Nominal Anchors and Real Consequences: Inflation Targeting, Debt, and Investment*

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Abstract

We investigate whether the conduct of monetary policy can explain secular trends in macro-finance since the 1980s: the decline in long-term real interest rates, the rise in private debt, the decline in capital investment, and the stability of risky returns to capital. We develop a general equilibrium model in which heterogeneous agents allocate savings between nominal bonds and real capital under borrowing constraints. When central banks commit to aggressive inflation stabilization, they compress the inflation risk premium embedded in long-term nominal bond returns. Lower real returns on nominal assets encourage private borrowing while crowding out capital investment, yet risky returns to capital remain stable, a cross-asset pattern difficult to reconcile with explanations based on real factors alone. The model provides a unified account of these trends as equilibrium consequences of credible inflation targeting regimes.

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1 Introduction

The persistent decline in long-term real interest rates represents one of the most consequential macroeconomic developments of the past four decades. Since the early 1980s, real yields on long-term bonds across advanced economies have fallen dramatically from approximately 6-7 percent in the early 1980s to around 2 percent in recent years. The existing literature has predominantly attributed this phenomenon to real factors—demographic transitions, productivity slowdowns, and shifts in the global supply of savings.[1] In addition, this decline in long-term real rates has been accompanied by three other striking trends: a substantial increase in private nominal debt, a persistent decline in the investment share of output, and a widening gap between the returns on safe nominal bonds and risky real capital.

This paper proposes that the conduct of monetary policy itself may play a more substantial role than previously recognized. We develop a theoretical framework connecting the progressive strengthening of inflation targeting regimes– one of the most distinctive macroeconomic transformations since the early 1980s– to these four secular trends. We show that a commitment to aggressive inflation stabilization provides a coherent general equilibrium mechanism that can contribute to the joint decline in long-term real rates, the rise in private nominal debt, the crowding out of physical investment, and the stability of risky returns to capital

The mechanism operates through the inflation risk premium embedded in nominal asset returns. When central banks credibly commit to aggressive inflation targeting, they effectively provide insurance against inflation fluctuations. Crucially, this insurance is valuable because inflation fluctuations covary with savers' marginal utility: inflation tends to be high precisely when savers value consumption most, so nominal bonds deliver poor real returns in exactly the states where savers need wealth most. This reduction in inflation volatility renders nominal bonds safer in real terms, lowering the compensation investors require for bearing inflation risk. Savers respond by reallocating their portfolios toward nominal bonds and away from risky real capital. In equilibrium, this portfolio reallocation lowers real returns on nominal bonds, expands the quantity of private nominal debt, and crowds out physical

[1] See, among others, Del Negro et al. (2017), Eggertsson et al. (2021), Mian et al. (2021), and Rachel and Summers (2019).

investment—while the expected return on capital remains stable or rises to compensate for its now relatively greater risk. In essence, credible nominal anchoring transforms the risk characteristics of nominal assets, triggering a portfolio reallocation that reshapes both asset prices and real quantities.

We formalize this mechanism in a three-period general equilibrium model with heterogeneous agents and aggregate risk. Patient savers allocate their portfolios between nominal bonds and real capital, while impatient borrowers issue nominal debt up to an endogenous borrowing limit. Aggregate uncertainty arises from preference shocks that generate inflation fluctuations, and the central bank follows a Taylor rule whose aggressiveness determines the pass-through of these shocks to inflation. The model is analytically tractable: we derive closed-form expressions for the inflation risk premium, the equilibrium quantities of debt and capital, and a crowd-out coefficient governing the displacement of capital by nominal bonds. Our analytical strategy expands around the stochastic zero-debt equilibrium rather than a deterministic steady state, which preserves a nonzero inflation risk premium at first order and allows us to capture the effects of policy regimes on risk premia that would vanish in a certainty-equivalent approximation.

Our analytical results establish three main findings. First, in a benchmark economy without debt, more aggressive inflation targeting unambiguously lowers the real return on nominal bonds by reducing the inflation risk premium. The entire effect operates through inflation volatility: the stochastic discount factor volatility and its correlation with inflation are invariant to the Taylor rule coefficient, so more aggressive targeting lowers expected real returns exclusively by compressing the inflation risk premium. Second, when we introduce borrowing and lending, the reduction in inflation risk increases the supply of savings in nominal assets. As nominal bonds become safer, savers are willing to hold more of them at a lower expected return, raising equilibrium private debt. Third, when we extend the model to include real capital as an alternative savings vehicle, the improved risk-return profile of nominal bonds crowds out capital investment. We derive a closed-form crowd-out coefficient that depends on the degree of diminishing returns to capital and the intertemporal elasticity of substitution, and show that stronger inflation targeting always reduces equilibrium capital.

These results provide a unified account of the macro-finance trends that have charac-

terized advanced economies since the adoption of credible inflation targeting. The decline in long-term real rates, the rise in private nominal debt, and the weakness of investment all emerge as equilibrium responses to a single underlying shift in monetary policy regime. if credible inflation targeting crowds out productive capital by making nominal bonds too attractive on a risk-adjusted basis, then the first-order real costs of successful stabilization policy may offset the second-order welfare gains from reduced inflation volatility.

Related Literature. Our paper contributes to several strands of the literature, and we view our work as complementary to existing contributions.

Inflation risk premia and bond returns. A growing body of work connects monetary policy to the inflation risk premium in bond returns. Campbell et al. (2020) document the transition of nominal Treasury bonds from bad hedges (positive bond-stock correlation, positive risk premium) in the pre-2000 period to good hedges (negative correlation, negative risk premium) post-2000, attributing this shift to more aggressive monetary policy. Miller et al. (2025) show that an exogenous reduction in inflation disaster risk can explain the secular decline in long-term rates. Gourio and Ngo (2024) use a nonlinear Phillips curve to connect inflation risk to bond premia: when inflation is high, the economy operates on the steep portion of the Phillips curve where inflation is more volatile and carries a larger risk premium; as inflation stabilizes, this premium compresses.[2] Our paper builds on these insights in two ways. First, we endogenize the reduction in inflation risk through the monetary policy regime rather than treating it as an exogenous shift, linking the decline in risk premia directly to the aggressiveness of the Taylor rule. Second, and more importantly, we embed the inflation risk channel in a heterogeneous-agent general equilibrium model with portfolio choice between nominal bonds and real capital. This allows us to derive not only the price effects that are the focus of the existing literature—lower real bond returns—but also quantity implications that these papers abstract from: the endogenous expansion of private nominal debt and the crowding out of physical investment.

Secular trends in interest rates, debt, and investment. A large literature seeks to explain

[2]Heaton and Lucas (1996) study asset pricing with heterogeneous agents making portfolio choices between stocks and bonds under aggregate and idiosyncratic risk with trading frictions, a framework our model shares in its emphasis on portfolio allocation across assets with different risk characteristics.

secular macro-finance trends using real structural forces. Farhi and Gourio (2018) provide an accounting framework showing that no single real factor—demographics, productivity, or risk aversion—can simultaneously match the decline in safe rates, stable risky returns, and rising price-dividend ratios. Kumhof et al. (2015); Mian et al. (2021) emphasize the role of rising income inequality, which increases the supply of savings and depresses interest rates. Crouzet and Eberly (2019, 2023) attribute weak physical capital investment to the rise of intangible capital and market rents, offering a complementary firm-side explanation for the investment drought that our portfolio-reallocation channel may reinforce. Eggertsson et al. (2021) and Jordà et al. (2017) document longer-run trends in the returns to capital and the accumulation of private credit. Our model offers a complementary mechanism that can jointly account for several of these patterns. In our framework, the supply of savings shifts specifically toward nominal bond markets and away from real capital markets as inflation risk declines: this generates simultaneously lower real bond returns, higher private debt, and lower capital investment without requiring separate shocks for each trend. A distinguishing prediction of our framework is that nominal bond returns and risky capital returns move in opposite directions—a cross-asset pattern that is difficult to reconcile with explanations based solely on real factors that shift the marginal utility of intertemporal consumption.

Safe asset creation and the crowding out of capital. Acharya and Dogra (2022) show that exogenous expansions of safe real government debt can crowd out risky capital in a dynamically efficient overlapping-generations economy. In their framework, more government debt raises the real interest rate as the reduced capital stock elevates the marginal product of capital. Di Tella (2020) shows that money, as a safe store of value, prevents interest rates from falling when risk premia rise, crowding out risky capital investment—a mechanism that shares our emphasis on the tension between safe assets and productive capital, though operating through the zero lower bound rather than the inflation risk premium. Our paper is related in spirit but differs in three respects. First, we endogenize the change in the effective supply of safe assets: monetary policy reduces inflation risk, thereby making existing private nominal debt safer rather than introducing new government liabilities. Second, the real interest rate on nominal bonds *falls* in our model as safer debt attracts greater demand from savers, in contrast to the rising real rate in their setting. Third, the crowding out in

our model operates through portfolio choice—savers reallocate from risky capital to safer nominal bonds—rather than through the absorption of real resources by government debt. More broadly, the normative implications of our crowding-out result connect to the literature on the welfare costs of government debt in incomplete-markets economies (Aiyagari and McGrattan, 1998), raising the question of whether the first-order costs of capital displacement may offset the gains from inflation stabilization.

Monetary policy, financial stability, and policy regimes. Our paper also contributes to work on how monetary policy affects financial conditions, including Borio and White (2004), Woodford (2012), Gourio et al. (2018), Borio et al. (2019), Cairó and Sim (2023), and Boissay et al. (2024). These papers examine how policy affects financial stability over business cycle and medium-run frequencies. We complement this work by identifying a channel through which policy *regimes*—as opposed to transitory policy shocks—affect secular trends in real rates, leverage, and capital formation. In our framework, nominal debt contracts interact with the inflation risk implied by the monetary regime to determine debt capacity, providing a transmission mechanism through which monetary credibility reshapes the real economy.

Organization. The remainder of the paper is organized as follows. Section 2 documents the five stylized facts motivating our analysis. Section 3 develops the three-period model and derives the main analytical results. Section 4 concludes.

2 Motivating Evidence

In this section, we present five stylized facts.

2.1 The Decline in Long-Term Real Rates

The downward trajectory of long-term real interest rates since the 1980s is now well documented across multiple estimation approaches and country samples. Figure 1 shows that expected real yields on U.S. nominal bonds—computed using inflation expectations from the Survey of Professional Forecasters—exhibit a secular decline from approximately 6–7 percentage points in the early 1980s to values near or below 2 percentage points by the late

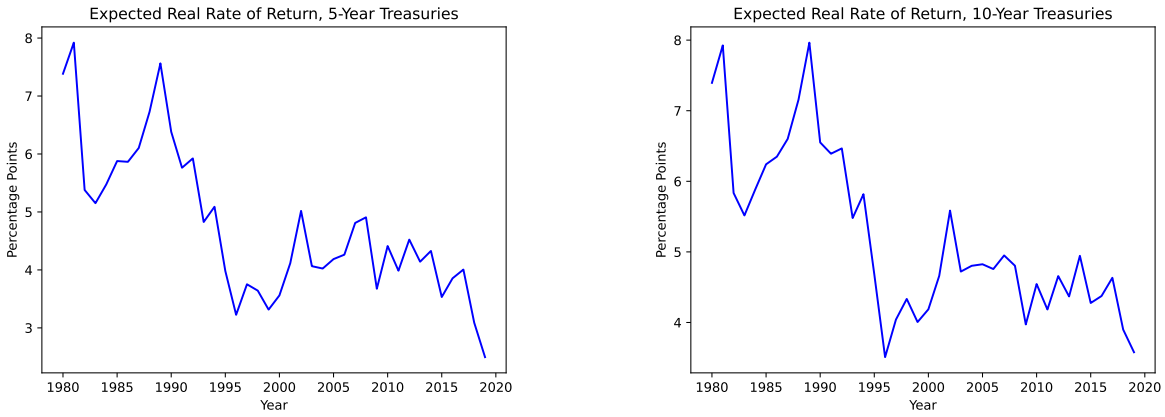


Figure 1: Expected real interest rate on 5-year and 10-year US government bonds

2010s. This pattern holds across maturities, with both five-year and ten-year expected real returns tracing similar downward paths over the four-decade sample.

2.2 The Rise of Private Debt

Coinciding with the decline in long-term real rates, private sector debt has increased substantially across advanced economies. Figure 2 shows that, in the United States, household debt relative to GDP rose from approximately 50 percent in 1980 to nearly 100 percent prior to the Global Financial Crisis. While some deleveraging occurred in the aftermath of 2008, debt ratios remain elevated relative to pre-1980 norms. Data from the Bank for International Settlements confirm that this pattern characterizes the broader set of advanced economies, with private credit-to-GDP ratios exhibiting sustained increases throughout the post-1980 period.

The composition of this debt accumulation merits attention. The growth has been concentrated in nominal debt instruments—mortgage obligations, corporate bonds, and other fixed-income securities denominated in domestic currency.

2.3 Stability of Risky Returns and the Investment Drought

In contrast to the pronounced decline in returns on bonds, returns to business capital have exhibited a remarkable increase over the same period. Figure 3 shows that, using data from Gomme et al. (2017), the pre-tax returns to business capital in the United States have

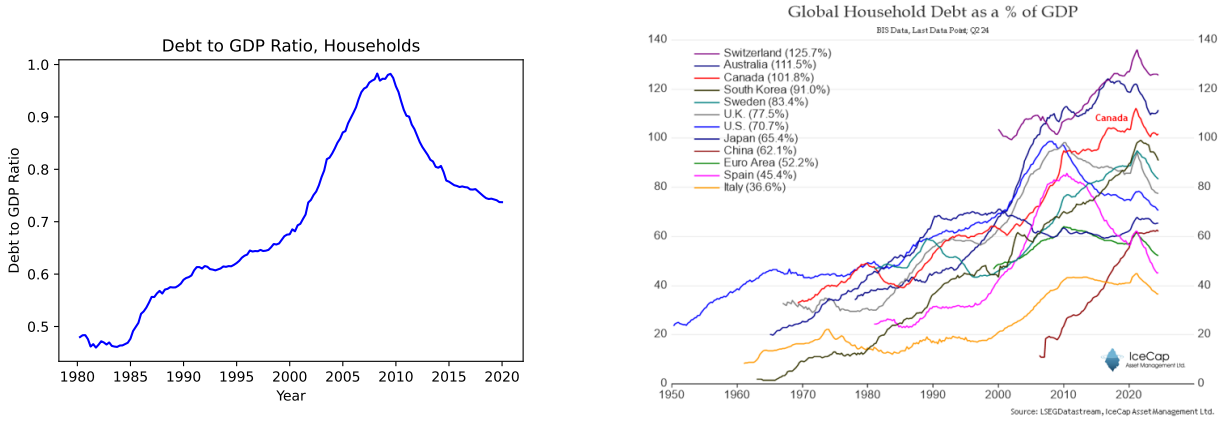


Figure 2: Household Debt to GDP ratios

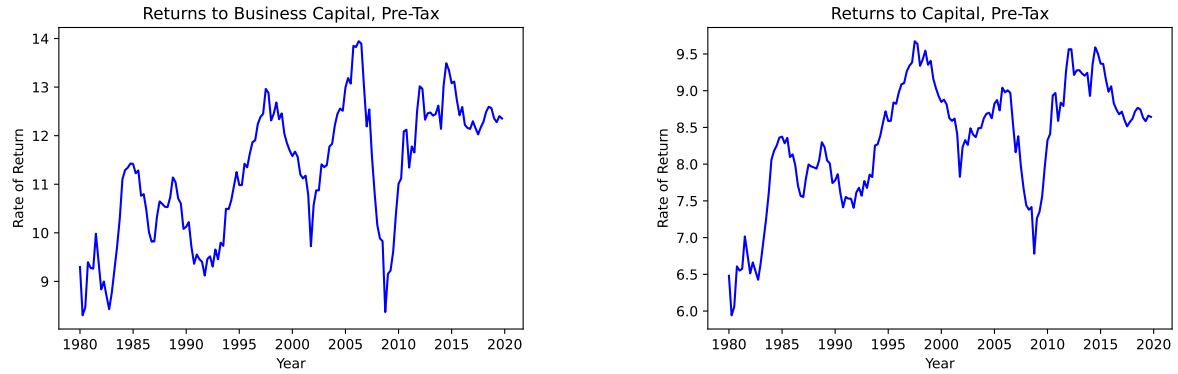


Figure 3: Returns to Capital (Gomme et al., 2017).

increased from around 9 percent to more than 12 percent since 1980. When measured more broadly to include all capital, pre-tax returns have similarly trended upward, from around 6 percent to 9 percent.

This divergence between bond returns and capital returns implies a substantial widening of the gap between safe and risky returns over the period. If the decline in bond rates reflected only shifts in the marginal utility of intertemporal consumption—through demographic factors or changes in time preference—one would expect parallel movements in both safe and risky returns. The observed pattern instead suggests a reallocation of savings across asset classes, with investors increasing their holdings of safer nominal assets at the expense of riskier real investments.

Consistent with this portfolio rebalancing, Figure 4 shows that net investment as a share of GDP has declined persistently since the early 1980s. Net fixed nonresidential invest-

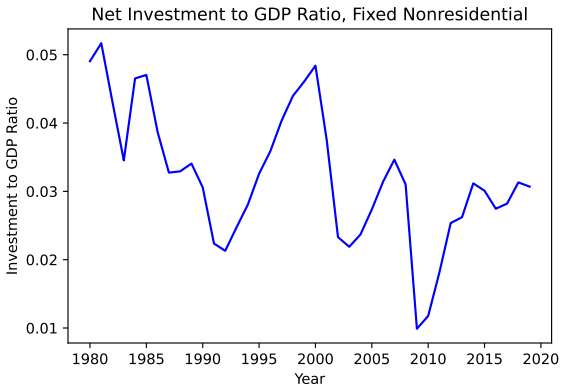


Figure 4: Decline in net investment

ment fell from approximately 5 percent of GDP in the early 1980s to roughly 3 percent in recent years. This investment drought has attracted considerable attention in the macroeconomic literature, with various explanations: declining relative prices of investment goods or increased market concentration. Our framework suggests that the shift in portfolio composition induced by safer nominal assets may contribute to this phenomenon.

2.4 The Strengthening of Inflation Targeting

The post-Volcker era witnessed a fundamental transformation in the conduct of monetary policy across advanced economies. Central banks progressively adopted explicit inflation targeting frameworks, committing to maintain price stability as the primary objective of monetary policy. Even where formal inflation targets were not adopted, the implicit responsiveness of policy rates to inflation increased substantially. This transformation was effective in reducing the level and volatility of inflation. Figure 5 shows that inflation itself declined from double-digit rates in the early 1980s to low and stable rates of 2-3 percent, while the volatility of inflation—measured as the rolling standard deviation—also fell dramatically.

Estimates of Taylor rule coefficients using rolling-window regressions document this policy shift quantitatively. Following Carvalho et al. (2021), we estimate:

$$r_t = \alpha_{aux} + \rho_{1,aux} r_{t-1} + \rho_{2,aux} r_{t-2} + \beta_{aux} \pi_t + \gamma_{aux} x_t + \epsilon_t$$

The implied long-run inflation coefficient is $\hat{\phi}_\pi = \beta_{aux}/(1 - \hat{\rho})$ where $\hat{\rho} = \rho_{1,aux} + \rho_{2,aux}$.

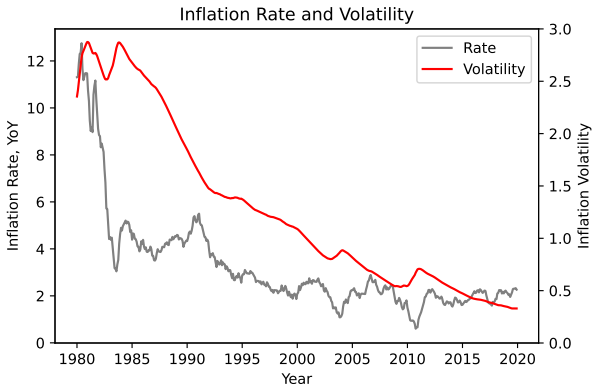


Figure 5: Inflation rate and volatility in the U.S.

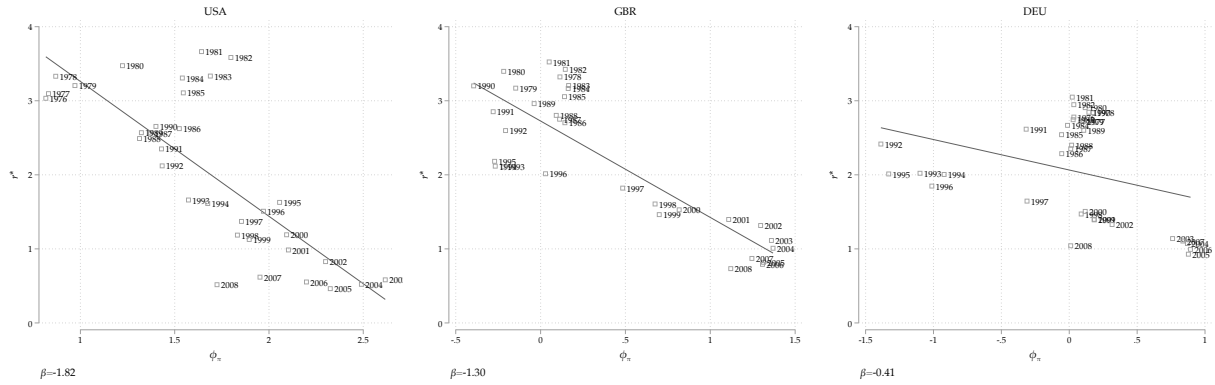


Figure 6: Taylor rule Inflation Coefficient vs Real rates in the U.S., the U.K. and Germany

For the United States, United Kingdom, and Germany over 1976-2008 using 20-year rolling windows, the estimated coefficient on inflation exhibits a sustained increase from the late 1970s through the 2000s. Figure 6 plots the estimated Taylor rule coefficients against estimates of trend real rates. For the United States, the regression coefficient relating trend real rates to $\hat{\phi}_\pi$ is approximately -1.82, indicating that a one-unit increase in the Taylor rule inflation coefficient is associated with a decline in real rates of nearly two percentage points. Similar negative relationships emerge for the United Kingdom and Germany, though with somewhat smaller magnitudes.

3 A Three-Period Economy

To formalize the mechanism linking inflation targeting to real rates, private debt, and capital investment, we develop a tractable three-period borrower-saver model. The model permits closed-form analytical results while capturing the essential trade-offs between nominal bonds and real capital under different monetary policy regimes.

3.1 Environment

Agents and Preferences. The economy extends over three periods, indexed by $t = 0, 1, 2$, and is populated by agents who differ in their patience. Specifically, there are two types $j \in \{b, s\}$ (borrowers and savers), each with mass one. Agents have standard time-separable preferences:

$$U_j = \mathbb{E}_0 \sum_{t=0}^{2} \beta_{j,t} u(C_{j,t}),$$

where $u(C_{j,t}) = \frac{C_{j,t}^{1-\gamma}}{1-\gamma}$ is the CRRA period utility function with risk aversion parameter $\gamma > 0$. The discount factors are $\beta_{b,t} = \beta_b^t$ for borrowers and $\beta_{s,t} = \beta_s^t \xi_t$ for savers, where $\beta_b < \beta_s \equiv \beta$ and ξ_t is a stochastic preference shock described below. The heterogeneity in patience generates equilibrium credit flows from patient savers to impatient borrowers, as in the borrower-saver model of Eggertsson and Krugman (2012).

Aggregate Uncertainty. The preference shock ξ_1 is realized at date 1 with $\xi_0 = 1$. We assume ξ_1 has bounded support $[\xi_L, \xi_H] \subset (0, \infty)$ with $\mathbb{E}_0[\xi_1] = 1$ and $\text{Var}(\xi_1) > 0$. At date 2 there is no new shock; the preference weight evolves deterministically as $\xi_2 = \xi_1^\rho$ with $\rho \in (0, 1)$. All uncertainty therefore resolves at date 1.

The preference shock serves as a demand shock that generates inflation fluctuations in equilibrium. Higher realizations of ξ_1 increase savers' marginal utility, raising desired saving and putting upward pressure on real interest rates. In equilibrium, the central bank's policy rule determines how these demand disturbances pass through to inflation. The degree of this pass-through, governed by ϕ_π, is the central object of our comparative statics.

Financial Markets. Agents can borrow or save using a one-period nominal bond, denoted by B_t. The bond is risk-free in nominal terms but not in real terms because of inflation risk. The nominal bond pays a rate of return equal to the policy rate i_t. Agents face a constraint limiting their debt capacity:

$$\mathbb{E}_t \left[\frac{(1+i_t)B_{j,t+1}}{P_{t+1}} \right] \geq -d_t,$$

where $d_t \geq 0$ represents the maximum expected real debt burden. Rewriting the constraint yields:

$$\frac{B_{j,t+1}}{P_t} \geq -\frac{d_t}{R_t^*}$$

where $R_t^* \equiv \mathbb{E}_t[\Pi_{t+1}^{-1}](1+i_t)$ is the expected real return on a nominal bond. The maximum real borrowing is decreasing in the expected real return.

Assuming households start with no debt, their budget constraints are:

$$\begin{aligned} P_0 C_{j,0} + B_{j,1} &\leq P_0 Y_0 \\ P_1 C_{j,1} + B_{j,2} &\leq P_1 Y_1 + (1+i_0)B_{j,1} \\ P_2 C_{j,2} &\leq P_2 Y_2 + (1+i_1)B_{j,2} \end{aligned}$$

No-Default Assumption. We maintain throughout that parameters $(d_0, \xi_L, \xi_H, \phi_\pi)$ are such that borrower consumption $C_{b,1} \geq 0$ in all states. Since real debt repayment $(1+i_0)b/\Pi_1$ is largest when inflation is lowest ($\xi_1 = \xi_L$), this imposes an upper bound on the debt limit d_0 that is increasing in the borrower's date-1 endowment and in ξ_L.

Monetary Policy. The central bank conducts monetary policy according to a Taylor-type interest rate rule:

$$1 + i_t = \frac{1}{\beta} \Pi_t^{\phi_\pi}, \tag{1}$$

where $\phi_\pi > 1$ and $\Pi_t \equiv P_t/P_{t-1}$ denotes gross inflation. The parameter ϕ_π captures the aggressiveness of the policy response to inflation deviations. The restriction $\phi_\pi > 1$ ensures that the Taylor principle is satisfied, generating a determinate rational expectations equilibrium. The value of the parameter ϕ_π characterizes the policy regime and represents the object of our comparative statics.

Following standard practice in the literature, we assume $\Pi_2 = 1$ since there are no shocks at $t = 2$. This assumption serves as the equilibrium selection device in the finite-horizon setting and corresponds to convergence to the steady-state in infinite-horizon extensions. We normalize $P_0 = 1$ without loss of generality.

Equilibrium. Next, we define an equilibrium for this economy.

Definition 1. An equilibrium consists of consumption allocations $\{C_{j,t}\}$, bond holdings $\{B_{j,t}\}$, prices $\{P_t\}$, and interest rates $\{i_t\}$ such that:

1. Households optimize given prices and policy
2. Bond markets clear: $B_{s,t} = -B_{b,t}$ for all $t \in \{1, 2\}$
3. The monetary authority sets interest rates according to the Taylor rule

We focus on equilibria in which borrowers remain constrained, an assumption satisfied when β_b is sufficiently small relative to β_s.

3.2 Equilibrium Analysis: Zero Debt Case

To develop intuition for the inflation insurance mechanism, we first analyze the benchmark case with zero debt limits: $d_0 = d_1 = 0$. In this configuration, no borrowing or lending occurs in equilibrium, and consumption equals the common endowment for all agents: $C_{j,t} = Y_t$ for all j and t. The model effectively reduces to a representative agent economy. To simplify notation, assume $Y_0 = Y_1 = Y_2 = Y$.

In this simple model, given the monetary policy rule (1), the inflation rate is determined by the sequence of savers' Euler equations:

$$1 = \beta \mathbb{E}_t \left[\frac{\xi_{t+1}}{\xi_t} \left(\frac{C_{s,t}}{C_{s,t+1}} \right)^{\gamma} \frac{1 + i_t}{\Pi_{t+1}} \right], \quad t \in \{0, 1\}.$$

We solve the model by backward induction.

Equilibrium inflation. At date 1, all uncertainty has resolved. The date-1 Euler equation combined with the Taylor rule (1) yields:

$$1 + i_1 = \frac{1}{\beta}\xi_1^{1-\rho}, \qquad \Pi_1 = \xi_1^a, \qquad a \equiv \frac{1-\rho}{\phi_\pi}.$$

The parameter a is the pass-through coefficient: it measures how aggregate shocks map into inflation. Higher ϕ_π reduces a, attenuating the pass-through of aggregate disturbances to prices.

Lemma 1 (Inflation volatility)**.** *The variance of log inflation satisfies:*

$$\mathrm{Var}(\log \Pi_1) = \left(\frac{1-\rho}{\phi_\pi}\right)^2 \mathrm{Var}(\log \xi_1),$$

which is strictly decreasing in ϕ_π for any distribution of ξ_1 with positive variance.

More aggressive inflation targeting reduces the volatility of inflation.

Date-0 equilibrium. At date 0, the bond Euler equation determines the nominal rate i_0, and the expected real return on bonds is $R_0^* \equiv (1 + i_0)\,\mathbb{E}_0[\Pi_1^{-1}]$. At $b = 0$, the stochastic discount factor is $M_{01} = \beta\xi_1$, so the entire dependence of R_0^* on ϕ_π comes through the inflation process. The real risk-free rate is $R^f \equiv 1/\mathbb{E}[M_{01}] = 1/\beta$ (using $\mathbb{E}[\xi_1] = 1$).

Proposition 1. *The expected real return on nominal bonds in the zero-debt economy is:*

$$R_0^* = R^f \cdot \frac{\mathbb{E}[\xi_1^{-a}]}{\mathbb{E}[\xi_1^{1-a}]}, \tag{2}$$

where $a = (1-\rho)/\phi_\pi$. This return is strictly decreasing in ϕ_π for any distribution of ξ_1 with positive variance. As $\phi_\pi \to \infty$ ($a \to 0$), $R_0^* \to R^f$.

Proof. The bond Euler at $b = 0$ gives $(1 + \bar{i}_0) = 1/(\beta\,\mathbb{E}[\xi_1^{1-a}])$. Multiplying by $\mathbb{E}[\Pi_1^{-1}] = \mathbb{E}[\xi_1^{-a}]$ yields (2). For the comparative static, write $\log(R_0^*/R^f) = f(-a) - f(1-a)$ where $f(s) \equiv \log \mathbb{E}[\xi_1^s]$. Differentiating with respect to a:

$$\frac{d}{da}\log(R_0^*/R^f) = f'(1-a) - f'(-a).$$

The function f is strictly convex: $f''(s) = \mathrm{Var}_s(\log \xi_1) > 0$ where Var_s denotes the variance under the tilted measure $d\mu_s = \xi_1^s \, dF / \mathbb{E}[\xi_1^s]$. Since $1 - a > -a$ and f' is strictly increasing, the derivative is strictly positive. The chain rule $\partial a / \partial \phi_\pi < 0$ then gives $\partial R_0^* / \partial \phi_\pi < 0$. Finally, as $a \to 0$: $\mathbb{E}[\xi_1^{-a}] / \mathbb{E}[\xi_1^{1-a}] \to 1 / \mathbb{E}[\xi_1] = 1$. □

This proposition establishes the core mechanism in the paper: more aggressive inflation targeting (higher ϕ_π) lowers the expected real return on nominal bonds. The intuition stems from inflation risk. When inflation is volatile (low ϕ_π), nominal bonds deliver low real returns precisely when demand is strong (ξ_1 high) and savers value consumption most. Savers demand compensation for this risk. Aggressive inflation targeting flattens the inflation response to shocks, reducing this covariance and lowering the required premium.

Since $R_0^* > R^f$ for all finite ϕ_π and $R_0^* \to R^f$ as $\phi_\pi \to \infty$, the *inflation risk premium* $R_0^* - R^f > 0$ is strictly decreasing in ϕ_π: perfect inflation targeting eliminates inflation risk and the associated premium entirely.

Volatility versus Correlation Channels The inflation risk premium, defined as the excess expected return on the nominal bond over the risk-free real rate, $\psi \equiv R_0^* - R^f$ where $R^f = 1/\mathbb{E}[M_{01}]$, can be written using the standard asset pricing identity $\mathbb{E}[R] - R^f = -R^f \, \mathrm{Cov}(M_{01}, R)$ as:

$$\psi = -R^f \, (1 + \bar{i}_0) \cdot \rho_{M,\Pi^{-1}} \cdot \sigma_M \cdot \sigma_{\Pi^{-1}},$$

where $\rho_{M,\Pi^{-1}}$ is the correlation between the stochastic discount factor and inverse inflation. This decomposition reveals three potential channels through which monetary policy could affect the risk premium: changes in the correlation, changes in SDF volatility, and changes in inflation volatility.

At the zero-debt equilibrium, $M \propto \xi_1$ and $\bar{\Pi}_1^{-1} \propto \xi_1^{-a}$. Since both are monotonic functions of the same underlying shock, we can evaluate each component:

Channel	Expression	Depends on ϕ_π?	Share of $\frac{\partial \log R_0^*}{\partial \phi_\pi}$
Correlation	$\rho_{M,\Pi^{-1}} = -1$	No	0%
SDF volatility	$\sigma_M \propto \mathrm{sd}(\xi_1)$	No	0%
Inflation volatility	$\sigma_{\Pi^{-1}} \propto a \, \mathrm{sd}(\log \xi_1)$	**Yes**	**100%**

In our model, the entire comparative static $\partial \log R_0^* / \partial \phi_\pi$ operates through the inflation volatility channel.[3] The *correlation* between the stochastic discount factor and inflation—which determines whether bonds are "good" or "bad" hedges—is invariant to the policy regime: nominal bonds are always bad hedges in the sense that they deliver low real returns when savers' marginal utility is high. However, more aggressive monetary policy reduces the sensitivity of inflation to demand shocks through the pass-through parameter $a = (1-\rho)/\phi_\pi$, compressing the volatility of real bond payoffs. This shrinks the *covariance* between the SDF and real bond returns, reducing the risk premium even though the adverse correlation persists. In short, bonds remain directionally bad hedges, but the magnitude of the inflation risk they carry falls as the central bank stabilizes prices more aggressively.

3.3 Positive Private Debt

We now extend the analysis to economies with positive private debt at date 0, maintaining $d_1 = 0$ for tractability. In equilibrium with binding constraints, the saver's consumption becomes:

$$\begin{aligned} C_{s,0} &= Y - b, \\ C_{s,1} &= Y + \frac{1 + i_0}{\Pi_1} b, \end{aligned}$$

where $b > 0$ denotes the saver's real bond holdings.

We characterize how the equilibrium responds to changes in ϕ_π by expanding around the stochastic zero-debt equilibrium. Since this expansion point features positive aggregate risk (inflation is stochastic even at $b = 0$), our approximation embeds a nonzero inflation risk premium from the outset. This contrasts with the standard approach of expanding around a deterministic steady state, where risk premia vanish at first order.

[3] The decomposition is exact in logs: $\mathrm{Var}(\log \Pi_1) = a^2 \, \mathrm{Var}(\log \xi_1)$ depends on ϕ_π only through a, while $\mathrm{Var}(\log M)$ and $\mathrm{Corr}(\log M, \log \Pi_1^{-1})$ are invariant to ϕ_π.

Perturbation around $b = 0$. To first order in b, saver consumption at date 1 is $C_{s,1} = Y + h\,b$ where

$$h \equiv \frac{1+\bar{i}_0}{\bar{\Pi}_1} = \frac{\xi_1^{-a}}{\beta\,\mathbb{E}[\xi_1^{1-a}]}.$$

Write $(1+i_0) = (1+\bar{i}_0) + i_b\,b$ for the endogenous nominal rate response. Log-linearizing the inflation mapping $\Pi_1^{\phi_\pi} = \xi_1^{1-\rho}(Y/C_{s,1})^\gamma$ around $C_{s,1} = Y$:

$$\Pi_1^{-1} = \bar{\Pi}_1^{-1}\left[1 + \frac{\gamma}{\phi_\pi}\frac{h}{Y}\,b\right].$$

Expanding $R_0^*(b,\phi_\pi) = (1+i_0)\,\mathbb{E}[\Pi_1^{-1}]$:

$$R_0^*(b,\phi_\pi) = \bar{R}_0^*(\phi_\pi) + \Psi(\phi_\pi)\,b, \tag{3}$$

where $\bar{R}_0^* = R_0^*(0,\phi_\pi)$ is the zero-debt return from Proposition 1.

Proposition 2. *For a small positive debt limit $d_0 > 0$, the slope $\Psi(\phi_\pi)$ in (3) is positive and strictly decreasing in ϕ_π. Consequently,*

$$\frac{\partial R_0^*}{\partial \phi_\pi} < 0$$

for all $b \geq 0$.

Proof. Write $M(s) \equiv \mathbb{E}[\xi_1^s]$ and $a = (1-\rho)/\phi_\pi$. Differentiating the bond Euler equation with respect to b at $b = 0$ yields i_b; substituting into the expansion gives

$$\Psi = \frac{\gamma\,\bar{R}_0^*}{Y}\left[1 + \frac{V(a)}{\beta(1-\rho)}\right] \equiv \frac{\gamma\,\bar{R}_0^*}{Y}\,Q(a),$$

where

$$V(a) \equiv (1-\rho-a)\,r_1(a) + a\,r_2(a), \qquad r_1(a) \equiv \frac{M(1-2a)}{M(1-a)^2}, \quad r_2(a) \equiv \frac{M(-2a)}{M(-a)\,M(1-a)}.$$

At $a = 0$: $r_1(0) = r_2(0) = 1$ (using $M(0) = M(1) = 1$), so $V(0) = 1-\rho$ and $Q(0) = 1 + 1/\beta > 0$.

We show $V'(a) > 0$. Set $f(s) \equiv \log M(s)$, which is strictly convex: $f''(s) = \mathrm{Var}_s(\log \xi_1) >$

0, where Var_s denotes the variance under the tilted measure $d\mu_s = \xi_1^s \, dF/M(s)$. Differentiating: $V' = (r_2 - r_1) + (1-\rho-a)\, r_1' + a\, r_2'$. Each term is positive:

1. $r_2 > r_1$: The pairs $(-2a,\, 1-a)$ and $(-a,\, 1-2a)$ share the midpoint $(1-3a)/2$ but the first has spread $1 + a > 1 - a$ (the second pair's spread). Since f is strictly convex, $f(m-d)+f(m+d)$ is strictly increasing in d, so $f(-2a)+f(1-a) > f(-a)+f(1-2a)$.

2. $r_1' > 0$: $(\log r_1)' = 2[f'(1-a) - f'(1-2a)] > 0$ since f' is strictly increasing and $1 - a > 1 - 2a$.

3. $r_2' > 0$: $(\log r_2)' = f'(-a) + f'(1-a) - 2f'(-2a) > 0$ since $-a > -2a$ and $1-a > -2a$.

Since $V'(a) > 0$: $V(a) > V(0) = 1 - \rho$, so $Q(a) = 1 + V(a)/[\beta(1-\rho)] > 1 + 1/\beta > 0$ and $Q'(a) = V'(a)/[\beta(1-\rho)] > 0$.

By the product rule, $d\Psi/da = (\gamma/Y)[(\bar{R}_0^*)'\, Q + \bar{R}_0^*\, Q']$. By Proposition 1, $\bar{R}_0^*$ is positive and strictly increasing in a. Both terms are strictly positive, so $d\Psi/da > 0$, i.e., $\Psi'(\phi_\pi) < 0$.

Since $\partial \bar{R}_0^*/\partial \phi_\pi < 0$ and $\Psi'(\phi_\pi) < 0$: $\partial R_0^*/\partial \phi_\pi = \partial \bar{R}_0^*/\partial \phi_\pi + b\, \Psi'(\phi_\pi) < 0$ for all $b \geq 0$. □

Proposition 2 establishes the key result: more aggressive inflation targeting lowers the expected real return on nominal bonds when debt is positive. With binding borrowing constraints, the cheaper cost of borrowing (lower R_0^*) allows agents to borrow more in equilibrium, so that $b_{s,1}$ increases with ϕ_π. The mechanism operates through the supply of savings in nominal assets: more aggressive inflation targeting reduces inflation volatility, making nominal bonds safer in real terms. Savers, facing lower inflation risk, increase their supply of savings in nominal instruments. With a demand curve pinned down by the binding borrowing constraint, greater supply translates into lower equilibrium real rates and higher equilibrium debt quantities.

3.4 Real Capital Investment

Section 2 showed that besides the increase in nominal private debt, the last forty years are characterized by a reduction in the investment-to-GDP ratio. To account for this fact, we now extend the framework to include real capital investment. This allows us to analyze

how inflation targeting affects the portfolio allocation between real and nominal assets, and assess the possibility of crowding out of capital formation.

Setup. In addition to nominal bonds, the saver can now invest in real capital K_1 at date 0. Capital produces output $\bar{A}K_1^\alpha$ at date 1, where $\alpha \in (0, 1]$ governs the degree of diminishing returns: $\alpha = 1$ is the constant-returns benchmark, while $\alpha < 1$ introduces curvature. The saver's budget constraints become:

$$C_{s,0} + K_1 + b = Y_0, \tag{4}$$

$$C_{s,1} = \bar{A}K_1^\alpha + \frac{1 + i_0}{\Pi_1}\, b, \tag{5}$$

$$C_{s,2} = Y_2,$$

where $b \equiv B_{s,1}/P_0$ denotes real bond holdings. All other primitives (preferences, borrowing constraint, monetary policy rule, shock structure) are as in Section 3.1.

First-order conditions. Define the stochastic discount factor from date 0 to date 1:

$$M_{01} \equiv \beta\, \xi_1 \left(\frac{C_{s,1}}{C_{s,0}}\right)^{-\gamma}. \tag{6}$$

The saver's optimality conditions yield the Euler equations:

$$1 = \mathbb{E}_0\left[M_{01}\, \alpha \bar{A} K_1^{\alpha-1}\right] \qquad \text{(capital)}, \tag{7}$$

$$1 = \mathbb{E}_0[M_{01}\, (1 + i_0)/\Pi_1] \qquad \text{(bonds)}. \tag{8}$$

Zero-debt equilibrium. At $b = 0$, date-1 consumption is deterministic: $\bar{C}_1 = \bar{A}\bar{K}_1^\alpha$. The capital Euler equation (7) with $\mathbb{E}[\xi_1] = 1$ gives:

$$\beta\, R_K \cdot \left(\frac{\bar{C}_1}{\bar{C}_0}\right)^{-\gamma} = 1 \qquad \Longrightarrow \qquad R_K = \frac{1}{\beta}\left(\frac{\bar{C}_1}{\bar{C}_0}\right)^{\gamma}, \tag{9}$$

where $R_K \equiv \alpha \bar{A} \bar{K}_1^{\alpha-1}$ is the marginal product of capital and $\bar{C}_0 = Y_0 - \bar{K}_1$. The real risk-free rate is $R^f = 1/\mathbb{E}[M_{01}] = R_K$.

Equilibrium inflation takes the same form as in the endowment economy:

$$\bar{\Pi}_1 = \left(\frac{Y_2}{\bar{C}_1}\right)^{\gamma/\phi_\pi} \xi_1^a, \qquad a \equiv \frac{1-\rho}{\phi_\pi}. \tag{10}$$

Proposition 3 (Expected real bond return with capital)**.** *The expected real return on nominal bonds at $b = 0$ is:*

$$R_0^* = R^f \cdot \frac{\mathbb{E}[\xi_1^{-a}]}{\mathbb{E}[\xi_1^{1-a}]}, \tag{11}$$

where $R^f = R_K = \alpha \bar{A} \bar{K}_1^{\alpha-1}$. In particular, the comparative static $\partial R_0^*/\partial \phi_\pi < 0$ holds by the same log-convexity argument, and $R_0^* \to R^f$ as $\phi_\pi \to \infty$.

Proof. Evaluating the bond Euler (8) at $b = 0$ with $\bar{\Pi}_1 \propto \xi_1^a$ and using the steady-state relation $\beta(\bar{C}_1/\bar{C}_0)^{-\gamma} = R_K^{-1}$ from (9): $R_0^* = (1 + \bar{i}_0)\,\mathbb{E}[\bar{\Pi}_1^{-1}] = R_K\,\mathbb{E}[\xi_1^{-a}]/\mathbb{E}[\xi_1^{1-a}]$. The proof of $\partial R_0^*/\partial \phi_\pi < 0$ is identical to that of Proposition 1. □

This is the same formula as in Proposition 1 with $R^f = R_K$ replacing $R^f = 1/\beta$. The reason is intuitive: at $b = 0$, all savings are invested in real capital, so date-1 consumption $\bar{C}_1 = \bar{A}\bar{K}_1^\alpha$ is deterministic. The stochastic discount factor therefore has the same structure as in the endowment economy—it loads only on the preference shock ξ_1, not on inflation—and the pricing of nominal bonds is governed by the same trade-off between ξ_1 and Π_1^{-1}. Two features of this result are worth highlighting. First, the return on capital R_K is entirely pinned down by real fundamentals—the production technology, the discount factor, and the consumption-smoothing motive—and is invariant to the monetary policy regime. This is because capital pays off in real terms and carries no inflation exposure. Second, the expected real return on nominal bonds R_0^* exceeds R_K for any finite ϕ_π and converges to R_K from above as $\phi_\pi \to \infty$. The wedge $R_0^* - R_K$ is a pure inflation risk premium: it compensates savers for the fact that nominal bond payoffs are eroded by inflation in precisely the states where marginal utility is high. As the central bank stabilizes inflation more aggressively, this wedge shrinks, making nominal bonds increasingly substitutable for real capital on a risk-adjusted basis. It is this narrowing of the risk-adjusted return gap that drives the portfolio reallocation when we introduce positive debt in the next section.

3.4.1 Portfolio and Savings Margins

We now introduce positive debt limits at date 0, maintaining $d_1 = 0$. The key question is: when savers increase their holdings of nominal bonds, does this come at the expense of real capital investment, or do savers simply save more overall? To answer this, write total savings as $S_0 \equiv Y_0 - C_{s,0} = K_1 + b$. A change in bond demand affects capital through two margins:

$$\left.\frac{dK_1}{db}\right|_{b=0} = \underbrace{-1}_{\substack{\text{portfolio}\\ \text{margin}}} + \underbrace{\left.\frac{dS_0}{db}\right|_{b=0}}_{\substack{\text{savings}\\ \text{margin}}}. \tag{12}$$

The portfolio margin is mechanical: holding total savings fixed, each unit of bonds displaces a unit of capital one-for-one. The savings margin captures the endogenous response of date-0 consumption: when bonds become more attractive, the saver may choose to increase total savings rather than simply reshuffling the existing portfolio. If the savings margin is zero, crowd-out is complete; if the savings margin equals one, the saver finances the entire bond position through additional saving, and capital is unaffected.

Derivation of the crowd-out coefficient. Parameterize $K_1 = \bar{K}_1 + k\,b$ and $C_{s,0} = \bar{C}_0 - (1+k)\,b$ for small b, so that k is the crowd-out coefficient and $1+k$ is the savings margin. Linearizing the capital Euler equation (7) around the stochastic zero-debt equilibrium and using the bond-pricing identity $(1+\bar{i}_0)\,\mathbb{E}[\xi_1 \bar{\Pi}_1^{-1}] = R_K$ (which holds at $b=0$ by dividing (7) by (8)), we obtain:[4]

$$0 = (k+1)\left(\frac{\gamma\alpha}{\bar{K}_1} + \frac{\gamma}{\bar{C}_0}\right) + \frac{1-\alpha}{\bar{K}_1}\,k. \tag{13}$$

The first term captures the CRRA consumption-smoothing motive, operating through both date-1 consumption (via $\gamma\alpha/\bar{K}_1$) and date-0 consumption (via $\gamma/\bar{C}_0$). The second term, proportional to the curvature $(1-\alpha)$, reflects diminishing returns: as capital falls, the rising marginal product makes each remaining unit of capital more valuable, partially offsetting the displacement.

[4]The key algebraic simplification is the identity $R_K/\bar{C}_1 = \alpha/\bar{K}_1$, which eliminates the dependence on γ and $\bar{C}_0$ from the production-curvature term.

Proposition 4 (Crowd-out coefficient)**.** *With CRRA preferences (parameter $\gamma > 0$) and production $F(K) = \bar{A}K^{\alpha}$ with $\alpha \in (0, 1]$, the crowd-out coefficient at $b = 0$ is:*

$$k \equiv \left.\frac{dK_1}{db}\right|_{b=0} = -\frac{\gamma\left(\alpha\bar{C}_0 + \bar{K}_1\right)}{\gamma\left(\alpha\bar{C}_0 + \bar{K}_1\right) + (1-\alpha)\bar{C}_0} \in [-1, 0). \tag{14}$$

The savings margin is:

$$\left.\frac{dS_0}{db}\right|_{b=0} = 1 + k = \frac{(1-\alpha)\bar{C}_0}{\gamma\left(\alpha\bar{C}_0 + \bar{K}_1\right) + (1-\alpha)\bar{C}_0} \in [0, 1). \tag{15}$$

Crowd-out is strict ($k < 0$) for all $\alpha \in (0, 1]$ and $\gamma > 0$, with $k = -1$ when $\alpha = 1$ (one-for-one displacement) and $k \in (-1, 0)$ when $\alpha < 1$.

The sign of crowd-out is completely robust: for all admissible α and γ, an increase in nominal bond holdings reduces capital. The magnitude, however, depends on two elasticities—the curvature of the production function and the intertemporal elasticity of substitution—whose roles are clarified by the following limiting cases.

Proposition 5 (Role of elasticities)**.** *The crowd-out coefficient satisfies the following limiting cases:*

(i) *$k \to -1$ as $\alpha \to 1$ (constant returns) or as $\gamma \to \infty$ (low IES).*

(ii) *$k \to 0$ as $\alpha \to 0$ (strong curvature) or as $\gamma \to 0$ (high IES).*

Moreover, holding the zero-debt steady-state quantities $\bar{K}_1$ and $\bar{C}_0$ fixed:

(iii) *$|k|$ is increasing in α: more concave production attenuates crowd-out.*

(iv) *$|k|$ is increasing in γ: lower intertemporal elasticity of substitution strengthens crowd-out.*

Proof. All limits follow by direct substitution into (14).[5] □

[5]When $\alpha = 1$ and $\gamma \to 0$ simultaneously, the formula is $0/0$; the order of limits matters. For any fixed $\alpha < 1$, $k \to 0$ as $\gamma \to 0$; for any fixed $\gamma > 0$, $k \to -1$ as $\alpha \to 1$.

The two forces that push toward full crowd-out ($k = -1$) are economically distinct. Constant returns ($\alpha \to 1$) eliminate the stabilizing price signal: the marginal product of capital does not rise as the capital stock shrinks, so there is no force pulling savers back toward capital. Low IES ($\gamma \to \infty$) shuts down the savings margin: date-0 consumption is rigid, so savers are unwilling to cut consumption to finance additional bond holdings and instead displace capital one-for-one. Conversely, the two forces that attenuate crowd-out ($k \to 0$) work through opposite margins. Strong curvature ($\alpha \to 0$) means that even a small decline in capital sharply raises its marginal product, making capital increasingly attractive and arresting the displacement. High IES ($\gamma \to 0$) means that savers readily reduce current consumption to finance bond purchases, absorbing the displacement through additional saving rather than through lower investment.

3.4.2 Expected Real Bond Return with Positive Debt

The previous results establish that, at the zero-debt benchmark, more aggressive inflation targeting lowers the expected real return on nominal bonds (Proposition 3) and that any increase in bond holdings crowds out capital with coefficient $k \in [-1, 0)$ (Proposition 4). A natural question is whether the bond-return channel survives when debt is positive: does the interaction between outstanding debt and inflation risk undo or reinforce the basic mechanism? We now show that the result extends: more aggressive inflation targeting lowers real bond returns for all levels of debt, not just at the zero-debt benchmark.

Perturbation of date-1 consumption and inflation. Let $(1+i_0) = (1+\bar{i}_0)+i_b\, b$ for the endogenous nominal rate response. To first order, date-1 saver consumption is $C_{s,1} = \bar{C}_1 + h\, b$ where

$$h \equiv R_K\, k + \frac{1+\bar{i}_0}{\bar{\Pi}_1} \tag{16}$$

captures the two sources of date-1 income variation: the change in capital income ($R_K\, k$, which is negative since crowd-out reduces the capital stock) and the real bond repayment ($(1+\bar{i}_0)/\bar{\Pi}_1$, which is state-dependent through inflation). The bond-pricing identity $(1+\bar{i}_0)\,\mathbb{E}[\xi_1 \bar{\Pi}_1^{-1}] = R_K$ gives $\mathbb{E}[\xi_1 h] = R_K(k+1)$, which vanishes when $\alpha = 1$ and is positive when $\alpha < 1$ — reflecting the active savings margin in the presence of diminishing returns.

Log-linearizing the inflation mapping (10) around $b = 0$:

$$\Pi_1^{-1} = \bar{\Pi}_1^{-1}\left[1 + \frac{\gamma}{\phi_\pi}\frac{h}{\bar{C}_1}\, b\right]. \tag{17}$$

This expression reveals an important feedback: positive debt makes inflation itself respond to the policy regime through the consumption channel. When savers hold bonds, their date-1 consumption depends on inflation, which in turn feeds back into inflation through the Phillips curve relationship. The strength of this feedback is governed by γ/ϕ_π: more aggressive monetary policy dampens it.

Expanding $R_0^*(b, \phi_\pi) = (1 + i_0)\,\mathbb{E}[\Pi_1^{-1}]$:

$$R_0^*(b, \phi_\pi) = \bar{R}_0^*(\phi_\pi) + \Psi(\phi_\pi)\, b, \tag{18}$$

where $\bar{R}_0^* = R_0^*(0, \phi_\pi)$ is the zero-debt return from Proposition 3 and

$$\Psi(\phi_\pi) \equiv i_b\,\mathbb{E}[\bar{\Pi}_1^{-1}] + \frac{\gamma(1 + \bar{i}_0)}{\phi_\pi \bar{C}_1}\,\mathbb{E}[\bar{\Pi}_1^{-1} h]. \tag{19}$$

The coefficient $\Psi(\phi_\pi)$ measures how the expected real bond return responds to the level of debt. The first term captures the direct effect of the endogenous nominal rate adjustment; the second captures the indirect effect operating through the inflation feedback.

Determination of i_b. Differentiating the bond Euler with respect to b at $b = 0$ and using $\mathbb{E}[\xi_1 \bar{\Pi}_1^{-1}] = R_K/(1 + \bar{i}_0)$:

$$i_b = \underbrace{\frac{\gamma(1 + k)(1 + \bar{i}_0)}{\bar{C}_0}}_{\substack{\text{savings-margin} \\ \text{correction}}} + \underbrace{\frac{\gamma(1 + \bar{i}_0)^2(\phi_\pi - 1)}{\phi_\pi R_K \bar{C}_1}\,\mathbb{E}[\xi_1 \bar{\Pi}_1^{-1} h]}_{\substack{\text{inflation-feedback} \\ \text{term}}}. \tag{20}$$

The savings-margin correction reflects the saver's desire to smooth consumption intertemporally: when crowd-out is incomplete ($k > -1$), the saver increases total savings, which requires a higher nominal rate to clear the bond market. The inflation-feedback term arises because positive debt makes consumption, and hence inflation, endogenous to bond quantities. When $\alpha = 1$, $k = -1$ and the savings-margin correction vanishes: the saver does not

adjust total savings, so bond market clearing requires only the inflation-feedback adjustment.

Substituting (20) into (19):

$$\Psi(\phi_\pi) = \frac{\gamma(1+k)}{\bar{C}_0}\,\bar{R}_0^* \;+\; \frac{\gamma(1+\bar{i}_0)}{\phi_\pi \bar{C}_1}\left[(\phi_\pi - 1)\frac{\bar{R}_0^*}{R_K}\,\mathbb{E}[\xi_1 \bar{\Pi}_1^{-1} h] + \mathbb{E}[\bar{\Pi}_1^{-1} h]\right]. \tag{21}$$

Proposition 6 (Expected real bond return with positive debt and capital)**.** *With CRRA preferences (parameter $\gamma > 0$), production $F(K) = \bar{A}K^\alpha$ with $\alpha \in (0,1]$, and any bounded distribution for ξ_1 with positive variance, the expected real return on nominal bonds satisfies*

$$R_0^*(b, \phi_\pi) = \bar{R}_0^*(\phi_\pi) + \Psi(\phi_\pi)\, b, \tag{22}$$

where $\bar{R}_0^*(\phi_\pi)$ is the zero-debt return (Proposition 3) and $\Psi(\phi_\pi)$ is defined by (21)*. Moreover, $\Psi'(\phi_\pi) < 0$, hence*

$$\frac{\partial R_0^*}{\partial \phi_\pi} < 0$$

for all $b \geq 0$.

Proof. The expansion (22) follows from (18). Write $M(s) \equiv \mathbb{E}[\xi_1^s]$ and $a = (1-\rho)/\phi_\pi$. Since $\bar{\Pi}_1 \propto \xi_1^a$ and the bond-pricing identity gives $(1+\bar{i}_0)(Y_2/\bar{C}_1)^{-\gamma/\phi_\pi} = R_K/M(1-a)$, definition (16) becomes $h = R_K[k + \xi_1^{-a}/M(1-a)]$. Substituting into (21) yields

$$\Psi = \gamma\,\bar{R}_0^*\left[\frac{1+k}{\bar{C}_0} + \frac{R_K}{(1-\rho)\bar{C}_1}\Big((1-\rho)k + V(a)\Big)\right] \equiv \gamma\,\bar{R}_0^*\,Q(a),$$

where $V(a)$, $r_1(a)$, $r_2(a)$ are defined exactly as in the proof of Proposition 2. At $a = 0$: $V(0) = 1-\rho$, so $(1-\rho)k + V(a)\big|_{a=0} = (1-\rho)(1+k) \geq 0$ since $k \in [-1, 0)$. By the same log-convexity argument as in Proposition 2, $V'(a) > 0$, so $V(a) > V(0) = 1-\rho$ and $Q(a) > 0$ with $Q'(a) = R_K\,V'(a)/[(1-\rho)\bar{C}_1] > 0$. Since $\Psi = \gamma\,\bar{R}_0^*\,Q$, the product rule gives $d\Psi/da = \gamma[(\bar{R}_0^*)'\,Q + \bar{R}_0^*\,Q']$. By Proposition 3, $\bar{R}_0^*$ is positive and strictly increasing in a. Both terms are strictly positive, so $d\Psi/da > 0$, i.e., $\Psi'(\phi_\pi) < 0$. Since $\partial\bar{R}_0^*/\partial\phi_\pi < 0$ and $\Psi'(\phi_\pi) < 0$: $\partial R_0^*/\partial\phi_\pi < 0$ for all $b \geq 0$. □

Proposition 6 is the central result of the paper: it unifies the pricing and quantity implications of the inflation risk channel. The result says that the negative effect of inflation

targeting on real bond returns is not an artifact of the zero-debt benchmark but persists—and is in fact reinforced—when debt is positive. The reinforcement ($\Psi'(\phi_\pi) < 0$) arises because positive debt introduces an additional channel through which monetary policy affects bond returns: outstanding nominal debt makes saver consumption sensitive to inflation, which feeds back into equilibrium inflation through the Euler equation and the Taylor rule. More aggressive monetary policy dampens this feedback, further reducing the real volatility of bond payoffs and compressing the risk premium.

The equilibrium implications follow directly. As ϕ_π rises, the expected real return on nominal bonds falls, making bonds more attractive relative to capital on a risk-adjusted basis. Savers reallocate their portfolios toward bonds: bond holdings rise and capital investment falls, with the magnitude of displacement governed by the crowd-out coefficient k from Proposition 4. With constant returns ($\alpha = 1$), crowd-out is one-for-one and the savings margin is inactive. With diminishing returns ($\alpha < 1$), the rising marginal product of the shrinking capital stock partially absorbs the displacement, but crowd-out remains strict for all parameter configurations. The expected return on capital, pinned down by real fundamentals, is invariant to the monetary regime—so the spread between risky capital returns and safe bond returns widens as inflation targeting becomes more aggressive.

Summary. Section 3 delivers three results. First, the expected real return on nominal bonds is strictly decreasing in the aggressiveness of monetary policy (ϕ_π), regardless of the level of nominal debt and regardless of whether the economy features real capital investment. Second, in the presence of real capital, an increase in bond holdings crowds out physical investment, with a crowd-out coefficient $k \in [-1, 0)$ that is bounded away from zero whenever diminishing returns are present ($\alpha < 1$). Third, at the zero-debt benchmark, the entire comparative static operates through inflation volatility: the correlation between the SDF and inflation, and the volatility of the SDF itself, are invariant to the policy regime. Away from the zero-debt benchmark, positive debt introduces additional margins—saver consumption and the SDF become endogenous to the policy regime through nominal debt repayments—but the overall sign of the effect is preserved and reinforced. These findings are manifestations of a single mechanism: by reducing inflation pass-through, aggressive mon-

etary policy compresses the inflation risk premium, which simultaneously lowers real bond returns, expands nominal credit, and displaces physical capital.

4 Conclusion

This paper develops a theoretical framework connecting the strengthening of inflation targeting regimes to secular trends in macro-finance since the 1980s. The mechanism operates through the risk characteristics of nominal assets. When central banks commit more aggressively to inflation stabilization, they reduce inflation volatility, lowering the risk premium investors require for holding nominal bonds. This decline in real rates on nominal assets encourages saving through private debt instruments while crowding out riskier real capital investment.

Our analytical results, derived in the three-period general equilibrium model, establish that long-term real rates decrease and private debt increases with stronger inflation targeting, while capital investment falls and the expected return on capital rises. These comparative statics align with the empirical observation that the strengthening of inflation targeting coincided with declining real rates, rising private debt, and persistent investment weakness, alongside stable returns on capital.

Several extensions merit investigation in this paper. First, quantitative implementation calibrated to match key moments of the post-1980 data could discipline the magnitude of effects attributable to the monetary policy channel relative to alternative explanations. Second, decomposing the decline in long-term real rates into contributions from policy regimes versus demographic, productivity, and other real factors would sharpen our understanding of relative importance. Third, welfare analysis incorporating both the benefits of price stability and the costs of reduced capital accumulation would provide normative guidance for optimal policy design.

The broader message of our analysis is that successful nominal anchoring—one of the signal achievements of modern central banking—may carry underappreciated consequences for the allocation of savings between safe nominal assets and risky real capital. Understanding these consequences is essential for evaluating the full effects of monetary policy regimes.

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Appendix A: Proofs for Positive Private Debt

This appendix provides detailed derivations for the linearized equilibrium with positive private debt in the three-period model.

A.1 Proof of Proposition 2

The equilibrium of the economy is given by

$$1 = \mathbb{E}_0 \left[\beta \xi_1 \left(\frac{Y - b_{s,1}}{Y + \frac{R_0^*}{\mathbb{E}_0\left[\Pi_1^{-1}\right]\Pi_1} b_{s,1}} \right)^{\gamma} \frac{R_0^*}{\mathbb{E}_0\left[\Pi_1^{-1}\right]\Pi_1} \right] \tag{23}$$

$$1 = \mathbb{E}_1 \left[\frac{\xi_2}{\xi_1} \left(\frac{Y + \frac{R_0^*}{\mathbb{E}_0\left[\Pi_1^{-1}\right]\Pi_1} b_{s,1}}{Y} \right)^{\gamma} \Pi_1^{\phi_\pi} \right]. \tag{24}$$

Date-1 Euler Equation. The saver's Euler equation in period 1 can be written as:

$$\xi_1^{1-\rho} = \overline{\Pi}_1^{\phi_\pi} e^{\phi_\pi \hat{\pi}_1} + \overline{\Pi}_1^{\phi_\pi - 1} e^{(\phi_\pi - 1)\hat{\pi}_1} \left(1 + \bar{i}_0\right) e^{\hat{i}_0} \hat{b}_{s,1}.$$

Log-linearizing around the zero-debt equilibrium, we obtain:

$$\hat{\pi}_1 = -\frac{\gamma}{\phi_\pi} \frac{\overline{R}_0^*}{\mathbb{E}_0\left[\overline{\Pi}_1^{-1}\right]\overline{\Pi}_1} \hat{b}_{s,1}.$$

From the zero-debt equilibrium, we know that:

$$\frac{\overline{R}_0^*}{\mathbb{E}_0\left[\overline{\Pi}_1^{-1}\right]\overline{\Pi}_1} = \frac{1}{\beta} e^{\frac{1-\rho}{\phi_\pi}\left(1-\frac{1-\rho}{\phi_\pi}\right)\frac{\sigma^2}{2}} \xi_1^{-\frac{1-\rho}{\phi_\pi}}.$$

Substituting this expression yields equation (**??**):

$$\hat{\pi}_1 = -\frac{1}{\beta}\frac{\gamma}{\phi_\pi} e^{\frac{1-\rho}{\phi_\pi}\left(1-\frac{1-\rho}{\phi_\pi}\right)\frac{\sigma^2}{2}} \xi_1^{-\frac{1-\rho}{\phi_\pi}} \hat{b}_{s,1}.$$

Date-0 Euler Equation. The saver's Euler equation in period 0 is:

$$\mathbb{E}_0\left[\overline{\Pi}_1^{-1} e^{-\hat{\pi}_1}\right] = \beta\left(1 - \hat{b}_{s,1}\right) \mathbb{E}_0\left[\overline{\Pi}_1^{\phi_\pi - 1} e^{(\phi_\pi - 1)\hat{\pi}_1}\right] \overline{R}_0^* e^{\hat{r}_0^*}.$$

Log-linearizing:

$$-\mathbb{E}_0\left[\overline{\Pi}_1^{-1}\hat{\pi}_1\right] = -\beta\mathbb{E}_0\left[\overline{\Pi}_1^{\phi_\pi-1}\right]\overline{R}_0^*\hat{b}_{s,1} + \beta\mathbb{E}_0\left[\overline{\Pi}_1^{\phi_\pi-1}(\phi_\pi-1)\hat{\pi}_1\right]\overline{R}_0^* + \beta\mathbb{E}_0\left[\overline{\Pi}_1^{\phi_\pi-1}\right]\overline{R}_0^*\hat{r}_0^*.$$

Using the zero-debt Euler equation $\mathbb{E}_0\left[\overline{\Pi}_1^{-1}\right] = \beta\mathbb{E}_0\left[\overline{\Pi}_1^{\phi_\pi-1}\right]\overline{R}_0^*$, this simplifies to:

$$\hat{b}_{s,1} = (\phi_\pi-1)\frac{\mathbb{E}_0\left[\overline{\Pi}_1^{\phi_\pi-1}\hat{\pi}_1\right]}{\mathbb{E}_0\left[\overline{\Pi}_1^{\phi_\pi-1}\right]} + \frac{\mathbb{E}_0\left[\overline{\Pi}_1^{-1}\hat{\pi}_1\right]}{\mathbb{E}_0\left[\overline{\Pi}_1^{-1}\right]} + \hat{r}_0^*.$$

Substituting $\hat{\pi}_1 = -\frac{1}{\phi_\pi}\frac{1}{\beta}e^{\frac{1-\rho}{\phi_\pi}\left(1-\frac{1-\rho}{\phi_\pi}\right)\frac{\sigma^2}{2}}\xi_1^{-\frac{1-\rho}{\phi_\pi}}\hat{b}_{s,1}$ and using $\overline{\Pi}_1 = e^{\frac{\epsilon_1}{\phi_\pi}}$ where $\epsilon_1 \sim N(-\sigma^2/2, \sigma^2)$, we can compute the expectations using lognormal moment formulas.

After evaluating the expectations, we obtain equation (3):

$$\hat{r}_0^* = \gamma\left[1 + \frac{1}{\beta}\frac{\phi_\pi-1}{\phi_\pi}e^{\left(\frac{1-\rho}{\phi_\pi}\right)^2\sigma^2} + \frac{1}{\beta}\frac{1}{\phi_\pi}e^{\frac{1-\rho}{\phi_\pi}\left(1+\frac{1-\rho}{\phi_\pi}\right)\sigma^2}\right]\hat{b}_{s,1}.$$

Market Clearing. The debt market clearing condition gives:

$$\frac{b_{s,1}}{Y_0} = \frac{\mathbb{E}_0\left[\Pi_1^{-1}\right]}{R_0^*}d_0.$$

Log-linearizing:

$$\hat{b}_{s,1} = \frac{\mathbb{E}_0\left[\overline{\Pi}_1^{-1}\right]}{\overline{R}_0^*}d_0 = \beta e^{-\frac{1-\rho}{\phi_\pi}\left(1-\frac{1-\rho}{\phi_\pi}\right)\frac{\sigma^2}{2}}d_0.$$

This completes the characterization of the linearized equilibrium. □

A.2 Proof that $\partial\hat{r}_0^*/\partial\phi_\pi < 0$

We differentiate the expression for $\hat{r}_0^*$ from Proposition 2 with respect to ϕ_π. Let $a = \frac{1-\rho}{\phi_\pi}$, so $\frac{da}{d\phi_\pi} = -\frac{a}{\phi_\pi}$. The coefficient on $\hat{b}_{s,1}$ is:

$$T(\phi_\pi) = \gamma\left[1 + \frac{1}{\beta}\frac{\phi_\pi-1}{\phi_\pi}e^{a^2\sigma^2} + \frac{1}{\beta}\frac{1}{\phi_\pi}e^{a(1+a)\sigma^2}\right].$$

Taking the derivative with respect to ϕ_π (dropping γ and $\frac{1}{\beta}$ as constants):

$$\frac{\partial T}{\partial\phi_\pi} = \frac{\partial}{\partial\phi_\pi}\left[\frac{\phi_\pi-1}{\phi_\pi}e^{a^2\sigma^2}\right] + \frac{\partial}{\partial\phi_\pi}\left[\frac{1}{\phi_\pi}e^{a(1+a)\sigma^2}\right].$$

For the first term:

$$\frac{\partial}{\partial \phi_\pi}\left[\frac{\phi_\pi - 1}{\phi_\pi} e^{a^2\sigma^2}\right] = \frac{1}{\phi_\pi^2} e^{a^2\sigma^2} + \frac{\phi_\pi - 1}{\phi_\pi} \cdot 2a\sigma^2 \cdot \left(-\frac{a}{\phi_\pi}\right) e^{a^2\sigma^2} = \frac{e^{a^2\sigma^2}}{\phi_\pi^2}\left[1 - 2(\phi_\pi - 1)a^2\sigma^2\right].$$

For the second term:

$$\frac{\partial}{\partial \phi_\pi}\left[\frac{1}{\phi_\pi} e^{a(1+a)\sigma^2}\right] = -\frac{1}{\phi_\pi^2} e^{a(1+a)\sigma^2} + \frac{1}{\phi_\pi} \cdot (1+2a)\sigma^2 \cdot \left(-\frac{a}{\phi_\pi}\right) e^{a(1+a)\sigma^2} = -\frac{e^{a(1+a)\sigma^2}}{\phi_\pi^2}\left[1 + a(1+2a)\sigma^2\right].$$

Therefore:

$$\frac{\partial T}{\partial \phi_\pi} = \frac{e^{a^2\sigma^2}}{\phi_\pi^2}\left[1 - 2(\phi_\pi - 1)a^2\sigma^2\right] - \frac{e^{a(1+a)\sigma^2}}{\phi_\pi^2}\left[1 + a(1+2a)\sigma^2\right].$$

Factoring out $\frac{e^{a^2\sigma^2}}{\phi_\pi^2}$:

$$\frac{\partial T}{\partial \phi_\pi} = \frac{e^{a^2\sigma^2}}{\phi_\pi^2}\left\{\left[1 - 2(\phi_\pi - 1)a^2\sigma^2\right] - e^{a\sigma^2}\left[1 + a(1+2a)\sigma^2\right]\right\}.$$

The first factor $\frac{e^{a^2\sigma^2}}{\phi_\pi^2}$ is strictly positive. For the bracketed term, since $e^{a\sigma^2} > 1$ for $a > 0$ and $\sigma^2 > 0$:

$$e^{a\sigma^2}\left[1 + a(1+2a)\sigma^2\right] > 1 + a(1+2a)\sigma^2.$$

Therefore:

$$\left[1 - 2(\phi_\pi - 1)a^2\sigma^2\right] - e^{a\sigma^2}\left[1 + a(1+2a)\sigma^2\right] < 1 - 2(\phi_\pi - 1)a^2\sigma^2 - [1 + a(1+2a)\sigma^2].$$

Simplifying the right-hand side:

$$1 - 2(\phi_\pi - 1)a^2\sigma^2 - 1 - a(1+2a)\sigma^2 = -2(\phi_\pi - 1)a^2\sigma^2 - a(1+2a)\sigma^2.$$

Since $\phi_\pi > 1$, $a = \frac{1-\rho}{\phi_\pi} > 0$ (because $\rho < 1$), and $\sigma^2 > 0$:

$$-2(\phi_\pi - 1)a^2\sigma^2 - a(1+2a)\sigma^2 < 0.$$

Thus the bracketed term is strictly negative, and therefore $\frac{\partial \hat{r}_0^*}{\partial \phi_\pi} < 0$. □

A.3 Proof that $\partial \hat{r}_0^*/\partial \phi_\pi < 0$ in the model with capital

Proof. From (3), the sign of $\partial \hat{r}_0^*/\partial \phi_\pi$ is determined by the sign of

$$\frac{\partial}{\partial \phi_\pi}\left[\frac{\phi_\pi - 1}{\phi_\pi}\left(e^{a^2\sigma^2} - 1\right) + \frac{1}{\phi_\pi}\left(e^{a(1+a)\sigma^2} - 1\right)\right] = B'(\phi_\pi),$$

where $a = (1-\rho)/\phi_\pi$. Using $da/d\phi_\pi = -a/\phi_\pi$ and differentiating the exponential terms yields

$$B'(\phi_\pi) = -\frac{1}{\phi_\pi^2}\left[\left(e^{a(1+a)\sigma^2} - e^{a^2\sigma^2}\right) + 2(\phi_\pi - 1)a^2\sigma^2 e^{a^2\sigma^2} + a(1+2a)\sigma^2 e^{a(1+a)\sigma^2}\right].$$

For $\sigma^2 > 0$, $\rho < 1$ (so $a > 0$), and $\phi_\pi > 1$, all three terms in brackets are strictly positive: (i) $e^{a(1+a)\sigma^2} > e^{a^2\sigma^2}$ since $a(1+a) > a^2$; (ii) the second and third terms are manifestly positive. Therefore $B'(\phi_\pi) < 0$, establishing the result. □