Optimal Asset Market Operations

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Abstract

We characterize governments' optimal responses to asset market disturbances across a broad class of models with financial frictions. We show that the Ramsey plan can be achieved by a policy rule targeting a specific relationship between asset returns, regardless of the underlying disturbances. This relationship is determined by asset supply and demand elasticities that can be estimated empirically with standard identification strategies. Absent financial frictions, the optimal policy stabilizes spreads across all assets. However, in the presence of financial frictions, the optimal rule prescribes time-varying spreads to facilitate financial intermediation. We apply our framework to compare optimal policy to historical outcomes, finding significant deviations between the two, particularly during the 2008–2009 financial crisis.

Keywords: monetary policy, financial frictions, asset demand estimation, sufficient statistics
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1 Introduction

How should governments respond to disturbances in asset markets? Since the Great Financial Crisis, governments have deployed increasingly sophisticated "asset market operations" beyond traditional monetary policy, including Quantitative Easing, Operation Twist, and the Troubled Asset Relief Program. However, to design these policies, existing macroeconomic models mostly rely on detailed microfoundations of financial frictions and focus on specific shocks to the economy. Policy prescriptions from such analysis inevitably depend on particular structural assumptions, raising questions about their broader applicability. Since deep parameters in these models are difficult to measure and shocks hard to observe, implementing their prescriptions proves challenging in practice. In this paper, we aim to provide a policy framework to overcome these challenges.

We study a broad class of models with financial frictions in which the government can trade assets in response to arbitrary shocks to the economy. We show that the Ramsey plan can be achieved (up to first order) by maintaining a target relationship between asset returns. This relationship depends on agents' asset supply and demand elasticities. These elasticities are sufficient statistics (Chetty, 2009; Kleven, 2021) that capture the invariant features of the economy: preferences, technology, and financial frictions. In equilibrium, asset returns fluctuate to reflect underlying economic disturbances under the target relationship. Crucially, the target relationship links observable asset returns with measurable elasticities and does not depend directly on underlying disturbances. Policymakers can therefore implement optimal policy by trading assets according to the target relationship without observing the specific shocks driving market conditions.

Our framework features a production economy with a general form of financial intermediation. A representative household has preferences over various forms of financial assets, while the financial sector issues (or holds) these financial assets and holds claims on physical capital. Financial frictions are described by a generic mapping between expected returns and the issuance (or holdings) of financial assets, similar to Chiang and Zoch (2023). This "financial block" allows for frictions arising from agency problems (Bernanke et al., 1999; Gertler and Karadi, 2011), interbank search and bargaining (Afonso and Lagos, 2015; Bianchi and Bigio, 2022), and heteroge-

neous intermediaries with complex interactions, as in Bellifemine et al. (2022). We use the sequence space approach (Auclert et al., 2023, 2025; McKay and Wolf, 2022; Dávila and Schaab, 2023) to abstract from these complications, and instead focus on analyzing how financial frictions shape the government's optimal policy design in general equilibrium.

We study a one-time unexpected shock that shifts preferences, technology, and financial frictions in arbitrary ways. The government can implement long-run macroprudential policy in the form of taxes on assets, but can only respond to shocks by trading financial assets, performing asset market operations. This captures the realistic constraint that policymakers often cannot rapidly adjust regulatory frameworks in response to unexpected events, unlike the state-contingent taxes in Farhi and Werning (2016) and Dávila and Korinek (2018), and instead rely on direct market interventions. We solve a Ramsey problem to characterize the government's optimal asset market operations in response to shocks around the steady state, providing a theory that complements the literature's focus on macroprudential taxes.

We show that the Ramsey plan can be achieved by a target rule under which the government trades assets so that returns on financial assets equal weighted averages of returns on capital and net worth, with weights determined by agents' asset supply and demand elasticities. The government implements this by committing to supply assets perfectly elastically at target returns, allowing quantities to adjust endogenously while maintaining the optimal relationship between returns. Under the rule, returns on capital and net worth adjust to incorporate information about the underlying disturbances, while returns on financial assets adjust accordingly to address externalities from financial frictions.

Our approach combines the design of policy rules from the New Keynesian tradition (Giannoni and Woodford, 2003) with insights from the sequence space approach, formulating the Ramsey problem as a planner facing an asset supply and demand system, similar to Auclert et al. (2024). In contrast to New Keynesian policy rules, where targets such as output gaps can be difficult to observe, our target rule specifies a relationship among readily observable asset returns, with weights determined by measurable elasticities, sufficient statistics that capture underlying preferences, technology, and financial frictions.

We illustrate how financial frictions shape optimal policy through a series of examples. In our framework, the Ramsey planner faces a key trade-off between satiating household asset demand and addressing pecuniary externalities: due to financial frictions, the planner can create welfare gains from influencing asset returns and financial intermediation (Lorenzoni, 2008). Without financial frictions, we show that the planner equalizes returns on all assets, implementing a version of the Friedman rule. However, financial frictions motivate the planner to deviate from this rule: When returns on capital are high, the planner lowers funding costs to maintain an optimal level of intermediation. Hence, the optimal policy rule prescribes time-varying spreads rather than equalizing returns as in a neoclassical benchmark.

We show how this mechanism alters the optimal target rule under various frictions, including those arising from costly state verification and asset diversion problems. The same intuition carries over to the state-of-the-art quantitative model of Bianchi and Bigio (2022), where search frictions in the interbank market shape portfolio choices across deposits, reserves, and government bonds. This emphasis on financial frictions distinguishes our work from studies emphasizing fiscal considerations for the optimal debt structures, including Angeletos (2002), Buera and Nicolini (2004), Aparisi de Lannoy et al. (2022), and Angeletos et al. (2023).

We demonstrate how to estimate key elasticities entering the optimal target rule directly from data using reduced-form methods from the asset demand literature (Gabaix and Koijen, 2021, 2024; Chaudhary et al., 2025). We focus on the U.S. banking sector and use balance sheets from the Call Reports to construct holdings across three asset categories: liquid assets (e.g., reserves, deposits, Treasury securities), illiquid assets (e.g., corporate bonds, agency securities), and capital (e.g., loans, goodwill). We apply the Granular Instrumental Variables (GIV) approach to isolate idiosyncratic shocks and construct instruments that allow us to identify the banking sector's asset supply and demand elasticities.

Building on our empirical estimates, we compute the optimal target rule that implements the Ramsey plan. The rule prescribes that when expected returns on capital are high, the government should target lower returns on liquid assets while simultaneously targeting higher returns on illiquid assets. To assess the practical relevance of our framework, we compare returns implied by the optimal rule to historically observed returns. Using observed returns and government asset positions together with

our estimated elasticities, we back out the implied asset market disturbances and compute the counterfactual returns that would have prevailed under optimal policy. The comparison reveals systematic deviations between actual and optimal outcomes, particularly during the 2008–2009 financial crisis. The pattern of disturbances during this period is suggestive of a "flight to liquidity," with excess demand for safe, liquid assets accompanied by excess supply of illiquid assets and capital. Under observed policy, liquid rates fell and illiquid rates rose by more than the optimal rule would prescribe, suggesting scope for more aggressive issuance of liquid liabilities and purchases of illiquid securities.

Finally, our work complements recent literature on monetary transmission through financial markets, including Piazzesi et al. (2019), Vayanos and Vila (2021), Pflueger and Rinaldi (2022), Kekre et al. (2024), and Caballero et al. (2024). While these works provide insights into specific transmission mechanisms, such as nominal rigidities and time-varying risk premia, their structural approach ties policy prescriptions to particular modeling assumptions and shock processes. Although our framework does not incorporate all the mechanisms in these models, our work is a first step toward providing policy guidance that is less dependent on specific structural assumptions.

2 Model

2.1 Setup

Time is discrete, with $t \in \{0, \ldots, \infty\}$. The economy comprises a representative household, a financial sector, and the government. There are three classes of assets: productive assets, financial assets, and the net worth of the financial sector. Productive assets consist of the value of physical capital, which can only be held by the financial sector, yielding returns R_t^A. Financial assets, denoted by set $\mathcal{B}$, are in zero net supply. They can be traded by all agents and generate returns $\boldsymbol{R}_t^B$. The net worth of the financial sector can be held only by the household, and its returns are denoted by R_t^N.

We study how the government can best respond to a one-time unexpected shock at time 0, $\{\boldsymbol{\omega}_t\}$, which is a sequence of aggregate states that shifts preferences, technol-

ogy, and the financial sector's ability to intermediate assets. With probability one, $\boldsymbol{\omega}_t = \bar{\boldsymbol{\omega}}$ for all t; this history defines the baseline on which macroprudential taxes are chosen. With probability zero, the economy experiences an unexpected shock at $t = 0$ that places $\boldsymbol{\omega}_t$ on a known deterministic path:

$$\boldsymbol{\omega}_t = \bar{\boldsymbol{\omega}} + \hat{\boldsymbol{\omega}}_t, \quad \forall t \geq 0.$$

We study the government's optimal asset market operations in this probability-zero event. Along this path, macroprudential taxes remain fixed at their baseline values. Throughout the paper, we use the convention that a generic variable $\boldsymbol{x}_t$ is a column vector and braces $\{x_s\}$ denote sequences from $s = 0, \ldots, \infty$. We use $\mathbf{0}_{\mathcal{B}}$ and $\mathbf{1}_{\mathcal{B}}$ to represent vectors of zeros and ones of dimension $|\mathcal{B}|$ (the number of financial assets).

Household

The representative household derives utility from consumption and holdings of financial assets, $\{c_t, \boldsymbol{b}_t\}$, and takes the financial sector's net worth n_t as given. The household solves the following maximization problem:

$$\max_{\{\boldsymbol{b}_t, c_t\}_{t=0}^{\infty}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t u\left(\{c_s, \boldsymbol{b}_s\}_{s \leq t}; \boldsymbol{\omega}_t\right) \tag{1}$$

subject to budget constraints

$$c_t + \mathbf{1}_{\mathcal{B}}^{\intercal} \boldsymbol{b}_t + n_t + \Psi\left(\{\boldsymbol{b}_s\}_{s \leq t}; \boldsymbol{\omega}_t\right) = w_t + (\boldsymbol{R}_t^B - \boldsymbol{\tau}_t^B)^{\intercal} \boldsymbol{b}_{t-1} + (R_t^N - \tau_t^N) n_{t-1} - T_t. \tag{2}$$

The household supplies one unit of labor inelastically for income w_t and pays lump-sum taxes T_t. Asset holdings are taxed at rate $\boldsymbol{\tau}_t^B$ and τ_t^N, and they can potentially incur a portfolio adjustment cost $\Psi\left(\{\boldsymbol{b}_s\}_{s \leq t}; \boldsymbol{\omega}_t\right)$. We allow the aggregate state $\boldsymbol{\omega}_t$ to shift the household's preferences and costs of holding assets. These shifts can generate "asset demand shocks" such as a flight to liquidity, which allows us to study policy responses to those scenarios. We use $\mathbb{E}$ to denote expectation before the realization of $\boldsymbol{\omega}_t$.

We denote by λ_t the Lagrange multiplier on (2).

Although we allow quite arbitrary preferences and technology in holding assets, we

maintain an assumption that there exists an asset (or a set of assets) that allows us to "span" the household's intertemporal valuation:

Assumption 1 *There exists a function $\boldsymbol{\ell}(\cdot)$ such that $R^*_{t+1} = \boldsymbol{\ell}(\boldsymbol{R}^B_{t+1}, R^N_{t+1})$, and*

$$\lambda_t = \beta \lambda_{t+1} R^*_{t+1},$$

where $\{\lambda_t\}$ are the Lagrange multipliers on the household's budget constraint.

Intuitively, the assumption holds when a "standard" Euler equation exists for the household with respect to some "neutral rate" R^*_{t+1}. This can occur if holding a particular asset (say, B_0) incurs no utility or portfolio adjustment costs. In this case, we have $R^*_{t+1} = \boldsymbol{R}^{B_0}_{t+1}$. Similarly, if holding equity is costless, then $R^*_{t+1} = \boldsymbol{R}^N_{t+1}$. Even when no single asset satisfies a standard Euler equation, the neutral rate may exist for a "synthetic" asset with return $R^*_{t+1} = \boldsymbol{\alpha}^\intercal \boldsymbol{R}^B_{t+1}$ for some portfolio weight $\boldsymbol{\alpha}$, as we demonstrate in Appendix B.3.

Another important class of models that satisfy this assumption trivially is when the household has quasi-linear preferences:

$$u\left(\{c_s, \boldsymbol{b}_s\}_{s \leq t}; \boldsymbol{\omega}_t\right) = c_t + \nu(\boldsymbol{b}_t; \boldsymbol{\omega}_t). \tag{3}$$

This class of models is common in the money search frameworks, and it is a useful simplification in models of financial frictions, as in Bianchi and Bigio (2022). In this case, $\lambda_t \equiv 1$ and $R^*_{t+1} = \beta^{-1}$, which satisfies Assumption 1.

Production

Production of output uses labor h_t and capital k_{t-1} subject to a constant-return-to-scale production function $F(h_t, k_{t-1}; \boldsymbol{\omega}_t)$. The aggregate state $\boldsymbol{\omega}_t$ shifts the production of output, allowing disturbances such as TFP shocks. A representative firm chooses labor input and capital rental to maximize profit, taking wage w_t and the rental rate of capital ϱ_t as given:

$$\max_{h_t, k_{t-1}} F(h_t, k_{t-1}; \boldsymbol{\omega}_t) - w_t h_t - \varrho_t k_{t-1}. \tag{4}$$

The accumulation of capital is governed by a capital production function $\Phi(\cdot)$:

$$k_t = \Phi(\iota_t; \boldsymbol{\omega}_t) k_{t-1}, \tag{5}$$

where ι_t is the investment rate per unit of capital and the aggregate state $\boldsymbol{\omega}_t$ shifts capital production, allowing for disturbances such as capital destruction shocks. Holding capital from period $t-1$ to period t is taxed at rate τ_t^A and generates returns R_t^A:

$$R_{t+1}^A + \tau_{t+1}^A \coloneqq \frac{1}{q_t}\Big[\varrho_{t+1} + \max_{\iota_{t+1}} \big\{q_{t+1}\Phi(\iota_{t+1}; \boldsymbol{\omega}_{t+1}) - \iota_{t+1}\big\}\Big]. \tag{6}$$

We denote the value of capital in period t by

$$a_t^K \coloneqq q_t k_t, \tag{7}$$

which comprises the supply of productive assets in the economy. Tax τ_t^A shifts the supply of capital in the economy through the gross return on capital. From CRS and labor market clearing ($h_t \equiv 1$), wages are determined as $w_t = w_t(\{R_{s+1}^A + \tau_{s+1}^A; \boldsymbol{\omega}_s\})$, and the capital supply is

$$a_t^K = a_t^K(\{R_{s+1}^A + \tau_{s+1}^A; \boldsymbol{\omega}_s\}). \tag{8}$$

Note that production-side objects depend on R_{t+1}^A and τ_{t+1}^A only through the sum $R_{t+1}^A + \tau_{t+1}^A$.

The Financial Sector

The financial sector holds productive assets, a_t^F, and can issue or hold financial assets, $\boldsymbol{b}_t^F$. Positive entries of $\boldsymbol{b}_t^F$ indicate issuance and constitute liabilities (e.g., deposits issued by banks), while negative entries correspond to assets held by the financial sector (e.g., Treasury bills or reserves at the central bank). Given the equity of the financial sector, n_t, the balance sheet is given by:

$$a_t^F = \mathbf{1}_{\mathcal{B}}^\intercal \boldsymbol{b}_t^F + n_t. \tag{9}$$

Frictions in the financial sector are described by how its asset holdings and issuance depend on returns $\{R^A_{s+1}, \boldsymbol{R}^B_{s+1}\}$ per unit of equity:

$$a^F_t = \theta_t n_t, \quad \boldsymbol{b}^F_t = \boldsymbol{\phi}_t n_t, \tag{10}$$

where

$$\boldsymbol{\phi}_t \coloneqq \boldsymbol{\phi}(\{R^A_{s+1}, \boldsymbol{R}^B_{s+1}\}, \boldsymbol{\omega}_t)$$

is a function of returns sequences. The financial sector balance sheet implies:

$$\theta_t = \mathbf{1}^\intercal_{\mathcal{B}} \boldsymbol{\phi}_t + 1. \tag{11}$$

The function $\boldsymbol{\phi}_t$ captures how the financial sector's ability to intermediate and supply assets depend on expected returns in any equilibrium. As we discuss in Section 2.3, this formulation of the financial sector allows us to nest a large class of financial intermediation models. The aggregate state $\boldsymbol{\omega}_t$ represents disturbances affecting the financial sector's ability to supply different assets.

Financial sector equity n_t represents the residual claim on holdings on its balance sheet, and the return on equity R^N_t satisfies:

$$R^N_t = R^A_t \theta_{t-1} - \boldsymbol{R}^{B\intercal}_t \boldsymbol{\phi}_{t-1}. \tag{12}$$

Exogenous net worth. We assume that net worth is given by an exogenous reduced-form process with the same argument domain as the household block:

$$n_t = n_t(\{R^A_{s+1}, \boldsymbol{R}^B_{s+1}, \tau^A_{s+1}, \boldsymbol{\tau}^B_{s+1}, \tau^N_{s+1}, T_s; \boldsymbol{\omega}_s\}). \tag{13}$$

This formulation nests a large class of financial intermediation models, including Gertler and Karadi (2011); Gertler and Kiyotaki (2010).

Government

The government can issue or hold financial assets with positions $\boldsymbol{b}^G_t$, where a positive entry stands for issuance and a negative entry for holdings. The total resource the government receives from the (net) issuance of assets is $\mathbf{1}^\intercal_{\mathcal{B}} \boldsymbol{b}^G_t$. The government budget

constraint is:

$$\mathbf{1}_{\mathcal{B}}^{\intercal}\boldsymbol{b}_t^G = \boldsymbol{R}_t^{B\intercal}\boldsymbol{b}_{t-1}^G - \tau_t^A a_{t-1}^K - \boldsymbol{\tau}_t^{B\intercal}\boldsymbol{b}_{t-1} - \tau_t^N n_{t-1} - T_t. \tag{14}$$

We assume that the government can operate in the asset markets by adjusting $\boldsymbol{b}_t^G$ contingent on the realizations of $\{\hat{\boldsymbol{\omega}}_t\}$. These operations include traditional tools, like open market operations, and unconventional tools such as quantitative easing and tightening, the Troubled Asset Relief Program, and Operation Twist. We allow the government to adjust T_t contingent on $\{\hat{\boldsymbol{\omega}}_t\}$.

Taxes $\tau_t^A, \boldsymbol{\tau}_t^B, \tau_t^N$ act as macroprudential policies: they allow the government to influence the incentive to issue and hold assets, thereby confronting frictions in the financial sector. We restrict these taxes to be non-state-contingent, reflecting the fact that macroprudential regulations typically require legislative or regulatory approval and cannot be adjusted swiftly in response to unexpected shocks.[1]

Definition of Equilibrium

Definition 1 *An equilibrium consists of allocations* $\{c_t, \boldsymbol{b}_t, n_t, h_t, k_t, \iota_t, a_t^K, a_t^F, \boldsymbol{b}_t^F\}$, *prices* $\{w_t, \varrho_t, q_t\}$, *returns* $\{R_t^A, \boldsymbol{R}_t^B, R_t^N\}$, *and policies* $\{\boldsymbol{b}_t^G, T_t, \tau_t^A, \boldsymbol{\tau}_t^B, \tau_t^N\}$ *such that:* $\{c_t, \boldsymbol{b}_t\}$ *solves the households' problem* (1) *given returns and policies;* $\{h_t, k_t, \iota_t\}$ *solves the firm's problem* (4) *given wage, rental rate, and capital price; supply of productive asset is given by* (7)*; the financial sector's supply and demand of assets,* $\{a_t^F, \boldsymbol{b}_t^F\}$*, is given by* (10)*; net worth is given by* (13)*; the labor and capital rental market clear; returns on productive assets and equity are* (6) *and* (12)*; asset markets clear:*

$$a_t^K = a_t^F, \quad \boldsymbol{b}_t^G + \boldsymbol{b}_t^F = \boldsymbol{b}_t; \tag{15}$$

and the government budget constraint (14) *holds. The goods market clears as a result of Walras' law.*

[1] This restriction is analogous to that on product market subsidies in the New Keynesian framework: those subsidies correct monopolistic distortions in the steady state but cannot respond to shocks, thus making room for monetary policy.

2.2 A Sequence Space Representation

We represent the private sector's consumption, saving, and production behavior as an intertemporal system of supply and demand for assets. This representation is useful because, given the system, any competitive equilibrium is characterized by prices and policies that clear markets and satisfy government budget constraints. As a result, this representation builds a connection between a Ramsey planner's constraints and the properties of the supply and demand system that can be measured empirically.

Lemma 1 *(i) Given* $\{R^A_{s+1}, \tau^A_{s+1}; \boldsymbol{\omega}_s\}$*, there exist functions* $w_t(\cdot), \varrho_t(\cdot), k_t(\cdot)$*, and* $q_t(\cdot)$ *that satisfy production optimality and clear factor markets. The capital supply is given by*

$$a^K_t(\{R^A_{s+1}, \tau^A_{s+1}; \boldsymbol{\omega}_s\}) \coloneqq q_t(\{R^A_{s+1}, \tau^A_{s+1}; \boldsymbol{\omega}_s\})\ k_t(\{R^A_{s+1}, \tau^A_{s+1}; \boldsymbol{\omega}_s\}).$$

(ii) Given returns and taxes $\{R^A_{s+1}, \boldsymbol{R}^B_{s+1}, \tau^A_{s+1}, \boldsymbol{\tau}^B_{s+1}, \tau^N_{s+1}, T_s; \boldsymbol{\omega}_s\}$*, there exists an asset demand function*

$$\boldsymbol{b}_t(\{R^A_{s+1}, \boldsymbol{R}^B_{s+1}, \tau^A_{s+1}, \boldsymbol{\tau}^B_{s+1}, \tau^N_{s+1}, T_s; \boldsymbol{\omega}_s\})$$

that solves the household's problem with welfare

$$v(\{R^A_{s+1}, \boldsymbol{R}^B_{s+1}, \tau^A_{s+1}, \boldsymbol{\tau}^B_{s+1}, \tau^N_{s+1}, T_s; \boldsymbol{\omega}_s\}).$$

These functions allow us to express production and household decisions as functions of returns and taxes. Taken together with the reduced-form financial-sector objects governing intermediation, they summarize the private sector's equilibrium responses and allow us to characterize an equilibrium with the *excess asset supply* $\boldsymbol{\epsilon}_t$ and the *excess capital supply* κ_t:

$$\boldsymbol{\epsilon}_t(\cdot) \coloneqq \boldsymbol{b}^G_t + \boldsymbol{\phi}_t(\cdot) n_t(\cdot) - \boldsymbol{b}_t(\cdot), \quad \kappa_t(\cdot) \coloneqq a^K_t(\cdot) - \theta_t(\cdot) n_t(\cdot). \tag{16}$$

Implementability

Given these excess asset supply functions, an equilibrium requires that

$$\boldsymbol{\epsilon}_t = \mathbf{0}_{\mathcal{B}}, \quad \kappa_t = 0,$$

and that the government budget holds. These conditions form the constraints faced by a Ramsey planner when choosing policies: the market-clearing conditions correspond

to the implementability constraint, and the government budget constraint replaces the feasibility constraint.

Our characterization of the planner's constraints as a requirement to clear an intertemporal asset supply and demand is similar to Auclert et al. (2024) in the context of a heterogeneous-agent model, where the complexity of the economy can be succinctly summarized by aggregate functions. In our context, the same idea allows us to represent a wide variety of financial frictions with potentially complex structures, extending the result in Chiang and Zoch (2023). We provide a few examples in the following section.

2.3 Nested Models of Financial Intermediation

We now illustrate how our sequence space representation can accommodate a wide variety of financial frictions. This flexibility illustrates the key advantage of our approach: complex financial intermediation structures can be summarized by reduced-form financial-sector objects, allowing us to remain agnostic about specific microfoundations while capturing their economic interaction with the aggregate economy. The exact microfoundation becomes irrelevant for policy analysis as long as we can measure properties of the asset supply and demand system.

For canonical macroeconomic models in which financial intermediaries issue only one asset ($|\mathcal{B}| = 1$), Chiang and Zoch (2023) show that the formulation contains intermediation frictions originating from asset diversion (Gertler and Karadi, 2011), costly state verification (Bernanke et al., 1999), costly leverage (Uribe and Yue, 2006), and collateral constraints similar to Kiyotaki and Moore (1997) and Bianchi and Mendoza (2018).

This paper extends the framework to allow for an arbitrary number of financial assets $\mathcal{B}$. The extension provides a general policy framework to study how the government can best operate in various asset markets. In Appendix C, we show that the formulation allows for key frictions in recent frameworks developed to study these policies. For example, by specializing to $|\mathcal{B}| = 3$, our formulation of the financial sector nests intermediation frictions in Sims and Wu (2021), Bianchi and Bigio (2022), Piazzesi et al. (2019), where the assets in $\mathcal{B}$ contain reserves, deposits, and government bonds. Similarly, by specializing to $|\mathcal{B}| = 2$, our framework nests the type of financial in-

termediation studied in Gertler et al. (2012) where assets in $\mathcal{B}$ contain deposits and outside equity.

One common feature of the frameworks above is that financial intermediaries do not feature persistent heterogeneity, and, as a result, the distribution of intermediary net worth does not have aggregate effects. However, our representation of the financial sector is not restricted by this property: the reduced-form financial-sector objects can also accommodate cases where heterogeneity within the financial sector does matter, as emphasized by Bellifemine et al. (2022). In Appendix C.4, we provide an example where financial intermediaries have heterogeneous abilities to hold and issue assets, and where the net worth distribution affects the aggregate financial sector response. Nevertheless, we can still derive reduced-form financial-sector objects that encapsulate the complex and heterogeneous frictions across different financial intermediaries. To the extent that the effects of heterogeneity are summarized by these reduced-form objects, our policy analysis remains unchanged regardless of the underlying heterogeneity.

3 The Ramsey Problem

We consider the policy problem of the government under full commitment. The government is assumed to have the same objective function as the household, and it sets policies to choose an equilibrium that maximizes the value of the objective function. We use the intertemporal asset supply and demand system in Section 2.2 to formulate the Ramsey problem. As we show in Section 4, this formulation allows us to characterize the solution to the planner's problem in terms of observable sufficient statistics.

3.1 The Planner's Problem

Under the history without any shock, the planner solves the Ramsey problem and commits to macroprudential taxes $\{\tau_t^A, \boldsymbol{\tau}_t^B, \tau_t^N\}$. In the probability-zero event in which the shock occurs, these taxes remain fixed at their baseline values; asset market operations $\boldsymbol{b}_t^G$ and lump-sum taxes T_t adjust in response to the realized shock path $\{\hat{\boldsymbol{\omega}}_s\}_{s\geq 0}$.

Acting as a Ramsey planner, the government solves the following optimization problem:[2]

$$\min_{\boldsymbol{\gamma}_t,\eta_t,\xi_t}\ \max_{R^A_{t+1},\boldsymbol{R}^B_{t+1},\tau^A_{t+1},\boldsymbol{\tau}^B_{t+1},\tau^N_{t+1},\boldsymbol{b}^G_t,T_t} v(\{R^A_{s+1},\boldsymbol{R}^B_{s+1},\tau^A_{s+1},\boldsymbol{\tau}^B_{s+1},\tau^N_{s+1},T_s;\boldsymbol{\omega}_s\}) \tag{17}$$

$$+\mathbb{E}\sum_{t=0}^{\infty}\beta^t\Big\{\underbrace{\boldsymbol{\gamma}_t^\intercal\boldsymbol{\epsilon}_t+\eta_t\kappa_t}_{\text{asset market clearing}}+\underbrace{\xi_t\big(\mathbf{1}_{\mathcal{B}}^\intercal\boldsymbol{b}^G_t+T_t+\tau^A_t a^K_{t-1}+\boldsymbol{\tau}^{B\intercal}_t\boldsymbol{b}_{t-1}+\tau^N_t n_{t-1}-\boldsymbol{R}^{B\intercal}_t\boldsymbol{b}^G_{t-1}\big)}_{\text{government budget constraint}}\Big\},$$

subject to initial conditions $\{\boldsymbol{R}^B_0,\boldsymbol{b}^G_{-1},\boldsymbol{b}_{-1},n_{-1},\theta_{-1},\boldsymbol{\phi}_{-1},k_{-1}\}$. Equation (17) states the full-tool Ramsey problem used to pin down baseline macroprudential taxes; in the probability-zero event we hold $\{\tau^A_t,\boldsymbol{\tau}^B_t,\tau^N_t\}$ fixed at their baseline values and allow only $\{\boldsymbol{b}^G_t,T_t\}$ to be state-contingent.

Here, $\boldsymbol{\gamma}_t$ and η_t denote the Lagrange multipliers associated with the asset market clearing conditions, representing the social value of excess asset supply. The multiplier ξ_t corresponds to the government budget constraint and represents the shadow value of governmental resources.

The government's concern when designing its asset market operation is represented by the first-order condition with respect to $\boldsymbol{b}^G$:

$$\beta\xi_{t+1}\boldsymbol{R}^B_{t+1}-\xi_t\mathbf{1}_{\mathcal{B}}=\boldsymbol{\gamma}_t. \tag{18}$$

The left-hand side represents the fiscal cost the government incurs from issuing the assets: the cost of repayment in period $t+1$ with returns $\boldsymbol{R}^B_{t+1}$, net of the proceeds from issuance in period t.[3] The government trades off the fiscal cost with the social value of generating excess asset supply, which is represented by $\boldsymbol{\gamma}_t$ on the right-hand side.

Social Value Accounting

This brings us to the question of how we can account for the social value of excess asset supply. To understand the welfare impact of an extra unit of asset supply, we

[2] We keep market clearing constraints and treat returns as choice variables, unlike the standard dual approach, which substitutes out prices. This links the planner's optimality conditions to measurable properties of asset supply and demand.

[3] Return $\boldsymbol{R}^B$ reflects household's preferences over assets and portfolio adjustment cost. For example, high marginal utility of holding assets would appear as low $\boldsymbol{R}^B$.

use the first-order condition with respect to returns to link the social values $\{\boldsymbol{\gamma}_t, \eta_t\}$ to how changes in returns affect welfare. For example, consider the first-order condition with respect to $\boldsymbol{R}^B_{t+1}$:[4]

$$\sum_{s=0}^{\infty} \beta^s \begin{bmatrix} \boldsymbol{\gamma}_s \\ \eta_s \end{bmatrix}^{\intercal} \begin{bmatrix} -\boldsymbol{\epsilon}_{s,\boldsymbol{R}^B_{t+1}} \\ -\kappa_{s,\boldsymbol{R}^B_{t+1}} \end{bmatrix} = v_{\boldsymbol{R}^B_{t+1}} + s_{\boldsymbol{R}^B_{t+1}}, \tag{19}$$

where $v_{\boldsymbol{R}^B_{t+1}}$ and $s_{\boldsymbol{R}^B_{t+1}}$ represent how changes in asset returns affect the household's welfare and the government budget:

$$\begin{aligned} v_{\boldsymbol{R}^B_{t+1}} \coloneqq{}& \beta^{t+1}\lambda_{t+1}\boldsymbol{b}_t^{G\intercal} + \sum_{s=0}^{\infty} \beta^{s+1}\lambda_{s+1} \underbrace{\left(R^A_{s+1}\theta_{s,\boldsymbol{R}^B_{t+1}} - \boldsymbol{R}^{B\intercal}_{s+1}\boldsymbol{\phi}_{s,\boldsymbol{R}^B_{t+1}}\right)}_{\text{pecuniary externality through portfolio}} n_s \\ &+ \underbrace{\sum_{s=-1}^{\infty} \beta^{s+1}\lambda_{s+1}\left[\left(R^N_{s+1} - \tau^N_{s+1}\right) n_{s,\boldsymbol{R}^B_{t+1}} - n_{s+1,\boldsymbol{R}^B_{t+1}}\right]}_{\text{pecuniary externality through net worth}}, \\ s_{\boldsymbol{R}^B_{t+1}} \coloneqq{}& -\beta^{t+1}\xi_{t+1}\boldsymbol{b}_t^{G\intercal} + \sum_{s=0}^{\infty} \beta^{s+1}\xi_{s+1}\left(\tau^A_{s+1}a^K_{s,\boldsymbol{R}^B_{t+1}} + \boldsymbol{\tau}^{B\intercal}_{s+1}\boldsymbol{b}_{s,\boldsymbol{R}^B_{t+1}} + \tau^N_{s+1}n_{s,\boldsymbol{R}^B_{t+1}}\right), \end{aligned}$$

with the convention $n_{-1,\boldsymbol{R}^B_{t+1}} = 0$.

To understand the formula, consider what happens when asset returns $\boldsymbol{R}^B_{t+1}$ increase by one unit in period $t+1$. First, the household gains on its holdings of assets issued by the government, while the government incurs losses; these effects are captured by the first terms in $v_{\boldsymbol{R}^B_{t+1}}$ and $s_{\boldsymbol{R}^B_{t+1}}$.[5]

As asset returns change, they also trigger endogenous responses in the economy. The second term in $v_{\boldsymbol{R}^B_{t+1}}$ captures one such response: how the financial sector adjusts its portfolio $(\theta, \boldsymbol{\phi})$ when returns change. Since households own the financial sector, these adjustments directly affect household welfare through equity income. Other endogenous responses, such as changes in household bond demand $\boldsymbol{b}_t$, have no first-order effect on welfare by the envelope theorem, as households had already made these decisions optimally. The third term in $v_{\boldsymbol{R}^B_{t+1}}$ captures a pecuniary externality through net worth: because n_t is an exogenous reduced-form object, changes in $\boldsymbol{R}^B_{t+1}$

[4]The first-order condition for returns is an accounting identity, not a characterization of optimal policy: it holds in any equilibrium and does not describe a trade-off faced by the planner.

[5]Redistribution between the household and the financial sector is inconsequential because the household is the residual claimant on the financial sector's net worth.

alter the reduced-form net-worth path $\{n_s\}$, and these induced responses affect future household equity income $(R^N_{s+1} - \tau^N_{s+1})n_s$ and reinvestment n_{s+1} with no household envelope-theorem suppression. Both the second and third terms are pecuniary externalities from financial frictions: if net worth were chosen optimally by the household, the envelope theorem would suppress the third term exactly as it suppresses the household's own bond demand. Finally, the second term in $s_{\boldsymbol{R}^B_{t+1}}$ represents how endogenous responses to asset returns affect the tax base and, consequently, tax revenue, which has the shadow value ξ_t.[6]

These effects of asset returns on household welfare and the government budget allow us to recover the social values of excess asset supply, $\boldsymbol{\gamma}_t$ and η_t: on the left-hand side of Equation (19), $\boldsymbol{\epsilon}_{s,\boldsymbol{R}^B_{t+1}}$ and $\kappa_{s,\boldsymbol{R}^B_{t+1}}$ are sensitivities that describe the effects of asset returns on excess asset supply. By inverting these sensitivities, we can derive the impact of excess supply on asset returns, and consequently recover the social value of excess asset supply through its impact on asset returns.

Given these considerations, the government designs the issuance of assets $\boldsymbol{b}^G_t$ along with macroprudential taxes τ^A_t, $\boldsymbol{\tau}^B_t$, and τ^N_t to maximize household welfare. In the following sections, we first characterize the Ramsey steady state that the government can achieve with the full set of tools in the absence of aggregate shocks. Afterwards, we characterize the government's optimal asset market operations in response to shocks when macroprudential taxes cannot respond.

3.2 Ramsey Steady State

Absent aggregate shocks, we define a Ramsey steady state as follows:

Definition 2 *Given $\boldsymbol{\omega}_t = \bar{\boldsymbol{\omega}}$ for all $t \geq 0$, a Ramsey steady state consists of returns $\bar{R}^A, \bar{\boldsymbol{R}}^B$, policies $\bar{\boldsymbol{b}}^G, \bar{T}, \bar{\tau}^A, \bar{\boldsymbol{\tau}}^B, \bar{\tau}^N$, and multipliers $\bar{\boldsymbol{\gamma}}, \bar{\eta}$, and $\bar{\xi}$, which solve the saddle-point problem (17) for all periods t, with the initial condition given by the steady-state returns, policies, and allocations.*

The following lemma characterizes the Ramsey steady state for the planner's problem described in (17):

[6] This term is often referred to as the *fiscal externality* in the sufficient statistics literature, see Kleven (2021) for details.

Lemma 2 [7] *In a Ramsey steady state, the multipliers and taxes are such that*

$$\bar{\boldsymbol{\gamma}} - \beta\bar{\xi}\bar{\boldsymbol{\tau}}^B = \mathbf{0}_{\mathcal{B}}, \quad \bar{\eta} + \beta\bar{\xi}\bar{\tau}^A = 0, \quad \textit{(Pigou tax)} \tag{20}$$

and $\bar{\xi} = \bar{\lambda}$. *Asset returns satisfy*

$$\beta(\bar{\boldsymbol{R}}^B - \bar{\boldsymbol{\tau}}^B) = \mathbf{1}_{\mathcal{B}}, \quad \beta(\bar{R}^A + \bar{\tau}^A) = 1, \quad \beta(\bar{R}^N - \bar{\tau}^N) = 1. \quad \textit{(Friedman rule)} \tag{21}$$

Proof. See footnote above. □

The optimal macroprudential tax policy balances welfare benefits against fiscal costs in Equation (20). For both financial assets and capital, the planner sets taxes so that the social welfare gain from reducing excess private demand equals the fiscal cost of shrinking the corresponding tax base. In the financial-asset market, this tradeoff is summarized by $\bar{\boldsymbol{\gamma}} - \beta\bar{\xi}\bar{\boldsymbol{\tau}}^B = \mathbf{0}_{\mathcal{B}}$; analogously, in the capital market it is summarized by $\bar{\eta} + \beta\bar{\xi}\bar{\tau}^A = 0$. When the planner increases tax to reduce excess asset demand by one unit, they incur a direct fiscal cost of $\beta\bar{\xi}\bar{\boldsymbol{\tau}}^B$ from losing the tax base. However, this reduction generates a social welfare gain $\bar{\boldsymbol{\gamma}}$, which captures how changing asset demand affects asset returns and welfare through two channels. First, changes in returns affect welfare due to pecuniary externalities from financial frictions — both through the financial sector's portfolio adjustments $(\boldsymbol{\phi}, \theta)$ and through the dividend channel, as the exogenous net worth process n_t responds to returns in ways that can potentially differ from household-optimal choices. Second, changes in returns create indirect fiscal effects as households adjust their asset demand in response. The optimality condition $\bar{\boldsymbol{\gamma}} - \beta\bar{\xi}\bar{\boldsymbol{\tau}}^B = \mathbf{0}_{\mathcal{B}}$ reflects the standard Pigou tax logic: optimal taxes equate the welfare gain from correcting externalities with the net fiscal cost, including the direct and indirect fiscal effects.

With the externality addressed by macroprudential taxes, the values of resources are equalized between the household and the government, $\bar{\xi} = \bar{\lambda}$, as a lump-sum tax $\bar{T}$ has no additional welfare effects. Moreover, all returns are effectively equalized with β^{-1} after adjusting for taxes, featuring a generalized version of the Friedman rule, as the government meets the household's demand for assets with the issuance of $\boldsymbol{b}^G$. However, as we show in the following section, this simple prescription about asset

[7] Appendix A.4–A.5 shows a more general result under which this steady-state characterization is a special case: under the full tool set, the wedge conditions $\xi_t = \lambda_t$, $\boldsymbol{\gamma}_t = \beta\xi_{t+1}\boldsymbol{\tau}^B_{t+1}$, and $\eta_t + \beta\xi_{t+1}\tau^A_{t+1} = 0$ hold for all t.

returns will no longer be optimal in the presence of aggregate shocks when the government cannot adjust macroprudential taxes. Without macroprudential taxes, the government will need to rely on asset market operations $\boldsymbol{b}^G$ to address any pecuniary externality that emerges as the consequence of the shocks.

4 Optimal Asset Market Operations

We analyze the government's optimal responses to shock $\hat{\boldsymbol{\omega}}_t$ around the Ramsey steady state up to first-order approximation. We denote deviations from the steady state as $\hat{\boldsymbol{x}}_t \coloneqq \boldsymbol{x}_t - \bar{\boldsymbol{x}}_t$ and use $\boldsymbol{f}_{t,\boldsymbol{x}_s}$ for the Jacobian of $\boldsymbol{f}_t(\boldsymbol{x}_s)$ with respect to $\boldsymbol{x}_s$ at steady state. We use column vectors discounted at rate β to represent multipliers and returns such as $\hat{\boldsymbol{\xi}} \coloneqq [\beta^t \hat{\xi}_t]$ and $\hat{\boldsymbol{R}}^B \coloneqq [\beta^{t+1} \hat{\boldsymbol{R}}^B_{t+1}]$, as well as the first difference of sequences such as $\Delta\hat{\boldsymbol{\xi}} \coloneqq [\beta^t(\hat{\xi}_{t+1} - \hat{\xi}_t)]$. We use matrices such as $\boldsymbol{\epsilon}_{\boldsymbol{R}^B} \coloneqq [\boldsymbol{\epsilon}_{t,\boldsymbol{R}^j_{s+1}}]$ to capture the sensitivities of supply and demand with respect to returns, where row t describes how $\boldsymbol{\epsilon}_t$ responds to returns $\boldsymbol{R}^B_{s+1}$ across different periods s.

The core objective of the planner is to address the reemergence of pecuniary externalities that arise due to aggregate shocks in the presence of financial frictions, using asset market operations as their instrument.

In principle, one might think that the planner needs to observe the underlying shocks and know about the detailed structure of the economy to design appropriate policy responses. However, this knowledge is unnecessary: for the class of models we study, asset returns contain all relevant information about the underlying shocks, and asset supply and demand elasticities contain everything the planner needs to know about the underlying structure of the economy. It is therefore sufficient for the planner to target a specific relationship between these assets to optimally mitigate the externality.

To see how it works, consider the optimality condition for asset market operations $\boldsymbol{b}^G$, Equation (18), which we approximate up to first order and stack in vector form. By adding the household's intertemporal valuation $\Delta\hat{\boldsymbol{\lambda}}$ to both sides of the equation and rearranging, we can represent the optimality condition as:

$$\Delta\hat{\boldsymbol{\lambda}} \otimes \mathbf{1}_{\mathcal{B}} + \bar{\lambda}\hat{\boldsymbol{R}}^B = \underbrace{\tilde{\boldsymbol{\gamma}}}_{\text{pecuniary externality gap}} + \underbrace{(\Delta\hat{\boldsymbol{\lambda}} - \Delta\hat{\boldsymbol{\xi}})}_{\text{government valuation gap}} \otimes \mathbf{1}_{\mathcal{B}}, \tag{22}$$

where $\otimes$ denotes the Kronecker product, and

$$\tilde{\boldsymbol{\gamma}} \coloneqq \left[\beta^t(\hat{\boldsymbol{\gamma}}_t - \beta\hat{\xi}_{t+1}\bar{\boldsymbol{\tau}}^B)\right].$$

The left-hand side represents the extent to which the household's intertemporal valuation deviates from asset returns. This deviation captures illiquidity effects from portfolio adjustment costs, net of any liquidity benefits embedded in household preferences. On the right-hand side, $\tilde{\boldsymbol{\gamma}}$ measures the welfare effects of increasing asset supply, which creates a pecuniary externality. This term emerges because the government cannot respond to shocks with macroprudential taxes; the steady-state tax rates are no longer sufficient to internalize the pecuniary externality when the economy faces unexpected disturbances. In particular, we assume that the macroprudential taxes are fixed at their Ramsey steady-state values, $\hat{\tau}^A \equiv 0$, $\hat{\boldsymbol{\tau}}^B \equiv 0$, $\hat{\tau}^N \equiv 0$. The final term, $\Delta\hat{\boldsymbol{\lambda}} - \Delta\hat{\boldsymbol{\xi}}$, represents a valuation gap between households and the government that also arises from this pecuniary externality.

Our key result is to show that both the pecuniary externality gap $\tilde{\boldsymbol{\gamma}}$ and the government valuation gap $\Delta\hat{\boldsymbol{\lambda}} - \Delta\hat{\boldsymbol{\xi}}$ can be expressed in terms of observable asset returns weighted by specific combinations of asset supply and demand elasticities. Combined with the condition $\Delta\hat{\boldsymbol{\lambda}} + \bar{\lambda}\hat{\boldsymbol{R}}^* = 0$ from Assumption 1, the optimality condition in Equation (22) provides us with an optimal target rule that relates asset returns to each other. Given the optimal target rule, we will demonstrate how the government can implement the Ramsey plan using this rule without requiring direct observation of the underlying shocks.

4.1 Social Value Accounting

The planner characterizes the welfare impact of its instruments by deriving first-order conditions for T_s, $\boldsymbol{R}^B_{s+1}$, and R^A_{s+1} around the Ramsey steady state. These conditions share a common channel structure, captured by a shared core $\mathcal{S}_{x_s}$, and differ only in an instrument-specific front block. Lemma 3 makes this structure precise.

Lemma 3 *Under the Ramsey plan, the shared core*

$$\mathcal{S}_{x_s} \coloneqq \underbrace{\sum_{t=0}^{\infty} \beta^t \tilde{\boldsymbol{\gamma}}_t^{\intercal} \bar{\boldsymbol{\epsilon}}_{t,x_s} + \sum_{t=0}^{\infty} \beta^t \tilde{\eta}_t \, \bar{\kappa}_{t,x_s}}_{\text{(i) externality gap}} + \underbrace{\sum_{t=0}^{\infty} \beta^{t+1} \tilde{\xi}_{t+1} \big(\bar{\tau}^A \bar{a}^F_{t,x_s} + \bar{\boldsymbol{\tau}}^{B\intercal} \bar{\boldsymbol{b}}^F_{t,x_s} + \bar{\tau}^N \bar{n}_{t,x_s} \big)}_{\text{(ii) valuation gap}}$$
$$+ \underbrace{\sum_{t=0}^{\infty} \beta^t \left[\beta \bar{\xi} \Big(\hat{R}^A_{t+1} \bar{a}^F_{t,x_s} - \hat{\boldsymbol{R}}^{B\intercal}_{t+1} \bar{\boldsymbol{b}}^F_{t,x_s} \Big) + (\hat{\lambda}_{t+1} - \hat{\lambda}_t) \bar{n}_{t,x_s} \right]}_{\text{(iii) pecuniary externality: financial sector}}.$$

The following conditions hold for all $s \geq 0$:

$$0 = \beta^s \tilde{\xi}_s + \mathcal{S}_{T_s}, \tag{23}$$
$$\mathbf{0} = -\beta^{s+1} \tilde{\xi}_{s+1} \bar{\boldsymbol{b}}^{G\intercal}_s + \mathcal{S}_{\boldsymbol{R}^B_{s+1}}, \tag{24}$$
$$0 = \underbrace{\sum_{t=0}^{\infty} \beta^t \Big(\hat{\lambda}_{t+1} - \hat{\lambda}_t + \beta \bar{\lambda} \hat{R}^A_{t+1} \Big) \zeta^K_{t,R^A_{s+1}}}_{\text{(iv) pecuniary externality: production sector}} + \mathcal{S}_{R^A_{s+1}}, \tag{25}$$

where

$$\zeta^K_{t,R^A_{s+1}} \coloneqq \sum_{\ell=t}^{\infty} \beta^{\ell-t+1} \bar{w}_{\ell+1,R^A_{s+1}} + \beta^{s-t+1} \bar{a}^K \, \mathbf{1}_{\{t \leq s\}} \tag{26}$$

is the discounted production-side stream generated by R^A_{s+1}, combining future wage adjustments and the direct capital reallocation at date s.

Proof. See Appendix A.6.4. □

The three labeled channels in $\mathcal{S}_{x_s}$ capture how each instrument reshapes the financial sector's balance sheet and the resulting general-equilibrium feedbacks. Channel (i), the *externality gap*, prices how the instrument shifts excess supply of financial assets and capital through the wedge between the social and private return to intermediation: $\tilde{\boldsymbol{\gamma}}_t$ and $\tilde{\eta}_t$ measure the welfare cost per unit of excess supply, and $\bar{\boldsymbol{\epsilon}}_{t,x_s}$ and $\bar{\kappa}_{t,x_s}$ are the corresponding Jacobians. Channel (ii), the *valuation gap*, captures how the instrument shifts the intermediary's tax revenue $(\bar{\tau}^A \bar{a}^F_{t,x_s} + \bar{\boldsymbol{\tau}}^{B\intercal} \bar{\boldsymbol{b}}^F_{t,x_s} + \bar{\tau}^N \bar{n}_{t,x_s})$, priced at the wedge-adjusted shadow cost of public funds $\tilde{\xi}_{t+1}$. Channel (iii), the *pecuniary externality through the financial sector*, captures the direct repricing of the intermediary's existing positions when returns deviate from the intertemporal price. Under perfect markets, $\hat{R}^A_{t+1}$ and $\hat{\boldsymbol{R}}^B_{t+1}$ would equalize to the neutral rate $\hat{R}^*_{t+1}$, leaving no

residual revaluation. Financial frictions break this equalization, so existing asset and liability positions are revalued at prices that diverge from their social cost, generating a welfare-relevant externality.

The three conditions in Lemma 3 differ only in their instrument-specific front blocks. The front block in (23), $\beta^s \tilde{\xi}_s$, is the valuation wedge at date s: the gap between the social and private value of a dollar transferred to the household. The front block in (24), $-\beta^{s+1}\tilde{\xi}_{s+1}\bar{\boldsymbol{b}}_s^{G\intercal}$, is the direct fiscal revaluation of the government's existing financial-asset liabilities when $\boldsymbol{R}_{s+1}^B$ changes. The front block in (25) constitutes channel (iv), the *pecuniary externality through the production sector*: when financial frictions drive a wedge between the capital return and the neutral rate, wages and capital reallocation respond, and these adjustments propagate through the discounted production-side stream $\zeta^K_{t,R^A_{s+1}}$. The two forces acting on this stream are the intertemporal shift $(\hat{\lambda}_{t+1} - \hat{\lambda}_t)$ and the capital return deviation $\beta\bar{\lambda}\hat{R}^A_{t+1}$, both of which are zero when $\hat{R}^A$ equals $\hat{R}^*$ in frictionless markets.

A key feature of Lemma 3 is that all Jacobians entering the conditions ($\bar{\boldsymbol{\epsilon}}_{t,x_s}$, $\bar{\kappa}_{t,x_s}$, $\bar{a}^F_{t,x_s}$, $\bar{\boldsymbol{b}}^F_{t,x_s}$, $\bar{n}_{t,x_s}$, and $\zeta^K_{t,R^A_{s+1}}$) are evaluated at the Ramsey steady state and are invariant to the underlying shocks $\hat{\boldsymbol{\omega}}$. Shock information enters the conditions only through three objects: the return disturbances $\hat{R}^A_{t+1}$ and $\hat{\boldsymbol{R}}^B_{t+1}$, and the intertemporal valuation term $(\hat{\lambda}_{t+1} - \hat{\lambda}_t)$. Under Assumption 1, the latter is equivalent to the neutral rate $\hat{R}^*_{t+1}$, so together these three objects contain all relevant information about the underlying disturbances. To first order, only return disturbances and the household's intertemporal valuation, not the underlying shock structure, determine the welfare impact of the government's instruments.

4.2 The Optimal Target Rule

Using the accounting result in Lemma 3, the optimality condition for asset market operations in Equation (22), along with Assumption 1, immediately imply an optimal target rule for asset returns:

Theorem 1 *The Ramsey plan can be achieved by a target rule of the following form:*

$$\hat{\boldsymbol{R}}^B - \hat{\boldsymbol{R}}^* \otimes \mathbf{1}_{\mathcal{B}} = \begin{bmatrix} \boldsymbol{\Gamma}^A \\ -\boldsymbol{\Gamma}^B \\ -\boldsymbol{\Gamma}^* \end{bmatrix}^{\intercal} \begin{bmatrix} \hat{\boldsymbol{R}}^A \\ \hat{\boldsymbol{R}}^B \\ \hat{\boldsymbol{R}}^* \end{bmatrix}, \tag{27}$$

where $\boldsymbol{\Gamma}^j \coloneqq \Gamma^j_{\boldsymbol{\epsilon}} + \Gamma^j_{\Delta T} \otimes \mathbf{1}^{\intercal}_{\mathcal{B}}$ *for* $j \in \{A,\, B,\, *\}$ *are loading matrices.*

Proof. See Appendix A.7. □

The left-hand side of the target rule contains the spreads between financial asset returns and the household's neutral rate, representing their intertemporal valuation of resources. Any spread above the neutral rate represents a welfare cost for the household to increase their asset holding, as a result of the portfolio adjustment cost, net of liquidity service.

The right-hand side is the sum of the externality gap $\tilde{\boldsymbol{\gamma}}$, and the valuation gap $\Delta\hat{\boldsymbol{\lambda}} - \Delta\hat{\boldsymbol{\xi}}$. Together, they capture the total welfare gain from mitigating the presence of pecuniary externality with an increase in asset supply.

The rule described in Theorem 1 has two properties that make it especially useful. First, the rule specifies a fixed relationship between asset returns, *regardless* of the underlying disturbances $\hat{\boldsymbol{\omega}}$. These asset returns, which are readily observable in the markets, summarize all relevant information for the prescription of optimal policy regardless of underlying disturbances. Second, this relationship, captured by loading matrices $\boldsymbol{\Gamma}^j$, is fully described by the reduced-form elasticities and response matrices. These reflect the *invariant* properties of the economy, including the underlying preferences, technology, and frictions. They can be estimated using standard identification strategies. The exact microfoundations of the model are irrelevant to the optimal policy rule as long as they imply the same matrices $\boldsymbol{\Gamma}^j$. This represents the spirit of the sufficient statistics approach (Chetty, 2009; Kleven, 2021).

4.3 Implementation

We now show that, given these supply and demand elasticities, Theorem 1 allows the government to implement the Ramsey plan with a simple asset supply schedule based on the target rule without any knowledge about the underlying disturbances,

following the spirit of "robust policy rules" discussed in Giannoni and Woodford (2003) and Woodford (2011).

We start by explaining why one might think that implementation could be a challenge. Given a specific sequence of asset market operations $\hat{\boldsymbol{b}}^G$, an equilibrium $\{\hat{\boldsymbol{R}}^A, \hat{\boldsymbol{R}}^B, \hat{\boldsymbol{T}}\}$ satisfies

$$\boldsymbol{\kappa}_{R^A}\hat{\boldsymbol{R}}^A + \boldsymbol{\kappa}_{R^B}\hat{\boldsymbol{R}}^B + \boldsymbol{\kappa}_T\hat{\boldsymbol{T}} + \boldsymbol{\kappa}_\omega\hat{\boldsymbol{\omega}} = 0, \tag{28}$$

$$\hat{\boldsymbol{b}}^G + \boldsymbol{\epsilon}_{R^A}\hat{\boldsymbol{R}}^A + \boldsymbol{\epsilon}_{R^B}\hat{\boldsymbol{R}}^B + \boldsymbol{\epsilon}_T\hat{\boldsymbol{T}} + \boldsymbol{\epsilon}_\omega\hat{\boldsymbol{\omega}} = 0, \tag{29}$$

and the government budget constraint. One way to think about the rule is to solve $\hat{\boldsymbol{R}}^A$ and $\hat{\boldsymbol{R}}^B$ as functions of shocks $\hat{\boldsymbol{\omega}}$ and $\hat{\boldsymbol{b}}^G$, calculate the implied $\hat{\boldsymbol{R}}^*$, and then choose $\hat{\boldsymbol{b}}^G$ so that the resulting equilibrium satisfies the optimal target rule. This approach is impractical because it requires the government to observe how all shocks from various sources disturb asset markets ($\boldsymbol{\kappa}_\omega\hat{\boldsymbol{\omega}}$ and $\boldsymbol{\epsilon}_\omega\hat{\boldsymbol{\omega}}$). Measuring all possible shocks and identifying how they affect asset markets is likely infeasible.

However, the target rule in Theorem 1 provides us with a simple way to circumvent this challenge. In Theorem 1, the target rule specifies a target relationship between asset returns that must hold under the Ramsey plan and that *does not* depend on the underlying shocks. Rather than inferring shocks and then solving backward for the required asset supply, the government can instead commit to a return-contingent asset supply schedule that adjusts $\hat{\boldsymbol{b}}^G$ in response to observed returns so that the equilibrium relationship in Equation (27) is satisfied. Under this asset supply schedule, the equilibrium will be given by $\{\hat{\boldsymbol{R}}^A, \hat{\boldsymbol{R}}^B, \hat{\boldsymbol{T}}, \hat{\boldsymbol{b}}^G\}$ that satisfies market clearing conditions (28) and (29), the government budget constraint, and Equation (27), given arbitrary realization of $\hat{\boldsymbol{\omega}}$. In other words, the quantities of assets $\hat{\boldsymbol{b}}^G$ adjust endogenously to absorb shocks while keeping equilibrium returns consistent with the Ramsey target rule.

To understand the implementation intuitively, note that at each date, the target rule imposes $|\mathcal{B}|$ restrictions on the vector of financial-asset returns relative to the neutral rate. The government, in turn, has a $|\mathcal{B}|$-dimensional asset-supply instrument, $\hat{\boldsymbol{b}}^G_t$. Relative to a passive asset-supply policy, the rule pins down these $|\mathcal{B}|$ return wedges, and asset market operations adjust to clear markets after shocks. In this sense, implementing the Ramsey plan does not require the government to identify

the underlying disturbances directly; it requires the government to move $\hat{\boldsymbol{b}}^G$ so that equilibrium returns satisfy the target relationship.

5 The Role of Financial Frictions

5.1 Neoclassical Benchmark

First, we provide a benchmark case where key features of the financial sector are absent. Specifically, we consider an economy where (1) financial intermediation does not respond to changes in returns: $\boldsymbol{\phi}_t(\cdot) \equiv \bar{\boldsymbol{\phi}}$ and $\theta_t(\cdot) \equiv \bar{\theta}$, where $\bar{\theta} \coloneqq \mathbf{1}_{\mathcal{B}}^{\intercal}\bar{\boldsymbol{\phi}}+1$; and (2) the reduced-form net-worth law coincides with the benchmark construction $n^{NGM}(\cdot)$ defined in Appendix B.1. In this benchmark, the latter condition is equivalent to $\Delta\hat{\boldsymbol{\lambda}} = -\bar{\lambda}\hat{\boldsymbol{R}}^N$ and, combined with Assumption 1, to $\hat{\boldsymbol{R}}^N = \hat{\boldsymbol{R}}^*$.

Under these assumptions, the general target rule in Theorem 1 simplifies to a simple benchmark prescription: the planner should maintain a constant spread between all returns for any realization of shocks $\hat{\boldsymbol{\omega}}$, thereby preserving the spirit of the Friedman rule:

Corollary 1 *If $\boldsymbol{\phi}_t(\cdot) \equiv \bar{\boldsymbol{\phi}}$, $\theta_t(\cdot) \equiv \bar{\theta}$, and the reduced-form net-worth law satisfies the benchmark condition $n_t(\cdot) = n_t^{NGM}(\cdot)$, then a target rule*

$$\hat{\boldsymbol{R}}^B = \hat{\boldsymbol{R}}^A \otimes \mathbf{1}_{\mathcal{B}}$$

achieves the Ramsey plan up to first order. Under the benchmark solution, $\tilde{\boldsymbol{\gamma}} = \mathbf{0}$ and $\hat{\boldsymbol{R}}^N = \hat{\boldsymbol{R}}^A = \hat{\boldsymbol{R}}^*$ in equilibrium.

Proof. See Appendix B.1. □

When all returns on financial assets move in tandem with returns on capital, we have $\hat{\boldsymbol{R}}^N = \hat{\boldsymbol{R}}^A = \hat{\boldsymbol{R}}^*$. In this case, households effectively value intermediary net worth as a direct claim on capital, as in a neoclassical growth model. At the same time, the planner issues the necessary amount of each type of financial asset to satiate household demand for these assets arising from any additional motives.

Corollary 1 allows us to isolate the role of financial frictions in designing the optimal policy rule: In the absence of these frictions, the planner would maintain constant

spreads between all assets. The presence of financial frictions upends this prescription, requiring the planner to implement a target rule that facilitates intermediation by maintaining variations between spreads in response to shocks.

5.2 Frictions in Financial Intermediation

To highlight the role of financial frictions in designing optimal policy, consider a special case where (1) the household has quasi-linear preferences without portfolio adjustment cost, with liquidity service from holding assets given by a concave function $\nu(\cdot)$:

$$u\left(\{c_s, \boldsymbol{b}_s\}_{s\leq t}; \boldsymbol{\omega}_t\right) = c_t + \nu(\boldsymbol{b}_t; \boldsymbol{\omega}_t),$$

(2) the capital supply from the production sector is perfectly elastic, and the returns on capital $\{\hat{R}^A_{t+1}\}$ are given exogenously, and (3) the financial sector net worth is fixed exogenously: $n_t = \bar{n}$.

Corollary 2 *With quasi-linear preference, perfectly elastic capital, and fixed net worth, the optimal target rule is given by*

$$\hat{\boldsymbol{R}}^B = \left(\bar{n}\boldsymbol{\theta}_{\boldsymbol{R}^B}\boldsymbol{b}^{-1}_{\boldsymbol{R}^B}\right)^{\intercal}\hat{\boldsymbol{R}}^A.$$

Proof. See Appendix B.2. □

To build intuition, we further simplify to the case where there is a single financial asset, $|\mathcal{B}| = 1$, and the level of intermediation per net worth is given by a function $\varphi(\cdot)$ of the spread between R^A_{t+1} and R^B_{t+1}:

$$\phi_t = \varphi(R^A_{t+1} - R^B_{t+1}), \quad \varphi'(\cdot) > 0.$$

Financial frictions of this form arise from microfoundations such as costly state verification (Bernanke et al., 1999) or costly leverage (Cúrdia and Woodford, 2011). Since the level of intermediation depends only on next-period returns, the financial sector's asset supply reduces to a static system: $\phi_{s,R^j_{t+1}} = \mathbf{0}$ if $s \neq t$, and the optimal target rule in Corollary 2 reduces to

$$\hat{R}^B_{t+1} = -\frac{\bar{n}\varphi'}{b_{R^B}}\hat{R}^A_{t+1}, \tag{30}$$

where φ' and b_{R^B} are evaluated at the steady state.

Because $\varphi' > 0$ and $b_{R^B} > 0$, the rule prescribes a negative relationship between $\hat{R}^A_{t+1}$ and $\hat{R}^B_{t+1}$. When $\hat{R}^A_{t+1} > 0$, pecuniary externality emerges because the endogenous response of $\hat{R}^B_{t+1}$ does not take into account the fact that a lower $\hat{R}^B_{t+1}$ would allow the financial sector to increase its holding of capital. As a result of the externality, the planner can create welfare gains by decreasing excess asset supply, lowering the financial sector's funding cost, and thereby increasing the level of intermediation. To decide how much excess asset supply to absorb, the planner trades off the welfare gain from addressing the pecuniary externality against the loss of liquidity service that the household values. A decrease in excess asset supply has a large impact on the level of intermediation when (1) the financial sector's intermediation capacity is sensitive to its funding cost (high φ'), and (2) the household asset demand is inelastic (low b_{R^B}). In these cases, the optimal target rule prescribes a larger cut in $\hat{R}^B_{t+1}$.

Forward-Looking Frictions

A similar logic extends to the case in which the financial sector features an intertemporal asset supply system, where the level of intermediation in one period depends on asset returns in all future periods. As an example, suppose that the level of financial friction is given by $\phi_t(\{R^A_{s+1} - R^B_{s+1}\}_{s \geq t})$, with responses to asset returns around the steady state given by:

$$\phi_{t,R^A_{s+1}} = -\phi_{t,R^B_{s+1}} = \phi_R \times \Xi^{s-t}, \quad \forall s \geq t,$$

and zero for all $s < t$, for some $\phi_R > 0$ and $\Xi \in [0, 1)$. As shown in Chiang and Zoch (2023), financial frictions of this form resemble those in asset diversion models as in Gertler and Kiyotaki (2010).

With the intertemporal asset supply system, Corollary 2 implies an optimal target rule of the form:

$$\hat{R}^B_{t+1} = -\frac{\bar{n}\phi_R}{b_{R^B}} \sum_{s=0}^{t} \beta^{s-t} \Xi^{t-s} \hat{R}^A_{s+1}.$$

Unlike the static case in Equation (30), a decrease in the financial sector's funding cost in period t, R^B_{t+1}, enhances the financial sector's intermediation capacity in all periods before t due to the forward-looking friction. As a result, the optimal target for $\hat{R}^B_{t+1}$ reflects disturbances in returns on capital in all periods before t: $\hat{R}^A_{s+1}$ for all

$s \leq t$, with the weight $\beta^{s-t}\Xi^{t-s}$ combining the geometric decay of the friction with the discounting convention in the linearized system.

6 Quantitative Application: Banking with Interbank Frictions

We now apply the target rule to the multi-asset banking model of Bianchi and Bigio (2022) (henceforth BB), where interbank search frictions generate a rich portfolio structure across deposits, reserves, and government bonds. The structural model provides a concrete mapping between microfoundations and the elasticities of $\boldsymbol{\phi}_t$ and θ_t that determine the target rule. The directional conclusion from Section 5 carries over: when capital returns are high, the planner targets lower returns on financial assets to facilitate intermediation. The multi-asset structure reveals that the optimal bond return responds much more strongly than the reserve or deposit rates.

6.1 The Financial Block

We briefly describe the BB financial block here; Appendix C.1 summarizes the key optimality and market-clearing conditions that determine the portfolio mappings.

In BB, a continuum of banks hold loans (return R^A_{t+1}), reserves (return $R^{B_1}_{t+1}$), and government bonds (return $R^{B_2}_{t+1}$), funded by deposits (return $R^{B_0}_{t+1}$). Banks face idiosyncratic withdrawal shocks and settle reserve imbalances in an OTC interbank market, with a discount window (rate R^W_{t+1}) as a backstop. The key channel for monetary policy: by altering the supply of reserves relative to deposits, the government changes the liquidity premium in the interbank rate, which propagates through portfolio first-order conditions to affect the composition of bank balance sheets and the lending spread.

In our framework, the BB financial block maps into portfolio-share functions $\boldsymbol{\phi}_t = \boldsymbol{\phi}(R^A_{t+1}, \boldsymbol{R}^B_{t+1}; R^W_{t+1})$ and loans per unit of equity $\theta_t = \mathbf{1}^\intercal_{\mathcal{B}}\boldsymbol{\phi}_t + 1$.[8] A distinctive feature of the BB calibration is that the deposit constraint binds ($\phi_{\text{dep},t} = \bar{\phi}_{\text{dep}}$), so deposits do not respond to returns.

[8] We treat the discount-window rate R^W_{t+1} parametrically.

The return on net worth is $R_t^N = R_t^A \theta_{t-1} - \boldsymbol{R}_t^{B\intercal} \boldsymbol{\phi}_{t-1}$. Net worth follows the law of motion $n_t = (1-f) R_t^N n_{t-1} + m$, where f is the exit rate and m represents entry by new intermediaries.

6.2 Calibration and Mapping

We calibrate the model to the 2006 U.S. banking sector using the parameters from Bianchi and Bigio (2022). All quantities and returns are in real terms at monthly frequency. The calibration proceeds in two stages. First, the portfolio Jacobians $\bar{\boldsymbol{\phi}}_{R^A}$, $\bar{\boldsymbol{\phi}}_{\boldsymbol{R}^B}$ depend only on the BB financial block: the interbank market parameters and the steady-state returns and balance sheet ratios. Second, to obtain the target rule we additionally require the household demand elasticities and the production-side objects. Table 4 in Appendix C.1 reports the full set of parameters.

Financial-sector Jacobians. The interbank market parameters and steady-state balance sheet ratios are taken directly from Table II of Bianchi and Bigio (2022). The steady-state loans-to-equity ratio is $\bar{\theta} = 9.61$, with deposits ($\bar{\phi}_{\text{dep}} = 8.87$) funded by equity and a small reserve position ($\bar{\phi}_{\text{res}} = -0.05$) and government bond holding ($\bar{\phi}_{\text{gov}} = -0.21$). Annualized steady-state returns are 2.35% on loans, -0.07% on deposits, -1.96% on reserves, and 1.84% on government bonds, with a discount-window rate of 3.25%. Given these inputs, the BB optimality and interbank market-clearing conditions determine the portfolio Jacobians reported in Table 1. Two features stand out. First, deposits are completely inelastic: the leverage constraint binds, so ϕ_{dep} does not respond to any return. Second, the government bond share is by far the most elastic margin: a 100bp increase in R^A raises ϕ_{gov} by 4.15, compared to only 0.26 for reserves. These magnitudes drive the structure of the optimal target rule.

Rest of the economy. We assume Cobb–Douglas production ($\alpha = 0.33$, $\delta = 0.08$ annual) with linear capital accumulation $\Phi(\iota_t) = 1 - \delta + \iota_t$. As in BB, the household has quasi-linear preferences over consumption and downward sloping demand for assets. The household's period utility is:

$$u(c_t, \boldsymbol{b}_t) = c_t - \sum_{j \in \mathcal{B}} \frac{\zeta_{b_j}}{2} \left(b_{j,t} - \bar{b}_j\right)^2,$$

Table 1: Portfolio Jacobians under the Bianchi and Bigio (2022) calibration.

	R^A	R^{B_0}	R^{B_1}	R^{B_2}
Loans (θ)	3.89	−0.007	−0.40	−3.16
Deposits (ϕ_{dep})	0	0	0	0
Reserves (ϕ_{res})	−0.26	0.0004	−0.005	0.25
Gov bonds (ϕ_{gov})	4.15	−0.008	−0.39	−3.41

Note: Each entry gives the change in the portfolio share (per unit of equity) when the column return increases by 100 annualized basis points, holding other returns fixed. Evaluated at the 2006 steady state.

where $\bar{b}_j$ is the bliss point for asset j and $\zeta_{b_j} > 0$ controls the slope of demand. This implies $\lambda_t \equiv 1$ and asset demand semi-elasticities $\partial b_j / \partial R^{B_j} = \beta / \zeta_{b_j}$. We calibrate the curvatures ζ_{b_j} to match BB's household demand semi-elasticities, which have absolute value $|\varepsilon| = 35$ (monthly) for each of the three assets. We set the household discount factor $\beta = 0.981$ as in BB. We calibrate the bliss points $\bar{b}_j$ by assuming the 2006 asset holdings reported by BB correspond to the Ramsey steady state. This also pins down the macroprudential taxes τ^A, $\boldsymbol{\tau}^B$ that support the planner's optimum.

6.3 The Optimal Target Rule

We compute the target rule from Theorem 1. Because the household has quasi-linear preferences, $\lambda_t \equiv 1$, so $\hat{\boldsymbol{R}}^* = 0$ and all $\boldsymbol{\Gamma}^*$ terms vanish. The target rule (27) reduces to:

$$\hat{\boldsymbol{R}}^B = \begin{bmatrix} \boldsymbol{\Gamma}^A \\ -\boldsymbol{\Gamma}^B \end{bmatrix}^{\intercal} \begin{bmatrix} \hat{\boldsymbol{R}}^A \\ \hat{\boldsymbol{R}}^B \end{bmatrix}, \tag{31}$$

where $\boldsymbol{\Gamma}^A$ and $\boldsymbol{\Gamma}^B$ are loading matrices from Theorem 1, evaluated at the steady state. Figure 1 plots the rule: the response of each target return $\hat{R}^{B_j}_{t+1}$ to a capital return disturbance $\hat{R}^A_{s+1}$ under the Ramsey plan.

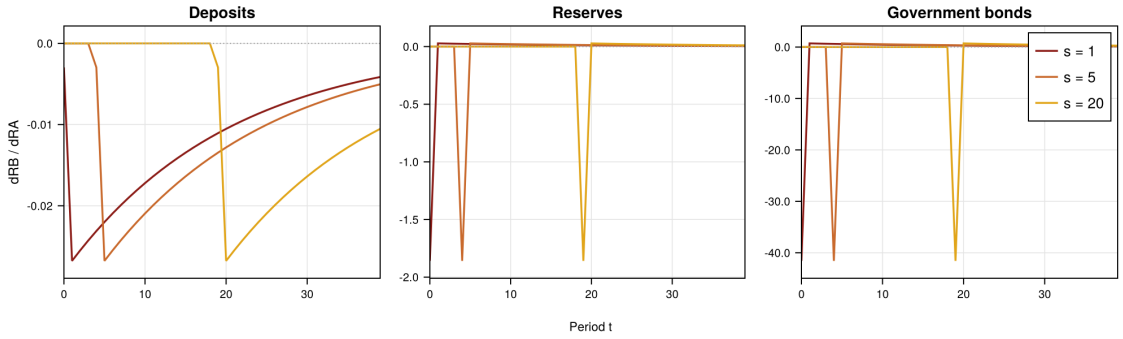


Figure 1: Target rule (31) for the Bianchi and Bigio (2022) calibration. Each panel shows the response of the target return $\hat{R}_t^{B_j}$ to a one-unit capital return disturbance $\hat{R}_s^A$ for one financial asset $j \in \{\text{deposits}, \text{reserves}, \text{gov bonds}\}$. Lines correspond to shock dates $s \in \{1, 5, 10, 20\}$. Monthly frequency.

The contemporaneous response ($t = s$) is negative for every financial asset: when capital returns rise, the planner targets lower funding costs, so banks reduce their holdings of government bonds and reserves, making room for additional loans. Unlike the static examples, persistent net worth generates responses at horizons $t \neq s$ as well. The government bond return is the dominant margin, with contemporaneous loadings on the order of -40, roughly 25 times larger in magnitude than reserves (around -1.5) and several hundred times larger than deposits (around -0.05). This hierarchy reflects the portfolio Jacobians in Table 1: because the leverage constraint binds, deposits are inelastic, so banks adjust primarily through their bond portfolio.

Comparative statics: the role of deposit elasticity

In practice, deposit issuance may also respond to returns (Chiang and Zoch, 2023). We explore this by varying the deposit sensitivities ϕ_{dep,R^A} and $\phi_{\text{dep},R^{B_0}}$ from the baseline (where both are zero) and recomputing the target rule. As deposit supply becomes more elastic, the deposit return enters the target rule and the government bond return responds less strongly (Figure 2). This illustrates that the structure of the optimal rule depends on which margins are active: on the portfolio Jacobians rather than the structural parameters that generate them.

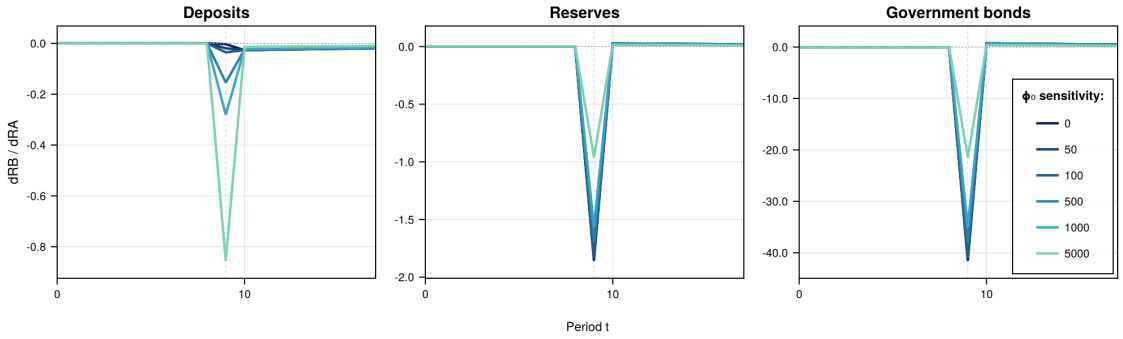


Figure 2: Sensitivity of the solved target rule to deposit elasticity for shock date $s = 10$. The symmetric deposit sensitivity ϕ_{dep} is varied from 0 (baseline, dark) to 5000 (light). As deposits become elastic, the planner redistributes the response away from government bonds toward deposits. Monthly frequency.

7 Taking the Model to the Data

Because the target rule depends on asset supply and demand elasticities rather than the structural model that generates them, one can obtain the target rule directly from data using established empirical methods. We estimate the portfolio sensitivities $\boldsymbol{\phi}_{\boldsymbol{R}^B}$ for the U.S. banking sector using Call Report balance sheets and expected returns. To address simultaneity, we identify the elasticities with the Granular Instrumental Variables (GIV) approach of Gabaix and Koijen (2024). We also use the constructed instruments to estimate capital supply elasticities.

7.1 Asset Supply and Demand Estimation

Empirical Specification

Throughout this section, the empirical objects $\mathbb{E}_t[R_{t+1}]$ denote time-t conditional expectations of one-period-ahead gross real returns, which correspond to the pricing objects in the theoretical model.

The financial sector consists of J entities. Let $b^F_{j,\ell,t}$ denote financial asset $\ell \in \mathcal{B}$ issued by the entity j in period t. The net worth of entity j is denoted by $n_{j,t}$. Issuance and holdings of assets per unit of net worth are defined as $\boldsymbol{\phi}_{j,t} \coloneqq \{\phi_{j,\ell,t}\}_{\ell=1}^{\mathcal{B}}$ where $\phi_{j,\ell,t} \coloneqq b^F_{j,\ell,t}/n_{j,t}$.

Entity j's holdings and issuance of assets per unit of net worth in period t are given

by a supply and demand system:

$$\boldsymbol{\phi}_{j,t} = \bar{\boldsymbol{\phi}}_j(t) + \boldsymbol{\phi}_{\boldsymbol{R}^B} \boldsymbol{S}_{t+1} + \boldsymbol{\Lambda}_j \boldsymbol{\eta}_t + \boldsymbol{\varepsilon}_{j,t}, \tag{32}$$

where $\bar{\boldsymbol{\phi}}_j(t)$ denotes entity-specific time trends of holdings and issuance, the sensitivity matrix $\boldsymbol{\phi}_{\boldsymbol{R}^B}$ parameterizes how they respond to spreads between asset returns (also calculated as deviations of from their trends):

$$\boldsymbol{S}_{t+1} \coloneqq \mathbb{E}_t[\boldsymbol{R}^B_{t+1}] - \mathbb{E}_t[R^A_{t+1}] \otimes \mathbf{1}_{\mathcal{B}},$$

$\boldsymbol{\eta}_t$ are unobserved aggregate shocks on which entity j has loadings $\boldsymbol{\Lambda}_j$, and $\boldsymbol{\varepsilon}_{j,t}$ are idiosyncratic shocks i.i.d. by entity-asset-time.

The specification above imposes several types of restrictions. First, to apply the standard granular instrumental variable approach, we assume entities in the financial sector respond only to expected returns one period ahead. Second, to reduce the number parameters to be estimated, we assume that the portfolio choice is determined by the spreads between $\mathbb{E}_t[\boldsymbol{R}^B_{t+1}]$ and $\mathbb{E}_t[R^A_{t+1}]$, rather than the returns separately. Finally, while we allow entities to have different loadings on the unobserved aggregate shocks, we assume that the sensitivity matrix is homogeneous across entities.

Identification

To obtain consistent estimates of $\boldsymbol{\phi}_{\boldsymbol{R}^B}$ in the presence of unobservable shocks $\boldsymbol{\eta}_t$ and $\boldsymbol{\varepsilon}_{j,t}$ that determine equilibrium values of spreads, we need instrumental variables. We follow the approach in Gabaix and Koijen (2024) to construct granular instrumental variables from idiosyncratic shocks $\boldsymbol{\varepsilon}_{j,t}$ which are "large enough" to generate non-trivial movements in equilibrium spreads. We provide details of our implementation in Appendix D.2.

Data

We construct our dataset using Call Report data for U.S. banking sector balance sheets, supplemented with CRSP data for market values of bank equity and TIC data for foreign countries' asset holdings.[9] Our sample spans 2003Q2–2023Q4. We build

[9]For the empirical application, we extend the model to include a foreign sector with asset demand $\boldsymbol{f}_t(\boldsymbol{R}^B_{t+1})$. The asset market clearing condition becomes $\boldsymbol{b}^G_t + \boldsymbol{b}^F_t = \boldsymbol{b}_t + \boldsymbol{f}_t$, and the government can tax foreign demand at rate $\boldsymbol{\tau}^E$. At the Ramsey steady state, the foreign demand taxes ad-

the GIV measure from both bank and foreign holdings data, focusing our analysis on estimating the banking sector's sensitivity matrix $\boldsymbol{\phi}_{\boldsymbol{R}^B}$. We provide an overview below, with full details in Appendices D.1 and D.2.

Entities: An entity corresponds to a bank holding company in the Call Report-CRSP data and a country in the TIC data. After sample selection, our data contains balanced panels of 130 bank holding companies and 48 countries.

Assets: We set $|\mathcal{B}| = 2$ and refer to the two net financial positions as "liquid" and "illiquid." The liquid position consists of cash and Treasury holdings, trading Treasury securities, and federal funds sold and securities purchased under agreement to sell, net of deposits and federal funds purchased and securities sold under agreement to repurchase. The illiquid position consists of other securities holdings and other trading securities, net of trading liabilities. Capital is then measured residually as market capitalization minus the bank's net liquid and net illiquid positions.

Returns: We use the real 3-month Treasury bill yield as the return on the liquid asset and the real BAA yield as the return on the illiquid asset. To calculate $\mathbb{E}_t[R^A_{t+1}]$ we proceed in two steps. For each bank, we construct the expected return on net worth, $\mathbb{E}_t[R^N_{t+1}]$, by regressing the realized excess return on a set of aggregate and bank-specific predictors. We then recover $\mathbb{E}_t[R^A_{t+1}]$ from each bank's balance sheet using Equation (12).

GIV: To isolate the idiosyncratic shocks for banks and countries, we proceed as follows: First, we remove entity-asset-specific quadratic trends and common time fixed effects (which contain responses to expected returns). Next, we extract principal components and their loadings, and calculate the residuals as $\boldsymbol{\varepsilon}_{j,t}$. We apply the procedure for banks and countries separately, constructing the granular instrumental variables as the weighted average of $\boldsymbol{\varepsilon}_{j,t}$ for each group. For banks, we use their net worth share as weights and construct GIV for their holdings of the two $\mathcal{B}$ assets jointly, using three principal components to capture aggregate shocks in our baseline specification. For foreign countries, we use their holding share as weights, focusing on their liquid asset

dress the externality from foreign holdings, and the structure of the target rule in Theorem 1 is unchanged—the rule continues to depend on the financial sector's portfolio Jacobians and aggregate demand elasticities, now inclusive of foreign demand. Full derivations are available upon request. In the estimation, foreign holdings serve a dual role: country-level idiosyncratic variation provides an additional instrument for the GIV construction, and we estimate foreign demand elasticities (Table 2, last two columns) to measure the aggregate demand side of the market.

holdings, and absorb aggregate shocks with one principal component.

Asset Supply and Demand of the U.S. Banking Sector

Table 2 presents our estimates of banking sector sensitivities and foreign demand elasticities. The first three columns show how net issuance and holdings of liquid and illiquid assets respond to spread changes per unit of net worth, with the third column reporting implied sensitivities of capital holdings derived from balance sheet constraints. The last two columns report the log semi-elasticities of foreign demand for liquid and illiquid assets.

Table 2: Effect of spreads on bank portfolio and foreign demand.

	Banking sector			Foreign demand	
	ϕ_{liq}	ϕ_{illiq}	θ_{cap}	$\log f_{\text{liq}}$	$\log f_{\text{illiq}}$
$R^B_{\text{liq}} - R^A$	-0.829	0.116	-0.713	0.090	-0.079
	(0.149)	(0.040)		(0.111)	(0.115)
$R^B_{\text{illiq}} - R^A$	0.521	-0.060	0.461	-0.350	0.091
	(0.159)	(0.048)		(0.244)	(0.269)

Note: Responses of $\boldsymbol{\phi}_t$ and θ_t to a one percentage point increase in annualized spreads. Robust standard errors in parentheses. Capital responses are implied by balance sheet constraints. Foreign demand responses are log semi-elasticities.

The estimated sensitivities have intuitive signs. When expected returns on an asset increase, the financial sector reduces its net issuance of that asset (equivalently, increases its holdings). Conversely, when the return on capital R^A rises, the financial sector increases its net issuance of liquid and illiquid assets (reduces its holdings) while increasing its capital holdings.

Capital Supply of the Production Sector

We estimate the elasticity of capital supply, focusing on the banking sector's capital holdings. Using bank holding company balance sheets, we calculate and aggregate total real capital holdings as:

$$k_t = \int (\mathbf{1}_{\mathcal{B}}^{\intercal} \boldsymbol{\phi}_{j,t} + 1) n_{j,t} dj.$$

We estimate the semi-elasticity of k_t with respect to R^A_{t+1} in a 2SLS regression, employing the GIV constructed in Section 7.1 as a source of exogenous variation in capital demand. The semi-elasticity of capital supply with respect to annualized return on capital is -0.02 with a standard error of (0.006).

7.2 Calibration

Household

We assume the household's utility function has the following form

$$u\left(c_t, \boldsymbol{b}_t; \boldsymbol{\omega}_t\right) = c_t \quad - \sum_{i \in \{\text{liquid,illiquid}\}} \varsigma_i \left(b_{i,t} - \bar{b}_i(\boldsymbol{\omega_t})\right)^2,$$

and there is no portfolio adjustment cost: $\Psi(\cdot) \equiv 0$. We set $\beta = 0.99$ and calibrate parameters of the utility function such that in the steady state the semi-elasticities with respect to annualized expected returns are equal to 0.1. We assume that the net worth of the financial sector is exogenous: $n_t = n(\boldsymbol{\omega}_t)$.

Steady State

We calculate the long-run averages of returns $\bar{\boldsymbol{R}}^B$ and $\bar{R}^A$, the portfolio shares of the financial sector $\bar{\boldsymbol{\phi}}$ and $\bar{\theta}$, along with household's holdings, as described in Appendix D.3. Treating these values as the Ramsey steady state, we compute the taxes $\bar{\tau}^A$, $\bar{\boldsymbol{\tau}}^B$ that are consistent with these asset return values in the Ramsey steady state, assuming the household's holdings are at their bliss points. We normalize the net worth of the financial sector to one. Table 3 shows the steady-state values of key variables.

Table 3: Steady-state values.

$\bar{R}^B_{\text{liquid}}$	$\bar{R}^B_{\text{illiquid}}$	$\bar{R}^A$	$\bar{\phi}_{\text{liquid}}$	$\bar{\phi}_{\text{illiquid}}$	$\bar{\theta}$	$\bar{b}_{\text{liquid}}$	$\bar{b}_{\text{illiquid}}$
1.004	1.014	1.002	4.774	-1.700	4.074	6.470	-1.700

Note: Asset returns are expressed as gross quarterly rates. Calculations of returns, portfolio shares, and holdings are provided in Appendix D.3.

8 Policy Implications

Building on our empirical estimates, we examine their implications for optimal policy design. We begin by deriving the optimal target rule that implements the Ramsey plan, which prescribes the relationship between asset returns that the government should maintain in response to economic disturbances. We then compare the returns implied by this rule to historically observed returns, assessing how actual policy outcomes have deviated from the optimal benchmark.

8.1 Optimal Target Rule

Using the asset supply and demand elasticities from our estimation and household calibration, Theorem 1 shows that the government can implement the Ramsey plan through the following target rule:

$$\begin{bmatrix} \hat{R}^B_{\text{liquid},t+1} \\ \hat{R}^B_{\text{illiquid},t+1} \end{bmatrix} = \begin{bmatrix} -0.11 \\ 0.22 \end{bmatrix} \hat{R}^A_{t+1}.$$

This optimal rule prescribes that when expected returns on capital are high, the government should target lower returns on liquid assets while simultaneously targeting higher returns on illiquid assets. Unlike the BB calibration in Section 6, where all financial asset returns move opposite to capital, here the sign pattern reflects the different relationship between each asset class and intermediation: liquid assets are substitutes for capital on bank balance sheets, while illiquid assets are complements.

The intuition behind these contrasting prescriptions stems directly from the asset supply and demand elasticities estimated in Section 7.1. The first row of Table 2 shows that higher liquid rates cause the financial sector's balance sheet to contract, reducing both liquid asset issuance and illiquid asset holdings. Since liquid asset issuance responds more strongly than illiquid asset holdings, capital holdings decline as liquid rates rise. In contrast, the second row of Table 2 shows that higher illiquid rates stimulate both liquid asset issuance and illiquid asset holdings, with liquid asset issuance again dominating the portfolio adjustment. This leads to increased capital holdings, indicating that capital holdings and illiquid assets are complementary.

As in Section 6, quasilinear preferences imply that returns on financial assets equal the

externality gap $\tilde{\boldsymbol{\gamma}}$ under optimal policy (Equation (22)), so the sign of the rule reveals how $\tilde{\boldsymbol{\gamma}}$ comoves with capital returns: it is positive for illiquid assets and negative for liquid assets.

8.2 Comparing Optimal and Historical Policy

To assess the practical relevance of our framework, we compare returns that would have prevailed under the optimal target rule to historically observed returns. This exercise allows us to evaluate how actual policy outcomes have deviated from the optimum and to identify periods where alternative interventions could have improved welfare.

Our approach exploits the structure of the asset market equilibrium conditions. From Equations (28) and (29), observed returns and government asset positions, combined with our estimated elasticities, allow us to back out the implied asset market disturbances $\boldsymbol{\kappa}_\omega \hat{\boldsymbol{\omega}}$ and $\boldsymbol{\epsilon}_\omega \hat{\boldsymbol{\omega}}$. Specifically, given the observed paths of returns $\{\hat{\boldsymbol{R}}_t^A, \hat{\boldsymbol{R}}_t^B\}$ and government quantities $\{\hat{\boldsymbol{b}}_t^G\}$, we can infer the disturbances that rationalize these outcomes as an equilibrium of the private sector's supply and demand system. We then compute the counterfactual returns that would have prevailed had the government instead followed the optimal target rule in response to these same disturbances.

8.2.1 Asset Market Disturbances

Figure 3 displays the implied asset market disturbances over our sample period, 2003Q2–2023Q4. The three panels show disturbances to the capital market ($\boldsymbol{\kappa}_\omega \hat{\boldsymbol{\omega}}$) and to the two financial asset markets ($\boldsymbol{\epsilon}_\omega \hat{\boldsymbol{\omega}}$ for liquid and illiquid assets), all expressed as multiples of financial sector net worth.

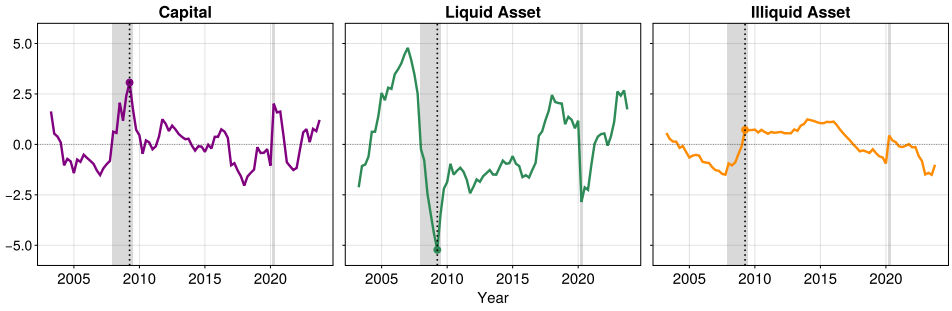


Figure 3: Asset market disturbances $\boldsymbol{\kappa}_\omega \hat{\boldsymbol{\omega}}$ and $\boldsymbol{\epsilon}_\omega \hat{\boldsymbol{\omega}}$ implied by the observed returns, quantities issued by the government, and the estimated elasticities; reported as multiples of net worth.

The disturbances exhibit substantial time variation, with several notable episodes. During the 2008–2009 financial crisis, all three series display sharp movements consistent with a "flight to liquidity": the disturbance to liquid assets turns sharply negative, indicating excess demand, while the disturbances to both illiquid assets and capital turn positive, reflecting excess supply.

A similar, though less pronounced, pattern emerges during the COVID-19 pandemic in early 2020. The initial shock again features a flight toward liquid assets and away from illiquid holdings, though the disturbances reverse more quickly, consistent with the rapid policy response during this episode.

8.2.2 Optimal versus Observed Returns

Figure 4 compares asset returns under the optimal policy to observed returns over the same sample period. The figure plots returns on capital, liquid assets, and illiquid assets as percentage point deviations from their steady-state values. For each asset class, we display both the historically observed return and the return that would have prevailed under the optimal target rule given the backed-out disturbances.

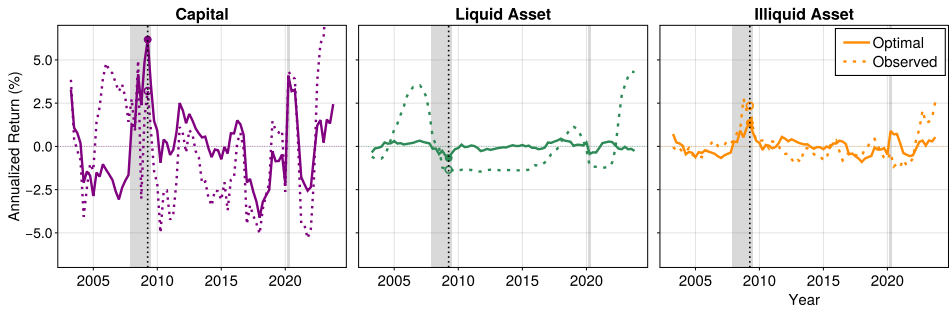


Figure 4: Asset returns under optimal policy given disturbances vs. observed returns, expressed as percentage point deviations from the steady state.

The comparison reveals systematic and often substantial deviations between actual and optimal outcomes. During the 2008–2009 crisis, observed returns on liquid assets fell sharply, more so than the optimal rule would prescribe. Returns on illiquid assets, by contrast, rose well above their optimal counterparts.

The gap between observed and optimal returns measures the deviation of $\hat{\boldsymbol{R}}^B$ from $\tilde{\boldsymbol{\gamma}}$, and hence unrealized welfare gains. During the financial crisis, the observed liquid rate fell below $\tilde{\gamma}_{\text{liquid}}$, while the observed illiquid rate rose above $\tilde{\gamma}_{\text{illiquid}}$. These wedges indicate that the private valuation of liquidity services exceeded the social cost of providing them, and that the private cost of holding illiquid assets exceeded their social value, precisely the pattern one would expect during a flight to liquidity.

The figure also reveals that deviations from optimality were not confined to crisis episodes. Throughout much of the sample, observed returns exhibit persistent gaps relative to the optimal benchmark, suggesting that the historical policy stance, whether through debt management, asset purchases, or other interventions, did not fully exploit the opportunities for welfare improvement identified by our framework.

8.2.3 A Closer Look: 2009Q2

To build further intuition, we examine the second quarter of 2009, a period of acute financial stress following the collapse of Lehman Brothers. Under the optimal policy, returns on financial assets should equal the externality gap: $\beta \hat{\boldsymbol{R}}^B_{t+1} = \tilde{\boldsymbol{\gamma}}_t$. The ob-

served returns and the implied externality gap for this period are (annualized):

$$\beta \begin{bmatrix} \hat{R}^B_{\text{liquid},t+1} \\ \hat{R}^B_{\text{illiquid},t+1} \end{bmatrix} = \begin{bmatrix} -1.33 \\ 2.33 \end{bmatrix}, \quad \begin{bmatrix} \tilde{\gamma}_{\text{liquid},t} \\ \tilde{\gamma}_{\text{illiquid},t} \end{bmatrix} = \begin{bmatrix} -0.51 \\ 0.57 \end{bmatrix}.$$

The wedge between observed returns and the externality gap is substantial for both asset classes. For liquid assets, the observed return deviation (-1.33) exceeds the externality gap (-0.51) in magnitude by a factor of more than two. This gap reflects the intensity of the flight to liquidity: households and financial institutions placed such a high premium on safe, liquid instruments that the private value of liquidity services far exceeded the social cost of expanding their supply. For illiquid assets, the observed return deviation (2.33) similarly dwarfs the externality gap (0.57), indicating that the private cost of holding illiquid securities, whether due to funding pressures, capital constraints, or heightened risk aversion, exceeded their social value by a wide margin.

These gaps point to unrealized welfare gains from more aggressive intervention. The wedge on liquid assets suggests that further issuance of liquid liabilities, expanding the supply of Treasury bills, reserves, or other safe assets, would have reduced the liquid rate toward its optimal level and captured additional welfare gains. The wedge on illiquid assets indicates that additional purchases of agency securities, corporate bonds, or mortgage-backed securities would have compressed spreads and facilitated intermediation. In both cases, the implied policy response aligns with the spirit of the interventions undertaken during this period, including the expansion of the Federal Reserve's balance sheet and the Treasury's various lending facilities. However, our analysis suggests that the scale of intervention fell short of the optimum implied by our framework, at least as measured by the persistent gap between observed returns and the externality benchmark.

9 Conclusion

In this paper, we characterize governments' optimal responses to asset market disturbances. We show that in a large class of models with financial frictions, the Ramsey plan entails a policy rule that targets a specific relationship between asset returns, regardless of the underlying disturbances. The relationship is determined by asset

supply and demand elasticities, which are measurable sufficient statistics capturing features of the economy most relevant for the design of optimal policy.

Our framework omits some key transmission mechanisms, such as time-varying risk premia or nominal rigidities. However, if these features can be characterized by properties of asset supply and demand, we believe it is possible to extend our approach to accommodate them. Another important avenue for future work concerns the empirical structure of the asset supply and demand elasticities. Current empirical works often impose strong restrictions on these elasticities, for example, assuming a static structure, which is not without loss of generality. Developing suitable empirical structures for these elasticities represents a promising avenue for bridging theoretical optimal policy with practical implementation.

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A Proofs and Derivations

A.1 Timing Conventions and Regularity Conditions

Derivative notation. For any composed mapping U_t of the planner's instruments, the notation $[U_t]_{x_s}$ denotes the partial derivative with respect to instrument x_s, holding all multipliers ($\boldsymbol{\gamma}_t$, η_t, ξ_t) fixed; the derivative acts only on allocations and returns.

Timing conventions. All date--1 stock objects $\{n_{-1}, a^F_{-1}, \boldsymbol{b}^F_{-1}, \boldsymbol{b}_{-1}, k_{-1}, q_{-1}\}$ are predetermined, so their derivatives with respect to any instrument x_s vanish. We adopt the convention $\eta_{-1} = 0$, which makes the date--1 boundary term in the Ω absorption well defined.

Regularity conditions. We assume throughout that the following transversality conditions hold for all instruments x_s:

(i) $\lim_{t\to\infty} \beta^{t+1}\xi_{t+1}n_{t+1,x_s} = 0$,
(ii) $\lim_{t\to\infty} k_{t+1}q_{t+1,x_s}/\prod_{\ell=0}^{t} R_{\ell+1}^{A,g} = 0$, where $R_{t+1}^{A,g} \coloneqq R_{t+1}^{A} + \tau_{t+1}^{A}$,
(iii) $\lim_{t\to\infty} \beta^{t+1}\bar{\xi}\,\hat{n}_{t+1,x_s} = 0$ (first-order analogue of (i)).

A.2 The Planner's FOCs

We derive the FOCs for the allocation instruments $x_s \in \{R_{s+1}^{A}, \boldsymbol{R}_{s+1}^{B}, \tau_{s+1}^{A}, \boldsymbol{\tau}_{s+1}^{B}, \tau_{s+1}^{N}, T_s\}$, having resolved $\boldsymbol{b}_t^G$ via (18).

Dividend channel. Because net worth is exogenous, the household's indirect utility v responds to instruments partly through equity income. Using $R_{t+1}^{N}n_t = R_{t+1}^{A}a_t^F - \boldsymbol{R}_{t+1}^{B\intercal}\boldsymbol{b}_t^F$ from (12)–(10), define:

$$\Omega_{x_s} \coloneqq \sum_{t=-1}^{\infty} \beta^{t+1}\lambda_{t+1}\left[R_{t+1}^{A}a_t^F - \boldsymbol{R}_{t+1}^{B\intercal}\boldsymbol{b}_t^F - \tau_{t+1}^{N}n_t - n_{t+1}\right]_{x_s}. \tag{33}$$

This captures the response of the household's net income from equity (dividends minus reinvestment), weighted by the marginal utility of wealth.

Fiscal-externality block. Differentiating the government budget constraint with respect to x_s yields tax-base response terms:

$$\text{feBlock}[x_s] \coloneqq \sum_{t=0}^{\infty} \beta^{t+1}\xi_{t+1}\left(\tau_{t+1}^{A}a_{t,x_s}^{K} + \boldsymbol{\tau}_{t+1}^{B\intercal}\boldsymbol{b}_{t,x_s} + \tau_{t+1}^{N}n_{t,x_s}\right). \tag{34}$$

Direct effects. Define $d_{x_s} \coloneqq v_{x_s}^{H} + v_{x_s}^{G}$, the sum of explicit household envelope derivatives and explicit government budget derivatives:

$$\begin{aligned}
d_{T_s} &= \beta^{s}(\xi_s - \lambda_s),\\
d_{\boldsymbol{\tau}_{s+1}^{B}} &= \beta^{s+1}(\xi_{s+1} - \lambda_{s+1})\boldsymbol{b}_s^{\intercal},\\
d_{\tau_{s+1}^{N}} &= \beta^{s+1}\xi_{s+1}n_s,\\
d_{\boldsymbol{R}_{s+1}^{B}} &= \beta^{s+1}\lambda_{s+1}\boldsymbol{b}_s^{\intercal} - \beta^{s+1}\xi_{s+1}\boldsymbol{b}_s^{G\intercal},\\
d_{R_{s+1}^{A}} &= \textstyle\sum_t \beta^{t}\lambda_t\, w_{t,R_{s+1}^{A}},\\
d_{\tau_{s+1}^{A}} &= \textstyle\sum_t \beta^{t}\lambda_t\, w_{t,\tau_{s+1}^{A}} + \beta^{s+1}\xi_{s+1}a_s^{K}.
\end{aligned} \tag{35}$$

Since production depends on R^A_{s+1} and τ^A_{s+1} only through the sum $R^A_{s+1} + \tau^A_{s+1}$, we have $w_{t,R^A_{s+1}} = w_{t,\tau^A_{s+1}}$.

Planner first-order condition. Differentiating (17) with respect to x_s and applying the envelope theorem:

$$0 = d_{x_s} + \Omega_{x_s} + \sum_{t=0}^{\infty} \beta^t \boldsymbol{\gamma}_t^\intercal \boldsymbol{\epsilon}_{t,x_s} + \sum_{t=0}^{\infty} \beta^t \eta_t \, \kappa_{t,x_s} + \text{feBlock}[x_s]. \tag{36}$$

A.3 Rearrangement and the Intermediary Block

We rearrange $\Omega_{x_s} + \sum_t \beta^t \eta_t \, \kappa_{t,x_s} + \text{feBlock}[x_s]$ in (36) into economically transparent blocks.

Definitions. The spread vector is:

$$\boldsymbol{\Delta}_{t+1} \coloneqq \Big(R^A_{t+1} - \frac{\eta_t}{\beta \xi_{t+1}}\Big)\mathbf{1}_{\mathcal{B}} - \big(\boldsymbol{R}^B_{t+1} - \boldsymbol{\tau}^B_{t+1}\big). \tag{37}$$

Define the planner's capital wedge:

$$\chi_t \coloneqq \beta \xi_{t+1} R^A_{t+1} - \eta_t - \xi_t, \tag{38}$$

so that $\beta^{t+1}\xi_{t+1}(R^A_{t+1} - \eta_t/(\beta\xi_{t+1})) = \beta^t(\xi_t + \chi_t)$. From (18): $\beta^{t+1}\xi_{t+1}\boldsymbol{R}^B_{t+1} = \beta^t(\xi_t \mathbf{1}_{\mathcal{B}} + \boldsymbol{\gamma}_t)$.

Rearrangement. Split $\kappa_t = a^K_t - a^F_t$ and absorb $-\sum \beta^t \eta_t \, a^F_{t,x_s}$ into Ω_{x_s} using the decomposition $\lambda_{t+1} = \xi_{t+1} - (\xi_{t+1} - \lambda_{t+1})$ and the convention $\eta_{-1} = 0$ (Section A.1). Apply the balance-sheet identity $a^F_t = \mathbf{1}^\intercal_{\mathcal{B}} \boldsymbol{b}^F_t + n_t$ to extract the spread $\boldsymbol{\Delta}_{t+1}$. Then absorb the three feBlock components: the τ^N term cancels the τ^N_{t+1} coefficient on n_{t,x_s} in the net-worth piece (with the explicit $d_{\tau^N_{s+1}}$ term offsetting when $x_s = \tau^N_{s+1}$); the $\boldsymbol{\tau}^B$ term shifts from $\boldsymbol{b}$ to $\boldsymbol{b}^G = \boldsymbol{b} - \boldsymbol{b}^F$; and the τ^A term combines with $\sum \beta^t \eta_t a^K_{t,x_s}$

into $\sum \beta^{t+1}\xi_{t+1}(\tau^A_{t+1} + \eta_t/(\beta\xi_{t+1}))a^K_{t,x_s}$. Collecting:

$$0 = d_{x_s} + \underbrace{\sum_{t=-1}^{\infty} \beta^{t+1}\xi_{t+1}\big(\boldsymbol{\Delta}^{\intercal}_{t+1}\boldsymbol{b}^F_t\big)_{x_s}}_{\text{spread block}} + \underbrace{\sum_{t=-1}^{\infty} \beta^{t+1}\xi_{t+1}\Big(\big(R^A_{t+1} - \tfrac{\eta_t}{\beta\xi_{t+1}}\big)n_t - n_{t+1}\Big)_{x_s}}_{\text{net-worth block}}$$

$$+ \underbrace{\sum_{t=0}^{\infty} \beta^{t+1}\xi_{t+1}\big(\tau^A_{t+1} + \tfrac{\eta_t}{\beta\xi_{t+1}}\big)a^K_{t,x_s}}_{\text{capital-wedge block}} - \underbrace{\sum_{t=-1}^{\infty} \beta^{t+1}(\xi_{t+1} - \lambda_{t+1})\big(R^A_{t+1}a^F_t - \boldsymbol{R}^{B}_{t+1}{}^{\intercal}\boldsymbol{b}^F_t - \tau^N_{t+1}n_t - n_{t+1}\big)_{x_s}}_{\text{discount-gap block}}$$

$$+ \sum_{t=0}^{\infty} \beta^t \boldsymbol{\gamma}^{\intercal}_t \boldsymbol{\epsilon}_{t,x_s} + \sum_{t=0}^{\infty} \beta^{t+1}\xi_{t+1}\boldsymbol{\tau}^{B}_{t+1}{}^{\intercal}\boldsymbol{b}^G_{t,x_s}. \tag{39}$$

Intermediary (FI) block. Combine the spread block and net-worth block into a single intermediary block:

$$\mathrm{FI}[x_s] \equiv \sum_{t=-1}^{\infty} \beta^{t+1}\xi_{t+1}\Big[\Big(R^A_{t+1} - \frac{\eta_t}{\beta\xi_{t+1}}\Big)a^F_t - \big(\boldsymbol{R}^B_{t+1} - \boldsymbol{\tau}^B_{t+1}\big)^{\intercal}\boldsymbol{b}^F_t - n_{t+1}\Big]_{x_s}. \tag{40}$$

We separate $\mathrm{FI}[x_s]$ into *response terms* (the derivative hits allocations $a^F_t, \boldsymbol{b}^F_t, n_{t+1}$) and *level terms* (the derivative hits the direct instrument terms $R^A_{t+1}, \boldsymbol{R}^B_{t+1}, \boldsymbol{\tau}^B_{t+1}$, plus the induced R^A_0 term).

FI response. Applying the return-wedge identities to the response part of (40) and using $a^F_{t,x_s} - \mathbf{1}^{\intercal}_{\mathcal{B}}\boldsymbol{b}^F_{t,x_s} = n_{t,x_s}$, the n_{t,x_s} terms telescope intertemporally: each $+\beta^t\xi_t\, n_{t,x_s}$ cancels against $-\beta^{t+1}\xi_{t+1}\, n_{t+1,x_s}$, with $n_{-1,x_s} = 0$ at the boundary and transversality (Section A.1) at infinity. Therefore:

$$\text{FI response}[x_s] = \sum_{t=0}^{\infty} \beta^t \Big[\chi_t\, a^F_{t,x_s} - \big(\boldsymbol{\gamma}_t - \beta\xi_{t+1}\boldsymbol{\tau}^B_{t+1}\big)^{\intercal}\boldsymbol{b}^F_{t,x_s}\Big]. \tag{41}$$

For allocation instruments ($\boldsymbol{b}^G_{t,x_s} = 0$), combining (41) with the $\boldsymbol{\gamma}\boldsymbol{\epsilon}$ and $\boldsymbol{\tau}^B\boldsymbol{b}^G$ terms from (39) and using $\boldsymbol{\epsilon}_{t,x_s} = \boldsymbol{b}^F_{t,x_s} - \boldsymbol{b}_{t,x_s}$ gives:

$$\begin{aligned} &\text{FI response}[x_s] + \sum_t \beta^t \boldsymbol{\gamma}^{\intercal}_t \boldsymbol{\epsilon}_{t,x_s} + \sum_t \beta^{t+1}\xi_{t+1}\boldsymbol{\tau}^{B}_{t+1}{}^{\intercal}\boldsymbol{b}^G_{t,x_s} \\ &\quad = \sum_{t=0}^{\infty} \beta^t \chi_t\, a^F_{t,x_s} - \sum_{t=0}^{\infty} \beta^{t+1}\xi_{t+1}\Big(\frac{\boldsymbol{\gamma}_t}{\beta\xi_{t+1}} - \boldsymbol{\tau}^B_{t+1}\Big)^{\intercal}\boldsymbol{b}_{t,x_s}. \end{aligned} \tag{42}$$

FI level. For $t \geq 0$, the instrument terms in (40) are independent ($R^A_{t+1,x_s} = \delta_{ts}\mathbf{1}_{\{x_s = R^A_{s+1}\}}$, etc.). However, R^A_0 is not an instrument: it is determined by the asset-pricing recursion, so $R^A_{0,x_s} \neq 0$ whenever x_s shifts $R^A_{s+1} + \tau^A_{s+1}$. The level terms are:

$$\text{FI level}[x_s] = \begin{cases} +\beta^{s+1}\xi_{s+1}\, a^F_s + \xi_0\, R^A_{0,x_s}\, a^F_{-1} & \text{if } x_s = R^A_{s+1}, \\ -\beta^{s+1}\xi_{s+1}\, \boldsymbol{b}^{F\intercal}_s & \text{if } x_s = \boldsymbol{R}^B_{s+1}, \\ \xi_0\, R^A_{0,\tau^A_{s+1}}\, a^F_{-1} & \text{if } x_s = \tau^A_{s+1}, \\ +\beta^{s+1}\xi_{s+1}\, \boldsymbol{b}^{F\intercal}_s & \text{if } x_s = \boldsymbol{\tau}^B_{s+1}, \\ 0 & \text{if } x_s \in \{T_s, \tau^N_{s+1}\}. \end{cases} \tag{43}$$

Here $R^A_{0,R^A_{s+1}} = R^A_{0,\tau^A_{s+1}}$ since R^A_0 depends on $R^A_{s+1} + \tau^A_{s+1}$. The capital-wedge block contains only the *response* of a^K_t to x_s; the explicit level term $\beta^{s+1}\xi_{s+1}a^K_s$ when $x_s = \tau^A_{s+1}$ is already captured in $d_{\tau^A_{s+1}}$.

A.4 Homogeneity of the Planner's FOCs

Substituting the results of Section A.3, the rearranged FOC becomes:

$$\begin{aligned} 0 = {} & d_{x_s} + \text{FI level}[x_s] \\ & + \underbrace{\sum_{t=0}^{\infty} \beta^t \chi_t\, a^F_{t,x_s} - \sum_{t=0}^{\infty} \beta^{t+1}\xi_{t+1}\big(\tfrac{\boldsymbol{\gamma}_t}{\beta\xi_{t+1}} - \boldsymbol{\tau}^B_{t+1}\big)^{\intercal}\boldsymbol{b}_{t,x_s}}_{\text{from (42)}} \\ & + \underbrace{\sum_{t=0}^{\infty} \beta^{t+1}\xi_{t+1}\big(\tau^A_{t+1} + \tfrac{\eta_t}{\beta\xi_{t+1}}\big)a^K_{t,x_s}}_{\text{capital-wedge block}} - \underbrace{\sum_{t=-1}^{\infty} \beta^{t+1}(\xi_{t+1} - \lambda_{t+1})(\cdots)_{x_s}}_{\text{discount-gap block}}. \end{aligned} \tag{44}$$

Four wedge objects. The following objects are zero at the first-best allocation:

(a) $(\xi_t - \lambda_t)$: private–public discount gap,
(b) $\chi_t = \beta\xi_{t+1}R^A_{t+1} - \eta_t - \xi_t$: capital wedge,
(c) $(\tau^A_{t+1} + \eta_t/(\beta\xi_{t+1}))$: capital-tax wedge,
(d) $(\boldsymbol{\gamma}_t/(\beta\xi_{t+1}) - \boldsymbol{\tau}^B_{t+1})$: bond-tax wedge.

The FOC is **homogeneous in these wedges** if every term in (44) is proportional to

at least one of (a)–(d). The last three blocks are manifestly homogeneous. It remains to verify that $d_{x_s} + \text{FI level}[x_s]$ is also proportional to a wedge.

Case 1: $x_s = T_s$**.** From (35): $d_{T_s} = \beta^s(\xi_s - \lambda_s)$. From (43): FI level$[T_s] = 0$. Therefore $d_{T_s} + \text{FI level}[T_s] = \beta^s(\xi_s - \lambda_s)$.

Case 2: $x_s = \boldsymbol{R}^B_{s+1}$**.** From (35) and (43):

$$\begin{aligned} d_{\boldsymbol{R}^B_{s+1}} + \text{FI level}[\boldsymbol{R}^B_{s+1}] &= \beta^{s+1}\lambda_{s+1}\boldsymbol{b}_s^\intercal - \beta^{s+1}\xi_{s+1}(\boldsymbol{b}_s^G + \boldsymbol{b}_s^F)^\intercal \\ &= -\beta^{s+1}(\xi_{s+1} - \lambda_{s+1})\boldsymbol{b}_s^\intercal. \end{aligned}$$

where the second equality uses bond market clearing $\boldsymbol{b}_s^G + \boldsymbol{b}_s^F = \boldsymbol{b}_s$.

Case 3: $x_s = R^A_{s+1}$ **vs.** τ^A_{s+1}**.** Consider the difference FOC$[R^A_{s+1}]$− FOC$[\tau^A_{s+1}]$. Production-side objects depend on R^A_{s+1} and τ^A_{s+1} only through the sum $R^A_{s+1} + \tau^A_{s+1}$, so $a^K_{t,R^A_{s+1}} = a^K_{t,\tau^A_{s+1}}$ and $w_{t,R^A_{s+1}} = w_{t,\tau^A_{s+1}}$. The nonproduction responses a^F_{t,x_s} and $\boldsymbol{b}_{t,x_s}$ need not coincide, but these enter only the manifestly homogeneous blocks, so the difference remains homogeneous.

For the residual $[d_{R^A_{s+1}} + \text{FI level}[R^A_{s+1}]] - [d_{\tau^A_{s+1}} + \text{FI level}[\tau^A_{s+1}]]$: the wage terms cancel ($w_{t,R^A_{s+1}} = w_{t,\tau^A_{s+1}}$), as do the R^A_0 terms ($R^A_{0,R^A_{s+1}} = R^A_{0,\tau^A_{s+1}}$). The surviving terms are $\beta^{s+1}\xi_{s+1}a^F_s - \beta^{s+1}\xi_{s+1}a^K_s = -\beta^{s+1}\xi_{s+1}\kappa_s = 0$ by capital market clearing.

Main result. Define the manifestly homogeneous part of (44):

$$\begin{aligned} \mathcal{H}[x_s] \coloneqq & \sum_{t=0}^{\infty} \beta^t \chi_t \, a^F_{t,x_s} - \sum_{t=0}^{\infty} \beta^{t+1}\xi_{t+1}\big(\tfrac{\gamma_t}{\beta\xi_{t+1}} - \boldsymbol{\tau}^B_{t+1}\big)^\intercal \boldsymbol{b}_{t,x_s} \\ & + \sum_{t=0}^{\infty} \beta^{t+1}\xi_{t+1}\big(\tau^A_{t+1} + \tfrac{\eta_t}{\beta\xi_{t+1}}\big)a^K_{t,x_s} \\ & - \sum_{t=-1}^{\infty} \beta^{t+1}(\xi_{t+1} - \lambda_{t+1})\big(R^A_{t+1}a^F_t - \boldsymbol{R}^B_{t+1}{}^\intercal \boldsymbol{b}^F_t - \tau^N_{t+1}n_t - n_{t+1}\big)_{x_s}. \end{aligned} \tag{45}$$

Since a^K_t depends on $R^A_{t+1} + \tau^A_{t+1}$ only, the capital-wedge block drops out of the three FOCs for T_s, $\boldsymbol{R}^B_{s+1}$, and the difference R^A_{s+1} vs. τ^A_{s+1}. Substituting the residuals from

Cases 1–3:

$$\text{(i)} \quad 0 = \beta^s(\xi_s - \lambda_s) + \mathcal{H}[T_s], \tag{46}$$

$$\text{(ii)} \quad 0 = -\beta^{s+1}(\xi_{s+1} - \lambda_{s+1})\boldsymbol{b}_s^{\intercal} + \mathcal{H}[\boldsymbol{R}_{s+1}^B], \tag{47}$$

$$\text{(iii)} \quad 0 = \mathcal{H}[R_{s+1}^A] - \mathcal{H}[\tau_{s+1}^A]. \tag{48}$$

Proposition 1 (Three-wedge homogeneity) *Conditions* (46)–(48) *constitute a system that is homogeneous in* $(\xi_t - \lambda_t)$, χ_t, *and* $(\boldsymbol{\gamma}_t/(\beta\xi_{t+1}) - \boldsymbol{\tau}_{t+1}^B)$. *If the associated coefficient matrix, evaluated at the equilibrium, has full rank, then* $\xi_t = \lambda_t$, $\chi_t = 0$, *and* $\boldsymbol{\gamma}_t = \beta\xi_{t+1}\boldsymbol{\tau}_{t+1}^B$ *for all* t.

Remark *The* $\boldsymbol{\tau}_{s+1}^B$ *FOC includes a nonzero FI-level term* $\text{FI level}[\boldsymbol{\tau}_{s+1}^B] = \beta^{s+1}\xi_{s+1}\boldsymbol{b}_s^{F\intercal}$ *and is not used in the homogeneity system* (46)–(48). *The* τ_{s+1}^N *FOC adds no independent restriction: under the conditions of Proposition 1, its non-homogeneous front terms reduce to* $\beta^{s+1}(\xi_{s+1} - \lambda_{s+1})n_s$, *and the remaining terms are proportional to the three wedges.*

A.5 The Fourth Wedge

Conditional on $\xi_t = \lambda_t$, $\chi_t = 0$, and $\boldsymbol{\gamma}_t = \beta\xi_{t+1}\boldsymbol{\tau}_{t+1}^B$ (Proposition 1), the FOC for τ_{s+1}^A pins down the remaining wedge $(\tau_{t+1}^A + \eta_t/(\beta\xi_{t+1}))$. Write $R_{t+1}^{A,g} \coloneqq R_{t+1}^A + \tau_{t+1}^A$.

CRS identity. From CRS production with $h_t \equiv 1$: $F(1, k_{t-1}) = w_t + \varrho_t k_{t-1}$. Differentiating with respect to τ_{s+1}^A and using $F_k = \varrho_t$:

$$\varrho_{t,\tau_{s+1}^A} \cdot k_{t-1} = -w_{t,\tau_{s+1}^A}. \tag{49}$$

Asset-pricing identity. Differentiating (6) with respect to τ_{s+1}^A, applying the envelope theorem, and telescoping the $k_t q_{t,\tau_{s+1}^A}$ terms using (49) and transversality (Section A.1):

$$a_{-1}^K R_{0,\tau_{s+1}^A}^A = \sum_{t=0}^{\infty} \frac{-w_{t+1,\tau_{s+1}^A}}{\prod_{\ell=0}^t R_{\ell+1}^{A,g}} - \frac{a_s^K}{\prod_{\ell=0}^s R_{\ell+1}^{A,g}}. \tag{50}$$

Multiplier recursion. From $\chi_t = \beta\xi_{t+1}R^A_{t+1} - \eta_t - \xi_t$, iterating:

$$\beta^{t+1}\xi_{t+1} = \frac{\xi_0}{\prod_{\ell=0}^{t} R^{A,g}_{\ell+1}} + \sum_{\ell=0}^{t} \frac{\beta^\ell\big(\chi_\ell + \eta_\ell + \beta\xi_{\ell+1}\tau^A_{\ell+1}\big)}{\prod_{m=\ell}^{t} R^{A,g}_{m+1}}. \tag{51}$$

Homogeneity of the τ^A FOC. Under $\xi_t = \lambda_t$ and $\chi_t = 0$, (44) reduces to:

$$0 = d_{\tau^A_{s+1}} + \text{FI level}[\tau^A_{s+1}] + \sum_{t=0}^{\infty} \beta^{t+1}\xi_{t+1}\big(\tau^A_{t+1} + \tfrac{\eta_t}{\beta\xi_{t+1}}\big)a^K_{t,\tau^A_{s+1}}. \tag{52}$$

The residual $d_{\tau^A_{s+1}} + \text{FI level}[\tau^A_{s+1}]$ consists of three pieces (using $a^F_{-1} = a^K_{-1}$):

(a) *Wages:* $\sum_{t=0}^{\infty} \beta^{t+1}\xi_{t+1}\, w_{t+1,\tau^A_{s+1}}$,
(b) *FI level:* $\xi_0\, R^A_{0,\tau^A_{s+1}}\, a^K_{-1}$,
(c) *Govt budget:* $\beta^{s+1}\xi_{s+1}\, a^K_s$.

Under $\chi = 0$, the multiplier recursion (51) reduces to $\beta^{t+1}\xi_{t+1} = \xi_0/\prod R^{A,g} + \text{terms} \propto (\eta + \beta\xi\tau^A)$. Substituting into (a) and (c), and applying (50) to (b), the ξ_0 pieces cancel pairwise: the wage sums in (a) and (b), and the $a^K_s/\prod R^{A,g}$ terms in (b) and (c). The bracket

$$\xi_0\left[\sum_{t=0}^{\infty} \frac{w_{t+1,\tau^A_{s+1}}}{\prod_{\ell=0}^{t} R^{A,g}_{\ell+1}} + R^A_{0,\tau^A_{s+1}}\, a^K_{-1} + \frac{a^K_s}{\prod_{\ell=0}^{s} R^{A,g}_{\ell+1}}\right] = 0$$

vanishes by the asset-pricing identity. The surviving terms are all proportional to $(\eta_\ell + \beta\xi_{\ell+1}\tau^A_{\ell+1})$.

Proposition 2 (Fourth wedge) *Under the conditions of Proposition 1 ($\xi_t = \lambda_t$, $\chi_t = 0$, $\boldsymbol{\gamma}_t = \beta\xi_{t+1}\boldsymbol{\tau}^B_{t+1}$), the FOC for τ^A_{s+1} (52) is homogeneous in $(\eta_t + \beta\xi_{t+1}\tau^A_{t+1})$. If the associated coefficient matrix, evaluated at the equilibrium, has full rank, then $\eta_t + \beta\xi_{t+1}\tau^A_{t+1} = 0$ for all t.*

The cancellation relies on three ingredients: CRS production (49), the asset-pricing identity (50), and the multiplier recursion (51).

A.6 First-Order Expansion

We linearize the planner FOCs around the optimal steady state. Let $\bar{z}$ denote the steady-state value of z_t and $\hat{z}_t \coloneqq z_t - \bar{z}$ the first-order deviation. We write $\bar{U}_{t,x_s}$ for total derivatives evaluated at steady state.

A.6.1 Steady-State Restrictions and Fixed-Tax Convention

Fixed-tax convention. Set $\hat{\tau}^A \equiv 0$, $\hat{\boldsymbol{\tau}}^B \equiv 0$, and $\hat{\tau}^N \equiv 0$: macroprudential taxes are held at their Ramsey values. The planner adjusts $\boldsymbol{b}_t^G$ and T_t in response to shocks.

Steady-state restrictions. Propositions 1 and 2 imply:

(i) $\bar{\xi} = \bar{\lambda}$ (discount gap vanishes),
(ii) $\bar{\chi} = 0$, equivalently $\beta \bar{R}^{A,g} = 1$ (capital wedge vanishes),
(iii) $\bar{\boldsymbol{\gamma}} = \beta \bar{\xi}\, \bar{\boldsymbol{\tau}}^B$ (bond-tax wedge vanishes),
(iv) $\bar{\eta} + \beta \bar{\xi}\, \bar{\tau}^A = 0$ (capital-tax wedge vanishes).

A.6.2 Tilde Wedge-Deviation Variables

Define three objects capturing first-order deviations of the wedges:

$$\tilde{\boldsymbol{\gamma}}_t \coloneqq \hat{\boldsymbol{\gamma}}_t - \beta \hat{\xi}_{t+1} \bar{\boldsymbol{\tau}}^B, \tag{53}$$

$$\tilde{\eta}_t \coloneqq \hat{\eta}_t + \beta \hat{\xi}_{t+1} \bar{\tau}^A, \tag{54}$$

$$\tilde{\xi}_t \coloneqq \hat{\xi}_t - \hat{\lambda}_t. \tag{55}$$

Under the fixed-tax convention, $\tilde{\boldsymbol{\gamma}}_t$ is the hat of $(\boldsymbol{\gamma}_t - \beta \xi_{t+1} \boldsymbol{\tau}_{t+1}^B)$, $\tilde{\eta}_t$ the hat of $(\eta_t + \beta \xi_{t+1} \tau_{t+1}^A)$, and $\tilde{\xi}_t$ the hat of $(\xi_t - \lambda_t)$; all vanish at steady state by (i)–(iv).

A.6.3 Linearized FOC for $\boldsymbol{b}^G$

Linearizing (18) and substituting $\bar{\boldsymbol{R}}^B = \beta^{-1} \mathbf{1}_{\mathcal{B}} + \bar{\boldsymbol{\tau}}^B$ (from conditions (i) and (iii)), then decomposing $\hat{\xi} = \tilde{\xi} + \hat{\lambda}$:

$$\tilde{\boldsymbol{\gamma}}_t = \beta \bar{\xi}\, \hat{\boldsymbol{R}}_{t+1}^B + (\tilde{\xi}_{t+1} - \tilde{\xi}_t) \mathbf{1}_{\mathcal{B}} + (\hat{\lambda}_{t+1} - \hat{\lambda}_t) \mathbf{1}_{\mathcal{B}}. \tag{56}$$

A.6.4 Linearized Allocation FOCs

Linearized first-order condition. Define $M_{t,x_s} \coloneqq \tau_{t+1}^A a_{t,x_s}^K + \boldsymbol{\tau}_{t+1}^{B\intercal} \boldsymbol{b}_{t,x_s} + \tau_{t+1}^N n_{t,x_s}$ and $E_{t,x_s} \coloneqq (R_{t+1}^A a_t^F - \boldsymbol{R}_{t+1}^{B\intercal} \boldsymbol{b}_t^F - \tau_{t+1}^N n_t - n_{t+1})_{x_s}$. Applying the product rule to

(36):

$$0 = \hat{d}_{x_s} + \hat{\Omega}_{x_s} + \sum_{t=0}^{\infty} \beta^t (\hat{\boldsymbol{\gamma}}_t^{\intercal} \bar{\boldsymbol{\epsilon}}_{t,x_s} + \bar{\boldsymbol{\gamma}}^{\intercal} \hat{\boldsymbol{\epsilon}}_{t,x_s}) + \sum_{t=0}^{\infty} \beta^t (\hat{\eta}_t \bar{\kappa}_{t,x_s} + \bar{\eta}\, \hat{\kappa}_{t,x_s}) + \sum_{t=0}^{\infty} \beta^{t+1} \left(\hat{\xi}_{t+1} \bar{M}_{t,x_s} + \bar{\xi}\, \hat{M}_{t,x_s} \right). \tag{57}$$

Elimination of level-multiplier terms. For allocation instruments $x_s \in \{T_s, \boldsymbol{R}^B_{s+1}, R^A_{s+1}\}$ ($\boldsymbol{b}^G_{t,x_s} = 0$), add and subtract the steady-state tax terms inside $\hat{E}_{t,x_s}$ and use $\bar{\lambda} = \bar{\xi}$. The terms $\bar{\boldsymbol{\gamma}}^{\intercal} \hat{\boldsymbol{\epsilon}}_{t,x_s}$ and $\bar{\eta} \hat{\kappa}_{t,x_s}$ combine with the feBlock and $\hat{\Omega}$ hats into

$$(\bar{\boldsymbol{\gamma}} - \beta \bar{\xi} \bar{\boldsymbol{\tau}}^B)^{\intercal} \hat{\boldsymbol{\epsilon}}_{t,x_s} + (\bar{\eta} + \beta \bar{\xi} \bar{\tau}^A) \hat{\kappa}_{t,x_s} + \text{gross-return and } \hat{n} \text{ terms.}$$

By steady-state conditions (iii)–(iv), the first bracket vanishes. The $\hat{n}_{t,x_s}$ terms telescope (using $\beta \bar{R}^{A,g} = 1$ and transversality), canceling all net-worth response terms from the linearized system.

Return-instrument identity. For return instruments $x_s \in \{R^A_{s+1}, \boldsymbol{R}^B_{s+1}\}$ and $t \geq 0$:

$$\bar{E}_{t,x_s} = \left(\bar{a}^F_t\, \mathbf{1}_{\{x_s = R^A_{s+1}\}} - \bar{\boldsymbol{b}}^{F\intercal}_t \mathbf{1}_{\{x_s = \boldsymbol{R}^B_{s+1}\}} \right) \mathbf{1}_{\{t=s\}} + (\beta^{-1} - \bar{\tau}^N) \bar{n}_{t,x_s} - \bar{n}_{t+1,x_s} - \bar{\tau}^A \bar{a}^F_{t,x_s} - \bar{\boldsymbol{\tau}}^{B\intercal} \bar{\boldsymbol{b}}^F_{t,x_s}. \tag{58}$$

The ζ^K object. Define

$$\zeta^K_{t,R^A_{s+1}} \coloneqq \sum_{\ell=t}^{\infty} \beta^{\ell-t+1} \bar{w}_{\ell+1,R^A_{s+1}} + \beta^{s-t+1} \bar{a}^K\, \mathbf{1}_{\{t \leq s\}}. \tag{59}$$

This is the discounted production-side stream from date t for a unit change in R^A_{s+1}. Abel summation converts the $\hat{\lambda}_t \bar{w}_{t,R^A_{s+1}}$ block into $(\hat{\lambda}_{t+1} - \hat{\lambda}_t) \zeta^K_{t,R^A_{s+1}}$ terms:

$$\sum_{\ell=0}^{\infty} \beta^{\ell} \hat{\lambda}_{\ell} \bar{w}_{\ell,R^A_{s+1}} + \beta^{s+1} \hat{\lambda}_{s+1} \bar{a}^K = \sum_{t=0}^{\infty} \beta^t (\hat{\lambda}_{t+1} - \hat{\lambda}_t) \zeta^K_{t,R^A_{s+1}} - \hat{\lambda}_0\, \overline{R^A_{0,R^A_{s+1}}}\, \bar{a}^K_{-1}. \tag{60}$$

The boundary term $-\hat{\lambda}_0 \overline{R^A_{0,R^A_{s+1}}}\,\bar{a}^K_{-1}$ is absorbed when combining with the date--1 terms. Similarly, the linearized asset-pricing identity yields

$$\sum_{\ell=0}^{\infty} \beta^\ell \hat{w}_{\ell,R^A_{s+1}} + \bar{a}^K_{-1}\widehat{R^A_{0,R^A_{s+1}}} = -\beta^{s+1}\hat{a}^K_s + \sum_{t=0}^{\infty} \beta^t \big(\beta \hat{R}^A_{t+1}\big)\zeta^K_{t,R^A_{s+1}}, \tag{61}$$

so both $\hat{\lambda}$ and $\hat{R}^A$ load on ζ^K: the first through Abel summation of the wage block, the second through the asset-pricing identity that offsets the direct wage hats against the capital-return revaluation channel.

Remark *Both $(\hat{\lambda}_{t+1} - \hat{\lambda}_t)$ and $\beta\bar{\lambda}\hat{R}^A_{t+1}$ multiply the same object $\zeta^K_{t,R^A_{s+1}}$. This is because the production side operates through a single discounted stream of wages and capital values, and any valuation force, whether from intertemporal consumption shifts or from return changes, acts on that same stream. Under CRS, the wage-side and capital-side effects offset except for the revaluation and repricing terms that load on ζ^K.*

Parallel summary. Define the shared core block

$$\begin{aligned}\mathcal{S}_{x_s} \coloneqq & \sum_{t=0}^{\infty} \beta^t \tilde{\boldsymbol{\gamma}}_t^{\intercal} \bar{\boldsymbol{\epsilon}}_{t,x_s} + \sum_{t=0}^{\infty} \beta^t \tilde{\eta}_t\, \bar{\kappa}_{t,x_s} + \sum_{t=0}^{\infty} \beta^{t+1}\tilde{\xi}_{t+1}\Big(\bar{\tau}^A \bar{a}^F_{t,x_s} + \bar{\boldsymbol{\tau}}^{B\intercal}\bar{\boldsymbol{b}}^F_{t,x_s} + \bar{\tau}^N \bar{n}_{t,x_s}\Big) \\ & + \sum_{t=0}^{\infty} \beta^t \Big[\beta\bar{\xi}\Big(\hat{R}^A_{t+1}\bar{a}^F_{t,x_s} - \hat{\boldsymbol{R}}^{B\intercal}_{t+1}\bar{\boldsymbol{b}}^F_{t,x_s}\Big) + \big(\hat{\lambda}_{t+1} - \hat{\lambda}_t\big)\bar{n}_{t,x_s}\Big].\end{aligned} \tag{62}$$

Then the three allocation FOCs can be written in parallel as

$$0 = \beta^s \tilde{\xi}_s + \mathcal{S}_{T_s}, \tag{63a}$$

$$0 = -\beta^{s+1}\tilde{\xi}_{s+1}\bar{\boldsymbol{b}}^{G\intercal}_s + \mathcal{S}_{\boldsymbol{R}^B_{s+1}}, \tag{63b}$$

$$0 = \sum_{t=0}^{\infty} \beta^t \Big(\hat{\lambda}_{t+1} - \hat{\lambda}_t + \beta\bar{\lambda}\hat{R}^A_{t+1}\Big)\zeta^K_{t,R^A_{s+1}} + \mathcal{S}_{R^A_{s+1}}. \tag{63c}$$

The three equations differ only in the instrument-specific front block: $\beta^s\tilde{\xi}_s$ for T_s, $-\beta^{s+1}\tilde{\xi}_{s+1}\bar{\boldsymbol{b}}^{G\intercal}_s$ for $\boldsymbol{R}^B_{s+1}$, and the ζ^K block for R^A_{s+1}.

A.7 Matrix-Form Target Rule

We derive the target rule in Theorem 1 from the parallel FOC system (63).

Stacked notation. Stack discounted sequences as infinite-dimensional vectors:

$$\tilde{\boldsymbol{\xi}} := [\beta^t \tilde{\xi}_t], \quad \tilde{\boldsymbol{\gamma}} := [\beta^t \tilde{\boldsymbol{\gamma}}_t], \quad \tilde{\boldsymbol{\eta}} := [\beta^t \tilde{\eta}_t], \quad \hat{\boldsymbol{R}}^A := [\beta^{t+1} \hat{R}^A_{t+1}], \quad \hat{\boldsymbol{R}}^B := [\beta^{t+1} \hat{\boldsymbol{R}}^B_{t+1}],$$

$$\hat{\boldsymbol{R}}^* := [\beta^{t+1} \hat{R}^*_{t+1}], \quad \Delta\hat{\boldsymbol{\lambda}} := [\beta^t (\hat{\lambda}_{t+1} - \hat{\lambda}_t)].$$

Define the lead operator $(Jz)_t := z_{t+1}$ and the wedge vector $\boldsymbol{m} := (\tilde{\boldsymbol{\xi}}, \tilde{\boldsymbol{\gamma}}, \tilde{\boldsymbol{\eta}})^\intercal$.

Operator definitions. For instrument block $x \in \{T, R^B, R^A\}$, define adjoint operators from the Jacobians in $\mathcal{S}_{x_s}$:

$$\left(\mathcal{E}_x^\intercal \tilde{\boldsymbol{\gamma}}\right)_s = \sum_t \bar{\boldsymbol{\epsilon}}^\intercal_{t,x_s} [\tilde{\boldsymbol{\gamma}}]_t, \quad \left(\mathcal{K}_x^\intercal \tilde{\boldsymbol{\eta}}\right)_s = \sum_t \bar{\kappa}_{t,x_s} [\tilde{\boldsymbol{\eta}}]_t, \quad \left(\mathcal{T}_x^\intercal z\right)_s = \sum_t \left(\bar{\tau}^A \bar{a}^F_{t,x_s} + \bar{\boldsymbol{\tau}}^{B\intercal} \bar{\boldsymbol{b}}^F_{t,x_s} + \bar{\tau}^N \bar{n}_{t,x_s}\right) z_t,$$

and $(\mathcal{A}_x^\intercal r)_s = \sum_t \bar{a}^F_{t,x_s} r_t$, $(\mathcal{B}_x^\intercal r^B)_s = \sum_t \bar{\boldsymbol{b}}^{F\intercal}_{t,x_s} r^B_t$, $(\mathcal{N}_x^\intercal d)_s = \sum_t \bar{n}_{t,x_s} d_t$, $((\mathcal{B}^G)^\intercal z)_s = \bar{\boldsymbol{b}}^G_s z_s$, $((\mathcal{Z}^K)^\intercal v)_s = \sum_m \zeta^K_{m,R^A_{s+1}} v_m$.

The linear system. The parallel FOCs (63) become $\mathcal{A}\boldsymbol{m} = -\boldsymbol{b}$, where

$$\mathcal{A} = \begin{bmatrix} I + \mathcal{T}_T^\intercal J & \mathcal{E}_T^\intercal & \mathcal{K}_T^\intercal \\ -(\mathcal{B}^G)^\intercal J + \mathcal{T}_B^\intercal J & \mathcal{E}_B^\intercal & \mathcal{K}_B^\intercal \\ \mathcal{T}_A^\intercal J & \mathcal{E}_A^\intercal & \mathcal{K}_A^\intercal \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} \boldsymbol{g}_T \\ \boldsymbol{g}_B \\ \boldsymbol{g}_A + (\mathcal{Z}^K)^\intercal (\Delta\hat{\boldsymbol{\lambda}} + \bar{\lambda}\hat{\boldsymbol{R}}^A) \end{bmatrix},$$

and the given-term vectors are $\boldsymbol{g}_x = \bar{\lambda}(\mathcal{A}_x^\intercal \hat{\boldsymbol{R}}^A - \mathcal{B}_x^\intercal \hat{\boldsymbol{R}}^B) + \mathcal{N}_x^\intercal \Delta\hat{\boldsymbol{\lambda}}$.

Eliminating $\tilde{\boldsymbol{\eta}}$. Define $\boldsymbol{h}_A := \boldsymbol{g}_A + (\mathcal{Z}^K)^\intercal (\Delta\hat{\boldsymbol{\lambda}} + \bar{\lambda}\hat{\boldsymbol{R}}^A)$. Assuming $(-\mathcal{K}_A^\intercal)$ is invertible, the R^A row gives

$$\tilde{\boldsymbol{\eta}} = (-\mathcal{K}_A^\intercal)^{-1} \left(\mathcal{E}_A^\intercal \tilde{\boldsymbol{\gamma}} + \mathcal{T}_A^\intercal J \tilde{\boldsymbol{\xi}} + \boldsymbol{h}_A\right). \tag{64}$$

Substituting (64) into the T and R^B rows, define the reduced operators ($x \in \{T, B\}$):

$$\check{\mathcal{E}}_x^\intercal := \mathcal{E}_x^\intercal + \mathcal{K}_x^\intercal (-\mathcal{K}_A^\intercal)^{-1} \mathcal{E}_A^\intercal, \qquad \check{\mathcal{T}}_x^\intercal := \mathcal{T}_x^\intercal + \mathcal{K}_x^\intercal (-\mathcal{K}_A^\intercal)^{-1} \mathcal{T}_A^\intercal,$$

$$\check{\boldsymbol{g}}_x := \boldsymbol{g}_x + \mathcal{K}_x^\intercal (-\mathcal{K}_A^\intercal)^{-1} \boldsymbol{h}_A.$$

The reduced two-equation system is

$$\underbrace{\begin{bmatrix} I + \check{\mathcal{T}}_T^\intercal J & \check{\mathcal{E}}_T^\intercal \\ -(\mathcal{B}^G)^\intercal J + \check{\mathcal{T}}_B^\intercal J & \check{\mathcal{E}}_B^\intercal \end{bmatrix}}_{\mathcal{A}_{\text{red}}} \begin{bmatrix} \tilde{\boldsymbol{\xi}} \\ \tilde{\boldsymbol{\gamma}} \end{bmatrix} = - \begin{bmatrix} \check{\boldsymbol{g}}_T \\ \check{\boldsymbol{g}}_B \end{bmatrix}.$$

Eliminating $\tilde{\boldsymbol{\xi}}$. Define

$$\mathcal{M}_T^\intercal := I + \check{\mathcal{T}}_T^\intercal J, \qquad \mathcal{M}_B^\intercal := -(\mathcal{B}^G)^\intercal J + \check{\mathcal{T}}_B^\intercal J.$$

From the first row, $\tilde{\boldsymbol{\xi}} = (-\mathcal{M}_T^\intercal)^{-1}(\check{\mathcal{E}}_T^\intercal \tilde{\boldsymbol{\gamma}} + \check{\boldsymbol{g}}_T)$. Substituting into the second row and defining

$$\check{\mathcal{E}}_{B|T}^\intercal := \check{\mathcal{E}}_B^\intercal + \mathcal{M}_B^\intercal(-\mathcal{M}_T^\intercal)^{-1}\check{\mathcal{E}}_T^\intercal, \qquad \check{\boldsymbol{g}}_{B|T} := \check{\boldsymbol{g}}_B + \mathcal{M}_B^\intercal(-\mathcal{M}_T^\intercal)^{-1}\check{\boldsymbol{g}}_T,$$

the system reduces to $\check{\mathcal{E}}_{B|T}^\intercal \tilde{\boldsymbol{\gamma}} + \check{\boldsymbol{g}}_{B|T} = \mathbf{0}$. If $\check{\mathcal{E}}_{B|T}^\intercal$ is invertible, then $\tilde{\boldsymbol{\gamma}} = -(\check{\mathcal{E}}_{B|T}^\intercal)^{-1}\check{\boldsymbol{g}}_{B|T}$.

Target rule. Using $\bar{\xi} = \bar{\lambda}$ and $\Delta\hat{\boldsymbol{\lambda}} = -\bar{\lambda}\hat{\boldsymbol{R}}^*$ (neutral-rate condition), the forcing vectors decompose linearly in $(\hat{\boldsymbol{R}}^A, \hat{\boldsymbol{R}}^B, \hat{\boldsymbol{R}}^*)$. Define

$$\check{\mathcal{A}}_x^\intercal := \mathcal{A}_x^\intercal + \mathcal{K}_x^\intercal(-\mathcal{K}_A^\intercal)^{-1}\big(\mathcal{A}_A^\intercal + (\mathcal{Z}^K)^\intercal\big),$$

$$\check{\mathcal{B}}_x^\intercal := \mathcal{B}_x^\intercal + \mathcal{K}_x^\intercal(-\mathcal{K}_A^\intercal)^{-1}\mathcal{B}_A^\intercal, \qquad \check{\mathcal{N}}_x^\intercal := \mathcal{N}_x^\intercal + \mathcal{K}_x^\intercal(-\mathcal{K}_A^\intercal)^{-1}\big(\mathcal{N}_A^\intercal + (\mathcal{Z}^K)^\intercal\big),$$

and the transfer-compensated versions:

$$\check{\mathcal{A}}_{B|T}^\intercal := \check{\mathcal{A}}_B^\intercal + \mathcal{M}_B^\intercal(-\mathcal{M}_T^\intercal)^{-1}\check{\mathcal{A}}_T^\intercal, \qquad \check{\mathcal{B}}_{B|T}^\intercal := \check{\mathcal{B}}_B^\intercal + \mathcal{M}_B^\intercal(-\mathcal{M}_T^\intercal)^{-1}\check{\mathcal{B}}_T^\intercal,$$

$$\check{\mathcal{N}}_{B|T}^\intercal := \check{\mathcal{N}}_B^\intercal + \mathcal{M}_B^\intercal(-\mathcal{M}_T^\intercal)^{-1}\check{\mathcal{N}}_T^\intercal.$$

The externality gap is then $\tilde{\boldsymbol{\gamma}} = \bar{\lambda}\big(\Gamma_\epsilon^A \hat{\boldsymbol{R}}^A - \Gamma_\epsilon^B \hat{\boldsymbol{R}}^B - \Gamma_\epsilon^* \hat{\boldsymbol{R}}^*\big)$, with

$$\Gamma_\epsilon^A := -(\check{\mathcal{E}}_{B|T}^\intercal)^{-1}\check{\mathcal{A}}_{B|T}^\intercal, \qquad \Gamma_\epsilon^B := -(\check{\mathcal{E}}_{B|T}^\intercal)^{-1}\check{\mathcal{B}}_{B|T}^\intercal, \qquad \Gamma_\epsilon^* := -(\check{\mathcal{E}}_{B|T}^\intercal)^{-1}\check{\mathcal{N}}_{B|T}^\intercal.$$

For the valuation gap, define

$$\check{\mathcal{A}}_{\Delta T}^\intercal := (-\mathcal{M}_T^\intercal)^{-1}\Big[\check{\mathcal{A}}_T^\intercal - \check{\mathcal{E}}_T^\intercal(\check{\mathcal{E}}_{B|T}^\intercal)^{-1}\check{\mathcal{A}}_{B|T}^\intercal\Big],$$

$$\check{\mathcal{B}}_{\Delta T}^\intercal := (-\mathcal{M}_T^\intercal)^{-1}\Big[\check{\mathcal{B}}_T^\intercal - \check{\mathcal{E}}_T^\intercal(\check{\mathcal{E}}_{B|T}^\intercal)^{-1}\check{\mathcal{B}}_{B|T}^\intercal\Big], \qquad \check{\mathcal{N}}_{\Delta T}^\intercal := (-\mathcal{M}_T^\intercal)^{-1}\Big[\check{\mathcal{N}}_T^\intercal - \check{\mathcal{E}}_T^\intercal(\check{\mathcal{E}}_{B|T}^\intercal)^{-1}\check{\mathcal{N}}_{B|T}^\intercal\Big],$$

so that $\Delta\hat{\boldsymbol{\lambda}} - \Delta\hat{\boldsymbol{\xi}} = \bar{\lambda}\big(\Gamma^A_{\Delta T}\hat{\boldsymbol{R}}^A - \Gamma^B_{\Delta T}\hat{\boldsymbol{R}}^B - \Gamma^*_{\Delta T}\hat{\boldsymbol{R}}^*\big)$, where $\mathcal{D} := \beta^{-1}J - I$ and

$$\Gamma^A_{\Delta T} := -\mathcal{D}\,\check{\mathcal{A}}^{\intercal}_{\Delta T}, \qquad \Gamma^B_{\Delta T} := -\mathcal{D}\,\check{\mathcal{B}}^{\intercal}_{\Delta T}, \qquad \Gamma^*_{\Delta T} := -\mathcal{D}\,\check{\mathcal{N}}^{\intercal}_{\Delta T}.$$

Define loading matrices

$$\boldsymbol{\Gamma}^A := \Gamma^A_{\epsilon} + \Gamma^A_{\Delta T} \otimes \mathbf{1}_{\mathcal{B}}, \qquad \boldsymbol{\Gamma}^B := \Gamma^B_{\epsilon} + \Gamma^B_{\Delta T} \otimes \mathbf{1}_{\mathcal{B}}, \qquad \boldsymbol{\Gamma}^* := \Gamma^*_{\epsilon} + \Gamma^*_{\Delta T} \otimes \mathbf{1}_{\mathcal{B}}.$$

Substituting into the stacked linearized $\boldsymbol{b}^G$ FOC $\bar{\lambda}(\hat{\boldsymbol{R}}^B - \hat{\boldsymbol{R}}^* \otimes \mathbf{1}_{\mathcal{B}}) = \tilde{\boldsymbol{\gamma}} + (\Delta\hat{\boldsymbol{\lambda}} - \Delta\hat{\boldsymbol{\xi}}) \otimes \mathbf{1}_{\mathcal{B}}$ yields the target rule:

$$\hat{\boldsymbol{R}}^B - \hat{\boldsymbol{R}}^* \otimes \mathbf{1}_{\mathcal{B}} = \begin{bmatrix} \boldsymbol{\Gamma}^A \\ -\boldsymbol{\Gamma}^B \\ -\boldsymbol{\Gamma}^* \end{bmatrix}^{\intercal} \begin{bmatrix} \hat{\boldsymbol{R}}^A \\ \hat{\boldsymbol{R}}^B \\ \hat{\boldsymbol{R}}^* \end{bmatrix}, \tag{65}$$

which is equation (27) of the main text.

B Benchmarks and Examples

B.1 Neoclassical Benchmark via n^{NGM}

Impose

$$\boldsymbol{\phi}_t \equiv \bar{\boldsymbol{\phi}}, \qquad \theta_t \equiv \bar{\theta} := \mathbf{1}^{\intercal}_{\mathcal{B}}\bar{\boldsymbol{\phi}} + 1. \tag{66}$$

Under (66), for any instrument $x_s \in \{T_s, \boldsymbol{R}^B_{s+1}, R^A_{s+1}\}$: $\bar{a}^F_{t,x_s} = \bar{\theta}\,\bar{n}_{t,x_s}$ and $\bar{\boldsymbol{b}}^F_{t,x_s} = \bar{\boldsymbol{\phi}}\,\bar{n}_{t,x_s}$.

Auxiliary construction of n^{NGM}. Consider the auxiliary household problem that maximizes (1) subject to (2), but with n_t treated as a choice variable. Under (66), any input sequence induces $R^N_t = \bar{\theta}R^A_t - \bar{\boldsymbol{\phi}}^{\intercal}\boldsymbol{R}^B_t$. Let $n^{NGM}_t(\cdot)$ denote the net-worth path solving this auxiliary problem. The auxiliary FOC is $\lambda_t = \beta\lambda_{t+1}(R^N_{t+1} - \tau^N_{t+1})$, which linearizes to

$$\hat{\lambda}_{t+1} - \hat{\lambda}_t = -\beta\bar{\lambda}\hat{R}^N_{t+1}. \tag{67}$$

By the neutral-rate definition, this is equivalent to $\hat{R}^N_{t+1} = \hat{R}^*_{t+1}$.

Cancellation of the balance-sheet pecuniary channel. If $n_t(\cdot) = n_t^{NGM}(\cdot)$, then in the shared core (62), the last bracket vanishes:

$$\beta\bar{\xi}\Big(\hat{R}^A_{t+1}\bar{a}^F_{t,x_s} - \hat{\boldsymbol{R}}^{B\intercal}_{t+1}\bar{\boldsymbol{b}}^F_{t,x_s}\Big) + (\hat{\lambda}_{t+1} - \hat{\lambda}_t)\bar{n}_{t,x_s} = \big(\beta\bar{\lambda}\hat{R}^N_{t+1} + \hat{\lambda}_{t+1} - \hat{\lambda}_t\big)\bar{n}_{t,x_s} = 0,$$

using (66), $\bar{\xi} = \bar{\lambda}$, and (67). The shared core reduces to

$$\mathcal{S}^{NGM}_{x_s} = \sum_t \beta^t \tilde{\boldsymbol{\gamma}}^{\intercal}_t \bar{\boldsymbol{\epsilon}}_{t,x_s} + \sum_t \beta^t \tilde{\eta}_t\, \bar{\kappa}_{t,x_s} + \sum_t \beta^{t+1}\tilde{\xi}_{t+1}\big(\bar{\tau}^A \bar{a}^F_{t,x_s} + \bar{\boldsymbol{\tau}}^{B\intercal}\bar{\boldsymbol{b}}^F_{t,x_s} + \bar{\tau}^N \bar{n}_{t,x_s}\big).$$

Proposition 3 (Neoclassical benchmark: existence) *Assume* (66) *and* $n_t(\cdot) = n_t^{NGM}(\cdot)$. *The candidate*

$$\hat{\boldsymbol{R}}^B_{t+1} = \hat{R}^A_{t+1}\mathbf{1}_{\mathcal{B}}, \qquad \tilde{\boldsymbol{\gamma}}_t = \mathbf{0}, \qquad \tilde{\eta}_t = 0, \qquad \tilde{\xi}_t = 0 \quad \forall t,$$

satisfies the linearized Ramsey system. Under this solution, $\hat{R}^N_{t+1} = \hat{R}^A_{t+1} = \hat{R}^*_{t+1}$ *for all* t.

Proof. Under the candidate, $\hat{R}^N_{t+1} = (\bar{\theta} - \mathbf{1}^{\intercal}_{\mathcal{B}}\bar{\boldsymbol{\phi}})\hat{R}^A_{t+1} = \hat{R}^A_{t+1}$, so $\hat{R}^A_{t+1} = \hat{R}^*_{t+1}$ and $\hat{\lambda}_{t+1} - \hat{\lambda}_t + \beta\bar{\lambda}\hat{R}^A_{t+1} = 0$. Setting all tilde wedges to zero gives $\mathcal{S}^{NGM}_{x_s} = 0$, so the parallel FOCs (63) are satisfied. Finally, (56) yields $\tilde{\boldsymbol{\gamma}}_t = \beta\bar{\lambda}\hat{R}^A_{t+1}\mathbf{1}_{\mathcal{B}} - \beta\bar{\lambda}\hat{R}^A_{t+1}\mathbf{1}_{\mathcal{B}} = \mathbf{0}$, confirming consistency. □

Remark (On uniqueness) *Proposition 3 is an existence result. Uniqueness requires an additional invertibility condition on the reduced constant-intermediation system.*

B.2 Quasi-Linear Demand and Fixed Net Worth

Assume quasi-linear preferences without portfolio adjustment costs,

$$u\left(\{c_s, \boldsymbol{b}_s\}_{s\le t}; \boldsymbol{\omega}_t\right) = c_t + \nu(\boldsymbol{b}_t; \boldsymbol{\omega}_t), \qquad \Psi \equiv 0, \tag{68}$$

and fixed net worth:

$$n_t \equiv \bar{n}. \tag{69}$$

Proposition 4 (Quasi-linear reduction) *Under* (68) *and* (69)*:*

(i) $\lambda_t \equiv 1$, *so* $\hat{\lambda}_t = 0$, $\Delta\hat{\lambda}_{t+1} = 0$, *and* $\hat{R}^*_{t+1} = 0$ *for all* t*;*

(ii) household asset demand is static in current returns: $\bar{\boldsymbol{b}}_{t,\boldsymbol{R}^B_{s+1}} = \mathbf{0}$ *for* $s \neq t$*, and* $\bar{\boldsymbol{b}}_{t,T_s} = \mathbf{0}$*,* $\bar{\boldsymbol{b}}_{t,R^A_{s+1}} = \mathbf{0}$*;*

(iii) $\bar{n}_{t,x_s} = 0$ *for all instruments, and for return instruments,* $\bar{a}^F_{t,x_s} = \bar{n}\,\bar{\theta}_{t,x_s}$*,* $\bar{\boldsymbol{b}}^F_{t,x_s} = \bar{n}\,\bar{\boldsymbol{\phi}}_{t,x_s}$*;*

(iv) $\tilde{\xi}_t = 0$ *and* $\tilde{\boldsymbol{\gamma}}_t = \beta\bar{\lambda}\,\hat{\boldsymbol{R}}^B_{t+1}$*.*

Proof. Under (68), $\lambda_t \equiv 1$. Static demand follows from linearity in c_t and the current-period structure of ν. Under (69), $\bar{n}_{t,x_s} = 0$. For $x_s = T_s$, all Jacobians in $\mathcal{S}_{T_s}$ vanish, so (63a) gives $\tilde{\xi}_s = 0$. Substituting into (56) yields (iv). □

For return instruments, the shared core $\mathcal{S}_{x_s}$ from (63) simplifies to

$$\mathcal{S}^{QL}_{x_s} = \sum_t \beta^t \tilde{\boldsymbol{\gamma}}_t^{\intercal}(\bar{n}\,\bar{\boldsymbol{\phi}}_{t,x_s} - \bar{\boldsymbol{b}}_{t,x_s}) + \sum_t \beta^t \tilde{\eta}_t(\bar{a}^K_{t,x_s} - \bar{n}\,\bar{\theta}_{t,x_s}) + \sum_t \beta^{t+1}\bar{\lambda}\Big(\hat{R}^A_{t+1}\bar{n}\,\bar{\theta}_{t,x_s} - \hat{\boldsymbol{R}}^{B\intercal}_{t+1}\bar{n}\,\bar{\boldsymbol{\phi}}_{t,x_s}\Big). \tag{70}$$

We now substitute the quasi-linear simplifications from Proposition 4 into (70). Using $\tilde{\boldsymbol{\gamma}}_t = \beta\bar{\lambda}\hat{\boldsymbol{R}}^B_{t+1}$ from Proposition 4(iv), the first and third sums combine so that the $\bar{\boldsymbol{\phi}}_{t,x_s}$ terms cancel inside the $\boldsymbol{R}^B$ equation. Similarly, $\bar{\boldsymbol{b}}_{t,R^A_{s+1}} = \mathbf{0}$ by quasi-linearity, so the $\bar{\boldsymbol{\phi}}_{t,R^A_{s+1}}$ terms cancel inside the R^A equation. This yields two independent conditions:

$$0 = \sum_{t=0}^{\infty} \beta^t \tilde{\eta}_t\big(\bar{a}^K_{t,\boldsymbol{R}^B_{s+1}} - \bar{n}\,\bar{\theta}_{t,\boldsymbol{R}^B_{s+1}}\big) - \sum_{t=0}^{\infty} \beta^{t+1}\bar{\lambda}\,\hat{\boldsymbol{R}}^{B\intercal}_{t+1}\bar{\boldsymbol{b}}_{t,\boldsymbol{R}^B_{s+1}} + \sum_{t=0}^{\infty} \beta^{t+1}\bar{\lambda}\,\hat{R}^A_{t+1}\bar{n}\,\bar{\theta}_{t,\boldsymbol{R}^B_{s+1}}, \tag{71}$$

$$0 = \sum_{t=0}^{\infty} \beta^t(\beta\bar{\lambda}\hat{R}^A_{t+1})\zeta^K_{t,R^A_{s+1}} + \sum_{t=0}^{\infty} \beta^t \tilde{\eta}_t\big(\bar{a}^K_{t,R^A_{s+1}} - \bar{n}\,\bar{\theta}_{t,R^A_{s+1}}\big) + \sum_{t=0}^{\infty} \beta^{t+1}\bar{\lambda}\,\hat{R}^A_{t+1}\bar{n}\,\bar{\theta}_{t,R^A_{s+1}}. \tag{72}$$

General target rule under perfectly elastic capital. Define $(\boldsymbol{b}^{\intercal}_{\boldsymbol{R}^B}\hat{\boldsymbol{R}}^B)_s := \sum_t \beta^{t+1}\hat{\boldsymbol{R}}^{B\intercal}_{t+1}\bar{\boldsymbol{b}}_{t,\boldsymbol{R}^B_{s+1}}$ and $(\boldsymbol{\theta}^{\intercal}_{\boldsymbol{R}^B}\hat{\boldsymbol{R}}^A)_s := \sum_t \beta^{t+1}\hat{R}^A_{t+1}\bar{\theta}_{t,\boldsymbol{R}^B_{s+1}}$.

Corollary 3 (General quasi-linear target rule with perfectly elastic capital) *Under (68) and (69), suppose capital supply becomes perfectly elastic along a family of economies indexed by* ν*, in the sense that the* R^A *block (72) can be written as*

$$\mathcal{M}^\nu_A\tilde{\eta}^\nu = -\mathcal{H}^\nu_A\hat{\boldsymbol{R}}^A,$$

where each $\mathcal{M}^\nu_A$ *is invertible,* $\sup_\nu \|\mathcal{H}^\nu_A\| < \infty$*, and* $\|(\mathcal{M}^\nu_A)^{-1}\| \to 0$ *as* $\nu \to \infty$*. Here*

ν indexes a family of economies with increasingly elastic capital supply; in the limit $\nu \to \infty$, the capital-market-clearing constraint ceases to bind on the planner ($\tilde{\eta}^\nu \to 0$). If $\boldsymbol{b}_{\boldsymbol{R}^B}$ is invertible, then in the perfectly elastic limit the optimal target rule is

$$\hat{\boldsymbol{R}}^B = \left(\bar{n}\, \boldsymbol{\theta}_{\boldsymbol{R}^B}\, \boldsymbol{b}_{\boldsymbol{R}^B}^{-1}\right)^{\intercal} \hat{\boldsymbol{R}}^A. \tag{73}$$

Proof. Define the excess capital supply operator $\mathcal{M}_A^\nu$, which maps the capital-market multiplier sequence $\tilde{\eta}^\nu$ to excess capital supply responses, and the capital-return forcing operator $\mathcal{H}_A^\nu$, which collects the capital-return forcing terms from $\hat{\boldsymbol{R}}^A$:

$$(\mathcal{M}_A^\nu \tilde{\eta}^\nu)_s \coloneqq \sum_{t=0}^{\infty} \beta^t \tilde{\eta}_t^\nu \left(\bar{a}^{K,\nu}_{t,R^A_{s+1}} - \bar{n}\, \bar{\theta}_{t,R^A_{s+1}}\right), \qquad (\mathcal{H}_A^\nu \hat{\boldsymbol{R}}^A)_s \coloneqq \sum_{t=0}^{\infty} \beta^{t+1} \bar{\lambda} \hat{R}^A_{t+1} \left(\zeta^{K,\nu}_{t,R^A_{s+1}} + \bar{n}\, \bar{\theta}_{t,R^A_{s+1}}\right).$$

Then (72) is exactly $\mathcal{M}_A^\nu \tilde{\eta}^\nu = -\mathcal{H}_A^\nu \hat{\boldsymbol{R}}^A$, so $\tilde{\eta}^\nu = -(\mathcal{M}_A^\nu)^{-1} \mathcal{H}_A^\nu \hat{\boldsymbol{R}}^A$. By assumption, $\|(\mathcal{M}_A^\nu)^{-1}\| \to 0$ while $\mathcal{H}_A^\nu$ remains bounded, hence $\tilde{\eta}^\nu \to 0$.

Production-side objects depend only on $R^A + \tau^A$, so $\bar{a}^{K,\nu}_{t,\boldsymbol{R}^B_{s+1}} = 0$ for all (t, s, ν). Taking the limit $\nu \to \infty$ in (71) yields $\boldsymbol{b}_{\boldsymbol{R}^B}^{\intercal} \hat{\boldsymbol{R}}^B = \bar{n}\, \boldsymbol{\theta}_{\boldsymbol{R}^B}^{\intercal} \hat{\boldsymbol{R}}^A$. If $\boldsymbol{b}_{\boldsymbol{R}^B}$ is invertible, solving gives (73). This confirms the target rule stated in Corollary 3. □

Static one-asset friction with finite capital elasticity. Impose $|\mathcal{B}| = 1$, $\phi_t = \varphi(R^A_{t+1} - R^B_{t+1})$ with $\varphi' > 0$, and linear capital accumulation $\Phi(\iota_t) = 1 + \iota_t$ (so $q_t = 1$ and production is static).

Proposition 5 (Static one-asset friction) *Under the above assumptions, with $a^K_{R^A} - \bar{n}\varphi' \neq 0$ and $b_{R^B} \neq 0$:*

$$\tilde{\eta}_s = -\beta \bar{\lambda} \hat{R}^A_{s+1} \frac{\bar{n}\varphi'}{a^K_{R^A} - \bar{n}\varphi'}. \tag{74}$$

The optimal target rule is

$$\hat{R}^B_{t+1} = -\frac{\bar{n}\varphi'}{b_{R^B}} \left(1 - \frac{\bar{n}\varphi'}{\bar{n}\varphi' - a^K_{R^A}}\right) \hat{R}^A_{t+1}. \tag{75}$$

Proof. Static production gives $\bar{a}^K_{t,R^A_{s+1}} = a^K_{R^A} \mathbf{1}_{\{t=s\}}$, $\zeta^K = 0$, and $\bar{\theta}_{t,R^A_{s+1}} = \varphi' \mathbf{1}_{\{t=s\}}$. The R^A equation reduces to $\tilde{\eta}_s(a^K_{R^A} - \bar{n}\varphi') + \beta\bar{\lambda}\hat{R}^A_{s+1}\bar{n}\varphi' = 0$, giving (74). The R^B equation becomes $\tilde{\eta}_s \bar{n}\varphi' - \beta\bar{\lambda}\hat{R}^B_{s+1} b_{R^B} - \beta\bar{\lambda}\hat{R}^A_{s+1}\bar{n}\varphi' = 0$ (using $\bar{a}^K_{s,R^B_{s+1}} = 0$). Substituting (74) yields (75). □

Corollary 4 (Perfectly elastic capital) *If capital supply is perfectly elastic, the target rule simplifies to* $\hat{R}^B_{t+1} = -(\bar{n}\varphi'/b_{R^B})\hat{R}^A_{t+1}$.

Forward-looking one-asset friction. Maintain (68)–(69) with $|\mathcal{B}| = 1$ and perfectly elastic capital ($\tilde{\eta}_t \to 0$). Let the friction kernel satisfy $\phi_{t,R^A_{s+1}} = -\phi_{t,R^B_{s+1}} = \phi_R \Xi^{s-t}$ for $s \geq t$ and zero otherwise, with $\phi_R > 0$ and $\Xi \in [0, 1)$. Then the R^B equation gives

$$\hat{R}^B_{t+1} = -\frac{\bar{n}\phi_R}{b_{R^B}} \sum_{s=0}^{t} \beta^{s-t}\Xi^{t-s}\hat{R}^A_{s+1}. \tag{76}$$

The optimal target for R^B_{t+1} depends on current and past capital-return disturbances with geometric weights $(\Xi/\beta)^{t-s}$.

B.3 Synthetic Intertemporal Price

This subsection gives a sufficient condition under which Assumption 1 is satisfied by a synthetic portfolio of financial assets.

Consider a household block with additively separable preferences and no portfolio adjustment cost,

$$u\left(\{c_s, \boldsymbol{b}_s\}_{s\leq t}; \boldsymbol{\omega}_t\right) = \tilde{u}(c_t; \boldsymbol{\omega}_t) + \nu(\boldsymbol{b}_t; \boldsymbol{\omega}_t), \qquad \Psi \equiv 0. \tag{77}$$

Proposition 6 (Synthetic intertemporal price) *Suppose there exists a constant vector* $\boldsymbol{\alpha} \in \mathbb{R}^{|\mathcal{B}|}$ *such that*

$$\boldsymbol{\alpha}^\intercal \mathbf{1}_{\mathcal{B}} = 1, \qquad \boldsymbol{\alpha}^\intercal \nu_{\boldsymbol{b}}(\boldsymbol{b}; \boldsymbol{\omega}) = 0$$

for all $(\boldsymbol{b}, \boldsymbol{\omega})$ *in a neighborhood of the reference allocation. Then the household Euler equation implies*

$$\lambda_t = \beta\lambda_{t+1}\boldsymbol{\alpha}^\intercal\left(\boldsymbol{R}^B_{t+1} - \boldsymbol{\tau}^B_{t+1}\right).$$

Hence Assumption 1 holds with the synthetic return

$$R^*_{t+1} = \boldsymbol{\alpha}^\intercal\left(\boldsymbol{R}^B_{t+1} - \boldsymbol{\tau}^B_{t+1}\right).$$

Under the fixed-tax convention used in the first-order analysis, this reduces to

$$\hat{R}^*_{t+1} = \boldsymbol{\alpha}^\intercal \hat{\boldsymbol{R}}^B_{t+1}, \qquad \Delta\hat{\lambda}_{t+1} = -\beta\bar{\lambda}\hat{R}^*_{t+1}.$$

Proof. Under (77), the household first-order condition for $\boldsymbol{b}_t$ is

$$\lambda_t \mathbf{1}_{\mathcal{B}} = \beta \lambda_{t+1} \big(\boldsymbol{R}^B_{t+1} - \boldsymbol{\tau}^B_{t+1} \big) + \nu_{\boldsymbol{b}}(\boldsymbol{b}_t; \boldsymbol{\omega}_t).$$

Left-multiplying by $\boldsymbol{\alpha}^\intercal$ and using $\boldsymbol{\alpha}^\intercal \mathbf{1}_{\mathcal{B}} = 1$ and $\boldsymbol{\alpha}^\intercal \nu_{\boldsymbol{b}}(\boldsymbol{b}_t; \boldsymbol{\omega}_t) = 0$ gives

$$\lambda_t = \beta \lambda_{t+1} \boldsymbol{\alpha}^\intercal \big(\boldsymbol{R}^B_{t+1} - \boldsymbol{\tau}^B_{t+1} \big),$$

which proves the exact synthetic Euler equation. The first-order statement follows immediately under the fixed-tax convention of Appendix A.6. □

Remark *A local sufficient condition for the orthogonality restriction* $\boldsymbol{\alpha}^\intercal \nu_{\boldsymbol{b}}(\boldsymbol{b}; \boldsymbol{\omega}) = 0$ *is that* $\nu_{\boldsymbol{bb}}$ *is rank deficient with left-null vector* $\boldsymbol{\alpha}^\intercal$*, together with* $\boldsymbol{\alpha}^\intercal \nu_{\boldsymbol{b}}(\bar{\boldsymbol{b}}; \bar{\boldsymbol{\omega}}) = 0$ *at the reference allocation.*

C Nested Models of Financial Intermediation

We show how our framework nests commonly used models of financial frictions by specifying the reduced-form financial-sector objects $\boldsymbol{\phi}_t(\cdot)$, $\theta_t(\cdot)$, and $n_t(\cdot)$. In each model below, we express the portfolio functions in terms of equilibrium return sequences $\{R^A_{s+1}, \boldsymbol{R}^B_{s+1}\}$. We write θ_t, $\phi_{\text{dep},t}$, etc. for portfolio shares per unit of net worth, reserving $\bar{z}$ for steady-state values.

C.1 Bianchi and Bigio (2022)

Bianchi and Bigio (2022) develop a banking model with reserves, deposits, and government bonds. Banks face idiosyncratic withdrawal shocks and settle balances with reserves in an interbank market with search frictions, allowing the government to affect relative returns by changing the supply of bonds and reserves.

There is a continuum of financial intermediaries with relative risk aversion $\gamma_{\text{IB}} \geq 0$. Each intermediary chooses portfolio shares of loans $\theta_t \geq 0$, government debt $\phi_{\text{gov},t} \leq 0$, reserves $\phi_{\text{res},t} \leq 0$, and deposits $\phi_{\text{dep},t} \geq 0$, with balance sheet $\theta_t = 1 + \phi_{\text{gov},t} + \phi_{\text{res},t} + \phi_{\text{dep},t}$. Returns are R^A_{t+1}, $R^B_{\text{gov},t+1}$, $R^B_{\text{res},t+1}$, and $R^B_{\text{dep},t+1}$.

Intermediaries face idiosyncratic withdrawal shocks υ with CDF F and $\mathbb{E}[\upsilon] = 0$. Intermediaries in deficit borrow from the interbank market or the discount window

(rate R^W_{t+1}); those in surplus lend. Let $\phi_{\text{liq},t} \coloneqq \phi_{\text{res},t} + \phi_{\text{gov},t}$. The liquidity-yield function is piecewise linear,

$$\chi_{t+1}(s) = \begin{cases} \chi^+_{t+1} s, & s \geq 0, \\ \chi^-_{t+1} s, & s < 0, \end{cases}$$

where the slope coefficients depend on the reserve return, the discount-window rate, and interbank market tightness:

$$\chi^+_{t+1} = (R^W_{t+1} - R^B_{\text{res},t+1}) \left(\frac{\bar{\vartheta}(\vartheta_t)}{\vartheta_t}\right)^{\eta_{\text{IB}}} \left(\frac{\vartheta_t^{\eta_{\text{IB}}} \bar{\vartheta}(\vartheta_t)^{1-\eta_{\text{IB}}} - \vartheta_t}{\bar{\vartheta}(\vartheta_t) - 1}\right),$$
$$\chi^-_{t+1} = (R^W_{t+1} - R^B_{\text{res},t+1}) \left(\frac{\bar{\vartheta}(\vartheta_t)}{\vartheta_t}\right)^{\eta_{\text{IB}}} \left(\frac{\vartheta_t^{\eta_{\text{IB}}} \bar{\vartheta}(\vartheta_t)^{1-\eta_{\text{IB}}} - 1}{\bar{\vartheta}(\vartheta_t) - 1}\right),$$

and

$$\bar{\vartheta}(\vartheta) = \begin{cases} 1 + (\vartheta - 1)\exp(\lambda_{\text{IB}}), & \vartheta > 1, \\ 1, & \vartheta = 1, \\ \left(1 + (\vartheta^{-1} - 1)\exp(\lambda_{\text{IB}})\right)^{-1}, & \vartheta < 1. \end{cases}$$

The optimal portfolio solves

$$\max_{\theta_t, \phi_{\text{gov},t}, \phi_{\text{res},t}, \phi_{\text{dep},t}} \mathbb{E}_\upsilon \left[\left(R^A_{t+1}\theta_t - R^B_{\text{res},t+1}\phi_{\text{liq},t} - R^B_{\text{dep},t+1}\phi_{\text{dep},t} + \chi_{t+1}(\phi_{\text{liq},t}, \phi_{\text{dep},t}, \upsilon) \right)^{1-\gamma_{\text{IB}}} \right]^{\frac{1}{1-\gamma_{\text{IB}}}}$$

subject to the balance sheet and $\phi_{\text{dep},t} \leq \bar{\phi}_{\text{dep}}$, where χ_{t+1} is piecewise linear in υ with slopes χ^+_{t+1} and χ^-_{t+1}.

Key first-order conditions. Proposition 8 of Bianchi and Bigio (2022) shows that the FOCs yield (whether or not the leverage constraint binds):

$$R^A_{t+1} - R^B_{\text{res},t+1} = \chi^+_{t+1} + (\chi^-_{t+1} - \chi^+_{t+1}) \cdot F(\upsilon^*) \cdot \frac{\mathbb{E}_\upsilon[(R^e_{t+1})^{-\gamma_{\text{IB}}} \mid \upsilon < \upsilon^*]}{\mathbb{E}_\upsilon[(R^e_{t+1})^{-\gamma_{\text{IB}}}]},$$
$$R^B_{\text{gov},t+1} - R^B_{\text{res},t+1} = \chi^+_{t+1},$$

where $\upsilon^* \coloneqq \phi_{\text{liq},t}/(\phi_{\text{dep},t} \cdot R^B_{\text{dep},t+1}/R^B_{\text{res},t+1})$ and R^e_{t+1} is the gross return on net worth. When the leverage constraint is slack, an additional condition determines $\phi_{\text{dep},t}$; when binding, $\phi_{\text{dep},t} = \bar{\phi}_{\text{dep}}$.

Market tightness ϑ_t is determined by the interbank market clearing condition as a

function of returns and portfolio shares. Solving the system for portfolio shares as functions of returns gives the mapping:

$$\begin{aligned}
\theta_t &= \theta(R^A_{t+1}, R^B_{\text{dep},t+1}, R^B_{\text{res},t+1}, R^B_{\text{gov},t+1}; R^W_{t+1}),\\
\phi_{\text{dep},t} &= \phi_{\text{dep}}(R^A_{t+1}, R^B_{\text{dep},t+1}, R^B_{\text{res},t+1}, R^B_{\text{gov},t+1}; R^W_{t+1}),\\
\phi_{\text{res},t} &= \phi_{\text{res}}(R^A_{t+1}, R^B_{\text{dep},t+1}, R^B_{\text{res},t+1}, R^B_{\text{gov},t+1}; R^W_{t+1}),\\
\phi_{\text{gov},t} &= \phi_{\text{gov}}(R^A_{t+1}, R^B_{\text{dep},t+1}, R^B_{\text{res},t+1}, R^B_{\text{gov},t+1}; R^W_{t+1}).
\end{aligned}$$

The discount window rate R^W_{t+1} is treated as an exogenous policy parameter.

Net worth. To align Bianchi and Bigio (2022) with the linear return representation (12), suppose the government uses a simple tax-transfer rule that reallocates discount-window spread revenue so as to offset its direct effect on intermediary net worth:

$$\frac{T^{\text{lin}}_t}{n_{t-1}} = -\left(R^W_t - R^B_{\text{res},t}\right)\varpi_{t-1},$$

where ϖ_{t-1} denotes discount-window borrowing per unit of lagged net worth. This removes the extra term generated by discount-window borrowing from the intermediary return. Under this tax-transfer rule, the after-tax return on intermediary net worth is

$$R^N_t = R^A_t\theta_{t-1} - R^B_{\text{dep},t}\phi_{\text{dep},t-1} - R^B_{\text{res},t}\phi_{\text{res},t-1} - R^B_{\text{gov},t}\phi_{\text{gov},t-1},$$

so the realized quantity of discount-window lending does not enter the reduced-form net-worth return separately. The discount-window rate R^W_t still affects portfolio choice through the liquidity-yield terms. Under the survival-entry structure, a fraction f of intermediaries exits each period and entrants receive a transfer proportional to the surviving stock. Hence $n_t = n_t(\{R^A_{s+1}, \boldsymbol{R}^B_{s+1}; \boldsymbol{\omega}_s\})$ as in the general specification (13).

Calibration details. We calibrate the model to the 2006 U.S. banking sector using the extended calibration from Bianchi and Bigio (2022). The interbank market parameters (matching efficiency λ_{IB}, bargaining power η_{IB}, and withdrawal volatility σ_v) are calibrated to match the discount-window borrower share, the effective federal funds rate, and the volume of discount-window lending, respectively. Bank risk

aversion $\gamma_{\text{IB}} = 10$ reflects the curvature of the intermediary's objective over return on equity.

Steady-state balance-sheet ratios are taken from Call Reports data for 2006. Deposits are the dominant liability ($\bar{\phi}_{\text{dep}} = 8.87$ per unit of equity), while reserves ($\bar{\phi}_{\text{res}} = -0.05$) and government bonds ($\bar{\phi}_{\text{gov}} = -0.21$) are held as assets. Steady-state returns are expressed as monthly gross real rates; the annualized loan–deposit spread is approximately 240 basis points.

On the household side, the discount factor $\beta = 0.981$ (monthly) is taken from Bianchi and Bigio (2022). The household demand semi-elasticity $\varepsilon = -35$ (monthly) is taken from Table II of Bianchi and Bigio (2022). Production parameters ($\alpha = 0.33$, $\delta = 0.007$ monthly) are standard. The exit rate is $f = 0.03$ (monthly).

Table 4 collects the full set of calibrated parameters and steady-state values.

Table 4: Key parameters and steady-state values for the Bianchi and Bigio (2022) calibration.

Parameter	Symbol	Value	Source
Interbank market			
Matching efficiency	λ_{IB}	7.93	Discount-window borrower share
Bargaining power	η_{IB}	0.15	Fed funds rate
Withdrawal volatility	σ_υ	0.12	Discount-window lending volume
Risk aversion (bank)	γ_{IB}	10	Standard (EZ preferences)
Bank balance sheet (per unit of equity)			
Deposits / equity	$\bar{\phi}_{\text{dep}}$	+8.87	Call Reports, 2006
Reserves / equity	$\bar{\phi}_{\text{res}}$	−0.05	Call Reports, 2006
Gov bonds / equity	$\bar{\phi}_{\text{gov}}$	−0.21	Call Reports, 2006
Loans / equity	$\bar{\theta}$	9.61	Balance sheet identity
Steady-state returns (monthly gross real)			
Loans	$\bar{R}^A$	1.00194	2.35% ann.
Deposits (B_0)	$\bar{R}^{B_0}$	0.99994	−0.07% ann.
Reserves/currency (B_1)	$\bar{R}^{B_1}$	0.99835	−1.96% ann.
Gov bonds (B_2)	$\bar{R}^{B_2}$	1.00152	1.84% ann.
Net worth			
Discount factor	β	0.981	Bianchi and Bigio (2022)
Exit rate (monthly)	f	0.03	
Household			
Discount factor	β	0.981	Same as intermediary
HH demand semi-elasticity	$\lvert\varepsilon\rvert$	35	Bianchi and Bigio (2022) Table II (monthly)
HH deposits ($\bar{b}_0/n$)		8.87	$= \bar{\phi}_{\text{dep}}$ (market clearing)
HH currency ($\bar{b}_1/n$)		0.74	U.S. currency data, 2006
HH gov bonds ($\bar{b}_2/n$)		0.27	56.3% HH share of total
Production			
Capital share	α	0.33	
Depreciation (monthly)	δ	0.007	

Note: Financial asset 1 (B_1) combines reserves (held by banks) and currency (held by households). Both earn the same return $\bar{R}^{B_1}$; banks settle withdrawal shocks using reserves, while households hold currency. Household holdings are expressed per unit of intermediary net worth. The bliss points $\bar{b}_j$ are backed out from the household Euler equations at the steady-state returns and taxes; see Section 6.2.

C.2 Gertler, Kiyotaki, Queralto (2012)

Gertler et al. (2012) extend the GKK framework by allowing intermediaries to issue outside equity. Outside equity affects the ability of intermediaries to divert assets: it

is more difficult to divert assets funded by deposits than by outside equity. Let θ_t be the portfolio share of loans, $\phi_{\text{dep},t}$ the portfolio share of deposits, and $\phi_{\text{oeq},t}$ the share of outside equity. The intermediary can divert fraction χ of assets, where

$$\chi \coloneqq \chi_0 \left(1 + \chi_1 \frac{\phi_{\text{oeq},t}}{\theta_t} + \frac{\chi_2}{2} \left(\frac{\phi_{\text{oeq},t}}{\theta_t} \right)^2 \right).$$

Returns are R^A_{t+1}, $R^B_{\text{dep},t+1}$, and $R^B_{\text{oeq},t+1}$. The Bellman equation is

$$\begin{aligned} \eta_t = \max_{\phi_{\text{oeq},t}, \theta_t} \Lambda_{t,t+1}(f + (1-f)\eta_{t+1}) \big[\theta_t (R^A_{t+1} - R^B_{\text{dep},t+1}) - \phi_{\text{oeq},t}(R^B_{\text{oeq},t+1} - R^B_{\text{dep},t+1}) + R^B_{\text{dep},t+1} \big] \\ + \lambda_t \big[\eta_t - \chi \theta_t \big], \end{aligned}$$

where η_t is the marginal value of net worth and λ_t is the multiplier on the incentive compatibility (IC) constraint. The discount factor $\Lambda_{t,t+1}$ is a function of returns only, as it depends on the stochastic discount factor of the household who owns the intermediary.

If the IC constraint binds, the FOC determines the ratio $\phi_{\text{oeq},t}/\theta_t = x(R^A_{t+1}, R^B_{\text{oeq},t+1}, R^B_{\text{dep},t+1})$. Using this together with the IC constraint and the Bellman equation gives $\eta_t = \eta(\{R^A_{s+1}, R^B_{\text{oeq},s+1}, R^B_{\text{dep},s+1}\}_{s \geq t})$, and hence

$$\begin{aligned} \theta_t &= \theta(\{R^A_{s+1}, R^B_{\text{oeq},s+1}, R^B_{\text{dep},s+1}\}_{s \geq t}), \\ \phi_{\text{oeq},t} &= \phi_{\text{oeq}}(\{R^A_{s+1}, R^B_{\text{oeq},s+1}, R^B_{\text{dep},s+1}\}_{s \geq t}), \\ \phi_{\text{dep},t} &= \phi_{\text{dep}}(\{R^A_{s+1}, R^B_{\text{oeq},s+1}, R^B_{\text{dep},s+1}\}_{s \geq t}). \end{aligned}$$

Net worth. A fraction f of intermediaries exit each period and new entrants receive a transfer. Net worth follows the GKK recursion $n_{t+1} = (1-f)R^N_{t+1} n_t + \text{transfers}$, giving $n_t = n_t(\{R^A_{s+1}, \boldsymbol{R}^B_{s+1}; \boldsymbol{\omega}_s\})$.

C.3 Sims and Wu (2021)

Sims and Wu (2021) extend the GKK framework with interest-bearing reserves and differential diversion rates for private loans versus government bonds. Let $\theta_t \geq 0$ be the portfolio share of private loans and $\phi_{\text{gov},t}, \phi_{\text{res},t} \leq 0$, $\phi_{\text{dep},t} \geq 0$ the shares of government bonds, reserves, and deposits. The balance sheet is $\theta_t = \phi_{\text{gov},t} + \phi_{\text{res},t} + \phi_{\text{dep},t} + 1$. The intermediary can divert fraction χ of private loans and $\chi\Delta$

of government bonds, where $0 < \Delta \leq 1$, and faces a reserve requirement $\phi_{\text{res},t} \leq -\varsigma\phi_{\text{dep},t}$.

The program is linear in portfolio shares, so the solution generically involves corner solutions. Define the effective funding rate

$$R_{\text{fund},t+1} \coloneqq R^B_{\text{dep},t+1} + \frac{\varsigma}{1-\varsigma}(R^B_{\text{dep},t+1} - R^B_{\text{res},t+1}).$$

Assume the IC constraint binds and $R^B_{\text{dep},t+1} \geq R^B_{\text{res},t+1}$, which requires

$$\max\{R^A_{t+1}, R^B_{\text{gov},t+1}\} \geq R_{\text{fund},t+1}.$$

If

$$R^B_{\text{gov},t+1} = (1-\Delta)R_{\text{fund},t+1} + \Delta R^A_{t+1},$$

the intermediary is indifferent between loans and government debt. Define the effective dominant return

$$R^*_{t+1} \coloneqq \begin{cases} R^A_{t+1}, & R^B_{\text{gov},t+1} \leq (1-\Delta)R_{\text{fund},t+1} + \Delta R^A_{t+1}, \\ R^B_{\text{gov},t+1}, & R^B_{\text{gov},t+1} > (1-\Delta)R_{\text{fund},t+1} + \Delta R^A_{t+1}. \end{cases}$$

Then the Bellman recursion can be written as

$$\eta_t = \Lambda_{t,t+1}(f + (1-f)\eta_{t+1}) \left[\left(R^*_{t+1} - R_{\text{fund},t+1}\right) \frac{\eta_t}{\chi} + R_{\text{fund},t+1} \right].$$

Since $\Lambda_{t,t+1}$ depends on returns only, this implies

$$\eta_t = \eta(\{R^A_{s+1}, R^B_{\text{gov},s+1}, R^B_{\text{res},s+1}, R^B_{\text{dep},s+1}\}_{s \geq t}).$$

Using the binding IC constraint, the intermediary's loan demand is

$$\theta_t \coloneqq \begin{cases} \eta_t/\chi, & (1-\Delta)R_{\text{fund},t+1} + \Delta R^A_{t+1} > R^B_{\text{gov},t+1}, \\ [0, \eta_t/\chi], & (1-\Delta)R_{\text{fund},t+1} + \Delta R^A_{t+1} = R^B_{\text{gov},t+1}, \\ 0, & (1-\Delta)R_{\text{fund},t+1} + \Delta R^A_{t+1} < R^B_{\text{gov},t+1}, \end{cases}$$

and government-debt issuance is pinned down by

$$\phi_{\text{gov},t} = \frac{1}{\Delta}\left(\theta_t - \frac{\eta_t}{\chi}\right).$$

If $R^B_{\text{dep},t+1} > R^B_{\text{res},t+1}$, the reserve requirement binds and the balance sheet implies

$$\phi_{\text{res},t} = -\frac{\varsigma}{1-\varsigma}(\theta_t - \phi_{\text{gov},t} - 1),$$
$$\phi_{\text{dep},t} = \frac{1}{1-\varsigma}(\theta_t - \phi_{\text{gov},t} - 1).$$

If $R^B_{\text{dep},t+1} = R^B_{\text{res},t+1}$, only the sum $\phi_{\text{res},t} + \phi_{\text{dep},t} = \theta_t - \phi_{\text{gov},t} - 1$ is pinned down.

Net worth. As in the other GKK-type models, net worth follows $n_{t+1} = (1 - f)R^N_{t+1} n_t + \text{transfers}$. Hence $n_t = n_t(\{R^A_{s+1}, \boldsymbol{R}^B_{s+1}; \boldsymbol{\omega}_s\})$.

C.4 Heterogeneous Intermediaries

There is a continuum of financial intermediaries indexed by $j \in [0, 1]$ with net worth $n_{j,t}$. For each unit of net worth, intermediary j can hold $\theta_{j,t}(\{R^A_{s+1}, \boldsymbol{R}^B_{s+1}\})$ units of capital and issue $\boldsymbol{\phi}_{j,t}(\{R^A_{s+1}, \boldsymbol{R}^B_{s+1}\})$ units of financial assets. The relative net worth of intermediary j is

$$\frac{n_{j,t}}{n_t} \coloneqq \mu_{j,t}(\{R^A_{s+1}, \boldsymbol{R}^B_{s+1}\}), \qquad \int_0^1 \mu_{j,t}(\cdot)\, dj = 1.$$

The aggregate reduced-form objects are

$$\theta_t(\{R^A_{s+1}, \boldsymbol{R}^B_{s+1}\}) \coloneqq \int_0^1 \theta_{j,t}(\cdot)\, \mu_{j,t}(\cdot)\, dj,$$
$$\boldsymbol{\phi}_t(\{R^A_{s+1}, \boldsymbol{R}^B_{s+1}\}) \coloneqq \int_0^1 \boldsymbol{\phi}_{j,t}(\cdot)\, \mu_{j,t}(\cdot)\, dj.$$

The Jacobians of the aggregate objects reflect both the intensive margin (individual portfolio adjustments) and the extensive margin (redistribution of net worth across intermediary types):

$$\theta_{t,\boldsymbol{R}^B_{k+1}} = \int_0^1 \left(\theta_{j,t,\boldsymbol{R}^B_{k+1}}\, \mu_{j,t} + \theta_{j,t}\, \mu_{j,t,\boldsymbol{R}^B_{k+1}}\right) dj,$$
$$\boldsymbol{\phi}_{t,\boldsymbol{R}^B_{k+1}} = \int_0^1 \left(\boldsymbol{\phi}_{j,t,\boldsymbol{R}^B_{k+1}}\, \mu_{j,t} + \boldsymbol{\phi}_{j,t}\, \mu_{j,t,\boldsymbol{R}^B_{k+1}}\right) dj.$$

The return on aggregate net worth is

$$R^N_t = R^A_t\, \theta_t - \boldsymbol{R}^{B\intercal}_t \boldsymbol{\phi}_t,$$

which follows from $R_t^N = \int_0^1 R_{j,t}^N \, \mu_{j,t} \, dj$.

Net worth. Define aggregate net worth as $n_t := \int_0^1 n_{j,t} \, dj$. The induced law of motion for n_t depends on the return-dependent distribution $\mu_{j,t}(\cdot)$: changes in returns alter not only individual intermediary net worth but also the cross-sectional distribution, so $n_t = n_t(\{R_{s+1}^A, \boldsymbol{R}_{s+1}^B; \boldsymbol{\omega}_s\})$ as in the general specification (13).

D Empirical Appendix

This appendix collects additional details for the empirical implementation discussed in the main text.

D.1 Data Construction

D.1.1 Balance Sheets of Banks

Our primary data source is the Reports of Condition and Income (Call Reports) U.S. Call Reports provided by the Federal Financial Institutions Examination Council. The dataset contains quarterly data on the income statements and balance sheets of all U.S. commercial banks. The raw Call Reports coverage spans 2001Q1 to 2024Q4. In the empirical analysis, however, we restrict attention to the balanced sample from 2003Q2 to 2023Q4.

For each reporting entity in the dataset we calculate the following items on the balance sheet:

- "Total assets" (RCFD2170);
- "Equity" (RCFD3210);
- "Cash" (sum of codes RCFD0071 "Interest Bearning Balances" and RCFD0081 "Non-interest Bearning Balances and Coin");
- "Net loans and leases" (RCFDB528);
- "Fed funds sold and securities purchased under agreement to sell" (sum of RCFDB987 "Federal Funds Sold" and RCFDB989 "Securities Purchased Under Agreement to Sell");

- "Trading assets" (RCFD3545);
- "Trading Treasury securities" (RCON3531);
- "Total securities holdings" (sum of RCFD1773 "Securities Available for Sale" and RCFD1754 "Held-to-maturity Securities", substituting RCFDJJ34 for RCFD1754 after December 31, 2018, when regulatory reporting codes changed);
- "Treasury securities holdings" (sum of codes RCON1287 "Fair Value of Available-for-Sale U.S. Treasury Securities" and RCON0211 "Amortized Cost of Held-to-Maturity U.S. Treasury Securities");
- "Deposits" (sum of codes RCON2200 "Domestic Deposits" and RCFN2200 "Foreign Deposits");
- "Fed funds purchased and securities sold under agreement to repurchase" (sum of RCFDB993 "Federal Funds Sold" and RCFDB995 "Securities Sold Under Agreement to Repurchase");
- "Trading liabilities" (RCFD3548);
- "Other borrowed money" (RCFD3190).

Banks without foreign offices have missing values of the RCFD codes. In this case, we use the RCON codes instead (i.e., RCON3190 for other borrowed money).

We merge Call Reports data with Center of Research in Security Prices (CRSP) on stock prices, dividends and returns (1980–2024) by using a crosswalk provided by the Federal Reserve Bank of New York. We retain only entities with a 1-1 mapping between "permco" in CRSP data and "bhcid" in Call Reports data.

For each "permco" we calculate products of the number of outstanding shares (code "SHROUT") and prices (code "PRC"), which we then label as "TCAP". Since one entity can have more than one type of stock outstanding, we calculate a sum of all "TCAP" corresponding to a given entity at each date. We define "Market value" as the first sum of "TCAP" of an entity in a given quarter.

We calculate "Other securities holdings" as "Total securities holdings" - "Treasury securities holdings." We calculate "Other trading securities" as "Trading assets" - "Trading Treasury securities."

We define "Financial assets" as: "Cash" + "Net loans and leases" + "Fed funds sold and securities purchased under agreement to sell" + "Other trading securities" + "Trading Treasury securities" + "Other securities holdings" + "Treasury securities holdings."

We define "Other liabilities" as: "Liabilities" - "Deposits" - "Fed funds purchased and securities sold under agreement to repurchase" - "Trading liabilities" - "Other borrowed money."

We define "Non-financial assets" as: "Market value" + "Liabilities" - "Financial assets."

We now group assets net of liabilities into three categories: "Liquid assets", "Illiquid assets", and "Capital":

- We define "Liquid assets" as: "Cash" + "Treasury securities holdings" + "Fed funds sold and securities purchased under agreement to sell" + "Trading Treasury securities" - "Deposits" - "Fed funds purchased and securities sold under agreement to repurchase;"
- We define "Illiquid assets" as: "Other securities holdings" + "Other trading securities" - "Trading liabilities;"
- We define "Capital" as: "Non-financial assets" + "Loans" - "Other borrowed money" - "Other liabilities."

D.1.2 Foreign

To construct foreign holdings of liquid and illiquid assets, we use information from the Treasury International Capital (TIC) system. We combine two TIC datasets: the Bertaut-Tryon (BT) estimates from Bertaut and Judson (2022) and the Securities Holdings and Transactions (SLT) survey data. From both sources, we extract country-level monthly holdings of Treasury securities (liquid assets) and Agency securities (illiquid assets). We aggregate to quarterly frequency using beginning-of-quarter values. For each country, we compute growth rates using the midpoint formula $2(x_t - x_{t-1})/(|x_t| + |x_{t-1}|)$, which is bounded and handles sign changes.

D.1.3 Returns

We use the 3-month Treasury Bills yield as expected nominal return on "Liquid assets" and Moody's Seasoned BAA Corporate Bond yield as expected nominal return on "Illiquid assets." We denote annualized rates of return as $E_t\left[i_{t+1}^{\text{Liquid}}\right]$ and $E_t\left[i_{t+1}^{\text{Illiquid}}\right]$.

In CRSP data we calculate daily dividends on each stock of an entity as returns (code "RET") minus returns without dividends (code "RETX") times the market value of a stock ("TCAP"). To calculate the total dividends of an entity we sum over all types of stocks. To calculate total returns of an entity, we sum "RET" multiplied by "TCAP" over all stocks of an entity and divide by the sum of "TCAP".

Because CRSP data is daily, this gives us a daily series of dividends and returns. To obtain quarterly variables from CRSP data, we use the following procedure. First, we define "Market value" as the first sum of "TCAP" of an entity within a given quarter. To calculate "Quarterly dividends," we sum all daily dividends of an entity in a quarter. To calculate "Quarterly returns," we compute the product of daily returns. We denote nominal "Quarterly returns" of an entity j in a quarter t as $i_{j,t+1}^N$. To obtain the "Dividend to price ratio" in a quarter t we divide the sum of "Quarterly dividends" in the four preceding quarters by the "Market value" in the first day of the quarter t. We denote the resulting series as $dp_{j,t}$.

We follow the same procedure with the aggregate S&P500 index to obtain the series of "Dividend to price ratio" $dp_{S\&P500,t}$

To obtain nominal expected quarterly returns $E_t\left[i_{j,t+1}^N\right]$ for each entity, we estimate:

$$\begin{aligned}\text{Excess return}_{j,t+1} = \alpha_{j,0} + \beta_{j,1} dp_{S\&P500,t} + \beta_{j,2} dp_{j,t} \\ +\beta_{j,3}\text{Term premium}_t + \beta_{j,4}\log\text{Market Value}_{j,t} + \epsilon_{j,t+1},\end{aligned}$$

where $\text{Excess return}_{j,t+1} \coloneqq i_{j,t+1}^N - E_t\left[i_{t+1}^{\text{Liquid}}\right]$, Term premium_t is the difference between the 10-year and 3-month Treasury yields. We use the predicted values of $\text{Excess return}_{j,t+1}$, $\widehat{\text{Excess return}}_{j,t+1}$, to construct

$$E_t\left[i_{j,t+1}^N\right] \coloneqq \widehat{\text{Excess return}}_{j,t+1} + E_t\left[i_{t+1}^{\text{Liquid}}\right].$$

To recover the nominal expected return on capital, $E_t\left[i^{\text{Capital}}_{j,t+1}\right]$ we use

$$E_t\left[i^{N}_{j,t+1}\right] = \frac{\text{Capital}_{j,t}}{\text{Market Value}_{j,t}} E_t\left[i^{\text{Capital}}_{j,t+1}\right] + \frac{\text{Liquid Assets}_{j,t}}{\text{Market Value}_{j,t}} E_t\left[i^{\text{Liquid}}_{j,t+1}\right] + \frac{\text{Illiquid Assets}_{j,t}}{\text{Market Value}_{j,t}} E_t\left[i^{\text{Illiquid}}_{j,t+1}\right].$$

We then calculate the aggregate return $E_t\left[i^{\text{Capital}}_{t+1}\right]$ as

$$E_t\left[i^{\text{Capital}}_{t+1}\right] := \sum_{j=1}^{J} \frac{\text{Capital}_{j,t}}{\sum_{i=1}^{J} \text{Capital}_{i,t}} E_t\left[i^{\text{Capital}}_{j,t+1}\right]$$

We calculate real expected returns by subtracting 1-year expected inflation series from Cleveland Fed.

Finally, we define:

$$\begin{aligned} E_t\left[R^{A}_{t+1}\right] &:= E_t\left[1 + \frac{i^{\text{Capital}}_{t+1}}{400}\right] \\ E_t\left[R^{B}_{\text{Liquid},t+1}\right] &:= E_t\left[1 + \frac{i^{\text{Liquid}}_{t+1}}{400}\right] \\ E_t\left[R^{B}_{\text{Illiquid},t+1}\right] &:= E_t\left[1 + \frac{i^{\text{Illiquid}}_{t+1}}{400}\right]. \end{aligned}$$

D.2 Construction of GIV

We describe how we construct Granular Instrumental Variables (GIVs) in our setting. We follow the procedure outlined in Gabaix and Koijen (2024).

We first obtain an instrument associated with foreign holdings. We work with logarithms of country-level holdings of Treasury securities, $\log \text{Treasury}_{c,t}$, where c represents a country and t is a time period. We postulate

$$\log \text{Treasury}_{c,t} = \overline{\log \text{Treasury}}_c(t) + \mathbf{\Lambda}_c \boldsymbol{\eta}^{C}_{t} + u_{c,t}.$$

Here $\overline{\log \text{Treasury}}_c(t)$ is a country-specific quadratic trend, $\boldsymbol{\eta}^{C}_{t}$ is a vector of aggregate factors, and $\mathbf{\Lambda}_c$ are the loadings on them. The last term, $u_{c,t}$, represents idiosyncratic

i.i.d. country-level shocks. For each country, we remove a country-specific quadratic trend and a common time fixed effect (we assume the loadings on the first element of $\boldsymbol{\eta}_t^C$ are equal for all countries). We then remove the first principal component from the panel of detrended country-level holdings. We call the residuals $\hat{u}_{c,t}$. They are proxies for $u_{c,t}$. We define

$$z_t^{\text{TIC}} \coloneqq \sum_c s_c \hat{u}_{c,t}, \quad s_c \coloneqq \frac{\sum_t \text{Treasury}_{c,t}}{\sum_c \sum_t \text{Treasury}_{c,t}}.$$

We next construct GIVs in Call Reports data. Let

$$\boldsymbol{\phi}_{j,t} \coloneqq -\frac{1}{\text{Market Value}_{j,t}} \begin{bmatrix} \text{Liquid Assets}_{j,t} \\ \text{Illiquid Assets}_{j,t} \end{bmatrix}.$$

We assume portfolio shares $\boldsymbol{\phi}_{j,t}$ are given by Equation (32), i.e:

$$\boldsymbol{\phi}_{j,t} = \bar{\boldsymbol{\phi}}_j(t) + \boldsymbol{\phi}_{\boldsymbol{R}^B} \boldsymbol{S}_{t+1} + \boldsymbol{\Lambda}_j \boldsymbol{\eta}_t + \boldsymbol{\varepsilon}_{j,t}, \tag{78}$$

where $\bar{\boldsymbol{\phi}}_j(t)$ are entity-specific quadratic time trends, $\boldsymbol{\phi}_{\boldsymbol{R}^B}$ parameterizes responses to deviations of spreads from trends,

$$\boldsymbol{S}_{t+1} \coloneqq \mathbb{E}_t[\boldsymbol{R}_{t+1}^B] - \mathbb{E}_t[R_{t+1}^A] \otimes \mathbf{1}_{\mathcal{B}},$$

$\boldsymbol{\eta}_t$ are unobserved aggregate shocks on which entity j has loadings $\boldsymbol{\Lambda}_j$, and $\boldsymbol{\varepsilon}_{j,t}$ are idiosyncratic shocks i.i.d. by entity-asset-time.

Our goal is to obtain a proxy for $\boldsymbol{\varepsilon}_{j,t}$. We remove entity-asset specific quadratic trends and an asset-specific time fixed effect common for all entities. We normalize the resulting detrended portfolio shares by dividing them by their standard deviation (common for all entities). We then remove three principal components from the panel of detrended and normalized entity-level portfolio shares. We are left with a vector of residuals $\hat{\boldsymbol{\varepsilon}}_{j,t}$ for each entity. We construct GIVs as

$$\boldsymbol{z}_t^{\text{Call Reports}} \coloneqq \sum_j s_j \hat{\boldsymbol{\varepsilon}}_{j,t}, \quad s_j \coloneqq \frac{\sum_t \text{Market Value}_{j,t}}{\sum_j \sum_t \text{Market Value}_{j,t}}.$$

We now estimate $\boldsymbol{\phi}_{\boldsymbol{R}^B}$ in Equation (32). The key difficulty is the endogeneity of $\boldsymbol{S}_{t+1}$: these returns are determined by equilibrium conditions. However, $\boldsymbol{\varepsilon}_{j,t}$ are by

construction i.i.d.. We use three instruments for $\boldsymbol{S}_{t+1}$: one from TIC and two from Call Reports,

$$\boldsymbol{z}_t := \begin{bmatrix} z_t^{\text{TIC}} \\ \boldsymbol{z}_t^{\text{Call Reports}} \end{bmatrix}.$$

We calculate simple cross-sectional averages of $\boldsymbol{\phi}_{j,t}$. We regress them on $\boldsymbol{S}_{t+1}$, with $\boldsymbol{z}_t$ as instruments, controlling for aggregate factors recovered by using principal component analysis (1 from TIC data and 3 from Call Reports data) and quadratic trend. We do it separately for liquid and illiquid assets. Table 2 in Section 7.1 shows the estimates.

Our instruments are strong: the value of the effective F-statistic is 28.1 and 11.5 for the spread between R^B_{liquid} and R^A and between R^B_{illiquid} and R^A, respectively.

D.3 Model Calibration

To calculate the steady-state portfolio shares $\bar{\boldsymbol{\phi}}$ and $\bar{\theta}$, we first compute portfolio shares for each period. We sum the negative of "Liquid Assets" and "Illiquid Assets" across all entities j in the Call Reports and divide by the sum of "Market Value." These variables are defined in Appendix D.1.1. This yields $\boldsymbol{\phi}_t$ for each period. We then calculate the sample average of these ratios over the empirical sample from 2003Q2 to 2023Q4 to obtain $\bar{\boldsymbol{\phi}}$. We compute $\bar{\theta}$ using the balance sheet identity: $\bar{\theta} = \bar{\phi}_{\text{liquid}} + \bar{\phi}_{\text{illiquid}} + 1$. We normalize the financial sector's net worth, $\bar{n}$, to one.

The steady-state real returns on assets are obtained as long-run averages of returns calculated in Appendix D.1.3.

To determine steady-state household holdings of liquid assets, $\bar{b}_{\text{liquid}}$, we calculate the long-run average of the financial sector's liquid *liabilities* using Call Report data. This category comprises two balance sheet positions: "Deposits" and "Fed funds purchased and securities sold under agreement to repurchase."

By construction, the sum of these liabilities exceeds the $\bar{\phi}_{\text{liquid}}$ calculated for the financial sector. The portfolio share $\bar{\phi}_{\text{liquid}}$ represents liquid asset issuance net of liquid asset holdings and therefore does not appropriately measure the value of assets held by households. The difference between these measures largely corresponds to "Cash," "Treasury security holdings," and "Fed funds sold and securities purchased

under agreement to sell," which we attribute to $\bar{b}^G_{\text{liquid}}$.

We set $\bar{b}_{\text{illiquid}}$ equal to $\bar{\phi}_{\text{illiquid}}$, which implies that $\bar{b}^G_{\text{illiquid}}$ equals zero. This assumption is justified because our measure of "Illiquid assets" excludes Treasury securities.

Steady-state levels of household asset holdings matter only insofar as they affect household asset demand sensitivities, given our target semielasticities with respect to returns of 0.1.

Finally, we assume that these long-run averages of returns and asset positions correspond to the Ramsey steady state. Under this assumption, the household asset positions calculated through this method correspond to the bliss point.