Expansionary Hikes

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Abstract

Expansionary hikes describe a non-conventional situation in which a monetary tightening, normally expected to curb inflation and output, leads instead to higher inflation and an expansion in economic activity. This paper develops a tractable analytical framework to explain when and why such episodes arise. Within an otherwise standard New Keynesian model augmented with an explicit fiscal block, we show that under an *output-targeting fiscal policy*, interest-rate hikes increase the government's interest bill without a compensating fiscal adjustment. When the fiscal authority instead *rolls over debt*, monetary tightening raises interest payments that are financed through new debt issuance. In both cases, the resulting expansion in nominal liabilities acts as a net transfer to the private sector, stimulating consumption through a *nominal wealth effect*. The analysis implies that in highly indebted economies, the effectiveness—and even the sign—of monetary policy depends on the fiscal regime, challenging conventional interpretations of rate hikes as unambiguously disinflationary.

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1 Introduction

The traditional view of monetary policy, rooted in the New Keynesian framework, holds that central banks raise policy interest rates to restrain aggregate demand and curb inflationary pressures. In this framework, systematic policy behavior is often captured by the *Taylor rule* (Taylor, 1993), which prescribes that nominal interest rates should rise more than one-for-one with inflation—the so-called *Taylor principle*. This ensures that real interest rates increase when inflation rises, dampening consumption and investment and stabilizing both prices and output.

Over the past four decades, adherence to this principle has been credited with securing the *Great Moderation*: stable inflation expectations and reduced macroeconomic volatility across advanced economies. However, the post-pandemic experience challenged this traditional narrative. In the wake of the COVID-19 shock, central banks—most notably the Federal Reserve—deviated substantially from the Taylor rule, allowing inflation to rise temporarily while avoiding a deep recession. Nakamura et al. (2025) document that during this episode, central banks with strong anti-inflation credibility deliberately "looked through" supply-driven inflation, achieving what they call an "immaculate disinflation." In their interpretation, the success of this approach reflected *anchored expectations*: credibility substituted for mechanical adherence to the Taylor principle.

At the same time, the pandemic response featured unprecedented fiscal expansions. Government deficits and public debt surged to levels unseen in peacetime, and large-scale transfers supported household balance sheets. These developments raise a question largely absent from pre-pandemic analyses: how does the effectiveness of monetary tightening depend on the fiscal position? When the public sector carries a large stock of interest-bearing liabilities, higher policy rates directly increase the interest bill on government debt, transferring income to bondholders and, *ceteris paribus*, expanding private sector wealth.

This mechanism is emphasized in Modern Monetary Theory (MMT), which argues that interest rate hikes may operate as fiscal injections rather than contractions. Kelton (2020) highlights that rising rates increase government interest payments—a form of income support for the private sector—potentially offsetting the contractionary demand effects of tighter credit conditions. Mosler (1995) and Mosler (2023) make a similar point: with large outstanding public debt, higher rates can add to aggregate demand through increased interest income, especially when wealth holders have nontrivial propensities to consume. The quantitative relevance of this *interest-income channel* grows with the size of public debt and depends on its distribution across agents.

Our argument. We argue that the post-pandemic combination of (i) a weak empirical link between policy rates and inflation, (ii) large-scale fiscal expansions, and (iii) historically high public debt ratios signals a shift toward a regime of fiscal dominance. In such a regime, fiscal variables—the path of primary surpluses, the maturity structure of public debt, and the scale of

interest payments—become central to the determination of inflation and output, even within an otherwise standard New Keynesian environment.

Our analytical framework extends the canonical New Keynesian model by introducing an explicit fiscal block that allows for debt rollover and feedback from fiscal policy to aggregate demand. The joint configuration of the monetary and fiscal rules determines whether the economy operates under monetary dominance, where the central bank anchors the price level, or fiscal dominance, where inflation is determined by the intertemporal resource budget constraint. Under monetary dominance, a higher policy rate lowers current consumption through the intertemporal substitution channel and stabilizes inflation, consistent with traditional theory. Fiscal policy in this regime accommodates monetary tightening by stabilizing the real value of government debt, ensuring that wealth effects are transitory and that real allocations remain unaffected in steady state.

Under fiscal dominance, by contrast, the government's response to interest payments becomes the primary driver of inflation dynamics. When higher rates are financed through additional nominal debt issuance—captured by a weak or negative fiscal feedback parameter, households perceive the increase in nominal liabilities as an expansion in their wealth. This *nominal wealth effect* stimulates consumption and raises inflation even in response to a contractionary rate hike. As a result, monetary tightening may become expansionary rather than contractionary, producing what we label expansionary hikes. This mechanism extends the tight money paradox of Loyo (1999) within a New Keynesian structure: when fiscal policy accommodates higher interest expenditures through debt rollover, the conventional transmission of monetary policy can invert, with higher policy rates fueling, rather than restraining, inflation and output.

Within this framework, the insight often associated with Modern Monetary Theory—that higher interest rates can be expansionary—could be framed within a particular manifestation of equilibrium interactions between monetary and fiscal policy. The model provides a formal rationale for this claim: in a high-debt environment, a monetary tightening unaccompanied by fiscal adjustment raises interest expenditures and thus acts as a net fiscal transfer to the private sector, offsetting or even reversing the standard contractionary effect of higher rates.

More specifically, in our framework, expansionary hikes can arise through two related fiscal mechanisms. The first occurs under an *output-targeting fiscal rule* , where fiscal policy responds to economic activity but not to the level of debt. In this case, the government's intertemporal constraint must be closed by revaluing its outstanding nominal liabilities through inflation. The stock of debt becomes a key state variable: larger debt implies a greater price-level adjustment in response to monetary tightening. The second mechanism operates when fiscal policy actively accommodates higher interest payments through *debt rollover* . Here, the expansion of nominal liabilities generates a *nominal wealth effect*—households perceive an increase in net nominal assets and raise current spending, producing an inflationary and expansionary response to higher rates.

Nominal rigidities amplify both mechanisms by slowing the adjustment of prices and wages,

prolonging the real effects of nominal revaluations. When prices are sticky, the higher interest bill on government debt translates into a persistent increase in perceived private wealth, reinforcing the positive feedback between demand, inflation, and nominal debt accumulation. Hence, the interaction between fiscal accommodation and nominal rigidity determines whether monetary tightening dampens or amplifies inflation.

These dynamics resonate with the current U.S. policy context, characterized by elevated public debt, persistent fiscal deficits, and large interest outlays following recent rate hikes. In such an environment, the disinflationary power of monetary policy depends crucially on whether fiscal policy stabilizes the real value of debt or accommodates it through continued issuance. The model therefore provides a structural interpretation of the ongoing debate about monetary–fiscal coordination.

This perspective complements the interpretation of Nakamura et al. (2025), who emphasizes expectations and credibility as central to monetary transmission beyond the Taylor rule. We emphasize instead that fiscal policy itself can sustain or undermine that credibility by shaping the intertemporal budget constraint of the economy. In doing so, we provide an alternative equilibrium characterization—one in which the boundary between monetary and fiscal policy becomes endogenous, and the effectiveness of interest-rate policy depends critically on the fiscal regime in place.

Related Literature This paper contributes to several strands of research that examine the interaction between monetary and fiscal policy, the transmission of interest rate changes, and the conditions under which monetary tightening can produce counterintuitive outcomes.

Monetary–Fiscal Interactions

Our framework builds on the monetary–fiscal literature initiated by Sargent and Wallace (1981) and formalized by Leeper (1991a). These contributions emphasize that the determinacy of equilibrium prices depends on the relative stance of fiscal and monetary policy—whether monetary policy actively stabilizes inflation or fiscal policy anchors debt sustainability. Recent work, such as Bianchi and Melosi (2019), revisits this interaction in the context of high-debt economies, showing that credibility shocks to fiscal adjustment can shift the economy from a regime of monetary to fiscal dominance. We extend this insight by providing an analytical decomposition of the transmission of monetary shocks under different fiscal configurations, focusing in particular on the role of debt rollover and output-targeting fiscal rules.

The Perverse Effects of Monetary Tightening

A related strand of research highlights circumstances in which higher policy rates may have perverse or inflationary effects. Ravenna and Walsh (2006) introduce the so-called *cost channel* of monetary policy, whereby firms' marginal costs rise with the policy rate because working capital or credit financing depends on short-term borrowing costs. As a result, a higher nominal interest rate can increase inflationary pressure in the short run, complicating stabilization.

More recently, Benigno (2023) has revived this argument in the context of the United Kingdom's inflation dynamics, emphasizing what he terms a *Catch-22 effect*: by raising policy rates to combat inflation, the central bank directly increases the rental and mortgage components of the consumer price index, thereby reinforcing the very price pressures it aims to suppress. This phenomenon underscores that monetary tightening can feed into the cost structure of the economy, generating self-reinforcing inflationary dynamics even in the absence of fiscal stimulus.

Our analysis complements these findings by showing that such "perverse" monetary effects can also emerge through the fiscal channel. When fiscal policy either targets output without stabilizing debt or accommodates interest payments through debt rollover, higher interest rates raise government outlays and expand private sector nominal wealth. This mechanism operates independently of the cost channel: even if firms' marginal costs remain unaffected, aggregate demand increases through higher fiscal interest transfers. In this sense, our model identifies a distinct yet complementary pathway through which monetary tightening can become inflationary.

Reinterpreting MMT through a New Keynesian Lens

In doing so, we reinterpret the key insight of Modern Monetary Theory (MMT)—that higher interest rates can be expansionary because they raise private income through interest payments—within a microfounded, equilibrium-consistent framework. Rather than treating this as an accounting identity, we embed it in a New Keynesian model with explicit fiscal dynamics and expectations. The result is a tractable environment in which debt dynamics, fiscal credibility, and monetary policy are jointly determined.

Our contribution thus bridges the gap between MMT intuitions and mainstream macroeconomics. The framework demonstrates that the MMT interest-income channel can be rationalized as an equilibrium outcome arising under fiscal dominance, high public debt, and non-Ricardian fiscal behavior. In this setting, the transmission of monetary policy is endogenously state-dependent: the same policy rate increase may be contractionary under monetary dominance but expansionary under fiscal dominance.

Summary

Taken together, these literatures underscore that the macroeconomic effects of interest rate changes depend crucially on the interaction between nominal rigidities, fiscal stance, and the structure of debt financing. By unifying these mechanisms within a single analytical model, we provide an equilibrium characterization of the conditions under which monetary tightening becomes expansionary—a phenomenon we label *expansionary hikes*.

2 Key Stylized Evidence

Recent macroeconomic developments in the United States reveal a set of stylized facts that motivate our analysis of *Expansionary Hikes*. These facts point to a structural shift in the interaction between monetary and fiscal policy—one characterized by a departure from traditional

monetary policy rules, an unprecedented accumulation of public debt, and a growing fiscal interest burden. Together, these developments provide the empirical background for the mechanisms explored in our model.

Deviation from the Taylor Principle

A central observation concerns the conduct of U.S. monetary policy in the post-pandemic period. Figure 1 contrasts the actual Federal Funds Rate with the implied rate from a conventional Taylor rule. The figure illustrates that, for much of the post-2020 period, the effective policy rate has remained below the level prescribed by the rule, even as inflation surged.

This persistent gap suggests that monetary policy has deviated from the Taylor principle—the requirement that the nominal interest rate rise more than one-for-one with inflation. This departure from rule-based behavior is an important precondition for the fiscal-monetary interactions that underpin the emergence of expansionary hikes.

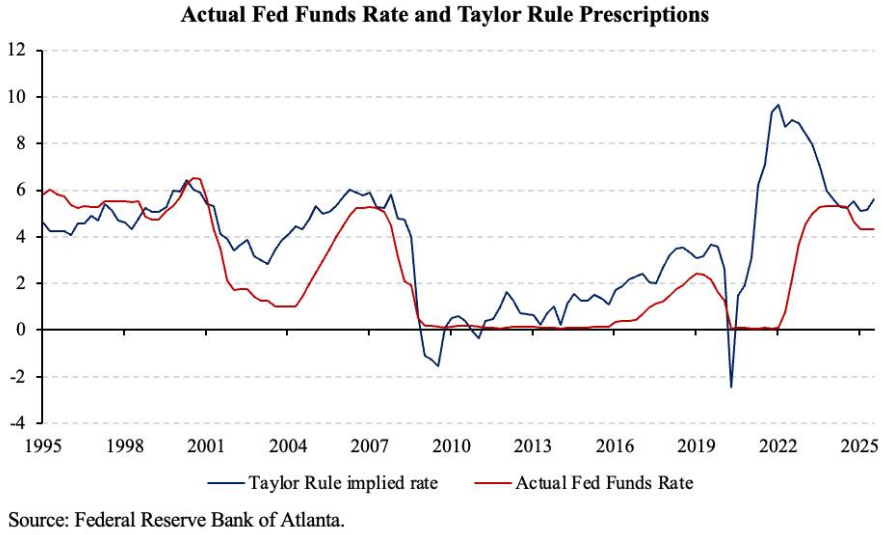


Figure 1: Effective Federal Funds Rate (red) and the Taylor-Rule-Implied Rate (blue). The implied rate is computed as $\hat{FFR}_t = \rho FFR_{t-1} + (1-\rho)[(r_t^* + \pi_t^*) + 1.5(\pi_t - \pi_t^*) + \beta \min\{gap_t, \theta\}]$, with $\rho = 0$, $\beta = 0.5$, $\pi_t^* = 2\%$. r_t^* is the Laubach–Williams (2003) one-sided estimate of the natural real rate; π_t is four-quarter core PCE inflation; and gap_t is the CBO output gap. Persistent deviations of the effective rate below the rule highlight the post-pandemic departure from Taylor-principle behavior.

Fiscal Expansion and the Rise of Public Debt

The second empirical development is the sustained rise in the federal debt-to-GDP ratio, shown in Figure 2. After a temporary decline in the late 2010s, public debt accelerated sharply following the pandemic, surpassing 120% of GDP by 2024. This increase reflects the joint impact of large-scale fiscal stimulus and pandemic-related spending, as well as structural imbalances in entitlement programs and interest payments.

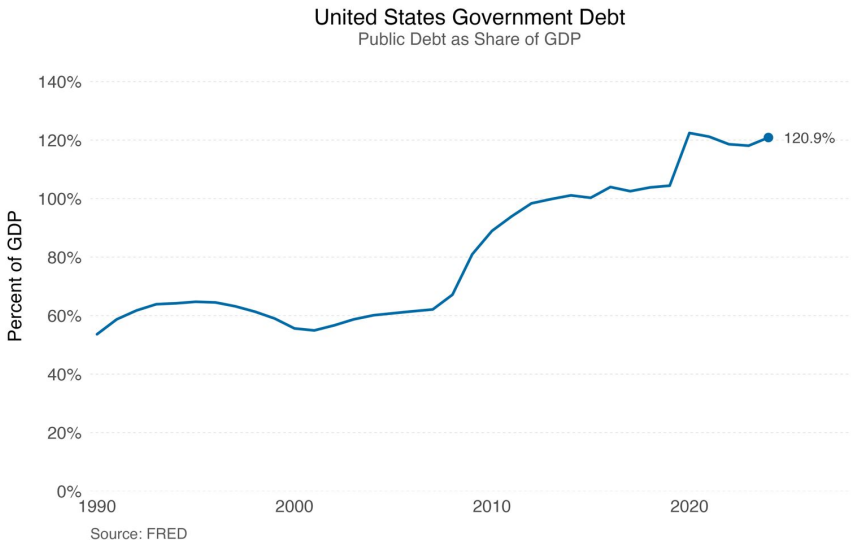


Figure 2: U.S. Federal Debt-to-GDP Ratio, 2000–2025.

High levels of public debt alter the transmission of monetary policy. When interest rates rise, the government's interest bill increases mechanically, transferring income to bondholders. In an environment where fiscal policy does not offset these higher payments through primary surpluses, monetary tightening can inadvertently stimulate demand—a key channel for the expansionary-hike mechanism developed later in the paper.

The Growing Fiscal Interest Burden

A related fact is the dramatic rise in net interest expenses and fiscal deficits. Figure 3 shows that net interest payments as a share of GDP have nearly doubled since 2021, while primary deficits have remained sizable despite strong nominal growth. This combination implies that higher policy rates now translate directly into fiscal expansions through the interest channel, especially in the absence of corresponding fiscal consolidation.

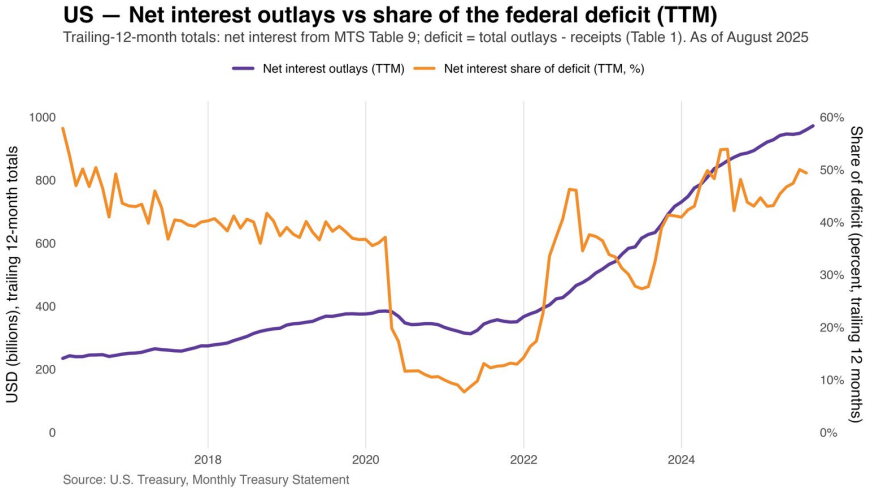


Figure 3: U.S. Net Interest Payments and Fiscal Deficit (% of GDP), 2010–2025.

Synthesis: A New Policy Regime

Taken together, these empirical patterns suggest that the U.S. economy may have transitioned toward a regime consistent with *fiscal dominance*. Monetary policy has become less Taylor rule-based, while fiscal policy remains expansionary amid high debt and rising interest obligations. We argue that this configuration implies that the standard monetary transmission mechanism—where higher interest rates dampen inflation through intertemporal substitution—can be attenuated or even reversed. Instead, higher rates may amplify aggregate demand through the fiscal channel, as the government's rising interest payments act as a transfer to the private sector.

These background facts motivate the analytical framework developed in the remainder of the paper. By embedding fiscal feedback rules within a New Keynesian structure, we show formally how deviations from the Taylor principle, high public debt, and debt rollover jointly give rise to the phenomenon of *expansionary hikes*.

3 The Model

We consider a standard New Keynesian model in discrete time, augmented to include an explicit fiscal authority. The economy is populated by a representative household and a continuum of firms producing intermediate and final goods. Final-good producers operate under perfect competition and combine a continuum of differentiated intermediate goods through a constant-elasticity-of-substitution (CES) technology with elasticity of substitution $\epsilon > 1$. Production uses labor as the only input, and intermediate-good firms operate under monopolistic competition with price rigidities of the Calvo type.

3.1 The Household

The representative household chooses sequences $\{C_t, N_t, B_t^d\}_{t=0}^{\infty}$ to maximize

$$\max_{\{C_t, N_t, B_t^d\}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) \right], \tag{1}$$

subject to the budget constraint

$$P_t C_t + Q_t B_t^d \;=\; B_{t-1}^d + W_t N_t + P_t T_t + D_t, \tag{2}$$

and a standard no-Ponzi condition.

Definitions: C_t denotes consumption and N_t hours worked. Parameters satisfy $0 < \beta < 1$, $\sigma > 0$ (inverse IES), and $\varphi > 0$ (inverse Frisch elasticity). P_t is the aggregate price level, W_t the nominal wage, T_t lump-sum transfers from the government expressed in real terms, and D_t nominal dividends from firms. The household trades one-period nominal government bonds

B_t^d at price Q_t, each paying one unit of currency next period. Hence the gross nominal interest factor is $1 + i_t \equiv 1/Q_t$, and gross inflation is $\Pi_{t+1} \equiv P_{t+1}/P_t$. We next report the optimality conditions for the consumer optimization problem

Intra-temporal optimality:

$$C_t^{\sigma} N_t^{\varphi} \;=\; \frac{W_t}{P_t}. \tag{3}$$

Intertemporal optimality (Euler equation).

$$Q_t \;=\; \beta\, \mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \Pi_{t+1}^{-1}\right], \tag{4}$$

equivalently,

$$1 \;=\; \beta\, \mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{1+i_t}{\Pi_{t+1}}\right]. \tag{5}$$

Intertemporal budget constraint. Let the real stochastic discount factor be

$$M_{t,t+1} \;\equiv\; \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \Pi_{t+1}^{-1}, \qquad M_{0,t} \;\equiv\; \prod_{j=0}^{t-1} M_{j,j+1},$$

which is consistent with the Euler equation (4)–(5). Forward-iterating the period budget constraint (2) and imposing the no-Ponzi condition yields the household's intertemporal budget constraint:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty} M_{0,t}\, C_t\right] \;=\; \frac{B_{-1}^d}{P_{-1}} \;+\; \mathbb{E}_0\left[\sum_{t=0}^{\infty} M_{0,t}\left(\frac{W_t}{P_t} N_t \;+\; T_t \;+\; \frac{D_t}{P_t}\right)\right]. \tag{6}$$

Transversality condition. To rule out Ponzi schemes, the discounted value of terminal wealth must vanish:

$$\lim_{T\to\infty} \mathbb{E}_0\left[\, M_{0,T}\, \frac{B_T^d}{P_T}\,\right] \;=\; 0. \tag{7}$$

Conditions (2)–(5), together with (6)–(7), characterize the household's optimal choices and intertemporal feasibility, given prices, transfers, and dividends.

3.2 The Firm

Final-good producers. A representative final-good firm operates under perfect competition and aggregates a continuum of differentiated intermediate goods $Y_{i,t}$, $i \in [0,1]$, using a CES

technology:

$$Y_t \;=\; \left(\int_0^1 Y_{i,t}^{\frac{\epsilon-1}{\epsilon}}\,\mathrm{d}i\right)^{\frac{\epsilon}{\epsilon-1}}, \qquad \epsilon > 1. \tag{8}$$

Cost minimization implies the standard demand for each variety and the corresponding Dixit–Stiglitz price index:

$$Y_{i,t} \;=\; \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t, \qquad P_t \;=\; \left(\int_0^1 P_{i,t}^{1-\epsilon}\,\mathrm{d}i\right)^{\frac{1}{1-\epsilon}}. \tag{9}$$

Intermediate-good producers and technology. Each intermediate firm produces a differentiated good under monopolistic competition using labor as the only input:

$$Y_{i,t} \;=\; N_{i,t}^{1-\alpha}, \qquad \alpha \in [0,1). \tag{10}$$

Given nominal wage W_t, total nominal cost is $W_t N_{i,t}$, and the nominal marginal cost is

$$\mathrm{MC}_{i,t} \;=\; \frac{\partial(W_t N_{i,t})}{\partial Y_{i,t}} \;=\; \frac{W_t}{1-\alpha}\,N_{i,t}^{\alpha} \;=\; \frac{W_t}{1-\alpha}\,Y_{i,t}^{\frac{\alpha}{1-\alpha}}. \tag{11}$$

The corresponding *real* marginal cost is $s_{i,t} \equiv \mathrm{MC}_{i,t}/P_t$.

Price setting à la Calvo and sales tax. In each period, a measure $1-\theta$ of intermediate firms can reset their price, while the remaining share θ keep their previous price unchanged. The government levies a proportional sales tax $\tau \in [0,1)$ on firms' revenues. Let P_t^* denote the optimal reset price chosen by firms that can reoptimize at time t. Under household discounting, the firm solves:

$$\max_{P_t^*}\; \mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+k}}\Big[(1-\tau)\,P_t^*\,Y_{i,t+k|t} \;-\; \mathrm{MC}_{t+k}\,Y_{i,t+k|t}\Big], \tag{12}$$

subject to the demand schedule faced by a firm that last reset at t:

$$Y_{i,t+k|t} \;=\; \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} Y_{t+k}. \tag{13}$$

Optimal reset price. The first-order condition with respect to P_t^* yields:

$$\sum_{k=0}^{\infty} (\theta\beta)^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+k}} Y_{i,t+k|t}\Big[(1-\tau)\,P_t^* \;-\; \mu\,\mathrm{MC}_{t+k}\Big] \;=\; 0, \qquad \mu \equiv \frac{\epsilon}{\epsilon-1}. \tag{14}$$

Equivalently, the optimal reset price can be expressed as:

$$P_t^* \;=\; \frac{\mu \sum_{k=0}^{\infty} (\theta\beta)^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+k}} \mathrm{MC}_{t+k}\, Y_{i,t+k|t}}{(1-\tau) \sum_{k=0}^{\infty} (\theta\beta)^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+k}} Y_{i,t+k|t}}. \tag{15}$$

Price index evolution. Aggregating across firms, the aggregate price level evolves according to the Calvo pricing equation:

$$P_t^{1-\epsilon} \;=\; \theta\, P_{t-1}^{1-\epsilon} \;+\; (1-\theta)\, (P_t^*)^{\,1-\epsilon}. \tag{16}$$

This equation captures the gradual adjustment of the aggregate price level: in each period, only a fraction $1-\theta$ of firms update their prices to P_t^*, while the remainder keep P_{t-1}.

Remarks. (i) When $\alpha = 0$, technology is linear, and nominal marginal cost simplifies to $\mathrm{MC}_t = W_t$. (ii) The parameter $\mu = \epsilon/(\epsilon-1)$ is the desired steady-state markup, while the sales tax τ reduces the effective markup on revenues. (iii) The law of motion for P_t underpins the New Keynesian Phillips Curve once the model is log-linearized around the steady state.

3.3 The Government

Budget constraint. The government finances its expenditures by issuing nominal one-period bonds B_t, collecting revenues from the proportional sales tax τ, and making lump-sum transfers T_t to households. Its real budget constraint is:

$$Q_t \frac{B_t}{P_t} \;=\; \frac{B_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t} \;-\; \tau Y_t \;+\; T_t. \tag{17}$$

The left-hand side represents the real value of newly issued debt, while the right-hand side includes the real value of outstanding debt carried forward, net of sales tax revenues, plus lump-sum transfers (negative if lump-sum taxes).

Fiscal policy rule. Fiscal policy is assumed to be *non-Ricardian*: primary surpluses do not automatically adjust to ensure intertemporal solvency. Following Leeper (1991b), the government sets lump-sum transfers as a function of the real value of public debt and the level of output:

$$T_t \;=\; \gamma_0 \;-\; \gamma_d \frac{B_t}{P_t} \;-\; \gamma_y Y_t \;+\; \varepsilon_t^f, \tag{18}$$

where ε_t^f denotes an exogenous, covariance-stationary *fiscal shock*.

Interpretation. The parameter γ_d captures the responsiveness of fiscal policy to the real value of government debt:

- $\gamma_d > 0$: debt-stabilizing policy—higher debt leads to lower transfers (higher primary surplus);
- $\gamma_d < 0$: debt-accommodating or procyclical policy—higher debt leads to higher transfers (continued debt rollover).

The coefficient γ_y captures the sensitivity of fiscal policy to the level of output:

- $\gamma_y > 0$: countercyclical response—transfers fall in booms;
- $\gamma_y < 0$: procyclical response—transfers rise in booms.

The constant term γ_0 ensures that the fiscal rule is consistent with the steady-state debt-to-output ratio $\xi_d \equiv B/(PY)$.

Monetary policy rule. The central bank sets the nominal interest rate in response to deviations of inflation and output from their steady states, following a Taylor-type rule in levels:

$$1 + i_t \;=\; (1+i)\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi}\left(\frac{Y_t}{Y}\right)^{\phi_y} e^{\nu_t}, \tag{19}$$

where i and Π denote steady-state nominal interest and inflation rates, respectively, and ν_t is an exogenous, mean-zero, covariance-stationary monetary policy shock.

$$i_t \;=\; \phi_\pi\,\pi_t \;+\; \phi_y\,y_t \;+\; \nu_t,$$

Interpretation. The parameter $\phi_\pi > 0$ measures the degree of policy aggressiveness toward inflation, while $\phi_y \geq 0$ captures the responsiveness to real activity. The shock ν_t represents unexpected deviations from the systematic policy rule, such as communication surprises or discretionary adjustments.

3.4 Market Clearing and Competitive Equilibrium

Market clearing. All markets clear each period:

- **Goods market:** output is entirely consumed,

$$Y_t = C_t. \tag{20}$$

- **Labor market:** labor demand by firms equals labor supplied by households,

$$N_t = \int_0^1 N_{i,t}\,\mathrm{d}i. \tag{21}$$

- **Asset market:** the supply of government bonds equals households' demand,

$$B_t = B_t^d. \tag{22}$$

Definition of equilibrium. Given an initial level of real government debt $b_{-1} \equiv B_{-1}/P_{-1}$ and sequences of monetary and fiscal shocks $\{\nu_t, \varepsilon_t^f\}_{t\geq 0}$, a *competitive equilibrium* is a set of sequences

$$\{C_t, N_t, Y_t, P_t, W_t, i_t, \Pi_t, P_t^*, B_t, T_t\}_{t\geq 0}$$

such that, for all $t \geq 0$ they satisfy:

1. **Household optimality:** the intratemporal labor–leisure condition (3), the intertemporal Euler equation (5), and the transversality condition (7);
2. **Firm behavior:** the final-good aggregation and demand system (20)–(16), the production technology (10), and the optimal price-setting condition with Calvo friction and sales tax (14)–(15);
3. **Government:** the budget constraint (17) and the policy rules for the fiscal and monetary authorities, given by (18) and (19);
4. **Market clearing:** the goods, labor, and asset market conditions (20)–(22).

The system of equations (3)–(19), together with the market-clearing relations (20)–(22), fully characterizes the equilibrium dynamics of the model economy.

3.5 Steady State

We focus on a zero-inflation steady state ($\Pi = 1$) with no shocks ($\nu_t = \varepsilon_t^f = 0$) and time-invariant allocations. Steady-state variables are denoted by the same symbols without time subscripts.

Interest rate and asset pricing. From the Euler equation (5) with $\Pi = 1$,

$$\beta(1 + i) = 1, \qquad Q = \beta. \tag{23}$$

Pricing and marginal cost. With no price dispersion ($P_i = P$), the steady-state reset price satisfies a static markup condition incorporating the sales-tax wedge:

$$(1 - \tau)P = \mu\,\mathrm{MC}, \qquad \Rightarrow \qquad s \equiv \frac{\mathrm{MC}}{P} = \frac{1 - \tau}{\mu}. \tag{24}$$

Technology and intratemporal optimality. Using the production function $Y = N^{1-\alpha}$ and the household condition $C^{\sigma}N^{\varphi} = W/P$, together with $s = \frac{W/P}{(1-\alpha)N^{\alpha}}$ and (24), one obtains the steady-state allocation (up to normalization) as a function of primitives $(\sigma, \varphi, \alpha, \epsilon, \tau)$:

$$C = Y = \left[(1-\alpha)\,\frac{(\epsilon-1)(1-\tau)}{\epsilon}\right]^{\frac{1-\alpha}{\sigma(1-\alpha)+\varphi+\alpha}}, \tag{25}$$

$$N = Y^{\frac{1}{1-\alpha}}. \tag{26}$$

Government and debt. Let $D \equiv B/P$ denote real government debt and $\xi_d \equiv D/Y$ the steady-state debt-to-output ratio. In steady state, from (17) and $\Pi = 1$,

$$T = \tau Y - (1-\beta)D, \qquad \xi_d = \frac{D}{Y}. \tag{27}$$

The steady-state levels of C, N, and Y in (25)–(26) are independent of D; instead, lump-sum transfers T adjust to ensure the government budget constraint holds for a given ξ_d.

Parameters relevant for calibration. Preferences and technology: $\{\beta, \sigma, \varphi, \alpha, \epsilon\}$; price setting: $\{\theta\}$; tax and markup wedge: $\{\tau\}$; steady-state policy targets: $\{i, \Pi(=1)\}$; fiscal stance: $\{\gamma_d, \gamma_y, \gamma_0, \xi_d\}$; monetary stance: $\{\phi_\pi, \phi_y\}$.

3.6 Log-Linear Equilibrium

We log-linearize equations (5), (14)–(15), (17)–(19),and the market-clearing conditions (20)–(22) around the steady state described above. Let lowercase letters denote log-deviations from steady state (e.g., $y_t \equiv \log(Y_t/Y)$, $\pi_t \equiv \log(\Pi_t)$, $b_t \equiv (B_t/P_t)/D - 1$). Since $C = Y$ in steady state, we have $c_t = y_t$.

Resulting system. The linearized equilibrium can be expressed in the standard three-equation New Keynesian form augmented with the debt accumulation condition:

$$\text{(NKPC)} \quad \pi_t = \beta\,\mathbb{E}_t\pi_{t+1} + \kappa\, y_t, \tag{28}$$

$$\text{(IS/Euler)} \quad \sigma\,\mathbb{E}_t y_{t+1} = \sigma\, y_t + i_t - \mathbb{E}_t\pi_{t+1}, \tag{29}$$

$$\text{(Taylor, linear)} \quad i_t = \phi_\pi\,\pi_t + \phi_y\, y_t + \nu_t, \tag{30}$$

$$\text{(Debt accumulation)} \quad (\beta+\gamma_d)\, b_t = b_{t-1} - \pi_t + \beta\, i_t + \left(\gamma_y - \tfrac{\tau}{\xi_d}\right) y_t + \varepsilon_t^f. \tag{31}$$

The Phillips-curve slope κ depends on structural parameters and Calvo price rigidities. With decreasing returns $Y = N^{1-\alpha}$ and a sales tax τ, one convenient expression is:

$$\kappa = \frac{(1-\beta\theta)(1-\theta)}{\theta}\left[\sigma + \frac{\varphi+\alpha}{1-\alpha}\right]\frac{1-\alpha}{1-\alpha+\epsilon\alpha}. \tag{32}$$

Compact characterization. Equations (28)–(31), together with the stochastic processes for the policy shocks (ν_t, ε_t^f) and the initial condition for b_{-1}, fully characterize the log-linear equilibrium. Substituting (30) into (31) yields the reduced two-forward-one-state system used in the determinacy analysis.

3.7 Calibration

To characterize the equilibrium dynamics and analyze the stability properties of the model, we adopt a standard calibration commonly used in the New Keynesian literature, following Galí (2015). The baseline structural parameters are reported in Table 1. These parameters govern preferences, technology, and nominal rigidities, and are set to conventional values that ensure realistic steady-state behavior of output, labor supply, and inflation dynamics.

Table 1: Baseline Parameter Calibration

Parameter	Description	Value
β	Discount factor	0.99
θ	Calvo probability of not changing prices	0.75
σ	Risk aversion (inverse IES)	1
φ	Inverse Frisch elasticity of labor supply	2
ϵ	Elasticity of substitution across varieties	6
α	Labor Elasticity	1/3

The remaining parameters—τ, ξ_d, ϕ_π, ϕ_y, γ_d, and γ_y—are associated with the policy rules and steady-state ratios that shape the macroeconomic adjustment of the model. The sales-tax rate τ determines the size of the fiscal wedge in firms' pricing decisions, while $\xi_d \equiv D/Y$ measures the steady-state ratio of government debt to output. The parameters (ϕ_π, ϕ_y) govern the monetary policy rule and describe how aggressively the central bank reacts to deviations of inflation and output from their steady states. Similarly, (γ_d, γ_y) define the fiscal transfer rule and capture the degree of fiscal adjustment to the level of real public debt and to the level of economic activity.

The main focus of the analysis is on the interaction between these policy-rule parameters. In particular, the joint configuration of (ϕ_π, ϕ_y) and (γ_d, γ_y) determines whether the economy operates under a regime of monetary or fiscal dominance and how the system responds to inflationary shocks. By varying these parameters systematically, we can trace how fiscal feedback to debt and monetary aggressiveness to inflation jointly affect equilibrium stability and the possibility of expansionary inflationary episodes. When not explicitly stated otherwise, the remaining parameters, such as the sales-tax rate τ and the debt-to-output ratio ξ_d, are held at empirically plausible levels that ensure the existence of a stable steady state.

Having established the baseline calibration, the next step is to examine how alternative combinations of monetary and fiscal parameters influence the stability and determinacy of the equilibrium. To this end, we analyze the system of log-linearized equations (28)–(31) using the Blanchard and Kahn (1980) conditions, which require that the number of unstable

eigenvalues matches the number of forward-looking variables in the model. By exploring the parameter space of (ϕ_π, ϕ_y) for monetary policy and (γ_d, γ_y) for fiscal policy, we identify the regions corresponding to monetary dominance, fiscal dominance, and indeterminacy. This analysis provides the foundation for understanding how different policy interactions shape the transmission of inflationary shocks and the conditions under which such shocks may generate expansionary outcomes.

4 Equilibrium Dynamics

To study the dynamics of the economy, we log-linearize the equilibrium conditions around the zero-inflation steady state described in Section 3, assuming a constant path for policy variables.[1]

The linearized equilibrium conditions are summarized by the following system:

$$\begin{cases} (\beta + \gamma_d)\, b_t = b_{t-1} - \pi_t + \beta i_t + \left(\dfrac{\gamma_y - \tau}{\xi_d}\right) y_t, \\ \pi_t = \beta\, \mathbb{E}_t \pi_{t+1} + k\, y_t, \\ \sigma\, \mathbb{E}_t y_{t+1} = \sigma\, y_t + i_t - \mathbb{E}_t \pi_{t+1}, \\ i_t = \phi_\pi\, \pi_t + \phi_y\, y_t + \nu_t, \end{cases} \tag{33}$$

where the slope of the Phillips Curve is given by

$$k = \frac{1-\alpha}{1-\alpha+\epsilon\alpha} \frac{(1-\beta\theta)(1-\theta)}{\theta} \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right).$$

4.1 Determinacy Analysis

The dynamic system (33) contains two forward-looking variables, inflation π_t and consumption (or output) y_t, and one predetermined variable, the real stock of government debt b_t. According to Blanchard and Kahn (1980), local determinacy requires that the number of eigenvalues of the transition matrix outside the unit circle equals the number of forward-looking variables (two in this case).

The following proposition characterizes the conditions under which the equilibrium is determinate.

Proposition 1 (Equilibrium Regimes and Determinacy)**.** *Let* (ϕ_π, ϕ_y) *denote the coefficients of the monetary policy rule and* (γ_d, γ_y) *the fiscal-response parameters. Then, under system* (33)*,*

[1] Lowercase variables denote log-linearized deviations from steady state.

the characteristic equation admits three eigenvalues:

$$\lambda_1 = \frac{1}{\beta + \gamma_d},$$
$$\lambda_2 = \frac{k + \sigma + \beta\phi_y + \beta\sigma - \sqrt{D}}{2\beta\sigma},$$
$$\lambda_3 = \frac{k + \sigma + \beta\phi_y + \beta\sigma + \sqrt{D}}{2\beta\sigma},$$

where

$$D = (k + \sigma + \beta\phi_y + \beta\sigma)^2 - 4\beta\sigma(k\phi_\pi + \sigma + \phi_y).$$

Equilibrium determinacy arises under the following parameter configurations:

(i) ***Monetary dominance (active monetary, passive fiscal policy):***

$$|\lambda_1| < 1, \quad |\lambda_2| > 1, \quad |\lambda_3| > 1,$$

which occurs when

$$\beta + \gamma_d < -1 \quad or \quad \beta + \gamma_d > 1, \qquad k(\phi_\pi - 1) + (1 - \beta)\phi_y > 0.$$

(ii) ***Fiscal dominance (active fiscal, passive monetary policy):***

$$|\lambda_1| > 1, \quad |\lambda_2| < 1, \quad |\lambda_3| > 1,$$

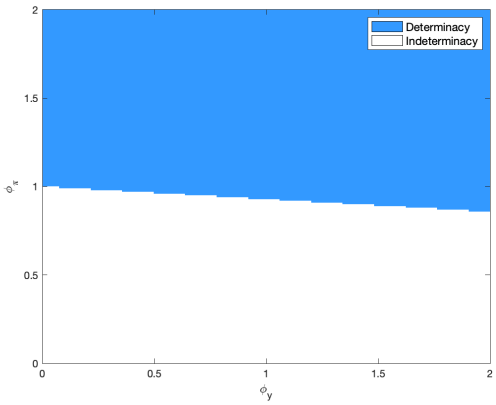
which occurs when

$$-1 < \beta + \gamma_d < 1, \qquad k(\phi_\pi - 1) + (1 - \beta)\phi_y < 0.$$

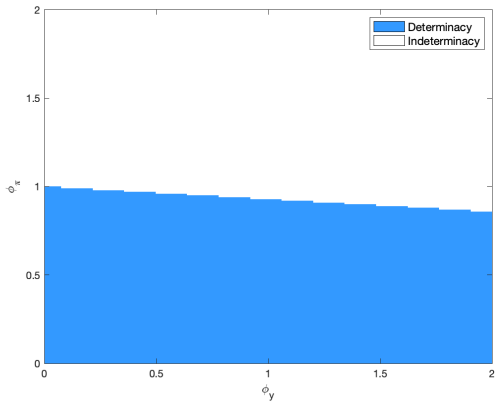
The discriminant $D > 0$ ensures that the eigenvalues are real. Parameters τ, ξ_d, and γ_y do not affect local stability directly and can be varied without altering determinacy conditions.

Proof (sketch). The proof follows from the linearized system (33). Writing it in matrix form and applying the Blanchard–Kahn conditions yields a cubic characteristic equation whose roots correspond to the eigenvalues reported above. Full derivations are in Appendix A. □

The two regimes identified in Proposition 1 partition the policy space into regions of monetary and fiscal dominance. Under monetary dominance, the price level is determined by monetary policy, and government debt is expected to be stabilized in real terms. Under fiscal dominance, fiscal policy determines the price level, and monetary policy must accommodate the fiscal stance to ensure solvency. The admissible regions for (ϕ_π, ϕ_y) under the baseline calibration are illustrated in Figures 4a and 4b, under the calibration reported in Table 1.



(a) Determinacy Region with $|\lambda_2| > 1$ and $|\lambda_3| > 1$



(b) Determinacy Region with $|\lambda_2| < 1$ and $|\lambda_3| > 1$

Figure 4: Determinacy regions under alternative monetary–fiscal configurations.

4.2 The Decomposition: Substitution and Wealth Effects

A monetary policy shock—specifically, an increase in the nominal interest rate—affects household consumption and inflation through two interacting channels: the intertemporal substitution effect and the wealth effect.

An increase in the nominal interest rate influences the real interest rate, thereby altering consumption decisions via the Euler equation. A higher real interest rate encourages households to postpone current consumption in favor of future consumption, as the return on savings increases. The resulting decline in current demand places downward pressure on inflation. Conversely, a lower real interest rate stimulates present consumption and pushes inflation upward. This mechanism constitutes the *intertemporal substitution effect*.

At the same time, changes in the nominal interest rate affect the general price level and the valuation of nominal assets, primarily through their impact on the government's interest bill. This channel operates through variations in household income and wealth and is referred to as the *wealth effect*. An increase in nominal asset holdings tends to boost consumption, as households perceive themselves to be wealthier due to higher interest income. Similarly, a decline in the price level—associated with lower inflation—increases the real value of nominal assets, further supporting consumption.

The wealth effect can be decomposed into two components: a *real wealth effect*, stemming from changes in real asset values, and a *nominal wealth effect*, which becomes relevant when variations in nominal variables are sufficiently persistent. While the real component is always present, the nominal component matters only when nominal adjustments are long-lasting.

Appendix B provides a simple analytical illustration of these transmission mechanisms using a standard intertemporal consumption model that incorporates both real and nominal dimensions.

The overall effect of a monetary policy shock on consumption and inflation depends on the interaction between these two channels. The following analysis disentangles the relative importance of the substitution and wealth effects across the two regimes identified in the

determinacy analysis (see Subsection 4.1 and Appendix A).

4.2.1 Monetary Dominance Regime

We begin by considering the system formed by the Euler equation and the Phillips Curve:

$$\begin{cases} \pi_t = \beta \mathbb{E}_t \pi_{t+1} + k y_t, \\ \sigma \mathbb{E}_t y_{t+1} = (\sigma + \phi_y) y_t + \phi_\pi \pi_t - \mathbb{E}_t \pi_{t+1} + \nu_t. \end{cases} \tag{34}$$

Substituting the first equation into the second yields a second-order expectational difference equation for inflation:

$$\mathbb{E}_t \pi_{t+2} - \frac{\sigma + k + \sigma\beta + \beta\phi_y}{\sigma\beta} \mathbb{E}_t \pi_{t+1} + \frac{\sigma + \phi_y + k\phi_\pi}{\sigma\beta} \pi_t = -\frac{k}{\sigma\beta} \nu_t. \tag{35}$$

Applying the lag operator L gives:

$$L^{-2} \pi_t - \frac{\sigma + k + \sigma\beta + \beta\phi_y}{\sigma\beta} L^{-1} \pi_t + \frac{\sigma + \phi_y + k\phi_\pi}{\sigma\beta} \pi_t = -\frac{k}{\sigma\beta} \nu_t. \tag{36}$$

Solving the homogeneous part of this equation yields the same eigenvalues as in the determinacy analysis, λ_2 and λ_3:

$$(L^{-1} - \lambda_2)(L^{-1} - \lambda_3) \pi_t = -\frac{k}{\sigma\beta} \nu_t. \tag{37}$$

Under monetary dominance, both eigenvalues satisfy $|\lambda_2| > 1$ and $|\lambda_3| > 1$ (with $\lambda_3 > \lambda_2$; see Appendix A). Rearranging gives:

$$\begin{aligned} \pi_t &= -\frac{k}{\sigma\beta\lambda_2\lambda_3} \cdot \frac{\nu_t}{(1 - (L\lambda_2)^{-1})(1 - (L\lambda_3)^{-1})} \\ &= -\frac{k}{\sigma\beta\lambda_2\lambda_3} \sum_{j=0}^{\infty} \left(\sum_{m=0}^{j} \left(\frac{1}{\lambda_2} \right)^{j-m} \left(\frac{1}{\lambda_3} \right)^{m} \right) \mathbb{E}_t \nu_{t+j}. \end{aligned}$$

Hence, inflation can be expressed compactly as:

$$\pi_t = \underbrace{-\frac{k}{\sigma\beta\lambda_2\lambda_3} \sum_{j=0}^{\infty} \lambda_2^{-j} \left(\frac{1 - (\lambda_2/\lambda_3)^{j+1}}{1 - \lambda_2/\lambda_3} \right) \mathbb{E}_t \nu_{t+j}}_{\textit{Intertemporal Substitution Effect}}. \tag{38}$$

Under monetary dominance, inflation is uniquely determined by the monetary policy shock ν_t. A contractionary monetary shock unambiguously lowers inflation. Because $|\lambda_1| < 1$, the fiscal authority stabilizes the real value of public debt, allowing the central bank to focus exclusively on its price stability objective. Firms, anticipating weaker demand, reset prices downward.

Consequently, the dynamics of inflation are governed solely by the strength of the intertemporal substitution channel.

We next derive the impact of monetary shocks on output (and, equivalently, on consumption):

$$y_t = \underbrace{-\frac{1}{\sigma\beta\lambda_2\lambda_3}\sum_{j=0}^{\infty}\lambda_2^{-j}\left(\frac{1-(\lambda_2/\lambda_3)^{j+1}}{1-\lambda_2/\lambda_3}\right)\mathbb{E}_t\nu_{t+j}}_{\textit{Intertemporal Substitution Effect}} + \underbrace{\frac{1}{\sigma\lambda_2\lambda_3}\sum_{j=0}^{\infty}\lambda_2^{-j}\left(\frac{1-(\lambda_2/\lambda_3)^{j+1}}{1-\lambda_2/\lambda_3}\right)\mathbb{E}_t\nu_{t+1+j}}_{\textit{Real Wealth Effect}}. \tag{39}$$

Equation (39) highlights the two channels through which monetary policy affects real activity. Higher interest rates raise the return on savings, inducing households to postpone consumption—the intertemporal substitution channel. At the same time, disinflation increases the real value of nominal assets, temporarily supporting consumption—the real wealth channel. The latter can partly offset the contractionary substitution effect, especially when monetary shocks are anticipated or highly persistent.

Figure 5 presents the impulse response functions for different degrees of persistence following a monetary policy shock. As shown, the monetary authority reduces inflation by contracting aggregate demand. When the persistence of the shock increases, the positive Real Wealth Effect becomes more relevant. This is reflected in the smaller variation in output responses across different values of ρ_ν, compared with the wider dispersion observed in inflation responses. This pattern arises because inflation dynamics are not directly affected by the Real Wealth Effect.

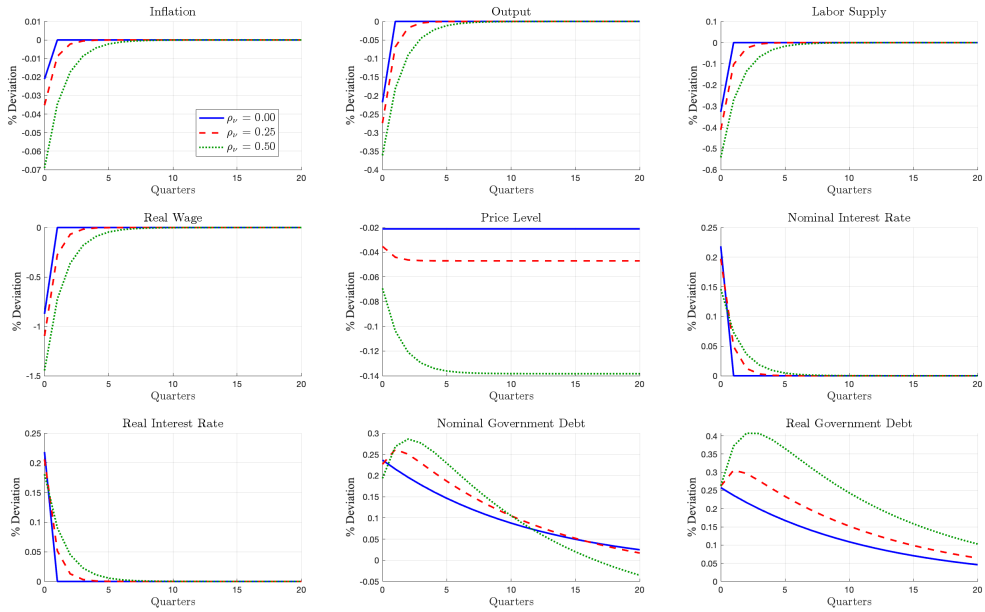


Figure 5: Impulse Responses to a 1% annualized monetary policy shock
Figure generated with parameters: $\phi_y = 0$, $\phi_\pi = 1.5$, $\gamma_y = 0$, $\gamma_d = 0.1$, $\tau = 0.2$, and $\xi_d = 1.0$.

On impact, a contractionary monetary shock lowers both inflation and output. However,

when future tightening is anticipated, nominal rigidities prompt firms to reduce prices in advance, raising real interest rates and inducing households to shift consumption forward. This intertemporal reallocation can generate a short-lived output expansion before the economy gradually converges back to steady state. As in standard New Keynesian dynamics, the interaction between substitution and wealth effects ensures a smooth adjustment path following the shock.

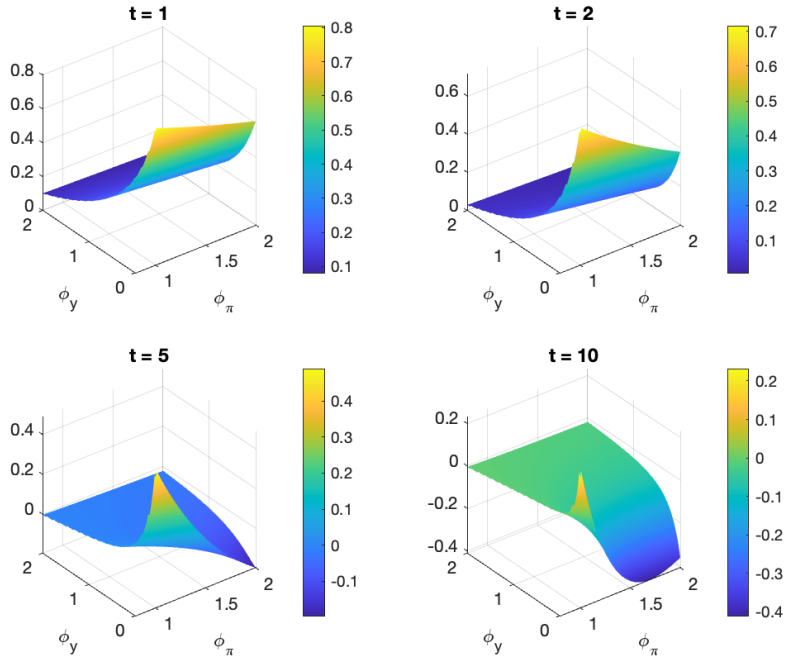


Figure 6: Impact of anticipated monetary policy shocks at time **t** on output.

Figure 6 shows that anticipated monetary contractions can be temporarily expansionary if the wealth effect dominates the substitution effect. The coefficients of monetary shocks are evaluated across different horizons. A higher ϕ_y in the policy rule enhances consumption stability and reduces macroeconomic volatility by weakening the real wealth channel. Consequently, incorporating output stabilization into the Taylor rule strengthens forward guidance and mitigates sensitivity to small changes in the inflation-response parameter ϕ_π. An extended version of this figure is provided in Appendix E (see Figure 13).

Such temporary expansions arise only when monetary shocks are highly persistent or when there is a structural shift in the policy regime.[2]

We now turn to the implications for government debt.

Proposition 2. *Under monetary dominance, the only admissible equilibrium path for government debt is one in which debt is expected to be repaid in real terms. In this regime, government debt*

[2]This observation suggests that adopting a global solution method to study monetary–fiscal interactions may uncover nonlinear equilibrium dynamics that remain underexplored in the current literature.

fully adjusts to past and current monetary policy shocks:

$$b_t = \frac{\beta\phi_\pi - 1}{\beta + \gamma_d} \sum_{k=0}^{\infty} \frac{\pi_{t-k}}{(\beta + \gamma_d)^k} + \left(\frac{\beta\phi_y + \frac{\gamma_y - \tau}{\xi_d}}{\beta + \gamma_d} \right) \sum_{k=0}^{\infty} \frac{y_{t-k}}{(\beta + \gamma_d)^k} + \frac{\beta}{\beta + \gamma_d} \sum_{k=0}^{\infty} \frac{\nu_{t-k}}{(\beta + \gamma_d)^k}.$$

Proof. Full derivations are in Appendix C.1. □

Proposition 2 establishes that, under monetary dominance, the intertemporal government budget constraint holds in real terms. Inflation does not erode nominal debt, and government liabilities adjust endogenously to absorb current and past monetary shocks.

In this regime, nominal wealth effects are transitory and exert no lasting influence on macroeconomic dynamics. They arise solely from temporary increases in real interest rates following monetary tightening, not from changes in the long-run trajectory of nominal debt. Although fiscal parameters—γ_y, τ, and ξ_d—shape the short-run adjustment of debt to monetary shocks, their quantitative effect is limited. Inflation and consumption dynamics remain independent of the speed at which nominal debt adjusts.

In equilibrium, the wealth effect is thus independent of fiscal determinants. Fiscal policy plays an accommodating role, ensuring that monetary policy retains full control over the price level—a mechanism commonly referred to as *fiscal backing*. In this sense, the fiscal authority stabilizes the real value of public debt while the central bank anchors inflation expectations and maintains nominal stability.

4.2.2 Fiscal Dominance Regime

Under *Fiscal Dominance*, the fiscal authority is active while monetary policy is passive. This regime is characterized by the configuration $|\lambda_1| > 1$, $|\lambda_2| < 1$, and $|\lambda_3| > 1$, implying that the equilibrium path of inflation and output is no longer pinned down by monetary policy alone but depends on the evolution of the predetermined state variable, real government debt b_t. Intuitively, the government's intertemporal budget constraint now determines the price level, while the central bank adjusts the nominal interest rate to stabilize real debt.

Starting from the second-order expectational difference equation for inflation,

$$(L^{-1} - \lambda_2)(L^{-1} - \lambda_3)\pi_t = -\frac{k}{\sigma\beta}\,\nu_t,$$

and using the fact that under Fiscal Dominance $|\lambda_2| < 1 < |\lambda_3|$, the forward-looking solution for inflation depends explicitly on the initial condition for debt. The inflation process therefore satisfies

$$\mathbb{E}_t\pi_{t+1} = \lambda_2\pi_t + \frac{k}{\sigma\beta\lambda_3} \sum_{j=0}^{\infty} \lambda_3^{-j}\nu_{t+j}.$$

Proposition 3 (Fiscal Intertemporal Budget Constraint)**.** *Under Fiscal Dominance, the equilibrium path for government debt is determined by the expected future trajectories of inflation,*

output, and monetary policy shocks. The intertemporal government budget constraint is given by

$$b_t = -(\beta\phi_\pi - 1)\sum_{k=0}^{\infty}(\beta+\gamma_d)^k\,\mathbb{E}_t\pi_{t+1+k} - \left(\beta\phi_y + \frac{\gamma_y-\tau}{\xi_d}\right)\sum_{k=0}^{\infty}(\beta+\gamma_d)^k\,\mathbb{E}_t y_{t+1+k} - \beta\sum_{k=0}^{\infty}(\beta+\gamma_d)^k\,\mathbb{E}_t\nu_{t+1+k}.$$

Proof. Full derivations are in Appendix C.1. □

This condition highlights that under Fiscal Dominance, monetary policy accommodates fiscal behavior by stabilizing the real value of debt, while the fiscal authority anchors the equilibrium paths of inflation and consumption. Nominal government debt can rise persistently—or even permanently—so long as the real value of debt remains bounded. The degree of debt accommodation is captured by the parameter γ_d, which determines how the government finances its interest bill: when $\gamma_d < 0$, interest payments are rolled over through new nominal issuance.

This mechanism generates what we term a *Nominal Wealth Effect*. A persistent increase in the nominal stock of government debt raises households' perceived nominal wealth, stimulating demand and, consequently, inflation. The equilibrium dynamics of inflation and output can be summarized as:

$$\pi_t = \underbrace{\left(1-(\beta+\gamma_d)L^{-1}\right)\Theta b_{t-1}}_{\textit{Nominal Wealth Effect}} + \underbrace{\beta\,\Theta\,\nu_t}_{\textit{Intertemporal Substitution Effect}}, \tag{40}$$

$$y_t = \underbrace{\frac{1-\beta L^{-1}}{k}\left(1-(\beta+\gamma_d)L^{-1}\right)\Theta b_{t-1}}_{\textit{Nominal Wealth Effect}} + \underbrace{\frac{\beta}{k}\Theta\nu_t}_{\textit{Intertemporal Substitution Effect}} - \underbrace{\frac{\beta^2}{k}\Theta\mathbb{E}_t\nu_{t+1}}_{\textit{Real Wealth Effect}}, \tag{41}$$

where

$$\Theta^{-1} = 1 - \beta\phi_\pi - \left(\beta\phi_y + \frac{\gamma_y-\tau}{\xi_d}\right)\frac{1-\beta L^{-1}}{k}.$$

Full derivations are provided in Appendix C.2.

Equations (40)–(41) provide a decomposition of the forces shaping equilibrium dynamics under Fiscal Dominance. The *Nominal Wealth Effect* captures the influence of nominal debt creation on aggregate demand and prices; the *Intertemporal Substitution Effect* represents the standard forward-looking response to monetary shocks; and the *Real Wealth Effect* reflects how higher inflation erodes the real value of future savings, dampening consumption. Figure 7 illustrates how these three mechanisms interact as the persistence of the monetary policy shock varies.

When monetary policy shocks are persistent, the rise in inflation triggered by higher nominal debt issuance can coexist with temporary output declines—especially when γ_d is only mildly negative and the initial nominal wealth response is limited. As γ_d becomes more negative,

households perceive a larger fraction of new debt as net wealth, making the Nominal Wealth Effect stronger and potentially generating expansionary inflationary episodes.

This mechanism mirrors the *tight money paradox* of Loyo (1999), whereby a restrictive monetary policy that increases the nominal supply of government debt can lead to higher, rather than lower, inflation. Within this framework, such episodes correspond to *Expansionary Hikes*—periods in which higher interest rates, by raising the nominal interest bill, ultimately fuel inflation and activity through the fiscal channel.

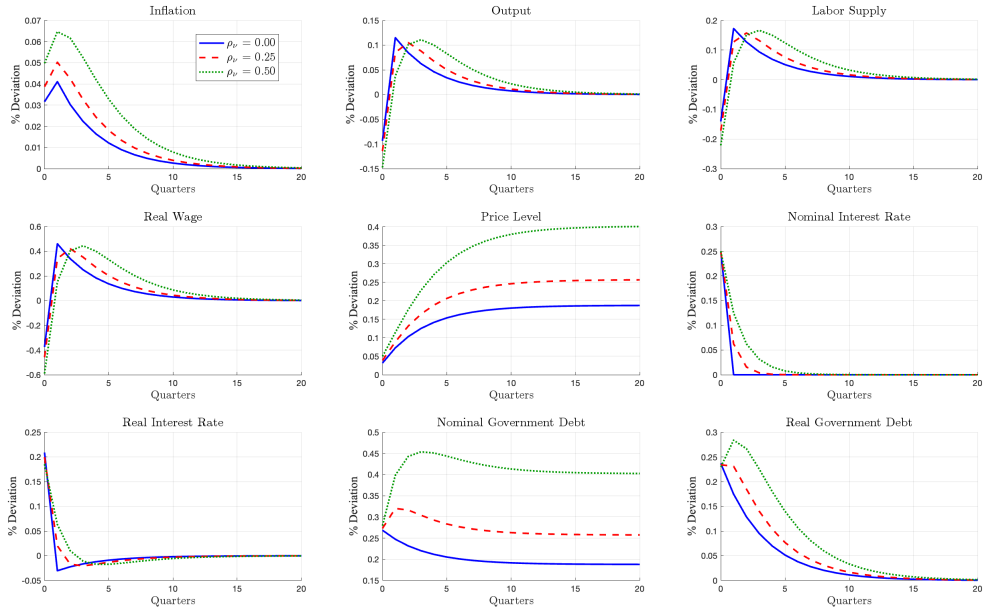


Figure 7: Impulse Responses to a 1% annualized monetary policy shock

Figure generated with parameters: $\phi_y = 0$, $\phi_\pi = 0$, $\gamma_y = 0$, $\gamma_d = -0.1$, $\tau = 0.2$, and $\xi_d = 1.0$.

The strength and sign of the Nominal Wealth Effect depend critically on the parameter γ_d, which governs how interest payments are financed. When γ_d is sufficiently negative—implying a high degree of debt rollover—households interpret new debt issuance as an increase in net nominal wealth, encouraging higher consumption and output. By contrast, for less negative or positive values of γ_d, the short-run effect may be contractionary, as households anticipate future fiscal adjustments.

The short-run dynamics of output and consumption therefore depend not only on monetary parameters (ϕ_π, ϕ_y) but also on fiscal financing behavior (γ_d, γ_y), which jointly determine whether interest-rate increases amplify or stabilize inflationary pressures.

4.3 Two Illustrative Special Cases

To clarify the mechanisms driving the interaction between monetary and fiscal policy, we analyze two special cases. The first examines an economy with perfectly flexible prices and a purely

inflation-targeting monetary rule, in which real allocations adjust instantaneously and nominal variables alone determine equilibrium dynamics. The second considers a fiscal rule that reacts only to output, without any feedback to the stock of debt. This configuration is particularly informative, as it reveals how the absence of fiscal backing transforms the stock of government debt into a key state variable governing the transmission of monetary shocks and the emergence of *Expansionary Hikes*.

4.3.1 Flexible Prices and Inflation Targeting Rule

We start by focusing on the special case with perfectly flexible prices and no output stabilization in the monetary rule:

$$\theta = 0, \qquad \phi_y = 0.$$

With $\theta = 0$, firms reset prices each period and there is no price dispersion. In the absence of real (cost-push) disturbances, output adjusts instantaneously to equate the marginal rate of substitution and the marginal product of labor, so real allocations are pinned down by technology and preferences. As a result, *nominal* interactions—between the interest-rate rule and the fiscal block—determine the price level, while *real* allocations are invariant to the policy regime.

Reduced system. Starting from (33) and imposing $\phi_y = 0$, flexible prices imply that the output gap is zero, and (with only nominal shocks) $y_t = 0$ in the linearized system.[3] Hence, inflation and real debt follow:

$$i_t = \phi_\pi \, \pi_t + \nu_t, \tag{42}$$

$$(\beta + \gamma_d) \, b_t = b_{t-1} - \pi_t + \beta \, i_t + \varepsilon_t^f, \tag{43}$$

which combine to

$$(\beta + \gamma_d) \, b_t \;=\; b_{t-1} + (\beta\phi_\pi - 1) \, \pi_t + \beta \, \nu_t + \varepsilon_t^f. \tag{44}$$

Under price flexibility, the real side, output, and labor jump to their (shock-free) efficient levels, so monetary policy cannot engineer real gaps through nominal rigidities. What remains is a *nominal anchor* problem. Equation (44) shows that the evolution of real debt and the inflation rate are tied by the *fiscal financing margin* $(\beta + \gamma_d)$ and the *monetary stance* ϕ_π. If monetary policy is sufficiently aggressive ($\phi_\pi > 1$) and fiscal policy stabilizes real debt ($|\beta + \gamma_d| > 1$), inflation is pinned down by the interest-rate rule (monetary dominance). If instead the fiscal block absorbs shocks by rolling over nominal liabilities ($|\beta + \gamma_d| < 1$), then the intertemporal government budget constraint determines the price level (fiscal dominance). Crucially, because $y_t = 0$, *determinacy boundaries are identical to those in the sticky-price case*: nominal rigidities affect propagation (the slope k), but not the regime map.

[3] Equivalently, the NKPC collapses to a static condition that does not constrain y_t when there are no real cost-push disturbances.

Proposition 4 (Flexible Prices, No Output Response)**.** *Consider the system* (42)–(44) *with* $\theta = 0$ *and* $\phi_y = 0$*. Then:*

(a) ***Determinacy boundaries.*** *The Blanchard–Kahn conditions coincide with those in Proposition 1 (with* $\phi_y = 0$*):*

$$\begin{aligned} \textit{Monetary dominance:}\ \ |\beta + \gamma_d| > 1 \ \ \textit{and}\ \ k(\phi_\pi - 1) > 0, \\ \textit{Fiscal dominance:}\ \ |\beta + \gamma_d| < 1 \ \ \textit{and}\ \ k(\phi_\pi - 1) < 0. \end{aligned}$$

Thus, determinacy is independent of θ*.*

(b) ***Inflation determination.*** *Inflation is uniquely determined by the policy regime and shocks. In particular, solving* (44) *forward yields*

$$\pi_t = \frac{1 - (\beta + \gamma_d)L^{-1}}{1 - \beta\phi_\pi} b_{t-1} + \frac{\beta}{1 - \beta\phi_\pi} \nu_t + \frac{1}{1 - \beta\phi_\pi} \varepsilon_t^f \tag{45}$$

under monetary dominance; under fiscal dominance, the forward solution is pinned down by the intertemporal budget constraint (see Proposition 3, specialized to $y_t = 0$*).*

(c) ***Real allocations.*** *In both regimes,* $y_t = 0$ *and* $c_t = 0$ *(in gaps): real allocations are regime-invariant in this special case.*

Proof (sketch). With $y_t = 0$, the (π_t, b_t) subsystem has the same characteristic roots as in Proposition 1 evaluated at $\phi_y = 0$. Item (a) follows by the same BK count. Item (b) is obtained by substituting (42) into (43) and imposing the appropriate boundary condition (backward for b_t when $|\beta + \gamma_d| > 1$, forward when $|\beta + \gamma_d| < 1$). Item (c) follows from flexible price clearing with only nominal shocks. Full derivations are in Appendix A. □

In sum, with $\theta = 0$ and $\phi_y = 0$ the policy regime continues to determine the *nominal anchor* (inflation and the price level), while the real allocation is unaffected. This confirms that the stickiness parameter governs *propagation*, not *determinacy*.

4.3.2 Output-Targeting Fiscal Rule ($\gamma_d = 0$)

We now consider a fiscal rule that reacts only to output fluctuations and ignores the level of public debt—a specification we refer to as an *output-targeting fiscal rule*. Under this configuration, the fiscal authority adjusts lump-sum transfers in response to economic activity, but provides no stabilizing feedback on debt dynamics. Consequently, the government's intertemporal budget constraint must be satisfied through changes in the price level, making the existing stock of debt a key determinant of the inflationary response to monetary shocks.

$$\gamma_d = 0, \qquad T_t = \gamma_0 - \gamma_y Y_t + \varepsilon_t^f.$$

Flexible prices ($\theta = 0$) and no output response ($\phi_y = 0$). Under the reduced system (42)–(43), setting $\gamma_d = 0$ yields

$$i_t = \phi_\pi \pi_t + \nu_t, \tag{46}$$

$$\beta\, b_t = b_{t-1} - \pi_t + \beta\, i_t + \varepsilon_t^f. \tag{47}$$

Combining gives

$$\beta\, b_t \;=\; b_{t-1} + (\beta\phi_\pi - 1)\pi_t + \beta\, \nu_t + \varepsilon_t^f. \tag{48}$$

Since $|\beta| < 1$, this configuration corresponds to fiscal dominance: b_t is forward-looking, and the price level must adjust to keep the forward solution for debt bounded. Solving for inflation,

$$\pi_t \;=\; \underbrace{\frac{1-\beta L^{-1}}{1-\beta\phi_\pi}}_{\Theta(\phi_\pi)}\, b_{t-1} \;+\; \underbrace{\frac{\beta}{1-\beta\phi_\pi}}_{\Theta_\nu(\phi_\pi)}\, \nu_t \;+\; \underbrace{\frac{1}{1-\beta\phi_\pi}}_{\Theta_f(\phi_\pi)}\, \varepsilon_t^f. \tag{49}$$

Equation (49) shows that, even with lump-sum taxes, the initial real debt b_{t-1} enters inflation directly: with no fiscal feedback to debt, the government's intertemporal constraint is closed by revaluing outstanding nominal liabilities through the price level.

Corollary 1 (No Debt Feedback under Flexible Prices)**.** *Let $\theta = 0$, $\phi_y = 0$, and $\gamma_d = 0$. Then:*

(a) The economy is in fiscal dominance ($|\beta| < 1$), and inflation is uniquely determined by (49)*.*

(b) The impact of a monetary tightening on inflation is increasing in the pre-shock debt stock b_{t-1}: the larger is b_{t-1}, the larger the price-level adjustment required to keep the forward solution for b_t bounded.

(c) Real allocations remain at their efficient levels ($y_t = 0$ in gaps).

Extension: sticky prices ($\theta > 0$). When nominal rigidities are present, the same mechanism strengthens. Setting $\gamma_d = 0$ in the general linear system (31)–(30) (with $\phi_y = 0$) yields the fiscal-dominance solution

$$\pi_t \;=\; \left(1-\beta L^{-1}\right)\Theta\, b_{t-1} \;+\; \beta\,\Theta\,\nu_t \;+\; \Theta\,\varepsilon_t^f, \qquad \Theta^{-1} \;=\; 1-\beta\phi_\pi \;-\; \left(\tfrac{\gamma_y-\tau}{\xi_d}\right)\frac{1-\beta L^{-1}}{k}.$$

Two observations follow. First, as in the flexible-price case, b_{t-1} loads directly into inflation because debt is not backed by future fiscal surpluses. Second, the coefficient γ_y now scales the transmission through Θ: a higher (more countercyclical) γ_y dampens the nominal wealth effect by reducing transfers when output rises, whereas a negative (procyclical) γ_y amplifies it. The steady-state debt ratio ξ_d further magnifies this channel by determining how interest income interacts with fiscal responses to output.

With $\gamma_d = 0$, fiscal policy does not commit to future primary surpluses to offset current liabilities. Households therefore perceive outstanding nominal debt as net wealth, and the government's intertemporal constraint is enforced by revaluing that stock through inflation. In the flexible-price limit, this shows up as a direct b_{t-1} term in (49); with sticky prices, the effect is amplified by both the steady-state debt ratio ξ_d and the fiscal output response γ_y. Thus, in the absence of debt feedback, the magnitude of an Expansionary Hike depends jointly on the size of public debt and the cyclical stance of fiscal policy.

To formalize these insights, we present two complementary corollaries. The first characterizes how the cyclical stance of fiscal policy affects the strength of Expansionary Hikes; the second establishes the conditions under which the stock of debt matters for monetary transmission.

Corollary 2 (Cyclical Fiscal Response and the Amplification of Expansionary Hikes)**.** *Under Fiscal Dominance with $\gamma_d = 0$ and $\phi_y = 0$, the sensitivity of inflation to monetary shocks depends on the sign and magnitude of the fiscal output coefficient γ_y. In the sticky-price case ($\theta > 0$), the transmission coefficient*

$$\Theta^{-1} \;=\; 1 - \beta\phi_\pi - \left(\tfrac{\gamma_y - \tau}{\xi_d}\right)\frac{1-\beta L^{-1}}{k}$$

implies:

(a) ***Countercyclical fiscal policy*** *($\gamma_y > 0$)**:** higher γ_y increases the denominator of Θ^{-1}, thereby reducing Θ and dampening the inflationary effect of a rate hike. Fiscal retrenchment in booms offsets the nominal wealth effect.*

(b) ***Procyclical fiscal policy*** *($\gamma_y < 0$)**:** lower γ_y decreases the denominator of Θ^{-1}, enlarging Θ and amplifying the inflationary response. Fiscal expansion in booms reinforces the nominal wealth channel.*

Hence, the strength of an Expansionary Hike is decreasing in γ_y:

$$\frac{\partial\left(\partial\pi_t/\partial\nu_s\right)}{\partial\gamma_y} \;<\; 0.$$

Sketch of intuition. When $\gamma_d = 0$, fiscal solvency requires that the price level adjust to revalue the outstanding nominal debt stock. A more countercyclical fiscal stance ($\gamma_y > 0$) offsets part of this revaluation by reducing transfers as output rises, mitigating the nominal wealth effect. Conversely, a procyclical stance ($\gamma_y < 0$) amplifies the fiscal expansion following a rate hike, strengthening the feedback from nominal debt to inflation. (See Figure 15 in Appendix.) □

Corollary 3 (Distortionary Taxation and the Dampening of Expansionary Hikes)**.** *Under Fiscal Dominance with $\gamma_d = 0$ and $\phi_y = 0$, the sensitivity of inflation to monetary shocks depends on*

the magnitude of the distortionary taxation coefficient τ. In the sticky-price case ($\theta > 0$), the transmission coefficient

$$\Theta^{-1} \;=\; 1 - \beta\phi_\pi - \left(\tfrac{\gamma_y - \tau}{\xi_d}\right)\frac{1-\beta L^{-1}}{k}$$

shows that higher distortionary taxation increases the denominator of Θ^{-1}, thereby reducing Θ and dampening the inflationary response. Intuitively, the reliance of the government on tax revenue partially offsets the nominal wealth channel: higher taxes on output reduce households' perceived wealth and dampen their incentive to increase consumption or production.

Hence, the strength of an Expansionary Hike decreases with τ:

$$\frac{\partial\,(\partial\pi_t/\partial\nu_s)}{\partial\tau} \;<\; 0.$$

Sketch of Intuition. When $\gamma_d = 0$, fiscal solvency requires that the price level adjust to revalue the outstanding nominal debt. Higher distortionary taxes (e.g., on sales or output) reduce households' net payoff from additional production and consumption, because increased output raises their expected tax burden. Anticipating this, households temper their consumption and production responses, weakening the nominal wealth effect. As a result, the inflationary and output responses to a monetary shock are smaller (see Figure 16 in Appendix). □

Corollary 4 (Debt–Stock Relevance Depends on Fiscal Feedback)**.** *Consider the flexible-price, no-output-response benchmark ($\theta = 0$, $\phi_y = 0$) and its sticky-price extension. Then:*

(a) If $\gamma_d = 0$ (no fiscal feedback to debt), inflation loads directly on the initial real debt stock:

$$\pi_t \;=\; (1-\beta L^{-1})\,\Theta\, b_{t-1} \;+\; \beta\,\Theta\,\nu_t \;+\; \Theta\,\varepsilon_t^f, \qquad \Rightarrow \quad \frac{\partial \pi_t}{\partial b_{-1}} \neq 0.$$

Hence, the size of an Expansionary Hike scales with the stock of debt (via b_{t-1} and, under sticky prices, via ξ_d through Θ).

(b) If $\gamma_d < 0$ (debt rollover), the forward solution selects a jump in the price level that keeps the real-debt path bounded. Starting from steady state ($b_{-1} = 0$ in deviations), the nominal-wealth term

$$\left(1 - (\beta + \gamma_d)L^{-1}\right)\Theta\, b_{t-1}$$

has zero impact on impulse responses, and the marginal response to a rate hike, $\partial\pi_t/\partial\nu_s$, is independent *of the pre-shock debt stock.*

Proof. With $\gamma_d = 0$, the government does not stabilize debt, so the price level must adjust to satisfy the intertemporal budget constraint. The larger the initial stock of debt, the stronger the required revaluation and hence the greater the inflationary response to a monetary tightening. When $\gamma_d < 0$, instead, debt rollover ensures that nominal liabilities adjust endogenously,

preventing the stock of debt from amplifying the shock. The debt path remains bounded without further price-level adjustments, rendering inflation independent of b_{t-1}. □

Taken together, Corollaries 2 and 4 demonstrate that the interplay between fiscal cyclicality and debt stabilization governs the transmission of monetary policy. When fiscal policy targets output but not debt, the stock of outstanding debt becomes a key state variable: inflation must revalue this stock to satisfy the government's intertemporal constraint, and its impact is amplified when fiscal behavior is procyclical. Conversely, when fiscal policy accommodates debt rollover, nominal liabilities adjust endogenously and the stock of debt ceases to shape short-run dynamics. These results underscore a central insight of the paper: in high-debt environments, the fiscal response to interest-rate changes—not nominal rigidities per se—determines whether monetary tightening is contractionary or expansionary.

Non-linear Amplification. Although the equilibrium characterization above is derived from a log–linear approximation, the underlying mechanism is inherently *nonlinear*. The fiscal impulse driving the nominal–wealth channel scales with nominal interest outlays,

$$\text{interest outlays}_t = i_t \cdot B_{t-1}.$$

Because both the policy rate i_t and the stock of nominal debt B_{t-1} typically rise during a tightening cycle in a high-debt regime, the resulting fiscal stimulus is multiplicative: it depends on the product $i_t \cdot B_{t-1}$. For small deviations, this appears as a linear link between b_{t-1} and the inflation response, but in the true nonlinear model the mapping from debt to inflation is *convex*,

$$\frac{\partial^2 \pi_t}{\partial b_{t-1}^2} > 0, \qquad \frac{\partial^2 \pi_t}{\partial b_{t-1}\, \partial \nu_t} > 0,$$

implying that the higher the initial debt, the stronger the inflationary effect of an equivalent interest-rate shock.

Intuitively, when the government carries substantial nominal liabilities and does not adjust primary surpluses ($\gamma_d = 0$), a rate hike raises nominal interest payments, effectively transferring income to bondholders. This expands household wealth and stimulates demand, which feeds back into inflation. As a result, monetary tightening is *state dependent*: its expansionary effect increases nonlinearly with the debt stock. The mechanism operates through the interaction of fiscal accommodation and nominal rigidities, and it becomes especially potent in environments—such as the current U.S. fiscal configuration—where high public debt and rising interest outlays coexist with limited fiscal adjustment.

5 Model Solution

To directly solve for the shock affecting the economy, we adopt the analytical approach developed by Tan and Walker (2015). This method allows us to derive a solution that is valid for a white-noise process influencing the economy, without requiring assumptions about initial conditions. In particular, it yields a moving average MA(∞) representation for inflation, consumption, and real debt, expressed entirely in terms of current and past shocks. This is especially convenient, as it enables us to isolate the impact of shocks without referencing initial values. Furthermore, the MA representation facilitates spectral analysis, which is particularly useful for examining the wealth effects of government debt—especially in relation to the growth of nominal debt.

5.1 An Analytical Approach

We employ the frequency-domain methodology developed by Tan and Walker (2015) to solve our model under rational expectations. Specifically, our analysis extends the framework of Tan (2017) to provide a more comprehensive explanation of the interaction between monetary and fiscal policy.

We now rewrite the system in equation (33) in the following form:

$$\left[\underbrace{\begin{pmatrix}\sigma & 1 & 0\\ 0 & \beta & 0\\ 0 & 0 & 0\end{pmatrix}}_{\Gamma_{-1}} L^{-1} + \underbrace{\begin{pmatrix}-(\sigma+\phi_\pi) & -\phi_\pi & 0\\ k & -1 & 0\\ -\left(\beta\phi_y + \frac{\gamma_y-\tau}{\xi_d}\right) & -\beta\phi_\pi + 1 & \beta+\gamma_d\end{pmatrix}}_{\Gamma_0} L^0 + \underbrace{\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{pmatrix}}_{\Gamma_1} L\right] \cdot \underbrace{\begin{pmatrix}y_t\\ \pi_t\\ b_t\end{pmatrix}}_{\hat{x}_t} = \underbrace{\begin{pmatrix}1\\ 0\\ \beta\end{pmatrix}}_{\psi_0} L^0 \nu_t + \underbrace{\begin{pmatrix}\sigma & 1 & 0\\ 0 & \beta & 0\\ 0 & 0 & 0\end{pmatrix}}_{\Gamma_{-1}} \underbrace{\begin{pmatrix}\eta^y_{t+1}\\ \eta^\pi_{t+1}\\ \eta^b_{t+1}\end{pmatrix}}_{\eta_{t+1}} \tag{50}$$

where L is the lag operator, such that $L^k \hat{x}_t = \hat{x}_{t-k}$, and ν_t is a white noise, normally distributed with mean zero and variance σ^2_ν. The matrices $\{\Gamma_{-1}, \Gamma_0, \Gamma_1, \psi_0\}$ are coefficients matrices and η_{t+1} denotes the vector of endogenous forecast errors, defined as $\eta_{t+1} = \hat{x}_{t+1} - \mathbb{E}_t\hat{x}_{t+1}$, so that $\mathbb{E}_t\eta_{t+1} = 0$

Following the methodology outlined above, we can express the solution to the system in a moving-average (MA) form:

$$\hat{x}_t = \sum_{k=0}^{\infty} C_k \nu_{t-k} = C(L)\nu_t \tag{51}$$

where ν_t is a covariance-stationary disturbance and $C(L)$ is a matrix polynomial in the lag operator. This representation provides a direct way to interpret the dynamic response of $\hat{x}_{t+k}(i)$ to a shock $\nu_t(j)$, which is given by the coefficient $C_k(i, j)$.

We derive the frequency-domain representation of the time-domain system (50) by evaluating the forecast errors η_{t+1}. Using (51) and the Wiener-Kolmogorov optimal prediction formula, we obtain:

$$\eta_{t+1} = \left\{ C(L)L^{-1} - \left[\frac{C(L)}{L} \right]_+ \right\} \nu_t = C_0 L^{-1} \nu_t \tag{52}$$

where $[\cdot]_+$ denotes the annihilation operator (discarding negative powers of L). This formulation assumes perfect observability of monetary shock histories up to period t.

Let $\Gamma(L) = \Gamma_{-1}L^{-1} + \Gamma_0 + \Gamma_1 L$. Substituting (51) and (52) into (50) yields:

$$\Gamma(L)C(L)\nu_t = (\psi_0 + \Gamma_{-1}C_0 L^{-1})\nu_t \tag{53}$$

Thus, the solution $\hat{x}_t$ can be expressed as:

$$\hat{x}_t = \begin{pmatrix} y_t \\ \pi_t \\ b_t \end{pmatrix} = \left[L\Gamma(L) \right]^{-1} (L\psi_0 + \Gamma_{-1}C_0)\nu_t \tag{54}$$

Equation (54) expresses the New Keynesian variables as a moving average of current and past monetary policy shocks. For the system to be stationary, C_0 must be chosen to exactly cancel the unstable roots contained in $S(z)$.

The matrix coefficients satisfy the z-transform identities:

$$z\Gamma(z)C(z) = z\psi_0 + \Gamma_{-1}C_0 \tag{55}$$

Applying the Smith canonical decomposition to $z\Gamma(z)$ gives:

$$z\Gamma(z) = \underbrace{U(z)^{-1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & z(z-z_1)(z-z_2) \end{pmatrix}}_{S(z)} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & (z-z_3) \end{pmatrix} V(z)^{-1}}_{T(z)}$$

where $S(z)$ separates roots inside (stable) and outside (unstable) the unit circle, while $U(z)$ and $V(z)$ are unimodular matrices. Following Tan and Walker (2015), roots in the frequency domain correspond to reciprocals of time-domain roots ($z = 1/\lambda$).

Multiplying the z-transform identities by $S(z)^{-1}$ yields:

$$T(z)C(z) = \begin{pmatrix} U_1(z) \\ U_2(z) \\ \frac{1}{z(z-z_1)(z-z_2)}U_3(z) \end{pmatrix} (z\psi_0 + \Gamma_{-1}C_0)$$

where $U_j(z)$ is the j-th row of $U(z)$. This implies the following necessary condition for C_0:

$$U_3(z)\left(z\psi_0 + \Gamma_{-1}C_0\right)\Big|_{z\,=\,0,\,z_1,\,z_2} = 0 \tag{56}$$

where $C_0 \in \mathbb{R}^{3\times 1}$ is the column vector of moving average coefficients:

$$C_0' = \begin{pmatrix} C_{01} & C_{02} & C_{03} \end{pmatrix} \tag{57}$$

This condition can be expressed in matrix form as:

$$RC_0 = -A \tag{58}$$

A solution exists if and only if A lies in the column space of R, i.e., $\text{col}(A) \subseteq \text{col}(R)$.

Let $Q = U_3(z_3^{-1})\Gamma^{-1}$. The solution is unique when the row space of Q is contained in the row space of R, i.e., $\text{row}(Q) \subseteq \text{row}(R)$.

These conditions established, we proceed to analyze the model under both monetary and fiscal dominance regimes.

5.1.1 Monetary Dominance Regime

Under the monetary dominance regime, and according to the determinacy analysis in Section 4.1, we have $z_1 = 1/\lambda_2$, $z_2 = 1/\lambda_3$ and $z_3 = 1/\lambda_1$.

This yields the following solutions[4]:

$$y_t = -\frac{1}{k\phi_\pi + \sigma + \phi_y}\nu_t \tag{59}$$

$$\pi_t = -\frac{k}{k\phi_\pi + \sigma + \phi_y}\nu_t \tag{60}$$

$$b_t = \left(\frac{k + \beta\sigma - \frac{\gamma_y - \tau}{\xi_d}}{(k\phi_\pi + \sigma + \phi_y)(\beta + \gamma_d)}\right)\sum_{j=0}^{\infty}\lambda_1^j \cdot \nu_{t-j} \tag{61}$$

Under the Monetary Dominance regime, when monetary policy shocks are not persistent, all adjustments in output and consumption occur within the first period, and the Real Wealth Effect discussed in Section 4.2.1 disappears. In contrast, when monetary policy shocks are persistent, a real wealth effect emerges due to the differential speed of price adjustment across time and

[4]Full derivations are in Appendix D.2.

firms caused by nominal rigidities. Nonetheless, real government debt dynamics continue to evolve over time, driven by the parameter λ_1, which captures the responsiveness of fiscal policy to monetary disturbances. Fiscal variables—specifically γ_d, γ_y, τ, and ξ_d—do not, however, exert a direct influence on the contemporaneous behavior of output or inflation.

5.1.2 Fiscal Dominance Regime

Under the fiscal dominance regime, and according to the determinacy analysis in Section 4.1, we have $z_1 = 1/\lambda_1$, $z_2 = 1/\lambda_3$ and $z_3 = 1/\lambda_2$.

This yields the following solutions[5]:

$$y_t = C_y \nu_t + \sum_{k=1}^{\infty} \left(\frac{1}{\lambda_2^{-1}(k\phi_\pi + \sigma + \phi_y)} + C_y \right) \lambda_2^k \cdot \nu_{t-k} \tag{62}$$

where

$$C_y = \frac{-\beta + \frac{\beta C_{02} \lambda_3^{-1}}{U_1(\lambda_3^{-1})} (1 - \beta\phi_\pi) \left\{ \frac{\gamma_y - \tau}{\xi_d} - (k\phi_\pi + \phi_y + \sigma)\lambda_3^{-1} + \beta\phi_y + \sigma \right\}}{\lambda_2^{-1}(k\phi_\pi + \sigma + \phi_y)}$$

$$\pi_t = C_\pi \nu_t + \sum_{k=1}^{\infty} \left(\frac{k}{\lambda_2^{-1}(k\phi_\pi + \sigma + \phi_y)} + C_\pi \right) \lambda_2^k \cdot \nu_{t-k} \tag{63}$$

where

$$C_\pi = \frac{\frac{\beta C_{02} \lambda_3^{-1}}{U_1(\lambda_3^{-1})} \left\{ k\sigma(1 - \beta\phi_\pi) + \left(\beta\phi_y + \frac{\gamma_y - \tau}{\xi_d} \right) (\beta\phi_y + \sigma\beta + k - (\sigma + \phi_y + k\phi_\pi)\lambda_3^{-1}) \right\}}{\lambda_2^{-1}(k\phi_\pi + \sigma + \phi_y)}$$

$$b_t = C_d \nu_t + \sum_{k=1}^{\infty} C_d \cdot \lambda_2^k \cdot \nu_{t-k} \tag{64}$$

where

$$C_d = \frac{\beta(k\phi_\pi + \sigma + \phi_y) + k(1 - \beta\phi_\pi) - \beta\phi_y - \frac{\gamma_y - \tau}{\xi_d}}{\lambda_2^{-1}(k\phi_\pi + \sigma + \phi_y)}$$

Figure 8 shows the time-zero responses of consumption, inflation, and real government debt to a monetary policy shock across the parameter space (ϕ_π, γ_d). Across the entire parameter space, the real value of government debt rises, indicating an increase in the perceived value of financial wealth. This occurs because the growth rate of nominal debt can exceed the rise in inflation. The resulting increase in perceived wealth, along with expectations of higher future prices, leads agents to raise their consumption.

The rise in the interest rate fails to suppress prices or reduce consumer spending because the fiscal authority does not offset the inflationary shock by reducing debt levels or committing to a

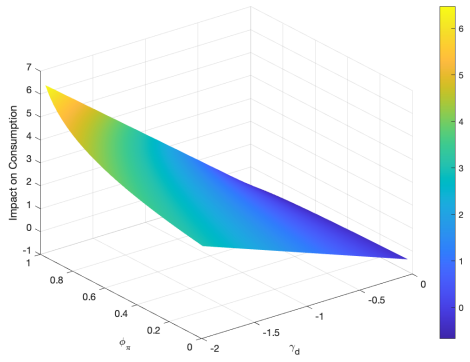
[5]Full derivations are in Appendix D.2.

repayment schedule for the newly issued debt. Instead, higher interest expenditures are financed through additional nominal debt issuance that is continually rolled over.

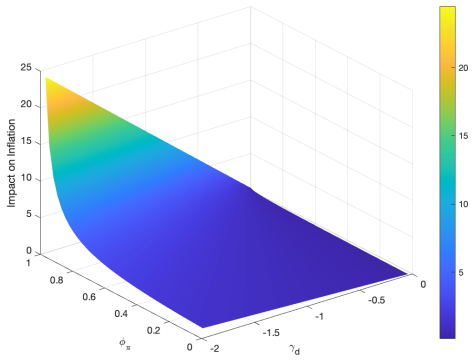
Importantly, the fiscal parameter γ_d—which measures the responsiveness of lump-sum taxes to changes in real debt—is either weakly positive around zero or negative. This implies that fiscal policy is either procyclical or acyclical. Consequently, households perceive an increase in their wealth and raise their consumption. As a result, the higher interest rate fails to reduce the price level or inflation.

Paradoxically, a weaker monetary policy response—characterized by a lower value of ϕ_π—leads to smaller movements in consumption, inflation, and real debt across the entire range of γ_d. This suggests that the parameter ϕ_π introduces a form of endogenous persistence in the dynamics of these variables, amplifying the effects of inflationary shocks when monetary policy reacts more aggressively (see Figure 14 in the Appendix).

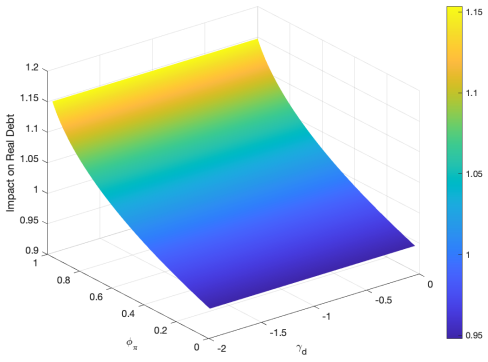
Indeed, the previous formulas (62), (63), and (64) exhibit endogenous persistence, which is governed by the eigenvalue λ_2. (See Figure 17 in Appendix.) The value of λ_2 increases with the monetary policy parameters ϕ_π and ϕ_y, indicating that a more aggressive monetary stance expands rather than dampens the expansionary effects. In other words, an increase in the nominal interest rate amplifies the Nominal Wealth Effect, which lies at the core of our analysis.



(a) Consumption



(b) Inflation



(c) Real Debt

Figure 8: Impact Responses following a Monetary Policy Shock

The figures are generated using the default parameter values: $\phi_y = 0$, $\gamma_y = 0$, $\tau = 0.2$, and $\xi_d = 1$.

Proposition 5. *Under the fiscal dominance regime, the equilibrium paths of output, inflation, and debt are independent of the steady-state debt-to-output ratio when the tax parameter τ equals the output elasticity of taxation, γ_y.*

Proof. Full derivations are in Appendix D.1. □

5.2 Spectral Analysis

In this section, we exploit the MA representation of the model solution to study the long-run dynamics of nominal debt growth. Under both the monetary dominance and fiscal dominance regimes, the real value of debt is stabilized. Therefore, understanding the dynamics of nominal debt growth is essential to better capture the wealth effects operating under both regimes.

Specifically, the spectral density allows us to decompose the impulse response function (IRF) into persistent (low-frequency) and transitory (high-frequency) components.

Proposition 6. *Under the monetary dominance regime, the spectral density of the growth rate of nominal debt is given by*

$$S_{\Delta b^{nom}}(\omega) = \left| \left(\frac{k + \sigma\beta - k(\beta + \gamma_d) - \frac{\gamma_y - \tau}{\xi_d}}{(k\phi_\pi + \sigma + \phi_y)(\beta + \gamma_d)} \right) e^{i\omega} + \left(\frac{\left(k + \sigma\beta - \frac{\gamma_y - \tau}{\xi_d}\right)(\lambda_1 - 1)}{(k\phi_\pi + \sigma + \phi_y)(\beta + \gamma_d)} \right) \left(\frac{1}{1 - \lambda_1 e^{-i\omega}} \right) \right|^2 \cdot \frac{\sigma_v^2}{2\pi}$$

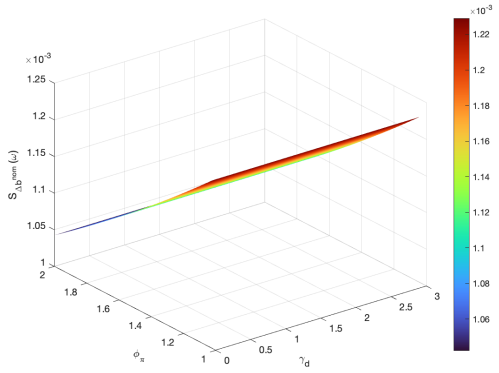
where $S_{\Delta b^{nom}}(\omega)$ denotes the spectral densities of the growth of nominal debt.

Proof. Full derivations are in Appendix D.1. □

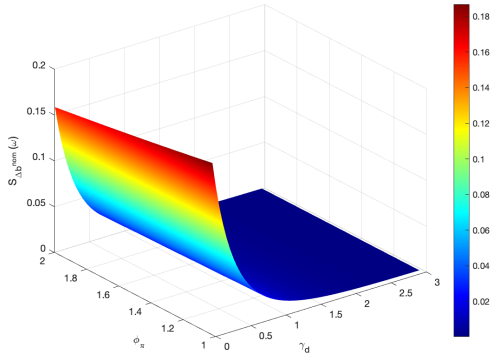
Figure 9 displays the spectral density of nominal debt growth across different frequencies. The figure reveals a pronounced spike at high frequencies, while the low-frequency plot remains almost flat, on the order of 10^{-3}. This indicates that the effects of shocks are absorbed almost entirely in the short run.

At medium and high frequencies, the spikes appear primarily when the fiscal policy parameter γ_d is low—i.e., when fiscal adjustment is sluggish. In these cases, for all values of the monetary policy parameter ϕ_π, the variability in nominal debt growth increases. Nevertheless, the system always converges toward full stabilization, allowing monetary policy to achieve its inflation target by setting the appropriate price level.

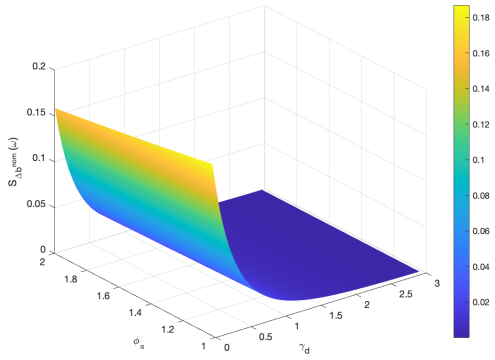
Figure 10 further shows that variability in nominal debt supply arises only in the short run, particularly when the fiscal authority adjusts debt issuance slowly (i.e., when γ_d is low). However, this variability remains uniformly distributed across short-term fluctuations, ensuring that changes in nominal debt supply do not accumulate over time.



(a) Low Frequency

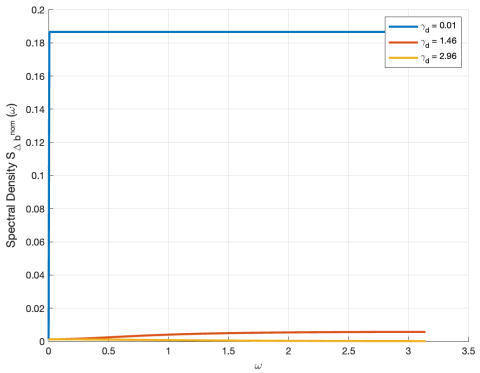


(b) Medium Frequency

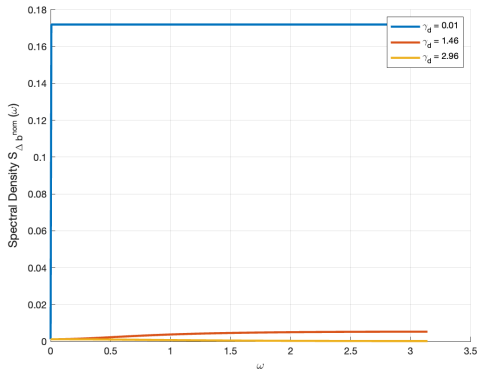


(c) High Frequency

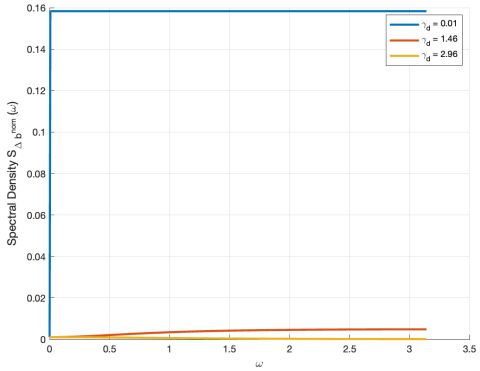
Figure 9: Spectral Density for different frequencies across the parameter space
The figures are generated using the default parameter values: $\phi_y = 0$, $\gamma_y = 0$, $\tau = 0.2$, and $\xi_d = 1$.



(a) $\phi_\pi = 1.02$



(b) $\phi_\pi = 1.50$



(c) $\phi_\pi = 2.00$

Figure 10: Spectral Density plots for different values of ϕ_π and γ_d
The figures are generated using the default parameter values: $\phi_y = 0$, $\gamma_y = 0$, $\tau = 0.2$, and $\xi_d = 1$.

Proposition 7. *Under the fiscal dominance regime, the spectral density of the growth rate of nominal debt is given by*

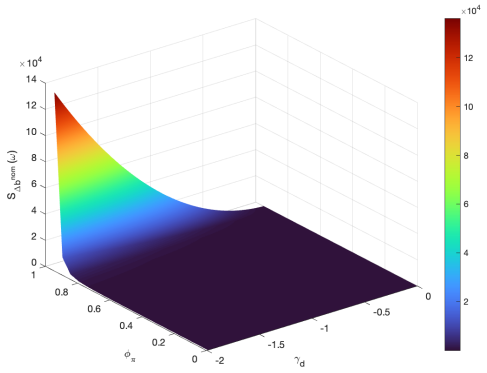
$$S_{\Delta b^{nom}}(\omega) = \left| (C_d + C_\pi)e^{i\omega} + \left[C_d(\lambda_2 - 1) + \left(\frac{k}{\lambda_2^{-1}(k\phi_\pi + \sigma + \phi_y)} + C_\pi \right) \lambda_2 \right] \left(\frac{1}{1 - \lambda_2 e^{-i\omega}} \right) \right|^2 \cdot \frac{\sigma_v^2}{2\pi}$$

where $S_{\Delta b^{nom}}(\omega)$ denotes the spectral densities of the growth of nominal debt.

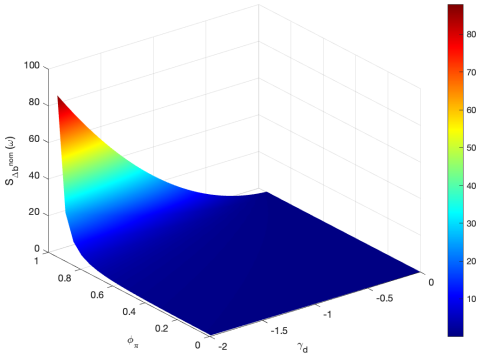
Proof. Full derivations are in Appendix D.1. □

Figure 11 displays the spectral density of nominal debt growth across different frequencies. The figure shows a pronounced spike at low frequencies, followed by a sharp decline at medium and high frequencies. This pattern indicates that the increase in nominal debt following a monetary policy shock is persistent rather than transitory. In other words, most of the variability of the IRF is concentrated in the long run, implying a permanent increase in nominal debt. The stabilization of the real value of government debt occurs only through a permanent rise in the price level.

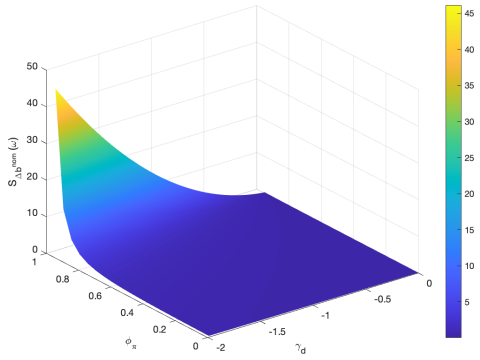
Figure 12 further illustrates that stronger monetary policy responses by the central bank—under



(a) Low Frequency



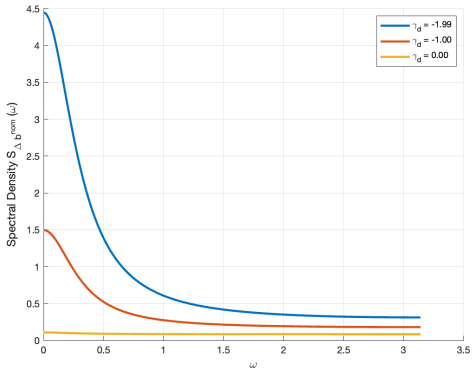
(b) Medium Frequency



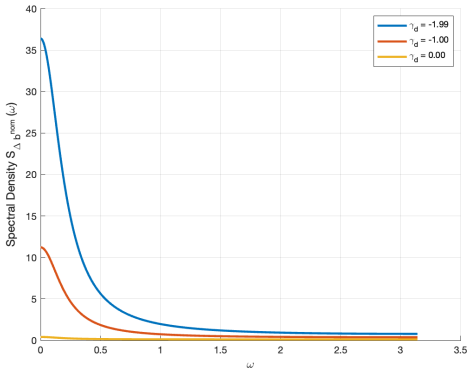
(c) High Frequency

Figure 11: Spectral Density for different frequencies across the parameter space
The figures are generated using the default parameter values: $\phi_y = 0$, $\gamma_y = 0$, $\tau = 0.2$, and $\xi_d = 1$.

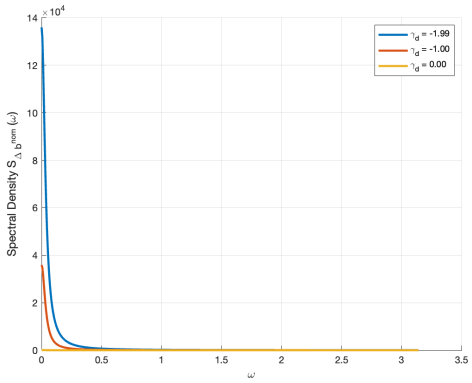
fiscal dominance—lead to a more permanent increase in nominal debt. In particular, while monetary tightening fails to contain short-run fluctuations in nominal debt, it results in an even larger long-run accumulation of nominal liabilities. The variation remains concentrated at low frequencies, confirming the enduring nature of the nominal debt expansion.



(a) $\phi_\pi = 0$



(b) $\phi_\pi = 0.48$



(c) $\phi_\pi = 0.98$

Figure 12: Spectral Density plots for different values of ϕ_π and γ_d

The figures are generated using the default parameter values: $\phi_y = 0$, $\gamma_y = 0$, $\tau = 0.2$, and $\xi_d = 1$.

6 Conclusion

In this paper, we develop a tractable analytical framework to analyze the conditions under which monetary tightening raises inflation and activity rather than curbing them. We show that when public debt is high and fiscal policy either targets output or rolls over debt, interest rate increases amplify nominal liabilities and stimulate demand through a Nominal Wealth Effect. In such environments, the effectiveness—and even the direction—of monetary policy depends on the fiscal regime.

One way to interpret our results is to highlight the importance of explicit policy coordination. In economies with elevated debt and persistent deficits, treating monetary and fiscal policy as independent instruments is no longer viable. Stabilization requires acknowledging their joint role in anchoring expectations and managing the fiscal implications of interest rate changes.

Future research should explore how debt maturity structure, the growth–interest rate differential, and heterogeneous fiscal instruments affect this interaction. Incorporating these features would clarify how the burden of adjustment between monetary and fiscal authorities evolves and help design policies that prevent tightening from becoming self-defeating.

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A Determinacy Analysis

The equilibrium conditions of the model are the are given by:

$$\begin{cases} \sigma \mathbb{E}_t y_{t+1} + \mathbb{E}_t \pi_{t+1} = (\sigma + \phi_y) y_t + \phi_\pi \pi_t + v_t \\ \pi_t = \beta \mathbb{E}_t \pi_{t+1} + k y_t \\ (\gamma_d + \beta) b_t = b_{t-1} + (\beta \phi_\pi - 1)\pi_t + \left(\beta \phi_y + \dfrac{\gamma_y - \tau}{\xi_d}\right) y_t + \beta v_t \end{cases} \tag{65}$$

We can express the system 65 in matrix form as follows:

$$\underbrace{\begin{bmatrix} \sigma & 1 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & (\gamma_d + \beta) \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \mathbb{E}_t y_{t+1} \\ \mathbb{E}_t \pi_{t+1} \\ b_t \end{bmatrix}}_{\mathbb{E}_t x_{t+1}} = \underbrace{\begin{bmatrix} (\sigma + \phi_y) & \phi_\pi & 0 \\ -k & 1 & 0 \\ \left(\beta \phi_y + \frac{\gamma_y - \tau}{\xi_d}\right) & (\beta \phi_\pi - 1) & 1 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} y_t \\ \pi_t \\ b_{t-1} \end{bmatrix}}_{x_t} + \underbrace{\begin{bmatrix} 1 \\ 0 \\ \beta \end{bmatrix}}_{C} v_t \tag{66}$$

In this model, there is one predetermined variable, b_t, and two forward-looking variables,π_t and y_t. Following Blanchard and Kahn (1980), the equilibrium is locally unique (i.e., determinate) if and only if two eigenvalues of the matrix $V = A^{-1}B$ lie outside the unit circle, while one lies inside.

The eigenvalues of the system are:

$$\lambda_1 = \frac{1}{\beta + \gamma_d}$$

$$\lambda_2 = \frac{k + \sigma + \beta \phi_y + \beta \sigma - \sqrt{D}}{2\beta\sigma}$$

$$\lambda_3 = \frac{k + \sigma + \beta \phi_y + \beta \sigma + \sqrt{D}}{2\beta\sigma}$$

$$\text{where} \quad D = \left(k + \sigma + \beta \phi_y + \beta \sigma\right)^2 - 4\beta\sigma(k\phi_\pi + \sigma + \phi_y)$$

As stated in Section 4.1, the system admits only two possible equilibria. We now demonstrate this result step by step.

Proposition 8. *λ_3 is always outside the unit circle in equilibrium.*

Proof. Suppose, for the sake of contradiction, that λ_3 lies inside the unit circle in equilibrium.

First, consider the case where λ_3 is a complex root. In this case, λ_2 and λ_3 are complex conjugate eigenvalues. Since complex conjugates have the same modulus, both eigenvalues must

lie either inside or outside the unit circle simultaneously. If both lie inside the unit circle, the equilibrium is indeterminate.

Next, consider the case where λ_3 is a real root. It is straightforward to verify that $\lambda_3 > 0$ and $\lambda_3 \geq \lambda_2$.

If $\lambda_3 = \lambda_2$, the same argument used for the case of complex conjugate eigenvalues applies.

If instead $\lambda_3 > \lambda_2$ and we want to verify the existence of an equilibrium with $|\lambda_3| < 1$, then, in order to satisfy the Blanchard-Kahn conditions for determinacy, it must be that:

$$\lambda_3 < 1 \quad \cap \quad \lambda_2 < -1$$

We now check whether these inequalities can simultaneously hold. Start with the condition $\lambda_3 < 1$:

$$\begin{aligned}
&\lambda_3 < 1 \\
&k + \sigma + \beta\phi_y + \beta\sigma + \sqrt{D} - 2\beta\sigma < 0 \\
&\Rightarrow k(\phi_\pi - 1) + (1 - \beta)\phi_y < 0 \quad \cap \quad \beta\sigma - k - \sigma - \beta\phi_y > 0
\end{aligned}$$

Similarly, for $\lambda_2 < -1$, we have:

$$\begin{aligned}
&\lambda_2 < -1 \\
&k + \sigma + \beta\phi_y + \beta\sigma - \sqrt{D} + 2\beta\sigma < 0 \\
&\Rightarrow k(1 + \phi_\pi) + 2\sigma + 2\beta\sigma + (1 + \beta)\phi_y < 0
\end{aligned}$$

However, since all parameters $k, \sigma, \beta, \phi_y, \phi_\pi$ are positive, the condition $k(1 + \phi_\pi) + 2\sigma + 2\beta\sigma + (1 + \beta)\phi_y < 0$ cannot be satisfied, as it requires that a sum of positive terms be strictly negative — a contradiction.

Therefore, the two conditions $\lambda_3 < 1$ and $\lambda_2 < -1$ cannot hold simultaneously. Their intersection is the empty set:

$$\lambda_3 < 1 \quad \cap \quad \lambda_2 < -1 \implies \varnothing$$

The contradiction implies that λ_3 cannot lie within the unit circle in equilibrium. Hence, λ_3 must lie outside the unit circle. □

□

Given that λ_3 is always outside the unit circle, we can state the **Proposition 1**.

Proof of Proposition 1

Proof. Following Blanchard and Kahn (1980) and the results in Proposition 8, the only possible equilibria are those in which $|\lambda_3| > 1$ and either $|\lambda_1| < 1$, $|\lambda_2| > 1$ or $|\lambda_1| > 1$, $|\lambda_2| < 1$.

Case 1: $|\lambda_1| < 1$ **and** $|\lambda_2| > 1$

In this case, the equilibrium is determined by the dynamics of y_t and π_t, as government debt remains stationary as long as y_t and π_t are stationary.

From the debt evolution equation (67):

$$b_t = \frac{b_{t-1}}{\beta + \gamma_d} + \frac{(\beta\phi_\pi - 1)\pi_t}{\beta + \gamma_d} + \left(\beta\phi_y + \frac{\gamma_y - \tau}{\xi_d}\right)\frac{y_t}{\beta + \gamma_d} + \frac{\beta}{\beta + \gamma_d}v_t \tag{67}$$

we see that if $|\lambda_1 + \gamma_d| > 1$, then the debt process is stationary provided that π_t and y_t are stationary.

We can reduce the system (66) to a 2×2 form:

$$\underbrace{\begin{bmatrix}\sigma & 1\\ 0 & \beta\end{bmatrix}}_{A'}\underbrace{\begin{bmatrix}\mathbb{E}_t y_{t+1}\\ \mathbb{E}_t \pi_{t+1}\end{bmatrix}}_{\mathbb{E}_t x'_{t+1}} = \underbrace{\begin{bmatrix}(\sigma + \phi_y) & \phi_\pi\\ -k & 1\end{bmatrix}}_{B'}\underbrace{\begin{bmatrix}y_t\\ \pi_t\end{bmatrix}}_{x'_t} + \underbrace{\begin{bmatrix}1\\ 0\end{bmatrix}}_{C'} v_t \tag{68}$$

Rewriting (68) in this form: $x'_t = (A'^{-1}B') \cdot \mathbb{E}_t x'_{t+1} - (A'^{-1}C') \cdot v_t$, we have:

$$\underbrace{\begin{bmatrix}y_t\\ \pi_t\end{bmatrix}}_{x'_t} = \underbrace{\frac{1}{\sigma + \phi_y + \phi_\pi k}\begin{bmatrix}\sigma & 1 - \beta\phi_\pi\\ \sigma k & k + \beta(\sigma + \phi_y)\end{bmatrix}}_{Q}\underbrace{\begin{bmatrix}\mathbb{E}_t y_{t+1}\\ \mathbb{E}_t \pi_{t+1}\end{bmatrix}}_{\mathbb{E}_t x'_{t+1}} - \frac{1}{\sigma + \phi_y + \phi_\pi k}\underbrace{\begin{bmatrix}1\\ 0\end{bmatrix}}_{C'} v_t \tag{69}$$

The system (69) admits a locally unique solution if and only if both eigenvalues of Q lie inside the unit circle. The characteristic polynomial of Q is:

$$\begin{aligned}
&|Q - \lambda I| = 0\\
&\left|\frac{1}{\sigma + \phi_y + \phi_\pi k}\begin{bmatrix}\sigma & 1 - \beta\phi_\pi\\ \sigma k & k + \beta(\sigma + \phi_y)\end{bmatrix} - \lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right| = 0\\
&\lambda^2 - \frac{\sigma + k + \beta(\sigma + \phi_y)}{\sigma + \phi_y + \phi_\pi k}\lambda + \frac{\sigma\beta}{\sigma + \phi_y + \phi_\pi k} = 0
\end{aligned}$$

Using standard stability criteria (see LaSalle (1986)), both roots lie inside the unit circle if and only if:

$$\left|\frac{\sigma\beta}{\sigma+\phi_y+\phi_\pi k}\right| < 1 \tag{70}$$

$$\left|\frac{\sigma+k+\beta(\sigma+\phi_y)}{\sigma+\phi_y+\phi_\pi k}\right| < 1+\frac{\sigma\beta}{\sigma+\phi_y+\phi_\pi k} \tag{71}$$

From the first inequality (70), we derive:

$$\sigma(1-\beta)+\phi_y+\phi_\pi k > 0 \tag{72}$$

which holds for all $\beta < 1$.

From the second inequality (71), we obtain:

$$k(\phi_\pi - 1)+(1-\beta)\phi_y > 0 \tag{73}$$

Hence, the monetary-dominance equilibrium exists if and only if

$$k(\phi_\pi - 1)+(1-\beta)\phi_y > 0 \quad \cap \quad |\beta+\gamma_d| > 1$$

.

Case 2: $|\lambda_1| > 1$ **and** $|\lambda_2| < 1$

In this case, for a locally unique equilibrium, we need:

$$|\lambda_2| < 1 \text{ and } |\lambda_3| > 1$$

To ensure $\|\lambda_2\| \neq \|\lambda_3\|$ we must rule out complex eigenvalues, i.e., assume $D > 0$ so that both roots are real.

From condition $\lambda_3 > 1$, we obtain:

$$\begin{aligned}
&\lambda_3 > 1 \\
&\sqrt{D} > 2\beta\sigma - (k+\sigma+\beta\phi_y+\beta\sigma) \\
&\Rightarrow k(\phi_\pi - 1)+(1-\beta)\phi_y < 0
\end{aligned}$$

From condition $-1 < \lambda_2 < 1$, we derive:

$$
\begin{aligned}
&-1 < \lambda_2 < 1 \\
&\lambda_2 > -1 \quad \cap \quad \lambda_2 < 1 \\
&k + \sigma + \beta\phi_y + \beta\sigma + 2\beta\sigma > \sqrt{D} \quad \cap \quad k + \sigma + \beta\phi_y + \beta\sigma - 2\beta\sigma < \sqrt{D} \\
&2\beta\sigma + k(1 + \phi_\pi) + 2\sigma + (1 + \beta)\phi_y > 0 \quad \cap \quad k(\phi_\pi - 1) + (1 - \beta)\phi_y < 0 \\
&\Rightarrow \quad k(\phi_\pi - 1) + (1 - \beta)\phi_y < 0
\end{aligned}
$$

Thus, we have that $|\lambda_2| < 1$ if and only if $k(\phi_\pi - 1) + (1 - \beta)\phi_y < 0$

Thus, a fiscal-dominance equilibrium exists if and only if:

$$
D > 0 \quad \cap \quad k(\phi_\pi - 1) + (1 - \beta)\phi_y < 0 \quad \cap \quad |\beta + \gamma_d| < 1
$$

This concludes the proof. □ □

B Intertemporal Consumption Model with Interest Rate Effects

In a standard perfect-foresight intertemporal consumption model, the effects of interest rate changes can be analyzed analytically through the lens of the Euler equation and the intertemporal budget constraint.

Given the Euler equation (analogous to 5), the consumption path evolves according to:

$$
C_{t+1} = \left[\frac{\beta \cdot (1 + i_t)}{\Pi_{t+1}} \right]^{\frac{1}{\sigma}} C_t
$$

Consumption growth is increasing in the real interest rate and in the intertemporal elasticity of substitution, which under CRRA preferences satisfies $EIS = \frac{1}{\sigma}$.

Given the budget constraint (2), let total real wealth be defined as

$$
\mathcal{W}_t = \frac{W_t}{P_t} N_t + T_t + \frac{D_t}{P_t}
$$

The intertemporal budget constraint can then be expressed as:

$$
\begin{aligned}
C_t + Q_t \frac{B_t^d}{P_t} &= \frac{B_{t-1}^d}{P_{t-1}} \frac{1}{\Pi_t} + \mathcal{W}_t \\
C_{t+1} + Q_{t+1} \frac{B_{t+1}^d}{P_{t+1}} &= \frac{B_t^d}{P_t} \frac{1}{\Pi_{t+1}} + \mathcal{W}_{t+1}
\end{aligned}
$$

By combining the two equations, we obtain the following relationship for consumption:

$$C_t = \left\{1 + \beta^{\frac{1}{\sigma}} \left(\frac{1+i_t}{\Pi_{t+1}}\right)^{\frac{1}{\sigma}-1}\right\}^{-1} \left[\mathcal{W}_t + \frac{\mathcal{W}_{t+1}}{1+i_t}\Pi_{t+1} + \frac{B^d_{t-1}}{P_{t-1}}\frac{1}{\Pi_t} - \frac{\Pi_{t+1}}{(1+i_t)(1+i_t+1)}\frac{B^d_{t+1}}{P_{t+1}}\right] \quad (74)$$

Consumption depends both on nominal quantities and on future relative prices and the real valuation of assets. The term $\mathcal{W}_t + \frac{\mathcal{W}_{t+1}}{1+i_t}\Pi_{t+1}$ shows that consumption increases with current and expected future income flows, including wages, dividends, and fiscal transfers (or taxes). The term $\frac{B^d_{t-1}}{P_{t-1}}\frac{1}{\Pi_t} - \frac{\Pi_{t+1}}{(1+i_t)(1+i_t+1)}\frac{B^d_{t+1}}{P_{t+1}}$ captures how consumption responds to the real return on existing debt and to the expected future value of debt in nominal terms. This mechanism gives rise to the nominal wealth effect, whereby higher interest payments on government debt increase household income and stimulate consumption. In particular, when nominal quantities (such as the interest rate or nominal debt) rise more rapidly than real adjustments, a positive wealth effect emerges. This mechanism is often referred to in the literature as the *Tight Money Paradox*, whereby tighter monetary policy—through its impact on nominal debt servicing—may temporarily boost household wealth and, paradoxically, increase consumption.

C Proofs and Derivations: Equilibrium Dynamics

C.1 Proofs

Proof of Proposition 2

Proof. As shown in Section 4.1, under monetary dominance, the equilibrium paths of inflation and output are determined independently of fiscal policy instruments. In this case, we have $|\lambda_1| < 1$, which implies $|\beta + \gamma_d| > 1$.

Using the lag operator L, the government's intertemporal budget constraint (IBC) can be written as:

$$(\beta + \gamma_d)b_t = Lb_t + (\beta\phi_\pi - 1)\pi_t + \left(\beta\phi_\pi + \frac{\gamma_y - \tau}{\xi_d}\right)y_t + \beta\nu_t.$$

Rearranging terms:

$$\left(1 - \frac{L}{\beta + \gamma_d}\right)b_t = \frac{\beta\phi_\pi - 1}{\beta + \gamma_d}\pi_t + \frac{\beta\phi_\pi + \frac{\gamma_y - \tau}{\xi_d}}{\beta + \gamma_d}y_t + \frac{\beta}{\beta + \gamma_d}\nu_t.$$

Applying the inverse of the lag polynomial operator $\left(1 - \frac{L}{\beta+\gamma_d}\right)^{-1}$, which is well-defined since $|\beta + \gamma_d| > 1$, yields:

$$b_t = \frac{\beta\phi_\pi - 1}{\beta + \gamma_d}\sum_{k=0}^{\infty}\left(\frac{\pi_{t-k}}{(\beta+\gamma_d)^k}\right) + \left(\frac{\beta\phi_\pi + \frac{\gamma_y-\tau}{\xi_d}}{\beta+\gamma_d}\right)\sum_{k=0}^{\infty}\left(\frac{y_{t-k}}{(\beta+\gamma_d)^k}\right) + \frac{\beta}{\beta+\gamma_d}\sum_{k=0}^{\infty}\left(\frac{\nu_{t-k}}{(\beta+\gamma_d)^k}\right).$$

This expression characterizes the unique bounded solution for b_t under monetary dominance.

□ □

Proof of Proposition 3

Proof. As shown in Section 4.1, under fiscal dominance, the equilibrium paths of inflation and output are determined by taking into account the initial level of government debt. In this case, we have $|\lambda_1| > 1$, which implies $|\beta + \gamma_d| < 1$.

Using the lag operator L, the government's intertemporal budget constraint (IBC) can be written as:

$$(\beta + \gamma_d)L^{-1}b_{t-1} = b_{t-1} + (\beta\phi_\pi - 1)\pi_t + \left(\beta\phi_y + \frac{\gamma_y - \tau}{\xi_d}\right) y_t + \beta\nu_t.$$

Rearranging terms:

$$\left(1 - (\beta + \gamma_d)L^{-1}\right) b_{t-1} = -\left(\beta\phi_\pi - 1\right)\pi_t - \left(\beta\phi_y + \frac{\gamma_y - \tau}{\xi_d}\right) y_t - \beta\nu_t.$$

Applying the inverse of the lag polynomial operator $\left(1 - (\beta + \gamma_d)L^{-1}\right)^{-1}$, which is well-defined since $|\beta + \gamma_d| < 1$, yields:

$$b_{t-1} = -(\beta\phi_\pi - 1)\sum_{k=0}^{\infty}(\beta+\gamma_d)^k \cdot \mathbb{E}_t\pi_{t+k} - \left(\beta\phi_y + \frac{\gamma_y - \tau}{\xi_d}\right)\sum_{k=0}^{\infty}(\beta+\gamma_d)^k \cdot \mathbb{E}_t y_{t+k} - \beta\sum_{k=0}^{\infty}(\beta+\gamma_d)^k \cdot \mathbb{E}_t\nu_{t+k}$$

This expression characterizes the equilibrium path for b_t under fiscal dominance, as follows:

$$b_t = -(\beta\phi_\pi - 1)\sum_{k=0}^{\infty}(\beta+\gamma_d)^k \cdot \mathbb{E}_t\pi_{t+1+k} - \left(\beta\phi_y + \frac{\gamma_y - \tau}{\xi_d}\right)\sum_{k=0}^{\infty}(\beta+\gamma_d)^k \cdot \mathbb{E}_t y_{t+1+k} - \beta\sum_{k=0}^{\infty}(\beta+\gamma_d)^k \cdot \mathbb{E}_t\nu_{t+1+k}$$

□ □

C.2 Mathematical Derivations

Derivations for Equations (40)**–**(41) **(Fiscal Dominance, Section 4.2.2)** The equilibrium path for government debt is given by:

$$b_{t-1} = -(\beta\phi_\pi - 1)\sum_{k=0}^{\infty}(\beta+\gamma_d)^k \cdot \mathbb{E}_t\pi_{t+k} - \left(\beta\phi_y + \frac{\gamma_y - \tau}{\xi_d}\right)\sum_{k=0}^{\infty}(\beta+\gamma_d)^k \cdot \mathbb{E}_t y_{t+k} - \beta\sum_{k=0}^{\infty}(\beta+\gamma_d)^k \cdot \mathbb{E}_t\nu_{t+k}$$

This expression can be rewritten entirely in terms of current inflation, the initial level of debt, and monetary policy shocks as follows:

$$\left(1-(\beta+\gamma_d)L^{-1}\right)b_{t-1} = -(\beta\phi_\pi - 1)\pi_t - \left(\beta\phi_y + \frac{\gamma_y - \tau}{\xi_d}\right)y_t - \beta\nu_t$$

From the system (33), output can be expressed as:

$$y_t = \frac{1-\beta L^{-1}}{k}\cdot\pi_t$$

Hence, inflation can be expressed as a function of government debt and monetary policy shocks:

$$\pi_t = \frac{1-(\beta+\gamma_d)L^{-1}}{1-\beta\phi_\pi - \left(\beta\phi_y + \frac{\gamma_y-\tau}{\xi_d}\right)\left(\frac{1-\beta L^{-1}}{k}\right)}b_{t-1} + \frac{\beta}{1-\beta\phi_\pi - \left(\beta\phi_y + \frac{\gamma_y-\tau}{\xi_d}\right)\left(\frac{1-\beta L^{-1}}{k}\right)}\nu_t$$

Finally, the equilibrium path for output is given by:

$$y_t = \frac{\left(\frac{1-\beta L^{-1}}{k}\right)\left(1-(\beta+\gamma_d)L^{-1}\right)}{1-\beta\phi_\pi - \left(\beta\phi_y + \frac{\gamma_y-\tau}{\xi_d}\right)\left(\frac{1-\beta L^{-1}}{k}\right)}b_{t-1} + \frac{\frac{\beta}{k}}{1-\beta\phi_\pi - \left(\beta\phi_y + \frac{\gamma_y-\tau}{\xi_d}\right)\left(\frac{1-\beta L^{-1}}{k}\right)}\nu_t - \frac{\frac{\beta}{k^2}}{1-\beta\phi_\pi - \left(\beta\phi_y + \frac{\gamma_y-\tau}{\xi_d}\right)\left(\frac{1-\beta L^{-1}}{k}\right)}\mathbb{E}_t\nu_{t+1}$$

D Proofs and Derivations: Model Solution

D.1 Proofs

Proof of Proposition 5

Proof. Consider the closed-form solutions for output (62), inflation (63), and real debt (64).

We begin by analyzing the coefficients C_y, C_π, and C_d under the assumption $\tau = \gamma_y$:

$$C_y = \frac{-\beta + \frac{\beta C_{02}\lambda_3^{-1}}{U_1(\lambda_3^{-1})}(1-\beta\phi_\pi)\left\{-(k\phi_\pi+\phi_y+\sigma)\lambda_2^{-1}+\beta\phi_y+\sigma\right\}}{\lambda_2^{-1}}$$

$$C_\pi = \frac{\frac{\beta C_{02}\lambda_3^{-1}}{U_1(\lambda_3^{-1})}\left\{k\sigma(1-\beta\phi_\pi)+\beta\phi_y(\beta\phi_y+\sigma\beta+k-(\sigma+\phi_y+k\phi_\pi)\lambda_3^{-1})\right\}}{\lambda_2^{-1}}$$

$$C_d = \frac{\beta + k(1-\beta\phi_\pi) - \beta\phi_y}{\lambda_2^{-1}}$$

It is immediately evident that the coefficient C_d, governing the dynamics of real debt, is independent of the steady-state debt-to-output ratio ξ_d, that is, $C_d \perp \xi_d$.

Next, we examine the dependence of C_y and C_π on ξ_d. To do so, consider the term $\frac{\beta C_{02}\lambda_3^{-1}}{U_1(\lambda_3^{-1})}$ under the same assumption ($\tau = \gamma_y$):

$$\frac{\beta C_{02}\lambda_3^{-1}}{U_1(\lambda_3^{-1})} = \frac{\beta(\beta+\gamma_d) - \beta\lambda_3^{-1} - \gamma_d\beta\phi_y + k(1-\beta\phi_\pi)(\beta+\gamma_d)}{(1-\beta\phi_\pi)\left\{-k\left[\beta\phi_y+\sigma(1-\beta\phi_\pi)\right]+\sigma\beta\phi_y(1-\beta)\right\}}$$

The right-hand side of this expression contains no term depending on ξ_d, which implies that $\frac{\beta C_{02}\lambda_3^{-1}}{U_1(\lambda_3^{-1})} \perp \xi_d$. Hence, both C_y and C_π are independent of the debt-to-output ratio as well.

We therefore conclude that under fiscal dominance and when $\tau = \gamma_y$, the dynamics of output, inflation, and real debt do not depend on the steady-state debt-to-output ratio. In other words:

$$C_y,\ C_\pi,\ C_d \perp \xi_d.$$

□ □

Proof of Proposition 6

Proof. [6]

We begin by deriving the $MA(\infty)$ representation for the growth of nominal debt.

By definition, the growth rate of nominal debt is given by:

$$\Delta b_{t+1}^{nom} = b_{t+1} - b_t + \pi_{t+1} \tag{75}$$

Through algebraic manipulation and substitution using the model's reduced-form solutions (60) and (61), this expression can be rewritten as

[6]The proof is based on well-established principles in time series analysis, rigorously demonstrated in Brockwell and Davis (1991), Hamilton (2020), and Kilian (2006).

$$\Delta b_{t+1}^{nom} = \left(\frac{k + \sigma\beta - k(\beta + \gamma_d) - \frac{\gamma_y - \tau}{\xi_d}}{(k\phi_\pi + \sigma + \phi_y)(\beta + \gamma_d)} \right) \nu_{t+1} + \sum_{k=0}^{\infty} \left[\frac{\left(k + \sigma\beta - \frac{\gamma_y - \tau}{\xi_d} \right)(\lambda_1 - 1)}{(k\phi_\pi + \sigma + \phi_y)(\beta + \gamma_d)} \right] \lambda_1^k \cdot \nu_{t-k} \quad (76)$$

where ν_t is the monetary policy shock and $\lambda_1 \in (0, 1)$ is the stable root associated with the fiscal dominance regime.

Let $C_{\Delta b^{nom},k}$ denote the MA coefficients of the process Δb_{t+1}^{nom}.

Stationarity: The process Δb_{t+1}^{nom} is a linear combination of lagged shocks ν_{t-k} with coefficients that decay geometrically due to the presence of λ_1^k. Since $\lambda_1 \in (0, 1)$, these coefficients are absolutely summable. Therefore, Δb_{t+1}^{nom} is covariance-stationary.

This result also follows from the fact that Δb_{t+1}^{nom} is constructed from a linear combination of stationary components—namely b_t, b_{t+1}, and π_{t+1}—each of which admits an MA representation with geometrically decaying coefficients under fiscal dominance (see Section 4.1).

Spectral Density: Since Δb_{t+1}^{nom} admits an $MA(\infty)$ representation and is stationary, we can apply the standard result for the spectral density of a linear process. If ν_t is a zero-mean, covariance-stationary process with absolutely summable autocovariances, the spectral density of Δb_{t+1}^{nom} is given by:

$$S_{\Delta b^{nom}}(\omega) = \left| \sum_{k=0}^{\infty} C_{\Delta b^{nom},k} e^{-i\omega k} \right|^2 \cdot S_\nu(\omega)$$

where $S_\nu(\omega)$ is the spectral density of the innovation process ν_t.

Substituting the closed-form expression for the MA coefficients yields:

$$S_{\Delta b^{nom}}(\omega) = \left| \left(\frac{k + \sigma\beta - k(\beta + \gamma_d) - \frac{\gamma_y - \tau}{\xi_d}}{(k\phi_\pi + \sigma + \phi_y)(\beta + \gamma_d)} \right) e^{i\omega} + \left(\frac{\left(k + \sigma\beta - \frac{\gamma_y - \tau}{\xi_d} \right)(\lambda_1 - 1)}{(k\phi_\pi + \sigma + \phi_y)(\beta + \gamma_d)} \right) \left(\frac{1}{1 - \lambda_1 e^{-i\omega}} \right) \right|^2 \cdot S_\nu(\omega)$$

Since the innovation process ν_t is white noise, its spectral density is constant and equal to $S_\nu(\omega) = \frac{\sigma_\nu^2}{2\pi}$.

This completes the proof. □

□

Proof of Proposition 7

Proof. [7]

[7]The proof is based on well-established principles in time series analysis, rigorously demonstrated in Brockwell and Davis (1991), Hamilton (2020), and Kilian (2006).

We begin by deriving the $MA(\infty)$ representation for the growth of nominal debt.

By definition, the growth rate of nominal debt is given by:

$$\Delta b_{t+1}^{nom} = b_{t+1} - b_t + \pi_{t+1} \tag{77}$$

Through algebraic manipulation and substitution using the model's reduced-form solutions (63) and (64), this expression can be rewritten as

$$\Delta b_{t+1}^{nom} = (C_d + C_\pi)\nu_{t+1} + \sum_{k=0}^{\infty} \left[C_d(\lambda_2 - 1) + \left(\frac{k}{\lambda_2^{-1}(k\phi_\pi + \sigma + \phi_y)} + C_\pi \right) \lambda_2 \right] \lambda_2^k \cdot \nu_{t-k} \tag{78}$$

where ν_t is the monetary policy shock and $\lambda_2 \in (0, 1)$ is the stable root associated with the fiscal dominance regime.

Let $C_{\Delta b^{nom},k}$ denote the MA coefficients of the process Δb_{t+1}^{nom}.

Stationarity: The process Δb_{t+1}^{nom} is a linear combination of lagged shocks ν_{t-k} with coefficients that decay geometrically due to the presence of λ_2^k. Since $\lambda_2 \in (0, 1)$, these coefficients are absolutely summable. Therefore, Δb_{t+1}^{nom} is covariance-stationary.

This result also follows from the fact that Δb_{t+1}^{nom} is constructed from a linear combination of stationary components—namely b_t, b_{t+1}, and π_{t+1}—each of which admits an MA representation with geometrically decaying coefficients under fiscal dominance (see Section 4.1).

Spectral Density: Since Δb_{t+1}^{nom} admits an $MA(\infty)$ representation and is stationary, we can apply the standard result for the spectral density of a linear process. If ν_t is a zero-mean, covariance-stationary process with absolutely summable autocovariances, the spectral density of Δb_{t+1}^{nom} is given by:

$$S_{\Delta b^{nom}}(\omega) = \left| \sum_{k=0}^{\infty} C_{\Delta b^{nom},k} e^{-i\omega k} \right|^2 \cdot S_\nu(\omega)$$

where $S_\nu(\omega)$ is the spectral density of the innovation process ν_t.

Substituting the closed-form expression for the MA coefficients yields:

$$S_{\Delta b^{nom}}(\omega) = \left| (C_d + C_\pi)e^{i\omega} + \left[C_d(\lambda_2 - 1) + \left(\frac{k}{\lambda_2^{-1}(k\phi_\pi + \sigma + \phi_y)} + C_\pi \right) \lambda_2 \right] \left(\frac{1}{1 - \lambda_2 e^{-i\omega}} \right) \right|^2 \cdot S_\nu(\omega)$$

Since the innovation process ν_t is white noise, its spectral density is constant and equal to $S_\nu(\omega) = \frac{\sigma_\nu^2}{2\pi}$.

This completes the proof. □

□

D.2 Mathematical Derivations

Derivations for Equations (59)**,** (60)**, and** (61) **(Monetary Dominance, Section 5.1.1)** The closed-form solution for the New Keynesian model presented in Section 5.1.1 under Monetary Dominance relies on identifying coefficients that eliminate the system's unstable root in the representation of the equilibrium processes as Moving Average Processes, derived from system (65).

A key step in this derivation is computing the coefficients C_0 subject to the restrictions in equation (56).

This leads to the following system of equations[8] to be solved:

$$\begin{aligned} U_{31}(\lambda_2^{-1})\left[\lambda_2^{-1} + \sigma C_{01} + C_{02}\right] + U_{32}(\lambda_2^{-1})\beta C_{02} = 0 \\ U_{31}(\lambda_3^{-1})\left[\lambda_3^{-1} + \sigma C_{01} + C_{02}\right] + U_{32}(\lambda_3^{-1})\beta C_{02} = 0 \end{aligned} \tag{79}$$

Here, $U_{3i}(z)$ denotes the i-th element of the third row of the matrix U, evaluated at z. The components of U_3[9] are given by:

$$\begin{aligned} U_{31}(z) &= \left(k(1-\beta\phi_\pi) - \beta\phi_y - \frac{\gamma_y - \tau}{\xi_d}\right) z^2 + \beta\left(\beta\phi_y + \frac{\gamma_y - \tau}{\xi_d}\right) z \\ U_{32}(z) &= \left(\sigma(1-\beta\phi_\pi) + \phi_y + \phi_\pi \cdot \frac{\gamma_y - \tau}{\xi_d}\right) z^2 - \left(\sigma(1-\beta\phi_\pi) + \beta\phi_y + \frac{\gamma_y - \tau}{\xi_d}\right) z \\ U_{33}(z) &= (\phi_\pi k + \sigma + \phi_y) z^2 - (\beta\sigma + \beta\phi_y + \sigma + k) z + \beta\sigma \end{aligned}$$

Finally, we derive the closed-form solution for the variables of interest as expressed in equation (54):

$$\begin{pmatrix} y_t \\ \pi_t \\ b_t \end{pmatrix} = \left[L\Gamma(L)\right]^{-1} (L\psi_0 + \Gamma_{-1} C_0)\nu_t$$

where the vector $L\psi_0 + \Gamma_{-1}C_0$ is given by:

$$L\psi_0 + \Gamma_{-1}C_0 = \begin{pmatrix} L + \sigma C_{01} + C_{02} \\ \beta C_{02} \\ L\beta \end{pmatrix}$$

and the inverse of the operator $L\Gamma(L)$ takes the form:

[8] Here we omit the restriction imposed by z=0 because it is non-restrictive.

[9] Note that $U_{33}(z) = (z - \lambda_2^{-1})(z - \lambda_3^{-1})$

$$[L\Gamma(L)]^{-1} = \begin{bmatrix} \frac{L-\beta}{-(k\phi_\pi+\sigma+\phi_y)(L-\lambda_2^{-1})(L-\lambda_3^{-1})} & \frac{1-\phi_\pi L}{-(k\phi_\pi+\sigma+\phi_y)(L-\lambda_2^{-1})(L-\lambda_3^{-1})} & 0 \\ \frac{kL}{-(k\phi_\pi+\sigma+\phi_y)(L-\lambda_2^{-1})(L-\lambda_3^{-1})} & \frac{(\phi_y+\sigma)L-\sigma}{-(k\phi_\pi+\sigma+\phi_y)(L-\lambda_2^{-1})(L-\lambda_3^{-1})} & 0 \\ \frac{\left(\beta\phi_y+\frac{\gamma_y-\tau}{\xi_d}\right)(L-\beta)-k(1-\beta\phi_\pi)L}{(k\phi_\pi+\sigma+\phi_y)(L-(\beta+\gamma_d))(L-\lambda_2^{-1})(L-\lambda_3^{-1})} & \frac{\sigma(1-\beta\phi_\pi)(1-L)+\frac{\gamma_y-\tau}{\xi_d}(1-L\phi_\pi)-\phi_y(L-\beta)}{(k\phi_\pi+\sigma+\phi_y)(L-(\beta+\gamma_d))(L-\lambda_2^{-1})(L-\lambda_3^{-1})} & -\frac{1}{L(L-(\beta+\gamma_d))} \end{bmatrix}$$

The crucial step is to arrange the term $[L\Gamma(L)]^{-1}(L\psi_0 + \Gamma_{-1}C_0)$ in such a way that the coefficients C_0 eliminate the contributions of the unstable roots λ_2^{-1} and λ_3^{-1} from the solution.

Let us begin by analyzing the consumption (output) variable:

$$\begin{aligned} y_t &= \frac{(L-\beta)(L+\sigma C_{01}+C_{02})+(1-\phi_\pi L)\beta C_{02}}{-(k\phi_\pi+\sigma+\phi_y)(L-\lambda_2^{-1})(L-\lambda_3^{-1})} \\ &= \frac{(L-\beta)(L-\lambda_2^{-1})+\beta\frac{C_{02}\lambda_2^{-1}}{U_{31}(\lambda_2^{-1})}(1-\beta\phi_\pi)\left[\frac{\gamma_y-\tau}{\xi_d}(L-\lambda_2^{-1})+(\beta\phi_y+\sigma-(k\phi_\pi+\phi_y+\sigma)\lambda_2^{-1})(L-\lambda_2^{-1})\right]}{-(k\phi_\pi+\sigma+\phi_y)(L-\lambda_2^{-1})(L-\lambda_3^{-1})} \\ &= \frac{(L-\beta)+\beta\frac{C_{02}\lambda_2^{-1}}{U_{31}(\lambda_2^{-1})}(1-\beta\phi_\pi)\left[\frac{\gamma_y-\tau}{\xi_d}+\beta\phi_y+\sigma-(k\phi_\pi+\phi_y+\sigma)\lambda_2^{-1}\right]}{(k\phi_\pi+\sigma+\phi_y)(L-\lambda_3^{-1})} \end{aligned}$$

Let's note that

$$\lambda_2^{-1} = -\lambda_3^{-1} + \frac{k+\sigma+\beta\phi_y+\beta\sigma}{\sigma+\phi_y+k\phi_\pi}$$

Next, we substitute this expression into the term $\frac{C_{02}\lambda_2^{-1}}{U_{31}(\lambda_2^{-1})}$:

$$\begin{aligned} \frac{C_{02}\lambda_2^{-1}}{U_{31}(\lambda_2^{-1})} &= \frac{U_{31}(\lambda_3^{-1})(\lambda_3^{-1}-\lambda_2^{-1})}{U_{32}(\lambda_2^{-1})U_{31}(\lambda_3^{-1})-U_{32}(\lambda_3^{-1})U_{31}(\lambda_3^{-1})} \\ &= \frac{\left[k(1-\beta\phi_\pi)\lambda_3^{-1}-\left(\beta\phi_y+\frac{\gamma_y-\tau}{\xi_d}\right)\lambda_3^{-1}+\beta\left(\beta\phi_y+\frac{\gamma_y-\tau}{\xi_d}\right)\right](\lambda_3^{-1}-\lambda_2^{-1})}{\left[\sigma k(1-\beta\phi_\pi)+\left(\sigma\beta-\sigma+k-\frac{\gamma_y-\tau}{\xi_d}\right)\left(\beta\phi_y+\frac{\gamma_y-\tau}{\xi_d}\right)\right](1-\beta\phi_\pi)(\lambda_2^{-1}-\lambda_3^{-1})} \end{aligned}$$

After simplifying this expression, we obtain the moving-average representation reported in Equation (59):

$$y_t = -\frac{1}{k\phi_\pi+\sigma+\phi_y}\nu_t$$

Next, let us analyze the inflation variable:

$$\pi_t = \frac{kL(L+\sigma C_{01}+C_{02})+(\phi_y L+\sigma L-\sigma)\beta C_{02}}{-(k\phi_\pi+\sigma+\phi_y)(L-\lambda_2^{-1})(L-\lambda_3^{-1})}$$
$$= \frac{kL(L-\lambda_2^{-1})+\beta\frac{C_{02}\lambda_2^{-1}}{U_{31}(\lambda_2^{-1})}\left[\sigma k(1-\beta\phi_\pi)(L-\lambda_2^{-1})+\left(\beta\phi_y+\frac{\gamma_y-\tau}{\xi_d}\right)(\beta\sigma+\beta\phi_y+k-(k\phi_\pi+\phi_y+\sigma)\lambda_2^{-1})(\right.}{-(k\phi_\pi+\sigma+\phi_y)(L-\lambda_2^{-1})(L-\lambda_3^{-1})}$$
$$= \frac{kL+\beta\frac{C_{02}\lambda_2^{-1}}{U_{31}(\lambda_2^{-1})}\left[\sigma k(1-\beta\phi_\pi)+\left(\beta\phi_y+\frac{\gamma_y-\tau}{\xi_d}\right)(\beta\sigma+\beta\phi_y+k-(k\phi_\pi+\phi_y+\sigma)\lambda_2^{-1})\right]}{-(k\phi_\pi+\sigma+\phi_y)(L-\lambda_3^{-1})}$$

Following the same derivation steps as before, we obtain the moving-average representation reported in Equation (60):

$$\pi_t = -\frac{k}{k\phi_\pi+\sigma+\phi_y}\nu_t$$

Finally, let us analyze the real debt variable:

$$b_t = -\frac{\beta}{L-(\beta+\gamma_d)} + [L\Gamma(L)]_{3,1}^{-1}(L+\sigma C_{01}+C_{02}) + [L\Gamma(L)]_{3,2}^{-1}\beta C_{02}$$
$$= -\frac{\beta}{L-(\beta+\gamma_d)} + [L\Gamma(L)]_{3,1}^{-1}(L-\lambda_2^{-1}) + \frac{\beta C_{02}\lambda_2^{-1}}{U_{31}(\lambda_2^{-1})}\left\{\frac{U_{31}(\lambda_2^{-1})}{\lambda_2^{-1}}[L\Gamma(L)]_{3,2}^{-1} - \frac{U_{32}(\lambda_2^{-1})}{\lambda_2^{-1}}[L\Gamma(L)]_{3,1}^{-1}\right\}$$

where $[L\Gamma(L)]_{i,j}^{-1}$ denotes the element in the row i and column j of the matrix $[L\Gamma(L)]^{-1}$.

After some tedious algebraic manipulation, the expression for b_t simplifies to:

$$b_t = \frac{\beta C_{02}\lambda_2^{-1}}{U_{31}(\lambda_2^{-1})}\left\{-k\left(\sigma(1-\beta\phi_\pi)+\beta\phi_y+\frac{\gamma_y-\tau}{\xi_d}\right)+\right.$$
$$\left.+\left(\beta\phi_y+\frac{\gamma_y-\tau}{\xi_d}\right)\left(\sigma(1-\beta)+\frac{\gamma_y-\tau}{\xi_d}\right)\right\}\frac{(1-\beta\phi_\pi)}{(k\phi_\pi+\sigma+\phi_y)(L-(\beta+\gamma_d))(L-\lambda_3^{-1})}+$$
$$+[L\Gamma(L)]_{3,1}^{-1}(L-\lambda_2^{-1}) - \frac{\beta}{L-(\beta+\gamma_d)}$$

Replacing the expression for $\frac{\beta C_{02}\lambda_2^{-1}}{U_{31}(\lambda_2^{-1})}$ allows us to eliminate the unstable root λ_3^{-1}. Then, by applying the geometric series expansion to the denominator, we arrive at the moving average representation reported in equation (61):

$$b_t = \left(\frac{k+\beta\sigma-\frac{\gamma_y-\tau}{\xi_d}}{(k\phi_\pi+\sigma+\phi_y)(\beta+\gamma_d)}\right)\sum_{k=1}^{\infty}\left(\frac{1}{\beta+\gamma_d}\right)^k\cdot\nu_{t-k}$$

Derivations for Equations (62)**,** (63)**, and** (64) **(Fiscal Dominance, Section 5.1.2)** The closed-form solution for the New Keynesian model presented in Section 5.1.2 under Fiscal Dominance relies on identifying coefficients that eliminate the system's unstable root in the representation of the equilibrium processes as Moving Average Processes, derived from system (65).

A key step in this derivation is computing the coefficients C_0 subject to the restrictions in equation (56).

This leads to the following system of equations[10] to be solved:

$$\begin{aligned} &U_{31}(\beta+\gamma_d)\left[\beta+\gamma_d+\sigma C_{01}+C_{02}\right]+U_{32}(\beta+\gamma_d)\beta C_{02}+U_{33}(\beta+\gamma_d)\beta(\beta+\gamma_d)=0 \\ &U_{31}(\lambda_3^{-1})\left[\lambda_3^{-1}+\sigma C_{01}+C_{02}\right]+U_{32}(\lambda_3^{-1})\beta C_{02}=0 \end{aligned} \tag{80}$$

Here, $U_{3i}(z)$ denotes the i-th element of the third row of the matrix U, evaluated at z.

Thus, we can define:

$$\begin{aligned} \frac{\beta C_{02}\lambda_3^{-1}}{U_{31}(\lambda_3^{-1})} &= \frac{U_{31}(\beta+\gamma_d)(\beta+\gamma_d-\lambda_3^{-1})+\beta U_{31}(\beta+\gamma_d)}{\frac{U_{32}(\lambda_3^{-1})}{\lambda_3^{-1}}U_{31}(\beta+\gamma_d)-U_{32}(\beta+\gamma_d)\frac{U_{31}(\lambda_3^{-1})}{\lambda_3^{-1}}} \\ &= -\frac{k(1-\beta\phi_\pi)(\beta+\gamma_d)-\left(\beta\phi_y+\frac{\gamma_y-\tau}{\xi_d}\right)(\beta+\gamma_d)+\beta\left(\beta\phi_y+\frac{\gamma_y-\tau}{\xi_d}\right)+\beta(k\phi_\pi+\sigma+\phi_y)(\beta+\gamma_d-\lambda_2^{-1})}{(1-\beta\phi_\pi)\left\{k\left[\sigma(1-\beta\phi_\pi)+\beta\phi_y+\frac{\gamma_y-\tau}{\xi_d}\right]-\left(\beta\phi_\pi+\frac{\gamma_y-\tau}{\xi_d}\right)\left(\sigma-\sigma\beta-\frac{\gamma_y-\tau}{\xi_d}\right)\right\}} \end{aligned} \tag{81}$$

Finally, we derive the closed-form solution for the variables of interest as expressed in equation (54):

$$\begin{pmatrix} y_t \\ \pi_t \\ b_t \end{pmatrix} = \left[L\Gamma(L)\right]^{-1}\left(L\psi_0+\Gamma_{-1}C_0\right)\nu_t$$

The crucial step is to arrange the term $\left[L\Gamma(L)\right]^{-1}\left(L\psi_0+\Gamma_{-1}C_0\right)$ in such a way that the coefficients C_0 eliminate the contributions of the unstable roots λ_1^{-1} and λ_3^{-1} from the solution.

Let us begin by analyzing the consumption (output) variable:

$$\begin{aligned} y_t &= \frac{(L-\beta)(L+\sigma C_{01}+C_{02})+(1-\phi_\pi L)\beta C_{02}}{-(k\phi_\pi+\sigma+\phi_y)(L-\lambda_2^{-1})(L-\lambda_3^{-1})} \\ &= \frac{(L-\beta)+\beta\frac{C_{02}\lambda_3^{-1}}{U_{31}(\lambda_3^{-1})}(1-\beta\phi_\pi)\left[\frac{\gamma_y-\tau}{\xi_d}+\beta\phi_y+\sigma-(k\phi_\pi+\phi_y+\sigma)\lambda_3^{-1}\right]}{\lambda_2^{-1}(k\phi_\pi+\sigma+\phi_y)(1-\lambda_2 L)} \end{aligned}$$

Applying the geometric series expansion to the denominator, we obtain the moving average

[10] Here we omit the restriction imposed by z=0 because it is non-restrictive.

representation reported in equation (62):

$$y_t = C_y \nu_t + \sum_{k=1}^{\infty} \left(\frac{1}{\lambda_2^{-1}(k\phi_\pi + \sigma + \phi_y)} + C_y \right) \lambda_2^k \cdot \nu_{t-k}$$

where the coefficient C_y is defined as:

$$C_y = \frac{-\beta + \frac{\beta C_{02} \lambda_3^{-1}}{U_1(\lambda_3^{-1})} (1 - \beta\phi_\pi) \left\{ \frac{\gamma_y - \tau}{\xi_d} - (k\phi_\pi + \phi_y + \sigma)\lambda_3^{-1} + \beta\phi_y + \sigma \right\}}{\lambda_2^{-1}(k\phi_\pi + \sigma + \phi_y)}$$

Next, let us analyze the inflation variable:

$$\begin{aligned}
\pi_t &= \frac{kL(L + \sigma C_{01} + C_{02}) + (\phi_y L + \sigma L - \sigma)\beta C_{02}}{-(k\phi_\pi + \sigma + \phi_y)(L - \lambda_2^{-1})} \\
&= \frac{kL + \beta \frac{C_{02}\lambda_3^{-1}}{U_{31}(\lambda_3^{-1})} \left[\sigma k (1 - \beta\phi_\pi) + \left(\beta\phi_y + \frac{\gamma_y - \tau}{\xi_d} \right) (\beta\sigma + \beta\phi_y + k - (k\phi_\pi + \phi_y + \sigma)\lambda_3^{-1}) \right]}{\lambda_2^{-1}(k\phi_\pi + \sigma + \phi_y)(1 - \lambda_2 L)}
\end{aligned}$$

Applying the geometric series expansion to the denominator, we obtain the moving average representation reported in equation (63)

$$\pi_t = C_\pi \nu_t + \sum_{k=1}^{\infty} \left(\frac{k}{\lambda_2^{-1}(k\phi_\pi + \sigma + \phi_y)} + C_\pi \right) \lambda_2^k \cdot \nu_{t-k}$$

where the coefficient C_π is defined as:

$$C_\pi = \frac{\frac{\beta C_{02} \lambda_3^{-1}}{U_1(\lambda_3^{-1})} \left\{ k\sigma(1 - \beta\phi_\pi) + \left(\beta\phi_y + \frac{\gamma_y - \tau}{\xi_d} \right) (\beta\phi_y + \sigma\beta + k - (\sigma + \phi_y + k\phi_\pi)\lambda_3^{-1}) \right\}}{\lambda_2^{-1}(k\phi_\pi + \sigma + \phi_y)}$$

Finally, let us analyze the real debt variable:

$$\begin{aligned}
b_t &= -\frac{\beta}{L - (\beta + \gamma_d)} + [L\Gamma(L)]_{3,1}^{-1} (L + \sigma C_{01} + C_{02}) + [L\Gamma(L)]_{3,2}^{-1} \beta C_{02} \\
&= -\frac{\beta}{L - (\beta + \gamma_d)} + [L\Gamma(L)]_{3,1}^{-1} (L - \lambda_3^{-1}) + \frac{\beta C_{02} \lambda_3^{-1}}{U_{31}(\lambda_3^{-1})} \left\{ \frac{U_{31}(\lambda_3^{-1})}{\lambda_3^{-1}} [L\Gamma(L)]_{3,2}^{-1} - \frac{U_{32}(\lambda_3^{-1})}{\lambda_3^{-1}} [L\Gamma(L)]_{3,1}^{-1} \right\}
\end{aligned}$$

where $[L\Gamma(L)]_{i,j}^{-1}$ denotes the element in the row i and column j of the matrix $[L\Gamma(L)]^{-1}$.

After some tedious algebraic manipulation, the expression for b_t simplifies to:

$$b_t = \frac{\beta C_{02}\lambda_3^{-1}}{U_{31}(\lambda_3^{-1})}\Bigg\{ -k\left(\sigma(1-\beta\phi_\pi)+\beta\phi_y+\frac{\gamma_y-\tau}{\xi_d}\right)+$$
$$+\left(\beta\phi_y+\frac{\gamma_y-\tau}{\xi_d}\right)\left(\sigma(1-\beta)+\frac{\gamma_y-\tau}{\xi_d}\right)\Bigg\}\frac{(1-\beta\phi_\pi)}{(k\phi_\pi+\sigma+\phi_y)(L-(\beta+\gamma_d))(L-\lambda_2^{-1})}+$$
$$+\left[L\Gamma(L)\right]_{3,1}^{-1}(L-\lambda_3^{-1})-\frac{\beta}{L-(\beta+\gamma_d)}$$

Which further simplifies to:

$$b_t = \frac{\beta C_{02}\lambda_3^{-1}}{U_{31}(\lambda_3^{-1})}\frac{\left\{-k\left(\sigma(1-\beta\phi_\pi)+\beta\phi_y+\frac{\gamma_y-\tau}{\xi_d}\right)+\left(\beta\phi_y+\frac{\gamma_y-\tau}{\xi_d}\right)\left(\sigma(1-\beta)+\frac{\gamma_y-\tau}{\xi_d}\right)\right\}(1-\beta\phi_\pi)}{(k\phi_\pi+\sigma+\phi_y)(L-(\beta+\gamma_d))(L-\lambda_2^{-1})}$$
$$+\frac{\left(\beta\phi_y+\frac{\gamma_y-\tau}{\xi_d}\right)(L-\beta)-Lk(1-\beta\phi_\pi)}{(k\phi_\pi+\sigma+\phi_y)(L-(\beta+\gamma_d))(L-\lambda_2^{-1})}-\frac{\beta}{L-(\beta+\gamma_d)}$$

Replacing the expression for $\frac{\beta C_{02}\lambda_3^{-1}}{U_{31}(\lambda_3^{-1})}$ allows us to eliminate the unstable root λ_1^{-1}. Then, by applying the geometric series expansion to the denominator, we arrive at the moving average representation reported in equation (64):

$$b_t = C_d\nu_t + \sum_{k=1}^{\infty} C_d\cdot\lambda_2^k\cdot\nu_{t-k}$$

where the coefficient C_d is defined as:

$$C_d = \frac{\beta(k\phi_\pi+\sigma+\phi_y)+k(1-\beta\phi_\pi)-\beta\phi_y-\frac{\gamma_y-\tau}{\xi_d}}{\lambda_2^{-1}(k\phi_\pi+\sigma+\phi_y)}$$

E Supplementary Figures

E.1 Impact of future expected monetary policy shocks on output under the Monetary Dominance Regime

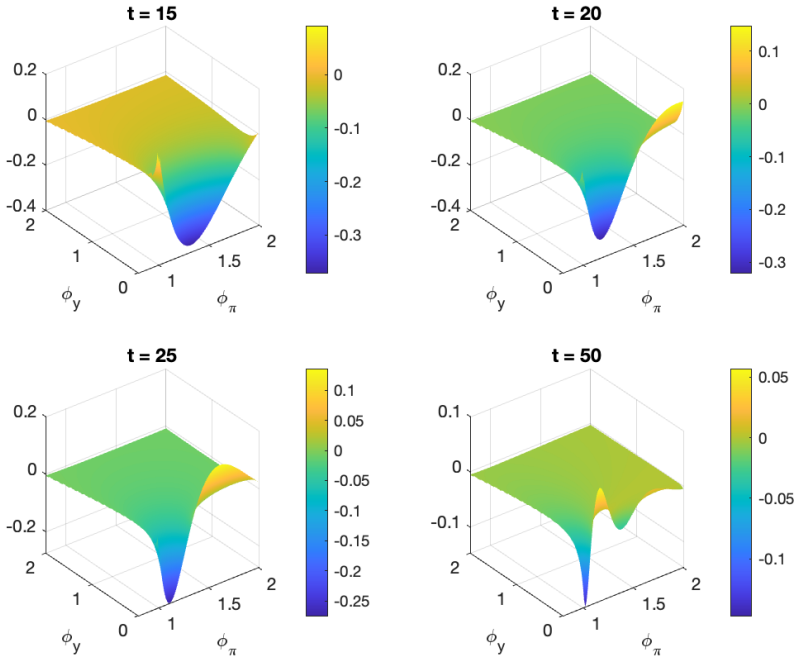


Figure 13: Impact of future monetary policy shocks occurring at t on output.

E.2 Fiscal Dominance Regime

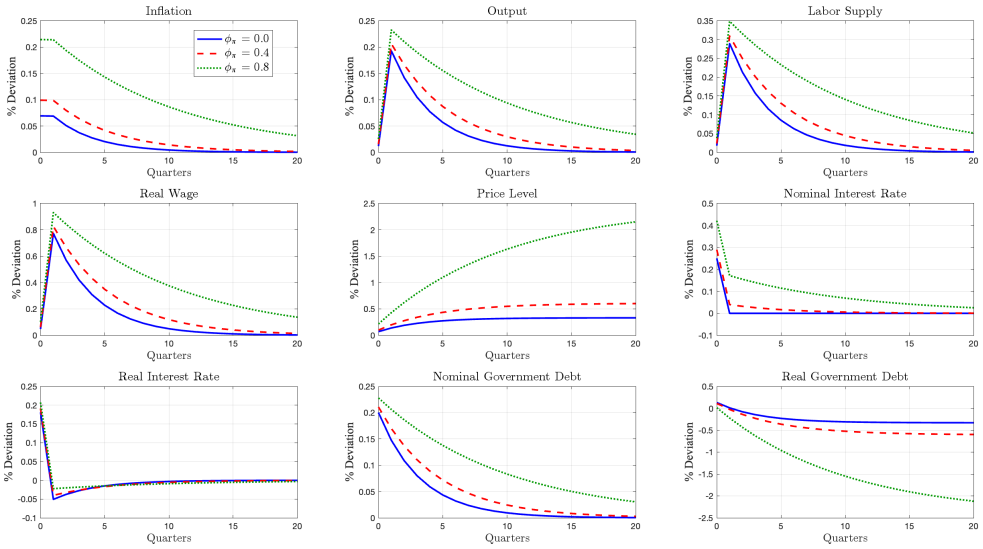


Figure 14: Impulse Responses to a 1% annualized monetary policy shock for different ϕ_π

The figures are generated using the parameter values: $\phi_y = 0$, $\gamma_y = 0$, $\gamma_d = -0.1$, $\tau = 0$, and $\xi_d = 1$.

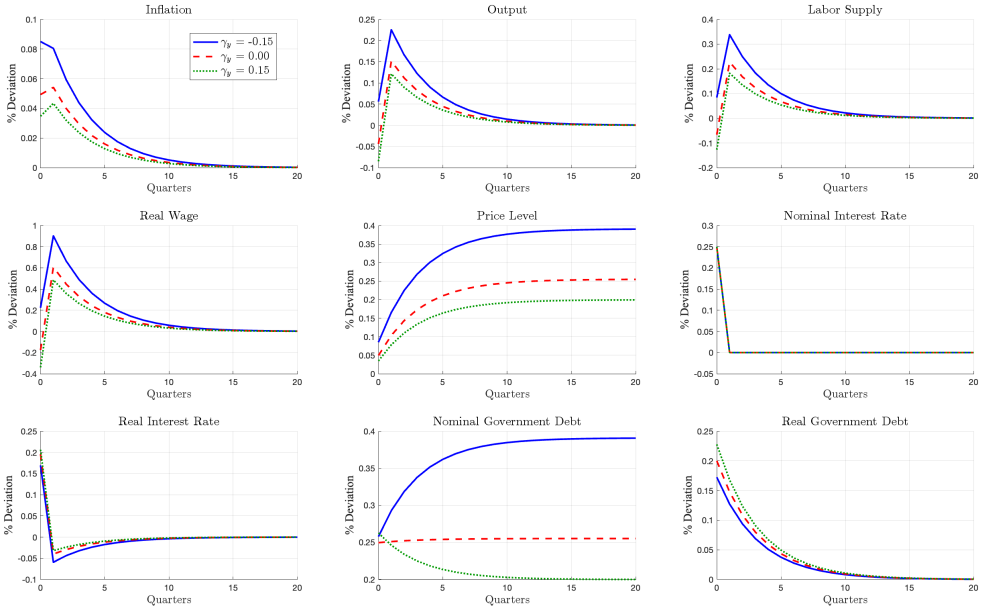


Figure 15: Impulse Responses to a 1% annualized monetary policy shock for different γ_y

The figures are generated using the parameter values: $\phi_\pi = 0$, $\phi_y = 0$, $\gamma_d = 0.0$, $\tau = 0.2$, and $\xi_d = 1$.

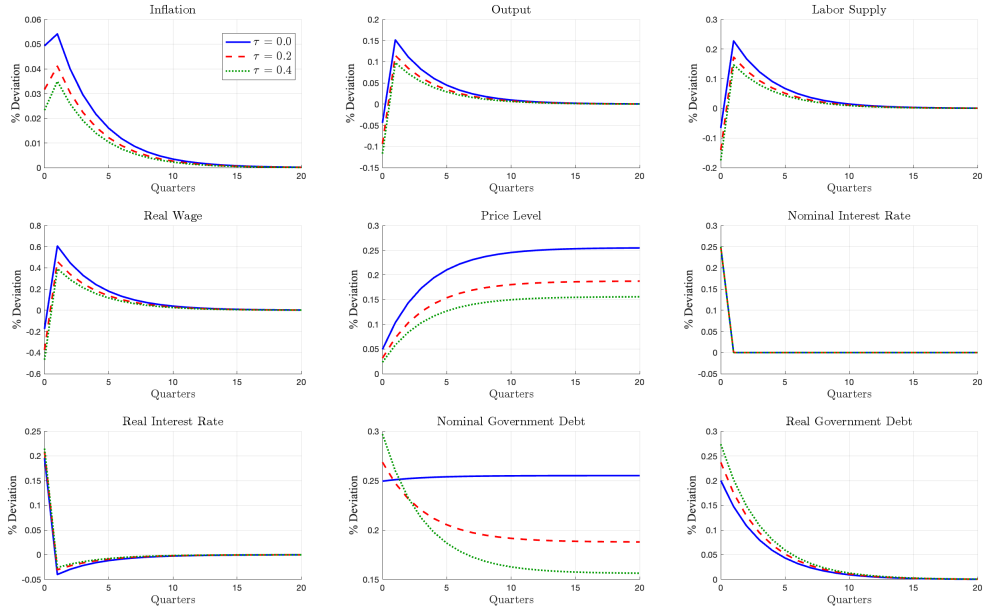


Figure 16: Impulse Responses to a 1% annualized monetary policy shock for different γ_y

The figures are generated using the parameter values: $\phi_\pi = 0$, $\phi_y = 0$, $\gamma_y = 0$, $\gamma_d = 0.0$, and $\xi_d = 1$.

E.3 Endogenous Persistency Parameter λ_2

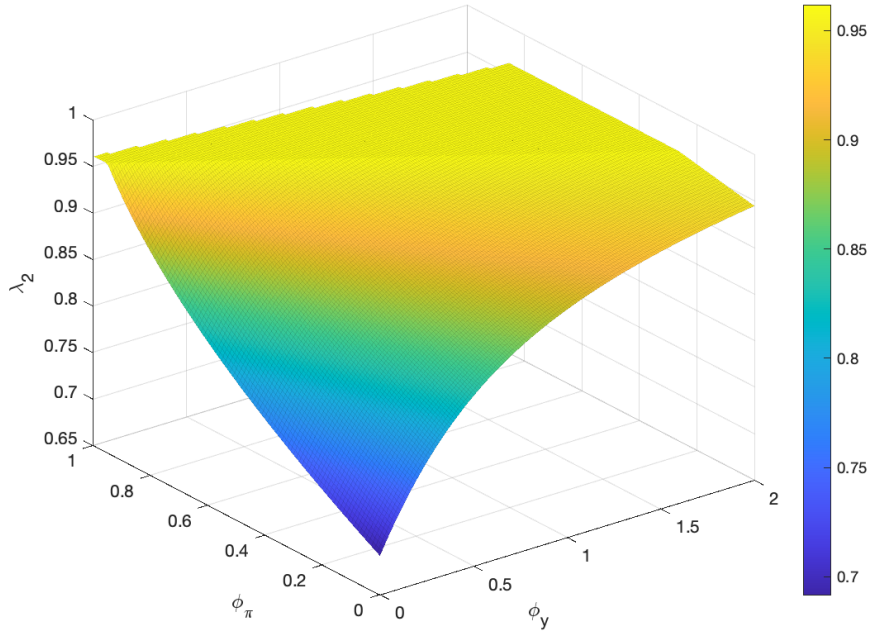


Figure 17: Variation of the eigenvalue λ_2 across the parameter space (ϕ_π, γ_d)