The Design and Business Models of Financial Ratings

Pierre Chaigneau* Queen's University

Nicolas Sahuguet† HEC Montréal

March 26, 2026

Abstract

We study how the funding structure of financial intermediaries affects the type of information they produce and its real effects. When ratings are sold to investors, they resolve uncertainty about firm fundamentals ("informational ratings"); when sold to issuers, they bundle signals about fundamentals and managerial effort ("motivational ratings"). Motivational ratings increase effort incentives but may be crowded out by informational ratings in highly liquid markets. With enough uncertainty, ratings can provide implicit incentives such that an insider's exit does not matter – the "time horizon irrelevance" result, which can explain some features of venture capital financing. More broadly, we provide a unified framework for analyzing the real effects of financial information design.

Keywords: corporate governance, crowding-out effect, information acquisition, ESG ratings, short-termism, socially responsible investment.

*Address: Smith School of Business, Goodes Hall, 143 Union Street West, K7L 2P3, Kingston, ON, Canada. Email: pierre.chaigneau@queensu.ca. Tel: 613 533 2312.

†Address: Applied Economics Department, HEC Montréal, 3000 chemin de la côte Sainte Catherine, H3T 2A7 Montréal, QC, Canada. Email: nicolas.sahuguet@hec.ca. Tel: 514 340 6031.

The production of financial information is often delegated to intermediaries such as credit rating agencies, equity research providers, and ESG rating agencies. Many financial information intermediaries are funded by the firms they assess rather than by the investors who use their assessments. Credit rating agencies rate debt securities issued by paying clients; auditors verify financial statements for the firms being audited; sustainability certifiers assess corporate environmental and social performance for fee-paying firms. This business model has long raised concerns about conflicts of interest. Issuer-funded ratings may be systematically inflated (Bolton, Freixas, and Shapiro (2012)), issuers may shop among rating agencies for favorable assessments (Skreta and Veldkamp (2009)), and the credibility of issuer-paid information is frequently questioned by regulators and market participants.

Yet issuer-paid models persist across contexts and over time. Credit ratings remain predominantly issuer-funded despite decades of documented problems and major regulatory reforms following the 2008 financial crisis. Sustainability certifications exhibit both issuer-funded models and investor-funded screening models serving different market segments. This persistence suggests that issuer-paid information may provide economic benefits alongside the well-documented costs of conflicts of interest. At the same time, investor-funded information, including ESG ratings and independent equity research, has proliferated. Why do different segments adopt different funding models, and what are their real effects?

We show that issuer-paid information intermediaries provide incentive benefits whereas investor-paid intermediaries do not, because issuers and investors value information for fundamentally different reasons: investors try to assess firm value, whereas issuers try to improve it.

We develop a unified framework for the design and funding of financial information. We study how a profit-maximizing rating agency chooses both the type of information produced and its funding structure. We characterize equilibrium rating designs and the conditions under which each arises. Informational ratings increase price efficiency but reduce output, whereas motivational ratings increase output by strengthening incentives. This identifies a novel inefficiency: the most informative rating design may nevertheless induce low effort.

Financial intermediaries typically observe information along several dimensions, including signals about insiders' actions and signals about firm fundamentals. In our model, issuer-paid intermediaries bundle signals about effort and fundamentals, making prices more sensitive to insiders' actions. This allows insiders to better internalize the effect of their actions even if they exit before cash flows are realized. Investor-paid intermediaries instead maximize informativeness about fundamentals to improve clients' trading profits, thereby providing minimal incentive for value-increasing actions.

Our main result shows that, under some conditions, ratings will eliminate short-termism entirely. When fundamental uncertainty is sufficiently high and ratings are issuer-funded, the equilibrium rating provides implicit incentives such that the insider's effort equals the level attainable when he can commit to effort – regardless of how likely he is to exit early. High

uncertainty amplifies the information content of the signal about firm fundamentals, which increases the sensitivity of the market price to any rating which depends on this signal. When the intermediary puts value-maximizing weights on effort and fundamentals, the market price is then sufficiently sensitive to actual effort, so even short-term insiders fully internalize long-run value creation through the sale price. This time horizon irrelevance result contrasts with standard models of exit and moral hazard (Faure-Grimaud and Gromb (2004), Chemla and Hennessy (2014)), where exit always weakens incentives relative to the commitment benchmark.

When investors pay, the rating agency optimally designs information to maximize their trading profits, which is achieved by resolving uncertainty about fundamentals. The resulting "informational rating" maximizes price informativeness but provides weak effort incentives.[1] When issuers pay, the agency instead designs a "motivational rating" sensitive to both effort and fundamentals, so that equilibrium prices better reflect insiders' actions. A rating that were uninformative about fundamentals would be ignored by market participants, would have no price impact, and would therefore generate no effort incentives. Thus, the agency optimally bundles signals about fundamentals and effort in a single signal.

Which business model emerges depends on market liquidity and corporate ownership structures. When stock markets are highly liquid or insiders have long horizons, the rating agency optimally caters to investors and sells informational ratings. In this case, "bad information", which maximizes trading profits but not output, can crowd out "good information." This highlights a new cost of stock market liquidity.[2] We also identify regimes where both business models coexist, as observed with ESG information.

We extend the baseline model to allow for competition between rating agencies. When markets are illiquid, entry is blockaded and the incumbent provides the firm value-maximizing motivational rating. For intermediate market liquidity, the incumbent strategically distorts its rating below the firm value-maximizing weight to deter entry. Thus, the *threat* of entry can be welfare-reducing. When markets are liquid, the incumbent accommodates entry by selling an informational ratings to investors, ceding the motivational role to the entrant. This extension reinforces the "bad information drives out good information" pattern: when informational ratings are cheap to produce, they can render motivational ratings ineffective because highly precise information about fundamentals swamps the price impact of effort-sensitive information. This has implications for the regulation of rating agencies.

We formalize these ideas in a model in which an insider chooses costly effort that increases expected firm value but may have to sell his stake before output is realized, as in Faure-Grimaud

[1] That effort is correctly anticipated is not crucial. What matters is the distinction between information that improves incentives and information that resolves uncertainty. In an extension in which the insider's effort is uncertain, we show that an informational rating can instead load on the effort signal but still fails to motivate effort. However, this is no longer true if speculators have an initial stake in the firm; see Appendix Section D.

[2] This is in contrast to the classic result that public disclosure crowds out private information acquisition (Verrecchia (1982)). The literature has identified other costs of liquidity, including inadequate internal monitoring (Bhide (1993)) and liquidations that contribute to financial contagion (Allen and Gale (2000)).

and Gromb (2004) – interpretable as stochastic exit by founders, managers, or blockholders such as venture capitalists. Output depends on effort and unknown firm fundamentals. Shares trade at an interim date in a financial market in the spirit of Kyle (1985). A rating agency observes noisy signals of effort and fundamentals and designs a single rating as a linear combination of the two, following Hörner and Lambert (2021); it chooses both the weights and the business model (selling to the issuer or to the speculator) to maximize its profits.

The theory extends beyond literal "ratings" to any setting in which intermediaries produce information about firms – investment banks in IPOs, auditors certifying private-company reports,[3] or ESG intermediaries. ESG ratings sold to investors may emphasize exposure to sustainability risks rather than motivating real social or environmental improvement, consistent with leading providers' methodologies. Issuer-oriented certification can instead function as a commitment device for beneficial actions but may be crowded out when informational products dominate. We also show that emerging "impact funds" can generate social impact not because of their investment decisions or their engagement, but through their influence on the design of financial information. This is a new channel in the literature on socially responsible investment (Broccardo, Hart, and Zingales (2022), Edmans, Levit, and Schneemeier (2023), Oehmke and Opp (2025), Bisceglia, Piccolo, and Schneemeier (2026)).

The time horizon irrelevance result sheds light on venture capital and private equity. Active investors with a short horizon can in principle finance long-term projects when uncertainty is high enough that information aggregation restores strong incentives; this helps rationalize how funds with 7–10 year horizons finance firms whose value unfolds over longer horizons, and why venture capitalists concentrate on early-stage, high-uncertainty ventures (Gompers and Lerner (2001), Janeway, Nanda, and Rhodes-Kropf (2021)). We can explain this narrow focus, which has been deplored by Lerner and Nanda (2020). Our result can also contribute to explain the apparently puzzling fact noted by Lerner and Nanda (2020) that the time horizons of venture capital funds do not vary depending on the type of firms financed, even though "great diversity across industries exists in terms of the typical project length."

Finally, when uncertainty is high enough that the commitment outcome is achievable, there is no additional role for debt as a disciplining device, in contrast to classical moral-hazard models (Innes (1990), Townsend (1979), Hart and Moore (1998)). This is consistent with evidence that firms with high market-to-book ratios and R&D intensity are more likely to be unlevered (Strebulaev and Yang (2013)).

[3]Audit reports for private companies are flexible relative to public companies: disclosure can combine GAAP and non-GAAP earnings (Leung and Veenman (2018)), risk assessment procedures vary (Knechel et al. (2013)), and so do the levels complexity and disaggregation (Chen, Miao, and Shevlin (2015), Hoitash and Hoitash (2018), Noordermeerb and Vorst (2024)). These degrees of freedom can shift the weight of different signals in the report.

1 Related literature

Our paper contributes to two strands of literature on information production in financial markets. The first, initiated by Admati and Pfleiderer (1986, 1988), studies how informed agents sell exogenously given information, with subsequent work analyzing how much information to produce (Admati, Pfleiderer, and Zechner, 1994; Peress, 2010; Ferreira, Manso, and Silva, 2014; Dasgupta and Matthews, 2024). The second strand studies information design problems where the designer cares about actions or outcomes (Bergemann and Morris, 2019). Most closely related, Hörner and Lambert (2021) analyze optimal ratings designed to maximize social surplus, which is equivalent to maximizing effort in their setting.

We extend both strands by analyzing not only how much information is produced but also what type of information emerges when the designer is profit-maximizing rather than welfare-maximizing. Critically, we show that who pays determines the type of information produced. In our model, equilibrium rating weights generally differ from those that induce first-best effort because the agency maximizes its profit; moreover, when uncertainty is large enough, the rating weights can induce effort above the first-best level. This differs from Pollrich and Strausz (2024), who show that the fee structure is irrelevant in a setting with one seller and one buyer where transparency does not affect efficiency, and from Saeedi and Shourideh (2024), who study optimal design when information is sold to a given third party but the information provider only observes one signal.

The distinction between motivational and informational uses of ratings connects to certification models where buyers and sellers use information for different purposes, such as inspection versus signaling (Stahl and Strausz, 2017). However, certification does not affect effort in their framework. Closer to our setup, Bouvard and Levy (2018) study a profit-maximizing certifier choosing accuracy, though their rating is one-dimensional (quality), whereas ours can depend on both effort and type. Several papers study ratings in contracting problems without insider effort (Faure-Grimaud, Peyrache, and Quesada, 2009; Daley, Green, and Vanasco, 2020; Kashyap and Kovrijnykh, 2016), and many analyze optimal weights on performance signals in principal-agent settings (Banker and Datar, 1989; Paul, 1992; Datar, Kulp, and Lambert, 2001). Our contribution is to show that optimal aggregation of signals depends on who pays for the information.

Despite multidimensional information, ratings are often uni-dimensional. Our framework helps explain this puzzle: as in Hörner and Lambert (2021), a rating depending only on effort would be ignored by the market, while one depending only on fundamentals would not improve aggregate surplus. Aggregation is thus essential for motivational ratings to function.[4]

A large literature studies credit rating agencies and conflicts of interest arising from issuer payment. Bolton, Freixas, and Shapiro (2012) show that issuer fees can lead to inflated ratings and that competition enables rating shopping; related work examines shopping behavior

[4]This relates to work on rating properties: discrete ratings can limit inflation (Goel and Thakor, 2015) or misinterpretation (Martel, van Wesep, and van Wesep, 2022).

(Sangiorgi and Spatt, 2017) and competition effects (Doherty, Kartasheva, and Phillips, 2012). Reputational concerns and disciplining mechanisms are studied in Mathis, McAndrews, and Rochet (2009), Bar-Isaac and Shapiro (2013), Fulghieri, Strobl, and Xia (2014), and Piccolo and Shapiro (2022), with empirical evidence of issuer catering in Griffin, Nickerson, and Tang (2013) and Kraft (2015).

A parallel literature studies ESG ratings, where both issuer-paid and investor-paid models coexist. Olivier and Lovo (2025) analyze who should pay for ESG ratings when investors have heterogeneous social responsibility preferences, while Azarmsa and Shapiro (2026) study whether competing ESG agencies generalize or specialize. An important distinction is that these papers do not model effort choices by rated firms.

Our contribution to this literature is to identify a new benefit of issuer payment, which suggests that issuer payment persists because it creates value through optimal information aggregation. The fact that ESG markets feature both business models is consistent with our prediction that investor-paid dominates when liquidity is high or ownership dispersed, while issuer-paid dominates when uncertainty is high and ownership concentrated.

Our result that investor-paid information can depress output while maximizing market efficiency connects to Dow and Gorton (1997), who show that informationally efficient markets can coincide with depressed real investment. The mechanism differs: in their paper, inefficiency arises through feedback effects when managers learn from prices, whereas in ours, prices matter for effort incentives through the insider's sale decision.

The model relates to the literature on short-termism. Existing work shows that short horizons or potential exit weaken incentives relative to commitment (Faure-Grimaud and Gromb, 2004; Chemla and Hennessy, 2014), and examines how to mitigate short-termism through contracting (Marinovic and Varas, 2019), monitoring (Von Thadden, 1995), or endogenous horizon choice (Gryglewicz, Mayer, and Morellec, 2020; Hackbarth, Rivera, and Wong, 2021; Thakor, 2021). Our contribution is to show that with proper information design, short horizons need not reduce effort: when uncertainty is sufficiently high and ratings are issuer-funded, equilibrium effort equals the commitment level regardless of the exit probability. This time horizon irrelevance result provides a new perspective on when and why short-termism emerges, highlighting information design as a mechanism that can restore efficient incentives even with short horizons.

2 Model

2.1 The firm

A firm is owned and managed by an "insider" who exerts effort to improve its output. The insider initially owns a fraction $1 - \delta$ of the shares, such that he has effective control of the firm. As in Faure-Grimaud and Gromb (2004), the insider can be viewed as an entrepreneur

who founded or purchased the firm, or as an involved shareholder such as a venture capitalist. The output of the firm, which is realized at $t = 2$, is given by:

$$\tilde{X} \quad = \quad A + \tilde{\theta}. \tag{1}$$

where A is the insider's effort, and θ is the firm's "type". The latter should be interpreted broadly: it includes any source of uncertainty about firm output, whether this uncertainty depends on the firm itself or the insider's characteristics such as his ability.[5] In any case, information about θ is symmetric.[6] Effort A is unobservable and optimally chosen by the insider at $t = 0$. The firm type θ, which is unknown at $t = 0$, is drawn from a normal distribution with mean $\bar{\theta}$ and variance σ_θ^2. The realization θ of the random variable $\tilde{\theta}$ is unobservable. Effort is privately costly for the insider, with a monetary cost of $C(A)$, with $C' > 0$, $C'' > 0$, $C(0) = 0$, and $C'(0) = 0$.

2.2 Financial market

Trading occurs at $t = 1$. The financial market is in the spirit of Kyle (1985). In addition to the insider, who may or may not trade as specified below, it involves three economic agents:

- Noise traders submit a random market order $\tilde{u}$, which is normally distributed with mean 0 and variance σ_u^2 and is independent of the firm type $\tilde{\theta}$.
- A risk-neutral speculator obtains information about the firm by observing a rating $\tilde{R}$, then she submits a market order $q(R)$.
- A competitive and risk-neutral market maker observes whether the insider is hit by a liquidity shock as well as the total order flow. He may observe the rating or not, depending on the business model chosen by the rating agency. He sets the price of the asset to its expected value given his information.

As in Faure-Grimaud and Gromb (2004), with probability $\pi \in (0, 1]$, the "insider" is hit by a publicly observable liquidity shock at $t = 1$ and sells his shares in the firm.[7] The model allows for $\pi = 1$, in which case the insider has a fixed investment horizon of $t = 1$. As in Faure-Grimaud and Gromb (2004), the insider's trade is observable and cannot be anonymous.

The game is sequential: the insider and the informed speculator submit market orders, then the market maker sets the market price p depending on the aggregate order flow.

[5] We interpret output as firm profits, but other interpretations are also possible. For example, if some economic agents also have nonpecuniary preferences, "effort" could represent actions that improve social and environmental outcomes.

[6] If θ is interpreted as the manager's ability, this is consistent with a standard model of career concerns, where the agent's ability is unknown ex-ante.

[7] As in Faure-Grimaud and Gromb (2004), this assumption allows to abstract from adverse selection issues that could lead to a market breakdown when initial owners have private information about firm value.

2.3 Rating agency

A rating agency observes two signals at $t = 1$, one about effort and the other about the firm type:

$$\tilde{S}_1 = A + \tilde{\varepsilon}_1 \quad (2)$$

$$\tilde{S}_2 = \theta + \tilde{\varepsilon}_2 \quad (3)$$

where θ is the realization of $\tilde{\theta}$; for $i \in \{1, 2\}$, $\tilde{\varepsilon}_i$ follows a standard normal distribution and is independent from other random variables. The unit variance assumption simplifies expressions and could be relaxed.

The rating agency designs a rating R to maximize its expected profits. It chooses the rating design at $t = 0$ and reveals the rating R at $t = 1$. Below we consider different business models for the rating agency, depending on whether it sells its rating to the firm (the issuer) or to the speculator.

We first show that selling two linearly independent ratings is equivalent to selling a single rating.

Lemma 1 *Suppose that the rating agency sells two ratings which are linear functions of S_1 and S_2. The same information will be revealed by selling a single rating which is a linear function of S_1 and S_2.*

The intuition is that a rating on S_1 alone is not valued by market participants, since the insider's effort is already known in equilibrium.[8] Market participants only value information about S_2. Thus, as in Hörner and Lambert (2021), we assume the rating agency designs a single rating:

$$\tilde{R} = k\tilde{S}_1 + (1 - k)\,\tilde{S}_2. \quad (4)$$

where $k \in [0, 1]$ is chosen by the rating agency and is common knowledge. The parameter k captures the relative weight on each signal: higher k makes the rating more sensitive to effort, lower k makes it more informative about the firm type. For a given k, the variance of the rating is:

$$\sigma_R^2 \equiv (1 - k)^2\,\sigma_\theta^2 + k^2 + (1 - k)^2\,. \quad (5)$$

The timeline of events is summarized in Figure 1.

[8] This is not a crucial assumption, as discussed in Appendix C.

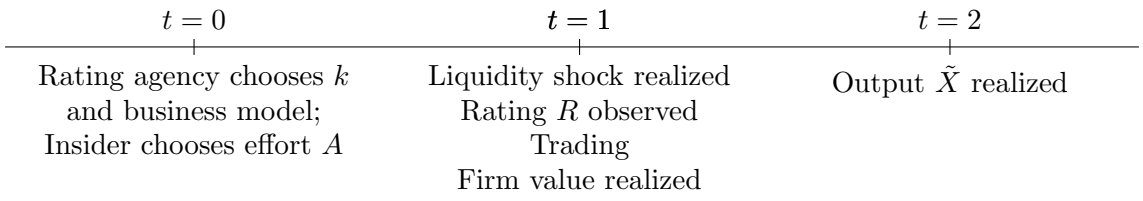


Figure 1: Timeline of the model.

3 Baseline: single rating

3.1 Market equilibrium

We characterize the market equilibrium for two information structures: when the rating is privately observed by the speculator, and when it is publicly disclosed to all market participants.

3.1.1 Private rating equilibrium

We characterize the linear rational expectations equilibrium following Kyle (1985). The speculator observes the rating R and submits a market order $q(R)$. The market maker observes the total order flow $y \equiv q+u$ (where u is noise trade) and sets a price $p(y)$. Both agents optimize given their information. Since the insider's order is driven only by an exogenous shock whose probability is independent of firm value, it does not affect beliefs about firm value.

Proposition 1 (equilibrium with private rating) *When the rating R is privately observed by the speculator, the unique linear equilibrium is characterized by:*
(i) Speculator's strategy:

$$q(R) = \alpha + \beta R \tag{6}$$
$$\alpha = \frac{\sigma_u}{\sigma_R}\left(k\hat{A} + (1-k)\,\bar{\theta}\right) \tag{7}$$
$$\beta = \frac{\sigma_u}{\sigma_R} \tag{8}$$

(ii) Market maker's pricing rule:

$$p(y) = \mu + \lambda y \tag{9}$$
$$\mu = \hat{A} + \bar{\theta} - \frac{(1-k)\,\sigma_\theta^2}{2\sigma_u\sigma_R} \tag{10}$$
$$\lambda = \frac{(1-k)\,\sigma_\theta^2}{2\sigma_u\sigma_R} \tag{11}$$

(iii) Speculator's expected profit:

$$\mathbb{E}[\Pi_{spec}] \quad = \quad \frac{\sigma_u \left(1-k\right) \sigma_\theta^2}{2\sigma_R} \tag{12}$$

(iv) Expected stock price as function of effort:

$$\mathbb{E}\left[p\middle|A,\hat{A}\right] \quad = \quad \hat{A} + \bar{\theta} + \frac{k(1-k)\sigma_\theta^2}{2\sigma_R^2}(A-\hat{A}) \tag{13}$$

Parameter λ, the sensitivity of the market price to the order flow, is a measure of market illiquidity. As in the Kyle (1985) model, it is increasing in σ_θ, which reflects the informational advantage of the speculator, and decreasing in σ_u, which reflects the random part of the order flow. In addition, market illiquidity λ is decreasing in σ_R. Intuitively, holding σ_θ constant, a higher σ_R makes the speculator's signal less precise, which makes her trade less informative, reducing the price sensitivity to the order flow.

Parameter β represents the trading aggressiveness of the speculator. As in Kyle (1985), it is increasing in the magnitude σ_u of noise trading and decreasing in the standard deviation σ_R of the rating.

3.1.2 Equilibrium with public rating

We now characterize the equilibrium when the rating is publicly disclosed. As the speculator has not private information, he does not tarde and the order flow is pure noise and is not taken into account by the market-maker.

Proposition 2 (public rating equilibrium) *When the rating R is publicly observed by all market participants, the equilibrium is characterized by:*
(i) Speculator's strategy:

The speculator does not trade, since she has no informational advantage: $q(R) = 0$ for all R.
(ii) Market maker's pricing rule:

The price fully reflects the rating:

$$p(R) \quad = \quad \mathbb{E}[X|R] = \hat{A} + \bar{\theta} + \frac{k(1-k)\sigma_\theta^2}{\sigma_R^2}(R - k\hat{A} - (1-k)\bar{\theta}) \tag{14}$$

(iii) Speculator's expected profit:

$$\mathbb{E}[\Pi_{spec}] \quad = \quad 0 \tag{15}$$

(iv) Expected stock price as function of effort:

$$\mathbb{E}\left[p\middle|A,\hat{A}\right] \;=\; \hat{A}+\bar{\theta}+\frac{k(1-k)\sigma_\theta^2}{\sigma_R^2}(A-\hat{A}) \tag{16}$$

Comparing Propositions 1 and 2, the sensitivity of the expected stock price to actual effort is exactly twice as large when the rating is public ($\frac{k(1-k)\sigma_\theta^2}{\sigma_R^2}$) versus private ($\frac{k(1-k)\sigma_\theta^2}{2\sigma_R^2}$). The speculator's strategic trading reduces price informativeness when the rating is private. As a result, for any given positive rating weight k, implicit effort incentives are higher when the rating is public information than when it is privately observed by the speculator.

3.2 Effort and implicit incentives

We now analyze the insider's effort incentives, focusing on the implicit incentives that arise from the rating's impact on the stock price. Define the expected stock price at $t=0$ as a function of the insider's actual effort A and his anticipated effort $\hat{A}$ as $\mathbb{E}\left[p\middle|A,\hat{A}\right]$. Although effort is correctly anticipated in equilibrium ($A=\hat{A}$), implicit incentives arise from the sensitivity of the expected stock price to a deviation in actual effort, as captured by $\mathbb{E}[p \mid A,\hat{A}]$ in Propositions 1(iv) and 2(iv): the insider internalizes that higher effort shifts the distribution of the rating and hence the market price at which he exits, even though markets correctly conjecture his equilibrium action.[9]

As a benchmark, we consider the effort that would be optimally chosen by the insider if he could commit to a level of effort at $t=0$. We denote this effort by A^{LT}. This level of effort solves:

$$\max_A\,(1-\delta)\left(\pi\mathbb{E}\left[p\middle|A,A\right]+(1-\pi)\left(A+\bar{\theta}\right)\right)-C(A) \quad\Leftrightarrow\quad \max_A\,\{(1-\delta)A-C(A)\} \tag{17}$$

This gives:

$$C'(A^{LT})=1-\delta \tag{18}$$

We call this level of effort optimally chosen under commitment A^{LT} because it is also the level of effort that would be achieved at the second-best if the insider were never hit by a liquidity shock, i.e. if he had a long-term ("LT") horizon with $\pi=0$.

Appendix A derives the first-best effort and the rating weights that elicit this level of effort.

Using Proposition 1(iv), we characterize the insider's effort when the rating is privately observed by the speculator.

[9] This is the standard mechanism in models of career concerns (Holmström, 1999). As we show in Appendix C, the mechanism has a natural reinterpretation when there is uncertainty about the insider's effort cost parameter $\bar{A}$: in that case the rating is informative about equilibrium effort in the cross-section, since effort varies with the privately observed $\bar{A}$, but the key incentive coefficient is identical to the one derived here.

Lemma 2 (effort with private rating) *When the rating is privately observed by the speculator, the insider's equilibrium effort satisfies:*

$$C'(A) = (1-\delta)\left[1-\pi+\pi\cdot\frac{k(1-k)\sigma_\theta^2}{2\sigma_R^2}\right] \tag{19}$$

The insider effort is equal to the level under commitment A^{LT} if and only if $\sigma_\theta \geq 2\sqrt{2+\sqrt{5}} \approx 4.12$ and $k = k^{LT} \equiv \frac{4+5\sigma_\theta^2 \pm \sqrt{\sigma_\theta^4-16-16\sigma_\theta^2}}{2(4+3\sigma_\theta^2)}$.

The insider's effort incentives depend on how effort affects the expected market value of the firm. When the rating is privately observed, the speculator's strategic trading reduces the transmission of information about effort into the stock price, creating a wedge between actual effort and market valuation. These implicit incentives are stronger when prior uncertainty about firm value (σ_θ) is higher, as the rating becomes more informative and has greater impact on firm valuation.

When uncertainty is sufficiently high, the insider can achieve the full commitment effort level A^{LT} – the effort he would choose if he never faced early exit ($\pi = 0$). This requires an appropriately designed rating weight k. There are two rating weights k^{LT} that achieve A^{LT}. For σ_θ above the threshold, the rating weight that maximizes effort induces effort above A^{LT}. To achieve exactly A^{LT}, the rating weight can be either increased (making the rating more sensitive to effort but less informative about θ, reducing price impact) or decreased (making the rating less sensitive to effort but more informative about θ, increasing price impact). Both adjustments achieve the same effort level through different information structures (see Figure 2).

Using Proposition 2(iv), we now characterize effort when the rating is publicly disclosed.

Lemma 3 (effort with public rating) *When the rating is publicly disclosed, the insider's equilibrium effort satisfies:*

$$C'(A) = (1-\delta)\left[1-\pi+\pi\cdot\frac{k(1-k)\sigma_\theta^2}{\sigma_R^2}\right] \tag{20}$$

The insider effort is equal to the level under commitment A^{LT} if and only if $\sigma_\theta \geq \sigma_\theta^d \equiv \sqrt{2(1+\sqrt{2})}$ and $k = k_d^{LT} \equiv \frac{3\sigma_\theta^2 \pm \sqrt{\sigma_\theta^4-4\sigma_\theta^2-4}+2}{4\sigma_\theta^2+4}$.

Comparing with the case of a private rating, the key difference is that implicit effort incentives are exactly twice as strong when the rating is public (compare the coefficients $\frac{k(1-k)\sigma_\theta^2}{\sigma_R^2}$ versus $\frac{k(1-k)\sigma_\theta^2}{2\sigma_R^2}$). This is because when the rating is public information, it is fully incorporated into the firm's market price, whereas strategic trading by the speculator reduces price informativeness when the rating is private. As a result, the minimum level of uncertainty needed to achieve full commitment effort is lower when the rating is public: $\sigma_\theta^d < \sigma_\theta^*$. The doubled sensitivity of firm value to effort makes it easier to provide adequate implicit incentives.

Proposition 3 (time horizon irrelevance) *When uncertainty about firm value is sufficiently high ($\sigma_\theta \geq \sigma_\theta^*$ for private ratings, $\sigma_\theta \geq \sigma_\theta^d$ for public ratings), there exists a rating weight k such that the insider achieves full commitment effort A^{LT} regardless of the probability π of early exit. The insider's investment horizon becomes irrelevant for effort provision when information is optimally designed and uncertainty is high enough.*

The time horizon irrelevance result has important theoretical and practical implications. At a theoretical level, it shows that the tension between short investment horizons and long-term value creation can be completely resolved through optimal information design when uncertainty is sufficiently high. This contrasts with the standard view in corporate finance that short horizons necessarily reduce effort and efficiency (Stein, 1988; Stein, 1989; Shleifer and Vishny, 1990). The key mechanism is that when fundamental uncertainty is large, properly aggregated information creates strong implicit incentives through market prices that exactly replicate the incentives an insider would have with permanent ownership. Practically, this result helps explain several empirical patterns. Most notably, it provides a foundation for understanding why venture capitalists with fund lives of 7-10 years can successfully finance companies requiring 15-20 years to mature. High uncertainty about startup prospects, combined with intensive information production through monitoring and governance, enables time horizon irrelevance to hold. More broadly, the result suggests that concerns about short-termism should be conditional on the uncertainty environment: they are most acute in mature, stable industries where σ_θ is low, but may be largely irrelevant in high-growth, high-uncertainty sectors. Perhaps most striking is that the threshold for time horizon irrelevance does not itself depend on the exit probability π. Once uncertainty exceeds the threshold, the insider achieves full commitment effort for any time horizon π. This makes the solution robust across different ownership structures and investment horizons, and suggests that rating design may be more important than contractual restrictions on trading for addressing horizon problems.

3.3 Optimal rating design: Insider-paid vs. speculator-paid ratings

We now determine the optimal rating design for different business models. The rating agency sets the rating weight k to maximize the willingness to pay of its client – either the speculator or the insider. When the rating agency sells the rating to the speculator, it sets k to maximize the speculator's willingness to pay, which equals her expected profit from Proposition 1(iii).

Proposition 4 *When the rating agency sells the rating to the speculator, it optimally sets:*

$$k = k^{**} \equiv 0 \tag{21}$$

A speculator who purchases the rating wants to maximize her informational advantage to generate trading profits. This is achieved by having a rating which is as informative as possible

about firm type $\tilde{\theta}$. Indeed, information about effort is useless for trading since effort can be correctly anticipated in equilibrium. As a result, the rating weight that maximizes the speculator's willingness to pay is $k = k^{**} = 0$, which puts all weight on the signal about firm type. In Appendix D, we show that when the speculator is endowed with shares, the rating that maximizes his willingness to pay is positive.

This is the business model of ESG rating agencies sold to investors, and some equity research conducted by investment banks and brokerage houses that is privately communicated to their investor clientele.

We now consider the business model where the firm pays the rating agency to communicate the rating privately to the speculator. The rating agency sets the rating weight to maximize the insider's willingness to pay. This case serves as a benchmark, though we show below (Proposition 7) that it is always dominated by the public rating.

Define:

$$k^* \equiv \frac{1 + \sigma_\theta^2 - \sqrt{1 + \sigma_\theta^2}}{\sigma_\theta^2}, \qquad \sigma_\theta^* \equiv 2\sqrt{2 + \sqrt{5}}. \tag{22}$$

It can be shown that k^* is the effort-maximizing rating weight.

Proposition 5 *When the insider pays the rating agency to communicate the rating to the speculator, the rating agency optimally sets:*

$$k = \begin{cases} k^* & \text{if } \sigma_\theta < \sigma_\theta^* \\ k^{LT} & \text{if } \sigma_\theta \geq \sigma_\theta^* \end{cases} \tag{23}$$

The rating agency chooses k to maximize the insider's willingness to pay. If the rating that induces full commitment effort A^{LT} is achievable (when $\sigma_\theta \geq \sigma_\theta^*$), this is the most valuable rating for the insider. Otherwise, the most valuable rating is the one that maximizes effort.

Figure 2 depicts the rating weights corresponding to the business model in Proposition 5. It shows that, for any σ_θ, we have $k^* > k^{**}$, i.e. the weight on the signal S_1, which is informative about effort, is always higher when the rating is designed to maximize the firm insider's willingness to pay. It also shows that, at $k = k^*$, the optimal weight on the signal S_1 is increasing in σ_θ. The greater the prior uncertainty about firm value, the more the rating is informative about the insider's actual effort. Intuitively, the greater the uncertainty about firm value, the more valuable is a rating informative about firm value, and the greater its price impact. As a result, as uncertainty about firm value increases, the rating agency can increase the weight on the signal S_1 about effort in the optimal rating. This allows the insider to commit to a higher level of effort while still maintaining a sufficiently high price impact of the rating.

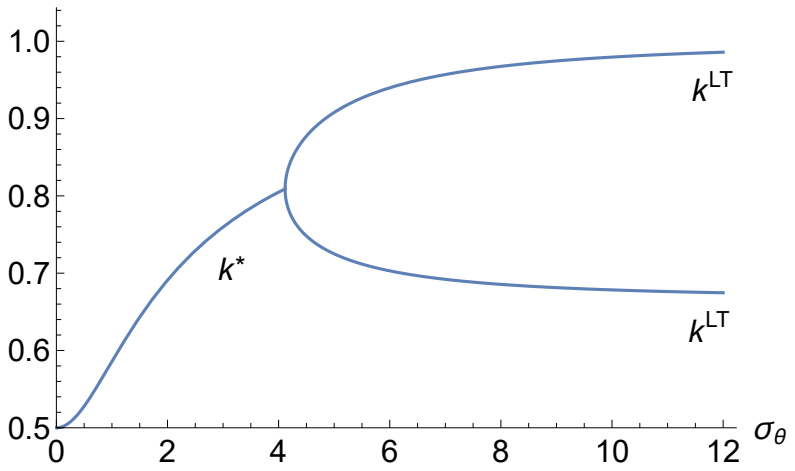


Figure 2: Optimal rating weight k from Proposition 5 as a function of σ_θ.

We now study the rating design when the firm pays the rating agency to give a public rating, observable by all market participants. Specifically, it sets the rating price and the rating weight, and makes a take-it-or-leave-it offer to the insider. This is the business model of credit rating agencies, for example. It is also the business model for some equity research conducted by investment banks, when it is made publicly available.

Proposition 6 *When the insider pays the rating agency to publicly disclose the rating, the rating agency optimally sets:*

$$k = \begin{cases} k^* & \text{if } \sigma_\theta < \sigma_\theta^d \\ k_d^{LT} & \text{if } \sigma_\theta \geq \sigma_\theta^d \end{cases} \tag{24}$$

The logic is similar to Proposition 5. If possible, the rating weight is the one that induces the level of effort under full commitment (k^{LT}), and if not it is the same rating weight k^* that maximizes the insider's effort. Intuitively, the information mix that maximizes effort incentives does not change when the rating is communicated to the speculator as in Proposition 5, and when it is publicly revealed as in Proposition 6. Moreover, since $\sigma_\theta^d < \sigma_\theta^*$, the optimal effort under full commitment is compatible with a lower level of uncertainty σ_θ when the rating is publicly disclosed, because of the higher sensitivity of firm value to the rating in this case. This has implications for economic efficiency.

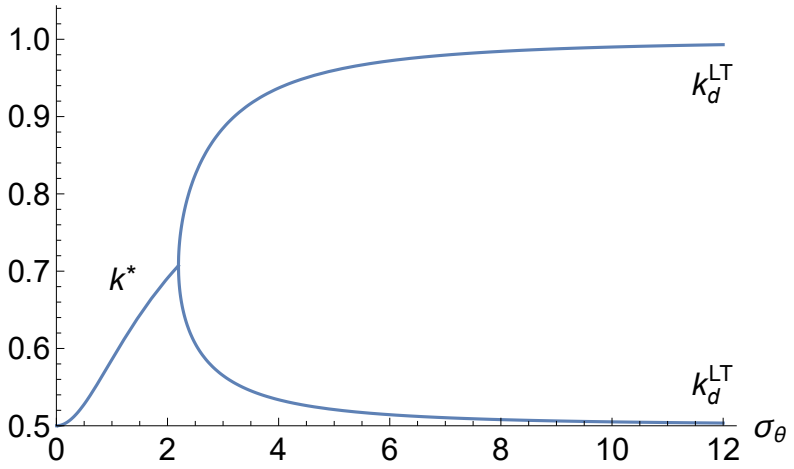


Figure 3: Optimal rating weight k from Proposition 6 as a function of σ_θ.

Corollary 1 *As σ_θ increases, one of the two optimal rating weights k_d^{LT} in Proposition 6 tends asymptotically to $\frac{1}{2}$:*

$$\lim_{\sigma_\theta \to \infty} \frac{3\sigma_\theta^2 - \sqrt{\sigma_\theta^4 - 4\sigma_\theta^2 - 4} + 2}{4\sigma_\theta^2 + 4} = \frac{1}{2} \tag{25}$$

The optimal rating corresponding to the lowest value of k_d^{LT} in Proposition 6 converges asymptotically to $k = \frac{1}{2}$ as uncertainty about firm value increases. Intuitively, when uncertainty about firm value is very high, there is a lot of scope for learning, so that the rating has a very high price impact. To not provide excessive effort incentives, the rating weight converges to either $\frac{1}{2}$ or to 1. For $k = \frac{1}{2}$, the rating is simply a linear transformation of a noisy version of output. Thus, as uncertainty about firm value is sufficiently high, it is approximately optimal to provide a rating which is simply informative about output. This rating will approximately induce the level of effort under full commitment.

3.4 Choice of rating and business model

We now determine the optimal profit maximizing rating for this agency. We show how the equilibrium business model depends on model parameters.

Proposition 7 *The rating agency optimally sells the rating to the speculator rather than the firm insider if σ_u is sufficiently high (i.e. $\sigma_u \geq \overline{\sigma}_u$) or π is sufficiently low. When it sells the rating to the firm insider, it discloses it publicly if $\sigma_\theta < \sigma_\theta^*$. Otherwise, if $\sigma_\theta \geq \sigma_\theta^*$, it is indifferent between disclosing it publicly or privately communicating it to the speculator.*

The business model described in Proposition 5 is always weakly dominated. Indeed, the insider's willingness to pay is higher when the rating is publicly disclosed rather than when it is

communicated privately to the speculator (it is only higher in a weak sense when $\sigma_\theta \geq \sigma_\theta^*$).

In an illiquid stock market with a low σ_u, the expected profit of the informed speculator is very low, and so is her willingness to pay for a rating. On the contrary, if the stock market is sufficiently liquid, the rating agency can sell its rating at a higher price to the speculator than to the firm insider. This highlights a new cost of financial market liquidity. Now consider the insider's liquidity shock, which proxies for his time horizon. If $\pi = 0$, the insider does not value the rating: he will never sell the firm and therefore does not care about its market value. More generally, when π is low, the insider has a low willingness to pay for the rating since he already largely internalizes the benefits of his effort. Therefore, the rating agency will rather sell a rating to the speculator.

This is the time horizon irrelevance result.

4 Competition between rating agencies

We now extend the analysis to consider competition between rating agencies. This extension addresses an important practical concern: in many markets, multiple rating agencies compete for business. Does competition improve or worsen the information environment? Our analysis reveals a surprising answer: competition can reduce welfare by creating strategic distortions in rating design.

4.1 Model setup

We modify the baseline model to allow for potential entry by a second rating agency. The timing is as follows. The first rating agency (incumbent) observes signals $S_1 = A + \varepsilon_1$ and $S_2 = \theta + \varepsilon_2$, designs rating $R_1 = k_1 S_1 + (1 - k_1) S_2$, and decides whether to sell publicly (to the market) or privately (to the speculator). A second rating agency (potential entrant) observes the first rating $R_1(k_1)$ and decides whether to acquire the same signals S_1 and S_2 at cost C. If the second agency enters, it designs rating $R_2 = k_2 S_1 + (1 - k_2) S_2$ and decides whether to sell publicly or privately. Trading occurs as in the baseline model, but market participants may observe multiple ratings.

A critical observation is that if an agent observes two ratings with different aggregation weights, she can recover the underlying signals.

Lemma 4 (signal recovery) *If an agent observes $R_1 = k_1 S_1 + (1 - k_1) S_2$ and $R_2 = k_2 S_1 + (1 - k_2) S_2$ with $k_1 \neq k_2$, then she can recover*

$$S_1 = \frac{R_1(1 - k_2) - R_2(1 - k_1)}{k_1(1 - k_2) - k_2(1 - k_1)}, \tag{26}$$

$$S_2 = \frac{R_1 k_2 - R_2 k_1}{k_2(1 - k_1) - k_1(1 - k_2)}. \tag{27}$$

This follows from solving the system of two linear equations in two unknowns. The denominator is non-zero when $k_1 \neq k_2$. This result has implications for equilibrium configurations. If the speculator observes two public ratings (or purchases two private ratings), she can recover the signals perfectly, making at least one rating redundant. Similarly, if the market maker observes two public ratings, the informational advantage of a private rating is eliminated.

Corollary 2 *In equilibrium, at most one public rating and one private rating can coexist.*

4.2 Equilibrium with public and private ratings

We focus on the configuration where the first agency provides a public rating R_1 and the second agency (if it enters) provides a private rating R_2 to the speculator. When a public rating R_1 and a private rating R_2 coexist, the market microstructure differs from the baseline model. The market maker observes R_1 and the order flow y, while the speculator observes both R_1 and R_2. As the speculator can recover the sgnals from observing the two ratings, the equilibrium with a public and a private rating does not depend on the specific design of the seconx rating as its informational value for the speculator is the same as that of a fully informational rating ($k_2 = 0$).

Proposition 8 (market equilibrium with two ratings) *There exists a unique linear equilibrium with the following properties:*

(i) Speculator's strategy:

$$q(R_1, R_2) = \beta(R_2 - \mathbb{E}[R_2]) + \gamma(R_1 - \mathbb{E}[R_1]), \tag{28}$$

where

$$\beta = \frac{\sigma_{R_1}\sigma_u}{k_1\sigma_{R_2}}, \tag{29}$$

$$\gamma = -\frac{(1-k_1)\sigma_{R_2}\sigma_u}{k_1\sigma_{R_1}}. \tag{30}$$

(ii) Market maker's pricing:

$$p(y, R_1) = \mu + \lambda(y - \mathbb{E}[y]) + \nu(R_1 - \mathbb{E}[R_1]), \tag{31}$$

where

$$\mu = \hat{A} + \bar{\theta}, \tag{32}$$

$$\lambda = \frac{\sigma_\theta^2 k_1}{2\sigma_u \sigma_{R_1} \sigma_{R_2}}, \tag{33}$$

$$\nu = \frac{\sigma_\theta^2 (1 - k_1)}{\sigma_{R_1}^2}. \tag{34}$$

(iii) Expected price conditional on effort:

$$\mathbb{E}[p|A, \hat{A}] = \hat{A} + \bar{\theta} + \frac{\sigma_\theta^2 (1 - k_1) k_1}{2\sigma_{R_1}^2}(A - \hat{A}). \tag{35}$$

(iv) Speculator's expected profit:

$$\mathbb{E}[Profit] = \lambda \sigma_u^2 = \frac{k_1 \sigma_\theta^2 \sigma_u}{2\sigma_{R_1} \sigma_{R_2}}. \tag{36}$$

Here $\sigma_{R_1}^2 = k_1^2 + (1 - k_1)^2(1 + \sigma_\theta^2)$ *and* $\sigma_{R_2}^2 = 1 + \sigma_\theta^2$.

Proposition 8 shows that the implicit incentives created by the public rating R_1 have the same form as in the baseline model with a private rating of weight k_1. The mechanism, however, comes from two opposing effects. The public rating directly affects the market maker's price (positive effect), but it also affects the speculator's trading strategy, which in turn influences price (negative effect). These two effects exactly offset to produce the familiar incentive structure.

In the two-rating equilibrium, the speculator's profit increases with the motivational content of the public rating.

Lemma 5 (monotonicity of speculator profit) *The speculator's expected profit,* $\frac{k_1 \sigma_\theta^2 \sigma_u}{2\sigma_{R_1} \sigma_{R_2}}$*, is strictly increasing in* k_1 *on* $[0, 1]$.

This monotonicity means that when the first agency makes its public rating more motivational (higher k_1), it makes the rating less informative about the firm's type θ. This increases the value of the second agency's private rating, which can reveal information about θ. Thus, paradoxically, better ratings (from the insider's perspective) make entry by a competing agency more attractive.

4.3 Competition regimes

The second agency's entry decision depends on whether the profit from selling a private rating to the speculator exceeds the cost C of acquiring information. Let $\Delta(k_1)$ denote the gain in expected profit to the speculator from observing both ratings versus only the public rating:

$$\Delta(k_1) = \frac{\sigma_u \sigma_\theta^2}{2\sqrt{1+\sigma_\theta^2}} - \frac{k_1 \sigma_\theta^2 \sigma_u}{2\sigma_{R_1}\sigma_{R_2}}. \tag{37}$$

Note that $\Delta(0) = 0$ (if $k_1 = 0$, the public rating already reveals type, so the private rating is redundant) and $\Delta'(k_1) < 0$ by Lemma 5.

Proposition 9 (competition regimes) *Define threshold values such that $\Delta(k^*) = C$ determines $\underline{\sigma}_u(k^*)$, and the first agency is indifferent between selling a public rating with $k_1 > 0$ to the insider versus selling a private rating with $k_1 = 0$ to the speculator at $\bar{\sigma}_u$. The equilibrium exhibits three regimes:*

(i) Blockaded Entry ($\sigma_u < \underline{\sigma}_u(k^*)$): The first agency sells a public rating with $k_1 = k^*$ (optimal for the insider). The second agency does not enter because $\Delta(k^*) < C$.

(ii) Deterred Entry ($\underline{\sigma}_u(k^*) \leq \sigma_u < \bar{\sigma}_u$): The first agency sells a public rating with $k_1 = \hat{k}_1 < k^*$, where $\hat{k}_1$ is implicitly defined by $\Delta(\hat{k}_1) = C$. This deters entry by making the second rating just unprofitable. There is strategic distortion: $\hat{k}_1 < k^*$.

(iii) Accommodated Entry ($\sigma_u \geq \bar{\sigma}_u$): The first agency sells a private rating with $k_1 = 0$ to the speculator. The second agency sells a public rating with $k_2 > 0$ to the insider. Both agencies operate.

4.4 Policy and empirical Implications

In the deterred entry regime, insider effort and welfare is strictly lower than with the optimal rating: $V(\hat{k}_1) < V(k^*)$. Recall that insider welfare is $V(k_1) = (1-\delta)\{(1-\pi)[A(k_1) + \bar{\theta}] + \pi\mathbb{E}[p|A(k_1)]\} - C(A(k_1))$, which is maximized at $k_1 = k^*$ in the absence of entry concerns. Since $\hat{k}_1 < k^*$ is chosen to deter entry rather than to maximize insider value, we have $V(\hat{k}_1) < V(k^*)$ by revealed preference. The first-order condition for maximizing V is violated at $\hat{k}_1$.

Moreover, this distortion creates efficiency loss: the effort level $A(\hat{k}_1)$ is below the insider-optimal level $A(k^*)$, reducing expected output. The second agency does not enter, so no offsetting information gain occurs. The distortion is pure waste driven by strategic entry deterrence.

This result highlights a new inefficiency in competitive information markets. Unlike standard models where competition improves outcomes by lowering prices or increasing quality, here competition (or the threat thereof) reduces welfare by distorting the information structure away from the efficient insider-optimal design. The incumbent agency distorts down its rating not because of collusion or bias, but to make entry less attractive.

The analysis yields several implications for the regulation of rating agencies. Competition is not always beneficial. Reducing barriers to entry for rating agencies can backfire when information acquisition costs are low. The threat of entry may induce the incumbent to provide

less motivational ratings, reducing effort incentives and output. The deterred entry regime is most harmful. When entry is either impossible (blockaded) or inevitable (accommodated), the information structure is undistorted. The welfare loss is concentrated in the intermediate regime where entry is deterred through strategic distortion. Information technology has ambiguous effects. Declining costs of information acquisition (as in Gao and Huang 2020) expand the deterred entry regime by making entry threats more credible. This can reduce insider's effort despite improving the raw efficiency of information production. Mandatory dual ratings may help. If regulation requires firms to obtain ratings from two agencies, this eliminates the strategic distortion: the incumbent has no incentive to deter entry that will occur anyway. Both agencies can then optimize their ratings for their respective clients.

Our model generates predictions consistent with recent empirical patterns. Empirical studies find mixed effects of competition on rating quality (Becker and Milbourn 2011; Xia 2014). Our model explains this: competition improves quality (reduces distortion) in the accommodated regime but worsens it (creates distortion) in the deterred regime. Our results are also consistent with the observed proliferation of ESG ratings. The ESG rating market exhibits both issuer-paid certification (motivational) and investor-paid screening (informational) models. Our framework predicts this coexistence in the accommodated regime, with each model serving different clients and information needs.

5 Applications

The theoretical results developed have implications for several important financial markets. We examine four applications that illustrate how optimal information aggregation affects real-world rating and certification systems: venture capital, credit ratings, ESG certifications, and impact investing.

5.1 Venture capital

Venture capitalists are shareholders with a short investment horizon who are actively involved in the firms in their investment portfolio (Hellmann and Puri (2002), Bernstein, Giroud, and Townsend (2016)).

Venture capital presents a striking puzzle: venture capital funds typically have lives of 7-10 years, yet the companies they finance often require 15-20 years to reach their full potential. How can short-horizon investors successfully finance long-duration projects? Standard theories of short-termism (Stein (1988, 1989) suggest this should lead to underinvestment and poor outcomes.

Our time horizon irrelevance result provides an explanation. Venture-backed companies typically face enormous uncertainty about their prospects due to new technologies, unproven business models, and uncertain market demand. For example, venture capitalists tend to finance

"technological revolutions" (Lerner and Nanda (2020)), in which the idiosyncratic uncertainty about each firm's potential is compounded by uncertainty about the space that the firm is trying to enter. This high fundamental uncertainty (σ_θ) is precisely the condition that enables time horizon irrelevance. When σ_θ exceeds the threshold σ_θ^*, optimal information design allows entrepreneurs to achieve full commitment effort A^{LT} regardless of the venture capitalist's exit probability π.

This suggests that, when financial information is designed optimally, the short investment horizon of some key investors is not necessarily costly. For example, venture capitalists have limited horizons, yet they and the firms they support are often very successful. Our model emphasizes that two conditions are especially important: the optimal design of financial information, and the substantial uncertainty about the assets financed. Thus, our model can help explain the narrow focus of venture capitalists noted by Lerner and Nanda (2020), and the finding that VC funding is especially beneficial for firms that face high uncertainty (Park and Steensma (2012)).

To this end, an important condition is that financial information about VC-backed firms be designed to cater to these firms rather than to investors. This can be achieved via private audits, which can be tailored to firm-specific needs. For example, VC contracts often include contractible milestones (Kaplan and Strömberg (2003)), whose verification creates an information environment where signals about effort are acknowledged. A similar effect arises when exit takes the form of an IPO, where disclosure is designed by an investment bank paid by the issuing firm.

5.2 Credit ratings

Credit ratings present a well-known puzzle. Despite extensive evidence of conflicts of interest in the issuer-paid business model (Bolton, Freixas, and Shapiro, 2012; Griffin, Nickerson, and Tang, 2013), and despite periodic regulatory proposals to mandate investor-paid ratings, the issuer-paid model has persisted for over a century. While rating shopping and selective disclosure (Sangiorgi and Spatt, 2017) partially explain rating inflation, they do not explain why equity holders commission debt ratings in the first place, nor why this business model survives competitive pressure.

Our framework provides a complementary explanation: issuer-paid credit ratings create value through their motivational effects on corporate decision-making. When a firm's creditworthiness depends on managerial effort, whether through operating decisions, capital allocation, or risk management, an issuer-paid rating that aggregates information about both effort and firm fundamentals ($k > 0$) can provide stronger effort incentives than an investor-paid rating focused solely on fundamentals ($k^{**} = 0$).

The mechanism operates through the refinancing channel. Credit ratings directly affect the yields firms must pay on new debt issuance. When a rating incorporates information about

managerial effort ($k > 0$), managers internalize that their actions affect not only current operations but also future financing costs. This creates implicit incentives for value-enhancing effort even when managers have short horizons, consistent with our time horizon irrelevance result (see Proposition 3).

This explains why equity holders commission debt ratings. Although ratings constrain debt issuance, they simultaneously provide discipline that increases expected equity value. The rating acts as a commitment device: by making refinancing costs sensitive to managerial actions, it induces effort that would otherwise be suboptimal when managers face liquidity shocks or have short horizons.

Our model predicts issuer-paid credit ratings should dominate when: (i) firm fundamentals involve high uncertainty (σ_θ large), making ratings valuable for both information and incentives; (ii) managerial actions significantly affect credit risk (effort is productive); (iii) managers face horizon constraints due to potential departures or liquidity needs. These conditions characterize many corporate borrowers, explaining the widespread adoption of issuer-paid ratings despite their well-known limitations.

This motivational channel complements but differs from existing explanations. Rating shopping (Bolton, Freixas, and Shapiro, 2012) explains inflation but not demand. Regulatory certification (Opp, Opp, and Harris, 2013) explains mandatory requirements but not voluntary purchases. Reputational concerns (Mathis, McAndrews, and Rochet, 2009) constrain bias but don't explain the business model. Our framework suggests issuer-paid ratings persist not despite conflicts but because optimal aggregation creates effort incentives that investor-paid ratings cannot replicate. This is consistent with evidence that rated firms subsequently improve operating performance (Kisgen, 2006) and that rating changes affect investment decisions (Kisgen, 2009).

Our model also predicts that highly liquid secondary markets for corporate debt could undermine issuer-paid ratings. When σ_u is very large, investor demand for informational ratings ($k^{**} = 0$) increases, potentially crowding out motivational ratings (Proposition 7). This suggests that the growth of liquid credit markets may shift the equilibrium toward investor-paid models, consistent with recent regulatory discussions about rating agency reform.

5.3 ESG and impact investing

The ESG information market features two coexisting business models. Investor-paid ESG ratings (provided by agencies such as MSCI and Sustainalytics) are sold to institutional investors for portfolio screening and emphasize financially material ESG risks. Issuer-paid certifications (such as B-Corp and LEED) are purchased by companies themselves; as Diez-Busto, Sanchez-Ruiz, and Fernandez-Laviada (2022) document, firms pursue them to "give visibility to commitment" and "encourage self-assessment for continuous improvement", which can be viewed as motivational rather than purely informational objectives.

Our framework explains this coexistence. Investor-paid ratings ($k^{**} = 0$) emerge when market

liquidity is high or insiders have long horizons, serving investors who need information about ESG risks for trading. Issuer-paid certifications ($k > 0$) emerge when insiders face exit risk and ESG uncertainty is high, creating incentives for ESG improvements by aggregating effort and outcome signals. The two models serve different clients and are complementary rather than competing. This also explains why ESG raters produce ratings with low correlation (Berg, Kölbel, and Rigobon, 2022): rather than reflecting market failure, this divergence reflects different business models optimized for different purposes.

Our framework also illuminates impact investing. Consider a setting where the insider's effort A generates social impact (reducing emissions, improving labor practices). Following Oehmke and Opp (2025), suppose the speculator maximizes expected wealth plus social impact, where the latter depends on actual effort A.

Corollary 3 *When the manager's effort improves social outcomes and the speculator is socially responsible, the rating that maximizes her willingness to pay involves a strictly positive weight k.*

The intuition parallels our baseline results but with a crucial difference. Socially responsible speculators value information about effort A not only because it affects firm value, but because they intrinsically value social outcomes. Their willingness to pay is maximized by a rating that both increases informational advantage about $\tilde{\theta}$ (for trading) and is sensitive to effort A (for impact). Importantly, impact arises not through the speculator's trading activity per se, but through her influence on information design. This contrasts with existing theories where impact arises through divestment or engagement (Edmans, Levit, and Schneemeier, 2023; Oehmke and Opp, 2025; Bisceglia, Piccolo, and Schneemeier, 2026): in our paper, impact operates through the information aggregation channel.

This analysis yields two broader implications. First, regulatory proposals to standardize ESG disclosure should distinguish between financially material ESG information (which should emphasize $k = 0$ for accurate risk assessment) and impact-oriented information (which should allow $k > 0$ to create incentives). Forcing both into a single framework reduces the effectiveness of each. Second, the real impact of socially responsible investment may operate primarily through information systems rather than capital allocation. When impact investors demand information with $k > 0$, they create incentives for social and environmental effort that would not exist with purely informational ratings ($k = 0$).

For example, suppose that the rating is $R = \zeta\left(kS_1 + (1-k)S_2\right)$, where ζ is observable but uncontractible, and could be inflated ex-post ("rating inflation") following a tacit agreement between the insider and the rating agency. This would not affect belief updating and stock price determination, since market participants correctly account for the effect of rating inflation when they update their beliefs, but it would increase the insider's contractual payoff.

6 Conclusion

The paper develops a framework in which the funding structure of financial ratings affects the information that ratings agencies produce. The rating design matters because it affects the firm's market valuation, which in turns affects the incentives of firm insiders who might sell their shares. In our model, aggregating multidimensional information about a firm in a single rating paid by the issuer maximizes firm value. However, it may not be the equilibrium outcome. When stock markets are highly liquid or when insiders have long-term horizons, the equilibrium will instead involve highly informed investors and low firm output.

The model generates important implications. First, we show that information design can fully offset the distortions associated with the insider's exit. When prior uncertainty is sufficiently high, the optimal motivational rating induces the same effort as if the insider could commit to long-term effort. Second, we identify a novel crowding-out effect of informational ratings. When investor-funded ratings are cheap to produce, they may eliminate the scope for issuer-funded ratings that increase effort incentives. Thus, more precise information can paradoxically reduce welfare by eliminating motivational information. Third, our framework explains why the interests of venture capitalists, whose investment horizon is significantly shorter than the horizon of the firms they invest in, can actually be closely aligned with these firms. However, this is only true if they invest in firms with a sufficiently uncertain outlook: only then can ratings be designed to provide adequate effort incentives.

Our analysis revisits debates on how rating agencies should be funded (Hayward and Boeker (1998), Benmelech and Dlugosz (2010), Bolton, Freixas, and Shapiro (2012), Kraft (2015)). Conventional wisdom stresses that issuer-paid models lead to bias; our model highlights a countervailing concern: investor-paid designs may be excessively informative about fundamentals and insufficiently motivational, depressing output. Policies that promote investor-funded ratings or mandate disclosure should take this effect into account.

Appendix

A First-best outcome

The first-best effort A^{FB} of the insider maximizes aggregate welfare, i.e. expected output minus the effort cost. It solves:

$$\max_{A} \{A - C(A)\} \tag{38}$$

which gives:

$$C'(A^{FB}) = 1 \tag{39}$$

The insider's optimally chosen effort is first-best if he owns the whole firm and is never hit by a liquidity shock, i.e. $\delta = 0$ and $\pi = 0$. In this case, he knows ex-ante that he will own the whole firm ex-post, and therefore fully internalizes the effect of his effort on firm value.

We now show that under some conditions a rating can lead to the insider to exert the first-best effort.

Lemma 6 *When the speculator privately observes the rating, the insider effort is* A^{FB} *if* $\sigma_\theta \geq \sqrt{4\sqrt{\frac{\delta^2-2\delta+5}{(\delta-1)^4}} + \frac{8}{(\delta-1)^2}}$ *and* $k = k^{FB} \equiv \frac{(\delta-5)\sigma_\theta^2 - 4 \pm \sqrt{(\delta-1)^2\sigma_\theta^4 - 16\sigma_\theta^2 - 16}}{2(\delta-3)\sigma_\theta^2 - 8}$.

Proof of Lemma 6:

The insider maximizes:

$$\max_{A} (1-\delta)\left((1-\pi)\left(A+\bar{\theta}\right) + \pi\left(\left(\hat{A}+\bar{\theta}\right) + \lambda\beta k\left(A-\hat{A}\right)\right)\right) - C(A) \tag{40}$$

The second-order condition for a maximization problem is satisfied, so that the optimal effort is given by the first-order necessary condition:

$$C'(A) = (1-\delta)(1-\pi+\pi\lambda\beta k) \tag{41}$$

From the solution to the insider's maximization problem in equation (41) and the definition of the first-best effort, the rating induces the first-best effort if and only if:

$$(1-\delta)(1-\pi+\pi\lambda\beta k) = 1 \quad \Leftrightarrow \quad \lambda\beta k = 1 + \frac{1}{\pi}\frac{\delta}{1-\delta} \tag{42}$$

Substituting for λ and β, the condition is:

$$\frac{k(1-k)\sigma_\theta^2}{2\sigma_R^2} = 1 + \frac{1}{\pi}\frac{\delta}{1-\delta} \qquad \Leftrightarrow \qquad k(1-k)\sigma_\theta^2 = 2\left((1-k)^2(\sigma_\theta^2+1) + k^2\right)\left(1 + \frac{1}{\pi}\frac{\delta}{1-\delta}\right)$$

Solving for k gives the optimal weights.

■

The insider's effort is first-best when the implicit incentives from the resale value of the firm are sufficiently high to outweigh the insider's partial ownership of the firm (when $\delta > 0$). This is only possible for a subset of parameter values.

First, insider ownership $1-\delta$ must be high enough. Otherwise, there is no rating design such that market pricing based on the rating delivers first-best incentives. Intuitively, the insider's incentives can be decomposed into two components. First, the explicit incentives from his (stochastic) long-term ownership. These incentives are proportional to $(1-\delta)(1-\pi)$. Second, the implicit incentives from the firm's market valuation when he needs to liquidate his stake. These incentives are proportional to $(1-\delta)\pi$. By contrast, first-best effort incentives are independent of the manager's ownership $1-\delta$. As a result, when the insider does not own the whole firm, i.e. $1-\delta < 1$, a necessary condition for first-best effort is that the implicit incentives from market valuation be sufficiently stronger than the explicit incentives from the insider's long-term ownership.

Second, prior uncertainty about firm value, as captured by σ_θ, must be sufficiently high. the rating is not very informative, so that the market value of the firm is not very sensitive to the rating, and therefore the implicit incentives for effort emanating from the effect of the rating on the resale value of the firm cannot be high enough, for any rating weight k on the signal $\tilde{S}_1$ which depends on the insider's effort.

If the first-best effort is achievable, there are two rating weights k^{FB} that induce it. Intuitively, there are two ways to deviate from the effort-maximizing rating weight, which induces a level of effort higher than at the first-best. The first is for the rating to put a high weight on effort, so that it is very sensitive to effort, but not very informative about firm value. Because of the latter, the price impact of the rating is small. The second possibility is for the rating to put a relatively low weight on effort, so that it is not very sensitive effort. However, the rating is then very informative about firm value, so that its price impact is large. The implicit incentives from the resale value of the firm are the same in these two cases.

Lemma 7 shows that, when the rating is publicly disclosed and when uncertainty about firm value is sufficiently high, there are rating weights that induce the first-best effort defined in equation (39).

Lemma 7 *When the rating is publicly disclosed, the insider effort is A^{FB} if $\sigma_\theta \geq \Sigma(\delta,\pi)$ and $k = k_d^{FB}$, where $\Sigma(\delta,\pi)$ and k_d^{FB} are defined in the proof of Lemma 7.*

Proof of Lemma 7:

The insider maximizes:

$$\max_A (1-\delta)\left((1-\pi)\left(A+\bar{\theta}\right)+\pi\left(\hat{A}+\bar{\theta}+k(1-k)\frac{\sigma_\theta^2}{\sigma_R^2}(A-\hat{A})\right)\right)-C(A) \qquad (43)$$

The second-order condition for a maximization problem is satisfied, so that the optimal effort is given by the first-order necessary condition:

$$C'(A) = (1-\delta)\left(1-\pi+\pi k(1-k)\frac{\sigma_\theta^2}{\sigma_R^2}\right) \tag{44}$$

From the solution to the insider's maximization problem in equation (44) and the definition of the first-best effort in equation (39), the rating induces the first-best effort if and only if:

$$(1-\delta)\left(1-\pi+\pi k(1-k)\frac{\sigma_\theta^2}{\sigma_R^2}\right) = 1 \quad \Leftrightarrow \quad k(1-k)\frac{\sigma_\theta^2}{\sigma_R^2} = \frac{1}{\pi}\frac{\delta}{1-\delta}+1 \tag{45}$$

Solving for k gives the optimal weights $k = k_d^{FB}$, which exist for $\sigma_\theta \geq \Sigma(\delta,\pi)$, where k_d^{FB} and $\Sigma(\delta,\pi)$ are defined as:

$$k_d^{FB} \equiv \frac{\frac{2\delta\sigma_\theta^2}{(1-\delta)\pi}+\frac{2\delta}{(1-\delta)\pi}+3\sigma_\theta^2+2 \pm \sqrt{-\frac{4\delta^2\sigma_\theta^2}{(1-\delta)^2\pi^2}-\frac{4\delta^2}{(1-\delta)^2\pi^2}-\frac{8\delta\sigma_\theta^2}{(1-\delta)\pi}-\frac{8\delta}{(1-\delta)\pi}+\sigma_\theta^4-4\sigma_\theta^2-4}}{2\left(\frac{\delta\sigma_\theta^2}{(1-\delta)\pi}+\frac{2\delta}{(1-\delta)\pi}+2\sigma_\theta^2+2\right)}$$

$$\Sigma(\delta,\pi) \equiv \left(\frac{2\left(\delta^2\pi^2-2\delta^2\pi+\delta^2-2\delta\pi^2+2\delta\pi+\pi^2\right)}{(\delta-1)^2\pi^2} + 2\sqrt{\frac{2\delta^4\pi^4-6\delta^4\pi^3+7\delta^4\pi^2-4\delta^4\pi+\delta^4-8\delta^3\pi^4+18\delta^3\pi^3-14\delta^3\pi^2+4\delta^3\pi+12\delta^2\pi^4-18\delta^2\pi^3+7\delta^2\pi^2-8\delta\pi^4+6\delta\pi^3+2\pi^4}{(\delta-1)^4\pi^4}}\right)^{\frac{1}{2}}$$

■

Lemma 7 describes the rating weights that induce the first-best optimal effort. As when ratings are communicated privately to the speculator (see Lemma 6 , insider ownership $1-\delta$ and prior uncertainty about firm value σ_θ, must be sufficiently high for these rating weights to exist. The difference is that these rating weights exist for a larger subset of parameter values when the rating is publicly disclosed. This is because a publicly disclosed rating will have a higher price impact. In particular, the market value of the firm will be more sensitive to the insider's actual effort, all else equal.

A rating that induces first-best effort with $k = k^{FB}$, if it exists, would be the rating used by a benevolent rating agency whose objective is to maximize social welfare. Of course, even if this rating exists, this is not necessarily the rating that arises at the second-best when the rating agency designs the rating to maximize its profits.

B Iso-effort curves

This Appendix depicts iso-effort curves for the two business models considered. Figures 4 and 5 illustrate that the minimum level of σ_θ required for a given effort is lower when the rating is publicly disclosed. They also illustrate how the rating weights that induce a given effort differ

depending on the business model.

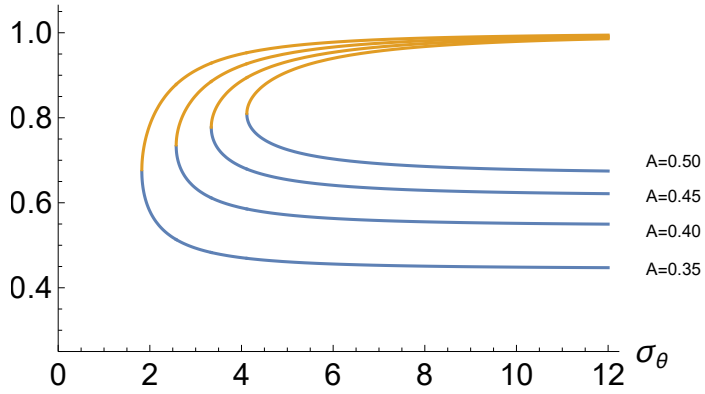


Figure 4: Iso-effort curves in the $\{\sigma_\theta, k\}$ plane that induce effort levels of $A = 0.35, 0.40, 0.45, 0.50$, for $\delta = 0.5$, $\pi = 0.5$, and $C(A) = \frac{A^2}{2}$, when the rating is privately communicated to the speculator. The rating weight k^{LT}, as defined in Lemma 2, induces $A = 0.5$. For each effort level depicted, the two parts of the iso-effort curve with different colors correspond to the two rating weights that induce this effort.

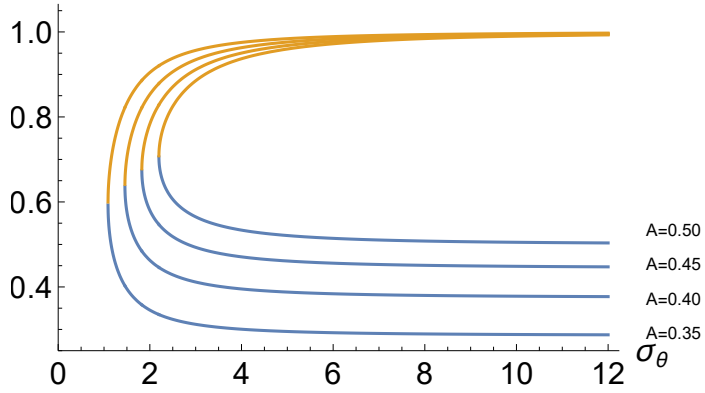


Figure 5: Iso-effort curves in the $\{\sigma_\theta, k\}$ plane that induce effort levels of $A = 0.35, 0.40, 0.45, 0.50$, for $\delta = 0.5$, $\pi = 0.5$, and $C(A) = \frac{A^2}{2}$, when the rating is publicly disclosed. The rating weight k_d^{LT}, as defined in Lemma 3, induces $A = 0.5$. For each effort level depicted, the two parts of the iso-effort curve with different colors correspond to the two rating weights that induce this effort.

C Uncertain effort

In our model, the agent's equilibrium effort is known, as is common in moral hazard models (Bolton and Dewatripont (2004)). This is not a crucial assumption.

Indeed, the model can be reinterpreted when there is uncertainty about the insider's action (i.e. the level of effort) rather than about the "firm type". Suppose that firm output is simply:

$$X = A, \tag{46}$$

and the cost of effort is now:

$$C(A) = \frac{c}{2}\left(A - \overline{A}\right)^2 \tag{47}$$

where c is common knowledge and $\overline{A}$, which is drawn from $\overline{A} \sim \mathcal{N}(\bar{\theta}, \sigma_\theta^2)$, is privately learned by the insider at the effort choice stage.[10] There is uncertainty about the parameter $\overline{A}$, which enters the effort cost function and affects the equilibrium effort. This is similar to the specification in Akerlof and Kranton (2005) and Fischer and Huddart (2008), where $\overline{A}$ would be interpreted as the "effort norm". This implies:

$$C'(A) = c\left(A - \overline{A}\right) \tag{48}$$

The first-order condition with respect to the insider's effort is now, given a first derivative of the insider's wealth with respect to effort ξ:[11]

$$\xi = c\left(A - \overline{A}\right) \quad \Leftrightarrow \quad A = \frac{\xi}{c} + \overline{A} \tag{49}$$

Thus, output, as defined in equation (46), can be decomposed as:

$$X = \underbrace{A - \overline{A}}_{\text{correctly anticipated}} + \underbrace{\overline{A}}_{\text{uncertain}} = \frac{\xi}{c} + \overline{A} \tag{50}$$

In this setting, the two signals which can be combined into a rating are $\tilde{S}_1 = A - \overline{A} + \tilde{\varepsilon}_1$, and $\tilde{S}_2 = \overline{A} + \tilde{\varepsilon}_2$. As in the main model, the first signal is sensitive to the level of effort A chosen by the insider, and the second signal reduces uncertainty about firm output. Note that the first signal is not informative in equilibrium: it it common knowledge that $A - \overline{A} = \frac{\xi}{c}$ (see equation (49)). Thus, as in the main model, S_1 is not informative in equilibrium but it can still be used to provide effort incentives since it is sensitive to the level of effort A actually chosen by the insider.

For comparison purposes, the first-order condition with respect to effort in the main model,

[10] This is for tractability and avoid issues that arise with asymmetric information at the contracting stage (Lizzeri (1999)).

[11] There are several possible expressions for ξ, depending on the business model used by the rating agency.

which relies on the more standard specification $C(A) = \frac{c}{2}A^2$, would be:

$$\xi = cA \quad \Leftrightarrow \quad A = \frac{\xi}{c} \quad \Rightarrow \quad X = \frac{\xi}{c} + \theta \tag{51}$$

Comparing the equalities on the right of equations (49) and (51) shows that the firm output X is additive in a constant ($\frac{\xi}{c}$) and either $\overline{A}$ (in the alternative setting) or $\tilde{\theta}$ (in the main model). In both cases, this latter variable is unknown, normally distributed, and the signal S_2 reduces uncertainty about it: $\tilde{S}_2 = \theta + \tilde{\varepsilon}_2$ in the main model, and $\tilde{S}_2 = \overline{A} + \tilde{\varepsilon}_2$ in this alternative setting.

In sum, the tradeoff remains essentially the same in this alternative setting: shall information motivate effort or resolve uncertainty about firm value? The difference lies in the interpretation of the source of uncertainty. In the main model, it is uncertainty about the "firm type" $\tilde{\theta}$. In the alternative setting outlined here, it is uncertainty about the baseline level of effort $\overline{A}$.

D Speculator endowed with shares

We assumed that the speculator did not initially own any shares in the firm. We now relax this assumption to allow the speculator to also be an initial shareholder.

Corollary 4 *When the speculator owns q_0 shares in the firm at $t = 0$, the rating that maximizes her willingness to pay involves a strictly positive weight k.*

Proof of Corollary 4:

The equilibrium of the financial market follows the one in Proposition 1. The speculator's expected profit is now equal to:

$$\underbrace{\frac{\sigma_u (1-k)\, \sigma_\theta^2}{2\sigma_R}}_{f(k)} + \underbrace{q_0(\hat{A} + \bar{\theta})}_{g(k)} \tag{52}$$

The objective function of the speculator is additive in two functions: $f(k)$ and $g(k)$.

The first derivative with respect to k is:

$$-\frac{\sigma_u \sigma_\theta^2}{2} \frac{k}{(k^2 + (k-1)^2(1+\sigma_\theta^2))^{\frac{3}{2}}} + q_0 \frac{d\hat{A}}{dk} \tag{53}$$

When $q_0 = 0$, the rating weight that maximizes the speculator's objective function is simply the one that maximizes $f(k)$. It is $k^{**} = 0$. Indeed, for $k > 0$, $f'(k) < 0$.

When $\sigma_u = 0$, the rating weight that maximizes the speculator's objective function is simply the one that maximizes $g(k)$. It is:

$$k = k^* \equiv \frac{1 + \sigma_\theta^2 - \sqrt{1 + \sigma_\theta^2}}{\sigma_\theta^2}$$

and $g'(k) < 0$ for $k < k^*$.

By the intermediate value theorem, the rating weight that maximizes $f(k) + g(k)$ is in-between k^{**} and k^*. Thus, the rating weight that maximizes the speculator's willingness to pay is strictly positive. ■

A speculator who initially owns shares in the firm does not only care about maximizing her informational advantage. She also cares, to an extent that depends on her stake, about maximizing the expected value of the firm. The latter is achieved by increasing effort incentives, which in turn involves increasing the rating weight above the level that maximizes the rating's informativeness, i.e. above $k^{**} = 0$.

This result has ambiguous implications for economic efficiency. On the one hand, if the rating agency sells the rating to the speculator, it will now sell a rating that is not purely informational but instead partly motivational. On the other hand, a speculator with a stake in the firm has a higher willingness to pay for the rating, which makes it more profitable for the rating agency to sell the rating to her rather than to the issuer – who would prefer a purely motivational rating.

E Proofs

Proof of Lemma 1:

Suppose that the speculator has access to two ratings, R and R', which are independent linear combinations of signals S_1 and S_2 with weights k and k'.

$$R = kS_1 + (1-k)S_2. \tag{54}$$
$$R' = k'S_1 + (1-k')S_2. \tag{55}$$

From observing R and R', the speculator can recover signals S_1 and S_2. We get:

$$S_1 = \frac{(1-k')R - (1-k)R'}{k-k'} \tag{56}$$
$$S_2 = \frac{kR' - k'R}{k-k'} \tag{57}$$

The speculator will then use only S_2 to update her beliefs about θ. This is equivalent to providing a single rating with $k = 0$. ■

Proof of Proposition 1:

The speculator solves:

$$\max_q \ \mathbb{E}\left[(\tilde{X} - p(\tilde{y}))q \middle| R\right]$$
$$\Leftrightarrow \qquad \max_q \ \Big(\mathbb{E}[\tilde{X}|R] - (\mu + \lambda(1+q))\Big)\, q \tag{58}$$

The second-order condition for a maximum is satisfied, so the optimum $q^*(R)$ is described by the first-order necessary condition:

$$q^*(R) = \frac{\mathbb{E}\left[\tilde{X}\middle|R\right] - (\mu + \lambda)}{2\lambda} \tag{59}$$

We have:

$$\begin{aligned}\mathbb{E}[\tilde{X}|R] &= \hat{A} + \bar{\theta} + \frac{cov(\tilde{X}, \tilde{R})}{var(\tilde{R})}\left(R - k\hat{A} - (1-k)\,\bar{\theta}\right) \\ &= \hat{A} + \bar{\theta} + \frac{(1-k)\,\sigma_\theta^2}{\sigma_R^2}\left(R - k\hat{A} - (1-k)\,\bar{\theta}\right)\end{aligned} \tag{60}$$

Denoting $b_I \equiv \frac{(1-k)\sigma_\theta^2}{\sigma_R^2}$, we have:

$$q^*(R) = \frac{\hat{A} + \bar{\theta} - (\mu + \lambda)}{2\lambda} + \frac{b_I\left(R - k\hat{A} - (1-k)\,\bar{\theta}\right)}{2\lambda} \tag{61}$$

So we get

$$\beta = \frac{b_I}{2\lambda} \tag{62}$$

$$\alpha = \frac{\hat{A} + \bar{\theta} - (\mu + \lambda)}{2\lambda} - \frac{b_I\left(k\hat{A} + (1-k)\,\bar{\theta}\right)}{2\lambda} \tag{63}$$

The price set by the market maker corresponds to the expected value of the firm given the total order flow y observed, including an order $\alpha + \beta R$ from the speculator and an order u from noise traders.

$$\begin{aligned}p(y) &= \mathbb{E}\left[\tilde{X}|y\right] \\ &= \mathbb{E}\left[\tilde{X}\right] + \frac{cov(\tilde{X}, \tilde{y})}{Var(\tilde{y})}\,(y - \mathbb{E}[\tilde{y}]) \\ &= \hat{A} + \bar{\theta} + \frac{\beta\,(1-k)\,\sigma_\theta^2}{\beta^2(\sigma_R^2) + \sigma_u^2}\,(y - \mathbb{E}[\tilde{y}])\end{aligned} \tag{64}$$

In particular:

$$\begin{aligned}p\left(\mathbb{E}[\tilde{y}]\right) &= \hat{A} + \bar{\theta} \\ &= \mu + \lambda\left(1 + \alpha + \beta\left(k\hat{A} + (1-k)\,\bar{\theta}\right)\right)\end{aligned} \tag{65}$$

We get:

$$\begin{aligned}\lambda &= \frac{\beta\left(1-k\right)\sigma_\theta^2}{\beta^2(\sigma_R^2)+\sigma_u^2}\\ \mu &= \hat{A}+\bar{\theta}-\lambda\left(1+\alpha+\beta\left(k\hat{A}+\left(1-k\right)\bar{\theta}\right)\right.\end{aligned}$$

Solving for α, β, λ and μ yields:

$$\beta = \frac{\sigma_u}{\sigma_R} \tag{66}$$

$$\lambda = \frac{\left(1-k\right)\sigma_\theta^2}{2\sigma_u\sigma_R} \tag{67}$$

$$\alpha = \beta\mathbb{E}[\tilde{R}] = \beta\left(k\hat{A}+\left(1-k\right)\bar{\theta}\right) \tag{68}$$

$$\mu = \mathbb{E}[\tilde{X}]-\lambda = \hat{A}+\bar{\theta}-\lambda \tag{69}$$

The expected stock price as a function of the expected effort of the insider, $\hat{A}$, and his actual effort, A, is:

$$\begin{aligned}\mathbb{E}\left[p|A,\hat{A}\right] &= \mathbb{E}\left[\mu+\lambda y\right]\\ &= \mu+\lambda+\lambda\beta\mathbb{E}\left[R-\mathbb{E}[\tilde{R}]\right]\\ &= \hat{A}+\bar{\theta}+\lambda\beta k\left(A-\hat{A}\right)\\ &= \hat{A}+\bar{\theta}+\frac{1}{2}\left(1-k\right)k\frac{\sigma_\theta^2}{\sigma_R^2}\left(A-\hat{A}\right)\end{aligned}$$

The expected (ex-ante) profit of the speculator who is trading after privately observing rating R is:

$$\begin{aligned}\mathbb{E}\left[\left(\tilde{X}-p(1+q+u)\right)q\right] &= \mathbb{E}\left[(\tilde{X}-\mu-\lambda-\beta(R-\mathbb{E}[\tilde{R}]))(\lambda\beta(R-\mathbb{E}[\tilde{R}])\right]\\ &= \beta\mathbb{E}\left[\tilde{X}(R-\mathbb{E}[\tilde{R}])\right]-\lambda\beta^2\mathbb{E}\left[\left(R-\mathbb{E}[\tilde{R}]\right)^2\right]\\ &= \frac{\sigma_u}{\sigma_R}\left(1-k\right)\sigma_\theta^2-\lambda\beta^2\sigma_R^2\\ &= \frac{\sigma_u\left(1-k\right)\sigma_\theta^2}{2\sigma_R}\end{aligned} \tag{70}$$

■

Proof of Proposition 2:

With a public rating, the market maker sets the market value of the firm equal to its expected output given the rating. The order flows are irrelevant as they do not include any additional

information about firm value. Given rating R, the market value of the firm is:

$$\begin{aligned} p &= \hat{A} + \mathbb{E}\left[\tilde{\theta}\middle|R\right] \\ &= \hat{A} + \mathbb{E}\left[\tilde{\theta}\right] + \frac{cov(\tilde{\theta},\tilde{R})}{var(\tilde{R})}\left(R - \mathbb{E}\left[\tilde{R}\right]\right) \\ &= \hat{A} + \bar{\theta} + \frac{(1-k)\sigma_\theta^2}{\sigma_R^2}\left(R - \mathbb{E}\left[\tilde{R}\right]\right) \end{aligned}$$

The expected market value conditional on actual effort A and expected effort $\hat{A}$ is:

$$\mathbb{E}\left[p\middle|A,\hat{A}\right] = \hat{A} + \bar{\theta} + k(1-k)\frac{\sigma_\theta^2}{\sigma_R^2}(A - \hat{A}) \tag{71}$$

■

Proof of Lemma 2:
The insider maximizes:

$$\max_A (1-\delta)\left((1-\pi)\left(A+\bar{\theta}\right) + \pi\left(\left(\hat{A}+\bar{\theta}\right) + \lambda\beta k\left(A - \hat{A}\right)\right)\right) - C(A) \tag{72}$$

The second-order condition for a maximization problem is satisfied, so that the optimal effort is given by the first-order necessary condition:

$$C'(A) = (1-\delta)\left(1 - \pi + \pi\lambda\beta k\right) \tag{73}$$

From the solution to the insider's maximization problem in equation (73) and the definition of the effort preferred by the insider, the rating induces the effort preferred by the insider if and only if:

$$(1-\delta)\left(1-\pi+\pi\lambda\beta k\right) = 1-\delta \quad \Leftrightarrow \quad \lambda\beta k = 1 \tag{74}$$

Substituting for λ and β, the condition is:

$$\frac{k\left(1-k\right)\sigma_\theta^2}{2\sigma_R^2} = 1 \qquad \Leftrightarrow \qquad k\left(1-k\right)\sigma_\theta^2 = 2\left(\left(1-k\right)^2\left(\sigma_\theta^2+1\right) + k^2\right) \tag{75}$$

Solving for k gives the optimal weights. ■

Proof of Lemma 3:
The insider maximizes:

$$\max_A (1-\delta)\left((1-\pi)\left(A+\bar{\theta}\right) + \pi\left(\hat{A} + \bar{\theta} + k(1-k)\frac{\sigma_\theta^2}{\sigma_R^2}(A-\hat{A})\right)\right) - C(A) \tag{76}$$

The second-order condition for a maximization problem is satisfied, so that the optimal effort is given by the first-order necessary condition:

$$C'(A) = (1-\delta)\left(1-\pi+\pi k(1-k)\frac{\sigma_\theta^2}{\sigma_R^2}\right) \tag{77}$$

From the solution to the insider's maximization problem in equation (77) and the definition of the effort preferred by the insider in equation (18), the rating induces the effort preferred by the insider if and only if:

$$\begin{aligned}(1-\delta)\left(1-\pi+\pi k(1-k)\frac{\sigma_\theta^2}{\sigma_R^2}\right) = 1-\delta \quad &\Leftrightarrow \quad k(1-k)\frac{\sigma_\theta^2}{\sigma_R^2} = 1 \\ &\Leftrightarrow \quad k\left(1-k\right)\sigma_\theta^2 = \left(\left(1-k\right)^2\left(\sigma_\theta^2+1\right)+k^2\right) \end{aligned} \tag{78}$$

Solving for k gives the optimal weights. ■

Proof of Proposition 4:

The rating weight that maximizes the speculator's willingness to pay is given by:

$$\max_k \left(\frac{\sigma_u\left(1-k\right)\sigma_\theta^2}{2\sigma_R}\right) \tag{79}$$

Since σ_u and σ_θ are constants, and substituting for σ_R, this is equivalent to:

$$\max_k \left(\frac{1-k}{\sqrt{(1-k)^2\left(\sigma_\theta^2+1\right)+k^2}}\right) \tag{80}$$

The first derivative with respect to k is:

$$-\frac{k}{\left(k^2+(k-1)^2(1+\sigma_\theta^2)\right)^{\frac{3}{2}}} \tag{81}$$

which is negative for any $\sigma_\theta > 0$ and $k \in (0,1)$. The optimal weight k^{**} is given by:

$$k = k^{**} \equiv 0 \tag{82}$$

■

Proof of Proposition 5:

Given equilibrium effort $\hat{A}$ induced by the contract, the insider's willingness to pay is maximized by solving:

$$\max_k (1-\delta)\left(\pi\left(\hat{A}+\bar{\theta}\right)+(1-\pi)\left(\hat{A}+\bar{\theta}\right)\right) - C(\hat{A}) \quad \Leftrightarrow \quad \max_k (1-\delta)\hat{A} - C(\hat{A}) \tag{83}$$

The first-order condition is:

$$(1-\delta-C'(\hat{A}))\frac{d\hat{A}}{dk}=0 \tag{84}$$

There are two cases.

First, if parameters are such that there exists a rating weight k that induces $1-\delta=C'(\hat{A})$, then the optimal rating weight is implicitly determined to induce $1-\delta=C'(\hat{A})$. This maximizes the insider's objective function in equation (83).

Second, if parameters are such that there does not exist a rating weight k that induces $1-\delta=C'(\hat{A})$, the level of k that maximizes the insider's willingness to pay maximizes the equilibrium effort. It is given by:

$$\max_k \lambda\beta k=\frac{k(1-k)}{\sigma_R^2}=\frac{k(1-k)}{(1-k)^2\left(\sigma_\theta^2+1\right)+k^2} \tag{85}$$

The solutions are thus the roots of:

$$k^2(\sigma_\theta^2)-2(\sigma_\theta^2+1)k+\left(\sigma_\theta^2+1\right) \tag{86}$$

We find:

$$k=k^*\equiv\frac{\sigma_\theta^2+1\pm\sqrt{1+\sigma_\theta}}{\sigma_\theta^2} \tag{87}$$

The maximum is reached for the smaller root. The larger root corresponds to a minimum. ■

Proof of Proposition 6:

The insider chooses effort to maximize:

$$\max_A (1-\delta)\left((1-\pi)\left(A+\bar{\theta}\right)+\pi\left(\hat{A}+\bar{\theta}+k(1-k)\frac{\sigma_\theta^2}{\sigma_R^2}(A-\hat{A})\right)\right)-C(A) \tag{88}$$

The second-order condition for a maximum is satisfied. At the time of exerting effort, the insider takes the rating k as given. The equilibrium effort, $\hat{A}$, is described by the first-order necessary condition:

$$C'(\hat{A})=(1-\delta)\left(1-\pi+\pi k(1-k)\frac{\sigma_\theta^2}{\sigma_R^2}\right) \tag{89}$$

The rating agency chooses k to maximize the insider's objective function and thus his willingness to pay for the rating.

$$\max_k\left((1-\delta)\hat{A}-C(\hat{A})\right) \tag{90}$$

The first-order condition is:

$$(1-\delta-C'(\hat{A}))\frac{d\hat{A}}{dk}=0 \tag{91}$$

There are two cases.

First, if parameters are such that there exists a rating weight k that induces $1-\delta = C'(\hat{A})$, then the optimal rating weight is implicitly determined to induce $1-\delta = C'(\hat{A})$. This maximizes the insider's objective function in equation (88).

Second, if parameters are such that there does not exist a rating weight k that induces $1-\delta = C'(\hat{A})$, the level of k that maximizes the insider's willingness to pay maximizes the equilibrium effort. It is given by:

$$\max_k \frac{k(1-k)}{\sigma_R^2} = \frac{k(1-k)}{(1-k)^2(\sigma_\theta^2+1)+k^2} \tag{92}$$

The first-order necessary conditions are

$$\frac{1+\sigma_\theta^2+k^2\sigma_\theta^2-2k(1+\sigma_\theta^2)}{(\sigma_R^2)^2} = 0 \tag{93}$$

The solutions are thus the roots of

$$1+\sigma_\theta^2+k^2\sigma_\theta^2-2k(1+\sigma_\theta^2) \tag{94}$$

We find:

$$k = k^* \equiv \frac{1+\sigma_\theta^2 \pm \sqrt{1+\sigma_\theta^2}}{\sigma_\theta^2} \tag{95}$$

The maximum is reached for the smaller root. The larger root corresponds to a minimum. ■

Proof of Proposition 7:

First, with $k = k^{**} = 0$, the expected profit of the speculator from Lemma 5 becomes:

$$\frac{\sigma_u \sigma_\theta^2}{2\sigma_R} = \frac{\sigma_u}{2}\frac{\sigma_\theta^2}{\sqrt{\sigma_\theta^2+1}}, \tag{96}$$

which is linearly increasing in σ_u. Since the rating agency makes a take-it-or-leave-it offer, it captures the surplus from the speculator's valuation. In addition, the insider's willingness to pay is independent from σ_u.

Second, the insider's valuation of any given rating is linear in π; with $\pi = 0$, the insider does not value any rating. In addition, the speculator's willingness to pay with $k = k^{**}$ is independent from π. ■

Proof of Proposition 8: We provide the complete derivation of the Kyle equilibrium when a public rating R_1 and a private rating R_2 coexist. We derive all coefficients (α, β, γ, μ, λ, ν) from first principles.

The market maker observes the public rating $R_1 = k_1 S_1 + (1-k_1)S_2$ and the total order flow $y = q + \tilde{u}$.

The speculator observes both ratings:

$$R_1 = k_1 S_1 + (1 - k_1) S_2 = k_1(A + \varepsilon_1) + (1 - k_1)(\theta + \varepsilon_2), \tag{97}$$

$$R_2 = k_2 S_1 + (1 - k_2) S_2 = k_2(A + \varepsilon_1) + (1 - k_2)(\theta + \varepsilon_2). \tag{98}$$

Since the speculator observes two independent linear combinations of S_1 and S_2, she can recover both signals. From Lemma 1, she behaves as if observing $S_2 = \theta + \varepsilon_2$ directly, regardless of k_2.

We conjecture linear strategies:

$$q(R_1, R_2) = \alpha + \beta(R_2 - \mathbb{E}[R_2]) + \gamma(R_1 - \mathbb{E}[R_1]), \tag{99}$$

$$p(y, R_1) = \mu + \lambda(y - \mathbb{E}[y]) + \nu(R_1 - \mathbb{E}[R_1]). \tag{100}$$

Assumptions: $\tilde{\theta}, \tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \tilde{u}$ are mutually independent with:

$$\mathrm{Var}(\theta) = \sigma_\theta^2, \quad \mathrm{Var}(\varepsilon_1) = \mathrm{Var}(\varepsilon_2) = 1, \quad \mathrm{Var}(u) = \sigma_u^2. \tag{101}$$

Key variance formulas:

$$\sigma_{R_1}^2 \equiv \mathrm{Var}(R_1) = k_1^2 + (1 - k_1)^2(1 + \sigma_\theta^2), \tag{102}$$

$$\sigma_{R_2}^2 \equiv \mathrm{Var}(R_2) = k_2^2 + (1 - k_2)^2(1 + \sigma_\theta^2) = 1 + \sigma_\theta^2 \quad (\text{when } k_2 = 0). \tag{103}$$

The speculator maximizes expected profit:

$$\max_q \mathbb{E}[(X - p(y, R_1))q \mid R_1, R_2]. \tag{104}$$

Expanding:

$$\max_q \mathbb{E}[X \mid R_1, R_2] \cdot q - \mathbb{E}[p(y, R_1) \mid R_1, R_2] \cdot q. \tag{105}$$

Substituting the conjectured price function (100) with $y = q + u$:

$$\mathbb{E}[p(q + u, R_1) \mid R_1, R_2] = \mu + \lambda \mathbb{E}[q + u \mid R_1, R_2] + \nu(R_1 - \mathbb{E}[R_1]) \tag{106}$$

$$= \mu + \lambda q + \nu(R_1 - \mathbb{E}[R_1]). \tag{107}$$

The first-order condition is:

$$\frac{\partial}{\partial q} \left\{ \mathbb{E}[X \mid R_1, R_2] \cdot q - [\mu + \lambda q + \nu(R_1 - \mathbb{E}[R_1])] \cdot q \right\} = 0. \tag{108}$$

Taking the derivative:

$$\mathbb{E}[X \mid R_1, R_2] - \mu - 2\lambda q - \nu(R_1 - \mathbb{E}[R_1]) = 0. \tag{109}$$

Solving for q:

$$q = \frac{\mathbb{E}[X \mid R_1, R_2] - \mu - \nu(R_1 - \mathbb{E}[R_1])}{2\lambda}. \tag{110}$$

Since the speculator recovers S_2 from (R_1, R_2):

$$\begin{aligned}
\mathbb{E}[X \mid R_1, R_2] &= \mathbb{E}[A + \theta \mid S_2] && (111)\\
&= \hat{A} + \mathbb{E}[\theta \mid \theta + \varepsilon_2] && (112)\\
&= \hat{A} + \bar{\theta} + \frac{\mathrm{Cov}(\theta, \theta + \varepsilon_2)}{\mathrm{Var}(\theta + \varepsilon_2)}(\theta + \varepsilon_2 - \bar{\theta}) && (113)\\
&= \hat{A} + \bar{\theta} + \frac{\sigma_\theta^2}{\sigma_\theta^2 + 1}(S_2 - \bar{\theta}). && (114)
\end{aligned}$$

Substituting (114) into (110):

$$q = \frac{\hat{A} + \bar{\theta} + \frac{\sigma_\theta^2}{1+\sigma_\theta^2}(S_2 - \bar{\theta}) - \mu - \nu(R_1 - \mathbb{E}[R_1])}{2\lambda}. \tag{115}$$

Since $S_2 = R_2$ (recovered signal) and $\mathbb{E}[R_2] = \bar{\theta}$ (effort correctly anticipated):

$$q = \frac{\hat{A} + \bar{\theta} - \mu}{2\lambda} + \frac{\sigma_\theta^2}{2(1 + \sigma_\theta^2)\lambda}(R_2 - \mathbb{E}[R_2]) - \frac{\nu}{2\lambda}(R_1 - \mathbb{E}[R_1]). \tag{116}$$

Matching coefficients with the conjectured form (99):

$$\begin{aligned}
\alpha &= \frac{\hat{A} + \bar{\theta} - \mu}{2\lambda}, && (117)\\
\beta &= \frac{\sigma_\theta^2}{2(1 + \sigma_\theta^2)\lambda}, && (118)\\
\gamma &= -\frac{\nu}{2\lambda}. && (119)
\end{aligned}$$

The market maker sets price equal to expected firm value given available information:

$$p(y, R_1) = \mathbb{E}[X \mid y, R_1]. \tag{120}$$

With multivariate normal variables, the conditional expectation is:

$$\mathbb{E}[X \mid y, R_1] = \mathbb{E}[X] + \mathrm{Cov}(X, [y, R_1])' \cdot \mathrm{Var}([y, R_1])^{-1} \cdot \begin{pmatrix} y - \mathbb{E}[y] \\ R_1 - \mathbb{E}[R_1] \end{pmatrix}. \tag{121}$$

We need: $\text{Cov}(X, y)$, $\text{Cov}(X, R_1)$, $\text{Var}(y)$, $\text{Cov}(y, R_1)$.
(i) $\mathbb{E}[X]$:

$$\mathbb{E}[X] = \mathbb{E}[A + \theta] = \hat{A} + \bar{\theta}. \tag{122}$$

(ii) $\text{Cov}(X, R_1)$:
Since $X = A + \theta = \hat{A} + \theta$ in equilibrium (A correctly anticipated):

$$\begin{aligned} \text{Cov}(X, R_1) &= \text{Cov}(\theta, k_1(A + \varepsilon_1) + (1 - k_1)(\theta + \varepsilon_2)) && (123)\\ &= \text{Cov}(\theta, (1 - k_1)\theta) && (124)\\ &= (1 - k_1)\sigma_\theta^2. && (125) \end{aligned}$$

(iii) Order flow:
From (116), the order flow is:

$$\begin{aligned} y &= q + u && (126)\\ &= \alpha + \beta(R_2 - \mathbb{E}[R_2]) + \gamma(R_1 - \mathbb{E}[R_1]) + u && (127)\\ &= \beta(\theta + \varepsilon_2 - \bar{\theta}) + \gamma(k_1\varepsilon_1 + (1 - k_1)(\theta + \varepsilon_2) - (1 - k_1)\bar{\theta}) + u + \alpha && (128)\\ &= [\beta + \gamma(1 - k_1)](\theta - \bar{\theta}) + \beta\varepsilon_2 + \gamma k_1\varepsilon_1 + \gamma(1 - k_1)\varepsilon_2 + u + \alpha. && (129) \end{aligned}$$

(iv) $\text{Cov}(X, y)$:
From (129):

$$\begin{aligned} \text{Cov}(X, y) &= \text{Cov}(\theta, y) && (130)\\ &= \text{Cov}(\theta, [\beta + \gamma(1 - k_1)](\theta - \bar{\theta})) && (131)\\ &= [\beta + \gamma(1 - k_1)]\sigma_\theta^2. && (132) \end{aligned}$$

(v) $\text{Var}(y)$:
From (129) and independence:

$$\text{Var}(y) = [\beta + \gamma(1 - k_1)]^2(1 + \sigma_\theta^2) + \gamma^2 k_1^2 + \sigma_u^2. \tag{133}$$

(vi) $\text{Cov}(y, R_1)$:
From (129):

$$\text{Cov}(y, R_1) = (1 - k_1)\beta(1 + \sigma_\theta^2) + \gamma\sigma_{R_1}^2. \tag{134}$$

From (121), the price coefficients are:

$$\lambda = \frac{1}{D}\left[\text{Cov}(X, y) \cdot \sigma_{R_1}^2 - \text{Cov}(X, R_1) \cdot \text{Cov}(y, R_1)\right], \tag{135}$$

$$\nu = \frac{1}{D}\left[\text{Cov}(X, R_1) \cdot \text{Var}(y) - \text{Cov}(X, y) \cdot \text{Cov}(y, R_1)\right], \tag{136}$$

where

$$D = \text{Var}(y) \cdot \sigma_{R_1}^2 - [\text{Cov}(y, R_1)]^2. \tag{137}$$

Substituting covariances into (135):

$$\lambda = \frac{\sigma_\theta^2}{D}\left\{[\beta + \gamma(1-k_1)]\sigma_{R_1}^2 - (1-k_1)[(1-k_1)\beta(1+\sigma_\theta^2) + \gamma\sigma_{R_1}^2]\right\} \tag{138}$$

$$= \frac{\sigma_\theta^2}{D}\left\{\beta[\sigma_{R_1}^2 - (1-k_1)^2(1+\sigma_\theta^2)]\right\} \tag{139}$$

$$= \frac{\sigma_\theta^2 \beta k_1^2}{D}. \tag{140}$$

Similarly, from (136), after substantial algebra:

$$\nu = \frac{\sigma_\theta^2}{D}\left[-\gamma\beta k_1^2 + (1-k_1)\sigma_u^2\right]. \tag{141}$$

We conjecture (based on Kyle model structure):

$$\beta = \frac{\sigma_{R_1}\sigma_u}{k_1\sigma_{R_2}}, \tag{142}$$

$$\lambda = \frac{\sigma_\theta^2 k_1}{2\sigma_u\sigma_{R_1}\sigma_{R_2}}. \tag{143}$$

Verifying $\beta = \frac{\sigma_\theta^2}{2(1+\sigma_\theta^2)\lambda}$:

$$\text{RHS} = \frac{\sigma_\theta^2}{2(1+\sigma_\theta^2)} \cdot \frac{2\sigma_u\sigma_{R_1}\sigma_{R_2}}{\sigma_\theta^2 k_1} \tag{144}$$

$$= \frac{\sigma_u\sigma_{R_1}\sigma_{R_2}}{(1+\sigma_\theta^2)k_1} = \frac{\sigma_u\sigma_{R_1}}{k_1\sigma_{R_2}} = \beta. \tag{145}$$

From (140):

$$\lambda = \frac{\sigma_\theta^2\beta k_1^2}{D} \quad \Rightarrow \quad D = \frac{\sigma_\theta^2\beta k_1^2}{\lambda} = 2\sigma_{R_1}^2\sigma_u^2. \tag{146}$$

From (141) with $\gamma = -\frac{\nu}{2\lambda}$:

$$\nu = \frac{\sigma_\theta^2}{D}\left[\frac{\nu\beta k_1^2}{2\lambda} + (1-k_1)\sigma_u^2\right]. \tag{147}$$

Rearranging:

$$\nu \left[1 - \frac{\sigma_\theta^2 \beta k_1^2}{2\lambda D}\right] = \frac{\sigma_\theta^2 (1-k_1)\sigma_u^2}{D}. \tag{148}$$

Computing the bracketed term using $D = 2\sigma_{R_1}^2 \sigma_u^2$:

$$\frac{\sigma_\theta^2 \beta k_1^2}{2\lambda D} = \frac{1}{2}. \tag{149}$$

Therefore:

$$\nu = \frac{2\sigma_\theta^2 (1-k_1)\sigma_u^2}{D} = \frac{2\sigma_\theta^2 (1-k_1)\sigma_u^2}{2\sigma_{R_1}^2 \sigma_u^2} = \frac{\sigma_\theta^2 (1-k_1)}{\sigma_{R_1}^2}. \tag{150}$$

From (119):

$$\gamma = -\frac{\nu}{2\lambda} = -\frac{\sigma_\theta^2 (1-k_1)/\sigma_{R_1}^2}{2 \cdot \sigma_\theta^2 k_1/(2\sigma_u \sigma_{R_1} \sigma_{R_2})} \tag{151}$$

$$= -\frac{(1-k_1)\sigma_u \sigma_{R_2}}{k_1 \sigma_{R_1}}. \tag{152}$$

The expected price conditional on effort is:

$$\mathbb{E}[p|A, \hat{A}] = \mu + \lambda \mathbb{E}[q|A, \hat{A}] + \nu \mathbb{E}[R_1|A, \hat{A}] \tag{153}$$

$$= \hat{A} + \bar{\theta} + (\lambda\gamma + \nu) k_1 (A - \hat{A}). \tag{154}$$

Therefore:

$$\lambda\gamma + \nu = -\frac{\sigma_\theta^2 (1-k_1)}{2\sigma_{R_1}^2} + \frac{\sigma_\theta^2 (1-k_1)}{\sigma_{R_1}^2} = \frac{\sigma_\theta^2 (1-k_1)}{2\sigma_{R_1}^2}. \tag{155}$$

So:

$$\mathbb{E}[p|A, \hat{A}] = \hat{A} + \bar{\theta} + \frac{\sigma_\theta^2 (1-k_1) k_1}{2\sigma_{R_1}^2} (A - \hat{A}). \tag{156}$$

For the speculator's expected profit:

$$\mathbb{E}[\text{Profit}] = \lambda \sigma_u^2 = \frac{\sigma_\theta^2 k_1}{2\sigma_u \sigma_{R_1} \sigma_{R_2}} \cdot \sigma_u^2 = \frac{k_1 \sigma_\theta^2 \sigma_u}{2\sigma_{R_1} \sigma_{R_2}}. \tag{157}$$

This completes the proof. ■

Proof of Lemma 5: We need to show that $\frac{\partial}{\partial k_1}\left[\frac{k_1}{\sigma_{R_1}}\right] > 0$.
Let $f(k_1) = \frac{k_1}{\sigma_{R_1}}$ where $\sigma_{R_1} = \sqrt{k_1^2 + (1-k_1)^2(1+\sigma_\theta^2)}$.

Taking the derivative:

$$f'(k_1) = \frac{\sigma_{R_1} - k_1 \cdot \frac{1}{2\sigma_{R_1}} \cdot \frac{\partial \sigma_{R_1}^2}{\partial k_1}}{\sigma_{R_1}^2} \tag{158}$$

$$= \frac{\sigma_{R_1}^2 - k_1 \cdot [k_1 - (1-k_1)(1+\sigma_\theta^2)]}{\sigma_{R_1}^3}. \tag{159}$$

The numerator simplifies to:

$$k_1^2 + (1-k_1)^2(1+\sigma_\theta^2) - k_1^2 + k_1(1-k_1)(1+\sigma_\theta^2) \tag{160}$$

$$= (1-k_1)(1+\sigma_\theta^2) > 0. \tag{161}$$

Therefore $f'(k_1) > 0$ for all $k_1 \in (0,1)$. ■

Proof of Proposition 9: The proof characterizes equilibrium by comparing the incumbent's profit across strategies.

We first define the thresholds used in the proposition.

From equation (140), $\Delta(k_1)$ is linear in σ_u:

$$\Delta(k_1) = \sigma_u \left[\frac{\sigma_\theta^2}{2\sqrt{1+\sigma_\theta^2}} - \frac{k_1 \sigma_\theta^2}{2\sigma_{R_1}\sigma_{R_2}} \right] \tag{162}$$

The entry condition is $\Delta(k_1) \geq C$. Rearranging:

$$\sigma_u \geq \frac{C}{\frac{\sigma_\theta^2}{2\sqrt{1+\sigma_\theta^2}} - \frac{k_1\sigma_\theta^2}{2\sigma_{R_1}\sigma_{R_2}}} \tag{163}$$

Define $\underline{\sigma}_u(k^*)$ as the value of σ_u where $\Delta(k^*) = C$:

$$\underline{\sigma}_u(k^*) = \frac{C}{\frac{\sigma_\theta^2}{2\sqrt{1+\sigma_\theta^2}} - \frac{k^*\sigma_\theta^2}{2\sigma_{R_1}(k^*)\sigma_{R_2}}} \tag{164}$$

For $\sigma_u < \underline{\sigma}_u(k^*)$, we have $\Delta(k^*) < C$, so entry is unprofitable even when the incumbent chooses the insider-optimal rating k^*.

Define $\bar{\sigma}_u$ as the value where the incumbent is indifferent between selling a public rating to the insider and a private rating to the speculator. This is the solution to:

$$V(k^*) = W(0) \tag{165}$$

where $V(k^*)$ is the insider's willingness to pay for the optimal public rating and $W(0)$ is the speculator's willingness to pay for a purely informational private rating.

Regime (i): Blockaded Entry ($\sigma_u < \underline{\sigma}_u(k^*)$)

Since $\Delta(k^*) < C$, entry is unprofitable even if the incumbent chooses k^*. The incumbent optimally chooses $k_1 = k^*$ to maximize insider willingness to pay. The second agency does not enter.

Regime (ii): Deterred Entry ($\underline{\sigma}_u(k^*) \leq \sigma_u < \bar{\sigma}_u$)

Here $\Delta(k^*) \geq C$, so entry would be profitable if the incumbent chose k^*. However, $V(k^*) > W(0)$ in this range, so the incumbent prefers to serve the insider rather than switch to serving the speculator.

The incumbent can deter entry by reducing k_1 to $\hat{k}_1 < k^*$ where $\Delta(\hat{k}_1) = C$. Since Δ is decreasing in k_1 (by Lemma 5), lowering k_1 makes entry less profitable.

The incumbent chooses $\hat{k}_1$ implicitly defined by:

$$\Delta(\hat{k}_1) = C \tag{166}$$

This is profitable for the incumbent as long as $V(\hat{k}_1) > W(0)$, which holds for $\sigma_u < \bar{\sigma}_u$.

Regime (iii): Accommodated Entry ($\sigma_u \geq \bar{\sigma}_u$)

In this range, $W(0) \geq V(k^*)$, so the incumbent prefers selling a private rating to the speculator over a public rating to the insider.

Deterring entry would require setting k_1 so low that $\Delta(k_1) < C$. But as $k_1 \to 0$, we have $V(k_1) \to 0$ while $W(0)$ remains positive. The incumbent finds it more profitable to switch to selling a private rating with $k_1 = 0$ to the speculator, earning $W(0)$.

The second agency then enters and sells a public rating with $k_2 > 0$ to the insider (assuming $V(k_2^*) > C$), earning $V(k_2^*) - C$. Both agencies operate. ■

References

Admati, A.R., and Pfleiderer, P., 1986. A monopolistic market for information. *Journal of Economic Theory*, 39(2), 400-438.

Admati, A.R., and Pfleiderer, P., 1988. Selling and trading on information in financial markets. *American Economic Review*, 78(2), 96-103.

Admati, A.R., Pfleiderer, P. and Zechner, J., 1994. Large shareholder activism, risk sharing, and financial market equilibrium. *Journal of Political Economy*, 102(6), 1097-1130.

Agrawal, A. and Hockerts, K., 2021. Impact investing: Review and research agenda. *Journal of Small Business & Entrepreneurship*, 33(2), 153-181.

Akerlof, G.A. and Kranton, R.E., 2005. Identity and the Economics of Organizations. *Journal of Economic Perspectives*, 19(1), 9-32.

Allen, F. and Gale, D., 2000. Financial contagion. *Journal of Political Economy*, 108(1), 1-33.

Azarmsa, E., Shapiro, J.D., 2023. The market for ESG ratings. *Journal of Finance*, forthcoming.

Banker, R.D. and Datar, S.M., 1989. Sensitivity, precision, and linear aggregation of signals for performance evaluation. *Journal of Accounting Research*, 27(1), 21-39.

Bar-Isaac, H. and Shapiro, J., 2013. Ratings quality over the business cycle. *Journal of Financial Economics*, 108(1), 62-78.

Becker, B. and Milbourn, T., 2011. How did increased competition affect credit ratings? *Journal of Financial Economics*, 101(3), 493-514.

Benmelech, E. and Dlugosz, J., 2010. The credit rating crisis. *NBER Macroeconomics Annual*, 24(1), 161-208.

Berg, F., Kölbel, J.F. and Rigobon, R., 2022. Aggregate confusion: The divergence of ESG ratings. *Review of Finance*, 26(6), 1315-1344.

Bergemann, D. and Morris, S., 2019. Information design: A unified perspective. *Journal of Economic Literature*, 57(1), 44-95.

Bernstein, S., Giroud, X. and Townsend, R.R., 2016. The impact of venture capital monitoring. *Journal of Finance*, 71(4), 1591-1622.

Bhide, A., 1993. The hidden costs of stock market liquidity. *Journal of Financial Economics*, 34(1), 31-51.

Bisceglia, M., Piccolo, A. and Schneemeier, J., 2026. Externalities of responsible investments. *Journal of Finance*, forthcoming.

Bolton, P. and Dewatripont, M., 2004. *Contract Theory*. MIT press.

Bolton, P., Freixas, X. and Shapiro, J., 2012. The credit ratings game. *Journal of Finance*, 67(1), 85-111.

Bouvard, M. and Levy, R., 2018. Two-sided reputation in certification markets. *Management Science*, 64(10), 4755-4774.

Broccardo, E., Hart, O. and Zingales, L., 2022. Exit versus voice. *Journal of Political Economy*, 130(12), 3101-3145.

Chemla, G. and Hennessy, C.A., 2014. Skin in the game and moral hazard. *Journal of Finance*, 69(4), 1597-1641.

Chen, S., Miao, B. and Shevlin, T., 2015. A new measure of disclosure quality: The level of disaggregation of accounting data in annual reports. *Journal of Accounting Research*, 53(5), 1017-1054.

Daley, B., Green, B. and Vanasco, V., 2020. Securitization, ratings, and credit supply. *Journal of Finance*, 75(2), 1037-1082.

Dasgupta, A. and Mathews, R.D., 2022. Delegated Blocks. Working paper, London School of Economics.

Datar, S., Kulp, S.C. and Lambert, R.A., 2001. Balancing performance measures. *Journal of Accounting Research*, 39(1), 75-92.

Diez-Busto, E., Sanchez-Ruiz, L. and Fernandez-Laviada, A., 2022. B Corp certification: Why? How? and What for? A questionnaire proposal. *Journal of Cleaner Production*, 372(20), 133801.

Doherty, N.A., Kartasheva, A.V. and Phillips, R.D., 2012. Information effect of entry into credit ratings market: The case of insurers' ratings. *Journal of Financial Economics*, 106(2), 308-330.

Dow, J. and Gorton, G., 1997. Stock market efficiency and economic efficiency: is there a connection? *Journal of Finance*, 52(3), 1087-1129.

Edmans, A., Levit, D. and Schneemeier, J., 2023. Socially responsible divestment. Working paper, London Business School.

Faure-Grimaud, A. and Gromb, D., 2004. Public trading and private incentives. *Review of Financial Studies*, 17(4), 985-1014.

Faure-Grimaud, A., Peyrache, E. and Quesada, L., 2009. The ownership of ratings. *RAND Journal of economics*, 40(2), 234-257.

Ferreira, D., Manso, G. and Silva, A.C., 2014. Incentives to innovate and the decision to go public or private. *Review of Financial Studies*, 27(1), 256-300.

Fischer, P. and Huddart, S., 2008. Optimal contracting with endogenous social norms. *American Economic Review*, 98(4), 1459-1475.

Foroughi, J., 2022. ESG Is Not Impact Investing and Impact Investing Is Not ESG. *Stanford Social Innovation Review*, Nov 10.

Fulghieri, P., Strobl, G. and Xia, H., 2014. The economics of solicited and unsolicited credit ratings. *Review of Financial Studies*, 27(2), 484-518.

Gao, M. and Huang, J., 2020. Informing the market: The effect of modern information technologies on information production. *Review of Financial Studies*, 33(4), 1367-1411.

Goel, A.M. and Thakor, A.V., 2015. Information reliability and welfare: A theory of coarse credit ratings. *Journal of Financial Economics*, 115(3), 541-557.

Gompers, P. and Lerner, J., 2001. The venture capital revolution. *Journal of Economic Perspectives*, 15(2), 145-168.

Griffin, J.M., Nickerson, J. and Tang, D.Y., 2013. Rating shopping or catering? An examination of the response to competitive pressure for CDO credit ratings. *Review of Financial Studies*, 26(9), 2270-2310.

Gryglewicz, S., Mayer, S. and Morellec, E., 2020. Agency conflicts and short-versus long-termism in corporate policies. *Journal of Financial Economics*, 136(3), 718-742.

Hart, O. and Moore, J., 1998. Default and renegotiation: A dynamic model of debt. *Quarterly Journal of Economics*, 113(1), 1-41.

Hayward, M.L. and Boeker, W., 1998. Power and conflicts of interest in professional firms: Evidence from investment banking. *Administrative Science Quarterly*, 43(1), 1-22.

Hellmann, T. and Puri, M., 2002. Venture capital and the professionalization of start-up firms: Empirical evidence. *Journal of Finance*, 57(1), 169-197.

Hoitash, R. and Hoitash, U., 2018. Measuring accounting reporting complexity with XBRL. *The Accounting Review*, 93(1), 259-287.

Holmström, B., 1999. Managerial incentive problems: a dynamic perspective. *Review of Economic Studies*, 66, 169-182.

Hörner, J. and Lambert, N.S., 2021. Motivational ratings. *Review of Economic Studies*, 88(4), 1892-1935.

Innes, R.D., 1990. Limited liability and incentive contracting with ex-ante action choices. *Journal of Economic Theory*, 52(1), 45-67.

Janeway, W.H., Nanda, R. and Rhodes-Kropf, M., 2021. Venture capital booms and start-up financing. *Annual Review of Financial Economics*, 13(1), 111-127.

Kaplan, S.N. and Strömberg, P., 2003. Financial contracting theory meets the real world: An empirical analysis of venture capital contracts. *Review of Economic Studies*, 70(2), 281-315.

Kashyap, A.K. and Kovrijnykh, N., 2016. Who should pay for credit ratings and how?. *Review of Financial Studies*, 29(2), 420-456.

Knechel, W.R., Krishnan, G.V., Pevzner, M., Shefchik, L.B. and Velury, U.K., 2013. Audit quality: Insights from the academic literature. *Auditing: A Journal of Practice & Theory*, 32(1), 385-421.

Kraft, P., 2015. Do rating agencies cater? Evidence from rating-based contracts. *Journal of Accounting and Economics*, 59(2-3), 264-283.

Kyle, A.S., 1985. Continuous auctions and insider trading. *Econometrica*, 53(6), 1315-1335.

Lerner, J. and Nanda, R., 2020. Venture capital's role in financing innovation: What we know and how much we still need to learn. *Journal of Economic Perspectives*, 34(3), 237-261.

Leung, E. and Veenman, D., 2018. Non-GAAP earnings disclosure in loss firms. *Journal of Accounting Research*, 56(4), 1083-1137.

Lizzeri, A., 1999. Information revelation and certification intermediaries. *RAND Journal of Economics*, 30(2), 214-231.

Marinovic, I., Varas, F., 2019. CEO Horizon, Optimal Pay Duration, and the Escalation of Short-Termism. *Journal of Finance*, 74, 2011-2053.

Martel, J., Van Wesep, E.D. and Van Wesep, R., 2022. Ratings and Cooperative Information Transmission. *Management Science*, 68(12), 9175-9197.

Mathis, J., McAndrews, J. and Rochet, J.C., 2009. Rating the raters: Are reputation concerns powerful enough to discipline rating agencies? *Journal of Monetary Economics*, 56(5), 657-674.

Martini, A., 2021. Socially responsible investing: from the ethical origins to the sustainable development framework of the European Union. *Environment, Development and Sustainability*, 23(11), 16874-16890.

Noordermeer, B. and Vorst, P., 2024. The Informativeness of Balance Sheet Disaggregations: Evidence from Forecasting Operating Assets. *Management Science*, forthcoming.

Olivier, J, Lovo, S., 2025. Who should pay for ESG ratings? Working paper, HEC Paris.

Oehmke, M. and Opp, M.M., 2025. A theory of socially responsible investment. *Review of Economic Studies*, 92(2), 1193-1225.

Opp, C.C., Opp, M.M. and Harris, M., 2013. Rating agencies in the face of regulation. *Journal of Financial Economics*, 108(1), 46-61.

Park, H.D. and Steensma, H.K., 2012. When does corporate venture capital add value for new ventures? *Strategic Management Journal*, 33(1), 1-22.

Paul, J.M., 1992. On the efficiency of stock-based compensation. *Review of Financial Studies*, 5(3), 471-502.

Peress, J., 2010. The tradeoff between risk sharing and information production in financial markets. *Journal of Economic Theory*, 145(1), 124-155.

Piccolo, A. and Shapiro, J., 2022. Credit ratings and market information. *Review of Financial Studies*, 35(10), 4425-4473.

Pollrich, M. and Strausz, R., 2024. The irrelevance of fee structures for certification. *American Economic Review: Insights*, 6(1), 55-72.

Saeedi, M. and Shourideh, A., 2024. Optimal rating design under moral hazard. Working paper, Carnegie Mellon University.

Sangiorgi, F. and Spatt, C., 2017. Opacity, credit rating shopping, and bias. *Management Science*, 63(12), 4016-4036.

Skreta, V. and Veldkamp, L., 2009. Ratings shopping and asset complexity: A theory of ratings inflation. *Journal of Monetary Economics*, 56(5), 678-695.

Stahl, K. and Strausz, R., 2017. Certification and market transparency. *Review of Economic Studies*, 84(4), 1842-1868.

Stein J. 1989. Efficient capital markets, inefficient firms: A model of myopic corporate behavior. *Quarterly Journal of Economics*, 104, 655-669.

Strebulaev, I.A., and B. Yang., 2013. The Mystery of Zero-Leverage Firms. *Journal of Financial Economics*, 109, 1-23.

Thakor, R.T., 2021. Short-termism, Managerial Talent, and Firm Value. *Review of Corporate Finance Studies*, 10(3), 473-512

Townsend, R.M., 1979. Optimal contracts and competitive markets with costly state verification. *Journal of Economic Theory*, 21(2), 265-293.

Verrecchia R.E. 1982. The use of mathematical models in financial accounting. *Journal of Accounting Research*, 20, 1-42.

Von Thadden E. 1995. Long-term contracts, short-term investment and monitoring. *Review of Economic Studies*, 62, 557-575.

Xia, H., 2014. Can investor-paid credit rating agencies improve the information quality of issuer-paid rating agencies? *Journal of Financial Economics*, 111(2), 450-468.

Online Appendix

Proof of Corollary 3:

The speculator solves:

$$\begin{aligned} & \max_q \ \mathbb{E}\left[A + (\tilde{\theta} - p(\tilde{y}))q \middle| R\right] \\ \Leftrightarrow \qquad & \max_q \ \mathbb{E}[A|R] + \left(\mathbb{E}[\tilde{\theta}|R] - (\mu + \lambda(1+q))\right) q \end{aligned} \tag{167}$$

The second-order condition for a maximum is satisfied, so the optimum $q^*(R)$ is described by the first-order necessary condition:

$$q^*(R) = \frac{\mathbb{E}\left[\tilde{\theta}\middle|R\right] - (\mu + \lambda)}{2\lambda} \tag{168}$$

We have:

$$\begin{aligned} \mathbb{E}\left[\tilde{\theta}\middle|R\right] &= \bar{\theta} + \frac{cov(\tilde{X}, \tilde{R})}{var(\tilde{R})}\left(R - k\hat{A} - (1-k)\bar{\theta}\right) \\ &= \bar{\theta} + \frac{(1-k)\sigma_\theta^2}{\sigma_R^2}\left(R - k\hat{A} - (1-k)\bar{\theta}\right) \end{aligned} \tag{169}$$

Denoting $b_I \equiv \frac{(1-k)\sigma_\theta^2}{\sigma_R^2}$, we have:

$$q^*(R) = \frac{\bar{\theta} - (\mu + \lambda)}{2\lambda} + \frac{b_I\left(R - k\hat{A} - (1-k)\bar{\theta}\right)}{2\lambda} \tag{170}$$

So we get

$$\beta = \frac{b_I}{2\lambda} \tag{171}$$

$$\alpha = \frac{\bar{\theta} - (\mu + \lambda)}{2\lambda} - \frac{b_I\left(k\hat{A} + (1-k)\bar{\theta}\right)}{2\lambda} \tag{172}$$

The price set by the market maker corresponds to the expected value of the firm given the total order flow observed, including an order $\alpha + \beta R$ from the speculator and an order u from

noise traders.

$$\begin{aligned} p(y) &= \mathbb{E}\left[\tilde{\theta}\middle|y\right] \\ &= \mathbb{E}\left[\tilde{\theta}\right] + \frac{cov(\tilde{\theta},\tilde{y})}{Var(\tilde{y})}\left(y - \mathbb{E}[\tilde{y}]\right) \\ &= \bar{\theta} + \frac{\beta\left(1-k\right)\sigma_\theta^2}{\beta^2(\sigma_R^2)+\sigma_u^2}\left(y - \mathbb{E}[\tilde{y}]\right) \end{aligned} \tag{173}$$

In particular:

$$\begin{aligned} p\left(\mathbb{E}[\tilde{y}]\right) &= \bar{\theta} \\ &= \mu + \lambda\left(1 + \alpha + \beta\left(k\hat{A} + \left(1-k\right)\bar{\theta}\right)\right) \end{aligned} \tag{174}$$

We get:

$$\begin{aligned} \lambda &= \frac{\beta\left(1-k\right)\sigma_\theta^2}{\beta^2(\sigma_R^2)+\sigma_u^2} \\ \mu &= \bar{\theta} - \lambda\left(1 + \alpha + \beta\left(k\hat{A} + \left(1-k\right)\bar{\theta}\right)\right. \end{aligned}$$

Solving for α, β, λ and μ yields:

$$\beta = \frac{\sigma_u}{\sigma_R} \tag{175}$$

$$\lambda = \frac{\left(1-k\right)\sigma_\theta^2}{2\sigma_u\sigma_R} \tag{176}$$

$$\alpha = \beta\mathbb{E}[\tilde{R}] = \beta\left(k\hat{A} + \left(1-k\right)\bar{\theta}\right) \tag{177}$$

$$\mu = \mathbb{E}[\tilde{\theta}] - \lambda = \bar{\theta} - \lambda \tag{178}$$

The expected stock price as a function of the expected effort of the insider, $\hat{A}$, and his actual effort, A, is:

$$\begin{aligned} \mathbb{E}\left[p\middle|A,\hat{A}\right] &= \mathbb{E}\left[\mu + \lambda y\right] \\ &= \mu + \lambda + \lambda\beta\mathbb{E}\left[R - \mathbb{E}[\tilde{R}]\right] \\ &= \bar{\theta} + \lambda\beta k\left(A - \hat{A}\right) \\ &= \bar{\theta} + \frac{1}{2}\left(1-k\right)k\frac{\sigma_\theta^2}{\sigma_R^2}\left(A - \hat{A}\right) \end{aligned}$$

From equations (168) and (178), we have:

$$\mathbb{E}[q^*(R)] \;=\; \mathbb{E}\left[\frac{\mathbb{E}\left[\tilde{\theta}\middle|R\right]-\bar{\theta}}{2\lambda}\right]=0 \tag{179}$$

Using equation (169), we also have:

$$\mathbb{E}\left[\tilde{\theta}\middle|R\right] \;=\; \bar{\theta}+\frac{(1-k)\,\sigma_\theta^2}{\sigma_R^2}\left(R-k\hat{A}-(1-k)\,\bar{\theta}\right) \tag{180}$$

The speculator's expected utility is now equal to:

$$\underbrace{\frac{\sigma_u\,(1-k)\,\sigma_\theta^2}{2\sigma_R}}_{f(k)}+\underbrace{A}_{g(k)} \tag{181}$$

The objective function of the speculator is additive in two functions: $f(k)$ and $g(k)$.

When $q_0=0$, the rating weight that maximizes the speculator's objective function is simply the one that maximizes $f(k)$. It is $k^{**}=0$. Indeed, for $k>0$, $f'(k)<0$.

When $\sigma_u=0$, the rating weight that maximizes the speculator's objective function is simply the one that maximizes $g(k)$, which is equivalent to maximizing the equilibrium effort. It is:

$$k=k^*\equiv\frac{1+\sigma_\theta^2-\sqrt{1+\sigma_\theta^2}}{\sigma_\theta^2}$$

and $g'(k)<0$ for $k<k^*$.

By the intermediate value theorem, the rating weight that maximizes $f(k)+g(k)$ is in-between k^{**} and k^*. Thus, the rating weight that maximizes the speculator's willingness to pay is strictly positive. ■