# Carbon Pricing and Redistribution *

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**Abstract**

We show that carbon pricing fundamentally alters the equity-efficiency trade-off, shifting optimal policy toward greater redistribution. We study optimal redistribution and carbon pricing in an optimal income taxation model with non-homothetic demand, where households differ in carbon footprints through both spending levels and the composition of their consumption baskets. Carbon pricing separates the effective marginal tax rate from the welfare-relevant labor wedge: labor-supply reductions reduce not only tax revenue, but also emissions. This lowers the welfare cost of redistribution. The optimal policy therefore increases redistribution, but not as much as a lump-sum carbon dividend would. Instead, the optimal allocation is implemented by a rebate schedule that rises with income. Low-income households are overcompensated relative to their carbon tax burden and high-income households are undercompensated, even though the rebate itself increases with income. In a quantitative application to Germany, the optimal rebate at the 90th income percentile is nearly three times that at the 10th percentile, whereas carbon tax payments are about seven times as large. The resulting carbon-pricing package is therefore strongly progressive.

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# 1 Introduction

Carbon pricing is one of the most effective instruments for reducing greenhouse gas emissions, but it is politically contested because it creates distributional conflict. The Yellow Vests protests in France illustrate the broader challenge: higher prices for fuel, heating, and other carbon-intensive necessities reduce real purchasing power, and because lower-income households devote larger budget shares to such goods, the burden can fall relatively more heavily on them (Grainger and Kolstad 2010, Bigio et al. 2026). Compensation is therefore widely viewed as central to socially and politically sustainable carbon pricing (Blanchard et al. 2023), and survey evidence suggests that support depends not only on the carbon price itself, but also on who bears the burden and how revenues are rebated (Dechezleprêtre et al. 2025).

This paper asks how fiscal policy should respond to the distributional conflict created by carbon pricing. Our central result is that carbon pricing does not merely create a compensation problem within the standard equity-efficiency trade-off; it relaxes the trade-off itself.

When output generates carbon emissions, reductions in labor supply lower output and therefore reduce emissions. Redistribution therefore becomes less costly in welfare terms, tilting optimal policy toward greater equity and more redistribution, holding the welfare criterion fixed.

We formalize and quantify this mechanism in an optimal income taxation model with carbon emissions and non-homothetic demand. The strength of the mechanism is governed by carbon inequality: the distribution of carbon footprints and carbon tax burdens across the income distribution. This carbon-income profile matters for incidence and incentives. The profile of total carbon tax burdens determines who pays how much under carbon pricing and how these burdens vary with income. The local derivative of emissions with respect to net expenditure – the marginal propensity to pollute – determines how labor-supply responses translate into emissions changes. The larger this marginal propensity to pollute, the more redistribution reduces emissions and the larger the optimal increase in redistribution.

A useful benchmark is a lump-sum carbon dividend, which pools carbon-tax revenue and pays the same rebate to every household, leaving the income gradient in carbon tax liabilities fully reflected in net burdens. The optimal policy generally does not go this far. It partially offsets this gradient through income-dependent rebates. As a result, the optimal rebate rises with income rather than being flat. Nevertheless, low-income households are overcompensated relative to their carbon tax burden, high-income households are undercompensated. The overall policy is more redistributive than the status quo but less redistributive than a carbon dividend.

We quantify the model for Germany using household expenditure microdata and input-output based carbon emission intensities. Carbon inequality is large: households at the 90th income percentile pay roughly seven times as much in carbon taxes as households at the 10th

percentile, while the optimal rebate is only about three times as large. Despite the upward-sloping rebate schedule, the resulting carbon-policy package is therefore strongly progressive.

**Theory.** We consider an economy in which households differ by skill, make labor-supply and consumption decisions, and have non-homothetic preferences that generate heterogeneous consumption baskets across the income distribution. Carbon emissions arise from the consumption of goods with different emission intensities. We compare two otherwise identical environments: a status-quo economy in which the government sets a nonlinear income tax to balance equity and efficiency, and a carbon-constrained economy in which it must additionally satisfy a binding emissions cap. To meet the emissions target, the government imposes a uniform carbon tax and adjusts the income tax schedule accordingly.[1] The resulting change in the income tax schedule can be interpreted as an income-dependent carbon rebate. This setup isolates the effect of carbon constraints on optimal redistribution in a clean ceteris paribus comparison.

Our central analytical result is that carbon pricing separates two objects that coincide in the standard benchmark. Absent a binding carbon constraint, the effective marginal tax rate plays two roles: it is the local measure of marginal redistribution and it coincides with the welfare-relevant labor wedge, the local measure of the efficiency cost of labor-supply distortions. Under carbon pricing, by contrast, these two objects diverge. The effective marginal tax rate continues to measure marginal redistribution, but the welfare-relevant labor wedge is smaller. The reason is that labor-supply reductions not only reduce tax revenue; they also reduce emissions and relax the carbon constraint. We call the resulting gap between the effective marginal tax rate and the welfare-relevant labor wedge the carbon redistribution wedge. It is the local measure of the redistributive scope created by carbon pricing. This makes the strength of the mechanism empirically disciplined: estimating the carbon-income profile, and in particular the marginal propensity to pollute, quantifies the additional redistributive scope. The wedge is larger when the carbon tax is higher and when this local carbon-income gradient is steeper.

This decomposition clarifies the role of alternative rebate schemes. A distributionally neutral rebate offsets each household" carbon tax liability and leaves effective marginal tax rates unchanged; it therefore leaves the new redistributive scope unused. A lump-sum carbon dividend is the opposite benchmark. It does not offset the income gradient in carbon tax liabilities, so the marginal carbon tax burden is fully reflected in higher effective marginal tax rates. In this sense, the carbon dividend exhausts the redistributive scope created by the carbon constraint. The optimal policy generally lies between these two cases. Where it lies depends on how quickly marginal distributional gains decline as redistribution increases. If these gains decline strongly, the optimum remains closer to distributionally neutral compensation; if they decline only weakly, it moves closer to the carbon dividend. In the polar case in which marginal distri-

[1] In our setting it is optimal to have a uniform carbon tax. For a recent analysis of how this changes under alternative assumptions on heterogeneity and production technology, see (Bierbrauer 2025).

butional gains are constant, the carbon dividend is optimal. More generally, the optimal policy increases redistribution relative to the status quo, but by less than a carbon dividend would. Equivalently, the optimal rebate schedule rises with income, but less steeply than carbon tax payments.

**Calibration.** We calibrate the model to Germany in 2018 using household expenditure microdata, product-level carbon emission intensities, administrative income tax records, and the German tax-transfer system. A central part of the exercise is the construction of household carbon footprints. We combine detailed expenditure data from the German Income and Consumption Survey with input-output based emission intensities, accounting for both direct emissions from household consumption and indirect emissions embodied in production. This allows us to measure carbon footprints and carbon tax burdens across the income distribution.

To estimate the marginal propensity to pollute and conduct counterfactual policy analysis, we embed these data in a structural demand system. Preferences are represented by a two-tier non-homothetic CES specification in which households allocate expenditure across broad consumption categories and, within each category, between brown and green varieties. The model is disciplined by expenditure patterns along the income distribution and by product-level emission intensities. It therefore estimates how an additional euro of net expenditure translates into additional emissions at different income levels.

The counterfactual response to carbon pricing operates through both demand and supply. On the demand side, households adjust their consumption baskets by substituting across goods and between brown and green varieties. On the supply side, we allow emission intensities to respond endogenously through a DICE-style convex abatement technology: higher carbon prices induce abatement and thereby reduce the residual emissions embodied in consumption. We combine this demand-and-abatement block with administrative income tax data and an estimated German tax-transfer system to quantify the redistributive scope created by carbon pricing and the optimal income-dependent rebate schedule.

**Quantitative Results.** We first evaluate a carbon cap that reduces aggregate emissions by 25%. The optimal rebate schedule is increasing in income: households at the 90th income percentile receive 3.33 times the rebate of households at the 10th percentile. Yet the overall carbon policy is strongly progressive, because carbon-tax payments rise much more steeply with income: households at the 90th percentile pay 7.92 times as much in carbon taxes as households at the 10th percentile. These results are robust across alternative parameter values.

We then trace the optimal policy along the decarbonization path by varying the emissions-reduction target. The relative incidence patterns remain remarkably stable: the 90-to-10 ratios of optimal rebates and carbon-tax payments stay close to their benchmark values. What changes is the quantitative importance of the policy. As the carbon cap tightens, carbon-tax

revenue initially rises, so the induced increase in redistribution becomes larger. This effect peaks around a 65% emissions reduction; beyond that point, the shrinking carbon tax base dominates and revenues begin to decline. Thus, compensation and rebate design become increasingly important over a substantial part of the decarbonization path, not only for small carbon-pricing reforms.

**Related Literature.** The interaction between income and carbon taxation was first analyzed in the so-called double-dividend literature. In representative agents models, it has been shown that the optimal carbon tax should be below the Pigouvian tax when distortionary income taxes are present, since carbon taxation erodes the income tax base (see, e.g., Bovenberg and De Mooij, 1994; Bovenberg and van der Ploeg, 1994; Goulder, 1995).[2]

Kaplow (2012) and Jacobs and De Mooij (2015) integrate externalities into a framework with heterogeneous agents and redistributive income taxation.[3] Both show that, under weak separability, the optimal carbon tax coincides with the Pigouvian tax, even in the presence of distortionary income taxation. Recent complementary work by Craig, Lloyd, and Moore (2025) asks when distributional concerns justify moving environmental taxes away from the Pigouvian benchmark. They show that the Pigouvian benchmark can fail when differences in carbon intensity across the income distribution are driven by preference heterogeneity rather than by pure income effects. Our paper complements this literature. We deliberately focus on the weak-separability environment in which the optimal carbon tax continues to equal the Pigouvian benchmark. We show that even in that setting, carbon pricing changes the equity-efficiency trade-off itself and thereby the optimal degree of redistribution as well as the schedule of income-dependent rebates.[4]

A closely related strand studies additivity and targeting in optimal tax problems with externalities. Sandmo (1975) established the classic additivity result: with an atmospheric externality, the externality alters the optimal tax on the dirty good, while the tax formulas for other instruments remain unchanged. Cremer, Gahvari, and Ladoux (1998) show that whether this logic extends to income taxation depends on the available commodity tax instruments, while Kopczuk (2003) generalizes the argument into a broader principle of targeting. In a Mirrleesian setting, Micheletto (2008) shows that this additivity logic extends to atmospheric externalities

[2] A recent modern quantitative treatment of this channel can be found in Barrage (2020) who finds that the optimal carbon tax should be 8-24% lower.

[3] Focusing on a linear income tax, Jacobs and Van der Ploeg (2019) show that if carbon Engel curves are linear, the optimal pollution tax follows a first-best rule. With non-linear carbon Engel curves, however, carbon taxes acquire a redistributive role that may push them above or below the Pigouvian level. Hänsel, Franks, Kalkuhl, and Edenhofer (2022) extend the analysis by adding horizontal inequality in carbon emissions and by considering subsidies for clean energy as an additional policy instrument.

[4] Ahlvik, Liski, and Mäkimattila (2025) study a Mirrleesian environment with multidimensional heterogeneity in earning ability and mitigation costs. They show that the optimal externality tax varies with income and apply their framework to vehicle use and electricity consumption. Their analysis focuses on income as a signal of heterogeneous mitigation costs and horizontal-equity concerns, whereas our mechanism operates through the effect of additional expenditure on emissions.

under asymmetric information. At the level of statutory tax formulas, our results remain consistent with the classic additivity logic. Our contribution is to show that this does not carry over to effective redistribution, because carbon pricing drives a wedge between the effective marginal tax rate and the welfare-relevant labor wedge.

Our paper also relates to recent quantitative work on carbon pricing and heterogeneous households. Goulder, Hafstead, Kim, and Long (2019) quantify the distributional incidence of carbon taxes across US households and show that the net burden depends crucially on how revenues are recycled. van der Ploeg, Rezai, and Tovar Reaños (2022) study how alternative recycling schemes shape equity and political support for green tax reform using German household survey data. Douenne, Hummel, and Pedroni (2026) analyze jointly optimal carbon and income taxes in a dynamic climate-economy model with heterogeneous households; Douenne, Dyrda, Hummel, and Pedroni (2025) extend the analysis to incomplete markets and uninsurable income risk. By contrast, our paper makes explicit and quantifies a mechanism through which carbon pricing affects redistribution: it opens a gap between the effective marginal tax rate and the welfare-relevant labor wedge.

**Structure.** In Section 2, we explain the main mechanism for our result in a simple model. We then introduce our model in Section 3 and characterize the optimal tax-transfer system in the absence of carbon policies in Section 4. In Section 5 we then turn to the optimal design of carbon rebate policies. In Section 6, we calibrate our model to German economy of 2018. Section 7 presents the quantitative results and Section **??** discusses further aspects. Section 8 concludes.

# 2 The Main Mechanism

This section provides a reduced-form explanation of the mechanism underlying our main result. Consider a representative household of productivity $\theta$ who chooses labor supply $l$. Gross labor income is then given by $y = l\theta$ and expenditure equals $e(y) = y - \mathcal{T}(y)$, where $\mathcal{T}(y)$ is the tax-transfer system. Consumption expenditure $e$ generates carbon emissions $f(e)$, which are taxed at rate $\tau_{\text{co2}}$ per unit of carbon. Total tax revenue raised from this household is given by:

$$\mathcal{R}(y) = \mathcal{T}(y) + \tau_{\text{co2}} f(e(y)).$$

**Effective Marginal Tax Rate** Define the effective marginal income tax rate (EMTR) as the increase in fiscal revenue induced by a one-unit increase in income:

$$\tau_{\text{eff}}(y) \equiv \frac{d\mathcal{R}(y)}{dy} = \mathcal{T}'(y) + \tau_{\text{co2}}(1 - \mathcal{T}'(y))MPP(y). \tag{1}$$

where $\mathcal{T}'(y)$ is the marginal income tax rate and $MPP(y)$ is the marginal propensity to pollute out of expenditure of an individual with income $y$, formally defined by $MPP(y) \equiv f'(e(y))$. It captures the increase in the carbon footprint that results from one more unit of spending. Hence, the EMTR exceeds the marginal income tax rate whenever higher expenditure raises emissions, because an additional unit of gross income generates not only income tax revenue but also carbon tax revenue. In general, a higher $\tau_{\text{eff}}(y)$ means that the total tax burden rises more steeply with gross income, implying that the tax system is more redistributive at the margin.

In the absence of externalities, the EMTR is the relevant metric for the welfare effects of changes in labor supply in response to policies. By the envelope theorem, a change in labor supply has no first-order effects on individual utility. To first order, the welfare effect of a change in labor supply operates through the induced change in government revenue (fiscal externalities) equal to $dy \times \tau_{\text{eff}}(y)$ (see e.g. Hendren (2016)).

**Welfare Wedge** In the presence of externalities, the EMTR no longer summarizes the welfare effects of labor supply responses because an income change also affects carbon emissions. To make this explicit, consider a government that maximizes a standard welfare function $\mathcal{W}$ subject to (i) a revenue requirement and (ii) a carbon emission cap $F \leq \bar{F}$, where $F$ denotes aggregate emissions in the economy and $\bar{F}$ is an exogenous limit on emissions.[5] Let $\mathcal{L}$ denote the Lagrangian of the government optimization problem, and let $\Lambda$ and $\mu$ be the multipliers on the revenue requirement and the emissions cap, respectively.

Consider the welfare effect of a marginal increase in gross labor income $y$ induced by some policy change:

$$\frac{\partial \mathcal{L}}{\partial y} dy = \Lambda \Bigg( \underbrace{\tau_{\text{eff}}(y)}_{\text{fiscal externality}} - \underbrace{\frac{\mu}{\Lambda}(1 - \mathcal{T}'(y)) MPP(y)}_{\text{carbon cap externality}} \Bigg) dy \equiv \Lambda \underbrace{\mathrm{T}(y)}_{\text{welfare wedge}} dy. \tag{2}$$

where we use $df/dy = (1 - \mathcal{T}'(y))MPP(y)$. The first term in (2) captures the fiscal gain from higher labor income, while the second term captures the welfare loss from tightening the emissions constraint. The welfare wedge $\mathrm{T}(y)$ is, by construction, the welfare-relevant wedge for labor supply changes:

$$\mathrm{T}(y) = \mathcal{T}'(y) + \left(\tau_{\text{co2}} - \frac{\mu}{\Lambda}\right)(1 - \mathcal{T}'(y))\, MPP(y).$$

Two implications follow immediately. First, if the carbon cap is slack ($\mu = 0$), then we obtain the usual result that this wedge equals the EMTR: $\mathrm{T}(y) = \tau_{\text{eff}}(y)$. In this case, the welfare effect of an income change is fully summarized by the EMTR. Second, if the cap binds

[5] Equivalently, one could incorporate the atomistic externality directly into the welfare function. The argument would be unchanged, see Section 3.3.

($\mu > 0$) and emissions rise with income ($MPP(y) > 0$), then $\mathrm{T}(y) < \tau_{\text{eff}}(y)$. In this case, a labor supply reduction has the positive side effect that it relaxes the carbon cap constraint.

As we show formally in Lemma 3 for our main model, optimal carbon pricing satisfies $\tau_{\text{co2}} = \mu/\Lambda$. In that case, $\mathrm{T} = \mathcal{T}'(y)$. Intuitively, the negative fiscal externality from lower carbon tax revenue exactly offsets the positive effect implied by the relaxation of the carbon cap constraint.

Under optimal carbon pricing, the gap between the EMTR and the welfare wedge is then given by:

$$\tau_{\text{eff}}(y) - \mathrm{T}(y) = \tau_{\text{co2}} \left(1 - \mathcal{T}'(y)\right) MPP(y).$$

This equation will be central for our results below. A larger gap implies a larger adjustment in the optimal degree of redistribution following an introduction of a binding exogenous limit on carbon emissions. Mechanically, the gap is increasing in (i) the level of the carbon tax and (ii) the marginal propensity to pollute. It decreases in the amount of redistribution as captured by the marginal income tax rate $\mathcal{T}'(y)$.

# 3 Model

## 3.1 Model Basics

**Heterogeneity.** We consider a static economy with a continuum of heterogeneous households of total mass one, indexed by their productivity $\theta \in [\underline{\theta}, \bar{\theta}]$. Let $H(\theta)$ denote the cumulative distribution function of productivity, with density $h(\theta)$. Households earn a gross labor income given by $y = l\theta$, where $l$ denotes labor supply.

**Preferences.** Households make a labor-leisure decision and allocate their income across a vector of consumption goods and services $\mathbf{c} = (c_1, c_2, \ldots, c_I)$. Preferences over consumption and labor supply are represented by

$$U(\mathbf{c}) - \frac{l^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}},$$

where $\varphi > 0$ denotes the Frisch elasticity of labor supply. The utility function is weakly separable in consumption and leisure. In the absence of externalities, this implies that uniform commodity taxation is optimal (Atkinson and Stiglitz 1976).

For parts of the theoretical analysis, we impose the following functional form assumption on preferences, which sharpens the theoretical analysis and isolates the main mechanism.

**Assumption 1.** *Consumption utility is given by a homothetic CES aggregator,*

$$U(\mathbf{c}) = \left( \sum_{i=1}^{I} \Omega_i^{\frac{1}{\sigma}} \, c_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

*with* $\sigma > 0$ *and weights* $\Omega_i > 0$.

Assumption 1 implies homothetic demands and, in particular, an indirect utility from consumption that is linear in total expenditure. Hence, the marginal utility of expenditure does not vary with expenditure implying zero income effects in labor supply. In the theoretical analysis, Assumption 1 serves two purposes. First, for the results on the equity-efficiency trade-off (Lemma 2 and Proposition 2), it is imposed only for expositional clarity; the corresponding results without this restriction are provided in Appendix A. Second, the sharp comparative statics results in Section 5.3 rely on this assumption.

**Government.** The government levies a nonlinear income tax schedule $\mathcal{T}(y)$ and a carbon tax $\tau_{\text{co2}}$ per unit of emission.[6]

**Carbon Emissions.** Absent abatement, consumption of one unit of good $i$ generates $\bar{f}_i > 0$ units of carbon emissions, where the emission intensity varies across goods. We then allow firms to reduce these baseline emissions through abatement, modeled in reduced form following the abatement block of the DICE model (Barrage and Nordhaus 2024). Firms choose abatement $a_i \in [0, \bar{f}_i]$ at convex resource cost $\kappa_i(a_i)$ per unit. The resulting endogenous carbon emissions intensity for one unit of good $i$ is $f_i = \bar{f}_i - a_i$.

Let $q_i$ denote the marginal resource cost of producing one unit of good $i$ before abatement costs and carbon taxes. Competitive firms take the carbon tax $\tau_{\text{co2}}$ as given and choose abatement to minimize the unit cost of good $i$. The resulting unit price is therefore

$$p_i(\tau_{\text{co}_2}) \equiv q_i + \min_{a_i \in [0, \bar{f}_i]} \left\{ \kappa_i(a_i) + \tau_{\text{co}_2}(\bar{f}_i - a_i) \right\},$$

and equals the marginal cost of production given optimal abatement. The optimal abatement $a_i^*(\tau_{\text{co}_2})$ is implicitly defined by $\kappa_i'(a_i^*(\tau_{\text{co}_2})) = \tau_{\text{co}_2}$: firms abate up to the point where the marginal cost of abatement equals the marginal savings in carbon tax payments. The resulting endogenous emissions intensity per unit of good $i$ is then given by

$$f_i(\tau_{\text{co}_2}) \equiv \bar{f}_i - a_i^*(\tau_{\text{co}_2}).$$

We adopt a consumption-based accounting perspective: the residual emissions generated along the production chain of one unit of good $i$ are attributed to the final consumption of

[6] In the Appendix A, all results are also derived for the case with a uniform commodity tax.

that unit. Hence, after firms optimally choose abatement, household consumption of one unit of good $i$ is associated with $f_i(\tau_{\text{co2}})$ units of emissions, even though these emissions physically arise during production. Under perfect competition, the carbon tax is fully passed through to consumer prices, so purchasing one unit costs $p_i(\tau_{\text{co}_2})$.

By the envelope theorem, the effect of the carbon tax on the consumer price is given by

$$p_i'(\tau_{\text{co}_2}) = f_i(\tau_{\text{co}_2}). \tag{3}$$

That is, a marginal increase in the carbon tax raises the price of good $i$ by exactly its residual emissions intensity. This identity will be useful below when characterizing how carbon taxes affect household carbon footprints and labor-supply incentives.

## 3.2 Household decision problem given taxes

We divide the household's decision problem into two parts. First, households choose their optimal consumption allocation given net income $e(\theta) = y(\theta) - \mathcal{T}(y(\theta))$. Second, households choose labor supply given the optimal consumption allocation.

The first stage is given by

$$u(e(\theta); \mathbf{p}, \tau_{\text{co}_2}) = \max_{\mathbf{c}} U(\mathbf{c}) \quad \text{s.t.} \quad \sum_{i=1}^{I} c_i \, p_i(\tau_{\text{co}_2}) = e(\theta). \tag{4}$$

Note that this consumption allocation problem – for a given level of expenditures $e$ – does not depend on productivity $\theta$ due to the separability of the utility function. The first-order-condition is given by:

$$\forall i: \ \lambda(e(\theta); \mathbf{p}, \tau_{\text{co}_2}) = \frac{U_i\left(\mathbf{c}\left(e(\theta); \mathbf{p}, \tau_{\text{co}_2}\right)\right)}{p_i(\tau_{\text{co}_2})}, \tag{5}$$

where $\lambda(e(\theta); \mathbf{p}, \tau_{\text{co}_2})$ is the Lagrangian multiplier on the household's budget constraint. By the envelope theorem, it follows that $u_e(e(\theta); \mathbf{p}, \tau_{\text{co}_2}) = \lambda(e(\theta); \mathbf{p}, \tau_{\text{co}_2})$. Let $c_i(e(\theta); \mathbf{p}, \tau_{\text{co}_2})$ denote the optimal consumption of good $i$, and define the resulting carbon footprint as

$$f(e(\theta); \mathbf{p}, \tau_{\text{co}_2}) \equiv \sum_{i=1}^{I} f_i(\tau_{\text{co}_2}) \, c_i(e(\theta); \mathbf{p}, \tau_{\text{co}_2}).$$

The second stage of the household's problem is given by

$$V(\theta; \mathbf{p}, \mathcal{T}, \tau_{\text{co}_2}-) = \max_{y} u(e; \mathbf{p}, \tau_{\text{co}_2}) - \frac{(y/\theta)^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \quad \text{s.t.} \quad e = y - \mathcal{T}(y). \tag{6}$$

The corresponding first-order condition of the labor supply is given by

$$u_e(e(\theta, \mathbf{p}, \mathcal{T}, \tau_{\text{co}_2}); \mathbf{p}, \tau_{\text{co}_2})\, \theta\, [1 - \mathcal{T}'(y(\theta, \mathbf{p}, \mathcal{T}, \tau_{\text{co}_2}))] = \left( \frac{y(\theta, \mathbf{p}, \mathcal{T}, \tau_{\text{co}_2})}{\theta} \right)^{\frac{1}{\varphi}}. \tag{7}$$

**Simplifying Notation.** In the following, we suppress the dependence of all variables on the exogenous price vector $\mathbf{p}$ and define the policy vector $\mathcal{P} \equiv (\mathcal{T}, \tau_{\text{co}_2})$. We denote the optimal income and expenditure choices as $y(\theta, \mathcal{P})$ and $e(\theta, \mathcal{P})$, respectively. We often use $u_e(\theta, \mathcal{P})$ as short-hand notation for $u_e(e(\theta, \mathcal{P}); \mathbf{p}, \tau_{\text{co}_2})$, and similarly we write $f(\theta, \mathcal{P}) \equiv f(e(\theta, \mathcal{P}); \mathbf{p}, \tau_{\text{co}_2})$. For ease of notation, we omit dependence on $\mathcal{P}$ when the context is clear.

**Comparative Statics.** We now define the parameters that govern how households adjust their carbon emissions in response to a change in the carbon tax. First, we define the marginal propensity to pollute (MPP) as the partial effect of net income $e$ on household emissions:[7]

$$MPP(\theta, \mathcal{P}) \equiv \frac{\partial f(\theta, \mathcal{P})}{\partial e}.$$

Second, we define the partial effect of the carbon tax $\tau_{\text{co}_2}$ on household emissions as

$$f_{\tau_{\text{co}_2}}(\theta, \mathcal{P}) \equiv \frac{\partial f(\theta, \mathcal{P})}{\partial \tau_{\text{co}_2}}.$$

Note that $f_{\tau_{\text{co}_2}}$ reflects both changes in the consumption allocation across goods and changes in emissions intensities through abatement. The total effect of a change in the carbon tax $\tau_{\text{co}_2}$ on household emissions is then given by (omitting dependence of $\mathcal{P}$):

$$\frac{df(\theta)}{d\tau_{\text{co}_2}} = f_{\tau_{\text{co}_2}}(\theta) + MPP(\theta)\, (1 - \mathcal{T}'(y(\theta)))\, \frac{\partial y(\theta)}{\partial \tau_{\text{co}_2}},$$

where the first part captures the direct effect of the carbon tax on carbon emissions, holding total expenditure constant. The second term captures the indirect effect through the labor supply response to the carbon tax and the resulting change in the level of net income.

In the following, we derive an expression for the impact of carbon taxes on labor supply, $\frac{\partial y(\theta)}{\partial \tau_{\text{co}_2}}$, which depends on classical labor supply elasticities. This helps to relate the labor supply effects of carbon taxes to the well-known labor supply effects of income taxes. First, the elasticity of income with respect to the retention rate $1 - \mathcal{T}'$ and the marginal propensity to earn out of unearned income:

$$\varepsilon_{y,\, 1-\mathcal{T}'}(\theta) \equiv \frac{d \log(y(\theta))}{d \log(1 - \mathcal{T}')} \qquad \eta(\theta) \equiv \frac{\partial y(\theta)}{\partial \mathcal{T}(0)}.$$

The exact formal expressions for both objects are standard and can be found in Appendix A.3.

[7] Under Assumption 1, demands are homothetic, so carbon footprints are proportional to expenditure. Hence $MPP(\theta, \mathcal{P})$ is constant across $\theta$ in that benchmark.

We now provide a result for the labor supply response to a carbon tax reform $d\tau_{co_2}$:[8]

**Lemma 1.** *The labor supply change resulting from a marginal increase in the carbon tax $d\tau_{co_2}$ is equivalent to the labor supply change resulting from an income tax reform*

$$d\mathcal{T}(y(\theta)) = f(\theta)d\tau_{co_2}. \tag{8}$$

*The change in income is given by:*

$$\frac{\partial y(\theta)}{\partial \tau_{co_2}} d\tau_{co_2} = -\frac{\varepsilon_{y,1-\mathcal{T}'}(\theta) y(\theta)}{1-\mathcal{T}'(y(\theta))} MPP(\theta)\ (1-\mathcal{T}'(y(\theta)))\, d\tau_{co_2} + \eta(\theta) f(\theta) d\tau_{co_2}. \tag{9}$$

*Proof.* See Appendix A.1. □

Lemma 1 decomposes the labor-supply effect of a carbon tax into two channels: (i) a substitution effect that operates through the marginal payment on additional earnings, and (ii) an income effect that operates through the reduction in disposable resources.

The first term in (9) captures the substitution effect of the carbon tax on labor supply. A marginal increase in the carbon tax changes labor supply incentives in exactly the same way as an increase in the marginal income tax rate of[9]

$$d\mathcal{T}'(y(\theta)) = MPP(\theta)\,(1-\mathcal{T}'(y(\theta)))\,d\tau_{co_2}. \tag{10}$$

To see this, note that one additional unit of gross income generates $(1-\mathcal{T}'(y(\theta)))$ units of extra expenditure. Since an extra unit of expenditure raises emissions by $MPP(\theta)$, the associated increase in carbon tax payments is $MPP(\theta)\,(1-\mathcal{T}'(y(\theta)))\,d\tau_{co_2}$, which is exactly the same wedge created by the income tax reform in (10). This is analogous to the labor-supply effects of savings taxation emphasized by Ferey, Lockwood, and Taubinsky (2024).

The second term in (9) captures the income effect. A marginal increase in the carbon tax reduces household income by $f(\theta)d\tau_{co_2}$, generating an income effect equivalent to that of the income tax reform in (8).

## 3.3 Government and Policies

**Welfare and Constraints.** The government maximizes a standard social welfare function,

$$\mathcal{W}(\mathcal{P}) = \int_{\underline{\theta}}^{\overline{\theta}} \mathcal{G}\left(V(\theta,\mathcal{P})\right)\omega(\theta)\,dH(\theta), \tag{11}$$

[8] This is equivalent to Lemma 1 of Saez (2002), who studies the desirability of non-uniform commodity taxes in the presence of preference heterogeneity.

[9] Equation (10) follows directly from differentiating (8).

where the weights $\omega(\theta)$ are unrestricted and normalized to integrate to one. We assume that either (i) $\mathcal{G}(x) = x$ or (ii) $\mathcal{G}'(x) > 0$, $\mathcal{G}''(x) < 0$. The latter captures aversion to inequality of utilities. In the latter case, we further assume that $\mathcal{G}(x)$ is such that the coefficient of relative utility-inequality aversion, $-\frac{\mathcal{G}''(V)}{\mathcal{G}'(V)}V$, is non-increasing in $V$.

We define the social marginal utility of type $\theta$ as[10]

$$g(\theta, \mathcal{P}) = \frac{\partial \mathcal{W}(\mathcal{P})}{\partial e(\theta)} \frac{1}{h(\theta)} = \mathcal{G}'\left(V(\theta, \mathcal{P})\right) \omega(\theta) u_e(\theta, \mathcal{P}). \tag{12}$$

The government budget constraint is given by:

$$\int_{\underline{\theta}}^{\overline{\theta}} \left(\mathcal{T}(y(\theta, \mathcal{P})) + \tau_{\text{co}_2} f(\theta, \mathcal{P})\right) dH(\theta) \geq \mathcal{R}, \tag{13}$$

where $\mathcal{R}$ is an exogenous revenue requirement. The associated Lagrangian multiplier is denoted by $\Lambda$.

Besides the government budget constraint, we assume that the government has to obey a carbon cap constraint

$$\int_{\underline{\theta}}^{\overline{\theta}} f(\theta, \mathcal{P}) dH(\theta) \equiv F \leq \bar{F}, \tag{14}$$

where $\bar{F}$ is an exogenous cap on aggregate carbon emissions. We denote the Lagrangian multiplier of the carbon cap constraint by $\mu$. This constraint captures, for example, limits imposed by international carbon reduction agreements. All our results below would carry over if, instead of a cap, we modeled carbon emissions as an atmospheric externality, with welfare net of the externality given by $\mathcal{W} - D(F)$, where the damage function $D(F)$ is monotonically increasing in aggregate emissions $F$.

**Effective Marginal Tax Rate.** We define the effective marginal tax rate as the increase in total tax revenue resulting from a one unit increase of household's gross labor income:

$$\tau_{\text{eff}}(\theta, \mathcal{P}) = \mathcal{T}'(y(\theta)) + \left(1 - \mathcal{T}'(y(\theta))\right) MPP(\theta)\, \tau_{\text{co}_2}. \tag{15}$$

Besides the marginal income tax rate $\mathcal{T}'(\cdot)$, it also accounts for the additional tax revenue generated through higher carbon emissions resulting from a one unit increase in gross household income.[11] The derivation of $\tau_{\text{eff}}(\theta)$ is provided in Appendix A.2. Note that $\tau_{\text{eff}}(\theta, \mathcal{P})$ is the relevant concept to capture the fiscal externalities that arise from changes in incomes of type $\theta$.

[10] This definition slightly differs from the standard definition of endogenous social welfare weights going back to Saez (2001), as we do not normalize them by the shadow value of public funds. In our optimal policy formulas below, the reader can interpret the social marginal utility $g(\theta, \mathcal{P})$ as being expressed in units of public funds, since all optimality conditions involve it only through relative terms.

[11] Under Assumption 1, $MPP(\theta)$ is constant across types. In that benchmark, cross-sectional variation in the carbon component of $\tau_{\text{eff}}(\theta, \mathcal{P})$ arises only through the net-of-tax factor $1 - \mathcal{T}'(y(\theta))$.

### 3.4 Incidence of Carbon Taxes

We briefly discuss the incidence of a carbon tax. As shown in Lemma 1, the labor supply effect of a carbon tax increase is equivalent to that of an income tax reform defined in (8), implying the same distributional impact. Hence, the two reforms are equivalent in terms of their impact on earnings and redistribution. The key difference between the two tax reforms lies in the carbon tax's additional effect on the allocation of expenditure across goods by consumers and on abatement choices by firms. This reallocation matters for the incidence since it alters household carbon footprints. Holding income and total expenditures fixed, the carbon footprint of household $\theta$ changes by $f_{\tau_{\text{co}_2}}(\theta)d\tau_{\text{co}_2}$. Hence, a carbon tax hike implies an additional fiscal externality given by

$$\tau_{\text{co}_2} \int_{\underline{\theta}}^{\overline{\theta}} f_{\tau_{\text{co}_2}}(\theta)\, dH(\theta)\, d\tau_{\text{co}_2}. \tag{16}$$

Since higher carbon taxes lower carbon emissions, $f_{\tau_{\text{co}_2}} < 0$, there is a loss of carbon tax revenue if $\tau_{\text{co}_2} > 0$. If the carbon cap constraint is not binding, (16) captures the welfare effect of jointly increasing the carbon tax by $d\tau_{\text{co}_2}$ and implementing the income tax reform defined in (8), but with the opposite sign. In this case, optimality of the policy $\mathcal{P}$ implies $\tau_{\text{co}_2} = 0$. Put differently, when the carbon cap constraint (14) is slack, raising the carbon tax is strictly inferior to implementing the equivalent income tax reform defined in (8).[12] In Section 4, we first consider this benchmark without a binding carbon cap and characterize the optimal 'status quo' policy $\mathcal{P}_{sq} = (\mathcal{T}_{sq}, 0)$.

In Section 5, we introduce a binding carbon cap, which makes it optimal for the government to impose a positive carbon tax. We characterize the resulting optimal policy as $\mathcal{P}_{co_2} = (\mathcal{T}_{co_2}, \tau_{\text{co}_2})$.[13] We interpret the difference between the two income tax schedules as the optimal carbon tax rebate:

$$R_{co_2}(y) \equiv \mathcal{T}_{\text{sq}}(y) - \mathcal{T}_{co_2}(y),$$

and derive its key properties. We also examine whether $\mathcal{P}_{co_2}$ is more or less redistributive than the status quo policy $\mathcal{P}_{sq}$.

[12] This logic mirrors the Atkinson-Stiglitz result on uniform commodity tax: differentiated commodity taxes cannot achieve distributional goals or influence labor supply incentives beyond what income taxes can, but they do introduce additional distortions on the consumption allocation (Atkinson and Stiglitz 1976).

[13] This involves as slight abuse of notation: for $\mathcal{P}$ and $\mathcal{T}$, the subscript $co2$ denotes optimality under the carbon cap constraint, whereas for the carbon tax $\tau_{\text{co}_2}$, the subscript was used earlier even when the carbon tax was not set optimally.

# 4 Normative Benchmark: No Carbon Cap

As a first step, we characterize the optimal tax-transfer system in the absence of a carbon cap constraint, which we refer to as the status-quo policy throughout the following. The government's optimization problem is then given by

$$\max_{\mathcal{T}(\cdot)} \int_{\underline{\theta}}^{\overline{\theta}} \mathcal{G}\left(V(\theta;\mathcal{P})\right)\omega(\theta)dH(\theta) \tag{17}$$

subject to a government budget constraint (13) and optimal household behavior (4) and (6). As shown in Section 3.4, the carbon tax is zero in the absence of a binding cap. Hence, only the first element of the optimal status-quo policy vector $\mathcal{P}_{sq} = (\mathcal{T}_{sq}, 0)$ – the income tax schedule – is endogenous.

The following lemma describes the optimal income tax system in the form presented by Heathcote and Tsujiyama (2021). Its derivation is standard in the literature. We focus on the case of Assumption 1 for the ease of exposition. In Appendix A.4, we present also the more general result.

**Lemma 2.** *Assume that Assumption 1 holds. Denote the optimal status quo policy by* $\mathcal{P}_{\text{sq}} = (\mathcal{T}_{\text{sq}}, \tau_{\text{co}_2} = 0)$. *The optimal income tax schedule* $\mathcal{T}_{\text{sq}}$ *that solves (17) satisfies*

$$\forall\, \theta^* :\ D(\theta^*, \mathcal{P}_{\text{sq}}) = E(\theta^*, \mathcal{P}_{\text{sq}}),$$

*where*

$$D(\theta^*, \mathcal{P}_{\text{sq}}) = 1 - \frac{\mathbb{E}[g(\theta, \mathcal{P}_{\text{sq}})|\theta \geq \theta^*]}{\mathbb{E}[g(\theta, \mathcal{P}_{\text{sq}})]} \tag{18}$$

*captures the distributional gains of raising the marginal tax rate* $\mathcal{T}'(y(\theta))$ *and*

$$E(\theta^*, \mathcal{P}_{\text{sq}}) = \frac{\tau_{\text{eff}}(\theta^*, \mathcal{P}_{\text{sq}})}{1 - \mathcal{T}'_{\text{sq}}(y(\theta^*))} \frac{\varphi}{1+\varphi} \frac{h(\theta^*)\theta^*}{1 - H(\theta^*)} \tag{19}$$

*captures the efficiency cost of raising the marginal tax rate.*

*In this benchmark,* $\tau_{\text{co2}} = 0$ *and thus* $\tau_{\text{eff}}(\theta, \mathcal{P}_{\text{sq}}) = \mathcal{T}'_{\text{sq}}(y(\theta))$. *This highlights the usual benchmark: absent a binding carbon constraint, the EMTR governs both marginal redistribution and marginal efficiency costs.*

*Proof.* See Appendix A.4. □

In this benchmark, $\tau_{\text{co2}} = 0$ and thus $\tau_{\text{eff}}(\theta, \mathcal{P}_{\text{sq}}) = \mathcal{T}'_{\text{sq}}(y(\theta))$. This highlights the usual benchmark: absent a binding carbon constraint, the EMTR governs both marginal redistribution and marginal efficiency costs. Concretely, the optimal marginal tax rate $\mathcal{T}'_{\text{sq}}(y(\theta^*))$ balances optimally the distributional gains $D(\theta^*, \mathcal{P}_{\text{sq}})$ with the efficiency cost $E(\theta^*, \mathcal{P}_{\text{sq}})$. The term $D(\theta^*, \mathcal{P}_{\text{sq}})$ measures the social marginal utility of individuals earning more than $y(\theta^*)$

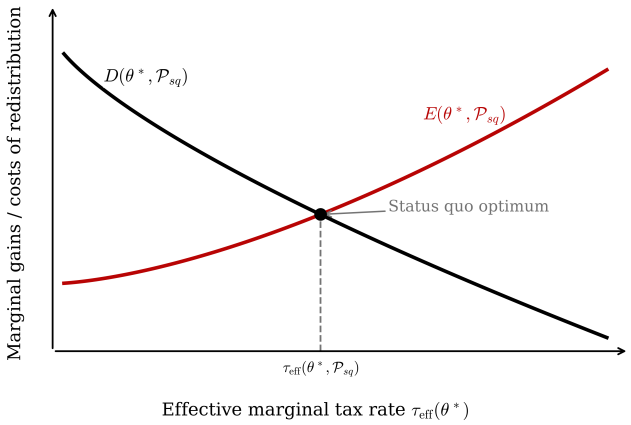



Figure 1: Status-quo equity–efficiency trade-off

*Notes:* In the absence of a binding carbon cap, marginal redistribution is governed by the standard Mirrleesian condition $D(\theta^*, \mathcal{P}_{sq}) = E(\theta^*, \mathcal{P}_{sq})$. The black curve shows the marginal gains from redistribution, the red curve the corresponding marginal efficiency costs. Their intersection determines the status-quo optimal tax system and the associated effective marginal tax rate $\tau_{\text{eff}}(\theta^*, \mathcal{P}_{sq})$.

relative to the population average. Thus, it captures the gains of redistributing incomes from individuals with income above $y(\theta^*)$ to those with income below $y(\theta^*)$.

The term $E(\theta^*, \mathcal{P}_{\text{sq}})$ captures the efficiency costs arising from changes in labor supply. Since changes in income affect welfare through fiscal externalities, they are weighted by the effective marginal tax rate $\tau_{\text{eff}}(\theta, \mathcal{P}_{\text{sq}})$, which measures the marginal fiscal effect of earning an additional unit of income. Since we abstract from income effects here, efficiency costs $E(\theta^*, \mathcal{P}_{\text{sq}})$ only reflect the standard substitution effect: a higher marginal tax rate for individuals with income $y(\theta^*)$ reduces their labor supply.

Figure 1 illustrates the trade-off for a given type $\theta$. The efficiency costs (red curve) convexly increase in $\tau_{\text{eff}}(\theta)$. The distributional gains term (black curve) decreases in $\tau_{\text{eff}}(\theta)$, which holds whenever we have that welfare weights $g(\theta)$ are decreasing in expenditure. Note that the distributional gains term depends on the lump sum element and the effective marginal tax rates at all other levels. The graph will be useful for the interpretation of our main comparative statics results in Section 5.3.

# 5 Optimal Carbon Rebate Policies

We now turn to the case in which the government faces a binding carbon constraint (14). The optimal policy consists of a nonlinear income tax schedule $\mathcal{T}(y)$ and a carbon tax $\tau_{\text{co2}}$ that together ensure compliance with the emission cap. The optimal implementation of the carbon cap requires solving the following problem:

$$\max_{\mathcal{T}(\cdot), \tau_{\text{co2}}} \int_{\underline{\theta}}^{\overline{\theta}} \mathcal{G}\left(V(\theta; \mathcal{P})\right) \omega(\theta)\, dH(\theta) \tag{20}$$

subject to the carbon cap constraint (14), the government budget constraint (13), and optimal household behavior (4) and (6). As opposed to (17), the government now optimally chooses both elements of the policy vector $\mathcal{P} = (\mathcal{T}, \tau_{\text{co2}})$. We denote the solution to this problem by $\mathcal{P}_{\text{co2}} = (\mathcal{T}_{\text{co2}}, \tau_{\text{co2}})$.

Note that the differences between $\mathcal{T}_{\text{co2}}$ and $\mathcal{T}_{\text{sq}}$ arise from the carbon cap constraint and the associated use of the carbon tax.

Equivalently, we can fix the income tax schedule at $\mathcal{T}_{\text{sq}}$ and introduce a carbon tax rebate function $R_{\text{co2}}(y)$ defined by

$$\forall y: \; R_{\text{co2}}(y) = \mathcal{T}_{\text{sq}}(y) - \mathcal{T}_{\text{co2}}(y).$$

This representation facilitates the analysis of how carbon tax revenues should be optimally rebated across the income distribution.

We present our results sequentially. Section 5.1 analyzes the optimal carbon tax $\tau_{\text{co2}}$. Section 5.2 examines the equity-efficiency trade-off. Finally, Section 5.3 explores the properties of the carbon rebate function and assesses whether the optimal policy $\mathcal{P}_{\text{co2}}$ is more redistributive than $\mathcal{P}_{\text{sq}}$.

## 5.1 Optimal carbon tax

**Lemma 3.** *The carbon tax rate that solves the optimization problem (20) is given by*

$$\tau_{\text{co}_2} = \frac{\mu}{\Lambda}, \tag{21}$$

*where $\Lambda$ is the Lagrangian multiplier on the government budget constraint (13) and $\mu$ is the Lagrangian multiplier on the carbon cap constraint (14).*

*Proof.* See Appendix A.5.1 □

This formula for the carbon tax represents a first-best rule: it holds even if the government could implement type-dependent lump-sum taxes. This result resembles Kaplow (2012) and Jacobs and De Mooij (2015), who show that – under the Atkinson-Stiglitz assumptions – the optimal Pigouvian tax follows a first-best rule, even in a second-best setting with income taxation.[14]

As established in Lemma 1 and further discussed in Section 3.4, a distributionally equivalent income tax reform (8) replicates both the redistributive and labor supply effects of a carbon tax. To derive the condition in Lemma 3, we consider a joint perturbation: a marginal increase in the carbon tax $d\tau_{\text{co}_2}$ combined with the distributionally equivalent income tax reform defined in (8), taken with the opposite sign, i.e. $d\mathcal{T}(y(\theta)) = -f(\theta)d\tau_{\text{co}_2}$. By construction, the reform

[14] As shown by Jacobs and De Mooij (2015), this no longer holds when the utility function is non-separable.

leaves the distribution of income and labor supply unchanged. The welfare effect therefore arises solely from changes in carbon emissions and the associated carbon tax revenue, which affects welfare via the budget constraint and the binding cap:

$$(\Lambda\tau_{\text{co2}} - \mu) \int_{\underline{\theta}}^{\bar{\theta}} f_{\tau_{\text{co2}}}(\theta)\, dH(\theta)\, d\tau_{\text{co2}} = 0. \tag{22}$$

Recall that $f_{\tau_{\text{co2}}}(\theta)$ includes both consumption reallocation and endogenous intensity reductions through abatement. Setting (22) equal to zero, then immediately yields Lemma 3.

## 5.2 Equity-Efficiency Trade-Off

We now derive the analogue of Lemma 2 under a binding emissions cap. The key difference will be that labor supply responses affect welfare not only through tax revenue, but also through their impact on the tightness of the emissions constraint. We first formalize this in Proposition 1 where we characterize the welfare effect of a marginal increase in gross labor income and compare it to the fiscal externality.

**Proposition 1.** *The welfare effect of a marginal increase in gross labor income $dy(\theta)$ is given by*

$$\Lambda \mathrm{T}(\theta, \mathcal{P}_{\text{co2}}) dy(\theta) < \Lambda \tau_{\text{eff}}(\theta, \mathcal{P}_{\text{co2}}) dy(\theta),$$

*where we denote $\mathrm{T}(\theta, \mathcal{P}_{\text{co2}}) \equiv \mathcal{T}'_{\text{co2}}(y(\theta))$ as the welfare wedge.*

*Proof.* The welfare effect of a change in labor supply through the fiscal externality is given by $\Lambda\tau_{\text{eff}}(\theta, \mathcal{P}_{\text{co2}})dy(\theta)$. In addition, there is a welfare effect through the carbon cap constraint, given by $-\mu\left(1 - \mathcal{T}'_{\text{co2}}(y(\theta))\right) MPP(\theta)dy(\theta)$. Using (21) and (15), these two effects add up to $\Lambda \mathrm{T}(\theta)dy(\theta)$. □

Combining (15) with Proposition 1 yields the key identity

$$\tau_{\text{eff}}(\theta, \mathcal{P}_{\text{co2}}) - \mathrm{T}(\theta, \mathcal{P}_{\text{co2}}) = \tau_{\text{co2}}\big(1 - \mathcal{T}'_{\text{co2}}(y(\theta))\big)\, MPP(\theta, \mathcal{P}_{\text{co2}}). \tag{23}$$

That is, the effective marginal tax rate exceeds the welfare-relevant wedge by the marginal carbon tax payment induced by an additional unit of gross income. The reason is that a change in income affects welfare through two constraints: it changes tax revenue, but it also changes emissions and hence the tightness of the cap. Ceteris paribus, this reduces the efficiency costs of redistribution: labor supply reductions become less costly as they relax the carbon cap constraint. In particular, by Lemma 3, it follows that the welfare effect on carbon tax revenue is exactly as large as the welfare effect through the impact on the carbon cap constraint. Accordingly, the efficiency term in the Mirrlees optimal income tax condition must weight labor supply responses by the wedge $\mathrm{T}(\theta, \mathcal{P}_{co2})$ rather than by the effective marginal tax rate $\tau_{\text{eff}}(\theta, \mathcal{P}_{co2})$.

We are now ready to present the counterpart to Lemma 2 under a binding emissions cap. As for Lemma 2, we focus on the case of Assumption 1 for the ease of exposition. In Appendix A.5.2, we present the more general results.

**Proposition 2.** *Assume that Assumption 1 holds. Consider a government that solves (20). The optimal carbon tax satisfies (21). The optimal income tax schedule $\mathcal{T}_{\text{co2}}$ satisfies*

$$\forall\, \theta^* :\ D(\theta^*, \mathcal{P}_{\text{co2}}) = E(\theta^*, \mathcal{P}_{\text{co2}}),$$

*where*

$$D(\theta^*, \mathcal{P}_{\text{co2}}) = 1 - \frac{\mathbb{E}[g(\theta, \mathcal{P}_{\text{co2}})|\theta \geq \theta^*]}{\mathbb{E}[g(\theta, \mathcal{P}_{\text{co2}})]} \tag{24}$$

*and*

$$E(\theta^*, \mathcal{P}_{\text{co2}}) = \frac{\mathrm{T}(\theta^*, \mathcal{P}_{\text{co2}})}{1 - \mathcal{T}'_{\text{co2}}(y(\theta^*))} \frac{\varphi}{1+\varphi} \frac{h(\theta^*)\theta^*}{1 - H(\theta^*)} \tag{25}$$

*Equivalently, using (23), the efficiency term can be written as*

$$E(\theta^*, \mathcal{P}_{\text{co2}}) = \left( \frac{\tau_{\text{eff}}(\theta^*, \mathcal{P}_{\text{co2}})}{1 - \mathcal{T}'_{\text{co2}}(y(\theta^*))} - \tau_{\text{co2}}\, MPP(\theta^*, \mathcal{P}_{\text{co2}}) \right) \frac{\varphi}{1+\varphi} \frac{h(\theta^*)\theta^*}{1 - H(\theta^*)}. \tag{26}$$

*Proof.* See Appendix A.5.2. □

The key difference to Lemma 2 lies in the efficiency term $E_{\text{co2}}(\theta^*, \mathcal{P}_{\text{co2}})$, which includes the labor wedge $\mathrm{T}(\theta, \mathcal{P}_{\text{co2}})$ rather than the effective marginal tax rate $\tau_{\text{eff}}(\theta, \mathcal{P}_{\text{co2}})$. For $\tau_{\text{co2}} > 0$ and $MPP(\theta) > 0$, we have $\mathrm{T}(\theta, \mathcal{P}_{\text{co2}}) < \tau_{\text{eff}}(\theta, \mathcal{P}_{\text{co2}})$, implying that, ceteris paribus, the efficiency costs of redistribution are lower than in standard calculations based on the effective marginal tax rate $\tau_{\text{eff}}(\theta, \mathcal{P}_{\text{co2}})$. By contrast, the formula for the welfare gains from redistribution $D_{\text{co2}}(\theta^*, \mathcal{P}_{\text{co2}})$ is not affected by the binding carbon cap.

Hence, Proposition 2 shows that the carbon tax $\tau_{\text{co2}}$ asymmetrically affects the equity-efficiency trade-off: for a given allocation, it reduces the efficiency costs of redistribution while leaving the welfare gains from redistribution unchanged.[15] It thus highlights that the presence of a carbon tax has nuanced implications for optimal redistribution. Overall, the presence of a binding carbon cap relaxes the equity-efficiency trade-off. Note that this result could easily be overlooked because $\mathcal{T}'(y(\theta)) = \mathrm{T}(\theta, \mathcal{P}_{\text{co2}})$. Substituting $\tau_{\text{eff}}(\theta, \mathcal{P}_{\text{sq}})$ into Lemma 2 and $\mathrm{T}(\theta, \mathcal{P}_{\text{co2}})$ into Proposition 2 could mistakenly suggest that optimal redistribution and optimal carbon pricing are tangential.[16]

[15] If uniform commodity taxation is allowed (Appendix A.5.2), it enters both the distributional and efficiency terms in the usual symmetric way; we abstract from it here to focus on carbon pricing and income taxation.

[16] The income-tax condition in Proposition 4 of Jacobs and de Mooij (2015, equation (38)) is a close counterpart to our Proposition 2. They show that the optimal nonlinear income-tax formula retains the standard Mirrlees–Saez form when expressed in terms of the marginal income tax rate. This corresponds to our result

At first glance, Lemma 2 and Proposition 2 share the familiar $D = E$ structure. The key difference is the object that governs the efficiency term. In the benchmark without a binding cap, the welfare-relevant wedge coincides with the effective marginal tax rate, $\mathrm{T}(\theta, \mathcal{P}_{\mathrm{sq}}) = \tau_{\mathrm{eff}}(\theta, \mathcal{P}_{\mathrm{sq}})$. Under a binding cap and optimal carbon pricing, instead, the welfare wedge is strictly below the EMTR:

$$\tau_{\mathrm{eff}}(\theta, \mathcal{P}_{\mathrm{co2}}) - \mathrm{T}(\theta, \mathcal{P}_{\mathrm{co2}}) = \tau_{\mathrm{co2}}\big(1 - \mathcal{T}'_{\mathrm{co2}}(y(\theta))\big)\, MPP(\theta, \mathcal{P}_{\mathrm{co2}}).$$

Hence, the EMTR continues to measure marginal redistribution, but it overstates marginal efficiency costs by exactly the EMTR–wedge gap. In the next subsection, we present comparative statics results, explicitly addressing how $\mathcal{T}_{\mathrm{co2}}$ differs from $\mathcal{T}_{\mathrm{sq}}$. This analysis (i) compares the level of redistribution with and without the carbon taxes, and (ii) derives key properties of the optimal carbon rebate function $R_{\mathrm{co2}}$.

For the graphical illustration in Figure 1, consider a given marginal income tax rate $T'_{\mathrm{co2}}(y(\theta))$. The efficiency costs are governed by this marginal income tax rate, while the effective marginal tax rate is higher by $\tau_{\mathrm{co2}}\left(1 - T'_{\mathrm{co2}}(y(\theta))\right) MPP(\theta, \mathcal{P}_{\mathrm{co2}})$. Thus, the corresponding point on the efficiency-cost curve is shifted to the right by exactly this amount. Since the gap depends on the marginal income tax rate, however, the horizontal shift is generally not constant along the curve. Moreover, the distributional-gains curve may shift as well because of the distributional implications of the carbon tax.

## 5.3 Comparative Statics

We now provide comparative statics for $\mathcal{P}$ to address two questions:

1. Is the optimal policy in the presence of a binding carbon cap $\mathcal{P}_{\mathrm{co2}}$ more or less redistributive than the status quo policy $\mathcal{P}_{\mathrm{sq}}$?

2. What are the properties of the carbon tax rebate function $R_{\mathrm{co2}}(y) = \mathcal{T}_{\mathrm{sq}}(y) - \mathcal{T}_{\mathrm{co2}}(y)$?

While the two questions are distinct, they are closely linked: both require comparing the optimal marginal tax rates $\mathcal{T}'_{\mathrm{sq}}(y(\theta))$ and $\mathcal{T}'_{\mathrm{co2}}(y(\theta))$, as characterized by Lemma 2 and Proposition 2. This comparison is subtle, because even the common terms differ in value due to their evaluation under distinct policies $\mathcal{P}_{\mathrm{sq}} \neq \mathcal{P}_{\mathrm{co2}}$.

To obtain tractable comparative statics, we impose two more simplifying assumptions. These assumptions apply only to the theoretical results in this section and are *not* imposed in the quantitative analysis in Section 6.

that, under optimal carbon pricing, the welfare-relevant labor wedge equals the marginal income tax rate. The additional implication in our analysis arises from distinguishing this wedge from the total EMTR, which also includes the marginal carbon-tax burden. This distinction permits us to compare effective redistribution across the carbon-constrained and unconstrained optima and to derive the optimal rebate schedule.

**Assumption 2.** *The carbon emission cap $\bar{F}$ is marginally binding: $\bar{F} = F^{\text{sq}} - \varepsilon$, where $\varepsilon \to 0$ and $F^{\text{sq}}$ denotes the aggregate carbon footprint under the status quo policy $\mathcal{P}_{\text{sq}}$.*

This assumption allows us to focus on the first-order effects of carbon taxes on individual utility and behavior as well as government revenue.

**Assumption 3.** *The primitives of the economy and the welfare function are such that the optimal status quo income tax schedule $\mathcal{T}_{\text{sq}}$ is affine, i.e. it features a constant marginal tax rate: $\mathcal{T}'_{\text{sq}}(y) = \mathcal{T}'_{\text{sq}}(y')$ for all $y, y'$.*

This assumption does *not* restrict the structure of $\mathcal{T}'_{\text{co2}}$; it merely implies that the social planner optimally chooses an affine income tax schedule in the absence of a carbon cap. It is helpful for deriving sharper results, as it ensures that a household's marginal income tax rate $\mathcal{T}'_{\text{sq}}(y)$ is unaffected by small income changes induced by the carbon tax and the corresponding adjustments in the lump-sum rebate.

### 5.3.1 The Effect of the Carbon Cap for the Optimal Level of Redistribution

We now show that the optimal level of redistribution is higher in the presence of the carbon cap:

$$\tau_{\text{eff}}(\theta, \mathcal{P}_{\text{co2}}) \geq \tau_{\text{eff}}(\theta, \mathcal{P}_{\text{sq}}) \; \forall \theta.$$

To establish this, we define a distributionally neutral carbon rebate as a benchmark:

$$\forall\, \theta: \; R_{\text{co2}}(y(\theta, \mathcal{P}_{\text{co2}})) = \tau_{\text{co2}}\, f(\theta, \mathcal{P}_{\text{co2}}). \tag{27}$$

Such an implementation of the carbon cap implies a rebate schedule that exactly offsets each household's carbon tax burden. Under this policy, the level of redistribution is the same as under the status quo, and it holds that $\tau_{\text{eff}}(\theta, \mathcal{P}_{\text{co2}}) = \tau_{\text{eff}}(\theta, \mathcal{P}_{\text{sq}}) \; \forall \theta$. We now demonstrate that the optimality condition in Proposition 2 would be systematically violated, in the sense that $E(\theta, \mathcal{P}_{\text{co2}}) < D(\theta, \mathcal{P}_{\text{co2}})$.

**Proposition 3.** *Assume that Assumptions 1-3 hold. The optimal policy $\mathcal{P}_{\text{co2}} = (\mathcal{T}_{\text{co2}}, \tau_{\text{co2}})$ cannot satisfy (27), because this would imply $E(\theta, \mathcal{P}_{\text{co2}}) < D(\theta, \mathcal{P}_{\text{co2}}) \; \forall \; \theta \; \in ]\underline{\theta}, \overline{\theta}[$ and hence the government would have an incentive to increase redistribution, i.e. decrease $R'_{\text{co2}}(y(\theta))$ (increase $\mathcal{T}'_{\text{co2}}(y(\theta))$ respectively).*

*Proof.* See Appendix A.6.1. □

We first consider the distributional gains term and examine how the social welfare weights $g(\theta, \mathcal{P})$ change. Since the distributionally neutral policy leaves the levels of utility unchanged,

only changes in the marginal utility of expenditure $u_e(\theta)$ can affect the distributional gains term. As shown in Appendix A.6.1,

$$\forall \theta : \quad u_e(\theta, \mathcal{P}_{\text{co2}}) = u_e(\theta, \mathcal{P}_{\text{sq}}) \left(1 - \tau_{\text{co2}} \, MPP(\mathcal{P}_{\text{sq}})\right). \tag{28}$$

Here $MPP(\mathcal{P}_{\text{sq}}) \equiv MPP(\theta, \mathcal{P}_{\text{sq}})$ is constant across $\theta$ under Assumption 1. Equation (28) implies that the relative change in marginal utilities is identical across households, leaving the distributional gains term unaffected: $D(\theta, \mathcal{P}_{\text{co2}}) = D(\theta, \mathcal{P}_{\text{sq}}) \ \forall \ \theta$, where $D(\theta, \mathcal{P}_{\text{co2}})$ denotes the distributional gains from marginally increasing $\mathcal{T}'_{\text{co2}}(y(\theta))$ starting from the distributional neutral policy. Put differently, starting from a distributionally neutral carbon rebate (27), the distributional gains are exactly the same as before the introduction of the carbon policy.

Next, we turn to the efficiency costs of redistribution. Note that the distributionally neutral policy (27) implies $\forall \ \theta \ \tau_{\text{eff}}(\theta, \mathcal{P}_{\text{sq}}) = \tau_{\text{eff}}(\theta, \mathcal{P}_{\text{co2}})$, i.e.

$$\mathcal{T}'_{\text{co2}}(y(\theta)) = \mathcal{T}'_{\text{sq}}(y(\theta)) - (1 - \mathcal{T}'_{\text{co2}}(y(\theta))) \, MPP(\theta, \mathcal{P}_{\text{co2}}) \tau_{\text{co2}}. \tag{29}$$

Substituting (29) into $\text{T}(\theta, \mathcal{P}_{\text{co2}})$ and (25) then yields:

$$E(\theta, \mathcal{P}_{\text{co2}}) < E(\theta, \mathcal{P}_{\text{sq}}) \ \forall \ \theta \ \in ]\underline{\theta}, \overline{\theta}[,$$

which implies that $E(\theta, \mathcal{P}_{\text{co2}}) < D(\theta, \mathcal{P}_{\text{co2}}) \ \forall \ \theta \ \in ]\underline{\theta}, \overline{\theta}[$. Hence, except at the boundaries $\theta^* = \underline{\theta}$ and $\theta^* = \overline{\theta}$, it is optimal to raise marginal tax rates from (29) implying $\tau_{\text{eff}}(\theta, \mathcal{P}_{\text{co2}}) > \tau_{\text{eff}}(\theta, \mathcal{P}_{\text{sq}})$. The optimal level of redistribution therefore increases, regardless of the form of the social welfare function. The rationale is akin to the arguments underlying the *degrowth* idea. Carbon externalities make, ceteris paribus, reductions in labor supply and consumption more desirable, as these directly lower the carbon footprint.

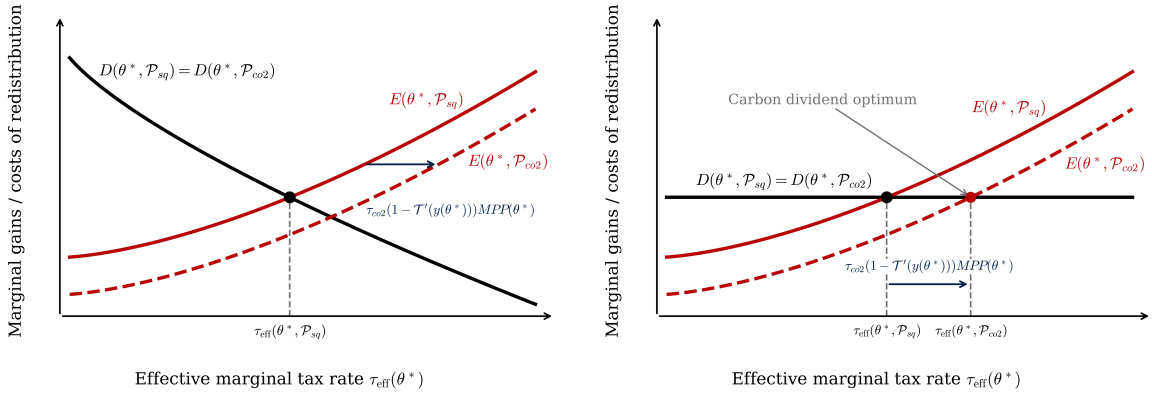
The result is illustrated in Figure 2(a). A carbon dividend implies $R'_{\text{co2}}(y) = 0$ and hence $T'_{\text{co2}}(y) = T'_{\text{sq}}(y)$. The welfare-relevant efficiency costs therefore remain unchanged. The effective marginal tax rate, by contrast, increases by $\tau_{\text{co2}} \left(1 - T'_{\text{co2}}(y(\theta))\right) MPP(\theta, \mathcal{P}_{\text{co2}})$, which is exactly the horizontal distance between the two intersections in Figure 2(a).

### 5.3.2 Properties of the Optimal Carbon Tax Rebate Schedule

We next show that a carbon dividend is optimal only in the special case in which marginal distributional gains do not decline with the overall level of redistribution.

**Proposition 4.** *Assume that Assumptions 1-3 hold and that $\mathcal{G}(V) = V$. Then, the optimal rebate is lump-sum; that is, a carbon dividend is optimal.*

The result is illustrated in Figure 2(b). Under $\mathcal{G}(V) = V$, the marginal gains from redistribution are constant: the distributional gains curve is horizontal and does not shift when the




(a) Distributionally neutral rebate

(b) Carbon dividend

Figure 2: Equity–efficiency trade-off in case of carbon cap

*Notes:* This figure illustrates the comparative static results of Proposition 3 and Corollary 4.

level of redistribution changes. Hence, once the carbon cap lowers the welfare-relevant efficiency cost of redistribution, there is no countervailing decline in marginal distributional gains that would limit the redistributive adjustment.

A carbon dividend implements this polar case. Since the rebate is lump-sum, $R'_{\text{co2}}(y) = 0$, none of the carbon-tax revenue is used to offset the income gradient in carbon tax liabilities. The marginal carbon tax burden is therefore fully translated into higher effective marginal tax rates. In this sense, the planner exhausts the additional redistributive scope created by the carbon cap: carbon-tax revenue is fully pooled and redistributed uniformly. This is optimal only because marginal distributional gains remain constant as redistribution increases.

When $\mathcal{G}''(V) < 0$, this polar case is no longer optimal. A carbon dividend increases redistribution relative to the status quo. With diminishing marginal utility, this reduces the marginal distributional gains from further redistribution. Hence, the carbon dividend overshoots the redistributive adjustment that is optimal under the carbon cap. We show this formally by considering a budget-neutral carbon dividend and verifying that it violates the optimality condition in Proposition 2.

**Corollary 1.** *Assume that Assumptions 1-3 hold and that* $\mathcal{G}''(V) < 0$. *The optimal policy* $\mathcal{P}_{\text{co2}} = (\mathcal{T}_{\text{co2}}, \tau_{\text{co2}})$ *cannot be consistent with a lump-sum rebate. A lump-sum rebate would imply*

$$E(\theta, \mathcal{P}_{\text{co2}}) > D(\theta, \mathcal{P}_{\text{co2}}) \qquad \forall \theta \in ]\underline{\theta}, \overline{\theta}[,$$

*and hence, starting from a budget-neutral carbon dividend, the government would have an incentive to reduce redistribution. Equivalently, it would increase* $R'_{\text{co2}}(y(\theta))$, *or decrease* $\mathcal{T}'_{\text{co2}}(y(\theta))$.

*Proof.* See Appendix A.6.2. □

Graphically, Corollary 1 can be read as a departure from the polar case in Figure 2(b). In Figure 2(b)(b), the carbon dividend is optimal because the distributional-gains curve is

horizontal. When $\mathcal{G}''(V) < 0$, the same carbon dividend reduces marginal distributional gains. Hence, at the carbon-dividend point in Figure 2(b), the distributional-gains curve would lie below the horizontal benchmark, while the marginal efficiency cost is unchanged. The planner therefore moves left from the carbon-dividend allocation, choosing less redistribution than under a lump-sum rebate.

The planner consequently reduces redistribution relative to the carbon dividend. In terms of the rebate schedule, this means that the optimal rebate cannot be lump-sum: it must increase with income.

Combining Proposition 3 and Corollary 1 pins down the slope of the optimal rebate schedule. A distributionally neutral rebate would fully offset the marginal carbon tax burden and therefore leave effective marginal tax rates unchanged. A carbon dividend would set $R'_{\text{co2}}(y) = 0$ and therefore translate the full marginal carbon tax burden into higher effective marginal tax rates. The optimum lies between these two polar cases. Hence, for all interior types,

$$0 < R'_{\text{co2}}(y(\theta)) < \tau_{\text{co2}}\big(1 - \mathcal{T}'_{\text{co2}}(y(\theta))\big) MPP(\theta, \mathcal{P}_{\text{co2}}).$$

Thus, the optimal rebate is increasing in income, but less steeply than carbon tax payments. Low-income households are overcompensated relative to their carbon tax burden, while high-income households are undercompensated.

# 6 Quantitative Model

We calibrate our model to the German economy in 2018 to quantify the optimal carbon policy. This requires specifying both the set of goods considered and the functional form of utility derived from these goods, which we introduce in Section 6.1. The main inputs to the model are (i) carbon emission intensities of different goods and services, (ii) household consumption expenditure shares, (iii) a gross labor income distribution, and (iv) the current German tax-transfer system. We describe the different datasets that we use in Section 6.2. We describe our calibration and present the model fit in Section 6.3.

## 6.1 Preferences

For the functional form of $U(\mathbf{c})$, we adopt a utility function with nested non-homothetic CES preferences. At the upper tier, there is a consumption basket $\mathbf{c} = (c_1, c_2, ..., c_I)$. At the lower tier, each good $i = 1, \dots I$ is a non-homothetic CES composite of a 'brown' and 'green' variety, capturing substitution possibilities between goods with different carbon emission intensities. The nested non-homothetic CES structure allows for varying expenditure shares along the income distribution in both goods (upper tier) and green and brown varieties (lower tier).

**Upper-Tier Utility Function.** We consider non-homothetic CES preferences defined over a consumption basket $\mathbf{c} = (c_1, c_2, ..., c_I)$. Following Comin, Lashkari, and Mestieri (2021), the functional form of $U(\mathbf{c})$ is given by[17]

$$U(\mathbf{c}) = \frac{\mathbb{C}(\mathbf{c})^{1-\gamma}}{1-\gamma}, \tag{30}$$

where $\gamma \geq 0$ denotes the coefficient of relative risk aversion and $\mathbb{C}$ is a consumption aggregator implicitly defined by

$$\sum_{i=1}^{I} (\Omega_i \mathbb{C}(\mathbf{c})^{\varepsilon_i})^{\frac{1}{\sigma}} c_i^{\frac{\sigma-1}{\sigma}} = 1 \tag{31}$$

with $\sigma > 0$, $\Omega_i > 0 \; \forall i$. We consider the case $\sigma < 1$, where these broader consumption categories are complements. In this case, $\varepsilon_i > 0 \; \forall i$. With non-homothetic preferences, the elasticity of substitution $\sigma$ is constant across consumption goods, but the expenditure shares households allocate to different goods vary with their net income $e(\theta)$.

The elasticity of consumption of good $i$ w.r.t. to overall expenditure $e$ is given by

$$\xi_i \equiv \frac{\partial \log c_i}{\partial \log e} = \sigma + (1-\sigma)\frac{\varepsilon_i}{\overline{\varepsilon}}, \tag{32}$$

where $\overline{\varepsilon} = \sum_i = \psi_i \varepsilon_i$ and $\psi_i$ is the expenditure share of good $i$. Note that goods with $\varepsilon_i > \overline{\varepsilon}$ are luxuries, while those with $\varepsilon_i < \overline{\varepsilon}$ are necessities.

**Lower-Tier Utility Function.** Each good $c_i$ is a composite of a 'brown' and 'green' variety, which differ in their carbon emission intensities. We denote these varieties by $c_{ij}$, with $j \in \{b, g\}$ indicating the 'brown' ($b$) and 'green' ($g$) variety, respectively.

$$\sum_{j \in \{b,g\}} (\Omega_{ij} {c_i}^{\varepsilon_{ij}})^{\frac{1}{\sigma_i}} c_{ij}^{\frac{\sigma_i - 1}{\sigma_i}} = 1,$$

where $\sigma_i$ denotes the elasticity of substitution between two varieties and $\Omega_{ij} > 0$. For the inner nest, we consider both cases: $\sigma_i > 1 \; \forall i$ and $\sigma_i < 1 \; \forall i$ implying $\varepsilon_{ij} < 0 \forall i, j$ and $\varepsilon_{ij} > 0 \forall i, j$ respectively. This preference structure allows for substitution from environmentally harmful to environmentally friendly goods within a consumption category $i$ when the relative price of brown varieties increases due to the introduction of a carbon tax.

## 6.2 Data

We base our calibration on three primary data sources: (i) German income and consumption expenditure survey (*Einkommens- und Verbrauchsstichprobe*; EVS), (ii) environmentally-

[17] To be precise, we follow the notation of an older working paper version of this paper (Comin, Lashkari, and Mestieri 2017).

extended multi-regional input-output tables from EXIOBASE, and (iii) German administrative tax records (*Lohn- und Einkommenssteuerstatistik*; LESt). We briefly describe each dataset below; further details are provided in Appendix B.2 for EXIOBASE and Appendix C for the micro data.

**EVS.** The *Einkommens- und Verbrauchsstichprobe* (EVS) is a representative cross-sectional household survey that provides detailed data on consumption, income and socioeconomic characteristics. We use the 2018 wave to measure household spending across different goods and services along the income distribution. The survey reports consumption expenditures across $N = 107$ categories, classified according to the COICOP system. We aggregate these goods and services into nine broad products: clothing, electricity, food, heating, household and personal care, housing, leisure, services, transport and vacation. In addition, we use detailed information on transfer payments to calibrate a transfer function with a gradual phase-out. Finally, we convert the data from the household level to the tax unit level, resulting in a sample of around 26 000 tax units. We assign each EVS tax unit to the corresponding percentile of the gross labor income distribution from the LESt and compute percentile level moments as survey-weighted averages within these bins.

**EXIOBASE.** We use the environmentally-extended multi-regional input-output tables from EXIOBASE to compute emission intensities $f_{ij}$ for the 'brown' and 'green' version of each of the nine goods. EXIOBASE links economic activity to environmental accounts, including greenhouse gas emissions, through a detailed global input-output framework. We use the product dimension of EXIOBASE, which distinguishes $P = 200$ products and encompasses $K = 49$ regions. To align these data with the COICOP classified consumption data in the EVS, we construct a crosswalk from EXIOBASE to COICOP products. This allows us to estimate the carbon footprint of each good, accounting for both direct and indirect emissions.

**LESt.** The *Lohn- und Einkommenssteuerstatistik* (LESt) is a 10% cross-sectional random sample of German taxpayers based on administrative income tax records. The unit of observation is the tax unit, defined as either an individual including single persons and married couples filing separately, or a married couple filing jointly. We use data from the 2018 wave on annual gross labor income to calibrate a gross income distribution. In addition, we use total income tax payments to calibrate the status-quo tax system. Our sample includes around 2.27 million tax units.

## 6.3 Calibration

The calibration of our model proceeds in two steps. First, we construct the auxiliary inputs to the model: the emission intensities of the 'brown' and 'green' varieties of each of the nine

consumption categories, the carbon abatement technology, the status quo tax-transfer system, and the gross labor income distribution. Second, we calibrate the parameters of the utility function, combining externally assigned values with parameters estimated to match expenditure patterns across the income distribution.

Throughout the analysis, we set $I = 9$, where each good $i$ corresponds to one of nine broad consumption categories: clothing, electricity, food, heating, household and personal care, housing, leisure, services, and transport and vacation.

### 6.3.1 Emission Intensities

We compute carbon emission intensities for 107 different COICOP goods and services using a consumption-based approach that attributes emissions to final household demand. For each product, the total emission intensity is the sum of a direct and indirect component. Direct emissions arise at the stage of household consumption (e.g. driving a gasoline-powered car), whereas indirect emissions occur throughout the production chain (e.g. production of a car).

**Indirect Emission Intensities.** We begin by computing indirect emission intensities at basic prices using EXIOBASE. Let $\mathbf{L} \in \mathbb{R}^{K \cdot P \times K \cdot P}$ denote the multi-regional Leontief inverse and let $\mathbf{E} \in \mathbb{R}^{1 \times K \cdot P}$ denote the row vector of product-region emission coefficients, where each entry gives the amount of carbon dioxide equivalent ($CO_2e$) emitted per monetary unit of output for a given product-region pair. To capture the regional composition of German household demand, we construct a sparse allocation matrix $\tilde{\mathbf{Y}} \in \mathbb{R}^{K \cdot P \times P}$. For each EXIOBASE product $p$, the $p$-th column of $\tilde{\mathbf{Y}}$ allocates one euro of German household final demand for product $p$, valued at basic prices, across regions according to the expenditure shares observed in EXIOBASE.[18] Indirect emission intensities at the EXIOBASE product level are then given by

$$\mathbf{f}_{\mathbf{ind}}^{\mathbf{EXIO}} = \mathbf{E} \cdot \mathbf{L} \cdot \tilde{\mathbf{Y}} \quad \in \mathbb{R}^{1 \times P},$$

where $f_{\text{ind},p}^{\text{EXIO}}$ measures the total upstream $CO_2e$ emissions embodied in one euro of German household demand for EXIOBASE product $p$. We then convert these intensities from basic to purchaser prices and map them from the EXIOBASE product classification into the COICOP classification to make the EXIOBASE based intensities consistent with the household expenditure data. This yields $\tilde{\mathbf{f}}_{\mathbf{ind}}^{\mathbf{COICOP}} \in \mathbb{R}^{1 \times N}$, the vector of indirect emission intensities at purchaser prices in the COICOP classification. Appendix B.3.2 and Appendix B.3.3 provides the details of the conversion from basic to purchaser prices and the mapping from the EXIOBASE to COICOP classification, respectively.

[18] Note that $\tilde{\mathbf{Y}}$ is a vertical stack of $K$ diagonal $P \times P$ blocks, where the $k$-th block places region $k$'s demand shares across all $P$ products on its diagonal. Appendix B.3.1 describes the construction of $\mathbf{E}$, $\mathbf{L}$ and $\tilde{\mathbf{Y}}$.

**Direct Emission Intensities.** We next compute direct emission intensities at the COICOP level using fuel specific physical emission factors. For each underlying fuel, we convert the physical emission factor into a monetary emission intensity using the corresponding 2018 consumer price and then aggregate these fuel specific intensities to the COICOP level using consumption weighted fuel shares from the 2013 wave of the German Residential Energy Consumption Survey (GRECS). This yields the vector of direct emission intensities, $\tilde{\mathbf{f}}_{\text{dir}}^{\text{COICOP}} \in \mathbb{R}^{N\times 1}$, where $\tilde{f}_{\text{dir},n}^{\text{COICOP}}$ measures the direct $CO_2e$ emissions associated with one euro of purchaser price expenditure in COICOP product $n$. Appendix B.3.4 provides details on the transformation of physical emission factors into monetary emission intensities.

**Total Emission Intensity.** The total emission intensity of COICOP product $n$ is the sum of its indirect and direct components:

$$f_n = \tilde{f}_{\text{ind},n}^{\text{COICOP}} + \tilde{f}_{\text{dir},n}^{\text{COICOP}}.$$

The resulting product level emission intensities are reported in Table 7 in Appendix C. As a validation check, combining these intensities with aggregate consumption expenditures implies aggregate emissions of 731.26 $MtCO_2e$ for Germany in 2018, close to the corresponding benchmark reported by the German Statistical Office (German Statistical Office 2025b). See Appendix B.4 for details.

**Green and Brown Varieties.** Each product is then classified as either 'brown' or 'green' within its product category $i$, based on its emission intensity. Specifically, we apply K-means clustering with $K = 2$ separately within each category, using the log emission intensity as the clustering variable. We label the cluster with the higher average emission intensity as 'brown' and the other as 'green'.[19] We then compute the expenditure-weighted average emission intensity of all goods classified as brown and green within each product category. This yields the emission intensity $f_{ij}$, where $j \in \{b, g\}$ indicates the brown or green variety of product category $i$. Table 1 summarizes the classification and the resulting average emission intensities $f_{ij}$. Note that electricity is not split into a brown or green variety, as this product category comprises only one product.

[19] Table 7 in Appendix C contains the classification of all 107 products into 'green' and 'brown' varieties.

| Product Category | Cutoff Emission Intensity | Average Emission Intensity | |
|---|---|---|---|
| | | Brown | Green |
| Clothing | 0.1849 | 0.2807 | 0.1199 |
| Electricity | – | 1.7913 | 1.7913 |
| Food | 0.4905 | 0.631 | 0.3944 |
| Heating | 5.9558 | 8.6185 | 3.8923 |
| Household and Personal Care | 0.2003 | 0.3001 | 0.1498 |
| Housing | 0.1416 | 0.3641 | 0.059 |
| Leisure | 0.1741 | 0.2571 | 0.1229 |
| Services | 0.1703 | 0.2737 | 0.1063 |
| Transport and Vacation | 0.4761 | 1.5732 | 0.218 |

Table 1: Carbon Footprint of Brown and Green Varieties

*Notes:* Emission intensities are measured in $kgCO_2e$ per euro of expenditure. Within each product category, products are classified into brown and green varieties using K-means clustering with $K = 2$ applied to product-level emission intensity. The second column reports the category-specific cutoff implied by this procedure. Columns 3 and 4 report the corresponding expenditure-weighted average emission intensities. Electricity is not split into brown and green varieties because this category contains only one product.

### 6.3.2 Carbon Footprint

We construct household carbon footprints by combining the COICOP level emission intensities with consumption expenditures from the EVS. Let $e_n(p)$ denote average annual consumption expenditure of percentile $p$ on COICOP product $n$. The carbon footprint of percentile $p$ is

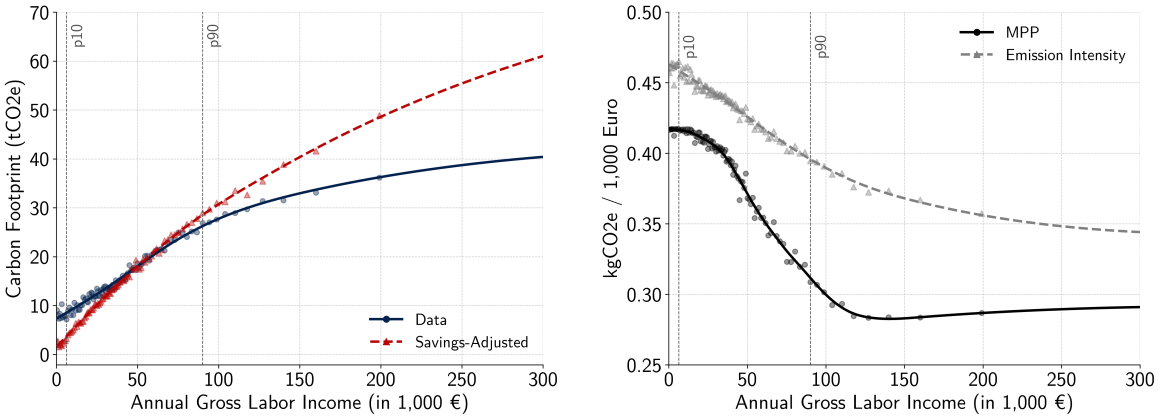
$$f(p) = \sum_{n=1}^{N} e_n(p) f_n,$$

which captures the direct and indirect emissions embodied in the percentile's consumption basket.[20]

Because our model is static, it abstracts from the intertemporal allocation of consumption across the life-cycle. In the model, disposable income is fully allocated to current consumption, whereas observed expenditures in the data need not equal disposable income because households may save, borrow, or dissave. We therefore apply a 'savings-adjustment': for each percentile, we assign the difference between disposable income and observed expenditure the average emission intensity of that percentile's observed consumption basket. Equivalently, the adjustment rescales total expenditure to disposable income while holding the composition of the observed consumption basket fixed. This reduced-form approach allows us to abstract from the timing of consumption over the life cycle.[21]

[20] In the quantitative model, consumption is defined over broad products and varieties rather than at the detailed COICOP level. We therefore construct the model counterpart of the carbon footprint by aggregating each tax unit's EVS expenditures to the corresponding product–variety cells, multiplying these expenditures by the associated expenditure weighted average emission intensities $f_{ij}$ and summing over all cells. Appendix D.1 shows that the approximation error from this aggregation is quantitatively small.

[21] This adjustment is similar to the one applied in Craig, Lloyd, and Moore (2025).




(a) Carbon Footprint

(b) MPP and Emission Intensity

Figure 3: Carbon Footprint, Marginal Propensity to Pollute and Emission Intensity

*Notes:* The left panel illustrates the carbon footprint $f(p)$ (blue dots) and the savings-adjusted carbon footprint (red triangles) for each income percentile $p$, both expressed in $tCO_2e$. The savings adjustment holds the composition of the consumption basket fixed and rescales all expenditures by a common factor such that adjusted total expenditure equals disposable income. The right panel shows the marginal propensity to pollute (MPP, blue dots) and the emission intensity (red triangles), both expressed in $kgCO_2e$ per 1 000 Euro. The MPP is defined as the derivative of the spline-fitted carbon footprint with respect to total expenditure, evaluated at each percentile: $\text{MPP}(p) = \hat{f}'(e(p))$, where $\hat{f}(\cdot)$ is a cubic smoothing spline of $f(p)$ on $e(p)$. The emission intensity is defined as $\iota(p) = \hat{f}(p)/e(p)$. In both panels, the lines show cubic smoothing spline fits to the respective data moments. The x-axis shows annual gross labor income of a tax unit (in 1,000 EUR). The grey vertical dashed-dotted lines correspond to the 10th and 90th percentile of the annual gross labor income distribution.

Figure 3 shows the carbon footprint, marginal propensity to pollute and emission intensity along the gross labor income distribution. Markers denote the percentile-level moments, while the solid and dashed lines are cubic spline fits. Panel (a) illustrates the carbon footprint in the data in blue and the 'savings-adjusted' carbon footprint in red. Carbon footprints increase with income, but at a decreasing rate, consistent with the literature on environmental Engel curves and household carbon footprints (Levinson and O'Brien 2019, Sager 2019, Hardadi, Buchholz, and Pauliuk 2021, among others). The saving adjustment lowers carbon footprints at the bottom of the income distribution, where observed expenditures exceed measured disposable income, and raises them for higher income tax units with positive saving. Quantitatively, the 90th percentile emits 7.62 times more than the 10th percentile after the savings adjustment. Panel (b) illustrates the corresponding emission intensity of consumption expenditure in red and marginal propensity to pollute out of expenditure in blue, both measured in $kgCO_2e$ per 1 000 Euros of expenditure. The MPP is computed as the local slope of the carbon Engel curve, $MPP(p) = \hat{f}'(e(p))$, where $\hat{f}(e(p))$ is a cubic smoothing spline fitted to the percentile-level relationship between the carbon footprint $f(p)$ and total expenditure $e(p)$.The emission intensity is defined as $\iota(p) = \hat{f}(p)/e(p)$. Both measures decline along the gross labor income distribution. This reflects the non-homotheticity of consumption: as income rises, spending

shifts toward less carbon-intensive goods and services, so an additional euro of expenditure generates fewer emissions than the average euro spent at lower income levels.[22]

We further decompose the MPP into a spending-level and a composition component:

$$MPP(e) = \iota(e) + e\,\frac{d\iota(e)}{de}.$$

The spending-level component captures the emissions generated by an additional euro of expenditure holding the consumption basket fixed and coincides with the average emission intensity $\iota(e)$. The composition component, $e\,d\iota/de$, captures change in emissions arising from changes in the composition of the consumption basket as total expenditure rises. In Figure 3(b), this component is simply the vertical difference between the MPP and the average emission intensity.[23] Because spending shifts toward less carbon-intensive goods as expenditure rises ($d\iota/de < 0$), the composition component is negative, and the total MPP lies below the spending-level component throughout most of the income distribution: shifts in marginal spending attenuate the emissions generated by an additional euro. This attenuation is strongest at low and middle incomes; above roughly EUR 120 000 the reallocation of marginal spending weakens and the total MPP converges toward the spending-level component. Figure 10 in Appendix D.2 illustrates the composition component.

**Anatomy of the income gradient in carbon footprints.** As just described, Figure 3 summarizes the main empirical objects that discipline the distributional effects of carbon pricing in the model. The level of the carbon Engel curve determines carbon tax liabilities across the income distribution, while its local slope with respect to expenditure disciplines the carbon redistribution wedge. We next examine the sources of this income gradient. Since a tax unit's carbon footprint aggregates expenditures across goods and emissions sources, the carbon-income gradient can reflect several distinct margins: the distinction between emissions released directly by households and emissions embodied in production, changes in the observable composition of tax units along the income distribution, and changes in consumption across product categories. We examine each margin in turn.

We first split total carbon footprints into direct emissions and indirect emissions. Indirect emissions account for most of the income gradient, whereas direct emissions are substantially flatter. This decomposition clarifies the scope of the benchmark policy. As described in Section 3 our model implements a consumption-based carbon price whose tax base is the household's carbon footprint, including both direct emissions and indirect emissions embodied in

[22] This is consistent with Aghion, Boppart, Peters, Schwartzman, and Zilibotti (2025) also provide evidence for the United States that demand shifts toward relatively clean, service-intensive goods as income rises.

[23] Total differentiation of the carbon footprint $f(e) = \iota(e)\,e$ with respect to expenditure $e$ gives $MPP = df/de = \iota(e) + e\,d\iota(e)/de$. In the data, we measure the spending-level component as the observed average emission intensity $\iota(e(p)) = f(p)/e(p)$ at each percentile $p$, and recover the composition component $e(p)\,d\iota(e(p))/de$ as the residual $\widehat{MPP}(p) - \iota(e(p))$.

purchased goods and services. By contrast, a policy that prices only direct household emissions would use the direct footprint $f^{dir}(e)$ and the corresponding $MPP^{dir}(e)$ in place of the total footprint $f(e)$ and total $MPP(e)$. Since the direct component is flatter, such a narrower policy would imply a smaller carbon tax liability gradient and, for a given carbon price, a smaller carbon redistribution wedge. Figure 11 in Appendix D.3 illustrates the carbon footprints decomposed into direct and indirect emissions along the gross labor income distribution.

We next consider observable horizontal heterogeneity. Carbon footprints may vary not only with income, but also across tax units with similar income, reflecting differences in household structure, demographics, housing characteristics, and location. If these characteristics vary systematically with income, the income gradient combines vertical carbon inequality with observable composition effects. To quantify this component, we construct covariate-standardized carbon footprints for each income percentile. Using EVS tax unit microdata, we estimate a linear projection of carbon footprints on a income percentile fixed effects and controls for household size, education, homeownership, heating fuel, rural residence, and age. We then recover the covariate-standardized percentile profile from the estimated percentile fixed effects, evaluating the controls at their weighted empirical distribution in the full estimation sample for every percentile. The resulting profile therefore captures how average carbon footprints would vary across the income distribution when the distribution of observed covariates is held fixed across percentiles.[24]

The adjustment attenuates, but does not eliminate, the income gradient in carbon footprints. Holding the distribution of observed covariates fixed across percentiles, the MPP is about 23% lower on average, but remains positive throughout the income distribution and declines over most of the income distribution. Since the carbon redistribution wedge is proportional to the MPP, taking out observed horizontal heterogeneity reduces the quantitative scope for additional redistribution, but leaves the qualitative mechanism unchanged. Figure 12 in Appendix D.3 illustrates the covariate-standardized carbon footprints and the corresponding MPP along the gross labor income distribution.

Finally, we decompose the percentile level footprint by broad product category. For each income decile, we compute the average product-specific carbon footprint. The income gradient is present across the consumption basket: category-specific footprints increase with income in each category. Transport and vacation is the largest single contributor, accounting for 41.2% of the gap in carbon emissions between the 90th and 10th percentile. The remaining categories jointly account for 58.8%. Hence, the income gradient in carbon emissions is not an artifact of a single high emission product. Figure 13 in Appendix D.3 shows the category level decomposition.

[24]Note that this exercise is purely descriptive. It removes the component of the income gradient associated with observed covariates, but it does not account for unobserved heterogeneity or isolate a causal effect of income on emissions. See Appendix D.3 for details.

### 6.3.3 Carbon Abatement Technology

We calibrate the abatement technology following the DICE literature. For product $i$ and variety $j$, abatement costs are given by

$$\kappa_{ij}(s_{ij}) = \bar{f}_{ij} \frac{k}{1+\psi} s_{ij}^{1+\psi} \quad \forall i, j,$$

where $\bar{f}_{ij}$ denotes the emission intensity absent abatement and $s_{ij} \equiv a_{ij}/\bar{f}_{ij} \in [0,1]$ is the abatement share. The scale parameter $k$ is the backstop price, i.e. the carbon tax that induces full abatement, while $\psi$ governs the curvature of abatement costs and equals the inverse of the elasticity of the abatement share with respect to the carbon tax. Both parameters are common across goods and varieties. The implied optimal abatement share at carbon price $\tau_{\text{co2}}$ is

$$s_{ij}(\tau_{\text{co2}}) = \min\left\{\left(\frac{\tau_{\text{co2}}}{k}\right)^{1/\psi}, 1\right\} \quad \forall i, j. \tag{33}$$

Residual emission intensities are therefore

$$f_{ij}(\tau_{\text{co2}}) = \bar{f}_{ij}\left[1 - s_{ij}(\tau_{\text{co2}})\right] \quad \forall i, j. \tag{34}$$

Following DICE-2023, we set $\psi = 1.6$ and $k = 600$ EUR/tCO$_2$e.[25] We then infer $\bar{f}_{ij}$ from the observed 2018 emission intensity in the data. Given the average effective carbon price on emissions embodied in German household consumption in 2018, $\tau_0 = 3.91$ EUR/tCO$_2$e[26], (33) and (34) imply then

$$s_0 = \left(\frac{\tau_0}{k}\right)^{1/\psi} \qquad \text{and} \qquad \bar{f}_{ij} = \frac{f_{ij}}{1 - s_0}.$$

### 6.3.4 Tax-Transfer System

We adopt a parametric specification for the tax-transfer system proposed by Ferriere, Grübener, Navarro, and Vardishvili (2023). Total tax payments are given by

$$\mathcal{T}(y) = \exp\left[\log(\lambda)(y/\bar{y})^{-2\tau}\right] y - \mathcal{T}_0(y),$$

where $\bar{y}$ denotes average gross labor income. The first term specifies a two-parameter tax schedule: the parameter $\lambda$ governs the overall level of taxation and the parameter $\tau$ controls

[25] Barrage and Nordhaus (2024, Appendix E) report a DICE-2023 backstop price of 695 USD/tCO$_2$ in 2020, measured in 2019 dollars. Backcasting this value to 2018 using DICE's assumed 1% annual decline gives 709 USD/tCO$_2$. Using the 2018 average market exchange rate of 1.181 USD per euro gives about 600 EUR/tCO$_2$. In Section 7.3, we consider $\psi = 1$ and $\psi = 2$ as a robustness exercises.

[26] We approximate the average effective carbon price on emissions embodied in German household consumption as the uniform carbon price that would generate the same aggregate carbon cost as the EU ETS component embodied in the 2018 consumption basket. See E.1 for details.

the degree of progressivity. The transfer function $\mathcal{T}_0(y)$ phases out with labor income and is given by

$$\mathcal{T}_0(y) = m\bar{y}\frac{2\exp\left[-\zeta\frac{y}{\bar{y}}\right]}{1+\exp\left[-\zeta\frac{y}{\bar{y}}\right]},$$

where $m$ captures the size of transfers given to households with zero income as a share/multiple of average labor income $\bar{y}$ and $\zeta$ determines the phase-out rate of transfers. This specification allows us to jointly fit the tax payments of the bottom and the top of the income distribution.

We estimate the tax parameters $(\lambda, \tau)$ using tax return data from the LESt, and the transfer parameters $(m, \zeta)$ using data on transfers from the EVS. We estimate the parameters using non-linear least squares. Table 2 presents the estimated parameters and Figure 4(a) illustrates the marginal rates obtained from the estimated tax-transfer function. The solid red line depicts the statutory marginal income tax rate, while the dashed gray line shows the transfer phase-out rate. Transfers decline steeply with gross labor income and are fully phased out at an annual gross labor income of around €40 000. The effective marginal income tax rate is the sum of the statutory marginal income tax rate and the transfer phase-out rate. As a result, the effective marginal tax rate exhibits a sharp drop at the point where transfers are fully phased out, after which it converges to the statutory marginal tax rate. Appendix E.2 provides further details on the estimation process.

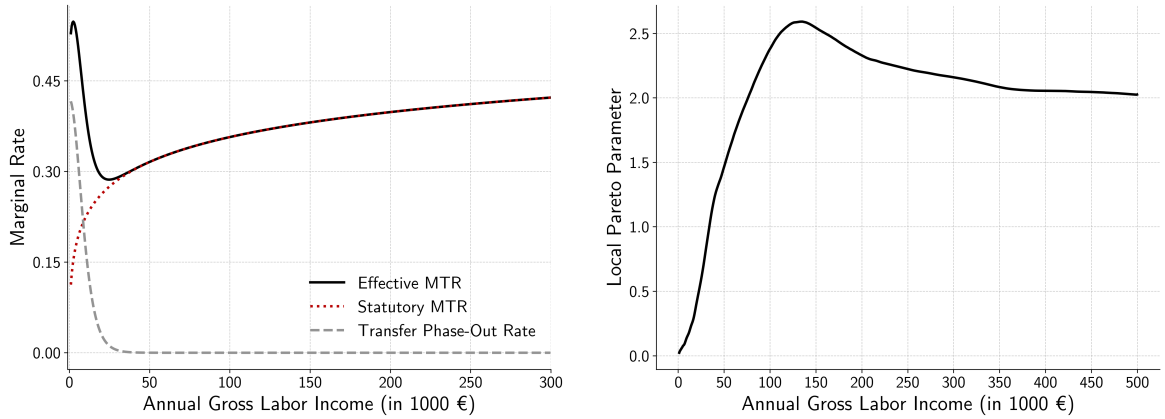
### 6.3.5 Income Distribution

To calibrate the distribution of gross labor income, we use administrative income tax records from the LESt for the year 2018. Gross labor income is defined as the sum of income from employment and self-employment. We estimate the income distribution $\tilde{h}(y)$ non-parametrically using a standard kernel density estimation. Figure 4(b) illustrates the Local Pareto parameter $\tilde{h}(y)y/(1-\tilde{H}(y)$ derived from the estimated estimated income distribution. At the top, the income distribution $\tilde{h}(y)y/(1-\tilde{H}(y)$ roughly converges consistent with a Pareto tail with a Pareto parameter of around 2. Appendix E.3 documents the estimation procedure in detail.

Given the estimated income distribution and the current German tax-transfer system, we infer the skill distribution $h(\theta)$ by inverting the labor supply first-order condition, following Saez (2001).

### 6.3.6 Utility Function

The calibration of the utility function, as represented by (30)-(32) requires a total of 43 parameters: $\{\Omega_{ig}\}_{i=1}^{8}$, $\{\epsilon_{ig}\}_{i=1}^{8}$ and $\{\sigma_i\}_{i=1}^{8}$ for the lower tier; $\{\Omega_i\}_{i=1}^{8}$, $\{\epsilon_i\}_{i=1}^{8}$ and $\sigma$ for the upper tier. Finally, the more standard Frisch elasticity parameter $\psi$ and the curvature parameter $\gamma$,




(a) Tax-Transfer System

(b) Income Distribution

Figure 4: Tax-Transfer System and Income Distribution

*Notes:* The left panel illustrates the statutory marginal income tax rate (dotted red line) and the transfer phase-out rate (dashed gray line) based on our estimates of the parametric function for the tax-transfer system proposed by Ferriere, Grübener, Navarro, and Vardishvili (2023). The effective marginal income tax rate (solid black line) is the sum of the statuary marginal income tax rate and the transfer phase-out rate. The right panel displays the local Pareto parameter $h_y(y)y/(1-H_y(y))$, where $H_y(y)$ is the cumulative distribution function and $h_y(y)$ is the probability density function of the gross labor income distribution. The density is estimated using a standard kernel density estimation. The x-axis shows gross labor income $y$ of a tax unit.

which would equal the coefficient of relative risk aversion case in case of homothetic preferences. Next, we explain which of those parameters we calibrate externally and which internally.

**Externally Calibrated Parameters.** We fix the Frisch elasticity of labor supply at $1/\varphi = 0.5$, in line with empirical estimates (Chetty, Guren, Manoli, and Weber 2011). We set the curvature parameter of consumption utility to $\gamma = 0.5$ and the elasticity of substitution for the upper tier utility function is set to $\sigma = 0.3$ (Comin, Lashkari, and Mestieri 2021). Finally, we assume that the elasticity of substitution of the lower-tier utility function, i.e. between 'brown' and 'green' varieties, to be identical across all products and set to $\sigma_i = 1.5$. Table 2 summarizes the set of externally calibrated parameters.[27]

**Internally Calibrated Parameters.** We calibrate the remaining preference parameters in two steps, exploiting how expenditure patterns in the EVS vary along the gross labor income distribution.

First, for each product $i$, we calibrate the lower-tier parameters governing the allocation between the 'brown' and 'green' varieties. We use the green variety as the reference and impose the normalization $\Omega_{ig} = 1$ and $\epsilon_{ig} = -1$ for all $i$. The parameters $\{\Omega_{ib}, \epsilon_{ib}\}_{i=1}^{I}$ on the 'brown' variety are then chosen by minimum distance to match the empirical within-product expenditure shares on the 'brown' variety across income deciles. Intuitively, the parameter $\Omega_{ib}$

[27]The chosen values for $\gamma$, $\sigma$ and $\varphi$ allow the model to replicated reasonable values of the income effect $\eta(\theta)$. Figure 15 in Appendix E.4.1 illustrates the income effect implied by the model. In Section 7.3 we provide robustness for these parameter choices.

| Parameter | Description | Value |
|---|---|---|
| | **Utility Function** | |
| $\varphi$ | Frisch elasticity of labor supply | 0.5 |
| $\gamma$ | Coefficient of relative risk aversion | 0.5 |
| $\sigma$ | Elasticity of substitution between products | 0.3 |
| $\sigma_i \ \forall i$ | Elasticity of substitution between varieties | 1.5 |
| | **Tax-Transfer System** | |
| $\lambda$ | Overall level of taxation | 0.25 |
| $\tau$ | Tax progressivity | 0.08 |
| $m$ | Transfers given to households with zero income | 0.1 |
| $\zeta$ | Transfer phase-out rate | 8.4 |
| $\bar{y}$ | Average gross labor income (in €) | 42 189 |

Table 2: Externally Calibrated Parameters

*Notes:* The table reports the set of externally calibrated parameters used in the model. The tax-transfer system parameters are chosen to match the German tax-transfer system in 2018, based on data from the LESt and EVS.

disciplines the average brown share within a category, while the lower-tier non-homotheticity parameter $\epsilon_{ib}$ disciplines how this share varies with income.

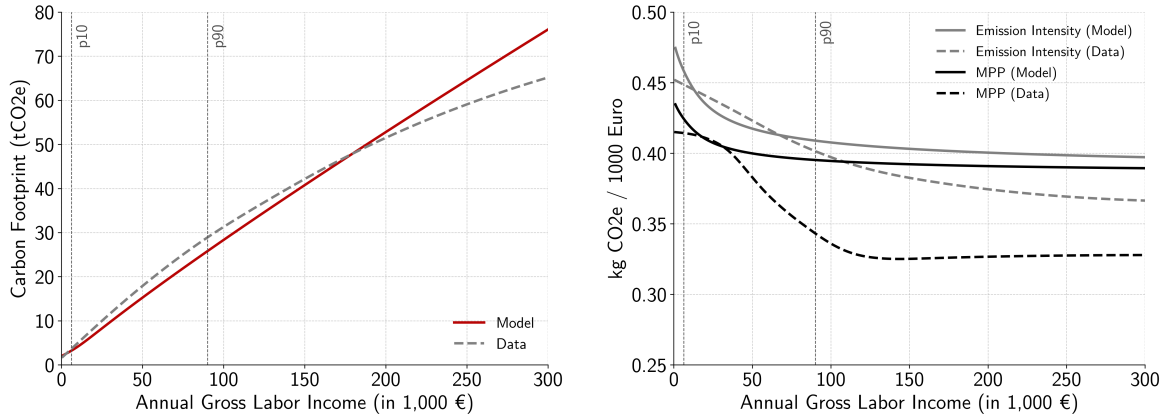
Second, conditional on the calibrated lower-tier nests, we calibrate the upper-tier parameters $\{\varepsilon_i, \Omega_i\}_{i=1}^{I-1}$, which govern the allocation of total expenditure across product categories. We normalize the parameters of one reference category and choose the remaining parameters $\Omega_i$ and non-homotheticity parameters $\epsilon_i$ by minimum distance, targeting the product-level expenditure shares across income deciles. The average level of a product's budget share primarily identifies $\Omega_i$, while the income gradient of the budget share identifies the non-homotheticity parameter $\varepsilon_i$. The internally calibrated parameters are reported in Table 8 in E.4.2. See Appendix E.4.2 for details on the internal calibration strategy.

**Model fit.** We first validate the fit of the calibrated demand system to the expenditure share moments used in the internal calibration. As shown in Figure 16 in Appendix E.4.2 the model closely tracks the empirical income gradient in both the allocation of expenditure across products and the within-category allocation between 'brown' and 'green' variates.

Second, we evaluate the model's fit to how carbon emissions vary with income. Figure 5(a) illustrates the model-implied carbon footprints along gross labor income (solid red line), together with the corresponding empirical moments from the EVS (grey lines). For the empirical moment, we apply the savings adjustment and aggregate goods into the same broad category–variety classification as in the model. The model captures well the overall income gradient in carbon emissions, but implies a less concave relationship than the data.[28] The the model-

[28]This partially reflects that utility parameters are disciplined by decile-level expenditure-share moments, which target average expenditure patterns within income bins.

implied carbon footprint of households at the 90th percentile is 7.92 times higher than of those at the 10th percentile, compared with 7.91 in the savings-adjusted aggregated data. Figure 5(b) shows the model-implied MPP and emission intensity profiles (solid lines), together with the corresponding empirical moments from the EVS (dashed lines). The model captures the declining pattern across the income distribution well, although the model-implied values are slightly higher than the empirical moments.




(a) Carbon Footprint

(b) Emission Intensity

Figure 5: Model Fit

*Notes:* The left panel plots the model-implied carbon footprint (red solid line) against a cubic smoothing spline fitted to the carbon footprint observed in the EVS data (grey dashed line). The right panel shows the model-implied average emission intensity, per Euro of consumption expenditure (grey solid line) and the marginal propensity to pollute (black solid line), both measured in kg$CO2_2$e per 1 000 Euro. The MPP is defined as the derivative of the carbon footprint with respect to total expenditure, and emission intensity is defined as the ratio of the carbon footprint to expenditure. In both figures, the corresponding empirical counterparts from the EVS are shown as dashed lines. To make the model and data comparable, the EVS based spline applies the same broad category–variety aggregation as the model and rescales expenditures to match disposable income. The x-axis shows gross labor income $y$ of a tax unit. The grey vertical dashed-dotted lines correspond to the 10th and 90th percentile of the annual gross labor income distribution.

# 7 Quantitative Policy Analysis

We now quantify how a binding cap on aggregate emissions reshapes redistribution in the calibrated German economy. The baseline is the 2018 calibration described in Section 6. For a given carbon cap, the government chooses the carbon price and an income-dependent rebate schedule, holding fixed the welfare criterion that rationalizes the status quo tax-transfer system. In the benchmark specification, $\mathcal{G}(x) = x$. The implied social marginal utilities and Pareto weights are shown in Figure 17 in Appendix E.5.

First, we consider a cap that implies a 25% reduction in carbon emissions as the baseline in Section 7.1. In Section 7.2 we then analyze the robustness of our results along the abatement path to zero emissions. Section 7.3 considers various robustness checks.

## 7.1 Baseline Results: 25% Carbon Emissions Reduction

Our benchmark imposes a 25% reduction in aggregate model emissions relative to the 2018 baseline, $\bar{F} = 0.75F_{2018}$. We adopt this target as a conservative policy benchmark. Reductions implied by current German and European climate targets imply reductions between 27% and 49% relative to 2018.[29] However, these policy targets are not directly comparable to the model cap. They apply to territorial emissions, whereas the model prices consumption-based household emissions, including both direct and indirect emissions. Section 7.2 studies the full abatement path to zero emissions.

**Carbon tax.** The optimal carbon tax amounts to 69.90 €/tCO2e. To gauge the fiscal scale, we express carbon tax revenue in VAT-equivalent terms. Holding the VAT base fixed, the carbon tax required to implement the benchmark cap raises the same revenue as a 2.33 percentage point increase in the statutory VAT rate. This highlights the substantial revenue potential and motivates the question of how these revenues should be rebated.

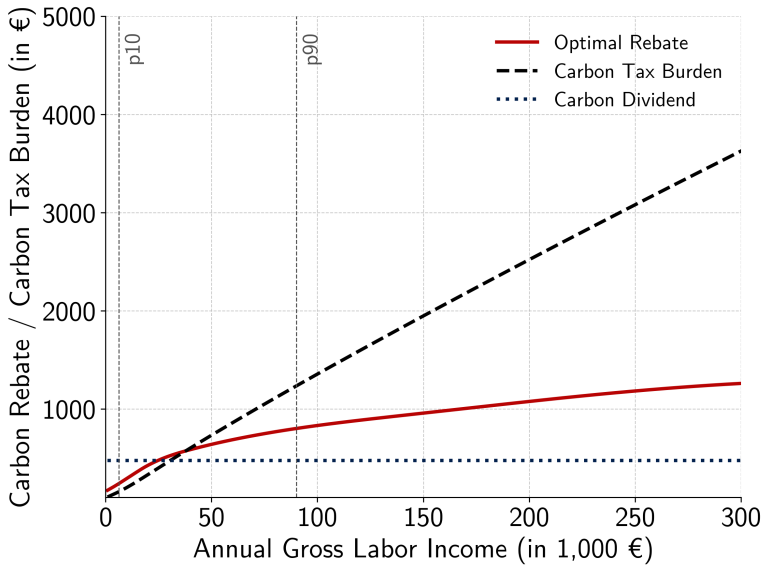



Figure 6: Optimal Carbon Rebate

*Notes:* The figure shows the optimal carbon rebate as a function of gross labor income (red solid line), the rebate under a carbon dividend policy (blue dotted line) and the carbon tax burden under the optimal carbon rebate (black dashed line). The grey vertical dashed-dotted lines correspond to the 10th and 90th percentile of the annual gross labor income distribution.

**Main results.** Figure 6 illustrates the optimal income-dependent carbon rebate schedule $R_{\mathrm{CO}_2}(y)$ (red solid line) as a function of gross labor income. The schedule is upward sloping:

[29] In particular, the 2030 target under the German Federal Climate Change Act (*Bundes-Klimaschutzgesetz*, KSG) corresponds to a reduction of about 49% relative to 2018. The 2030 target under the European Union Effort Sharing Regulation (ESR) implies a reduction of about 45% relative to 2018. The target under the European Union's second emissions trading system for buildings and road transport (ETS2) corresponds to a reduction of about 27% relative to 2018. Appendix E.6 provides the underlying calculations.

the annual rebate is about € 200 per tax unit at the bottom of the income distribution and about € 1 200 at an annual gross labor income of € 300 000. To assess the distributional pattern of the rebate policy, we introduce a *regressivity ratio*, defined as the ratio of the rebate received by the 90th percentile to that received by the 10th percentile of the income distribution. Formally,

$$RR_{90/10}^{\text{rebate}} \equiv \frac{R_{\text{CO}_2}(y_{90})}{R_{\text{CO}_2}(y_{10})},$$

where $y_p$ denotes annual gross labor income at percentile $p$, with $y_{90} = 91,909$ and $y_{10} = 6,495$. For the optimal rebate schedule, we find $RR_{90/10}^{\text{rebate}} = 3.31$, indicating that the tax units in the 90th percentile receive a 3.31 times higher rebate than those in the 10th percentile. This implies that the rebate policy in isolation is regressive. By contrast, the *progressivity ratio* of the carbon tax is $PR_{90/10}^{\text{carbon tax}} = 7.88$, indicating that the 90th percentile pays 7.88 times more in carbon taxes than the 10th percentile. Recall that this latter result directly rests on how carbon footprints vary along the income distribution.

Although the optimal rebate rises with income in absolute amounts, it rises much less steeply than carbon tax payments. The overall policy is therefore progressive. Figure 7(a) shows the effective marginal tax rate along the gross income distribution. The optimal carbon policy (red solid line) increases effective marginal tax rates relative to the status quo (black dashed line) for incomes above € 13 000. For the highest incomes, this increase is around 1.5 percentage points.

To assess the overall incidence of the reform, Figure 7(b) reports changes in effective average tax rates. The red solid line illustrates the net burden of the carbon policy, defined as the change in the effective average tax rate, as a function of the percentile of the income distribution.[30] The net burden rises from approximately −10 percentage points at the very bottom to 1 percentage points at the top of the distribution. Further, tax units until the 50th percentile are net beneficiaries of the carbon policy, receiving more in rebates than they pay in carbon taxes.

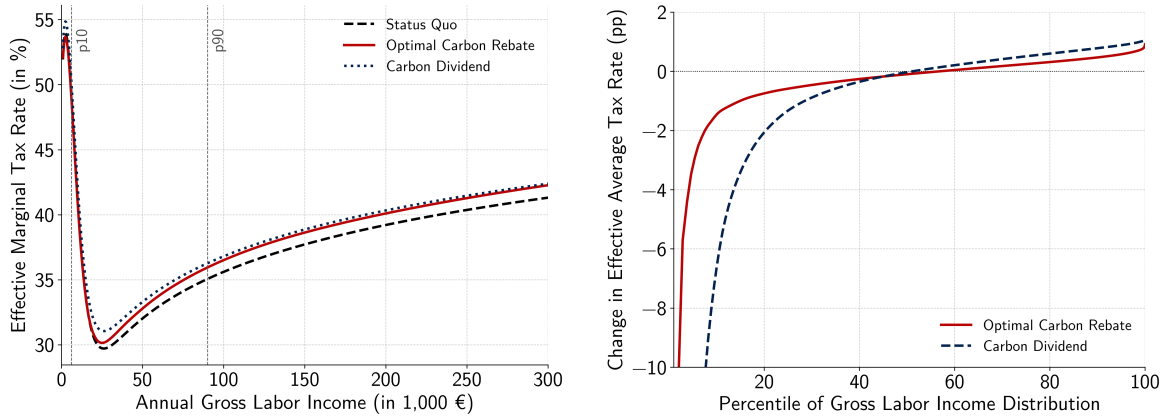
To summarize the redistributive effect of the carbon policy in a single statistic, we compare effective average tax rates (EATR) at the 90th and 10th percentiles of the gross labor income distribution. For policy $x \in \{\text{sq}, \text{co2}\}$, define the EATR gap as

$$\Pi_{90/10}^{x} \equiv \bar{\tau}_{\text{eff}}^{x}(y_{90}) - \bar{\tau}_{\text{eff}}^{x}(y_{10}),$$

$\bar{\tau}_{\text{eff}}^{x}(y_p)$ denotes the effective average tax rate evaluated at that income under policy $x$. We measure the shift in redistribution as the change in the EATR gap:

$$\Delta\Pi_{90/10} \equiv \Pi_{90/10}^{\text{co2}} - \Pi_{90/10}^{\text{sq}}.$$

[30] In model notation, we define the EATR as $\bar{\tau}_{\text{eff}}(\theta) = \frac{\mathcal{T}(y(\theta)) + \tau_{co_2} f(\theta)}{y(\theta)}$.




(a) Effective Marginal Tax Rates

(b) Net Burden of Carbon Policy

Figure 7: Redistributive Implications of Carbon Policy

*Notes:* The left panel shows the effective marginal tax rate $\tau_{\text{eff}}$ as a function of gross labor income. The right panel shows the change in the effective average tax rate $\bar{\tau}_{\text{eff}}$ across percentiles of the gross labor income distribution, defined as $\bar{\tau}_{\text{eff}}(\theta) = (\mathcal{T}(y(\theta)) + \tau_{co_2} f(\theta))/y(\theta)$. The vertical dashed-dotted line indicates the percentile with no change in the effective average tax rate. The black dashed line represents the status quo without carbon policies, the red solid line the optimal carbon rebate policy, and the blue dotted line the carbon dividend policy. The grey vertical dashed-dotted lines mark to the 10th and 90th percentile of the annual gross labor income distribution.

We find $\Delta\Pi_{90/10} = 1.91$ percentage points, indicating an economically meaningful increase in redistribution under the optimal carbon policy. Table 10 in Appendix F reports the underlying EATRs and implied differences.

**Comparison with carbon dividend.** We next compare the optimal rebate schedule with a lump-sum carbon dividend policy that achieves the same aggregate emissions reduction. The carbon dividend is illustrated by the blue dotted line in Figure 6. Relative to the optimal policy, the dividend provides larger rebates to tax units in the bottom 45% of the income distribution and smaller rebates to tax units above that threshold. Figure 7(a) shows the corresponding effective marginal tax rates (blue dotted line). Because the dividend is lump-sum, it does not offset the income gradient in carbon tax liabilities and therefore raises effective marginal tax rates relative to the optimal policy. For example, for gross labor income below € 25 000, effective marginal tax rates are more than 1 percentage point higher under the carbon dividend. The resulting increase in redistribution is also higher with $\Delta\Pi_{90/10} = 7.31$.[31]

## 7.2 Along the Abatement Path

We next trace the optimal policy as the emissions cap tightens from the 2018 baseline to zero emissions. For five illustrative levels of emission reduction targets, Table 3 reports the carbon

[31] See Table Table 10 in Appendix F for the underlying EATRs.

| Carbon reduction | 10% | 25% | 50% | 75% | 90% |
|---|---|---|---|---|---|
| Carbon tax | 26.44 | 69.90 | 182.73 | 355.36 | 492.85 |

Table 3: Carbon tax by emissions-reduction target

*Notes:* This table illustrates the different levels of carbon taxes needed to achieve a carbon reduction of X% relative to 2018 carbon emissions.

tax rate that implements the corresponding cap.[32] Figure 8(a) shows that the relative incidence of the policy is remarkably stable along this path. Both the regressivity ratio $RR_{90/10}^{\text{rebate}}$ and the progressivity ratio $PR_{90/10}^{\text{carbon tax}}$ vary little with the emission reduction target. The increase in redistribution, however, is non-monotone. As shown in Figure 8(b), the change in the EATR gap $\Delta\Pi_{90/10}$ rises as the cap tightens, reaches its maximum at an emissions reduction of about 60%, and declines thereafter. At the peak, the optimal policy increases the change in the EATR gap by about 4 percentage points. A carbon dividend generates a much larger increase, about 15 percentage points.

The same hump shaped pattern is reflected in carbon tax revenue. Figure 8(c) reports revenue in VAT-equivalent terms. The VAT-equivalent increase peaks at nearly 5 percentage points when aggregate emissions are reduced by about 60%, before declining as the carbon tax base shrinks.

Finally, Figure 8(d) reports the implied GDP loss along the abatement path, driven by the labor supply reductions to higher effective marginal tax rates. The loss follows a similar hump-shaped profile. Across the abatement path, the GDP loss is substantially larger under a carbon dividend than under the optimal rebate, consistent with the larger increase in marginal tax rates induced by the dividend policy.

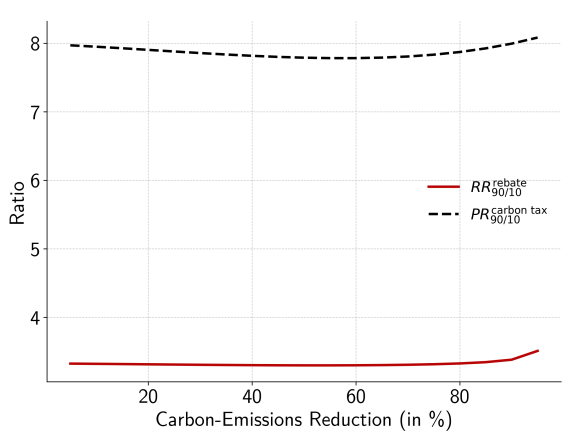
## 7.3 Robustness Analysis

We now consider eight alternative parameter specifications, with the results summarized in Table 4.[33]. Overall, the findings are highly robust. The optimal steepness of the rebate schedule varies slightly across specifications, consistent with the insights from the theoretical analysis. The same holds for the implications regarding the increase in redistribution.

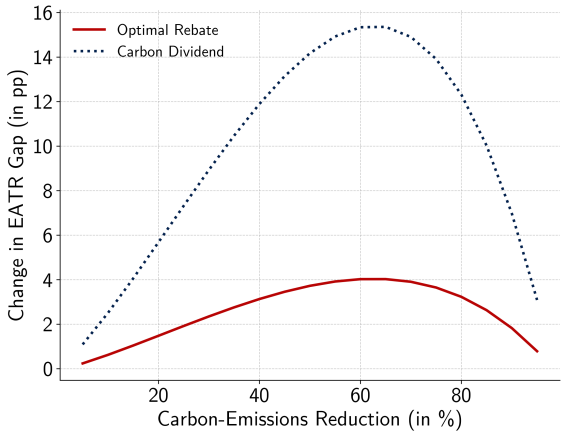
**Lower Elasticity.** In this scenario, we set the Frisch elasticity parameter to half its benchmark value, i.e. $\varphi = 0.25$. The optimal rebate becomes more regressive, with our regressivity measure increasing from 3.31 to 4.45. The progressivity ratio of the carbon tax is almost un-

[32] Figure 18 in Appendix F.1 shows the optimal carbon rebate, effective marginal tax rate and the net burden of the carbon policy along the gross labor income distribution for an emission reduction target of 10%, 25%, 50%, 75% and 95%.

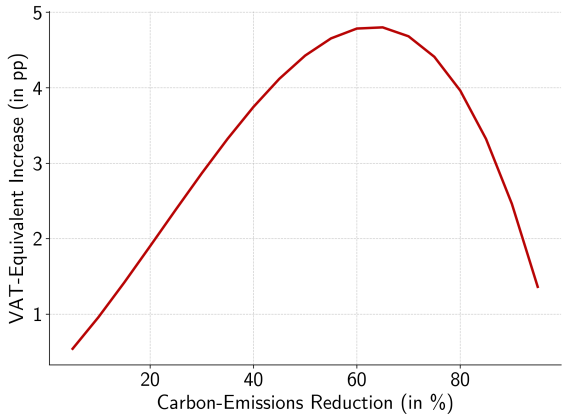
[33] Appendix F.2 documents the optimal carbon rebate, effective marginal tax rate and the net burden of the carbon policy along the gross labor income distribution for all seven alternative parameter specifications.



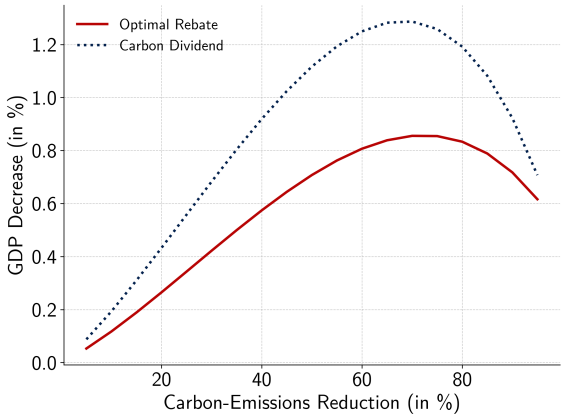

(a) $RR^{\text{rebate}}_{90/10}$ and $PR^{\text{carbon tax}}_{90/10}$




(b) $\Delta\Pi_{90/10}$




(c) Revenue-equivalent VAT increase




(d) GDP reduction through labor supply responses

Figure 8: Redistributive, Revenue, and GDP Implications of Carbon Policy along the Abatement Path

*Notes:* The upper left panel shows the regressivity ratio $RR^{\text{rebate}}_{90/10}$ (red solid line) and progressivity ratio $PR^{\text{carbon tax}}_{90/10}$ (black dashed line) for different values of the targeted carbon emission reduction. The upper right panel shows the change in the effective average tax rate (EATR) gap $\Delta\Pi_{90/10}$ for different values of the targeted carbon emission reduction. The red corresponds to the EATR gap under the optimal carbon rebate and the black dashed line under a carbon dividend. The lower left panel shows the percentage point revenue-equivalent VAT increase for different values of the targeted carbon emission reduction. The lower right panel shows the percentage reduction in the GDP under the optimal rebate (red solid line) and carbon dividend (black dashed line) for different values of the targeted emission reduction.

changed. Consequently, the increase in redistribution of the carbon policy is smaller: the EATR gap rises by only 1.14 percentage points.

At first sight, this may seem surprising, as a lower elasticity typically implies higher optimal taxes and thus more redistribution. However, in our setup, lowering the elasticity also changes the structure of Pareto weights in such a way that the German tax-transfer system of 2018 remain optimal. For this to hold, the Pareto weights must become less 'pro-poor', offsetting the potential redistribution gains from higher optimal taxes.

**Higher Curvature.** In this scenario, we increase the curvature of consumption utility to $\gamma = 1.5$. The optimal rebate again becomes more regressive. As the government expands redis-

Table 4: Robustness of Policy Implications

| Case | $RR_{90/10}^{\text{rebate}}$ | $PR_{90/10}^{\text{carbon tax}}$ | $\Delta\Pi_{90/10}$ | $\tau_{\text{co2}}$ |
|---|---|---|---|---|
| Baseline | 3.31 | 7.88 | 1.91 | 63.82 |
| Lower elasticity ($\varphi = 0.25$) | 4.45 | 7.93 | 1.14 | 64.44 |
| Higher curvature parameter ($\gamma = 1.5$) | 4.05 | 7.92 | 1.41 | 64.28 |
| Lower product substitution ($\sigma = 0.5$) | 3.49 | 8.25 | 1.83 | 61.93 |
| Lower brown-green substitution ($\sigma_i = 0.5\ \forall i$) | 3.33 | 7.75 | 2.09 | 70.32 |
| Lower heating and housing brown-green substitution | 3.41 | 7.82 | 1.89 | 66.55 |
| Curvature in welfare function ($\gamma_G = 3$) | 4.31 | 7.93 | 1.20 | 64.02 |
| Lower curvature of abatement costs ($\psi = 1$) | 3.30 | 7.78 | 3.59 | 113.31 |
| Higher curvature of abatement costs ($\psi = 2$) | 3.31 | 7.91 | 1.40 | 48.60 |

*Notes:* The table reports distributional patterns of the optimal rebate policy for the baseline specification and eight alternative parameter settings. The case *lower heating and housing brown-green substitution* refers to a scenario in which the elasticity of substitution between the brown and green varieties for housing and heating is set to $\sigma_i = 0.5$, while holding $\sigma_i = 1.5$ for all other products. This allows richer households to substitute more easily from brown to green varieties. The carbon tax $\tau_{\text{co2}}$ is measured in $tCO_2/€$ and the change in the EATR gap $\Delta\Pi_{90/10}$ in percentage points (pp). All cases refer to the 25% emission reduction target.

tribution, the gains from redistribution (as captured by the distributional gains term) diminish more quickly than in the benchmark case with a lower curvature. Consequently, it is optimal to increase redistribution by less, and the EATR gap amounts to only 1.41 percentage points. The change in the curvature also affects income effects. Below, we consider an alternative scenario in which these income effects are held constant.

**Lower Elasticity of Substitution between products** We increase the elasticity of substitution across products to $\sigma = 0.5$. Greater sustainability allows households to adjust their expenditure more strongly in response to the carbon tax. This reduced the carbon price slightly. The relative incidence of the policy changes only modestly. The rebate ratio and progressivity ratio slightly increase and the EATR gap declines from 1.91 to 1.83 percentage points.

**Lower Elasticity of Substitution between Varieties.** In this scenario, a higher carbon tax is required to achieve the same reduction in the aggregate carbon footprint. The regressivity of the optimal rebate and the progressivity of the carbon tax are similar to the benchmark. The increase in redistribution, $\Delta\Pi_{90/10}$, is slightly larger, however, this is primarily driven by the fact that both the level of the rebate and the carbon tax rate are higher in this scenario.

**Lower Substitution for Housing and Heating.** We lower the elasticity of substitution between brown and green varieties to $\sigma_i = 0.5$ for heating and housing, while retaining $\sigma_i = 1.5$ for all other products. In these categories, substitution toward the green variety typically requires replacing the heating system, retrofitting the dwelling, or moving, rather than marginal reallo-

cations of expenditure.[34] Because the expenditure shares on heating and housing decline with income, reducing $\sigma_i$ in these categories constraints adjustment more strongly for low income households, giving high income households relatively greater scope to substitute toward green varieties. Nevertheless, the rebate and carbon-tax incidence ratios and the implied increase in redistribution remain virtually unchanged.

**Curvature in Welfare Function.** We assume $\mathcal{G}(x) = \frac{x^{1-\gamma_\mathcal{G}}}{1-\gamma_\mathcal{G}}$ with $\gamma_\mathcal{G} = 3$ for the calibration of the inverse-optimum weights. This substantial increase in curvature implies a markedly smaller increase in redistribution and a rebate schedule that rises more steeply with income. The effect is similar to the second robustness check with $\gamma = 1.5$, but here the curvature effect on the social marginal utility is isolated because income effects are held constant.

**Curvature of Abatement Costs.** We vary the curvature parameter of abatement costs by setting $\psi = 1$ and $\psi = 2$. Holding the backstop price fixed, a lower (higher) value of $\psi$ implies that firms initially abate less (more) and hence a larger carbon tax is needed to acheive the 25% reduction: it amounts to EUR 113.31/t$CO_2$e under $\psi = 1$ and EUR 48.60/t$CO_2$e under $\psi = 2$. Consequently, the EATR gap increases to 3.59 percentage points under $\psi = 1$, compared with 1.40 percentage points under $\psi = 2$. The relative incidence of the policy, however, is virtually unchanged: the 90–10 ratios of optimal rebates and carbon tax payments remain close to 3.3 and 7.9, respectively.

# 8 Conclusion

In this paper we study the interaction between carbon taxation and redistribution. A comparative statics analysis shows that the introduction of a carbon cap increases the optimal level of redistribution, independent of the social welfare function. However, the optimal increase in redistribution is smaller than what a carbon dividend would imply. Implementing the new desired allocation through carbon tax rebates requires rebates rise with income, a result that holds independently of the welfare function. Only in a polar case, requiring, among other assumptions, constant social marginal utility of consumption, is it optimal to implement a lump-sum carbon tax rebate.

We quantify our model using German administrative and household microdata, matched with environmentally extended input-output tables to estimate carbon footprints as a function of income. Our results show that the optimal rebate is roughly three times larger for households at the 90th percentile than for those at the 10th percentile of the income distribution. While

[34] Descriptive evidence supports limited substitution in the short to medium run. Roughly half of German households rent and do not control the the installed heating system. Moreover, evidence from the RWI Residential Energy Consumption Survey (GRECS) further indicates that, among non-moving households, only 1.7% switch from fossil heating to district heating or a heat pump over five years. See Appendix F.2.1 for details.

such a rebate schedule appears regressive, the overall policy of carbon taxation combined with carbon tax rebates significantly increases redistribution: carbon tax payments are about seven times higher at the 90th percentile than at the 10th percentile of the income distribution. The optimal policy therefore makes more households better off than a carbon dividend would, suggesting that it could also enjoy greater political support.

# References


Abrell, J. (2024): "Database for the European Union Transaction Log," .

Aghion, P., T. Boppart, M. Peters, M. Schwartzman, and F. Zilibotti (2025): "A theory of endogenous degrowth and environmental sustainability," Discussion paper, National Bureau of Economic Research.

Atkinson, A., and J. Stiglitz (1976): "The design of tax structure: Direct versus indirect taxation," *Journal of Public Economics*, 6(1), 55–75.

Barrage, L. (2020): "Optimal dynamic carbon taxes in a climate–economy model with distortionary fiscal policy," *The Review of Economic Studies*, 87(1), 1–39.

Barrage, L., and W. Nordhaus (2024): "Policies, projections, and the social cost of carbon: Results from the DICE-2023 model," *Proceedings of the National Academy of Sciences*, 121(13), e2312030121.

Bierbrauer, F. (2025): "Is a Uniform Price on Carbon Desirable?," .

Bigio, S., D. R. Känzig, P. Sánchez, and C. Walsh (2026): "Carbon Pricing and Inequality: A Normative Perspective," *The Economic Journal*.

Blanchard, O. J., C. Gollier, and J. Tirole (2023): "The Portfolio of Economic Policies Needed to Fight Climate Change," *Annual Review of Economics*, 15(1), 693–720.

Blank, P., B. Wickert, A. Obermeier, and R. Friedrich (1999): "Erstellung eines Emissionskatasters für Feuerungsanlagen in Haushalt und Kleinverbrauch: 9 Anlagen," Anhang, Umweltbundesamt, Appendix volume; accessed 2026-04-12.

Bovenberg, A. L., and R. A. De Mooij (1994): "Environmental levies and distortionary taxation," *The American Economic Review*, 84(4), 1085–1089.

Bovenberg, A. L., and F. van der Ploeg (1994): "Environmental policy, public finance and the labour market in a second-best world," *Journal of Public Economics*, 55(3), 349–390.

Brockjan, K., L. Maier, K. Kott, and N. Sewald (2021): "Umwelt, Energie und Mobilität. Kapitel 13 des Datenreports 2021," Discussion paper, Statistisches Bundesamt (Destatis), Accessed 2026-04-12.

Bundesagentur für Arbeit (2018): "Ausgaben für aktive und passive Leistungen im SGB II (Jahreszahlen)," .

Chetty, R., A. Guren, D. Manoli, and A. Weber (2011): "Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive mMrgins," *The American Economic Review*, 101(3), 471–475.

Choné, P., and G. Laroque (2010): "Negative Marginal Tax Rates and Heterogeneity," *American Economic Review*, 100(5), 2532–47.

Comin, D., D. Lashkari, and M. Mestieri (2017): "Structural Change With Long-Run Income and Price Effects," *Working Paper*.

——— (2021): "Structural Change With Long-Run Income and Price Effects," *Econometrica*, 89(1), 311–374.

Craig, A. C., T. Lloyd, and D. T. Moore (2025): "Do Distributional Concerns Justify Lower Environmental Taxes?," CESifo Working Paper 12317, CESifo.

Cremer, H., F. Gahvari, and N. Ladoux (1998): "Externalities and Optimal Taxation," *Journal of Public Economics*, 70(3), 343–364.

Douenne, T., S. Dyrda, A. J. Hummel, and M. Pedroni (2025): *Optimal Climate Policy with Incomplete Markets*. Centre for Economic Policy Research.

Douenne, T., A. J. Hummel, and M. Pedroni (2026): "Optimal Fiscal Policy in a Climate-Economy Model with Heterogeneous Households," *The Economic Journal*, p. ueag006.

European Environment Agency (2026): "European Union Emissions Trading System (EU ETS) data," European Environment Agency Datahub, Accessed: 2026-07-02.

Familienkasse Direktion SR1 (2018): "Kindergeld / Kinderzuschlag Jahreszahlen 2018," .

Ferey, A., B. B. Lockwood, and D. Taubinsky (2024): "Sufficient statistics for nonlinear tax systems with general across-income heterogeneity," *American Economic Review*, 114(10), 3206–3249.

Ferriere, A., P. Grübener, G. Navarro, and O. Vardishvili (2023): "On the Optimal Design of Transfers and Income Tax Progressivity," *Journal of Political Economy Macroeconomics*, 1(2), 276–333.

German Statistical Office (2022): "Input-Output-Rechnung – Fachserie 18 Reihe 2," Revision 2019, Stand: August 2022. Reference year: 2018. Published 2022-12-22.

——— (2024a): "VGR des Bundes - Konsumausgaben der privaten Haushalte," .

——— (2024b): "Wohngeld - Ausgaben insgesamt nach Bundesländern im Zeitvergleich am 31.12.," .

——— (2025a): "Bruttoausgaben, Einnahmen, Nettoausgaben der Sozialhilfe: Deutschland, Jahre, Sozialhilfearten (22111-0003)," .

——— (2025b): "CO2 footprint of private households by areas of demand," .

Golosov, M., M. Graber, M. Mogstad, and D. Novgorodsky (2023): "How Americans Respond to Idiosyncratic and Exogenous Changes in Household Wealth and Unearned Income," *The Quarterly Journal of Economics*, 139(2), 1321–1395.

Goulder, L. H. (1995): "Environmental taxation and the double dividend: a reader's guide," *International tax and public finance*, 2, 157–183.

Goulder, L. H., M. A. C. Hafstead, G. Kim, and X. Long (2019): "Impacts of a Carbon Tax across U.S. Household Income Groups: What Are the Equity-Efficiency Trade-Offs?," *Journal of Public Economics*, 175, 44–64.

Grainger, C. A., and C. D. Kolstad (2010): "Who Pays a Price on Carbon?," *Environmental and Resource Economics*, 46(3), 359–376.

Hänsel, M. C., M. Franks, M. Kalkuhl, and O. Edenhofer (2022): "Optimal Carbon Taxation and Horizontal Equity: A Welfare-Theoretic Approach with Application to German Household Data," *Journal of Environmental Economics and Management*, 116, 102730.

Hardadi, G., A. Buchholz, and S. Pauliuk (2021): "Implications of the distribution of German household environmental footprints across income groups for integrating environmental and social policy design," *Journal of Industrial Ecology*, 25(1), 95–113.

Heathcote, J., and H. Tsujiyama (2021): "Optimal income taxation: Mirrlees meets Ramsey," *Journal of Political Economy*, 129(11), 3141–3184.

Hendren, N. (2016): "The policy elasticity," *Tax Policy and the Economy*, 30(1), 51–89.

IPCC (2006a): "Chapter 2: Stationary Combustion," in *2006 IPCC Guidelines for National Greenhouse Gas Inventories. Volume 2: Energy*. Institute for Global Environmental Strategies (IGES), for the IPCC, Accessed 2026-04-12.

——— (2006b): "Chapter 3: Mobile Combustion," in *2006 IPCC Guidelines for National Greenhouse Gas Inventories. Volume 2: Energy*. Institute for Global Environmental Strategies (IGES), for the IPCC, Accessed 2026-04-12.

Jacobs, B., and R. A. De Mooij (2015): "Pigou meets Mirrlees: On the irrelevance of tax distortions for the second-best Pigouvian tax," *Journal of Environmental Economics and Management*, 71, 90–108.

Kaplow, B. L. (2012): "Optimal control of externalities in the presence of income taxation," *International Economic Review*, 53(2), 487–509.

Kopczuk, W. (2003): "A Note on Optimal Taxation in the Presence of Externalities," *Economics Letters*, 80(1), 81–86.

Levinson, A., and J. O'Brien (2019): "Environmental Engel Curves: Indirect Emissions of Common Air Pollutants," *The Review of Economics and Statistics*, 101(1), 121–133.

Lockwood, B. B. (2020): "Optimal Income Taxation with Present Bias," *American Economic Journal: Economic Policy*, 12(4), 298–327.

Micheletto, L. (2008): "Redistribution and Optimal Mixed Taxation in the Presence of Consumption Externalities," *Journal of Public Economics*, 92(10–11), 2262–2274.

Research Data Centre of the Federal Statistical Office and the statistical offices of the Länder (RDC) (2018a): "Einkommens- und Verbrauchsstichprobe (HB) 2018, On-Site-Zugang," .

——— (2018b): "Einkommens- und Verbrauchsstichprobe (NGT) 2018, On-Site-Zugang," .

——— (2018c): "Lohn- und Einkommenssteuerstatistik 2018, Gastwissenschaftsarbeitsplatz," .

RWI, and forsa (2015): "The German Residential Energy Consumption Survey 2011–2013," Project report, RWI – Leibniz Institute for Economic Research, Essen.

Saez, E. (2001): "Using Elasticities to Derive Optimal Income Tax Rates," *The Review of Economic Studies*, 68(1), 205–229.

——— (2002): "The desirability of commodity taxation under non-linear income taxation and heterogeneous tastes," *Journal of Public Economics*, 83(2), 217–230.

Sager, L. (2019): "Income inequality and carbon consumption: Evidence from Environmental Engel curves," *Energy Economics*, 84, 104507, Eighth Atlantic Workshop on Energy and Environmental Economics.

Sandmo, A. (1975): "Optimal Taxation in the Presence of Externalities," *The Swedish Journal of Economics*, 77(1), 86–98.

Stadler, K., R. Wood, T. Bulavskaya, C.-J. Södersten, M. Simas, S. Schmidt, A. Usubiaga, J. Acosta-Fernández, J. Kuenen, M. Bruckner, S. Giljum, S. Lutter, S. Merciai, J. H. Schmidt, M. C. Theurl, C. Plutzar, T. Kastner, N. Eisenmenger, K.-H. Erb, A. de Koning, and A. Tukker (2018): "EXIOBASE 3: Developing a Time Series of Detailed Environmentally Extended Multi-Regional Input-Output Tables," *Journal of Industrial Ecology*, 22(3), 502–515.

Statistisches Bundesamt (2025): "Greenhouse gas emissions1 (residents concept)," .

Trading Economics (2026): "EU Carbon Permits: Index Price, Live Quote and Historical Chart," .

Umweltbundesamt (2025a): "Carbon Dioxide Emissions for the German Atmospheric Emission Reporting, 1990 - 2024," .

——— (2025b): "Emissionen sinken 2018 um mehr als 31 Prozent gegenüber 1990," .

United Nations Climate Change (2026): "Global Warming Potentials (IPCC Fourth Assessment Report)," Accessed: 2026-01-14.

van der Ploeg, F., A. Rezai, and M. Tovar Reaños (2022): "Gathering Support for Green Tax Reform: Evidence from German Household Surveys," *European Economic Review*, 141, 103966.

# A Theoretical Appendix

The main text states the theoretical results for the benchmark without a uniform commodity tax and, often, without income effects. In the appendix, we keep the more general expressions that allow for an exogenous uniform commodity tax $t_c$ and non-zero income effects as captured by $\eta(\theta)$. The proofs below use these more general objects and at the end of the proofs, it is shown how they simplify in the case of $t_c = 0$ and no income effects.

## A.1 Proof of Lemma 1

Recall the first-order condition for labor supply (7):

$$u_e(\theta, \mathcal{P})\theta\left(1 - \mathcal{T}'(y(\theta))\right) = \frac{y(\theta)}{\theta}^{\frac{1}{\varphi}}. \tag{35}$$

We proof the result by showing that both tax reforms affect the left-hand side (LHS) in the same way.

**Carbon Tax Reform.** A marginal increase in the carbon tax by $d\tau_{\text{co2}}$ affects the optimality condition through $u_e(\theta, \mathcal{P})$. Denote its change due to the carbon tax by $du_e(\theta)$. It is given by:

$$\begin{aligned} du_e(\theta) =& \frac{\partial u_e(\theta)}{\partial \tau_{\text{co2}}} d\tau_{\text{co2}} = \frac{\partial \left(u_{\tau_{\text{co2}}} d\tau_{\text{co2}}\right)}{\partial e} \\ =& \frac{\partial \left(-u_e(\theta) \sum_{i=1}^{n} f_i c_i(\theta) d\tau_{\text{co2}}\right)}{\partial e} \\ =& - u_e(\theta) f_e(\theta) d\tau_{\text{co2}} - u_{ee} f(\theta) d\tau_{\text{co2}}. \end{aligned}$$

Hence, the relative change of the LHS of (35) is given by:

$$-f_e(\theta)d\tau_{\text{co2}} - \frac{u_{ee}}{u_e} f(\theta) d\tau_{\text{co2}}. \tag{36}$$

**Income Tax Reform.** Next, consider an equivalent income tax reform that satisfies

$$d\mathcal{T}(y(\theta)) = f(\theta) d\tau_{\text{co2}},$$

and hence also

$$d\mathcal{T}'(y(\theta)) = \left(1 - \mathcal{T}'(y(\theta))\right. f_e(\theta) d\tau_{\text{co2}}.$$

The change in the LHS of (35) is given by

$$-u_{ee} f(\theta) d\tau_{\text{co2}} \theta \left(1 - \mathcal{T}'(y(\theta))\right) - u_e \theta \left(1 - \mathcal{T}'(y(\theta))\right. f_e(\theta) d\tau_{\text{co2}}.$$

Dividing this expression by $u_e\theta\left(1-\mathcal{T}'(y(\theta))\right)$ yields that the same relative effect on the LHS of (35) as in the carbon tax case (36).

## A.2 Effective marginal tax rate

Total tax revenue raised from a household of type $\theta$ is given by

$$\mathcal{T}(y(\theta))+\frac{t_c}{1+t_c}e(\theta)+\frac{\tau_{\text{co2}}}{1+t_c}f(\theta).$$

We define $\tau_{\text{eff}}(\theta,\mathcal{P})$ as the increase in government revenue resulting from a one unit increase in the gross labor income of a household of type $\theta$. Differentiating total tax revenue from a household $\theta$ with respect to its gross household income $y(\theta)$ gives

$$\tau_{\text{eff}}(\theta,\mathcal{P})=\mathcal{T}'(y(\theta))+\left(1-\mathcal{T}'(y(\theta))\right)\left(\frac{t_c}{1+t_c}+\frac{\tau_{\text{co2}}}{1+t_c}\frac{\partial f(\theta)}{\partial e(\theta)}\right),$$

which implies

$$\tau_{\text{eff}}(\theta,\mathcal{P})=\frac{\mathcal{T}'(y(\theta))+t_c+\left(1-\mathcal{T}'(y(\theta))\right)\tau_{\text{co2}}\frac{\partial f(\theta)}{\partial e}}{1+t_c}.$$

## A.3 Labor supply responses

$$\varepsilon_{y,\,1-\mathcal{T}'}(\theta)\equiv\frac{d\log(y(\theta))}{d\log(1-\mathcal{T}')}=\frac{1}{\frac{1}{\varphi}+\gamma(\theta)\frac{y(\theta)}{e(\theta)}\left(1-\mathcal{T}'\left(y(\theta)\right)\right)+\frac{\mathcal{T}''(y(\theta))y(\theta)}{1-\mathcal{T}'(y(\theta))}},\tag{37}$$

where $\gamma(\theta)=-\frac{u_{ee}(e(\theta))e(\theta)}{u_e(\theta)}$ is the coefficient of relative risk aversion. Second, the income effect parameter is defined as

$$\eta(\theta)\equiv\frac{\partial y(\theta)}{\partial\mathcal{T}(0)}=\frac{\gamma(\theta)\frac{y(\theta)}{e(\theta)}}{\frac{1}{\varphi}+\gamma(\theta)\frac{y(\theta)}{e(\theta)}\left(1-\mathcal{T}'\left(y(\theta)\right)\right)+\frac{\mathcal{T}''(y(\theta))y(\theta)}{1-\mathcal{T}'(y(\theta))}}.\tag{38}$$

Both, (37) and (38) can be obtained from implicit differentiation of (7).

## A.4 Proof of Lemma 2

The Lagrangian of the government's problem is given by

$$\mathcal{L} = \int_{\underline{\theta}}^{\overline{\theta}} \left(u(e(\theta); \mathbf{p}, t_c, \mathcal{T}(\cdot)) - v(l(\theta))\right) w(\theta) h(\theta) d\theta$$
$$+ \lambda \int_{\underline{\theta}}^{\overline{\theta}} \left(\mathcal{T}(y(\theta)) + \frac{t_c}{1+t_c}(y(\theta) - \mathcal{T}(y(\theta))\right) h(\theta) d\theta - \lambda \mathcal{R},$$

where $v(\cdot) = \frac{l^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$ is the disutility of labor effort. Following Saez (2001), consider a small increase in the marginal income tax rate $d\mathcal{T}'$ within a small interval $[y(\theta^*); y(\theta^*) + dy]$. The mass of people affected by this increase in the marginal income tax rate is approximately given by $\tilde{h}(y(\theta^*)) \times dy$, where $\tilde{h}(\cdot)$ is the pdf of the endogenous income distribution defined by $\tilde{H}(\theta^*) = H(y(\theta^*))$. Hence, $h(\theta^*) = \tilde{h}(y(\theta^*)) y_\theta(\theta^*)$ and

$$\tilde{h}(y(\theta^*)) \times dy = \frac{h(\theta^*) dy}{y_\theta(\theta^*; \mathbf{p}, T(\cdot), t_c)} = \frac{h(\theta^*)\theta^* dy}{\varepsilon_{y,\theta}(\theta^*) y(\theta^*)}.$$

We now collect the three effects that such a reform has on welfare: (i) a substitution effect for those individuals in the interval, (ii) a mechanical tax increase for those to the right of the interval and (iii) an income effect for those same individuals. The sum of the effects must equal zero. We now briefly discuss these three effects which is standard exercise in the literature.

**Substitution Effect.** Individuals with an income of $y \in [y(\theta^*); y(\theta^*) + dy]$ change their earnings in response to a small change in the marginal tax rate. The change in earnings is given by

$$\frac{\partial y(\theta^*)}{\partial \mathcal{T}'} d\mathcal{T}' = -\varepsilon_{y,1-\mathcal{T}'}(\theta^*) \frac{y(\theta^*)}{1 - \mathcal{T}'(y(\theta))} d\mathcal{T}',$$

where $\varepsilon_{y,1-\mathcal{T}(\theta^*)}$ denotes the elasticity of earnings with respect to the retention rate. Multiplying this earnings response by the effective marginal tax rate and the mass of individuals in the intervals yields the substitution effect on welfare

$$dS(\theta^*) = -\Lambda \frac{\tau_{\text{eff}}(\theta^*)}{1 - \mathcal{T}'(y(\theta^*))} \frac{\varepsilon_{y,1-\mathcal{T}'}(\theta^*)}{\varepsilon_{y,\theta}(\theta^*)} \theta^* d\mathcal{T}' h(\theta^*) dy, \tag{39}$$

which can be simplified to

$$dS(\theta^*) = -\Lambda \frac{\tau_{\text{eff}}(\theta^*)}{1 - \mathcal{T}'(y(\theta^*))} \frac{\varphi}{1+\varphi} \theta^* d\mathcal{T}' h(\theta^*) dy. \tag{40}$$

**Mechanical Effect.** Households with $\theta > \theta^*$ pay $d\mathcal{T}'dy$ more taxes leading to a mechanical effect:

$$dM(\theta^*) = d\mathcal{T}'dy \int_{\theta^*}^{\bar{\theta}} \left( \frac{\Lambda}{1+t_c} - g(\theta) \right) dH(\theta). \tag{41}$$

Note that the revenue term is scaled by $\frac{1}{1+t_c}$ because for each dollar of additional income taxes, the government looses $\frac{t_c}{1+t_c}$ in the form of VAT tax revenue.

**Income Effect.** Households with $\theta > \theta^*$ face a reduction in their income by $d\mathcal{T}'dy$ and thus, adjust their income. The revenue effect of this income effect is given by

$$dI(\theta^*) = d\mathcal{T}'dy\Lambda \int_{\theta^*}^{\bar{\theta}} \tau_{\text{eff}}(\theta)\eta(\theta)dH(\theta). \tag{42}$$

Finally, note that the marginal value of public funds is given by[35]

$$\Lambda = \frac{\int_{\underline{\theta}}^{\bar{\theta}} g(x)h(x)dx}{\frac{1}{1+t_c} + \int_{\underline{\theta}}^{\bar{\theta}} \tau_{\text{eff}}(x)\eta(x)h(x)dx}.$$

**Optimal Tax Schedule.** If the tax schedule is optimal, all three welfare effects must sum to zero: $dS(\theta^*) + dM(\theta^*) + dI(\theta^*) = 0$. Rearranging and inserting the expression for the marginal value of public funds yields .

$\forall\, \theta^* :\ D(\theta^*, \mathcal{P}_{sq}) = E(\theta^*, \mathcal{P}_{sq})$, where

$$D(\theta^*, \mathcal{P}_{sq}) = 1 - \frac{E[g(\theta, \mathcal{P}_{sq}) | \theta \geq \theta^*]}{E[g(\theta, \mathcal{P}_{sq})]}$$

captures the distributional gains of raising the marginal tax rate $\mathcal{T}'(y(\theta))$ and

$$E(\theta^*, \mathcal{P}_{sq}) = 1 - \frac{\frac{1}{1+t_c} - \frac{\tau_{eff}(\theta^*, \mathcal{P}_{sq})}{1-\mathcal{T}'_{sq}(y(\theta^*))} \frac{\varphi}{1+\varphi} \frac{h(\theta^*)\theta^*}{1-H(\theta^*)} + \int_{\theta^*}^{\bar{\theta}} \tau_{eff}(\theta, \mathcal{P}_{sq})\eta(\theta, \mathcal{P}_{sq}) \frac{dH(\theta)}{1-H(\theta^*)}}{\frac{1}{1+t_c} + \int_{\underline{\theta}}^{\bar{\theta}} \tau_{eff}(\theta, \mathcal{P}_{sq})\eta(\theta, \mathcal{P}_{sq})dH(\theta)}$$

where the optimal marginal tax rate on income is implicitly defined through the effective marginal tax rate and the uniform VAT:

$$\tau_{eff}(\theta, \mathcal{P}_{sq}) = \frac{\mathcal{T}'_{sq}(y(\theta)) + t_c}{1+t_c}. \tag{43}$$

In the case of a zero VAT $t_c = 0$ and Assumption 1 (implying zero income effects), we obtain Lemma 2.

[35] The expression follows from a marginal variation of the lump-sum component of the tax schedule.

## A.5 Optimal Policies under a Carbon Cap

In the case of the carbon cap constraint, the Lagrangian of the government problem reads as:

$$\begin{aligned}\mathcal{L} = &\int_{\underline{\theta}}^{\bar{\theta}} \left(u(e(\theta); \mathbf{p}, t_c, \tau_{\text{co2}}, \mathcal{T}(\cdot)) - v(l(\theta))\right) w(\theta) h(\theta) d\theta \\ &\lambda \int_{\underline{\theta}}^{\bar{\theta}} \left(\mathcal{T}(y(\theta)) + \frac{t_c}{1+t_c}(y(\theta) - \mathcal{T}(y(\theta)) + \frac{\tau_{\text{co2}}}{1+t_c} f(\theta)\right) h(\theta) d\theta - \lambda \mathcal{R} \\ &- \mu \left(\int_{\underline{\theta}}^{\bar{\theta}} f(\theta) dH(\theta) - \bar{F}\right)\end{aligned}$$

We first derive the expression for the optimal carbon tax in Appendix A.5.1, and then turn to the characterization of the optimal rebate schedule in Appendix A.5.2.

### A.5.1 Proof of Lemma 3

Consider a marginal policy variation consisting of a small increase in the carbon tax by $d\tau_{\text{co2}}$, accompanied by an offsetting reform of the income tax schedule $\mathcal{T}(y(\theta))$ as defined in (8), but with the opposite sign, i.e.

$$\forall \theta \; d\mathcal{T}(y(\theta)) = -f(\theta) d\tau_{\text{co2}}.$$

By Lemma 1, this joint reform leaves labor supply decisions and utility levels of all households unaffected. The only effects on welfare therefore arise through changes in carbon footprints, which operate via the government budget constraint and the carbon cap constraint. These effects sum to

$$\int_{\underline{\theta}}^{\bar{\theta}} f_{\tau_{\text{co2}}}(\theta) \left(\Lambda \frac{\tau_{\text{co2}}}{1+t_c} - \mu\right) dF(\theta).$$

In the optimum, this effect must equal zero, which yields the result in Lemma 3:

$$\tau_{\text{co2}} = (1+t_c)\frac{\mu}{\Lambda}.$$

### A.5.2 Proof of Proposition 2

The proof closely follows the proof of Lemma 2 in Appendix A.4 and builds on Proposition 1. There are, however, two differences:

1. Changes in labor supply matter not only through their fiscal externalities but also through their impact on the carbon cap constraint. By Proposition 1, this implies that income changes must be weighted by $\mathrm{T}(\theta)$ rather than by $\tau_{\text{eff}}(\theta)$.

2. Even when labor supply is constant, changes in the level of tax payments for households richer than $y(\theta^*)$ can have income effects on their consumption composition and thereby

their carbon footprint. These consumption composition changes, however, do not affect the government's objective: (i) for households themselves, the envelope theorem applies, and (ii) the resulting changes in carbon footprints have no welfare effect at the margin, since the induced change in carbon tax revenue is valued exactly as the corresponding change in the slackness of the carbon cap constraint. This follows from Lemma 3, in the same way that Proposition 1 follows from Lemma 3.

**Substitution Effect.** The substitution effect corresponding to (40) now takes the form

$$dS(\theta^*) = -\Lambda \frac{\mathrm{T}(\theta^*)}{1-\mathcal{T}'(y(\theta^*))} \frac{\varepsilon_{y,1-\mathcal{T}'}(\theta^*)}{\varepsilon_{y,\theta}(\theta^*)} \theta^* d\mathcal{T}' h(\theta^*) dy., \tag{44}$$

which follows directly from Proposition 1.

**Income Effect.** Equivalently, the corresponding income effect, analogous to (42), is now given by

$$dI(\theta^*) = d\mathcal{T}' dy \Lambda \int_{\theta^*}^{\overline{\theta}} \mathrm{T}(\theta^*)\eta(\theta) dH(\theta). \tag{45}$$

**Mechanical Effect.** The mechanical effect $dM(\theta^*)$ remains unchanged compared to (41).

The marginal value of public funds is now given by

$$\Lambda = \frac{\int_{\underline{\theta}}^{\overline{\theta}} g(x)h(x)dx}{\frac{1}{1+t_c} + \int_{\underline{\theta}}^{\overline{\theta}} \mathrm{T}(x)\eta(x)h(x)dx}. \tag{46}$$

**Optimal Tax Schedule.** Setting $dS(\theta^*) + dM(\theta^*) + dI(\theta^*) = 0$ and substituting (46) yields

$$\forall\ \theta^* :\ D(\theta^*, \mathcal{P}_{co_2}) = E(\theta^*, \mathcal{P}_{co_2}),$$

where

$$D(\theta^*, \mathcal{P}_{co_2}) = 1 - \frac{E[g(\theta, \mathcal{P}_{co_2}) | \theta \geq \theta^*]}{E[g(\theta, \mathcal{P}_{co_2})]}$$

$$E(\theta^*, \mathcal{P}_{co_2}) = 1 - \frac{\frac{1}{1+t_c} - \frac{\mathrm{T}(\theta^*, \mathcal{P}_{co_2})}{1-\mathcal{T}'_{co_2}(y(\theta^*))} \frac{\varphi}{1+\varphi} \frac{h(\theta^*)\theta^*}{1-H(\theta^*)} + \int_{\theta^*}^{\overline{\theta}} \mathrm{T}(\theta, \mathcal{P}_{co_2})\eta(\theta, \mathcal{P}_{co_2}) \frac{dH(\theta)}{1-H(\theta^*)}}{\frac{1}{1+t_c} + \int_{\underline{\theta}}^{\overline{\theta}} \mathrm{T}(\theta, \mathcal{P}_{co_2})\eta(\theta, \mathcal{P}_{co_2}) dH(\theta) d\theta}.$$

The optimal marginal income tax rates $\mathcal{T}'_{co_2}(y(\theta)$ follow implicitly from the labor wedge:

$$\mathrm{T}(\theta, \mathcal{P}_{co_2}) = \frac{\mathcal{T}'_{co_2}(y(\theta)) + t_c}{1 + t_c}.$$

In the case of a zero VAT $t_c = 0$ and Assumption 1 (implying zero income effects), we obtain Proposition 2.

## A.6 Comparative statics

### A.6.1 Proof of Proposition 3

We first examine how a distributionally neutral implementation of the carbon cap affects the distributional gains term. Since such a policy utility levels unchanged, only changes in the marginal utility of expenditure can affect the distributional gains term. From the proof of Lemma 1 in Appendix A.1, the change in marginal utility induced by a carbon tax increase is given by:[36]

$$\frac{\partial u_e(\theta)}{\partial \tau_{\text{co2}}} d\tau_{\text{co2}} = - u_e(\theta) f_e(\theta) \tau_{\text{co2}} - u_{ee} f(\theta) \tau_{\text{co2}}.$$

Next, consider the change in the marginal utility due the distributionally neutral rebate policy satisfying $R(y(\theta)) = \tau_{\text{co2}} f(\theta)$:[37]

$$\frac{\partial u_e(\theta)}{\partial R(y(\theta))} dR(y(\theta)) = u_{ee} f(\theta) \tau_{\text{co2}}.$$

Hence, the overall change in marginal utility is given by

$$du_e(\theta) = \frac{\partial u_e(\theta)}{\partial \tau_{\text{co2}}} \tau_{\text{co2}} + \frac{\partial u_e(\theta)}{\partial R(y(\theta))} dR(y(\theta)) = -u_e(\theta) f_e(\theta) \tau_{\text{co2}}.$$

Under Assumption **??**, $f_e(\theta) = \alpha \, \forall \theta$, which gives (28).[38]

The remainder of the proof, concerning the efficiency costs term, is provided in the main text.

### A.6.2 Proof of Proposition 1

We first consider how the distributional gains term is affected by implementing the carbon cap with a carbon dividend. Under Assumption 1, which implies both linear utility in expenditure $u(e)$ and linear carbon Engel curves, the change in marginal utility simplifies to

$$du_e(\theta) = -u_e(\theta) \alpha \tau_{\text{co2}}. \tag{47}$$

[36] Since we consider here an infinitesimal increase of the carbon tax from zero, we have $\tau_{\text{co2}} = d\tau_{\text{co2}}$.

[37] Since we consider here an infinitesimal increase of the carbon rebate from zero, we have $R(y(\theta)) = dR(y(\theta))$.

[38] Note that due to the linearity of $u(e; \mathcal{P})$ in $e$, we could have gotten to this results with fewer steps since $u_{ee} = 0$. Yet, we kept it more general because this result about the distributional gains term holds also for utility functions with curvature in $e$.

This resembles the result from Appendix A.6.1 for the distributionally neutral carbon rebate policy. Yet, under a carbon dividend there is a crucial difference: while the ratio of marginal utilities remains unchanged, the ratio of welfare weights does not, since utility levels shift. Households that receive more in carbon dividends than they pay in carbon taxes experience an increase in utility, whereas households that pay more than they receive experience a decrease in utility. As a result, inequality in utilities falls, thereby reducing the distributional gains term. To show this formally, we substitute the social marginal utility $g(\theta, \mathcal{P})$ into the expression for the distributional gains term:

$$D(\theta^*, \mathcal{P}_{\text{co2}}) = 1 - \frac{E[\mathcal{G}'\left(V(\theta, \mathcal{P})\right)\omega(\theta)u_e(\theta, \mathcal{P})|\theta \geq \theta^*]}{E[\mathcal{G}'\left(V(\theta, \mathcal{P})\right)\omega(\theta)u_e(\theta, \mathcal{P})]}. \tag{48}$$

Formally, the carbon dividend affects the utility level of type $\theta$ by

$$du(\theta) = u_e(\theta)\left(\bar{R}_{\text{co2}} - \tau_{\text{co2}}\alpha e(\theta)\right),$$

implying that the social marginal utility of type $\theta$ changes by

$$d\mathcal{G}'\left(V(\theta, \mathcal{P})\right) = \mathcal{G}''\left(V(\theta, \mathcal{P})\right)u_e(\theta)\left(\bar{R}_{\text{co2}} - \tau_{\text{co2}}\alpha e(\theta)\right).$$

Hence, the relative change of the numerator of (48) (also accounting for (47)), is given by:

$$\frac{E[\mathcal{G}''\left(V(\theta, \mathcal{P})\right)\omega(\theta)u_e(\theta)\left(\bar{R}_{\text{co2}} - \tau_{\text{co2}}\alpha e(\theta)\right) - \mathcal{G}'\left(V(\theta, \mathcal{P})\right)\omega(\theta)u_e(\theta)\alpha\tau_{\text{co2}}|\theta \geq \theta^*]}{E[\mathcal{G}'\left(V(\theta, \mathcal{P})\right)\omega(\theta)u_e(\theta, \mathcal{P})|\theta \geq \theta^*]}.$$

Analogously, for the denominator, it is

$$\frac{E[\mathcal{G}''\left(V(\theta, \mathcal{P})\right)\omega(\theta)u_e(\theta)\left(\bar{R}_{\text{co2}} - \tau_{\text{co2}}\alpha e(\theta)\right) - \mathcal{G}'\left(V(\theta, \mathcal{P})\right)\omega(\theta)u_e(\theta)\alpha\tau_{\text{co2}}]}{E[\mathcal{G}'\left(V(\theta, \mathcal{P})\right)\omega(\theta)u_e(\theta, \mathcal{P})]}.$$

We now define the expectation operator

$$E_{\mathcal{G}'\times\omega\times u_e\times h}\left[\frac{\mathcal{G}''}{\mathcal{G}'}\left(\bar{R}_{\text{co2}} - \tau_{\text{co2}}\alpha e(\theta)\right) - \alpha\tau_{\text{co2}}|\theta \geq x\right],$$

where the subscript captures the weighting in the expectation. The second part in the expectation operator is independent of $\theta^*$. We therefore define

$$\xi(x) = E_{\mathcal{G}'\times\omega\times u_e}\left[\Psi(\theta)|\theta \geq x\right],$$

where

$$\Psi(\theta) = \frac{\mathcal{G}''}{\mathcal{G}'}\left(\bar{R}_{\text{co2}} - \tau_{\text{co2}}\alpha e(\theta)\right).$$

A necessary condition for the distributional gains term to decrease due to the carbon dividend is

$$\xi(\theta^*) > \xi(\underline{\theta}), \tag{49}$$

since $\xi(\theta^*)$ captures the relative change of the numerator and $\xi(\underline{\theta})$ the relative change of the denominator. There exists a threshold $\tilde{\theta}$ such that $\Psi(\theta) > (<)0$ if $\theta < (>)\tilde{\theta}$. Intuitively, $\tilde{\theta}$ is the productivity type for whom the carbon dividend equals exactly the carbon tax burden. For $\theta^* > \tilde{\theta}$, condition (49) always holds. For $\theta^* < \tilde{\theta}$, a sufficient condition for (49) to hold is $\Psi'(\theta) > 0$. To check for this condition, rewrite $\Psi(\theta)$ as

$$\Psi(\theta) = \underbrace{-\frac{\mathcal{G}''(V(\theta))V(\theta)}{\mathcal{G}'(V(\theta))}}_{\equiv CRUIA(\theta)} \times \left(-\frac{1}{V(\theta)}\left(\bar{R}_{\text{co2}} - \tau_{\text{co2}}\alpha e(\theta)\right)\right).$$

Differentiating yields

$$\begin{aligned}\Psi'(\theta) = &- CRUIA'(\theta)\frac{1}{V(\theta)}\left(\bar{R}_{\text{co2}} - \tau_{\text{co2}}\alpha e(\theta)\right) \\ &+ CRUIA(\theta)\frac{1}{V(\theta)^2}V'(\theta)\left(\bar{R}_{\text{co2}} - \tau_{\text{co2}}\alpha e(\theta)\right) \\ &+ CRUIA(\theta)\frac{1}{V(\theta)}\tau_{\text{co2}}\alpha e'(\theta).\end{aligned}$$

By assumption, the aversion to relative utility inequality is not increasing, $CRUIA'(\theta) \leq 0$. Hence, the first line is non-negative. The second line is positive for all $\theta < \theta^*$, and the third line is also positive. Together, this establishes that $\Psi'(\theta) > 0$ thus that (49) holds.

We have therefore shown that the distributional gains term is lower under the carbon dividend than under the status quo. As explained in the main text, since the efficiency cost term remains unchanged compared to the status quo policies, the government optimally reduces redistribution compared to a carbon dividend.

# B Computation of Emission Intensities

We compute emission intensities by combining two data sources: (i) environmentally extended multi-regional input-output (EE-MRIO) data from EXIOBASE v3.9.6 (Stadler et al. 2018) and (ii) German supply-use and input-output tables (SUT/IOT) from the national accounts provided by the German Federal Statistical Office (German Statistical Office 2022). EXIOBASE provides the global input-output framework needed to compute emission intensities at basic prices, but it does not provide supply-use tables required to convert them to purchaser prices. We therefore use the German SUT/IOT to recover product-level trade margins and net product taxes, which we then map into the EXIOBASE framework.

Section B.1 describes the German SUT/IOT data and Section B.2 describes the EXIOBASE data and explains how we map the German trade margins and net product taxes into the EXIOBASE framework. Section B.3 then describes the computation of emission intensities and Section B.4 validates these intensities by comparing the aggregate emissions implied by our approach with official estimates reported by German Statistical Office (2025b).

## B.1 German Supply-Use and Input-Output Tables

### B.1.1 Data

The *supply table* of the German SUT reports total supply at both basic and purchaser prices, together with trade margins and taxes on products less subsidies.[39] For household final consumption, the *use table* is reported only at purchaser prices and therefore does not separately identify basic prices, trade margins, and net product taxes. We recover this decomposition by combining the SUT with the IOT, which report household final consumption expenditure at basic prices. The SUT and IOT are published in the CPA classification, but at different levels of disaaggregation: the SUT distinguish 85 CPA products, whereas the IOT distinguish 72 CPA products. We work at the 85-product SUT level.[40]

In the following, we recover household final consumption at basic prices, trade margins, and net product taxes in the 85-product CPA classification. In Section B.2, we then map these components to the ISIC classification used in EXIOBASE, thereby obtaining an EXIOBASE-consistent use table.

### B.1.2 Construction of German Use Table

We recover household final consumption at basic prices, trade margins, and net product taxes in four steps. We first harmonize the 72-product IOT and 85-product SUT classifications. We then recover the total wedge between purchaser and basic prices for household consumption, decompose this wedge into trade margins and net product taxes, and finally balance trade margins so that they satisfy the accounting identity that they sum to zero.

**Harmonizing IOT and SUT classifications.** Household final consumption expenditure at basic prices is available only in the IOT, which distinguish 72 products, whereas the SUT distinguish 85 products. To reconcile the two classifications, we define a binary aggregation

[39] By construction, supply at purchaser prices equals supply at basic prices plus trade margins plus net taxes on products.

[40] An alternative would be to work at the 72-product IOT classification and aggregate the SUT from 85 to 72 CPA products. We do not perform the conversion from purchaser to basic prices at that level because aggregation to 72 products removes within-group heterogeneity in the wedge between purchaser and basic prices. In particular, products that belong to the same 72-product category would inherit the same average wedge, even when trade margins and net product taxes differ substantially across the underlying 85-product categories. Working at the 85-product level preserves this variation. Our results are robust using this alternative approach.

matrix $\mathbf{S} \in \{0,1\}^{72\times 85}$, where $S_{ik} = 1$ if product $k$ in the 85 product classification belongs product group $i$ in the 72-product classification, and $S_{ik} = 0$ otherwise. Each 85-product category maps to exactly one 72-product category. For each 72-product category $c$, let $\mathcal{I}_c := \{k \in \{1,\dots,85\} : S_{ck} = 1\}$ denote the set of corresponding 85-product categories. The aggregation matrix $\mathbf{S}$ aggregates any 85-product vector to the 72-product classification by summing within categories.[41]

**Recovering the wedge between purchaser and basic prices.** Let $\mathbf{y}_{pp}^{SUT,85} \in \mathbb{R}^{85\times 1}$ denote household final consumption expenditure at purchaser prices from the SUT *use table*, and let $\mathbf{y}_{bp}^{IOT,72} \in \mathbb{R}^{72\times 1}$ denote household final consumption expenditure at basic prices from the IOT. Aggregating the SUT use table to the 72-product classification gives

$$\mathbf{y}_{pp}^{SUT,72} = \mathbf{S}\mathbf{y}_{pp}^{SUT,85}.$$

The implied wedge between purchaser and basic prices in household consumption at the 72-product level is then

$$\boldsymbol{\delta}^{SUT,72} = \mathbf{y}_{pp}^{SUT,72} - \mathbf{y}_{bp}^{IOT,72}.$$

This wedge reflects both trade margins and net product taxes.

We disaggregate $\boldsymbol{\delta}^{SUT,72}$ to the 85-product classification using information from the SUT *supply table*. Let $\mathbf{s}_{pp}^{SUT,85}$ and $\mathbf{s}_{bp}^{SUT,85}$ denote total supply at purchaser and basic prices, and define the corresponding supply-side wedge as $\boldsymbol{\delta}_{supply}^{SUT,85} = \mathbf{s}_{pp}^{SUT,85} - \mathbf{s}_{bp}^{SUT,85}$. We assume that, within each 72-product category, the household wedge is distributed across 85-product categories in proportion to the supply-side wedge. Accordingly, for each $k \in \mathcal{I}_c$

$$w_{\delta,k}^{SUT,85} = \frac{\delta_{supply,k}^{SUT,85}}{\sum_{j\in\mathcal{I}_c} \delta_{supply,j}^{SUT,85}}.$$

By construction, these weights sum to one within each 72-product category.[42] The disaggregated household wedge is given by

$$\boldsymbol{\delta}^{SUT,85} = (\mathbf{S}^{\intercal}\boldsymbol{\delta}^{SUT,72}) \odot \mathbf{w}_{\boldsymbol{\delta}}^{SUT,85},$$

where $\odot$ denotes the Hadamard product.

To ensure nonnegative household consumption at basic prices, we impose the admissibility constraint $\delta_k^{SUT,85} \le y_{pp,k}^{SUT,85}$. Whenever the initial allocation violates this bound for some

[41] Formally, aggregation is given by $\mathbf{x}^{72} = \mathbf{S}\mathbf{x}^{85}$. Conversely, a 72-product vector can be disaggregated to 85 products using weights $\mathbf{w}^{85} \in \mathbb{R}_+^{85\times 1}$ satisfying $\mathbf{S}\mathbf{w}^{85} = \mathbf{1}_{72}$, so that $\mathbf{x}^{85} = (\mathbf{S}^{\intercal}\mathbf{x}^{72}) \odot \mathbf{w}^{85}$, where $\odot$ denotes the Hadamard product.

[42] Whenever $\sum_{j\in\mathcal{I}_c} \delta_{supply,j}^{SUT,85} = 0$, we set $w_{\delta,k}^{SUT,85} = 0$ for all $k \in \mathcal{I}_c$.

product $k$, we set $\delta_k^{SUT,85} = y_{pp,k}^{SUT,85}$, and reallocate the residual wedge within the same 72-product group across the remaining products in proportion to their renormalized weights.[43]

We then recover household final consumption expenditure at basic prices in the 85-product SUT classification as

$$\mathbf{y}_{bp}^{SUT,85} = \mathbf{y}_{pp}^{SUT,85} - \boldsymbol{\delta}^{SUT,85}.$$

This procedure recovers household consumption at basic prices in the 85-product classification while remaining consistent with the published SUT and IOT totals. Finally, we define the corresponding basic-to-purchaser price conversion factor as

$$\tilde{\boldsymbol{\delta}}^{SUT,85} = \mathbf{y}_{pp}^{SUT,85} \oslash \mathbf{y}_{bp}^{SUT,85},$$

where $\oslash$ denotes element-wise division. Each element $\tilde{\delta}_k^{SUT,85}$ converts household consumption expenditure from basic to purchaser prices for product $k$.

**Trade margins and net product taxes.** We next decompose the household wedge between purchaser and basic prices $\boldsymbol{\delta}^{SUT,85}$ into trade margins and net product taxes using the decomposition of the corresponding wedge from the SUT *supply table*. Let $\mathbf{m}_{supply}^{SUT,85}$ denote trade margins and $\mathbf{t}_{supply}^{SUT,85}$ denote net product taxes for total supply at the 85-product level, so that $\boldsymbol{\delta}_{supply}^{SUT,85} = \mathbf{m}_{supply}^{SUT,85} + \mathbf{t}_{supply}^{SUT,85}$. For each product $k = 1, \ldots, 85$ with $\delta_{supply,k}^{SUT,85} \neq 0$, define the supply-side decomposition coefficients

$$\tilde{m}_{supply,k}^{SUT,85} = \frac{m_{supply,k}^{SUT,85}}{\delta_{supply,k}^{SUT,85}}, \qquad \tilde{t}_{supply,k}^{SUT,85} = \frac{t_{supply,k}^{SUT,85}}{\delta_{supply,k}^{SUT,85}}.$$

By construction, $\tilde{m}_{supply,k}^{SUT,85} + \tilde{t}_{supply,k}^{SUT,85} = 1$. If $\delta_{supply,k}^{SUT,85} = 0$, we set $\tilde{m}_{supply,k}^{SUT,85} = \tilde{t}_{supply,k}^{SUT,85} = 0$.[44]

We then assume that, the wedge from the *use table* is split between trade margins and net product taxes in the same way as the supply-side wedge. Household trade margins and household net product taxes are therefore given by

$$\mathbf{m}^{SUT,85} = \tilde{\boldsymbol{m}}_{supply}^{SUT,85} \odot \boldsymbol{\delta}^{85}, \qquad \mathbf{t}^{SUT,85} = \tilde{\boldsymbol{t}}_{supply}^{SUT,85} \odot \boldsymbol{\delta}^{85}.$$

By construction, $\mathbf{m}_h^{SUT,85} + \mathbf{t}_h^{SUT,85} = \boldsymbol{\delta}^{85}$, so that $\mathbf{y}_{bp}^{SUT,85} = \mathbf{y}_{pp}^{SUT,85} - \mathbf{m}^{SUT,85} - \mathbf{t}^{SUT,85}$.

**Balancing trade margins.** The decomposition of the wedge between purchaser and basic prices need not satisfy the accounting identity that trade margins sum to zero in aggregate.

[43] In the data, this adjustment affects only two products, and the reallocated amount is quantitatively small.

[44] Whenever $y_{pp,k}^{SUT,85} = 0$ *and* $\tilde{t}_{pp,k}^{SUT,85} > 0$, we set the tax component to zero and assign the entire wedge between basic and purchaser prices to trade margins, i.e. $\tilde{m}_{supply,k}^{SUT,85} = 1$ and $\tilde{t}_{supply,k}^{SUT,85} = 0$. This prevents the allocation of net product taxes to products with zero household final consumption expenditure at purchaser prices while preserving the accounting identity $\tilde{m}_k^{SUT,85} + \tilde{t}_k^{SUT,85} = 1$. *In the data, this adjustment affects only two products and is quantitatively negligible.*

We therefore rebalance trade margins and net product taxes to impose that restriction while preserving the wedge between purchaser and basic prices $\boldsymbol{\delta}^{SUT,85}$.

Let $P := \{k : m_k^{SUT,85} > 0\}$ and $N := \{k : m_k^{SUT,85} < 0\}$ denote the sets of products with positive and negative preliminary trade margins. Define the scaling factor

$$\lambda^{SUT,85} \equiv \frac{\sum_{k \in N} |m_k^{SUT,85}|}{\sum_{k \in P} m_k^{SUT,85}}.$$

We then rescale positive trade margins proportionally and leave negative trade margins unchanged:

$$\hat{m}_k^{SUT,85} = \begin{cases} \lambda^{SUT,85} m_k^{SUT,85}, & k \in P, \\ m_k^{SUT,85}, & \text{otherwise.} \end{cases}$$

The amount removed from positive trade margins is added to net product taxes:

$$\hat{t}_k^{SUT,85} = t_k^{SUT,85} + \left(m_k^{SUT,85} - \hat{m}_k^{SUT,85}\right).$$

By construction, $\hat{m}_k^{SUT,85} + \hat{t}_k^{SUT,85} = \delta_k^{SUT,85}$ for every product $k$ and $\sum_{k=1}^{85} \hat{m}_k^{SUT,85} = 0$.

## B.2 EXIOBASE

### B.2.1 Data

EXIOBASE provides monetary input-output tables together with physical extensions, including detailed emission accounts. The air emissions extension covers 32 distinct pollutants, disaggregated by products and final demand. EXIOBASE covers 44 individual countries plus five 'rest of the world' regions giving a total of $K = 49$ regions. We use the product-by-product dimension of EXIOBASE, which distinguishes $P = 200$ products, giving $K \times P = 49 \times 200 = 9,800$ region-product combinations. The tables are provided in basic prices, with products classified according to the EXIOBASE product classification (Stadler et al. 2018).

### B.2.2 Construction of EXIOBASE Use Table

We use the German household final-consumption decomposition constructed constructed in Section B.1.2 to convert household final consumption expenditure in EXIOBASE from basic to purchaser prices. Specifically, we map the basic-to-purchaser price conversion factors, as well as the decomposition of the purchaser-basic price wedge into trade margins and net product taxes, from the CPA classification to the EXIOBASE product classification. Applying these mapped objects to EXIOBASE household final consumption at basic prices yields a purchaser-price household final-consumption vector together with its decomposition into basic prices, trade margins, and net product taxes.

**Correspondence from CPA to EXIOBASE.** To map the basic-to-purchaser price conversion factor and the decomposition coefficients from the CPA classification to the EXIOBASE product classification, we use a correspondence matrix provided by Stadler et al. (2018) .[45] Let $\mathbf{C} \in \{0,1\}^{200\times 85}$ denote the binary correspondence matrix, where $C_{pk} = 1$ if CPA product $k$ is mapped to EXIOBASE product $p$, and $C_{pk} = 0$ otherwise. Because several CPA products may map to the same EXIOBASE product, we aggregate the CPA-level basic-to-purchaser price conversion factors and decomposition coefficients using household final consumption expenditure at basic prices, $\mathbf{y}_{bp}^{SUT,85}$, as weights. We therefore compute each EXIOBASE-level value as a weighted average over the CPA products associated with that EXIOBASE product. For any CPA-level vector $\mathbf{x}^{SUT,85} \in \mathbb{R}^{85\times 1}$, the corresponding vector in the EXIOBASE classification $\mathbf{x}^{EXIO,200} \in \mathbb{R}^{200\times 1}$ is given by:

$$x_p^{EXIO,200} = \frac{\sum_{k=1}^{85} C_{pk}\, y_{bp,k}^{SUT,85}\, x_k^{85}}{\sum_{k=1}^{85} C_{pk}\, y_{bp,k}^{SUT,85}} \qquad \forall\ p = 1,\ldots,200.$$

**Household final consumption expenditure at purchaser prices.** We first map the basic-to-purchaser price conversion factor $\tilde{\boldsymbol{\delta}}^{SUT,85}$ from the CPA classification to the EXIOBASE product classification, yielding $\tilde{\boldsymbol{\delta}}^{EXIO,200} \in \mathbb{R}^{200\times 1}$. Applying this factor to household final consumption expenditure at basic prices in EXIOBASE $\mathbf{y}_{bp}^{EXIO,200} \in \mathbb{R}^{200\times 1}$, gives household final consumption expenditures at purchaser prices:

$$\mathbf{y}_{pp}^{EXIO,200} = \mathbf{y}_{bp}^{EXIO,200} \odot \tilde{\boldsymbol{\delta}}^{EXIO,200}.$$

The corresponding wedge between basic and purchaser prices is then given by

$$\boldsymbol{\delta}^{EXIO,200} = \mathbf{y}_{pp}^{EXIO,200} - \mathbf{y}_{bp}^{EXIO,200}.$$

**Trade margins and net product taxes.** Finally, we decompose the wedge between purchaser and basic prices $\boldsymbol{\delta}^{EXIO,200}$ into trade margins and net product taxes. Starting from the CPA-level decomposition $\hat{\mathbf{m}}^{SUT,85}$ and $\hat{\mathbf{t}}^{SUT,85}$, we define

$$r_{m,k}^{SUT,85} = \frac{\hat{m}_k^{SUT,85}}{|\delta_k^{SUT,85}|}, \qquad r_{t,k}^{SUT,85} = \frac{\hat{t}_k^{SUT,85}}{|\delta_k^{SUT,85}|},$$

for $|\delta_k^{SUT,85}| > 0$, and set them to zero otherwise. We normalize by the absolute wedge because some products carry negative trade margins, reflecting their role as suppliers of trade services.

[45] Strictly speaking, Stadler et al. (2018) provides a NACE-to-EXIOBASE correspondence matrix. Since each CPA product can be assigned to a single NACE category, the same concordance can be applied to CPA products.

These normalized components therefore preserve the sign of each margin while scaling by the magnitude of the purchaser-basic price wedge. By construction,

$$r_{m,k}^{SUT,85} + r_{t,k}^{SUT,85} = \operatorname{sgn}(\delta_k^{SUT,85}).$$

We then map $\mathbf{r}_m^{SUT,85}$ and $\mathbf{r}_t^{SUT,85}$ to the EXIOBASE product classification using the correspondence described above, yielding $\mathbf{r}_m^{EXIO,200} \in \mathbb{R}^{200\times 1}$ and $\mathbf{r}_t^{EXIO,200} \in \mathbb{R}^{200\times 1}$. Let $\mathbf{s}^{EXIO,200} \equiv \mathbf{r}_m^{EXIO,200} + \mathbf{r}_t^{EXIO,200}$. We recover EXIOBASE trade margins and net product taxes as

$$m_p^{EXIO,200} = \frac{r_{m,p}^{EXIO,200}}{s_p^{EXIO,200}}\,\delta_p^{EXIO,200}, \qquad t_p^{EXIO,200} = \frac{r_{t,p}^{EXIO,200}}{s_p^{EXIO,200}}\,\delta_p^{EXIO,200}.$$

Whenever $s_p^{EXIO,200} = 0$, we set $m_p^{EXIO,200} = t_p^{EXIO,200} = 0$. This yields a decomposition of the purchaser-basic price wedge into trade margins and net product taxes.

**Balancing trade margins.** As in the German *use table*, the decomposition of the purchaser-basic price wedge in the EXIOBASE product classification need not satisfy the accounting identity that trade margins sum to zero in the aggregate. We therefore apply the balancing procedure described in Section B.1.2 to the EXIOBASE vectors $\mathbf{m}^{EXIO,200}$ and $\mathbf{t}^{EXIO,200}$. Positive trade margins are scaled down proportionally so that they exactly offset negative trade margins, and the removed amount is added product by product to net product taxes. We denote the balanced tax vector and trade margin by $\hat{\mathbf{m}}^{EXIO,200}$ and $\hat{\mathbf{t}}^{EXIO,200}$, respectively. By construction, $\hat{\mathbf{m}}^{EXIO,200} + \hat{\mathbf{t}}^{EXIO,200} = \boldsymbol{\delta}^{EXIO,200}$ and $\mathbf{1}_{200}^{\top}\hat{\mathbf{m}}^{EXIO,200} = 0$.

## B.3 Computation of Emission Intensities

In this section we describe the details of the computation of indirect and direct carbon emission intensities.

**Carbon Dioxide Equivalents.** In line with the Kyoto Protocol, we measure greenhouse gas emissions in carbon dioxide equivalents ($CO_2e$).[46] Emissions of carbon dioxide ($CO_2$), methane ($CH_4$), nitrous oxide ($N_2O$), sulfur hexafluoride ($SF_6$), Hydrofluorocarbons (HFC), and Perfluorocarbons (PFC) are converted into $CO_2e$ using 100-year global warming potentials (GWP100) from the IPCC Fourth Assessment Report (AR4) (United Nations Climate Change 2026).[47]

[46] Measuring emissions in ($CO_2e$) rather than in physical units of ($CO_2$) does not affect our quantitative results as $CO_2$ accounts for 88% of total greenhouse gas emissions in Germany (Umweltbundesamt 2025b, Statistisches Bundesamt 2025) and the share relative to other greenhouse gases is approximately constant along the income distribution.

[47] HFC and PFC emissions are aggregated in EXIOBASE v3.9.6 and reported directly in kg $CO_2e$.

### B.3.1 Indirect Emission Intensities

**Leontief Inverse Matrix.** Let $\mathbf{I} \in \mathbb{R}^{K \cdot P \times K \cdot P}$ denote the identity matrix, $\mathbf{A} \in \mathbb{R}^{K \cdot P \times K \cdot P}$ the matrix of technical input coefficients from EXIOBASE, $\mathbf{x} \in \mathbb{R}^{K \cdot P \times 1}$ total economic output, and $\mathbf{y} \in \mathbb{R}^{K \cdot P \times 1}$ final demand. The standard input-output system is

$$\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{y}.$$

Assuming $(\mathbf{I} - \mathbf{A})$ is invertible, this implies

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{y} \equiv \mathbf{L}\mathbf{y},$$

where $\mathbf{L} \equiv (\mathbf{I} - \mathbf{A})^{-1}$ is the Leontief inverse. The Leontief inverse describes the total input required from each sector-region combination to produce one unit of final demand in another sector-region combination.

**Emission Coefficients.** We derive product-region emission coefficients from EXIOBASE's satellite accounts. Let $\mathbf{F} \in \mathbb{R}^{20 \times K \cdot P}$ collect total emissions of the 20 greenhouse gases falling under the Kyoto protocol by sector-region combination and let $\mathbf{g} \in \mathbb{R}^{1 \times 20}$ denote the vector of GWP100 characterization factors, where entry $g_i$ converts emissions of greenhouse gas $i$ into $CO_2$e equivalents. The vector of emission coefficients is given by

$$\mathbf{E} = \mathbf{g} \cdot \left(\mathbf{F} \cdot \mathrm{diag}(\mathbf{x})^{-1}\right) \quad \in \mathbb{R}^{1 \times K \cdot P},$$

where $\mathrm{diag}(\mathbf{x})$ denotes the diagonal matrix with the entries of $\mathbf{x}$ on its main diagonal. The entry $E_{kp}$ gives the total $CO_2$e emissions per unit of output for sector-region pair $(k, p)$.[48]

We set $E_r = 0$ for all sector-region combinations $r$ with $x_r < 1\text{e}6$ to avoid division by zero or unrealistically high emission intensities arising from very low levels of economic output. We interpret these cases as likely driven by measurement error rather than genuine production relationships (Hardadi, Buchholz, and Pauliuk 2021).

[48] We set $E_r = 0$ for all sector-region combinations $r$ with $x_r < 1\,000\,000$ to avoid division by zero or unrealistically high emission intensities arising from very low levels of economic output. We interpret these cases as likely driven by measurement error rather than genuine production relationships (Hardadi, Buchholz, and Pauliuk 2021).

**Matrix of Regional Demand Shares.** Let $\mathbf{Y^{HH}} = [y_{kp}^{HH}] \in \mathbb{R}^{K\times P}$ denote the matrix of final household demand in Germany, where $y_{kp}^{HH}$ is household final demand for product $p$ in region $k$. For each product $p$, define the regional demand shares as

$$\omega_{kp} \equiv \begin{cases} \frac{y_{kp}^{HH}}{\sum_{k'=1}^{K} y_{k'p}^{HH}} & \text{if } \sum_{k'=1}^{K} y_{k'p}^{HH} > 0 \\ 0 & \text{else} \end{cases}.$$

Intuitively, $\omega_{kp}$ gives region $k$'s share of Germany's household final demand for product $p$. Let $\mathbf{\Omega} = [\omega_{kp}] \in \mathbb{R}^{K\times P}$ denote the resulting matrix of regional demand shares. We then transform $\mathbf{\Omega}$ into a stacked diagonal-block matrix

$$\tilde{\boldsymbol{Y}} \equiv \mathbf{\Omega} * \mathbf{I} = \begin{bmatrix} \operatorname{diag}(\mathbf{\Omega}_{1,:}) \\ \operatorname{diag}(\mathbf{\Omega}_{2,:}) \\ \vdots \\ \operatorname{diag}(\mathbf{\Omega}_{K,:}) \end{bmatrix} \in \mathbb{R}^{KP\times P},$$

where $\mathbf{I} \in \mathbb{R}^{P\times P}$ is the identity matrix and $*$ denotes the Khatri-Rao product.

### B.3.2 Conversion from Basic to Purchaser Prices

Household consumption expenditures in the EVS are reported at purchaser prices, whereas $\mathbf{f_{ind}^{ISIC}}$ is defined at basic prices. To align the two concepts, we convert ISIC-level indirect emission intensities from basic to purchaser prices using the valuation structure implied by the EXIOBASE supply-use tables.[49]

For each product $p$, one euro of expenditure at purchaser prices can be decomposed into net product taxes, trade margins, and the basic price value of the product:

$$1 = \tau_p + (1-\tau_p)\mu_p + (1-\tau_p)(1-\mu_p),$$

where $\tau_p$ denotes the product tax rate (net of subsidies) and $\mu_p$ the trade margin ratio.[50] Thus, $\tau_p$ is the share of one euro of expenditure at purchaser prices that goes to taxes, $(1-\tau_p)\mu_p$ is the share paying for wholesale and retail trade services, and $(1-\tau_p)(1-\mu_p)$ is the share accruing to the producer at basic prices. Since taxes are a pure price wedge, they do not generate emissions. The remaining two components instead inherit the indirect emissions intensities, of, repetitively trade services and the production of the product $p$.

[49] The most recent version of EXIOBASE does not contain supply-use tables. We therefore reconstruct the valuation information required for the purchaser-price conversion using the German input-output model of the Federal Statistical Office. See Appendix B.2.2 for details.

[50] We construct both the net product tax rate $\tau_p$ and the trade margin ratio $\mu_p$ from the EXIOBASE use tables. Specifically, we compute $\tau_p = \hat{t}_p^{ISIC,200}/y_{pp,p}^{ISIC,200}$ and $\mu_p = \hat{m}_p^{ISIC,200}/(y_{pp,p}^{ISIC,200} - \hat{t}_p^{ISIC,200})$, where $\hat{t}_p^{ISIC,200}$ dentoes net product taxes, $\hat{m}_p^{ISIC,200}$ trade margins, and $y_{pp,p}^{ISIC,200}$ household final demand at purchaser prices, respectively. We set $\tau_p = 0$ and $\mu_p = 0$ whenever the corresponding denominator is zero.

Let $\mathcal{S}$ denote the set of EXIOBASE products corresponding to wholesale and retail trade services.[51] For products outside this set, $p \notin \mathcal{S}$, the indirect emission intensity per euro of purchaser price expenditure is given by

$$\tilde{f}^{\text{ISIC}}_{\text{ind},p} = (1-\tau_p)\left[(1-\mu_p)f^{\text{ISIC}}_{\text{ind},p} + \mu_p \bar{f}_{\text{trade}}\right] \quad \text{if } p \notin \mathcal{S},$$

where

$$\bar{f}_{\text{trade}} = \sum_{s\in\mathcal{S}} \gamma_s f^{\text{ISIC}}_{\text{ind},s} \qquad \text{with} \quad \gamma_s = \frac{|\hat{m}_s|}{\sum_{s'\in\mathcal{S}}|\hat{m}_{s'}|},$$

denotes the trade margin weighted average indirect emission intensity of wholesale and retail trade services, and $m_s$ denotes the trade margin attributed to trade service product $s$[52]. The intuition is straightforward: out of one euro spent on product $p$ at purchaser prices, the share $\tau_p$ is absorbed by product taxes and is therefore emission neutral. Of the remaining post-tax expenditure, the fraction $(1-\mu_p)$ accrues to the producer of product $p$ at basic prices, implying that the share $(1-\tau_p)(1-\mu_p)$ of the original purchaser price euro inherits the emission intensity $f^{ISIC}_{\text{ind},p}$. The complementary share $(1-\tau_p)\mu_p$ pays for the wholesale and retail trade services embodied in the trade margin and therefore inherits the average trade service emission intensity $\bar{f}^{\text{ISIC}}_{\text{trade}}$. Hence, $\tilde{f}^{\text{ISIC}}_{\text{ind},p}$ is a weighted average of the indirect emission intensities embodied in the production of product $p$ and in the associated trade services, with weights given by their respective shares in one euro of purchaser-price expenditure.

For EXIOBASE products corresponding to wholesale and retail trade services $p \in \mathcal{S}$, only the part of household demand that reflects direct final consumption should be assigned to the product's own indirect emission intensity. Define the intermediation share of trade-service product $p$ as

$$\alpha_p = \frac{|\hat{m}_p|}{\sum_{k'=1}^{K} y^{HH}_{k'p}}.$$

Intuitively, $\alpha_p$ measures the share of household final demand for product $p$ that is already embodied in the trade margins allocated to other products. The purchaser price indirect emission intensity of product $p \in \mathcal{S}$ is then given by

$$\tilde{f}^{\text{ISIC}}_{\text{ind},p} = (1-\alpha_p)(1-\tau_p)(1-\mu_p)f^{\text{ISIC}}_{\text{ind},p} \quad \text{if } p \in \mathcal{S},$$

The factor $1-\alpha_p$ captures the part of products $p$'s output that reflects direct household final demand rather than trade intermediation services already embodied in the trade margins

[51] Wholesale and retail trade service products are defined as EXIOBASE products with negative balanced trade margins, i.e. $\hat{m}^{EXIO,200}_p < 0$. This yields five products: "Retail trade services of motor fuel", "Wholesale trade and commission trade services", "Sale, maintenance & repair of motor vehicles, parts & motorcycles", "Retail trade services; repair of personal & household goods" and "Distribution services of gaseous fuels through mains". The last three products play a dual role, as they provide trade services while also exhibiting positive household final demand.

[52] See Section B.2.2 for details on the construction of the trade margin in EXIOBASE.

assigned to other products. This distinction is relevant for trade service sectors that are not pure intermediaries. If a trade service sector used purely as an intermediary, then $\alpha_p = 1$, and its purchaser price indirect emission intensity is zero, since all of its supply-chain emissions are already attributed through the trade margins of the producing products.

### B.3.3 Mapping form EXIOBASE to COICOP Classification

Household consumption expenditures in the EVS are recorded in the COICOP classification, whereas EXIOBASE is organized by according to the ISIC classification. To combine the two data sources, we map indirect emission intensities from the EXIOBASE to the COICOP classification using a correspondence matrix. Let $\mathbf{M} \in \mathbb{R}^{P \times N}$ denote the correspondence matrix, where the $n$-th column allocates one euro of purchaser price expenditure in COICOP category $n$ across EXIOBASE products. The resulting vector of indirect emission intensities in the COICOP classification is then given by

$$\tilde{\mathbf{f}}_{\text{ind}}^{\text{COICOP}} = \tilde{\mathbf{f}}_{\text{ind}}^{\text{ISIC}} \cdot \mathbf{M} \quad \in \mathbb{R}^{1 \times N},$$

where $\tilde{f}_{\text{ind},n}^{\text{COICOP}}$ measures the indirect $CO_2e$ emissions embodied in one euro of purchaser price expenditure in COICOP category $n$.

We construct the correspondence matrix $\mathbf{M}$ in three steps. First, we define an initial binary correspondence between EXIOBASE and COICOP products. Second, we apply the RAS method to adjust this matrix so that its row and column totals match household final demand in each EXIOBASE and COICOP product, respectively. Third, we normalize the resulting matrix column-wise so that each column sums to one. The final step converts the adjusted expenditure flows into purchaser price expenditure weights, where each entry $M_{pn}$ gives the weight placed on EXIOBASE product $p$ when constructing the indirect emission intensity of COICOP category $n$. The resulting matrix therefore defines a many-to-many mapping, whereby each COICOP category can be associated with multiple EXIOBASE products.

**Initial Correspondence Matrix.** Let $\mathbf{A}_0 = [a_{ij}] \in \mathbb{R}^{P \times N}$ denote the initial binary correspondence matrix, where $a_{ij} = 1$ if COICOP product $j$ is assigned to EXIOBASE product $i$ (based on product similarities), and 0 otherwise. We construct $\mathbf{A}_0$ from the correspondence proposed by Stadler et al. (2018) and Hardadi, Buchholz, and Pauliuk (2021) with additional modifications to ensure convergence of the RAS algorithm.[53]

[53]Specifically, we augmented the support of $A_0$ as follows. We manually added the links required for block feasibility, meaning that row and column totals balance within each connected support block. We then added further links identified by a maximum-flow feasibility test, which checks whether EXIOBASE household final demand for each ISIC product can be allocated across the admissible COICOP categories while simultaneously matching EVS expenditure in every COICOP category. Each of these added links was reviewed case by case for economic plausibility.

**RAS Method.** Let $\mathbf{r} \in \mathbb{R}^{P\times 1}$ denote household final demand by EXIOBASE product, expressed at purchaser prices, and let $\mathbf{s} \in \mathbb{R}^{N\times 1}$ denote aggregate household expenditure by COICOP category in the EVS.[54] The RAS method adjusts $\mathbf{A}_0$ iteratively as follows:

1. **Row Adjustment**: The rows of the correspondence matrix $\mathbf{A}^{k-1}$ at iteration $k$ are scaled to match the final demand in each EXIOBASE product, given by the column vector $\mathbf{r} \in \mathbb{R}^{P\times 1}$. The updated correspondence matrix is given by

$$\mathbf{A}^k = \hat{\mathbf{r}}^k \mathbf{A}^{k-1},$$

where the diagonal matrix of row scaling factors $\hat{\mathbf{r}}^k \in \mathbb{R}^{P\times P}$ is defined as

$$\hat{\mathbf{r}}^k = \text{diag}\left(\left[\frac{r_i}{\sum_{j=1}^{N} \mathbf{A}_{ij}^{k-1}}\right]_{i=1}^{P}\right)$$

with $r_i$ being the $i$-th element of vector $\mathbf{r}$ and $\sum_{j=1}^{N} \mathbf{A}_{ij}^{k-1}$ being the row sum of matrix $\mathbf{A}^{k-1}$.[55] Note that at the first iteration ($k = 1$), the matrix $\mathbf{A}^{k-1}$ is given by the initial correspondence matrix $A_0$.

2. **Column Adjustment**: The columns of the correspondence matrix $\mathbf{A}^{k-1}$ at iteration $k$ are scaled to match the total expenditures in each COICOP category from EVS, given by the column vector $\mathbf{s} \in \mathbb{R}^{N\times 1}$. The updated correspondence matrix is given by

$$\mathbf{A}^{k+1} = \mathbf{A}^k \hat{\mathbf{s}}^k = \hat{\mathbf{r}}^k \mathbf{A}^{k-1} \hat{\mathbf{s}}^k,$$

where the diagonal matrix of column scaling factors $\hat{\mathbf{s}}^k \in \mathbb{R}^{N\times N}$ is defined as

$$\hat{\mathbf{s}}^k = \text{diag}\left(\left[\frac{s_j}{\sum_{i=1}^{P} \mathbf{A}_{ij}^{k}}\right]_{j=1}^{N}\right)$$

with $s_j$ being the $j$-th element of vector $\mathbf{s}$ and $\sum_{i=1}^{P} \mathbf{A}_{ij}^{k}$ being the column sum of the matrix $\mathbf{A}^k$.[56]

These two steps are repeated iteratively until convergence. Specifically, convergence is achieved when

$$\max\left\{\max_{i=1,\ldots,P}\left|\frac{\sum_{j=1}^{N} \mathbf{A}_{ij}^{k} - r_i}{r_i}\right|_{i=1}^{P}, \max_{j=1,\ldots,N}\left|\frac{\sum_{i=1}^{P} \mathbf{A}_{ij}^{k} - s_j}{s_j}\right|_{j=1}^{N}\right\} < \epsilon_{tol},$$

[54] Because EXIOBASE reports final demand at basic prices, we first convert it to purchaser prices using EXIOBASE use table. See Appendix B.2.2 for details on the construction of the EXIOBASE use table. We scale aggregate EXIOBASE household final demand to match the corresponding national accounts aggregate, so that the row and column targets are mutually consistent.

[55] Note that this requires $\sum_{j=1}^{N} \mathbf{A}_{ij}^{k-1} \neq 0 \; \forall \; i$.

[56] Note that this requires that $\sum_{i=1}^{P} \mathbf{A}_{ij}^{k} \neq 0 \; \forall \; j$.

where $\epsilon_{tol} = 10^{-6}$. This ensures that both the row and column sums of $\mathbf{A}^{k+1}$ match $r$ and $s$. Once the algorithm converges, we define the resulting correspondence matrix as $\tilde{\mathbf{A}} \in \mathbf{R}^{P \times N}$.

**Normalization of Correspondence Matrix.** Finally, we normalize each column of $\tilde{\mathbf{A}}$ to sum to one:

$$\mathbf{M} = \tilde{\mathbf{A}} \oslash (\mathbf{1}^{\top} \tilde{\mathbf{A}}),$$

where $\oslash$ denotes element-wise division and $\mathbf{1} \in \mathbb{R}^{P \times 1}$ is a column vector of ones. Conceptually $M_{pn}$ gives the share of one euro spent in COICOP category $n$ that is allocated to EXIOBASE product $p$. The matrix $\mathbf{M}$ therefore defines a many-to-many mapping from COICOP categories to EXIOBASE products.

### B.3.4 Direct Emission Intensities

We compute direct carbon emission intensities for each COICOP product by converting physical emission factors for the underlying fuels into monetary emission intensities. Direct emissions are assigned only to COICOP categories that generate emissions at the point of household use: gas (c04.5.2), liquid fuels (c04.5.3), solid fuels (c04.5.4), and fuels and lubricants for personal transport equipment (c07.2.2). For each COICOP product $n$, let $K_n$ denote the set of underlying fuels and let $k \in K_n$ index individual fuels. We then compute the monetary direct emission intensity for each fuel $k$ as

$$\tilde{f}_{dir,k} = \frac{f_{dir,k}^{physical}}{p_k^{2018}},$$

where $f_{dir,k}^{physical}$ is the direct emission factor per physical unit consumed and $p_k^{2018}$ is the corresponding 2018 consumer price per physical unit. This conversion yields direct emissions per euro of expenditure. Table 5 reports, for each fuel within the relevant COICOP categories, the physical emission factor measured in $tCO_2e/TJ$ and the corresponding monetary emission intensity.[57]

Finally, we aggregate fuel specific monetary emission intensities to the COICOP level by taking a weighted average across the fuels $k \in K_n$ underlying COICOP product $n$:

$$\tilde{f}_{dir,n}^{COICOP} = \sum_{k \in K_n} s_{k,n} \tilde{f}_k^{dir},$$

where $s_{k,n}$ denotes the share of fuel $k$ in household fuel consumption within COICOP product $n$. For gas and solid fuels, we construct these shares using the 2011–2013 German Residential Energy Consumption Survey (GRECS), provided by RWI – Leibniz-Institut für Wirtschafts-

[57] For each fuel, we collect greenhouse gas specific physical emission factors for $CO_2$, $CH_4$, and $N_2O$. We then convert $CH_4$ and $N_2O$ into $CO_2$-equivalents using the corresponding characterization factors and sum across gases to obtain the total physical emission factor in $tCO_2e/TJ$. We exclude $SF_6$, HFCs, and PFCs, as these greenhouse gases do not arise as direct household emissions for the fuels considered.

forschung.[58] For fuels and lubricants for personal transport equipment, we use the corresponding fuel shares from the German Statistical Office (Brockjan, Maier, Kott, and Sewald 2021). For COICOP categories that contain only one relevant fuel type, we set $s_{k,n} = 1$. Table 5 reports the resulting consumption shares. The resulting vector of direct emission intensities is denoted by $\tilde{\mathbf{f}}_{dir}^{COICOP} \in \mathbb{R}^{N\times 1}$. For all COICOP categories that do not generate direct emissions at the household level, the corresponding entries of $\tilde{\mathbf{f}}_{dir}^{COICOP}$ are set to zero.

| **Energy Source** | **Physical Emission Factor** ($tCO_2e/TJ$) | **Emission Intensity** ($kgCO_2e/EUR$) | **Share** |
|---|---|---|---|
| **Gas (c04.5.2)** | | **3.00** | |
| Liquefied Petroleum Gas (LPG) | 66.45 | 3.37 | 0.045 |
| Natural Gas | 55.82 | 2.98 | 0.955 |
| **Liquid Fuels (c04.5.3)** | | **3.87** | |
| Heating Oil | 74.18 | 3.87 | 1.000 |
| **Solid Fuels (c04.5.4)** | | **8.00** | |
| Hard Coal | 102.58 | 5.57 | 0.010 |
| Lignite | 104.59 | 7.95 | 0.016 |
| Wood | 105.60 | 8.14 | 0.852 |
| Wood Pellets | 101.14 | 7.22 | 0.122 |
| **Fuels for Personal Transport Equipment (c07.2.2)** | | **1.85** | |
| Diesel Fuel | 76.04 | 2.09 | 0.422 |
| Gasoline | 75.09 | 1.68 | 0.578 |

Table 5: Physical and Monetary Direct Emission Intensities

*Notes:* Physical emission factors for $CO_2$ are taken from Umweltbundesamt (2025a). Physical emission factors for $CH_4$ and $N_2O$ are taken from Blank, Wickert, Obermeier, and Friedrich (1999), except for liquefied petroleum gas (LPG), for which we use IPCC (2006a), and for diesel fuel and gasoline, for which we use IPCC (2006b). We convert $CH_4$ and $N_2O$ into $CO_2$-equivalents using the corresponding characterization factors and sum across gases to obtain the total physical emission factor in $tCO_2e/TJ$. Fuel shares for gas (c04.5.2) and solid fuels (c04.5.4) are based on the 2013 wave of the 2011–2013 German Residential Energy Consumption Survey (GRECS) (RWI and forsa 2015), provided by RWI – Leibniz-Institut für Wirtschaftsforschung, and are computed as consumption-weighted shares using standardized household fuel consumption measured in kWh and the corresponding fuel-specific survey weights. Fuel shares for fuels and lubricants for personal transport equipment (c07.2.2) are taken from the German Statistical Office (Brockjan, Maier, Kott, and Sewald 2021). For liquid fuels (c04.5.3), the fuel share is set to one.

## B.4 Validation of Emission Intensities

Table 6 reports the implied aggregate emissions of German households and compares them with the estimates reported by German Statistical Office (2025b). Since the benchmark statistics are reported for $CO_2$, while our main estimates are expressed in $CO_2$ equivalents, the table reports both $CO_2e$ and $CO_2$ estimates. Our estimates imply total annual household emissions of 731.3 $MtCO_2e$, of which 32% are attributable to direct emissions. Dividing aggregate emissions by the number of tax units gives an average carbon footprint of 16.9 $tCO_2e$ per tax unit. Our estimates are of the same order of magnitude as the estimates reported by the German statistical office.

[58] The GRECS data is a representative panel on the energy consumption of German households (RWI and forsa 2015). We use the 2013 wave to construct consumption-weighted fuel shares based on standardized household fuel consumption measured in kWh and the corresponding fuel-specific survey weights.

| | Own Estimates | | German Statistical Office |
|---|---|---|---|
| | $CO_2e$ | $CO_2$ | $CO_2$ |
| **Total Emissions** | **731.26 Mt** | **603.57 Mt** | **592.70 Mt** |
| Indirect Emissions | 500.12 Mt | 375.33 Mt | 376.72 Mt |
| Direct Emissions | 231.14 Mt | 228.23 Mt | 215.97 Mt |
| **Carbon Footprint per Tax Unit** | **16.92 t** | **13.96 t** | **13.71 t** |

Table 6: Aggregate GHG emissions of German households.

*Notes:* Own estimates are obtained by multiplying COICOP-level emission intensities by aggregate household consumption expenditures in purchaser prices obtained from the EVS and summing across COICOP categories. The estimates of the German Statistical office are obtained from German Statistical Office (2025b). Indirect emissions refer to emissions generated along the production chain, while direct emissions are released directly by households during consumption. Per tax unit values are obtained by dividing aggregate emissions by the number of tax units.

# C Micro Data

We use two primary data sources: (i) detailed income and consumption expenditure data from the *Einkommens- und Verbrauchsstichprobe* (EVS), provided by the German Statistical Office (Research Data Centre of the Federal Statistical Office and the statistical offices of the Länder (RDC) 2018b, Research Data Centre of the Federal Statistical Office and the statistical offices of the Länder (RDC) 2018a) and (ii) German administrative income tax data from the *Lohn- und Einkommenssteuerstatistik* (LESt), provided by the German Federal Statistical Office (Research Data Centre of the Federal Statistical Office and the statistical offices of the Länder (RDC) 2018c). Both datasets refer to the year of 2018. We limit our sample to the working-age population, defined as tax units in which the main earner is between 20 and 60 years old.

**EVS.** The EVS provides quarterly household-level data on consumption expenditures, gross labor income, and socioeconomic characteristics. We convert all monetary variables to an annual level by multiplying the quarterly values by four. The unit of observation is the household. To ensure comparability with income tax data from the LESt, we convert the data from the household level to the tax unit level. Specifically, we split non-married couples into separate tax units by allocating household consumption expenditures according to each person's share of household labor income. This leaves us with a sample of around 26 000 tax units.

Consumption expenditures are reported in purchaser's prices and classified at the five-digit COICOP system. For tractability, we aggregate expenditures to the four-digit COICOP level by summing across all corresponding five-digit subcategories, resulting in 107 different products

and services.[59] Table 7 documents the aggregation and provides the relevant variables in the EVS.

We define gross labor income as the sum of income from employment and self-employment. Accordingly, we assign each tax unit to the corresponding percentile of the gross labor income distribution obtained from the LESt. Transfer payments include unemployment benefits (*Arbeitslosengeld II; Sozialgeld nach Sozialgesetzbuch II*), employment promotion transfers, housing allowance, social assistance and child supplement.[60]

To address the well-known issue of underreporting in household survey data, we scale consumption expenditures in the EVS by a constant factor such that the population-weighted aggregate consumption in the EVS aligns with national accounts data from German Statistical Office (2024a). Similarly, we adjust transfer receipts by applying transfer specific scaling factors to ensure consistency with administrative aggregates. Specifically, unemployment benefits are scaled to match total reported spending from Bundesagentur für Arbeit (2018), housing allowances to aggregates from German Statistical Office (2024b), social assistance to aggregates from German Statistical Office (2025a), and child supplements to aggregates from Familienkasse Direktion SR1 (2018).[61]

**LESt.** The LESt comprises a 10% cross-sectional random sample of German taxpayers based on administrative income tax records. We use data on annual gross labor income to calibrate the income distribution. Note that the unit of observation is the tax unit. This implies that if a married couple files taxes separately, we cannot identify them as belonging to the same household in our data. In 2018, around 8.1% of married couples filed taxes separately. Labor income of a tax unit is defined as the sum of income from employment and self-employment. Tax payments are calculated as the sum of the assessed income tax (*festzusetzende Einkommenssteuer*), the solidarity surcharge (*SolidaritÃâ¬tszuschlag*) and the church tax.[62] Our sample includes around 2.78 million tax units.

| Code | Product | EVS Variables | Brown/Green | Emission Intensity |
|---|---|---|---|---|

[59] Detailed expenditure data for food (c01.1), non-alcoholic (c01.2) and alcoholic beverages (c02.1) is not available in the main EVS file (gf5). Therefore, we rely on supplementary data from the gf4 file, which contains detailed recording booklet for food, beverages and tobacco products, covering roughly 20% of the tax units included in the main EVS file (gf5). We compute the expenditure share of each product within the COICOP groups food (c01.1), non-alcoholic beverages (c01.2) and alcoholic beverages (c02.1) at the tax unit level. These shares are then averaged across tax units within net income bins, using survey weights. Finally, we impute consumption expenditures in the gf5 file by multiplying total expenditure in each COICOP group (EVS variables EF242, EF243, and EF244) by the corresponding average expenditure share.

[60] Gross labor income is computed as the sum of the following EVS variables: EF109U - EF115U, EF118U - EF137U, and EF176. Transfer payments are computed as the sum of the following EVS variables: EF149U, EF150U1, EF151U, EF152U, EF154U - EF157U, and EF168U.

[61] We do not scale employment promotion transfer due to lack of aggregate data.

[62] Gross labor income is calculated as the sum of the following LESt variables: C65120, C65100, C65140, C65163 and C65164. Income tax payments are computed as the sum variables C65613, C66104 and C66105. The solidarity surcharge is computed based on the assessed income tax, taking into account the exemption thresholds applicable in 2018.

| | | | | |
|---|---|---|---|---|
| | | **Food** | | |
| c01.1.1 | Cereals and cereal products | EF58U2 - EF87U2 | Green | 0.3927 kgCO2/€ |
| c01.1.2 | Live animals, meat and other parts of slaughtered land animals | EF88U2 - EF111U2 | Brown | 0.6383 kgCO2/€ |
| c01.1.3 | Fish and other seafood | EF112U2 - EF125U2 | Brown | 0.5273 kgCO2/€ |
| c01.1.4 | Milk, other diary products and eggs | EF126U2 - EF144U2 | Brown | 0.6412 kgCO2/€ |
| c01.1.5 | Oils and fats | EF145U2 - EF153U2 | Brown | 0.6366 kgCO2/€ |
| c01.1.6 | Fruits and nuts | EF154U2 - EF177U2 | Green | 0.4076 kgCO2/€ |
| c01.1.7 | Vegetables, tubers, plantains, cooking bananas and pulses | EF178U2 - EF208U2 | Green | 0.4078 kgCO2/€ |
| c01.1.8 | Sugar, confectionery and desserts | EF209U2 - EF222U2 | Brown | 0.6271 kgCO2/€ |
| c01.1.9 | Ready-made food and other food products | EF224U2 - EF249U2 | Green | 0.4014 kgCO2/€ |
| c01.2.1 | Coffee, tea and cocoa | EF250U2 - EF255U2 | Green | 0.4210 kgCO2/€ |
| c01.2.2 | Mineral waters, soft drinks, fruit and vegetable juices | EF256U2 - EF267U2 | Green | 0.3714 kgCO2/€ |
| c02.1.1 | Spirits | EF268U2 - EF271U2 | Green | 0.3767 kgCO2/€ |
| c02.1.2 | Wine | EF272U2 - EF283U2 | Green | 0.3743 kgCO2/€ |
| c02.1.3 | Beer | EF284U2 - EF288U2 | Green | 0.3853 kgCO2/€ |
| | | **Clothing** | | |
| c03.1.1 | Clothing materials | EF247 | Brown | 0.3240 kgCO2/€ |
| c03.1.2 | Garments | EF248 - EF250 | Brown | 0.2857 kgCO2/€ |
| c03.1.3 | Other articles of clothing and clothing accessories | EF251 | Brown | 0.2914 kgCO2/€ |
| c03.1.4 | Cleaning, repair and hire of clothing | EF252, EF253 | Green | 0.1199 kgCO2/€ |
| c03.2.1 | Shoes and other footwear | EF254 - EF257 | Brown | 0.2530 kgCO2/€ |
| c03.2.2 | Repair and hire of footwear | EF258 | Green | 0.1199 kgCO2/€ |
| c12.3.1 | Jewelery, clocks and watches | EF443, EF444 | Brown | 0.2557 kgCO2/€ |
| c12.3.2 | Other personal effects | EF445 | Brown | 0.3083 kgCO2/€ |
| | | **Housing** | | |
| c04.1.1 | Actual rentals paid by tenants | EF259 - EF262 | Green | 0.0751 kgCO2/€ |
| c04.1.2 | Other actual rentals | EF263 - EF267 | Green | 0.0462 kgCO2/€ |
| c04.2.1 | Imputed rentals of owner-occupiers | EF268 - EF271 | Green | 0.0450 kgCO2/€ |
| c04.2.2 | Other imputed rentals | EF272 - EF276 | Green | 0.0462 kgCO2/€ |
| c04.3.1 | Materials for the maintenance and repair of the dwelling | EF277 - EF280 | Brown | 0.4301 kgCO2/€ |
| c04.4.1 | Water supply | EF285 - EF287 | Brown | 0.3581 kgCO2/€ |
| c04.4.2 | Refuse collection | EF288 - EF290 | Brown | 0.4023 kgCO2/€ |
| c04.4.3 | Sewage collection | EF291 - EF293 | Brown | 0.4023 kgCO2/€ |
| c04.4.5 | Cold operating costs, additional costs | EF297 - EF307 | Brown | 0.3465 kgCO2/€ |
| | | **Electricity** | | |
| c04.5.1 | Electricity | EF313 | - | 1.7913 kgCO2/€ |
| | | **Heating** | | |
| c04.5.2 | Gas | EF314 - EF317 | Green | 3.4322 kgCO2/€ |
| c04.5.3 | Liquid fuels | EF318 - EF320 | Green | 4.4227 kgCO2/€ |
| c04.5.4 | Solid fuels | EF321 | Brown | 8.6185 kgCO2/€ |
| c04.5.5 | Heat energy | EF322 - EF324 | Green | 4.5929 kgCO2/€ |
| | | **Household and Personal Care** | | |
| c05.1.1 | Furniture and furnishing | EF326, EF327 | Brown | 0.2890 kgCO2/€ |
| c05.1.2 | Carpets and other floor coverings | EF328, EF329 | Brown | 0.2994 kgCO2/€ |
| c05.1.3 | Repair of furniture, furnishings and floor coverings | EF330 | Green | 0.1010 kgCO2/€ |
| c05.2.0 | Household textiles | EF331, EF332 | Brown | 0.3470 kgCO2/€ |
| c05.3.1 | Major household appliances whether electric or not | EF333 - EF336 | Green | 0.1741 kgCO2/€ |
| c05.3.2 | Small electric household appliances | EF337 | Brown | 0.3188 kgCO2/€ |
| c05.3.3 | Repair of household appliances | EF338 | Brown | 0.2692 kgCO2/€ |
| c05.4.0 | Glassware, tableware and household utensils | EF339, EF340 | Brown | 0.2791 kgCO2/€ |
| c05.5.1 | Major tools and equipment | EF341, EF342 | Green | 0.1126 kgCO2/€ |
| c05.5.2 | Small tools and miscellaneous accessories | EF343 - EF345 | Brown | 0.4548 kgCO2/€ |
| c05.6.1 | Non-durable household goods | EF346 | Brown | 0.2872 kgCO2/€ |
| c06.1.1 | Pharmaceutical products | EF349 - EF353 | Brown | 0.2765 kgCO2/€ |
| c06.1.2 | Other medical products | EF354 - EF358 | Brown | 0.2860 kgCO2/€ |
| c06.1.3 | Therapeutic appliances and equipment | EF359 - EF362 | Green | 0.1384 kgCO2/€ |
| c09.5.4 | Stationery and drawing materials | EF422 | Brown | 0.2916 kgCO2/€ |
| c12.1.2 | Electric appliances for personal care | EF438 | Green | 0.1453 kgCO2/€ |
| c12.1.3 | Other appliances, articles and products for personal care | EF439 - EF441 | Brown | 0.2905 kgCO2/€ |
| | | **Services** | | |
| c04.3.2 | Services for the maintenance and repair of the dwelling | EF281 - EF284 | Brown | 0.3295 kgCO2/€ |
| c04.4.4 | Other services relating to the dwelling | EF294 - EF296 | Brown | 0.3418 kgCO2/€ |
| c05.6.2 | Domestic services and household services | EF347, EF348 | Brown | 0.2182 kgCO2/€ |
| c06.2.1 | Medical services | EF363 | Green | 0.1048 kgCO2/€ |
| c06.2.2 | Dental services | EF364 | Green | 0.1048 kgCO2/€ |
| c06.2.3 | Paramedical services | EF365, EF366 | Green | 0.1241 kgCO2/€ |
| c06.3.0 | Hospital services | EF367 | Green | 0.1267 kgCO2/€ |
| c07.2.4 | Other services in respect of personal transport equipment | EF378 | Green | 0.1557 kgCO2/€ |
| c08.1.0 | Postal services | EF385 | Green | 0.0866 kgCO2/€ |
| c08.3.0 | Telephone and telefax services | EF387 - EF392 | Green | 0.0890 kgCO2/€ |
| c09.1.5 | Repair of audio-visual, photographic and information processing equipment | EF398 | Brown | 0.3380 kgCO2/€ |
| c10.1.0 | Pre-primary and primary education | EF425 - EF427 | Green | 0.0661 kgCO2/€ |
| c10.2.0 | Secondary education | EF428 | Green | 0.0661 kgCO2/€ |
| c10.5.0 | Education not definable by level | EF429, EF430 | Green | 0.0661 kgCO2/€ |

| | | | | |
|---|---|---|---|---|
| c12.1.1 | Hairdressing salons and personal grooming establishments | EF434 - EF437 | Green | 0.0807 kgCO2/€ |
| c12.4.0 | Social protection (private) | EF446 - EF449 | Green | 0.1155 kgCO2/€ |
| c12.6.2 | Other financial services | EF451 | Green | 0.0999 kgCO2/€ |
| c12.7.0 | Other services | EF452 | Green | 0.1030 kgCO2/€ |
| | **Transport and Vacation** | | | |
| c07.1.1 | Motor cars | EF368, EF369 | Green | 0.1965 kgCO2/€ |
| c07.1.2 | Motor cycles | EF370 | Green | 0.1959 kgCO2/€ |
| c07.1.3 | Bicycles | EF371 | Green | 0.3053 kgCO2/€ |
| c07.1.4 | Animal drawn vehicles | EF372 | Green | 0.2466 kgCO2/€ |
| c07.2.1 | Spare parts and accessories for personal transport equipment | EF374, EF375 | Green | 0.3088 kgCO2/€ |
| c07.2.2 | Fuels and lubricants for personal transport equipment | EF376 | Brown | 2.1892 kgCO2/€ |
| c07.2.3 | Maintenance and repair of personal transport equipment | EF377 | Green | 0.1696 kgCO2/€ |
| c07.3.1 | Passenger transport by railway | EF379 | Green | 0.2455 kgCO2/€ |
| c07.3.2 | Passenger transport by road | EF380 | Green | 0.2383 kgCO2/€ |
| c07.3.3 | Passenger transport by air | EF381 | Brown | 0.8358 kgCO2/€ |
| c07.3.4 | Passenger transport by sea and inland waterways | EF382 | Brown | 1.6285 kgCO2/€ |
| c07.3.5 | Combined passenger transport | EF383 | Brown | 0.6750 kgCO2/€ |
| c07.3.6 | Other purchased transport services | EF384 | Green | 0.1809 kgCO2/€ |
| c09.6.1 | Package holidays (Germany) | EF423 | Brown | 0.6527 kgCO2/€ |
| c09.6.2 | Package holidays (foreign country) | EF424 | Brown | 0.8357 kgCO2/€ |
| c11.2.0 | Accommodation services | EF433 | Green | 0.1914 kgCO2/€ |
| | **Leisure** | | | |
| c02.2.0 | Tobacco | EF245 | Green | 0.1695 kgCO2/€ |
| c08.2.0 | Telephone and telefax equipment | EF386 | Green | 0.0896 kgCO2/€ |
| c09.1.1 | Equipment for the reception, recording and reproduction of sound and pictures | EF393, EF394 | Brown | 0.3037 kgCO2/€ |
| c09.1.2 | Photographic and cinematographic equipment and optical instruments | EF395 | Brown | 0.3052 kgCO2/€ |
| c09.1.3 | Information processing equipment | EF396 | Brown | 0.2578 kgCO2/€ |
| c09.1.4 | Recording media | EF397 | Brown | 0.2979 kgCO2/€ |
| c09.2.1 | Major durables for outdoor recreation | EF399 | Brown | 0.2758 kgCO2/€ |
| c09.2.3 | Maintenance and repair of other major durables for recreation and culture | EF400 | Green | 0.1521 kgCO2/€ |
| c09.3.1 | Games, toys and hobbies | EF401 | Brown | 0.2412 kgCO2/€ |
| c09.3.2 | Equipment for sport, camping and open-air recreation | EF402, EF403 | Brown | 0.2664 kgCO2/€ |
| c09.3.3 | Gardens, plants and flowers | EF404, EF405 | Brown | 0.4167 kgCO2/€ |
| c09.3.4 | Pets and related products | EF406 | Brown | 0.2964 kgCO2/€ |
| c09.4.1 | Recreational and sporting services | EF407 - EF409 | Brown | 0.1809 kgCO2/€ |
| c09.4.2 | Cultural services | EF410 - EF416 | Green | 0.0897 kgCO2/€ |
| c09.4.3 | Games of chance | EF417 | Green | 0.1017 kgCO2/€ |
| c09.5.1 | Books | EF418 | Green | 0.0986 kgCO2/€ |
| c09.5.2 | Newspapers and periodicals | EF419, EF420 | Green | 0.1173 kgCO2/€ |
| c09.5.3 | Miscellaneous printed matter | EF421 | Green | 0.0986 kgCO2/€ |
| c11.1.1 | Restaurants, cafés and the like | EF431 | Brown | 0.2391 kgCO2/€ |
| c11.1.2 | Canteens | EF432 | Brown | 0.2391 kgCO2/€ |

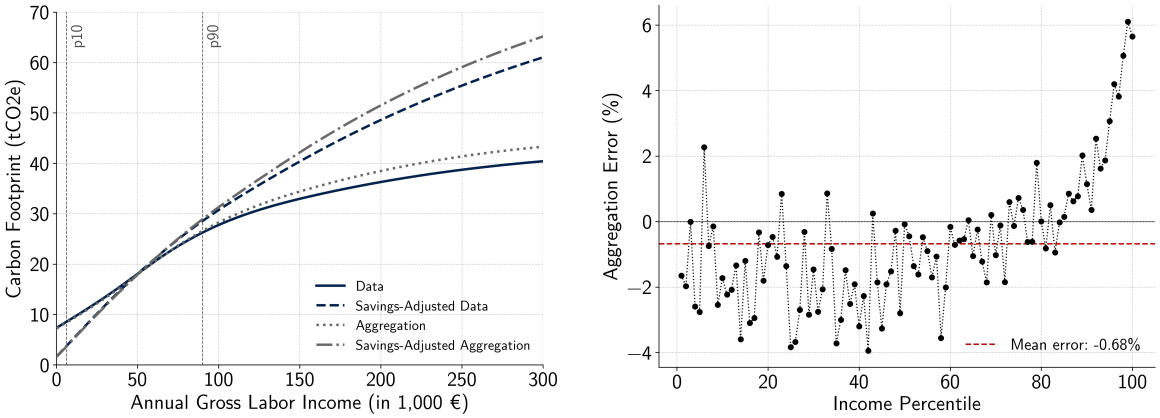
Table 7: Classification of Products

*Note:* The COICOP groups c01.1 (food), c01.2 (non-alcoholic beverages), and c02.1 (alcoholic beverages) are based on item-level expenditure data from the EVS recording booklet for food, beverages and tobacco products (gf4). All remaining COICOP groups are based on data from the EVS main file (gf5). Emission intensities are expressed in kgCO2/€per euro of expenditure. Within each product category, products are classified as either 'brown' or 'green' based on whether their emission intensity is above or below the cutoff emission intensity for that category. The cutoff emission intensity is determined by K-means clustering. The methodology for computing carbon footprints is described in Appendix B.

# D Household Carbon Footprint

## D.1 Aggregation Error

Figure 9 quantifies the aggregation error induced by aggregating the detailed COICOP categories into the model's broader product–variety cells. The left panel compares the carbon footprint calculated at the detailed COICOP level (blue line) with the model counterpart ob-




(a) Carbon Footprint

(b) Aggregation Error

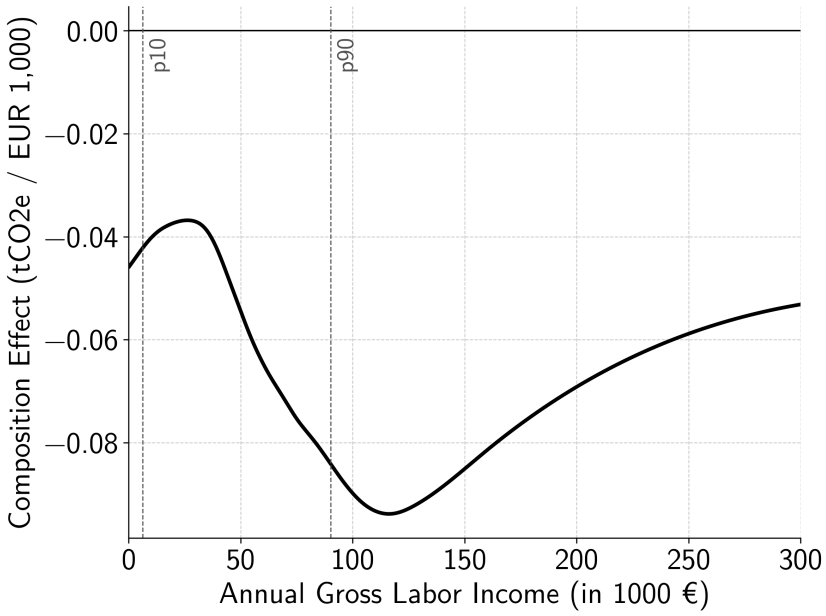
Figure 9: Carbon Footprint and Aggregation Error by Labor Income

*Notes:* The left panel shows carbon footprint estimates based on detailed COICOP categories (blue line) and on the aggregated good classification into broad products and varieties (grey line), for both the raw and savings-adjusted carbon footprints. All series show a cubic smoothing spline fit to the respective data moments. The right panel shows the aggregation error (black solid line), defined as the percentage difference between the aggregated estimate and the detailed COICOP estimate. The black dots correspond to percentile-level observations. The x-axis shows gross labor income $y$ of a tax unit in the left panel and the gross labor income percentile in the right panel. The grey vertical dashed-dotted lines mark to the 10th and 90th percentile of the annual gross labor income distribution.

tained by aggregating COICOP products to broad product–variety cells (grey line), for both the raw and savings-adjusted carbon footprints. Note that the savings adjustment does not affect the aggregation error because the savings adjustment is a proportional scalar applied identically to both the detailed and the aggregate estimate. The right panel shows the resulting aggregation error, defined as the percentage difference between the aggregated and detailed estimates, across income percentiles. The two estimates overlap closely up to around the 95th percentile of the gross labor income distribution. Above the 90th percentile, the aggregated estimate increasingly overstates the carbon footprint calculated based on the detailed COICOP products. Quantitatively, the aggregation error is small with an average across the income distribution of -0.68%. The overestimation at the top reflects within-category composition effects: high income tax units spend relatively more on goods within broad categories whose detailed COICOP emission intensities differ from the category-level average used in the aggregated classification.

## D.2 Decomposition of Marginal Propensity to Pollute

Figure 10 plots the composition component of the marginal propensity to pollute (MPP) along the income distribution.




(a) Composition Effect

Figure 10: Composition Component of the Marginal Propensity to Pollute

*Notes:* The figure shows the composition component of the marginal propensity to pollute, defined as the part of the change in emissions from an additional unit of total expenditure that is due to the induced change in the composition of the consumption basket. It is given by difference between the MPP, defined as $\text{MPP}(p) = \hat{f}'(e(p))$ and average emission intensity $\iota(p) = \hat{f}(p)/e(p)$, where $\hat{f}(\cdot)$ is a cubic smoothing spline of $\tilde{f}(p)$ on $e(p)$. The difference is computed based on the cubic smoothing spline of both object. The $x$-axis shows annual gross labor income of a tax unit. The grey vertical dashed-dotted lines mark to the 10th and 90th percentile of the annual gross labor income distribution.

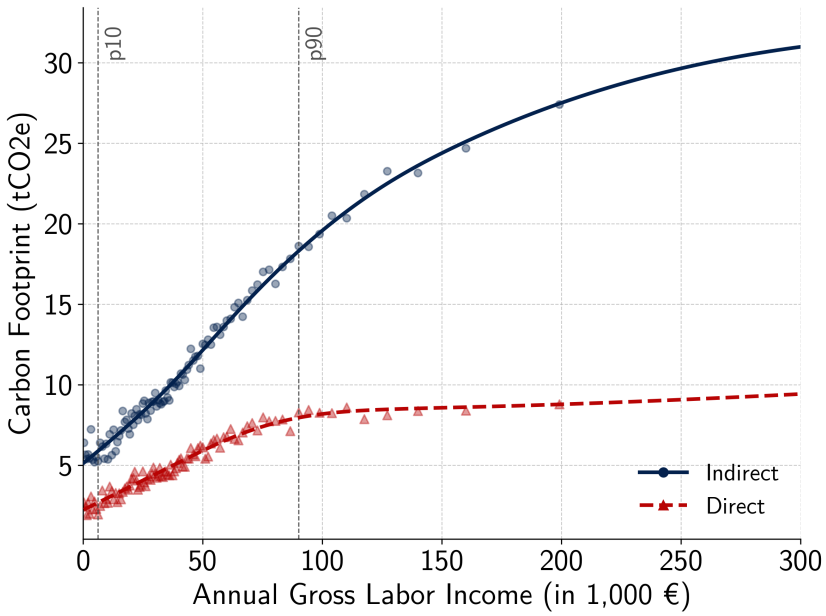



Figure 11: Direct and Indirect Carbon Emissions

*Notes:* The figure shows the carbon footprint of indirect emissions (blue dots) and direct emissions (red triangles) for each income percentile, both expressed in $tCO_2e$. The lines show cubic smoothing spline fits to the respective data moments. The x-axis shows annual gross labor income of a tax unit. The grey vertical dashed-dotted lines mark to the 10th and 90th percentile of the annual gross labor income distribution.

## D.3 Anatomy of the Income Gradient in Carbon Emissions

### D.3.1 Direct vs. Indirect Emissions

Figure 11 decomposes the percentile level carbon footprint into indirect and direct emissions. In the aggregate, indirect emissions account for 69% of total household emissions and account for 68% of the gap in carbon emissions between the 90th and 10th percentile of the income distribution.

### D.3.2 Horizontal Carbon Inequality

To assess the role of observable horizontal heterogeneity in carbon footprints, we compute a covariate-standardized carbon footprints for each percentile of the gross labor income distribution. We first estimate the following survey-weighted linear projection at the tax-unit level:

$$f_i = \alpha + \sum_{p \neq p_0} \gamma_p \mathbf{1}[p_i = p] + \mathbf{X}_i' \boldsymbol{\beta} + u_i,$$

where $f_i$ denotes the carbon footprint of tax unit $i$ and $p_i$ denotes its income percentile. The vector $\mathbf{X}_i$ contains observed characteristics, including household size, education, homeownership status, heating fuel type, rural residence status and age.

We then compute the covariate-standardized carbon footprint for each income percentile as

$$f_{std}(p) = \hat{\alpha} + \hat{\gamma}_p + \overline{\boldsymbol{X}}'_w \hat{\boldsymbol{\beta}},$$

where $\overline{\boldsymbol{X}}'_w$ is the survey-weighted mean of the covariates in the full estimation sample.Thus, every income percentile is evaluate at the same empirical distribution of observed characteristics.

In contrast, the raw carbon footprint profile is computed as the survey-weighted average carbon footprint among tax units observed in income percentile $p$ and therefore uses the actual covariate distribution of each income percentile.

Figure 12(a) illustrates the raw (blue solid line) and covariate-standardized adjusted carbon footprints (grey dashed line) along the gross labor income distribution. The gap in carbon emissions between the 90th and 10th percentile falls by about 24%; equivalently, 76% of the raw gap remains. The covariate-standardized profile therefore remains upward sloping, indicating that observable horizontal heterogeneity accounts for part of carbon inequality observed in the data, but a substantial vertical component remains. Figure 12(b) shows the corresponding marginal propensity to pollute, computed from the raw and standardized carbon footprint profiles.

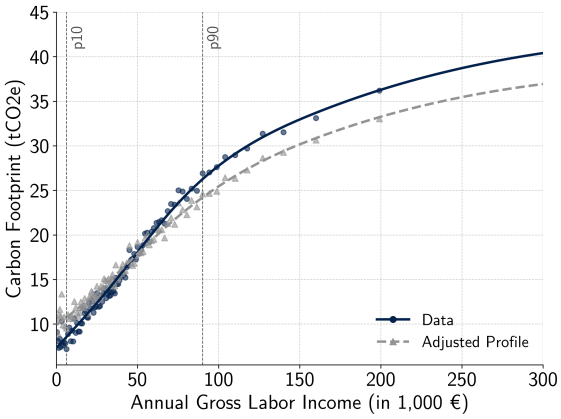
#### D.3.3 Carbon Footprint by Product Category

Figure 13(a) shows the average carbon footprint by income decile of the annual gross labor income distribution and broad product category. Figure 13(b) reports for each product category $i$ the contribution to the gap in the carbon footprint between the 90th and 10th income percentile, e.g. $\frac{\tilde{f}_i(p90)-\tilde{f}_i(p10)}{\tilde{f}(p90)-\tilde{f}(p10)}$, where $\tilde{f}_i(p)$ is the carbon footprint of product category $i$ of percentile $p$.

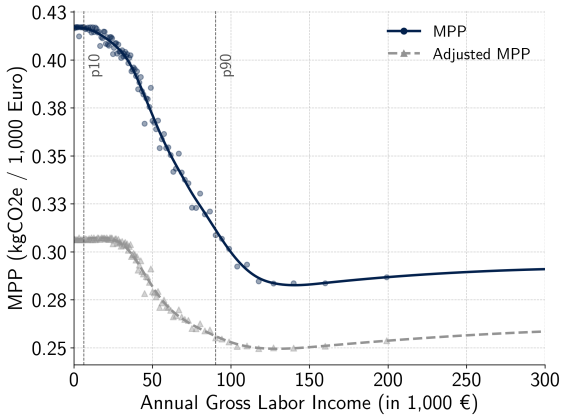
# E Quantitative Model

## E.1 Initial Effective Carbon Price

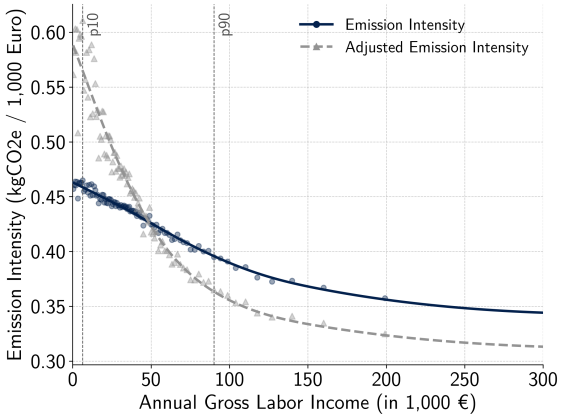
We approximate the initial effective carbon price on emissions embodied in German household consumption in 2018 as the uniform carbon price that would generate the same aggregate carbon cost as the EU ETS component embodied in the 2018 consumption basket. Since only part of embodied emissions is covered by the EU ETS, this effective price is given by the 2018 average EU ETS allowance price multiplied by the emission-weighted ETS coverage share of




(a) Carbon Footprint



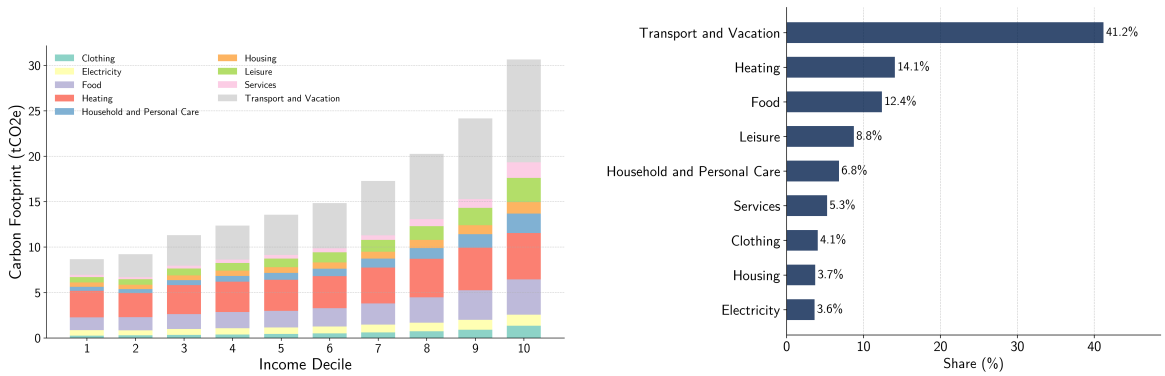

(b) Marginal Propensity to Pollute




(c) Emission Intensity

Figure 12: Covariate-Standardized Carbon Footprint and Marginal Propensity to Pollute

*Notes:* The left panel illustrates the carbon footprint $\tilde{f}(p)$ in the data (blue dots) and the covariate-standardized carbon footprint (grey triangles) for each income percentile $p$, both expressed in $tCO_2e$. The covariate-standardized profile is constructed from a linear projection of tax unit carbon footprints on income percentile fixed effects and observed characteristics, evaluating the controls at their common distribution in the EVS sample for every percentile. The right panel shows the corresponding marginal propensity to pollute (MPP), both expressed in $kgCO_2e$ per 1 000 Euro. The MPP is defined as the derivative of the spline-fitted carbon footprint with respect to total expenditure, evaluated at each percentile: $\text{MPP}(p) = \hat{f}'(e(p))$, where $\hat{f}(\cdot)$ is a cubic smoothing spline of $\tilde{f}(p)$ on $e(p)$. The bottom panel shows the corresponding emission intensity $\iota(p) = \hat{f}(p)/e(p)$, both expressed in $kgCO_2e$ per 1 000 Euro. In all panels, the lines show cubic smoothing spline fits to the respective data moments. The x-axis shows annual gross labor income of a tax unit. The grey vertical dashed-dotted lines mark to the 10th and 90th percentile of the annual gross labor income distribution.




(a) Carbon Footprint by Product Category

(b) Product Contribution to the p90–p10 Gap

Figure 13: Decomposition of Carbon Footprints

*Notes:* The left panel shows the average carbon footprint by income decile of the annual gross labor income distribution and broad product category. The right panel shows each product category's contribution to the gap in the carbon footprint between the 90th and 10th income percentile.

embodied household emissions. Using an average EU ETS price of $\tau_{2018}^{ETS} = 15.96$ EUR/tCO$_2$[63] and an effective embodied-emissions coverage share of $\bar{\rho}^{ETS} = 24.5\%$, we obtain

$$\tau_0 = \tau_{2018}^{ETS} \bar{\rho}^{ETS} = 15.96 \times 0.245 = 3.91 \text{ EUR/tCO}_2e.$$

To compute the share of embodied emissions covered by the EU ETS, we construct an ETS-covered $CO_2$ satellite account in the EXIOBASE product classification and apply it to the 2018 input-output model. Section E.1.1 describes this construction.

### E.1.1 EU ETS Coverage

**Data** We use installation-level data on verified emissions under the EU Emissions Trading System (EU ETS) compiled by Abrell (2024). The data include a match between installations and NACE Rev. 2 sectors, allowing us to aggregate verified emissions at the country and sector level.[64] Since aviation emissions are not contained in the installation-level data, we add aviation emissions from the EU ETS database (European Environment Agency 2026) and assign them to NACE Rev. 2 division 51, corresponding to air transport. The resulting dataset provides country-sector ETS emissions that can be mapped to the EXIOBASE country-product classification.

**Mapping NACE sectors to EXIOBASE products.** We next construct an ETS-covered $CO_2$ satellite account in the EXIOBASE product classification. Let $M$ denote the number of

[63] We compute the average EU ETS price in 2018 as the arithmetic mean of the daily closing prices for EU Allowances (EUAs), obtained from Trading Economics (2026).

[64] In 2018 around 5% of installations have no NACE code. Restricting the sample to installations with a non-missing NACE code leaves aggregate verified emissions virtually unchanged. The excluded installations account about 1% of total verified emissions.

NACE Rev. 2 sectors. We map NACE-level ETS emissions to EXIOBASE products using correspondence provided by Stadler et al. (2018). Let $\mathbf{C} \in \{0,1\}^{P\times M}$ denote the binary correspondence matrix, where $C_{pm} = 1$ if NACE sector $m$ is mapped to EXIOBASE product $p$, and $C_{pm} = 0$ otherwise.

For each country $k$, we allocate ETS emissions from NACE sectors to the corresponding EXIOBASE products. If a NACE sector maps to multiple EXIOBASE products, we split its ETS emissions in proportion to direct $CO_2$ emissions in the EXIOBASE satellite account. Let $F_{km}^{ETS,NACE}$ denote ETS emissions in country $k$ and NACE sector $m$, and let $F_{kp}^{CO2}$ denote direct $CO_2$ emissions in EXIOBASE in country $k$ and product $p$. Define the allocation weight

$$\omega_{kpm} = \frac{C_{pm} F_{kp}^{CO2}}{\sum_{q=1}^{P} C_{qm} F_{kq}^{CO2}}.$$

The ETS emissions distributed to EXIOBASE product $p$ are then given by

$$F_{kp}^{ETS,EXIO} = \sum_{m=1}^{M} \omega_{kpm} F_{km}^{ETS,NACE} \qquad \forall\ k = 1,\dots,K, \quad p = 1,\dots,P.$$

The resulting matrix $\mathbf{F}^{ETS,EXIO} \in \mathbb{R}^{K\times P}$ is then vectorized, using the same country–product ordering as in the input-output system, to obtain the ETS-covered $CO_2$ satellite account $\mathbf{F}^{ETS} \in \mathbb{R}^{1\times KP}$.

**ETS coverage share.** We compute ETS-covered indirect emission intensities by replacing the EXIOBASE emission satellite account $\mathbf{F}$ with the ETS-covered $CO_2$ satellite account $\mathbf{F}^{ETS} \in \mathbb{R}^{1\times KP}$, and otherwise applying the same procedure as in Section B.3. This yields an ETS-covered indirect emission intensity $f_n^{ETS}$ for each COICOP category $n$, measured in kg $CO_2$ per euro of household expenditure.

The ETS coverage share of embodied indirect $CO_2$ emissions in COICOP category $n$ is then defined as

$$\rho_n^{ETS} = \frac{f_n^{ETS}}{f_n},$$

where $f_n$ is the corresponding emission intensity computed from the standard EXIOBASE emission satellite account.[65] The resulting emission weighted effective ETS coverage share of embodied emissions is $\bar{\rho}^{ETS} = 24.5\%$ in 2018.

[65] For COICOP classes c04.5.5 (heat energy) and c04.5.1 (electricity), the raw mapped ETS coverage shares exceed one. This reflects imperfect reconciliation between verified EU ETS and EXIOBASE emissions at the country–product level, rather than true coverage above 100%. Since the purpose of the exercise is to approximate the EU ETS price embodied in household consumption, we impose the natural upper bound $\rho_n^{ETS} \leq 1$ and set the effective coverage share for these two classes equal to one.

## E.2 Tax-Transfer System

We estimate the parameters of the tax and transfer function separately using nonlinear least squares. The estimation sample is restricted to tax units with positive labor income. To ensure a good fit across the entire income distribution, we group observations into percentiles of the gross labor income distribution prior to estimation. The parameters are then estimated using these percentile-level data.

**Tax Function.** We estimate the parameters of the tax function $(\theta, \tau)$ using the following regression equation:

$$y_i^{atax} = \left(1 - \exp\left[\log(\lambda)(y_i/\bar{y})^{-2\tau}\right]\right) y_i + u_i,$$

where $y_i^{atax}$ denotes after-tax labor income, $y_i$ is gross labor income of tax unit $i$, $\bar{y}$ is average gross labor income, and $u_i$ is the error term.

**Transfer Function.** We estimate the parameters of the transfer function $(m, \zeta)$ using the following regression equation

$$\mathcal{T}_{0_i} = \hat{m}\bar{y}\frac{2\exp\left[-\zeta\frac{y_i}{\bar{y}}\right]}{1+\exp\left[-\zeta\frac{y_i}{\bar{y}}\right]} + u_i,$$

where $\mathcal{T}_{0_i}$ is the received transfer of tax unit $i$.

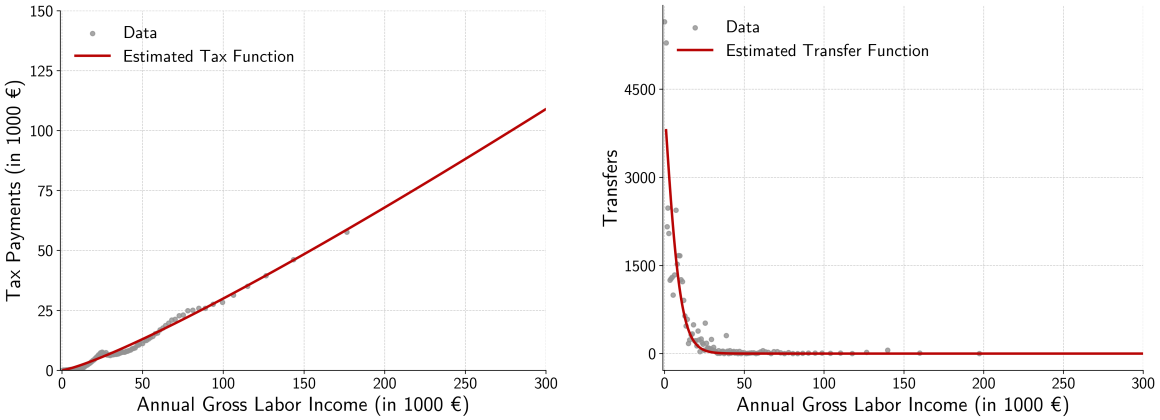
**Estimation Fit.** Figure 14 illustrates the estimation fit by comparing estimated taxes and transfers with their data counterparts. Overall, the tax-transfer function provides a good fit of the observed tax-transfer schedule in Germany.

## E.3 Income Distribution

**Kernel Density Estimation.** To obtain a smooth estimate of the income distribution from the LESt, we apply a standard kernel density estimation over log labor income using a Gaussian kernel and survey weights. The sample is restricted to tax units with positive labor income. We use an evenly spaced log income grid with 5000 nodes ranging from €0.8 to €10 470 000.

We select a bandwidth of approximately 0.07 on the log income scale, which implies smoothing over a local neighborhood of about 7% around each income level. Because the bandwidth is constant in log space, the corresponding smoothing window becomes wider in absolute income units as income rises.[66]

[66] Using a large bandwidth is a common approach in the optimal income taxation literature, as it helps to produce smooth schedules of optimal policies (Lockwood 2020, Choné and Laroque 2010).




(a) Tax Function (b) Transfer Function

Figure 14: Estimated Tax-Transfer Function

*Notes:* The left panel shows the estimated tax function and the right panel the estimated transfer function (red solid lines). The gray dots correspond to the observed average tax payments and transfer payments, respectively. Observations are grouped into percentiles of the gross labor income distribution. The x-axis shows gross labor income $y$ of a tax unit.

## E.4 Calibration of Utility Function

### E.4.1 Income Effects

Figure 15 illustrates the income effect $\eta(\theta)$ implied by the calibrated model along the gross labor income distribution. The shape and magnitude of the income effect are broadly consistent with the empirical estimates reported by Golosov, Graber, Mogstad, and Novgorodsky (2023).

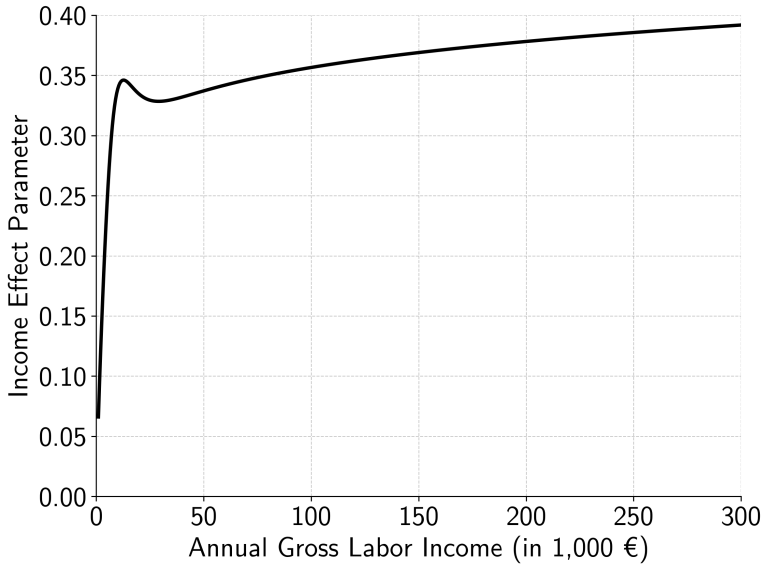



Figure 15: Income Effects

*Notes:* The figure shows the calibrated income effect $\eta(\theta)$. The x-axis shows gross labor income $y$ of a tax unit.

### E.4.2 Internally Calibrated Parameters

**Parameters of Lower-Tier Utility Function.** For each product $i$, we calibrate the lower-tier parameters $\boldsymbol{\theta}_i = (\epsilon_{ig}, \Omega_{ig})^\top$ by minimizing the squared distance between the model-implied and empirical within-good expenditure share on the brown variety across income deciles:

$$\hat{\boldsymbol{\theta}}_i = \arg\min_{\boldsymbol{\theta}_i} \big(\mathbf{s}_{ib}(\boldsymbol{\theta}_i) - \bar{\mathbf{s}}_{ib}\big)^\top \big(\mathbf{s}_{ib}(\boldsymbol{\theta}_i) - \bar{\mathbf{s}}_{ib}\big), \tag{50}$$

where $\mathbf{s}_{ib}(\boldsymbol{\theta}_i) = \big(s_{ib}(c_{i,1};\boldsymbol{\theta}_i), \ldots, s_{ib}(c_{i,D};\boldsymbol{\theta}_i)\big)^\top$ summarizes the model-implied moments for income deciles $d = 1, \ldots, D$. The model-implied within-good expenditure share on the brown variety in decile $d$ is

$$s_{ib,d}(\boldsymbol{\theta}_i) = \frac{\Omega_{ib} p_{ib}^{1-\sigma_i} c_{i,d}(\boldsymbol{\theta}_i)^{\epsilon_{ib}}}{\Omega_{ib} p_{ib}^{1-\sigma_i} c_{i,d}(\boldsymbol{\theta}_i)^{\epsilon_{ib}} + \Omega_{ig} p_{ig}^{1-\sigma_i} c_{i,d}(\boldsymbol{\theta}_i)^{\epsilon_{ig}}}. \tag{51}$$

We compute the $c_{i,d}(\boldsymbol{\theta}_i)$ from the expenditure level on product $i$ in income decile $d$. Let $e_{i,d} = \omega_{i,d}\bar{e}_d$ denote the model-consistent expenditure on product $i$ in decile $d$, where $\bar{e}_d$ is average disposable income in decile $d$ and $\omega_{i,d}$ is the corresponding EVS expenditure share of category $i$. For each candidate parameter vector $\boldsymbol{\theta}_i$, the consumption index $c_{i,d}(\boldsymbol{\theta}_i)$ is defined as the solution to

$$e_{i,d} = e_i(c_{i,d}(\boldsymbol{\theta}_i)) = \left[\sum_{j\in\{b,g\}} \Omega_{ij} p_{ij}^{1-\sigma_i} c_{i,d}(\boldsymbol{\theta}_i)^{\epsilon_{ij}}\right]^{\frac{1}{1-\sigma_i}}.$$

Substituting the resulting $c_{i,d}(\boldsymbol{\theta}_i)$ into (51) yields the model-implied expenditure share on the brown variety $s_{ib,d}(\boldsymbol{\theta}_i)$ used in (50). The empirical counterpart is $\bar{\mathbf{s}}_{ib} = (\bar{s}_{ib,1}, \ldots, \bar{s}_{ib,D})^\top$, where $\bar{s}_{ib,d}$ is the survey-weighted average expenditure share of the brown variety in total expenditure on category $i$ among tax units in income decile $d$.[67] We solve (50) by nonlinear least squares.

**Parameters of Upper-Tier Utility Function.** We calibrate the upper-tier parameters $\boldsymbol{\Phi} = \{\varepsilon_i, \Omega_i\}_{i=2}^I$ by minimizing the squared distance between the model-implied and empirical product expenditure shares across income deciles:

$$\hat{\boldsymbol{\Phi}} = \arg\min_{\boldsymbol{\Phi}} \ \big(\mathbf{s}(\boldsymbol{\Phi}) - \bar{\mathbf{s}}\big)^\top \big(\mathbf{s}(\boldsymbol{\Phi}) - \bar{\mathbf{s}}\big), \tag{52}$$

[67] We smooth the empirical expenditure share profiles across deciles with a locally weighted (LOESS) regression prior to calibration to limit the influence of sampling noise.

where $\mathbf{s}(\mathbf{\Phi}) = (s_{2,1}(\mathbf{\Phi}), \ldots, s_{I,1}(\mathbf{\Phi}), \ldots, s_{2,D}(\mathbf{\Phi}), \ldots, s_{I,D}(\mathbf{\Phi}))^\top$ summarizes the model-implied expenditure shares for products $i = 2, \ldots, I$ and income deciles $d = 1, \ldots, D$. The model implied moment for product $i$ and income decile $d$ is

$$s_{i,d}(\mathbf{\Phi}) = \frac{\Omega_i P_i^{1-\sigma} \mathbb{C}_d(\mathbf{\Phi})^{\epsilon_i}}{\sum_{k=1}^{I} \Omega_k P_k^{1-\sigma} \mathbb{C}_d(\mathbf{\Phi})^{\epsilon_k}} \tag{53}$$

where $P_{i,d}$ denotes the price index of product $i$, implied by the calibrated lower-tier nest. In particular,

$$P_{i,d} \equiv \frac{e_{i,d}}{c_{i,d}} = \frac{\left[\sum_{j \in \{b,g\}} \Omega_{ij} c_{i,d}^{\epsilon_{ij}} p_{ij}^{1-\sigma_i}\right]^{\frac{1}{1-\sigma_i}}}{c_{i,d}}.$$

We compute the aggregate upper-tier consumption index $\mathbb{C}_d(\mathbf{\Phi})$ from the average overall expenditure $E_d$ of each income decile $d$. For each candidate parameter vector $\mathbf{\Phi}$, the consumption index $\mathbb{C}_d(\mathbf{\Phi})$ is defined implicitly by the upper-tier expenditure function

$$E_d = E(\mathbb{C}_d(\mathbf{\Phi})) = \left[\sum_{k=1}^{I} \Omega_k P_k^{1-\sigma} \mathbb{C}_d(\mathbf{\Phi})^{\varepsilon_k}\right]^{\frac{1}{1-\sigma}}.$$

Substituting the resulting $\mathbb{C}_d(\mathbf{\Phi})$ into (53) yields the model-implied product expenditure shares $s_{i,d}(\mathbf{\Phi})$. The empirical counterpart is $\bar{\mathbf{s}} = (\bar{s}_{2,1}, \ldots, \bar{s}_{I,1}, \ldots, \bar{s}_{2,D}, \ldots, \bar{s}_{I,D})^\top$, where $\bar{s}_{i,d}$ is the survey-weighted expenditure share of product category $i$ in total expenditure among tax units in income decile $d$. We solve (52) by nonlinear least squares.

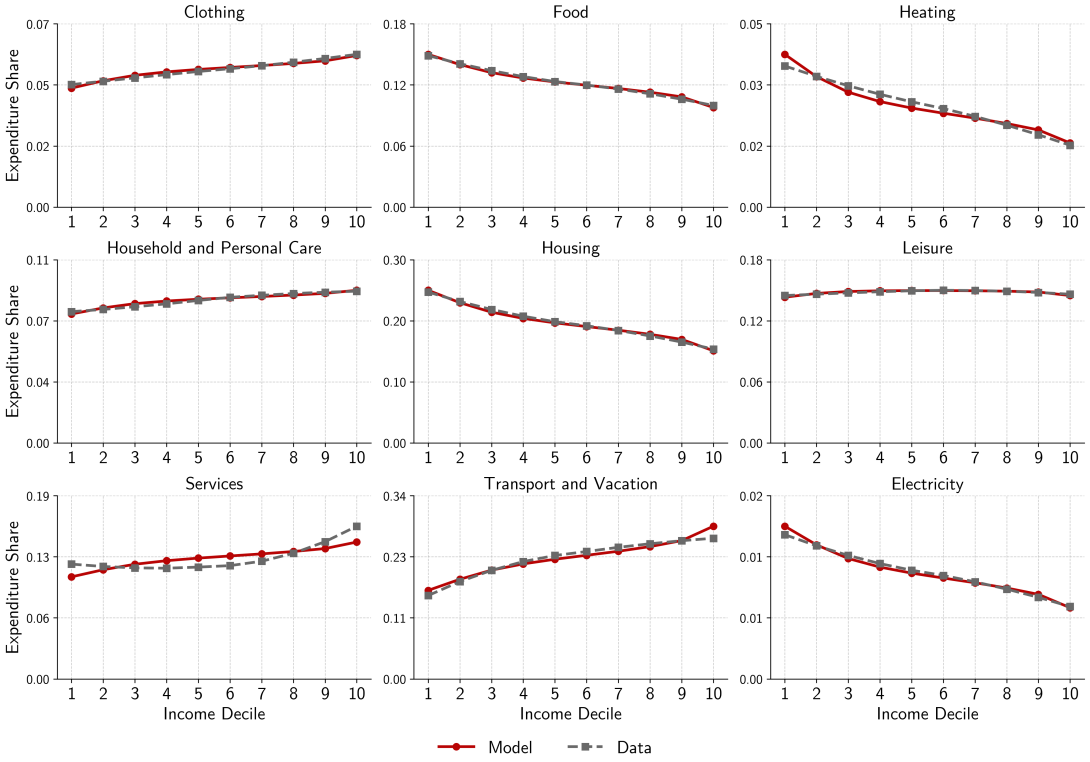
**Estimated parameters.** Table 8 reports the calibrated parameters of the lower-tier and upper-tier utility.

| **Product** | **Lower-Tier Utility** | | **Upper-Tier Utility** | |
|---|---|---|---|---|
| | $\Omega_{ig}$ | $\epsilon_{ig}$ | $\Omega_i$ | $\epsilon_i$ |
| Clothing | 0.013 | -0.983 | 1.000 | 1.000 |
| Food | 1.604 | -1.022 | 6.016 | 0.620 |
| Heating | 9736.949 | -1.830 | 32.290 | 0.113 |
| Household and Personal Care | 0.122 | -0.940 | 1.604 | 0.967 |
| Housing | 48.742 | -1.332 | 23.555 | 0.428 |
| Leisure | 15.161 | -1.713 | 4.523 | 0.691 |
| Services | 9999.515 | -1.868 | 29.722 | 0.596 |
| Transport and Vacation | 1.701 | -0.925 | 4.321 | 1.225 |
| Electricity | – | – | 2.555 | 1.081 |

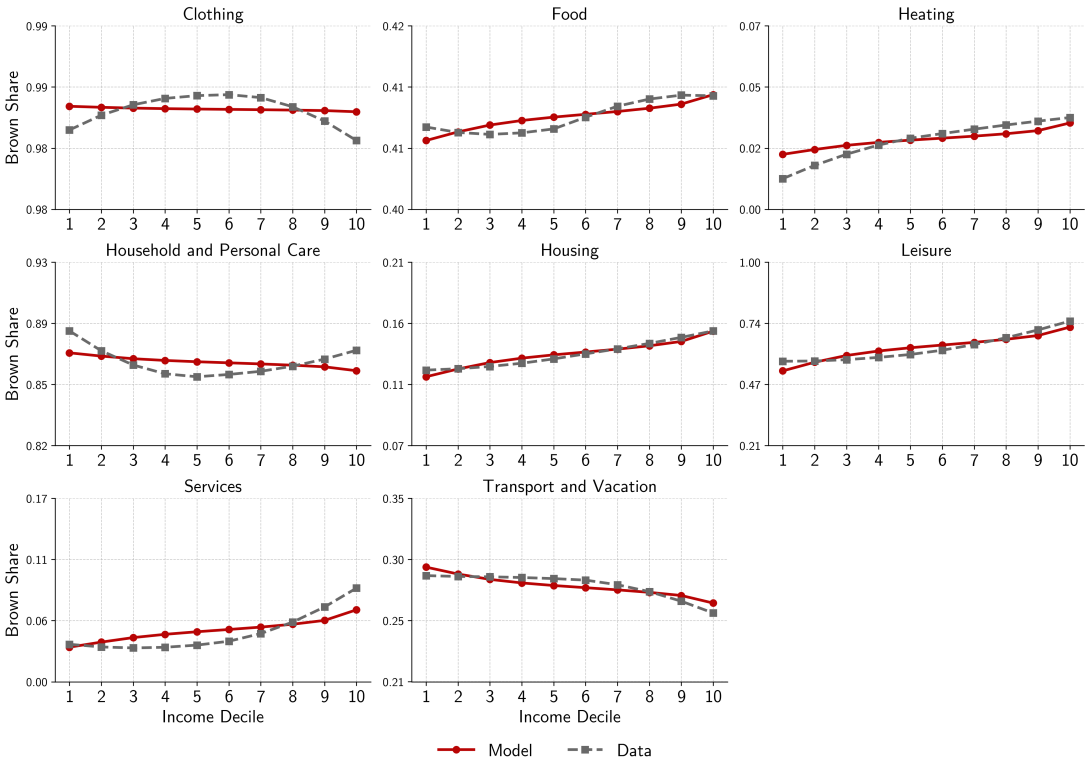
Table 8: Internally Calibrated Parameters

*Notes:* The table reports the internally calibrated parameters of the utility function. The moments of parameters of the lower-tier utility function are the within-good brown expenditure shares and of the upper-tier utility function are the good-specific expenditure shares. The lower-tier CES weight and non-homotheticity parameter of electricity is normalized to one and minus one, respectively. The upper-tier CES weight and non-homotheticiy parameter of clothing is normalized to one.

**Model fit.** Figure 16(a) illustrates the expenditure shares of each of the nine products and Figure 16(b) shows the expenditure share of the brown variety within the corresponding category across deciles of the gross labor income distribution. The solid red lines represent the results from the model, while the dashed grey lines are the corresponding empirical moments from the EVS. The model captures both the overall levels and the distributional patterns of expenditures across income goods well.




(a) Expenditure Share (Products)



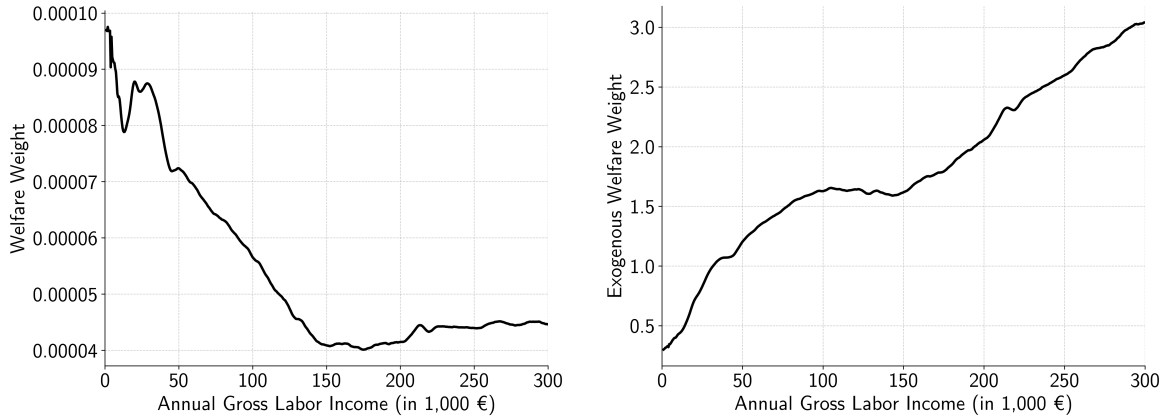

(b) Expenditure Share (Varieties)

Figure 16: Model Fit

*Notes:* The upper panel shows the model implied expenditure share for each of the nine products and the lower panel shows the expenditure share on the brown variety within the corresponding product category (solid line). This is compared to the dashed line which shows the expenditure shares in the EVS data (dashed line). The x-axis shows the decile of the annual gross labor income distribution.

## E.5 Welfare Weights

Figure 17(a) shows the calibrated social marginal utility and Figure 17(b) shows the Pareto weights along the gross labor income distribution.




(a) Social Marginal Utility (b) Exogenous Welfare Weights

Figure 17: Calibrated Welfare Weights

*Notes:* The left panel shows the social marginal utility $g(\theta, \mathcal{P}_{\text{sq}})$ and the right panel shows the Pareto weights $\omega(\theta)$ as a function of the percentile of the gross labor income distribution.

## E.6 Calibration of Emission Reduction Targets

This appendix describes how we map the 2030 emission targets under the German Federal Climate Change Act (*Bundes-Klimaschutzgesetz*, KSG), the EU Effort Sharing Regulation (ESR), and the EU's second emissions trading system for buildings and road transport (ETS2) into reductions relative to the model's 2018 baseline.[68]

For policy $j$, let $q_j$ denote the target emissions level as a share of emissions in its statutory base year $b_j$, and let $E_t^j$ denote the corresponding official emissions measure. The target emissions level relative to 2018 is then

$$\widetilde{\kappa}_j = q_j \frac{E_{b_j}^j}{E_{2018}^j},$$

implying a reduction of $1 - \widetilde{\kappa}_j$ relative to 2018. Table 9 reports the resulting mappings.

[68] Our imposes caps on the aggregate carbon footprint generated by household consumption, whereas the statutory targets are defined in terms of territorial emissions or selected territorial source categories. The calculations should therefore be interpreted as proportional benchmarks rather than as exact equivalents to the model's emission cap.

| Policy target | Baseline year | Target year | Implied 2018 emissions level | Reduction |
|---|---|---|---|---|
| KSG | 1990 | 2030 | $\frac{0.35 \times 1252}{852.9} \simeq 0.51$ | 49% |
| ESR | 2005 | 2030 | $\frac{0.50 \times 477.8}{434.0} \simeq 0.55$ | 45% |
| ETS2 | 2005 | 2030 | $\frac{0.58 \times 344.5}{271.9} \simeq 0.73$ | 27% |

Table 9: Mapping 2030 Emission Targets to the 2018 Baseline

*Notes:* KSG and ESR emissions are measured in $MtCO_2e$. Because ETS2 covers $CO_2$ emissions from fuel combustion in selected sectors, its mapping uses German $CO_2$ emissions from road transport and residential and commercial fuel combustion as an approximate measure of ETS2-covered emissions.

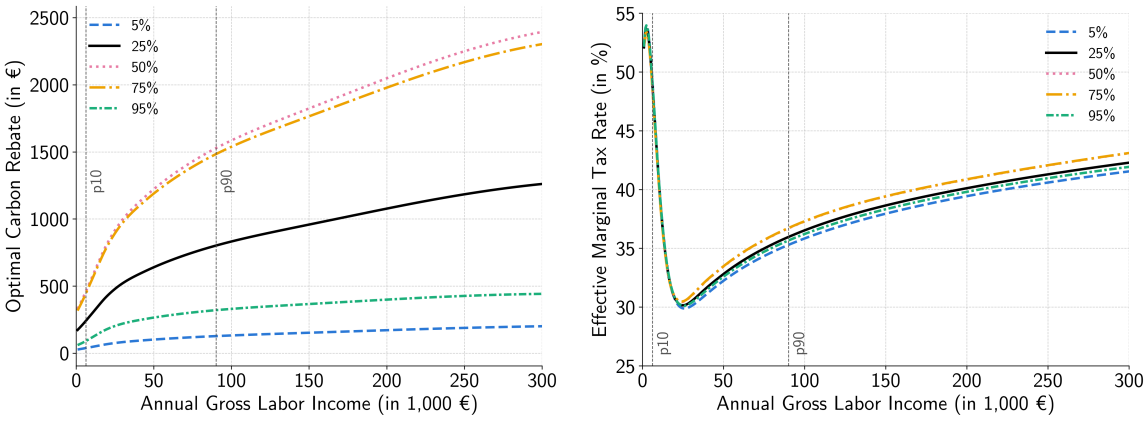
# F Quantitative Results

| | $\bar{\tau}_{\text{eff}}(y_{10})$ | $\bar{\tau}_{\text{eff}}(y_{90})$ | $\Pi_{90/10}$ |
|---|---|---|---|
| Status quo | −9.99 | 30.06 | 40.06 |
| Optimal carbon policy | −11.44 | 30.53 | 41.97 |
| Carbon dividend | −16.52 | 30.85 | 47.37 |
| Optimal carbon policy − status quo | −1.45 | 0.47 | 1.91 |
| Carbon dividend − status quo | −6.53 | 0.78 | 7.31 |

Table 10: Change in Redistribution

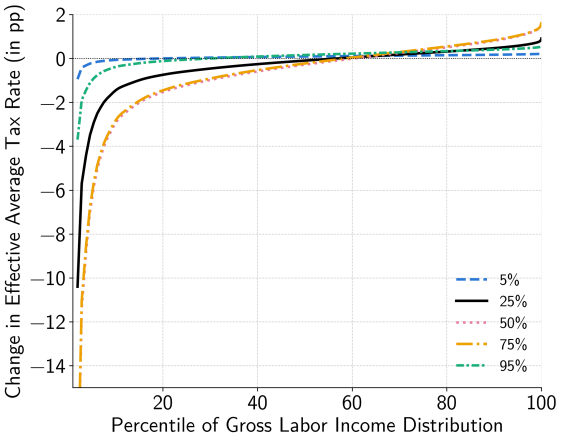
*Notes:* The table reports the effective average tax rate (EATR) at the 10th percentile, $\bar{\tau}_{\text{eff}}(y_{10})$, and the 90th percentile, $\bar{\tau}_{\text{eff}}(y_{90})$, of the gross labor income distribution, defined as $\bar{\tau}_{\text{eff}}(\theta) = \left(\mathcal{T}(y(\theta)) + \tau_{co_2} f(\theta)\right)/y(\theta)$. The EATR gap is defined as$\Pi_{90/10} = \bar{\tau}_{\text{eff}}(y_{90}) - \bar{\tau}_{\text{eff}}(y_{10})$. The final two rows report the changes relative to the status quo. EATRs are reported in percent; differences are reported in percentage points.

## F.1 Along the Abatement Path




(a) Optimal Carbon Rebate

(b) Effective Marginal Tax Rates

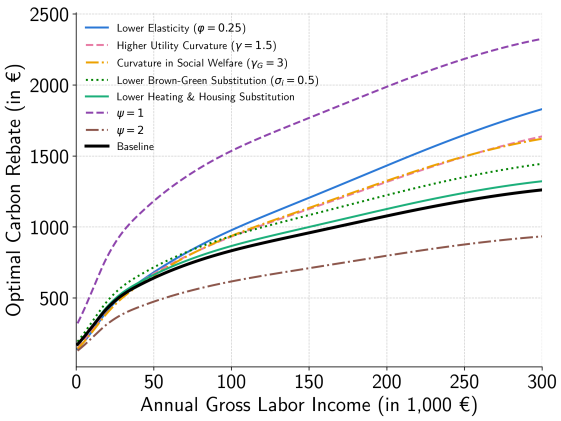



(c) Net Burden of Carbon Policy

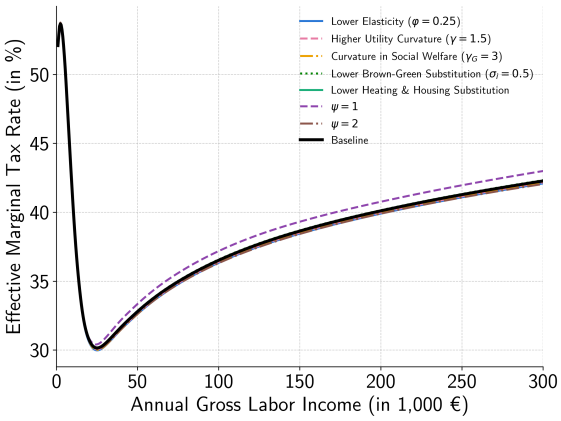
Figure 18: Different Emission Reduction Targets

*Notes:* The upper left panel plots the optimal carbon rebate and the upper right panel shows the effective marginal tax rate $\tau_{eff}$, both as functions of gross labor income. The lower panel shows the change in the effective average tax rate $\bar{\tau}_{eff}$ across percentiles of the gross labor income distribution, defined as $\bar{\tau}_{\text{eff}}(\theta) = \left(\mathcal{T}(y(\theta)) + \tau_{co_2} f(\theta)\right)/y(\theta)$. Each curve corresponds to an aggregate emissions-reduction target of 5%, 25%, 50%, 75%, or 95% relative to the model's 2018 baseline emissions. The grey vertical dashed-dotted lines mark to the 10th and 90th percentile of the annual gross labor income distribution.

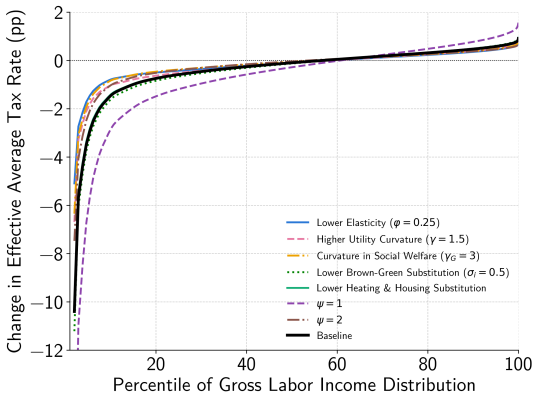
## F.2 Robustness




(a) Optimal Carbon Rebate




(b) Effective Marginal Tax Rates




(c) Net Burden of Carbon Policy

Figure 19: Robustness Preference Parameters

*Notes:* The upper left panel plots the optimal carbon rebate and the upper right panel shows the effective marginal tax rate $\tau_{eff}$, both as functions of gross labor income. The lower panel shows the change in the effective average tax rate $\bar{\tau}_{eff}$ across percentiles of the gross labor income distribution, defined as $\bar{\tau}_{\text{eff}}(\theta) = (\mathcal{T}(y(\theta)) + \tau_{co_2} f(\theta))/y(\theta)$. Each curve corresponds to a different robustness check. The grey vertical dashed-dotted lines mark to the 10th and 90th percentile of the annual gross labor income distribution.

### F.2.1 Persistence of Heating Choices

We use the panel component of the RWI German Residential Energy Consumption Survey (GRECS), linking households across the 2006–08, 2008–11, and 2011–13 survey waves; see RWI and forsa (2015). To separate changes in heating systems from changes associated with relocation, we restrict the sample to households that did not move between observations.

We record a switch when a household reports oil, gas, coal or wood, or stove heating in the earlier wave and district heating or a heat pump in the later wave. The transition rate is 0.84% between 2006–08 and 2008–11 and 0.95% between 2008–11 and 2011–13. Over the approxi-

mately five-year interval from 2006–08 to 2011–13, it is only 1.71%.[69] These low transition rates indicate substantial persistence in residential heating choices over the short to medium run. They motivate, but do not identify, a lower substitution elasticity for heating.

[69]Because the 2011–13 wave does not allow to identify moves in 2009–10, the five-year transition rate is, if anything, an upper bound for households that did not move.