Asset Pricing with Dynamic and Static Investors

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ABSTRACT

I analytically characterize the stock prices resulting from the interaction between dynamic investors, who adjust their asset allocation to news, and static investors, who allocate wealth across asset classes to meet pre-specified targets. Static investors' asset allocation strategy, largely insensitive to earnings news, generates price pressure. Dynamic investors, though unconstrained, do not regard such price pressure as an arbitrage opportunity. They rationally expect it to persist, because static investors consistently adhere to their strategy over time. The equilibrium stock market valuation reflects the level of wealth of static investors, over and above wealth flows, dividends, and discount rates.

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1 Introduction

What determines the level of stock prices? Standard asset pricing theories explain stock prices as the outcome of agents' dynamic optimization—e.g., by households (Merton, 1973), firms (Cochrane, 1991), or intermediaries (He and Krishnamurthy, 2013). In the traditional view, stocks are priced at the present discounted value of their expected future dividends by agents engaging in dynamic asset allocation strategies who control their policies in response to news. Gabaix and Koijen (2023) offer a new perspective, by observing that investors adopting static asset allocation strategies keep their capital allocation across asset classes close to target weights that exhibit little to no variation in response to news. In this more recent view, wealth flows to the stock market lead to price pressure because too few investors are able to engage in dynamic optimization to accommodate the resulting flow-induced demand for equity assets.

Financial markets include investors who adopt dynamic asset allocation strategies as well as investors who adhere to static asset allocation strategies.[1] This coexistence suggests that, while both perspectives discussed above capture important forces, neither of the two is complete in isolation. This paper integrates them by developing and solving in closed form an Intertemporal Capital Asset Pricing Model (ICAPM) with heterogeneous agents. *Dynamic Investors*, such as sophisticated households and hedge funds, optimize expected utility from consumption by adjusting their asset allocation upon the arrival of news, as in Merton (1973). *Static Investors*, such as balanced mutual funds and exchange-traded funds (ETFs), are professionally managed funds that adhere to a pre-specified policy of capital allocation across asset classes, as in Gabaix and Koijen (2023). Static investors receive stochastic wealth flows from inertial households (Brunnermeier and Nagel, 2008), who buy fund shares and hold them over diverse periods, over which time their capital is invested according to static investors' asset allocation strategy.

The central finding of this paper is that the equilibrium price of the equity asset class exceeds the risk-adjusted present discounted value of its dividend stream by a term that is proportional to the capital managed by static investors. The idea runs as follows. Static investors' asset allocation is strongly predictable: they can be expected not to reduce too much their target equity exposure in bad times, nor to increase it in good times. This intertemporal consistency leads dynamic investors to rationally anticipate the demand of static investors for equity assets and impound it into equity prices. Thus, the practice of static investors to invest wealth in target proportions across asset classes impacts security valuations, even in frictionless capital markets with unconstrained dynamic investors engaging in intertemporal optimization. I refer to this asset pricing mechanism as the *Asset Classification Effect*.

[1] Dynamic asset allocation strategies are observed among hedge funds (Fung and Hsieh, 1997), mutual funds that engage in market timing (Kacperczyk et al., 2014), and households that are wealthy (Giglio et al., 2021) and have higher cognitive ability (Grinblatt et al., 2012). Static asset allocation strategies are characteristic of, though not universal among, institutional investors such as pension funds (Andonov and Rauh, 2022) and balanced mutual funds (Dahlquist and Ibert, 2024), as frequently formalized in their prospectuses. They are also widely recommended by advisors to retail investors (Canner et al., 1997).

The asset classification effect can explain (at least partly) the evidence on the aggregate effect of wealth flows into mutual funds on stock prices first documented by Warther (1995) and Edelen and Warner (2001) and corroborated by a growing subsequent literature. At the same time, it points to wealth *flows* as part of a broader quantity that impacts asset prices: the *level* of wealth invested under static asset allocation strategies, which reflects wealth flows, capital revaluations, and dividend distributions. A quantitative evaluation of the model suggests that this distinction is important, because the level of wealth managed by funds with static equity weights exhibits an upward trend, hovering around $7 trillion in 2025. The net wealth flows they receive, computed following Lou (2012), are stationary and much smaller in magnitude. In other words, the model suggests that the stock market features persistent price pressure driven by the level of wealth invested in stocks under static asset allocation strategies, which is significantly larger than the wealth flows emphasized by the previous literature.

Strikingly, the large and persistent pressure exerted by static investors on equilibrium prices is compatible with the presence of unconstrained dynamic investors in asset markets free of borrowing limits and short-sale restrictions. The key ingredient that sets the model apart from standard theories with perfect capital markets is the practice of static investors to invest in target proportions across asset classes, allowing at most limited deviations from target allocations in response to news about dividends. This simple feature leads dynamic investors to revise their expectations of stock prices, given their expectations of future dividends. A rational asset price bubble emerges, defined as a difference between the equilibrium price of the equity asset class and the present discounted value of its dividends (Tirole, 1985).[2] The bubble component of stock prices is uniquely pinned down by static investors' assets under management and target equity exposure and can be measured using portfolio holdings, and is not an extraneous sunspot that has nothing to do with the economy. For intuition, consider a trust that is required by its charter to invest a constant fraction of its wealth in a tree, regardless of the fruit it bears. Other investors may sell into that demand at a higher price than the present value of its fruits, but they cannot rationally expect the deviation to unwind, because the trust keeps investing in the tree persistently over time. Notably, this price pressure is compatible with the workings of arbitrage, rather than arising from limits to arbitrage as in the inelastic markets literature.

Distinguishing investors by their strategies offers a new perspective on how the stock market value responds to capital flows, a quantity labeled the stock market macro-elasticity in Gabaix and Koijen (2023). Building on their paper, a growing literature estimates how the market responds when an investor sells $1 worth of bonds and buys $1 worth of stocks. However, this approach treats all flows equally, without considering them as part of a broader strategy. The model presented in this paper suggests that a flow into equities from dynamic investors that is motivated by news should not be regarded as price pressure. Otherwise, one might incorrectly

[2]Hence, the equilibrium violates the transversality condition that the discounted asset price converge to zero as the horizon tends to infinity.

interpret the stock price response to a wealth flow by dynamic investors reacting to earnings news as evidence of inelastic markets, rather than of efficient markets. By contrast, a flow into equities from static investors that shapes rational expectations of future asset demand should indeed be regarded as price pressure. Hence, the model suggests that before assessing how the stock market responds to a wealth flow, one should consider the *strategy* behind that investment.

After examining the equilibrium stock market valuation, three interrelated questions remain. First, how should the portfolio choice of dynamic investors account for the market presence of static investors? Addressing this question, I find that the presence of static investors induces a tilt in the portfolio choice of dynamic investors. Beyond the myopic component that reflects the contemporaneous trade-off between risk and return, the optimal portfolio of dynamic investors also contains an intertemporal hedging component, proportional to the number of equity shares held by static investors. Intuitively, when static investors become larger owners of the stock market, stock prices become more driven by their wealth rather than by dividends. This makes stocks less attractive as vehicles to finance future consumption, because high ownership from static investors tends to coincide with lower expected returns and greater exposure to price volatility. Thus, as the number of equity shares held by static investors rises, dynamic investors tilt their portfolios away from equity assets in order to better smooth consumption over time.

Second, when in the time series does the capital invested under static asset allocation strategies exert a larger price pressure? The analysis reveals that static investors' capital becomes relatively more important for equilibrium stock prices when risk-adjusted expected returns on stocks are low, at which point dynamic investors shift their portfolio toward the bond market. With dynamic investors taking flight to the safety of bonds, stock prices increasingly reflect static investors' equity exposure rather than their underlying dividend streams. This is also the effect of unfavorable news about earnings, which lead dynamic investors to reduce their stock holdings. Generally speaking, static investors' wealth is more important for equilibrium stock prices in bad times than it is in good times.

Third, which security prices in the cross-section are most influenced by the capital invested under static asset allocation strategies? To study the cross-section of stock prices, it is necessary to examine not only static investors' asset allocation, but also their stock selection. To parsimoniously extend the model in this direction, static investors are divided into two groups: *Index Trackers* and *Stock Pickers*. Index trackers following static asset allocation strategies (such as passive balanced mutual funds and ETFs) continuously track the performance of an index composed of a subset of stocks. Stock pickers following static asset allocation strategies (such as active balanced mutual funds and ETFs) select optimally the mix of stocks in their portfolio at each point in time. Despite optimal stock selection, even stock pickers end up exerting aggregate price pressure on the equity asset class, by maintaining their exposure to the stock market close to a static asset allocation target.

In the cross section, the equilibrium price of each stock exceeds the risk-adjusted present discounted value of its dividend stream by a term that is proportional to the stock's weight in the portfolio of investors following static asset allocation strategies, whether stock pickers or index trackers. The *index inclusion* of a stock influences its price, as it makes that stock part of the asset demand of index trackers. Accordingly, the price pressure on index stocks relative to non-index ones rises with the level of wealth managed by index trackers, as more capital is deployed to track index stocks. The *asset classification* of a stock also influences its price, as it makes that stock part of the asset demand of static investors with target exposure to the equity asset class. Accordingly, the aggregate price pressure on the equity asset class rises with the level of wealth managed by static investors, as more capital is allocated to equity assets in fixed proportions. The economic intuition is straightforward: just as cross-sectional constraints imposed by index-tracking policies affect the prices of stocks included in an index, constraints imposed by static asset allocation strategies affect the prices of the securities classified within target asset classes. This analysis sheds some light on the relationship between the price pressure at the individual stock level (Harris and Gurel, 1986; Shleifer, 1986) and the aggregate price pressure at the asset class level (Da et al., 2018; Hartzmark and Solomon, 2025), a relationship that appears to have been neglected in the literature thus far.

I examine the robustness of my theory to various considerations without qualitative changes to my findings. For instance, I consider correlation between wealth flows and earnings news, flexible asset allocation targets (Buffa et al., 2022), alternative preference specifications for dynamic investors, and survival of agents (Kogan et al., 2006). Taken together, these exercises suggest that the findings of this paper can be attributed to the interactions between dynamic and static asset allocation strategies, rather than to specific modeling assumptions. The remainder of this paper is organized as follows. Section 2 surveys the related literature. Section 3 presents an intertemporal asset pricing model with dynamic and static investors. Section 4 solves for the equilibrium in the cross-section of stocks. Section 5 examines the empirical relevance of the theory. Section 6 offers concluding remarks.

2 Literature Review

To the best of my knowledge, this paper is the first to show that the level of wealth invested under static asset allocation strategies leads to a rational bubble in the stock market valuation, despite the presence of unconstrained investors engaging in dynamic optimization.

The effect of demand forces on the aggregate value of the stock market was first explored by Warther (1995) and Edelen and Warner (2001). Recent studies in this field include Da et al. (2018), Parker et al. (2023), and Hartzmark and Solomon (2025). This paper proposes a theory that is consistent with the evidence from this literature. Interest in the effect of demand forces on the aggregate value of the stock market has been rekindled by Gabaix and Koijen (2023),

who develop a Lucas tree model in which a representative household sector delegates its investments to professional funds facing asset allocation portfolio constraints. In that setup, wealth flows to the stock market give rise to aggregate stock price fluctuations, due to the absence of investors adopting dynamic optimization strategies ready to accommodate the resulting asset demand. This paper departs from this mechanism by showing that the effect of demand forces on the aggregate value of the stock market is consistent with the presence of investors adopting dynamic asset allocation strategies, such as hedge funds and mutual funds engaging in market timing. The heterogeneous-agent perspective presented in this paper also addresses how the presence of investors adopting static asset allocation strategies shapes the portfolio choice implied by dynamic asset allocation strategies.

By recognizing the importance of investors' heterogeneous strategies for asset prices, this paper ties in well with the literature on the macrostructure of financial markets, surveyed by Haddad and Muir (2025). Notably, Koijen and Yogo (2019) helped draw considerable attention to investors trading policies by estimating asset demand systems. Although the demand system approach to asset pricing has yielded fundamental insights, it has also raised points of debate. For instance, Fuchs et al. (2023) caution against interpreting estimated demand elasticities as structural parameters, van Binsbergen et al. (2025) point out that the path of future trading prices represents an omitted asset characteristic, and He et al. (2025) emphasize that demand shocks might violate the exclusion restriction in a dynamic economy. By developing an ICAPM where investors follow heterogeneous asset allocation strategies, which specify their plan of action at each point in time, this paper incorporates key insights from the literature on asset demand systems while remaining robust to the aforementioned considerations.

Related literature on asset pricing theory in intertemporal settings and through models with heterogeneous agents includes Veronesi (1999), Basak and Chabakauri (2010), Chabakauri (2013), and Kimball et al. (2020). This literature provides tractable characterizations of intertemporal equilibria primarily in models where all agents engage in dynamic optimization.[3] In Basak and Cuoco (1998), some agents do not engage in stock market participation. In this paper, some agents forego dynamic portfolio choice, rather than stock market participation. These agents access equity exposure by delegating wealth to professional investment funds. Relatedly, Basak and Pavlova (2013) examine how institutional investors affect asset prices. Their analysis centers on benchmarking within the equity market; mine examines static allocation across asset classes and its implications for the aggregate stock market valuation.

Notwithstanding the widespread adoption of the static asset allocation strategy, theoretical exploration of its impact on asset prices remains limited. Exceptions include Chien et al. (2011)

[3] A related strand of this literature studies the interaction of newswatchers and trend followers in financial markets (Hong and Stein, 1999; Barberis and Shleifer, 2003; Barberis et al., 2015). Although dynamic investors resemble newswatchers, trend followers form beliefs about future price changes by extrapolating past price changes. In sharp contrast, static investors maintain a pre-specified exposure to the stock market even during time periods when bonds represent more efficient investment opportunities, generating entirely different market outcomes.

and Chien et al. (2012), who show that heterogeneous asset allocation strategies help to match asset prices and the distribution of household wealth. However, agents in their frameworks trade state-contingent finite-maturity bonds, which rule out the presence of a rational asset price bubble that is instead central to this paper. Duarte et al. (2025) examine the dynamics of the aggregate stock market elasticity within a Lucas economy with non-storable consumption and endogenous risk-free rate populated by active and passive investors. Their findings emphasize that unconstrained arbitrage eliminates price pressure, which arises only by introducing risk misallocation and leverage constraints. However, their analysis relies on a local perturbation around a stationary equilibrium, which abstracts from non-stationary rational bubbles. Compared with the above papers, this paper is the first to show analytically that, above and beyond the market impact of flows, the level of wealth managed by static investors is priced through a rational bubble compatible with no-arbitrage, even absent leverage or short-selling constraints. Moreover, this paper provides a unified framework to examine price pressure from static asset allocation strategies both at the aggregate level and in the cross-section of stocks in frictionless capital markets.

3 The Model

Time is continuous and goes from zero to infinity. The investors have rational expectations and identical information, and are grouped into two categories. Dynamic investors optimally adjust their asset allocation in response to news. Static investors maintain a constant asset allocation, without regard to the revelation of information. The environment is otherwise standard.

3.1 Assets

The economy features a risk-free bond and a set of stocks that distribute stochastic dividends. The bond is elastically supplied and yields the constant risk-free real rate of return r. The set of stocks forms the equity asset class. The ex-dividend price of the equity asset class, P_t, is the sum of the ex-dividend prices of the individual stocks. Its drift and diffusion are denoted by μ_t and σ_t, respectively. The aggregate earnings of the listed firms, denoted by E_t and expressed in real terms, follow a stochastic differential equation with drift m and diffusion ω. That is,

$$dE_t = mdt + \omega dB_t, \tag{1}$$

where B_t is a Brownian motion that generates the filtration $\{\mathscr{F}_t\}$. Equation (1) is motivated by the linear growth of real earnings. The aggregate dividend rate per share, net of rights issues, is proportional to the aggregate earnings, $D_t = aE_t$, with the payout ratio normalized to $a = 1$.[4]

[4]Rights issues, which reduce the value of the shares outstanding without affecting their supply to the public, can account for negative dividends.

3.2 Dynamic Investors

Dynamic investors optimize the utility function $U(c_t) = -\exp(-\gamma c_t)$, where c_t denotes consumption, and δ and γ represent the time-preference and risk-aversion parameters, respectively. Their portfolio choice changes continuously as news reach the market, in the style of Merton (1973).[5] Dynamic investors control their consumption and investment policies to maximize their derived utility from consumption over an infinite time horizon, subject to their budget constraint and transversality condition. They are *not* subject to any leverage and short-selling constraints. The wealth of dynamic investors is denoted by W_t, and their derived utility from consumption

$$J(W_t, V_t, t) \equiv \max_{\{c_t, X_t\}} \quad \mathbb{E}_t\left[\int_t^\infty e^{-\delta(s-t)} U(c_s)\, ds\right], \tag{2}$$

$$\text{s.t.} \quad dW_t = (rW_t - c_t)dt + X_t dY_t, \qquad \lim_{h\to\infty} \mathbb{E}_t\Big[J(W_{t+h}, V_{t+h}, t+h)\Big] = 0. \tag{3}$$

In the above, $dY_t = (D_t - rP_t)dt + dP_t$ is the return of a share of the equity asset class financed at the risk-free rate, and X_t is the number of such shares held by dynamic investors. This optimization program is similar to that in Veronesi (1999) and others, with the key distinction that in this paper dynamic investors interact with static investors, described below.

3.3 Static Investors

Static investors are professional investment funds that allocate their wealth in fixed proportions across asset classes. They continuously rebalance their portfolios to maintain target exposures, without regard to current or prospective market prices. For expositional simplicity, these funds are aggregated into a representative fund whose wealth, V_t, is allocated in a constant proportion, θ, to the stock market.[6] It is assumed that $\theta \in [0, 1)$, since pure equity funds such as equity ETFs invest primarily in stocks, while balanced mutual funds only allocate a fraction of their wealth to equities. By adhering to static asset allocation targets, static investors present a downward sloping demand for the equity asset class, reminiscent of the downward sloping demand for individual securities in Harris and Gurel (1986) and Shleifer (1986).

Naturally, static investors are investment funds acting on behalf of underlying retail investors. These retail investors make consumption and portfolio choices, as discussed in the micro-foundation of the economy provided in Appendix A. Although each retail investor plans to eventually exit the fund and consume their wealth, the wealth under management of static investors is continuously replenished by new retail investors. Thus, static investors operate on

[5] Chien et al. (2012) examine the asset pricing implications of dynamic but intermittent portfolio choice.

[6] For instance, two funds with wealth $V_x = \$100$ and $V_y = \$200$ and equity targets $\theta_x = 0.5$ and $\theta_y = 0.75$ aggregate into a representative fund with wealth $V = \$300$ that allocates to equities the wealth-weighted average of the two targets, $\theta = 0.67$. The subsequent wealth flows into each of the funds are scaled equivalently.

an infinite horizon. For equilibrium analysis, it suffices to track the asset holdings and wealth dynamics of static investors. Static investors hold Q_t shares of the equity asset class, where

$$Q_t = \theta V_t / P_t. \tag{4}$$

Taking the perspective of assets under management, and assuming for simplicity that static investors reinvest dividends, static investors' wealth dynamics are driven by both price revaluations and inflows of capital

$$dV_t = V_t\big[(1-\theta)r dt + \theta(dP_t + D_t dt)/P_t\big] + \pi dF_t \tag{5}$$

where π is a loading parameter and F_t is a Brownian motion adapted to $\{\mathcal{F}_t\}$ that describes wealth flows to the static asset allocation fund net of share redemptions.[7] Those wealth flows are assumed uncorrelated with earnings news, an assumption relaxed in Internet Appendix A.

3.4 Market Clearing

The shares of the equity asset class are in fixed supply, $\overline{S}$, normalized to one without loss of generality.[8] The market clearing condition is

$$X_t(P_t, D_t, V_t) + Q_t(P_t, V_t) = \overline{S}. \tag{6}$$

3.5 Equilibrium

The equilibrium consists of a price, P_t, of the equity asset class such that the supply of stock shares, $\overline{S}$, is equal to their demand, $X_t + Q_t$. Dynamic investors maximize their indirect utility from consumption, given their wealth, corporate earnings, and market prices. Static investors receive stochastic wealth flows and allocate a fixed share θ of their wealth to the equity asset class. The following three equalities characterize the equilibrium.

$$X_t + Q_t = \overline{S}, \qquad 0 = \max_{\{c_t, X_t\}} U(c_t) + \frac{\mathbb{E}_t[dJ(W_t, V_t, t)]}{dt}, \qquad Q_t = \theta V_t / P_t. \tag{7}$$

The first equality imposes market clearing. The second solves the Hamilton-Jacobi-Bellman (HJB) equation for dynamic investors. The third is the static investors' asset allocation strategy.

[7] Following extremely negative wealth outflows, V_t could potentially turn negative, an event ruled out by assumption. Its dynamics near this barrier can be disciplined by a term ηdL, where η is the speed of reflection and L the local time of V_t at zero, without changing the economics.

[8] The economic mechanism emphasized in this paper sets it apart from standard models with stochastic supply. There, unlike in this model, assets typically become effectively worthless when, even temporarily, it is inefficient to hold them.

3.6 Equilibrium without Static Investors

For illustrative purposes, I begin by analyzing the equilibrium price of the equity asset class in the absence of static investors. Throughout the paper, $\mathbb{E}_t^*$ denotes conditional expectations taken with respect to the probability measure adjusted for the dynamic investors' preferences toward risk. Optimal policies are instead denoted with a star superscript; e.g., $X_t^\star$.

Lemma 1. Equilibrium without Static Investors.

$$P_t = \underbrace{\mathbb{E}_t^* \left[\int_t^\infty e^{-r(s-t)} D_s \, ds \right]}_{Fundamentals}.$$

Proof. Special case of Proposition 1. See also Veronesi (1999).

Substituting the process and dividend into dynamic investors' HJB yields the representation

$$P_t = \text{PDV}_t(D_t). \tag{8}$$

$\text{PDV}_t(D_t) = p_g + p_D D_t + p_m m$ denotes the present discounted value of the expected aggregate dividend stream net of the risk adjustment required by dynamic investors for the exposure of the equity asset class to earnings risk. Moreover, $p_g = -\frac{\gamma\omega^2}{r^2}$, $p_D = \frac{1}{r}$, and $p_m = \frac{1}{r^2}$.[9] Lemma 1 describes a standard dividend discount valuation model. This result can alternatively be obtained by iterating forward the no-arbitrage restriction that $P_t = e^{-rdt}\mathbb{E}_t^*[P_{t+dt} + D_t dt]$ and imposing the following transversality condition:

$$\lim_{h\to\infty} e^{-rh}\mathbb{E}_t^*[P_{t+h}] = 0. \tag{9}$$

Absent the transversality condition in Equation (9), the stock price would exceed the discounted dividend stream, leading to a rational bubble. A rational bubble is defined here as a difference between an asset's price and the risk-adjusted present value of its expected cash flows that does not constitute an arbitrage opportunity. This condition could be looked at another way. A rational bubble, if present, impacts both the current level of the asset price and its risk-adjusted expected future value equally. This definition follows the literature on rational expectations in present-value models (Tirole, 1985; Froot and Obstfeld, 1991; Santos and Woodford, 1997). It does not require that the asset's cash flows be replicable, but that it bears the same yield as alternative assets (Stiglitz, 1990). Indeed, when the asset's risk-adjusted expected return equals that on alternative assets, the asset price deviation from its fundamental value is merited.

[9] Throughout the paper, it is assumed that the risk correction is not too large, so that equilibrium prices remain strictly positive after the additive risk adjustment. An analogous restriction is maintained in Veronesi (1999). The opposite case corresponds to dynamic investors being sufficiently risk averse to attach a negative certainty-equivalent value to equity payoffs making them willing to pay to avoid their risk.

Tirole (1982) argues that rational asset price bubbles are not present when agents optimize their actions dynamically. Lemma 1 is consistent with this view, because all of the agents are dynamic investors, the asset price equals the discounted dividend stream, and the transversality condition is respected. Namely,

$$\lim_{h\to\infty} \mathbb{E}_t^* e^{-rh}[P_{t+h}] = \lim_{h\to\infty} e^{-rh}\mathbb{E}_t^*[\text{PDV}_{t+h}(D_{t+h})] = 0. \tag{10}$$

3.7 Equilibrium with Dynamic and Static Investors

Static investors induce a stark departure from the benchmark equilibrium in Section 3.6. This is because static investors *consistently* depart from dynamic optimization of their asset allocation. Proposition 1 below examines the equilibrium with dynamic and static investors, demonstrating that static investors lead to an asset price deviation from the present value of its dividend stream.

Proposition 1. Equilibrium with Dynamic and Static Investors.

$$P_t = \underbrace{\text{PDV}_t(D_t)}_{Fundamentals} + \underbrace{p_f + \theta V_t}_{Asset\ Classification\ Effect}.$$

Where $p_f = -\gamma\theta\pi^2$ is the risk adjustment induced by stochastic wealth flows to equity assets.

Proof. Appendix B presents the proof of this statement and the associated optimal portfolio and consumption rules that maximize the HJB equation for dynamic investors. To complement the equilibrium analysis, Appendix C derives the corresponding Stochastic Discount Factor (SDF).

Proposition 1 shows that static investors impact the aggregate valuation of the stock market through an *Asset Classification Effect*. This closed-form representation captures the two distinct effects that static investors exert on stock prices: (i) the flow risk adjustment, p_f, induced by the exposure of stock prices to stochastic wealth flows to static investors; (ii) the equity exposure of static investors, θV_t, which leads to upward pressure on stock prices. Importantly, the asset classification effect impacts the equilibrium price of the equity asset class, above and beyond the effect of the dividends, D_t, the risk-free rate, r, and the compensation for earnings risk required by dynamic investors, p_g. A detailed economic interpretation of Proposition 1 follows after the next remarks, which characterize central properties of the equilibrium.

Remark 1. The Asset Classification Effect is a Discounted Risk-adjusted Martingale. In financial markets free of arbitrage opportunities, the risk-adjusted expected return on stocks coincides with the rate of return on the risk-free asset. Thus, under the risk-adjusted measure, the conditional expectation of static investors' wealth dynamics in Equation (5) is

$$\mathbb{E}_t^*[dV_t] = rV_t dt + \mathbb{E}_t^*[\pi dF_t]. \tag{11}$$

Moreover, static investors' equity exposure, θ, is known in advance and constant over time. These observations can be used to show that, for every positive time horizon h,

$$\theta V_t + p_f = e^{-rh}\mathbb{E}_t^*[\theta V_{t+h} + p_f]. \tag{12}$$

Thus, the asset classification effect is a discounted risk-adjusted martingale that impacts both the current level of the asset price and its risk-adjusted expected future value equally.

Proof. See Appendix D.

Remark 1 shows that the asset classification effect cannot be turned into arbitrage profits. The economic interpretation of this result is that short-selling stocks to accommodate static investors' demand for equity assets yields no risk-adjusted expected excess returns. Thus, any impact of static investors' equity exposure on the level of the equity asset price cannot be arbitraged away. In other words, unconstrained dynamic investors cannot regard the prospect to force the asset price back to its fundamentals as an arbitrage opportunity.[10] The following result shows that the price pressure of static investors does not die out over time.

Remark 2. Transversality Condition. The equilibrium valuation of the stock market in Proposition 1 violates the transversality condition.

Proof.

$$\lim_{h\to\infty} e^{-rh}\mathbb{E}_t^*[P_{t+h}] = \lim_{h\to\infty} e^{-rh}\mathbb{E}_t^*[\text{PDV}_{t+h}(D_{t+h}) + p_f + \theta V_{t+h}] = p_f + \theta V_t. \tag{13}$$

Here, the first equality uses Proposition 1 and the second equality uses Equations (10) and (12).

Remark 2 shows that the asset classification effect survives discounting. Investors do not rationally expect the market price to eventually converge to the fundamental. This is due to the infinite maturity of equity assets and the perpetual nature of the expected demand pressure. Dynamic investors are not subject to any portfolio constraint. Yet, maintaining a short position indefinitely without ever realizing any gains is not optimal. Static investors' equity exposure is thus fully consistent with the workings of arbitrage by dynamic investors.[11]

Remarks 1 and 2 demonstrate that the presence of static investors leads to a rational bubble. Having established this, one may wonder whether this bubble is uniquely determined. As is usual in the stochastic control literature, the equilibrium price is obtained by ansatz and verification, which establishes the validity of the solution but does not guarantee its uniqueness.

[10] This argument continues to work when dynamic investors are risk-neutral. In this case, expected stock returns coincide with the risk-free rate. The price pressure from static investors drifts at the same rate and does not offer any arbitrage opportunities.

[11] Somewhat relatedly, Hugonnier (2012) notes that portfolio constraints can generate a rational bubble, but their finite-horizon setup is markedly different. There, a bubble can arise as prices may diverge further from fundamentals before terminal convergence, exhausting investors' capital. By contrast, in this setting, the bubble arises in infinite horizon from the predictability of static investors' equity exposure.

Fortunately, the rational bubble resulting from the equity exposure of static investors has a clear interpretation as an objective feature of the economy. This sets it apart from an extraneous sunspot that has nothing to do with the economy, as those questioned by Woodford (1987). In Proposition 1, the rational bubble term is uniquely determined by static investors' assets under management and target equity exposure. As such, it can be *measured* using portfolio holdings. Accordingly, this bubble does not burst due to an extraneous sunspot, but rather if investors adopting static asset allocation strategies (such as balanced mutual funds and ETFs) drop their equity allocation targets or are forced into liquidation by fund outflows.

Corollary 1. Stock Prices, Market Fundamentals, and the Asset Classification Effect.

$$P_t = \underbrace{\mathbb{E}_t^* \left[\int_t^\infty e^{-r(s-t)} D_s \, ds \right]}_{Fundamentals} + \underbrace{\lim_{h \to \infty} e^{-rh} \mathbb{E}_t^* \left[\theta V_{t+h} \right]}_{Asset\ Classification\ Effect} \tag{14}$$

Proof. The statement follows from Proposition 1 and Remarks 1 and 2.

Corollary 1 offers a transparent interpretation of the equilibrium with dynamic and static investors. It shows that the aggregate price of the equity asset class reflects its *Fundamentals*, the risk-adjusted present value of its expected cash flows given the discount rates required by dynamic investors—the marginal agents, whose optimality conditions determine asset prices. Above and beyond its fundamentals, the aggregate price of the equity asset class reflects an *Asset Classification Effect*—a rational bubble that reflects an observable characteristic of the economy, the continuation value of the equity exposure of static investors. The interpretation of this effect is that the aggregate value of equity assets rationally rises with the level of wealth invested in that asset class under static asset allocation strategies, which shape expectations of future asset prices.

As an illustration of the asset classification effect, consider the following thought experiment. Suppose that bonds are temporarily expected to outperform stocks. Dynamic investors then swiftly reallocate their entire wealth to the bond market. Absent static investors, stocks would temporarily be reduced to worthless pieces of paper. This constitutes a serious limitation of the dividend discount model in Lemma 1—bonds *are* sometimes expected to outperform stocks. For instance, Gómez-Cram (2022) documents that stock returns are predictably lower than the risk-free rate for several months after the onset of recessions. How can stocks have positive prices, when the risk-free asset is expected to outperform them and investors are risk-averse? The asset classification effect offers a way forward: above and beyond the discounted dividend stream, stocks price the abundance of cash invested under static asset allocation strategies.

Is the presence of an asset classification effect an artifact of restrictive assumptions? First, investment mandates often allow for limited flexibility (Buffa et al., 2022). Internet Appendix

B allows static asset allocation targets to flexibly respond to the revelation of news. Second, wealth flows into investment funds following static asset allocation strategies are likely to be partly correlated with earnings news, as discussed in Internet Appendix A. Third, wealth flows may feature a drift term, for instance reflecting the shift from direct to delegated asset management. Appendix E extends the results to this case. This robustness exercise also makes clear that taking wealth flows to be a Brownian motion entails no loss of generality, because a drift can be accommodated. Fourth, some of the static investors may adjust their holdings of individual stocks and potentially engage in market timing. The cross section of stocks is discussed in Section 4. Fifth, the CARA utility of dynamic investors prevents wealth effects. Appendix F extends the results to a case in which dynamic investors have CRRA utility. The asset classification effect remains robust to all of these considerations, suggesting that the economic force underneath it can be attributed to the static asset allocation strategy.

Does the passage of time affect the relative wealth of dynamic and static investors? Because the portfolio choice of dynamic investors is relatively more efficient, their wealth share can be expected to rise over time. However, agents' survival and their price impact are two independent concepts (Kogan et al., 2006). Dynamic investors may accumulate wealth faster over time, but their asset demand is state contingent and thus cannot be anticipated. Static investors, by contrast, commit to a predetermined asset allocation—given their current wealth, their future demand for equities is predictable. It is precisely this predictability that allows static investors' asset demand to be reflected into current prices. Thus, the asset classification effect depends on the level of static investors' wealth, not on their share of aggregate wealth.

There is considerable evidence that wealth flows to investment funds impact equity asset prices (Coval and Stafford, 2007; Da et al., 2018). Proposition 1 is consistent with this evidence, but embeds wealth *flows* within a broader state variable: the *level* of wealth under management by static investors, whose evolution is described by Equation (5). In doing so, it raises the possibility that price pressure may be much larger than previously thought. Furthermore, it offers a perspective consistent with the evidence of price pressure originating from dividend distributions (Hartzmark and Solomon, 2025) and portfolio rebalancing (Harvey et al., 2025), discussed in Appendix G.

A broader implication of the equilibrium with dynamic and static investors is that the price impact of a trade depends on the intertemporal investment approach of its initiator. Stock purchases by dynamic investors are state contingent and do not generate any price pressure—they merely impound news about the investment opportunities into the equilibrium price. Stock purchases generate price pressure that cannot be justified by news if and only if they are initiated by static investors. This is because static investors are committed to maintaining a static asset allocation, regardless of the investment opportunities. This observation proposes a new perspective on the literature estimating the aggregate stock market elasticity, namely, the price

increase resulting from each dollar of equity purchases (Gabaix and Koijen, 2023). Hence, my findings suggest that before assessing how the stock market responds to an investment, we should consider the *strategy* behind that investment.

Having characterized the equilibrium price, I turn to the equilibrium portfolio holdings. In Appendix B, it is shown that

$$X_t^\star(P_t, D_t, V_t) = \frac{\mu_t - rP_t + D_t}{r\gamma\sigma_t^2} - (1-\theta)\frac{Q_t p_D^2 \omega^2 + \theta\pi^2}{(1-\theta Q_t)^2\sigma_t^2}. \tag{15}$$

Dynamic investors hold a number of stock shares, $X_t^\star$, which increases with the expected returns per unit of variance and decreases with the risk-free rate. Beyond the instantaneous risk–return tradeoff, their optimal portfolio also contains an intertemporal hedging component, increasing in the number of equity shares held by static investors, Q_t. When static investors become larger owners of the stock, prices become more driven by their wealth rather than by dividends. This makes the stock less attractive as a vehicle to finance future consumption, because high static ownership tends to coincide with lower future expected returns and greater exposure to non-fundamental price swings. Thus, as the number of equity shares held by static investors rises, dynamic investors tilt their portfolios away from equity assets in order to better smooth consumption over time.

Static investors hold a number of stock shares that adheres to their strategy; namely, $Q_t(P, V) = \theta V_t / P_t$. The market clearing condition in Equation (6), together with Proposition 1 and the asset allocation strategy of static investors, implies that the equilibrium holdings of dynamic investors can be characterized as follows.

$$X_t^\star(P_t, D_t, V_t) = \frac{\text{PDV}_t(D_t) + p_f}{P_t}. \tag{16}$$

Equation (16) shows that, as the present discounted value of the dividend stream increases, so does the fraction of the equity asset class held by dynamic investors. Thus, in good times, the asset classification effect represents a relatively lower fraction of the risky asset price. All else equal, when the risk-return ratio is higher, the scope for static investors' equity exposure to affect asset prices is more limited, because dynamic investors take comparatively more aggressive positions. The implication is that how closely the price of the equity asset class tracks the aggregate stream of dividends depends on its risk and expected return.[12] For stock prices to accurately reflect future dividends, it is essential that dynamic investors find it attractive to allocate resources to the equity asset class.[13] By contrast, when dynamic investors find it profitable

[12] Indeed, under the earnings process of Equation (1), $\text{PDV}_t(D_t)$ is the best linear unbiased estimator of the dividend stream. Equation (16) shows that $P_t = \text{PDV}_t(D_t) + p_f$ when $X_t = 1$, at which point price forecast accuracy is maximized. More generally, the forecast accuracy of the price of the equity asset class for the aggregate stream of dividends rises when dynamic investors have stronger incentives to allocate resources to the equity asset class, which occurs as the expected return to risk ratio of equity assets rises.

[13] The effort to connect price forecast accuracy to observable market conditions is broadly related to Dávila and Parlatore (2023).

to reduce their equity exposure, the influence of static investors' wealth becomes a relatively more important determinant of equity asset prices. Thus, in bad times, the asset classification effect represents a relatively larger fraction of the price of equity assets.

Overall, this section has demonstrated the presence of the asset classification effect under fairly general conditions. According to this effect, the aggregate market value of equity assets increases with the equity exposure of investors following static asset allocation strategies. In the time series, this effect becomes relatively more important when expected excess returns on stocks are low and volatile.

4 The Cross-Section of Stocks

The model presented thus far, which characterizes the aggregate stock market in the time series, opens the door to an important question: Which stocks exhibit the strongest price responses, when the wealth of static investors rises? To address this question, I distinguish investors based on their degrees of freedom in portfolio management. Dynamic investors are treated as unconstrained: they engage in both optimal asset allocation and optimal stock selection.[14] Static investors are instead divided into two groups: *Stock pickers*, which select stocks optimally while adopting a static asset allocation strategy; *Index trackers*, which replicate the performance of an index composed by a subset of stocks and adopt a static asset allocation strategy.

4.1 Assets

There are I firms in the economy. The real earnings, E_{it}, of firm i at time t follow dynamics

$$dE_{it} = m_i dt + \omega_i dB_{it}. \tag{17}$$

Here, m_i denotes the expected growth of earnings and ω_i their volatility, and B_{it} is a Brownian motion. The pairwise instantaneous correlation between earnings news is $dB_{it}dB_{jt} = \varrho_{ij}dt$. Firms distribute the real dividends per share $D_{it} = aE_{it}$ with a constant payout ratio $a = 1$. Stock shares trade at the real price P_{it}, with price drift and diffusion coefficients denoted by μ_{it} and σ_{it}, respectively. The covariance between the prices of stocks i and j is denoted by σ_{ijt}. To ensure that the real earnings of individual firms aggregate to the total earnings in the economy in Equation (1), impose the restrictions $m = \sum_{i=1}^{I} m_i$ and $\omega = \sqrt{\sum_i \sum_j \omega_i \omega_j \varrho_{ij}}$.

The number of shares outstanding for the i-th stock, $\overline{S}_i$, is constant over time and normalized to unity for every stock. The aggregate valuation of the equity asset class from Section 3 is thus

$$P_t = \sum_{i=1}^{I} P_{it}. \tag{18}$$

[14] For simplicity, I abstract from cross-sectional portfolio constraints on dynamic investors, treating all of them as unconstrained.

There is a stock market index consisting of a collection of $N \leq I$ stocks. The index level is

$$P_t^{\text{IDX}} = \sum_{i=1}^{I} N_i P_{it}. \tag{19}$$

The dummy variable N_i equals 1 if stock i is included in the index and 0 otherwise. As the supply of shares is held constant across stocks, float-adjusted market capitalization index weights coincide with price weights. Thus, each stock has a weight in the index that corresponds to its price relative to the index level and is denoted by $\lambda_{it} = N_i P_{it} / P_t^{\text{IDX}}$.

4.2 Dynamic Investors

The wealth of dynamic investors evolves as in Merton (1973).

$$dW_t = (rW_t - c_t)dt + \sum_{i=1}^{I} X_{it}[(D_{it} - rP_{it})dt + dP_{it}]. \tag{20}$$

Their derived utility function and related transversality condition were specified in Section 3.2. Dynamic investors optimally select the number of shares, X_{it}, in the cross-section of stocks. The aggregate number of shares of risky assets held by dynamic investors is $X_t = \sum_i X_{it}$.

4.3 Static Investors

Static investors allocate a proportion θ of their wealth in the stock market. There are two groups of static investors: *Stock Pickers* (active investors), and *Index Trackers* (passive investors). The wealth of static investors is the sum of the wealth of stock pickers and index trackers. Namely, $V_t = V_t^A + V_t^P$, where V_t^A and V_t^P denote the wealth of stock pickers and index trackers, respectively. Aggregate wealth flows to static investors, dF_t, are split in two constant fractions: stock pickers receive π^A, and index trackers $\pi^P = \pi - \pi^A$. The number of shares of the i-th stock held by static investors is denoted by $Q_{it} = Q_{it}^A + Q_{it}^P$, where Q_{it}^A and Q_{it}^P denote the number of shares held by stock pickers and index trackers, respectively.

4.3.1 Stock Pickers

Stock pickers adopting a static asset allocation strategy have target allocation θ to equity assets. Their wealth evolves as follows.

$$dV_t^A = rV_t^A(1-\theta)dt + \sum_{i=1}^{I} Q_{it}^A \left(dP_{it} + D_{it}dt\right) + \pi^A dF_t. \tag{21}$$

Equation (21) makes clear that stock pickers do not regard the risk-free rate as the opportunity cost of their individual stock positions, as their allocation to equity assets is predetermined. Stock pickers select their cross-sectional stock holdings, Q_{it}, as a solution to the constrained

mean-variance portfolio optimization problem:

$$\max_{\{Q_{it}\}} \mathbb{E}_t[dV_t^A] - 0.5\gamma\mathbb{E}_t[(dV_t^A)^2] \quad \text{s.t.} \quad \sum_{i=1}^{I} P_{it}Q_{it}^A = \theta V_t^A. \tag{22}$$

In Equation (22), the parameter capturing the degree of risk aversion of stock pickers, γ, is the same risk-aversion parameter used for dynamic investors.[15] Hence, stock pickers adopting a static asset allocation strategy differ from dynamic investors solely because the proportion of their wealth invested in stocks, θV_t^A, is fixed rather than sensitive to news. The optimal number of shares of the i-th stock held by stock pickers is denoted by $Q_{it}^{A\star}$. The associated equity-sleeve weight is denoted by $q_{it}^\star$ and results from the accounting identity $Q_{it}^{A\star} = q_{it}^\star \theta V_t^A / P_{it}$.

4.3.2 Index Trackers

Index trackers adopting a static asset allocation strategy have target allocation θ to equity assets. The number of shares of the i-th stock held by index trackers is proportional to the index weight of that stock, λ_{it}, and thus equal to $Q_{it}^P = \lambda_{it}\theta V_t^P / P_{it}$. Their wealth evolves as follows.

$$dV_t^P = rV_t^P(1-\theta)dt + \sum_{i=1}^{I} Q_{it}^P \left(dP_{it} + D_{it}dt\right) + \pi^P dF_t. \tag{23}$$

4.4 Market Clearing

There are I market clearing conditions, one for each individual stock. For the i-th stock,

$$\underset{\downarrow}{\text{Dynamic Investors}} \qquad \underset{\downarrow}{\text{Stock Pickers}} \qquad \underset{\downarrow}{\text{Index Trackers}}$$
$$X_{it} \quad + \quad Q_{it}^A \quad + \quad Q_{it}^P \quad = \quad \overline{S}_i. \tag{24}$$

The next proposition characterizes the cross-section of equilibrium stock prices by imposing the market clearing conditions and solving for the stock prices that equate agents' asset demands.

Proposition 2. The cross-section of Equilibrium Stock Prices.

$$P_{it} = \underbrace{\mathbb{E}_t^*\left[\int_t^\infty e^{-r(s-t)} D_{is}\, ds\right]}_{Fundamentals} + \underbrace{p_{f_{it}}^A + q_{it}^\star \theta V_t^A + \overbrace{p_{f_{it}}^P + \lambda_{it}\theta V_t^P}^{Index\ Inclusion}}_{Asset\ Classification}.$$

Where $p_{f_{it}}^A = -\gamma\pi\theta\pi^A q_{it}^\star$ and $p_{f_{it}}^P = -\gamma\pi\theta\pi^P\lambda_{it}$ are the risk adjustments induced by stochastic wealth flows to active stock pickers and passive index trackers, respectively.

[15] This choice is a parsimonious normalization that aims to level the playing field between the preferences toward risk of dynamic investors and static stock pickers, so that differences in the portfolio choices of these agents can be attributed solely to their asset allocation strategies.

Proof. See Appendix H. The appendix also reports the optimal portfolio and consumption choices maximizing dynamic investors' HJB and solves stock pickers' mean-variance problem.

Proposition 2 is the second contribution of the paper. Clearly, the price of each stock reflects its *Fundamentals*, the expected cash flows and the discount rates required by dynamic investors—which adjust for the correlations with other stocks and flow risk, as discussed later. Above and beyond its fundamentals, the price of each stock reflects its *Asset Classification*, and the resulting price pressure from static investors—both stock pickers and index trackers. The importance of this effect for the price of each stock corresponds to its weight in the portfolio of static investors. Static investors hold two categories of stocks: (i) index stocks, attracting investments from index trackers; and (ii) stocks with favorable risk-return profiles, attracting investments from stock pickers.

There is little doubt that stocks included in an index tend to trade at high valuations relative to their fundamentals due to the price pressure from index trackers, since the seminal papers of Harris and Gurel (1986) and Shleifer (1986). In my model, the price pressure from *Index Inclusion* is due to the tracking constraint, λ_{it}, preventing index trackers from stock selection in the cross-section. Index trackers adhere to a policy of passively investing in each stock in proportion to its index weight, leading index stocks to trade at high valuations relative to non-index stocks with an equivalent dividend stream.

Proposition 2 finds that a similar constraint on portfolio choice impacts the price of a security due to its classification within the equity asset class. The price pressure from *Asset Classification* is due to the static asset allocation constraint, θ, preventing static investors from adjusting their exposure to equity assets over time. Static investors adhere to a policy of allocating a given fraction of their wealth in the cross section of equity assets, leading them to trade at high valuations relative to their dividend stream (from the unconstrained perspective of dynamic investors). The asset classification encompasses all price pressure from static asset allocation strategies, of which the index inclusion is a special case applying to index stocks. Yet, the literature has only begun to characterize this effect.

Appendix H further shows that the i-th stock price admits the following representation:[16]

$$P_{it} = \text{PDV}_{it}(D_{it}) + p_{f_{it}} + \theta\big[q^{\star}_{it} V^A_t + \lambda_{it} V^P_t\big], \tag{25}$$

where $\text{PDV}_{it}(D_{it}) = p_{\gamma_i} + p_D D_{it} + p_m m_i$ denotes the present discounted value of the dividend stream of the i-th stock net of the risk adjustment $p_{\gamma_i} = -\frac{\gamma}{r^2}\sum_j \omega_i \omega_j \varrho_{ij}$. Each stock has a heterogeneous exposure to the risk of aggregate fund flows, which increases with its weight

[16] The right-hand sides of Proposition 2 and Equation (25) themselves depend on the equilibrium price through λ_{it}, and $q^{\star}_{it}$, whose expressions are provided in Equations (H.5) and (H.16), respectively. Thus, these representations of equilibrium are fixed points rather than closed-form functions of the primitives. Nonetheless, they establish equilibrium existence. The fixed-point representation is merely notational and helps illustrating clearly that static investors induce a stock's price to exceed its risk-adjusted dividend stream.

in the portfolio of static investors (Dou et al., 2025). The corresponding risk adjustment is $p_{f_{it}} = -\gamma\pi\theta(\pi^A q^\star_{it} + \pi^P \lambda_{it})$. Together, p_{γ_i} and $p_{f_{it}}$ adjust the price of a stock for the variance of its dividends, its correlation with other stocks' dividends, and its exposure to aggregate flows.

The i-th stock's price thus exceeds the risk-adjusted present value of its dividends. This does not constitute an arbitrage opportunity, because the i-th stock is priced consistently with other assets. This remains true even if the cross-sectional weights $q^\star_{it}$ and λ_{it} are time varying, because investors' optimal portfolio choices ensure that each stock's risk return profile is equalized in the cross-section. Individual stocks contribute to the aggregate asset classification effect in proportion to their weight in the portfolio of static investors. This can be seen from the aggregation condition in Equation (18). Indeed,

$$\sum_i \left[\text{PDV}_{it}(D_{it}) + p_{f_{it}} + \theta\left[q^\star_{it} V^A_t + \lambda_{it} V^P_t\right]\right] = \text{PDV}_t(D_t) + p_f + \theta V_t. \tag{26}$$

Contrary to conventional wisdom, both groups of static investors exert pressure on stock prices. *Index trackers* are understood to do so, as they hold the index constituents in fixed proportions, regardless of the returns offered by non-index stocks or by the risk-free asset. *Stock pickers* also exert price pressure, even with their active approach to portfolio management, because they operate under a static asset allocation strategy—this may be surprising. Stock pickers may excel at identifying a relative mispricing in the cross-section of stocks, and they do use their stock selection to engage in market timing. However, they also maintain a static wealth allocation to stocks, without regarding the risk-free asset as an alternative investment opportunity. Relative to an unconstrained dynamic investor who would scale down equity in bad times, the static allocation constraint compels stock pickers hold too much equity—and thus, excessively large positions in their best stock ideas.

This is clear from investors' optimal portfolio choices. Appendix Equation (H.29) shows that the portfolio choice maximizing the HJB for dynamic investors is

$$X^\star_{it} = \frac{\mu_{it} - rP_{it} + D_{it}}{r\gamma\sigma^2_{it}} - \frac{\sum_j X^\star_{jt}\sigma_{ijt}}{\sigma^2_{it}} - \frac{(1-\theta)}{\sigma^2_{it}}\frac{\text{Cov}_t(dP_{it}, dV_t)}{dt}. \tag{27}$$

The number of shares maximizing stock pickers' constrained mean-variance problem is

$$Q^{A\star}_{it} = \frac{\mu_{it} - \xi_t P_{it} + D_{it}}{\gamma\sigma^2_{it}} - \sum_{j\neq i} \frac{Q^{A\star}_{jt}\sigma_{ijt}}{\sigma^2_{it}}, \tag{28}$$

where ξ_t is the Lagrange multiplier on stock pickers' asset allocation constraint ensuring that

$$\sum_{i=1}^{I} Q_{it}^{A\star} P_{it} = \theta V_t^A. \tag{29}$$

Stock pickers may take short positions in some stocks. However, the constraint in Equation (29) is binding, because stock pickers are static investors. Increasing Q_{it}^A by one share necessarily forces an offsetting reduction in other equity holdings to satisfy the asset allocation target. Stock pickers thus regard ξ_t as the shadow price of investing in a stock. Economically, this corresponds to the marginal change in their mean–variance objective from relaxing Equation (29) by one unit of wealth. Even with a fixed equity allocation, stock pickers can time the market factor by forming long–short positions that reduce their portfolio's exposure to aggregate equity risk, while remaining net long equities in value.

Comparing Equations (27) and (28) shows that the portfolio choice of dynamic investors differs from that of stock pickers in two regards: (i) Dynamic investors compare the investment in each stock against the opportunity cost offered by the risk-free rate, r, reflecting their flexibility to invest in the risk-free asset. Stock pickers instead face a purely cross-sectional problem, and compare the investment in each stock against the opportunity cost, represented by the shadow cost of the asset allocation constraint ξ_t.[17] (ii) Dynamic investors present a hedging demand proportional to the term $\text{Cov}_t(dP_{it}, dV_t)$, which captures the contribution of each stock to the aggregate tilt in the equity exposure of dynamic investors induced by static investors. These two aspects are sources of inefficiency for stock pickers. By adopting static asset allocation strategies, stock pickers end up exerting price pressure precisely on stocks they optimally select, leading these stocks to be priced coherently with other equity assets, but overpriced relative to the risk-free asset.

The analysis of the cross section is deliberately kept simple to yield analytical tractability, but the key economic forces identified here are likely to hold more broadly. As discussed in the literature, stock pickers could evaluate their performance relative to the index (Basak and Pavlova, 2013). They could also engage in style investing (Boyer, 2011), generating price pressure on a groups of stocks with those style label. There could be several indexes (Pavlova and Sikorskaya, 2023). The fraction of the aggregate flows directed to passive investors could feature a time trend due to the rise in passive investment (Jiang et al., 2025). These extensions would be natural and, I expect, would preserve the core findings. I contribute to this literature by exploring how price pressure in the cross-section of individual stocks relates to the aggregate demand for the equity asset class, a question that has received limited attention in prior work.

[17] The Lagrange multiplier ξ_t generally differs from the risk-free rate r. When the target equity exposure forces stock pickers to hold more stocks than they optimally would, $\xi_t > r$. Conversely, when it forces them to hold fewer stocks than they optimally would, $\xi_t < r$. Equality $\xi_t = r$ obtains only when the target equity exposure corresponds to the unconstrained optimum.

Overall, this section has examined the relationship between stock-level price pressure and aggregate price pressure at the equity asset-class level. In equilibrium, aggregate price pressure has a stronger impact on index constituents, which attract the investment of index trackers, and on stocks with high and stable expected returns, which attract the investment of stock pickers.

5 Empirical Relevance

This section examines the empirical relevance of the model presented in this paper. There is considerable evidence that dynamic asset allocation strategies can generate large alphas by reducing risk when volatility is high (Moreira and Muir, 2017) and after the onset of recessions (Gómez-Cram, 2022). This evidence supports the presence of dynamic investors in the model.[18] Static investors represent the key element of novelty of the model. The empirical counterpart of static investors are institutional funds operating under static asset allocation strategies. I restrict attention to mutual funds and ETFs, a subset of institutional investors whose asset allocation can be measured with relatively high precision. Other institutional investors, such as pension funds and insurance companies, may also follow static allocation strategies but are harder to observe with comparable granularity. As a result, my measurement of the importance of static investors is likely to be conservative.

The data on mutual funds and ETFs are drawn from the CRSP Survivor-Bias-Free US Mutual Fund Database Summary Files.[19] As this study focuses on the stock market, funds with CRSP objective codes beginning with "O" (Other), "I" (Fixed Income), "MT" (Mixed Target), or "EF" (Equity Foreign) are excluded.[20] The sample is further restricted to funds open to investment. To mitigate reporting errors, observations with Total Net Assets (TNA) below $5 million are excluded. Following standard practices, share-class data are aggregated into data on unique portfolios identified by the crsp_portno variable, available from 2004 onwards. Following Lou (2012), fund-level variables, such as the fraction of the portfolio allocated to common stocks, are computed as the TNA-weighted average of the underlying share classes. Quality screens are applied to mitigate reporting errors: observations are excluded from the sample if the sum of fixed income and equity allocations exceeds 110%, or if the total reported fraction of the portfolio allocated across asset classes falls outside the [75%, 125%] range. The resulting sample comprises 431,079 fund-quarter observations covering the period 2004:Q1–2024:Q4.

Funds are then classified based on the stability of their asset allocation strategies. To this end, the standard deviation of the allocation of each fund to common stocks (per_com) is computed. The computation uses only the fund's allocation history available up to that quarter;

[18] The normative literature on dynamic portfolio choice is extensive (see, e.g., Campbell and Viceira, 1999; Brandt and Santa-Clara, 2006).

[19] CRSP Summary Files are not affected by the data quality problems documented by Schwarz and Potter (2016), which primarily pertain to CRSP Holdings Files prior to 2008.

[20] Target-date funds are excluded from the sample for consistency with the theoretical model, which does not explicitly consider deterministic glide paths in static asset allocation targets. Again, this exclusion induces a conservative measurement of the importance of static investors.

subsequent quarters are excluded to avoid look-ahead bias. To ensure reliable estimation and avoid incubation bias, the standard deviation is computed only when at least eight prior quarters of observations are available. As mentioned in the introduction, mutual funds and ETFs are classified as 'static' if the standard deviation of their asset allocation is less than or equal to 5%. At this threshold, equity weights exhibit very limited time-series variation, rather than being state-contingent, consistent with the static asset allocation strategy modeled in the paper. This classification identifies 11,893 funds above the threshold, which display meaningful time-series variation in their equity allocations, and 6,789 unique funds below the threshold, which are classified as static funds.

TABLE 1: **Summary Statistics of Static Funds.** The table reports annual statistics for static funds from the CRSP Mutual Fund Database. Static funds are defined as mutual funds and ETFs with standard deviation of the fraction of the portfolio allocated to common stocks $\leq 5\%$ over the sample period. Wealth flows are computed closely following Lou (2012). TNA is the aggregate total net assets in trillions of dollars. U.S. Equity Exposure is the sum of TNA weighted on fund-specific Equity Allocation. Mkt Cap is total U.S. stock market capitalization. Fraction Held by Static Investors is the ratio of Static Investors' U.S. Equity Exposure to U.S. Stock Market Capitalization. Data are drawn from the CRSP Mutual Fund Summary Files for the period 2004–2024.

Model Counterpart	F_t	V_t	θV_t	P_t	Q_t
Year	Wealth Flows ($Bn)	TNA ($T)	U.S. Equity Exposure ($T)	U.S. Stock Market Cap ($T)	% U.S. Stock Market Held by Static Investors
2004	25.09	2.38	2.04	14.27	14.30
2005	50.48	2.43	2.16	14.80	14.63
2006	-13.75	2.65	2.42	16.46	14.72
2007	-9.76	2.73	2.50	16.49	15.13
2008	-23.49	1.72	1.56	9.98	15.67
2009	-14.11	2.13	1.92	12.56	15.32
2010	-28.35	2.81	2.42	14.48	16.69
2011	-38.10	2.70	2.26	14.04	16.07
2012	-25.55	3.00	2.46	15.72	15.67
2013	116.11	3.78	3.18	20.65	15.41
2014	-32.81	3.98	3.39	22.57	15.02
2015	-109.85	3.94	3.39	21.55	15.75
2016	-34.50	4.20	3.60	23.15	15.56
2017	-40.34	4.03	3.59	27.28	13.14
2018	-84.87	4.41	3.87	24.96	15.49
2019	-186.41	5.27	4.67	31.59	14.78
2020	-240.79	5.94	5.32	39.10	13.60
2021	35.29	6.69	6.05	49.04	12.34
2022	-23.90	5.75	5.14	37.95	13.54
2023	-128.53	6.50	5.89	46.63	12.63
2024	63.78	7.26	6.64	57.36	11.58
Average	-39.91	4.04	3.53	26.03	14.54

The sample is then restricted to static funds. Wealth flows to these funds are computed by comparing the change in each fund's TNA with the change implied by fund returns and

mergers; any residual is attributed to net flows. This procedure closely follows Lou (2012). For each quarter, the net wealth flows to static funds, their TNA, and their equity exposure, are then aggregated across funds. The share of the U.S. stock market held by static investors is computed as the ratio of static funds' aggregate U.S. equity exposure to total U.S. stock market capitalization, measured using CRSP. Table 1 reports summary statistics for the sample of static funds. The data suggest that wealth flows are stationary, while the TNA of static funds exhibits an upward trend, consistent with the rise in delegated asset management over the sample period.

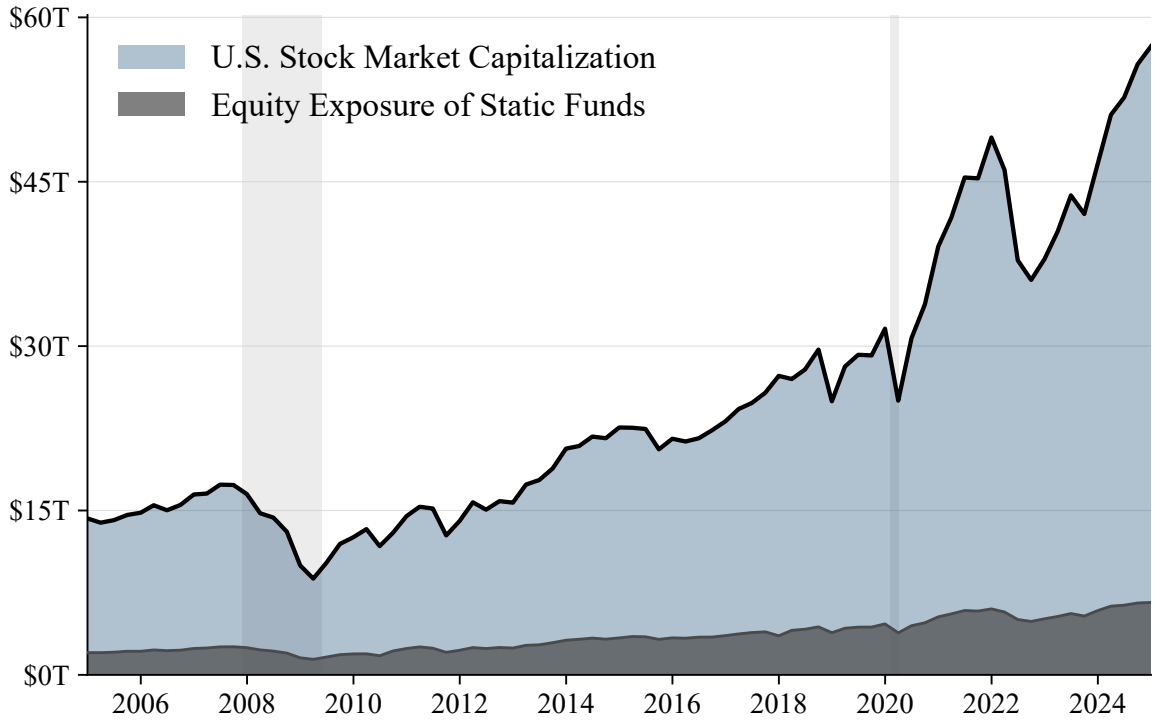


FIGURE 1: **U.S. stock market capitalization and exposure of funds with static equity allocation**. This figure plots the time series of aggregate U.S. stock market capitalization alongside the equity exposure of Mutual Funds and ETFs classified as following static asset allocation strategies. Funds are classified as static if their percentage wealth allocated to common equity has a standard deviation below 5%. To avoid look-ahead bias, the standard deviation is computed using only the fund's allocation history available up to each quarter. Gray shaded areas denote NBER recessions. Source: quarterly 2005-2025 CRSP data.

The model delivers a key prediction. It shows that the aggregate valuation of equity assets rises with equity investments operated under static asset allocation strategies (Proposition 1). The economic rationale of this effect is that stocks experience price pressure because they are classified as equity assets. Conceptually, the asset classification effect is reminiscent of the index inclusion effect, whereby stocks experience price pressure because of their inclusion in an index. However, testing for the asset classification effect is challenging, because exogenous events that influence asset classification are hard to find.

The literature on aggregate price pressure overcomes this challenge by exploiting variation in the level of wealth allocated to an asset class in settings plausibly unrelated to information about fundamentals. This literature presents strong evidence that unexpected mutual funds wealth flows are closely related to aggregate security returns (Warther, 1995; Edelen and Warner, 2001; Ben-Rephael et al., 2011). This source of exogenous variation corresponds to a shift in fund flows, dF_t. Da et al. (2018) take a different approach. They examine the reallocation of wealth between investment funds with different target equity allocation, such as a pension plan participant reallocating wealth from a 40% to a 60% equity mandate, documenting that it results in aggregate price pressure on the stock market. This source of exogenous variation corresponds to a shift in the equity target of static investors, θ. The model presented in this paper delivers both sets of findings: wealth flows and shifts in the equity target of static investors generate aggregate price pressure on the stock market.

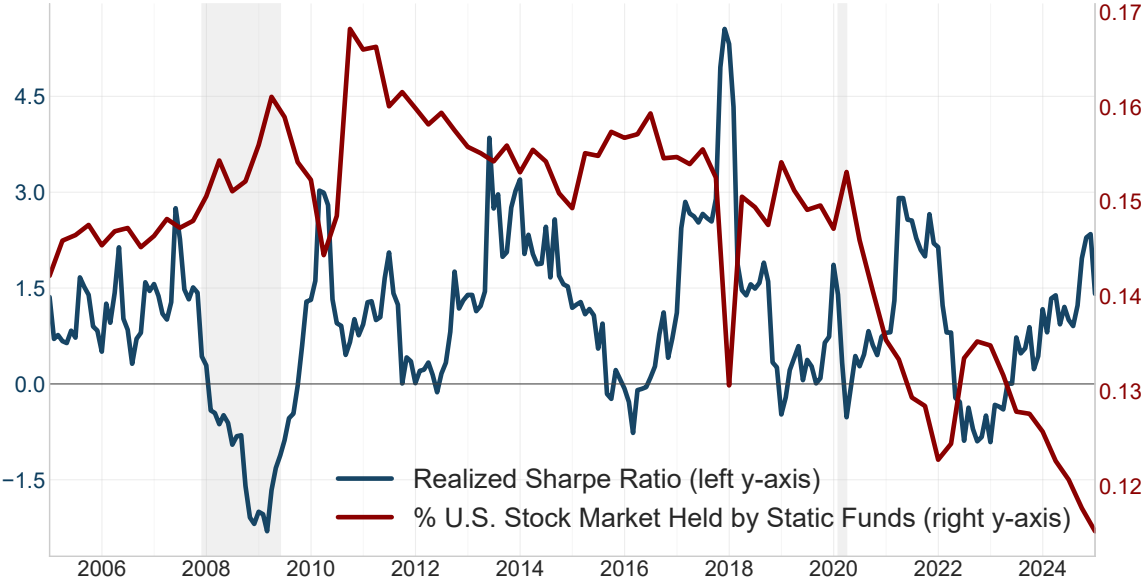


FIGURE 2: **U.S. stock market Sharpe ratio and ownership by funds with static equity allocation**. This figure displays the realized annual Sharpe ratio of the U.S. stock market (left y-axis) and the fraction of U.S. stock market capitalization held by Mutual Funds and ETFs with static allocation to equity assets (right y-axis). Funds are classified as implementing a static allocation to equity assets and labeled as "static funds" if their percentage wealth allocated to common equity has a standard deviation below 5%. To avoid look-ahead bias, the standard deviation is computed using only the fund's allocation history available up to each date. The two time series in the figure have a correlation of -0.21 in levels and -0.65 in first differences. Gray shaded areas denote NBER recessions. Source: quarterly 2005-2025 CRSP data. The data cleaning procedure is standard and described in Section 5.

The asset classification effect can also be viewed through the lens of a rational bubble generated by investors adhering to static asset allocation strategies. Detecting a rational bubble in the data or rejecting its presence is no easy matter (Stiglitz, 1990).[21] Relative to much of the

[21]The pricing of claims on dividends at specific horizons could potentially offer a way forward. Unfortunately, claims on dividends are not

literature on rational bubbles, however, my model presents an advantage: the bubble term is pinned down by quantities that are observable in the data. In the model, static investors' TNA and target equity exposure map into a deviation between stock prices and discounted fundamentals. Under the assumptions maintained by the model, a rational bubble exists and the data can be used to gauge its magnitude. This approach is not a formal test for the presence of a bubble, but rather a model-implied quantification exercise.

Figure 1 illustrates the model-implied bubble component, θV_t, which is measured in the data by static investors' aggregate equity exposure. This exposure rises from about \$2 trillion in 2004 to \$6.6 trillion in 2024, while U.S. stock market capitalization grows from \$14.3 trillion to \$57.4 trillion. Hence, the implied wedge between prices and discounted fundamentals is trending alongside the stock market. This offers some evidence in support of Remark 1, which shows that the asset classification effect is a martingale under the risk-adjusted measure.

Figure 2 shows the share of the U.S. stock market held by mutual funds and ETFs that maintain a stable allocation to stocks over time, as defined in the caption. On average, these funds hold about 15% of the U.S. stock market capitalization, a sizeable but non-exhaustive market share. The figure also illustrates the realized Sharpe ratio on the U.S. stock market. Interestingly, these two time series exhibit a negative comovement. This stylized fact, while descriptive, suggests that funds that maintain a stable allocation to stocks tend to hold a larger fraction of the stock market when its risk-adjusted returns deteriorate. The interaction of agents in the model rationalizes this pattern, because dynamic investors control their exposure to equity assets to time stock market risk-adjusted returns, while static investors keep a stable proportion of capital in equity assets.

6 Conclusion

This paper examines the importance of heterogeneous asset allocation strategies for the valuation of equity assets. The central insight from the paper is that static asset allocation strategies drive the aggregate value of stock prices above their discounted dividend stream, yet do not create any arbitrage opportunities. Analytical model solutions show that strategies that invest in static proportions across asset classes exert a price pressure on the securities within these asset classes. Given the persistence of this price pressure, attempting to reverse its price effects does not constitute an arbitrage opportunity. Unconstrained investors adopting dynamic asset allocation strategies regard the presence of assets priced above their fundamental values as consistent with arbitrage, under the rational expectation of high prices going forward. This tractable theory thus highlights a mechanism through which price pressure on perpetual assets is consistent with the absence of arbitrage.

available at long maturities. Giglio et al. (2024) develop a model to estimate the price of such claims, but their approach rules out rational bubbles by assumption, through the imposition of a transversality condition.

This paper offers a theory that rationalizes existing empirical results on the aggregate price pressure of wealth flows to the stock market. Meanwhile, equilibrium analysis points to wealth flows as part of a broader quantity that impacts asset prices: the level of wealth invested under static asset allocation strategies. A broader message from this paper is that the classification of assets within an asset class matters for asset prices, insofar as static asset allocation strategies specify target exposures to that asset class. Policymakers appear inclined to harness this effect to support domestic equity markets, as they often encourage investment vehicles under their influence to raise their domestic equity allocation targets in order to stimulate domestic financial conditions. Static asset allocation targets may also shape valuations in other asset classes, such as private equity and commodities, opening new avenues for future research.

Appendix

A Micro-foundation of the Economy

A.1 Environment

The economy is populated by overlapping generations of a continuum of retail agents. Agents' lifetimes are exponentially distributed, as in the perpetual youth models of Yaari (1965) and Blanchard (1985). Each agent discounts the future at a constant rate, δ, which incorporates time preferences as well as a constant probability of survival. The j-th retail agent has wealth W_{jt} and permanently adopts one of two investment strategies: *Dynamic* or *Static*. Agents have no bequest motive. Upon death, each agent is replaced by a newborn who adopts the same strategy as her predecessor and inherits their wealth, including stock and fund holdings. This environment facilitates the aggregation of agents and silences life-cycle considerations, which are beyond the scope of this paper.

A.2 Dynamic Investors

Retail agents adopting the dynamic investment strategy continuously observe financial markets and optimize their consumption and portfolio rules accordingly. They are professionals whose wealth is financial; for simplicity, their labor income is set to zero.[22] The value function for an agent adopting the dynamic strategy, denoted by J^D, solves the following HJB equation:

$$J^D(W_{jt}, V_t, t) \equiv \max_{\{c_{js}, X_{js}\}} \quad \mathbb{E}_t\left[\int_t^\infty e^{-\delta(s-t)} U(c_{js})\, ds\right], \tag{A.1}$$

$$\text{s.t.} \quad dW_{jt} = (rW_{jt} - c_{jt})dt + X_{jt}dY_t, \qquad \lim_{h\to\infty} \mathbb{E}_t\left[J^D(W_{jt+h}, V_{t+h}, t+h)\right] = 0. \tag{A.2}$$

Let $\mathcal{D}_t$ denote the set of agents alive at time t who adopt the dynamic strategy. As lifetimes are exponentially distributed and heirs inherit financial wealth, the aggregate wealth, consumption, and equity holdings of dynamic investors evolve continuously over generations, with no discontinuity at the points of generational turnover. Collectively, dynamic investors have aggregate wealth $W_t = \int_{\mathcal{D}_t} W_{jt}\, dj$, aggregate consumption $c_t = \int_{\mathcal{D}_t} c_{jt}\, dj$, and aggregate equity holdings $X_t = \int_{\mathcal{D}_t} X_{jt}\, dj$. Because each individual agent's optimal policies coincide with those of the representative dynamic investor evaluated at her own wealth level, the value function,

[22] Labor income of agents adopting the dynamic strategy could be incorporated, but would not induce a rational bubble on equity assets, because agents adopting the dynamic strategy have a state-contingent demand for equity assets. Instead, the labor income of agents adopting the static strategy is delegated to a fund that allocates it to equities in proportion θ, without regard to the state of the economy. This channel transforms non-tradeable income into a persistent equity demand that is not governed by dynamic optimization, leading to a rational bubble.

the aggregate budget constraint and the transversality condition in the optimization program in Equation (2) characterize the aggregate consumption and investment paths of the dynamic investors. Since the individual wealth dW_{jt} follows linear dynamics, integration across $\mathcal{D}_t$ yields the wealth dynamics in Equation (3).

A.3 Static Investors

Retail agents adopting the static strategy do not engage in continuous market observation. Rather, they work in the real economy and earn non-tradeable labor income, y_{jt}, with dynamics

$$dy_{jt} = \bar{y}_j \, dt + \zeta dF_t + \tilde{\zeta} d\epsilon_{jt}, \tag{A.3}$$

where dF_t is an aggregate shock and $d\epsilon_{jt}$ is an idiosyncratic labor income shock. Agents adopting the static strategy invest through the aggregate investment fund sector described in Section 3.3, which maintains a static asset allocation.[23] Furthermore, they do not adjust their consumption and investment plan continuously, otherwise they would not avoid continuous market observation. Instead, they are inertial households à la Brunnermeier and Nagel (2008), who adjust their fund exposure only infrequently, each at random dates, accepting to disregard intertemporal optimality considerations. The value function for an agent adopting the static regime, J^S, is determined by infrequently setting the consumption c_{jt} and exposure to the static investment fund, n_{jt}

$$J^S(W_{jt}, t) \equiv \max_{\{c_{js}, n_{js}\} \in \mathcal{A}_j^S} \mathbb{E}_t \left[\int_t^\infty e^{-\delta(s-t)} U(c_{js}) \, ds \right], \tag{A.4}$$

$$\text{s.t.} \quad dW_{jt} = n_{jt} \frac{W_{jt}}{V_t} dV_t - c_{jt} dt + dy_{jt}, \qquad \lim_{h \to \infty} \mathbb{E}_t \Big[J(W_{jt+h}, t+h) \Big] = 0. \tag{A.5}$$

Equation (A.4) is the value function of the retail investors adopting the static regime, free from the flow cost but without intertemporal hedging for state variables. Investors adopting the static regime adjust their consumption and investment plan $\{c_{js}, n_{js}\}$ only infrequently at investor-specific review dates. Accordingly,

$$\mathcal{A}_j^S \equiv \{(c_{js}, n_{js})_{s \geq t} : c_{js} = c_{j,k}, \; n_{js} = n_{j,k} \text{ for } s \in [t_{j,k}, t_{j,k+1})\}, \tag{A.6}$$

is a sequence of investor-specific review dates. Between review dates, the consumption rate and fund exposure are held constant. Equation (A.5) presents the wealth dynamics and the transversality condition for each agent adopting the static regime. Specifically, their wealth

[23] Asset allocation targets can mitigate the agency costs of delegated portfolio management (He and Xiong, 2013).

grows by the number of their shares of the static fund, $n_{jt}\frac{W_{jt}}{V_t}$, times its capital revaluation, dV_t, minus their consumption, $dc_{jt}dt$, plus their labor income, dy_{jt}.

Let $\mathcal{S}_t$ denote the set of agents alive at time t who adopt the static strategy. Because fund balances are inherited rather than liquidated at death, no wealth enters or exits the fund due to mortality. On aggregate, the investment fund sector manages $V_t = \int_{\mathcal{S}_t} v_{jt}\, dj$, where $v_{jt} = n_{jt}W_{jt}$ is the fund balance of the j-th investor. Changes in V_t arise from investment returns on existing capital and from wealth flows, determined by labor income deposits, consumption withdrawals, and changes in fund exposure at review dates. By integration across $\mathcal{S}_t$,

$$dV_t = \underbrace{V_t\Big[(1-\theta)r dt + \theta(dP_t + D_t dt)/P_t\Big]}_{\text{capital revaluation}} + \underbrace{\int_{\mathcal{S}_t} n_{jt}dy_{jt} - n_{jt}c_{jt}\, dt + W_{jt}\, dn_{jt}\, dj}_{\text{fund flows}}. \tag{A.7}$$

Substituting the income process in Equation (A.3), the idiosyncratic components; namely, the income shocks $\tilde{\zeta}d\epsilon_{jt}$ and the timing of individual review dates in dn_{jt}, wash out across the continuum $\mathcal{S}_t$ by the law of large numbers. The deterministic terms collect into a drift term,

$$\nu = \int_{\mathcal{S}_t} n_{jt}\bar{y}_j - n_{jt}c_{jt}dj + W_{jt}dn_{jt}dj, \tag{A.8}$$

corresponding to the expected flow income deposits net of consumption withdrawals and fund share redemptions. Under the perpetual youth structure, the cross-sectional distribution of ages is time-invariant, so ν is a constant. The diffusion component ζdF_t, originating from the aggregate component of the aggregate income shock that retail investors in the static regime deposit into the fund, survives integration. Equation (A.7) thus reduces to

$$dV_t = V_t\big[(1-\theta)r\, dt + \theta(dP_t + D_t\, dt)/P_t\big] + \nu\, dt + \pi\, dF_t, \tag{A.9}$$

where $\pi\, dF_t$ originates from the aggregate component of the labor income that static investors earn and deposit into the fund, and $\pi = \zeta\int_{\mathcal{S}_t} n_{jt}dj$. Under the restriction $\nu = 0$, maintained for ease of notation and subsequently relaxed in Appendix E, this expression corresponds to Equation (5).

Each retail agent adopting the static strategy satisfies her individual transversality condition. Since labor income is accumulated into W_{jt} through the budget constraint in Equation (A.5), the fund balance of the j-th investor, defined as $v_{jt} = n_{jt}W_{jt}$, reflects all past income deposits. Each agent is mortal, with survival probability $e^{-\tau h}$ over horizon h, where τ is the instantaneous probability of survival. Due to the lack of bequest motives, her investment plan respects

$$\lim_{h\to\infty} \mathbb{E}_t[e^{-(r+\tau)h}v_{j,t+h}] = 0, \qquad \forall j \in \mathcal{S}_t \tag{A.10}$$

Each retail investor thus plans to consume the present value of her wealth, and the individual transversality conditions are respected. Importantly, however, the investment fund sector is a perpetual institution whose shareholder base turns over. Its wealth at any future date $t+h$ can be decomposed as

$$V_{t+h} = \underbrace{\int_{\mathcal{S}_t \cap \mathcal{S}_{t+h}} v_{j,t+h}\, dj}_{\text{survivors from date } t} + \underbrace{\int_{\mathcal{S}_{t+h} \setminus \mathcal{S}_t} v_{j,t+h}\, dj}_{\text{born after date } t}. \tag{A.11}$$

The first term reflects the balances of agents who were alive at date t and survive to $t+h$. Their individual transversality conditions constrain these balances. The second term reflects the balances of agents born after date t. These agents earn non-tradeable labor income. Their future deposits into the fund are therefore not constrained by the transversality conditions of any agent alive at date t. The arrival of new agents with non-traded sources of wealth creates the condition under which a rational asset price bubble can arise (Weil, 1989). As new cohorts continually arrive and deposit non-tradeable labor income into the fund, the individual transversality conditions of the fund's current shareholders do not determine the long-run continuation value of the aggregate fund's wealth (characterized in Remark 2).

B Proof of Proposition 1

As is standard in the literature, the proof proceeds by postulating that the price function is as guessed and verifies that the HJB equation, the market clearing condition, and the transversality conditions are satisfied in equilibrium. The state variables follow the dynamics:

$$dD_t = m dt + \omega dB_t, \tag{B.1}$$

$$dV_t = rV_t(1-\theta)dt + Q_t(dP_t + D_t dt) + \pi dF_t. \tag{B.2}$$

Replacing the dynamics of the state variable into the guess yields

$$P_t = p_\gamma + p_D D_t + p_m m + \theta V_t, \tag{B.3}$$

where $p_\gamma = -\gamma\big(p_D^2\omega^2 + \theta\pi^2\big)$, $p_D = \frac{1}{r}$, and $p_m = \frac{1}{r^2}$. The price dynamics are as follows.

$$dP_t = p_D dD_t + \theta\big[rV_t(1-\theta)dt + Q_t(dP_t + D_t dt) + \pi dF_t\big]. \tag{B.4}$$

Equivalently,

$$dP_t = \frac{\frac{m}{r} + \theta[rV_t(1-\theta) + Q_t D_t]}{1-\theta Q_t}dt + \frac{p_D\omega}{1-\theta Q_t}dB_t + \frac{\theta\pi}{1-\theta Q_t}dF_t. \tag{B.5}$$

The price drift coefficient is given by

$$\mu_t = \frac{\frac{m}{r} + \theta[rV_t(1-\theta) + Q_t D_t]}{1 - \theta Q_t}. \tag{B.6}$$

The price diffusion is the projection of the two independent Brownian increments, increments dB_t and dF_t, on the vector of their two respective loadings. Equivalently and more compactly, the price diffusion coefficient can be expressed as the corresponding scalar,

$$\sigma_t = \frac{\sqrt{p_D^2\omega^2 + \theta^2\pi^2}}{1 - \theta Q_t}. \tag{B.7}$$

The HJB equation is

$$\begin{aligned} 0 &= \max_{\{c_t, X_t\}} U(c_t) + \frac{\mathbb{E}_t[dJ]}{dt} \\ &= \max_{\{c_t, X_t\}} U(c_t) + J_t + J_W \mathbb{E}_t[dW_t] + J_V \mathbb{E}_t[dV_t] \\ &+ \frac{1}{2} J_{WW} \mathbb{E}_t[dW_t^2] + \frac{1}{2} J_{VV} \mathbb{E}_t[dV_t^2] + J_{WV} \mathbb{E}_t[dW_t dV_t]. \end{aligned} \tag{B.8}$$

The laws of motion of the wealth of dynamic and static investors' wealth satisfy

$$dW_t = \big[rW_t - c_t + X_t(\mu_t - rP_t + D_t)\big]dt + \frac{X_t p_D \omega}{1 - \theta Q_t} dB_t + \frac{X_t \theta \pi}{1 - \theta Q_t} dF_t, \tag{B.9}$$

$$dV_t = \big[rV_t(1-\theta) + Q_t(\mu_t + D_t)\big]dt + \frac{Q_t p_D \omega}{1 - \theta Q_t} dB_t + \frac{\pi}{1 - \theta Q_t} dF_t. \tag{B.10}$$

The conditional first moment of dynamic and static investors' wealth are as follows.

$$\mathbb{E}_t[dW_t] = [rW_t - c_t + X_t\big(\mu_t - rP_t + D_t\big)]dt, \tag{B.11}$$

$$\mathbb{E}_t[dV_t] = [rV_t(1-\theta) + Q_t(\mu_t + D_t)]dt. \tag{B.12}$$

The conditional second moments of dynamic and static investors' wealth are as follows.

$$\mathbb{E}_t[dW_t^2] = \mathbb{E}_t[(X_t dP_t)^2] = (X_t\sigma_t)^2 dt, \tag{B.13}$$

$$\mathbb{E}_t[dV_t^2] = \frac{Q_t^2 p_D^2 \omega^2 + \pi^2}{(1 - \theta Q_t)^2} dt, \tag{B.14}$$

$$\mathbb{E}_t[dV_t dW_t] = \frac{X_t(Q_t p_D^2 \omega^2 + \theta\pi^2)}{(1 - \theta Q_t)^2} dt. \tag{B.15}$$

By substituting the above expressions in the HJB equation,

$$\begin{aligned}
0 = \max_{\{c_t, X_t\}} & U(c_t) + J_t + J_W \mathbb{E}_t[dW_t] + J_V \mathbb{E}_t[dV_t] + \frac{1}{2} J_{WW} \mathbb{E}_t[dW_t^2] \\
& + \frac{1}{2} J_{VV} \mathbb{E}_t[dV_t^2] + J_{WV} \mathbb{E}_t[dW_t dV_t] \\
= \max_{\{c_t, X_t\}} & U(c_t) + J_t + J_W \big[rW_t - c_t + X_t(\mu_t - rP_t + D_t) \big] \\
& + J_V \big[rV_t(1-\theta) + Q_t(\mu_t + D_t) \big] + \frac{1}{2} J_{WW} X_t^2 \sigma_t^2 \\
& + \frac{1}{2} J_{VV} \frac{Q_t^2 p_D^2 \omega^2 + \pi^2}{(1-\theta Q_t)^2} + J_{WV} \frac{X_t (Q_t p_D^2 \omega^2 + \theta \pi^2)}{(1-\theta Q_t)^2}.
\end{aligned} \tag{B.16}$$

The first order conditions (FOCs) are

$$U'(c_t^\star) = J_W, \tag{B.17}$$

$$X_t^\star = -\frac{J_W}{J_{WW} \sigma_t^2} (\mu_t - rP_t + D_t) - \frac{J_{WV}}{J_{WW}} \frac{Q_t p_D^2 \omega^2 + \theta \pi^2}{(1-\theta Q_t)^2 \sigma_t^2}. \tag{B.18}$$

As dynamic investors have CARA utility, I conjecture that the value function takes the form

$$J(W_t, V_t, t) = -e^{-\delta t - r\gamma W_t - g(V_t) - \beta} \tag{B.19}$$

Throughout the paper, subscripts on J denote partial derivatives. The partial derivatives of the value function are given by $J_t = -\delta J$, $J_W = -r\gamma J$, $J_{WW} = (r\gamma)^2 J$, $J_V = -g'(V_t) J$, $J_{VV} = \big(g'(V_t)^2 - g''(V_t)\big) J$, and $J_{WV} = r\gamma g'(V_t) J$. Thus, the FOCs become

$$c_t^\star(W_t, V_t) = rW_t + \frac{1}{\gamma} \big(g(V_t) + \beta - \log r \big), \tag{B.20}$$

$$X_t^\star(P_t, D_t, V_t) = \frac{\mu_t - rP_t + D_t}{r\gamma\sigma_t^2} - \frac{g'(V_t)}{r\gamma} \frac{Q_t p_D^2 \omega^2 + \theta \pi^2}{(1-\theta Q_t)^2 \sigma_t^2}. \tag{B.21}$$

At this stage, it is standard to replace in the HJB the FOCs paired with the market clearing condition, which is normally given by $X_t = 1$. However, the presence of static investors complicates the problem, as the market clearing condition now requires $X_t + Q_t = 1$. By replacing the expression for Q_t,

$$X_t^\star(P_t, D_t, V_t) = \frac{P_t - \theta V_t}{P_t}, \qquad Q_t(P_t, V_t) = \frac{\theta V_t}{P_t}, \tag{B.22}$$

replacing the guess for the equilibrium price yields the following expressions.

$$X_t^\star(P_t, D_t, V_t) = \frac{p_\gamma + p_D D_t + p_m m}{p_\gamma + p_D D_t + p_m m + \theta V_t}, \qquad Q_t(P_t, V_t) = \frac{\theta V_t}{p_\gamma + p_D D_t + p_m m + \theta V_t}. \tag{B.23}$$

The equilibrium price must ensure consistency between the market clearing condition and the FOCs of the optimization program of the dynamic investors. Substituting the market clearing condition, $X_t + Q_t = 1$, into the portfolio FOC and rearranging,

$$\mu_t - rP_t + D_t = r\gamma\sigma_t^2(1 - Q_t) + g'(V_t)\frac{Q_t p_D^2\omega^2 + \theta\pi^2}{(1 - \theta Q_t)^2}. \tag{B.24}$$

In the constrained equilibrium without static investors, $\theta = 0$, and the Sharpe ratio equals the supply of bonds normalized to 1 (see Veronesi, 1999). In general, however, a portion of investors may exert price pressure unrelated to fundamentals. To solve for the general case, I first work out the left-hand side of Equation (B.24). After algebraic rearrangements,

$$\mu_t - rP_t + D_t = -\frac{1}{1 - \theta Q_t} r p_\gamma, \tag{B.25}$$

Turning to the right-hand side of Equation (B.24), substitute σ_t^2 from Equation (B.7) to obtain

$$r\gamma\sigma_t^2(1 - Q_t) + g'(V_t)\frac{Q_t p_D^2\omega^2 + \theta\pi^2}{(1 - \theta Q_t)^2} = \frac{r\gamma(1 - Q_t)(p_D^2\omega^2 + \theta^2\pi^2) + g'(V_t)(Q_t p_D^2\omega^2 + \theta\pi^2)}{(1 - \theta Q_t)^2}. \tag{B.26}$$

Equating Equations (B.25) and (B.26) and multiplying both sides by $(1 - \theta Q_t)^2$ yields

$$-rp_\gamma(1 - \theta Q_t) = r\gamma(1 - Q_t)(p_D^2\omega^2 + \theta^2\pi^2) + g'(V_t)(Q_t p_D^2\omega^2 + \theta\pi^2). \tag{B.27}$$

Both sides of Equation (B.27) are linear in Q_t, and the equality must hold for all states. Matching the constant terms,

$$-rp_\gamma = r\gamma(p_D^2\omega^2 + \theta^2\pi^2) + g'(V_t)\theta\pi^2. \tag{B.28}$$

Matching the coefficients of Q_t,

$$rp_\gamma\theta = -r\gamma(p_D^2\omega^2 + \theta^2\pi^2) + g'(V_t)p_D^2\omega^2. \tag{B.29}$$

Adding θ times Equation (B.28) to Equation (B.29) yields

$$0 = r\gamma(1-\theta)(p_D^2\omega^2 + \theta^2\pi^2) - g'(V_t)(p_D^2\omega^2 + \theta^2\pi^2), \tag{B.30}$$

whose unique solution is $g'(V_t) = (1-\theta)r\gamma$, satisfied by the equation

$$g(V_t) = K + (1-\theta)r\gamma V_t, \tag{B.31}$$

for some constant K to be determined, which implies $g'(V_t) = (1-\theta)r\gamma$ and $g''(V_t) = 0$. Substituting Equation (B.31) back into Equation (B.28) yields

$$p_\gamma = -\gamma\left(\frac{\omega^2}{r^2} + \theta\pi^2\right), \tag{B.32}$$

which corresponds to the definition of p_γ, confirming that FOC and market clearing hold jointly. In Lemma 1, static investors are absent, $\theta = 0$, and the risk adjustment reduces to $p_g = -\gamma\frac{\omega^2}{r^2}$. In Proposition 1, static investors are present, and the compensation for risk, $p_\gamma = p_g + p_f$, also incorporates the flow-risk adjustment, $p_f = -\gamma\theta\pi^2$. Next, replace $U(c_t^\star)$ and $X_t^\star$ into the HJB.

$$\begin{aligned}
0 &= \frac{1}{J}U(c_t^\star) + \frac{1}{J}\frac{\mathbb{E}_t[dJ]}{dt} = r + \frac{1}{J}\frac{\mathbb{E}_t[dJ]}{dt} \\
&= r - \delta - r\gamma\left[\frac{1}{\gamma}(\log r - g(V_t) - \beta) + X_t^\star(\mu_t - rP_t + D_t)\right] + \frac{1}{2}(r\gamma)^2 X_t^{\star 2}\sigma_t^2 \\
&\quad - g'(V_t)\big[rV_t(1-\theta) + Q_t(\mu_t + D_t)\big] + \frac{1}{2}g'(V_t)^2\frac{Q_t^2 p_D^2\omega^2 + \pi^2}{(1-\theta Q_t)^2} + r\gamma g'(V_t)\frac{X_t^\star(Q_t p_D^2\omega^2 + \theta\pi^2)}{(1-\theta Q_t)^2}.
\end{aligned} \tag{B.33}$$

We have used the fact that $g''(V) = 0$. The portfolio FOC in Equation (B.21) implies that

$$r\gamma(\mu_t - rP_t + D_t) = (r\gamma)^2 X_t^\star\sigma_t^2 + r\gamma g'(V_t)\frac{Q_t p_D^2\omega^2 + \theta\pi^2}{(1-\theta Q_t)^2}. \tag{B.34}$$

Multiplying both sides of Equation (B.34) by $X_t^\star$ shows that the three terms involving $X_t^\star$ in Equation (B.33) combine into $-\frac{1}{2}(r\gamma X_t^\star\sigma_t)^2$. The HJB thus simplifies to

$$\begin{aligned}
0 &= r - \delta - r(\log r - g(V_t) - \beta) - \frac{(r\gamma X_t^\star\sigma_t)^2}{2} \\
&\quad - g'(V_t)\big[rV_t(1-\theta) + Q_t(\mu_t + D_t)\big] + \frac{g'(V_t)^2}{2}\frac{Q_t^2 p_D^2\omega^2 + \pi^2}{(1-\theta Q_t)^2}.
\end{aligned} \tag{B.35}$$

Rewriting $Q_t(\mu_t + D_t) = Q_t(\mu_t - rP_t + D_t) + rQ_tP_t$ and using $rQ_tP_t = r\theta V_t$ shows that

$$g'(V_t)\big[rV_t(1-\theta) + Q_t(\mu_t + D_t)\big] = (1-\theta)r\gamma\big[rV_t + Q_t(\mu_t - rP_t + D_t)\big]. \tag{B.36}$$

Substituting Equation (B.36) and $g(V_t) = K + (1-\theta)r\gamma V_t$ into the HJB Equation (B.35) yields

$$\begin{aligned} 0 = r - \delta - r\log r + rK + r\beta - \frac{(r\gamma)^2(1-Q_t)^2\sigma_t^2}{2} \\ - (1-\theta)r\gamma Q_t(\mu_t - rP_t + D_t) + \frac{(1-\theta)^2r^2\gamma^2}{2}\frac{Q_t^2p_D^2\omega^2 + \pi^2}{(1-\theta Q_t)^2}, \end{aligned} \tag{B.37}$$

which again uses $X_t^\star = 1 - Q_t$ from market clearing. Next, substituting σ_t^2 from Equation (B.7) and $\mu_t - rP_t + D_t$ from Equations (B.25) and (B.32), the terms in Q_t in Equation (B.37) collect as

$$-\frac{(r\gamma)^2}{2(1-\theta Q_t)^2}\Big(\big[p_D^2\omega^2 + (2\theta-1)\pi^2\big](1-\theta Q_t)^2\Big), \tag{B.38}$$

which simplifies to a constant term free of Q_t. Hence, Equation (B.37) reduces to

$$0 = r - \delta - r\log r + rK + r\beta - \frac{(r\gamma)^2}{2}\Big[p_D^2\omega^2 + (2\theta-1)\pi^2\Big]. \tag{B.39}$$

After replacing $p_D = \frac{1}{r}$ and $\beta = \frac{(\gamma\omega)^2}{2r} + \frac{\delta}{r} + \log(r) - 1$, the HJB reduces to

$$0 = rK - \frac{r^2\gamma^2(2\theta-1)\pi^2}{2}. \tag{B.40}$$

Thus, the guess satisfies the requisite FOC and market clearing conditions for $K = \frac{r\gamma^2(2\theta-1)\pi^2}{2}$. The transversality condition is also respected. Given Equation (B.19) and the wealth dynamics,

$$\lim_{h\to\infty}\mathbb{E}_t\Big[J(W_{t+h}, V_{t+h}, t+h)\Big] = \lim_{h\to\infty}\mathbb{E}_t\Big[-e^{-\delta(t+h) - r\gamma W_{t+h} - K - r\gamma(1-\theta)V_{t+h} - \beta}\Big] = 0. \tag{B.41}$$

This concludes the proof of Proposition 1. Lemma 1 follows as a special case when $\theta = 0$, which prevents static investors from allocating their wealth into equity assets. Q.E.D.

C The Stochastic Discount Factor

Proposition 1 is consistent with the SDF $M_t = e^{-\delta t}U'(c_t^\star)$. Equation (B.20) shows that the equilibrium consumption is

$$c_t^\star = rW_t + \frac{1}{\gamma}\big(g(V_t) + \beta - \log r\big), \tag{C.1}$$

With CARA utility, $U'(c) = \gamma e^{-\gamma c}$. Substituting the equilibrium consumption yields

$$M_t = \gamma r e^{-\delta t - r\gamma W_t - g(V_t) - \beta} = -\gamma r\, J_t, \tag{C.2}$$

As $g(V_t) = K + (1-\theta) r\gamma V_t$, the function $\log M_t$ is affine in W_t and V_t. Applying Itô's lemma,

$$d(\log M_t) = -\delta dt - r\gamma dW_t - (1-\theta) r\gamma dV_t. \tag{C.3}$$

Substituting Equations (3) and (5) evaluated at the equilibrium into Equation (C.3) yields

$$\begin{aligned} d(\log M_t) = &-\delta dt - r\gamma \Big[(rW_t - c_t^\star)dt + X_t^\star [(D_t - rP_t)dt + dP_t] \Big] \\ &- (1-\theta) r\gamma \Big[rV_t(1-\theta)dt + Q_t^\star (dP_t + D_t dt) + \pi dF_t \Big]. \end{aligned} \tag{C.4}$$

Collecting terms,

$$\begin{aligned} d(\log M_t) = &\Big[-\delta - r\gamma [rW_t - c_t^\star + X_t^\star (D_t - rP_t)] - (1-\theta) r\gamma (rV_t(1-\theta) + Q_t^\star D_t) \Big] dt \\ &- r\gamma \big(X_t^\star + (1-\theta) Q_t^\star \big)\, dP_t - (1-\theta) r\gamma\pi\, dF_t. \end{aligned} \tag{C.5}$$

The market clearing condition implies that $X_t^\star + (1-\theta) Q_t^\star = 1 - \theta Q_t^\star$. Hence,

$$\begin{aligned} d(\log M_t) = &\Big[-\delta - r\gamma [rW_t - c_t^\star + X_t^\star (D_t - rP_t)] - (1-\theta) r\gamma (rV_t(1-\theta) + Q_t^\star D_t) \Big] dt \\ &- r\gamma (1 - \theta Q_t^\star)\, dP_t - (1-\theta) r\gamma\pi\, dF_t. \end{aligned} \tag{C.6}$$

Next, express the price dynamics in Equation (B.4) as follows.

$$(1 - \theta Q_t^\star) dP_t = p_D dD_t + \theta \big(rV_t(1-\theta) + Q_t^\star D_t \big)\, dt + \theta\pi dF_t. \tag{C.7}$$

Substituting Equation (C.7) into (C.6) yields

$$\begin{aligned} d(\log M_t) = &\Big[-\delta - r\gamma [rW_t - c_t^\star + X_t^\star (D_t - rP_t)] - (1-\theta) r\gamma (rV_t(1-\theta) + Q_t^\star D_t) \Big] dt \\ &- r\gamma \Big[p_D dD_t + \theta \big(r(1-\theta)V_t + Q_t^\star D_t \big)\, dt + \theta\pi dF_t \Big] - (1-\theta) r\gamma\pi\, dF_t. \end{aligned} \tag{C.8}$$

Collecting terms and substituting c^* from Equation (C.1),

$$\begin{aligned} d(\log M_t) = &\Big[-\delta + r\big(g(V_t) + \beta - \log r \big) - r\gamma \big(X_t^\star (D_t - rP_t) + r(1-\theta)V_t + Q_t^\star D_t \big) \Big] dt \\ &- r\gamma p_D\, dD_t - r\gamma\pi\, dF_t. \end{aligned} \tag{C.9}$$

Using market clearing $X_t^\star + Q_t^\star = 1$, rewrite

$$X_t^\star(D_t - rP_t) + Q_t^\star D_t = D_t - rP_t(1 - Q_t^\star) = (D_t - rP_t) + rP_tQ_t^\star. \tag{C.10}$$

The static asset allocation policy, $Q_t^\star = \theta V_t/P_t$, implies that $rP_tQ_t^\star = r\theta V_t$. Hence,

$$X_t^\star(D_t - rP_t) + Q_t^\star D_t = (D_t - rP_t) + r\theta V_t. \tag{C.11}$$

Substituting Equation (C.11) into (C.9) yields

$$\begin{aligned} d(\log M_t) = \Big[&- \delta + r\big(g(V_t) + \beta - \log r\big) - r\gamma\big(D_t - rP_t + rV_t\big)\Big]dt \\ &- r\gamma p_D\, dD_t - r\gamma\pi\, dF_t. \end{aligned} \tag{C.12}$$

Recall that $p_D = 1/r$ and $dD_t = mdt + \omega dB_t$. Hence,

$$-r\gamma p_D dD_t = -\gamma dD_t = -\gamma m dt - \gamma\omega dB_t. \tag{C.13}$$

Thus, Equation (C.12) can be written as

$$d(\log M_t) = \Delta_t\, dt - \gamma\omega\, dB_t - \gamma r\pi\, dF_t, \tag{C.14}$$

where $\Delta_t = -\delta + r\big(g(V_t) + \beta - \log r\big) - r\gamma\big(D_t - rP_t + rV_t\big) - \gamma m$. Applying Itô's lemma to $M_t = \exp(\log M_t)$ yields

$$\frac{dM_t}{M_t} = \Big(\Delta_t + \tfrac{1}{2}\gamma^2\omega^2 + \tfrac{1}{2}\gamma^2 r^2\pi^2\Big)dt - \gamma\omega\, dB_t - \gamma r\pi\, dF_t. \tag{C.15}$$

Next, recall from Equation (B.19) that the value function is given by

$$J = -\exp\{-\delta t - r\gamma W_t - g(V_t) - \beta\}. \tag{C.16}$$

Thus, $M_t = \gamma r e^{-\delta t - r\gamma W_t - g(V_t) - \beta} = -\gamma r J_t$. Under the optimal policy, the HJB implies

$$0 = e^{-\delta t}U(c_t^\star) + \frac{E_t[dJ_t]}{dt}. \tag{C.17}$$

Moreover,

$$e^{-\delta t}U(c_t^\star) = -\exp(-\delta t - \gamma c_t^\star) = -\exp(-\delta t - r\gamma W_t - g(V_t) - \beta + \log r) = rJ. \tag{C.18}$$

Thus,

$$\frac{1}{M_t}\frac{E_t[dM_t]}{dt} = \frac{1}{J_t}\frac{E_t[dJ_t]}{dt} = -r. \tag{C.19}$$

Comparing Equation (C.19) with Equation (C.15) implies that

$$\Delta_t + \frac{1}{2}(\gamma\omega)^2 + \tfrac{1}{2}(\gamma r\pi)^2 = -r. \tag{C.20}$$

Replacing Equation (C.20) into Equation (C.15) yields

$$\frac{dM_t}{M_t} = -r dt - \gamma\omega dB_t - \gamma r\pi\, dF_t. \tag{C.21}$$

Define the constant market prices of risk by

$$\lambda_B = \gamma\omega, \qquad \lambda_F = \gamma r\pi. \tag{C.22}$$

Equation (C.21) induces the risk-adjusted measure under which

$$dB_t^* = dB_t + \lambda_B dt, \qquad dF_t^* = dF_t + \lambda_F dt. \tag{C.23}$$

The stock price satisfies the SDF pricing identity

$$P_t = E_t^*\left[\int_t^\infty e^{-r(s-t)} D_s ds\right] + \lim_{T\to\infty} e^{-r(T-t)} E_t^*[P_T]. \tag{C.24}$$

Equivalently, under the standard transversality and integrability conditions, the discounted cum-dividend stock price is a discounted risk-adjusted martingale, so

$$E_t^*[dP_t + D_t dt] = r P_t dt. \tag{C.25}$$

First, recover p_g. Since $dD_t = m dt + \omega dB_t$, Equation (C.23) implies

$$dD_t = (m - \omega\lambda_B)dt + \omega dB_t^* = (m - \gamma\omega^2)dt + \omega dB_t^*. \tag{C.26}$$

Hence, for $s \geq t$,

$$E_t^*[D_s] = D_t + (m - \gamma\omega^2)(s - t). \tag{C.27}$$

Therefore,

$$E_t^*\left[\int_t^\infty e^{-r(s-t)}D_s ds\right] = \int_t^\infty e^{-r(s-t)}\left(D_t + (m-\gamma\omega^2)(s-t)\right)ds$$
$$= \frac{D_t}{r} + \frac{m-\gamma\omega^2}{r^2}. \tag{C.28}$$

It follows that the SDF-implied risk adjustment to the discounted dividend claim is

$$p_g = -\gamma\frac{\omega^2}{r^2}. \tag{C.29}$$

Second, recover p_f from the law of motion for V_t,

$$dV_t = rV_t(1-\theta)dt + Q_t(dP_t + D_t dt) + \pi dF_t. \tag{C.30}$$

From Equation (C.23),

$$E_t^*[\pi dF_t] = \pi E_t^*[dF_t^* - \lambda_F dt] = -\pi\lambda_F dt = -\gamma r\pi^2 dt. \tag{C.31}$$

Hence,

$$E_t^*[dV_t] = rV_t(1-\theta)dt + Q_t r P_t dt - \gamma r\pi^2 dt = (rV_t - \gamma r\pi^2)dt. \tag{C.32}$$

Next, using the affine equilibrium price representation established in Proposition 1,

$$dP_t = p_D dD_t + \theta dV_t. \tag{C.33}$$

Evaluating the risk-adjusted expectation of the above Equation (C.33), and using Equations (C.26) and (C.32), yields

$$E_t^*[dP_t] = \left[p_D(m-\gamma\omega^2) + \theta(rV_t - \gamma r\pi^2)\right]dt. \tag{C.34}$$

Substituting Equation (C.34) into Equation (C.25) gives

$$p_D(m-\gamma\omega^2) + \theta(rV_t - \gamma r\pi^2) + D_t = rP_t. \tag{C.35}$$

Using $p_D = \frac{1}{r}$, $p_g = -\gamma\frac{\omega^2}{r^2}$, and $p_f = -\gamma\theta\pi^2$, and rearranging terms,

$$P_t = p_g + \frac{m}{r^2} + \frac{D_t}{r} + p_f + \theta V_t. \tag{C.36}$$

Thus, the SDF approach reproduces the equilibrium stock price of Proposition 1. Furthermore, it allows to determine the market prices of risk factors and evaluate risk-adjusted expectations.

D Proof of Remark 1

In the absence of arbitrage opportunities, the risk-adjusted expected return on stocks coincides with the rate of return on the risk-free asset. Thus, under the risk-adjusted measure, the conditional expectation of static investors' wealth dynamics in Equation (5) is

$$\mathbb{E}_t^*[dV_t] = rV_t dt + \mathbb{E}_t^*[\pi dF_t]. \tag{D.1}$$

Multiplying both sides of Equation (11) by the constant θ preserves the equality. Namely,

$$\mathbb{E}_t^*[d\theta V_t] = r\theta V_t dt + \theta\mathbb{E}_t^*[\pi dF_t]. \tag{D.2}$$

From Equation (C.31), it follows that

$$p_f = \frac{\theta}{r}\frac{\mathbb{E}_t^*[\pi dF_t]}{dt}. \tag{D.3}$$

Replacing Equation (D.3) into Equation (D.2),

$$\mathbb{E}_t^*[d\theta V_t] = r(\theta V_t dt + p_f). \tag{D.4}$$

Since p_f is constant, the previous expression is equivalent to

$$\mathbb{E}_t^*[d(\theta V_t + p_f)] = r(\theta V_t dt + p_f). \tag{D.5}$$

Hence, for every positive time horizon h,

$$\theta V_t + p_f = e^{-rh}\mathbb{E}_t^*[\theta V_{t+h} + p_f]. \tag{D.6}$$

This shows that the asset classification effect is a discounted risk-adjusted martingale. Q.E.D.

E Wealth Flows with Drift

This appendix considers the robustness of a case in which wealth flows into the static fund have a constant drift term. Suppose that the flow process satisfies

$$dF_t = \nu\, dt + d\widetilde{F}_t, \tag{E.1}$$

where $\widetilde{F}_t$ is a Brownian motion adapted to $\{\mathcal{F}_t\}$, and ν is a constant drift term. We want to show that this environment is equivalent to the benchmark after a change of variables. Substituting Equation (E.1) into the static investors' wealth dynamics in Equation (5) yields

$$dV_t = rV_t(1-\theta)dt + Q_t(dP_t + D_t dt) + \pi\nu dt + \pi d\widetilde{F}_t. \tag{E.2}$$

As the risk-adjusted expected return on stocks must equal the risk-free rate,

$$\mathbb{E}_t^*[dV_t] = rV_t\,dt + \pi\nu\,dt. \tag{E.3}$$

Define the auxiliary wealth process

$$\widetilde{V}_t \equiv V_t + \frac{\pi\nu}{r}. \tag{E.4}$$

Since $\frac{\pi\nu}{r}$ is constant, $d\widetilde{V}_t = dV_t$, and Equation (E.3) implies

$$\mathbb{E}_t^*[d\widetilde{V}_t] = r\widetilde{V}_t\,dt, \tag{E.5}$$

and $e^{-rt}\widetilde{V}_t$ is a risk-adjusted martingale. Accordingly, conjecture that the equilibrium price is

$$P_t = p_\gamma + p_D D_t + p_m m + \theta\widetilde{V}_t. \tag{E.6}$$

The verification in Appendix B carries over verbatim, with $\widetilde{V}_t$ in place of V_t. In particular, $d\widetilde{V}_t = dV_t$ ensures identical dynamics, and the martingale property in Equation (E.5) ensures that the auxiliary wealth process evolves as a rational bubble term. Expressing the solution in terms of V_t,

$$P_t = p_\gamma^\nu + p_D D_t + p_m m + \theta V_t, \qquad p_\gamma^\nu \equiv p_\gamma + \frac{\theta\pi\nu}{r}. \tag{E.7}$$

Setting $\nu = 0$ recovers the benchmark of Proposition 1. The asset classification effect thus naturally extends to this environment. Q.E.D.

F Preferences of Dynamic Investors

The baseline model of Section 3 assumes that dynamic investors' preferences are described by a CARA utility function. This section shows that the results continue to hold when preferences are described by a CRRA utility function. Assume that dynamic investors have utility

$$U(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}. \tag{F.1}$$

Internet Appendix C shows that the resulting equilibrium price of the equity asset class is

$$P_t = \bar{p} + \bar{p}_\gamma W_t + p_D D_t + p_m m + \theta V_t, \tag{F.2}$$

for two constant scalars $\bar{p} < 0$ and $\bar{p}_\gamma > 0$. Equation (F.2), where static investors' equity exposure, θV_t, enters additively, again indicates the presence of an asset classification effect. Thus, the key asset pricing mechanism of the model is robust to the specification of dynamic investors' preferences toward risk, and continues to hold under both CARA and CRRA utility. The asset classification effect arises because the equity exposure of static investors generates a rational bubble, independently of the curvature of dynamic investors' utility function.

G Asset Price Dynamics

The price of the equity asset class characterized in Proposition 1 follows the dynamics

$$dP_t = p_D dD_t + \theta dV_t. \tag{G.1}$$

Asset price dynamics are driven by the evolution of dividends and the wealth of static investors. By replacing Equation (5) in the above expression,

$$dP_t = p_D dD_t + \theta\big[rV_t(1-\theta)dt + Q_t(dP_t + D_t dt) + \pi dF_t\big]. \tag{G.2}$$

From Equation (G.2), it is observed that the equity holdings of static investors, Q_t, give rise to price amplification effects.[24] The intuition for this effect is as follows. Price changes, dP_t, affect the wealth of both dynamic and static investors. Static investors reinvest into the equity asset class a proportion θ of their capital gains, exerting upward price pressure. This result contrasts with common wisdom, since static investors rebalance their portfolio in the opposite direction of price changes. To illustrate this mechanism, consider the example below.

Example 1. Suppose $P_t = \$100$, $X_t = Q_t = 0.5$, and $\theta = 0.4$. As firms report earnings, the present value of the dividend stream increases by \$20. Dynamic investors demand more shares and the price rises to $P_t + dP_t = \$120$. Dynamic investors realize capital gains $dW_t = X_t dP_t = \$10$. Static investors also realize capital gains $dV_t = Q_t dP_t = \$10$. Their actions can be understood through two alternative interpretations.

1) Static investors reinvest $\theta dV_t = \$4$ into the stock market leading to a price of \$124, which has a positive second-round effect on wealth and prompts further investment. 2) Static investors keep their capital gains in the stock market, leading to a price of \$130. Equities are now overweight in their portfolio and they divest $(1-\theta)dV_t = \$6$ from stocks, leading to a price of

[24] This wealth amplification effect shares similarities with Kyle and Xiong (2001) and Basak and Pavlova (2013), while stemming from a different group of investors following static asset allocation strategies.

$124. Equities are now cheaper, and thus slightly underweight, prompting further investment. Under both interpretations, this process leads to an amplification of $\frac{1}{1-\theta Q_t} = 1.25$. In the new equilibrium, $P_t = \$125$, $X_t = 0.56$, and $Q_t = 0.44$. Following fundamental news of $20 increase in the present value of dividends, the price changed by $25. Absent static investors, the price would have closed at $120.

Example 1 shows that as the price of the equity asset class increases, static investors sell some of their equity shares, trading counter-cyclically against the price change. The first interpretation is perhaps more intuitive, but the two are equivalent. Recently, Harvey et al. (2025) have highlighted the quantitative importance of this pattern in the data, documenting that portfolio rebalancing from institutional investors adopting static asset allocation strategies has a market-wide price impact. The model reproduces this mechanism through the amplification term implied by Example 1, interpretation 2. Finally, this example clarifies that static investors' equity exposure in levels is pro-cyclical—even though their trades are contrarian—and highlights the counterfactual price that would obtain in the absence of static investors.

H Proof of Proposition 2

Before presenting the formal proof, let me present three illustrative benchmark cases. First, consider the equilibrium with only *dynamic investors*, by setting $\theta = 0$. The optimality of dynamic investors implies that the price of the i-th stock takes the following standard form:

$$P_{it} = \mathbb{E}_t^* \left[\int_t^\infty e^{-r(s-t)} D_{is} \, ds \right] = p_{\gamma_i} + p_D D_{it} + p_m m_i. \tag{H.1}$$

There is no aggregate price pressure on the equity asset class. Second, consider the equilibrium with only *static stock pickers*, by letting $r \to \infty$ to lure dynamic investors to the risk-free asset and by setting $N_i = 0$ for each stock, muting the presence of index trackers. Then, by market clearing, the price of the i-th stock is

$$P_{it} = q_{it}^\star \theta V_t^A. \tag{H.2}$$

The aggregate price pressure on the equity asset class is given by $\sum P_{it} = \theta V_t^A$. Third, consider the equilibrium with *static stock pickers and index trackers*. This benchmark can be obtained either by setting dynamic investors' wealth to zero, or by considering the limiting case in which the risk-free rate becomes arbitrarily high, so that dynamic investors optimally invest in the risk-free asset. By market clearing, the price of the i-th stock is

$$P_{it} = q_{it}^\star \theta V_t^A + \lambda_{it} \theta V_t^P. \tag{H.3}$$

The aggregate price pressure on the equity asset class is now given by $\sum P_{it} = \theta V_t$.

Formally, consider the equilibrium with *dynamic investors, stock pickers, and index trackers*. Guess that the price of the i-th stock takes the following form.

$$P_{it} = p_{\gamma_i} + p_{f_{it}} + p_D D_{it} + p_m m_i + \theta\left[q_{it}^{\star} V_t^A + \lambda_{it} V_t^P\right]. \tag{H.4}$$

The portfolio weights of index trackers are given by

$$\lambda_{it} = P_{it} N_i / P_t^{\text{IDX}}. \tag{H.5}$$

Stock pickers choose holdings $\{Q_{it}^A\}_{i=1}^I$ to solve:

$$\max_{\{Q_{it}^A\}} \sum_{i=1}^{I} Q_{it}^A(\mu_{it} + D_{it}) - \frac{\gamma}{2} \sum_{i=1}^{I} \sum_{j=1}^{I} Q_{it}^A Q_{jt}^A \sigma_{ijt} \tag{H.6}$$

subject to:

$$\sum_{i=1}^{I} Q_{it}^A P_{it} = \theta V_t^A. \tag{H.7}$$

The Lagrangian is:

$$\mathcal{L} = \sum_{i=1}^{I} Q_{it}^A(\mu_{it} + D_{it}) - \frac{\gamma}{2} \sum_{i=1}^{I} \sum_{j=1}^{I} Q_{it}^A Q_{jt}^A \sigma_{ijt} - \xi_t \left(\sum_{i=1}^{I} Q_{it}^A P_{it} - \theta V_t^A \right). \tag{H.8}$$

The FOC is:

$$\frac{\partial \mathcal{L}}{\partial Q_{it}^A} = (\mu_{it} + D_{it}) - \gamma \sum_{j=1}^{I} Q_{jt}^A \sigma_{ijt} - \xi_t P_{it} = 0. \tag{H.9}$$

Rearranging terms,

$$Q_{it}^{A\star} = \frac{\mu_{it} - \xi_t P_{it} + D_{it}}{\gamma \sigma_{it}^2} - \sum_{j \neq i} \frac{Q_{jt}^{A\star} \sigma_{ijt}}{\sigma_{it}^2}, \tag{H.10}$$

where ξ_t is a Lagrange multiplier such that:

$$\sum_{i=1}^{I} Q_{it}^{A\star} P_{it} = \theta V_t^A. \tag{H.11}$$

Stock pickers' problem can be reformulated in terms of the individual weights, $q_{it}^{\star}$. Let R_t be the vector of expected returns on the I stocks, Σ_t be the covariance matrix of their returns, $\mathbf{1}$ a

$I \times 1$ vector of ones, and q_t a $I \times 1$ vector of individual portfolio weights. Stock pickers solve

$$\max_{q_t} \quad q_t^\top R_t - \frac{\gamma}{2} q_t^\top \Sigma_t q_t \qquad s.t. \quad \mathbf{1}^\top q_t = 1. \tag{H.12}$$

The Lagrangian is

$$\mathcal{L} = q_t^\top R_t - \frac{\gamma}{2} q_t^\top \Sigma_t q_t - \chi_t \left(\mathbf{1}^\top q_t - 1 \right). \tag{H.13}$$

The first-order condition with respect to the vector q_t implies that

$$q_t^\star = \frac{1}{\gamma} \Sigma_t^{-1} \left(R_t - \chi_t \mathbf{1} \right). \tag{H.14}$$

Imposing the constraint $\mathbf{1}^\top q_t = 1$ yields the expression for the Lagrange multiplier χ_t,

$$\chi_t = \frac{\mathbf{1}^\top \left(\Sigma_t \right)^{-1} R_t - \gamma}{\mathbf{1}^\top \Sigma_t^{-1} \mathbf{1}}. \tag{H.15}$$

Substituting Equation (H.15) into (H.14), the optimal within-equity weights are

$$q_t^\star = \frac{1}{\gamma} \Sigma_t^{-1} R_t - \frac{1}{\gamma} \left(\frac{\mathbf{1}^\top \Sigma_t^{-1} R_t - \gamma}{\mathbf{1}^\top \Sigma_t^{-1} \mathbf{1}} \right) \Sigma_t^{-1} \mathbf{1}. \tag{H.16}$$

Stock pickers ensure that index stocks and non-index stocks are priced consistently in the cross-section. This greatly simplifies the problem of the dynamic investors, who simply keep track of the aggregate wealth of static investors, V_t, as a state variable. The sum of the wealth of static stock pickers and index trackers is the wealth of static investors $V_t = V_t^A + V_t^P$, which follows Equation (5). Namely,

$$\begin{aligned} dV_t &= rV_t(1-\theta)dt + Q_t(dP_t + D_t dt) + \pi dF_t \\ &= rV_t(1-\theta)dt + \sum_i Q_{it}(dP_{it} + D_{it}dt) + \pi dF_t. \end{aligned} \tag{H.17}$$

Equation (H.17) implies that $Q_t P_t = \sum Q_{it} P_{it}$ and $Q_t D_t = \sum Q_{it} D_{it}$. Analogously, Equation (20) implies that $X_t P_t = \sum X_{it} P_{it}$ and $X_t D_t = \sum X_{it} D_{it}$. These conditions state that the aggregate exposure of investors to the equity asset class is the sum of their exposures to individual stocks. Finally, by definition of $P_t = \sum P_{it}$, we have $\mu_t = \sum \mu_{it}$ and $\sigma_t^2 = \frac{1}{dt}\mathbb{E}_t\big[\big(\sum dP_{it}\big)^2\big]$.

As in Appendix B, we guess and then verify that the value function of dynamic investors is $J(W_t, V_t, t) = -e^{-\delta t - r\gamma W_t - g(V_t) - \beta}$, with $g(V_t) = K + (1-\theta) r\gamma V_t$. Dynamic investors choose

$\{c_t, X_{1t}, \ldots, X_{It}\}$ to maximize the HJB

$$0 = \max_{\{c_t, X_{it}\}} U(c_t) + J_t + J_W \mathbb{E}_t[dW_t] + J_V \mathbb{E}_t[dV_t] + \frac{1}{2} J_{WW} \mathbb{E}_t[dW_t^2] + \frac{1}{2} J_{VV} \mathbb{E}_t[dV_t^2] + J_{WV} \mathbb{E}_t[dW_t dV_t]. \tag{H.18}$$

The aggregation identities $\sum_i X_{it} dP_{it} = X_t dP_t$ and $\sum_i Q_{it} dP_{it} = Q_t dP_t$ imply that the diffusion coefficients of dW_t and dV_t depend on individual holdings only through their aggregates, X_t and Q_t. Thus, substituting the aggregate price decomposition, the laws of motion are

$$dW_t = \left[rW_t - c_t + \sum_i X_{it}(\mu_{it} - rP_{it} + D_{it}) \right] dt + \frac{X_t p_D \omega}{1 - \theta Q_t} dB_t + \frac{X_t \theta \pi}{1 - \theta Q_t} dF_t, \tag{H.19}$$

$$dV_t = \big[rV_t(1-\theta) + Q_t(\mu_t + D_t) \big] dt + \frac{Q_t p_D \omega}{1 - \theta Q_t} dB_t + \frac{\pi}{1 - \theta Q_t} dF_t. \tag{H.20}$$

The conditional first moments are

$$\mathbb{E}_t[dW_t]/dt = rW_t - c_t + \sum_i X_{it}\big(\mu_{it} - rP_{it} + D_{it}\big), \tag{H.21}$$

$$\mathbb{E}_t[dV_t]/dt = rV_t(1-\theta) + Q_t(\mu_t + D_t). \tag{H.22}$$

The conditional second moments follow from $dB_t^2 = dF_t^2 = dt$ and $dB_t dF_t = 0$:

$$\mathbb{E}_t[dW_t^2]/dt = X_t^2 \sigma_t^2, \tag{H.23}$$

$$\mathbb{E}_t[dV_t^2]/dt = \frac{Q_t^2 p_D^2 \omega^2 + \pi^2}{(1 - \theta Q_t)^2}, \tag{H.24}$$

$$\mathbb{E}_t[dW_t dV_t]/dt = \frac{X_t(Q_t p_D^2 \omega^2 + \theta \pi^2)}{(1 - \theta Q_t)^2}. \tag{H.25}$$

These expressions are identical to those in Appendix B. Substituting into the HJB,

$$\begin{aligned} 0 &= \frac{1}{J} U(c_t^\star) + \frac{1}{J} \frac{\mathbb{E}_t[dJ]}{dt} = r + \frac{1}{J} \frac{\mathbb{E}_t[dJ]}{dt} \\ &= r - \delta - r\gamma \left[\frac{1}{\gamma}(\log r - g(V_t) - \beta) + \sum_i X_{it}^\star \big(\mu_{it} - rP_{it} + D_{it}\big) \right] + \frac{1}{2}(r\gamma)^2 X_t^{\star 2} \sigma_t^2 \\ &\quad - (1-\theta) r\gamma \big[rV_t(1-\theta) + Q_t(\mu_t + D_t) \big] + \frac{(1-\theta)^2 r^2 \gamma^2}{2} \frac{Q_t^2 p_D^2 \omega^2 + \pi^2}{(1 - \theta Q_t)^2} \\ &\quad + (1-\theta) r^2 \gamma^2 \frac{X_t^\star (Q_t p_D^2 \omega^2 + \theta \pi^2)}{(1 - \theta Q_t)^2}. \end{aligned} \tag{H.26}$$

Equation (H.26) coincides with the HJB in Equation (B.33) if and only if

$$\sum_i X_{it}^\star\big(\mu_{it} - rP_{it} + D_{it}\big) = X_t^\star(\mu_t - rP_t + D_t). \tag{H.27}$$

The relationship $\sum X_{it}P_{it} = X_tP_t$ directly implies $r\sum_i X_{it}^\star P_{it} = rX_t^\star P_t$. Moreover, as $\sum_i X_{it}^\star \mathbb{E}_t[dP_{it}] = X_t^\star \mathbb{E}_t[dP_t]$, we have $\sum_i X_{it}^\star \mu_{it} = X_t^\star \mu_t$. Finally, $\sum X_{it}D_{it} = X_tD_t$. Thus, Equation (H.27) holds. Consequently, Equation (H.26) corresponds to the HJB in Equation (B.33). The arguments of Appendix B apply, and Equation (H.26) is solved by the optimal choice of dynamic investors, the transversality condition is respected, and the guess for the price in Equation (25) is verified. The individual portfolio FOC follows from differentiating the HJB with respect to X_{it}, which yields

$$0 = J_W(\mu_{it} - rP_{it} + D_{it}) + J_{WW}\sum_j X_{jt}\sigma_{ijt} + J_{WV}\frac{\text{Cov}(dP_{it}, dV_t)}{dt}, \tag{H.28}$$

Substituting the partial derivatives from Equation (B.19) and using $g'(V_t) = (1-\theta)r\gamma$.

$$X_{it}^\star = \frac{\mu_{it} - rP_{it} + D_{it}}{r\gamma\sigma_{it}^2} - \frac{\sum_{j\neq i} X_{jt}^\star \sigma_{ijt}}{\sigma_{it}^2} - \frac{\text{Cov}(dP_{it}, dV_t)}{dt}\frac{(1-\theta)}{\sigma_{it}^2}. \tag{H.29}$$

It remains to show that Equation (25) corresponds to the expression in Proposition 2. Using Equation (17) to replace the dividends process, we have

$$\mathbb{E}_t\left[\int_t^\infty e^{-r(s-t)}D_{is}ds\right] = p_D D_{it} + p_m m_i. \tag{H.30}$$

Replacing this expression into Equation (25) yields

$$P_{it} = p_{\gamma_i} + p_{f_{it}} + \mathbb{E}_t\left[\int_t^\infty e^{-r(s-t)}D_{is}ds\right] + \theta\big[q_{it}^\star V_t^A + \lambda_{it}V_t^P\big], \tag{H.31}$$

where

$$p_{\gamma_i} + p_{f_{it}} = -\left[\frac{\gamma}{r^2}\sum_j \omega_i\omega_j\varrho_{ij} + \gamma\theta\pi\big(\pi^A q_{it}^\star + \pi^P\lambda_{it}\big)\right] \tag{H.32}$$

represents the compensation for risk, so that

$$\mathbb{E}_t^*\left[\int_t^\infty e^{-r(s-t)}D_{is}\,ds\right] = p_{\gamma_i} + p_{f_{it}} + \mathbb{E}_t\left[\int_t^\infty e^{-r(s-t)}D_{is}ds\right]. \tag{H.33}$$

Consequently, Equation (25) corresponds to the equilibrium price in Proposition 2. Q.E.D.

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Internet Appendix to
"Asset Pricing with Dynamic and Static Investors"

Ruggero Jappelli

Internet Appendix A: Wealth Flows and Earnings News

Thus far, wealth flows, dF_t, were assumed to be uncorrelated with earnings news, dB_t, and thus with changes in dividends, dD_t. (Recall that, since the payout ratio is constant and normalized to one, the dividend innovations are driven by earnings news). However, wealth invested under static allocation strategy investment funds may partly reflect information about earnings, as some agents could use static funds interchangeably with direct exposure to stocks. For example, positive news about the stock market may attract wealth inflows into static investment funds; conversely, negative news may prompt outflows. To extend the model in this direction, consider the following specification:

$$dF_t = \rho dB_t + \sqrt{1-\rho^2} dU_t, \tag{IA.A1}$$

where U_t is a Brownian motion uncorrelated with earnings news. I assume that $0 < \rho < 1$, which is economically plausible, considering that some educated fund shareholders may time their exposure to the fund in response to earnings news, while other fund shareholders are inertial and unresponsive to earnings news.

Guess that the equilibrium price of the equity asset class is

$$P_t = p_\gamma + p_D D_t + p_m m + \theta V_t. \tag{IA.A2}$$

Using $dP_t = p_D\, dD_t + \theta\, dV_t$ and the static investors' wealth dynamics,

$$dV_t = rV_t(1-\theta)\, dt + Q_t(dP_t + D_t dt) + \pi\, dF_t, \tag{IA.A3}$$

and substituting Equation (IA.A1) yields

$$dP_t = p_D\, dD_t + \theta\big[rV_t(1-\theta)\, dt + Q_t(dP_t + D_t dt) + \pi \overbrace{(\rho dD_t + \sqrt{1-\rho^2} dU_t)}^{dF_t}\big]. \tag{IA.A4}$$

Using Equation (1) for dD_t and collecting terms in dP shows that the price drift and diffusion

coefficients are:

$$\mu_t = \frac{p_D m + \theta\big(rV_t(1-\theta) + Q_t D_t\big)}{1-\theta Q_t}, \tag{IA.A5}$$

$$\sigma_t = \frac{\sqrt{(p_D\omega + \theta\pi\rho)^2 + \theta^2\pi^2(1-\rho^2)}}{1-\theta Q_t}. \tag{IA.A6}$$

Comparing this environment with the benchmark of Proposition 1 shows that the only change is a reallocation of diffusion loadings across the two independent shocks: the loading on the earnings shock dB_t becomes $(p_D\omega + \theta\pi\rho)$, while the loading on the orthogonal flow shock dU_t becomes $\theta\pi\sqrt{1-\rho^2}$. Define the auxiliary parameters

$$\widehat{\omega} = \omega + \frac{\theta\pi\rho}{p_D}, \qquad \widehat{\pi} = \pi\sqrt{1-\rho^2}. \tag{IA.A7}$$

Clearly, setting $\rho = 0$ yields $\widehat{\omega} = \omega$ and $\widehat{\pi} = \pi$, reverting the environment to the benchmark of Proposition 1. Moreover, the correlation coefficient only shows up in the diffusion coefficient in Equation (IA.A5). Such diffusion coefficient can be written in the same form as in Proposition 1, with ω replaced by $\widehat{\omega}$ and π replaced by $\widehat{\pi}$, as U_t plays the role of the uncorrelated flow shock. With this adjustment, the problem is traced back to the baseline model derived in Proposition 1, and the derivations carry over verbatim. Hence, the guess is verified by Appendix B. Q.E.D.

Internet Appendix B: Flexible Asset Allocation Targets

This section extends the environment of Section 3 by allowing static investors to adopt an asset allocation strategy with flexible asset allocation targets, defined as the convex combination of a fixed benchmark $\bar{\theta}$ and a state-contingent component $\hat{\theta}$. Namely,

$$\theta_t = (1-\tau)\bar{\theta} + \tau\hat{\theta}_t, \qquad \tau \in [0,1]. \tag{IA.B1}$$

The parameter τ governs flexibility over a continuum of cases, with $\tau = 0$ reproducing constant portfolio weights and $\tau = 1$ corresponding to fully state-contingent targets. For simplicity, the state-contingent asset allocation target, $\hat{\theta}$, is specified so as to replicate the optimal policy of dynamic investors. Namely,

$$\hat{\theta}_t = X_t^\star P_t / W_t. \tag{IA.B2}$$

This policy is always feasible, because there are no borrowing or short-selling constraints. Equation (IA.B2) implies that a proportion τ of the wealth of static investors is invested optimally, as is the wealth of the dynamic investors. In this case, it can be shown that the equilibrium price of the equity asset class is given by:

$$P_t = \underbrace{\mathbb{E}_t^* \left[\int_t^\infty e^{-r(s-t)} D_s \, ds \right]}_{\text{Fundamentals}} + \underbrace{(1-\tau)\bar{\theta} V_t}_{\text{Asset classification effect}}. \tag{IA.B3}$$

Thus, allowing mandates to adjust with states does not eliminate the asset-classification effect, which persists precisely to the extent that targets retain a fixed element, i.e., whenever $\tau < 1$.

Internet Appendix C: Dynamic Investors with CRRA Utility

The proof proceeds by verification that the conjectured price in Section F is an equilibrium. Differentiating the conjecture and using the dividend dynamics yields

$$dP_t = \bar{p}_\gamma dW_t + p_D dD_t + \theta dV_t. \tag{IA.C4}$$

Substituting the dynamics of dividends, dD_t, of dynamic investors' wealth, dW_t, and of static investors' wealth, dV_t, and collecting dP terms on the left-hand side yields

$$dP_t = \frac{\bar{p}_\gamma \left[(rW_t - c_t) + X_t(D_t - rP_t) \right] dt + p_D(mdt + \omega dB_t) + \theta[rV_t + Q_t(D_t - rP_t)]dt + \pi\theta dF_t}{(1 - \bar{p}_\gamma X_t - \theta Q_t)}.$$

The price drift coefficient is:

$$\mu_t = \frac{\bar{p}_\gamma \left[(rW_t - c_t) + X_t(D_t - rP_t) \right] + \frac{m}{r} + \theta \left[rV_t + Q_t(D_t - rP_t) \right]}{(1 - \bar{p}_\gamma X_t - \theta Q_t)}. \tag{IA.C5}$$

The price diffusion coefficient is:

$$\sigma_t = \frac{\sqrt{\frac{\omega^2}{r^2} + \theta^2 \pi^2}}{(1 - \bar{p}_\gamma X_t - \theta Q_t)}. \tag{IA.C6}$$

For later use, it is convenient to derive the following expression from the conjectured price.

$$rP_t = r(\bar{p} + \bar{p}_\gamma W_t) + D_t + \frac{m}{r} + r\theta V_t, \tag{IA.C7}$$

so that

$$D_t - rP_t = -r(\bar{p} + \bar{p}_\gamma W_t) - \frac{m}{r} - r\theta V_t. \tag{IA.C8}$$

Substituting this expression into Equation (IA.C5) and simplifying gives the auxiliary equation

$$\mu_t - rP_t + D_t = -\frac{\bar{p}_\gamma c_t + r\bar{p}}{1 - \bar{p}_\gamma X_t - \theta Q_t}. \tag{IA.C9}$$

The HJB equation is

$$\begin{aligned} 0 = \max_{\{c_t, X_t\}} & e^{-\delta t} U(c_t) + J_t + J_W \mathbb{E}_t[dW_t] + J_V \mathbb{E}_t[dV_t] \\ & + \frac{1}{2} J_{WW} \mathbb{E}_t[dW_t^2] + \frac{1}{2} J_{VV} \mathbb{E}_t[dV_t^2] + J_{WV} \mathbb{E}_t[dW_t dV_t]. \end{aligned} \tag{IA.C10}$$

Guess a homothetic value function of the form

$$J(W_t, V_t, t) = e^{-\delta t} \Phi\left(V_t + \phi W_t\right), \tag{IA.C11}$$

where Φ is a twice continuously differentiable, strictly increasing, and strictly concave function, and $\phi = \frac{1-\bar{p}_\gamma}{1-\theta}$ is a constant scalar. For ease of notation, define the positive state variable

$$s_t \equiv V_t + \phi W_t. \tag{IA.C12}$$

Under Equation (IA.C11), $J = e^{-\delta t}\Phi(s_t)$. The partial derivatives of the value function are

$$\begin{aligned} J_t = -\delta e^{-\delta t}\Phi(s_t), \quad J_W = e^{-\delta t}\phi\Phi'(s_t), \quad J_V = e^{-\delta t}\Phi'(s_t), \\ J_{WW} = e^{-\delta t}\left(\phi\right)^2\Phi''(s_t), \quad J_{VV} = e^{-\delta t}\Phi''(s_t), \quad J_{WV} = e^{-\delta t}\phi\Phi''(s_t). \end{aligned} \tag{IA.C13}$$

Substituting the partial derivatives in Equation (IA.C13) into the HJB in Equation (IA.C10), the first-order terms combine as

$$J_W \mathbb{E}_t[dW_t] + J_V \mathbb{E}_t[dV_t] = e^{-\delta t}\Phi'(s_t)\big(\phi \mathbb{E}_t[dW_t] + \mathbb{E}_t[dV_t]\big) = e^{-\delta t}\Phi'(s_t)\mathbb{E}_t[ds_t], \tag{IA.C14}$$

and the second-order terms combine as

$$\frac{1}{2} J_{WW} \mathbb{E}_t[dW_t^2] + \frac{1}{2} J_{VV} \mathbb{E}_t[dV_t^2] + J_{WV} \mathbb{E}_t[dW_t dV_t] = \frac{1}{2} e^{-\delta t}\Phi''(s_t)\mathbb{E}_t[ds_t^2], \tag{IA.C15}$$

where $\mathbb{E}_t[ds_t^2] = \phi^2 \mathbb{E}_t[dW_t^2] + \mathbb{E}_t[dV_t^2] + 2\phi \mathbb{E}_t[dW_t dV_t]$. Thus, the HJB reduces to

$$0 = \max_{\{c_t, X_t\}} e^{-\delta t} U(c_t) - \delta e^{-\delta t}\Phi(s_t) + e^{-\delta t}\Phi'(s_t)\frac{\mathbb{E}_t[ds_t]}{dt} + \frac{1}{2} e^{-\delta t}\Phi''(s_t)\frac{\mathbb{E}_t[ds_t^2]}{dt}. \tag{IA.C16}$$

The conditional drift of s_t is

$$\frac{\mathbb{E}_t[ds_t]}{dt} = rs_t - \phi c_t + (\phi X_t + Q_t)(\mu_t - rP_t + D_t). \tag{IA.C17}$$

The conditional quadratic variation of s_t is

$$\begin{aligned}\frac{\mathbb{E}_t[ds_t^2]}{dt} &= \phi^2 \frac{\mathbb{E}_t[dW_t^2]}{dt} + \frac{\mathbb{E}_t[dV_t^2]}{dt} + 2\phi \frac{\mathbb{E}_t[dW_t dV_t]}{dt} \\ &= \frac{\phi^2 X_t^2 (p_D^2\omega^2 + \theta^2\pi^2) + Q_t^2 p_D^2 \omega^2 + \pi^2(1-\bar{p}_\gamma X_t)^2 + 2\phi X_t \left[Q_t p_D^2 \omega^2 + \theta\pi^2(1-\bar{p}_\gamma X_t)\right]}{(1-\bar{p}_\gamma X_t - \theta Q_t)^2} \\ &= \frac{(\phi X_t + Q_t)^2 p_D^2 \omega^2 + \pi^2 (1 - \bar{p}_\gamma X_t + \phi\theta X_t)^2}{(1-\bar{p}_\gamma X_t - \theta Q_t)^2}. \end{aligned} \tag{IA.C18}$$

Furthermore, since

$$1 - \bar{p}_\gamma X_t + \phi\theta X_t = (1 - \bar{p}_\gamma X_t - \theta Q_t) + \theta(\phi X_t + Q_t), \tag{IA.C19}$$

the quadratic variation in Equation (IA.C18) can be decomposed as follows.

$$\frac{\mathbb{E}_t[ds_t^2]}{dt} = (\phi X_t + Q_t)^2 \sigma_t^2 + \frac{2\theta\pi^2(\phi X_t + Q_t)}{1 - \bar{p}_\gamma X_t - \theta Q_t} + \pi^2, \tag{IA.C20}$$

where σ_t is the price diffusion coefficient in Equation (IA.C6). Differentiating the HJB Equation (IA.C16) with respect to c_t yields the FOC for consumption,

$$c_t^\star = \left(\phi \Phi'(s_t)\right)^{-1/\gamma}. \tag{IA.C21}$$

Differentiating Equation (IA.C16) with respect to X_t yields the FOC for the portfolio choice. A complication arises because price drift and diffusion themselves depend on X_t. Specifically, differentiating Equation (IA.C5) yields

$$\frac{\partial \mu_t}{\partial X_t} = \frac{\bar{p}_\gamma(\mu_t - rP_t + D_t)}{1 - \bar{p}_\gamma X_t - \theta Q_t}, \tag{IA.C22}$$

and differentiating Equation (IA.C6) yields

$$\frac{\partial \sigma_t}{\partial X_t} = \frac{\bar{p}_\gamma \sigma_t}{1 - \bar{p}_\gamma X_t - \theta Q_t}. \tag{IA.C23}$$

The FOC of Equation (IA.C16) with respect to X_t is thus

$$0 = \left[\Phi'(s_t)(\mu_t - rP_t + D_t) + \Phi''(s_t)\left((\phi X_t + Q_t)\sigma_t^2 + \frac{\theta\pi^2}{1 - \bar{p}_\gamma X_t - \theta Q_t}\right)\right] \times \left[\phi + \frac{\bar{p}_\gamma(\phi X_t + Q_t)}{1 - \bar{p}_\gamma X_t - \theta Q_t}\right]. \tag{IA.C24}$$

As the last factor is nonzero, the portfolio FOC reduces to

$$0 = \Phi'(s_t)(\mu_t - rP_t + D_t) + \Phi''(s_t)\left[(\phi X_t + Q_t)\sigma_t^2 + \frac{\theta\pi^2}{1 - \bar{p}_\gamma X_t - \theta Q_t}\right]. \tag{IA.C25}$$

Equivalently,

$$0 = \Phi'(s_t)(\mu_t - rP_t + D_t) + \frac{\Phi''(s_t)\left[(\phi X_t + Q_t)p_D^2\omega^2 + \theta\pi^2(1 - \bar{p}_\gamma X_t + \phi\theta X_t)\right]}{(1 - \bar{p}_\gamma X_t - \theta Q_t)^2}. \tag{IA.C26}$$

Substitute $\mu_t - rP_t + D_t$ from Equation (IA.C9) into Equation (IA.C26) and rearrange terms:

$$\frac{\Phi''(s_t)}{\Phi'(s_t)} = \frac{(1 - \bar{p}_\gamma X_t - \theta Q_t)(\bar{p}_\gamma c_t + r\bar{p})}{(\phi X_t + Q_t)p_D^2\omega^2 + \theta\pi^2(1 - \bar{p}_\gamma X_t + \phi\theta X_t)}. \tag{IA.C27}$$

Next, substituting the market clearing condition, $X_t + Q_t = 1$, into Equation (IA.C19) yields

$$1 - \bar{p}_\gamma X_t + \phi\theta X_t = \phi X_t + Q_t, \tag{IA.C28}$$

the definition of $\phi = \frac{1-\bar{p}_\gamma}{1-\theta}$ implies that $\phi\theta - \bar{p}_\gamma = \phi - 1$. The denominator in the FOC in Equation (IA.C27) thus simplifies to

$$(\phi X_t + Q_t)p_D^2\omega^2 + \theta\pi^2(\phi X_t + Q_t) = (\phi X_t + Q_t)(p_D^2\omega^2 + \theta\pi^2). \tag{IA.C29}$$

Moreover, using again the definition of ϕ,

$$\frac{1 - \bar{p}_\gamma X_t - \theta Q_t}{\phi X_t + Q_t} = 1 - \theta. \tag{IA.C30}$$

Substituting Equations (IA.C28), (IA.C29), and (IA.C30) into the FOC in Equation (IA.C27) yields

$$\frac{\Phi''(s_t)}{\Phi'(s_t)} = \frac{(1 - \theta)(\bar{p}_\gamma c_t + r\bar{p})}{\frac{\omega^2}{r^2} + \theta\pi^2}. \tag{IA.C31}$$

Replacing $c_t^\star$ from Equation (IA.C21) yields the differential equation for Φ':

$$\frac{\Phi''(s_t)}{\Phi'(s_t)} = \frac{\kappa\gamma}{r\bar{p}}\left[\bar{p}_\gamma\left(\phi\Phi'(s_t)\right)^{-\frac{1}{\gamma}} + r\bar{p}\right], \tag{IA.C32}$$

where κ is a constant scalar given by $\kappa \equiv \frac{1-\theta}{\frac{\omega^2}{r^2}+\theta\pi^2}\frac{r\bar{p}}{\gamma}$. The solution to Equation (IA.C32) is

$$\Phi'(s_t) = \left(C\exp\left(\kappa s_t\right) - \frac{\bar{p}_\gamma}{r\bar{p}}(\phi)^{-\frac{1}{\gamma}}\right)^\gamma, \tag{IA.C33}$$

for a constant C chosen so that the term in parentheses is positive. To verify that Equation (IA.C33) solves Equation (IA.C32), define $\Psi(s_t) \equiv C\exp(\kappa s_t) - \frac{\bar{p}_\gamma}{r\bar{p}}(\phi)^{-\frac{1}{\gamma}}$ so that $\Phi'(s_t) = [\Psi(s_t)]^\gamma$. Differentiating yields $\Phi''(s_t) = \gamma[\Psi(s_t)]^{\gamma-1}C\kappa\exp(\kappa s_t)$. Thus, Φ is concave if and only if $C\kappa < 0$. For $C > 0$, this is equivalent to $\kappa < 0$, which holds because $r > 0$ and $\bar{p} < 0$. Moreover, the left-hand side of Equation (IA.C32) equals

$$\frac{\Phi''(s_t)}{\Phi'(s_t)} = \frac{\gamma C\kappa\exp(\kappa s_t)}{\Psi(s_t)}. \tag{IA.C34}$$

Since $\left(\phi\Phi'(s_t)\right)^{-\frac{1}{\gamma}} = [\Psi(s_t)]^{-1}\phi^{-\frac{1}{\gamma}}$, the right-hand side of Equation (IA.C32) becomes

$$\frac{\kappa\gamma}{r\bar{p}}\left[\bar{p}_\gamma[\Psi(s_t)]^{-1}\phi^{-\frac{1}{\gamma}} + r\bar{p}\right] = \frac{\kappa\gamma}{r\bar{p}}\Psi(s_t)^{-1}\left[\bar{p}_\gamma\phi^{-\frac{1}{\gamma}} + r\bar{p}\Psi(s_t)\right]. \tag{IA.C35}$$

Equating these two conditions and multiplying both sides by $\Psi(s_t)$ yields

$$\gamma C\kappa\exp(\kappa s_t) = \frac{\kappa\gamma}{r\bar{p}}\left[\bar{p}_\gamma\phi^{-\frac{1}{\gamma}} + r\bar{p}C\exp(\kappa s_t) - \bar{p}_\gamma\phi^{-\frac{1}{\gamma}}\right], \tag{IA.C36}$$

which is verified, confirming that Equation (IA.C33) satisfies the differential Equation (IA.C32). Thus, Equation (IA.C33) pins down Φ' (and hence Φ'') given the conjectured constants $\bar{p}$ and $\bar{p}_\gamma$. I now verify that the candidate value function satisfies the HJB. Evaluating Equation (IA.C16) at the controls $(c_t^\star, X_t^\star)$ and imposing market clearing, the quadratic variation in Equation (IA.C18) simplifies via Equations (IA.C28) and (IA.C30) to

$$\frac{\mathbb{E}_t[ds_t^2]}{dt} = \frac{\frac{\omega^2}{r^2} + \pi^2}{(1-\theta)^2}. \tag{IA.C37}$$

The HJB evaluated at the optimal choices becomes

$$0 = e^{-\delta t}U(c_t^\star) - \delta e^{-\delta t}\Phi(s_t) + e^{-\delta t}\Phi'(s_t)(rs_t - \phi c_t^\star) \\ + e^{-\delta t}(\phi X_t + Q_t)\Phi'(s_t)(\mu_t - rP_t + D_t) + \frac{1}{2}e^{-\delta t}\Phi''(s_t)\frac{\frac{\omega^2}{r^2} + \pi^2}{(1-\theta)^2}. \tag{IA.C38}$$

The reduced portfolio FOC at market clearing, from Equations (IA.C26) and (IA.C28), is

$$0 = \Phi'(s_t)(\mu_t - rP_t + D_t) + \frac{\Phi''(s_t)(\phi X_t + Q_t)(\frac{\omega^2}{r^2} + \theta\pi^2)}{(1 - \bar{p}_\gamma X_t - \theta Q_t)^2}. \tag{IA.C39}$$

Multiplying Equation (IA.C39) by $(\phi X_t + Q_t)$ and using Equation (IA.C30),

$$(\phi X_t + Q_t)\Phi'(s_t)(\mu_t - rP_t + D_t) = -\Phi''(s_t)\frac{\frac{\omega^2}{r^2} + \theta\pi^2}{(1-\theta)^2}. \tag{IA.C40}$$

Substituting Equation (IA.C40) into (IA.C38) yields

$$0 = e^{-\delta t}U(c_t^\star) - \delta e^{-\delta t}\Phi(s_t) + e^{-\delta t}\Phi'(s_t)(rs_t - \phi c_t^\star) \\ + \frac{1}{2}e^{-\delta t}\Phi''(s_t)\frac{(1-2\theta)\pi^2 - \frac{\omega^2}{r^2}}{(1-\theta)^2}. \tag{IA.C41}$$

Replace $c_t^\star$ using the consumption FOC in Equation (IA.C21). This yields the HJB expressed solely in terms of $\Phi(s_t)$.

$$0 = \frac{\gamma}{1-\gamma}\left(\phi\Phi'(s_t)\right)^{\frac{\gamma-1}{\gamma}} - \delta\Phi(s_t) + rs_t\Phi'(s_t) + \frac{1}{2}\Phi''(s_t)\frac{(1-2\theta)\pi^2 - \frac{\omega^2}{r^2}}{(1-\theta)^2}. \tag{IA.C42}$$

Equation (IA.C42) determines the *level* of Φ once its *slope* is pinned down by Equation (IA.C32). To see this, define

$$G(s_t) \equiv \frac{\gamma}{1-\gamma}\left(\phi\Phi'(s_t)\right)^{\frac{\gamma-1}{\gamma}} + rs_t\Phi'(s_t) + \frac{1}{2}\Phi''(s_t)\frac{(1-2\theta)\pi^2 - \frac{\omega^2}{r^2}}{(1-\theta)^2}. \tag{IA.C43}$$

Equation (IA.C42) is equivalent to the linear equation

$$\delta\Phi(s_t) = G(s_t). \tag{IA.C44}$$

Equation (IA.C32) determines $\Phi'(s_t)$, and hence $\Phi''(s_t)$, by requiring that the FOC and market clearing hold simultaneously. Once $\Phi'(s_t)$ is fixed, $\Phi(s_t)$ is determined up to a constant, C_0.

Namely,

$$\Phi(s_t) = C_0 + \int_0^{s_t} \Phi'(u)du. \tag{IA.C45}$$

Equation (IA.C42) is the condition that makes the evaluated HJB equal to zero. Once $\Phi'(s_t)$ is fixed by Equation (IA.C32), we choose the unique additive constant C_0 in $\Phi(s_t)$ so that $\delta\Phi(s_t) = G(s_t)$ holds for all s, implying that the right-hand side of the HJB vanishes. Therefore, there exists a function $\Phi(s_t)$, with $\Phi'(s_t) > 0$ and $\Phi''(s_t) < 0$, satisfying Equation (IA.C42). This completes the verification, showing that the candidate value function solves the HJB under the conjectured equilibrium price.

It remains to verify that the candidate value function satisfies the transversality condition. Under the candidate value function, $J(W_t, V_t, t) = e^{-\delta t}\Phi(s_t)$, with $s_t = V_t + \phi W_t$, it suffices to show that

$$\lim_{h\to\infty} \mathbb{E}_t\left[e^{-\delta(t+h)}\Phi(s_{t+h})\right] = 0. \tag{IA.C46}$$

Using the budget dynamics for V_t and W_t and the definition $s_t \equiv V_t + \phi W_t$, we have

$$ds_{t+h} = dV_{t+h} + \phi\, dW_{t+h}. \tag{IA.C47}$$

Taking conditional expectations,

$$\mathbb{E}_t[ds_{t+h}] = \mathbb{E}_t[dV_{t+h}] + \phi\, \mathbb{E}_t[dW_{t+h}]. \tag{IA.C48}$$

From the static investors' wealth dynamics,

$$\frac{1}{dh}\mathbb{E}_t[dV_{t+h}] = rV_{t+h} + Q_{t+h}(\mu_{t+h} - rP_{t+h} + D_{t+h}) \tag{IA.C49}$$

Similarly, from the dynamic investors' budget equation,

$$\frac{1}{dh}\mathbb{E}_t[dW_{t+h}] = rW_{t+h} - c_{t+h} + X_{t+h}(\mu_{t+h} - rP_{t+h} + D_{t+h}) \tag{IA.C50}$$

Combining these identities yields

$$\begin{aligned}\frac{d}{dh}\mathbb{E}_t[s_{t+h}] = \frac{d}{dh}\mathbb{E}_t[V_{t+h} + \phi W_{t+h}] &= \frac{1}{dh}\mathbb{E}_t[dV_{t+h}] + \phi\frac{1}{dh}\mathbb{E}_t[dW_{t+h}] \\ &= r\,\mathbb{E}_t[V_{t+h}] + r\phi\,\mathbb{E}_t[W_{t+h}] - \phi\,\mathbb{E}_t[c_{t+h}] \\ &\quad + \mathbb{E}_t[(Q_{t+h} + \phi X_{t+h})(\mu_{t+h} - rP_{t+h} + D_{t+h})]\,.\end{aligned} \tag{IA.C51}$$

By definition of s,

$$\frac{d}{dh}\mathbb{E}_t[s_{t+h}] = r\,\mathbb{E}_t[s_{t+h}] - \phi\,\mathbb{E}_t[c_{t+h}] + \mathbb{E}_t[(\phi X_{t+h} + Q_{t+h})(\mu_{t+h} - rP_{t+h} + D_{t+h})]\,. \tag{IA.C52}$$

By Equations (IA.C9) and (IA.C30),

$$(\phi X_{t+h} + Q_{t+h})(\mu_{t+h} - rP_{t+h} + D_{t+h}) = -\frac{\bar{p}_\gamma c_{t+h} + r\bar{p}}{1-\theta}. \tag{IA.C53}$$

Substituting this identity into the conditional drift of s derived above yields

$$\frac{d}{dh}\mathbb{E}_t[s_{t+h}] = r\,\mathbb{E}_t[s_{t+h}] - \phi\,\mathbb{E}_t[c_{t+h}] - \frac{\bar{p}_\gamma}{1-\theta}\mathbb{E}_t[c_{t+h}] - \frac{r\bar{p}}{1-\theta}. \tag{IA.C54}$$

Recalling that $\phi = \frac{1-\bar{p}_\gamma}{1-\theta}$, it follows that $\phi + \frac{\bar{p}_\gamma}{1-\theta} = \frac{1}{1-\theta}$. Equation (IA.C54) simplifies to

$$\frac{d}{dh}\mathbb{E}_t[s_{t+h}] = r\mathbb{E}_t[s_{t+h}] - \frac{1}{1-\theta}\mathbb{E}_t[c_{t+h}] - \frac{r\bar{p}}{1-\theta} \le r\mathbb{E}_t[s_{t+h}] - \frac{r\bar{p}}{1-\theta}, \tag{IA.C55}$$

where the inequality uses $c_{t+h} \ge 0$. Let $y(h) \equiv \mathbb{E}_t[s_{t+h}]$ and define the shifted process

$$\widetilde{y}(h) \equiv y(h) - \frac{\bar{p}}{1-\theta}. \tag{IA.C56}$$

Equation (IA.C55) implies

$$\frac{d}{dh}\widetilde{y}(h) = y'(h) \le ry(h) - \frac{r\bar{p}}{1-\theta} = r\widetilde{y}(h). \tag{IA.C57}$$

Since the function $\widetilde{y}(\cdot)$ is absolutely continuous and satisfies the differential inequality

$$\frac{d}{dh}\widetilde{y}(h) \le r\widetilde{y}(h). \tag{IA.C58}$$

Grönwall's inequality implies that, for all $h \ge 0$,

$$\widetilde{y}(h) \le e^{rh}\widetilde{y}(0). \tag{IA.C59}$$

Using $\widetilde{y}(0) = y(0) - \frac{\bar{p}}{1-\theta} = s_t - \frac{\bar{p}}{1-\theta}$ and $y(h) = \widetilde{y}(h) + \frac{\bar{p}}{1-\theta}$ yields

$$\mathbb{E}_t[s_{t+h}] \le e^{rh}s_t - \frac{\bar{p}}{1-\theta}\left(e^{rh} - 1\right), \qquad h \ge 0. \tag{IA.C60}$$

In particular, this implies the simpler inequality

$$\mathbb{E}_t[s_{t+h}] \le e^{rh}\left(s_t + \frac{|\bar{p}|}{1-\theta}\right), \qquad h \ge 0. \tag{IA.C61}$$

Next, since Φ is increasing and concave, for any $s_t \geq 0$,

$$\Phi(s_t) \leq \Phi(0) + \Phi'(0)\, s_t, \tag{IA.C62}$$

Thus,

$$\begin{aligned}\mathbb{E}_t\left[e^{-\delta(t+h)}\Phi(s_{t+h})\right] &\leq e^{-\delta(t+h)}\left(\Phi(0) + \Phi'(0)\mathbb{E}_t[s_{t+h}]\right) \\ &\leq e^{-\delta t}\left(e^{-\delta h}\Phi(0) + \Phi'(0)e^{-(\delta-r)h}\left(s_t + \frac{|\bar{p}|}{1-\theta}\right)\right).\end{aligned}$$

Hence, provided that the standard impatience condition in infinite-horizon continuous-time economies holds, namely $\delta > r$,

$$\lim_{h\to\infty} \mathbb{E}_t\left[e^{-\delta(t+h)}\Phi(s_{t+h})\right] = 0. \tag{IA.C63}$$

This guarantees that the transversality condition is respected and proves that the candidate value function is admissible and solves the HJB under the conjectured equilibrium price. Q.E.D.