced by changes in building costs are amplified toward the bottom of the housing ladder, consistent with our first empirical fact. Our model also rationalizes our second and third empirical facts.

While we focus on the effect of changes in building costs throughout the paper, our model yields further results relevant for housing policy. We find that changes in land value induce strictly more amplification than changes in building costs. This insight is useful in contrasting the effects on prices at the bottom of the housing ladder of policies which relax zoning (Glaeser and Gyourko 2002) with those of policies which target building costs. In response to increases in minimum quality occupancy standards, both the affordability effect and the maintenance effect push in the same direction, leading to large increases in rental prices of old units at the bottom of the housing ladder. We discuss how results from the housing assignment literature (Nikolakoudis 2025; Landvoigt et al. 2015; Määttänen and Terviö 2014) on the effect of changes in the income distribution are affected by filtering. Lastly, we discuss how our model can be extended to account for the balanced growth – or decline – of a city's population.

Quantitative Model. We enrich our model and solve it numerically. Comparative statics results are consistent with our empirical facts. The quantitative model yields further insights: the combination of the affordability effect, which disproportionately increases rent paid by low-income households, and of the maintenance effect, which re-assigns those households to lower quality units, leads to disproportionately large welfare losses for households at the bottom of the income distribution. Studying transition dynamics, we find that the economy initially exhibits resilience in response to an increase in building costs. This is due to housing durability and to the economy's ability to substitute from providing housing services with new construction to providing housing services with maintenance of the existing housing stock. Resilience *delays*, but does not prevent the affordability effect. In the long run, the economy must build new housing units at higher building costs, leading to a large affordability effect. This in turn leads to larger welfare losses for households at the bottom

of the income distribution.

Related Literature. Our empirical facts and our theory connect to an empirical literature studying how a given housing unit is occupied by lower-income households over time. This phenomenon is often also referred to as filtering. For clarity, we label it *income filtering* throughout the paper, and reserve the word filtering for the physical process by which a housing units loses quality as it ages. Rosenthal (2014) finds annual income filtering rates of 2.5% for rental housing[1]. Liu et al. (2022) find substantial heterogeneity across space in income filtering rates and show that supply constraints are negatively correlated with income filtering rates. Spader (2025) finds a recent decline in income filtering, which coincides in the time series with lower housing affordability. We view these findings as consistent with our second empirical fact, which documents that counties with higher building costs feature a flatter relationship between housing unit ages and household income levels.

In modeling a housing market with filtering, our paper connects to an early theoretical literature. Sweeney (1974) models housing filtering in a setting with a finite set of housing qualities and shows how a stationary equilibrium with filtering can feature housing segments where prices are not fully-determined by building costs. Ohls (1975) performs numerical simulations of a housing filtering model to study demand-side policies such as rental voucher programs. Braid (1984) studies those policies while modeling filtering along a continuum of housing qualities. Arnott and Braid (1997) model a housing maintenance technology which allows for units to be maintained at a constant quality. We build upon this literature to study supply-side policies in a model with filtering. Our model allows us to derive comparative statics with respect to building costs and rationalize our empirical facts relating building costs to outcomes along the housing ladder. We solve a quantitative version of our model to provide further insights. Recent work by Hong and Macek (2025) also studies a quantitative model with filtering, focusing on the effect of a tear-down tax on housing outcomes in Chicago.

Our model features assignment of households by income to housing units by quality as in work by Määttänen and Terviö (2014), Landvoigt et al. (2015), and Nikolakoudis (2025). Their work focuses on the demand-side of housing and for the most part holds the supply of housing in the economy fixed. Instead, we fully endogenize the housing supply side. While doing so would eliminate the possibility of long-run demand-driven changes in relative prices across the quality ladder in a standard model where construction determines prices at all quality levels, this is not the case in our model with filtering. To highlight this, we discuss

[1]Rosenthal (2014) finds lower annual income filtering rates of 0.5% for owner-occupied housing. However, this can be rationalized with a lower income elasticity of demand for renters and does not necessarily imply lower physical filtering for owner-occupied units.

how a key result on the – absence of – trickling down of changes in the income distribution (see Nikolakoudis 2025) is affected in the presence of filtering.

Our work connects to papers studying the effect of exogenous increases in new construction on prices along the housing ladder. Mense (2025) provides empirical evidence exploiting weather shocks. The effect of added construction on prices at the bottom of the housing ladder is attributed to moving chains, by which households moving into added units free up old units at the bottom of the housing ladder. Nathanson (2025) studies those moving chains in a model calibrated to Los Angeles. He highlights how external migration can limit the effectiveness of moving chains in lowering prices at the bottom of the housing ladder. Hauck and Kluser (2025), Bratu et al. (2023), and Mast (2023) provide direct evidence of moving chains. By considering exogenous perturbations to a fixed housing supply, those papers study a housing market in partial equilibrium. Instead, we endogenize housing supply to study a housing market in equilibrium. This allows us to study the effect of changes in supply-side policy, instead of the effect of changes in supply. Because we endogenize the supply side and abstract from short-term frictions which may limit re-assignment of households to housing units, our results can be thought of as a longer-run equilibrium counterpart to the partial equilibrium results cited above. Mense (2025) estimates that exogenous changes in new construction have *smaller* effects on rental prices at the bottom of the housing ladder. By contrast, the empirical facts we document in US cross-sectional data suggest that in equilibrium, changes in building costs can have *larger* effects on rental prices at the bottom of the housing ladder. Our model rationalizes this finding.

Lastly, a large literature has found building costs to depend on housing regulation (see Glaeser and Gyourko 2025; Molloy et al. 2020). This motivates us to study the effect of a fundamental change in building costs, which we think of as induced by a change in regulation. Studying Los Angeles, Soltas and Gruber (2026) find that permitting costs can account for as much as 36% of building costs and are largely the result of long permitting times. We can therefore think of the counterfactual changes in building costs we study as resulting from changes in permitting times, consistent with policies such as California's AB 130 and New York's Let Them Build program.

Outline. The rest of the paper proceeds as follows. In Section 2, we document three facts connecting building costs to outcomes along the housing ladder. In Section 3, we introduce our model of a housing market with filtering. We study comparative statics of the model which rationalize all three of our empirical facts in Section 4. In Section 5, we study a quantitative version of the model. We conclude in Section 6.

2 Three Empirical Facts

We present three facts connecting building costs to outcomes along the housing ladder. A key feature of housing markets is that new housing units are expensive and become affordable as they age. Therefore, we view age as a determinant of where a housing unit lies on the housing ladder. Our first fact is motivated by the relationship between housing prices and housing age observed in Figure 1a. We compare how rental prices vary with housing unit age in high and low building costs markets. Our second fact further explores the income sorting observed in Figure 1b by comparing the assignment of households to housing units in high and low building costs markets. Our third fact considers how building costs affect the rate of new construction and maintenance spending.

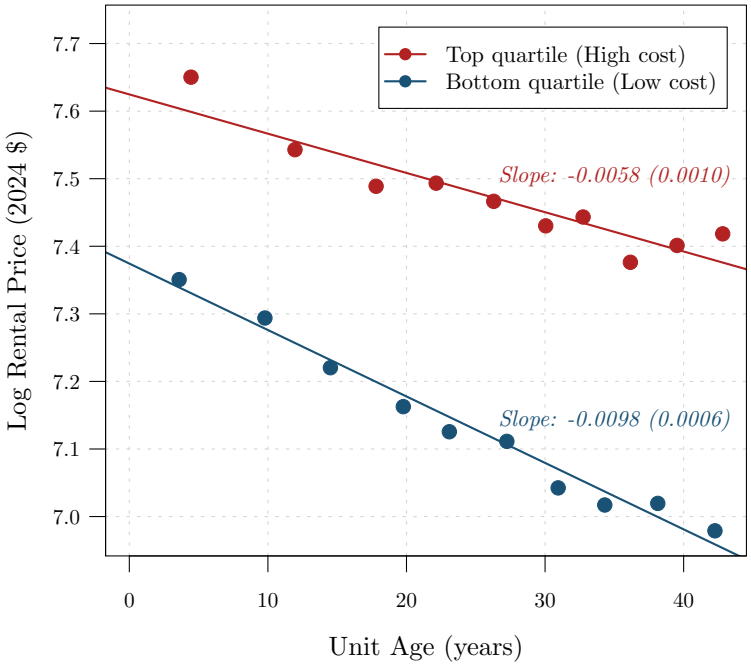
Data. We exploit cross-sectional variation in building costs across US counties. We treat counties as housing markets (Nikolakoudis 2025; Landvoigt et al. 2015), but note that our results are qualitatively similar when instead treating metropolitan statistical areas (MSAs) as housing markets. We source data on building costs from the Building Permit Survey (BPS). The BPS surveys the number of residential construction permits issued by a permitting office and the total value of the resulting properties, net of land. We estimate building costs in a given county by dividing the total value of the resulting properties by the number of permits issued. We source rental prices, household income, and housing age from the American Community Survey (ACS). Our sample runs from 2005 to 2024. The relationship between rental prices and housing age may be blurred by large-scale housing remodeling for very old housing units (see Randolph 1988). Indeed, large-scale remodeling effectively resets a housing units age. Because we do not observe large-scale remodeling, we partially address this concern by restricting our sample to housing units with an age lower than 45 years, with younger units less likely to have undergone large-scale remodeling. Figure A3 provides support for this approach by showing how the relationship between housing unit age and rental prices in our data is nearly flat beyond age 45. We provide details and further discuss our data collection in Appendix C.1.

2.1 Building Costs and Rental Prices Along the Housing Ladder

Our first fact connects building costs to rental prices along the housing ladder. We start by comparing the rent–age gradient in counties with high and low building costs. Figure 2 shows a binscatter of log rental prices against housing unit ages for the top and bottom quartile of counties by building costs. First, we observe that older units have lower rental prices. This is consistent with Figure 1a, and with filtering. Second, we observe that counties with

higher building costs feature higher rental prices. Splitting by building costs also reveals that counties with higher building costs feature a substantially flatter gradient of rental prices to housing age. For each year a housing unit ages, the associated decline in rental prices is 0.6% in high building costs counties compared to 1% in low building costs counties. This provides suggestive evidence for *amplification*: higher building costs increase rental prices of old units at the bottom of the housing ladder more than rental prices of new units.

Figure 2: Age and Rental Prices for High and Low Building Costs Counties.



Note: residualized by year fixed effects. Standard errors clustered at the MSA level.

We estimate the regression analog to Figure 2. With h indexing a housing unit, t a year, and c a county, we consider the specification

$$\log p_{h,c,t} = \beta_0 \text{age}_{h,c,t} + \beta_1(\text{age}_{h,c,t} \cdot \log \kappa_{c,t}) + \alpha_{c,t} + \varepsilon_{h,c,t}, \tag{1}$$

where p denotes rental prices, α are fixed effects, and $\varepsilon_{h,c,t}$ are errors which are potentially correlated across time, units and counties. To account for serial and spatial correlation, we cluster standard errors by MSA. The coefficient of interest is β_1. Column 1 of Table A1 reports ordinary least squares coefficient estimates, with $\hat{\beta}_1$ positive and significant at the standard levels.

Our specification uses log rental prices and allows for county fixed effects. The county fixed effects absorb any components of rental prices which may systematically correlate with the interaction term $\text{age}_{h,c,t} \cdot \log \kappa_{c,t}$ to the extent that they enter rental prices multiplicatively.

This allows us to account for land prices, which one could expect to be on average higher in highly developed counties with older housing stocks and high building costs [2]. A limit of the fixed effects specification, however, is that some counties cover large areas (e.g. LA county) with large within-county variation in characteristics including land prices.

To address the limitations described above and other potential measurement error or endogeneity concerns, we use terrain ruggedness as an instrument for building costs. Relevance stems from ruggedness increasing costs of land preparation and laying foundation. We discuss exogeneity of the instrument and provide further details in Appendix C. We instrument $\text{age}_{h,c,t} \cdot \log \kappa_{c,t}$ with $\text{age}_{h,c,t}$ interacted with ruggedness[3]. Column 2 of Table A1 shows our results. The coefficient on the interaction term $\text{age}_{h,c,t} \cdot \log \kappa_{c,t}$ is estimated to be positive and significant at the standard levels. A 1% increase in building costs increases rental prices by 0.46 percentage points more for a 40 year old housing unit than for a 10 year old unit. This suggests that amplification may not only hold qualitatively but also be quantitatively relevant. We document our first empirical fact below.

> **Fact 1 (Amplification):** Housing markets with higher building costs feature disproportionately higher rental prices for old units.

We have examined the relationship between rental prices and (i) building costs and (ii) variation in building costs induced by terrain ruggedness. While recognizing the endogeneity of housing regulation, we find it instructive to test whether rental prices are similarly related to (iii) variation in building costs induced by housing regulation. Soltas and Gruber (2026) show that permitting times can explain a large fraction of building costs. Using data on permitting times from Gyourko et al. (2008), we investigate their role in shaping the rent–age gradient. In Appendix A, Figure A1 shows the positive association between permitting times and building costs, and Figure A2 shows the relationship between housing unit age and rental prices for fast and slow-to-issue permits counties. The pattern documented with Fact 1 emerges: housing markets with longer permitting times feature disproportionately higher rental prices for old units.

Our empirical exercise therefore connects with prior observations made about the effects of housing regulation on the distribution of rental prices. Malpezzi and Green (1996) document that higher regulated areas feature higher prices for low quality housing units relative to high quality units. Molloy et al. (2020) discuss this fact and interprets it as suggestive that regulation reduces the supply of low quality units relative to high quality units, conflicting with survey evidence from Emrath (2016) which finds that the regulatory component of

[2] Land is typically assumed to enter hedonic models of housing prices as a multiplicative fixed effect.

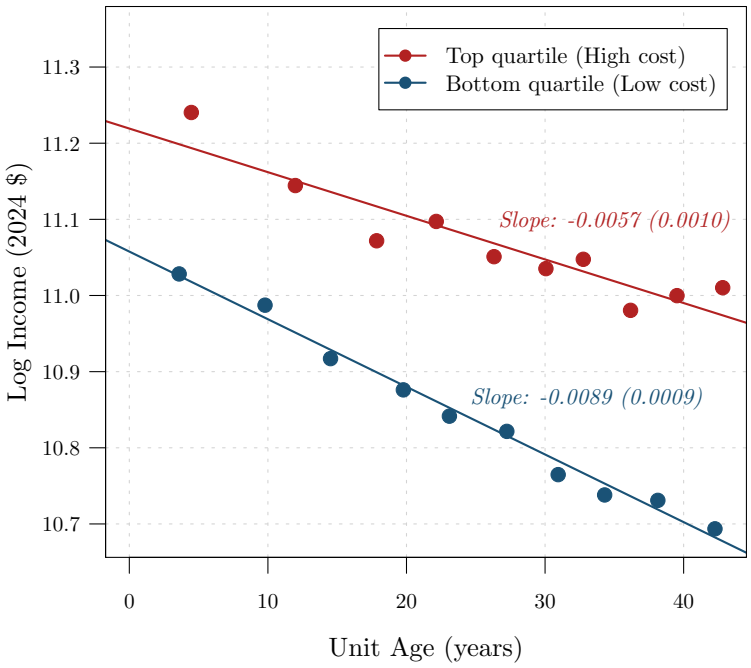
[3] Further details are in Appendix C.2.

building costs is larger for high quality units than low quality units. Our empirical facts put housing unit age at the center of the analysis. We find that markets with higher building costs feature higher prices for old housing units relative to new housing units, despite, in what will become clear from our third empirical fact, having a higher relative supply of old units. Our model rationalizes this observation in Section 4.

2.2 Building Costs and Income Along the Housing Ladder

Our second fact connects building costs to income sorting along the housing ladder. We start by comparing the income–age gradient in high and low building costs counties. Figure 2 plots a binscatter of log income against housing unit ages for the top and bottom quartile of counties by building costs.

Figure 3: Age and Household Income for High and Low Building Costs Counties.



Note: residualized by year fixed effects. Standard errors clustered at the MSA level.

Consistent with Figure 1b, we find that lower income households occupy older units in both high and low building costs counties. We also see that higher building costs counties feature a flatter gradient of income to housing age. For each year a housing unit ages, the associated decline in income of the occupants is 0.6% in high building costs counties compared to 0.9% in low building costs counties.

The fact that the income – age gradient is negative is consistent with income filtering (Rosenthal 2014). The fact that it is flatter in high building costs counties suggests that

income filtering may be slower in those counties. Our finding is thus connected to the observation by Liu et al. (2022) that supply constraints are negatively correlated with income filtering rates. Our finding is also consistent with evidence by Spader (2025) of a recent decline in income filtering, coinciding with rising building costs (Baum-Snow and Duranton 2025).

Since labor makes up a large fraction of building costs, it is natural for counties with higher incomes to feature higher building costs. Figure 2 suggests this is indeed the case. To account for potential differences in the first and second moments of the distribution of income across housing markets, in the regression analog of Figure 2 we measure relative income by standardizing log income within markets. We regress relative log income on age, building costs interacted with age and county fixed effects. We again instrument for building costs using terrain ruggedness. Further details can be found in Appendix C.2. Column 2 of Table A2 shows our results. The coefficient on the building costs – age interaction is positive and significant at the standard levels. Estimates imply that a 1 percent increase in building costs is associated with an increase in the log income of the occupants of a 40 year old unit of 0.55 standard deviations higher than the increase in the log income of the occupants of a 10 year old unit. We document our second empirical fact below.

> **Fact 2 (Assignment):** Housing markets with higher building costs feature old units occupied by relatively higher income households.

2.3 Building Costs, New Construction, and Maintenance

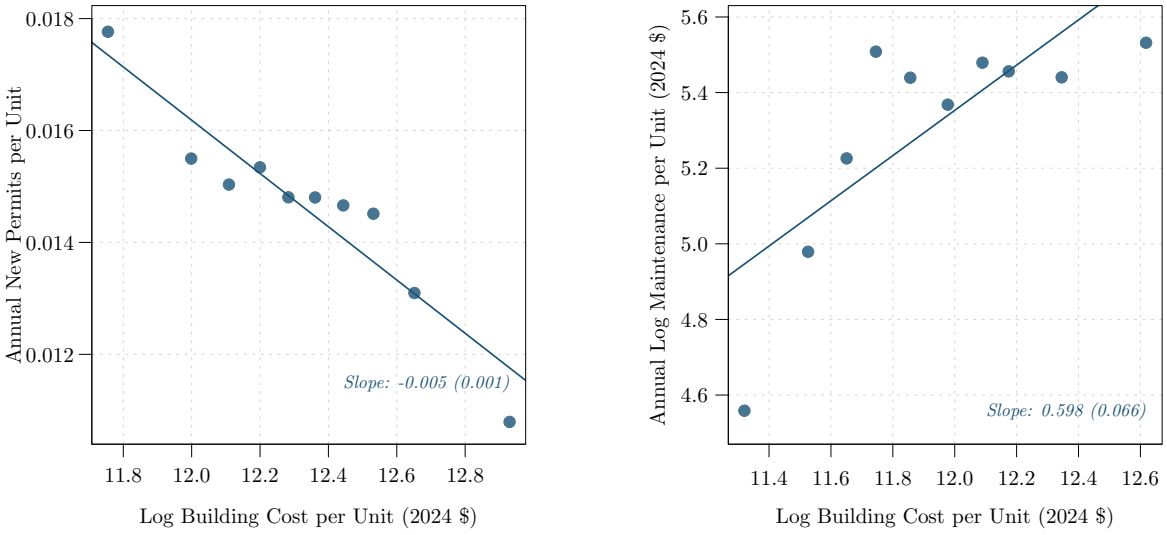
Our third fact connects building costs to new construction and maintenance spending. Both new construction and maintenance are ways in which economies can provide housing services. New construction provides housing services by adding new housing units to the economy, while maintenance provides housing services by extending the life of existing housing units. This suggests that economies with high building costs may substitute from providing housing services with new construction to providing housing services with maintenance. We test this prediction in the data.

To measure new construction, we use permits issued per housing unit, combining data from the BPS and from the ACS. As we do not observe housing maintenance spending in the ACS, we complement our data with the American Housing Survey (AHS). The AHS provides data on annual maintenance spending, unit characteristics, and household income. Due to coarse geographic data in the AHS beyond 2013, we restrict our attention to years

1991 to 2013. Because counties are not reported in the AHS, we use MSAs as the unit of geography.

Figure 4 plots binscatters of the rate of new construction and the rate of maintenance against building costs. Consistent with the view that housing markets with higher building costs shift from providing housing services through new construction to providing housing services through maintenance of the existing housing stock, we find that MSAs with higher building costs feature a lower rate of new construction and higher maintenance spending. This results in a median age of the housing stock of 42 years in the top quartile of counties by building costs, compared to a median of 36 years in the bottom quartile.

Figure 4: Relationships between new construction, maintenance and building costs.



We present OLS regression results in columns 1 and 3 of Table A3. We control for household income, bedrooms and square footage of housing units when regressing maintenance on building costs. A 1 percent increase in building costs is associated with 5 fewer permits per year per every 1,000 housing units and 0.3 percent more annual maintenance spending. As before, we may be concerned about endogeneity of building costs. Coarse geographic data in the AHS however weakens the terrain ruggedness instrument, rendering IV estimates insignificant. One endogeneity concern is that MSAs with high rates of new construction may feature higher building costs due to upward sloping construction supply curves. This, however, would dampen the result reported above. Nevertheless, we document our third empirical fact below with the caveat of potential endogeneity.

> **Fact 3 (Construction and Maintenance):** Housing markets with higher building costs feature less new construction and more maintenance.

3 A Housing Equilibrium Model with Filtering

In this section, we introduce our housing equilibrium model with filtering. We model an endowment economy in continuous time. Households are endowed with heterogeneous income. They purchase a consumption good and rent housing units. Landlords are endowed with land. They choose whether to build new housing units on their land *and* whether to maintain existing housing units. Maintained housing units gradually filter down the housing ladder. We focus on steady-state equilibria of the economy and therefore omit time subscripts. In Section 4, we use the model to study the propagation of changes in building costs down the housing ladder. We study a quantitative version of the model and dynamic equilibria in Section 5.

3.1 Household Problem

A unit mass of households inhabits the economy. Households are heterogeneous in their income endowment w. We assume that w is distributed according to the CDF $F(\cdot)$ with support $[\underline{w}, \overline{w}]$ and density $f(\cdot)$. All households are renters and allocate their income between consumption of a numéraire good and rental of a housing unit. Housing units are differentiated along a single quality dimension which we denote by q. This results in the following intratemporal problem: a household with income w chooses $c \geq 0$ and the quality q of its housing unit to solve

$$\max_{c,q} u(c, q) \tag{2}$$

subject to the budget constraint

$$c + p(q) = w, \tag{3}$$

where $p(\cdot)$ is the rental price schedule, with $p(q)$ the price of renting a unit of quality q. The function $u(c, q)$ is flow utility over consumption and housing quality. Importantly, $p(\cdot)$ need not be linear. Contrary to models of housing *quantity*, where there exists a single price per unit of housing quantity, our model features a continuum of prices. This is crucial to understand how prices respond to changes in building costs along the housing ladder. Because we assume a constant level of income and study steady-state equilibria, there is no loss of generality from treating the household problem as static instead of embedding it in a two-state budgeting problem where households also solve an intertemporal consumption-

saving problem[4].

Whenever the price schedule is differentiable, the household problem yields the first-order conditions

$$\frac{u_q(c,q)}{u_c(c,q)} = p'(q), \quad c = w - p(q) \tag{4}$$

for a household with income w. Given their income, households choose a level of housing quality such that the marginal rate of substitution of housing quality for consumption is equal to the gradient of the rental price schedule. This condition is standard in housing assignment models (e.g. Landvoigt et al. 2015).

So far, we have remained agnostic as to what constitutes housing unit quality, but have implicitly imposed that all households agree on orderings of housing units by assuming the existence of a quality index [5]. We have further assumed that quality enters flow utility identically for all households. In our model, housing age and maintenance spending will be the sole determinants of housing quality. While age and maintenance are key components of housing quality, they are by no means the only ones. Indeed, other housing characteristics such as size and location explain large fractions of housing prices. Because our model does not explicitly model size or location, it can be thought of as representing the housing market for a given size of housing units in a given location.

In what follows, we assume that the flow utility of households admits the Cobb-Douglas form

$$u(c,q) = cq. \tag{5}$$

Equal Cobb-Douglas weights on consumption and housing quality are without loss of generality up to an increasing transformation of quality. We denote the demand function for housing quality by $\tilde{q} : [\underline{w}, \overline{w}] \to [\underline{q}, \overline{q}]$. The first-order conditions of the household problem can then be combined as

$$\frac{\tilde{q}^{-1}(q) - p(q)}{q} = p'(q) \tag{6}$$

whenever the inverse of $\tilde{q}(\cdot)$ exists, characterizing a linear ordinary differential equation for $p(q)$.

[4]To see this, note that in the two-state budgeting formulation of the household problem, any steady-state requires that the household discount rate equal the interest rate. As a result, households choose to spend a constant amount in every period, as in our formulation of the intratemporal household problem.

[5]While limiting, this assumption is crucial for tractability in housing assignment models (Määttänen and Terviö 2014).

3.2 Housing Technology

New housing units are built at the top of the housing ladder, with quality $\overline{q}$. This is in line with our observation that 82% of newly built rental housing units are rented above median rental prices and with the well documented fact that housing markets typically feature very little building activity at the bottom of the housing ladder (e.g. Mense 2025, Ma 2025). Because equilibrium requires for the price schedule $p(q)$ to be strictly increasing, this modeling choice is equivalent to assuming that new housing units can be built at any quality $q \in [\underline{q}, \overline{q}]$, but at a constant cost. The latter assumption leads to housing units being built only at quality $\overline{q}$ as an equilibrium outcome. In Section 5, we extend the model to allow for construction in an interval of housing qualities.

Housing filtering is endogenous in our model. Given an amount of maintenance spending m, a housing unit of quality q filters down the housing ladder at a rate given by $\mathcal{D}(m, q)$. We model filtering to capture two properties. First, that the filtering rate is decreasing in maintenance spending: better maintained housing units filter down the housing ladder at a slower rate. Second, that the maintenance spending required for a given rate of filtering $\mathcal{D}$ is non-increasing in q: as housing units go down the housing ladder, more maintenance is required to slow down their deterioration. Specifically, we impose

$$\mathcal{D}(m, q) = \begin{cases} \infty & \text{if } m < \mathcal{M}(q), \\ \delta & \text{if } m \geq \mathcal{M}(q), \end{cases} \tag{7}$$

where $\mathcal{M}(q)$, the maintenance cost function, is an arbitrary and exogenously given differentiable function satisfying $\mathcal{M}(\overline{q}) = 0$ and $\mathcal{M}(\cdot)$ non-increasing. This implies that improperly maintained housing units lose their quality instantly, while maintained units progressively lose their quality at the rate δ. As becomes clear upon introducing the landlord problem, this modeling choice generates an endogenous extensive margin maintenance decision, with landlords deciding whether or not to maintain units of different qualities. It however abstracts from any intensive margin maintenance decision which would allow landlords to further slow down or interrupt the filtering process (Arnott and Braid 1997). We extend the model in this direction in Section 5.

Finally, $\underline{q} < \overline{q}$ is the minimal housing quality, with housing units ceasing to provide housing services when they reach quality $\underline{q}$. The minimal quality $\underline{q}$ can be interpreted either as reflecting a physical constraint, capturing that housing units even when they reach a certain quality, or as reflecting a minimal quality occupancy standard (Braid 1984). While we treat $\underline{q}$ as exogenous, whether housing units are maintained until they reach this quality level or not is an endogenous outcome of the model.

3.3 Landlord Problem

Landlords are endowed with land. We normalize the size of a land unit so that one housing unit occupies exactly one land unit. The total mass of land in the economy is $L > 1$, implying there is sufficient land to house the unit mass of households. Land has an exogenous outside option value P_0, which can be thought of as the value from using land for agriculture or another commercial purpose. We can also think of P_0 as reflecting minimum lot size policies, which mechanically increase the amount – and hence the cost – of land required to build one housing unit[6]. In Section 4.5 we examine how comparative statics with respect to P_0 differ from those with respect to building costs κ.

We let $P(q)$ denote housing value, the value of a land unit with a quality q housing unit[7]. We assume that landlords can tear down housing units at no cost and denote by $\underline{P}$ the value of a vacant land unit. Landlords take as given the rental price schedule $p(q)$. Housing value $P(q)$ solves the Hamilton–Jacobi–Bellman (HJB) equation

$$rP(q) = \max_{m} \mathbf{1}_{m<\mathcal{M}(q)} \left(r\underline{P} - m\right) + \mathbf{1}_{m\geq\mathcal{M}(q)} \left(p(q) - m - \delta P'(q)\right), \tag{8}$$

where r denotes the exogenous interest rate at which landlords discount their profits. The first term of the HJB corresponds to the value of improperly maintained housing units, which are torn-down, yielding the value of a vacant land unit. The second term of the HJB corresponds to the value of maintained housing units, which provide landlords with flow profits $p(q)-m$ and filter down the quality ladder at a rate δ. The solution to the HJB is the maintenance policy $m(q)$, which landlords optimally set to zero for housing units they decide to abandon and tear-down and to $\mathcal{M}(q)$ for housing units they decide to maintain. Hence, filtering in our model generates an endogenous extensive margin maintenance decision.

It is profitable for landlords to build a new housing unit on vacant land if

$$P(\overline{q}) - \kappa \geq P_0. \tag{9}$$

Because the economy has sufficient land to house the unit mass of households, landlord free entry into building guarantees

$$\underline{P} = P(\overline{q}) - \kappa = P_0 \tag{10}$$

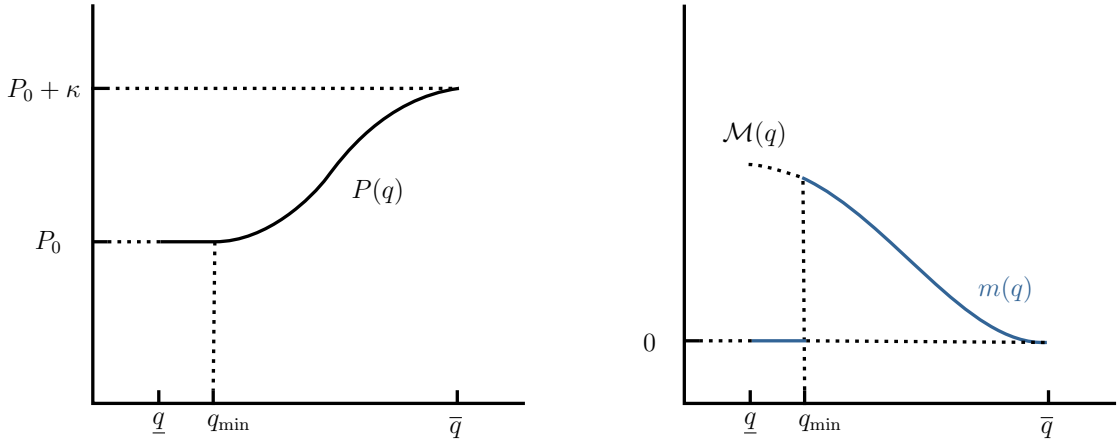
in equilibrium.

[6]Other forms of land-use regulation have also been shown to significantly affect land value (e.g. Glaeser and Gyourko 2002).

[7]In a model with home-ownership, $P(q)$ would be the owner-occupied housing price schedule.

Figure 5a shows housing value for an arbitrary increasing rental price schedule $p(q)$. The value of a new unit is $P_0+\kappa$, but falls as the unit filters down the housing ladder. Because $p(q)$ is increasing while maintenance costs $\mathcal{M}(q)$ are non-increasing, it is optimal for landlords to tear-down all housing units below a quality cut-off $q_{\min} \geq \underline{q}$. Housing value $P(q)$ is constant and equal to land value below $q_{\min}$. Figure 5b shows the associated maintenance policy. It is non-increasing and given by $\mathcal{M}(q)$ above $q_{\min}$, with older housing units further down the quality ladder requiring more maintenance. It is zero below $q_{\min}$, where housing units are torn down.

Figure 5: Landlord Problem.



(a) $P(q)$ **(b)** $m(q)$

From the HJB, we can solve for the housing value of a new unit

$$P(\overline{q}) = \frac{1}{\delta}\int_{q_{\min}}^{\overline{q}} \exp\left(-r\frac{\overline{q}-q}{\delta}\right)(p(q)-\mathcal{M}(q))dq + \exp\left(-r\frac{\overline{q}-q_{\min}}{\delta}\right)P_0. \tag{11}$$

The first term is an integral of rental prices net of maintenance costs over the housing ladder. The integral is discounted based on the interest rate r and the inverse of the filtering rate of maintained units δ, which captures the time spent by housing units in any part of the housing ladder. The second term captures leftover land value after the new unit is eventually torn down.

3.4 Housing Market Clearing

We impose housing market clearing at every quality q. Let C denote the aggregate rate of new housing unit construction in the economy. At any quality q such that housing is

maintained for all $q' \in [q, \overline{q})$, the condition

$$\int_{\underline{w}}^{\overline{w}} \mathbf{1}_{\tilde{q}(w) \geq q} dF(w) = C \frac{\overline{q} - q}{\delta} \tag{12}$$

must be satisfied. The condition states that the mass of households demanding housing of quality above q must equal the mass of housing units filtering through qualities above q. The aggregate construction rate is an endogenous object which together with landlords' maintenance policy pins down the total mass of housing in the economy.

From housing market clearing, we see how filtering is key to understanding equilibrium rental prices toward the bottom of the housing ladder. In housing assignment models with an exogenous supply side, equilibrium prices along the housing ladder interact only due to demand-side forces. In our housing assignment model with an endogenous supply side, this is no longer the case. Equilibrium prices along the housing ladder interact due to demand-side forces but also due to supply-side forces. Indeed, quality q housing is supplied to the economy only if new construction is profitable and if maintenance at all qualities $[q, \overline{q})$ is profitable. Given new construction and landlords' maintenance policy, filtering then pins down the distribution of housing supply. This distribution is uniform in a steady-state equilibrium. In Section 4.5, we discuss how the model can be extended to study a balanced growth path with population growth at a rate g. We show how a uniform distribution of housing supply corresponds to the case $g = 0$ of a more general distribution of housing supply, with $g > 0$ leading to more mass at the top and $g < 0$ leading to more mass at the bottom of the housing ladder.

3.5 Equilibrium

We now define a steady state equilibrium of the economy.

Definition 1 (Steady State Equilibrium)**.** *A steady state equilibrium of the economy is characterized by a price schedule $p(q)$, a housing value function $P(q)$, a maintenance policy $m(q)$, an aggregate construction rate C, and allocations $\{(c_w, q_w)\}_w$ such that:*

1. *Given $p(q)$, households with income w choose (c_w, q_w) optimally according to the intratemporal problem.*
2. *Given $p(q)$, landlords choose $m(q)$ optimally according to the HJB for $P(q)$.*
3. *Free entry into building pins down $P(\overline{q})$.*

4. *Given the aggregate construction rate C and the housing demand function $\tilde{q}(w) = q_w$, the housing market clears at all qualities $q \in [\underline{q}, \overline{q}]$.*

5. *The consumption good market clears by Walras' law.*

We restrict our attention to equilibria featuring a continuous rental price schedule[8]. Our characterization of the landlord problem implied the existence of a quality cut-off $q_{\min}$ below which landlords tear down housing units. We derive the following proposition.

Proposition 1. *Any equilibrium features a cut-off q_{min} such that housing is maintained and occupied in the interval $[q_{min}, \overline{q}]$. Whenever $q_{min} > \underline{q}$,*

$$p(q_{min}) - \mathcal{M}(q_{min}) = rP_0 \tag{13}$$

and whenever $q_{min} = \underline{q}$,

$$p(\underline{q}) - \mathcal{M}(\underline{q}) \geq rP_0. \tag{14}$$

Proof. See appendix D.1. □

Equation A6 implicitly characterizes the quality cut-off $q_{\min}$ in an equilibrium with $q_{\min} > \underline{q}$. The left-hand side of equation A6 corresponds to the flow profits from maintaining a unit of quality $q_{\min}$. When $q_{\min} > \underline{q}$, they must equal rP_0, the flow opportunity cost of using a land unit for housing rental. In an equilibrium with $q_{\min} = \underline{q}$, maintenance of housing units is profitable to landlords even at the minimal quality $\underline{q}$, as stated by equation A8.

We assume that the economy's income endowment is such that all households can afford housing[9]. We can then establish existence of a unique equilibrium. As is common in housing assignment models, the equilibrium features positive assortative matching. We define the assignment function $\tilde{w} : [\underline{q}, \overline{q}] \to [\underline{w}, \overline{w}]$ as

$$\tilde{w}(q) = \tilde{q}^{-1}(q) \tag{15}$$

when the inverse of the demand function for housing quality $\tilde{q}(\cdot)$ exists. The assignment function maps a housing unit quality to the income of households occupying units of that quality. We derive the following proposition.

[8]We do so because we notice that starting from any equilibrium featuring a continuous rental price schedule, one can raise the price schedule discontinuously at a finite number of qualities without affecting market clearing. This is the case because we impose market clearing as an integral condition, which cannot be affected by measure 0 re-assignments of households.

[9]In doing so, we abstract away from homelessness, which we view as beyond the scope of our current analysis. O'Flaherty (1995) studies the determinants of homelessness in a model with housing filtering.

Proposition 2. *There exists a unique equilibrium. It features the positive assortative matching rule*

$$\tilde{w}(q) = F^{-1}\left(\frac{q - q_{min}}{\overline{q} - q_{min}}\right). \tag{16}$$

Proof. See appendix D.2. □

Contrary to housing assignment models with an exogenous housing supply side (Nikolakoudis 2025; Landvoigt et al. 2015; Määttänen and Terviö 2014) the assignment function is not fully pinned-down by the distribution of income endowments $F(\cdot)$. Instead, it also depends on $q_{\min}$, an equilibrium outcome. From equation A9, we can derive

$$\frac{d\tilde{w}(q)}{dq_{\min}} = \frac{-1}{f(\tilde{w}(q))} \frac{\overline{q} - q}{(\overline{q} - q_{\min})^2} \leq 0. \tag{17}$$

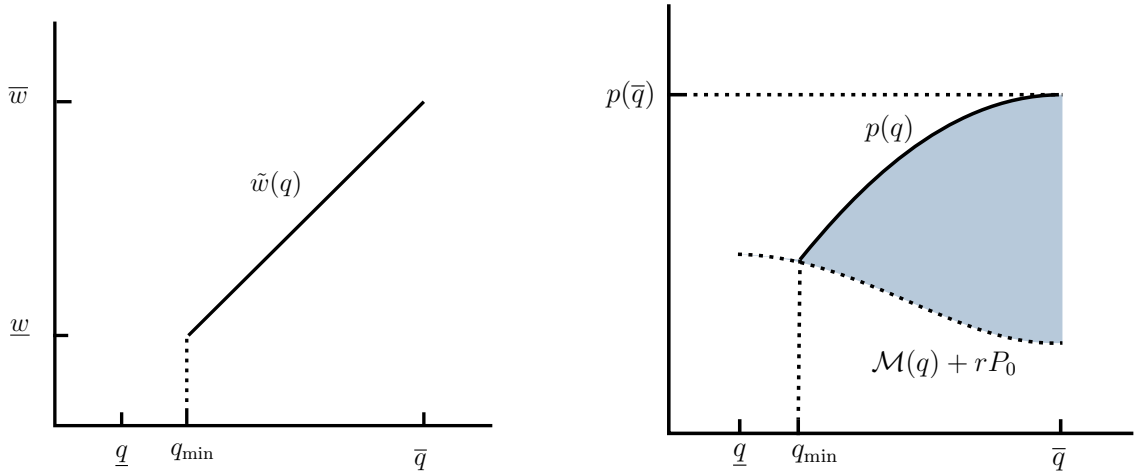
An economy with a higher quality cut-off $q_{\min}$ concentrates households at the top of the quality ladder, with lower-income households assigned to housing of every quality. In response to a decrease in $q_{\min}$, caused by landlords further maintaining housing units at the bottom of the housing ladder, richer households are re-assigned to units, with the extent of re-assignment depending inversely on the local density of the income distribution $f(\tilde{w}(q))$.

We illustrate the equilibrium of the economy in Figure 6. Figure 6a shows the assignment function. Consistent with positive assortative matching, higher income households are assigned to housing units of higher quality. The gradient of the assignment function depends on $q_{\min}$. Figure 6b shows the rental price schedule $p(q)$. The rental price is highest for new units of quality $\overline{q}$. Rental prices fall for older units towards the bottom of the housing ladder. In an equilibrium with $q_{\min} > \underline{q}$, the rental price schedule intersects the sum of the maintenance cost function, $\mathcal{M}(q)$, and of the flow opportunity cost of land, rP_0, at $q_{\min}$. As stated in proposition 1, these curves need not intersect when $q_{\min} = \underline{q}$. We illustrate the case $q_{\min} = \underline{q}$ in Figure A7. The shaded area in Figure 6b represents landlord flow profits net of the opportunity cost of land. In equilibrium, properly discounted flow profits net of the opportunity cost of land must exactly compensate landlords for building costs κ.

4 The Propagation of Changes in Building Costs

How do changes in building costs propagate down the housing ladder? To shed light on this question, we study the comparative statics of our model's steady state equilibrium with respect to building costs. We rationalize all three of our empirical facts from Section 2 and

Figure 6: Equilibrium Characterization.



(a) $\tilde{w}(q)$ **(b)** $p(q)$

interpret them through the lens of the model. Lastly, we discuss differences and similarities between the effect of changes in building costs and the effect of other changes to the economy's fundamentals.

We view a change in building costs as a change in κ, holding the maintenance cost function $\mathcal{M}(q)$ fixed. While building and maintenance have cost components in common, other cost components are unique to building. This is the case for costs induced by terrain ruggedness, which we used as exogenous variation in building costs to document our empirical facts in Section 2. This is also the case for costs induced by permitting times. Therefore, we hold the maintenance cost function fixed to be consistent with our empirical facts and to be consistent with policies of interest attempting to accelerate permitting processes. The model then rationalizes our third empirical fact, with increases in building costs leading to substitution from construction to maintenance. In Section 4.5, we discuss the effect of changes in the maintenance cost function.

4.1 Changes in Building Costs

We can characterize the effect of changes in building costs on the equilibrium of the economy as follows.

Lemma 1. *Comparative statics with respect to building costs κ satisfy*

$$1 = \frac{1}{\delta} \int_{q_{min}}^{\overline{q}} \exp\left(-r \frac{\overline{q} - q}{\delta}\right) \frac{dp(q)}{d\kappa} dq, \tag{18}$$

$$\frac{dp(q)}{d\kappa} = \frac{\overline{q}}{q} \frac{dp(\overline{q})}{d\kappa} - \frac{1}{q} \int_{q}^{\overline{q}} \frac{d\tilde{w}(x)}{d\kappa} dx, \tag{19}$$

$$\frac{d\tilde{w}(q)}{d\kappa} = \frac{-1}{f(\tilde{w}(q))} \frac{\overline{q} - q}{(\overline{q} - q_{min})^2} \frac{dq_{min}}{d\kappa}, \tag{20}$$

and, whenever $q_{min} > \underline{q}$,

$$0 = \frac{dp(q_{min})}{d\kappa} + \left(p'(q_{min}) - \mathcal{M}'(q_{min})\right) \frac{dq_{min}}{d\kappa}. \tag{21}$$

Proof. See appendix D.3. □

In equilibrium, there must be new construction in the economy to sustain the housing stock. For new construction to remain profitable for landlords after an increase in building costs, the increase in building costs must be offset by increases in rental prices in parts of the housing ladder. This is captured by equation 18, which discounts increases in rental prices based on the interest rate r and the inverse of the filtering rate of maintained units δ. Discounting implies that increases in rental prices at the bottom of the housing ladder have a *lower* effect on landlords' incentives to build new units than increases in rental prices at the top of the housing ladder. Indeed, forward-looking landlords building new housing units anticipate that increases in rental prices at the bottom of the housing ladder will generate higher flow profits only at a more distant time horizon.

Rental prices must increase in parts of the housing ladder to compensate landlords for higher building costs, but they must do so in a way consistent with housing market clearing. Equation 19 imposes housing market clearing by connecting changes in rental prices at any point along the housing ladder to changes in rental prices at the top of the housing ladder. This expression is simple when changes in building costs leave the distribution of housing supply unchanged so that there is no re-assignment of households to housing units ($d\tilde{w}(x)/d\kappa = 0$). However, this is not the case in general in our economy with an endogenous supply side.

Equation 20 characterizes re-assignment of households to housing units. A given quality unit is re-assigned to a lower-income household if a change in building costs induces a decrease in maintenance at the bottom of the housing ladder, reducing the stock of old, low quality housing units. The opposite happens if a change in building costs induces an increase in

maintenance. Households assigned to the highest level of quality, with income $\tilde{w}(\overline{q}) = \overline{w}$, are never re-assigned. This is because the highest-income households rent the new housing units that must always be built for the economy to sustain its housing stock.

Finally, equation 21 characterizes the response of the cut-off $q_{\min}$ to changes in building costs. This response depends on changes in the rental price at the bottom of the housing ladder as well as on the gradient of both the rental price schedule and the maintenance cost function at the bottom of the housing ladder. Because the rental price schedule is increasing and the maintenance cost schedule is non-increasing, we see that increases in prices at the bottom of the housing ladder lower the quality cut-off $q_{\min}$ at which landlords decide to tear-down housing units.

From Lemma 1, we can derive the following result.

Corollary 1. *Comparative statics with respect to building costs κ satisfy*

$$\frac{dp(q)}{d\kappa} > 0, \tag{22}$$

$$\frac{dq_{min}}{d\kappa} \leq 0. \tag{23}$$

Proof. See appendix D.4. □

Corollary 1 states that increases in building costs lead to increases in rental prices in every part of the housing ladder. In particular, increases in building costs increase the rental price of housing units of quality $q_{\min}$ at the bottom of the housing ladder. This leads to further maintenance of those housing units by landlords, lowering the cut-off $q_{\min}$ where landlords tear-down housing units. The flip side of this observation is that *decreases* in building costs lead landlords to tear-down old housing units at the bottom of the housing ladder. Housing activists have raised this concern with respect to supply-side policies (Been et al. 2025). The concern is valid: as we discuss in Section 4.2, landlords tearing-down old housing units which were previously maintained limits the extent to which decreases in building costs can lower rental prices at the bottom of the housing ladder.

4.2 Fact 1: Amplification

In Section 2.1, we observed that housing markets with higher building costs feature disproportionately higher rental prices for old housing units. We referred to this result as amplification. Specifically, we documented the following pattern in the data:

$$\frac{d}{d\text{age}}\left(\frac{d\log p(\text{age})}{d\log\kappa}\right) > 0. \tag{24}$$

In our model, housing age determines the quality of maintained housing units through the negative and strictly monotonic relationship

$$q = \overline{q} - \delta \cdot \text{age}. \tag{25}$$

Hence, the model rationalizes Fact 1 if

$$\frac{d}{dq}\left(\frac{d\log p(q)}{d\log \kappa}\right) < 0, \tag{26}$$

meaning that increases in building costs disproportionately raise the prices of lower quality units. To understand how amplification can occur in our model, it is useful to re-write equation 19 as

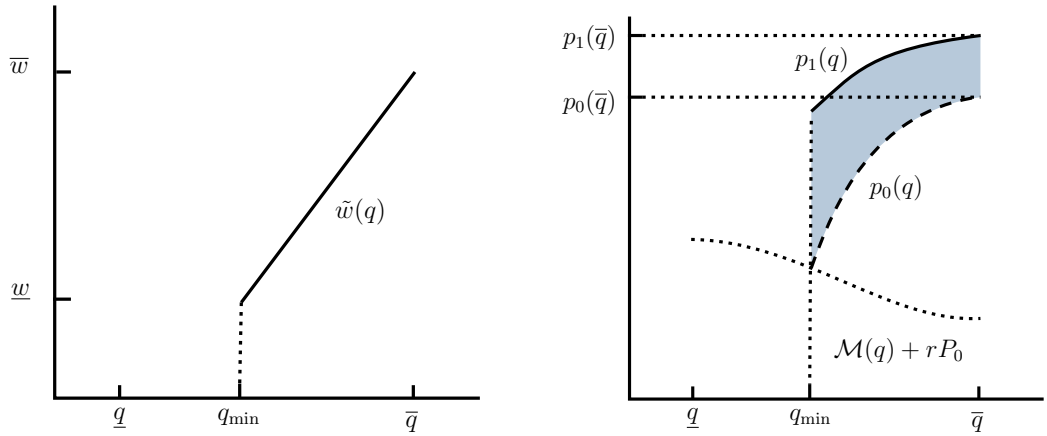
$$\frac{dp(q)}{d\kappa} = \underbrace{\frac{\overline{q}}{q}}_{\text{Affordability}} \overbrace{\frac{dp(\overline{q})}{d\kappa}}^{\text{Substitutes}} - \underbrace{\frac{1}{q}\left(\int_q^{\overline{q}} \frac{-dw(x)}{dq_{\min}}dx\right)\frac{-dq_{\min}}{d\kappa}}_{\text{Maintenance}}. \tag{27}$$

First, we consider the effect of an increase in building costs on rental prices while holding the quality cut-off $q_{\min}$ constant. This eliminates the second term in equation 27, allowing us to focus on the first. The coefficient in front of $dp(\overline{q})/d\kappa$ is positive: rental price increases along the housing ladder are positively related to the rental price increase of new housing units. This is because old housing units are substitutes for new housing units. Hence, when the rental price of new units increases to compensate the increase in building costs, demand for old units increases. This leads to increases in rental prices of old units. We refer to this as the *substitutes* effect. By how much do rental prices of old units increase? An intuitive view is as follows: because old units are differentiated in quality from new units and hence only imperfect substitutes, their prices should increase by less than those of new units in response to an increase in building costs. This logic fails in our model because of the *affordability* effect. Lemma 1 establishes that increases in building costs raise rental prices all along the housing ladder. The resulting decline in housing affordability lowers each households' consumption, causing each households' marginal rate of substitution of housing quality for consumption to decline. This pushes housing demand down the housing ladder toward more affordable housing of lower quality, increasing the demand for old units relative to the demand for new units. However, filtering pins down the supply of old units relative to the supply of new units. Indeed, in equilibrium, there is exactly one unit of quality $\overline{q} - \delta$ for every unit of quality $\overline{q}$ built one unit of time ago. For housing markets to clear, rental prices

of old units at the bottom of the housing ladder must therefore increase by *more* than those of new units, consistent with amplification. The affordability effect is visible in equation 27, where the coefficient in front of $dp(\overline{q})d\kappa$ is $\overline{q}/q$, which is decreasing in quality and therefore highest for old units at the bottom of the housing ladder.

We represent the affordability effect graphically in Figure 7, where we consider an increase in building costs from κ_0 to $\kappa_1 > \kappa_0$. Holding $q_{\min}$ fixed, there is no re-assignment of households in response to the increase in building costs, as evident from Figure 7a. To compensate landlords for the higher cost of building new housing units, rental prices must go up. The shaded area in Figure 7b represents increases in rental prices along the housing ladder, which must equal $\kappa_1 - \kappa_0$ once properly discounted. In which part of the housing ladder do rental prices increase the most? Higher rental prices lower affordability in the economy, making households prefer lower quality units. Housing market clearing then requires prices to go up by more at the bottom of the housing ladder, as shown in figure 7b. The figure also shows that with higher rental prices at the bottom of the housing ladder, the rental price schedule no longer intercepts the sum of the maintenance cost curve and of the flow opportunity cost of land, $\mathcal{M}(q) + rP_0$, at $q_{\min}$. In equilibrium, $q_{\min}$ must therefore adjust.

Figure 7: Affordability Effect.



(a) $\tilde{w}(q)$ **(b)** $p(q)$

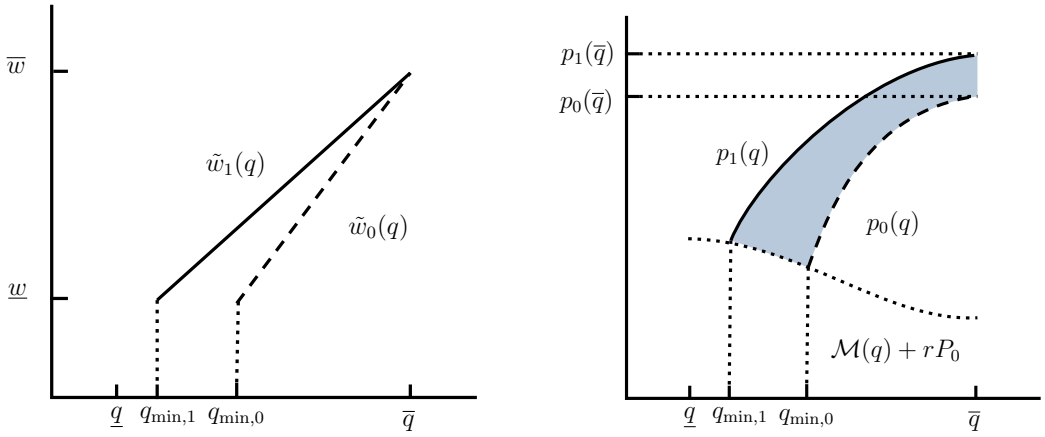
Note: Effect of an increase in building costs from κ_0 to $\kappa_1 > \kappa_0$, holding $q_{\min}$ fixed.

Second, we consider the effect on rental prices of the adjustment of the quality cut-off $q_{\min}$. From lemma 1, we know that $q_{\min}$ shifts to the left, with landlords maintaining older housing units. This decreases pressure on rental prices by allowing households to move into more affordable housing units. We refer to this as the *maintenance effect*. In which part of the housing ladder is the maintenance effect largest? The integral in the second term of equation 27 shows that it is zero at the top of the housing ladder, negative everywhere else,

and builds up toward the bottom of the housing ladder. Intuitively, allowing households to move into more affordable housing units increases households' marginal rate of substitution of housing quality for consumption, steepening the rental price schedule $p(q)$. Steepening the rental price schedule leads to large decreases in rental prices at the bottom of the housing ladder.

We represent the maintenance effect graphically in Figure 8. Figure 8a shows how the maintenance effect leads to a re-assignment of households to lower quality units by shifting the quality cut-off $q_{\min}$ to the left. Figure 8b shows how this spreads the shaded area required to offset the building cost increase to the left and decreases the pressure on rental prices at the bottom of the housing ladder.

Figure 8: Maintenance Effect.



(a) $\tilde{w}(q)$ **(b)** $p(q)$

It is useful to contrast our results with partial equilibrium results on price changes induced by exogenous changes in new construction at the top of the housing ladder (Mense 2025; Nathanson 2025). In a partial equilibrium where housing supply is taken as given, a perturbation of housing supply at the top of the housing ladder can generate decreases in prices at the bottom of the housing ladder through moving chains that re-assign households to housing units. In our model, the equilibrium response of rental prices at the bottom of the housing ladder to a change in building costs does not rely on moving chains. In fact, the affordability effect, which generates large changes in prices at the bottom of the housing ladder, occurs absent any re-assignment of households to housing units. The maintenance effect, which does generate re-assignment, dampens the extent to which changes in building costs propagate down the housing ladder.

When does amplification occur in our model? Our discussion above suggests that increases in building costs lead to larger increases in prices at the bottom than at the top of

the housing ladder when the affordability effect dominates the maintenance effect. We derive the following corollary[10].

Corollary 2 (Amplification)**.** *Comparative statics with respect to building costs κ satisfy*

$$\frac{d}{dq}\left(\frac{d\log p(q)}{d\log \kappa}\right) < 0 \iff \frac{d}{d\kappa}\left(\frac{p(q)}{\tilde{w}(q)}\right) > 0. \tag{28}$$

A sufficient condition on parameters for

$$\frac{d}{dq}\left(\frac{d\log p(q)}{d\log \kappa}\right) < 0 \tag{29}$$

to hold globally is

$$\left(\frac{\kappa\delta}{\overline{q}-\underline{q}} - \left(\overline{\mathcal{M}} + rP_0\right)\right)\left(|\overline{\mathcal{M}}'|\underline{q} + 2\left(\underline{w} - \left(\overline{\mathcal{M}} + rP_0\right)\right)\right) > \frac{1}{\underline{f}}\frac{\overline{q}}{\underline{q}}\overline{w}. \tag{30}$$

Proof. See appendix D.5. □

The first part of Corollary 2 connects amplification to the rent-to-income ratio. Amplification occurs in our model if and only if increases in building costs increase the rent-to-income ratio of households at every quality q. We have discussed how the affordability effect amplifies rental price responses to changes in building costs toward the bottom of the housing ladder and how the maintenance effect dampens those rental price responses by re-assigning households to older housing units. For a given quality q, the higher its rental price ($p(q)$) increase, the higher the affordability effect it generates for housing units with quality $q' < q$. For the same quality q, the more it is re-assigned to households of income $w' > \tilde{w}(q)$, the higher the maintenance effect it generates for housing units with quality $q' < q$. Hence, the rent-to-income ratio captures the relative strength of the affordability and maintenance effects along the housing ladder.

Exploiting the connection between amplification and the rent-to-income ratio, we derive a sufficient condition on model parameters which guarantees that amplification holds globally along the housing ladder. The LHS of equation 30 is strictly increasing in building costs κ, which don't appear on the RHS. When building costs are high, housing is expensive in the economy and households have a low marginal rate of substitution of housing quality for consumption. Further increases in building costs then lead to a large affordability effect.

[10]Variables which appear in Corollary 2 with an upper-bar or under-bar respectively represent maximums and minimums. In particular, $\overline{M}$ is the maximum of the maintenance cost function, $\overline{\mathcal{M}}'$ is the maximum of it's derivative, and $\underline{f}$ is the minimum of the density $f(\cdot)$.

High building costs κ can therefore induce amplification. The LHS of equation 30 is also strictly increasing in $|\overline{\mathcal{M}}'|$. When the gradient of the maintenance cost function is high, it is expensive for landlords to further maintain housing units. This limits potential re-assignment of households to older housing units in response to an increase in building costs. This in turn limits the maintenance effect and can therefore inducing amplification. The sufficient condition in equation 30 imposes stringent requirements on additional model parameters such as the minimum of the income distribution density $\underline{f}$. Our quantitative model in Section 5 reveals the condition is by no means necessary, with amplification typically occurring even when it is violated.

A special case of amplification occurs when $q_{\min} = \underline{q}$ and

$$p(\underline{q}) - \mathcal{M}(\underline{q}) > rP_0, \tag{31}$$

so that landlords are not marginally indifferent to changes in the quality cut-off $q_{\min}$. Marginal changes in building costs then leave $q_{\min}$ unchanged, shutting off the maintenance effect. As a result, the affordability effect trivially dominates, with increases in building costs leading to large increases in rental prices at the bottom of the housing ladder. This special case occurs when $\underline{q}$ is high, potentially reflecting a binding minimal quality occupancy standard. We provide a graphical illustration in Figure A8.

Due to the affordability effect, our model is able to rationalize our first empirical fact on amplification. We turn to our second empirical fact.

4.3 Fact 2: Assignment

In Section 2.2, we observed that housing markets with higher building costs feature old units occupied by relatively higher income households. Specifically, we documented the following pattern in the data:

$$\frac{d}{d\text{age}}\left(\frac{d\log\tilde{w}(\text{age})}{d\log\kappa}\right) > 0. \tag{32}$$

Hence, the model must satisfy

$$\frac{d}{dq}\left(\frac{d\log\tilde{w}(q)}{d\log\kappa}\right) \leq 0 \tag{33}$$

to rationalize Fact 2. We derive the following corollary.

Corollary 3 (Assignment)**.** *Comparative statics with respect to building costs κ satisfy*

$$\frac{d\tilde{w}(\overline{q})}{d\kappa} = 0, \quad \frac{d\tilde{w}(q)}{d\kappa} \geq 0 \quad \forall q < \overline{q}. \tag{34}$$

A sufficient condition on parameters for

$$\frac{d}{d\kappa}\left(\frac{d\log\tilde{w}(q)}{dq}\right) \leq 0 \tag{35}$$

to hold globally is

$$\frac{d}{dw} w f(w) \geq 0 \quad \forall w. \tag{36}$$

Proof. See appendix D.6. □

The first part of Corollary 3 states that increases in building costs lead to a re-assignment of households to lower quality units, except at the top of the housing ladder, where the richest households always rent new housing units. Intuitively, higher building costs then stretch out and flatten the assignment function $\tilde{w}(q)$, consistent with the change in the assignment pattern we documented with Fact 2. This is clear graphically from Figure 8a.

The second part of the corollary provides a sufficient condition which guarantees that the assignment function is flattened globally, implying that changes in building costs induce the largest re-assignment of households to housing units at the bottom of the housing ladder. The sufficient condition in equation 36 is satisfied when the density of the income distribution ($f(w)$) decreases at most at a rate $1/w$. This is trivially satisfied by any income distribution with a non-decreasing density over the support $[\underline{w}, \overline{w}]$, such as a uniform distribution. The condition is violated by other distributions such as Pareto distributions. While Pareto distributions are known to accurately portray income distributions, they do so by capturing their fat right tail, which corresponds to very high-income households unlikely to occupy housing units whose size and location is identical to those occupied by low-income households. Instead, distributions satisfying the condition in equation 36 – distributions with high density at the top of the distribution – may better portray households in rental markets for housing unit sizes and locations typically occupied by low-income households. Our quantitative model in Section 5 reveals that the condition is not necessary[11].

Our model hence is able to rationalize our second empirical fact on household assignment to housing units. We turn to our third empirical fact.

[11] In the quantitative model, we parametrize the income distribution with a log-normal distribution.

4.4 Fact 3: Construction and Maintenance

In Section 2.3, we observed that housing markets with higher building costs shift from providing housing services through the construction of new housing units to providing housing services through the maintenance of existing housing units. We now study how the aggregate construction rate C and aggregate maintenance expenditure M respond to changes in building costs in our model. Aggregate maintenance expenditure is given by

$$M = \int_{\underline{w}}^{\overline{w}} \mathcal{M}(\tilde{q}(w))dF(w) \tag{37}$$

and can be expressed, using a change of variables and the positive assortative matching rule derived in Proposition 2, as

$$M = \frac{\int_{q_{\min}}^{\overline{q}} \mathcal{M}(q)dq}{\overline{q} - q_{\min}}. \tag{38}$$

The RHS of equation 38 simply corresponds to average maintenance expenditure for housing units in the economy. We derive the following corollary.

Corollary 4 (Construction and Maintenance)**.** *Comparative statics with respect to building costs κ satisfy*

$$\frac{dC}{d\kappa} \leq 0, \tag{39}$$

$$\frac{dM}{d\kappa} \geq 0. \tag{40}$$

Proof. See appendix D.7. □

In Lemma 1, we established that increases in building costs lower the quality cut-off $q_{\min}$ at which housing units are torn-down. This expands the housing stock in the economy, reducing the need for construction of new housing units. This in turns lowers the aggregate construction rate C. However, maintaining old housing units for longer is costly for landlords. Maintenance expenditure is non-increasing in quality, with old housing units requiring more maintenance than new housing units. As a result, the decrease in $q_{\min}$ resulting from an increase in building costs increases maintenance expenditure M in the economy. Hence, our model rationalizes our third empirical fact on construction and maintenance.

4.5 Further Results

We now discuss further results from the model.

Changes in Land Value. So far, we have studied the effect of changes in building costs but have held land value (P_0) fixed. How do changes in land value propagate down the housing ladder? This question is relevant for policy. Indeed, minimum lot size requirements mechanically increase the cost of land required to build one housing unit. Other forms of land-use regulation have also been shown to significantly affect land value (Glaeser and Gyourko 2002). We derive the following proposition.

Proposition 3. *Comparative statics with respect to land value P_0 satisfy*

$$\frac{dp(q)}{dP_0} > 0 \tag{41}$$

and

$$\frac{dq_{min}}{d\kappa} \leq \frac{dq_{min}}{dP_0} \leq 0. \tag{42}$$

Furthermore,

$$\frac{d}{dq}\left(\frac{d\log p(q)}{d\log \kappa}\right) < 0 \implies \frac{d}{dq}\left(\frac{d\log p(q)}{d\log P_0}\right) < 0. \tag{43}$$

Proof. See appendix D.8. □

The first part of the proposition establishes that similar to building costs, increases in land value raise rental prices in every part of the housing ladder. Furthermore, while increases in land value increase maintenance and lower the quality cut-off $q_{\min}$, they do so by less than increases in building costs. The intuition for this result is that increases in land value increase the opportunity cost of landlords using a land unit for housing rental. This lowers landlords' incentive to maintain housing units at the bottom of the housing ladder, which generate small flow profits due to low rental prices and high maintenance costs. This dampens the maintenance effect resulting from changes in land value, implying the second part of the proposition: when the affordability effect dominates the maintenance effect and amplification occurs in response to changes in building costs, then it also does in response to changes in land values. This result suggests that policies targeting minimum lot size requirements may be more effective than policies targeting building costs in lowering rental prices at the bottom of the housing ladder. Figure A9 illustrates the effect of a change in land value graphically.

Minimal Quality Occupancy Standard. We derive the following result summarizing the effect of changes in a minimal quality occupancy standard (Braid 1984) in our model.

Proposition 4 (Minimal Quality Occupancy Standard)**.** *Comparative statics with respect to a minimal quality occupancy standard* $\underline{q}$ *satisfy*

$$\frac{dp(q_{min})}{d\underline{q}} \geq 0, \quad \frac{d}{dq}\left(\frac{d\log p(q)}{d\underline{q}}\right)\Bigg|_{q=q_{min}} \leq 0 \tag{44}$$

with strict inequalities whenever $p(\underline{q}) - \mathcal{M}(\underline{q}) > rP_0$.

Proof. See appendix D.9. □

When $q_{\min} > \underline{q}$ a marginal change in the minimal quality occupancy standard has no effect as the standard is not binding. When $q_{\min} = \underline{q}$ and $p(\underline{q}) - \mathcal{M}(\underline{q}) > rP_0$, so that landlords would not tear-down housing units of quality $q_{\min}$ absent the standard, increasing the standard leads to large increases in prices at the bottom of the bottom of the housing ladder. This is because an increase in the standard generates an affordability effect and a maintenance effect which *both* lead to amplification. An increase in the standard causes landlords to raise rental prices along the housing ladder to compensate the loss in flow profits from previously profitable old units which must now be torn-down. This decreases housing affordability and generates an affordability effect: to clear markets, rental prices must increase by more at the bottom of the housing ladder. In response to an increase in building costs, the affordability effect was dampened by maintenance of old units and re-assignment of households to those units. An increase in the minimal quality occupancy standard instead forces households to be re-assigned to newer higher quality units, generating a maintenance effect which also increases prices the most at the bottom of the housing ladder[12]. This suggests that raising minimal quality occupancy standards can result in particularly large increases in rental prices at the bottom of the housing ladder.

Changes in Maintenance Costs. While studying the effect of changes in building costs in the model, we held maintenance costs fixed. We did so for two reasons. First, to be consistent with our empirical facts, which exploit terrain ruggedness – a variable unlikely to affect maintenance costs – to isolate exogenous variation in building costs. Second, because supply-side policies of interest such as California's AB 130 or New York's Let Them Build program, which target building costs through permitting times, are also unlikely to affect maintenance costs. However, there is substantial evidence that building costs in general co-move with maintenance costs (Gyourko and Saiz 2004). Figure A9 helps understand how

[12]Note that contrary to our other results, Proposition 4 establishes amplification not globally, but locally at the bottom of the housing ladder. Establishing amplification globally in response to changes in a minimal quality occupancy standard requires restrictions on the shape of the income distribution.

more amplification results from increases in building costs when they are combined with increases in maintenance costs.

Changes in the Income Distribution. Nikolakoudis (2025) studies a housing assignment model which holds housing supply fixed. A key feature of the model is that increases in income at the top of the income distribution do not trickle-down to affect housing prices at the bottom of the housing ladder. Our model with endogenous housing supply can provide new insights. Increases in income at the top of the distribution increases the demand for new housing units. New housing units eventually filter down the housing ladder, increasing supply at the bottom. Therefore, in addition to not increasing prices at the bottom of the housing ladder, increases in income at the top of the income distribution can contribute to lowering those prices through filtering. However, the extent to which prices fall at the bottom of the housing ladder depends on the changes in $q_{\min}$ induced by changes in the income distribution, in line with concerns about increases in income at the top of the income distribution leading to tear-downs at the bottom of the housing ladder (Been et al. 2025).

Owner-Occupied Prices. Our model does not feature owner-occupied housing. However, a natural starting point to study the effect of building costs on owner-occupied prices along the housing ladder is to consider their effect on housing value $P(q)$. In appendix E.1, we show how amplification of changes in rental prices toward the bottom of the housing ladder leads to similar amplification of changes in housing value. The intuition for this result is simply that the housing value of a unit of quality q contains an integral of discounted rental prices of units of qualities $q' \leq q$, as shown in equation 11.

Balanced Growth Paths. It is useful to ask how our model could be extended to feature population growth at a rate $g > 0$, or population decline at a rate $g < 0$. The housing supply CDF of an economy on a balanced population growth path is given by

$$H(q) = \frac{\exp\left(-g\frac{\overline{q}-q}{\delta}\right) - \exp\left(-g\frac{\overline{q}-q_{\min}}{\delta}\right)}{1 - \exp\left(-g\frac{\overline{q}-q_{\min}}{\delta}\right)}. \tag{45}$$

This distribution depends on the filtering rate δ and on the population growth rate g. Applying L'Hôpital's rule in the limit where $g \to 0$, we see how it collapses collapses to the uniform housing supply distribution implicit in our housing market clearing conditions from Section 3. Economies with growing populations must rely on construction to expand their housing supply and feature a relatively scarcer supply of older and lower quality units. This leads to high housing prices at the bottom of the housing ladder. The opposite is true for

economies with declining populations, whose supply of older and lower quality units is large, as in Glaeser and Gyourko (2005). Extending our results on the propagation of changes in building costs to cases where $g \neq 0$ remains work in progress.

5 A Quantitative Model

In this section, we study a quantitative housing equilibrium model with filtering. We extend our model from Section 3 to allow for construction on an interval of housing qualities and for an intensive margin for maintenance. We solve the quantitative model numerically to gain further insights into the propagation of changes in building costs and its welfare implications. We consider both comparative statics and transition dynamics. In what follows, we introduce time $t \geq 0$ subscripts.

5.1 Model Extensions

We relax two assumptions from Section 3 on the housing technology.

New Construction. We denote the rate of new construction of housing of quality q at time t by $C_t(q)$. The building cost $\mathcal{K}_t$ for a given quality q is given by the inverse supply function

$$\mathcal{K}_t(C_t(q), q) = \kappa C_t(q) + \gamma(q) C_t(q)^{1+\lambda}, \tag{46}$$

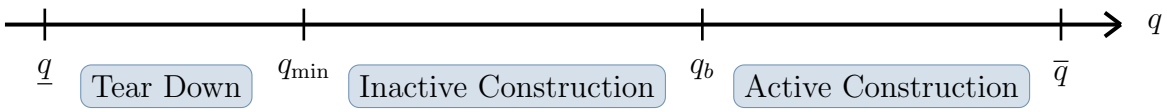
where $\kappa > 0$ and $\lambda \geq 0$ are fixed parameters and $\gamma(q)$ allows for higher building costs for higher quality housing units.

The parameter κ reflects the linear component of building costs. We think of costs induced by terrain ruggedness, our source of variation in building costs from Section 2, as entering construction costs linearly[13]. We also think of permitting times, a key housing supply-side policy outcome, as entering building costs linearly. Hence, in what follows, we again study the effect of changes in κ. The parameter λ introduces convexity in building costs and therefore generates construction of housing units of different quality. We again model new construction of housing as instantaneous and note that in the special case $\gamma(q) = 0$, the housing technology for new construction in our quantitative model collapses to that in Section 3. We denote the housing distribution at any time t by $H_t(q)$ and its density by $h_t(q)$.

[13]This is because terrain ruggedness affects the costs of land preparation and laying foundation.

An implication of the convex building costs $\mathcal{K}_t(C_t(q), q)$ is that the marginal building cost of the first housing unit built at any quality is κ. In an equilibrium with strictly increasing housing values, the housing ladder is then characterized by three regions, represented in Figure 9. The first region is the quality interval $[\underline{q}, q_{\min})$ where housing units are torn-down by landlords. The second region is the interval $[q_{\min}, q_b)$, where housing units are maintained by landlords, but there is no active construction, with marginal building costs equal to κ and *strictly* higher than housing values $P(q)$. This region can exist in an equilibrium of our model due to filtering: even if there are no housing units built at qualities below q_b, housing units built at higher qualities filter down the housing ladder into the second region. The quality cut-off q_b corresponds to the lowest quality at which there is new construction and satisfies $P(q_b) = \kappa$. The third region is the interval $[q_b, \overline{q}]$, where new construction is active and housing value is pinned down by building costs. With construction only at the top of the housing ladder, our model from Section 3 corresponds to the special case $q_b = \overline{q}$.

Figure 9: Housing Ladder.



Filtering and Maintenance. As in Section 3, the filtering rate is decreasing in maintenance spending and the maintenance spending required for a given rate of filtering is non-increasing in quality q. We again assume that improperly maintained units lose their quality instantly, generating an extensive margin for maintenance. We now further assume that maintained housing units gradually filter down the housing ladder at a rate which depends continuously on maintenance. Specifically, we now model the filtering rate $\mathcal{D}$ as satisfying

$$\mathcal{D}(m_t, q) = \begin{cases} \infty & \text{if } m_t < \mathcal{M}(q), \\ \delta e^{-\phi(m_t - \mathcal{M}(q))} & \text{if } m_t \geq \mathcal{M}(q). \end{cases} \tag{47}$$

The parameter $\phi \geq 0$ generates an intensive margin for maintenance. For maintained housing units with $m_t \geq \mathcal{M}(q)$, the filtering rate is decreasing in maintenance spending m_t. Furthermore, for $\phi > 0$,

$$\lim_{m \to \infty} \mathcal{D}(m, q) = 0 \tag{48}$$

and filtering can be brought to a halt with sufficient maintenance. As before, however, filtering cannot be reversed. In the special case where $\phi = 0$, the housing technology for maintenance in our quantitative model collapses to that in Section 3.

Dynamic Equilibrium. We characterize and define a dynamic equilibrium of the economy in Appendix F.

5.2 Parameterization

While a full calibration of model parameters remains work in progress, we now describe a preliminary yearly level parametrization under which we solve our quantitative model.

We normalize housing quality such that $\underline{q} = 1$ and $\overline{q} = 2$. We set the exogenous interest rate to $r = 0.05$ and the income distribution to a truncated log-normal distribution with a lower bound of 1, an upper bound of 10, a mean of 3, and a standard deviation of 0.5. We set $\lambda = 1$ for quadratic building costs. We set $\delta = 0.01$ so that a minimally maintained housing unit provides housing services for 100 years before reaching the minimal quality $\underline{q}$.

For functional forms, we again assume Cobb-Douglas flow utility, but given our normalization of housing quality, allow for non-equal exponents with $u(c, q) = c^{\alpha} q^{1-\alpha}$. We set $\mathcal{M}(q) = m_0(\overline{q} - q)$ and set $\gamma(q) = \gamma$.

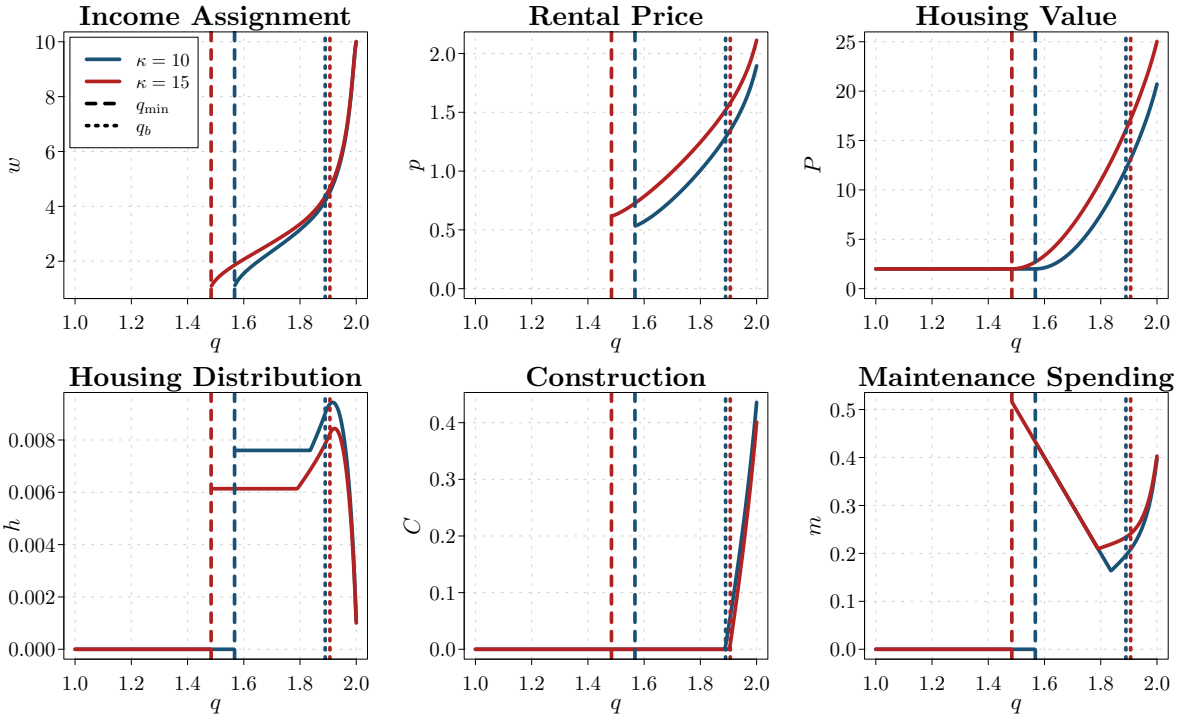
We set the values of remaining parameters to generate plausible values for key moments of the model. We use the rate of new construction in the economy and the quality percentile of the housing stock at which median new construction occurs to inform the building cost parameters κ and γ. We use total spending on maintenance relative to total value of the housing stock and the average lifespan of housing units to inform parameters of the maintenance technology m_0 and ϕ. Finally, we use the median rent-to-income ratio to inform α and the share of land in housing value to inform P_0. We report resulting parameters and values for key moments of the model in Table 2. Table A4 in Appendix F.8 reports values for further moments of the model.

Table 2: Parameterization

Parameter	Value	Moment	Model Value
κ	10.0	Construction Rate	0.021
γ	10.0	Median Construction Pctile. in q Dist.	97.6
m_0	1.0	Maintenance–to–Value	0.038
ϕ	2.0	Average Lifespan	48.15
α	0.3	Median Rent–to–Income	0.43
P_0	2.0	Land Share of Value	0.275

Figure 10 presents in blue our baseline steady state equilibrium. The horizontal axis represents quality, with the dotted line corresponding to the lowest quality with active construction (q_b) and the dashed line representing the quality cut-off at which housing units are torn-down ($q_{\min}$). The equilibrium features positive assortative matching, with higher income households assigned to higher quality housing units. Both rental prices and housing value are strictly increasing above $q_{\min}$. Housing value is flat below $q_{\min}$, reflecting the outside option value of land. The housing distribution features a hump-shape, with few very high-quality units but many high-quality housing units. Construction is concentrated at the top of the housing ladder. Landlords slow down the filtering rate of high-quality units through maintenance, causing the hump-shaped housing distribution.

Figure 10: Equilibrium and Comparative Statics.



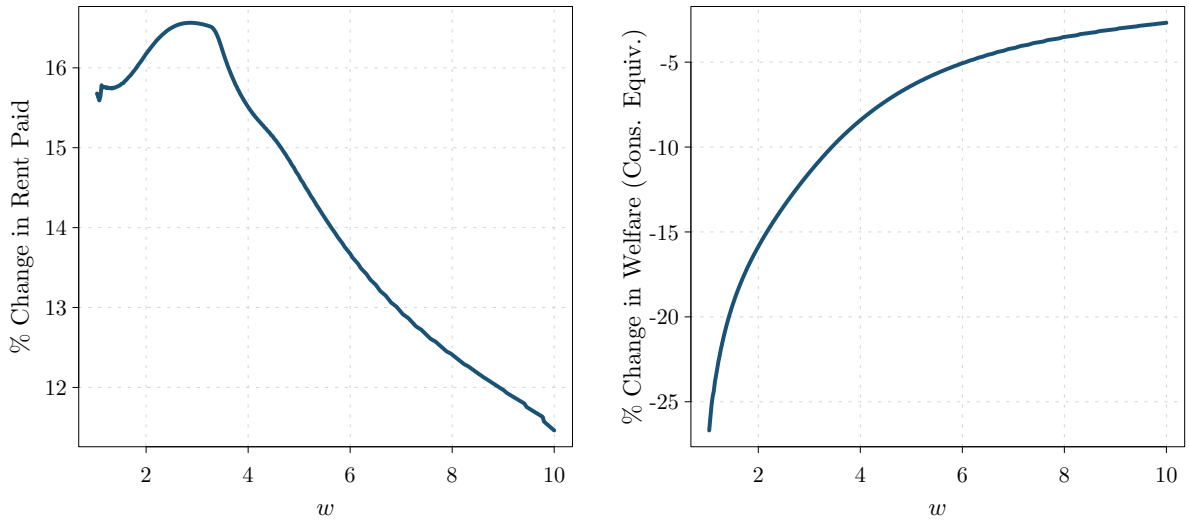
5.3 Changes in Building Costs: Comparative Statics

We now study the comparative statics induced by an increase in the linear component of building costs κ. Starting from our baseline steady state equilibrium, we consider a 50% increase in κ. We show key values of the resulting steady state equilibrium in red in Figure 10. We see that $q_{\min}$ shifts to the left and that q_b shifts to the right in the new equilibrium. Facing higher building costs, the economy substitutes from providing housing services with new construction to providing housing services with maintenance, as can be seen from lower

construction and higher maintenance spending in all parts of the housing ladder, consistent with our third empirical fact on construction and maintenance. This increases the mass of housing at the bottom of the housing ladder and decreases the mass of housing at the top of the housing ladder. Households are re-assigned to older housing units, with the largest re-assignment occurring at the bottom of the housing ladder, consistent with our second empirical fact on assignment. The resulting maintenance effect is not large enough to dominate the affordability effect: consistent with our first empirical fact on amplification, increases in rental prices are largest not for new housing units in the segment of the housing ladder where construction is active, but rather for old housing units at the bottom of the housing ladder where construction is inactive.

Our quantitative model allows us to study implications for housing affordability and welfare along the income distribution. As shown in Figure 11, all households face higher rental prices as a result from the increase in building costs. For the most part, increases in rent paid are larger for low-income households. This is due to positive assortative matching and amplification: low-income households occupy the housing units at the bottom of the housing ladder whose rental prices increase the most. At the very bottom of the income distribution, this pattern is reversed. This is because the lowest-income households in the economy undergo the largest re-assignment to lower quality units. Accounting for both increases in rent paid and re-assignment to lower quality units, we see that the consumption equivalent welfare loss from the increase in building costs is largest for low-income households.

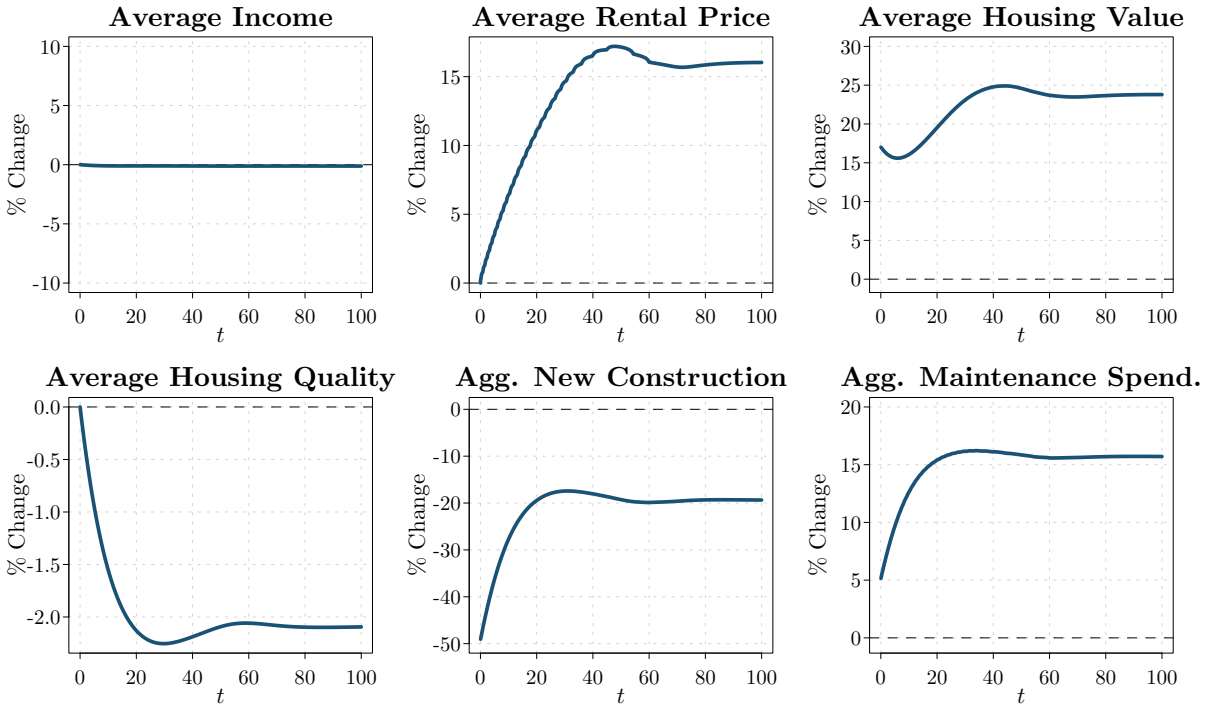
Figure 11: Comparative Statics Along the Income Distribution.



5.4 Changes in Building Costs: Transition Dynamics

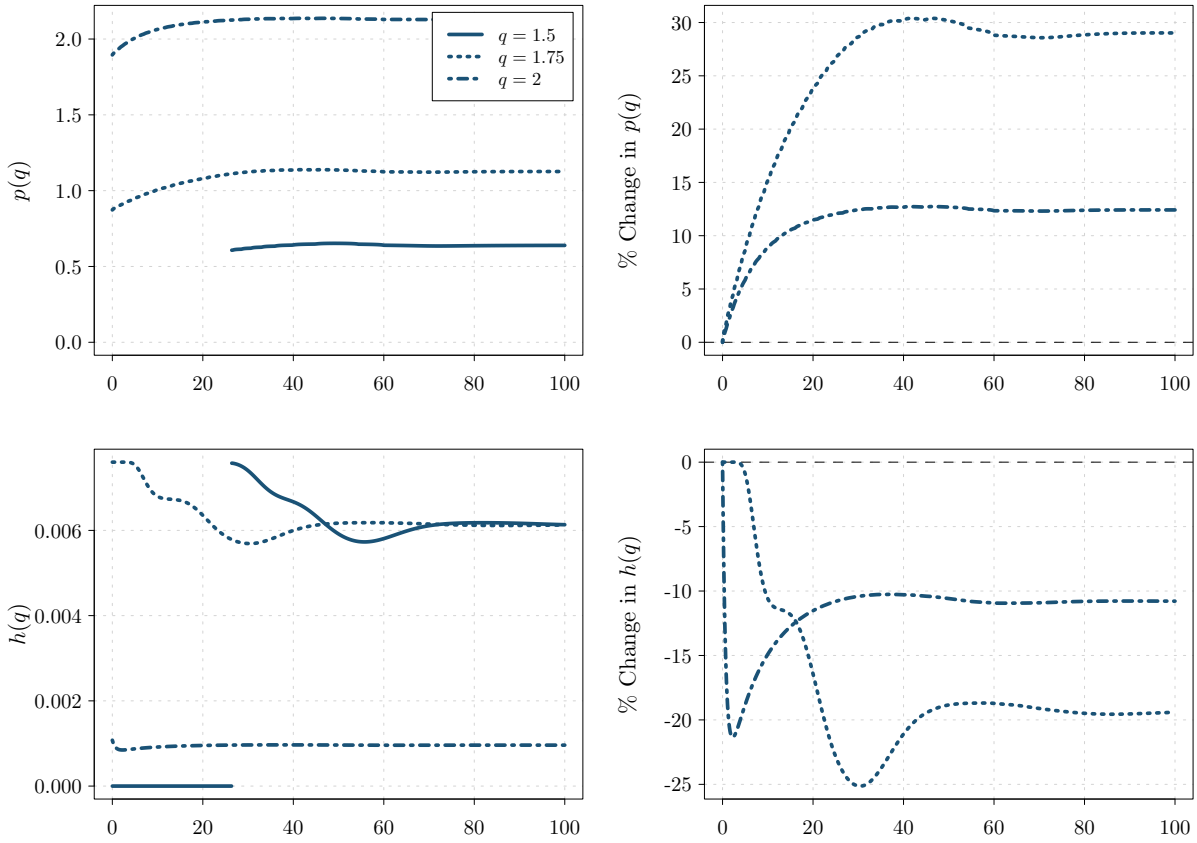
We now study the transition dynamics induced by an unanticipated and permanent 50% increase in the linear component of building costs κ. In Figure 12, we show the resulting aggregate transition dynamics. The combination of housing durability and of maintenance acting as a substitute to new construction in the provision of housing services make our model economy initially resilient to the shock. Indeed, the economy first responds to the shock by slowing down new construction and maintaining the pre-shock housing stock. As a result, rental prices do not jump on impact. Over time, the housing units built before the shock inevitably filter down the housing ladder and the economy must increase its rate of new construction. Rental prices then gradually increase. In anticipation of higher future rental prices, housing value jumps on impact of the shock.

Figure 12: Transition Dynamics.



In Figure 13, we show how transition dynamics differ along the housing ladder. Rental prices increase gradually in all parts of the housing ladder. At first, relative increases in rental prices at the bottom and at the top of the housing ladder track each other. This is because the economy's ability to initially absorb the shock by slowing down new construction and maintaining the pre-shock housing stock effectively *delays* the affordability effect. It is only after several decades, when the economy must begin building housing units at a high rate again, that the affordability effect achieves its full force. This then leads to much larger

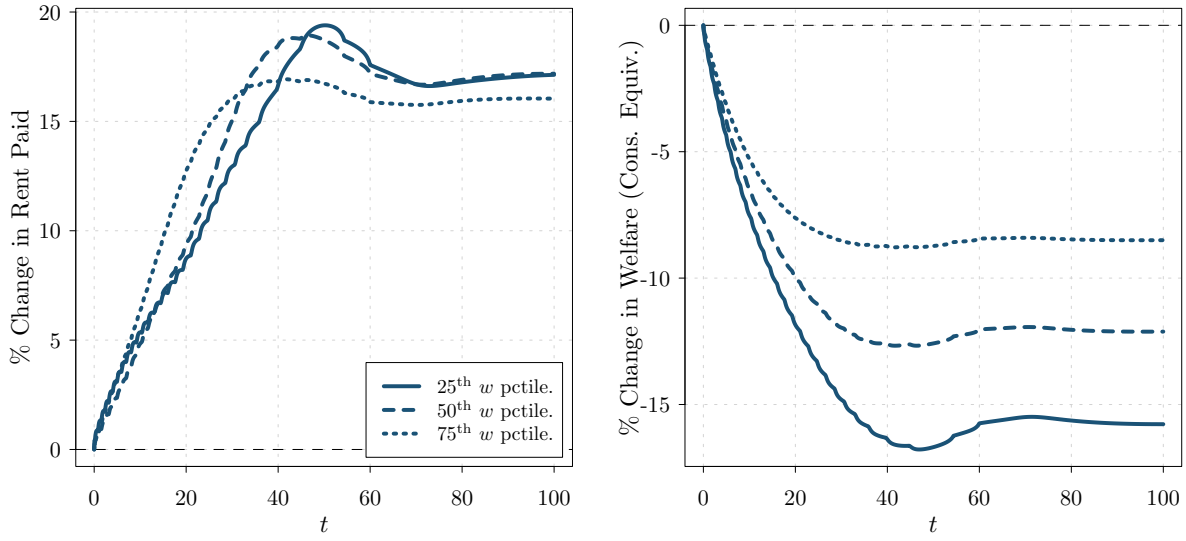
Figure 13: Transition Dynamics Along the Housing Ladder.



increases in rental prices at the bottom of the housing ladder. We also see that while the quality segment $q = 1.5$ was inactive before the shock, it becomes active roughly 25 years after the shock, with landlords having maintained housing units further down the housing ladder. Instead, in higher quality segments, the mass of housing goes down along the transition, due to lower new construction and extended maintenance increasing the relative supply of lower quality units.

In Figure 14, we show how transition dynamics differ along the income distribution. Because of the delayed affordability effect, higher increases in rent paid by households at the bottom of the distribution occur only after several decades. While the consumption equivalent welfare loss induced by the increase in building costs is largest for households at the bottom of the income distribution throughout the transition, the welfare loss gap widens after several decades, when the affordability effect achieves its full force.

Figure 14: Transition Dynamics Along the Income Distribution.



6 Conclusion

Recent policy initiatives in the US have embraced a supply-side approach to tackling housing affordability. To what extent can supply-side policies which lower building costs improve affordability at the bottom of the housing ladder?

In this paper, we document three empirical facts connecting building costs to outcomes along the housing ladder. Housing markets with higher building costs feature (i) disproportionately higher rental prices for old units, (ii) old units occupied by relatively higher income households, and (iii) less new construction as well as more maintenance of the housing stock. We rationalize all three facts in a housing equilibrium model with filtering. In the model, as building costs increase, rental prices increase in every part of the housing ladder. The magnitude of the increase in rental prices of older, lower quality housing units depends on the size of two opposing effects: the affordability effect and the maintenance effect. When the affordability effect dominates, the largest increases in rental prices occur at the bottom of the housing ladder.

We consider a quantitative version of the model with richer features. Numerical results are consistent with our empirical facts. We find that changes in building costs affect the welfare of households at the bottom of the income distribution the most. Studying transition dynamics, we find that the economy is resilient to changes in building costs in the short run. Resilience delays the affordability effect, and therefore postpones large welfare changes for households at the bottom of the income distribution.

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A Figures

Figure A1: Relationship between time to permit and building costs.

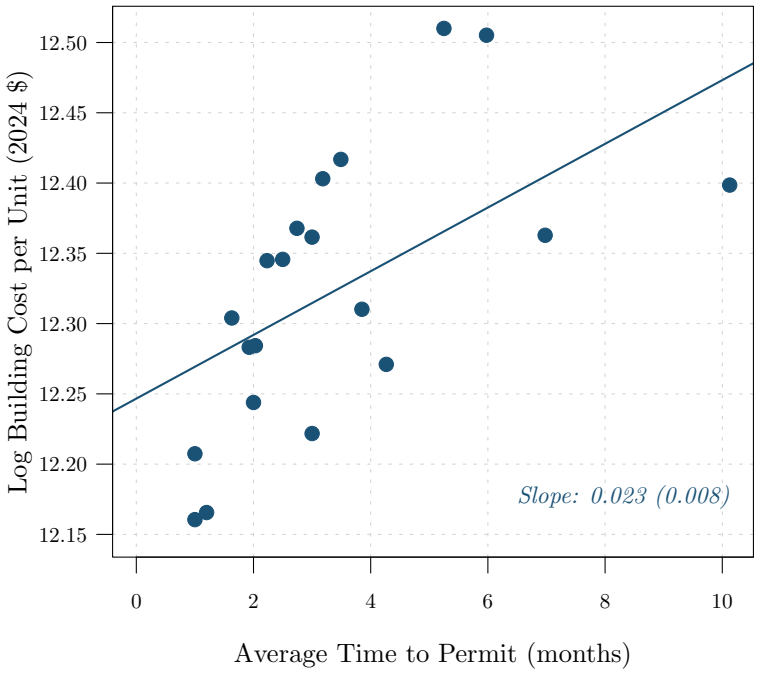


Figure A2: Relationship between rental price and age for fast and slow to issue permit housing markets.

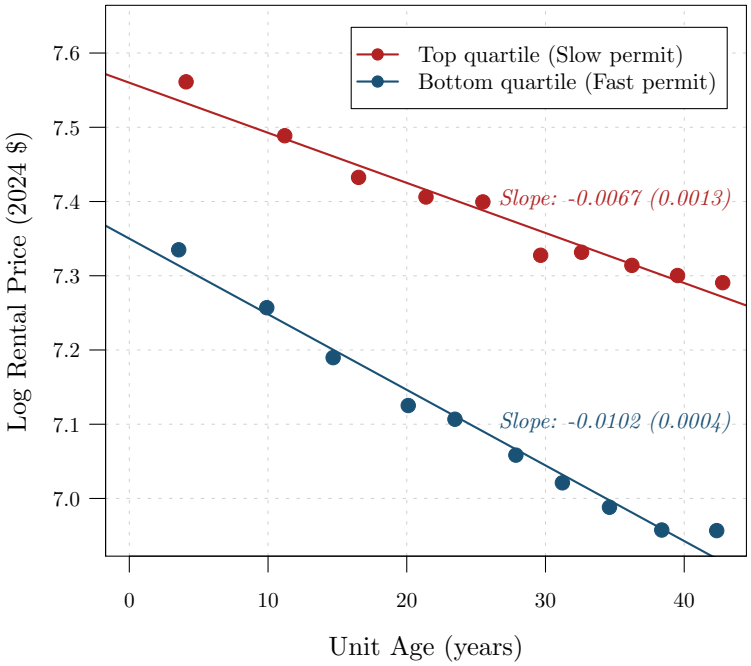
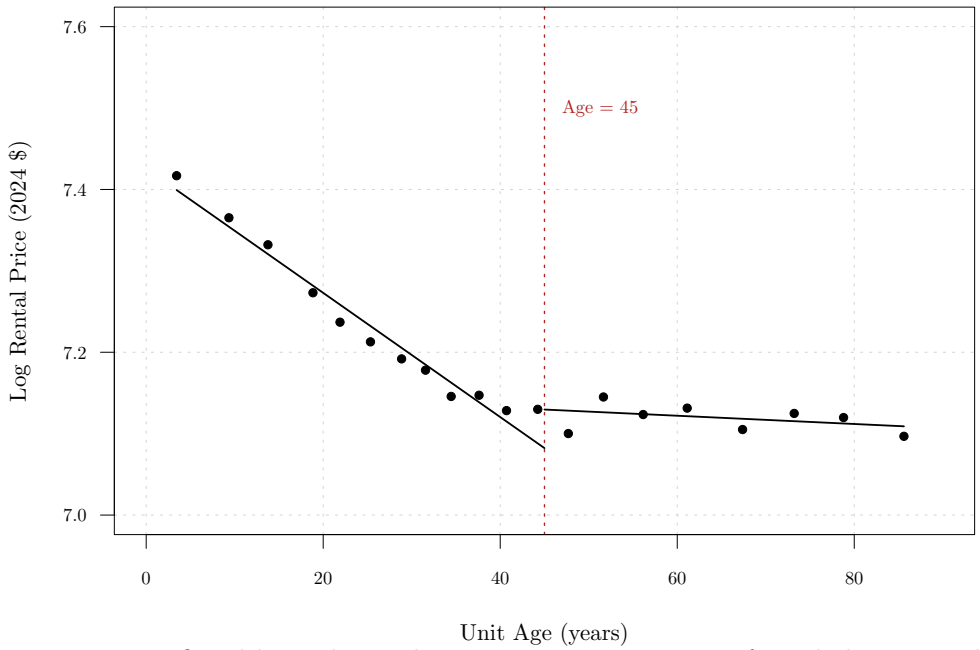


Figure A3: Relationship between unit age and rental price.



Note: Regressions fitted by ordinary least squares using units of age below 45 and above 45 separately.

Figure A4: Distribution of building costs across counties in 2024.

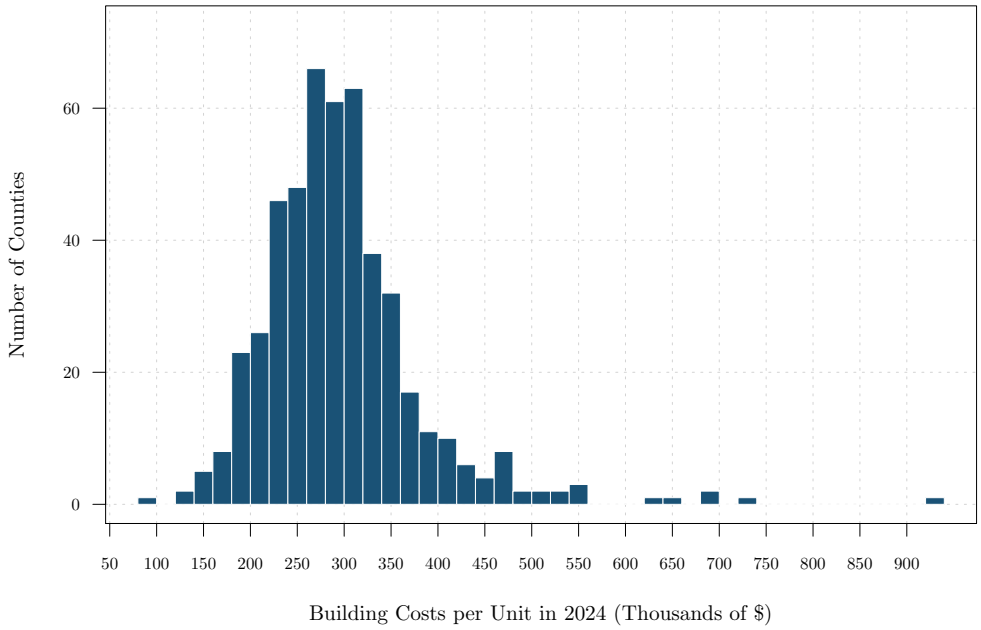
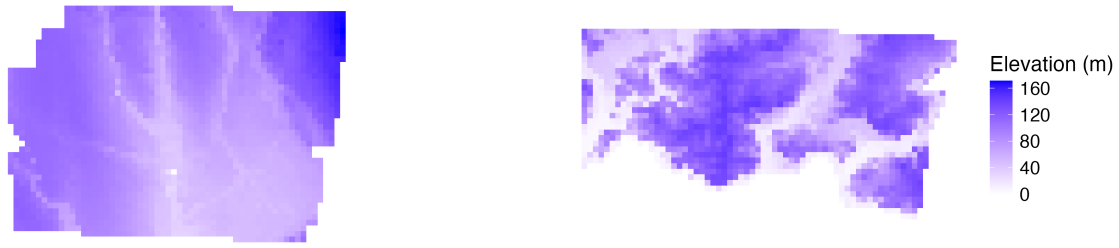
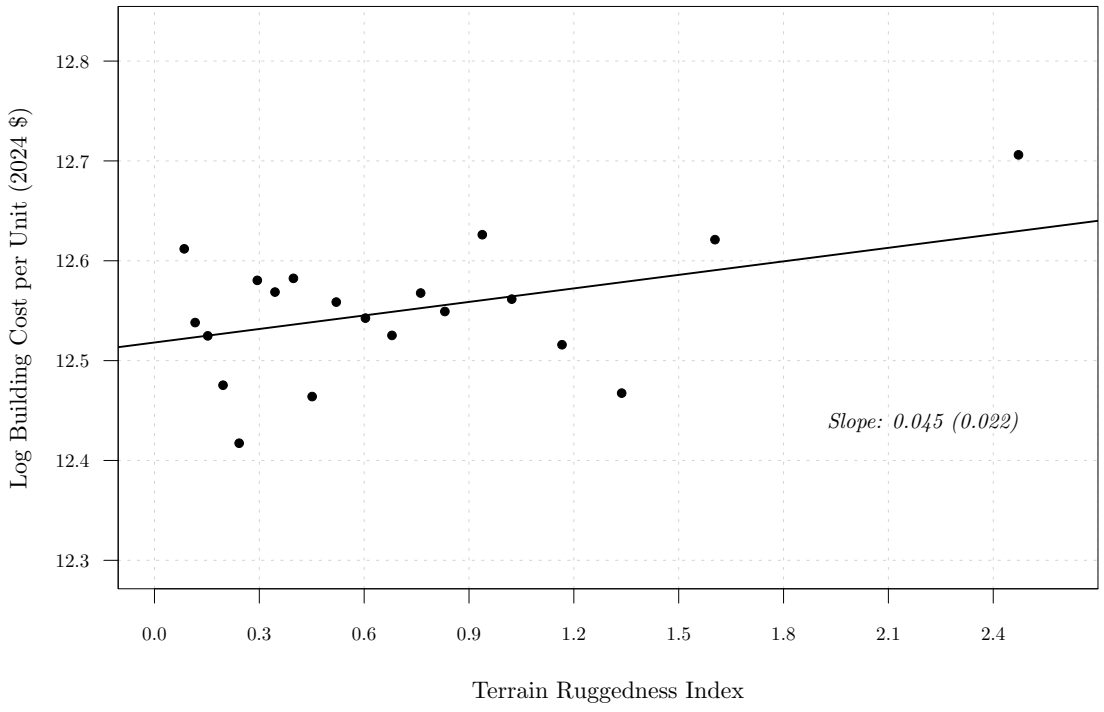


Figure A5: Elevation across Franklin, OH, and Hamilton, OH.



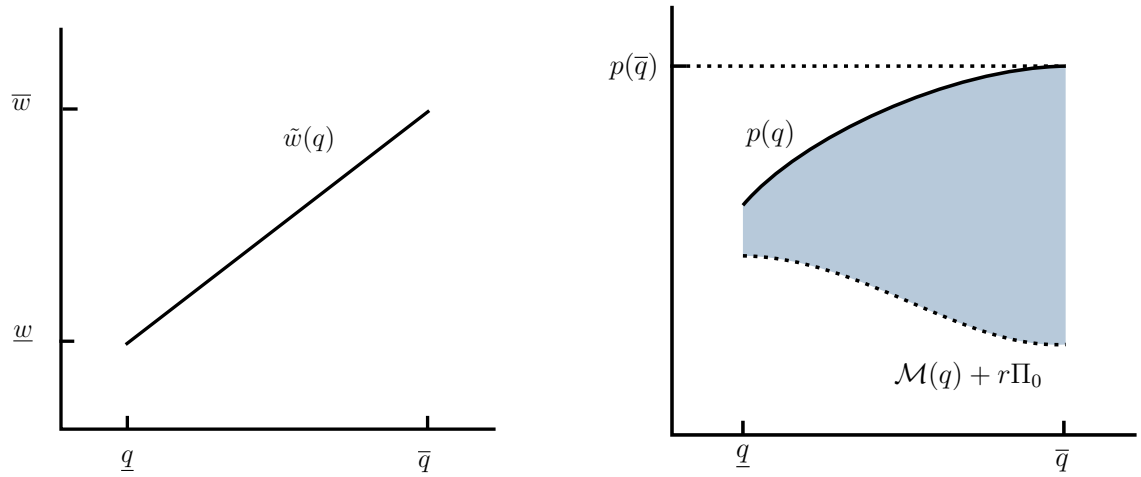
Note: Franklin, OH, County corresponds to the left panel. It has an elevation range of 171m, average ruggedness index of 0.339, and average 2024 building costs of $396,329. Hamilton, OH, County corresponds to the right panel. It has an elevation range of 145m, average ruggedness index of 1.181, and average 2024 building costs of $599,589. Elevation values are normalized such that the lowest point in the county corresponds to 0m.

Figure A6: Building costs and terrain ruggedness across US counties.



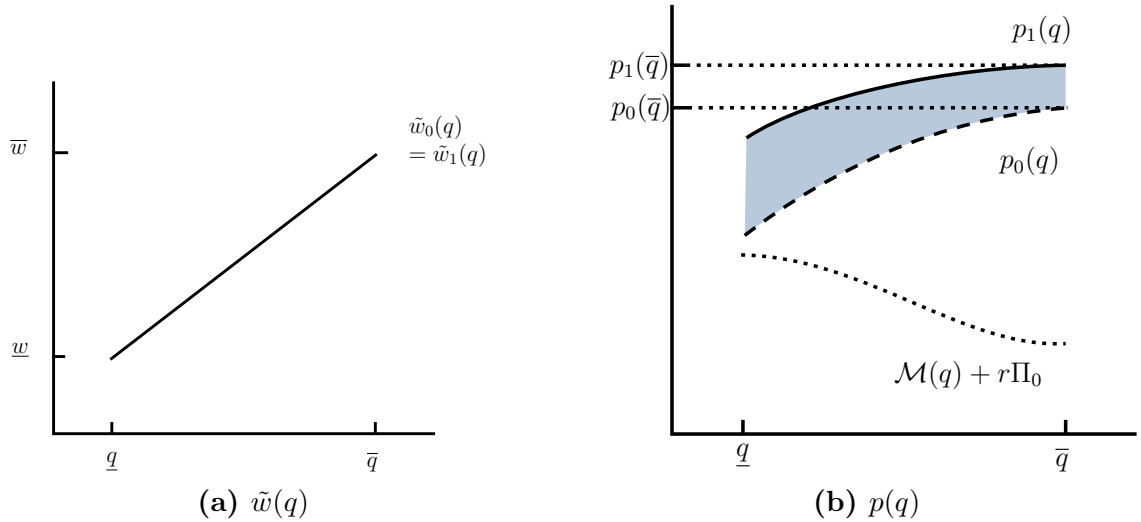
Note: Binscatter of building costs residualized by year fixed effects.

Figure A7: Equilibrium characterization when $q_{\min} = \underline{q}$.



(a) $\tilde{w}(q)$ **(b)** $p(q)$

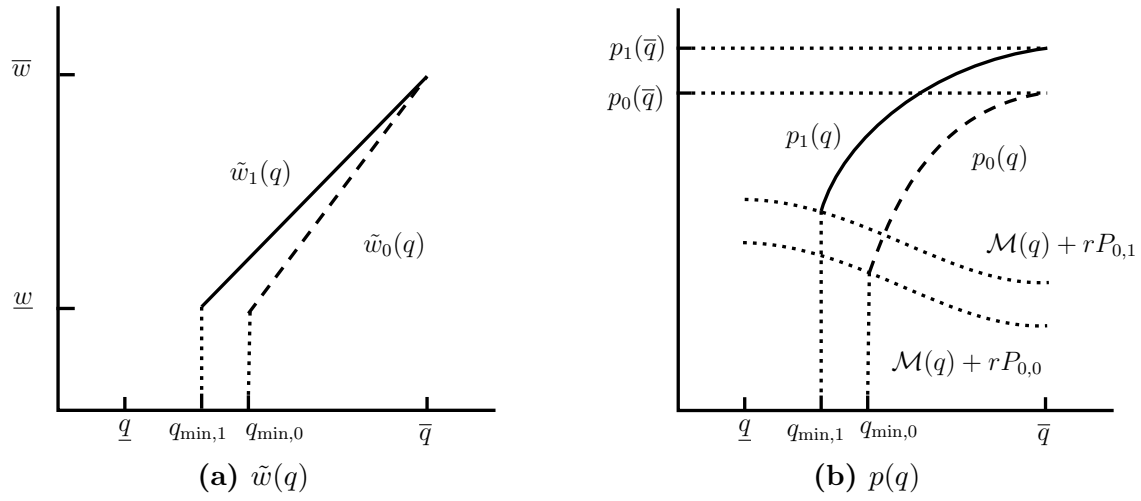
Figure A8: Effect of a change in building costs when $q_{\min} = \overline{q}$ and $p(\underline{q}) - \mathcal{M}(\underline{q}) > rP_0$.



(a) $\tilde{w}(q)$ **(b)** $p(q)$

Note: Effect of an increase in building costs from κ_0 to $\kappa_1 > \kappa_0$.

Figure A9: Effect of a Change in Land Value P_0.



(a) $\tilde{w}(q)$ **(b)** $p(q)$

Note: Effect of an increase in land values from P_0 to $P_1 > P_0$.

B Tables

Table A1: Regression results for the relationship between rental price and building costs across unit ages.

Dep. Variable:	Log Rental Price				
	OLS	Main IV	Saiz Control	SF-units	All Ages
	(1)	(2)	(3)	(4)	(5)
Variables					
Age	-0.0326***	-0.1987***	-0.2208***	-0.1643***	-0.0575
	(0.0104)	(0.0437)	(0.0750)	(0.0504)	(0.0367)
Age × Log Cost	0.0019**	0.0153***	0.0172***	0.0126***	0.0043
	(0.0008)	(0.0035)	(0.0061)	(0.0041)	(0.0029)
Fixed-effects					
County-year	Yes	Yes	Yes	Yes	Yes
Fit statistics					
Observations	1,635,688	1,635,688	1,506,908	446,913	2,852,858
R^2	0.30599	0.30049	0.29035	0.36154	0.28310
1st-stage F		13.2	9.3	11.3	11.2

Note: Clustered (MSA) standard-errors in parentheses. Signif. Codes: ***: 0.01, **: 0.05, *: 0.1. Instrument is Age × Terrain Ruggedness Index.

Table A2: Regression results for the relationship between occupant incomes and building costs across unit ages.

Dep. Variable:	Relative Income				
	OLS	Main IV	Control Saiz	Single-unit	All Ages
	(1)	(2)	(3)	(4)	(5)
Variables					
Age	-0.0258*	-0.2329***	-0.3012***	-0.1583**	-0.0832**
	(0.0127)	(0.0663)	(0.1128)	(0.0725)	(0.0407)
Age × Log Cost	0.0014	0.0182***	0.0239**	0.0122**	0.0065**
	(0.0010)	(0.0053)	(0.0092)	(0.0059)	(0.0033)
Fixed-effects					
County-year	Yes	Yes	Yes	Yes	Yes
Fit statistics					
Observations	1,602,951	1,602,951	1,476,831	440,073	2,793,271
R^2	0.01056	0.00768	0.00546	0.03024	0.00466
1st-stage F		13.3	9.3	11.4	11.3

Note: Clustered (MSA) standard-errors in parentheses. Signif. Codes: ***: 0.01, **: 0.05, *: 0.1. Instrument is Age × Terrain Ruggedness Index.

Table A3: Regression results for the relationship between new construction & maintenance and building costs.

Dependent Variables:	Permits per household		Log Maintenance	
	OLS	IV	OLS	IV
Model:	(1)	(2)	(3)	(4)
Variables				
Log Cost per Unit	-0.0048***	-0.0175	0.2697***	-0.1494
	(0.0010)	(0.0114)	(0.0919)	(0.3593)
Square Feet			3.51×10^{-5}***	3.46×10^{-5}***
			(6.7×10^{-6})	(6.85×10^{-6})
Log Household Income			0.4447***	0.4543***
			(0.0173)	(0.0197)
Bedrooms			0.3379***	0.3322***
			(0.0216)	(0.0204)
Fixed-effects				
Year	Yes	Yes	Yes	Yes
Fit statistics				
Observations	7,917	6,645	87,881	87,881
R^2	0.01780	0.00281	0.09252	0.09106
1st-stage F		5.9		3.3

Note: Standard errors clustered by county (columns (1) & (2)) and MSA (columns (3) & (4)). Signif. Codes: ***: 0.01, **: 0.05, *: 0.1. Instrument is Age × Terrain Ruggedness Index.

C Details on Facts

In this section we provide further detail on the empirical facts. We define a county as a housing market, with an observation comprising of households in a county-year pair. Central to our facts is variation in building costs. We exploit variation in building costs across housing markets. To alleviate concerns in measurement error and endogeneity we also consider terrain ruggedness as an instrument for building costs. We first document the data sources and then provide further details on the motivating facts and corresponding regressions.

C.1 Data

American Community Survey. We use the American Community Survey (ACS) for rental prices, building age and household income. We restrict our analysis to counties in Metropolitan Statistical Areas (MSAs). Based on the identifiability of geography in the ACS, our resulting sample is 490 counties, across 293 MSAs. We use housing age as a negative proxy of house quality. This can be problematic for two reasons (Randolph, 1988). First, cohort effects may break down the age – quality relationship. For example units built in the 1980's may have been built with different characteristics (e.g. size or location) than in the 2000's. Second, as units age they may be remodeled. We partly address these concerns by primarily focusing on housing units with an age lower than 45. Figure A3 plots a binscatter motivating this threshold. After age 45, the slope of rental prices on age is flatter. Age no longer has such a strong relationship with prices, potentially due to large-scale remodeling.

Building Permits Survey. The source of building costs for our analysis is the Building Permits Survey (BPS). The BPS surveys permit issuing places about how many permits were issued in a given period and what was the residential *structure* value of those permits. Approximately 99% of new construction occurs in permit issuing places, which the BPS then aggregates to the county level, where necessary. The advantage of using the building permits data is that it gives estimates on the construction cost of the structure itself, not including land prices. We estimate construction costs in a given county each year by taking the total value of permits issued divided by the number of permits issued[14]. Figure A4 plots substantial heterogeneity in building costs across counties. There are several potential concerns with measurement error in the data. First, we cannot observe the characteristics of the units being built, so we cannot account for composition effects. Second, the data does not include developer markups, which may vary across counties. To address these concerns we consider ruggedness as an instrument for building costs (Hsiao, 2023).

U.S. Geological Survey. We source elevation data at the 30 meter resolution from the United States Geological Survey. We compute the terrain ruggedness index (Riley et al., 1999) for each 30 square meter pixel by calculating

$$\text{tri}_i = \sqrt{\sum_{j \in N(i)} (z_j - z_i)^2}$$

[14]We use data for single unit structures; results remain qualitatively the same when using data on all kinds of units.

where i denotes a pixel, N its set of neighbors and z denotes elevation. We then average terrain ruggedness across each county. The motivation for using ruggedness as an instrument is that more rugged terrain requires costlier land preparation. Figure A5 plots elevation for the Ohio Franklin and Hamilton Counties. These encompass large parts of similarly sized metros, Columbus and Cincinnati. While the counties have similar elevation ranges, Hamilton (average tri of 0.339) is substantially more rugged than Franklin (average tri of 1.181). Not only is this evident by the ruggedness index, but also by the fact that the Hamilton elevation plot is much more jagged. The building costs in Hamilton are about two thirds larger than in Franklin. While these are just two particular examples, Figure A6 plots a binscatter for our sample of US counties. It shows a positive relationship between counties' average building costs over our sample and ruggedness.

American Housing Survey. For our fact on housing maintenance we source data from the biennial American Housing Survey (AHS). The AHS follows a panel of housing units across the US, with a much more extensive questionnaire than the ACS. Unfortunately the finest level of geography available in the public use AHS is at the MSA level. Furthermore from 2015 onwards a smaller set of MSAs is listed in the public use file. We therefore use the survey years from 1991 to 2013. We only start from 1991 (and not before) because our permit data from the BPS goes back only to 1990.

C.2 Empirical Strategy

In the main text we present associations in the form of figures for the relationship between building costs and (i) the rent–age gradient, (ii) the income–age gradient & (iii) maintenance and new construction. In this section we detail corresponding regressions and how we address concerns of measurement error or endogeneity.

Fact 1 concerns the slope of rental price to housing age for counties split into the bottom and top quartiles of building costs. The slope in high building cost counties is almost half of that in low building cost counties, suggesting that amplification is indeed present in the cross-section. To quantify this amplification we regress rental prices on unit age interacted with county building costs. Specifically, with h indexing a unit, t a year, c a county we consider the following specification

$$\log p_{h,c,t} = \beta_0 \text{age}_{h,c,t} + \beta_1 (\text{age}_{h,c,t} \cdot \log \kappa_{c,t}) + \alpha_{c,t} + \varepsilon_{h,c,t} \tag{A1}$$

where p denotes rental prices, α denotes fixed effects and $\varepsilon_{h,c,t}$ denotes errors which are potentially correlated across time, units and markets. To account for correlation we cluster

standard errors by MSA. The coefficient of interest is β_1, which Figure 2 suggests should be positive. Column 1 of Table A1 reports results from estimating (A1) by ordinary least squares. We observe that as age increases rental prices decrease, consistent with Figure 1a. Furthermore, the estimate of β_1 is positive and statistically significant, providing supporting evidence to Figure 2 in the main text. However, this regression suffers from potential measurement error, as discussed in the description of the data, and endogeneity. One potential source of endogeneity is the following. Suppose that across housing markets there is differential preference for new housing which both pushes up the price of new units and the cost of construction. For example there may be an unobserved component $u_{h,c,t}$ to prices such that

$$u_{h,c} = \begin{cases} z_c & \text{if age}_{h,c} = 0 \\ 0 & \text{if age}_{h,c} > 0 \end{cases}$$

and $\text{cov}(z_c, \kappa_c) > 0$. Then the estimate in regression (A1) will be biased downward. To alleviate these measurement error and endogeneity concerns we instrument $\text{age}_{h,c,t} \cdot \log \kappa_{c,t}$ by $\text{age}_{h,c,t} \cdot \text{tri}_c$. The results for our baseline instrumental variable estimation by two-stage least squares are in Column 2 of Table A1. We observe that not only the coefficient of interest, β_1, increases, but the ratio $\hat{\beta}_1/\hat{\beta}_0$ goes from -0.058 to -0.770. This suggests that the least squares estimate had downward bias. We can interpret these results as a 1% increase in building costs increases rental prices by 0.46 p.p. more for a 40 year old unit than a 10 year old unit.

There are two further concerns with the instrument which we discuss. First, terrain ruggedness can correlate with land unavailability at the metro level (Burchfield et al., 2006). For example if more rugged terrain is associated with mountainous regions which can box in a city providing physical limits to land where one can construct new housing. To account for this we use the land unavailability measure computed in Saiz (2010), and control for this in the instrumented regression (specifically we control for $\text{age}_{h,c,t} \cdot \text{land unaval}_c$). Column 3 of Table A1 describes the result. It is quantitatively comparable with our main instrumental variable specification. The second concern is that the instrument, terrain ruggedness, may shift the outside option value of land; rugged terrain may be associated with lower land prices. Through the lens of our model in Section 3 this would imply a downwards bias in the coefficient β_1. Finally, Columns 4 and 5 of Table A1 present results on the instrumented regression when using single family units and when using all housing ages respectively.

Fact 2 concerns the slope of income to housing age in high and low building cost markets. We conduct a similar analysis to Fact 1. Here we provide further details on the regression.

In particular we construct the measure of relative income of household h in county c in year t as the standardized log income within that county year

$$\text{rel_income}_{h,c,t} = \frac{\log \text{income}_{h,c,t} - \text{mean}\left(\log \text{income}_{h',c,t}\right)}{\text{sd}\left(\log \text{income}_{h',c,t}\right)}.$$

We then regress

$$\text{rel_income}_{h,c,t} = \beta_0 \text{age}_{h,c,t} + \beta_1(\text{age}_{h,c,t} \cdot \log \kappa_{c,t}) + \alpha_{c,t} + \varepsilon_{h,c,t} \tag{A2}$$

and instrument for $\text{age}_{h,c,t} \cdot \log \kappa_{c,t}$ by $\text{age}_{h,c,t} \cdot \text{tri}_c$. We discuss the results in the main text.

D Proofs

D.1 Proof of Proposition 1

Proof. First, it is useful to establish that $p(q)$ must be strictly increasing over $[q_{\min}, \overline{q}]$. Suppose that it is not. Then there exists $q_0 < q_1$ such that $p(q_0) \geq p(q_1)$ and by continuity of $p(q)$, no households choose housing in the neighborhood of q_0, violating market clearing at q_0.

Note that the landlord HJB rewrites as

$$rP(q) = \max\{r\underline{P}, p(q) - \mathcal{M}(q) - \delta P'(q)\}, \tag{A3}$$

with landlords maintaining only as much as necessary.

Consider the decision of maintaining housing for an additional instant $\Delta > 0$ or of tearing down and recouping land value P_0. It is optimal to maintain to first-order if

$$\Delta(p(q) - \mathcal{M}(q)) + \exp(-r\Delta)P_0 \geq P_0, \tag{A4}$$

which to first-order is

$$p(q) - \mathcal{M}(q) \geq rP_0. \tag{A5}$$

Because $p(q)$ is strictly increasing and $\mathcal{M}(q)$ is non-increasing, there exists at most one $q_{\min}$ such that

$$p(q_{\min}) - \mathcal{M}(q_{\min}) = rP_0. \tag{A6}$$

It is optimal to maintain housing of quality $q \geq q_{\min}$ and optimal to tear-down housing of quality $q < q_{\min}$.

If there exists no such $q_{\min}$, then either

$$p(\underline{q}) - \mathcal{M}(\underline{q}) \geq rP_0, \tag{A7}$$

and it is never optimal for landlords to tear-down housing, with $q_{\min} = \underline{q}$. Or,

$$p(\overline{q}) - \mathcal{M}(\overline{q}) < rP_0, \tag{A8}$$

and no housing is built implying the economy is not in equilibrium.

In any equilibrium, housing is therefore maintained in an interval $[q_{\min}, \overline{q}]$. Market clearing then requires for this housing to be occupied. This completes the proof. □

D.2 Proof of Proposition 2

Proof. Suppose there exists an equilibrium. First, households cannot bunch at extremities of the quality ladder by market clearing. Second, we must have strict positive assortative matching: indeed, suppose that w prefers $q' > q$ to q. We know $p(q') > p(q)$ from Section D.1. For any $w' > w$, we therefore have

$$\frac{w' - p(q')}{w' - p(q)} > \frac{w - p(q')}{w - p(q)}. \tag{A9}$$

This ensures

$$q'(w' - p(q')) > q'(w - p(q'))\frac{w' - p(q)}{w - p(q)} > q(w - p(q))\frac{w' - p(q)}{w - p(q)} = q(w' - p(q)), \tag{A10}$$

where the first inequality follows from equation A9 and the second inequality follows by assumption on the preferences of w. This ensures that w' strictly prefers q' to q and establishes strict positive assortative matching.

Due to positive assortative matching, housing market clearing can be re-expressed as

$$\int_{\tilde{w}(q)}^{\overline{w}} dF(w) = C\frac{\overline{q} - q}{\delta}. \tag{A11}$$

The market clearing condition evaluated at $q = q_{\min}$ requires

$$1 = C\frac{\overline{q} - q_{\min}}{\delta}, \tag{A12}$$

pining down

$$C = \frac{\delta}{\overline{q} - q_{\min}}. \tag{A13}$$

Plugging in for C and integrating the LHS of the housing market clearing condition then yields

$$1 - F(\tilde{w}(q)) = \frac{\overline{q} - q}{\overline{q} - q_{\min}}, \tag{A14}$$

which simplifies as

$$F(\tilde{w}(q)) = \frac{q - q_{\min}}{\overline{q} - q_{\min}} \tag{A15}$$

which we can invert to obtain out result on $\tilde{w}(q)$.

We now need to show existence and uniqueness. $p(q)$ is continuous by assumption and strictly monotonic as established in Section D.1. Therefore, it must be almost everywhere differentiable by Lebesgue's theorem for monotone functions. Housing market clearing rules out bunching of households at quality levels and therefore rules out kinks in $p'(q)$, implying $p(q)$ is everywhere differentiable.

The first-order condition in equation 6 is therefore well defined and using our expression for $\tilde{w}(q)$, can be expressed as

$$p'(q) = \frac{\tilde{w}(q) - p(q)}{q}. \tag{A16}$$

We verify that this condition is sufficient for household optimality. The household objective can be expressed as

$$q(w - p(q)). \tag{A17}$$

The second derivative with respect to q is then given by

$$-p'(q) - p''(q)q - p'(q), \tag{A18}$$

provided $p''(q)$ is well defined.

We obtain

$$p''(q) = \frac{q(\tilde{w}'(q) - p'(q)) - (\tilde{w}(q) - p(q))}{q^2} \tag{A19}$$

from equation A16. Therefore, the second derivative of the household objective is

$$
\begin{aligned}
&- p'(q) - p''(q)q - p'(q) && \text{(A20)}\\
&= -2p'(q) - \tilde{w}'(q) + p'(q) + \frac{\tilde{w}(q) - p(q)}{q} && \text{(A21)}\\
&= -\tilde{w}'(q) < 0, && \text{(A22)}
\end{aligned}
$$

guaranteeing strict concavity. Indeed, $\tilde{w}'(q) > 0$ as is clear from equation A15.

Given a cut-off $q_{\min}$, we can establish that a unique solution exists to the linear ordinary differential equation A16, together with the integral constraint

$$
\frac{1}{\delta}\int_{q_{\min}}^{\overline{q}} \exp\left(-r\frac{\overline{q}-q}{\delta}\right)(p(q) - \mathcal{M}(q))dq + \exp\left(-r\frac{\overline{q}-q_{\min}}{\delta}\right)P_0 = \kappa + P_0, \tag{A23}
$$

which guarantees free entry into building. Indeed, using the integrating factor q, the differential equation admits the general solution

$$
p(q) = \frac{1}{q}\left(K + \int_{q_{\min}}^{q} \tilde{w}(s)\,ds\right), \tag{A24}
$$

for some constant $K \in \mathbb{R}$. Substituting this expression into the integral constraint from equation A40 yields the linear equation in K

$$
\begin{aligned}
\kappa + P_0 = \frac{1}{\delta}\int_{q_{\min}}^{\overline{q}} \exp\left(-r\frac{\overline{q}-q}{\delta}\right)\left(\frac{K}{q} + \frac{1}{q}\int_{q_{\min}}^{q}\tilde{w}(s)\,ds - \mathcal{M}(q)\right)dq && \text{(A25)}\\
+ \exp\left(-r\frac{\overline{q}-q_{\min}}{\delta}\right)P_0, && \text{(A26)}
\end{aligned}
$$

with a unique solution for K.

We have established that given a cut-off $q_{\min} < \overline{q}$, there is a unique solution to the linear ordinary differential equation A16 together with the integral constraint A40. This solution satisfies household optimality and free entry into building as well as, by construction, housing market clearing. The consumption good market always clears by Walras' law. We therefore only need to show that there exists a unique $q_{\min}$ which solves the landlord HJB.

From Section D.1, this is the case if $q_{\min}$ satisfies the condition

$$
rP_0 = p(q_{\min}) - \mathcal{M}(q_{\min}) \tag{A27}
$$

or

$$rP_0 < p(\underline{q}) - \mathcal{M}(\underline{q}). \tag{A28}$$

For existence, we must show that $p(q_{\min})$ can be made to be high enough. Indeed, we don't need to show that it can be made to be low enough, because equation A28 can also pin down $q_{\min} = \underline{q}$.

Consider

$$p(q_{\min}) = p(\overline{q}) - \int_{q_{\min}}^{\overline{q}} \frac{\tilde{w}(x) - p(x)}{x} dx. \tag{A29}$$

From equation A40, we know that when $q_{\min} \to \overline{q}$, then $p(q_{\min}) \to \infty$ and

$$rP_0 < p(q_{\min}) - \mathcal{M}(q_{\min}). \tag{A30}$$

This is so because prices need to blow up for landlords to break even on building and because the slope of prices with respect to quality is bounded above by $\overline{w}/\underline{q}$ so that $p(q_{\min})$ becomes close to $p(\overline{q})$ as $q_{\min} \to \overline{q}$. Note that this requires for $\underline{w}$ to be high enough so that households are able to afford housing. This establishes existence and we turn to uniqueness.

Uniqueness is violated if there exists at least two values $q_{\min,0} < q_{\min,1}$ such that

$$rP_0 = p(q_{\min,0}) - \mathcal{M}(q_{\min,0}) = p(q_{\min,1}) - \mathcal{M}(q_{\min,1}). \tag{A31}$$

Without loss of generality, we consider the two closest such values.

Consider changing $q_{\min}$ while still satisfying the linear ordinary differential equation problem near either value.

First we argue that $p(q_{\min})$ is continuous with respect to changes in $q_{\min}$. We have from equation A24

$$K = q_{\min} p(q_{\min}) \tag{A32}$$

and

$$p(q) = \frac{1}{q}\left(q_{\min} p(q_{\min}) + \int_{q_{\min}}^{q} \tilde{w}(s)\, ds\right) \tag{A33}$$

Plugging into the integral constraint A40 defines $p(q_{\min})$ as the unique solution to a linear equation in $p(q_{\min})$ with coefficients that depend continuously on $q_{\min}$. Hence, $p(q_{\min})$ varies

continuously with $q_{\min}$.

Second, we observe that the integral constraint in equation A40, requires

$$\frac{1}{\delta}\int_{q_{\min}}^{\overline{q}} \exp(-r\frac{\overline{q}-q}{\delta})\frac{dp(q)}{dq_{\min}}dq - \frac{1}{\delta}\exp(-r\frac{\overline{q}-q_{\min}}{\delta})(p(q_{\min}) - \mathcal{M}(q_{\min}) - rP_0) = 0. \quad \text{(A34)}$$

which simplifies when evaluated at $q_{\min} = q_{\min,0}$ or $q_{\min} = q_{\min,1}$ as

$$\frac{1}{\delta}\int_{q_{\min}}^{\overline{q}} \exp(-r\frac{\overline{q}-q}{\delta})\frac{dp(q)}{dq_{\min}}dq = 0. \quad \text{(A35)}$$

Differentiating equation A24 with respect to $q_{\min}$ yields

$$\frac{dp(q)}{dq_{\min}} = \frac{1}{q}\left(p(q_{\min}) + q_{\min}\frac{dp(q_{\min})}{dq_{\min}} - \tilde{w}(q_{\min})\right) + \frac{1}{q}\int_{q_{\min}}^{q}\frac{d\tilde{w}(x)}{dq_{\min}}dx \quad \text{(A36)}$$

$$= \frac{q_{\min}}{q}\left(\frac{dp(q_{\min})}{dq_{\min}} - p'(q_{\min})\right) + \frac{1}{q}\int_{q_{\min}}^{q}\frac{d\tilde{w}(x)}{dq_{\min}}dx. \quad \text{(A37)}$$

To satisfy equation A35, when evaluated at $q_{\min} = q_{\min,0}$ or $q_{\min} = q_{\min,1}$, we must have

$$\frac{dp(q_{\min,0})}{dq_{\min}} > 0 \quad \text{(A38)}$$

and

$$\frac{dp(q_{\min,1})}{dq_{\min}} > 0. \quad \text{(A39)}$$

If not, then the LHS of A35 would be strictly negative because $p'(q) > 0$ and $\frac{d\tilde{w}(x)}{dq_{\min}} < 0$.

But this contradicts the continuity of $p(q_{\min})$ as a function of $q_{\min}$, therefore establishing uniqueness. □

D.3 Proof of Lemma 1

Proof. Landlords breaking even on development requires requires

$$\frac{1}{\delta}\int_{q_{\min}}^{\overline{q}} \exp\left(-r\frac{\overline{q}-q}{\delta}\right)(p(q) - \mathcal{M}(q))dq + \exp\left(-r\frac{\overline{q}-q_{\min}}{\delta}\right)P_0 = \kappa + P_0. \quad \text{(A40)}$$

Applying the Leibnitz rule yields

$$\exp\left(-r\frac{\overline{q}-q_{\min}}{\delta}\right)\left(\frac{-dq_{\min}}{d\kappa}\right)\left(\frac{p(q_{\min})-\mathcal{M}(q_{\min})}{\delta}-\frac{rP_0}{\delta}\right) \tag{A41}$$

$$+\frac{1}{\delta}\int_{q_{\min}}^{\overline{q}}\exp\left(-r\frac{\overline{q}-q}{\delta}\right)\frac{dp(q)}{d\kappa}dq=1. \tag{A42}$$

When $q_{\min}=\underline{q}$ and $p(\underline{q})-\mathcal{M}(\underline{q})-rP_0>0$, then the landlord maintenance policy does not change in response to small changes in κ and $dq_{\min}/d\kappa=0$. When $q_{\min}>\underline{q}$, then $p(q_{\min})-\mathcal{M}(q_{\min})-rP_0=0$. Either way, we obtain

$$\frac{1}{\delta}\int_{\underline{q}}^{\overline{q}}\exp\left(-r\frac{\overline{q}-q}{\delta}\right)\frac{dp(q)}{d\kappa}dq=1, \tag{A43}$$

the first equation in the lemma.

Next, we differentiate equation 6 from household optimality, expressed as

$$p'(q)=\frac{\tilde{w}(q)-p(q)}{q}. \tag{A44}$$

We let $y(q)=dp(q)/d\kappa$. This yields the differential equation

$$y'(q)+\frac{1}{q}y(q)=\frac{1}{q}\frac{d\tilde{w}(q)}{d\kappa}, \tag{A45}$$

which we can solve using the integrating factor q to obtain

$$y(q)=\frac{\overline{q}}{q}y(\overline{q})-\frac{1}{q}\int_{q}^{\overline{q}}\frac{d\tilde{w}(x)}{d\kappa}dx. \tag{A46}$$

This gives the second equation in the lemma.

We derive the third equation in the lemma directly from the assignment function in Proposition 2, applying the inverse function theorem.

Lastly, we derive the fourth expression for the lemma by differentiating

$$p(q_{\min})-\mathcal{M}(q_{\min})-rP_0=0. \tag{A47}$$

□

D.4 Proof of Corollary 1

Proof. The proof relies on noting that $dp(q)/d\kappa$ must be continuous in q. Indeed, from Lemma 1, we have the conditions

$$\frac{dp(q)}{d\kappa} = \frac{\overline{q}}{q}\frac{dp(\overline{q})}{d\kappa} - \frac{1}{q}\int_q^{\overline{q}} \frac{d\tilde{w}(x)}{d\kappa}dx \tag{A48}$$

and

$$1 = \frac{1}{\delta}\int_{q_{\min}}^{\overline{q}} \exp\Big(-r\frac{\overline{q}-q}{\delta}\Big)\frac{dp(q)}{d\kappa}\,dq \tag{A49}$$

which characterize $dp(q)/d\kappa$ as the unique solution to a well defined linear ordinary differential equation problem.

We start by showing $dq_{\min}/d\kappa \leq 0$. Suppose for a contradiction that $dq_{\min}/d\kappa > 0$. Then it must be that

$$p(q_{\min}) - \mathcal{M}(q_{\min}) = rP_0, \tag{A50}$$

otherwise, $q_{\min}$ would not adjust in response to small changes in κ. From the condition

$$0 = \frac{dp(q_{\min})}{d\kappa} + (p'(q_{\min}) - \mathcal{M}'(q_{\min}))\frac{dq_{\min}}{d\kappa}, \tag{A51}$$

in Lemma 1, it follows that we must have $\frac{dp(q_{\min})}{d\kappa} < 0$. Indeed $p'(q)$ is strictly positive from Section D.1 and $\mathcal{M}'(q)$ is weakly negative by assumption.

We know there must exist some q' such that $\frac{dp(q')}{d\kappa} > 0$ due to equation A49. There must then also exist a $q'' < q'$ such that $\frac{dp(q'')}{d\kappa} = 0$. Consider the lowest such q''. We have

$$\frac{d^2p(q'')}{dqd\kappa} = \frac{1}{q''}\left(\frac{d\tilde{w}(q'')}{d\kappa} - \frac{dp(q'')}{d\kappa}\right) = \frac{1}{q''}\frac{d\tilde{w}(q'')}{d\kappa} < 0 \tag{A52}$$

because Lemma 1 requires

$$\frac{d\tilde{w}(q'')}{d\kappa} = \frac{-1}{f(\tilde{w}(q''))}\frac{\overline{q}-q''}{(\overline{q}-q_{\min})^2}\frac{dq_{\min}}{d\kappa} < 0, \tag{A53}$$

when $dq_{\min}/d\kappa > 0$. This is a contradiction of $\frac{dp(q_{\min})}{d\kappa} < 0$. Hence we must have $dq_{\min}/d\kappa \leq 0$.

We now show that $dp(q)/d\kappa > 0$. From equation A48 in Lemma 1, we see that when

$dq_{\min}/d\kappa = 0$ so that $d\tilde{w}(q)/d\kappa = 0$,

$$\frac{dp(q)}{d\kappa} > 0 \tag{A54}$$

must hold for any q.

When $dq_{\min}/d\kappa < 0$, suppose there exists a q' such that $\frac{dp(q')}{d\kappa} \leq 0$. Again to satisfy equation A49, there must exist a q'' with $\frac{dp(q'')}{d\kappa} = 0$. Consider the lowest such q'' with $q'' < \overline{q}$. We have

$$\frac{d^2p(q'')}{dqd\kappa} = \frac{1}{q''}\left(\frac{d\tilde{w}(q'')}{d\kappa} - \frac{dp(q'')}{d\kappa}\right) = \frac{1}{q''}\frac{d\tilde{w}(q'')}{d\kappa} > 0, \tag{A55}$$

a contradiction since

$$dp(q_{\min})/d\kappa > 0. \tag{A56}$$

Furthermore, we cannot have $dp(\overline{q})/d\kappa = 0$ as this would imply $dp(q)/d\kappa \leq 0$ for every q by equation A48. Hence we must have $dp(q)/d\kappa > 0$. □

D.5 Proof of Corollary 2

Proof. First we note that

$$\frac{d}{dq}\left(\frac{d\log p(q)}{d\log \kappa}\right) < 0 \iff \frac{d}{dq}\left(\frac{d\log p(q)}{d\kappa}\right) < 0. \tag{A57}$$

We study the second condition in the proof. We re-write

$$\frac{d}{dq}\left(\frac{d\log p(q)}{d\kappa}\right) \tag{A58}$$

as

$$\frac{d^2\log p(q)}{d\kappa dq} \tag{A59}$$

for the remainder of the proof.

We note that

$$p(q) = p(\overline{q}) - \int_q^{\overline{q}} p'(x)dx \tag{A60}$$

and observe that

$$\frac{d\log p(q)}{d\kappa} = \frac{1}{p(q)}\left(\frac{dp(\overline{q})}{d\kappa} - \int_q^{\overline{q}} \frac{dp'(x)}{d\kappa}dx\right). \tag{A61}$$

Using the Leibnitz rule, we then derive

$$\frac{d^2\log p(q)}{d\kappa dq} < 0 \tag{A62}$$

$$\iff p(q)\left(\frac{dp'(q)}{d\kappa}\right) - \frac{dp(q)}{d\kappa}p'(q) < 0. \tag{A63}$$

Using our the household optimality condition

$$p'(q) = \frac{\tilde{w}(q) - p(q)}{q}, \tag{A64}$$

we obtain

$$\frac{d^2\log p(q)}{d\kappa dq} < 0 \tag{A65}$$

$$\iff p(q)\left(\frac{d\tilde{w}(q)}{d\kappa} - \frac{dp(q)}{d\kappa}\right) - \frac{dp(q)}{d\kappa}(\tilde{w}(q) - p(q)) < 0 \tag{A66}$$

$$\iff \frac{d\tilde{w}(q)}{d\kappa} < \frac{dp(q)}{d\kappa}\frac{\tilde{w}(q)}{p(q)} \tag{A67}$$

$$\iff \frac{d\log\tilde{w}(q)}{d\kappa} < \frac{d\log p(q)}{d\kappa}. \tag{A68}$$

We re-write this as

$$\frac{d}{d\kappa}\left(\log\frac{p(q)}{\tilde{w}(q)}\right) > 0. \tag{A69}$$

We note that

$$\frac{d}{d\kappa}\left(d\log\frac{p(q)}{\tilde{w}(q)}\right) > 0 \iff \frac{d}{d\kappa}\left(\frac{p(q)}{\tilde{w}(q)}\right) > 0. \tag{A70}$$

This concludes the first part of the proof.

For the second part of the proof, we start from the condition

$$\frac{d\tilde{w}(q)}{d\kappa} < \frac{dp(q)}{d\kappa}\frac{\tilde{w}(q)}{p(q)} \tag{A71}$$

and note that it always holds when $dq_{\min}/d\kappa = 0$. We derive the sufficient condition on

parameters for when $dq_{\min}/d\kappa < 0$. Following the proof of Lemma 1, we obtain the differential equation

$$\frac{dp(q)}{d\kappa} = \frac{q_{\min}}{q}\left(\frac{dp(q_{\min})}{d\kappa} - p'(q_{\min})\frac{dq_{\min}}{d\kappa}\right) + \frac{1}{q}\int_{q_{\min}}^{q} \frac{d\tilde{w}(x)}{d\kappa}dx. \tag{A72}$$

From Lemma 1, we also note

$$\frac{dp(q_{\min})}{d\kappa} = \left(p'(q_{\min}) - \mathcal{M}'(q_{\min})\right)\frac{-dq_{\min}}{d\kappa} \tag{A73}$$

and

$$\frac{d\tilde{w}(q)}{d\kappa} = \frac{-1}{f(\tilde{w}(q))}\frac{\overline{q}-q}{(\overline{q}-q_{\min})^2}\frac{dq_{\min}}{d\kappa}. \tag{A74}$$

Substituting into condition A68 and simplifying the $-dq_{\min}/d\kappa > 0$ terms yields

$$\frac{d^2\log p(q)}{d\kappa dq} < 0 \tag{A75}$$

$$\iff \frac{1}{f(\tilde{w}(q))}\frac{\overline{q}-q}{(\overline{q}-q_{\min})^2} \tag{A76}$$

$$< \frac{\tilde{w}(q)}{p(q)}\frac{1}{q}\left(q_{\min}\left(2p'(q_{\min}) - \mathcal{M}'(q_{\min})\right) + \int_{q_{\min}}^{q}\frac{1}{f(\tilde{w}(x))}\frac{\overline{q}-x}{(\overline{q}-q_{\min})^2}dx\right). \tag{A77}$$

We proceed to find an upper bound for the LHS and a lower bound for the RHS of this condition. Starting with the RHS, we note that because we focus on equilibria where households can afford housing, $\tilde{w}(q)/p(q) \geq 1$. We obtain

$$\frac{\tilde{w}(q)}{p(q)}\frac{1}{q}\left(q_{\min}\left(2p'(q_{\min}) - \mathcal{M}'(q_{\min})\right) + \int_{q_{\min}}^{q}\frac{1}{f(\tilde{w}(x))}\frac{\overline{q}-x}{(\overline{q}-q_{\min})^2}dx\right) \tag{A78}$$

$$> \frac{1}{q}\left(q_{\min}2p'(q_{\min}) - q_{\min}\mathcal{M}'(q_{\min}) + \int_{q_{\min}}^{q}\frac{1}{f(\tilde{w}(x))}\frac{\overline{q}-x}{(\overline{q}-q_{\min})^2}dx\right) \tag{A79}$$

$$> \frac{1}{\overline{q}}\left(q_{\min}2p'(q_{\min}) - q_{\min}\mathcal{M}'(q_{\min})\right) \tag{A80}$$

$$= \frac{1}{\overline{q}}(2(\underline{w} - p(q_{\min})) - \underline{q}\overline{\mathcal{M}}') \tag{A81}$$

$$= \frac{1}{\overline{q}}(2(\underline{w} - (\mathcal{M}(q_{\min}) + rP_0)) - \underline{q}\overline{\mathcal{M}}'), \tag{A82}$$

where the last equality follows from the implicit definition of $q_{\min}$. Assuming an upper bound

$\overline{\mathcal{M}}$ on maintenance costs $\mathcal{M}(q)$ then yields that the RHS is bounded below by

$$\frac{1}{\overline{q}}(2(\underline{w} - (\overline{\mathcal{M}} + rP_0)) - \underline{q}\overline{\mathcal{M}}'). \tag{A83}$$

For the LHS, we start by computing an upper bound on $1/(\overline{q} - q_{\min})$. To do so, we first note that the condition

$$\frac{1}{\delta}\int_{q_{\min}}^{\overline{q}} \exp\left(-r\frac{\overline{q}-q}{\delta}\right)(p(q) - \mathcal{M}(q))dq + \exp\left(-r\frac{\overline{q}-q_{\min}}{\delta}\right)P_0 = \kappa + P_0. \tag{A84}$$

and the fact that prices are strictly decreasing implies

$$\frac{(\overline{q}-\underline{q})}{\delta}p(\overline{q}) \geq \frac{1}{\delta}\int_{q_{\min}}^{\overline{q}} \exp\left(-r\frac{\overline{q}-q}{\delta}\right)(p(q) - \mathcal{M}(q))dq = P_0(1 - \exp\left(-r\frac{\overline{q}-q_{\min}}{\delta}\right)) + \kappa \tag{A85}$$

so that

$$p(\overline{q}) \geq \frac{\kappa\delta}{\overline{q}-\underline{q}}. \tag{A86}$$

Therefore, we have that

$$\begin{aligned}
rP_0 &= p(q_{\min}) - \mathcal{M}(q_{\min}) && \text{(A87)}\\
&= p(\overline{q}) - \int_{q_{\min}}^{\overline{q}} p'(x)dx - \mathcal{M}(q_{\min}) && \text{(A88)}\\
&\geq \frac{\kappa\delta}{\overline{q}-\underline{q}} - \int_{q_{\min}}^{\overline{q}} p'(x)dx - \mathcal{M}(q_{\min}) && \text{(A89)}\\
&\geq \frac{\kappa\delta}{\overline{q}-\underline{q}} - (\overline{q} - q_{\min})\frac{\overline{w}}{\underline{q}} - \mathcal{M}(q_{\min}) && \text{(A90)}\\
&\geq \frac{\kappa\delta}{\overline{q}-\underline{q}} - (\overline{q} - q_{\min})\frac{\overline{w}}{\underline{q}} - \overline{\mathcal{M}} && \text{(A91)}\\
& && \text{(A92)}
\end{aligned}$$

implying

$$\frac{1}{\overline{q} - q_{\min}} \leq \frac{\overline{w}}{\underline{q}}\frac{1}{\frac{\kappa\delta}{\overline{q}-\underline{q}} - (\overline{\mathcal{M}} + rP_0)}. \tag{A93}$$

It follows that the LHS is bounded above by

$$\frac{1}{\underline{f}}\frac{\overline{w}}{\underline{q}}\frac{1}{\frac{\kappa\delta}{\overline{q}-\underline{q}}-(\overline{\mathcal{M}}+rP_0)}. \tag{A94}$$

Putting everything together, we obtain the sufficient condition

$$(2(\underline{w}-(\overline{\mathcal{M}}+rP_0))-\underline{q}\overline{\mathcal{M}}')\left(\frac{\kappa\delta}{\overline{q}-\underline{q}}-(\overline{\mathcal{M}}+rP_0)\right)>\frac{1}{\underline{f}}\frac{\overline{q}}{\underline{q}}\overline{w}. \tag{A95}$$

Noting that $-\underline{q}\overline{\mathcal{M}}'=\underline{q}|\overline{\mathcal{M}}'|$ since $\mathcal{M}(q)$ is decreasing, we re-express this as

$$\left(\frac{\kappa\delta}{\overline{q}-\underline{q}}-\left(\overline{\mathcal{M}}+rP_0\right)\right)\left(|\overline{\mathcal{M}}'|\underline{q}+2\left(\underline{w}-\left(\overline{\mathcal{M}}+rP_0\right)\right)\right)>\frac{1}{\underline{f}}\frac{\overline{q}}{\underline{q}}\overline{w}, \tag{A96}$$

completing the proof. □

D.6 Proof of Corollary 3

Proof. From Lemma 1, we have the following expression for changes in the assignment in response to changes in building costs:

$$\frac{d\tilde{w}(q)}{d\kappa}=\frac{-1}{f(\tilde{w}(q))}\frac{\overline{q}-q}{(\overline{q}-q_{\min})^2}\frac{dq_{\min}}{d\kappa}. \tag{A97}$$

It follows immediately that

$$\frac{d\tilde{w}(\overline{q})}{d\kappa}=0. \tag{A98}$$

From Corollary 1, we know that $\frac{dq_{\min}}{d\kappa}\leq 0$. It follows that

$$\frac{d\tilde{w}(q)}{d\kappa}\geq 0 \quad \forall q<\overline{q}. \tag{A99}$$

This concludes the first part of the proof.

Using the positive assortative matching rule from Proposition 2, we derive

$$\frac{d\log\tilde{w}(q)}{dq}=\frac{1}{\tilde{w}(q)\,f(\tilde{w}(q))\,(\overline{q}-q_{\min})}. \tag{A100}$$

It follows that

$$\frac{d}{d\kappa}\left(\frac{d\log\tilde{w}(q)}{dq}\right)\leq 0 \iff \frac{d}{d\kappa}\left(\tilde{w}(q)\,f(\tilde{w}(q))\,(\overline{q}-q_{\min})\right)\geq 0. \tag{A101}$$

We derive

$$\frac{d}{d\kappa}\left(\tilde{w}(q)f(\tilde{w}(q))\,(\overline{q}-q_{\min})\right)=\frac{d}{dw}\left(wf(w)\right)\frac{d\tilde{w}(q)}{d\kappa}(\overline{q}-q_{\min})+\tilde{w}(q)f(\tilde{w}(q))\frac{-dq_{\min}}{d\kappa}. \tag{A102}$$

The first term is non-negative under our sufficient condition and the second term is always non-negative. This completes the proof. □

D.7 Proof of Corollary 4

Proof. We have obtained

$$C=\frac{\delta}{\overline{q}-q_{\min}}, \tag{A103}$$

$$M=\frac{\int_{q_{\min}}^{\overline{q}}\mathcal{M}(x)dx}{\overline{q}-q_{\min}}. \tag{A104}$$

We derive

$$\frac{dC}{d\kappa}=\delta\frac{1}{(\overline{q}-q_{\min})^2}\frac{dq_{\min}}{d\kappa}\leq 0. \tag{A105}$$

Using the Leibnitz rule, we derive

$$\frac{dM}{d\kappa}=\frac{d}{d\kappa}\frac{\int_{q_{\min}}^{\overline{q}}\mathcal{M}(x)dx}{\overline{q}-q_{\min}} \tag{A106}$$

$$=\frac{1}{(\overline{q}-q_{\min})^2}\left(-\mathcal{M}(q_{\min})(\overline{q}-q_{\min})+\int_{q_{\min}}^{\overline{q}}\mathcal{M}(x)dx\right)\frac{dq_{\min}}{d\kappa} \tag{A107}$$

$$=\frac{1}{\overline{q}-q_{\min}}\left(\mathcal{M}(q_{\min})-M\right)\frac{-dq_{\min}}{d\kappa}\geq 0. \tag{A108}$$

The last step follows from the fact that $\mathcal{M}(q)$ is non-decreasing and hence the maintenance cost at $q_{\min}$ is always higher than the average maintenance cost. □

D.8 Proof of Proposition 3

Proof. Once again, either $\frac{dq_{\min}}{dP_0} = 0$ or changes $q_{\min}$ can be expressed by differentiating

$$p(q_{\min}) - \mathcal{M}(q_{\min}) = rP_0, \tag{A109}$$

which now yields

$$\frac{dp(q_{\min})}{dP_0} + (p'(q_{\min}) - \mathcal{M}'(q_{\min}))\frac{dq_{\min}}{dP_0} = r. \tag{A110}$$

Free entry into building now requires, applying the Leibnitz rule,

$$\frac{1}{\delta}\int_{q_{\min}}^{\overline{q}} \exp\left(-r\frac{\overline{q}-q}{\delta}\right)\frac{dp(q)}{dP_0}dq = 1 - \exp\left(-r\frac{\overline{q}-q_{\min}}{\delta}\right). \tag{A111}$$

We start by showing $\frac{dq_{\min}}{dP_0} \leq 0$ and $\frac{dp(q)}{dP_0} > 0$. Suppose for a contradiction that $\frac{dq_{\min}}{dP_0} > 0$. Following the proof of Corollary 2, we can derive

$$\frac{dp(q)}{dP_0} = \frac{q_{\min}}{q}\left(\frac{dp(q_{\min})}{dP_0} - p'(q_{\min})\frac{dq_{\min}}{dP_0}\right) + \frac{1}{q}\int_{q_{\min}}^{q}\frac{d\tilde{w}(x)}{dP_0}dx. \tag{A112}$$

Substituting in using equation A111, we obtain

$$\frac{dp(q)}{dP_0} = \frac{q_{\min}}{q}\left(r - 2p'(q_{\min})\frac{dq_{\min}}{dP_0} + \mathcal{M}'(q_{\min})\frac{dq_{\min}}{dP_0}\right) + \frac{1}{q}\int_{q_{\min}}^{q}\frac{d\tilde{w}(x)}{dP_0}dx. \tag{A113}$$

Since we have assumed $\frac{dq_{\min}}{dP_0} \geq 0$, it follows that $\frac{d\tilde{w}(q)}{dP_0} \leq 0$. We can then bound

$$\frac{dp(q)}{dP_0} \leq \frac{q_{\min}}{q}r. \tag{A114}$$

We can use this to note that

$$\begin{aligned}
\frac{1}{\delta}\int_{q_{\min}}^{\overline{q}} \exp\left(-r\frac{\overline{q}-q}{\delta}\right)\frac{dp(q)}{dP_0}dq &\leq \frac{1}{\delta}\int_{q_{\min}}^{\overline{q}} \exp\left(-r\frac{\overline{q}-q}{\delta}\right)\frac{q_{\min}}{q}rdq &\text{(A115)}\\
&< \frac{1}{\delta}\int_{q_{\min}}^{\overline{q}} \exp\left(-r\frac{\overline{q}-q}{\delta}\right)rdq &\text{(A116)}\\
&= 1 - \exp\left(-r\frac{\overline{q}-q_{\min}}{\delta}\right) &\text{(A117)}
\end{aligned}$$

which contradicts equation A111. Hence $\frac{dq_{\min}}{dP_0} \leq 0$. It follows that $\frac{d\tilde{w}(q)}{dP_0} \geq 0$ and $\frac{dp(q_{\min})}{dP_0} \geq 0$

and since

$$\frac{dp(q)}{dP_0} = \frac{q_{\min}}{q}\left(\frac{dp(q_{\min})}{dP_0} - p'(q_{\min})\frac{dq_{\min}}{dP_0}\right) + \frac{1}{q}\int_{q_{\min}}^{q} \frac{d\tilde{w}(x)}{dP_0}dx, \tag{A118}$$

that $\frac{dp(q)}{dP_0} > 0$.

Next, we show that

$$\frac{dq_{\min}}{dP_0} \geq \frac{dq_{\min}}{d\kappa}. \tag{A119}$$

For a contradiction, suppose otherwise, that

$$\frac{dq_{\min}}{dP_0} < \frac{dq_{\min}}{d\kappa}. \tag{A120}$$

Then we have

$$\frac{d\tilde{w}(q)}{dP_0} \geq \frac{d\tilde{w}(q)}{dP_0}. \tag{A121}$$

Using the fourth condition from Lemma 1 and equation A110, we also have

$$\frac{dp(q_{\min})}{dP_0} \geq \frac{dp(q_{\min})}{d\kappa}. \tag{A122}$$

Comparing equation A113 with the equation

$$\frac{dp(q)}{d\kappa} = \frac{q_{\min}}{q}\left(\frac{dp(q_{\min})}{d\kappa} - p'(q_{\min})\frac{dq_{\min}}{d\kappa}\right) + \frac{1}{q}\int_{q_{\min}}^{q} \frac{d\tilde{w}(x)}{d\kappa}dx. \tag{A123}$$

from the proof of Corollary 2 then yields

$$\frac{dp(q)}{dP_0} \geq \frac{dp(q)}{d\kappa}. \tag{A124}$$

But this implies

$$\frac{1}{\delta}\int_{q_{\min}}^{\overline{q}} \exp\left(-r\frac{\overline{q}-q}{\delta}\right)\frac{dp(q)}{dP_0}dq \tag{A125}$$

$$\geq \frac{1}{\delta}\int_{q_{\min}}^{\overline{q}} \exp\left(-r\frac{\overline{q}-q}{\delta}\right)\frac{dp(q)}{d\kappa}dq \tag{A126}$$

$$=1 \tag{A127}$$

$$>1-\exp\left(-r\frac{\overline{q}-q_{\min}}{\delta}\right), \tag{A128}$$

where the third equality follows from Lemma 1, contradicting equation A111.

For the last part of the proof, we note again that differentiating

$$p(q_{\min})-\mathcal{M}(q_{\min})=rP_0 \tag{A129}$$

yields

$$\frac{dp(q_{\min})}{dP_0}=\left(p'(q_{\min})-\mathcal{M}'(q_{\min})\right)\frac{-dq_{\min}}{dP_0}+r. \tag{A130}$$

From the proof of Corollary 2 we had that

$$\frac{d}{dq}\left(\frac{d\log p(q)}{d\log\kappa}\right)<0 \tag{A131}$$

$$\iff \frac{1}{f(\tilde{w}(q))}\frac{\overline{q}-q}{(\overline{q}-q_{\min})^2} \tag{A132}$$

$$<\frac{\tilde{w}(q)}{p(q)}\frac{1}{q}\left(q_{\min}\left(2p'(q_{\min})-\mathcal{M}'(q_{\min})\right)+\int_{q_{\min}}^{q}\frac{1}{f(\tilde{w}(x))}\frac{\overline{q}-x}{(\overline{q}-q_{\min})^2}dx\right). \tag{A133}$$

Following the same proof, we find

$$\frac{d}{dq}\left(\frac{d\log p(q)}{d\log P_0}\right)<0 \tag{A134}$$

$$\iff \frac{1}{f(\tilde{w}(q))}\frac{\overline{q}-q}{(\overline{q}-q_{\min})^2} \tag{A135}$$

$$<\frac{\tilde{w}(q)}{p(q)}\frac{1}{q}\Big(q_{\min}\left(2p'(q_{\min})-\mathcal{M}'(q_{\min})+\frac{r}{-dq_{\min}/dP_0}\right) \tag{A136}$$

$$+\int_{q_{\min}}^{q}\frac{1}{f(\tilde{w}(x))}\frac{\overline{q}-x}{(\overline{q}-q_{\min})^2}dx\Big), \tag{A137}$$

a strictly weaker condition. □

D.9 Proof of Proposition 4

Proof. Landlords breaking even on development requires, applying the Leibniz rule,

$$\exp\left(-r\frac{\overline{q}-q_{\min}}{\delta}\right)\left(\frac{dq_{\min}}{d\underline{q}}\right)\left(\frac{p(q_{\min})-\mathcal{M}(q_{\min})}{\delta}-\frac{rP_0}{\delta}\right) \tag{A138}$$

$$=\frac{1}{\delta}\int_{q_{\min}}^{\overline{q}}\exp\left(-r\frac{\overline{q}-q}{\delta}\right)\frac{dp(q)}{d\underline{q}}dq. \tag{A139}$$

A change in $\underline{q}$ when $q_{\min}>\underline{q}$ has no effect. When

$$\left(\frac{p(q_{\min})-\mathcal{M}(q_{\min})}{\delta}-\frac{rP_0}{\delta}\right)>0, \tag{A140}$$

then we have $q_{\min}=\underline{q}$ so that

$$\frac{dq_{\min}}{d\underline{q}}=1>0\implies\frac{d\tilde{w}(q)}{d\underline{q}}\leq 0. \tag{A141}$$

and prices must increase on average.

Following the proof of Lemma 1, we obtain the differential equation

$$\frac{dp(q)}{d\underline{q}}=\frac{q_{\min}}{q}\left(\frac{dp(q_{\min})}{d\underline{q}}-p'(q_{\min})\right)+\frac{1}{q}\int_{q_{\min}}^{q}\frac{d\tilde{w}(x)}{d\underline{q}}dx. \tag{A142}$$

For prices to increase on average, the term

$$\left(\frac{dp(q_{\min})}{d\underline{q}}-p'(q_{\min})\right) \tag{A143}$$

must be positive and hence $dp(q_{\min})/d\underline{q}$. Locally at $q_{\min}$, we obtain

$$\frac{d}{dq}\left(\frac{d\log p(q)}{d\underline{q}}\right)\Bigg|_{q=q_{\min}}=\frac{1}{p(q_{\min})}\left(-\frac{1}{q_{\min}}\left(\frac{dp(q_{\min})}{d\underline{q}}-p'(q_{\min})\right)+\frac{1}{q_{\min}}\frac{d\tilde{w}(q_{\min})}{d\underline{q}}\right) \tag{A144}$$

$$-\frac{p'(q_{\min})}{p(q_{\min})^2}\frac{dp(q_{\min})}{d\underline{q}}\leq 0. \tag{A145}$$

□

E Additional Results

E.1 Housing Value $P(q)$

Proposition 5. *For every q, there exists $\overline{P}_0(q) \geq 0$ such that if $P_0 < \overline{P}_0(q)$, then*

$$\frac{d}{dq}\left(\frac{\log p(q)}{d\log\kappa}\right) < 0 \implies \frac{d}{dq}\left(\frac{\log P(q)}{d\log\kappa}\right) < 0. \tag{A146}$$

Proof. We wish to show that whenever $\frac{d}{dq}\left(\frac{\log p(q)}{d\log\kappa}\right) < 0$,

$$\frac{d}{dq}\left(\frac{\frac{dP(q)}{d\kappa}}{P(q)}\right) = \frac{\frac{d^2P(q)}{d\kappa dq}P(q) - \frac{dP(q)}{d\kappa}\frac{dP(q)}{dq}}{P(q)^2} < 0. \tag{A147}$$

Since the denominator $P(q)^2$ is strictly positive, the sign is entirely determined by the numerator. We therefore must show that

$$\frac{d^2P(q)}{d\kappa dq}P(q) - \frac{dP(q)}{d\kappa}\frac{dP(q)}{dq} < 0. \tag{A148}$$

First recall that

$$P(q) = \frac{1}{\delta}\int_{q_{min}}^{q} \exp\left(-r\frac{q-x}{\delta}\right)(p(x) - \mathcal{M}(x))dx + \exp\left(-r\frac{q-q_{min}}{\delta}\right)P_0 \tag{A149}$$

and that

$$\frac{dP(q)}{d\kappa} = \frac{1}{\delta}\int_{q_{\min}}^{q} \exp\left(-r\frac{q-x}{\delta}\right)\frac{dp(x)}{d\kappa}dx. \tag{A150}$$

We can derive

$$\frac{dP(q)}{dq} = \frac{1}{\delta}(p(q) - \mathcal{M}(q)) - \frac{r}{\delta}P(q) \tag{A151}$$

and

$$\frac{d^2P(q)}{d\kappa dq} = \frac{1}{\delta}\frac{dp(q)}{d\kappa} - \frac{r}{\delta}\frac{dP(q)}{d\kappa}. \tag{A152}$$

Substituting back into equation A148, we obtain

$$\left(\frac{1}{\delta}\frac{dp(q)}{d\kappa} - \frac{r}{\delta}\frac{dP(q)}{d\kappa}\right)P(q) - \frac{dP(q)}{d\kappa}\left(\frac{1}{\delta}(p(q) - \mathcal{M}(q)) - \frac{r}{\delta}P(q)\right) < 0 \tag{A153}$$

which is equivalent to

$$\frac{dp(q)}{d\kappa}P(q) - \frac{dP(q)}{d\kappa}(p(q) - \mathcal{M}(q)) < 0. \tag{A154}$$

Substituting in for $P(q)$ and $dP(q)/d\kappa$ yields the condition

$$\frac{1}{\delta}\int_{q_{min}}^{q} \exp\left(-r\frac{q-x}{\delta}\right)\left(\frac{dp(q)}{d\kappa}(p(x) - \mathcal{M}(x)) - \frac{dp(x)}{d\kappa}(p(q) - \mathcal{M}(q))\right)dx \tag{A155}$$

$$+\frac{dp(q)}{d\kappa}\exp\left(-r\frac{q-q_{min}}{\delta}\right)P_0 < 0. \tag{A156}$$

We first re-write the integrand as,

$$\frac{d\log p(q)}{d\kappa}p(q)(p(x) - \mathcal{M}(x)) - \frac{d\log p(x)}{d\kappa}p(x)(p(q) - \mathcal{M}(q). \tag{A157}$$

We note that

$$p(q)(p(x) - \mathcal{M}(x)) < p(x)(p(q) - \mathcal{M}(q)) \iff p(x)\mathcal{M}(q) < p(q)\mathcal{M}(x) \tag{A158}$$

which is true for $x < q$ because rental prices $p(q)$ are strictly increasing as shown in Section D.1 while maintenance costs $\mathcal{M}(q)$ are non-increasing.

Further, we note that

$$\frac{d}{dq}\left(\frac{d\log p(q)}{d\log\kappa}\right) < 0 \implies \frac{d\log p(q)}{d\kappa} < \frac{d\log p(x)}{d\kappa} \tag{A159}$$

for $x < q$.

Hence the integrand in equation A157 is strictly negative which implies that the integral in equation A155 is strictly negative. Therefore for every q, there exists $\overline{P}_0(q) \geq 0$ such that if $P_0 < \overline{P}_0(q)$, then

$$\frac{d}{dq}\left(\frac{\log p(q)}{d\log\kappa}\right) < 0 \implies \frac{d}{dq}\left(\frac{\log P(q)}{d\log\kappa}\right) < 0, \tag{A160}$$

which proves the proposition. □

F Quantitative Model

In this Section we present the remainder of the details of the quantitative model. Time is continuous, denoted by $t \geq 0$.

F.1 Households

There is a unit mass of households in the economy, endowed with identically and independently distributed period income $w_t \sim F$. We assume that the support of F is $[\underline{w}, \overline{w}]$, with derivative f. Households choose non-housing consumption c_t and rent a housing unit of quality q_t to solve

$$\max_{c_t, q_t} \; u(c_t, q_t) \tag{A161}$$

subject to the flow budget constraint

$$c_t + p_t(q_t) = w_t, \tag{A162}$$

where $p_t(\cdot)$ is the rental price schedule that households face. Households do not have access to a savings vehicle, hence their problem is static. We assume $u(c_t, p_t) = c_t^{\alpha} p_t^{\beta}$.

F.2 Housing

Housing units are of heterogeneous quality, $q \in [\underline{q}, \overline{q}]$. There is a distribution of housing supply, with cumulative density function defined by $H_t(q)$, and probability density function $h_t(q)$, which evolves through two margins, filtering and new construction. As highlighted in Section 5, the quantitative model features potential for construction at every quality level. Denoting the rate of new construction at time t of quality q as $C_t(q)$, the cost of new construction follows an inverse supply function

$$\mathcal{K}_t(C_t(q), q) = \kappa C_t(q) + \gamma(q) C_t(q)^{1+\lambda}. \tag{A163}$$

with $\kappa, \lambda > 0$ parameters fixed across qualities and $\gamma(q)$ potentially varying across qualities. Production of new construction is instantaneous.

The quantitative model features both an extensive and intensive margin role for maintenance. In particular we impose

$$\mathcal{D}(m_t, q) = \begin{cases} \infty & \text{if } m_t < \mathcal{M}(q), \\ \delta e^{-\phi(m_t - \mathcal{M}(q))} & \text{if } m_t \geq \mathcal{M}(q), \end{cases} \tag{A164}$$

where $\mathcal{M}(q)$ is differentiable, non-increasing and satisfies $\mathcal{M}(\overline{q}) = 0$. Housing quality there-

fore drifts downward according to

$$\dot{q} = -\mathcal{D}\big(m_t(q), q\big). \tag{A165}$$

When housing units reach $\underline{q}$ they may no longer provide housing services and are torn down at no cost.

F.3 Landlords

Landlords are endowed with a mass $L > 1$ of land. They choose whether to and by how much to maintain existing units, and what qualities of new construction to supply. In this model we focus on equilibria without homelessness, where the exogenous income of households is always sufficient to pay for rents. Hence we restrict our attention to the case where the mass of housing is equal to the mass of households. We denote the value of a unit of land with a unit of housing of quality q as $P_t(q)$, and the value of vacant plot of land as $\underline{P}_t$. Landlords discount the future at an exogenous rate r. As before, we denote the exogenous outside value of land as P_0. This will be equal to $\underline{P}_t$ in equilibrium.

F.3.1 Landlords' Maintenance Problem

The value of a housing unit of quality q, $P_t(q)$ now satisfies the Hamilton–Jacobi–Bellman equation

$$rP_t(q) = \mathbf{1}_{m<\mathcal{M}(q)}\Big(r\underline{P}_t - m\Big) + \mathbf{1}_{m\geq\mathcal{M}(q)}\Big(p_t(q) - m + \partial_t P_t(q) - \mathcal{D}(m,q)\,\partial_q P_t(q)\Big). \tag{A166}$$

In the case where maintenance is insufficient such that housing loses all quality instantaneously, the flow payoff is given by the flow value of land $r\underline{P}_t$ net of any maintenance m.[15] When maintenance is sufficient, the flow payoff consists of rental income $p_t(q)$, minus maintenance cost m, plus the capital gain $\partial_t P_t(q)$ from changes in the value of the housing unit over time, minus the capital loss $\mathcal{D}(m,q)\,\partial_q P_t(q)$ from the unit filtering down the quality ladder at rate $\mathcal{D}$. At an interior optimum, the first order condition is

$$1 = -\partial_m \mathcal{D}(m_t^*(q), q)\,\partial_q P_t(q) = \phi\,\mathcal{D}(m_t^*(q), q)\,\partial_q P_t(q). \tag{A167}$$

This equates the marginal cost of maintenance, one unit of the numéraire, to the marginal benefit; slowing depreciation by $-\mathcal{D}_m$ saves $\partial_q P_t(q)$ per unit of quality preserved. Solving

[15] The solution for m in this case is naturally 0.

for the optimal depreciation rate,

$$\mathcal{D}_t^*(q) \equiv \mathcal{D}(m_t^*(q), q) = \min\left(\delta,\ \frac{1}{\phi\, \partial_q P_t(q)}\right). \tag{A168}$$

The optimal depreciation rate is decreasing in $\partial_q P_t(q)$; landlords maintain more when the housing value gradient is steep. The constraint $\mathcal{D} \leq \delta$ binds when the value gradient is too flat to justify extra maintenance ($\phi\, \partial_q P_t(q) \leq 1/\delta$), in which case $m_t^*(q) = \mathcal{M}(q)$. Substituting (A164) into the first order condition,

$$m_t^*(q) - \mathcal{M}(q) = \max\left(0,\ \frac{1}{\phi} \ln\big(\phi\, \delta\, \partial_q P_t(q)\big)\right). \tag{A169}$$

Endogenous $q_{\min}$. At $q = q_{\min,t}$, the landlord is indifferent between maintaining and tearing down

$$P_t(q_{\min,t}) = \underline{P}_t. \tag{A170}$$

This pins down the endogenous minimal housing quality in the economy. Whenever $q_{\min,t} > \underline{q}$

$$p_t(q_{\min,t}) - \mathcal{M}(q_{\min,t}) = r\underline{P}_t - \partial_t P_t(q_{\min,t}). \tag{A171}$$

In steady state $\partial_t P = 0$, such that this condition reduces to $p(q_{\min}) - \mathcal{M}(q_{\min}) = r\underline{P}$. In the case where $q_{\min,t} = \underline{q}$ the analogue to (A171) is

$$p_t(q_{\min,t}) - \mathcal{M}(q_{\min,t}) \geq r\underline{P}_t - \partial_t P_t(q_{\min,t}). \tag{A172}$$

F.3.2 Landlord's Development Problem

A landlord with vacant land develops a quality q unit if the value exceeds the marginal cost of construction plus land value

$$P_t(q) \geq \mathcal{K}'(C_t(q), q) + \underline{P}_t. \tag{A173}$$

Free entry of landlords drives profits to zero at each quality where construction is active

$$P_t(q) - \underline{P}_t = \kappa + (1+\lambda)\gamma(q)\, C_t(q)^{\lambda} \quad \text{for } q \in (q_{b,t}, \overline{q}]. \tag{A174}$$

In equilibrium this pins down the price of land $\underline{P}_t = P_0$. Inverting (A174) yields the construction rate

$$C_t(q) = \left[\frac{P_t(q) - \underline{P}_t - \kappa}{(1+\lambda)\gamma(q)}\right]^{1/\lambda} \quad \text{for } q > q_{b,t}. \tag{A175}$$

Endogenous q_b. Below $q = q_{b,t}$, $P_t(q) - \underline{P}_t$ is insufficient to cover κ, and hence $C_t(q) = 0$. This determines the endogenous building quality cut-off

$$P_t(q_{b,t}) - \underline{P}_t = \kappa. \tag{A176}$$

F.4 Evolution of the Housing Distribution

Given the maintenance policy $m_t(q)$, which determines $\mathcal{D}_t^*(q)$, and the construction rate $C_t(q)$, the housing density $h_t(q)$ evolves according to the Kolmogorov Forward Equation

$$\partial_t h_t(q) = \partial_q\big[\mathcal{D}_t^*(q)\, h_t(q)\big] + C_t(q). \tag{A177}$$

The first term captures the inflow of units filtering down from qualities above and the outflow of units filtering to qualities below. The second term captures new construction. Note that the total stock is $\int_{q_{\min}}^{\overline{q}} h_t(q)\, dq = 1$ by the fact that we focus on equilibria without homelessness, where the exogenous income of households is always sufficient to pay for rents.

Steady State Density. In steady state $\partial_t h_t(q) = 0$ and (A177) integrates to

$$\mathcal{D}^*(q)\, h(q) = \int_q^{\overline{q}} C(s)\, ds, \tag{A178}$$

so the density at quality q is

$$h(q) = \frac{1}{\mathcal{D}^*(q)} \int_q^{\overline{q}} C(s)\, ds. \tag{A179}$$

Housing accumulates at qualities where filtering is slow.

Transition Density. During a transition, the maintenance policy and the resulting depreciation rate $\mathcal{D}_t^*(q)$ vary over time. To find the density of housing at quality q at time t, we track the arrival of past construction. Let $Q(u; t, q)$ denote the quality at historical time $u \leq t$ of a unit that is currently at quality q at time t. This satisfies the backward ordinary

differential equation

$$\frac{\partial Q(u;t,q)}{\partial u} = -\mathcal{D}_u^*(Q(u;t,q)), \quad \text{with terminal condition} \quad Q(t;t,q) = q. \tag{A180}$$

Let $\tau(q,s,t) \leq t$ denote the historical time at which this unit was originally constructed at quality $s > q$, implicitly defined by the intersection $Q(\tau(q,s,t);t,q) = s$. The density $h_t(q)$ is obtained by integrating the historical construction arrivals over all higher qualities $s \in (q, \overline{q}]$, adjusted for the time-varying rate of filtering

$$h_t(q) = \frac{1}{\mathcal{D}_t^*(q)} \int_q^{\overline{q}} C_{\tau(q,s,t)}(s) ds. \tag{A181}$$

In steady state, where filtering is constant over time, (A181) recovers the steady-state density (A179).

F.5 Market Clearing

Housing market clearing requires that at every quality q at which housing is occupied, the mass of households demanding quality above q equals the mass of housing above q

$$\int_{\underline{w}}^{\overline{w}} \mathbf{1}\{\tilde{q}_t(w) \geq q\} \, dF(w) = \int_q^{\overline{q}} h_t(s) \, ds \quad \text{for all } q \in [q_{\min,t}, \overline{q}]. \tag{A182}$$

Differentiating with respect to q yields

$$f(\tilde{w}_t(q)) \, \tilde{w}_t'(q) = h_t(q). \tag{A183}$$

F.6 Equilibrium

In this section we define the equilibrium of the quantitative model. The economy is characterized by a coupled system of differential equations: an ordinary differential equation for household assignment, a Hamilton-Jacobi-Bellman equation for housing unit values, and a Kolmogorov Forward Equation for the housing distribution. The definition of equilibrium is standard, however we highlight that we impose housing market clearing at all qualities. A stationary equilibrium is defined accordingly.

Definition 2 (Dynamic Equilibrium)**.** *Given an initial housing distribution $h_0(q)$, a dynamic equilibrium is a path of prices $\{p_t(q), \underline{P}_t\}_{t\geq 0}$, value functions $\{P_t(q)\}_{t\geq 0}$, policies $\{c_t(w), \tilde{q}_t(w), m_t^*(q), C_t(q)\}_{t\geq 0}$, cut-offs $\{q_{\min,t}, q_{b,t}\}_{t\geq 0}$, and a housing distribution $\{h_t(q)\}_{t\geq 0}$ such that for all $t \geq 0$:*

1. *Optimality*

   (a) *Households maximize utility, with demand* $\tilde{q}_t(w)$ *satisfying* (6) *whenever its inverse exists.*

   (b) $P_t(q)$ *solves the HJB equation* (A166)*, yielding the optimal maintenance policy* $m_t^*(q)$ *in* (A169)*.*

   (c) *Landlords satisfy the free-entry condition* (A174)*, yielding the construction rate* $C_t(q)$ *in* (A175)*.*

2. *Quality Cut-off Conditions.*

   (a) *The tear-down cut-off* $q_{\min,t}$ *satisfies conditions* (A170) *and* (A171) *or* (A170) *and* (A172) *when* $q_{\min,t} = \underline{q}$*.*

   (b) *The building cut-off* $q_{b,t}$ *satisfies* (A176)*.*

3. *Distributional Consistency.*

   (a) *Given the optimal maintenance and new construction policies, the housing density* $h_t(q)$ *evolves according to the KFE* (A177)*.*

4. *Market Clearing*

   (a) *The housing market clears at all qualities respecting* (A182)*.*

   (b) *The numéraire goods market clears by Walras' law.*

Definition 3 (Stationary Equilibrium)**.** *A stationary equilibrium is a dynamic equilibrium in which all aggregate variables, prices, policies, and distributions are time-invariant.*

F.7 Solution Algorithm

We now present the algorithm we use to solve the model transition. We discretize housing qualities and time into evenly spaced grids, with Δq and Δt steps respectively. The algorithm iterates on a path of price vectors $\{P_t(q)\}_{t=0}^T$. Each iteration consists of three steps:

1. Forward Pass (given $\{P_t\}$, compute $\{h_t\}$): Starting from $h_0 = h_{\text{ss}}^{\text{old}}$, use P_t to compute C_t and D_t, then evolve h_t forward with the discretized equation

$$h_t(q_i) = h_{t-1}(q_i) + \Delta t \left[\underbrace{C_t(q_i)\,\Delta q}_{\text{construction}} + \underbrace{\frac{D_{t-1}(q_{i+1})}{\Delta q}\, h_{t-1}(q_{i+1})}_{\text{filtering in}} - \underbrace{\frac{D_{t-1}(q_i)}{\Delta q}\, h_{t-1}(q_i)}_{\text{filtering out}} \right]. \quad \text{(A184)}$$

2. Static Equilibrium (given $\{h_t\}$, compute $\{p_t\}$): At each t, compute the assignment w_t using $w_t(q) = F_w^{-1}(H_t(q))$ and rental prices p_t from h_t.

3. Backward Pass (given $\{p_t, D_t, m_t\}$, compute $\{\tilde{P}_t\}$): Starting from $P_T = P_{\text{ss}}^{\text{new}}$, compute the implied asset values $\tilde{P}_t$ backward.

The residual is $G_t = \tilde{P}_t - P_t$, and P_t is updated via a damped fixed-point iteration.

F.8 Further Moments From Model Parameterization

Table A4: Further Model Moments

Diagnostic	Value
Rent–to–Income p.10	0.5176
Rent–to–Income p.25	0.4677
Rent–to–Income p.50	0.4319
Rent–to–Income p.75	0.3989
Rent–to–Income p.90	0.3595
Price p.25	3.116
Price p.50	6.157
Price p.75	10.7
Maint. Share p.25	0.1266
Maint. Share p.50	0.04454
Maint. Share p.75	0.01437
New Construction Quality p.25	1.94
New Construction Quality p.50	1.967
New Construction Quality p.75	1.987
Stock Bottom 10%	0.1046
Stock Bottom 25%	0.2713
Stock Top 25%	0.2008
New Construction Above Median	1.0
Lifespan New p.25	45.65
Lifespan New p.50	48.68
Lifespan New p.75	51.15