The Zero-Beta Rate and the Transmission of Monetary Policy*

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Abstract

Central banks set the federal funds rate, but the rate that determines household saving and spending is the zero-beta rate, the risk-free return implied by the price of risky assets rather than by Treasury yields. The two differ because Treasuries embed a convenience premium for their liquidity services. Following a monetary contraction, the zero-beta rate rises 3.6 times more than the policy rate, as falling bond prices reduce the supply of liquid assets and raise the marginal value of liquidity. When the Euler equation uses the zero-beta rate, it accounts for the observed response of consumption to monetary policy; the textbook specification understates the response by roughly eighty percent. We develop an analytical heterogeneous-agent New Keynesian model with rare disasters and liquidity services that matches the term premium, equity premium, and convenience yield. If Treasuries lost their convenience value, the output response to monetary policy would fall by twenty-one percent.

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1 Introduction

The consumption Euler equation lies at the heart of monetary transmission: it provides the mechanism through which changes in the policy rate affect aggregate demand. Yet empirically, this link appears remarkably weak. Estimates of the consumption Euler equation consistently imply a small elasticity of intertemporal substitution, and the predicted comovement between interest rates and consumption growth is largely absent from the data. A large literature has interpreted this evidence as a failure of the Euler equation itself, questioning the empirical relevance of intertemporal optimization for aggregate consumption dynamics. Recent evidence suggests that this failure may reflect a misspecification: Di Tella, Hébert, Kurlat and Wang (2023) show that the zero-beta rate, a convenience-free risk-free rate inferred from risky asset prices, satisfies the aggregate Euler equation remarkably well, in contrast to Treasury rates.

This paper embeds the zero-beta rate into a New Keynesian model that jointly matches key asset pricing moments and the transmission of monetary policy to the real economy. When calibrated to micro estimates of the elasticity of intertemporal substitution, the model generates quantitative dynamics consistent with the data, but the intertemporal substitution channel plays a minor role: the bulk of the consumption response arises from movements in risk premia and in the zero-beta rate.

This amplification of the zero-beta rate arises because monetary policy affects not only the yield on U.S. Treasuries but also the liquidity services they provide, so that the convenience-free rate that enters the Euler equation moves by more than the policy rate. Treasuries embed a convenience premium in their yields (Krishnamurthy and Vissing-Jorgensen, 2012; van Binsbergen, Diamond and Grotteria, 2022a), and when the central bank tightens, long-term bond prices fall, reducing the market value of liquid assets. Because the marginal value of liquidity is decreasing in supply, this contraction raises the convenience premium and pushes the zero-beta rate up further. The resulting feedback loop between bond prices and liquidity premia amplifies intertemporal incentives in a way that is invisible when one looks only at Treasury yields.

We begin by documenting two empirical facts using high-frequency identification and

local projections. A contractionary monetary shock raises both market-implied tail risk, as measured by the option-based rare disaster concern index of Gao, Gao and Song (2018), and the convenience yield on long-term U.S. Treasuries by approximately 25 basis points on impact. These effects are robust across three identification strategies (Bauer and Swanson, 2023; Gürkaynak, Sack and Swanson, 2005; Romer and Romer, 2004) and are economically significant: the average convenience yield in our sample is 88 basis points, so a 25-basis-point movement represents a shift of nearly one standard deviation.

Guided by this evidence, we build an analytical heterogeneous-agent New Keynesian model with rare disasters and liquidity services from government debt. Households differ in their beliefs about the likelihood of a crisis state and in their preferences for liquidity services, generating endogenous, time-varying risk premia and convenience yields. Despite this rich structure, the model admits closed-form solutions for asset prices and aggregate dynamics. We calibrate the model to match the levels of the term premium, equity premium, and convenience yield, as well as their responses to monetary policy. Following a monetary contraction, the zero-beta rate rises more than three times as much as the federal funds rate. When the Euler equation is evaluated at this rate and accounts for the precautionary motive from tail risk, it generates consumption responses roughly five times larger than in a specification that uses the Treasury rate. The Euler equation does not fail; it has been fed the wrong input. The same mechanism provides a structural explanation for the excess sensitivity of long-term interest rates documented by Hanson and Stein (2015): the feedback between bond prices and liquidity premia, together with movements in risk premia, accounts for the bulk of the yield response that the expectations hypothesis cannot explain. In a counterfactual where government bonds cease to provide liquidity services, the output response to monetary policy falls by twenty-one percent. This result speaks directly to recent evidence by Jiang, Richmond and Zhang (2025), who document a secular decline in Treasury convenience yields: if Treasuries become less "special," the Federal Reserve may need larger rate movements to achieve the same macroeconomic effects.

A central theoretical contribution is that the model breaks the *macro-finance separation* that characterizes many monetary models. In most existing frameworks, the dynamics of

the real economy can be solved independently of asset prices (Tallarini Jr., 2000; Pflueger and Rinaldi, 2022a; Bianchi, Lettau and Ludvigson, 2022). In our model, this separation fails because bond prices determine the supply of liquidity, which determines the zero-beta rate, which governs aggregate demand. Asset prices are not merely a reflection of monetary shocks but an active amplification mechanism.

The model also provides a unified account of the response of equity prices to monetary policy. Nagel and Xu (2024) document that movements in bond yields account for most of the stock price response to monetary shocks. In our framework, equity prices respond to both the zero-beta rate and the price of risk, the same forces driving bond yields. The convenience yield also embeds a *convenience risk premium* that captures the differential risk government and corporate bonds face in the crisis state, generating the hedging properties of convenience yields documented by Acharya and Laarits (2023).

Related literature. This paper connects several strands of literature. On convenience yields and safe assets, we build on the foundational evidence of Krishnamurthy and Vissing-Jorgensen (2012), the framework of Greenwood, Hanson and Stein (2015), and the global perspective of Caballero, Farhi and Gourinchas (2008) and Caballero, Farhi and Gourinchas (2017). Nagel (2016) shows that short-term convenience yields comove with the opportunity cost of holding money. Our contribution to this literature is showing how convenience yields respond to monetary policy and embedding them in a general equilibrium framework where they amplify real transmission.

On rare disasters and asset pricing, we build on Rietz (1988), Barro (2006), Gabaix (2012), Wachter (2013), and Gourio (2012). Our approach to heterogeneous beliefs follows Chen, Joslin and Tran (2012) and extends the monetary framework of Caramp and Silva (2025) to include liquidity services.[1]

On the zero-beta rate, the concept originates with Black (1972), who showed that in a CAPM without a risk-free asset, the expected return of a zero-beta portfolio exceeds Treasury yields. Di Tella et al. (2023) construct a time-varying zero-beta rate from equity

[1]Our empirical approach connects to work on variance and tail risk pricing (Dew-Becker, Giglio, Le and Rodriguez, 2017). The rare disaster concern index captures the option-implied probability of extreme negative returns, a related object to the variance risk premium studied in that literature.

returns and show that it satisfies the aggregate consumption Euler equation remarkably well. van Binsbergen et al. (2022a) use option prices to estimate convenience-yield-free interest rates without relying on a specific asset pricing model, finding a Treasury convenience yield of approximately 40 basis points that quadruples during the financial crisis. We embed the zero-beta rate in a New Keynesian framework, where it governs not only asset pricing but also the transmission of monetary policy to real activity.

Several recent papers study how monetary policy affects risk premia. Pflueger and Rinaldi (2022b) show that monetary shocks move risk aversion in a model with external habits, generating large asset price responses. Bianchi et al. (2022) study monetary policy and asset valuation with recursive preferences, emphasizing the role of long-run risks. Kekre, Lenel and Mainardi (2023) highlight the interaction between heterogeneous MPCs and portfolio choice in shaping monetary transmission. David and Zeke (2024) show that monetary policy affects resource allocation through its impact on firm-level risk premia when firms differ in their exposure to aggregate risk. Our model differs from these papers by focusing on how risk premia affect household behavior, both through precautionary savings and the pricing of convenience.

Finally, this paper contributes to the literature on heterogeneous-agent New Keynesian models. Following Kaplan, Moll and Violante (2018), a growing body of work emphasizes that household heterogeneity shapes the transmission of monetary policy. Auclert (2019) decomposes transmission into income and substitution effects across the wealth distribution. Werning (2015) and Bilbiie (2020) provide analytical characterizations of HANK dynamics. Our contribution is an analytical HANK model that incorporates both disaster risk and convenience yields while remaining tractable, generating realistic asset pricing moments that standard HANK models do not attempt while preserving closed-form solutions for aggregate dynamics.

The rest of the paper is organized as follows. Section 2 documents the empirical response of tail risk and convenience yields to monetary shocks. Section 3 presents the model and characterizes the equilibrium. Section 4 analyzes the transmission of monetary policy to asset prices. Section 5 studies the transmission to the real economy and the counterfactual experiment.

2 Motivating Evidence

This section presents an empirical investigation of how monetary policy affects the market-implied tail risk and the convenience yield. To do so, we rely on three different monetary policy shocks identified in the literature, and we focus on both a broad market measure of tail risk as well as a more granular measure that zooms in on the financial sector. We also consider different specifications for the convenience yield to account for the fact that it is not directly observable.

2.1 Empirical Setup

The first monetary shock we rely on is the orthogonalized shock from Bauer and Swanson (2023) (BS). The second one is the "target" factor in Gürkaynak et al. (2005) (GSS), which captures the unexpected change in the current target for the federal funds rate. The third one is the narrative measure from Romer and Romer (2004) (RR).[2]

For the market-implied tail risk measure, we follow Gao et al. (2018) and employ their rare disaster concern index (RIX), which is given by:

$$\text{RIX} = \frac{2e^{r\tau}}{\tau} \int_{K<S_t} \frac{\ln\left(S_t/K\right)}{K^2} P\left(S_t; K, T\right) dK,$$

where S_t is the price of an asset at time t, $P\left(.; K, T\right)$ is the price of a put option with strike K and maturity date T, and $\tau = T - t$ is the time-to-maturity. The RIX measure isolates abnormally large price movements of the asset and thus is a market-implied measure of tail risk. We are interested in broad market measures of tail risk, but also in more granular measures that focus on the financial sector. Thus, for the former, we rely on the S&P 500 index for S_t (SPX), and for the latter we rely on the Financial Select Sector Index (XLF) from the S&P 500. We also follow Gao et al. (2018) and normalize the RIX measure by a

[2]For the first measure we rely on the updated series provided in Michael Bauer's website: https://www.michaeldbauer.com/publication/mps/. For the other two measures, we rely on the updated series provided by Miguel Acosta on his website. For the GSS shock see Acosta, Brennan and Jacobson (2024) and https://dataverse.harvard.edu/dataset.xhtml?persistentId=doi:10.7910/DVN/WUXWHS. For the RR shock see https://www.acostamiguel.com/data/fomc_data.html.

portfolio that assigns larger weights to more deeply out-of-the-money put options:

$$V^{-} = \frac{2e^{r\tau}}{\tau} \int_{K<S_t} \frac{1 - \ln\left(K/S_t\right)}{K^2} P\left(S_t; K, T\right) dK.$$

For the convenience yield, we follow Cieslak, Li and Pflueger (2024) and use the same measure as Krishnamurthy and Vissing-Jorgensen (2012), who rely on the spread between the Aaa-rated corporate bond yields and long-term Treasury bond yields. For the Aaa rates we use the Moody's seasoned Aaa corporate bond index from FRED (AAA). For the long-term Treasuries we use the average yield on long-term government bonds from FRED (LTGOVTBD) for years before 1999, and the 20-year Tresury bonds, also from FRED (GS20), from 2000 onwards. We will also control for the Baa-Aaa spread in some specifications, constructed as the spread between Moody's seasoned Baa corporate bond index from FRED and the same Aaa-rated corporate bond yields from the Aaa-Treasury spread.

For both type of outcome variables, we follow Jordà, Schularick and Taylor (2015) and estimate local projections-IV (LP-IV) specifications of the following form:

$$y_{t+h} - y_{t-1} = \alpha_h + \beta_h \Delta R_t + \delta_{1h} X_{t-1} + \delta_{2h} B_{t-1} + \epsilon_{t+h},$$

where y_t denotes either the (normalized) RIX measure or the convenience yield, measured in percentage points. All the specifications we estimate rely on monthly data, so $h = 0, 1, ...$ denotes a month. For our measure of monetary policy stance, ΔR_t, we use the change in the Fed funds rate. The vector X_{t-1} includes lags of different control variables, such as the first differenced log industrial production index (ΔIP_t) and the consumer price index (ΔCPI_t), and B_{t-1} includes lags of the (first differenced) outcome variables. Given the reliance on a single time series, the baseline specifications are chosen to minimize the number of control variables that deliver a consistent pattern across specifications, and we detail precisely our approach for each shock in the results below. The appendix presents additional results and robustness tests in which we estimate more demanding specifications for the three shocks. Standard errors for the impulse responses are Newey-West with $h + 1$ lags.

2.2 Market-Implied Tail Risk

Figure 1 presents the baseline results for the RIX measure using each of the shocks for the first 5 quarters. Figure A.1 in Appendix A shows the same results for the full first 16 quarters. For ease of exposition we further normalize the RIX measures by their standard deviation. The left columns in both figures show the RIX measure based on the SPX index, whereas the right columns focus instead on the XLF index. In the left columns, for the BS specification we employ 6 quarters of lagged ΔIP_t, ΔCPI_t, Aaa-Treasury spread, Baa-Aaa spread, 1 quarter of the lagged shock, and 1 month of the lagged RIX measure. For the GSS shock we employ 5 quarters of lagged ΔIP_t, ΔCPI_t and lagged shock, and 1 month of the lagged Aaa-Treasury spread and the Aaa-Treasury spread. For the RR shock we use 6 quarters of lagged ΔIP_t, ΔCPI_t and lagged shock, and 1 month of the lagged RIX measure. Controls for the XLF index are similar and are reported in the notes to the figures.

As the left columns show, a 1 percent increase in the Fed funds rate generates on impact a response of more than one standard deviation on the RIX-SPX measure, specifically 1.44 for the more precisely estimated IRFs (RR). This response increases further during the subsequent quarter, although there is insufficient statistical power to reject the possibility that the increase is negligible. After a quarter, the responses start to diminish before they vanish in between 7 to 18 months. For the RIX-XLF measure on the right columns, the behavior is similar, although they are much more precisely estimated. The impact responses in this case range from 1.14 (RR) to 1.52 (GSS) standard deviations. The responses continue to increase during the first quarter, and they begin to decline shortly thereafter before fully dissipating within 6 to 15 months.

These results are strong evidence that monetary contractions significantly increase the probability of a crisis. The results support this notion both when looking at broad market measures like the S&P 500, but also when looking at more granular sectors such as the Financial Select Sector Index. The fact that the results are consistent not only across the two RIX measures employed, but that they display the same behavior when using three different monetary shocks provides a lot of confidence that we are indeed capturing a systematic pattern in the data. Moreover, the results for the RIX-XLF being in line with a

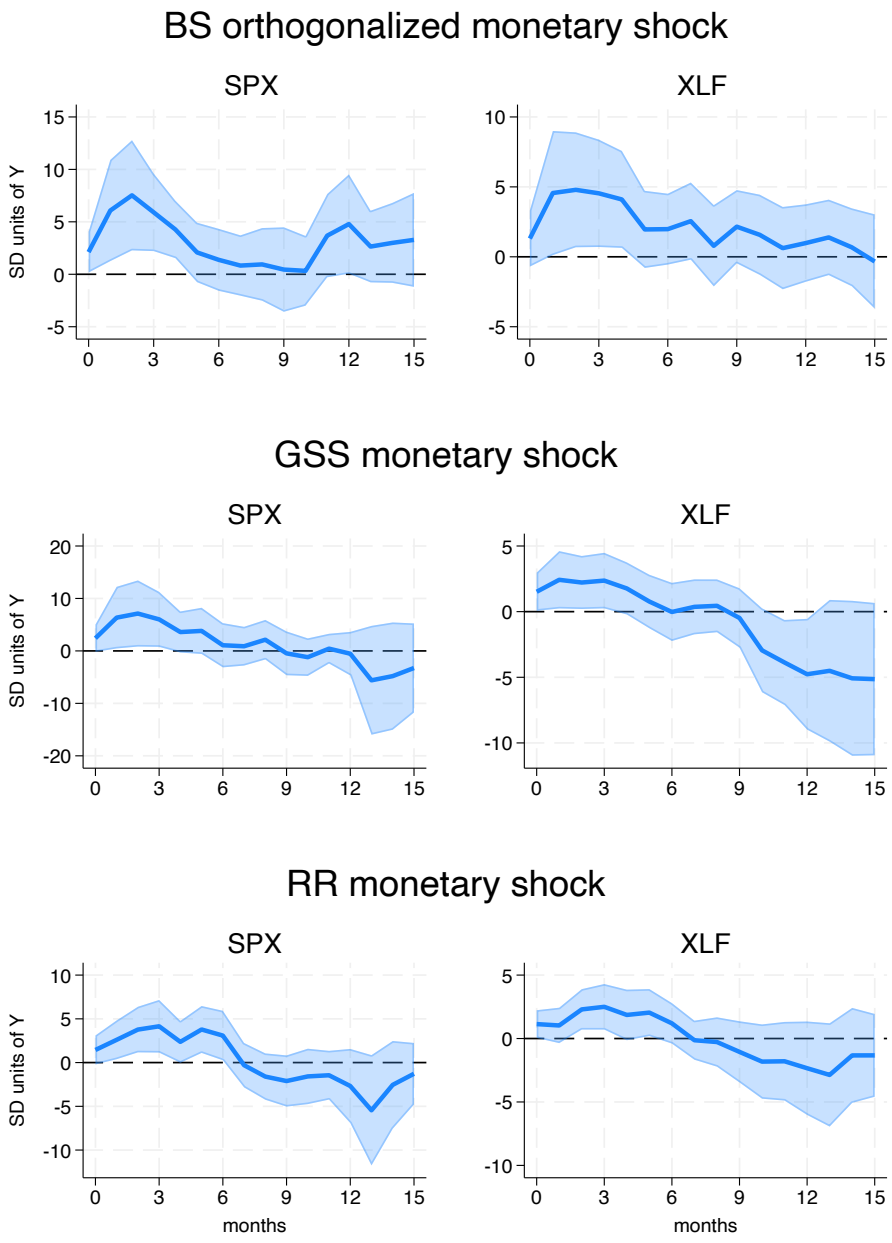


Figure 1: RIX response to a monetary shock (BS, GSS, and RR).
The figures show the impulse responses of the RIX measures, normalized by their standard deviations, to a BS monetary shock (top panel), a GSS shock (middle panel), and a RR shock (bottom panel), using the Jordà local projection method. The left column shows the RIX-SPX measure, while the right column shows the RIX-XLF measure. The frequency is monthly and the sample period is 1996m1-2018m12 in the left column, and 1999m2-2018m12 in the right column. Controls in the right column for each panel are, respectively: 5 quarters of lagged outcome variable, ΔIP_t, ΔCPI_t, and the shock; 1 month of lagged outome, 4 quarters of lagged ΔIP_t, ΔCPI_t, 3 quarters of the lagged shock; 1 month of lagged outome, 6 quarters of lagged ΔIP_t, ΔCPI_t, and the shock. Shaded areas represent 90% confidence bands.

broad market measure is reassuring since one would expect the financial sector to be particularly sensitive to a crisis state. The model we build in Section 3 provides a theoretical framework that builds on these empirical results.

2.3 Convenience Yield

Figure 2 turns the attention to the convenience yield and presents the baseline specifications for the first 5 quarters. Figure A.2 in Appendix A shows the same results for the full first 16 quarters. The left columns in both figures show the convenience yield for the baseline specification, and the right columns add the contemporaneous Baa-Aaa spread (its first difference) to the baseline. The baseline specification, for the BS shock, controls for 6 quarters of the lagged outcome variable, ΔIP_t, ΔCPI_t, first-differenced Baa-Aaa spread, and 1 quarter of the shock. For the GSS shock, it controls for 1 month of the lagged outcome variable, and 5 quarters of lagged ΔIP_t, ΔCPI_t, and the shock. For the bottom panel, it controls for 1 quarter of the lagged outcome variable, and 6 quarters of lagged ΔIP_t, ΔCPI_t, and the shock.

As the left columns show, an increase of 1 percent in the Fed funds rate increases the convenience yield by roughly 25 basis points (bps) on impact, and the response is statistically significant for the three shocks. The full range of the contemporaneous impact is 18 bps (RR) to 39 bps (GSS). To put these numbers into context, it is useful to compare them against an average Aaa-Treasury spread of 88 bps with a standard deviation of 31 bps in our sample. Thus, the initial responses are economically significant as well. As with the RIX measures of the previous sections, these initial responses continue to increase during the first quarter, before reverting back and vanishing in around 6 to 18 months.

To account for the fact the convenience yield is only one piece of the Aaa-Treasury spread, we follow Cieslak et al. (2024) and reestimate the three specifications adding the contemporaneous Baa-Aaa spread as a control. These impulse responses are presented in the right columns of Figures 2 and A.2. This side-by-side presentation facilitates comparison across specifications, as the only difference between the left and right columns is

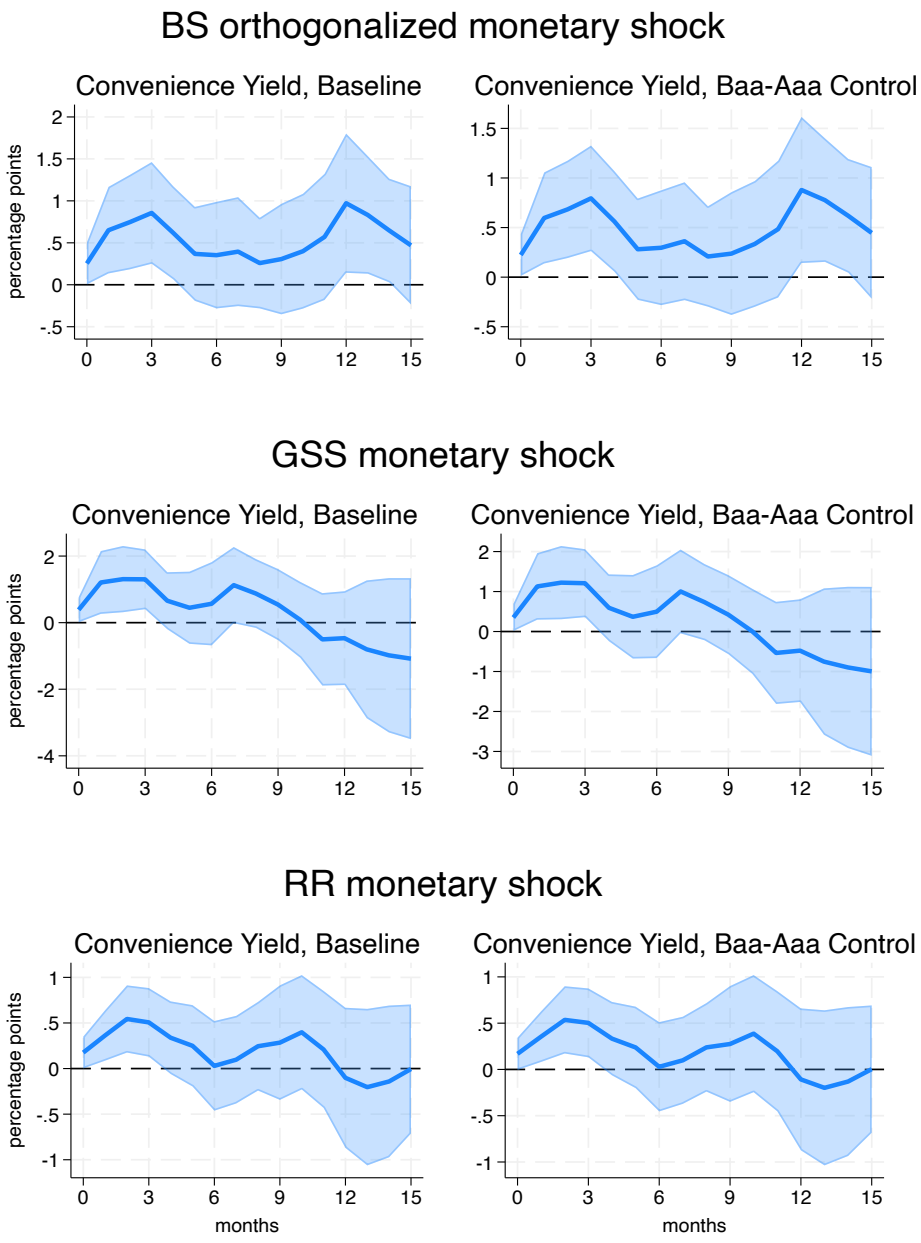


Figure 2: Convenience yield response to a monetary shock (BS, GSS, and RR).
The figures show the impulse responses of the convenience yield to a BS monetary shock (top panel), a GSS shock (middle panel), and a RR shock (bottom panel), using the Jordà local projection method. The frequency is monthly and the sample period is 1988m2-2018m12 in the top and bottom panels, and 1995m2-2018m12 in the middle panel. Shaded areas represent 90% confidence bands.

the inclusion of the Baa-Aaa spread. As the side-by-side comparison shows, the inclusion of the Baa-Aaa spread dampens the estimates only slightly, leaving the results essentially unchanged. This is consistent with the findings of Cieslak et al. (2024).

Overall, these results point to a strong and statistically significant effect of monetary policy on the convenience yield, with the estimates pointing to approximately a 1 standard deviation effect on impact. This effect increases during the first quarter and lasts for about 1 year on average, with the exact persistence varying across the different monetary shocks employed. Similar to the market-implied tail risk measure of the previous section, the consistency of the estimates across three distinct monetary shocks and different samples significantly bolsters the confidence in these patterns.

The convenience yield's economically meaningful reaction suggests a distinct channel through which monetary policy affects the economy, complementing those explored more thoroughly in the previous literature. The next sections are devoted to studying this channel in detail.

2.4 Additional Results and Robustness Checks

Additional Results We begin by showing that the market-implied tail risk effects identified previously are also evident in more granular RIX measures. Figure 3 presents results for the RIX-KBE, based on the S&P Banks Select Industry Index, on the left column, and the RIX-HYG, based on the iShares iBoxx High Yield Corporate Bond ETF, on the right column. These specifications are constrained by data availability, as the RIX-KBE and RIX-HYG measures only start in September and July 2007, respectively. Consequently, these results rely on a substantially shorter sample period than previous estimates, leading to a noticeable reduction in precision. We focus our analysis on the GSS and RR shocks, as these remain robust despite the limited sample size.

Notably, despite the loss in precision, both panels mirror the patterns observed for the RIX-SPX and RIX-XLF measures. We find a positive, economically significant effect on impact of, e.g. following a GSS shock, 0.73 and 1.22 standard deviations for the RIX-KBE and RIX-HYG, respectively. This effect persists for 6 to 12 months before receding.

These results underscore that the same reaction to a crisis state can be detected within both the banking sector and corporate yields. These patterns are consistent with a market-wide phenomenon, where sectors with high sensitivity to systemic distress clearly reflect

the escalation in perceived tail risk.

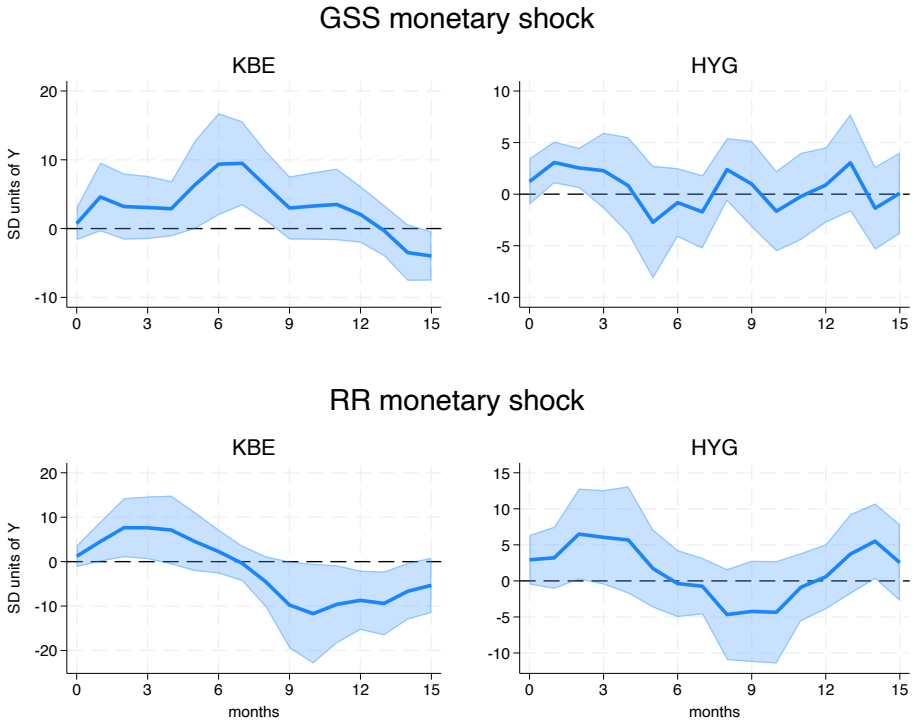


Figure 3: RIX response to a monetary shock (GSS, RR).
The figures show the impulse responses of the RIX measures, normalized by their standard deviations, to a GSS monetary shock (top panel) and a RR shock (bottom panel), using the Jordà local projection method. Left column: RIX-KBE measure. Right column: RIX-HYG measure. Frequency: monthly. Sample period: 2007m9-2018m12 (left column), 2007m7-2018m12 (right column). 7 missing values from 2008m5 to 2008m11 in the RIX-HYG measure are imputed by linear interpolation for the analysis. Controls (all lagged): 1 month of outcome, 2 months of ΔR_t, 5 quarters of ΔIP_t, ΔCPI_t, 2 quarters of the shock (left col, top panel); 1 month of outcome, 2 quarters of ΔIP_t, ΔCPI_t, 4 quarters of shock (left col, bottom panel); 1 quarter of outcome, ΔR_t, ΔIP_t, ΔCPI_t, 5 quarters of the shock (right col, top panel); 1 quarter of outcome, 2 quarters of ΔIP_t, 6 quarters of shock (right col, bottom panel). Shaded areas represent 90% confidence bands.

Robustness Checks Figure 4 estimates the main specifications from Figure 1 for the BS monetary shock using additional lags of the controls. For ease of comparison, the top row reproduces the baseline specification. The bottom row, for both columns, shows specifications that control for 6 quarters of the lagged outcome variable, ΔIP_t, ΔCPI_t, and the shock. Comparing the top and bottom rows, we see that the estimates remain essentially unchanged.

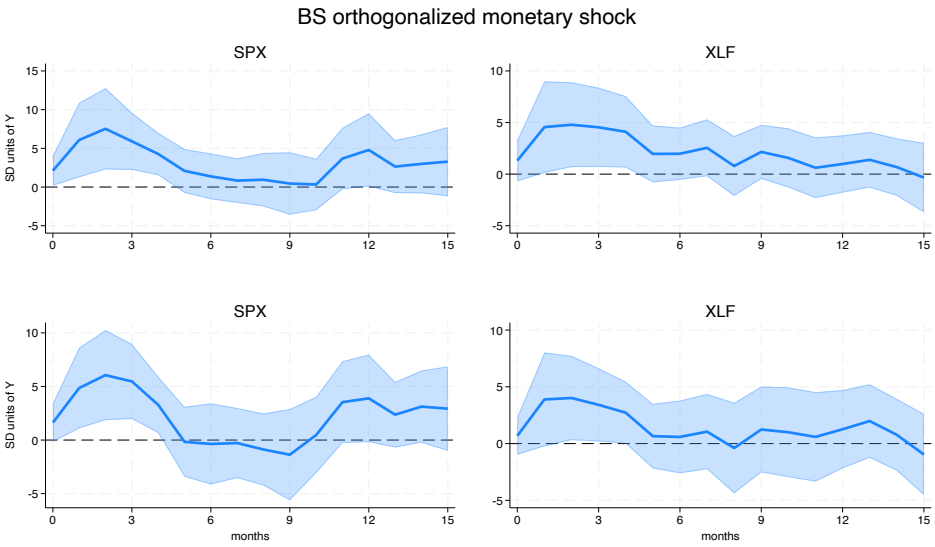


Figure 4: RIX response to a monetary shock (BS).
The figures show the impulse responses of the RIX measures, normalized by their standard deviations, to a BS monetary shock, using the Jordà local projection method. The left column shows the RIX-SPX measure, while the right column shows the RIX-XLF measure. The frequency is monthly and the sample period is 1996m1-2018m12 in the left column, and 1999m2-2018m12 in the right column. Shaded areas represent 90% confidence bands.

Figure 5 repeats the analysis for the convenience yield, using 6 quarters of all controls in the specifications of Figure 2 for the BS monetary shock. As before, the top row reproduces the baseline specification for ease of comparison. As we can see, there is no meaningful difference from including the additional controls.

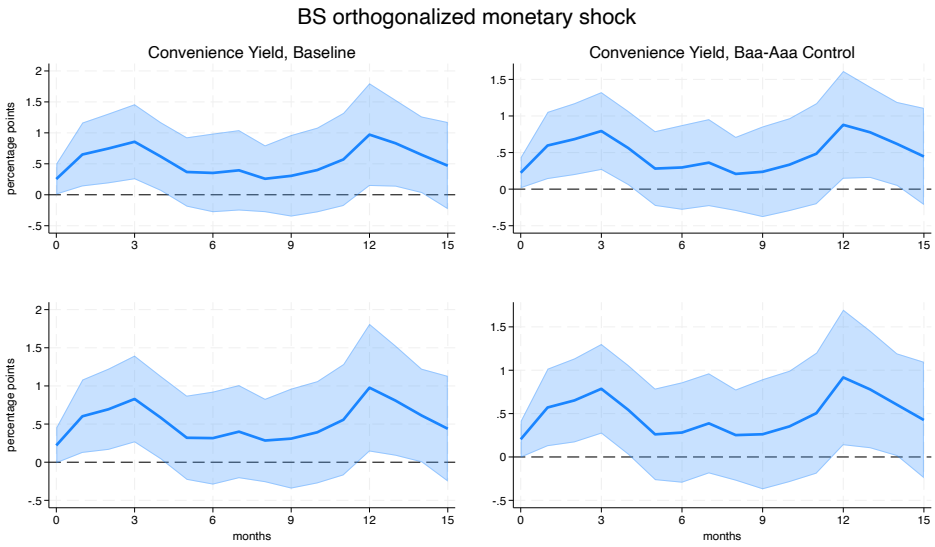


Figure 5: Convenience yield response to a monetary shock (BS).
The figures show the impulse responses of the convenience yield to a BS monetary shock using the Jordà local projection method. The frequency is monthly and the sample period is 1988m2-2018m12. Shaded areas represent 90% confidence bands.

3 A Monetary Model with Risk and Convenience Yields

Building on Caramp and Silva (2025), we consider a heterogeneous agent New Keynesian (HANK) model with aggregate risk and convenience yields. Investors differ in both their liquidity needs and beliefs about the probability of switching to a crisis state. This heterogeneity is key to capturing the response of market-implied tail risks and convenience yields to monetary policy shocks documented in Section 2.

3.1 The Model

Environment. Time is continuous and denoted by $t \in \mathbb{R}_+$. The economy is populated by households, firms, and a government. There is a continuum of households of three types: *workers*, *optimistic savers*, and *pessimistic savers* (denoted by w, o and p, respectively). These households differ in their discount rates, beliefs about the probability of aggregate shocks, and their preferences for government bond holdings. We let $\mu_j \geq 0$ denote the mass of households of type $j \in \{w, o, p\}$, where $\mu_w + \mu_o + \mu_p = 1$.

Households can borrow or lend at a riskless rate, subject to borrowing constraints, and

they can save in short- and long-term nominal government bonds and equity. For simplicity, the borrowing limit is set to zero. Workers are the only ones who supply labor and are relatively impatient, so their borrowing constraint is binding in equilibrium. Moreover, savers derive utility from the liquidity services provided by government bonds.

Firms produce final or intermediate goods. Final-goods producers are competitive and combine intermediate goods using a CES aggregator with elasticity of substitution $\epsilon > 1$. Intermediate-goods producers use only labor and face pricing adjustment costs.

Intermediate-goods producers are subject to a large aggregate productivity shock: with Poisson intensity $\overline{\lambda} \geq 0$, their productivity is permanently reduced.[3] We refer to periods prior to the realization of the shock as *normal times*, and periods that follow the shock as the *crisis state*. The crisis state is absorbing, and there are no further shocks after the crisis is realized. Our focus is on the economy's behavior during normal times.

Importantly, we assume that the liquidity services of government bonds fall in the crisis state. Therefore, a crisis triggers a loss of convenience from bonds. We interpret the loss of liquidity services in a crisis as capturing reduced market depth during financial stress, consistent with evidence that Treasury market functioning deteriorates in tail events (He, Nagel and Song, 2022).

The government conducts both fiscal and monetary policy. Fiscal policy comprises transfers to workers and savers, while monetary policy is specified by an interest rate rule subject to monetary shocks.

Financial assets. The economy features three types of financial assets: short-term government bonds, long-term government bonds, and corporate equity. Short-term bonds pay the nominal interest rate i_t. A long-term bond issued in period t trades at a nominal market price $Q_{L,t}$ in normal times and promises to pay coupons $e^{-\psi_L(s-t)}$ at all dates $s \geq t$. Because of the coupon structure, bonds issued at previous dates trade at proportional prices: a bond issued in s trades at $Q_{L,t}e^{-\psi_L(t-s)}$ in period t. The rate of decay ψ_L is inversely related to the bond's duration, where a consol corresponds to $\psi_L = 0$ and the limit $\psi_L \to \infty$ corresponds to short-term bonds. We denote by $Q^*_{L,t}$ the price of the bond

[3] In Appendix C.1, we show that our results are unchanged if we allow productivity to partially recover after the shock, consistent with empirical evidence (see, e.g., Barro, Nakamura, Steinsson and Ursúa 2013).

in the crisis state. Then, the nominal return on the long-term bond is given by

$$\frac{dR_{L,t}}{R_{L,t}} = \left[\frac{1}{Q_{L,t}} + \frac{\dot{Q}_{L,t}}{Q_{L,t}} - \psi_L\right] dt + \frac{Q^*_{L,t} - Q_{L,t}}{Q_{L,t}} d\mathcal{N}_t,$$

where $\mathcal{N}_t$ is a Poisson process with arrival rate $\overline{\lambda}$ (under the objective measure), and $R_{L,t}$ is the cumulative gross return up to time t, given $R_{L,0} = 1$.

The price of a claim on aggregate corporate profits is denoted by $P_t Q_{E,t}$, where P_t is the price level and $Q_{E,t}$ is the real equity price, and the nominal equity return is given by

$$\frac{dR_{E,t}}{R_{E,t}} = \left[\frac{\Pi_t}{Q_{E,t}} + \frac{\dot{Q}_{E,t}}{Q_{E,t}} + \pi_t\right] dt + \frac{Q^*_{E,t} - Q_{E,t}}{Q_{E,t}} d\mathcal{N}_t,$$

where Π_t denotes real dividends, $\pi_t = \frac{\dot{P}_t}{P_t}$ is the inflation rate, and $Q^*_{E,t}$ is the real equity price in the crisis state. A superscript $*$ will throughout denote variables in the crisis state.

Savers' problem. Savers differ along two dimensions. First, they have heterogeneous beliefs regarding the crisis probability. Subjective beliefs about the arrival rate of the productivity shock are given by λ_j, for $j \in \{o, p\}$, where $\lambda_o \leq \lambda_p$. Following Chen et al. (2012), we assume that savers are dogmatic in their beliefs about crisis risk, abstracting from any learning process. This heterogeneity generates time-varying risk premia after a monetary shock. Second, savers have different preferences for liquidity services. This generates time-varying convenience yields.

We follow Schmitt-Grohé and Uribe (2003) and assume that savers' subjective discount rate is a function of their consumption share, $\rho_{j,t} = \rho_j\left(\frac{C_{j,t}}{C_{s,t}}\right)$, where $C_{s,t} = \frac{\mu_o}{\mu_o+\mu_p} C_{o,t} + \frac{\mu_p}{\mu_o+\mu_p} C_{p,t}$ denotes savers' aggregate consumption. Though not essential for our main results, this feature is helpful as it ensures a unique stationary wealth distribution.

Let $B_{j,t} = B^S_{j,t} + B^L_{j,t} + B^E_{j,t}$ denote the real value of the net worth of a type-j saver, where $B^S_{j,t}$, $B^L_{j,t}$, and $B^E_{j,t}$ represent holdings of short-term bonds, long-term bonds, and equity, respectively.

A type-j saver chooses consumption $C_{j,t}$, long-term bonds $B^L_{j,t}$, and equity holdings

$B^E_{j,t}$, given initial net worth $B_{j,0} > 0$, to solve the following problem:

$$V_{j,t}(B_{j,t}) = \max_{[C_{j,z}, B^L_{j,z}, B^E_{j,z}]_{z \geq t}} \mathbb{E}_{j,t} \left[\int_t^{t^*} e^{-\int_t^z \rho_{j,u} du} \left[\frac{C_{j,z}^{1-\sigma}}{1-\sigma} + \xi_j \frac{L_{j,z}^{1-\nu}}{1-\nu} \right] dz + e^{-\int_t^{t^*} \rho_{j,u} du} V^*_{j,t^*}(B^*_{j,t^*}) \right], \quad (1)$$

where $L_{j,t}$ is an aggregator of short- and long-term government bond holdings:

$$L_{j,t} = B^S_{j,t} + \alpha B^L_{j,t},$$

subject to the flow budget constraint

$$dB_{j,t} = \left[(i_t - \pi_t) B_{j,t} + r_{L,t} B^L_{j,t} + r_{E,t} B^E_{j,t} + T_{j,t} - C_{j,t} \right] dt + \left[B^*_{j,t} - B_{j,t} \right] d\mathcal{N}_t,$$

and borrowing constraint $B_{j,t} \geq 0$, given $B_{j,0} > 0$.

Here, $B^*_{j,t} = B_{j,t} + B^L_{j,t} \frac{Q^*_{L,t} - Q_{L,t}}{Q_{L,t}} + B^E_{j,t} \frac{Q^*_{E,t} - Q_{E,t}}{Q_{E,t}}$ denotes savers' net worth in a crisis. The excess return, conditional on no crisis, is defined as $r_{L,t} \equiv \frac{1}{Q_{L,t}} + \frac{\dot{Q}_{L,t}}{Q_{L,t}} - \psi_L - i_t$ for long-term bonds and $r_{E,t} \equiv \frac{\Pi_t}{Q_{E,t}} + \frac{\dot{Q}_{E,t}}{Q_{E,t}} - (i_t - \pi_t)$ for equities. $T_{j,t}$ denotes fiscal transfers.

The random arrival time t^* denotes when the aggregate shock hits the economy. In the crisis state, the value function V^*_{j,t^*} solves a deterministic version of Problem (1) with $\xi^*_j < \xi_j$, reflecting the loss of convenience from government bonds in a crisis.

A key feature of this problem is that government bonds provide liquidity services, as captured by $L_{j,t}$. The parameter $\xi_j \geq 0$ is a liquidity multiplier, which is allowed to vary across savers. The parameter $\nu \geq 0$ controls the degree of intertemporal substitution of liquidity services. The parameter $\alpha \in [0, 1]$ corresponds to the fraction of the liquidity services of short-term bonds provided by long-term bonds. Appendix B considers an extension where short- and long-term bonds are imperfect substitutes.

Workers' problem. Workers supply labor and have GHH preferences over consumption and labor. Unlike savers, workers do not derive utility from government bonds. Their

problem is given by:

$$V_{w,t}(B_{w,t}) = \max_{[C_{w,z},N_{w,z}]_{z\geq t}} \mathbb{E}_{w,t}\left[\int_t^{t^*} \frac{e^{-\rho_w(z-t)}}{1-\sigma}\left(C_{w,z}-\frac{N_{w,z}^{1+\phi}}{1+\phi}\right)^{1-\sigma} dz + e^{-\rho_w(t^*-t)}V^*_{w,t^*}(B_{w,t^*})\right],$$

subject to $\dot{B}_{w,t} = (i_t - \pi_t)B_{w,t} + \frac{W_t}{P_t}N_{w,t} + T_{w,t} - C_{w,t}$, and the borrowing constraint $B_{w,t} \geq 0$, where W_t is the nominal wage and $T_{w,t}$ denotes fiscal transfers to workers.

We focus on the case where $B_{w,0} = 0$ and ρ_w is sufficiently large, so workers are constrained at all times. As workers are constrained, their subjective crisis probability plays no direct role in equilibrium. Labor supply satisfies $\frac{W_t}{P_t} = N_{w,t}^{\phi}$.

Firms' problem. Final-goods producers are price takers and combine differentiated intermediate goods with a CES aggregator, implying the price index $P_t = \left(\int_0^1 P_{i,t}^{1-\epsilon}\, di\right)^{\frac{1}{1-\epsilon}}$ and demand $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t$.

Intermediate-goods producers are monopolistically competitive and produce with linear technology $Y_{i,t} = A_t N_{i,t}$, where $A_t = A$ in normal times and $A_t = A^*$ in the crisis state, with $0 < A^* < A$. Real profits are

$$\overline{\Pi}_{i,t} = \frac{P_{i,t}}{P_t}Y_{i,t} - \frac{W_t}{P_t}N_{i,t} - \frac{\varphi}{2}\,\pi_{i,t}^2, \tag{2}$$

where $\pi_{i,t} \equiv \dot{P}_{i,t}/P_{i,t}$ and $\varphi > 0$ is the Rotemberg adjustment-cost parameter. Adjustment costs are rebated to shareholders, i.e., dividends are given by $\Pi_{i,t} = \overline{\Pi}_{i,t} + \frac{\varphi}{2}\,\pi_{i,t}^2$, so adjustment costs do not represent a real deadweight loss. Each firm chooses $\pi_{i,t}$, given $P_{i,0}$, to maximize the expected discounted value of profits subject to demand and technology.

Government. The government is subject to a flow budget constraint:

$$\dot{D}_{G,t} = (i_t - \pi_t)D_{G,t} + r_{L,t}D^L_{G,t} + \sum_{j\in\{w,o,p\}} \mu_j T_{j,t},$$

and a No-Ponzi condition, $\lim_{T\to\infty} \mathbb{E}_t[\eta_T P_T D_{G,T}] \leq 0$, where $D_{G,t} = D^S_{G,t} + D^L_{G,t}$ denotes the real value of government debt, and η_t denotes the stochastic discount factor (SDF).

Transfers to workers follow the policy rule $T_{w,t} = T_w(Y_t)$, capturing a form of automatic stabilizers. We assume $T_{o,t} = T_{p,t}$, and the government adjusts lump-sum transfers to savers to satisfy its intertemporal budget constraint.

In normal times, monetary policy is determined by an inertial Taylor rule:

$$\dot{i}_t = \theta_m \left[r_n + \phi_\pi \pi_t + u_t - i_t \right], \tag{3}$$

where $\phi_\pi > 1$, u_t is a monetary shock, and r_n denotes the real rate when $\pi_t = u_t = 0$ at all periods. The parameter $\theta_m \geq 0$ controls the degree of inertia in monetary policy. We recover the non-inertial case as $\theta_m \to \infty$. We assume that in the crisis state there are no monetary shocks, that is, $\dot{i}^*_t = \theta_m \left[r^*_n + \phi_\pi \pi^*_t - i^*_t \right]$. By abstracting from the policy response after a crisis, we isolate the impact of changes in monetary policy during normal times.

Market clearing. The market-clearing conditions for consumption, short-term bonds, long-term bonds, and equity are given by:

$$\sum_{j \in \{w,o,p\}} \mu_j C_{j,t} = Y_t, \quad \sum_{j \in \{w,o,p\}} \mu_j B^S_{j,t} = D^S_{G,t}, \quad \sum_{j \in \{w,o,p\}} \mu_j B^L_{j,t} = D^L_{G,t}, \quad \sum_{j \in \{w,o,p\}} \mu_j B^E_{j,t} = Q_{E,t},$$

and $\mu_w N_{w,t} = N_t$, where $Y_t = \left(\int_0^1 Y_{i,t}^{\frac{\epsilon}{\epsilon-1}} di \right)^{\frac{\epsilon-1}{\epsilon}}$ and $N_t = \int_0^1 N_{i,t} di$.

3.2 Equilibrium characterization

We now characterize the equilibrium by deriving savers' Euler equations, the optimality conditions of firms, and the government intertemporal budget constraint.

Euler equations and zero-beta rate. Jump risk and liquidity services modify the standard Euler equations. The savers' Euler equation for short-term bonds is given by:

$$\frac{\dot{C}_{j,t}}{C_{j,t}} = \underbrace{\sigma^{-1}(i_t - \pi_t - \rho_{j,t})}_{\text{intertemporal substitution}} + \underbrace{\frac{\lambda_j}{\sigma} \left[\left(\frac{C_{j,t}}{C^*_{j,t}} \right)^\sigma - 1 \right]}_{\text{precautionary motive}} + \underbrace{\sigma^{-1} \xi_j C^\sigma_{j,t} L^{-\nu}_{j,t}}_{\text{liquidity services}}.$$

This expression highlights three channels shaping consumption growth. The first term captures the intertemporal substitution effect—the primary channel through which monetary policy affects consumption in textbook models (see, e.g., Galí 2015). The second term reflects the precautionary savings motive induced by crisis risk. The third term captures the marginal liquidity benefit from holding an additional unit of short-term bonds.

One way to interpret the short-bond Euler equation is that the relevant rate for saving decisions is not the nominal interest rate i_t, but a liquidity-adjusted rate, which we refer to as the *zero-beta rate*. Formally,

$$\frac{\dot{C}_{j,t}}{C_{j,t}} = \sigma^{-1}\left(i^{zb}_{j,t} - \pi_t - \rho_{j,t}\right) + \frac{\lambda_j}{\sigma}\left[\left(\frac{C_{j,t}}{C^*_{j,t}}\right)^{\sigma} - 1\right], \tag{4}$$

where the zero-beta rate for saver j is defined as

$$i^{zb}_{j,t} \equiv i_t + \xi_j C^{\sigma}_{j,t} L^{-\nu}_{j,t}.$$

Expressed in terms of $i^{zb}_{j,t}$, the Euler equation takes the same form as in an economy without liquidity services. This observation highlights that the behavior of the zero-beta rate is central for how monetary policy affects consumption and savings decisions.

Intuitively, the zero-beta rate corresponds to the riskless return implied by the prices of risky assets (see, e.g., Black 1972; Frazzini and Pedersen 2014). Because risky assets do not deliver liquidity services, the return inferred from their pricing reflects the compensation investors require on a riskless instrument that delivers no additional convenience. Short-term government bonds, in contrast, yield below this benchmark because they embed a *liquidity premium*. Thus, the spread $i^{zb}_{j,t} - i_t$ measures the value of liquidity services.

The connection between the zero-beta rate and the price of risky assets can be seen from the Euler equation for equities, which provide no liquidity services:

$$r_{E,t} = \underbrace{\lambda_j \left(\frac{C_{j,t}}{C^*_{j,t}}\right)^{\sigma}}_{\text{price of risk}} \underbrace{\frac{Q_{E,t} - Q^*_{E,t}}{Q_{E,t}}}_{\text{quantity of risk}} + \underbrace{\xi_j C^{\sigma}_{j,t} L^{-\nu}_{j,t}}_{i^{zb}_{j,t} - i_t}. \tag{5}$$

The first term is the (conditional) risk premium, which equals the product of the price of risk (the compensation per unit exposure to jump risk) and the quantity of risk (the fall in equity value upon a crisis).[4] The second term, $i^{zb}_{j,t} - i_t$, implies that the investor's required risk compensation corresponds to the excess return relative to the zero-beta rate. Consequently, the spread between equity returns and the short rate — the observed equity premium — includes not only the required risk compensation but also the liquidity premium on government bonds.

As the quantity of risk (or aggregate beta) approaches zero, expected returns converge to the zero-beta rate. This property matches how the zero-beta rate is estimated empirically (see, e.g., van Binsbergen, Diamond and Grotteria 2022b; Di Tella et al. 2023).[5] The zero-beta rate deviates from the risk-free rate because it corresponds to a return on a portfolio of risky assets, which does not provide liquidity services.

The Euler equation for long-term bonds is given by:

$$r_{L,t} = \lambda_j \left(\frac{C_{j,t}}{C^*_{j,t}} \right)^{\sigma} \frac{Q_{L,t} - Q^*_{L,t}}{Q_{L,t}} + (1-\alpha)\xi_j C^{\sigma}_{j,t} L^{-\nu}_{j,t}. \tag{6}$$

As in the case of equities, the risk premium on long-term bonds is the product of the price of risk and the quantity of risk. The second term captures the *inconvenience* of holding long-term bonds relative to short-term bonds. While equities provide no liquidity services—so their inconvenience is $i^{zb}_{j,t} - i_t$—long-term bonds provide a fraction α of short-bond liquidity services, implying a correspondingly smaller inconvenience.

The representative saver's SDF. In principle, the price of risk and the zero-beta rate could differ across investors. The next lemma shows that, generically, trading equalizes these two objects in equilibrium.

Lemma 1 (Risk–Liquidity Equalization)**.** *Generically, Equations* (5)–(6)*, for* $j \in \{o, p\}$*, imply*

[4] This is conditional on no crisis. The unconditional premium subtracts the expected crisis loss.

[5] In Appendix C.2, we introduce options into the model and show that the zero-beta rate coincides with the box rate (van Binsbergen et al. 2022b), the risk-free rate implied by the put-call parity, at short maturities.

the following equalities:[6]

$$\lambda_o \left(\frac{C_{o,t}}{C^*_{o,t}}\right)^{\sigma} = \lambda_p \left(\frac{C_{p,t}}{C^*_{p,t}}\right)^{\sigma}, \qquad \xi_o C^{\sigma}_{o,t} L^{-\nu}_{o,t} = \xi_p C^{\sigma}_{p,t} L^{-\nu}_{p,t}.$$

Lemma 1 implies that, even though savers disagree on the probability of a crisis, they agree on the value of a unit of consumption in the crisis state. This requires optimistic investors to be more exposed to jump risk than pessimistic investors in equilibrium. Similarly, even if savers differ in their liquidity needs—as captured by the multipliers ξ_j—the marginal value of liquidity services is equalized. A key implication is that the zero-beta rate is the same for all investors, and thus uniquely pinned down in equilibrium.

The equalization of the price of risk and the marginal value of liquidity enables us to derive an aggregation result. This result will play an important role in the analysis, as it allows us to obtain an SDF in terms of a representative saver.

Proposition 1 (Representative Saver's SDF). *Define* $\eta_t = e^{-\int_0^t \rho_{s,z} dz} C^{-\sigma}_{s,t} / P_t$ *and let* $\mathbb{E}_t[\cdot]$ *denote the expectation operator associated with the market-implied crisis probability* λ_t*:*

$$\lambda_t \equiv \left[\frac{\mu_o C_{o,t}}{\mu_o C_{o,t} + \mu_p C_{p,t}} \lambda_o^{\frac{1}{\sigma}} + \frac{\mu_p C_{p,t}}{\mu_o C_{o,t} + \mu_p C_{p,t}} \lambda_p^{\frac{1}{\sigma}}\right]^{\sigma}. \tag{7}$$

Then, for an appropriately chosen $\rho_{s,t}$*,* η_t *is a valid SDF, i.e., it satisfies*

$$\mathbb{E}_t\left[\frac{d(\eta_t R_{j,t})}{\eta_t R_{j,t}}\right] = -\alpha_j (i^{zb}_t - i_t) dt, \qquad \text{for } j \in \{S, L, E\}, \tag{8}$$

where $dR_{S,t} = i_t R_{S,t} dt$*, and* $\alpha_S = 1, \alpha_L = \alpha, \alpha_E = 0$*. The zero-beta rate is given by*

$$i^{zb}_t = i_t + \xi_t C^{\sigma}_{s,t} L^{-\nu}_{s,t},$$

[6] When the rank condition $(Q_{L,t} - Q^*_{L,t})/Q_{L,t} \neq (1-\alpha)(Q_{E,t} - Q^*_{E,t})/Q_{E,t}$ holds, there is a unique pair $\left(\lambda_j (C_{j,t}/C^*_{j,t})^{\sigma}, \xi_j C^{\sigma}_{j,t} L^{-\nu}_{j,t}\right)$ that solves Equations (5)–(6). If this condition fails, portfolios are indeterminate.

with the market-implied liquidity multiplier ξ_t defined as

$$\xi_t = \left[\frac{\mu_o C_{o,t}}{\mu_o C_{o,t} + \mu_p C_{p,t}} \left(\frac{C_{o,t}}{C_{s,t}} \right)^{\frac{\sigma-\nu}{\nu}} \xi_o^{\frac{1}{\nu}} + \frac{\mu_p C_{p,t}}{\mu_o C_{o,t} + \mu_p C_{p,t}} \left(\frac{C_{p,t}}{C_{s,t}} \right)^{\frac{\sigma-\nu}{\nu}} \xi_p^{\frac{1}{\nu}} \right]^{\nu}. \tag{9}$$

Proposition 1 shows that the economy behaves *as if* there is a representative saver who prices all assets. Importantly, the representative saver's beliefs, captured by the *market-implied probability* λ_t, are time-varying and endogenous whenever investors have heterogeneous beliefs ($\lambda_o < \lambda_p$).

Moreover, the zero-beta rate depends on aggregate liquidity, $L_{s,t} \equiv \frac{\mu_o}{\mu_o+\mu_p} L_{o,t} + \frac{\mu_p}{\mu_o+\mu_p} L_{p,t}$, and on the market-implied liquidity multiplier ξ_t. The liquidity multiplier, like the market-implied probability, is endogenous and time-varying. In the special case $\nu = \sigma$, the market-implied liquidity multiplier reduces to a CES aggregator of the individual multipliers, directly analogous to the expression for the market-implied crisis probability.

The pricing conditions in Equation (8) are reminiscent of models with stochastic Euler equation wedges, which are commonly used in international finance to capture convenience yield effects. However, while these wedges are typically treated as exogenous in such models (see, e.g., Jiang, Krishnamurthy and Lustig 2021), in our setting they are endogenous and adjust in response to monetary policy. This endogeneity—and the resulting dependence of the zero-beta rate on policy—is crucial for understanding how convenience yields shape the monetary transmission mechanism.

Non-linear NKPC. Firms maximize the expected discounted value of profits, given in Equation (2), discounted by the SDF η_t. The resulting optimality condition yields the non-linear New Keynesian Phillips curve (NKPC):

$$\dot{\pi}_{i,t} = \left(i_t^{zb} - \pi_t + \lambda_t \frac{\eta_t^*}{\eta_t} \right) \pi_{i,t} - \frac{\epsilon}{\varphi A} \left(\frac{W_t}{P_t} - \left(1 - \epsilon^{-1}\right) A \right) Y_t, \tag{10}$$

assuming $P_{i,0} = P_0$ and $\pi_{i,t}^* = 0$; under symmetry, $\pi_{i,t} = \pi_t$ for all i.

Government intertemporal budget constraint. From the No-Ponzi condition, which holds with equality in equilibrium, and the pricing conditions in Equation (8), we obtain

the government intertemporal budget constraint:

$$D_{G,0} = \mathbb{E}_0 \left[\int_0^\infty \frac{\eta_t}{\eta_0} \frac{P_t}{P_0} \left((i_t^{zb} - i_t)(D_{G,t}^S + \alpha D_{G,t}^L) - \mu_w T_w(Y_t) - \mu_s T_{s,t} \right) dt \right], \tag{11}$$

where $\mu_s \equiv \mu_o + \mu_p$ and $T_{s,t} = \frac{\mu_o}{\mu_o+\mu_p} T_{o,t} + \frac{\mu_p}{\mu_o+\mu_p} T_{p,t}$ is the transfer to savers in period t.

In the absence of liquidity frictions, we recover the standard government budget constraint: the real value of debt equals the present discounted value of future primary surpluses—equal to minus the value of transfers in our setting. Equation (11) also features an additional source of revenue: the *seigniorage from liquidity provision*, which depends on the spread between the zero-beta rate and the nominal interest rate, $i_t^{zb} - i_t$, and on the total supply of liquidity, $D_{G,t}^S + \alpha D_{G,t}^L = \mu_s L_{s,t}$.

3.3 Log-linear dynamics

In line with much of the monetary economics literature, we study a log-linear approximation of the equilibrium conditions. Unlike standard approaches that expand around the non-stochastic steady state, we follow Caramp and Silva (2025) and linearize around the stochastic stationary equilibrium. This choice ensures that the approximation captures key unconditional moments—such as a positive term spread and equity premium—and allows risk premia and precautionary savings to shape the dynamics of the economy even in the linearized system.[7] This approach is also crucial for capturing the hedging properties of convenience yields and the impact of changes in the risk of losing convenience.

Stationary equilibrium. We define a stationary equilibrium as one in which all variables remain constant within each aggregate state. Formally, assume $u_t = \pi_t = 0$ for all $t \geq 0$, so that there is no shock to the policy rule (3) and the monetary authority implements a zero-inflation policy.[8] Since variables are constant in each state, we drop time

[7] In standard approximations, a third-order expansion is required to capture risk premia effects (Schmitt-Grohé and Uribe 2004), and even higher-order approximations are needed to capture the effects of time-varying disaster probabilities (Andreasen 2012).

[8] By abstracting from uncertainty due to monetary shocks, we can solve for the stationary equilibrium in closed form. In contrast, a risky steady-state approximation (Coeurdacier, Rey and Winant 2011) requires numerical methods to characterize the approximation point.

subscripts and write, for instance, $C_{j,t} = C_j$ and $C^*_{j,t} = C^*_j$. For ease of exposition, we assume that fiscal transfers implement $C_w = Y$ and $C^*_w = Y^*$, which implies $C_s = Y$ and $C^*_s = Y^*$, and that the crisis-state allocation is symmetric: $C^*_w = C^*_o = C^*_p$.

In a stationary equilibrium with both savers unconstrained, Equation (4) requires $\rho_o + \lambda_o = \rho_p + \lambda_p$. Since $\rho_j(\cdot)$ depends on the consumption share, this condition pins down the unconditional consumption and wealth distribution. For simplicity, we assume that it holds when both types have the same net worth, $B_o = B_p$, which implies $C_o > C_p$.

The natural interest rate—the real rate in the stationary equilibrium—is

$$r_n = \rho_s - \lambda \left[\left(\frac{C_s}{C^*_s} \right)^{\sigma} - 1 \right] - \xi \, C_s^{\sigma} L_s^{-\nu},$$

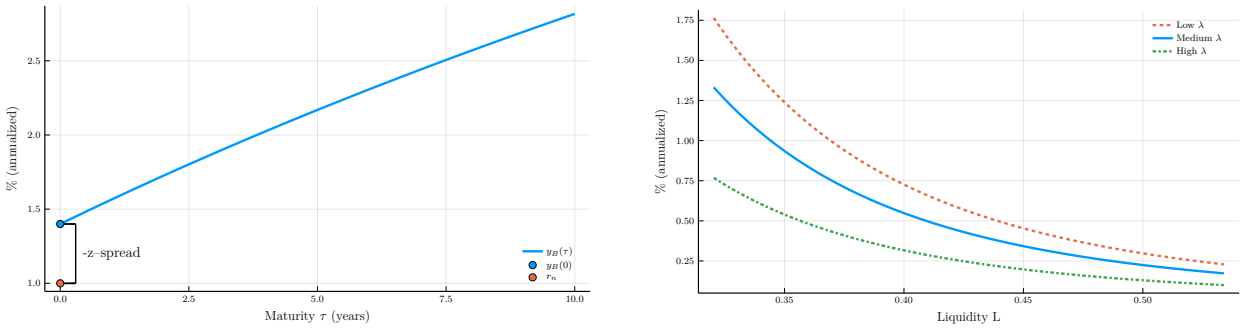
where (ρ_s, λ, ξ) denote the stationary value of $(\rho_{s,t}, \lambda_t, \xi_t)$. We assume $r_n > 0$. Relative to the textbook model, the natural rate is lower for two reasons: (i) crisis risk induces a precautionary motive that reduces r_n; and (ii) the liquidity services of short-term bonds further depress r_n. The dependence of the natural rate on the supply of liquid assets is consistent with evidence in Ferreira and Shousha (2023). Note that the zero-beta rate,

$$r^{zb}_n = r_n + \xi \, C_s^{\sigma} L_s^{-\nu},$$

is determined solely by impatience and risk, while the natural rate is also shaped by the value of liquidity services.

We now examine the behavior of long-term bonds. To characterize the entire yield curve, let $Q_B(\tau)$ denote the price of a zero-coupon bond of maturity τ in the stationary equilibrium. Liquidity services are given by $\alpha_B(\tau) = \alpha$ for all maturities. Hence, the government bond with exponentially decaying coupons can be viewed as a portfolio of zero-coupon bonds: $Q_L = \int_0^{\infty} e^{-\psi_L \tau} Q_B(\tau) \, d\tau$.

Proposition 2 (Yield curve). *Suppose $\lambda > 0$, $r^{zb}_n \leq r^{zb*}_n$, and $r^{zb}_n - r_n \geq r^{zb*}_n - r^*_n$, with at least one strict inequality. Let $y_b(\tau) \equiv -\frac{1}{\tau} \log Q_B(\tau)$ be the yield on a zero-coupon bond of maturity*



(a) Yield curve **(b)** Convenience yield

Figure 6: Yield curve and convenience yield in the stationary equilibrium.

τ. *Then, in the stationary equilibrium, the yield curve is upward sloping, i.e.*

$$\frac{\partial y_b(\tau)}{\partial \tau} > 0 \qquad \textit{for all } \tau > 0,$$

and the short-end limit satisfies

$$\lim_{\tau \to 0} y_b(\tau) = r_n^{zb} - \alpha \xi C^{\sigma} L^{-\nu} \; > \; r_n.$$

Proposition 2 shows that the yield curve is upward sloping. Intuitively, long-term bonds are more exposed to crisis risk, in which case their prices fall either because they lose convenience services or because they are discounted at a higher zero-beta rate.

Moreover, Proposition 2 establishes that the short end of the yield curve, for bonds with $\alpha_B(\tau) = \alpha$, lies above the natural rate, as shown in Panel (a) of Figure 6. This pattern matches the evidence in Greenwood et al. (2015), who measure a "z-spread" by comparing the observed yields on T-bills with those of short-term bonds extrapolated from the fitted yield curve of Gürkaynak, Sack and Wright (2007)—which relies mainly on longer-term bonds. In our framework, the corresponding spread is

$$r_n - y_B(0) = -(1-\alpha)\, \xi\, C_s^{\sigma} L_s^{-\nu}, \tag{12}$$

which is negative, in line with their findings. Moreover, Greenwood et al. (2015) show that the z-spread rises with the supply of short-term bonds, consistent with Equation (12).

Following Krishnamurthy and Vissing-Jorgensen (2012), we define the convenience yield as the difference between the yield on a (default-free) corporate bond, $i_C \equiv Q_C^{-1} - \psi_C$, and the yield on the government bond, $i_L \equiv Q_L^{-1} - \psi_L$, where the two bonds have identical promised cash flows ($\psi_C = \psi_L$) but the corporate bond provides no liquidity services (i.e., $\alpha_C = 0$):

$$cy \equiv i_C - i_L = \underbrace{\alpha(r_n^{zb} - r_n)}_{\substack{\text{liquidity} \\ \text{differential}}} + \underbrace{\lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \left[\frac{Q_L^*}{Q_L} - \frac{Q_C^*}{Q_C}\right]}_{\text{convenience risk premium}}.$$

The first term reflects the liquidity differential: government bonds deliver liquidity services that corporate bonds do not, which depresses their returns. This term declines with the supply of liquidity L_s, as shown in Panel (b) of Figure 6, consistent with the evidence in Krishnamurthy and Vissing-Jorgensen (2012). The second term, the *convenience risk premium*, captures the fact that government bonds are exposed to the risk of losing their liquidity services in a crisis, whereas corporate bonds are not exposed to this risk. Although the two securities have identical cash flows and therefore no difference in cash-flow risk, they nonetheless command different risk premia. Acharya and Laarits (2023) show that the convenience yield is lower when the correlation between stocks and bonds increases. In our setting, an increase in the market-implied probability λ makes it more likely that stocks and bonds will underperform, as both suffer losses in a crisis, consistent with their evidence.[9] Thus, our convenience risk premium captures a mechanism that links liquidity services to risk premia in a way that is reminiscent of the liquidity-adjusted CAPM of Acharya and Pedersen (2005). Overall, this highlights how risk and liquidity jointly shape both the yield curve and the convenience yield.

Finally, the equity premium conditional on no crisis is

$$r_E = \left(r_n^{zb} - r_n\right) + \lambda \left(\frac{C_s}{C_s^*}\right)^{-\sigma} \frac{Q_E - Q_E^*}{Q_E}.$$

[9]Formally, the model cannot generate time-varying correlations with a single shock. In the appendix, we add a shock that affects stocks but not bonds, and show that an increase in λ corresponds to an increase in the correlation between stocks and bonds in that setting.

The first component reflects the inconvenience of equities relative to short-term bonds, while the second component is the risk premium, investor's required risk compensation. Hence, excess returns over short-term government bonds embed not only compensation for risk but also for illiquidity.

The linear system. Next, we linearize the equilibrium conditions around the stationary equilibrium. Let lower-case variables denote log-deviations from the stationary equilibrium, e.g., $y_t \equiv \log Y_t/Y$. Workers' consumption is

$$c_{w,t} = \frac{WN_w}{PY}(w_t - p_t + n_{w,t}) + T'_w(Y)y_t,$$

which simplifies to $c_{w,t} = \chi_y y_t$ with $\chi_y \equiv \frac{WN_w}{PY}(1+\phi) + T'_w(Y)$, using $w_t - p_t = \phi y_t$ and $n_{w,t} = y_t$. We focus on the case $0 < \chi_y < \mu_w^{-1}$, such that savers' consumption, $c_{s,t} = \frac{1-\mu_w\chi_y}{1-\mu_w} y_t$, is also increasing in y_t.

Linearizing the Euler equation (4) around the stationary equilibrium yields

$$\dot{c}_{s,t} = \sigma^{-1}(\hat{i}^{zb}_t - \pi_t - r_n) + \sigma^{-1} p_{r,t},$$

after aggregating across savers, where we used the linearized discount-rate function $\rho_{j,t} = \rho_j + \sigma\psi_c(c_{j,t} - c_{s,t})$, which we assume has a common slope, $\sigma\psi_c$.

The linearized price of risk and the zero-beta rate are given by

$$p_{r,t} = \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \left(\hat{\lambda}_t + \sigma c_{s,t}\right), \qquad \hat{i}^{zb}_t = i_t + \xi C_s^{\sigma} L_s^{-\nu} \left(\hat{\xi}_t + \sigma c_{s,t} - \nu l_{s,t}\right),$$

with $\hat{\lambda}_t \equiv \log(\lambda_t/\lambda)$ and $\hat{\xi}_t \equiv \log(\xi_t/\xi)$, and we used the fact that $c^*_{s,t} = 0$

Proposition 3 (Aggregate dynamics)**.** *Given $[\hat{i}^{zb}_t, p_{r,t}]_{t\geq 0}$, output and inflation are given by:*

i. AGGREGATE EULER EQUATION:

$$\dot{y}_t = \tilde{\sigma}^{-1}(\hat{i}^{zb}_t - \pi_t - r_n + p_{r,t}), \tag{13}$$

where $\tilde{\sigma}^{-1} \equiv \frac{1-\mu_w}{1-\mu_w\chi_y}\sigma^{-1}$.

ii. NEW KEYNESIAN PHILLIPS CURVE:

$$\dot{\pi}_t = \rho\pi_t - \kappa y_t, \tag{14}$$

with $\rho \equiv \rho_s + \lambda$ *and* $\kappa \equiv \varphi^{-1}(\epsilon - 1)\phi Y$.

Proposition 3 shows that, despite heterogeneity, risk, and liquidity frictions, the aggregate dynamics resemble the textbook New Keynesian system. The aggregate Euler equation retains its familiar form but with two key differences: (i) the relevant rate is the zero-beta rate, $\hat{i}_t^{zb}$, which depends on both the nominal rate and marginal liquidity services; and (ii) output dynamics also depend on the price of risk, $p_{r,t}$, which captures compensation for crisis risk. Quantitatively, $p_{r,t}$ is driven primarily by the market-implied probability $\hat{\lambda}_t$, which is time-varying in the presence of heterogeneous beliefs.

Proposition 3 also highlights the role of heterogeneous marginal propensities to consume (MPCs). In particular, the aggregate elasticity of intertemporal substitution (EIS), $\tilde{\sigma}^{-1}$, is not simply a weighted average of the EIS of workers (i.e., effectively zero) and savers. Consider the case $\chi_y = 1$, consistent with recent evidence in Bilbiie, Galaasen, Gürkaynak, Mæhlum and Molnar (2025). Even if $\mu_w \approx 1$, so that only a small fraction of agents substitute intertemporally, the aggregate EIS coincides with that of savers.[10] Thus, a small fraction of agents reacting to the zero-beta rate suffices for the whole economy to behave as if everyone reacts, owing to the amplification created by hand-to-mouth households.

Finally, Proposition 3 shows that the New Keynesian Phillips curve retains the same form as in the standard model. Hence, inflation equals the present discounted value of future output gaps, but here the discount rate is ρ, which exceeds the natural interest rate r_n due to presence of risk and liquidity frictions.

Market-implied crisis probability and liquidity multiplier. Relative to the standard New Keynesian model, aggregate dynamics now depend on both the *price of risk*, a function of $\hat{\lambda}_t$, and the *zero-beta rate*, which is shaped by $\hat{\xi}_t$ and by the supply of liquid assets

[10] This mechanism is well understood in the HANK literature, see, e.g., Werning (2015) and Bilbiie (2019).

$l_{s,t}$. To close the log-linear system, we must therefore characterize the joint dynamics of $(\hat{\lambda}_t,\ \hat{\xi}_t,\ l_{s,t})_{t\geq 0}$.

Because $l_{s,t}$ depends on the quantity of government debt that delivers liquidity services, the response of the economy to a monetary shock necessarily depends on fiscal policy. In particular, what matters is how the supply of liquid assets adjusts when monetary policy moves interest rates.

To keep the analysis tractable, we assume that the fiscal authority chooses transfers so that the *par value* of government liabilities remains constant:

$$D^S_{G,t} = D^S_G, \qquad D^L_{G,t} = \frac{Q_{L,t}}{Q_L} D^L_G.$$

Although the quantities of short- and long-term debt are held fixed at par, the market value of long-term debt still responds endogenously to interest rate movements. Consequently, the supply of liquidity varies with bond prices:

$$l_{s,t} = \gamma_L\, q_{L,t}, \qquad \gamma_L \equiv \frac{\alpha D^L_G}{\mu_s L_s}.$$

Given this fiscal policy rule, it remains to determine $(\hat{\lambda}_t,\ \hat{\xi}_t,\ q_{L,t})_{t\geq 0}$.

Lemma 2 (Consumption, net worth, and bond prices). *Let $\mu_{c,j} \equiv \frac{\mu_j C_j}{\mu_o C_o + \mu_p C_p}$ for $j \in \{o, p\}$. Then the market-implied crisis probability and liquidity multiplier satisfy*

$$\hat{\lambda}_t = \chi_{\lambda,\Delta c}\,(c_{p,t} - c_{o,t}), \qquad \hat{\xi}_t = \chi_{\xi,\Delta c}\,(c_{p,t} - c_{o,t}).$$

Moreover, the joint dynamics of relative consumption, bond prices, and relative net worth obey

$$\begin{bmatrix} \dot{c}_{p,t} - \dot{c}_{o,t} \\ \dot{q}_{L,t} \\ \dot{b}_{p,t} - \dot{b}_{o,t} \end{bmatrix} = \begin{bmatrix} \chi_{\Delta c,\Delta c} & 0 & 0 \\ \chi_{q_L,\Delta c} & \chi_{q_L,q_L} & 0 \\ \chi_{\Delta b,\Delta c} & \chi_{\Delta b,q_L} & \chi_{\Delta b,\Delta b} \end{bmatrix} \begin{bmatrix} c_{p,t} - c_{o,t} \\ q_{L,t} \\ b_{p,t} - b_{o,t} \end{bmatrix} + \begin{bmatrix} 0 \\ i_t - r_n + r_L \sigma c_{s,t} \\ \chi_{\Delta b,c_s}\, c_{s,t} \end{bmatrix}, \tag{15}$$

with coefficients given in the appendix.

Lemma 2 shows that the *only* variable driving both the market-implied probability of a

crisis and the liquidity multiplier is the *consumption gap* between pessimists and optimists, $c_{p,t} - c_{o,t}$. The behavior of $(c_{p,t} - c_{o,t},\ q_{L,t},\ b_{p,t} - b_{o,t})$ is governed by the linear system (15), whose coefficients depend solely on portfolios and returns in the stationary equilibrium.

An important implication is that, once this system is characterized, the dynamics of the market-implied crisis probability and liquidity multiplier can be solved independently of aggregate output and inflation.

Proposition 4 (Market-implied probability and liquidity multiplier)**.** *The market-implied crisis probability and liquidity multiplier evolve according to*

$$\hat{\lambda}_t = e^{-\psi_c t}\,\hat{\lambda}_0, \qquad \hat{\xi}_t = e^{-\psi_c t}\,\hat{\xi}_0,$$

where the initial values depend on the path of nominal interest rates and on the degree of redistribution across savers, $b_{p,0} - b_{o,0}$.

Proposition 4 implies that the market-implied crisis probability and the liquidity multiplier both decay at rate ψ_c. The parameter ψ_c is the slope of the discount-rate function,

$$\rho_{j,t} = \rho_j + \sigma\psi_c(c_{j,t} - c_{s,t}),$$

and equals zero when subjective discount rates are constant.

The intuition is straightforward. A monetary tightening redistributes wealth away from optimistic savers, who temporarily reduce consumption to rebuild their wealth. The strength of this rebalancing motive is governed by ψ_c: the larger is ψ_c, the faster relative consumption (and hence relative wealth) reverts to its stationary level. This illustrates the role of the Uzawa preferences, which ensure that the wealth distribution is stationary and eventually returns to its stationary equilibrium value after a temporary shock.[11]

The four-equation system. By combining the results of Proposition 3 and Lemma 2, we obtain a system describing the economy's aggregate dynamics. Given $b_{p,0} - b_{o,0}$, which

[11] Uzawa preferences play the same role in the small open economy model of Schmitt-Grohé and Uribe (2003), where they ensure that the country's foreign net asset position is stationary.

determines $\{\hat{\lambda}_0, \hat{\xi}_0\}$, we can solve the system given by the textbook New Keynesian core — the policy rule (3), the aggregate Euler equation (13), and the New Keynesian Phillips curve (14) — and an additional equation capturing the the dynamics of long-term bond prices. Then, using the boundary condition $b_{p,0} - b_{o,0} = \sum_{k\in\{L,E\}} \left(\frac{B_p^k}{B_p} - \frac{B_o^k}{B_o}\right) q_{k,0}$, we get the solution to the system.

A key property of the full dynamic system is that the evolution of real variables (i_t, y_t, π_t) is intertwined with the dynamics of long-term government bond prices $q_{L,t}$. In many macro-finance environments, a form of *macro-finance separation* holds: the dynamics of the real economy are invariant to the behavior of asset prices.[12] In contrast, in our setting the behavior of long-term yields directly feeds back into, and shapes, the transmission of monetary policy to output and inflation.

4 Monetary Policy and Asset Prices

This section studies how monetary policy affects asset prices through its joint impact on long-term interest rates, convenience yields, and equity valuations. The key mechanism is that monetary policy simultaneously alters (i) the price of risk, through wealth redistribution between pessimistic and optimistic savers, and (ii) the liquidity value of government debt, primarily through changes in the value of long-term government bonds. We first characterize analytically how these channels shape the response of long-term interest rates and convenience yields, and then examine the quantitative implications for asset prices.

4.1 Long-Term Interest Rates

We begin by characterizing the response of long-term government bond yields to monetary policy. Let $i_{L,t} \equiv Q_{L,t}^{-1} - \psi_L$ denote the yield on the long-term government bond.

[12] See, e.g., Tallarini Jr. (2000), Pflueger and Rinaldi (2022a), and Bianchi et al. (2022) for examples of models exhibiting macro-finance separation.

Linearizing the pricing condition for $Q_{L,t}$ gives the decomposition

$$i_{L,0} - i_L = \underbrace{\mathcal{E}_0}_{\substack{\text{expected} \\ \text{interest rates}}} + \underbrace{\mathcal{R}_0}_{\text{risk premium}} + \underbrace{\mathcal{L}_0}_{\text{liquidity}},$$

where $i_{L,0} - i_L = -Q_L^{-1} q_{L,0}$. Long-term yields therefore reflect three forces: expected short rates, a crisis-risk premium, and the liquidity value of government debt.

(i) Expectations component:

$$\mathcal{E}_0 = (i_L + \psi_L) \int_0^\infty e^{-(\rho - \alpha(r_n^{zb} - r_n) + \psi_L)t} (i_t - r_n)\, dt.$$

In the absence of risk, the long-term yield reduces to

$$i_L = \rho - \alpha(r_n^{zb} - r_n),$$

and so $\mathcal{E}_0$ equals the average expected future short rate. This is the familiar *expectations-hypothesis* term and is the only channel present in the textbook New Keynesian model.

(ii) Risk premium:

$$\mathcal{R}_0 = (i_L + \psi_L) \int_0^\infty e^{-(\rho - \alpha(r_n^{zb} - r_n) + \psi_L)t} \frac{Q_L - Q_L^*}{Q_L} p_{r,t}\, dt,$$

where recall that $p_{r,t} = \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma (\hat{\lambda}_t + \sigma c_{s,t})$ is the price of risk. The quantity of risk, $\frac{Q_L - Q_L^*}{Q_L}$, is determined by the bond's payoff in a crisis, while movements in $p_{r,t}$ reflect shifts in the wealth distribution. A contractionary monetary policy shock raises the price of risk, increasing long-term yields. This channel is consistent with the excess sensitivity of long-term interest rates to monetary policy shocks documented, e.g., by Hanson and Stein (2015).

(iii) Liquidity component:

$$\mathcal{L}_0 = (i_L + \psi_L) \int_0^\infty e^{-(\rho - \alpha(r_n^{zb} - r_n) + \psi_L)t} (1 - \alpha) (\hat{i}_t^{zb} - i_t)\, dt,$$

where

$$\hat{i}_t^{zb} - i_t = \xi C_s^{\sigma} L_s^{-\nu} (\hat{\xi}_t + \sigma c_{s,t} - \nu l_{s,t})$$

is the change in the deviation of the zero-beta rate from the policy rate. This term will be present to the extent that long-term government bonds provide less liquidity services than short-term bonds, which is the case in our model.

To connect this expression to the convenience yield, let $i_{C,t} \equiv Q_{C,t}^{-1} - \psi_C$ denote the yield on a AAA corporate bond with the same duration as $Q_{L,t}$ (so $i_L + \psi_L = i_C + \psi_C$) and with zero liquidity services ($\alpha_C = 0$). In the absence of risk, the convenience yield satisfies

$$cy_0 = (i_L + \psi_L) \int_0^{\infty} e^{-(i_L+\psi_L)t} \alpha \, (\hat{i}_t^{zb} - i_t) \, dt,$$

where $cy_0 \equiv i_{C,0} - i_C - (i_{L,0} - i_L)$.

Matching the empirical evidence requires that the zero-beta rate respond more than one-for-one to movements in i_t. The log-linearized expression shows two primary channels:[13]

$$\hat{i}_t^{zb} - i_t \approx \xi C_s^{\sigma} L_s^{-\nu} (\hat{\xi}_t - \nu l_{s,t}).$$

The first term reflects the response of the liquidity multiplier $\hat{\xi}_t$, which moves similarly to the price of risk $\hat{\lambda}_t$. The second reflects the response of the supply of liquidity $l_{s,t} = \gamma_L q_{L,t}$.

This second channel generates an important feedback effect. A contractionary monetary policy lowers the price of long-term bonds, contracting the supply of liquidity. Because the zero-beta rate increases when liquidity is scarce, the decline in bond prices raises the liquidity spread, which further depresses bond prices. This feedback loop is stronger when ν is larger, that is, when liquidity services are difficult to substitute intertemporally. Such a situation naturally arises when liquidity demand is driven by regulatory or institutional requirements. This mechanism is essential for matching the empirical evidence in Section 2.

[13] We ignore the term $\xi C_s^{\sigma} L_s^{-\nu} \sigma c_{s,t}$, which is quantitatively small. A similar observation applies to the $\sigma c_{s,t}$ component of the price of risk.

4.2 Equity Prices

The pricing condition for equities is given by

$$q_{E,0} = \underbrace{\frac{\Pi}{Q_E}\int_0^\infty e^{-\rho t}\,\hat{\Pi}_t\,dt}_{\text{dividends}} - \underbrace{\int_0^\infty e^{-\rho t}\left[\hat{i}_t^{zb} - \pi_t - r_n + \frac{Q_E - Q_E^*}{Q_E}p_{r,t}\right]dt}_{\text{discount rate}}.$$

Changes in equity valuations therefore depend on two components: (i) the path of dividends and (ii) the discount rate. The dividend channel is standard. The discount rate channel, however, is shaped by two forces that are specific to our environment.

First, the term $\frac{Q_E - Q_E^*}{Q_E}p_{r,t}$ shows that the discount rate moves with the price of risk $p_{r,t}$. A contractionary monetary policy shock increases $p_{r,t}$ by redistributing wealth toward pessimistic savers, raising the discount rate and lowering equity prices.

Second, the relevant short-term discount rate is the zero-beta rate $\hat{i}_t^{zb}$, rather than the nominal policy rate i_t. As discussed above, the convenience-yield channel implies that $\hat{i}_t^{zb}$ rises more than one-for-one with i_t. This amplifies the effect of monetary policy on discount rates and therefore on equity prices.

This mechanism is consistent with the evidence of Nagel and Xu (2024), who document that movements in bond yields account for a large share of the response of equity prices to monetary policy shocks. In our model, both the movement in the zero-beta rate and the change in the price of risk affect bond yields, and these yield movements feed directly into equity valuations.

4.3 Calibration

We calibrate the model at a quarterly frequency. Table 1 summarizes the parameter values.

Preferences and demographics. We set the savers' subjective discount rate in the stationary equilibrium to match a natural interest rate of $r_n = 1\%$ annually. The risk-aversion coefficient is set to $\sigma = 4$, a value within the range considered reasonable by Mehra and Prescott (1985), but substantially larger than $\sigma = 1$, a value often adopted in macroeco-

Table 1: Calibrated parameters

Parameter	Description	Value	Target / Source
Preferences			
σ	Risk aversion	4	Mehra and Prescott (1985); EIS $= 0.25$
ρ_s	Discount rate (savers)	0.033	$r_n = 1\%$ annually
μ_w	Fraction of workers	0.30	Kaplan, Violante and Weidner (2014)
Government debt			
d_G^S	Short-term debt / GDP	0.075	Reserves / GDP
d_G^L	Long-term debt / GDP	0.28	Public debt held by private sector
ψ_L	Bond decay rate	0.044	Duration of 5 years
Disaster risk			
λ	Disaster probability	0.017	Barro (2006)
ζ_A	Disaster size (average)	0.29	Barro (2006)
Price rigidity and policy			
κ	Phillips curve slope	0.30	3-quarter price duration
ψ_m	Monetary shock persistence	0.33	Half-life of 2 quarters
ψ_r	Forward rate decay	0.57	Hanson and Stein (2015)
Asset pricing (estimated)			
ξ	Liquidity parameter	4.75×10^{-3}	
ν	Liquidity substitution	150	
α	Long-bond liquidity services	0.70	
ρ_s^*	Crisis discount rate	0.01	
ξ^*	Crisis liquidity parameter	0.0	
$\hat{\lambda}_0$		300	
$\hat{\xi}_0$		100	

Note: Parameters in the "Asset pricing" block are jointly estimated to match six moments. See Table 2 for model fit.

nomic models. This choice implies a micro EIS of $\sigma^{-1} = 0.25$, consistent with the estimate of 0.1 in Best, Cloyne, Ilzetzki and Kleven (2020) and the estimates for asset holders of 0.3 in Havránek (2015). The fraction of workers is set to $\mu_w = 30\%$, in line with the share of (poor and wealthy) hand-to-mouth agents in the U.S. estimated by Kaplan et al. (2014).

Government debt and fiscal policy. The stock of short-term debt, d_G^S, is chosen to match the ratio of reserves to GDP of 7.5% in the data. The stock of long-term debt, d_G^L, is set to

match a ratio of the market value of public debt held by the private sector to GDP of 28%. The parameter ψ_L is chosen to match a duration of five years, roughly in line with the historical average between 1962 and 2007 for the United States (Hall and Sargent, 2011).

The parameter $T'_w(Y)$ is chosen such that $\chi_y = 1$, which requires countercyclical transfers to balance procyclical wage income. A value of $\chi_y = 1$ is consistent with the evidence in Cloyne, Ferreira and Surico (2020) on the monetary policy impact on the income of borrowers (proxing for hand-to-mouth agents) and savers, where they show that the net income of mortgagors and non-mortgagors reacts similarly to monetary shocks.

Disaster risk. We follow Barro (2006) closely in calibrating the disaster risk parameters. The steady-state disaster intensity λ is set to match an annual disaster probability of 1.7%. To better map the model to the data, we consider an extension where the magnitude of the productivity drop, $\zeta_A \equiv 1 - A_*/A$, to be stochastic, drawn from the empirical distribution estimated by Barro (2006). In that distribution, ζ_A ranges from 15% to 64%, with an average of 29%. Introducing a random disaster size has only a minor effect on the analytical expressions, with the term $(C_{s*})^{-\sigma}$ typically replaced by $\mathbb{E}[(C_{s*})^{-\sigma}]$, where the expectation is taken over ζ_A.[14]

Price rigidity and monetary policy. The pricing cost parameter φ is chosen to match a slope of the Phillips curve of $\kappa = 0.30$, the value implied by the textbook model with an average price duration of three quarters and $\sigma = 4$.

We focus on a monetary shock that generates an exponentially decaying path for the nominal interest rate: $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$. We set $\psi_m = 0.33$, which gives a half-life of roughly two quarters. From the response of the forward curve around FOMC meetings estimated by Hanson and Stein (2015), we obtain $\psi_r = 0.57$.

Asset-pricing parameters. Seven parameters govern the asset-pricing implications of the model: $(\nu, \alpha, \xi, \rho_s^*, \xi_*, \hat{\xi}_0, \hat{\lambda}_0)$. We calibrate these to match six moments: the term premium, equity premium, and convenience yield in the stationary equilibrium, and the

[14] With $\sigma = 4$ and the estimated distribution of disaster sizes, the expected change in marginal utility conditional on a disaster is $\mathbb{E}[(1 - \zeta_A)^{-\sigma}] = 7.69$.

responses of the five-year yield, stock prices, and the convenience yield, to a monetary shock. The calibration implies a zero-beta rate of 2.9% in the stationary equilibrium.

The calibrated value of $\nu = 150$ implies that liquidity services are very difficult to substitute intertemporally. This high value is consistent with liquidity demand being driven by regulatory or institutional requirements—such as reserve requirements or collateral constraints—rather than by smoothly substitutable preferences. As discussed above, a high ν strengthens the feedback loop between bond prices and convenience yields: when long-term bond prices fall, the supply of liquidity contracts, raising the zero-beta rate and further depressing bond prices.

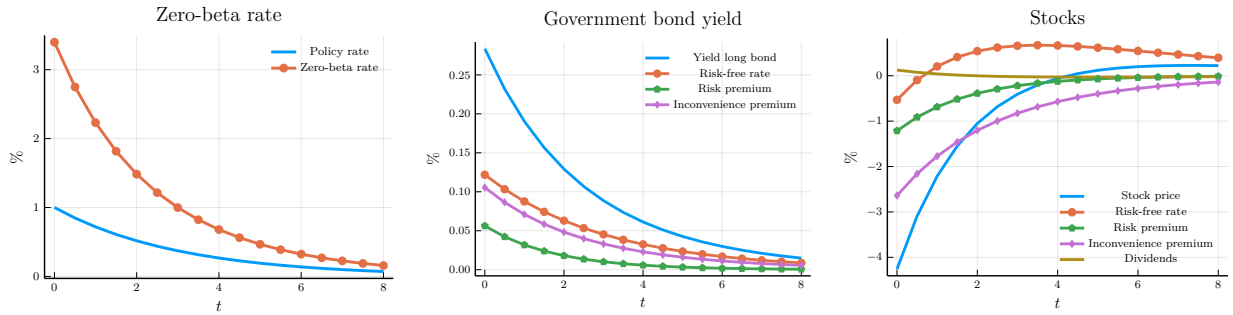
The parameter $\alpha = 0.7$ indicates that long-term government bonds provide a fair amount of liquidity services but significantly lower than short-term bonds. This value is broadly consistent with evidence that Treasury securities of different maturities are close substitutes for many institutional investors, though short-term bills retain a liquidity advantage.

The calibration implies an initial response of the market-implied disaster probability, $\hat{\lambda}_0 = 300$, that is substantially larger than the initial response of the liquidity multiplier, $\hat{\xi}_0 = 100$. This indicates that the redistribution of wealth toward pessimistic investors following a monetary contraction has a larger effect on perceived tail risk than on the marginal value of liquidity services.

Fiscal backing. For the response of fiscal variables to monetary shocks, we follow Jiang, Lustig, Van Nieuwerburgh and Xiaolan (2019) and estimate a VAR augmented with fiscal variables, applying the recursiveness assumption to construct empirical impulse responses. The present value of interest payments increases by 36 basis points and the initial value of government debt drops by 18 basis points.[15] The present value of primary surpluses increases by just 9 basis points.

[15]The present discounted value of interest payments is calculated as $\sum_{t=0}^{\mathcal{T}} \left(\frac{1+g}{1+i_L}\right)^{4t} [\hat{i}_{L,t} - \hat{\pi}_t]$, where $\mathcal{T}$ is the truncation period, $\hat{i}_{L,t}$ is the IRF of the 5-year rate, and $\hat{\pi}_t$ is the IRF of inflation. We set $g = 0.02$, $i_L = 0.043$, and $\mathcal{T} = 60$ quarters.

Figure 7: Asset-price response to a monetary shock



Note: The figure shows the response of asset prices to a 100-basis-point contractionary monetary shock. The left panel decomposes the zero-beta rate response. The middle panel decomposes the long-term government bond yield. The right panel decomposes the stock price response.

4.4 Quantitative Asset-Pricing Implications

We now examine the model's quantitative implications for asset prices. Figure 7 presents the impulse responses to a 100-basis-point contractionary monetary shock, along with decompositions that isolate the contribution of each channel.

The zero-beta rate. The shock raises the zero-beta rate by 340 basis points on impact. Since the nominal policy rate rises by only 100 basis points, the remaining 240 basis points reflect the endogenous response of convenience yields. This amplification arises mainly because the decline in long-term bond prices reduces the market value of liquid assets, further increasing the marginal value of liquidity services. The magnitude of this effect depends on ν, the intertemporal elasticity of substitution for liquidity services: when ν is high, as it is in our calibration, liquidity is difficult to substitute over time, and the feedback from bond prices to the zero-beta rate is stronger. The contribution of the change in the liquidity multiplier ξ_t is quantitatively negligible.

Long-term bond yields. The yield on long-term government bonds rises by 28 basis points on impact. The decomposition in the middle panel of Figure 7 reveals that this response reflects contributions from all three channels identified above: the expectations channel (movements in expected future short rates), the risk-premium channel (an increase in the market-implied probability of a crisis), and the liquidity channel (a rise in the

Table 2: Moments in the data and in the model (%)

Moment	Data	Full Model	No Convenience
Term premium	1.50	1.55	0.98
Equity premium	5.00	4.53	3.43
Convenience yield	0.72	0.83	0.00
Δ Long bond yield	0.32	0.28	0.18
Δ Stock prices	-4.00	-4.26	-2.37
Δ Convenience yield	0.25	0.24	0.00

marginal value of liquidity services). The risk-premium and liquidity channels account for the bulk of the yield response, consistent with the evidence on the excess sensitivity of long-term rates to monetary policy documented by Hanson and Stein (2015).

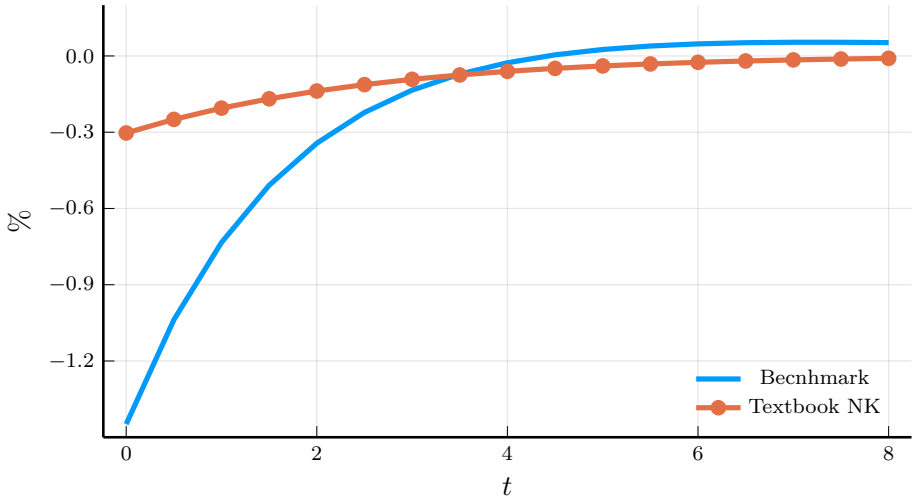
Stock prices. Stock prices decline by 4.26% on impact. The right panel of Figure 7 decomposes this decline into the different channels. The increase in the risk-free component of the discount rate contributes 53 basis points to the decline. The rise in the risk premium—driven by the redistribution of wealth toward pessimistic investors—adds 1.21 percentage points. The widening of the convenience spread contributes 2.64 percentage points. Thus, movements in risk premia and convenience yields account for the vast majority of the equity-price response, with the convenience channel being quantitatively the most important.

This decomposition is consistent with the findings of Nagel and Xu (2024), who show that movements in bond yields account for a large share of the stock-price response to monetary policy. In our model, the same forces that move bond yields—changes in the zero-beta rate and the price of risk—also drive equity valuations, providing a unified account of these patterns.

5 Monetary Policy and the Real Economy

We now turn to the response of output and inflation to monetary policy. The model's Phillips curve is standard, but the aggregate Euler equation is modified in two important ways. First, the response of output depends on the zero-beta rate and on the price of risk.

Figure 8: Output response to a monetary shock



Note: The figure shows the response of aggregate output to a 100-basis-point contractionary monetary shock.

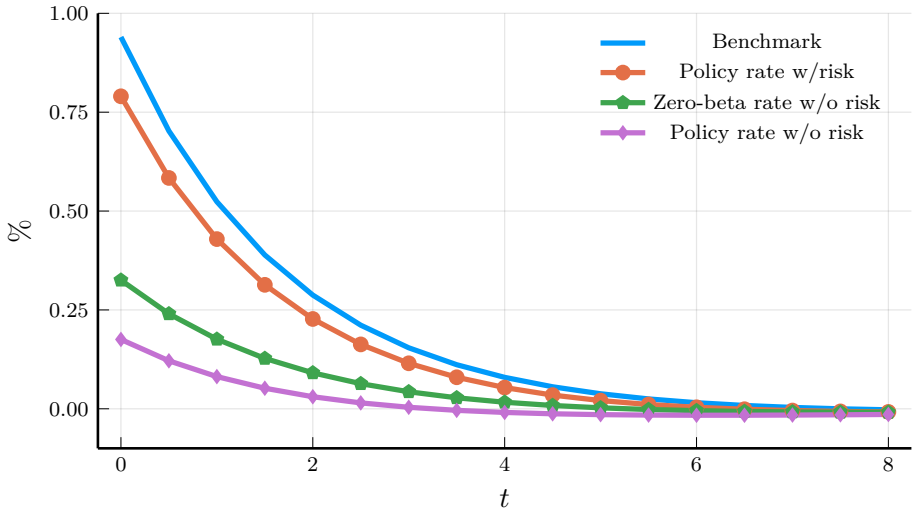
Second, because the difference between the zero-beta rate and the policy rate depends on the value of government bonds, aggregate demand responds directly to movements in long-term government bond yields. This contrasts with other monetary models with asset pricing, which often feature a form of *macro-finance separation*, whereby the response of the real economy to monetary policy is independent of the response of asset prices.

5.1 Quantitative Results

Figure 8 presents the impulse response of aggregate output to a contractionary monetary shock under three specifications: the full model, a version without aggregate risk ($\lambda = 0$), and the textbook New Keynesian model.

In the full model, aggregate output falls by 1.55% on impact and gradually returns to its steady-state level over 4–6 quarters. This response is substantially larger than in the textbook New Keynesian model, where the standard intertemporal substitution channel—operating through the policy rate alone—generates a much smaller decline. The gap between the full model and the textbook model illustrates the quantitative importance of the mechanisms emphasized in this paper.

Figure 9: The Aggregate Euler equation



5.2 The Aggregate Euler Equation

A key implication of our framework is that the standard Euler equation may substantially understate the effects of monetary policy on consumption. This section examines two features that distinguish our model from the textbook specification: (i) the relevant interest rate for intertemporal substitution is the zero-beta rate, not the policy rate; and (ii) the presence of disaster risk introduces non-linearities and precautionary motives that affect the mapping from interest rates to consumption growth.

Figure 9 illustrates these points by comparing the benchmark model's prediction against several alternative specifications that correspond to different "mistakes" an econometrician might make when estimating or interpreting the Euler equation.

The benchmark specification. In our model, the aggregate Euler equation takes the form

$$\dot{y}_t = \tilde{\sigma}^{-1}\left(\hat{i}_t^{zb} - \pi_t - r_n + p_{r,t}\right),$$

where $\hat{i}_t^{zb}$ is the zero-beta rate and $p_{r,t}$ captures the precautionary motive arising from time-varying disaster risk. The benchmark line in Figure 9 shows the implied path of output growth following a 100-basis-point monetary contraction.

Using the policy rate instead of the zero-beta rate. The line labeled "Policy rate w/risk" shows the prediction when the policy rate i_t is used in place of the zero-beta rate $\hat{i}_t^{zb}$, while still correctly accounting for the precautionary motive. The response is substantially attenuated—about 15 percent smaller than the benchmark. This gap reflects the amplification from convenience yields: because the zero-beta rate rises by more than the policy rate, using the policy rate understates the true incentive to postpone consumption.

Ignoring disaster risk. The lines labeled "w/o risk" show predictions when the precautionary term $p_{r,t}$ is omitted, corresponding to estimation on samples that exclude disaster realizations. The "Policy rate w/o risk" specification—which corresponds most closely to a textbook Euler equation—predicts an impact response less than 20% of the benchmark.

These results suggest that empirical estimates of the Euler equation may understate the true interest-rate sensitivity of consumption due to the use of incorrect interest rates and the omission of disaster risk.

5.3 Loss of Convenience Yields

We now conduct a counterfactual exercise to isolate the contribution of convenience yields to monetary transmission. We eliminate the convenience value of government bonds by setting $\xi = 0$ while keeping all other parameters at their calibrated values. This exercise is motivated by the recent evidence in Jiang et al. (2025), who document a secular decline in Treasury convenience yields. It allows us to address a key question: if Treasury bonds were to lose their "specialness," how much less effective would monetary policy become?

Asset prices. Without convenience yields, the term premium falls modestly from 1.50% to 1.38%, reflecting the fact that the term premium primarily compensates for crisis risk rather than for the inconvenience of holding long-term bonds. The equity premium falls from 5.23% to 4.36%, a 17% decline, because the convenience spread between short-term bonds and equities contributes to the measured equity premium.

The most striking difference is in the stock-price response, which falls from -5.10% to -2.72%, a reduction of 47%. This attenuation occurs because the convenience channel

amplifies the discount-rate response to monetary policy. In the baseline model, the zero-beta rate rises by 360 basis points when the policy rate rises by 100 basis points; without convenience yields, the zero-beta rate moves one-for-one with the policy rate.

Output. The output response falls from -1.45% to -1.15%, a decline of 21%. This attenuation operates through two channels. First, the zero-beta rate—which governs intertemporal substitution—rises by less when government bonds do not provide liquidity services, directly weakening the intertemporal substitution channel. Second, without liquidity services, the time-varying liquidity channel disappears entirely.

Implications. A complete loss of Treasury convenience would reduce the output response to monetary policy by roughly one-fifth. If Treasury bonds become less "special"—whether due to increased supply, changes in financial regulation, or competition from alternative safe assets—the Federal Reserve may need to move policy rates by more to achieve the same macroeconomic effects. The zero-beta rate, which captures both the policy rate and the convenience spread, provides a more complete picture of monetary conditions than the federal funds rate alone.

6 Conclusion

This paper shows that convenience yields and risk premia are central to the transmission of monetary policy. Empirically, we document that contractionary monetary shocks raise both market-implied tail risk and the convenience yield on long-term U.S. Treasuries, revealing two channels that operate alongside the traditional expectations mechanism.

To account for these facts, we develop a tractable heterogeneous-agent New Keynesian model with rare disasters and liquidity services from government debt. The framework delivers closed-form expressions for the zero-beta rate, convenience yields, and risk premia, and matches the empirical behavior of the term, equity, and convenience premia as well as their responses to monetary policy shocks. The model highlights how monetary policy redistributes wealth across investors with different beliefs and liquidity needs,

generating endogenous movements in tail risk and the marginal value of liquidity.

A key implication is that convenience yields amplify monetary policy: when government bonds provide liquidity services, monetary contractions raise the zero-beta rate and strengthen the intertemporal substitution channel. Quantitatively, removing liquidity services from government bonds reduces the output response to monetary policy by roughly twenty-one percent.

More broadly, our results suggest that changes in the supply and valuation of safe and liquid assets have first-order consequences for monetary transmission. Understanding the joint dynamics of convenience yields, risk premia, and long-term interest rates is therefore essential for evaluating the effects of monetary policy in environments where liquidity demand and perceptions of tail risk vary over time.

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Appendix: Proofs

Proof of Lemma 1. Assuming $\Delta \equiv (1-\alpha)\frac{Q_{E,t}-Q^*_{E,t}}{Q_{E,t}} - \frac{Q_{L,t}-Q^*_{L,t}}{Q_{L,t}} \neq 0$, we can uniquely solve Equations (5)-(6) for the price of risk and the marginal value of liquidity:

$$\lambda_j \left(\frac{C_{j,t}}{C^*_{j,t}}\right)^{-\sigma} = \frac{(1-\alpha)r_{E,t} - r_{L,t}}{\Delta}, \qquad \xi_j C^{\sigma}_{j,t} L^{-\nu}_{j,t} = \frac{r_{L,t}\frac{Q_{L,t}-Q^*_{L,t}}{Q_{L,t}} - r_{E,t}\frac{Q_{E,t}-Q^*_{E,t}}{Q_{E,t}}}{\Delta}.$$

As the right-hand side of both equations is independent of j, the solution is the same for both types. In the knife-edge case where $\Delta = 0$, portfolios are indeterminate, and there are multiple solutions to the system of equations.

□

Proof of Proposition 1. To ensure that η_t correctly prices long-term bonds and equities, consistent with Equations (5)-(6), it is sufficient to show that $\lambda_t \left(\frac{C_{s,t}}{C^*_{s,t}}\right)^{\sigma} = \lambda_j \left(\frac{C_{j,t}}{C^*_{j,t}}\right)^{\sigma}$ and that $\xi_t C^{\sigma}_{s,t} L^{-\nu}_{s,t} = \xi_j C^{\sigma}_{j,t} L^{-\nu}_{j,t}$. The first expression implies that $C^*_{j,t} = \left(\frac{\lambda_j}{\lambda_t}\right)^{\frac{1}{\sigma}} \frac{C^*_{s,t}}{C_{s,t}} C_{j,t}$. Plugging this into the definition of savers' average consumption in the crisis state, $C^*_{s,t} \equiv \frac{\mu_o}{\mu_o+\mu_p} C^*_{o,t} + \frac{\mu_p}{\mu_o+\mu_p} C^*_{p,t}$, and rearranging, we obtain Equation (7). The second expression implies that $L_{j,t} = \left(\frac{\xi_j}{\xi_t}\right)^{\frac{1}{\nu}} \left(\frac{C_{j,t}}{C_{s,t}}\right)^{\frac{\sigma}{\nu}} L_{s,t}$. Plugging this into the definition of savers' average liquidity holdings, $L_{s,t} \equiv \frac{\mu_o}{\mu_o+\mu_p} L_{o,t} + \frac{\mu_p}{\mu_o+\mu_p} L_{p,t}$, and rearranging, we obtain Equation (9). By setting $\rho_{s,t} \equiv \sum_{j\in\{o,p\}} \frac{\mu_j C_{j,t}}{\mu_o C_{o,t} + \mu_p C_{p,t}} (\rho_{j,t} + \lambda_j) - \lambda_t$, we ensure that η_t correctly prices risk-free bonds, i.e., $\mathbb{E}_t[d\eta_t]/\eta_t = -i^{zb}_t dt$. □

Proof of Proposition 2. As shown in Appendix B.4, the yield on the zero-coupon bond is given by:

$$y_B(\tau) = r^{zb}_n - \alpha\xi C^{\sigma} L^{-\nu} + \lambda \left(\frac{C_s}{C^*_s}\right)^{\sigma} - \frac{1}{\tau}\log\left[1 + \lambda\left(\frac{C_s}{C^*}\right)^{\sigma} \frac{e^{(\rho_B - \rho^*_B)\tau} - 1}{\rho_B - \rho^*_B}\right].$$

The limit as $\tau \to 0$ is given by

$$\lim_{\tau\to 0} y_B(\tau) = r^{zb}_n - \alpha\xi C^{\sigma} L^{-\nu}.$$

Define $u(\tau) \equiv 1 + \lambda K \frac{e^{\Delta\tau}-1}{\Delta}$, where $K = \left(\frac{C_s}{C_s^*}\right)^{\sigma} \geq 1$ and $\Delta = \rho_B - \rho_B^* = r_n^{zb} - r_n^{zb,*} - [\alpha\xi C_s^{\sigma} L_s^{-\nu} - \alpha^*\xi^*(C_s^*)^{\sigma}(L_s^*)^{-\nu}] + \lambda K$, and $g(\tau) \equiv \log u(\tau)$. Then,

$$y_B'(\tau) = \frac{g(\tau) - \tau g'(\tau)}{\tau^2}, \qquad g'(\tau) = \frac{\lambda K e^{\Delta\tau}}{u(\tau)}.$$

Hence, $y_B'(\tau) > 0$ if $h(\tau) \equiv g(\tau) - \tau g'(\tau) > 0$. Notice that $h(0) = 0$ and $h'(\tau) = -\tau g''(\tau)$. The second derivative is given by:

$$g''(\tau) = \frac{\lambda K e^{\Delta\tau}(\Delta u(\tau) - \lambda K e^{\Delta\tau})}{u(\tau)^2} = \frac{\lambda K e^{\Delta\tau}\tilde{\Delta}}{u(\tau)^2} < 0,$$

where $\tilde{\Delta} \equiv \Delta - \lambda K$, given the assumptions in the proposition ensure $\tilde{\Delta} < 0$.

In the limit as $\tau \to 0$, we obtain:

$$y_B'(0) = -\frac{1}{2}\lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \tilde{\Delta} > 0.$$

As $h(\tau) > 0$ for $\tau > 0$, we obtain $y_B'(\tau) > 0$. □

Proof of Proposition 3. Consider the New Keynesian Phillips curve (10). Linearizing this expression, and using $\frac{W}{P} = (1 - \epsilon^{-1})A$, we obtain

$$\dot{\pi}_t = \left(r_n^{zb} + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma}\right) \pi_t - \varphi^{-1}(\epsilon - 1) Y (w_t - p_t).$$

Using the fact that $w_t - p_t = \phi y_t$, we obtain $\dot{\pi}_t = (\rho_s + \lambda)\pi_t - \kappa y_t$, where $\kappa \equiv \varphi^{-1}(\epsilon - 1)\phi Y$ and we used that $r_n^{zb} + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} = \rho_s + \lambda$.

Consider next the aggregate Euler equation. From the market-clearing condition for goods and workers' consumption, we obtain $c_{s,t} = \frac{1-\mu_w\chi_y}{1-\mu_w} y_t$. Combining this condition with savers' Euler equation, and rearranging, we obtain $\dot{y}_t = \tilde{\sigma}^{-1}(\hat{i}_t^{zb} - \pi + p_{r,t})$, where the constant $\tilde{\sigma}^{-1}$ is defined in the proposition. □

Proof of Lemma 2. Linearizing Equation (7) around a stationary equilibrium, we obtain

$$\frac{\lambda^{\frac{1}{\sigma}}}{\sigma}\hat{\lambda}_t = \mu_{c,o}\mu_{c,p}\left(\lambda_o^{\frac{1}{\sigma}} - \lambda_p^{\frac{1}{\sigma}}\right)(c_{o,t} - c_{p,t}),$$

where $\mu_{c,j} \equiv \frac{\mu_j C_j}{\mu_o C_o + \mu_p C_p}$ and $c_{j,t} \equiv \log C_{j,t}/C_j$ for $j \in \{o, p\}$. Moreover, linearizing Equation (9), we get

$$\frac{\xi^{\frac{1}{\nu}}}{\nu}\hat{\xi}_t = \mu_{c,o}\mu_{c,p}\left[\left(\frac{C_o}{C_s}\right)^{\frac{\sigma-\nu}{\nu}}\xi_o^{\frac{1}{\nu}} - \left(\frac{C_p}{C_s}\right)^{\frac{\sigma-\nu}{\nu}}\xi_p^{\frac{1}{\nu}}\right]\frac{\sigma}{\nu}\left(c_{o,t} - c_{p,t}\right).$$

Rearranging these expressions, we obtain

$$\hat{\lambda}_t = \chi_{\lambda,\Delta c}(c_{p,t} - c_{o,t}), \qquad \hat{\xi}_t = \chi_{\xi,\Delta c}(c_{p,t} - c_{o,t}),$$

where the coefficients are given by

$$\begin{aligned}
\chi_{\lambda,\Delta c} &= \sigma\mu_{c,o}\mu_{c,p}\lambda^{-\frac{1}{\sigma}}(\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}),\\
\chi_{\xi,\Delta c} &= \nu\mu_{c,o}\mu_{c,p}\xi^{-\frac{1}{\nu}}\left[\left(\tfrac{C_p}{C_s}\right)^{\frac{\sigma-\nu}{\nu}}\xi_p^{\frac{1}{\nu}} - \left(\tfrac{C_o}{C_s}\right)^{\frac{\sigma-\nu}{\nu}}\xi_o^{\frac{1}{\nu}}\right].
\end{aligned}$$

Relative consumption. Linearizing savers' Euler equation, we obtain

$$\dot{c}_{j,t} = \sigma^{-1}\left(\hat{i}_t^{zb} - \pi_t\right) + \frac{\lambda}{\sigma}\left(\frac{C_s}{C_s^*}\right)^{\sigma} p_{r,t} - \psi_c(c_{j,t} - c_{s,t}).$$

where $j \in \{o, p\}$. Taking the difference of the Euler equation for the two types, we obtain

$$\dot{c}_{p,t} - \dot{c}_{o,t} = \chi_{\Delta c,\Delta c}(c_{p,t} - c_{o,t}), \qquad \text{where } \chi_{\Delta c,\Delta c} \equiv -\psi_c.$$

Bond price. The price of the long-term government bond satisfies the condition:

$$\frac{\dot{Q}_{L,t}}{Q_{L,t}} = i_t + (1-\alpha)\xi_t C_{s,t}^{\sigma} L_{s,t}^{-\nu} + \lambda_t\left(\frac{C_{s,t}}{C_{s,t}^*}\right)^{\sigma} + \psi_L - \left[1 + \lambda_t\left(\frac{C_{s,t}}{C_{s,t}^*}\right)^{\sigma} Q_{L,t}^*\right]\frac{1}{Q_{L,t}}.$$

Linearizing the expression above, we obtain

$$\dot{q}_{L,t} = i_t - r_n + r_L\sigma c_{s,t} + (1-\alpha)\xi C_s^{\sigma} L_s^{-\nu}(\hat{\xi}_t - \nu l_{s,t}) + \left[1 + \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma} Q_L^*\right]\frac{1}{Q_L}q_{L,t} + \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma}\frac{Q_L - Q_L^*}{Q_L}\hat{\lambda}_t.$$

Using $l_{s,t} = \chi_{l,q} q_{L,t}$, $\hat{\lambda}_t = \chi_{\lambda,\Delta c}(c_{p,t} - c_{o,t})$, and $\hat{\xi}_t = \chi_{\xi,\Delta c}(c_{p,t} - c_{o,t})$, we obtain

$$\dot{q}_{L,t} = \chi_{q,\Delta c}(c_{p,t} - c_{o,t}) + \chi_{q,q} q_{L,t} + i_t - r_n + r_L \sigma c_{s,t},$$

where

$$\chi_{q,\Delta c} = (1-\alpha)\xi C_s^{\sigma} L_s^{-\nu} \chi_{\xi,\Delta c} + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{Q_L - Q_L^*}{Q_L} \chi_{\lambda,\Delta c}$$
$$\chi_{q,q} = \rho_L + \psi_L,$$

and $\rho_L \equiv \rho - \xi C_s^{\sigma} L_s^{-\nu} [\alpha + (1-\alpha)\nu\gamma_L]$.

Relative net worth. The relative net worth of optimistic and pessimistic savers evolves according to

$$\frac{\dot{B}_{p,t}}{B_{p,t}} - \frac{\dot{B}_{o,t}}{B_{o,t}} = \sum_{k \in \{L,E\}} r_{k,t} \left(\frac{B_{p,t}^k}{B_{p,t}} - \frac{B_{o,t}^k}{B_{o,t}}\right) - \left(\frac{C_{p,t} - T_{s,t}}{B_{p,t}} - \frac{C_{o,t} - T_{s,t}}{B_{o,t}}\right).$$

Linearizing the expression above, we obtain

$$\dot{b}_{p,t} - \dot{b}_{o,t} = \sum_{k \in \{L,E\}} r_k \left[\hat{r}_{k,t} \left(\frac{B_p^k}{B_p} - \frac{B_o^k}{B_o}\right) + \frac{B_p^k}{B_p}\left(b_{p,t}^k - b_{p,t}\right) - \frac{B_o^k}{B_o}\left(b_{o,t}^k - b_{o,t}\right)\right] - \left(\frac{C_p}{B_p} c_{p,t} - \frac{C_o}{B_o} c_{o,t}\right) - \left(\frac{1}{B_o} - \frac{1}{B_p}\right)(T_{s,t} - T_s) + \frac{C_p - T_p}{B_p} b_{p,t} - \frac{C_o - T_o}{B_o} b_{o,t},$$

where $r_k \hat{r}_{k,t} = \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{Q_k - Q_k^*}{Q_k} \left(\hat{\lambda}_t + \sigma c_{s,t} + \frac{Q_k^*}{Q_k - Q_k^*} q_{k,t}\right) + \xi C_s^{\sigma} L_s^{-\nu} (1 - \alpha_k) \left(\hat{\xi}_t + \sigma c_{s,t} - \nu l_{s,t}\right)$. Using the fact that $\frac{C_j - T_j}{B_j} = r_n + \sum_{k \in \{L,E\}} r_k \frac{B_j^k}{B_j}$, and $B_o = B_p$, we can write the expression above as follows:

$$\dot{b}_{p,t} - \dot{b}_{o,t} = \sum_{k \in \{L,E\}} \left(\frac{B_p^k}{B_p} - \frac{B_o^k}{B_o}\right) \left[\lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \left(\frac{Q_k - Q_k^*}{Q_k} \hat{\lambda}_t + \frac{Q_k^*}{Q_k} q_{k,t}\right) + (1 - \alpha_k) \xi C_s^{\sigma} L_s^{-\nu} \left(\hat{\xi}_t - \nu l_{s,t}\right) + r_k \sigma c_{s,t}\right] + \sum_{k \in \{L,E\}} r_k \left[\frac{B_p^k}{B_p} b_{p,t}^k - \frac{B_o^k}{B_o} b_{o,t}^k\right] - \left(\frac{C_p}{B_p} c_{p,t} - \frac{C_o}{B_o} c_{o,t}\right) + r_n \left(b_{p,t} - b_{o,t}\right). \quad (16)$$

We need to solve next for the portfolios $b^k_{j,t}$. First, consider the relative net worth in the crisis state at $t = t^*$ is given by

$$\frac{B^*_p}{B_p} b^*_{p,t^*} - \frac{B^*_o}{B_o} b^*_{o,t^*} = b_{p,t^*} - b_{o,t^*} - \sum_{k\in\{o,p\}} \left[\left(\frac{B^k_p}{B_p} - \frac{B^k_o}{B_o} \right) \frac{Q^*_k}{Q_k} q_{k,t^*} - \frac{Q^*_k - Q_k}{Q_k} \left(\frac{B^k_p}{B_p} b^k_{p,t^*} - \frac{B^k_o}{B_o} b^k_{o,t^*} \right) \right].$$

Consumption of savers in the crisis state is given by $c^*_{j,t} = \frac{r^*_n B^*_j}{C^*_s} b^*_{j,t}$, so we obtain that $c^*_{p,t} - c^*_{o,t} = \frac{r^*_n}{C^*_s} \left(B^*_p b^*_{p,t} - B^*_o b^*_{o,t} \right)$. Using this expression and the fact that $c^*_{p,t} - c^*_{o,t} = c_{p,t} - c_{o,t}$, we obtain:

$$\sum_{k\in\{o,p\}} \frac{Q_k - Q^*_k}{Q_k} \left(\frac{B^k_p}{B_p} b^k_{p,t} - \frac{B^k_o}{B_o} b^k_{o,t} \right) = b_{p,t} - b_{o,t} - \frac{C^*_s}{r^*_n B_s} \left(c_{p,t} - c_{o,t} \right) - \sum_{k\in\{o,p\}} \left(\frac{B^k_p}{B_p} - \frac{B^k_o}{B_o} \right) \frac{Q^*_k}{Q_k} q_{k,t}. \tag{17}$$

Next, consider the liquidity holdings. Using the definition of $L_{j,t}$, we have $L_{j,t} = B_{j,t} - \sum_{k\in\{L,E\}} (1 - \alpha_k) B^k_{j,t}$. Linearizing this expression, we obtain:

$$\frac{L_j}{B_j} l_{j,t} = b_{j,t} - \sum_{j\in\{L,E\}} (1 - \alpha_k) \frac{B^k_j}{B_j} b^k_{j,t}.$$

Taking the difference of the expression above for the two types, we obtain:

$$\sum_{j\in\{L,E\}} (1 - \alpha_k) \left(\frac{B^k_p}{B_p} b^k_{p,t} - \frac{B^k_o}{B_o} b^k_{o,t} \right) = b_{p,t} - b_{o,t} - \left[\frac{L_p}{B_p} \left(l_{p,t} - l_{s,t} \right) - \frac{L_o}{B_o} \left(l_{o,t} - l_{s,t} \right) + \left(\frac{L_p}{B_p} - \frac{L_o}{B_o} \right) l_{s,t} \right].$$

The deviation of individual savers' liquidity holdings from average can be written as

$$l_{p,t} - l_{s,t} = \frac{\mu_o L_o}{\mu_o L_o + \mu_p L_p} \left(l_{p,t} - l_{o,t} \right), \quad l_{o,t} - l_{s,t} = -\frac{\mu_p L_p}{\mu_o L_o + \mu_p L_p} \left(l_{p,t} - l_{o,t} \right).$$

Using the expressions above, as well as the optimality condition $l_{p,t} - l_{o,t} = \frac{\sigma}{\nu}(c_{p,t} -$

$c_{o,t}$), and the expression for aggregate liquidity, $l_{s,t} = \chi_{l,q} q_{L,t}$, we obtain:

$$\sum_{j\in\{L,E\}} (1-\alpha_k)\left(\frac{B_p^k}{B_p} b_{p,t}^k - \frac{B_o^k}{B_o} b_{o,t}^k\right) = b_{p,t} - b_{o,t} - \left[\left(\frac{L_p}{B_p}\frac{\mu_o L_o}{\mu_o L_o + \mu_p L_p} + \frac{\mu_p L_p}{\mu_o L_o + \mu_p L_p}\right)\frac{\sigma}{\nu}\left(c_{p,t} - c_{o,t}\right) + \left(\frac{L_p}{B_p} - \frac{L_o}{B_o}\right)\chi_{l,q} q_{L,t}\right]. \quad (18)$$

Finally, combining Equation (17) and Equation (18) with Equation (16), we obtain:

$$\dot{b}_{p,t} - \dot{b}_{o,t} = \chi_{\Delta b,\Delta c}(c_{p,t} - c_{o,t}) + \chi_{\Delta b,\Delta b}(b_{p,t} - b_{o,t}) + \chi_{\Delta b,\Delta q}(q_{L,t} - q_{L,t}^*) + \chi_{\Delta b,c_s} c_{s,t}, \quad (19)$$

where

$$\begin{aligned}
\chi_{\Delta b,\Delta c} &= \sum_{k\in\{L,E\}}\left(\frac{B_p^k}{B_p} - \frac{B_o^k}{B_o}\right)\left[\lambda\left(\frac{C_s}{C_s^*}\right)^\sigma \frac{Q_k - Q_k^*}{Q_k}\chi_{\lambda,\Delta c} + (1-\alpha_k)\xi C_s^\sigma L_s^{-\nu}\chi_{\xi,\Delta c}\right] - \lambda\left(\frac{C_s}{C_s^*}\right)^\sigma \frac{C_s^*}{r_n^* B_s} \\
&\quad - \xi C_s^\sigma L_s^{-\nu}\left(\frac{L_p}{B_p}\frac{\mu_o L_o}{\mu_o L_o + \mu_p L_p} + \frac{L_o}{B_o}\frac{\mu_p L_p}{\mu_o L_o + \mu_p L_p}\right)\frac{\sigma}{\nu} - \left(\frac{C_p}{B_p}\mu_{c,o} + \frac{C_o}{B_o}\mu_{c,p}\right), \\
\chi_{\Delta b,q} &= -\left[\left(\frac{L_p}{B_p} - \frac{L_o}{B_o}\right) + \sum_{k\in\{L,E\}}\left(\frac{B_p^k}{B_p} - \frac{B_o^k}{B_o}\right)(1-\alpha_k)\nu\right]\xi C_s^\sigma L_s^{-\nu}\gamma_L, \\
\chi_{\Delta b,\Delta b} &= r_n + \lambda\left(\frac{C_s}{C_s^*}\right)^\sigma + \xi C_s^\sigma L_s^{-\nu} = \rho_s + \lambda, \\
\chi_{\Delta b,c_s} &= \sum_{k\in\{L,E\}}\left(\frac{B_p^k}{B_p} - \frac{B_o^k}{B_o}\right) r_k(\sigma - 1).
\end{aligned}$$

□

Proof of Proposition 4. Suppose $r_k \sigma c_{s,t}$ is second order, i.e. $\mathcal{O}(||i_t - r_n||^2)$ for $k \in \{L, E\}$. This implies that $r_L \sigma c_{s,t}$ and $\chi_{\Delta b,c_s} c_{s,t}$ are second order, so we can drop $c_{s,t}$ from the system (19). The eigenvalues of the system are given by:

$$\omega_1 = -\psi_c, \qquad \omega_2 = \rho_L + \psi_L, \qquad \omega_3 = \rho.$$

Given our assumption on parameters, the system has one negative and two positive eigenvalues, so there is a unique bounded solution, given an initial condition for relative

net worth. The associated matrix of eigenvectors, and its inverse, is given by:

$$V = \begin{bmatrix} \frac{(\rho+\psi_c)(\rho_L+\psi_c)}{\Delta_1} & 0 & 0 \\ -\frac{(\rho+\psi_c)\chi_{q_L,\Delta c}}{\Delta_1} & \frac{\rho_L-\rho}{\chi_{\Delta b,q_L}} & 0 \\ 1 & 1 & 1, \end{bmatrix}, \qquad V^{-1} = \begin{bmatrix} \frac{\Delta_1}{(\rho+\psi_c)(\rho_L+\psi_c)} & 0 & 0 \\ \frac{\chi_{\Delta b,q_L}\chi_{q_L,\Delta c}}{(\rho_L-\rho)(\rho_L+\psi_c)} & \frac{\chi_{\Delta b,q_L}}{\rho_L-\rho} & 0 \\ -\frac{\Delta_1(\rho_L-\rho)+(\rho+\psi_c)\chi_{\Delta b,q_L}\chi_{q_L,\Delta c}}{(\rho_L-\rho)(\rho+\psi_c)(\rho_L+\psi_c)} & -\frac{\chi_{\Delta b,q_L}}{\rho_L-\rho} & 1, \end{bmatrix},$$

where $\Delta_1 \equiv \chi_{q_L,\Delta c}\chi_{\Delta b,q_L} - (\rho_L+\psi_c)\chi_{\Delta b,\Delta c}$.

Let $Z_t = [c_{p,t} - c_{o,t}, q_{L,t}, b_{p,t} - b_{o,t}]'$. The dynamic system can be written as $\dot{Z}_t = AZ_t + B(i_t - r_n)$, where $A = V\Omega V^{-1}$, Ω is a diagonal matrix of eigenvalues, and $B = [0,1,0]'$. Rotating the system into the eigenvector basis, we obtain $\dot{z}_t = \Omega z_t + b(i_t - r_n)$, where $b = V^{-1}B$ and $z_t = V^{-1}Z_t$. The solution is given by

$$z_{1,t} = e^{\omega_1 t}z_{1,0} + \int_0^t e^{\omega_1(t-s)}b_1(i_s - r_n)ds, \qquad z_{k,t} = -\int_t^\infty e^{-\omega_k(s-t)}b_k(i_s - r_n)ds,$$

for $k = 2,3$. Rotating back to the original basis, we obtain

$$Z_t = V\begin{bmatrix} e^{\omega_1 t}z_{1,0} + \int_0^t e^{\omega_1(t-s)}b_1(i_s - r_n)ds \\ -\int_t^\infty e^{-\omega_2(s-t)}b_2(i_s - r_n)ds \\ -\int_t^\infty e^{-\omega_3(s-t)}b_3(i_s - r_n)ds \end{bmatrix}.$$

This implies that relative consumption is given by:

$$c_{p,t} - c_{o,t} = e^{-\psi_c t}(c_{p,0} - c_{o,0}),$$

using the fact that $\omega_1 = -\psi_c$ and $b_1 = 0$, and the initial condition is given by

$$c_{p,0} - c_{o,0} = \frac{(\rho+\psi_c)(\rho_L+\psi_c)}{\Delta_1}\left[b_{p,0} - b_{o,0} + \frac{\chi_{\Delta b,q_L}}{\rho_L-\rho}\int_0^\infty \left(e^{-\rho_L t} - e^{-\rho t}\right)(i_t - r_n)ds\right].$$

Relative net worth is given by:

$$b_{p,t} - b_{o,t} = e^{-\psi_c t}\left(b_{p,0} - b_{o,0}\right) - \left(g_{b,t} - e^{-\psi_c t}g_{b,0}\right),$$

where $g_{b,t} = \frac{\chi_{\Delta b,q_L}}{\rho_L-\rho}\int_t^\infty \left(e^{-\rho_L(s-t)} - e^{-\rho(s-t)}\right)(i_s - r_n)ds$.

The bond price is given by:

$$q_{L,t} = -\frac{(\rho+\psi_c)\chi_{q_L,\Delta c}}{\Delta_1} e^{-\psi_c t}\left[b_{p,0} - b_{o,0} + g_{b,0}\right] - \int_t^\infty e^{-\rho_L(s-t)}(i_s - r_n)ds,$$

Hence, the initial condition is given by:

$$q_{L,0} = -\frac{(\rho+\psi_c)\chi_{q_L,\Delta c}}{\Delta_1}\left[b_{p,0} - b_{o,0} + g_{b,0}\right] - \frac{1}{\rho_L - \rho}\int_0^\infty e^{-\rho_L s}(i_s - r_n)ds.$$

Initial relative net worth. The market-implied disaster probability is given by:

$$b_{p,0} - b_{o,0} = \sum_{k\in\{L,E\}}\left(\frac{B_p^k}{B_p} - \frac{B_o^k}{B_o}\right)q_{k,0}.$$

□

Derivation of the D-HANK output and inflation decomposition. We can write dynamic system in matrix form as $\dot{Z}_t = AZ_t + B\nu_t$, where $B = [1,0]'$. Applying the eigendecomposition to matrix A, we obtain $A = V\Omega V^{-1}$ where $V = \begin{bmatrix} \frac{\rho-\overline{\omega}}{\kappa} & \frac{\rho-\underline{\omega}}{\kappa} \\ 1 & 1 \end{bmatrix}$, $V^{-1} = \frac{\kappa}{\overline{\omega}-\underline{\omega}}\begin{bmatrix} -1 & \frac{\rho-\underline{\omega}}{\kappa} \\ 1 & -\frac{\rho-\overline{\omega}}{\kappa} \end{bmatrix}$, and $\Omega = \begin{bmatrix} \overline{\omega} & 0 \\ 0 & \underline{\omega} \end{bmatrix}$. Decoupling the system, we obtain $\dot{z}_t = \Omega z_t + b\nu_t$, where $z_t = V^{-1}Z_t$ and $b = V^{-1}B$.

Solving the equation with a positive eigenvalue forward and the one with a negative eigenvalue backward, and rotating the system back to the original coordinates, we obtain

$$y_t = V_{12}\left(V^{21}y_0 + V^{22}\pi_0\right)e^{\underline{\omega}t} - V_{11}V^{11}\int_t^\infty e^{-\overline{\omega}(z-t)}\nu_z dz + V_{12}V^{21}\int_0^t e^{\underline{\omega}(t-z)}\nu_z dz$$
$$\pi_t = V_{22}\left(V^{21}y_0 + V^{22}\pi_0\right)e^{\underline{\omega}t} - V_{21}V^{11}\int_t^\infty e^{-\overline{\omega}(z-t)}\nu_z dz + V_{22}V^{21}\int_0^t e^{\underline{\omega}(t-z)}\nu_z dz,$$

where $V^{i,j}$ is the (i,j) entry of matrix V^{-1}. Integrating $e^{-\rho t}y_t$, we obtain $\Omega_0 = V_{12}\left(V^{21}y_0 + V^{22}\pi_0\right)\frac{1}{\rho-\underline{\omega}} - \frac{1}{\rho-\overline{\omega}}V_{11}V^{11}\int_0^\infty\left(e^{-\overline{\omega}t} - e^{-\rho t}\right)\nu_t dt + \frac{1}{\rho-\underline{\omega}}V_{12}V^{21}\int_0^\infty e^{-\rho t}\nu_t dt$. Rearranging the above expression, we obtain $V_{12}\left(V^{21}y_0 + V^{22}\pi_0\right) = (\rho-\underline{\omega})\Omega_0 + \frac{\rho-\underline{\omega}}{\rho-\overline{\omega}}V_{11}V^{11}\int_0^\infty e^{-\overline{\omega}t}\nu_t dt$, where we used the fact $\frac{V_{11}V^{11}}{\rho-\overline{\omega}} + \frac{V_{12}V^{21}}{\rho-\underline{\omega}} = 0$. Output is then given

by $y_t = \tilde{y}_t + (\rho - \underline{\omega})e^{\underline{\omega}t}\Omega_0$, where $\tilde{y}_t = -\frac{\overline{\omega}-\rho}{\overline{\omega}-\underline{\omega}}\int_t^\infty e^{-\overline{\omega}(z-t)}\nu_z dz + \frac{\overline{\omega}-\delta}{\overline{\omega}-\underline{\omega}}\int_0^t e^{\underline{\omega}(t-z)}\nu_z dz - \frac{\rho-\underline{\omega}}{\overline{\omega}-\underline{\omega}}e^{\underline{\omega}t}\int_0^\infty e^{-\overline{\omega}z}\nu_z dz$. Inflation is given by $\pi_t = \tilde{\pi}_t + \kappa e^{\underline{\omega}t}\Omega_0$, where $\tilde{\pi}_t = \frac{\kappa}{\overline{\omega}-\underline{\omega}}\int_t^\infty e^{-\overline{\omega}(z-t)}\nu_z dz + \frac{\kappa}{\overline{\omega}-\underline{\omega}}\int_0^t e^{\underline{\omega}(t-z)}\nu_z dz - \frac{\kappa}{\overline{\omega}-\underline{\omega}}e^{\underline{\omega}t}\int_0^\infty e^{-\overline{\omega}z}\nu_z dz$.

If $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, then

$$\nu_t = \left(\tilde{\sigma}^{-1} + \frac{\chi_{cy}\nu\gamma_L}{\rho_L + \psi_L + \psi_m}\right)e^{-\psi_m t}(i_0 - r_n) + \left(\chi_{p_d} + \lambda\left(\frac{C_s}{C_s^*}\right)^\sigma \frac{Q_L - Q_L^*}{Q_L}\frac{\chi_{cy}\nu\gamma_L}{\rho_L + \psi_L + \psi_r}\right)e^{-\psi_r t}\hat{\lambda}_0 + \left(\chi_{cy} + \xi C_s^\sigma L_s^{-\nu}(1-\alpha)\frac{\chi_{cy}\nu\gamma_L}{\rho_L + \psi_L + \psi_r}\right)e^{-\psi_r t}\hat{\xi}_0.$$

Then, $\tilde{y}_t = \chi_{ISE}\hat{y}_{m,t} + \chi_{TVP}\hat{y}_{r,t} + \chi_{TVL}\hat{y}_{r,t}$ and $\tilde{\pi}_t = \chi_{ISE}\hat{\pi}_{m,t} + \chi_{TVP}\hat{\pi}_{r,t} + \chi_{TVL}\hat{\pi}_{r,t}$, where $\chi_{ISE} \equiv \tilde{\sigma}^{-1}$, $\chi_{TVP} \equiv \chi_{p_d}$, $\chi_{TVL} \equiv \chi_{cy} + \frac{\chi_{cy}\nu\gamma_L}{\rho_L+\psi_L+\psi_m} + \lambda\left(\frac{C_s}{C_s^*}\right)^\sigma \frac{Q_L-Q_L^*}{Q_L}\frac{\chi_{cy}\nu\gamma_L}{\rho_L+\psi_L+\psi_r} + \xi C_s^\sigma L_s^{-\nu}(1-\alpha)\frac{\chi_{cy}\nu\gamma_L}{\rho_L+\psi_L+\psi_r}$, $\hat{y}_{k,t} = \frac{(\rho-\underline{\omega})e^{\underline{\omega}t} - (\rho+\psi_k)e^{-\psi_k t}}{(\psi_k+\overline{\omega})(\psi_k+\underline{\omega})}(i_0 - r_n)$, and $\hat{\pi}_{k,t} = \frac{\kappa(e^{\underline{\omega}t} - e^{-\psi_k t})}{(\underline{\omega}+\psi_k)(\overline{\omega}+\psi_k)}(i_0 - r_n)$. Note that $\int_0^\infty e^{-\rho t}\hat{y}_{k,t}dt = 0$, $\frac{\partial \hat{y}_{k,0}}{\partial i_0} = -\frac{1}{\psi_k+\overline{\omega}} < 0$, and $\lim_{t\to\infty}\hat{y}_{k,t} = 0$. Moreover, $\hat{\pi}_0 = 0$, $\frac{\partial \hat{\pi}_{k,t}}{\partial i_0} \geq 0$ with strict inequality if $t > 0$.

□

Internet Appendix

A Additional Figures

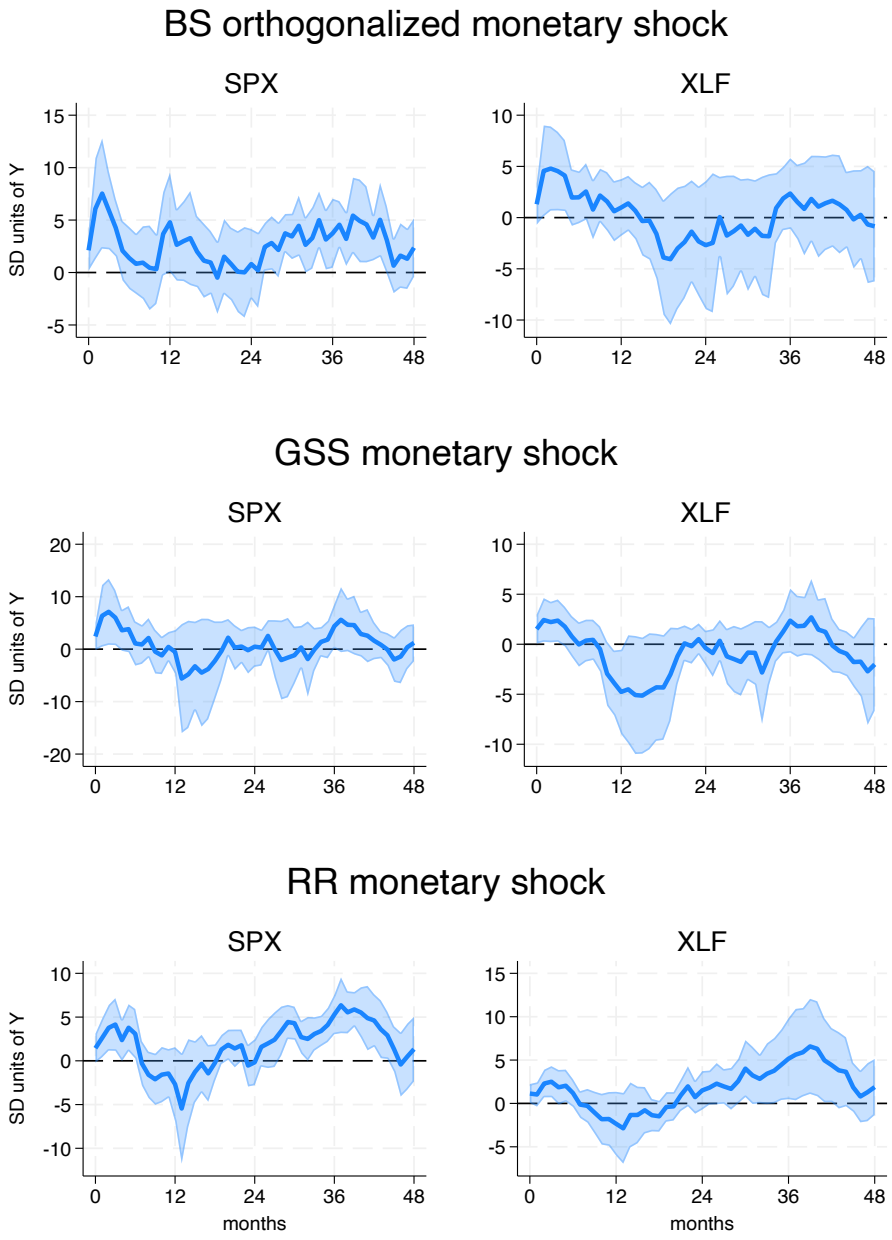


Figure A.1: RIX response to a monetary shock (BS, GSS, and RR).
The figures show the impulse responses of the RIX measures, normalized by their standard deviations, to a BS monetary shock (top panel), a GSS shock (middle panel), and a RR shock (bottom panel), using the Jordà local projection method. The left column shows the RIX-SPX measure, while the right column shows the RIX-XLF measure. The frequency is monthly and the sample period is 1996m1-2018m12 in the left column, and 1999m2-2018m12 in the right column. Controls in the right column for each panel are, respectively: 5 quarters of lagged outcome variable, ΔIP_t, ΔCPI_t, and the shock; 1 month of lagged outome, 4 quarters of lagged ΔIP_t, ΔCPI_t, 3 quarters of the lagged shock; 1 month of lagged outome, 6 quarters of lagged ΔIP_t, ΔCPI_t, and the shock. Shaded areas represent 90% confidence bands.

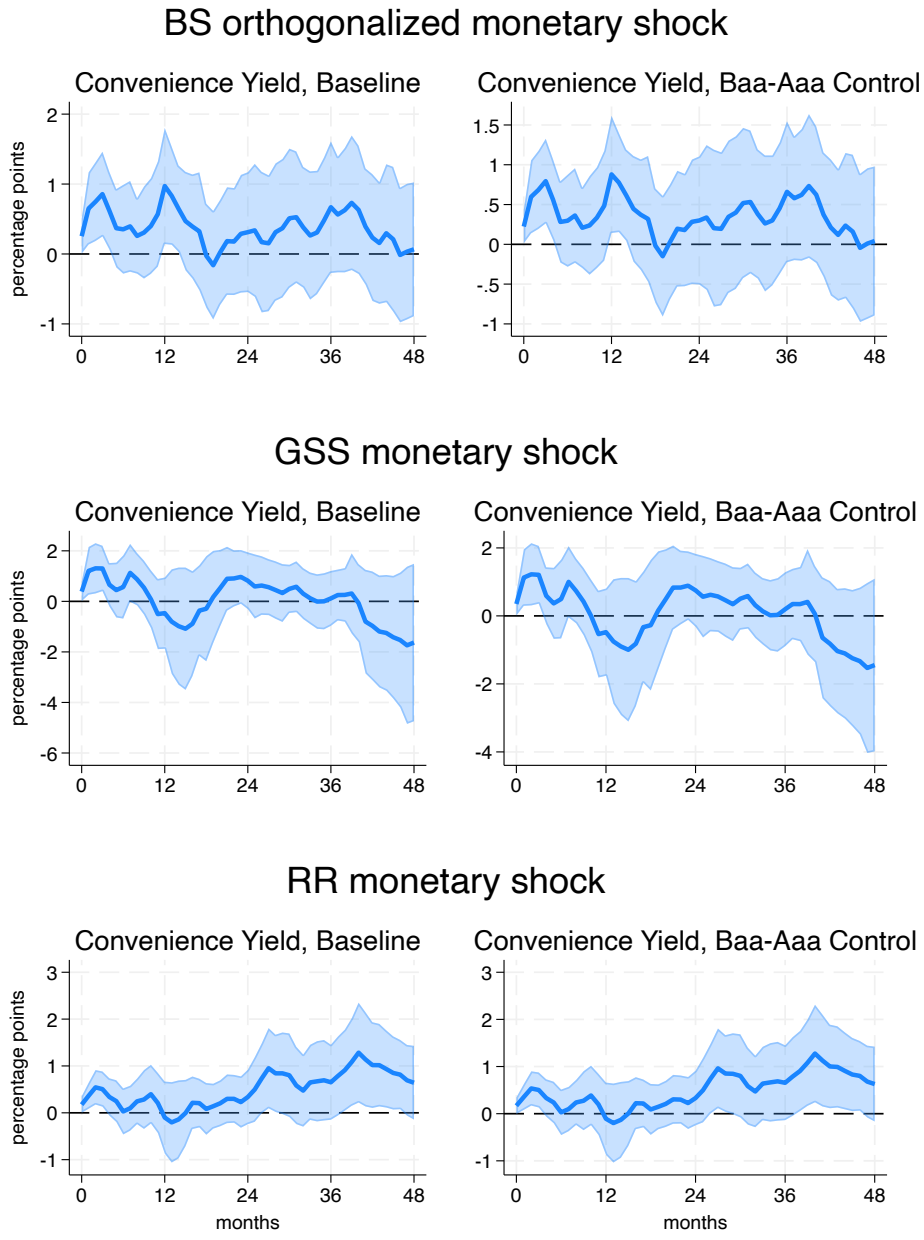


Figure A.2: Convenience yield response to a monetary shock (BS, GSS, and RR).
The figures show the impulse responses of the convenience yield to a BS monetary shock (top panel), a GSS shock (middle panel), and a RR shock (bottom panel), using the Jordà local projection method. The frequency is monthly and the sample period is 1988m2-2018m12 in the top and bottom panels, and 1995m2-2018m12 in the middle panel. Shaded areas represent 90% confidence bands.

B Derivations for Section 3

B.1 The non-linear model

In this section, we provide the detailed derivations for the non-linear model. We focus on an extension where short- and long-term bonds are imperfect substitutes. In particular, we assume that the liquidity aggregator is given by

$$L_{j,t} = \left[(B^S_{j,t})^{1-\theta} + \alpha_j (B^L_{j,t})^{1-\theta} \right]^{\frac{1}{1-\theta}},$$

where θ controls the degree of substitution between short- and long-term bonds. The baseline case corresponds to $\theta = 0$.

Savers' problem. The HJB for the savers' problem is given by

$$\rho_{j,t} V_{j,t} = \max_{C_{j,t}, B^L_{j,t}, B^E_{j,t}} \frac{C_{j,t}^{1-\sigma}}{1-\sigma} + \xi_j \frac{L_{j,t}^{1-\nu}}{1-\nu} + \frac{\partial V_{j,t}}{\partial t} + \lambda_j \left[V^*_{j,t} - V_{j,t} \right] + \frac{\partial V_{j,t}}{\partial B_{j,t}} \left[(i_t - \pi_t) B_{j,t} + r_{L,t} B^L_{j,t} + r_{E,t} B^E_{j,t} + T_{j,t} - C_{j,t} \right],$$

where $V^*_{j,t}$ is evaluated at $B^*_{j,t} = B_{j,t} + B^L_{j,t} \frac{Q^*_{L,t} - Q_{L,t}}{Q_{L,t}} + B^E_{j,t} \frac{Q^*_{E,t} - Q_{E,t}}{Q_{E,t}}$, and $B_{j,t} > 0$.

The corresponding HJB in the crisis state is given by

$$\rho^*_{j,t} V^*_{j,t} = \max_{C^*_{j,t}, B^{L,*}_{j,t}, B^{E,*}_{j,t}} \frac{(C^*_{j,t})^{1-\sigma}}{1-\sigma} + \xi^*_j \frac{\left(L^*_{j,t} \right)^{1-\nu}}{1-\nu} + \frac{\partial V^*_{j,t}}{\partial t} + \frac{\partial V^*_{j,t}}{\partial B^*_{j,t}} \left[(i^*_t - \pi^*_t) B^*_{j,t} + r^*_{L,t} B^{L*}_{j,t} + r^*_{E,t} B^{E*}_{j,t} + T^*_{j,t} - C^*_{j,t} \right],$$

where we assume that

$$L^*_{j,t} = \left[(B^{S*}_{j,t})^{1-\theta} + \alpha^*_j (B^{L*}_{j,t})^{1-\theta} \right]^{\frac{1}{1-\theta}}.$$

The first-order conditions for the savers' problem (using that $B^S_{j,t} = B_{j,t} - B^L_{j,t} - B^E_{j,t}$)

are:

$$
\begin{aligned}
C_{j,t}^{-\sigma} &= \frac{\partial V_{j,t}}{\partial B_{j,t}}, \\
\frac{\partial V_{j,t}}{\partial B_{j,t}} r_{L,t} &= \lambda_j \frac{\partial V_{j,t}^*}{\partial B_{j,t}^*} \frac{Q_{L,t} - Q_{L,t}^*}{Q_{L,t}} + \xi_j L_{j,t}^{\theta-\nu} \left[\left(B_{j,t}^S \right)^{-\theta} - \alpha_j \left(B_{j,t}^L \right)^{-\theta} \right], \\
\frac{\partial V_{j,t}}{\partial B_{j,t}} r_{E,t} &= \lambda_j \frac{\partial V_{j,t}^*}{\partial B_{j,t}^*} \frac{Q_{E,t} - Q_{E,t}^*}{Q_{E,t}} + \xi_j L_{j,t}^{\theta-\nu} \left(B_{j,t}^S \right)^{-\theta},
\end{aligned}
$$

and in the crisis state:

$$
\begin{aligned}
(C_{j,t}^*)^{-\sigma} &= \frac{\partial V_{j,t}^*}{\partial B_{j,t}^*}, \\
\frac{\partial V_{j,t}^*}{\partial B_{j,t}^*} r_{L,t}^* &= \xi_j^* \left(L_{j,t}^* \right)^{\theta-\nu} \left[\left(B_{j,t}^{S*} \right)^{-\theta} - \alpha_j^* \left(B_{j,t}^{L*} \right)^{-\theta} \right], \\
\frac{\partial V_{j,t}^*}{\partial B_{j,t}^*} r_{E,t}^* &= \xi_j (L_{j,t}^*)^{\theta-\nu} \left(B_{j,t}^{S*} \right)^{-\theta}.
\end{aligned}
$$

From these expressions, we obtain equations (6) and (5). Differentiating the HJB in normal times with respect to $B_{j,t}$, we obtain the envelope condition:

$$
\rho_{j,t} \frac{\partial V_{j,t}}{\partial B_{j,t}} = \xi_j L_{j,t}^{\theta-\nu} (B_{j,t}^S)^{-\theta} + \frac{\partial V_{j,t}}{\partial B_{j,t}} (i_t - \pi_t) + \frac{\mathbb{E}_{j,t} \left[d \left(\frac{\partial V_{j,t}}{\partial B_{j,t}} \right) \right]}{dt}.
$$

Using the optimality condition for consumption and the condition above, we get

$$
i_t - \pi_t - \rho_{j,t} + \xi_j \frac{L_{j,t}^{\theta-\nu} (B_{j,t}^S)^{-\theta}}{C_{j,t}^{-\sigma}} = -\frac{\mathbb{E}_{j,t} \left[d \left(C_{j,t}^{-\sigma} \right) \right]}{C_{j,t}^{-\sigma} dt} = \sigma \frac{\dot{C}_{j,t}}{C_{j,t}} - \lambda_j \left[\left(\frac{C_{j,t}^*}{C_{j,t}} \right)^{-\sigma} - 1 \right],
$$

using the fact that $dC_{j,t} = \dot{C}_{j,t} dt + \left[C_{j,t}^* - C_{j,t} \right] d\mathcal{N}_t$ and Ito's lemma. Rearranging the expression above, we obtain the version of Equation (4) for the case with imperfect substitution between bonds in the liquidity aggregator. A similar envelope condition holds

in the crisis state, which gives the Euler equation for the crisis state

$$\frac{\dot{C}^*_{j,t}}{C_{j,t}} = \sigma^{-1}(i^*_t - \pi^*_t - \rho^*_{j,t}) + \frac{\tilde{\xi}^*_j}{\sigma}\left(C^*_{j,t}\right)^{\sigma}\left(L^*_{j,t}\right)^{\theta-\nu}(B^{S*}_{j,t})^{-\theta}.$$

The relative net worth of optimistic and pessimistic savers evolves according to

$$\frac{\dot{B}_{o,t}}{B_{o,t}} - \frac{\dot{B}_{p,t}}{B_{p,t}} = \sum_{k\in\{L,E\}} r_{k,t}\left(\frac{B^k_{o,t}}{B_{o,t}} - \frac{B^k_{p,t}}{B_{p,t}}\right) - \left(\frac{C_{o,t}-T_{s,t}}{B_{o,t}} - \frac{C_{p,t}-T_{s,t}}{B_{p,t}}\right). \tag{B.1}$$

Workers' problem. The HJB for the workers' problem is given by

$$\rho_w V_{w,t} = \max_{\tilde{C}_{w,t},N_{w,t},B^L_{w,t}} \frac{\tilde{C}^{1-\sigma}_{w,t}}{1-\sigma} + \frac{\partial V_{w,t}}{\partial B_{w,t}}\left[(i_t-\pi_t)B_{w,t} + r_{L,t}B^L_{w,t} + \frac{W_t}{P_t}N_{w,t} + T_{w,t} - \tilde{C}_{w,t} - \frac{N^{1+\phi}_{w,t}}{1+\phi}\right] + \frac{\partial V_{w,t}}{\partial t} + \lambda_w\left[V^*_{w,t} - V_{w,t}\right],$$

subject to the state-constraint boundary condition

$$\frac{\partial V_{w,t}(0)}{\partial B_{w,t}} \geq \left(\frac{W_t}{P_t}N_{w,t} - \frac{N^{1+\phi}_{w,t}}{1+\phi} + T_{w,t}\right)^{-\sigma},$$

where we adopted the change of variables $\tilde{C}_{w,t} \equiv C_{w,t} - \frac{N^{1+\phi}_{w,t}}{1+\phi}$.

For simplicity, we have already imposed that $B^E_{w,t} = 0$. We show below that $B^L_{w,t} = 0$ and a similar argument shows that workers would be against the short-selling constraint for equities when $B^E_{w,t}$ is a choice variable.

The optimality condition for labor supply is given by

$$N^{\phi}_{w,t} = \frac{W_t}{P_t}.$$

We focus on an equilibrium where workers are always constrained. To derive the conditions that ensure this is indeed the case, we start by considering a stationary equilibrium where all variables are constant conditional on the state. If workers are constrained in the stationary equilibrium, then they will also be constrained if fluctuations are small enough.

In a stationary equilibrium, net consumption $\tilde{C}_w$ in normal times is given by

$$\tilde{C}_w = \frac{W}{P} N_w - \frac{N_w^{1+\phi}}{1+\phi} + T_w,$$

and an analogous expression holds in the crisis state. Notice that this expression does not depend on ρ_w or λ_w.

For workers to be unconstrained, the following condition would have to hold:

$$\frac{\dot{\tilde{C}}_{w,t}}{\tilde{C}_{w,t}} = \sigma^{-1}(r_n - \rho_w) + \frac{\lambda_w}{\sigma}\left[\left(\frac{\tilde{C}_{w,t}}{\tilde{C}^*_{w,t}}\right)^{\sigma} - 1\right].$$

For ρ_w sufficiently large, workers would want a declining path of consumption, so current consumption would be above $\frac{W}{P}N_w - \frac{N_w^{1+\phi}}{1+\phi} + T_w$, which would violate the state-constraint Hence, the constraint must be binding for ρ_w sufficiently large.

If the workers hold a positive amount of long-term bonds, then the following condition must hold:

$$r_L = \lambda_w \left(\frac{\tilde{C}_w}{\tilde{C}^*_w}\right)^{\sigma} \frac{Q_L - Q^*_L}{Q_L}.$$

As C_w and C^*_w are independent of λ_w, the equation above would hold only if λ_w equals the value $\overline{\lambda}_w \equiv \frac{r_L}{\left(\frac{\tilde{C}_w}{\tilde{C}^*_w}\right)^{\sigma}\frac{Q_L - Q^*_L}{Q_L}}$. For $\lambda_w > \overline{\lambda}_w$, borrowers would want a smaller dispersion between C_w and C^*_w, which requires holding less risky bonds, violating the non-negativity constraint on long-term bonds. Therefore, borrowers will hold zero long-term bonds for λ_w sufficiently large.

Firms' problem. The intermediate-goods producers' problem is given by

$$Q_{i,t}(P_i) = \max_{[\pi_{i,s}]_{s\geq t}} \mathbb{E}_t\left[\int_t^{t^*} \frac{\tilde{\eta}_s}{\tilde{\eta}_t}\left(\frac{P_{i,s}}{P_s}Y_{i,s} - \frac{W_s}{P_s}\frac{Y_{i,s}}{A_s} - \frac{\varphi}{2}\pi^2_{i,s}\right) ds + \frac{\tilde{\eta}_{t^*}}{\tilde{\eta}_t} Q^*_{i,t^*}(P_{i,t^*})\right],$$

subject to $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t$ and $\dot{P}_{i,t} = \pi_{i,t}P_{i,t}$, given $P_{i,t} = P_i$, where $\tilde{\eta}_t \equiv \eta_t P_t$ is the real SDF, and η_t is given in Proposition 1.

The HJB equation for this problem is

$$0 = \max_{\pi_{i,t}} \tilde{\eta}_t \left(\frac{P_{i,t}}{P_t} Y_{i,t} - \frac{W_t}{P_t} \frac{Y_{i,t}}{A_t} - \frac{\varphi}{2} \pi_{i,t}^2 \right) dt + \mathbb{E}_t \left[d \left(\tilde{\eta}_t Q_{i,t}(P_{i,t}) \right) \right]$$

where $\frac{\mathbb{E}_t\left[d\left(\tilde{\eta}_t Q_{i,t}(P_{i,t})\right)\right]}{\tilde{\eta}_t dt} = -\left(i_t^{zb} - \pi_t\right) Q_{i,t} + \frac{\partial Q_{i,t}}{\partial t} + \frac{\partial Q_{i,t}}{\partial P_{i,t}} \pi_{i,t} P_{i,t} + \lambda_t \frac{\tilde{\eta}_t^*}{\tilde{\eta}_t} \left[Q_{i,t}^* - Q_{i,t} \right]$.

The first order condition is given by

$$\frac{\partial Q_{i,t}}{\partial P_{i,t}} P_{i,t} = \varphi \pi_{i,t}.$$

The change in π_t in normal times is then given by

$$\left(\frac{\partial^2 Q_{i,t}}{\partial P_{i,t} \partial t} + \frac{\partial^2 Q_{i,t}}{\partial P_{i,t}^2} \pi_{i,t} P_{i,t} + \frac{\partial Q_{i,t}}{\partial P_{i,t}} \pi_{i,t} \right) P_{i,t} = \varphi \dot{\pi}_{i,t}. \tag{B.2}$$

The envelope condition with respect to $P_{i,t}$ is given by

$$\begin{aligned} 0 = \left((1-\epsilon) \frac{P_{i,t}}{P_t} + \epsilon \frac{W_t}{P_t A_t} \right) \left(\frac{P_{i,t}}{P_t} \right)^{-\epsilon} \frac{Y_t}{P_{i,t}} - \left(i_t^{zb} - \pi_t \right) \frac{\partial Q_{i,t}}{\partial P_{i,t}} + \frac{\partial^2 Q_{i,t}}{\partial t \partial P_{i,t}} + \\ \frac{\partial^2 Q_{i,t}}{\partial P_{i,t}^2} \pi_{i,t} P_{i,t} + \frac{\partial Q_{i,t}}{\partial P_{i,t}} \pi_{i,t} + \lambda_t \frac{\eta_t^*}{\eta_t} \left[\frac{\partial Q_{i,t}^*}{\partial P_{i,t}} - \frac{\partial Q_{i,t}}{\partial P_{i,t}} \right] \end{aligned}$$

Multiplying the expression above by $P_{i,t}$ and using Equation (B.2), we obtain

$$0 = \left((1-\epsilon) \frac{P_{i,t}}{P_t} + \epsilon \frac{W_t}{P_t A_t} \right) \left(\frac{P_{i,t}}{P_t} \right)^{-\epsilon} Y_t - \left(i_t^{zb} - \pi_t \right) \varphi \pi_t + \varphi \dot{\pi}_{i,t} + \lambda_t \varphi \frac{\eta_t^*}{\eta_t} \left(\pi_t^* - \pi_t \right),$$

where $\eta_t^* / \eta_t = \tilde{\eta}_t^* / \tilde{\eta}_t$, given the price level is predetermined.

Rearranging this expression, we obtain the non-linear New Keynesian Phillips curve

$$\dot{\pi}_{i,t} = \left(i_t^{zb} - \pi_t + \lambda_t \frac{\eta_t^*}{\eta_t} \right) \pi_t - \frac{\epsilon}{\varphi A} \left(\frac{W_t}{P_t} - \left(1 - \epsilon^{-1} \right) A \right) Y_t,$$

where we have assumed that $P_{i,t} = P_t$ for all $i \in [0,1]$ and that $\pi_t^* = 0$.

B.2 The stationary equilibrium

Next, we solve for the stationary equilibrium. For simplicity, we focus on the case of perfect substitution between bonds in the liquidity aggregator, $\theta = 0$.

Aggregate output. Consider a stationary equilibrium with zero inflation. From the New Keynesian Phillips curve, we obtain:

$$\frac{W}{P} = (1-\epsilon^{-1})A, \qquad \frac{W^*}{P} = (1-\epsilon^{-1})A^*.$$

Combining the expressions above with the labor supply condition, we obtain:

$$Y = \mu_w(1-\epsilon^{-1})^{\frac{1}{\phi}} A^{\frac{1+\phi}{\phi}}, \qquad Y^* = \mu_w(1-\epsilon^{-1})^{\frac{1}{\phi}} (A^*)^{\frac{1+\phi}{\phi}}.$$

Crisis state. We assume that the government chooses fiscal transfers so workers have a given share $0 < \mu^*_{Y,w} < 1$ of output, so $C^*_w = \mu^*_{Y,w}\frac{Y^*}{\mu_w}$. The parameter $\mu_{Y,w}$ captures the government's preference for redistribution. This requires that the government sets $T^*_w = \left[\frac{\mu^*_{Y,w}}{\mu_w} - \frac{1-\epsilon^{-1}}{\mu_w}\right] Y^*$. In the main text, we focus on the case $\mu^*_{Y,w} = \mu_w$.

For simplicity, we assume that $\rho^*_j(\cdot)$ is constant so that $\rho^*_o = \rho^*_p = \rho^*_s$. In this case, the real interest rate in the crisis state is given by $r^*_n = \rho^*_s - \xi^*(C^*_s)^{\sigma}(L^*_s)^{-\nu}$. The zero beta rate is given by $r^{*,zb}_n = \rho^*_s$. We focus on a symmetric allocation where $C^*_o = C^*_p$, then

$$\xi^* = \left[\frac{\mu_o}{\mu_o+\mu_p}(\xi^*_o)^{\frac{1}{\nu}} + \frac{\mu_p}{\mu_o+\mu_p}(\xi^*_p)^{\frac{1}{\nu}}\right]^{\nu}.$$

The excess return on equities and long-term bonds are given by

$$r^*_E = \xi^*(C^*_s)^{\sigma}(L^*_s)^{-\nu}, \qquad r^*_L = (1-\alpha^*)\xi^*(C^*_s)^{\sigma}(L^*_s)^{-\nu}.$$

Then, the equity price is given by

$$Q^*_E = \frac{\Pi^*}{\rho^*_s},$$

and the long-term bond price is given by

$$Q_L^* = \frac{1}{r_n^{*,zb} + \psi_L - \alpha^* \xi^* (C_s^*)^{\sigma} (L_s^*)^{-\nu}},$$

where $L_s^* = \frac{\overline{D}_G^{S,*}}{\mu_s} + \alpha^* \frac{\overline{D}_G^{L,*}}{\mu_s}$, with $\overline{D}_G^{L,*} = Q_L^* N_G^{L,*}$, for a fixed number of bonds $N_G^{L,*}$. Given the number of bonds, the equation above pins down the value of Q_L^*.

The consumption of borrowers is given by

$$C_w^* = (1 - \epsilon^{-1}) \frac{Y^*}{\mu_w} + T_w^*.$$

Savers' consumption is given by

$$C_j^* = r_n^* B_j^* + r_L^* B_j^{*,L} + r_E^* B_j^{*,E} + T_j^*,$$

where $B_j^* = B_j + B_j^L \frac{Q_L^* - Q_L}{Q_L} + B_j^E \frac{Q_E^* - Q_E}{Q_E}$.

Aggregate consumption of savers is given by

$$C_s^* = r_n^* \frac{\overline{D}_G^*}{\mu_s} + r_L^* \frac{\overline{D}_G^{*,L}}{\mu_s} + \frac{\Pi^*}{\mu_s} + T_s.$$

Transfers to savers must satisfy $T_s^* = (1 - \mu_{Y,w} - \epsilon^{-1}) \frac{Y^*}{\mu_s} - r_n^* \frac{\overline{D}_G^*}{\mu_s} - r_L^* \frac{\overline{D}_G^{*,L}}{\mu_s}$ such that the government's budget constrain is satisfied. This implies that the aggregate consumption of savers is given by $C_s^* = (1 - \mu_{Y,w}) \frac{Y^*}{\mu_s}$.

We focus on a symmetric allocation in the crisis state, so we assume that $T_{o,t}^* - T_{p,t}^* = -r_n^*(B_o^* - B_p^*) - r_L^*(B_o^{*,L} - B_p^{*,L}) - r_E^*(B_o^{*,L} - B_p^{*,E})$, for $t \geq t^*$. This implies that $C_j^* = C_s^*$.

From the optimality condition for long-term bonds and equities, we have that:

$$\xi_o^* (C_o^*)^{\sigma} (L_o^*)^{-\nu} = \xi_p^* (C_p^*)^{\sigma} (L_p^*)^{-\nu} \Rightarrow L_p^* = \left(\frac{\xi_p^*}{\xi_o^*} \right)^{\frac{1}{\nu}} L_o^*.$$

The market clearing condition for bonds imply the condition:

$$\frac{\mu_o}{\mu_o+\mu_p}L_o^* + \frac{\mu_p}{\mu_o+\mu_p}L_p^* = L_s^*.$$

Combining the previous two equations, we obtain:

$$L_o^* = \frac{\mu_s(\xi_o^*)^{\frac{1}{\nu}}}{\mu_o(\xi_o^*)^{\frac{1}{\nu}}+\mu_p(\xi_p^*)^{\frac{1}{\nu}}}L_s^*, \qquad L_p^* = \frac{\mu_s(\xi_p^*)^{\frac{1}{\nu}}}{\mu_o(\xi_o^*)^{\frac{1}{\nu}}+\mu_p(\xi_p^*)^{\frac{1}{\nu}}}L_s^*.$$

As heterogeneity in liquidity multiplier has no aggregate implication in the crisis state, we focus on the simpler case where $\xi_o^* = \xi_p^*$.

Given the net worth in the crisis state, the portfolios satisfy the following condition:

$$B_j^* = L_j^* + (1-\alpha^*)B_j^{*,L} + B_j^{*,E}.$$

Normal times: λ and ξ. The consumption of workers is given by

$$C_w = \left[(1-\epsilon^{-1})A\right]^{\frac{1+\phi}{\phi}} + T_w.$$

As in the crisis state, the government chooses fiscal transfers so workers have a given share $0 < \mu_{Y,w} < 1$ of output, so $C_w = \mu_{Y,w}\frac{Y}{\mu_w}$ and $C_s = (1-\mu_{Y,W})\frac{Y}{\mu_s}$. This requires that the government sets $T_w = \left[\frac{\mu_{Y,w}}{\mu_w} - \frac{1-\epsilon^{-1}}{\mu_w}\right]Y$. In the special case where $\mu_{Y,w} = \mu_w$, we have $T_w = \epsilon^{-1}Y$.

From the market clearing condition for assets, we obtain

$$B_s = \frac{\overline{D}_G^S + \overline{D}_G^L + Q_E}{1-\mu_w}, \quad B_s^S = \frac{\overline{D}_G^S}{1-\mu_w}, \quad B_s^L = \frac{\overline{D}_G^L}{1-\mu_w}, \quad B_s^E = \frac{Q_E}{1-\mu_w}.$$

The consumption of individual savers is given by

$$C_j = r_n B_j + r_L B_j^L + r_E B_j^E + T_j.$$

From the Euler equation for short-term bonds to be satisfied for both types of savers,

the following condition must be satisfied: $\rho_o - \rho_p = \lambda_p - \lambda_o$, where ρ_j is an increasing function of $\frac{C_j}{C_s}$. As the consumption of type-j savers is increasing in B_j, $\rho_o - \rho_p$ is increasing in B_o. Hence, there is a unique value of B_o such that $\rho_o - \rho_p = \lambda_p - \lambda_o$. We assume the function $\rho_j(\cdot)$ is such that this equality is achieved when $B_o = B_p$.

Using the fact that $B_o = B_p$ and $T_o = T_p$ in a stationary equilibrium, we can write the consumption of optimistic and pessimistic savers as follows:

$$C_o = C_s \left[1 + \frac{\mu_p}{\mu_o + \mu_p}\left(r^L \frac{B_o^L - B_p^L}{C_s} + r_E \frac{B_o^E - B_p^E}{C_s}\right)\right],$$
$$C_p = C_s \left[1 - \frac{\mu_o}{\mu_o + \mu_p}\left(r^L \frac{B_o^L - B_p^L}{C_s} + r_E \frac{B_o^E - B_p^E}{C_s}\right)\right],$$

where $r_L \equiv \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{Q_L - Q_L^*}{Q_L} + \xi C_s^{\sigma} L_s^{-\nu}(1-\alpha)$ and $r_E \equiv \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{Q_E - Q_E^*}{Q_E} + \xi C_s^{\sigma} L_s^{-\nu}$, and we used that $C_o^* = C_p^* = C_s^*$. Using Lemma 1, we obtain

$$\left(1 + \frac{\mu_p}{\mu_o + \mu_p}\left(r^L \frac{B_o^L - B_p^L}{C_s} + r_E \frac{B_o^E - B_p^E}{C_s}\right)\right) = \left(\frac{\lambda_p}{\lambda_o}\right)^{\frac{1}{\sigma}} \left(1 - \frac{\mu_o}{\mu_o + \mu_p}\left(r^L \frac{B_o^L - B_p^L}{C_s} + r_E \frac{B_o^E - B_p^E}{C_s}\right)\right), \tag{B.3}$$

and

$$B_s - (1-\alpha)B_o^L - B_o^E = \left(\frac{\xi_o}{\xi_p}\frac{\lambda_p}{\lambda_o}\right)^{\frac{1}{\nu}} \left[B_s - (1-\alpha)B_p^L - B_p^E\right].$$

Rearranging Equation (B.3), we obtain

$$r^L \frac{B_o^L - B_p^L}{C_s} + r_E \frac{B_o^E - B_p^E}{C_s} = \frac{\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}}{\frac{\mu_o}{\mu_o+\mu_p}\lambda_p^{\frac{1}{\sigma}} + \frac{\mu_p}{\mu_o+\mu_p}\lambda_o^{\frac{1}{\sigma}}}.$$

Then, we can solve for the consumption share of optimistic savers:

$$\frac{\mu_o C_o}{\mu_o C_o + \mu_p C_p} = \frac{\mu_o}{\mu_o + \mu_p}\left[1 + \frac{\mu_p\left(\lambda_o^{-\frac{1}{\sigma}} - \lambda_p^{-\frac{1}{\sigma}}\right)}{\mu_o \lambda_o^{-\frac{1}{\sigma}} + \mu_p \lambda_p^{-\frac{1}{\sigma}}}\right].$$

Given this expression, we can obtain the market-implied disaster probability:

$$\lambda = \left[\frac{\mu_o C_o}{\mu_o C_o + \mu_p C_p}\lambda_o^{\frac{1}{\sigma}} + \frac{\mu_p C_p}{\mu_o C_o + \mu_p C_p}\lambda_p^{\frac{1}{\sigma}}\right]^{\sigma} \implies \lambda = \left[\frac{\mu_o \lambda_o^{-\frac{1}{\sigma}}}{\mu_o + \mu_p} + \frac{\mu_p \lambda_p^{-\frac{1}{\sigma}}}{\mu_o + \mu_p}\right]^{-\sigma}.$$

Moreover, we can obtain the market-implied convenience:

$$\xi = \left[\frac{\mu_o \lambda_p^{\frac{1}{\nu}}}{\left(\mu_o \lambda_p^{\frac{1}{\sigma}} + \mu_p \lambda_o^{\frac{1}{\sigma}}\right)^{\frac{\sigma}{\nu}}} \frac{\xi_o^{\frac{1}{\nu}}}{\left(\mu_o + \mu_p\right)^{\frac{\nu-\sigma}{\nu}}} + \frac{\mu_p \lambda_o^{\frac{1}{\nu}}}{\left(\mu_o \lambda_p^{\frac{1}{\sigma}} + \mu_p \lambda_o^{\frac{1}{\sigma}}\right)^{\frac{\sigma}{\nu}}} \frac{\xi_p^{\frac{1}{\nu}}}{\left(\mu_o + \mu_p\right)^{\frac{\nu-\sigma}{\nu}}}\right]^{\nu}.$$

If $\nu = \sigma$, this expression simplifies to

$$\xi = \left[\frac{\mu_o \lambda_o^{-\frac{1}{\sigma}}}{\mu_o \lambda_o^{-\frac{1}{\sigma}} + \mu_p \lambda_p^{-\frac{1}{\sigma}}}\xi_o^{\frac{1}{\sigma}} + \frac{\mu_p \lambda_p^{-\frac{1}{\sigma}}}{\mu_o \lambda_o^{-\frac{1}{\sigma}} + \mu_p \lambda_p^{-\frac{1}{\sigma}}}\xi_p^{\frac{1}{\sigma}}\right]^{\sigma}.$$

Normal times: asset prices. Given λ and ξ, we can solve for the interest rate and the price of risky assets. The natural interest rate is given by:

$$r_n = \rho_s - \lambda\left[\left(\frac{C_s}{C_s^*}\right)^{-\sigma} - 1\right] - \xi C_s^{\sigma} L_s^{-\nu}.$$

where $L_s = \frac{\overline{D}_G^S + \alpha \overline{D}_G^L}{1-\mu_w}$.

From the definition of the returns we have that $r_L = \frac{1}{Q_L} - \psi_L - r_n$ and $r_E = \frac{\Pi}{Q_E} - r_n$, where $\Pi = \epsilon^{-1} Y$. Then using Equation (6), we can solve for Q_L:

$$\frac{1}{Q_L} - \psi_L - r_n = \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma}\left(1 - \frac{Q_L^*}{Q_L}\right) + \xi C_s^{\sigma} L_s^{-\nu}(1-\alpha)$$

$$\implies Q_L = Q_L^* \frac{\rho_s^* + \psi_L - \alpha^* \xi^* (C_s^*)^{\sigma} (L_s^*)^{-\nu} + \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma}}{\rho + \psi_L - \alpha \xi C_s^{\sigma} L_s^{-\nu}},$$

where $Q_L > Q_L^*$. The loss in long-term bonds in the crisis state is given by

$$\frac{Q_L - Q_L^*}{Q_L} = \frac{r_n^{zb,*} - r_n^{zb} + \alpha\xi C_s^\sigma L_s^{-\nu} - \alpha^*\xi^*(C_s^*)^\sigma(L_s^*)^{-\nu}}{\rho_s^* + \psi_L - \alpha^*\xi^*(C_s^*)^\sigma(L_s^*)^{-\nu} + \lambda\left(\frac{C_s}{C_s^*}\right)^\sigma}.$$

Bond prices fall in the crisis state if either the zero-beta increases in a crisis, or bonds lose liquidity (or a combination of the two). We focus on the case where the zero-beta rate increases s and long-term bonds lose liquidity in a crisis, so $Q_L^* < Q_L$.

The equity price is given by

$$\frac{\Pi}{Q_E} - r_n = \lambda\left(\frac{C_s^*}{C_s}\right)^{-\sigma}\left(1 - \frac{Q_E^*}{Q_E}\right) + \xi C_s^\sigma L_s^{-\nu} \implies Q_E = \frac{\Pi + \lambda\left(\frac{C_s^*}{C_s}\right)^{-\sigma} Q_E^*}{r_n^{zb} + \lambda\left(\frac{C_s^*}{C_s}\right)^{-\sigma}},$$

so the loss on equity in the crisis state is given by

$$\frac{Q_E - Q_E^*}{Q_E} = \frac{r_n^{*,zb}\zeta_\Pi + \left(r_n^{*,zb} - r_n^{zb}\right)(1 - \zeta_\Pi)}{r_n^{*,zb} + \lambda\left(\frac{C_s}{C_s^*}\right)^\sigma(1 - \zeta_\Pi)},$$

where $\zeta_\Pi \equiv 1 - \frac{\Pi^*}{\Pi}$ is the size of the drop in profits. As this expression is positive, the equity premium is positive in the stationary equilibrium.

We can then compute the spreads:

$$r_L = \lambda\left(\frac{C_s}{C_s^*}\right)^\sigma \frac{Q_L - Q_L^*}{Q_L} + (1-\alpha)\xi C_s^\sigma L_s^{-\nu}, \qquad r_E = \lambda\left(\frac{C_s}{C_s^*}\right)^{-\sigma} \frac{Q_E - Q_E^*}{Q_E} + \xi C_s^\sigma L_s^{-\nu}.$$

Normal times: asset allocations. It remains to determine the asset allocations of savers: $(B_o^L, B_o^E, B_p^L, B_p^E)$. From the equality of the price of risk, we have that

$$r^L(B_o^L - B_p^L) + r_E(B_o^E - B_p^E) = \frac{\lambda_o^{-\frac{1}{\sigma}} - \lambda_p^{-\frac{1}{\sigma}}}{\omega_o\lambda_o^{-\frac{1}{\sigma}} + \omega_p\lambda_p^{-\frac{1}{\sigma}}} C_s,$$

where $\omega_o \equiv \mu_o/\mu_s$ and $\omega_p \equiv \mu_p/\mu_s$.

From the equality of the marginal value of liquidity, we have that

$$\left(\frac{\lambda_o}{\xi_o}\right)^{\frac{1}{\nu}}\left[B_s-(1-\alpha)B_o^L-B_o^E\right]=\left(\frac{\lambda_p}{\xi_p}\right)^{\frac{1}{\nu}}\left[B_s-(1-\alpha)B_p^L-B_p^E\right],$$

given $B_s=\overline{B}_S+\overline{B}_L+\overline{B}_E$, and the definitions $\overline{B}_S\equiv\frac{\overline{D}_G^S}{\mu_s}$, $\overline{B}_L\equiv\frac{\overline{D}_G^L}{\mu_s}$, and $\overline{B}_E\equiv\frac{Q_E}{\mu_s}$.

We need two more conditions to solve for the asset allocations. From the market clearing conditions for bonds and equities, we have that

$$\omega_o B_o^L+\omega_p B_p^L=\overline{B}_L,\quad \omega_o B_o^E+\omega_p B_p^E=\overline{B}_E.$$

We can write the system of equations as follows:

$$\begin{bmatrix} r_L & -r_L & r_E & -r_E \\ (1-\alpha)A_o & -(1-\alpha)A_p & A_o & -A_p \\ \omega_o & \omega_p & 0 & 0 \\ 0 & 0 & \omega_o & \omega_p \end{bmatrix}\begin{bmatrix} B_o^L \\ B_p^L \\ B_o^E \\ B_p^E \end{bmatrix}=\begin{bmatrix} \frac{\lambda_o^{-\frac{1}{\sigma}}-\lambda_p^{-\frac{1}{\sigma}}}{\omega_o\lambda_o^{-\frac{1}{\sigma}}+\omega_p\lambda_p^{-\frac{1}{\sigma}}}C_s \\ \left(A_o-A_p\right)B_s \\ \overline{B}_L \\ \overline{B}_E \end{bmatrix}, \tag{B.4}$$

where $A_j\equiv\left(\frac{\lambda_j}{\xi_j}\right)^{1/\nu}$ for $j=\{o,p\}$.

From the market clearing conditions for bonds and equities, we have that

$$B_p^L=\overline{B}_L-\omega_o(B_o^L-B_p^L),\quad B_p^E=\overline{B}_E-\omega_o(B_o^E-B_p^E).$$

Plugging these into the first two rows of (B.4), we have that

$$\begin{bmatrix} r_L & r_E \\ (1-\alpha)H & H \end{bmatrix}\begin{bmatrix} B_o^L-B_p^L \\ B_o^E-B_p^E \end{bmatrix}=\begin{bmatrix} \frac{\lambda_o^{-\frac{1}{\sigma}}-\lambda_p^{-\frac{1}{\sigma}}}{\omega_o\lambda_o^{-\frac{1}{\sigma}}+\omega_p\lambda_p^{-\frac{1}{\sigma}}}C_s \\ -\left(A_o^{-1}-A_p^{-1}\right)L_s \end{bmatrix},$$

where $H\equiv\omega_p A_p^{-1}+\omega_o A_o^{-1}$.

If $H=0$, as it would be the case in the model without liquidity frictions, $\xi_0=\xi_p=0$, then portfolios would be indeterminate. Similarly, portfolios are indeterminate in the

knife-edge case where $r_L = (1-\alpha)r_E$. We proceed by assuming the rank conditions $r_L \neq (1-\alpha)r_E$ and $H \neq 0$ hold. Then we can solve for $B_o^L - B_p^L$ and $B_o^E - B_p^E$ as follows:

$$B_o^L - B_p^L = \frac{\frac{\lambda_o^{-1/\sigma} - \lambda_p^{-1/\sigma}}{\omega_o \lambda_o^{-1/\sigma} + \omega_p \lambda_p^{-1/\sigma}} C_s + r_E \frac{A_o^{-1} - A_p^{-1}}{\omega_o A_o^{-1} + \omega_p A_p^{-1}} L_s}{r_L - (1-\alpha) r_E}.$$

$$B_o^E - B_p^E = -\frac{(1-\alpha)\frac{\lambda_o^{-1/\sigma} - \lambda_p^{-1/\sigma}}{\omega_o \lambda_o^{-1/\sigma} + \omega_p \lambda_p^{-1/\sigma}} C_s + r_L \frac{A_o^{-1} - A_p^{-1}}{\omega_o A_o^{-1} + \omega_p A_p^{-1}} L_s}{r_L - (1-\alpha) r_E}.$$

For $k \in \{E, L\}$, we can recover the individual asset allocations using:

$$B_p^k = \overline{B}_k - \omega_o (B_o^k - B_p^k), \qquad B_o^k = \overline{B}_k + \omega_p (B_o^k - B_p^k).$$

B.3 Pricing conditions

Consider the following law of motion for the SDF:

$$\frac{d\eta_t}{\eta_t} = -i_t^{zb} dt + \frac{\eta_t^* - \eta_t}{\eta_t} \left[d\mathcal{N}_t - \lambda_t dt \right],$$

given the arrival rate λ_t.

The pricing conditions for the three assets are given by:

$$\mathbb{E}_t \left[\frac{d(\eta_t R_{j,t})}{\eta_t R_{j,t}} \right] = -\alpha_j (i_t^{zb} - i_t) dt,$$

for $j \in \{E, L, S\}$, where $\alpha_E = 0, \alpha_L = \alpha$, and $\alpha_S = 1$, and $dR_{S,t} = i_t dt$.

The left-hand side of the pricing condition for the short-term bond then be written as:

$$\mathbb{E}_t \left[\frac{d(\eta_t R_{S,t})}{\eta_t R_{S,t}} \right] = \mathbb{E}_t \left[\frac{d\eta_t}{\eta_t} \right] + i_t dt = -(i_t^{zb} - i_t) dt,$$

so the pricing condition is satisfied, as $\alpha_S = 1$.

The left-hand side of the pricing condition for the equity can be written as:

$$\begin{aligned}\mathbb{E}_t\left[\frac{d(\eta_t R_{E,t})}{\eta_t R_{E,t}}\right] &= -\left[i_t^{zb} + \lambda_t \frac{\eta_t^* - \eta_t}{\eta_t}\right] dt + [i_t + r_{E,t}]dt + \lambda_t \left[\frac{\eta_t^*}{\eta_t}\frac{Q_{E,t}^*}{Q_{E,t}} - 1\right] dt \\ &= \left[r_{E,t} - \lambda_t \frac{\eta_t^*}{\eta_t}\frac{Q_{E,t} - Q_{E,t}^*}{Q_{E,t}} - (i_t^{zb} - i_t)\right] dt,\end{aligned}$$

which is equal to zero as long as $\lambda_t \frac{\eta_t^*}{\eta_t} = \lambda_j \left(\frac{C_{j,t}}{C_{j,t}^*}\right)^{\sigma}$ from Equation (5), consistent with the pricing condition for equities, as $\alpha_E = 0$.

The left-hand side of the pricing condition for the long-term bond can be written as:

$$\begin{aligned}\mathbb{E}_t\left[\frac{d(\eta_t R_{L,t})}{\eta_t R_{L,t}}\right] &= -\left[i_t^{zb} + \lambda_t \frac{\eta_t^* - \eta_t}{\eta_t}\right] dt + [i_t + r_{L,t}]dt + \lambda_t \left[\frac{\eta_t^*}{\eta_t}\frac{Q_{L,t}^*}{Q_{L,t}} - 1\right] dt \\ &= \left[r_{L,t} - \lambda_t \frac{\eta_t^*}{\eta_t}\frac{Q_{L,t} - Q_{L,t}^*}{Q_{L,t}} - (i_t^{zb} - i_t)\right] dt,\end{aligned}$$

which is equal to $-\alpha(i_t^{zb} - i_t)dt$ if $\lambda_t \frac{\eta_t^*}{\eta_t} = \lambda_j \left(\frac{C_{j,t}}{C_{j,t}^*}\right)^{\sigma}$ from Equation (6), consistent with the pricing condition for long-term bonds, as $\alpha_L = \alpha$.

B.4 Bond pricing and convenience yield.

Zero-coupon bonds. Let $Q_{B,t}(\tau)$ denote the price of a zero-coupon bond with maturity τ at time t in normal times, and $Q_{B,t}^*(\tau)$ denote the corresponding price in the crisis state. The bond yield is defined as $y_{B,t}(\tau) \equiv -\frac{1}{\tau}\log Q_{B,t}(\tau)$. The pricing condition for the zero-coupon bond in normal times is given by:

$$r_{B,t}(\tau) = \lambda_t \left(\frac{C_{s,t}}{C_{s,t}^*}\right)^{\sigma} \frac{Q_{B,t}(\tau) - Q_{B,t}^*(\tau)}{Q_{B,t}(\tau)} + (1 - \alpha(\tau))\xi_t C_{s,t}^{\sigma} L_{s,t}^{-\nu},$$

where

$$r_{B,t}(\tau) \equiv \frac{1}{Q_{B,t}(\tau)}\left[-\frac{\partial Q_{B,t}(\tau)}{\partial \tau} + \frac{\partial Q_{B,t}(\tau)}{\partial t}\right] - i_t.$$

The price of the zero-coupon bond in the crisis state is given by:

$$Q_{B,t}^*(\tau) = e^{-\rho_B^* \tau},$$

where $\rho_B^* = r_n^{zb,*} - \alpha^* \xi^* (C_s^*)^\sigma (L_s^*)^{-\nu}$.

We focus on the case where $\alpha(\tau) = \alpha$ is constant. In a stationary equilibrium, $Q_{B,t}(\tau) = Q_B(\tau)$. In this case, we can write the pricing condition as follows:

$$\frac{\partial Q_B(\tau)}{\partial \tau} = -\rho_B Q_B(\tau) + \lambda \left(\frac{C_s}{C^*}\right)^\sigma Q_B^*(\tau),$$

where $\rho_B \equiv r_n^{zb} + \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma - \alpha \xi C_s^\sigma L_s^{-\nu} = \rho_s + \lambda - \alpha \xi C_s^\sigma L_s^{-\nu}$.

Solving the differential equation above, we obtain:

$$Q_B(\tau) = e^{-\rho_B \tau} + \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma \frac{e^{-\rho_B^* \tau} - e^{-\rho_B \tau}}{\rho_B - \rho_B^*}.$$

Notice that $Q_L = \int_0^\infty e^{-\psi_L \tau} Q_B(\tau) d\tau$.

The bond yield is given by:

$$y_B(\tau) = r_n^{zb} - \alpha \xi C^\sigma L^{-\nu} + \lambda \left(\frac{C_s}{C^*}\right)^\sigma - \frac{1}{\tau} \log \left[1 + \lambda \left(\frac{C_s}{C^*}\right)^\sigma \frac{e^{(\rho_B - \rho_B^*)\tau} - 1}{\rho_B - \rho_B^*}\right].$$

Convenience yield. The price of the long-term government bond is given by

$$Q_L = \frac{1 + \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma Q_L^*}{\rho_B + \psi_L},$$

where $Q_L^* = 1/(\rho_B^* + \psi_L)$ is the price of the long-term government bond in the crisis state.

Similarly, the price of the corporate bond with the same promised coupons is given by

$$Q_C = \frac{1 + \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma Q_C^*}{\rho_B + \psi_C + \alpha \xi C_s^\sigma L_s^{-\nu}},$$

where $Q_C^* = 1/(\rho_B^* + \psi_C + \alpha^* \xi^* (C_s^*)^\sigma (L_s^*)^{-\sigma})$ is the price of the corporate bond in the crisis state.

The yield on the government bond is given by $y_L = Q_L^{-1} - \psi_L$, and the yield on the

corporate bond is given by $y_C = Q_C^{-1} - \psi_C$. The convenience yield is then given by

$$cy \equiv y_C - y_L = \frac{\rho_B + \psi_L + \alpha\xi C_s^{\sigma} L_s^{-\nu}}{1 + \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma} Q_C^*} - \frac{\rho_B + \psi_L}{1 + \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma} Q_L^*}.$$

We can then write the convenience yield as follows:

$$\begin{aligned}
cy &= \frac{\alpha\xi C_s^{\sigma} L_s^{-\nu}}{1 + \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma} Q_C^*} + (\rho_B + \psi_L)\frac{\lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma}(Q_L^* - Q_C^*)}{(1 + \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma} Q_C^*)(1 + \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma} Q_L^*)} \\
&= \frac{\alpha\xi C_s^{\sigma} L_s^{-\nu}}{1 + \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma} Q_C^*} + \frac{\lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma} Q_C^*}{1 + \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma} Q_C^*}\left[\frac{\rho_B + \psi_L}{1 + \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma} Q_L^*}\frac{\alpha^*\xi^*(C_s^*)^{\sigma}(L_s^*)^{-\nu}}{\rho_B^* + \psi_L}\right] \\
&= \frac{\alpha\xi C_s^{\sigma} L_s^{-\nu}}{1 + \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma} Q_C^*} + \frac{\lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma} Q_C^*}{1 + \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma} Q_C^*}\left[\frac{\rho_B + \psi_L}{\rho_B^* + \psi_L + \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma}}(\alpha^*\xi^*(C_s^*)^{\sigma}(L_s^*)^{-\nu})\right]
\end{aligned}$$

B.5 Equilibrium determinacy

Using the fact that $\sigma r_k c_{s,t}$ is second order, we can write the dynamic system describing the economy's equilibrium as follows:

$$\begin{bmatrix} \dot{c}_{p,t} - \dot{c}_{o,t} \\ \dot{q}_{L,t} \\ \dot{b}_{p,t} - \dot{b}_{o,t} \\ \dot{y}_t \\ \dot{\pi}_t \end{bmatrix} = \begin{bmatrix} \chi_{\Delta c,\Delta c} & 0 & 0 & 0 & 0 \\ \chi_{q_L,\Delta c} & \chi_{q_L,q_L} & 0 & r_L\tilde{\sigma} & \phi_\pi \\ \chi_{\Delta b,\Delta c} & \chi_{\Delta b,q_L} & \chi_{\Delta b,\Delta b} & \chi_{\Delta b,y} & 0 \\ \chi_{y,\Delta c} & \chi_{y,q_L} & 0 & \delta & \tilde{\sigma}^{-1}(\phi_\pi - 1) \\ 0 & 0 & 0 & -\kappa & \rho \end{bmatrix} \begin{bmatrix} c_{p,t} - c_{o,t} \\ q_{L,t} \\ b_{p,t} - b_{o,t} \\ y_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} 0 \\ u_t \\ 0 \\ \tilde{\sigma}^{-1}u_t \\ 0 \end{bmatrix},$$

where $\chi_{\Delta b,y} = \chi_{\Delta b,c_s}\frac{1-\mu_w\chi_y}{1-\mu_w}$, and

$$\chi_{\Delta c,\Delta c} = -\psi_c, \qquad \chi_{q_L,q_L} = \rho + \psi_L - \xi C_s^{\sigma} L_s^{-\nu}\left[\alpha + (1-\alpha)\nu\chi_{l,q}\right] > 0, \qquad \chi_{\Delta b,\Delta b} = \rho.$$

and

$$\chi_{y,\Delta c} \equiv \tilde{\sigma}^{-1}\xi C_s^{\sigma} L_s^{-\nu}\chi_{\xi,\Delta c} + \chi_{p_r}\chi_{\lambda,\Delta c},$$
$$\chi_{y,q_L} \equiv -\tilde{\sigma}^{-1}\xi C_s^{\sigma} L_s^{-\nu}\nu\chi_{l,q_L}$$
$$\delta \equiv \left[\chi_{p_r} + \tilde{\sigma}^{-1}\xi C_s^{\sigma} L_s^{-\nu}\right]\sigma\chi_{c_s,y} = \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma} + \xi C_s^{\sigma} L_s^{-\nu}.$$

To compute the eigenvalues of the matrix above, we use Schur's formula, that is, for a given matrix M,

$$M = \begin{bmatrix} A & B \\ C & D \end{bmatrix},$$

then $det(M) = det(A)det(D - CA^{-1}B) = det(D)det(A - BD^{-1}C)$, assuming A and D are invertible.

The 5×5 matrix is block diagonal with the first block being a 1×1 matrix, corresponding to $[\chi_{\Delta c,\Delta c}]$. This implies that the first eigenvalue is given by $\omega_1 = -\psi_c < 0$. Let A denote the corresponding 4×4 matrix:

$$A = \begin{bmatrix} \chi_{q,q} & 0 & 0 & \phi_\pi \\ \chi_{\Delta b,q} & \chi_{\Delta b,\Delta b} & 0 & 0 \\ \chi_{y,q_L} & 0 & \delta & \tilde{\sigma}^{-1}(\phi_\pi - 1) \\ 0 & 0 & -\kappa & \rho \end{bmatrix} \equiv \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}.$$

Notice that $\omega_2 = \chi_{\Delta b,\Delta b} = \rho$ is an eigenvalue of A associated with the eigenvector $[0, 1, 0, 0]'$. It can be shown that $\chi_{q,q}$ is not an eigenvalue of A if $\chi_{y,q_L} \neq 0$. Suppose $\lambda \neq \chi_{\Delta b,\Delta b}$ and also $\lambda \neq \chi_{q,q}$, then the remaining eigenvalues satisfy the equation:

$$det(A - \lambda I_4) = det(A_{11} - \lambda I_2)det(A_{22} - \lambda I_2 - A_{21}(A_{11} - \lambda I_2)^{-1}A_{12}) = 0,$$

where I_k is the $k \times k$ identity matrix.

Given our assumptions, this condition will be satisfied only if $det(A_{22} - \lambda I_2 - A_{21}(A_{11} -$

$\lambda I_2)^{-1} A_{12}) = 0$. We can write the second condition as $det(B) = 0$, where

$$B = \begin{bmatrix} \delta - \lambda & \tilde{\sigma}^{-1}(\phi_\pi - 1) \\ -\kappa & \rho - \lambda \end{bmatrix} - \begin{bmatrix} 0 & \frac{\chi_{y,q_L}\phi_\pi}{(\chi_{q,q}-\lambda)} \\ 0 & 0 \end{bmatrix}.$$

The remaining eigenvalues satisfy the equation:

$$(\rho - \lambda)(\delta - \lambda) + \kappa \left[\tilde{\sigma}^{-1}(\phi_\pi - 1) - \chi_{y,q_L}\phi_\pi(\chi_{q,q} - \lambda)^{-1}\right] = 0.$$

Multiplying by $\lambda - \chi_{q,q}$, we obtain:

$$(\lambda - \chi_{q,q})(\lambda - \rho)(\lambda - \delta) + \kappa\tilde{\sigma}^{-1}(\phi_\pi - 1)(\lambda - \chi_{q,q}) + \kappa\chi_{y,q_L}\phi_\pi = 0.$$

The solution to this cubic equation gives the remaining eigenvalues. Expanding this equation, we obtain:

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$$

where

$$\begin{aligned} S_1 &= \rho + \delta + \chi_{q,q}, \\ S_2 &= \rho\delta + \chi_{q,q}(\rho + \delta) + \kappa\tilde{\sigma}^{-1}(\phi_\pi - 1), \\ S_3 &= \chi_{q,q}\left(\rho\delta + \kappa\tilde{\sigma}^{-1}(\phi_\pi - 1)\right) - \kappa\chi_{y,q_L}\phi_\pi. \end{aligned}$$

To ensure that all roots of the cubic equation are positive, the following conditions must be satisfied:

$$S_1 > 0, \qquad S_2 > 0, \qquad S_3 > 0, \qquad S_1 S_2 > S_3.$$

The S_3 is positive if the following condition is satisfied:

$$\phi_\pi > 1 - \frac{|\chi_{y,q_L}|}{\tilde{\sigma}^{-1}\chi_{q,q} + |\chi_{y,q_L}|} - \frac{\rho\delta\chi_{q,q}}{\kappa(\tilde{\sigma}^{-1}\chi_{q,q} + |\chi_{y,q_L}|)}.$$

We can rewrite the inequality above as:

$$\phi_\pi > \left(1 - \frac{\rho\delta}{\kappa\tilde{\sigma}^{-1}}\right) \frac{\chi_{q,q}}{\chi_{q,q} + \tilde{\sigma}|\chi_{y,q_L}|}.$$

The condition $S_1 S_2 > S_3$ holds if the following condition is satisfied:

$$(\rho + \delta + \chi_{q,q}) \left[\rho\delta + \chi_{q,q}(\rho + \delta) + \kappa\tilde{\sigma}^{-1}(\phi_\pi - 1)\right] > \chi_{q,q} \left(\rho\delta + \kappa\tilde{\sigma}^{-1}(\phi_\pi - 1)\right) + \kappa|\chi_{y,q_L}|\phi_\pi.$$

Rewriting the inequality above, we obtain:

$$\chi_{q,q}(\rho + \delta) + \frac{\rho + \delta}{\rho + \delta + \chi_{q,q}} \left[\rho\delta + \kappa\tilde{\sigma}^{-1}(\phi_\pi - 1)\right] > \frac{\kappa|\chi_{y,q_L}|\phi_\pi}{\rho + \delta + \chi_{q,q}}.$$

Therefore, ϕ_π must satisfy the following condition:

$$\phi_\pi > 1 - \frac{\rho\delta}{\kappa\tilde{\sigma}^{-1}} - \frac{\chi_{q,q}(\rho + \delta + \chi_{q,q})}{\kappa\tilde{\sigma}^{-1}} + \frac{|\chi_{y,q_L}|\phi_\pi}{\tilde{\sigma}^{-1}(\rho + \delta)},$$

where

$$\begin{aligned}
\chi_{q_L,q_L} &= \rho + \psi_L - \xi C_s^\sigma L_s^{-\nu} \left[\alpha + (1 - \alpha)\nu\chi_{l,q}\right] \\
\tilde{\sigma}|\chi_{y,qL}| &= \xi C_s^\sigma L_s^{-\nu} \nu\chi_{l,q_L}.
\end{aligned}$$

Finally, to ensure the coefficient χ_{q_L,q_L} is positive we need the condition:

$$\rho + \psi_L - \xi C_s^\sigma L_s^{-\nu} \left[\alpha + (1 - \alpha)\nu\chi_{l,q}\right] > 0,$$

and $\rho + \delta$ is given by

$$\rho + \delta$$

B.6 Log-linearization

We consider next the effects of an unexpected monetary shock for an economy starting at the stationary equilibrium described above.

Crisis state. As there is no monetary shock in the crisis state, inflation is equal to zero, $\pi_t^* = 0$, and output is kept at the stationary-equilibrium level, $y_t^* = 0$. Wages and hours are unchanged, so $c_{w,t}^* = 0$. Savers' aggregate consumption is also the same as in the stationary equilibrium, $c_{s,t}^* = 0$. Savers' flow budget constraint is given by $\mu_s C_{s,t}^* = r_{n,t}^* D_{G,t}^S + (r_{n,t}^* + r_{L,t}^*) D_{G,t}^L \frac{Q_{L,t}^*}{Q_{L,t}} + (r_{n,t}^* + r_{E,t}^*) Q_{E,t}^* + T_{s,t}^*$. Notice that $r_{n,t}^* = r_n^*$, $r_{L,t}^* = r_L^*$, $r_{E,t}^* = r_E^*$, $Q_{L,t}^* = Q_L^*$, and $Q_{E,t}^* = Q_E^*$. For simplicity, we further assume that the government chooses transfers in normal times such that $D_{G,t}^L = D_G^L Q_{L,t}$, so transfers must satisfy $T_{s,t}^* = T_s^*$.

The budget constraint of saver j is given by:

$$C_{j,t}^* = r_n^* B_{j,t}^* + r_{E,t}^* \left[(1 - \alpha^*) B_{j,t}^{*,L} + B_{j,t}^{*,E} \right] + T_{j,t}^*,$$

using the fact that $r_{L,t}^* = (1 - \alpha^*) r_E^*$. Given $B_{j,t}^* = L_{j,t}^* + (1 - \alpha^*) B_{j,t}^{*,L} + B_{j,t}^{*,E}$, we obtain:

$$C_{j,t}^* = r_n^* B_{j,t}^* + r_{E,t}^* \left[B_{j,t}^* - L_{j,t}^* \right] + T_{j,t}^*$$

Linearizing the previous equation, we obtain:

$$C_j^* c_{j,t}^* = (r_n^* + r_E^*) B_j^* b_{j,t}^* - r_E^* L_j^* l_{j,t}^* + T_{j,t}^* - T_j^*.$$

Linearizing the equalization of the marginal value of liquidity, we obtain:

$$\sigma (c_{o,t}^* - c_{p,t}^*) = \nu (l_{o,t}^* - l_{p,t}^*).$$

Combining the expressions above, we obtain:

$$c_{o,t}^* - c_{p,t}^* = \frac{r_n^* + r_E^*}{C_s^* + \frac{\sigma}{\nu} r_E^* L_s^*} \left[B_o^* b_{o,t}^* - B_p^* b_{p,t}^* \right] + \frac{T_{o,t}^* - T_{p,t}^*}{C_s^* + \frac{\sigma}{\nu} r_E^* L_s^*} + \frac{T_o^* - T_p^*}{C_s^* + \frac{\sigma}{\nu} r_E^* L_s^*}.$$

We focus on the case where $C_j^* c_{j,t} = r_n^* B_j^* b_{j,t}^*$, so we obtain:

$$T_{o,t}^* - T_o^* - (T_{p,t}^* - T_p^*) = -r_E^* \left[B_o^* b_{o,t}^* - B_p^* b_{p,t}^* - L_s^* (l_{o,t}^* - l_{p,t}^*) \right].$$

Market-based disaster probability and convenience yield. Linearizing Equation (7) around a stationary equilibrium, we obtain

$$\frac{\lambda^{\frac{1}{\sigma}}}{\sigma}\hat{\lambda}_t = \mu_{c,o}\mu_{c,p}\left(\lambda_o^{\frac{1}{\sigma}} - \lambda_p^{\frac{1}{\sigma}}\right)\left(c_{o,t} - c_{p,t}\right),$$

where $\mu_{c,j} \equiv \frac{\mu_j C_j}{\mu_o C_o + \mu_o C_o}$ and $c_{j,t} \equiv \log C_{j,t}/C_j$ for $j \in \{o, p\}$. Moreover, linearizing Equation (9), we get

$$\frac{\xi^{\frac{1}{\nu}}}{\nu}\hat{\xi}_t = \mu_{c,o}\mu_{c,p}\left[\left(\frac{C_o}{C_s}\right)^{\frac{\sigma-\nu}{\nu}}\xi_o^{\frac{1}{\nu}} - \left(\frac{C_p}{C_s}\right)^{\frac{\sigma-\nu}{\nu}}\xi_p^{\frac{1}{\nu}}\right]\frac{\sigma}{\nu}\left(c_{o,t} - c_{p,t}\right).$$

Euler equation for short-term bonds. Using the fact that $\lambda_j\left(\frac{C_{j,t}}{C^*_{j,t}}\right)^{\sigma} = \lambda_t\left(\frac{C_{s,t}}{C^*_{s,t}}\right)^{\sigma}$ and $\xi_j\frac{L_{j,t}^{-\nu}}{C_{j,t}^{-\sigma}} = \xi_t\frac{L_{s,t}^{-\nu}}{C_{s,t}^{-\sigma}}$, we can write the Euler equation for short-term bonds as follows:

$$\dot{c}_{j,t} = \sigma^{-1}\left(i_t - \pi_t - \left(\rho_{j,t} + \lambda_j\right)\right) + \frac{\lambda_t}{\sigma}\left(\frac{C_{s,t}}{C^*_{s,t}}\right)^{\sigma} + \frac{\xi_t}{\sigma}C_{s,t}^{\sigma}L_{s,t}^{-\nu}.$$

Linearizing the discount-rate function, we obtain $\rho_{j,t} = \rho_j + \sigma\psi_c(c_{j,t} - c_{s,t})$, where we assumed a common slope for both types $\sigma\psi_c$, so we obtain the linearized Euler equation

$$\dot{c}_{j,t} = \sigma^{-1}\left(\hat{i}_t^{zb} - \pi_t - r_n\right) + \frac{\lambda}{\sigma}\left(\frac{C_s}{C^*_s}\right)^{\sigma}p_{r,t} - \psi_c(c_{j,t} - c_{s,t}),$$

where

$$p_{r,t} = \hat{\lambda}_t + \sigma c_{s,t}, \qquad \hat{i}_t^{zb} = i_t + \xi C_s^{\sigma}L_s^{-\nu}(\hat{\xi}_t + \sigma c_{s,t} - \nu l_{s,t}),$$

with $\hat{\lambda}_t \equiv \log(\lambda_t/\lambda)$ and $\hat{\xi}_t \equiv \log(\xi_t/\xi)$, and we used the fact that $c^*_{s,t} = 0$

Aggregating this expression using that $c_{s,t} = \sum_{j\in\{o,p\}}\mu_{c,j}c_{j,t}$, we obtain

$$\dot{c}_{s,t} = \sigma^{-1}\left(\hat{i}_t^{zb} - \pi_t\right) + \frac{\lambda}{\sigma}\left(\frac{C_s}{C^*_s}\right)^{\sigma}p_{r,t}.$$

Relative consumption. Given Lemma 1, we have

$$\lambda_o^{\frac{1}{\sigma}} \frac{C_{o,t}}{C_{o,t}^*} = \lambda_p^{\frac{1}{\sigma}} \frac{C_{p,t}}{C_{p,t}^*} \implies c_{p,t} - c_{o,t} = c_{p,t}^* - c_{o,t}^*.$$

Relative consumption in normal times evolves according to

$$\dot{c}_{p,t} - \dot{c}_{o,t} = -\psi_c (c_{p,t} - c_{o,t}).$$

Relative net worth. Linearizing Equation (B.1), we obtain

$$\dot{b}_{p,t} - \dot{b}_{o,t} = \sum_{k \in \{L,E\}} r_k \left[\hat{r}_{k,t} \left(\frac{B_p^k}{B_p} - \frac{B_o^k}{B_o} \right) + \frac{B_p^k}{B_p} \left(b_{p,t}^k - b_{p,t} \right) - \frac{B_o^k}{B_o} \left(b_{o,t}^k - b_{o,t} \right) \right] - \left(\frac{C_p}{B_p} c_{p,t} - \frac{C_o}{B_o} c_{o,t} \right) + \frac{C_p - T_p}{B_p} b_{p,t} - \frac{C_o - T_o}{B_o} b_{o,t},$$

where $r_k \hat{r}_{k,t} = \lambda \left(\frac{C_s}{C_s^*} \right)^{\sigma} \frac{Q_k - Q_k^*}{Q_k} \left(\hat{\lambda}_t + \sigma c_{s,t} + \frac{Q_k^*}{Q_k - Q_k^*} q_{k,t} \right) + \xi C_s^{\sigma} L_s^{-\nu} (1 - \alpha_k) \left(\hat{\xi}_t + \sigma c_{s,t} - \nu l_{s,t} \right)$. Using the fact that $\frac{C_j - T_j}{B_j} = r_n + \sum_{k \in \{L,E\}} r_k \frac{B_j^k}{B_j}$, we can write the expression above as follows:

$$\dot{b}_{p,t} - \dot{b}_{o,t} = \sum_{k \in \{L,E\}} r_k \left[\hat{r}_{k,t} \left(\frac{B_p^k}{B_p} - \frac{B_o^k}{B_o} \right) + \frac{B_p^k}{B_p} b_{p,t}^k - \frac{B_o^k}{B_o} b_{o,t}^k \right] - \left(\frac{C_p}{B_p} c_{p,t} - \frac{C_o}{B_o} c_{o,t} \right) + r_n \left(b_{p,t} - b_{o,t} \right).$$

The relative net worth in the crisis state at $t = t^*$ is given by

$$\frac{B_p^*}{B_p} b_{p,t^*}^* - \frac{B_o^*}{B_o} b_{o,t^*}^* = b_{p,t^*} - b_{o,t^*} - \sum_{k \in \{o,p\}} \left[\left(\frac{B_p^k}{B_p} - \frac{B_o^k}{B_o} \right) \frac{Q_k^*}{Q_k} q_{k,t^*} - \frac{Q_k^* - Q_k}{Q_k} \left(\frac{B_p^k}{B_p} b_{p,t^*}^k - \frac{B_o^k}{B_o} b_{o,t^*}^k \right) \right].$$

Relative risk exposure. Consumption of savers in the crisis state is given by $c_{j,t}^* = \frac{r_n^* B_j^*}{C_s^*} b_{j,t}^*$, so we obtain that $c_{p,t}^* - c_{o,t}^* = \frac{r_n^*}{C_s^*} \left(B_p^* b_{p,t}^* - B_o^* b_{o,t}^* \right)$. Using this expression and

the fact that $c^*_{p,t} - c^*_{o,t} = c_{p,t} - c_{o,t}$, we can solve for the relative risk exposure:

$$\sum_{k\in\{o,p\}} \frac{Q_k - Q_k^*}{Q_k}\left(\frac{B_p^k}{B_p}b_{p,t}^k - \frac{B_o^k}{B_o}b_{o,t}^k\right) = b_{p,t} - b_{o,t} - \frac{C_s^*}{r_n^* B_s}\left(c_{p,t} - c_{o,t}\right) - \sum_{k\in\{o,p\}}\left(\frac{B_p^k}{B_p} - \frac{B_o^k}{B_o}\right)\frac{Q_k^*}{Q_k}q_{k,t}. \tag{B.5}$$

Liquidity holdings. Savers' liquidity holdings can be written as

$$\frac{L_j}{B_j}l_{j,t} = b_{j,t} - \sum_{j\in\{L,E\}}\left(1-\alpha_k\right)\frac{B_j^k}{B_j}b_{j,t}^k,$$

thus

$$\sum_{j\in\{L,E\}}\left(1-\alpha_k\right)\left(\frac{B_p^k}{B_p}b_{p,t}^k - \frac{B_o^k}{B_o}b_{o,t}^k\right) = b_{p,t} - b_{o,t} - \left[\frac{L_p}{B_p}\left(l_{p,t} - l_{s,t}\right) - \frac{L_o}{B_o}\left(l_{o,t} - l_{s,t}\right) + \left(\frac{L_p}{B_p} - \frac{L_o}{B_o}\right)l_{s,t}\right].$$

The deviation of individual savers' liquidity holdings from the average can be written as

$$l_{p,t} - l_{s,t} = \frac{\mu_o L_o}{\mu_o L_o + \mu_p L_p}\left(l_{p,t} - l_{o,t}\right), \quad l_{o,t} - l_{s,t} = -\frac{\mu_p L_p}{\mu_o L_o + \mu_p L_p}\left(l_{p,t} - l_{o,t}\right).$$

Combining these two expressions, we obtain

$$\sum_{j\in\{L,E\}}\left(1-\alpha_k\right)\left(\frac{B_p^k}{B_p}b_{p,t}^k - \frac{B_o^k}{B_o}b_{o,t}^k\right) = b_{p,t} - b_{o,t} - \left[\left(\frac{L_p}{B_p}\frac{\mu_o L_o}{\mu_o L_o + \mu_p L_p} + \frac{\mu_p L_p}{\mu_o L_o + \mu_p L_p}\right)\left(l_{p,t} - l_{o,t}\right) + \left(\frac{L_p}{B_p} - \frac{L_o}{B_o}\right)l_{s,t}\right]. \tag{B.6}$$

Given Lemma 1, we have

$$l_{p,t} - l_{o,t} = \frac{\sigma}{\nu}\left(c_{p,t} - c_{o,t}\right),$$

and from the definition of $l_{s,t}$

$$l_{s,t} = \gamma_L q_{L,t},$$

where $\gamma_L = \alpha\frac{\overline{D}_G^L}{(1-\mu_w)L_s}$ and we used that the quantity of debt is constant. Combining these

expressions with Equation (B.6), we get

$$\sum_{j\in\{L,E\}}(1-\alpha_k)\left(\frac{B_p^k}{B_p}b_{p,t}^k-\frac{B_o^k}{B_o}b_{o,t}^k\right)=b_{p,t}-b_{o,t}-\left[\left(\frac{L_p}{B_p}\frac{\mu_o L_o}{\mu_o L_o+\mu_p L_p}+\frac{\mu_p L_p}{\mu_o L_o+\mu_p L_p}\right)\frac{\sigma}{\nu}\left(c_{p,t}-c_{o,t}\right)+\left(\frac{L_p}{B_p}-\frac{L_o}{B_o}\right)\gamma_L q_{L,t}\right]. \quad \text{(B.7)}$$

The dynamic system. Using equations (B.5) and (B.7), the relative net worth in normal times is given by

$$\dot{b}_{p,t}-\dot{b}_{o,t}=\sum_{k\in\{L,E\}}\left(\frac{B_p^k}{B_p}-\frac{B_o^k}{B_o}\right)\left[\lambda\left(\frac{C_s}{C_s^*}\right)^\sigma\frac{Q_k-Q_k^*}{Q_k}\hat{\lambda}_t+\xi C_s^\sigma L_s^{-\nu}\left(1-\alpha_k\right)\hat{\xi}_t+r_k\left(\sigma-1\right)c_{s,t}\right]-\left[r_n+\frac{T_s}{B_s}+\lambda\left(\frac{C_s}{C_s^*}\right)^\sigma\frac{C_s^*}{r_n^*B_s}+\xi C_s^{-\sigma}L_s^{-\nu}\left(\frac{L_p}{B_p}\frac{\mu_o L_o}{\mu_o L_o+\mu_p L_p}+\frac{\mu_p L_p}{\mu_o L_o+\mu_p L_p}\right)\frac{\sigma}{\nu}\right]\left(c_{p,t}-c_{o,t}\right)-\sum_{k\in\{L,E\}}r_k\left(\frac{B_p^k}{B_p}\left(c_{p,t}-c_{s,t}\right)-\frac{B_o^k}{B_o}\left(c_{o,t}-c_{s,t}\right)\right)+\rho\left(b_{p,t}-b_{o,t}\right)-\xi C_s^\sigma L_s^{-\nu}\left[\nu\sum_{k\in\{L,E\}}\left(1-\alpha_k\right)\left(\frac{B_p^k}{B_p}-\frac{B_o^k}{B_o}\right)+\left(\frac{L_p}{B_p}-\frac{L_o}{B_o}\right)\right]\gamma_L q_{L,t},$$

using that $r_k\hat{r}_{k,t}=\lambda\left(\frac{C_s}{C_s^*}\right)^\sigma\frac{Q_k-Q_k^*}{Q_k}\hat{\lambda}_t+\xi C_s^\sigma L_s^{-\nu}\left(1-\alpha_k\right)\hat{\xi}_t+r_k\sigma c_{s,t}+\lambda\left(\frac{C_s}{C_s^*}\right)^\sigma\frac{Q_k^*}{Q_k}q_{k,t}-\xi C_s^\sigma L_s^{-\nu}\left(1-\alpha_k\right)\nu\gamma$ $\frac{C_j}{B_j}=r_n+\frac{T_s}{B_s}+\sum_{k\in\{L,E\}}r_k\frac{B_j^k}{B_j}$, and $r_n=\rho-\lambda\left(\frac{C_s}{C_s^*}\right)^\sigma-\xi C_s^\sigma L_s^{-\nu}$.

The deviation of consumption from average can be written as

$$c_{p,t}-c_{s,t}=\mu_{c,o}(c_{p,t}-c_{o,t}),\quad c_{o,t}-c_{p,t}=-\mu_{c,p}(c_{p,t}-c_{o,t}).$$

Combining the expressions above, we can write $\dot{b}_{p,t}-\dot{b}_{o,t}$ as follows:

$$\dot{b}_{p,t}-\dot{b}_{o,t}=\rho\left(b_{p,t}-b_{o,t}\right)-\chi_{b,c}\left(c_{p,t}-c_{o,t}\right)-\chi_{b,q}q_{L,t}+\chi_{b,c_s}c_{s,t},$$

where $\chi_{b,c_s}\equiv\left(\sigma-1\right)\sum_{k\in\{L,E\}}\left(\frac{B_p^k}{B_p}-\frac{B_o^k}{B_o}\right)r_k$, $\chi_{b,q}\equiv\xi C_s^\sigma L_s^{-\nu}\left[\nu\sum_{k\in\{L,E\}}\left(1-\alpha_k\right)\left(\frac{B_p^k}{B_p}-\frac{B_o^k}{B_o}\right)+\left(\frac{L_p}{B_p}-\frac{L_o}{B_o}\right)\right.$

and

$$\chi_{b,c} \equiv \sigma\mu_{c,o}\mu_{c,p}\left(\frac{\lambda_o^{\frac{1}{\sigma}}-\lambda_p^{\frac{1}{\sigma}}}{\lambda^{\frac{1}{\sigma}}}\right)\lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma}\sum_{k\in\{L,E\}}\frac{Q_k-Q_k^*}{Q_k}\left(\frac{B_p^k}{B_p}-\frac{B_o^k}{B_o}\right)+$$
$$\sigma\mu_{c,o}\mu_{c,p}\left[\frac{\left(\frac{C_o}{C_s}\right)^{\frac{\sigma-\nu}{\nu}}\xi_o^{\frac{1}{\nu}}-\left(\frac{C_p}{C_s}\right)^{\frac{\sigma-\nu}{\nu}}\xi_p^{\frac{1}{\nu}}}{\xi^{\frac{1}{\nu}}}\right]\xi C_s^{\sigma}L_s^{-\nu}\sum_{k\in\{L,E\}}(1-\alpha_k)\left(\frac{B_p^k}{B_p}-\frac{B_o^k}{B_o}\right)+$$
$$\left(r_n+\frac{T_s}{B_s}+\lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma}\frac{C_s^*}{r_n^*B_s}\right)+\sum_{k\in\{L,E\}}r_k\left(\mu_{c,o}\frac{B_p^k}{B_p}+\mu_{c,p}\frac{B_o^k}{B_o}\right)+$$
$$\frac{\sigma}{\nu}\xi C_s^{-\sigma}L_s^{-\nu}\left(\frac{L_p}{B_p}\frac{\mu_oL_o}{\mu_oL_o+\mu_pL_p}+\frac{\mu_pL_p}{\mu_oL_o+\mu_pL_p}\right).$$

Note that $r_n+\frac{T_s}{B_s}=\frac{C_j}{B_j}-\sum_{k\in\{L,E\}}r_k\frac{B_j^k}{B_j}$, so $r_n+\frac{T_s}{B_s}=\mu_{c,p}\frac{C_p}{B_p}+\mu_{c,o}\frac{C_o}{B_o}-\sum_{k\in\{L,E\}}r_k\left(\mu_{c,p}\frac{B_p^k}{B_p}+\mu_{c,o}\frac{B_o^k}{B_o}\right)$. We can then write $\chi_{b,c}$ as follows:

$$\chi_{b,c} \equiv \sigma\mu_{c,o}\mu_{c,p}\left(\frac{\lambda_o^{\frac{1}{\sigma}}-\lambda_p^{\frac{1}{\sigma}}}{\lambda^{\frac{1}{\sigma}}}\right)\lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma}\sum_{k\in\{L,E\}}\frac{Q_k-Q_k^*}{Q_k}\left(\frac{B_p^k}{B_p}-\frac{B_o^k}{B_o}\right)+$$
$$\sigma\mu_{c,o}\mu_{c,p}\left[\frac{\left(\frac{C_o}{C_s}\right)^{\frac{\sigma-\nu}{\nu}}\xi_o^{\frac{1}{\nu}}-\left(\frac{C_p}{C_s}\right)^{\frac{\sigma-\nu}{\nu}}\xi_p^{\frac{1}{\nu}}}{\xi^{\frac{1}{\nu}}}\right]\xi C_s^{\sigma}L_s^{-\nu}\sum_{k\in\{L,E\}}(1-\alpha_k)\left(\frac{B_p^k}{B_p}-\frac{B_o^k}{B_o}\right)+$$
$$\mu_{c,p}\frac{C_p}{B_p}+\mu_{c,o}\frac{C_o}{B_o}+\lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma}\frac{C_s^*}{r_n^*B_s}+$$
$$\frac{\sigma}{\nu}\xi C_s^{-\sigma}L_s^{-\nu}\left(\frac{L_p}{B_p}\frac{\mu_oL_o}{\mu_oL_o+\mu_pL_p}+\frac{\mu_pL_p}{\mu_oL_o+\mu_pL_p}\right).$$

The price of long-term government bonds is given by

$$\dot{q}_{L,t}=\frac{1}{Q_L}q_{L,t}+r_L\hat{r}_{L,t}+(i_t-r_n).$$

Using the expression for $r_L\hat{r}_{L,t}$, we obtain

$$\dot{q}_{L,t}=\chi_{q,q}q_{L,t}+\chi_{q,c}\left(c_{p,t}-c_{o,t}\right)+r_L\sigma c_{s,t}+(i_t-r_n),$$

where $\chi_{q,q} \equiv \frac{1}{Q_L} + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{Q_L^*}{Q_L} - \xi C_s^{\sigma} L_s^{-\nu} (1-\alpha_L) \nu \gamma_L$, and

$$\chi_{q,c} \equiv \sigma \mu_{c,o} \mu_{c,p} \left[\frac{\lambda_p^{\frac{1}{\sigma}} - \lambda_o^{\frac{1}{\sigma}}}{\lambda^{\frac{1}{\sigma}}} \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{Q_L - Q_L^*}{Q_L} + \frac{\left(\frac{C_p}{C_s}\right)^{\frac{\sigma-\nu}{\nu}} \xi_p^{\frac{1}{\nu}} - \left(\frac{C_o}{C_s}\right)^{\frac{\sigma-\nu}{\nu}} \xi_o^{\frac{1}{\nu}}}{\xi^{\frac{1}{\nu}}} \xi C_s^{\sigma} L_s^{-\nu} (1-\alpha) \right].$$

In general, we would have to simultaneously solve for the aggregate variables and the relative net worth and relative consumption of pessimistic savers, which would increase the dimensionality of the problem relative to the standard New Keynesian. We assume that $r_k c_{s,t} = \mathcal{O}(||i_t - r_n||^2)$, so this term is small and can be ignored in our approximate solution. This implies that the system is no *block recursive*, where we can solve for the dynamics of relative consumption and relative net worth before fully characterizing the behavior of other aggregate variables. Under this assumption, we can write the joint dynamics of $c_{p,t} - c_{o,t}$, $b_{p,t} - b_{o,t}$ and $q_{L,t}$ as follows:

$$\begin{bmatrix} \dot{c}_{p,t} - \dot{c}_{o,t} \\ \dot{b}_{p,t} - \dot{b}_{o,t} \\ \dot{q}_{L,t} \end{bmatrix} = \begin{bmatrix} -\psi_r & 0 & 0 \\ -\chi_{b,c} & \rho & -\chi_{b,q} \\ \chi_{q,c} & 0 & \chi_{q,q} \end{bmatrix} \begin{bmatrix} c_{p,t} - c_{o,t} \\ b_{p,t} - b_{o,t} \\ q_{L,t} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ i_t - r_n \end{bmatrix}.$$

Persistence of $\hat{\lambda}_t$ and $\hat{\xi}_t$. The system above has two positive and one negative eigenvalue $(-\psi_r, \rho, \chi_{q,q})$, so there is a unique bounded solution. The associated matrix of eigenvectors is

$$V = \begin{bmatrix} \frac{\rho+\xi}{\chi_{b,c} - \frac{\chi_{b,q}\chi_{q,c}}{\chi_{q,q}+\xi}} & 0 & 0 \\ 1 & 1 & 1 \\ -\frac{\chi_{q,c}}{\chi_{q,q}+\xi} \frac{\rho+\xi}{\chi_{b,c} - \frac{\chi_{b,q}\chi_{q,c}}{\chi_{q,q}+\xi}} & 0 & \frac{\rho-\chi_{q,q}}{\chi_{b,q}} \end{bmatrix}, \quad V^{-1} = \begin{bmatrix} \frac{\chi_{b,c} - \frac{\chi_{b,q}\chi_{q,c}}{\chi_{q,q}+\xi}}{\rho+\xi} & 0 & 0 \\ -\frac{\chi_{b,c} - \frac{\chi_{b,q}\chi_{q,c}}{\chi_{q,q}+\xi}}{\rho+\xi} - \frac{\chi_{q,c}}{\chi_{q,q}+\xi} \frac{\chi_{b,q}}{\rho-\chi_{q,q}} & 1 & -\frac{\chi_{b,q}}{\rho-\chi_{q,q}} \\ \frac{\chi_{q,c}}{\chi_{q,q}+\xi} \frac{\chi_{b,q}}{\rho-\chi_{q,q}} & 0 & \frac{\chi_{b,q}}{\rho-\chi_{q,q}} \end{bmatrix}.$$

In this solution

$$c_{p,t} - c_{o,t} = e^{-\psi_r t}(c_{p,0} - c_{o,0}),$$

where

$$c_{p,0} - c_{o,0} = \frac{\rho + \xi}{\chi_{b,c} - \frac{\chi_{b,q}\chi_{q,c}}{\chi_{q,q}+\xi}} \left[\left(b_{p,0} - b_{o,0} \right) + \frac{\chi_{b,q}}{\rho - \chi_{q,q}} \int_t^\infty \left(e^{-\chi_{q,q}(s-t)} - e^{-\rho(s-t)} \right) \left(i_s - r_n \right) ds \right].$$

We can then write the market-implied disaster probability as follows:

$$\hat{\lambda}_t = e^{-\psi_r t} \hat{\lambda}_0,$$

where

$$\hat{\lambda}_0 = \sigma \mu_{c,o} \mu_{c,p} \left(\frac{\lambda_o^{\frac{1}{\sigma}} - \lambda_p^{\frac{1}{\sigma}}}{\lambda^{\frac{1}{\sigma}}} \right) \frac{\rho + \xi}{\chi_{b,c} - \frac{\chi_{b,q}\chi_{q,c}}{\chi_{q,q}+\xi}} \left[\left(b_{p,0} - b_{o,0} \right) + \frac{\chi_{b,q}}{\rho - \chi_{q,q}} \int_t^\infty \left(e^{-\chi_{q,q}(s-t)} - e^{-\rho(s-t)} \right) \left(i_s - r_n \right) ds \right],$$

and the market-implied convenience as

$$\hat{\xi}_t = e^{-\psi_r t} \hat{\xi}_0,$$

where

$$\hat{\xi}_0 = \sigma \mu_{c,o} \mu_{c,p} \left[\frac{\left(\frac{C_o}{C_s} \right)^{\frac{\sigma-\nu}{\nu}} \xi_o^{\frac{1}{\nu}} - \left(\frac{C_p}{C_s} \right)^{\frac{\sigma-\nu}{\nu}} \xi_p^{\frac{1}{\nu}}}{\xi^{\frac{1}{\nu}}} \right] \frac{\rho + \xi}{\chi_{b,c} - \frac{\chi_{b,q}\chi_{q,c}}{\chi_{q,q}+\xi}} \left[\left(b_{p,0} - b_{o,0} \right) + \frac{\chi_{b,q}}{\rho - \chi_{q,q}} \int_t^\infty \left(e^{-\chi_{q,q}(s-t)} - e^{-\rho(s-t)} \right) \left(i_s - r_n \right) ds \right].$$

Hence, ψ_r captures the persistence of $\hat{\lambda}_t$ and $\hat{\xi}_t$. If $\psi_r = 0$, then changes in λ_t and ξ_t are permanent. For positive values of ξ, the effects on λ_t and ξ_t are transitory and xi controls de speed of convergence.

Wealth revaluation, $\hat{\lambda}_0$ and $\hat{\xi}_0$. The revaluation of the relative net worth is given by

$$b_{p,0} - b_{o,0} = \sum_{k \in \{L,E\}} \left(\frac{B_p^k}{B_p} - \frac{B_o^k}{B_o} \right) q_{k,0}.$$

The price of the long-term bond satisfies the condition

$$-\frac{1}{Q_L} q_{L,t} + \dot{q}_{L,t} - (i_t - r_n) = \chi_r \frac{Q_L - Q_L^*}{Q_L} \hat{\lambda}_t + \chi_l (1-\alpha) \hat{\xi}_t + \chi_r \frac{Q_L^*}{Q_L} q_{L,t} - \chi_l (1-\alpha) \nu \gamma_L q_{L,t},$$

where we have assumed that $r_L \sigma c_{s,t} = \mathcal{O}(||i_t - r_n||^2)$. Rearranging this expression, we obtain

$$\dot{q}_{L,t} - (\rho_L + \psi_L) q_{L,t} = (i_t - r_n) + \chi_r \frac{Q_L - Q_L^*}{Q_L} \hat{\lambda}_t + \chi_l (1-\alpha) \hat{\xi}_t,$$

where $\rho_L \equiv \rho - \chi_l [\alpha + (1-\alpha) \nu \gamma_L]$. Solving the differential equation, we obtain

$$q_{L,0} = -\int_0^\infty e^{-(\rho_L + \psi_L)t} (i_t - r_n) \, dt - \chi_r \frac{Q_L - Q_L^*}{Q_L} \int_0^\infty e^{-(\rho_L + \psi_L)t} \hat{\lambda}_t dt - \chi_l (1-\alpha) \int_0^\infty e^{-(\rho_L + \psi_L)t} \hat{\xi}_t dt.$$

Suppose $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$. Then

$$q_{L,0} = -\frac{i_0 - r_n}{\rho_L + \psi_L + \psi_m} - \chi_r \frac{Q_L - Q_L^*}{Q_L} \frac{\hat{\lambda}_0}{\rho_L + \psi_L + \psi_r} - \chi_l (1-\alpha) \frac{\hat{\xi}_0}{\rho_L + \psi_L + \psi_r}.$$

The price of equity satisfies

$$\frac{\Pi}{Q_E} \left(\hat{\Pi}_t - q_{E,t} \right) + \dot{q}_{E,t} - (i_t - \pi_t - r_n) = \lambda \left(\frac{C_s}{C_s^*} \right)^\sigma \frac{Q_E - Q_E^*}{Q_E} \hat{\lambda}_t + \xi C_s^\sigma L_s^{-\nu} \hat{\xi}_t + r_E \sigma c_{s,t} + \lambda \left(\frac{C_s}{C_s^*} \right)^\sigma \frac{Q_E^*}{Q_E} q_{E,t} - \xi C_s^\sigma L_s^{-\nu} \nu \gamma_L q_{L,t}.$$

Rearranging the expression above, we obtain

$$\dot{q}_{E,t} - \rho q_{E,t} = -\frac{\Pi}{Q_E} \hat{\Pi}_t + (i_t - \pi_t - r_n) + \lambda \left(\frac{C_s}{C_s^*} \right)^\sigma \frac{Q_E - Q_E^*}{Q_E} \hat{\lambda}_t + \xi C_s^\sigma L_s^{-\nu} \hat{\xi}_t + r_E \sigma c_{s,t} - \xi C_s^\sigma L_s^{-\nu} \nu \gamma_L q_{L,t}.$$

Solving the differential equation and assuming that $r_E \sigma c_{s,t} = \mathcal{O}(||i_t - r_n||^2)$, we obtain

$$q_{E,0} = \frac{\Pi}{Q_E} \int_0^\infty e^{-\rho t} \hat{\Pi}_t dt - \int_0^\infty e^{-\rho t} \left[(i_t - \pi_t - r_n) + \lambda \left(\frac{C_s}{C_s^*} \right)^\sigma \frac{Q_E - Q_E^*}{Q_E} \hat{\lambda}_t \right.$$
$$\left. + \xi C_s^\sigma L_s^{-\nu} \hat{\xi}_t - \xi C_s^\sigma L_s^{-\nu} \nu \gamma_L q_{L,t} \right] ds.$$

Assuming that $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, we obtain

$$q_{E,0} = \left[\frac{1 - \left(1 - \epsilon^{-1}\right)(1+\phi)}{Q_E} Y + \frac{\kappa}{\rho - \underline{\omega}} \right] \Omega_0 - \left(1 + \frac{\chi_l \nu \gamma_L}{\rho_L + \psi_L + \psi_m} - \kappa \frac{\chi_{ISE}}{(\rho - \underline{\omega})(\overline{\omega} + \psi_m)} \right) \frac{i_0 - r_n}{\rho + \psi_m} -$$
$$\left[\left(\frac{Q_E - Q_E^*}{Q_E} + \frac{Q_L - Q_L^*}{Q_L} \frac{\chi_l \nu \gamma_L}{\rho_L + \psi_L + \psi_r} \right) \chi_r - \frac{\kappa \chi_{TVP}}{(\rho - \underline{\omega})(\overline{\omega} + \psi_r)} \right] \frac{\hat{\lambda}_0}{\rho + \psi_r} -$$
$$\left[\left(1 + (1 - \alpha) \frac{\chi_l \nu \gamma_L}{\rho_L + \psi_L + \psi_r} \right) \chi_l - \frac{\kappa \chi_{TVL}}{(\rho - \underline{\omega})(\overline{\omega} + \psi_r)} \right] \frac{\hat{\xi}_0}{\rho + \psi_r}.$$

C Extensions

In this section, we discuss some extensions to the baseline model. First, we allow for partial recovery of productivity in the crisis state. Second, we introduce options into the model and discuss the connection between the box rate and the zero-beta rate.

C.1 Partial recovery

In the baseline model, productivity is permanently reduced in the crisis state. We now allow for partial recovery, assuming productivity evolves according to

$$A_t^* = e^{-\psi_A (t - t^*)} A^* + \left(1 - e^{-\psi_A (t - t^*)}\right) A_{lr}^*,$$

where A_{lr}^* is the long-run level of productivity in the crisis state.

Since there are no monetary shocks in the crisis state, output is supply-determined:

$$Y_t^* = \mu_w (1 - \epsilon^{-1})^{\frac{1}{\phi}} (A_t^*)^{\frac{1+\phi}{\phi}},$$

and, with the same fiscal policy assumptions as in the baseline model, $C_{w,t}^* = C_{s,t}^* = Y_t^*$.

The baseline case corresponds to $A^*_{lr} = A^*$, implying constant output in the crisis state. Empirically, Barro et al. (2013) find that roughly half of the initial drop in output is subsequently reversed. However, because investors have time-separable preferences, the price of risk depends only on the contemporaneous drop in consumption and is unaffected by whether recovery occurs.

Partial recovery instead influences asset pricing in the crisis state. For simplicity, assume bonds provide no liquidity services in the crisis, $\xi^*_j = 0$ for $j \in \{o, p\}$. The risk-free rate is then

$$r^*_{n,t} = \rho^*_s + \sigma \frac{\dot{C}^*_{s,t}}{C^*_{s,t}} = \rho^*_s + \sigma \frac{1+\phi}{\phi} \psi_A \frac{A^*_{lr} - A^*_t}{A^*_t}.$$

The long-term bond price satisfies

$$Q^*_{L,0} = \int_0^\infty \exp\left(- \int_0^t (r^*_{n,s} + \psi_L)\, ds \right) dt = \frac{1}{y^*_{L,0} + \psi_L}.$$

Thus, the decline in $Q^*_{L,0}$ reflects the average risk-free rate over the crisis period. By calibrating ρ^*_s so that $y^*_{L,0}$ matches the long-term bond yield in the baseline model, one obtains the same drop in bond prices.

Overall, allowing for partial recovery alters the time profile of productivity and interest rates in the crisis state, but does not materially change the model's main implications.

C.2 The box rate

We consider the model implications for option prices. Let $p_t(K, \tau)$ $(c_t(K, \tau))$ denote the price of an European put (call) option with strike price K and maturity τ at time t,

Assuming that options provide no liquidity services, and using the SDF from Proposition 1, we obtain:

$$c_t(K, \tau) = \mathbb{E}_t \left[\frac{\eta_{t+\tau}}{\eta_t} \max(0, S_{t+\tau} - K) \right], \qquad p_t(K, \tau) = \mathbb{E}_t \left[\frac{\eta_{t+\tau}}{\eta_t} \max(0, K - S_{t+\tau}) \right],$$

where $S_t \equiv P_t Q_{E,t}$ is the nominal stock price.

Taking the difference between the put and call option prices, we obtain the version of

the *put-call parity* in our setting:

$$p_t(K,\tau) - c_t(K,\tau) = \mathbb{E}_t\left[\frac{\eta_{t+\tau}}{\eta_t}(K - S_{t+\tau})\right] = Q_B^{br}(\tau)K - \left[S_t - \mathbb{E}_t\left[\int_t^\tau \frac{\eta_z}{\eta_t} P_z \Pi_z dz\right]\right]$$

using the fact that $S_t = \mathbb{E}_t\left[\int_t^\tau \frac{\eta_z}{\eta_t} P_z \Pi_z dz\right] + \mathbb{E}_t\left[\frac{\eta_{t+\tau}}{\eta_t} S_{t+\tau}\right]$.

The bond price $Q_B^{br}(\tau)$ corresponds to the value of a zero-coupon bond maturing at time $t+\tau$, but that provides no liquidity services. The *box rate* corresponds to the yield on this bond:

$$y_B^{br}(\tau) \equiv -\frac{1}{\tau}\log Q_B^{br}(\tau).$$

In the limit as $\tau \to 0$, the box rate coincides with the zero-beta rate:

$$y_B^{br}(0) = i_t^{zb}.$$

C.3 Time-varying stock-bond correlation

To capture fluctuations in the stock-bond correlation, we introduce an additional shock into model. With Poisson intensity $\lambda^\dagger$ the economy switches to a recession state, where productivity falls to $A^\dagger = (1 - \zeta_A^\dagger)A$. Liquidity services are not affected by the recession shock. For simplicity, we assume that there is no disagreement about the arrival rate of the recession shock and there is no monetary shock in the recession state. We also assume that $C_w^\dagger = Y^\dagger$.

The recession shock is recurring, that is, the economy may receive another recession shock where productivity falls by the factor $\zeta_A^\dagger$ again. The economy may switch to the crisis state after it switches to the recession state, which is an absorbing state. This implies that the only effect of the recession shock is to reduce productivity, but uncertainty remains the same. In a stationary equilibrium, interest rates and bond prices are not affected by the recession shock. Equity prices fall by the factor $\zeta_A^\dagger$ in the recession state. Therefore, this shock affects stocks, but not bonds.

We assume that savers have access to a derivative that pays off $r_{D,t}$ at the normal state for each unit of exposure to the recession shock. There is no cost to enter into this

derivative, and the derivative provides no liquidity services.

Savers' problem. The savers' problem is given by:

$$V_{j,t} = \max_{C_{j,t}, B^L_{j,t}, B^E_{j,t}} \mathbb{E}_{j,t}\left[\int_t^{t^*} e^{-\int_t^z \rho_{j,u} du}\left[\frac{C_{j,z}^{1-\sigma}}{1-\sigma} + \xi_j \frac{L_{j,z}^{1-\nu}}{1-\nu}\right] dz + e^{-\int_t^{t^*} \rho_{j,u} du} V^*_{j,t^*}(B_{j,t^*})\right], \qquad s.t.$$

$$dB_{j,t} = \left[(i_t - \pi_t)B_{j,t} + r_{L,t}B^L_{j,t} + r_{E,t}B^E_{j,t} + r_{D,t}B^D_{j,t} + T_{j,t} - C_{j,t}\right] dt + \left[B^*_{j,t} - B_{j,t}\right] d\mathcal{N}_t + \left[B^\dagger_{j,t} - B_{j,t}\right] d\mathcal{N}^\dagger_t,$$

given $B_{j,0} > 0$, where $\mathbb{E}_j[d\mathcal{N}_t] = \lambda_j dt$ and $\mathbb{E}_j[d\mathcal{N}^\dagger_t] = \lambda^\dagger dt$. The loss of wealth in the recession state is given by $B^\dagger_{j,t} - B_{j,t} = B^E_{j,t}\frac{Q^\dagger_{E,t} - Q_{E,t}}{Q_{E,t}} + B^L_{j,t}\frac{Q^\dagger_{L,t} - Q_{L,t}}{Q_{L,t}} - B^D_{j,t}$.

We can rewrite the flow budget constraint as:

$$dB_{j,t} = \left[(i_t - \pi_t)B_{j,t} + \tilde{r}_{L,t}B^L_{j,t} + \tilde{r}_{E,t}B^E_{j,t} - r_{D,t}[B^\dagger_{j,t} - B_{j,t}] + T_{j,t} - C_{j,t}\right] dt + \left[B^*_{j,t} - B_{j,t}\right] d\mathcal{N}_t + \left[B^\dagger_{j,t} - B_{j,t}\right] d\mathcal{N}^\dagger_t,$$

where $\tilde{r}_{k,t} = r_{k,t} - r_{D,t}\frac{Q_{k,t} - Q^\dagger_{k,t}}{Q_{k,t}}$ for $k \in \{L, E\}$.

The pricing condition for the derivative is given by:

$$r_{D,t} = \lambda^\dagger \left(\frac{C_{j,t}}{C^\dagger_{j,t}}\right)^\sigma.$$

The pricing condition for equities and long-term bonds are given by:

$$\tilde{r}_{E,t} = \lambda_j \left(\frac{C_{j,t}}{C^*_{j,t}}\right)^\sigma \frac{Q_{E,t} - Q^*_{E,t}}{Q_{E,t}} + \xi_j C^\sigma_{j,t} L^{-\nu}_{j,t},$$
$$\tilde{r}_{L,t} = \lambda_j \left(\frac{C_{j,t}}{C^*_{j,t}}\right)^\sigma \frac{Q_{L,t} - Q^*_{L,t}}{Q_{L,t}} + (1-\alpha)\xi_j C^\sigma_{j,t} L^{-\nu}_{j,t}.$$

A version of Lemma 1 holds in this setting, where the price of disaster risk and marginal value of liquidity are equalized, as well as the price of recession risk:

$$\lambda^\dagger \left(\frac{C_{o,t}}{C^\dagger_{o,t}}\right)^\sigma = \lambda^\dagger \left(\frac{C_{p,t}}{C^*_{p,t}}\right)^\sigma.$$

D The response of asset prices to monetary shocks

In this section, we derive the response of bond prices to monetary shocks.

D.1 Bond price

Stationary equilibrium. We start by summarizing the pricing condition for the long-term government bond in the stationary equilibrium. We assume that a bond issued at date s pays coupons $\zeta_L e^{-\psi_L(t-s)}$ at all dates $t \geq s$, where we will choose the constant ζ_L so the bond is issued at par.

The price of the bond in the crisis state is given by:

$$Q_L^* = \frac{\zeta_L}{r_n^{*,zb} + \psi_L - \alpha^* \xi^* (C_s^*)^\sigma (L_s^*)^{-\nu}}.$$

The price of the bond in normal times satisfies the condition:

$$\frac{\zeta_L}{Q_L} - \psi_L - r_n = \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma \left(1 - \frac{Q_L^*}{Q_L}\right) + \xi C_s^\sigma L_s^{-\nu} (1-\alpha).$$

Rearranging the expression above, we obtain:

$$Q_L = \zeta_L \frac{1 + \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma Q_L^*/\zeta_L}{r_n + \psi_L + \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma + (1-\alpha)\xi C_s^\sigma L_s^{-\nu}}.$$

The bond price in normal times is equal to one if

$$\zeta_L = r_n + \psi_L + (1-\alpha)\xi C_s^\sigma L_s^{-\nu} + \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma (1 - Q_L^*).$$

The bond yield $i_{L,t}$ satisfies the condition:

$$Q_{L,t} = \int_0^\infty e^{-i_{L,t}s} (\zeta_L e^{-\psi_L s}) ds = \frac{\zeta_L}{i_{L,t} + \psi_L} \Rightarrow i_{L,t} = \frac{\zeta_L}{Q_{L,t}} - \psi_L.$$

Bond price. The price of the long-term government bond satisfies the condition:

$$\frac{\dot{Q}_{L,t}}{Q_{L,t}} = i_t + (1-\alpha)\xi_t C_{s,t}^{\sigma} L_{s,t}^{-\nu} + \lambda_t \left(\frac{C_{s,t}}{C_{s,t}^*}\right)^{\sigma} + \psi_L - \left[\zeta_L + \lambda_t \left(\frac{C_{s,t}}{C_{s,t}^*}\right)^{\sigma} Q_{L,t}^*\right] \frac{1}{Q_{L,t}}.$$

Linearizing the expression above, we obtain

$$\dot{q}_{L,t} = i_t - r_n + (1-\alpha)\xi C_s^{\sigma} L_s^{-\nu}(\hat{\xi}_t - \nu l_{s,t}) + \left[1 + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} Q_L^*\right] \frac{1}{Q_L} q_{L,t} + \frac{Q_L - Q_L^*}{Q_L} p_{r,t}.$$

Using $l_{s,t} = \chi_{l,q} q_{L,t}$, $\hat{\lambda}_t = \chi_{\lambda,\Delta c}(c_{p,t} - c_{o,t})$, and $\hat{\xi}_t = \chi_{\xi,\Delta c}(c_{p,t} - c_{o,t})$, we obtain

$$\dot{q}_{L,t} = \chi_{q,\Delta c}(c_{p,t} - c_{o,t}) + \chi_{q,q} q_{L,t} + i_t - r_n + r_L \sigma c_{s,t},$$

where

$$\chi_{q,\Delta c} = (1-\alpha)\xi C_s^{\sigma} L_s^{-\nu} \chi_{\xi,\Delta c} + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{Q_L - Q_L^*}{Q_L} \chi_{\lambda,\Delta c}$$
$$\chi_{q,q} = \left[1 + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} Q_L^*\right] \frac{1}{Q_L} - (1-\alpha)\xi C_s^{\sigma} L_s^{-\nu} \nu \chi_{l,q}.$$

References