The Clean Transition in a Network Economy*

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Abstract

Governments increasingly provide sector-specific subsidies to support the adoption of clean technologies. Given that interventions in one sector may have knock-on effects on upstream and downstream sectors, how should governments optimally target these subsidies? To address this question, I develop a static model featuring emissions, climate damages, endogenous choice between clean and dirty technologies, and intermediate inputs in production. I find that the total effect of a sector-level intervention on emissions and welfare depends on a direct (i.e., non-network) channel and three network channels. I show that the presence of these network channels may cause well-meaning interventions to backfire and increase aggregate emissions. I calibrate my model to a bloc of 23 advanced OECD economies and find that network channels significantly dampen or reverse the direct channel of clean technology subsidies. As a result, accounting for the network meaningfully changes the set of sectors to target.

Keywords: Climate Change Mitigation, Production Networks, Industrial Policy
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1 Introduction

In pursuit of net zero emissions, many large economies have adopted spending-based measures to encourage the adoption and deployment of technologies that directly use little or no fossil fuels (henceforth clean technologies). For example, the US Inflation Reduction Act, signed into law in 2022, provides tax credits for electric vehicles and a wide range of renewable energy systems such as solar, wind, geothermal, and energy storage[1]. At the same time, economies face significant fiscal constraints due to elevated debt levels and weak growth prospects (IMF, 2023). Given the urgency of climate change mitigation and the pressure on public funds, how should governments optimally target these subsidies?

In this paper, I take an endogenous network perspective to answer this question. To motivate this approach, it is useful to discuss a simplified example. Consider the steel industry, which accounts for 7% of global carbon dioxide emissions (IEA, 2020). Traditionally, steel is produced by combining coal, iron ore and limestone in a blast furnace, releasing a significant amount of greenhouse gases as a byproduct[2]. An alternative production technology involves using hydrogen instead of coal, producing far fewer emissions at the point of production[3]. In this setting, how would a subsidy for hydrogen-based steel affect total emissions? First of all, the subsidy may encourage firms in the steel sector to change their production technology by switching from the traditional production method that uses coal to the hydrogen-based method. This would increase the steel sector's demand for hydrogen and reduce its demand for coal i.e., change the input-output linkages between the sectors. While this leads to a decrease in the emissions generated at the point of production in the steel sector, the change in aggregate emissions depends on the nature of hydrogen production. If the hydrogen is extracted from natural gas, the most common production technique today, then the adoption of hydrogen-based technology in the steel sector would lead to an increase in production in the hydrogen sector and, importantly, the emissions generated at the point of production in the hydrogen sector. These additional emissions may increase total emissions. If the hydrogen is produced via electrolysis[4] and, in turn, electricity is produced from renewables, then the increase in hydrogen demand would lead to no change in the emissions generated at the point of production in upstream sectors and, thus, a decrease in total emissions. This example highlights how the structure of an economy's production network is not only shaped by technology subsidies, but also mediates their effects.

With this intuition in mind, I develop a novel and analytically tractable model that nests an endogenous production network inside a simple integrated assessment model. To the best

[1] https://www.epa.gov/green-power-markets/summary-inflation-reduction-act-provisions-related-renewable-energy

[2] Coal plays three roles in steel-making: it is burned to generate high temperatures, it acts a chemical reductant to extract the iron from iron ore, and it provides the carbon that alloys with iron to produce steel

[3] This process is referred to as hydrogen-based direct reduction of iron (H2-DRI). In this process, hydrogen acts as the fuel to power the process, and as the chemical reductant to extract the iron from the iron ore

[4] Electrolysis involves the passing of an electrical current through water to extract hydrogen and oxygen

of my knowledge, my paper is the first to bring an endogenous network framework to study climate change mitigation policy, specifically green industrial policy. The model is static and considers the global economy as a single entity. There is a representative household, a final good sector, multiple intermediate sectors (including fossil fuel sectors), a single factor of production (labour), and a balanced-budget government. The representative household supplies labour inelastically and consumes the final good. The final good is produced by a representative firm that combines the output from intermediate sectors. Within each intermediate sector, there are a continuum of heterogeneous firms that produce firm-specific varieties that are imperfect substitutes. These firm-specific varieties are combined into a composite sectoral good that can be sold as an intermediate input to other intermediate sectors or the final good sector. The firms in each intermediate sector make an endogenous choice whether to produce with a sector-specific dirty technology or a sector-specific clean technology[5]. Both technologies combine labour with intermediate inputs via a Cobb-Douglas production function, but differ in the set of inputs used. Specifically, the input set for each sector-specific dirty technology includes fossil fuels while the sector-specific clean technologies do not. As firms are heterogeneous, they may make different technology choices allowing for both technologies to co-exist in a sector. The government provides sector- and technology-specific ad valorem subsidies, and balances its budget via a lump sum tax on the representative household. Finally, on the climate side, the use of fossil fuels as an intermediate input generates emissions. For simplicity, I model the negative effect of these emissions (henceforth, climate damages or, simply, damages) as a loss of the final good.

Within this setup, I characterise the effect of marginal changes in sector-specific subsidies for clean technologies on technology choices, emissions, and aggregate output. My theoretical findings are threefold. First, I show that aggregate TFP can be decomposed into two parts: damage-related efficiency, which accounts for the direct damage from emissions to productivity, and non-damage efficiency, which accounts for technical and allocative efficiency given the level of damages.

Second, I show that the impact of a sector-specific subsidy on total emissions can be decomposed into a direct channel (i.e., non-network) and three network channels, which capture different ways by which the subsidy change propagates along intersectoral linkages. The intuition behind the network channels is as follows. The subsidy induces a reduction in marginal cost in the target sector that propagates downstream along existing input-output linkages. Sectors with the largest direct and indirect exposure to the target sector, given the existing production network, experience the largest decrease in their marginal cost. The subsidy and resulting change in sectoral prices affect the relative competitiveness of the clean and dirty

[5]For instance, in the steel sector the dirty technology may correspond to coal-based production while the clean technology corresponds to hydrogen-based production; in the electricity sector the dirty technology may correspond to natural gas-based generation while the clean technology corresponds to solar power; in the heavy duty transport sector the dirty technology may correspond to diesel-based vehicles while the clean technology corresponds to hydrogen-based vehicles.

technologies across sectors, inducing firms to change their technology choices. The direction of this effect in a given sector (i.e., whether it induces an increase or decrease in the share of firms in the sector that use the clean technology) depends on the direct and indirect reliance of each sector-specific technology on the target sector, while the size of the effect depends on the mass of firms in the sector at the margin of switching between the two technologies. The changes in technology (and input) choices then propagate upstream affecting the total demand for fossil fuels and, thus, the level of emissions. While the direct channel is generally negative (i.e., a reduction in total emissions), the sign and magnitude of the network channels are ambiguous as they depend on the interplay of firm technology choices, the input requirements of different technologies, and the existing structure of production. To illustrate the importance of these network channels, I provide several stylised examples where an increase in a sector-specific clean technology subsidy can, perversely, increase total emissions.

Third, and finally, I show that the change in aggregate output depends on the change in emissions as well as the change in non-damage efficiency. The latter captures how changes in subsidies interact with existing subsidies. I discuss several examples where an increase in a clean technology subsidy generates a double dividend i.e., a reduction in emissions and an improvement in non-damage efficiency. This analysis suggests that a policymaker interested in improving welfare should exploit the network structure to find interventions with the largest improvement in aggregate output, taking into account the changes in both damage-related and non-damage efficiency.

I then quantify my results by calibrating my model to a bloc of 23 advanced economies. I use the OECD's Inter-Country Input-Output tables to derive plausible values for the input shares for each sector-specific clean and dirty technology. For each sector, I calibrate the clean (dirty) technology input shares to those of the country with the lowest (highest) expenditure share on fossil fuel sectors. My quantitative analysis reveals three further results. First, the network channels are quantitatively significant and lead to a dampening or reversal of the direct channel of a subsidy increase for 24 of the 27 industrial sectors under study. Second, the semi-elasticity of total emissions with respect to a subsidy in a given sector is poorly correlated with other sectoral characteristics such as value added or Domar weights, underlining the importance of explicitly modelling technology choice and intermediate inputs in production for evaluating policy. Finally, accounting for network channels changes the set of sectors that a social planner should target. In fact, some sectors which would be targeted when not considering network channels would actually yield an increase in total emissions.

1.1 Literature

In this subsection, I describe how my paper builds on, and contributes to, several areas of research. First, my work relates to the broad literature on climate change mitigation. This literature typically studies the impact of carbon taxes using Integrated Assessment Models

(IAMs) - general equilibrium models featuring an economy, emissions, and damages - on aggregate emissions, output, and welfare. The most closely related studies include Cavalcanti et al. (2022) and King et al. (2019), both of which consider the impact of carbon taxation in a production network. Cavalcanti et al. study the aggregate and distributional impacts of a carbon tax while King et al. study the impact of marginal changes to sector-specific carbon taxes. I contribute to this literature in two keys ways. First, I focus on green industrial policy (i.e., subsidies to encourage clean technology adoption) rather than carbon taxes. Second, I investigate the impact of such policies within an endogenous production network, which contrasts with the exogenous production structure assumed in prior work. The endogeneity in my model arises from firms' technology choices, which are shown to play an important role in determining the effects of green industrial policy on aggregate outcomes.

Second, this paper relates to the literature on the propagation of shocks along input-output linkages. Early studies focused on the transmission of sector-level productivity shocks to aggregate output e.g., Acemoglu et al. (2012), Baqaee and Farhi (2019), and Carvalho et al. (2021)). More recent work has studied how sector-level distortions compound along intersectoral linkages to depress aggregate productivity. Bigio and La'O (2020) characterise how frictions compounded over the network, driving prices away from marginal cost and the economy away from productive efficiency. Similarly, Liu (2019) describes how the distortionary effects of market imperfections compound along backward demand linkages, leading to the largest distortions in upstream sectors. Baqaee and Farhi (2020) decompose changes in output into changes in technical and allocative efficiency. This allows the authors to trace out how distortions mediate the impact of productivity shocks. I contribute to this literature by considering a setting with both sector-level distortions (i.e., technology subsidies) and aggregate-level distortions (i.e., negative externalities from emissions). I also show how aggregate TFP in such an economy can be decomposed into two components: non-damage efficiency, which corresponds to the existing results in the literature, and damage-related efficiency, which is a novel component related to the presence of emissions and climate damages.

Third, my work builds on research studying endogenous production networks. Early research in production networks took the structure of input-output linkages as given (e.g., Acemoglu et al. (2012), Acemoglu et al. (2015), Baqaee and Farhi (2019), Carvalho et al. (2021))[6]. More recent studies consider the formation of intersectoral linkages and how this process influences macroeconomic outcomes. For example, Lim (2018) and Huneeus (2020) explore how frictions to the formation and modification of customer-supplier relationships affect the propagation of shocks. The closest paper in this literature to my work is Acemoglu and Azar (2020). The authors consider a multi-sector economy with endogeneous choice over the set of intermediate inputs used in production. They then study how changes in productivity or distortions affect the structure of production, specifically the density of the network. My paper differs along several dimensions. First, I study a continuum of heterogeneous firms in each sector rather

[6]See survey by Carvalho and Tahbaz-Salehi (2019) for further discussion

than representative firms. This allows for multiple technologies to co-exist in my framework. Second, I include a climate module to track emissions and damages. Third, I study the impact of sector- and technology-specific subsidies while Acemoglu and Azar consider only sector-level distortions. Further, I focus on emissions and aggregate output rather than the density of the production network.

Fourth, I contribute to the recent literature on industrial policy in production networks. For example, Liu (2019) shows how governments should target subsidies in a production network featuring sector-level distortions. He shows that sector-level subsidies can be a second-best tool to address the aggregate consequences of these distortions and that such subsidies should be targeted at the sectors that are furthest away from their efficient level of production. Buera and Trachter (2024) extend Liu's results by studying how firm-level technology adoption decisions shape the aggregate effects of sector level subsidies. I contribute to this field by studying green industrial policy i.e., the role of subsidies for the adoption of clean technologies in the presence of emissions and climate damages. I show that welfare depends on how subsidies influence aggregate emissions (the damage-related component of aggregate TFP) as well as how the subsidy change interacts with existing subsidies to shape technical and allocative efficiency (the non-damage component of aggregate TFP).

Finally, to the best of my knowledge, there is one other contemporary paper that also considers green industrial policy in endogenous production networks: Aghion et al. (2024). The authors study green R&D subsidies in the presence of endogenous technical change in simple chains. My paper differs in several areas. Most importantly, while Aghion et al. employ a dynamic model with R&D subsidies, my analysis is based on a static model with production subsidies. This allows me to study any network structure, unlike the simplified chains considered by Aghion et al. Furthermore, my results reveal how changes in clean technology subsidies can interact with existing subsidies to amplify or dampen the aggregate welfare implications of subsidies.

1.2 Outline

The remainder of the paper is structured as follows. In Section 2, I describe the model. In Section 3, I characterise the equilibrium of the model. In Section 4, I present my key results, including a general characterisation of the effect of a marginal increase in a sector-specific technology subsidy on technology choices, emissions and aggregate output. This Section also includes several stylised examples to illustrate the channels and the potential for well-meaning clean technology subsidies to generate an increase in total emissions. In Section 5, I study how a social planner should target subsidies at the margin. Section 6 describes how I calibrate the model while Section 7 contains the results from my quantitative analysis. Section 8 concludes and discusses avenues for future research.

2 Model

In this section, I describe my model, which combines elements of integrated assessment models and endogenous production network models. I begin with an overview of the key features before explaining each element in more detail. At the end of the section, I define the equilibrium concepts to study this economy.

2.1 Overview

I model the global economy as a single entity composed of multiple intermediate sectors, indexed $i \in \mathcal{N}$, and a final good sector. A subset of intermediate sectors, $\mathcal{N}_F \subset \mathcal{N}$, are fossil fuel sectors. Within each sector, there is a continuum of firms that can choose to produce their firm-specific variety with a dirty technology $\theta = d$, which combines labour with intermediate inputs including some fossil fuel inputs, or a clean technology $\theta = c$, which combines labour with a different set of intermediate inputs. Importantly, clean technologies do not use any fossil fuels.

The use of fossil fuels generates emissions, which are a global negative externality. For simplicity and tractability, I model the negative effect of emissions as a reduction in labour productivity across the economy. To focus on the role of the production network and technology choice in shaping aggregate outcomes, I model the consumption side of the economy with a representative household. There are an exogenous set of sector- and technology- specific subsidies, σ_i^θ, which will ultimately influence technology choices, emissions and aggregate output. There is a lump sum tax levied on the household to balance the government's budget. In the following subsections, I discuss each element of the model.

2.2 Climate

In my model, the use of fossil fuels generates emissions, which impose a negative externality on the economy. I assume the quantity of emissions generated by the use of one unit of fossil fuel $i \in N_F$ is exogenous, homogeneous across sector of use, and denoted by ϵ_i. Thus, the total level of emissions E can be expressed as

$$E = \sum_{i \in \mathcal{N}_F} \epsilon_i Y_i \tag{1}$$

where Y_i is the output of fossil fuel sector i. This specification allows for heterogeneity in emissions content across different fossil fuels, such as coal, oil and natural gas.

I model the negative effect of emissions as a loss of the final good. This can be interpreted as the negative impact of high temperature and extreme weather events on production. Specifically, the proportion of the final good lost D is defined as

$$D = \delta(E), \tag{2}$$

where $\delta(E)$ is continuous, differentiable, and increasing in emissions E. I normalise $\delta(0) = 0$ and assume damages approach 100% as emissions rise, $\lim_{E\to\infty} \delta(E) = 1$.

2.3 Production

2.3.1 Firms and Technologies

Each sector is composed of a unit mass of firms, indexed $v \in [0, 1]$. Each firm produces a firm-specific variety that is imperfectly substitutable. For example, consider the steel industry where each firm may produce different qualities of steel or the automotive sector where firms produce different styles of vehicles. To focus on the role of the network, I assume markets are perfectly competitive and model firms as profit-maximising price-takers.

Each firm can produce with a sector-specific dirty technology or a sector-specific clean technology, denoted by $\theta = d$ and $\theta = c$, respectively. These technologies combine labour and intermediate inputs in production but, importantly, differ in the set of intermediate inputs used. Specifically, the dirty technology uses some fossil fuel inputs, while the clean technology does not.

I now provide some examples to illustrate this setup. In the hydrogen industry, the dirty technology could refer to the process of natural gas reformation, which combines natural gas with high-temperature steam to produce hydrogen, while the clean technology could refer to the process of electrolysis, where an electric current splits water into hydrogen and oxygen. In the case of cement production, the dirty technology could refer to the process of using coal to heat a kiln with limestone and gypsum, while the clean technology could refer to a production process using hydrogen or biofuels to power the kiln. Finally, in the steel sector, the dirty technology can denote the traditional method of using coal to power the blast furnace, while the clean technology could refer to the use of hydrogen instead of coal to power the process.

For analytical tractability, I assume both technologies are Cobb-Douglas with constant returns to scale. Formally, the production function for variety v in sector i using technology $\theta \in \{c, d\}$ is

$$y_i^\theta(v) = \frac{1}{A_i^\theta} q_i^\theta(v) \left(l_i^\theta(v)\right)^{\tilde{a}_i^\theta} \prod_{j\in\mathcal{N}} \left(x_{ij}^\theta(v)\right)^{\tilde{m}_{ij}^\theta} \tag{3}$$

where A_i^θ is a normalising contant[7], $q_i^\theta(v)$ is a firm- and technology-specific Hicks-neutral productivity shifter, $l_i^\theta(v)$ is the quantity of labour employed by firm v, $x_{ij}^\theta(v)$ is the quantity of good j purchased by firm v, $\tilde{a}_i^\theta \in (0, 1]$ is the labour share for technology θ, $\tilde{m}_{ij}^\theta \in [0, 1)$ is the intermediate input share of good j for technology θ, and $a_i^\theta + \sum_j \tilde{m}_{ij}^\theta = 1 \,\forall\, i, \theta$. Within each sector, the labour and intermediate input shares differ across technologies, reflecting differences in the inputs required for different production processes.

[7]The constant is defined as $A_i^\theta \equiv \left(\tilde{a}_i^\theta\right)^{\tilde{a}_i^\theta} \prod_{j\in\mathcal{N}} \left(\tilde{m}_{ij}^\theta\right)^{\tilde{m}_{ij}^\theta}$. This simplifies the subsequent algebra, but does not affect any of my results.

2.3.2 Technology Choice

Given that production is subject to sector- and technology-specific subsidies, denoted by σ_i^θ, the marginal cost $\kappa_i^\theta(v)$ for variety v in sector i using technology θ is given by

$$\kappa_i^\theta(v) \equiv \frac{1-\sigma_i^\theta}{q_i^\theta(v)} W^{\tilde{a}_i^\theta} \prod_{j\in\mathcal{N}} P_j^{\tilde{m}_{ij}^\theta} \tag{4}$$

where W is the wage and P_j is the price of good j.

Profit maximisation implies that the representative firm in each variety will choose the technology with a lower marginal cost

$$\theta_i^*(v) = \arg\min_{\theta\in\Theta_i} \kappa_i^\theta(v), \tag{5}$$

and that price $p_i(v)$ equals marginal cost

$$p_i(v) = \min_{\theta\in\Theta_i} \kappa_i^\theta(v) \equiv \kappa_i^{\theta_i^*(v)}(v). \tag{6}$$

Let $\gamma_i(v) \equiv q_i^d(v)/q_i^c(v)$ denote the technology gap between the dirty and clean technologies for firm iv. We can re-express a firm's technology choice, $\theta_i^*(v)$, as a threshold function

$$\theta_i^*(v) = \begin{cases} c \text{ if } \log\gamma_i(v) < \log\Gamma_i \\ d \text{ otherwise} \end{cases} \tag{7}$$

where

$$\log\Gamma_i \equiv \sum_j (\tilde{m}_{ij}^d - \tilde{m}_{ij}^c)\log\overline{P}_j + \big(\log(1-\sigma_i^d) - \log(1-\sigma_i^c)\big), \tag{8}$$

denotes the competitiveness threshold in sector i and

$$\overline{P}_i \equiv P_i/W \tag{9}$$

denotes the normalised sectoral price in sector i. In words, when the technology gap $\gamma_i(v)$ is sufficiently small (i.e., the productivity shifter of the clean technology for variety iv is sufficiently large relative to that of the dirty technology), the firm chooses the clean technology.

Equation (8) reveals that the threshold depends on subsidy rate for each technology as well as the (normalised) price of the intermediate input bundle required for each technology. Note that the absolute level of the wage has no impact on technology choice after accounting for normalised prices. Intuitively, we get this result as labour is the only factor of production. This means all inputs are embodied labour. If a variety is produced via a technology with a low labour input share, $\tilde{a}_i^\theta$, then it is exposed to the wage via the labour embodied in its high intermediate input share. This means all sector-technology pairs are equally exposed to labour

when we factor in their direct and indirect exposure to labour. As a result, a change in the wage does not change the relative competitiveness of the clean and dirty technology in a given sector.

2.3.3 Clean Technology Share

Let G_i denote the share of firms in sector i that use the clean technology. Formally, this is

$$G_i \equiv \int_0^1 \mathbb{1}(\theta_i^*(v) = c)\, dv. \tag{10}$$

As with the technology choice decision, it is useful to re-express this in terms of technology gaps. Let $\mathcal{G}_i(X) \equiv \int_0^1 \mathbb{1}(\log \gamma_i(v) < X)\, dv$ denote the cumulative distribution function of the technology gaps in sector i. Then (10) can be re-written as

$$G_i = \mathcal{G}_i(\log \Gamma_i). \tag{11}$$

2.3.4 Sectoral Goods

In each sector, the unit mass of varieties are aggregated into a composite good that is sold to other sectors as an intermediate input in production. I model this aggregation with a price-taking sectoral good producer using the following Cobb-Douglas production function

$$\log Y_i = \int_0^1 \log x_i(v)\, dv \tag{12}$$

where Y_i denotes the output of good i, and $x_i(v)$ is the quantity of variety iv.

2.3.5 Final Good

There is a competitive final good sector populated by a representative firm with the following constant returns to scale Cobb-Douglas production function[8]

$$Y = (1 - D)\frac{\prod_{i \in \mathcal{N}} X_i^{b_i}}{\prod_{i \in \mathcal{N}} b_i^{b_i}} \tag{13}$$

where Y is the output of the final good, D is the proportion of the final good lost due to climate damages (defined in (2)), X_i is the quantity of sectoral good i, b_i denotes the share of sectoral good i, and $\sum_i b_i = 1$.

[8]The denominator $\prod_{i \in \mathcal{N}} b_i^{b_i}$ simplifies the algebra, but does not change any results

2.4 Consumption

As the focus of this paper is the role of the network in mediating policy interventions, I simplify the consumption side of my model. There is a representative household endowed with L units of labour, which are supplied inelastically. Preferences are given by

$$U = \log C \tag{14}$$

where C is consumption of the final good. Setting the final good as the numeraire, the household budget constraint is given by

$$C \leq WL - T \tag{15}$$

where W is the wage and T is the lump sum tax to balance the government's budget. Note that the household does not earn any profits as markets are perfectly competitive throughout the model.

2.5 Equilibrium Concept

I end this section by defining the equilibrium concept I will use to study this economy.

Definition 1 (Equilibrium)**.** *An equilibrium is a set of prices, quantities, technologies choices, emissions and damages such that:*

- *(Household) The household chooses consumption C to maximise utility (14) subject to its budget constraint (15)*
- *(Final Good) The final good producer chooses quantities $\{X_i\}_{i\in\mathcal{N}}$ to maximise profit given sectoral prices $\{P_i\}$*
- *(Sectoral Goods) The sectoral good producers choose quantities $x_i(v)$ to maximise profit given variety prices $p_i(v)$*
- *(Varieties) Firms choose a technology and quantities to minimise cost given the wage W, sectoral good prices $\{P_i\}$, and damages D*
- *(Emissions) Emissions E satisfies (1)*
- *(Damages) Damages D satisfies (2)*
- *(Market clearing) The quantities satisfy market clearing for varieties, sectoral goods, the final good, and labour*

$$x_i(v) = y_i(v) \tag{16}$$

$$X_i + \sum_{j\in N} \int_0^1 x_{ji}(v)\, dv = Y_i \tag{17}$$

$$C = Y \tag{18}$$

$$\sum_{i \in N} \int_0^1 l_i(v)\, dv = L \tag{19}$$

3 Equilibrium Characterisation

In this section, I characterise the equilibrium of my model. The section is organised as follows. First, following Baqaee and Farhi (2020), I introduce some useful notation that aggregates firm-level decisions to sector- and aggregate-level. After this, I establish the existence and uniqueness of equilibrium. There are several challenges to establish this result, which I discuss in the subsection. Following this, I characterise the equilibrium relationships between prices, quantities, and emissions in terms of the equilibrium values of other endogenous objects. I finish the section with a decomposition of aggregate output and TFP into damage-related and non-damage components. Taken together, these results provide the foundation for me to study the impact of subsidy changes in Section 4. All proofs are relegated to the Appendix.

3.1 Useful Notation

I now introduce some notation to aggregate firm-level decisions and quantities to sector-level. First, let Q_i denote the average productivity shifter in sector i

$$Q_i \equiv \int_0^1 \log q_i^{\theta_i^*(v)}(v)\, dv. \tag{20}$$

Note that while the productivity shifters for each variety-technology pair are exogenous, the average productivity shifter in a sector is an endogenous variable that depends on the technology choices of firms. Next, let S_i denote the average subsidy rate in sector i

$$\log(1 - S_i) \equiv \int_0^1 \log(1 - \sigma_i^{\theta_i^*(v)}(v))\, dv. \tag{21}$$

Similarly to sectoral productivity, average markdowns are an endogenous quantity that depend on the exogenous subsidy rates, given by σ_i^d and σ_i^c, and the endogenous technology choice of firms.

Next, let ξ denote the wage premium. This object captures the difference between the wage and the average product of labour, which is equivalent to the firm-level pre-subsidy marginal product of labour. Formally, the wage premium is defined as

$$1 + \xi \equiv \frac{W}{Y/L}. \tag{22}$$

Following Baqaee and Farhi (2020), I introduce some notation to describe the economy's equilibrium production network. First, let the $N \times N$ matrix $\tilde{\boldsymbol{M}}^{\theta} \equiv [\tilde{m}_{ij}^{\theta}]$ denote the cost-based intermediate input shares matrix for technology θ. Next, let $\tilde{\boldsymbol{\Omega}}_{N\times N}$ denote the economy's cost-based input-output matrix, where

$$\tilde{\Omega}_{ij} = \int_0^1 \tilde{m}_{ij}^{\theta_i^*(v)} \, dv. \tag{23}$$

I define the economy's cost-based Leontief inverse matrix as $\tilde{\boldsymbol{\Psi}}$ where

$$\tilde{\boldsymbol{\Psi}} = (\mathbf{I} - \tilde{\boldsymbol{\Omega}})^{-1}. \tag{24}$$

Intuitively, each element $\tilde{\Psi}_{ij}$ denotes the importance of sector j as a supplier to sector i given the existing production network. Finally, let the $N \times 1$ column vector $\tilde{\boldsymbol{\lambda}}$ denote the economy's cost-based Domar weights, where

$$\tilde{\boldsymbol{\lambda}}' \equiv \mathbf{b}'\tilde{\boldsymbol{\Psi}} \equiv \mathbf{b}'(\mathbf{I} - \tilde{\boldsymbol{\Omega}})^{-1}. \tag{25}$$

This object measures the direct and indirect importance of each sector to the production of the final good.

I now define a similar set of revenue-based objects. These broadly describe the direct and indirect expenditure of a given sector on intermediate inputs from other sectors as a share of the former's revenue.

Let $N \times N$ matrix $\mathbf{M}^{\theta} \equiv (\mathbf{I} - \text{diag}(\boldsymbol{\sigma}^{\theta}))^{-1}\tilde{\mathbf{M}}^{\theta}$ denote the revenue-based intermediate input shares for technology θ i.e., the ijth element is the share of a firm's revenue in sector i spent on intermediate inputs from sector j. I use the $N \times N$ matrix $\boldsymbol{\Omega}$ to denote the economy's revenue-based input-output matrix, where each element is defined as

$$\Omega_{ij} \equiv \int_0^1 \frac{\tilde{m}_{ij}^{\theta_i^*(v)}}{1 - \sigma_i^{\theta_i^*(v)}} \, dv. \tag{26}$$

This aggregates the unit of firms' expenditure on good j to find the share of sector i's revenue spent directly on good j. I define the economy's revenue-based Leontief inverse matrix as $\boldsymbol{\Psi}$ where

$$\boldsymbol{\Psi} = (\mathbf{I} - \boldsymbol{\Omega})^{-1}. \tag{27}$$

Each element measures the direct and indirect expenditure of sector i on sector j. Finally, let the $N \times 1$ column vector $\boldsymbol{\lambda}$ denote the economy's (revenue-based) Domar weights, defined as

$$\boldsymbol{\lambda}' \equiv \mathbf{b}'\boldsymbol{\Psi} \equiv \mathbf{b}'(\mathbf{I} - \boldsymbol{\Omega})^{-1}. \tag{28}$$

The jth element of this vector measures the direct and indirect expenditure of the final good

sector on sector j.

The next set of notation describes the direct and indirect reliance of sector-specific technology on other sectors given the technology's input requirements and the existing structure of the production network. Let the $N \times N$ matrix $\tilde{\mathbf{\Psi}}^{\theta} \equiv \tilde{\mathbf{M}}^{\theta}\mathbf{\Psi}$ denotes the cost-based Leontief inverse via technology θ. Intuitively, the ijth entry of this matrix measures the elasticity of the marginal cost of technology θ in sector i with respect to the price of good j given the equilibrium production network. Similarly, let the $N \times N$ matrix $\mathbf{\Psi}^{\theta} \equiv \mathbf{M}^{\theta}\mathbf{\Psi}$ denote the revenue-based Leontief inverse via technology θ. The ijth entry of this matrix measures the direct and indirect expenditure on sector j if a firm uses technologyθ in sector i

Finally, I introduce notation to extract a specific row or column vector from the matrix objects above. For example, consider the $N \times N$ revenue-based Leontief inverse matrix $\mathbf{\Psi}$. I use $\mathbf{\Psi}^{\theta}_{k\bullet}$ to denote the $1 \times N$ row vector equal to the kth row of $\mathbf{\Psi}^{\theta}$. Similarly, I use $\mathbf{\Psi}^{\theta}_{\bullet k}$ to denote the $N \times 1$ column vector equal to the kth column of $\mathbf{\Psi}^{\theta}$.

3.2 Existence and Uniqueness

There are two key challenges in establishing these results. First of all, the availability of different technologies creates a discrete choice problem in each variety alongside the continuous choice problem over the quantity of labour and intermediate inputs. Second, there is feedback between prices, technology choice, emissions, and output. Despite this, I show that an equilibrium exists and is unique.

Theorem 1 (Existence and uniqueness of equilibrium)**.** *Given the exogenous set of productivity shifters* $\{q_i^{\theta}(v)\}_{i\in\mathcal{N},\theta\in\{c,d\}}$*, subsidy rates* $\{\sigma_i^{\theta}\}_{i\in\mathcal{N},\theta\in\{c,d\}}$*, and technology input requirements* $\{\tilde{a}_i^{\theta}, \tilde{m}_{ij}^{\theta}\}_{i,j\in\mathcal{N},\theta\in\{c,d\}}$*, there exists a unique equilibrium that satisfies Definition* (1)*.*

The intuition for this result can be broken down into two steps. First, I show that there is a unique set of normalised sectoral prices that satisfy equilibrium. Intuitively, this is the problem remains convex despite the discrete choice of technology. This derives from the assumptions of constant returns to scale and Cobb-Douglas production. On a technical note, since the sectoral prices depend on firm-variety prices, which in turn depend on sectoral prices, firms' optimisation behaviour creates a mapping from sectoral prices onto itself. I show that this mapping is a contraction mapping, guaranteeing existence and uniqueness of an equilibrium set of prices.

The second step of the proof maps these normalised prices to all the other endogenous variables. For a given set of equilibrium normalised prices, there is a unique set of technology choices. This pins down the economy's production network from which we can derive all other elements of an equilibrium for a given level of emissions.

3.3 Equilibrium Relationships

As part of the proof for the existence and uniqueness of equilibrium, I derive characterisations of key endogenous variables in terms of the equilibrium values of other endogenous variables. I now discuss these corollaries in turn in order to support the discussion of my comparative statics in Section 4.

Corollary 1 (Equilibrium normalised prices)**.** *Given the equilibrium values for sectoral productivity shifters* $\boldsymbol{Q}$*, sectoral markdowns* $(1-\boldsymbol{S})$*, and cost-based Leontief matrix* $\tilde{\boldsymbol{\Psi}}$*, the equilibrium set of normalised prices is given by*

$$\log \overline{\boldsymbol{P}} = \tilde{\boldsymbol{\Psi}}(-\log \boldsymbol{Q} + \log(1-\boldsymbol{S})) \tag{29}$$

This result reveals that the normalised price of a good is a function of the average productivity shifter and markdown in its sector as well as the productivity shifters and markdowns of its direct and indirect inputs, as measured by the economy's cost-based Leontief inverse matrix. Each element of this matrix, $\tilde{\Psi}_{ij}$, measures the importance of the price of good j to the price of good i.

The following two results characterise the equilibrium wage and its relationship with the average product of labour.

Corollary 2 (Equilibrium wage)**.** *Given the equilibrium level of damages* D*, cost-based Domar weights* $\tilde{\boldsymbol{\lambda}}$*, sectoral productivity shifters* $\boldsymbol{Q}$*, and sectoral markdowns* $(1-\boldsymbol{S})$*, the equilibrium wage is given by*

$$\log W = \log(1-D) + \tilde{\boldsymbol{\lambda}}'(\log \boldsymbol{Q} - \log(1-\boldsymbol{S})) \tag{30}$$

The intuition for this expression is as follows. Increases in average sectoral productivity shifters or subsidies increase the marginal revenue product of labour, bidding up the wage. The size of this effect depends on a sector's cost-based Domar weight $\tilde{\lambda}_i$. Intuitively, this measures the sector's direct and indirect importance to the final good sector and, therefore, aggregate output. On the other hand, the wage is decreasing in damages as higher damages reduce labour productivity.

The following corollary shows the determinants of the wage premium.

Corollary 3 (Equilibrium wage premium)**.** *Given the equilibrium values for clean technology shares* $\boldsymbol{G}$ *and revenue-based Domar weights* $\boldsymbol{\lambda}$*, the equilibrium wage premium is given by*

$$\xi = \sum_i \left(\lambda_i G_i \frac{\sigma_i^c}{1-\sigma_i^c} + \lambda_i (1-G_i) \frac{\sigma_i^d}{1-\sigma_i^d} \right) \tag{31}$$

Intuitively, this relationships occurs as subsidies raise the wage above the marginal product of labour. While subsidised sectors are able to afford these higher wages, other sectors reduce

their demand for labour. Thus subsidies distort the allocation of labour away from their most efficient use given the level of damages.

Next, I present an expression for the equilibrium level of emissions.

Corollary 4 (Equilibrium emissions)**.** *Given the equilibrium values for the revenue-based Domar weights* ($\boldsymbol{\lambda}$)*, normalised prices* ($\overline{\boldsymbol{P}}$)*, and the wage premium* ξ*, total emissions* (E) *is given by*

$$E = \sum_i E_i = \sum_i \frac{\lambda_i \epsilon_i}{\overline{P}_i(1+\xi)} L \tag{32}$$

The output of fossil fuel sectors, and thus emissions, depends on features of other sectors. For example, holding all else equal, an increase in the sectoral productivity shifter of an upstream sector to a fossil fuel sector will lead to an increase in fossil fuel output and emissions. Further, the Domar weight of a sector depends on the economy's input-output network. Therefore, technology and input choices in other sectors influences the Domar weight and, thus, size of fossil fuel sectors.

I finish this section with a characterisation of aggregate output in the following corollary.

Corollary 5 (Equilibrium aggregate output)**.** *Given the equilibrium values for damages* D*, sectoral productivity shifters* $\boldsymbol{Q}$*, sectoral markdowns* $(1 - \boldsymbol{S})$*, cost-based Domar weights* $\tilde{\boldsymbol{\lambda}}$*, and the wage premium* ξ*, equilibrium aggregate output is given by*

$$\log Y = \overbrace{\underbrace{\log(1-D)}_{\textit{Damage-related efficiency}} + \underbrace{\tilde{\boldsymbol{\lambda}}' \log \boldsymbol{Q} - \tilde{\boldsymbol{\lambda}}' \log(1-\boldsymbol{S}) - \log(1+\xi)}_{\textit{Non-damage efficiency}}}^{\textit{Aggregate TFP}} + \log L \tag{33}$$

The corollary above shows that aggregate TFP can be decomposed into two components: damage-related efficiency, which measures the negative impact of emissions on labour productivity across the economy, and non-damage efficiency, which measures how the allocation of labour and intermediate inputs across sectors influence output holding the level of damages constant. From the discussion in this section, it is easy to see that changes in exogenous subsidy rates will influence both components of TFP and, therefore, aggregate output. In the next section, I investigate this response in more detail.

4 Comparative Statics

In this section, I investigate the effect of a marginal change in the sector-specific subsidy rate for the clean technology. The first set of results show how prices, technology choices, Domar weights, and the wage premium change in response to the subsidy change. These results provide the building blocks for my two main results: Theorem 7, which characterises the impact on total emissions, and Theorem 8, which characterises the impact on aggregate output. Throughout, I show that subsidy changes propagate along intersectoral linkages to affect other sectors and,

ultimately, the total impact of the intervention. I provide several stylised examples to illustrate my results. As before, all proofs are relegated to the Appendix.

4.1 Normalised Prices

The following result describes how normalised prices, a key determinant of technology choice, change in response to a marginal change in a sector-specific clean technology subsidy.

Theorem 2 (Change in normalised prices)**.** *The semi-elasticity of the normalised price of sectoral good j ($\overline{P}_j$) with respect to the clean technology subsidy in sector $k \in \mathcal{N}$ is given by*

$$\frac{d \log \overline{P}_j}{d\sigma_k^c} = -\tilde{\Psi}_{jk} \frac{G_k}{1-\sigma_k^c}. \tag{34}$$

The result shows that the effect is entirely determined by the existing structure of the production network and the existing clean technology share in sector k. Specifically, the change in the normalised price of good j depends on its direct and indirect reliance on good k, as measured by the cost-based Leontief inverse matrix $\tilde{\boldsymbol{\Psi}}$, and the change in the price of good k due to the subsidy change, measured by $\frac{G_k}{1-\sigma_k^c}$.

The intuition is as follows. The marginal increase in the clean technology subsidy in sector k immediately reduces the price of good k given that the proportion G_k of firms in the sector use the clean technology. The reduction in the price of good k propagates to downstream sectors that use it as an intermediate input, either directly or indirectly. This propagation is captured by the cost-based Leontief inverse. While the change in prices causes some varieties to switch to another production technology, this switching has no effect on prices at the margin. Consequently, the effect on normalised prices is entirely determined by existing technology choices.

4.2 Technology Choices

I now discuss the implication of a sector-specific subsidy on technology choices and, thus, clean technology shares across the economy.

Theorem 3 (Change in technology shares)**.** *The derivative of the vector of clean technology shares ($\boldsymbol{G}$) with respect to the clean technology subsidy in sector $k \in \mathcal{N}$ is given by*

$$\frac{d\boldsymbol{G}}{d\sigma_k^c} = \underbrace{\boldsymbol{g}}_{\mathcal{G}'(\log \boldsymbol{\Gamma})} \circ \underbrace{\left(\boldsymbol{e}_k \frac{1}{1-\sigma_k^c} + \left(\tilde{\boldsymbol{M}}^d - \tilde{\boldsymbol{M}}^c \right) \frac{d \log \overline{\boldsymbol{P}}}{d\sigma_k^c} \right)}_{\frac{d \log \boldsymbol{\Gamma}}{d\sigma_k^c}} \tag{35}$$

where $\circ$ denotes the Hadamard product operator and $\frac{d \log \overline{\boldsymbol{P}}}{d\sigma_k^c}$ is given by (34).

We see that the effect is composed of two parts: the density of firms in each sector at the margin of switching, measured by the vector $\mathbf{g}$, and the change in the competitiveness threshold in each sector due to the change in the subsidy. This latter component decomposes further into two interpretable components. First, holding normalised prices constant, the subsidy in sector k makes the clean technology more attractive. Second, the subsidy induces a reduction in normalised prices across the economy, as described in Theorem (2). This lowers the marginal cost of production across sectors for both technologies with the degree depending on the specific inputs used by the technology. This affects the relative cost of the two technologies, prompting some firms to change their technology choice. In more concrete terms, if the clean technology is more exposed to good k than the dirty technology (i.e., $\tilde{\Psi}^c_{ik} > \tilde{\Psi}^d_{ik}$), then its marginal cost will fall relative to the dirty technology. As a result, more firms in sector i will choose to produce with the clean technology.

4.3 Input-Output Matrix

The following Theorem concerns how the economy's input-output matrix evolves in response to a sector-specific clean technology subsidy.

Theorem 4 (Change in revenue-based IO matrix)**.** *The derivative of the revenue-based input-output matrix* ($\mathbf{\Omega}$) *with respect to the clean technology subsidy in sector* $k \in \mathcal{N}$ *is given by*

$$\frac{d\mathbf{\Omega}}{d\sigma^c_k} = \underbrace{\frac{d\boldsymbol{G}}{d\sigma^c_k} \circ \left(\boldsymbol{M}^c - \boldsymbol{M}^d\right)}_{\textit{Technology switching}} + \underbrace{\frac{G_k}{1-\sigma^c_k}\,\mathrm{diag}(\boldsymbol{e}_k)\boldsymbol{M}^c}_{\textit{Increased expenditure}}, \tag{36}$$

where $\boldsymbol{M}^\theta \equiv (\boldsymbol{I} - \mathrm{diag}(\boldsymbol{\sigma}^\theta))^{-1}\tilde{\boldsymbol{M}}^\theta$ *denotes the revenue-based expenditure shares for technology* θ*.*

The result reveals that the change in the revenue-based input-output matrix can be decomposed into two components. The first term is the impact of firms changing their technology choices. For concreteness, suppose the subsidy change induces some firms in sector j to switch from the dirty technology to the clean technology. As a result, these firms will divert their spending from intermediate inputs required for the dirty technology in sector j, which is encoded in jth row of matrix $\mathbf{M}^d$, to those required by the clean technology in sector j, which is encoded in jth row of matrix $\mathbf{M}^c$. This effect is increasing in the measure of firms in sector j that switch to the clean technology, which is described by $\frac{dG_j}{d\sigma^c_k}$.

The second term is the impact of the subsidy increase on firms in sector k that already use the clean technology; for a given level of revenue, these firms are able to spend more on the intermediate inputs required by the clean technology.

4.4 Domar weights

The next result shows that the change in sectoral Domar weights in response to a sector-specific subsidy can be decomposed into three components: a direct channel, an upstream channel, and a cascade channel.

Theorem 5 (Change in Domar weights)**.** *The derivative of the Domar weight of sector j (λ_j) with respect to the clean technology subsidy in sector $k \in \mathcal{N}$ is given by*

$$\frac{d\lambda_j}{d\sigma_k^c} = \frac{d\lambda_j^{dir}}{d\sigma_k^c} + \frac{d\lambda_j^{up}}{d\sigma_k^c} + \frac{d\lambda_j^{casc}}{d\sigma_k^c} \tag{37}$$

where

$$\frac{d\lambda_j^{dir}}{d\sigma_k^c} \equiv \lambda_k \frac{G_k}{1-\sigma_k^c} M_{kj}^c + \lambda_k \frac{g_k}{1-\sigma_k^c}\left(M_{kj}^c - M_{kj}^d\right), \tag{38}$$

$$\frac{d\lambda_j^{up}}{d\sigma_k^c} \equiv \lambda_k \frac{G_k}{1-\sigma_k^c}(\Psi_{kj}^c - M_{kj}^c) + \lambda_k \frac{g_k}{1-\sigma_k^c}\left((\Psi_{kj}^c - M_{kj}^c) - (\Psi_{kj}^d - M_{kj}^d)\right), \tag{39}$$

$$\frac{d\lambda_j^{casc}}{d\sigma_k^c} \equiv \sum_i \lambda_i g_i \left(\tilde{\Psi}_{ik}^c - \tilde{\Psi}_{ik}^d\right) \frac{G_k}{1-\sigma_k^c}\left(\Psi_{ij}^c - \Psi_{ij}^d\right), \tag{40}$$

$\boldsymbol{M}^\theta$ denotes the revenue-based expenditure shares for technology θ, and $\boldsymbol{\Psi}^\theta$ ($\tilde{\boldsymbol{\Psi}}^\theta$) denotes the revenue-based (cost-based) Leontief matrix for technology θ.

The intuition for this result is as follows. The direct channel, given by (38), captures the change in demand for sector j given the subsidy-induced changes in the direct expenditure of sector k on sector j. This channel can be broken down into two subparts: the first captures the additional expenditure by existing clean technology firms on their intermediate inputs, while the second captures the change in expenditure by firms that adopt the clean technology. If $M_{kj}^c < M_{kj}^d$, i.e., the clean technology in sector k uses less of good j than the dirty technology, then there is a reduction in demand for good j.

The upstream channel is the upstream propagation of the direct channel to suppliers's suppliers and so on.

Finally, the cascade channel is the implication of price-induced switching on the total demand for good j. This captures the downstream propagation of the marginal cost reductions (measured by the cost-based Leontief matrix $\tilde{\boldsymbol{\Psi}}$), the change in technology choice of these downstream sectors (measured by the density of firms at the margin of switching, $\mathbf{g}$), and the subsequent upstream propagation of demand for direct and indirect intermediate inputs (measured by the revenue-based Leontief inverse matrix $\boldsymbol{\Psi}$).

4.5 Wage Premium

In this subsection, I turn to the wage premium i.e., the difference between the wage and the average product of labour. As shown in Section 3, the wage premium is a key determinant of total emissions (32) and aggregate output (33). The result below describes how this object responds to a change in a sector-specific subsidy.

Theorem 6 (Change in wage premium)**.** *The derivative of the wage premium ξ with respect to the clean technology subsidy in sector k is given by*

$$\begin{aligned}\frac{d\xi}{d\sigma_k^c} = \sum_j \Bigg(& \frac{d\lambda_j}{d\sigma_k^c}\Big(G_j \frac{\sigma_j^c}{1-\sigma_j^c} + (1-G_j)\frac{\sigma_j^d}{1-\sigma_j^d}\Big) \\ & + \lambda_j \frac{dG_j}{d\sigma_k^c}\Big(\frac{\sigma_j^c}{1-\sigma_j^c} - \frac{\sigma_j^d}{1-\sigma_j^d}\Big) \\ & + \frac{\lambda_k G_k}{(1-\sigma_j^c)^2}\mathbb{1}_{j=k}\Bigg)\end{aligned} \tag{41}$$

where $\frac{d\lambda_j}{d\sigma_k^c}$ is given by (37) and $\frac{dG_j}{d\sigma_k^c}$ is given by (35).

Intuitively, the increase in the clean subsidy in sector k changes the demand for labour across the economy. This change in demand depends on the size of the target sector (line three), changes in Domar weights across all sectors (line one), and changes in clean technology shares across all sectors (line two).

In general, the value and sign of the change in the wage premium is ambiguous. In the following corollary, I consider the special case of no existing subsidies where we can derive a more precise result.

Corollary 6 (Change in wage premium with no existing subsidies)**.** *Suppose there are no existing subsidies, $\sigma_i^\theta = 0 \, \forall \, i \in \mathcal{N}, \theta \in \{d, c\}$. Then the derivative of the wage premium ξ with respect to the clean technology subsidy in sector k is given by*

$$\frac{d\xi}{d\sigma_k^c} = \tilde{\lambda}_k G_k \tag{42}$$

In this setting, the change in the wage premium is always positive. This effect is increasing in the Domar weight of sector k and the clean technology share in sector k.

4.6 Emissions

I now present one of my key results: the semi-elasticity of total emissions with respect to a sector-specific clean technology subsidy.

Theorem 7 (Change in emissions)**.** *The semi-elasticity of total emissions* (E) *with respect to the clean technology subsidy in sector* $k \in \mathcal{N}$ *is given by*

$$\frac{d\log E}{d\sigma_k^c} = \sum_i \frac{E_i}{E}\frac{d\log E_i}{d\sigma_k^c} \tag{43}$$

where

$$\frac{d\log E_i}{d\sigma_k^c} = \underbrace{\frac{1}{\lambda_i}\frac{d\lambda_i^{dir}}{d\sigma_k^c} - \frac{1}{1+\xi}\frac{d\xi}{d\sigma_k^c}}_{Direct} + \overbrace{\underbrace{\frac{1}{\lambda_i}\frac{d\lambda_i^{up}}{d\sigma_k^c}}_{Upstream} - \underbrace{\frac{d\log \overline{P}_i}{d\sigma_k^c}}_{Downstream} + \underbrace{\frac{1}{\lambda_i}\frac{d\lambda_i^{casc}}{d\sigma_k^c}}_{Cascade}}^{Network\ Channels}, \tag{44}$$

$\frac{d\lambda_i^{dir}}{d\sigma_k^c}$ *is given by* (38), $\frac{d\lambda_i^{up}}{d\sigma_k^c}$ *is given by* (39), $\frac{d\lambda_i^{casc}}{d\sigma_k^c}$ *is given by* (40), $\frac{d\log \overline{P}_i}{d\sigma_k^c}$ *is given by* (34)*, and* $\frac{d\xi}{d\sigma_k^c}$ *is given by* (41)*.*

Equation (43) shows that the semi-elasticity of total emissions with respect to a subsidy change is the average of the semi-elasticities of each sector's emissions weighted by their contribution to total emissions. Equation (44) shows that the sectoral semi-elasticity can be decomposed into four components. The first component is the direct (i.e., non-network) channel of an intervention. This can be decomposed further into two parts. The first subpart is the change in sector i's Domar weight due to changes in sector k's direct expenditure on sector i. Holding prices constant, the increase in sector i's Domar weight leads to an increase in real output and, thus, sectoral emissions. The second subpart of the direct channel relates to the misallocation created by the subsidy. The subsidy to sector k raises the demand for labour and, thus, the wage premium ξ, as shown in Theorem 6. This reduces labour demand and output in sector i, providing a downward force on emissions.

The remaining terms in (44) relate to how the intervention in sector k propagates along intersectoral linkages. The third component of (44) is the upstream channel. This captures the impact on sector i's Domar weight due to changes in sector k's indirect expenditure on sector i. In other words, this captures the upstream propagation of the intervention. The fourth component of (44) captures the downstream propagation of the intervention. Specifically, the subsidy induces a reduction in the marginal cost of good k that propagates downstream to sector i, leading to an increase in output for the same level of revenue. The fifth, and final, component of (44) is the cascade channel. This channel captures the downstream propagation of marginal cost reductions, the changes in downstream sectors' technology choices and direct intermediate input expenditure, and the subsequent upstream propagation of this expenditure.

Below, I re-express the semi-elasticity of total emissions in terms of direct, upstream, downstream, and cascade channels. This version of the Theorem is more convenient for the following examples and quantitative analysis.

Corollary 7 (Decomposition of change in emissions)**.** *The semi-elasticity of total emissions* (E) *with respect to the clean technology subsidy in non-fossil fuel sector* $k \in \mathcal{N}/\mathcal{N}_F$ *can be*

decomposed as

$$\frac{d\log E}{d\sigma_k^c} = \frac{d\log E^{dir}}{d\sigma_k^c} + \frac{d\log E^{up}}{d\sigma_k^c} + \frac{d\log E^{down}}{d\sigma_k^c} + \frac{d\log E^{casc}}{d\sigma_k^c} \tag{45}$$

where

$$\frac{d\log E^{dir}}{d\sigma_k^c} = \sum_i \frac{E_i}{E}\left\{\frac{1}{\lambda_i}\frac{d\lambda_i^{dir}}{d\sigma_k^c} - \frac{1}{1+\xi}\frac{d\xi}{d\sigma_k^c}\right\}, \tag{46}$$

$$\frac{d\log E^{up}}{d\sigma_k^c} = \sum_i \frac{E_i}{E}\frac{1}{\lambda_i}\frac{d\lambda_i^{up}}{d\sigma_k^c}, \tag{47}$$

$$\frac{d\log E^{down}}{d\sigma_k^c} = -\sum_i \frac{E_i}{E}\frac{d\log \overline{P}_i}{d\sigma_k^c}, \tag{48}$$

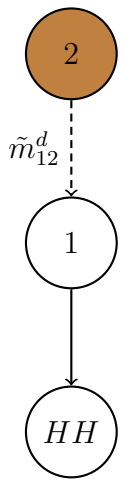
and

$$\frac{d\log E^{casc}}{d\sigma_k^c} = \sum_i \frac{E_i}{E}\frac{1}{\lambda_i}\frac{d\lambda_i^{casc}}{d\sigma_k^c}. \tag{49}$$

4.6.1 Examples

To illustrate the above result, I discuss several examples of subsidy changes in stylised economies where different network channels operate and under which conditions well-meaning policies can backfire leading to an increase in total emissions. For simplicity, these examples all assume there are no existing technology subsidies.

FIGURE I. STYLISED ECONOMY A



Direct Channel only Consider the two-sector economy depicted in Figure I where sector 2 is a fossil fuel sector. The final good is produced using good 1 only. In sector 1, the clean production technology uses labour only while the dirty technology uses good 2 and labour with input shares of $\tilde{m}_{12}^d$ and $1-\tilde{m}_{12}^d$, respectively. In sector 2, both the clean and dirty technologies

use labour only. This simplifies the exposition but does not affect the key insight. All subsidies are initially set to zero. This economy is formally defined in Definition 3 (Appendix).

The different arrows denote intermediate inputs for different production technologies. More specifically, the solid arrows denote inputs for the final good, the dashed arrows denote intermediate inputs for each sector's dirty technology, and the dotted arrows denote intermediate inputs for each sector's clean technology. The edge labels denote the value of the intermediate input shares.

Suppose that there is a marginal increase in the clean technology subsidy in sector 1. This encourages firms in sector 1 to switch from the dirty technology, which uses labour and good 2 (the fossil fuel sector) to the clean technology, which uses labour only. Lemma 1 decomposes the impact of this subsidy change on total emissions.

Lemma 1 (Example: Direct Channel only)**.** *Consider the economy described by Definition 3. The semi-elasticity of total emissions E with respect to the clean technology subsidy in sector* 1 *is given by*

$$\frac{d\log E}{d\sigma_k^c} = \frac{d\log E^{dir}}{d\sigma_k^c} + \frac{d\log E^{up}}{d\sigma_k^c} + \frac{d\log E^{down}}{d\sigma_k^c} + \frac{d\log E^{casc}}{d\sigma_k^c} \tag{50}$$

where

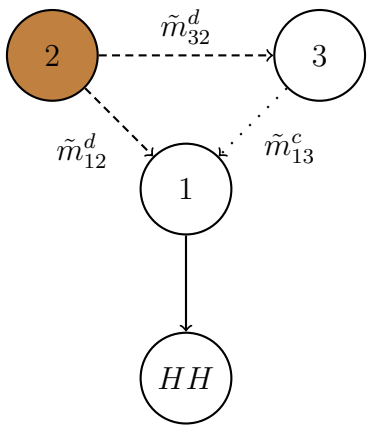
$$\frac{d\log E^{dir}}{d\sigma_k^c} = \underbrace{-\frac{g_1}{1-G_1}}_{\frac{1}{\lambda_2}\frac{d\lambda_2^{dir}}{d\sigma_1^c}} \underbrace{-\ G_1}_{\frac{1}{1+\xi}\frac{d\xi^{dir}}{d\sigma_1^c}} < 0, \tag{51}$$

and

$$\frac{d\log E^{up}}{d\sigma_1^c} = \frac{d\log E^{down}}{d\sigma_1^c} = \frac{d\log E^{casc}}{d\sigma_1^c} = 0. \tag{52}$$

In this very simple setting, only the direct channel is operational and the clean technology subsidy always reduces total emissions. The remaining examples study settings where at least one network channel is operational.

FIGURE II. STYLISED ECONOMY B



Direct and Upstream Channels Consider the three-sector economy depicted in Figure II where sector 2 is the fossil fuel sector. The final good is produced used good 1 only. In sector 1, the clean technology uses good 3 and labour with input shares of $\tilde{m}^c_{13}$ and $1-\tilde{m}^c_{13}$, respectively, while the dirty technology uses good 2 and labour with input shares of $\tilde{m}^d_{12}$ and $1-\tilde{m}^d_{12}$, respectively. In sector 2, the clean and dirty technologies use labour only. Finally, in sector 3, the clean technology uses labour only while the dirty technology uses good 2 and labour with input shares of $\tilde{m}^d_{32}$ and $1-\tilde{m}^d_{32}$, respectively. This economy is formally defined in Definition 4 (Appendix). Lemma 2 applies Corollary 7 to evaluate the impact of a marginal increase to the clean technology subsidy in sector 1 on total emissions.

Lemma 2 (Example: Direct and Upstream Channels)**.** *Consider the economy described by Definition 4. The semi-elasticity of total emissions E with respect to the clean technology subsidy in sector* 1 *is given by*

$$\frac{d\log E}{d\sigma^c_k} = \frac{d\log E^{dir}}{d\sigma^c_k} + \frac{d\log E^{up}}{d\sigma^c_k} + \frac{d\log E^{down}}{d\sigma^c_k} + \frac{d\log E^{casc}}{d\sigma^c_k} \tag{53}$$

where

$$\frac{d\log E^{dir}}{d\sigma^c_k} = \underbrace{\frac{-g_1\tilde{m}^d_{12}}{(1-G_1)\tilde{m}^d_{12} + G_1\tilde{m}^c_{13}(1-G_3)\tilde{m}^d_{32}}}_{\frac{1}{\lambda_2}\frac{d\lambda_2^{dir}}{d\sigma^c_1}} - \underbrace{G_1}_{\frac{1}{1+\xi}\frac{d\xi^{dir}}{d\sigma^c_1}} < 0, \tag{54}$$

$$\frac{d\log E^{up}}{d\sigma^c_k} = \underbrace{\frac{(g_1+G_1)\tilde{m}^c_{13}(1-G_3)\tilde{m}^d_{32}}{(1-G_1)\tilde{m}^d_{12} + G_1\tilde{m}^c_{13}(1-G_3)\tilde{m}^d_{32}}}_{\frac{1}{\lambda_2}\frac{d\lambda_2^{up}}{d\sigma^c_1}} > 0, \tag{55}$$

and

$$\frac{d\log E^{down}}{d\sigma^c_1} = \frac{d\log E^{casc}}{d\sigma^c_1} = 0. \tag{56}$$

In addition to the direct channel, which featured in the previous example, there is also an upstream channel. As before, the direct channel is negative as the subsidy increase leads to a reduction in demand for sector 2 (the fossil fuel) and an increase in labour misallocation. The upstream channel captures the change in fossil fuel production due to the increased demand for good 3, which is an intermediate input for the clean technology in sector 1. As the direct and upstream channels have opposite signs, the total effect depends on which channel is larger. Proposition 1 characterises when a clean technology subsidy increase in sector 1 leads to an increase in total emissions.

Proposition 1 (Policy backfiring with Direct and Upstream Channels)**.** *Consider the economy described by Definition 4. A marginal increase in the clean technology subsidy in sector* 1 *has*

a positive effect on emissions E if and only if

$$\underbrace{\tilde{m}^c_{13}(1-G_3)\tilde{m}^d_{32}}_{\tilde{\Psi}^c_{12}} > \underbrace{\tilde{m}^d_{12}}_{\tilde{\Psi}^d_{12}}. \tag{57}$$

In words, if the indirect reliance of sector 1's clean technology on the fossil fuel sector is larger than the direct and indirect reliance of sector 1's dirty technology on the fossil fuel sector, then the positive upstream channel will outweigh the negative direct channel, leading to an increase in total emissions.

Direct and Cascade Channels Let us continue with the three-sector economy depicted in Figure II and described in the previous example. Instead of targeting sector 1, I now consider a marginal increase to the clean technology subsidy in sector 3. Lemma 3 evaluates the impact of such an intervention on emissions.

Lemma 3 (Example: Direct and Cascade Channels)**.** *Consider the economy described by Definition 4. The semi-elasticity of total emissions E with respect to the clean technology subsidy in sector 3 is given by*

$$\frac{d\log E}{d\sigma^c_3} = \frac{d\log E^{dir}}{d\sigma^c_3} + \frac{d\log E^{up}}{d\sigma^c_3} + \frac{d\log E^{down}}{d\sigma^c_3} + \frac{d\log E^{casc}}{d\sigma^c_3} \tag{58}$$

where

$$\frac{d\log E^{dir}}{d\sigma^c_3} = \underbrace{\frac{G_1\tilde{m}^c_{13}g_3(-\tilde{m}^d_{32})}{(1-G_1)\tilde{m}^d_{12}+G_1\tilde{m}^c_{13}(1-G_3)\tilde{m}^d_{32}}}_{\frac{1}{\lambda_2}\frac{d\lambda^{dir}_2}{d\sigma^c_3}} - \underbrace{G_1\tilde{m}^c_{13}G_3}_{\frac{1}{1+\xi}\frac{d\xi^{dir}}{d\sigma^c_3}} < 0, \tag{59}$$

$$\frac{d\log E^{casc}}{d\sigma^c_k} = \underbrace{\frac{G_3\tilde{m}^c_{13}g_1(\tilde{m}^c_{13}(1-G_3)\tilde{m}^d_{32}-\tilde{m}^d_{12})}{(1-G_1)\tilde{m}^d_{12}+G_1\tilde{m}^c_{13}(1-G_3)\tilde{m}^d_{32}}}_{\frac{1}{\lambda_2}\frac{d\lambda^{casc}_2}{d\sigma^c_3}}, \tag{60}$$

and

$$\frac{d\log E^{up}}{d\sigma^c_3} = \frac{d\log E^{down}}{d\sigma^c_3} = 0. \tag{61}$$

Unlike in the previous example, there is now a cascade channel. This network channel occurs because the subsidy leads to a reduction in the price of good 3. This propagates downstream and affects the technology decision in sector 1. More specifically, the clean technology becomes more cost competitive relative to the dirty technology, encouraging firms in sector 1 to switch over to the clean technology. As a result, the intervention leads to an increase in the clean share in sectors 1 and 3. Despite this, it is still possible for an intervention to increase total emissions.

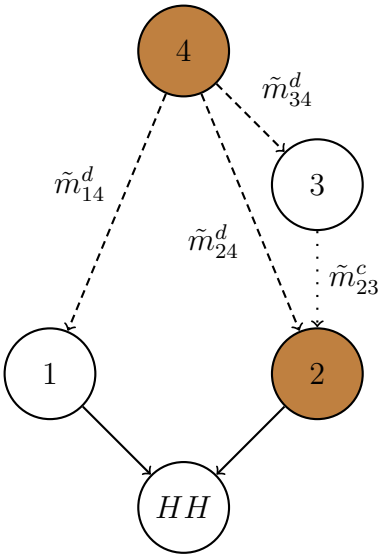
Proposition 2 characterises when a clean technology subsidy increase in sector 3 leads to an increase in total emissions.

Proposition 2 (Policy backfiring with Direct and Cascade Channels)**.** *Consider the economy described by Definition 4. A marginal increase in the clean technology subsidy in sector* 3 *has a positive effect on emissions* E *if and only if the following two conditions hold:*

1. $\tilde{\Psi}^c_{12} > \tilde{\Psi}^d_{12}$
2. $g_1 > \frac{\tilde{\lambda}_3 g_3 \tilde{m}^d_{32} + \tilde{\lambda}_2 \tilde{\lambda}_3 G_3}{G_3 \tilde{\Psi}^c_{13}(\tilde{\Psi}^c_{12} - \tilde{\Psi}^d_{12})}$

The intuition for this result is as follows. While the direct channel is negative, the cascade channel is ambiguous. The direction of the technology cascade channel depends on the relative size of $\tilde{\Psi}^c_{13}$ and $\tilde{\Psi}^d_{13}$ - in words, the difference in the direct and indirect use of fossil fuels by sector 1's clean and dirty technologies given the existing production network. If the dirty technology's reliance is higher ($\tilde{\Psi}^c_{13} < \tilde{\Psi}^d_{13}$), then the technology cascade channel - and the overall emissions change - is negative. On the other hand, if the clean technology's reliance is higher ($\tilde{\Psi}^c_{13} > \tilde{\Psi}^d_{13}$), then the technology cascade channel is positive. The density of firms in sector 1 at the margin of switching, g_1, determines the magnitude of this channel. If this value is large enough, it will outweigh the negative direct channel.

FIGURE III. STYLISED ECONOMY C



Direct and Downstream Channels I now construct an example where only the direct and downstream channels are present. Consider the four-sector economy depicted in Figure III where sectors 2 and 4 are fossil fuel sectors and sector 3 is the target for intervention. The final good is produced used good 1 and good 2 with input shares b_1 and b_2, respectively. In sector 1, the clean technology uses labour only while the dirty technology uses good 4 and labour with input shares of $\tilde{m}^d_{14}$ and $1 - \tilde{m}^d_{14}$, respectively. In sector 2, the clean technology uses good 3 and labour with input shares of $\tilde{m}^c_{23}$ and $1 - \tilde{m}^c_{23}$, respectively, while the dirty technology uses good 4 and labour with input shares of $\tilde{m}^d_{24}$ and $1 - \tilde{m}^d_{24}$, respectively. In sector

3, the clean technology uses labour only while the dirty technologies use good 4 and labour with input shares of $\tilde{m}_{34}^d$ and $1-\tilde{m}_{34}^d$, respectively. In sector 4, the clean and dirty technologies use labour only. All sectors use both the clean and dirty technologies in equilibrium. To turn off the cascade channel, I set the adoption elasticity for all sectors, except sector 3, to zero i.e., $g_1 = g_3 = g_4 = 0$. This economy is formally defined in Definition 5 (Appendix).

Lemma 4 applies Corollary 7 to evaluate the impact of a marginal increase to the clean technology subsidy in sector 3 on total emissions.

Lemma 4 (Example: Direct and Downstream Channels)**.** *Consider the economy described by Definition 5. The semi-elasticity of total emissions E with respect to the clean technology subsidy in sector 3 is given by*

$$\frac{d\log E}{d\sigma_3^c} = \frac{d\log E^{dir}}{d\sigma_3^c} + \frac{d\log E^{down}}{d\sigma_3^c} \tag{62}$$

where

$$\frac{d\log E^{dir}}{d\sigma_3^c} = -\frac{E_4}{E}\frac{1}{\tilde{\lambda}_4}\tilde{\lambda}_3 g_3 \tilde{m}_{34}^d - \tilde{\lambda}_3 G_3 < 0 \tag{63}$$

and

$$\frac{d\log E^{down}}{d\sigma_3^c} = \frac{E_2}{E}\tilde{\Psi}_{23} G_3 > 0. \tag{64}$$

Under the assumptions, we see that two channels are operational. The first is the direct channel whereby the subsidy induces some firms in sector 3 to switch to the clean technology and stop purchasing good 4, which is used in the dirty technology. This channel is always negative. The second is the downstream channel whereby the subsidy induces a reduction in the marginal cost of good 3 that, in turn, induces a reduction in the marginal cost of good 2, leading to an increase in production of good 2 (a fossil fuel). We can see that this channel is always positive. Thus, the direction of the total effect depends on which of these forces is larger.

Proposition 3 characterises when a clean technology subsidy increase in sector 3 leads to an increase in total emissions.

Proposition 3 (Policy backfiring with Direct and Downstream Channels)**.** *Consider the economy described by Definition 5. A marginal increase in the clean technology subsidy in sector 3 has a positive effect on emissions E if and only if*

$$\frac{\epsilon_2/P_2}{\epsilon_4/P_4} > \frac{\tilde{\lambda}_3[g_3\tilde{m}_{34}^d + \tilde{\lambda}_4 G_3]}{\tilde{\lambda}_2(1-\tilde{\lambda}_2)\tilde{\Omega}_{23} G_3}. \tag{65}$$

In words, the Proposition shows that if the emissions per dollar in sector 2 is sufficiently large relative to sector 4, then an intervention in sector 3 causes an overall increase in emissions. This is because the decrease in emissions from the use of good 2 are more than offset by the increase in emissions from the use of good 4.

4.7 Aggregate Output

I now present my second key theoretical result: a general characterisation of the impact of a subsidy change on aggregate output in terms of the existing production network structure, the input requirements of different technologies, and firms' technology decisions.

Theorem 8 (Change in aggregate output)**.** *The semi-elasticity of aggregate output* (Y) *with respect to the clean technology subsidy in sector* $k \in \mathcal{N}$ *is given by*

$$\frac{d\log Y}{d\sigma_k^c} = \frac{d\log Y^D}{d\sigma_k^c} + \frac{d\log Y^{ND}}{d\sigma_k^c} \tag{66}$$

where

$$\frac{d\log Y^D}{d\sigma_k^c} = -\frac{\delta'(E)E}{1-\delta(E)}\frac{d\log E}{d\sigma_k^c}, \tag{67}$$

$$\frac{d\log Y^{ND}}{d\sigma_k^c} = \tilde{\lambda}_k \frac{G_k}{1-\sigma_k^c} - \frac{1}{1+\xi}\frac{d\xi}{d\sigma_k^c}, \tag{68}$$

$\frac{d\log E}{d\sigma_k^c}$ *is given by* (43)*, and* $\frac{d\xi}{d\sigma_k^c}$ *is given by* (41)*.*

The results reveals that the change in aggregate output, which is equivalent to aggregate TFP, depends on two components: the change in damage-related efficiency and the change in non-damage efficiency.

The first component captures how the subsidy affects TFP via damages, which depends on the semi-elasticity of total emissions with respect to the subsidy (given by Theorem 7) and the responsiveness of damages to emissions, given by $\frac{\delta'(E)E}{1-\delta(E)}$. If a sector-specific subsidy induces a large reduction in total emissions and damages are very responsive to emissions (for example, if the derivative of damages with respect to emissions is large or the level of damages is already high), then a subsidy would generate a large increase in damage-related efficiency.

The second component of Theorem 8 captures how the subsidy affects allocative and technical efficiency given the current level of damages. In turn, this component is determined by two objects. The first term is the increase in aggregate output due to the increase in the output of sector k, which is directly or indirectly used in the production of the final good. Intuitively, $G_k/(1-\sigma_k^c$ is the size of the increase in sector k's output and $\tilde{\lambda}_k$ measures the importance of sector k to the final good. The second component of non-damage efficiency accounts for the change in output due to the change in the wage premium term. If a subsidy generates a large increase in the endogeneous wage premium, then there may be a decrease in non-damage efficienct and aggregate output.

An important policy implication of Theorem 8 is that interventions that reduce total emissions may not always be welfare-enhancing. In particular, the subsidy increase may interact with existing subsidies, leading to a reduction in non-damage efficiency that could offset the improvement in damage-related efficiency. Consequently, a policymaker wishing to find welfare-

maximising interventions must consider the impact of interventions on both components of aggregate TFP.

4.7.1 Examples

I now present three examples to illustrate how the existing structure of the economy determines the welfare impact of clean technology subsidies. The first example considers the case when there are no existing subsidies. In this case, the welfare impact is entirely determined by the emissions impact of a subsidy. The second example concerns an economy with existing subsidies where the clean technology subsidies leads to a reduction in emissions at the margin, but still decrease welfare due to the non-damage component of output. The third and final example studies an economy with existing subsidies whereby a clean technology subsidies leads to a double dividend i.e., a reduction in emissions and distortions. In this case, aggregate output (and welfare) increases.

No Subsidies In the follow corollary, I apply the above Theorem to study a simpler setting with no existing subsidies.

Corollary 8 (Change in aggregate output with no existing subsidies)**.** *Suppose there are no existing subsidies,* $\sigma_i^\theta = 0 \, \forall \, i \in \mathcal{N}, \theta \in \{d, c\}$*. Then the semi-elasticity of non-damage TFP is given by*

$$\frac{d \log Y^{ND}}{d\sigma_k^c} = 0, \tag{69}$$

and the semi-elasticity of aggregate output reduces to

$$\frac{d \log Y}{d\sigma_k^c} = \frac{d \log Y^D}{d\sigma_k^c}, \tag{70}$$

where $\frac{d \log Y^D}{d\sigma_k^c}$ *is given by* (68)*.*

In this setting, there is no change in the non-damage component of TFP as the positive effect on aggregate output of the increase in sector k output is perfectly offset by the reduction in output across the economy due to the wage premium. Intuitively, this occurs because, given the level of damages, resources are allocated to their most efficient use in the absence of subsidies, which are distortionary. As a result, the change in output is entirely determined by the change in damage-related efficiency.

Backfiring with Existing Subsidies *Work in progress*

Double Dividend with Existing Subsidies *Work in progress*

5 Targeting (Incomplete)

In the previous section, I characterised the impact of a change in a sector-specific clean technology subsidy on several sector- and aggregate-level outcomes, including emissions and output. In this section, I use those results to provide insight on how governments should target subsidies across sector-technology pairs in some stylised economies. This section is structured as follows. Subsection 5.1 introduces the concept of normalised emissions impact, which accounts for differences in expenditure requirements across sector-technology pairs. I then apply this concept to investigate optimal targeting in two stylised economies: a horizontal economy (Subsection 5.2) and a vertical economy (Subsection 5.3).

5.1 Normalised Emissions Impact

Theorem (7) in Section 4 expressed the aggregate emissions impact of a marginal change in a sector-specific clean technology subsidy, σ_k^c. This is not a useful metric for comparing across sectors as the lump sum tax requirements to pay for the subsidy change are heterogeneous across sector-technology pairs. For example, a change in the subsidy rate for a sector with a large Domar weight and high clean technology share requires a much larger increase in total subsidy expenditure than for a sector with a low Domar weight and low clean technology share. To resolve this, we can normalise the semi-elasticity of emissions by the tax impact of the subsidy rate change.

This procedure introduces a further problem: the overall impact of a subsidy change on the necessary lump sum tax depends on the exogenous subsidy rate as well as three endogenous objects, namely aggregate output Y, Domar weights $\boldsymbol{\lambda}$, and clean technology shares $\mathbf{G}$. To avoid this complexity, I will consider the change in tax ignoring the general equilibrium effects of the subsidy change.

Definition 2 below formalises this discussion and will be used in my subsequent targeting analyses.

Definition 2 (Normalised emissions impact)**.** *The normalised emissions impact of a change in the clean technology subsidy in sector sector k, denoted by $\overline{\frac{d\log E}{d\sigma_k^c}}$ is defined as*

$$\overline{\frac{d\log E}{d\sigma_k^c}} \equiv \frac{(1-\sigma_k^c)^2}{Y\lambda_k G_k}\frac{d\log E}{d\sigma_k^c}, \tag{71}$$

where $\frac{d\log E}{d\sigma_k^c}$ is given by (43).

5.2 Horizontal Economy

Work in Progress

Table I. Summary of Functional Forms

Function	Description	Form	Overview
$\mathcal{G}_i(X)$	Sector-specific cumulative distribution function of technology gaps	$e^{-e^{-\frac{X-\mu_i}{\beta}}}$	Sector-specific Gumbel distribution with sector-specific location μ_i and economy-wide scale β
$\delta(E)$	Damage function	$\overline{\delta}E^2$	Captures convex damages from emissions

5.3 Vertical Economy

Work in Progress

6 Calibration

In this section, I explain how I calibrate the model to a bloc of 23 advanced economies[9]. Throughout, I assume there are no initial subsidies. In Subsection 6.1, I describe the challenge of deriving clean and dirty technology input shares and present an approach to derive plausible parameters for my quantitative analysis. In Subsection 6.2, I explain how I calibrate the sector-specific technology gap distributions. This is a key object for determining how firms choose their technology and respond to changes in subsidies for clean technologies. Subsection 6.3 describes my approach for the remaining objects, such as the final demand shares and damage function. Finally, Subsection 6.4 presents the calibrated baseline economy. For a summary of my functional form assumptions and parameter calibration, I refer the reader to Table I and Table II, respectively.

6.1 Technology Input Shares

A key problem for my calibration is that sector- and technology-specific input shares cannot be directly extracted from observed industry-to-industry or firm-to-firm transaction data. This is because such data are an equilibrium outcome where the observed expenditure shares are a convex combination of the input shares for the sector's clean and dirty technologies with weights depending on the equilibrium share of firms using the clean and dirty technology, respectively.

Instead, for each sector, I calibrate the clean (dirty) technology input shares to the intermediate input expenditure shares of the country with the lowest (highest) expenditure share on fossil fuel sectors. Under the very strong identification assumption that the country with the

[9]The list of countries is: AUS AUT BEL CAN CZE DNK EST FIN FRA DEU ITA JPN KOR LTU NLD NOR PRT SVN ESP SWE CHE GBR USA

TABLE II. SUMMARY OF PARAMETER CALIBRATION

Parameter	Description	Value	Source
$\boldsymbol{\sigma}^c, \boldsymbol{\sigma}^d$	$N \times 1$ column vectors of clean and dirty technology subsidies, respectively	$\mathbf{0}$	Simplifying assumption
$\tilde{\mathbf{M}}^c$, $\tilde{\mathbf{M}}^d$	Clean and Dirty IO matrices	Sector-specific	Derived from inter-country input-output tables
$\tilde{\mathbf{a}}^c$, $\tilde{\mathbf{a}}^d$	Clean and Dirty labour shares	Sector- and tech-specific	Remainder from total intermediate input shares
$\boldsymbol{\mu}$	$N \times 1$ column vector of technology gap location parameters	Sector-specific	Calibrated to match sector's total expenditure share on fossil fuel inputs
β	Economy-wide scale parameter for technology gap distributions	10	Placeholder value. Further work needed
$\mathbf{b}$	$N \times 1$ column vector of final demand shares	Sector-specific	Calibrated to expenditure shares for consumption, investment, and government spending
$\boldsymbol{\epsilon}$	$N \times 1$ column vector of emissions content	Sector-specific	Calibrated to match each fossil fuel's contribution to global emissions
$\overline{\delta}$	Coefficient in damage function	0.01	Placeholder value. Further work needed

lowest (highest) fossil fuel share for a given sector has an equilibrium clean technology share of 100% (0%), then the shares derived by this approach will be those of the true clean (dirty) technology. However, this assumption is unlikely to be true as clean production technologies have, in general, low penetration rates across countries and sectors. This means the expenditure shares of the lowest fossil fuel share country-sector pair would still be a convex combination of the true unobserved dirty and clean input shares. As a result, the clean input shares derived by the procedure are likely to overestimate the total input share for fossil fuels while over- or underestimating the input shares for non fossil fuel inputs.

I now describe my procedure to generate the clean and dirty input share matrices:

1. I extract the inter-country input-output data for the latest available year.

2. I aggregate each country-sector pair's expenditure on domestic and foreign intermediate inputs to obtain its total expenditure on each intermediate input.

3. I calculate the expenditure on each intermediate input as a share of the sector's total nominal output. If a country-sector pair has no output, I discard the data.

4. For each country-sector pair, I calculate the total expenditure share on fossil fuel sectors (C19 and D35)

5. For each sector, I identify the country with the lowest total expenditure share on fossil fuel sectors and the country with the highest.

6. To generate the clean input matrix, I collate the intermediate input shares from the lowest fossil fuel share country for each sector. For the dirty input matrix, I collate the intermediate input shares from the highest fossil fuel share country for each sector

To implement the approach, I use the OECD's Inter-Country Input-Output tables[10]. These tables shows the intermediate input expenditure by a sector in one country on any given sector-country pair. The data cover 76 countries (and a bloc for the rest of the world) and 45 industries, which are based on the International Standard Industrial Classification Revision 4 (ISIC Rev. 4). Expenditure is in current prices and denoted in millions of US dollars. I note that this data is still relatively aggregated. Consequently, differences in an industry's intermediate expenditure shares across countries may be due to differences in the sub-industry mix across countries rather than differences in the intermediate inputs used for the same set of sub-industries.

Ideally, I would observe expenditure on sectors C19 (Manufacture of coke and refined petroleum products) and D352 (Manufacture of gas, distribution of gaseous fuels through mains). While sector C19 is delineated in the OECD's ICIO data, sector D352 is not. Instead, sector D352 is included in Sector D35, which also includes electricity, steam and air conditioning. For the purpose of my quantitative analysis, I use sectors C19 and D35 as fossil

[10]https://www.oecd.org/en/data/datasets/inter-country-input-output-tables.html

fuel sectors. This approach overestimates fossil fuel usage as it assigns unobserved electricity expenditure, which does not directly generate emissions, as dirty.

Given the issue discussed above, my derived parameter values should be interpreted as indicative and useful for a first pass answer at the role of the network in policy propagation. However, more granular data would be required to truly identify the clean and dirty input matrices.

6.2 Technology gap distribution

I assume the technology gap distribution in each sector follows a Gumbel distribution with sector-specific location parameters $\boldsymbol{\mu} = [\mu_1, \mu_2, ..., \mu_N]'$ and an economy-wide scale parameter β. I have chosen the Gumbel distribution as its right skew replicates the empirical distribution estimated by Acemoglu et al. (2016). Intuitively, this captures the following three ideas: first, even with high input prices and low subsidies, there is already a small share of varieties within each sector for which the clean technology is ahead of the dirty technology; second, there is a large share for which the dirty technology is only marginally more productive than the clean technology; and third, there is a long tail of varieties for which the dirty technology is far ahead of the clean technology. For my baseline set of results, I set the scale parameters $\beta = 1$. Later, I check the robustness of my findings by using a range of smaller and larger values. I calibrate the sector-specific location parameters such that the model-predicted expenditure share on fossil fuels for each sector matches the observed expenditure share.

6.3 Remaining objects

I calibrate the final demand shares by aggregating expenditure across household consumption, government spending, and gross fixed capital formation from the inter-country input-output data. For now, I normalise the emissions content of all fossil fuels to 1. I will re-visit this assumption in the next draft of the paper. Finally, I set the damage function as $\delta(E) \equiv \overline{\delta} E^2$, which captures the convex damages to emissions. For this version of the paper, I set $\overline{\delta} = 0.01$. As with the emissions content parameters, I will revisit this assumption in the next version of the paper.

6.4 Baseline Economy

Table VIII shows the model output for sectoral value added, Domar weights, and clean technology shares given the baseline calibration and no technology subsidies. Sectors are listed in descending order of value added.

7 Quantitative Analysis

In this section, I present my quantitative findings. In Subsection 7.1, I address my first research question on the role of the production network in mediating the effect of sector-specific clean technology subsidies. I use my calibrated model to evaluate the semi-elasticity of total emissions to a sector-specific clean technology subsidy (Theorem 7) for all industrial sectors (excluding energy-related sectors). I discuss the contribution of the four channels introduced in the previous section and find that the three network channels, in general, dampen or reverse the direct channel of clean subsidies. In Subsection 7.2, I show that these semi-elasticities are not well correlated with other sectoral features such as size and value added, underlining the importance of explicitly modelling the network structure of production. In Subsection 7.3, I consider my second research question: how should governments target clean technology subsidies in the absence of carbon pricing. I show that the top ten sectors with the largest semi-elasticities is different when excluding or including the three network channels.

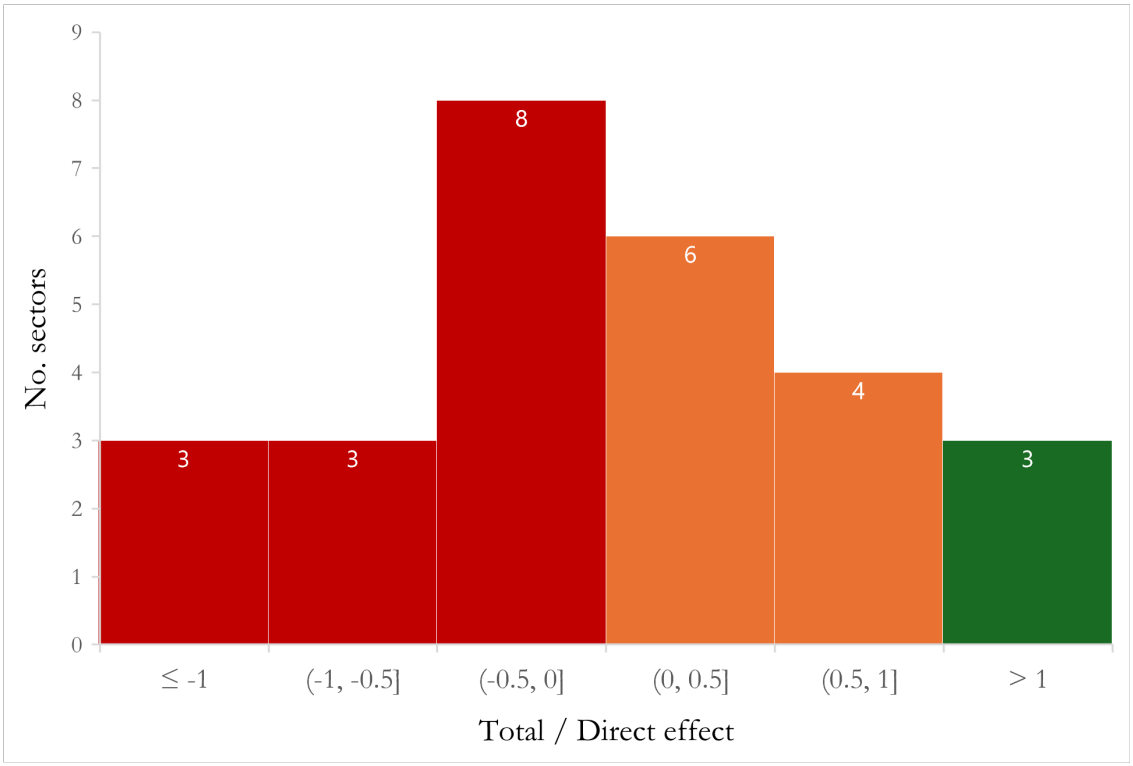
7.1 Role of the Network

Table IX summarises the effect of a sector-specific clean technology subsidy increase on total emissions. The first four columns shows the four components of the total effect, namely the direct channel, upstream channel, downstream channel, and cascade channel. The fifth column shows the total effect. The six column shows the ratio of the total effect to the direct channel. Sectors are listed in ascending order of the total effect i.e., a clean technology subsidy increase in the top ranked sector generates the largest decrease in total emissions, while the bottom ranked sector generates the largest increase.

To understand the contribution of the network channels to the total effect, it is useful to consider the ratio of the total effect over the direct channel. When this ratio is above one, the network amplifies the direct channel of the policy. When this ratio is between zero and one, the network channels are positive, dampening the direct channel. Finally, when the ratio is below one, the network channels are positive and sufficiently large as to offset the direct channel of a policy and lead to an overall increase in emissions.

Figure IV shows the number of sectors by different intervals of the ratio of the total effect to the direct channel. There are two key features of the data. First, for the vast majority of sectors in our exercise (24 out of 27), the network channels offsets the direct channel of an intervention on emissions. Second, for a large share of sectors (14 out of 27), the network channels are sufficiently positive and large as to reverse the direct channel of an intervention and lead to an overall increase in emissions. These results underline the importance of considering the full network of intersectoral linkages when designing subsidies for clean technologies.

FIGURE IV. NO. SECTORS BY LEVEL OF NETWORK AMPLIFICATION



7.2 Relationship with other Network Measures

Figure V plots the relationship between the total emissions effect of a sector-specific subsidy increases and various characteristics of the sector, namely the direct channel, the difference between the total fossil fuel share for the clean and dirty technology in the sector, the value added in a sector, and the Domar weight of the sector. The total emissions effect is poorly correlated with any of these measures indicating that it cannot be easily proxied by using observable network features.

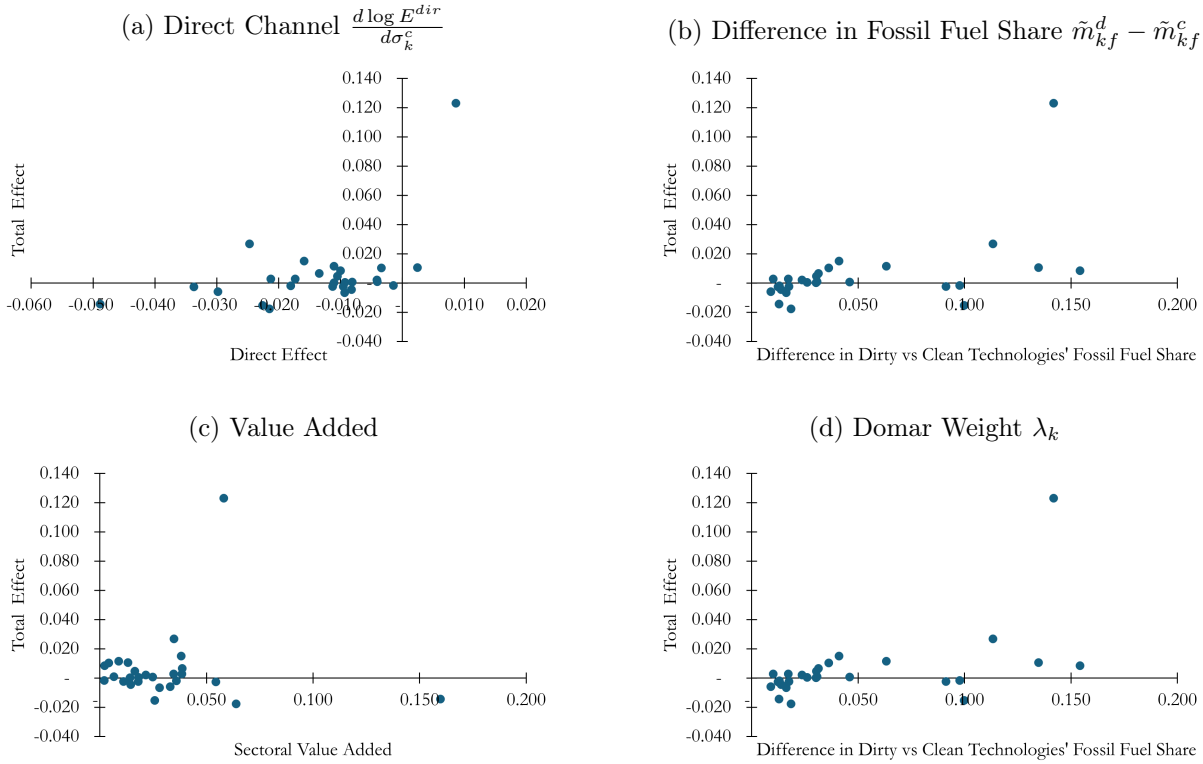
7.3 Optimal Targeting

I now study how the interplay of firm choice, technology input requirements, and existing network structure changes the optimal sector for targeting. Table III lists the top ten sectors with the largest negative direct emissions channel (i.e., ignoring network channels) alongside their ranking when considering the total emissions effect.

The implications of these results are twofold. First, the direct channel is a poor guide for optimal policy. This is because only five of the top ten sectors appear in the top ten when considering the total emissions effect. Furthermore, four of the sectors have a positive total effect indicating that targeting sectors by their direct channel can be harmful to emissions and the economy.

Second, the discrepancy between a sector's rank according to its direct channel and its total

FIGURE V. RELATIONSHIP BETWEEN $\frac{d \log E}{d\sigma_k^c}$ AND SECTORAL CHARACTERISTICS



effect is driven by the network. More specifically, under this calibration, the network channels are generally positive and counteract the negative direct channel of interventions. For six of the ten sectors, the network channels are sufficiently small that they do not completely offset the negative direct channel of an intervention. However, for four of the sectors, the network channels are so large that they outweigh the direct channel and cause an increase in total emissions.

Table IV lists the top ten sectors with the largest negative total emissions effect (i.e., including the network channels) and their ranking when considering the direct channel only. This table reveals that the key to effective targeting is finding sectors that may not have the largest direct channel, but are in a position in the network such that the network channels do not offset the direct channel of an intervention.

8 Conclusion

In this paper, I applied a novel endogenous network perspective to understand how changes in sector-specific technology subsidies affect technology choices, the level of emissions, and aggregate output. To do this, I developed an analytically tractable model featuring multiple sectors, heterogeneous firms, endogenous technology choice, and climate damages.

I showed how aggregate output can be decomposed into damage-related and non-damage efficiency. I then provided a general characterisation of the impact of a subsidy change on

TABLE III. TARGETING BASED ON DIRECT CHANNEL: TOP TEN SECTORS

Rank (Direct)	**Sector**	**Direct Channel**	**Total Effect**	**Rank (Total)**
1	(F) Construction	-0.05	-0.01	3
2	(C10-12) Food products, bever...	-0.03	-0.00	7
3	(C29) Motor vehicles, trai...	-0.03	-0.01	5
4	(C20) Chemical and chemica...	-0.02	0.03	26
5	(H53) Postal and courier a...	-0.02	-0.02	2
6	(C26) Computer, electronic...	-0.02	-0.02	1
7	(C25) Fabricated metal pro...	-0.02	0.00	18
8	(C31-33) Manufacturing nec; r...	-0.02	-0.00	10
9	(C28) Machinery and equipm...	-0.02	0.00	17
10	(H52) Warehousing and supp...	-0.02	0.02	25

TABLE IV. TARGETING BASED ON TOTAL EFFECT: TOP TEN SECTORS

Rank (Total)	**Sector**	**Direct Channel**	**Total Effect**	**Rank (Direct)**
1	(C26) Computer, electronic...	-0.02	-0.02	6
2	(H53) Postal and courier a...	-0.02	-0.02	5
3	(F) Construction	-0.05	-0.01	1
4	(C21) Pharmaceuticals, med...	-0.01	-0.01	18
5	(C29) Motor vehicles, trai...	-0.03	-0.01	3
6	(C30) Other transport equi...	-0.01	-0.00	20
7	(C10-12) Food products, bever...	-0.03	-0.00	2
8	(C13-15) Textiles, textile pr...	-0.01	-0.00	12
9	(C23) Other non-metallic m...	-0.01	-0.00	17
10	(C31-33) Manufacturing nec; r...	-0.02	-0.00	8

aggregate output in terms of the structure of the production network, the input requirements of different technologies, the profile of firms at the margin of switching, and other macroeconomic variables. To illustrate the importance of the network, I discussed several examples where well-meaning policies that increase clean technology subsidies lead, perversely, to an increase in total emissions and a reduction in aggregate output.

I then calibrate my model to a bloc of 23 advanced OECD countries to quantify my analysis. I show that network channels are quantitatively significant and can dampen or even reverse the direct channel of interventions. Next, I show that the impact of a sector-level subsidy on total emissions is poorly correlated with other sectoral characteristics, such as value added, underlining the importance of explicitly modelling the endogenous technology choice and intermediate inputs in production to accurately evaluate policy. Finally, I show that accounting for intersectoral linkages meaningfully changes the set of sectors that a welfare-maximising social planner should target.

Taken together, my results have three important policy implications. First, the existing structure of the production network determines whether, and by how much, an increase in a sector-specific clean technology subsidy reduces total emissions. Conditional on finding an intervention that reduces total emissions (and thus improves damage-related efficiency), policy makers may face a trade-off with a decrease in non-damage efficiency. Third, and finally, a policymaker interested in improving welfare should exploit the network structure to find interventions with the largest improvement in aggregate output, taking into account the changes in both damage-related and non-damage efficiency.

I plan to develop this paper along several dimensions. First, I aim to compare the constrained efficient outcome using subsidies with the optimal outcome using a carbon tax. Second, I will study how a social planner should optimally allocate an additional dollar of tax revenue across sectors. Third, I will introduce a second time period, which will allow me to study the implications of capital investment and/or learning-by-doing. Fourth, I plan to extend the analysis from a closed economy to a multi-country trade setting. Finally, I aim to improve my quantitative analysis by drawing on more granular intersectoral transaction data and refining my calibration approach.

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A Other equilibrium conditions

In this appendix section, I derive the equilibrium conditions that were not discussed in Section 2.

A.1 Household's Optimisation Problem

It is trivial to see that the household consumes all of its income

$$C = WL - T \tag{72}$$

A.2 Final Good Producer's Optimisation Problem

Recall that the final good is the numeraire. Solving the final good producer's optimisation problem gives the following equilibrium conditions:

$$1 = \frac{1}{1-D}\prod_i P_i^{b_i} \tag{73}$$

$$P_i X_i = b_i Y \tag{74}$$

A.3 Sectoral Good Producer's Optimisation Problem

Solving the sectoral good producer's optimisation problem gives the following conditions regarding prices and quantities:

$$\log P_i = \int_0^1 \log p_i(v)\, dv \tag{75}$$

$$p_i(v)x_i(v) = P_i Y_i \tag{76}$$

A.4 Variety Producer's Optimisation Problem

For each variety $i \in \mathcal{N}, v \in [0,1]$, solving the variety producer's optimisation problem gives the following conditions regarding prices, intermediate input expenditure, and labour expenditure:

$$P_j x_{ij}(v) = \frac{\tilde{m}_{ij}^{\theta_i^*(v)}}{1-\sigma_j^{\theta_i^*(v)}} p_i(v) y_i(v) \,\forall\, j \in \mathcal{N} \tag{77}$$

$$W l_i(v) = \tilde{a}_i^{\theta_i^*(v)} p_i(v) y_i(v) \tag{78}$$

B Additional Tables and Figures

B.1 Calibration

Table V. Lowest and highest fossil fuel share country by sector

		Lowest		Highest	
No.	**Sector Name**	**Country**	**Share**	**Country**	**Share**
1	(A01-02) Agriculture, hunting, forestry	USA	0.024	CAN	0.056
2	(A03) Fishing and aquaculture	USA	0.016	KOR	0.114
3	(B05-06) Mining and quarrying, energy producing products	NOR	0.006	AUS	0.030
4	(B07-08) Mining and quarrying, non-energy producing products	AUS	0.039	USA	0.063
5	(B09) Mining support service activities	GBR	0.008	NOR	0.044
6	(C10-12) Food products, beverages and tobacco	NLD	0.008	FRA	0.020
7	(C13-15) Textiles, textile products, leather and footwear	FRA	0.009	BEL	0.027
8	(C16) Wood and products of wood and cork	FRA	0.008	ESP	0.038
9	(C17-18) Paper products and printing	ITA	0.026	FRA	0.056
10	(C19) Coke and refined petroleum products	USA	0.027	BEL	0.226
11	(C20) Chemical and chemical products	USA	0.043	KOR	0.157
12	(C21) Pharmaceuticals, medicinal chemical and botanical products	USA	0.004	ESP	0.020
13	(C22) Rubber and plastics products	FRA	0.016	KOR	0.042
14	(C23) Other non-metallic mineral products	AUS	0.019	ESP	0.110
15	(C24) Basic metals	AUS	0.021	FRA	0.084
16	(C25) Fabricated metal products	NLD	0.011	KOR	0.028
17	(C26) Computer, electronic and optical equipment	USA	0.003	ITA	0.021
18	(C27) Electrical equipment	CZE	0.009	CHE	0.039
19	(C28) Machinery and equipment, nec	DNK	0.005	CHE	0.015
20	(C29) Motor vehicles, trailers and semi-trailers	USA	0.004	ESP	0.013
21	(C30) Other transport equipment	USA	0.002	KOR	0.016

Continued on next page

Table V – continued from previous page

No.	Sector Name	Lowest		Highest	
		Country	Share	Country	Share
22	(C31-33) Manufacturing nec; repair and installation of machinery and equipment	FRA	0.005	CAN	0.018
23	(D) Electricity, gas, steam and air conditioning supply	CAN	0.010	AUT	0.525
24	(E) Water supply; sewerage, waste management and remediation activities	USA	0.014	KOR	0.060
25	(F) Construction	FRA	0.006	JPN	0.019
26	(G) Wholesale and retail trade; repair of motor vehicles	USA	0.013	KOR	0.028
27	(H49) Land transport and transport via pipelines	NLD	0.055	KOR	0.197
28	(H50) Water transport	CHE	0.023	ITA	0.178
29	(H51) Air transport	USA	0.082	KOR	0.217
30	(H52) Warehousing and support activities for transportation	ITA	0.007	USA	0.048
31	(H53) Postal and courier activities	FRA	0.007	KOR	0.107
32	(I) Accommodation and food service activities	CAN	0.013	JPN	0.042
33	(J58-60) Publishing, audiovisual and broadcasting activities	USA	0.002	FRA	0.034
34	(J61) Telecommunications	USA	0.004	ESP	0.044
35	(J62-63) IT and other information services	AUS	0.001	GBR	0.006
36	(K) Financial and insurance activities	FRA	0.003	KOR	0.008
37	(L) Real estate activities	ITA	0.001	USA	0.030
38	(M) Professional, scientific and technical activities	NLD	0.005	KOR	0.050
39	(N) Administrative and support services	ITA	0.004	JPN	0.026
40	(O) Public administration and defence; compulsory social security	FRA	0.009	JPN	0.026
41	(P) Education	AUS	0.004	KOR	0.040

Continued on next page

Table V – continued from previous page

		Lowest		Highest	
No.	**Sector Name**	**Country**	**Share**	**Country**	**Share**
42	(Q) Human health and social work activities	AUS	0.005	KOR	0.040
43	(R) Arts, entertainment and recreation	AUS	0.005	JPN	0.052
44	(S) Other service activities	USA	0.008	ITA	0.057
45	(T) Activities of households as employers; undifferentiated goods- and services-producing activities of households for own use	KOR	0.000	FRA	0.000

TABLE VI. CALIBRATED VALUES FOR TECHNOLOGY GAP LOCATION PARAMETERS μ

No.	**Sector Name**	μ_i
1	(A01-02) Agriculture, hunting, forestry	-1.187
2	(A03) Fishing and aquaculture	-1.007
3	(B05-06) Mining and quarrying, energy producing products	0.385
4	(B07-08) Mining and quarrying, non-energy producing products	-0.307
5	(B09) Mining support service activities	-0.375
6	(C10-12) Food products, beverages and tobacco	-0.357
7	(C13-15) Textiles, textile products, leather and footwear	-0.469
8	(C16) Wood and products of wood and cork	-0.195
9	(C17-18) Paper products and printing	-1.062
10	(C19) Coke and refined petroleum products	-2.301
11	(C20) Chemical and chemical products	-1.673
12	(C21) Pharmaceuticals, medicinal chemical and botanical products	-0.887
13	(C22) Rubber and plastics products	-0.467
14	(C23) Other non-metallic mineral products	-0.713
15	(C24) Basic metals	-0.869
16	(C25) Fabricated metal products	-0.902
17	(C26) Computer, electronic and optical equipment	-1.404
18	(C27) Electrical equipment	-1.210
19	(C28) Machinery and equipment, nec	-0.393
20	(C29) Motor vehicles, trailers and semi-trailers	-0.990
21	(C30) Other transport equipment	-1.443
22	(C31-33) Manufacturing nec; repair and installation of machinery and equipment	-0.764
23	(D) Electricity, gas, steam and air conditioning supply	-1.549
24	(E) Water supply; sewerage, waste management and remediation activities	-1.175
25	(F) Construction	0.007
26	(G) Wholesale and retail trade; repair of motor vehicles	-1.193
27	(H49) Land transport and transport via pipelines	-2.249
28	(H50) Water transport	-0.671
29	(H51) Air transport	-1.800

Continued on next page

Table VI – continued from previous page

No.	Sector Name	μ_i
30	(H52) Warehousing and support activities for transportation	-0.196
31	(H53) Postal and courier activities	-1.447
32	(I) Accommodation and food service activities	-0.832
33	(J58-60) Publishing, audiovisual and broadcasting activities	-2.242
34	(J61) Telecommunications	-2.000
35	(J62-63) IT and other information services	-0.054
36	(K) Financial and insurance activities	-0.578
37	(L) Real estate activities	-0.361
38	(M) Professional, scientific and technical activities	-2.143
39	(N) Administrative and support services	-0.914
40	(O) Public administration and defence; compulsory social security	0.194
41	(P) Education	-0.605
42	(Q) Human health and social work activities	-2.125
43	(R) Arts, entertainment and recreation	-0.651
44	(S) Other service activities	-1.609
45	(T) Activities of households as employers; undifferentiated goods- and services-producing activities of households for own use	-

TABLE VII. CALIBRATED VALUES FOR FINAL DEMAND SHARES b

No.	Sector Name	b_i
1	(A01-02) Agriculture, hunting, forestry	0.009
2	(A03) Fishing and aquaculture	0.001
3	(B05-06) Mining and quarrying, energy producing products	0.002
4	(B07-08) Mining and quarrying, non-energy producing products	0.000
5	(B09) Mining support service activities	0.002
6	(C10-12) Food products, beverages and tobacco	0.037
7	(C13-15) Textiles, textile products, leather and footwear	0.010
8	(C16) Wood and products of wood and cork	0.001

Continued on next page

Table VII – continued from previous page

No.	Sector Name	b_i
9	(C17-18) Paper products and printing	0.002
10	(C19) Coke and refined petroleum products	0.010
11	(C20) Chemical and chemical products	0.006
12	(C21) Pharmaceuticals, medicinal chemical and botanical products	0.011
13	(C22) Rubber and plastics products	0.003
14	(C23) Other non-metallic mineral products	0.001
15	(C24) Basic metals	0.000
16	(C25) Fabricated metal products	0.005
17	(C26) Computer, electronic and optical equipment	0.018
18	(C27) Electrical equipment	0.006
19	(C28) Machinery and equipment, nec	0.017
20	(C29) Motor vehicles, trailers and semi-trailers	0.029
21	(C30) Other transport equipment	0.007
22	(C31-33) Manufacturing nec; repair and installation of machinery and equipment	0.013
23	(D) Electricity, gas, steam and air conditioning supply	0.012
24	(E) Water supply; sewerage, waste management and remediation activities	0.005
25	(F) Construction	0.089
26	(G) Wholesale and retail trade; repair of motor vehicles	0.089
27	(H49) Land transport and transport via pipelines	0.013
28	(H50) Water transport	0.002
29	(H51) Air transport	0.005
30	(H52) Warehousing and support activities for transportation	0.004
31	(H53) Postal and courier activities	0.001
32	(I) Accommodation and food service activities	0.038
33	(J58-60) Publishing, audiovisual and broadcasting activities	0.015
34	(J61) Telecommunications	0.014
35	(J62-63) IT and other information services	0.021
36	(K) Financial and insurance activities	0.042
37	(L) Real estate activities	0.117
38	(M) Professional, scientific and technical activities	0.029

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Table VII – continued from previous page

No.	Sector Name	b_i
39	(N) Administrative and support services	0.010
40	(O) Public administration and defence; compulsory social security	0.105
41	(P) Education	0.058
42	(Q) Human health and social work activities	0.113
43	(R) Arts, entertainment and recreation	0.013
44	(S) Other service activities	0.015
45	(T) Activities of households as employers; undifferentiated goods- and services-producing activities of households for own use	0.002

TABLE VIII. SUMMARY OF BASELINE ECONOMY

Sector	VA_i	λ_i	G_i
(L) Real estate activities	0.37	0.15	0.39
(G) Wholesale and retail trade; repair of motor vehicles	0.33	0.19	0.71
(O) Public administration and defence; compulsory social security	0.26	0.11	0.25
(Q) Human health and social work activities	0.25	0.12	0.85
(M) Professional, scientific and technical activities	0.20	0.13	0.90
(K) Financial and insurance activities	0.17	0.12	0.64
(F) Construction	0.16	0.11	0.36
(P) Education	0.14	0.06	0.56
(N) Administrative and support services	0.13	0.08	0.76
(J62-63) IT and other information services	0.08	0.05	0.41
(I) Accommodation and food service activities	0.08	0.05	0.61
(D) Electricity, gas, steam and air conditioning supply	0.07	0.04	0.59
(C26) Computer, electronic and optical equipment	0.06	0.03	0.63
(J58-60) Publishing, audiovisual and broadcasting activities	0.06	0.03	0.88
(H49) Land transport and transport via pipelines	0.06	0.04	0.92
(C10-12) Food products, beverages and tobacco	0.05	0.06	0.53
(J61) Telecommunications	0.05	0.03	0.85
(B05-06) Mining and quarrying, energy producing products	0.04	0.02	0.14

Continued on next page

Table VIII – continued from previous page

Sector	VA_i	λ_i	G_i
(A01-02) Agriculture, hunting, forestry	0.04	0.03	0.72
(C25) Fabricated metal products	0.04	0.03	0.64
(S) Other service activities	0.04	0.02	0.82
(H52) Warehousing and support activities for transportation	0.04	0.03	0.52
(C31-33) Manufacturing nec; repair and installation of machinery and equipment	0.04	0.03	0.65
(C20) Chemical and chemical products	0.03	0.03	0.75
(C28) Machinery and equipment, nec	0.03	0.03	0.57
(C29) Motor vehicles, trailers and semi-trailers	0.03	0.05	0.67
(R) Arts, entertainment and recreation	0.03	0.02	0.68
(C21) Pharmaceuticals, medicinal chemical and botanical products	0.03	0.02	0.57
(H53) Postal and courier activities	0.03	0.02	0.62
(E) Water supply; sewerage, waste management and remediation activities	0.02	0.01	0.73
(B07-08) Mining and quarrying, non-energy producing products	0.02	0.01	0.37
(C19) Coke and refined petroleum products	0.02	0.03	0.80
(C22) Rubber and plastics products	0.02	0.02	0.47
(C13-15) Textiles, textile products, leather and footwear	0.02	0.02	0.54
(C17-18) Paper products and printing	0.02	0.02	0.73
(C30) Other transport equipment	0.01	0.01	0.69
(C27) Electrical equipment	0.01	0.02	0.72
(H51) Air transport	0.01	0.01	0.75
(C23) Other non-metallic mineral products	0.01	0.01	0.58
(C24) Basic metals	0.01	0.02	0.55
(C16) Wood and products of wood and cork	0.01	0.01	0.48
(T) Activities of households as employers; undifferentiated goods- and services-producing activities of households for own use	0.01	0.00	0.37
(B09) Mining support service activities	0.00	0.00	0.46
(H50) Water transport	0.00	0.01	0.71
(A03) Fishing and aquaculture	0.00	0.00	0.40

B.2 Results

Table IX. Change in emissions: total and decomposition

Rank	Sector	Dir.	Up.	Down.	Casc.	Tot.	Tot. / Dir
1	(C26) Computer, electronic and optical equipment	-0.021	-0.000	0.004	0.001	-0.018	0.816
2	(H53) Postal and courier activities	-0.023	0.000	0.007	0.000	-0.015	0.679
3	(F) Construction	-0.049	0.023	0.013	-0.001	-0.014	0.295
4	(C21) Pharmaceuticals, medicinal chemical and botanical products	-0.009	0.001	0.001	0.001	-0.007	0.714
5	(C29) Motor vehicles, trailers and semi-trailers	-0.030	0.019	0.003	0.002	-0.006	0.201
6	(C30) Other transport equipment	-0.008	0.002	0.002	0.000	-0.004	0.540
7	(C10-12) Food products, beverages and tobacco	-0.034	0.027	0.004	0.000	-0.003	0.076
8	(C13-15) Textiles, textile products, leather and footwear	-0.011	0.007	0.002	0.000	-0.002	0.219
9	(C23) Other non-metallic mineral products	-0.010	0.005	0.002	0.000	-0.002	0.252
10	(C31-33) Manufacturing nec; repair and installation of machinery and equipment	-0.018	0.008	0.007	0.001	-0.002	0.108
11	(A03) Fishing and aquaculture	-0.001	-0.000	0.000	0.000	-0.002	1.069
12	(C27) Electrical equipment	-0.011	0.007	0.003	0.001	0.000	-0.023
13	(C22) Rubber and plastics products	-0.009	0.005	0.004	0.001	0.001	-0.060
14	(E) Water supply; sewerage, waste management and remediation activities	-0.008	0.004	0.005	0.001	0.001	-0.073

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Table IX – continued from previous page

Rank	Sector	Dir.	Up.	Down.	Casc.	Tot.	Tot. / Dir
15	(C16) Wood and products of wood and cork	-0.004	0.003	0.001	0.000	0.001	-0.227
16	(B07-08) Mining and quarrying, non-energy producing products	-0.004	0.001	0.004	0.001	0.002	-0.536
17	(C28) Machinery and equipment, nec	-0.017	0.012	0.008	0.001	0.003	-0.158
18	(C25) Fabricated metal products	-0.021	0.013	0.011	0.001	0.003	-0.138
19	(C17-18) Paper products and printing	-0.011	0.009	0.006	0.001	0.005	-0.443
20	(A01-02) Agriculture, hunting, forestry	-0.013	0.017	0.004	-0.000	0.007	-0.498
21	(H50) Water transport	-0.010	0.008	0.012	-0.002	0.008	-0.839
22	(B09) Mining support service activities	-0.003	0.001	0.013	-0.000	0.010	-3.013
23	(H51) Air transport	0.002	0.003	0.005	0.000	0.011	4.370
24	(C24) Basic metals	-0.011	0.017	0.005	0.001	0.012	-1.046
25	(H52) Warehousing and support activities for transportation	-0.016	0.010	0.020	0.001	0.015	-0.953
26	(C20) Chemical and chemical products	-0.025	0.015	0.031	0.006	0.027	-1.082
27	(H49) Land transport and transport via pipelines	0.009	0.042	0.075	-0.002	0.123	14.261

C Examples

Definition 3 (Stylised Economy A)**.** *There are two sectors $\mathcal{N} = \{1, 2\}$, of which one is a fossil fuel sector $\mathcal{N}_F = \{2\}$. The final demand shares $\boldsymbol{b}$, clean technology input shares $\tilde{\boldsymbol{M}}^c$, dirty technology input shares $\tilde{\boldsymbol{M}}^d$, and emissions content vectors $\boldsymbol{\epsilon}$ are given, respectively, by*

$$\boldsymbol{b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \tilde{\boldsymbol{M}}^c = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \tilde{\boldsymbol{M}}^d = \begin{bmatrix} 0 & \tilde{m}^d_{12} \\ 0 & 0 \end{bmatrix}, \quad \textit{and} \quad \boldsymbol{\epsilon} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

All subsidies are set to zero

$$\sigma_i^\theta = 0, \quad \forall\, i, \theta.$$

The set of technology gap distributions $\{\mathcal{G}_i(X)\}_{i\in\mathcal{N}}$ *is such that the equilibrium clean technology shares are given by*

$$\boldsymbol{G} = [G_1,\, G_2]',$$

where $G_1, G_2 \in (0,1)$ *and the equilibrium adoption elasticities are given by*

$$\boldsymbol{g} = [g_1,\, g_2]',$$

where $g_1, g_2 \geq 0$.

Definition 4 (Stylised Economy B)**.** *There are three sectors* $\mathcal{N} = \{1,2,3\}$*, of which one is a fossil fuel sector* $\mathcal{N}_F = \{2\}$*. The final demand shares* $\boldsymbol{b}$*, clean technology input shares* $\tilde{\boldsymbol{M}}^c$*, dirty technology input shares* $\tilde{\boldsymbol{M}}^d$*, and emissions content vectors* $\boldsymbol{\epsilon}$ *are given, respectively, by*

$$\boldsymbol{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \tilde{\boldsymbol{M}}^c = \begin{bmatrix} 0 & 0 & \tilde{m}_{13}^c \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \tilde{\boldsymbol{M}}^d = \begin{bmatrix} 0 & \tilde{m}_{12}^d & 0 \\ 0 & 0 & 0 \\ 0 & \tilde{m}_{32}^d & 0 \end{bmatrix}, \quad \textit{and} \quad \boldsymbol{\epsilon} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

All subsidies are set to zero

$$\sigma_i^\theta = 0, \quad \forall\, i, \theta.$$

The set of technology gap distributions $\{\mathcal{G}_i(X)\}_{i\in\mathcal{N}}$ *is such that the equilibrium clean technology shares are given by*

$$\boldsymbol{G} = [G_1,\, G_2,\, G_3]',$$

where $G_1, G_2, G_3 \in (0,1)$ *and the equilibrium adoption elasticities are given by*

$$\boldsymbol{g} = [g_1,\, g_2,\, g_3]',$$

where $g_1, g_2, g_3 \geq 0$.

Definition 5 (Stylised Economy C)**.** *There are four sectors* $\mathcal{N} = \{1,2,3,4\}$*, of which two are fossil fuel sectors* $\mathcal{N}_F = \{2,4\}$*. The final demand shares* $\boldsymbol{b}$*, clean technology input shares* $\tilde{\boldsymbol{M}}^c$*, and dirty technology input shares* $\tilde{\boldsymbol{M}}^d$ *are given, respectively, by*

$$\boldsymbol{b} = \begin{bmatrix} b_1 \\ b_2 \\ 0 \\ 0 \end{bmatrix}, \quad \tilde{\boldsymbol{M}}^c = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \tilde{m}_{23}^c & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \textit{and} \quad \tilde{\boldsymbol{M}}^d = \begin{bmatrix} 0 & 0 & 0 & \tilde{m}_{14}^d \\ 0 & 0 & 0 & \tilde{m}_{24}^d \\ 0 & 0 & 0 & \tilde{m}_{34}^d \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

All subsidies are set to zero

$$\sigma_i^\theta = 0, \quad \forall\, i, \theta.$$

The set of technology gap distributions $\{\mathcal{G}_i(X)\}_{i\in\mathcal{N}}$ *is such that the equilibrium clean technology shares are given by*

$$\boldsymbol{G} = [G_1,\, G_2,\, G_3,\, G_4]',$$

where $G_1, G_2, G_3, G_4 \in (0,1)$ *and the equilibrium adoption elasticities are given by*

$$\boldsymbol{g} = [0,\, g_2,\, 0,\, 0]',$$

where $g_2 > 0$.

D Main Proofs (Incomplete)

Proof of Theorem (1) This proof proceeds in three steps. First, I show that the equilibrium concept (1) implies a fixed point problem in normalised sectoral prices. Second, I show that there exists a unique set of normalised prices that satisfies this fixed point problem. Finally, I show that this set of normalised prices maps uniquely to all other endogenous variables in the model.

Step 1: Fixed Point Problem in Normalised Prices Taking logs of (6) gives

$$\log p_i(v) = \min_{\theta\in\Theta_i} \log \kappa_i^\theta(v), \tag{79}$$

where $\kappa_i^\theta(v)$ is given by (4). Similarly, price-taking, profit-maximising behaviour by sectoral good producers implies that sectoral prices can be expressed in terms of variety-level prices

$$\log P_i = \int_0^1 \log p_i(v)\, dv. \tag{80}$$

Using $\overline{p}_i(v) \equiv p_i(v)/W$ and $\overline{P}_i \equiv P_i/W$, the above expressions imply

$$\log \overline{p}_i(v) = \min_{\theta\in\Theta_i} \phi_{iv}^\theta(\log \overline{\mathbf{P}}), \tag{81}$$

where function ϕ_{iv}^θ is defined as

$$\phi_{iv}^\theta : \mathbb{R}^{|\mathcal{N}|} \to \mathbb{R} \quad \text{where} \quad \phi_{iv}^\theta(\log \overline{\mathbf{P}}) \equiv \log(1-\sigma_i^\theta) - \log q_i^\theta(v) + \sum_j \tilde{m}_{ij}^\theta \log \overline{P}_j, \tag{82}$$

and

$$\log \overline{P}_i = \int_0^1 \log \overline{p}_i(v)\, dv. \tag{83}$$

Substituting (81) into (83) implies that the equilibrium vector of normalised prices satisfies

the following fixed point problem

$$\log \overline{\mathbf{P}} = \Phi(\log \overline{\mathbf{P}}), \tag{84}$$

where the ith element of function $\Phi : \mathbb{R}^{|\mathcal{N}|} \to \mathbb{R}^{|\mathcal{N}|}$ is defined as

$$\Phi_i(\log \overline{\mathbf{P}}) \equiv \int_0^1 \min_{\theta \in \Theta_i} \phi_{iv}^{\theta}(\log \overline{\mathbf{P}})\, dv. \tag{85}$$

Step 2: Existence and Uniqueness of Normalised Prices As all technologies require labour by assumption, the total intermediate input share for any sector-technology pair is strictly less than one. This implies the largest intermediate input share across sector-technology pairs is strictly less than one, i.e.

$$\max_{j \in \mathcal{N}, \theta \in \Theta_j} 1 - \tilde{a}_j^{\theta} < 1. \tag{86}$$

Combined with Lemma (8), this means Φ is a contraction mapping on the metric space $(\mathbb{R}^{|\mathcal{N}|}, \|.\|_\infty)$, where $\|.\|_\infty$ is the ∞-norm operator[11]. The existence and uniqueness of the set of normalised sectoral prices follows directly from this.

Step 3: Mapping to all other endogenous variables Given the exogenous set of productivity shifters $\{q_i^{\theta}(v)\}_{i \in \mathcal{N}, \theta \in \{c,d\}}$, subsidy rates $\{\sigma_i^{\theta}\}_{i \in \mathcal{N}, \theta \in \{c,d\}}$, and technology input requirements $\{\tilde{a}_i^{\theta}, \tilde{m}_{ij}^{\theta}\}_{i,j \in \mathcal{N}, \theta \in \{c,d\}}$, I show that the set of normalised sectoral prices that satisfy the fixed point problem (84) uniquely determines all other equilibrium objects in the model: $\theta_i^*(v), p_i(v), y_i(v), x_{ij}(v), l_i(v), P_i, Y_i, x_i(v), X_i, E, Y, W, C, T \ \forall \ i, j \in \mathcal{N}, v \in [0, 1]$.

Technology Choice. By equation (8), the set of normalised prices uniquely determines the set of sectoral competitiveness thresholds $\mathbf{\Gamma}$. These thresholds determine variety-level technology choices $\theta_i^*(v)$ via equation (7), and sectoral clean technology shares $\mathbf{G}$ by equation (11).

Network. The technology choices $\theta_i^*(v)$ derived above determine the cost-based input-output matrix $\tilde{\mathbf{\Omega}}$, cost-based Leontief inverse $\tilde{\mathbf{\Psi}}$, and cost-based Domar weights $\tilde{\boldsymbol{\lambda}}$ via equations (23), (24), and (25), respectively. Similarly, the technology choices determine the revenue-based input-output matrix $\mathbf{\Omega}$, revenue-based Leontief inverse $\mathbf{\Psi}$, and revenue-based Domar weights $\boldsymbol{\lambda}$ via equations (26), (27), and (28), respectively.

Wage Premium. I now show that the wage premium ξ is determined by the set of normalised prices. First, I derive an expression for nominal sectoral output in terms of aggregate output Y and objects pinned down by $\overline{\mathbf{P}}$. Multiply the market clearing condition for sectoral good i (17) by P_i to obtain

$$P_i X_i + \sum_j \int_0^1 P_i x_{ji}(v)\, dv = P_i Y_i. \tag{87}$$

[11] The ∞-norm operator returns the absolute value of the largest element of a vector

Use (74), (76), (77), and the definition of the revenue-based input-output matrix (26) to get

$$b_i Y + \sum_j \Omega_{ji} P_j Y_j = P_i Y_i. \tag{88}$$

Collating these equations for all $i \in \mathcal{N}$, solving for nominal sectoral output, and using the definition of revenue-based Domar weights (28) leads to

$$P_i Y_i = \lambda_i Y. \tag{89}$$

Next, recall that the government levies a lump sum tax T on the representative household to pay for its total subsidy expenditure. This implies

$$T = \sum_i \int_0^1 \frac{\sigma_i^{\theta_i^*(v)}}{1 - \sigma_i^{\theta_i^*(v)}} p_i(v) y_i(v)\, dv. \tag{90}$$

Substitute in (76) and (89) to obtain

$$T = Y \sum_i \lambda_i \int_0^1 \frac{\sigma_i^{\theta_i^*(v)}}{1 - \sigma_i^{\theta_i^*(v)}}\, dv. \tag{91}$$

Use the definition of the sectoral clean technology share G_i (10) to re-write this expression as

$$T = Y \sum_i \lambda_i \Big(G_i \frac{\sigma_i^c}{1 - \sigma_i^c} + (1 - G_i) \frac{\sigma_i^d}{1 - \sigma_i^d} \Big). \tag{92}$$

Now, recall the market clearing condition for the final good (18). Substitute in (72) and (92) to obtain

$$WL - Y \sum_i \lambda_i \Big(G_i \frac{\sigma_i^c}{1 - \sigma_i^c} + (1 - G_i) \frac{\sigma_i^d}{1 - \sigma_i^d} \Big) = Y. \tag{93}$$

Rearrange and substitute into the definition of the wage premium (22) to get the result

$$\xi \equiv \frac{W}{Y/L} - 1 = \sum_i \lambda_i \Big(G_i \frac{\sigma_i^c}{1 - \sigma_i^c} + (1 - G_i) \frac{\sigma_i^d}{1 - \sigma_i^d} \Big). \tag{94}$$

Sectoral Output. Use the revenue-based Domar weight to re-express sectoral output as

$$Y_i = \frac{P_i Y_i}{P_i} = \frac{\lambda_i Y}{P_i}. \tag{95}$$

Substitute in the definition of normalised sectoral price and rearrange to get

$$Y_i = \frac{\lambda_i}{P_i} \frac{Y/L}{W} L. \tag{96}$$

Substitute in the expression for the wage premium (94) to get sectoral output in terms of exogenous objects and the set of normalised sectoral prices

$$Y_i = \frac{\lambda_i L}{\overline{P}_i(1+\xi)}. \tag{97}$$

Emissions. Substitute (97) into (1) to get the result

$$E = \sum_i E_i = \sum_i \frac{\lambda_i \epsilon_i}{\overline{P}_i(1+\xi)} L. \tag{98}$$

Damages. Substitute (98) into the damage function (2).

Wage. Start with equation (73). Substitute in the definition for normalised sectoral price, take logs, and rearrange to obtain

$$\log W = \log(1-D)) - \sum_i b_i \log \overline{P}_i. \tag{99}$$

Aggregate Output. Rearrange the definition of the wage premium and take logs to get

$$\log Y = \log W + \log(1+\xi) + \log L. \tag{100}$$

Substitute in (99) to obtain the following expression where each object is determined by the set of normalised sectoral prices

$$\log Y = \log(1-D) - \sum_i b_i \log \overline{P}_i + \log(1+\xi) + \log L. \tag{101}$$

Remaining objects. It is trivial to show that the remaining objects ($p_i(v)$, $y_i(v)$, $x_{ij}(v)$, $l_i(v)$, P_i, $x_i(v)$, X_i, C, T) are determined by normalised sectoral prices. □

Proof of Corollary (1) Take logs of (9), then substitute in (75), (6), and (4) to obtain

$$\log \overline{P}_i = \int_0^1 \log(1-\sigma_i^{\theta_i^*(v)}) - \log q_i^{\theta_i^*(v)}(v) + \sum_j \tilde{m}_{ij}^{\theta_i^*(v)} \log(P_j/W)\, dv. \tag{102}$$

Use definitions (21), (20), (23), and (9) to re-write this as

$$\log \overline{P}_i = \log(1-S_i) - \log Q_i + \sum_j \tilde{\Omega}_{ij} \log \overline{P}_j. \tag{103}$$

Collating for all sectors and using matrix notation gives

$$(\mathbf{I} - \tilde{\mathbf{\Omega}}) \log \overline{\mathbf{P}} = -\log \mathbf{Q} + \log(\mathbf{1} - \mathbf{S}). \tag{104}$$

Multiply both sides by the cost-based Leontief matrix (24) to get the result. □

Proof of Corollary (2) Re-write (99) in matrix notation to get

$$\log W = \log(1 - D) - \mathbf{b}' \log \overline{\mathbf{P}}. \tag{105}$$

Substitute in the expression for equilibrium normalised prices (29) and the definition of cost-based Domar weights (25) to get the result. □

Proof of Corollary (3) See derivation of Equation (94) in the proof of Theorem (1). □

Proof of Corollary (4) See derivation of Equation (98) in the proof of Theorem (1). □

Proof of Corollary (5) Re-write (101) in matrix notation to get

$$\log Y = \log(1 - D) - \mathbf{b}' \log \overline{\mathbf{P}} + \log(1 + \xi) + \log L. \tag{106}$$

Substitute in the expression for equilibrium normalised sectoral prices from Corollary (1) and use the definition of cost-based Domar weights (25) to obtain the result. □

Proof of Theorem (2) Differentiating (29) with respect to σ_k^c, I obtain

$$\frac{d \log \overline{\mathbf{P}}}{d\sigma_k^c} = \frac{d\tilde{\mathbf{\Psi}}}{d\sigma_k^c}\Big(- \log \mathbf{Q} + \log(1 - \mathbf{S})\Big) + \tilde{\mathbf{\Psi}}\Big(- \frac{d \log \mathbf{Q}}{d\sigma_k^c} + \frac{d \log(1 - \mathbf{S})}{d\sigma_k^c}\Big) \tag{107}$$

I now derive the three derivative terms on the right hand side. First, we re-express the cost-based IO matrix as

$$\tilde{\mathbf{\Omega}} = \text{diag}(\mathbf{G})\tilde{\mathbf{M}}^c + (\mathbf{I} - \text{diag}(\mathbf{G}))\tilde{\mathbf{M}}^d \tag{108}$$

Taking the derivative with respect to σ_k^c, we obtain

$$\frac{d\tilde{\mathbf{\Omega}}}{d\sigma_k^c} = \text{diag}\left(\frac{d\mathbf{G}}{d\sigma_k^c}\right)\left(\tilde{\mathbf{M}}^c - \tilde{\mathbf{M}}^d\right). \tag{109}$$

Thus, the derivative of the cost-based Leontief inverse is given by

$$\frac{d\tilde{\mathbf{\Psi}}}{d\sigma_k^c} = \tilde{\mathbf{\Psi}}\frac{d\tilde{\mathbf{\Omega}}}{d\sigma_k^c}\tilde{\mathbf{\Psi}} = \tilde{\mathbf{\Psi}}\,\text{diag}\left(\frac{d\mathbf{G}}{d\sigma_k^c}\right)\left(\tilde{\mathbf{M}}^c - \tilde{\mathbf{M}}^d\right)\tilde{\mathbf{\Psi}}. \tag{110}$$

Second, I turn to $\frac{d \log \mathbf{Q}}{d\sigma_k^c}$. Use the expression for the technology gap $\gamma_i(v)$ to re-write the sectoral productivity shifter as

$$\begin{aligned} \log Q_i &= \int_0^1 \log q_i(v)\, dv \\ &= \int_0^1 \log q_i^c(v) + \mathbb{1}_{\theta_i^*(v)=c}\Big(\log q_i^d(v) - \log q_i^c(v)\Big)\, dv \\ &= \log Q_i^c + \int_0^1 \mathbb{1}_{\theta_i^*(v)=c} \log \gamma_i(v)\, dv. \end{aligned} \tag{111}$$

Now, re-writing the integral component using the change of variables $u = \log \gamma_i(v)$, we obtain

$$\log Q_i = \log Q_i^c + \int_{\log \Gamma_i}^{\infty} u \mathcal{G}_i'(u)\, du. \tag{112}$$

Taking the derivative with respect to σ_k^c, substituting in (8), and writing in vector form gives

$$\frac{d \log \mathbf{Q}}{d\sigma_k^c} = \text{diag}\left(\frac{d\mathbf{G}}{d\sigma_k^c}\right)\left(\left(\tilde{\mathbf{M}}^c - \tilde{\mathbf{M}}^d\right)\log \overline{\mathbf{P}} + \log(1 - \boldsymbol{\sigma}^c) - \log(1 - \boldsymbol{\sigma}^d)\right). \tag{113}$$

Third, I derive $\frac{d \log(1-\mathbf{S})}{d\sigma_k^c}$. Re-write the average sectoral markdown as

$$\log(1 - S_i) = G_i \log(1 - \sigma_i^c) + (1 - G_i)\log(1 - \sigma_i^d). \tag{114}$$

Taking the derivative with respect to respect to σ_k^c and writing in vector form, we obtain

$$\frac{d \log(1 - \mathbf{S})}{d\sigma_k^c} = \text{diag}\left(\frac{d\mathbf{G}}{d\sigma_k^c}\right)\left(\log(1 - \boldsymbol{\sigma}^c) - \log(1 - \boldsymbol{\sigma}^d)\right) - \frac{G_k}{1 - \sigma_k^c}\mathbf{e}_k. \tag{115}$$

Substituting (110), (113), (115), and (29) into (111) and rearranging gives the result. □

Proof of Theorem (3) Taking the derivative of (8) and $\mathbf{G}$ with respect to σ_k^c and writing in vector form, we obtain

$$\frac{d \log \boldsymbol{\Gamma}}{\sigma_k^c} = \mathbf{e}_k \frac{1}{1 - \sigma_k^c} + \left(\tilde{\mathbf{M}}^d - \tilde{\mathbf{M}}^c\right)\frac{d \log \overline{\mathbf{P}}}{d\sigma_k^c} \tag{116}$$

and

$$\frac{d\mathbf{G}}{d\sigma_k^c} = \mathbf{g} \circ \frac{d \log \boldsymbol{\Gamma}}{\sigma_k^c}. \tag{117}$$

Substituting (34) and (116) into (117) leads to

$$\frac{d\mathbf{G}}{d\sigma_k^c} = \mathbf{g} \circ \left(\mathbf{e}_k \frac{1}{1 - \sigma_k^c} + \left(\tilde{\mathbf{M}}^d - \tilde{\mathbf{M}}^c\right)\tilde{\boldsymbol{\Psi}}\mathbf{e}_k \frac{G_k}{1 - \sigma_k^c}\right). \tag{118}$$

Using the definition of $\boldsymbol{\Psi}_{\bullet k}^{\theta}$ i.e., the kth column of the cost-based Leontief inverse via technology θ, gives the necessary result. □

Proof of Theorem (4) First, we re-write the ijth element of the revenue-based input-output matrix (26) as

$$\begin{aligned} \Omega_{ij} &= \int_0^1 \mathbb{1}(\theta_i^*(v) = c)\frac{\tilde{m}_{ij}^c}{1 - \sigma_i^c} + \mathbb{1}(\theta_i^*(v) = d)\frac{\tilde{m}_{ij}^d}{1 - \sigma_i^d}\, dv \\ &= G_i \frac{\tilde{m}_{ij}^c}{1 - \sigma_i^c} + (1 - G_i)\frac{\tilde{m}_{ij}^d}{1 - \sigma_i^d}. \end{aligned} \tag{119}$$

Using the above, we can re-write the revenue-based IO matrix as

$$\boldsymbol{\Omega} = \text{diag}(\mathbf{G})(\mathbf{I} - \text{diag}(\boldsymbol{\sigma}^c))^{-1}\tilde{\mathbf{M}}^c + (\mathbf{I} - \text{diag}(\mathbf{G}))(\mathbf{I} - \text{diag}(\boldsymbol{\sigma}^d))^{-1}\tilde{\mathbf{M}}^d \tag{120}$$

Differentiating this with respect to to σ_k^c, we obtain

$$\begin{aligned}\frac{d\boldsymbol{\Omega}}{d\sigma_k^c} = \text{diag}\left(\frac{d\mathbf{G}}{d\sigma_k^c}\right)\left((\mathbf{I} - \text{diag}(\boldsymbol{\sigma}^c))^{-1}\tilde{\mathbf{M}}^c - (\mathbf{I} - \text{diag}(\boldsymbol{\sigma}^d))^{-1}\tilde{\mathbf{M}}^d\right) \\ + \text{diag}(\mathbf{G})(\mathbf{I} - \text{diag}(\boldsymbol{\sigma}^c))^{-1}\,\text{diag}(\mathbf{e}_k)(\mathbf{I} - \text{diag}(\boldsymbol{\sigma}^c))^{-1}\tilde{\mathbf{M}}^c.\end{aligned} \tag{121}$$

Using $\mathbf{M}^\theta \equiv (\mathbf{I} - \text{diag}(\boldsymbol{\sigma}^\theta))^{-1}\tilde{\mathbf{M}}^\theta$, this simplifies to

$$\frac{d\boldsymbol{\Omega}}{d\sigma_k^c} = \text{diag}\left(\frac{d\mathbf{G}}{d\sigma_k^c}\right)(\mathbf{M}^c - \mathbf{M}^d) + \frac{G_k}{1-\sigma_k^c}\,\text{diag}(\mathbf{e}_k)\mathbf{M}^c. \tag{122}$$

Using the Hadamard product operator ($\circ$) to rewrite the first term gives the result. □

Proof of Theorem (5) Differentiate the revenue-based Leontief inverse (27) with respect to σ_k^c, substitute in the derivative of the revenue-based input-output matrix with respect to σ_k^c (4), and use $\boldsymbol{\Psi}^\theta \equiv \mathbf{M}^\theta\boldsymbol{\Psi}$ to obtain

$$\begin{aligned}\frac{d\boldsymbol{\Psi}}{d\sigma_k^c} &= \boldsymbol{\Psi}\frac{d\boldsymbol{\Omega}}{d\sigma_k^c}\boldsymbol{\Psi} \\ &= \boldsymbol{\Psi}\left(\frac{G_k}{1-\sigma_k^c}\,\text{diag}(\mathbf{e}_k)\mathbf{M}^c + \text{diag}\left(\frac{d\mathbf{G}}{d\sigma_k^c}\right)(\mathbf{M}^c - \mathbf{M}^d)\right)\boldsymbol{\Psi} \\ &= \boldsymbol{\Psi}\left(\frac{G_k}{1-\sigma_k^c}\,\text{diag}(\mathbf{e}_k)\boldsymbol{\Psi}^c + \text{diag}\left(\frac{d\mathbf{G}}{d\sigma_k^c}\right)(\boldsymbol{\Psi}^c - \boldsymbol{\Psi}^d)\right).\end{aligned} \tag{123}$$

Now, differentiate the revenue-based Domar weights (28) with respect to σ_k^c, substitute in the above expression to get

$$\begin{aligned}\frac{d\boldsymbol{\lambda}'}{d\sigma_k^c} &= \mathbf{b}'\frac{d\boldsymbol{\Psi}}{d\sigma_k^c} \\ &= \boldsymbol{\lambda}'\left(\frac{G_k}{1-\sigma_k^c}\,\text{diag}(\mathbf{e}_k)\boldsymbol{\Psi}^c + \text{diag}\left(\frac{d\mathbf{G}}{d\sigma_k^c}\right)(\boldsymbol{\Psi}^c - \boldsymbol{\Psi}^d)\right).\end{aligned} \tag{124}$$

Next, substitute in the derivatives (with respect to σ_k^c) of the clean technology shares (35) and

the set of normalised prices (34), and rearrange to obtain

$$\begin{aligned}\frac{d\boldsymbol{\lambda}'}{d\sigma_k^c} &= \boldsymbol{\lambda}'\Bigg(\frac{G_k}{1-\sigma_k^c}\,\mathrm{diag}(\mathbf{e}_k)\boldsymbol{\Psi}^c \\ &+ \Bigg(\,\mathrm{diag}(\mathbf{e}_k)\frac{g_k}{1-\sigma_k^c} + \mathrm{diag}(\mathbf{g})\,\mathrm{diag}\left(\tilde{\boldsymbol{\Psi}}^c_{\bullet k} - \tilde{\boldsymbol{\Psi}}^d_{\bullet k}\right)\frac{G_k}{1-\sigma_k^c}\Bigg)(\boldsymbol{\Psi}^c - \boldsymbol{\Psi}^d)\Bigg) \\ &= \frac{\lambda_k G_k}{1-\sigma_k^c}\boldsymbol{\Psi}^c_{k\bullet} + \frac{\lambda_k g_k}{1-\sigma_k}(\boldsymbol{\Psi}^c_{k\bullet} - \boldsymbol{\Psi}^d_{k\bullet}) + \boldsymbol{\lambda}'\,\mathrm{diag}(\mathbf{g})\,\mathrm{diag}\left(\tilde{\boldsymbol{\Psi}}^c_{\bullet k} - \tilde{\boldsymbol{\Psi}}^d_{\bullet k}\right)\frac{G_k}{1-\sigma_k^c}(\boldsymbol{\Psi}^c - \boldsymbol{\Psi}^d)\end{aligned} \tag{125}$$

Using $\boldsymbol{\Psi}^\theta_{k\bullet} \equiv \mathbf{M}^\theta_{k\bullet} + (\boldsymbol{\Psi}^\theta_{k\bullet} - \mathbf{M}^\theta_{k\bullet})$, this can be rewritten as

$$\begin{aligned}\frac{d\boldsymbol{\lambda}'}{d\sigma_k^c} &= \frac{\lambda_k G_k}{1-\sigma_k^c}\mathbf{M}^c_{k\bullet} + \frac{\lambda_k g_k}{1-\sigma_k}(\mathbf{M}^c_{k\bullet} - \mathbf{M}^d_{k\bullet}) \\ &+ \frac{\lambda_k G_k}{1-\sigma_k^c}(\boldsymbol{\Psi}^c_{k\bullet} - \mathbf{M}^c_{k\bullet}) + \frac{\lambda_k g_k}{1-\sigma_k}\Big(((\boldsymbol{\Psi}^c_{k\bullet} - \mathbf{M}^c_{k\bullet}) - (\boldsymbol{\Psi}^d_{k\bullet} - \mathbf{M}^d_{k\bullet})\Big) \\ &+ \boldsymbol{\lambda}'\,\mathrm{diag}(\mathbf{g})\,\mathrm{diag}\left(\tilde{\boldsymbol{\Psi}}^c_{\bullet k} - \tilde{\boldsymbol{\Psi}}^d_{\bullet k}\right)\frac{G_k}{1-\sigma_k^c}(\boldsymbol{\Psi}^c - \boldsymbol{\Psi}^d)\end{aligned} \tag{126}$$

Post multiply this expression by column vector $\mathbf{e}_j$ and use the definitions of $\frac{d\lambda_j^{dir}}{d\sigma_k^c}$, $\frac{d\lambda_j^{up}}{d\sigma_k^c}$ and $\frac{d\lambda_j^{casc}}{d\sigma_k^c}$ in Theorem (5) to obtain the necessary result. □

Proof of Theorem (6) Start with the equilibrium wage premium in Corollary (3). Differentiating with respect to σ_k^c gives

$$\begin{aligned}\frac{d\xi}{d\sigma_k^c} &= \sum_i\left(\frac{d\lambda_i}{d\sigma_k^c}G_i\frac{\sigma_i^c}{1-\sigma_i^c} + \lambda_i\frac{dG_i}{d\sigma_k^c}\frac{\sigma_i^c}{1-\sigma_i^c} + \mathbb{1}_{i=k}\frac{1}{(1-\sigma_k^c)^2}\right) \\ &+ \sum_i\left(\frac{d\lambda_i}{d\sigma_k^c}(1-G_i)\frac{\sigma_i^d}{1-\sigma_i^d} - \lambda_i\frac{dG_i}{d\sigma_k^c}\frac{\sigma_i^d}{1-\sigma_i^d}\right).\end{aligned} \tag{127}$$

Rearrange the expression to obtain the result. □

Proof of Corollary (6) Substitute $\sigma_i^\theta = 0\,\forall\, i \in \mathcal{N}, \theta \in \{d, c\}$ into Theorem (6) to obtain the result. □

Proof of Theorem (7) There are two parts to this result. To obtain the semi-elasticity of total emissions in terms of the semi-elasticity of sectoral emissions (43), take logs of $E = \sum_i E_i$, differentiate with respect to σ_k^c, and apply the chain rule.

To obtain the semi-elasticity of sectoral emissions (44), start with the expression for sectoral emissions (32)

$$E_i = \frac{\lambda_i \epsilon_i}{\overline{P}_i(1+\xi)}L. \tag{128}$$

Take logs and differentiate with respect to σ_k^c to obtain

$$\frac{d\log E_i}{d\sigma_k^c} = \frac{1}{\lambda_i}\frac{d\lambda_i}{d\sigma_k^c} - \frac{d\log \overline{P}_i}{d\sigma_k^c} - \frac{1}{1+\xi}\frac{d\xi}{d\sigma_k^c}. \tag{129}$$

Substitute in the expression for the change in a sector's Domar weight (37) and rearrange to get the result. □

Proof of Corollary (7) Follows trivially by rearranging (43) and (44) from Theorem (7). □

Proof of Lemma (1) Applying Corollary (7) to the economy in Definition 3 using $k = 1$ yields the result. □

Proof of Lemma (2) Applying Corollary (7) to the economy in Definition 4 using $k = 1$ yields the result. □

Proof of Proposition (1) Work in progress □

Proof of Lemma (3) Applying Corollary (7) to the economy in Definition 4 using $k = 3$ yields the result. □

Proof of Proposition (2) Work in progress □

Proof of Lemma (4) Applying Corollary (7) to the economy in Definition 5 using $k = 3$ yields the result. □

Proof of Proposition (3) Work in progress □

Proof of Theorem (8) The expression for aggregate output (33) reveals that there are two components to the semi-elasticity of aggregate output with respect to σ_k^c. To obtain the first component - the change in the damage-related component of aggregate TPF (67) - substitute the damage function (2) into the damage-related component of aggregate output (33), differentiate with respect to σ_k^c, and apply the chain rule.

$$\begin{aligned}\frac{d}{d\sigma_k^c}\log(1-D) &= \frac{d}{d\sigma_k^c}\log(1-\delta(E)) \\ &= \frac{-\delta'(E)}{1-\delta(E)}\frac{dE}{d\sigma_k^c} \\ &= \frac{-\delta'(E)E}{1-\delta(E)}\frac{d\log E}{d\sigma_k^c}\end{aligned} \tag{130}$$

To obtain the second component, start with the expression for non-damage component of efficiency.

$$\log Y^{ND} \equiv \tilde{\boldsymbol{\lambda}}'\log \mathbf{Q} - \tilde{\boldsymbol{\lambda}}'\log(1-\mathbf{S}) - \log(1+\xi) \tag{131}$$

Re-write this using the expression for the cost-based Domar weights in terms of the final demand shares and the cost-based Leontief inverse (25).

$$\begin{aligned}\log Y^{ND} &= \mathbf{b}'\tilde{\mathbf{\Psi}}\log\mathbf{Q} - \mathbf{b}'\tilde{\mathbf{\Psi}}\log(1-\mathbf{S}) - \log(1+\xi) \\ &= -\mathbf{b}'\tilde{\mathbf{\Psi}}(-\log\mathbf{Q} + \log(1-\mathbf{S})) - \log(1+\xi)\end{aligned} \tag{132}$$

Substitute in the expression for the vector of log normalised prices (29) to get

$$\log Y^{ND} = -\mathbf{b}'\log\overline{\mathbf{P}} - \log(1+\xi) \tag{133}$$

Differentiating with respect to σ_k^c, using the semi-elasticity of normalised prices (34), and re-applying the definition of cost-based Domar weights (25) gives

$$\begin{aligned}\frac{d}{d\sigma_k^c}\log Y^{ND} &= \frac{d}{d\sigma_k^c}\big(-\mathbf{b}'\log\overline{\mathbf{P}} - \log(1+\xi)\big) \\ &= \mathbf{b}'\tilde{\mathbf{\Psi}}\mathbf{e}_k\frac{G_k}{1-\sigma_k^c} - \frac{1}{1+\xi}\frac{d\xi}{d\sigma_k^c} \\ &= \tilde{\boldsymbol{\lambda}}'\mathbf{e}_k\frac{G_k}{1-\sigma_k^c} - \frac{1}{1+\xi}\frac{d\xi}{d\sigma_k^c} \\ &= \frac{\tilde{\lambda}_k G_k}{1-\sigma_k^c} - \frac{1}{1+\xi}\frac{d\xi}{d\sigma_k^c}\end{aligned} \tag{134}$$

Combining the above objects gives the result. □

Proof of Corollary (8) Applying the no subsidies assumption ($\sigma_i^\theta\ \forall\ \theta \in \{c,d\}, i \in \mathcal{N}$) to Corollary (3), Theorem (6), and Theoreom (8) gives $\xi = 0$, $\frac{d\xi}{d\sigma_k^c} = \lambda_k G_k$, and $\frac{d\log Y^{ND}}{d\sigma_k^c} = \tilde{\lambda}_k G_k - \frac{1}{1+\xi}\frac{d\xi}{d\sigma_k^c}$, respectively. Substituting the first two results into the third, we obtain

$$\frac{d\log Y^{ND}}{d\sigma_k^c} = G_k(\tilde{\lambda}_k - \lambda_k). \tag{135}$$

Next, apply the no subsidies assumption to the definition of the revenue-based input-output matrix (26) to see

$$\Omega_{ij} = \tilde{\Omega}_{ij}. \tag{136}$$

Given the definitions of the cost-based Domar weights (25) and the revenue-based Domar weights (28), equation (136) implies

$$\lambda_k = \tilde{\lambda}_k. \tag{137}$$

Substituting this into (135) gives the result. □

E Carbon Tax Proofs

Proof of Carbon Tax To model the impact of a carbon tax, I make the following assumptions

- There is one fossil fuel sector, $i = f$
- Let the clean technology represent the production function of the fossil fuel, with $G_f = 1$ and $g_f = 0$ i.e., effectively a single production function
- The fossil fuel sector is subject to a negative subsidy $\sigma_f^c = -\tau$. I will think of τ as a carbon tax (more precisely, it is an ad-valorem tax on fossil fuel production)

I now proceed to study $\frac{d\log E}{d\tau}$ and $\frac{d\log Y}{d\tau}$. Let's start with emissions

$$\frac{d\log E}{d\tau} = \frac{d\log E}{d\sigma_f^c} \times \frac{d\sigma_f^c}{d\tau} = -\frac{d\log E}{d\sigma_f^c} \tag{138}$$

From Theorem 7, we know that

$$\frac{d\log E}{d\sigma_f^c} = \frac{1}{\lambda_f}\frac{d\lambda_f}{d\sigma_f^c} - \frac{d\log \overline{P}_f}{d\sigma_f^c} - \frac{1}{1+\xi}\frac{d\xi}{d\sigma_f^c} \tag{139}$$

Substituting this into $\frac{d\log E}{d\tau}$, we get

$$\frac{d\log E}{d\tau} = -\frac{1}{\lambda_f}\frac{d\lambda_f}{d\sigma_f^c} + \frac{d\log \overline{P}_f}{d\sigma_f^c} + \frac{1}{1+\xi}\frac{d\xi}{d\sigma_f^c} \tag{140}$$

Now, we will derive the derivatives on the RHS, starting with $\frac{d\lambda_f}{d\sigma_f^c}$. From Theorem (5), we know

$$\begin{aligned}\frac{d\lambda_j}{d\sigma_k^c} &= \lambda_k \frac{G_k}{1-\sigma_k^c}\Psi_{kj}^c + \lambda_k \frac{g_k}{1-\sigma_k^c}(\Psi_{kj}^c - \Psi_{kj}^d) \\ &+ \sum_i \lambda_i g_i \left(\tilde{\Psi}_{ik}^c - \tilde{\Psi}_{ik}^d\right)\frac{G_k}{1-\sigma_k^c}\left(\Psi_{ij}^c - \Psi_{ij}^d\right)\end{aligned} \tag{141}$$

Applying this formula for the fossil fuel sector ($k = f, \sigma_f^c = -\tau, G_f = 1, g_f = 1$) gives us

$$\frac{d\lambda_j}{d\sigma_f^c} = \lambda_f \frac{1}{1+\tau}\Psi_{ff}^c + \sum_i \lambda_i g_i \left(\tilde{\Psi}_{if}^c - \tilde{\Psi}_{if}^d\right)\frac{1}{1+\tau}\left(\Psi_{if}^c - \Psi_{if}^d\right) \tag{142}$$

The key difference with the equation above is there is no local switching effect. Dividing this through by λ_f gives us

$$\frac{1}{\lambda_f}\frac{d\lambda_j}{d\sigma_f^c} = \frac{1}{1+\tau}\Psi_{ff}^c + \sum_i \lambda_i g_i \left(\tilde{\Psi}_{if}^c - \tilde{\Psi}_{if}^d\right)\frac{1}{1+\tau}\left(\frac{\Psi_{if}^c - \Psi_{if}^d}{\lambda_f}\right) \tag{143}$$

Next, I derive $\frac{d\log \overline{P}_f}{d\sigma_f^c}$. Using Theorem (2), we get

$$\begin{aligned}\frac{d\log \overline{P}_f}{d\sigma_f^c} &= -\tilde{\Psi}_{ff}\frac{G_f}{1-\sigma_f^c}\\ &= -\tilde{\Psi}_{ff}\frac{1}{1-(-\tau)}\\ &= -\tilde{\Psi}_{ff}\frac{1}{1+\tau}\end{aligned} \tag{144}$$

Finally, I derive $\frac{d\xi}{d\sigma_f^c}$. Using Theorem (6), we get

$$\begin{aligned}\frac{d\xi}{d\sigma_f^c} = \sum_j \Bigg(&\frac{d\lambda_j}{d\sigma_f^c}\Big(G_j\frac{\sigma_j^c}{1-\sigma_j^c}+(1-G_j)\frac{\sigma_j^d}{1-\sigma_j^d}\Big)\\ &+\lambda_j\frac{dG_j}{d\sigma_f^c}\Big(\frac{\sigma_j^c}{1-\sigma_j^c}-\frac{\sigma_j^d}{1-\sigma_j^d}\Big)\\ &+\frac{\lambda_f G_f}{(1+\tau)^2}\mathbb{1}_{j=f}\Bigg)\end{aligned} \tag{145}$$

Combining the above equations gives us

$$\begin{aligned}\frac{d\log E}{d\tau} &= -\frac{1}{\lambda_f}\frac{d\lambda_f}{d\sigma_f^c}+\frac{d\log \overline{P}_f}{d\sigma_f^c}+\frac{1}{1+\xi}\frac{d\xi}{d\sigma_f^c}\\ &= -\frac{1}{1+\tau}\Psi_{ff}^c-\sum_i \lambda_i g_i\left(\tilde{\Psi}_{if}^c-\tilde{\Psi}_{if}^d\right)\frac{1}{1+\tau}\left(\frac{\Psi_{if}^c-\Psi_{if}^d}{\lambda_f}\right)-\tilde{\Psi}_{ff}\frac{1}{1+\tau}+\frac{1}{1+\xi}\frac{d\xi}{d\sigma_f^c}\end{aligned} \tag{146}$$

□

F Additional Proofs

Lemma 5. *Let $\overline{\boldsymbol{P}}'', \overline{\boldsymbol{P}}' \in \mathbb{R}_+^{|\mathcal{N}|}$ denote two different vectors of normalised sectoral prices. Let function ϕ_{iv}^{θ} be defined as in (82). Let $|.|$ denote the absolute value operator. Then the following condition holds.*

$$\left|\min_{\theta\in\Theta_i}\phi_{iv}^{\theta}(\log\overline{\boldsymbol{P}}'')-\min_{\theta'\in\Theta_i}\phi_{iv}^{\theta'}(\log\overline{\boldsymbol{P}}')\right| \leq \max_{\theta\in\Theta_i}\left|\phi_{iv}^{\theta}(\log\overline{\boldsymbol{P}}'')-\phi_{iv}^{\theta}(\log\overline{\boldsymbol{P}}')\right|. \tag{147}$$

Proof of Lemma (5) This follows from the result that the magnitude of the difference between the respective nth order static of two vectors is weakly smaller than the infinity norm of the difference between the two vectors. □

Lemma 6. *Let $\overline{\boldsymbol{P}}'', \overline{\boldsymbol{P}}' \in \mathbb{R}_+^{|\mathcal{N}|}$ denote two different vectors of normalised sectoral prices. Let*

function ϕ_{iv}^{θ} *be defined as in* (82). *Let* $|.|$ *and* $\|.\|_{\infty}$ *denote the absolute value operator and* ∞*-norm operator, respectively. Then the following condition holds.*

$$\max_{\theta\in\Theta_i}\left|\phi_{iv}^{\theta}(\log\overline{\boldsymbol{P}}'')-\phi_{iv}^{\theta}(\log\overline{\boldsymbol{P}}')\right|\leq\Big(\max_{j\in\mathcal{N},\theta\in\Theta_j}1-\tilde{a}_j^{\theta}\Big)\left\|\log\overline{\boldsymbol{P}}''-\log\overline{\boldsymbol{P}}'\right\|_{\infty} \tag{148}$$

Proof of Lemma (6) Define $\bar{\theta}$ as

$$\bar{\theta}=\arg\max_{\theta\in\Theta_i}\left|\phi_{iv}^{\theta}(\log\overline{\mathbf{P}}'')-\phi_{iv}^{\theta}(\log\overline{\mathbf{P}}')\right| \tag{149}$$

Starting from the LHS of the equation (148), we establish the inequality as follows

$$\begin{aligned}
LHS&=\max_{\theta\in\Theta_i}\left|\phi_{iv}^{\theta}(\log\overline{\mathbf{P}}'')-\phi_{iv}^{\theta}(\log\overline{\mathbf{P}}')\right|\\
&=\left|\phi_{iv}^{\bar{\theta}}(\log\overline{\mathbf{P}}'')-\phi_{iv}^{\bar{\theta}}(\log\overline{\mathbf{P}}')\right|\\
&=\left|\sum_{j\in\mathcal{N}}\tilde{m}_{ij}^{\bar{\theta}}\big(\log\overline{P}_j''-\log\overline{P}_j'\big)\right|\\
&\leq\sum_{j\in\mathcal{N}}\tilde{m}_{ij}^{\bar{\theta}}\left|\big(\log\overline{P}_j''-\log\overline{P}_j'\big)\right|\\
&=(1-\tilde{a}_i^{\bar{\theta}})\left|\big(\log\overline{P}_j''-\log\overline{P}_j'\big)\right|\\
&\leq(1-\tilde{a}_i^{\bar{\theta}})\left\|\log\overline{\mathbf{P}}''-\log\overline{\mathbf{P}}'\right\|_{\infty}\\
&\leq\Big(\max_{j\in\mathcal{N},\theta\in\Theta_j}1-\tilde{a}_j^{\theta}\Big)\left\|\log\overline{\mathbf{P}}''-\log\overline{\mathbf{P}}'\right\|_{\infty}\\
&=RHS,
\end{aligned} \tag{150}$$

The second line follows from the definition of $\bar{\theta}$. The third line follows from the definition of $\phi_{iv}^{\bar{\theta}}(.)$. The fourth line applies the triangle inequality. The fifth line follows from $\tilde{a}_i^{\bar{\theta}}=1-\sum_j\tilde{m}_{ij}^{\bar{\theta}}$. The sixth line follows from $|\log\overline{P}_j''-\log\overline{P}_j'|\leq\max_i|\log\overline{P}_i''-\log\overline{P}_i'|\equiv\|\log\overline{\mathbf{P}}''-\log\overline{\mathbf{P}}'\|_{\infty}$. The seventh line follows from the fact that the intermediate input share for a given sector-technology pair is weakly smaller than the largest intermediate input share across sectors and technologies. □

Lemma 7. *Let* $\overline{\boldsymbol{P}}'',\overline{\boldsymbol{P}}'\in\mathbb{R}_+^{|\mathcal{N}|}$ *denote two different vectors of normalised sectoral prices. Let function* ϕ_{iv}^{θ} *be defined as in* (82). *Let function* Φ_i *be defined as in* (85). *Let* $|.|$ *and* $\|.\|_{\infty}$ *denote the absolute value operator and* ∞*-norm operator, respectively. Then the following condition holds.*

$$\left|\Phi_i(\log\overline{\boldsymbol{P}}'')-\Phi_i(\log\overline{\boldsymbol{P}}')\right|\leq\Big(\max_{j\in\mathcal{N},\theta\in\Theta_j}1-\tilde{a}_j^{\theta}\Big)\left\|\log\overline{\boldsymbol{P}}''-\log\overline{\boldsymbol{P}}'\right\|_{\infty} \tag{151}$$

Proof of Lemma (7) Starting from the LHS, we establish the inequality as follows

$$\begin{aligned}
LHS &= \left|\Phi_i(\log \overline{\mathbf{P}}'') - \Phi_i(\log \overline{\mathbf{P}}')\right| \\
&= \left| \int_0^1 \min_{\theta\in\Theta_i} \phi_{iv}^{\theta}(\log \overline{\mathbf{P}}'')\, dv - \int_0^1 \min_{\theta'\in\Theta_i} \phi_{iv}^{\theta'}(\log \overline{\mathbf{P}}')\, dv \right| \\
&= \left| \int_0^1 \min_{\theta\in\Theta_i} \phi_{iv}^{\theta}(\log \overline{\mathbf{P}}'') - \min_{\theta'\in\Theta_i} \phi_{iv}^{\theta'}(\log \overline{\mathbf{P}}')\, dv \right| \\
&\leq \int_0^1 \left| \min_{\theta\in\Theta_i} \phi_{iv}^{\theta}(\log \overline{\mathbf{P}}'') - \min_{\theta'\in\Theta_i} \phi_{iv}^{\theta'}(\log \overline{\mathbf{P}}') \right| dv \\
&\leq \int_0^1 \max_{\theta\in\Theta_i} \left|\phi_{iv}^{\theta}(\log \overline{\mathbf{P}}'') - \phi_{iv}^{\theta}(\log \overline{\mathbf{P}}')\right| dv \\
&\leq \int_0^1 \left(\max_{j\in\mathcal{N},\theta\in\Theta_j} 1-\tilde{a}_j^{\theta}\right) \left\| \log \overline{\mathbf{P}}'' - \log \overline{\mathbf{P}}' \right\|_{\infty} dv \\
&= \left(\max_{j\in\mathcal{N},\theta\in\Theta_j} 1-\tilde{a}_j^{\theta}\right) \left\| \log \overline{\mathbf{P}}'' - \log \overline{\mathbf{P}}' \right\|_{\infty} \\
&= RHS.
\end{aligned} \tag{152}$$

The second line follows from the definition of function Φ (85). The third line rearranges the integrals. The fourth line follows from the triangle inequality applied to integrals. The fifth line applies Lemma (5). The sixth line applies Lemma (6). The integral is dropped from line seven as none of the terms depend on variety v. □

Lemma 8. *Let $\overline{\boldsymbol{P}}'', \overline{\boldsymbol{P}}' \in \mathbb{R}_+^{|\mathcal{N}|}$ denote two different vectors of normalised sectoral prices. Let function ϕ_{iv}^{θ} be defined as in (82). Let functions Φ and Φ_i be defined as in (85). Let $|.|$ and $\|.\|_{\infty}$ denote the absolute value operator and ∞-norm operator, respectively. Then the following condition holds.*

$$\left\|\Phi(\log \overline{\boldsymbol{P}}'') - \Phi(\log \overline{\boldsymbol{P}}')\right\|_{\infty} \leq \left(\max_{j\in\mathcal{N},\theta\in\Theta_j} 1-\tilde{a}_j^{\theta}\right) \left\| \log \overline{\boldsymbol{P}}'' - \log \overline{\boldsymbol{P}}' \right\|_{\infty} \tag{153}$$

Proof of Lemma (8) Starting from the LHS, we establish the result as follows

$$\begin{aligned}
LHS &= \left\|\Phi(\log \mathbf{P}'') - \Phi(\log \mathbf{P}')\right\|_{\infty} \\
&= \max_{i\in\mathcal{N}} \left|\Phi_i(\log \mathbf{P}'') - \Phi_i(\log \mathbf{P}')\right| \\
&\leq \max_{i\in\mathcal{N}} \left(\max_{j\in\mathcal{N},\theta\in\Theta_j} 1-\tilde{a}_j^{\theta}\right) \left\| \log \overline{\mathbf{P}}'' - \log \overline{\mathbf{P}}' \right\|_{\infty} \\
&= \left(\max_{j\in\mathcal{N},\theta\in\Theta_j} 1-\tilde{a}_j^{\theta}\right) \left\| \log \overline{\mathbf{P}}'' - \log \overline{\mathbf{P}}' \right\|_{\infty} \\
&= RHS.
\end{aligned} \tag{154}$$

The second line follows from the definition of infinity norm. The third line applies claim three. The fourth line drops the first maximisation component as the RHS does not depend on i. □