No Regret Fiscal Reforms

Pierre-Edouard Collignon*

Université Laval, Québec, Canada

March 26, 2026

Click here for most recent version

Click here for Python code (Ramsey)

Abstract

This paper studies time inconsistency in fiscal policy. If the government can commit to delivering the promised rewards only to the extent necessary to ensure that households do not regret their past decisions (rather than fully committing to a sequence of future policies or to strict promise-keeping), then the second-best policy remains implementable. When commitment is restricted to non-contingent rewards, simulations of a Ramsey model and of a Mirrlees model with heterogeneous wealth and skills show that shock-coping performance (measured in welfare) nearly matches that of second-best policies in response to public spending shocks, but is weaker for shocks to redistributive preferences. This welfare differential, the No-Regret score, provides a unified metric for quantifying the role of contingent rewards in the government's response to shocks. It applies to a broad class of models featuring time inconsistency and shocks, and can help guide real-world policies.

Keywords: Time inconsistency, Fiscal policy, Ramsey model, Mirrlees approach.
JEL Classification: E61, E62, H21, H63

*Contact: pierre-edouard.collignon.1@ulaval.ca - I am grateful to Facundo Piguillem and Jean-Baptiste Michau for all our helpful discussions. I thank the audience at CREST seminars, Paris Taxation Workshop, AFSE 2021, IIPF 2021, ADRES 2022, T2M 2022, NTA 2023, CPEG 2025, CIRANO. This research was supported by a grant of the French National Research Agency (ANR), "Investissements d'Avenir" (LabEx Ecodec/ANR-11-LABX-0047)

1 Introduction

Many economic models exhibit time-inconsistent public policies, where the *present* government and some *future* governments hold differing views on what constitutes optimal public policy at a given future time. If future governments have full discretion, they might implement policies that the present government deems undesirable. Consequently, the present government may benefit from restrictions on future governments' discretion – whether by fully committing to future policies, imposing legally binding fiscal rules, or delegating decision-making authority to an independent body. Such restrictions may come at a cost: future governments have less discretion to adapt policies to shocks. Overall, reducing future governments' discretion may reduce the welfare losses due to time inconsistencies, but, with reduced discretion, future governments may be less able to effectively cope with shocks. This is the standard trade-off between rule and discretion.

This paper focuses on time-inconsistency steaming from the temptation to renege on past promises, in the spirit of Kydland and Prescott (1977) and Lucas Jr and Stokey (1983). When anticipated *future* policies influence *present* households' decisions, the present government takes this into account when designing the *ex-ante* optimal future policies. In contrast, future governments (rightfully) treat past agents' decisions as sunk, and implement policies without aiming to reward (or punish) agents' past decisions. Thus, without restrictions to future governments' discretion, the *present* government cannot credibly provide agents with rewards (or off-path punishments) *in the future*. Such a government must rely solely on rewards (or punishments) *in the present*, leading to a welfare loss when future rewards are 'cheaper' to provide than present ones.[1] This paper is built on a key observation: the ability to credibly incentivize *current* agents' decisions with rewards (or punishments) delivered *in the future* is sufficient to fully address time-inconsistency issues. The following three approaches achieve this and can implement the Second-Best policy and allocation. First, the full commitment: the initial government commit to the full sequence of future policies. Second, the promise-keeping rule: successive governments can commit to exactly delivering the rewards and punishments promised by past governments, thereby retaining some discretion for themselves. Third, the No Regret rule: successive governments commit to delivering the rewards and

[1]The fact that future policies influence current household decisions is a necessary but not sufficient condition for time-inconsistent policies. For example, in the Ramsey model with lump-sum taxes, the first-best allocation can still be achieved under full discretion.

punishments promised by past governments only to the extent necessary to ensure that households do not regret their past actions, thereby granting weakly greater discretion to the government. There are two reasons to prefer the No Regret rule over the promise-keeping rule. First, the No Regret rule allows for strictly greater discretion in both the following introductory child model and the Mirrlees model, while delivering the same level of discretion in the Ramsey model. Second, in many settings, rewards (and even more off-equilibrium punishments) are multi-dimensional and identifying the rewards and punishments variables that are necessary to uphold to address time-inconsistency is a challenge. E.g. in the Ramsey model one reward variable is enough: the current value of wealth. I find the No Regret rule less ambiguous and thus more generally applicable.

To illustrate the main idea of this paper, consider the following child-parent model. A parent would like his child to work hard and get good grades. Before the semester starts, the optimal policy for the parent is to incentivize effort by promising a toy car against good grades. After the exams, the child effort will be sunk, and it will be optimal for the parent to save time and money by not buying the toy car. This parental policy is time-inconsistent: the parent and his *future* self hold differing views on what constitutes the optimal reward policy after the exams. If the parent is committed to reward good grades with a toy car, then the child studies hard, gets good grades because he knows he will be rewarded. Without commitment, the child (rightfully) anticipates never getting a toy car, does not study and does not get good grades, resulting in lower welfare for the parent who values good grades above all. Importantly, to avoid this parental welfare loss, it is sufficient for the parent to commit to the *incentives* associated with the reward rather than to the *exact nature* of the reward. If the parent commits to rewarding good grades but retains the discretion to replace the toy car with any other toy that is sufficiently desired to induce effort, the child will still exert effort and achieve good grades in equilibrium. The discretion offered by this looser commitment does not entail any welfare losses due to time-inconsistency, but it allows the parent to adapt to shocks, for instance, by taking advantage of a temporary reduction in the price of toy planes. This change of toys does not make the child regret his effort and exemplifies the key object of this paper: No Regret reforms. However, not all shocks are equally well managed by such reforms. For instance, if all toy prices move together (and remain equal), then No Regret reforms are ineffective in coping with these shocks.

I now turn to the paper's main contribution: a novel fiscal rule – the No-Regret fiscal rule – which constrains governments to implement only No-Regret reforms. Here is how the No Regret rule works. At each point in time, each successive government under the No Regret rule inherits a *status quo* policy plan from the last government. It may choose to reform the *status quo* policy plan in response to economic shocks. This *reformed* policy plan then becomes the *status quo* policy plan of future governments, until it gets itself reformed. Most importantly, a reform of the *status quo* fiscal plan is allowed by the No Regret rule when it does not cause households to regret the decisions they made prior to the reform. A reform causes a household to regret a past decision if the household's past self would have chosen differently had it known about the reform in advance. In other words, under the No Regret rule, agents' optimal past decisions under the *status quo* policy plan and agents' optimal past decisions under the *reformed* policy plan must be identical. Under the No Regret rule, successive governments are required to uphold promises of rewards (or punishments) embedded in the inherited *status quo* policy plans announced by past governments, to the extent necessary to prevent regret. Beyond this No Regret constraint, current governments retain full discretion to respond to shocks. I formally define No Regret reforms and show that successive governments under the No Regret rule implement the Second-Best policy in two very different environments: a model *à la* Ramsey (1927) (Definition 2 and Proposition 1), and a model *à la* Mirrlees (1971) (Definition 3 and Proposition 3). These two results establish that it is not necessary to commit to the full sequence of future contingent policies to implement Second-Best policies and allocations, thereby making them more realistic – and thus more useful – benchmarks.

Under the No Regret rule, successive governments are required to sufficiently uphold promises of rewards (or punishments) conveyed in the inherited *status quo* policy plans. Such inherited policy plans and such promises of rewards may be contingent, which may be viewed as unrealistic. I therefore run simulations of the Ramsey model and the Mirrlees model in which governments under the No-Regret rule are restricted to using non-contingent policies. I study how they cope with shocks using No Regret reforms and compare the resulting policies and allocations to the Second-Best benchmark in the Ramsey model and in the Mirrlees model.

I first study a benevolent government in a closed economy with a representative agent and linear taxation *à la* Ramsey (1927). The representative agent consumes, provides labor, and owns capital and sovereign debt. The benevo-

lent government issues debt and sets tax rates on capital, outstanding sovereign debt and labor to fund a stream of endogenous public expenditures. I consider shocks to the government's preferences for public spending. I provide numerical illustrations, calibrated using standard parameter values, showing that, without contingent policies, optimal No Regret reforms achieve 99.6% of the shock-coping performance of the optimal contingent (Second-Best) policy. I refer to this figure as the No Regret score of the model. Specifically, the Second-Best policy delivers a welfare gain (in consumption equivalent) of 3.766% over the *non-contingent* optimal policy plan, while the optimal No Regret reforms achieve a welfare gain of 3.751% without relying on contingent policies. In this Ramsey model, without contingent policies the discretion allowed by the No Regret rule is insufficient to achieve the Second-Best welfare level because contingent taxes on capital and bonds, which enable perfect smoothing of tax distortions across different shock realizations in the Second-Best policy, generate regrets and thus cannot be replicated under the No Regret rule.

I then study a two-period model with a government that relies on a mechanism-design approach *à la* Mirrlees (1971) to finance public expenditures and to redistribute across households with both initial wealth heterogeneity and persistent skill heterogeneity. This model exhibits two time-inconsistency issues: the temptation to fully redistribute initial wealth revealed in the first period, and the temptation to use first-period labor supplies to infer second-period skills so as to make high-skill agents work harder without any consumption reward.

Within the Mirrlees framework, I study three distinct types of shocks. First, I study shocks to the government's preferences for public spending, and the main simulation result is that the No Regret Score is 97% – similar to the outcome observed in the Ramsey model. In other words, optimal No Regret reforms achieve 97% of the shock-coping performance of the optimal contingent (Second-Best) mechanism.

Second, I study shocks to the government's preferences for redistribution *along the skill dimension*. The main simulation result is that, in contrast to shocks on public spending, the No Regret Score is low: optimal No Regret reforms achieve only a small fraction – just a few percent – of the shock-coping performance of the Second-Best mechanism. Privately-observed skills give rise to information rents that increase with skill level. Without making agents regret revealing their skills in the first period, the government can change *ex-post* the shape of the information rent to cope with shocks to its redistribution preferences. However, most of the shock-coping achieved by the Second-Best mech-

anism is done with *contingent* information rents. Contingent rents are particularly useful to cope with shocks to redistribution preferences: high-skill agents receive low rents when the government wants to redistribute towards the low-skill, otherwise they receive high rents. Contingent rents cannot be replicated under the No Regret rule because they make agents have regrets.

This reasoning applies also to the third type of shocks I study: shocks to the government's preferences for redistribution *along the wealth dimension*. Then the No Regret Score is even lower: optimal No Regret reforms achieve less than 1% of the shock-coping performance of the Second-Best mechanism – and this figure even drops to zero in the absence of skill heterogeneity. The intuition is that the information rent associated with privately-observed initial wealth cannot be changed *ex-post* without making agents regret revealing their initial wealth. Indeed, when a No Regret reform is implemented, households with identical skills but different initial wealth should all see their expected utility levels shifted by *exactly* the same utility amount (Lemma 4). Thus, the No Regret rule prevents *any* redistribution across households that differ in wealth but share an identical skill level. In other words, No Regret reforms abide by the Equal Sacrifice principle within each skill group. This implies that the discretion granted under the No Regret rule is largely ineffective for coping with shocks to the government's redistributive preferences along the wealth dimension. Redistribution can occur only indirectly – and only to a limited extent – through redistribution along the skill dimension.

Related Literature

This paper studies the optimal fiscal policies. Several approaches have been considered in the literature.

A common approach is to allow the initial government to commit to a contingent policy so that the government can implement the *ex-ante* optimal contingent (Second-Best) policy (Chari, Christiano, & Kehoe, 1994; Lucas Jr & Stokey, 1983; Zhu, 1992). Some researchers acknowledge that optimal contingent policies have unrealistic features and make more realistic assumptions. Aiyagari, Marcet, Sargent, and Seppälä (2002) restrict government borrowing to one-period non-contingent bonds. Farhi (2010) adds capital and also restricts capital taxes to be known one period in advance and so cannot be used to cope with future shocks. In a model without capital, Bhandari, Evans, Golosov, and Sargent (2017) allow for more securities and derive the optimal policies using second-

order approximations. This paper has incomplete markets and capital taxes cannot provide full insurance when the government is under the No Regret rule and cannot use contingent policies.

The opposite approach is full discretion: the government cannot commit to any policy in the future. Martin (2010) finds that private capital is heavily taxed.[2] In a different setup with a government taxing capital owners to redistribute towards workers, Krusell (2002) finds that the capital is *not* heavily taxed because neither workers nor the government can hold capital, so wealthy capital owners are necessary. In an economy without capital nor contingent bonds, Debortoli, Nunes, and Yared (2017) study the time-inconsistency problem associated with bond price manipulation. They show that flat maturity structures are used by the government because they are successful in mitigating this time-inconsistency problem. Flat maturity structures, however, fail to provide insurance against shocks. Similarly, the No Regret rule addresses time inconsistency problems but fails to provide insurance, the government can only deal with shocks by using discretion.

A compromise approach is to combine limited commitment with limited discretion. In Klein and Ríos-Rull (2003), the government can commit to the next-period capital tax and has the discretion to choose the current labor tax. Barthélemy and Mengus (2024) allow the government to renege if it pays a fixed welfare cost: the higher this cost, the higher the government's commitment ability. In Martin (2010) and Domínguez and Feng (2017) the government may commit to a capital tax cap: at any period, capital taxes cannot be higher than this cap. Debortoli and Nunes (2010) study 'loose commitment': they allow governments to have full discretion and to renege at each period with a (possibly very small) positive probability. In this paper, the government never reneges. We also adopt a compromise approach: the government commits to abide by the No Regret rule but has some discretion to choose from all the available No Regret reforms. In the Ramsey model, I show that being under the No Regret rule implies respecting successive constraints on *wealth in utility terms*, as done in Chari, Nicolini, and Teles (2020). Their Proposition 5 is equivalent to my Proposition 1. They focus more on saving distortions and fiscal implementations, whereas I focus on the 'Rule vs Discretion' debate, in particular how governments use No Regret reforms to cope with shocks without contingent policies.

My paper assumes that governments have full commitment ability. Never-

[2]See also Cohen and Michel (1988).

theless, a key question in the literature is how governments can obtain (some) commitment ability. One avenue is reputation *à la* Kreps and Wilson (1982), as in Amador and Phelan (2021) on optimal sovereign default. Another is trigger strategies *à la* Chari and Kehoe (1990), where a government that reneges faces reversion to the worst equilibrium, as in Dovis (2019) on optimal sovereign default, and in Dovis and Kirpalani (2020) on local governments finance. Studying Markov perfect equilibrium in a bipartisan setting *à la* Alesina and Tabellini (1990) that feature endogenous spending bias, Piguillem and Riboni (2021) allow fiscal rules to be overridden by mutual agreement of both parties, which then act as off-equilibrium threats that limit the discretion of current governments.

Other well studied time-inconsistencies are those steaming from time-inconsistent preferences. Successive governments hold different views on optimal future policies because they have different objectives. Athey, Atkeson, and Kehoe (2005) model a present-biased benevolent government that uses surpluses and deficits to spend a fixed stream of government revenues. As is supposed here, the usefulness of public spending is stochastic but, in contrast, tax revenues are endogenous in the framework presented here.[3] They find optimal fiscal rules that restrict the amount the government may spend.

The approach *à la* Mirrlees is also used in a dynamic setting in the New Dynamic Public Finance (see Golosov, Tsyvinski, Werning, Diamond, and Judd (2006)). For simplicity, this paper features only initial heterogeneity and excludes any *ex-post* idiosyncratic shocks to skills or tastes. Similarly, Ábrahám, Koehne, and Pavoni (2016) consider unobserved wealth, while Golosov, Kocherlakota, and Tsyvinski (2003) feature unobserved financial trades at all periods. In contrast, this paper only allows for unobserved wealth at $t = 0$.

As in Savage (1951), regret is the difference between the utility level that could have been obtained had a household known the realized path of the economy in advance and the actual utility level he obtained. In the regret theory of Loomes and Sugden (1982), decision-makers choosing among two actions have a utility penalty (boost) called regret (rejoicing) when, for the realized state of the world, non-chosen actions would have yielded higher (lower) payoffs. In this setup, households' regret do not influence their decisions.

The remainder of the paper is organized as follows. Section 2 describes the Ramsey model. Section 3 presents the main contribution: No Regret fiscal reforms and the No Regret rule in the Ramsey model. Section 4 provides numer-

[3] Halac and Yared (2014) allow for persistent government taste shocks.

ical illustrations of No Regret fiscal reforms. Section 5 introduces a model *à la* Mirrlees (1971) with wealth and skill heterogeneity. Section 6 analyzes No Regret fiscal reforms in this model and provides numerical illustrations. Section 7 concludes.

2 The Ramsey model

2.1 Preferences and technology

The economic environment is a neo-classical growth model with endogenous government expenditures.[4] Time is discrete and infinite. At the beginning of any period t, the marginal usefulness of public spending is affected by a shock θ_t that is publicly observable. $\theta \equiv (\theta_t)_{t\in\mathbb{N}}$ follows a time-homogeneous, irreducible and aperiodic Markov chain that can take N different positive values and that has a non-decreasing transition probability matrix denoted M.[5] The representative household's preferences are represented by the following *ex-ante* utility function

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\ \left[\ u(C_t(\theta^t)) - v(L_t(\theta^t)) + \theta_t w(G_t(\theta^t))\ \right]\right]$$

where $C_t(\theta^t)$, $L_t(\theta^t)$ and $G_t(\theta^t)$ are consumption, labor and government expenditures at time t when shocks $\theta^t \equiv \{\theta_s\}_{s=0}^t$ realized. $\beta \in (0,1)$ is the discount factor. The functions $u(\cdot)$ and $w(\cdot)$ are differentiable, strictly increasing and strictly concave, while the function $v(\cdot)$ is differentiable, strictly increasing and strictly convex.

The economy is closed and a representative firm has a neo-classical production function $F(\cdot,\cdot)$ with constant returns to scale in capital and labor. Output can be used either for consumption, government expenditures or next-period capital. Thus, the resource constraint of the economy at period t when θ^t realized is

$$C_t(\theta^t) + G_t(\theta^t) + K_{t+1}(\theta^t) = F(K_t(\theta^{t-1}), L_t(\theta^t)) \tag{1}$$

[4] Similar to Zhu (1992) and Chari et al. (1994).

[5] Non-decreasingness means that if $\theta' \geq \theta$, then for all x, $\mathbb{P}[\theta_{t+1} \geq x \mid \theta_t = \theta'] \geq \mathbb{P}[\theta_{t+1} \geq x \mid \theta_t = \theta]$. As a consequence, a high shock today implies that high shocks are more likely at all future periods, which is undeniably bad news for public finances.

where $K_t(\theta^{t-1})$ is current-period capital and $K_{t+1}(\theta^t)$ is next-period capital.[6] $\{C, L, G, K\} \equiv \{C_t(\theta^t), L_t(\theta^t), G_t(\theta^t), K_t(\theta^{t-1})\}_{\theta^t}$ is called an *allocation*.

At each period t, the government sells at price $P_t(\theta^t)$ a single divisible one-period non-contingent (i.e. risk-free) bond with payoff $B_{t+1}(\theta^t)$, raises linear taxes on labor incomes $\tau_t^L(\theta^t)$, on capital (including returns) $\tau_t^K(\theta^t)$, and on maturing bonds $\tau_t^B(\theta^t)$ in order to finance its expenditures $G_t(\theta^t)$ and to pay back the non-contingent payoff $B_t(\theta^{t-1})$ of the maturing bond.[7] Thus, at any period t, when any shocks θ^t realized, the government's budget constraint is

$$G_t(\theta^t) + B_t(\theta^{t-1}) - P_t(\theta^t) \tag{2}$$
$$= \tau_t^L(\theta^t) L_t(\theta^t) F_L(K_t(\theta^{t-1}), L_t(\theta^t)) + \tau_t^K(\theta^t) K_t(\theta^{t-1}) F_K(K_t(\theta^{t-1}), L_t(\theta^t)) + \tau_t^B(\theta^t) B_t(\theta^{t-1})$$

$\{G, B, \tau^L, \tau^K, \tau^B\} \equiv \{G_t(\theta^t), B_t(\theta^{t-1}), \tau_t^L(\theta^t), \tau_t^K(\theta^t), \tau_t^B(\theta^t)\}_{\theta^t}$ is called a *policy plan*. All markets are perfectly competitive, si in equilibrium, at any period t, when shocks θ^t realized, net-of-tax wage is $W_t(\theta^t) \equiv (1-\tau_t^L(\theta^t)) F_L(K_t(\theta^{t-1}), L_t(\theta^t))$, and net-of-tax capital returns are denoted $R_t^K(\theta^t) \equiv (1-\tau^K(\theta)) F_K(K_t(\theta^{t-1}), L_t(\theta^t))$.[8] Perfect competition and constant returns to scale in production imply the absence of profit, so firm ownership is irrelevant.

The representative household takes prices as given and chooses consumption $C_t(\theta^t)$, labor $L_t(\theta^t)$, capital investment $K_{t+1}(\theta^t)$, and the *share* $b_t(\theta^{t-1})$ of the divisible bond to hold in order to maximize its *ex-ante* expected utility subject to the following budget constraint

$$C_t(\theta^t) + K_{t+1}(\theta^t) + P_t(\theta^t) b_t(\theta^{t-1}) \tag{3}$$
$$= R_t^K(\theta^t) K_t(\theta^{t-1}) + (1 - \tau_t^B(\theta^t)) b_{t-1}(\theta^{t-1}) B_t(\theta^{t-1}) + W_t(\theta^t) L_t(\theta^t).$$

and to an arbitrarily large negative value limit $-D$, i.e.

$$K_t + P_t(\theta^t) b_t(\theta^{t-1}) \geq -D \tag{4}$$

At any period t when θ^t realized, optimal labor effort implies

$$v'(L_t(\theta^t)) = W_t(\theta^t) u'(C_t(\theta^t)). \tag{5}$$

[6] Without loss of generality, depreciated capital is included in F.

[7] For simplicity, I assume incomplete markets, since the Second-Best allocation can be implemented without contingent bonds or taxes on bonds (see Farhi (2010)). However, such taxes on bonds are required in No Regret reforms.

[8] Capital *and* capital incomes are taxed. A tax with a rate $\frac{\tau^K F_K(K(\theta), L(\theta))}{F_K(K(\theta), L(\theta)) - 1}$ applied to capital incomes *only* is exactly equivalent.

Optimal saving in capital investment and in bonds yield the following Euler Equations

$$u'(C_t(\theta^t)) = \mathbb{E}_t \left[\beta R^K_{t+1}(\theta^{t+1}) u'(C_{t+1}(\theta^{t+1})) \right] \tag{6}$$

$$P_t(\theta^t) u'(C_t(\theta^t)) = \mathbb{E}_t \left[\beta (1 - \tau^B_{t+1}(\theta^{t+1})) B_{t+1}(\theta^t)(\theta^{t+1}) u'(C_{t+1}(\theta^{t+1})) \right] \tag{7}$$

where $\mathbb{E}_t[\,\cdot\,] \equiv \mathbb{E}[\,\cdot \mid \theta^t\,]$. I introduce an arbitrarily large sequence $(M_t)_{t\in\mathbb{N}}$ such that $M_t \xrightarrow[t\to\infty]{} 0$, and restrict attention to equilibria such that the following transversality condition holds.[9]

$$\mathbb{E}_0 \left[\beta^t u'(C_t(\theta^t)) \Big[K_{t+1}(\theta^t) + b_t(\theta^t) P_t(\theta^t) + D \Big] \right] \;\leq\; M_t \tag{8}$$

Standard proofs of the necessity of the first order conditions (5), (6), (7), and of the sufficiency of first order conditions (5), (6), (7), and the transversality condition (8) are provided in the Supplemental Appendix.

2.2 Equilibrium

Definition 1. ***(Competitive Equilibrium)***
A competitive equilibrium is composed of the household's decisions $\{C, L, K, b\}$*, a policy plan* $\{G, B, \tau^L, \tau^K, \tau^B\}$*, and prices* $\{W, R^K, P\}$ *such that:*

1. *The household chooses* $\{C, L, K, b\}$ *to maximize her ex-ante utility s.t. her budget constraints (3), while taking net-of-tax prices* $\{W, R^K, P\}$ *as given.*

2. *The firm maximizes its profit while taking net-of-tax prices* $\{W, R^K, P\}$*, and taxes* $\{\tau^L, \tau^K, \tau^B\}$ *as given. This implies* $W_t(\theta^t) = (1-\tau^L_t(\theta^t)) F_L(K_t(\theta^{t-1}), L_t(\theta^t))$ *and* $R^K_t(\theta^t) = (1 - \tau^K_t(\theta^t)) F_K(K_t(\theta^{t-1}), L_t(\theta^t))$.

3. *All bond markets clear, i.e., for all shocks* θ^t, $b_t(\theta^t) = 1$.

4. *All the resource constraints (1) are met.*[10]

An allocation $\{C, L, K, G\}$ or a policy plan $\{G, B, \tau^L, \tau^K, \tau^B\}$ that is part of an equilibrium is called *implementable* and we then say that the policy plan $\{G, B, \tau^L, \tau^K, \tau^B\}$ *implements* the allocation $\{C, L, K, G\}$ and *induces* the household's decisions $\{C, L, K, b\}$.

[9] This restriction prevents governments to delay rewards or punishments indefinitely. One could restrict primitive instead, e.g. this transversality condition is *necessary* for optimality when utility functions are bounded above.

[10] By Walras' law, the government's budget constraints (2) hold as well.

A government that can commit at time $t = 0$ to any contingent policy can implement any implementable allocation.[11]

Combining equations (5), (6), (7), and (8) with the household's budget constraint (3) yields the following equality, referred to as an *implementability constraint*, for any realization of the shocks θ^t

$$u'(C_t(\theta^t)) \left[R_t^K(\theta^t) K_t(\theta^t) + (1 - \tau_t^B(\theta^t)) B_t(\theta^t) \right] \tag{9}$$

$$= \quad \mathbb{E}_t \left[\sum_{s=t}^{\infty} \beta^{s-t} \left[u'(C_s(\theta^s)) C_s(\theta^s) - v'(L_s(\theta^s)) L_s(\theta^s) \right] \right]$$

where the left-hand side term can be interpreted as the value of the household's wealth at time t and the right-hand side term can be interpreted as the expected value of consumption rewards net of work efforts from t onward. At time $t = 0$, a limit on the taxation of initial capital and bond holdings is imposed by assuming that the household's initial net-of-tax wealth cannot be below some exogenous value IW.[12]

$$IW \quad \leq \quad R_0^K K_0 + (1 - \tau_0^B) B_0$$

which yields the *initial implementability constraint*

$$u'(C_0) IW \quad \leq \quad \mathbb{E}_0 \left[\sum_{s=0}^{\infty} \beta^s \left[u'(C_s(\theta^s)) C_s(\theta^s) - v'(L_s(\theta^s)) L_s(\theta^s) \right] \right] \tag{10}$$

This inequality is a necessary and sufficient condition for any resource-feasible allocation to be implementable, therefore an initial government that can fully commit to any contingent policy plan solves the following problem.

$$\max_{\substack{\{C_t(\theta^t), L_t(\theta^t), \\ G_t(\theta^t), K_{t+1}(\theta^t)\}_\theta}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left[u(C_t(\theta^t)) - v(L_t(\theta^t)) + \theta_t w(G_t(\theta^t)) \right] \right] \tag{11}$$

$$\text{s.t.} \quad C_t(\theta^t) + G_t(\theta^t) + K_{t+1}(\theta^t) = F(K_t(\theta^{t-1}), L_t(\theta^t)) \quad \text{for all } \theta^t$$

$$\text{and to} \quad u'(C_0) IW \leq \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left[u'(C_t(\theta^t)) C_t(\theta^t) - v'(L_t(\theta^t)) L_t(\theta^t) \right] \right]$$

[11] Such a commitment ability is sufficient but not necessary. For example, the equilibrium allocation when government has full discretion is implementable without any commitment ability.

[12] A less standard approach could have been to follow Chari et al. (2020) and to impose an minimum initial *value of wealth* (in utility terms) instead. Both approaches yield very similar results.

This problem is referred to as the Second-Best problem. Its solution is the Second-Best allocation, which is implemented by the Second-Best policy, or *ex-ante* optimal contingent policy plan.

3 The No Regret fiscal rule in the Ramsey model

3.1 Reforms and time-inconsistency

The term *fiscal reform* will be used extensively going forward. A fiscal reform at period t^R, after the realization of the shocks θ^{t_R}, takes place when the government announces a change of the *status quo* policy plan $\pi \equiv \{G, B, \tau^L, \tau^K, \tau^B\}$. This change of policy plan starts at period t^R, after the realization of θ^{t_R}. The *reformed* policy plan $\tilde{\pi} \equiv \{\tilde{G}, \tilde{B}, \tilde{\tau}^L, \tilde{\tau}^K, \tilde{\tau}^B\}$ will serve as the *status quo* policy plan in all subsequent periods $t > t_R$, until it is, in turn, reformed. Of course, a fiscal reform at period t^R, after the realization of the shocks θ^{t_R}, can change neither the policies implemented before period t^R nor the policies that would have been implemented after other realizations of the shocks $\tilde{\theta}^{t_R} \neq \theta^{t_R}$. In other words, a fiscal reform that follows the realization of θ^{t_R} changes only one 'branch' of the 'tree' π: the branch that starts at θ^{t_R}. Whether or not there is a reform at time t_R, we call the policy plan currently faced by the household the *current* policy plan.

Lemma 5 in the Supplemental Appendix shows that the Second-Best policy plan is time-inconsistent: there exists a realizations of shocks θ^t after which the government would like to reform the Second-Best policy plan.[13] Restrictions on future governments' discretion, such as fiscal rules, are therefore relevant in this environment. We now turn to the study of the No Regret fiscal rule.

3.2 The No Regret fiscal rule

When the government is fully committed to a policy plan, it cannot be reformed. In contrast, when the government has full discretion, it can reform

[13] This model exhibits several well-known time-inconsistency problems. Most notably, there is the temptation to tax capital and repudiate public debt in order to ease the tax burden on elastic labor income. In addition, as in Lucas Jr and Stokey (1983), the government may promise artificially high future taxes and public spending to stimulate current saving and sell bonds at a low price today – only to renege on this promise once the bonds have been sold.

status quo policy plans at any time without any constraint. When the government adopts the No Regret fiscal rule – the key novelty of this paper – it can reform *status quo* policy plans at all time with *some* constraints. Thus, by imposing some constraints on the government's discretion, the No Regret fiscal rule offers a middle ground between full commitment and full discretion. The fiscal reforms prohibited by the No Regret fiscal rule are the ones that, if there were unanticipated by the household, cause the household to regret the decisions he made prior to the reform.[14] Unanticipated reforms cause the household to regret his past decisions when, had he anticipated these reforms, he could have achieved a higher *ex-ante* expected utility level by making different decisions prior to the time of the reform.[15] The fiscal reforms allowed by the No Regret rule are called No Regret fiscal reforms. Here is the formal definition of a No Regret fiscal reform specific to the Ramsey model.

Definition 2. ***(No Regret fiscal reform in the Ramsey model)***
A fiscal reform at time t_R after the realization of the shocks θ^{t_R} is a No Regret fiscal reform when $\{C, L, K\}$, the household's decisions induced by the status quo policy plan π, and $\{\tilde{C}, \tilde{L}, \tilde{K}\}$, the household's decisions induced by the reformed policy plan $\tilde{\pi}$, are identical prior the reform. I.e., for all $t < t^R$,

$$C_t(\theta^t) = \tilde{C}_t(\theta^t) \quad , \quad L_t(\theta^t) = \tilde{L}_t(\theta^t) \quad \textit{and} \quad K_{t+1}(\theta^t) = \tilde{K}_{t+1}(\theta^t).$$

Lemma 1 establishes that when the government is under the No Regret fiscal rule, a dominant strategy for the household is to make decisions identical to those induced by the *current* policy plan. Not only this strategy optimal for a rational household who anticipates all reforms, butit is dominant in the sense that it is optimal regardless of the No Regret reforms that will be announced by the government. In other words, it is a dominant strategy for the household to make decision *as if* no fiscal reform will ever occur, and to (rightfully) trust the government not to implement future reforms that alter the future incentives for current actions embedded in current policy plans.

Lemma 1. ***(The household's optimal decisions under the No Regret fiscal rule)***
When the government is under the No Regret fiscal rule, a dominant strategy for the household is to make decisions identical to those induced by the current policy plan.

The proof is in the Appendix.

[14] We limit our analysis to equilibria in which households correctly anticipate all forthcoming reforms (see Definition 1).

[15] This notion of regret was introduced by Savage (1951).

Because the time-horizon is infinite, backward induction is not possible and the proof relies on the transversality condition (8) instead. Without the transversality condition (8), this strategy may not be optimal, and such a counter-example of Lemma 1 is provided in the Appendix.[16]

Under the No Regret rule, and using Lemma 1, the value V^{NR} of each successive government's problem solves

$$V^{\text{NR}}(\theta^{t_R}, K_{t_R}, \pi) = \max_{\tilde{\pi}, C_{t_R}, L_{t_R}, K_{t_R+1}} \; u(C_{t_R}) - v(L_{t_R}) + \theta_{t_R} w(G_{t_R}) + \beta \mathbb{E}_{t_R} \left[V^{\text{NR}}(\theta^{t_R+1}, K_{t_R+1}, \tilde{\pi}) \right] \tag{12}$$

s.t. $\tilde{\pi}$ is a No Regret fiscal reform of π after θ_{t^R}

and s.t. $C_{t_R}, L_{t_R}, K_{t_R+1}$ are induced by the policy plan $\tilde{\pi}$

In the Problem (12), the inherited policy plan π constrains the choices of the current government. The following Lemma establishes that this constraint effectively reduces to a single real variable.

Lemma 2. ***(Partial Characterization of No Regret reforms)***
Consider a No Regret fiscal reform at time t_R after the realization of the shocks θ^{t_R}. The household's decisions $\{C, L, K\}$ induced by the status quo policy plan π, and the household's decisions $\{\tilde{C}, \tilde{L}, \tilde{K}\}$ induced by the reformed policy plan $\tilde{\pi}$ are such that

$$\mathbb{E}\left[\sum_{t=t_R}^{\infty} \beta^t \left[u'(C_t(\theta^t)) C_t(\theta^t) - v'(L_t(\theta^t)) L_t(\theta^t) \right] \,\middle|\, \theta^{t_R} \right] \tag{13}$$

$$= \mathbb{E}\left[\sum_{t=t_R}^{\infty} \beta^t \left[u'(\tilde{C}_t(\theta^t)) \tilde{C}_t(\theta^t) - v'(\tilde{L}_t(\theta^t)) \tilde{L}_t(\theta^t) \right] \,\middle|\, \theta^{t_R} \right]$$

Proof is in the Appendix.

The two sides of Equation (13) represent the consumption rewards net of work efforts offered by the two policy plans π and $\tilde{\pi}$. By Equation (9), in equilibrium these terms are equal to the value of wealth at time t_R. Consequently, the reformed policy plan $\tilde{\pi}$ keeps the rewards for past saving efforts unchanged (conditional on the realization θ_t) and Equation (13) can therefore be interpreted as a promise-keeping constraint: rewards (and punishments) for past decisions

[16] This counter-example meets this weaker (and more standard) transversality condition that for all t, $\mathbb{E}_t \left[\beta^r u'(C_r(\theta^r)) \left[K_{r+1}(\theta^r) + b_r(\theta^r) P_r(\theta^r) + D \right] \right] \xrightarrow[r \to \infty]{} 0$, which illustrates why the strong transversality condition (8) is needed to obtain Lemma 1.

must be preserved.

In general, a reform that keeps rewards (and punishments) for past decisions unchanged is a No Regret reform. By Lemma 2, the converse holds in this Ramsey model. However, it fails in both the introductory child model and the Mirrlees model.

Using Lemma 2 and the fact that $(\theta_t)_{t\in\mathbb{N}}$ is a Markov process, the value V^{NR} of each successive government's problem can be written instead

$$
\begin{aligned}
V^{\text{NR}}(\theta, K, A) = \max_{\substack{C,L,G,K_{+1} \\ \{A_{+1}(\theta_{+1})\}_{\theta_{+1}}}} \quad & u(C) - v(L) + \theta w(G) \; + \; \beta\mathbb{E}\left[\, V^{\text{NR}}\big(\theta_{+1}, K_{+1}, A_{+1}(\theta_{+1})\big) \mid \theta \,\right] \\
\text{s.t.} \quad & C + G + K_{+1} = F(K, L) \qquad (14) \\
\text{and to} \quad & A = u'(C)C - v'(L)L + \beta\mathbb{E}\left[\, A_{+1}(\theta_{+1}) \mid \theta \,\right]
\end{aligned}
$$

We show in the Lemma 6 in the Supplemental Appendix that each admissible $\{C, L, G, K_{+1}, \{A_{+1}(\theta_{+1})\}_{\theta_{+1}}\}$ can be implemented by a No Regret reform, so the Problem (14) is not a relaxed version of Problem (12).

The recursive Problem (14) can be re-written as a sequential problem. Furthermore, at $t = 0$, the government under the No Regret rule faces no constraint except that the household's initial net-of-tax wealth must be above an exogenous value IW (see Equation (10)). We get that the value of the initial government is

$$
\begin{aligned}
&\max_{\substack{\{C_t(\theta^t), L_t(\theta^t), \\ G_t(\theta^t), K_{t+1}(\theta^t)\}_\theta}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \; \left[\, u(C_t(\theta^t)) - v(L_t(\theta^t)) + \theta_t w(G_t(\theta^t)) \,\right] \right] \qquad (15) \\
&\text{s.t.} \quad C_t(\theta^t) + G_t(\theta^t) + K_{t+1}(\theta^t) = F(K_t(\theta^{t-1}), L_t(\theta^t)) \;\; \text{for all } \theta^t \\
&\text{and to} \quad u'(C_0)IW \leq \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \; \left[\, u'(C_t(\theta^t))C_t(\theta^t) - v'(L_t(\theta^t))L_t(\theta^t) \,\right] \right]
\end{aligned}
$$

The Problem (15) solved by a government under the No Regret fiscal rule is exactly identical to the Second-Best Problem (11).

Proposition 1. ***(The No Regret fiscal rule and Second-Best)***
A government under the No Regret fiscal rule implements the Second-Best policy and allocation.

Proposition 1 establishes two results in the Ramsey model. First, the discretion available to governments under the No Regret fiscal rule does not give rise to time-inconsistency problems. Second, implementing the Second-Best allocation does not require full commitment to a contingent policy plan; it is sufficient

to commit to honoring promises of rewards embedded in the inherited status quo policy plans. Consequently, the Second-Best allocation and the associated policy are more realistic – and therefore more useful – benchmarks.

One implementation of the Second-Best policy and allocation under the No Regret rule is for the initial government to announce the Second-Best policy, with all subsequent governments choosing not to implement any No Regret reform. In this sense, fiscal policy is time-consistent under the No Regret fiscal rule.

3.3 Non-contingent No Regret fiscal reforms

Up to this point, all policies have been allowed to be contingent, which may be viewed as unrealistic. The main concerns are contractual complexity, asymmetrically observed shocks, and the presence of unforeseen contingencies. In this subsection, I therefore restrict governments to using *non-contingent* policies.

A *non-contingent* fiscal reform of the *status quo* policy plan $\pi \equiv \{G, B, \tau^L, \tau^K, \tau^B\}$ after the realization of θ^{t_R}, is when the government replaces the 'branch' of the policy plan π that starts at the node θ^{t_R} by a *non-contingent* policy plan.

After the realization of any shocks θ^{t_R}, a non-contingent No Regret reform induces non-contingent household's decisions $\{\tilde{C}_t, \tilde{L}_t, \tilde{K}_t\}_{t \geq t_R}$. As a result, for any realizations $\theta_{t_R+1} \neq \theta'_{t_R+1}$ following θ^{t_R}, Lemma 2 implies that the following *non-contingent* version of Equation (13) must hold

$$\sum_{t=t_R}^{\infty} \beta^t \left[u'(\tilde{C}_t(\theta^t))\tilde{C}_t(\theta^t) - v'(\tilde{C}_t(\theta^t))\tilde{C}_t(\theta^t) \right] \tag{16}$$

$$= \sum_{t=t_R}^{\infty} \beta^t \left[u'(\tilde{C}_t(\theta^t))\tilde{C}_t(\theta^t) - v'(\tilde{C}_t(\theta^t))\tilde{C}_t(\theta^t) \right].$$

By following the same steps that lead to Problem (15), we obtain that, when successive governments under the No Regret rule are restricted to using non-contingent policies, the value of the initial government is given by

$$\max_{\substack{\{C_t(\theta^t),L_t(\theta^t),\\ G_t(\theta^t),K_{t+1}(\theta^t)\}_\theta}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left[u(C_t(\theta^t)) - v(L_t(\theta^t)) + \theta_t w(G_t(\theta^t)) \right] \right] \tag{17}$$

$$\text{s.t.} \quad C_t(\theta^t) + G_t(\theta^t) + K_{t+1}(\theta^t) = F(K_t(\theta^{t-1}), L_t(\theta^t)) \quad \text{for all } \theta^t$$

$$\text{and to} \quad u'(C_0)IW \leq \sum_{t=0}^{\infty} \beta^t \left[u'(C_t(\theta^t))C_t(\theta^t) - v'(L_t(\theta^t))L_t(\theta^t) \right] \quad \text{for all } \{\theta^t\}_t$$

The Problem (11) of a government that can fully commit to a contingent policy plan, and the Problem (17) of a government under the No Regret rule that can only use *non-contingent* policies are very similar. The only difference is that for Problem (11) the initial implementability constraint must be met in expectation, whereas, for the Problem (17), it must be met for each realization of the shocks $(\theta_t)_t$. Clearly, the optimal solution of the Problem (17) is also an admissible solution of the Problem (11), but the reverse is not necessarily true (unless there is no shock).

4 Simulations

In this section, I turn to simulations of the Ramsey model. I study how governments restricted to using non-contingent policies cope with shocks through No Regret reforms, and compare the resulting policies and allocations both to the ex-ante optimal contingent policy and allocation (the Second-Best benchmarks) and to the ex-ante optimal non-contingent policy and allocation.

4.1 Specifications

Technology and stochastic preferences. We use a Cobb-Douglas production function with capital share α and with a depreciation rate of capital δ.

$$F(K, L) = K^\alpha L^{1-\alpha} + (1-\delta)K$$

Preferences regarding consumption, labor and government spending are iso-elastic

$$u(C) - v(L) + \theta w(G) = \frac{C^{1-\sigma} - 1}{1-\sigma} - \kappa \frac{L^{1+\psi} - 1}{1+\phi} + \theta \frac{G^{1-\xi} - 1}{1-\xi}$$

The shock process θ is modeled as a two-state Markov chain with a constant transition matrix. The two states are denoted H and L (High and Low). The

transition matrix is

TRANSITION MATRIX

$\mathbb{P}[\,\theta_{t+1} = H \mid \theta_t = H\,] \quad = p_{HH}$	$\mathbb{P}[\,\theta_{t+1} = L \mid \theta_t = H\,] \quad = 1 - p_{HH}$
$\mathbb{P}[\,\theta_{t+1} = L \mid \theta_t = L\,] \quad = p_{LL}$	$\mathbb{P}[\,\theta_{t+1} = H \mid \theta_t = L\,] \quad = 1 - p_{LL}$

4.2 Parameters

I use a yearly time period as is standard in fiscal policy. The parameters are in Table 1. The discount factor β is 0.96, the intensive Frisch elasticity $1/\psi$ is 0.5, the private relative risk aversion σ is 1, which are standard values. $\sigma = 1$ corresponds to the log utility which balances wealth and substitution effects. I choose to set the public relative risk aversion ξ to 1, equal to the private one. The capital share α in the Cobb-Douglas production function is 0.33. I set p_{HH} to $3/4$ and p_{LL} to $5/6$ so that the average length of high and low shocks are 4 and 6 years respectively. This implies that, on average, the economy is in the low state 60% of the time. The depreciation rate δ, the disutility of labor parameter κ and the *expected* marginal utility of government spending $\mathbb{E}[\theta_t]$ are respectively estimated to 6.83%, 0.74 and 0.33 in order to have a capital to output ratio equal to 3, labor equal to 1 and a government spending share of the output equal to 20% in a steady-state, unshocked economy. The standard deviation of the shocks θ is estimated so that the standard deviation of the government spending is 10% of GDP.

CALIBRATED PARAMETERS		
Discount factor β	0.96	-
Intensive Frisch elasticity $1/\psi$	0.5	Chetty, Guren, Manoli, and Weber (2011)
Private relative risk aversion σ	1	Calvet, Campbell, Gomes, and Sodini (2021)
Public relative risk aversion ξ	1	Identical to σ
Capital share α	0.33	-
p_{HH}	$3/4$	4 year expected length for High
p_{LL}	$5/6$	6 year expected length for Low
ESTIMATED PARAMETERS		
δ	6.83 %	K/Y=3
κ	0.74	Normalization $L_{SS} = 1$
$\mathbb{E}[\theta_t]$	0.33	Gov spending unshocked SS = 20 % of GDP
$\sigma[\theta_t]/\mathbb{E}[\theta_t]$	0.5	σ(Gov spending) $\approx$ 10 % of GDP

Table 1: Parameters

4.3 Optimal non-contingent No Regret reforms

The main result of numerical illustrations with standard parameters is that No Regret reforms achieve 99.6% of the shock-coping performance of the optimal contingent policy (Second-Best). I call this number the *No Regret score* of the model. More precisely, a government under the No Regret fiscal rule that can only announce non-contingent policy plans achieves a 3.751% consumption-equivalent welfare gain over the optimal non-contingent policy plan. In comparison, the Second-Best policy achieves a slightly higher 3.766% welfare gain. In this economy, the discretion allowed by the No Regret fiscal rule allows the government to cope with public spending shocks almost as well as the Second-Best policy. I now briefly describe the optimal policy implemented by a government under the No Regret rule that is restricted to using non-contingent policies.

Sample path. Figures 1, 2, 3, 4, 5, 6, 7, and 8 illustrate the realized policy for one sampled path, *i.e.* one realization of the shock process θ. The economy starts with the capital of the unshocked steady state. The initial shocks are low (*i.e.* $\theta_t = L$) for 6 periods which is the expected length of low shocks. Subsequent shocks are high for 4 periods (grey zone in the figures) which is the expected length of high shocks. Finally, shocks are low until the end of time.

Note that the government does not know the realizations of future shocks at any time.

Government expenditures. As displayed in Figure 1, a government under the No Regret rule has sufficient discretion to spend more (less) when preferences for public spending are high (low). The Second-Best spending is very close.

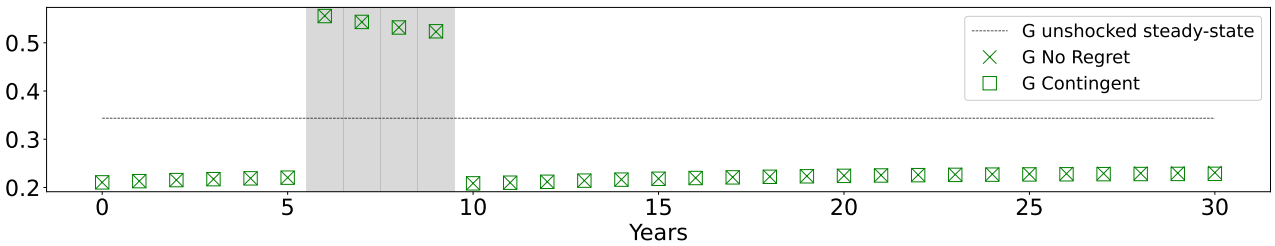


Figure 1: Public spending

Consumption and Labor. As displayed in Figures 2 and 3, a government under the No Regret rule has sufficient discretion to induce higher work effort and less private consumption when public spending is high. Compared to the Second-Best allocation, the higher consumption is explained by higher capital stock and the higher labor effort is explained by lower labor tax.

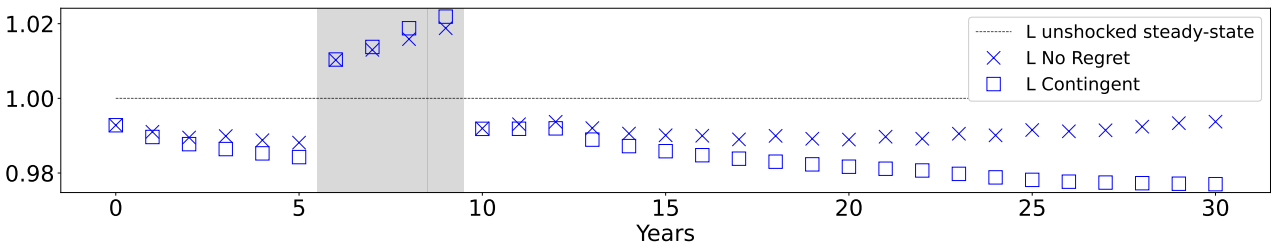


Figure 2: Labor

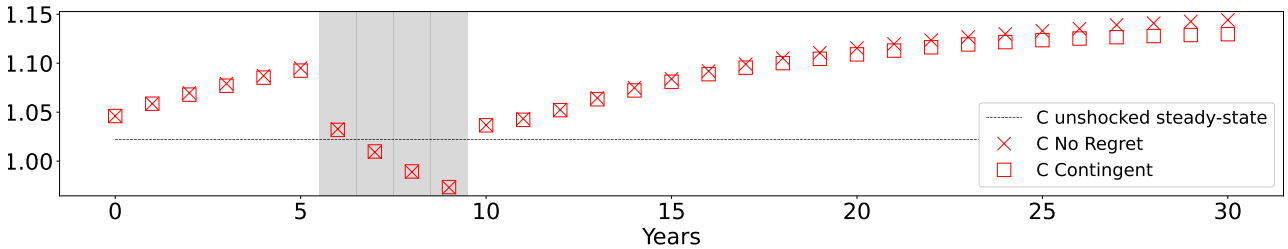


Figure 3: Consumption

Labor tax. The main tool used by the government to raise revenues is the labor tax. Shocks are persistent, so when the shock θ goes from low to high, the government learns that *expected* public spending will be high in the near future. As a consequence, the government anticipates these high future expenditures and, as displayed in Figure 4, raises the labor tax significantly in order to smooth the labor tax distortion. The labor tax also increases when the shock stays high but less dramatically. The size of the change in labor tax rate is related to the 'size of the news': because shocks are persistent, tax changes are more dramatic when the shock goes from low to high or from high to low compared to when it stays high or stays low. Under the No Regret rule, the tax rate on labor is rather smooth (the maximum change in tax rate is when the shock goes from low to high or from high to low) because the government uses debt to smooth tax distortions. In contrast, the labor tax under the Second-Best policy is constant across periods and across realizations of shocks.[17]

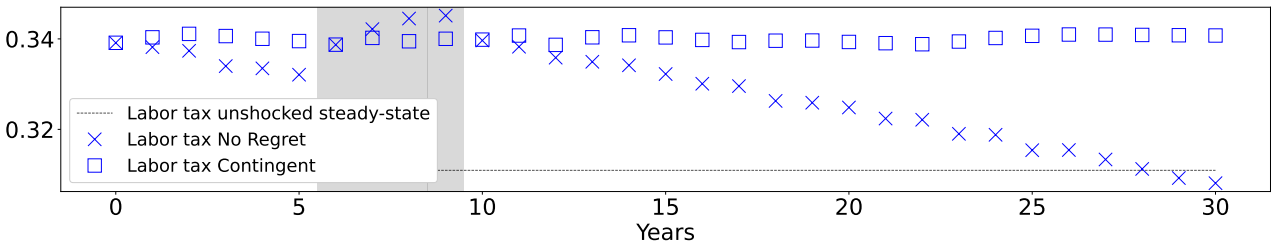


Figure 4: Labor tax

Capital. As displayed in Figure 5, a government under the No Regret rule uses the capital stock as a buffer. Compared to the Second-Best allocation, since labor becomes higher (because it is less taxed), the marginal product of capital becomes higher, which commands higher capital stock.

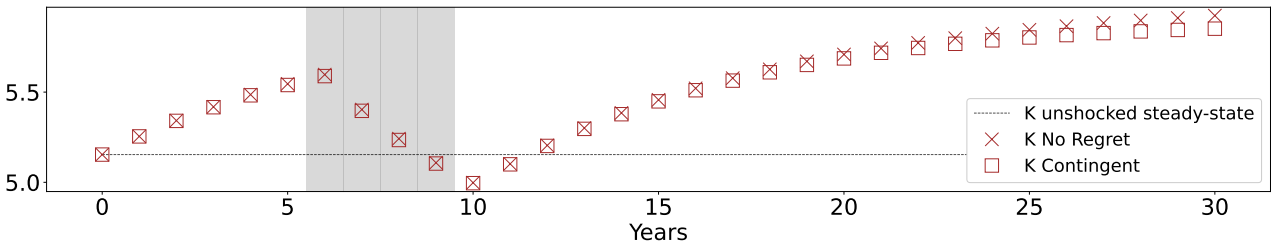


Figure 5: Capital

[17] With iso-elastic preferences, theory predicts that the Second-Best labor tax is constant across time and shocks.

Public debt. As shown in Figure 6, when shocks remain low, after-tax public debt declines, whereas it increases when shocks remain high. However, when the shock switches from low to high, after-tax public debt decreases. This occurs because consumption falls, prompting the government to raise taxes on capital and debt in order to prevent households from experiencing regret. Symmetrically, when the shock shifts from a high to a low state, the government issues a substantial amount of debt to finance large one-off subsidies on capital and bonds. Put simply, public debt level is low (high) when public spending is high (low), and the same holds for interest payments. Consequently, taxes on capital and debt play an insurance role, allowing a government under the No-Regret rule to smooth labor tax distortions across shocks. This insurance role is even more pronounced in the Second-Best allocation: after-tax public debt responds more strongly when the shock switches states and varies less when it remains unchanged.

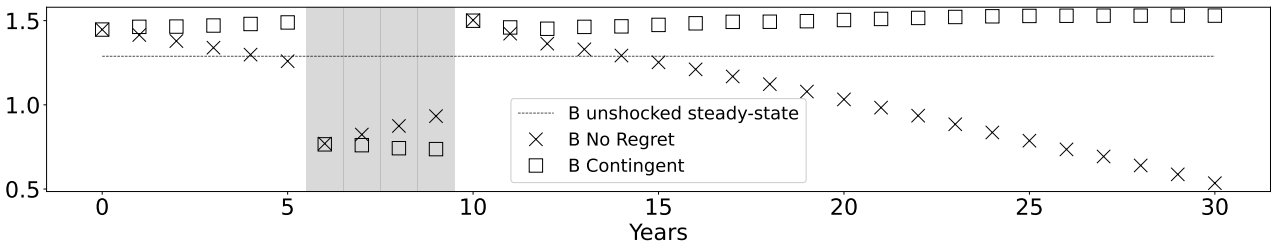


Figure 6: After-tax public debt

Debt and capital taxes. Figures 7 and 8 show that taxes on capital and bonds indeed play a more important insurance role under the Second-Best policy .[18]

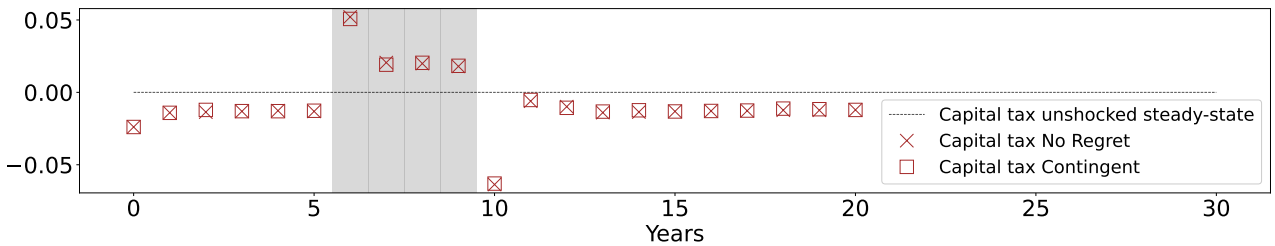


Figure 7: Capital tax

[18] As shown by Zhu (1992), the Second-Best policy exhibits fiscal indeterminacy, so I pin down the Second-Best capital tax rate to avoid households' regret.

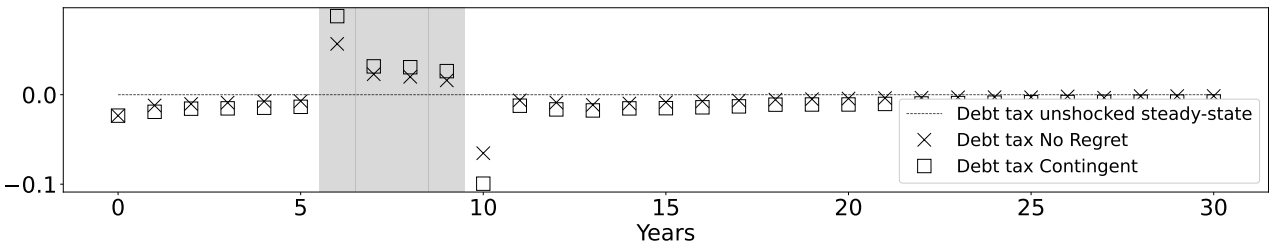


Figure 8: Debt tax

To summarize, there are four channels through which governments can cope with public spending shocks: (i) using the capital stock as a buffer; (ii) increasing labor taxes when government spending rises; (iii) issuing public debt to smooth tax distortions over time; and (iv) using contingent taxation on savings to smooth tax distortions across shock realizations. Of these four mechanisms, only the first three are available to a government operating under the No Regret rule when restricted to non-contingent policies.

5 A model à la Mirrlees

The three preceding sections examined the Ramsey model. This section turns to a different framework: a two-period open-economy model *à la* Mirrlees (1971), featuring initial wealth heterogeneity and persistent skill differences across households.

5.1 Heterogeneous households

Heterogeneous households. At the beginning of the first period $t = 0$, each household has some initial wealth $a \in \mathbb{R}^+$. Each household's skill $\omega \in \mathbb{R}^+$ is the same in periods $t = 0$ and $t = 1$. The pair (a, ω) is referred to as the household's *type*. These types are distributed according to a c.d.f $F(a, \omega)$. Each household's initial wealth a and skill ω are privately observed. This information asymmetry prevents the government from perfectly redistributing either along the initial wealth dimension or the skill dimension.[19]

[19]The role of unobserved initial wealth is to model heterogeneous expected utility levels among otherwise identical households. Another modeling choice could have been to add a period $t = -1$ with idiosyncratic shocks to skill or taste that would endogenously produce 'heterogeneous promises' at $t = 0$. This is more micro-founded but more complex.

The government. At $t = 1$, a shock to the usefulness of public expenditures θ, is publicly observed. θ takes values in a finite set $\Theta \subseteq \mathbb{R}_+$. The government objective is to maximize

$$\int U(a, \omega)\, dF(\omega, a) \;+\; \mathbb{E}\,[\,\theta w(G_\theta)\,] \tag{18}$$

where $U(a, \omega)$ is the expected utility level of households with initial wealth a and skill ω, and where G_θ represents public expenditures at time $t = 1$ when θ is realized. We assume that the function $w(\cdot)$ is strictly increasing, and strictly concave. Households with initial wealth a and skill ω who consume $c_0(a, \omega)$ at $t = 0$ and $c_\theta(a, \omega)$ at time $t = 1$ when θ is realized, and who produce effective labor $y_0(a, \omega)$ at time $t = 0$ and $y_\theta(a, \omega)$ at time $t = 1$ when θ is realized have the following expected utility level

$$U(a, \omega) \;\equiv\; u(c_0(a, \omega)) - v(y_0(a, \omega)/\omega) + \mathbb{E}\left[\, u(c_\theta(a, \omega)) - v(y_\theta(a, \omega)/\omega)\,\right]$$

The function $u(\cdot)$ is differentiable, strictly increasing and strictly concave, while the function $v(\cdot)$ is differentiable, strictly increasing and strictly convex.

Technology. Each unit of effective labor produces one unit of output. Output and wealth can be allocated to either consumption or public spending. Goods can be stored (or borrowed) between periods $t = 0$ and $t = 1$ at a gross interest rate of $R = 1$. Thus, for any realization θ, the intertemporal resource constraint of this small open economy is

$$\int [\, c_0(a, \omega) + c_\theta(a, \omega)\,] \;\; dF(\omega, a) \;+\; G_\theta \tag{19}$$

$$= \int [\, a + y_0(a, \omega) + y_\theta(a, \omega)\,] \;\; dF(\omega, a)$$

5.2 Contingent fiscal plans with heterogeneous households

Mechanism design. Each household's initial wealth a and skill ω are privately observed and the government relies on mechanism design to elicit them. According to the Revelation Principle, we can use a *direct* and *truthful* mechanism without loss of generality. A mechanism is *direct* when each household reports only an initial wealth level a' and a skill ω'. A household with initial wealth a and skill ω who reports initial wealth a' and skill ω' (i) pays taxes $T(a', \omega')$ at

time $t = 0$ and so consumes $a - T(a', \omega')$ at time $t = 0$, (ii) produces (for the government) effective labor $y_0(a', \omega')$ at time $t = 0$ and $y_\theta(a', \omega')$ at time $t = 1$ when θ is realized, and (iii) receives consumption $c_\theta(a', \omega')$ at time $t = 1$ when θ is realized.[20] A direct mechanism is *truthful* when any household with initial wealth a and skill ω chooses to report truthfully, i.e. the following incentive compatibility constraint holds for any pair of types $(a, \omega), (a', \omega')$,

$$u(a - T(a', \omega')) - v(y_0(a', \omega')/\omega) + \mathbb{E}\left[\, u(c_\theta(a', \omega')) - v(y_\theta(a', \omega')/\omega) \,\right] \tag{20}$$

$$\leq \quad u(a - T(a, \omega)) - v(y_0(a, \omega)/\omega) + \mathbb{E}\left[\, u(c_\theta(a, \omega)) - v(y_\theta(a, \omega, \theta)/\omega) \,\right]$$

Taking into account that $c_0(a, \omega) = a - T(a, \omega)$, for any realization θ, the resource constraint (19) is equivalent to the following government budget constraint

$$\int c_\theta(a, \omega)\, dF(\omega, a) \;+\; G_\theta \tag{21}$$

$$= \int \left[\, T(a, \omega) + y_0(a, \omega) + y_\theta(a, \omega) \,\right]\, dF(\omega, a)$$

The Second-Best mechanism. The *ex-ante* optimal contingent (Second-Best) mechanism is the mechanism $\{T(a, \omega), y_0(a, \omega), c_\theta(a, \omega), y_\theta(a, \omega), G_\theta\}_{a,\omega,\theta}$ that maximizes the government's objective (18) subject to the incentive compatibility constraints (20) and to the government budget constraints (21).

$$\max_{\substack{T(\cdot,\cdot), y_0(\cdot,\cdot), U(\cdot,\cdot), \\ \{c_\theta(\cdot,\cdot), y_\theta(\cdot,\cdot), G_\theta\}_\theta}} \quad \int U(a, \omega)\, dF(\omega, a) + \mathbb{E}\left[\, \theta w(G_\theta) \,\right] \tag{22}$$

$$\text{s.t.} \quad U(a, \omega) = u(a - T(a, \omega)) - v(y_0(a, \omega)/\omega) + \mathbb{E}\left[\, u(c_\theta(a, \omega) - v(y_\theta(a, \omega)/\omega) \,\right] \quad \text{for all } a, \omega$$

$$\text{and to} \quad \int c_\theta(a, \omega)\, dF(\omega, a) \;+\; G_\theta = \int \left[\, T(a, \omega) + y_0(a, \omega) + y_\theta(a, \omega) \,\right]\, dF(\omega, a) \quad \text{for all } \theta$$

$$\text{and to} \quad u(a - T(a', \omega')) - v(y_0(a', \omega')/\omega) + \mathbb{E}\left[\, u(c_\theta(a', \omega')) - v(y_\theta(a', \omega')/\omega) \,\right] \leq U(a, \omega) \quad \text{for all } (a, \omega), (a', \omega')$$

In the next subsection, we introduce mechanism reforms and see that the Second-Best mechanism – the solution of Problem (22) – is time-inconsistent.

[20] Households cannot save on their own between times $t = 0$ and $t = 1$.

5.3 Mechanism reforms

In this model, a *reform* at time $t = 1$ after the realization θ is a change of consumption $\{c_\theta(a,\omega)\}_{a,\omega}$, effective labor $\{y_\theta(a,\omega)\}_{a,\omega}$, or public expenditures G_θ. We first show that, at $t = 1$ after any realization θ, the government is tempted to reform the Second-Best mechanism, i.e. the Second-Best mechanism is time-inconsistent.

Proposition 2. ***(The Second-Best mechanism is time-inconsistent)*** *Under the specification* $v : x \to \frac{x^{1+\psi}-1}{1+\psi}$ *with* $\psi > 0$*, the Second-Best mechanism is time-inconsistent.*

See the proof in the Supplmental Appendix.[21]

Time-inconsistency. Since initial wealth a and skill ω are privately observed, the Second-Best mechanism mechanism grants households information rents.[22] The Second-Best mechanism smooths these rents across periods because it is less costly to do so. Specifically, consumption heterogeneity is persistent: households with higher initial wealth or higher skill consume more in both periods, and labor supplies are distorted in both periods. At $t = 1$, the government is tempted to renege on the portion of information rents scheduled for delivery at $t = 1$ but promised at $t = 0$, which were designed to incentivize type revelation. Indeed, the government is tempted to equalize consumption across initial wealth a and skills ω, and to use the skill information gathered at $t = 0$ to infer skills at $t = 1$, so that all households supply labor without distortions – typically forcing high-skill households to work harder without consumption rewards.

Revelation Principle and time-inconsistency. When mechanisms are time-inconsistent and reforms are possible, the Revelation Principle no longer necessarily holds, i.e. restricting the government to direct and truthful mechanisms can shrink the set of implementable allocations. For instance, in our Mirrlees model, when the government has full discretion to reform at $t = 1$, any full revelation of types at $t = 0$ will be used at $t = 1$ to perfectly equalize consumption across all households, and to make all households supply labor without distortions.

[21] This specification for v rules out differences in labor supplies between $t = 0$ and $t = 1$ for type (a,ω) when there is no shock.

[22] An information rent is the utility gain an agent obtains from possessing private information, compared to a situation in which that information is public.

Anticipating this, rational high-skill or high-wealth households choose not to reveal their types at $t = 0$. Thus, when the government has full discretion, direct and truthful mechanisms cannot implement allocations with consumption heterogeneity and distorted labor supplies at $t = 1$, although other mechanisms can, *e.g.* pulling of all types at $t = 0$ and a linear tax on labor income at $t = 1$. Nevertheless, in the next subsection, we focus exclusively on direct and truthful mechanisms (and their reforms) because the Revelation Principle holds under the No Regret rule. This non-trivial result is based on the fact that mechanisms are time-consistent under the No Regret rule and is established in the Supplemental Appendix.

5.4 No Regret reforms with heterogeneous households

With direct mechanisms and perfectly persistent skills, the only households' decisions are the skill ω' and the initial wealth a' they report at $t = 0$.

The promise-keeping rule. One way to ensure that households choose to truthfully report is that any *reformed* mechanism $\{\tilde{c}_\theta(a,\omega), \tilde{y}_\theta(a,\omega), \tilde{G}_\theta\}_{a,\omega}$ offers truthful households rewards and punishments at least as large as those promised under the *status quo* mechanism $\{c_\theta(a,\omega), y_\theta(a,\omega), G_\theta\}_{a,\omega}$. Formally, for any shock θ and for any pair of types $(a,\omega), (a',\omega')$, the following promise-keeping constraint must hold[23]

$$\begin{aligned} &\left[\, u(c_\theta(a,\omega)) - v(y_\theta(a,\omega)/\omega) \,\right] \;-\; \left[\, u(c_\theta(a',\omega')) - v(y_\theta(a',\omega')/\omega) \,\right] \qquad (23)\\ \leq\quad &\left[\, u(\tilde{c}_\theta(a,\omega)) - v(\tilde{y}_\theta(a',\omega')/\omega) \,\right] \;-\; \left[\, u(\tilde{c}_\theta(a',\omega')) - v(\tilde{y}_\theta(a',\omega')/\omega) \,\right] \end{aligned}$$

The No Regret rule. However, for most type pairs $(a,\omega), (a',\omega')$, it is likely that the incentive compatibility constraint (20) associated with the *status quo* mechanism $\{T(a,\omega), y_0(a,\omega), c_\theta(a,\omega), y_\theta(a,\omega), G_\theta\}_{a,\omega,\theta}$ is slack. That is, the 'avoided loss' from truthfully reporting defined as

$$\Delta((a,\omega),(a',\omega')) \equiv \qquad U(a,\omega) \qquad (24)$$

[23] Imposing an equality rather than an inequality, i.e. delivering to truthful households rewards and punishments *exactly* as promised under the *status quo* mechanism, would further reduce discretion under the promise-keeping rule.

$$- \Big[\;\; u(a - T(a', \omega')) - v(y_0(a', \omega')/\omega) + \mathbb{E}\left[u(c_\theta(a', \omega')) - v(y_\theta(a', \omega')/\omega) \right] \;\; \Big],$$

is strictly positive. When $\Delta((a, \omega), (a', \omega')) > 0$, honoring the entire promise embedded in the *status quo* mechanism by respecting the constraint (23) is unnecessarily strong to avoid regrets: part of this promise could remain unfulfilled without inducing households to have regrets.[24] Indeed, as long as the reform does not reduce the 'avoided utility loss' by more than $\Delta((a, \omega), (a', \omega')) > 0$, households will not regret having reported truthfully at $t = 0$. Thus, in this model *à la* Mirrlees (1971), a No Regret reform can be formally defined as follows:

Definition 3. ***(No Regret reform with heterogeneous households)***
Consider the direct and truthful status quo mechanism $\{T(a, \omega), y_0(a, \omega), c_\theta(a, \omega), y_\theta(a, \omega), G_\theta\}_{a,\omega,\theta}$. *A reform* $\{\tilde{c}_\theta(a, \omega), \tilde{y}_\theta(a, \omega), \tilde{G}_\theta\}_{a,\omega}$ *at time* $t = 1$ *after the realization* θ *is a No Regret reform when the following No Regret constraint is met for all pairs of types* $(a, \omega), (a', \omega')$

$$\left[u(c_\theta(a, \omega)) - v(y_\theta(a, \omega)/\omega) \right] \; - \; \left[u(c_\theta(a', \omega')) - v(y_\theta(a', \omega')/\omega) \right] \qquad (25)$$

$$\leq \quad \left[u(\tilde{c}_\theta(a, \omega)) - v(\tilde{y}_\theta(a', \omega')/\omega) \right] - \left[u(\tilde{c}_\theta(a', \omega')) - v(\tilde{y}_\theta(a', \omega')/\omega) \right] + \Delta((a, \omega), (a', \omega'))$$

By construction, truthful reporting at time $t = 0$ is a dominant strategy for all households as formally stated by the next Lemma.

Lemma 3. ***(Households' optimal strategy under the No Regret rule with heterogeneity)***
When a government under the No Regret rule announces a direct and truthful mechanism at $t = 0$, *truthful reporting of types at* $t = 0$ *is a dominant strategy for all households.*

Note that Lemma 3 is a special case of Lemma 7 in the Supplemental Appendix established with general mechanisms.

No Regret rule and Second-Best. At $t = 1$ after the realization θ, a government under the No Regret rule must choose $\{\tilde{c}_\theta(a, \omega)\}_{a,\omega}$,$\{\tilde{y}_\theta(a, \omega)\}_{a,\omega}$ and $\tilde{G}_\theta$ to maximize its objective (18) subject to the budget constraint (21), and the No

[24] Here is an illustration of unfulfilled promise that does not cause regret: a teacher promises a student $ 2 million for attending the next class. The student shows up in class, but the teacher gives only $ 1 million. Despite the reduced reward, the student does not regret attending.

Regret constraint (25).

$$\max_{\tilde{c}_\theta(\cdot,\cdot),\tilde{y}_\theta(\cdot,\cdot),\tilde{G}_\theta} \int \left[u(\tilde{c}_\theta(a,\omega)) - v(\tilde{y}_\theta(a,\omega))/\omega\right] dF(\omega,a) + \theta w(\tilde{G}_\theta) \tag{26}$$
$$\text{s.t.} \quad (21) \quad \text{and} \quad (25)$$

When the *status quo* mechanism is the Second-Best mechanism (i.e. the solution to the Problem (22)), then the government under the No Regret rule can not benefit from any reform at time $t = 1$. Otherwise, the Second-Best mechanism could be bettered, which is a contradiction. Thus, a government under the No Regret rule implements the Second-Best mechanism and allocation.

Proposition 3. ***(The No Regret fiscal rule and Second-Best with heterogeneity)*** *A government under the No Regret fiscal rule implements the Second-Best mechanism and allocation.*

As in the Ramsey model (Proposition 1), the discretion available to governments under the No Regret fiscal rule does not give rise to time-inconsistency problems, and implementing the Second-Best mechanism and allocation does not require full commitment to a contingent mechanism; it suffices to commit to *sufficiently* honoring the promises of rewards and punishments embedded in the status quo mechanism. Consequently, the Second-Best mechanism and allocation are more realistic – and therefore more useful — benchmarks.

Promise-keeping vs. No Regret. A government constrained to fully honor past promises has less discretion than under the No Regret rule (the promise-keeping constraint (23) is more restrictive than the No Regret constraint (25)). Consequently, such a government would also choose not to reform the Second-Best policy at $t = 1$. With contingent mechanisms, discretion has no value. In the next subsection, I restrict attention to non-contingent mechanisms, in which case the additional discretion afforded by the No Regret rule relative to the promise-keeping rule becomes valuable.

5.5 Non-contingent No Regret reforms with heterogeneous agents

In this section, I study how governments restricted to using non-contingent mechanism cope with shocks through No Regret reforms.

At $t = 1$ after any realization θ, the government under the No Regret rule inherits a *non-contingent status quo* mechanism $\{T(a,\omega), y_0(a,\omega), c_1(a,\omega), y_1(a,\omega), G_1\}_{a,\omega}$

that is direct and truthful. The 'avoided utility loss' from truthfully reporting, as per Equation (24), is

$$\Delta((a,\omega),(a',\omega')) = u(a - T(a,\omega)) - v(y_0(a,\omega)/\omega) + u(c_1(a,\omega)) - v(y_1(a,\omega)/\omega)$$

$$- \left[\; u(a - T(a',\omega')) - v(y_0(a',\omega')/\omega) + u(c_1(a',\omega')) - v(y_1(a',\omega')/\omega) \; \right],$$

and the No Regret constraint (25) can be simplified as

$$u(a - T(a',\omega')) - v(y_0(a',\omega')/\omega) + u(\tilde{c}_1(a',\omega')) - v(\tilde{y}_1(a',\omega')/\omega) \tag{27}$$

$$\leq \quad u(a - T(a,\omega)) - v(y_0(a,\omega)/\omega) + u(\tilde{c}_1(a,\omega)) - v(\tilde{y}_1(a,\omega)/\omega)$$

This inequality (27) is a standard incentive compatibility constraint, which means that the reformed mechanism $\{T(a,\omega), y_0(a,\omega), \tilde{c}_1(a,\omega), \tilde{y}_1(a,\omega), \tilde{G}_1\}_{a,\omega}$ must be truthful from the $t = 0$ perspective.[25]

Thus, at time $t = 1$ after any realization θ, the government under the No Regret rule inherits the non-contingent *status quo* mechanism $\{T(a,\omega), y_0(a,\omega), c_1(a,\omega), y_1(a,\omega), G_1\}_{a,\omega}$ and maximizes welfare subject to its budget constraint and to an incentive compatibility constraint.

$$\max_{\substack{\tilde{G}_1, \tilde{c}_1(\cdot,\cdot), \\ \tilde{y}_1(\cdot,\cdot)}} \quad \int \left[\; u(\tilde{c}_1(a,\omega)) - v(\tilde{y}_1(a,\omega)/\omega) \; \right] \, dF(\omega,a) + \theta w(\tilde{G}_1) \tag{28}$$

$$\text{s.t.} \qquad \int \tilde{c}_1(a,\omega) \, dF(\omega,a) \; + \; \tilde{G}_1$$

$$= \quad \int \left[\, T(a,\omega) + y_0(a,\omega) + \tilde{y}_1(a,\omega)) \, \right] \, dF(\omega,a)$$

and to $\quad u(a - T(a',\omega')) - v(y_0(a',\omega')/\omega) + u(\tilde{c}_1(a',\omega')) - v(\tilde{y}_1(a',\omega')/\omega)$

$$\leq u(a - T(a,\omega)) - v(y_0(a,\omega)/\omega) + u(\tilde{c}_1(a,\omega)) - v(\tilde{y}_1(a,\omega)/\omega) \quad \forall \, (a,\omega),(a',\omega')$$

Finally, by backward induction, a government under the No Regret rule that can only announce a non-contingent direct and truthful mechanism at $t = 0$ but that has some discretion to reform at $t = 1$ after any realization θ solves the following Problem.

[25] The promise-keeping rule would not yield (27).

$$\max_{\substack{T(\cdot,\cdot),y_0(\cdot,\cdot),U(\cdot,\cdot),\\ \{\tilde{c}_\theta(\cdot,\cdot),\tilde{y}_\theta(\cdot,\cdot),\tilde{G}_\theta\}_\theta}} \int U(a,\omega)\, dF(\omega,a) + \mathbb{E}\left[\, \theta w(G_\theta)\, \right] \tag{29}$$

$$\text{s.t.} \qquad U(a,\omega) = u(a - T(a,\omega)) - v(y_0(a,\omega)/\omega) + \mathbb{E}\left[\, u(\tilde{c}_\theta(a,\omega) - v(\tilde{y}_\theta(a,\omega)/\omega)\, \right] \quad \forall\, a,\omega$$

$$\text{and to} \qquad \int \tilde{c}_\theta(a,\omega)\, dF(\omega,a) \;+\; \tilde{G}_\theta \leq \int \left[\, T(a,\omega) + y_0(a,\omega) + \tilde{y}_\theta(a,\omega)\, \right]\, dF(\omega,a) \qquad \forall\, \theta$$

$$\text{and to} \quad u(a - T(a',\omega')) - v(y_0(a',\omega')/\omega) + u(\tilde{c}_\theta(a',\omega')) - v(\tilde{y}_\theta(a',\omega')/\omega) \leq u(a - T(a,\omega)) - v(y_0(a,\omega)/\omega) + u(\tilde{c}_\theta(a,\omega) - v(\tilde{y}_\theta(a,\omega)/\omega) \;\; \forall\, (a,\omega),(a',\omega'),\theta$$

As in the Ramsey model, the Second-Best Problem (22) and the Problem (29) of a government under the No Regret rule restricted to using *non-contingent* mechanism are similar. The only difference is that in Problem (22), the incentive compatibility constraints must be satisfied in expectation, whereas in Problem (29), they must be satisfied for each realization of the shock θ. Clearly, the optimal solution of Problem (29) is also an admissible solution of Problem (22), but the reverse is not necessarily true (except if there is no shock).

To implement the solution of Problem (29), the government can announce at $t = 0$ a non-contingent mechanism $\{T(\cdot,\cdot), y_0(\cdot,\cdot), U(\cdot,\cdot), c_{\hat{\theta}}(\cdot,\cdot), y_{\hat{\theta}}(\cdot,\cdot), G_{\hat{\theta}}\}$ where $\hat{\theta}$ is is an arbitrary realization. Then, at $t = 1$, for any realization $\theta \neq \hat{\theta}$, the government can implement the No Regret reform $\{\tilde{c}_\theta(\cdot,\cdot), \tilde{y}_\theta(\cdot,\cdot), \tilde{G}_\theta\}$.

6 Simulations

This section turns to simulations of the Mirrlees model. I study how governments restricted to using non-contingent mechanisms cope with shocks through No Regret reforms, and compare the resulting mechanism and allocation to the ex-ante optimal contingent mechanism and allocation (the Second-Best benchmarks).

Specifications Preferences over consumption, labor, and government spending are iso-elastic:

$$u(C) - v(L) + \theta w(G) = \frac{C^{1-\sigma} - 1}{1-\sigma} - \frac{L^{1+\psi} - 1}{1+\psi} + \theta \frac{G^{1-\xi} - 1}{1-\xi}$$

There are two shocks, Low and High i.e. $\Theta = \{L, H\}$. There is a mass m_0 of households that have no wealth, and the other households are distributed over $[\underline{a}, +\infty[$ following a Pareto distribution of a density $f_a(a) = \frac{\alpha}{\underline{a}}(\underline{a}/a)^{\alpha}$. Households' skills follow a log-normal distribution of density $f_\omega(\omega) = \frac{1}{\omega\sigma\sqrt{2\pi}} e^{-\frac{(\ln(\omega)-\mu)^2}{2\rho^2}}$. Wealth and skill are independently distributed. The parameters used are presented in Table 2:

CALIBRATED PARAMETERS		
$1/\psi$	0.5	Chetty et al. (2011)
σ	1	Calvet et al. (2021)
ξ	1	Identical to σ
m_0	50%	2022 Survey of Consumer Finances (SCF)
α	1.65	Saez and Zucman (2016)
$\underline{a}$	Varies	Average positive wealth=1
p_L	0.6	Low shock 60% of the time
p_H	0.4	High shock 40% of the time
JOINTLY ESTIMATED PARAMETERS		
μ	-0.7	Labor share = 67% of total consumption
ρ	0.65	Coefficient of variation of labor income=0.65
θ_L	0.35	$\mathbb{E}[G_\theta]$ = 20 % of total consumption
θ_H	0.65	$\sigma(G_\theta)$ = 5 % of GDP

Table 2: Parameters

Results. The main simulation result is that what I call the *No Regret score* of this model is 97.6%, i.e. No Regret reforms achieve 97.6% of the shock-coping performance of the Second-Best mechanism. Specifically, the Second-Best mechanism delivers a welfare gain (in consumption equivalent) of 0.8356% over the non-contingent optimal mechanism, while the optimal No Regret reform yields a welfare gain of 0.8156% without relying on contingent mechanisms. The two mechanisms are very similar, but the key difference lies in contingent labor supply: under the Second-Best mechanism, households are

required to work more (less) when public spending is high (low) compared to the No Regret reforms. This enables the contingent mechanism to adjust public spending more effectively in response to shocks and to better smooth consumption across shocks, particularly for low-skill, low-wealth households.

6.1 Labor income redistribution

The previous subsections studied a shock to the government's preferences regarding public expenditures θ. In a model with heterogeneous households, shocks to the government's preferences regarding *redistribution* are also interesting to study. Introducing stochastic Pareto welfare weights $\Omega(a, \omega)$ yields the following objective

$$\mathbb{E}\left[\int \Omega(a,\omega)\ [\,U_\Omega(a,\omega)\,]\ dF(\omega,a)\right] + \mathbb{E}\,[\,\theta w(G_\Omega)\,]$$

where $U_\Omega(a,\omega) \equiv\ u(c_0(a,\omega)) - v(y_0(a,\omega)/\omega) + u(c_\Omega(a,\omega)) - v(y_\Omega(a,\omega)/\omega)$ is the utility level of type (a,ω) when $\Omega(\cdot,\cdot)$ is realized.

First, we study a shock to the preference for redistribution along the skill dimension. At time $t = 1$, the government has a 50% chance of being utilitarian, with $\Omega(a,\omega) = 1$, and a 50% chance of placing more weight on low-skill households regardless of their initial wealth a–that is, $\Omega(a,\omega) = \overline{\Omega}(\omega)$, where $\overline{\Omega}(\omega)$ is decreasing in ω. The main simulation result is that, in contrast to shocks on public spending, the No Regret score of this model is low, i.e. No Regret reforms achieve only a small fraction – just a few percentage points – of the shock-coping performance of the Second-Best mechanism. The intuition is that privately-observed skills give rise to information rents that increase with skill level, and since a portion of information rents are delivered at $t = 1$, the government can still slightly change the shapes of information rents without making households regret revealing their skills at $t = 0$. However, most of the shock-coping achieved by the Second-Best mechanism is done by delivering contingent rents: high-skill households receive low rents when the government wants to redistribute towards the low-skill, which clearly make households regret their choices, otherwise they receive high rents.

6.2 Wealth redistribution and the Equal Sacrifice principle

We now study a shock to the preference for redistribution along the initial wealth dimension. At time $t = 1$, the government has a 50% chance of being utilitarian, with $\Omega(a, \omega) = 1$, and a 50% chance of placing more weight on low-wealth households regardless of their skill ω–that is, $\Omega(a, \omega) = \overline{\Omega}(a)$, where $\overline{\Omega}(a)$ is decreasing in a. The main result is that the No Regret score of this model is very low, i.e. No Regret reforms achieve less than 1% of the shock-coping performance of the Second-Best mechanism. This figure even drops to zero in the absence of skill heterogeneity.

This can be explained by the following partial characterization of No Regret reforms. For any skill ω, consider the set of households with skill ω. Any No Regret reform necessarily shifts their utility levels by a common value $H(\omega)$.

Lemma 4. ***(Wealth heterogeneity and Equal Sacrifice)***
Consider any status quo mechanism $\{T(a,\omega), y_0(a,\omega), c_\theta(a,\omega), y_\theta(a,\omega), G_\theta\}_{a,\omega,\theta}$ *and* $\{\tilde{c}_\theta(a,\omega), \tilde{y}_\theta(a,\omega), \tilde{G}_\theta\}$*, a No Regret reform at time* $t = 1$ *after the realization* θ*.*
For any skill ω*, there exists a value* $H(\omega)$ *such that, for all wealth levels* a*,*

$$u(\tilde{c}_\theta(a,\omega)) - v(\tilde{y}_\theta(a,\omega)/\omega) = u(c_\theta(a,\omega)) - v(y_\theta(a,\omega)/\omega) + H(\omega)$$

See the proof in the Appendix.

The intuition is that reporting any given initial wealth a rather than a' yields some expected utility difference. If a reform at $t = 1$ alters some of these utility differences, then some households regret their initial wealth report at $t = 0$.

Lemma 4 is reminiscent of the Equal Sacrifice principle (Mill, 1848; Young, 1988) according to which households facing a common burden – such as an increase in government expenditures – should share it in a way that equalizes utility losses across households. This principle has a clear normative appeal in terms of equity. Lemma 4 provides another rationale: the Equal Sacrifice principle should be respected along the wealth dimension to ensure households do not regret their past saving efforts.[26]

Going back to redistribution along the wealth dimension, if at time $t = 1$ the government suddenly places greater weight on wealth-poor (or wealth-rich) households, then – as Lemma 4 shows – it cannot shift resources toward these

[26] A symmetric Equal Benefit principle should apply when there is a common benefit to be shared.

households through a No Regret reform. When skill heterogeneity is present, No Regret reforms can redistribute along the wealth dimension only indirectly by redistributing along the skill dimension.[27]

7 Conclusion

The main contribution of this paper is to introduce No Regret fiscal reforms, the No Regret rule and the No Regret score. This rule is non-contingent, time-invariant, and model-invariant, and it does not specify in advance the exact set of policies available to the government.

When governments' preferences are time-consistent, time-inconsistent polices are detrimental to welfare because governments cannot credibly promise households future rewards for present actions. The key insight is that avoiding such welfare losses requires only that the government commit to the incentives embodied in those rewards, rather than their exact natures. This is precisely what the No Regret rule achieves; consequently, the discretion it allows entails no welfare losses from time-inconsistency (Propositions 1 and 3). Without contingent policies, a government under the No Regret rule uses discretion to cope with shocks. Nevertheless, it cannot achieve Second-Best welfare levels, as it lacks the ability to employ contingent rewards – such as contingent net-of-tax returns on savings in the Ramsey model or contingent information rents in the Mirrlees model. The No Regret score provides a measure of the shock-coping performance of optimal non-contingent No Regret reforms relative to the optimal contingent (Second-Best) policy.

In numerical simulations of the two models, governments cope with shocks to public expenditures almost as effectively as the Second-Best policies, i.e. No Regret scores are high. By contrast, No Regret scores are low when the government's preferences for redistribution are shocked. These results show that contingent rewards – used by contingent policies only – are superfluous for some combinations of shocks and time-inconsistency issues, yet essential for others.

The No Regret scores can be computed in many models and can inform the design of real-world policies as follows. If contingent rewards are found to be essential, then to reap their benefits, one could (i) commit to contingent policies that are simple enough to be implemented, (*e.g.* sovereign (partial) default trig-

[27] This indirect redistribution is dampened in these simulations because I specified wealth and skill as independently distributed.

gered by some events, escape clauses), or (ii) delegate decision-making to an independent body.[28] In contrast, if contingent rewards are found to be superfluous, then simple fiscal rules – such as "never default" or "never tax capital" – could address time inconsistency issues very well while still allowing sufficient discretion to cope well with shocks.[29]

The advantage of proceeding in this way is that computing the optimal fiscal No Regret policies is straightforward: under the No Regret rule, fiscal policies are time-consistent, so can be computed as if all policies were chosen by the initial government. By contrast, evaluating the performance of fiscal rules, partially contingent policies, or delegation to an independent body requires solving a complex sequential game played by households and successive governments (or independent bodies).

Of course, in a real (macro)economy, No Regret reforms are not feasible: fiscal tools are not sufficiently complex to eliminate regret for millions of households.[30] However, the No Regret rule can be relaxed by allowing reforms that incur limited regrets. This relaxed No Regret rule allows some reforms. What would be the optimal regret limit, and would the equilibrium allocation change significantly? We leave these questions for future research.

Another real-world implementation of the No Regret rule could be based on a characterization of No Regret reforms: when announced in advance, households do not change their current behaviors. A relaxed No Regret rule could forbid, for instance, any reform that makes average consumptions within subpopulations jump when announced. What would be the choice of subpopulations? We leave these questions for future research.

Finally, let us discuss the $t = 0$ problem. The standard approach in optimal dynamic fiscal policy is to suppose that past actions are sunk and do not enter the government's problem. Optimality typically entails heavily taxing savings at $t = 0$, but never again. The fact that the government yields to temptation and heavily taxes savings now, is a threat to the credibility of the promise never

[28] This is also the case for an independent central bank in the monetary policy literature. In the one-period textbook model of monetary policy used by (Flood & Isard, 1988; Lohmann, 1992; Rogoff, 1985), deviations around expected inflation are the only way to cope with shocks. Unfortunately, any such deviation induces regret of price-setting decisions, so a monetary authority under the No Regret rule cannot cope with shocks at all, and its optimal policy is to keep zero inflation.

[29] Studying Markov perfect equilibria of a Ramsey model, Martin (2010) shows that bounds on capital tax increase welfare. In Domínguez and Feng (2017), best sequential equilibria may suffer from bounds on capital tax as they make worst sequential equilibria better.

[30] When considering only limited parts of the economy, some policy reforms can be No Regret, e.g., *ex-post* adjustments to mispriced public procurements.

to do it again. The standard way around this, as adopted in this paper, is to impose *ad hoc* initial tax limits. A second approach, used by Chari et al. (2020), is to restrict the policy space at $t = 0$ to policies that do not cause households to regret their past actions, i.e., policies that constitute No Regret reforms of the policy announced in the past. The appeal of No Regret reforms for $t > 0$ stems from the fact that they address time-inconsistency problems and respond well to shocks (when the No Regret score is high). The appeal of No Regret reforms at $t = 0$ is of a very different nature. First, some readers may find that avoiding households' regret is normatively appealing *per se*, much like keeping a promise. Second, if the No Regret rule is used at $t > 0$, then $t = 0$ is not treated as a special period, which enhances the credibility of the No Regret rule. Third, it guarantees that households' past decisions were rational.[31] Forbidding any regret at $t = 0$ may seem overly restrictive to some readers. Nevertheless, when comparing the merits of policies, the distribution of regrets across agents should play a role. For example, for a given GHG reduction target in transport, taxing ICE cars may cause recent EV buyers to experience less regret than would subsidizing future EV buyers. While one could attempt to minimize average regret, some regrets may be more normatively acceptable than others. For instance, inducing regret among investors in recent carbon-intensive projects, given the urgency of the climate crisis, may appear relatively acceptable to some readers.

References

Ábrahám, Á., Koehne, S., & Pavoni, N. (2016). Optimal income taxation when asset taxation is limited. *Journal of Public Economics*, *136*, 14–29.

Aiyagari, S. R., Marcet, A., Sargent, T. J., & Seppälä, J. (2002). Optimal taxation without state-contingent debt. *Journal of Political Economy*, *110*(6), 1220–1254.

Alesina, A., & Tabellini, G. (1990). A positive theory of fiscal deficits and government debt. *The Review of Economic Studies*, *57*(3), 403–414.

Amador, M., & Phelan, C. (2021). Reputation and sovereign default. *Econometrica*, *89*(4), 1979–2010.

[31] A third approach used by Jensen and McCallum (2010) is to restrict the policy space to time-invariant policies that, by definition, treat all periods identically. A fourth approach is to (artificially) include past actions as non-sunk in the government's problem, *i.e.*, adopt the timeless perspective.

Athey, S., Atkeson, A., & Kehoe, P. J. (2005, September). The Optimal Degree of Discretion in Monetary Policy. *Econometrica*, *73*(5), 1431-1475.

Barthélemy, J., & Mengus, E. (2024). Time-consistent implementation in macroeconomic games. *Theoretical Economics*, *19*(3), 1119–1150.

Bhandari, A., Evans, D., Golosov, M., & Sargent, T. J. (2017). Fiscal policy and debt management with incomplete markets. *The Quarterly Journal of Economics*, *132*(2), 617–663.

Calvet, L. E., Campbell, J. Y., Gomes, F., & Sodini, P. (2021). The cross-section of household preferences.

Chari, V. V., Christiano, L. J., & Kehoe, P. J. (1994). Optimal fiscal policy in a business cycle model. *Journal of Political Economy*, *102*(4), 617–652.

Chari, V. V., & Kehoe, P. J. (1990). Sustainable plans. *Journal of Political Economy*, *98*(4), 783–802.

Chari, V. V., Nicolini, J. P., & Teles, P. (2020). Optimal capital taxation revisited. *Journal of Monetary Economics*, *116*, 147–165.

Chetty, R., Guren, A., Manoli, D., & Weber, A. (2011). Are micro and macro labor supply elasticities consistent? A review of evidence on the intensive and extensive margins. *American Economic Review*, *101*(3), 471–75.

Cohen, D., & Michel, P. (1988). How should control theory be used to calculate a time-consistent government policy? *The Review of Economic Studies*, *55*(2), 263–274.

Debortoli, D., & Nunes, R. (2010). Fiscal policy under loose commitment. *Journal of Economic Theory*, *145*(3), 1005–1032.

Debortoli, D., Nunes, R., & Yared, P. (2017). Optimal time-consistent government debt maturity. *The Quarterly Journal of Economics*, *132*(1), 55–102.

Domínguez, B., & Feng, Z. (2017). An Evaluation of Constitutional Constraints on Capital Taxation. *Macroeconomic Dynamics*, *21*(7), 1519–1544.

Dovis, A. (2019). Efficient sovereign default. *The Review of Economic Studies*, *86*(1), 282–312.

Dovis, A., & Kirpalani, R. (2020). Fiscal rules, bailouts, and reputation in federal governments. *American Economic Review*, *110*(3), 860–888.

Farhi, E. (2010). Capital taxation and ownership when markets are incomplete. *Journal of Political Economy*, *118*(5), 908–948.

Flood, R. P., & Isard, P. (1988). Monetary policy strategies. *NBER Working Papers*.

Golosov, M., Kocherlakota, N., & Tsyvinski, A. (2003). Optimal indirect and capital taxation. *The Review of Economic Studies*, *70*(3), 569–587.

Golosov, M., Tsyvinski, A., Werning, I., Diamond, P., & Judd, K. L. (2006). New Dynamic Public Finance: A User's Guide [with Comments and Discussion]. *NBER macroeconomics annual*, *21*, 317–387.

Halac, M., & Yared, P. (2014). Fiscal rules and discretion under persistent shocks. *Econometrica*, *82*(5), 1557–1614.

Jensen, C., & McCallum, B. T. (2010). Optimal continuation versus the timeless perspective in monetary policy. *Journal of Money, Credit and Banking*, *42*(6), 1093–1107.

Klein, P., & Ríos-Rull, J.-V. (2003). Time-consistent optimal fiscal policy. *International Economic Review*, *44*(4), 1217–1245.

Kreps, D. M., & Wilson, R. (1982). Reputation and imperfect information. *Journal of Economic Theory*, *27*(2), 253–279.

Krusell, P. (2002). Time-consistent redistribution. *European Economic Review*, *46*(4-5), 755–769.

Kydland, F. E., & Prescott, E. C. (1977). Rules rather than discretion: The inconsistency of optimal plans. *Journal of Political Economy*, *85*(3), 473–491.

Lohmann, S. (1992). Optimal commitment in monetary policy: credibility versus flexibility. *The American Economic Review*, *82*(1), 273–286.

Loomes, G., & Sugden, R. (1982). Regret Theory: An Alternative Theory of Rational Choice Under Uncertainty. *The Economic Journal*, *92*(368), 805–824.

Lucas Jr, R. E., & Stokey, N. L. (1983). Optimal fiscal and monetary policy in an economy without capital. *Journal of Monetary Economics*, *12*(1), 55–93.

Martin, F. M. (2010). Markov-perfect capital and labor taxes. *Journal of Economic Dynamics and Control*, *34*(3), 503–521.

Mill, J. S. (1848). Principles of political economy. *J. Laurence Laughlin. New York: D. Appleton*.

Mirrlees, J. A. (1971). An exploration in the theory of optimum income taxation. *The Review of Economic Studies*, *38*(2), 175–208.

Piguillem, F., & Riboni, A. (2021). Fiscal rules as bargaining chips. *The Review of Economic Studies*, *88*(5), 2439–2478.

Ramsey, F. P. (1927). A Contribution to the Theory of Taxation. *The Economic Journal*, *37*(145), 47–61.

Rogoff, K. (1985). The optimal degree of commitment to an intermediate monetary target. *The Quarterly Journal of Economics*, *100*(4), 1169–1189.

Savage, L. J. (1951). The theory of statistical decision. *Journal of the American Statistical association*, *46*(253), 55–67.

Young, H. P. (1988). Distributive justice in taxation. *Journal of Economic Theory*, *44*(2), 321–335.

Zhu, X. (1992). Optimal fiscal policy in a stochastic growth model. *Journal of Economic Theory*, *58*(2), 250–289.

Appendix

Proof of Lemma 1 (The household's optimal decisions under the No Regret fiscal rule)

Let us show that, when the government is under the No Regret fiscal rule, it is optimal for households to make decisions identical to those induced by the *current* policy plan. First, the household is able to follow this strategy because, by definition, No Regret reforms do not change past induced decisions.

Consider any competitive equilibrium under the No Regret fiscal rule. Denote by $\tilde{\pi}$ the equilibrium contingent policy plan that is actually implemented (obtained after all No Regret fiscal reforms). The decisions $\{\tilde{C}, \tilde{L}, \tilde{K}, \tilde{b}\}$ induced by this policy plan $\tilde{\pi}$ are optimal. Denote $\{C, L, K, b\}$ the decisions that are induced by the successive current policies. Let us show that these decisions are also optimal. Denote the welfare difference before T

$$\Delta_T \equiv \mathbb{E}_0[\ \sum_{t=0}^{T} \beta^t\ [\ u(\tilde{C}_t) - v(\tilde{L}_t)\ ]\ ]\ -\ \mathbb{E}_0[\ \sum_{t=0}^{T} \beta^t\ [\ u(C_t) - v(L_t)\ ]\ ]$$

By concavity of u and $-v$,

$$\Delta_T \leq \mathbb{E}_0[\ \sum_{t=0}^{T} \beta^t\ [\ u'(C_t)(\tilde{C}_t - C_t) - v'(L_t)(\tilde{L}_t - L_t)\ ]\ ]$$

Denote by π^T the contingent policy plan that is announced at time T. π^T and $\tilde{\pi}$ are identical for all $t \leq T$, and $\{C, L, K, b\}$ and $\{\tilde{C}, \tilde{L}, \tilde{K}, \tilde{b}\}$ respect the budget constraints (3) for all $t \leq T$. Replacing C_t and $\tilde{C}_t$ yields

$$\Delta_T \leq \mathbb{E}_0[\ \sum_{t=0}^{T} \beta^t\ [\ u'(C_t)(R_t^K(\tilde{K}_t - K_t) + (1 - \tau_t^B)B_t(\tilde{b}_{t-1} - b_{t-1}) - (\tilde{K}_{t+1} - K_{t+1})$$

$$-(\tilde{b}_t - b_t)P_t + W_t(\tilde{L}_t - L_t)) - v'(L_t)(\tilde{L}_t - L_t)\ ]\ ]$$

Decisions $\{C, L, K, b\}$ and $\{\tilde{C}, \tilde{L}, \tilde{K}, \tilde{b}\}$ respect the first order condition (5), (6), (7) for all $t \leq T$. Using the fact that $\tilde{K}_0 = K_0$ and $\tilde{b}_{-1} = b_{-1}$ yields

$$\Delta_T \leq -\mathbb{E}_0[\ \beta^T u'(C_T)\{(\tilde{K}_{T+1} - K_{T+1}) + (\tilde{b}_T - b_T)P_T\}\ ]$$

Using the debt limit (4) we get

$$\Delta_T \leq \mathbb{E}_0[\ \beta^T u'(C_T)\{K_{T+1} + b_T P_T + D\}\ ]$$

Since the (contingent) decisions C_T, K_{T+1}, and b_T are induced by the policy plan π^T obtained after all No Regret fiscal reforms weakly before T are announced, they are part of an equilibrium and so they respect the transversality condition (8) at time T

$$\Delta_T \leq \mathbb{E}_0[\ \beta^T u'(C_T)\{K_{T+1} + b_T P_T + D\}\ ] \leq M_T$$

This yields

$$\overline{\lim_{T\to\infty}}\ \Delta_T \leq 0$$

which means that $\{C, L, K, b\}$ is optimal. □

Counter-example of Lemma 1 without the transversality condition (8)

In an economy without shocks, denote C_{SS}, L_{SS}, K_{SS} the laissez-faire steady state values (i.e. $G_t = b_t = \tau_t^L = \tau_t^K = \tau_t^B = 0$ for all t). Assume $K_0 = K_{SS}$ and $b_{-1} = 0$. Facing the laissez-faire policy for ever, C_{SS}, L_{SS}, K_{SS} are the optimal decisions at all times.

For a small $\varepsilon > 0$, denote $C_0 = C_{SS} - \varepsilon$. Then, select L_0 such that $u'(C_0)F_L(K_0, L_0) = v'(L_0)$, and $K_1 \equiv F(K_0, L_0) - C_0$. Then, at the next period $t = 1$, select C_1 and L_1 such that $u'(C_0) = \beta F_K(K_1, L_0)u'(C_1)$ and $u'(C_1)F_L(K_1, L_1) = v'(L_1)$ and $K_1 \equiv F(K_1, L_1) - C_1 > K_0$. By doing this for all periods t, we construct $(C_t)_t, (L_t)_t$ and $(K_t)_t$.

For any period $T > 0$. Denote L_{SS}^T the solution to

$$u'(C_0)F_K(K_0, L_0)K_0 = \sum_{t=0}^{T} \beta^t\ [\ u'(C_t)C_t - v'(L_t)L_t\ ] + \sum_{t=T+1}^{\infty} \beta^t\ \left[\ u'(C_{SS})C_{SS} - v'(L_{SS}^T)L_{SS}^T\ \right]$$

Then, set $K_t^T = K_{SS}$ for all $t \geq T+1$ and select $(G_t^T)_{t\geq T+1}$ such that all the resource constraints (3) hold after time T. The initial implementability constraint (10) and the budget constraints (3) are met, so $(C_t)_{t\leq T}, (L_t)_{t\leq T}$ and $(K_t)_{t\leq T}$ are induced by some policy plan denoted π^T that implements the laissez-faire policy for all $t \leq T$ (but not after). At any time $T+1$, π^{T+1} is a No Regret reform of π^T since both induce the same decisions $(C_t)_{t\leq T}, (L_t)_{t\leq T}$ before T.

One can show that for all t, $C_t \leq C_{SS} - \varepsilon$, and $L_t \geq L_0 > L_{SS}$, which

implies that the induced decisions bring lower welfare to the household than the optimal decisions. Furthermore, combining the first order conditions (5), (6), and (7) with the household's budget constraint (3), which are met for all $t \leq T$, yields

$$u'(C_0)F_K(K_{SS}, L_0)K_{SS} = \sum_{t=0}^{T} \beta^t \left[u'(C_t)C_t - v'(L_t)L_t \right] + \beta^T u'(C_T)K_{T+1}$$

$$u'(C_{SS})F_K(K_{SS}, L_{SS})K_{SS} = \sum_{t=0}^{T} \beta^t \left[u'(C_{SS})C_{SS} - v'(L_{SS})L_{SS} \right] + \beta^T u'(C_{SS})K_{SS}$$

Furthemore, $v'(L_t)L_t \geq v'(L_0)L_0 > v'(L_{SS})L_{SS}$ and $u'(C_0)F_K(K_{SS}, L_0) > u'(C_{SS})F_K(K_{SS}, L_{SS})$. Assuming $u(C) = \frac{C^{1-\sigma}}{1-\sigma}$ with $0 < \sigma < 1$, yields $u'(C_t)C_t \leq u'(C_0)(C_0) < u'(C_{SS})C_{SS}$ and so

$$\beta^T u'(C_T)K_{T+1} \geq \beta^T u'(C_{SS})K_{SS} + \frac{1-\beta^{T+1}}{1-\beta} \left[u'(C_{SS})(C_{SS}) - u'(C_0)(C_0) - v'(L_{SS})L_{SS} + v'(L_0)L_0 \right]$$

and so

$$\lim_{T\to\infty} \beta^T u'(C_T)K_{T+1} \geq \frac{1}{1-\beta} \left[u'(C_{SS})(C_{SS}) - u'(C_0)(C_0) - v'(L_{SS})L_{SS} + v'(L_0)L_0 \right] > 0$$

This means that there exists a time T such that equilibrium decisions induced by the policy plan π^T violate the transversality condition (8). However, note that these induced decisions are constant after T and so meet this alternative (and more standard) transversality condition that, for all t

$$\mathbb{E}_t \left[\sum_{s=r}^{+\infty} \beta^s \Big[u'(C_s(\theta^s))C_s(\theta^s) - v'(L_s(\theta^s))L_s(\theta^s) \Big] \right] + \beta^r D$$

$$= \quad \mathbb{E}_t \left[\beta^r u'(C_r(\theta^r)) \Big[K_{r+1}(\theta^r) + b_r(\theta^r)P_r(\theta^r) + D \Big] \right] \quad \xrightarrow[r\to\infty]{} \quad 0$$

□

Proof of Lemma 2 (Partial Characterization of No Regret reforms)

Both $\{C, L, K\}$ and $\{\tilde{C}, \tilde{L}, \tilde{K}\}$ are implementable. Thus, they both meet the implementability constraint (9) at time $t = 0$

$$u'(C_0) \left[R_0^K K_0 + (1 - \tau_0^B)B_0 \right] = \mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t \left[u'(C_t(\theta^t))C_t(\theta^t) - v'(L_t(\theta^t))L_t(\theta^t) \right] \right]$$

$$u'(C_0)\left[\ R_0^K K_0 + (1-\tau_0^B)B_0\ \right] = \mathbb{E}\left[\ \sum_{t=0}^{\infty}\beta^t\ \left[\ u'(\tilde{C}_t(\theta^t))\tilde{C}_t(\theta^t) - v'(\tilde{L}_t(\theta^t))\tilde{L}_t(\theta^t)\ \right]\ \right]$$

The two right-hand-side terms can be decomposed into three terms as follows.

$$\mathbb{E}\left[\ \sum_{t=0}^{\infty}\beta^t\ \left[\ u'(C_t(\theta^t))C_t(\theta^t) - v'(L_t(\theta^t))L_t(\theta^t)\ \right]\ \right]$$

$$= \mathbb{P}\left[\ non\ \theta^{t_R}\ \right]\mathbb{E}\left[\ \sum_{t=0}^{\infty}\beta^t\ \left[\ u'(C_t(\theta^t))C_t(\theta^t) - v'(L_t(\theta^t))L_t(\theta^t)\ \right]\ \middle|\ non\ \theta^{t_R}\ \right]$$

$$+\mathbb{P}\left[\ \theta^{t_R}\ \right]\mathbb{E}\left[\ \sum_{t=0}^{t_R-1}\beta^t\ \left[\ u'(C_t(\theta^t))C_t(\theta^t) - v'(L_t(\theta^t))L_t(\theta^t)\ \right]\ \middle|\ \theta^{t_R}\ \right]$$

$$+\mathbb{P}\left[\ \theta^{t_R}\ \right]\mathbb{E}\left[\ \sum_{t=t_R}^{\infty}\beta^t\ \left[\ u'(C_t(\theta^t))C_t(\theta^t) - v'(L_t(\theta^t))L_t(\theta^t)\ \right]\ \middle|\ \theta^{t_R}\ \right]$$

By definition of a No Regret reform, all household's decisions taken before the realization of θ^{t_R} are unchanged by the reform. This implies

$$\mathbb{E}\left[\ \sum_{t=0}^{t_R-1}\beta^t\ \left[\ u'(C_t(\theta^t))C_t(\theta^t) - v'(L_t(\theta^t))L_t(\theta^t)\ \right]\ \middle|\ \theta^{t_R}\ \right]$$

$$= \mathbb{E}\left[\ \sum_{t=0}^{t_R-1}\beta^t\ \left[\ u'(\tilde{C}_t(\theta^t))\tilde{C}_t(\theta^t) - v'(\tilde{L}_t(\theta^t))\tilde{L}_t(\theta^t)\ \right]\ \middle|\ \theta^{t_R}\ \right]$$

Consider any realization $\hat{\theta}^{t_R} \neq \theta^{t_R}$. Let us denote by $t_C \leq t_R$ the first time at which $\theta^{t_C} \neq \hat{\theta}^{t_C}$. By definition of a No Regret reform after the realization θ^{t_R}, all induced decisions made strictly before time t_R are unchanged by this No Regret reform. Thus, with or without a No Regret reform after the realization θ^{t_R}, the household after $\hat{\theta}^{t_C}$ inherits the same debt holding and capital from the past. In addition, with or without a No Regret reform after the realization θ^{t_R}, this household faces the same policy plan after $\hat{\theta}^{t_C}$ since it is not changed by a reform after θ^{t_R}. Identical debt holding, capital and policy plan yield identical decisions after $\hat{\theta}^{t_C}$. This holds for any realization $\hat{\theta}^{t_R} \neq \theta^{t_R}$ so

$$\mathbb{P}\left[\ non\ \theta^{t_R}\ \right]\mathbb{E}\left[\ \sum_{t=0}^{\infty}\beta^t\ \left[\ u'(C_t(\theta^t))C_t(\theta^t) - v'(L_t(\theta^t))L_t(\theta^t)\ \right]\ \middle|\ non\ \theta^{t_R}\ \right]$$

$$= \mathbb{P}\left[\ non\ \theta^{t_R}\ \right]\mathbb{E}\left[\ \sum_{t=0}^{\infty}\beta^t\ \left[\ u'(\tilde{C}_t(\theta^t))\tilde{C}_t(\theta^t) - v'(\tilde{L}_t(\theta^t))\tilde{L}_t(\theta^t)\ \right]\ \middle|\ non\ \theta^{t_R}\ \right]$$

The last three equalities imply

$$\mathbb{E}\left[\sum_{t=t_R}^{\infty}\beta^t\ \left[\ u'(C_t(\theta^t))C_t(\theta^t)-v'(L_t(\theta^t))L_t(\theta^t)\ \right]\ \middle|\ \theta^{t_R}\right]$$

$$=$$

$$\mathbb{E}\left[\sum_{t=t_R}^{\infty}\beta^t\ \left[\ u'(\tilde{C}_t(\theta^t))\tilde{C}_t(\theta^t)-v'(\tilde{L}_t(\theta^t))\tilde{L}_t(\theta^t)\ \right]\ \middle|\ \theta^{t_R}\right]$$

Proof of Lemma 4 (Wealth heterogeneity and Equal Sacrifice)

The envelope conditions for the optimization problems of agents of skill ω facing $\{T(a,\omega), y_0(a,\omega), c_\theta(a,\omega), y_\theta(a,\omega), G_\theta\}_{a,\omega,\theta}$, the status quo direct and truthful mechanism, can be expressed in its integral form with $s \in [0,a]$

$$u(a-T(a,\omega))-v(y_0(a,\omega)/\omega)+\mathbb{E}\left[\ u(c_\theta(a,\omega))-v(y_\theta(a,\omega)/\omega)\ \right]$$

$$\begin{aligned}=\quad & u(0-T(0,\omega))-v(y_0(0,\omega)/\omega)+\mathbb{E}\left[\ u(c_\theta(0,\omega))-v(y_\theta(0,\omega)/\omega)\ \right]\\ &+\quad \int_{s=0}^{a}u'(s-T(s,\omega))\,ds\end{aligned}$$

By Lemma 3, the reformed mechanism is also truthful, so the envelope condition is

$$\begin{aligned}& u(a-T(a,\omega))-v(y_0(a,\omega)/\omega)+\sum_{\theta'\neq\theta}\mathbb{P}\left[\ \theta'\ \right]\left[\ u(c_{\theta'}(a,\omega))-v(y_{\theta'}(a,\omega)/\omega)\ \right]\\ &\qquad +\quad \mathbb{P}\left[\ \theta\ \right]\left[\ u(\tilde{c}_\theta(a,\omega))-v(\tilde{y}_\theta(a,\omega)/\omega)\ \right]\\ =\quad & u(0-T(0,\omega))-v(y_0(0,\omega)/\omega)+\sum_{\theta'\neq\theta}\mathbb{P}\left[\ \theta'\ \right]\left[\ u(c_{\theta'}(0,\omega))-v(y_{\theta'}(0,\omega)/\omega)\ \right]\\ &\qquad +\quad \mathbb{P}\left[\ \theta\ \right]\left[\ u(\tilde{c}_\theta(0,\omega))-v(\tilde{y}_\theta(0,\omega)/\omega)\ \right]\\ &\qquad +\quad \int_{s=0}^{a}u'(s-T(s,\omega))\,ds\end{aligned}$$

Subtracting both equalities yield after, some simplifications,

$$\mathbb{P}\left[\ \theta\ \right]\left[\ u(c_\theta(a,\omega))-v(y_\theta(a,\omega)/\omega)\ \right]-\mathbb{P}\left[\ \theta\ \right]\left[\ u(\tilde{c}_\theta(a,\omega))-v(\tilde{y}_\theta(a,\omega)/\omega)\ \right]$$

$$=\quad \mathbb{P}\left[\ \theta\ \right]\left[\ u(c_\theta(0,\omega))-v(y_\theta(0,\omega)/\omega)\ \right]-\mathbb{P}\left[\ \theta\ \right]\left[\ u(\tilde{c}_\theta(0,\omega))-v(\tilde{y}_\theta(0,\omega)/\omega)\ \right]$$

Using the fact that $\mathbb{P}[\,\theta\,] \neq 0$, and setting

$$H(\omega) \equiv \left[\, u(\tilde{c}_\theta(0,\omega)) - v(\tilde{y}_\theta(0,\omega)/\omega) \,\right] - \left[\, u(c_\theta(0,\omega)) - v(y_\theta(0,\omega)/\omega) \,\right]$$

We get, that for any wealth level a,

$$\left[\, u(\tilde{c}_\theta(0,\omega)) - v(\tilde{y}_\theta(0,\omega)/\omega) \,\right] = \left[\, u(c_\theta(0,\omega)) - v(y_\theta(0,\omega)/\omega) \,\right] + H(\omega)$$

□

Online Supplemental Appendix

Necessity of (5), (6), and (7)

Assume $\{C, L, K, b\}$ is optimal. It is budget-feasible so (3) holds.
We build small deviations of $\{C, L, K, b\}$ to establish (5),(6),(7) by contradiction.
(5) : If $v'(L_t) < W_t u'(C_t)$ then there is $\varepsilon > 0$ such that a deviation $\tilde{L}_t = L_t + \varepsilon$ and $\tilde{C}_t = C_t + W_t\varepsilon$ yields higher welfare to the households and the budget constraint (3) still holds, contradicting optimality. The reverse inequality can be ruled out with $\varepsilon < 0$.
(6) : If $u'(C_t) < \mathbb{E}_t[\, \beta R_{t+1} u'(C_{t+1})\,]$ then there is $\varepsilon > 0$ such that a deviation with $\tilde{K}_{t+1} = K_{t+1} + \varepsilon$, $\tilde{C}_t = C_t - \varepsilon$ and $\tilde{C}_{t+1} = C_{t+1} + R_{t+1}\varepsilon$ yields higher welfare to the households and the budget constraint (3) still holds, contradicting optimality. The reverse inequality can be ruled out with $\varepsilon < 0$.
(7) : $P_t < \frac{\mathbb{E}_t[\, \beta B_{t+1} u'(C_{t+1})\,]}{u'(C_t)}$ then there is $\varepsilon > 0$ such that a deviation with $\tilde{b}_t = b_t + \varepsilon$, $\tilde{C}_t = C_t - \varepsilon P_t$ and $\tilde{C}_{t+1} = C_{t+1} + B_{t+1}\varepsilon$ yields higher welfare to the households and (3) still holds, contradicting optimality. The reverse inequality can be ruled out with $\varepsilon < 0$. □

Sufficiency of (5), (6), (7), and (8)

Assume household's decisions $\{C, L, K, b\}$ are such that (5), (6), (7), and (8) hold. Denote $\{\tilde{C}, \tilde{L}, \tilde{K}, \tilde{b}\}$ some optimal decisions. They must also satisfy the necessary conditions (5), (6), (7). Denote the welfare difference before T

$$\Delta_T \equiv \mathbb{E}_0[\ \sum_{t=0}^{T} \beta^t\ [\ u(\tilde{C}_t) - v(\tilde{L}_t)\ ]\ ]\ -\ \mathbb{E}_0[\ \sum_{t=0}^{T} \beta^t\ [\ u(C_t) - v(L_t)\ ]\ ]$$

By concavity of u and $-v$,

$$\Delta_T \leq \mathbb{E}_0[\ \sum_{t=0}^{T} \beta^t\ [\ u'(C_t)(\tilde{C}_t - C_t) - v'(L_t)(\tilde{L}_t - L_t)\ ]\ ]$$

and using the budget constraints (3) to replace C_t and $\tilde{C}_t$

$$\Delta_T \leq \mathbb{E}_0[\ \sum_{t=0}^{T} \beta^t\ [\ u'(C_t)(R_t^K(\tilde{K}_t - K_t) + (1 - \tau_t^B)B_t(\tilde{b}_{t-1} - b_{t-1}) - (\tilde{K}_{t+1} - K_{t+1})$$

$$-(\tilde{b}_t - b_t)P_t + W_t(\tilde{L}_t - L_t)) - v'(L_t)(\tilde{L}_t - L_t)\ ]\ ]$$

Using (5), (6), (7), and the fact that $\tilde{K}_0 = K_0$ and $\tilde{b}_{-1} = b_{-1}$, we obtain

$$\Delta_T \leq -\mathbb{E}_0[\ \beta^T u'(C_T)\{(\tilde{K}_{T+1} - K_{T+1}) + (\tilde{b}_T - b_T)P_T\}\ ]$$

$$\Delta_T \leq \mathbb{E}_0[\ \beta^T u'(C_T)\{K_{T+1} + b_T P_T + D\}\ ]$$

By assumption, the transversality condition (8) holds, and so

$$\overline{\lim_{T\to\infty}}\ \Delta_T \leq 0$$

which means that $\{C, L, K, b\}$ is optimal. □

Lemma 5. *In the Ramsey model, the Second-Best policy is time-inconsistent.*

Proof:

Let us assume *ad absurdo* that it is time-consistent, i.e. that any future government with full discretion to reform will choose noy to reform this policy. In particular, at time $t = 1$, when shock θ^1 realized, a government with full discretion may tax capital and bonds without any limit and therefore can set household's wealth $R_1^K(\theta^1)K_1(\theta^1) + (1 - \tau_1^B(\theta^1))B_1(\theta^1)$ at any value, making the implementabilty constraint (9) non-binding at time $t = 1$. Consequently, it solves the following First Best problem:

$$\max_{\substack{\{C_t(\theta^t), L_t(\theta^t),\\ G_t(\theta^t), K_{t+1}(\theta^t)\}_\theta}} \mathbb{E}_1\left[\sum_{t=1}^{\infty}\beta^t\ \left[\ u(C_t(\theta^t)) - v(L_t(\theta^t)) + \theta_t w(G_t(\theta^t))\ \right]\right] \tag{30}$$

$$\text{s.t.}\quad C_t(\theta^t) + G_t(\theta^t) + K_{t+1}(\theta^t) = F(K_t(\theta^{t-1}), L_t(\theta^t))\ \text{ for all } \theta^t \text{ that follows } \theta^1$$

When comparing the first order conditions for $C_1(\theta^1)$ and $L_1(\theta^1)$ in the two problems (11) and (30), we find:

$$\left[\ u'(C_1(\theta^1)) + \mu\left[\ u'(C_1(\theta^1)) + u''(C_1(\theta^1))C_1(\theta^1)\ \right]\right]F_L(K_1(\theta^0), L_1(\theta^1))$$

$$= \quad v'(L_1(\theta^1)) + \mu\left[\ v'(L_1(\theta^1)) + v''(L_1(\theta^1))L_1(\theta^1)\ \right]$$

$$\text{and}\quad u'(C_1(\theta^1))F_L(K_1(\theta^0), L_1(\theta^1)) = v'(L_1(\theta^1))$$

where μ is the Lagrange multiplier associated to the initial implementability constraint (10), so, unless it is non-binding, theses two first order conditions cannot hold for identical $C_1(\theta^1)$ and $L_1(\theta^1)$. This is a contradiction.

□

Lemma 6. ***(Existence of a No Regret reform satisfying Equation (13))***
Consider a fiscal reform at time t_R after the realization of the shocks θ^{t_R}. Denote by $\{C, L, K\}$ the household's decisions induced by the status quo policy plan π, and by $\{\tilde{C}, \tilde{L}, \tilde{K}\}$ the household's decisions induced by the reformed policy plan $\tilde{\pi}$.
If Equation (13) holds, then there exists a No Regret fiscal reform of the status quo policy plan π after the realization of θ^{t_R} such that the reformed policy plan $\overline{\pi}$ induces household's decisions $\{\overline{C}, \overline{L}, \overline{K}\}$ that are identical to $\{\tilde{C}, \tilde{L}, \tilde{K}\}$ after the realization of θ^{t_R}.

Proof: Assume that the fiscal reform is such that the following equality holds

$$\mathbb{E}\left[\sum_{t=t_R}^{\infty}\beta^t\ \left[\ u'(C_t(\theta^t))C_t(\theta^t) - v'(L_t(\theta^t))L_t(\theta^t)\ \right]\ \middle|\ \theta^{t_R}\right]$$

$$=$$

$$\mathbb{E}\left[\sum_{t=t_R}^{\infty}\beta^t\ \left[\ u'(\tilde{C}_t(\theta^t))\tilde{C}_t(\theta^t) - v'(\tilde{L}_t(\theta^t))\tilde{L}_t(\theta^t)\ \right]\ \middle|\ \theta^{t_R}\right]$$

Let us build a No Regret fiscal reform after the realization of θ^{t_R}, i.e. a reform such that the new policy plan $\overline{\pi}$ induces household's decisions $\{\overline{C}, \overline{L}, \overline{K}\}$ that are identical to $\{\tilde{C}, \tilde{L}, \tilde{K}\}$ after the realization of θ^{t_R} and identical to $\{C, L, K\}$ everywhere else, which implies that the fiscal reform is a No Regret fiscal reform.

For all times $t \geq t_R$, and for all realizations of θ^t that follows the realization of θ^{t_R}, we define the policy plan $\overline{\pi} \equiv \{\overline{G}, \overline{T}, \overline{B}, \overline{\tau}^L, \overline{\tau}^K, \overline{\tau}^B\}$ as follows.
1) Public spending is set in order to meet all the resource constraints.

$$\overline{G}_t(\theta^t) \equiv F(\overline{K}_t(\theta^{t-1}), \overline{L}_t(\theta^t)) - \overline{C}_t(\theta^t) - \overline{K}_{t+1}(\theta^t)$$

2) When $t = t_R$, *i.e.* just after the realization of θ^{t_R}, we set

$$\overline{\tau}^K_{t_R}(\theta^{t_R}) \equiv 1 - (1 - \tau^K_{t_R}(\theta^{t_R}))\frac{u'(C_{t_R}(\theta^{t_R})F_K(K_{t_R}(\theta^{t_R-1}), L_{t_R}(\theta^{t_R}))}{u'(\overline{C}_{t_R}(\theta^{t_R})F_K(\overline{K}_{t_R}(\theta^{t_R-1}), \overline{L}_{t_R}(\theta^{t_R}))}$$

$$\overline{\tau}^B_{t_R}(\theta^{t_R}) \equiv 1 - (1 - \tau^B_{t_R}(\theta^{t_R}))\frac{u'(C_{t_R}(\theta^{t_R})}{u'(\overline{C}_{t_R}(\theta^{t_R})}$$

$$\overline{R}^K_{t_R}(\theta^{t_R}) \equiv (1-\overline{\tau}^K_{t_R}(\theta^{t_R}))F_K(\overline{K}_{t_R}(\theta^{t_R-1}), \overline{L}_{t_R}(\theta^{t_R})) \quad ; \quad \overline{R}^B_{t_R}(\theta^{t_R}) \equiv \frac{1 - \overline{\tau}^B_{t_R}(\theta^{t_R})}{P_{t_R-1}}$$

Under the *status quo* policy plan π, the Euler equation for capital investment one period prior to the realization of θ^{t_R} is

$$u'(C_{t_R-1}(\theta^{t_R-1})) = \mathbb{E}_{t_R-1}\left[\ \beta R^K_{t_R}(\theta^{t_R})u'(C_{t_R}(\theta^{t_R}))\ \right]$$

$$= \sum_{\hat{\theta}_{t_R}\neq\theta_{t_R}} \mathbb{P}(\ \hat{\theta}_{t_R}\mid\theta^{t_R-1}\ )\beta R^K_{t_R}((\theta^{t_R-1},\hat{\theta}_{t_R}))u'(C_{t_R}((\theta^{t_R-1},\hat{\theta}_{t_R})))+\mathbb{P}(\ \theta_{t_R}\mid\theta^{t_R-1}\ )\beta R^K_{t_R}(\theta^{t_R})u'(C_{t_R}(\theta^{t_R}))$$

$\overline{\tau}^K_{t_R}(\theta^{t_R})$ and $\overline{R}^K_{t_R}(\theta^{t_R})$ were defined in order to have

$$R^K_{t_R}(\theta^{t_R})u'(C_{t_R}(\theta^{t_R})) = \overline{R}^K_{t_R}(\theta^{t_R})u'(\overline{C}_{t_R}(\theta^{t_R}))$$

π and $\{C, L, K\}$ are identical to $\overline{\pi}$ and $\{\overline{C}, \overline{L}, \overline{K}\}$ for all realizations of $\hat{\theta}_{t_R} \neq \theta_{t_R}$. Thus,

$$u'(C_{t_R-1}(\theta^{t_R-1})) = u'(\overline{C}_{t_R-1}(\theta^{t_R-1})) = \mathbb{E}_{t_R-1}\left[\ \beta\overline{R}^K_{t_R}(\theta^{t_R})u'(\overline{C}_{t_R}(\theta^{t_R}))\ \right]$$

which means that $\{\overline{C}, \overline{L}, \overline{K}\}$ meet the Euler equation for capital investment one period prior to the realization of θ^{t_R}. Similarly, $\overline{\tau}^B_{t_R}(\theta^{t_R})$ and $\overline{R}^B_{t_R}(\theta^{t_R})$ were defined in order to meet the Euler equation for bond investment one period prior to the realization of θ^{t_R}.

3) When $t > t_R$, in the periods following the realization of θ^{t_R}, we set taxes on labour and capital

$$\overline{\tau}^L_t(\theta^t) \equiv 1-\frac{v'(\overline{L}_t(\theta^t))}{F_L(\overline{K}_t(\theta^{t-1}),\overline{L}_t(\theta^t))u'(\overline{C}_t(\theta^t))} \quad \text{and} \quad \overline{\tau}^K_t(\theta^t) \equiv 1-\frac{u'(\overline{C}_{t-1}(\theta^{t-1}))}{\beta F_K(\overline{K}_t(\theta^{t-1}),\overline{L}_t(\theta^t))u'(\overline{C}_t(\theta^t))}$$

and the net-of-tax prices

$$\overline{W}_t(\theta^t) \equiv (1-\overline{\tau}^L_t(\theta^t))F_L(\overline{K}_t(\theta^{t-1}),\overline{L}_t(\theta^t)) \quad \text{and} \quad \overline{R}^K_t(\theta^t) \equiv (1-\overline{\tau}^K_t(\theta^t))F_K(\overline{K}_t(\theta^{t-1}),\overline{L}_t(\theta^t))$$

in order to meet the intra-period optimality condition and the Euler equation for capital investment. Finally, we set bond net-of-tax contingent payoffs

$$\overline{B}_t(\theta^{t-1})(1-\overline{\tau}^B_t(\theta^t)) = \frac{\mathbb{E}_t[\ \sum_{s=0}^{\infty}\beta^s\ [\ u'(\overline{C}_{t+s}(\theta^{t+s}))\overline{C}_{t+s}(\theta^{t+s}) - v'(\overline{L}_{t+s}(\theta^{t+s}))\overline{L}_{t+s}(\theta^{t+s})\ ]\ ]}{u'(\overline{C}_t(\theta^t))}$$

$$-(1-\overline{\tau}^K_t)F_K(\overline{K}_t(\theta^{t-1}),\overline{L}_t(\theta^t))\overline{K}_t(\theta^{t-1})$$

and also bond prices and taxes

$$\overline{P}_t(\theta^t) \equiv \frac{\mathbb{E}_t[\ \beta \overline{B}_{t+1}(\theta^t)(1-\overline{\tau}^B_{t+1}(\theta^{t+1}))u'(\overline{C}_{t+1}(\theta^{t+1}))\ ]}{u'(\overline{C}_t(\theta^t))}$$

in order to meet the Euler equation for bond investment and all the budget constraints. Indeed, without the reform, we have

$$u'(C_{t_R}(\theta^{t_R}))\ [\ R^K_{t_R}(\theta^{t_R})K_{t_R}(\theta^{t_R}) + (1-\tau^B_{t_R}(\theta^{t_R}))B_{t_R}(\theta^{t_R-1})\ ]$$
$$= \mathbb{E}\left[\ \sum_{t=t_R}^{\infty} \beta^t\ \ [\ u'(C_t(\theta^t))C_t(\theta^t) - v'(L_t(\theta^t))L_t(\theta^t)\ ]\ \ \middle|\ \theta^{t_R}\right]$$

Since

$$R^K_{t_R}(\theta^{t_R})u'(C_{t_R}(\theta^{t_R})) = \overline{R}^K_{t_R}(\theta^{t_R})u'(\overline{C}_{t_R}(\theta^{t_R}))$$
$$(1-\tau^B_{t_R}(\theta^{t_R}))u'(C_{t_R}(\theta^{t_R})) = (1-\overline{\tau}^B_{t_R}(\theta^{t_R}))u'(\overline{C}_{t_R}(\theta^{t_R}))$$

we have

$$u'(C_{t_R}(\theta^{t_R}))\ [\ R^K_{t_R}(\theta^{t_R})K_{t_R}(\theta^{t_R}) + (1-\tau^B_{t_R}(\theta^{t_R}))B_{t_R}(\theta^{t_R-1})\ ]$$
$$= u'(\overline{C}_{t_R}(\theta^{t_R}))\ [\ R^K_{t_R}(\theta^{t_R})\overline{K}_{t_R}(\theta^{t_R}) + (1-\overline{\tau}^B_{t_R}(\theta^{t_R}))\overline{B}_{t_R}(\theta^{t_R})\ ]$$

and so, by assumption,

$$A_{t_R} \equiv u'(\overline{C}_{t_R}(\theta^{t_R}))\ [\ R^K_{t_R}(\theta^{t_R})\overline{K}_{t_R}(\theta^{t_R}) + (1-\overline{\tau}^B_{t_R}(\theta^{t_R}))\overline{B}_{t_R}(\theta^{t_R})\ ]$$
$$= \mathbb{E}\left[\ \sum_{t=t_R}^{\infty} \beta^{t-t_R}\ \ [\ u'(\overline{C}_t(\theta^t))\overline{C}_t(\theta^t) - v'(\overline{L}_t(\theta^t))\overline{L}_t(\theta^t)\ ]\ \ \middle|\ \theta^{t_R}\right]$$
$$A_{t_R} = u'(\overline{C}_{t_R}(\theta^{t_R}))\ [\ \overline{C}_{t_R}(\theta^{t_R}) - \overline{W}_{t_R}(\theta^{t_R})\ ]$$
$$+\beta\mathbb{E}\left[\ \sum_{t=t_R+1}^{\infty} \beta^{t-t_R-1}\ \ [\ u'(\overline{C}_t(\theta^t))\overline{C}_t(\theta^t) - v'(\overline{L}_t(\theta^t))\overline{L}_t(\theta^t)\ ]\ \ \middle|\ \theta^{t_R}\right]$$

Let us study the last term using the definitions of $\overline{B}_{t_R+1}(\theta^{t_R+1})$ and $P_{t_R}(\theta^{t_R})$

$$\beta\mathbb{E}\left[\ \sum_{t=t_R+1}^{\infty} \beta^{t-t_R-1}\ \ [\ u'(\overline{C}_t(\theta^t))\overline{C}_t(\theta^t) - v'(\overline{L}_t(\theta^t))\overline{L}_t(\theta^t)\ ]\ \ \middle|\ \theta^{t_R}\right]$$

$$= \beta\mathbb{E}\left[\ u'(\overline{C}_{t_R+1}(\theta^{t_R+1}))\ [\ \overline{B}_{t_R+1}(\theta^{t_R+1}) + (1-\overline{\tau}^K_{t_R+1})F_K(\overline{K}_{t_R+1}(\theta^{t_R}), \overline{L}_{t_R+1}(\theta^{t_R+1}))\overline{K}_{t_R+1}(\theta^{t_R})\ ]\ \ \middle|\ \theta^{t_R}\ \right]$$

$$= u'(\overline{C}_{t_R}(\theta^{t_R}))\ \left[\ P_{t_R}(\theta^{t_R}) + \overline{K}_{t_R+1}(\theta^{t_R})\ \right]$$

Thus, we get

$$u'(\overline{C}_{t_R}(\theta^{t_R}))\ \left[\ R^K_{t_R}(\theta^{t_R})\overline{K}_{t_R}(\theta^{t_R}) + (1-\overline{\tau}^B_{t_R}(\theta^{t_R}))\overline{B}_{t_R}(\theta^{t_R-1})\ \right]$$

$$= u'(\overline{C}_{t_R}(\theta^{t_R}))\ \left[\ \overline{C}_{t_R}(\theta^{t_R}) - \overline{W}_{t_R}(\theta^{t_R}) + P_{t_R}(\theta^{t_R}) + \overline{K}_{t_R+1}(\theta^{t_R})\ \right]$$

Simplifying by $u'(\overline{C}_{t_R}(\theta^{t_R}))$ yields the budget constraint at time t_R. Similarly, all the budget constraints after time t_R can be shown to be met.

$\{\tilde{C}, \tilde{L}, \tilde{K}\}$ is optimal for the households, so the transversality condition holds for $\{\overline{C}, \overline{L}, \overline{K}\}$. □

Proof of Proposition 2 (The Second-Best mechanism is time-inconsistent)

Assume *ad absurdo* that $\{T(a,\omega), y_0(a,\omega), c_\theta(a,\omega), y_\theta(a,\omega), G_\theta\}_{a,\omega,\theta}$, the optimal contingent mechanism, is time-consistent. At $t=1$ after any realization θ, and since all the types (a,ω) were revealed at $t=0$, the government wants to implement the solution to the following problem.

$$\max_{\substack{\{\tilde{c}_\theta(a,\omega)\}_{a,\omega},\\ \{\tilde{y}_\theta(a,\omega)\}_{a,\omega},\\ \tilde{G}_\theta}} \int \left[u(\tilde{c}_\theta(a,\omega)) - v(\tilde{y}_\theta(a,\omega)/\omega)\right]\ dF(\omega,a) + \theta w(\tilde{G}_\theta)$$

$$\text{s.t.} \quad \int \tilde{c}_\theta(a,\omega)\ dF(\omega,a)\ +\ \tilde{G}_\theta \leq \int \left[\ T(a,\omega) + y_0(a,\omega) + \tilde{y}_\theta(a,\omega)\ \right]\ dF(\omega,a)$$

Since the optimal contingent mechanism is time-consistent, it must meet the the following first order conditions for any type (a,ω)

$$\theta w'(\tilde{G}_\theta) = u'(c_\theta(a,\omega)) = \frac{1}{\omega} v'(y_\theta(a,\omega)/\omega)$$

So all agents consume the same amount $\overline{c}_\theta$ and have the same marginal disutility of production. Since $v : x \to \frac{x^{1+\chi}-1}{1+\chi}$, we note that $y_\theta(a,\omega) = \overline{y}(\omega) = \omega^{1+1/\chi} u'(\overline{c}_\theta)^{1/\chi}$, whic is increasing in ω.

Now we show that it is optimal to smooth labor efforts, *i.e.* that, for any type (a,ω),

we have

$$\frac{1}{\omega}v'(y_0(a,\omega)/\omega) = \mathbb{E}\left[\frac{1}{\omega}v'(y_\theta(a,\omega)/\omega)\right]$$

Assume *ad absurdo* that there exists A, a positive measure subset of types (a,ω), such that (the case $>$ is similar)

$$\frac{1}{\omega}v'(y_0(a,\omega)/\omega) < \mathbb{E}\left[\frac{1}{\omega}v'(y_\theta(a,\omega)/\omega)\right] \quad \textit{i.e.} \quad y_0(a,\omega)^\chi < \mathbb{E}\left[\, y_\theta(a,\omega)^\chi \,\right]$$

Consider the following resource-feasible deviation from the optimal contingent mechanism, aimed at restoring the previous equality.

For all $(a,\omega) \in A \quad y_0(a,\omega) \to y_0(a,\omega)(1+\varepsilon) \qquad y_\theta(a,\omega) \to y_\theta(a,\omega)(1-\eta_\varepsilon(a,\omega))$

$$G_\theta \to G_\theta + \int_A y_0(a,\omega)\,\varepsilon\,dF(\omega,a) - \int_A y_\theta(a,\omega)\,\eta_\varepsilon(a,\omega)\,dF(\omega,a)$$

and then we pin down $\eta_\varepsilon(a,\omega)$ as a function of $\varepsilon > 0$ to make sure expected utilities are unchanged, *i.e.*

$$v(y_0(a,\omega)/\omega)+\mathbb{E}\left[\, v(y_\theta(a,\omega)/\omega)\,\right] = v(y_0(a,\omega)(1+\varepsilon)/\omega)+\mathbb{E}\left[\, v(y_\theta(a,\omega)(1-\eta_\varepsilon(a,\omega))/\omega)\,\right]$$

$$\textit{i.e.} \quad y_0(a,\omega)^{1+\chi}+\mathbb{E}\left[\, y_\theta(a,\omega)^{1+\chi}\,\right] = \quad y_0(a,\omega)^{1+\chi}(1+\varepsilon)^{1+\chi}+\mathbb{E}\left[\, y_\theta(a,\omega)^{1+\chi}\,\right](1-\eta_\varepsilon(a,\omega))^{1+\chi}$$

and so

$$(1-\eta_\varepsilon(a,\omega)) = \left[\frac{y_0(a,\omega)^{1+\chi} + \mathbb{E}\left[\, y_\theta(a,\omega)^{1+\chi}\,\right] - y_0(a,\omega)^{1+\chi}(1+\varepsilon)^{1+\chi}}{\mathbb{E}\left[\, y_\theta(a,\omega)^{1+\chi}\,\right]}\right]^{1/(1+\chi)} < 1$$

So

$$\eta_\varepsilon(a,\omega) = \frac{y_0(a,\omega)^{1+\chi}}{\mathbb{E}\left[\, y_\theta(a,\omega)^{1+\chi}\,\right]}\varepsilon + \circ(\varepsilon)$$

We notice that, thanks to the assumption on $v(\cdot)$, the utility levels of types (a',ω') who misreport (a,ω) are unchanged as well, so this small deviation of the optimal contingent mechanism is incentive compatible. Finally, we need to show that the first order effect (in ε) on the government's objective of this small deviation is positive *i.e.*

$$\mathbb{E}\left[\, \theta w'(G_\theta)\Big[\, \int_A y_0(a,\omega)\,\varepsilon\,dF(\omega,a) - \int_A y_\theta(a,\omega)\,\eta_\varepsilon(a,\omega)\,dF(\omega,a)\,\Big]\right] \quad > 0$$

Using the fact that, for any type (a,ω), $\theta w'(G_\theta) = \frac{1}{\omega}v'(y_\theta(a,\omega)/\omega) = \frac{1}{\omega^{1+\chi}}y_\theta(a,\omega)^\chi$,

we need to show

$$O \equiv \mathbb{E}\left[\int_A \frac{1}{\omega^{1+\chi}} y_\theta(a,\omega)^\chi \left[\; y_0(a,\omega) - y_\theta(a,\omega) \, \frac{y_0(a,\omega)^{1+\chi}}{\mathbb{E}\left[\, y_\theta(a,\omega)^{1+\chi} \, \right]} \right] dF(\omega,a) \; \right]$$

By definition of the subset A, for all $(a,\omega) \in A$, $y_0(a,\omega)^\chi < \mathbb{E}\left[\, y_\theta(a,\omega)^\chi \, \right]$, so

$$O \quad > \quad \int_A \frac{y_0(a,\omega)}{\omega^{1+\chi}} \left[\; \mathbb{E}\left[\, y_\theta(a,\omega)^\chi \, \right] - \frac{\mathbb{E}\left[\, y_\theta(a,\omega)^{1+\chi} \, \right] \mathbb{E}\left[\, y_\theta(a,\omega)^\chi \, \right]}{\mathbb{E}\left[\, y_\theta(a,\omega)^{1+\chi} \, \right]} \right] dF(\omega,a) = 0$$

Thus, we have shown $\frac{1}{\omega} v'(y_0(a,\omega)/\omega) = \mathbb{E}\left[\, \frac{1}{\omega} v'(y_\theta(a,\omega)/\omega) \, \right]$, for almost all types (a,ω). This equality pins down $y_0(a,\omega)$ as a function of $\{y_\theta(a,\omega)\}_\theta$. Since for all (a,ω) and θ, $y_\theta(a,\omega) = \overline{y}_\theta(\omega)$, $y_0(a,\omega)$ is invariant with a and we denote $\overline{y}_0(\omega) \equiv y_0(a,\omega)$.

$T(a,\omega)$ cannot be the only variable varying with a (no agents would choose to report an a associated with high taxes $T(a,\omega)$) so we denote $\overline{T}(\omega) \equiv T(a,\omega)$. Since, $\overline{y}_0(\omega)$ and $\overline{y}_\theta(\omega)$ are increasing in ω, truthful reporting implies that $\overline{T}(\omega)$ is decreasing in ω. Thus, $\overline{T}(\omega)$, $\overline{y}_0(\omega)$, and $\overline{y}_\theta(\omega)$ are monotone and therefore differentiable for almost all ω. Take such an ω and consider $a^- < a^+$. The first order condition w.r.t the reported ω' for type (a,ω) is

$$-\overline{T}'(\omega) u'(a - \overline{T}(\omega)) - \frac{\overline{y}_0'(\omega)}{\omega} v'(\overline{y}_0(\omega)/\omega) - \mathbb{E}\left[\frac{\overline{y}_\theta'(\omega)}{\omega} v'(\overline{y}_\theta(\omega)/\omega) \right] = 0$$

This equality holds for any $a^- \neq a^+$, so $\overline{T}'(\omega) u'(a^- - \overline{T}(\omega)) = \overline{T}'(\omega) u'(a^+ - \overline{T}(\omega))$. This requires $\overline{T}'(\omega) = 0$ for almost all ω, which is a contradiction since $\overline{T}'(\omega)$ is decreasing. □

The Revelation Principle under the No Regret rule

We consider a general mechanism such that agents can send messages $m_0 \in M_0$ at $t = 0$ and messages $m_\theta \in M_\theta$ at $t = 1$ after any realization θ. The functions of the mechanism are denoted at $t = 0$

$$T : \left\{ \begin{array}{ccc} \mathbb{M}_0 & \to & \mathbb{R} \\ m_0 & \mapsto & T(m_0) \end{array} \right. \qquad y_0 : \left\{ \begin{array}{ccc} \mathbb{M}_0 & \to & \mathbb{R} \\ m_0 & \mapsto & y(m_0) \end{array} \right.$$

and at $t = 1$ after any realization θ

$$c_\theta : \begin{cases} \mathbb{M}_0 \times \mathbb{M}_\theta & \to & \mathbb{R} \\ (m_0, m_\theta) & \mapsto & c_\theta(m_0, m_\theta) \end{cases} \qquad y_\theta : \begin{cases} \mathbb{M}_0 \times \mathbb{M}_\theta & \to & \mathbb{R} \\ (m_0, m_\theta) & \mapsto & y_\theta(m_0, m_\theta) \end{cases}$$

This mechanism induces agents to choose their message-functions at $t = 0$ and at $t = 1$ after any realization θ to maximize their expected utility depending on their type (a, ω) and on their past messages:

$$m_0 : \begin{cases} [0, \overline{a}] \times [0, \overline{\omega}] & \to & \mathbb{M}_0 \\ (a, \omega) & \mapsto & m_0(a, \omega) \end{cases} \qquad m_\theta : \begin{cases} [0, \overline{a}] \times [0, \overline{\omega}] \times \mathbb{M}_0 & \to & \mathbb{M}_\theta \\ (a, \omega, m_0) & \mapsto & m_\theta(a, \omega, m_0) \end{cases}$$

For any agent type (a, ω), it is optimal to send messages $m_0(a, \omega)$ and $m_\theta(a, \omega, m_0(a, \omega))$ for any realization θ. Other messages m'_0 and $\{m'_\theta\}_\theta \in \mathbb{M}_1 \equiv \mathbb{M}_{\theta_1} \times \mathbb{M}_{\theta_2} \times \ ...$ would yield a welfare loss. We denote this avoided utility loss by

$$\Delta_{(a,\omega)}\Big(m_0(a,\omega), \{m_\theta(a,\omega,m_0(a,\omega))\}_\theta, m'_0, \{m'_\theta\}_\theta\Big) \equiv u\big(a - T(m_0(a,\omega))\big) - v\big(y_0(m_0(a,\omega))/\omega\big)$$

$$+ \quad \mathbb{E}\Big[\, u\big(c_\theta(m_0(a,\omega), m_\theta(a,\omega,m_0(a,\omega)))\big) - v\big(y_\theta(m_0(a,\omega), m_\theta(a,\omega,m_0(a,\omega)))/\omega\big) \,\Big]$$

$$- \quad \Big[\, u\big(a - T(m'_0)\big) - v\big(y_0(m'_0)/\omega\big) \,\Big]$$

$$- \quad \mathbb{E}\Big[\, u\big(c_\theta(m'_0, m'_\theta)\big) - v\big(y_\theta(m'_0, m'_\theta)/\omega\big) \,\Big]$$

A reform of the mechanism at $t = 1$ after any realization θ takes place when these two last functions are replaced by two other functions, possibly with two new message spaces.

$$\tilde{c}_\theta : \begin{cases} \mathbb{M}_0 \times \tilde{\mathbb{M}}_\theta & \to & \mathbb{R} \\ (m_0, \tilde{m}_\theta) & \mapsto & \tilde{c}_\theta(m_0, m_\theta) \end{cases} \qquad \tilde{y}_\theta : \begin{cases} \mathbb{M}_0 \times \tilde{\mathbb{M}}_\theta & \to & \mathbb{R} \\ (m_0, \tilde{m}_\theta) & \mapsto & \tilde{y}_\theta(m_0, m_\theta) \end{cases}$$

Facing this reformed mechanism, agents use a new message-function at time $t = 1$ after the realization θ

$$\tilde{m}_\theta : \begin{cases} [0, \overline{a}] \times [0, \overline{\omega}] \times \mathbb{M}_0 & \to & \tilde{\mathbb{M}}_\theta \\ (a, \omega, m_0) & \mapsto & \tilde{m}_\theta(a, \omega, m_0) \end{cases}$$

This reform at $t = 1$ after the realization θ is called a No Regret reform when,

for any type (a, ω), and for any messages $(m_0', \{m_\theta\}_\theta) \in \mathbb{M}_0 \times \mathbb{M}_1$, we have

$$\Big[\, u\big(\tilde{c}_\theta(m_0', \tilde{m}_\theta(a, \omega, m_0'))\big) - v\big(\tilde{y}_\theta(m_0', \tilde{m}_\theta(a, \omega, m_0'))/\omega\big) \,\Big]$$

$$- \quad \Big[\, u\big(\tilde{c}_\theta(m_0(a, \omega), \tilde{m}_\theta(a, \omega, m_0(a, \omega)))\big) - v\big(\tilde{y}_\theta(m_0(a, \omega), \tilde{m}_\theta(a, \omega, m_0(a, \omega)))/\omega\big) \,\Big]$$

$$- \quad \Big[\, u\big(c_\theta(m_0', m_\theta(a, \omega, m_0'))\big) - v\big(y_\theta(m_0', m_\theta(a, \omega, m_0'))/\omega\big) \,\Big]$$

$$+ \quad \Big[\, u\big(c_\theta(m_0(a, \omega), m_\theta)\big) - v\big(y_\theta(m_0(a, \omega), m_\theta)/\omega\big) \,\Big]$$

$$\leq \quad \Delta_{(a,\omega)}\Big(m_0(a, \omega), \{m_\theta(a, \omega, m_0(a, \omega))\}_\theta, m_0', \{m_\theta\}_\theta\Big)$$

This inequality means that the reformed mechanism can 'punish' less than the *status quo* mechanism any deviation from $m_0(a, \omega)$ but the difference in 'punishments' must be smaller than the avoided utility loss given by $\Delta_{(a,\omega)}(\,\cdot\,)$. This guarantees that the reformed mechanism induces the same message-function $m_0(\cdot)$ at time $t = 0$. In other words, when agents discover unanticipated No Regret reforms, they do not regret their $t = 0$ message.

Lemma 7. ***(Households' optimal messages under the No Regret rule)***
Consider any status quo mechanism $\{\ T(\cdot), y_0(\cdot), \{c_\theta(\cdot), y_\theta(\cdot)\}_\theta\ \}$ *and any reformed mechanism* $\{\ T(\cdot), y_0(\cdot), \{\tilde{c}_\theta(\cdot), \tilde{y}_\theta(\cdot)\}_\theta\ \}$ *such that, after any realization* θ*,* $(\tilde{c}_\theta(\cdot), \tilde{y}_\theta(\cdot))$ *is a No Regret reform.*
The status quo mechanism and the reformed mechanism induces the same message-function at time $t = 0$*.*
Proof: Assume *ad absurdo* that there exists an agent of type (a, ω) for whom it is optimal to send $\tilde{m}_0$ instead of $m_0(a, \omega)$ when facing the reformed mechanism $\{\ T(\cdot), y_0(\cdot), \{\tilde{c}_\theta(\cdot), \tilde{y}_\theta(\cdot)\}_\theta\ \}$. This means that

$$u(a - T(m_0(a, \omega))) - v(y_0(m_0(a, \omega))/\omega)$$

$$+ \quad \mathbb{E}\,[\, u(\tilde{c}_\theta(m_0(a, \omega), \tilde{m}_\theta(a, \omega, m_0(a, \omega)))) - v(\tilde{y}_\theta(m_0(a, \omega), \tilde{m}_\theta(a, \omega, m_0(a, \omega)))/\omega)\,]$$

$$< \quad u(a - T(\tilde{m}_0)) - v(y_0(\tilde{m}_0)/\omega)$$

$$+ \quad \mathbb{E}\,[\, u(\tilde{c}_\theta(\tilde{m}_0, \tilde{m}_\theta(a, \omega, \tilde{m}_0))) - v(\tilde{y}_\theta(\tilde{m}_0, \tilde{m}_\theta(a, \omega, \tilde{m}_0))/\omega)\,]$$

Thanks to the No Regret constraints (with $m_\theta = m_\theta(a, \omega, \tilde{m}_0)$), and by definition of the avoided utility loss $\Delta_{(a,\omega)}(\,\cdot\,)$ when facing the *status quo* mecha-

nism $\{\ T(\cdot), y_0(\cdot), \{c_\theta(\cdot), y_\theta(\cdot)\}_\theta\ \}$, we get for every realization θ

$$\left[\ u\big(\tilde{c}_\theta(\tilde{m}_0, \tilde{m}_\theta(a,\omega,\tilde{m}_0))\big) - v\big(\tilde{y}_\theta(\tilde{m}_0, \tilde{m}_\theta(a,\omega,\tilde{m}_0))/\omega\big)\ \right]$$

$$- \quad \left[\ u\big(\tilde{c}_\theta(m_0(a,\omega), \tilde{m}_\theta(a,\omega,m_0(a,\omega)))\big) - v\big(\tilde{y}_\theta(m_0(a,\omega), \tilde{m}_\theta(a,\omega,m_0(a,\omega)))/\omega\big)\ \right]$$

$$- \quad \left[\ u\big(c_\theta(\tilde{m}_0, m_\theta(a,\omega,\tilde{m}_0))\big) - v\big(y_\theta(\tilde{m}_0, m_\theta(a,\omega,\tilde{m}_0))/\omega\big)\ \right]$$

$$+ \quad \left[\ u\big(c_\theta(m_0(a,\omega), m_\theta(a,\omega,m_0(a,\omega)))\big) - v\big(y_\theta(m_0(a,\omega), m_\theta(a,\omega,m_0(a,\omega)))/\omega\big)\ \right]$$

$$\leq \quad \Delta_{(a,\omega)}\Big(m_0(a,\omega), \{m_\theta(a,\omega,m_0(a,\omega))\}_\theta, \tilde{m}_0, \{m_\theta(a,\omega,\tilde{m}_0)\}_\theta\Big)$$

$$=$$

$$u(a - T(m_0(a,\omega))) - v(y_0(m_0(a,\omega))/\omega)$$

$$+ \quad \mathbb{E}\left[\ u(c_\theta(m_0(a,\omega), m_\theta(a,\omega,m_0(a,\omega)))) - v(y_\theta(m_0(a,\omega), m_\theta(a,\omega,m_0(a,\omega)))/\omega)\ \right]$$

$$- \quad \left[\ u(a - T(\tilde{m}_0)) - v(y_0(\tilde{m}_0)/\omega)\ \right]$$

$$- \quad \mathbb{E}\left[\ u(c_\theta(\tilde{m}_0, m_\theta(a,\omega,\tilde{m}_0))) - v(y_\theta(\tilde{m}_0, m_\theta(a,\omega,\tilde{m}_0))/\omega)\ \right]$$

Taking the expectation over θ of this inequality and summing it with the first strict inequality, all the terms simplify and we get $0 < 0$, which is a contradiction. □

We can now state the main result of this Appendix.

Theorem 1. ***(The Revelation Principle under the No Regret rule)***
Consider any status quo mechanism $\left\{ T(\cdot), y_0(\cdot), \{c_\theta(\cdot), y_\theta(\cdot)\}_\theta \right\}$ *and any reformed mechanism* $\left\{ T(\cdot), y_0(\cdot), \{\tilde{c}_\theta(\cdot), \tilde{y}_\theta(\cdot)\}_\theta \right\}$ *such that, after any realization* θ, $(\tilde{c}_\theta(\cdot), \tilde{y}_\theta(\cdot))$ *is a No Regret reform. The resulting allocation can also be implemented by a status quo mechanism* $\left\{T^d(\cdot), y_0^d(\cdot), \{c_\theta^d(\cdot), y_\theta^d(\cdot)\}_\theta\right\}$ *and a reformed mechanism* $\left\{ T^d(\cdot), y_0^d(\cdot), \{\tilde{c}_\theta^d(\cdot), \tilde{y}_\theta^d(\cdot)\}_\theta \right\}$ *such that*

- *Direct: at* $t = 0$*, the message space is the type space (i.e.* $\mathbb{M}_0 = [0, \overline{a}] \times [0, \overline{\omega}]$*)*
  at $t = 1$*, there is no message sent (i.e.* $\mathbb{M}_\theta = \emptyset$*)*
- *Truthful: the message-function at* $t = 0$ *is the identity (i.e. for all types* (a, ω)*,* $m_0(a, \omega) = (a, \omega)$*).*
- *No Regret: after any realization* θ*,* $(\tilde{c}_\theta^d(\cdot), \tilde{y}_\theta^d(\cdot))$ *is a No Regret reform*

Proof: Let us build direct mechanisms and then show they are truthful.

$$T^d : \begin{cases} [0,\overline{a}] \times [0,\overline{\omega}] & \to & \mathbb{R} \\ (a,\omega) & \mapsto & T(m_0(a,\omega)) \end{cases} \qquad y_0^d : \begin{cases} [0,\overline{a}] \times [0,\overline{\omega}] & \to & \mathbb{R} \\ (a,\omega) & \mapsto & y_0(m_0(a,\omega)) \end{cases}$$

and at $t = 1$ after any realization θ

$$c_\theta^d : \begin{cases} [0,\overline{a}] \times [0,\overline{\omega}] & \to & \mathbb{R} \\ (a,\omega) & \mapsto & c_\theta(m_0(a,\omega), m_\theta(a,\omega, m_0(a,\omega))) \end{cases}$$

$$y_\theta^d : \begin{cases} [0,\overline{a}] \times [0,\overline{\omega}] & \to & \mathbb{R} \\ (a,\omega) & \mapsto & y_\theta(m_0(a,\omega), m_\theta(a,\omega, m_0(a,\omega))) \end{cases}$$

$$\tilde{c}_\theta^d : \begin{cases} [0,\overline{a}] \times [0,\overline{\omega}] & \to & \mathbb{R} \\ (a,\omega) & \mapsto & \tilde{c}_\theta(m_0(a,\omega), m_\theta(a,\omega, \tilde{m}_0(a,\omega))) \end{cases}$$

$$\tilde{y}_\theta^d : \begin{cases} [0,\overline{a}] \times [0,\overline{\omega}] & \to & \mathbb{R} \\ (a,\omega) & \mapsto & \tilde{y}_\theta(m_0(a,\omega), m_\theta(a,\omega, \tilde{m}_0(a,\omega))) \end{cases}$$

Let us show that the direct reformed mechanism $\left\{ T^d(\cdot), y_0^d(\cdot), \{\tilde{c}_\theta^d(\cdot), \tilde{y}_\theta^d(\cdot)\}_\theta \right\}$ is truthful by assuming *ad absurdo* that there exists an agent with type (a, ω) who gains by reporting another type (a', ω') *i.e.*

$$u(a - T^d(a,\omega)) - v(y_0^d(a,\omega)/\omega) + \mathbb{E}\left[\, u(\tilde{c}_\theta^d(a,\omega)) - v(\tilde{y}_\theta^d(a,\omega)/\omega) \,\right]$$

$$< \quad u(a-T^d(a',\omega'))-v(y_0^d(a',\omega')/\omega)+\mathbb{E}\left[\, u(\tilde{c}_\theta^d(a',\omega'))-v(\tilde{y}_\theta^d(a',\omega')/\omega)\,\right]$$

Using the definitions of this direct mechanism, we get

$$u(a-T(m_0(a,\omega)))-v(y_0(m_0(a,\omega))/\omega)$$

$$+ \quad \mathbb{E}\left[\, u(\tilde{c}_\theta(m_0(a,\omega),\tilde{m}_\theta(a,\omega,\tilde{m}_0(a,\omega))))-v(\tilde{y}_\theta(m_0(a,\omega),\tilde{m}_\theta(a,\omega,\tilde{m}_0(a,\omega)))/\omega)\,\right]$$

$$< \quad u(a-T(m_0(a',\omega')))-v(y_0(m_0(a',\omega'))/\omega)$$

$$+ \quad \mathbb{E}\left[\, u(\tilde{c}_\theta(m_0(a',\omega'),\tilde{m}_\theta(a',\omega',\tilde{m}_0(a',\omega'))))-v(\tilde{y}_\theta(m_0(a',\omega'),\tilde{m}_\theta(a',\omega',\tilde{m}_0(a',\omega')))/\omega)\,\right]$$

$$\leq \quad u(a-T(m_0(a',\omega')))-v(y_0(m_0(a',\omega'))/\omega)$$

$$+ \quad \mathbb{E}\left[\, u(\tilde{c}_\theta(m_0(a',\omega'),\tilde{m}_\theta(a,\omega,\tilde{m}_0(a',\omega'))))-v(\tilde{y}_\theta(m_0(a',\omega'),\tilde{m}_\theta(a,\omega,\tilde{m}_0(a',\omega')))/\omega)\,\right]$$

where the last inequality is obtained with the fact that type (a,ω) is weakly better off sending the message $\tilde{m}_\theta(a,\omega,\tilde{m}_0(a',\omega'))$ instead of $\tilde{m}_\theta(a',\omega',\tilde{m}_0(a',\omega'))$.
By Lemma 3, we know that $m_0(a,\omega)$ is the optimal message for type (a,ω) when facing the reformed mechanism $\left\{\, T(\cdot), y_0(\cdot), \{\tilde{c}_\theta(\cdot),\tilde{y}_\theta(\cdot)\}_\theta \,\right\}$, so this inequality cannot be true.
The direct *status quo* mechanism is a special case of a direct reformed mechanism and so it is also truthful.
Let us now show that, after any realization θ, $(\tilde{c}_\theta^d(\cdot),\tilde{y}_\theta^d(\cdot))$ is a No Regret reform.
After any realization θ, $(\tilde{c}_\theta(\cdot),\tilde{y}_\theta(\cdot))$ is a No Regret reform, so, for $m_0 = m_0(a',\omega')$ and, for every realization θ, $m_\theta = m_\theta(a',\omega',m_0(a',\omega'))$ the No Regret constraint is

$$\left[\, u\big(\tilde{c}_\theta(m_0(a',\omega'),\tilde{m}_\theta(a,\omega,m_0(a',\omega')))\big)-v\big(\tilde{y}_\theta(m_0(a',\omega'),\tilde{m}_\theta(a,\omega,m_0(a',\omega')))/\omega\big)\,\right]$$

$$- \quad \left[\, u\big(\tilde{c}_\theta(m_0(a,\omega),\tilde{m}_\theta(a,\omega,m_0(a,\omega)))\big)-v\big(\tilde{y}_\theta(m_0(a,\omega),\tilde{m}_\theta(a,\omega,m_0(a,\omega)))/\omega\big)\,\right]$$

$$- \quad \left[\, u\big(c_\theta(m_0(a',\omega'),m_\theta(a',\omega',m_0(a',\omega')))\big)-v\big(y_\theta(m_0(a',\omega'),m_\theta(a',\omega',m_0(a',\omega')))/\omega\big)\,\right]$$

$$+ \quad \left[\, u\big(c_\theta(m_0(a,\omega),m_\theta(a,\omega,m_0(a,\omega)))\big)-v\big(y_\theta(m_0(a,\omega),m_\theta(a,\omega,m_0(a,\omega)))/\omega\big)\,\right]$$

$$\leq \quad \Delta_{(a,\omega)}\Big(m_0(a,\omega),\{m_\theta(a,\omega,m_0(a,\omega))\}_\theta,m_0(a',\omega'),\{m_\theta(a',\omega',m_0(a',\omega'))\}_\theta\Big)$$

$$=$$

$$u(a-T(m_0(a,\omega)))-v(y_0(m_0(a,\omega))/\omega)$$

$$+ \quad \mathbb{E}\left[\, u(c_\theta(m_0(a,\omega), m_\theta(a,\omega,m_0(a,\omega)))) - v(y_\theta(m_0(a,\omega), m_\theta(a,\omega,m_0(a,\omega)))/\omega) \,\right]$$

$$- \quad \left[\, u(a - T(m_0(a',\omega'))) - v(y_0(m_0(a',\omega'))/\omega) \,\right]$$

$$- \quad \mathbb{E}\left[\, u(c_\theta(m_0(a',\omega'), m_\theta(a',\omega',m_0(a',\omega')))) - v(y_\theta(m_0(a',\omega'), m_\theta(a',\omega',m_0(a',\omega')))/\omega) \,\right]$$

Using the fact that type (a,ω) is weakly better off sending the message $\tilde{m}_\theta(a,\omega,\tilde{m}_0(a',\omega'))$ instead of $\tilde{m}_\theta(a',\omega',\tilde{m}_0(a',\omega'))$ and using the definitions of this direct mechanism, we get

$$\left[\, u(\tilde{c}^d_\theta(a',\omega')) - v(\tilde{y}^d_\theta(a',\omega')/\omega) \,\right] - \left[\, u(\tilde{c}^d_\theta(a,\omega)) - v(\tilde{y}^d_\theta(a,\omega)/\omega) \,\right]$$

$$-\left[\, u(c^d_\theta(a',\omega')) - v(y^d_\theta(a',\omega')/\omega) \,\right] + \left[\, u(c^d_\theta(a,\omega)) - v(y^d_\theta(a,\omega)/\omega) \,\right]$$

$$\leq$$

$$\left[\, u(a - T^d(a,\omega)) - v(y^d_0(a,\omega)/\omega) + \mathbb{E}\left[\, u(\tilde{c}^d_\theta(a,\omega)) - v(\tilde{y}^d_\theta(a,\omega)/\omega) \,\right] \,\right]$$

$$-\left[\, u(a - T^d(a',\omega')) - v(y^d_0(a',\omega')/\omega) + \mathbb{E}\left[\, u(\tilde{c}^d_\theta(a',\omega')) - v(\tilde{y}^d_\theta(a',\omega')/\omega) \,\right] \,\right]$$

$$\equiv \Delta^d((a,\omega),(a',\omega'))$$

which means that for, any realization θ, $(\tilde{c}^d_\theta(\cdot), \tilde{y}^d_\theta(\cdot))$ is a No Regret reform of the direct *status quo* mechanism. □